DERIVATION OF SLOPE DEFLECTION EQUATIONS FOR STRAIGHT TRUSSES OF CONSTANT DEPTH

By

DALE E. MORRISETT

Bachelor of Science

Oklahoma Agricultural and Mechanical College

Stillwater, Oklahoma

1954

Submitted to the Faculty of the Graduate School of
the Oklahoma State University in partial
fulfillment of the requirements
for the degree of
MASTER OF SCIENCE
August, 1957

DERIVATION OF SLOPE DEFLECTION EQUATIONS FOR

STRAIGHT TRUSSES OF CONSTANT DEPTH

Report Approved:

____

Report Advisor

____

____

Dean of the Graduate School

PREFACE

The purpose of this report is the derivation of slope deflection equations for trusses of constant depth and their application to the analysis of structures containing truss-members. Slope deflection equations and moment distribution constants for truss-members of constant or variable depth were developed by J. M. Haynes,[2] E. R. Jacobsen,[3] and L. C. Maugh.[4] The basic structure used in their investigations was a simple beam-truss. The writer's contribution is the application of elastic center to these derivations.

The normally slow procedure for evaluating load and truss constants has been eliminated by deriving general formulas using power series. The evaluation of constants by power series was first introduced by J. J. Tuma in his 1956-57 extension class held in Oklahoma City. The power series expressions were applied by him and W. Sullivan to the analysis of truss frames for the C. A. C. building in Oklahoma City. The computation of elastic constants for three typical cases were added by the writer.

The nomenclature used in this report is explained either in the chapter they are used or in a preceding chapter.

Indebtedness is acknowledged to Professor Tuma for his valuable guidance and assistance in the preparation of this report, and to my wife, Mrs. Dolores Morrisett, for her care in the typing of this report.

---

1. It is a general reference number in the bibliography.

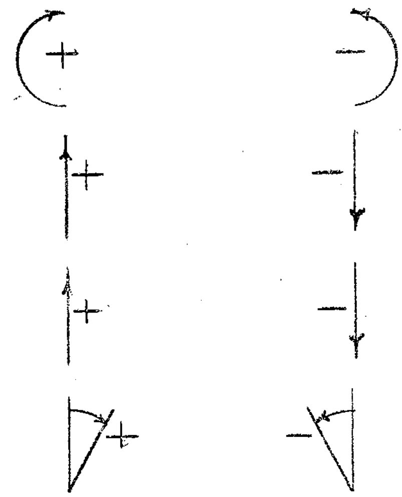
TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

Sign Convention

1. Moments
2. Vertical forces
3. Vertical displacements
4. Angular rotations



CHAPTER I

DERIVATION OF SLOPE DEFLECTION EQUATIONS

A. Statics

A typical truss beam removed from a continuous elastic system, loaded by a general system of forces, is considered (Fig. 1). The truss has constant depth and is fixed at both ends.

In the analysis of this truss, the following assumptions have been made:

1. All members are connected by frictionless hinges.
2. All members are subjected to axial forces only, and the influence of shear and bending moment is neglected.
3. The truss and the loads are forming a coplanar system.
4. All loads are applied at joints.
5. The deformations of the truss are elastic and small.

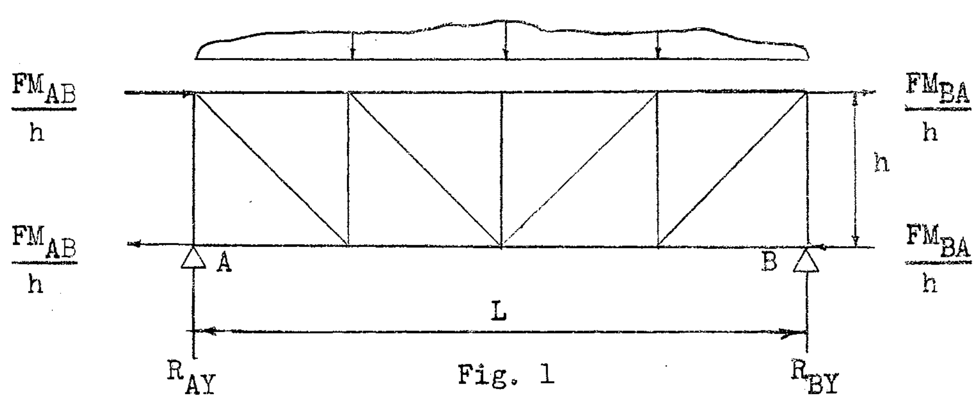


Fig. 1

Truss Beam with General System of Loading

The structure has four reactions: two reactive forces, R_{AY} and R_{BY}, and two reactive moments, FM_{AB} and FM_{BA}. The problem is statically indeterminate to the second degree and its solution requires two equations of deformation.

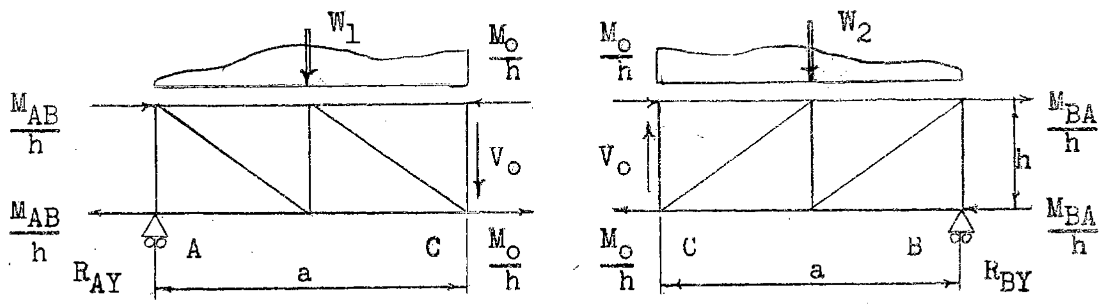


Fig. 2

Free - body Trusses AC and CB

General displacements of supports $\triangle_{AY}$, $\triangle_{BY}$, θ_A and θ_B are introduced. The given system is resolved into free body sketches AC and BC as shown in Fig. 2. The resultant of loads corresponding to part AC and CB is denoted by W_1 and W_2 respectively. The forces at the central cross-section are V_o and M_o/h. Assuming all displacements and reactions to be positive and using conditions of static equilibrium, the end reactions of parts AC and CB are:

$$\left.\begin{aligned} R_{AY} &= W_1 + V_o, & M_{AB} &= M_o - aV_o - CM_{AC} \\ R_{BY} &= W_2 - V_o, & M_{BA} &= -M_o - aV_o + CM_{BC} \end{aligned}\right\}(1)$$

where CM_{AC} and CM_{BC} represent the cantilever moment due to W_1 and W_2 respectively.

The normal force for any member in the truss in terms of the applied loads and the redundants is:

$$N_i = SN_i + \alpha_i M_o + \beta_i V_o \tag{2}$$

where SN_i = normal force in any member due to loading

α_i = normal force in any member due to $M_o = 1$

and β_i = normal force in any member due to $V_o = 1$

B. Least Work

The Principle of Conservation of Energy states that:

$$U_i = U_e \tag{3}$$

where U_e = the external work

and U_i = the internal work

The internal work is formed by:

$$U_i = U_s + U_v \tag{3a}$$

where U_s = the strain energy of the structure

U_v = the strain energy due to volume change

The energy due to volume change is neglected and equation (3a) becomes:

$$U_i = U_s = \sum_A^B \frac{N_i^2 L_i}{2A_i E}$$

where L_i = length of any member

A_i = cross-sectional area of any member

and E = modulus of elasticity

In reduced form:

$$U_i = U_s = \sum_A^B \frac{N_i^2 \lambda_i}{2} \tag{3b}$$

where $\lambda_i = \dfrac{L_i}{A_i E}$

The external work is expressed as:

$$U_e = U_1 + U_r \tag{3c}$$

where $U_1 = \sum_A^B W\Delta + \sum_A^B W\theta$ = work due to loads

and $U_r = \sum_A^B R\Delta + \sum_A^B M\theta$ = work due to reactions

The work of supports in terms of displacements and reactions defined by equation (1) is:

$$\begin{aligned} U_r &= R_{AY}\Delta_{AY} + M_{AB}\theta_A + R_{BY}\Delta_{BY} + M_{BA}\theta_B \\ &= (W_1 + V_o)\Delta_{AY} + (M_o - aV_o - CM_{AC})\theta_A \\ &+ (W_2 - V_o)\Delta_{BY} + (-M_o - aV_o + CM_{BC})\theta_B \end{aligned} \tag{3d}$$

According to Castigliano's theorems, the first partial derivative of the strain energy of a truss with unyielding supports, with respect to a redundant, is equal to zero. Allowing displacement of supports, we have:

$$\frac{\partial U_r}{\partial M_o} = \frac{\partial U_s}{\partial M_o} \tag{3e}$$

$$\frac{\partial U_r}{\partial V_o} = \frac{\partial U_s}{\partial V_o} \tag{3f}$$

The partial derivatives of equation (3b) with respect to each redundant are:

$$\frac{\partial U_s}{\partial M_o} = \sum_A^B N_i \frac{\partial N_i}{\partial M_o} \lambda_i = \sum_A^B N_i \alpha_i \lambda_i \tag{3g}$$

$$\frac{\partial U_s}{\partial V_o} = \sum_A^B N_i \frac{\partial N_i}{\partial V_o} \lambda_i = \sum_A^B N_i \beta_i \lambda_i \tag{3h}$$

The partial derivatives of equation (3d) with respect to each redundant are:

$$\frac{\partial U_r}{\partial M_o} = \theta_A - \theta_B \tag{3i}$$

$$\frac{\partial U_r}{\partial V_o} = \Delta_{AY} - a\theta_A - a\theta_B - \Delta_{BY}$$

$$= a\theta_A - a\theta_B + \Delta_Y \tag{3j}$$

where $\Delta_Y = \Delta_{AY} - \Delta_{BY}$

C. Deformation Equations

Equations (3e, 3f) in terms of equations (3g, 3h, 3i, 3j) becomes:

$$\left.\begin{aligned} \Delta_A - \theta_B &= \sum_A^B SN_i\alpha_i\lambda_i + M_o\sum_A^B \alpha_i^2\lambda_i + V_o\sum_A^B \beta_i\alpha_i\lambda_i \\ \Delta_Y - a\theta_A - a\theta_B &= \sum_A^B SN_i\beta_i\lambda_i \\ &\quad + M_o\sum_A^B \alpha_i\beta_i\lambda_i + V_o\sum_A^B \beta_i^2\lambda_i \end{aligned}\right\} \tag{4}$$

From symmetry:

$$\sum_A^B \alpha_i\beta_i\lambda_i = \sum_A^B \beta_i\alpha_i\lambda_i = 0$$

Thus the simplified equations are:

$$\left.\begin{aligned} \theta_A - \theta_B &= \sum_A^B SN_i\alpha_i\lambda_i + M_o\sum_A^B \alpha_i^2\lambda_i \\ \Delta_Y - a\theta_A - a\theta_B &= \sum_A^B SN_i\beta_i\lambda_i + V_o\sum_A^B \beta_i^2\lambda_i \end{aligned}\right\} \tag{4a}$$

Denoting:

$$D_1 = \sum_A^B SN_i\alpha_i\lambda_i, \qquad D_2 = \sum_A^B SN_i\beta_i\lambda_i$$

$$C_1 = \sum_A^B \alpha_i^2 \lambda_i \qquad\qquad C_2 = \sum_A^B \beta_i^2 \lambda_i$$

the deformation equations become:

$$\begin{aligned} \theta_A - \theta_B &= D_1 + M_o C_1 \\ \Delta_Y - a\theta_A - a\theta_B &= D_2 + V_o C_2 \end{aligned} \tag{4b}$$

Solving these two equations, the redundants are:

$$\begin{aligned} M_o &= \frac{-D_1}{C_1} + \frac{\theta_A}{C_1} - \frac{\theta_B}{C_1} \\ V_o &= \frac{-D_2}{C_2} - \frac{a\theta_A}{C_2} - \frac{a\theta_B}{C_2} + \frac{\Delta_Y}{C_2} \end{aligned} \tag{4c}$$

Substituting the results of equation (4c) into equation (1):

$$\begin{aligned} M_{AB} = {} & \left(\frac{a^2}{C_2} + \frac{1}{C_1}\right)\theta_A + \left(\frac{a^2}{C_2} - \frac{1}{C_1}\right)\theta_B \\ & - \frac{a\Delta_Y}{C_2} - \frac{D_1}{C_1} + \frac{aD_2}{C_2} - CM_{AC} \end{aligned} \tag{5}$$

$$\begin{aligned} M_{BA} = {} & \left(\frac{a^2}{C_2} + \frac{1}{C_1}\right)\theta_B + \left(\frac{a^2}{C_2} - \frac{1}{C_1}\right)\theta_A \\ & - \frac{a\Delta_Y}{C_2} + \frac{D_1}{C_1} + \frac{aD_2}{C_2} + CM_{BC} \end{aligned} \tag{6}$$

CHAPTER II

SERIES EVALUATION OF CONSTANTS

FOR SPECIAL CASES

In this chapter, three straight trusses of constant depth, are considered. Nomenclature used in this chapter is:

a	=	half-length of truss
A_{Bi}	=	cross-sectional area of bottom bars
A_{BMD}	=	bending moment area of basic structure
A_{Di}	=	cross-sectional area of diagonal members
A_{Ti}	=	cross-sectional area of top members
b	=	panel length
BMD	=	bending moment diagram
c	=	length of a diagonal member
h	=	height of truss
i	=	any truss member
L	=	length of truss beam
n	=	number of panels in full-truss
s	=	number of panels in half-truss
λ_{Di}	=	λ of any bottom member
λ_{Hi}	=	λ of horizontal members $= \frac{b}{A_H E}$
λ_{Ti}	=	λ of any top member

A. Case I - Pratt Truss

A Pratt Truss of constant depth and loaded by a general system of forces as shown in Fig. 3 is considered.

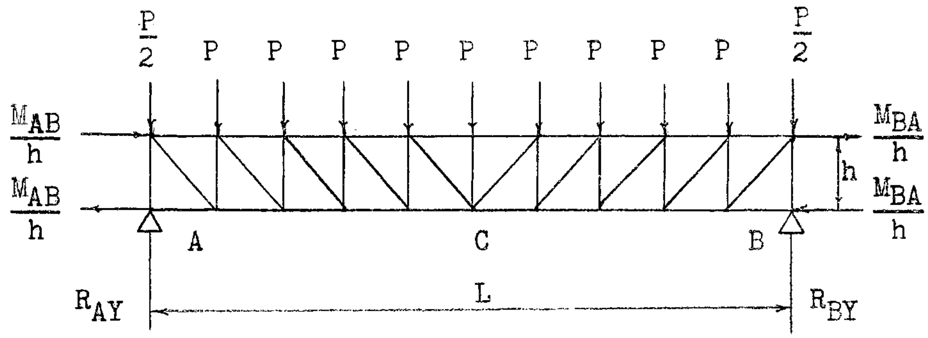


Fig. 3

Typical Pratt Truss

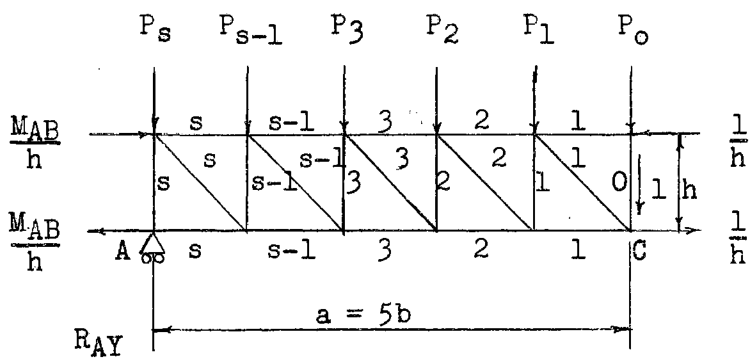


Fig. 4

Truss Element AC

TABLE I

TRUSS AND LOAD CONSTANTS

i		α_i	β_i	SN_i
Top	1	$-\frac{1}{h}$	0	$+\frac{M_o}{h}$
	2	$-\frac{1}{h}$	$+\frac{b}{h}$	$+\frac{M_1}{h}$
	3	$-\frac{1}{h}$	$+2\frac{b}{h}$	$+\frac{M_2}{h}$
	s-1	$-\frac{1}{h}$	$+(s-2)\frac{b}{h}$	$+\frac{M_{s-2}}{h}$
	s	$-\frac{1}{h}$	$+(s-1)\frac{b}{h}$	$+\frac{M_{s-1}}{h}$
Bottom	1	$+\frac{1}{h}$	$-\frac{b}{h}$	$-\frac{M_1}{h}$
	2	$+\frac{1}{h}$	$-2\frac{b}{h}$	$-\frac{M_2}{h}$
	3	$+\frac{1}{h}$	$-3\frac{b}{h}$	$-\frac{M_3}{h}$
	s-1	$+\frac{1}{h}$	$-(s-1)\frac{b}{h}$	$-\frac{M_{s-1}}{h}$
	s	$+\frac{1}{h}$	$-s\frac{b}{h}$	$-\frac{M_s}{h}$
Vertical	0	0	0	
	1	0	-1	
	2	0	-1	
	3	0	-1	
	s-1	0	-1	
	s	0	-1	
Diagonal	1	0	$+c/h$	
	2	0	$+c/h$	
	3	0	$+c/h$	
	s-1	0	$+c/h$	
	s	0	$+c/h$	

The truss of Fig. 3 is resolved into two free-bodies, AC and CB, with AC shown in Fig. 4. The elastic constants are tabulated in Table I. From this table, the value of the constant:

$$C_1 = \sum_A^B \alpha_i \lambda_i = \frac{2n}{h^2} \lambda_H \tag{7}$$

where $\lambda_H = \lambda_T = \lambda_B$

The computation of the constant C_2 is apparently more complicated. From definition:

$$\begin{aligned} C_2 = & \sum_A^B \beta_{Ti}^2 \lambda_{Ti} + \sum_A^B \beta_{Bi}^2 \lambda_{Bi} \\ & + \sum_A^B \beta_{Vi}^2 \lambda_{Vi} + \sum_A^B \beta_{Di}^2 \lambda_{Di} \end{aligned} \tag{8a}$$

The first term of eq. (8a) is:

$$\sum_A^B \beta_{Ti}^2 \lambda_{Ti} = 2\frac{b^2}{h^2}\left[1^2 + 2^2 + \ldots + (s-1)^2\right] \lambda_T \tag{8b}$$

The expression in the bracket of eq. (8b) is a power series. Evaluating the power series, eq. (8b) becomes:

$$\sum_A^B \beta_{Ti}^2 \lambda_{Ti} = 2\frac{b^2}{h^2}\frac{(s-1)}{6}(s)(2s-1)\lambda_T \tag{8c}$$

Similarly:

$$\sum_A^B \beta_{Bi} \lambda_{Bi} = \frac{b^2 s}{h^2}(s+1)(2s-1)\lambda_B \tag{8d}$$

The corresponding expressions for the vertical and diagonal members are:

$$\sum_A^B \beta_{Vi}^2 \lambda_{Vi} = 2s\lambda_V \tag{8e}$$

$$\sum_{A}^{B} \beta_{Di}^{2} \lambda_{Di} = \frac{c^2}{h^2} s \lambda_D \tag{8f}$$

Combining eqs. (8c, 8d, 8e, and 8f), the constant:

$$C_2 = \frac{n}{h^2}\left[b^2 \frac{(n^2 + 2)}{6} \lambda_H + h^2 \lambda_V + c^2 \lambda_D\right] \tag{8}$$

Finally, the load constants D_1 and D_2 are derived. If the system of loading is symmetrical with respect to the axis of symmetry of the truss, the constant:

$$D_2 = 0 \tag{9}$$

and the constant D_1 may be expressed in a very simple form. From definition:

$$\begin{aligned} D_1 = & \sum_{A}^{B} SN_{Ti} \alpha_{Ti} \lambda_{Ti} + \sum_{A}^{B} SN_{Bi} \alpha_{Bi} \lambda_{Bi} \\ & + \sum_{A}^{B} SN_{Vi} \alpha_{Vi} \lambda_{Vi} + \sum_{A}^{B} SN_{Di} \alpha_{Di} \lambda_{Di} \end{aligned} \tag{10a}$$

The third and fourth terms of eq. (10a) are equal to zero, and the first and second terms (Table I) are:

$$\sum_{A}^{B} SN_{Ti} \alpha_{Ti} \lambda_{Ti} = \frac{-2}{h^2}(0 + M + \ldots + M_{s-1}) \lambda_T \tag{10b}$$

$$\sum_{A}^{B} SN_{Bi} \alpha_{Bi} \lambda_{Bi} = \frac{-2}{h^2}(M_1 + M_2 + \ldots + M_s) \lambda_B \tag{10c}$$

Combining the first and second terms (eq. 10a) and introducing a new function:

A_{BMD} = bending moment area of the basic structure (both parts)

$$= b(2M_1 + 2M_2 + \ldots + 2M_{s-1} + M_s) \tag{10d}$$

$$D_1 = \sum_{A}^{B} SN_i \alpha_i \lambda_i = \frac{-2 A_{BMD} \lambda_H}{bh^2} \tag{10}$$

In cases of unsymmetrical loading, the general procedure for the evaluation of load constants is more convenient.

B. Case II - Warren Truss

A Warren Truss of constant depth and loaded by a general system of forces is considered (Fig. 5).

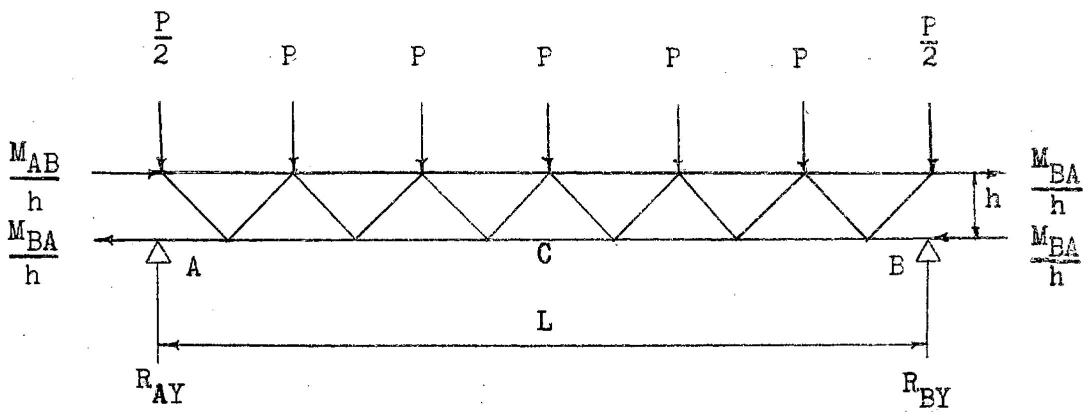


Fig. 5

Typical Warren Truss

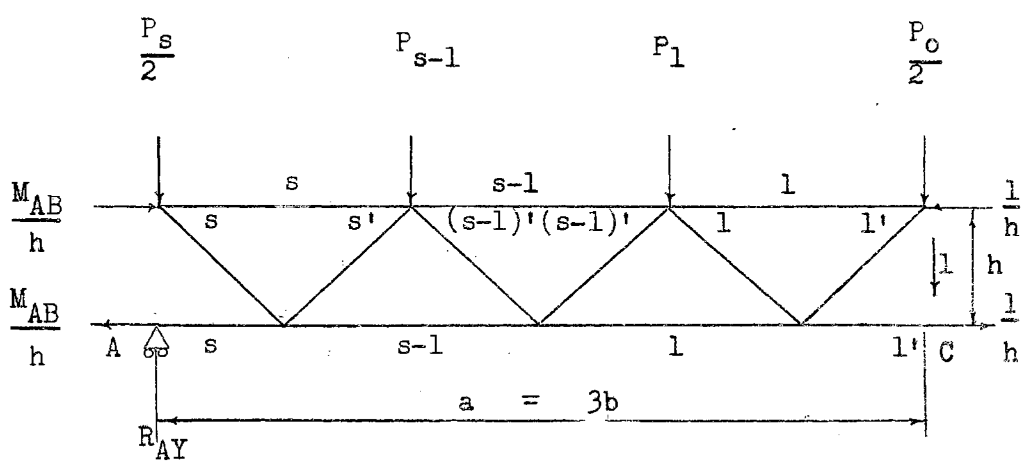


Fig. 6

Truss Element AC

TABLE II

TRUSS AND LOAD CONSTANTS

i		α_i	β_i	SN_i
Top	1	$-\frac{1}{h}$	$+\frac{b}{2h}$	$+\frac{M'_1}{h}$
	s-1	$-\frac{1}{h}$	$+\frac{3b}{h}$	$+\frac{M'_{s-1}}{h}$
	s	$-\frac{1}{h}$	$+\frac{5b}{2h}$	$+\frac{M'_s}{h}$
Bottom	1'	$+\frac{1}{h}$	-0	$-\frac{M_o}{h}$
	1	$+\frac{1}{h}$	$-1\frac{b}{h}$	$-\frac{M_1}{h}$
	s-1	$+\frac{1}{h}$	$-2\frac{b}{h}$	$-\frac{M_{s-1}}{h}$
	s	$+\frac{1}{h}$	$-3\frac{b}{h}$	$-\frac{M_s}{h}$
Diagonal	1'	0	$-c/h$	
	1	0	$+c/h$	
	(s-1)'	0	$-c/h$	
	(s-1)	0	$+c/h$	
	s'	0	$-c/h$	
	s	0	$+c/h$	

The Warren Truss of Fig. 5 is resolved into two free-bodies, AC and CB, with AC being shown in Fig. 6. In Table II, the truss and load constants are shown. From observation, the constant:

$$C_1 = \sum_A^B \alpha_i^2 \lambda_i = \frac{2n}{h^2}\lambda_H \qquad (11)$$

where $\lambda_H = \lambda_B = \lambda_T$

The expression for C_2 is more difficult to determine. By definition:

$$C_2 = \sum_A^B \beta_{Ti}^2 \lambda_{Ti} + \sum_A^B \beta_{Bi}^2 \lambda_{Bi} + \sum_A^B \beta_{Di}^2 \lambda_{Di} \qquad (12a)$$

Using power series, the first term is:

$$\sum_A^B \beta_{Ti}^2 \lambda_{Ti} = 2\frac{b^2}{4h^2}\left[1^2 + 3^2 + \ldots + (2s-1)^2\right] \lambda_T$$

$$= \frac{b^2 n}{h^2 12}(n-1)(n+1)\lambda_T \qquad (12b)$$

The second term is similar to the first, however a different power series formula is used. As shown:

$$\sum_A^B \beta_{Bi}^2 \lambda_{Bi} = 2\frac{b^2}{h^2}(1^2 + 2^2 + \ldots + (s-1)^2 + s^2 - \frac{s^2}{2})\lambda_B$$

$$= \frac{b^2}{h^2} n(\frac{n^2+2}{12})\lambda_B \qquad (12c)$$

By inspection:

$$\sum_A^B \beta_{Di} \lambda_{Di} = \frac{4sc^2}{h^2}\lambda_D = \frac{2nc^2}{h^2}\lambda_D \qquad (12d)$$

The final form of eq. (12a) is:

$$C_2 = \frac{n}{h^2}\left[b^2(\frac{2n^2+1}{12})\lambda_H + 2c^2\lambda_D\right] \qquad (12)$$

From a previous discussion, if the system of loading is symmetrical to the axis of symmetry, the load constants:

$$D_2 = 0 \qquad (13)$$

and

$$D_1 = \sum_A^B SN_{Ti}\alpha_{Ti}\lambda_{Ti} + \sum_A^B SN_{Bi}\alpha_{Bi}\lambda_{Bi} \qquad (14a)$$

From Table II and Fig. 7, the terms of eq. (14a) are:

$$\sum_{A}^{B} SN_{Ti}\,\alpha_{Ti}\,\lambda_{Ti} = \frac{-2}{h^2}\left(M'_1 + M'_2 + \dots + M'_{s-1} + M'_s\right)\lambda_T \qquad (14b)$$

$$\sum_{A}^{B} SN_{Bi}\,\alpha_{Bi}\,\lambda_{Bi} = \frac{-2}{h^2}\left(M_1 + M_2 + \dots + M_{s-1} + \frac{M_s}{2}\right)\lambda_B \qquad (14c)$$

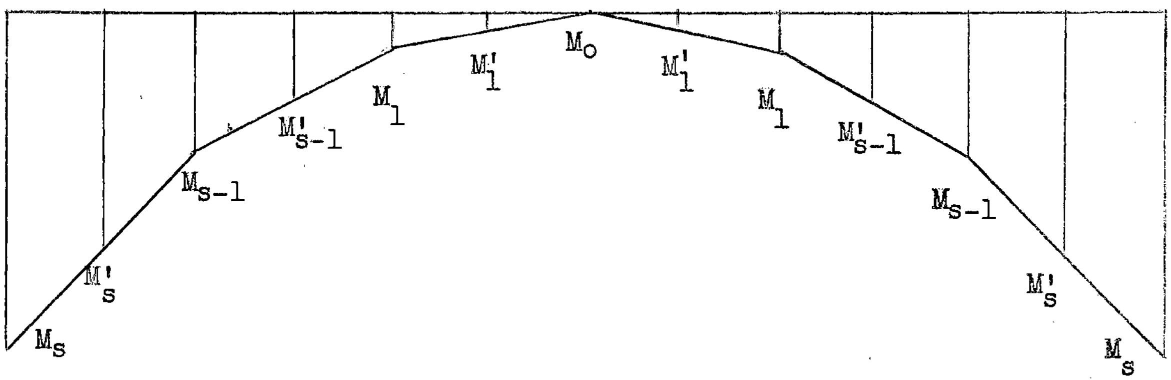


Fig. 7

The Bending Moment Diagram

of Parts AC and BC

The area of the bending moment diagram of Fig. 7:

$$A_{BMD} = 2M'_1\left(\frac{b}{2}\right) + 2M_1\left(\frac{b}{2}\right) + \dots + M_s\left(\frac{b}{2}\right) \qquad (14d)$$

Combining equations (14b, 14c, and 14d), the final expression is:

$$D_1 = \frac{-2A_{BMD}}{bh^2}\lambda_H \qquad (14)$$

C. Case III - Warren Truss (with verticals)

In Fig. 8 is shown the third and last special case to be considered. It is a Warren Truss (with verticals) of constant depth, and loaded by a general system of forces.

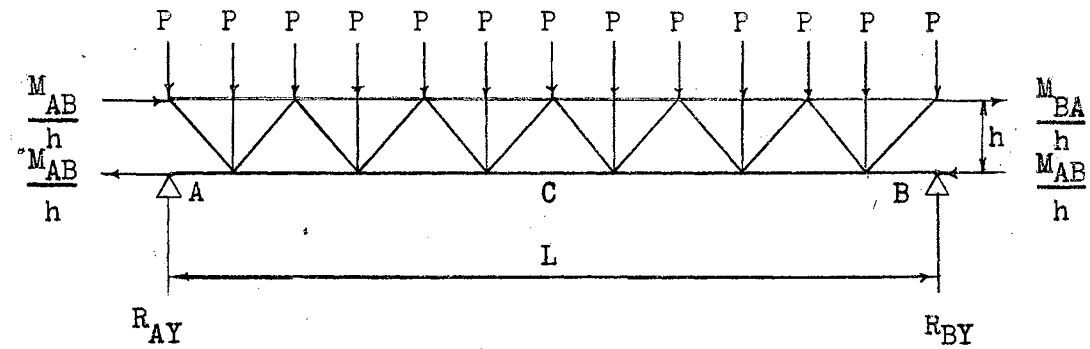


Fig. 8

Warren Truss (with verticals)

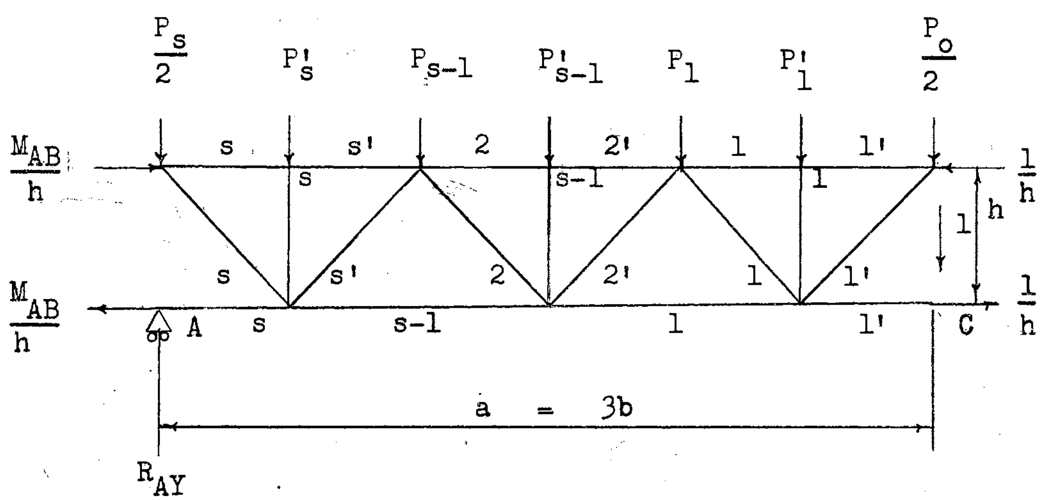


Fig. 9

Truss Element AC

TABLE III

TRUSS AND LOAD CONSTANTS

i		α_i	β_i	SN_i
Top	1'	$-\frac{1}{h}$	$+\frac{b}{2h}$	$+\frac{M'1}{h}$
	1	$-\frac{1}{h}$	$+\frac{b}{2h}$	$+\frac{M'1}{h}$
	2'	$-\frac{1}{h}$	$+\frac{3b}{2h}$	$+\frac{M'2}{h}$
	2	$-\frac{1}{h}$	$+\frac{3b}{2h}$	$+\frac{M'2}{h}$
	s'	$-\frac{1}{h}$	$+\frac{(2s-1)b}{2h}$	$+\frac{M's}{h}$
	s	$-\frac{1}{h}$	$+\frac{(2s-1)b}{2h}$	$+\frac{M's}{h}$
Bottom	1'	$+\frac{1}{h}$	0	$-\frac{M_o}{h}$
	1	$+\frac{1}{h}$	$-\frac{b}{h}$	$-\frac{M_1}{h}$
	s-1	$+\frac{1}{h}$	$-\frac{(s-2)b}{h}$	$-\frac{M_{s-1}}{h}$
	s	$+\frac{1}{h}$	$-\frac{(s-1)b}{h}$	$-\frac{M_s}{h}$
Vertical	1	0	0	
	s-1	0	0	
	s	0	0	
Diagonal	1'	0	$-$ c/h	
	1	0	$+$ c/h	
	2'	0	$-$ c/h	
	2	0	$+$ c/h	
	s'	0	$-$ c/h	
	s	0	$+$ c/h	

The truss shown in Fig. 8 is resolved into two parts, AC and CD. The part AC is shown in Fig. 9. The axial forces due to loads and unit-redundants are computed and tabulated. From Table III:

$$C_1 = \sum_A^B \alpha_1^2 \lambda_1 = \frac{2n\lambda_H}{h^2} \tag{15}$$

where
$$\lambda_H = \lambda_B = 2\lambda_T$$

The expression for C_2 is almost identical to the expression of C_2 (eq. 12a). By definition:

$$C_2 = \sum_A^B \beta_{Ti}^2 \lambda_{Ti} + \sum_A^B \beta_{Bi}^2 \lambda_{Bi} + \sum_A^B \beta_{Di}^2 \lambda_{Di} + \sum_A^B \beta_{Vi}^2 \lambda_{Vi} \tag{16a}$$

From observation, the first term of eq. (16a) is identical to the first term of eq. (12a) if λ_T of eq. (16a) is replaced by $2\lambda_T$. Accordingly:

$$\sum_A^B \beta_{Ti} \lambda_{Ti} = \frac{b^2}{h^2} \frac{n}{6}(n-1)(n+1)\lambda_T \tag{16b}$$

The second term of eq. (16a) is equal to the second term of eq. (12a). Therefore:

$$\sum_A^B \beta_{Bi} \lambda_{Bi} = \frac{b^2}{h^2} n \frac{(n^2+2)}{12} \lambda_B \tag{16c}$$

From Table III, the third and fourth terms of eq. (16a) are observed to be:

$$\sum_A^B \beta_{Di} \lambda_{Di} = 2\frac{c^2}{h^2} n \lambda_D \tag{16d}$$

$$\sum_A^B \beta_{Vi} \lambda_{Vi} = 0 \tag{16e}$$

Combining the terms of eq. (16a), the expression for the constant:

$$C_2 = \frac{n}{h^2}\left[b^2\frac{(2n^2+1)}{12}\lambda_H + 2c^2\lambda_D\right] \qquad (16)$$

The expression for the load constant D_1 is defined by a previous discussion:

$$D_1 = \sum_A^B SN_{Ti}\alpha_{Ti}\lambda_{Ti} + \sum_A^B SN_{Bi}\alpha_{Bi}\lambda_{Bi} \qquad (17a)$$

From Table III, the first and last terms of eq. (17a) are respectively:

$$\sum_A^B SN_{Ti}\alpha_{Ti}\lambda_{Ti} = -\frac{4}{h^2}(M'_1 + M'_2 + \ldots + M_{s-1} + M_s)\lambda_T \qquad (17b)$$

$$\sum_A^B SN_{Bi}\alpha_{Bi}\lambda_{Bi} = -\frac{2}{h^2}(M_1 + M_2 + \ldots + M_{s-1} + M_s)\lambda_B \qquad (17c)$$

Combining eq. (17b, 17c) and A_{BMD}, the final expression for D_1 is:

$$D_1 = \frac{-2A_{BMD}\lambda_H}{bh^2} \qquad (17)$$

If the system of loading is symmetrical with respect to the axis of symmetry of the truss, the constant:

$$D_2 = 0 \qquad (18)$$

CHAPTER III

PROCEDURE AND EXAMPLES

A. Procedure of Analysis

The procedure of application of the slope deflection equations to the analysis of structures with trusses of constant depth is:

1. Determine geometry of truss

   a. external dimensions

   b. cross-sectional areas and dimensions of members

2. Compute constants C_1, C_2, D_1, and D_2

3. Compute fixed-end moments

   a. due to loads

   b. due to displacements

4. Write slope deflection equations for the structure

5. Write equilibrium equations

6. Solve equilibrium equations for unknown Δ's and θ's by substituting constants into the slope deflection equations

7. Compute final moments by substituting Δ's and θ's into the slope deflection equations

B. Example I - Warren Truss (with verticals)

A three span Warren Truss is considered. The structure (Fig. 10) is symmetrical and symmetrically loaded.

1. Geometry of truss

The dimensions of the truss are indicated in Fig. 10, the cross-sectional areas are:

$A_{Ti} = 10$ inches $\qquad A_{Bi} = 10$ inches

$A_{Vi} = 2$ inches $\qquad A_{Di} = 4$ inches

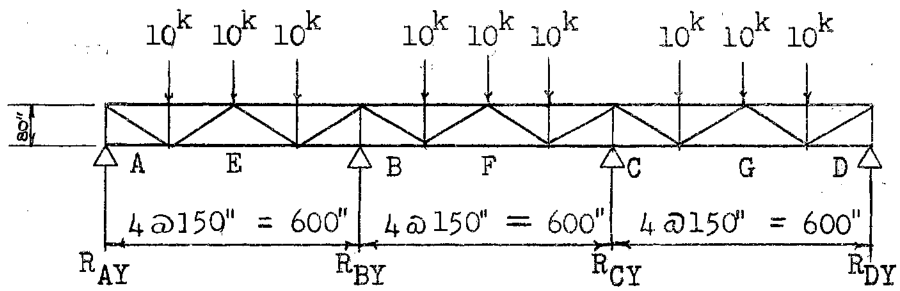


Fig. 10

A Continuous Truss

2. Computation of constants

Using the general expressions, eqs. (15, 16, 17, and 18), the constants are:

$$C_1 = \frac{2n\lambda_H}{h^2} = \frac{(2)(2)(300)}{(80)^2(10)E} = +\frac{.01875}{E}$$

$$C_2 = \frac{n}{h^2}\left[b^2\frac{(2n^2+1)}{12}\lambda_H + 2c^2\lambda_D\right]$$

$$= \frac{2}{(80)^2}\left[(300)^2\frac{(8+1)}{12}\frac{300}{10(E)} + 2(170)^2\frac{170}{4E}\right] = +\frac{1400}{E}$$

$$D_1 = \frac{-2AB_{BMD}\lambda_H}{h^2}$$

$$= \frac{-2(750+750+3000)(150)(300)}{(300)(80)^2\ (10)E} = \frac{-21.1}{E}$$

$$D_2 = 0$$

The error in applying the general constant formulas without modification is small, and no correction is needed for the truss of Fig. 10.

3. Conditions of symmetry

$$\theta_A = -\theta_D, \qquad \theta_B = -\theta_C$$

$$FM_{AB} = FM_{BC} = FM_{CD}$$

$$= -FM_{BA} = -FM_{CB} = -FM_{DC}$$

4. Fixed-end moments due to loads

$$FM_{AB} = \frac{-D_1}{C_1} + \frac{aD_2}{C_3} - CM_{AC}$$

$$= \frac{+\frac{21.1}{E}}{\frac{.01875}{E}} - 3000 = -1875 \text{ kip-inches}$$

5. Moment equations

$$M_{AB} = -M_{DC} = \left(\frac{a^2}{C_2} + \frac{1}{C_1}\right)\theta_A + \left(\frac{a^2}{C_2} - \frac{1}{C_1}\right)\theta_B + FM_{AB}$$

$$= (64.3 + 53.3)E\theta_A + (64.3 - 53.3)E\theta_B - 1875$$

$$= 117.6E\theta_A - 11E\theta_B - 1875$$

$$M_{BA} = -M_{CB} - \left(\frac{a^2}{C_2} + \frac{1}{C_1}\right)\theta_B + \left(\frac{a^2}{C_2} - \frac{1}{C_1}\right)\theta_A + FM_{BA}$$

$$= 117.6E\theta_B + 11.0E\theta_A + 1875$$

$$M_{BC} = -M_{CB} = \left(\frac{a^2}{C_2} + \frac{1}{C_1}\right)\theta_B + \left(\frac{a^2}{C_2} - \frac{1}{C_1}\right)\theta_C + FM_{BC}$$

$$= 106.6E\theta_B - 1875$$

6. Equilibrium equations

$$M_{AB} = 0, \qquad M_{BA} + M_{BC} = 0$$

7. Deformation

Simultaneous solution of the equilibrium equations gives the following results:

$$E\theta_A = + 16$$

$$E\theta_B = - 0.784$$

8. Final end moments

Substituting the values of $E\theta$ into the moment equations, the final moments are:

$$M_{AB} = - M_{DC} = 0$$

$$M_{BA} = - M_{CB} = - 1959 \text{ kip-inches}$$

$$M_{BC} = - M_{CB} = - 1959 \text{ kip-inches}$$

8. Example II - Pratt Truss - Frame

A two span truss-frame (Fig. 11) is considered. The structure is symmetrical and symmetrically loaded.

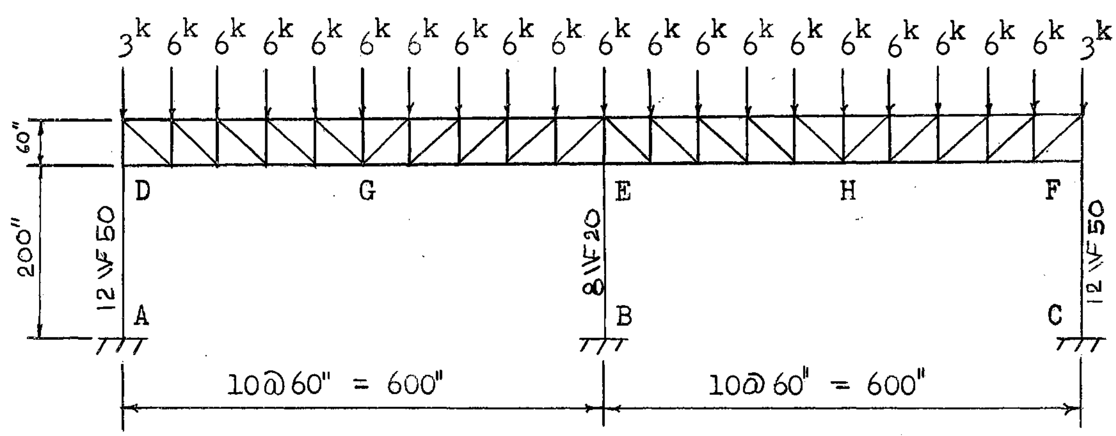


Fig. 11

A Loaded Truss-Frame

1. Geometry of truss

Dimensions of the structure and sizes of the columns are indicated in Fig. 11. The cross-sectional areas of the truss members are:

$$A_{Hi} = 10 \text{ inches}, \quad A_{Vi} = 4 \text{ inches}, \quad A_{Di} = 6 \text{ inches}$$

2. Computation of constants

Using eqs. (7, 8, 9, 10), the constants are:

$$C_1 = \frac{2n\lambda_H}{h^2 E} = +\frac{2(10)(60)}{(60)^2(10)E} = +\frac{0.033}{E}$$

$$C_2 = \frac{n}{h^2}\left[b^2\frac{(n^2+2)}{6}\lambda_H + h^2\lambda_V + c^2\lambda_D\right]$$

$$= \frac{10}{(60)^2}\left[\frac{(60)^2(102)\,60}{(6)(10)(E)} + \frac{(60)^2(60)}{4E} + \frac{(85)^2 85}{6E}\right] = +\frac{1454}{E}$$

$$D_1 = \frac{-2A_{BMD}\lambda_H}{bh^2} = \frac{-2(15,300)(60)}{(60)^2(10)E} = \frac{-51}{E}$$

$$D_2 = 0$$

The difference between the basic truss of Chapter II and the truss of Fig. 11 is insignificant. Accordingly, no correction to the constant formulas are used.

3. Conditions of symmetry

$$FM_{DE} = FM_{EF} = -FM_{ED} = -FM_{FE}$$

$$\theta_A = \theta_B = \theta_C = \theta_E = 0$$

$$\theta_D = -\theta_F$$

4. Fixed - end moments due to loads

$$FM_{AB} = -\frac{D_1}{C_1} + \frac{aD_2}{C_2} - CM_{AC}$$

$$= \frac{+\frac{51}{E}}{\frac{0.033}{E}} + 0 - 4500 = 2970 \text{ kip-inches}$$

5. Moment equations

$$M_{AD} = -M_{CF} = 4E\frac{I}{L}\theta_A + 2E\frac{I}{L}\theta_D + FM_{AD}$$

$$= 2E\frac{I}{L}\theta_D = \frac{2(394.5)}{230}\theta_D = +3.42E\theta_D$$

$$M_{DA} = -M_{FC} = 4E\frac{I}{L}\theta_D + 2E\frac{I}{L}\theta_A + FM_{DA}$$

$$= \frac{4(394.5)}{230}E\theta_D = +6.85E\theta_D$$

$$M_{DE} = -M_{FE} = \left(\frac{a^2}{C_2} + \frac{1}{C_1}\right)\theta_D + \left(\frac{a^2}{C_2} - \frac{1}{C_1}\right)\theta_E + FM_{DE}$$

$$= (61.8 + 30)E\theta_D + (61.8-30)E\theta_E - 2{,}970$$

$$= 91.8E\theta_D + 31.8E\theta_E - 2970$$

$$M_{ED} = -M_{EF} = \left(\frac{a^2}{C_2} + \frac{1}{C_1}\right)\theta_E + \left(\frac{a^2}{C_2} - \frac{1}{C_1}\right)\theta_D + FM_{ED}$$

$$= 91.8E\theta_E + 31.8E\theta_D + 2970$$

$$M_{EB} = 4E\frac{I}{L}\theta_E + 2E\frac{I}{L}\theta_B + FM_{EB}$$

$$\doteq 0$$

$$M_{BE} = 4E\frac{I}{L}\theta_B + 2E\frac{I}{L}\theta_E + FM_{BE}$$

$$= 0$$

6. Equilibrium equation

$$M_{DA} + M_{DE} = 0$$

7. Deformation

Solving the equilibrium equation:

$$E\theta_D = +30.1$$

8. Final end moments

Substituting $E\theta_D$ into the moment equations:

$$M_{AD} = -M_{CF} = 3.42(30.1) = +103$$

$$M_{DA} = -M_{FC} = 6.85(30.1) = +208$$

$$M_{DE} = -M_{FE} = 91.8(30.1) - 2970 = -208$$

$$M_{ED} = -M_{EF} = 31.8(30.1) + 2970 = +3,927$$

$$M_{EB} = M_{BE} = 0$$

All moments have units of kip-inches

A SELECTED BIBLIOGRAPHY

1. Grinter, Linton E. Theory of Modern Steel Structures. New York: The MacMillan Co., 1950, pp. 297-344.

2. Haynes, John M. A New Approach to the Rigid Joint Method. Bulletin No. 134, Purdue Engineering Experiment Station, 1957.

3. Jacobsen, E. R. "Moment Distribution and the Analysis of a Continuous Truss of Varying Depth." The Engineering Journal, 1941.

4. Maugh, L. C. Statically Indeterminate Structures. New York: John Wiley and Sons, Inc., 1946, pp. 167-183.

5. Parcel, John I. and Moorman, Robert B. B. Analysis of Statically Indeterminates Structures. New York: John Wiley and Sons, Inc., 1955, 141-248.

6. Seely, Fred B. and Smith, James O. Advanced Mechanics of Materials. New York: John Wiley and Sons, Inc., 1955, pp. 421-510.

7. Spofford, Charles M. The Theory of Structures. New York: McGraw-Hill Co., Inc., 1939, pp. 427-454.

8. Timoshenko, S. and Young, D. H. Theory of Structures. New York: McGraw-Hill Book Co., 1945, pp. 258-264.

VITA

Dale Edward Morrisett

Candidate for the Degree of

Master of Science

Report: DERIVATION OF SLOPE DEFLECTION EQUATIONS FOR STRAIGHT TRUSSES OF CONSTANT DEPTH

Major Field: Civil Engineering

Biographical:

Personal data: Born near Guthrie, Oklahoma, the son of Horace L. and Myrtle F. Morrisett.

Education: Attended grade school near Guthrie, Oklahoma; graduated from Guthrie High School in 1950; received the Bachelor of Science degree from the Oklahoma Agricultural and Mechanical College, with a major in Civil Engineering; completed requirements for the Master of Science degree in August 1956.

Professional experience: United States Army Corps of Engineers, 1954-1956, Commissioned Officer