CONSTRUCTIONS AND CLASSROOM USES FOR THE FIVE REGULAR AND RELATED POLYHEDRONS

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Preface

In our modern world of missles, satellites, moon probes, atomic and hydrogen bombs, the need for improving our mathematics program is imperative. Thus, the emphasis has shifted to an understanding of mathematical concepts rather than rote learning.

The National Science Foundation with its Academic and Summer Institutes has been very successful in its program of introducing mathematics teachers to these newer ideas. Their program of textbook revisions should improve the mathematical offerings of our schools.

However, one problem remains, that is to interest our more capable students into entering the field of mathematics. I believe that by gradually introducing the students to the more abstract concepts, we will have a better opportunity to accomplish this result. Many times, I have seen students entering my high school classroom for the first time show amazement at the wide variety of mathematical models on display. The multi-colored polyhedrons make a lasting impression on these young students, even if these solids are of questionable modern mathematical value. In my opinion, if these figures would only induce one gifted student to undertake mathematics as a career, the effort involved in constructing them would not have been in vain.

Since the time of the early Greeks and Egyptians, mathematics has been considered an integral part of our cultural heritage. Regular polyhedrons were discovered very early in the history of western civilization. I believe our students of today can profit both from the method of making these constructions and also, from a brief history of the men responsible for first making them.

Another feature of these constructions is their easy adaptability for use in our Mathematical Clubs, especially if the teacher does not feel justified in devoting classroom time to these projects. Since these clubs generally attract the abler students a more detailed study of the concepts involved may be pursued. Even students who see no more in the figures than a possible Christmas tree decoration get a certain amount of enjoyment from observing them.

In my report I plan to include full page drawings of the five regular polyhedrons with a thorough description on how to duplicate them. For the geometry teacher, who still insists on constructions using only a straight edge and a compass, I will give some of the basic constructions necessary to make models.

A wide variety of materials may be used in building these polyhedrons, but since in many schools the cost of some material may be prohibitive, I would recommend the use of construction paper. For example, a plastic model purchased from a dealer for approximately fifteen dollars will cost less than ten cents if construction paper is employed. For the benefit of teachers with an adequate budget a list of materials and their appropriate cost will be given.

Archimedean or semi-regular polyhedrons are solids whose polyhedral angles are equal and whose faces are composed of two or more different kinds of regular polygons.[1] As the pupils gain skills in the easier constructions, many of them may desire to attempt some of the more difficult figures. I shall give a brief description how some of these may be made.

For further simplification, I shall divide the remainder of my report into the following categories:

1. A brief history of the discovery and development of the five regular polyhedrons, including the men responsible for their discoveries will be discussed.

2. Detailed information on how to construct the five regular polyhedrons, including full page patterns will be given.

3. Information on Archimedean Polyhedrons and Star Polyhedrons in which I will present an easier method of making the constructions than that given in most textbooks.

4. Approximate cost of the various materials available to the instructor and their adaptability for use in the classroom will be listed.

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[1]The National Council of Teachers of Mathematics, <u>Multi-Sensory Aids in the Teaching of Mathematics</u>, Eighteenth Yearbook, (New York, Teacher's College, Columbia University, 1945), p. 45.

ACKNOWLEDGEMENTS

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CHAPTER I

A BRIEF HISTORY OF THE FIVE REGULAR POLYHEDRONS

The history of mathematics has many valuable suggestion for mathematics teachers, as well as for students. This is true, since most learning situations are merely repetitions of the cultural evolution of our race. We often wonder how certain concepts and ideas developed. The discussions in this chapter will be limited to those concerning the development of the five regular polyhedrons.

Even on these limited concepts, authorities disagree upon whom should be given credit for the initial discoveries. The absence of adequate writing material and the ancient practice of teaching by word of mouth probably explains the dissension. All mathematical historians seem to agree that all of the regular polyhedrons were discovered and thoroughly understood before Euclid presented his master structure of geometry.

Intuitive geometry developed mainly in Egypt and was centered upon the measurement of areas of land; both for taxation and the relocation of boundary markers washed away by the flood waters of the Nile. The pyramids of ancient Egypt prove the existence of elementary mathematical knowledge. However, it remained for the Greeks to organize the study of geometry into a science independent of its practical applications.

To illustrate the difficulty of crediting someone with the discovery of any particular solid one could examine Coxeter's viewpoint when he stated, "The early history of these polyhedra is lost in the shadows of antiquity. To ask who first constructed them is almost as futile as to ask who first discovered fire."[1]

On the other hand, Burnet goes so far as to identify one man, Teaetetus, as the discoverer of solid geometry. He stated:

> At any rate, it is a fact that the theory of the five regular solids were completed for the first time by Theaetetus. We are told in the scholia to Euclid that in Book XIII we have what are called the five figures of Plato, which are not his; but three of them the cube, the pyramid, and the dodecahedron belong to the Pythagorans, while the octahedron and the icosahedron belong to Theaetetus, at a time when he was already a member of the Academy founded by Plato, and he also showed that there were only five. From what has been said it is obvious that Theaetetus of Athens was the real founder of mathematics as we understand that word.[2]

The history of demonstrative geometry begins with Thales who lived from 640 to 546 B. C. He studied the applied geometry of the Egyptians and then introduced deductive demonstration into geometry. Thales suggested a study of lines with the idea of making the study abstract.

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[1]H. S. M. Coxeter, Regular Polytops (New York, 1929), p. 13.

[2]John F. Burnet, Platonism (Berkley, 1928), p. 102.

This great idea seems to be his major contribution to the study of mathematics. As Smith so aptly phrases it: "Without Thales there could not have been a Pythagoras--or such a Pythagoras; and without Pythagoras there would not have been a Plato--or such a Plato."[3]

Pythagoras and Thales were separated by about fifty years. Pythagoras should be given credit for transforming geometry into a more scientific study. In order to accomplish this he laid down certain principles, including definitions and proceeded to build up an orderly set of propositions. Major emphasis was placed upon basing a new theorem upon previously proved statements.

Pythagoras and the other members of his society, the Pythagoreans, are credited with quite a few theorems or new ideas in mathematics. These include the theorem on the sum of the angles of a triangle, the one on the square of the hypotenuse of a right triangle, theorems on the applications of areas and the discovery of irrational numbers. Their discovery of the incommensurable caused a great sensation at the time and was surpressed by them, since many of their proofs were based upon an arithmetical theory of proportion.

Further evidence that the Pythagoreans were aquainted with the five regular solids stems from the mysticism attached to these figures. They believed the universe to be composed of five elements, one for each of these solids. In their

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[3]David E. Smith, History of Mathematics, Vol. I, (New York, 1923), p. 68.

belief the earth arose from the cube, fire from the tetrahedron, air from the octahedron, water from the icosahedron, and the sphere of the universe from the dodecahedron.

Pythagoras or members of his society probably knew that the plane space around a point could be filled by six equilateral triangles, four squares, or three regular hexagons. These configurations could readily be observed in the mosaic pavements of the era. The badge of their society was the mystic pentagon or five pointed star. Thus, it is evident that they possessed the basic knowledge for the building of the solids. Merely, by making random arrangements they would have succeeded eventually in constructing all five of the figures.

The cube, tetrahedron, and the octahedron occur in nature as crystals. A fourth, the dodecahedron, of Etruscan origin has been discovered proving a very early knowledge of a primitive construction procedure. One could conclude that Pythagoras and his followers possessed all the basis knowledge necessary for the constructions.

Plato's Academy flourished about 375 B. C.; and he is more famous as a philosopher than as a mathematician. He insisted that constructions should be made by using only an unmarked ruler and a compass. This novel idea led to the three famous problems of antiquity--the trisection of any given angle, the construction of a cube with volume double that of a given cube, and the construction of a square with an area equal to that of a given circle.

Although, later these constructions were proved impossible under Plato's restrictions, investigation of the possibilities led to many mathematical discoveries.

The Platonic solids were definitely not Plato's discovery for they had been partly investigated by the Pythagoreans and more thoroughly by Theaetetus. These figures were called Platonic because of their appearance in the writings of Plato.

Euclid's major contribution to mathematics seems to be his logical organization of nearly all mathematical knowledge of his time. In his famous Elements, which consisted of thirteen books he founded the basis of geometrical thought that persists even today. Book XIII gives the method of constructing the sphere which circumscribes each solid. Of course, to do this he studied the relation between an edge of the solid and the radius of the sphere.

Many authors have refined and improved Euclid's construction methods since he first introduced them. One major improvement had to wait until an adequate coordinate system was discovered. After this, it was an easy matter to locate the position of the vertices and to study the polyhedrons analytically.

CHAPTER II

MATERIALS AND TOOLS FOR MATHEMATICAL MODELS

One of the principal advantages in the making of mathematical models is the wide range and adaptability of the available construction materials. Some of the factors, which limit their use and selection, include initial cost of the material, time allotment for the project, ease or difficulty of handing, useful life of the model, and the concepts or relations the object is to display.

In some cases the teacher will assign a project to the entire class, while at other times individual projects will be more desirable. Thus, every member of the class should possess all of the basic tools and materials. A basic list of tools and material should include:

1. A straight-edge or ruler
2. A compass
3. A pair of scissors
4. A 3-H drawing pencil
5. Airplane glue
6. Cardboard
7. Multi-colored construction paper
8. Razor blade
9. Thumb tacks
10. Paper clips
11. Protractor

Other students depending upon their talents and ingenuity may find the following list very helpful.

1. Plastic sheets
2. Wooden applicators
3. Plywood

4. Balsa wood
5. Colored elastic string
6. Plaster of Paris
7. Stapler
8. Sheet metal
9. Soldering iron
10. Wire brads
11. Metal shears
12. Gummed letters
13. Paints, assorted colors
14. Small paint brushes
15. Masking tape
16. Drawing instruments
17. India ink

The author does not wish to convey the impression that all or even any of the material in the second list should be acquired by the individual student. This report is concerned only with the basic constructions possible using only construction paper and the additional information is primarily for the teacher or student who wishes to construct the more advanced models. This list is certainly not complete and may be added on to as the need arises.

Each srudent should acquire a small cardboard box to keep his materials readily available for use in the classroom or as a container to carry the material to his home for outside assignments.

CHAPTER III

CONSTRUCTION OF THE REGULAR POLYHEDRONS

As these constructions can be utilized at a variety of age and grade levels the individual teacher will have to decide at what point to initate the studies. A knowledge of some basic construction procedures is necessary before the actual constructions are started.

Figures 1 to 6 contain six basic geometric constructions. The teacher should give students, who have not had a previous course in plane geometry, a brief explanation and a considerable amount of practice on these figures.

In Figure 1 a line segment AB is drawn first. Then a line segment CE longer than AB is drawn. Place the compass point at A and open it until it touches the point B. With the same radius and the point of the compass at C, draw an arc intersecting CE at D.

In Figure 2 draw a line segment AB of some convenient length. Place the point of the compass at A and B respectively with a radius longer than one-half AB, and draw arcs intersecting at C and D. Draw the line CD. Depending upon the maturity of the class, the teacher may wish to prove that CD is the perpendicular bisector of AB.

In Figure 3 again draw a line segment AB of some convenient length. Select a point D on the given line.

With the compass point at D and a convenient radius less than AD or BD, draw arcs intersecting AB at a and b. With a and b as centers draw arcs intersecting at C. Draw the line CD. A proof may or may not be given at the discretion of the teacher.

In Figure 4 begin with a line segment AB. With the point of the compass at A, measure the length AB. Then with A and B as centers and the same radius, draw arcs intersecting at C. Draw AC and BC.

In Figure 5 draw line segment AB and extend it a convenient distance to the right and left. Place the compass point at A and draw arcs intersecting AB or AB extended at a and b. Repest the same procedure at B obtaining arcs at b and c. With AB as a radius and the compass point at a, b, and c, draw arcs intersecting at d and c. Draw AC, CD and BD making all three equal to AB.

In Figure 6 draw a convenient angle A. Place the compass at A and draw an arc intersecting the sides of the angle at B and C. With B and C as centers draw arcs intersecting at D. Draw AD.

The students should practice these first six basic constructions until they have acquired sufficient skill to make them easily and proficiently.

A straight edge and compass construction of the pentagon is possible only after the students have learned how to divide a line into an extreme and mean ratio. Figures 7, 8, and 9 give the essential knowledge for making these drawings.

In Figure 7 the line AB is drawn and extended to F. At B

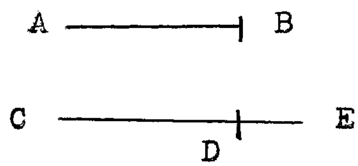

Figure 1. To construct a line equal to a given line.

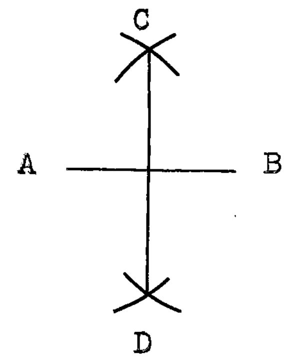

Figure 2. To construct a perpendicular bisector of a given line.

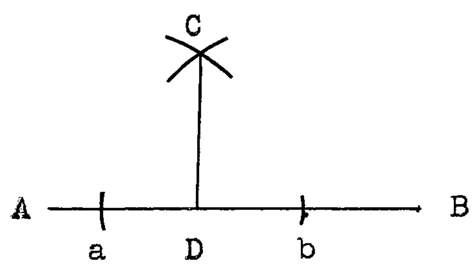

Figure 3. To construct a line perpendicular to a given line at a point on the line.

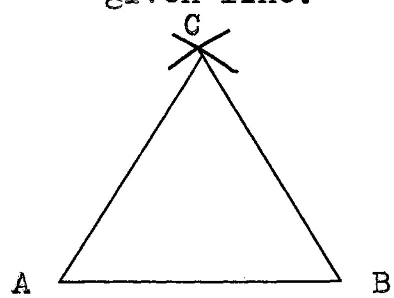

Figure 4. To construct an equilateral triangle given a side.

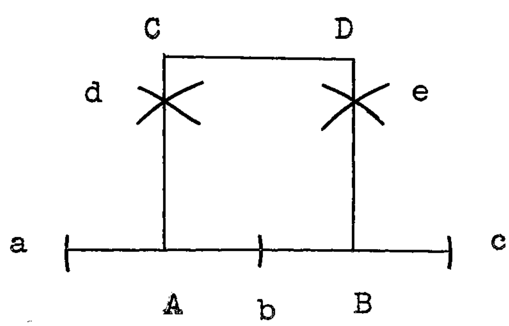

Figure 5. To construct a square given a side.

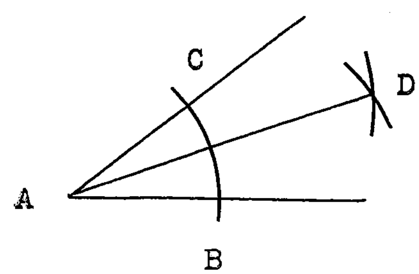

Figure 6. To bisect a given angle.

a perpendicular BC is erected to the line AB. The line AB is bisected at E. Mark off BO equal to AE. Draw the line AO and take OD equal to OB. With A as a center and AD as a radius draw the arc DC. Then C is the required point of division. A proof of this problem will not be given, as it may be found in any Plane Geometry textbook.

The next problem is to construct an angle of 72 degrees. In Figure 8 a line AB was drawn equal to the AB of the previous figure and the point C was located. With A as a center and a radius equal to AB, an arc was drawn above the line. With B as a center and a radius equal to AC an arc was drawn above the line intersecting the first arc, thereby locating the point D. The line CD may be drawn, but it is not necessary for our purposes unless the teacher desires a proof of the construction. It is not difficult to show that the angle ABD is equal to 72 degrees.

The actual construction of the regular pentagon can be accomplished by the class at this time. As is shown in Figure 9 an angle of 72 degrees is constructed at O, which will also be the center of the circumscribed circle.

Thus far the pupils have not been shown how to construct an angle equal to a given angle. This procedure can easily be demonstrated at this time. Draw the line segment OB. In Figure 8 with B as a center draw the arc ab. Then with O as a center in Figure 9 and the same radius draw arc cd. Using your compass, measure across arc ab and there use d as a center with the same radius draw the arcs intersecting at C. The radius of the circle is determined by the size one wishes

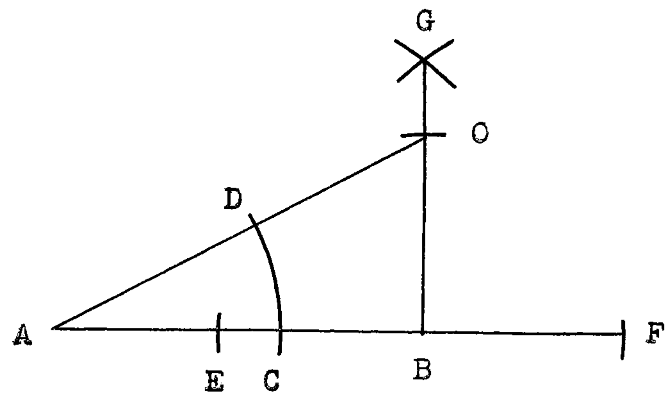

Figure 7. To divide a given line in an extreme and mean ratio.

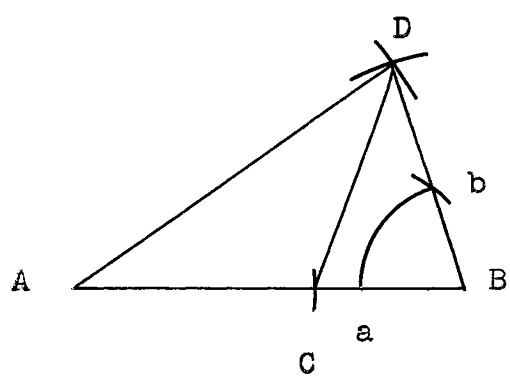

Figure 8. To construct an angle of 72 degrees.

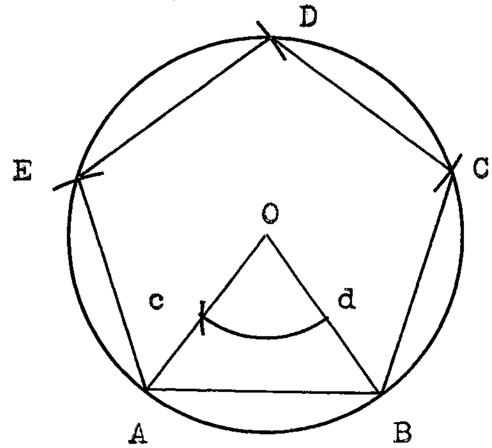

Figure 9. To inscribe a pentagon in a given circle.

to make sides of the pentagon. After the chord AB has been drawn, its arc is measured with a compass and the other equal arcs of the circle are determined. The pentagon is completed by joining the points of division of the arcs.

The teacher may decide to make the constructions using a ruler and a protractor. If this is the case, then some time may have to be spent in teaching the scale markings on the ruler and the degree markings on the protractor. This author will continue to discuss the straight edge and compass procedures although the other method will achieve the same result.

The tetrahedron may be constructed following the pattern given in Figure 10. First construct the equilateral triangle ABC and then, bisect the three sides at D, E, and F. The glue tabs are added on the sides DC, CE, and AF. The size of the tabs do not affect the appearance of the completed figure. The drawing is cut out on the solid lines and folded on the dotted lines. Apply a small amount of airplane glue on the tabs and carefully press the corresponding parts together. Some difficulty may be encountered with the last tab to be glued, but with a little practice this difficulty will be overcome. Patience should result in a pleasing figure.
If one desires different colors for each face of the tetrahedron, an equilateral triangle should be constructed with a glue tab on each side. It will take a little longer to glue the figure together, but the more pleasing appearance may justify the additional effort.

As the cube or hexahedron is merely a combination of con-

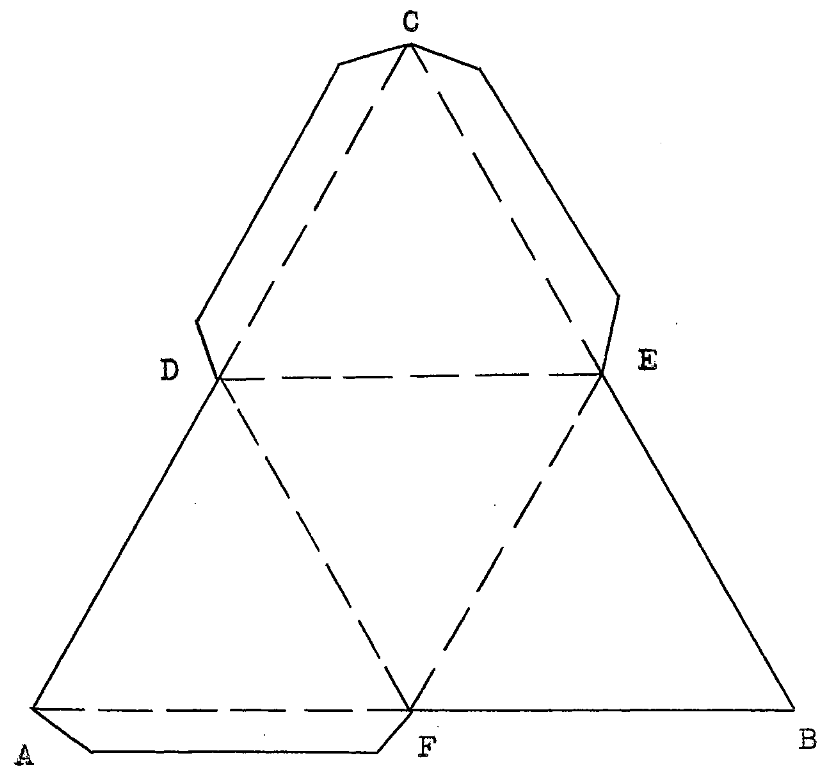

Figure 10. The Tetrahedron

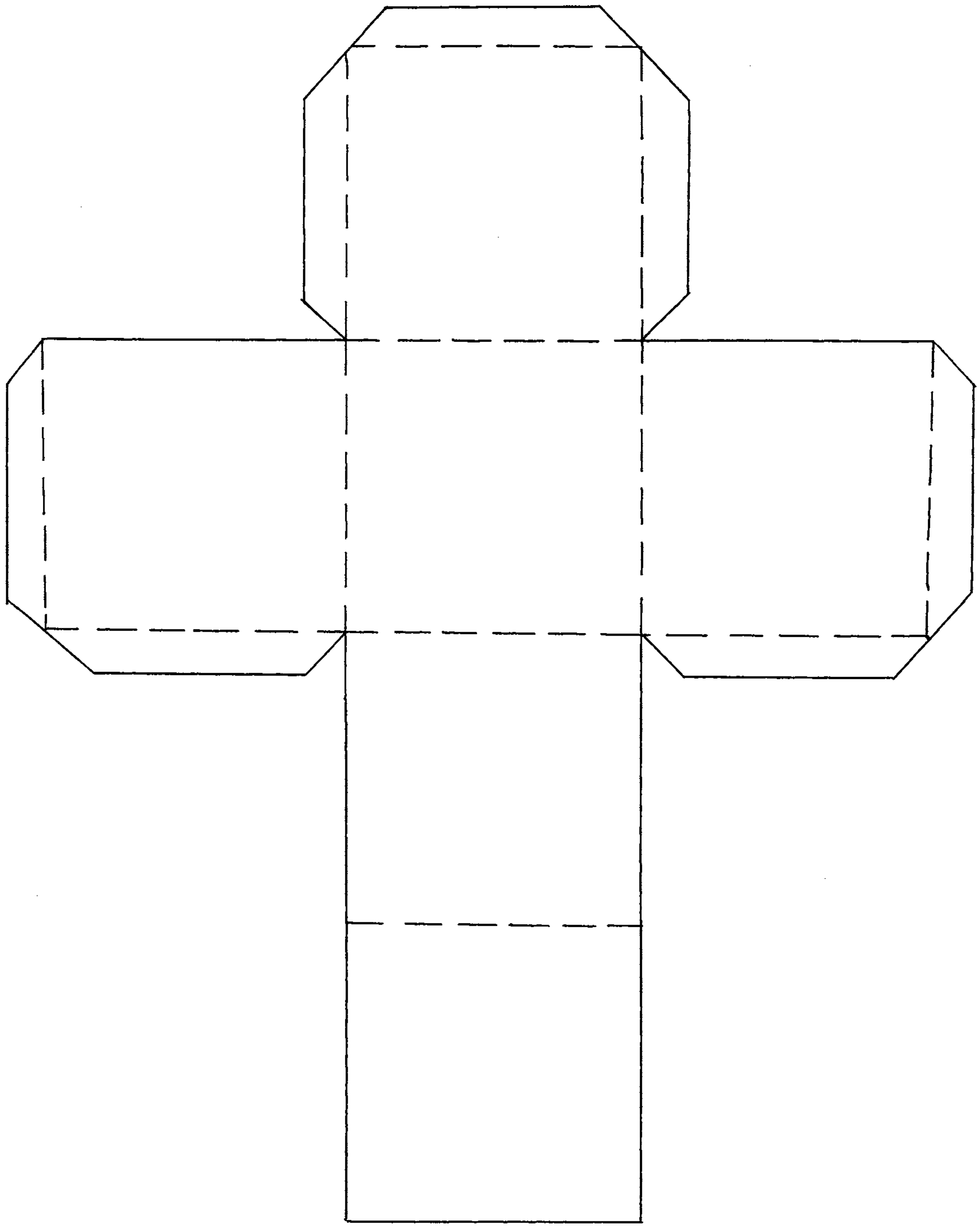

Figure 11. The Cube or Hexahedron

gruent squares, the figure is very quickly constructed. Again, it should be cut on the solid lines and folded on the dotted lines. The same gluing technique should be followed. Different colors may be used for each square, but if this is done, then each square should be constructed separately and glue surfaces left on each side.

The octahedron or eight sided solid is a series of equilateral triangles. If only one color is desired, the drawing in Figure 12 may be followed by cutting the solid lines and folding the dotted ones. Individual triangles with three glue tabs will have to be used if more than one color is wanted.

The icosahedron or twenty sided regular solid is also a combination of equilateral triangles. For convenience and because of the space limitations, only one half of the pattern was drawn in Figure 13. Thus, two copies of the drawing must be made. The two figures may be cut out on the solid lines, but before folding the glue tabs labeled AB must be joined, A to B and B to A. To do this one of the figures must be turned over. After this has been glued the same steps are followed for the rest of the assembly.

The dodecahedron or twelve sided regular solid has faces composed of regular pentagons. As was the case in the icosahedron, only one half of the drawing is given. One of the two required figures should be turned over and the tab AB glued as was done in Figure 13. After the size of the central pentagon has been determined, the outer pentagons may be constructed by using the side and diagonal of the first.

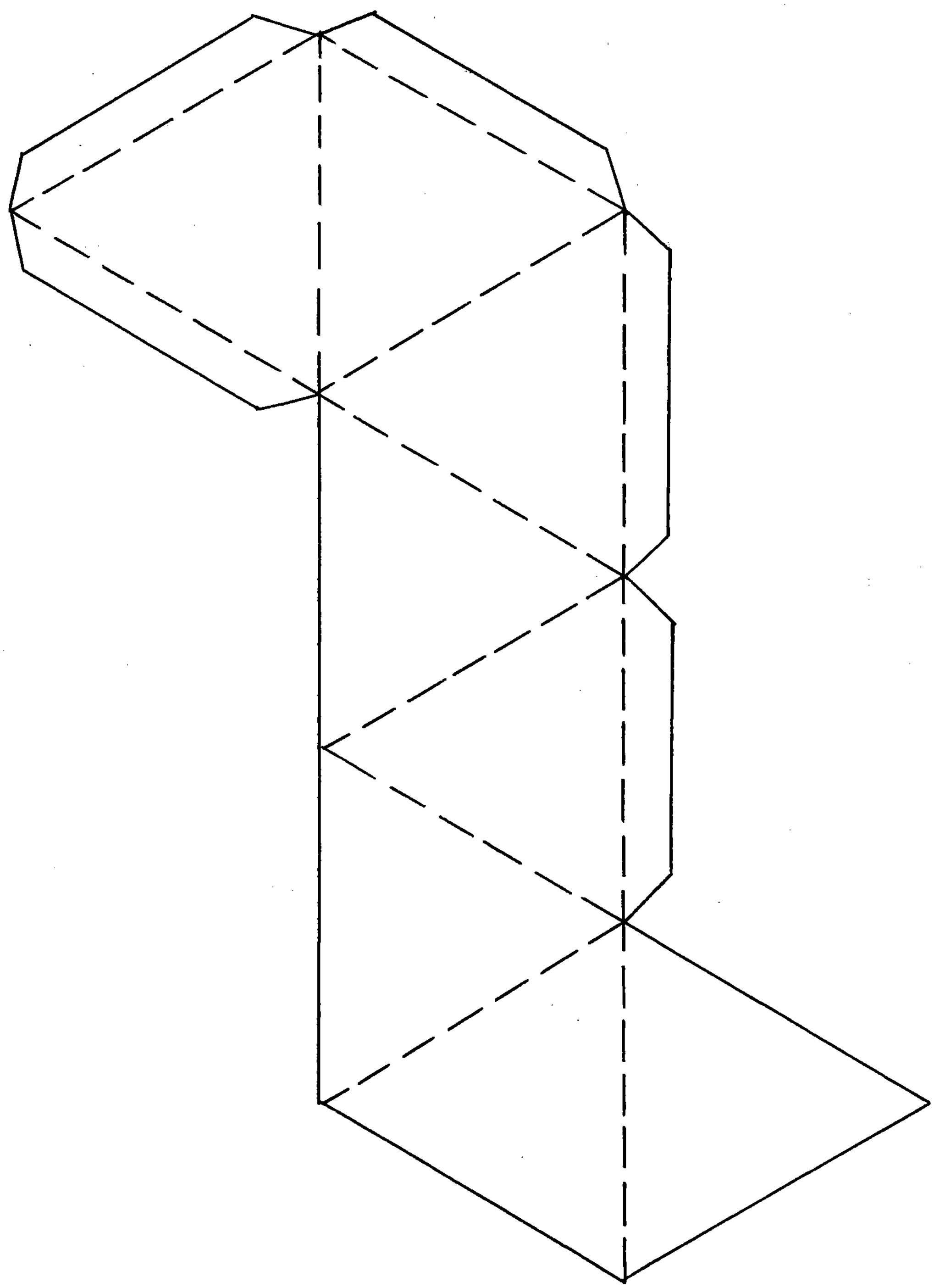

Figure 12. The Octahedron

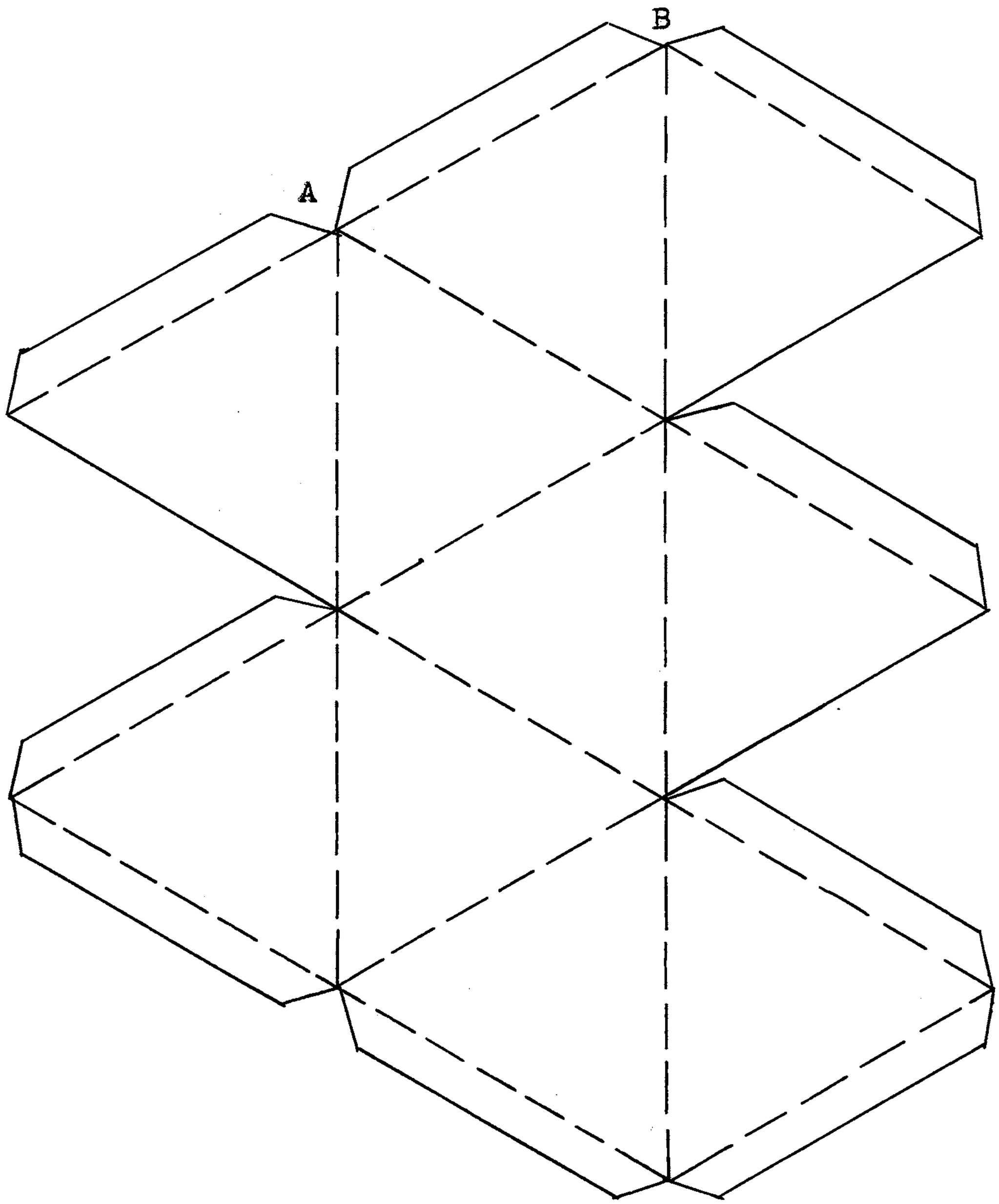

Figure 13. The Icosahedron

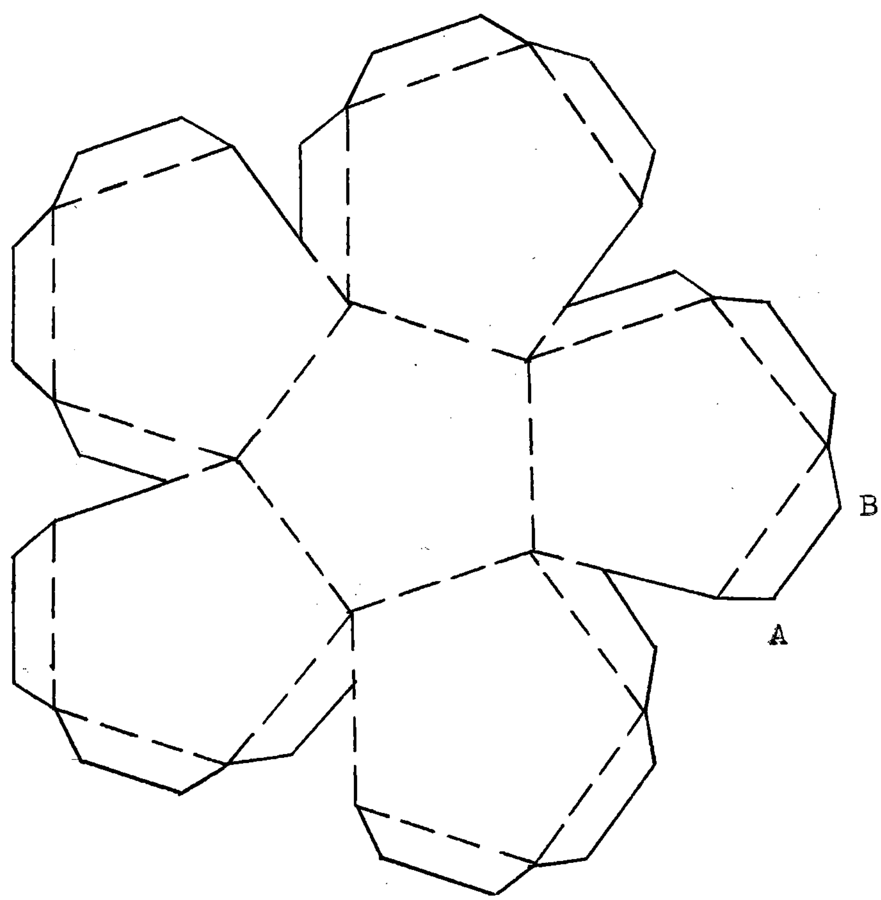

Figure 14. The Dodecahedron

CHAPTER IV

CLASSROOM USES FOR THE POLYHEDRONS

The completed polyhedrons may be used in a variety of classroom situations. The inner satisfaction of actually creating a pleasing symetrical geometrical figure should buid up the morale of many of the children who may have grown weary of the repeated manipulation of numbers.

At this time the names of the five solids should easily be learned by the class. Mere possession of the solids will make the learning seem less of a chore.

Various terms, together with the appropiate definitions could be introduced at this time. These would include edges, planes, vertices, faces, dihedral angle, face angle, and polyhedral angles. As the investigation of the various properties progresses other terms, some elementary and others more advanced, may be given.

Early in the discussion, a demonstration of the number of regular polygons required to fill the space around a point should be shown. Naturally, there will be six equilateral triangles, four squares, or three hexagons. The formula for finding the interior angle of any regular polygon could be given at this time.

At this point, some energetic students will probably want to begin construction of the next regular polyhedron. As the pupils have already acquired some knowledge of polyhedral angles a demonstration, with or without proof, that the sum of the face angles must be less than 360 degrees will be easy. A combination of 3, 4, or 5 equilateral triangles at one vertex, 3 squares at one vertex, or 3 regular pentagons at one vertex are all the possibilities. The student has already completed the constructions of these solids.

The more mature students, possibly members of the school's mathematics club, may be interested in investigating the altitudes of the regular tetrahedron. Whether or not the altitudes meet in a point should prove to be a very interesting problem. Court's article in the Mathematics Teacher is an excellent reference for this investigation.[1]

The tetrahedron and the cube may be used for elementary problems involving permutations and combinations. First select six basic colors of construction paper. If each face of a cube is made of different colored paper and all possible arrangements of the colors are made, thirty cubes can be constructed. As the average mathematics class consists of about thirty members, each student could be asked to make one square out of each color with glue tabs on each side, after a standard size for the squares has been determined. The

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[1]Court, N. A. "A Historical Puzzle," The Mathematics Teacher, Vol. 52, 1959, pp. 31-32.

teacher could predetermine the arrangements and assign one to each class member. This procedure should result in the thirty required cubes.

The question will quite naturally arise on why there are 30 different cubes or if there is a mathematical way to determine the correct number without actually constructing them. One method involves the arbitrary selection of one color for the first face. This leaves a choice of five colors for the opposite face. After these two faces have been chosen, the arrangement of the other four faces constitutes a circular permutation of four things. This can be done in 3 factorial or 6 different ways. The total number of ways to arrange the colors will be 6 times 5 or 30.

While it is theoretically possible to construct the other remaining polyhedrons using this idea, the tetrahedron is the only other practical one to construct. In this case select four basic colors and construct the congruent equilateral triangles with glue tabs on each side. If one color is permuted in 2 factorial or 2 ways. The total number of ways would be 1 times 2 or 2 different tetrahedrons.

Another method, which is more easily adapted to the three remaining polyhedrons depends on the number of ways the solid may be turned over or rotated. The number of rotations can be found by multiplying the number of edges per face. These numbers can be determined by the individual students.

The tetrahedron could be colored 4 factorial ways if the number of rotations were not considered. Thus, the number of tetrahedrons that can be painted in different ways with 4 colors is 4 factorial divided by 12 which equals 2.

If we let F equal to the number of faces, n the number of edges per face, R the number of rotations, and P the number of permutations of F colors, then P equals F!/R. Since R equals Fn, reduce this to (F-1!/n.[2]

The student, after calculating the possibilities with the octahedron, dodecahedron, and icosahedron, will see the futility of attempting the actual constructions.

The students could make a table listing the number of faces, the number of edges, and the number of vertices. It should be interesting to see if some of them could empirically derive Euler's formula by studying the table.

The various uses listed in this chapter are certainly not the only ones that may be utilized. Many teachers and perhaps some students will be able to think of many more applications.

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[2] Perishs, C. R. "Colored Polyhedra:A Permutation Problem." The Mathematics Teacher. Vol. 53, 1960, pp. 253-255.

CHAPTER V

ARCHIMEDEAN POLYHEDRONS

An Archimedean or semi-regular polyhedron is a solid whose polyhedral angles are equal and whose faces are composed of two or more regular polygons. These solids derived their name from Archimedes, who described fourteen of them which are also inscribable in a sphere. Some of his descriptions were later proved to be incorrect.[1]

An interesting feature of these constructions stems from the discoveries the individual student can make by experimenting with combinations of equilateral triangles, squares, regular pentagons and other polygons with equal sides.

The usual method for making an acceptable pattern is to take the regular polygons, glue them carefully, and see if a pleasing solid is formed. Then cut the model apart marking the adjoining edges so that glue tabs may be added to the pattern. In some cases, a student may be able to visualize the combinations without following this procedure and be able to make the pattern at once. For the convenience of the

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[1]Heath, Sir Thomas L., _A Manual of Greek Mathematics_, London, 1931, pp. 176-177.

teacher some of the possible combinations are given in Table I.[2]

The drawing for the canted cube or cuboctahedron is included in the text because of its historical signifance and ease of construction. The ancient Greeks were aware of this solid, as they obtained it by bisecting the edges of a cube and then cutting off the eight corners along the lines joining the points of bisection.

Composite and star polyhedrons can be made by gluing regular solids on the faces of the solids that have been previously described. It is necessary that the base of one solid and the face of the other solid be congruent.

An alternate and perhaps easier method of making these constructions is to replace one face of the solid with glue tabs on each edge, and then join these tabs to the corresponding tabs on the adjoining solid. As the base is hidden in the completed figure, one would be unable to differentiate the method used. An example of this procedure is given in Figure 15.

A very pleasing effect will be obtained in Figure 15 if the four required tetrahedrons are cut from different colored construction paper. As was done in the previous examples, cut the figure out on the solid lines and fold on the dotted lines. Next glue the tab AB to the edge BC forming the four

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[2]Hartley, Miles C. "A Laboratory Approach to Solid Geometry." Eighteenth Yearbook, National Council of Teachers of Mathematics, New York, 1945, pp. 56-57.

TABLE I

ARCHIMEDEAN POLYHEDRONS

Units of Each Polyhedral Angle	Number of Faces	Name of Polyhedron
1. Two 120° angles, one 60° angle.	Four hexagons and four equilateral triangles.	Truncated tetrahedron
2. Two 135° angles, one 60° angle.	Six octagons and eight equilateral triangles.	Truncated cube
3. Two 120° angles, one 90° angle.	Eight hexagons and six squares.	Truncated octahedron
4. Two 144° angles, one 60° angle.	Twelve decagons and twenty equilateral triangles.	Truncated dodecahedron
5. Two 120° angles, one 108° angle.	Twelve pentagons and twenty hexagons.	Truncated icosahedron
6. Two 90° angles, one 60° angle.	Three squares and two equilateral triangles.	Archimedean prism
7. Two 90° angles, two 60° angles.	Six squares and eight equilateral triangles.	Cuboctahedron
8. Three 90° angles, one 60° angles.	Twelve squares and eight equilateral triangles.	Small rhombicuboctahedron
9. Two 108° angles, two 60° angles.	Twelve pentagons and twenty equilateral triangles.	Icosidodecahedron
10. Three 60° angles, one $\frac{180^\circ (n-2)}{n}$ angle.	Two polygons with n sides and 2n equilateral triangles.	Archimedean prismatoid
11. One 90° angle, four 60° angles.	Six squares and thirty-two equilateral triangles.	Snub cube
12. One 108° angle, four 60° angles.	Twelve pentagons and eighty equilateral triangles.	Snub dodecahedron
13. One 90° angle, one 120° angle, one 135° angle.	Six octagons and eight hexagons and twelve squares.	Great rhombicuboctahedron
14. One 90° angle, one 120° angle, one 144° angle.	Twelve decagons and twenty hexagons and thirty squares.	Great rhombicosidodecahedron
15. One 108° angle, one 90° angle, one 60° angle.	Twelve pentagons and thirty squares and twenty equilateral triangles.	Small rhombicosidodecahedron

tetrahedrons. Then glue the corresponding tabs together to complete the required figure. A little difficulty may be experienced with the last tabs to be glued.

Many other pleasing figures may be obtained by following the basic procedures given for Figure 15.

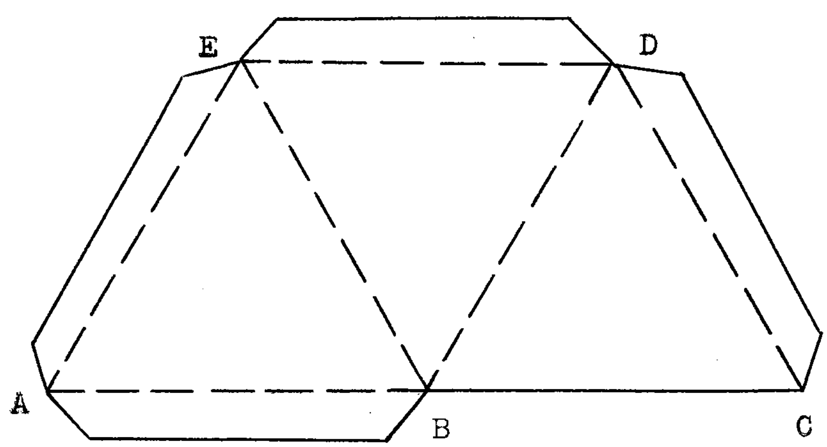

Figure 15. Tetrahedrons on the Faces of Tetrahedrons.

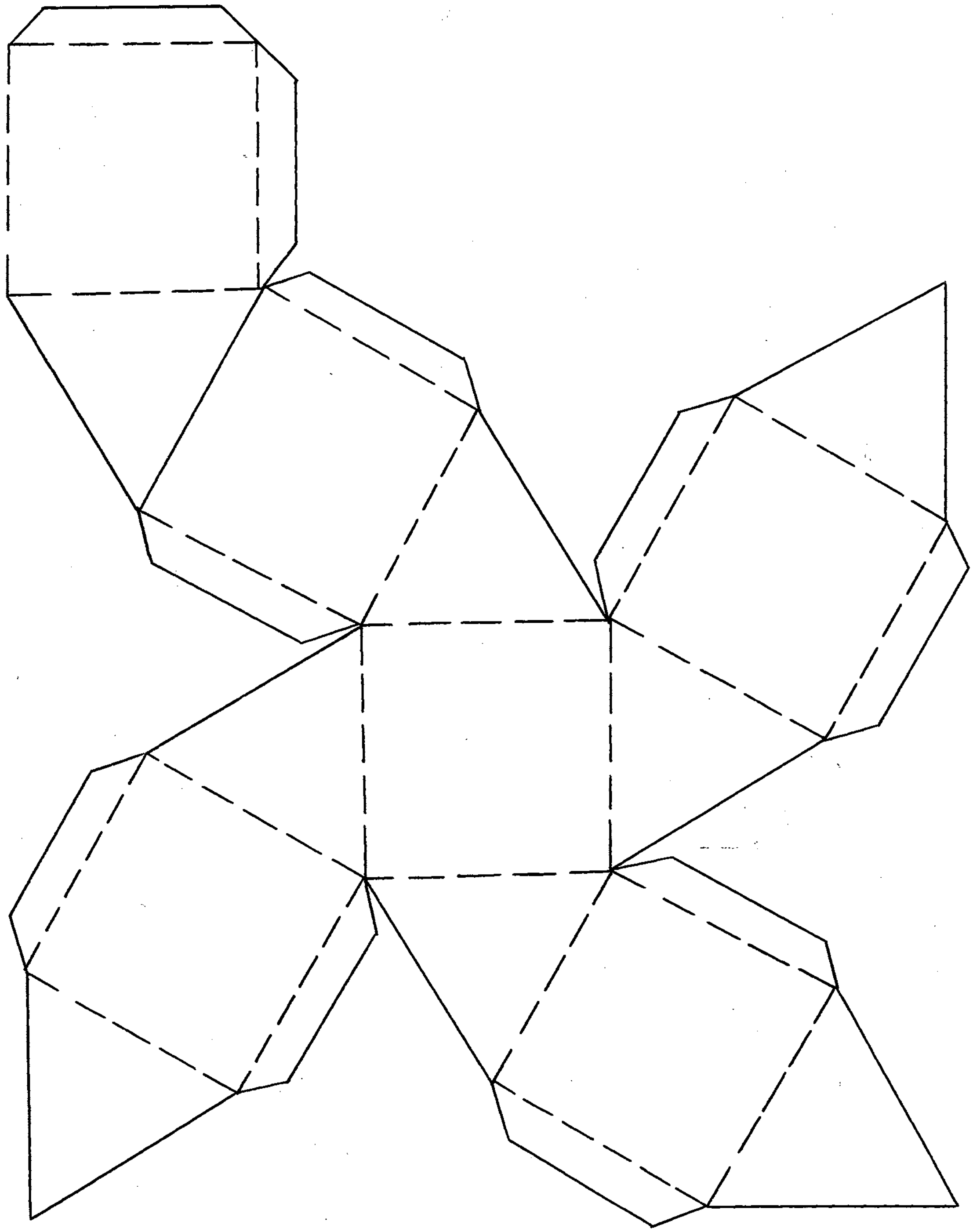

Figure 16. Canted Cube

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