UNIVERSITY OF OKLAHOMA

GRADUATE COLLEGE

AN INTEGRATED MULTI-ECHELON MULTI-OBJECTIVE PROGRAMMING ROBUST CLOSED-LOOP SUPPLY CHAIN UNDER DYNAMIC UNCERTAINTY SETS AND IMPERFECT QUALITY PRODUCTION

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AN INTEGRATED MULTI-ECHELON MULTI-OBJECTIVE PROGRAMMING ROBUST CLOSED-LOOP SUPPLY CHAIN UNDER DYNAMIC UNCERTAINTY SETS AND IMPERFECT QUALITY PRODUCTION

A DISSERTATION APPROVED FOR THE
SCHOOL OF INDUSTRIAL AND SYSTEMS ENGINEERING

BY

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To my parents Madinah and Ibrahim

To my lovely wife Hawra, and sweety kid Yaseen

To the soul of my grandmother Khatoon

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ABSTRACT

Modeling robust closed- loop supply chain under multiple uncertainties and multiple criteria where imperfect quality production is incorporated is a new research trend in this area. Such integration is essential as it provides meaningful solutions to the practical problems of supply chain management. In this dissertation, we develop three models. In the first model, we consider a novel closed loop supply chain design consisting of multiple periods and multiple echelons. In addition, we assume that the screening is not always perfect, and inspection errors are more likely to take place in practice. We measure the amount of quality loss as conforming products deviate from the specification (target) value. In this model, we develop three robust counterparts models based on box, polyhedral, and combined interval and polyhedral uncertainty sets. We utilize different a priori probability bounds to approximate probabilistic constraints and provide a safe solution. The objective is to minimize the total cost of the supply chain network.

As an extension to the first model, the second model considers a robust multi-objective mixed integer linear programming model which includes three objectives simultaneously. The first objective function minimizes the total cost of the supply chain. The second objective function seeks to minimize the environmental influence, and the third objective function maximizes the social benefits. The augmented weighted Tchebycheff method is used to aggregate the three objectives into one objective function and produce the set of efficient solutions. Robust optimization, based on the extended Mulvey et al. (1995) approach, is used to obtain a set of solutions that are robust against the future fluctuation of parameters.

In the third model, the affinely adjustable robust formulation based on "wait and see" decisions is presented. That is, the decisions are made over two sequential stages where multiple uncertainties are included. Moreover, we propose a budget dynamic uncertainty set to mimic the dynamic behavior of the market demand over time. The introduced dynamic uncertainty set is formulated according to Vector Autoregressive (VAR) models where the temporal and spatial correlations of customer demand zones are captured. Also, we utilize different a priori probability bounds to approximate probabilistic constraints and provide safe solutions.

Finally, numerical examples have been presented to test and analyze the tradeoff between solution robustness and models robustness. The results reveal valuable managerial views. Our proposed models are compatible with several types of industries including steel making, electronic and automobile manufacturing, and various plastic products where return products (either defective or used) can be reused as a raw material, and when environmental and social issues become a company concern.

Keywords: uncertainty sets, robust counterpart, a priori probabilistic bound, closed loop- supply chain, imperfect quality production, Tchebycheff method, dynamic uncertainty set, adjustable robust counterpart.

CHAPTER 1: INTRODUCTION

In this chapter, we briefly explore the different approaches used to deal with uncertainty in the modeling optimization problems. Then, we introduce the concept of robust optimization through simple examples. In the motivation section, we propose our novel robust optimization approach for inventory optimization problems with uncertainty sets. Finally, the structure for the rest of dissertation chapters is presented.

1.1 Relative Background:

One assumption of the parameter values in optimization problems is that they are usually assumed to be precisely known. However, this is not always the case in practical real- life problems. Parameter uncertainties might have a significant influence on the solution optimality and model feasibility if they are ignored. Therefore, the uncertainties have to be considered in both modeling and analysis stages. Thus, the current research streams tends to tackle the problems raised in an uncertain environment.

1.1.1 Approaches Used to Deal with Uncertainty

Although there are several different approaches to deal with uncertainties in the optimization problems, researchers recently have utilized four main approaches depending on the level of uncertainties and information availability in the problems: dynamic programming, fuzzy, stochastic, and robust optimization, see figure 1.1.

Dynamic programming was developed by Richard Bellman in the 1953 and has found applications in numerous fields. In its traditional version, if large problems can be broken into sub-problems and then recursively finding the optimal solutions to the sub-problems, then dynamic programming methods are applicable. This is done through a mathematical relationship which is known in the optimization literature as Bellman equation. However, one major issue with dynamic programming is the curse of dimensionality resulted from medium to large scale problems. Therefore, several techniques have been developed to address this issue and commonly known as approximate dynamic programming. Interested readers may refer to the book titled " Approximate Dynamic Programming" by Powell.

The concept of a fuzzy set was originally published in 1965 by Lotfi Zadeh. Since that time, the fuzzy set theory has been applied with great success in many different fields when uncertainty associated with data exists. The model is formulated based on the generalization of the classical concepts of set and its characteristic membership function. However, with Fuzzy logic, a well-defined set of rules is needed, and these rules are not capable of handling indeterminate relations that exist in the data., (Kumar and Yadav, 2015). Flexible programming and possibilistic programming are classified as special cases of fuzzy set theory programming.

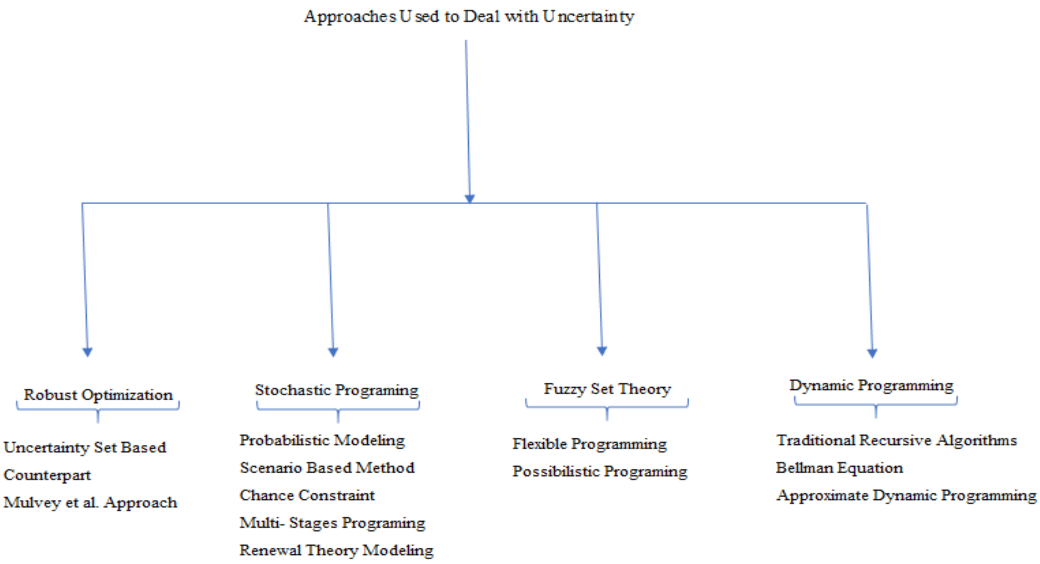


Figure 1.1: Approaches used to deal with uncertainty in Operations Researches.

When the probability distribution of an uncertain parameter is known, the appropriate modeling approach is stochastic programming. This approach is one of the most important approaches used to deal with uncertainty in operations research and has applications in a broad range of areas. In terms of methods, stochastic programming appears in various forms such as probabilistic modeling, scenario based method, chance constraint (sometimes known as probabilistic guarantees on constraint satisfaction), two stage (recourse modeling) programming, and renewal theory modeling. The stochastic programming seeks to optimize the expected value or some other suitable utility function, and thus the solution is not robust. Also, this approach results in huge optimization problems with several assumptions and heavy data requirements. When the probability distribution of the uncertain parameter is unknown, robust optimization is the appropriate modeling approach. In the next section, we introduce the definition of robust optimization with simple examples.

1.1.2 Definition of Robust Optimization and Examples

Robust optimization was first proposed in the early 1970s in order to provide a decision-making framework when probabilistic models are either unavailable or intractable, and has been the focus of significant research attention from the 1990s onwards, (Sozuer and Thiele, 2016). Robust optimization is an important methodology for dealing with optimization problems with data uncertainty. Although no distribution assumption is made on uncertain parameters, the availability of data information can be utilized beneficially.

Before we introduce the mathematical concept of robust optimization, consider the example of " cancer treatment" by Chu, Zinchenko, Henderson, and Sharpe (2014). Studies show that about 1.3 million new cancer cases in the U.S. each year, and nearly 60% receive radiation therapy (in conjunction with surgery, chemotherapy etc.). In external beam radiation therapy, radiation is delivered by a linear accelerator. Because cancer cells are more susceptible than normal cells, overlay beams are released from different angles, see figure 1.2.

Figure 1.2: External beam radiation therapy.

Each radiotherapy beam is divided into many small beamlets that can vary the intensity of radiation. This allows different doses of radiation to be given across the tumor. Intensity-modulated radiation therapy (IMRT) can be very helpful in areas such as the head and neck, for example to avoid the spinal cord or salivary glands. The objective is to choose beam angles and beamlet intensities that deliver enough radiation to kill all tumor cells, while avoiding healthy organs and tissue as much as possible. The sources of uncertainties are setup errors, patient motion, and structural changes during treatment. Therefore, robust optimization is critical to achieve a safe treatment planning.

Robust optimization assumes that the uncertain data belongs to a convex and bounded set, called uncertainty set, (Sozuer and Thiele, 2016). The uncertainty set is defined as the set of all possible realizations of the uncertain parameter that will be considered in the robust problem. The known uncertainty sets in the literature are,

1. Box Uncertainty Set
2. Ellipsoidal Uncertainty Set
3. Polyhedral Uncertainty Set
4. Interval + Ellipsoidal Uncertainty Set
5. Interval + Polyhedral Uncertainty Set
6. Box + Ellipsoidal Uncertainty Set
7. Box+ Polyhedral Uncertainty Set
8. Interval + Ellipsoidal + Polyhedral Uncertainty Set
9. Box + Ellipsoidal + Polyhedral Uncertainty Set

The proposed uncertainty sets are formulated based on different norms of the perturbation variables. Moreover, the shape of the selected uncertainty set will affect the tractability of the resulting robust optimization counterpart, figure 1.3. The characteristics of used uncertainty sets will be explored in the later chapters.

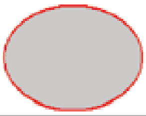
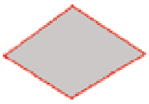
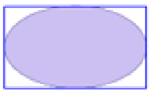
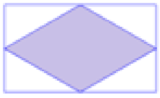
Illustration	Type	Adjustable Parameter
	Box	Ψ
	Ellipsoidal	Ω
	Polyhedral	Γ
	Interval + Ellipsoidal	$\Psi = 1$ Ω
	Interval + Polyhedral	$\Psi = 1$ Γ

Figure 1.3: Illustration of different types of bounded and convex uncertainty sets.

Robust optimization has many application areas including supply chain and logistics problems, combinatorial optimization, scheduling, and facility layout location. Some examples of finance applications are general portfolio problems and risk measures. In machine learning and statistics, the incorporation of robust optimization is a growing field, (Petros Xanthopoulos, Pardalos, and Trafalis, 2013). Another area that has seen significant growth recently is robust optimization in energy such as renewable energy, wireless network, and electricity markets. In health care applications, robust optimization is considered as an effective approach to IMRT treatment planning for different types of cancers.

1.2 Motivation: A Robust Optimization Approach for Inventory Problem

Since uncertainty is an essential issue in inventory production management, it has been recently discussed extensively by researchers and industry practitioners. The approach commonly used in their work is stochastic programming, where a specific probability distribution of the uncertain parameters is assumed. The multi-period inventory control problem under uncertainty has been also addressed using dynamic and fuzzy programming. In this work, we develop two robust counterpart inventory models based on the box and ellipsoidal uncertainty sets using a different approach than the one used in the literature. In our work, we utilize a priori probability bounds which can be used to approximate probabilistic constraints and provide safe solutions. Different upper probability bounds for both bounded and unbounded uncertainty, with and without detailed probability distribution information under different probability constraint violations are considered, and useful insights are gained for their corresponding robust solutions.

1.2.1 Literature Review

Although there are several different approaches to deal with uncertainties in the production systems and inventory control problems, researchers recently have utilized four main approaches depending on the level of uncertainties and information availability in the problems: dynamic programming, fuzzy, stochastic, and robust optimization.

In the dynamic programming approach (e.g., marketing demand) , for example, Mandel (2009) discussed a set of models and algorithms for inventory control with uncertainty and dynamic nature following the methodology of adaptive control theory and the theory of expert-statistical data processing. Kastsian and Martin (2011), however, focused on the so-called normal vector method which was developed for solving optimization problems in which stability or related dynamical properties of the systems have to be insured with uncertain parameters. They showed that this optimization method can be successfully applied for solving supply chain optimal design problems. On the other side, Song, Dong, and Xu (2014) considered a manufacturing supply chain with multiple suppliers and multiple uncertainties such as uncertain material supplies, production times, and customer demands. This integrated system was formulated using the stochastic dynamic programming approach. Chuang and Chiang (2016) also studied the dynamic and stochastic behavior of the coefficient of demand uncertainty incorporated with economic order quantity (EOQ) variables. They applied this approach to a finished-goods inventory from General Motors' dealerships. Recently Qiu, Sun, and Fong (2017) discussed a finite-horizon single-product periodic-review inventory management problem with demand distribution uncertainty. The problem was formulated as a robust dynamic program where the box and the ellipsoid uncertainty sets were used to formulate the corresponding robust counterpart.

As another approach to deal with uncertainties, many researchers have begun to analyze various problems related to inventory management models by incorporating fuzzy set theories. Interested readers may refer to (Shekarian, Kazemi, & Abdul-rashid, 2017). They conducted a comprehensive and systematic literature review in the field of fuzzy inventory management. One interesting study in this research stream is (Guillaume, Kobyla, & Zieli, 2012). They considered a lot sizing problem with uncertain demands modeled by fuzzy intervals. They also provided some algorithms for determining optimal robust production plans under fuzzy demands. Chen and Ho (2013) focused on an optimal inventory policy for the fuzzy newsboy problem with quantity discounts where the proposed solution was based on the ranking of fuzzy numbers and optimization theory. Treating demand in terms of fuzzy sets was also considered by Sadeghi, Sadeghi, Taghi, and Niaki (2014) with a vendor-managed inventory (VMI) policy in supply chain management. However, the solution was based on an improved particle swarm optimization algorithm. Recently, Farrokh, Azar, Jandaghi, and Ahmadi (2017) developed a novel robust fuzzy stochastic programming approach for closed loop supply chain network design under a hybrid uncertainty.

In some uncertain models, parameters follow known probability distributions. However, in many cases the available information about the probability distributions is limited or not known. When the probability distribution of an uncertain parameter is known, the appropriate modeling approach is Stochastic Programming. This approach is one of the most important approaches used to deal with uncertainty in production systems and inventory control, (Masih-tehrani, Xu, Kumara, & Li, 2011), (Z. L. Zhang, Li, & Huang, 2014), and (Wang, Qin, & Kar, 2015). However, when the probability distribution of the uncertain parameter is unknown, robust optimization is the appropriate modeling approach.

Making decisions in inventory control problems under uncertainty has been recently addressed using robust optimization. Commonly, marketing demand is treated as an uncertain parameter (Bertsimas & Thiele, 2006), (Bai, Alexopoulos, Ferguson, & Tsui, 2012), (Caglayan, Maioli, & Mateut, 2012), (Qiu, Shang, & Huang, 2014), (Carrizosa, Olivares-nadal, & Ramírez-cobo, 2016). However, Ammar et al. (2013) reviewed extensively some of the existing literature of supply planning under uncertainty of lead times.

Several studies on robust multi-period inventory problems have been recently discussed in the literature, (Giarr, Giarr`e, & Pesenti, 2008), (Aharon, Boaz, & Shimrit, 2009), (Aharon et al., 2009), (See & Sim, 2010), (Quansheng, 2015), (Vahdani, Soltani, Yazdani, & Mousavi, 2017), (Zhaolin Li & Grace, 2017).

Other extensions to the previous works were done by considering different parameters subject to uncertainty in production systems and inventory problems. For example, Al-e-hashem, Malekly, and Aryanezhad (2011) considered cost parameters of the supply chain and demand fluctuations subject to uncertainty for multi-product multi-site aggregate production planning. Their work was a generalization of Rahmani, Ramezanian, Fattahi, and Heydari (2013), Xin, Xi, Yu, and Wu (2013), and Hatefi and Jolai (2014) studies. In their papers, network design costs and customer demand were uncertain. Wei, Li, and Cai (2011), on the other hand, studied robust optimal policies of production and inventory with uncertain returns and demand. Pishvaee, Rabbani, and Torabi (2011) included the uncertainty of customer demands and transportation costs in a closed-loop supply chain network design. Similarly, Kisomi, Solimanpur, and Doniavi (2016) treated transportation costs, processing costs and customers' demand as uncertain where the counterpart was formulated based on three different uncertain sets namely, box, polyhedral and interval plus polyhedral uncertainty sets.

1.2.2 Inventory Problem Formulation

The model of the inventory problem at a single station and finite discrete horizons of T periods is considered to minimize a given cost function. The notation is defined as follows:

For, $k = 0, \dots \dots . T$

I_k: Quantity of stock available at the beginning of the kth period,

Q_k: Stock of goods ordered at the beginning of the kth period,

D_k: Demand during the kth period,

It can be noticed that,

$I_{k+1} = I_k + Q_k - D_k$, where $k = 0,1, \dots . T - 1$.

Thus, the closed form of I_{k+1} can be written as:

$$I_{k+1} = I_0 + \sum_{i=0}^{k} (Q_i - D_i),\ k = 0,1, \dots . T - 1 \quad (1.1)$$

We will consider that the stock available and the quantity ordered at each period is not subject to upper bounds. In their model they consider two types of costs; namely purchasing, and a holding/shortage cost. The purchasing cost $C(Q_k)$ is defined as follows:

$$C(Q_k) = \begin{cases} K + c.Q_k & if\ Q_k > 0 \\ 0 & Q_k = 0, \end{cases} \tag{1.2}$$

where c is the unit variable cost, and K is the fixed cost. The holding/shortage cost represents the cost associated with having either excess inventory, h (positive stock) or unfilled demand p (negative stock). Specifically, we consider a convex, piecewise linear holding/shortage cost $R(I)$ with:

$$R(I) = \max(hI, -pI), \tag{1.3}$$

where h and p are nonnegative real numbers, and $p > c$ is assumed so that the ordering stock remains a possibility up to the last period. Therefore, the mixed-integer programming modeling of the inventory problem can be formulated as:

$$minimize \sum_{k=0}^{T-1} c\, Q_k + Kv_k + y_k \tag{1.4}$$

Subject to

$$y_k \geq h\left(I_0 + \sum_{i=0}^{k}\left(Q_i - \widetilde{D}_i\right)\right) \qquad k = 0,1,\dots.T-1 \tag{1.5}$$

$$y_k \geq -p\left(I_0 + \sum_{i=0}^{k}\left(Q_i - \widetilde{D}_i\right)\right) \qquad k = 0,1,\dots.T-1 \tag{1.6}$$

$$0 \leq Q_k \leq Mv_k, \quad v_k \in \{0,1\}, \quad k = 0,1,\dots.T-1 \tag{1.7}$$

Where y_k is a variable which needs to be minimized according to (1.5) and (1.6) and M is a large positive number. In Bertsimas and Thiele (2006) model, the polyhedral plus interval uncertainty set was utilized where the uncertain parameter, $\widetilde{D}_i$, is defined as follows; $\widetilde{D}_i = D_i + \widehat{D_i}.\zeta_i$. Note that D_i is the nominal value and $\widehat{D}_i$ represents the deviation magnitudes from the nominal value of the uncertain parameter D_i. In addition, ζ_i is a variable that takes values in the interval [-1, 1]. Actually, this variable provides perturbations to the uncertain parameter.

1.2.3 A Novel Robust Counterparts Approach Based on Uncertainty Sets

Next, we describe a novel robust optimization approach for inventory optimization problems with box and ellipsoidal uncertainty sets. In order to ensure the computational tractability of robust optimization problems, the parameter uncertainty should be defined carefully. Specifically, the uncertainty set should be specified by the decision maker, (Gorissen, Yan, & Hertog, 2015). The size and shape of the uncertainty set reflect the degree of conservativeness and the preferences of the decision maker, respectively. Typically applied uncertainty sets are box, ellipsoidal, polyhedral or combinations of them, (Zukui Li, Ding, & Floudas, 2011).

Suppose, without loss of generality, that only the right-hand-side parameters in the constraints of (1.15-1.6) model have uncertain data. This assumption is valid because of the following:

- If uncertain data exists in the objective function as coefficients, then the objective function can be written as a constraint.
- In any constraint k, if the right-hand-side parameter is subject to uncertainty, then it can be written as,

$$y_k - h\left(I_0 + \sum_{i=0}^{k}(Q_i - D_i)\right) \geq 0, and\ y_k + p\left(I_0 + \sum_{i=0}^{k}(Q_i - D_i)\right) \geq 0$$

Therefore, we end up with a constraint that has uncertain parameters on the left-hand side only.

Assuming that only the parameter D_i is subject to uncertainty in the previous model, then in order to acquire control of the conservativeness degree of the robust formulation, the true value of the uncertain parameter $\widetilde{D}_i$ is represented as follows:

$$\widetilde{D}_i = D_i + \zeta_i\, \widehat{D}_i \tag{1.8}$$

In our work, we will use two different uncertainty sets to formulate the inventory counterpart problem; namely, box and ellipsoidal uncertainty sets. In addition, we will use a different approach than the one used previously in the literature. Our approach is based on probabilistic guarantees on constraint satisfaction. To immunize against uncertainty, we apply the robust counterpart approach to the original constraint (1.5) and (1.6) under the uncertainty set (1.8).

This is based on Soyster's approach (1973a). Then, the resulting optimization problem is as follows:

$$minimize \sum_{k=0}^{T-1} c\, Q_k + Kv_k + y_k \tag{1.9}$$

$$y_k - h\left(I_0 + \sum_{i=0}^{k} Q_i - \left(\sum_{i=0}^{k}(D_i) + max_{\zeta_i}\left[\sum_{i=0}^{k}(\zeta_i\, \widehat{D}_i)\right]\right)\right) \geq 0 \tag{1.10}$$

$$y_k + p\left(I_0 + \sum_{i=0}^{k} Q_i - \left(\sum_{i=0}^{k}(D_i) + max_{\zeta_i}\left[\sum_{i=0}^{k}(\zeta_i\, \widehat{D}_i)\right]\right)\right) \geq 0 \tag{1.11}$$

$$0 \leq Q_k \leq Mv_k, \quad v_k \in \{0,1\}, \quad k = 0,1,\ldots.T-1 \tag{1.12}$$

1.2.3.1 Robust Counterpart Based on Box Uncertainty Set:

Li et al. (2011) provided a comprehensive study on the robust counterpart formulation for linear and MILP. They gave the mathematical proof of the robust counterpart to linear and MILP using different uncertainty sets. The proposed uncertainty sets are formulated based on different norms of the perturbation variables.

The box uncertainty set is formulated based on the Chebyshev norm of the perturbation variables and it is presented as follows:

$$U_\infty = \{\zeta_i \,|\, \|\zeta_i\|_\infty \leq \Psi\} \;, \tag{1.13}$$

where Ψ is the adjustable parameter that controls the uncertainty set size, and hence controls the degree of conservatism, (see figure 1.4). If Ψ = 1, then the resulting uncertainty set is a unit sphere with respect to the Chebyshev norm which is a special case of the box uncertainty set.

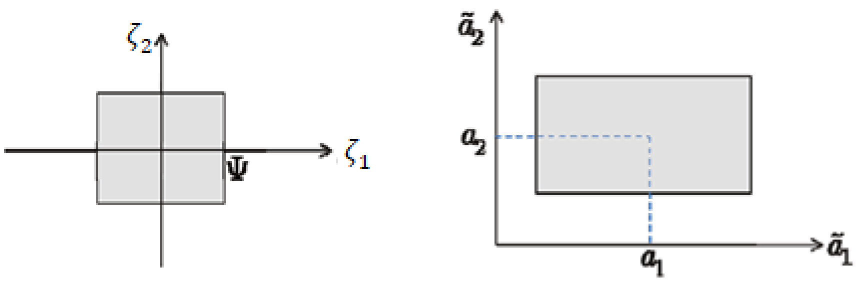


Figure 1.4: Illustration of box uncertainty set where a_1 and a_2 are the nominal values of the uncertain parameters $\tilde{a}_1$ and $\tilde{a}_2$, respectively.

The robust counterpart of the inventory problem model under the box uncertainty set (1.13) is given as follows:

$$minimize \sum_{k=0}^{T-1} c\, Q_k + K v_k + y_k \tag{1.14}$$

$$y_k - h\left(I_0 + \sum_{i=0}^{k} Q_i - \left(\sum_{i=0}^{k} (D_i) + \Psi_k \left(\left[\sum_{i=0}^{k} (\,\widehat{D}_i) \right] \right) \right) \right) \geq 0 \tag{1.15}$$

$$y_k + p\left(I_0 + \sum_{i=0}^{k} Q_i - \left(\sum_{i=0}^{k} (D_i) + \Psi_k \left(\left[\sum_{i=0}^{k} (\,\widehat{D}_i) \right] \right) \right) \right) \geq 0 \tag{1.16}$$

$$0 \leq Q_k \leq M v_k, \quad v_k \in \{0,1\}, \quad k = 0,1,\ldots.T-1 \tag{1.17}$$

1.2.3.2 Robust Counterpart Based on Ellipsoidal Uncertainty Set:

The ellipsoidal uncertainty set is defined as follows:

$$U_2 = \{\zeta_i \,|\, \|\zeta_i\|_2 \leq \Omega\}\,, \tag{1.18}$$

where Ω is the radius of the uncertainty set; it also represents the degree of conservatism. The ellipsoidal uncertainty set is formulated based on the 2-norm of the perturbation variables, (see figure 1.5).

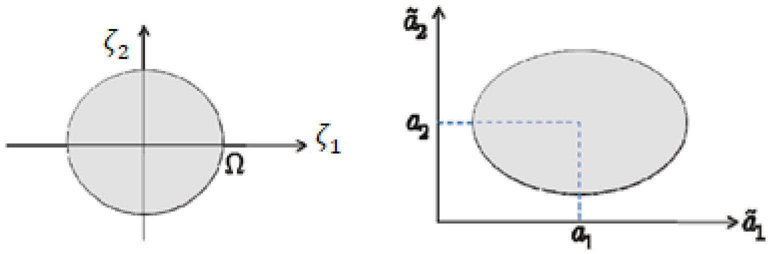


Figure 1.5: Illustration of ellipsoidal uncertainty set where a_1 and a_2 are the nominal values of the uncertain parameters $\tilde{a}_1$ and $\tilde{a}_2$, respectively.

$$minimize \sum_{k=0}^{T-1} c\, Q_k + K v_k + y_k \tag{1.19}$$

$$y_k - h\left(I_0 + \sum_{i=0}^{k} Q_i - \left(\sum_{i=0}^{k} (D_i) + \Omega_k \sqrt{\left[\sum_{i=0}^{k} \left(\,\widehat{D}_i^{\,2}\right) \right]} \right) \right) \geq 0 \tag{1.20}$$

$$y_k + p\left(I_0 + \sum_{i=0}^{k} Q_i - \left(\sum_{i=0}^{k} (D_i) + \Omega_k \sqrt{\left[\sum_{i=0}^{k} \left(\widehat{D}_i^{\,2} \right) \right]} \right) \right) \geq 0 \qquad (1.21)$$

$$0 \leq Q_k \leq M v_k, \quad v_k \in \{0,1\}, \quad k = 0,1, \ldots . T-1 \qquad (1.22)$$

Note that as the robust counterpart is formulated for each constraint, different uncertainty set size parameters values can be applied for different constraints.

1.2.4 Probabilistic Guarantees of Robust Counterpart Optimization

In many practical problems, the uncertainty set is defined by the decision maker. What makes robust optimization (RO) different from stochastic programming is that RO does not require a known probability distribution for the uncertainty. However, probabilistic guarantees (chance constraint approach) can be used to evaluate the lower bound on constraint satisfaction based on the desired constraint violation.

Li, et al. (2012) and Guzman, et al. (2016) considered probabilistic guarantees on constraint satisfaction employed in the literature for different uncertainty set robust counterpart optimization models, for both bounded and unbounded uncertainty, with and without a detailed probability distribution information.

In general, two different methods can be used in evaluating the probabilistic guarantees: a priori and a posteriori probability bounds. In this work, we will focus on the first type of methods which uses the uncertainty set information to derive the probability bound before we solve the problem.

1.2.4.1 Priori Probabilistic Guarantees Based on Uncertainty Set Information:

The a priori probabilistic guarantees approach is used as a traditional way to compute the size of the uncertainty set necessary to ensure that the degree of constraint violation does not exceed a certain level. Therefore,

$$\Pr\{\sum_j a_{ij} x_j + \sum_{j \in J_i} \zeta_j \, \hat{a}_{ij} x_j > b_i\} \leq \Pr\{\sum_{j \in J_i} \zeta_j \, a\delta_j > \Delta\} \qquad (1.23)$$

where the parameter Δ is the uncertainty set parameter (i.e. Ψ, or Ω), and J_i is the number of uncertain parameters in the ith constraint. Note that δ is a vector with its δ_j components satisfying $-1 \leq \delta_j \leq 1$. Moreover, $\sum_{j \in J_i} \delta_j \leq 1$, and $\sum_{j \in J_i} \delta^2{}_j = 1$ for box and ellipsoidal uncertainties sets respectively.

The poof of (1.23) is available in Li et al. (2011). The summary of different upper bounds on the probability of constraint violation is presented in Table 1.1. It is important to mention that the a priori probability bounds apply to bunded probability distributions such as in the case of uniform or triangle distributions. If the uncertainty probability distribution of the random variable is unbounded as in the cases of exponential or normal distributions, then a priori probability bounds do not apply, (Li, et al., 2012).

upper bounds on the probability of constraint violation	Assumption on Uncertainty distribution	Robust Counterpart Applicable	Proposed by
B1: $\exp(-\frac{\Delta^2}{2})$	Independent, symmetric, bounded	B, E, IE, P, IP	(Ben-tal & Nemirovski, 2000)
B2: $\exp(-\frac{\Delta^2}{2\lvert J_i\rvert})$	Independent, symmetric, bounded	B, E, IE, P, IP	(Bertsimas & Sim, 2004b)
B3:$\exp(min_{\theta>0}\{-\theta\Delta+\sum_{j\in J_i} In\ E[e^{\theta\zeta_j}]\})$	It has known probability distribution.	B, E, IE, P, IP	(Paschalidis & Kang, 2005)
B4: $\exp(min_{\theta>0}\{-\theta\Delta+\sum_{j\in J_i} In\ G_j(\theta)\})$	known bounds on $E[\zeta_j]$	B, E, IE, P, IP	(Guzman, Matthews, & Floudas, 2016)
B5: $\exp(min_{\theta>0}\left\{-\theta\Delta+\lvert J_j\rvert\sum_{j\in J_i} In\ \bar{G}_j\left(\theta/\sqrt{\lvert J_j\rvert}\right)\right\})$	known bounds on $E[\zeta_j]$	E, IE	(Guzman et al., 2016)

Table 1.1: The summary of different upper bounds on the probability of constraint violation.

Note that in Table 1.1 we follow the following abbreviations; B: Box, E: Ellipsoidal, IE: Interval and Ellipsoidal, P: Polyhedral, IP: Interval and Polyhedral. The proof of upper bounds on the probability of constraint violation provided by Table 1 is available in (Ben-tal & Nemirovski, 2000), (Bertsimas & Sim, 2004b), (Paschalidis and Kang, 2005), and (Guzman et al., 2016).

1.2.4.2 The Characteristics of The Introduced Probability Bounds:

From Table 1, it is observed that for the different types of robust counterparts, bounding the probability of constraint violation corresponds to the evaluation of the expression $\Pr\{\sum_{j\in J_i}\zeta_j\,\delta_j > \Delta\}$. The given probability bounds in Table 1 are bounded, symmetric and independent. Moreover, different bounds can be derived if the full probability distribution information of the uncertainty is provided. The following characteristics of the introduced probability bounds can be listed as follows:

1. If $\{\zeta_j\}_{j\epsilon J_i}$ are independent and subject to a bounded and symmetric probability distribution supported on [-1, 1], then B1 and B2 apply. That is;

$$\Pr\{\sum_j a_{ij}x_j + \sum_{j\in J_i}\zeta_j\,\hat{a}_{ij}x_j > b_i\} \le \exp(-\frac{\Delta^2}{2}) \tag{1.24}$$

$$\Pr\{\sum_j a_{ij}x_j + \sum_{j\in J_i}\zeta_j\,\hat{a}_{ij}x_j > b_i\} \le \exp(-\frac{\Delta^2}{2|J_i|}) \tag{1.25}$$

However, B1 only applies for the box (B), ellipsoidal (E), and interval plus ellipsoidal (IE) uncertainty sets induced robust counterparts.

2. If $\{\zeta_j\}_{j\epsilon J_i}$ are independent and subject to a symmetric probability distribution, then B3 applies such that,

$$\Pr\left\{\sum_j a_{ij}x_j + \sum_{j\in J_i} \zeta_j \,\hat{a}_{ij}x_j > b_i\right\} \leq \exp\left(min_{\theta>0}\left\{-\theta\Delta + \sum_{j\in J_i} In\ E\left[e^{\theta\zeta_j}\right]\right\}\right) \quad (1.26)$$

where $E\left[e^{\theta\zeta_j}\right]$ refers to the moment generation function of probability density function $f(\zeta_j)$. Moreover, it needs the solution of the following additional nonlinear nonconvex optimization problem (1.27):

$$\begin{aligned} &\min \Delta \\ &\text{s.t.} \\ &-\theta\Delta + \sum_{j\in J_i} In\ E\left[e^{\theta\zeta_j}\right] \leq \ln(\varepsilon) \\ &\Delta, \theta \geq 0 \end{aligned} \quad (1.27)$$

3. For B4 and B5 the uncertain parameters have known lower and upper bounds and their means are known only to within some range of values. Hence, a single expected value cannot be confidently imposed. Thus, we have the following expressions:

$$\Pr\left\{\sum_j a_{ij}x_j + \sum_{j\in J_i} \zeta_j \,\hat{a}_{ij}x_j > b_i\right\} \leq \exp\left(min_{\theta>0}\left\{-\theta\Delta + \sum_{j\in J_i} In\ G_j(\theta)\right\}\right) \quad (1.28)$$

$$\Pr\left\{\sum_j a_{ij}x_j + \sum_{j\in J_i} \zeta_j \,\hat{a}_{ij}x_j > b_i\right\} \leq \exp\left(min_{\theta>0}\left\{-\theta\Delta + |J_j| \sum_{j\in J_i} In\ \overline{G_j}\left(\theta/\sqrt{|J_j|}\right)\right\}\right) \quad (1.29)$$

where $G_j(\theta) = \mu_j \sinh\theta + \cosh\theta$, and $\overline{G_j}(\theta) = (\max \mu_j)\sinh\theta + \cosh\theta$. Note that B5 is applicable only to ellipsoidal (E) and interval and ellipsoidal (IE) uncertainty sets. Also, we may notice that (1.28) and (1.29) require the solution of the additional nonlinear nonconvex optimization problems (1.30) and (1.31), respectively.

For (1.27), we need to solve the following optimization problem;

$$\begin{aligned} &\min \Delta \\ &\text{s.t.} \\ &-\theta\Delta + \sum_{j\in J_i} In\ G_j(\theta) \leq \ln(\varepsilon) \\ &\Delta, \theta \geq 0 \end{aligned} \quad (1.30)$$

and for (1.30),

$$\min \Delta$$
s.t.
$$-\theta\Delta + |J_j| \sum_{j \in J_i} In\ \bar{G}_J \left(\theta / \sqrt{|J_j|}\right)$$
$$\Delta, \theta \geq 0 \tag{1.31}$$

In B4 and B5 instead of the nominal value of $\hat{a}_{ij}$ representing the mean, yielding $E[\zeta_{ij}] = 0$, the nominal value is chosen such that $|E[\zeta_{ij}]| \leq \mu_{ij}$.

1.2.5 Solution Methodology and Computational Results

1.2.5.1 Traditional Robust Approach using Priori Probabilistic Bound:

Traditional framework steps (Li et al., 2012) of applying robust optimization for a probabilistically constrained optimization problem can be summarized as follows:

1. The probabilistic constraint violation ε is set.
2. The uncertainty set is selected by the distribution of the uncertainty.
3. The uncertainty set size parameter is computed based on the a priori probability bounds.
4. The problem can be solved using the above uncertainty set size parameter and the solution obtained satisfies the desired probability $1-\varepsilon$.

Therefore, the framework for robust optimization under box uncertainty set can be formulated as follows,

$$minimize \sum_{k=0}^{T-1} c\ Q_k + Kv_k + y_k \tag{1.32}$$

$$\Pr\left\{-y_k + h\left(I_0 + \sum_{i=0}^{k} Q_i - \left(\sum_{i=0}^{k}(D_i) + \Psi_k\left(\left[\sum_{i=0}^{k}(\widehat{D}_i)\right]\right)\right)\right) < 0\right\} \leq \varepsilon \tag{1.33}$$

$$\Pr\left\{-y_k - p\left(I_0 + \sum_{i=0}^{k} Q_i - \left(\sum_{i=0}^{k}(D_i) + \Psi_k\left(\left[\sum_{i=0}^{k}(\widehat{D}_i)\right]\right)\right)\right) < 0\right\} \leq \varepsilon \tag{1.34}$$

$$0 \leq Q_k \leq Mv_k, \quad v_k \in \{0,1\}, \quad k = 0,1, \dots .T-1 \tag{1.35}$$

Similarly, the framework for robust optimization under an ellipsoidal uncertainty set can be formulated as follows,

$$\Pr\left\{ y_k - h\left(I_0 + \sum_{i=0}^{k} Q_i - \left(\sum_{i=0}^{k} (D_i) + \Omega_k \sqrt{\left[\sum_{i=0}^{k} \left(\widehat{D}_i^{\,2} \right) \right]} \right) \right) < 0 \right\} \leq \varepsilon \qquad (1.36)$$

$$\Pr\left\{ y_k + p\left(I_0 + \sum_{i=0}^{k} Q_i - \left(\sum_{i=0}^{k} (D_i) + \Omega_k \sqrt{\left[\sum_{i=0}^{k} \left(\widehat{D}_i^{\,2} \right) \right]} \right) \right) < 0 \right\} \leq \varepsilon \qquad (1.37)$$

$$0 \leq Q_k \leq M v_k, \quad v_k \in \{0,1\}, \quad k = 0,1, \ldots . T-1 \qquad (1.38)$$

1.2.5.2 Numerical Examples:

To illustrate the application of the robust optimization framework based on the two different uncertainty sets which are box and ellipsoidal uncertainty sets, we solve the production and inventory problem introduced earlier. We will utilize the five different probability bounds including those bounds which require solving additional nonlinear nonconvex optimization problems. In addition, we will evaluate the robust solutions at different probability constraint violations ε.

- Inventory Problem Based on Box Uncertainty Set:

In this example, we consider the following data:

$T = 20$ months; $I_0 = 1200$ units; $h = 4$; $p = 6$; and $\widehat{D} = 0.1D$. The nominal values D_i and the deviation magnitudes from the nominal values $\widehat{D}_i$ of the uncertain parameter D_i are provided in Table 1.2.

The computations of MILP were run using the branch and bound algorithm accessed via LINGO16.0 on a PC -3GHzand; 4 GB RAM and under win 10. While computations of the nonlinear nonconvex optimization problems were run using BARON solver which is offered by GAMS modeling languages.

We solve the above problem using the five probability bounds at five different probability constraint violations ε: 0.05, 0.1, 0.15, 0.20, and 0.25. Also, we will perform further analysis to compare and study the robust solutions obtained by these probability bounds. You can refer to tables (a-e) in the appendix to observe the uncertainty set sizes under the probability bounds.

k	The Expected Demand (D_k)	The Number of Uncertain Parameters J_k
0	1500	1
1	1800	2
2	2200	3
3	1300	4
4	3500	5
5	950	6
6	680	7
7	1050	8
8	750	9
9	1200	10
10	930	11
11	1400	12
12	1600	13
13	1850	14
14	1500	15
15	1700	16
16	1370	17
17	1000	18
18	750	19
19	450	20

Table 1.2: The uncertain parameter D_k values and their corresponding deviation magnitudes $\widehat{D}_k$

- **Bound 1(B1):** The robust solutions ($\boldsymbol{Q_k^*}$) obtained by this probability bound at five different probability constraint violations are provided by the appendix in table 3.
- **Bound 2(B2):** The robust solutions ($\boldsymbol{Q_k^*}$) obtained by this probability bound at different five probability constraint violations are provided by the appendix in table 4.
- **Bound 3(B3):** The robust solutions ($\boldsymbol{Q_k^*}$) obtained by this probability bound at different five probability constraint violations are provided by the appendix in table 5. This requires solving additional nonlinear nonconvex optimization problems provided in (1.26) to obtain the uncertainty set size parameter. You can refer to (f-j) in the appendix to observe the corresponding values of θ.

Note that in this case, it is assumed that each ζ_k is subject to the uniform distribution in $[-1, 1]$, and hence the box uncertainty set applies. For the uniform distribution $U(a, b)$, the moment generation function is $E\left(e^{\theta\zeta}\right) = \frac{e^{\theta b} - e^{\theta a}}{\theta(b-a)}$.

- **Bound 4(B4):** The robust solutions($\boldsymbol{Q_k^*}$) obtained by this probability bound at five different probability constraint violations are provided by the appendix in Table 6. This requires solving additional nonlinear nonconvex optimization problems to obtain the uncertainty set size parameter. You can refer to tables (f-j) in the appendix to observe the corresponding values of θ. The expected values of the parameters are only known to be within 1% of their nominal values. Therefore, $E[\widetilde{D}_i] \epsilon\ [D_i - 0.01D_i,\ D_i + 0.01D_i]$and $E[\zeta_i]\ \epsilon\ [-0.1, 0.1]$ that is equivalent to $|E[\zeta_i]| \leq 0.1 = \mu_i$

It should be noted that Bound 5 is not applicable for box uncertainty set. In section VI we discuss the conservatism of the obtained robust solutions over the proposed different scenarios.

- Inventory Problem Based on Ellipsoidal Uncertainty Set:

In this example, we consider data given in previous example. We solve the above problem using the five probability bounds at different five probability constraint violations ε: 0.05, 0.1, .15, 0.20, and 0.25. The robust solutions ($\boldsymbol{Q_k^*}$) obtained by the five probability bounds at five different probability constraint violations are provided in the appendix in Tables 1.7-1.11.

- **Bound 3(B3):** This requires solving additional nonlinear nonconvex optimization problems provided in (1.26) to obtain the uncertainty set size parameter. Note that in this case, it is also assumed that each ζ_k is subject to the uniform distribution in [−1, 1].
- **Bound 4(B4):** This requires solving additional nonlinear nonconvex optimization problems provided in (1.30) to obtain the uncertainty set size parameter. The expected values of the parameters are only known to be within 1% of their nominal values. Therefore,

$E[\widetilde{D}_i] \epsilon\ [D_i - 0.01D_i,\ D_i + 0.01D_i]$and $E[\zeta_i]\ \epsilon\ [-0.1, 0.1]$ that is equivalent to $|E[\zeta_i]| \leq 0.1 = \mu_i$

- **Bound 5(B5):** This requires solving additional nonlinear nonconvex optimization problems provided in (1.31) to obtain the uncertainty set size parameter. As for B4, the expected values of the parameters are only known to be within 1% of their nominal values.

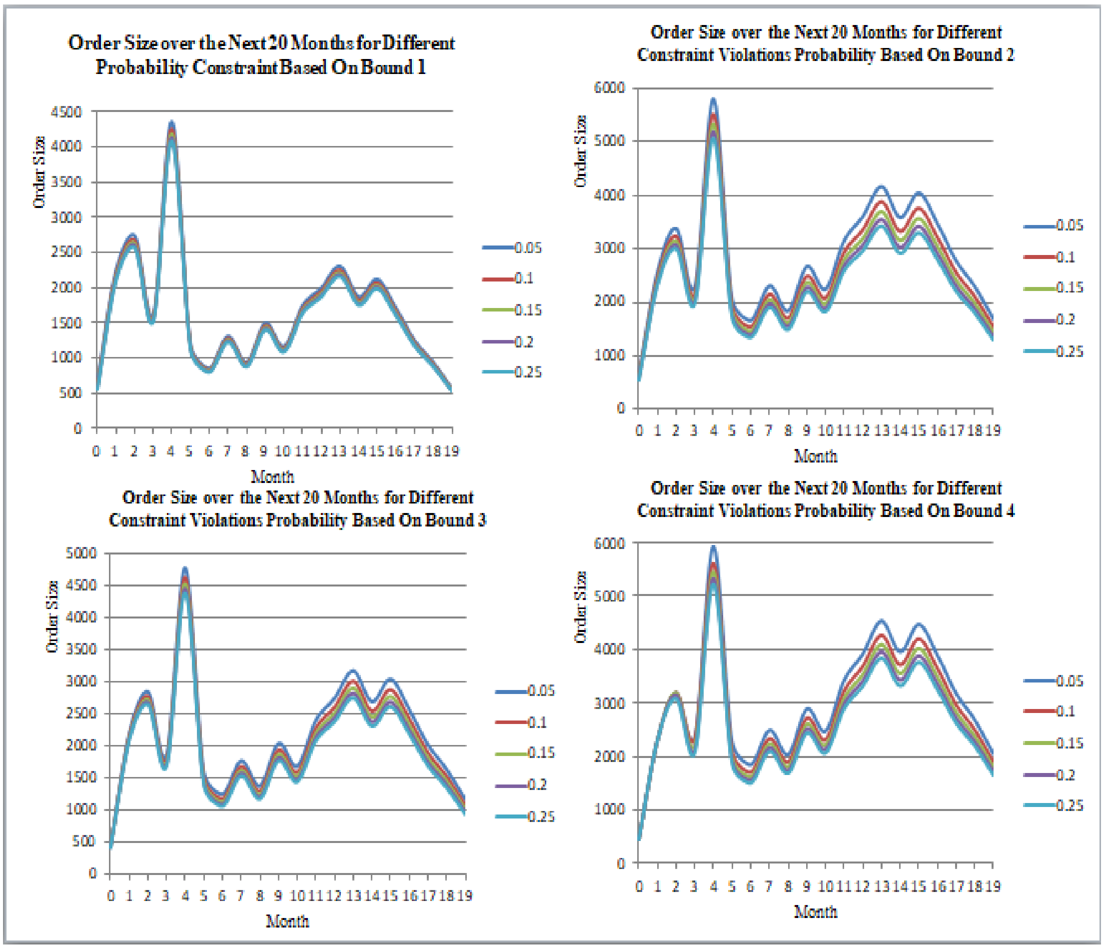


Figure 1.6: Order size over the next 20 months for different probability constraints based on different bounds under box uncertainty set.

1.2.5.3 Discussion:

In this section, we discuss the sensitivity and conservatism of the obtained robust solutions based on the box and ellipsoidal counterparts formulation. In our discussion, we refer to figures 1.8 and 1.9 which explain how the objective functions behave as the probability constraint violations increase for the five different bounds. The figures provide to the decision maker an overview of a conservatism comparison between the introduced uncertainty sets under different probability bounds. Note that B5 is not applicable to the case of box uncertainty set and, therefore it is not included in figure 1.6.

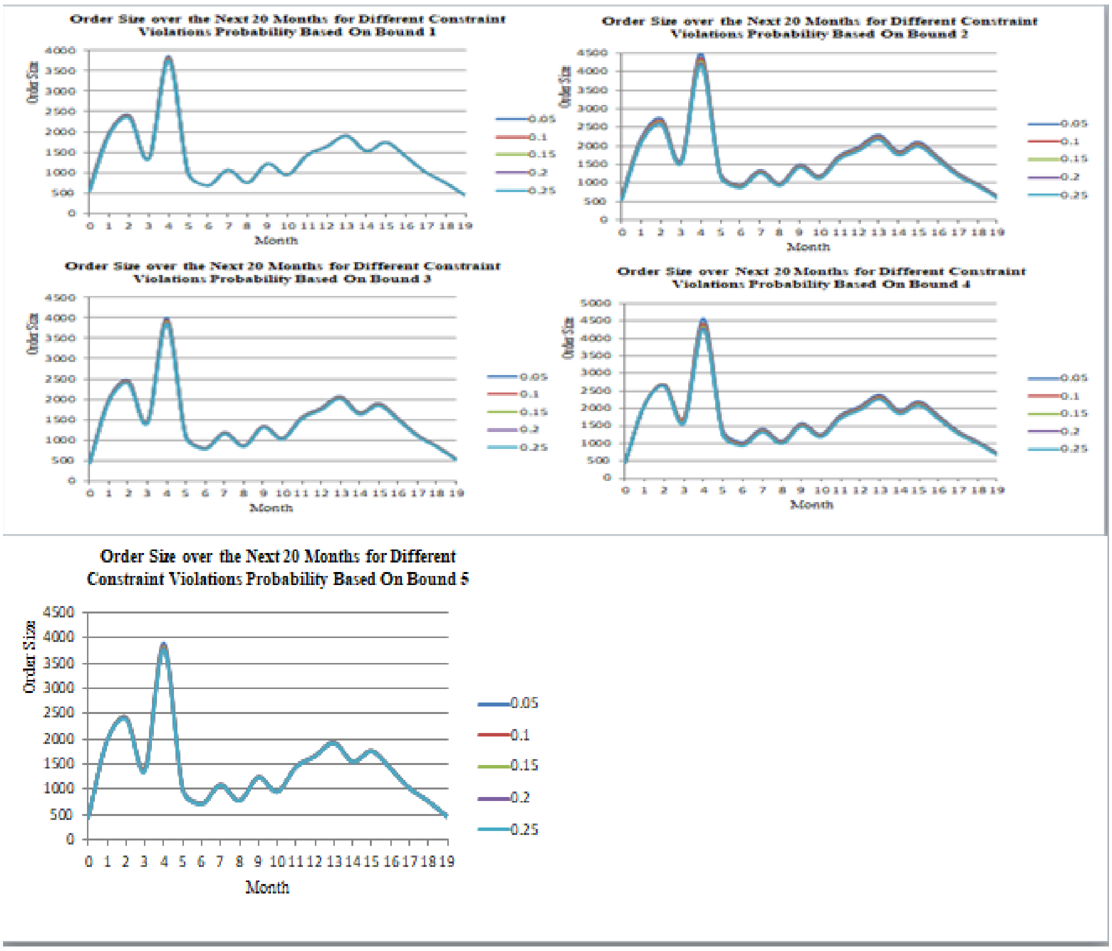


Figure 1.7: Order size over the next 20 months for different probability constraints based on different bounds under ellipsoidal uncertainty set.

While we compare the size of the different types of uncertainty sets, a conservatism recommendation could be made based on the following fact: the larger the uncertainty set is, the more conservative the solution are obtained. Thus, the model's conservatism increases in the following order: box, ellipsoidal, polyhedral, (Li et al., 2012). However, this is true only and only if the bounded uncertainty is within the suggested range such that the adjustable uncertainty set parameters are $\Psi_k \leq 1$, and $\Omega_k \leq \sqrt{|J_k|}$ for box and ellipsoidal uncertainty sets, respectively (Li et al., 2011). Therefore, the robust solution based on the ellipsoidal uncertainty counterpart is less conservative than the box uncertainty counterpart.

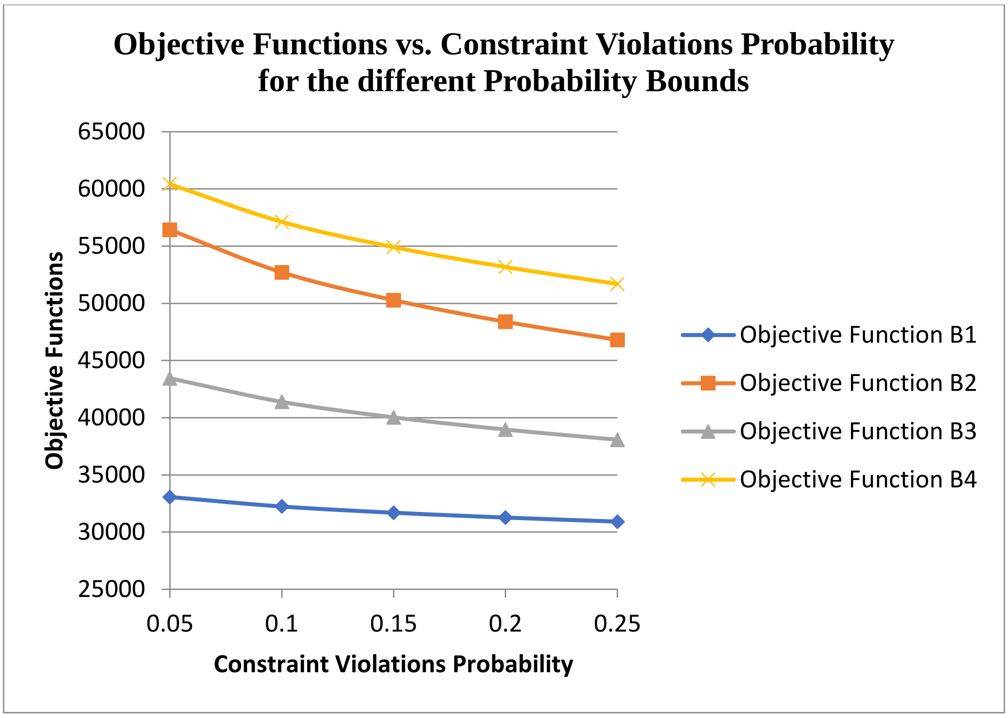


Figure 1.8: The behavior of the robust objective functions when different upper bounds are applied based on box counterpart.

From figures 1.8 and 1.9, we make the following observations:

- In all probability upper bounds as the probability constraint violations increase, the robust objective functions tend to be less conservative. This is valid since we allow for high constraint violations, and hence we make the performance of objective function to get improved.
- In figures 1.8 and 1.9, the robust solution obtained by B1 is less conservative (better solution) among the other probability bounds. However, practically B1is not the best probability bound to be applied in the discussed inventory problem. This is because B1 assumes that the amount of uncertainty, $|J_k|$, is constant over the months which contradicts with the nature of the model where the uncertainty increases as the period increases.
- In figure 1.8, the robust solution obtained by B3 is less conservative (and hence better solution) comparing with B2 and B4. This would be a better choice due to full probability distribution information. If such information is available, it can be utilized beneficially which makes the solution less conservative.

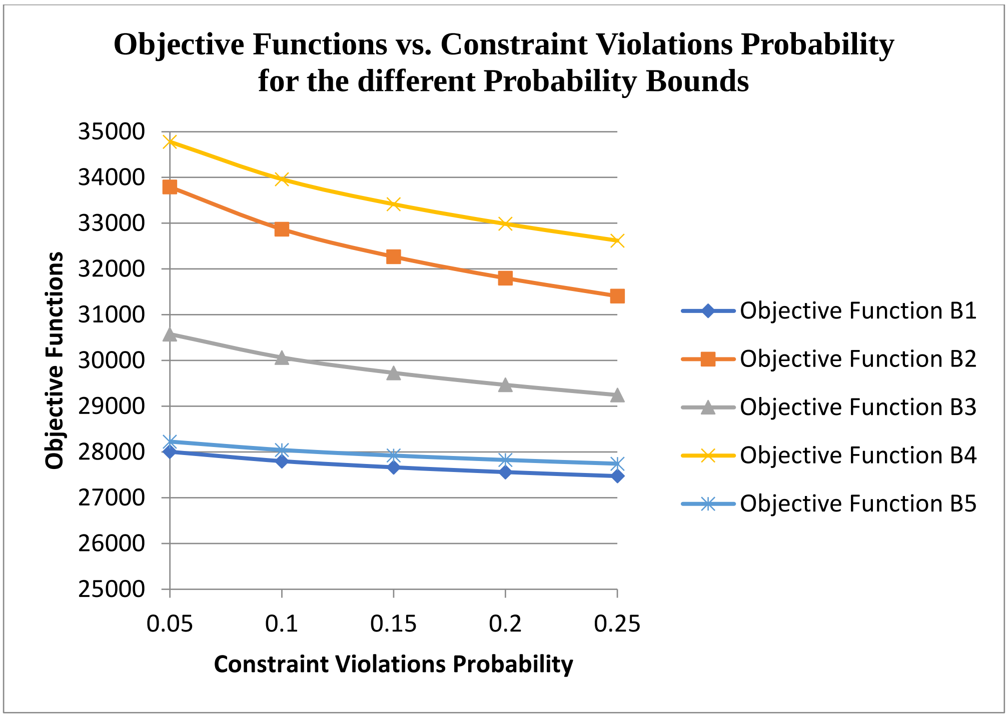


Figure 1.9: The behavior of the robust objective functions when different upper bounds are applied based on ellipsoidal counterpart.

- In both figures, B2 and B4 show high conservatism. In both probability bounds, the amount of uncertainty, $|J_k|$ over a period of months is considered. However, the rapid increase in the uncertainty set size parameter makes the robust solution obtained by B4 to be more conservative comparing with B2, B3, and B5 (in case of ellipsoidal counterpart).
- In figure 1.9, if we omit B1, B5 is the tightest probability bound since it is specifically derived to be applicable in ellipsoidal and ellipsoidal plus interval uncertainty sets. Remember that bounds B4 and B5 permit robust counterpart optimization even in the case where the mean of the distribution of an uncertain parameter is not known exactly but is assumed to lie within a range of values.

In our inventory problem, we come up with following conclusion: depending on the uncertainty information such as whether the uncertain parameter has bounded and symmetric distribution or it has a known probability distribution, the decision maker will identify a better choice in constructing the robust counterpart model.

In addition, the probability constraint violation should be set properly if the decision maker seeks for low or high conservative robust solutions (e.g. high risk involved in the decision making).

1.2.6 Conclusion and Future Work

Different uncertainty approaches have been used to undress the multi-period inventory problem . In our study, we have developed two robust counterpart inventory models based on box and ellipsoidal uncertainty sets. Moreover, we use a different approach based on uncertainty set-based robust optimization. In this work, we have utilized a priori probability bound which can be used to approximate probabilistic constraints and provide safe solutions. Different upper probability bounds for both bounded and unbounded uncertainty, with and without detailed probability distribution information in the literature under different probability constraint violations are considered in our work, and useful insights are gained for their corresponding robust solutions.

In future work, a posteriori probabilistic guarantees approach can be also used to improve the robust solutions. Also, we will apply the approaches discussed in this paper to higher classes of production systems and inventory control with a dynamic uncertainty set and imperfect quality models where the uncertainties may be considered in different system's parameters. These future studies will provide more insights in improving the production systems under uncertainties.

1.3 Dissertation Structure:

For more practical and effective decision making, we carry out the optimization over the whole supply chain under multiple uncertainties rather than focusing only on the inventory problem. In the literature, the supply chain networks activities are divided into two general groups: 1) Forward network (forward flow): dealing only with the supply chain activities from suppliers up to customers, 2) Reverse network (returned flow): focusing on the activities returned from customers. The concept of closed-loop supply chains (CLSC) is now widely garnering attention as a result of the recognition that both the forward and reverse supply chains need to be managed jointly. From the previous brief introduction, we develop the following research questions:

- Does considering (CLSC) under imperfect quality production with multiple uncertainties make it more interesting, realistic, and worthwhile study?
- What are the different approaches used in the literature to deal with the uncertainty in the above (CLSC) problem?

- Are those approaches effective, and tractable formulations especially with the lack of information?
- Although the robust optimization is the most modern and appealing uncertainty approach, how can the conservatism issue be addressed?
- Can robust formulations based on different uncertainty sets and sizes improve the quality of robust solutions?

In addition to chapter 1, three papers are provided in this dissertation, one in each chapter. In chapter 2, we consider a novel closed loop supply chain design consisting of multiple periods and multiple echelons. The models are considered under imperfect quality production with multiple uncertainties to provide meaningful solutions to practical problems. In addition, we assume that the screening is not always perfect, and inspection errors are more likely to take place in practice. We measure the amount of quality loss as conforming products deviate from the specification (target) value. In our study, we develop three robust counterparts models based on box, polyhedral, and combined interval and polyhedral uncertainty sets. We utilize different a priori probability bounds to approximate probabilistic constraints and provide a safe solution. The objective is to minimize the total cost of the supply chain network. Finally, numerical examples are provided to illustrate the proposed models.

As an extension to chapter 2, chapter 3 considers robust multi-objective mixed integer linear programming model which includes three objectives simultaneously. The first objective function minimizes the total cost of the supply chain. The second objective function seeks to minimize the environmental influence, and the third objective function maximizes the social benefits. The augmented weighted Tchebycheff method is used to aggregate the three objectives into one objective function and produce the set of efficient solutions. Robust optimization, based on Mulvey et al. (1995) approach, is used to obtain a set of solutions that are robust against the future fluctuation of parameters. Finally, numerical examples have been presented to test and analyze the tradeoff between solution robustness and model robustness.

In chapter 4, the affinely adjustable robust formulation based on "wait and see" decisions is presented. That is, the decisions are made over two sequential stages where multiple uncertainties are included. Moreover, we propose a budget dynamic uncertainty set to mimic the dynamic behavior of the market demand over time. The introduced dynamic uncertainty set is formulated according to Vector Autoregressive (VAR) models where the temporal and spatial correlations of customer demand zones are captured. Also, we utilize different a priori probability bounds to approximate probabilistic constraints and provide a safe solution. The objective is to minimize the total cost of the supply chain network. Finally, numerical examples are provided to illustrate the proposed models.

Conclusions and possible future research directions are provided in Chapter 5.

CHAPTER 2: AN INTEGRATED MULTI-ECHELON ROBUST CLOSED-LOOP SUPPLY CHAIN UNDER IMPERFECT QUALITY PRODUCTION

In this chapter, we consider a novel closed loop supply chain design consisting of multiple periods and multiple echelons. The models are considered under imperfect quality production with multiple uncertainties to provide meaningful solutions to practical problems. In addition, we assume that the screening is not always perfect, and inspection errors are more likely to take place in practice. We measure the amount of quality loss as conforming products deviate from the specification (target) value. In our study, we develop three robust counterparts models based on box, polyhedral, and combined interval and polyhedral uncertainty sets. We utilize different a priori probability bounds to approximate probabilistic constraints and provide a safe solution. The objective is to minimize the total cost of the supply chain network. Finally, numerical examples are provided to illustrate the proposed models. The paper is expected to provide more insights in managing this important problem.

2.1 Introduction

The uncertainty modelling is an important topic in supply chain management, and has been recently discussed extensively by researchers and industry practitioners. Modeling and solving closed-loop supply chains (CLS) under uncertainty is now widely taking attention because both the forward and reverse supply chains need to be managed simultaneously. A common assumption of the supply chain inventory model is that the produced items are perfect. We consider the imperfect quality production to provide meaningful solutions to practical supply chain management problems.

Our modeling investigates the integrated multi-echelon, multi-period under multiple uncertainties models, where the most recent techniques of robust optimization are used as solution approaches. In addition, we assume that the screening is not always perfect, and inspection errors are more likely to take place in practice. Thus, some errors are committed in the inspection process. In addition, we measure the amount of quality loss as conforming products deviate from the specification (target) value.

2.2 Literature Review:

The literature is reviewed with two different ideas in mind. The first section is about the most recent studies in the area of robust supply chain under uncertainty. The second section discusses incorporating imperfect production quality and scenarios of reworking and recycling in the robust supply chain.

2.2.1 The Most Recent Studies in the Area of Robust Supply Chain under Uncertainty:

A literature survey conducted recently by Govindan, Fattahi, and Keyvanshokooh (2017), shows that four main approaches in recent decades are adopted to handle the uncertainty environment in the supply chain. These four approaches are dynamic programming, stochastic programming, fuzzy programming, robust optimization, or the combination of any two of these approaches. Consideration of uncertainties in the model dynamic parameters (i.e. market demand) will represent a more realistic problem situation. This explains the special attention is recently paid to stochastic and dynamic market demand. On the other side, fuzzy programming is a popular approach applied recently by many researchers along with the supply chain area under uncertainty, (Shekarian, Kazemi, and Abdul-rashid, 2017). When the probability distribution of an uncertain parameter is known, the appropriate modeling approach is Stochastic Programming. This approach is one of the most important approaches used to handle the uncertainty in production supply chain and inventory control, (Masih-tehrani, Xu, Kumara, and Li, 2011), (Zhang, Li, and Huang, 2014), and (Wang, Qin, and Kar, 2015). Several extensions of previous studies with supply chain uncertainty make stochastic programming an increasingly important modeling approach.

Robust optimization is the most recent approach for dealing with optimization problems with data uncertainty. Although no distribution assumption is made on uncertain parameters, the availability of data information can be utilized beneficially. The development of robust optimization is based on uncertainty sets approach and is summarized in Table 1. The uncertainty set is defined as the set of all possible realizations of the uncertain parameter that will be considered in the robust optimization problem (Ben-Tal and Nemirovski, 2000). Table 2.1 lists different developed uncertainty sets.

One of the first early work where robust optimization is incorporated with logistics and supply chain is (Yu and Li, 2000). They reformulate robust programming methods proposed by Mulvey et al. (1995), into a linear program which only requires adding n + m variables (where n and m are the number of scenarios and total control constraints, respectively).

Author	Contribution	Year
Soyster	• Simple perturbations in the data are considered in the linear programming problem to make the solution feasible under all perturbations. • Introduces interval set.	1973
Ben-tala, Nemirovski and coworkers	• The ellipsoidal set robust counterpart is proposed to formulate the linear and quadratic programming problems under uncertain parameters.	1998-2004
El-Ghaoui and coworkers	• Study the uncertain least-squares problems with the robust solutions. • Study uncertain semidefinite problems.	1997,1998
Lin et al. Janak et al.	• Extend RO for (LP) to MILP • The robust optimization framework for different bounded known probability distributions are developed.	2004, 2007
Verderame and Floudas	• Investigate both continuous (general, bounded, uniform, normal) and discrete (general, binomial, Poisson) uncertainty distributions.	2009
Bertsimas, Sim and coworkers	• Introduce the uncertainty budgets set (combined interval and polyhedral uncertainty set) in the LP. • A new approach is proposed to deal with uncertain parameters in the discrete network optimization problems.	2003-2004
Bertsimas and Thiele	• Extend previous work to address inventory control problems to minimize total costs.	2006
Soyster Li et al. Ben-Tal and Nemirovski Bertsimas and Sim	• Interval Uncertainty Set • Pure Box, Ellipsoidal, and Polyhedral Uncertainty Sets • Combined interval and ellipsoidal set • Combined interval and polyhedral set	1973 2011 2000 2004

Table 2.1: Robust optimization approaches in operations research based on uncertainty sets.

The adapted Mulvey approach has been widely used in supply chain for the sake of uncertainty management. Some of these recent studies are (Al-e-hashem, Malekly, and Aryanezhad, 2011; Ma, Yao, Jin, Ren, and Lv, 2016; F. Mohammed et al., 2017; Pishvaee, Rabbani, and Torabi, 2011; Rahmani, Ramezanian, Fattahi, and Heydari, 2013; Safaei, Roozbeh, and Paydar, 2017).

These models are based on the approach introduced by Mulvey et al. (1995), named robust stochastic optimization or scenario-based robust approach. Mulvey et al. (1995) extend scenario-based stochastic programming by defining the objective function as a mean-variance function incorporating and risk measures and decision makers' preferences in their model formulation.

The solution obtained by the scenario-based robust model is strongly dependent on the defined scenarios accuracy and their probabilities of occurrence. Thus, solving such models is more difficult because as the number of scenarios increases, the computational complexity increases too. A more popular approach is the uncertainty set based robust modelling which enables determining the desirable robust decisions without the need to consider different scenarios and their occurrence probabilities.

The uncertainty set is defined as the set of all possible realizations of the uncertain parameter that will be considered in the robust optimization problem (Ben-Tal and Nemirovski, 2000). See Table 2.1 for different developed uncertainty sets. Recently, many researchers apply the uncertainty set based approach to manage the multiple uncertainties associated with the robust supply chain optimization, (Aharon, Boaz, and Shimrit, 2009; Baghalian, Rezapour, and Zanjirani, 2013; Hatefi and Jolai, 2014; Kisomi, Solimanpur, and Doniavi, 2016; Ma et al., 2016; Pishvaee et al., 2011; Wei, Li, and Cai, 2011; Xin, Xi, Yu, and Wu, 2013; Y. Zhang and Jiang, 2017; Zokaee, Jabbarzadeh, Fahimnia, and Jafar, 2017) . Table 2.2 summarizes some of the current supply chain models that study the parameter uncertainty in their models using a robust optimization approach.

2.2.2 Incorporating Imperfect Production Quality and Scenarios of Reworking and Recycling in the Robust Supply Chain

A common assumption of the closed loop supply chain model is that the produced items are perfect. In several real- life situations, this assumption may not be valid. In various inventory problems such as economic order quantity (EOQ)models, many researchers relaxed this assumption to provide meaningful solutions to practical problems. Khan, Jaber, Guiffrida,and Zolfaghari (2011) make an extensive literature review of the extensions of a modified EOQ model for imperfect quality items. They also include a fuzzy set theory approach in these investigated studies.

Paper	**Supply Chain Network Open/Closed**	**Single/ Multiple Echelons- Period**	**Main Contribution**	**Optimization Problem**
(Yu & Li, 2000)	Open SC	Multi-echelons multi-period and multi-product	Developing a robust optimization model for stochastic logistic problems	Robust Stochastic programming
(Aharon, Boaz, & Shimrit, 2009)	Open SC	Multi-echelons, and multi-period	Modeling, analyzing and testing an extension of the AARC method known as the Globalized Robust Counterpart (GRC) in order to control inventories in serial supply chains.	MILDP
(Pishvaee, Rabbani, &Torabi, 2011)	Closed-loop	Multiple Echelons, and Periods	Introducing a robust optimization approach to closed-loop supply chain network design under uncertainty	MILP
(Al-e-hashem, Malekly, & Aryanezhad, 2011)	Open SC	Multi-echelons multi-period and multi-product	Developing a supply chain addressing multi-product aggregate production planning (APP) problem	MINLP
(Rahmani, Ramezanian, Fattahi, & Heydari, 2013)	Open SC	Three-echelons multi-period and multi-product	Developing model for multi-product two-stage capacitated production planning under uncertainty	MILP
(Baghalian, Rezapour, & Zanjirani, 2013)	Open SC	Multi-echelons multi-period and multi-product	Supply chain network design with service level against disruptions and demand uncertainties	MILP
(Hatefi & Jolai, 2014)	Closed-loop	Multi- Echelons	Reliable forward–reverse logistics network design	MILP

			under demand uncertainty and facility disruptions	
(Science et al., 2016)	Closed-loop	Three -echelons and multi-product	Environmental closed-loop supply chain design under uncertainty	MINLP
(Kisomi, Solimanpur, & Doniavi, 2016)	Closed-loop	Multiple Echelons and Products	An integrated supply chain configuration model and procurement management under uncertainty	MILP
(Safaei, Roozbeh, & Paydar, 2017)	Closed-loop	Multiple Echelons, and Periods	Developing a model for the design of a cardboard closed- loop supply chain	MILP
(Vahdani, Soltani, Yazdani, & Mousavi, 2017)	Closed-loop	Three Echelons	A three level joint location-inventory problem with correlated demand, shortages and periodic review system	MINLP
(Mohammed, Selim, Hassan, & Naqeebuddin, 2017)	Closed-loop	Multiple Echelons, Periods and Products	Proposing an optimization model for design and planning supply chain with carbon footprint consideration	S-MILP
(Zhang & Jiang, 2017)	Open SC	Three Echelons	Addressing the design of a Waste cooking oil for-biodiesel-for-biodiesel supply chain at both strategic and tactical levels.	MILP
(Bairamzadeh, Saidi-mehrabad, & Pishvaee, 2018)	Open SC	Multiple Echelons, and Products	Modelling different types of uncertainty in biofuel supply network design and planning	MILP

Table 2.2: Summary of the most recent studies robust supply chain under uncertainty

There are very few studies which recognize incorporation of the imperfect quality production to the supply chain modelling, (Ahmadi, Khoshalhan, and Glock, 2016; Masoudipour, Amirian, and Sahraeian, 2017; Sana, 2011). However, these studies consider deterministic models.

Modeling supply chain under uncertainty where imperfect quality production is incorporated is also studied by few researchers. For example, Hu, Zheng, Xu, Ji, and Guo (2010) study coordination of supply chain for the fuzzy random newsboy problem with imperfect quality in the decentralized and centralized systems. Quality uncertainty from a supply chain coordination perspective is addressed by Hwan, Rhee, and Cheng (2013). Rad, Khoshalhan, and Glock (2014), however, use the renewal-reward theorem as a stochastic approach in optimizing inventory and sales decisions in a two-stage supply chain.

2.3 Problem Definition and Mathematical Formulation

2.3.1 Problem Definition

In this study, we consider a closed loop supply chain system consisting of multiple periods, products, and echelons. The flow of materials can be described as follows: the network is managed by a manufacturer such that the required quantity of raw materials is ordered for production. Then, the produced lot size is sent to the distribution center and finally moved to the customer zone according to customer demands. The location of the customer zone is supposed to be predefined and fixed. In the reverse network, the activities start from the collection center at which the returned products (defective or used products) are shipped to the inspection facility within the collection center. Subsequently after separation, the recyclable items are sent for recycling while the defective items are subject to another inspection that classifies them to either reworkable or not reworkable. However, we assume that the screening is not always perfect, and inspection errors are more likely to take place in practice. Thus, two types of errors are committed in the inspection process. Type I, is committed when a conforming item is classified as non-conforming and type II error, is committed when a non-conforming item is classified as conforming.

The recyclable items are used to cover the market demand while the non-recyclable items are disposed. For those items that are apparently reworkable, they will be reworked and become as good as new ones and will be sent back to the original plant to cover the demand otherwise they will be disposed. Although the perfect items are supposed to be within the specification limits and fall within a certain acceptance range, we measure the amount of quality loss as conforming products deviate from the specification (target) value. We describe the activities associated to each supply chain component as follows (Fig.2.1):

- **Suppliers:**
  According to the order received from the manufacturer, the suppliers prepare and process the required quantity of raw material necessary to produce the lot size used to cover the market demand. Several costs are considered including, ordering cost per lot size, purchasing, processing and transportation costs. Note that the supplier can be either national or overseas supplier. Also, we make a restriction on the capacity of raw material of any product type for each supply center.

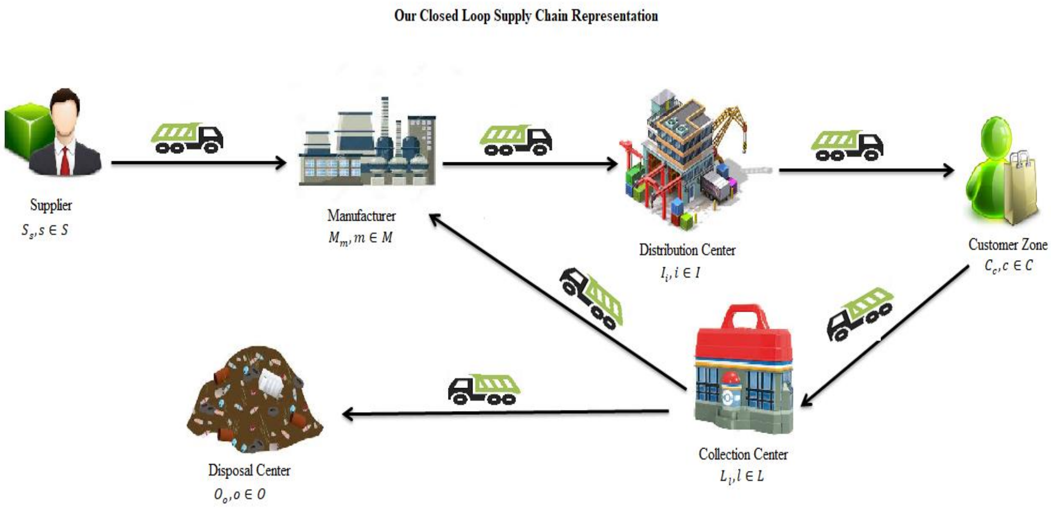


Figure 2.1: Illustration of Our Multi- Echelon Closed Loop Supply Chain

- **Manufacturers:**
  The network is managed by manufacturers which include three main facilities:

➢ *Manufacturing new products facility*: within this facility the ordered raw material is used to produce the lot size for covering the demand. A common assumption of the supply chain manufacturer model is that the produced items are perfect. However, we consider the imperfect quality production to provide meaningful solutions to practical problems. Thus, a proportion of produced items is assumed defective. Moreover, this proportion of defective items are treated as an uncertain parameter. For practical reasons, we restrict the production of any product type in each manufacturer to a specific capacity. The considered costs here are manufacturing and transportation costs. Also, we penalize for producing defects.

- *Recycling facility:* for an effective use of supply chain resources, part of customers' returns can be recycled and used to meet the market demand. Thus, within this facility the recyclable items sent from the collection center are recycled at a cost. However, we consider this cost as an uncertain random variable depending on the items condition. Also, these recycled items may not be used completely to cover the demand and hence a portion of it may be used after recycling.
- *Reworking facility:* unlike the recycling process where only the used items can be recycled, in reworking process, those defective items which are identified by customers and sent from the collection center will be reworked at a cost in the reworking facility. Similar to the recycling cost, reworking cost is subject to uncertainty. After reworking, the lot or part of it will contribute in covering the market demand. The transportation cost is considered as well in both recycling and reworking facilities.

One important question here is why that recycling and reworking costs are treated as uncertain parameters in this model? The answer to this question is that any cost in a supply chain model can be considered as either deterministic or uncertain parameter depending on the model assumptions. However, we assume here that the recycling and reworking costs are uncertain because the condition of each individual returned item is not necessarily the same, and hence the cost of recycling or reworking process needed for each item is not certain.

- **Distribution Centers:**

  The distribution centers consist of three facilities:

- *Inspection facility:* based on our assumption referring to the imperfect production quality, inspection and screening process is carried on the whole lot transported from the manufacturer. To make it more realistic we assume that the inspection process is not always perfect. We consider two types of inspection errors: type I, is committed when a conforming item is classified as non-conforming and type II error, is committed when a non-conforming item is classified as conforming. Moreover, these two types of errors are treated as uncertain values. Also, we carry out another type of inspection to ensure that the produced lot is close enough to the target value according to Taguchi Quality approach. The costs included here are inspection and quality loss.

- *Inventory facility*: after the lot being inspected and screened by the inspection facility, the lot is placed in the inventory. Note that the items contained in the lot are considered apparently conforming because of the uncertainties in the inspection process. An inventory holding cost is assigned for each item/lot.
- *Distributing facility:* The lot size is prepared and packed to be delivered to customer zone as requested. Processing and transportation costs are included here. A limit for aggregated capacity of these facilities at each distribution center and product type is assumed.

- **Customer Zones:**

  As a final destination in the forward network, the lot is transported to customer zones based on the expected market demand over time periods. Because the demand is subject to uncertainty as well, the shortage is allowed in this model. Since this is a closed loop supply chain, we expect some returned products from our customers in the form of used or defective products. For interesting practical issues, we treat the returned products in either form as uncertain parameters as well. Like the supplier, the customer can be either national or overseas.

- **Collection Centers:**

  In the collection centers, further classification of the returned products is performed to either classify them to recyclable or reworkable; otherwise they will be disposed through the disposal center. Recyclable and reworkable items are stored in collection center inventory facility at a reduced cost in order to be shipped later to the manufacturer for further processing. Also, we assume a capacity of each product type at any collection center.

- **Disposal Center:**

We assume that any return products which can not be reworked or recycled are disposed through the capacitated disposal centers. The disposal fraction of products is treated as uncertain.

2.3.2 Some Applications

The proposed models have wide industrial applications including high-technology, car manufacturing, some grocery store products, plastic products industries, various types of machine parts, etc. Here we provide one suitable application for the proposed model related to steel making process.

Steel can be produced using different methods such as blast furnace (BF) and direct reduction (DR). BF represents more than 66% of global steel production. Principal raw materials consist of iron ore and coke. Several types of iron ore can be provided; i.e. iron ore is mined and prepared as concentrate which are sold as separate products. Steel products can be also produced in different qualities and various rolling types (beam and bar and so on) upon users' request, (Soltany, Sayadi, Monjezi, and Hayati, 2013).

- The single or multiple suppliers can be either overseas or nationals.
- The manufacturers produce different quantities, qualities and various rolling types which are uncertain. In addition, the production is not always perfect which means an uncertain portion of produced steel products is defective.
- The screening and inspection process, which is done by the facility of quality assurance, is subject to uncertain inspection error.
- The distributers are centered in multiple sites. The inventory holding cost is assigned to each steel product according to its type.
- According to the expected customer demands, the orders are transported to multiple customer zones. The uncertain amount of returned steel is collected by different collection centers. The returned products can be either used or defective. Due to uncertainties in the product condition the recycling and reworking costs are uncertain. Also, the recycled and reworked steel can be used to satisfy the demand.
- The amount of the steel production residues, i.e. by-products and waste, reflect the amount of produced steel. In 2015, iron and steel production in Sweden generated just over two million tons of residual products. This total

can roughly be divided into three groups ("Steel production residues," 2017):

1. 39 % is used externally, e.g. sold on as products.
2. 40 % is used internally, e.g. reused as a raw material in the production processes.
3. 21 % is waste that is sent to landfill

2.3.3 Notation

- The following sets are used:

T Set of periods, with $t \in T$.

S Set of possible supplier center locations, with $s \in S$.

M Set of manufactures centers locations, with $m \in M$.

I Set of potential distribution center locations, with $i \in I$.

C Set of customer zones, with $c \in C$.

L Set of potential collection/disassembly center locations, with $l \in L$

O Set of potential disposal center locations, with $o \in O$.

P Set of products, with $p \in P$.

- The parameters are defined as follows:

$\widetilde{D}_{tpc}$: Market demand for product p for customer zone c at period t which is subject to uncertainty.

$\tilde{R}_{tpc}$: Returned of amount product p as used items form customer zone c at period t which is subject to uncertainty.

$\widetilde{Rw}_{tpc}$: Returned of amount product p as defective items form customer zone c at period t which is subject to uncertainty.

$\widetilde{Rc}_{tpm}$: Recycling cost/unit for product p at manufacturer m and period t which is subject to uncertainty.

$\widetilde{REc}_{tpm}$: Rework costs for items produced below and above the specification limits for product p at manufacturer m and period t, respectively, which is subject to uncertainty.

$\tilde{e}_{1t}$: Uncertain proportion of type I error at period t.

$\tilde{e}_{2t}$: Uncertain proportion of type II error at period t.

$\tilde{\beta}_p$: Uncertain disposal fraction of product p.

FS_s: Fixed cost of selecting supplier s.

FD_i: Fixed cost of opening distribution i.

FC_l: Fixed cost of opening collection/disassembly l.

FO_o: Fixed cost of opening disposal o.

Sc_{ps}: Manufacturing cost/unit for product p by the supplier s.

Mc_{pm}: Manufacturing cost/unit for product p by the manufacturer m.

Ic_{pi} : Inspection cost/ unit for product p the distribution i.

Dc_{pi}: Processing cost/unit of product p at the distribution i.

Cc_{pl}: Collection cost/unit for the returned product p at the collection center l.

h_{pi}: Holding cost of apparent good items for product p in distribution center i.

hw_{pl}: Holding cost associated with quantity of product p returned from the customer zone to the collection l.

$\hat{\pi}_{pc}$: Shortage (penalty) cost for product p and customer zone c.

W_{pm}: Ordering cost per lot size of product p at manufacturer m.

P_{ps}: Purchasing cost/ unit for product p from supplier s.

Io_{po}: Disposal cost/unit of non-recyclable items of product p at the disposal center o

TMc_{psm}: Transportation cost of the raw materials of product for product p from supplier s to manufacturer m.

TPc_{pmi}: Transportation cost of product p from manufacturer m to distribution i.

TZc_{pic}: Transportation cost of the product p from distribution i to customer zone c.

TOc_{pcl}: Transportation cost of product p from the customer zone c to collection center l.

$TOPc_{plm}$: Transportation cost of product p from collection center l to manufacturer m.

TIc_{plo}: Transportation cost of product p from collection center l to disposal center o.

CS_{ps}: Capacity of raw material of product p for supply center s.

CP_{pm}: Capacity for production of product p in manufacturer m.

CI_{pi}: Capacity of product p in distribution center i.

CL_{pl}: Capacity of product p in collection center l.

CO_{po}: Capacity of product p in disposal center o.

USL_p: Upper specification limit of product p.

LSL_p: Lower specification limit of product p.

K: loss parameter

X_p: Actual value of the quality characteristic of product p.

$L(x)$: Loss of poor quality per unit product.

μ_p : Target quality characteristic of product p.

σ_p : Standard deviation of quality characteristic of product p.

ψ: Deviation from the target value.

- The decision variables are defined as follows:

QSM_{tpsm}: Quantity of raw material of product p ordered from supplier s to manufacturer m at period t.

QMD_{tpmi}: Quantity of product p sent from manufacturer m to distribution center i at period t.

QDC_{tpic}: Quantity of product p planned to be sent from distribution center i to customer zone c at period t.

QNS_{tpc}: Quantity of non-satisfied demand of product p for customer zone c at period t.

QCO_{tpcl}: Quantity of product p returned from customer zone c to collection center l at period t.

QRP_{tplm}: Quantity of recyclable product p shipped from collection center l to manufacturer m at period t.

QEP_{tplm}: Quantity of reworkable product p shipped from collection center l to manufacturer m at period t.

QIP_{tplo}: Quantity of disposal product p shipped from collection center l to disposal center o at period t.

v_{tpsm}: 1 if the order of product p is placed by manufacturer m at period t and 0 otherwise.

S_{ts}: 1 if a supplier is selected at location *s* at period t , 0 otherwise.

DT_{ti}: 1 if a distribution is opened at location *i* at period t, 0 otherwise.

CT_{tl}: 1 if a collection/disassembly is opened at location *l* at period t, 0 otherwise.

DO_{to}: 1 if a disposal is opened at location *o* at period t, 0 otherwise.

2.3.4 Mathematical Formulation:

The objective function, Z minimizes the total cost of the supply chain network. The included costs are:

- Facility opening costs:

$$\sum_{t\in T}\sum_{s\in S} FS_s S_{ts} + \sum_{t\in T}\sum_{i\in I} FD_i DT_{ti} + \sum_{t\in T}\sum_{l\in L} FC_l CL_{tl} + \sum_{t\in T}\sum_{o\in O} FO_o DO_{to}$$

- Purchasing cost:

$$\sum_{t\in T}\sum_{p\in P}\sum_{s\in S}\sum_{m\in M} W_{pm} v_{tpsm} + P.\,QSM_{tpsm}$$

- Ordering costs:

$$\sum_{t\in T}\sum_{p\in P}\sum_{s\in S}\sum_{m\in M} QSM_{tpsm}\left(Sc_{ps} + TMc_{psm}\right)$$

- Cost incurred in the manufacturers:

$$\sum_{t\in T}\sum_{p\in P}\sum_{m\in M}\sum_{i\in I} QMD_{tpmi}\left(Mc_{pm} + TPc_{pmi} + Ic_{pi} + \tilde{d}_t PR_p\right)$$

- Cost incurred in the distributor centers:

$$\sum_{t\in T}\sum_{p\in P}\sum_{i\in I}\sum_{c\in C} QDC_{tpic}\left(Dc_{pi} + TZc_{pic} + h_{pi} + F_p\left(x_p;\mu_p\right)\right)$$

- Cost incurred in the collection centers:

$$\sum_{t\in T}\sum_{p\in P}\sum_{c\in C}\sum_{l\in L} QCO_{tpcl}\left(Cc_{pl} + TOc_{pcl}\right) + \sum_{t\in T}\sum_{p\in P}\sum_{c\in C}\sum_{l\in L} QCO_{tpcl} hw_{pl}$$

- Costs related to recycling and reworking respectively:

$$\sum_{t\in T}\sum_{p\in P}\sum_{l\in L}\sum_{m\in M} QRP_{tplm}\left(\widetilde{Rc}_{tpm} + TOPc_{plm}\right) + \sum_{t\in T}\sum_{p\in P}\sum_{l\in L}\sum_{m\in M} QEP_{tplm}\left(\widetilde{REc}_{tpm} + TOPc_{plm}\right)$$

- Cost incurred in the disposal center:

$$\sum_{t\in T}\sum_{p\in P}\sum_{l\in L}\sum_{o\in O} QIP_{tplo}\left(Io_{po} + TIc_{plo}\right)$$

- Shortage cost:

$$\sum_{t\in T}\sum_{p\in P}\sum_{c\in C} QNS_{tpc}\hat{\pi}_{pc}$$

The total cost of the supply chain network optimization problem can be defined as follows:

$$
\begin{aligned}
Minimize\ Z = {} & \sum_{t\in T}\sum_{s\in S} FS_s S_{ts} + \sum_{t\in T}\sum_{i\in I} FD_i DT_{ti} + \sum_{t\in T}\sum_{l\in L} FC_l CL_{tl} \\
& + \sum_{t\in T}\sum_{o\in O} FO_o DO_{to} + \sum_{t\in T}\sum_{p\in P}\sum_{s\in S}\sum_{m\in M} W_{pm} v_{tpsm} + P_{ps}.QSM_{tpsm} \\
& + \sum_{t\in T}\sum_{p\in P}\sum_{s\in S}\sum_{m\in M} QSM_{tpsm}\left(Sc_{ps} + TMc_{psm}\right) \\
& + \sum_{t\in T}\sum_{p\in P}\sum_{m\in M}\sum_{i\in I} QMD_{tpmi}\left(Mc_{pm} + TPc_{pmi} + Ic_{pi} + \tilde{d}_t PR_p\right) \\
& + \sum_{t\in T}\sum_{p\in P}\sum_{i\in I}\sum_{c\in C} QDC_{tpic}\left(Dc_{pi} + TZc_{pic} + h_{pi} + F_p\left(x_p;\mu_p\right)\right) \\
& + \sum_{t\in T}\sum_{p\in P}\sum_{c\in C}\sum_{l\in L} QCO_{tpcl}\left(Cc_{pl} + TOc_{pcl}\right) \\
& + \sum_{t\in T}\sum_{p\in P}\sum_{l\in L}\sum_{m\in M} QRP_{tplm}\left(\widetilde{Rc}_{tpm} + TOPc_{plm}\right) \\
& + \sum_{t\in T}\sum_{p\in P}\sum_{l\in L}\sum_{m\in M} QEP_{tplm}\left(\widetilde{REc}_{tpm} + TOPc_{plm}\right) \\
& + \sum_{t\in T}\sum_{p\in P}\sum_{c\in C}\sum_{l\in L} QCO_{tpcl} hw_{pl} \\
& + \sum_{t\in T}\sum_{p\in P}\sum_{l\in L}\sum_{o\in O} QIP_{tplo}\left(Io_{po} + TIc_{plo}\right) + \sum_{t\in T}\sum_{p\in P}\sum_{c\in C} QNS_{tpc}\hat{\pi}_{pc}
\end{aligned}
\tag{2.1}
$$

Subject to:

$$\sum_{i\in I} QDC_{tpic} + QNS_{tpc} \geq \widetilde{D}_{tpc}\,, \qquad \forall\, t \in T, p \in P, c \in C \tag{2.2}$$

$$\sum_{l\in L} QCO_{tpcl} \leq \tilde{R}_{tpc} + \widetilde{Rw}_{tpc}, \forall\, t \in T, p \in P, c \in C \tag{2.3}$$

$$\sum_{m\in M} QMD_{tpmi}\left(1 - \tilde{d}_t\right) \geq \sum_{c\in C} QDC_{tpic}\,, \forall\, t \in T, p \in P, i \in I \tag{2.4}$$

$$\sum_{c\in C}\sum_{p\in P} \tilde{\beta}_p.QCO_{tpcl} \leq \sum_{o\in O}\sum_{p\in P} QIP_{tplo}\,, \forall\, t \in T, l \in L \tag{2.5}$$

$$\sum_{o\in O} QIP_{tplo} + \sum_{m\in M} QRP_{tplm} + \sum_{m\in M} QEP_{tplm} = \sum_{c\in C} QCO_{tpcl}\,, \forall\, t \in T, p \in P, l \in L \tag{2.6}$$

$$\sum_{s\in S}\sum_{p\in P} QSM_{tpsm} + \sum_{l\in L}\sum_{p\in P} QRP_{tplm} + \sum_{l\in L}\sum_{p\in P} QEP_{tplm} = \sum_{i\in I}\sum_{p\in P} QMD_{tpmi}\,, \forall\, t \in T, m \in M \tag{2.7}$$

$$\sum_{s\in S} QSM_{tpsm} \le B.v_{tpsm}, \forall\, t\in T, p\in P, s\in S, m\in M \tag{2.8}$$

$$\sum_{m\in M} QSM_{tpsm} \le CS_{ps}S_{ts}, \forall\, t\in T, p\in P, s\in S \tag{2.9}$$

$$\sum_{i\in I} QMD_{tpmi} \le CP_{pm}, \forall\, t\in T, p\in P, m\in M \tag{2.10}$$

$$\sum_{m\in M} QMD_{tpmi} \le CI_{pi}DT_{ti}, \forall\, t\in T, p\in P, i\in I \tag{2.11}$$

$$\sum_{c\in C} QCO_{tpcl} \le CL_{pl}CT_{tl}, \forall\, t\in T, p\in P, l\in L \tag{2.12}$$

$$\sum_{l\in L} QIP_{tplo} \le CS_{po}DO_{to}, \forall\, t\in T, p\in P, o\in O \tag{2.13}$$

$$v_{tsm}, S_{ts}, DT_{ti}, CT_{tl}, DO_{to} \in \{0,1\} \quad \forall t,p,s,m,i,l,o \tag{2.14}$$

Non-negativity constraints (2.15)

Constraint (2.2) ensures the customer demand satisfaction. Constraint (2.3) states that the returned items are not all necessarily collected from the customer zones. Constraint (2.4) makes sure that apparent produced good items quantity is larger than the quantity transported to the customer zone. Constraint (2.5) limits the quantity of disposed products shipped from the collection centers. Constraints (2.6) and (2.7) confirm the movement equilibrium between all the echelons. Constraint (2.8) assigns cost whenever the order is placed .Constraints (2.9-2.13) are based on capacity restriction for the facilities.

Two types of errors are committed in the inspection process. Type I error, $\tilde{\boldsymbol{e}}_{\boldsymbol{1}}$, is committed when a conforming item is classified as non-conforming and Type II error, $\tilde{\boldsymbol{e}}_{\boldsymbol{2}}$, is committed when a non-conforming item is classified as conforming. The apparent conforming items fraction can be determined as follows:

$$(1-\boldsymbol{d})(1-\tilde{\boldsymbol{e}}_{\boldsymbol{1}}) + \boldsymbol{d}\,\tilde{\boldsymbol{e}}_{\boldsymbol{2}} = 1 - \tilde{\boldsymbol{e}}_{\boldsymbol{1}} - \boldsymbol{d}(1-\tilde{\boldsymbol{e}}_{\boldsymbol{1}}-\tilde{\boldsymbol{e}}_{\boldsymbol{2}}) = 1 - \tilde{\boldsymbol{d}}, \text{ with } 0 \le \tilde{\boldsymbol{d}} \le 1$$

where,

$$\tilde{\boldsymbol{d}} = \tilde{\boldsymbol{e}}_{\boldsymbol{1}} + \boldsymbol{d}(1-\tilde{\boldsymbol{e}}_{\boldsymbol{1}}-\tilde{\boldsymbol{e}}_{\boldsymbol{2}}), \tag{2.16}$$

and the vectors $\tilde{\boldsymbol{e}}_{\boldsymbol{1}}$ and $\tilde{\boldsymbol{e}}_{\boldsymbol{2}}$ are both uncertain.

The quality loss function is proposed by Taguchi (1986). It states that for given specification limits not all values falling within them are equal and create equal loss because of poor quality. Quality loss function $L(x)$ is defined as follows:

$$L(x) = K(x-\mu)^2 \qquad LSL \leq x \leq USL$$

The quadratic term indicates that if the difference between actual value and target value is large, the loss would be more where K is the loss parameter,

$$K = \frac{V}{\psi^2}$$

and,

$$\psi = (USL-\mu) = (\mu - LSL)$$

Thus, the amount of loss is expressed as follows:

$$QDC_{tpic} \int_{LSL}^{USL} \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}} K(x-\mu)^2 dx = QDC_{tpic}\left(1-\tilde{d}_t\right)F(x;\mu) \qquad (2.17)$$

Equation (2.17) states that the apparent conforming quantity of product p planned to be sent from distribution center i to customer zone c at period t is subject to an inspection to ensure that the produced lot is close enough to the target value according to Taguchi Quality approach. This loss is included in the objective function under the cost incurred in the distribution centers.

2.4 Robust Counterpart Formulations

2.4.1 Definition 1: Counterpart Formulation for Linear Programming

Consider the following linear programming £,

$$Min \sum_j \tilde{c}_j x_j$$

s.t. $\sum_j \tilde{a}_{ij} x_j \leq \tilde{b}_i \quad \forall i$

where $\tilde{a}_{ij}$, $\tilde{b}_i$, and $\tilde{c}_j$, represent the true value of the parameters which are subject to uncertainty and defined as follows:

$$\tilde{a}_{ij} = a_{ij} + \zeta_{ij}\hat{a}_{ij} \quad \forall j \in J_i$$

$$\tilde{b}_i = b_i + \zeta_{ij}\hat{b}_i$$

$$\tilde{c}_j = c_j + \zeta_j \hat{c}_j \qquad \forall j \in J_i$$

where a_{ij}, b_i, and c_j represent the nominal (expected) value of the parameters; $\hat{a}_{ij}$, $\hat{b}_i$, and $\hat{c}_j$ represent constant perturbation; ζ_{ij} is a random variable that takes values in the interval [-1, 1]. Without loss of generality, we make the following assumptions:

• If uncertain data exists in the objective function as coefficients, then the objective function can be written as a constraint.

• In any constraint j, if the right-hand-side parameter is subject to uncertainty, then model £ can be written as:

$Min\ Z$

s.t. $\sum_j \tilde{c}_j x_j \leq Z$

$$\tilde{b}_i - \sum_j \tilde{a}_{ij} x_j \leq 0 \quad \forall i$$

Therefore, we end up with a constraint that has uncertain parameters on the left-hand-side only.

2.4.2 Definition 2: Box Uncertainty Set

The box uncertainty set is formulated based on the Chebyshev (infinity) norm of the perturbation variables (Figure 2.2). It is presented as follows:

$$U_\infty = \{\zeta \mid \|\zeta\|_\infty \leq \Psi\} = \{\zeta \mid |\zeta| \leq \Psi\}$$
(2.18)

where Ψ is the adjustable parameter that controls the uncertainty set size, and hence controlling the degree of conservatism (Figure 2.1). If $\Psi = 1$, then the resulting uncertainty set is the interval uncertainty set which is a special case of the box uncertainty set.

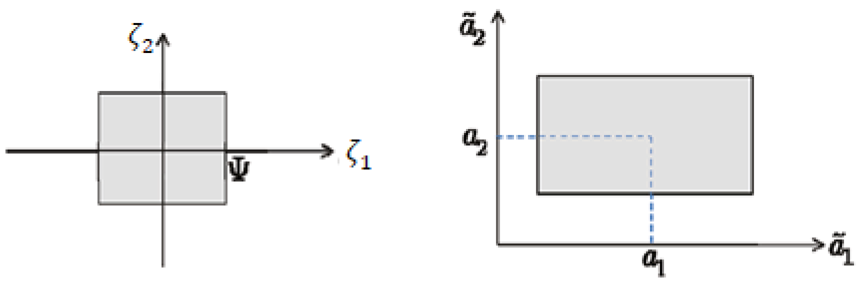


Figure 2.2: Illustration of box uncertainty set where a_1 and a_2 are the nominal values of the uncertain parameters $\tilde{a}_1$ and $\tilde{a}_2$, respectively.

Ben–Tal and Nemirovski (2000) introduced a tractable form of a model with box uncertainty sets which is given as follows, derived from Model £:

$Min\ Z$

s.t. $\sum_j c_j x_j + \Psi\left[\sum_j \hat{c}_j |x_j|\right] \leq Z$

$$\sum_j a_{ij} x_j + \Psi\left[\sum_j \hat{a}_{ij}|x_j| + \hat{b}_i\right] \leq b_i \quad \forall i$$

The box uncertainty set is less conservative in comparison with the other bounded uncertainty sets. However, if Ψ_i is not within the suggested range such that the adjustable uncertainty set parameters $\Psi_i \geq 1$, the box uncertainty becomes more conservative than the original linear programming. Proof is provided by (Zukui Li, Ran Ding, and Christodoulos A. Floudas, 2011).

The corresponding robust counterpart formulation for Model (2.1) – (2.15) is given as follows,

$$
\begin{aligned}
Minimiz\ Z = &\sum_{t\in T}\sum_{s\in S} FS_s S_{ts} + \sum_{t\in T}\sum_{i\in I} FD_i DT_{ti} + \sum_{t\in T}\sum_{l\in L} FC_l CL_{tl} \\
&+ \sum_{t\in T}\sum_{o\in O} FO_o DO_{to} + \sum_{t\in T}\sum_{p\in P}\sum_{s\in S}\sum_{m\in M} W_{pm} v_{tpsm} + P.\, QSM_{tpsm} \\
&+ \sum_{t\in T}\sum_{p\in P}\sum_{s\in S}\sum_{m\in M} QSM_{tpsm}\left(Sc_{ps} + TMc_{psm}\right) \\
&+ \sum_{t\in T}\sum_{p\in P}\sum_{m\in M}\sum_{i\in I} QMD_{tpmi}\left(Mc_{pm} + TPc_{pmi} + Ic_{pi}\right) \\
&+ \sum_{t\in T}\sum_{p\in P}\sum_{i\in I}\sum_{c\in C} QDC_{tpic}\left(Dc_{pi} + TZc_{pic} + h_{pi} + F_p\left(x_p;\mu_p\right)\right) \\
&+ \sum_{t\in T}\sum_{p\in P}\sum_{c\in C}\sum_{l\in L} QCO_{tpcl}\left(Cc_{pl} + TOc_{pcl}\right) \\
&+ \sum_{t\in T}\sum_{p\in P}\sum_{l\in L}\sum_{m\in M} QRP_{tplm}\left(TOPc_{plm}\right) \\
&+ \sum_{t\in T}\sum_{p\in P}\sum_{l\in L}\sum_{m\in M} QEP_{tplm}\left(TOPc_{plm}\right) \\
&+ \sum_{t\in T}\sum_{p\in P}\sum_{c\in C}\sum_{l\in L} QCO_{tpcl} hw_{pl} \\
&+ \sum_{t\in T}\sum_{p\in P}\sum_{l\in L}\sum_{o\in O} QIP_{tplo}\left(Io_{po} + TIc_{plo}\right) + \sum_{t\in T}\sum_{p\in P}\sum_{c\in C} QNS_{tpc}\hat{\pi}_{pc} \\
&+ y_d + y_{RC} + y_{REc} \qquad (2.19)
\end{aligned}
$$

Subject to:

$$\sum_{t\in T}\sum_{p\in P}\sum_{m\in M}\sum_{i\in I} PR_p QMD_{tpmi}\, d_t + \Psi_d \sum_{t\in T}\sum_{p\in P}\sum_{m\in M}\sum_{i\in I} QMD_{tpmi}\, \hat{d}_t \le y_d \qquad (2.20)$$

$$\sum_{t\in T}\sum_{p\in P}\sum_{l\in L}\sum_{m\in M} QRP_{tplm} Rc_{tpm} + \Psi_{RC} \sum_{t\in T}\sum_{p\in P}\sum_{l\in L}\sum_{m\in M} QRP_{tplm}\widehat{Rc}_{tpm} \le y_{RC} \qquad (2.21)$$

$$
\begin{aligned}
&\sum_{t\in T}\sum_{p\in P}\sum_{l\in L}\sum_{m\in M} QEP_{tplm} REc_{tpm} + \Psi_{REc} \sum_{t\in T}\sum_{p\in P}\sum_{l\in L}\sum_{m\in M} QEP_{tplm}\widehat{REc}_{tpm} \\
&\qquad \le y_{REc} \qquad (2.22)
\end{aligned}
$$

$$\sum_{i\in I} QDC_{tpic} + QNS_{tpc} \ge D_{tpc} + \ \Psi_D \widehat{D}_{tpc}, \qquad \forall\, t \in T, p \in P, c \in C \qquad (2.23)$$

$$\sum_{l\in L} QCO_{tpcl} -\ \Psi_R \hat{R}_{tpc} - \Psi_{Rw}\widehat{Rw}_{tpc} \le R_{tpc} + Rw_{tpc}, \forall\, t \in T, p \in P, c \in C \quad (2.24)$$

$$\sum_{m\in M} QMD_{tpmi}\left(1-d_t-\Psi_Q\hat{d}_t\right) \geq \sum_{c\in C} QDC_{tpic}\,, \forall\, t\in T, p\in P, i\in I \qquad (2.25)$$

$$\sum_{c\in C}\sum_{p\in P}\beta_p.QCO_{tpcl} + \Psi_\beta\sum_{o\in O}\sum_{p\in P}\hat{\beta}_p.QCO_{tplo} \leq \sum_{o\in O}\sum_{p\in P} QIP_{tplo}\,, \forall\, t\in T, l \in L \qquad (2.26)$$

Given constraints (2.6)- (2.15).

2.4.3 Definition 3: Polyhedral Uncertainty Set

The polyhedral uncertainty set that is described using the 1-norm of the uncertain data vector is presented as follows:

$$U_1 = \{\zeta \mid \|\zeta\|_1 \leq \Gamma\} = \left\{\zeta \mid \textstyle\sum_{j\in J_i}|\zeta_i| \;\leq \Gamma\right\} \qquad (2.27)$$

where Γ is the adjustable parameter controlling the size of the uncertainty set, Figure 2.3.

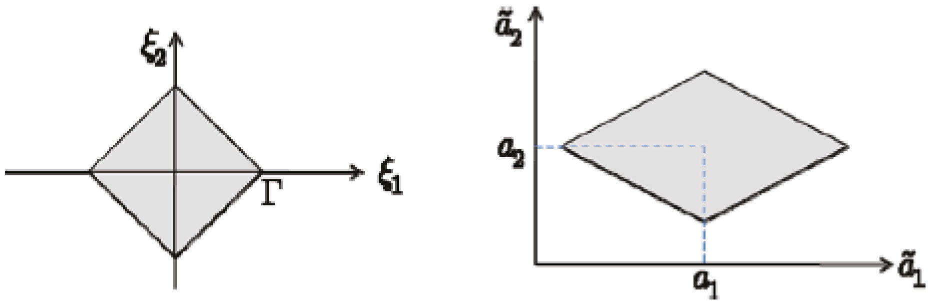


Figure 2.3: Illustration of polyhedral uncertainty set where a_1 and a_2 are the nominal values of the uncertain parameters $\tilde{a}_1$ and $\tilde{a}_2$, respectively.

Bertsimas and Sim introduced the polyhedral uncertain set which has the equivalent tractable form, based on Model £:

$Min\ Z$

s.t. $\sum_j c_j x_j + \Gamma U \leq Z$

$U \geq \hat{c}_j|x_j|, \quad \forall j\in J$

$$\sum_j a_{ij}x_j + \Gamma u_i \;\leq b_i \quad \forall i$$

$u_i \geq \hat{a}_{ij}|x_j|, \qquad \forall i, j \in J$

$u_i \geq \hat{b}_i, \qquad \forall i$

In the case where the uncertain parameter is subject to an unbounded distribution, it is recommended to use the polyhedral uncertainty set because of its flexibility to design a set size that leads to the desired robust solution. Unlike the bounded distribution, where the combined interval and polyhedral uncertainty sets are considered such that the bounds can not be exceeded by the designed set.

The corresponding robust counterpart formulation based on the polyhedral uncertainty sets for model (2.1) – (2.15) is given as follows,

$$\sum_{t \in T} \sum_{p \in P} \sum_{m \in M} \sum_{i \in I} PR_p QMD_{tpmi}\, d_t + u_d \Gamma_d \leq y_d \tag{2.28}$$

$$u_d \geq \hat{d}_t QMD_{tpmi}, \qquad \forall\, t \in T, p \in P, m \in M, i \in I \tag{2.29}$$

$$\sum_{t \in T} \sum_{p \in P} \sum_{l \in L} \sum_{m \in M} QRP_{tplm} Rc_{tpm} + u_{RC} \Gamma_{RC} \leq y_{RC} \tag{2.30}$$

$$u_{RC} \geq \widehat{Rc}_{tpm} QRP_{tplm}, \quad \forall\, t \in T, p \in P, l \in L, m \in M \tag{2.31}$$

$$\sum_{t \in T} \sum_{p \in P} \sum_{l \in L} \sum_{m \in M} QEP_{tplm} REc_{tpm} + u_{REc} \Gamma_{REc} \leq y_{REc} \tag{2.32}$$

$$u_{REc} \geq \widehat{RE}c_{tpm} QEP_{tplm}, \quad \forall\, t \in T, p \in P, l \in L, m \in M \tag{2.33}$$

$$\sum_{i \in I} QDC_{tpic} + QNS_{tpc} \geq D_{tpc} + \Gamma_D u_D\,, \qquad \forall\, t \in T, p \in P, c \in C \tag{2.34}$$

$$u_D \geq \widehat{D}_{tpc}, \qquad \forall t \in T, p \in P, c \in C \tag{2.35}$$

$$\sum_{l \in L} QCO_{tpcl} - R_{tpc} - Rw_{tpc} \leq \Gamma_{R+W} + \hat{R}_{tpc} + \widehat{Rw}_{tpc}, \quad \forall\, t \in T, p \in P, c \in C \tag{2.36}$$

$$\sum_{m \in M} QMD_{tpmi}\big(1 - \tilde{d}_t\big) - u_Q \Gamma_Q \geq \sum_{c \in C} QDC_{tpic}, \quad \forall\, t \in T, p \in P, i \in I \tag{2.37}$$

$$u_Q \geq \hat{d}_t QMD_{tpmi}, \qquad \forall\, t \in T, p \in P, m \in M, i \in I \tag{2.38}$$

$$\sum_{c \in C} \sum_{p \in P} \beta_p . QCO_{tpcl} + u_\beta \Gamma_\beta \leq \sum_{o \in O} \sum_{p \in P} QIP_{tplo}\,, \forall\, t \in T, l \in L \tag{2.39}$$

$$u_\beta \geq \hat{\beta}_p . QCO_{tpcl} \qquad \forall\, t \in T, p \in P, c \in C, l \in L \tag{2.40}$$

Given equation (2.19) and constraints (2.6) - (2.15).

2.4.4 Definition 4: Combined Interval and Polyhedral Uncertainty Set

This type of uncertainty set is the intersection between the polyhedral and the interval set defined with both 1-norm and infinite norm as follows:

$$U_{1\cap\infty} = \{\zeta_i \,|\sum_{j\in J_i}|\zeta_i| \le \Gamma, |\zeta_i| \le 1, \forall j \in J_i\} \tag{2.41}$$

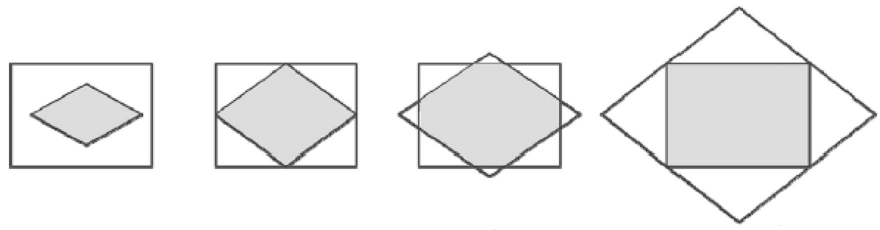


Figure 2.4: Illustration of the combined interval and polyhedral uncertainty set.

Bertsimas and Sim introduced the combined interval and polyhedral uncertain set which has the following equivalent tractable form, based on Model £:

$$Min\ Z$$

$$\text{s.t. } \sum_j c_j x_j + \Gamma U + \sum_{j\in J_i} \varphi_{j0} \le Z$$

$$U + \varphi_{j0} \ge \hat{c}_j |x_j|, \quad \forall j \in J$$

$$U, p_{j0} \ge 0$$

$$\sum_j a_{ij} x_j + \Gamma u_i + \sum_{j\in J_i} \varphi_{ij} + \varphi_{i0} \le b_i \quad \forall i$$

$$u_i + \varphi_{ij} \ge \hat{a}_{ij} |x_j|, \qquad \forall i, j \in J$$

$$u_i + \varphi_{i0} \ge \hat{b}_i, \qquad \forall i$$

The complete derivation of the above model can be seen in (Li, Tang, and Floudas, 2012). The corresponding robust counterpart formulation based on the combined interval and polyhedral uncertainty sets for model (2.1) – (2.15) is given as follows,

$$\sum_{t\in T}\sum_{p\in P}\sum_{m\in M}\sum_{i\in I} PR_p QMD_{tpmi}\, d_t + u_d \Gamma_d + \sum_{t\in T} \varphi_t^d \le y_d \tag{2.42}$$

$$u_d + \varphi_t^d \geq \hat{d}_t QMD_{tpmi}, \qquad \forall\, t \in T, p \in P, m \in M, i \in I \tag{2.43}$$

$$\sum_{t\in T}\sum_{p\in P}\sum_{l\in L}\sum_{m\in M} QRP_{tplm} Rc_{tpm} + u_{RC}\Gamma_{Rc} + \sum_{t\in T}\sum_{p\in P}\sum_{m\in M} \varphi_{tpm} \leq y_{RC} \tag{2.44}$$

$$u_{RC} + \varphi_{tpm}^{Rc} \geq \widehat{Rc}_{tpm} QRP_{tplm}, \quad \forall\, t \in T, p \in P, l \in L, m \in M \tag{2.45}$$

$$\sum_{t\in T}\sum_{p\in P}\sum_{l\in L}\sum_{m\in M} QEP_{tplm} REc_{tpm} + u_{REc}\Gamma_{REc} + \sum_{t\in T}\sum_{p\in P}\sum_{m\in M} \varphi_{tpm} \leq y_{REc} \tag{2.46}$$

$$u_{REc} + \varphi_{tpm}^{REc} \geq \widehat{RE}c_{tpm} QEP_{tplm}, \quad \forall\, t \in T, p \in P, l \in L, m \in M \tag{2.47}$$

$$\sum_{i\in I} QDC_{tpic} + QNS_{tpc} \geq D_{tpc} + \Gamma_D u_D\,, \qquad \forall\, t \in T, p \in P, c \in C \tag{2.48}$$

$$\sum_{l\in L} QCO_{tpcl} - R_{tpc} - Rw_{tpc} \leq \Gamma_{R+W} + \hat{R}_{tpc} + \widehat{Rw}_{tpc}, \quad \forall\, t \in T, p \in P, c \in C \tag{2.49}$$

$$\sum_{m\in M} QMD_{tpmi}\left(1 - \hat{d}_t\right) - u_Q\Gamma_Q - \sum_{t\in T} \varphi_t^Q \geq \sum_{c\in C} QDC_{tpic}, \quad \forall\, t \in T, p \in P, i \in I \tag{2.50}$$

$$u_Q + \varphi_t^Q \geq \hat{d}_t QMD_{tpmi}, \qquad \forall\, t \in T, p \in P, m \in M, i \in I \tag{2.51}$$

$$\sum_{c\in C}\sum_{p\in P} \beta_p . QCO_{tpcl} + u_\beta\,\Gamma_\beta + \sum_{p\in P} \varphi_p^\beta \leq \sum_{o\in O}\sum_{p\in P} QIP_{tplo}\,, \forall\, t \in T, l \in L \tag{2.52}$$

$$u_\beta + \varphi_p^\beta \geq \hat{\beta}_p . QCO_{tpcl} \qquad \forall\, t \in T, p \in P, c \in C, l \in L \tag{2.53}$$

Given equation (2.19) and constraints (2.6) - (2.15).

2.5 Probabilistic Guarantees of Robust Counterpart Optimization:

In many practical problems the uncertainty set is defined by the decision maker. What makes robust optimization (RO) different from stochastic programming is that RO does not require a known probability distribution for the uncertainty. However, probabilistic guarantees (chance constraint approach) can be used to evaluate the lower bound on constraint satisfaction based on the desired constraint violation.

Li, Tang, and Floudas (2012) and Guzman, Matthews, and Floudas (2016) considered probabilistic guarantees on constraint satisfaction employed in the literature for different uncertainty set robust counterpart optimization models, for both bounded and unbounded uncertainty, with and without a detailed probability distribution information.

In general, two different methods can be used in evaluating the probabilistic guarantees: a priori and a posteriori probability bound, (Li, et al., 2012). In this work we will focus on the first type of methods which uses the uncertainty set information to derive the probability before we solve the problem.

2.5.1 Priori Probabilistic Guarantees Based on Uncertainty Set Information

The a priori approach is used as a traditional way to compute the size of the uncertainty set necessary to ensure that the degree of constraint violation does not exceed a certain level. Therefore,

$$\Pr\{\sum_j a_{ij}x_j + \sum_{j\in J_i} \zeta_j\, \hat{a}_{ij}x_j > b_i\} \leq \Pr\{\sum_{j\in J_i} \zeta_j\, a\delta_j > \Delta\} \qquad (2.54)$$

where the parameter Δ is the uncertainty set parameter (i.e. Ψ, or Γ), and J_i is the number of uncertain parameters in the ith constraint. Note that $\boldsymbol{\delta}$ is a vector with its δ_j components satisfying $-1 \leq \delta_j \leq 1$. Moreover, $\sum_{j\in J_i} \delta_j \leq 1$, and $0 \leq \delta_j \leq 1$ for the box and combined interval and polyhedral uncertainties sets respectively.

The poof of (2.54) is available in Li et al. (2011). The summary of different upper bounds on the probability of constraint violation is presented in Table 2.3. Note that in Table 2.3 we follow the following abbreviations; B: Box, E: Ellipsoidal, IE: Interval and Ellipsoidal, P: Polyhedral, IP: Interval and Polyhedral. The proof of upper bounds on the probability of constraint violation provided by Table 2.3 is available in (Ben-tal & Nemirovski, 2000), (Bertsimas & Sim, 2004b), (Paschalidis and Kang, 2005), and (Guzman et al., 2016).

upper bounds on the probability of constraint violation	Assumption on Uncertainty distribution	Robust Counterpart Applicable	Proposed by
B1: $\exp(-\frac{\Delta^2}{2})$	Independent, symmetric, bounded	B, E, IE	(Ben-tal & Nemirovski, 2000)
B2: $\exp(-\frac{\Delta^2}{2\lvert J_i\rvert})$	Independent, symmetric, bounded	B, E, IE, P, IP	(Bertsimas & Sim, 2004b)
B3:$\exp(min_{\theta>0}\{-\theta\Delta + \sum_{j\in J_i} In\, E[e^{\theta\zeta_j}]\})$	It has known probability distribution.	B, E, IE, P, IP	(Paschalidis & Kang, 2005)
B4: $\exp(min_{\theta>0}\{-\theta\Delta + \sum_{j\in J_i} In\, G_j(\theta)\})$	known bounds on $E[\zeta_j]$	B, E, IE, P, IP	(Guzman et al., 2016)
B5: $\exp(min_{\theta>0}\left\{-\theta\Delta + \lvert J_j\rvert \sum_{j\in J_i} In\, \bar{G}_j\left(\theta/\sqrt{\lvert J_j\rvert}\right)\right\})$	known bounds on $E[\zeta_j]$	E, IE	(Guzman et al., 2016)

Table 2.3: The summary of different upper bounds on the probability of constraint violation.

2.5.2 The Characteristics of The Introduced Probability Bounds

From Table 2.3, it is observed that for the different types of robust counterparts, bounding the probability of constraint violation corresponds to the evaluation of the expression $\Pr\{\sum_{j\in J_i} \zeta_j\, \delta_j > \Delta\}$. The given probability bounds in Table 2.3 are bounded, symmetric and independent. Moreover, different bounds can be derived if the full probability distribution information of the uncertainty is provided. The following characteristics of the introduced probability bounds can be listed as follows:

1. If $\{\zeta_j\}_{j\epsilon J_i}$ are independent and subject to a bounded and symmetric probability distribution supported on [-1, 1], then B1 and B2 apply. That is;

$$\Pr\{\sum_j a_{ij}x_j + \sum_{j\in J_i} \zeta_j\, \hat{a}_{ij}x_j > b_i\} \leq \exp(-\frac{\Delta^2}{2}) \qquad (2.55)$$

$$\Pr\{\sum_j a_{ij}x_j + \sum_{j\in J_i} \zeta_j\, \hat{a}_{ij}x_j > b_i\} \leq \exp(-\frac{\Delta^2}{2|J_i|}) \qquad (2.56)$$

However, B1 only applies for the box (B), ellipsoidal (E), and interval plus ellipsoidal (IE) uncertainty sets induced robust counterparts.

2. If $\{\zeta_j\}_{j\epsilon J_i}$ are independent and subject to symmetric probability distribution, then B3 applies such that,

$$\Pr\{\sum_j a_{ij}x_j + \sum_{j\in J_i} \zeta_j\, \hat{a}_{ij}x_j > b_i\} \leq \exp(min_{\theta>0}\{-\theta\Delta + \sum_{j\in J_i} In\, E[e^{\theta\zeta_j}]\}) \quad (2.57)$$

where $E[e^{\theta\zeta_j}]$ refers to the moment generation function of probability density function $f(\zeta_j)$. Moreover, it needs the solution of the following additional nonlinear nonconvex optimization problem (2.58):

$$\begin{aligned} &\min \Delta \\ &\text{s.t.} \\ &-\theta\Delta + \sum_{j\in J_i} In\, E[e^{\theta\zeta_j}] \leq \ln(\varepsilon) \\ &\qquad \Delta, \theta \geq 0 \end{aligned} \qquad (2.58)$$

3. For B4 and B5 the uncertain parameters have known lower and upper bounds and their means are known only to within some range of values. Hence, a single expected value cannot be confidently imposed. Thus, we have the following expressions:

$$\Pr\{\sum_j a_{ij}x_j + \sum_{j\in J_i} \zeta_j\, \hat{a}_{ij}x_j > b_i\} \leq \exp(min_{\theta>0}\{-\theta\Delta + \sum_{j\in J_i} In\, G_j(\theta)\}) \quad (2.59)$$

$\Pr\{\sum_j a_{ij}x_j + \sum_{j\in J_i} \zeta_j \hat{a}_{ij}x_j > b_i\} \leq \exp(min_{\theta>0}\left\{-\theta\Delta + |J_j| \sum_{j\in J_i} In\ \overline{G}_J \left(\theta/\sqrt{|J_j|}\right)\right\}$ (2.60)

where $G_j(\theta) = \mu_j \sinh\theta + \cosh\theta$, and $\overline{G}_J(\theta) = (\max\mu_j)\sinh\theta + \cosh\theta$. Note that B5 is applicable to only ellipsoidal (E) and interval and ellipsoidal (IE) uncertainty sets. Also, we may notice that (2.59) and (2.60) require the solution of additional nonlinear nonconvex optimization problems (2.61) and (2.62), respectively.

For (2.53), we need to solve the following optimization problem;

$$\min \Delta$$
s.t.
$$-\theta\Delta + \sum_{j\in J_i} In\ G_j(\theta) \leq \ln(\varepsilon)$$
$$\Delta, \theta \geq 0 \qquad (2.61)$$

and for (2.54),

$$\min \Delta$$
s.t.
$$-\theta\Delta + |J_j| \sum_{j\in J_i} In\ \overline{G}_J\left(\theta/\sqrt{|J_j|}\right)$$
$$\Delta, \theta \geq 0 \qquad (2.62)$$

In B4 and B5 instead of the nominal value of $\hat{a}_{ij}$ representing the mean, yielding $E[\zeta_{ij}] = 0$, the nominal value is chosen such that $|E[\zeta_{ij}]| \leq \mu_{ij}$.

Traditional framework steps (Li et al., 2012) of applying robust optimization for a probabilistically constrained optimization problem can be summarized as follows:

1. The probabilistic constraint violation ε is set.
2. The uncertainty set is selected by the distribution of the uncertainty.
3. The uncertainty set size parameter is computed based on the a priori probability bounds.
4. The problem can be solved using the above uncertainty set size parameter and the solution obtained satisfies the desired probability $1-\varepsilon$.

2.6 Numerical Example and Computational Results

To illustrate the application of robust optimization framework based on the three different uncertainty sets which are box, polyhedral, and the combined interval and polyhedral, we solve our proposed model. We utilize four different probability bounds including those bounds which require solving additional nonlinear nonconvex optimization problems. In addition, we evaluate the robust solutions at different probability constraint violations, ε, for three problem sizes. The sizes of the problem are explained in Table 2.4.

Size problem No.	No. of periods	No. of potential supplier centers	No. of plant centers	No. of potential distribution centers	No. of customer zones	No. of potential collection centers	No. of potential disposal centers
1	12	3	2	3	5	3	2
2	12	5	3	5	10	5	3
3	12	7	5	7	20	7	5

Table 2.4: Test Problem Sizes.

Three random numerical examples of different sizes are considered, and specifications of the test problems are presented next. The nominal values of the following uncertain parameters: $\widetilde{D}_{tpc}$, $\tilde{R}_{tpc}$, $\widetilde{Rw}_{tpc}$, $\widetilde{Rc}_{tpm}$, $\widetilde{REc}_{tpm}$, $\tilde{\beta}_p$, and $\tilde{d}_t$ are generated randomly using uniform distribution at $t = 1$, Table 2.5, and then the nominal values for the rest of the periods are generated as explained in Fig.2.5. For instance, it shows that in Fig.2.5 the nominal values at period $t = 2$, is higher than the nominal values of $t = 1$ by 10%. This increase continues until it reaches to $t = 6$, at which the nominal values decrease by 10% of $t = 5$. Then, the values keep going down by 10% until it reaches the end of the year $t = 12$.

This behavior is projected on the assumption that the market demand growth for some products would increase gradually at the beginning of the cycle until it reaches to its highest sales in the mid of the cycle. After that the customers lose their interests in these products because other companies in the market offer competitive products with reasonable prices. In addition, the company decides to shift to new products with new features which means low sales of old products at the end of the cycle.

Note that the deviation magnitudes of the uncertain parameters are always set to be 0.1 of the nominal values. The random generated data of the proposed model parameters are given in Tables 2.5 and 2.6.

	Nominal Values for Product *p*		
Uncertain Parameter	1	2	3

$\widetilde{D}_{tpc}$	U (65, 165)	U (55, 147)	U (70, 170)
$\tilde{R}_{tpc}$	U (44, 85)	U (38, 95)	U (61, 110)
$\widetilde{Rw}_{tpc}$	U (10, 36)	U (13, 43)	U (9, 26)
$\widetilde{Rc}_{tpm}$	U (9, 12)	U (6.5, 9)	U (6, 8)
$\widetilde{REc}_{tpm}$	U (4, 6)	U (4, 6.5)	U (3.5, 6)
$\tilde{\beta}_p$	0.2	0.175	0.18
$\tilde{d}_t$		0.05	

Table 2.5: The nominal values of the model uncertain parameters at period $t = 1$,for each product p.

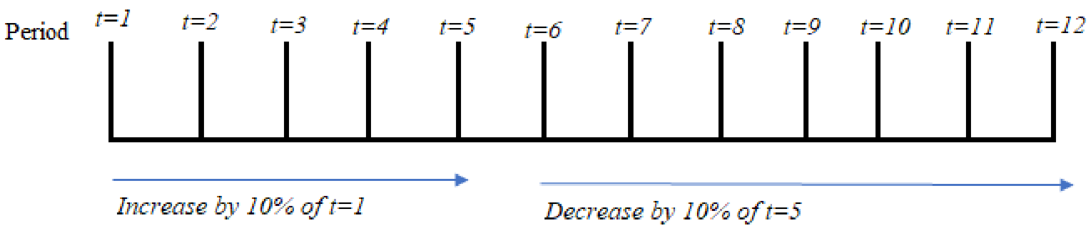


Figure 2.5: Generating the nominal values for the entire year based on period *t=1*.

	Values				Values		
Parameter	Product 1 (p_1)	Product 2 (p_2)	Product 3 (p_3)	Parameter	Product 1(p_1)	Product 2	Product 3
Sc_{ps}	~U(12.5, 15)	~U(10,12)	~U(8,13)	CI_{pi}	~U(575, 660)	~U(580,645)	~U(550,630)
Mc_{pm}	~U(40,45)	~U(38,42)	~U(43,45)	CL_{pl}	~U(235, 280)	~U(200, 245)	~U(220, 265)
Ic_{pi}	~U(5,6)	~U(3.75,5.75)	~U(4.5,5.5)	CO_{po}	~U(345,350)	~U(295,300)	~U(315, 320)
Dc_{pi}	~U(10,12)	~U(10,11)	~U(9.5,10.5)	TMc_{psm}	~U(5, 8)		
Cc_{pl}	~U(8,9.5)	~U(7,8)	~U(7.75,8.75)	TPc_{pmi}	~U(3, 4.75)		
h_{pi}	~U(3,4)	~U(4,4.5)	~U(4,5)	TOc_{pcl}	~U(4, 8)	0.75 +Values of (p_1)	1.2 +Values of (p_1)
P_{ps}	~U(6.5,10)	~U(5,6)	~U(3,7)	TZc_{pic}	~U(3, 5)		
Io_{po}	~U(3,3.5)	~U(3, 3.75)	~U(3,5)	$TOPc_{plm}$	~U(3.25, 5)		
CS_{ps}	~U(685, 800)	~U(720, 840)	~U(750, 780)	Tic	~U(4,5)		
CP_{pm}	~U(540, 650)	~U(500,600)	~U(590,620)				

Table 2.6: The randomly generated data of the proposed model parameters.

Parameter	Values	Parameter	Values
FS_s	~U(65000,81000)	USL_p	4.8
FD_i	~U(40000, 55000)	LSL_p	5.2
FC_l	~U(35000, 45000)	K	120
FO_o	~U(20000, 30000)	μ_p	5
hw_{pl}	~U(2, 2.5)	σ_p	0.05
$\hat{\pi}_{pc}$	~U(70000, 95000)		
W_{pm}	1000		

Table 2.7: Design of the data set.

The computations of MILP were run using the branch and bound algorithm accessed via LINGO16.0 on a PC -3GHzand; 4 GB RAM and under win 10. While computations of the nonlinear nonconvex optimization problems were run using BARON solver which is offered by GAMS modeling languages.

Prior to solving the robust models, the deterministic model is solved where the uncertain parameters in model (2.1-2.15) are set at their expected values, Table 2.8. The optimal uncertainty set sizes (Ψ, Γ) using four probability bounds at five constraint violations ε are provided in Tables 2.9 and 2.10.

Test Problem Size	The Objective Function of The Deterministic Model
1	3771306
2	15576814
3	61765418

Table 2.8: The solutions of the deterministic model.

	The Optimal Values of Δ				
$\Delta = \Psi, \Gamma$	B1	B2	B3	B4	Constraint Violations
Δ_d	2.44775	8.47924	4.77114	9.04779	0.05
Δ_{Rc}, Δ_{REc}		20.76982	11.9414	27.5186	
Δ_β		4.23962	2.18631	3.00241	
Δ_D, Δ_R, Δ_{Rw}		2.44775	0.96321	1.00356	
Δ_d	2.14597	7.43384	4.20847	8.18640	0.1
Δ_{Rc}, Δ_{REc}		18.20913	10.4793	25.0654	
Δ_β		3.71692	1.97231	3.00054	
Δ_D, Δ_R, Δ_{Rw}		2.14597	0.92642	1.00214	
Δ_d	1.94788	6.74766	3.83349	7.59772	0.15
Δ_{Rc}, Δ_{REc}		16.52832	9.51744	23.4452	
Δ_β		3.37383	1.81918	3.00005	
Δ_D, Δ_R, Δ_{Rw}		1.94788	0.88964	1.00179	
Δ_d	1.79412	6.21502	3.53967	7.12964	0.2
Δ_{Rc}, Δ_{REc}		15.22363	8.76969	22.1824	
Δ_β		3.10751	1.69421	2.92976	
Δ_D, Δ_R, Δ_{Rw}		1.79412	0.85285	1.00115	
Δ_d	1.66511	5.76811	3.29144	6.73008	0.25
Δ_{Rc}, Δ_{REc}		14.12892	8.14161	21.1197	
Δ_β		2.88405	1.58565	2.80700	
Δ_D, Δ_R, Δ_{Rw}		1.66511	0.81606	1.00081	

Table 2.9: The optimal values of uncertainty set size parameters for the four upper probability bounds at different ε for problem size 1.

Since Δ_d, Δ_β, Δ_D, Δ_R, and Δ_{Rw} have the same number of uncertain parameters, $|J_j|$, for the three problem sizes, only Δ_{Rc}, and Δ_{REc} are presented in

Table 2.10. Note that in case B3, it is assumed that each ζ_j is subject to the uniform distribution in [−1, 1], and hence the three uncertainty sets apply. For the uniform distribution $U(a,b)$, the moment generation function is $E\left(e^{\theta\zeta}\right) = \frac{e^{\theta b}-e^{\theta a}}{\theta(b-a)}$. Also, in B4 the expected values of the parameters are only known to be within 1% of their nominal values. Therefore,

$E[\tilde{a}_i]\ \epsilon\ [a_i - 0.01a_i,\ a_i + 0.01a_i]$and $E[\zeta_j]\ \epsilon\ [-0.1, 0.1]$ that is equivalent to $|E[\zeta_i]| \leq 0.1 = \mu_i$.

	The Optimal Values of $\Delta = \Psi, \Gamma$			
Problem Size	B2	B3	B4	Constraint Violations
2	25.43773	14.6457	35.7908	0.05
3	32.83997	18.9286	50.3834	
2	22.30153	12.8483	32.7558	0.1
3	28.79116	16.6013	46.4313	
2	20.24297	11.6667	30.755	0.15
3	26.13356	15.0723	43.8297	
2	18.64507	10.7487	29.1974	0.2
3	24.07068	13.8848	41.8065	
2	17.30432	9.97782	27.8877	0.25
3	22.33978	12.8879	40.1065	

Table 2.10: The optimal values of uncertainty set size parameters of Δ_{Rc} and Δ_{REc} for the four upper probability bounds at different ε for problem sizes 2.2 and 2.3.

The computational time in seconds (CPU) is presented in Table 2.11 where CPU column indicates the average computational time taken for each probability bound at the five constraint violations. The obtained robust solutions under the three uncertainty sets at different constraint violations are provided in Table set 2.12.

Test Problem	Deterministic	Probability bound	Average CPU Time in Seconds		
			Box	Polyhedral	Interval +Polyhedral
1	15	B1	261.8	-	-
		B2	515.33	10097.38	13842.51
		B3	1731.47	10280.39	14460.46
		B4	92.16	8969.28	11397.14
2	9547.12	B1	18111.07	-	-
		B2	18472.67	23445.18	26303.43
		B3	17811.24	25398.47	25803.61
		B4	15256.9	22945.36	28256.72

3	21796.43	B1	26547.83	-	-
		B2	28327.78	32364.41	36222.67
		B3	27563.04	31804.27	38214.36
		B4	26325.66	33062.41	35189.02

Table 2.11: Average CPU time in seconds for the three robust counterparts and deterministic models.

Obviously, Table 2.11 shows that as the problem size gets bigger, the CPU time becomes higher. Moreover, among the three robust models, the combined interval and polyhedral uncertainty set has the highest computational time due to its large number of variables and constraints. Although the number of variables is slightly smaller in the polyhedral uncertainty set, it shows a higher CPU time comparing to the box uncertainty set because it has a higher number of constraints (i.e. almost two times of the box uncertainty set). Finally, because of the complexity of robust models, the deterministic model always shows the lowest computational time. Figure 2.6 depicts this issue clearly.

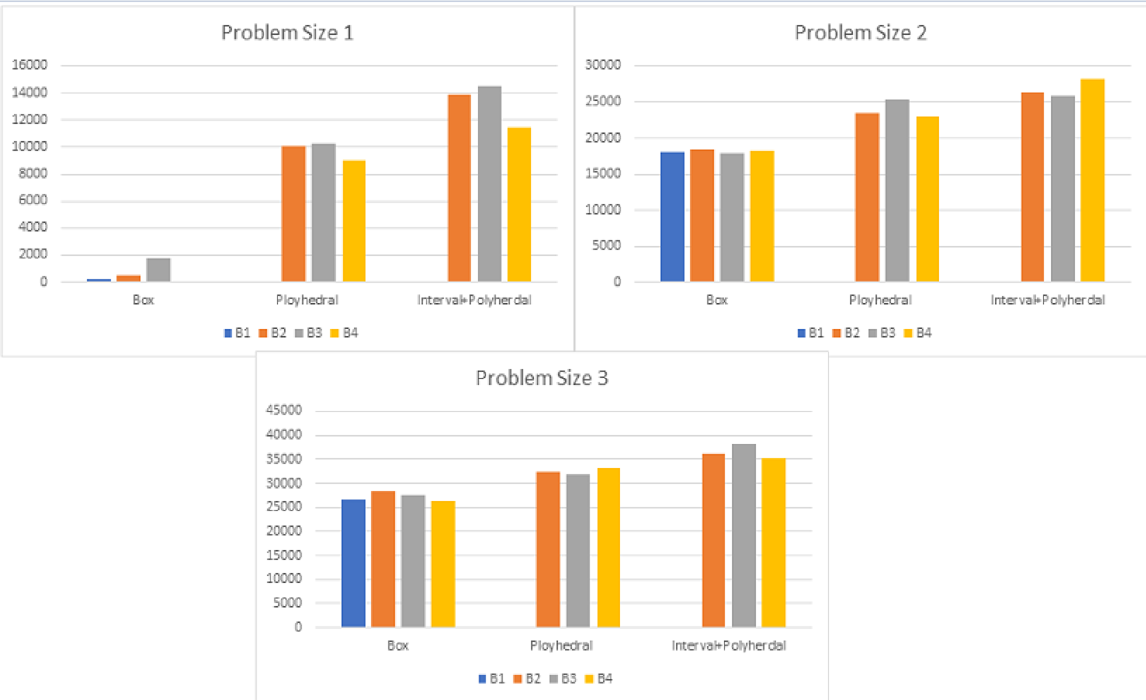


Figure 2.6: The average computational time in seconds (CPU) for the three problems sizes.

Constraint violation ε =0.05

Test Problem	Probability bound	Objective function under the three-uncertainty sets		
		Box	Polyhedral	Interval +Polyhedral
1	B1	6592058	-	-
	B2	8009469	4729108	4544201
	B3	4537171	4685844	4538036
	B4	5046554	4735464	4556286
2	B1	47514058	-	-
	B2	54963465	18902103	17100948
	B3	28226301	18008611	17042753
	B4	32520896	19038609	17128582
3	B1	173098213	-	-
	B2	192446017	67542110	65321472
	B3	109144300	65655012	65278114
	B4	122666354	67851320	65378330

Constraint violation ε =0.1

Test Problem	Probability bound	Objective function under the three-uncertainty sets		
		Box	Polyhedral	Interval +Polyhedral
1	B1	5920070	-	-
	B2	6935649	4716817	4532741
	B3	4440835	4635722	4529285
	B4	4909418	4726066	4547893
2	B1	42426511	-	-
	B2	48363623	18648395	17055214
	B3	27410419	17884535	17011472
	B4	31734072	18829480	17087106
3	B1	156638354	-	-
	B2	173927244	67017146	65320129
	B3	106143139	65365550	65121678
	B4	120144300	67419121	65346831

Constraint violation ε =0.15

Test Problem	Probability bound	Objective function under the three-uncertainty sets		
		Box	Polyhedral	Interval +Polyhedral
1	B1	5525777	-	-
	B2	6342092	4709063	4530214
	B3	4358074	4610032	4521069
	B4	4775322	4719112	4539261
2	B1	39034525	-	-
	B2	44347860	18480642	17031856
	B3	26639880	17806288	16925471
	B4	31207772	18688012	17055987

3	B1	146209120	-	-
	B2	161181194	66676940	65318098
	B3	103617411	65176014	64921648
	B4	118451247	67120520	65328432

Constraint violation ε =0.2

Test Problem	Probability bound	Objective function under the three-uncertainty sets		
		Box	Polyhedral	Interval +Polyhedral
1	B1	5279842	-	-
	B2	5927165	4703231	4523512
	B3	4306092	4606827	4515345
	B4	4740978	4713609	4537736
2	B1	36614050	-	-
	B2	41296110	18350532	16982734
	B3	26016644	17744765	16825694
	B4	30772114	18574604	17015872
3	B1	138494312	-	-
	B2	151772013	66407740	65307263
	B3	101245176	65030806	64910547
	B4	117099157	66859850	65316835

Constraint violation ε =0.25

Test Problem	Probability bound	Objective function under the three-uncertainty sets		
		Box	Polyhedral	Interval +Polyhedral
1	B1	5084526	-	-
	B2	5607536	4698142	4513441
	B3	4263312	4604356	4509232
	B4	4711869	4708867	4529678
2	B1	34638133	-	-
	B2	38734567	18244110	16874382
	B3	25421019	17693080	16647985
	B4	30404775	18477380	16970146
3	B1	131955214	-	-
	B2	144099242	66152348	65274602
	B3	99063534	64907270	64899431
	B4	116007325	66644866	65300292

Table 2.12: The robust solutions under the three uncertainty sets at different constraint violations.

2.7 Discussion and Analysis

In this section we discuss the sensitivity and conservatism of the obtained robust solutions based on the box, polyhedral, and combined interval and polyhedral counterparts formulations. In our discussion, we refer to figures 2.7, 2.8, and 2.9 which explain how the objective functions behave as the probability constraint violations increase for the four different bounds under three test problems. The figures provide to the decision maker an overview of a conservatism comparison between the introduced uncertainty sets under different probability bounds. Note that B1 is not applicable for the case of the polyhedral, and the combined interval and polyhedral uncertainty sets and, therefore it is not included in figures 2.8 and 2.9.

While we compare the size of the different types of uncertainty sets, a conservatism recommendation could be made based on the following fact: the larger the uncertainty set is, the more conservative the solution are obtained. Thus, the model's conservatism increases in the following order: box, polyhedral, (Li et al., 2012). However, this is true if and only if the bounded uncertainty is within the suggested range such that the adjustable uncertainty set parameters are $\Psi_j \leq 1$, and $\Gamma_j \leq |J_j|$ for box and polyhedral uncertainty sets, respectively (Li et al., 2011). Therefore, the robust solution based on the polyhedral uncertainty counterpart is less conservative than the box uncertainty counterpart.

Comparing the combined interval and polyhedral and the polyhedral set based models, the polyhedral model is more conservative since the combined interval and polyhedral set is always inside the polyhedral set with same parameter defining the

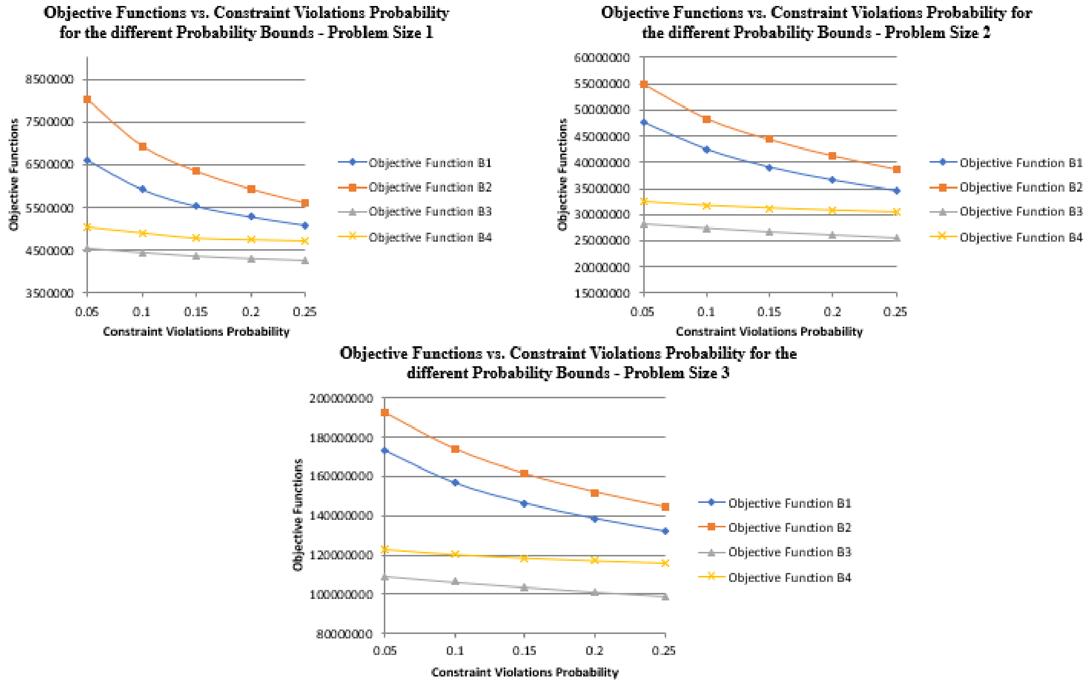


Figure 2.7: The behavior of the robust objective functions when different upper bounds are applied based on box counterpart.

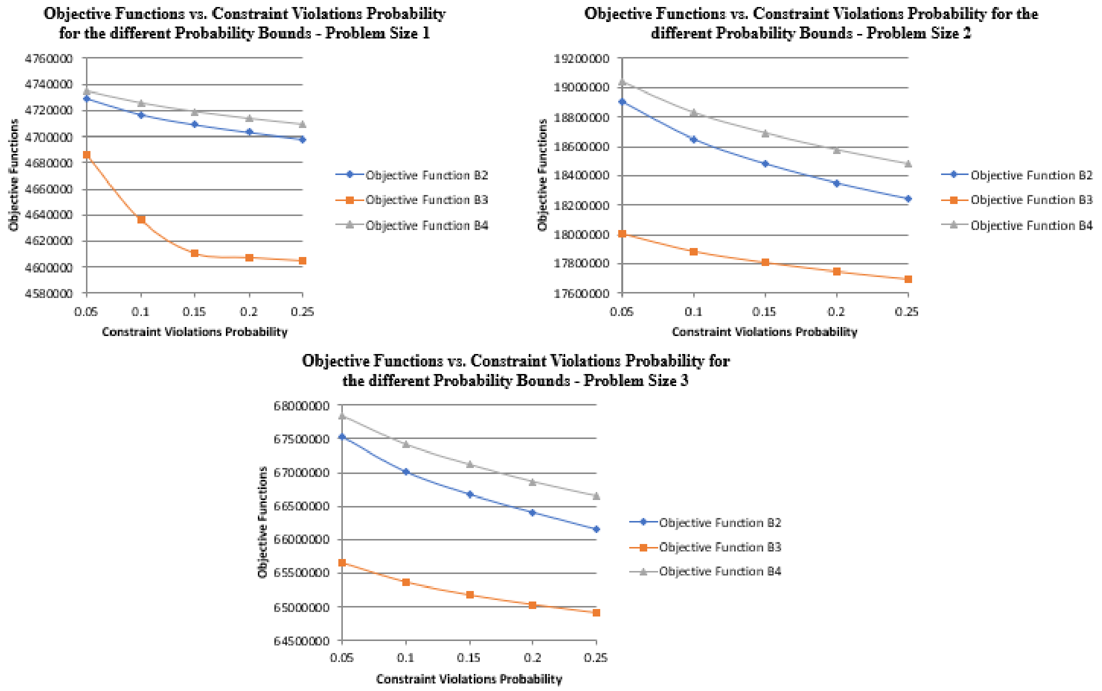


Figure 2.8: The behavior of the robust objective functions when different upper bounds are applied based on polyhedral counterpart.

set. From the results, it can be observed that the solution of the different models is consistent with the above recommendation on robust counterpart optimization models' conservatism. Therefore, we conclude that for our proposed model the robust solutions based on the combined interval and polyhedral is the least conservative and robust solutions.

From figures 2.7, 2.8, and 2.9 we make the following observations:

- In all probability upper bounds as the probability constraint violations increase, the robust objective functions tend to be less conservative. This is valid since we allow for a higher constraint violation, and hence we make the performance of objective function to get improved.
- In all the figures, the robust solution obtained by B3 is the least conservative (and hence the best solution) comparing with the other probability bounds. This would be a better choice due to full probability distribution information. If such information is available, it can be utilized beneficially which makes the solution less conservative.

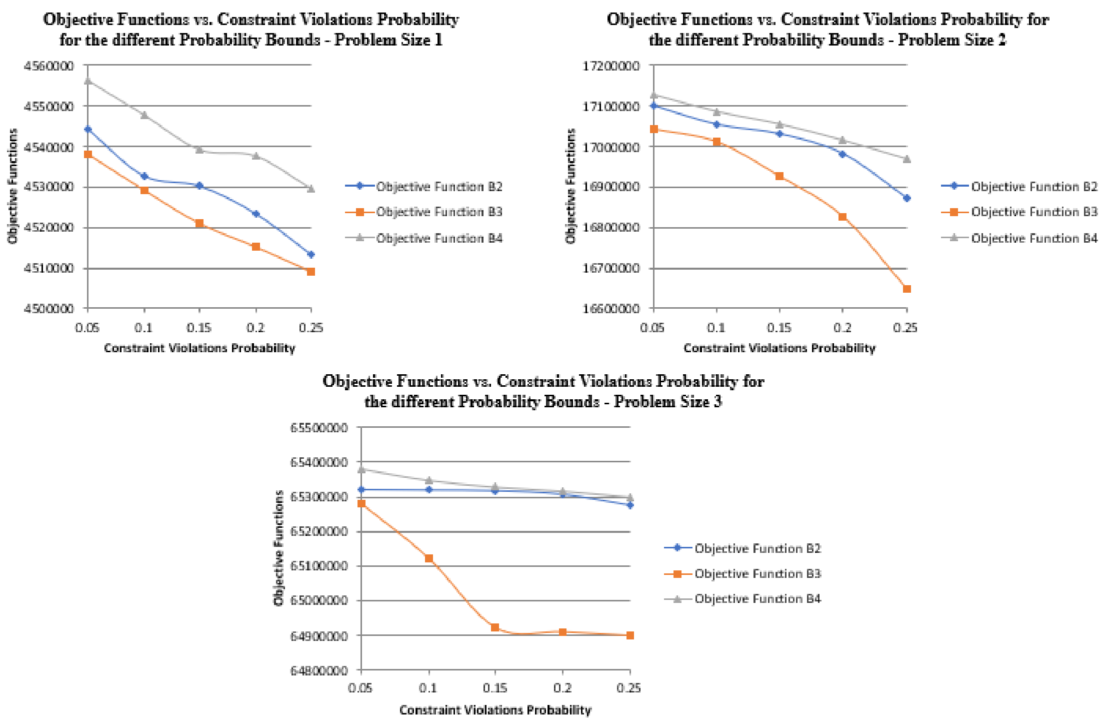


Figure 2.9: The behavior of the robust objective functions when different upper bounds are applied based on the combined interval and polyhedral counterpart.

- In figure 2.7, the robust solution obtained by B1 is less conservative (better solution) comparing with B2 . However, practically B1is not a good probability bound to be applied in the discussed multi periods closed – loop supply chain problem. This is because B1 assumes that the amount of uncertainty, $|J_j|$, is constant over the course of time which contradicts with the nature of the model where the uncertainty increases as the period increases.

- When we compare B2 with B4, we can not reach to a definite conclusion for which one gives the tightest probability bound. As indicated by figures 2.8 and 2.9, the objective functions attained using B2 are better than those attained at B4, since the uncertainty levels are almost lower in B2 (see Tables 2.9 and 2.10) while in the box uncertainty set formulation B4 provides better solution.

To display the impact of model parameters, we perform a sensitivity analysis for deterministic and robust models. As our proposed models have several parameters, our focus is on: shortage, and inventory holing costs. However, the other parameters such as transportation, and processing costs can also be tested, and the models behavior can be easily inferred. Note that the sensitivity analysis is tested over fixed parameters because the uncertain parameters are insensitive to the variation.

For consistency purposes, the robust counterparts models are solved where the constraint violation is set at $\varepsilon = 0.05$, and a priori probability bound, B3, is used. Figures 2.10 and 2.11 depict the sensitivity analysis for the shortage and inventory costs, respectively. Practically, the shortage cost is set relatively high by decision makers

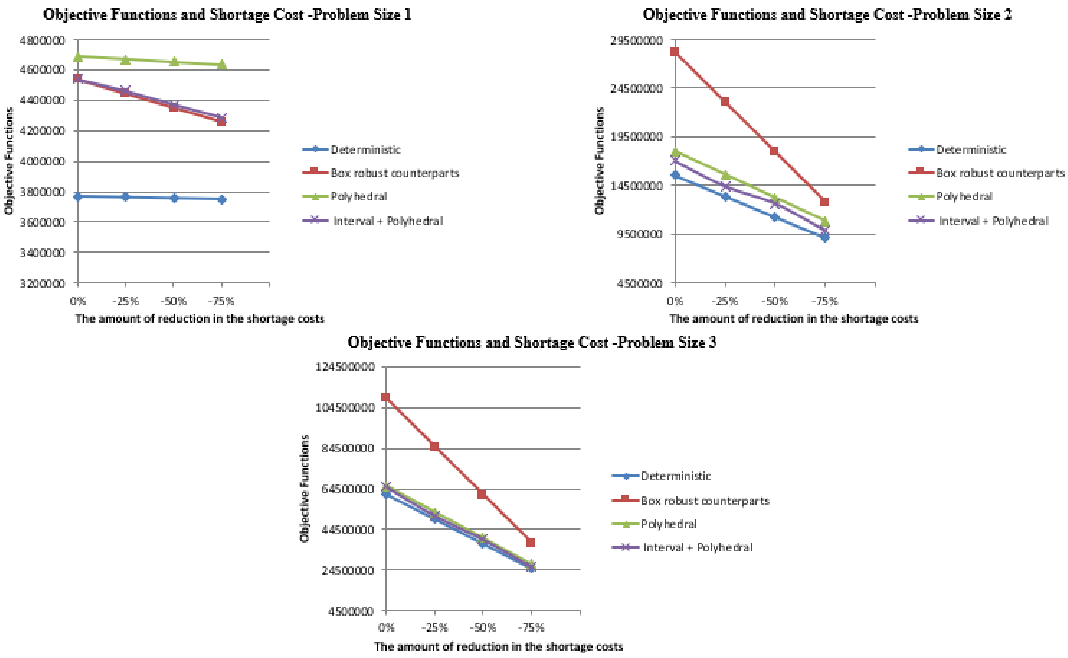


Figure 2.10: Objective function values and shortage costs for deterministic and robust models.

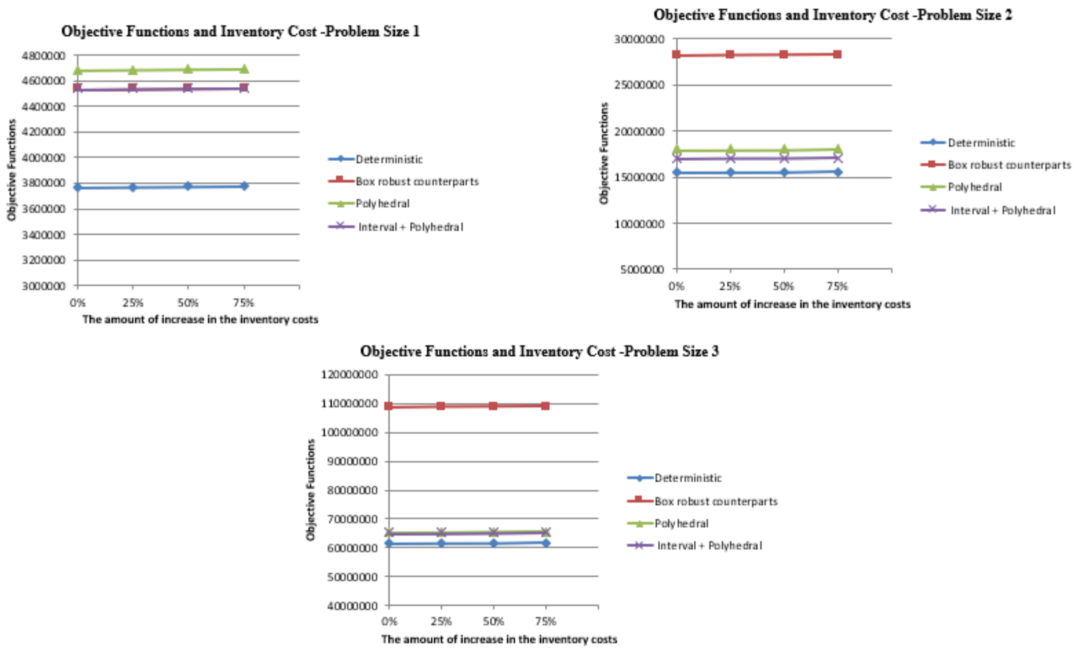


Figure 2.11: Objective function values and inventory costs for deterministic and robust models.

because it may result in loss in goodwill. Figure 2.10 shows a dramatic decrease in the objective function values with a steeper slope as the shortage cost reduces for both deterministic and robust models. For example, when the shortage cost is reduced only by 25% in problem size 3, the average reduction in the robust objective functions is 21% ,and 19.6% reduction in the deterministic objective function. On the other hand, the inventory costs, (h_{pi}, hw_{pl}), which include holding cost of apparent good and returned items respectively, show a slight impact on the objective function values, figure 2.11. As shown in this figure, by increasing the value of inventory costs, the objective function value for all the models increases in an insignificant manner.

2.8 Conclusion

In this chapter we have developed three robust counterparts formulations based on the box, polyhedral, and combined interval and polyhedral uncertainty sets to address our multi-echelon robust closed- loop supply chain under imperfect quality production model. The characteristics of each of the selected uncertainty sets provide the decision maker a flexibility to design his own robust model based on his favorable robustness. For example, if the uncertainty has a bounded distribution (as in our case), then the combined interval and polyhedral uncertainty set give the least conservative solution. However, if he assumes that the uncertainty levels over periods are generally low (i.e. $\Psi_j \leq 1$), then he will implement the box uncertainty set, otherwise he can apply the polyhedral uncertainty set.

Our proposed model is compatible with several types of industries including steel making, electronic and automobile manufacturing, and various plastic products where return products (either defective or used) can be reused as a raw material. Moreover, in this model the imperfect quality production, inspection errors and quality loss function have been taken into consideration to provide meaningful solutions.

In future work, a posteriori probabilistic guarantees approach can be also used to improve the robust solutions. Also, besides to minimizing the total supply chain network costs, the model can consider multiple objective functions under uncertainty, such as minimizing environmental influences and maximizing social benefits. In addition, the market demand can be treated as an uncertain dynamic parameter.

CHAPTER 3: AN INTEGRATED MULTI-ECHELON AND MULTI-OBJECTIVE PROGRAMMING ROBUST CLOSED- LOOP SUPPLY CHAIN UNDER IMPERFECT QUALITY PRODUCTION

In this chapter, we propose a novel robust multi-objective mixed integer linear programming model considering the optimization of three objectives simultaneously. The first objective function minimizes the total cost of the supply chain. The second objective function seeks to minimize the environmental influence, and the third objective function maximizes the social benefits. The augmented weighted Tchebycheff method is used to aggregate the three objective functions into one objective and produce the set of efficient solutions. Robust optimization, based on Mulvey et al. (1995) approach, is used to obtain a set of solutions that are robust against the future fluctuation of parameters. Finally, numerical examples have been presented to test and analyze the tradeoff between solution robustness and model robustness.

3.1 Introduction and Literature Review

The integration of uncertainty is an important topic in the supply chain management. Many researchers and industry practitioners have extensively discussed modeling and solving closed-loop supply chains (CLS) under uncertainty because both the forward and reverse supply chains need to be managed simultaneously. Moreover, the optimal decisions under uncertainty need to be taken in the presence of trade-offs between two or more conflicting objectives to provide meaningful solutions to the current practical problems.

A common assumption of the supply chain model is that the produced items are perfect. However, in real application this does not hold. To address this practical issue, we consider the imperfect quality production modeling scenario. We assume that the screening is not always perfect, and inspection errors are more likely to take place in practice. Thus, some errors are committed in the inspection process. We measure the amount of quality loss as conforming products deviate from the specification (target) value.

There are very few studies which recognize incorporation of the imperfect quality production to the supply chain modelling, (Ahmadi, Khoshalhan, and Glock, 2016; Masoudipour, Amirian, and Sahraeian, 2017; Sana, 2011). These studies consider deterministic models.

Modeling supply chain under uncertainty where imperfect quality production is incorporated is also studied by few researchers. For example, Hu, Zheng, Xu, Ji, and Guo (2010) study coordination of supply chain for the fuzzy random newsboy problem with imperfect quality in the decentralized and centralized systems. Quality uncertainty from a supply chain coordination perspective is addressed by Hwan, Rhee, and Cheng (2013).

One of the most important issues is designing a green supply chain network which guarantees the product delivery from a manufacturer to a customer, or vice versa, in an environmentally friendly manner (Ma et al., 2016). The growing awareness of green supply chain activities aspects is now greatly recognized by academic and industrial communities. Thus, in this study we attempt to address the environmental issues where one of the objective functions aims to minimize carbon emission and environmental waste. Because of environmental concerns many nations devise incentives and penalties to lower their carbon footprints. Particularly, CO_2 and greenhouse gas emissions (GHG) resulting from transportation activities and power generation in supply chains have a significant impact on the global climate change. A survey conducted in 2016 shows that 26% of CO_2 emissions are generated by transportation activities, (U.S. Environmental Protection Agency, 2016)

In recent years, social benefits are widely taking attention besides to environmental factors in the design of CLSC, (Tsao, Thanh, Lu, and Yu, 2017). This new impact dimension considers the number of job opportunities created and hazardous products while minimizing the total supply chain costs.

Modeling the supply chain while the above three objectives are taken into consideration simultaneously (the economic, environmental, and social aspects) is the current research trend in this area. Table 3.1 shows several studies of supply chain optimization under imperfect quality production over the past decade. For the sake of comparison different features are set across each work where mark (×) in

			Objectives Criteria					
Author(s)	CLSC	Imperfect Quality Production	Economic	Environmental	Social	Uncertainty in The Model	Robust Model	Multi-Objective Approach
(Al-e-hashem, Malekly, & Aryanezhad,2011)			*			*	*	LP-metrics method
(Pishvaee & Razmi, 2012)	*		*	*		*		Interactive fuzzy approach
(Datta, 2012)			*	*		*		Heuristics
(Beheshtifar & Alimoahmmadi, 2014)			*		*			-
(Garg, Kannan, Diabat, & Jha, 2015)	*		*	*				Interactive programming approach
(Govindan, Jha, & Garg, 2016)	*		*	*	*			Interactive programming approach
(Ma et al., 2016)	*		*	*		*	*	LP-metrics method
(Pal & Mahapatra, 2017)	*	*	*			*		-
(Masoudipour et al., 2017)	*	*	*					Simple weighted method
(Tsao et al., 2017)			*	*	*	*		Interactive fuzzy approach
(Govindan, Dhingra, Agarwal, & Jha, 2017)	*		*	*		*		Weighted max-min
(Soleimani, Govindan, Saghafi, & Jafari, 2017)	*		*	*		*		E-Constraint
(Puji, Carvalho, & Costa, 2017)	*		*	*				Augmented weighted Tchebycheff
(Imran, Kang & Babar, 2018)			*			*		Interactive fuzzy approach
(Govindan, Jafarian, & Nourbakhsh, 2018)			*	*	*			weighting method
(Ghaderi, Moini, & Pishvaee, 2018)	*		*	*	*	*	*	Interactive fuzzy approach

This paper	*	*	*	*	*	*	*	Augmented weighted Tchebycheff

Table 3.1: Some of the studies in the field of supply chain optimization under imperfect quality production. Mark (*) in this table means that an article in a row has the feature mentioned in that column.

this table means that an article in a row has the feature mentioned in that column. These features include modeling the supply chain with closed-loop (CLSC), incorporating imperfect quality production, multi-criteria optimization considering existence of the uncertainty, and finally a robust framework optimization.

Our proposed model is based on the approach introduced by Mulvey et al. (1995), namely robust stochastic optimization or scenario-based robust approach. Mulvey et al. (1995) extend scenario-based stochastic programming by defining the objective function as a mean-variance function incorporating the risk measures and decision makers' preferences in their model formulation.

An adapted Mulvey approach has been widely used in supply chain for the sake of uncertainty management. In this approach, both solution robustness and model robustness are taken into consideration .Some of these recent studies are (Al-e-hashem, Malekly, and Aryanezhad, 2011; Ma, Yao, Jin, Ren, and Lv, 2016; F. Mohammed et al., 2017; Pishvaee, Rabbani, and Torabi, 2011; Rahmani, Ramezanian, Fattahi, and Heydari, 2013; Safaei, Roozbeh, and Paydar, 2017). The solution obtained by the scenario-based robust model is strongly dependent on the defined scenarios accuracy and their probabilities of occurrence.

The rest of the chapter is organized as follows. Section 3.2 provides the problem definition and mathematical formulation, section 3.3 discusses the robust formulation, section 3.4 introduces the multi-objective solution considering the augmented weighted Tchebycheff method, section 3.5 is about numerical examples and computational results. Finally, section 3.6 concludes the paper.

3.2 Mathematical Formulation

3.2.1 Notation

- The following sets are used:

T Set of periods, with $t \in T$.

S Set of possible supplier center locations, with $s \in S$.

M Set of manufactures centers locations, with $m \in M$.

I Set of potential distribution center locations, with $i \in I$.

C Set of customer zones, with $c \in C$.

L Set of potential collection/disassembly center locations, with $l \in L$

O Set of potential disposal center locations, with $o \in O$.

P Set of products, with $p \in P$.

- Parameters Subjected to Uncertainty:

First Objective Function (f_1): Minimizing the total cost across the supply chain network:

${D^{\zeta}}_{tpc}$: Market demand for product p for customer zone c at period t and scenario ζ.

${R^{\zeta}}_{tpc}$: Returned amount of product p as used items form customer zone c at period t and scenario ζ.

${Rw^{\zeta}}_{tpc}$: Returned amount of product p as defective items form customer zone c at period t and scenario ζ.

${Rc^{\zeta}}_{tpm}$ Recycling cost/unit for product p at manufacturer m and period t for scenario ζ.

${REc^{\zeta}}_{tpm}$: Rework costs for items produced below and above the specification limits for product p at manufacturer m and period t for scenario ζ, respectively.

${e^{\zeta}}_{1t}$: Type I error at period t and scenario ζ.

${e^{\zeta}}_{2t}$: Type II error at period t and scenario ζ.

${\beta^{\zeta}}_{p}$: Disposal fraction of product p and scenario ζ.

Second Objective Function (f_2): Minimizing the environmental Influence Costs:

EMc_{mp}^{ζ}: Environmental impact (CO_2 equivalent emission per unit product) of producing one unit of product p by manufacturer m and scenario ζ.

ERc_{mp}^{ζ}: Environmental impact of recycling one unit of product p by manufacturer m and scenario ζ.

ERW_{mp}^{ζ}: Environmental impact of reworking one unit of product p by manufacturer m and scenario ζ.

EOc_{op}^{ζ}: Environmental impact of handling one unit of product p in disposal center o and scenario ζ.

ETc_{p}^{ζ}: Environmental impact of transporting one unit of product p per km and scenario ζ.

Third and Forth Objective Functions (f_3, f_4): Maximizing the Social Benefits

GD_{i}^{ζ}: Number of job opportunities created for a distribution center i and scenario ζ.

GC_{l}^{ζ}: Number of job opportunities created for a collection center l and scenario ζ.

GO_{o}^{ζ}: Number of job opportunities created for a disposal center o and scenario ζ.

HS_{m}^{ζ}: Average fraction of potentially hazardous products manufactured by plant m and scenario ζ.

- The following fixed parameters are defined:

FS_s: Fixed cost of selecting supplier s.

FD_i: Fixed cost of opening distribution i.

FC_l: Fixed cost of opening collection/disassembly l.

FO_o: Fixed cost of opening disposal o.

Sc_{ps}: Manufacturing cost/unit for product p by the supplier s.

Mc_{pm}: Manufacturing cost/unit for product p by the manufacturer m.

Ic_{pi} : Inspection cost/ unit for product p the distribution center i.

Dc_{pi}: Processing cost/unit of product p at the distribution center i.

Cc_{pl}: Collection cost/unit for the returned product p at the collection center l.

h_{pi}: Holding cost of apparent good items for product p at distribution center i.

hw_{pl}: Holding cost associated with quantity of product p returned from the customer zone to the collection l.

$\hat{\pi}_{pc}$: Shortage (penalty) cost for product p and customer zone c.

W_{pm}: Ordering cost per lot size of product p at manufacturer m.

P_{ps}: Purchasing cost/ unit for product p from supplier s.

Io_{po}: Disposal cost/unit of non-recyclable items of product p at the disposal center o.

Bc_{ms}: Abatement cost of manufacturer m by material from s per unit of product p.

TMc_{psm}: Transportation cost of the raw materials of product p from supplier s to manufacturer m.

TPc_{pmi}: Transportation cost of product p from manufacturer m to distribution center i.

TZc_{pic}: Transportation cost of the product p from distribution center i to customer zone c.

TOc_{pcl}: Transportation cost of product p from the customer zone c to collection center l.

$TOPc_{plm}$: Transportation cost of product p from collection center l to manufacturer m.

TIc_{plo}: Transportation cost of product p from collection center l to disposal center o.

γ_{sm}: The distance between supplier s to manufacturer m generated based on the Euclidean distance.

γ_{mi}: Euclidean distance between manufacturer and distributer.

γ_{ic}: Euclidean distance between distributer and customer zone.

γ_{cl}: Euclidean distance between customer zone and collection center.

γ_{lm}: Euclidean distance between collection center and manufacturer.

γ_{lo}: Euclidean distance between collection center and disposal center.

CS_{ps}: Capacity of raw material of product p for supply center s.

CP_{pm}: Capacity of production for product p in manufacturer m.

CI_{pi}: Capacity of product p in distribution center i.

CL_{pl}: Capacity of product p in collection center l.

CO_{po}: Capacity of product p in disposal center o.

USL_p: Upper specification limit of product p.

LSL_p: Lower specification limit of product p.

K: loss parameter

X_p: Actual value of the quality characteristic of product p.

L(x): Loss of poor quality per unit product.

μ_p : Target quality characteristic of product p.

σ_p : Standard deviation of quality characteristic of product p.

ψ: Deviation from the target value.

- The following decision variables are defined as follows:

QSM_{tpsm}: Quantity of raw material of product p ordered from supplier s to manufacturer m at period t.

QMD_{tpmi}: Quantity of product p sent from manufacturer m to distribution center i at period t.

QDC_{tpic}: Quantity of product p planned to be sent from distribution center i to customer zone c at period t.

QNS_{tpc}: Quantity of non-satisfied demand of product p for customer zone c at period t.

QCO_{tpcl}: Quantity of product p returned from customer zone c to collection center l at period t.

QRP_{tplm}: Quantity of recyclable product p shipped from collection center l to manufacturer m at period t.

QEP_{tplm}: Quantity of reworkable product p shipped from collection center l to manufacturer m at period t.

QIP_{tplo}: Quantity of disposal product p shipped from collection center l to disposal center o at period t.

v_{tpsm}: 1 if the order of product p is placed by manufacturer m at period t and 0 otherwise.

S_{ts}: 1 if a supplier is selected at location s at period t , 0 otherwise.

DT_{ti}: 1 if a distribution is opened at location i at period t, 0 otherwise.

CT_{tl}: 1 if a collection/disassembly is opened at location l at period t, 0 otherwise.

DO_{to}: 1 if a disposal is opened at location o at period t, 0 otherwise.

The objective function, f_1 minimizes the total cost of the supply chain network. The included costs are:

- Facility opening costs:

$$\sum_{t\in T}\sum_{s\in S} FS_s S_{ts} + \sum_{t\in T}\sum_{i\in I} FD_i DT_{ti} + \sum_{t\in T}\sum_{l\in L} FC_l CL_{tl} + \sum_{t\in T}\sum_{o\in O} FO_o DO_{to}$$

- Purchasing cost:

$$\sum_{t\in T}\sum_{p\in P}\sum_{s\in S}\sum_{m\in M} W_{pm} v_{tpsm} + P.QSM_{tpsm}$$

- Ordering costs:

$$\sum_{t\in T}\sum_{p\in P}\sum_{s\in S}\sum_{m\in M} QSM_{tpsm}\left(Sc_{ps} + TMc_{psm}\right)$$

- Cost incurred in the manufacturers:

$$\sum_{t\in T}\sum_{p\in P}\sum_{m\in M}\sum_{i\in I} QMD_{tpmi}\left(Mc_{pm} + TPc_{pmi} + Ic_{pi} + d_t^{\zeta} PR_p\right)$$

- Cost incurred in the distributor centers:

$$\sum_{t\in T}\sum_{p\in P}\sum_{i\in I}\sum_{c\in C} QDC_{tpic}\left(Dc_{pi} + TZc_{pic} + h_{pi} + F_p(x_p;\mu_p)\right)$$

- Cost incurred in the collection centers:

$$\sum_{t\in T}\sum_{p\in P}\sum_{c\in C}\sum_{l\in L} QCO_{tpcl}(Cc_{pl} + TOc_{pcl}) + \sum_{t\in T}\sum_{p\in P}\sum_{c\in C}\sum_{l\in L} QCO_{tpcl}hw_{pl}$$

- Costs related to recycling and reworking respectively:

$$\sum_{t\in T}\sum_{p\in P}\sum_{l\in L}\sum_{m\in M} QRP_{tplm}(Rc^{\zeta}{}_{tpm} + TOPc_{plm}) + \sum_{t\in T}\sum_{p\in P}\sum_{l\in L}\sum_{m\in M} QEP_{tplm}(REc^{\zeta}{}_{tpm} + TOPc_{plm})$$

- Cost incurred in the disposal center:

$$\sum_{t\in T}\sum_{p\in P}\sum_{l\in L}\sum_{o\in O} QIP_{tplo}(Io_{po} + TIc_{plo})$$

- Shortage cost:

$$\sum_{t\in T}\sum_{p\in P}\sum_{c\in C} QNS_{tpc}\hat{\pi}_{pc}$$

- Abatement cost:

$$\sum_{t\in T}\sum_{s\in S}\sum_{m\in M} Bc_{ms}S_{ts}$$

The objective function, f_2 minimizes the environmental impact. The included impacts are represented in Figure 3.1 and defined as follows:

- Suppliers:

$$\sum_{t\in T}\sum_{p\in P}\sum_{s\in S}\sum_{m\in M} ETc_p^{\zeta}\gamma_{sm}QSM_{tpsm}$$

- Manufacturers:

$$\sum_{t\in T}\sum_{p\in P}\sum_{m\in M}\sum_{i\in I}(EMc_{mp}^{\zeta} + ETc_{p}^{\zeta}\,\gamma_{mi})QMD_{tpmi}$$

- Destitution Centers:

$$\sum_{t\in T}\sum_{p\in P}\sum_{i\in I}\sum_{c\in C} QDC_{tpic}\widetilde{ET}c_p\gamma_{ic}$$

- Collection centers:

$$\sum_{t\in T}\sum_{p\in P}\sum_{l\in L}\sum_{m\in M}(ERc_{mp}^{\zeta} + ETc_{p}^{\zeta}\gamma_{lm})QRP_{tplm} + \sum_{t\in T}\sum_{p\in P}\sum_{l\in L}\sum_{m\in M}(ERW_{mp}^{\zeta} + ETc_{p}^{\zeta}\gamma_{lm})QEP_{tplm}$$

- Disposal Centers:

$$\sum_{t\in T}\sum_{p\in P}\sum_{l\in L}\sum_{o\in O}\left(EOc_{op}^{\zeta} + ETc_{p}^{\zeta}\gamma_{lo}\right)QIP_{tplo}$$

The objective functions, f_3 and f_4, maximize the social benefits and their terms are defined as follows:

- The number of jobs created in the distributions, collections and disposals centers, respectively:

$$\sum_{t\in T}\sum_{i\in I} GD_{i}^{\zeta}DT_{ti} + \sum_{t\in T}\sum_{l\in L} GC_{l}^{\zeta}CL_{tl} + \sum_{t\in T}\sum_{o\in O} GO_{o}^{\zeta}DO_{to}$$

- Average fraction of potentially hazardous products manufactured:

$$\sum_{t\in T}\sum_{s\in S} HS_{s}^{\zeta}S_{ts}$$

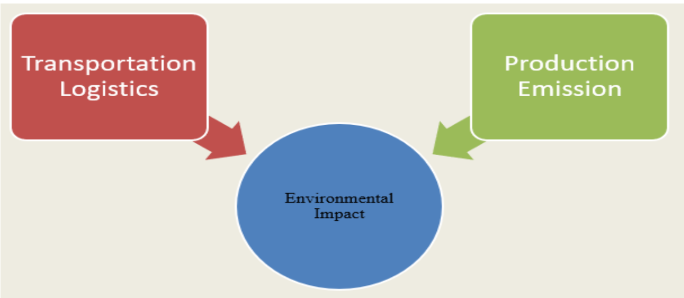


Figure 3.1: Breakdown of the environmental impacts.

3.2.2 The Multi-Objectives MILP Model

Thus, the multi-Objectives MILP model of proposed closed-loop supply chain is:

$$\boldsymbol{Minimize\ f_1} = \sum_{t\in T}\sum_{s\in S} FS_s S_{ts} + \sum_{t\in T}\sum_{i\in I} FD_i DT_{ti} + \sum_{t\in T}\sum_{l\in L} FC_l CT_{tl}$$

$$+ \sum_{t\in T}\sum_{o\in O} FO_o DO_{to} + \sum_{t\in T}\sum_{p\in P}\sum_{s\in S}\sum_{m\in M} W_{pm} v_{tpsm} + P_{ps}.QSM_{tpsm}$$

$$+ \sum_{t\in T}\sum_{p\in P}\sum_{s\in S}\sum_{m\in M} QSM_{tpsm}\left(Sc_{ps} + TMc_{psm}\right)$$

$$+ \sum_{t\in T}\sum_{p\in P}\sum_{m\in M}\sum_{i\in I} QMD_{tpmi}\left(Mc_{pm} + TPc_{pmi} + Ic_{pi} + d_t^{\zeta} PR_p\right)$$

$$+ \sum_{t\in T}\sum_{p\in P}\sum_{i\in I}\sum_{c\in C} QDC_{tpic}\left(Dc_{pi} + TZc_{pic} + h_{pi} + F_p\left(x_p;\mu_p\right)\right)$$

$$+ \sum_{t\in T}\sum_{p\in P}\sum_{c\in C}\sum_{l\in L} QCO_{tpcl}\left(Cc_{pl} + TOc_{pcl}\right)$$

$$+ \sum_{t\in T}\sum_{p\in P}\sum_{l\in L}\sum_{m\in M} QRP_{tplm}\left(Rc^{\zeta}{}_{tpm} + TOPc_{plm}\right)$$

$$+ \sum_{t\in T}\sum_{p\in P}\sum_{l\in L}\sum_{m\in M} QEP_{tplm}\left(REc^{\zeta}{}_{tpm} + TOPc_{plm}\right)$$

$$+ \sum_{t\in T}\sum_{p\in P}\sum_{c\in C}\sum_{l\in L} QCO_{tpcl} hw_{pl}$$

$$+ \sum_{t\in T}\sum_{p\in P}\sum_{l\in L}\sum_{o\in O} QIP_{tplo}\left(Io_{po} + TIc_{plo}\right) + \sum_{t\in T}\sum_{p\in P}\sum_{c\in C} QNS_{tpc}\hat{\pi}_{pc}$$

$$+ \sum_{t\in T}\sum_{s\in S}\sum_{m\in M} Bc_{ms} S_{ts}$$

$$\boldsymbol{Minimize f_2} = \sum_{t\in T}\sum_{p\in P}\sum_{s\in S}\sum_{m\in M} ETc_p^{\zeta}\gamma_{sm}QSM_{tpsm}$$
$$+\sum_{t\in T}\sum_{p\in P}\sum_{m\in M}\sum_{i\in I}(EMc_{mp}^{\zeta} + ETc_p^{\zeta}\,\gamma_{mi})QMD_{tpmi}$$
$$+\sum_{t\in T}\sum_{p\in P}\sum_{i\in I}\sum_{c\in C} QDC_{tpic}ETc_p^{\zeta}\gamma_{ic}$$
$$+\sum_{t\in T}\sum_{p\in P}\sum_{l\in L}\sum_{m\in M}(ERc_{mp}^{\zeta} + ETc_p^{\zeta}\gamma_{lm})QRP_{tplm}$$
$$+\sum_{t\in T}\sum_{p\in P}\sum_{l\in L}\sum_{m\in M}(ERW_{mp}^{\zeta} + ETc_p^{\zeta}\gamma_{lm})QEP_{tplm}$$
$$+\sum_{t\in T}\sum_{p\in P}\sum_{c\in C}\sum_{l\in L} ETc_p^{\zeta}\gamma_{cl}\,QCO_{tpcl}$$
$$+\sum_{t\in T}\sum_{p\in P}\sum_{l\in L}\sum_{o\in O}(EOc_{op}^{\zeta} + ETc_p^{\zeta}\,\gamma_{lo})QIP_{tplo}$$

$$\boldsymbol{Maximize\ f_3} = \sum_{t\in T}\sum_{i\in I} GD_i^{\zeta}DT_{ti} + \sum_{t\in T}\sum_{l\in L} GC_l^{\zeta}\,CL_{tl} + \sum_{t\in T}\sum_{o\in O} GO_p^{\zeta}\,DO_{to}$$

$$\boldsymbol{Maximize\ f_4} = -\sum_{t\in T}\sum_{p\in P}\sum_{m\in M}\sum_{i\in I} HS_m^{\zeta}\,QMD_{tpmi} \tag{3.1}$$

Subject to:

$$\sum_{i\in I} QDC_{tpic} + QNS_{tpc} \geq D_{tpc}^{\zeta}, \qquad \forall\, t\in T, p\in P, c\in C, \zeta \tag{3.2}$$

$$\sum_{l\in L} QCO_{tpcl} \leq\ R_{tpc}^{\zeta} + Rw_{tpc}^{\zeta}, \forall\, t\in T, p\in P, c\in C, \zeta \tag{3.3}$$

$$\sum_{m\in M} QMD_{tpmi}\left(1 - d_t^{\zeta}\right) \geq \sum_{c\in C} QDC_{tpic}, \forall\, t\in T, p\in P, i\in I, \zeta \tag{3.4}$$

$$\sum_{c\in C}\sum_{p\in P} \beta_p^{\zeta}.\,QCO_{tpcl} \leq \sum_{o\in O}\sum_{p\in P} QIP_{tplo}, \forall\, t\in T, l\in L, \zeta \tag{3.5}$$

$$\sum_{o\in O} QIP_{tplo} + \sum_{m\in M} QRP_{tplm} + \sum_{m\in M} QEP_{tplm} = \sum_{c\in C} QCO_{tpcl}, \forall\, t\in T, p\in P, l \in L \tag{3.6}$$

$$\sum_{s\in S}\sum_{p\in P} QSM_{tpsm} + \sum_{l\in L}\sum_{p\in P} QRP_{tplm} + \sum_{l\in L}\sum_{p\in P} QEP_{tplm} = \sum_{i\in I}\sum_{p\in P} QMD_{tpmi}, \forall\, t \in T, m\in M \tag{3.7}$$

$$\sum_{s \in S} QSM_{tpsm} \leq B.\, v_{tpsm}, \forall\, t \in T, p \in P, s \in S, m \in M \tag{3.8}$$

$$\sum_{m \in M} QSM_{tpsm} \leq CS_{ps} S_{ts}, \forall\, t \in T, p \in P, s \in S \tag{3.9}$$

$$\sum_{i \in I} QMD_{tpmi} \leq CP_{pm}, \forall\, t \in T, p \in P, m \in M \tag{3.10}$$

$$\sum_{m \in M} QMD_{tpmi} \leq CI_{pi} DT_{ti}, \forall\, t \in T, p \in P, i \in I \tag{3.11}$$

$$\sum_{c \in C} QCO_{tpcl} \leq CL_{pl} CT_{tl}, \forall\, t \in T, p \in P, l \in L \tag{3.12}$$

$$\sum_{l \in L} QIP_{tplo} \leq CS_{po} DO_{to}, \forall\, t \in T, p \in P, o \in O \tag{3.13}$$

$$v_{tsm}, S_{ts}, DT_{ti}, CT_{tl}, DO_{to} \in \{0,1\} \quad \forall t, p, s, m, i, l, o \tag{3.14}$$

Non-negativity constraints (3.15)

Constraint (3.2) ensures the customer demand satisfaction. Constraint (3.3) states that the returned items are not all necessarily collected from the customer zones. Constraint (3.4) makes sure that the apparent produced good items quantity is larger than the quantity transported to the customer zone. Constraint (3.5) limits the quantity of disposed products shipped from the collection centers. Constraints (3.6) and (3.7) confirm the movement equilibrium between all the echelons. Constraint (3.8) assigns cost whenever the order is placed .Constraints (3.9-3.13) are based on capacity restriction for the facilities.

Two types of errors are committed in the inspection process. Type I error, $\boldsymbol{e}_1^{\zeta}$, is committed when a conforming item is classified as non-conforming and Type II error, $\boldsymbol{e}_2^{\zeta}$, is committed when a non-conforming item is classified as conforming. The apparent conforming items fraction can be determined as follows:

$$(1-\boldsymbol{d})(1-\boldsymbol{e}_1^{\zeta}) + \boldsymbol{d}\, \boldsymbol{e}_2^{\zeta} = 1 - \boldsymbol{e}_1^{\zeta} - \boldsymbol{d}(1 - \boldsymbol{e}_1^{\zeta} - \boldsymbol{e}_2^{\zeta}) = 1 - \boldsymbol{d}^{\zeta}, \text{ with } 0 \leq \boldsymbol{d}^{\zeta} \leq 1$$

where,

$$\boldsymbol{d}^{\zeta} = \boldsymbol{e}_1^{\zeta} + \boldsymbol{d}(1 - \boldsymbol{e}_1^{\zeta} - \boldsymbol{e}_2^{\zeta}), \tag{3.16}$$

and the vectors $\boldsymbol{e}_1^{\zeta}$ and $\boldsymbol{e}_2^{\zeta}$ are both uncertain.

The quality loss function is proposed by Taguchi (1986). It states that for given specification limits LSL, USL not all values falling within them are equal and create equal loss because of poor quality. Quality loss function $L(x)$ is defined as follows:

$$L(x) = K(x-\mu)^2 \qquad LSL \leq x \leq USL$$

The quadratic loss function indicates that if the difference between the actual and the target value is large, the loss would be more where K is the loss parameter,

$$K = \frac{V}{\psi^2}$$

and,

$$\psi = (USL - \mu) = (\mu - LSL)$$

Thus, the amount of loss is expressed as follows:

$$QDC_{tpic} \int_{LSL}^{USL} \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}} K(x-\mu)^2 dx = QDC_{tpic} \left(1 - d_t^{\zeta}\right) F(x;\mu) \qquad (3.17)$$

Equation (3.17) states that the apparent conforming quantity of product p planned to be sent from distribution center i to customer zone c at period t is subject to an inspection to ensure that the produced lot is close enough to the target value according to Taguchi Quality approach. This loss is included in the objective function, f_1 under the cost incurred in the distribution centers. Next, we describe the robust optimization formulation.

3.3 Robust Formulation:

The robust framework introduced by Mulvey et al. (1995) addresses two types of robustness: solution robustness which means that the solution remains nearly optimal under all realizations (scenarios), and model robustness which refers to the solution feasibility under all realizations. This approach of robust optimization is an extension of stochastic programming (scenario-based method) where the cost variability is addressed instead of minimizing /maximizing the expected value of the objective function.

3.3.1 Preliminaries

Consider the following linear programming with uncertain parameters:

$$\min_{,x,y\geq 0}\left\{ c_x^T x + c_y^T y: Ax \leq b, Bx + Dy \leq e, \forall\zeta = [B, D, e] \in \mathcal{Z}\right\}$$

(3.18)

where $\boldsymbol{x}$ is the vector of decision variables determined under the uncertainty of model parameters denoted by $\boldsymbol{B}, \boldsymbol{D},$ and $\boldsymbol{e}$, respectively. $\mathcal{Z}$ is assumed a finite scenario set, with $\mathcal{Z} = \{1, 2, \ldots, \zeta\}$. Thus, we associate a scenario $\zeta \in \mathcal{Z}$ to model the uncertain parameters, $[\boldsymbol{B}^\zeta, \boldsymbol{D}^\zeta, \boldsymbol{e}^\zeta]$, where the probabilities of the scenarios $\sum_\zeta \rho_\zeta = 1$. The above model is a general case where $\boldsymbol{y}$ denotes a vector of control variables which are determined and adjusted after the realization of the uncertain parameters. Thus, $\boldsymbol{y}$ can be represented by $\boldsymbol{y}_\zeta$ for each scenario.

Due to the parameters uncertainty, the model infeasibility may occur at some scenarios. Therefore, the uncertainty amount under scenario ζ can be represented by δ_ζ, where $\delta_\zeta > 0$ indicates an infeasible model, and 0 otherwise. Model (3.18) becomes,

$$\min_{x,y_\zeta,\delta_\zeta\geq 0}\left\{\sum_\zeta \rho_\zeta f^\zeta + \omega \sum_\zeta \rho_\zeta \delta_\zeta : Ax \leq b, B^\zeta x + D^\zeta y \; + \; \delta_\zeta \leq e^\zeta, \forall\zeta = [B, D, e] \in \mathcal{Z}\right\} \qquad (3.19)$$

In model (3.19), the first term in the objective function refers to solution robustness while the second term presents the model robustness which penalizes the infeasibility in the model by the infeasibility parameter ω. The infeasibility is resulted from the constraint violations. In other words, a low change of the uncertain parameters values can cause a high change in the objective function.

To represent solution robustness, Mulvey et al. (1995) develop the following formulation:

$$Z = \sum_\zeta \rho_\zeta f^\zeta + \lambda \sum_\zeta \rho_\zeta \left(f^\zeta - \sum_\zeta \rho_\zeta f^\zeta \right)^2 \qquad (3.20)$$

where λ is the weighting scale to measure the tradeoff between sensitivity and robustness (i.e. if λ is a relatively high, the model becomes insensitive to the uncertain model variation). Because of the quadratic term in equation (3.20), the issue of computational complexity arises. Yu and Li (2000) proposed an absolute deviation instead of the quadratic term as shown in the following formulation:

$$Z = \sum_{\zeta} \rho_{\zeta} f^{\zeta} + \lambda \sum_{\zeta} \rho_{\zeta} \left| f^{\zeta} - \sum_{\zeta} \rho_{\zeta} f^{\zeta} \right| \tag{3.21}$$

As can be seen, there is a nonlinear term in equation (3.21) denoted by the absolution deviation term. However, the above formulation can be optimized through converting this term into linear by introducing two non-negative deviational variables. Yu and Li (2000) extend equation (3.20) as follows:

$$Z = min \sum_{\zeta} \rho_{\zeta} f^{\zeta} + \lambda \sum_{\zeta} \rho_{\zeta} \left[\left(f^{\zeta} - \sum_{\zeta} \rho_{\zeta} f^{\zeta} \right) + 2\theta_{\zeta} \right] \tag{3.22}$$

Subject to

$$f^{\zeta} - \sum_{\zeta} \rho_{\zeta} f^{\zeta} + \theta_{\zeta} \geq 0, \qquad \forall \zeta \tag{3.23}$$

$$\theta_{\zeta} \geq 0, \qquad \forall \zeta \tag{3.24}$$

where the following relation can be interpreted as follows:

$$\theta_{\zeta} = \begin{cases} 0, & f^{\zeta} \geq \sum_{\zeta} \rho_{\zeta} f^{\zeta} \\ \sum_{\zeta} \rho_{\zeta} f^{\zeta} - f^{\zeta}, & otherwise \end{cases} \tag{3.25}$$

Finally, the trade-off between solution robustness measured from the first term in equation (3.22) and model robustness measured from the penalty term, the weight ω, is included as follows:

$$Z = min \sum_{\zeta} \rho_{\zeta} f^{\zeta} + \lambda \sum_{\zeta} \rho_{\zeta} \left[\left(f^{\zeta} - \sum_{\zeta} \rho_{\zeta} f^{\zeta} \right) + 2\theta_{\zeta} \right] + \omega \sum_{\zeta} \rho_{\zeta} \delta_{\zeta} \tag{3.26}$$

Subject to constraints (3.22) and (3.24), which presents the extended Mulvey et al. (1995) approach of robust optimization.

3.3.2 Robust Model Formulation

According to the previous discussion, our novel multi-objective robust optimization model is based on the extended Mulvey's approach where the uncertainty is expressed through a set of discrete scenarios (ζ):

$$Z_1 = \ min \sum_{\zeta} \rho_\zeta {f_1}^{\zeta} + \lambda_1 \sum_{\zeta} \rho_\zeta \left[\left({f_1}^{\zeta} - \sum_{\zeta} \rho_\zeta \, {f_1}^{\zeta} \right) + 2\theta_{1\zeta} \right] + \omega \left[\sum_{\zeta,t,p,c} \rho_\zeta \delta^{D}_{tpc\zeta} + \sum_{\zeta,t,p,c} \rho_\zeta \delta^{R,Rw}_{tpc\zeta} + \sum_{\zeta,t} \rho_\zeta \delta^{d}_{t\zeta} + \sum_{\zeta,p} \rho_\zeta \delta^{\beta}_{p\zeta} \right]$$

$$Z_2 = \ min \sum_{\zeta} \rho_\zeta {f_2}^{\zeta} + \lambda_2 \sum_{\zeta} \rho_\zeta \left[\left({f_2}^{\zeta} - \sum_{\zeta} \rho_\zeta \, {f_2}^{\zeta} \right) + 2\theta_{2\zeta} \right]$$

$$Z_3 = \ max \sum_{\zeta} \rho_\zeta {f_3}^{\zeta} - \lambda_3 \sum_{\zeta} \rho_\zeta \left[\left({f_3}^{\zeta} - \sum_{\zeta} \rho_\zeta \, {f_3}^{\zeta} \right) + 2\theta_{3\zeta} \right]$$

$$Z_4 = \ max \sum_{\zeta} \rho_\zeta {f_4}^{\zeta} - \lambda_4 \sum_{\zeta} \rho_\zeta \left[\left({f_4}^{\zeta} - \sum_{\zeta} \rho_\zeta \, {f_4}^{\zeta} \right) + 2\theta_{4\zeta} \right]$$

Subject to:

$${f_1}^{\zeta} - \sum_{\zeta} \rho_\zeta {f_1}^{\zeta} + \theta_{1\zeta} \geq 0, \qquad \forall \zeta \tag{3.27}$$

$${f_2}^{\zeta} - \sum_{\zeta} \rho_\zeta {f_2}^{\zeta} + \theta_{2\zeta} \geq 0, \qquad \forall \zeta \tag{3.28}$$

$${f_3}^{\zeta} - \sum_{\zeta} \rho_\zeta {f_3}^{\zeta} + \theta_{3\zeta} \geq 0, \qquad \forall \zeta \tag{3.29}$$

$${f_4}^{\zeta} - \sum_{\zeta} \rho_\zeta {f_4}^{\zeta} + \theta_{4\zeta} \geq 0, \qquad \forall \zeta \tag{3.30}$$

$$\sum_{i \in I} QDC_{tpic} + QNS_{tpc} \geq {D^{\zeta}}_{tpc} - \delta^{D}_{tpc\zeta}, \qquad \forall\, t \in T, p \in P, c \in C, \zeta \tag{3.31}$$

$$\sum_{l \in L} QCO_{tpcl} \leq \ {R^{\zeta}}_{tpc} + {Rw^{\zeta}}_{tpc} + \delta^{R,Rw}_{tpc\zeta}, \qquad \forall\, t \in T, p \in P, c \in C, \zeta \tag{3.32}$$

$\sum_{m \in M} QMD_{tpmi} \left(1 - {d^{\zeta}}_t\right) \geq \sum_{c \in C} QDC_{tpic} - \delta^{d}_{t\zeta}, \ \forall\, t \in T, p \in P, i \in I, \zeta$ (3.33)

$$\sum_{c \in C} \sum_{p \in P} \beta^{\zeta}_{p}. QCO_{tpcl} \leq \sum_{o \in O} \sum_{p \in P} (QIP_{tplo} + \delta^{\beta}_{p\zeta})\,, \forall\, t \in T, l \in L, \zeta \tag{3.34}$$

$\theta_\zeta \geq 0, \ \forall \zeta$ (3.35)

Constraints (3.6) – (3.15)

The objective functions set, $[Z_1, Z_2, Z_3, Z_4]$ are the robust formulations of the original objective functions set given in model (3.1-3.15), respectively. The non- negative decision variables vector $\grave{\boldsymbol{\theta}}_{\boldsymbol{\zeta}} = [\theta_{1\zeta}, \theta_{2\zeta}, \theta_{3\zeta}, \theta_{4\zeta}]$, are described by constraints (3.27-3.30) according to the relation given in (3.25). Because of uncertain parameters, the model infeasibility may occur at some scenarios, ζ. Therefore, constraints (3.31-3.34) are included to consider any potential violations. Next, we discuss the application of the augmented weighted Tchebycheff method in our robust optimization model.

3.4 Multi-Objective Solution Approach: The Augmented Weighted Tchebycheff Method

The augmented weighted Tchebycheff is a special case of compromise programming performed through scalarization. Thus, the multi-objective optimization problem is converted into a single objective with some parameters. However, the limitation of other scalarization methods (i.e. methods with a priori articulation of preferences) such as the weighted sum method is that it can not reach to solutions in non-convex regions of the Pareto-optimal frontier. A solution is called Pareto-optimal if there are no other solutions that dominates it, and therefore none of the objectives can be improved without deteriorating at least one of the other objectives. Moreover, weighted Tchebycheff method has a limitation which does not guarantee that all solutions obtained are Pareto, and therefore the augmented weighted Tchebycheff approach is used.

The use of augmentation terms is to avoid weakly nondominated points and allows to handle non-convexity of the Pareto-optimal frontier. Miettinen, Makela, and Kaario (2006) study through an experimental comparison of methods with or without augmentation terms and they conclude that the methods with augmentation term significantly outperform equivalent methods without such a term with respect to computational costs. We refer to the work of (Steuer & Choo, 1983) to formulate our multi-objective model according to the augmented weighted Tchebycheff approach. The solution methodology can be outlined as follows:

Step (1): the multi-objective robust method based on extended Mulvey approach is formulated and provided by (3.6)- (3.15) and (3.27)- (3.35).

Step(2): set the vectors of solution robustness, $\boldsymbol{\lambda}$, and model robustness, $\boldsymbol{\omega}$, as follows:

$$\lambda = \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \end{bmatrix}, \quad \omega = \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \\ \omega_4 \end{bmatrix}$$

Step (3): solve $Z_i,\ \forall i = 1,\ 2,\ 3,\ 4$ relative to its constraints independently to obtain the optimal values, Z_i^* (a utopia point) corresponding to each objective function. The utopia vector $\boldsymbol{Z}^*$, is described as follows:

$$\boldsymbol{Z}^* = [Z_1^*,\ Z_2^*,\ Z_3^*,\ Z_4^*]^T$$

Step (4): apply the augmented weighted Tchebycheff method:

$$min\ \nu + \tau \textstyle\sum_{i=1}^{4} [Z_i - Z_i^*]$$

Subject to

$$w_1(Z_1 - Z_1^*) \leq \nu$$

$$w_2(Z_2 - Z_2^*) \leq \nu$$

$$w_3(Z_3 - Z_3^*) \geq \nu$$

$$w_4(Z_4 - Z_4^*) \geq \nu$$

(3.6)- (3.15) and (3.27)- (3.35).

where τ is a small positive number roughly $(0.001 \leq \tau \leq 0.01)$.

Step(5): solve the model of step(4) with different weight combinations which are generated randomly using uniform distribution (URG):

$$w_1 = URG(0,1)$$

$$w_2 = URG(0,1 - w_1)$$

$$w_3 = URG(0,1 - (w_1 + w_2))$$

$$w_4 = 1 - (w_1 + w_2 + w_3)$$

where $w_1 + w_2 + w_3 + w_4 = 1$

Step(6): report the efficient solutions obtained by step (5). Adjust $\boldsymbol{\lambda}$, and $\boldsymbol{\omega}$ from step(2) as needed, otherwise stop. Note that the vectors $\boldsymbol{\lambda}$, and $\boldsymbol{\omega}$ are selected such that $\boldsymbol{\theta_\zeta}$ and $\boldsymbol{\delta_\zeta}$ are minimum, respectively.

An appropriate choice of the parameter, τ, is critical when the complete set of nondominated solutions has to be obtained. If τ is too small, this may cause numerical issues because the augmentation term weight in the objective function may lose significance with respect to the primary objective. On the other hand, selecting τ to be very large may result in the situation that some of the nondominated points are not reachable. The proof is available in (Dachert, Gorski, & Klamroth, 2012).

3.5 Numerical Example and Computational Results

In this section, we illustrate the application of our novel multi-objective robust optimization model. The size of our artificial numerical example is explained next. The closed-loop supply chain system consists of 12 periods, and 3 products, where the network is managed by 3 manufacturers. The required quantity of raw materials is ordered for production from 5 potential suppliers. Then, the produced lot size is sent to 5 potential distribution centers and finally moved to 10 customer zones according to customer demands. In the reverse network, the returned products (defective or used products) are shipped to 5 potential collection centers. The non-recyclable and non-reworkable items are disposed through 3 potential disposal centers.

3.5.1 Illustrated Numerical Example

Three scenarios are considered in this study with probabilities of 0.3, 0.5, and 0.2, respectively. Note that for scenarios 2 and 3, the estimations are always multiplied by 1.3 and 1.5, respectively. The values of scenario 1 for the uncertain parameters associated with the first objective function (${D^\zeta}_{tpc}$, ${R^\zeta}_{tpc}$, ${Rw^\zeta}_{tpc}$, ${Rc^\zeta}_{tpm}$, ${REc^\zeta}_{tpm}$, ${d^\zeta}_t$, ${\beta^\zeta}_p$) are generated randomly using the uniform distribution at $t = 1$, table 2, and then the values for the rest of the periods are generated as explained in figure 3.2. It shows that the value at period $t = 2$, is higher than the values of $t = 1$ by 10%. This increase continues until it reaches to $t = 6$, at which the value decrease by 10% of $t = 5$. Then, the value keeps going down by 10% until it reaches the end of the year $t = 12$.

	Values for Product p		
Uncertain Parameter	1	2	3
$\widetilde{D}_{tpc}$	U (65, 165)	U (55, 147)	U (70, 170)
$\tilde{R}_{tpc}$	U (44, 85)	U (38, 95)	U (61, 110)
$\widetilde{Rw}_{tpc}$	U (10, 36)	U (13, 43)	U (9, 26)
$\widetilde{Rc}_{tpm}$	U (9, 12)	U (6.5, 9)	U (6, 8)
$\widetilde{REc}_{tpm}$	U (4, 6)	U (4, 6.5)	U (3.5, 6)
$\tilde{\beta}_p$	0.2	0.175	0.18
$\tilde{d}_t$		0.05	

Table 3.2: The values of the uncertain parameters associated with the first objective function at period $t = 1$, and scenario 1.

This behavior is projected on the assumption that the market demand growth for some products would increase gradually at the beginning of the cycle until it reaches to its highest sales in the mid of the cycle. After that the customers lose their interests in these products because other companies in the market offer competitive products with reasonable prices. In addition, the company decides to shift to new products with new features which means low sales of old products at the end of the cycle.

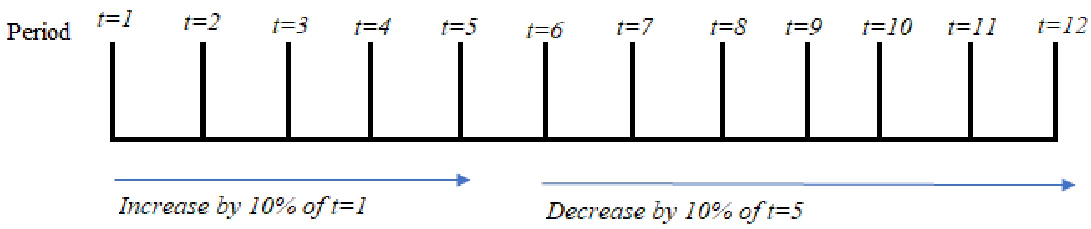


Figure 3.2: Generating the values for the entire year based on period *t=1*.

Data related to environmental impact are estimated as follows. We assume that the manufacturing centers (including reworking and recycling facilities), and the disposal centers would consume electrical energy beside to gasoline. Thus, the amount of CO_2 emissions is estimated according to (ECTA, 2012; McKinnon, 2007), table 3.3. Note that gasoline is used for transportation delivery.

Utility	CO_2	Unit of measure
Electricity	0.7306	kg/kwh
Gasoline	2.392	kg/m^3

Table 3.3: CO_2 per utility consumption.

The proportion of each utility usage depends on the facility. For example, the power for manufacturing new products may require 35% gasoline and 65% electricity. This percentage of gasoline consumption would be lower in products recycling and reworking with 15% and 25%, respectively, while it is 100% in the transportation, table 3.4. Practically, the distance between the echelons can be obtained from "Google Map". In our artificial example we assume these distances are generated randomly according to matrix distance in a metric space.

Parameter	Amount of CO_2 emissions
EMc_{mp}^{ζ}	$65\% \times$ Electricity $+ 35\% \times$ Gasoline
ERc_{mp}^{ζ}	$85\% \times$ Electricity $+ 15\% \times$ Gasoline
ERW_{mp}^{ζ}	$75\% \times$ Electricity $+ 25\% \times$ Gasoline
EOc_{op}^{ζ}	$50\% \times$ Electricity $+ 50\% \times$ Gasoline
ETc_{p}^{ζ}	Gasoline

Table 3.4: : The values of the uncertain parameters associated with the second objective function for p_1, m_1, o_1, and first scenario.

Note that for p_2 and p_3, the estimations are multiplied by 1.15 and 1.25, respectively. Also, the estimations for m_2, and m_3 as well as the disposal centers are multiplied by 1.25 and 1.35, respectively.

The number of jobs created depends on the number of facilities and their capacity. Also, in a region with high unemployment, the weight assigned to the number of jobs created should be higher than the weights assigned to other objective functions. The values of scenario 1 for the uncertain parameters associated with the third and fourth objective functions are provided in table 3.5. The random generated data of the fixed model parameters are given in tables 3.6 and 3.7. Note that in table 3.6, the values of parameters listed from TMc_{psm} to *Tic* for products p_2, and p_3 are estimated to be 0.75 +Values of (p_1) , and 1.2 +Values of (p_1), respectively.

Uncertain Parameter	Expected values
GD_i^{ζ}	*U(9,35)*
GC_l^{ζ}	*U(15,45)*
GO_o^{ζ}	*U(9, 25)*
HS_m^{ζ}	*U(0.05,0.1)*

Table 3.5: The values of the uncertain parameters associated with the third and fourth objective functions for scenario 1.

Parameter	Values: Product 1 (p_1)	Product 2 (p_2)	Product 3 (p_3)	Parameter	Values: Product 1(p_1)	Product 2	Product 3
Sc_{ps}	~U(12.5, 15)	~U(10,12)	~U(8,13)	CI_{pi}	~U(575, 660)	~U(580,645)	~U(550,630)
Mc_{pm}	~U(40,45)	~U(38,42)	~U(43,45)	CL_{pl}	~U(235, 280)	~U(200, 245)	~U(220, 265)
Ic_{pi}	~U(5,6)	~U(3.75,5.75)	~U(4.5,5.5)	CO_{po}	~U(345,350)	~U(295,300)	~U(315, 320)
Dc_{pi}	~U(10,12)	~U(10,11)	~U(9.5,10.5)	TMc_{psm}	~U(5, 8)		
Cc_{pl}	~U(8,9.5)	~U(7,8)	~U(7.75,8.75)	TPc_{pmi}	~U(3, 4.75)		
h_{pi}	~U(3,4)	~U(4,4.5)	~U(4,5)	TOc_{pcl}	~U(4, 8)	0.75 +Values of (p_1)	1.2 +Values of (p_1)
P_{ps}	~U(6.5,10)	~U(5,6)	~U(3,7)	TZc_{pic}	~U(3, 5)		
Io_{po}	~U(3,3.5)	~U(3, 3.75)	~U(3,5)	$TOPc_{plm}$	~U(3.25, 5)		
CS_{ps}	~U(685, 800)	~U(720, 840)	~U(750, 780)	Tic	~U(4,5)		
CP_{pm}	~U(540, 650)	~U(500,600)	~U(590,620)				

Table 3.6: The randomly generated data of the proposed model parameters.

Parameter	Values	Parameter	Values
FS_s	~U(65000,81000)	USL_p	4.8
FD_i	~U(40000, 55000)	LSL_p	5.2
FC_l	~U(35000, 45000)	K	120
FO_o	~U(20000, 30000)	μ_p	5
hw_{pl}	~U(2, 2.5)	σ_p	0.05
$\hat{\pi}_{pc}$	~U(70000, 95000)		
W_{pm}	1000		

Table 3.7: Design of the data set.

The computations of MILP were run using the branch and bound algorithm accessed via LINGO16.0 on a PC -3GHzand; 4 GB RAM and under win 10. The ideal solution for each objective function is calculated before performing the computational processes, see table 3.8. This ideal point is used as reference point for the augmented weighted Tchebycheff approach. Note that in table 3.8, Z_i^* refers to the robust objective function of f_i^{ζ}, at each scenario ζ. Considering different values for weights of the objective functions by uniformly varying the weights, different Pareto solution are produced, and the results are presented in table 3.9. The solutions are computed at $\boldsymbol{\lambda} = 1,\ \boldsymbol{\omega} = [1000, 100,10000,2000]$, and $\tau = 0.01$.

Utopia Point Z_i^*	Value of Z_i^*	Objective function (f_i^{ζ}) at Z_i^*	$\zeta = 1$	$\zeta = 2$	$\zeta = 3$
Z_1^*	26857316	f_1^{ζ}	7924407	7927961	7930330
Z_2^*	46764950	f_2^{ζ}	23095187	30022209	34638172
Z_3^*	22214.4	f_3^{ζ}	3468	4572	5256
Z_4^*	1658.21	f_4^{ζ}	809.2205	1047.751	1276.928

Table 3.8: The ideal solution of the robust objective function, Z_i^* and its corresponding optimal function value, f_i^{ζ} at each scenario, ζ.

Weights Combinations				Robust Objective Functions			
w_1	w_2	w_3	w_4	Z_1	Z_2	Z_3	Z_4
0.7	**0.1**	**0.1**	**0.1**	31557288	79664791	2094.78	6989.774
0.6	0.2	0.1	0.1	34788564	70558702	2108.13	5388.623
0.5	0.2	0.2	0.1	35577058	68564314	4259.34	5111.073
0.4	0.3	0.2	0.1	38030300	61662246	4266.34	3899.628
0.3	0.4	0.2	0.1	40025672	56641218	4314.76	3126.003
0.2	0.4	0.2	0.2	41089851	53881219	4357.84	2733.508
0.2	0.3	0.2	0.3	40365119	55770150	4329.12	3011.481
0.1	0.5	0.3	0.1	42806945	49954872	6614.91	2182.152
0.1	0.5	0.1	0.3	42806945	49954872	2204.97	2182.152
0.25	0.25	0.25	0.25	39112397	59020407	5355.65	3460.824
0.5	0.25	0.15	0.1	36526536	66103432	3209.43	4751.669
0.2	0.1	0.4	0.3	36475576	66001477	8529.36	4689.249
0.1	0.2	0.4	0.3	41113755	53893164	8715.68	2730.917
0.1	**0.7**	**0.1**	**0.1**	43185878	49097576	2207.64	2002.309
0.1	**0.1**	**0.7**	**0.1**	39059363	58967000	15082.97	3520.784
0.1	**0.1**	**0.1**	**0.7**	39060739	58968379	2150.2	2103.234
0.5	0.4	0.05	0.05	38272151	61033501	1070.175	4806.619
0.4	0.4	0.1	0.1	39063694	58971324	2150.2	3440.517
0.2	0.2	0.3	0.3	39122157	59029800	6464.13	3519.653
0.1	0.2	0.3	0.4	41312944	53992754	6581.91	2857.06

Table 3.9: Robust objective functions value of numerical example through augmented weighted Tchebycheff approach.

3.5.2 Discussion and Analysis

The solutions provided in table 3.9 validate the proposed model. The weights indicated in bold refer to the highest priority assigned to each objective function. Thus, the resulted robust objective functions at these priorities tend to be close to the utopia vector provided in table 3.8. Referring to table 3.9, when we assign the highest weight to the second robust objective function, Z_2 (the amount of CO_2 emissions is minimum), this leads to a significant reduction in the production lots. Consequently, the first robust objective function, Z_1(the total cost across the CLSC network) tends to be high because the market demand is partially met, and therefore the penalty term sharply increases. Indeed, the minimum of hazardous products manufactured, Z_4 is achieved. On the other hand, if the third robust objective function, Z_3 (maximizing job opportunities created) is given the highest weight, the maximum total cost across the CLSC network is obtained. Practically speaking, this conclusion is valid because more facilities have to be operational to increase the number of jobs which in turn leads to an undesired strategic planning due to the high facilities opening costs.

Next, we study the behavior of the performance of the robust objective functions as the weighting scale to measure the tradeoff between sensitivity and robustness, $\boldsymbol{\lambda}$ changes. Generally speaking, $\boldsymbol{\lambda}$ should be small enough. However, if $\boldsymbol{\lambda}$ is chosen to be a relatively high value, the model becomes insensitive to the uncertain

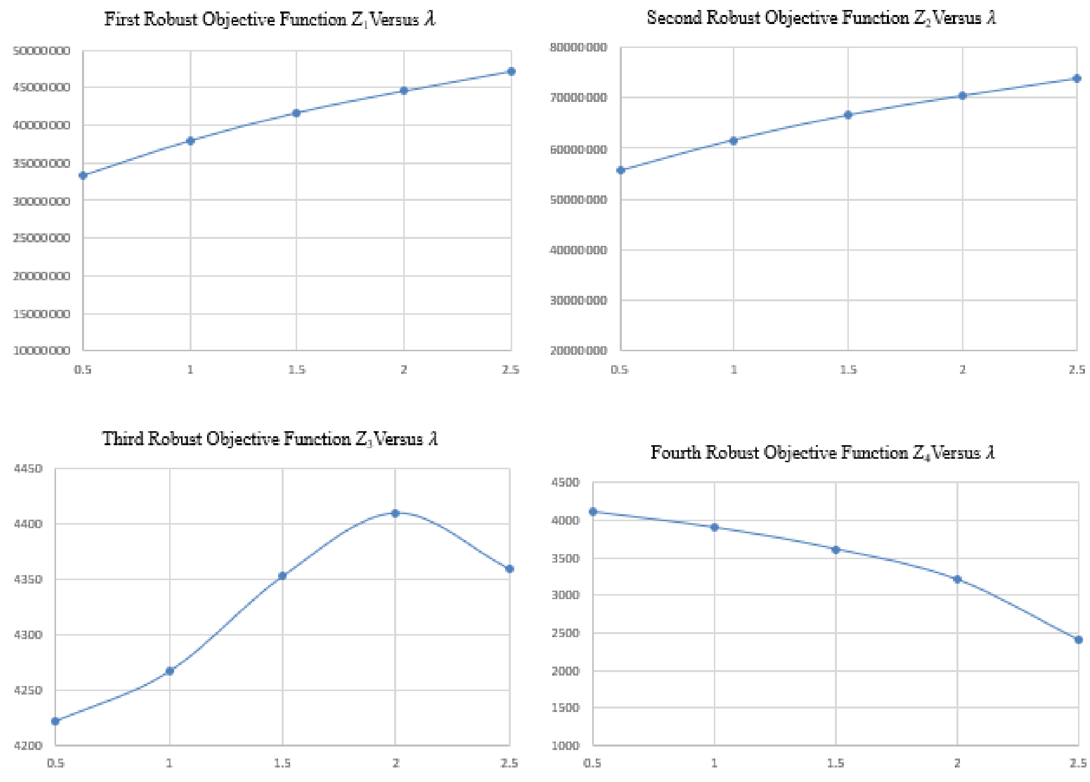


Figure 3.3: The behavior of the robust objective functions as vector $\boldsymbol{\lambda}$ increases.

model variation which means more conservative. Therefore, choosing $\boldsymbol{\lambda}$ properly can control the degree of conservatism and improve the quality of the robust solution. In this regard, we test the sensitivity of the model for two different cases. In the first case, we study the behavior of the four robust objective functions as the vector $\boldsymbol{\lambda}$ increases, see figure 3.3, while in the second test, we study the behavior of each robust objective function as its corresponding value of λ_i increases, see figure 3.4. It should be noted that in this analysis the weights of four objectives are set at (0.4, 0.3, 0.2, 0.1), respectively.

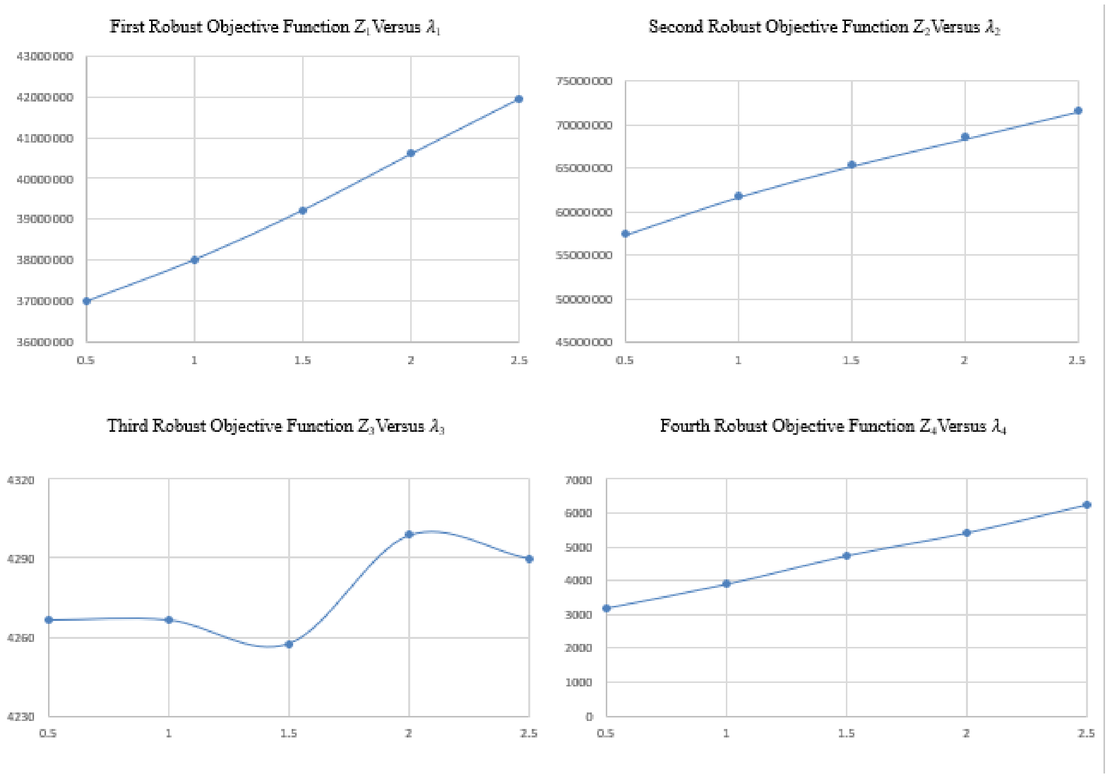


Figure 3.4: The behavior of each robust objective function as its corresponding value of λ increases.

In figure 3.3, as $\boldsymbol{\lambda}$ increases from 0.5 to 2.5, only the values of robust objectives Z_1, and Z_2, increase while in figure 3.4, Z_1, Z_2, and Z_4 increase. Moreover, the increase in Z_1, and Z_2 is relatively higher in figure 3.3. Note that in figure 3.3 because of interactions between the four objectives, Z_4 decreases as $\boldsymbol{\lambda}$ increases. However, the average value of Z_4 in figure 3.3 ($\bar{Z}_4 = 3443.4016$) is less than the average value of Z_4 in figure 3.4 ($\bar{Z}_4 = 4690.2066$). Therefore, we make the following observation: to reduce the conservatism and improve the robust solutions quality of Z_1, and Z_2, the decision maker should change their corresponding values of λ_i individually (case 2), while changing the value of $\boldsymbol{\lambda} = [\lambda_1, \lambda_2, \lambda_3, \lambda_4]^T$, simultaneously (case 1), leads to improve Z_4. Also, the difference in Z_2 is not significant for the two cases. The situation is different in robust objective Z_3 since the goal here is maximization. In this case, we seek to maximize the expected value of Z_3 but at the same time its variance term must be minimized. Due to this conflict, we can not draw a conclusion when λ_3 increases. As depicted in both figures, it seems that the optimal value of Z_3 is achieved at $\lambda_3 = 2$.

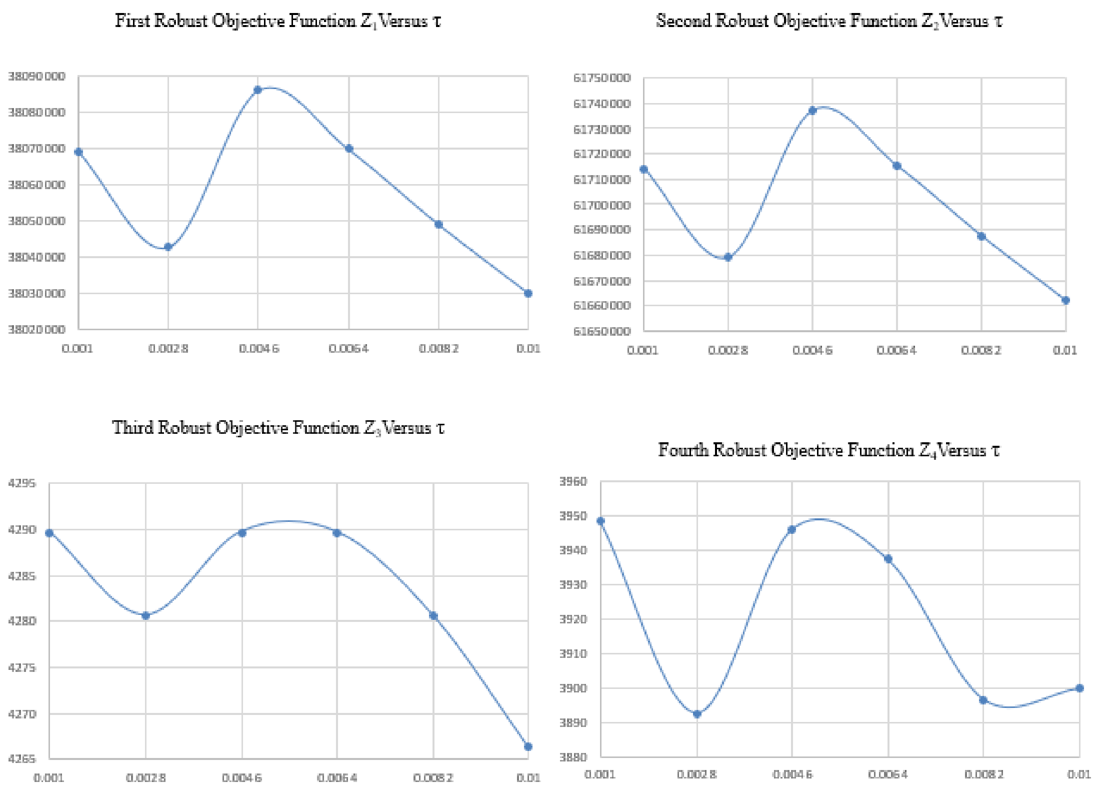


Figure 3.5: The behavior of the robust objective functions as τ increases.

Figure 3.5 shows the effect of the penalty parameter, τ associated with augmented weighted Tchebycheff method on the solutions. It can be observed that as τ changes from 0.001 to 0.01 (Steuer and Choo, 1983), there is a slightly change in the robust objective functions. As shown in figure 3.5, the ranges of Z_1, Z_2, Z_3, and Z_4 remain approximately at 38×10^6, 61×10^6, 4200, and 3900, respectively.

3.6 Conclusion

This chapter proposes a novel robust multi objective closed-loop supply chain model to accommodate the gaps in the previous researches in mathematical modeling concerning CLSC. Some of the features of the proposed model are as follows: (i) Investigating the imperfect quality production to provide meaningful solutions to practical problems; (ii) Considering that the inspection is not free of errors such that types I and II errors are associated with the inspection, and the amount of quality loss as conforming products deviate from the specification (target) value is measured; (iii) Exploring multiple periods, echelons, and uncertainties; (iv) Modeling MILP of the supply chain, while four objectives are taken into consideration simultaneously (the economic, environmental, and social aspects) and the augmented weighted Tchebycheff method is used to aggregate the four objective functions and produce the set of efficient solutions; (v) Robust optimization, based on Mulvey et al. (1995) approach, is used to obtain a set of solutions that are robust against the future fluctuation of parameters. Our proposed model is compatible with several types of industries including steel making, electronic and automobile manufacturing, and various plastic products where return products (either defective or used) can be reused as a raw material, and when environmental and social issues become a company concern.

Several research directions read considerations in the area of CLSC under uncertainty. One possible future extension is treating the market demand as an uncarting dynamic parameter. For real input datasets, integrating this model with design of control charts can be a subject of future research. In the case of large scale problems, this MILP robust optimization model is NP-hard and requires an effective algorithm to handle large scale real problems.

CHAPTER 4: A ROBUST CLOSED- LOOP SUPPLY CHAIN UNDER IMPERFECT QUALITY PRODUCTION: AFFINELY ADJUSTABLE ROBUST OPTIMIZATION APPROACH UNDER DYNAMIC UNCERTAINTY SET

In this chapter, the affinely adjustable robust formulation based on "wait and see" decisions is presented. That is, the decisions are made over two sequential stages where multiple uncertainties are included. Moreover, we propose a budget dynamic uncertainty set to mimic the dynamic behavior of the market demand over time. The introduced dynamic uncertainty set is formulated according to Vector Autoregressive (VAR) models where the temporal and spatial correlations of customer demand zones are captured. Also, we utilize different a priori probability bounds to approximate probabilistic constraints and provide a safe solution. The objective is to minimize the total cost of the supply chain network. Finally, numerical examples are provided to illustrate the proposed models.

4.1 Introduction and Literature Review

The uncertainty in the supply chain modeling has been recently discussed extensively by researchers and industry practitioners. When both the forward and reverse supply chains are considered, then the network modeling becomes a closed-loop supply chain (CLSC) which is now widely taking attention. A common assumption of the supply chain inventory model is that the produced items are perfect. We consider here the imperfect quality production to provide meaningful solutions to practical supply chain management problems.

Our modeling investigates the integrated multi-echelon, multi-period under multiple uncertainties models, where the most recent techniques of robust optimization are used as solution approaches. Many researches have addressed the issues of the uncertainty of the supply chain using robust optimization under a single stage decision (here and now decision). In this chapter, the affinely adjustable robust formulation based on "wait and see" decision is presented over two sequential stages.

The traditional uncertainty set in robust optimization assumes that the uncertain parameter lies within a static uncertainty set which may not be the case for some real applications. To make this model more practical, we assume that the uncertainty of the market demand in the CLSC is subject to a dynamic uncertainty set in which the temporal and spatial correlations of customer demand zones are captured. In addition, to determine the uncertainty set size parameters we utilize different a priori probability bounds to approximate probabilistic constraints and provide a safe solution.

Recently, Govindan et al. (2017) conducted a literature survey showing that four main approaches in recent decades have been adopted to handle the uncertainty environment in the supply chain modeling. The four approaches are: dynamic programming, stochastic programming, fuzzy programming, robust optimization, or the combination of any two of these approaches. With existing uncertainty in the dynamic modeling, the dynamic parameters (i.e. market demand) will represent a more realistic problem, and hence there is a special attention recently paid to stochastic dynamic market demand. On the other side, fuzzy programming is a popular approach applied recently by many researchers to the supply chain area under uncertainty, (Shekarian, Kazemi, and Abdul-rashid, 2017). When the probability distribution of an uncertain parameter is known, the appropriate modeling approach is stochastic programming. This approach is one of the most important approaches used to handle the uncertainty in production supply chain and inventory control, (Masih-tehrani, Xu, Kumara, and Li, 2011), (Zhang, Li, and Huang, 2014), and (Wang, Qin, and Kar, 2015). Several extensions of previous studies with supply chain uncertainty make stochastic programming an increasingly important modeling approach.

Robust optimization is a modeling approach where an uncertainty set is considered to describe the possible values of uncertain parameters of an optimization model. This optimization approach seeks to find the best feasible solution for all uncertain parameters inscribed in the uncertainty set. The formulation is originally proposed by Soyster (1972), but the proposed solution is very conservative. Ben-Tal and Nemirovski (1998, 1999, 2000), Ghaoui and Lebret (1997) and Ghaoui et al. (1998) propose a robust counterpart (RC) with tractable solution approaches based on ellipsoidal uncertainty set (conic quadratic problems). The developed RC formulation produced a less conservative solution. Although no distribution assumption is made on uncertain parameters, the availability of data information can be utilized beneficially. The development of robust optimization is based on uncertainty sets approach and is summarized in Table 4.1.

Author	Contribution	Year
Soyster	• Simple perturbations in the data are considered in the linear programming problem to make the solution feasible under all perturbations. • Introduces interval set.	1973
Ben-tala, Nemirovski and coworkers	• The ellipsoidal set robust counterpart is proposed to formulate the linear and quadratic programming problems under uncertain parameters.	1998-2004
El-Ghaoui and coworkers	• Study the uncertain least-squares problems with the robust solutions. • Study uncertain semidefinite problems.	1997,1998
Lin et al. Janak et al.	• Extend RO for (LP) to MILP • The robust optimization framework for different bounded known probability distributions are developed.	2004, 2007
Verderame and Floudas	• Investigate both continuous (general, bounded, uniform, normal) and discrete (general, binomial, Poisson) uncertainty distributions.	2009
Bertsimas, Sim and coworkers	• Introduce the uncertainty budgets set (combined interval and polyhedral uncertainty set) in the LP. • A new approach is proposed to deal with uncertain parameters in the discrete network optimization problems.	2003-2004
Bertsimas and Thiele	• Extend previous work to address inventory control problems to minimize total costs.	2006
Soyster Li et al. Ben-Tal and Nemirovski Bertsimas and Sim	• Interval Uncertainty Set • Pure Box, Ellipsoidal, and Polyhedral Uncertainty Sets • Combined interval and ellipsoidal set • Combined interval and polyhedral set	1973 2011 2000 2004

Table 4.1: Robust optimization approaches in operations research based on uncertainty sets.

Recently, many researchers apply the uncertainty set based approach to manage the multiple uncertainties associated with the robust supply chain optimization, (Aharon, Boaz, and Shimrit, 2009; Baghalian, Rezapour, and Zanjirani, 2013; Hatefi and Jolai, 2014; Kisomi, Solimanpur, and Doniavi, 2016;

Ma et al., 2016; Pishvaee et al., 2011; Wei, Li, and Cai, 2011; Xin, Xi, Yu, and Wu, 2013; Y. Zhang and Jiang, 2017; Zokaee, Jabbarzadeh, Fahimnia, and Jafar, 2017).

There are very few studies which recognize incorporation of the imperfect quality production to the supply chain modelling, (Ahmadi, Khoshalhan, and Glock, 2016; Masoudipour, Amirian, and Sahraeian, 2017; Sana, 2011). However, these studies consider deterministic models.

Author(s)	Closed Loop- SC	Imperfect Quality Production	Uncertainty in The Model	Robust Framework	Multistage Formulation	Dynamic Uncertainty Set
Hu, Zheng, Xu, Ji, and Guo (2010)		×	×		×	
Sana (2011)		×				
Hwan, Rhee, and Cheng (2013)		×	×			
Rad, Khoshalhan, and Glock (2014)		×	×		×	
Ahmadi, Khoshalhan, and Glock (2016)		×	×		×	
Masoudipour, Amirian, and Sahraeian (2017)	×	×				
Manna, Das, Dey, and Mondal (2017)	×	×	×			
This paper	×	×	×	×	×	×

Table 4.2: Some of the studies in the field of supply chain under imperfect quality production. Mark (×) in this table means that an article in a row has the feature mentioned in that column.

Modeling supply chain under uncertainty where imperfect quality production is incorporated is also studied by few researchers. For example, Hu, Zheng, Xu, Ji, and Guo (2010) study coordination of supply chain for the fuzzy random newsboy problem with imperfect quality in the decentralized and centralized systems. Quality uncertainty from a supply chain coordination perspective is addressed by Hwan, Rhee, and Cheng (2013). Rad, Khoshalhan, and Glock (2014), however, use the renewal-reward theorem as a stochastic approach in optimizing inventory and sales decisions in a two-stage supply chain. Table 4.2 presents some of the studies in the field of supply chain under imperfect quality.

In summary, our contributions are the integration of the following:

- We propose a novel closed loop supply chain design with multiple periods and echelons. The considered CLS model is under imperfect quality production. Also, we assume that the inspection is not free of errors.
- The modelling is with multiple uncertainties including market demand, returned of amount product as either used or defective, recycling and reworking costs, and types I and II errors associated with the inspection.
- The affinely adjustable robust formulation based on "wait and see" decision is presented over two sequential stages.
- We propose a budget dynamic uncertainty set to mimic the dynamic behavior of market demand over time, and it is formulated according to Vector Autoregressive (VAR) models where the temporal and spatial correlations of customer demand zones are captured.
- We utilize a priori probability bounds to approximate probabilistic constraints and provide a safe solution. Then, we will evaluate the robust solutions at different probability constraint violations.

The rest of the chapter is organized as follows. Section 4.2 discusses the adjustable robust formulation, section 4.3 introduces a budget dynamic uncertainty set, section 4.4 proposes the integrated model formulation, followed by a solution methodology and numerical examples in section 4.5. Finally, section 4.6 concludes the paper.

4.2 The Adjustable Robust Formulation

The usual RC formulation is used to treat "here and now" decisions. That is, all decision variables values are determined before the realization of uncertain parameters. However, in many practical real life problems some variables, including auxiliary variables such as slack or surplus variables, could be decided after realization of (some of) the uncertain parameters. We refer to this as "wait and see” decisions.

To the best of our knowledge, the first work that addressed this type of robust formulation was done by (Ben-Tal et al., 2004). They proposed an adjustable robust counterpart (ARC) approach for models such that the adjustable variables reveal themselves with uncertainty. Moreover, the developed ARC tackled two types of recourses; fixed, where the coefficients of adjustable variables are deterministic, and uncertain, otherwise. However, the computational tractability of their model was a major concern. Therefore, they proposed an affinely adjustable robust counterpart (AARC) approach to approximate the ARC by restricting the adjustable variables to be affine functions of the uncertain parameters. Next, we describe the AARC approach for the case of linear programming.

Consider a linear program (LP):

$$\min_{w\geq 0} c^T w: \quad Aw \leq b, \tag{4.1}$$

where $w \in \mathbb{R}_+^n, c \in \mathbb{R}^n, A \in \mathbb{R}^{m\times n}, b \in \mathbb{R}^m$. The RC was proposed by Ben-Tal et al. (2004) as follows:

$$\min_{w\geq 0} \max_{\zeta\in\mathcal{Z}} \{c^T w: \quad Aw - b \leq 0, \quad \forall \zeta = [c, A, b] \in \mathcal{Z}\},$$

where $\mathcal{Z} \subset \mathbb{R}^n \times \mathbb{R}^{m\times n} \times \mathbb{R}^m$ is a given uncertainty set.

In fact, the decision variables w can be decomposed into non-adjustable variables x and adjustable variables y. In addition, if the costs of some non-adjustable variables are affected by uncertainty then we reformulate the problem as follows to move all uncertainty to the constraints:

$$\min_{u,x,y\geq 0} \left\{u: c_x^T x + c_y^T y - u \leq 0, Ax + Dy \leq b, \forall \zeta = [c, A, D, b] \in \mathcal{Z}\right\}, \tag{4.2}$$

where $x \in \mathbb{R}_+^{n-p}, y \in \mathbb{R}_+^p, A \in \mathbb{R}^{m\times(n-p)}, D \in \mathbb{R}^{m\times p}, b \in \mathbb{R}^m, \mathcal{Z} \subset \mathbb{R}^n \times \mathbb{R}^{m\times(n-p)} \times \mathbb{R}^{m\times p} \times \mathbb{R}^m$.

Upon this formulation (if necessary) we can state the robust counterpart as:

$$Z_{RC} = \min_{x,y\geq 0} \left\{ c_x^T x + c_y^T y, Ax + Dy \leq b, \forall \zeta = [c, A, D, b] \in \mathcal{Z}\right\}. \tag{4.3}$$

Therefore, we assume that all uncertain parameters appear in the constraints. The ARC corresponding to (4.3), where the adjustable variable y is decided after realization of the uncertain parameters, is:

$$Z_{ARC} = \min_{x,y(\zeta)\geq 0,\forall\zeta\in\mathcal{Z}} \left\{ c_x^T x + \max_{\zeta\in\mathcal{Z}} c_y^T y(\zeta)\,, Ax + Dy(\zeta) \leq b, \forall\zeta = [A,D,b] \in \mathcal{Z} \right\} \quad (4.4)$$

Ben-Tal et al. (2004) assume, without loss of generality, that the uncertainty set $\mathcal{Z}$ is affinely parameterized by a perturbation vector ζ varying in a given non-empty convex compact perturbation set , $\chi \subset \mathbb{R}^L$:

$$\mathcal{Z} = \{[A,D,b] = [A^0,\ D^0, b^0] + \textstyle\sum_{l=1}^{L} \zeta^l [A^l,\ D^l, b^l] : \zeta \in \chi\} \quad (4.5)$$

In the case of fixed recourse, the coefficients of the adjustable variables are deterministic. the RC formulation with fixed recourses is as follows:

$$Z_{RC} = \min_{x,y\geq 0} \{ c_x^T x + c_y^T y, \left(a_i^0 + \textstyle\sum_{l=1}^{L} \zeta^l a_i^l\right)x + d_i y \leq b_i^0 + \sum_{l=1}^{L} \zeta^l b_i^l\,, \forall\zeta \in \chi, i = 1, \dots m \}, \quad (4.6)$$

and the fixed recourse version of ARC is:

$$Z_{ARC} = \min_{x,y(\zeta)\geq 0,\forall\zeta\in\chi} \left\{ c_x^T x + \max_{\zeta\in\chi} c_y^T y(\zeta) : \left(a_i^0 + \textstyle\sum_{l=1}^{L} \zeta^l a_i^l\right)x + d_i y(\zeta) \leq b_i^0 + \sum_{l=1}^{L} \zeta^l b_i^l\,, \forall\zeta \in \chi, i = 1, \dots m \right\} \quad (4.7)$$

The AARC is an approximation of the ARC in which the adjustable variables are restricted to be affine functions of the uncertain parameters. In this approximation, if $\mathcal{Z}$ is affinely parameterized as defined in equation (4.5), the adjustable variables y are restricted to affinely depend on ζ :

$$y = \pi^0 + \textstyle\sum_{l=1}^{L} \zeta^l \pi^l \geq 0, \quad (4.8)$$

where $\pi^l \in \mathbb{R}^p$ for $l = 0, \dots, L$. The fixed recourse AARC formulation corresponding to (4.7) is:

$$Z_{AARC} = \min_{x,\pi} \left\{ c_x^T x + \max_{\zeta\in\chi} c_y^T (\pi^0 + \sum_{l=1}^{L} \zeta^l \pi^l) : \left(a_i^0 + \sum_{l=1}^{L} \zeta^l a_i^l \right) x + d_i \left(\pi^0 + \sum_{l=1}^{L} \zeta^l \pi^l \right) \leq b_i^0 + \sum_{l=1}^{L} \zeta^l b_i^l\,, \forall\zeta \in \chi, i = 1, \dots m;\ \pi^0 + \sum_{l=1}^{L} \zeta^l \pi^l \geq 0\,, \forall\zeta \in \chi \right\} \quad (4.9)$$

$u_i,$

4.3 Budget Dynamic Uncertainty Set:

The traditional uncertainty sets used in robust optimization assume that the uncertain parameter lies within a convex and static uncertainty set in which all values of the uncertainty set are realized.

The budget (polyhedral) uncertainty set is described using the 1-norm of the uncertain data vector and is presented as follows:

$$U_1 = \{\zeta \mid \|\zeta\|_1 \leq \Gamma\} = \left\{\zeta \mid \textstyle\sum_{j \in J_i} |\zeta_i| \ \leq \Gamma\right\} \tag{4.10}$$

where Γ is the adjustable parameter controlling the size of the uncertainty set, see figure 4.1

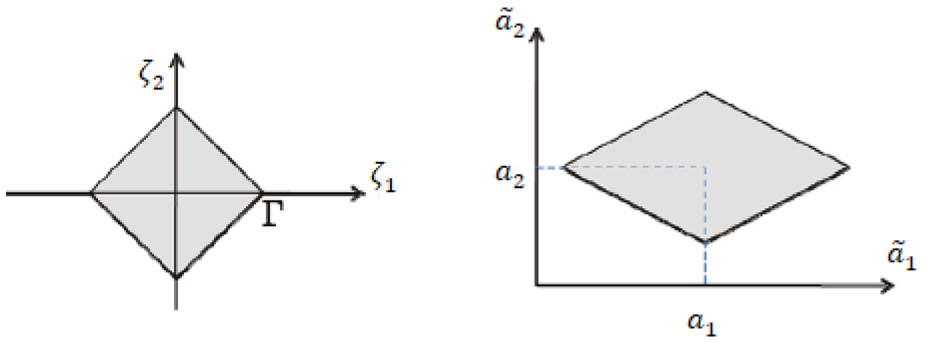


Figure 4.1: Illustration of a polyhedral uncertainty set where a_1 and a_2 are the nominal values of the uncertain parameters $\tilde{a}_1$ and $\tilde{a}_2$, respectively.

Bertsimas and Sim (2004) introduced the polyhedral uncertain set which has the equivalent tractable form:

$Min\ Z$

s.t. $\sum_j c_j x_j + \Gamma U \leq Z$

$U \geq \hat{c}_j |x_j|, \quad \forall j \in J$

$$\sum_j a_{ij} x_j + \Gamma u_i \ \leq b_i \quad \forall i$$

$u_i \geq \hat{a}_{ij} |x_j|, \qquad \forall i, j \in J$

$u_i \geq \hat{b}_i, \qquad \forall i$

where a_{ij}, b_i, and c_j represent the nominal (expected) value of the parameters; $\hat{a}_{ij}$, $\hat{b}_i$, and $\hat{c}_j$ represent constant perturbation; ζ_{ij} is a random variable that takes values in the interval [-1, 1].

In some practical problems, however, this may not be the case; the uncertainty depends on the previous stage and hence the bounds of the uncertainty set dynamically change over the course of time.

4.3.1 The Formulation of Budget Dynamic Uncertainty Set

These correlations can be explicitly modeled by introducing the so -called dynamic uncertainty set. To the best of our knowledge Lorca and Sun (2015) proposed a linear budgeted dynamic uncertainty set. Specifically, they constructed a dynamic uncertainty set for wind power using linear systems to capture the temporal and spatial correlations of wind speeds at adjacent wind farms at time t.

In this model we will propose a polyhedral dynamic uncertainty set to mimic the dynamic behavior of market demand over time. Also, the construction of such dynamic set captures the correlation of the demand at each customer zone. Consider the following general form of dynamic uncertainty set:

$$\mathbf{Z}_{\boldsymbol{t}}\left(\boldsymbol{\zeta}_{[\mathbf{1}:\boldsymbol{t}-\mathbf{1}]}\right) = \left\{\boldsymbol{\zeta}_{\boldsymbol{t}} : \exists u_{[t]} \text{ s.t.} f(\boldsymbol{\zeta}_{[\boldsymbol{t}]}, \boldsymbol{\epsilon}_{[t]}) \leq 0\right\} \;\; \forall t \tag{4.11}$$

where $\boldsymbol{\zeta}_{[\boldsymbol{t_1}:\,\boldsymbol{t_n}]} \triangleq \left(\boldsymbol{\zeta}_{\boldsymbol{t_1}}, \dots, \boldsymbol{\zeta}_{\boldsymbol{t_n}}\right)$ and in shorthand $\boldsymbol{\zeta}_{[\boldsymbol{t}]} \triangleq \boldsymbol{\zeta}_{[\mathbf{1}:\boldsymbol{t}]}$, and the uncertainty vector $\boldsymbol{\zeta}_{\boldsymbol{t}}$ are functions of uncertainty realizations in previous time periods. The error term is denoted by $\boldsymbol{\epsilon}_{\boldsymbol{t}}$. To make the model computational tractable, we model $f(\boldsymbol{\zeta}_{[\boldsymbol{t}]}, \boldsymbol{\epsilon}_{[t]})$ as semi-definite representable. Therefore, f can described through a linear dynamic uncertainty set.

To construct the dynamic uncertainty set for the market demand, we define the uncertain demand vector $\boldsymbol{D}_{\boldsymbol{t}}$ as:

$$\mho_{\boldsymbol{t}}\left(\boldsymbol{D}_{[\boldsymbol{t}-\boldsymbol{\Pi}:\boldsymbol{t}-\mathbf{1}]}\right) = \{\boldsymbol{D}_{\boldsymbol{t}} : \exists\, \widetilde{\boldsymbol{D}}_{[\boldsymbol{t}-\boldsymbol{\Pi}:\boldsymbol{t}]}, \boldsymbol{\epsilon}_{t} \;\; \text{s.t.}$$

$$\widetilde{\boldsymbol{D}}_{\boldsymbol{t}} = \sum_{r=1}^{\Pi} \boldsymbol{A}_{\boldsymbol{r}} \widetilde{\boldsymbol{D}}_{\boldsymbol{t}-\boldsymbol{r}} + \Gamma^{\epsilon} \boldsymbol{u}_{\boldsymbol{t}}^{\epsilon}, \qquad \forall\, t \in T \tag{4.12}$$

$$|u_{ct}^{\epsilon}| \geq \epsilon_{ct} \qquad \forall\, t \in T, c \in C \tag{4.13}$$

$$\widetilde{\boldsymbol{D}}_{\boldsymbol{t}} \geq 0 \tag{4.14}$$

The temporal and spatial correlations of customer demand zones at time t is represented by Eq.(4.12) where the vector $\boldsymbol{D}_{\boldsymbol{t}} = (D_{1t},\; D_{2t}, \dots, D_{ct})\acute{}$ denotes the uncertain market demand for each customer zone c at time t. The temporal and spatial correlation coefficients are denoted by matrix A. The error vector ϵ_t consists of random variables defined by the dynamic budget uncertainty set which is controlled by the parameter Γ^{ϵ} in Eq.(4.12). Finally, the non-negativity constraint of $\boldsymbol{D_t}$ is provided by (4.14).

4.3.2 Estimating the Parameters of the Dynamic Uncertainty Set

The introduced dynamic uncertainty set is formulated according to Vector Autoregressive (VAR) models and includes parameters which need to be estimated. The preliminaries and definitions of the multivariant of time series model is provided in Appendix A.

Consider the vector autoregressive model given by Eq(4.12). It can be expanded as follows:

$$\begin{cases} \widetilde{D}_{1,t} = \alpha_1 + \rho_{11}\widetilde{D}_{1,t-1} + \rho_{12}\widetilde{D}_{2,t-1} + \cdots + \rho_{1k}\widetilde{D}_{c,t-\Pi} + \epsilon_{1,t} \\ \widetilde{D}_{2,t} = \alpha_2 + \rho_{21}\widetilde{D}_{1,t-1} + \rho_{22}\widetilde{D}_{2,t-1} + \cdots + \rho_{2k}\widetilde{D}_{c,t-\Pi} + \epsilon_{2,t} \\ \qquad\qquad\qquad\qquad\qquad . \\ \qquad\qquad\qquad\qquad\qquad . \\ \qquad\qquad\qquad\qquad\qquad . \\ \widetilde{D}_{c,t} = \alpha_c + \rho_{k1}\widetilde{D}_{1,t-1} + \rho_{k2}\widetilde{D}_{2,t-1} + \cdots + \rho_{kk}\widetilde{D}_{c,t-\Pi} + \epsilon_{c,t} \end{cases} \tag{4.15}$$

The correlation coefficients, ρ_{ij},given by (4.15) refers to the *i*^th^ row and *j*^th^ column element of the $k \times k$ *cross-correlation* matrix *A.* Each variable is a linear function of the lag Π values for all variables in the set. Also, $\boldsymbol{\alpha} = (\alpha_1, \alpha_1, \ldots \alpha_c\acute{)}$ is a fixed $c \times 1$ vector of intercept terms. Note that the first equation in the recursive formulations given by (4.15), we have run the regression $\widetilde{D}_{1,t}$ on $\widetilde{D}_{1,t-1}, \ldots, \widetilde{D}_{c,t-1}, \ldots, \widetilde{D}_{1,t-\Pi}, \ldots, \widetilde{D}_{c,t-\Pi}$, and in the second equation, we regress $\widetilde{D}_{2,t}$ and so on.

Using statistical inference techniques developed for time series, the parameters of the autoregressive component namely the *cross-correlation* matrix, *A*, and the matrix of cross-covariance Σ can be estimated. We use R software package to estimate VAR model parameters. The function for estimating a VAR(Π) model is VAR(). It consists of seven arguments such as a data matrix, the appropriate lag-order, a desired information criterion, and the type of deterministic regressors. The details of these determinations can be found in (Pfaff & Taunus, 2008).

4.4 The Model Based on AARC and Budget Dynamic Uncertainty Set

In this section we discuss the formulation of our tractable closed-loop supply chain network under imperfect quality production that is introduced in chapter 2. Our proposed model assumes a single stage decision making or "here and now decision". In this model, however, we consider "wait and see decision". That is, the decisions are made over two sequential stages: the first stage variables determine long-term facility configurations which includes the number of selected suppliers, number of opened distribution centers, collection centers, and disposal centers. Thus, v_{tsm}, S_{ts}, DT_{ti}, CT_{tl}, and DO_{to} represent the "here and now" decision variables. Since our model includes multiple uncertain parameters, the first stage decision variables values are determined before the realization of these uncertain parameters.

The second stage decisions concern a plan for the product flows among facilities after realization of the uncertain parameters which include market demand, returned amount of product as used items and defective, recycling and reworking costs, and inspection errors. Thus, the "wait and see" decision variables are denoted by QSM_{tsm}, QDC_{tpic}, QRP_{tplm}, and QEP_{tplm}.

We assume that the quantity of raw material ordered from the suppliers, QSM_{tsm}, must be determined after the market demand is realized, while the quantity of product planned to be sent from the distribution centers to the customer zones, QDC_{tpic}, must be determined after the proportion of apparent defective items is realized. Finally, quantity of recyclable and reworkable products shipped from the collection centers to the manufacturers (QRP_{tplm}, QEP_{tplm}) must be determined before the realization of the returned amount of product as either used or defective items form the customer zones are realized, respectively. Thus,

$$QSM_{tsm} = \pi_{tsm(0)}^{QSM} + \widetilde{D}_{tc}\pi_{tsm(1)}^{QSM} \tag{4.16}$$

$$QDC_{tpic} = \pi_{tpic(0)}^{QDC} + \tilde{d}_{t}\pi_{tpic(1)}^{QDC} \tag{4.17}$$

$$QRP_{tplm} = \pi_{tplm(0)}^{QRP} + \tilde{R}_{tpc}\pi_{tplm(1)}^{QRP} \tag{4.18}$$

$$QEP_{tplm} = \pi_{tplm(0)}^{QEP} + \widetilde{Rw}_{tpc}\pi_{tplm(1)}^{QEP} \tag{4.19}$$

In Eq.(4.16-4.19), the adjustable variables are restricted to be affine functions of the uncertainties, where π_0 and $\pi_1$are non-adjustable variables which allow the adjustable decision variables to depend on the uncertain parameters.

Therefore, the corresponding AARC objective of model (2.1-2.15) under budget dynamic uncertainty set is given as follows:$Minimize\ Z_{AARC}$= Facility opening costs determined before the realization of the uncertainty at the first stage + the product flows among facilities after realization of the uncertainty at the second stage. Thus,

$$
\begin{aligned}
Minimize\ Z_{AARC} &= \sum_{t\in T}\sum_{s\in S} FS_s S_{ts} + \sum_{t\in T}\sum_{i\in I} FD_i DT_{ti} + \sum_{t\in T}\sum_{l\in L} FC_l CL_{tl} \\
&+ \sum_{t\in T}\sum_{o\in O} FO_o DO_{to} + \sum_{t\in T}\sum_{s\in S}\sum_{m\in M} W_{pm} v_{tsm} + P_s \pi^{QSM}_{tsm(0)} \\
&+ \sum_{t\in T}\sum_{s\in S}\sum_{m\in M} \pi^{QSM}_{tsm(0)} (Sc_s + TMc_{sm}) \\
&+ \sum_{t\in T}\sum_{p\in P}\sum_{m\in M}\sum_{i\in I} QMD_{tpmi}\big(Mc_{pm} + TPc_{pmi} + Ic_{pi}\big) \\
&+ \sum_{t\in T}\sum_{p\in P}\sum_{i\in I}\sum_{c\in C} \pi^{QDC}_{tpic(0)} \Big(Dc_{pi} + TZc_{pic} + h_{pi} + F_p\big(x_p;\mu_p\big)\Big) \\
&+ \sum_{t\in T}\sum_{p\in P}\sum_{c\in C}\sum_{l\in L} QCO_{tpcl}\big(Cc_{pl} + TOc_{pcl}\big) \\
&+ \sum_{t\in T}\sum_{p\in P}\sum_{l\in L}\sum_{m\in M} \pi^{QRP}_{tplm(0)}\big(TOPc_{plm}\big) \\
&+ \sum_{t\in T}\sum_{p\in P}\sum_{l\in L}\sum_{m\in M} \pi^{QEP}_{tplm(0)}\big(TOPc_{plm}\big) \\
&+ \sum_{t\in T}\sum_{p\in P}\sum_{c\in C}\sum_{l\in L} QCO_{tpcl} hw_{pl} \\
&+ \sum_{t\in T}\sum_{p\in P}\sum_{l\in L}\sum_{o\in O} QIP_{tplo}\big(Io_{po} + TIc_{plo}\big) + \sum_{t\in T}\sum_{p\in P}\sum_{c\in C} QNS_{tpc}\hat{\pi}_{pc} \\
&+ y^{QSM(1)} + y^{QSM(2)} + y^{d} + y^{QDC} + y^{QRP} + y^{QEP} \qquad (4.20)
\end{aligned}
$$

$$\sum_{t\in T}\sum_{s\in S}\sum_{c\in C}\sum_{m\in M} P_s.(\widetilde{D}_{tc}\pi^{QSM}_{tsm(1)}) \le y^{QSM(1)} \qquad (4.21)$$

$$\sum_{t\in T}\sum_{s\in S}\sum_{c\in C}\sum_{m\in M} \widetilde{D}_{tc}\pi^{QSM}_{tsm(1)}(Sc_s + TMc_{sm}) \le y^{QSM(2)} \qquad (4.22)$$

$$\sum_{t\in T}\sum_{p\in P}\sum_{m\in M}\sum_{i\in I} PR_p QMD_{tpmi}\, d_t + u^d\Gamma^d \le y^d \qquad (4.23)$$

$$u^{d} \geq \hat{d}_t QMD_{tpmi}, \qquad \forall\, t \in T, p \in P, m \in M, i \in I \tag{4.24}$$

$$\sum_{t\in T}\sum_{p\in P}\sum_{i\in I}\sum_{c\in C} d_t \pi^{QDC}_{tpic(1)} \left(Dc_{pi} + TZc_{pic} + h_{pi} + F_p\left(x_p;\mu_p\right)\right) + u^{QDC}\Gamma^{QDC} \leq y^{QDC} \tag{4.25}$$

$$u^{QDC} \geq \hat{d}_t\, \pi^{QDC}_{tpic(1)}, \qquad \forall\, t \in T, p \in P, i \in I, c \in C \tag{4.26}$$

$$\sum_{t\in T}\sum_{p\in P}\sum_{c\in C}\sum_{l\in L}\sum_{m\in M} Rc_{tpm}\pi^{QRP}_{tplm(0)} + Rc_{tpm}R_{tpc}\pi^{QRP}_{tplm(1)} + TOPc_{plm}R_{tpc}\pi^{QRP}_{tplm(1)} + u^{QRP}\Gamma^{QRP} \leq y^{QRP} \tag{4.27}$$

$$u^{QRP} \geq \widehat{Rc}_{tpm}\pi^{QRP}_{tplm(0)}, \qquad \forall\, t \in T, p \in P, l \in L, m \in M \tag{4.28}$$

$$u^{QRP} \geq \widehat{Rc}_{tpm}\hat{R}_{tpc}\pi^{QRP}_{tplm(1)}, \quad \forall\, t \in T, p \in P, c \in, C\ l \in L, m \in M \tag{4.29}$$

$$\sum_{t\in T}\sum_{p\in P}\sum_{l\in L}\sum_{m\in M} REc_{tpm}\pi^{QEP}_{tplm(0)} + REc_{tpm}Rw_{tpc}\pi^{QEP}_{tplm(1)} + TOPc_{plm}Rw_{tpc}\pi^{QEP}_{tplm(1)} + u^{QEP}\Gamma^{QEP} \leq y^{QEP} \tag{4.30}$$

$$u^{QEP} \geq \widehat{REc}_{tpm}\pi^{QEP}_{tplm(0)}, \qquad \forall\, t \in T, p \in P, l \in L, m \in M \tag{4.31}$$

$$u^{QEP} \geq \widehat{REc}_{tpm}\widehat{Rw}_{tpc}\pi^{QEP}_{tplm(1)}, \quad \forall\, t \in T, p \in P, l \in L, m \in M \tag{4.32}$$

$$\sum_{i\in I}\pi^{QDC}_{tpic(0)} + d_t\pi^{QDC}_{tpic(1)} + QNS_{tpc} + u^{QDC(2)}\Gamma^{QDC(2)} \geq \widetilde{D}_{tc}, \quad \forall\, t \in T, p \in P, c \in C \tag{4.33}$$

$$u^{QDC(2)} \geq \hat{d}_t\, \pi^{QDC(2)}_{tpic(1)}, \qquad \forall\, t \in T, p \in P, i \in I, c \in C \tag{4.34}$$

$$\sum_{l\in L} QCO_{tpcl} - R_{tpc} - Rw_{tpc} \leq \Gamma^{R+W} + \hat{R}_{tpc} + \widehat{Rw}_{tpc}, \quad \forall\, t \in T, p \in P, c \in C \tag{4.35}$$

$$\sum_{m\in M} QMD_{tpmi}(1-d_t) - u^{QMD}\Gamma^{QMD} \geq \sum_{c\in C}\pi^{QDC}_{tpic(0)} + d_t\pi^{QDC}_{tpic(1)}, \quad \forall\, t \in T, p \in P, i \in I \tag{4.36}$$

$$u^{QMD} \geq \hat{d}_t QMD_{tpmi}, \qquad \forall\, t \in T, p \in P, m \in M, i \in I \tag{4.37}$$

$$\sum_{s\in S}\left(\pi^{QSM}_{tsm(0)} + \widetilde{D}_{tc}\pi^{QSM}_{tsm(1)}\right) \leq B.\, v_{tsm}, \forall\, t \in T, s \in S, c \in C\ m \in M \tag{4.38}$$

$$\sum_{o\in O} QIP_{tplo} + \sum_{m\in M}\pi^{QRP}_{tplm(0)} + R_{tpc}\pi^{QRP}_{tplm(1)} + \sum_{m\in M}\pi^{QEP}_{tplm(0)} + Rw_{tpc}\pi^{QEP}_{tplm(1)} + u^{R+Rw}\Gamma^{R+Rw} \leq \sum_{c\in C} QCO_{tpcl}, \forall\, t \in T, p \in P, l \in L, c \in C \tag{4.39}$$

$$u^{R+Rw} \geq \hat{R}_{tpc}\, \pi^{QRP}_{tplm(1)}, \qquad \forall\, t \in T, p \in P, l \in L, c \in C\ m \in M \tag{4.40}$$

$$u^{R+Rw} \geq \widehat{Rw}_{tpc}\, \pi^{QEP}_{tplm(1)}, \qquad \forall\, t \in T, p \in P, l \in L, c \in C\ m \in M \tag{4.41}$$

$$\sum_{s\in S}\left(\pi^{QSM}_{tsm(0)} + \widetilde{D}_{tc}\pi^{QSM}_{tsm(1)}\right) + \sum_{l\in L}\sum_{p\in P}\pi^{QRP}_{tplm(0)} + R_{tpc}\pi^{QRP}_{tplm(1)} + \sum_{l\in L}\sum_{p\in P}\pi^{QEP}_{tplm(0)} + Rw_{tpc}\pi^{QEP}_{tplm(1)} + u^{RRw}\Gamma^{RRw} \leq \sum_{i\in I}\sum_{p\in P} QMD_{tpmi}\ , \forall\, t \in T, c \in C\ m \in M \tag{4.42}$$

$$u^{RRw} \geq \hat{R}_{tpc}\, \pi^{QRP}_{tplm(1)}, \qquad \forall\, t \in T, p \in P, l \in L, c \in C\ m \in M \tag{4.43}$$

$$u^{RRw} \geq \widehat{Rw}_{tpc}\, \pi^{QEP}_{tplm(1)}, \qquad \forall\, t \in T, p \in P, l \in L, c \in C\ m \in M \tag{4.44}$$

$$\sum_{c\in C}\sum_{p\in P}\beta_p . QCO_{tpcl} +\ u^{\beta}\Gamma^{\beta} \leq \sum_{o\in O}\sum_{p\in P} QIP_{tplo}\, , \forall\, t \in T, l \in L \tag{4.45}$$

$$u^{\beta} \geq \hat{\beta}_p . QCO_{tpcl}, \qquad \forall\, t \in T, p \in P, c \in C, l \in L \tag{4.46}$$

$$\sum_{m\in M}\pi^{QSM}_{tsm(0)} + \widetilde{D}_{tc}\pi^{QSM}_{tsm(1)} \leq CS_s S_{ts}, \qquad \forall\, t \in T, s \in, c \in C \tag{4.47}$$

Given constraints (2.10-2.15), and (4.12-4.14).

Note that $\widetilde{D}_{tc}$ is assumed to be subject to a budget dynamic uncertainty set described by (4.12-4.14). The description of the robust counterpart formulation is provided in Appendix B.

4.5 Numerical Example and Computational Results:

In this section, we illustrate the application of our affinely adjustable robust optimization framework where the market demand is subject to a dynamic polyhedral budget uncertainty set. We utilize three different probability bounds including those bounds which require solving additional nonlinear nonconvex optimization problems. In addition, we evaluate the robust solutions at different probability constraint violations, ε. Finally, we discuss the sensitivity and conservatism of the obtained robust solutions.

4.5.1 Numerical Example

The size of our artificial numerical example is explained next. The closed-loop supply chain system consisting of 12 periods, and 3 products, where the network is managed by 3 manufacturers. The required quantity of raw materials is ordered for production from 5 potential suppliers. Then, the produced lot size is sent to 5 potential distribution centers and finally moved to 10 customer zones according to customer demands. In the reverse network, the returned products (defective or used products) are shipped to 5 potential collection centers. The non-recyclable and non-reworkable items are disposed through 3 potential disposal centers.

Assume a data set of *C* market demand zones taken between years [2003, 2018] with a frequency of 12 periods (months). To estimate the correlation coefficients, ρ_{ij} of the $k \times k$ *cross-correlation* matrix *A*, given by (32), we need first to estimate the lag Π. The lag length for the VAR(Π) model may be determined using model selection criteria. The general approach is to fit VAR(Π) models with orders $r = 0, \dots$ Π, and choose the value of Π which minimizes some model selection criteria. The three most common information criteria are the Akaike (AIC), Schwarz-Bayesian (BIC) and Hannan-Quinn (HQ). For more information on the use of model selection criteria in VAR models see Lutkepohl (1991), chapter four.

Once the parameters of the autoregressive components are estimated, the recursive formulations given by (4.15) are generated. The details of these determinations are provided in the supplementary document.

The nominal values of the following uncertain parameters: $\tilde{R}_{tpc}$, $\widetilde{Rw}_{tpc}$, $\widetilde{Rc}_{tpm}$, $\widetilde{REc}_{tpm}$, $\tilde{\beta}_p$, ϵ_{ct}, and $\tilde{d}_t$ are generated randomly using the uniform distribution, as shown in Table 4.2. Note that the deviation magnitudes of the uncertain parameters are always set to be 0.1 of the nominal values. The random generated data of the proposed model parameters are given in Tables 4.3 and 4.4.

	Nominal Values for Product *p*		
Uncertain Parameter	1	2	3
$\tilde{R}_{tpc}$	*U* (44, 85)	*U* (38, 95)	*U* (61, 110)
$\widetilde{Rw}_{tpc}$	*U* (10, 36)	*U* (13, 43)	*U* (9, 26)
$\widetilde{Rc}_{tpm}$	*U* (9, 12)	*U* (6.5, 9)	*U* (6, 8)
$\widetilde{REc}_{tpm}$	*U* (4, 6)	*U* (4, 6.5)	*U* (3.5, 6)
$\tilde{\beta}_p$	0.2	0.175	0.18
ϵ_{ct}		*U(5, 30)*	
$\tilde{d}_t$		0.05	

Table 4.2: The nominal values of the model uncertain parameters for each product p.

Parameter	Values			Parameter	Values		
	Product 1 (p_1)	Product 2 (p_2)	Product 3 (p_3)		Product 1(p_1)	Product 2	Product 3
Sc_{ps}	~*U*(12.5, 15)	~*U*(10,12)	~*U*(8,13)	CI_{pi}	~*U*(575, 660)	~*U*(580,645)	~*U*(550,630)
Mc_{pm}	~*U*(40,45)	~*U*(38,42)	~*U*(43,45)	CL_{pl}	~*U*(235, 280)	~*U*(200, 245)	~*U*(220, 265)
Ic_{pi}	~*U*(5,6)	~*U*(3.75,5.75)	~*U*(4.5,5.5)	CO_{po}	~*U*(345,350)	~*U*(295,300)	~*U*(315, 320)
Dc_{pi}	~*U*(10,12)	~*U*(10,11)	~*U*(9.5,10.5)	TMc_{psm}	~*U*(5, 8)		
Cc_{pl}	~*U*(8,9.5)	~*U*(7,8)	~*U*(7.75,8.75)	TPc_{pmi}	~*U*(3, 4.75)		
h_{pi}	~*U*(3,4)	~*U*(4,4.5)	~*U*(4,5)	TOc_{pcl}	~*U*(4, 8)	0.75 +Values of (p_1)	1.2 +Values of (p_1)
P_{ps}	~*U*(6.5,10)	~*U*(5,6)	~*U*(3,7)	TZc_{pic}	~*U*(3, 5)		
Io_{po}	~*U*(3,3.5)	~*U*(3, 3.75)	~*U*(3,5)	$TOPc_{plm}$	~*U*(3.25, 5)		
CS_{ps}	~*U*(685, 800)	~*U*(720, 840)	~*U*(750, 780)	*Tic*	~*U*(4,5)		
CP_{pm}	~*U*(540, 650)	~*U*(500,600)	~*U*(590,620)				

Table 4.3: The randomly generated data of the proposed model parameters.

Parameter	Values	Parameter	Values
FS_s	~*U*(65000,81000)	USL_p	4.8
FD_i	~*U*(40000, 55000)	LSL_p	5.2
FC_l	~*U*(35000, 45000)	*K*	120
FO_o	~*U*(20000, 30000)	μ_p	5
hw_{pl}	~*U*(2, 2.5)	σ_p	0.05
$\hat{\pi}_{pc}$	~*U*(70000, 95000)		
W_{pm}	1000		

Table 4.4: Design of the data set.

The computations of MILP were run using the branch and bound algorithm accessed via LINGO16.0 on a PC -3GHzand; 4 GB RAM and under win 10. While computations of the nonlinear nonconvex optimization problems were run using BARON solver which is offered by GAMS modeling languages. The optimal uncertainty set sizes (Γ) using three probability bounds at five constraint violations ε are provided in Table 4.5. Note that in case B3, it is assumed that each ζ_j is subject to the uniform distribution in [−1, 1], and hence the three uncertainty sets apply. For the uniform distribution $U(a, b)$, the moment generating function is $E\left(e^{\theta\zeta}\right) = \frac{e^{\theta b} - e^{\theta a}}{\theta(b-a)}$. Also, in B4 the expected values of the parameters are only known to be within 1% of their nominal values. Therefore, $E[\tilde{a}_i] \, \epsilon \, [a_i - 0.01a_i, \, a_i + 0.01a_i]$and $E[\zeta_j] \, \epsilon \, [-0.1, 0.1]$ that is equivalent to $|E[\zeta_i]| \leq 0.1 = \mu_i$. The obtained robust solutions under different constraint violations are provided in table 4.6. Note that ($\Gamma^{\epsilon(1)}$,$\Gamma^{\epsilon(2)}$) are associated with the dynamic budget uncertainty set, and they are corresponding to constraints (4.21-4.22) and (4.38, 4.42, 4.47), respectively.

Γ^{Δ}	The Optimal Values of Γ^{Δ} B2	B3	B4	Constraint Violations
$\Gamma^{\epsilon(1)}$	26.81372	15.4422	38.3662	0.05
$\Gamma^{\epsilon(2)}$	2.44775	0.96321	1.00356	
Γ^{d}, Γ^{QDC}	8.47924	4.77114	9.04779	
Γ^{QRP}, Γ^{QEP}	52.95286	30.5528	99.2308	
$\Gamma^{QDC(2)}$, Γ^{QMD}, Γ^{R+W}, Γ^{R+Rw}	3.46164	1.67096	2.00196	
Γ^{RRw}	6.47613	3.57414	6.39249	
Γ^{β}	4.23962	2.18631	3.00241	
$\Gamma^{\epsilon(1)}$	23.50788	13.5462	35.1603	0.1
$\Gamma^{\epsilon(2)}$	2.14597	0.92642	1.00214	
Γ^{d}, Γ^{QDC}	7.43384	4.20847	8.18640	
Γ^{QRP}, Γ^{QEP}	46.42434	26.7899	92.7998	
$\Gamma^{QDC(2)}$, Γ^{QMD}, Γ^{R+W}, Γ^{R+Rw}	3.03485	1.53458	2.00150	
Γ^{RRw}	5.67769	3.16769	5.84108	
Γ^{β}	3.71692	1.97231	3.00054	
$\Gamma^{\epsilon(1)}$	21.33797	12.3	33.0476	0.15
$\Gamma^{\epsilon(2)}$	1.94788	0.88964	1.00179	
Γ^{d}, Γ^{QDC}	6.74766	3.83349	7.59772	
Γ^{QRP}, Γ^{QEP}	42.13911	24.3192	88.746	
$\Gamma^{QDC(2)}$, Γ^{QMD}, Γ^{R+W}, Γ^{R+Rw}	2.75472	1.42956	2.00097	
Γ^{RRw}	5.15361	2.89326	5.44232	
Γ^{β}	3.37383	1.81918	3.00005	
$\Gamma^{\epsilon(1)}$	19.65363	11.3318	31.4034	0.2
$\Gamma^{\epsilon(2)}$	1.79412	0.85285	1.00115	
Γ^{d}, Γ^{QDC}	6.21502	3.53967	7.12964	
Γ^{QRP}, Γ^{QEP}	38.81281	22.4009	85.2886	
$\Gamma^{QDC(2)}$, Γ^{QMD}, Γ^{R+W}, Γ^{R+Rw}	2.53727	1.34031	2.00100	
Γ^{RRw}	4.74680	2.67654	5.11654	
Γ^{β}	3.10751	1.69421	2.92976	
$\Gamma^{\epsilon(1)}$	18.24036	10.5189	30.021	0.25
$\Gamma^{\epsilon(2)}$	1.66511	0.81606	1.00081	
Γ^{d}, Γ^{QDC}	5.76811	3.29144	6.73008	
Γ^{QRP}, Γ^{QEP}	36.02182	20.791	82.531	
$\Gamma^{QDC(2)}$, Γ^{QMD}, Γ^{R+W}, Γ^{R+Rw}	2.35482	1.26069	2.00013	
Γ^{RRw}	4.40546	2.49243	4.83361	
Γ^{β}	2.88405	1.58565	2.80700	

Table 4.5: The optimal values of uncertainty set size parameters for the three upper probability bounds at different ε.

	Objective Function under Probability Bounds		
Constraint Violation	B2	B3	B4
0.05	5,645,527	5,618,405	5,637,858
0.1	5,640,151	5,607,357	5,629,852
0.15	5,634,591	5,601,234	5,621,852
0.2	5,628,473	5,594,248	5,614,877
0.25	5,621,476	5,581,637	5,606,974

Table 4.6: The robust solutions under different constraint violations.

4.5.2 Analysis and Discussion

In this section we discuss the sensitivity and conservatism of the obtained robust solutions under the three probability bounds. We refer to figure 4.2 which explains how the objective functions behave as the probability constraint violations increase for the three different bounds. The figure provides to the decision maker an overview of a conservatism comparison between the introduced uncertainty set under different probability bounds.

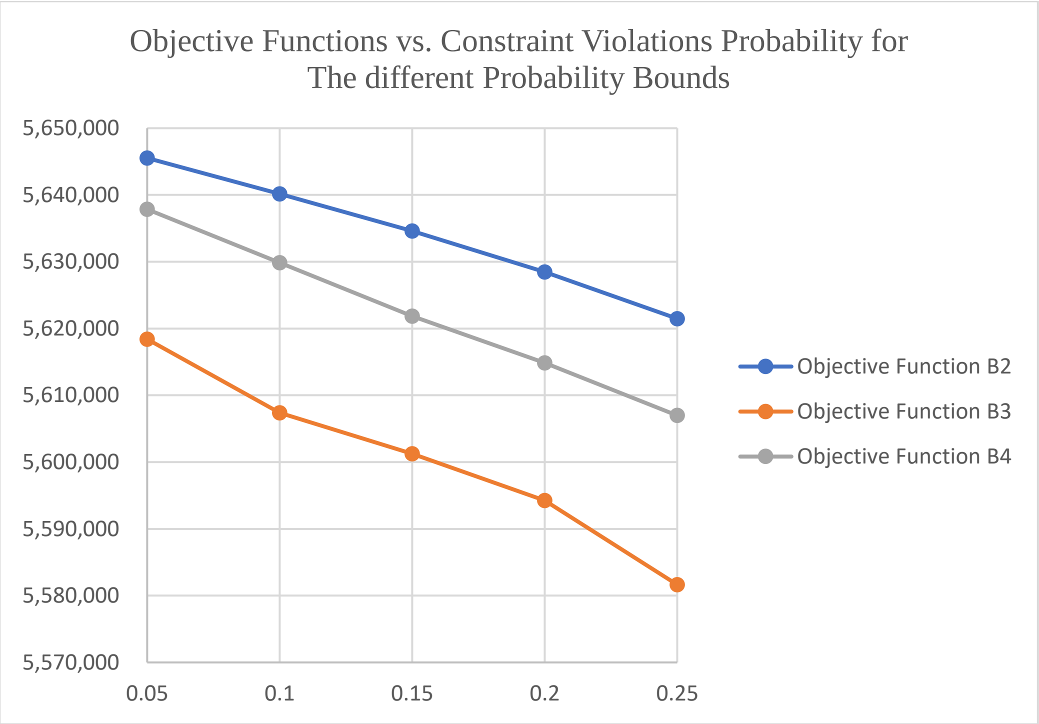


Figure 4.2: The behavior of the robust objective functions when different upper bounds are applied.

From figure 4.2, we make the following observations. In all probability upper bounds as the probability constraint violations increase, the robust objective functions tend to be less conservative. This is valid since we allow for a higher constraint violation, and hence we improve the performance of objective function. Also, the robust solution obtained by B3 is the least conservative (and hence the best solution) comparing with the other probability bounds. This would be a better choice due to full probability distribution information. If such information is available, it can be utilized beneficially which makes the solution less conservative. Besides to the affinely adjustable robust optimization framework, incorporating a budget dynamic uncertainly set can significantly improve the market demand forecasting and produce less conservative robust solutions.

To display the effectiveness of our closed -loop supply chain model under imperfect quality production, we consider the open version of our supply chain model. Unlike in the closed-loop or reverse supply chain, in the open-loop system, materials (products) are not returned and collected through the collection centers. Moreover, the scenarios of recycling and reworking products which can be as either defective or used are not considered in the open -loop case. As a result, the disposal centers are always not operational, see figure 4.3.

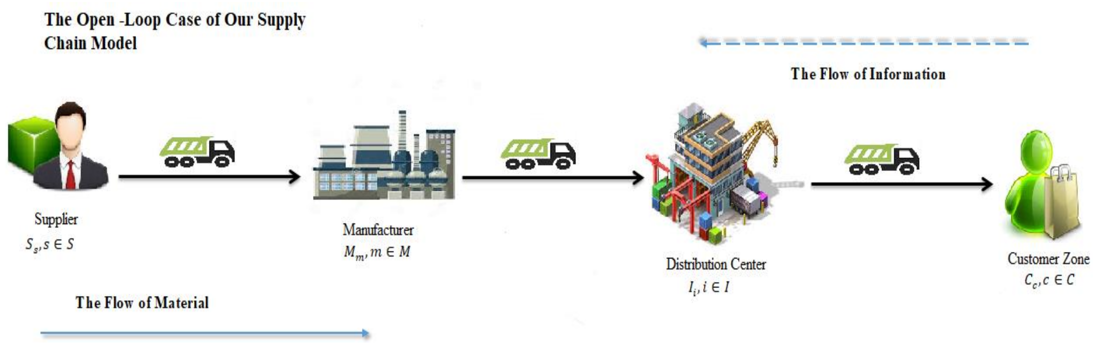


Figure 4.3: The open -loop system of our model.

To address the open loop system, models (2.10-2.15) and (4.20-4.47) developed in the previous sections and chapter 2 are modified such that all collection and disposal centers are closed and the associated opening facilities costs are omitted from the model. Note that the constraints referring to the return products are not considered.

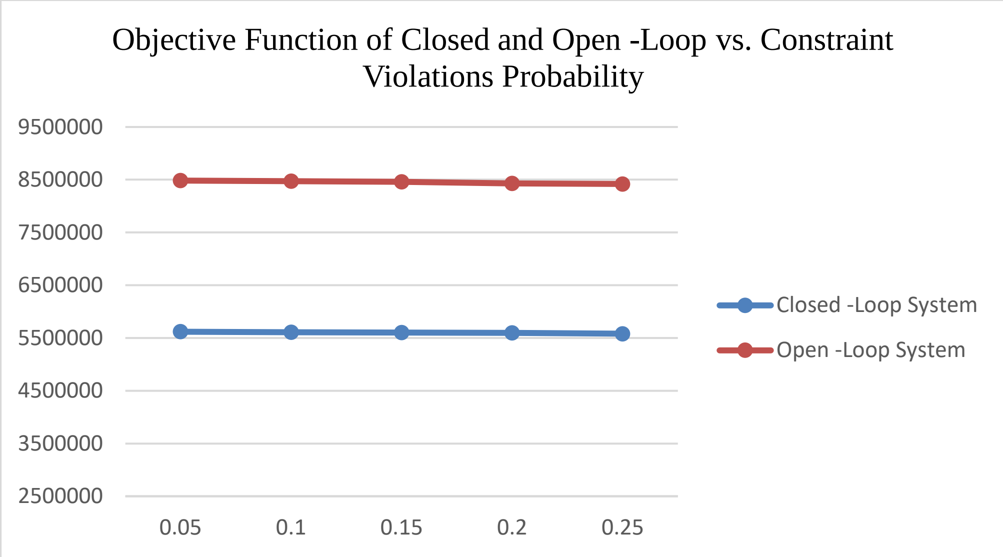


Figure 4.4: the total costs incurred in the open and closed- loop systems.

In figure 4.4, it is illustrated that the total costs incurred in the open- loop system is higher by at least 60% than the closed -loop system. In the scenario of the open -loop supply chain model, almost all the suppliers are selected, and distribution centers are operational. This is a necessary strategic planning as there are no returned products that can be used to satisfy the market demands although it would lead to high costs incurred due to opening facilities. In addition, manufacturing new products typically is more expensive than recycling used or reworking defective ones. We can see those aspects in a wide range of industries including steel making, electronic and automobile manufacturing, and various plastic products where the return products (either defective or used) can be reused as a raw material.

4.6 Conclusion

In this chapter, a robust optimization approach is applied to a novel closed loop supply chain design with multiple periods, echelons and uncertainties. The assumptions of imperfect quality production and that the inspection is not free of errors is practically sound. In the traditional uncertainty set-based robust approach, the uncertainty set is assumed static. We propose a budget dynamic uncertainty set to mimic the dynamic behavior of market demand over time, and the proposed approach is formulated according to Vector Autoregressive (VAR) models where the temporal and spatial correlations of customer demand zones are captured. In addition, the formulation is based on the affinely adjustable robust formulation.

Those aspects can significantly improve the market demand forecasting and produce less conservative robust solutions. Through the utilization of three different probability bounds at different probability constraint violations, ε, the robust solutions are evaluated. The results reveal valuable managerial views.

There are some interesting directions to extend this work. Besides to minimizing the total supply chain network costs, the model can consider multiple objective functions under uncertainty, where the economic, environmental, and social aspects are taken into consideration simultaneously. The problem may turn to a more complex, but of course, more interesting ,realistic, and worthwhile study. In addition, the market demand can be treated as an uncertain dynamic parameter. Another possible future work is to develop robust counterparts formulations based on different dynamic uncertainty sets such box and ellipsoidal uncertainty sets. The characteristics of each of the selected uncertainty sets provide to the decision maker a flexibility to design his own robust model based on his favorable robustness.

Appendix A: *Multivariate Time Series Analysis* :

Time series analysis comprises methods for analyzing time series data in order to extract meaningful statistics and other characteristics of the data. Multivariate Autoregressive models extend this approach to multiple time series so that the vector of current values of all variables is modelled as a linear sum of previous activities.

Let $\boldsymbol{D_t} = (D_{1t},\ D_{2t}, \dots, D_{ct})\acute{}$, $t = 0, \mp 1, \mp 2, \dots$, denote a *c*-dimensional time series vector of random variables of interest. The process $\{\boldsymbol{D_t}\}$ is a *stationary* if the probability distributions of the random vectors $\boldsymbol{D_t} = (D_{1t},\ D_{2t}, \dots, D_{ct})\acute{}$, and $\boldsymbol{D_t} = (\boldsymbol{D_{t_1+\Pi}},\ \boldsymbol{D_{t_2+\Pi}}, \dots, \boldsymbol{D_{t_n+\Pi}})\acute{}$ are the same for arbitrary times $t_1, t_2, \dots, t_n$, all *n*, and all lags or leads $\Pi = 0, \mp 1, \mp 2, \dots$. Thus, for a stationary process we must have $\boldsymbol{E}(\boldsymbol{D_t}) = \boldsymbol{\mu}$, constant for all *t*, where $\boldsymbol{\mu} = (\mu_1, \mu_2, \dots, \mu_c\ )\acute{}$ is the mean vector of the process. Also, the vectors $\boldsymbol{D_t}$ must have a constant covariance matrix for all *t*, which we denoted by Σ_D.

A MAR model predicts the next value in a *c*-dimensional time series, $\boldsymbol{D_t}$ as a linear combination of the Π previous vector values:

$$\widetilde{\boldsymbol{D}}_{\boldsymbol{t}} = \boldsymbol{\alpha} + \sum_{r=1}^{\Pi} \boldsymbol{A_r}\, \widetilde{\boldsymbol{D}}_{\boldsymbol{t-r}} + \boldsymbol{\epsilon_t} \tag{4.48}$$

In (64), the vector, ϵ_t, represents a residual term which is assumed uncertain in our model. Note that the vectors ϵ_t are independent across different time periods.

In addition, for a stationary process $\{\boldsymbol{D_t}\}$ the covariance between D_{it} and $D_{j,t+\Pi}$ must depend only on the difference in times $t+\Pi$ and t of the observations, that is, the time Π, not on time t, for $i, j = 1, \ldots, k, \Pi = 0, \mp 1, \mp 2, \ldots$. Hence, we let

$$\gamma_{ij}(\Pi) = \mathrm{Cov}\left(D_{it}, D_{j,t+\Pi}\right) = E[(D_{it} - \mu_i)(D_{j,t+\Pi} - \mu_j)]$$

(4.49)

and denote the $k \times k$ matrix of cross-covariance at lag Π as

$$\Sigma_D = \Gamma(\Pi) = E[(\boldsymbol{D_t} - \boldsymbol{\mu})(\boldsymbol{D_{t+\Pi}} - \boldsymbol{\acute{\mu}})] = \begin{bmatrix} \gamma_{11}(\Pi) & \gamma_{12}(\Pi) & . & . & \gamma_{1k}(\Pi) \\ \gamma_{21}(\Pi) & \gamma_{22}(\Pi) & . & . & \gamma_{2k}(\Pi) \\ . & . & . & . & . \\ . & . & . & . & . \\ \gamma_{k1}(\Pi) & \gamma_{k2}(\Pi) & . & . & \gamma_{kk}(\Pi) \end{bmatrix} \quad (4.50)$$

Also, the corresponding *cross-correlation* matrix at lag Π is denoted by

$$A = \rho(\Pi) = \begin{bmatrix} \rho_{11}(\Pi) & \rho_{12}(\Pi) & . & . & \rho_{1k}(\Pi) \\ \rho_{21}(\Pi) & \rho_{22}(\Pi) & . & . & \rho_{2k}(\Pi) \\ . & . & . & . & . \\ . & . & . & . & . \\ \rho_{k1}(\Pi) & \rho_{k2}(\Pi) & . & . & \rho_{kk}(\Pi) \end{bmatrix}, \text{ given that}$$

$$\rho_{ij}(\Pi) = \mathrm{Corr}\left(D_{it}, D_{j,t+\Pi}\right) = \frac{\gamma_{ij}(\Pi)}{[\gamma_{ii}(0)\,\gamma_{jj}(0)]^{1/2}} \quad (4.51)$$

Appendix B: *The Definition of Robust Counterpart Formulation*:

Consider the following linear programming,

$$Min \sum_j \tilde{c}_j x_j$$

$$\text{s.t.} \sum_j \tilde{a}_{ij} x_j \leq \tilde{b}_i \quad \forall i$$

where $\tilde{a}_{ij}$, $\tilde{b}_i$, and $\tilde{c}_j$, represent the true value of the parameters which are subject to uncertainty and defined as follows:

$$\tilde{a}_{ij} = a_{ij} + \zeta_{ij}\hat{a}_{ij} \quad \forall j \in J_i$$

$$\tilde{b}_i = b_i + \zeta_{ij}\hat{b}_i$$

$$\tilde{c}_j = c_j + \zeta_j \hat{c}_j \qquad \forall j \in J_i$$

where a_{ij}, b_i, and c_j represent the nominal (expected) value of the parameters; $\hat{a}_{ij}$, $\hat{b}_i$, and $\hat{c}_j$ represent constant perturbation; ζ_{ij} is a random variable that takes values in the interval [-1, 1]. Without loss of generality, we make the following assumptions:

- If uncertain data exists in the objective function as coefficients, then the objective function can be written as a constraint.
- In any constraint j, if the right-hand-side parameter is subject to uncertainty, then the model can be written as:

$Min\ Z$

s.t. $\sum_j \tilde{c}_j x_j \leq Z$

$$\tilde{b}_i - \sum_j \tilde{a}_{ij} x_j \leq 0 \quad \forall i$$

Therefore, we end up with a constraint that has uncertain parameters on the left-hand-side only.

Supplementary

- **Data set used in the chapter 4 :**

ï..Zone1	Zone2	Zone3	Zone4	Zone5	Zone6	Zone7	Zone8	Zone9	Zone10
Jan 2003	86	82	122	86	80	91	137	179	128	103
Feb 2003	85	96	73	147	107	101	99	128	82	82
Mar 2003	104	97	83	141	104	68	117	102	117	119
Apr 2003	161	67	75	190	86	86	89	117	93	73
May 2003	87	101	143	133	112	82	99	181	177	116
Jun 2003	114	64	132	98	94	64	95	140	142	78
Jul 2003	75	83	123	97	129	109	88	90	105	82
Aug 2003	102	91	90	110	80	54	113	102	153	137
Sep 2003	71	119	147	185	141	126	127	183	121	78
Oct 2003	79	70	98	89	146	56	131	84	90	86
Nov 2003	145	128	84	178	152	51	128	166	104	120
Dec 2003	118	137	113	163	135	74	118	160	66	106
Jan 2004	69	144	92	176	93	90	89	137	174	99
Feb 2004	115	120	145	118	97	71	134	137	64	132
Mar 2004	111	119	85	175	100	76	97	138	182	85
Apr 2004	140	109	167	164	100	115	119	84	150	103
May 2004	138	130	144	91	119	95	104	176	143	113
Jun 2004	145	146	116	183	127	130	125	172	83	98
Jul 2004	68	111	82	180	119	112	124	145	58	137
Aug 2004	165	136	118	89	146	90	118	164	154	80
Sep 2004	67	74	109	158	135	52	95	131	66	137
Oct 2004	82	110	163	138	80	83	117	129	131	130
Nov 2004	150	121	112	185	135	122	137	59	133	72
Dec 2004	144	61	77	165	91	79	144	136	162	106
Jan 2005	150	146	137	156	151	64	117	162	117	127
Feb 2005	96	96	157	146	154	130	117	147	105	80
Mar 2005	120	128	122	142	112	92	140	62	105	89

Apr 2005	93	133	129	142	89	68	106	141	183	86
May 2005	153	66	94	146	92	103	125	65	92	132
Jun 2005	72	59	131	96	126	59	149	78	179	101
Jul 2005	139	75	121	87	120	125	128	50	63	103
Aug 2005	124	84	163	111	147	92	91	139	128	83
Sep 2005	76	97	114	147	102	105	124	157	55	110
Oct 2005	158	118	149	128	101	47	124	69	132	100
Nov 2005	112	115	126	111	132	71	137	69	175	108
Dec 2005	127	78	157	120	152	94	101	84	76	112
Jan 2006	92	100	131	159	152	61	104	60	190	134
Feb 2006	130	92	163	134	124	52	137	71	94	93
Mar 2006	101	108	126	185	86	73	146	121	55	116
Apr 2006	97	73	129	107	133	87	129	123	78	94
May 2006	92	116	162	143	99	47	143	127	124	73
Jun 2006	147	64	76	186	117	56	107	163	145	73
Jul 2006	128	118	79	151	83	111	149	128	163	76
Aug 2006	164	139	109	143	144	54	110	107	76	85
Sep 2006	73	88	156	149	115	75	108	184	99	103
Oct 2006	146	74	163	94	121	92	111	89	128	75
Nov 2006	125	103	83	163	112	123	149	150	115	88
Dec 2006	86	69	86	145	122	94	139	171	124	73
Jan 2007	70	146	85	150	146	68	143	169	166	133
Feb 2007	138	119	92	92	134	95	88	170	95	88
Mar 2007	93	86	147	171	113	74	124	165	84	135
Apr 2007	102	114	93	176	82	73	135	142	142	113
May 2007	127	124	146	173	109	48	139	144	87	102
Jun 2007	76	73	102	117	94	108	111	78	86	107
Jul 2007	113	126	111	112	111	94	131	184	74	117
Aug 2007	163	142	89	107	96	121	113	140	136	127
Sep 2007	111	137	139	119	134	72	134	102	151	123

Oct 2007	110	102	99	111	85	128	84	179	118	87
Nov 2007	91	133	163	110	154	55	114	180	61	119
Dec 2007	121	59	123	148	122	52	84	152	96	89
Jan 2008	155	76	148	115	112	105	85	119	108	78
Feb 2008	111	107	147	105	103	124	99	87	67	125
Mar 2008	161	117	94	175	117	74	98	59	70	88
Apr 2008	135	83	103	156	150	113	91	158	78	130
May 2008	95	68	83	121	111	119	104	81	190	103
Jun 2008	91	81	133	136	138	97	147	102	153	75
Jul 2008	125	58	84	182	130	84	144	66	132	137
Aug 2008	83	99	156	88	114	105	127	88	124	135
Sep 2008	153	119	167	106	99	89	91	90	170	94
Oct 2008	161	113	152	161	84	111	114	81	178	104
Nov 2008	133	56	170	112	151	128	101	164	83	130
Dec 2008	157	90	93	189	95	104	116	67	146	85
Jan 2009	141	116	134	114	147	82	99	124	82	113
Feb 2009	76	147	156	187	145	68	110	167	92	130
Mar 2009	162	147	166	160	140	109	87	167	127	85
Apr 2009	165	63	125	104	148	92	139	103	101	116
May 2009	111	117	141	169	104	79	105	132	157	114
Jun 2009	80	114	164	187	104	90	82	156	168	129
Jul 2009	90	101	134	143	100	92	123	167	178	128
Aug 2009	102	60	137	91	131	112	107	75	189	104
Sep 2009	145	132	79	100	96	57	141	93	87	96
Oct 2009	90	138	130	93	81	87	132	109	104	82
Nov 2009	130	80	157	135	138	129	113	77	59	124
Dec 2009	132	109	139	154	145	112	108	117	133	99
Jan 2010	109	127	128	110	133	75	95	77	170	119
Feb 2010	107	55	139	181	106	96	141	62	154	113
Mar 2010	162	69	92	91	103	129	87	87	60	105

Apr 2010	83	88	91	175	105	101	117	144	152	89
May 2010	95	123	170	152	100	59	130	74	69	123
Jun 2010	76	113	166	136	122	62	129	51	128	73
Jul 2010	148	58	86	176	117	80	133	172	88	122
Aug 2010	65	77	112	149	142	46	137	153	122	77
Sep 2010	66	58	104	115	146	76	93	88	163	127
Oct 2010	96	142	88	98	86	60	109	177	174	137
Nov 2010	97	136	112	177	108	83	128	127	134	123
Dec 2010	128	139	166	167	151	66	117	77	184	134
Jan 2011	122	81	90	172	119	52	121	76	162	80
Feb 2011	88	133	87	88	100	96	107	166	62	112
Mar 2011	126	131	85	177	139	78	98	71	112	92
Apr 2011	144	132	131	135	113	126	122	106	147	85
May 2011	144	72	85	166	87	128	111	152	114	133
Jun 2011	151	116	134	163	86	109	127	133	140	112
Jul 2011	156	141	99	154	82	74	132	144	112	75
Aug 2011	136	138	133	98	126	73	109	59	89	136
Sep 2011	135	56	106	136	139	109	84	117	112	104
Oct 2011	147	86	106	131	115	117	116	118	63	85
Nov 2011	96	118	73	175	126	89	102	61	162	110
Dec 2011	78	142	105	109	82	113	148	67	130	99
Jan 2012	131	111	95	110	137	105	123	64	129	88
Feb 2012	105	73	103	130	130	57	106	122	118	94
Mar 2012	133	72	115	181	152	87	84	120	152	110
Apr 2012	131	116	112	158	118	85	88	123	109	97
May 2012	70	147	170	91	87	71	85	148	57	101
Jun 2012	127	63	163	85	98	102	91	62	102	79
Jul 2012	73	131	150	85	105	81	85	156	142	74
Aug 2012	81	105	120	168	102	70	94	58	183	107
Sep 2012	140	137	167	108	115	45	96	180	106	116

Oct 2012	125	137	88	115	91	71	121	74	115	139
Nov 2012	163	129	74	91	152	77	102	90	189	108
Dec 2012	104	105	134	93	146	103	101	123	71	114
Jan 2013	163	135	145	129	103	116	85	114	84	103
Feb 2013	95	104	109	187	151	126	126	169	71	113
Mar 2013	89	71	146	177	121	95	141	89	109	103
Apr 2013	137	68	162	110	126	71	98	84	104	73
May 2013	153	123	168	136	146	60	126	153	75	95
Jun 2013	84	100	113	113	94	119	143	148	74	120
Jul 2013	105	76	169	91	115	128	119	83	70	77
Aug 2013	127	56	100	141	98	55	143	147	141	134
Sep 2013	96	94	161	100	133	82	93	161	137	138
Oct 2013	149	115	117	102	149	68	92	80	88	129
Nov 2013	128	105	155	173	140	116	131	82	189	128
Dec 2013	71	88	72	89	81	123	142	125	88	103
Jan 2014	114	90	127	86	104	89	139	51	179	107
Feb 2014	102	123	130	168	95	117	114	170	137	136
Mar 2014	132	103	110	138	91	71	132	127	120	105
Apr 2014	65	140	135	168	139	86	95	93	66	92
May 2014	151	147	96	97	140	104	92	77	127	101
Jun 2014	158	119	80	153	86	112	93	181	106	121
Jul 2014	103	135	116	165	86	118	118	163	84	123
Aug 2014	134	115	157	166	83	66	82	184	146	100
Sep 2014	81	90	168	120	87	101	92	164	171	81
Oct 2014	115	127	133	111	116	105	97	74	140	112
Nov 2014	131	55	102	128	109	72	121	164	127	134
Dec 2014	160	120	133	117	131	106	91	165	169	100
Jan 2015	159	79	155	108	99	55	82	75	73	93
Feb 2015	131	107	76	171	135	45	126	50	139	74
Mar 2015	157	87	147	157	132	59	122	142	187	84

Apr 2015	117	93	167	165	144	55	134	96	101	108
May 2015	144	56	151	153	125	115	127	159	160	124
Jun 2015	147	60	73	148	101	77	135	156	146	99
Jul 2015	112	115	98	178	97	130	138	57	67	109
Aug 2015	69	115	168	166	83	120	122	74	177	114
Sep 2015	129	107	119	109	136	92	130	185	137	133
Oct 2015	123	137	137	132	90	63	105	90	129	97
Nov 2015	165	73	121	98	115	80	99	66	160	136
Dec 2015	85	119	133	98	139	118	128	118	129	137
Jan 2016	79	115	125	162	130	83	117	180	185	123
Feb 2016	164	72	130	111	96	111	89	162	166	132
Mar 2016	155	134	95	115	142	106	123	100	97	81
Apr 2016	81	131	113	125	141	59	113	129	178	131
May 2016	157	130	139	127	148	53	88	165	117	103
Jun 2016	165	95	94	123	129	78	112	178	166	126
Jul 2016	76	108	106	127	109	69	144	122	156	89
Aug 2016	141	59	157	181	151	115	140	62	142	99
Sep 2016	151	97	130	158	129	83	87	165	164	131
Oct 2016	110	126	130	157	93	49	136	168	104	115
Nov 2016	118	103	128	86	107	49	102	56	180	111
Dec 2016	100	104	134	165	134	92	85	155	142	127
Jan 2017	141	102	92	174	124	51	107	117	140	100
Feb 2017	160	116	157	142	145	96	120	62	128	132
Mar 2017	113	142	127	104	130	72	139	149	163	91
Apr 2017	103	120	145	140	84	120	147	95	114	126
May 2017	76	66	121	120	112	69	102	116	161	124
Jun 2017	95	80	158	101	106	116	126	56	134	103
Jul 2017	121	83	81	168	139	127	127	172	143	113
Aug 2017	87	115	156	106	154	63	96	180	119	97
Sep 2017	75	97	118	95	96	62	111	175	71	98

Oct 2017	161	81	129	88	151	90	137	175	62	134
Nov 2017	97	62	140	173	138	84	83	54	183	118
Dec 2017	144	61	129	87	97	88	102	149	139	92
Jan 2018	74	113	71	179	133	113	133	133	165	116
Feb 2018	161	70	105	126	80	82	108	51	147	114
Mar 2018	82	62	74	119	105	72	120	89	91	130
Apr 2018	75	60	134	179	114	80	116	166	155	89
May 2018	113	66	74	142	154	63	145	161	127	120
Jun 2018	75	60	145	181	128	128	121	134	98	79
Jul 2018	105	96	137	96	87	106	135	157	188	90
Aug 2018	132	118	71	149	116	127	101	122	151	121
Sep 2018	104	144	168	160	155	55	148	107	106	112
Oct 2018	65	137	114	167	144	108	136	121	110	126
Nov 2018	140	66	77	139	137	56	90	134	149	112
Dec 2018	84	100	156	111	148	60	130	110	113	137

- The behavior of the market demand at each zone and period:

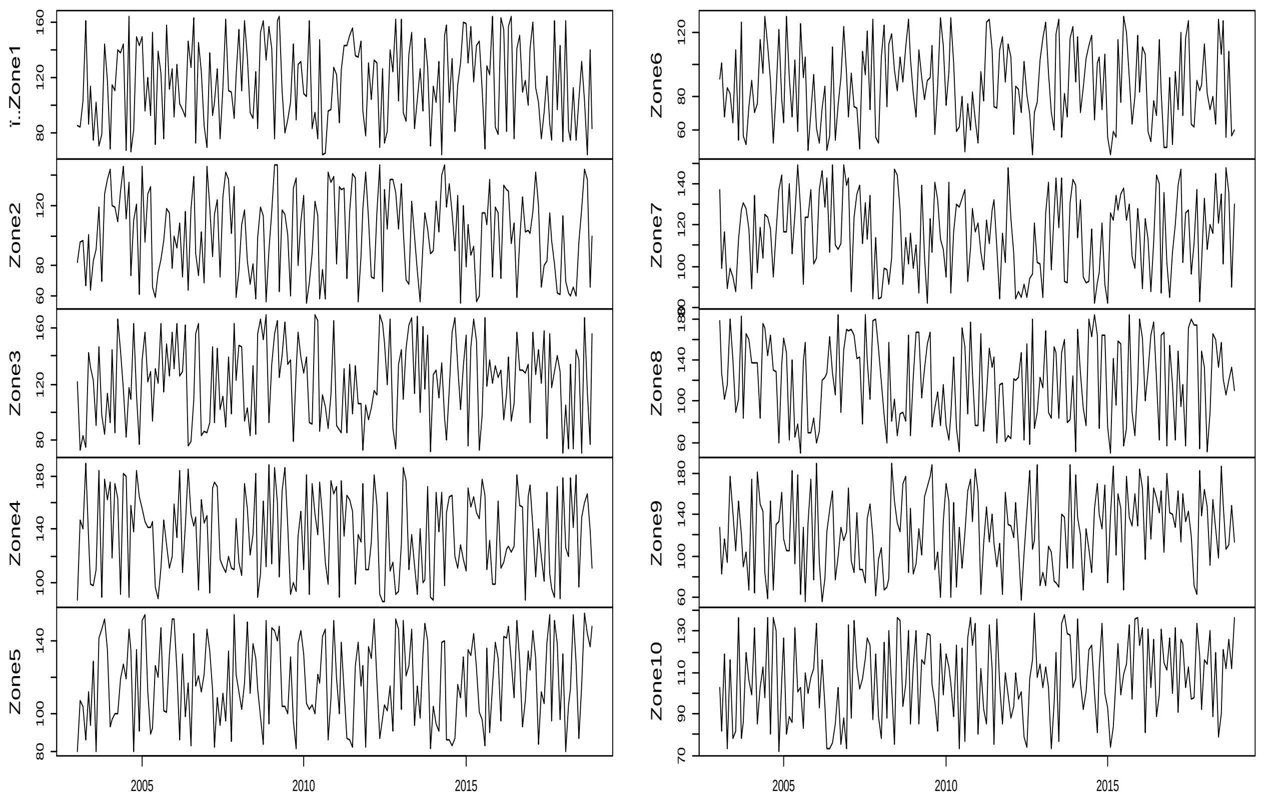


Figure 4.5: The behavior of the market demand at each zone and period.

- Lag Criteria selection:

VARselect(data2, lag.max = 5, type = "both")

AIC(n) HQ(n) SC(n) FPE(n)

2 1 1 2

$criteria

	1	2	3	4	5
AIC(n)	6.655835e+01	6.715162e+01	6.757084e+01	6.799316e+01	6.832601e+01
HQ(n)	6.739851e+01	6.869191e+01	6.981126e+01	7.093372e+01	7.196669e+01
SC(n)	6.863179e+01	7.095292e+01	7.310001e+01	7.525020e+01	7.731091e+01
FPE(n)	8.066597e+28	1.473431e+29	2.293038e+29	3.655019e+29	5.481780e+29

- Generating dynamic recursive equations:

$$\begin{aligned}
\widetilde{D}_{1,t} = {} & 78.878294 - 0.086340\,\widetilde{D}_{1,t-1} + 0.109930\,\widetilde{D}_{2,t-1} + 0.119004\,\widetilde{D}_{3,t-1} \\
& + 0.023933\,\widetilde{D}_{4,t-1} - 0.054209\,\widetilde{D}_{5,t-1} + 0.073547\,\widetilde{D}_{6,t-1} \\
& - 0.039347\widetilde{D}_{7,t-1} - 0.042888\,\widetilde{D}_{8,t-1} + 0.092220\,\widetilde{D}_{9,t-1} \\
& + 0.045914\,\widetilde{D}_{10,t-1} - 0.021736\,\widetilde{D}_{1,t-2} + 0.108956\,\widetilde{D}_{2,t-2} \\
& + 0.099983\,\widetilde{D}_{3,t-2} - 0.030266\,\widetilde{D}_{4,t-2} + 0.091980\,\widetilde{D}_{5,t-2} \\
& - 0.077802\,\widetilde{D}_{6,t-2} - 0.005297\widetilde{D}_{7,t-2} + 0.020798\,\widetilde{D}_{8,t-2} \\
& - 0.039784\,\widetilde{D}_{9,t-2} - 0.025672\,\widetilde{D}_{10,t-2}
\end{aligned}$$

$$\begin{aligned}
\widetilde{D}_{2,t} = {} & 95.321945 + 0.068930\,\widetilde{D}_{1,t-1} + 0.1796600\,\widetilde{D}_{2,t-1} - 0.164288\,\widetilde{D}_{3,t-1} \\
& - 0.017699\,\widetilde{D}_{4,t-1} - 0.093021\,\widetilde{D}_{5,t-1} + 0.029761\,\widetilde{D}_{6,t-1} \\
& - 0.200601\,\widetilde{D}_{7,t-1} + 0.010177\,\widetilde{D}_{8,t-1} - 0.018151\,\widetilde{D}_{9,t-1} \\
& + 0.118296\,\widetilde{D}_{10,t-1} + 0.001663\,\widetilde{D}_{1,t-2} - 0.045539\,\widetilde{D}_{2,t-2} \\
& - 0.032239\,\widetilde{D}_{3,t-2} + 0.072001\,\widetilde{D}_{4,t-2} + 0.110241\,\widetilde{D}_{5,t-2} \\
& + 0.121883\,\widetilde{D}_{6,t-2} - 0.036194\,\widetilde{D}_{7,t-2} + 0.012520\,\widetilde{D}_{8,t-2} \\
& - 0.047897\,\widetilde{D}_{9,t-2} - 0.051237\,\widetilde{D}_{10,t-2}
\end{aligned}$$

$$\begin{aligned}
\widetilde{D}_{3,t} = {} & 91.021949 + 0.006942\,\widetilde{D}_{1,t-1} - 0.012449\,\widetilde{D}_{2,t-1} + 0.031256\,\widetilde{D}_{3,t-1} \\
& + 0.060516\,\widetilde{D}_{4,t-1} - 0.137204\,\widetilde{D}_{5,t-1} - 0.172722\,\widetilde{D}_{6,t-1} \\
& - 0.156500\widetilde{D}_{7,t-1} - 0.013626\,\widetilde{D}_{8,t-1} - 0.011580\,\widetilde{D}_{9,t-1} \\
& + 0.171030\,\widetilde{D}_{10,t-1} + 0.187244\,\widetilde{D}_{1,t-2} + 0.009811\,\widetilde{D}_{2,t-2} \\
& + 0.050655\,\widetilde{D}_{3,t-2} - 0.069318\,\widetilde{D}_{4,t-2} - 0.018403\,\widetilde{D}_{5,t-2} \\
& + 0.054601\,\widetilde{D}_{6,t-2} + 0.212725\,\widetilde{D}_{7,t-2} + 0.047883\,\widetilde{D}_{8,t-2} \\
& - 0.0177257\,\widetilde{D}_{9,t-2} + 0.037576\,\widetilde{D}_{10,t-2}
\end{aligned}$$

$$\begin{aligned}
\widetilde{D}_{4,t} = {} & 155.4 + 0.1587\,\widetilde{D}_{1,t-1} + 0.06195\,\widetilde{D}_{2,t-1} - 0.008855\,\widetilde{D}_{3,t-1} \\
& - 0.002605\,\widetilde{D}_{4,t-1} + 0.1063\,\widetilde{D}_{5,t-1} - 0.04936\,\widetilde{D}_{6,t-1} \\
& + 0.06932\widetilde{D}_{7,t-1} + 0.04060\,\widetilde{D}_{8,t-1} - 0.09607\,\widetilde{D}_{9,t-1} \\
& - 0.037533\,\widetilde{D}_{10,t-1} - 0.05686\,\widetilde{D}_{1,t-2} - 0.098981\,\widetilde{D}_{2,t-2} \\
& - 0.06733\,\widetilde{D}_{3,t-2} - 0.02262\,\widetilde{D}_{4,t-2} - 0.007478\,\widetilde{D}_{5,t-2} \\
& + 0.1243\,\widetilde{D}_{6,t-2} - 0.1003\,\widetilde{D}_{7,t-2} - 0.07455\,\widetilde{D}_{8,t-2} \\
& + 0.0008324\,\widetilde{D}_{9,t-2} - 0.08390\,\widetilde{D}_{10,t-2}
\end{aligned}$$

$$\begin{aligned}
\widetilde{D}_{5,t} = {} & 113.7 + 0.04913\,\widetilde{D}_{1,t-1} - 0.002244\,\widetilde{D}_{2,t-1} - 0.06855\,\widetilde{D}_{3,t-1} \\
& - 0.05505\,\widetilde{D}_{4,t-1} + 0.1033\,\widetilde{D}_{5,t-1} - 0.05684\,\widetilde{D}_{6,t-1} \\
& + 0.1387\widetilde{D}_{7,t-1} - 0.01224\,\widetilde{D}_{8,t-1} - 0.008922\,\widetilde{D}_{9,t-1} \\
& + 0.002732\,\widetilde{D}_{10,t-1} - 0.03780\,\widetilde{D}_{1,t-2} + 0.03813\,\widetilde{D}_{2,t-2} \\
& - 0.00005268\,\widetilde{D}_{3,t-2} - 0.01241\,\widetilde{D}_{4,t-2} - 0.03020\,\widetilde{D}_{5,t-2} \\
& - 0.09132\,\widetilde{D}_{6,t-2} + 0.03495\,\widetilde{D}_{7,t-2} + 0.001576\,\widetilde{D}_{8,t-2} \\
& - 0.004235\,\widetilde{D}_{9,t-2} + 0.03766\,\widetilde{D}_{10,t-2}
\end{aligned}$$

$$
\begin{aligned}
\widetilde{D}_{6,t} = 75.79478 + {} & 0.08578\,\widetilde{D}_{1,t-1} - 0.02812\,\widetilde{D}_{2,t-1} + 0.04961\,\widetilde{D}_{3,t-1} \\
& - 0.06831\,\widetilde{D}_{4,t-1} - 0.10721\,\widetilde{D}_{5,t-1} + 0.05984\,\widetilde{D}_{6,t-1} \\
& + 0.11032\,\widetilde{D}_{7,t-1} + -0.01548\,\widetilde{D}_{8,t-1} + 0.02285\,\widetilde{D}_{9,t-1} \\
& + 0.05964\,\widetilde{D}_{10,t-1} + 0.08563\,\widetilde{D}_{1,t-2} + 0.05921\,\widetilde{D}_{2,t-2} \\
& - 0.01844\,\widetilde{D}_{3,t-2} - 0.04296\,\widetilde{D}_{4,t-2} + 0.02846\,\widetilde{D}_{5,t-2} \\
& - 0.03872\,\widetilde{D}_{6,t-2} - 0.09358\,\widetilde{D}_{7,t-2} - 0.04732\,\widetilde{D}_{8,t-2} \\
& - 0.07069\,\widetilde{D}_{9,t-2} + 0.11808\,\widetilde{D}_{10,t-2}
\end{aligned}
$$

$$
\begin{aligned}
\widetilde{D}_{7,t} = 77.108989 + {} & 0.008744\,\widetilde{D}_{1,t-1} - 0.046470\,\widetilde{D}_{2,t-1} - 0.033333\,\widetilde{D}_{3,t-1} \\
& + 0.047098\,\widetilde{D}_{4,t-1} - 0.008751\,\widetilde{D}_{5,t-1} + 0.067757\,\widetilde{D}_{6,t-1} \\
& + 0.141424\,\widetilde{D}_{7,t-1} - 0.007236\,\widetilde{D}_{8,t-1} + 0.030365\,\widetilde{D}_{9,t-1} \\
& - 0.085690\,\widetilde{D}_{10,t-1} - 0.027492\,\widetilde{D}_{1,t-2} - 0.023492\,\widetilde{D}_{2,t-2} \\
& + 0.078597\,\widetilde{D}_{3,t-2} + 0.053659\,\widetilde{D}_{4,t-2} + 0.046921\,\widetilde{D}_{5,t-2} \\
& - 0.009055\,\widetilde{D}_{6,t-2} + 0.017410\,\widetilde{D}_{7,t-2} - 0.027098\,\widetilde{D}_{8,t-2} \\
& + 0.023197\,\widetilde{D}_{9,t-2} + 0.064786\,\widetilde{D}_{10,t-2}
\end{aligned}
$$

$$
\begin{aligned}
\widetilde{D}_{8,t} = 109.275745 + {} & 0.111486\,\widetilde{D}_{1,t-1} + 0.012849\,\widetilde{D}_{2,t-1} + 0.012574\,\widetilde{D}_{3,t-1} \\
& + 0.117133\,\widetilde{D}_{4,t-1} + 0.104027\,\widetilde{D}_{5,t-1} + 0.049200\,\widetilde{D}_{6,t-1} \\
& - 0.133409\,\widetilde{D}_{7,t-1} + 0.057492\,\widetilde{D}_{8,t-1} + 0.049676\,\widetilde{D}_{9,t-1} \\
& + 0.002191\,\widetilde{D}_{10,t-1} - 0.207120\,\widetilde{D}_{1,t-2} + 0.118808\,\widetilde{D}_{2,t-2} \\
& - 0.051294\,\widetilde{D}_{3,t-2} - 0.077960\,\widetilde{D}_{4,t-2} + 0.147847\,\widetilde{D}_{5,t-2} \\
& + 0.107461\,\widetilde{D}_{6,t-2} + 0.214762\,\widetilde{D}_{7,t-2} - 0.067192\,\widetilde{D}_{8,t-2} \\
& - 0.068820\,\widetilde{D}_{9,t-2} - 0.385775\,\widetilde{D}_{10,t-2}
\end{aligned}
$$

$$
\begin{aligned}
\widetilde{D}_{9,t} = 104.88455 {} & - 0.08667\,\widetilde{D}_{1,t-1} - 0.10556\,\widetilde{D}_{2,t-1} + 0.02747\,\widetilde{D}_{3,t-1} \\
& + 0.06802\,\widetilde{D}_{4,t-1} + 0.02598\,\widetilde{D}_{5,t-1} + 0.06135\,\widetilde{D}_{6,t-1} \\
& + 0.04938\,\widetilde{D}_{7,t-1} - 0.08392\,\widetilde{D}_{8,t-1} - 0.03958\,\widetilde{D}_{9,t-1} \\
& + 0.38943\,\widetilde{D}_{10,t-1} - 0.05604\,\widetilde{D}_{1,t-2} - 0.06069\,\widetilde{D}_{2,t-2} \\
& + 0.03549\,\widetilde{D}_{3,t-2} + 0.01084\,\widetilde{D}_{4,t-2} - 0.15354\,\widetilde{D}_{5,t-2} \\
& - 0.01805\,\widetilde{D}_{6,t-2} - 0.09828\,\widetilde{D}_{7,t-2} - 0.10270\,\widetilde{D}_{8,t-2} \\
& - 0.04820\,\widetilde{D}_{9,t-2} + 0.05920\,\widetilde{D}_{10,t-2}
\end{aligned}
$$

$$
\begin{aligned}
\widetilde{D}_{10,t} = 83.285753 {} & + 0.028729\,\widetilde{D}_{1,t-1} + 0.077074\,\widetilde{D}_{2,t-1} - 0.019566\,\widetilde{D}_{3,t-1} \\
& - 0.027475\,\widetilde{D}_{4,t-1} + 0.024747\,\widetilde{D}_{5,t-1} + 0.004009\,\widetilde{D}_{6,t-1} \\
& - 0.045474\,\widetilde{D}_{7,t-1} - 0.008409\,\widetilde{D}_{8,t-1} + 0.074345\,\widetilde{D}_{9,t-1} \\
& - 0.104962\,\widetilde{D}_{10,t-1} - 0.023839\,\widetilde{D}_{1,t-2} + 0.058262\,\widetilde{D}_{2,t-2} \\
& - 0.018108\,\widetilde{D}_{3,t-2} - 0.089708\,\widetilde{D}_{4,t-2} + 0.058820\,\widetilde{D}_{5,t-2} \\
& + 0.065851\,\widetilde{D}_{6,t-2} + 0.116664\,\widetilde{D}_{7,t-2} - 0.015468\,\widetilde{D}_{8,t-2} \\
& + 0.077614\,\widetilde{D}_{9,t-2} + 0.008399\,\widetilde{D}_{10,t-2}
\end{aligned}
$$

CHAPTER 5: GENERAL CONCLUSION

Robust optimization approach for closed-loop supply chain under uncertain environments and imperfect quality production is the focus of this dissertation. It integrates three areas together namely, operations research, production systems, and quality engineering, and is a key to come up with theses sustainable, robust, and realistic design of CLSC models.

The proposed CLSC network design problems in this dissertation include multiple periods, echelons, objectives, and uncertainties. The robust optimization with uncertainty set- based approach , and Mulvey et al. (1995) approach are used to obtain a set of solutions that are robust against the future fluctuation of parameters.

In the motivation section of chapter 1, a novel robust model for the inventory problem at a single station and finite discrete horizons of T periods is proposed. The robust counterparts are based on box and ellipsoidal uncertainty sets. The box uncertainty set is formulated based on the Chebyshev norm of the perturbation variables, while the ellipsoidal uncertainty set is formulated based on the 2-norm of the perturbation variables. The a priori probabilistic guarantees approach is used to compute the size of the uncertainty set necessary to ensure that the degree of constraint violation does not exceed a certain level. The problem is solved using five probability bounds at five different probability constraint violations. The results reveal the following conclusion: the robust solution based on the ellipsoidal uncertainty counterpart is less conservative than the box uncertainty counterpart. In addition, depending on the uncertainty information such as whether the uncertain parameter has bounded and symmetric distribution or it has a known probability distribution, the decision maker will identify a better choice in constructing the robust counterpart model.

In chapter 2, modeling CLSC under uncertainty with incorporation of imperfect quality production is addressed. The uncertainties are associated with each component of the network and include market demand, return of amount products used and defective items, recycling and reworking costs, types I and II errors, and disposal fraction of products. The objective of the MILP model is to minimize the total cost of the supply chain network. To address the uncertainties, three robust counterparts formulations based on the box, polyhedral, and combined interval and polyhedral uncertainty sets are developed. The polyhedral uncertainty set is described using the 1-norm of the uncertain data vector, while combined interval and polyhedral uncertainty set is the intersection between the polyhedral and the interval set defined with both 1-norm and infinite norm. To illustrate the application of the robust optimization framework based on the three different uncertainty sets, four different probability bounds are utilized. Also, the robust solutions at different probability constraint violations, ε, for three problem sizes are evaluated. The solutions and analysis show that for our proposed model, the robust solutions based on the combined interval and polyhedral is the least conservative robust solutions.

Chapter 3 extends chapter 2 such that the robust multi-objective mixed integer linear programming model is developed and includes three objectives simultaneously. The first objective function minimizes the total cost of the supply chain. The second objective function seeks to minimize the environmental influence, and the third objective function maximizes the social benefits. The limitation of scalarization methods (i.e. methods with a priori articulation of preferences) is that it can not reach to solutions in non-convex regions of the Pareto-optimal frontier. In this work, the augmented weighted Tchebycheff method is used to aggregate the three objective functions and produce the set of efficient solutions. Robust optimization, based on Mulvey et al. (1995) approach, is used. The robust framework introduced by Mulvey et al. (1995) addresses two types of robustness: solution robustness which means that the solution remains nearly optimal under all realizations (scenarios), and model robustness which refers to the solution feasibility under all realizations. Considering different values for weights of the objective functions by uniformly varying the weights, different Pareto solution are produced. Also, the behavior of the performance of the robust objective functions as the weighting scale to measure the tradeoff between sensitivity and robustness, $\boldsymbol{\lambda}$ changes is studied.

In chapter 4, the affinely adjustable robust formulation based on "wait and see" decision is presented over two sequential stages. In this robust optimization approach, the adjustable variables reveal themselves with uncertainty. Thus, the first stage variables determine long-term facility configurations which includes the number of selected suppliers, number of opened distribution centers, collection centers, and disposal centers. The second stage decisions concern a plan for the product flows among facilities after realization of the uncertain parameters which include market demand, returned of amount product as used items and defective, recycling and reworking costs, and inspection errors. Moreover, a polyhedral dynamic uncertainty set is proposed to mimic the dynamic behavior of market demand over time. Also, the construction of such dynamic set captures the correlation of the demand at each customer zone. The introduced dynamic uncertainty set is formulated according to Vector Autoregressive (VAR) models. Besides to the affinely adjustable robust optimization framework, incorporating a budget dynamic uncertainly set can significantly improve the market demand forecasting and produce less conservative robust solutions. Finally, in the comparison between open and closed -loop systems, the total costs incurred in the open- loop system is higher by at least 60% than the closed -loop system.

We summarize the future research directions as follows:

- Integration of Robust Optimization and Stochastic Programming: in the hybrid robust/stochastic optimization approach, the model is formulated over multi sequential stages. In the first stage, binary decisions variables for facility configuration are determined. The second stage decisions are determining the expected values of product flows after realization of random variables which follow some probability distributions. The third stage decisions are unit transportation capacities that should be decided after realization of the uncertain parameters.
- Robust Counterparts Formulations Based on Different Dynamic Uncertainty Sets: another possible future work is to develop robust counterparts formulations based on different dynamic uncertainty sets such box and ellipsoidal uncertainty sets. The characteristics of each of the selected uncertainty sets provide the decision maker a flexibility to design his own robust model based on his favorable robustness.
- Multi-Stage Adjustable Robust Optimization with Uncertainty-Affected Recourse: in this dissertation we consider the fixed recourse case. Also,

nonlinear adjustable robust optimization can be considered as a future research, but it would require more computational complexity.

- Larger-Scale Instances: they may require new decomposition approaches.

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