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CALCULUS INSTRUCTORS' RESPONSES TO PRIOR KNOWLEDGE ERRORS

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DEPARTMENT OF MATHEMATICS

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DEDICATION

This body of work is dedicated to my loving parents,

Gracie Lee Talley and Wilmon Odell Talley

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TABLE OF CONTENTS

LIST OF TABLES

ABSTRACT

This study investigates the responses to prior knowledge errors that Calculus I instructors make when assessing students. Prior knowledge is operationalized as any skill or understanding that a student needs to successfully navigate through a Calculus I course. A two part qualitative study consisting of student exams and instructor interviews was employed to examine how instructors approach prior knowledge mistakes when they are evaluating students. Analysis of these interviews revealed that calculus instructors agree that algebra and trigonometry are essential components of prior knowledge within a calculus course. Additionally, perceived inconsistencies in instructor grading were reconciled using a sensible system framework.

Chapter 1

Introduction

The Problem

My six years of experience as a graduate teaching assistant (GTA) includes the instruction of several freshman level pre-calculus courses at the University of Oklahoma. The topics covered in these courses usually include (but are not limited to) a review of algebra, characteristics of basic functions, systems of equations, and introductory trigonometry. Though I was responsible for lesson planning and assessment of assignments, the four multiple choice exams that comprised a large percentage of the course grade were written by a coordinator and administered to over 1000 students as a standardized test. Often the students who did poorly on these exams found that many of their errors were due to mistakes made in algebra, not the topics they were taught in my course. Though unfortunate, this issue did not become of interest to me until I began teaching calculus discussion and lecture courses. It was at that time that I realized my calculus students were also making algebra mistakes similar to those my pre-calculus students made.

Intrigued by this situation, I informally polled other GTAs and colleagues only to find that they have encountered comparable situations with their own calculus students. Knowing that algebra mistakes could be detrimental to the pre-calculus students taking multiple choice exams, I began to wonder just how the same mistakes affect calculus students who are generally assessed using partial credit techniques. Additionally, I became concerned with all mistakes that calculus instructors may consider to be attributed to prerequisite courses or experiences. For the purposes of this inquiry, such mistakes will be referred to as prior knowledge errors. More specifically, the term prior knowledge is operationalized as any skill, ability, or understanding that calculus instructors feel students should have before entering a calculus course. With this definition in mind I wanted to know the influence that prior knowledge had over calculus students' grades. However, I was not clear on the grading patterns of calculus instructors. Nor was I familiar with the conceptions they had concerning prior knowledge within the context of a calculus course. Thus, I embarked on the current study.

Purpose of Research

The ease with which a student learns a mathematical concept often depends on that student's knowledge base. For instance, in basic mathematics, to understand multiplication one must have an understanding of addition. In trigonometry, one must understand the geometric properties of triangles to apply the Law of Cosines. In calculus, to apply the rate of change formula, it is necessary to be able to manipulate algebraic expressions. The examples of the need for prior knowledge in mathematical understanding are endless. Nevertheless, students often find themselves in classes for which their understandings of prerequisite skills are insufficient. Students may on one hand understand the concepts of change that give life to calculus, but on the other hand lack the prior knowledge skills needed to apply said concepts to the very problems in physics and engineering that calculus was created to solve.

Understandably, this dilemma presents a unique set of considerations that calculus instructors must take into account on a regular basis. Besides the obvious difficulties of teaching students that lack prerequisite skills, instructors must assess both calculus and prior knowledge abilities as well. Currently, little research has been conducted that speaks to the way that instructors of calculus courses respond to the prior knowledge errors made by their students. The goal of this study is to better understand the views that calculus instructors have about student prior knowledge errors. Additionally, the aim is to uncover how those views influence the assessment techniques that calculus instructors apply to class assignments and exams. To achieve these goals the following questions were addressed:

1. How do calculus instructors define prior knowledge?
2. How do calculus instructors view prior knowledge in the context of a calculus course?
3. How do calculus instructors assess prior knowledge errors?

These research questions are exploratory in nature. More specifically, they seek a description of the phenomenon of instructors dealing with prior knowledge errors in a calculus course. Therefore, a qualitative study is in order. Denzin and Lincoln (1994) give the following definition of qualitative research:

> Qualitative research is multi-method in its focus, involving an interpretive, naturalistic approach to its subject matter. This means that qualitative researchers study things in their natural setting, attempting to make sense of, or interpret, phenomena in terms of the meaning people bring to them. (p. 2)

As the aim of this study is to understand what prior knowledge means to calculus instructors and how those meanings influence assessment strategies, a qualitative research approach is most fitting. Further explanation of the research design is included in the last two sections of this chapter.

The outcomes of this study may be useful in various ways. The analysis of instructor thinking alone will inevitably encourage teacher self-awareness of opinions and how these opinions influence their grading policies. This reflection may evoke necessary adjustments to assessment strategies in future teaching experiences. Also, once department administrators better understand the point of view of calculus instructors they may decide to align the educational objectives of their programs accordingly. On the other hand, administrators may feel the need to provide professional development to instructors whose assessment practices do not support the department's goals for student performance. In terms of student benefits, understanding the opinions that instructors have may help calculus students realize the value in prior knowledge skills and adjust their study habits accordingly. Further research in this area may include inquiries into teacher responses to specific prior knowledge areas within various mathematics courses as well as investigations of prior knowledge assessment strategies within other academic disciplines such as chemistry or engineering.

Subjectivity Statement

To reveal my perspective on this topic I must first describe my background that relates to the field of Research in Undergraduate Mathematics Education (RUME) as well as the relationship I have with this particular topic. My two undergraduate degrees are bachelors' in mathematics and secondary mathematics education. I entered the doctoral program in mathematics at the University of Oklahoma immediately after attaining these degrees in 2003. Therefore, my dealings with junior high and high school teaching are limited to a semester's worth of student teaching. I have, however, been teaching freshman and sophomore level mathematics course as a GTA since I entered graduate school. In particular, most of the classes I've taught were faculty coordinated pre-calculus and calculus courses. It was in teaching these classes and interacting with students that I developed an interest in prior knowledge errors.

Within a calculus course my definition of prior knowledge includes several topics. First, I feel a student's prior knowledge should be comprised of any concept covered in the basic algebra, geometry, and trigonometry classes. Additionally, I expect students to be well versed in graphing, evaluating and defining functions; applying the rate of change formula; and using mathematical notation appropriately. I realize that my own conception of what prior knowledge entails within a calculus course could be quite similar or vary widely in comparison to other instructors. Therefore, it is imperative that I seek out the definitions of prior knowledge that my instructor participants use. Hence research question one: 'How do calculus instructors define prior knowledge?' Considering each instructor's own definition of prior knowledge can, to some extent, prevent my own opinions from coloring the data collection and analysis process.

My own approach to assessing prior knowledge errors has been consistent throughout my teaching career. In a calculus course I generally deduct points for any error that I consider to be prior knowledge. However, I tend to be lenient with such errors in comparison to those I identify as being caused by misunderstandings of calculus. I would expect other instructors to do the same, however some may disregard prior knowledge altogether when grading. On the other hand, some might go as far as to allot zero points to students who make prior knowledge mistakes. The basis for research question three, 'How do calculus instructors grade prior knowledge errors?' is to find out where instructors' grading patterns fall along this spectrum.

One limitation that could hinder the success of this project is my limited research experience. Nevertheless, to prepare myself to work in the field of RUME I have completed coursework in both the mathematics and educational psychology departments that outlined the fundamentals of educational research. Particularly useful to conducting the current study was a qualitative research course. While taking this course I was able to conduct a small qualitative study that provided me with a great deal of insight into how to collect data, prepare interviews, and analyze participant responses among other things. Additionally, the class included discussions about epistemologies, theoretical frameworks, and social research methodologies. I found that of the three epistemologies (objectivism, constructionism, and subjectivism) outlined in Crotty (1998), constructionism appeals to me the most. Crotty asserts that constructionism:

> Is the view that all knowledge, and therefore all meaningful reality as such, is contingent upon human practices, being constructed in and out of

> interaction between human beings and their world, and developed and transmitted within an essentially social context. (p. 42)

He goes on to explain that "According to constructionism, we do not create meaning, we construct meaning" (p. 44) which supports my own opinions about how people understand the world around them. Notably, when it comes to mathematics or any physical science I take an objectivist stance; asserting that truth and reality are discovered. However, to deal with social issues and to understand people I believe that the epistemology of constructionism is most appropriate. In particular, to understand how instructors define prior knowledge and use said definitions to guide their assessment techniques, a useful interpretation of the instructor's viewpoint must be obtained. Belief that such an interpretation exists is aligned with a constructionist epistemology in that it takes into account that there are no *true* interpretations of reality; rather there are only interpretations that are useful to the people or groups being studied. With this epistemological stance, I went on to design the following study based on the theoretical perspective of phenomenology.

Theoretical Perspective/Methodology

One philosophy associated with the epistemology of constructionism is Husserl's Phenomenology. As described by Mohanty (1954),

> Husserl's treatment of objectivity starts basically from the recognition of *intentionality* as the principal feature of conscious life. This doctrine of intentionality had been handed down through Brentano and Meinong. The distinguishing feature of conscious life had been recognized as lying in the

> fact that all consciousness is of some object...The straightforward doctrine of intentionality is transformed by Husserl into the more complex doctrine... (p. 3)

Though Husserl's philosophy is quite extensive, the core tenets of the theoretical perspective of phenomenology are based mainly in this idea of intentionality. To distinguish between the two, the philosophy will be referred to as Husserl's Phenomenology and the theoretical perspective as Phenomenology. Intentionality hinges on the claim that our consciousness is always directed towards some object of experience or phenomenon. Crotty (1998) explains that "intentionality means referentiality, relatedness, directedness, 'aboutness'." Positing that all acts and/or experiences have some underlying object that they are 'about', Phenomenology is concerned with describing the essence of an experience or phenomenon. Thus, the corresponding phenomenological research methodology fundamentally "describes the meaning for several individuals of their *lived experiences* of a concept or a phenomenon" (Creswell, 2007). According to Creswell (2007):

> Phenomenologists focus on describing what all participants have in common as they experience a phenomenon. The basic purpose of phenomenology is to reduce individual experiences with a phenomenon to a description of the universal essence. (p 58)

This study is concerned with the phenomenon of grading prior knowledge errors within a calculus course and how calculus instructors conceptualize and tend to this issue. Fittingly, Husserl's Phenomenology, Phenomenology (the theoretical perspective), and the phenomenological research methodology all work together to describe this phenomenon and the participants who encounter it.

Chapter Two

Review of Literature

This study of calculus instructors and prior knowledge errors is exploratory in nature because similar investigations have not been found in previous research. Of course, the teaching and learning of calculus has been the focus of a multitude of studies. Similarly, prior knowledge has anything but been neglected by researchers in all levels of education. However, attention to calculus instructors' grading patterns in response to prior knowledge errors has been overlooked. That is not to say that this study has not been informed by the efforts of researchers with adjacent and complimentary interests. Studies of assessment, particularly in higher education, support the necessity of understanding all aspects of instructor grading techniques. In addition, this discussion begins with prior knowledge and its importance in education.

Prior Knowledge

As described in the previous chapter, I have operationalized the term *prior knowledge* to refer to any skill, ability, or understanding that a calculus instructor feels a student should possess prior to enrolling in a first calculus course. As history would have it, prior knowledge has been defined in a variety of ways; including some similar to the definition I've provided. The terms used to refer to prior knowledge include "prestorage, permanent stored knowledge, prestored knowledge, knowledge store, prior knowledge state, prior knowledge state in the knowledge base, implicit knowledge, or archival memory, not to mention exper[i]ential knowledge, background knowledge, world knowledge, pre-existing knowledge, or personal knowledge" (Dochy & Anderson, 1995, p. 227). Joseph and Dwyer(1984) made use of the term 'entering behavior' and later Cox (2001) employed the term 'probable preparedness' to refer to prior knowledge. Obviously, such a large selection of terms may cause some confusion among researchers and educators about what constitutes prior knowledge. Dochy and Anderson (1995) point out that, unfortunately, many researchers overlook the importance of carefully laying out a definition for prior knowledge or comparable term utilized in their research reports.

Dochy, Segers, & Pletinckx (2002), on the other hand, provided the following definition:

> We define prior knowledge as the whole of a person's actual knowledge that is available before a certain task; that is structured in schemata; that is declarative and procedural; that is partly explicit and partly tacit; which contains content knowledge and metacognitive knowledge; which is dynamic in nature and part of the prior knowledge base, being the total collection of prior knowledge. (p. 33)

In this definition the terms declarative and procedural refer to knowing facts or concepts and knowing actions or processes, respectively. In other words, declarative knowledge is characterized by 'knowing that' and procedural knowledge by 'knowing how' (Anderson, 1995). In contrast, Pritchard, Lee, and Bao (2008) simply define prior knowledge as "what he [the student] does or does not know when he encounters each new thing to be learned" (p. 1). This is an example of what Dochy and Alexander (1995) called a nominal definition which is "largely cast in terms such as…what [the student] knows already beforehand" (p. 226). In contrast to the nominal definition of prior knowledge, Dochy and Alexander refer to definitions of prior knowledge that describe the specific characteristics that make up a student's prior knowledge (e.g. understands rate of change, able to factor binomials) as a real definition. A nominal definition is what is used in the current study to solicit conceptions of the real definition of prior knowledge from each calculus instructor participant.

The types of mistakes that calculus students make range widely. From basic mathematics to proofs to graphing, the difficulties that calculus students encounter have been well documented (Durand – Guerrier & Arsac, 2005; Orton, 1983a; Orton 1983b; Ubuz, 2007). Therefore, the areas to focus concern are plentiful. After deciding to study prior knowledge, I found it useful to investigate just how important prior knowledge has been to teaching and learning. I found that prior knowledge greatly influences the learning process. More so, prior knowledge has been found to be a key factor in student success and learning styles across all ages and subjects (Carmichael & Hayes, 2001; Degroff, 1987; Henry, 1990; Hmelo, Nagarajan & Day, 2000; Lambiotte & Dansereau, 1992). For instance, Joseph and Dwyer's 1984 study of tenth grade students enrolled in a mandatory health class determined the effectiveness of visual learning aids on students with varying levels of 'entering behavior'. These researchers found that the level of prior knowledge was a determining factor in how well students respond to various types of visual concept representations in biology.

Researchers within science education have found that prior knowledge is often the cause of student difficulty (Brumby, 1984; Gunstone & White, 1981; Selman et al., 1982). Tariq (2008) found that basic mathematical calculations were problematic for an overwhelming percentage of university students. She studied how these basic mathematical calculations influenced first-year bioscience undergraduates (Tariq, 2008). The errors the bioscience majors exhibited were attributed to fractions, exponents, long division, negative integers, and more. Clearly, the results of Tariq's study demonstrate how prior knowledge can hinder the success of students in science, mathematics, engineering, and technology fields. Also within a university setting, Hailikari, Nevgi, and Komulainen (2008) investigated the interplay of prior knowledge, academic self-beliefs, and previous study success on predicting mathematical achievement in a university obligatory mathematics course. Their study revealed that "prior knowledge predicted student achievement over and above other variables" (p. 67). Moreover, domain-specific prior knowledge was the determining factor in predicting final course grade. Dochy (1992) defines domain-specific knowledge as the knowledge a person has about a particular subject area. In addition to the two studies just discussed, previous research has observed that domain-specific prior knowledge sets the stage for learning in various fields of study (Alexander, Kulikowich & Schulze, 1994; De Bock, Van Dooren, Janssens & Verschaffel, 2002).

Particularly in calculus, the errors that students make have been attributed to algebraic misunderstandings in previous research (Edge & Friedberg, 1984; Orton, 1983a; Scofield, 2003; White & Mitchelmore, 1996). One cause of difficulty found by White and Mitchelmore (1996) is students' tendencies to misinterpret the use of variables in calculus problems. They refer to students that manipulate symbols without an understanding of what they are doing as having an 'abstract-apart' concept of variables whereas students who generalize, symbolize, and abstract variables as having an 'abstract-general' concept. They concluded that "a prerequisite to a successful study of calculus is an abstract-general concept of a variable..." (p. 93). Orton's study (1983a) confirms that problems with algebra (in addition to ratio and proportion) hinder calculus students when dealing with differentiation. "It certainly appeared that algebraic difficulties could be obscuring the ideas of calculus" (p. 245). *The Journal of Experimental Education* published an article that reported the investigation of three groups of Calculus I students at Illinois State University (Edge & Freidberg, 1984). The goal of this study was to determine the factors of success in a first calculus course. Edge and Friedberg used regression models to find that for all three groups calculus success could be predicted by algebraic skills.

The findings outlined here align with the observations that motivated the current study. Namely, I've encountered numerous students that have difficulties algebraically simplifying after appropriately applying calculus concepts. Research documenting similar situations in calculus, as well as other disciplines, and in varying degrees supports the continued study of this issue. Furthermore, the lack of specific focus on instructors' grading patterns in relation to prior knowledge errors requires that the current study take place.

Assessment Issues

The concern with instructor grading practices partly developed out of existing research on assessment and student reactions to assessment. Wanting to understand what messages instructors send to students via grading patterns and in turn how students react to said patterns led to research of what Snyder (1970) referred to as 'the hidden curriculum'. The hidden curriculum is characterized by the unstated expectations instructors have of students' performance. Snyder's study of Massachusetts Institute of Technology (MIT) students revealed the disparities between the formal curriculum and the hidden curriculum. The formal curriculum, being that which is set by standards of deep intellectually inquiry and creative endeavors, is often the ideal backdrop of university life. The passage below is an example of the disconnect between the hidden curriculum and the formal curriculum. It describes a quiz given to a class at MIT. Note that at the beginning of the term the instructor gave the students the impression that being creative and engaged was the most important aspect of the course.

> The students found that they were asked to return a large amount of information that they could only have mastered by memorization. There was a considerable discrepancy between the students' expectations for the course and what they were in fact expected to learn in order to pass the quiz. In spite of the professor's opening pronouncement, the hidden but required task was not to be imaginative or creative but to play a specific, tightly circumscribed academic game." (Snyder, 1970, p. 17)

How often do students take an exam or quiz only to realize that they completely misjudged how to prepare? What do these situations relay to students about what is important in their classes? And what actions do students take in terms of exam preparation once they have, or think they have, determined the expectations of instructors? Each of these questions demonstrates just how much impact assessment can have on students. In light of this observation, the main topic in literature to be addressed in this section is how assessment methods influence student learning habits.

Though evaluation accounts for only a small portion of class time for most courses, it should be no surprise that assessment is an influential factor in student study habits. Research supports that students structure their study time in accordance with their perceptions of the course assessment (Crooks, 1988; Gibbs & Simpson, 2004; Snyder, 1970; Struyven, Dochy, & Janssens, 2005). Several aspects of assessment influence the way students prepare for classroom evaluations. Included in this list are time and energy constraints, type of assessments, and perception of important concepts (Crooks, 1988; Gibbs & Simpson, 2004; Miller & Parlett, 1974; Snyder, 1970). In consideration of time and energy constraints, even the most dedicated students sometimes cut corners to get by in a class. As stated by Gibbs and Simpson (2004), "[Students] are strategic in their use of time and 'selectively negligent' in avoiding content that they believe is not likely to be assessed" (p. 6). The student accounts in *The Hidden Curriculum* repeatedly demonstrate the affects that limited time and energy have on the study patterns of university students (Snyder, 1970). One student even expressed the desire to "go deeper" into a particular course concept but was unable to afford the time it would take to do so. After a stressful first year of attempting to adhere to the formal curriculum, another student referred to college life as an "exercise in time budgeting" (p. 62).

In addition to time and energy constraints, assessment type relays a message to students about the attention that should be given to courses. Struyven, Dochy, and Janssens (2005) argue that traditional assessment measures (like multiple choice or essay questions) are viewed by students as less relevant to their learning than alternative measures (like portfolios). In preparing for traditional assessments students tend to use study methods that promote only shallow understandings of course material whereas alternative testing measures encourage more rigorous study. Gibbs (1999) also explained that preparation for learning tasks is dependent upon the type of assessment being presented. Brown (1999), like Struyven et al., purports that higher-level thinking is not promoted through traditional assessments. He states that "if we reward, through our marking, information recall and repetition of what has been taught (as much traditional assessment does), then this is what students think we want and perform accordingly" (p.7).

Finally the perceptions that students have about which course concepts are and are not important directly influence the techniques used to study. Again, a student account from Snyder (1970) beautifully exemplifies this notion:

> …You had to filter out what was really important in each course, regardless of whether you were worried about the grade or not. You

couldn't physically do it all. I found out that if you did a good job of filtering out what was important you could do well in every course. (p. 62)

This student makes a conscious effort to disregard portions of the course material based on his perception of what is important. In a study of science majors, Miller and Parlett (1974) were able to classify students into three groups: cue-seekers, cue-conscious, and cue-deaf. These classifications described the students level of belief that there is a technique to taking exams and that this technique could be mastered by observing cues from course instructors and other staff. At one extreme, cue-seekers probed instructors for clues about exam techniques while cue-deaf students, at the other extreme, were oblivious to the possibility that there was a method to being assessed. The passage below comes from the interview of a cue-seeker:

> The technique involves knowing what's going to be in the exam and how it's going to be marked. You can acquire these techniques from sitting in the lecturer's class, getting ideas from his point of view, the form of his notes, and the books he has written...(p. 61)

Miller found that cue-seekers, like the student above, and cue conscious students had distinctively higher scores than cue-deaf students. That is to say, that the messages we send our students, whether intentional or not, are often received and used effectively to manage classes.

The literature on assessment and its influence on student study habits have overwhelmingly focused on pre-assessment issues such as evaluation methods, assessment types, and student preparation for exams. Missing from this discussion is the influence that assessment results and grading techniques have on students. The current study aims to fill that void in the arena of calculus instructors and prior knowledge errors.

Sensible Belief Systems

Throughout educational research there is an abundance of explorations into teacher beliefs (Kagan, 1992; Munby, 1982; Pajares, 1992). More so, the connection between teacher beliefs and teacher practices are of great concern to researchers (Hsu, Wu, & Hwang, 2007; She, 2000; Trumbull, Scarano, & Bonney, 2006). The aim of these studies tends toward development of teacher belief categories. In addition, these investigations often employ interview and observation data collection techniques in an attempt to compare beliefs to instructional practices (Richardson et al, 1991; Skott, 2009; Solomon, Battistich, & Hom, 1996; Thompson, 1984). Particularly in mathematics, research has shown that there are connections between belief and practice (Beswick, 2007; Philipp, 2007;Skott, 2009; Thompson, 1984). For instance, Thompson's (1984) case study of three junior high mathematics instructors utilized interviews, observations, and belief revealing tasks to compare teacher beliefs with classroom practices. Thompson found that the overall views that each teacher held about mathematics, teaching mathematics, and their own experiences with learning mathematics were influential in the instructors approach to classroom instructional techniques. Despite findings such as Thompson's, reports that teacher beliefs contradict teaching practices do exist. A study of six beginning elementary school teachers is a clear example (Raymond, 1997). The representative case, Joanna, was reported as having traditional beliefs about mathematics, but primarily nontraditional beliefs about learning and teaching mathematics. Even more, Joanna's classroom practice was described as primarily traditional. Needless to say, Raymond concluded that the cases studied demonstrated inconsistencies between teacher beliefs and teaching practices.

In response to reports such as Raymond's, claims have been made that the means by which such research was conducted facilitated inaccurate conclusions to be made concerning teacher beliefs and practices (Philip, 2007; Skott, 2009; Speer, 2005; Speer, 2008). These claims assert that, among other discrepancies, the findings of inconsistencies between teacher beliefs and practices are due to (a) a lack of shared language and understandings between researcher and teacher, (b) failure to consider the context under which beliefs are analyzed, and (c) disregard of belief systems. Belief systems are defined by Phillip (2007) as follows:

> [A belief system is a] metaphor for describing the manner in which one's beliefs are organized in a cluster, generally around a particular idea or object. Beliefs systems are associated with three aspects: (a) Beliefs within a beliefs system may be primary or derivative; (b) beliefs within a beliefs system may be central or peripheral; (c) beliefs are never held in isolation and might be thought of as existing in clusters. (p. 259)

Leatham's (2006) attempt to reconcile the perceived inconsistencies found in previous reports leans heavily on the notion of belief systems. He asserts that, as observers, the perceived beliefs of another are assumed consistent or contradictory based on our own perspective. "The sensible system framework attempts to minimize these assumptions" (p. 95). As an example, Leatham applies the sensible system framework to Raymond's 1997 case study, Joanna. He points out that because the only beliefs of Joanna's that were investigated were beliefs about mathematics, her complete system of beliefs was not considered. Joanna's beliefs concerning time management, instructional resources, and student behavior were disregarded when they could have very well had an impact on her teaching practices. Leatham goes on to describe how some of the perceived inconsistencies could have been reasoned away through a more thorough investigation of Joanna's sensible belief system. An adaption of the approach suggested here was used to make sense of contradictions found in the current study's data. As stated earlier, the details of the use of a sensible system framework is reserved for an explanation within the analysis chapter.

Chapter Three

Pilot Study

As the goal of this study is to better understand the perspective of calculus instructors, it was imperative to conduct interviews with available teachers. Student exams, both graded and ungraded, were also necessary to provide additional evidence of instructor perspectives through observing assessment techniques. Though this study began in the summer of 2008, the only available calculus instructors at that time were two graduate teaching assistants. Therefore, a two part qualitative approach (consisting of a pilot study and a primary study) was employed to include additional instructors that were interviewed in the fall of 2008. The instructors that participated in the primary study were faculty members so that a more seasoned perspective on prior knowledge and calculus could be taken into consideration. This chapter is devoted to the details of the pilot study conducted during the summer of 2008.

Participants and Setting

This study was conducted at the University of Oklahoma (OU), a four year research institution in which students enrolled in a calculus course must have passed a prerequisite pre-calculus course or have received a satisfactory score on a department administered placement exam. The calculus sequence (for engineering majors) consists of four courses. The first course, which is the only one of interest here, may include but is not limited to the following topics: "equations of straight lines, conic sections, functions, limits and continuity, differentiation, maximum-minimum theory, and curve sketching" (http://catalog.ou.edu/courses/mathematics_course.htm, accessed by author on June 8, 2008).

The investigation included the participation of 31 students enrolled in two summer calculus I courses (17 in course A and 14 in course B) and their instructors (to be referred to as Instructor A and Instructor B respectively). Both instructors were contacted prior to the first day of class and given a description of the study goals and requirements for participation. These instructors were GTAs and the only instructors teaching calculus at the time.

Data Collection

The instructors allowed me to solicit participation of their students during a small portion of class time as well as interview them following each class exam. Once informal consent was gained from each instructor, I solicited student participants during a regularly scheduled class period. At that time the instructor was asked to leave the room while the purpose of the study was briefly explained and informed consent forms were distributed. The students were assured that their consent or denial to participate would not be revealed to the instructor and would therefore have no influence on their standing in the course. I explained that the student's informed consent allows each of their original and graded exams to be copied and that no other action would be required of them. Once the informed consent forms were obtained from the students, the student names were randomly shuffled using a list randomizer. This allowed a number to be assigned to each student for future coding purposes. All informed consent forms were then placed in a locked office for safe storage.

Data collection began with the gathering of the students' first exam. Both instructors conducted in-class exams approximately three weeks after the start date of the course. Immediately following each class, the exams were collected from the instructors and copies of each student exam were made. After returning the original exams to the instructor, I set aside the exam copies of the students who signed informed consent forms. All other exam copies were promptly destroyed. Immediately after grading, the instructors returned the exams and another set of copies were created. It should be noted that Instructor B decided to re-grade exam 1 after deciding to adjust his grading technique based on the advice of a senior faculty member. These set of graded exams were also copied.

After returning the graded exams, the student work was reviewed. Common errors and possible instructor grading patterns were identified. Two types of Student Error Examples (SEEs) were identified among the students' work. One type of SEE among the students' exams was characterized by those containing what I determined to be prior knowledge errors. These were errors in the student's ability to use basic mathematics (such as addition or multiplication), algebra (such as factoring or rationalizing roots), trigonometry (such as evaluating the sine function at a particular value), geometry (such as applying the formula for the area of a rectangle), or pre-calculus (such as sketching the graphs of functions). The other type of SEEs consisted of errors that appeared to be caused by a lack of skill that should have been acquired in the current calculus course. These SEEs included mistakes in the use of the chain rule or product rule formulas, incorrect evaluation of a limit, or setting up the limit definition of the derivative incorrectly; just to name a few.

Interviews

Individual interviews for Instructor A and Instructor B were scheduled approximately one week after the exams were administered to the students. During the first interview the instructors were asked to review and sign an Informed Consent Form that explained the purpose, goal, and requirements for participation in the study. This form also requested that the instructor confirm allowance or denial to be audio taped during each interview. Instructor A agreed to be audio taped whereas Instructor B did not. Each interview was conducted in a conveniently located campus classroom and lasted approximately two hours. The interviews consisted of two sections (see Appendix A). Section I was designed to gain general information about how the instructor viewed their role in the classroom and to address the first two research questions:

1. How do instructors define prior knowledge?
2. How do calculus instructors view prior knowledge within the context of a calculus course?

The instructors were asked open-ended questions that inquired about their role as calculus instructors, how they felt prior knowledge fits into the context of a calculus course, and how they assess prior knowledge errors in a calculus course. To address the final research question, 'How do instructors grade prior knowledge errors?' the interview also included a task-based section.

In Section II the instructors were asked to explain the following for each exam question:

- What work were the students expected to show
- How were points assigned to each portion of the question
- What were the most common error types found in the problem

They were also presented with SEEs (both graded and ungraded) that represented common errors among particular questions as well as SEEs that demonstrated instances of what seemed to be unusual grading. In these cases the instructors were asked to give more detail about their grading. Immediately following each interview I debriefed by expanding the field notes. This was done for organizational purposes as well as to include my general thoughts and observations about the instructor's responses. To begin searching for possible themes among the interview data summaries of the interview item responses were generated. The interviews for Exam II and the Final Exam were conducted in a similar manner. However, during the subsequent interviews the instructor participants were provide with the response summaries of the previous interviews as they consisted of nearly the same questions. The participants were asked to confirm, amend, and/or add to any summaries as they saw fit given the new material and interactions with their students since the previous interview. The process of allowing the instructor participants to confirm or amend the summaries of their interview responses added to the validity of the interviews through participant check. The summaries for each instructor's set of interviews were combined to help in identifying the themes found throughout their responses. Using the summaries of all three interviews I compared the responses for each interview item. I first looked for patterns within each instructors set of interviews separately. Next, I compared the summaries of each item from both instructors together. This allowed me to identify possible patterns in item responses.

Useful Results

Several themes were found throughout the two instructors' interviews; many of which are worthy of further discussion. However, the purpose of this chapter is to highlight the aspects of the pilot that were useful in developing the primary study. Thus, this section will be devoted to describing those useful results and explaining how they informed the methodology and analysis of the current investigation. Here I will discuss three major aspects of the pilot study: the interview experience, the student error examples (SEEs), and the selection of error types.

First, the six interviews proved to be of great benefit to me as a new researcher. The experience alone aided in developing my skills in controlling the pace of the interviews, probing the participants for details, and managing field notes while interviewing. Additionally, I was able to develop systems for organizing the data collection, expanding field notes and transcription of interview audio. Probably the most helpful aspect of conducting the pilot interviews was the identification of necessary interview protocol adjustments. Particularly, I found that there were items that needed to be added, omitted, and amended to ensure the participants had ample opportunity to share as much of their experience as calculus instructors as possible. All in all, the interview experience during the pilot was a key component in maturing my interview skills.

Secondly, the collection of student exams provided me with a plethora of SEEs to choose from when deciding which examples to have the primary study participants view and assess. The student exams collected during the pilot made it possible to present the participants in the primary study with unfamiliar student work. I found that the pilot instructors would sometimes explain that their grading techniques were influenced by their interactions with individual students. Therefore, using the pilot exam papers as examples of student errors allowed the primary participants to more objectively describe their opinion about prior knowledge and their strategies for grading specific SEEs.

Lastly, the decisions made about which SEEs to use in the primary study interviews was largely influenced by the responses of the pilot instructor participants. Knowing the type of skills calculus instructors classify as prior knowledge enabled me to select SEEs that reflect errors in these areas. This ultimately optimized the primary study interview time as I was able to narrow the SEEs presented to the participants. The pilot interviews allowed me to identify some of the topics that calculus instructors consider to be prior knowledge in two ways.

First, during Section I of each of the interviews the following question was asked:

"What prior knowledge do you believe students need in order to be successful in a calculus course?"

Instructor A and Instructor B both responded to this question by expressing that facility with algebra is necessary for any student to do well in a calculus course. They also agreed that a student's ability to organize their written work and solutions should be considered an asset in successfully completing the class. In addition to algebra and organization skills, Instructor B made a point to emphasize that calculus students should know how to use proper mathematical language to navigate through the course with as little confusion as possible. With these three skills in mind I was able to conclude that the SEEs I present to the primary participants should include errors in each area.

The other way that the pilot helped me to decide which SEEs should be used in the primary study came out of Instructor A's and Instructor B's responses to Section II of the interviews. In Section II the instructors described which type of errors were most common for each exam question as well as classified errors for the SEEs I presented to them. Table 1 outlines the most common errors found throughout all three of each instructor's exams. The SEE error classifications made by Instructor A and Instructor B are shown in Table 2a and Table 2b respectively.

	Most Common Error Type						
	CALC	PK	ALG	ML	C/S	TRIG	O/S
Inst. A	13	11	6	0	0	0	0
Inst. B	17	8	7	2	6	0	0
TOTAL	30	18	13	2	6	0	0

Table 1: Most Common Error Types
CALC = calculus, PK = prior knowledge, ALG = algebra, ML = mathematical language, C/S = careless/silly, TRIG = trigonometry, O/S = organization/sloppy.

Table 1, above, shows that among all the exam questions 30 of the most common errors were attributed to mistakes in calculus and 18 attributed to prior knowledge. More importantly, algebra, mathematical language, and silly/careless errors were also recognized as the most common errors for 12, 2, and 5 exams questions respectfully. Table 1 prompted the addition of careless/silly as a type of error considered for SEEs to be used for the primary study.

Inst. A	# of SEEs Presented	Error Type						
		CALC	PK	ALG	ML	C/S	TRIG	O/S
Exam 1	5	2	1	1	0	0	0	0
Exam 2	19	7	4	1	1	1	1	1
Final	25	13	15	5	0	1	0	1
Total	49	21	20	7	1	2	1	2

Table 2a: Classification of SEE error types for each of Instructor A's exams.

Inst. B	# of SEEs Presented	Error Type						
		CALC	PK	ALG	ML	C/S	TRIG	O/S
Exam 1	20	8	5	5	1	3	1	1
Exam 2	33	15	9	8	0	7	0	0
Final	47	17	1	1	2	18	0	2
Total	100	40	15	14	3	23	1	3

Table 2b: Classification of SEE error types for each of Instructor B's exams.

Tables 2a and 2b show the categorization that the instructors made when reviewing SEEs during Section II of their interviews. These tables show that Instructor A and Instructor B classified errors as algebra, mathematical language, careless/silly, organization/sloppy, and trigonometry. The additional category of trigonometry was therefore added to the list of error types chosen to be included in the SEEs for the primary investigation.

In summary, the pilot study presented here was helpful in many ways. It provided much needed data collection and interviewing experience. Also, the pilot made it possible to prevent bias responses from the primary study participants through the acquisition of anonymous student error examples. Lastly, the pilot study results helped to shape the primary interviews through the collection of error types described by Instructor A and Instructor B. Namely, I was able to identify five types of errors (algebra, organization/sloppy, mathematical language, careless/silly, and trigonometry) that would guide the selection of SEEs used during the interviews of the primary investigation.

Chapter Four

Methods

Participants and Settings

The participants of this study were solicited among all tenured faculty members who taught the class recently (recently defined as within the last five years). Of the fourteen faculty members who met these criteria, five were willing and able to take part in the study. The research areas of these faculty members included algebra, analysis, and topology. Among these participants, the range of experience with the Engineering Calculus I course at the University of Oklahoma spanned from teaching the class only once to teaching the class several times. More than one instructor confirmed that they previously taught a small honors calculus course in addition to the large lecture course addressed in this study. The large lecture course usually enrolls approximately 120 students for a three hour lecture. It includes a one hour per week discussion section facilitated by graduate teaching assistants under the direction of the faculty instructor.

Data Collection.

The interview protocol (See Appendix B) was quite similar to that of the pilot study instructor participants' in that it included two sections: one consisting of standard interview style questions and the other a task based section. The first section was essentially identical to that of the Pilot interviews. However, the task based section differed in several ways. The SEE's presented to the instructors were chosen from the pilot study student participants' work. Therefore the instructors were not familiar with the individual students. This strengthened the confidence in their responses as they would have no bias based on their interactions with the students. The number of SEE's presented to the instructors was dramatically less than the pilot study. One reason for the restricted number of SEE's presented was to ensure that the interviews stayed under the two hour interview limit outlined in the informed consent form. Also, of the twenty-two question types found among the pilot study exams only seven types were chosen to present to the instructors. These were chosen to represent test items commonly found on OU exams as identified by myself and several other instructors within the department. The questions types chosen for focus were as follows:

- Find the derivative using the limit definition
- Find intervals of continuity
- Find the derivative using rules
- Find tangent lines
- Implicit differentiation
- Related rates
- Maximum/minimum application

Among the nineteen SEEs (See Appendix C) selected for use in the interviews, a variety of error types were chosen as well. In light of the vast amount of SEEs collected during the pilot study it was imperative to further limit those chosen as part of the primary study interviews. Therefore, taking the results of the pilot study into account as it regards to student error types became key. To ensure a reasonable interview length, the SEEs selected for review by the faculty instructors were those that fell into the categories that the pilot instructors identified most often as being cause for student error. In the pilot study we found that the most identified prior knowledge and/or non-calculus mistakes were algebra, carelessness, poor use of mathematical language, poor organization, and trigonometry. The examples chosen for inclusion in the primary study interviews represented the most identified types of errors listed as well as some calculus errors (as identified by the investigator). Therefore, the interviewees had the opportunity to respond to example styles that they would both be familiar with and reflected mistakes in prior knowledge as well as calculus. For each SEE presented to the instructors they were asked to address the following questions:

- If possible identify any errors.
- Classify the error as a calculus error or prior knowledge error.
- Classify, if possible, the type of prior knowledge error.
- How would you score this question given the stated point value?

Each of the five interviews took place in a closed room on the OU campus and was held at the participant's convenience. The audio recorded interviews were approximately two hours long for each instructor. Immediately following each interview the field notes taken throughout the session were expanded for future reference. Included in the expanded field notes were quotes made by the participants that appeared to directly address the research questions. Also included were descriptions of the participants' demeanor during the interview and summaries of question item responses. Upon completion of all interviews each was transcribed for further analysis.

Analysis

Reflective analysis.

To begin analysis of the interviews the expanded field notes were reviewed. In keeping with the phenomenological research methodology this investigation was designed after, reflective analysis was employed to locate patterns among the instructor responses. The themes that emerged from this process were primarily found in the first section of the interviews and addressed the first two research questions. There were four initial themes identified for the first research question, *How do instructor's define prior knowledge*?. The second research question, *How do instructor's view prior knowledge in a calculus course*?, yielded six initial themes from the expanded field notes. Next, the transcribed interviews were used to further identify themes relating to research questions one and two. In particular, all instructor comments relating to each theme found during the reflective analysis were categorized and collectively evaluated to determine their relevance to the research questions. This process further reduced the number of themes associated with the first two research questions to two and five respectively.

Wide score ranges.

The third research question, *How do instructors grade prior knowledge errors*?, was intended to be answered by finding patterns in the participant responses. However, the data collection did not allow for such patterns to be identified. Admittedly, answering question three in its intended form would be better served by an alternate research plan; possibly a quantitative design. Attempting to answer this question required a new approach that did not utilize reflective analysis but hinged on the score responses the participants provided concerning the eighteen SEEs they were presented with during interviews. Each score was logged into a spreadsheet (See Appendix D) that allowed the investigator to compare the scores for each SEE across participants. This spreadsheet was also used to log each error type reported by the participants, the average score for each question, and the range over which the participants scored the SEEs. Review of the spreadsheet quickly revealed that several of the SEEs had interesting score ranges. More specifically, these scores ranged from a grade below 70% to a grade of 80% or above. The interest in what was termed *wide score ranges* (WSR) stems from the idea that one instructor could assign a failing grade to a student who according to another instructor is performing above average work. Thus, this occurrence became the focus of the participant's score responses.

Sensible belief systems.

In light of the observed variance among the participants' assessment of SEEs, the goal of this study was extended to better understand the WSRs. To tackle this issue, Leatham's (2006) use of Sensible Belief Systems to explain the assumed discrepancies between teacher beliefs and teacher actions was examined. As discussed in Chapter 2, Leatham viewed beliefs as constructs that existed within a system known as belief systems. He considered the entire system when analyzing the actions of teachers who appeared to contradict their own beliefs recorded in interview sessions. Rather than concluding that the teacher contradicted themselves when observed instructional behavior did not align with a stated belief, Leatham resolved that there were other beliefs existing within the teacher's belief system that took precedence at the time of the perceived contradictory action. He as well as Speer (2005) acknowledged that the descriptions of beliefs provided by instructor participants are not always well communicated to the investigator. In addition, the terminology used by researchers in order to question instructors may at times be misconstrued by participants, further clouding the interview data gathered pertaining to instructor beliefs.

Adapting this process to the data presented in this report required that the WSRs be viewed as the "inconsistencies" to be explained through *sensible systems* rather than instructor behavior compared to a single instructor's stated beliefs. For each SEE that was identified as having a WSR we used the interview transcripts to find justification behind each instructors score. Therefore, in this study I am viewing the WSRs as the "inconsistencies" and the group of instructors' justifications for grades as the sensible systems. I uncovered the sensible systems by highlighting instructor comments that supported their grading decisions throughout each interview. After compiling grade supporting comments for each SEE a summary was created to describe each instructor's justification for their grading decision.

Chapter Five

Analysis

Research Question One

Of the three research questions that guided this study two were primarily captured in Section I of the participant interviews. Section I of the interviews prompted the instructors to give their general opinion about prior knowledge and its influence on the Calculus I course. Research question one (RQ1) "How do Calculus instructors define prior knowledge", was initially investigated using the following standard interview questions:

- What prior knowledge do you believe students need in order to be successful in a calculus course?
- Describe the prior knowledge skills that are needed to master the topics covered in your Calculus course?

The above questions provided instructors with an opportunity to describe what prior knowledge means to them in a Calculus course. Additionally, several other interview items yielded responses that were directly and indirectly related to RQ1. These responses were also used to identify themes among the instructors that would help me to discover a set of abilities, understandings, and/or skills that faculty expect their students to possess prior to entering a first calculus course. Preliminary analysis of interview recordings and expanded field notes uncovered four areas that instructors focused on when describing their expectations of students entering their Calculus course.

The first and most obvious skill that instructors expressed as important was proficiency in algebra. Instructors commented that the ability to manipulate algebraic expressions and equations was foundational to success in a calculus course. They felt that without an understanding of algebra and its processes students would have difficulty handling the calculations necessary to complete calculus problems. Additionally, instructors remarked that students are at an increased risk of failing to understand the basic concepts of calculus if not adept in algebra.

Trigonometry was the next topic that instructors tended to feel passionately that students have some understanding of before entering a calculus course. Some considered an understanding of trigonometric functions to be an absolute necessity. While others also noted that students should at least be familiar with the trigonometric values for the special angles (30, 60 and 90 degrees). The instructors showed some understanding of students who were not fluent in trigonometry, one going as far as noting that it is not being emphasized enough in high school. Instructor remarks made clear the belief that basic trigonometric skills were important not only because of the many applications introduced in the first calculus course, but also because of the applications of trigonometry in higher level mathematics (i.e. differential equations).

Another skill normally discussed in pre-calculus courses that the faculty instructors valued was an *understanding of functions*. Those who perceived understanding functions to be a key prerequisite to taking a calculus course pointed out two main reasons for its importance. One is that the study of change simply depends on the ability one has to, among other things, recognize, evaluate, and arithmetically combine functions. As one instructor expressed it "*If I tell you something and you don't know what a function is, what am I going to be able to teach you"?* The other aspect is the role geometric interpretations of a function play in understanding change and rate of change. In particular, the use of tangent lines as a visual representation of derivatives requires that students understand how to relate the formula of a function to its graphical counterpart.

Lastly, several comments that instructors made about the skills and/or abilities they desired of their students were put into a category that could not be attributed to any particular course or topic in mathematics. These responses were classified as *scholarly enthusiasm* because they referred to student study skills, interest in the course, and maturity in problem solving, just to name a few. Some comments that represent this group of prior knowledge skills are listed here.

> *Instructor D: I think the way I teach calculus [students] don't need a whole lot of facility with calculators. I guess I would just say, well there are broader skills like just good study skills and organizational skills. I'm not sure if we want to get into all that.* ***I****: Yeah, if you feel they should come to the class with that that's included in prior knowledge.* ***P****: Let me put it this way, I think if students have enough organizational discipline that they work on the homework assignments in a timely fashion I think that's obviously, you can view that as a skill which I think can aide them greatly in succeeding in the course.*

Instructor E: But you do see, there are cases, of students who come in seeming quite weak but with work and persistence they get it. A student can come in with a not very strong background [in mathematics] but the key thing is if you're willing to work, I try to explain this in class that persistence is the key. Mathematics is not the sort of thing you put aside and study once a week or just the night before an exam.

Instructor G: One thing I would like to see more of, and maybe this isn't prior knowledge and not everybody has this, but coming to the course with maybe kind of an attitude where they are not thinking of it as just learning to following recipes but where they come in to it prepared to learn something. I don't think this is prior knowledge that is absolutely necessary. But this is something that I would ideally like to see because a lot of times students will arrive; they're use to high school style math where all it is about is following little recipes to get the answer. Where if you ask them why something works or what's going on its completely strange to them. So sort of an openness to understanding how things work. So that would be maybe one kind of knowledge that in an ideal world it would be nice to have.

These types of comments were grouped together because they spoke to the effectiveness of being a good student without reference to any form of mathematical thought. Though, this idea of scholarly enthusiasm appears to be rich in further research-worthy investigations, I have determined that it should be omitted from the current study. I feel that it deserves a more intentional line of questioning to be understood to any reasonable degree. Unfortunately, the scope of this study did not allow me to pursue such further inquiry.

Of the four types of prior knowledge described above, only the first two were mentioned by all five of the interviewed instructors. Therefore, I conclude that at the very least, students are expected to be efficient in the use of algebraic manipulations and processes as well as trigonometric functions. Proficiency in algebra was spoken of as the foundation needed to develop student understanding of the concepts of calculus. Additionally, a student who has no understanding of trigonometric functions is seen by these calculus instructors as being at a disadvantage in the course. What follows is a discussion of instructor remarks to make clear the essence of their opinions about the need for algebra and trigonometry in a calculus course.

Algebra.

When asked about the importance of prior knowledge in a calculus course Instructor D replied:

> *~It's absolutely essential, I think there is a pretty high correlation between student skills in algebra and their success in at least the basic calculus courses, maybe the first and second semesters of calculus. If their struggling with algebra, its difficult to see how they're going to master calculus is how I look at it…I think algebra is the most crucial…If I had to rank order [the necessary skills] I would say algebra skills are the most important fundamental skills students need to succeed in calculus.*

Instructor E agreed by stating that:

> *~They need basic algebra. They need to be able to compute fractional expressions. They need to be able to do those sorts of calculations and say you wanted to compute x plus y squared or cubed, that's something they should be able to do or with a little reminding become quickly comfortable with.*

He goes on to state that a struggle with algebra in a calculus course can distract the student from the concepts they are trying to learn. He explains that "*You'll see a student who makes certain errors in a calc class but if they are totally at sea with those errors they just won't be able to stay with the material.* Instructor F goes as far to say that "*if you cannot handle basic things like [algebra] then you shouldn't be in a calculus course period ....*"

Several of the pilot participants' comments point directly to algebra as a tool for completing calculus problems as the reason why it is seen as a prerequisite. These comments include:

> *~I do see it a lot for example when I'm talking to a student in office hours, they seem to understand the particular calculus idea we're talking about but if they're stumbling with algebra, I keep saying algebra but I guess there are other kinds of prior knowledge that I'm not thinking of; but sure it can trip them up and they can end up missing lots of problems or thinking they don't really understand the calculus when that might not really be the problem.*

~And being somewhat proficient in algebra is important in order to do any calculations or to follow reasoning.

~Maybe they've forgotten how to factor polynomials or there are plenty of sort of set skills that they are expected, or algebra. Ideally they should be comfortable solving for something or basic algebra. Like with grading problems a lot of times in calculus it ends up being algebra they get a problem wrong and not so much a misunderstanding of calculus.

Instructor C echoed this attitude by remarking that:

~They have to be comfortable enough with the algebra to run examples until this thing becomes second nature...So yeah they need the familiarity with algebra to kind of make it second nature ...

His discussion not only refers to algebra skills as tools for successful calculation of calculus problems, but also as necessities in understanding the concepts of the course. For example, the following two comments from Instructor C for example.

~ So I think deep conceptual understanding requires that oh yeah moment but it also requires that you are comfortable enough with the algebra that you can work a bunch of examples yourself.

~ Yes to have a conceptual understanding of what calculus is about you need the geometry and the relationship to algebra, that a difference quotient means something geometrically, it means rise over run. I will tell them these things.

Though each instructor viewed algebra as necessary, their comments reveal two distinct reasons. The first, and most obvious, is that algebraic operations and processes are dense in the computation of most calculus problems. Therefore it is seen as a tool for students in the course. The other reason algebra is seen as a prerequisite is because of its usefulness in conceptually understanding calculus. Without facility in algebra, students struggle with ideas like the difference quotient, that are key in mastering calculus.

Trigonometry.

Other than algebra, the only other topic all five of the instructors stated that they wanted their calculus students to be comfortable with was trigonometry. Ironically, all but one instructor listed trigonometry as an important prerequisite, second only to algebra. The exception, Instructor F, felt trigonometry was second to fluency in computing values without a calculator.

When asked to list the prior knowledge that students need to be successful in a first calculus course Instructor C listed, among other things, "*good solid knowledge of trigonometry, the geometric intuition behind trigonometry, trigonometry in terms of right angles, similar triangles, knowing how to manipulate trig functions, knowing sine of x plus y is not sine of x plus sine of y.*" Instructor G points out below that students are often less than proficient in identifying the most common values for trigonometric functions.

> *~Another one that comes up a lot is not knowing the values of trig functions. So like cosine of pi or sine of zero. That happens a lot. That I would count as prior knowledge; basic knowledge of the trig functions and the standard values. They just need to remember them.*

Instructor F also found student forgetfulness of trigonometric values to be a problem. He notes that:

> *~...perhaps the most egregious omits of high school is the fact that they don't understand or know very little of trigonometry. They've heard the term sine, cosine, tangent, but they have no clue of what it means. They don't even remember the sine, cosine, tangent of your basic angles: 30 degrees, 60 degrees and 90 degrees.*

Interestingly, several comments were made regarding the instructors' willingness to review topics in trigonometry despite their belief that students should already be proficient in the subject. For instance, the following passage is Instructor D's response the question 'How do you feel prior knowledge influences the teaching of calculus.'

> *~In terms of the teaching of calculus, if one makes the blanket assumption that students are properly prepared in algebra then it can greatly simplify the teaching of calculus because you can concentrate on the concepts of calculus and not worry about reviewing basic topics in algebra or trigonometry or things like that ... Do I always assume that, no! For instance, if I'm getting ready to talk about the differentiation of the trigonometric functions I will normally include a brief review of the trig functions, talk about radian measure. So I do in fact review topics here and there b/c based on my prior experience students aren't as sharp in these skills as they should be.*

Sticking with the belief that students should know something about trigonometry, Instructor E explains:

> *~They need to have seen some trig and have some recollections of it. But I tend to review that stuff but you can't just start cold. They need to have some understanding of what those functions mean and where they come from.*

The agreement among all participants is that trigonometry plays a key role in a calculus course. Therefore, students should have some understanding of the topic prior to enrolling in calculus. Most instructors expressed the attitude of Instructor F when he says, "*I'm not necessarily saying you should be prepared to do research in these topics but at the very least know what they are.*" However, it would seem that overall his position could be summed up by his statement that "*As a mathematician I cannot imagine doing calculus without trig.*"

Research Question Two

Research question two, "How do instructors view prior knowledge in the context of a calculus course?" was analyzed in the same fashion as research question one. In particular, initial themes were identified through expanded field notes and later confirmed or denied using the full interview transcripts. The initial themes found in the expanded field notes were:

- Ideal versus reality of student preparedness
- Conceptual understanding and prior knowledge
- Assessment by student comparison
- Comparison of instructor background to student background

The first theme, *Ideal versus reality of student preparedness*, refers to the level of skill the instructors wish their students had and the level they normally encounter in a calculus course. The second theme, *Conceptual understanding and prior knowledge,* deals with the role that instructors feel prior knowledge plays in both the teaching and learning of calculus concepts. The third theme, *Assessment by student comparison*, was found mostly in Section II of the interviews where instructors were asked to assess samples of student work. I found that instructors not only considered the types of errors when assigning grades, but that they also compared students in order to weight grades fairly throughout the grading process. Notably, I was not able to link these comparisons to prior knowledge in any way. Therefore, the exploration of this theme will be reserved for future studies. Lastly, C*omparison of instructor background to student background* was evident in comments like:

> *Instructor F: Where I went to school, and I did my college education in the U.S., they actually spent, in my first year calculus course, half a semester on logic. That may be a luxury given the tight knit courses that we have here but that is something that hurts them throughout their mathematical career.*
>
> *Instructor C: A level of maturity to be able to abstract from a word problem to a mathematical model and do things. I'm not sure to what extent they do that in high school. I did see that in high school, when you do quadratic equations you would be able to do baby optimization*

problems. I remember seeing farmers with fences…I'm assuming if they've seen things like that they will be in good shape.

Instructor F: I am always sensitive to the fact that I shouldn't come off like "well in my time it was like this but in your time… ". It isn't. We all struggle through these things. But what is clear to me is that these students simply do not practice in high school or in college. I give them literally six or seven homework problems and they complain that it is too much work. And I'm saying this without exaggeration. I use to get 60 to 70 problems a week. Sometimes 100. I'm not saying that's what they should be doing – no I take that back I am saying that's what they should be doing. You want to understand the stuff; it's like music or sports. If you're on a team the coach is going to tell you to spend an hour shooting hoops. I can't think of anything more boring but you do it because you want to get good at it. It is exactly the same thing. They just don't drill enough.

Of these four themes, only the first two were mentioned by all five of the interviewed instructors. They are discussed further below.

Ideal versus reality of student preparedness.

The participants' ideal class would consist of students who have all the necessary prior knowledge mastered. However, they often find that their actual students do not live up to their expectations which require adjustments in instructional planning. Particularly, instructors include reviews of prior knowledge topics into their lectures as needed. In addition, they often feel obligated to "water-down" their lectures because student backgrounds do not allow a rigorous calculus course to be feasible.

Throughout the interviews instructors painted a picture of the type of classroom they hoped for. Descriptions like the following, provided by Instructor D, help to understand what is ideal:

> *~In terms of the teaching of calculus if one makes the blanket assumption that students are properly prepared in algebra then it can greatly simplify the teaching of calculus because you can concentrate on the concepts of calculus and not worry about reviewing basic topics in algebra or trigonometry or things like that. It can allow the instructor to put a greater concentration on the material at hand, maybe do a few more challenging example because you're not having to spend time in class reviewing. So from that point of view if you have a well prepared group of students I think you can get more done, you can do the material in more depth, I think all that comes out as being a better course...*

Instructor E gave a similar account when asked to describe his role as an instructor:

> *~Ideally, in an ideal world your students are very well prepared and you spend your time focusing on concepts and but that's not what happens in practice. First of all a large calculus class has over 100 students, many not very prepared.*

(It is worth noting that both instructors quickly go from describing the ideal of prepared students to the reality that they cannot count on the backgrounds of their students to be up to par.) Though Instructor G specifically feels that "*Ideally they should be comfortable solving for something or basic algebra.*"; below he also describes a similar dilemma as Instructor D and Instructor E with ill-prepared students but in terms of student attitude.

> *~One thing I would like to see more of, and maybe this isn't prior knowledge and not everybody has this, but coming to the course with maybe kind of an attitude where they are not thinking of it as just learning to following recipes but where they're prepared to think of it as learning to understand something. I don't think this is prior knowledge that is absolutely necessary. But this is something that I would ideally like to see because a lot of times students will arrive, they're use to high school style math where all it is about is following little recipes to get the answer. Where if you ask them why something works or what's going on its completely strange to them. So sort of an openness to understanding how things work. So that would be maybe one kind of knowledge that in an ideal world it would be nice to have.*

In response to the issue of student under-preparedness instructors developed their lesson plans accordingly. Take Instructor F's comment, for example:

> *~I've only taught this once but I've seen this across the board wherever I've taught. Around twenty percent of them are ill prepared. For the remaining two-thirds who understand the basic notions; what I mean by basic notions is that I do water down the course to meet the students. They*

don't all know these basic notions but I can't just say that none of you is going to learn calculus. That would be unacceptable. So I do water down my course.

After Instructor D and Instructor E describe the characteristics they would hope to find in their students they also explain how they handle the reality of the situation:

> *Instructor D: Do I always assume [students are prepared], no! For instance, if I'm getting ready to talk about the differentiation of the trigonometric functions I will normally include a brief review of the trig functions, talk about radian measure. So I do in fact review topics here and there b/c based on my prior experience students aren't as sharp in these skills as they should be.*
>
> *Instructor E: So what happens in practice is students learn a lot of basic algebra in a calculus course. They are supposed to know this beforehand but that's often not the case...*

Instructor C approaches the class by starting the semester with some reveiw. He states that "*I'm assuming if they've seen things like that they will be in good shape. I don't assume they've seen that. I tend to go from scratch and say look you need to be able to abstract, how do you choose a variable.*"

The above remarks clearly outline the desired types of classrooms that calculus instructors hope for. More specifically, in relation to prior knowledge it's obvious that the instructors interviewed often do not see the level of student preparedness they feel is necessary for the most productive calculus course to be held. Therefore, to accommodate their students, the instructors tend to review basic topics and limit the depth in which they cover the course material.

Conceptual understanding and prior knowledge.

Part of the guiding curiosities of this study was 'How is it that students could understand (or appear to understand) the concepts I taught them in calculus yet still struggle with seemingly simple algebraic processes?' Naturally, this question was woven into the inquiry of the current research. Specifically, the line of questioning pursued during the interviews included the following item:

"What influence or importance does prior knowledge have on the understanding of calculus?"

Responses to the above question yielded a new theme: conceptual understanding and prior knowledge. Interestingly, I found that there are two opinions under this category. The first stance was that students can understand some of the concepts of calculus without necessarily being fluent in all the prior knowledge skills, such as algebra or trigonometry, used to complete calculus exercises. Instructor G pointed out that "…*with grading problems a lot of times in calculus it ends up being algebra they get a problem wrong and not so much a misunderstanding of calculus."* He goes on to say that:

> *~I do see it a lot for example when I'm talking to a student in office hours, they seem to understand the particular calculus idea we're talking about but if they're stumbling with algebra, I keep saying algebra but I guess there are other kinds of p.k. that I'm not thinking of; but sure it can trip them up and they can end up missing lots of problems or thinking they*

don't really understand the calculus when that might not really be the problem.

When describing grading techniques Instructor D and Instructor E point out that they distinguish between conceptual understanding and prior knowledge:

> *Instructor D: So if I can discern that the student is showing knowledge of the calculus but is somehow impeded by poor algebra skills I try to weigh that and give them some credit for knowing the calculus but unfortunately sometimes the algebra is getting in the way.*
>
> *Instructor E: So if a student shows that they have some conceptual understanding I do give them some credit. But calculus is about calculations and you need to get the calculation right. So even if it's a question of a deficiency in prior knowledge it's still something their responsible for.*

Interestingly, though they both provide partial credit for evidence of conceptual understanding, they each still consider the deficiencies in prior knowledge to be a problem. Instructor C goes as far to say that

> *~The most frustrating one is when someone comes up with the equation of a tangent line and its just perfect and then they go and they simplify it more and there is an algebra mistake there and its just, I will usually give that full credit because I think that stuff is irrelevant. You know it's more important that they get the concepts…*

Though it is not clear from the transcripts, however his generosity in points may depend on the particular type of prior knowledge.

Despite Instructor C's leniency with students who sometimes fail to apply prior knowledge appropriately, his stance on conceptual understanding is quite different than Instructor G, Instructor D, and Instructor E. That is, along with Instructor F he feels that certain areas of prior knowledge must be mastered before a student can conceptually understand calculus. For example, consider the passage below from Instructor C's interview:

> *~So I think deep conceptual understanding requires that oh yeah moment but it also requires that you are comfortable enough with the algebra that you can work a bunch of examples yourself... They have to be comfortable enough with the algebra to run examples until this thing becomes second nature. I think everybody, you can sit them down and explain quite sophisticated notions. Its like if I go to a seminar somebody can explain something and I can be really enlightened but unless I work with it an hour later its gone. So yeah they need the familiarity with algebra to kinda make it second nature..to know the difference quotient is measuring this...*

Instructor F echoes the need for comfort with prior knowledge in stating that

> *~Mathematics is a cumulative process. Again, I repeat, calculus is not easy. That's why it is spread out over four semesters. And it's a full load each semester. You can not spend time relearning that stuff, you need to already be quite comfortable with anything from fractions to functions, limits, and trigonometric identities...*

According to Instructor F "*Prior knowledge is absolutely essential to understanding the concepts of calculus.*" More specifically, Instructor C feels that

"Yes to have a conceptual understanding of what calculus is about you need the geometry and the relationship to algebra, that a difference quotient means something geometrically, it means rise over run."

Overall the instructors agreed that prior knowledge is necessary within a calculus course. The opinions differed in degree and perceived usefulness of prior knowledge. Whether it be in terms of its usefulness in completing exercises and calculations or usefulness as a foundation for conceptual understanding each instructor expressed that without prior knowledge skills students will struggle with calculus.

Wide Score Ranges

The data collected in Section II of the interviews was organized into the Participant Score Sheet, as describe in the previous chapter (See Appendix C). This sheet was used to compare and contrast the instructor responses. In particular, it allowed me to cross reference the responses from each instructor with each SEE. The score sheet was later extended to reflect the SEE score ranges. It quickly became clear to me that several of the score ranges reflected distinct differences among the instructors. Recall from the previous chapter that wide score range (WSR) is the term used to describe the scores of an SEE that ranged from a failing grade of below 70% to above average grades of 80% or above.

After careful consideration I found that the seven WSRs represented four of the seven question types. Namely, they belonged to the following question type categories: Finding derivatives using the limit definition, identifying intervals of continuity, finding derivatives using derivative rules, and finding tangent lines.

The SEEs that had WSRs were: IA-E1-Q2-SP#9, IB-E1-Q5iii-SP#4, IB-E1-Q4ii-SP#10, IB-E3-Q2i-SP#14, IB-E2-Q2i-SP#5, IB-E2-Q3i-SP#7, and IB-E2-Q3i-SP#9 (See Appendix B).

To better understand this issue of WSRs, sensible systems were applied as described in Chapter Four. For the purposes of this study, the inconsistencies were those identified by the WSRs. The WSRs were viewed as inconsistencies in the grading technique of the instructor group as a whole. To make sense of these perceived discrepancies I considered each instructor's entire interview to find supporting comments that give understanding as to why they assigned each SEE score.

Below the WSRs are outlined along with the comments that "make sense of" each instructors grade decisions. For each SEE the instructors' comments are listed in order of the highest grade to the lowest grade. Additionally, a brief summary of the instructor's comments is included for clarity and ease of reading. Note that the SEEs can be found in Appendix C for reference.

IA-E1-Q2-SP9.

The first SEE with a WRS, IA-E1-Q2-SP9, required the student to find the derivative using the limit definition. This question was out of twenty points. Instructor C assigned the highest score of 18/20. In his opinion, the mistake is of little importance compared to the student's ability to demonstrate conceptual understanding.

~The most frustrating one is when someone comes up with the equation of a tangent line and its just perfect and then they go and they simplify it more

and there is an algebra mistake there and its just, I will usually give that full credit because I think that stuff is irrelevant. You know it's more important that they get the concepts.

~There are examples where the calculus is all true and people organize things and make mistakes. I'm not worried, I'll usually circle it and write some type of comment and give them nine out of ten.

426-436~*...Yeah, um I would kind of, depending on where they were in the course, if they also just learned rules of derivatives and knew there was another way and that this should be 2x + 1 then I would probably be a little bit more harsh. ... Again it's only a cancellation error or its only an algebra error. The definition here is perfect. The way that they input this into a function is all done perfectly. The idea of taking a limit afterwards is all very nicely done.*

<u>Sensible System</u>: Instructor C felt he could be lenient on this problem for three main reasons. The first is that he can clearly see that the student understands the process and was able to set everything up correctly. This is the most important consideration. Also, when everything else is correct, the algebra mistakes are seen as irrelevant. Therefore, such errors warrant only minimal point deduction if any at all. Additionally this instructor took into account that this was a first exam and noted that later in the course a similar mistake would be worth more points.

Instructor D's scoring of IA-E1-Q2-SP9 was 15/20. Though still a passing score, it is considerably lower than Instructor C's. Instructor D seemed to look for opportunities to allot points throughout the students work.

~ So if I can discern that the student is showing knowledge of the calculus but is somehow impeded by poor algebra skills I try to weigh that and give them some credit for knowing the calculus but unfortunately sometimes the algebra is getting in the way.

~...Out of twenty points, let's see; we have the correct definition of the derivative, we have the function x squared plus x being evaluated correctly on x plus h, that's a good thing. I could see maybe giving fifteen out of twenty.

Sensible System: Instructor D acknowledged several things that the student did correctly. These correct steps were in the overall process of the problem; particularly the calculus part of the work is done correctly. He recognizes an error in manipulation of fractions which is getting in the way of the work.

Instructor G was not able to provide an exact score for SEE IA-E1-Q2-SP9, but he did provide an estimated range of 12/15 to 15/20. The reason the score may be below average is because Instructor G considers the prior knowledge mistake to be a serious one.

~ *I take the calculus mistakes more seriously. Prior knowledge mistakes, for example a small arithmetic error, I don't take very seriously at all. Problems with algebra or with trig functions, those can be serious and I expect them to be able to do that aspect of it... So basically arithmetic I don't take very seriously at all. I'll take off a point or something. I don't think it means that they don't understand something. But with algebra or trig I do want them to do that correctly so I take that somewhat seriously.*

*~ ...**P**: I guess I would consider that an algebra error. **I**: So this is out of twenty points. **P**:[long pause] So they are setting it up correctly and they make a fairly serious algebra error. I don't know maybe I would give it twelve points, fifteen points, something like that...*

Sensible System: Instructor G has no issue with the mathematical syntax errors. He feels that they can be overlooked in terms of grading, but are worth pointing out to the student. He is pleased with the calculus part of the solution however; he recognizes that there is a "fairly serious algebra error". His score reflects the attitude that algebra errors can hinder a students' progress in calculus and expectation that students carry out algebra processes efficiently.

Only half credit or slightly more (10/20 – 12/20) was given to IA-E1-Q2-SP9 by Instructor F. The importance he places on prior knowledge is clearly the reason this SEE received such a low grade.

~Now, and this is going to come off harsh, if a calculus problem requires that you are able to add fractions or factor a polynomial or find the root of something and you cannot do it, I am very generous with partial credit, but it is a fairly serious lapse on your part in my mind if you cannot handle basic things like this then you shouldn't be in a calculus course period... Prior knowledge is absolutely essential to understanding the concepts of calculus.

~ ...I look at a given problem and break it up into several parts, and then depending on my assessment on how important it is that they know each part I assign point values to each part. That's how I assign partial

credit...So here I would assign five points out of twenty for knowing and writing down correctly the limit definition of the derivative. I would assign another five points for correctly plugging in the function value here of x plus h and opening up the parenthesis. So I would assign this somewhere between ten or twelve points...

<u>Sensible System</u>: First breaking the problem into sections, Instructor F assigned grades by determining the students' performance in each portion of the exercise. Therefore this student earned half the credit for the portions that were done correctly. Instructor F feels that students who don't know how to do basic algebra or arithmetic should not be in the calculus course which explains the low grading.

The lowest score, 10/20, was assigned by Instructor E. He simply recognized that the student was unable to work through the entire problem. Thus, SEE IA-E1-Q2-SP9 warranted only ten of the possible twenty points.

~*... But when it comes to doing concrete problems these skills that they might not have such a solid grasp on that can really hurt them.*

~*So if a student shows that they have some conceptual understanding I do give them some credit. But calculus is about calculations and you need to get the calculation right. So even if it's a question of a deficiency in p.k. its still something their responsible for.*

~*So if students don't get a hold of certain basic manipulations they won't do well in the course.*

~*...So they wrote down the derivative correctly and wrote down the correct limit and put in the function and expanded the functions. So the whole* *problem was in the manipulating...They did that incorrectly and then they got the right limit. So I'd give maybe 10 points. It would depend if it was given towards the end of the course versus close to the beginning. ...*

Sensible System: Instructor E felt that "calculus is about doing calculations". He sees the prior knowledge as part of the work of calculus. Partial credit is assigned when students demonstrate good work, which he was able to identify in this case.

IB-E1-Q5iii-SP4.

The next SEE with a WSR is IB-E1-Q5iii-SP4. Its question type is also finding the derivative using the limit definition. This SEE was scored out of 10 points. The highest score, 9/10, was assigned by Instructor E who attributes the student's mistake to a small lapse in focus.

> ~...*So I take back what I said. This is better than what I said. This student has a very clear idea of what's going on. So there are some very minor slips. So they would get almost full points...maybe 9 points. I: Are those minor slips in calculus or prior knowledge.* ***P****: Oh umm, concentration. I mean the student is skilled enough I think they made a quick error. I'm assuming in giving them nine points they took the limit correctly...yeah its nine.*
>
> ~...Yeah that's something I wouldn't be so harsh with. You just assume they were rushing. They did everything correctly up to then it seems. So yeah, that's just a slip of concentration. Then did they work everything correctly?

Sensible System: Instructor E considers slips or lapses in concentration to be somewhat insignificant. He is able to recognize that the student "has a clear idea of what's going on" and therefore deserves close to full credit.

Instructor F felt that SEE IB-E1-Q5iii-SP4 earned between 8/10 and 9/10 points. Like Instructor E, he suggested that the student's errors were not serious but due to unorganized work.

> *~The thing that I don't like but I tend to forgive is this person is very sloppy which is why they make these mistakes. You know not putting parentheses or keeping track of their steps. Right in the beginning they write one definition of the derivative and then switch to a. So I would call this student in and say look you understand this stuff pretty well you need to write better. So I would give this 8 or 9.*
>
> *~...I mean there is being sloppy but knowing what to do and there is being sloppy and not understanding why certain things are important. And I tend not to be too hard on that in calc I.*

Sensible System: Though the student's sloppiness was irritating him, Instructor F was willing to overlook it in the instance that the student shows that they understand the problem. In this case, the students errors are viewed as small errors and not worthy of much point deduction.

Instructor G gave SEE IB-E1-Q5iii-SP4 a score of 7/10 and cited an arithmetic error as not being serious. However, he does have concern that the student takes inappropriate steps to complete the problem.

~ ...So the four for a two that's not very serious but it does kind of mess things up. ***I:*** *do you consider that prior knowledge or calculus?* ***P:*** *Yeah is it prior knowledge just arithmetic. Then they made the second error to compensate. So the second error, so it looks to me they know the h is supposed to cancel maybe from doing practice problems. I'm not sure what kind of error this should be. Overall everything is not so bad. I don't like the second error b/c they are sort of pretending things work out when it doesn't. I might give seven points for this, something like that.*

Sensible System: Instructor G notices a small arithmetic error that he does not consider to be serious. This is the type of mistake that he may not even take points off for. He recognizes that the student makes another error, in compensation for the first less serious one. Displeased that the student at this point is "pretending things work out when it doesn't". Instructor G felt this may have been a sign that the student cannot carry out the process of the problem appropriately.

The first failing grade for SEE IB-E1-Q5iii-SP4 came from Instructor D. He assigned a score of 6/10 supported by his comments that "*any mistake in factoring or adding two fractions or anything silly like...those things I view as fairly serious mistakes which could account for losing half credit on a problem.*"

~So if I can discern that the student is showing knowledge of the calculus but is somehow impeded by poor algebra skills I try to weigh that and give them some credit for knowing the calculus but unfortunately sometimes the algebra is getting in the way.

~For prior knowledge, any mistake in factoring or adding two fractions or anything silly like…those things I view as fairly serious mistakes which could account for losing half credit on a problem. I do try to take off enough, I guess what I'm trying to do is get the students attention that this is a serious mistake and it is something you need to work on.

~…There is some attempt to algebraically simplify the expression; there looks like a spare term, two square root of four plus h, but it might be a result of the students thinking process about how to add those two fractions. On the next line that seems to be okay. The student is attempting to rationalize the numerator I guess. So I see there is one algebraic mistake: there should be a four minus quantity four plus h and the student wrote it as two. That is an unfortunate arithmetic mistake and it could happen to anyone but obviously it makes the problem hard to work through. These are things that I think if the student had a lot of experience with doing this thing then they might realize that at the end the h should cancel… ***I****: this is actually their first exam.* ***P****: I'm looking at half credit maybe more. It looks like the student has it how to set up the formula, there may be a little bit of struggle at first but we have this unfortunate arithmetic mistake when trying to push the problem through. I don't like it when students try to cancel a factor that is not a common factor; I see that as a serious mistake. I'm looking at something that would be maybe six out of ten.*

Sensible System: Though Instructor D was able to locate points of good work in the student's calculus, he was concerned with the algebraic/prior knowledge mistakes that threw the problem off. He considered such mistakes as serious and therefore warranted a significant reduction in points to get the student's attention.

Instructor C gave the lowest score range of 5/10 to 6/10. Though he held this students work in higher regards than a previous student, the errors are considered to be quite serious.

> *~Its not clear that they are comfortable using the algebra or that they have any clue as to what's going on conceptually. Then you have a discussion with the student afterwards...*
>
> *~Oh God, see they would get more than the previous person just because of the order.*
>
> *~This is a hard one because they're making really horrible errors like two squared is two but they're actually quite sophisticated with the adding of these two. I don't know maybe 6 and a conversation or maybe five and a conversation. This student is closer to salvation than [the previous student]*

Sensible System: One component of this grade can be attributed to the comparison between this student's and the previous student's work. The instructor felt that this student's organization was better than the previous student so the allotment of points reflected these differences. This aligns with the idea that he should be consistent in grading. Also, the instructor felt it was not "clear [the student was] comfortable using algebra" which warranted a discussion with the student about their work. Furthermore, conceptual understanding, which he views as most important, is not proven in this case.

IB-E1-Q4ii-SP10.

The next two SEEs ask the student to identify the points on a graph where the given (piecewise) function is discontinuous. The most favorable score for the first SEE of this question type, IB-E1-Q4ii-SP10, was given by Instructor G. He justified a score of 8/10 by calling attention to the student's ability to demonstrate an understanding of the overall process used to solve the problem.

> ~...So I like they are trying to draw the graph...okay so the explanations, neither of those is quite right but they got the right conclusions. Is that right?... their wrong pictures are throwing me off. [looks through the problem again]...The second is definitely calculus, well I guess they are both calculus. Seven or eight points. The second explanation is not totally wrong it's just saying something...Maybe eight points overall. The first one is wrong to say it is undefined.
>
> ~ Anyway, they are putting the wrong numbers in and they're using funny notation. But they are following more or less the right procedure.
>
> ...There is certainly stuff here that they are doing correctly but the overall method seems to be more or less [unwise]. This is a case where the simple errors are serious. I might give this half credit.
>
> ~ And that's moderately serious but they were more or less doing the right procedure. This last step of moving the three across I don't think it was very serious. Out of fifteen points I might give twelve points for that.

Sensible System: Here Instructor G sees that the student is trying to use the graph to make conclusions, which would be acceptable but the graph is off. Also, he acknowledges that the answers are correct but the explanations are not appropriate. He puts value in "following more or less the right procedure".

For SEE IB-E1-Q4ii-SP10 there were no scores of 70%. Instructor C assigned the next lowest score of 6/10 by considering the percentage of correct work.

> ~I really will grade most harshly for someone who will write down an answer with no explanation even if it's the correct answer. So it's probably that one, organizing and layout the results carefully.
>
> *~Again conceptually, they need to brush up on how to draw graphs but conceptually they have a good bit down here. Unfortunately they didn't give a reason here and they give a vague reason here. I don't know. That's a lot they have to do for only ten points. Umm, okay, [reads instructions again]; so they have to do six things for ten points. They've already got four of those six things correct so they should at least be given two-thirds of the points even though they've drawn an incorrect graph. I don't know maybe 6 out of ten I'm not sure. I get more upset when things are geometrically off. That's a personal bias.*
>
> *~What is that, power, product, trig, quotient. Four out of ten, two points each. So minus two, minus three. Maybe six or seven, seven If I was being generous.*

Sensible System: Instructor C approached this problem by allotting points based on the proportion of parts of it that were correct. He saw the grade as a percentage of correctness. He notes that the answers are correct but not supported by clear and/or appropriate explanations, which are key. Additionally, Instructor C is bias against (or unhappy with) things that are "geometrically off".

Still a failing score, 5/10 was the grade Instructor D gave SEE IB-E1-Q4ii-SP10. He also created a rubric-like grading scheme by breaking the problem into parts and allotting points accordingly.

> ~The reason is...obviously doesn't make a whole lot of sense but I do like the fact that there is a sketch there. I would probably give the student three out of five for that. At 1 the...this part of the sketch is incorrect, let's see (pause to review problem). If this part of the problem is five points I would probably give the student one point for recognizing discontinuity, well...that's actually okay; two out of five for that. **I**: Would you classify those as calculus or prior knowledge mistakes? **P**: The graph is not quite correct but if it, were would that have really helped? I would say more of a calculus mistake.

Sensible System: The student was given credit for creating and attempting to use a sketch. Instructor D saw that the student recognizes continuity; however the sketch is off and the explanations are poor. He viewed this as a calculus error and questioned if a correct graph would have been helpful to the student.

An even lower score of 3/10 was assigned to IB-E1-Q4ii-SP10 by Instructor E who felt the student's failure to explain the given answers warranted few points.

> *~...so their first answer is totally off. Well they're right that it is discontinuous at zero but it's not because it's undefined from the left. So supposed you split it as five and five. So the first one they got the right answer so that counts for something so maybe two points but their explanation, they're just writing something so at the most two out of five for that first part. The second point they are saying that it is discontinuous but that's false. Its continuous at one. No, I've got it wrong. Its two x squared so it's two from the left and one from the right. Again their explanation is wrong. So again maybe two points for getting the right answer but they don't seem to have any idea why. Their explanations are completely off. Maybe I would revise that because I'm giving them almost half. So maybe three out of ten...These are conceptual errors. It seems the student doesn't have an understanding of what continuity means. Its maybe not a prior knowledge I'm not sure...it seems they just don't have an intuitive understanding of what continuity means. They didn't draw the graph correctly. It's a combination. They don't have a conceptual understanding of the continuity and at the same time they didn't get the right graph. So the graph would be prior knowledge.*

Sensible System: Even though the student got the right answer their inability to explain showed a misunderstanding of the concepts. Therefore, Instructor E felt that less than half credit was appropriate.

Instructor F applied a grade of 3/10 to IB-E1-Q4ii-SP10, just like Instructor E. Unlike several of the other instructors who split up the problem to assign a grade, Instructor F gave what seem to be mercy points for the little work that was determined to be correct.

> *~To really get you need to understand the limit notion and most students don't understand limits. I don't blame them it is a difficult concept but you need to get it.*
>
> *~So I do this thing where if they write something down I give them maybe one or two points…*
>
> *~ …Yeah this is a calculus error again because the notion of continuity is only explained in a calculus course. This is a disaster. I would give this person three points for having identified the points where the function is potentially discontinuous. They've at least drawn a graph to indicate that there is a break in the graph but from their answers it is clear that they do not understand left and right continuity.*
>
> *~This is a disaster. I would give two points for writing something down I guess…*

Sensible System: Instructor F quickly assessed the student as not understanding what was going on. He felt the student's work was a disaster and only deserved a few points. He calls these mercy points.

IB-E3-Q2i-SP14.

IB-E3-Q2i-SP14 is also an SEE that required the student to find the points of discontinuity. Like the other SEE of the same question type, the highest score was 8/10. However, in this case Instructor C (as well as Instructor D) assigned the 80% grade. The errors Instructor C identified are in mathematical language which, in terms of grading exams, is not considered to be very important to him.

~There are examples where the calculus is all true and people organize things and make mistakes. I'm not worried, I'll usually circle it and write some type of comment and give them nine out of ten.

~ ...So honestly, my feeling here is that this student has a good intuitive concept. They have problems with the language but they have an intuitive concept of the limit. I would be happier with this than with someone who has no idea what's going on. Yes this person needs just mathematical language and just the English language as well...

~Maybe not practicing enough to get used to the language. Otherwise I don't see an error conceptually.

~...Its not that important. The language, people will find a way to talk to each other eventually.

~I would again give them eight out of ten. I know the expressions over there on the left are incorrect but they're also no equal signs there. Strictly speaking these are mental computations the student is using to [consider the limit]. Again I'd be pretty serious here. ***I:*** *okay.*

Sensible System: Instructor C gave two reasons why this problem was wrong. One is that the student used inappropriate notation. However, language and notation is not worth much of a deduction. It is more important that conceptual understanding is shown. Also, he reasoned that the student was using inappropriate notation to keep track of their work which seems to be of no concern to Instructor C when grading.

Instructor D's reasons for assigning 8/10 are also related to mathematical language.

> *~Oh I see they don't carry the limit notation through. I normally will take a point off anytime a student does that. Let's see, that mistake was made four times so I would probably take off two points.*
>
> *~...I guess the problem here is we're making statements like f of zero is two or f of zero is one...The student is just evaluating the function at the limiting value of x which is sort of okay if you know which piece you're using. I would probably give the student eight out of ten. The most serious for me is that the student is saying that f of zero is two and f of zero is one at the same time, that's a bit disturbing...*

Sensible System: It does not appear that Instructor D noticed that the student used inappropriate values to determine continuity of the function. He recognized that the student was equating two and one by saying they both are equivalent to f(0). He sees this as the main issue with this problem. Again he docks one point for each instance of misuse of mathematical language.

Again for SEE IB-E3-Q2i-SP14 the grades jump from an above average score to a failing score. The first failing score, assigned by Instructor G, is 5/10.

> ~...*Anyway, they are putting the wrong numbers in and they're using funny notation. But they are following more or less the right procedure... That might be prior knowledge. They are writing f of zero and putting a different number in, both cases I just have no idea where those numbers are coming from. This is one of those answers that's hard to understand. There is certainly stuff here that they are doing correctly but the overall method seems to be more or less unwise this is a case where the simple errors are serious. I might give this half credit...*

<u>Sensible System</u>: Instructor G viewed the student's work as nonsense which he felt was worth only half credit. Though he recognized that some parts of it were correct, the structure of the work was inappropriate which warranted a failing score.

Instructor E gave a score of 3/10 or 4/10. He felt the student was disorganized.

> ~...*So they are computing the left limits...So they just get it wrong...I don't know what's going on there...They are doing something at minus one, I don't know. So this student is much more adrift. They know the function is discontinuous because the limits from the left and right are unequal but after that point, um....This student just doesn't seem to understand how to compute a limit...They write down the correct function but...this might be a prior knowledge, a mathematical maturity type error.*

Sensible System: The student appeared to Instructor E to be lost and disorganized. He sees a misunderstanding in computing limits, a large part of the problem.

The lowest score was 2/10. Instructor F saw little work worth awarding points to IB-E3-Q2i-SP14.

> 676-682~...*This is a disaster. I would give two points for writing something down I guess. It is just littered with both types of errors...*

Sensible System: Again Instructor F assigned mercy points to disastrous work.

> *IB-E2-Q2i-SP5.*

The next two question types are finding the equation of the tangent line. Instructor C assigned a grade of 8/10 which reflects his stance that when grading he should focus on the student's ability to demonstrate that they've mastered the concepts of the course.

> ~T*here is an algebra mistake there and its just, I will usually give that full credit because I think that stuff is irrelevant. You know it's more important that they get the concepts...*
>
> ~*There are examples where the calculus is all true and people organize things and make mistakes. I'm not worried, I'll usually circle it and write some type of comment and give them nine out of ten.*
>
> ~T*he sign error, I would be lenient with that because it's just a mistake they made with the signs. They got the quotient rule right. The two x bit [with the cosine] is very ambiguous. They are actually understanding some things here, namely sine squared plus cosine squared is one...So they*

made two sign mistakes…They know enough basic trig to see things cancelling out here. Eight out of ten. This is all pre calc errors…

Sensible System: All of the calculus was right in this problem according to Instructor C. He showed concern for the things that are being tested. Therefore the sign errors, which are noted to be pre-calculus errors, are not that important. Also, the error with manipulating cosine was considered. However, it may be the student's ability to later demonstrate that they know some basic trig that persuaded him to overlook this error in assigning a grade.

An average score of 7/10 was given to SEE IB-E2-Q2i-SP5 by Instructor D. He attributes the errors to small mistakes in algebra and trigonometry.

~Simple arithmetic mistakes I normally do not view as being very serious in nature and I try to take off only token points because that is usually just a result of a time pressure exam situation.

~So if I can discern that the student is showing knowledge of the calculus but is somehow impeded by poor algebra skills I try to weigh that and give them some credit for knowing the calculus…

~…I would probably take off three points, actually the student seems to recover from this sign error, two wrongs make a right. But I would still ding the student for that so I would take off a point for that and the x cosine x mistake I would take off a couple points for that. I'm thinking seven out of ten. There are some good things here and [the errors are] mostly prior knowledge.

Sensible System: Instructor D's responses seem to confuse algebraic and arithmetic mistakes. [Notice that he does clearly distinguish between sign errors attributed to calculus and those attributed to algebra/arithmetic]. Other instances of arithmetic error classifications confirm that in this problem he is referring to an arithmetic mistake which he does not take very seriously and warrants a ding, namely a deduction of only one point. The error in trigonometry, which the instructor was not sure how to classify, was weighted more heavily. This can be supported by his belief that trigonometry is somewhat serious in a calculus course because of its use and importance in subsequent courses, like differential equations. Though the errors are mostly prior knowledge in nature, the instructor sees that the student does "some good things [in calculus]".

7/10 was the score given to SEE IB-E2-Q2i-SP5 by Instructor E. As in previous problems he determined the point score by dividing the problem into parts and assigning points accordingly.

> ~...*This is mostly correct. If you were to give five for getting the right shape of the derivative and five for computing things out appropriately. I mean I'd give them five for getting the derivative mostly correct. And then these manipulations, they make a series of errors. I mean x cos x, writing it as cos 2x is a pretty serious error. That shows they really don't understand...*

Sensible System: Instructor E demonstrates a pattern of attempting to allot a certain amount of points to each portion of the problem. Here he saw that the student set up the problem and understood the general idea which seems to be most important to him.

Instructor G felt that the SEE IB-E2-Q2i-SP5 was borderline average and designated a score between 6/10 and 7/10. The multitude of errors in simplifying lowered the grade.

> *~Problems with algebra or with trig functions, those can be serious and I expect them to be able to do that aspect of it. I am definitely more lenient with a simple algebra mistake as opposed to setting up the problem wrong or mixing up integrals and derivatives. So basically arithmetic I don't take very seriously at all. I'll take off a point or something. I don't think it means that they don't understand something. But with algebra or trig I do want them to do that correctly so I take that somewhat seriously.*
>
> *~...So that would also be a prior knowledge, or just sloppiness. Then this thing with cosine two x; I guess I would call it prior knowledge. So what exactly are they saying. They are saying that x cosine x is cosine 2 x, so some mix of algebra and trig. Its hard to know how they got that. So this first thing they wrote down is almost correct and then they just dug a whole by trying to simplify it. I don't know what their instructions were. If I gave a problem like this I would tell them they could stop here [after applying the rule] I wouldn't always want them to simplify it all the way down.*

<u>Sensible System</u>: Instructor G feels that students can dig "...a whole by trying to simplify...". In this case the student's errors were attributed to both algebra and trigonometry which are moderately serious to this instructor.

Instructor F felt the SEE IB-E2-Q2i-SP5 deserved between 5/10 and 6/10 points. His comments clearly depict a strong belief that trigonometry is a major part of calculus.

~And the third and perhaps the most egregious omits of high school is that fact that they don't understand or know very little of trigonometry...

*~**I**: And out of these topics what prior knowledge skills do you feel the students absolutely need to be successful. **P**: Absolutely need this notion of being able to handle computation by hand and have an idea of quantity and size. The second is trigonometry.*

*~**I**: Can you describe which areas you penalize most harshly for? **P**: Trigonometry.*

~But as a mathematician I cannot imagine doing calculus without trig.

~...No but this business of writing x times cosine x as cosine two x, that is a very bad mistake, as prior knowledge as it gets. The sign error in the first problem doesn't bother me too much. It is a prior knowledge. error but in the scale of errors, these things happen. But the [cosine error] is very bad. And the second error in the second step is writing sine x times sine x is sine of x squared. Now this is not an error per se in they have eventually done it correctly in all the mess that they've made but I always tell my students 'when you write sine of x put the x in parenthesis'...very bad prior knowledge. error again it speaks to the point I was making is that they don't understand trigonometric functions. They see the cosine and they somehow feel they can float variables in and out of there....Umm,

I would give this five or six. Because they have carefully applied the quotient rule and got most of it correct and in the rest they did some calculations and did okay.

Sensible System: Again Instructor F applied approximately half credit for setting things up correctly. However, he felt the problem with trigonometry (which he admittedly grades most harshly) was quite serious.

IB-E2-Q3i-SP7.

The other SEE that dealt with finding the equation of the tangent line was IB-E2-Q3i-SP7. Instructor C's pattern of grading mainly based on student demonstration of conceptual understanding is again confirmed here. He gave this SEE between 14 and 15 points out of 15.

~...Really a small algebra error I won't...I really will grade most harshly for someone who will write down an answer with no explanation even if it's the correct answer.

~Clearly the student knows how to use the power rule and subtract fractions even when it's a negative fraction. The things that are being tested here the student knows how to do.

~They differentiated correctly, they used a power rule, a chain rule...that's all very good. They had to plug in a number, except five plus four is not five times four. Was everything else done correctly after that?... Maybe fifteen depending on how I was feeling.

Sensible System: This was a simple error that the instructor saw as something anyone could do. This may even warrant full credit since everything else was correct. Again the material that is being tested is all correct. Therefore, Instructor C was content with the students work.

Instructor D's grade of 12/15, an above average score, reflects his lenience in arithmetic errors.

> *~My view is that if I'm teaching them calculus that should be the main thing. So if I can discern that the student is showing knowledge of the calculus but is somehow impeded by poor algebra skills I try to weigh that and give them some credit for knowing the calculus but unfortunately sometimes the algebra is getting in the way.*
>
> *~Most lenient would be a simple arithmetic mistake..*
>
> *~...so five plus four is twenty. I would say that is certainly a prior knowledge mistake and I would put that in the category of dumb arithmetic mistake...I think the student knows how to find the derivative, knows the derivative has to be evaluated at two. The five plus four is twenty is a silly arithmetic mistake. The problems in trying to simplify that expression are a little more serious.*

<u>Sensible System</u>: Instructor D recognizes a "silly arithmetic" mistake that was previously shown to be worth only a one-point deduction. Again the more serious prior knowledge errors in simplification are accounted for but do not overshadow the calculus that the student does correctly.

Straddling the line between a below and an above average grade, Instructor F assigned a score between 10/15 and 11/15 to IB-E2-Q3i-SP7. He found small errors, but their quantity was reason to deduct up to five points.

~...its sort of littered with errors...Anyway, the first is of course a very common mistake. I wouldn't call it a prior knowledge error it's just an arithmetic error that a lot of people make. The prior knowledge error is in writing down the equation of the line, where they wrote the x minus three instead of x minus two. That could have been an oversight...So the five plus four is not so serious, these things happen. What is serious is this business with four over two root twenty becoming ten root five. I would give this a ten or eleven.

Sensible System: Several errors were classified as common by Instructor F. He seems to not take them very seriously as they are errors that "could have been an oversight".

Both Instructor G and Instructor E gave the lowest score of 10/15. Instructor G's displeasure with the multitude of mistakes and sloppiness justified a 5 point deduction.

~...If everything is correct and they didn't write this last line then I might not take off anything. That's an easy error, that's a silly arithmetic error. I'd call that prior knowledge and if everything else were correct I might just take off one point for that. in the end I would take off another point for writing three instead of two. That's not a serious error, I'm not sure what it is. I think that's just sloppiness. I'm not sure if I would call that prior knowledge and then changing to ten root five doesn't look very good. And there is nothing wrong with the simplifying, in my opinion. In fact this five plus four is twenty, I mean I've been calling them prior knowledge but

maybe it's just sloppiness. I don't think if you asked this student what five plus four is they would say twenty. Out of fifteen, so they are basically doing the right thing but there are lots of little mistakes all over the place. I might give ten point for that.

Sensible System: Instructor G thinks that the mistakes individually may not warrant any point deduction. He saw them as a result of the student being sloppy but not evidence of any misunderstanding of calculus on the student's part. However, as there were several mistakes, a deduction of five out of fifteen points was made.

Instructor E's response on this problem was brief and did not allow me to determine a specific reason behind his scoring. I can only point out that partial credit was given.

~If the student gets some of the problem right I would give them some credit

~That's a calculus error for sure. I'd probably give ten points.

Sensible System: It is not clear hear why ten points are allotted. It can only be discussed in terms of the student receiving partial credit for getting some of the problem right.

IB-E2-Q3i-SP9.

The last WSR is also found in a tangent line question. 13/15 was the highest grade given to this SEE. Instructor D seemed to be comfortable with the student's formulation of the problem despite the difficulties the student had in finding the actual equation of the tangent line.

~...Right I see that they differentiated x squared and got two. So that resulted in a derivative of one-third, it looks like the set up of the tangent line equation is okay...

<u>Sensible System</u>: Though the error is classified as a calculus error Instructor D gave a score of 13/15. He recognizes that the student knows how to set up the problem. It may be that he feels this error is not very serious. Instructor D seems to have a pattern of assigning an average or above average score to calculus errors that do not impede on the overall structure/process/format of the problem.

Almost the same as Instructor D, Instructor C assigned a grade between 12/15 and 13/15. He approaches grading in this case by giving the student the benefit of the doubt. More specifically, he assumes that the student's errors are not sincere reflections of misunderstandings but a brief lapse in judgment.

~Its not an error they are consistently doing. I would know the student...

~...If I knew the student and this was just a blip I would cut the student some slack. If I knew the student struggled with this I might be a little more strict. This is certainly a calc error. I'd have to know the student. Twelve or thirteen...

<u>Sensible System</u>: Despite the fact that Instructor C identified this to be a calculus error he reasoned that it may just be a 'blip'. Therefore he could let the student slide. Also, he commented that the grade would depend on what else he knows about the student. Particularly, if the student had a problem with this specific error in the past Instructor C would not be as lenient.

Just as Instructor D, Instructor F felt 12/15 was an appropriate score because the errors (in calculus) were considered to be small.

> ~...*Oh I see so there is just a very small error in the very last step. Is that right? ...In terms of prior knowledge I would say there isn't really a prior knowledge error in my opinion. There is a simple calculus error with the chain rule. Which could happen to me I guess...*

<u>Sensible System</u>: Instructor F recognized both a small error and an error that he sympathized with. He felt he could have made the same mistakes and that the errors did not indicate that the student misunderstood the concepts this problem aimed to assess.

Again, 12/15 was the score given to SEE IB-E2-Q3i-SP9 by Instructor G. He, like Instructor C, believed that the student demonstrated understanding of the process of finding the equation of a tangent line.

> ~...And that's moderately serious but they were more or less doing the right procedure...

<u>Sensible System</u>: Instructor G viewed the student's error in the chain rule as a calculus error but not an error in understanding the calculus process or application. So the error was not very serious but still deserved to be penalized.

The lowest score, 10/15, was assigned to SEE IB-E2-Q3i-SP9 by Instructor E who may have been more lenient but the student's sloppiness was such that a low grade was deemed appropriate.

> ~...*Yeah that's something I wouldn't be so harsh with. You just assume they were rushing. They did everything correctly up to then it seems. So*

yeah, that's just a slip of concentration. Then did they work everything correctly?... So it seems they have a good understanding. So certainly I'd give ten out of fifteen points maybe a little bit more....probably a prior knowledge error. But I am guessing that looking at the student's work that they are just very sloppy. But who knows. You have to go off of what is written. I have the impression that the student has a very good grasp but that's just a slip like the five plus four is twenty.

Sensible System: Though Instructor E found several errors, he classified these errors as a lapse in concentration or problems with being sloppy. Instructor E believed this student "has a very good grasp" on the concept addressed in this problem. Therefore, the student deserved 10/15 or maybe even more.

The instructor comments outlined above illuminate how the perceived inconsistencies among the set of instructors; namely, the WSRs, can be explained by a sensible system perspective. For each SEE with a WSR the grade assigned depended upon the individual instructors focus. Some instructors focused on students' demonstration of conceptual understanding, others on student ability to lay out a logical progression through the problem, and still others on the technical accuracy of student work. These focuses also varied across SEE, not just across instructors, which makes the sensible system approach even more useful in explaining the perceived inconsistencies. Further discussion of the results follow in Chapter Six.

Chapter 6

Conclusion

The presented report describes the motivation and background for inquiry into calculus instructors and prior knowledge. The methodology outlines the phenomenological approach used to access the perspective of instructors when considering prior knowledge errors. Particularly, the methods used set out to answer the following questions:

1. How do calculus instructors define prior knowledge?
2. How do calculus instructors view prior knowledge in the context of a calculus course?
3. How do calculus instructors assess prior knowledge errors?

The analysis of collected data proved to provide a more informed view about the first two questions. After preliminary analysis of the collected data I determined that the third research question, however; would likely require the use of a quantitative data collection process. However, there are two points related to research question three that I would like to highlight. First, the implication that no grading pattern in relation to prior knowledge exists is worth some discussion. Particularly, we as educators and researchers should consider what this means to students. What might students gather from the fact that a set of instructors can differ widely in the grading of prior knowledge while at the same time all hold a general belief that prior knowledge is essential to a course. More importantly, how might students react to such observations? I would suggest, as Snyder (1970) has, that students will disregard any claims their professor makes about the important aspects of a course whenever those claims are unsupported by grading practices. Unfortunately, for many students it can be quite problematic to even identify the skills or concepts that instructors deem important; let alone make conclusions about the skills or concepts that are assigned the most weight in the grading of assignments or exams. More troubling for students is the possibility that they must decipher this "hidden curriculum" for several professors who each hold distinct beliefs and/or practices. Specifically in reference to prior knowledge, students may begin to feel that the importance of prerequisite skills is a matter of opinion and therefore an unnecessary requirement of academic endeavors.

Secondly, despite the absence of a prior knowledge-specific grading pattern in my data, the scoring of the WSRs does seem to follow a rank order. Namely, the order of instructors from the highest to the lowest grades appears to be fairly consistent. With the exception of IB-E1-Q5iii-SP4, Instructor C and Instructor D tend to assign the highest scores, whereas Instructors E, F, and G alternate among the middle to lowest scores. Instructor F notably assigned the lowest score more often than any other instructor. It is also worth mentioning that excluding IB-E1-Q4iii-SP10 as well would make these observations much more likely to be evidence of a pattern in grading for this set of instructors. Specifically, Instructor C and Instructor D consistently assigned the top two scores and Instructors E, F, and G the bottom three when only IA-E1-Q2-SP9, IB-E3-Q2i-SP14, IB-E2-Q2i-SP5, IB-E2-Q3i-SP7, and IB-E2- Q3i-SP9 are considered.

To understand what aspects of IB-E1-Q5iii-SP4 and IB-E1-Q4iii-SP10 that could have allowed them to break this possible pattern I again used the instructors' comments to compare their scores. IB-E1-Q5iii-SP4 is particularly interesting because it is not only opposite of the pattern with Instructor C and Instructor D as the two lowest scores, but it is also the exact mirror image of IA-E1-Q2-SP9. The order of instructors for IA-E1-Q2-SP9 from highest to lowest is C, D, G, F, E. The order for IB-E1-Q5iii-SP4 is the reverse: E, F, G, D, C. As awkward as this may seem, a closer look at the sensible systems reveals that each instructor who switched rank position for these two SEEs generally used the same considerations to evaluate each student's work. Consider Instructor C's sensible systems for these two SEEs. In both cases he looked for the student to show conceptual understanding of calculus as well as the ability to work through the correct process of the problem. Instructor E, for example, also emphasized an understanding of calculus in his grading, but he distinguished between perceived "slips" in student work and algebra errors that are cause for a large point deduction. Interestingly, the same rationales the instructors used to evaluate SEE IA-E1-Q2-SP9 and SEE IB-E1-Q5iii-SP4 can be found in the sensible systems of SEE IB-E1-Q4iii-SP10 as well. Just as an example, consider Instructor G's sensible systems for these three SEEs. He appears to focus on the students' ability to demonstrate an understanding of the overall process needed to complete the problem despite possible difficulties in carrying out each necessary step. Similar comparisons can be made for the remaining instructors in regards to SEE IB-E1-Q4iii-SP10. So, yes, IB-E1-Q5iii-SP4 and IB-E1-Q4iii-SP10 did not follow the ranking pattern of instructor scores found in the entire set of WSRs. However, the above observations suggest that sensible systems can be used to begin an investigation into instructor profiles.

Keep in mind that though these considerations are noteworthy, they still do not directly address the main premise behind research question three as it relates to prior knowledge. Therefore, it is set aside for future exploration. Nonetheless, the responses that the instructors provided for the interview portion that intended to answer the third research question did instigate interest into perceived inconsistencies across instructors. Namely, the WSRs motivated a closer look into the sensible system framework. A summary of the findings concerning research question one, research question two, and the WSRs is below.

How do Calculus Instructors Define Prior Knowledge

The interview data revealed that the real definitions (see Chapter Two) for prior knowledge that calculus instructors hold vary widely. Common themes, however, were identified that may very well represent components of prior knowledge that all instructors would agree upon. The components of prior knowledge that were consistent across the entire set of instructor participants were algebra and trigonometry. Algebra was described as a foundational topic for which students needed to master for two main reasons. Instructors felt that in order for students to understand the most basic concept of calculus, rate of change, a solid grasp on algebraic processes, such as manipulating fractional expressions, must be attained. Also, facility with algebra was deemed important because it allows students to work through a multitude of problems without getting tripped up with mistakes. Besides algebra, the other collectively defined component of prior knowledge was trigonometry. Instructors emphasized the difficulties that arise when students are unable to compute trigonometric values of special angles or recall trigonometric identities.

How do instructors view prior knowledge within the context of a calculus course?

The comments that exemplified how instructors viewed prior knowledge were narrowed to two major themes. The first, *ideal versus reality of student preparedness*, represents the instructors' issues concerning the gap between the preferred and actual skill levels of their students. Each instructor expressed that this gap prevented students from staying abreast with course concepts, required that less material be covered within the semester, or stunted the depth with which the course concepts were explored. The only other theme that was shared by all instructors and shed light onto the topic of prior knowledge was *conceptual understanding and prior knowledge*. The remarks attributed to this theme were two fold. On one hand, instructors felt that students could understand concepts without necessarily being able to apply prior knowledge skills to calculus problems. On the other hand, the opinion of other instructors upheld that in order to develop a deep understanding of calculus students cannot be bogged down with issues of algebra or trigonometry.

Wide Score Ranges

The wide variance of scores assigned to several SEEs initially appeared to represent inconsistencies among the instructors grading patterns. However, using an adaption of Leatham's (2006) sensible belief systems, I was able to identify instructor remarks that supported the differences in grading. More specifically, when allocating scores one instructor may have considered a student's ability to outline the procedures for solving problems as worthy of an above average score. At the same time, another instructor may assign points based on the quantity of correct steps the student made toward completing the problem. Though the WSRs initially appear to represent inconsistent grading patterns, these differences in assessment strategy are justified after considering the perspectives of the individual instructors using the sensible systems framework.

Limitations

One obvious limitation of this study was the inability to answer research question three. That is, I was unable to address the ways that instructors grade prior knowledge errors. In retrospect it appears that a more systematic presentation of SEEs during instructor interviews was needed. Additionally, quantitative methods of inquiry would possibly be better suited to address the issue of how prior knowledge errors are graded. Despite this setback, the interview items that I expected would produce data relevant to this question illuminated another avenue of inquiry that proved fruitful in considering variations across instructors grading decisions. An additional drawback of the research design was the limited number of participants available for the pilot study. Because only two instructors were teaching the course during the data collection period, I was unable to include a wide variety of instructor interviews to aid in the development of the primary study. Furthermore, the pilot participants were GTAs instructing their own calculus courses for the first time. So their conceptions of prior knowledge and assessment may differ from those of the experienced faculty instructors'.

Participants with closely comparable backgrounds related to assessment and calculus instruction would likely have yielded more dependable results.

Further Research

The reported findings have revealed useful results pertaining to the perceptions that calculus instructors have about prior knowledge. But this is by no means an exhaustive report. Further exploration is needed to confirm and possibly expand upon the elements of prior knowledge that calculus instructors deem necessary for their students to possess.

In dealing with WSRs, the current study may well be extended to include individual instructor profiles based on grading priorities identified through the sensible system framework. Such profiles could be developed to determine possible relationships with teacher characteristics such as amount of teaching experience or educational background. Additionally, the profiles of instructors across academic disciplines could be compared.

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APPENDIX A

Pilot Interview Protocol

Interview Protocol

Introduction

Thank you for your time and willingness to participate. As you know, I am interested in the skills and performance of calculus students. Particularly, I am trying to understand how instructors respond to prior knowledge errors made by calculus students and how the prerequisite skills that students bring to calculus may affect their performance in the course. If the questions are general and abstract, you may volunteer any detail you wish. You also have the option of declining to answer – passing – on any of the questions. Do you have any questions before we start?

Interview Questions

What does being a calculus instructor entail?

1) Please explain your role as the instructor of an Engineering Calculus course at the University of Oklahoma.
2) Outside of your official role as instructor, what other duties do you perform that have a direct or indirect effect on your students and their performance? (such as extra office hours or supplemental course materials)

How does prior knowledge fit into the context of calculus?

3) How do you feel prior knowledge influences the teaching and learning of calculus?
4) What prior knowledge do you believe students need in order to be successful in a calculus course?
5) What influence/importance does prior knowledge have on the performance and understanding of calculus by students?
6) What concepts are included in your calculus course?

How do instructors weigh mistakes in prior knowledge when assessing the performance of students?

7) In what ways, if any, do you distinguish between calculus mistakes and prior knowledge mistakes when grading and/or assessing your students?
8) Other than calculus mistakes, how would you characterize the mistakes that calculus students make?
   a) How do you judge which subject areas student have trouble with?
   b) Within which subject areas do those mistakes reflect skill deficiencies? (basic math, algebra, geometry, trigonometry, etc.)

c) Which of these subject areas would you say that calculus students have the most trouble with?

9) How do you believe calculus mistakes compare to prior knowledge mistakes (mistakes in basic math, algebra, geometry, trigonometry, etc.) in a calculus course? In other words, what importance do calculus and prior knowledge have when considering grading, student understanding, etc?
10) Which areas of prior knowledge do you penalize most harshly for mistakes when grading?
11) Which areas of prior knowledge do you penalize least harshly for mistakes when grading?
12) What other comments would you like to provide that you believe would inform the topic of prior knowledge deficiencies in calculus students.

What grading policies were applied to grading this exam?

13) What kind of discretion, if any, was used in assigning point values? (For example, was any leniency or strictness applied to certain problems or certain types of mistakes?)
14) Did you find that any exams were graded incorrectly? If so, please explain.

Please provide explanation concerning specific Exam III questions.

15) What work/explanations were expected of the students on each question?
16) How did you decide what point value to assign to each portion of each question?
17) What were the most common errors for this problem
    a) How would you classify these errors; prior knowledge or calculus?

<u>Closing</u>

Now that we are done, do you have any questions you'd like to ask me about this research project? If you want to contact me later, here is my contact information (supply participant with a card). Also, I may need to contact you later for additional questions or clarification. Can I also have your follow-up contact information?

Participant Name:______________________________

Participant Phone:______________________________

Participant Email:______________________________

Participant Address: ___________________________

APPENDIX B

Interview Protocol

Interview Protocol

Introduction

Thank you for your time and willingness to participate. As you know, I am interested in the skills and performance of calculus students. Particularly, I am trying to understand how instructors respond to prior knowledge errors made by calculus students and how the prerequisite skills that students bring to calculus may affect their performance in the course. If the questions are general and abstract, you may volunteer any detail you wish. You also have the option of declining to answer – passing – on any of the questions. Do you have any questions before we start?

Interview Questions

1) What does being a calculus instructor entail?
2) Please explain your role as the instructor of an Engineering Calculus course at the University of Oklahoma.
3) Outside of your official role as instructor, what other duties would you perform that have a direct or indirect effect on your students and their performance? (such as extra office hours or supplemental course materials)
4) How does prior knowledge fit into the context of calculus?
5) How do you feel prior knowledge influences the teaching and learning of calculus?
6) What prior knowledge do you believe students need in order to be successful in a calculus course?
7) What influence/importance does prior knowledge have on the (a) performance in and (b) understanding of calculus by students?
8) What topics are normally included in your calculus course?
9) Describe the prior knowledge skills that are needed to master these topics?
10) How do instructors weigh mistakes in prior knowledge when assessing the performance of students?
11) In what ways, if any, do you distinguish between calculus mistakes and prior knowledge mistakes when grading and/or assessing your students?
12) Other than calculus mistakes, how would you characterize the calculus students' mistakes?
    a) How do you judge which subject areas students have trouble with?
    b) Within which subject areas do those mistakes reflect skill deficiencies? (basic math, algebra, geometry, trigonometry, etc.)

c) Which of these subject areas would you say that calculus students have the most trouble with?

13) How do you believe calculus mistakes compare to prior knowledge mistakes (mistakes in basic math, algebra, geometry, trigonometry, etc.) in a calculus course? In other words, what importance do calculus and prior knowledge have when considering grading, student understanding, student performance, etc?

14) Which areas of prior knowledge do you penalize most harshly for mistakes when grading?

15) Which areas of prior knowledge do you penalize least harshly for mistakes when grading?

16) What other comments would you like to provide that you believe would inform the topic of prior knowledge deficiencies in calculus students.

Elaborate on your approach to grading specific student exam questions.

17) Review the following student exam examples and describe how you would grade it using the following questions as a guide:
    a) If possible identify any errors.
    b) Classify the error as a calculus error or prior knowledge error.
    c) Classify, if possible, the type of prior knowledge error.
    d) How would you score this question given the stated point value?

I. Find the derivative using the limit definition.
    a. IA-E1-Q2-SP#9 (20 pts):
    b. IB-E1-Q5iii-SP#3 (10 pts):
    c. IB-E1-Q5iii-SP#4 (10 pts):

II. Identify points of discontinuity or intervals of continuity
    a. IA-E1-Q3-SP#9 (20 pts):
    b. IB-E1-Q4ii-SP#10 (10 pts):
    c. IB-E3-Q2i-SP#4 (10 pts):
    d. IB-E3-Q2i-SP#14 (10 pts):

III. Find the derivative using rules
    a. IA-E2-Q1B-SP#9 (5 pts):
    b. IA-E2-Q1D-SP#10 (5 pts):
    c. IA-E2-Q2B-SP#15 (5 pts):
    d. IB-E2-Q2i-SP#5 (10pts):

IV. Find tangent lines
    a. IB-E2-Q3i-SP#7 (15 pts):

b. IB-E2-Q3i-SP#9 (15 pts):

c. IB-E2-Q3i-SP#14 (15 pts):

V. Implicit Differentiation

a. IB-E2-Q4ii-SP#3 (10 pts):

b. IB-E2-Q4ii-SP#9 (10 pts):

VI. Related Rates

a. IA-E2-Q6-SP#1 (15 pts):

b. IA-E2-Q6-SP#3 (15 pts):

c. IB-E3-Q3-SP#1 (10 pts):

VII. Max/min applications

a. IB-E3-Q7-SP#9 (10 pts): problem solving

b. IB-E3-Q7-SP#10 (10 pts): wrong perimeter formula

Background information

18) What is your title? (GTA, assoc. professor, asst. professor…)

19) How long have you taught college mathematics?

20) How many times have you taught this calculus course?

21) What is your area of research?

Closing

Now that we are done, do you have any questions you'd like to ask me about this research project? If you want to contact me later, here is my contact information (supply participant with a card). Also, I may need to contact you later for additional questions or clarification. Can I also have your follow-up contact information?

Participant Name:______________________________

Participant Phone:______________________________

Participant Email:______________________________

Participant Address: ___________________________

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APPENDIX C

Student Error Examples (SEEs)

IA-E1-Q2-sp#9

Problem 2. Let f be the function defined by $f(x) = x^2 + x$. Find the derivative $f'(x)$ using the limit definition.

$f(x) = x^2 + x$

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$\Rightarrow \lim_{h \to 0} \frac{((x+h)^2 + (x+h)) - (x^2 + x)}{h}$

$\Rightarrow \lim_{h \to 0} \frac{x^2 + 2xh + h^2 + x + h - x^2 - x}{h}$

$\Rightarrow \lim_{h \to 0} x^2 + 2x + h + x - x^2 - x$

$\Rightarrow \boxed{2x}$

IB - E1 - Q5iii - SP#3

(iii) Use the limit definition of derivative to find $f'(2)$ for the following function. (10 points)

$f(x) = \frac{1}{\sqrt{x+2}} = \frac{\sqrt{x+2}}{x+2}$

$\lim_{h \to 0} \left[\frac{\frac{\sqrt{x+h+2}}{x+h+2} - \frac{\sqrt{x+2}}{x+2}}{h} \right] = \lim_{h \to 0} \frac{\sqrt{x-h+2} - \sqrt{x+2} + h}{x+2}$

$\lim_{h \to 0} \frac{\sqrt{x+h+2} - \sqrt{x+2} + h}{x+2} \lim_{h \to 0} \frac{\sqrt{x+2} - \sqrt{x+2}}{x+2} = 0$

$F'(x) = 0$ so $F'(2) = 0$

IB-E1-Q5iii-SP #4

(iii) Use the limit definition of derivative to find $f'(2)$ for the following function. (10 points)

$$f(x) = \frac{1}{\sqrt{x+2}}$$

$\lim_{y \to x} \frac{f(y)-f(x)}{y-x} = \lim_{h \to 0} \frac{(a+h) - (a)}{h}$

$\lim_{h \to 0} \frac{\frac{1}{\sqrt{a+h+2}} - \frac{1}{\sqrt{a+2}}}{h}$

$a = 2$

$\lim_{h \to 0} \left(\frac{\frac{1}{\sqrt{4+h}} - \frac{1}{2}}{h} \right) \cdot 2 \cdot \sqrt{4+h}$

$\lim_{h \to 0} \frac{2 - \sqrt{4+h}}{2h\sqrt{4+h}} \cdot \frac{2+\sqrt{4+h}}{2+\sqrt{4+h}}$

$\lim_{h \to 0} \frac{2-(4+h)}{4h\sqrt{4+h} + 2h(4+h)}$

$\frac{-2+h}{4h\sqrt{4+h} + 8h + h^2}$

$\lim_{h \to 0} \frac{-2}{4\sqrt{4+h} + 8 + h}$

$\lim_{h \to 0} \frac{-2}{4\sqrt{4+0} + 8 + 0}$

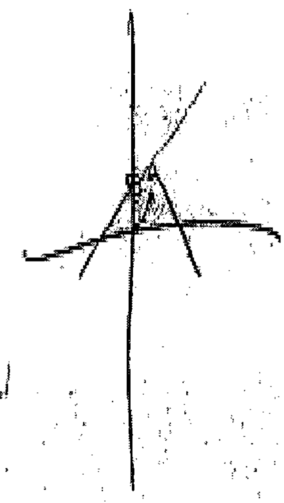
$\lim_{h \to 0} \frac{-2}{16} = -\frac{1}{8}$ $\lim \frac{1}{\sqrt{x+2}}$

8

IB-E1-Q4ii-SP#10

(ii) Find the numbers at which f is discontinuous. At which of these numbers is f continuous from the right, from the left, or neither? Explain your solution in detail. (10 points)

$$f(x) = \begin{cases} x+2 & \text{if } x < 0 \\ 2x^2 & \text{if } 0 \leq x \leq 1 \\ 2-x & \text{if } x > 1 \end{cases}$$



At zero, f(x) is discontinuous.

it is discontinuous from the left because it is undefined from that direction.

at 1, f(x) is also discontinuous, but from the right. if you approach 1 from the right, you will not reach it.

IB-E3-02i-sp#4

Problem 2.(i) Find the numbers at which f is discontinuous. Explain your solution in detail by computing the left and right limits at relevant points. (Note: You don't have to discuss right and left continuity) (10 points)

$$f(x) = \begin{cases} 1+x^2 & \text{if } x \le 0 \\ 2-x & \text{if } 0 < x \le 2 \\ (x-2)^2 & \text{if } x > 2 \end{cases}$$

$x = 0$

$\lim_{x \to 0^-} f(x) = 1 + x^2 = 1 + 0^2 = 1$

$\lim_{x \to 0^+} f(x) = 2 - x = 2 - 0 = 2$

$\lim_{x \to 0^+} f(x) \ne \lim_{x \to 0^-} f(x)$ x is discontinuous at 0

$x = 2$

$\lim_{x \to 2^-} f(x) = 2 - x = 2 - 2 = 0$

$\lim_{x \to 2^+} f(x) = (x-2)^2 = (2-2)^2 = 0$

$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^-} f(x) = f(2) = 0$

IB-E3-Q2i-SP#14

Problem 2.(i) Find the numbers at which f is discontinuous. Explain your solution in detail by computing the left and right limits at relevant points. (Note: You don't have to discuss right and left continuity) (10 points)

$$f(x) = \begin{cases} 1 + x^2 & \text{if } x \leq 0 \\ 2 - x & \text{if } 0 < x \leq 2 \\ (x-2)^2 & \text{if } x > 2 \end{cases}$$

$f(0)\ 1+(-1)^2 = 1+1 = 2 \quad \lim_{x \to 0^-} 1+x^2 = 2$

$f(0)\ 2-1 = 1 \quad \lim_{x \to 0^+} 2-x = 1$ so it is discontinuous at 0

$f(2)\ 2-1 = 1 \quad \lim_{x \to 2^-} 2-x = 1$

$f(2)\ (3-2)^2 = 1 \quad \lim_{x \to 2^+} (x-2)^2 = 1$

so it is continuous at 2

3

/

JA-E2-Q1B-SP#9

9

Calculus 1 MATH 1823 Section 001, Summer 2008
Exam 2 Monday, July 14, 2008, 9:30-10:20AM

In order to get full credit, all answers must be accompanied by appropriate justifications.

Name: **ID#:**

1(20)	3(20)	5(15)		
2(20)	4(10)	6(15)		total

100 points possible

Problem 1. Find the derivatives of the following

(1A) $f(x) = \sec(x) + \cos(x) + \tan(x) + \cot(x)$

$f'(x) = \sec(x)\tan(x) - \sin(x) + \sec^2(x) - \csc(x)$

(1B) $y = \sqrt[5]{t^2} + t^3 + \frac{1}{8}t^8 + \pi t^{-4} + 27 = t^{-\frac{2}{5}} + t^3 + \frac{1}{8}t^8 + \pi t^{-4} + 27$

$y' = -\frac{2}{5}t^{-\frac{7}{5}} + 3t^2 + \frac{9}{8}t^7 - 4\pi t^{-5}$

(1C) $H(y) = \dfrac{\sin(y) + y^2}{y^4}$

$H'(y) = \dfrac{(\cos(y) + 2y)(y^4) + (\sin(y) + y^2)(4y^3)}{(y^4)^2} = \dfrac{y^4\cos(y) + 2y^5 + 4y^3\sin(y) + y^2}{y^4}$

$\Rightarrow \dfrac{(y^3)[y\cos(y) + 2y + 4\sin(y) + 4y^2]}{(y^4)^2} = \dfrac{y\cos(y) + 2y + 4\sin(y) + 4y}{(y^4)^2}$

$x^7\sin(x)$

$x(\sin(x)) - 7x^7(\cos(x))$

(1D) $g(x) = \dfrac{3x - 7x^7\sin(x)}{x}$

$g'(x) = \dfrac{3x(49x(\sin(x)) - 7x^7(\cos(x))) + 3(7x^7\sin(x))}{x^2}$

IA-E2-Q1D-SP#10

10

Calculus 1 MATH 1823 Section 001, Summer 2008
Exam 2 Monday, July 14, 2008, 9:20-10:20AM

In order to get full credit, all answers must be accompanied by appropriate justifications.

Name: **ID#:**

1(20)	3(20)	5(15)		
2(20)	4(10)	6(15)		total

100 points possible

Problem 1. Find the derivatives of the following

(1A) $f(x) = \sec(x) + \cos(x) + \tan(x) + \cot(x)$

$f'(x) = \sec(x)\cdot\tan(x) + (-\sin(x)) + \sec^2(x) + (-\csc^2(x))$

(1B) $y = \sqrt[5]{t^2} + t^3 + \frac{1}{8}t^8 + \pi t^{-4} + 27$

$y' = t^{2/5} + t^3 + 1/8\, t^8 + \pi t^{-4} + 27$

$y' = 2/5\, t^{-3/5} + 3t^2 + t^7 + (-4\pi t)^{-5}$

(1C) $H(y) = \dfrac{\sin(y) + y^2}{y^4}$

$H' = \dfrac{\sin(y) + y^2}{y^4} = \dfrac{\sin(y) + y^2(4y^3) - y^4(\cos(y) + 2y)}{(y^4)^2}$

$H' = \dfrac{\sin(y) + 4y^5 - y^4\cos(y) - 2y^5}{y^8}$

(1D) $g(x) = \dfrac{3x - 7x^7\sin(x)}{x}$

$\frac{3x}{x} - \frac{7x^7\sin(x)}{x} = 3 - 7x^6\sin(x)$

$g' = \dfrac{3x - 7x^7\sin(x)}{x} = 3 - 7x^6\sin(x) = -42x^5\cos(x)$

$g' = \dfrac{3x - 7x^7\sin(x) - x(3 - 49x^6\cos(x))}{(x)^2}$

IA-E2-Q2B-SP#15

Problem 2. Find the derivative of the following

(2A) $h(x) = \sqrt{\frac{x}{\cos^3(x)+1}}$

$$h(x) = \left(\frac{x}{\cos^3(x)+1}\right)^{\frac{1}{2}}$$

$$h'(x) = \frac{1}{2}\left(\frac{x}{\cos^3(x)+1}\right)^{-\frac{1}{2}} \cdot \frac{(\cos^3(x)+1)(1) - (x)(-\sin^3(x))}{(\cos^3(x)+1)^2}$$

$$= \frac{1}{2}\left(\frac{x}{\cos^3(x)+1}\right)^{-\frac{1}{2}} \cdot \frac{\cos^3(x)+1+x\sin^3(x)}{\cos^5(x)+1}$$

(2B) $f(x) = \frac{\sin^2(x)}{(\sqrt{x}+3)^3}$

$$f(x) = \frac{\sin^2(x)}{x+27}$$

$$f'(x) = \frac{(x+27)(\cos^2(x)) - (\sin^2(x))(1)}{(x+27)^2}$$

$$= \frac{x\cos^2(x) + 27\cos^2(x) - \sin^2(x)}{(x+27)^2}$$

IB-E2-Q2i-SP#5

$$\frac{gf' - fg'}{g^2}$$

Quotient rule

Problem 2.(i) Find $f'(x)$ for the following function. (10 points)

$$f(x) = \frac{1+\sin(x)}{x+\cos(x)}$$

$f(x) = 1+\sin(x)$

$g(x) = x+\cos(x)$

$f'(x) = \cos(x)$

$g'(x) = -1+\sin(x)$

$$f'(x) = \frac{(x+\cos(x))(\cos(x)) - (1+\sin(x))(-1+\sin(x))}{(x+\cos(x))^2}$$

$$f'(x) = \frac{(\cos 2x) + (\cos(x))^2 - (1-\sin x^2)}{(x+\cos(x))^2}$$

$$f'(x) = \frac{\cos 2x + \cos x^2 - 1 + \sin x^2}{(x+\cos(x))^2}$$

$$f'(x) = \frac{\cos 2x}{(x+\cos(x))^2}$$

IB-E2-Q3i-SP#7

Problem 3. (i) Find an equation of the tangent line to the curve at the given point. (15 points)

$y = \sqrt{5+x^2}$ at the point (2,3)

$$\frac{1}{2\sqrt{5+x^2}} \cdot 2x$$

$$\frac{1}{2\sqrt{5+(2)^2}} \cdot 2(2)$$

$$\frac{1}{2\sqrt{5+4}} \cdot 4$$

$$\frac{1}{2\sqrt{20}} \cdot 4$$

$$= \frac{4}{2\sqrt{20}} = 2\sqrt{20} \rightarrow 2 \cdot 2\sqrt{5} = 10\sqrt{5}$$

$$y-3 = \frac{4}{2\sqrt{20}}(x-3)$$

$$y-3 = 10\sqrt{5}\,(x-3)$$

IB-E2-Q3i-SP#9

Problem 3. (i) Find an equation of the tangent line to the curve at the given point. (15 points)

$y = \sqrt{5 + x^2}$ at the point (2,3)

$\rightarrow y = (5+x^2)^{\frac{1}{2}}$

$\rightarrow y' = \frac{1}{2}(5+x^2)^{-\frac{1}{2}}(2)$

$\rightarrow y' = (5+x^2)^{-\frac{1}{2}}$

$\rightarrow y' = \frac{1}{\sqrt{5+x^2}}$

$\rightarrow \frac{1}{\sqrt{5+(2)^2}} \rightarrow \frac{1}{\sqrt{5+4}} \rightarrow \frac{1}{\sqrt{9}} = \frac{1}{3}$

$\rightarrow y - 3 = \frac{1}{3}(x-2)$

$\rightarrow y - 3 = \frac{1}{3}x - \frac{2}{3}$

$\rightarrow \boxed{y = \frac{1}{3}x + \frac{1}{3}}$

(ii) Find $\frac{dy}{dx}$ by implicit differentiation. (10 points)

$x^4y^2 - x^3y + 2xy^3 = 0$

~~$x^4(y(x))^2 - x^3(y(x)) + 2x(y(x))^3 = 0$~~

~~$\frac{d}{dx}\left[x^4(2y(x)) + 4x^3(y(x))^2\right] - \left[x^3(1) + 3x^2(y(x))\right] + \left[2x\left(3(y(x))^2\right) + 2(y(x))^3\right]$~~

$x^4y^2 - x^3y + 2xy^3 = 0$

$\frac{dy}{dx} = \left[x^4(2y') + 4x^3(y^2)\right] - (x^3y' + 3x^2y) + \left[2x\,3(y')^2 + 2y^3\right]$

$\frac{dy}{dx} = 2x^4y' + 4x^3y^2 - x^3y' - 3x^2y + 6xy'y' + 2y^3 = 0$

$\frac{dy}{dx} = 2x^4y' - x^3y' + 6x(y')^2 = -4x^3y^2 + 3x^2y - 2y^3$

$\frac{dy}{dx} = y'(2x^4 - x^3 + 6x) = -4x^3y^2 + 3x^2y - 2y^3$

$y' = \frac{-4x^3y^2 + 3x^2y - 2y^3}{2x^4 - x^3 + 6x}$

IB-E2-Q4ii-SP#9

(ii) Find $\frac{dy}{dx}$ by implicit differentiation. (10 points)

$x^4y^2 - x^3y + 2xy^3 = 0$

$\rightarrow x^4 2y \cdot y' + y^2 4x^3 - x^3 y' + y 3x^2 + 2x 3y^2 \cdot y' + y^3(1) = 0$

$\rightarrow x^4 2y \cdot y' - x^3 y' + 2x 3y^2 \cdot y' = -y^2 4x^3 - y 3x^2 - y^3$

$\rightarrow y' = \dfrac{-y^2 4x^3 - y 3x^2 - y^3}{x^4 2y - x^3 + 2x 3y^2}$

$\rightarrow y' = \dfrac{-y(y 4x^3 + 3x^2 + y^2)}{x(x^3 2y - x^2 + 6y^2)}$

IA-E2-Q6-SP#1

Problem 6. The volume of a sphere is decreasing at a rate of 25 $\frac{cm^3}{s}$. How fast is the surface area changing when the radius is 50 cm.

nun $\left\{ \begin{array}{l} \frac{dV}{dt} = 25\ cm^3/s = \text{Rate of decrease in volume} \\ R = 50\ cm = \text{Radius} \\ t = \text{time} \end{array} \right.$

nknown $\left\{ \frac{ds}{dt} = \text{change of surface area.} \right.$

uation $\left\{ \begin{array}{l} S = V\pi R^2 \\ \frac{ds}{dt} = \frac{dV}{dt}\pi 2R\frac{dR}{dt} \end{array} \right.$

$\frac{ds}{dt} = 25\pi 2(50)\frac{1}{400}$

$\boxed{\frac{ds}{dt} = \frac{25}{4}\pi}$

$V = \frac{4}{3}\pi R^3$

$\frac{dV}{dt} = 4\pi R^2 \cdot \frac{dR}{dt}$

$\frac{dR}{dt} = \frac{\frac{dV}{dt}}{4\pi R^2} = \frac{25}{4\pi(50)^2} = \frac{25}{4\pi 2500} =$

$\frac{1}{400\pi} = \frac{dR}{dt}$

IA-E2-Q6-S#3

Problem 6. The volume of a sphere is decreasing at a rate of 25 $\frac{cm^3}{s}$. How fast is the surface area changing when the radius is 50 cm.

$V = 4\pi r^3$ $\quad A = \frac{4}{3}\pi r^2$ $\quad \frac{dV}{dt} = 25\ cm\frac{3}{s}$

radius = 50 cm

we want $\frac{dA}{dt}$

I need to relate the two equations for volume and surface area, take $\frac{d}{dt}$ of both and solve for $\frac{dA}{dt}$.

take $\frac{d}{dt}$

$$\frac{dV}{dt} = 4\pi\, 3r^2 \frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{\frac{dV}{dt}}{4\pi 3r^2}$$

$$\frac{dr}{dt} = \frac{25}{1200\pi}$$

Take $\frac{d}{dt}$ $\quad A = \frac{4}{3}\pi r^2$

$$\frac{dA}{dt} = \frac{4}{3}\pi\, 2r\, \frac{dr}{dt}$$

$$\frac{dA}{dt} = \frac{4\pi}{3} \cdot 100 \cdot \frac{25}{1200\pi}$$

$$\frac{dA}{dt} = \frac{400(25)}{3(1200)} = \frac{10000}{3600}$$

$$\boxed{\frac{dA}{dt} = \frac{100}{36}\ \frac{cm^2}{s}}$$

IB-E3-05-3#1

Problem 3. The length of a rectangle is increasing at the rate of 8 cm/s and it's width is increasing at the rate of 3 cm/s. When the length is 20 cm and the width is 10 cm, how fast is the area of the rectangle increasing? (10 points)

$\frac{dl}{dt} = 8 \quad \frac{dw}{dt} = 3$

$\frac{dA}{dt} = dl\,dw$

$\Rightarrow l \frac{dw}{dt} + w \frac{dl}{dt}$

$= 20 \cdot 3 + 10 \cdot 8$

$= 60 + 80 = \boxed{140 \text{ cm/s}^2}$

Problem 7. Find the dimensions of a rectangle with area $1000m^2$ whose perimeter is as small as possible. (10 points)

Perimeter - $2(l+w)$

$1000 = l \cdot w$

$1000 = 2(l+w)$
$500 = (l+w)$

$\frac{500}{l} = w$; $\frac{500}{w} = l$

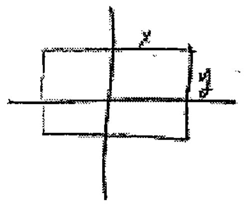
$\to \frac{500}{100} = 5$ $\to \frac{500}{5} = 100$

Width = 5
length = 100

1B-E3-Q7-SP#10

25 · 40

Problem 7. Find the dimensions of a rectangle with area $1000m^2$ whose perimeter is as small as possible. (10 points)



$A = 1000\ m^2$ $A = l \cdot w$

$P = 2(l+w)$

$A = 2x\,2y$

$P = 2(x+y)$

$\frac{P}{2} = x+y$ $A = 4x(\frac{P}{2} - x)$

$\frac{P}{2} - x = y$ $1000 = \frac{4xP}{2} - 4x^2$

$1000 = 4xy$ $1000 = 2x(P) - 4x^2$

$y = \frac{1000}{4x}$ $0 = 2P \cdot P' - 8x$

$y = \frac{250}{x}$ $8x = 2P \cdot P'$

$x = \frac{PP'}{4}$

$y' = \frac{-1 \cdot 250}{\sqrt{x}}$

The dimensions are

25 m × 40 m

APPENDIX D

Participant Score Sheet

	Find the derivative using the limit definition		
	1A-E1-Q2-SP#9	IB-E1-Q5iii-SP#3	IB-E1-Q5iii-SP#4
Instructor C	18/20	4/10 ~ 3/10	6/10 ~ 5/10
Error Type	PK, CALC		
Instructor D	15/20	0/10	6/10
Error Type	PK		PK
Instructor E	10/20	3/10	9/10
Error Type	PK		O/S
Instructor F	10/20 ~ 12/20	2/10	8/10 ~ 9/10
Error Type	PK		PK
Instructor G	12/20 ~ 15/20	2/10	7/10
Error Type	PK		PK
Average score	14/20 = 70%	11/20 = 22%	7.4/10 = 74%
Range	10/20 - 18/20	0/10 - 4/10	5/10 - 9/10

	Identify points of discontinuity or intervals of continuity		
	IB-E1-Q4ii-SP#10	IB-E3-Q2i-SP#4	IB-E3-Q2i-SP#14
Instructor C	6/10	8/10	8/10
Error Type	PK		ML
Instructor D	5/10	7/10	8/10
Error Type	CALC		CALC
Instructor E	3/10	9/10	3/10 ~ 4/10
Error Type	PK, CALC		PK
Instructor F	3/10	10/10	2/10
Error Type	CALC		PK
Instructor G	8/10	10/10	5/10
Error Type	CALC		PK, ML
Average score	5/10 = 50%	8.8 /10 = 88%	5.4/10 = 54%
Range	3/10 - 8/10	7/10 - 10/10	2/10 - 8/10

	Find the derivative using rules			
	IA-E2-Q1B-SP#9	IA-E2-Q1D-SP#10	IA-E2-Q2B-SP #15	IB-E2-Q2i-SP#5
Instructor C	4/5 ~ 3/5	3/5 ~ 7/10	1/5 ~ 2/5	8/10
Error Type		ML, CALC	PK	PK, CALC
Instructor D	4/5	1/5	1/5	7/10
Error Type		ML, CALC	PK, CALC	PK
Instructor E	4/5	2.5/5 ~ 3/5	3/5	7/10
Error Type		CALC	PK, CALC	PK
Instructor F	3/5	2/5	0/5 ~ 1/5	5/10 ~ 6/10
Error Type		ML, CALC	PK, CALC	PK
Instructor G	3/5	2/5	1/5 ~ 2/5	6/10 ~ 7/10
Error Type		CALC	PK, CALC	PK
Average score	3.6/5 = 72%	2.2/5 = 44%		7/10 = 70%
Range	3/5 - 4/5	1/5 - 3/5		5/10 - 8/10

	Find the tangent line	
	IB-E2-Q3i-SP#7	IB-E2-Q3i-SP#9
Instructor C	14/15	12/15 ~ 13/15
Error Type		CALC
Instructor D	12/15	13/15
Error Type	PK	CALC
Instructor E	10/15	10/15
Error Type	PK	CALC
Instructor F	10/15 ~ 11/15	12/15
Error Type	PK	CALC
Instructor G	10/15	12/15
Error Type	PK	CALC
Average score	11.4/15 = 76%	12/15 = 80%
Range	10/15 - 14/15	10/15 - 13/15

	Implicit Differentiation	
	IB-E2-Q4ii-SP#3	IB-E2-Q4iii-SP#9
Instructor C	4/10 ~ 5/10	9/10
Error Type		
Instructor D	4/10	9/10
Error Type		
Instructor E	7/10	9/10 ~9.5/10
Error Type		
Instructor F	6/10 ~ 7/10	8/10 ~ 9/10
Frror Type		
Instructor G	4/10	9/10
Error Type		
Average score	5.4/10 = 54%	9.1/10 = 91%
Range	4/10 - 7/10	8/10 - 9/10

	Related Rates		
Instructor C	IA-E2-Q6-SP#1 //	IA-E2-Q6-SP#3 14/15	IB-E3-Q3-SP#1 9/10
Error Type			
Instructor D	8/15	11/15	9/10
Error Type			
Instructor E	9/15	13/15	9/10
Error Type			
Instructor F	8/15 ~ 9/15	13/15	10/10
Error Type			
Instructor G	10/15 ~ 12/15	13/15 ~ 14/15	9/10 ~ 10/10
Error Type			
Average score	9.5/15 = 64%	13/15 = 87%	9.2/10 = 92%
Range	8/15 - 12/15	11/15 - 14/15	9/10 - 10/10

	Max/Min Applications	
	IB-E3-Q7-SP#9	IB-E3-Q7-SP#10
Instructor C	3/10	7/10
Error Type		
Instructor D	1/10	3/10
Error Type		
Instructor E	2/10	6/10
Error Type		
Instructor F	0/10	4/10
Error Type		
Instructor G	2/10	2/10 ~ 3/10
Error Type		
Average score	1.6/10 = 16%	4.6/10 = 46%
Range	0/10 - 3/10	2/10 - 3/10