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Scope and Method of Study: This study was written in an attempt to convey to the high school mathematics teacher some of the connotations of "modern mathematics". It discusses some extremely simple, but basic, concepts which the high school student should have even before he reaches high school, but which are not usually taught below the college level today. The preparation of the study necessitated a review of literature dealing with the teaching of mathematics from about 1945 to the present time. This literature was chiefly mathematics periodicals since up-to-date mathematics textbooks are just now beginning to emerge.

Findings and Conclusions: Mathematics, as taught in the nation's high schools today, is from 200 to 2000 years old. Modern mathematics, however, is definitely on its way to the secondary schools; the movement in this direction is from the college downward. It is chiefly an attempt to make mathematics more meaningful to the student by fostering an understanding of the basic fundamentals of mathematics and of the unified structure which rests upon these fundamentals. Mathematics can no longer be divided into the separate little compartments of arithmetic, algebra, geometry, etc.; these compartments are now mixed.

There are many concepts which are so simple that they are taken for granted and, hence, never mentioned to the high school student. This study is primarily concerned with these basic concepts since the concensus of opinion among authorities in the field of mathematics education seems to be that they should be taught explicitly.

ADVISER'S APPROVAL James H. Zant

THE RELATION OF SOME OF THE MODERN CONCEPTS OF MATHEMATICS TO TEACHING IN THE SECONDARY SCHOOLS

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PREFACE

Momentous achievements are being made at the frontiers of mathematics today, but, as is always the case with new developments, several years will be required before these achievements reach the secondary school. In the meantime, the high school mathematics teacher must continue to teach the conventional subjects in the conventional type of curriculum. There are many new concepts, however, which will make mathematics more meaningful and which can be introduced into the conventional courses. Many of these concepts are so simple that they have been taken for granted for ages but so important that they should be a part of the mathematical knowledge of every student.

This study was undertaken in an attempt to present to the high school teacher some idea of the meaning of "modern mathematics". In trying to reach this goal the dissatisfaction with our present curriculum is discussed quite extensively in the introductory chapter; then, in the following chapters, a few of the more elementary concepts which the high school student should have are discussed from, it is hoped, a modern point of view. It is impossible to include in a study of this type very many of the connotations of "modern mathematics", but it is hoped that what is included in this study will stimulate the reader to investigate the field for himself.

Indebtedness is acknowledged to Dr. James H. Zant for his invaluable guidance and assistance in the pursuit of this study.

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CHAPTER I

INTRODUCTION

"Nearly all of the topics in the first year algebra course of today were included in Ray's Algebra published in 1876."[1] Has nothing worthy of presentation to the high school student been discovered during the last eighty years? If this be the case mathematics is a dead subject! On the other hand, if there are new discoveries in mathematics which should be included in the high school curriculum, then the youth of America are not receiving the type of mathematics education which they should have, and furthermore, they will not receive it until the high school curriculum is modernized.

On all sides one hears criticism of the kind of mathematics which is currently being taught in the nation's high schools. MacLane of the University of Chicago, for example, takes a dim view of mathematics as taught at the present time when he says:

> The lively modern development of mathematics has had no impact on the content or on the presentation of secondary school mathematics. Algebra and geometry, as covered in schools, consist exclusively of ideas already well

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[1]James H. Zant, Address to the Oklahoma Council of Teachers of Mathematics, Oklahoma City, October 28, 1955.

> known two hundred years ago --- many of them two thousand years ago. No matter how much better these particular ideas are taught to more and more pupils, their presentation leaves school mathematics in a state far more antiquarian than that of any other part of the curriculum. The pupils can conclude only that there is no such thing as a new mathematical idea.[2]

Seegar of the National Science Foundation takes much the same viewpoint when he states:

> It is generally agreed that modern mathematics began in the seventeenth century with Descartes' conception of analytic geometry in 1637 and with Newton's invention of calculus in 1671. Yet many high school courses in mathematics are regarded as being quite satisfactory if the students know merely the binomial theorem in algebra, which was discovered by Newton in 1676 and have studied the first four books of Euclid, which were compiled about 325 B.C.[3]

The foregoing are truly serious indictments of current mathematics teaching, and it is thought-provoking to realize that many other such statements are currently being made. One has only to glance through a current mathematics periodical to see a multitude of similar statements.

High school geometry, in particular, seems to receive the most unfavorable criticism. As a concrete illustration, consider the words of E. T. Bell: "I should like to see

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[2]Saunders MacLane, "The Impact of Modern Mathematics," Bulletin of the National Association of Secondary School Principals, XXXVIII (1954), p. 66.

[3]Raymond J. Seegar, "Mathematical Science and the Manpower Problem," The Mathematics Teacher, L (1957), p. 16.

elementary geometry, as at present taught in all but a few schools in the United States, pitched neck and crop out of education."[4]

This writer's purpose, however, is not to dwell at length on the shortcomings of secondary school mathematics but to attempt to offer something constructive in this field. The above criticisms were included only in an attempt to show the gravity of the situation.

Upon entering the teaching profession, the high school teacher incurs a profound responsibility for the future of his students and, in a sense, for America itself. There is no room for complacency in the teaching profession. The needs of the student and of the nation must always be kept in mind.

These needs have changed since Ray's Algebra was published. Indeed, the change is now so swift that it is noticeable, at least, from decade to decade.

The present age is known as the "Age of Science", and it is an age which requires untold numbers of engineers, scientists, and mathematicians. It can be safely asserted that these engineers, scientists, and mathematicians must acquire their basic introduction to mathematics in the secondary school, and also, that in the secondary school students are either motivated to pursue mathematics still further in college, or they are repelled from all subsequent

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[4]E. T. Bell, "Buddha's Advice to Students and Teachers of Mathematics," The Mathematics Teacher, XXXIII (1940), p. 254.

contact with mathematics. Truly, much depends upon the teacher.

Many people are working to improve the present situation. One hears proposals for curriculum revision being offered constantly. A great many of these proposals embody a shift of some type in the traditional courses or in the sequence of these courses. One proposal should be noted especially, and that is the inclusion of analytic geometry and introductory calculus in the high school. This is being done in several school systems at the present time, and, in this writer's opinion, it is a valuable revision where circumstances permit its adoption. However, perhaps its value lies to a great extent in the fact that, in order to make room for analytic geometry and calculus in the curriculum, much of the dead material in the conventional geometry courses must be discarded. Many advocates of a curriculum change, though, are proposing something much more profound than the inclusion of the above two courses in the curriculum. Their proposal may be labeled "modern mathematics".

Only within this decade have some of the modern developments in mathematics begun to trickle down to the secondary school, and so far only a very small minority of the schools have received even a trickle of the developments of modern mathematics.

Perhaps an indication of the extent of these modern mathematical developments is best expressed in the statement that, "--- more mathematics has been produced since 1900 than in all previous time."[5] Certainly, not all of this new mathematics could or should be taught in the high school, but there is no doubt that some of it <u>should</u> be there.

There has been an explosive development of mathematical knowledge in recent years according to Northrop of the University of Chicago,[6] and a survey of current literature convinces one that this explosion is definitely on its way to the high school. The secondary school teacher will undoubtedly be hearing of modern mathematics very soon.

Modern mathematics may be defined in various ways by different individuals. It certainly includes the concepts which have been formulated recently, but it also must include a great many concepts which have been in use for hundreds of years. These old concepts, however, may be presented from a different point of view --- one which will give the student a much greater insight into the processes of mathematics.

The aim of this study is to review some of the basic concepts of elementary high school mathematics from a modern point of view. Hence, the study deals primarily with numbers and with the fundamental operations with numbers.

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[5]Zant.

[6]E. P. Northrop, "Modern Mathematics and the Secondary School Curriculum," <u>The Mathematics Teacher</u>, XLVIII (1955), p. 387.

CHAPTER II

SOME BASIC ASPECTS OF THE NUMBER SYSTEM

Numbers

In order to discuss numbers the concept of set should be introduced. It is one of the simplest and yet one of the most important ideas of modern mathematics. The committee on the Undergraduate Program of the Mathematics Association of America says, in regard to this important topic:

> The concept of set, whose fundamental role in mathematics was brought to light by Cantor, has revolutionized the structure and language of modern advanced mathematics. However, this concept has not penetrated very far into the elementary curriculum. This is not due to its difficulty for the fundamental ideas are so obvious that at first sight they hardly seem worth thinking about.[1]

What is a set? A <u>set</u> <u>S</u>, according to Meserve, is "---a collection into a whole of distinct, perceived, or considered objects called the <u>elements</u> of <u>S</u>."[2]

The high school student has thought in terms of sets all his life, and the idea should be very easy to introduce

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[1]Committee on the Undergraduate Program of the Mathematics Association of America, <u>Universal Mathematics</u>, Part II (New Orleans, 1955), p. I-1.

[2]Bruce E. Meserve, <u>Fundamental Concepts of Algebra</u> (New York, 1953), p. 1.

to him. His family is a set, and he is one of the elements of that particular set. Other sets with which he is familiar are: the football team, his class, the desks in his classroom, a week, a month, a year, and others too numerous to mention.

The idea of subset or set within a set naturally follows from the idea of set itself. Formally, a subset may be defined thus: "A set of elements B is called a subset of a set A if each element of B is an element of A."[3] Hence, the student is a subset of his family; the month is a subset of the year; and the week is a subset of both the month and the year.

No doubt, primitive man must have used the idea of set quite extensively for he had to have some method of "counting" his possessions. For example, he might have cut one notch on a stick for each sheep that he owned. He then had a set of notches which corresponded to his set of sheep, and furthermore, there was a one-to-one correspondence between his sheep and the notches on the stick (i.e., one sheep corresponded to exactly one notch on the stick, and one notch on the stick corresponded to exactly one sheep).

This idea of one-to-one correspondence is described by Meserve as one of the fundamental concepts of mathematics,[4] and Jones says, "--- there is little more to this process we

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[3] Meserve, p. 4.

[4] Ibid., p. 2.

call counting than establishing a one-to-one correspondence."[5] The small child establishes this correspondence between the objects under consideration and his fingers, but as he grows older he establishes the correspondence with a set of symbols called numerals and with the sounds associated with these symbols. This, very briefly and very simply, is the process of counting. It should be emphasized, however, that counting the elements of a set is a process entirely independent of the order in which the elements are considered.

The numbers which are used in counting are called the <u>natural numbers</u> or <u>positive integers</u>, and it is naturally this set of positive integers which the student first encounters in mathematics. A few of the other sets which he may often use are: the set of non-negative integers (i.e., the positive integers and zero), the set of negative integers, the set of integers, the set of rational numbers, the set of points on a line, the set of all right triangles, and so on <u>ad infinitum</u>.

Normally, only ten digits (0,1,2,3,4,5,6,7,8,9) are used in counting any finite collection of objects. But, in most instances the numbers with which a student works will be greater than 9. Hence, he must understand the meaning of place value. It is absolutely essential that he understand the meaning of each 3 in the number 3903, for example. The

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[5] Burton W. Jones, <u>Elementary Concepts of Mathematics</u> (New York, 1947), p. 27.

number 3903 is in fact a type of shorthand used to represent the sum

$$3000 + 900 + 3$$

or to be even more exact

$$3 \cdot 10^3 + 9 \cdot 10^2 + 0 \cdot 10 + 3.$$

The positional aspect of notation is used quite extensively in the ordinary methods of addition and multiplication. In adding 671 and 349, for example, one usually proceeds column-wise, obtaining first the subsidiary sums

$$10 = 9 + 1, \quad 12 = 7 + 4 + 1, \quad 10 = 6 + 3 + 1$$

and, from these, arriving at the required sum, 1020. This method avoids the more fundamental process illustrated by the following equations:

$$\begin{array}{rrrr} & (1) & (1) & \\ & 3 & 4 & 9 \\ & 6 & 7 & 1 \\ \hline 1 & 0 & 2 & 0 \end{array}$$

$$\begin{aligned}
671 + 349 &= (6 \cdot 10^2 + 7 \cdot 10 + 1) + (3 \cdot 10^2 + 4 \cdot 10 + 9) \\
&= (6 \cdot 10^2 + 3 \cdot 10^2) + (7 \cdot 10 + 4 \cdot 10) + (1 + 9) \\
&= (6 + 3) \cdot 10^2 + (7 + 4) \cdot 10 + (10) \\
&= (6 + 3) \cdot 10^2 + (7 + 4 + 1) \cdot 10 \\
&= 9 \cdot 10^2 + 12 \cdot 10 \\
&= 9 \cdot 10^2 + (10 + 2) \cdot 10 \\
&= 9 \cdot 10^2 + 10^2 + 2 \cdot 10 \\
&= (9 + 1) \cdot 10^2 + 2 \cdot 10 \\
&= 10 \cdot 10^2 + 2 \cdot 10 \\
&= 10^3 + 2 \cdot 10 \\
&= 1000 + 20 \\
&= 1020
\end{aligned}$$

It is readily seen from this one example that the positional aspect of notation is indeed a great labor-saving device.

A student who has worked with the base 10 number system all his life may at first find it difficult to visualize a system in any other base. Other bases have been used, however, at various times and places during the course of history.

"The number system in which addition and multiplication are easiest is the <u>binary</u> or <u>dyadic</u> <u>system</u>, or number system to the base two."[6] Only two digits, specifically 0 and 1, are used in this system, and this means that addition and multiplication tables become much shorter. The tables listed below should in fact appeal to the student because of their simplicity.

-	0	1
0	0	1
1	1	10

•	0	1
0	0	0
1	0	1

The student should have no difficulty at all in memorizing these tables!

A typical base 2 number might be 1101 which would represent

$$1 \cdot 2^3 + 1 \cdot 2^2 + 0 \cdot 2 + 1$$

just as 1101 in base 10 would represent

$$1 \cdot 10^3 + 1 \cdot 10^2 + 0 \cdot 10 + 1.$$

Upon examination, however, the student will find these two numbers to be radically different in value, for 1101 in base

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[6]Jones, p. 49.

2 notation is equal to only 13 in base 10 notation. Thus, it is readily apparent that, even though the binary system is extremely simple, the expression of large quantities in such a system becomes exceedingly cumbersome. For example, the number which is written as 144 in base 10 becomes 10010000 in base 2 notation.

The basic operations are performed in this system exactly as they are performed in the base 10 system (using, of course, the base 2 addition and multiplication tables). The following examples of addition and multiplication should make the processes clear. (In the addition example carries are indicated in parentheses at the top.)

$$
\begin{array}{ccccc}
 & (1) & (1) & (1) & \\
 & 1 & 0 & 1 & 1 \\
 & & 1 & 0 & 1 \\
\hline
1 & 0 & 0 & 0 & 0
\end{array}
\qquad\qquad
\begin{array}{cccccc}
 & & & 1 & 1 & 1 \\
 & & & 1 & 0 & 1 \\
\hline
 & & & 1 & 1 & 1 \\
 & & 0 & 0 & 0 & \\
 & 1 & 1 & 1 & & \\
\hline
1 & 0 & 0 & 0 & 1 & 1
\end{array}
$$

Until recent years the binary system was of little use, but today it is of paramount importance. The fact that only two numbers exist in the system fits perfectly with electrical circuits which are either on or off or with tubes which either conduct or do not conduct. For this reason the binary system is used in the modern high speed digital computers.

MacLane states that the binary system should be introduced to every high school pupil since "--- it promotes real understanding of what the ordinary symbols for numbers mean."[7]

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[7]MacLane, p. 69.

Equivalence Relations

Very early in his study of mathematics the student encounters equivalence relations, but does he understand the basic properties of such relations? An <u>equivalence</u> <u>relation</u> is defined to be: "Any relation having the three properties:

reflexive, $\underline{a} = \underline{a}$

symmetric, $\underline{a} = \underline{b}$ implies $\underline{b} = \underline{a}$

transitive, $a = b$ and $b = c$ imply $a = c$."[8]

These properties are perhaps best understood in terms of one-to-one correspondences. The reflexive property is immediately apparent, for the elements of any set may certainly be placed into one-to-one correspondence with themselves. The symmetric property is obtained almost as simply, since, if there exists a one-to-one correspondence between the elements of sets <u>A</u> and <u>B</u>, there must also exist a one-to-one correspondence between the elements of sets <u>B</u> and <u>A</u>. The validity of the transitive property may be established in much the same manner, because, if there exists a one-to-one correspondence between the elements of sets <u>A</u> and <u>B</u> and between the elements of sets <u>B</u> and <u>C</u>, it is evident that there also exists a one-to-one correspondence between the elements of sets <u>A</u> and <u>C</u>.

Closely related to the three properties just considered are those encountered in the fundamental operations of mathematics which are discussed in the following chapter.

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[8]Meserve, p. 7.

CHAPTER III

BASIC OPERATIONS OF MATHEMATICS

> Inadequate mastery of fundamental terminology, concepts, and skills is probably the most outstanding cause for the difficulty encountered by individuals of all ages in dealing with anything of a mathematical nature.[1]

Mathematics is too often taught as a set of rules and manipulations to be memorized without teaching the basic concepts which underly these rules. What can be done to help correct this situation? It seems logical that one of the starting points could be a consideration of the basic operations of addition, subtraction, multiplication, and division. In the set of positive integers the following basic properties exist.

1. <u>The commutative property of addition and multiplication</u>.

This property simply means that the order of addition of two numbers makes no difference to the sum, and that the order of multiplication of two numbers makes no difference to the product. Symbolically this property may be expressed as:

$$a + b = b + a$$

and

$$a \cdot b = b \cdot a$$

[1] F. Lynwood Wren, "Secondary Mathematics," *Encyclopedia of Educational Research*, ed. Walter S. Monroe (New York, 1950), p. 720.

That addition is commutative is readily apparent, and one might use innumerable simple examples to illustrate this property. The commutativity of multiplication, however, is not as easily seen, but perhaps an array of symbols similar to the following could be used to illustrate the property.

```
x x x x x x x
x x x x x x x
x x x x x x x
x x x x x x x
```

If a is used to designate the number of symbols in a row, and b is used to designate the number of symbols in a column, it can be clearly seen that $a \cdot b = b \cdot a$ $(7 \cdot 4 = 28 = 4 \cdot 7)$.

2. The associative property of addition.

This property is similar to the commutative property in that it is concerned with the order in which an operation is performed. Symbolically it is written thus:

$$(a + b) + c = a + (b + c)$$

Associativity of addition, like commutativity of addition, may be illustrated with many simple examples. In statement form this property means that the order of addition of a series of numbers does not affect the sum.

3. The associative property of multiplication.

In symbolic notation this property has the form:

$$(a \cdot b) \cdot c = a \cdot (b \cdot c)$$

It is very easily illustrated by an arrangement such as this:

```
x x x x x  x x x x x  x x x x x
x x x x x  x x x x x  x x x x x
x x x x x  x x x x x  x x x x x
```

If a is used to designate the number of elements in one row of one of the three sets, b is used to designate the number of elements in a column, and c represents the number of sets, it is readily apparent that $(a \cdot b) \cdot c = a \cdot (b \cdot c)$, for $(5 \cdot 3) \cdot 3 = 45 = 5 \cdot (3 \cdot 3)$. From the illustration it can also be seen that this product is equal to $(a \cdot c) \cdot b$. This result could have been predicted on the basis of the associative and commutative laws, for:

$$\begin{aligned} a \cdot (b \cdot c) &= a \cdot (c \cdot b), \text{ by the commutative law;} \\ &= (a \cdot c) \cdot b, \text{ by the associative law.} \end{aligned}$$

4. The distributive property.

This property states that multiplication distributes over addition, or that

$$a \cdot (b + c) = (a \cdot b) + (a \cdot c).$$

Of all the basic properties, this particular one is probably violated most by the high school student. He should definitely understand that the reverse of this property is not true (i.e. that addition does not distribute over multiplication).

It was stated at the beginning of the above-mentioned properties that they existed in the set of positive integers. The student would find it both interesting and informative to look for other sets in which these properties exist.

A study of addition and multiplication leads naturally to a consideration of the inverse operations of subtraction and division. Two operations are inverse operations if they are opposite in effect (i.e., if their successive application with the same number leaves the number unchanged). Thus, subtraction is the inverse of addition, and division is the inverse of multiplication; for

$$(a + b) - b = a$$

and

$$(a \cdot b) \div b = a.$$

Addition and multiplication are always possible, but this is not necessarily true for subtraction and division. For example, if $b > a$ then $a - b$ has no solution in the set of positive integers but does have a solution in the set of negative integers. If a and b are integers division may or may not be possible in the set of integers. Often it will be possible in the set of rational numbers only (e.g., $2 \div 3 = 2/3$).

Other basic concepts which are of importance in helping the high school student understand mathematics are: (1) closure, (2) identity elements, (3) inverse elements.

1. Closure. A set S is closed with respect to an operation if the result of that operation is also an element of the set S. For example, the set of positive integers is closed with respect to addition since the sum of any two positive integers is a third positive integer.

2. Identity element. The identity element for a given operation is that element which, when applied to any element of the given set under the given operation, leaves the original element unchanged. The identity element for addition is 0, and the identity element for multiplication is 1 since

$$a + 0 = a = 0 + a$$

and

$$a \cdot 1 = a = 1 \cdot a.$$

3. Inverse elements. One element is the inverse of another element under a particular operation if, upon application to the first element, it yields the identity element for that specific operation. Under addition a and $-a$ are inverse elements because $a + (-a) = 0$, and under multiplication a and $1/a$ are inverse elements since $a \cdot (1/a) = 1$.

A study of groups involves the three concepts just considered. The notion of a group appears throughout mathematics, mechanics, and physics; and it is described by MacLane as "---one instance of a simple and exciting new mathematical concept."[2]

A set of elements forms a group under a given operation if it exhibits the following properties:

1. The set is closed under the operation.
2. The operation is associative.
3. The set contains the identity element for the operation.
4. The set contains the inverse of each of its elements under the operation.

Probably the most familiar set which forms a group under addition is the set of integers. However, if the student examines the face of a clock, he will find that numbers

[2]MacLane, p. 68.

on a circle form a group under addition also.

Addition of these numbers on a circle possesses all the formal properties of addition enumerated earlier (i.e., commutativity, associativity, etc.), but addition of this type is also quite different from the usual type of addition. For example, seven hours past ten o'clock is five o'clock, and eight hours past five o'clock is one o'clock. These sums may be expressed as: $7 + 10 = 5$, and $8 + 5 = 1$. Addition of this type is addition modulo 12; it differs from ordinary addition in that the multiples of 12 are discarded.

Some time devoted to numbers on a circle would, no doubt, be well worthwhile in arousing interest and in teaching the properties of the fundamental operations. Some modulus other than 12 should be used also---particularly one which is a prime number since multiplication using a prime modulus presents such a striking contrast to multiplication using a composite modulus.

Only a very few of the extremely elementary concepts of mathematics have been discussed in this chapter, and yet these are the concepts which form the cornerstones of mathematics. It is felt that the whole of mathematics will be much more meaningful and much more interesting to the student if he grasps these basic fundamentals.

CHAPTER IV

SUMMARY

A study of this type cannot even begin to discuss the many topics in modern mathematics with which the teacher should be familiar. Volumes of textbooks would be required for such a goal. There are many other modern ideas which follow directly from the topics which have been discussed, however. As a concrete illustration, consider the idea of a variable---an idea which is encountered so often in mathematics.

Webster defines a variable as, "A quantity that may assume a succession of values, which need not be distinct." In the light of this definition consider a problem such as this: "The length of a rectangle is 3 units greater than its width, and its area is 54 square units. Find its length and width." The student arrives at the equation:

$$x^2 + 3x - 54 = 0.$$

Upon solving the equation he finds that $x = 6$ or $x = -9$. Both of these values satisfy the equation; but which is correct, or, are both of them correct? Last week he may have solved equations involving temperatures in which negative answers were permitted. Why then, can he not use a negative answer this time? This question could have been answered before the roots of the equation were found by specifying the set to which the answer must belong (i.e., the set of positive integers).

No attempt has been made in this study to discuss the modern ideas of those courses which are normally considered to be the higher mathematics courses of the secondary school (i.e., trigonometry, geometry, etc.). The reader may be interested, however, in this list of the most needed trigonometric concepts as compiled by Andree, of the University of Oklahoma:[1]

1. The graphs of the trigonometric functions.
2. Identities.
3. Trigonometric equations, especially of higher degree.
4. The inverse trigonometric functions.
5. Definitions and applications involving polar coordinates, De Moivre's theorem, and complex numbers.
6. Practical problems involving equations and identities.

Andree also makes the statement that: "One of the startling changes in modern trigonometry is that the student needs to think of the trigonometric functions of a number."[2] This is especially apparent during the present day when it is known that the behavior of so many natural phenomenon may be graphed as a periodic wave which is similar to the sine wave.

Something very closely akin to geometry is entering the field of mathematics today---topology. At this time very little seems to have been written about topology in the secondary school. This writer has so far found only one arti-

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[1]Richard V. Andree, "Modern Trigonometry," <u>The Mathematics Teacher</u>, XLVIII (1955), p. 83.

[2]Ibid., p. 82.

cle dealing with the subject in current literature.[3] Some of the topics encountered in topology seem to have been on the scene for years, however. The Möbius strip, a strip of paper which has been given a half-twist and the ends then glued together, is an example of one such topic. The Mobius strip is interesting in that it has only one surface. This the student can verify by attempting to color one side of it. The Königsberg bridge problem[4] is another example of one of the puzzle type problems which are quite interesting to the high school student. Probably the chief importance of problems of this sort is the fact that they stimulate interest and introduce the pupils to the new horizons in mathematics.

The ideas discussed herein have been very elementary but absolutely necessary in any study of mathematics. Each of the topics which was considered received only very brief treatment, and, consequently, nothing was added to the field of mathematical knowledge. It is hoped, however, that the reader will be stimulated to delve further into the new frontiers of mathematics after reading this study, and that he will thereby be better prepared to teach mathematics as it should be taught to the nation's youth.

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[3]Bruce E. Meserve, "Topology for Secondary Schools," <u>The Mathematics Teacher</u>, XLVI (1953), pp. 465-474.

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Candidate for the Degree of

Master of Science

Report: THE RELATION OF SOME OF THE MODERN CONCEPTS OF MATHEMATICS TO TEAVHING IN THE SECONDARY SCHOOLS

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