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TWO ESSAYS IN PRICE DISCRIMINATION

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Abstract

This study answers two questions brought by internet technology improvement in industrial organization literature. The first essay, “Customer Poaching, Coupon Trading and Consumer Arbitrage”, relaxes the no consumer arbitrage assumption and studies the impacts of coupon trading on equilibrium prices, promotion intensities (frequency and depth) and profits. The results show that: (i) firms never have incentive to distribute defensive coupons; (ii) a larger fraction of coupon traders among consumers or higher distribution costs reduce the attractiveness of couponing, and firms respond by lowering their (offensive) promotion frequency and depth; (iii) when the cost of distributing coupons increases, firms respond by sending fewer offensive coupons, but of higher face value; (iv) increase in the fraction of coupon traders and increase in coupon distribution cost both lead to higher equilibrium prices and profits.

The second essay, “Post-Sale Low Price Guarantee and Price Fluctuation”, explains the cyclical price fluctuation by the combination of firm’s self post-sale low price guarantees and its intertemporal pricing policy. An empirical analysis based on weekly price data from Best Buy and Circuit City shows that there is a negative relationship between each firm’s current price change and its previous price change. This is consistent with my theory that firms may use an intertemporal pricing policy that causes price fluctuation over time comparing to the existing literature that usually predicts a monotonic price decreasing over time.

CHAPTER I

INTRODUCTION

Industrial organization is usually defined as a field of economics that studies the strategic behavior of firms, structure of firms and markets, and their interactions. An important topic of industrial organization is price discrimination. The commonly observed types are second and third degree price discrimination.[1] In the past fifteen years[2], technology development, especially internet development, has had dramatic effects on both sides of a market (firms and consumers). This development not only has significantly changed firm price discrimination strategies, enriched firm instruments of price discrimination and information collecting; but also has influenced consumer market behavior, as well as market structure.

On the firm side, the development of the internet may either increase market efficiency or decrease market efficiency. On the one hand, better information allows firms to offer products that are better suited for consumers, which in turns increases market efficiency. Examples can be found in product customization literature like Bernhardt et al. (2006). On the other hand, internet development facilitates firms to price discriminate, and this price discrimination sometimes is associated with inefficiency. Examples can be found in coupon literature such as Liu and Serfes (2004).

On the consumer side, one advantage brought by the development of technology is that it enables consumers to acquire abundant information at a much lower cost. This

[1] Extended literature and definitions are given in Chapter II.

[2] Commercialization of internet started in mid 1990s.

information can be categorized into two types: price information and non-price information. Better price information enables consumers to find better prices; and better non-price information enables consumers to find better matched products. As a consequence, consumer welfare usually increases.

Chapter II of this study provides a more extensive theoretical and related empirical literature review.

I.1. Importance of the Study

In spite of extensive studies on price discrimination, there are still some open questions, especially when considering new firm and consumer behavior brought by technology improvement. Due to the development of the internet, both sides of the market (firms and consumers) can now acquire abundant information at a lower cost compared to decades ago. This has significantly affected firm strategies and consumer behavior, and has brought new questions in the study of price discrimination. The purpose of this dissertation is to address two of these issues: coupon competition when consumer arbitrage is allowed, and cyclical price fluctuation over time when post-sale low price guarantee is adopted.

Theoretical economists have explored firm promotion strategies through couponing or other similar ways from a wide range of perspectives. However, coupon trading, an important phenomenon that may affect firms' couponing strategies, is ignored in this literature. In the past, the time cost or hassle cost of collecting coupons and especially trading coupons[3] was prohibitively high. Coupon trading activity among consumers is not as popular as today and thus can be ignored without causing a significant inconsistency with the real world. However, due to the improvement of the internet, for following reasons, coupon trading has become more and more popular among consumers, and thus cannot be ignored any more.

1st. The cost of collecting coupons has been dramatically lowered.

Comparing to the past when coupons are usually distributed in the form of printed paper, nowadays, many firms distribute coupons through the internet, and coupons are usually in the form of electronic documents (usually PDF files), or even just a set of code. The innovation in the forms of coupon has significantly eased the way of collecting coupons.

2nd. The cost of trading coupons has been lowered.

Following the innovation of coupon formats, consumers do not have to trade the printed physical coupon face to face any more, instead, they can finish the trading by a simple email. This not only lowers the transaction cost of finishing the trading, but also releases the limitation of finding potential buyers and sellers. A potential coupon buyer/seller doesn’t have to be a local resident near a seller/buyer; rather she could live anywhere in the world since they can finish the trading process online.

3rd. Many websites have been developed to facilitate trading.

In the past fifteen years, many websites and online forums have been set up to facilitate trading among consumers. Besides the most well known website E-Bay, there

[3] This may come from the cost of finding a potential coupon buyer or seller, and cost of negotiating a transaction price.

are numerous websites and forums that help consumers to trade coupons and other products, such as E-Junkie.com, Craigslist.org, and so on. The emergence of these websites has diminished the cost of finding potential traders dramatically. Overall, the improvement in the internet has made coupon trading very easy today. And coupon trading, an important behavior among some consumers, which may affect firm competition strategies, certainly cannot be ignored in coupon competition or other similar promotion literature any more.

Based on existing coupon competition literature, my thesis incorporates consumer coupon trading behavior into firm coupon competition. This change allows us to study the effect of coupon trading on firm coupon competition strategies. In short, this study fills a gap in the coupon competition literature.

The other important consequence of the development of internet is that consumers can get abundant information at low cost nowadays. Since many stores (brick & motor stores or pure online stores) have started their online business, it's much easier for consumers to collect product information than ever before, especially for the price information. In order to compare prices across stores or track prices over time, consumers can simplify click some buttons on a computer, rather than drive or walk to the brick & motor stores. This has significant effect on firms' pricing strategies.

A variety of literature has been developed to explain price dispersion in a vertical dimension that is over time.[4] A crucial assumption made in this literature is that consumers can only buy early at a high price or late at a low price. This assumption, though it may still be true in some circumstances, should be revised since many firms

[4] See Stokey (1979) and Nair (2007) for example.

today adopt a low price guarantee policy (Hereafter LPG). One way to categorize LPG is to divide it into two groups: LPG across stores and LPG over time.[5] The LPG across stores has been extensively studied from different aspects, theoretically and empirically. Salop (1986) uses theoretical model to explain how LPG could be used as an instrument to achieve collusion among stores. Manez (2006) empirically shows that LPG could be used as a signal of low price. However, the latter, LPG over time, has drawn relatively little attention from economists. My thesis fills this gap by examining LPG over time, and its effect on firms' pricing strategy.

I.2. Objectives of the Study

In order to fill these two gaps, my study is presented in two separate essays. The first essay develops a theoretical model, in which coupon trading among consumers is allowed. The effects of coupon trading on firm competitive behavior are examined. The second essay first presents a theoretical model to examine firms' post-sale self LPG policy and its effect on firms' intertemporal pricing strategy, profit, consumer surplus and social welfare. Then I use weekly data from Best Buy and Circuit City to empirically examine the theoretical model.

I.3. Results of the Study

The third chapter, "Customer Poaching, Coupon Trading and Consumer Arbitrage", deals with firm coupon competition when coupon trading among consumers is presented. The results indicate that coupon trading among consumers has significant

[5] There are some stores adopt a LPG which is a combination of these two. For example, Staples claims if consumer finds a lower price anywhere else within 14 days of purchase, it will refund the difference.

effect on firm competitive behavior. Specifically, when the fraction of coupon traders increases, and when the cost of distributing coupons increases, competition is released. Competition through coupon distribution is a prisoner's dilemma game.

The fourth chapter, "Post-Sale Low Price Guarantees and Price Fluctuation", investigates firms' LPG policy and its effect on firms' pricing strategies. The essay finds that it is profitable for firms to adopt post-sale LPG policy. And further, with this policy firms may change their prices periodically to price discriminate the consumers with high cost of requesting price matching after purchase. This intertemporal pricing policy causes the commonly observed phenomenon of price fluctuation over time. Weekly data from Best Buy and Circuit City indicate that each firm's price change is significantly correlated with its own previous change. This provides some empirical evidence for the theoretical prediction at some degree.

I.4. Organization of the Study

Chapter II presents a summary of the existing theoretical and empirical literature of industrial organization. Chapter III presents the first essay, "Customer Poaching, Coupon Trading and Consumer Arbitrage". Chapter IV presents the second essay, "Post-Sale Low Price Guarantees and Price Fluctuation". Finally, Chapter V lays out a summary of the study and suggests possible extensions. In the next chapter, I summarize the theoretical and empirical literature related to this study.

CHAPTER II

LITERATURE REVIEW

This chapter provides an overview of the landmark theoretical and related empirical studies of industrial organization that focus on the price discrimination literature in the following areas: coupon distribution competition, price dispersion, intertemporal price change, and low-price guarantee. Section II.1 gives an overview of price discrimination that provides the conceptual background for the study. Section II.2 outlines the literature related to coupon distribution competition. Section II.3 introduces the literature related to price dispersion, intertemporal price strategy and LPG.

II.1. Price Discrimination

Overview and Taxonomy

Price discrimination has existed and has been studied for a long time. The possible earliest economist to study price discrimination is Jules Dupuit, a French economist-engineer. As pointed out by F.Y.Edgeworth (1910) and emphasized by Robert Ekelund Jr. (1970), Dupuit is believed to be the earliest, and the highest authority on the theory of price discrimination at that time. Regardless of its different types, price discrimination can be simply defined as identical products (goods or services) provided at different prices. Based on the degree to which product providers can charge different prices, Pigou (1932) categorize price discrimination into first, second and third degree price discrimination.

In the first degree price discrimination, price varies by customers. This is not very commonly observed in real world, because it requires the product provider has the ability to identify every single customer and the power to charge different prices to different customers. These two conditions are difficult to satisfy in many circumstances. One possible example could be repeated auctions for the same products. Thinking about an E-Bay seller selling multiple identical products through auction, each winning bid for the product could be different.[6] Therefore during this process, the same products are provided at different prices.

Second degree price discrimination is when providers are incapable of differentiating different types of consumers. Therefore they provide different supply schedules and rely on consumers to sort themselves by choosing different options. The most commonly observed second degree price discrimination is that price varies according to quantity sold. As one can easily find, the price of one-gallon milk is usually less than the twice of the price of half-gallon milk.

Third degree price discrimination occurs when providers are capable of differentiating and charging different prices between consumer classes (in the most extreme case, price varies by individual customer, and this becomes first degree price discrimination). Examples of third degree price discrimination could be senior discount or member discount.

As one can easily see from the above descriptions, these categories are not mutually exclusive or collectively exhaustive. Therefore Ivan Png (2002) suggests an

[6] Here the seller still cannot extract the entire surplus from the consumers. She can only charge different prices depending on the willingness to pay.

alternative taxonomy. Nevertheless, Pigou's taxonomy is still the prevailing taxonomy in industrial organization literature. Among the three types of price discriminations defined by Pigou, the most commonly seen and studied are second and third degree price discrimination. I will start with second degree price discrimination.

Second Degree Price Discrimination

For second degree price discrimination, providers offer different schedules to let consumers sort themselves into different groups. Consumers information plays a crucial role in firms' optimal price schedule determination. Stole (1995) discusses second degree price discrimination in oligopoly market where firms' products are spatially differentiated and consumers may have brand preference or quality preference. The paper finds that the optimal pricing schedules depend importantly on the type of private information the consumer has. When competition increases, quality distortion, price, and quality dispersion decrease. Armstrong (1999) discusses a model in which multi-products monopoly facing consumers with unobservable tastes.

A common seen second price discrimination is quantity discount. Goldman and Sibley (1984) give an example of non-uniform price based on quantity purchased. Before their work, existing literature about quantity discount usually focus on two-part non-uniform pricing. Their paper goes further by discussing arbitrary non-uniform pricing policies. In their paper, consumer demands are heterogeneous, and firms only know the distribution of the demand but not the demand of any exact consumer. The firm thus chooses an optimal non-uniform pricing policy based on quantity purchased. They find that this discrimination will in general increase welfare levels, and non-uniform price schedules will differ qualitatively when income effects are important from when they are not.

Besides the quantity discount discussed above, other characteristics could also be used as a sorting instrument. Chiang and Spatt (1982) develop a model, in which consumers are different in their valuations of waiting time to get the product. Knowing this, the monopoly price discriminates part of the market by bundling its products with different waiting time. They find that deadweight loss, consumer surplus and output between single-price monopoly and imperfect discrimination are ambiguous.

Sometimes, it is profitable for providers to offer different schedules to price discriminate part of the market; sometimes it is not profitable to do so. Diamond (1987) looks at a model with two classes of consumers—divided according to their willingness to pay—high type and low type. The model is in continuous time with a unit of flow of new consumers into the market. He finds that the equilibrium can have a single price or a pair of prices. In a two-price equilibrium, the lower price equals the lower willingness to pay of the low type, and the higher price is the reservation price of the high type.

Third Degree Price Discrimination

For third degree price discrimination, providers are able to sort different groups of consumers based on some of their characteristics. This difference can be categorized into two types: vertical differentiation and horizontal differentiation.

In vertical differentiation, consumers have the same preference ordering but different valuations of some characteristics of the product. A typical example is quality. Most (if not all) agree higher quality product is preferable, but only the consumers who value the quality high enough are willing to pay a premium to purchase a high quality product, while others do not value the quality high enough will pay low price and get low quality. This quality difference market could be automobile market (BMW versus Hyundai) or software market (Professional edition versus home edition). Shaked and Sutton (1982) look at a three stage model in which number of firms, product qualities, and prices are endogenous determined sequentially. They find the only perfect equilibrium exists when there are exact two firms, and they offer distinct qualities.

In horizontal differentiation, consumers have different tastes. An obvious example can be found in the soft drink market—Pepsi versus Coke. This horizontal differentiation is generally captured by consumers' heterogeneous locations. A benchmark model in horizontal differentiation literature is Hotelling's model (1929). In the paper, Hotelling consider a "linear city" of length 1. Consumers are uniformly distributed along the line. Two shops, located at the two ends of the line, sell identical products. Consumers have unit demand. Besides the price of the product, they have to pay a transportation cost, which is a linear function of the distance between the consumer's location and the location of the firm that the consumer shops at. Hotelling's model has been broadly discussed and cited. Generally speaking, there are two streams of results: minimum differentiation, in which firms will minimize their difference; and maximum differentiation, in which firms will maximize their difference.

Eaton and Lipsey (1975) examine the principle of minimum differentiation from Hotelling's model under various conditions. They find that of the models they studied, minimum differentiation is a property only of those in which firms pursue a strategy of zero conjectural variation and where the number of firms is restricted to two. When a new firm enters a market, or when an existing firm relocates, there is a strong tendency for that firm to locate as close as possible to another firm. This behavior tends to create local clusters of firms in many equilibrium and disequilibrium situations. The principle of minimum differentiation is a special case of the principle of local clustering when the number of firms in the market is restricted to two. D'Aspremont et al. (1979) reexamine Hotelling's model and the so-called principle of minimum differentiation, and find that no equilibrium price solution will exist when both sellers are not far enough from each other when using linear transportation cost as in Hotelling's model. They then find with quadratic transportation cost, firms will move away from each other as far as possible. Economides (1986) under the Hotelling's model structure, discusses a set of different transportation costs by let the transportation cost $f(d)=d^a$, where $1<=a<=2$. He finds that in general neither minimal nor maximal differentiation is correct; rather there exists a range of a value such that the perfect equilibrium locations are interior points of the product space.

<u>Other Price Discrimination Literature</u>

There is also many literature related to intertemporal price discrimination. A general idea of this intertemporal price discrimination is that firms charge different prices over time to price discriminate part of the market.[7] As contrary to the usually predicted monotonical price decreasing over time in existed literature, my second essay predict a price fluctuation over time.

While major literature discusses second or third degree price discrimination in a final good market, Katz (1987) looks at third degree price discrimination in

[7] Detailed information about this literature is provided in II.3.

intermediate good market. His paper develops a model in which there is a monopolist who is an intermediate good supplier. The intermediate good is an input into the production of a homogeneous final good that is sold in a set of independent local markets. The author finds that results are different from the final goods market. The factor that price discrimination may raise prices charged to both types of buyers shows that the demand interdependencies and possibility of integration can have powerful effects on the equilibrium outcome. Price discrimination can decrease consumer surplus and welfare but it can also prevent socially inefficient integration.

II.2. Coupon Trading Literature

Coupon Competition

Coupons have been used as an instrument to achieve second or third degree price discrimination for a long time. Based on the distribution method, coupons can be categorized as mass distribution or target coupon. For mass distribution coupons, each consumer generally has equal chance to get the coupon, and based on his/her characteristics (for example, time cost or hassle cost of using the coupon) decides whether to use the coupon or not. In this case, coupons are used as an instrument to facilitate second degree price discrimination. For target coupons, firms send coupons to a specific group or specific groups of consumers. These consumers, with the coupon, can get the product at a lower price. In this case, coupon is used as an instrument to achieve third degree price discrimination.

Early studies consider the use of coupons as a device to create market segmentation due to consumer self-selection (mass media coupons). For example, in

Narasimhan (1984), couponing enables price discrimination providing a lower price to a particular segment of consumers while keeping the price high for others. With the availability of more consumer information, firms can rely less on consumer self-selection and more on targeted coupons (see Shaffer and Zhang (1995) for examples of such practices). Targeted coupons are mostly modeled as offensive coupons, i.e., to poach rival firms' loyal customers (Shaffer and Zhang (1995), Bester and Petrakis (1996), Fudenberg and Tirole (2000) etc.).

The study most relevant to my dissertation chapter three is Bester and Petrakis (1996). They look at a duopoly model where consumers have preferences for one brand over the other. Each firm can send out coupons to consumers who prefer the other firm's product. They show that couponing intensifies competition between the firms, and the equilibrium prices and profits are lower than when no coupons are offered. Since couponing leads to a prisoners' dilemma game, an increase in the cost of coupon distribution would lead to higher prices and lower consumer surplus.

Fudenberg and Tirole (2000) look at a two-period game where in the second period. Firms can separate consumers who bought from them in period 1 from those consumers who did not. Consequently, each firm can poach the customers of their competitors by sending them coupons to induce them to switch. They also find that poaching leads to lower prices. They also investigate the efficiency of long-term contracts or short-term contracts.

In the previous two papers, a poaching firm sends the same coupons to all of its rival's customers. The coupons are different in Liu and Serfes (2004) and Shaffer and Zhang (1995, 2002). In Liu and Serfes, both firms send coupons of different face value.

This is because firms have detailed information which enables them to segment consumers into various groups. In Shaffer and Zhang (1995), each firm offers only one type of coupons, but it can choose to send coupons to only a portion of the customers (partial couponing), since each firm has the ability to identify and target each individual consumer. The reason firms do partial couponing is because some consumers' preferences are so strong that the poaching firm can't attract them by sending coupons. The game is still a prisoners' dilemma game in which the net effect of coupon targeting is the coupon distribution cost plus the discounts given to redeemers. Only when firms are asymmetric, the game may not be a prisoners' dilemma with one-to-one promotions (Shaffer and Zhang 2002), since there is also a market share effect. The firm with the higher-quality product may gain from one-to-one promotions at the cost of the lower-quality product firm.

Consumer Arbitrage

There have been few studies analyzing resale or consumer arbitrage, and they typically consider only monopoly. In Anderson and Ginsburg (1999), consumers differ in two dimensions: willingness to pay and arbitrage cost. In their setup, a monopolist can sell its product in two countries. It may sell in the second country even if there is no local demand, with the sole purpose of discriminating across consumers with different arbitrage costs in the first country. Calzolari and Pavan (2006) consider a monopolist's problem of designing revenue-maximizing mechanisms when resale is possible. They find that the revenue-maximizing mechanism may require a stochastic selling procedure. The auction literature has also considered how resale affects bidding. In particular, the information revealed in a primary auction market and changes in bidder participation patterns can create inefficiencies that affect the revenue ranking of standard auction formats when there is an option to resale in a secondary market. Zheng (2002) investigates the design of seller- optimal auctions when winning bidders can attempt to resell the good, and characterizes the sufficient and necessary condition for sincere bidding with resale. Haile (2003) considers how resale opportunities affect bidders' valuations and finds that the secondary market can benefit the initial seller if the resale seller can extract a sufficient share of the resale surplus. In our case, there is no pricing decision that is conditioned on the bidding outcome of the auction. The efficient re-allocation of coupons results via English auctions held with no incentive distortions.

Empirical and Experimental Coupon Literature

There are many experiments in the literature exploring various effects of firm promotion or couponing on consumer psychology and purchasing behavior. Some literature discusses adverse effects of promotion or couponing on firm profitability and consumer purchasing behavior. For example, Shor and Oliver (2006), from an experiment, find that the diminished likelihood of purchase has adverse effects on profitability and offsets any gains from market segmentation. Gonul and Srinivasan (1996) find that consumers' perception of future coupons affects purchase using 158 weeks disposable diaper data. Some researcher explore the factors that may affect coupon usage. Bawa et al. (1997) argue that a person's coupon usage behavior will depend not only on his or her inherent coupon proneness or desire to use coupons, but also on the attractiveness of the coupons encountered. In Mittal's (1994) paper, he claims that consumers' characteristics can determine their attitudes to coupon usage. Delvecchio (2005) uses two experiments to explore the effects of relative and absolute promotion value. Buckinx et al. (2003) develop two models to explore factors that affect manufacturer and retailer coupon redemption rates.

II.3. Online Price Dispersion, Over Time Price Change and LPG

Online Price Dispersion

Price dispersion always exists in Brick & Motor stores. One major explanation is that the price dispersion comes from consumer search cost. Salop (1977) develops a model, in which consumers have an expectation of a subjective price distribution and heterogeneous search costs. A consumer will calculate an optimal reserve price from the price distribution and search cost, and then search the local store until she finds a price equals to her reserve price. Knowing this, a monopolist can choose an optimal price distribution and allow price discrimination. As pointed out by Bakos (1991, 1997) and Smith et al. (1999), since the search cost dramatically drops for online shopping, we should observe a price convergence. However, we still observe consistent price dispersion online.

There are four major explanations for online price dispersion.

(1) Immaturity of markets. Brynjolfsson and Smith (2000) find price dispersion in books and CDs market and one of their explanations is immaturity of online market. Brown and Goolsbee (2002) investigate the online life insurance market and find that price dispersion is large at the beginning but falls as the use of internet spreads. However Baylis & Perloff (2002) find that the pattern of price dispersion in their data on electronic markets rarely changes over the course of more than one year, which does not support price convergence as the market matures.

(2) Search cost heterogeneity. After observing 32 online bookstores, Clay et al. (2001) find consistent price dispersion, and their results suggest that many consumers may not be engaging in search, despite its low cost and significant payoff.

(3) Mixed price strategy. Clemons et al. (2002) claim that firms can ease price competition by adopting a mixed price strategy after they observed the online travel market. But Baylis & Perloff (2002) find that online price rankings in their sample do not change their strategy much.

(4) Service premium. As pointed out by Lynch & Ariely (2000), online shopping lowers the cost of acquiring price information, which will increase price sensitivity. They develop an experiment for online wine shopping, and find that for differentiated products like wines, lowering the cost of search for quality information reduces price sensitivity. Grover & Ramanlal (1999) also find that providing more detailed product information could increase the reservation price for some consumers. However, Pan et al. (2002) find that online service quality accounted for only a small percentage of price dispersion based on an empirical analysis of 105 online retailers comprising 6,739 price observations for 581 items in eight product categories.

When we observe the price patterns of Best Buy, Circuit City, Office Depot, Office Max, and Staples, we found prices are not only different from store to store, but also from time to time. There is a large body of literature studying price change over time.

Over Time Price Change

A main theory used to explain each store's own price change over time is intertemporal pricing strategy. As pointed out by Elmaghraby & Keskinocak (2003), in recent years, due to technological improvement, there has been an increasing adoption of dynamic pricing policies in retail and other industries. Stokey (1979) proposed a monopoly model when both sides of the market have complete information about prices. Given that consumers differ from each other in terms of initial reservation prices and discount rates, intertemporal pricing can be used as a profitable price discrimination instrument if consumers' reservation prices fall proportionately faster than the firm's production costs. When costs don't decrease over time or they decrease at the same rate as reservation prices, a monopolist prefers to use a single price.

With incomplete information, Landsberger & Meilijson (1985) considered a monopoly market, in which the firm only knows consumers' distribution rather than the exact reservation prices. They find that in order to make intertemporal price discrimination profitable, consumers with high reservation prices must be discouraged from waiting until low prices are offered. To generate these incentives, the monopolist may have to delay considerably the lowered-price sales. The larger the consumer discount rate, the easier it is to discourage consumers with high reservation prices from waiting. Rustichini and Villamil (1996) go even further in the strand of incomplete information. They discuss a one buyer/one seller market, in which the buyer's value is private information and changes randomly through time according to a Markov chain with positive serial correlation. A firm uses its price to explore the buyer's valuation over time. Its intertemporal pricing strategy weighs the cost of attempting to learn the buyer's value (which may decrease current profit) versus its benefit (which may increase its future profit). In this literature, the profitability of intertemporal pricing strategy relies on the different discount rates between firms and consumers.

In the strand of literature that both firms and consumers share the same discount factor, Rodriguez and Locay (2002) present two different models in which both firm and consumers share the same discount factor. In their first model, demand is stochastic but consumers are uncertain about product availability in the future, so that those with higher willingness to pay prefer to pay higher prices today than face the risk that the product may sellout. Their second model uses Leibenstein's "snob effect"[8] to generate a desire on the part of the more enthusiastic consumers to buy early, when the good is more exclusive. Nair (2007) presents an empirical application to the market for video-games in the US. The results indicate that consumer forward-looking behavior has a significant effect on a firm's optimal pricing. Hosken and Reiffen (2004) examine grocery retail price variation across a range of goods and regions of the United States. They find that typical grocery product has a regular price and most deviations from that regular price are downward.

Contrary to this literature relying on monopoly market, Levin et al. (2009) present an intertemporal pricing model in an oligopolistic market. Nevertheless, all this intertemporal pricing literature is based on an assumption that some consumers are willing to wait and buy in the future; while this assumption should be revised if firms use post-sale self LPGs. Because with the post-sale self LPGs, consumers can buy early

[8] The satisfaction a consumer derives from a product decreases with the number of consumers that have consumed the good at the moment of purchase

and request a price match later if the price is lowered. This gives consumer no incentive to wait and buy. [9]

Low Price Guarantee

A first impression about LPG is that it must favor consumers in the sense that consumers are given an opportunity to buy from their favorite store at a lower price. However as a number of researches have shown, LPG sometimes may hurt consumers. Existing literature explains LPG in three streams: collusive device, price discrimination tool, and low price signal[10]. Salop (1986) points out the LPG could be used as an instrument to facilitate collusion. A simple version of his model has two firms selling a homogeneous product to fully informed consumers. Without LPG, Bertrand competition leads to both firms pricing at marginal cost. With LPG, however, both firms price at the monopoly level. Logan and Lutter (1989) find that the result is robust to heterogeneous products and an asymmetric duopoly. Empirically, Arbatskaya et al. (2006), using data on retail tire prices, find firms with price-matching guarantees tend to have weakly higher advertised prices than firms with no guarantees; whereas firms with price-beating guarantees tend to have weakly lower advertised prices than firms with no guarantees. In the strand of literature using LPG as price discrimination tool, Png and Hirshleifer (1987) look at a duopoly market, in which there are two types of consumers: Locals with elastic demand who will request price matching if possible, and tourists, who have perfect inelastic demand as long as the price is below their reservation price, and will not request price match. They find that in equilibrium, LPG is used as an

[9] Some consumers may still wait and buy either because the hassle cost of requesting price match is too high for them, or they think the price will not be lowered within the period of price matching.

[10] Moorthy and Winter (2006) summarize a nice table for the comparison among these three.

instrument to price discriminate the tourists. Corts (1996) looks at an oligopolistic market, and finds LPG is used as a price discrimination tool rather than a collusion device. LPG may raise or lower equilibrium prices. The relatively new explanation about LPG is using it as a signal of low price. Moorthy and Zhang (2006) investigate LPG under vertical differentiation structure, and find offering LPG is a signal of low price and low service, which is a way of branding the retailer to uninformed consumers. Moorthy and Winter (2006) develop a duopoly model in which firms marginal costs are random draws. They find LPG may be used as an instrument to signal low price. They provide empirical evidence from 46 retailers in U.S. and Canada to support the signaling explanation. Manez (2006) collects prices data from three superstores, and finds that LPG is offered by the firm with the lowest price. This indicates the LPG here is used as a low price signal. There are some researches investigating firm post-sale self low price guarantee policy (hereafter SLPG). For example, Chen and Liu (2009) study the effects of SLPG on price competition among major consumer electronics retailers and find SLPG is pro-competitive. Cooper (1986) find SLPG soften competition in a two-period duopoly heterogeneous products model. Schnitzer (1994) examines SLPG on collusion in a two-period homogeneous durable goods market and finds SLPG can facilitate collusion at limited degree. [11]

My second essay provides a theoretical model to explain the relationship between firm pricing strategy and firm post-sale low price guarantee. I find SLPG can be used as a price discrimination tool.

[11] In those paper, they call the self LPG most-favored customer.

CHAPTER III

CUSTOMER POACHING, COUPON TRADING AND CONSUMER ARBITRAGE

III.1. Introduction

There is a large literature on price discrimination.[12] A common assumption made in this literature (except for a few studies) is that consumers cannot engage in arbitrage.[13] In traditional markets, it is costly for a consumer to locate another potential buyer of the same product and then trade. In that sense, the no-arbitrage assumption, while not entirely true, may be realistic. However, this assumption is increasingly violated in the digital economy. First, it is easier to buy products cheap and resell for a profit online (e.g. Ebay.com), since the direct "consumer-to-consumer" markets are more developed and information can be easily exchanged on the internet. Moreover, one of the commonly used methods to achieve price discrimination is to target consumers with coupons, and coupons can be easily traded. In the case of online shopping, all that consumers need is a coupon code. Not surprisingly, more and more coupons are traded online. For a simple example, go to Ebay.com and search under

[12] A search in EconLit results in 521 papers with "price discrimination" in the titles of the papers.

[13] If a firm offers a menu of various qualities of products available to all consumers, and relies on consumers to self-select based on their characteristics, then consumers have no incentive to engage in arbitrage and the assumption is not needed. However, if the menu includes offers of different quantities, or if the firm give different options to different consumers, then the no arbitrage assumption is needed.

"coupon," one can find over 20,000 (not counting multiple coupons in one listing) of them for sale.[14]

In this paper we relax the no consumer arbitrage assumption by allowing coupons to be traded. Specifically, we assume that some consumers have low hassle cost of selling or buying coupons, and we call them coupon traders.[15] Other consumers have prohibitively high cost of trading coupons and are called non-traders. We assume that firms have information (e.g. purchase history) to differentiate between their own loyal customers and their rivals', and thus can price discriminate between them by sending coupons to only one group or by sending different coupons to different groups. We develop a location model of oligopolistic third-degree price discrimination to investigate how prices, promotion intensities and profits change as the fraction of coupon traders increases.

Depending on the method of distribution, coupons can be divided into two types: mass media coupons and targeted coupons. Mass media coupons are distributed randomly by the firms, and consumers, based on their characteristics, self-select as to whether to collect and use the coupons (Narasimhan 1984). However, with the availability of more and more data on actual consumer transactions, and better technology to utilize such data, firms do not need to rely exclusively on consumer self-selection. Instead, they can select shoppers with specific characteristics, and send

[14] These are only listings of coupons for auction or sale, and not all of them are sold. To get a sense of how many are actually sold, we searched for a specific coupon (Staples coupon), and checked the 10 listings with the earliest expiration time. We found that 6 of them had bids. One of the 6 coupons being auctioned is a Staples $20 off $100 coupon. It has 4 bids with less than 2 hours left for the auction, and the highest bid is $3.75.

[15] For example, some consumers may be familiar with Ebay and have various accounts already set up for transactions there, so the incremental transaction cost of trading coupons on Ebay is minimal.

targeted coupons (Shaffer and Zhang 1995). A popular form of targeted coupon is an offensive coupon (also called a poaching coupon). Firms send offensive coupons to poach rival firm's loyal customers, i.e., those who will purchase from the rival firm if prices are the same.[16] The opposite is a defensive coupon. A firm can distribute defensive coupons to retain its own loyal customers, who may be poached by rival firm's offensive coupons.

In this paper, we focus on coupons which are tradable, i.e., carrying no restriction on who can use them.[17] We allow firms to send offensive coupons and/or defensive coupons, but find that firms have incentive to send only offensive coupons. When a coupon trader receives a poaching coupon, he will sell this coupon back to the couponing firm's loyal customers, since the latter value the coupon more. In the symmetric pure strategy equilibrium of this model we find that when the fraction of coupon traders increases, firms will promote (send coupons) less frequently and with lower coupon face value. This reduces competition, leading to higher prices and profits. When coupon distribution cost increases, firms respond by promoting less frequently, but with higher coupon face value. Equilibrium prices and profits also go up. Increase in the fraction of coupon traders and increase in coupon distribution cost both

[16] This is somewhat similar to "reciprocal dumping" in the trade literature (e.g. Brander and Krugman (1983) and Deltas et. al. (2008)). In both settings, each firm has disadvantage in one market, whether it is due to weaker preferences of consumers in that market (our case) or higher transportation cost to serve consumers in that market (the reciprocal dumping case). Firms poach each other's strong markets, leading to lower profits for both firms, prisoners' dilemma.

[17] In practice, some coupons carry restrictions in terms of who can use the coupons. For example, if you receive an offer of a onetime bonus by switching long distance call service to AT&T, you qualify for the offer only if you are not currently with AT&T. However, many (if not most) coupons carry no restrictions in terms of who can use the coupons. In the extension, we allow firms the choice to distribute non-tradable coupons. Such coupons are tied to the targeted customers and cannot be used by other customers.

discourage firms from sending coupons. Since price discrimination with coupons constitutes a prisoners' dilemma game, equilibrium prices and profits increase.

The rest of the paper is organized as follows. Our model is presented in Section 2, and Section 3 contains our main results. In Section 4 we offer some extensions of our model and we conclude in Section 5. Section 6 is Appendix.

III.2. Description of the Model

Two firms—1and 2—produce competing goods with constant marginal cost, which we normalize to zero. Each consumer buys at most one unit of the good and is willing to pay V. We assume that V is sufficiently high and therefore the market is always covered. Consumers are heterogeneous with respect to the premium they are willing to pay for their favorite brand. This heterogeneity is captured by a parameter l, which represents the consumer's degree of loyalty. Specifically, a consumer located at l is indifferent between buying from the two firms if and only if $l=p_1-p_2$. We assume that l is uniformly distributed in the interval $[-L, L]$ with density 1.[18] When two firms charge the same prices, consumers located at $l > 0$ will buy from firm 1 and are called firm 1's loyal customers. Similarly, customers with $l < 0$ are firm 2's loyal customers.

The interval $[-L, L]$ is partitioned into two segments: $[-L, 0]$ and $[0, L]$, corresponding to firm 2's and firm 1's loyal customers respectively. Firms know which segment each consumer is located in, but do not know exactly where in the corresponding segment. For example, for someone located at L/2, firms will know that

[18] A similar model has been used in Shaffer and Zhang (2002) and Liu and Serfes (2006).

she is located in the segment [0, L], but not that she is located at L/2.[19] There are two types of pricing strategies that a firm can adopt in our context:

Uniform Pricing

Each consumer on the [−L, L] interval receives the same price. This price is also called the regular price, which is the price consumers pay without coupons. Let p_1 and p_2 denote the regular prices of the two firms. Without loss of generality, assume that $p_1 \geq p_2$.

Segment Couponing

Recall that the two segments are [−L, 0] and [0, L]. Firms can send coupons to consumers in one segment but not those in the other segment, or send different coupons to different segments. We allow firms to distribute both offensive and defensive coupons.[20] Let segment 1 denote the interval [−L, 0], and segment 2 the interval [0, L]. Let (λ_{11}, r_{11}) denote firm 1's couponing intensity in segment 1, where firm 2's loyal customers are located.[21] This means that every consumer in segment 1 has an equal probability λ_{11} of receiving coupons of face value r_{11} from firm 1. λ_{11} is called the promotion frequency and r_{11} is the promotion depth. Following the literature, we

[19] We assume this exogenous information structure and do not investigate how this structure emerges. One can think of a two-period model where firms can observe purchasing history but not consumers' exact willingness to pay for a good, and such information structure would emerge endogenously after consumers make purchasing decisions in the first period (e.g. Fudenberge and Tirole). In a symmetric equilibrium of this two-period model, consumers located on the interval [−L, 0] will buy from firm 2 in the first period, and those located on the interval [0, L] will buy from firm 1. Then at the beginning of the second period, for each consumer, firms know whether she bought from it in period 1. However, firms only know whether consumers bought from them in period 1, but do not know their exact locations.

[20] Recall that if firms send coupons to poach rival firm's loyal customers, such coupons are called offensive coupons. Defensive coupons are sent to retain a firm's own loyal customers.

[21] The first (second) subscript refers to firm (segment).

consider dollars-off coupons instead of percentage-off coupons. Similarly, let (λ_{12}, r_{12}) denote firm 1's defensive couponing intensity in segment 2. Note that, if $\lambda_{11} > 0$ and λ_{12}=0, then firm 1 is sending offensive coupons alone. Similarly firm 2's offensive couponing intensities are given by (λ_{22}, r_{22}) and its defensive couponing intensity are given by (λ_{21}, r_{21}). When consumers do not use coupons, they will pay either p_1 or p_2, depending on which firm they buy from. With coupons, consumers located at segment j=1, 2 will pay $p_1 - r_{1j}$ if they buy from firm 1, or pay $p_2 - r_{2j}$ if they buy from firm 2.

Next we introduce the cost of distributing coupons. We assume that coupon distribution cost is increasing and convex in the promotion effort and the size of the segment. In particular, it takes the form of $k(\lambda_{ij}L)^2$for firm i's promotion effort at segment j. If firm i sends out both offensive and defensive coupons, then its total promotion cost is $k(\lambda_{i1}L)^2 + k(\lambda_{i2}L)^2$. Consumers incur no cost when using coupons.[22] However, they differ in whether they trade (buy/sell) coupons. A fraction α of them have zero cost of trading coupons. We call them coupon traders. If these consumers receive coupons, they will either use the coupons or sell them to other consumers who value the coupons more. They may also buy coupons from other customers. The remaining $1 - \alpha$ fraction have infinite cost of trading coupons and are called coupon non-traders.[23]

[22] We relax this assumption by introducing coupon non-users in Section 5.1. Our results do not change qualitatively with the introduction of coupon non-users.

[23] It's certainly more realistic to assume a smooth distribution of coupon trading costs and endogenize the fraction of coupon traders. Consumers with trading costs below certain level are willing to trade coupons—coupon traders, and those with higher coupon trading costs will not—non-traders. The cutoff coupon trading cost will determine the fraction of coupon traders α. Note that, in this setup α and equilibrium prices and promotion strategies are interdependent. For tractability, we assume that α is exogenous in this paper, and we reserve the endogenization of α for future research.

Our models are related to those in Bester and Petrakis, and Fudenberg and Tirole. If we set α=0 and rule out defensive coupons, our model becomes the one in Bester and Petrakis (with uniform distribution). If we set $\alpha = 0$ and $k = 0$, our model becomes the second period of Fudenberg and Tirole (short-term contracts with uniform distribution).

The game we will study can be described as follows.

- Stage 1. Firms, simultaneously and independently, decide their regular prices (p_i), promotion frequency (λ_{ij}) and depth (r_{ij}), i, j = 1, 2.[24]
- Stage 2. Coupon distribution is realized. Coupon trading then takes place.
- Stage 3. Consumers make purchasing decisions. If they use coupons, they will pay regular price minus the coupon face value.

We assume that firms are risk neutral and maximize their expected profits. So firm i's problem is to choose p_i, λ_{ij}, r_{ij}, $i, j = 1, 2$, to maximize its profit,

$$\max \pi_i(p_i, \lambda_{ij}, r_{ij}, p_{-i}, \lambda_{-ij}, r_{-ij}),\ i, j = 1, 2.$$

III.3. Analysis

We first provide a road map for how we solve the game. The consumers can be segmented into various groups, depending on whether they are coupon traders and whether they receive coupons. We calculate firms' profits from each group. Then we aggregate profits over all groups of consumers net of the coupon distribution cost.

[24] Similar to Bester and Petrakis, we model the price and promotion strategies as a simultaneous game. An alternative way of modeling is a sequential-move game where firms choose one strategy (say price) before they choose the other strategy (say promotion strategy). However, it is unclear to us whether firms should choose price strategy or promotion strategy first. On the one hand, it is often viewed that regular price is a higher level managerial decision and is relatively slow to adjust in practice than promotions. On the other hand, we often observe regular price changes while promotion strategy (e.g. coupon face value) is relatively stable.

Solving the first order conditions, we obtain the equilibrium price, promotion frequency and depth.[25]

Consumers can be divided into the following groups:

Type (a): non-traders with neither firm's coupon;

Type (b): non-traders with only offensive coupons;

Type (c): non-traders with only defensive coupons;

Type (d): non-traders with both firms' coupons;

Type (e): traders.

Based on firm i's promotion effort λ_{ij} $(i, j = 1, 2)$, each consumer in segment j has an equal probability, λ_{ij} , of receiving the coupon from firm i. Firms maximize their expected profits, thus we only need to consider, on average, how many consumers are of each type. We start by calculating each firm's demand and profit from each type of consumers, and then add them up to obtain each firm's overall demand and profit.

(1) Type (a): non-traders with neither firm's coupons

We start with type (a) consumers, who are depicted in Figure III.1. Consumer density are different in [−L, 0] and in [0, L]. First consider consumers on the interval [−L, 0]. The fraction of non-traders is $1 - \alpha$. The probability of not receiving firm 1's (offensive) coupon is $1-\lambda_{11}$, and the probability of not receiving firm 2's (defensive) coupons is $1-\lambda_{12}$. Overall the probability of being a non-trader and receiving neither

[25] First order conditions are necessary but not sufficient. We need to make sure that the solutions that we obtain constitute equilibrium strategies (p∗, r∗ and λ∗). Instead of checking whether the Hessian is negative semi-definite (which is quite messy), we show that no firm has incentive to unilaterally deviate from this profile of strategies. Details are provided in the proof of Proposition 1 in the Appendix.

firm's coupon is $(1-\alpha)(1-\lambda_{11})(1-\lambda_{21})$. Similarly the fraction of non-traders with neither firm's coupon on the interval [0, L] is$(1-\alpha)(1-\lambda_{12})(1-\lambda_{22})$.

Let l_a denote the location of the marginal consumer, which is defined by $l_a = p_1 - p_2 \geq 0$, since $p_1 \geq p_2$. Every consumer to the left of la will buy from firm 2 at price p_2, and those to the right will buy from firm 1 at price p_1. Therefore, firms 1 and 2 make sales of

$$d_{1a} = (1-\alpha)(1-\lambda_{12})(1-\lambda_{22})(L-l_a)$$
$$= (1-\alpha)(1-\lambda_{12})(1-\lambda_{22})(L-p_1+p_2)$$

$$d_{2a} = (1-\alpha)(1-\lambda_{11})(1-\lambda_{21})L + (1-\alpha)(1-\lambda_{12})(1-\lambda_{22})l_a$$
$$= (1-\alpha)(1-\lambda_{11})(1-\lambda_{21})L + (1-\alpha)(1-\lambda_{12})(1-\lambda_{22})(p_1-p_2)$$

Their profits are

$$\pi_{1a} = p_1 d_{1a} = p_1(1-\alpha)(1-\lambda_{12})(1-\lambda_{22})(L-p_1+p_2)$$

$$\pi_{2a} = p_2 d_{2a} = p_2(1-\alpha)(1-\lambda_{11})(1-\lambda_{21})L$$
$$+ p_2(1-\alpha)(1-\lambda_{12})(1-\lambda_{22})(p_1-p_2)$$

(2) Type (b): non-traders with only offensive coupons

These consumers are depicted in Figure III.2. Let's start with consumers on the interval [−L, 0]. Firm 1 sends offensive coupons while firm 2 sends defensive coupons there. The probability of being a non-trader, receiving firm 1's (offensive) coupons but not firm 2's (defensive) coupons is $(1-\alpha)\lambda_{11}(1-\lambda_{21})$. Firms' roles are reversed for the interval [0, L]. The density of consumers receiving only offensive coupons there is $(1-\alpha)(1-\lambda_{12})\lambda_{22}$.

Let l_{b1} and l_{b2} denote the marginal consumer in segment 1 and 2 respectively. The left marginal consumer, located at l_{b1}, is indifferent between buying from firm 1 with a coupon (thus paying $p_1 - r_{11}$) and buying from firm 2 without a coupon (thus paying p_2).[26] Similarly, the right marginal consumer (located at l_{b2}) is indifferent between buying from firm 1 at p_1 and buying from firm 2 at $p_2 - r_{22}$. The exact locations of these two marginal consumers are

$$l_{b1} = (p_1 - r_{11}) - p_2, \quad l_{b2} = p_1 - (p_2 - r_{22}).$$

It's easy to see that $l_{b1} < 0$ and $l_{b2} > 0$. Otherwise, these coupons do not get the firms any extra customers, and firms would be better off not to send offensive coupons. Consumers located in the interval $[-\mathrm{L}, l_{b1}]$ receive coupons from firm 1, but the face value of firm 1's coupon is not enough to compensate for their strong preferences for firm 2's product. As a result, they will buy from firm 2 at p_2. Since they are non-traders, they will not sell firm 1's coupons. However, for consumers located in $(l_{b1}, 0]$, they only have a weak preference for firm 2's product. With firm 1's coupons, they will choose to buy from firm 1 and pay $p_1 - r_{11}$. Similarly, consumers located in $[0, l_{b2})$ will buy from firm 2 at a price of $p_2 - r_{22}$, and consumers in $[l_{b2}, \mathrm{L}]$ will buy from firm 1 at the price p_1. Consequently, firms' profits are

$$\pi_{1b} = (p_1 - r_{11})(1-\alpha)\lambda_{11}(1-\lambda_{21})(0 - l_{b1}) + p_1(1-\alpha)(1-\lambda_{12})\lambda_{22}(L - l_{b2})$$

$$= (p_1 - r_{11})(1-\alpha)\lambda_{11}(1-\lambda_{21})[p_2 - (p_1 - r_{11})]$$

$$+ p_1(1-\alpha)(1-\lambda_{12})\lambda_{22}[L - p_1 + (p_2 - r_{22})]$$

[26] We assume that firms do not match each other's coupons. If coupons are matched, then firms would have no incentive to send poaching coupons unless some consumers do not request coupon-matching.

$$\pi_{2b} = p_2(1-\alpha)\lambda_{11}(1-\lambda_{21})(l_{b1}+L) + (p_2 - r_{22})(1-\alpha)(1-\lambda_{12})\lambda_{22}l_{b2}$$

$$= p_2(1-\alpha)\lambda_{11}(1-\lambda_{21})[(p_1 - r_{11}) - p_2 + L]$$

$$+ (p_2 - r_{22})(1-\alpha)(1-\lambda_{12})\lambda_{22}[p_1 - (p_2 - r_{22})]$$

So far we have considered non-traders who either receive offensive coupons only or no coupons. Since we allow firms to distributed both offensive and defensive coupons, next we analyze non-traders who receive defensive coupons. In Lemma 1, we will show that firms will never distribute both types of coupons.

(3) Type (c): non-traders with only defensive coupons

The third type of consumers are non-traders who receive defensive coupons only. That is, non-traders on [−L, 0] who receive only firm 2's coupons and those on [0, L] who receive only firm 1's coupons. Their densities are $(1-\alpha)(1-\lambda_{11})\lambda_{21}$ and $(1-\alpha)\lambda_{12}(1-\lambda_{22})$ respectively.

Let's start with consumers on the interval [−L, 0]. These consumers prefer firm 2's products and we assumed that $p_1 \geq p_2$. Moreover, they receive coupons from their preferred firm but not the other firm. Thus, they will all buy from firm 2 and pay $p_2 - r_{21}$. Next we consider consumers on [0, L]. All these consumers prefer firm 1's product. Although $p_1 \geq p_2$, they receive firm 1's coupon. Let l_{c2} denote the marginal customer who is indifferent between buying from firm 1 with coupon and buying from firm 2 without coupon. Then, $l_{c2} = (p_1 - r_{12}) - p_2$. Depending on the sign of l_{c2}, there are two cases. In the first case, $l_{c2} \leq 0$, i.e., all consumers on [0, L] buy from firm 1. This case is depicted in Figure III. 3. Firms' profits are,

$$\pi_{1c} = (p_1 - r_{12})(1-\alpha)\lambda_{12}(1-\lambda_{22})L$$

$$\pi_{2c} = (p_2 - r_{21})(1-\alpha)(1-\lambda_{11})\lambda_{21}L$$

In the other case, l_{c2}>0. Then consumers on [0, l_{c2}) buy from firm 2 while those in [l_{c2}, L] buy from firm 1. Firms' profits become

$$\pi_{1c} = (p_1 - r_{12})(1-\alpha)\lambda_{12}(1-\lambda_{22})(L - l_{c2})$$
$$= (p_1 - r_{12})(1-\alpha)\lambda_{12}(1-\lambda_{22})(L - (p_1 - r_{12}) + p_2)$$

$$\pi_{2c} = (p_2 - r_{21})(1-\alpha)(1-\lambda_{11})\lambda_{21}L + p_2(1-\alpha)\lambda_{12}(1-\lambda_{22})l_{c2}$$
$$= (p_2 - r_{21})(1-\alpha)(1-\lambda_{11})\lambda_{21}L + p_2(1-\alpha)\lambda_{12}(1-\lambda_{22})[(p_1 - r_{12}) - p_2]$$

(4) Type (d) non-traders with both firms' coupons

Next, we move on to type (d) consumers—non-traders with both firms' coupons. These consumers are depicted in Figure III.4. Their densities are $(1-\alpha)\lambda_{11}\lambda_{21}$ on [−L, 0] and $(1-\alpha)\lambda_{12}\lambda_{21}$ on [0, L] respectively.

Let l_{d1} and l_{d2} denote the marginal consumer in segment 1 and 2 respectively. The left marginal consumer, located at l_{d1}, is indifferent between buying from firm 1 with a coupon (thus paying $p_1 - r_{11}$) and buying from firm 2 also with a coupon (thus paying $p_2 - r_{21}$). Similarly, the right marginal consumer (located at l_{d2}) is indifferent between buying from firm 1 at a price of $p_1 - r_{12}$ and buying from firm 2 at a price of $p_2 - r_{22}$. The exact locations of these two marginal consumers are

$$l_{d1} = (p_1 - r_{11}) - (p_2 - r_{21}), \qquad l_{d2} = (p_1 - r_{12}) - (p_2 - r_{22})$$

Consumers located in the interval [−L, l_{d1}] receive coupons from both firms, and the face value of firm 1's coupon is not enough to compensate for their strong preferences for firm 2's product. As a result, they will use firm 2's coupons and buy from firm 2 at $p_1 - r_{12}$. Since they are non-traders, they will not sell firm 1's coupons. However, for consumers located in $(l_{d1}, 0]$, they only have a weak preference for firm 2's product. With firm 1's coupons, they will choose to buy from firm 1 and pay $p_1 - r_{11}$. Similarly, consumers located in $[0, l_{d2})$ will buy from firm 2 at $p_2 - r_{22}$, and consumers in $[l_{d2}, L]$ will buy from firm 1 at $p_1 - r_{12}$. Consequently, firms' profits are

$$\pi_{1d} = (p_1 - r_{11})(1-\alpha)\lambda_{11}\lambda_{21}(0 - l_{d1}) + (p_1 - r_{12})(1-\alpha)\lambda_{12}\lambda_{22}(L - l_{d2})$$

$$= (p_1 - r_{11})(1-\alpha)\lambda_{11}\lambda_{21}[(p_2 - r_{21}) - (p_1 - r_{11})]$$

$$+ (p_1 - r_{12})(1-\alpha)\lambda_{12}\lambda_{22}[L - (p_1 - r_{12}) + (p_2 - r_{22})]$$

$$\pi_{2d} = (p_2 - r_{21})(1-\alpha)\lambda_{11}\lambda_{21}(l_{d1} + L) + (p_2 - r_{22})(1-\alpha)\lambda_{12}\lambda_{22}l_{d2}$$

$$= (p_2 - r_{21})(1-\alpha)\lambda_{11}\lambda_{21}[(p_1 - r_{11}) - (p_2 - r_{21}) + L]$$

$$+ (p_2 - r_{22})(1-\alpha)\lambda_{12}\lambda_{22}[(p_1 - r_{12}) - (p_2 - r_{22})]$$

(5) Type (e): traders

The last type of consumers is traders, with or without coupon. Their density is α on both segments. If coupons are auctioned, they are expected to go to the bidders who value them the most, since there is a continuum of traders in our model. So we make the assumption that the outcomes of coupon trading are efficient.

Let's first consider segment 1 ([−L, 0]), where firm 2's loyal customers are located. These customers may receive firm 1's offensive coupons and firm 2's defensive coupons. Since $p_1 \geq p_2$, they will buy from firm 2 in the absence of coupons. Therefore, they value firm 2's coupons at their face value, but may value firm 1's coupons at less than the face value. Therefore, they will use firm 2's coupons, and sell firm 1's coupons to traders who are firm 1's most loyal customers, i.e., those close to

L.[27] The intended objective of offensive coupons is to poach a rival firm's loyal customers, but since these poached customers generally value coupons less than the promoting firm's loyal customers do, those coupons reaching traders will end up in the hands of the promoting firm's loyal customers. That is, a fraction of the offensive coupons (those that reach traders) become somewhat similar to defensive coupons.[28]

Next consider segment 2 ([0, L]), where firm 1's loyal customers are located. These customers will buy from firm 1 in the absence of coupons, except those located close to zero if $p_1 > p_2$. Specifically, anyone located to the right of $l_e = p_1 - p_2$ will buy from firm 1 in the absence of coupons. They will use firm 1's coupons (if they receive these coupons), and sell firm 2's coupons (again if they receive such coupons) to traders located near -L—firm 2's most loyal customers. The rest of the consumers [-L, l_e] will buy from firm 2 in the absence of coupons. Therefore, if they receive firm 2's coupons they will use such coupons. However, if they receive firm 1's coupons, such coupons will be traded to consumers located near L—firm 1's most loyal customers.

Intuitively, when α is too large, no firm will distribute coupons. We assume throughout the paper that $\alpha < \frac{1}{2}$, i.e., there are fewer coupon traders than non-traders.[29] After coupon trading takes place, all coupons reaching traders will be traded to consumers who would buy from the coupon-issuing firm with or without the coupons

[27] We are not concerned with the exact transaction prices of coupons, i.e., how the surpluses will be divided between coupon buyers and sellers. In general, traders who receive poaching coupons (from their less preferred firm) will choose to sell them back to the promoting firm's most loyal customers.

[28] These traded offensive coupons are not exactly the same as defensive coupons. First, the face value of defensive coupons and traded offensive coupons can be different. Second, offensive coupons reaching traders are traded to the distributing firm's most loyal customers (so the less loyal ones will not buy such coupons), while all of the distributing firm's loyal customers have an equal probability of receiving the defensive coupons.

[29] Our results show that when α=1, firms stop sending coupons altogether ($\lambda_{ij}=0$, i, j=1, 2).

(thus they value the coupons at their face value—the maximum value). Therefore, after the coupon distribution and trading, there is a marginal consumer l_e near the middle, who does not have a coupon and is indifferent between both products at their regular prices,[30]

$$l_e = p_1 - p_2$$

To the left of l_e, all consumers buy from firm 2. Those close to -L will use firm 2's coupon whether they receive it or buy it. Those to the right of l_e all buy from firm 1, with consumers close to L using firm 1's coupon. Consumers in the neighborhood of l_e will not have coupons to use, since there is more demand than supply for coupons.

Firms' profits from the traders are,

$$\pi_{1e} = p_1\alpha(L - l_e) - r_{11}\alpha\lambda_{11}L - r_{12}\alpha\lambda_{12}L$$
$$= p_1\alpha(L - p_1 + p_2) - r_{11}\alpha\lambda_{11}L - r_{12}\alpha\lambda_{12}L$$
$$\pi_{2e} = p_2\alpha(L + l_e) - r_{21}\alpha\lambda_{21}L - r_{22}\alpha\lambda_{22}L$$
$$= p_2\alpha(L + p_1 - p_2) - r_{21}\alpha\lambda_{21}L - r_{22}\alpha\lambda_{22}L$$

Aggregating firms' profits over all types of consumers, and subtracting the cost of distributing coupons, we can obtain firm i's overall profit

$$\pi_i = \sum_{j=a}^{e} \pi_{ij} - k(\lambda_{i1}L)^2 - k(\lambda_{i2}L)^2, \qquad i = 1{,}2.$$

with *j* being the segment.

[30] This requires that there is more demand than supply for each firm's coupons, and the consumers in the neighborhood of l_e will not have coupons. Intuitively this holds if distributing coupons is sufficiently costly (k is large) so that λ_{ij} is significantly less than 1.

Firm i's problem is

$$\max \pi_i(p_i, \lambda_{ij}, r_{ij}, p_{-i}, \lambda_{-i,j}, r_{-i,j}), \; i, j = 1, 2.$$

We will divide the solution to this problem into three steps. First, in Lemma 1, we show that a firm never has incentive to distribute both offensive and defensive coupons, whether on equilibrium or off equilibrium (deviation). Second, in Lemma 2, we prove that firms do not distribute defensive coupons in a pure strategy equilibrium, with or without offensive coupons. This leaves the only possibility of distributing offensive coupons alone (distributing no coupons at all cannot be optimal given the quadratic coupon distribution cost). Last, in Proposition 1, we derive the equilibrium where both firms distribute offensive coupons only.

Lemma 1. *When coupon distribution is costly (k>0), firms have no incentive to distribute both offensive and defensive coupons, whether on equilibrium or off equilibrium (deviation).*

Proof. See Appendix.

The intuition is as follows. If a firm distributes coupons to both segments, it implies that regular price is not optimal in either segment and in particular lower price improves profits (even after accounting for coupon distribution cost and coupon trading). Since lower price is better in both segments, the firm is better off lower its regular price and get rid of couponing in one segment. This way, it saves on coupon distribution cost from that segment, and also improves profits from both segments.

It is interesting to compare our results with those in Shaffer and Zhang (1995), where firms mix the use of offensive and defensive coupons in equilibrium. In Shaffer and Zhang (1995), firms know the exact location of each individual customer. Consequently, they can decide whether or not to send targeted coupon to each individual consumer. For those consumers whose preferences are too strong to be induced to switch (loyals), firms will not send them coupons. For the rest of the consumers (brand-switchers), there is no pure strategy and firms play mixed strategies with both firms sending targeted coupons at positive probabilities. On the contrary, in our paper, firms do not know the exact locations of customers. Instead, they only know whether the customers prefer their products or their rival's products if prices are the same. As a result, firms can only tailor their coupon decisions toward each segment instead of each individual customer. If it sends defensive coupons to consumers who prefer its product, then the consumers who have strong preferences (loyals as in Shaffer and Zhang (1995)) and those who have weak preferences (brand-switchers) have an equal probability of receiving its coupons. While it may enhance profit if the brand-switchers receive coupons, couponing leads to pure losses if the coupons reach consumers with strong preferences. Therefore, firms do not want to distribute defensive coupons in our model.

Lemma 1 explains that firms will never send both offensive and defensive coupons. And intuitively since it's optimal for firms to be more aggressive when competing for the other firm's loyal customers, they will send only offensive coupons, instead of sending defensive coupons only. This is confirmed in the next Lemma.

Lemma 2. *There is no pure strategy equilibrium where firms distribute defensive coupons only.*

Proof. See Appendix.

The intuition is as follows. Suppose that firm 2 distributes only defensive coupons, thus charging lower price to its loyal customers in segment 1 ([-L,0]) than in segment 2. Now consider firm 1. It faces lower price from its competitor in segment 1, and consumers in segment 1 also prefer firm 2's products. Its best response should be to charger lower price in segment 1 than in segment 2. That is, it should distribute offensive coupons instead of defensive coupons.

Proposition 1. *There exists a symmetric pure strategy equilibrium in which firms send only offensive coupons, when k is not too small relative to L. This equilibrium is characterized by the following:*

(i) *The regular prices are*

$$p_1 = p_2 = p^* = \frac{\left(\frac{2}{3}A - \frac{3}{2}\frac{-\frac{4}{9}\alpha^2 + \frac{16}{3}k}{A} - \frac{1}{3}\alpha\right)L}{-1+\alpha} \quad (1)$$

where

$$A = \left(\alpha^3 + 36k\alpha - 27k + 3\sqrt{12\alpha^4 k + 96\alpha^2 k^2 + 192k^3 - 6k\alpha^3 - 216k^2\alpha + 81k^2}\right)^{\frac{1}{3}}$$

(ii) *The Promotion depths are*

$$r_{11} = r_{22} = r^* = \frac{1}{2}\left(p^* - \frac{\alpha L}{1-\alpha}\right), \quad r_{12} = r_{21} = 0 \quad (2)$$

(iii) *The promotion frequencies are*

$$\lambda_{11} = \lambda_{22} = \lambda^* = \frac{(\alpha L - p^* + \alpha p^*)^2}{8(1-\alpha)kL^2}, \quad \lambda_{12} = \lambda_{21} = 0 \quad (3)$$

(iv) *Firms' equilibrium profits are*

$$\pi_1 = \pi_2 = \pi^* \tag{4}$$

$$= \frac{\alpha^4 L^4 + 64(1-\alpha)^2 k L^3 p^* - 6(1-\alpha)^2 \alpha^2 L^2 (p^*)^2 + 8(1-\alpha)^3 \alpha L (p^*)^3 - 3(1-\alpha)^2 (p^*)^4}{64(1-\alpha)^2 k L^2}$$

Proof. See Appendix.

A numerical example

From equations (1), (2), (3) and (4), we can see that p^* and r^* are linear in L, π^* is quadratic in L, and λ^* is independent of L. Thus we normalize L=1. We choose k=1/2 and further set α=1/5, i.e., 20% of the consumers are traders. Then the equilibrium is

$$p^* = 0.9525, r^* = 0.3512, \lambda^* = 0.0987, \pi^* = 0.9309.$$

Recall that the coupon distribution cost is $k(\lambda L)^2$. Plugging the value of k, λ and L, this cost is about 0.005, or about 0.5% of the regular price.

Prisoners' dilemma

The model without coupons is essentially a standard Hotelling model (with the measure of consumers being 2 instead of 1). It can be easily verified that the equilibrium price is p=1. Each firm takes half of the market and enjoys a profit of π=1. Sending coupons to consumers first reduces firms' regular prices (seeing now that p^* <1). This is because, when a firm's loyal customers are poached by the rival firm, it responds by lowering its regular price to try to retain these loyal customers. Lower regular prices lead to lower profits. The discounts which some consumers get by using coupons and the coupon distribution cost will lower firms' profits even further ($\pi^* < p^*$).

III.3.1. Comparative statics

Proposition 1 provides the expressions of the equilibrium price and promotion variables (p,r,λ and π). If we normalize L=1, these variables are only functions of k and α. Therefore, we can analyze how they vary when we change either α or k, one at a time. The expressions for the relevant partial derivatives are very lengthy for reporting.[31] We tried various parameter value of α and k, and found that the qualitative comparative statics results do not depend on the choice of parameter value. Below, we assign parameter value and report the results in graphs.

Fix k and vary α

As long as k is sufficiently large so that firms have no incentive to deviate (details in Proof of Proposition 1), our results are robust to the choice of k. Results for $k = \frac{1}{2}$ are reported in Figure III.5.

From the figure we can see that, when the fraction of coupon traders (α) increases, firms promote less frequently (λ ↓) and with lower promotion depth (r ↓). They set higher prices and their profits increase. The intuition is as follows. Optimal promotion effort balances the following:

$$\text{benefit of couponing} = (1-\alpha)\lambda r(p-r)$$

$$\text{loss of couponing} = \alpha\lambda L r$$

$$\text{coupon distribution cost} = k(\lambda L)^2$$

[31] The Maple file which contains all the expressions is available upon request.

A firm reaps benefit when its coupons reaches non-traders, and the benefit is given by $(1-\alpha)\lambda r(p-r)$. $(1-\alpha)\lambda r$ measures the extra consumers the firm can attract, at the discounted price of $(p-r)$. However, a loss is realized when the coupons reach traders in the form of $\alpha\lambda Lr$. $\alpha\lambda L$ is the proportion of consumers affected, and r is the loss of revenue for each of these consumers. There is also a cost of distributing coupons in the form of $k(\lambda L)^2$. An increase in α lowers the benefit and increases the loss. To re-balance the benefit, loss, and distribution cost (λ affects all three terms), λ needs to go down. This is because, the benefit and loss are linear in λ, while the cost of distributing coupons is quadratic in λ.

Now let's see why an increase in α also puts downward pressure on coupon face value r. When α increases, the benefit decreases and the loss increases. To re-balance the benefit and loss (r affects the benefit and loss directly), r needs to decrease. While r does not enter into the distribution cost term, there is an indirect tradeoff effect between promotion frequency and depth. That is, a firm can poach more of a rival's customers by either sending more coupons with the same face value or sending the same number of coupons but with larger face value. This indirect effect implies that, when a firm reduces its promotion frequency, it increases its promotion depth. Our result suggests that, this indirect tradeoff effect is dominated by the direct effect of downward pressure on promotion depth. With fewer poaching coupons of less value there is less competition; thus price and profits go up. Obviously, consumers become worse off.

In a model with covered market and inelastic demand like ours, welfare analysis is not very informative. Nevertheless, we would like to point out an effect which coupon trading has on efficiency. Customer poaching leads to inefficient brand switching (consumers buy products they like less). If we fix firms' promotion intensities, allowing coupons to be traded implies less brand switching, thus improves efficiency. On top of this, coupon trading also reduces firms' promotion intensities, which leads to even less brand switching.

Fix α and vary k

We tried various value of $\alpha \in [0, 1/2)$, and the results do not change qualitatively. Results when $\alpha = 0.2$ are plotted in Figure III.6. These results are similar to those in Bester and Petrakis.

From the figure, we can see that when k increases, firms respond by promoting less frequently ($\lambda \downarrow$) but with higher promotion depth ($r \uparrow$). Prices (even net of coupon face value) and profits go up. These results are quite similar to the results when we fix k and vary α, and so is the intuition. Both coupon trading (α) and distribution cost (k) work against sending coupons, and firms have fewer incentives to promote. However, the implications on promotion depth are different. When firms promote less frequently due to larger cost of distributing coupons, they respond by increasing the promotion depth (tradeoff effect). This is because, while an increase in α applies a direct downward pressure on r, an increase in k does not directly affect the benefit and loss of promotion, but only indirectly through λ and r. Thus, when k increases, only the indirect tradeoff effect (higher promotion depth to go with lower promotion frequency) exists. Consequently, promotion depth increases with k. Since sending coupons constitutes a prisoners' dilemma game, less promotion reduces competition intensity, which leads to higher prices (including prices net of coupons). There are two opposite effects governing the effects of an increase in k on profits. First, the cost of distributing coupons increases, affecting profits negatively. Second, when k increases, competition is less intense which will improve profits. Our results show that the second effect dominates the first one.

III.4. Extensions

To check the robustness of our results, we extend our model in the following directions.

III.4.1 Introducing coupon non-users

In our model, we have assumed that all consumers are coupon users. This is unrealistic and the sole purpose of this is to simplify the analysis. Nevertheless, here we show that our results do not change qualitatively if we introduce consumers who do not use coupons. Assume that there is a fraction, $1-\gamma$, of consumers who do not use coupons. They are also uniformly distributed on the interval [-L, L], but they are allowed to have different price sensitivity than the coupon users do. Specifically, we assume that a coupon non-user located at *l* is indifferent between buying from either firm if and only if $l = \frac{p_1 - p_2}{t}$.[32] The remaining γ fraction of consumers are the same as in our model. We then analyze two setups, depending on whether or not the firms can distribute mass media coupons, in addition to the poaching coupons. We find that in both setups, our comparative statics results stay qualitatively the same as those in the main model.

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[32] If we want coupon non-users to be less price sensitive, then we need $t \geqslant 1$. Recall that for coupon users the marginal consumer is $l = p_1 - p_2$.

In the first one, firms cannot distribute mass media coupons. Following analysis similar to that in the main model, we look for a symmetric equilibrium ($p_2 = p_1, r_2 = r_1, \lambda_2 = \lambda_1$). We can then study how the equilibrium price, promotion intensity and profit change with respect to α or k. The comparative statics results are the same as those in Section III.3.1.

With the introduction of coupon non-users, especially when they are less price sensitive than the coupon users are, it is natural to consider not just poaching coupons, but mass media coupons as well. In the second setup, we assume that firms can distribute mass media coupons to all consumers costlessly. We further assume that mass media coupons and poaching coupons can be combined. When firms send mass-media coupons, all coupon users enjoy the discount of mass media coupons while coupon non-users pay regular price. This is equivalent to firms charging one price for coupons users, and another price for coupon non-users. Therefore, we can treat coupon users and coupon non-users as in separate markets. Then the coupon non-users market is the same as the whole market in our initial model (of course with different market sizes) and the comparative statics results are qualitatively the same as those in Section III.3.1.

III.4.2 Non-tradable coupons

We assumed that coupons are tradable in our model. But why would firms allow their coupons to be traded? With online coupons, firms can certainly tie coupon codes to the consumers they are targeting, and refuse to honor the coupons if used by others.[33] To analyze the issue of non-tradable coupons, we introduce another stage to our three-

[33] For firms to do this, they must be able tie coupons with consumers. That is, they can identify each customer (e.g., name, address) without knowing his/her location on the interval [-L, L] (preferences).

stage game. In particular, in stage 0, firms first simultaneously and independently decide whether they want their coupons to be tradable. We assume that tradable and non-tradable coupons cost the same to distribute. Once decisions of coupon types are made, the rest of the game proceeds in the same fashion as in Section III.2. There are a total of four subgames after the coupon type decisions are made, depending on whether the coupons are tradable or not. In the first subgame (T, T), both firms' coupons are tradable. This is the same as our main model. In the second subgame, neither firm's coupons are tradable (NT, NT). This is similar to imposing $\alpha = 0$ in our main model. Subgame 3 (T, NT) and 4 (NT, T) are symmetric to each other, where one firm's coupons are tradable but not the other firm's. We find that in general, firms want to mimic each other's decisions on coupon types. That is, a firm wants to make its coupons tradable if the other firm sends tradable coupons as well.[34] Both subgame 1 and 2 can be supported in subgame perfect Nash equilibria. However, equilibrium profits are higher in subgame 1, justifying our use of tradable coupons.[35]

III.4.3 Asymmetric Firms

When firms are symmetric, the equilibrium is symmetric and both firms always promote ($\lambda_i > 0$). In this subsection, we allow firms to be asymmetric by introducing a measurement of asymmetry, q. Consumers are distributed on the interval [-L+ q, L+q], with $q \geq 0$. Let firm 1's segment to be L+q, and firm 2's segment to be -L+ q. We then

[34] An exception is that when both α and k are small, we have asymmetric equilibria where only one firm chooses to issue tradable coupons.

[35] Note that the (NT, NT) subgame is the same as the (T, T) subgame but with α=0. From our comparative statics results, we have shown that equilibrium profits increases with α. Thus (T, T) leads to higher profits for firms than (NT, NT) does. Making coupons tradable may also have other benefits which we do not model here. For example, verifying non-tradable coupons adds hassle costs to both the firm and customers; denying customers' rights to use coupons may upset the customers and firms may lose their business.

look for asymmetric equilibrium. Specifically we are interested in finding whether one or both firms stop promoting can emerge as equilibrium. For simplicity, we assume firms only send offensive coupons. There are 4 possible promotion strategy combinations as in the table III.1.

Next we explain how we check for possible equilibrium in each of these four types, starting with type 4 where both firms promote. Firms' profits are given as followings:

$$\pi_1 = p_1\big[L - (1-\alpha)r_{22}\lambda_2 + \big((\alpha-1)\lambda_{11} - 1\big)(p_1 - p_2) + q\big] - (1-\alpha)r_{11}^2\lambda_{11}$$
$$+r_{11}\lambda_{11}[2p_1 - p_2 + \alpha(-L - 2p_1 + p_2 + q)] - k\lambda_{11}^2(L-q)^2$$
$$\pi_1 = p_2[L + p_1 - p_2 - (1-\alpha)\lambda_{11}\,(r_{11} - p_1 + p_2) - q] - (1-\alpha)r_{22}^2\lambda_{22}$$
$$+r_{22}\lambda_{22}[p_1 - 2p_2 + \alpha(L - p_1 + 2p_2 + q)] - k\lambda_{22}^2(L+q)^2$$

The first order conditions are

$$\frac{\partial \pi_i}{\partial \lambda_i} = \frac{\partial \pi_i}{\partial d_i} = \frac{\partial \pi_i}{\partial p_i} = 0, i = 1, 2.$$

Using $\frac{\partial \pi_i}{\partial \lambda_i} = 0$ and $\frac{\partial \pi_i}{\partial d_i} = 0$, we can obtain λ_i and d_i (i = 1, 2) as functions of p_1 and p_2.[36] We can then plug them into $\frac{\partial \pi_i}{\partial p_i}$, which are polynomials of degree 3 in p_1 and p_2, as in the symmetric case. However, $\frac{\partial \pi_i}{\partial p_i}$ are also polynomials of q, specifically $\frac{\partial \pi_2}{\partial p_2}$ is a polynomial of degree 5 in q. As a result, we cannot solve p_1 and p_2 analytically and we have to rely on numerical analysis instead. Since the results are not sensitive to k qualitatively, we fix $k = \frac{1}{2}$ for this section.[37] Pick any $\alpha \in \left[0, \frac{1}{2}\right)$, $q > 0$ and normalize

[36] $\lambda_i \leq 1$ is required and imposed throughout the paper.

[37] We checked our results for various k's and confirmed that changing k does not change our results qualitatively.

L = 1, we can solve for p_1 and p_2 numerically, and then use them to get λ_i, r_i. [38] If the resulting $\lambda_i, r_i > 0$, we say there is no violation, and we only need to check whether one or both firms have incentive to deviate by not promoting. For some parameter combinations, the solution to FOC leads to violations, for example, $r_{22} < 0$. In this case, we assume that firm 2 will not promote ($\lambda_{22} = r_{22} = 0$), and solve the new FOCs.[39] Then we check whether firms have incentives to deviate.

Next we present a series of lemmas which establish part of Proposition 2.

Lemma 3. *When the degree of firm asymmetry q is large, firm 2 will not promote. Moreover, when $q \to L$, there is no pure strategy equilibrium.*

Proof. See Appendix.

The intuition of why the smaller firm does not promote when q is large is as follows. For simplicity, assume that $\alpha = 0$, i.e., no coupon traders. If the smaller firm sends coupon to the large firm's customers, all these customers have an equal probability of receiving the coupon. Note that the smaller firm charges a lower price, so the larger firm's less loyal customers will switch to the smaller firm even without the smaller firm's coupons. Sending them coupons will not change their purchasing decisions, but it will lead to lower final price. So the smaller firm has no incentive to send coupons. The larger firm does not have this problem, since without its coupon, the smaller firm's loyal customers will never purchase from the larger firm.

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[38] We have multiple solutions of p_1, p_2, and we pick the only one with $p_1, p_2 \in (0, 2L)$.

[39] If there are violations for both firms, we assume that neither firm will promote.

When q is very large ($q \to$ L), i.e., if two firms charge the same price then almost all consumers will buy from one firm, there is no pure strategy equilibrium. This is because it is in the higher quality firm's best interest to send out coupons and grab all its rival's customers who receive coupons and are not coupon traders. This leads to a corner solution. But then firm 2 will always have incentive to either increase or decrease its price, and there is no pure strategy equilibrium. To guarantee the existence of a pure strategy equilibrium, we make the following assumption:

Assumption 1: q is not too large $\left(q < \bar{q}(\alpha, k)\right)$.

The cutoff $\bar{q}$ depends on α and k. For example, set k = $\frac{1}{2}$, we find $\bar{q} \approx 0.82$ when α = 0 and $\bar{q} \approx 0.98$ when $\alpha = 0.49$.

Lemma 4. *The equilibrium is of Type 4 (both firms promote), if and only if q is small.*

Proof. Whether a firm promotes or not depends on the cost and benefit of promoting. We have showed that when $q = 0$, both firms promote in the symmetric equilibrium. A sufficiently small but positive q is not going to change the cost/benefit of promotion much. Therefore, when q is small, although the equilibrium is asymmetric, both firms will promote.

When q is not small, our results show that the solutions to FOCs lead to violations (either $\lambda_{11} < 0$ or $r_{22} < 0$), which implies that there is no type 4 equilibrium.

We have now analyzed the case when q is small (type 4 equilibrium) or large (no PSNE). When q takes intermediate values, we will end up with the other three types of equilibria. The following proposition summarizes our results.

Proposition 2. *Fix k =* $\frac{1}{2}$ *. The equilibria we find are plotted in Figure III.7. Depending on α, there are 4 patterns when q increases:*

(i) α < 0.333: the pattern is Type 4 → Type 1 → No PSNE.

(ii) $\alpha \in (0.334, 0.338)$*: the pattern is Type 4 → Type 1 → Type 2 → Type 1 → No PSNE.*

(iii) $\alpha \in (0.339, 0.381)$*:: the pattern is Type 4 → Type 3 → Type 2 → Type 1 → No PSNE.*

(iv) $\alpha \in (0.382, 0.499)$*: the pattern is Type 4 → Type 1 → Type 2 → No PSNE.*

Proof. See Appendix.

<u>Discussion of comparative statics results when α increases</u>

Due to the numerical nature of our results, we are not sure how an increase in α would affect equilibrium price, promotion frequency and depth and profits, since this depends on the values of q and α in general. The general theme in Figure III.7 is that, for any $q < \bar{q}(\alpha, k)$, when α increases, firms are less likely to promote. This result has the same spirit as the benchmark symmetry case, that is, more coupon trading makes promotion less attractive, and firms respond by reducing their promotion frequency and intensity, although at different paces. From the figure, the area for Type 4 equilibrium shrinks, i.e., it's less likely to have both firms promote. The area for Type 2 grows. It's empty when α is small, but its size increases when α increases about certain threshold, implying that it's more likely to have no firm promoting as an equilibrium. When α increases, the smaller firm may stop promoting first. This is the case for most q 's, and the pattern is Type 4 → Type 1 → Type 2. However, when q is small, the larger firm may stop promoting first, and the pattern is Type 4 → Type 3 → Type 2. Intuitively, when firms reduce their promotion frequency and depth, they would increase equilibrium prices as well, which will leads to higher profits, just as in the symmetric case. However since firms are asymmetric, an increase in α may affect the two firms differently, in terms of magnitude or even direction of the effects.

III.5. Conclusion

There is a large literature on price discrimination, which has typically maintained the assumption that consumer arbitrage is not feasible. This assumption is increasingly violated when price discrimination is achieved through couponing, and coupons are increasingly traded online. We relax the no-arbitrage assumption by allowing coupons to be traded to and used by consumers not initially targeted by the firm. In particular, we assume that a fraction of consumers are coupon traders who can buy/sell coupons. We then analyze the impact of coupon trading on firms' decisions to promote (in terms of promotion frequency and promotion depth), the equilibrium prices and profits. We find that when the fraction of coupon traders increases, firms respond by promoting less frequently (sending fewer coupons out) and reducing the face value of coupons. This reduces competition and leads to higher equilibrium prices and profits. When the cost of distributing coupons increases, the results on promotion frequency, prices and profits are similar to the results when the fraction of coupon traders increases. This is because both coupon trading and distribution costs work against coupon promotions, reducing a firm's incentives to promote. The only difference is that while coupon face value decreases with the fraction of coupon traders, it increases with coupon distribution cost.

In both cases, consumers are all worse off since prices increase. Our results are robust to several extensions including the introduction of coupon non-users, non-tradable coupons and asymmetric firms.

III.6. Appendix

Proof of Lemma 1.

Without loss of generality due to symmetry, we only show that firm 1 has no incentive to distribute both offensive and defensive coupons. Suppose not, fix firm 2's price and promotion strategies, and let $p_1, \lambda_{11}, \lambda_{12}, r_{11}$ and r_{12} denote firm 1's best response to firm 2's strategy. We call this the *initial strategy*. Since firm 1 distributes both offensive and defensive coupons, thus $\lambda_{1j} > 0$ and $r_{1j} > 0, j$ =1, 2.

Next we will rank r_{11} and r_{12}. Intuitively firms are more aggressive and charge lower prices in the other firm's turf, due to best-response asymmetry (One firm's weak market is the other firm's strong market). This implies $r_{11} > r_{12}$. We will show that firm 1 can improve its profit by playing the following strategy instead

$$p_1' = p_1 - r_{12}, \lambda_{11}' = \lambda_{11}, r_{11}' = r_{11} - r_{12}, \lambda_{12}' = r_{12}' = 0.$$

We call this strategy the *alternative strategy*, where firm 1 distributes offensive coupons only. Next, we prove that compared to the initial strategy, under the alternative strategy, firm 1 (i) earns weakly higher profit from the non-traders; (ii) earns weakly higher profit from the traders and (iii) saves on coupon distribution cost.

Higher profit from non-traders

We first calculate firm 1's profit from non-traders under the initial strategy. Start with consumers on the left interval [-L, 0]. Let $\pi_{1L}(p)$ denote firm 1's expected profit in the left segment when its effective price is p. Firm 1's profit in the left segment is

$$(1-\lambda_{11})\pi_{1L}(P_1)+\lambda_{11}\pi_{1L}(P_1-r_{11}).$$

Similarly firm 1's profit in the right segment is

$$(1-\lambda_{12})\pi_{1R}(P_1)+\lambda_{12}\pi_{1R}(P_1-r_{12}).$$

Note that $\pi_{1L}(P_1)<\pi_{1L}(P_1-r_{11})$ and $\pi_{1R}(P_1)<\pi_{1R}(P_1-r_{12})$ must hold. Otherwise, firm 1 would be better off not to distribute the coupons. Moreover, since the initial strategy is a best-response to firm 2's strategy, $p_1 - r_{11}$ maximizes firm 1's profits. This implies that

$$\pi_{1L}(P_1-r_{11})>\pi_{1L}(P_1-r_{12})>\pi_{1L}(P_1),$$

since $p_1 - r_{11} < p_1 - r_{12} < p_1$, and $\pi_{1L}(p)$ is concave in p (demand is linear in p), and $p_1 - r_{11}$ is a maximum.

Firm 1's profit from non-traders under the initial strategy (with both types of coupons) is

$$\pi_1=(1-\lambda_{11})\pi_{1L}(P_1)+\lambda_{11}\pi_{1L}(P_1-r_{11})+(1-\lambda_{12})\pi_{1R}(P_1)+\lambda_{12}\pi_{1R}(P_1-r_{12})$$

Similarly, we can show that firm 1's profit from non-traders under the alternative strategy (with offensive coupons only) is

$$\pi_1'=(1-\lambda_{11})\pi_{1L}(P_1-r_{12})+\lambda_{11}\pi_{1L}(P_1-r_{11})+\pi_{1R}(P_1-r_{12}).$$

We have shown that $\pi_{1L}(P_1 - r_{12}) > \pi_{1L}(P_1)$ and $\pi_{1R}(P_1 - r_{12}) > \pi_{1R}(P_1)$. Thus

$$\pi_1' \geq \pi_1,$$

and firm 1 earns higher profit from the non-traders under the alternative strategy. The inequality is strict unless $\lambda_{11} = \lambda_{12} = 0$.

Higher profit from traders

Next we compare firm 1's profit from traders under the initial strategy and that under the alternative strategy. In both segments, firm 1's final prices after coupons are the same under either strategy, while the regular price is lower under the alternative strategy. For traders who receive firm 1's coupons, lower regular price means that they are more likely to buy from firm 1, instead of buying from firm 2 and selling firm 1's coupons. This improves profits. Moreover, when coupons are traded, lower coupon face value also implies lower loss for firm 1. Therefore, firm 1's profit from traders is higher under the alternative strategy.

Save on coupon distribution cost

The coupon distribution cost is $k(\lambda_{11}L)^2 + k(\lambda_{11}L)^2$ under the initial strategy, while it's only $k(\lambda_{11}L)^2$ under the alternative strategy. Whenever $k>0$, coupon distribution cost is strictly lower under the alternative strategy.

To summarize, firm 1 earns higher profits from traders and non-traders and saves on coupon distribution cost under the alternative strategy. Thus the initial strategy, where firm 1 distributes both offensive and defensive coupons, cannot be a best-response to firm 2's strategy. Therefore, it will never distribute both types of coupons on equilibrium or off equilibrium (in deviation).

<u>Proof of Lemma 2.</u>

We start by deriving firm 1's best response in each segment, assuming that, hypothetically, it can choose an individual price for each segment. We use L to denote the left segment or segment 1, and use R to denote the right segment (segment 2). We use p_{ij} and π_{ij}, I = 1, 2, j = L, R to denote firm i's price at and profit from segment j. Then

$$\pi_{1L} = P_{1L}[0 - (P_{1L} - (P_{2L})], \quad \pi_{1R} = P_{1R}[L - (P_{1R} - (P_{2R})].$$

The first-order conditions are

$$\frac{\partial \pi_{1L}}{\partial p_{1L}} = -2p_{1L} + p_{2L} = 0 \Rightarrow p_{1L} = \frac{1}{2}p_{2L} .$$

$$\frac{\partial \pi_{1R}}{\partial p_{1R}} = L - 2p_{1R} + p_{2R} = 0 \Rightarrow p_{1R} = \frac{1}{2}L + \frac{1}{2}p_{2R} .$$

For firm 1 to have incentive to send defensive coupons only, it must be that

$$p_{1R} < p_{1L} \Leftrightarrow p_{2L} - p_{2R} > L .$$

However, this is impossible when firm 2 is sending defensive coupons only. For consumers who do not receive its coupons, the effective prices are the same, or $p_{2L} = p_{2R}$. For those who do receive firm 2's defensive coupons (in the left segment), we have $p_{2L} < p_{2R}$.

Therefore, whenever firm 2 sends defensive coupons only, firm 1's best response is to choose $p_{1L} < p_{1R}$, i.e., to send offensive coupons instead of defensive coupons.

Proof of Proposition 1.

We divide this proof into three parts. In part 1, we assume that firms do not distribute defensive coupons, and we derive the optimal prices and offensive couponing strategies. This is the equilibrium candidate. Then in part 2, we show that neither firm has incentive to deviate from this without distributing defensive coupons.[40] In part 3, we show that neither firm has incentive to deviate and distribute defensive coupons only. Recall that Lemma proved that neither firm has incentive to distribute both offensive and defensive coupons, on or off equilibrium.

Part 1: Equilibrium candidate: Firms distribute offensive coupons only

When firms distribute offensive coupons only, $\lambda_{12} = \lambda_{21} = r_{12} = r_{21} = 0$, and type (c) and (d) customers do not exist. Firms' profit functions become

$$\pi_1 = P_1(1-\alpha)(1-\lambda_{22})(L-P_1+P_2) + P_1(1-\alpha)\lambda_{22}(L-P_1+P_2-r_{22}) + (P_1-r_{11})(1-\alpha)\lambda_{11}(P_2-P_1+r_{11}) + P_1\alpha(L-P_1+P_2) - r_{11}\alpha\lambda_{11}L - K(\lambda_{11}L)^2 \quad (5)$$

$$\pi_2 = P_2(1-\alpha)(1-\lambda_{11})L + P_2(1-\alpha)(1-\lambda_{22})(P_1-P_2) + P_2(1-\alpha)\lambda_{11}(L+ P1- r11- p2+ P2- r221- \alpha\lambda 22P1- P2+ r22+ P2\alpha L+ P1- P2- r_{22}\alpha\lambda_{22}L - K(\lambda_{22}L)^2 \quad (6)$$

[40] A companion Maple file for Part 1 and 2 is available for download at http://faculty-staff.ou.edu/L/Qihong.Liu-1/research.html.

We can use the FOCs for both firms, or use FOC for either firm and then impose symmetry conditions. Both lead to the same solutions. We will report the latter method here.

Taking derivative of π_1 with respect to p_1, r_{11} and λ_{11} respectively, then imposing the symmetry conditions ($p_2 = p_1$, $r_{22} = r_{11}$ and $\lambda_{22}=\lambda_{11}$), we can obtain

$$\frac{\partial \pi_1}{\partial r_{11}} = -\lambda_{11}(\alpha L - p_1 - 2r_{11}\alpha + 2r_{11} + \alpha p_1) = 0. \tag{7}$$

$$\frac{\partial \pi_1}{\partial \lambda_{11}} = -2k\lambda_{11}L^2 - r_{11}\alpha L - \alpha p_1 r_{11} + r_{11}^2\alpha + p_A r_{11} - r_{11}^2 = 0. \tag{8}$$

$$\frac{\partial \pi_1}{\partial p_1} = L - p_1 - \lambda_{11}r_{11}\alpha + \lambda_{11}\alpha p_1 + r_{11}\lambda_{11} - \lambda_{11}p_1 = 0. \tag{9}$$

Since the cost of coupon distribution is quadratic in λ, and the rest is roughly linear in λ, it must be that the optimal $\lambda_{11}>0$. Then, equation (7) implies,

$$r_{11} = \frac{\alpha L - p_1 + \alpha p_1}{2(\alpha - 1)} = \frac{1}{2}\left(p_1 - \frac{\alpha L}{1-\alpha}\right). \tag{10}$$

From this expression, we can see that $p_1 > r_{11}$.

Next, we substitute the expression of r_{11} into equation (8) and solve for λ_{11}. We obtain

$$\lambda_{11} = \frac{(\alpha L - p_1 + \alpha p_1)^2}{8(1-\alpha)kL^2}. \tag{11}$$

Using r_{11} and λ_{11} in equation (9), we can solve for the equilibrium price,[41]

[41] There are 3 solutions. We pick the one that is real and positive.

$$p_1 = \frac{\left(\frac{2}{3}A - \frac{3}{2}\frac{-\frac{4}{9}\alpha^2 + \frac{16}{3}k}{A} - \frac{1}{3}\alpha\right)L}{-1 + \alpha}.$$

Where

$$A = \left(\alpha^3 + 36k\alpha - 27k + 3\sqrt{12\alpha^4 k + 96\alpha^2 k^2 + 192k^3 - 6k\alpha^3 - 216k^2\alpha + 81k^2}\right)^{\frac{1}{3}}$$

We can substitute this expression of p_1 back into the expressions of r_{11} and λ_{11}. The final expressions are too lengthy to report.

So far, we have used first order conditions to solve for the optimal choices of prices and promotion intensities. However, first order conditions are necessary but not sufficient. We need to make sure that the solutions we obtained indeed constitute an equilibrium. Instead of checking whether the Hessian matrix is negative semidefinite (which is quite messy), we show that this is an equilibrium by verifying that neither firm has an incentive to unilaterally deviate from this pair of strategies (Bester and Petrakis use a similar method). Without loss of generality, we fix firm 2's price and promotion strategies as given in Proposition 1, and allow firm 1 to deviate from these strategies.

Part 2: Firm 1 deviates without sending defensive coupons

Since neither firm sends defensive coupons, there is still no type (c) and (d) customers in deviation. Note that the demand/profit functions depend on the locations of marginal consumers and there are two cases. In the first case, $p_1 \geqslant p_2$ still holds and thus $l_a \geq 0$. In the second case, $p_1 < p_2$. In both cases, we assume that $l_{b1} < 0$ and $l_{b2} > 0$.[42]

Start with case 1 where $p_1 \geqslant p_2$ still holds. Firm 1's deviation profit is given by equation (5), with $p_2 = p^*$, $r_{22} = r^*$ and $\lambda_{22} = \lambda^*$. We normalize L=1. Optimal choice requires that

$$\frac{\partial \pi_1^{dev}}{\partial r_{11}} = \frac{\partial \pi_1^{dev}}{\partial \lambda_{11}} = 0$$

Solving the first order conditions, we obtain

$$r_{11}^{dev} = \frac{2(1-\alpha)p_1 - (1-\alpha)p^* - \alpha}{2(1-\alpha)},$$

$$\lambda_{11}^{dev} = \frac{\alpha^2 + \alpha^2(p^*)^2 + 4\alpha^2 p_1 - 2p^*\alpha^2 + 2\alpha p^* - 4\alpha p_1 - 2\alpha(p^*)^2 + (p^*)^2}{(1-\alpha)k}.$$

The first order conditions are necessary and sufficient. We leave the p^* term in the expression, as clearly the expressions will be too lengthy to report if we substitute the value of p^*. Now firm 1's deviation profit depends only on p_1^{dev}, α and k. We want to check whether firm 1 can increase its profit by choosing $p_1^{dev} \neq p^*$, i.e., to have

$$\pi_1^{dev}(p_1^{dev}) > \pi^*, \forall\, \alpha, k.$$

We tried various combinations of α and k, and we found that firm 1 can never increase its profit by choosing a price different than p^*. Therefore, firm 1 has no incentive to deviate when no defensive coupons are used in the deviation. We then

[42] If $l_{b2} \leq 0$, then our formula of d_{1b} would be exaggerated. This is because the relevant demand is capped at L while our formula leads to $d_{1b} \geq L$. Since we show that firm 1 has no incentive to deviate under the exaggerated demand function, it surely has no incentive to deviate under the correct demand function. Thus we ignore the case of $l_{b2} < 0$. Note that $l_{b1} < 0$ must hold. This is because, as the deviating firm, firm 1 must be able to sell to some of firm 2's loyal customers, i.e., $l_{b1} < 0$.

proceed to the case of $l_a < 0$ (i.e. p_1<p_2). The steps are similar and we find that firm 1 has no incentive to deviate if k is not too small relative to L.[43]

Next we want to show that firm 1 has no incentive to deviate and send defensive coupons only.

Part 3: Firm 1 deviates by sending defensive coupons only

In proof of Lemma 2, we have shown that firm 1's best response is to choose $p_{1L} < p_{1R}$ as long as $p_{2L} - p_{2R} < L$. In the proposed equilibrium candidate, when consumers (in the right segment or segment 2) receive firm 2's offensive coupons, we have

$$p_{2L} = p^*, p_{2R} = p^* - r^* \Longrightarrow p_{2L} - p_{2R} = r^* \leq L.$$

When consumers in segment 2 do not receive firm 2's coupons, firm 2's prices are the same in the two segments

$$p_{2L} = p_{2R} = p^*.$$

In both cases, we have

$$p_{2L} - p_{2R} \leq L \Longrightarrow p_{1L} < p_{1R}.$$

Therefore, firm 1 should not send defensive coupons only.

[43] Details are available in the companion Maple file. For L=1 and α=0, the threshold value for k is around k=0.159. Technically, there is another constraint on k. That is, when k is sufficiently small, our λ^* formula leads to λ^*>1, which should be replaced by λ^*=1 (Probability cannot be greater than 1). However, we find that this constraint on k is never binding since the threshold k is smaller than the threshold k for firms not to have incentive to lower prices and deviate. For example, when α=0, while the threshold k for no deviation is k=0.159 (i.e., no deviation if k>0.159), that for for λ^*=1 is k=0.033.

Proof of Lemma 3

First we show that when q is large, firm 2 does not promote. Let's look at non-traders with coupons when firm 2 promotes. Firm 2 will make sale at $[0, p_1 - p_2 + r_{22}]$. If firm 2 does not promote, it can only sell in the interval $[0, p_1 - p_2]$. The gain in marginal revenue is $\lambda_2(p_2 - r_{22})r_{22}$, i.e., a gain of $\lambda_2 r_{22}$ extra customers at price $p_2 - r_{22}$. The loss in inframarginal revenue is $\lambda_2 r_{22}(p_1 - p_2)$, because a measure of $\lambda_2(p_1 - p_2)$, customers now enjoy discount and pay $p_2 - r_{22}$ instead of p_2. The results show that, $(p_1 - p_2) - (p_2 - r_{22})$ increases with q and becomes positive when q is large. Thus when q is large, gain is less than loss, and firm 2 will never promote. If we include the coupon distribution cost and the possibility of coupon trading, then firm 2 will have even less incentive to promote.

Next we show that when q is sufficiently close to L, there is no pure strategy equilibrium. We do this in two steps. In step 1, we show that fixing p_2, firm 1 has incentive to promote, and grab all firm 2's loyal customers who receive coupons. In step 2, we show that given p_1, r_{22} as best reply of p_2, firm 2 will always have incentive to change p_2.

Step 1: We know that firm 2 will not promote. Suppose that firm 1 does not promote as well. Then $\lambda_{11} = \lambda_{22} = r_{11} = r_{22} = 0$. The solution to FOCs is

$$p_1 = 1 + \frac{q}{3}, p_2 = 1 - \frac{q}{3}$$

Now consider the following deviation, firm 1 promotes and set r_{11}. When $\lambda_{11} \to 0$, the marginal cost of coupon distribution is 0, since this cost is quadratic in λ1. Therefore, if we assume $\lambda_{11} \to 0$, coupon distribution cost can be ignored.

The gain is new sales to firm 2's loyal customers who are non-traders but with firm 1's coupons,

$$gain = (1-\alpha)\lambda_{11}(p_1 - r_{11})(p_2 - p_1 + r_{11})$$

The loss happens when some coupons reach traders and are traded back to firm 1's loyal customers,

$$loss = \alpha\lambda_{11}(L-q)r_{11}$$

Let Δ = gain−loss. It can be showed that when q is sufficiently large, firm 1 has incentive to set d1 such that $p_1 - r_{11} = p_2 - (1-q)$, i.e., $r_{11} = p_1 - p_2 + (1-q) = 1-q$. In this case, for all firm 2's loyal customers who receive firm 1's coupons, they will buy from firm 1. Then,

$$\Delta = \frac{\lambda_{11}}{3}(1-q)(2q - q\alpha - 3\alpha)$$

Obviously $\Delta > 0$ when $q > \frac{3\alpha}{2-\alpha}$, with $\frac{3\alpha}{2-\alpha} < 1$ since $\alpha < \frac{1}{2}$.

If we allow p_2 to be different from $1-q$, following similar steps as above, we can obtain

$$\Delta = \frac{\lambda_{11}}{3}(1-q)(2q - q\alpha - 3 + 3p_2 - 3\alpha)$$

It can be easily checked that $\Delta > 0$ when $q > \frac{3(1+\alpha-p_2)}{3-\alpha}$. We need $\frac{3(1+\alpha-p_2)}{3-\alpha} < 1$, which is satisfied if $p_2 > \frac{4\alpha}{3}$.[44]

[44] If p_2 is small, firm 1 will not try to take all firm 2's loyal customers who receive coupons. But in this case, firm 2 is always better off raising p_2. In all the equilibria we find, p_2 > 1/2 is always satisfied. Moreover, p_2 increases with α, and when α→1/2, p_2→1. Thus we assume that p_2 > 4α is satisfied.

Therefore, when p_2>4α, firm 1 will always promote, and choose r_{11} such that last consumer (located at −1+q) is exactly indifferent between buying from firm 1 with coupon, and buying from firm 2 without coupon.

Step 2: Pick p_2>4α. Now we explain that firm 2 does not promote, firm 1 promotes and set $r_{11} = p_1 - p_2 + (1 - q)$ can't be an equilibrium. Let $\pi_2 = \pi_{2b} + \pi_{2,other}$, where π_{2b} denotes firm 2's profit from its loyal customers who receive firm 1's coupons, and $\pi_{2,other}$ denotes its profit from all other customers. Note that $\pi_{2,other}$ is continuously differentiable in p_2. If firm 2 increases p_2 slightly, π_{2b} will still be zero. Assume that firm 2 has no incentive to increase p_2, then,

$$\frac{\partial \pi_2}{\partial p_2^+} \leq 0 \Rightarrow \frac{\partial \pi_{2,other}}{\partial p_2} \leq 0$$

If firm 2 decreases p_2 slightly, it will gain some of its own loyal customer who receive firm 1's coupons. This is because previously the last consumer is exactly indifferent between buying from firm 1 with coupon, and buying from firm 2 at p_2. Therefore,

$$\frac{\partial \pi_{2b}}{\partial p_2^-} < 0$$

Then

$$\frac{\partial \pi_2}{\partial p_2^-} = \frac{\partial \pi_{2b}}{\partial p_2^-} + \frac{\partial \pi_{2,other}}{\partial p_2} < 0$$

This implies that firm 2 will always have incentive to lower p_2, if we assume that it has no incentive to increase p_2.

Proof of Proposition 2.

Since these are numerical results, we present details for specific parameter combinations of α and q, for type 1, 2 and 3 equilibria, and explain why they are equilibria. Type 4 equilibrium is explained in Lemma 4.

Type 1: Larger firm promotes, smaller firm does not.

This is an equilibrium with various (α, q) combinations (including $\alpha = 0$). Set $\alpha = 0.2$, $q = 0.5$ for example. Assume that both firms promote, the solution for FOCs lead to $r_{22} < 0$. So we assume that firm 2 does not promote, but firm 1 does. The equilibrium is p_1=1.112, p_2=0.741, λ_{11}=0.155, λ_{22}=0, r_{11}=0.679, r_{22}=0, and π_1=1.258, π_2=0.617. In this case, q is not small, so that firm 2 will not promote, as explained after Lemma 3. But q is not too large either, so firm 1 will make sales to part but not all of firm 2's loyal customers who receive firm 1's coupons. If firm 1 deviates and stop promoting, we show that its profit will go down. If firm 2 deviates, we find that the unique interior solution has r_{22}=< 0, so firm 2 will not deviate either. So neither firm has incentive to deviate.

Type 3: Smaller firm promotes but not the larger firm.

This happens when α is large and q is intermediate. Set $\alpha = 0.4$, $q = 0.15$. Assume both firms promote, the solution to FOCs leads to λ_{11}< 0. So we assume that firm 1 does not promote, firm 2 promotes, and we look for a type 3 equilibrium. The equilibrium is p_1=1.05, p_2=0.95, λ_{11}=r_{11}=0, λ_{22}=0.0007, r_{22}= 0.041 π_1=1.102, π_2=0.902. In this case, if firm 1 deviates and starts promoting, we find that λ_{11}< 0, which is a violation. If firm 2 deviates and set λ_{22}=r_{22}=0, its profit will go down. So neither firm has incentive to deviate.

The key difference between this case and the corresponding case with similar q but without coupon trading is that, the larger firm has no incentive to promote. The intuition is as follows. The larger firm sets a higher regular price than the smaller firm does ($p_1 > p_2$), and with sufficient firm asymmetry, this difference $p_1 - p_2$ can be quite large. For firm 1 to make a sale to its rival's loyal customers, it has to send coupons with significant face values ($r_{11} > p_1 - p_2$). Firm 1's gain is $p_1 - r_{11}$when the coupon reaches a switching non-trader (0 when they don't switch), and the loss is $p_1 - p_2$ when the coupon reaches a coupon trader. When d1 is small (but still greater than $p_1 - p_2$), firm 1 cannot attract many firm 2's loyal customers. But if r_{11} is large, the loss of coupons reaching traders is large too. As a result, there is no r_{11} that can attract enough of firm 2's loyal customers to compensate for the cost of coupon trading. However, firm 2 does not have this problem. It can set any r_{22}> 0, arbitrarily small if necessary, to attract more of firm 1's loyal customers. So it will promote when q is not too large.

Type 2: Neither firm promotes.

This happens only when α and q are large. Set $\alpha = 0.4, q = 0.5$ for example. Assume both firms promote, the solution to FOCs lead to violations for both firms (λ_{11}< 0, r_{22}< 0). So we assume that neither firm will promote, and look for a type 2 equilibrium. The equilibrium we find is p_1=1.166, p_2=0.833, λ_{11}=r_{11}=λ_{22}=r_{22}= 0, π_1=1.361, π_2=0.694. If firm 1 deviates and starts promoting, we find that λ_{11}< 0 which is a violation. If firm 2 deviates, we find that r_{22}< 0, also a violation. So neither firm has incentive to deviate. The intuition why firm 1 does not promote is similar to above. Firm 2 does not promote because q is large, and the intuition is explained in Lemma 3.

Table III.1. Four Types of Equilibrium

	Firm 1 promotes	Firm 1 does not promote
Firm 2 promotes	Type 4	Type 3
Firm 2 does not promote	Type 1	Type 2

Figure III.1. Type a) Non-traders with neither firm's coupons

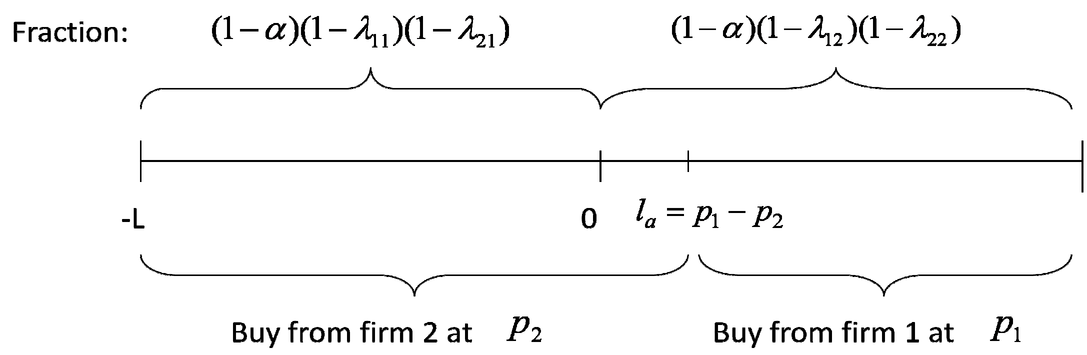


Figure III.2. Type b) Non-traders with offensive coupons only

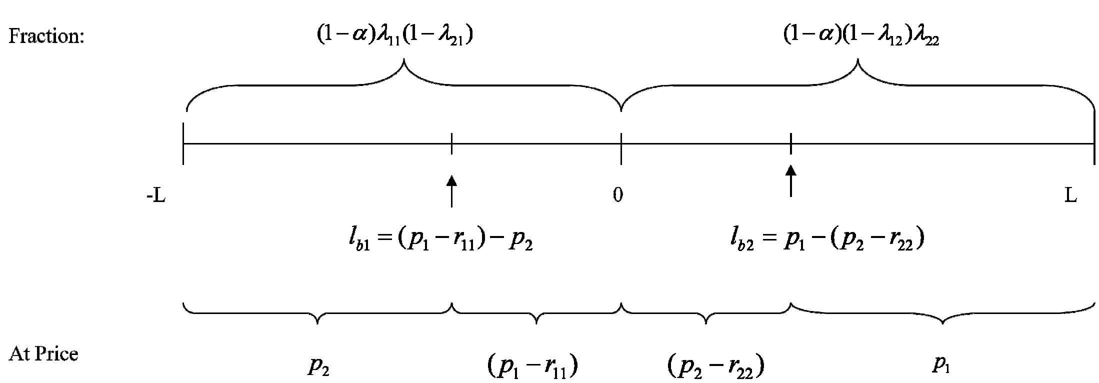


Figure III.3. Type c) Non-traders with defensive coupons only

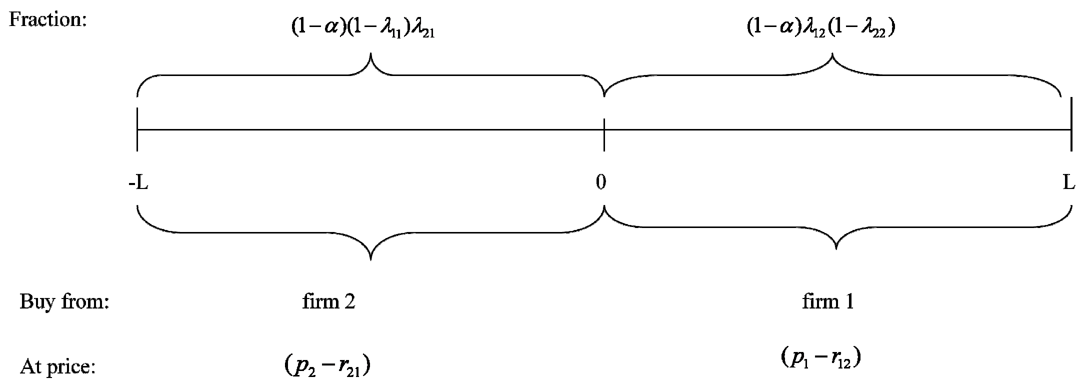


Figure III.4. Type d) Non-traders with both firms' coupons

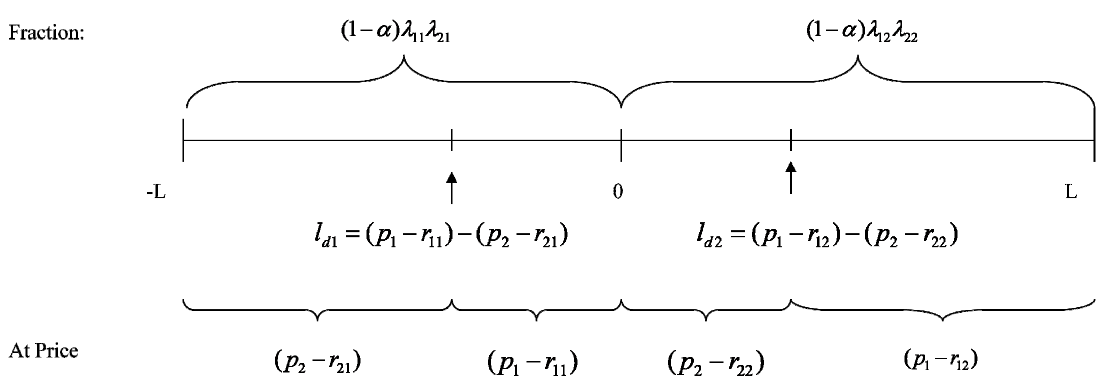


Figure III.5. Comparative statics when α varies

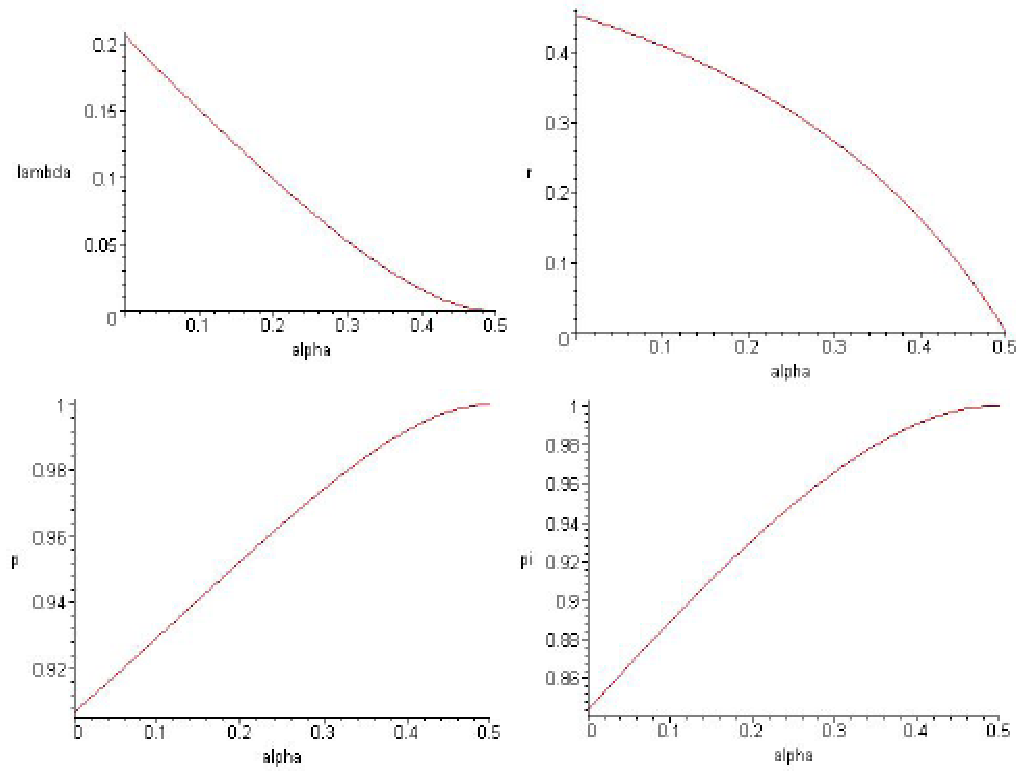


Figure III.6. Comparative statics when k varies

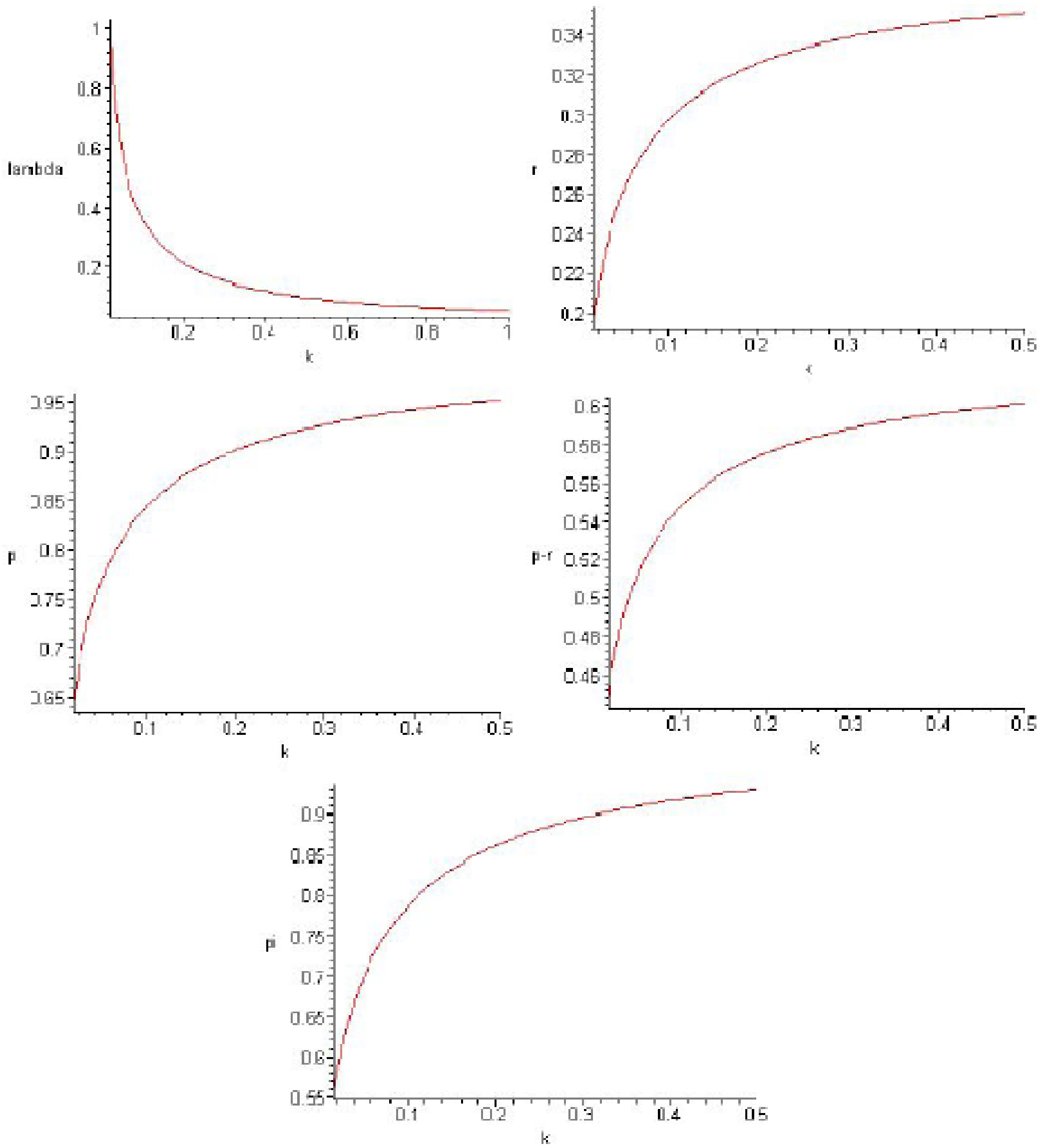
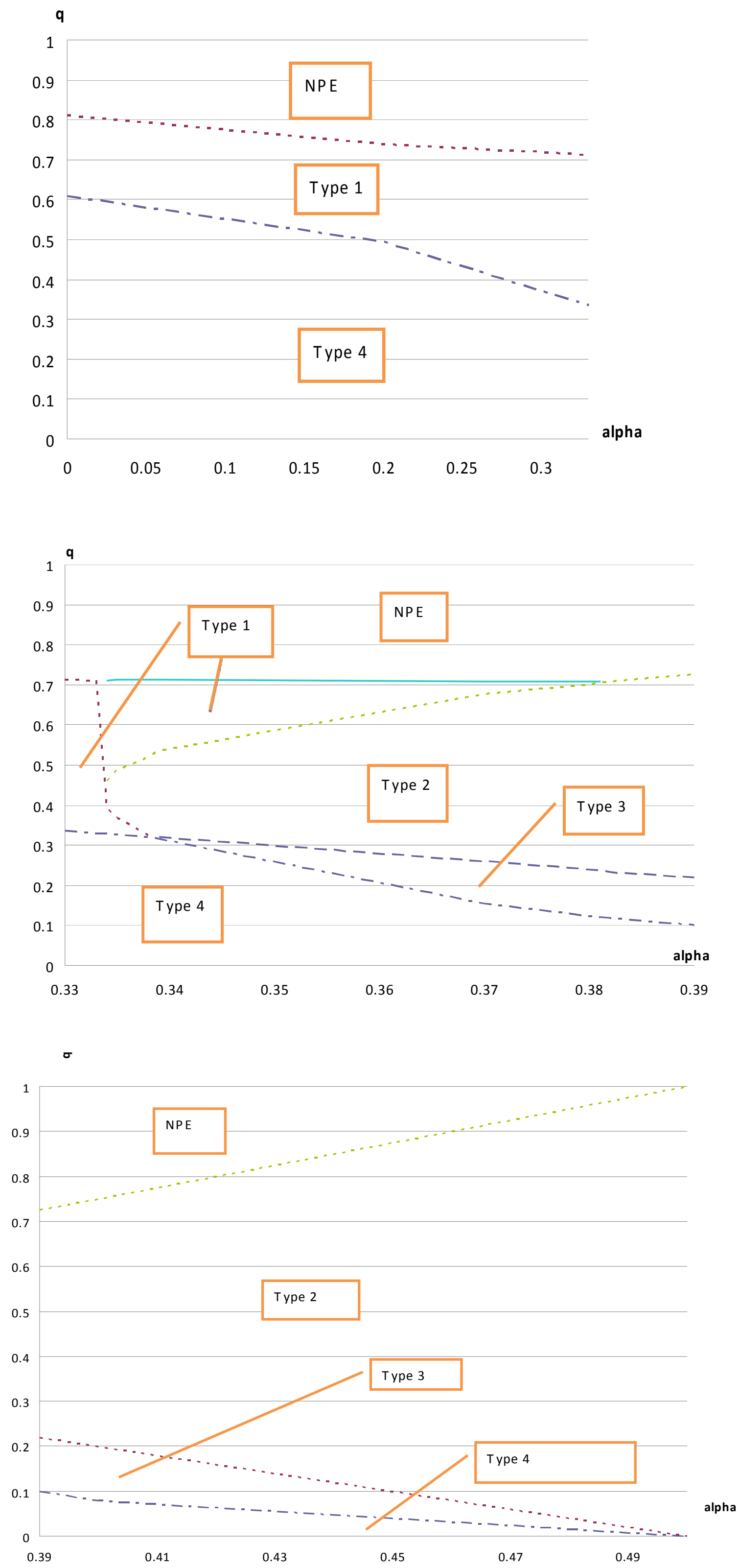


Figure III.7. Patterns of equilibria depending on α and q.



CHAPTER IV

POST-SALE LOW PRICE GUARANTEE AND PRICE FLUCTUATION

IV.1. Introduction

As widely observed, the online prices not only differ from store to store, but also from time to time[45]. Recent research on online price dispersion focuses on the price dispersion among stores. Very few studies[46] look at price fluctuation over time. A popular explanation for price changes over time is intertemporal pricing strategy. Firms use an intertemporal pricing strategy to either increase demand or to price discriminate part of the market. The intertemporal pricing strategy can explain many market situations very well. However, it does not fit our observation of price patterns from Best Buy and Circuit City well for two reasons. First, the intertemporal pricing strategy usually predicts prices decrease monotonically over time[47], but observed price changes often indicate a cyclical movement. Second, a common assumption in the intertemporal pricing literature is that some consumers are willing to wait and buy the product in the future at a lower price. However, as commonly observed, many retailers today adopt a post-sale low price guarantee policy, which allows consumers to get refund if the price

[45] This difference over time is not monotonically decreasing; rather the price goes up and down over time.

[46] There are many literatures look at price change driven by intertemporal pricing strategy. But in most of them the price is monotonically decreased over time, which is a typical feature of intertemporal pricing theory.

[47] Usually firm lowers its price gradually to extract maximum profits from groups of consumers with different demand elasticity or reservation prices.

is lowered within limited time period after purchase. Taking this into consideration, only few consumers will want to wait and buy in the future.[48]

A simple general description of the low price guarantee policy is the following: if you find a lower price than the store's price, they will refund you the difference or more than the difference. If the store only refunds the price difference, it is usually called price matching; and if the store refunds more than the difference[49], it is called price beating. According to the target of LPG, this policy can be divided into two groups: LPG targeting at the store's competitors price, called competitors low price guarantee (hereafter CLPG); and LPG targeting at the store's own post-sale price, called self low price guarantee (hereafter SLPG). These two LPG policies are widely used by various stores. Some stores only use CLPG, and examples could be found in airline companies, Buy.com etc.; some stores only use SLPG, like Amazon.com, Dell etc.; while some use both, for example Best Buy and Staples.

In this paper we explain the cyclical price change over time by firms using SLPG to price discriminate across consumers. Specifically we start with a two-period duopoly market, in which we show both firms will adopt the low price guarantee policy. At each period there are new customers entering into the market. Some with inelastic demand have prohibitively high cost of requesting price match; the others with elastic demand have low cost of requesting price match, and this cost is normalized to zero. We then extend the two-period model to a dynamic model. We find the optimal strategy for

[48] Those consumers may have very high hassle cost of requesting price match.

[49] Usually 110% of the difference as Best Buy, or 115% of the difference as Skinstore.com.

firms is to use a cyclical price strategy to price discriminate the inelastic demand consumers.

Using online price data from Best Buy and Circuit City, we test our theoretical prediction. We find a significant negative relationship between the firm's price change and its own previous price change. This means if one firm increases (decreases) its price at this period, it will very likely decrease (increase) its price in the following period.

The rest of the paper is organized as follows. Section 2 provides a review of related literature. Section 3 proposes a general theoretical model to explain cyclical price change, then, a specific model to examine the consumer surplus and social surplus, and at the end we extend the two-period model to a dynamic model. Section 4 offers empirical analysis of Best Buy and Circuit City's price change. Section 5 concludes.

IV.2. Theoretical model

IV.2.1 Overview

In this section we discuss the details of the SLPG with intertemporal pricing model and related notion of equilibrium. Existing intertemporal price discrimination literature usually predicts a monotonic price decrease. However, in practice, most of the time, we observe that price changes are not monotonic; rather prices fluctuate. With firms adopting SLPG policy, we can explain this cyclical price fluctuation over time. We first present a two-period general theoretical model of how duopolies selling homogeneous product to a population of heterogeneous consumers should adopt SLPG policy and set their optimal sequence of prices over time.[50] Similar to other intertemporal pricing models, this model takes demand parameters as given. Consumers have complete information of firm pricing policies.[51] We then present a more specific theoretical model and discuss the firm's profits, consumer surplus and social surplus with or without SLPG and intertemporal pricing policy. At the end, we extend the two-period model to a dynamic model.

The equilibrium concepts we using here is Nash equilibrium and Sub-game perfect Nash equilibrium.

IV.2.2 General theoretical model

We consider a two-period duopoly game, with two firms—1 and 2—produce homogeneous goods with constant marginal costs. Without loss of generality, we normalize the marginal cost to zero and assume there is no fixed cost or discount factor over time. Firms decide their prices simultaneously in each period. In each time period, new customers enter the market, and only new customers make purchases in that period. This assumption excludes the possibility that consumers can wait and buy at a lower price. In each period, consumers enter the market and choose whether to buy the product or not. If they choose not to buy, they leave the market and will not re-enter the market in the future.[52] So each consumer lives in the market in two periods and can only make purchase in the first period.

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[50] Since there is no consumer waits and buys in the second period, discount factor doesn't matter in our model. We therefore assume there is no discount factor over time.

[51] These are general assumption made in intertemporal pricing literature. For example, Stokey (1979).

[52] Later, with SLPG, we can relax this assumption. And one can easily show that under SLPG no consumer wants to wait and buy since they can request a price match.

There are three types of consumers, namely high, medium and low consumers.[53] Each consumer will buy at most one unit of the product from either firm. The high type and medium type consumers are willing to pay up to V for the product. We assume that the reservation price V is sufficiently high and thus the high type and medium type consumers will buy exactly one unit of the product. The high type consumers are divided into two groups; each firm embraces a group of high type consumers as their loyal consumers. Within each group the high type consumers are willing to pay a homogeneous premium to buy at their favorite firm. This premium is high enough that no high type consumer will buy at the firm other than her favorite firm. Therefore for each firm, their own high type consumer's demand is perfectly inelastic under the reservation price. The medium type consumers are heterogeneous with respect to the premium they are willing to pay for their favorite brand. We use a parameter l to measure this heterogeneity, which can be interpreted as the consumer's degree of loyalty. Specifically, a consumer located at l is indifferent between buying from the two firms if and only if $l = p_1 - p_2$. We can introduce the asymmetry in this market by allowing firm 1(or firm 2) to have a larger medium market share when prices of the two firms are equal. The last type is low type consumers, who are heterogeneous in the reservation price. This reservation price is not high enough to guarantee each low type consumer will buy exactly one unit of the product from either firm. Therefore, for both

[53] The model with only two types of consumers—high type and low type, or medium type and low type—will not change our result qualitatively. The reason we choose three types of consumers is because we think this is closer to the real world situation. The high type consumers represent the consumers who value the product high enough and are willing to pay a very high premium to shop under a familiar environment; the medium type consumers represent the consumers who value the product high enough but the premium they are willing to pay is not as high as the high type and heterogeneous; the low type consumers are those value the product not high enough. The other reason I adopt three types of consumers is that the two types of consumers model will generate a uniform price in the high price period (which is not consistent with what we observed in the data) as we will see later when we go to the solution of the model.

firms, the low type consumers' demand is elastic. The low type consumers are indifferent from buying from either firm, and thus $\left|\frac{\partial D_l(p_1,p_2)}{\partial p_1}\right| = \left|\frac{\partial D_l(p_1,p_2)}{\partial p_2}\right|$ [54], where D_l is low type's demand, and p_1, p_2 are firm 1 and firm 2's prices respectively. The introduction of low type consumers allows our model to be able to fit in a more general market rather than a strict duopoly market. For example, we can interpret our model as a market with two leading firms and a number of other small firms. The two leading firms have some brand advantages over the small firms, and each has certain amount of loyal customer who will only buy from their preferred firm. This is the high type consumers in my model. There are some consumers who are willing to buy from one of the leading firms, but when the price difference is large, they will switch to the other leading firm. This is the medium type consumers in my model. There are also some consumers who are willing to buy from the leading firms, but will also switch to the small firms if the small firms charge much lower prices. This is the low type consumers in my model.

The two firms compete in prices, and at each stage, they cannot price discriminate among customers because they are incapable of charging different prices to different customers. The firms have an option to choose whether adopting a SLPG policy or not in the first period. If they adopt SLPG, when prices are lowered in the second period, the consumers who bought the product in the first period at a higher price can request a price match, and the firm will refund the difference, which is equivalent to knowing some consumers buy the product in period 1 at period 2's price. For example, if firm 1 adopts SLPG, and it lowers its price from $p_{1,1}$ to $p_{1,2}$ from period 1 to period 2, then the

[54] Introducing firms preference in low type will not change our result qualitatively.

consumers who request price match will get a refund of $p_{1,1} - p_{1,2}$; after price matching, these consumers actually pay $p_{1,2}$ for the product. This SLPG policy only applies to the firm's own price one period after sale (i.e. firm 1 will not price match firm 2's price). For simplicity, we assume there is no discount factor over time.

Following Stokey (1979)[55], we assume that consumers know the entire price schedules over time of both firms and firms know all relevant characteristics of the potential market. Although all period 1 consumers know period 2 prices $p_{i,2}$(i=1,2) in period 1, not all of them will request a price match in period 2 even if firms adopt SLPG. We assume the high type and medium type consumers have very high hassle cost of requesting price matching, and without loss of generality, we assume this hassle cost is prohibitively high so that no high type or medium type consumers will request price matching. For low type consumers, we assume their hassle cost of requesting a price match is very low and normalized to zero. Therefore, if a firm adopts SLPG and its price is lowered in period 2, only the low type consumers who make purchases in period 1 will request a price match. The firms know the fraction and distribution of each type of consumers and their reservation prices. We assume the fraction and reservation price of high type consumer are not too large, and thus both firms want to serve all three types of customers when they maximize their profits.

Next, we discuss some conditions under which there exists pure strategy Nash equilibrium. When firms do not adopt SLPG, there is no linkage between periods. Each firm's objective is to maximize its profit at each period regardless of its previous or following periods' strategies. Therefore we can analyze this as a static model.

[55] It's a monopoly market in her paper, my model here is duopoly market.

Condition 1: $\frac{\partial^2 \pi_i}{\partial {p_i}^2} < 0$, for $i = 1, 2$, representing firm 1 or firm 2's profit respectively.[56]

If condition 1 is satisfied, there exists a unique pure strategy Nash equilibrium (p_1^*, p_2^*) in the static game when SLPG is not adopted by firms. Each firm will charge its optimal single price over time, and there is no price discrimination over time or among consumers.

Since the low type consumers can request price matching one period after sale, when firms adopt SLPG, there is a linkage between the two periods. By adopting the SLPG policy and intertemporal pricing policy, firms are capable of price discriminating the high and medium type consumers in the first period. They can set a high price in the first period and then a low price in the second period. The high and medium type consumers in the first period will pay the high price; while the low type consumers in the first period and all consumers in the second period will pay the low price. Let's denote $\pi_{i,j,t}$ as firm i's profit from type j customers in period t, where i=1,2, and j=h,m,l. $D_{i,j,t}$ are firms' corresponding demand defined in similar way.

Condition2: $\frac{\partial^2 \pi_{i,1}}{\partial {p_{i,1}}^2} < 0, \frac{\partial^2 \pi_{i,2}}{\partial {p_{i,2}}^2} < 0$, where $\pi_{i,1} = \pi_{i,h,1} + \pi_{i,m,1}$; $\pi_{i,2} = \pi_{i,h,2} + \pi_{i,m,2} + \pi_{i,l,1} + \pi_{i,l,2}$, for $i = 1, 2$.

If condition 2 is satisfied, there exists a pure strategy Nash equilibrium in the two-period game when SLPG is adopted. With firm i choosing $p_{i,1}^*$ in period 1, and $p_{i,2}^*$ in the period 2, consumers in these two periods can be divided into two groups according

[56] Png & Hirshleifer (1987) and Corts (1996) made similar assumption to assure strategic complementarities of prices and guarantee uniqueness of equilibrium in the game.

to the price they paid. Group 1 consists of high and medium type consumers in period 1 who pay $p^*_{i,1}$ to firm i; group 2 consists of high type, medium type consumers in period 2 and all low type consumers, who pay $p^*_{i,2}$ to firm i. With this price scheme, each group's profit is maximized, therefore the total profit is also maximized, and no firm wants to deviate. By doing this, firms can get maximum profits from high type and medium type consumers in the first period, and get maximum profits from the rest of consumers in the second periods.

Condition 3: $\left|\frac{\partial D_{i,m}}{\partial p_i}\right| < \left|\frac{\partial D_l}{\partial p_i}\right|$; $D_{i,l} < D_{i,m} + D_{i,h}$ for any $V > p_i > 0$. Where $D_{i,h}$ is firm i's demand from high type consumers, $D_{i,m}$ is defined in the similar way for medium type and D_l is demand for low type consumers. p_i is firm i's price.

Condition 3 means the low type consumers are more price sensitive than medium type and the total demand from high type and medium type consumers is always greater than that of low type consumers. With condition 3, firms can get higher profits by adopting SLPG and intertemporal pricing policy compared to single price.

Proposition 1: *When all three conditions are satisfied and firms can choose SLPG and an intertemporal pricing strategy, it's optimal for firms to adopt SLPG and charge a high price in the first period and a low price in the following period. The single pricing strategy is not a Nash equilibrium. The optimal single price (when firms can not choose SLPG or intertemporal pricing strategy) is between the two intertemporal prices (When firms can choose SLPG and an intertemporal pricing strategy).*

Proof. See Appendix.

As proposition 1 shows, when possible, firms will adopt SLPG and intertemporal pricing strategy. Based on the structure of our model, social surplus solely depends on the number of consumers served in the market. Under an intertemporal pricing strategy, social surplus increases because more low type consumers are served due to the lower price in second period compared to the game with a single price restriction. Firm's profit from group 1 consumers will be increased, but profit from group 2 consumers may or may not increase; therefore the total profit change is ambiguous. Consumer surplus change is ambiguous. There are two opposite effects on consumer surplus. The positive effect is that the lower price in the second period increases the consumer surplus of high and medium type consumers in the second period and the surplus of all low type consumers. The negative effect is the increased price for high and medium type consumers in the first period. Following, we propose a specific model, in which we can examine profits, consumer surplus, and social surplus.

IV.2.3 Specific model

Following our general model setup, at each stage, both firms decide their prices simultaneously and they have complete information about consumers' type and their corresponding fractions. There is a total of 2L new consumers entering the market each period, and the consumers can only make purchases in that period. Firm 1 and firm 2 each has βL of high type consumers. There is a total of $2\phi L$ medium type consumers. The premium they are willing to pay for their favorite store is measured by l, which is uniformly distributed on $[-L+q_t, L+q_t]$. And the interval $[-L+q_t, L+q_t]$ is partitioned into two segments: [$-L+q_t$, 0] and [0, $L+q_t$].[57] When two firms charge the same prices, consumers locate at [0, $L+q_t$] will buy from firm 1, and we call them firm 1's medium type loyal customers; consumers locate at [$-L+q_t$, 0] will buy from firm 2, and we call them firm 2's medium type loyal customers. The q here is the measurement for degree of asymmetry. When $q_t=0$, these two firms are symmetric in the medium type consumer market; when $q_t>0$, firm 1 is the strong firm with more market share; when $q_t<0$, firm 2 is the strong firm. If this asymmetry is constant over time, then q_t is constant over time, otherwise it varies with time. For simplicity, we assume q_t is constant within the two periods when we analyze the model. High type and medium type consumers' reservation price is high enough to guarantee one unit demand from each of them. Firm i's demand from low type consumers is $(1-\beta-\phi)L - a_t - \alpha p_i + \alpha p_j$.[58] $\alpha > 1$ means the low type consumers are more demand elastic than medium type. a_t measures the demand shock, which has a mean zero; and we assume it is constant within each two period we study. The low type consumers have complete information about both firms' current and next period prices[59].

Condition 1 and condition 2 in general model are satisfied for any parameter values. Condition 3 is satisfied if following inequality is true.

$$(2(\beta+\phi)-1)L-(\frac{2}{3}\alpha+\frac{1}{3}\phi)q$$

[57] Nowadays many firms have some kinds of reward programs to let consumers sign up. Through these programs, firms can get some information about consumers' purchasing history and may extract some information about their types and preferences.

[58] We can allow firm demand to have different sensitivity to its own price and its competitor price, e.g. introducing $\alpha_{own}\ and\ \alpha_{cross}$ for its own price and competitor price respectively. This adds another parameter without changing the qualitative results. For simplicity, we assume that the two sensitivities are the same.

[59] Since we assume q_t and a_t are constant within these two periods, the asymmetry and demand shock uncertainty will not affect consumers' complete information about next period prices.

Therefore if the fraction of high type and medium type consumers is large enough, and the degree of asymmetry is not too large, condition 3 is satisfied.[60]

First we look at firms' optimal strategy when they do not adopt SLPG. Under this setup, since there is no connection between each two periods, firms will choose an optimal price to maximize their profits in each period. Firm 1's profit is as follow:

$$\pi_1 = \pi_{1,h} + \pi_{1,m} + \pi_{1,l}$$

$$\pi_{1,h} = p_1 * \beta L$$

$$\pi_{1,m} = p_1 \phi (L + q_t - p_1 + p_2)$$

$$\pi_{1,l} = p_1\big((1 - \beta - \phi)L - a_t - \alpha p_1 + \alpha p_2\big)$$

We can get firm 2's profit in similar way. We then take first order conditions, set it equal to zero and solve for optimal prices. There is only one set of solution, and the second order condition is negative, therefore this game has a unique pure strategy Nash equilibrium with firms' prices as follow:

$$p_1 = \frac{1}{3}\frac{\phi q + 3L - 3a}{\phi + \alpha}, \qquad p_2 = \frac{1}{3}\frac{-\phi q + 3L - 3a}{\phi + \alpha}$$

Without assigning parameters values, firms' profits are too lengthy to display.

Next, we study firms' optimal strategies when SLPG is adopted. We denote firm i's profit from type j consumers in period t as $\pi_{i,j,t}$, firm i's price in period t as $p_{i,t}$. Then we can write firm 1 and firm 2's profits as follow:

$$\pi_1 = \pi_{1,h,1} + \pi_{1,m,1} + \pi_{1,l,1} + \pi_{1,h,2} + \pi_{1,m,2} + \pi_{1,l,2}$$

$$\pi_2 = \pi_{2,h,1} + \pi_{2,m,1} + \pi_{2,l,1} + \pi_{2,h,2} + \pi_{2,m,2} + \pi_{2,l,2}$$

[60] When the two firms are symmetric, condition 3 is satisfied if $\beta + \phi > 0.5$.

$\pi_{1,h,1} = p_{1,1}\beta L, \pi_{2,h,1} = p_{2,1}\beta L$

$\pi_{1,m,1} = p_{1,1}\phi\left(L + q - p_{1,1} + p_{2,1}\right), \pi_{2,m,1} = p_{2,1}\phi\left(L - q + p_{1,1} - p_{2,1}\right)$

$\pi_{1,l,1} = \min\left(p_{1,1}, p_{1,2}\right)\left((1 - \beta - \phi)L - a_1 - \alpha \min\left(p_{1,1}, p_{1,2}\right) + \alpha \min\left(p_{2,1}, p_{2,2}\right)\right)$

$\pi_{2,l,1} = \min\left(p_{2,1}, p_{2,2}\right)((1 - \beta - \phi)L - a_1 - \alpha\min\,(p_{2,1}, p_{2,2}) + \alpha\min\,(p_{1,1}, p_{1,2}))$

$\pi_{1,h,2} = p_{1,2}\beta L, \pi_{2,h,2} = p_{2,2}\beta L$

$\pi_{1,m,2} = p_{1,2}\phi\left(L + q - p_{1,2} + p_{2,2}\right), \pi_{2,m,2} = p_{2,2}\phi\left(L - q + p_{1,2} - p_{2,2}\right)$

$\pi_{1,l,2} = p_{1,2}\left((1 - \beta - \phi)L - a_2 - \alpha p_{1,2} + \alpha p_{2,2}\right)$

$\pi_{2,l,2} = p_{2,2}((1 - \beta - \phi)L - a_2 - \alpha p_{2,2} + \alpha p_{1,2})$

Since it is a two stage game, we use backward induction to solve for both firms' second stage prices and then the first stage prices. For each firm there exists a unique set of solutions. Therefore there exists a unique pure strategy sub-game perfect Nash equilibrium. Firm 1 and firm 2's prices ($p_{i,t}$) are as follow:

$$p_{1,1} = \frac{1}{3}\frac{3L\beta + 3L\phi + \phi q}{\phi}, p_{2,1} = \frac{1}{3}\frac{3L\beta + 3L\phi - \phi q}{\phi}$$

$$p_{1,2} = \frac{1}{3}\frac{6L - 3\beta L - 3L\phi - 6a + \phi q}{\phi + 2\alpha}$$

$$p_{2,2} = \frac{1}{3}\frac{6L - 3\beta L - 3L\phi - 6a - \phi q}{\phi + 2\alpha}$$

When q=0, the two firms are symmetric, their optimal prices are symmetric as well.

Without loss of generality, we normalize L=1. Next we compare firms' prices, profits, consumer surplus and social surplus under single pricing strategy with those under intertemporal pricing strategy. Since we have five parameters in the model, without assigning parameters values, they are too complicated to compare. Therefore we assume the two firms are symmetric and there is no demand shock. By these assumption, we can set $q = 0, a = 0$. We denote $p_{d,h}$ as the firm's high price in the intertemporal pricing subtracting firm's price in single pricing strategy; $p_{d,l}$ as the firm's low price in intertemporal pricing subtracting firm's single price; π_d as every two periods firm's profit under intertemporal pricing subtracting firm's profit under single pricing; cs_d as consumer surplus difference and defined in the same way. We have,

$$p_{d,h} = \frac{\phi^2+\phi\beta+\phi\alpha-\phi+\beta\alpha}{(\phi+\alpha)\phi},\ p_{d,l} = -\frac{\phi^2+\phi\beta+\phi\alpha-\phi+\beta\alpha}{(\phi+\alpha)(\phi+2\alpha)}$$

$$\pi_d = 2\frac{(\phi^2+\phi\beta+\phi\alpha-\phi+\beta\alpha)^2}{(\phi+\alpha)(\phi+2\alpha)\phi},\ cs_d = -4\frac{\gamma(\beta+\phi)(\phi^2+\phi\beta+\phi\alpha-\phi+\beta\alpha)}{(\phi+\alpha)(\phi+2\alpha)\phi}$$

Proposition 2: *When $q = 0$, and $a = 0$ firms' profits and social surplus increase with intertemporal pricing strategy under SLPG, and consumer surplus decreases with intertemporal pricing strategy under SLPG. The single price without SLPG is between the high price and low price under SLPG.*

Since $\alpha > 1$, proposition 2 is easy to see.

We can look at a numerical example. Setting $q = 0, a = 0, \beta = \frac{1}{3}, \phi = \frac{1}{3}$, and $\alpha = 2$, the profits, consumer surplus, social surplus and prices are described in Table IV.1. This table shows that profits and social surplus increase under SLPG; consumer surplus decreases under SLPG; and the single price without SLPG is between the high price and low price under SLPG.

IV.2.4 Extend to dynamic game

Next we develop a dynamic model based on the two-period specific model. There are infinite periods in the model. The two firms are symmetric and there is no demand shock. In each period, firms decide their prices simultaneous and independently. Therefore each firm doesn't know its competitor's current price when they decide their price, but they know all prices before current period. Firms will match its own price one period after sale.

In each period, new consumers enter the market and exist in the market for two periods. Consumer type, fraction, and demand are the same as those of the specific two-period model. Consumer will only make purchase in the period they enter the market (first period). In the second period, low type consumer can request price match if price is lowered. Therefore, in each period, there exist two kinds of consumers in the market: old and new. The old consumers who entered the market in the last period will not make a purchase, but the low type consumers may request a price match. The new consumers who enter the market in current period may make a purchase. When they enter the market, consumers have complete information about firm pricing for the two periods they exist in the market. For example, consumers entering the market in period t know both firms' price in that period (period t) and the following period (t+1). Since low type may request price if price is lowered in following period, when they make purchasing decision, they will not only consider firms' prices in period t but also prices in period t+1.

Firms choose optimal pricing strategy to maximize their profits over time. Firms have complete information about consumer fractions and demand parameters. Therefore if their price is lowered in period t+1 comparing to period t, they know how many low type consumers will request price match and pay the actual price of p_{t+1}, and they will take this into account when they choose the optimal pricing strategy.

Firms try to maximize their profits. Since two firms are symmetric, we can write out firm 1's profit and firm 2's profit is similar. We denote firm 1's profit from type j consumers in period t as $\pi_{1,j,t}$, firm 1's price in period t as $p_{1,t}$. Then we can write firm 1's profits in three different conditions as follow:

First, $p_{1,t+1} > p_{1,t} > p_{1,t-1}$, then in period t, firm 1 tries to maximize its profit $\pi_{1,t,1}$ which can be expressed as follow:

$$\pi_{1,t,1} = \pi_{1,h,t} + \pi_{1,m,t} + \pi_{1,l,t}$$

$$\pi_{1,h,t} = p_{1,t}\beta L, \qquad \pi_{1,m,t} = p_{1,1}\phi\left(L - p_{1,t} + p_{2,t}\right),$$

$$\pi_{1,l,t} = p_{1,t}\left((1-\beta-\phi)L - \alpha p_{1,t} + \alpha \min\left(p_{2,t}, p_{2,t+1}\right)\right)$$

Second, $p_{1,t} > p_{1,t-1}$ and $p_{1,t} > p_{1,t+1}$, then firm 1 price in period t has no effect on low type consumers in that period, therefore, in period t firm 1 will set price to maximize its profit from high and medium type consumers. We have $\pi_{1,t,2}$ as follow:

$$\pi_{1,t,2} = \pi_{1,h,t} + \pi_{1,m,t}$$

$$\pi_{1,h,t} = p_{1,t}\beta L, \qquad \pi_{1,m,t} = p_{1,1}\phi\left(L - p_{1,t} + p_{2,t}\right),$$

Third, $p_{1,t} < p_{1,t-1}$ and $p_{1,t} > p_{1,t+1}$, then firm 1 price in period t has no effect on low type consumers in that period but will decide the demand of low type from last period, therefore firm 1 will set price to maximize its profit from low type of last period and high and medium type of this period. We have $\pi_{1,t,3}$ as follow:

$$\pi_{1,t,3} = \pi_{1,h,t} + \pi_{1,m,t} + \pi_{1,l,t-1}$$

$$\pi_{1,h,t} = p_{1,t}\beta L, \qquad \pi_{1,m,t} = p_{1,1}\phi\left(L - p_{1,t} + p_{2,t}\right),$$

$$\pi_{1,l,t-1} = p_{1,t}\left((1-\beta-\phi)L - \alpha p_{1,t} + \alpha \min\left(p_{2,t-1}, p_{2,t}\right)\right)$$

Fourth, $p_{1,t} < p_{1,t-1}$ and $p_{1,t} < p_{1,t+1}$, then firm 1 price in that period will not only decide its profit from current period consumers but also the low type from last period. We have $\pi_{1,t,4}$ as follow:

$$\pi_{1,t,3} = \pi_{1,h,t} + \pi_{1,m,t} + \pi_{1,l,t} + \pi_{1,l,t-1}$$

$$\pi_{1,h,t} = p_{1,t}\beta L, \qquad \pi_{1,m,t} = p_{1,1}\phi\left(L - p_{1,t} + p_{2,t}\right),$$

$$\pi_{1,l,t} = p_{1,t}\left((1-\beta-\phi)L - \alpha p_{1,t} + \alpha \min\left(p_{2,t}, p_{2,t+1}\right)\right)$$

$$\pi_{1,l,t-1} = p_{1,t}\left((1-\beta-\phi)L - \alpha p_{1,t} + \alpha \min\left(p_{2,t-1}, p_{2,t}\right)\right)$$

We then solve for firm optimal strategies.

Proposition 3: *In the dynamic game, one optimal strategy is to charge a high price in one period and a low price in the following period, and repeat this price scheme, in which*

$$p_t = p_h = \frac{L\beta + L\phi}{\phi},\ p_{t+1} = p_l = \frac{2L - \beta L - L\phi}{\phi + 2\alpha}$$

Proof. See Appendix.

Since there is no disturbance, each firm charges the same high price and low price for each two periods. Then each firm's current price change equals to the reverse of its previous price change (coefficient is negative one). If the parameters (q or a) are non constant but highly stable, then each firm's high price and low price will be highly stable for every periods. We would observe that the correlation between each firm's current price change and its previous price change is not negative one but close to negative one. As a consequence, each firm's price changes may show a cyclical price fluctuation over time. Specifically, a firm may charge a high price in one period, and a low price in the following period, then high price, low price and so on. As a consequence, we will observe a more frequent price adjustment over time.[61]

Usually, the two firms in the market are not exactly symmetric ($q \neq 0$). We therefore allow $q \neq 0$ and explore to what degree each firm's price change is affected by its competitor's price change; we use firm 1 for example. Since firm 1's period t price is set to maximize its profits from group 1 consumers, and its period t+1 price is set to maximize its profits from group 2 consumers; we can solve its period t and t+1 optimal price as a function of firm 2's price. We have:

$$p_{1,t} = A + \frac{p_{2,t}}{2}, \text{ where } A = \frac{\beta L + L\phi + \phi q}{2\phi}$$

$$p_{1,t+1} = B + \frac{p_{2,t+1}}{2}, \text{ where } B = \frac{2L - \beta L - L\phi - 2\alpha + \phi q}{2(\phi + 2\alpha)}$$

$$\Delta p_1 = B - A + \frac{p_{2,t+1} - p_{2,t}}{2} = B - A + \frac{1}{2}\Delta p_2$$

From above equation, we can see when there is no other disturbance, the correlation between firm 1 price change and firm 2 price change is half. However, since these two firms move simultaneous and independently, it's not proper to say firm 2 price change causes firm 1 price change or vice verse. When there is other disturbance (for example, demand shock), but the disturbance is relatively small, we would expect

[61] This is consistent with the findings in Chen and Liu (2007)

the coefficient to be close to one half. Overall, both firms' prices will move to the same direction, either increase or decrease.

IV.3. Empirical evidence

To test our theoretical model, we collected weekly online price data from Best Buy and Circuit City. Best Buy and Circuit City were the largest two electronic products retailers in the country during the period of collecting data, and both companies adopt a SLPG policy within 30 days after sale[62]. Further, these two companies competed directly against each other for the period of analysis[63]. In the next section, we will describe our data first and then propose our empirical model and results.

IV.3.1 Data description

We have weekly price observations, rebate values if there are any, and shipping rates recorded every Sunday from Best Buy online and Circuit City online. The observations were collected between March 2nd 2008 and Jan 11th 2009, which is a total of 46 weeks[64]. There are total of 49 different products in our data set, covering the following nine major categories: mouse, MP3 player, printer, router, TV, SD card, data storage device[65], digital camera, and camcorder. For each category, we record two sub-categories of products; the first sub-category consists of low end products or mainly

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[62] Best Buy's policy has changed over time. Currently as stated on their website, for computer, monitors, notebook computers, projectors, camcorders, digital cameras, and radar detectors the SLPG period is 14 days, for all other products except software, movies, music and video games, the period is 30 days.

[63] Shares of *Best Buy* were up almost 9 percent the afternoon of January 16th 2009 after *Circuit City* claimed that it would liquidate all of its 567 U.S. stores. 82 percent of *Circuit City*'s domestic stores are within 5 miles of a *Best Buy*.

[64] We do not have observation on June 22nd 2008 though.

[65] For this category, we choose flash drive and external hard disk drive.

products for home use; while the other sub-category consists of high end products or products for business and professional use. Within each sub-category, we include two or three products. For example, for the mouse category, we use Logitech optical mice as the low end product, recording the prices of LX3, LX5, and LX7; and Logitech laser mice as the high end product including VX Revolution, MX Revolution, and MX Air. For printer category, we use All-In-One inkjet printers as our home use products, while laser black-white printers as business use products. Not all products exist in the market for the whole 46 weeks, and we do not have data for all products on June 22nd 2008, so this is an unbalanced panel data with a gap. For 12 products we collected data for the entire period of 45 weeks, while for two products we have only 6 weeks of data collected over time. The average number of observations per product over time is about 28.33. We only use the data in the regression when we have prices from both companies at the same week, and we have 1339 pairs of prices from Best Buy and Circuit City. Among these 1339 pairs, in 535(39.96%) of them Best Buy charges higher price than Circuit City; in 427(31.89%) of them Circuit City charges higher price; and in the remaining 377(28.16%) pairs they charge the same price. This indicates Best Buy's price is slightly more likely to be higher than Circuit City within our observations. A positive price change means the price increased from last period; a negative price change means the price decreased. Table 2 is a summary of the price change in Best Buy and Circuit City.

From the table 2 we can see that prices are fluctuating consistently over time. Moreover, the price change is not monotonically decreasing over time: about one fifth of the time prices increased and about one fourth of the time prices decreased. The reason we observe more decreasing than increasing prices may be due to depreciation in product value over time caused by either cost reduction or new products entering the market. A comparison across stores indicates that Circuit City's prices have less fluctuation than Best Buy's prices.

IV.3.2 Estimation

The purpose of the empirical analysis is to investigate the negative relationship within each firm's own price change. In the empirical model, we have the following notations:

$P_{b,t}$: Best Buy's price in period (week) t

$P_{c,t}$: Circuit City's price in period (week) t

$\Delta P_{b,t} = P_{b,t} - P_{b,t-1}$

$\Delta P_{c,t} = P_{c,t} - P_{c,t-1}$

Due to firm's SLPG policy, each firm's price is a function of its previous price and its competitor's current price and previous price. Since each company adopt a SLPG 30 days after sale, which is approximately 4 weeks. We have[66]:

$$P_{b,t} = \beta_{0,b} + \beta_{1,b}P_{c,t} + \beta_{2,b}P_{c,t-1} + \beta_{3,b}P_{c,t-2} + \beta_{4,b}P_{b,t-1} + \beta_{5,b}P_{b,t-2} + \beta_{6,b}P_{b,t-3} + \varepsilon_{b,t}$$

$$P_{c,t} = \beta_{0,c} + \beta_{1,c}P_{b,t} + \beta_{2,c}P_{b,t-1} + \beta_{3,c}P_{b,t-2} + \beta_{4,c}P_{c,t-1} + \beta_{5,c}P_{c,t-2} + \beta_{6,c}P_{c,t-3} + \varepsilon_{c,t}$$

Since our interest is the relationship among price changes, we take the difference of period t and period t-1 for each firm's model, and get our two regression models are[67]:

[66] Later, we will show how we chose the optimal number of lag terms.

$$\Delta P_{b,t} = \beta_{0,b}{}' + \beta_{1,b}\Delta P_{c,t} + \beta_{2,b}\Delta P_{c,t-1} + \beta_{3,b}\Delta P_{c,t-2} + \beta_{4,b}\Delta P_{b,t-1} + \beta_{5,b}\Delta P_{b,t-2} + \beta_{6,b}\Delta P_{b,t-3} + \Delta\varepsilon_{b,t}$$

$$\Delta P_{c,t} = \beta_{0,c}{}' + \beta_{1,c}\Delta P_{b,t} + \beta_{2,c}\Delta P_{b,t-1} + \beta_{3,c}\Delta P_{b,t-2} + \beta_{4,c}\Delta P_{c,t-1} + \beta_{5,c}\Delta P_{c,t-2} + \beta_{6,c}\Delta P_{c,t-3} + \Delta\varepsilon_{c,t}$$

We also include a set of time dummy variables to capture the hidden demand shock or other effect that may affect firm's price change. Since the model estimates price change over price change, the price differencing process cancels the product individual effects. This allows us to estimate the model via pooled OLS. Next, we estimate our model with pure price change first, and then as a robustness check we estimate our model with effective price change, in which the price are calculated from pure price minus possible rebates value plus possible shipping rates, and estimate pure price model with a relative price indicator.

Estimation with pure price change

Our model is estimated under three different setups: without time dummies, with time dummies for certain periods[68] [69], and with all time dummies. We discuss the model selection process for model with all dummies in detail here. And for models with no dummy or part dummies, the process is similar and will be discussed briefly.

In order to select the optimal price difference lags, we first estimate the model with different lags and then compute the corresponding Akaike Information Criterion

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[67] We include a constant because the prices of electronic products usually decrease over time.

[68] We include time dummies from Oct 26th 2008 to Jan 11th 2009.

[69] The reason we include time dummies for this period is that on Nov. 3rd, Circuit City closed its 155 brick and mortar stores; and on Jan 16th, Circuit City closed all its stores, including online store.

(AIC) and Bayesian Information Criterion (BIC). Table 3 and table 4 show the AIC and BIC scores of corresponding models.

From table 3 we can see that AIC scores and BIC scores choose different models to estimate Best Buy's price change. Comparing to AIC, the way the BIC score is computed puts more penalties on additional independent variable when the time dimension is large. In that sense, BIC usually favors a more parsimonious model. When the sample size is large, due to the asymptotic property of BIC, it usually outperforms AIC; but when the time dimension is small, BIC may over penalize for the additional independent variable. And in that case, AIC outperforms BIC. Here, for two reasons we choose the model indicated by BIC. First, a general belief is that including fewer independent variables will make the model more efficient. Second, from the IO Economist point of view, if Best Buy wants to response to Circuit City's price change, the effect of a change implemented three weeks later is expected to be small. We suspect that it may be too late to have significant effect on market. Based on these considerations, we choose the fourth model.

Table 4 shows AIC and BIC scores for Circuit City model. Both AIC and BIC procedures point to the selection of the fourth model. Therefore both regression models lead to the selection of three lags of its own price changes and two lags of its competitor's price changes.

For models without time dummy variables and with partial time dummy variables, we have similar results to those of models with full set of time dummy variables[70]. In Best Buy model, AIC and BIC point to the selection of different models, as we

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[70] AIC and BIC scores are reported in parentheses in table 3 and table 4.

discussed before, we choose the model with lowest BIC score. In Circuit City model, both AIC and BIC point to the selection of model with three lags of Circuit City's price changes and two lags of Best Buy's price changes.

Table 5 gives results for pooled OLS estimation for Best Buy's pure price changes, and Table 6 gives results for pooled OLS estimation for Circuit City's pure price changes. From our estimation results presented in table 5 and table 6, we can see there is no qualitative difference among these three[71]. The coefficients on price changes, which we focus on, are qualitatively the same and quantitatively close. Therefore we will focus our discussion on the model with part dummies. From the results we can see that each company's price change is significantly negatively correlated with its own previous three weeks price changes, and positively correlated with the competitor's current and previous price change.

For model estimating Best Buy's price changes, the coefficient for one lagged Best Buy's price change is -0.702 which means on average if Best Buy's price was increased by one dollar last week, then this week Best Buy will lower its price by about seventy cents. It's not exactly negative one as theory predicted, but considering that the one period in theory is actually four weeks in empirical analysis, thus Best Buy may lower its price gradually, and the summation of all lagged coefficients is close to negative one (-1.167)[72], the empirical findings are at some degree consistent with theoretical prediction.[73] The coefficient for current Circuit City's price change is

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[71] Except the constant and some time dummies, which are not our primary interests.

[72] At 1% significance level we fail to reject the null that it equals to -1.

[73] Calculated impulse response from four weeks ago shock is 0.013, this mean four weeks ago price change doesn't have very large effect on current price change. One explanation is that firms are

0.430[74], which means the price change correlation between Best Buy and Circuit is 0.430[75]. This is also at some degree consistent with our theory prediction. Best Buy's price change is more affected by its own previous changes than Circuit City's price changes. For higher lags, the scale of the coefficients decrease, which means last week's price change has a larger effect on current price change than that of the week before last week.

Circuit City's price change follows the same pattern as Best Buy's. The difference is the coefficients are smaller. This indicates that Circuit City's current price is affected less by its previous price changes and Best Buy's price changes. This is, to some degree, consistent with our observation in the data description that Circuit City's price is less volatile. The time dummy coefficients are not significant for most dates except three dates for Best Buy and one date for Circuit City listed in the table. The three significant dummy coefficients[76] for Best Buy indicates Best Buy's price was increased in these three weeks, which is not caused by its own previous price change nor Circuit City's price change. However all these three price increases are consistent with Circuit City's problems in these periods. On November 3rd 2008, Circuit City announced it plans to close 155 stores and lay off 17 percent of its workforce in the U.S., the dummy

more likely to response to a price change in the first three weeks, rather than wait until the last week of price matching period (30 days).

[74] At 1% significance level, we fail to reject the null that it equals to 0.5.

[75] In the OLS estimation, since the coefficients of competitor current price change are significant in both model (Best Buy and Circuit City), it's not clear whether Circuit City price change causes Best Buy price change or vice verse. However, as later we will see, the coefficient of competitor current price change is significant in Circuit City model but not in Best Buy model. In that case, it means Circuit City may decide its price according to Best Buy price, but Best Buy doesn't decide its price based on Circuit City price. One possible explanation is Best Buy has a larger market share, therefore it has larger market power and plays as a leader in the market.

[76] In model with part dummies, two more dummies for Nov 16th and 23rd are negative significant at 5% level. This may because Best Buy lowered its price for the coming Thanksgiving and Black Friday.

coefficient for November 2nd indicates Best Buy's price was increased in that week. Exactly one week later, Circuit City filed for bankruptcy protection. The dummy coefficient for one day before that indicates Best Buy's price increased again in that week. On January 16th 2009, Circuit City said it failed to find a buyer and will liquidate its 567 U.S. stores. Our result indicates that two weeks before that Best Buy increased its price. The only significant dummy for Circuit City's price change is for November 16th 2008, which is the following Sunday after it filed the bankruptcy protection.

Although the Hausman specification test suggests using pooled OLS estimation, we still present the instrumented variable estimation results here.[77] For Best Buy model, $\Delta P_{c,t-3}$ is used as an instrumented variable for the potentially endogenous variable $\Delta P_{c,t}$; and for Circuit City model, $\Delta P_{b,t-3}$ is used as an instrumented variable for the potentially endogenous variable $\Delta P_{b,t}$. Let's look at the Best Buy model first: In all three models, Best Buy's price change is still negatively correlated with its previous price changes, but at a higher degree compared to pooled OLS estimation. However, in IV estimation, Circuit City's price changes have no effect on Best Buy's price changes. Since Best Buy is the largest firm in the electronic products retail market, this means Best Buy plays as a leader in the market. It decides its price not based on its competitor price. One noticeable point here is that the R^2 is much smaller than that of the pooled OLS. This may suggest that IV estimation doesn't fit the framework very well. IV estimation for the Circuit City model pretty much yields the same results as pooled OLS with two exceptions. First, the coefficients on price changes have a larger scale in IV estimation. Second, constant and time dummies are not significant in all three models in

[77] Table 7 and Table 8.

IV estimation. Circuit City price change is still significantly correlated with Best Buy current price change. This indicates Circuit City may play as a follower in that market.

Estimation with effective price change and with indicator

For robustness check we also calculate the effective price which is the retail price minus rebates[78] plus shipping rates. We then calculate the price changes and follow AIC and BIC scores to select the proper estimation model. Table 9 and Table 10 show that both AIC and BIC lead to the selection of the estimation models with the same lags as that of the pure price change estimation. Since we still estimate a price difference model, the product individual effects are cancelled out when differencing the prices; we can estimate the model again via pooled OLS.

We first estimate the model under three different setups via pooled OLS. The estimation results are reported in table 11 and table 12. Comparing these results with the model with pure prices, the relationship between price changes are qualitatively the same in both the Best Buy and Circuit City models. This confirms our claim that each firm's price change is negatively related to its previous price changes and positively related to its competitor's price changes. Quantitatively, the effect of previous price changes on current price change is smaller in models with the effective price compared to the models with pure price. The major difference is in the dummy coefficients in the Best Buy model including all dummies. For November 2nd 2008, the dummy coefficient is not significant in the effective price estimation anymore; one possible explanation is that in that week, Best Buy increased the prices but at the mean time it offered discount shipping rates. Considering that one day later Circuit City closed its 155 stores, this

[78] When there is no rebate, this is zero.

eases the competition at brick and mortar places but not online. Therefore Best Buy had an incentive to increase the regular price face by consumers at the brick and mortal store; and at the same time it lowered its shipping rates to keep its delivered price the same for online shoppers. For November 9th 08, the dummy coefficient is still significant and positive, but with a larger coefficient in the effective price estimation. For November 16th and 23rd 2008, the dummy coefficients are significant and negative in effective price estimation, but not significant in the pure price estimation. Since the effective price is the price including shipping rates and rebates, this may be explained by either a decreasing in shipping rate or an increasing in rebate amount. An examination at the shipping rates and rebates shows that the decrease in shipping rates can explain the significant negative dummy coefficients for these dates. The average shipping rate drops from $12.13 on November 9th to $2.80 on 16th and $1.96 on 23rd. One possible explanation is that since Circuit City has filed for bankruptcy protection, Best Buy may expect liquidation by Circuit City and thus an intensified online competition. In response it lowered its shipping rates and thus the delivery price for online shoppers. For January 4th 09, the dummy coefficient is negative and significant at 5% level in pure price model but insignificant in effective price model. By examining the rebates and shipping rates, there is no evidence to support that the difference is caused by these two factors. This difference is still an open question, but does not affect our main results. The dummy coefficient in Circuit City model is qualitatively consistent between the pure price model and effective price model. Looking at Circuit City's shipping policy, we find that the shipping rates for Circuit City were very consistent over time. It offered free shipping for orders over $24.99, and there is almost no change in shipping rates for orders under $24.99.

IV estimation results are reported in Table 13 and Table 14. In Table 13, all three Best Buy models show that Circuit City's price changes have no significant effect on Best Buy's price changes. Moreover, the model with all dummies indicates more dummy coefficients are significant compared to other estimation results. In Table 14, Circuit City model doesn't present any qualitative difference on the price change coefficients. Nevertheless, each company's price change is still significantly correlated with its previous price changes.

Next, based on the pure price model, we add a relative price indicator as independent variable. The relative price indicator is generated as follow:

$$i = \begin{cases} 1, if\ p_{b,t-1} > p_{c,t-1} \\ -1, if\ p_{b,t-1} < p_{c,t-1}, \\ 0, otherwise \end{cases}$$

where $p_{b,t-1}$is Best Buy last week price, $p_{c,t-1}$ is Circuit City last week price.

We estimate the model with no time dummy variable, and results are reported in Table 15. As we can see, there is no qualitative difference between the model with indicator and without indicator.

Overall, the empirical evidence provides some investigation into our theory, but we have to point out the limitation here that the empirical analysis is not very rigorous. Therefore the empirical analysis cannot be used as a strong support to our theory.

IV.4. Conclusion

The traditional intertemporal pricing literature usually predicts a monotonic price decrease. In the real world, however, we usually observe price fluctuations. Existing literature on low price guarantee (LPG) typically focuses on competition among stores and not over time. In this paper, we develop a theoretical model using SLPG to explain the price fluctuation over time. In a theoretical model, we show that in equilibrium firms use SLPGs and intertemporal pricing strategies. This will cause price fluctuations over time. By examine a specific model, we find that when firms are symmetric and the low type consumers' demand respond to both firms prices at the same degree, firms' profits and social surplus increase with SLPG, but consumer surplus decreases with SLPG. Moreover, if we assume the parameters are constant over time, a firm's one period price change is negatively correlated with its previous period price change (-1), and positively correlated with its competitor's current price change (1/2). With weekly data collected from Best Buy and Circuit City, we find that there is a negative correlation between each firm price change and its previous periods price changes. An interesting empirical extension is to do a comparison between industry with SLPG and industry without SLPG. Future work could be done in this direction. If the work can show the industry without SLPG doesn't embrace this negative correlation between price changes, then it will provide a stronger empirical support to the theory.

IV.5. Appendix

Proof of Proposition 1.

We first denote $p_h$as the optimal high price in the intertemporal pricing strategy, p_l as the optimal low price in the intertemporal pricing strategy, and p_s as the optimal price in the single pricing strategy. Therefore we have

$$(1)\ \frac{\partial \pi_{i,1}}{\partial p_i}\bigg|_{p_\mathrm{i}=p_{i,h}} = 0, \qquad (2)\ \frac{\partial \pi_{i,2}}{\partial p_i}\bigg|_{p_i=p_{i,l}} = 0, \qquad (3)\ \frac{\partial \pi_{i}}{\partial p_i}\bigg|_{p_i=p_{i,s}} = 0$$

Since $\pi_{i,1} = p_i(D_{i,h} + D_{i,m})$, we can rewrite equation (1) as

$$D_{i,h} + D_{i,m} + \frac{\partial D_{i,m}}{\partial p_i} p_{i,h} = 0 \ \Rightarrow\ p_{i,h} = \frac{D_{i,h} + D_{i,m}}{\left|\frac{\partial D_{i,m}}{\partial p_i}\right|}$$

Similar from equation (2) and equation (3) we have

$$p_{i,l} = \frac{D_{i,h} + D_{i,m} + 2D_{i,l}}{\left|\frac{\partial D_{i,m}}{\partial p_i}\right| + 2\left|\frac{\partial D_{i,l}}{\partial p_i}\right|}$$

$$\mathrm{p}_{i,s} = \frac{D_{i,h} + D_{i,m} + D_{i,l}}{\left|\frac{\partial D_{i,m}}{\partial p_i}\right| + \left|\frac{\partial D_{i,l}}{\partial p_i}\right|}$$

From condition 3 we have

$$p_{i,h} = \frac{D_{i,h} + D_{i,m}}{\left|\frac{\partial D_{i,m}}{\partial p_i}\right|} > \frac{D_{i,h} + D_{i,m} + D_{i,l}}{2\left|\frac{\partial D_{i,m}}{\partial p_i}\right|} > \frac{D_{i,h} + D_{i,m} + D_{i,l}}{\left|\frac{\partial D_{i,m}}{\partial p_i}\right| + \left|\frac{\partial D_{i,l}}{\partial p_i}\right|} = p_{i,s}$$

$$\left(D_{i,h} + D_{i,m}\right)\left|\frac{\partial D_{i,l}}{\partial p_i}\right| > D_{i,l}\left|\frac{\partial D_{i,m}}{\partial p_i}\right|$$

$$\Rightarrow \left(D_{i,h} + D_{i,m} + D_{i,l}\right)\left|\frac{\partial D_{i,l}}{\partial p_i}\right| > D_{i,l}\left(\left|\frac{\partial D_{i,m}}{\partial p_i}\right| + \left|\frac{\partial D_{i,l}}{\partial p_i}\right|\right)$$

$$\Rightarrow \left(D_{i,h}+D_{i,m}+D_{i,l}\right)\left(\left|\frac{\partial D_{i,m}}{\partial p_i}\right|+2\left|\frac{\partial D_{i,l}}{\partial p_i}\right|\right)$$

$$> \left(D_{i,h}+D_{i,m}+2D_{i,l}\right)\left(\left|\frac{\partial D_{i,m}}{\partial p_i}\right|+\left|\frac{\partial D_{i,l}}{\partial p_i}\right|\right)$$

$$\Rightarrow p_{i,s}=\frac{D_{i,h}+D_{i,m}+D_{i,l}}{\left|\frac{\partial D_{i,m}}{\partial p_i}\right|+\left|\frac{\partial D_{i,l}}{\partial p_i}\right|}>p_{i,l}=\frac{D_{i,h}+D_{i,m}+2D_{i,l}}{\left|\frac{\partial D_{i,m}}{\partial p_i}\right|+2\left|\frac{\partial D_{i,l}}{\partial p_i}\right|}$$

So far, we have shown that $p_{i,h}>p_{i,s}$ and $p_{i,s}>p_{i,l}$, and from condition 2 we have $\frac{\partial \pi_{i,1}}{\partial p_i}\Big|_{p_i=p_{i,s}}>0$; therefore when firms adopt SLPG, it can deviate to increase its profit by increase its first period price a infinite small amount and keep its second period price the same. By doing this, firm can increase its first period profit and keep its second period profit the same. Therefore, when firms can adopt SLPG, it will always deviate from the single pricing strategy to intertemporal pricing strategy.

<u>Proof of Proposition 3.</u>

We prove proposition 3 is an equilibrium by checking deviation. If no firm will deviate from the equilibrium pattern, then the high-low pricing schedule is an equilibrium. Since the two firms are symmetric, we only need to show one firm has no incentive to deviate. Without loss of generality, we look at firm 1.

For any period in the dynamic game, firm 1 sets its price $p_{1,T}$ to maximize its profit as:

$$\pi_{1,T}=\pi_{1,h,T}(p_T)+\pi_{1,m,T}(p_T)+\pi_{1,l,T}(p_T|p_T<p_{T+1})+\pi_{1,l,T-1}(p_T|p_T<p_{T-1})$$

As a preliminary to the proof, we define two equilibrium prices under specific conditions that will be met later in the proof.

Suppose both firms follow the high-low pricing schedule, and assume that in period T firm 1 charges high price. Therefore we have $p_T > p_{T-1}$, and $p_T > p_{T+1}$.

When $p_T > p_{T-1}$, period T-1 low type consumers cannot request price match in period T, therefore their profits are determined by p_{T-1}, and $\pi_{1,l,T-1}(p_T|p_T < p_{T-1})$ is zero in $\pi_{1,T}$. When $p_T > p_{T+1}$, period T low type consumers will request price match in period T+1, therefore their profits are determined by p_{T+1}, and $\pi_{1,l,T}(p_T|p_T < p_{T+1})$ is zero in $\pi_{1,T}$.

Overall, firm 1 profit in period T can be simplified as:

$$\pi_{1,T} = \pi_{1,T,1} = \pi_{1,h,T} + \pi_{i,m,T},$$

Firm 1 corresponding optimal price in period T is:

$$p_{1,T} = p_h, \text{ such that } \left.\frac{\partial \pi_{1,T,1}}{\partial p_1}\right|_{p_1=p_h} = 0.$$

Alternatively, suppose it charges a low price, and thus $p_T < p_{T-1}$, and $p_T < p_{T+1}$. Therefore $\pi_{1,l,T-1}(p_T|p_T < p_{T-1})$ and $\pi_{1,l,T}(p_T|p_T < p_{T+1})$ are not zero in $\pi_{1,T}$. Overall firm 1 profit in period T can be simplified as:

$$\pi_{1,T} = \pi_{1,T,2} = \pi_{T,h,T} + \pi_{1,m,T} + \pi_{1,l,T} + \pi_{1,l,T-1},$$

and firm 1 corresponding optimal price in period T is:

$$p_{1,T} = p_l, \text{ such that } \left.\frac{\partial \pi_{1,T,2}}{\partial p_1}\right|_{p_1=p_l} = 0.$$

When firm 1 chooses p_l to maximize $\pi_{1,2}$, it has to consider two groups of low type consumers; whereas when it chooses p_h to maximize $\pi_{1,1}$, it only considers the high and medium type consumers. Since low type consumer demand is more elastic, we have $p_l < p_h$.

To prove Proposition 3, we examine any two periods for firm 1, namely period t and period t+1. Before period t we assume firm 1 chooses SLPG and the high-low pricing schedule, and firm 1 price in period t-1, $p_{1,t-1} = p_l$. After period t+1 firm 1 chooses SLPG and the high-low pricing schedule. Firm 1's price in period t+2, $p_{1,t+2} = p_h$.

Next, we provide a road map for the remaining proof. We first show that these two periods can be separated from their previous and following periods, and thus can be analyzed separately. We then use similar analysis in the two-period specific game to find the optimal strategies for firms.

In order to show these two periods can be separated from the previous and following periods, we need to show first that $p_{1,t} \geq p_{1,t-1}$, and thus firm 1 period t-1 profit is unrelated to its period t price and second that $p_{1,t+1} \leq p_{1,t+2}$ and thus firm 1 period t+1 profit is unrelated to its period t+2 price.

We first show that $p_{1,t} \geq p_{1,t-1}$. In period t, firm 1 can choose $p_{1,t} \geq p_{1,t-1}$ or $p_{1,t} < p_{1,t-1}$. If $p_{1,t} < p_{1,t-1}$, then the low type consumers who made purchase in period t-1 will request price match in period t. We have two situations depending on the relationship between $p_{1,t}$ and $p_{1,t+1}$.

First $p_{1,t} \leq p_{1,t+1}$, then in period t firm chooses the optimal price to maximize the total profits from its current period customers and last period low type customers. We have,

$$\pi_{1,t} = \pi_{1,h,t} + \pi_{1,m,t} + \pi_{1,l,t} + \pi_{1,l,t-1}.$$

As we have shown the optimal low price is p_l, therefore firm 1 will not choose a price lower than p_l, which is $p_{1,t} \geq p_{1,t-1} = p_l$.

Second $p_{1,t} > p_{1,t+1}$, then in period t firm 1 chooses the optimal price to maximize the total profits from its current period high and medium customers and last period low type customers. We have

$$\pi_{1,t}' = \pi_{1,h,t} + \pi_{1,m,t} + \pi_{1,l,t-1}$$

Firm 1 corresponding optimal price is $p_{1,t}'$. Its last period price is p_l, which is chosen to maximize $\pi_{1,2,t-1}$. Since $\pi_{1,2,t-1}$ contains more low type consumers than $\pi_{1,t}'$, the corresponding optimal price p_l should also be lower than $p_{1,t}'$ ($p_{1,t} > p_{1,t-1} = p_l$), and we have a contradiction here.

Combining these two cases, we have $p_{1,t} \geq p_{1,t-1}$. Therefore, low type consumers in period t-1 will not request price match. Firm 1 period t price has no effect on its previous period consumers.

We then show that $p_{1,t+1} \leq p_{1,t+2}$. Let's look at period t+1. In period t+1, firm 1 can choose $p_{1,t+1} \leq p_{1,t+2}$ or $p_{1,t+1} > p_{1,t+2}$. If firm 1 chooses $p_{1,t+1} > p_{1,t+2}$, the low type consumers will request price match in the next period. We have two situations depending on the relationship between $p_{1,t+1}$ and $p_{1,t}$.

First, if $p_{1,t+1} > p_{1,t}$, then $p_{1,t+1}$ has no effect on low type consumers in period t, and since $p_{1,t+1} > p_{1,t+2}$, $p_{1,t+1}$doesn't determine the profit from low type consumers in period t+1 as well. Therefore, firm 1 chooses $p_{1,t+1}$to maximize its period t+1 profit:

$$\pi_{1,t+1} = \pi_{1,h,t+1} + \pi_{i,m,t+1} = \pi_{1,1}$$

The optimal price is p_h, which is equal to $p_{1,t+2}$. Therefore in this situation, $p_{1,t+1} = p_{1,t+2}$.

Second, if $p_{1,t+1} < p_{1,t}$, then $p_{1,t+1}$ has effect on the low type consumers in period t, and since $p_{1,t+1} > p_{1,t+2}$, $p_{1,t+1}$doesn't determine the profit from low type consumers in period t+1. Therefore, firm 1 chooses $p_{1,t+1}$to maximize its period t+1 profit:

$$\pi_{1,t+1}{}' = \pi_{1,h,t} + \pi_{1,m,t} + \pi_{1,l,t-1}.$$

The corresponding optimal price is $p_{1,t+1}{}'$. The period t+2 price is p_h, which is chosen to maximize $\pi_{1,1}$. Since $\pi_{1,1}$ contains no low type consumer compared to $\pi_{1,t+1}{}'$, the corresponding optimal price p_h is higher than $p'_{1,t+1}$. Therefore, we have $p_{1,t+2} > p_{1,t+1}$, which creats a contradiction.

Combining these two cases, we have $p_{1,t+1} \leq p_{1,t+2}$. Therefore, low type consumer in period t+1 will not request price a match in period t+2. Firm 1 period t+1 profit is unrelated to period t+2 price.

Overall, the price in period t has no effect on the price in period t-1, and the price in period t+2 price has no effect on the price in period t+1. Therefore, these two periods, t and t+1, can be looked at separately and consider the game in which these prices appear in a recurrent pattern.

Next, we examine two periods of the dynamic game seperately.

In these two periods, firm 1 can either set $p_{1,t} > p_{1,t+1}$ or $p_{1,t} \leq p_{1,t+1}$. If firm 1 set $p_{1,t} \leq p_{1,t+1}$, then period t+1 price has no effect on period t, therefore in each period, firm 1 chooses a price to maximize its profits from the customer of that period. Which is

$$\pi_1 = \pi_{1,h} + \pi_{1,m} + \pi_{1,l}.$$

This is equivalent to when there is not SLPG and firm adopts a single optimal price.

If firm 1 set $p_{1,t} > p_{1,t+1}$, then period t+1 price has an effect on period t. Firm 1 chooses a price schedule to maximize its total profits from these two periods. Using the same functional form as in the two-period specific game in section IV.2.2.3., we have

$$\pi_1 = \pi_{1,t} + \pi_{1,t+1},$$

$$\pi_{1,t} = \pi_{1,1} = \pi_{1,h,t} + \pi_{i,m,t}, \qquad \pi_{1,t+1} = \pi_{1,h,t+1} + \pi_{1,m,t+1} + \pi_{1,l,t+1} + \pi_{1,l,t},$$

$$\pi_{1,h,t} = p_{1,t}\beta L,\ \pi_{1,m,t} = p_{1,t}\phi\left(L + q - p_{1,t} + p_{2,t}\right),$$

$$\pi_{1,l,t} = p_{1,t+1}\left((1 - \beta - \phi)L - a_1 - \alpha p_{1,t+1} + \alpha \min\left(p_{2,t}, p_{2,t+1}\right)\right),$$

$$\pi_{1,h,t+1} = p_{1,t+1}\beta L, \pi_{1,m,t+1} = p_{1,t+1}\phi\left(L + q - p_{1,t+1} + p_{2,t+1}\right),$$

$$\pi_{1,l,t+1} = p_{1,t+1}\left((1 - \beta - \phi)L - a_2 - \alpha p_{1,2} + \alpha p_{2,t+1}\right).$$

This is the same as in the specific two-period model. Firm 1 sets $p_{1,t}$ to maximize $\pi_{1,1} = \pi_{1,h,t} + \pi_{1,m,t}$ and $p_{1,t+1}$ to maximize $\pi_{1,2} = \pi_{1,l,t} + \pi_{1,h,t+1} + \pi_{1,m,t+1} + \pi_{1,l,t+1}$.

We calculate the first order conditions as follows:

$$\frac{\partial \pi_{1,1}}{\partial p_{1,t}} = \beta L + \phi\left(L - p_{1,t} + p_{2,t}\right) - \phi p_{1,t},$$

$$\frac{\partial \pi_{1,2}}{\partial p_{1,t+1}} = 2(1-\beta-\phi)L - 4\alpha p_{1,t+1} + 2\alpha p_{2,t+1} + \beta L - \phi p_{1,t+1} + \phi\big(L - p_{1,t+1} + p_{2,t+1}\big).$$

Using the first order conditions to derive an explicit expression of the optimal strategies, we get:

$$p_{1,t} = p_h = \frac{L\beta + L\phi}{\phi}, p_{1,t+1} = p_l = \frac{2L - \beta L - L\phi}{\phi + 2\alpha}.$$

The second order condition is negative definite.

As we have already shown in Proposition 2, the profit from single pricing strategy is lower than the profit from intertemporal pricing strategy. Firm will deviate from the single pricing strategy, therefore the equilibrium will be the intertemporal pricing strategy.

Table IV.1. Comparison of single pricing and intertemporal pricing[a]

	Single pricing strategy	Intertemporal pricing strategy
π	0.4286	1.7436
cs	2.6667v-1.0317	2.6667v-2.4444
ss	1.3333v+0.6825	2.6667v+1.0427
p	0.4286	2
		0.3077

[a]All numerical values are approximate to four decimal places.

Table IV.2. Price statistics summary

	Best Buy		Circuit City	
Price Change	Frequency	Maximum	Frequency	Maximum
Decrease	314	$500	295	$400
The same	660		721	
Increase	277	$400	235	$800
Total	1,251		1,251	
Price Range	Minimum	Maximum	Minimum	Maximum
	$9.99[a]	$4199.99[b]	$6.99[c]	$4199.99[d]

[a] SanDisk 1G SD card
[b,d] Panasonic TV (model: 58PZ700U)
[c] SanDisk 1G flash drive

Table IV.3. AIC and BIC for Best Buy pure price model

	AIC	BIC
$\Delta P_{b,t} = \Delta P_{c,t} + \Delta P_{c,t-1} + \Delta P_{b,t-1}$	12068.38[a] (12066.17)[b] (12052.06)[c]	12285.94 (12086.41) (12133.01)
$\Delta P_{b,t} = \Delta P_{c,t} + \Delta P_{c,t-1} + \Delta P_{b,t-1} + \Delta P_{b,t-2}$	11088.65 (11086.74) (11071.16)	11297.89 (11111.65) (11156.15)
$\Delta P_{b,t} = \Delta P_{c,t} + \Delta P_{c,t-1} + \Delta P_{c,t-2} + \Delta P_{b,t-1} + \Delta P_{b,t-2}$	11078.69 (11078.25) (11061.96)	11292.91 (11107.14) (11151.63)
$\Delta P_{b,t} = \Delta P_{c,t} + \Delta P_{c,t-1} + \Delta P_{c,t-2} + \Delta P_{b,t-1} + \Delta P_{b,t-2} + \Delta P_{b,t-3}$*	10038.71 (10030.92) (10017.17)	10244.5 (10065.21) (10110.27)
$\Delta P_{b,t} = \Delta P_{c,t} + \Delta P_{c,t-1} + \Delta P_{c,t-2} + \Delta P_{c,t-3} + \Delta P_{b,t-1} + \Delta P_{b,t-2} + \Delta P_{b,t-3}$	10037.06 (10029.15) (10015.61)	10247.75 (10068.34) (10113.6)

[a] Models with all time dummy variables.
[b] Models without time dummy variable.
[c] Models with part time dummy variables.
*Selected model.

Table IV.4. AIC and BIC for Circuit City pure price model

	AIC	BIC
$\Delta P_{c,t} = \Delta P_{b,t} + \Delta P_{c,t-1} + \Delta P_{b,t-1}$	12236.23[a] (12190.54)[b] (12209.53)[c]	12453.8 (12210.78) (12290.49)
$\Delta P_{c,t} = \Delta P_{b,t} + \Delta P_{b,t-1} + \Delta P_{c,t-1} + \Delta P_{c,t-2}$	11254.5 (11213.45) (11231.13)	11463.74 (11238.36) (11315.82)
$\Delta P_{c,t} = \Delta P_{b,t} + \Delta P_{c,t-1} + \Delta P_{c,t-2} + \Delta P_{b,t-1} + \Delta P_{b,t-2}$	11256.44 (11215.14) (11232.92)	11470.66 (11245.03) (11322.59)
$\Delta P_{c,t} = \Delta P_{b,t} + \Delta P_{c,t-1} + \Delta P_{c,t-2} + \Delta P_{b,t-1} + \Delta P_{b,t-2} + \Delta P_{c,t-3}$*	9982.173 (9940.935) (9955.981)	10187.96 (9975.233) (10049.08)
$\Delta P_{c,t} = \Delta P_{b,t} + \Delta P_{c,t-1} + \Delta P_{c,t-2} + \Delta P_{c,t-3} + \Delta P_{b,t-1} + \Delta P_{b,t-2} + \Delta P_{b,t-3}$	9983.473 (9942.551) (9957.303)	10194.16 (9981.749) (10055.3)

[a] Models with all time dummy variables.
[b] Models without time dummy variable.
[c] Models with part time dummy variables.
*Selected model.

Table IV.5. Pure Price Pooled OLS Estimation Results (Best Buy)

Variable	$\Delta P_{b,t}$ [a]	$\Delta P_{b,t}$ [b]	$\Delta P_{b,t}$ [c]
Constant	-3.744* (1.226)[d]	-4.009* (1.331)	-9.595 (10.033)
$\Delta P_{b,t-1}$	-0.693* (0.029)	-0.702* (0.029)	-0.701* (0.029)
$\Delta P_{b,t-2}$	-0.353* (0.034)	-0.349* (0.034)	-0.340* (0.035)
$\Delta P_{b,t-3}$	-0.124* (0.028)	-0.116* (0.028)	-0.112* (0.028)
$\Delta P_{c,t}$	0.436* (0.030)	0.430* (0.030)	0.427* (0.030)
$\Delta P_{c,t-1}$	0.308* (0.030)	0.307* (0.030)	0.307* (0.030)
$\Delta P_{c,t-2}$	0.038 (0.028)	0.040 (0.028)	0.040 (0.028)
Time dummies[e]			
11/02/08		19.045*** (9.767)	24.867*** (13.939)
11/09/08		34.000* (9.774)	39.516* (13.965)
11/16/08		-22.476** (9.835)	
11/23/08		-24.681** (9.811)	
1/04/08		24.077** (10.090)	29.737** (14.173)
R^2	0.489	0.508	0.520
Wald Test			
$\Delta P_{b,t-1} + \Delta P_{b,t-2} + \Delta P_{b,t-3} = -1$	P>|t|=0.020	P>|t|=0.024	
$\Delta P_{c,t} = 0.5$	P>F=0.035	P>F=0.019	

[a]Model with no time dummy variable.
[b]Model with part time dummy variables.
[c]Model with all time dummy variables.
[d]Standard errors are in parentheses.
[e]Only significant time dummies are reported.
*1% significance level; **5% significance level; ***10% significance level.

Table IV.6. Pure Price Pooled OLS Estimation Results (Circuit City)

Variable	$\Delta P_{c,t}$	$\Delta P_{c,t}$	$\Delta P_{c,t}$
Constant	-2.316** (1.164)	-2.202** (1.285)	-2.084 (9.736)
$\Delta P_{b,t}$	0.407* (0.027)	0.413* (0.028)	0.411* (0.028)
$\Delta P_{b,t-1}$	0.203* (0.032)	0.204* (0.033)	0.199* (0.034)
$\Delta P_{b,t-2}$	0.053*** (0.028)	0.051*** (0.027)	0.041 (0.029)
$\Delta P_{c,t-1}$	-0.562* (0.026)	-0.560* (0.027)	-0.559* (0.027)
$\Delta P_{c,t-2}$	-0.192* (0.030)	-0.192* (0.031)	-0.189* (0.031)
$\Delta P_{c,t-3}$	-0.078* (0.026)	-0.075* (0.026)	-0.071* (0.027)
Time dummies			
11/16/08		23.413** (9.498)	23.513*** (13.583)
R^2	0.412	0.418	0.429
Wald Test			
$\Delta P_{c,t-1} + \Delta P_{c,t-2} + \Delta P_{c,t-3} = -1$	P>|t|=0.01	P>|t|=0.009	
$\Delta P_{b,t} = 0.5$	P>F=0.001	P>F=0.002	

Table IV.7. Pure Price IV Estimation Results (Best Buy)

Variable	$\Delta P_{b,t}$	$\Delta P_{b,t}$	$\Delta P_{b,t}$
Constant	-7.453** (3.003)	-7.843** (3.277)	-12.970 (17.139)
$\Delta P_{b,t-1}$	-0.783* (0.072)	-0.803* (0.082)	-0.817* (0.097)
$\Delta P_{b,t-2}$	-0.469* (0.091)	-0.476* (0.102)	-0.486* (0.120)
$\Delta P_{b,t-3}$	-0.203* (0.065)	-0.198* (0.070)	-0.201** (0.078)
$\Delta P_{c,t}$	-0.377 (0.557)	-0.403 (0.599)	-0.499 (0.699)
$\Delta P_{c,t-1}$	-0.101 (0.282)	-0.110 (0.302)	-0.157 (0.352)
$\Delta P_{c,t-2}$	-0.092 (0.096)	-0.093 (0.102)	-0.107 (0.118)
Time dummies			
11/09/08		37.691* (13.363)	42.910*** (21.856)
11/23/08		-35.266** (15.182)	
1/04/08		34.281** (15.375)	40.247** (20.241)
R^2	0.111	0.116	0.047
First Stage	$\Delta P_{c,t}$	$\Delta P_{c,t}$	$\Delta P_{c,t}$
$\Delta \mathrm{P}_{\mathrm{c,t-3}}$	-0.066** (0.029)	-0.061** (0.029)	-0.056** (0.030)
F-statistics	65.56	23.69	10.33
Wald Test			
$\Delta P_{b,t-1} + \Delta P_{b,t-2} + \Delta P_{b,t-3} = -1$	P>|t|=0.036	P>|t|=0.05	
$\Delta P_{c,t} = 0.5$	P>F=0.115	P>F=0.131	

[a] Other variables are exogenous presented in 2nd stage.

Table IV.8. Pure Price IV Estimation Results (Circuit City)

Variable	$\Delta P_{c,t}$	$\Delta P_{c,t}$	$\Delta P_{c,t}$
Constant	-1.858 (1.389)[a]	-1.512 (1.557)	3.336 (11.822)
$\Delta P_{b,\mathrm{t}}$	0.502* (0.158)	0.548* (0.171)	0.554* (0.179)
$\Delta P_{b,t-1}$	0.270** (0.113)	0.301** (0.124)	0.301** (0.130)
$\Delta P_{b,t-2}$	0.082 (0.055)	0.093 (0.059)	0.084 (0.061)
$\Delta P_{c,t-1}$	-0.571* (0.030)	-0.573* (0.032)	-0.574* (0.033)
$\Delta P_{c,t-2}$	-0.188* (0.031)	-0.187* (0.031)	-0.185* (0.032)
$\Delta P_{c,t-3}$	-0.078* (0.026)	-0.075* (0.027)	-0.071* (0.027)
Time dummies[a]			
R^2	0.405	0.403	0.414
First Stage	$\Delta P_{b,\mathrm{t}}$	$\Delta P_{b,\mathrm{t}}$	$\Delta P_{b,\mathrm{t}}$
$\Delta \mathrm{P}_{\mathrm{b,t-3}}$	-0.170* (0.031)	-0.162* (0.031)	-0.157* (0.031)
F-statistics	100.79	38.80	16.77
Wald Test			
$\Delta P_{\mathrm{c},t-1} + \Delta P_{c,t-2} + \Delta P_{c,t-3} = -1$	P>|t|=0.014	P>|t|=0.015	
$\Delta P_{b,t} = 0.5$	P>F=0.9881	P>F=0.777	

[a] No dummy variable is significant in IV estimation.
[b] Other variables are exogenous presented in 2nd stage.

Table IV.9. AIC and BIC for Best Buy effective price model

	AIC	BIC
$\Delta P_{b,t} = \Delta P_{c,t} + \Delta P_{c,t-1} + \Delta P_{b,t-1}$	12238.13[a] (12243.32)[b] (12218.55)[c]	12455.69 (12263.56) (12299.51)
$\Delta P_{b,t} = \Delta P_{c,t} + \Delta P_{c,t-1} + \Delta P_{b,t-1} + \Delta P_{b,t-2}$	11261.46 (11265.86) (11241.25)	11470.7 (11290.77) (11325.94)
$\Delta P_{b,t} = \Delta P_{c,t} + \Delta P_{c,t-1} + \Delta P_{c,t-2} + \Delta P_{b,t-1} + \Delta P_{b,t-2}$	11256.76 (11262.44) (11236.95)	11470.98 (11292.33) (11326.63)
$\Delta P_{b,t} = \Delta P_{c,t} + \Delta P_{c,t-1} + \Delta P_{c,t-2} + \Delta P_{b,t-1} + \Delta P_{b,t-2} + \Delta P_{b,t-3}$*	10212.22 (10211.62) (10188.85)	10418.01 (10245.92) (10281.95)
$\Delta P_{b,t} = \Delta P_{c,t} + \Delta P_{c,t-1} + \Delta P_{c,t-2} + \Delta P_{c,t-3} + \Delta P_{b,t-1} + \Delta P_{b,t-2} + \Delta P_{b,t-3}$	10213.58 (10212.88) (10190.25)	10424.27 (10252.07) (10288.24)

[a] Models with all time dummy variables.
[b] Models without time dummy variable.
[c] Models with part time dummy variables.
*Selected model.

Table IV.10. AIC and BIC for Circuit City effective price model

	AIC	BIC
$\Delta P_{c,t} = \Delta P_{b,t} + \Delta P_{c,t-1} + \Delta P_{b,t-1}$	12236.07[a] (12191.18)[b] (12209.66)[c]	12453.64 (12211.41) (12290.61)
$\Delta P_{c,t} = \Delta P_{b,t} + \Delta P_{b,t-1} + \Delta P_{c,t-1} + \Delta P_{c,t-2}$	11260.98 (11221.05) (11238.01)	11470.23 (11245.96) (11322.7)
$\Delta P_{c,t} = \Delta P_{b,t} + \Delta P_{c,t-1} + \Delta P_{c,t-2} + \Delta P_{b,t-1} + \Delta P_{b,t-2}$	11262.59 (11222.21) (11239.4)	11476.81 (11252.1) (11329.07)
$\Delta P_{c,t} = \Delta P_{b,t} + \Delta P_{c,t-1} + \Delta P_{c,t-2} + \Delta P_{b,t-1} + \Delta P_{b,t-2} + \Delta P_{c,t-3}$*	10003.31 (9963.795) (9977.418)	10209.1 (9998.093) (10070.51)
$\Delta P_{c,t} = \Delta P_{b,t} + \Delta P_{c,t-1} + \Delta P_{c,t-2} + \Delta P_{c,t-3} + \Delta P_{b,t-1} + \Delta P_{b,t-2} + \Delta P_{b,t-3}$	10004.37 (9965.043) (9978.257)	10215.06 (10004.24) (10076.25)

[a] Models with all time dummy variables.
[b] Models without time dummy variable.
[c] Models with part time dummy variables.
*Selected model.

Table IV.11. Effective price Pooled OLS Estimation Results (Best Buy)

Variable	$\Delta P_{b,t}$	$\Delta P_{b,t}$	$\Delta P_{b,t}$
Constant	-3.319** (1.338)	-3.943* (1.450)	0.262 (10.951)
$\Delta P_{b,t-1}$	-0.652* (0.029)	-0.664* (0.029)	-0.662* (0.029)
$\Delta P_{b,t-2}$	-0.365* (0.033)	-0.362* (0.034)	-0.356* (0.034)
$\Delta P_{b,t-3}$	-0.139* (0.028)	-0.131* (0.028)	-0.128* (0.029)
$\Delta P_{c,t}$	0.453* (0.033)	0.446* (0.033)	0.444* (0.033)
$\Delta P_{c,t-1}$	0.286* (0.033)	0.287* (0.033)	0.287* (0.033)
$\Delta P_{c,t-2}$	0.007 (0.031)	0.010 (0.030)	0.012 (0.031)
Time dummies			
11/02/08		27.642** (10.645)	
11/09/08		39.194* (10.672)	39.931** (15.258)
11/16/08		-28.601* (10.746)	-33.006** (15.332)
11/23/08		-27.966* (10.725)	-32.272** (15.218)
1/04/08		28.143** (11.015)	
R^2	0.456	0.481	0.493
Wald Test			
$\Delta P_{b,t-1} + \Delta P_{b,t-2} + \Delta P_{b,t-3} = -1$	P>|t|=0.031	P>|t|=0.038	
$\Delta P_{c,t} = 0.5$	P>F=0.1585	P>F=0.0.0987	

Table IV.12. Effective price Pooled OLS Estimation Results (Circuit City)

Variable	$\Delta P_{c,t}$	$\Delta P_{c,t}$	$\Delta P_{c,t}$
Constant	-2.684** (1.175)	-2.501** (1.298)	-1.530 (9.851)
$\Delta P_{b,t}$	0.359* (0.025)	0.367* (0.026)	0.365* (0.026)
$\Delta P_{b,t-1}$	0.164* (0.029)	0.165* (0.030)	0.162* (0.031)
$\Delta P_{b,t-2}$	0.053** (0.026)	0.050*** (0.027)	0.043 (0.027)
$\Delta P_{c,t-1}$	-0.544* (0.026)	-0.542* (0.027)	-0.542* (0.027)
$\Delta P_{c,t-2}$	-0.179* (0.031)	-0.178* (0.031)	-0.178* (0.031)
$\Delta P_{c,t-3}$	-0.070* (0.026)	-0.067** (0.027)	-0.063** (0.027)
Time dummies			
11/16/08		25.050* (9.624)	24.270*** (13.744)
R^2	0.400	0.406	0.418
Wald Test			
$\Delta P_{c,t-1} + \Delta P_{c,t-2} + \Delta P_{c,t-3} = -1$	P>|t|=0.002	P>|t|=0.001	
$\Delta P_{b,t} = 0.5$	P>F=0.0000	P>F=0.0000	

Table IV.13. Effective price IV Estimation Results (Best Buy)

Variable	$\Delta P_{b,t}$	$\Delta P_{b,t}$	$\Delta P_{b,t}$
Constant	-5.039** (2.560)	-5.607** (2.758)	-26.843** (13.973)
$\Delta P_{b,t-1}$	-0.688* (0.055)	-0.703* (0.061)	-0.706* (0.067)
$\Delta P_{b,t-2}$	-0.411* (0.067)	-0.409* (0.074)	-0.409* (0.081)
$\Delta P_{b,t-3}$	-0.176* (0.054)	-0.166* (0.058)	-0.166* (0.060)
$\Delta P_{c,t}$	0.075 (0.470)	-0.091 (0.491)	0.049 (0.542)
$\Delta P_{c,t-1}$	0.095 (0.238)	0.109 (0.247)	0.089 (0.273)
$\Delta P_{c,t-2}$	-0.055 (0.083)	-0.047 (0.086)	-0.052 (0.094)
Time dummies			
4/13/08			26.322*** (15.186)
4/20/08			32.624** (14.238)
4/27/08			25.971*** (15.413)
6/01/08			26.941*** (15.339)
6/15/08			27.245*** (15.537)
10/05/08			30.233*** (16.938)
11/02/08		29.934** (11.711)	51.398* (17.623)
11/09/08		41.031* (11.585)	62.290 (17.794)
11/23/08		-32.469** (12.945)	
12/27/08			38.354** (17.991)
1/04/08		33.015** (13.481)	54.660* (16.596)
R^2	0.383	0.418	0.416
First stage	$\Delta P_{c,t}$	$\Delta P_{c,t}$	$\Delta P_{c,t}$
$\Delta \mathrm{P}_{c,t-3}$	-0.069** (0.029)	-0.064** (0.029)	-0.060** (0.030)
F-statistics	65.32	23.63	10.27
Wald Test			
$\Delta P_{b,t-1} + \Delta P_{b,t-2} + \Delta P_{b,t-3} = -1$	P>|t|=0.098	P>|t|=0.132	
$\Delta P_{c,t} = 0.5$	P>F=0.3654	P>F=0.0.4045	

[a] Other variables are exogenous presented in 2nd stage.

Table IV.14. Effective price IV Estimation Results (Circuit City)

Variable	$\Delta P_{c,t}$	$\Delta P_{c,t}$	$\Delta P_{c,t}$
Constant	-2.161 (1.336)	-1.719 (1.521)	-17.251 (11.153)
$\Delta P_{b,t}$	0.475* (0.138)	0.521* (0.149)	0.508* (0.155)
$\Delta P_{b,t-1}$	0.238** (0.092)	0.267* (0.102)	0.256** (0.106)
$\Delta P_{b,t-2}$	0.089*** (0.049)	0.097*** (0.053)	0.086 (0.054)
$\Delta P_{c,t-1}$	-0.550* (0.028)	-0.551* (0.028)	-0.550* (0.029)
$\Delta P_{c,t-2}$	-0.168* (0.033)	-0.165* (0.034)	-0.164* (0.034)
$\Delta P_{c,t-3}$	-0.066** (0.027)	-0.061** (0.028)	-0.058* (0.034)
Time dummies[a]			
10/12/08			26.740*** (14.321)
10/19/08			35.794** (15.339)
11/16/08		29.145* (10.547)	44.516** (17.200)
R^2	0.387	0.385	0.400
First Stage	$\Delta P_{b,t}$	$\Delta P_{b,t}$	$\Delta P_{b,t}$
$\Delta P_{b,t-3}$	-0.183* (0.031)	-0.173* (0.031)	-0.170* (0.032)
F-statistics	89.12	35.27	15.20
Wald Test			
$\Delta P_{c,t-1} + \Delta P_{c,t-2} + \Delta P_{c,t-3} = -1$	P>|t|=0.001	P>|t|=0.001	
$\Delta P_{b,t} = 0.5$	P>F=0.8570	P>F=0.8903	

[a] No dummy variable is significant in IV estimation.
[b] Other variables are exogenous presented in 2nd stage.

Table. 15. Model with relative price indicator

Variable	$\Delta P_{b,t}$	$\Delta P_{c,t}$
Constant	-3.245* (1.226)	-2.716** (1.158)
$\Delta P_{b,t}$		0.415* (0.027)
$\Delta P_{b,t-1}$	-0.666* (0.029)	0.181* (0.032)
$\Delta P_{b,t-2}$	-0.337* (0.034)	0.043 (0.028)
$\Delta P_{b,t-3}$	-0.117* (0.028)	
$\Delta P_{c,t}$	0.448* (0.030)	
$\Delta P_{c,t-1}$	0.290* (0.031)	-0.531* (0.027)
$\Delta P_{c,t-2}$	0.028 (0.028)	-0.173* (0.030)
$\Delta P_{c,t-3}$		-0.068* (0.026)
Indicator	-5.326* (1.496)	6.146** (1.424)
R^2	0.495	0.423

CHAPTER V

CONCLUSION

Price discrimination has been and is being extensively studied. The development of technology, especially internet, has changed firms and consumers behavior significantly. With the development of technology, consumers can get abundant of information at a much lower cost.[79] This change has effectively changed consumers' behavior and firms' competitive behavior. The study answers two changes brought by technology improvement at some degree. The first is coupon trading, which is hard to observe decades ago and ignored by IO literature, but now is becoming more and more popular and cannot be ignored anymore. The second is firms post-sale LPG. Existing literature usually focus on LPG among stores. But with the technology improvement, it is more and more easy for consumers to track price after purchase happens. Post-sale LPG thus is practically important in markets.

The first essay, "Customer Poaching, Coupon Trading and Consumer Arbitrage", incorporates consumer arbitrage by introducing coupon trading among consumers into coupon competition model. Specifically the essay assumes that a fraction of consumers are coupon traders who can trade coupons at a very low cost (normalized to zero). On the other side of the market, firms compete in regular prices, promotion depths, and promotion frequencies. The results show that when the fraction of coupon traders increases, firms respond by promoting less frequently and reducing the promotion depth. When the cost of distributing coupons increases, firms promote less frequently but with

[79] Some papers point out that current problem is information overload rather than information shortage. See Anderson and de Palma (2009) for example.

higher coupon face value. In both cases, competitions are released, and firms charge higher regular prices and get higher profits. Consumers are worse off since prices increase. The results are robust to several extensions including the introduction of coupon non-users and non-tradable coupons. Regarding to the extension of asymmetric firms, results are more complicated, but it has the same spirit of the symmetric benchmark model in the sense of that more coupon traders and higher coupon distribution cost reduce competition.

The second essay, “Post-Sale Low Price Guarantees and Price Fluctuation”, examines firms’ SLPG policy and its effect on consumers purchasing behavior and firms’ competitive behavior. In the theoretical model, part of the consumers are assumed to have very low cost (normalized to zero) to request price match after sale. Knowing this, firms can adopt SLPG and intertemporal pricing policy to discriminate the consumers with high cost of requesting SLPG. In the empirical part, week price data from Best Buy and Circuit City are used to estimate each firm’s price changes. The empirical results indicate that each firm’s price change is significantly and negatively correlated with its own previous price changes. This coincides with the theoretical model’s prediction, and thus supports the theoretical model at some degree.

V.1. Limitations

The limitations of the first essay arise from complexity of the model. Since the model has four endogenous variables, it is impossible to examine the model under a more general setting. The essay discusses several extensions based on the benchmark model. This, at some degree, may overcome the limitation.

The limitations of the second essay mainly come from the data. The sample size is not very large in terms of time dimension and products dimension. This may cause some skepticism about empirical results. The other shortage regarding to the data is that there are only two firms. This may not be strong enough to claim that SLPG combining intertemporal pricing policy is a common strategy adopted by firms. Another issue is that since the data contains only firms with SLPG, I cannot do comparison between firms with SLPG and firms without SLPG. If I have data from firms with and without SLPG, and can observe the periodical price fluctuation in the firms with SLPG but not in the firms without SLPG, then I could be more confident about the theoretical results.

V.2. Implications for Future Research

In the strand of coupon trading literature, future studies could investigate consumers' arbitrage behavior. Especially what factors may affect consumers' willingness to trade coupon. Another extension could be instead of assuming the fraction of coupon traders exogenously, it is endogenously determined. These studies could provide useful information on firms' promotion strategies.

In the strand of SLPG and price fluctuation literature, future studies could start from a richer data set, especially data with prices from firms with SLPG and firms without SLPG. Another extension could be examining the difference between SLPG and LPG among firms.

V.3. Conclusion

Overall, this study answers two questions brought by development of technology. The first essay shows that consumer arbitrage behavior has significant effect on firms' promotion behavior, but it doesn't change the fact that competition through couponing is a prisoner's dilemma.

The second essay shows that firms may use SLPG and intertemporal pricing policy to price discriminate part of the market. Low type consumers benefit from this strategy, but high type consumers hurts by this strategy. In the specific model, when firms are symmetric and the low type consumers' demand respond to both firms prices at the same degree, firms' profits and social surplus increase with SLPG, but consumer surplus decreases with SLPG.

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