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THE MEDIEVAL LATIN TRANSMISSION OF THE MENELAUS THEOREM

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Abstract

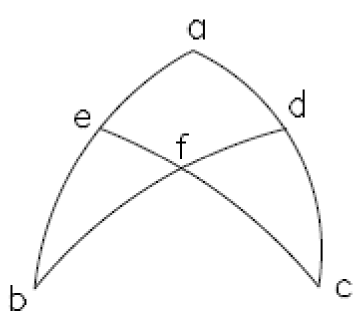
The Menelaus Theorem, which involves the ratios between the chords of arcs arranged in a certain manner on the surface of a sphere, was the fundamental proof for spherical astronomy in antiquity and the middle ages. In this dissertation, I trace the history of this theorem throughout the middle ages. Medieval scholars did not merely copy their ancient and Islamic sources, but instead they modified and added proofs and applications. Two important trends can be seen in medieval works on the Menelaus Theorem, most of which have received little or no scholarly attention. One involves systematization. Medieval scholars molded the astronomical content of their sources into the mold of Euclid's Elements, which was a model of a deductive mathematical science. Instead of arranging the work into chapters and using examples as Ptolemy did, medieval astronomers arranged the work into structured proofs and used general proofs that avoided specific values. Astronomy retained a connection to the practice of astronomy by including general rules for calculating values. The other trend seen in the works containing the Menelaus Theorem involves quantification. The theorem uses compound ratios, which were understood by medieval mathematicians in two ways. One view saw compounding as being analogous to addition, while the other saw it as the multiplication of the numbers or fractions that correspond to ratios. While the second way was more prevalent in the medieval texts containing the Menelaus Theorem, both concepts are found in them—often mixed confusedly—and both involve seeing ratios, which were normally understood as relationships between

quantities, as if they were quantities. These texts are part of a larger medieval trend of quantification of non-quantities. Editions of several works are included as appendices.

INTRODUCTION

The Menelaus Theorem

The spherical geometry of Ptolemaic astronomy was based largely upon one proposition, called the Menelaus Theorem. This theorem states that given a situation in which two arcs of great circles (the circles on a sphere whose centers are the sphere's center) meet at one point and from their other endpoints two other arcs of great circles of the same sphere cross and are terminated at the original two arcs, then the ratio of the chord of the double of the lower portion of one of the original two arcs to the chord of the double of the upper portion of the same arc is composed of the ratio of the chord of the double of the lower portion of the crossing arc that shares an endpoint with the first arc to the double of the upper portion and of the ratio of the chord of the double of the lower portion of the other



original arc to the chord of the double of this whole arc. To make this clearer, let us examine an example. Two arcs *ab* and *ac* meet at point *a*, and the two arcs *bd* and *ce* come from their other endpoints, cross at point *f*, and end in the original two arcs *ab* and *ac*. The Menelaus Theorem proves that the ratio of the chord of double arc *be* to the chord of double arc *ea* is composed of the ratio of the chord of double arc *bf* to the chord of double arc *fd* and of the ratio of the chord of double arc *cd* to the chord of double arc *ac*. If the theorem is applied with *ac* as the first arc, then it shows that the ratio of the chord of double arc *cd* to the chord of double arc *ad* is composed of the ratio of the chord of double arc *cf* to the chord of double arc *ef* and of the ratio of the chord of double arc *be* to the chord of double arc *ab*.

While the name "Menelaus Theorem" implies one singular proposition, it is used to refer to the statement discussed above and another related one. In terms of our example, if *ab* is the principal arc, this second proposition is that the ratio of the chord of double arc *eb* to the chord of double arc *ab* is composed of the ratio of the chord of double arc *bf* to the chord of double arc *bd* and of the ratio of the chord of double arc *cf* to the chord of double arc *ce*, or if *ac* is the principal arc, then the ratio of the chord of double arc *cd* to the chord of double arc *ac* is composed of the ratio of the chord of double arc *cf* to the chord of double arc *ce* and of the ratio of the chord of double arc *bf* to the chord of double arc *bd*. The Menelaus Theorem was often referred to as the "sector figure" because the arcs cut each other. "Kata" (also spelled in a variety of ways, including "katha" or "catha") was a transliterated word from Arabic that was also used to name the Menelaus Theorem. The first proposition of the Menelaus Theorem was often called the "disjoined sector figure/kata" because the terms of the first two ratios are separated parts of two of the original arcs, while the second proposition was referred to as the "conjoined sector figure/kata" because the first two ratios each have a term that is made up of the combined parts of an original arc.

This theorem proved of great use in astronomy.[1] Ancient and medieval astronomers, as well as many early modern astronomers before Kepler's ellipses were accepted, conceived of the universe as a series of spheres, one inside of another, and the quantities involved in calculation were generally not absolute distances, but arcs. For a person on the earth, the most obvious way of measuring the movement of the sun, moon, planets, and fixed stars is not by tracing their change in location in absolute space because without the means to detect parallax,[2] their absolute positions cannot be determined. Without the ability to detect parallax, their positions against a sphere centered around the earth is a measurement that can be found and that still offers enough material for sophisticated and accurate models to be created. Since the Menelaus Theorem gives a statement about the ratios of six chords, if five of these chords are known, the sixth can be found.

The Meneleaus Theorem's first known appearance is in Menelaus of Alexandria's *Sphaerica*, a work on spherical geometry written in the late first century, although the proof probably originated earlier and may have even been

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[1] Glen Van Brummelen, *The Mathematics of the Heavens and the Earth: The Early History of Trigonometry*, (Princeton, Oxford: Princeton University Press, 2009) provides an introduction to the theorem, its role in spherical geometry and astronomy, and the history of alternative theorems that could be used in place of the Menelaus Theorem.

[2] Parallax is the difference in angle to an object measured from two different locations, and it can be used to determine an object's distance from the observer. For example, we are able to judge distances by the slightly different viewpoints of our two eyes. Even for celestial objects for which the parallax can be found, the initial observations that lead to a calculation of distance are measurements of arc.

known and used by Hipparchus in the second century B.C.E.[3] The earliest existing work that applies the Menelaus Theorem to astronomical situations is Ptolemy's *Almagest*, which was written in the second century C.E. Ptolemy provides a proof of the Menelaus Theorem and uses it for most calculations involving spheres (as opposed to ones that only involve arcs along one circle). Given the importance of the *Almagest* in the field of astronomy, Ptolemy's use of the theorem ensured that scholars would learn and discuss it for almost one and a half millenia. Theon, who wrote a commentary on the *Almagest* in the fourth century, added several proofs involving different combinations of the terms in plane versions of the Menelaus Theorem and gave proofs for another case of the disjoined spherical sector figure and for two cases of the conjoined spherical sector figure.[4]

The original Greek version of the *Sphaerica* is lost and it only exists now in Arabic, Hebrew, and Latin translations.[5] Menelaus' *Sphaerica* was translated into Syriac in the eighth century or earlier and into Hebrew by Jacob ben Machir ibn

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[3] Nathan Sidoli, "The Sector Theorem Attributed to Menelaus," *SCIAMVS* 7 (2006): 43-79 argues that Menelaus was not the author of the theorem and that Hipparchus probably used it. Sidoli also gives a brief, but excellent summary of the history of the transmission of Menelaus' *Sphaerica*.

[4] Sidoli, pp. 47-8. Adolphe Rome, "Les explications de Théon d'Alexandrie sur le théorème de Ménélas," *Annales de la Société Scientifique de Bruxelles* Série A 53 (1933): 39-50, here p. 46.

[5] A Latin printed edition was produced in 1758, but it is not a critical edition. Instead it is a Latin paraphrase made from consulting Hebrew, Latin, and Arabic manuscripts. For the history and analysis of the contents of Menelaus' *Sphaerica*, see Sidoli, "The Sector Theorem" and Axel Anthon Björnbo, "Studien über Menelaos' *Sphärik*," *Abhandlungen zur Geschichte der mathematischen Wissenschaften* 14 (1902): 1-154. An edition and German translation of an Arabic version is found in Max Krause, *Die Sphärik von Menelaos aus Alexandrien in der Verbesserung von Abū Nasr Mansūr b. 'Alī b. 'Irāq: Mit Untersuchungen zur Geschicthe des Textes bei den Islamischen Mathematikern*, (Berlin: Weidmannsche, 1936).

Tibbon in the thirteenth century.[6] More important for our investigation into the medieval Western treatment and use of the theorem, the *Sphaerica* was translated into Arabic three times and revised several times,[7] and the *Almagest* was translated into the language at least three times.[8] Islamic mathematicians wrote several original works that treated the Menelaus Theorem. Important works include the ninth-century mathematician Thabit ibn Qurra's *On the Sector Figure*, Ahmad ibn Yusuf's *Epistola de proportione et proportionalitate* from around 900, ibn Sina's commentary on the *Almagest* from the early eleventh century, and the thirteenth-century mathematician al-Tusi's edition of and commentary upon the *Sphaerica*. Islamic mathematicians also created several theorems that could be used in place of the Menelaus Theorem.[9]

In order to understand the medieval works written on the theorem, it is necessary to closely examine their sources, the four works containing the Menelaus Theorem that were translated into Latin: Menelaus' *Sphaerica*, Ptolemy's *Almagest*, Ahmad ibn Yusuf's *Epistola*, and Thabit's *On the Sector Figure*. Part I

[6] Sidoli, pp. 48, 50. Ivor Bulmer-Thomas, "Menelaus of Alexandria," in *Complete Dictionary of Scientific Biography*, vol. 9, Detroit: Charles Scribner's Sons, 2008, pp. 296-302, here pp. 301-2.

[7] Sidoli, pp. 48-50.

[8] Olaf Pedersen, *A Survey of the Almagest: With Annotation and New Commentary by Alexander Jones*, (New York: Springer, 2011), here pp. 14-16.

[9] Sidoli, pp. 179-185. Also see H. Bürger and K. Kohl, "Zur Geschichte des Transversalensatzes, des Ersatztheorems, der Regel der vier Grössen und des Tangentensatzes," which is given as a substantial appendix on pp. 40-91 in Axel Anthon Björnbo, "Thabits Werk über den Transveralensatz (*liber de figura sectora*)," ed. by H. Bürger and K. Kohl, *Abhandlungen zur Geschichte der Naturwissenschaften und der Medizin* 7 (1924): 1-91. Bürger and Kohl focus on the Islamic history of the Menelaus Theorem, related proofs, and alternative theorems; they skip over the Latin medieval world with only a brief reference on p. 46 to Jordanus de Nemore and Leonardo of Pisa's use of compound ratios.

of this dissertation covers the treatment of the Menelaus Theorem in these four works. The *Almagest* is the only one of these four to apply the Menelaus Theorem to astronomical problems. I examine these astronomical uses of Ptolemy as well as the cases in which he uses plane approximations instead of spherical geometry. Menelaus gives the theorem in a geometric setting. Thabit's work gives several proofs of cases of the Menelaus Theorem that neither Ptolemy nor Menelaus proved. He also introduces a more comprehensive study of compound ratios, especially the study of the modes, which are the possible arrangements of the six terms in a statement that one ratio is composed of two others. Ahmad also focuses on proportion theory and treats the modes although in a different manner. This section concludes with an examination of the twelfth-century mathematician Jabir ibn Aflah's *Correction of the Almagest*. While this work does not contain the Menelaus Theorem, it contains powerful alternatives.

In Part II I examine original Latin works that treat the Menelaus Theorem. Many of these are commentaries. Marginalia written in manuscripts containing the *Sphaerica* or the *Almagest* reveal how medieval readers of these works understood them.[10] While there are several sets of notes found in the marginalia, a set written by Campanus is particularly significant. The autonomous commentaries include a summary of the *Almagest* known as the *Almagestum parvum*, an anonymous commentary on the first two books of the *Almagest* (which I call the "Erfurt

[10] The usefulness of marginalia in understanding medieval mathematical and scientific works can be seen in Danielle Jacquart and Charles Burnett, *Scientia in margine: études sur les marginalia dans les manuscrits scientifiques du moyen âge à la renaissance*, (Genève: Droz, 2005).

Commentary" because two of the four existing manuscripts are now held at the Universitätsbibliothek Erfurt), yet another anonymous commentary on the whole *Almagest* (which I call the "Vatican Commentary" because it exists in two manuscripts that are now in the Biblioteca Apostolica Vaticana), and a commentary by Simon of Bredon on the first three books of the *Almagest*. I also examine a few works that cannot be considered commentaries. Richard of Wallingford's *Quadripartitum* deals extensively with compound ratio and the sector figure although Richard took most of it from other works with only minor changes. Arzachel's canons on the Toletan Tables and some related works use the Menelaus Theorem and give a further glimpse into the practice of astronomers.[11]

Axiomatization

Although each of these works has its own approach to the Menelaus Theorem, two intriguing trends emerge. One is a movement of systematizing the content related to the theorem. In the *Posterior Analytics*, Aristotle describes how axiomatic sciences begin with principles known through themselves such as definitions, postulates, axioms, etc. and how they proceed through syllogism to conclusions. Although not agreeing with Aristotle in every matter about how a scientific discipline should proceed, Euclid provides an excellent model of an

[11] Due to the limits of a doctoral project, I am not attempting to include a detailed study of every medieval work that involves the Menelaus Theorem. Relevant works that I unfortunately will not discuss at length include works by Fibbonacci, Roger Bacon, other *Almagest* commentaries, and several short works.

axiomatic science in his *Elements*. Euclid lists definitions, postulates, and common notions, and then argues logically from these to conclusions. Generally, his theorems and problems can be divided into parts: an enunciation that lays out in general terms what it is that he is proving or doing, a restatement of the given situation and what is to be proved or done in terms of a diagram drawn and lettered in a certain way, a construction of any additional geometric objects that may be needed for the proof, the argument in which he shows how to proceed from the given knowledge to the conclusion through the use of the principles and propositions that he has already proved, and a conclusion in which he explains how he has done what he set out to do.[12]

Unlike Euclid, Ptolemy did not structure his content as an axiomatic science. Even in the introductory sections on the geometry of arcs and chords, he did not organize the mathematical content in the manner that Aristotle described and that Euclid put into practice. His proofs often have some of the parts of a Euclidean proof—particular givens and things to be proved, a construction, an argument, and a conclusion, but he does not give general enunciations or conclusions and any similarities that there are appear to be coincidental. In the astronomical portions of the text, there is even less similarity with the format of Euclid's *Elements*. That the format is not identical to that of the *Elements* is not

[12] The medieval versions of Euclid's *Elements* do show differences in their emphases on organizational structure and some divide the portions of each theorem or problem more clearly than others, but on the whole the medieval *Elements* versions were organized as I have described. See John Murdoch, "The Medieval Euclid: Salient Aspects of the Translations of the *Elements* by Adelard of Bath and Campanus of Novara," *Revue de Synthèse* 89 (1968): 67-94.

surprising since Ptolemy's astronomy is not a purely theoretical discipline—it involves the construction of instruments, observation, measurement, and approximation.[13] Another crucial difference between Ptolemy's astronomy and theoretical geometry is that the former discusses particular celestial bodies, while the latter stays on the level of universals.[14] One of the main goals of Ptolemy's astronomy is to find the particular parameters that will enable the astronomer to accurately model the particular motion of the sun, moon, the five visible planets, and the sphere of the fixed stars. Even when describing mathematics that will apply for several different situations (as is often done when describing how to find the values for astronomical tables), Ptolemy gives particular examples of calculations instead of a general account.[15]

Most of the medieval Latin commentaries written on the *Almagest* put the content of their source into the format of an axiomatic science. Some list principles at the beginning of the different books into which they are divided. Unlike the principles in the *Elements*, these principles often concern particular objects such as the equator and the ecliptic. The particular examples that Ptolemy gives are replaced with general proofs that make little or no mention of particular values and parameters. Some commentators even try to fit Ptolemy's explanations of how to

[13] The particularity involved in Ptolemy's astronomy contrasts with the goals of universality that some other ancient philosophers held. For example, Plato writes that true astronomy is not about the physical heavens and that there is no sense in expending a great amount of effort trying to observe and calculate the values of physical phenomena (*Republic*, VII, 528d-530c).

[14] A study of the particular heavenly bodies could still be considered scientific since each heavenly body has its own nature.

[15] E.g., *Almagest* I.14.

make and use instruments into the format of mathematical proofs. Although the matter is still not settled definitively, I suggest that the axiomatizers of astronomy used the *Elements* as a model.

Examining this topic of axiomatization in astronomy will shed light upon what it means to be an axiomatic science. We see that there is not only one way of organizing an axiomatic science—instead there are several different axiomatic formats. This point has been recognized by philosophers of mathematics; e.g., they have long distinguished the axiomatic method of Euclid from that of Hilbert.[16] But, a more careful and historical distinction between axiomatic formats that are not as dramatically different has not been made for medieval mathematical works.[17]

Another part of this trend of systematizing manifests itself in an effort to prove the Menelaus Theorem more completely. As we will see there are several different cases of the Menelaus Theorem in which the proof needs to be altered because arcs and lines have a different arrangement. While only one of the sources translated into Latin proves the theorem universally, medieval scholars often proved several or all of the cases in order to at least approach a universal knowledge of the theorem. Similarly, the classification, ennumeration, and proofs of the modes of compound ratio that Thabit and Ahmad discussed remained topics of interest. Systematic treatments of a subject attempted to be complete and self-

[16] Ian Mueller, "Euclid's *Elements* and the Axiomatic Method," *The British Journal for the Philosophy of Science*, 20.4 (Dec., 1969): 289-309.

[17] Murdoch, "The Medieval Euclid" does examine the differences in structure of versions of the *Elements*, but this is only a start towards an understanding of the variations across all of the mathematical disciplines.

sufficient, so once the topics of the Menelaus Theorem or of compound ratio arose, medieval scholars often decided to provide more complete and autonomous coverage of these subjects.

Quantification of Ratios

A second trend involves treating ratios as if they were quantities. According to Euclid and other authorities, ratios are relationships between quantities.[18] Ratios thus fall into a strange situation—they are studied in mathematics, but they are technically not quantities. As is clear from Euclid and Boethius, ratios can be treated in mathematics without being treated obviously like quantities. The concept of compound ratio, however, makes it difficult to avoid doing so. Few of the authorities of medieval mathematics defined compound ratio, so there was confusion over what exactly it meant for a ratio to be composed of others. Two different ideas of what this meant were formulated in ancient mathematics, and both of these are found in our medieval works on the Menelaus Theorem. Both of these ideas of compound ratio involve treating ratios in certain ways as if they were quantities.

One of these two sees compounding of ratios as being similar to addition of quantities. When a line is added to a line, the two are made continuous by making

[18] Euclid, *Elements* V def. 3. Boethius also defines ratio as a relation; see Michael Masi, ed., *Boethian Number Theory: A Translation of the De Institutione Arithmetica*, (Amsterdam: Rodopi, 1983).

the endpoint of one the starting point of the other. A line is said to be made up or composed of others, when a point is taken between its two endpoints; for example, given line *ac*, if point *b* is placed between *a* and *c*, then *ac* is said to be composed of the component parts *ab* and *bc*. Likewise with numbers, we add by starting a number at the end of another and we can split a number into component parts by counting part of the way and then counting from where we stopped to the end of the original number. A ratio can be "added" to a ratio by making the consequent of one ratio the antecedent of the other. For example, the ratio sesquialterate (the ratio of 1 1/2) can be "added" to the sesquitertiate ratio (the ratio of 1 1/3 to one) by putting these ratios in the terms 6 to 4 and 4 to 3. The consequent of one is the antecedent of the other, and the ratio of the antecedent of the first ratio to the consequent of the second, 6 to 3 or double, is the whole ratio or the sum of this addition of ratios. Using the same ratios, the whole ratio 6 to 3 can be split into its component parts by inserting a term between 6 and 3. If we choose to insert 4, it is clear that double is composed of sesquialterate and sesquitertiate ratios. Ratios can thus be considered as wholes and parts as if they were quantities, and so they can be "added" and "subtracted."

This first way of understanding compounding relies upon analogies between ratios and quantities such as lines and numbers. While there is a close similarity between ratios and quantities—e.g., we explained "addition" using similar concepts for ratios and for quantities—the analogy can only be carried so far. For example, medieval authors often have a ratio of greater inequality having a ratio of lesser inequality as one of its "component parts." We can speak of the ratio of 5 to 3 as a whole composed of component parts, the ratios of 5 to 2 and of 2 to 3; however, one of the parts, the ratio of 5 to 2, is greater than the whole, 5 to 3. This difficulty suggests that ratios are not wholes and parts in the same way that a line and numbers are. While most of the works that I will examine do not enter into discussions about the comparisons between ratio and quantity, medieval scholars were consciously or unconsciously making comparison between the two categories by transferring concepts from the category to which they are most properly related to another category to which they fit imperfectly. While entering into this deeper discussion is not one of my primary goals, my research on these medieval works should provide some of the historical matter that should inform future research on this subject.

The other way of thinking of compounding ratios relies upon the use of denominations. A denomination of a ratio is the number that gives the name to that ratio, and it is found by dividing the antecedent by the consequent. Since this division is an operation of numbers, this way of dealing with ratios does not work well for incommensurable quantities.[19] For a ratio to be composed of other ratios is for its denomination to be the product that results from the multiplication of the denominations of other ratios. Taking our example from above, the ratio double is

[19] The question of how to denominate a ratio such as that of the diagonal of a square to the side is troublesome. Nicole Oresme is one who attempted to answer this question in his work on ratios and proportionality. Edward Grant, *Nicole Oresme. De proportionibus proportionum, and Ad pauca respicientes. Edited with introductions, English translations, and critical notes by Edward Grant*, (Madison: University of Wisconsin Press, 1966).

composed of sesquialterate and sesquitertiate ratios because the denomination of double, which is the number 2, is the product of the multiplication of the two fractions that denominate sesquialterate and sesquitertiate, 3/2 and 4/3. While this method does not treat ratios as quantities that can be added and subtracted analogously to the way in which magnitudes or numbers can be (and thus the terminology of composition does not make as much sense as with the other conception), the distinction between a ratio and its denomination was often blurred. Ratios thus are treated as if they are fractions or numbers.

These two ideas of compound ratios are not necessarily incompatible. If one takes the idea of compound ratio that depends upon continuity, the ratios need to be put in terms such that they share terms. The process of converting ratios into suitable terms can be equivalent to the process of finding denominations. For example if a sesquialterate (3 to 2) and a sesquitertiate ratio (4 to 3) are to be compounded, suitable terms can be found by multiplying the terms of the first ratio by the antecedent of the second ratio, resulting in 12 and 8, and by multiplying the terms of the second ratio by the consequent of the first ratio, resulting in 8 and 6. Then since the ratios have been made continuous, the ratio of 12 to 6 is composed of our two original ratios. The important steps are the multiplication of the antecedent of the first ratio by the antecedent of the second ratio and the multiplication of the consequent of the first ratio and the consequent of the second ratio. These are the same critical operations as if we found the denominations of our two ratios, 3/2 and 4/3 and multiplied them together. Similarly, if one begins with the denominational concept of compound ratio, it can be proved that if ratios are continuous, then the ratio of the extreme terms is composed of the ratio of the first to the middle and of the ratio of that middle to the last.

While these two ideas of compound ratio are not contradictory, different mathematicians define compound ratio according to one or the other. Frequently mathematicians used compound ratio without defining it. In these cases, it can be difficult or impossible to see which definition the mathematician had in mind. In some cases, the mathematician was probably unsure what the definition of compound ratio was even though he uses compound ratio in his proofs.

Historians have long realized the importance of ratio theory in medieval mathematics and natural philosophy. The seminal article on the medieval study of ratios is John Murdoch's "The Medieval Language of Proportions."[20] Murdoch explains how medieval scholars had difficulty understanding the Eudoxean theory of proportion of Book V of Euclid's *Elements*, largely because of some faulty passages in the medieval translation, and consequently proportion theory came to rely upon the concept of denominations. He also points out that compound ratio became an important subject.

[20] John Murdoch, "The Medieval Language of Proportions," in *Scientific Change: Historical Studies in the Intellectual, Social and Technical Conditions for Scientific Discovery and Technical Invention, from Antiquity to the Present*, ed. by A. C. Crombie, (London: Heinemann, 1963), pp. 237-271; reply to responses, pp. 334-343. A commentary on Murdoch's talk by L. Minio-Paluello is found on pp. 306-311; and H. L. Crosby, Jr. critiques the talk on pp. 324-7.

Much of the discussion of medieval ratios has focused upon Thomas Bradwardine's use of compound ratios to explain the relationship of power, resistance, and velocity in motion. His *Tractatus de proportionibus* was edited and translated by H. Lamar Crosby, Jr.[21] In this work, Bradwardine examines several possible interpretations of Aristotle's claims about the relationship between velocity, motive power, and resistive power, and he finds that the only reasonable interpretation is, "The proportion of the speeds of motions varies in accordance with the proportion of motive to resistive forces, and conversely."[22] By this, Bradwardine means that the ratio of a velocity of one motion to the velocity of another motion is the same as the ratio of the ratio of the first motive power to the first resistive power to the ratio of the second motive power to the second resistive power.[23] The possible sources of Bradwardine's rule have been examined by several historians,[24] as has been the question of whether Bradwardine's rule is an

[21] Henry L. Crosby, ed., *Thomas of Bradwardine, His Tractatus De Proportionibus; Its Significance for the Development of Mathematical Physics*, (Madison: University of Wisconsin Press, 1955).

[22] Ibid., p. 113.

[23] In abbreviated form, $V_1:V_2::(M_1:R_1):(M_2:R_2)$. Bradwardine does not use the terminology of ratios of ratios, so his formulation is difficult to decipher.

[24] Marshall Clagett, *The Science of Mechanics in the Middle Ages*, (Madison: University of Wisconsin Press, 1959), p. 439 n. 35 argued that Bradwardine's rule was influenced by al-Kindi's idea of the power of mixed medicines. Michael McVaugh, "Arnald of Villanova and Bradwardine's Law," *Isis* 58 (1967): 56-64 argued that Bradwardine knew of al-Kindi's theories through the work of Arnald of Villanova. Stillman Drake, "Medieval Ratio Theory vs. Compound Medicines in the Origin of Bradwardine's Rule," *Isis*, 1973: 67-77 expanded our knowledge of the context of Bradwardine's rule by examining the ideas of denomination and duplicate and triplicate ratio in Campanus' edition of Euclid's *Elements*, and he shows that there is no strong connection between Arnald and Bradwardine because the ideas they share in common are one common to medieval proportion theory.

early example of a law or function.[25] Since Bradwardine's rule depends upon the idea of ratios of ratios, it also involves compound ratios. Ratios are relationships between quantities of the same kind, and each of the terms must be able to be increased to exceed the other.[26] For a ratio to exceed another means that it is a whole and that the other ratio and some ratio that is the difference between them are the parts that make up or compose the whole.

The proportion theory in Bradwardine's successors has also received a great deal of treatment. A group of mathematically inclined scholars at Oxford in the first half of the fourteenth century that included Richard Swineshead, John Dumbleton, Roger Swineshead, and William Heytesbury, used Bradwardine's rule and expanded the study of ratios to new problems in natural philosophy as did Nicole Oresme and Jean Buridan.[27] Edward Grant wrote several works treating

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[25] Many who have written on the subject call the rule a law or a function. Drake, "Medieval Ratio," rightly points out that neither Aristotle nor Bradwardine thought of the relation as $F_2/R_2 = (F_1/R_1)^\wedge V_2/V_1$, but he incorrectly thinks that Bradwardine only meant his rule to apply in the cases of halving and doubling the velocity and thus that Bradwardine's rule is not meant to work for any values and therefore is not a universal law (p. 73). Jean Celeyrette, "Bradwardine's Rule: a Mathematical Law ?," in *Mechanics and Natural Philosophy before the Scientific Revolution*, Walter Roy Laird and Sophie Roux (eds), (Dordrecht: Springer, 2008), pp. 51-66, here p. 52 attempts to answer whether Bradwardine's rule is a "truly mathematized law of motion" and whether it is a function. He answers in the negative, but the matter is still unresolved in my opinion over whether or not Bradwardine intended his rule to be general.

[26] Euclid, *Elements* V def. 3-4.

[27] Edith Sylla, *The Oxford Calculators and the Mathematics of Motion, 1320-1350: Physics and the Measurement of Latitudes*, (New York: Garland, 1991), which is a reprint of her 1970 doctoral dissertation, contains a detailed exposition of the use of ratios by the Calculators to examine natural phenomena (or imagined natural phenomena). She treats ratio in particular on pp. 308-327. Her outlines of their works (pp. 471-714) are especially useful. M. A. Hoskin and A. G. Molland, "Swineshead on Falling Bodies: An Example of Fourteenth-Century Physics," *The British Journal for the History of Science* 3.2 (Dec., 1966): 150-182 contains an edition, translation, and explanation of a passage in which Richard Swineshead uses Bradwardine's rule and more propositions about compound ratios to examine how a falling body acts as it approaches the center of the world. Ernest Moody, *The Rise of Mechanism in 14th Century Natural Philosophy*, (New York: Columbia

Oresme's proportion theory found in his *De proportionibus proportionum*, which was written around 1360, and his *Algorismus proportionum*.[28] Work has also been done on later thinkers who discussed Bradwardine's use of ratios including Albert of Saxony, Blasius of Parma, and Alvarus Thomas.[29]

Many of the medieval texts that treat compound ratio but not Bradwardine's rule have been edited and/or analyzed. Among these works are Bradwardine's *Geometria speculativa*,[30] Roger Bacon's *Communia mathematica*,[31] Fibonacci's

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University, 1950), especially on pp. 30, 42-52 contains translated excerpts from works treating physical problems mathematically by Jean Buridan and John Dumbleton, some of which use compound ratios. Edith Sylla "Fate of Oxford Calculatory Tradition," in *L'homme et son univers au moyen âge: actes du septième congrès international de philosophie médiévale (30 août-4 septembre 1982)*, ed. by C. Wenin, 692-698 (Louvain-la-Neuve: Editions de l'Institut Supérieur de Philosophie, 1986), has examined the popularity of the work of the Calculators and the decline of interest in them. Their popularity sprang from their use in the undergraduate study of logic, so as teaching methods changed, these works became less popular. The study of Bradwardine's rule has taken a new direction with Sabine Rommevaux, "Magnetism and Bradwardine's Rule of Motion in Fourteenth- and Fifteenth-Century Treatises," *Early Science and Medicine*, 15.6 (2010): 618-647.

[28] Edward Grant, *Nicole Oresme*; "The Mathematical Theory of Proportionality of Nicole Oresme," (PhD diss., University of Wisconsin, 1957); and "Part I of Oresme's *Algorismus Proportionum*," *Isis* 56.3 (Autumn, 1965): 327-341.

[29] Analysis and transcription of Albert's work is found in H. L. L. Busard, *Der Tractatus proportionum von Albert von Sachsen*, (Wien: Springer in Komm, 1971). For analysis and edition of Blasius' work, see Joel Biard and Sabine Rommevaux, *Quaestiones circa tractatum proportionum magistri Thome Braduardini*, (Paris: Vrin, 2006). Edith Sylla, "Mathematics in the *Liber de triplici motu* of Alvarus Thomas of Lisbon," in *The Practice of Mathematics in Portugal*, Luís Saraiva and Henrique Leitão (eds.), (Coimbra: Acta Universitatis Conimbrigensis, 2004), pp. 109-161, and Edith Sylla, "Alvarus Thomas and the Role of Logic and Calculations in Sixteenth Century Natural Philosophy," in *Studies in Medieval Natural Philosophy*, ed. by Stefano Caroti, (Florence: Olschki, 1989), pp. 257-298, analyze Alvarus' work.

[30] George Molland, *Thomas Bradwardine, Geometria Speculativa: Latin Text and English Translation with an Introduction and a Commentary*, (Stuttgart: F. Steiner Verlag Wiesbaden, 1989). Also see Molland, "An Examination of Bradwardine's Geometry," *Archive for the History of the Exact Sciences*, 19.2 (1978): 113-75. A section of this article examines Bradwardine's use of denominations, his idea of compounding ratios, and his treatment of ratios as quantities (pp. 150-160).

[31] Roger Bacon, *Communia mathematica fratris Rogeri, partes prima et secunda*, Roger Steele, ed., (Oxford: Clarendon Press, 1940).

Liber abbaci and *Practica geometriae*,[32] Jordanus de Nemore's *De elementis arithmetice artis*,[33] pseudo-Jordanus' treatise on compound ratio,[34] Campanus' *De sectore figura*[35] and treatise on ratios,[36] and Richard of Wallingford's *Quadripartitum*,[37] as well as Latin translations of works such as Ametus' *Epistola de proportionibus*[38] and Thabit ibn Qurra's *On the Sector Figure*.[39]

There are several problems with the way that many historians understood compound ratio. This is most apparent in the work on Bradwardine and Oresme. Crosby thought that the historical importance of Bradwardine's rule was that it was the first application of "what may be called a logarithmic, exponential, or geometric function" to physical phenomena, and he expressed it as $n^v = (F/R)$.[40] Although

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[32] Leonardo of Pisa (Fibonacci), *Scritti di Leonardo Pisano, matematico del secolo decimoterzo*, Baldassarre Boncompagni, ed., 1857, 1862. Translations are L. E. Sigler, *Fibonacci's Liber Abaci: A Translation into Modern English of Leonardo Pisano's Book of Calculation*, (New York: Springer, 2002); and Barnabas Hughes, *Fibonacci's De Practica Geometrie*, (New York: Springer, 2008).

[33] H. L. L. Busard, ed., *Jordanus De Nemore, De Elementis Arithmetice Artis: A Medieval Treatise on Number Theory*, (Stuttgart: F. Steiner, 1991).

[34] H. L. L. Busard, "Die Traktate *De Proportionibus* von Jordanus Nemorarius und Campanus," *Centaurus*, 15 (1971): 193-227. I argue that this work is not by Jordanus de Nemore in Henry Zepeda, "Compound Ratios in the Work of Jordanus de Nemore," (M.A. Thesis, University of Oklahoma, 2008).

[35] Richard Lorch, *Thabit ibn Qurra: On the Sector-Figure and Related Texts*, (Augsburg: Rauner, 2008), pp. 426-45.

[36] Busard, "Die Traktate."

[37] John North, *Richard of Wallingford: An Edition of His Writings*, (Oxford: Clarendon Press, 1976).

[38] M. Walter Reginald Schrader, *The Epistola De Proportione Et Proportionalitate of Ametus Filius Iosephi*, (PhD diss., University of Wisconsin, 1961).

[39] Axel Anthon Björnbo, "Thabits Werk." Lorch, *Thabit*, has editions of two Latin versions of Thabit's work, Campanus' work on the sector figure, and several short notes on the sector figure and compound ratio.

[40] Crosby, p. 12-3.

Bradwardine's rule is conceptually similar to that equation, there are a number of crucial differences. First, equations convey a sense of measurement and equivalency of units of different types that Bradwardine did not have in mind. Secondly, it is not clear what it might mean to take something to a power that is a velocity. Thirdly, while Bradwardine would admit that an agent power could have a ratio to a resistive power, it is doubtful that he would have admitted the division of one by the other.[41] When Bradwardine applies the language of multiples and fractions to ratios, Crosby does not understand these words to apply the same way as they do in normal usage. Crosby believes that when Bradwardine uses the phrase "medietas duplae proportionis" referring to the ratio of the diagonal of a square to the side, the only possible meaning is that "the square root of the proportion of two to one," and he denies that Bradwardine's could plainly mean "the half of the ratio double."[42] Similarly Grant treats compound ratio as if it were about exponents, and while Oresme compounds ratios according to both the continuous ratio way and the denominative way, Grant always understands him to be using the latter.[43] As Crosby has trouble taking the language associated with

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[41] While dividing a number by a number was common practice and some medieval mathematicians divided surfaces by lines, I know of none who divided lines by lines, surfaces by surfaces, or any one magnitude by one of the same kind, at least not in theoretical mathematics.

[42] Crosby's commentary on Murdoch's "The Medieval Language," p. 325. Similar misunderstandings are found throughout Crosby's work, so I will give only a few examples. In Crosby, pp. 20-1, he understands a doubled ratio to be a squared ratio and tries to make a linguistic distinction between "dupla" and "duplicata." A similar translation of Bradwardine's meaning into an anachronistic understanding of compound ratio is seen in Crosby's translation of Bradwardine's explanation of a ratio being the double of another, pp. 78-9.

[43] "Part I," pp. 330-1, n. 11 and pp. 340-1, n. 41.

compound ratio that Bradwardine uses at face value, Grant interprets words in strange ways.[44] These same mistakes have continued to be made even recently.[45]

Some historians have put more emphasis on understanding medieval thinkers in their own terms and have realized the mistakes of understanding compound ratios according to modern terms. While according to modern ideas of ratios, it is difficult to see how compound ratio could mean anything else than the multiplication of ratios, the continuity conception of compound ratio was explained in two works published in 1978. Molland briefly explained the two ways that compound ratios were understood, as did Murdoch and Sylla, who also examined the problems in understanding Bradwardine's rule according to modern notation and exponents.[46] Among other things, they argue that understanding that some medieval scholars thought of compounding ratios in terms of continuity avoids the problem of words being used in strange ways such as "doubled" (*duplicata*)

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[44] Grant, "Part I," pp. 340-1, n. 41 has great trouble explaining why Oresme talks of the compounding of ratios as "additio." If compounding is the multiplication of denominations, it seems that Oresme uses "additio" when he really means "multiplicatio." A particularly striking instance of modern ideas leading to strange translations is found in Drake, p. 72; he translates "medietas duplae proportionis" as the "proportionality of the ratio 2/1."

[45] E.g., see Edward Grant, "Reason and Authority in the Middle Ages: The Latin West and Islam", in *Scientific Values and Civic Virtues*, ed. by Noretta Koertge, 40-58, (Oxford, New York: Oxford University Press, 2005). Also, Sabine Rommevaux, "Aperçu sur la notion de dénomination d'un rapport numérique au Moyen Âge et à la Renaissance," *Methodos* 1 (2001): 223-243, here pp. 230-1, continues to make a similar mistake in understanding all duplicate ratios as squares of ratios when she writes, "Il faut noter que lorsque Bradwardine parle ici de la « moitié » du rapport double, il considère le rapport, qui composé par lui-même, donne le rapport double (2 : 1), c'est-à-dire le rapport (A : B) tel que $(A : B)^2 :: (2 : 1)$."

[46] Molland, "An Examination," pp. 113-75; John E. Murdoch and Edith D. Sylla, "The Science of Motion," in *Science in the Middle Ages*, ed. by David C. Lindberg, 206-264, (Chicago: University of Chicago Press, 1978).

meaning “squared” when applied to ratios.[47] Sylla made an initial attempt to examine the history of the two definitions of compound ratio from the middle ages to their appearance in different versions of Newton’s *Principia*, which she followed with a more detailed account of the two definitions and the development of and reaction to Bradwardine’s rule.[48] Recently, Oscar João Abdounur has examined the medieval history of compound ratio and connected it to the problem of dividing the tone (8:9) in music.[49] Music theory is in large part the composition and division of ratios into parts, so surely there were many influences between the study of compound ratios in music and in the other mathematical sciences. Unfortunately, only the very surface of this subject has been scratched.[50] The earlier history of compound ratio has also been the object of study.[51]

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[47] Ibid., p. 225.

[48] Edith D. Sylla, “Compounding Ratios: Bradwardine, Oresme, and the First Edition of Newton’s *Principia*,” in *Transformation and Tradition in the Sciences: Essays in Honor of I. Bernard Cohen*, ed. by Everett Mendelsohn, 11-43, (Cambridge: Cambridge University Press, 1984); Edith D. Sylla, “The Origin and Fate of Thomas Bradwardine’s *De Proportionibus Velocitatum in Motibus* in Relation to the History of Mathematics,” in *Mechanics and Natural Philosophy Before the Scientific Revolution,* ed. by Walter Roy Laird and Sophie Roux, 67-119, (Dordrecht, The Netherlands: Springer, 2008).

[49] Oscar João Abdounur, “Ratios and Music in the Late Middle Ages: A Preliminary Survey,” *Circumscribere* 7 (2009): 1-8.

[50] Andre Goddu’s “Harmony, Whole-Part relationship, and the Logic of Consequences,” in *Musik und die Geschichte der Philosophie und Naturwissenschaften im Mittelalter: Fragen zur Wechselwirkung von ‘Musica’ und ‘Philosophia’ im Mittelalter*, ed. by Frank Henstschel, 325-338, (Leiden: Brill, 1998), examines the connection between ratios as wholes and parts and the logical arguments of Copernicus.

[51] Ken Saito, “Compounded Ratio in Euclid and Apollonius,” *Historia Scientiarum* 31 (1986): 26-59; and “Duplicate Ratio in Book VI of Euclid’s *Elements*,” *Historia Scientiarum* 38 (1993): 116-135. Also relevant to the terminology and understanding of ratio from ancient Greece to the Renaissance is Wilbur R. Knorr, “On the Term Ratio in Early Mathematics,” in *Ratio: VII Colloquio Internazionale, Roma, 9-11 gennaio 1992*, eds. M. Fattori and M. L. Bianchi, (Firenze: Leo S. Olschki, 1994), pp. 1-35.

While a few historians have noted that the Menelaus Theorem does have a role in the history of proportion theory, the exact nature of this role is still uncertain. That the theorem was the locus for discussions of compound ratio in Greek and Arabic contexts has been established.[52] Both Murdoch and Sylla have observed that the sector figure texts were accompanied by discussion of compound ratio, and Sylla claims that the use of denominations spread through their influence.[53] In North's notes to Richard of Wallingford's *Quadripartitum*, he discusses several works that he sees as influential for what they contain about compound ratio. The works he lists include two that contain the Menelaus Theorem: Thabit's *On the Sector Figure* and Ametus' *Epistola*.[54] Some of the texts containing the sector figure, which are needed for an understanding of the role of the Menelaus Theorem in proportion theory, have been edited, but the bulk of medieval works treating the Menelaus Theorem have been left unedited and unexamined. One of my primary purposes of examining both the edited and unedited works containing the Menelaus Theorem is to observe how different works treat compound ratios.

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[52] E.g., Rome, "Les explications de Théon." Much of the work on the Arabic history of the theorem and its ties to compound ratio focuses on Thabit; see Lorch, *Thabit*; Hélène Bellosta, "Le Traité De Thabit Ibn Qurra Sur La Figure Secteur," *Arabic Sciences and Philosophy* 14.1 (2004): 145-68; Sabine Koelblen, "Une pratique de la composition des raison dans un exercice de combinatoire," *Revue d'Histoire des Sciences* 47 (1994): 209-247; Sabine Koelblen, "Un exercice de combinatoire: Les relations issues de la figure sécante de Ptolémée, ou les règles des six quantities en proportion," in *Un Parcours en Histoire des Mathématiques: Travaux et Recherches*, (Nantes: Université de Nantes, 1993), pp. 1-21; Pascal Crozet, "Thabit ibn Qurra et la Composition des Rapports," *Arabic Sciences and Philosophy* 14.2 (2004): 175-211.

[53] Murdoch, "The Medieval Language," pp. 263-4; Sylla, "Compounding Ratios," p. 23.

[54] North, *Richard of Wallingford*, Vol. 2, pp. 57-8. ;

The tendency to deal with compound ratios by treating ratios as quantities is part of a larger medieval trend of quantifying attributes that were not technically quantities. The treatment of the intensity or latitude of qualities that is found in the writings of the Oxford Calculators and Nicole Oresme are the most striking examples of this movement.[55] While this trend has been noticed especially in the fourteenth century, the study of compound ratios shows that medieval scholars were treating ratios as quantities much earlier. Whether and exactly how the quantification of ratios opened the door to the quantification of other non-quantitites remains to be seen. It is intriguing that many of the medieval thinkers who were most involved in this quantification movement in the fourteenth century were closely connected to Bradwardine.

Other Issues

Often following the lead of Thabit, some medieval scholars gave new proofs of the Menelaus Theorem, and these different proofs will be examined. As mentioned previously, Jabir developed alternative theorems that he used in place of the Menelaus Theorem. Some medieval scholars knew of these alternatives and used them to find astronomical values, and some even added their own alternative theorems. These new proofs and applications will be examined, as will the manner in which medieval scholars dealt with having more than one method to solve the same problem.

[55] Sylla, *The Oxford Calculators and the Mathematics of Motion*; Marshall Clagett, *Nicole Oresme and the Medieval Geometry of Qualities and Motions* (Madison: Univ. of Wisconsin Pr., 1968).

Besides examination of the two trends mentioned above, a major goal of this dissertation is to introduce medieval works that historians of science have not researched or have only briefly discussed. Of the thirteen chapters of the body of the dissertation, only four are primarily on texts that have been edited critically. Of the remaining nine, two are on works that were printed in the sixteenth century, and another chapter treats both manuscripts and critical editions. Transcriptions and critical editions of these unexamined works are included as appendices.[56]

[56] For many of these, I have limited myself to portions relevant to the Menelaus Theorem.

PART I: THE TRANSLATED WORKS

Chapter 1: Ptolemy's *Almagest*

The *Almagest*, Ptolemy's *magnum opus*, was translated four times into Latin in the middle ages. Gerard of Cremona made the only Arabic-to-Latin translation ca. 1150-1175, and it was the most popular version of the *Almagest*.[57] It exists in at least thirty-two manuscripts and was printed in 1515.[58] Another translation was made from Greek by a Sicilian translator ca. 1160 and exists in four manuscripts.[59] This version does not vary from Gerard's in any significant manner, so it will not be treated. The third translation, which only exists in one manuscript, should perhaps be considered more a commentary than a translation. Its treatment of the Menelaus Theorem is based upon Thabit's work on the Menelaus Theorem.[60] A fourth translation was made probably in Spain in the thirteenth century, but only fragments of it survive.[61] Because the Gerard of Cremona version was so much more widely used than the other translations, I will examine it in detail. Although

[57] The year 1175 has been supposed to be when Gerard finished this edition, but given that he was supposed to have gone to Spain in order to read the *Almagest*, a date nearer his arrival in Spain, which occurred in 1144 at the latest, seems more likely. See Richard Lemay, "Gerard of Cremona," *Complete Dictionary of Scientific Biography*, vol. 15, 173-192, (Detroit: Charles Scribner's Sons, 2008), here p. 174.

[58] The number comes from Francis J. Carmody, *Arabic Astronomical and Astrological Sciences in Latin Translation; A Critical Bibliography*, (Berkeley: University of California Press, 1956), p. 15. Ptolemy, *Almagestum Cl. Ptolemei Pheludiensis Alexandrini astronomorum principis: opus ingens ac nobile omnes coelorum motus continens*, (Venice: Peter Liechtenstein, 1515).

[59] Pedersen, *Survey of the Almagest*, p. 16.

[60] This work, which is found in Dresden, Sächsische Landesbibliothek, Db. 87, is discussed in Lorch, *Thabit*, pp. 355-9. He edited the passage on the Menelaus Theorem and made a mathematical paraphrase, pp. 363-375.

[61] Charles H. Haskins, *Studies in the History of Mediaeval Science*, (Cambridge: Harvard University Press, 1927), pp. 106-8. An exhaustive study of the exact number of Latin translations and the number of manuscripts of each has not been accomplished although it will likely be done in the next decade.

the most common of the works translated into Latin containing the Menelaus Theorem, the *Almagest* appears to have been studied only by a relatively small number of scholars. While astronomy was required in the arts faculties of medieval universities, the requirement was usually to attend classes on the *De sphaera*. That does not mean that the *Almagest* was not taught at universities, but that only those most interested in astronomy would attempt to study it.

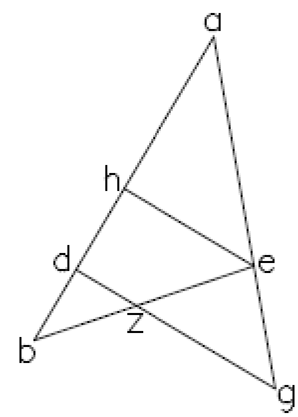
The Menelaus Theorem

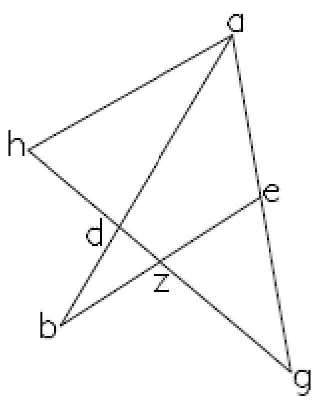
The treatment of the Menelaus Theorem in the Gerard of Cremona translation is located in the twelfth chapter of the first book.[62] After giving instructions for constructing an instrument in order to find the greatest declination of the ecliptic, Ptolemy proceeds to prove six preliminary theorems or lemmas for his proof of the Menelaus Theorem.

He first demonstrates what were called in the Middle Ages the "conjoined" (*coniuncta*) and "disjoined" (*disiuncta*) rectilinear sector figures. The given situation is the same in both. Two lines meet at an angle at a point, and from their other endpoints come two other lines that cross each other and terminate in the original two lines. Ptolemy first proves that the ratio of *ga* to *ae* is composed (*aggregatur*) of the ratio of *gd* to *dz* and of the ratio of *zb* to *be*. Ptolemy produces the line *eh* parallel to *gd*, and because of similar triangles *gad* and *eah*, *ga* is to *ae*

[62] All references will be to the 1515 printed edition of the *Almagest* unless noted. The chapter numbers sometimes differ slightly from those of other versions of the *Almagest*.

as *gd* is to *eh*. But, with *dz* placed as a middle between *gd* and *eh*, the ratio of *gd* to *eh* is composed of *gd* to *dz* and *dz* to *eh*. Therefore *ga* to *ae* is composed of *gd* to *dz* and *zd* to *eh*. But, because *eh* and *zd* are parallel, *zd* to *eh* is as *bz* to *be*. Therefore, *ga* to *ae* is composed of *gd* to *dz* and z*b* to *be*.[63]





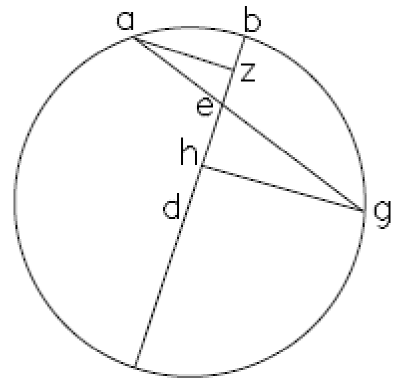
Secondly Ptolemy proves that the ratio of *ge* to *ea* is composed of the ratio of *gz* to *zd* and of the ratio of *db* to *ba*. He produces *ah* parallel to *ez*, and extends *gd* to *h*. With *dz* placed between *gz* and *zh*, the proportion of *gz* to *zh* is composed of *gz* to *zd* and of *zd* to *zh*. And the ratio of *dz* to *zh* is as *db* to *ba* because *ab* and *zh* fall between parallels *ah* and *be*. Therefore *gz* to *zh* is composed of *gz* to *zd* and *db* to *ba*. But *ge* is to *ea* as *gz* to *hz* because *ah* and *ez* are parallel, so *ge* to *ea* is composed of *gz* to *zd* and *bd* to *ba*.[64]

In the third lemma Ptolemy proves that given two continuous arcs taken on a circle, each less than a semicircle, if a diameter is produced from their shared point, then the line joining their other endpoints is divided by that diameter in the same ratio that is between the chords of the doubles of the given arcs. *Ab* and *bg* are the given arcs, and diameter *bd* and line *ag* are drawn. Lines *az* and *gh* are

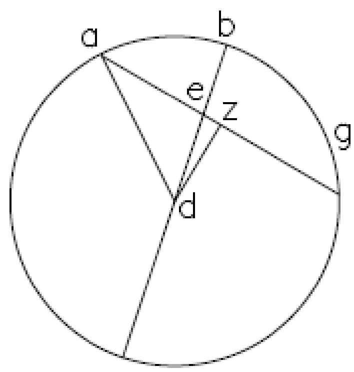
[63] Ibid., fol. 9v.

[64] Ibid.

dropped perpendicularly upon *bd*. Because *az* and *gh* are parallel and *aeg* falls on them, *az* is to *gh* as *ae* is to *eg*. But *az* and *gh*, which are the halves of the chords of double arcs *ab* and *bg*, are in the same ratio as crd. arc 2*ab* to crd. arc 2*bg*, therefore *ae* is to *eg* as crd. arc 2*ab* is to crd. arc 2*bg*.[65]



The fourth shows that if an arc combined from two others and the ratio of the chords of the doubles of the two smaller arcs are known, then each of the two smaller arcs will be known. In terms of the diagram, if arc *ag* and the ratio of the chords of double arcs *ab* and *bg* and known, then arcs *ab* and *bg* will be known.

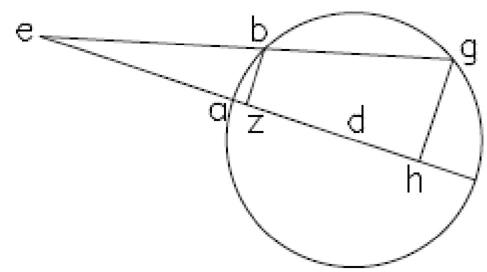


Again with diameter *bd* and line *ag* drawn, *a* and *d* are joined and *dz* is dropped perpendicularly from *d* to *ag*. Because arc *ag* is known, angle *adz* is known and right triangle *adz* will be known. Because from the last proposition, *ae* is to *eg* as the known ratio of crd. arc 2*ab* and crd. arc 2*bg*, line *ae* is also known; therefore *ez* is known. *Dz* will be known because of this, and then we know angle *edz* of right triangle *edz*. So,

[65] Ibid. I use the abbreviation "crd. arc 2__" to stand for "the chord of the double of arc __." Although we run the risk of losing the original meaning of the mathematics by using abbreviations, writing out "the chord of the double of arc __" in full every time can make some sentences very long and almost incomprehensible.

angle *adb* is known, and thus we know arc *ab*. Because arcs *ag* and *ab* are known, arc *bg* will also be known.[66]

In the fifth theorem, if two continuous arcs are taken on a circle, each less than a semicircle, and the diameter through the unshared endpoint of the first arc and the chord of the second arc are extended until they meet, then the whole line that cuts off the second arc and meets the diameter will be to its part outside the circle as the chord of double the combined arcs to the chord of double the first arc. In terms of Ptolemy's figure, *ab* and *bg* are the first and second arcs, and lines *ad* and *bg* are extended until they meet at *e*. It is to be proved that *eg* is to *eb* as crd. arc 2*ag* to crd. arc 2*ab*. *Bz* and *gh* are drawn perpendicular to *ad*. Because they are parallel, *eg* is to *eb* as *gh* to *bz*, which are the halves of the chords of double arcs *ag* and *ab*. Therefore *eg* to *eb* is as crd. arc 2*ag* to the chord of double *ab*.[67]

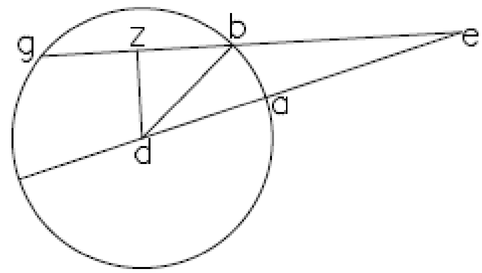


The sixth theorem proves that when there are two continuous arcs, each less than a semicircle, and one of the arcs and the ratio of the chord of double the combined arc to the chord of double the unknown arc are known, then this unknown arc will be known. In terms of Ptolemy's figure, if arc *gb* and the ratio of crd. arc 2*ag* to crd. arc 2*ab* are known, then arc *ab* will also be known. Line *gb* and radius *ad* are extended until they meet at *e*, line *dz* is dropped perpendicularly upon

[66] Ibid.

[67] Ibid., 9v-10r.

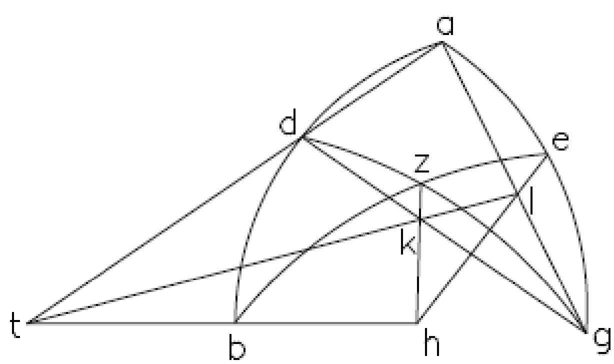
gb, and line *db* is made. Angle *bdz* will be known because it is the angle corresponding to half of the known arc *gb*, therefore whole right triangle *bdz* will be known. From the last theorem, the ratio of *ge* to *eb* is known, and chord *gb* is also known, so line *eb* and whole line *ge* can be determined. Because sides *ez* and *dz* of right triangle *dez* are known, angle *edz* will be known. Therefore angle *edb*, the difference between two known angles *zdb* and *edz,* is known, and therefore arc *ab* is known.[68] The fourth lemma and this sixth one are not necessary for the Menelaus Theorem. They are useful for certain applications, but not for any of the Ptolemy's applications. When the Menelaus Theorem is applied, the immediate result is that a ratio is composed of two others. If five of the six quantities involved are known, then one of the terms in this newfound ratio is known, so the other can be found easily by finding a fourth proportional. This is how Ptolemy always proceeds. These two lemmas show how to find a sought term if neither of the two terms that are in the newfound ratio are known, but their sum or the difference is known. From this inclusion of material that he does not use, it can be inferred that "the computation lemmas must have been written by some other mathematical astronomer who actually used them in his spherical astronomy."[69]



[68] Ibid, 10v.

[69] Sidoli, p. 60.

After the lemmas, Ptolemy demonstrates the Menelaus Theorem itself. The figure involves the intersection of four arcs of great circles, each less than a semicircle. The first two arcs form an angle, and from their other endpoints the two other arcs are reflected across each other and terminate in the first two arcs. The ratio of the chord of double the lower portion of one of the original arcs to the



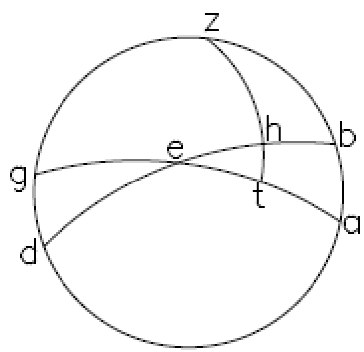
chord of double the upper part is composed of the ratio of the chord of double the lower portion of the reflected arc that is conterminal to that first arc to the chord of double the upper portion of that reflected arc and of the ratio of the chord of double the lower portion of the other original arc to the chord of double the whole other original arc. In particular terms, crd. arc 2*ge* is to crd. arc 2*ea* in the ratio composed of the ratio of crd. arc 2*gz* to crd. arc 2*zd* and of the ratio of crd. arc 2*bd* to crd. arc 2*ba*. Lines are drawn from the center of the sphere, *h*, to points *b*, *e*, and *z*. The chord *ad* and line *hb* are extended until they meet at *t*. In Ptolemy's diagram these meet on the left side of the diagram, but it is also possible that *ad* and *hb* are parallel or that they meet on the right side of the diagram. Ptolemy does not prove these other cases, but as we will see, many of his commentators do. The chords of arcs *gzd* and *gea*, *gd* and *ga* respectively, are drawn, and they cut *hz* and *he* at points *k* and *l* respectively. Points *t*, *k*, and *l* are in one straight line because they are all in the planes of both circle *bze* and of triangle *adg*. The rectilinear sector figure is thus produced with lines *ag* and *at* meeting at point *a* and the lines *gd* and *tl* coming from their other endpoints and cutting each other at *k*. From the second lemma, the ratio of *gl* to *la* is composed of the ratio of *gk* to *kd* and of the ratio of *td* to *ta*. But from the third lemma, *gl* is to *la* as crd. arc 2*ge* to crd. arc 2*ea*, and *gk* is to *kd* as crd. arc 2*gz* to crd. arc 2*zd*. From the fifth lemma, *td* is to *ta* as crd. arc 2*bd* to crd. arc 2*ba*. Therefore, the ratio of crd. arc 2*ge* to crd. arc 2*ea* is composed of the ratio of crd. arc 2*gz* to crd. arc 2*zd* and of the ratio of crd. arc 2*bd* to crd. arc 2*ba*, which is what he intended to prove. As stated in the introduction, this statement was later known as the "disjoined sector figure." Ptolemy also adds that with the same given situation of arcs on the surface of a sphere, another statement can be proved by using the first lemma—that the ratio of crd. arc 2*ga* to crd. arc 2*ae* is composed of the ratio of crd. arc 2*gd* to crd. arc 2*dz* and of the ratio of crd. arc 2*bz* to crd. arc 2*be*, but he does not go through the steps needed to prove this statement, which later was known as the "conjoined sector figure."[70]

One feature of Ptolemy's proofs is that their enunciations are given in terms of the specific diagram and not in general form. In my summary of these theorems, I have generalized his particular formulations. On the other hand, most of the commentaries give general enunciations, which follow more closely the style of enunciations found in systematic mathematical works such as the *Elements* or Jordanus de Nemore's *Arithmetica*.

[70] 1515 edition, 10r.

The Applications of the Sector Figure in the *Almagest*

After proving the Menelaus Theorem, Ptolemy immediately uses it to find the declination of points of the ecliptic. If a great circle is passed through a given point of the ecliptic and the poles, then that point's declination is the arc of that great circle between the point and the equator. Let circle *abgd* be the circle through

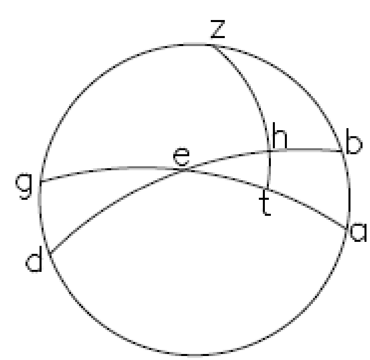


the poles of the ecliptic and equator, circle *aeg* the equator, and circle *bed* the ecliptic. The equator and ecliptic intersect at *e* which is the vernal equinox, and *b* and *d* are the winter and summer tropics. Let arc *eh* be taken as 30 degrees of the ecliptic, and from *z*, a pole of the equator, draw arc *zht* of a great circle. Arc *ht* is the sought declination. A sector figure is formed by arcs *az*, *ae*, *zt*, and *eb*. If we apply the conjoined sector figure, the ratio of crd. arc 2*za* to crd. arc 2*ab* is composed of the ratio of crd. arc 2*zt* to crd. arc 2*th* and of the ratio of crd. arc 2*he* to crd. arc 2*eb*. But, we already know that double arc *za* is 180 degrees and its chord is 120,[71] and from observations we have found that double arc *ab* is 47 degrees, 42 minutes, and 40 seconds, so its chord is 48 parts, 31 minutes, and 55 seconds. And we also know that double arc *he* is 60 degrees, so its chord is 60 parts. Double arc *eb* is 180 degrees and its chord is 120 parts. When we subtract the ratio of 60 to 120 from the ratio of 120 parts to 48 parts, 31 minutes, and 55 seconds, the remainder is the

[71] Note that Ptolemy uses "partes" for the units of arcs and chords, but he means the parts by which the circumference is 360 for arcs and the parts by which the diameter is 120 for chords. For the sake of simplicity I will use "degrees" for the units of arcs and "parts" for the units of chords.

ratio of crd. arc 2*zt* to crd. arc 2*th*, which is the ratio of 120 to 24 parts, 15 minutes, and 57 seconds. Double arc *zt* is 180 degrees, and its chord is 120 parts, therefore the chord of arc double arc *th* is 24 parts, 15 minutes, and 57 seconds, and from the table of chords double arc *th* will be 23 degrees, 19 minutes, and 59 seconds. Arc *th* is thus found to be 11 degrees and approximately 40 minutes.[72] Ptolemy repeats this process with *he* as 60 degrees.[73]

After a table of declinations for each degree of the ecliptic, Ptolemy shows how to find the right ascension of arcs of the ecliptic. The right ascension is the arc of the equator, here *et*, that rises with a given arc of the ecliptic, *eh* in the diagram, starting from an equinoctial point. Right ascensions give the time that it takes for arcs of the ecliptic to rise in the right sphere (when the horizon passes through the poles of the equator).[74] The figure is the same as for finding declinations, but now he wants to find arc *et* instead of *ht*. He does this by applying the disjoined sector figure, which gives him that the ratio of crd. arc 2*zb* to crd. arc 2*ba* is composed of the ratio of crd. arc 2*zh* to crd. arc 2*ht* and of the ratio of crd. arc 2*te* to crd. arc 2*ea*. The chords of double arcs *zb*, *ba*, *zh*, and *ht* are known, so when we subtract the



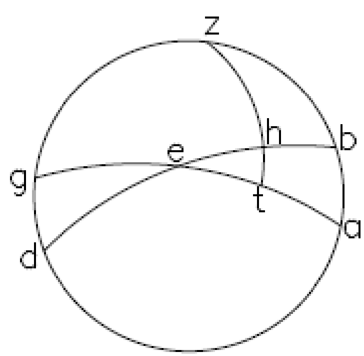
[72] 1515 edition, 10r.

[73] Ibid., 10r-v.

[74] He talks of the arc of the equator as the "tempus elevationis" and refers to the degrees of the equator as "tempora," which illustrates that he is primarily concerned with finding how long it takes different arcs of the ecliptic to move with the daily motion of the ecliptic past the meridian or any great circle through the equator's poles since the values are mirrored in the other three quadrants.

known composing ratio from the known composed ratio, the formerly unknown ratio crd. arc 2*te* to crd. arc 2*ea* remains and is now known. Double arc *ea* is known, so Ptolemy puts the ratio in terms such that the second term is the same quantity as the known quantity. Again Ptolemy first goes through the calculation when eh is 30 degrees and then when it is 60 degrees.[75] Ptolemy then gives the values of *ht* for 10 degree increments of *eh* in one quadrant.[76] The variations in smaller sections are apparently negligible.

Because for those not living on the equator, the horizon is not a great circle through the pole, understanding matters only in the right sphere is insufficient.



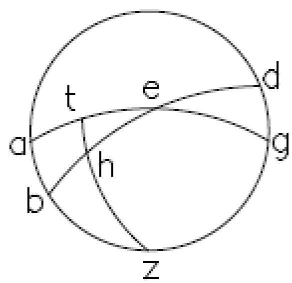
Book II of the *Almagest* is concerned with how things appear in the "oblique sphere." In Chapter 2 Ptolemy shows through an example how to find the size of the arc on the horizon between the equator and the ecliptic. The figure remains similar except that circle *bed* is now the horizon, point *e* is the intersection of the equator and the horizon, and point *h* is where a given point of the ecliptic meets the horizon as it moves with the daily motion of the heavens. He

[75] Ibid., 11r.

[76] Ibid., 11r-v.

uses the conjoined sector figure's inverse, i.e. the antecedents in all of the ratios become the consequents and vice versa.[77]

In the next chapter, he shows how with the same situation, but with the distance *he* along the horizon between the equator and the ecliptic given, one can find the height of the pole above the horizon, i.e. arc *bz*. He uses the disjoined sector figure (here crd. arc 2*et*:crd. arc 2*ta* comp. crd. arc 2*eh*:crd. arc 2*hb* and crd. arc 2*bz*:crd. arc 2*za*) and performs the usual subtraction of a known composing



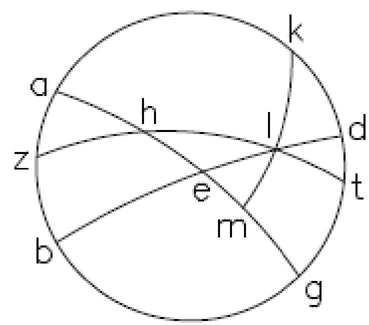
ratio (crd. arc 2*eh*:crd. arc 2*hb*) from the known compound ratio (crd. arc 2*et*:crd. arc 2*ta*) to find the unknown ratio (crd. arc 2*bz*:crd. arc 2*za*). He knows the term (crd. arc 2*za*) because arc *za* is a quadrant, so he is able to find the value for the other (crd. arc 2*bz*). Ptolemy also shows how to find the difference between the lengths of the longest or shortest day and the equinoctial day when given the height of the pole above the horizon. Again, time corresponds to arcs of the equator, so twice arc *et* is sought.[78] He uses the disjoined sector figure. He then briefly goes over how to find the value of the arc on the horizon between the points on the horizon where points of the ecliptic and the equator pass if the height of the pole and the length of the longest day are known. He uses the conjoined sector figure and does not lay out the argument with values, but only

[77] Ibid., 12r. In the 1515 edition, the initial formulation of the composed ratio is incorrect—the first two terms are in the wrong order, but this is corrected in the remainder of the argument.

[78] Arc *te* is the difference between the time *h* takes to travel from rising to the meridian and the six hours that it takes the equinoctial point to rise. There is an arc equal to *te* for the latter half of the day.

says generally that arc *eh* is known since two ratios and one of the terms of the other are known. In all of these examples in this chapter, Ptolemy only considers the distance between an equinox and the winter solstice, but using the same general proof, the distance could be found for any point on the ecliptic using the table of obliquity to find the value for *ht*.[79]

After three chapters that do not utilize the sector figure, Ptolemy returns to it in Ch. 7, which is concerned with oblique ascensions, the arcs of the equator that rise with a section of the ecliptic when the zenith is not on the equator. After two preliminary theorems that allow Ptolemy to easily calculate the co-ascensions for the whole 360 degrees of the ecliptic once they are known for one quadrant, he reaches the main proof. Ptolemy finds the co-ascensions of the sign of Aries and the combined 60 degrees of Aries and Taurus at the latitude of Rhodes. Circle *abgd* is the great circle through the poles of both the equator *aeg* and ecliptic *zhlt*. Arc *klm* is part of a great circle through the pole *k* and point *l*, which lies upon horizon *bed*.

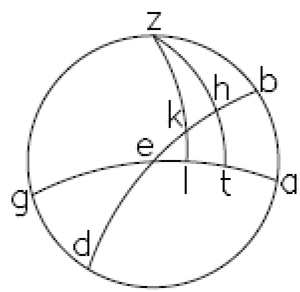


While arc *hl* rises over the horizon, arc *he* of the equator rises, so it is arc *hl*'s oblique ascension. Arc *hm* is arc *hl*'s right ascension, so arc *em* is the difference between the right and oblique ascensions. He uses the disjoined sector figure and subtracts to find the unknown ratio. Here as in a few other places, where Ptolemy subtracts a ratio from a ratio, his resulting ratio does not have either term the same

[79] 1515 edition, 12r-v.

as any of the four terms of the whole ratio and the subtracted ratio. He gives the result in terms such that the consequent is 120 parts, which the consequent is known to be. Then, he finds the antecedent, crd. arc 2*em*, and finds arc *em*, which he subtracts from the right ascension to find arc *he*.[80]

Ptolemy gives a second way of finding the co-ascensions for sections of the ecliptic. Given a figure where *k* and *h* are the points on the horizon *bed* where a given point of the ecliptic and a tropic pass through the horizon respectively and with great circles drawn from the pole *z* of the equator *aeg* through these points to make points *l* and *t* on the equator, the ratio of arc *te* to arc *el* can be found regardless of the latitude. From the inverse of the conjoined sector figure, the ratio of crd. arc 2*th* is to crd. arc 2*hz* is composed of the ratio of crd. arc 2*te* to crd. arc 2*el* and the ratio of crd. arc 2*lk* to crd. arc 2*kz*.[81] The first ratio is known because arc *th* is the greatest declination of the ecliptic and arc *hz* is its complement. The third ratio is also known because for arc *lk*, the obliquity of a given arc on the ecliptic, can be found for any given arc of the ecliptic through the table of obliquity, and arc *kz* is *lk*'s complement. By subtraction of one known ratio from the other known ratio, Ptolemy finds the ratio of crd. arc 2*te* to crd. arc 2*el* for 10-degree increments of the ecliptic. Given a latitude, arc *te* is easily found since it is half of the difference between the longest day at that latitude and 12



[80] Ibid., 16r-v.

[81] Ptolemy reverses the order of the composing ratios from his normal sequence.

equal hours, and therefore arc *el* can be found through the simple process of finding a fourth proportional. When Ptolemy performs the subtractions of ratios, the steps are not given and the resulting ratios are all given with the antecedent as 60, which Ptolemy presumably selected for ease in the calculation with sexagesimal system to find a fourth proportional, crd. arc 2*el*, when crd. arc 2*te* is known for a particular latitude.[82] Although initially more complicated than the first method of finding oblique ascensions, this second way leads to a simpler method for performing a large number of calculations such as are needed for making Ptolemy's Table of Oblique Ascensions, which follows this seventh chapter and which requires the calculation of 99 oblique ascensions.[83]

In the ninth chapter Ptolemy explains how to use the Table of Oblique Ascensions to find the length of any day or night throughout the year at a given latitude, to find the number of equal hours in that day, to find the length of that day's seasonal hours, to convert between the two types of hours, and to find the part of the ecliptic which is rising, setting, or passing through the meridian at any given time at a given latitude.[84] Using this table, which relies upon the sector figure, one can perform all of these astronomical calculations with only the simple operations of addition, subtraction, multiplication, and division.

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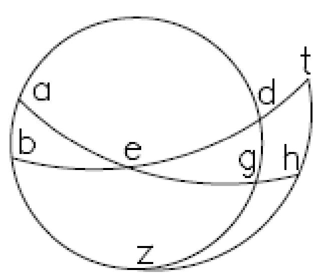
[82] 1515 edition, 16v-17v.

[83] Ibid., 17v-18v. The table gives 396 values, but three quarters of them can be easily derived from the calculation of oblique ascensions for one quarter of the ecliptic.

[84] Ibid., 19r.

Ptolemy's next three chapters deal with spherical angles contained between the ecliptic and the meridian, the horizon, and great circles passing through the zenith. Ptolemy points out that these angles are necessary for determining the parallax of the moon, among other things. For each of these types of angles, Ptolemy first performs general proofs that permit him to find the angles for the whole ecliptic if he finds those for one quarter. The first theorem that uses the sector figure concerns finding the angle between the meridian and the ecliptic when a given degree of the ecliptic passing through the meridian is provided. He does this with two different values of the ecliptic, and starting with the disjoined sector figure, he proceeds as normal to find the unknown arc, which he then uses to find the sought angle.[85]

Ptolemy breaks from his normal use of the sector figure in the eleventh chapter while finding the angle between a given horizon and the ecliptic. With meridian *abgd*, Ptolemy considers the situation in which intersection *e* of horizon *bed* and ecliptic *aeg* is the beginning of Taurus. With point *e* as pole, he draws a



great circle *zht*, (notice the interesting representation of three dimensions used to make more than a hemisphere visible)[86] and seeks the arc *ht*, which determines the sought angle *het*. He states that the ratio of crd. arc 2*gd*

[85] Ibid., 20r. He considers an angle known when the arc that subtends it of the great circle that has the vertex of the angle as its pole is known.

[86] This "unwrapping" of the surface of the sphere is found in the manuscripts as well. The representation of three dimensions in spherical geometry is an intriguing topic that I unfortunately am not able to treat at length at present.

to crd. arc 2*dz* is composed of the ratio of crd. arc 2*ge* to crd. arc 2*eh* and the ratio of crd. arc 2*ht* to crd. arc 2*tz*, which is not either of the conclusions of the sector figure that Ptolemy gives.[87] This statement is easily derived from the conjoined sector figure if it is assumed that given a ratio composed of two others, it follows that one of the composing ratios is composed of the original composed ratio and the inverse of the other composing ratio. In simpler terms, given that A:B is composed of C:D and E:F, then C:D is composed of A:B and F:E or E:F is composed of A:B and D:C. The statement is then inverted, so D:C is composed of B:A and E:F. Interestingly, Ptolemy could have worked directly from the conjoined sector figure, which, applied here, would state that the ratio of crd. arc 2*tz* to crd. arc 2*ht* is composed of the ratio of crd. arc 2*dz* to crd. arc 2*gd* and the ratio of crd. arc 2*ge* to crd. arc 2*eh*. The four terms of the composing ratios are known, so he would merely have to add or compound the ratios to find the composed ratio, but instead Ptolemy transforms his statement and inverts it in order to have the unknown term be the antecedent and not the consequent of the unknown ratio. Perhaps since the majority of his uses of the sector figure result in the unknown ratio being one of the composing ratios and the unknown term being the antecedent, Ptolemy decided that it was worth keeping this uniformity although it required the use of a transformation of the unproved statement of the sector figure. The making of a new statement of composition by rearranging the terms in another statement of composition that Ptolemy does in this chapter, the next chapter, and in Book VIII

[87] Ibid., 20v-21r.

would point commentators towards the study of how many different arrangements of the six terms there are, and which of these are necessarily true when the first is true.

In Chapter 12, Ptolemy shows how to find the angle at different points on the ecliptic between the ecliptic and great circles passing through the zenith at different latitudes. Again, he begins with a series of general propositions that do not use the sector figure but that allow him to extrapolate the angles over points of the whole ecliptic from angles in one quadrant. Unlike in previous uses, he applies the sector figure twice in order to find this angle. He uses it first to find the arc of a great circle between the zenith and a given point of the ecliptic (I will later refer to this arc as the "zenith distance arc"), and then he uses a sector figure involving that arc to find the arc subtending the sought angle. The actual arguments from the sector figure are fairly typical. In the first, he uses the inverse of the conjoined sector figure, and in the second, he uses the disjoined. In both cases he follows the typical pattern of subtraction to find the unknown ratio and the unknown term.[88] Ptolemy then finishes the second book with his table of angles between the ecliptic and great circles through the zenith at different latitudes, which will eventually be used in Book V for finding the moon's parallax.[89]

The Menelaus Theorem sees little explicit use until Book VIII. In III.10 Ptolemy says to use the table of right ascensions in order to find the differences in

[88] Ibid., 21v-22r.

[89] Ibid., 22v-25v.

the length of solar days throughout the year,[90] but the Menelaus Theorem's use is seen more clearly in lunar and eclipse theory. He uses the theorem (or a simplified version of it)[91] in V.8 for finding the latitude of the moon from the ecliptic.[92] Ptolemy treats the moon's longitudinal movements as if they were in the plane of the ecliptic. However, because the moon moves in a plane that is tilted with respect to the ecliptic, the longitudinal values calculated on the ecliptic vary slightly from the true values in the other plane, but Ptolemy's choice to treat the two planes as identical with regards to the moon's longitudinal motions is justifiable since the maximum error in longitude caused by this approximation is relatively minor.[93] The latitudinal variation, on the other hand, needs to be considered when locating the exact position of the moon since the moon can be a very noticeable 5^{o} north or south of the ecliptic. Ptolemy does not go through the proof of how to find these distances from the moon's positions to the ecliptic, but he merely adds the values of the moon's latitude in the seventh column of the accompanying table of the general lunar anomaly. Giving some clue though as to how this is done, he states, "And we

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[90] Ibid., 35r.

[91] Pedersen, *Survey of the Almagest*, p. 201 claims that Ptolemy uses a simplified version that treats one small arc as being equal to its chord..

[92] 1515 edition, 51r-v. The next few paragraphs deal with the very complicated lunar theory of Ptolemy. In order to not bog the reader down with a lengthy exposition of this, I have assumed familiarity with it. I suspect few readers will know the details of the theory, so I suggest that the reader turn to Pedersen, *Survey of the Almagest*, Chapters 6-7, or to read quickly through the next few paragraphs or skip them entirely, merely noting that the zenith distance arc, the angle formed by the zenith distance arc and the ecliptic, and the table of declinations are used to calculate the latitude of the moon from the ecliptic, the parallax of the moon and sun, the sun and moon's sizes and distance from the earth, and the occurrence and length of eclipses.

[93] Pedersen, *Survey of the Almagest*, pp. 199-200, shows that the maximum error is only 7 minutes of arc.

will use in the proof of this the chapter in which the arcs of the circle described through [the equator's] poles that are between the equator and the ecliptic are shown."[94]

In V.13 he uses values from both the table of latitudes of the moon and the table of obliquity in order to compare the observed latitude of the moon from the ecliptic to the calculated latitude of the moon from the ecliptic, which gives him the latitudinal difference of aspects of the moon, or the lunar parallax in altitude.[95] Then in V.14, he uses the moon's latitude from the ecliptic, which is found in the table of the general lunar anomaly (V.8), to find the ratio of the radius of the earth's shadow to the radius of the moon.[96] He then uses this last value in V.15 to find the distance of the sun, which is then used in the following chapter to find the size of the sun and moon compared to the earth.[97]

In V.17, he tells how to construct the parallax table (V.18) for the sun and the moon for different lengths (6^{o} apart) of the arc from the zenith to points on the ecliptic.[98] Since these zenith distance arcs are assumed, he does not need to calculate their values at this point. In V.19, however, he tells how to use the table

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[94] 1515 edition, 51v. The text reads, "Et utemur in declaratione illius capitulo quo demonstrantur arcus qui sunt inter orbem equationis diei et inter orbem signorum orbis descripti super polos eorum." That last "eorum" is a mistake since in I.13 arc *zht* does not pass through the ecliptic's poles.

[95] Ibid., 54r.

[96] Ibid., 55r-v.

[97] Ibid., 55v-56r.

[98] Ibid., 56v-58r.

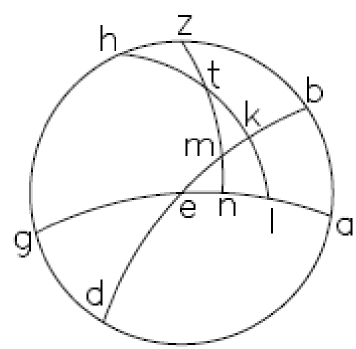
of parallaxes at any given longitudinal value for the sun or the moon.[99] From the sun or moon's longitude, one must find the zenith distance using the table at the end of Book II. This is not completely accurate since the moon can stray approximately 5^{o} from the ecliptic. However, because he is concerned with eclipses which occur only near the nodes, when the moon is on or near the ecliptic, the table in Book II is sufficiently accurate for his purposes. The arc of parallax that he finds through his table of parallaxes is measured along the circle passing through the zenith; he next shows how to convert this value to terms of longitude along and latitude from the ecliptic. He does this by many substitutions of known values for values that differ in reality but in practice are approximately equal and by treating a small spherical triangle as a rectilinear triangle. In one small right-angled spherical triangle, he knows the hypotenuse, which is the parallax measured along the zenith distance arc, and one of the non-right angles, because it is approximately the angle formed by the zenith distance arc and the ecliptic (from II.12).[100] He says that therefore the other two sides of the triangle are able to be known. Since Ptolemy nowhere tells how to find the legs of a right angled spherical triangle through knowledge of only the hypotenuse and another angle, he must intend that the spherical triangle be treated as rectilinear. Ptolemy uses the parallax values that he has shown how to find in Book V extensively in Book VI,

[99] Ibid., 58v-60r.

[100] Ptolemy knows that this angle is not in fact the angle between the ecliptic and the zenith distance arc, so he provides some further adjustments to partially account for the slight difference.

which is on eclipses. He also uses rectilinear approximations of spherical figures several times.[101]

Ptolemy's next explicit use of the theorem is in Book VIII, which treats the fixed stars. In Chapter 5, he tells how to find the equatorial longitude and latitude of a star whose ecliptical longitude and longitude are known. In this proof and in the other ones of Book VIII, he does not use specific values and he departs from the pattern of steps that he had followed almost without fail in the first and second books. With *abgd* the great circle through the poles *z* and *h* respectively of equator *aeg* and ecliptic *bed*, the latitude from the equator of the given star is arc *tn*, which Ptolemy finds with the conjoined sector figure. He does not talk about the subtraction of ratios; he merely states how five of the terms are known, and that thus the sixth will also be known. Then a variation of the disjoined sector figure is applied to find the equatorial longitude of the star, which determines the star's rising time. The



conclusion of the sector figure is stated in a form that Ptolemy does not give in his chapter on the Menelaus Theorem. His statement is that crd. arc 2*zh* is to crd. arc 2*ha* in the ratio composed of crd. arc 2*zt* to crd. arc 2*tn* and the ratio of crd. arc 2*nl* to crd. arc 2*la*, which is taken from the application of the statement that one of the composing ratios is composed from the original composed ratio and the inverse of the other composing ratio to the disjoined sector figure. Again, he merely states

[101] E.g., Book VI. 7 and 11, 1515 edition, 67r-68v, 71v-72r.

that with five of the terms known, the sixth will be known. Although Ptolemy does not go through the steps of the subtraction of ratios and the finding of the sixth arc, he seems to have decided to use this altered version of the sector figure statement with that process in mind. He could have given the normal disjoined sector figure and added the two known composing ratios to find the unknown composed ratio, but he apparently prefers to have the unknown ratio be one of the composing ratios. [102]

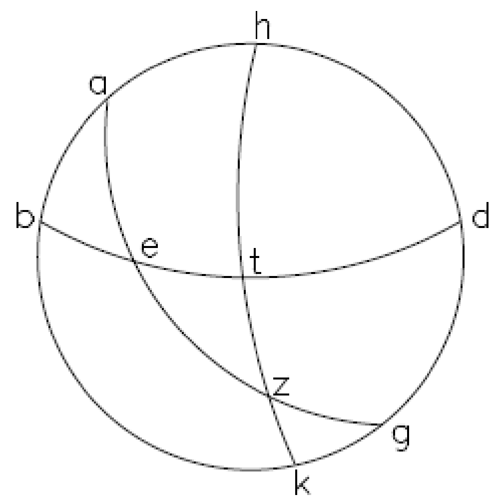
With the equatorial longitude and latitude found, Ptolemy is then able to apply the sector figure once more to find the rising time of a fixed star at a given latitude. This is done with the disjoined sector figure and without going through the steps of finding the unknown arc from the five known terms. Because the time that it takes to travel from the meridian to the horizon is equal to the time that it takes to rise from the horizon to the meridian, the setting time is also known. In this chapter, the second and third applications of the sector figure rely upon the first.[103]

In Chapter 6, Ptolemy discusses the appearance and disappearance of fixed stars. By this he means the times in a year when a star near the sun moves far enough from it that it becomes visible right after sunset and when it is last seen near the eastern horizon before it becomes too close to the sun to be seen. Of course, this varies with the brightness of the star, the latitude on the earth, the sun's

[102] Ibid., 92r.

[103] Ibid.

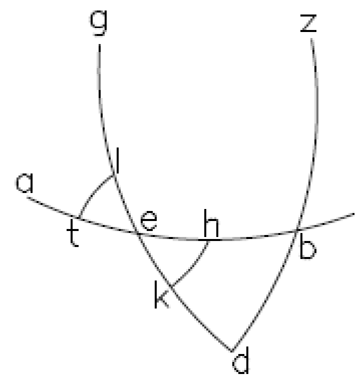
position in the ecliptic, and the latitude of the star with regard to the sun. Ptolemy uses the sector figure twice to find his sought value. *Bed* is the horizon, *aeg* is the ecliptic, *h* and *k* are the zenith and the opposite point on the sphere, and arc *zt* is the unknown distance below the horizon at which the sun *z* is when the star first appears. First, he assumes that arc *ez* is known through observation at some latitude, and he uses an altered version of the disjoined sector figure to find the value of arc *zt* for this star. As he has done before, he has one of the composing ratios of the original statement be composed of the original composed ratio and of the inverse of the other composing ratio. With five terms known, we find the sixth, the distance of the sun below the horizon when the star becomes visible or stops being visible.



Then for different latitudes, the star will appear or disappear when the sun is that distance below the horizon, but this will take longer or shorter from sunrise or sunset depending on the latitude. Using the sector figure again, here the inverse of the altered version of the disjoined sector figure that he has just used, the value of arc *ez* for the new latitude can be found, and this gives the time of the apparition or occultation of the sun.[104]

[104] Ibid. 92v-93r.

In Book XIII in Ch. 7-13 (after the first several chapters which are on the latitudes of the planets, which he treats without the Menelaus Theorem), Ptolemy once again uses values derived from the Menelaus Theorem to find the apparitions and occultations of the planets, a task similar to VIII.6, which we have just discussed. However, while the problems of the apparitions and occultations of the fixed stars and of the planets are very similar, Ptolemy does not need to use the Menelaus Theorem as he did in VIII.6, and instead he treats spherical triangles as rectilinear. This approximation is allowable since these triangles are relatively small. Also Ptolemy realized that because of the great differences in brightness of the planets (unlike the constant brightness of the fixed stars) these calculations cannot be very accurate anyway. In Ch. 7-9, treating right angled triangles such as *bed* as rectilinear, he knows right angle *b*, one of the sides, and angle *bed*, the angle formed by the horizon and the ecliptic which was found from the Menelaus Theorem in II.11.[105]



Format of the *Almagest*

Since one trend in medieval works is a movement of modifying the astronomical content of the *Almagest* into the form of an axiomatic, deductive science, the form in which this content is originally found needs to be examined.

[105] Ibid., 149v-152r.

The *Almagest* is divided into thirteen books and each of these books is divided into chapters. Many chapters involve the solution of several different astronomical problems. While we will be looking at several proofs given by Ptolemy, here let us look at a chapter that has been discussed but now with our attention upon the format instead of the mathematical content.

In Chapter 13 of Book I, Ptolemy describes how to find the declinations of various arcs of the ecliptic. The chapter begins with a heading: "Chapter 13. About the knowledge of the quantities of the arcs which are between the equator and the ecliptic, which are the declinations."[106] In some of the manuscripts, this chapter numbering and heading is not found at the beginning of the chapter but only in a list of chapter topics given at the beginning of Book I.[107] The text of the chapter begins with a statement introducing the chapter, but in a nonspecific manner: "And afterwards we put forward this chapter. We will show first proofs about these arcs as I will describe and exemplify."[108] He then describes how the figure for the finding of the declination is constructed and what astronomical objects the geometrical objects represent:

> I will describe therefore a circle [upon] which revolve two poles, the pole of the equator and the pole of the ecliptic, and I designate it *abg*. And I will describe on it half of the equator *aeg* and half of the ecliptic *bed*, which cut each other at point *e*, which is the vernal equinox. And let point *b* be the

[106] Ibid., 10r. "Capitulum tredecimum. De scientia quantitatum arcuum qui sunt inter orbem equationis diei et orbem medii signorum qui sunt declinationis."

[107] E.g., Città del Vaticano, Biblioteca Apostolica Vaticana, Barb. lat. 182.

[108] 1515 edition, 10r. "Et postquam premisimus hoc capitulum. Ostendemus primum demonstrationes super hos arcus quemadmodum narrabo et exemplificabo."

> winter tropic, and point *d* the summer tropic. And I will place point *z* of arc *abg* as the pole of the equator. And I will posit that arc *eh*, which is on the ecliptic is 30 according to the quantity by which a great circle is 360. And I will draw arc *zht*, which is part of a great circle, and I will search for the knowledge of arc *ht*.[109]

So far, in the text of the chapter, he has not pointed out that he is seeking the declination. Also notice that he has posited specific values and is taking a particular instance—when the arc of the ecliptic is 30 degrees. After a passage explaining the concept of degrees of arc and parts of chords, he then gives the main argument of how to get from what is already known to what he wants to know:

> And because in the figure of these great circles there are two arcs *zt* and *eb* between the two arcs *az* and *ae*, cutting each other upon *h*, the ratio of the chord of double arc *za* to the chord of double arc *ab* will be composed from two ratios—from the ratio of the chord of double arc *zt* to the chord of double arc *th* and from the ratio of the chord of double arc *he* to the chord of double arc *eb*. But we already knew that double arc *za* is 180 and its chord is 120, and double arc *ab* according to that which we considered and found . . . is 47 42' 40'', and its chord is 48 31' 55''. And double arc *he* is 60 and its chord is 60, and double arc *eb* is 180 and its chord is 120. Therefore when we subtract the ratio of 60 to 120 from the ratio of 120 to 48 31' 55'', there will remain the ratio of the chord of double arc *zt* to the chord of double arc *th*, which is the ratio of 120 to 24 15' 57''. But double arc *zt* is 180 and its chord 120, so the line which subtends double arc *th* according to those parts is 24 15' 57'', and similarly double arc *th* will be 23 19' 59'', and arc *th* will be according to these parts approximately 11 40'.[110]

[109] Ibid., 10r. "Describam ergo orbem quem revolvunt duo poli simul, polus equationis diei et polus medii orbis signorum. Et signabo super ipsum *a b g*, et describam in eo medietatem orbis equationis diei, supra quam sint *a e g*, et medietatem orbis medii signorum, supra quam sint *b e d*, qui se secent supra punctum *e*, quod sit punctum equationis diei vernale. Et sit tropicus hiemalis punctum *b*, et tropicus estivalis punctum *d*. Et ponam ut polus orbis equationis diei sit punctum *z* arcus *abg*, et ponam ut arcus *eh* qui est orbis signorum sit triginta secundum quantitatem qua orbis maior est 360. Et describam arcum *zht* qui sit orbis magni, et investigabo scientiam arcus *ht*."

[110] Ibid., 10r-v. "Et quoniam in forma horum orbium maiorum inter duos arcus *az* et *ae* sunt duo arcus *zt* et *eb* sese supra *h* secantes, sit ut proportio chorde dupli arcus *za* ad chordam dupli arcus *ab* aggregetur ex duabus proportionibus, ex proportione chorde dupli arcus *zt* ad chordam dupli arcus *th*

Ptolemy is outlining the way of finding the sought value of a specific declination by using the specific values that are known beforehand. What he finds is a particular result—here it was found that the declination, arc *ht*, of a 30 degree arc of the ecliptic is approximately 11 degrees 40 minutes. Ptolemy continues to give another example, in which the given arc of the ecliptic is 60 degrees instead of 30 degrees. He then follows this with a table of the declinations calculated for every degree of the ecliptic.[111] So, he does not start with a general description of what it is that he is finding and he does not give a universal proof. Instead he gives examples of finding specific values, and leaves it to the reader to abstract the general proof or method of finding declinations.

Compound Ratios

Because Ptolemy introduces compounding by the introduction of a quantity between two quantities and because he thinks that it is clear that the ratio of the extremes is composed of the ratio of one extreme to the middle and of the ratio of the middle to the other extreme, it is tempting to conclude that he must have held

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et ex proportione chorde dupli arcus *he* ad chordam dupli arcus *eb*. Iam autem scivimus quod duplum arcus *za* est 180, et chorda eius 120, et duplum arcus *ab* secundum quod consideravimus et convenimus supra ipsum secundum proportionem 11 ad 83 est 47 partes et 42 minuta et 40 secunda, et est eius chorda 48 partes et 31 minuta et 55 secunda. Et duplum arcus *he* est 60 partes, et eius chorda est 60 partes, et duplum arcus *eb* est 180 partes, et eius chorda 120 partes. Cum ergo nos proiecerimus ex proportione 120 partium ad quadragintaocto partes et 31 minuta et 55 secunda proportionem 60 ad 120, remanebit proportio chorde dupli arcus *zt* ad chordam dupli arcus *th* que est proportio 120 ad 24 partes et 15 minuta et 57 secunda. Duplum vero arcus *zt* est 180, et eius chorda 120. Linea ergo que subtenditur duplo arcus *th* secundum illas partes est 24 partes et 15 minuta et 57 secunda. Et similiter erit duplum arcus *th* 23 partes et 19 minuta et 59 secunda, et erit arcus *th* secundum illas partes undecim partes et 40 minuta vicinius.”

[111] Ibid., 10v.

the continuous concept of compounding. This, however, is not necessarily true. Even if he accepted the other concept of compounding, it is still possible that he could assume that his readers would be familiar enough with compound ratios to know that this followed from the second conception of compounding. It seems then more likely that his idea of compound ratio was based upon continuity, but it is not certain that he thought this way. As we will see, this ambiguity about Ptolemy's understanding contributed to both understandings of compound ratio being used by medieval commentators to explain him.

While he may have not been clear on the essence of compound ratios, he was clearer on how to work with compound ratios. Again and again, Ptolemy uses a statement that one ratio is composed from two others from the sector figure (I will be referring to these kinds of statements as "statements of composition") to find an unknown. He selects his sector figures and manipulates them such that he always knows the composed ratio and one of the composing ratios. He then subtracts to find the other composing ratio. In all cases, Ptolemy's sought quantity is known to be in that ratio to another known quantity, so the unknown is able to be found.

Ptolemy rarely gives enough steps for us to find the exact steps that he follows to subtract one known ratio from another, but some idea of how he does it can be surmised. He usually only gives the four terms of the two given ratios and then gives the two terms of the resulting ratio without going through the intermediate steps. A few of the times that Ptolemy subtracts ratios, the antecedents or consequents of both the ratio to be subtracted and the one from which it is to be subtracted are the same quantity or the terms of the ratios can be easily changed to make this the case.[112] In these instances, the simplest way for Ptolemy to subtract ratios and to reach the answer in these terms would be for him to merely put the composed ratio in new terms that are the halves of the current ones. In the first subtraction of ratios in I.14, Ptolemy subtracts the ratio of 117 31' 55'' to 24 15' 57'' from the ratio of 109 44' 53'' to 48 31' 55'', and he gives the resulting ratio in the terms of 54 52' 26'' to 117 31' 15''.[113] By halving the terms of the composing ratio,[114] we have the same quantity as the consequent of both this composed ratio and the ratio to be subtracted, so the ratios are continuous in a way such that taking the antecedent of the composed ratio to the antecedent of the ratio to be subtracted is the remaining ratio.

Except for the three easy examples that work out this way, the other subtractions of ratios cannot be done as easily. In the second example of calculation found in I.14, Ptolemy reveals his method more than in others (but it is still rather hidden). Ptolemy subtracts the ratio 112 23' 56'' to 42 1' 41'' from the ratio 109 44' 53'' to 48 31' 55''. He first gives the resulting remainder as the ratio

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112 Some of these instances can be found in his chapters on finding of declinations of arcs of the ecliptic (I.13, ibid., 10r-v), the finding of right ascensions (I.14, ibid., 11r), and the second way of finding arcs of the horizon between the equator and given points on the ecliptic (II.3, ibid., 12v). Of these, only the last necessarily works out this easily. Ptolemy gives examples for the first two that work out in this easy manner, but he follows these with more typical examples that do not work out so nicely.

113 1515 edition., 11r. Ptolemy compounds ratios of greater inequality and lesser inequality together. Later there would be debate over whether this was proper, but this does not appear to have appeared contentious to Ptolemy or his commentators.

114 Half of 109p 44' 53'' is 54' 52' 26'' 30''', and half of 48p 31' 55'' is 24p 15' 57'' 30''', but Ptolemy rounds down to the nearest seconds.

of 95 2’ 41’’ to 112 23’ 56’’. He then puts this into terms such that the consequent is 120, which is the chord of the double of one of the known arcs, so that the unknown arc can be found.[115] That he gives the remaining ratio first in the form that immediately results from whatever his operation of subtraction entails gives us some idea of his procedure. Because the antecedent of the ratio to be subtracted is the consequent of the ratio resulting from the subtraction, we are able to determine the likely way that Ptolemy proceeds. He seems to be performing a multiplication and division to put the composed ratio in terms such that it can be made continuous with the ratio to be subtracted. He does this by multiplying the antecedent of the composed ratio by the consequent of the ratio to be subtracted and divides that by the consequent of the composed ratio. Since the consequents of the composed and the composing ratio are now identical (the ratios are continuous but in “opposite directions”), the difference between them is the ratio of the antecedent of the composed ratio to the antecedent of the subtracted ratio. Since there are other ways—although not quite as simple as this way—to get to the same result, it is impossible to ascertain exactly what steps Ptolemy takes, but he seems to be performing this order of steps or equivalent ones in some order.[116] This way has the added appeal of uniting Ptolemy’s idea of compounding. If he had several completely different ways of subtracting ratios, it would suggest that he lacked a

[115] 1515 edition, 11r.

[116] In II.3’s first example of finding the difference between the longest day and twelve hours (ibid., 12v), he also gives the ratio that immediately results from his subtraction. The same steps seem to be followed there except that the two ratios are made continuous by putting the composing ratio into new terms such that the antecedents of the two ratios are identical, and then by taking the ratio of the consequent of the composing ratio to the consequent of the composed ratio.

clear definition of subtraction of ratios. If he subtracts ratios in the way that I have suggested, then all of his subtractions amount to putting ratios continuous in a certain way (i.e. with either the consequents or antecedents of the two ratios identical) although sometimes the continuity is already there or can be made without much work and at other times the continuity is reached by multiplying and dividing terms.

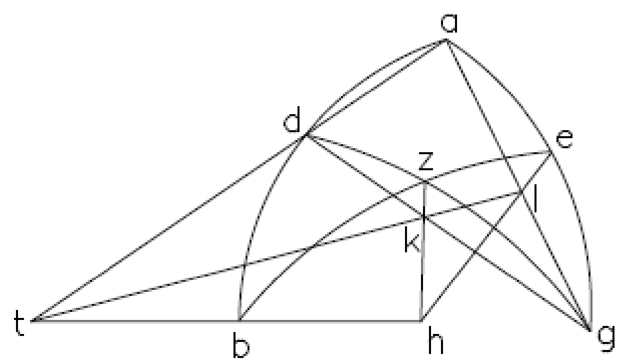
What is clear from this example is that Ptolemy is not dividing the denomination of the composed ratio by the denomination of the subtracted ratio. If he were to find the denominations of the composing ratios and the ratio to be subtracted and divided the first by the second, he would reach the denomination of the remaining ratio, not the ratio in terms of one quantity to another. He would then have to add an extra, completely unnecessary step of putting the ratio in the first terms that he reaches before putting them into the terms that are immediately useful for finding the sought quantity.

As mentioned above, in three applications of the sector figure, Ptolemy does not use the statements of composition given in I.12. In II.11, he gives a statement of composition that is derived from the conjoined sector figure, but now the terms are rearranged such that, naming our terms by the order they would be in from the statement from I.12, the ratio of the fourth term to the third is composed of the ratios of the fifth to the sixth and of the second to the first.[117] In VIII.5-6, he uses

[117] Ibid., 20v

another reordering of the terms in a statement of composition: the ratio of the fifth term to the sixth is composed of the ratios of the first to the second and of the fourth to the third.[118] Both of these can be justified with the proposition, unstated and unproved by Ptolemy, that if a ratio is composed of two others, then one of the composing ratios is composed of the composed ratio and the inverse of the other composing ratio.

Ptolemy's *Almagest* was the most common of the works translated into Latin that contain the Menelaus Theorem. It left ample room for further treatment of the sector figure and compound ratio. Ptolemy gives a proof of only a single case of one of the two statements that together were known as the Menelaus Theorem. Looking again at his diagram, we see that lines *ad* and *hb* could be parallel or could meet on the other side of the diagram. No proof at all was given for the spherical conjoined sector figure, although a hint of the way in which Ptolemy expected it to be proved is found in his inclusion of the plane conjoined sector figure. He probably understood compound ratio by continuity of ratios since he introduces compound ratio by the insertion of quantities "between"



[118] Ibid., 92r-v.

two others and since he most likely subtracted ratios through making ratios continuous; however, he never clearly defines compound ratio and as we will see, some commentators interpret him according to their denominative idea of compound ratio. Ptolemy's applications of the Menelaus Theorem show its versatility in finding a number of values related to spherical astronomy, but because of its difficulty he chooses to avoid using it and to use plane approximations for some spherical problems involving small arcs. While it is a powerful theorem, Ptolemy does not judge it worth the trouble for some scenarios.

Chapter 2: Menelaus of Alexandria' *Sphaerica*

Although the first known appearance of the Menelaus Theorem is in the *Sphaerica* of the first-century mathematician Menelaus of Alexandria, I am treating it after the *Almagest* since its version of the Menelaus Theorem is less developed and harder to follow than the proof in the *Almagest*. Also, the *Sphaerica* had less of an influence upon the medieval treatment of the theorem than the *Almagest* did. It appears, however, to have been a fairly popular geometric text—there are at least 26 extant manuscripts of the Latin translation made by Gerard of Cremona sometime between his arrival in Toledo around 1140 and his death in 1187.[119] He made the translation from an Arabic edition of the *Sphaerica* that appears to have been a compilation made from two sources. The first portion of this version seems to have been derived from al-Māhānī's ninth-century revision of an eighth-century Arabic translation of a Syriac version of the *Sphaerica*. The second portion of the compilation, which is more interesting to us since it contains the Menelaus Theorem, had Isḥāq ibn Ḥunayn's translation from the Greek (ca. 900) as its source.[120] While the original version was not necessarily divided in the same manner as Gerard of Cremona's translation, the *Sphaerica* is divided into three books, and the Menelaus Theorem is found at the beginning of the third of these.

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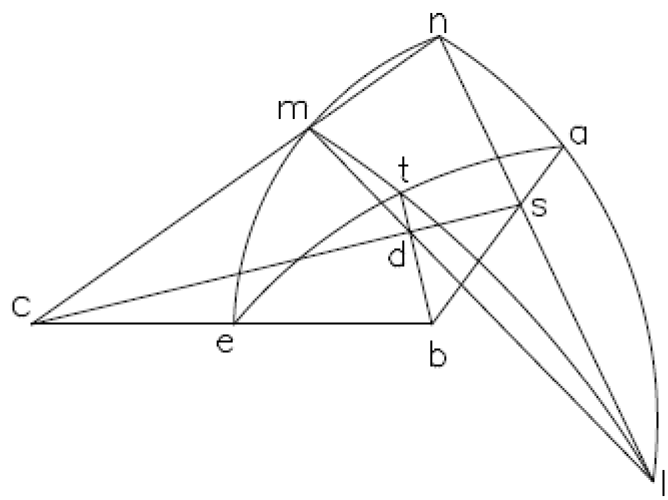
[119] Björnbo, "Studien," pp. 137-152 describes thirteen manuscripts containing Gerard's translation. He lists five others that he had not examined. Of these one contains not Gerard's translation but the later translation of Maurolycus. I have found nine more manuscripts containing this translation. This brings the number of manuscripts to twenty-five. See Appendix C on the *Sphaerica* for more information.

[120] Sidoli, pp. 48-52.

Somewhat surprisingly, the treatment of the Menelaus Theorem in the *Sphaerica* is different in several ways from that found in the *Almagest* and in almost every medieval treatment of the theorem. Gerard's version of the proof reads:

> Let there be on the surface of a sphere two arcs of great circles upon which are *ne* and *ln*. And I draw between them two arcs *eta* and *ltm*, and they intersect at point *t*. I say, therefore, that the ratio of the nadir of arc *an* to the nadir of arc *al* is composed of the ratio of the nadir of arc *ne* to the nadir of arc *me* and of the ratio of the nadir of arc *mt* to the nadir of arc *tl*. And indeed I do not signify anything when I say 'the nadir of an arc' except the line which is subtended by the double of that arc according to which that arc is less than a semicircle. This is the demonstration of it. I will locate the center of the sphere point *b*, and I will draw lines *nl*, *nm*, *lm*, *tb*, *eb*, *asb*, and *sd*. And first the two lines *nm* and *sd* will meet when extended at point *c* according to what is in the first figure, and I draw line *ec*. Therefore, point *c* will be in each of the two planes of the two arcs *ate* and *nme*. And each of the two points *e* and *b* again is in those two planes, therefore *ceb* is one straight line. And because the form is thus, then the ratio of *ns* to *sl* is as the ratio composed of the ratio of *nc* to *cm* and of the ratio of *md* to *dl*. But the ratio of *nc* to *cm* is as the ratio of the perpendicular falling from point *n* upon *ceb* to the perpendicular falling from point *m* upon line *ceb* again. But the perpendicular falling from point *n* upon line *bec* is half of crd. arc 2*ne*, and the perpendicular falling from point *m* upon that line is half of crd. arc 2*em*. Therefore the ratio of *nc* to *cm* is as the ratio of the nadir of arc *ne* to the nadir of arc *me*. And similarly it is made known that the ratio of *ns* to *sl* is as the ratio of the nadir of arc *na* to the nadir of arc *al* and that the ratio of



> *md* to *dl* is as the ratio of the nadir of arc *mt* to the nadir of arc *tl*. Therefore, the ratio of the nadir of arc *na* to the nadir of arc *al* is as the ratio composed of the ratio of the nadir of arc *ne* to the nadir of arc *me* and of the ratio of the nadir of arc *mt* to the nadir of arc *tl*.[121]

There were many different Arabic versions and redactions of the *Sphaerica*, and in these the proof of the sector figure differs.[122] It is, therefore, unclear how close this version of Gerard's is to the original proof of Menelaus. Van Brummelen has translated into English part of the proof from what Lorch argues is the text closest to the original version,[123] and the proof is essentially the same one that is found in Gerard's translation although Gerard's version is longer and gives more justification for certain steps.

One unique characteristic of Menelaus' proof is that he gives a different formulation of the statement of composition. He proves that the ratio of the crd. arc 2*na* to crd. arc 2*al* is composed of the ratios of crd. arc 2*ne* to crd. arc 2*me* and of crd. arc 2*mt* to crd. arc 2*tl*, but (in terms of this diagram) Ptolemy and all the chord others prove that the ratio of the crd. arc 2*al* to crd. arc 2*na* is composed of the ratios of crd. arc 2*tl* to crd. arc 2*mt* and crd. arc 2*me* to crd. arc 2*ne*. In the order of the normal statement of composition, Menelaus proves that the second term to the first is composed of the sixth to the fifth and the fourth to the third. He does, however, give the normal formulation of the disjoined sector figure, but only at the end of III.1; there he inverts the ratios of the statement of the proposition to reach

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[121] Appendix C, lines 1-75.

[122] Björnbo, "Studien," pp. 15-6; Sidoli; and Van Brummelen, p. 56.

[123] Van Brummelen, p. 57-8. See Lorch, *Thabit*, p. 327-35.

the statement of composition that Ptolemy and others prove immediately.[124] Gerard's proof is also different from any other existing version of the *Sphaerica* (or than any proof of the sector figure that I have seen) in that that it first extends lines *sd* and *nm* until they meet at *c* and then argues that points *b*, *e*, and *c* are in a straight line. All other proofs of the first case of the disjoined sector figure first extend lines *nm* and *be* until they meet at *c* and then show that points *s*, *d*, and *c* are in a straight line. Connected with this, Gerard's version is the only version that distinguishes the cases of the sector figure by the way that lines *sd* and *nm* meet or do not meet.[125] Also, this proof jumps immediately from the formation of the plane sector figure to a statement of composition but Menelaus has not given any proof for this jump. This proof in this version of the *Sphaerica* is also unique in that it gives in the proof the reasons for moving from the ratios of the sector figure to the ratios of chords of double arcs;[126] almost every other proof of this proposition proves these steps as separate, preliminary proofs (e.g., Ptolemy proves them as Lemmas 3 and 5). Gerard's translation also uses a rare term for the chords of double arcs: 'nadir" (or 'nadyr' or 'nadair' as it was sometimes spelled).[127]

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[124] Appendix C, lines 135-8.

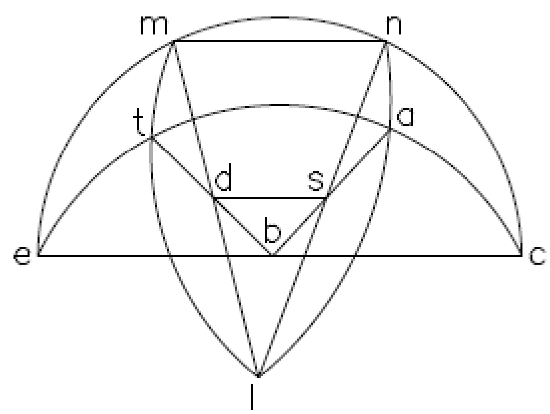
[125] It is unclear whether Menelaus' original proof had this unique feature of Gerard's proof. While Lorch, *Thabit*, p. 208, uses the fact that all other versions of the proof do not have this characteristic to argue that the original *Sphaerica* did not have it either, Sidoli, p. 58, argues that all other versions were influenced by the *Almagest* and so their way of dealing with the lines may come from a modification made originally by Ptolemy. He concludes that we have no basis to determine whether the original *Sphaerica*'s proof worked as Gerard's or as in the other versions.

[126] Presumably, Gerard's Arabic source had these same characteristics.

[127] Sidoli, p. 57 explains how this word is a transliteration of the ibn Ḥunayn's phrase "the correspondent to arc", which was his way of referring to the chords of double arcs.

Fully realizing that this is a proof of only one of three cases, Menelaus goes on to prove the case in which lines *nm* and *sd* are parallel:

> And again we will place line *sd* parallel to line *nm*, and we will complete the two half circles *etc* and *enc* according to what is in the second figure. And because in the two surfaces *enc* and *etc* there are two parallel lines, which are *sd* and *mn*, the intersection of those planes, which is line *ec*, will be parallel to the two lines *sd* and *mn*. And because a perpendicular falling from point *n* upon line *cbe* is half of the chord of double arc *cn*, and again half of the chord of double arc *en*, the nadir of arc *en* will be equal to the nadir of arc *em*. And because line *mn* is parallel to line *ds*, the ratio of *ns* to *sl*, which is as the ratio of the nadir of arc *na* to the nadir of arc *al*, will be as the ratio of *md* to *dl*, which is as the ratio of the nadir of arc *mt* to the nadir of arc *tl*. Therefore, the ratio of the nadir of arc *na* to the nadir of arc *al* is as the ratio composed from the ratio of the nadir of arc *mt* to the nadir of arc *tl* and from the ratio of the nadir of arc *ne* to the nadir of arc *em*, because it is equal to *ae*.[128]

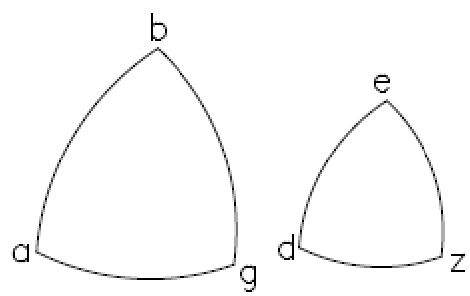


His proof is relatively straightforward and is similar to other proofs of this case. One difference is that while the others assume that *nm* and *ec* are parallel and then show that *nm* and *sd* must be parallel, this proof starts with the assumption that *mn* and *sd* are parallel and then argues that *mn* and *ec* must be parallel. Like many of the other proofs of this case, the difficult steps of seing why certain lines are parallel and that a statement of composition emerges from a proportion and a ratio of equality are not explained at length. Interestingly, Menelaus does not mention

[128] Appendix C, lines 76-120.

that there is a third case of this proposition, the one that has *nm* and *sd* meeting on the right side of the diagram.

In the next few theorems, Menelaus uses III.1 to prove simpler propositions about spherical triangles. For example, III.2 shows that given that an angle in one spherical triangle equals an angle in another triangle, and given that a second angle of the first triangle is equal to another angle of the second triangle (or if these add up to two right angles), then ratios of the nadirs of the arcs subtending those angles are the same. For example, given two triangles *abg* and *dez* with angle *a* equal to angle *d* and with angle *g* and *z* equal or equal to two right angles, then the ratio of crd. arc 2*ab* to crd. arc 2*bg* is as the ratio of crd. arc 2*de* to crd. arc 2*ez*.[129] Since this theorem only involves a proportion between the nadirs of four arcs, it is simpler to use it when possible instead of the Menelaus Theorem, which involves a statement of composition of the ratios of the nadirs of six arcs. In other words, III.2 can be used to find an unknown quantity from three known quantities while III.1 requires five known quantities to find an unknown value.



Interestingly, Ptolemy and the commentators do not give this proof of a simpler proposition or ever reference it. Some of the commentators on the

[129] For a description of the mathematics of this proof, see Björnbo, "Studien," pp. 92-4.

Almagest provide similar simplified statements, but they prove them in different manners. The lack of influence of III.2 and other simpler propositions that Menelaus proves by III.1 suggests that there was little direct influence of the *Sphaerica* upon astronomical works. It seems likely that even the *Almagest*'s treatment of the Menelaus Theorem did not come directly from the *Sphaerica* because of the omission of III.2 and because of Sidoli's argument mentioned before that the *Almagest*'s fourth and sixth lemmas to the sector figure came from an unknown astronomical work.[130]

In the *Sphaerica* of Menelaus, we find the Menelaus Theorem in its proper discipline of spherical geometry. Although a fairly popular book, Gerard's translation's version of the proof did not seem to be as influential on the medieval history of the theorem as the other works containing the proof that were translated into Latin. Perhaps a partial reason for this phenomenon is that Menelaus' proof skips many logical steps in its arguments and assumes more previous knowledge than the proofs found in Ptolemy's *Almagest*. He assumes knowledge of compound ratios and of the plane sector figure, so a medieval reader would not have found this text accessible unless he already possessed knowledge of these topics. As we will see in Part II, the *Sphaerica* came to be accompanied by commentary and notes

[130] Sidoli, p. 60.

written during the middle ages, and so from the middle of the thirteenth century onward, its proof was more self-contained.

Chapter 3: Thabit's *On the Sector Figure*

Thabit ibn Qurra, the ninth-century mathematician, wrote two works on the Menelaus Theorem and on compound ratio. One of these, *On the Sector Figure*, was translated into Latin three different times.[131] One translation, which Lorch calls the "Grecising translation," exists in four manuscripts, while the one that he calls the "Inter universas translation" exists in a sole manuscript. A third translation was made by Gerard of Cremona and is found in four manuscripts.[132] Two of these manuscripts with the work bear attributions to Campanus and none attribute it to Gerard of Cremona;[133] however, it is clearly a close translation of Thabit's work, as can be seen from a comparison to Lorch's edition and translation of the original Arabic, and it matches closely the style of Gerard of Cremona, who is known to have translated the work.[134] I will be examining Gerard's translation in detail.

Unlike the *Almagest* and the *Sphaerica*, *On the Sector Figure*, which is directed towards an unnamed correspondent who has asked some questions about the Menelaus Theorem, focuses completely on the Menelaus Theorem and the related subject of composed ratios. More specifically, the first main topic of the

[131] Lorch, *Thabit*, pp. 30-6. All three have been edited. The first is found in Björnbo, *Thabits Werk*, pp. 1-24. Unfortunately, Björnbo omits the text of several of the proofs. Editions of the other two versions are found in Lorch, *Thabit*, pp. 124-153. An edition, translation, and explanation of the other related work of Thabit, *On the Composition of Ratios*, is found in Lorch, *Thabit*, pp. 167-326. Since it was not translated into Latin, it is not particularly relevant here.

[132] Carmody, p. 123 lists eleven manuscripts, but Lorch, *Thabit*, p. 31, n. 4, says that none of them besides the ones that he lists have the work and that they either have another work by Thabit or another work on proportions.

[133] Björnbo, *Thabits Werk*, p. 6, no. 1.

[134] Lorch, *Thabit*, p. 31.

work is the enumeration and proofs of the different cases of the disjoined and conjoined spherical sector figures, and the second is the “modes” of compound ratios, which are the valid rearrangements of the six quantities that stand given one statement of composition—that a ratio is composed from two others.

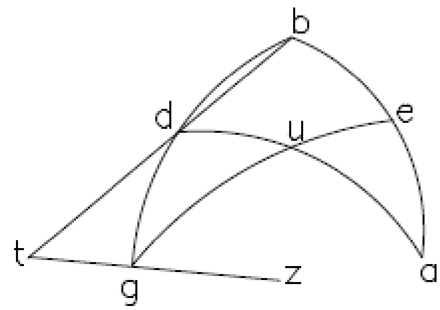
The Many Cases of the Menelaus Theorem

After repeating the two conclusions that Ptolemy gains from the Menelaus Theorem, Thabit points out that these are two of many other conclusions that can be reached. Thabit’s aim here is to complete the science of this figure by proving the cases that Ptolemy does not prove. He says that a unnamed colleague had divided the Menalaus Theorem into 30 different cases—three for the disjoined and 27 for the conjoined. He had told his correspondent that there were many different ways of dividing and proving the Menelaus Theorem, and the correspondent had worked on ways to simplify them and wanted to know whether his methods of simplifying the theorem were the same as ones done by Thabit and whether Ptolemy intended for all the cases to be proven in this same manner. Thabit informs him that he, Thabit, had come up with the same operations, but that Ptolemy had not intended them.[135]

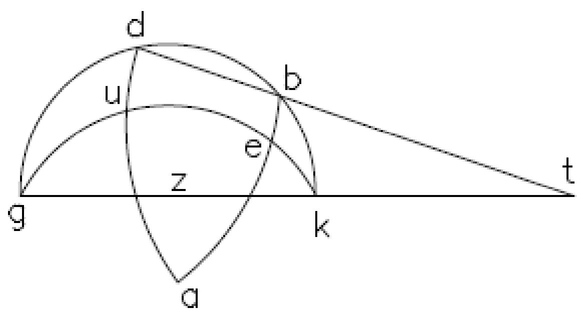
Thabit then enters into the different cases and proofs of the theorem. He starts by setting up the figure as Ptolemy does, but with different letters. He begins

[135] Björnbo, *Thabits Werk,* pp. 8-9.

with the spherical disjoined sector figure and does not prove any of the preliminary theorems of Ptolemy. The first case is when lines *zg* and *bd* meet on the side of *d* and *g*. Since this is the case that Ptolemy does



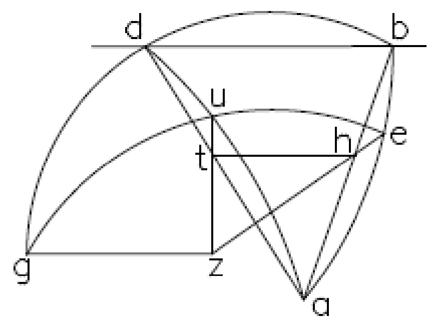
prove, Thabit moves on to the next case, in which those two lines meet on the other side of the diagram. He extends arcs *gb* and *ge* until they meet at *k*. Arcs *gbk* and *gek* will be semicircles and the line *gk* will be a diameter of the sphere passing



through center *z*. He then has the case of the sector figure that Ptolemy proved—the arcs *dbk* and *dua* meet at point *d*, while two other arcs, *aeb* and *keu*, come from their other endpoints and cross each other, and line *db* and *zk* meet at *t* on the side of points *b* and *k*. Therefore, the ratio of crd. arc 2*au* to crd. arc 2*ud* is composed of the ratio of crd. arc 2*ae* to crd. arc 2*eb* and of the ratio of crd. arc 2*bk* to crd. arc 2*kd*. Given 6 quantities such that the ratio of the first to the second is composed of the ratios of the third to the fourth and the fifth to the sixth, it is true that the ratio of the third to the fourth is composed of the ratios of the first to the second and of the sixth to the fifth—he will prove this later on in his discussion of the 18 valid modes. Applying this mode, the ratio of crd. arc 2*ae* to crd. arc 2*eb* is composed of the ratio of crd. arc 2*au* to crd. arc 2*ud* and the ratio of crd. arc 2*kd* to crd. arc 2*bk*.

But these last two chords are the same as the chords of double arcs *gd* and *gb* because the arc *gdbk* is a semicircle. Therefore, the desired conclusion is reached. By using Ptolemy's first case, Thabit avoided the complicated business of dealing with the planes of the figure.[136]

Thabit then proves the case when lines *gz* and *bd* are parallel. He states that line *ht* will also be parallel to *bd*. His reasoning for this is hard to follow; he writes, "Then line *ht* will be parallel to line *bd* because if it were not parallel to it, then *zg* would not be parallel to *bd*—but *zg* was parallel to *bd*. And if line *ht* were not parallel to the two lines *bd zg*, it would meet them, and it would be in one plane with them. The matter, however, is not thus. Therefore, the two lines *ht* and *bd* are parallel."[137] Although this argument is not clearly stated, what he is trying to prove is true since line *ht* is in a plane with line *gz*, the plane of circle *gue*, while it is also in a plane with line *bd*, the plane of triangle *abd*. Once he has shown that lines *bd* and *ht* are parallel, it is clear that in triangle *abd*, *ah* is to *hb* as *at* is to *dt*. But *ah* to *hb* is as crd. arc 2*ae* to crd. arc 2*eb*, and *at* to *dt* is as crd. arc 2*au* to crd. arc 2*du*. Therefore, the ratios of crd. arc 2*ae* to crd. arc 2*eb* is as crd. arc 2*au* to crd. arc 2*ud*. Crd. arc 2*gd* is the same as crd. arc 2*bg* because lines *bd* and *gz* are parallel. Thabit

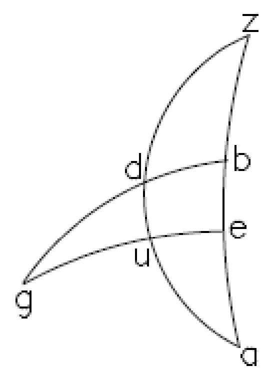


[136] Ibid., p. 9 line 10 to p. 10 line 15.

[137] Ibid., p. 10, lines 16-20. "Quod si fuerit linea *bd* aequidistans lineae *zg*, tunc linea *ht* erit aequidistans lineae *bd*, quoniam si non fuerit aequidistans ei, tunc *zg* non aequidistabit *bd*, sed *zg* fuit aequidistans *bd*. Et si esset linea *ht* non aequidistans duabus lineis *bd zg*, concurreret eis, et esset cum eis in superficie una."

then concludes that the ratio of crd. arc 2*ae* to crd. arc 2*eb* is composed of the ratio of crd. arc 2*au* to crd. arc 2*ud* and the ratio of crd. arc 2*gd* to crd. arc 2*bg*. He does not explain, but this works because a ratio of equality does not change a ratio with which it is compounded.

Thabit then tackles the conjoined sector figure or in his words the "modum compositionis." He first extends arcs *ad* and *ab* until they meet at *z*, forming semicircles. Applying the disjoined conclusion that he has already proved for all cases to the sector figure *gez*, the ratio of crd. arc 2*zb* to crd. arc 2*be* is composed of the ratio of crd. arc 2*zd* to crd. arc 2*du* and of the ratio of crd. arc 2*ug* to crd. arc 2*ge*. But crd. arc 2*zb* is the same as crd. arc 2*ab*, and crd. arc 2*zd* is the same as crd. arc 2*ad*. Therefore, the ratio of crd. arc 2*ab* to crd. arc 2*be* is composed of the ratio of crd. arc 2*ad* to crd. arc 2*du* and of the ratio of crd. arc 2*ug* to crd. arc 2*ge*.[138]



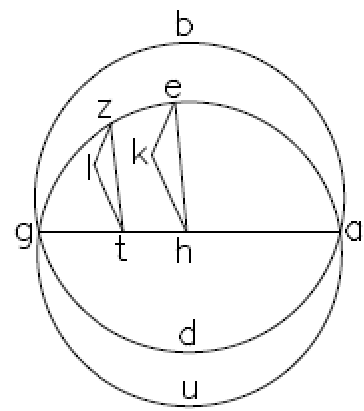
By using the disjoined sector figure in this way, Thabit is able to prove all the cases of the conjoined sector figure in a single proof, and he is able to avoid making lines from the center of the sphere and dealing with determining planes and lines inside the circle as was required for two of the three cases of the disjoined sector figure. Because this also means that not all of Ptolemy's lemmas are needed and because Ptolemy says that the conjoined sector figure can be proved similarly

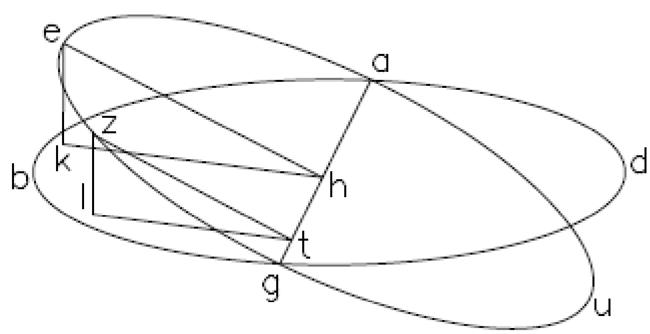
[138] Ibid., p. 10 line 29 to p. 11 line 8.

to the way that the disjoined is proved, Thabit points out that Ptolemy must not have intended the reader to prove the conjoined sector as Thabit does.[139]

Alternate Proofs of the Menelaus Theorem

Thabit not only proves cases that Ptolemy does not; he offers simpler proofs for the two conclusions of the Menelaus Theorem. In order to do this, Thabit gives a preliminary proof, which is that if two great circles cutting each other are described on the surface of a sphere, and if two arcs are taken on one great circle from a point of intersection of the great circles, and if perpendiculars are dropped from the other endpoints of these two arcs to the other circle (remember circles are plane figures, not just circumferences), then the ratio of the chord of double one arc to the chord of double the other is as the ratio of the perpendicular dropped from the endpoint of the first arc to the perpendicular dropped from the endpoint of the other arc. The great circles



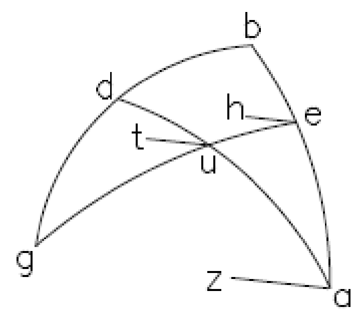


are *abgd* and *aegu* meeting at points *a* and *g*, and on circle *aegd*, arcs *ae* and *az* are taken. Diameter *ag*, which is the section of the two circles, is drawn, and perpendiculars *eh* and *zt* are dropped to it. If these two lines are also the

[139] Ibid., p. 11, lines 9-30.

perpendiculars to circle *abgd*, then the conclusion follows since lines *eh* and *zt* are the halves of the chords of double arcs *ae* and *az*. If they are not perpendicular to circle *abgd*, then perpendiculars *ek* and *zl*, which will be parallel to each other, are dropped to circle *abgd*, and lines *lt* and *kh* are drawn. Because lines *zl* and *zt* are parallel to *ek* and *eh*, the angles that they contain, angle *z* and *e*, are equal. Also angles *ekh* and *zlt* are both right, so the triangles *ekh* and *zlt* are similar. Therefore, *eh* is to *zt* as *ek* is to *zl*. But *eh* is to *zt* as crd. arc 2*ae* is to crd. arc 2*az*, so *ek* is to *zl* as crd. arc 2*ae* is to crd. arc 2*az*. Thabit points out that this can be proved even if point *e* or *z* is taken on the half of the circle that point *d* is on.[140]

With this theorem proved, Thabit is able to show the proposition proved by the conjoined sector figure. The situation is that of the sector figure—here arcs *ab* and *bg* of great circles meeting at a point and two other arcs coming from their other endpoints and crossing each other, *ad* and *ge*, and all the arcs are less than a semicircle. Thabit drops perpendiculars

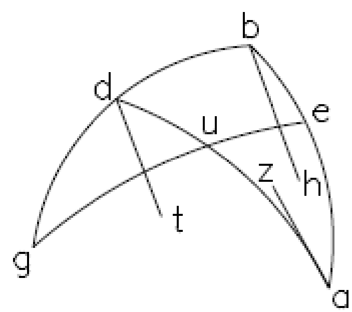


eh, *az*, and *ut*, from points *e*, *a*, and *u* to the circle of arc *bg*. Take line *ut* as a middle between *az* and *eh*, so the ratio of *az* to *eh* is composed of the ratio of *az* to *ut* and the ratio of *ut* to *eh*. But, using the theorem just proved three times, we see that the ratio of *az* to *eh* is the same as the ratio of crd. arc 2*ab* to crd. arc 2*be*, the ratio of *az* to *ut* is the same as the ratio of crd. arc 2*ad* to crd. arc 2*du*, and the ratio of *ut* to *eh* is the same as the ratio of crd. arc 2*gu* to crd. arc 2*ge*. Therefore, the

[140] Ibid., p. 12 line 9 to p. 13 line 13.

ratio of crd. arc 2*ab* to crd. arc 2*be* is composed of the ratio of crd. arc 2*ad* to crd. arc 2*du* and of the ratio of crd. arc 2*gu* to crd. arc 2*ge*. Since this theorem does not rely on the meeting of a line through two of the points on the surface of the sphere and a line from the center to one of the points on the sphere, there is no division into three different cases as in the Menelaus Theorem.[141]

Thabit next proves the proposition of the disjoined sector figure. Perpendiculars *az*, *bh*, and *dt* are dropped from points *a*, *b*, and *d* to the circle of arc *gue*. Line *dt* is placed as a middle between *az* and *bh*, so the ratio of *az* to *bh* is composed of the ratio of *az* to *dt* and the ratio of *dt* to *bh*. From the preliminary proof, *az* is to *bh* as crd. arc 2*ae* is to crd. arc 2*eb*, *az* is to *dt* as crd. arc 2*au* is to crd. arc 2*ud*, and *dt* is to *bh* as crd. arc 2*gd* is to crd. arc 2*gb*. Therefore, the ratio of crd. arc 2*ae* to crd. arc 2*eb* is composed of the ratio of crd. arc 2*au* to crd. arc 2*du* and of the ratio of crd. arc 2*gd* to crd. arc 2*gb*.[142]



The Modes

The rest of the work is on Thabit's second main concern, the possible rearrangements of the six quantities in a statement of composition (a statement that

[141] Ibid., p. 13 line 14-33.

[142] Ibid., p. 13 line 33 to p. 14 line 17.

a ratio is composed of two other ratios). He first explains the eighteen valid modes. For each, he gives a proof except for the first which is assumed to be true. After all, the other seventeen modes are the arrangements of six quantities in the statement "the ratio of __ to __ is composed of the ratio of __ to __ and the ratio of __ to __" that make true sentences when it some combination of those six terms in that statement is known to be true.

In organizing the modes, Thabit follows a general order. Starting with the six quantities in the order *a*, *b*, *g*, *d*, *e*, *u*, he follows alphabetical order (but counting *g* as the third letter) except in a couple of places. He strays by having Modes 5, 7, and 8 where they are instead of having them come respectively after Modes 6, 12, and 17, where they would occur if following a strict alphabetical order. The reason that Modes 7 and 8 come early is that Thabit uses them to prove other modes.[143] In the following list, each line shows the order of terms in the statement "the ratio of __ to __ is composed of the ratios of __ to __ and of __ to __."[144]

[143] Mode 5 is used to prove Mode 6, but Thabit could have easily proved the sixth first and then proved the fifth from it. The sixth could have been proved by rearranging the order of the composing ratios in the first mode, and applying the fourth mode. Then applying the second to this resulting mode would give us the fifth mode.

[144] Thabit's table of modes differs from this simplified table I give (Björnbo, *Thabits Werk*, p. 18 lines 16-36). Thabit's table has seven columns and 24 rows (Ibid., p. 20). The first row gives six numbers such that the ratio of the first to the second is composed of the ratios of the third to the fourth and of the fifth to the sixth. These are 1, 2, 3, 4, 6, and 9. In the seventh column the number of the mode found in that row is given. There are 24 rows instead of only 18 because Thabit includes all of the combinations of the six numbers in the first two columns even if they have no valid mode. In the rows with modeless numbers in the first two columns, the rest of the row is filled in with zeros.

a	*b*	*g*	*d*	*e*	*u*	1
a	*b*	*g*	*u*	*e*	*d*	2
a	*g*	*b*	*d*	*e*	*u*	3
a	*g*	*b*	*u*	*e*	*d*	4
a	*e*	*b*	*u*	*g*	*d*	5
a	*e*	*b*	*d*	*g*	*u*	6
g	*d*	*a*	*b*	*u*	*e*	7
e	*u*	*a*	*b*	*d*	*g*	8
b	*d*	*a*	*g*	*u*	*e*	9
b	*d*	*a*	*e*	*u*	*g*	10
b	*u*	*a*	*g*	*d*	*e*	11
b	*u*	*a*	*e*	*d*	*g*	12
g	*d*	*a*	*e*	*u*	*b*	13
g	*u*	*a*	*b*	*d*	*e*	14
g	*u*	*a*	*e*	*d*	*b*	15
d	*e*	*b*	*a*	*g*	*u*	16
d	*e*	*b*	*u*	*g*	*a*	17
e	*u*	*a*	*g*	*d*	*b*	18

Thabit proves most of the modes in one of two ways. Some of the modes are able to be proved by the application of proved modes to other modes. For example, Mode 3 is that the ratio of *a* to *g* is composed of the ratios of *b* to *d* and *e* to *u*. When Mode 2 is applied to this statement of composition as if it were the original Mode 1, the result is that the ratio of *a* to *g* is composed of the ratios of *b* to *u* and *e* to *d,* which is Mode 4.[145]

The other common crucial step in his proofs is the insertion of middles to produce compound ratios. An example of this is his proof for Mode 2. Given that the ratio of *a* to *b* is composed of the ratio of *g* to *d* and the ratio of *e* to *u,* the second mode is that the ratio of *a* to *b* is composed of the ratio of *g* to *u* and the

[145] Björnbo, *Thabits Werk,* p. 16 lines 3-8.

ratio of *e* to *d*. Thabit considers *d* and *e* as "middles" between *g* and *u*. Therefore, the ratio of *g* to *u* is composed of the ratio of *g* to *d* and the ratio of *d* to *e* and the ratio of *e* to *u*. From this Thabit concludes that the ratio composed of *g* to *u* and *e* to *d* is the same as the ratio composed of the ratios of *g* to *d*, *d* to *e*, *e* to *u*, and *e* to *d*. What he effectively did there was to add the same, the ratio of *e* to *d*, to two equals, the ratio of *g* to *u* and a series of ratios that together compose *g* to *u*. Since *d* to *e* and *e* to *d* compose a ratio of equality which does not change ratios with which it is compounded, he is left with the ratio composed of *g* to *u* and *e* to *d* being the same as the ratio composed of *g* to *d* and *e* to *u*, which is known to be *a* to *b* from the first mode. Therefore, the ratio of *a* to *b* is the ratio that is composed of *g* to *u* and *e* to *d*. Several of the proofs of the other sixteen modes are fairly similar to this. Thabit places some quantity or quantities between two others and then almost algebraically rearranges the terms, adds or subtracts equals, cancels out ratios of equality until he reaches the desired permutation.[146]

Although Thabit clearly defined compound ratio according to continuous ratios in his work on compound ratios[147] and here utilizes the idea of putting middles between two quantities to get a compound ratio, this does not mean that a reader would necessarily understand compound ratios the same way. Thabit does not define compound ratios in *De figura sectore*, and his reader could reasonably

[146] Lorch, *Thabit*, pp. 158-161.

[147] Lorch, *Thabit*, pp. 14-5.

think that Thabit inserted middles to reach compound ratios but that this practice ultimately relied upon denominations.[148]

The proof of Mode 7 is unique in that Thabit introduces a seventh quantity to prove it. He starts by finding a quantity *z* such that *d* is to *z* as *e* is to *u*. Therefore, substituting ratios in the first mode, the ratio of *a* to *b* is composed of the ratios of *g* to *d* and of *d* to z. But the ratio of *g* to *z* is composed from the same ratios *g* to *d* and *d* to *z*, so the ratio of *a* to *b* is the same as the ratio of *g* to *z*. But the ratio of *g* to *d* is composed of *g* to *z* and *z* to *d*. Also from the inverse of our first step, *z* is to *d* as *u* is to *e*. Therefore, the ratio *g* to *d* is composed of the ratios of *a* to *b* and *u* to *e*.[149]

Thabit demonstrates that these eighteen modes are the only valid ones. Of the fifteen possible combinations of the six quantities in the first two positions, he already showed that nine of them have valid modes. He proceeds to show that the other six combinations do not have valid modes. These pairs are the first and the fourth, the first and the sixth, the second and the third, the second and the fifth, the third and the fifth, and the fourth and the sixth. He also shows how one would demonstrate the invalidity of ten of the twelve combinations of the last four

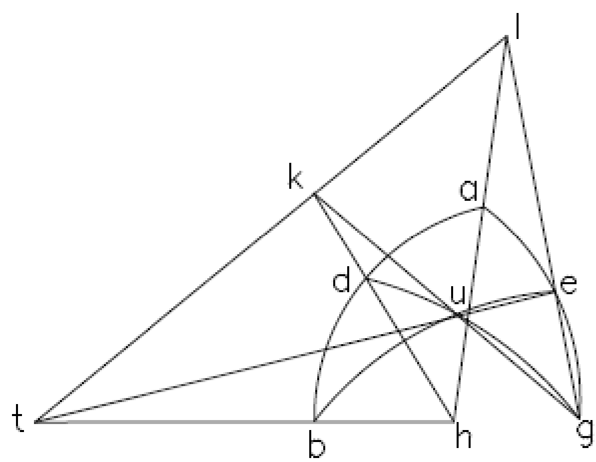
[148] For example, the Vatican Commentary relies heavily upon Thabit, but it still uses denominations and the multiplication of ratios extensively. See Part II, ch. 6 below.

[149] Björnbo, *Thabits Werk*, p. 17 lines 2-16.

positions. There are, therefore, eighteen, and only eighteen, valid arrangements or modes of the six quantities in a statement of composition.[150]

Maslama's Note

A note attributed to the Spanish mathematician Maslama al-Majrīṭī (fl. tenth century) following the text of Thabit is found in two of the Arabic manuscripts, in three of the manuscripts containing the Gerard of Cremona translation, and the one containing the "Inter universas" version.[151] This short text gives a different proof of the conjoined sector figure that uses the plane conjoined sector figure. The arcs *gud* and *bue* cross each other between arcs *aeg* and *adb*. The letters of the diagram correspond to Ptolemy's version of the Menealus Theorem rather than Thabit's. The center of the sphere is found, and lines *ha* and *ge* are drawn and extended until they meet at point *l*. Likewise, lines *hd* and *gu* are drawn meeting at *k*, and *hb* and *eu* at *t*. Points *t*, *k*, and *l* are both in the plane of triangle *geu* and in the plane of the circle *bda*, so they must be in one straight line. A plane



[150] Ibid., pp. 18-23.

[151] Lorch, *Thabit*, pp. 33, 35. The note can be found on Björnbo, *Thabits Werk*, pp. 23-4. A transcription of the note appended to the "Inter universas" version can be found on Lorch, *Thabit*, pp. 152-3.

sector figure is now formed by lines *lg*, *lt*, *gk*, and *te*, so the ratio of *gl* to *le* is composed of the ratio of *gk* to *ku* and the ratio of *tu* to *te*. From Lemma 5 of the *Almagest*, the ratio of *gl* to *le* is as crd. arc 2*ga* to crd. arc 2*ae*. Lemma 5 also applies to the lines meeting at points *k* and *l*. Therefore, the ratio of crd. arc 2*ga* to crd. arc 2*ae* is composed of the ratios of crd. arc 2*gd* to crd. arc 2*du* and of crd. arc 2*bu* to crd. arc 2*be*.

Maslama's proof only considers one of a large number of possible situations. He assumes that lines *ha* and *ge* meet on the side of points *a* and *e*, but depending upon the size of arcs *ae* and *eg*, lines *ha* and *ge* could be parallel or could meet on the side of points *h* and *g*. Similarly, lines *hd* and *gu* do not have to meet on the side of points *d* and *u*, nor do lines *hb* and *eu* have to meet on the side of points *u* and *h*. The reader of Thabit and Maslama, therefore, has a choice of how to prove the conjoined sector figure. He can follow Thabit's proof, which only has one case, but it does not utilize the plane sector and consequently is not what Ptolemy envisioned. On the other hand, he can follow Maslama's proof, which does use the plane sector figure, but he is then confronted by a large number of cases to be proved (or ignored).

Thabit's *De figura sectore* was the second most influential work translated into Latin that contains the Menelaus Theorem. Its importance lies in its exposition of the insufficiencies of Ptolemy's proof and its linking of the modes to the

Menelaus Theorem. Thabit made apparent the vast number of cases left unproved by Ptolemy and gave proofs that applied to all cases of both the conjoined and disjoined sector figure. In addition, he provided another set of proofs that proved the same propositions but without any recourse to the plane sector figures. While Ptolemy uses a few modes, Thabit connected a complete investigation of the number of possible modes and proofs of all valid modes to the sector figure. Another difference between the treatment in Thabit's work and in Ptolemy's is that here the treatment stays mostly on the general level—besides the table of modes, the text does not bring in specific values.

Chapter 4: Ametus' *Epistola de proportione et proportionalitate*

Gerard of Cremona translated a treatise on ratios and proportion, which was originally written around 900 C.E. by Ahmad ibn Yûsuf ibn Ibrâhîm ibn al-Dâya al-Misrî, who was known to the West as "Ametus filius Josephi."[152] This translation is found in ten manuscripts, and there are also two manuscripts that contain works derived from the *Epistola* but not the work itself.[153] Although the work was translated into Latin by Gerard sometime before his death in 1187, the oldest surviving manuscript is from the fourteenth century or perhaps the late thirteenth.

The work is in the form of a letter in three parts with an introduction and conclusion. The prologue focuses on the definitions of ratio and proportionality, and the first part continues to discuss these as well as to offer some basic properties of proportions and proofs of the fifth definition of *Elements* V and related statements. In the introduction to the second part Ametus treats certain properties of proportions of three, four, and six terms, lists various kinds of ratios, and states fourteen properties or conclusions of proportions. The twenty-one propositions of Part II are on how to find missing quantities in various proportions. Part III is on the different modes of compound ratio, and this treatment is tied to the Menelaus Theorem because the modes are not proved by abstract ratio theory but as different

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[152] This work is edited, translated, and discussed in Schrader, *The Epistola*.

[153] Schrader, p. 41-8. She lists nine manuscripts with the work, but Wolfenbüttel, Herzog August Bibliothek, Co. Guelf. 24 Aug. quart. has the work on 1r-12v although it may be missing the short conclusion. Oxford, Bodleian Library, Digby 168 has a paraphrase of the *Epistola*, and Paris, Bibliothèque nationale de France, lat. 7377B has a work that borrows some passages from the *Epistola*.

propositions concerning plane sector figures. While this work contains no proof of the spherical Menelaus Theorem, its handling of the plane sector figure and compound ratios make it an important part of the transmission of the Menelaus Theorem.

Compound Ratio

Ametus' treatment of compound ratios is not uniform. While he sometimes uses the normal language of compounding, he often relies more upon the concept of "iteration" or "repetition." As we will see, his idea of compounding is closely tied to the idea of ratio *ex aequali*, which will be explained shortly. In the introduction to Part II, while discussing different properties of proportions with three terms, Ametus states:

> And the fifth [property] is their [i.e. the quantities in a proportion] repetition beginning with their separation from the antecedent. But the sixth is their repetition beginning with their separation from the consequent. Repetition is found, however, in the two extremes according to the condition which is in division. Therefore if the division is similar, the repetition will be similiar, and if it is different, it will be different.[154]

Although the wording is incredibly obscure, as both Schrader and I read this passage, the meaning is that given a proportion such as a to b as b to c, two statements about the division and repetition of these ratios are true. The first starts with the antecedent of the first ratio and is that the ratio of a to c is compounded

[154] Schrader., p. 109-111. My translation. "Et quintus est iteratio earum incipiens cum divisione sua ab antecedenti. Sed sextus est iteratio earum incipiens cum divisione sua, a consequenti. Iteratio autem duabus accidit extremitatibus secundum habitudinem que est in divisione. Si ergo divisio fuerit similis, reiteratio erit similis, et si fuerit diversa, erit diversa."

from *a* to *b* and *b* to *c*. The second starts with the consequent of the second ratio and is that *c* to *a* is composed of *c* to *b* and *b* to *a*. Although the statements amount to compounding according to continuous ratios, the usual terminology of compounding is not to be found in this passage. Also, "repetition" usually implies that the same thing is found twice, which applies to the two statements in which a ratio is repeated, but the last clause makes it clear that Ametus allows "repetition" to be of dissimilar ratios. In other words, he apparently thinks that we can talk about *a* to *c* being the "repetition" of *a* to *b* and *b* to *c* even when *a* to *b* and *b* to *c* are not the same ratio. This strange terminology is even odder when one considers that Ametus is an expert on the theory of ratios. Perhaps what he has in mind is related to ratio *ex aequali*—a concept defined in Euclid's *Elements* V, definition 17. Given that the ratio of *a* to *b* is as *e* to *f* and the ratio of *b* to *c* is as *f* to *g*, it follows *ex aequali* that the ratio of *a* to *c* is as *e* to *g*. Ametus may be trying to describe what is happening with one of the two sets of quantities (*a*/*b*/*c* or *e*/*f*/*g*). The fifth and sixth properties that he is defining here are then the special cases in which the two ratios that are "repeated" are the same.

Soon after, Ametus gives four statements about proportions with six terms, and although these are not explicitly about compound ratio, they are about *ex aequali* and perturbed ratio.[155] Because these are not entirely clear from Schrader's translation, the first two of these four cases are ones that follow when given six quantities such that *a* to *b* to *g* as *d* to *e* to *u*. The first is the normal use of ratio *ex*

[155] Ibid., p. 113-115.

aequali, as is defined in the seventeenth definition of Book V of Euclid's *Elements*. Again, it says that since *a* is to *b* as *d* is to *e* and *b* is to *g* as *d* to *u*, then *a* is to *g* as *d* to *u*. Euclid and most others did not explicitly connect *ex aequali* and perturbed ratios to compounding, but they are closely related. Like compounding, *ex aequali* is a process that involves taking the extreme terms of continuous ratios and dropping out the intermediate terms. Although this was rarely expressed explicitly, taking a ratio *ex aequali* can be seen as stating that a whole ratio is equal to another whole ratio when the parts of the first whole are equal to the parts of the second whole.[156] The term "*ex aequali*" brings to mind the axiom of all quantities that equals added to equals make equals.[157] Ametus' second statement here is *ex aequali* with inverted ratios, i.e. that since *g* is to *b* as *u* to *e* and *b* is to *a* as *e* to *d*, then *g* is to *a* as *u* to *d*. The other two follow when given six quantities such that *a* is to *b* as *e* to *u* and *b* is to *g* as *d* to *e*. The third is that since *a* is to *b* as *e* to *u* and *b* is to *g* as *d* to *e*, then *a* is to *g* as *d* to *u*. The fourth is that since *g* is to *b* as *e* to *d* and *b* is to *a* as *u* to *e*, then *g* to *a* is as *u* to *d*. Although these last two are what Euclid calls perturbed proportion, Ametus describes them as being cases of

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[156] Although in many ways, compounding and ratio *ex aequali* are very similar conceptually—they both can be understood through treating ratios as wholes and parts—ancient and medieval mathematicians did not explicitly make this connection. It seems that *ex aequali* was usually understood as a process or as the result of a process of eliminating "middles."

[157] Euclid apparently did not believe that this axiom applies perfectly to ratios. He does not take the truth of ratio *ex aequali* as evident from the axiom, but proves it in V.22. One reason he might have done this is that ratio was not understood technically to be a quantity. Ratios are similar to quantities, e.g. we are able to talk about equal ratios even though equality is properly only said of quantity according to Aristotle in Ch. 6 of the *Categories*; however, ratios are defined by Euclid as a kind of relation, not as kinds of quantity. One reason for the confusion about compound ratios comes from precisely this, that we are quantifying something that is not itself a quantity. Parts, wholes, composition, addition, subtraction are applied to ratios not properly but by analogy.

aequalitas, i.e. *ex aequali*. This makes sense since both are essentially Euclid's second common notion—that equals added to equals are equal—applied to ratios.

While listing "the different species of proportionality," Ametus connects *ex aequali*, which he here calls "*aequalitas*", and "*reiteratio*" more explicitly.[158] While he does not define these terms, he explains, "Equality, moreover, and repetition are proportionality of extremes, but there are differences between them. For equality is not in less than six quantitites, but repetition exists in six and in less."[159] Although once again Ametus does not express himself very clearly or fully, he is pointing out that *ex aequali* and *reiteratio* both involve the taking of extremes from sets of terms of ratios. Ratio *ex aequali* involves doing this with the ratios on both sides of proportions, but since *reiteratio* can be done with ratios that are not in proportions, it only requires three terms.[160]

In the process of listing useful propositions of Euclid and others near the end of the introduction of Part II, Ametus again uses the language of repetition while giving the propositions of Euclid about squares and cubes being respectively in the duplicate and triplicate ratios of their sides; they read, "The ratio of any one

[158] Especially confusing here is that Ametus also uses the word "reiteratio" for proportionality. He states, "[P]roportionalitas autem est huius forme reiteratio in eo super quod cadit ex quantitatibus cum augmentatione aliarum super alias, aut diminutione aliarum ab aliis, quarum quantitates dicuntur proportionales" (Schrader, pp. 77-9).

[159] Schrader, p. 117. My translation. "Equalitas autem et reiteratio sunt extremitatum proportionalitas inter quas tamen existit differentia. Equalitas namque non est in quantitatibus paucioribus sex, reiteratio vero existit in eis et in illis que sunt preter eas."

[160] Gregg De Young, "*Ex Aequali* Ratios in the Greek and Arabic Euclidean Traditions," *Arabic Sciences and Philosophy*, 6 (1996): 167-213, makes clear that the ideas of *ex aequali*, continuous ratio, and compound ratio are all interconnected although he does not explain the similarities as I have through the ideas of wholes and parts.

of two squares to the other is as the ratio of its side to the the side of the other, duplicated by repetition. . . And the ratio of any one of two cubes to the other is as the ratio of the side of one to the side of the other when repeated three times."[161]

The sixth, seventh, eight, and ninth useful propositions also concern compound ratios, but from this point on in the *Epistola*, compounding no longer relies only upon the somewhat vague concept of repetition but uses more normal terminology of compounding. They read:

> [6.] And the ratio of the areas, one to the other, of every pair of parallelograms of which one angle is equal to an angle of the other, is composed of the [ratios] of their sides. [7.] And when there are numbers continued according to one ratio, their squares are continued according to one ratio. [8.] And when a line is placed between two lines, whatever way it falls between them, the ratio of one of them to the other is composed of the ratio of their antecedent to the middle and of the middle to the consequent. [9.] And likewise when two lines or more fall between them, the ratio of the antecedent of the two lines to the consequent of them will be composed of the ratio of the antecedent to the first middle and the ratio of the first middle to the second middle and the ratio of the second middle to the consequent of the two lines.[162]

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[161] Schrader, p. 123-4. "Que sunt omnium duorum quadratorum unius ad alterum proportio est sicut proportio lateris eius ad latus illius duplicata cum iteratione. . . Et omnium duorum cuborum unius ad alterum proportio est sicut proportio lateris unius ad latus alterius cum iteratione triplicata." These correspond to Euclid, *Elements* VIII.11-12, but while Euclid's propositions are about square and cube numbers, Ametus' language is unclear enough to apply to numbers and continuous magnitudes. While the propositions are true for both, Euclid does not prove them for geometric squares and cubes.

[162] Schrader, p. 125. My translation and added numbering. For the sixth, I read "proportione" as a mistake for "proportionibus." "Et omnium duarum superficieorum equidistantium laterum quarum unius angulus equatur angulo alterius unius ad alterum proportio est composita ex proportione [sic] laterum ipsarum. Et cum fuerint numeri secundum proportionem unam continui, quadrati eorum erunt continui secundum proportionem unam. Et cum linea inter duas lineas posita fuerit quocuiusque modo cadat inter eas erit unius earum ad alteram proportio composita ex proportione [sic] antecedentis earum ad mediam et medie ad consequentur[sic]. Et similiter cum ceciderint inter eas due linee aut plures eis erit proportio antecedentis duarum linearum ad consequentem earum composita ex proportione antecedentis ad mediam primam et proportione medie prime ad mediam secundam et proportione medie secunde ad consequentem duarum linearum."

The sixth is from *Elements* VI.23, and the seventh is a formulation of *Elements* VIII.13. The eight and ninth give the continuous ratio concept of compounding ratios. Since the statements alone are given, it is conceivable that Ametus sees them not as the definition of compounding but as something to be proved from another definition of compound ratio. On the other hand, although Ametus discusses denominations of ratios,[163] his treatment of compound ratios does not rely upon them in any way, so it is fairly certain that Ametus holds to the continuous ratio idea of compounding. It is noteworthy, however, that Ametus does not define compound ratio anywhere and that he only includes these more normal statements of composition in a list of useful propositions, which do not necessarily match his own thought perfectly. His penchant for discussing compounding in terms of repetition, which is a concept closely tied to continuity, is further evidence that he holds to the continuous ratio idea, but perhaps Ametus, who shows his hesitation to settle for a definition of proportionality in his prologue,[164] could think of no specific formulation that adequately expresses the essence of compounding.

The concept of compounding is so closely related to *ex aequali* and quantities in continuous proportion that some of Ametus' propositions that do not make any mention of compounding could be put into terms of compounding. For example, the twentieth of the twenty-one proofs in Part II implicitly concerns

[163] Schrader p. 121-3. "Sed proportio scita est denominans aliquem numerum. . . . Sed cum voluerimus proportionem numeri ad numerum dividemus unum duorum numerorum per alterum."

[164] The prologue of the *Epistola* is a dialogue in which characters struggle to provide a definition of proportionality.

compounding. It shows that if *a* to *b* is as *d* to *e* and *b* to *g* is as *e* to *u*, and if the sum of *a*, *b*, and *g* is known while each is unknown, and if *d*, *e*, and *u* are known, then *a*, *b*, and *g* can be found.[165] Although Ametus does not mention compound ratios here, another way of conceiving of this would be to try to find *a*, *b*, and *g* when the composed ratio *a* to *g* and the composing ratios *a* to *b* and *b* to *g* are known (but not in terms of quantities *a*, *b*, and *g*) and the sum of *a*, *b*, and *g* is known.[166] Propositions that concern *ex aequali* like this one and also ones that concern quantities in continuous ratio can be put in terms of compounding but the standard treatment of them leaves out any references to compounding. This can be seen not only in the *Epistola*, but also in the works of Euclid, Jordanus, and many others.[167]

Ametus includes information useful for actual application of compound ratio for calculation. In Proposition 21 of Part II, Ametus gives sets of directions for how to find various unknown terms in a statement of composition of ratios

[165] Schrader, pp. 177-9.

[166] Ametus jumps immediately from the givens to the statement that (a+b+g):g::(d+e+u):u, but this step requires the composition of ratios (in the sense of Euclid V.def. 14) twice with an *ex aequali* between them.Schrader, p. 177-9. A note in the margins of Wien, Österreichische Nationalbibliothek 5277, f. 316v justifies Ametus' leap as I have. It reads, "Quod ita probatur per proportionem compositionis et aequalitatis. Nam proportio *ad* [sic] ad *b* sicut *d* ad *e*. Componam igitur et erit proportio *a b* ad *b* sicut *d e* ad *e*. Sed proportio *b* ad *g* quae *e* ad *u*. Igitur secundum equalitatem proportio *a b* ad *g* sicut *d e* ad *u*. Per compositionem igitur proportio *a b g* ad *g* sicut *d e u* ad *u*." Note that this commentator uses "componere" and "compositio" in the sense of *Elements* V.def. 14.

[167] Many of the propositions in the Euclid's *Elements* and Jordanus' *Elements of Arithmetic* involve quantities in continuous ratio.

when the composing ratios are not the same.[168] Little justification is given in these directions; they are mainly sequences of operations to find the unknown quantities. After postulating that *a* to *b* is composed of *g* to *d* and *e* to *u*, Ametus says:

> Therefore, I will place the quantities together of the two ratios which are separated, which is that I multiply known [quantity] *e* into known *b*, and I divide that by known *a*. Therefore, there results quantity *h*, which is known. Then I multiply known *h* into known quantity *d*, and I divide that by known *g*. Therefore, there results from it *u*, which will be known."[169]

Although there are a few different ways that he could be proceding to reach these steps, here is a likely method. By rearranging the first statement to indicate that *d* to *g* is composed of *b* to *a* and *e* to *u*, and by compounding the right side as far as possible, we have that *d* to *g* is similar to *be*/*a* to *u*. Multiply *b* by *e* and divide by *a* to reach a number, which we call *h*. Therefore *d* to *g* is as *h* to *u*, and *u* can be found by multiplying *g* times *h* and dividing by *u*. If this recreation is correct, Ametus' series of directions rely fundamentally on the ability to rearrange a statement of composition into one of the valid modes. In that little phrase "I will place the quantities together" is a lot of hidden theory on the valid rearrangements of a statement of composition. Ametus continues to give the directions for finding the other unknowns, except for the fifth term.[170]

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[168] For the case in which they are the same, one could use Euclid VIII.11, which he has already given.

[169] Schrader, p. 181. My translation. "Ponam ergo quantitates duarum proportionum que sunt divise coniunctas quod est ut multiplicem E notam in B notam et dividam eam per A notam proveniet ergo quantitas H que est nota. Deinde multiplicabo H notam in quantitatum [sic] D notam et dividam eam per G notam proveniet ergo ex ea U que erit nota."

[170] Ibid., p.179-183. When *d* is the unknown, he multiplies *e* by *b*, divides by *a*. This quotient then divides the product of *u* and *g*. When *g* is unknown, he multiplies *e* by *b*, divides by *a*, multiplies by

In Part III before he enters into a discussion of the modes, Ametus describes how to compound ratios. He writes:

> Indeed when we place ratios of known quantities, whether they [the ratios] are similar or different, and after we multiply the first of the antecedents by the second of them and what results by the third and we continue thus until the number of antecedents is finished, and then we do the same with the consequents until they are finished, the ratio of the product of the multiplication of the antecedents to that which results from the multiplication of the consequents is composed of all those ratios whether they are similiar or dissimilar.[171]

In short, Ametus teaches the reader to compound by taking the ratio of the product of all the antecedents to the product of all the consequents of the composing ratios.[172]

His directions for subtracting a ratio are similar: "Moreover when we wish to remove some ratio from this composition, we multiply their antecedents by the consequent of that which we wish to take away from them and the antecedent of it which we are to remove by their consequents. And the ratio of one of these

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d, and divides by *u*. When *b* is unknown, he multiplies *u* by *g*, divides by *d*, multiplies by *a*, and divides by *e*. When *a* is unknown, he multiplies *u* by *g*, and divides by *d*. This divides the product of *e* and *b*.

[171] Ibid., p. 185. My translation. "Cum enim posuerimus proportiones quantitatum notarum sive sint similes sive diverse et post multiplicaverimus primam antecedentium in secundam earum et quod agregabitur in tertiam et fecerimus ita donec antecedentum numeratio finiatur, et deinde fecerimus illud in consequentibus donec finiantur, erit proportio eius quod ex multiplicatione antecedentium agregabitur ad id quod proveniet ex multiplicatione consequentium composita ex omnibus illius proportionibus sive similibus sive dissimilibus."

[172] Ametus gives essentially the same directions for compounding again soon after on Schrader, p. 187. "Cum ergo necessere [sic] fuerit nobis componere proportiones multiplicabimus antecedentes earum omnes ad invicem et consequentes etiam earum ad invicem et videbimus quid agregetur ex antecedentibus et illud quod agregabitur consequentibus. Erit namque proportio unius eorum ad alterum proportio composita ex summa illarum proportionum."

[products] to the other is the ratio of the remainder to remainder."[173] Although the operation is achieved through multiplication, it is phrased in terms of subtraction (Ametus uses the words *removere* and *residuum*). The last phrase in his directions, "the ratio of the remainder to remainder," is strange in that the ratio is the remainder of a subtraction or removal, but it is not the ratio of remainders. The way Ametus speaks of the ratio being removed from a set of composing ratios (he uses the plural "antecedents" and "consequents") is also odd since the statement can be correctly understood as saying that if one wants to remove the ratio of *e* to *f* from the composed ratio *ace* to *bdf*, then the remainder is the ratio of the product of *ace* times *f* to the product of *bdf* times *e*, or in modern terminology for ease of understanding, "$\frac{ace}{bdf} \cdot \frac{f}{e} = \frac{ac}{bd}$." when *a* to *b*, *c* to *d*, and *e* to *f* are ratios that are compounded together.[174] No terms are able to be made known through this statement as it stands. The statement could be made useful by a slightly modification to read that the remaining ratio from the subtraction is the ratio of the product of the antecedent (singular) of the composed ratio and the consequent of the ratio to be subtracted to the product of the consequent (singular) of the composed ratio and the antecedent of the ratio to be subtracted. That the leap from

[173] Ibid., p. 187. My translation. "Cum autem ex compositione illa aliquam voluerimus removere proportionem multiplicabimus antecedentes earum in consequentem eius quod removere voluimus ex eis et antecedentem eius quod remoturi sumus in consequentes earum. Et erit proportio unius earum ad alteram proportio residui ad residuum."

[174] Ibid., p. 187, n. 31.

the unuseful, literal meaning of the text to the more useful rule was made by at least one reader is clear from a note in one manuscript.[175]

Ametus also adds another statement about compound ratios:

> And when the ratio of one of two quantities to the other is composed of two ratios of quantities, whether they are similar or dissimilar, the ratio of the product of the multiplication of their [i.e. the original two quantities] antecedents by the consequent of one of the quantities of the two ratios to the product of the multiplication of the consequent by the antecedent of [the ratio] of which the consequent was taken will be as the ratio of the antecedent of the other ratio to its consequent.[176]

In other words, given that a ratio composed of two others, one of the composing ratios is similar to the ratio of the product of the antecedent of the composed ratio and the consequent of the other composing ratio to the product of the consequent of the composed ratio and the antecedent of the other composing ratio. In particulars, given that *a* to *b* is composed of *c* to *d* and *e* to *f*, then *ad* to *bc* is as *e* to *f*, or *af* to *be* is as *b* to *c*. Because composed ratios are made by taking the ratio of the product of the antecedents of the composing ratios to the product of their consequents, from this proposition, one can conclude that *e* to *f* is composed of the ratio of *a* to *b* and

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[175] Wien, Österreichische Nationalbibliothek, 5277 has a note in the margin at this point on f. 317r: "Verbi gratia, proportio 6 ad 2 componitur ex proportione 2 ad 1 et ex proportione 3 ad 2. Cum igitur removere voluerimus ex proportione 6 ad 2 proportionem 3 ad 2, multiplicabimus antecedens compositae quod est 6 in consequentem removendae qui est 2 et proveniet 12. Et consequens compositae in antecedentem removendae, proportionis et proveniet 6, et erit proportio 12 ad 6 sicut 2 ad 1."

[176] Schrader, p. 186-7. My translation. "Et cum proportio unius duarum quantitatum ad alteram fuerit composita ex duabus proportionibus similibus sive diversis quantitatum erit proportio eius quod agregatur ex multiplicatione antecedentis earum in consequens unius quantitatum duarum proportionum ad id quod agregatur ex multiplicatione consequentis in antecedentem cuius consequens fuit acceptus sicut proportio antecedentis proportionis alterius ad consequens eius." Schrader, p. 187 no. 31 understands this in this way: "If $\frac{a}{b} = \frac{e}{d} \cdot \frac{e}{f}, then \frac{ce \cdot f}{df \cdot e} = \frac{c}{d} or \frac{ce \cdot d}{df \cdot c} = \frac{e}{f}$." The passage is not easy to read, but to understand the passage in this way, she has to translate a few singular words plurally.

d to *c* and that *b* to *c* is composed of *a* to *b* and *f* to *e*. In other words, this statement of Ametus' is equivalent to giving two of the modes, but similarly to the way that propositions about *ex aequali* and numbers in continuous ratio do not need to have any mention of compounding although their subjects are closely linked, this proposition has a close connection to the modes, but is not put in terms of the modes.

Ametus' approach to compound ratios is thus not as clear as we might wish, but we can gather some general points about it. He deals with compound ratios in two different ways. First he talks about them in terms of repetition, but then he talks about them in a way that matches more closely the ways in which other mathematicians spoke of them. Although he does not clearly define compound ratios, his earlier and later treatments of them seem to accord with the continuous ratio conception, not the denominative. He does discuss the multiplications and divisions of quantitites that are to be done in order to compound or subtract ratios, but this interest in the algorism of ratios is still based ultimately upon continuity of ratios, not denominations. Ametus' loose terminology and his habit of giving propositions and statements without proofs and without any clear indication of whether they are logically prior to or dependent upon other statemets make it difficult to determine with certainty what he thinks it means for a ratio to be compounded of others.

The Modes

Part III of the *Epistola* provides the knowledge of the valid modes or combinations given a statement of composition. His coverage of the modes is rather different from Thabit's. While Thabit's treatment is almost wholly within theoretical proportion theory, Ametus' approach is geometric and more closely tied to the particular statements of composition given in the plane sector figure. Instead of giving universal proofs of the modes, he decided to discuss the modes in the context of Ptolemy's first use of compound ratios—the plane sector figures.

He begins with an ennumeration of the possible arrangements of the six terms in a statement of composition. While this treatment is fairly similar to Thabit's, Ametus treats them in the context of the plane sector figure, so he has two different statements of composition, one for the conjoined and one for the disjoined, which he calls "secundum compositionem" and "secundum divisionem." Thabit, on the other hand, treats one, general statement of composition that stands in for any particular statement of composition. Ametus then establishes that since there are fifteen different combinations of the six quantities in the first two positions, and since there are two combinations of the remaining four terms for each of these, there are thirty combinations. He gives a short justification for the second valid arrangement of the last four quantities; he writes, "... the two rectangles of the two antecedents and of the two consequents in the four ratios [the two ratios of the last four terms in the two valid modes] are equal."[177] In other

[177] Schrader, p. 189. My translation. I have inserted some punctuation and a "quoniam" that Schrader misread as a "quam": "Et quia proportionis unius duorum numerorum earum ad alterum

words, given *a* to *b* composed of *c* to *d* and *e* to *f*, it is also true that *a* to *b* is composed of *c* to *f* and *e* to *d*, because when the last two ratios are compounded by multiplication of antecedents and of consequents, the first statement involves the ratio of the product of *c* and *e* to the product of *d* and *f*, and the second involves the ratio of the product of *c* and *e* and of *f* and *d*. This argument from equivalencies of operations, here multiplication or making rectangles from lines, shows some similarities to algebraic thinking.[178]

Furthermore, because his thirty combinations include the six of the original fifteen combinations of the first two quantities that do not have any valid combinations of the last four terms, he subtracts twelve from thirty and reaches eighteen valid reconfigurations of a statement of compounding. He proves each of these eighteen modes for the conjoined plane sector figure and then again for the disjoined, so there are thirty-six cases of which he proves the validity. To this number, Ametus adds the six cases for each sector figure where the pair of quanitities in the first two positions does not have a valid configuration of the

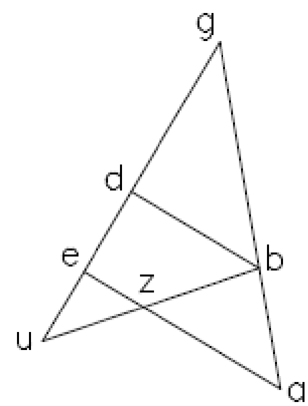
compositio in duobus queritur modis propter hoc quod cum proportio cuiusqueque duarum sex quantitatum ad alteram earum fuerit composita ex duabus proportionibus reliquarum quattuor quantitatum, erit etiam composita ex proportione antecedentis unius duarum proportionum ad consequentem alterius et ex proportione antecedentis alterius ad consequentem prime, quoniam duo quadrata duorum antecedentium et duorum consequentium in quattuor proportionibus equantur, demonstratur quod sunt triginta combinationes."

[178] In Madrid, Biblioteca Nacional, 10010 the *Epistola* has the rubrication "Ameti fili Josephi de Algebra," which seems applicable to this section on the modes if not to the entire work since many of the proofs related to the modes involve adding, subtracting, dividing, multiplying on the two "sides" of an equation or proportion.

remaining four quantities. This brings him to forty-eight total cases, although he gives proofs only of the valid modes, not for the invalidity of the others.[179]

Each of of the eighteen modes of the conjoined sector figure and of the eighteen modes of the disjoined sector figure is proved geometrically and without reference to the others. He notes (with one exception) when this sequence comes to one of the pairs of the first two terms that have no valid modes.[180] The proof of the first mode of the conjoined sector figure reads:

> I place therefore two lines *ag* and *ug* meeting upon point *g*. And let there be two lines *ae* and *ub* intersecting at point *z*. I say, therefore, that the ratio of line *ag* the first to *gb* the second is composed of the ratio of *ae* the third to *ez* the fourth and the ratio of *zu* the fifth to *ub* the sixth, which is proved thus. Indeed I draw line *bd* from point *b* parallel to *ae*. And I place line *ez* as a middle in the ratio between *ae* and *bd*. Therefore the ratio of *ae* to *bd* is composed of the ratio of *ae* to *ze* and the ratio of *ze* to *bd*. But the ratio of *ze* to *bd* is as the ratio of *zu* to *bu*. Therefore the ratio of *ae* to *bd* is composed of the ratio of *ae* to *ze* and the ratio of *zu* to *bu*. But the ratio of *ae* to *bd* is as the ratio of *ag* to *bg*. Therefore, the ratio of *ag* to *bg* is composed of the ratio of *ae* to *ze* and of the ratio of *zu* to *bu*. And that is what we wished to show.[181]

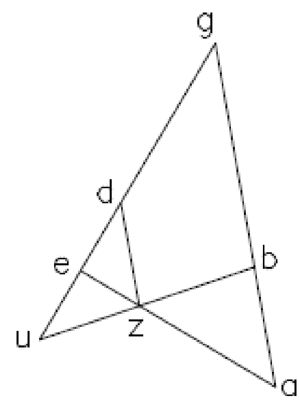


[179] Schrader, pp. 187-191.

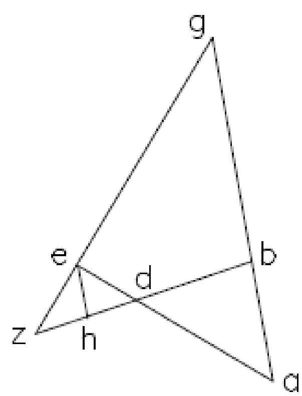
[180] The exception for the sequences of the modes of both the conjoined and the disjoined sector figure is the pair of the 2nd and 3rd terms from the original statement. Schrader, pp. 23-5 lists the cases neatly in a table on, but it has a couple of mistakes. She states that Ametus omits the pair of the 1st and 4th quantities, which has no valid modes, but he notes that it has no valid modes right after the fourth proposition. Also, her 27th case should read, "$\frac{AB}{AD} = \frac{BG}{DE} \cdot \frac{EZ}{GZ}$."

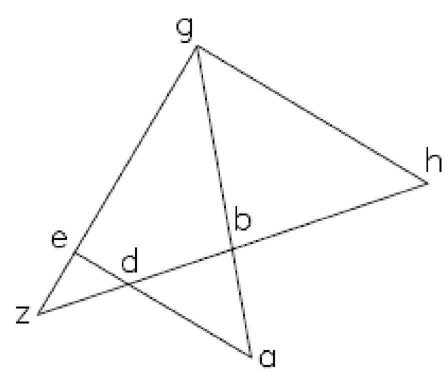
[181] Schrader, p. 191. My translation. "Incipiam itaque prius a compositione. Sitque composito in lineis. Ponam ergo duas lineas AG, UG supra punctum G concurrentis. Et sint due linee AE, UB sese in puncto Z secantes, dico ergo quod proportio linee AG prime ad GB secundam componitur ex proportione AE tertie ad EZ quartam et proportione ZU quinte ad UB sextam. Quod sic probatur.

Although most of the *Epistola* is almost conversational (and the prologue is a conversation), here Ametus uses the format and style of theoretical mathematics; he gives the assumed givens, explains what he is going to prove, makes any necessary constructions, proceeds step by step to his conclusion, and then he makes it clear that the proof is finished. He uses some words and phrases (e.g., “dico,” “quod sic probatur,” “et illud est quod demonstrare voluimus”) to mark the separate parts of the proof, but unlike the authors of many theoretical works, he does not justify his steps with references to earlier propositions or to ones from other works.



Ametus’ proofs are based largely on the use of inserting a middle between two quantities to produce a statement of compounding and on similar triangles. Different modes require the addition of new lines, and Ametus has two geometrical diagrams for the conjoined sector figure proofs and two for the disjoined . The first is essentially that





Protraham enim a puncto B lineam BD equidistantem AE. Et ponam lineam EZ mediam in proportione inter AE et BD. Est ergo proportio AE ad BD composita ex proportione AE ad ZE et proportione ZE ad BD. Sed proportio ZE ad BD est sicut proportio ZU ad BU. Ergo proportio AE ad BD componitur ex proportione AE ad ZE et proportione ZU ad BU. Sed proportio AE ad BD est sicut proportio AG ad BG. Ergo proportio AG ad BG est composita ex proportione AE ad ZE et ex proportione ZU ad BU. Et illud est quod demonstrare voluimus.”

of Ptolemy, but the other three differ.[182] Unlike Ptolemy's plane sector figure proofs, some of Ametus' proofs require the insertion of two middles and statements of composition in which one ratio is composed of three others. In these, he finds another compound ratio, which he then modifies by substituting similar ratios. He next substitutes this second composed ratio for two of the composing ratios in the first statement of composition. This gives him a more manageable statement of composition with only two composing ratios.[183]

Ametus' choice to prove 36 propositions geometrically and without any of them relying upon any of the other 35 has two main benefits. Since each proof is given its own geometrical proof, the cases are able to be given in an orderly sequence, and a reader can find the mode he needs for one of the sector figures without reading any of the other proofs; with a series of proofs of the modes that build upon preceding modes, such as Thabit has done, one has to go through the proofs of several modes to prove the one that is needed. The geometric nature of these proofs also makes them easier to follow than the more abstract proportion theory proofs of Thabit. In fact, Ametus states, "I will make it so that there is a figure for each of the cases of which the proof is according to the mode which Ptolemy used in the figure that is called "alchata" so that the consideration of it [the

[182] The fourth diagram is the mirror image of Ptolemy's diagram for the disjoined sector figure, but I consider it to be unique since the side on which the statement of composition occurs matters. In Ptolemy's figure, the reflected line whose parts are in the statement of composition is extended while in Ametus' the reflected line that is not in the statement of composition is extended.

[183] The first of these is Prop. 6, Schrader, p. 197-8.

sector figure] may be nearer and the imagination easier."[184] Ametus is clearly concerned with making his work accessible and states that his work is not wholly theoretical (which may explain why it is not arranged as a deductive science), that it is a work for beginners, and that he is concerned with teaching how to find values of unknown quantitities from known quantities.[185] For these goals, a geometric and non-universal approach works.

Ametus' approach, however, has its weaknesses. By using geometric proofs, he cannot prove each of the modes and then apply it to any statement of composition; Ametus has to prove thirty-six applications to cover the valid possibilities of the conjoined and disjoined sector figures. Also, while Thabit proves eight of the modes by applying the second mode to eight other modes modes, Ametus starts from scratch each time.

Despite its lack of the spherical sector figure, Ametus' coverage of compound ratios, the plane sector figures, and the modes of their statements of composition make the *Epistola* an integral part of the transmission of the transmission of the Menelaus Theorem. Although its coverage of compound ratios aligns much more with the continuous ratio conception of compounding, it introduces its readers to denominations, and like many of the sources available in

[184] Schrader, p. 187-9. My translation. Following Wien, Österreichische Nationalbibliothek 5277, I have corrected an "et" before "Ptolomeus" in Schrader's edition to an "est". "Quorum species secundam compositionem et divisionem comprehendam et faciam ut unicuique casuum sit figura cuius probatio sit secundum modum quo usus est Ptolomeus in figura que vocatur alchata [sectore] ut sit eius consideratio propinquior et ymaginatio facilior."

[185] Schrader, p. 117, 121.

the middle ages, it does not clearly define compounding. This work also shares responsibility with Thabit’s *On the Sector Figure* for connecting the study of the modes closely to the Menelaus Theorem, but Ametus’ approach towards the modes differs strikingly from Thabit’s.

Chapter 5: Gebir's *Correction of the Almagest*

Jabir ibn Aflah's *Correction of the Almagest* provided several theorems that could be used in place of the Menelaus Theorem.[186] Jabir, or Gebir as he was known in the Latin world, was a mathematician from Seville who appears to have lived in the first half of the twelfth century.[187] His *Correction* consists of nine books covering much of the material covered in the *Almagest*. The first two books of Gebir's *Correction* are the ones most concerned with spherical trigonometry. Parts of his work follow the *Almagest* very closely, and many of the diagrams are essentially the same, but he has many problems with the *Almagest*. Some of these are about inaccuracies or astronomical models, but a major one is the unnecessary complexity of using the Menelaus Theorem and compound ratios. Gebir also criticizes Ptolemy for combining the theoretical aspects of astronomy with the practical, which is confusing for students. The *Correction* focuses on the theoretical and has no tables and hardly any particular values. Perhaps Gebir's attempt to write a theoretical work on astronomy influenced medieval commentators on the *Almagest* to generalize its contents and to structure them as an axiomatic and deductive discipline.

[186] See Richard Lorch, "The Astronomy of Jabir ibn Aflah," *Centaurus* 19.2 (June 1975): 85-107 for a good overview of this work and its history.

[187] See Lorch, "The Astronomy," pp. 85-86.

The *Correction* was translated into Latin by Gerard of Cremona in the second half of the twelfth century. [188] This translation survives in approximately thirty manuscripts,[189] and it was also published along with Peter Apian's *Instrumentum primi mobilis* in 1534.[190] Interestingly, while Gebir did away with the Menelaus Theorem in his own work, many of the manuscripts containing the *Correction* also have works that prove and use it.

Gebir's complaint against the sector figure may not have been original,[191] but it did introduce a critique of the theorem and a replacement for it that many in the West found appealing. He lays out the problems with Ptolemy's use of the Menelaus Theorem in his introduction:

> He [Ptolemy] uses in several of his proofs the sector figure, which is difficult, and divided into several cases. Also in it compound ratio is varied extraneously. Because of this, it is difficult for the reader to remember and understand it and to make conclusions from it. And also because in his demonstrations he uses the books of Theodosius and Mileus [i.e., Menelaus], which are both so difficult and burdensome that it takes a student at least a whole year to have an understanding of and training in

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[188] Ibid., p. 90. The work was also translated into Hebrew twice about a century later. Gebir also seems to have written short commentaries on Thabit's treatise on the Menelaus Theorem and the eighteen modes and on Menelaus' *Spherics*, but these survive only in Hebrew manuscripts (Ibid., pp. 92-4).

[189] Lorch, The Manuscripts of Jābir b. Aflaḥ's Treatise," (as Appendix 1 to Item VI) in *Arabic Mathematical Sciences: Instruments, Texts, Transmission*, (Aldershot: Variorum, 1995), pp. 1-2. Kraków, Biblioteka Jagiellońska 1964 also contains the work. Palermo, Conv. di S. Francesco 4 may contain the first book of the *Correction*.

[190] Apian, Peter, Jabr ibn Aflah, *Instrumentum primi mobilis a Petro Apiano nunc primum et inventum et in Iucem editum... Accedunt iis Gebri filii Affla Hispalensis ... libri IX de Astronomia...*, (Nuremberg, 1534). It can be found online at http://www.cervantesvirtual.com/obra-visor/instrumentum-primi-mobilis-a-petro-apiano-accedunt-ijs-gebri-filii-affla-hispalensis-libri-ix-de-astronomia-per-girardum-cremonensem-latinitate-donati--0/html/.

[191] Lorch, "Jābir ibn Aflaḥ and the Establishment of Trigonometry in the West," (as Appendix 2 to item VI) in *Arabic Mathematical Sciences*, here p. 9.

> them and the sector figure, sometimes therefore the student hangs back or he loses that time in starting the book [the *Almagest*].[192]

And he later states:

> Therefore through the grace of God and the goodness of His help, we have easy and short propositions, by which we are excused from needing the book of Mileus [Menelaus], the sector figure, and much of the book of Theodosius, and by which an unknown [quantity] is found from knowns though four proportional numbers, instead of through [a statement of composition involving] six composed numbers as is done with the sector figure. For that reason finding an unknown from what is known is easy because fewer knowns are necessary, and the comprehension of it is easy through them, and the procedure is of little intricacy and density. And because of these propositions' easiness of what is known in them and the absence of the variety of compound ratios in them, whenever they are used, they lead to the finding of what is sought.[193]

His main complaint is that Ptolemy's reliance upon Menelaus and Theodosius, especially the Menelaus Theorem, makes it necessary to spend about a year just learning the spherical geometry necessary to fully understand the *Almagest*. Also, the many distinct cases of the Menelaus Theorem are difficult for students to understand and retain, as are compound ratios' various forms, which may refer either to the extensive consideration of possible modes such as is found in Thabit's

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[192] Apian-Gebir, p. 1. "[I]pse utitur in plurimo suarum probationum figura sectore, quae est difficilis, et partitur in ramos plurimos, et diversificatur in ea compositio proportionis varietate extranea, quapropter fit difficilis aspicienti in ipso rememoratio eius, et ipsius comprehensio, et concludere ea, quae concluduntur ex ea. Et de eis est etiam, quod ipse procedit in demonstrationibus suis secundum librum Theodosii et Milei, qui ambo sunt difficiles et graves, ita quod non praeparatur quaerenti et studenti cognitio eorum, et exercitatio in eis et in figura sectore, in minore spacio unius anni integri, quare quandoque pigritatur post illud, aut abscidit ipsum tempus ab introitu in librum."

[193] Ibid., p. 2. "Acciderunt ergo nobis per gratiam Dei et bonitatem auxilii eius, propositiones faciles et breves, quibus excusamus a libro Milei, et a figura sectore, et a plurimo libri Theodosii. Et quibus extrahitur ignotum ex noto per quatuor numeros proportionales, non per sex numeros compositos, sicut praeparantur in figura sectore. Quamobrem sit facilis extractio ignoti ex noto, cum indigeamus in ea notis paucioribus, et sit per illis comprehensio eius facilis, incessus paucae involutionis et consolidationis. Et accidit in istis propositionibus quae diximus, de facilitate notorum in eis, et paucitate diversitatis in compositione proportionis earum, quod ipseae perducunt ad verificationem in omni quaesito, in quo administrant."

work, or more simply to the two statements of the sector figure—the disjoined and conjoined—and the different orderings of the six terms in each. Gebir promises to provide an alternative that will only use four quantities instead of six and that will be learned more easily. Because his alternatives are easier to apply, Gebir will not use all of the rectilinear approximations that Ptolemy uses in astronomical applications such as the determination of lunar parallax in ecliptical coordinates.[194] A broader critique of the *Almagest* is that it inappropriately mixes the theoretical with the practical.[195] Gebir seems to want to have a scientific astronomy, which is universal and relies upon given first principles. In contrast to the *Almagest*, in the *Correction*, Gebir claims to provide in Book I the mathematics that he will need for his astronomy (although an understanding of Euclid's *Elements* is assumed). He claims that this mathematical training is concise enough "that it is possible that one studying it can learn it in one week," instead of the year that it could take to learn the texts of Theodosius and Menelaus.[196] After this separate mathematical section, Gebir enters into the astronomical applications in Book II.

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[194] Ibid. "Et non est necessarium cum eis uti lineis rectis, et angulis eorum, loco arcuum et angulorum suorum, sicut fecit Ptolemaeus in suo libro."

[195] Ibid, p. 1. "At vero est difficilis studenti in ipso, propter intentiones diversas de quibus est, quod ipse aggregat scientiam et operationem. Quia sit necessarium ex via operationis multiplicare numeros quosdam in alios et dividere alios per alios et invenire radices eorum et decenter praeparare tabulas, quae in operatione exercentur. Quapropter prolongatur liber, et dividitur scientia in ipso, et permiscetur cum operatione. Quare sit difficilis legenti ipsum." I have changed punctuation, capitalization to make this passage more comprehensible.

[196] Ibid., p. 2. "Et ad omnia illa fecimus singularem tractatum, quem posuimus primum, et est adeo propinquus et facilis, quod possibile est consideranti in eo, ut sciat ipsum in hebdomada una."

Gebir's Alternative Theorems

While most of Book I is on spherical geometry, the most relevant proofs for our purposes are Propositions 11-15. In Proposition 11, Gebir states that most of the Books I and II concern finding unknown arcs and angles and that to find them he resolves the figures into triangles, which in turn are resolved into right triangles. He then describes how to determine whether a side of a right triangle composed from great circles is less than, equal to, or greater than a quarter circle and whether an angle is less than, equal to or greater than a right angle. First, in a right triangle, the sides correspond to the angles that they subtend and vice versa—right angles are subtended by quarter circles, obtuse angles are subtended by arcs greater than quarter circles, arcs less than quarter circles subtend acute angles, etc. Then, Gebir goes through the different possible combinations of sides and angles that can occur in right triangles.[197]

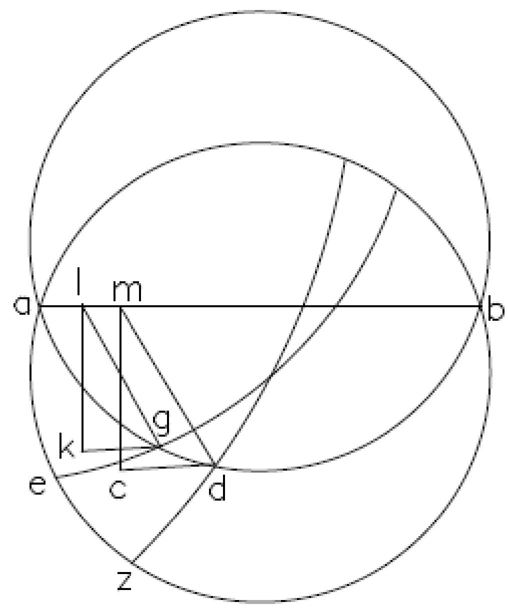
In Proposition 12, Gebir proves:

> When there are two great circles on a sphere and one of them does not pass through the poles of the other and if two points on the circumference of one of them or a point on the circumference of each of them is designated, however they fall, and two arcs are produced from each of these two points to the second circle, each of which makes a right angle with the arc of the circle to which it is produced, then the ratio of the sine of the arc which is between one of the two points and between one of the two intersections of the two circles to the sine of the arc produced from that point to the second circle is as the ratio of the sine of the arc which is between the second point

[197] Ibid., p. 9-10.

and between one of the two intersections to the sine of the arc produced from that point to the second circle.[198]

In this proposition, Gebir makes three conclusions with the two points taken in different positions and with different arcs taken between those two points and the two intersections of the two original great circles. In the first case, the two points are taken on the same great circle and the arcs of the great circles can be between the given points and the same intersection of the great circle. In the second, they are on the same great circle but the arcs can be between them and different intersections. In the third case, the two points are on different great circles and arcs are taken from the same intersection point. For the first case, *agdb* and *aezb* are the two great circles, and *g* and *d* are the two designated points. Through *g* and *d* and the pole of circle *aezb*, make great circles, of which circles arcs *ge* and



[198] Ibid., p. 10. "Cum sint duo circuli magni super sphaeram, et non transit unus eorum per polum alterius, et signantur super circumferentiam unius eorum duo puncta, aut super circumferentiam uniuscuiusque ipsorum punctum, qualitercunque cadant, et producuntur ex unoquoque illorum duorum punctorum duo arcus ad circulum secundum, quorum unusquisque continuat cum arcu circuli ad quem ipse producitur angulum rectum, tunc proportio sinus arcus, quae est inter unum duorum punctorum, et inter unum duorum punctorum sectionis duorum circulorum ad sinum arcus producti ex illo puncto ad circulum secundum, est sicut proportio sinus arcus, quae est inter punctum secundum et inter unum duorum punctorum sectionis ad sinum arcus producti ex illo puncto ad circulum secundum."

Gebir uses sines instead of the chords of double arcs. Since the sine of an arc is half of the chord of the double of that arc, the ratios between sines and those between chords are the same. While the use of sines may be important in other aspects of trigonometry, it does not have much of an effect upon the treatment of the Menelaus Theorem and its alternatives.

dz are the parts between the two original great circles. Arcs *ge* and *dz* form right angles with circle *aez*. Gebir proves that the sine of *ag* is to the sine of *ge* as the sine of *ad* is to the sine of *dz* (notice that Gebir uses sines instead of the chords of double arcs). First, from points *g* and *d*, he drops the two perpendiculars *gk* and *dc* to the plane of circle *aezb*. Also draw two perpendiculars to line *ab* from *g* and *d*, which let be *gl* and *dm*.[199] Draw lines *kl* and *cm*. Line *gk* is parallel to *dc*, and *gl* is parallel to *dm*, so angle *lgk* equals angle *mdc*. Also, because there are right angles at *k* and *c*, triangles *lgk* and *mdc* are similar. Therefore *gl* is to *gk* as *dm* is to *dc*. But *gl* is the sine of *ag*, *gk* is the sine of *ge*, *dm* is the sine of *ad*, and *dc* is the sine of *dz*, so the sine of *ag* is to the sine of *ge* as the sine of *ad* is to the sine of *dz*.[200]

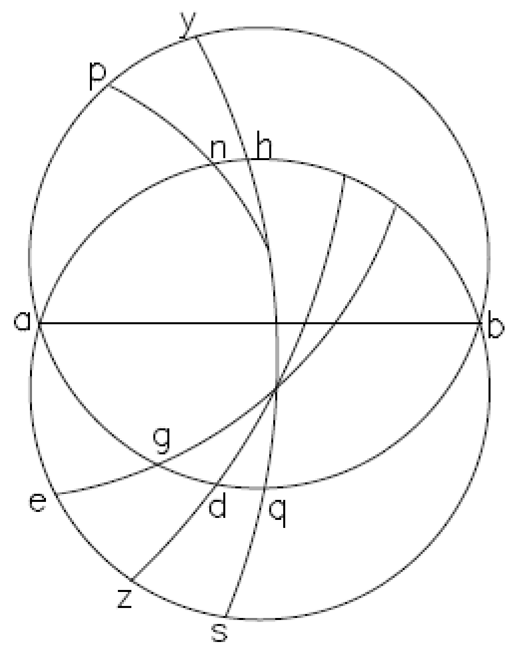
The second conclusion follows very easily. Since the sine of *ag* is the sine of *bg* and the sine of *ad* is the sine of *bd*, substitutions of equals in the first conclusion gives us that the sine of *bg* is to the sine of *ge* as the sine of *bd* is to the sine of *dz*, and that the sine of *ag* is to the sine of *ge* as the sine of *bd* is to the sine of *dz*.[201]

The third conclusion involves taking the two first points *g* and *n* on different great circles. First, make the great circle that goes through the poles of great circles *aezb* and *agdb*, and let it be *yhqs*. It will divide the two great circles in half and

[199] In the diagram in the printed edition, points *l* and *m* are mistakenly drawn as one point.

[200] Apian-Gebir, pp. 10-11.

[201] Ibid., p. 11.



will make right angles with them.[202] Arcs *aq*, *as*, *bq*, *bs*, *ah*, *ay*, *bh*, and *by* will all be quarter circles. Also arc *yh* will equal arc *sq*, so the ratio of the sine of any of the eight quarter circles to arc *yh* or *sq* will be constant. From the first conclusion, the sine of *an* is to the sine of *np* as the sine of *ah* is to the sine of *yh*, and also for the same reason, the sine of *ag* is to the sine of *ge* as the sine of *aq* is to the sine of *qs*. But, the sine of *ah* is to the sine of *yh* as the sine of *aq* is to the sine of *qs*, so the sine of *ag* is to the sine of *ge* as the sine of *an* is to the sine of *np*, which is Gebir's third conclusion.[203]

Proposition 13 is one of the most used propositions of Gebir's spherical trigonometry. It reads, "I say that in every triangle of arcs of great circles, the ratio of the sine of any side to the sine of the arc of the angle which it subtends is one ratio…"[204] Note that Gebir does not use sines of angles, which would not make sense with his definition of sine, but the sine of the arc of an angle. By "the arc of an angle" he means the sine of the arc of a great circle that has the vertex of the

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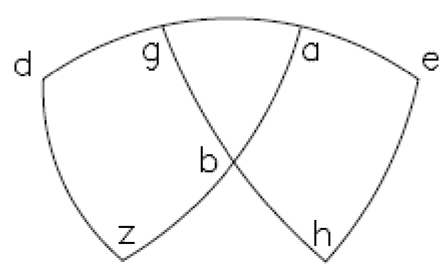
[202] Through his Propositions 5 and 9 (ibid., pp. 5-6, 7-8).

[203] Ibid., p. 11.

[204] Ibid.. p. 11. "[D]ico quod omnis trianguli ex arcubus circulorum magnorum proportio sinus cuiusque laterum ad sinum arcus anguli, cui subtensum est, est proportio una…"

angle as its pole that subtends the angle. In other words, the arc of an angle is the portion of the angle's "equator" that subtends the angle. Gebir divides this theorem into four different possible cases: three right angles, two right angles, one right angle, and no right angles in the given triangle. In the first case, all of the angles are rights, and from Proposition 11 all of the sides must be quarter circles. Therefore, the conclusion follows immediately. In the second case, the vertex of the non-right angle must be the pole of the arc between the two rights, so that arc is the arc of the non-right angle. Therefore, the ratio of the sine of the arc of that angle and the side subtending it is one of equality. The sides subtending the right angles are both quarter circles through Proposition 11, so also the sines of the arcs of the right angles are equal to the subtending sides. Therefore, the ratios are of equality.[205]

The third case, in which there is only one right angle, is more complicated. Given triangle *abg* with right angle *b*, he needs to show that the ratio of the sine of arc *ab* to the sine of the arc of angle *g* is the same as the ratio of the sine of arc *bg* to the sine of the arc of angle *a* and the ratio of the sine of arc *ag* to the sine of the arc of angle *b*.



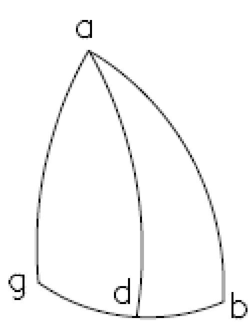
Make arcs *ge*, *ad*, *gh*, and *az* all equal to quarter circles. Pass a great circle through points *e* and *h*, and another through points *d* and *z*.[206] Point *g* is the pole of *eh*, and

[205] Ibid.

[206] Through his Proposition 4 (ibid., p. 5).

point *a* is the pole of *dz* through Proposition 11. From Proposition 12, the sine of *ag* is to the sine of *ge* as the sine of *ab* is to the sine of *eh*, and the sine of *ag* is to the sine of *ad* as the sine of *gb* is to the sine of *dz.* Arcs *ge* and *ad* are both quarter circles, so their sines are equal to the sine of the arc of angle g*ba*. Also, the sine of *eh* is the sine of the arc of angle a*gb*, and the sine of *dz* is the sine of the arc of angle *bag*. Therefore, the sine of *ag* is to the sine of the arc of angle *gba* as the sine of *ab* is to the sine of the arc of angle *agb*, and the sine of *ag* is to the sine of the arc of angle *gba* is also as the sine of *gb* is to the sine of the arc of angle *bag*. Because they are the same as the same ratio, also the ratio of the sine of *ab* is to the sine of the arc of angle *agb* is the same as the ratio of the sine of arc *bg* to the sine of the arc of angle *bag*.[207]

The fourth case is when the triangle has no right angles. Pass a great circle through point *a* and the pole of arc *gb*. This will make right angles at point *d*.[208] Gebir first assumes that point *d* falls on arc *gb* within the triangle. Applying what



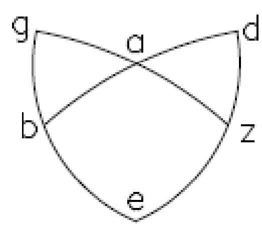
was just proved for the third case to the right triangles and alternating, Gebir finds that the sine of *ag* is to the sine of *ad* as the sine of the arc of angle *adg* is to the sine of the arc of angle *g*, and that the sine of *ad* is to the sine of *ab* as the sine of the arc of angle *b* is to the sine of the arc of angle *adg*. *Ex aequali* in the perturbed proportion,[209] the sine of *ag* is to the sine of *ab* as

[207] Ibid., pp. 11-12.

[208] Through his Proposition 5 (ibid., pp. 5-6).

the sine of the arc of angle *b* is to the sine of the arc of angle *g*. With the proportion alternated, the sine of *ag* is to the sine of the arc of angle *b* as the sine of *ab* is to the sine of the arc of angle *g*. Gebir does not prove all the steps in the proof, but states that if what he just did is repeated with a great circle passed through point *b*[210] and the pole of arc *ag*, we will likewise find that the sine of *ab* to the sine of the arc of angle *g* is as the sine of *bg* to the sine of the arc of angle *a*. From this, the conclusion follows easily. Gebir ends the proposition by outlining the argument if the great circle through point *a* does not fall within the triangle.[211]

In Proposition 14, Gebir proves, "That in every triangle composed from arcs of great circles in which one angle is right, the ratio of the sine of the arc of one of the remaining angles to the sine of the arc of the right angle is as the ratio of the sine of the arc of the complement of the remaining angle to the sine of the complement of the side subtending that angle."[212] In terms of the given triangle



abg, Gebir wants to prove that the sine of the arc of angle *a* is to the sine of the arc of right angle *b* as the sine of the arc of the complement of angle *g* is to the sine of the complement of arc *ab*. Quarter circle *bd* is made, and arc *dz* is made perpendicular

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[209] Gebir's wording is "in proportione aequalitatis secundum proportionem muthrariba" (ibid., p. 12). "Perturbed proportion" is defined in Euclid V def. 18 and proved in V.23. In other words, given A:B::C:D and B:E::F:C, then A:E::F:D.

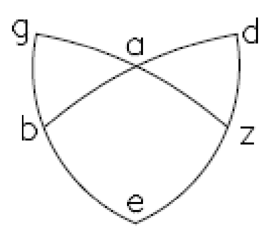
[210] The printed version has *d* for *b*, but that is clearly a mistake.

[211] Apian-Gebir. p. 12.

[212] Ibid., p. 12. "[Q]uod in omni triangulo ex arcubus circulorum magnorum, in quo est angulus unus rectus, est proportio sinus arcus unius duorum reliquorum ad sinum arcus anguli recti, sicut proportio sinus arcus complementi anguli reliqui, ad sinum arcus complementi lateris subtensi ei." The diagram is incorrectly labeled in the printed edition.

to *ag*, and point *e* is where it intersects arc *bg*. Through Proposition 12, because two arcs *ag* and *ab* intersect at *a* and the two points *g* and *d* are on them with perpendiculars *gb* and *dz*, the sine of *bg* is to the sine of *ag* as the sine of *dz* is to the sine of *ad*. But from Proposition 13, the sine of *bg* is to the sine of *ag* as the sine of the arc of angle *gab* is to the sine of the arc of right angle *gba*. Also, arc *dz* is the arc of the complement of angle *g*, and arc *ad* is the complement of arc *ab*. Therefore, the sine of the arc of angle *gab* is to the sine of right angle *gba* as the sine of the arc of the complement of angle *g* is to the sine of the complement of arc *ab*, which is what he wanted to prove.[213]

Proposition 15 proves that "the ratio of the sine of the complement of the arc subtending the right angle to the sine of one of the complements of the two arcs containing the right angle is as the ratio of the sine of the complement of the remaining side to the sine of a quarter circle."[214] From Proposition 12, the sine of arc *ad* is to the sine of *db* as the sine of *az* is to the sine of *eb*. But, arc *az* is the complement of arc *ag*, arc *eb* is the complement of arc *bg*, arc *ad* is the complement of arc *ab*, and arc *db* is a quarter circle. Therefore, the sine of the complement of arc *ag* is to the sine of the complement of *gb* as the sine



[213] Ibid., pp. 12-3.

[214] Ibid., p. 13. "Et dico iterum, quod proportio sinus complementi arcus subtensi recto ad sinum unius complementi duorum continentium ipsum, est sicut proportio sinus complementi lateris reliqui ad sinum quartae circuli. . ."

of complement of *ab* is to the sine of a quarter circle, which is what Gebir wanted to prove.[215]

Propositions 13, 14, and 15 are alternatives to the Menelaus Theorem. Gebir points out their importance by saying, "Therefore from these three theorems, the unknown is extracted from the known in a right triangle of arcs of great circles, namely because when three of the sides and angles of it are given, then the three remaining sides and angles are known by four proportional lines through these three theorems, and that will exempt us from the sector figure."[216] At the end of the spherical geometry section of Book I, Gebir once again repeats that he does not require his readers to refer to Menelaus or Theodosius and that unlike Ptolemy's, his book stands alone because he has proved all the theorems that he will need.[217]

Astronomical Applications

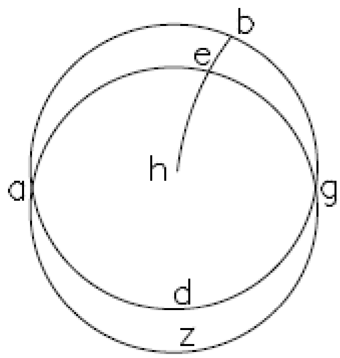
In the remainder of his *Correction*, Gebir shows that he can do everything that Ptolemy does with the Menelaus Theorem and that he can do it more simply. As an example of the ease of use of the alternatives, let us examine Gebir's proof of

[215] Ibid.

[216] Ibid., p. 13. "Ex istis ergo tribus theorematibus extrahitur ignotum ex noto trianguli arcuum circulorum magnorum orthogonii, scilicet, quia cum ponuntur eius tria laterum et angulorum eius nota, tunc cum istis tribus theorematibus scientur tria reliqua laterum et angulorum ipsius per quatuor lineas proportionales, et excusabit illud a figura sectore..."

[217] Ibid., p. 16. "Haec est ergo summa, quam necesse est praemittere eorum quibus consistit excusatio a figura sectore, et a libro Theodosii, et a libro Milei, et quibus declarantur, quae ipse dixit in libro suo sine demonstratione. Quare est liber iste noster stans per se, non egens alio sicut praemissimus."

how to find the declination of an arc of the ecliptic. The equator is circle *abgd*, the ecliptic is *aegz*, and the pole is *h*. The given point on the ecliptic is *e*, so arc *ae* is known. A great circle is passed through *h* and *e* to *b*. Because triangle *abe* is composed of great circles, I.13 can be applied. The sine of *ae* is to the sine of the arc of angle *b* as the sine of *eb* is to the sine of the arc of angle *a*. The arc of angle



a is known because it is the maximum declination, and arc *ae* is given, and angle *b* is a right angle. Therefore, the sine of *eb* can be found, and arc *eb* can also be found because it is known to be the shorter of the two arcs that share that sine.[218] Gebir does not say how to find the fourth proportional, but it would be clear to anyone with the barest rudiments of a mathematical education that this can be done by the rule of three or another easy method. This proof is evidently much easier than Ptolemy's use of the Menelaus Theorem—the diagram is simpler, there is a proportion instead of a statement of composition, and only four quantities are important instead of six. Gebir's constant boasting about the benefits of his alternative are justified.

In the *Correction* Gebir generally follows the order of Ptolemy, and he gives a proof using his alternatives for almost every one of Ptolemy's applications of the Menelaus Theorem although there is not a perfect one-to-one correspondence

[218] Ibid., p. 28, which is incorrectly numbered 36. The printed edition has some mistakes in page numbering. The page after 26 is numbered 35, so what should be pp. 27-30 are numbered 35-38. P. 31 is numbered correctly.

with Ptolemy's applications of the sector figure. Gebir has no proofs that match the second and third applications of the Menelaus Theorem in the *Almagest* II.3. This, however, is no deficiency on Gebir's part. In the first of these two missing applications, Ptolemy shows how to find the difference of the longest day and a twelve-hour day, but both Ptolemy and Gebir show how to find this value in another way after they show how to find oblique ascensions.[219] In the second application, Ptolemy shows how to find the arc of the horizon between the equator and a point of the ecliptic, but he had already given a way to find this value, and Gebir had given an alternative proof for this application. The omitted proofs are, therefore, not indications of the weakness of Gebir's alternatives, but are a conscious effort by Gebir to streamline astronomy and to remove redundancies that are found in the *Almagest*.

The other major deviation from what Ptolemy proves with the Menelaus Theorem is that Gebir does not find the apparitions and occultations of the fixed stars. Instead he states that these can be found in the way that the planets' apparitions and occultations are found, which he shows how to do at the end of the entire work.[220] Ptolemy had also treated the planets' apparitions and occultations, but he approximated by treating the arcs involved as equivalent to their chords. That Gebir gives one technique that applies to different types of celestial objects

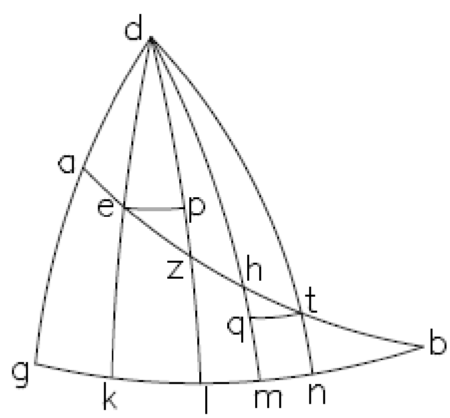
[219] *Almagest* II.9, 19r; Apian-Gebir, p. 32.

[220] Apian-Gebir, pp. 143-6.

suggests that his concern with theoretical as opposed to practical astronomy may involve a preference for universal proofs rather than particular ones.

Gebir not only uses his alternatives for propositions for which Ptolemy uses the Menelaus Theorem, but he uses these alternative proofs in two proofs in Book I of propositions that Ptolemy does not prove at all, for four propositions of Book II that Ptolemy proves without the sector figure, and for some statements in lunar theory for which Ptolemy uses rectilinear approximations.

The two proofs in Book I that have no corresponding passages in the *Almagest* are Propositions 17 and 18. In these theorems, he gives the enunciations in astronomical terms although the proofs themselves are purely mathematical (the astronomical terms show a small failure in his attempt to completely separate astronomy from the geometry needed for it). The first is "that the excess of the declination of parts of the ecliptic from the equator is greater near the two equinoxes than near the two tropic points."[221] Two quarter circles *ab* and *bg* meet at an acute angle. *D* is the pole of *bg*, and two equal arcs *ez* and *ht* are taken on arc *ab*. Great circles *dek*, *dzl*, *dhm*, and *dtn* are made. Perpendiculars *ep* and *tq* are drawn. Gebir wants to prove that the excess of *ek* over *zl* is less than the excess of *hm* over *tn.* Arc *hd* is



[221] Ibid., p. 14. "[Q]uod declinationum partium orbis signorum ab aequatore diei est apud duo puncta duarum aequalitatum plus quam sit apud duo puncta duorum tropicorum. . ."

greater than arc *dz*, so the ratio of the sine of *hd* to the sine of *da*[222] is greater than the ratio of the sine of *dz* to the sine of *da*. From Proposition 12, the sine of *hd* is to the sine of *da* as the sine of *ht* is to the sine of *tq*,[223] and the sine of *ez* is to the sine of *ep* as the sine of *dz*[224] is to the sine of *da*. Therefore, the ratio of the sine of *ht* to the sine of *tq* is greater than the ratio of the sine of *ez* to the sine of *ep*. It was given that arc *ez* equals *ht*, so putting that into our statement of inequality, arc *ep* must be greater than arc *tq*. Therefore, the complement of arc *ep* is less than the complement of arc *tq*.[225] Because of this and because the sine of *ht* is equal to the sine of *ez*, the ratio of the sine of the complement of *ez* to the sine of the complement of *ep* is greater than the ratio of the sine of the complement of *ht* to the sine of the complement of *tq*. From Proposition 15, the first half of that inequality is the same ratio as that of the sine of the complement of *pz* to the sine of a quarter circle, and the second half is the same as the sine of the complement of *hq* to the sine of a quarter circle. The complement of *pz* is therefore greater than the complement of *hq*, so *hq* is greater than *pz*. Also, arc *dt* is greater than arc *dq* through Prop. 11, so *qm* is greater than *tn*. Gebir skips over several steps here. Since *qm* is greater than *tn*, the excess of *hm* over *tn* is greater than the excess of *hm* over *qm*, which is *hq*. It was just shown that *hq* was greater than *pz*, so the

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[222] The print edition mistakenly has *dm* instead of *da*. This is the first of several mistakes in this proof.

[223] The edition also has a mistake here; both of the middle terms are missing from the proportionality.

[224] The edition has *ez*, but that clearly is incorrect.

[225] Mistakenly *ptq* in the text.

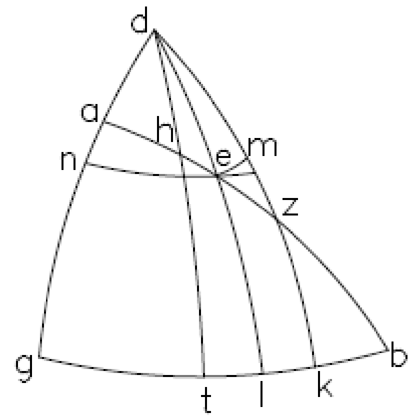
excess of *hm* over *tn* must be even greater than *pz*. Using similar steps, Gebir then shows that *pz* is greater than the excess of *ek* over *zl*. Therefore, the excess of *hm* over *tn* is even greater than the excess of *ek* over *zl*.[226] If this were put back into astronomical terms, which Gebir does not do, and *ab* were the ecliptic and *bg* were the equator, then it would be shown that given equal arcs of the ecliptic, the change in declination from the equator is greater for the arc nearer the equinox.

Proposition 18 demonstrates where on the ecliptic there occurs the greatest difference between an arc of the ecliptic and the right ascension of that arc. Quarters of great circles *bg* and *ab* meet, and *gad* is made a quarter of a great circle. Point *d* is therefore the pole of *gb*. A line is found that is the mean between the sine of arc *dg* and the sine of arc *da*. Let this be the sine of an arc *dn*. Measure *dn* along *dag*, and then using *d* as a pole and *dn* as a distance, make a circle (note that this is not a great circle). Let its intersection with circle *ab* be point *e*,[227] and therefore arc *de* equals arc *dn*. Make great circle *del* through point *d* and *e*. Take any two points on either side of *e* on *ab*, which let be *h* and *z*, and make great circles *dht* and *dzk*. It is to be proved that the excess of *be* over *bl* is greater than the excess of *bz* over *bk* and that the excess of *gl* over *ae* is greater than the excess of *gt* over *ah*. First, make arc *em* perpendicular to *dz* from point *e*. From Proposition 12, the sine of *lk* is to the sine of *em* as the sine of *dl* is to the sine of

[226] Apian-Gebir, pp. 14-5.

[227] There is an omission in the printed edition. At "qui secet circulum," it should read "secet circulum *ab* supra punctum *e* et faciamus transire super duo puncta *d e* circulum magnum qui sit circulus *del* et secet circulum."

de. But because of the way the sine of *dn* was made as a mean between the sine of *da* and the sine of a quarter circle, the sine of *dl* is to the sine of *de* as the sine of *de* is to the sine of *da*. Also, because *dz* is greater than *de*, the ratio of the sine of *dz* to the sine of *da* is greater than the ratio of the sine of *de* to the sine of *da*. Therefore, the ratio of the sine of *dz* to the sine of *da* is greater than the ratio of the sine of *lk* to the sine of *em*. Again from Prop. 12, the sine of *dz* is to the sine of *da* as the sine of *ez* is to the sine of *em*. From those last two statements, the ratio of the sine of *ez* to the sine of *em* is greater than the ratio of the sine of *lk* to the sine of *em*. Therefore, the sine of *ez* is greater than *lk*, and *ez* is greater than *lk*. Gebir concludes that the difference between *be* and *bl* is therefore greater than the difference between *bz* and *bk*.[228] Gebir then proves the second part of his proposition in a similar manner, using Proposition 12 twice.[229]

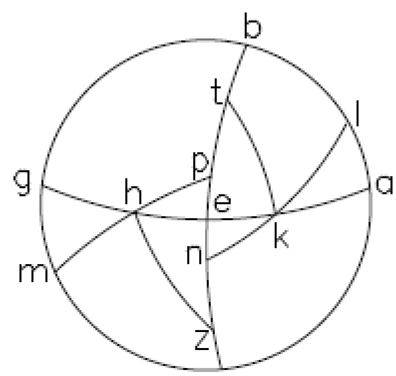


His proof of the first preliminary proposition for finding oblique ascensions is the first of the Book II propositions that use the alternatives to prove propositions of Ptolemy that do not use the Menelaus Theorem. It shows that arcs of the ecliptic equidistant from the same equinox ascend with equal arcs of the equator, and it is

[228] Gebir is skipping many steps of the argument. His result can be seen because the difference between *be* + *bl* can be seen as (*bz* + *ze*) – (*bk* and *kl)*, so it is the same as *bz -bk* + *ze* - *kl*. Therefore, the difference between (*be* – *bl*) and (*bz* – *bk*) is (*bz* – *bk*) + (*ze* – *kl*) – (*bz* – *bk*), which simplifies to *ze* – *kl*. Since *ze* was shown to be greater than *kl*, the difference between them is positive, and thus (*be* – *bl*) is greater than (*bz* – *bk*).

[229] Apian-Gebir, pp. 15-6.

much more complex than Ptolemy's proof. Gebir apparently saw this as one of the places where Ptolemy tacitly assumed knowledge of spherical geometry that he had not shown to be true. Circle *abg* is the meridian, *be* is the equator, *aeg* is the horizon, and *h* and *k* are the two points of the ecliptic as they rise (arcs' locations at two different times are represented in this one figure). *Hz* and *kt*[230] are arcs of the ecliptic (again, these arcs are not in these positions simultaneously). Points *t* and *z* are actually the same equinoctial point but at different times. From poles *l* and *m*, great circles are made to pass through *h* and *k* meeting the diameter at *p* and *n*. Points *h* and *k* are equally distant from an equinox, so their declinations, arcs *hp* and *kn* are equal, as are the arcs of the horizon *eh* and *ek*. From I.15, in right triangles *ekn* and *ehp,* the sine of the complement of *ek* is to the sine of the complement of *kn* as the sine of the complement of *en* is to the sine of a quarter circle, and also the sine of the complement of *eh* is to the sine of the complement of *hp* as the sine of the complement of *ep* is to the sine of a quarter circle. Also because *ek* equals *eh* and *kn* equals *hp*, the sine of the complement of *ek* is to the sine of the complement of *kn* as the sine of the complement of *eh* is to the sine of the complement of *hp*. Therefore, the sine of the complement of *ne* is to the sine of a quarter circle as the sine of the complement of *ep* is to the sine of a quarter circle. Because of this and



[230] In this proposition point *t* is mistakenly referred to as point *d* several times in the print edition.

because arcs *ne* and *ep* are both known to be less than quarter circles, they are known to be equal. Also, arcs *kt* and *hz* are given as equal arcs of the ecliptic on opposite sides of an equinox, so it is known that their co-ascensions in the right sphere, which are *zp* and *tn*, are equal. Therefore, *zn* equals *pt*. Adding equals *zn* and *pt* to equals *ne* and *ep* results in equal arcs *ez* and *et*, which are the arcs of the equator that rise with the given arcs of the ecliptic. [231]

Similarly, for two of the preliminary propositions about the angles between the meridian and the ecliptic, Gebir uses one of the alternatives where Ptolemy does not use the Menelaus Theorem. The first preliminary proposition, about angles of the ecliptic and meridian at points on the ecliptic equidistant from an equinox, is similar to that in the *Almagest* II.10. While both Gebir and Ptolemy use two equal triangles, Ptolemy merely states that the sides of one triangle equal those of the other, so the angles are the same. Gebir uses I.13 to take that step from the equality of the sides to the equality of the angles. In the second preliminary proposition, about the angles at points equidistant from a tropic point, Ptolemy had jumped from the statement that the arcs of great circles from the two given points to the pole are equal, to the statement that the angles at these points facing each other are equal. Gebir uses I.13 to bridge Ptolemy's unsubstantiated step.[232] For the first preliminary proposition regarding the angles between the ecliptic and the horizon, which is about the angles at points on the ecliptic equidistant from an equinox,

[231] Apian-Gebir, p. 31.

[232] Ibid., pp. 33-4.

Gebir once again uses I.13 in an elaboration of Ptolemy's step from triangles of equal sides to similarity of triangles.[233]

While Ptolemy does not directly use the Menelaus Theorem after Book II until Book VIII where he deals with the fixed stars, Gebir applies his alternative proofs to spherical problems of the latitude of the moon. For example, Gebir applies I.12 to find the center of the moon's longitude from a node during an eclipse.[234] When converting from lunar parallax along the circle of altitude to ecliptical longitude and latitude of parallax, Gebir first gives Ptolemy's plane approximation from *Almagest* V.19,[235] but then he adds, "It is possible to know that according to truth through that which I tell."[236] Applying I.13 and I. 15 to spherical triangles instead of plane ones, he finds the wanted values for the longitudinal and latitudinal parallax. A little farther on in his work, Gebir uses I.13 and I.12 for spherical problems of the duration of eclipses, which Ptolemy had approximated with rectilinear triangles.[237] Gebir later faults Ptolemy's method for finding the

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[233] Ibid., p. 34.

[234] Ibid., p. 51. This is Gebir's explanation of how Ptolemy could have found this value from eclipse records that he uses in IV.9 of the *Almagest* (1515, 44r-45v).

[235] 1515 edition, 58v-60r.

[236] Apian-Gebir, p. 70. "Et hanc quidem operationem ingreditur approximatio in utendo lineis rectis et angulis eorum loco arcuum et angulorum eorum, praecipue in arcubus transeuntibus per zenith capitis et lunam, et transeuntibus per zenith capitis et locum lunae, et per locum lunae verum in orbe signorum unumquodque, quorum possibile est pervenire prope quartam circuli, et est possibile scire illud secundum veritatem per illud quod narro."

[237] Ibid., pp. 76-7. See 1515 edition of the *Almagest*, 67r-v to compare.

point on the horizon opposite the darkened point during an eclipse as being difficult and inaccurate. He then finds it using I.13 twice.[238]

Gebir gives alternate proofs for the propositions that Ptolemy proves in VIII.5 of the *Almagest*. Gebir uses I.13, I.14, and I.15 in his conversion from a star's ecliptical coordinates to its equatorial coordinates.[239] He uses I.14 in the second theorem, which is on finding the points of the equator and ecliptic that rise and set with a star given the point on the equator that mediates the heavens with that star.[240] At this time, he does not prove how to find the first appearances and the occultations of stars, but merely states that it will be clear when he covers the same phenomena with the planets.[241]

Like Ptolemy, Gebir uses plane approximations to find the latitudinal variations of the planets, but unlike Ptolemy, he treats the apparitions and occultations of the planets spherically. Because the spherical triangles involved are relatively small, Ptolemy approximates by ignoring the difference between arcs and chords. He does use the angle between the horizon and the ecliptic that was derived from the Menelaus Theorem, but he treats it as a rectilinear angle.[242] Gebir follows basically the same steps but applies I.13 and I.15 to spherical triangles

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[238] Apian-Gebir, p. 83.

[239] Ibid., p. 102. He phrases the problem as finding the point of the equator that mediates the heavens along with the given star, what can also be called the co-culmination

[240] Ibid., pp. 102-3.

[241] Ibid., p. 103.

[242] 1515, 149v-152r.

instead of dealing with the ratios of the sides of plane right angle triangles as Ptolemy does.[243]

Gebir's alternative theorems, although not easy to prove, were much easier to apply to astronomical situations than the Menelaus Theorem. Introduced to the Latin West at approximately the same time as the *Almagest*, Gebir's *Correction* made apparent some of the disadvantages of the sector figure and offered replacements that made the practice of astronomy easier. Gebir was influential and many medieval astronomers realized the value of his work, but his new basis for spherical astronomy, which obviated the need for the Menelaus Theorem and compounding ratios, did not replace Ptolemy's approach. As we will see in Part II, medieval scholars studied these two topics in great detail even when incorporating material from Gebir's spherical geometry.

[243] Apian-Gebir, pp. 143-6.

PART II: ORIGINAL MEDIEVAL WRITING ON THE MENELAUS THEOREM

Chapter 1: The Marginalia of the Gerard of Cremona *Almagest* Manuscripts

Many of the manuscripts with Gerard of Cremona's translation of the *Almagest* contain a significant amount of marginalia. Of the nineteen manuscripts from which I have transcribed passages, ten contain substantial, paragraph-length notes and two more contain a large number of short notes of only phrases or sentences.[244] Because some of the sets of notes are found in more than one manuscript (and some manuscripts contain more than one set of notes), students of the *Almagest* must have copied marginalia from one manuscript into others.[245] While much work on the sets of marginalia needs to be done to examine their dating, their contents, and their relationships to each other, I will give a general overview of the notes and their treatment of the Menelaus Theorem and compounding. Later I will conduct a more thorough examination of one set of notes that was written by Campanus de Novara.

Some of the longer notes provide divisions of the text. For example, one note given at the beginning of the lemmas for the sector figure reads,

> In this part, with the greatest declination having been found by observation, he [Ptolemy] teaches how to find demonstratively any point of the ecliptic's declination from the equator. It is divided into two parts. In the first he

[244] The following manuscripts contain sets of lengthy notes as well as short notes: Città del Vaticano, Biblioteca Apostolica Vaticana, Barb. lat. 173 (MS M); Barb. lat. 336 (MS N); Pal. lat. 1365 (MS S); Vat. lat. 2057 (MS L); Vat. lat. 6788 (MS F); Kraków, Biblioteka Jagiellońska 590 (MS R); Kraków, BJ 619 (MS K); and Paris, Bibliothèque nationale de France, lat. 7257 (MS W); Paris, BnF, lat. 7256 (MS X). Paris, BnF, lat. 16200 (MS H). Short phrases and sentences can be found frequently in the margins of Paris, Bibliothèque nationale de France, lat. 7254 (MS Z) and Paris, BnF, lat. 7255 (MS Y). Although I have not transcribed any of it, the marginalia found in Erfurt, Universitätsbibliothek Erfurt, Dep. Erf. CA 2^{o} 375 has a substantial amount of notes.

[245] One set is found in MSS LMR, and another is found in MSS KH. There are two different sets of notes in MSS NX. MS R also has at least two sets of notes.

> places certain lemmas for finding those declinations. In the second, he demonstrates the proposition by these lemmas—at "et postquam praemisimus hoc capitulum." The first [part is divided] into two—into a preface and a treatment. The second [starts] at "describam ergo." This second [is divided] into two. In the first he posits certain common lemmas removed or mediated [i.e., to be used indirectly]. In the second he places particular, proximate, or immediate ones that are demonstrated by the prior ones—at "et postquam praemisimus haec antecedentia." The first [is divided] into two. In the first he gives certain general antecedents. In the second [he gives] certain others that are more particular—at "describam etiam." The first part is divided into two. In the first he gives a lemma according to conjoined proportionality. In the second [he gives] another according to disjoined proportionality—at "similiter declarabitur." That part "describam etiam" in which he gives certain more particular lemmas is similarly [divided] in two. In the first he gives a lemma according to disjoined proportionality. In the second [he gives] another according to conjoined proportionality—at "describam etiam. Each of these [is divided] into two because in the first of each he demonstates the truth of the proportion from the side of things. In the second, [he demonstrates] the knowledge of the extremes from our side. The second [part] of the first [is] at "hoc autem superest." The second of the second at: "sequitur nota hoc." That part "et postquam premisimus hec antecedentia" in which he gives the proper and immediate lemmas is divided into two. In the first he gives a lemma according to disjoined proporitionality. In the second [he gives] one according to conjoined proportionality—at "ex eo autem quod demonstratum est etc."[246]

And in this note the commentator continues to divide the next chapter. The inclusion of divisions of the text like this suggests that these manuscripts were involved in classroom teaching. One of the first stages of a lecture was the division of the text, which provided a rough summary of the text and how its parts fit together.

[246] Appendix B, NX edition, lines 215-248.

Many of the notes are concerned with supplying what the *Almagest* needs to become more axiomatic and deductive. One way of doing this is to explain steps in Ptolemy's arguments. Very frequently steps in Ptolemy's arguments were justified with references to propositions or definitions in Euclid's *Elements*. For example, between the lines and in the margins of the text giving the Menelaus Theorem's first lemma, the plane conjoined sector figure, there are notes saying "through the 31st [proposition] of the first [book] of Euclid," "through the first common notion of the sixth of the *Geometry* and … through the fourth proposition of the sixth and through the ninth of the fifth."[247] Since the *Elements* were the main authority of mathematics, notes such as "through the fourth of the sixth" were understood to mean "through the fourth proposition of the sixth book of Euclid's *Elements*." While Euclid is most often referenced, Theodosius, Menelaus, Ametus, Jordanus, and others are sometimes cited. Some of the notes refer to principles and propositions while others give only the numbers of the principles and propositions. For example, notes reading "because of similar triangles through the fourth and the penultimate of the sixth" and "because the ratio of the extremes is composed of the ratios of the intermediates through the 19th definition of the seventh of Euclid" are also found alongside the text of the first lemma.[248]

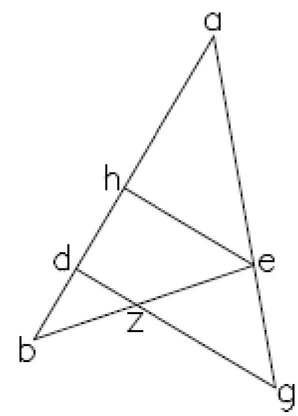
Some of the explanations of parts of Ptolemy's arguments are more than just a phrase or short sentence. A note that explains how some of the proportions

[247] Appendix A, *Almagest* edition, line 17's apparatus.

[248] Ibid., apparatus for lines 17, 20.

are reached in the first lemma goes thoroughly through many steps that Ptolemy passed over:

> For here there are two triangles *agd* and *aeh*. And angle *agd* of the first triangle is equal to angle *aeh* of the second triangle through the second part of the 29th of the first of Euclid, and angle *a* is common to both triangles. Therefore the third is equal to the third. Or the third is equal to the third through the second part of the 29th of the first of Euclid. Therefore through the fourth of the sixth of Euclid, the sides are proportional. Therefore the ratio of *ga* to *ea* etc.[249]



This gives a much more detailed explanation of how the proportion *ga* to *ea* as *gd* to *dh* is known than Ptolemy's text, which only gives the explanation "because *eh* and *gd* are parallel."[250] By referring to the second part of *Elements* I.29, the commentator is explaining how it is known that there are similar triangles, and by referring to *Elements* VI.4, he explains how the proportion of sides results from the similarity of triangles. Also, by referring to the *Elements*, which reaches conclusions through arguing deductively from first principles, he is implicitly connecting Ptolemy's argument to first principles of geometry.

A more extreme restructuring of the way the text was read (albeit in the margins, not *in* Ptolemy's text) is found in some sets of notes. Some of the longer add enunciations and proofs restating in a different format and mathematical style what Ptolemy writes in his text. Some manuscripts contain series of notes that are

[249] Appendix B, LMR edition, lines 7-13.

[250] 1515 edition, 9v.

very closely related to the *Almagestum parvum* and the work I call the "Erfurt Commentary," which are works that I will discuss after the marginalia. One manuscript contains much of the *Almagestum parvum* as a set of notes.[251] The notes in this manuscript give first principles at the beginning of some of the books, as well as enunciations and proofs restating in general terms what Ptolemy does in particular terms. Another manuscript shares some of these notes, but without giving as many proofs.[252] Two other manuscripts contain a set of notes that gives enunciations, rules, but only occasional proofs that are not identical to either the *Almagestum parvum* or the Erfurt Commentary but that are rather similar.[253] Usually general enunciations are given without proofs. An example of the enunciations found in these notes reads:

> 9. With two lines descending from one angle, if two lines are reflected from their endpoints and cut each other between them (the original two lines), the ratio of one of those descending lines to its part between the point where it is cut and the angle [where the first two lines meet] will be composed from a two-fold ratio: namely from the ratio of the reflected line conterminal with it to the upper part of that line and from the ratio of the lower part of the other reflected line to the whole.[254]

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[251] MS H. A closer examination of this manuscript and the *Almagestum parvum* may prove fruitful. Although this manuscript was written in 1263 and the marginalia probably later, this set of notes may show a stage in the genesis of the development of the *Almagestum parvum* from marginalia to a stand-alone work. On the other hand, the notes may be the result of a reader of the *Almagest* using the existing *Almagestum parvum* to understand Ptolemy's work. So, these notes may illuminate our understanding of the development of the *Almagestum parvum* or the manner in which scholars used stand-alone commentaries to study the originals.

[252] MS K.

[253] MSS NX. In the following chapter, I will examine another set of notes contained in these manuscripts—the commentary of Campanus de Novara on the *Almagest*.

[254] Appendix B, NX edition, lines 250-5.

While this is a much more wordy statement of what is being proved than is found in the *Almagest*, its virtue is that it is universal knowledge, not merely a statement about a figure involving particular values or lettered in a particular manner.

In these four manuscripts, enunciations are found not only for the introductory mathematics of Book I, but also for the astronomical content throughout the *Almagest*. These are sometimes accompanied by rules or corollaries that give the steps for finding sought astronomical values. For example, accompanying Ptolemy's chapter on finding the declinations of arcs of the ecliptic, the following note is given:

> 16. Given a point of the ecliptic, to find its declination from the equator. Whence it is clear that if the sine of the arc of the ecliptic which is between the equator and the given point is multiplied by the sine of the maximum declination, and the product is divided by the sine of a quarter circle, there will result the sine of the declination of the given point."[255]

These notes alter the way that the text was read. While Ptolemy organizes his works in chapters and gives particular examples of calculations, these commentators divide the work into numbered propositions stated in universal terms, not in terms of a particular example or even in terms of the letters of the diagram. While Ptolemy expected his reader to abstract the general way of performing a certain calculation from the one or two examples that he gives, the commentators give a series of generalized rules of simple mathematical operations that will lead to the desired quantity, and sometimes these rules and the enunciations are shown through proofs given in general terms, not with specific

[255] Appendix B, NX edition, lines 378-382.

values. [256] For the applications of the sector figure to astronomical problems, some of these notes explain where the sector figure is in Ptolemy's diagram, whether the conjoined or disjoined sector figure will be used, and whether a regular, inverted, or alternate version is utilized (i.e., one of the modes is used instead of the exact statement given in the *Almagest* I.12).[257] Probably following the example of the *Almagestum parvum*, which will be discussed later, the authors of these sets of notes even give enunciations to Ptolemy's chapters on instruments and observation. Although these chapters are clearly not theoretical mathematics, these notes attempt to give them some semblance of the structure of a Euclidean science.[258]

While many of the notes on the sector figure are about the basic geometry of parallel lines and similar triangles, a major topic in the notes in *Almagest* manuscripts is compound ratio. Ptolemy merely gives a statement of composition with the short explanation that a quantity has been placed as a middle between two others, and while this implies the continuous conception of compound ratio, it is left to commentators to explain to other readers exactly what compound ratio is. Some of the notes lay out what they take as a definition or first principle—that the ratio of extremes is composed of the ratio of the first quantity to a middle and the

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[256] While there are many examples of the enunciations, rules, and proofs, the note numbered 20 in Appendix B, NX edition, lines 780-81 6, provides an example in which these different parts of the note are especially clear.

[257] E.g., MS N has a note in the diagram for the *Almagest* I.13 that states, "Inquirit per katam coniunctum arcum *th* qui est declinatio arcus *eh*. Descendit autem kata ab *a* in *z e*." (Appendix B, NX edition, lines 383-4.)

[258] E.g., Appendix B, NX edition, lines 164-5, gives an enunciation for a section on the construction and use of an instrument.

ratio of that middle to the second extreme.[259] These notes support this view with references to arithmetical works and give arithemetical examples, which is surprising given that one of the advantages of the continuous conception of compound ratios is that it does not only apply to numbers and commensurable quantities as the denominative method does.[260] A note in Città del Vaticano, Biblioteca Apostolica Vaticana, Pal. lat. 1365 (MS S) reads, "For as Jordanus declares in the *Arithmetica*, the ratio of the extremes is composed of the intermediate ratios, as is clear in these three numbers 18, 12, and 6."[261] Another note in this same manuscript claims, "And it is generally true in all quantitites that the ratio of any quantity to another is composed of the ratio of that quantity to its part and of that part to the other quantity."[262] In other manuscripts, Ptolemy's introduction of compound ratios is explained by a reference to the nineteenth definition of *Elements* VII.[263] In one manuscript, we read, "Because the ratio of extremes is composed from the ratios of the intermediate [quantities] through the nineteenth definition of the seventh [book] of Euclid."[264] And in another manuscript a similar explanation is given: "Through the definition of the seventh

[259] E.g., see Appendix B, F edition, lines 41-54; HK edition lines 6-13; and S edition, lines 14-25.

[260] Nicole Oresme discusses how not all ratios can be represented directly by denominations in his *De proportionibus proportionum* (Grant, pp. 161-7).

[261] Appendix B, S edition, lines 14-6.

[262] Appendix B, S edition, lines 41-3.

[263] Although the definition that these commentators are referring to is not found in the modern versions of the *Elements* or in all medieval versions, it is found in Campanus' edition: "Cum continuate fuerint eedem vel diverse proportiones, dicetur proportio primi ad ultimum ex omnibus composita" (H. L. L. Busard, ed., *Campanus of Novara and Euclid's Elements*, (Stuttgart: Steiner, 2005), p. 230).

[264] MS. K. Appendix A, *Almagest* edition, apparatus to line 20.

[book] of Euclid which is that when there are continuous ratios that are the same or different, the ratio of the first to the last is said to be composed from the others."[265] In this same set of notes, this concept of compound ratios is explained with an example from music theory: the octave, which is the ratio of 2 to 1, is composed from the intervals of a fifth, which is the ratio of 3 to 2, and of a fourth, which is the ratio of 4 to 3.[266] Since it is common in music to hear of certain intervals being composed of others or to hear of intervals being added together, the continuous conception of compound ratio was more common in music than in other mathematical disciplines.

The denominative understanding of compound ratios is presented in other manuscripts. A note in one gives the statement that a ratio of a first quantity to another is composed of the ratio of the first to a quantity placed between them and the ratio of that middle to the other quantity; however, this is not given as a principle but as a proposition proved elsewhere. The commentator writes that it is the second proposition of the "book of ratios," which probably refers to the work on compound ratios by Campanus that will be discussed later, and he states that this is also the thirty-sixth proposition of Jordanus de Nemore's *De numeris datis*.[267]

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[265] Appendix B, F edition, lines 23-5.

[266] Ibid., lines 43-4.

[267] Appendix B, HK edition, lines 6-13. The 36th proposition of *De numeris datis* is II.7, of which part reads, "Denominatio enim proportionis primi ad secundum, in denominationem proportionis secundi ad tertium ducatur, et fiet proportio primi ad tertium" (Barnabas Hughes, *Jordanus de Nemore, De numeris datis: A Critical Edition and Translation*, (Berkeley; London: University of California Press, 1981), p. 72). This work does not contain theoretical proofs, so the reference to it is not as revealing as the one to the Campanus work.

Another set of notes includes a lengthier explanation of compound ratios in terms of denominations: “That a ratio is composed of ratios is nothing except that the denomination of the composed ratio is produced from multiplying the denomination of one of the composing [ratios] into the denomination of the other. Moreover, the denomination of a ratio is what results from the division of one of the terms of the ratio by the other.”[268] As we saw in Part I, Ptolemy did not define compound ratios or clearly explain them, but his use of them was more consistent with the continuous idea of them than with the denominative idea. Despite this, some medieval readers understood the uses of compound ratio in the *Almagest* through the denominative concept.

The treatment of compound ratios is not always clear or consistent. A prime example of a confused treatment of compound ratios is found in the notes of a particular commentator who writes that the statements of composition of the first two lemmas come from a rule given in commentary on Euclid’s *Elements* that states that given three quantities, the ratio of the first to the second multiplied by the ratio of the second to the third results in the ratio of the first to the third.[269] Seeing a problem with multiplying ratios, this commentator adds that it is more exact to speak of the multiplication of the denominations of ratios, not of ratios

[268] Appendix A, note to line 131. Interestingly, he claims that rules from the *Almagestum parvum* make dealing with compound ratios easier. As we will see, that work does contain rules for finding astronomical values, but these rules depend upon the reader’s prior knowledge of dealing with statements of composition.

[269] Appendix B, LMR edition, lines 38-43. Strangely the reference is to the commentary on *Elements* VI.24, but Campanus’ edition has nothing of the sort for IV.24. The commentary that this commentary is referencing is probably marginalia in an *Elements* manuscript.

themselves.[270] Although the commentator has these passages laying out the denominational idea of compounding ratios, in others he seems to still conceive of a composed ratio as being a whole made up of component parts similarly to the way in which a quantity is made up of component parts. In another note the commentator tries to explain that Ptolemy's statement of composition in the first lemma is "more a separation than a gathering together because with the ratio of the second [term] to the third subtracted from the ratio of the first to the second, the ratio of the first to the third remains. And in these terms laid out in this way, the rule has no truth, namely to multiply the ratio of the first to the second by the ratio of the second to the third."[271] The commentator has difficulty understanding Ptolemy's statement of composition because it states that a ratio of greater inequality (the antecedent is greater than the consequent) is composed of a ratio of greater inequality and one of lesser inequality (the consequent is greater than the antecedent). The ratio of lesser inequality does not seem to be as much a component that helps add up to or make a whole than as something that takes away or subtracts from the whole. That the commentator sees some difficulty in having a ratio of lesser inequality be a component of a ratio of greater inequality suggests that he thinks of composing ratios not merely as factors but as components. This understanding of compound ratios through parts and wholes is also displayed in the commentator's habit of explaining whenever Ptolemy subtracts a ratio from another

[270] Ibid., lines 33-6.

[271] Ibid., lines 26-31.

that the other composing ratio will be the remainder “because if anything is made up from two and one of them is subtracted from it, the other remains.”[272]

Some of the notes address the question of what it means to place a quantity as a middle. To make this clear, they rightly point out that in Ptolemy’s first use of the insertion of a middle to reach a statement of composition, the middle is less than either of the extremes so it cannot be a middle in terms of quantity. They explain that the middle does not have to be an intermediate in terms of size, but that it is a middle in the order that ratios are taken between the quantities.[273] Another commentator explains that even in the case of the numbers 18, 6, and 12, the ratio of the first to the last is composed of the intermediate ratios because “as much as a ratio of triple exceeds sesquialterate (the ratio of 3 to 2), by so much does the ratio of subduple fall short.”[274]

Because it is not clear how the continuous ratio conception is to be used in the subtraction of ratios in actual calculations of values, some of these commentators added directions for the subtraction of ratios. One of the sets of notes, which incidentally suscribes to the continuous idea of compound ratio, describes how to arrange the known quantities (two above two others) and how to multiply “in the form of a cross as the *Algorismus proportionum* teaches.”[275]

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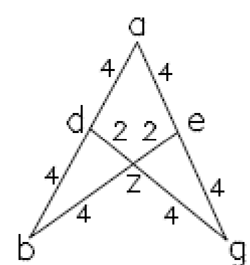
[272] Ibid., lines 144-5.

[273] Appendix B, HK edition, lines 36-50.

[274] Appendix B, S edition, lines 22-3.

[275] Appendix B, HK edition, lines 47-8. The *Algorismus proportionum* that this note references is probably that of Nicole Oresme. Like this commentator, Oresme defined compounding by

Others give directions for finding an unknown term when the other five in a statement of composition are known. For example, one set of notes gives a proof of how the unknown sixth term can be found through using the process of finding the fourth proportional twice.[276] And, another set of notes gives a similar set of rules depending on where in the statement of composition the unknown term appears, along with a "proof" by numerical example; e.g., if the sixth term is unknown, "Multiply the first by the fourth, divide by the third, and there will result something which has a ratio to the second as the fifth to the sixth... Take the fourth proportional of these."[277] This last commentator claims, "This [process] is [what it means] to subtract a ratio from a ratio."[278] This claim that a series of acts of multiplication and division is a type of subtraction means that he is not taking "subtraction" in anything like its normal use of the word.



Some of the sets of notes give numerical examples to explain the sector figure.[279] Instead of relying solely upon proofs, these notes attempt to show the truth in another manner that makes the difficult propositions more easily

continuity but also described the multiplication and divisions needed to use compounding in calculation. See Edward Grant, "Part I," for an English translation, or for an edition of Part I see Grant, "The Mathematical Theory." For an edition of the entire work see E. L. W. M. Curtze, ed., *Der Algorismus proportionum des Nicolaus Oresm. Zum ersten Male nach der Lesart der Handschrift R.4°.2. der Königlichen Gymnasial-Bibliothek zu Thorn*, (Berlin: S. Calvary & co, 1868).

[276] Appendix B, NX edition, lines 558-566.

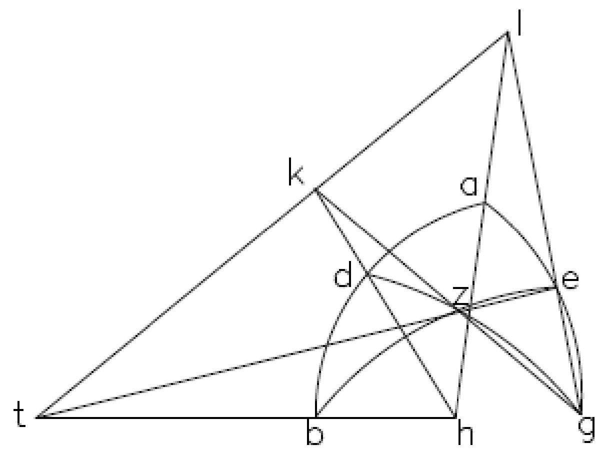
[277] Appendix B, S edition, lines 162-4.

[278] Ibid., lines 185-6.

[279] Appendix B, HK edition, lines 124-142; S edition, lines 11-2, 36-7.

understandable and memorable. While medieval scholars were interested in scientific knowledge derived deductively, there were also strains of thought that placed an emphasis on experience of mathematical objects. Perhaps because of this way of thinking, a mixture of universal, theoretical proofs with particular examples given in notes and in diagrams is found in many medieval mathematical works.

Another aspect of the *Almagest* that prevents it from being a systematic treatment of astronomy is that Ptolemy merely states that the conjoined spherical sector figure can be proved but he gives no proof. A few of the notes attempt to remedy this situation by giving an enunciation for it, and one gives a proof taken or derived from the *Almagestum parvum* (which again will be discussed later).[280] Even when the proof is not given, the diagram for the proof of the conjoined sector figure is sometimes given.[281]



That information about the sector figure was brought in from other sources is clear. Although Gerard does not use the phrases "sector figure," "catha disiuncta," or "catha coniuncta," several of commentators note that these are the

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[280] Appendix B, HK edition, lines 279-304 has an enunciation and proof while an enunciation only is found in NX edition, lines 370-6.

[281] E.g., MS K.

names for the plane and spherical versions of the Menelaus Theorem and some explain the reason behind the names.[282] Curiously, although at least one commentator knew of Gebir's work, no mention of his alternative theorems is to be seen in the margins of the *Almagest* manuscripts.[283]

While this is only a brief introduction to the marginalia of the *Almagest* manuscripts, a few intriguing points emerge. At least some of the sets of notes seem to have pedagogical purposes. The commentators' explanations appear as attempts to fill in some of the logical gaps that are to be found in the *Almagest* since Ptolemy did not attempt to write a wholly axiomatic and deductive astronomy. While several commentators attempted to explain Ptolemy's use of compound ratio, we have seen that there was not consensus on a definition of compounding. It is also clear from these manuscripts that many medieval scholars read the *Almagest* with ideas of compound ratio and of the Menelaus Theorem gained from other works, although the *Almagest* was one of the main ways by which the theorem was initially introduced to Latin scholars. In the sets of notes, more interest is shown in the calcalatory and algorismic aspects of compound ratios.

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[282] E.g.,Appendix B, F edition, lines 138-9; HK edition, lines 33-4, 90-2; NX edition, lines 411-3.

[283] Appendix B, NX edition, line 109.

Chapter 2: Campanus' *De Figura Sectore* and *Almagest* Commentary

Campanus de Novara (born early twelfth cent., died 1296) is among the few commentators on the *Almagest* who are known to us.[284] The chaplain of several popes, he wrote several works on mathematics including a version of the *Elements* with much commentary, a *Theorica planetarum*, works on astronomical instruments, and his *Computus maior*. While he does treat compound ratio in his version of the *Elements*, he deals more extensively with it and with the sector figure in a work on compound ratio and the sector figure and a set of notes giving a commentary on the *Almagest*.

De Figura Sectore

Although the work on compound ratios and the sector figure was divided into two portions which were often transmitted individually, it is fairly clear that they were both written by Campanus and were originally meant to be together.[285] The first portion defines terms related to compound ratio, proves four statements about compound ratio, and then lists and proves the valid modes. Among the definitions, we find a definition of denomination as the quotient of the division of

[284] For information on his life, see F. S. Benjamin and G. J. Toomer, eds., *Campanus of Novara and Medieval Planetary Theory: Theorica Planetarum*, Madison, University of Wisconsin Press, 1971; and G. J. Toomer, "Campanus of Novara," *Complete Dictionary of Scientific Biography*, vol. 15, Detroit; Charles Scribner's Sons, 2008, p. 70.

[285] Lorch gives the most complete argument for Campanus' authorship and the unity of the "two" works in *Thabit*, pp. 426-433. The first half of the work, on compound ratio and the modes was edited by Busard in "Die Traktate," pp. 213-222. An edition of the second half made by Lorch is found in *Thabit*, pp. 436-442.

the antecedent of a ratio by the consequent. Campanus then defines what it means to compound and divide ratios: “That a ratio is produced or composed of ratios is that the denomination is produced from the denominations. That a ratio be divided by a ratio or for a dividing [ratio] to be cast out from [a ratio] that is to be divided is for the denomination of the [ratio] that is to be divided to be divided by the denomination of the dividing [ratio].”[286] Among the theorems, Campanus proves that if a quantity is placed between two original quantities, then the ratio of one of these two quantities to the other is composed of the ratio of the first of these original quantities to the one placed between and of the ratio of that “middle” to the second original quantity. The proof hinges upon showing that the product of the multiplication of the two ratios’ denominations is the denomination of a third ratio. Because of the way he has defined compounding, that means that the third ratio is the ratio composed of the other two ratios.[287] Similarly he also shows that if more than one middle is placed between two extremes, the ratio of the extremes is composed of the ratios of the first extreme to the first middle, of the first middle to the second middle, etc.[288] Through the denominative method of compounding, he has proved that continuous ratios involve compounding.

[286] Busard, “Die Traktate,” p. 213. My translation. “Proportionem <autem> produci aut componi ex proportionibus est denominationem produci ex denominationibus. Proportionem dividi per denominationem aut dividentem abiici ex dividenda est denominationem dividende divide per denominationem dividentis.”

[287] Ibid., pp. 213-4.

[288] Ibid., p. 214.

Similar to the way that Thabit proceeded, Campanus enumerates the possible combinations of six terms, proves the valid modes that can be derived from a statement of composition, and proves the invalidity of other combinations. His proofs of the validity of the eighteen modes rely upon the placement of quantitites between others, but unlike Thabit, he has used the denominational understanding of compounding to prove that the interposition of quantities leads to a statement of composition.

The second half of Campanus' work addresses the sector figure. After two lemmas about the equality of the chords of the doubles of certain arcs,[289] he addresses the disjoined spherical sector figure. He points out that there are three different cases of the theorem depending on how and whether certain lines meet, and he proves the two cases that Ptolemy did not prove. Although he describes the formation of a plane sector figure for the case in which the arcs that correspond to Ptolemy's arcs *ad* and *hb* meet on the side of points *a* and *h*, his proof does not rely upon the plane sector figure. Instead, he proves it as Thabit does, i.e, by completing semicircles and forming a new spherical sector figure.[290] His proof of the case in which lines *ad* and *hb* (or their corresponding lines) are parallel is similar to Thabit's although he does give a more detailed explanation of why all

[289] The first proves that an arc and its supplement have the same chord of their doubles. The second proves that if a line cuts a circle parallel to a diameter, then the chord of the double of the arc from one endpoint of the diameter to the near end of the other line and the chord of the double of the arc from that endpoint of the diameter to the other endpoint of the other line are equal. Lorch, *Thabit*, pp. 436-7.

[290] See pp. 65-6 above.

three parallel lines in the figure are parallel.[291] He also uses the disjoined sector figure to prove the conjoined in one universal proof as Thabit does.[292]

Authorship of the *Almagest* Commentary

Campanus' commentary on the *Almagest* is found in a set of notes in two manuscripts.[293] These notes are attributed to Campanus; each note is preceded or followed with a reference, "Campanus."[294]

Any possible doubt about the attribution is removed by references in two notes. In the first of these, which is on the spherical sector figure, the author points out that Ptolemy proves neither the conjoined sector figure nor two of the three cases of the disjoined. He then adds, "And thus Thabit made one treatise which is titled 'Thabit de figura sectore,' in which he proves all these. I also wrote another treatise about the same, I think more clearly and evidently."[295] This is almost certainly a reference to the *De figura sectore*. While the attribution of *De figura sectore* to Campanus has been fairly well established, this reference to it in a note

[291] See pp. 67 above. [crosscheck]

[292] See pp. 67-8 above. [crosscheck]

[293] Città del Vaticano, Biblioteca Apostolica Vaticana, Barb. lat. 336 (MS N); and Paris, Bibliothèque nationale de France, lat. 7256 (MS X)

[294] The name is abbreviated in most instances, but is clear. The references were noticed by Theodore Silverstein, *Medieval Latin Scientific Writings in the Barberini Collection: A Provisional Catalogue*, (Chicago: University of Chicago Press, 1957), p. 102: "In addition text is heavily glosed [sic] in marg. (esp. through dictio 3a, fol. 72) with extensive passages from Campanus (many marked Campanus, Camp., or C): see, e.g., fols. 15-21, 26-29, 33v, 40-43."

[295] Appendix B, NX edition, lines 365-7.

attributed to Campanus makes it even more certain. In the other note, which is in Book II, Campanus cites the "book on ratio and the combinations of ratios which I wrote."[296] This is clearly the treatise on compound ratio and the valid modes that has been attributed to Campanus. These references confirm that Campanus is the author of these three works, as it is highly doubtful that three different works written by one person would have each been linked falsely to Campanus. However, it is not clear that this set of notes is not merely a set of excerpts from a longer, unknown commentary on the *Almagest* by Campanus.

General Description

While the Campanus notes cover many textual, physical, mathematical concerns, there are only two on the geometry of the lemmas and one on the sector figure. Campanus notes that Ptolemy only proves one of the three cases of the disjoined sector figure and none of the cases of the conjoined. While he does not give any proofs of these unproved cases, he references Thabit's and his own works that contain these proofs.[297] The Campanus proofs have a general enunciation, often the phrase "verbi gratia," a setting up in particular terms (i.e. setting up

[296] Ibid., lines 1107-8.

[297] Ibid., lines 365-7.

lettered quantities), a statement of what he is trying to prove in terms of the lettered quantities, the proof, and the phrase "quod est propositum."[298]

The relation of these notes to the other notes in manuscripts NX is unclear. Interestingly, he provides a reference to Ptolemy's proof of the sector figure as "the 15th of the first."[299] This matches the numbering found in another set of notes that is similar to the *Almagestum parvum*. Campanus could have had these notes as he wrote, or the reference could have been added by someone else and incorporated into the text, or possibly that numbering of propositions of the *Almagest* could have existed in other works. One of the notes in this other set of notes has the attribution to Campanus, but it seems to be a mistake. The note is clearly part of the set of numbered notes—it is given a number that fits with the surrounding notes and its style is similar to the other notes in this collection. On the other hand, the style of this note (and of all the numbered notes) is rather different than the style of the other notes marked as Campanus', and this note is essentially repeated in one of the notes that is clearly in the Campanus set. This suggests that the attribution of this one note to Campanus is a mistake made by a scribe who may have taken an "etc." mark as a capital C for "Campanus" as he copied the marginalia of another *Almagest* manuscript. Until a more careful examination of these two sets of notes

[298] This matches the format of Campanus' version of the *Elements*, which rarely uses the phrase "verbi gratia" but ends almost every theorem with "quod est propositum."

[299] Appendix B, NX edition, line 1099.

can be made, we should consider these as two sets of notes by two different authors.[300]

Ratios and Compounding

While Campanus does not give much attention in these notes to the geometry of the sector figure and its preliminaries, he uses the Menelaus Theorem, its preliminaries, and its applications as opportunities to give a detailed treatment of ratios and especially of compound ratios. Campanus gives notes on how to find missing terms given ratios and some terms. In a note on the fourth lemma to the spherical sector figure, he shows when given a known quantity divided into two and the known ratio of those unknown parts, that there is a way to find the parts.[301] Later, he directs the reader how to find an unknown term in a known ratio when the other term is known.[302]

He provides definitions of compounding and dividing of ratios that are similar to the definitions in the treatise on proportion. In the *Almagest* notes, he writes:

> Proportionem produci ex proportionibus est denominationem produci ex denominationibus. Proportionem componi vel aggregari ex proportionibus est ipsam produci ex componentibus. Proportionem dividi per proportionem

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[300] Ibid., lines 1054-1070.

[301] Ibid., lines 315-323.

[302] Ibid., lines 437-441.

est denominationem dividi per denominationem. Proportionem abici ex proportione est proportionem dividi per proportionem."[303]

These are almost identical to definitions 3 and 4 of the treatise on proportions, which read, "Proportionem produci aut componi ex proportionibus est denominationem produci ex denominationibus. Proportionem dividi per proportionem aut dividentem abiici ex dividenda est denominationem dividende dividi per denominationem dividentis."[304] Campanus' definition of compounding in the *Almagest* notes adds that "aggregari" is also a synonym for "componi" and "produci." While one definition is given in the treatise on proportions for "dividi" and "abiici" and here two are given, the difference is not significant since "casting out" a ratio from a ratio is defined as dividing a ratio by a ratio, which is defined as dividing a denomination by a denomination, which agrees with the treatise on proportions.

Campanus uses denominations to explain how in the first two lemmas to the sector figure Ptolemy is able to reach a statement of composition by inserting a middle between two quantities. Campanus' proof is conceptually identical with one found in the treatise on compound ratios and the modes that has been attributed to Campanus. The terms *b* and *c* have been switched, but the other letters are the same. The terminology is very similar, but not identical. In the treatise on proportions, it reads:

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[303] Ibid., lines 386-391.

[304] Busard, "Die Traktate," p. 213.

> Duobus quibuslibet interposito medio, cuius ad utrumque eorum duorum fit aliqua proportio, componetur primi ad tertium ex primi ad medium et medii ad tertium proportionibus. Sit enim inter *a* et *c* *b* medium sitque ipsius *b* ad utrumque eorum aliqua proportio, erunt ergo ex prima diffinitione *a b c* eiusdem generis quarum per eandem inter *a* et *c* erit aliqua proportio, dico ergo eam componi ex ea que est *a* ad *b* et ex ea que est *b* ad *c*. Sit enim *d* denominatio eius que est *a* ad *b* et *e* eius que est *b* ad *c*, *f* vero eius que est inter *a* et *c*, quia ergo ex *f* in *c* fit *a* et ex *e* in *c* fit *b* per primam propositionem, erit *f* ad *e* ut *a* ad *b* quare *d*, cum sit denominatio *a* ad *b*, erit etiam denominatio *f* ad *e* quare per eandem ex *d* in *e* fit *f*, quia ergo denominatio *a* ad *c* producitur ex denominatione *a* ad *b* et ex denominatione *b* ad *c* erit per tertiam diffinitionem *a* ad *c* composita ex *a* ad *b* et *b* ad *c*.[305]

The Campanus note in these *Almagest* manuscripts reads:

> Supponit hanc propositionem hic et infra in multis locis que est quasi quedam conceptio. Si inter quaslibet duas quantitates eiusdem generis, alia quantitas quantalibet eiusdem generis ponatur media, erit proportio prime ad ultimam composita ex proportione prime ad secundam et secunde ad tertiam. Sint due quantitates eiusdem generis *a* et *b* inter quas ponatur *c* eiusdem generis. Dico quod proportio *a* ad *b* componitur ex proportione *a* ad *c* et *c* ad *b*. Quod sic probatur. Sit *d* denominatio proportionis *a* ad *c*, et *e* denominatio proportionis *c* ad *b*, et *f* *a* ad [*b*]. Quia ergo ex *b* in *f* fit *a*, et ex *b* in *e* fit *c*, erit *a* ad *c* ut *f* ad *e*. Igitur cum *d* sit denominatio *a* ad *c*, erit denominatio *f* ad *e*. Quare ex *e* in *d* fit *f*, quod est propositum.[306]

While the same terminology is used throughout, the phrasing is also very similar from the sentence starting with "Sit (enim) *d* denomination…" In its content, this note falls clearly in the denominative group. Because of his idea of compounding, Campanus feels the need to prove what Ptolemy takes "as a certain axiom." This comment seems to indicate that Campanus realized that he and Ptolemy had different ideas of compound ratio; Ptolemy takes the insertion of a middle to create

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[305] Busard, "Die Traktate," pp. 213-4. I have removed some words that Busard added in brackets.

[306] Appendix B, NX edition, lines 257-267.

compound ratio as a principle, while for Campanus it is something to be proved. Also, Campanus' denominative conception of ratios is apparent when he thinks that he has reached the desired conclusion when he finds that *e*, the denomination of *c* to *b*, multiplied by *d*, the denomination of *a* to *c*, makes *f*, the denomination of *a* to *b*. Because of his understanding of compounding as the multiplication of denominations, saying this is the same as knowing that the ratio of *a* to *b* is composed of the ratios *a* to *c* and *c* to *b*. Campanus goes on to prove several other statements about compound ratio. Two are of special interest since they prove the validity of operations (multiplication and division) to compound and divide ratios. The first is, "A ratio compounded from any number of ratios is that of the product of all the antecedents to the product of the consequents."[307] He proves that this is true in the case when a ratio *a* to *b* is composed of two ratios *c* to *d* and *e* to *f*. He continues:

> And let *c* be multiplied by *e* and *g* is made, and let *d* be multiplied in *f* and *h* is made. I say that the ratio of *a* to *b* is between *g* and *h*. For let *d* be multiplied into *e* and *l* results. Therefore, because from *c* and *d* in *e*, come *g* and *l*, *g* will be to *l* as *c* to *d*. And because again from *d* in *e* and *f* comes *l* and *h*, *l* to *h* will be as *e* to *f*. Therefore, *g* to *h* is composed of *c* to *d* and *e* to *f*, but also *a* to *b* [is composed of the same ratios] as was given. Therefore *a* to *b* is as *g* to *h*, which was proposed.[308]

Using this proof, Campanus proves the proposition for the case of three composing ratios and explains how one can thus prove it for however many composing ratios there may be. Interestingly, the crucial step in this theorem is seeing that the ratios

[307] Appendix B, NX edition, lines 391-2.

[308] Ibid., lines 394-399.

g to *l* and *l* to *h* compose the ratio *g* to *h*. Although Campanus defines compounding by the multiplication of denominations, his proof relies not directly upon that but upon the insertion of a middle to reach a compound ratio. He has already proved that inserting middles produces compound ratios, and proving by using this proposition is more in line with the similar proof in the *Elements* VIII.5; however, converting the basic enunciation into Campanus' understanding of denominations and compounding, one expects him to show that the quotient of the product of *c* and *e* divided by the product of *d* and *f* (i.e. the denomination of the ratio of the products of the antecedents and consequents) is equal to the product of the quotient of *c* divided by *d* and *e* divided by *f* (i.e. is equal to the product of the denominations of the composing ratios).[309] The study of quotients or of fractions, which are conceptually similar, were usually not seen as belonging to strictly theoretical mathematics, so approaching the problem in this way would be to strike out onto untrodden territory by bringing algebraic or algorismic conceptions into theoretical mathematics—a step that Campanus was apparently not prepared to take.

Campanus' second proof about operations is, "The ratio which remains with however many ratios having been subtracted from one is between the product of the antecedent of that from which they are to be subtracted and the consequents of all those being subtracted and the product of the consequent of that from which they

[309] Put into more modern language, he is trying to show that $\frac{ce}{df} = \frac{c}{d} \cdot \frac{e}{f}$ which follows immediately using basic algebra, but which is not immediately obvious in the medieval, non-algebraic formulation.

are subtracted and the antecedents of all those to be subtracted."[310] In particular terms, given the ratio *c* to *d* subtracted from *a* to *b*, he shows that the remaining ratio is found between the product of *a* and *d* and the product of *b* and *c*, which products he renames *e* and *f*. He multiplies *b* by *d* and calls the product *g*. Because *a* and *b* are multiplied into *d* to reach *e* and *g*, *e* to *g* is as *a* to *b*. Similarly, *f* is to *g* as *c* is to *d*. The ratio of *e* to *g* is composed of *e* to *f* and *f* to *g*, so *e* to *g* is composed of *e* to *f* and *c* to *d*. The ratio *a* to *b* is as *e* to *g*, so *a* to *b* is composed from *c* to *d* and *e* to *f*. Therefore, *e* to *f* is the remaining ratio when *c* to *d* is taken from *a* to *b*. He goes on to prove the proposition when there are two ratios subtracted. Again, the proofs for this proposition rely upon the continuous ratio property of compounding (I say "property" since it is not the fundamental definition of compounding).[311]

Campanus then gets into even more practical issues of how to find an unknown ratio in a statement of composition. After all, this is how Ptolemy uses the results of the Menelaus Theorem to find unknown values in the *Almagest*. Campanus writes:

> If any known ratio is composed of two of which one is known, the other will be known. Let known [ratio] *a* to *b* be composed of known [ratio] *c* to *d* and unknown [ratio] *e* to *f*. And let *a* be multiplied by *d* and *g* results, and *b* by *c* and *h* results. And the ratio *g* to *h* will be known. And because it is as *e* to *f*, because the same [ratio] is composed from it and *c* to *d* as that from *e* to *f* and *c* to *d*, *e* to *f* will be known, which is what we wanted.[312]

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[310] Ibid., lines 415-8.

[311] Ibid., lines 419-429.

[312] Ibid., lines 431-5.

He then shows how to find a missing term in that ratio which was just found:

> If of any two [quantities] of which the ratio is known, one is known, then the other will be known. Let there be between *a* and *b* a known ratio and let *a* be known. And let the least numbers in the ratio of *a* to *b* be *c* and *d*. Because therefore *a* to *b* is as *c* to *d*, the product of *a* and *d* of which each is known is equal to the product of *c* and *b* of which *c* is known. Therefore, with that [*ad*] divided by *c*, comes forth *b*.[313]

While he has shown how Ptolemy's process of subtracting a known ratio from an unknown ratio works, Campanus provides rules for directly finding an unknown term in a statement of composition. He then gives five different ways (he gives two, then divides the second into four methods) for finding the unknown sixth term in a statement of composition:

> When any known ratio is composed of one known and the other unknown of which unknown ratio one term is known, to find that unknown ratio and its unknown term. Let the ratio of *a* to *b* be composed of *c* to *d* and *e* to *f*. And let the two first ratios and quantity *e* be known. And let it be proposed to find the ratio of *e* to *f* and quantity *f*. And this is done in two ways. For with the products *g* and *h* from *a* multiplied by *d* and *b* multiplied by *c*, *g* will be to *h* as *e* to *f*. Therefore, with *e* muliplied by *h* and the product divided by *g*, *f* results, and thus the proposition is established.
>
> Another way is that the product of *b* and *c*, which is *h*, is divided by *a* and *l* results. Therefore, because from *a* the first in *l* the fourth comes *h*, and again from *b* the second in *c* the third comes the same, *a* will be to *b* as *c* to *l*. Therefore *c* to *l* will be composed of *c* to *d* and *e* to *f*. But that is also composed of *c* to *d* and *d* to *l*. Therefore *d* to *l* is as *e* to *f*. Therefore, with *e* multiplied by *l* and the product divided by *d*, *f* results, which was proposed.
>
> But it is necessary to pay attention because that second way is able to be done in four ways....[314]

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[313] Ibid., lines 437-441.

[314] Ibid., lines 443-457.

In the outline of these ways of finding the unknown quantity, Campanus does not only relate the order of multiplication and division to follow, but he gives the theory as well. In fact as he gives the other ways of finding the unknown, he drops out the rules and assumes his readers can reach conclusions themselves. The first of these four ways of the second way has already been given. The second is to find a quantity *m* such that *a* is to *b* as *m* is to *d*. *M* to *d* is then composed of *c* to *d* and *e* to *f*, but it is also composed of *m* to *c* and *c* to *d*; therefore, *m* to *c* is as *e* to *f*. *F* can then be found. The others are similar. Campanus concludes by listing the ratios that each of the last four ways uses to find the ratio of *e* to *f*. Putting everything in terms of the six quantities of the original statement of composition, they are in modern formulation:

Method 2A) $d:(bc/a)::e:f$

Method 2B) $(ad/b)/c::e:f$

Method 2C) $(ad/c):b::e:f$

Method 2D) $a:(bc/d)::e:f$.

Finding the rules for finding a fourth proportional, these could easily have been turned into directions for the reader.

In a note in Book II, Campanus explains an instance when Ptolemy deviates from his normal treatment of compound ratios. Given a statement of compounding, Campanus writes, “The ratio of the third to the fourth will be composed of the ratios of the first to the second and the sixth to the fifth through the book about ratio and the combinations of ratios which I composed."[315] He explains that Ptolemy uses this mode because the unknown quantity is in the composed ratio, not in a composing ratio as it is in his normal *modus operandi*, and he adds, "If he wanted to compound those two known [ratios] and have that [ratio] known which is between *tz* and *ht* of which one term—namely *tz*—is known, the remaining [term] would be known, but in this way the work would be more difficult and lengthy."[316] Using a known valid mode and subtracting is easier for Ptolemy than compounding or adding ratios.

While Campanus' understanding of the Menelaus Theorem and of compounding is found in both the *De figura sectore* (and the accompanying treatise on proportions) and his commentary on the *Almagest*, we have seen that parts of these works are almost identical; however, since the subject matter of the two works diverges, neither work can be merely a variant of the other, but clearly one was consulted in the writing of the other. Campanus offers a treatment of the Menelaus Theorem that is similar to Thabit's. Interestingly, he proves the Menelaus Theorem only in the *De figura sectore*, not in the set of notes. In both works, we find that he adds a new theoretical basis to Thabit and Ptolemy's use of compound ratios. While they seem to understand compounding by the insertion of middles, Campanus sees the insertion of middles as a secondary feature of

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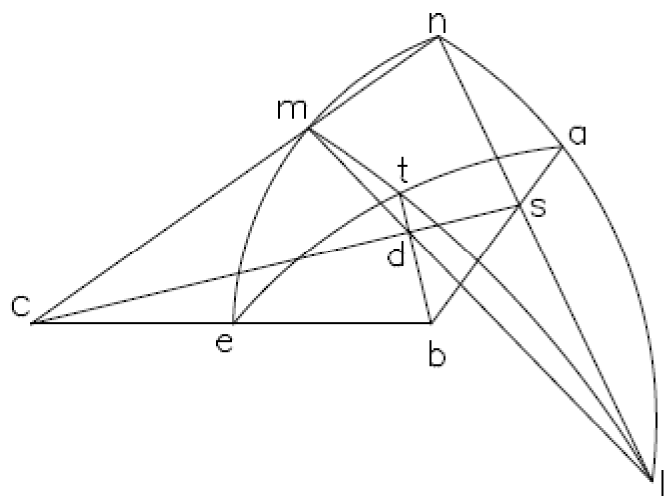
315 Ibid., lines 1107-8.

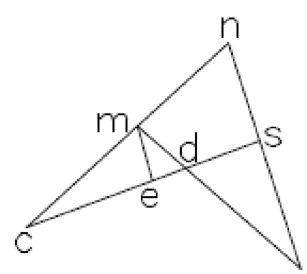
316 Ibid., lines 1114-7.

compounding. In his view, compounding is fundamentally the multiplication of ratios. While the theoretical understanding of compounding is the same in both works, we gain some insight from expanding our investigation to the commentary on the *Almagest*. We learn that he does not merely focus on the theoretical bases of compounding, but that he was also concerned with justifying the practice and operations of finding unknown values from known values.

Chapter 3: The Additions to Menelaus' *Sphaerica*

There are several notes that were added to Menelaus' *Spherica* in the middle ages.[317] Many were written by Campanus de Novarra.[318] They seem to have originally appeared in the margins, but in one family of texts they have been transferred into the main text with no marks that they were added.[319] These additions explain various steps of Menelaus' proof. The first more clearly states how points *s* and *d* are determined.[320] The second refers to Euclid XI.1 to argue that point *c* must be in the plane of circle *ate*.[321] The third proves the plane sector figure that Menelaus assumes.[322] The diagram and proof are similar mathematically to the first case of the





[317] See Part I, ch. 2 above.

[318] Björnbo, "Studien," pp. 152-4.

[319] Città del Vaticano, Biblioteca Apostolica Vaticana, Reg. lat. 1261 (MS R) and Città del Vaticano, Biblioteca Apostolica Vaticana, Vat. lat. 4571 (MS Y) have these notes in the main text; however, Milan, Biblioteca Ambrosiana, Q 69 sup. (MS O); Wien, Österreichische Nationalbibliothek, 5277 (MS W); and Città del Vaticano, Biblioteca Apostolica Vaticana, Pal. lat. 1351 (MS X) have them in the margins. Schweinfurt, Stadtbibliothek, H 81; Milan (MS L); and Venezia, Biblioteca Nazionale Marciana, Marc. lat. VIII 32 (MS M) have some of the notes in the margin and some in the text.

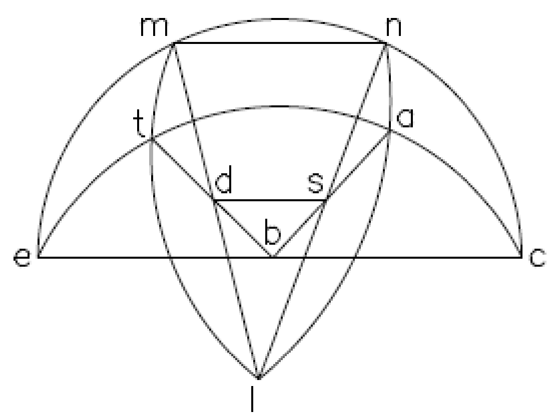
[320] Appendix C, lines 12-5.

[321] Ibid., lines 22-6.

[322] Ibid., lines 33-43.

disjoined sector figure in Ametus' *Epistola*—unlike Ptolemy's proof which starts by constructing a line from point *n* parallel to *cs*, Ametus and this commentator draw line *me* (note that the points marked *e* in this figure and the previous one are not the same point) from point *m* parallel to line *nl*.[323] This note is also remarkable in that it contains one of the rare personal addresses to the reader to be found in the medieval texts containing the sector figure. The author writes, "Therefore it will be clear to you, if you are not sleeping, that . . ."[324]

In the fourth note, the commentator provides a detailed proof that perpendiculars dropped from points *n* and *l* to line *ab* are in the same ratio as *ns* to *sl*. The bulk of the proof shows that the perpendiculars cannot both fall on the same side of point *s*. Once that is settled, the proposition is clear from similar triangles.[325]



In the sixth note, Campanus explains why in the parallel case of the disjoined sector figure, line *ec* must be parallel to both *mn* and *sd*. This commentator provides one of the few proofs of the parallel case that shows clearly why if there are three lines of which two are parallel and there is another line that is

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[323] Schrader, pp. 211-3.

[324] Appendix C, line 34 and apparatus.

[325] Ibid., lines 52-68.

in a plane with one of the two lines and in a plane with the other, then this third line must be parallel to each of the original two. His proof is a *reductio ad absurdum* that is conceptually the same argument as is in his *De figure sectore*.[326] He starts with the assumption that the third line, here *ebc*, meets each of the two other lines, here *mn* and *sd*. He first assumes that *bc* meets the other two lines at the same point. These two lines then meet, which goes against the assumption that they are parallel. If *ebc* meets them at different points, then the line that connects these two points must be in the same plane with lines *mn* and *sd*. This line is part of line *ebc*, but line *ebc* was assumed to not be in the same plane shared by both of these lines. Therefore *ebc* must be parallel to both line *mn* and *sd*.[327]

The last of Campanus' notes is on compound ratios and the explanation of compounding with a ratio of equality. In it, Campanus writes:

> Because according to what Jordanus defines in the comment of the eighth proposition of the ninth book of his *Arithmetica*, multiplying one ratio by another is nothing other than multiplying that which denominates one of these ratios by that which denominates the other, and because the nadir of arc *ne* is equal to the nadir of arc *me*, therefore [the number] one denominates the ratio of the nadir of arc *ne* to the nadir of arc *me*. Moreover, one multiplied by anything produces nothing except that by which it was multiplied.[328]

He is explaining compound ratio through the multiplication of denominations.

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[326] Lorch, *Thabit*, pp. 439-440.

[327] Ibid., lines 97-112.

[328] Ibid., lines 142-8.

Another addition that was not written by Campanus affected the way that Menelaus III.1 was understood by readers. In many manuscripts of the *Sphaerica*, the text is immediately followed by a fragment translated from Arabic by Gerard of Cremona.[329] This passage explains some aspects of compound ratios. It consists of the proofs of three statements:

> That of any three proportional lines, the ratio of the first to the second doubled is the ratio of the first to the third…. [and] that of any three lines, in whatever manner they may be, the ratio of the first to the third is as the ratio of the product of the first multiplied by the second to the product of the second multiplied by the the third.[330]
>
> That of any three lines in whatever way they are, the ratio of the first to the third is as the ratio of the first to the second multiplied by the ratio of the second to the third.[331]
>
> That of any four proportional lines following one another according to one ratio, the ratio of the first to the fourth is as the ratio of the first to the second tripled.[332] …

The proof of the second of these three propositions is similar to Campanus' proof that the interposition of a quantity produces a statement of composition, but this

[329] This fragment was also joined to the *Liber de triangulis Iordani* as Propositions IV.26-8. Marshall Clagett, *Archimedes in the Middle Ages, Vol. 5: Quasi-Archimedean Geometry in the Thirteenth Century, Parts I-III*, (Philadelphia: The American Philosophical Society, 1984), discusses this fragment on pp. 329-331, provides edition of it on pp. 425-9, and gives a translation on pp. 476-7.

[330] Ibid., p. 425. My translation. "Volo ostendere quod omnium trium linearum proportionalium proportio prime ad secundam duplicata est proportio prime ad tertiam. Quando ergo volumus ostendere illud, ostendemus quod omnium trium linearum, quocumque modo sint, proportio prime ad tertiam est sicut proportio aggregati ex multiplicatione prime in secundam ad aggregatum ex multiplicatione secunde in tertiam."

[331] Ibid., p. 426. My translation. "Et quia iam ostendi hanc propositionem, tunc ostendam iterum quod omnium trium linearum, quocumque modo sint, proportio prime ad tertiam est sicut proportio prime ad secundam multiplicata in proportionem secunde ad tertiam."

[332] Ibid., p. 428. My translation. " Quod omnium quatuor linearum consequentium secundum proportionem unam proportio prime ad quartam est sicut proportio prime ad secundam triplicata."

proof is geometrical. In it, the normal divisions between discrete and continuous quantity are blurred, as is the distinction between ratios and quantities; lines are multiplied by lines, ratios are identified both with lines and with the act of division, and ratios are multiplied by each other.[333] While ratios and quantities were sometimes conflated in Latin works, few, if any, go so far as to describe quantities as lines (while ratios were often represented in diagrams by lines, they were not said to be lines) or as the operation of division.

Although Menelaus does not explain compound ratios in the *Sphaerica*, many medieval readers of the work would have understood his use of compounding as being fundamentally about the multiplication of ratios since that is the understanding of compounding that is found in the additions to the work. The added notes that were often transmitted with could dramatically alter the way in which a text was read.

[333] Ibid., p. 427, contains the clauses "quoniam proportio est divisio," "ponam proportionem que est GD ad ET lineam T'," and "quod proportio … est sicut proportio … multiplicata in proportionem …"

Chapter 4: The *Almagestum Parvum*

The *Almagestum parvum* is an original Latin work which covers the solar and lunar theory found in the *Almagest*. Although clearly based upon the *Almagest*, it is its own work and not merely a collection of excerpts from the *Almagest*. In fact, it is unclear upon which translation of the *Almagest* it is based.[334] The proofs do not provide much explanatory material, and the proofs are often sparser than those in the *Almagest*. But, the content of the *Almagestum parvum* is more general. The work is divided into books that match material from the first six books of the *Almagest*, and its order of theorems is very close to that of Ptolemy. The work does not attempt to paraphrase the remainder of the *Almagest*, so it does not address the fixed stars or the planets. The author did not rely only upon Ptolemy; he relies heavily upon Albategni (al-Battānī) and also cites Theodosius, Thabit ibn Qurra, Arzachel (al-Zarqālī), and the Toledan tables.[335]

There is no definitive picture of the author or the date of composition. Because this work is described in Richard de Fournival's *Biblionomia*, it must have been completed by around 1250. It was assumed by some, such as Birkenmajer, that the *Almagestum parvum* must have been written after Gerard of Cremona's

[334] The terminology is rather different than that of any of the three Latin versions of the *Almagest* although the diagram letters and the proofs generally follow those of the Gerard of Cremona version.The same diagrams letters are used, so there may be a connection. However, Richard Lorch, "Some Remarks on the *Almagestum parvum*," in *Amphora: Festschrift Für Hans Wussing Zu Seinem 65. Geburtstag = Festschrift for Hans Wussing on the Occasion of his 65th Birthday*, ed. by S. S. Demidov, M. Folkerts, et. al., 407-437, (Basel: Birkhäuser Verlag, 1992.), p. 430 argues, "In general, if one of the GERARD texts is the basis of the *Almagestum parvum*, the compiler must have been at some pains to change the terminology as much as possible."

[335] Lorch, "Some Remarks," pp. 409-10.

translation because it seems to follow it closely, and thus the *terminus post quem* was argued to be the supposed translation date of that work, 1175.[336] Lorch, however, points out that while there are many similarities between the two works, they are different enough that it could have been based on another version that has the same letters for the diagrams. Lorch suggests that perhaps parts of the work and the propositions were "the work of a scholar in the HERMANN—ROBERT circle in the mid-twelfth century and that the treatise was filled out later on the basis of a form of GERARD's translation of the *Almagest*."[337] This timeline is complicated by uncertainty over the 1175 date that has traditionally been given for Gerard's translation of the *Almagest*.[338] North tentatively offered that the *Almagestum parvum* was written around 1085, which is far too early if it is an original Latin work.[339] If it is a translation of an Arabic work, it possibly could be from the late eleventh century since the latest sources it references are the Toledan Tables and Arzachel, who died in 1087, but based on the style, the lack of transliterations, and no knowledge of an Arabic version, it seems more likely that the work was authored in Latin. The manuscripts containing the *Almagestum parvum* attribute the work to Albategni, Geber, Albertus Magnus, Thomas Aquinas,

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[336] Aleksander Birkenmajer, "La Bibliothèque de Richard de Fournival," *Studia Copernicana*, 1 (1970): 118–210, here p. 144; Haskins, *Studies in the History of Medieval Science*, p. 104.

[337] Lorch, "Some Remarks," 434.

[338] LeMay, "Gerard of Cremona," p. 174.

[339] North, *Richard of Wallingford*, Vol. 2, p. 140.

and Campanus.[340] The manuscripts with these attributions are all from the fourteenth and fifteenth centuries, so their attributions are extremely questionable. The attributions to Albert and Thomas are almost certainly nonsense. The *Almagestum parvum* cites Albategni frequently, which would be very strange if he were the author. The attributions to Geber are probably caused by confusion with Geber's *Correction of the Almagest*. There are two other plausible attributions. The *Biblionomia* attributes the work to a "Galterum de Insulla,"[341] and a note in Oxford, Bodleian Library, Ashmole 424 from the fourteenth or fifteenth century, states that Campanus is the author of the *Almagestum parvum*.[342] Birkenmajer examined different Walters that Richard de Fournival could have been referring to and found none of them, including the poet Walter of Châtillon, likely authors. He prefers Campanus, but he and Lorch discount him as the author since they put his writing years later than the *Biblionomia*.[343] Pereira argues that Campanus' career could possibly have started early enough for him to have been the author by around 1250.[344] Evidence that Campanus is not the author is found in two manuscripts of the *Almagest*, Città del Vaticano, Biblioteca Apostolica Vaticana, Barb. lat. 336 and Paris, Bibliothèque nationale de France, lat 7256 (MSS NX), that have

[340] Lorch, "Some Remarks," pp. 416-9. On p. 416 n. 31 Lorch states that he was not able to find the supposed reference to Campanus in Firenze, Biblioteca Laurenziana, Conv. soppr. 414.

[341] "Liber extractionis elementorum astrologie ex libro Almagesti Ptolomei per Galterum de Insulla usque ad finem sexti libri ex eo." Birkenmajer, p. 169.

[342] It reads, "[I]pse [William of Moerbeke] autem socius fuit magistri Campani qui fecit Parvum Almagesti et commentavit Geometriam Euclidis." Birkenmajer, 145.

[343] Birkenmajer, p. 145, Lorch, "Some Remarks," p. 431.

[344] Michela Pereira, "Campano da Novara autore dell'Almagestum parvum," *Studi Medievali* 19 (1978): 769–776.

marginalia including both a set of enunciations apparently derived from the *Almagestum parvum* and a set of notes written by Campanus. The notes attributed to Campanus each are marked with the name "Campanus" or an abbreviation of his name to note that Campanus is the author. The excerpts from the *Almagestum parvum* are not marked in this way although they appear to be written in the same hand, so whoever added these notes consciously did not attribute them to Campanus. Also, although some terminology is common to Campanus' works and the *Almagestum parvum*, the latter does not have the same words and phrases marking the parts of theorems and problems as Campanus' version of the *Elements* or his *Almagest* commentary.[345]

The whole matter is rather inconclusive, and there is not more than the slightest evidence for attributing this work to any author. The issue of whether the work is a translation or not stretches its possible date range to almost two centuries. It seems, however, most likely that this work was written in Latin in either the last quarter of the twelfth or the first half of the thirteenth century—late enough for its sources to be readily available in Latin and early enough to be included in the *Biblionomia*.

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[345] See Appendix H.

Versions and Related Texts

The text of the *Almagestum parvum* is relatively consistent in about twenty of the manuscripts containing it, but some manuscripts contain versions that differ greatly either by only containing excerpts of the normal text or by having text that does not match that of the normal version. There are three manuscripts, Basel, Universitätsbibliothek, F.II.33; Firenze, Biblioteca Riccardiana 885; and Toledo, Biblioteca de la Santa Iglesia, 98-22, that contain the normal enunciations of the *Almagestum parvum*, but give some proofs in entirely different wording. The variant passages in these are unique. Lastly, in four manuscripts, there is a commentary on the first two books of the *Almagest*, which I call the "Erfurt Commentary," that has enunciations that are very similar to those of the *Almagestum parvum*, but that shows little similarity elsewhere to the *Almagestum parvum* in terminology or content.

Enunciations and occasionally proofs from the *Almagestum parvum* are occasionally found written as explanatory marginalia in manuscripts containing the *Almagest*. In the margins of Paris, Bibliothèque nationale de France, lat. 16200 and Erfurt, Universitätsbibliothek Erfurt, Dep. Erf. CA 2° 375, we find excerpts taken directly from the *Almagestum parvum* with few drastic changes. Likewise, Victoria, State Library of Victoria, Australia, Sinclair 224 has a list of enunciations on a flyleaf that are similar in wording to ones from Book I. The enunciations are also found in the marginalia of two other *Almagest* manuscripts, Città del Vaticano, Biblioteca Apostolica Vaticana, Barb. lat. 336 and Paris, Bibliothèque nationale de

France, lat. 7256, but without proofs for some and with different texts for the proofs of others.

Euclidian Style

The most obvious difference between the *Almagest* and the *Almagestum parvum* is that the latter is in a Euclidean style. Indeed the author of the *Speculum astronomiae* describes the work thus: "Ex hiis quoque duobus libris collegit quidam vir librum *secundum stilum Euclidis*, cuius commentarium continet sententiam utriusque, Ptolemaei scilicet atque Albategni, qui sic incipit: Omnium recte philosophantium etc."[346] Lorch takes “according to the manner of writing of Euclid” to mean that it is organized into propositions or enunciations and proofs and also that the propositions are numbered,[347] but there are other similarities to Euclid’s style.

As in the *Elements*, definitions or basic principles precede the theorems in some of the books. For example, the first book starts by listing several statements about the sphericity of the universe and of its motion, the practically infinite smallness of the earth compared to the universe, and the principal motions of the whole universe, the sun, moon, and planets. The author adds, “Faith in these things is to be taken firmly such that if some challenger unjustly opposes this, he is to be

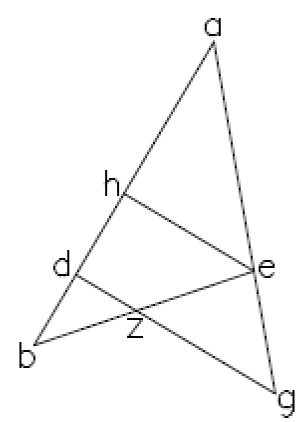
[346] My emphasis. Stefano Caroti, Michela Pereira, and Paola Zambelli (eds.), *Speculum astronomiae*, (Pisa: Domus Galilaeana, 1977), p. 9.

[347] Lorch, “Some Remarks,” 407-8.

not unworthily estimated as a sophist denying the truth knowingly or insane."[348] In other words, these statements are taken as first principles that are unquestioned truths in the discipline and that provide the basis for conclusions.

Moving on to the theorems and problems, the *Almagestum parvum* gives generalized enunciations of propositions, not particularized enunciations as in the *Almagest*. For example, in the seventh proposition, where the author reaches the sector figure material, the enunciation is given in general terms before it is rephrased in the terms of the particular diagram:

> With two straight lines descending from one angle and with two others cutting each other reflected from the remaining endpoints of these descending lines into the same, each of the reflected lines thus cuts the line sharing a point with the other such that the ratio of that pierced line to its part that is above the cutting point is produced from two ratios—from, I say, one ratio which the reflected line sharing a point with it to its part which lies between the intersection and the cutting point and another ratio which the portion of the other reflected line under the intersection has to that whole line of which it is a part. For example, the ratio of line *ga* to *ea* is produced from the ratio of line *gd* to line *zd* and the ratio of line *bz* to line *be*.[349]



By comparison, the *Almagest*'s enunciation is given in terms of the letters in the diagram and not in general terms: "Therefore I will describe the two lines *ab* and *ag* and extend *be* and *ge* cutting themselves at *z* into the area that is between those two lines. Therefore I say that the ratio of *ga* to *ea* is compounded from two

[348] Appendix D, lines 18-20.

[349] Ibid., lines 154-163.

ratios—from the ratio of *gd* to *dz* and from the ratio of *zb* to *be*."[350] While the enunciations of the *Almagest* can be applied to different situations and to figures with different lettering without much difficulty, the author of the *Almagestum parvum* decided to follow the mathematical convention (as in the *Elements*) of first stating propositions generally without referring to any letters assigned to diagrams.

Even sections of the text that are not strictly mathematical are fitted into this Euclidean format. For example, Proposition I.15 is about using instruments to find the maximum declination of the ecliptic, but the author starts this section with an enunciation as if it were a mathematical theorem: "To find the greatest declination through the making and observation of an instrument."[351] The instructions for making the instrument and observing with it are not given in mathematical terms.

The Euclidean feel of the *Almagestum parvum* also comes from its use of words and phrases to define the different parts of each theorem. Unlike the medieval versions of Euclid's *Elements*, the *Almagestum parvum* does not always use these organizational words consistently, but in the first book we find "corollarium," "ratio," "quod erat propositum," "exempli gratia," and "evidentiae gratia."[352] As we will see, many of the commentaries on the *Almagest* systematize the content, which had been presented rather differently by Ptolemy.

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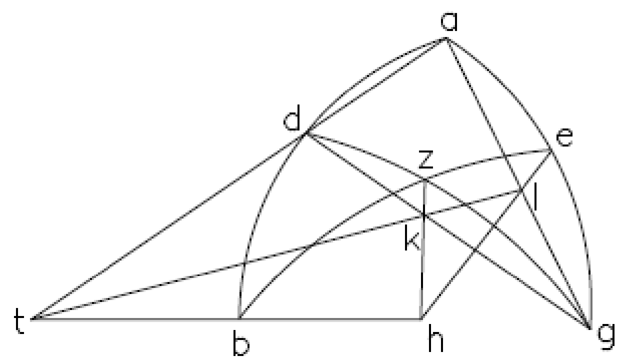
[350] 1515 *Almagest*, 9v.

[351] Appendix D, lines 267-8.

[352] E.g., ibid., lines 23, 29, 66-7, 162, 233.

The Menelaus Theorem

Most of the proofs concerning the Menelaus Theorem do not dramatically differ from those in the *Almagest*. Propositions 7-12, which are the lemmas for the sector figure, and the thirteenth proposition, which is the disjoined sector figure, are proved similarly to the corresponding propositions in the *Almagest*. Unlike Ptolemy, the author gives the propositions in general terms before setting up the diagrams and presenting the enunciation in the diagram's particular terms.

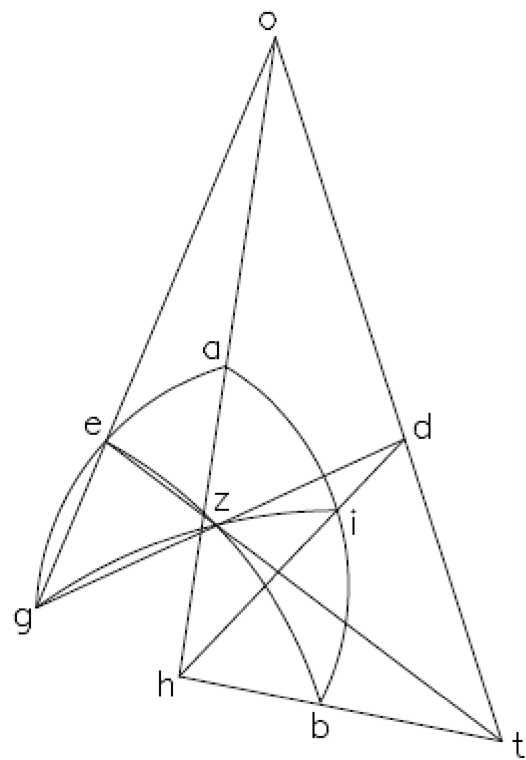


The proofs he gives are very skeletal. In the thirteenth proof, after establishing that the points *t*, *k*, and *l* are in one line that is the section of the plane of triangle *agd* and circle *bze*, the author merely states the rest can be proved by using the disjoined plane sector figure, the ninth theorem of the work twice, and the eleventh once, which agrees with Ptolemy's proof. While this gives the justifications for the remaining steps, these steps are left unwritten. This incomplete outline of a proof is reminiscent of the outlines of proofs that are given in the version of the *Elements* known as "Version II."[353]

While Ptolemy merely states that the conjoined spherical sector figure can be proved, the *Almagestum parvum* has it as a separate proposition and gives the

[353] H. L. L. Busard and Menso Folkerts, (eds.), *Robert of Chester's (?) Redaction of Euclid's Elements, the So-Called Adelard II Version*, (Basel: Birkhäuser, 1992).

proof, albeit in skeleton form with an incomplete construction and even an incorrect diagram in some of the manuscripts.[354] After stating the proposition in general and particular terms, lines are extended from the center of the sphere *h* through points *a*, *b*, and *i* until they meet with the extended chords *ge*, *gz*, and *ez* at points *o*, *d*, and *t*.[355] Points *o*, *d*, and *t* are in a line because they are in the planes of both triangle *gze* and circle *aig*. A difficulty that the author does not mention is that there is a variety of different ways that the points *o*, *d*, and *t* could be positioned depending on how the lines of triangle *gez* are inclined or not towards the plane of circle *abi*. For example, line *gz* could be parallel to line *hi* or they could meet on the other side of the diagram. The author outlines how the rest of the argument can be made with the conjoined rectilinear sector figure and the eleventh proposition of this work. To elaborate, the lines *geo*, *odt*, *tze*, and *gzd* form a



[354] Eg., Città del Vaticano, Biblioteca Apostolica Vaticana, Reg. lat. 1261 (MS F) and Wien, Österreichische Nationalbibliothek 5266 (MS V).

[355] In this proof, it is not clearly stated how points *o*, *d*, and *t* are derived. The author writes, "A sphaerae centro *h* lineae per sectiones circulorum *a b i* educantur donec singulae cum singulis praeter centrum transeuntibus ad notas *o d t* conveniant quas tres notas in eadem esse linea conveniet." The meaning must be that which I have explained, but with a corrupt diagram and this passage, a reader may have struggled to figure this out.

rectilinear sector figure, so through the plane conjoined sector figure, the ratio of *go* to *eo* is composed of the ratio of *gd* to *zd* and of *tz* to *te*. Because of the eleventh proposition, which is Ptolemy's fifth lemma, ratios of the chords of double arcs can replace all three of these ratios in this statement of composition to reach the sought conclusion. Unlike Thabit and Campanus, this author uses the plane conjoined sector figure. Maslama's note had a similar proof, but there are not enough similarities to confirm that this was the source.

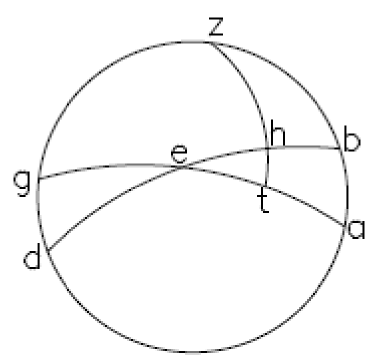
While the *Almagestum parvum* is, as its name suggests, an abbreviation of the *Almagest*, it still is able to cover the same mathematical content as Ptolemy's work by following a rigid structure modeled apparently after Euclid's *Elements* and by giving sparse instructions for how to prove each proposition instead of giving each and every step. It even is able to include some material that Ptolemy skips and still live up to its title.

Application of the Menelaus Theorem

In the applications of the sector figure, the author of the *Almagestum parvum* follows a regular pattern. His format invariably consists of an enunciation, a corollary or rule that gives the sought quantity by multiplying and dividing known quantities, and a general proof that shows how this rule is derived. Ptolemy's practice lacks general enunciation and also general directions; he almost always instructs with specific examples of calculations that serve as patterns for other particular cases a reader will need to perform. The proofs in the *Almagestum parvum* are often very threadbare, but they are almost all given in universal terms, not the particular ones of the *Almagest*.[356] The rules are reminiscent of the rules found in Albategni's *De motu stellarum*, which does use many numerical examples but has clearer and more general rules than the *Almagest* has.[357]

Each time the author moves from a statement of composition given by the Menelaus Theorem to one of these rules, he proceeds according to one of two ways. The first way is exemplified in the Proposition I.16, which shows how to find the declination of any point on the ecliptic. After the construction of the figure, which is essentially that of Gerard of Cremona's *Almagest*, the author continues:

> Because therefore in this sort of figure two arcs *az* and *ae* descend from a common endpoint between which two other arcs *zt* and *eb* intersect at point *h*, and quadrant *zt* is equal to quadrant *eb*, through the conjoined catha the ratio of crd. arc 2*he* to crd. arc 2*ht* will be that which is of crd. arc 2*az* to crd. arc 2*ab*.[358]



While this is once again a rather sparse argument—in one note a reader mentions that it has confused many—yet it does supply enough information to recreate the missing steps.[359] The conjoined sector

[356] In Book II, the author slips from his generalized approach a few times such as in II.3 and II.26.

[357] Carlo Alfonso Nallino, ed., *Al-Battani sive Albatenii*: *Opus astronomicum*, (Hildesheim: G. Olms, 1977).

[358] Appendix D, lines 330-5.

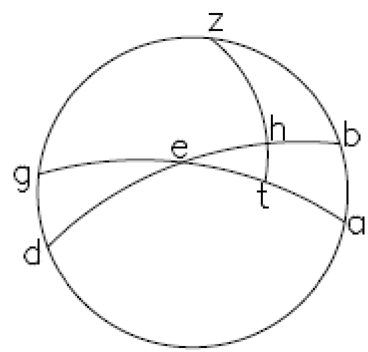
[359] Ibid., line 342 apparatus.

figure applied to this figure results in the conclusion that the ratio of crd. arc 2*az* to crd. arc 2*ab* is composed of crd. arc 2*zt* to crd. arc 2*ht* and the ratio of crd. arc 2*eh* to crd. arc 2*eb*. Because crd. arc 2*zt* is equal to crd. arc 2*eb* since both are quadrants, the two composing ratios can be rearranged so that the consequent of the first ratio is equal to the antecedent of the second. The chord of the double of a quadrant is here a middle between two extremes, crd. arc 2*eh* and crd. arc 2*ht*, so the ratio of the extremes, crd. arc 2*eh* to crd. arc 2*ht*, is composed of the ratios of the first extreme to the middle and of the middle to the second extreme, the ratios crd. arc 2*eh* to the diameter and of the diameter to crd. arc 2*ht*. Now the two ratios crd. arc 2*eh* to crd. arc 2*ht* and crd. arc 2*az* to crd. arc 2*zb* are composed from the same ratios. Therefore, these two composed ratios are equal, and from the standard rule of three for finding an unknown quantity in a proportion, the rule can be derived; here, the sine of the sought declination is found by multiplying the sine of the arc of the ecliptic of which the declination is sought by the sine of the greatest declination and by then dividing by the sine of a quadrant. Ptolemy had used the same statement from the sector figure, but he subtracted one of the composing ratios from the composed ratio to find the unknown ratio and then found the unknown term in that ratio.[360] The process here is rather finding a ratio that is composed of the same ratios as the original composed ratio, which can be easily done when one of the antecedents of one composing ratio is the consequent of the

[360] See pp. 36-7 above.

other, and seeing that the two composed ratios are the same. From this proportionality, the unknown term can be found through the rule of three.

The other way of establishing a rule from a statement of composition is first seen in I.17, which teaches how to find the right ascension of an arc of the ecliptic. From the disjoined sector figure, the ratio of the sine of *zb* to *ba* is composed of the ratio of the sine of *zh* to the sine of *ht* and the ratio of the sine of *et* to the sine of *ea*. Taking a very large leap over many unstated steps, the author then concludes that



it follows that if the sine of *zb* is multiplied by the sine of *ht*, and the product is divided by the sine of *zh*, the result will be a line which has the same ratio to the sine of *ba* as the sine of *et* has to the sine of *ea*. Readers did not comment on this step; its justification may have been assumed from another work or perhaps the readers were able to understand the missing steps but did not write them in the margins.[361]

[361] It appears that the rule is derived according to the following or perhaps from some equivalent justification. From the statement of the disjoined sector figure, the composing ratio of the sine of *et* to the sine of *ea* is composed of the inverse of the other composing ratio (so the sine of *ht* to the sine of *zh*) and the original composed ratio, which is the sine of *zb* to the sine of *ba*. In order to find a ratio that is compounded of the two composing ratios, a middle term is sought—the sine of *zb*. The first composing ratio has to be put into terms such that the antecedent is the consequent of the second composing ratio, so they can be compounded. To do this we find a line that is to the sine of *zb* as the sine of *ht* is to the sine of *zh*. The value of this line can be found through the rule of three applied to this proportion. The statement of the sector figure can now be altered to the ratio of the sine of *et* to the sine of *ea* is composed of the ratio of the line just found to the sine of *zb* and the ratio of the sine of *zb* to the sine of *ba*. It is known that the ratio of the found line to the sine of *ba* is also composed of those two composing ratios because the common term, the sine of *zb*, is a middle between the extremes. Therefore, the two ratios composed of the same ratios are the same, so the sine of *et* is to the sine of *ea* as the found line is to the sine of *ba*. The sine of *et* can then be found

The author uses the method from I.16 to establish rules for finding unknown values for the cases in which one composing ratio's antecedent is equal to the other's consequent, but for all other cases the rule he gives seems to have been found by a method similar to the one in I.17.[362] In a few places, he makes small variations in these methods. For example in II.4, he follows the pattern of I.16 but inverts the statement of composition, and in II.36 he converts the statement of composition so that one of the original composing ratios is the composed ratio because this lets him follow the familiar I.16 pattern.

While his first way of getting to a rule relies upon the idea that given three quantities, the ratio of the first to the last is composed of the ratio of the first to the second and of the second to the third, this does not mean that the author sees that as the definition of compound ratio. This proposition also follows if one defines compound ratio by the multiplication of denominations of ratios. The author does not give enough information here for a definitive statement about his conception of compound ratio to be made. This vagueness about the nature of compound ratio is also seen in his terminology, which utilizes verbs that were commonly associated with both understandings of compound ratio.[363]

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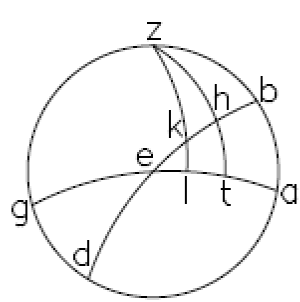
by the rule of three. Combining the two uses of the rule of three that are used to find the sine of *et* amounts to the multiplications and divisions that are given as the rule to find the elevation of a part of the ecliptic starting from an equinox.

[362] This second method is so abbreviated that it is impossible to confirm with certainty that the omitted steps are identical, but working through the steps as I have recreated them produces the same rules that the author gives.

[363] He most often uses forms of "producere" and "componere."

Book II generally follows the order of the *Almagest* although the material is arranged in 36 propositions instead of the twelve chapters of the *Almagest* and there are a few rearrangements of problems. Unlike Ptolemy, the author finds the difference between a twelve-hour day and longest day at a given latitude before he has found the arc of the horizon between the rising points of the equator and a point on the ecliptic and before he has found the height of the pole.

Also, in II.17-18, the author strays a little from the corresponding section of the *Almagest*. The seventeenth and eighteenth propositions give a method of finding the ascension of an arc of the ecliptic in a declined sphere that roughly corresponds to the second method that Ptolemy gives in II.7 of the *Almagest*. Proposition II.17 is essentially an explanation of the geometrical figure of Proposition II.18, where the sector figure is used to find the desired ascension. Given the *Almagestum parvum*'s penchant for remaining on the general level with few numerical values given, it is not surprising that this theorem strays somewhat from Ptolemy's version of this method, where he finds the values of the ratios between the arcs *el* and *et* for different points on the ecliptic that do not take the latitude into consideration. He then uses these values to make tables for the different parts of the ecliptic for different latitudes. The



Almagestum parvum uses the disjoined sector figure with the same arcs as Ptolemy's version, but then the two theorems deviate. In the *Almagestum parvum*, the author states that the ratio of the sine of *zh* to the sine of *th* is composed of the ratio of the sine of *zk* to the sine of *kl* and the ratio of the sine of *el* to the sine of *et*. He then applies I.17, which states that the same ratio of the sine of *zh* to the sine of *th* is composed of the ratio of the sine of *zk* to the sine of *kl* and the ratio of the sine of the rising in the right sphere of arc *ek*, which is not an arc in the diagram of I.18, to the sine of a quadrant. From this, the author states that the ratio of the sine of the rising in the right sphere of arc *ek* to the sine of a quadrant is the same as the ratio of the sine of *el* to the sine of *et*. This relies on the concept that if ratio A and ratio B make up ratio C and ratios A and D make up ratio C, then ratios B and D are the same. This is basically the application to ratios of the axiom that equals subtracted from equals leave equals.

While the *Almagestum parvum* does not have any corresponding passage to Chapters 5-6 of Book VIII of the *Almagest*, in which Ptolemy uses the Menelaus Theorem, its author does use the Menelaus Theorem in one place where Ptolemy does not. This theorem finds the longitudinal and latitudinal parallax of the moon when the moon is on the ecliptic by using the Menelaus Theorem three times.[364] Like Ptolemy, however, the author uses rectilinear approximations instead of using the Menelaus Theorem for the other spherical problems after Book II.

[364] Because of inconsistencies in dividing the last two books and dividing propositions, this proposition is given different numbers in different manuscripts, e.g. in Paris, BnF, lat. 7399 it is V.21 but it is the 47th proposition of Book V in Reg. lat. 1012.

The author of the *Almagestum parvum* wrote a condensed, systematized version of the mathematics in the first half of the *Almagest*. He universalized the mathematics and put it into a format similar to that of the prime model of mathematics, Euclid's *Elements.* He modified the content of the *Almagest* to better match the ideal of an axiomatic, deductive science. Also keeping an eye on the practical side of astronomy, he formulated clear, general directions for how to find different unknown values in astronomy. Despite this emphasis on extracting the theory and the easily applicable rules from the *Almagest*'s calculations of actual values, the author did not pay as much attention to either the theory or the practice of the Menelaus Theorem or of compound ratios as did other commentators such as the author of the Erfurt Commentary, a work that shares some enunciations and rules with the *Almagestum parvum*.

Chapter 5: The Erfurt Commentary

Another commentary on the *Almagest* is found in four manuscripts, all of which were written in the middle or second half of the fourteenth century. Because two of the four manuscripts containing this work are at Erfurt, I call it the "Erfurt Commentary."[365] This work is a commentary on and reworking of the first two books of the *Almagest*. Its incipit is "Data circuli dyametro latera decagoni pentagoni hexagoni tetragoni et trianguli omni ab eodem circulo circumscriptorum reperire. Pro probatione...," and the explicit is "... de aliis signis in quolibet climate etc. Et sic est expleta dictio secunda Almagesti."

The Manuscripts

Erfurt, Universitätsbibliothek Erfurt, Dep. Erf. CA 2º 375 (MS G).[366] This manuscript of 170 sheets of paper from the middle of the fourteenth century contains a large number of arithmetical, geometric, and astronomical works. These include Thomas Bradwardine's *Geometria*, his *Arithmetica*, a commentary on Sacrobosco's *Sphaera*, and works on the planets, the astrolabe, and right triangles. The first two books of Gerard of Cremona's translation of the *Almagest* are found on folios 85r-88r and 99r-112v. Many of the enunciations that are common to the *Almagestum parvum* and our anonymous commentary are found in the margins.

[365] By the name, I do not intend to imply that I think that this work originated in Erfurt.

[366] Described in Wilhelm Schum, *Beschreibendes Verzeichniss der Handschriften-Sammlung zu Erfurt*, (Berlin: Weidman, 1887), pp. 259-61; and also in Menso Folkerts, *Euclid in Medieval Europe*, (Winnipeg: The Benjamin Catalogue, 1989), p. 39.

Folios 113r to 126v contain the Erfurt Commentary, which in turn is followed by another commentary. This second set of commentary introduces several of its notes with a few words from the first commentary, so it is a second-order commentary. The last few propositions of the Erfurt Commentary are not found in this manuscript. The text ends with "... erit angulus *dea* notus orientalis super orizontem, quod est propositum."[367]

Erfurt, Universitätsbibliothek Erfurt, Dep. Erf. CA 2º 393 (MS H).[368] The contents of this manuscript composed of 80 folios of parchment from the later fourteenth century consist of works in the mathematical disciplines: Bacon's *Perspectiva*, Bernard of Verdun's *Astronomia*, a work of chronology, Dominic of Clavasio's *De practica geometrie*, and this commentary on ff. 63r-80v. The text stops short of the last few propositions in the same place as in MS G, so the explicit is essentially the same as in that manuscript.[369] In this manuscript and MS D, a section about the sources of imprecision that affect the determination of the maximum declination of the sun is located before the lemmas for the sector figure as in the *Almagest*, while in the other manuscripts it located after the first two lemmas.

Dijon, Bibliothèque municipale, 441 (MS D).[370] This manuscript from the end of the fourteenth century consists of 233 sheets of paper, and it was owned by the

[367] Appendix E, line 1856.

[368] This manuscript is described in Schum, pp. 275-6.

[369] Appendix E, line 1856.

[370] This manuscript is described in the *Catalogue général des Manuscrits des Bibliothèques Publiques de France. Départements, Bd.5*, (Paris: Imprimerie nationale, 1889), p. 104; Gustav

Convent of Chartreux. It contains Campanus' version of the *Elements*, Theodosius' *Sphaerica*, Witelo's *Optica*, Johannes Fusoris' *Libellus de sectione mukesi,* a short work on mirrors, and then this commentary on the first two books of the *Almagest* on folios 212r-233v in a slightly different version than in the other manuscripts. It has a prologue beginning with "Quaelibet circumferentia circuli..." that is not found in the other manuscripts. Also, while the text in the other manuscripts stops shortly before the end of Book II, this manuscript contains the last propositions about the angles between the ecliptic and a great circle passing through the zenith, but this completion of the contents of Book II shows signs of being written by a later commentator.[371] Confusingly, on f. 218r-219r right before the spherical Menelaus Theorem, the text is interrupted by a summary of and notes on the commentary up to that point. Also, on f. 220r the text jumps back to the section on how to approximate the chord of one degree, which had been touched upon on f. 214v. From this point, the text follows the general order found in Erfurt, Fol. 393. The

Haenel, *Catalogi Librorum Manuscriptorum, Qui in Bibliothecis Galliae, Helvetiae, Belgii, Britanniae M., Hispaniae, Lusitaniae Asservantur,* (Leipzig: I.C. Hinrichs,1830), p. 146; and in Marshall Clagett, *Archimedes in the Middle Ages Vol. 4*, (Philadelphia: American Philosophical Society, 1980), p. 167.

[371] Dijon, Bibliothèque municipale, 441 is the only manuscript to contain the material of *Almagest* II.12 (on finding the arc between the zenith and a point of the ecliptic and on finding the angle made by that arc and the ecliptic). This last section of the text varies from the rest of the commentary in content and stylistically. In the commentary on Ptolemy's preliminary propositions in that chapter, the commentator uses Gebir (Appendix D, line 1952), who is not used elsewhere in the commentary, to justify several steps. Also, in the main proofs of the chapter, the author uses "erit aggregata" (Appendix D, line 1975-6) in statements of composition instead of the "componitur" that had been used routinely up to this point. The text also reintroduces chords of double arcs, although sines had been used for much of Book I and throughout Book II. One of the subtractions of ratios also is given only in outline, not in the detailed, standard order that the original commentator had repeated over and over.

section on imprecision does not appear in the middle of the propositions required for the sector figure proof, but precedes it.

Città del Vaticano, Biblioteca Apostolica Vaticana, Pal. lat. 1380 (MS P).[372] This manuscript, which consists of 282 sheets of paper, appears to have been written in 1356 since it contains an astronomical table for the years 1356 to 1390. This manuscript contains works—too numerous to list here—on astronomy, astrology, music, geometry, and arithmetic. Among these works are Bernard of Verdun's *Astronomia* (here called the "*Tractatus super totam astrologiam*"), which is also in MS H, and Thomas Bradwardine's *Geometria speculativa*, which is in MS G. The manuscript has five different hands, so parts of the manuscript could have been written earlier or later. The commentary on the *Almagest* is written in two different hands. The first, a *cursiva currens*, extends from the beginning of the commentary on f. 116r to 123r and is thought to be the hand of Reimbotus Eberhardi de Castro, a physician of Emperor Charles IV. Reimbotus probably copied this commentary during his education at Bologna.[373] The second hand, a Gothic cursive, begins on f. 123v and continues to the end of the work. The astronomical table with the 1356 date is also in Reimbotus' hand, so the folios with the *Almagest* commentary can be dated with some degree of confidence. The work ends on f. 138v near the beginning of the commentary on Book II with "... et *z* erit polus articus et proveniet

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[372] This manuscript is described in Ludwig Schuba, *Die Quadriviums-Handschriften der Codices Palatini Latini in der Vatikanischen Bibliothek*, (Wiesbaden: Ludwig Reichert), 1992, pp. 111-4.

[373] Ludwig Schuba, "Reimbotus de Castro, Leibarzt Kaiser Karls IV. und Scholastikus an St. German vor den Toren der Stadt Speyer," *Miscellanea Biblioteca Apostolica Vaticana V: Palatina Studien*, pp. 287-294.

sector."[374] In this manuscript, few diagrams accompany the text, which would have made it difficult for the reader to understand many of the proofs unless he had the *Almagest* open in front of him.

General Description

There are no attributions for this work in these manuscripts, and there is not enough internal evidence to help ascertain the identity of the author. The dating of this work is also very unclear. The work's connection to the *Almagestum parvum* and its reference to Ametus' *Epistola* necessitates that the *terminus post quem* for its composition is in the late twelfth century, and the date is pushed further back by the reference to *De sphaera*, probably Sacrobosco's, which was written around 1220,[375] but given that all four manuscripts are from the fourteenth century, it is likely that it was written much later than that. The *terminus ante quem* is 1356, which is when MS P was written. As we will see, the work appears to have been influenced by Campanus, so the range of possible dates is narrowed to the period between the middle of the thirteenth and the middle of the fourteenth centuries.

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[374] Appendix E, line 1159.

[375] The reference to *De sphaera* is on line 950 of Appendix E. The commentary also refers there to Alfraganus. His work was translated into Latin first in 1135 by John of Seville, so the reference does not help date this commentary. Also, MS D has a note (Appendix E, line 515 apparatus) attributing a section of the text as an addition of Campanus, but not much weight can be put upon this. The only similarity with any known texts of Campanus is that this text contains a proof of something that Campanus proves in his *De figura sectore* (see Lorch, *Thabit*, 439-441).

Like the *Almagestum parvum*, the Erfurt Commentary is structured as a systematic mathematical work. Instead of chapters, the author states general propositions, which are followed by proofs, but unlike in the *Almagestum parvum*, first principles are not given at the start of each book. The parts of each theorem and problem are often marked by phrases such as "gratia exempli," "verbi gratia," "quod est propositum," and "unde colligitur corollarium."[376] Often between an enunciation and its proof, the author explains or proves propositions that will be used in the main proof of the theorem. For example, before proving the first lemma for the Menelaus Theorem, he lists three suppositions about ratios and compound ratios.[377] While rearranging Ptolemy's content as a deductive science, the format is not identical to the *Elements* or the *Almagestum parvum*—this work has proofs and principles between propositions and their proofs instead of being placed before the propositions that rely upon them.

This commentary clearly shows some influence from the *Almagestum parvum*.[378] Many of the enunciations in this commentary are almost identical to those in the earlier work. [Table 1] The proofs and explanations, however, vary considerably. For example, the proof of the plane conjoined sector figure is only

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[376] Appendix H.

[377] Appendix E, lines 12-39.

[378] In fact, this work has been confused with the *Almagestum parvum*. In a list of manuscripts that she claims have the *Almagestum parvum*, Olga Weijers, *Le travail intellectuel à la Faculté des arts de Paris: textes et maîtres (ca. 1200-1500). 2, Répertoire des noms commençant par C-F,* (Turnhout: Brepols, 1994), p. 33, includes MSS G and H. Lorch, "Some Remarks," pp. 421-2, very briefly mentions the connection between the *Almagestum parvum* and the contents of the Erfurt manuscripts and Pal. lat. 1380, but he does not realize that the texts in these three manuscripts are all the same work.

five lines long in the edition of the *Almagestum parvum*, but this commentary adds several preliminary proofs and expands the treatment of Lemma I to over 80 lines.[379] This commentary is also closely related to the set of commentaries on the *Almagest* found in the margins of the Gerard of Cremona translation of the *Almagest* in MS G, Città del Vaticano, Biblioteca Apostolica Vaticana, Barb. lat. 336, and Paris, Bibliothèque nationale de France, lat. 7256. The proofs in the gloss vary considerably from this commentary, but some of the enunciations are practically identical. For example, the enunciations of the plane conjoined sector figure share many words and phrasings, and the enunciation of the plane disjoined sector figure differs only as much as can be expected for the same text in two manuscripts. [Table 1] The examples of this marginal commentary in these manuscripts are also from the fourteenth century, so the order of influence is unclear. Lorch suggests that the *Almagestum parvum's* enunciations, preface, and some of the early proofs were written in the middle of the twelfth century and that the work was completed later in the century. Perhaps the common enunciations survived on their own from their original formulation in the middle of the twelfth century, separate from any proofs, and different mathematicians added proofs at different times. Some evidence for this theory is found in a thirteenth-century manuscript, State Library of Victoria, Sinclair 224, where the enunciations are

[379]Appendix E, lines 12-95 of my critical edition of this commentary, and Appendix D, lines 162-166 for the *Almagestum parvum*.

Table 1

Almagestum parvum	Erfurt Commentary	NX *Almagest* Commentary
Duabus rectis lineis ab angulo uno descendentibus aliisque duabus sese secantibus ab earum descendentium reliquis terminis in easdem reflexis utralibet | reflexarum alterius conterminalem sic figet ut proportio ipsius fixe ad eam sui partem que supra fixionem est producatur ex duabus proportionibus ex una dico proportione quam habet sibi conterminalis reflexa ad eam sui partem que sectioni interiacet et fixioni et alia proportione quam habet alterius reflexe inferior sub sectione portio ad eam totam cuius pars est lineam.[380]	Duabus rectis lineis ab uno angulo descendentibus aliisque duabus se secantibus a descendentium terminis reliquis in easdem reflexis, utralibet reflexarum alterius conterminalem sic figet ut proporcio fixe ad eam sui partem que supra fixionem est producatur ex duabus proporcionibus ex una quam habet conterminalis reflexa ad eam sui partem que sectioni interiacet et fixioni et ex ea proporcione quam habet alterius reflexe in inferiori sub sectione porcio ad eam totam cuius pars est linea.[381]	Duabus lineis ab angulo uno descendentibus si ab earum terminis due linee inter eas sese secantes super eas reflectantur, erit utriuslibet descendentium ad eam sui partem que est inter punctum reflexionis et angulum proportio ex duplici proportione composita, ex ea videlicet que est sue conterminalis reflexe ad eam sui partem superiorem et ea que est inferioris partis alterius reflexarum ad totam.[382]

written without proofs in the margins.[383] The *Almagestum parvum* and these related works have a parallel in the Adelard I and II Versions of the *Elements*, which share enunciations and definitions but have different proofs. The

[380] Appendix D, lines 154-161.

[381] Appendix E, lines 6-11.

[382] Appendix B, NX edition, lines 250-5.

[383] Lorch, "Some Remarks," p. 422. For a description of this manuscript, see Keith V. Sinclair, *Descriptive Catalogue of Medieval and Renaissance Western Manuscripts in Australia*, (Sydney: Sydney University Press, 1969), pp. 382-386.

"AdelardII" version of the *Elements* also may have first circulated as a list of enunciations to which outlines of proofs, and complete proofs were gradually added.[384]

This commentary generally follows the order and content found in the first two books of the *Almagest*. It focuses on the mathematical aspects of that work, and it does not contain tables. Like the *Almagestum parvum*, it generalizes the mathematics of Ptolemy, who had almost always used specific, numerical examples. The text is clearly divided into separate proofs that are introduced by enunciations expressed in general terms. The commentator reworks the proofs, supplying missing steps and adding justifications for many steps that might not be immediately apparent to his reader. He adds several alternate proofs, but almost always after explaining Ptolemy's proof. The commentator apparently meant his work to be read as a supplement to the *Almagest*, not as an autonomous work, since he does not give the constructions for many of the theorems that are in the *Almagest*. Many of the diagrams are not given, especially in MS P; the reader was presumably expected to use the diagrams from the *Almagest.*

There are a few sizeable sections that stray from Ptolemy's content. After the proofs of the plane sector figure, the text describes two ways of determining the

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[384] Busard and Folkerts, *Robert of Chester*, pp. 26-30. Another possibility is that the enunciations and proofs were written together but that the proofs were later considered to be separable. Some medieval scholars considered the enunciations of the *Elements* to be the work of Euclid but thought that the proofs were additions. With this mindset, keeping enunciations from an authoritative work but replacing the proofs would have allowed due deference to be shown to the authoritative work while allowing for the creativity of modifying proofs.

range to distant objects using the plane sector figure theorems.[385] Another long section in Book I discusses four different reasons that the observed values for the maximum declination of the sun, which is found using instruments, cannot be found with perfect precision.[386] Yet another very long section added at the beginning of the commentary on Book II deals with several issues about the world's sphericity.[387] The author discusses the shape and size of the dry portion of the earth and its greatest difference in longitude along the east-west diameter. He also shows that arcs of circles parallel to the equator have a greater longitudinal difference the closer they are to the North Pole, and that these longitudinal differences approach 180^o. After this, he argues that the methods of Alfraganus and the "author of the *Sphaera*" for calculating the diameter of the earth are wrong, and he shows how to find the distance between two locations on earth given their longitudinal difference and their latitudes. Then, given the longitudinal difference and the distance between them, he demonstrates how the direction from one to the other can be determined. Given the distance and the latitudes, he finds the longitudinal difference. Many of these proofs rely upon the Menelaus Theorem or its corollary, which will be discussed shortly.

[385] Appendix E, lines 118-153.

[386] Ibid., lines 155-246.

[387] Ibid., lines 820-1067.

Compound Ratio

Even more than most commentaries on the *Almagest*, this commentary gives close and detailed attention to the Menelaus Theorem and compound ratios. Commenting upon the first plane sector figure in which Ptolemy introduces compound ratios, the commentator tries to justify Ptolemy's insertion of a middle to produce a statement of composition. To do this, he supposes that if the denomination of a ratio of one quantity to another is multiplied by the second quantity, then the product will be the first quantity. This is similar to the first proposition of Campanus' treatise on proportion,[388] but he defines the denomination of a ratio differently than Campanus does. He writes that a denomination is "the number or fraction or fractions or number with a fraction or fractions denoting how many times the greater contains the lesser or what fraction or fractions the lesser is of the greater for denominations of ratios of lesser inequality."[389] A second supposition is "whenever two numbers or two fractions or two numbers with fractions are multiplied by some third, the ratio of the two products is as the ratio of the multiplying quantities."[390] His third supposition provides the definition of compound ratio according to the denomination concept.

[388] Busard, "Die Traktate," p. 213.

[389] Appendix E, lines 15-18.

[390] Ibid., lines 25-8. "

Conceptually, it is similar to Campanus' definition. The wording is also quite similar—too similar to be a coincidence.[391]

The commentator understands compound ratio as the multiplication of the denominations of ratios. He justifies this by citing "the authority of Euclid and others."[392] Our commentator explains his definition of compound ratio with an example from music theory: "As a diapason in music is said to be composed of a diatesseron and diapente because the denomination of a sesquitertiary [4:3] ratio, which is one and a third, multiplied in the denomination of a sesquialter [3:2] ratio, which is one and a half, produces two, which is the denomination of diapason or the double ratio."[393] This contrasts with the usual medieval way of talking about compound ratio in music theory, which is based upon taking the ratio of the extremes of continuous ratios.[394]

With these three suppositions, the commentator proves that with a middle placed between two quantities, the ratio of the first to the last is composed of the ratios of the first to the middle and the middle to the last. *A* and *b* are taken as the two original quantities and *c* is placed as middle. *D*, *e*, and *f* are set out as the

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[391] See Busard, "Die Traktate," p. 213; and Appendix E, lines 31-3.

[392] Appendix E, lines 33-4. Euclid did not explicitly define compound ratio, and his use actually tends towards the continuous ratio method of compounding, not the denominational method. See definitions 9, 10, and 17 in Book V of the *Elements* and VI.26. The use of continuous ratios in these suggests that Euclid understood compound ratio as taking the ratio of the extremes of continuous ratios. Some versions, however, of the *Elements* contain an explicit definition of compound ratio according to the denominational concept that was probably added by Theon of Alexandria. See Edith Sylla's "Compounding Ratios," pp. 11-44; and her "The Origin and Fate," pp. 67-119.

[393] Appendix E, lines 34-8.

[394] See Sylla, "The Origin and Fate," pp. 72-3.

denominations of the ratios of *a* to *c*, *c* to *b*, and *a* to *b*. Through the first supposition, *e* times *b* makes *c*, and *f* times *b* makes *a*. Through the second, therefore, *a* is to *c* as *f* is to *e*. And *d* is the denomination of *a* to *c*, so it is also the denomination of *f* to *e*. From the first supposition, *d* times *e* is *f*, and because these are denominations and because of the third supposition, the ratio of *a* to *b* is composed of *a* to *c* and *c* to *d*, which is what he wanted to prove. The commentator then proves that this works with more than one middle interposed and illustrates this with a numerical example.[395] These proofs are conceptually the same as the ones in Campanus' *De figura sectore* and his notes on the *Almagest*, and while the language is different, the letters assigned to quantities in the proof for one interposed middle are identical in the latter and the Erfurt Commentary.[396]

The commentator's use of the multiplication of denominations is seen in his explanation of the statement that any ratio is composed of itself and a ratio of equality, which is needed for the proof of one of the cases of the disjoined sector figure. His argument for this is that multiplying any denomination by the number one will result in that same denomination.[397]

While the treatment of compounding ratios is detailed, the commentator proves only one mode that follows from a statement of composition, not all

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[395] Appendix E, lines 40-80. The numbers used show that this commentator thought it acceptable to have the middles be larger or smaller than the extremes, e.g. 3, 5, 2, 14, 12. Some medieval mathematicians such as Thomas Bradwardine objected to the use of compounding ratios of lesser and greater inequality together.

[396] See Appendix B, NX edition, lines 356-367.

[397] Appendix E, line 524-8.

eighteen valid modes as Thabit and Ametus do. In his proof of the spherical conjoined sector figure, he shows that the ratio of the third quantity to the fourth is composed of the ratios of the first to the second and of the sixth to the fifth.[398] He justifies this by citing the "little book on ratio and proportionality," which is probably Ametus' *Epistola*, and by providing a numerical illustration.[399] He later refers to the "book about ratio and proportionality," which is almost surely Ametus' work, to justify the inverse of this mode, i.e. that the ratio of the fourth to the third is composed of the ratios of the fifth to sixth and of the second to the first.[400]

When he begins the proofs of the various astronomical applications of the Menelaus Theorem, the commentator supplies more details of dealing with the practice concerning compound ratios. Given that five quantities are known in a statement that a ratio is composed of two others, ways of determining the unknown quantity are needed. Throughout the *Almagest*, Ptolemy does this by "subtracting" one known composing ratio from a known composed ratio to find the previously unknown other composing ratio. While Ptolemy does not explain exactly how one goes about subtracting a ratio from another, the commentator does in a demonstration that proves that if one of two composing ratios is known and the composed ratio is known, the other is known by the subtraction of the known composing ratio from the composed ratio. He shows that the remaining ratio "... is

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[398] Ibid., lines 483-493.

[399] Ibid., line 487-493.

[400] Ibid., line 1817.

between the product of the antecedent of the composite ratio and the consequent of the known ratio that is subtracted and the product of the consequent of the same composed ratio and the antecedent of the subtracted ratio."[401] After giving this rule, he demonstrates that these steps do in fact give the desired result. He lets the known composed ratio be *a* to *b*, and the known composing ratio be *c* to *d*. The other composing ratio is sought. First, three multiplications are performed. *A* times *d* makes *e*, *b* times *c* makes *f*, and *b* times *d* makes *g*. From the first and the last of these multiplications, it is clear that *e* is to *g* as *a* is to *b*, and from the second and third it is clear that *f* is to *g* as *c* is to *d*. But with *f* placed as a middle between *e* and *g*, the ratio of *e* to *g* is composed of the ratios of *e* to *f* and of *f* to *g*. Substituting ratios in that last statement of composition of ratios, the ratio of *a* to *b* is composed of the ratios of *c* to *d* and of *e* to *f*. The conclusion is then clear because of the way that *e* and *f* were produced.[402] This rule and the proof are conceptually identical to one of Campanus' notes on the *Almagest*, the proofs use the same letters, and the language used shows some similarities.[403]

Curiously, the commentator later presents another way to show essentially the same conclusion (that an unknown composing ratio can be found when the other composing ratio and the composed ratios are known). He starts with the statement that the ratio of *a* to *b* is composed from the ratios of *c* to *d* and of *e* to *f*.

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[401] Ibid., lines 670-2.

[402] Ibid., lines 673-684.

[403] SeeAppendix B, NX edition, lines 415-429.

First, he multiplies *a* by *d* and divides the product by *b* to reach a number that he calls *g*. Calling upon Euclid VI.16 (which states that if the rectangle contained by two lines is equal to one contained by two others, then there is a proportion between the four)[404] since the product of *b* and *g* is the product of *a* and *d*, the resulting number will be to *d* as *b* is to *a*. Alternated, *a* is to *b* as *g* is to *d*. The third number is inserted as a middle between *g* and *d*, so the ratio of *a* to *b* is composed of the ratios of *g* to *c* and of *c* to *d*. The ratios *g* to *d* and *e* to *f* must be the same since with the ratio *c* to *d*, each composes the ratio of *a* to *b*.[405]

The commentator also shows that if the two composing ratios are known, then the composed ratio will also be known. The sought composed ratio is the one between the product of the two antecedents and the product of the two consequents. The unknown composed ratio is set out as *a* to *b*, and the known composing ratios are *c* to *d* and *e* to *f*. Again, three multiplications are performed. *C* times *e* makes *g*, *c* times *f* makes *l*, and *d* times *f* makes *h*. From the first two multiplications, *g* is to *l* as *e* is to *f*, and from the second two, *l* is to *h* as *c* is to *d*. The ratio of *g* to *h* is composed of the two ratios *g* to *l* and *l* to *h*, which are same as the two original composing ratios. Because the two ratios *g* to *h* and *a* to *b* are composed of equal ratios, they are equal to each other. The rule is then seen because of how *g* and *h*

[404] This proposition is numbered VI.15 in the numbering of Version II of the *Elements*. This proposition of Euclid is about lines and parallelograms, not numbers, but the two processes of using lines to determine parallelograms and of multiplying numbers to form plane numbers were often blurred in medieval mathematics (as the terminology of "plane" and "solid" numbers attests). Although the commentator here multiplies and divides, he only refers to "quantities" not numbers.

[405] Appendix E, lines 763-774. That the commentator gives two different proofs suggests that he had two different sources dealing with compound ratios.

were formed. The commentator points out that this can be done with more than two composing ratios.[406] He later gives a few examples of similar rules for finding unknown compound ratios from the quantities of the two known composing ratios.[407]

Because in most cases when the Menelaus Theorem is used, the goal is to find one unknown quantity from the five known ones, not merely to find one ratio from the two others, the commentator proceeds to show how to find the unknown quantity from the five known quantities. First, he gives rules for finding the unknown quantity when it is one of the terms in one of the composing ratios. The unknown composing ratio can be found, but it is not necessarily in terms that include an appropriate term from the original statement of composition. Using commonly known methods of finding an unknown quantity in a proportion, the unknown quantity is found. For example, given that the ratio of a quantity to a second is composed from the ratio of a third quantity to a fourth and of the unknown ratio of a fifth to a sixth, and all of these quantities are known except for the fifth or the sixth, the ratio of the fifth to the sixth can be found. Then since one of these is known, the other can easily be found. [408]

He ends this section by saying, "Therefore, we obtain from all these premises, how a ratio is able to be subtracted from another ratio through two ways,

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[406] Ibid., lines 685-697.

[407] Ibid., lines 717-723.

[408] Ibid., lines 698-716.

and also how it may be added to another, without which all uses of the sector figure would not be able to be had in calculations."[409] In other words, the algorisms of compound ratios are crucial for all the applications of the Menelaus Theorem, and without the knowledge of how to move from the statement of composition of ratios to the determination of specific, unknown quantities, the theorem would be useless in the actual practice of astronomy.

The Menelaus Theorem and Its Cases

Given the amount of attention paid to compound ratios, it is not surprising that the commentator expands upon Ptolemy's treatment of the Menelaus Theorem. In the six lemmas, the commentator proceeds slowly and deliberately. He has many long asides and provides numerical examples to help the reader understand concepts like ratios *coniunctim*.[410] Much of the material on compound ratios in this work is brought in to explain Ptolemy's use of compound ratios in the plane sector figures. The lemmas have detailed proofs and the same enunciations as those in the *Almagestum parvum* except the disjoined plane sector figure, which has a similar but not identical enunciation.[411]

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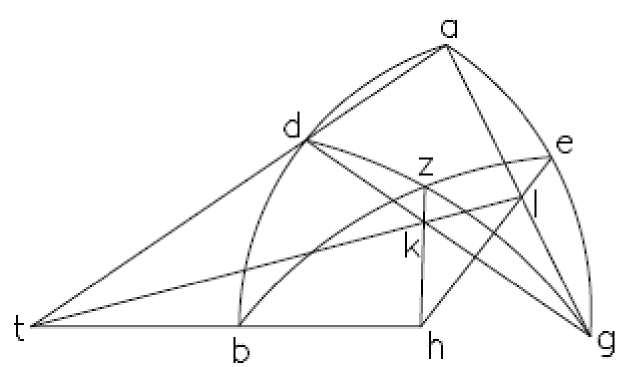
[409] Ibid., lines 723-726.

[410] See Euclid, *Elements* V. def. 14.

[411] See Table 1 on p. 191 above.

The spherical disjoined sector figure, which he calls the "catha divisa,"[412] has the same enunciation as in the *Almagestum parvum* and gives essentially the same proof as the one in the *Almagest*.[413] Unlike Ptolemy, the commentator attempts to prove the case of the conjoined sector figure where the lines meet as in the case of the disjoined that Ptolemy proves, but unlike in the *Almagestum parvum*, there is no generalized statement of what the conjoined sector figure is.[414] As in the *Almagestum parvum*, this proof of the conjoined only works for one very specific case.[415] A reader would likely find this short proof very difficult; no construction is given and only the Dijon manuscript has a diagram, which has mistakes, so a reader trying to make sense of this proof of the conjoined sector figure by using the construction for the proof of the disjoined would be led to a nonsensical reading of the text.



He then enters a lengthy section on the different cases of the Menelaus Theorem. He begins with the case where lines *da* and *bh* do not meet on the

[412] Ibid., line 776.

[413] Ibid., line 418-454.

[414] Ibid., lines 456-470.

[415] Lines *ge*, *gz*, and *ez* are assumed to meet their respective diameters of circle *adb* respectively at *l* on the side of *e*, at *k* on the side of *z*, and at *t* on the side of *z*. As we will later see when discussing the Vatican Commentary, there are many other ways that these three lines could meet or not meet their respective diameters.

side of points *b* and *d* as in Ptolemy's proof, but meet on the other side of the diagram. His proof is essentially that of Thabit, which extends two of the arcs until they are semicircles and reasons from a newly formed spherical sector figure,[416] but he begins his construction with the diagram lettered as in the *Almagest*. Also, unlike Thabit, he first separately proves two suppositions, that the sine of an arc is the same as the sine of the supplement of that arc, and that, as mentioned before, it is known from a statement of composition that the ratio of the third to the fourth is composed of the ratios of the first to the second and of the sixth to the fifth.[417]

The commentator then proves the disjoined sector figure when lines *da* and *bh* are parallel. This section is marked in the Dijon manuscript as "another addition of Campanus," but there is little correspondence of this text to Campanus' proof of this case in his *De figura sectore*.[418] He starts by giving three suppositions. The first is that given an arc with its chord parallel to a diameter, the sines of the arcs from either end of that diameter to either end of that arc will be the same. The second is that if a surface cuts two intersecting surfaces in such a way that its intersection with one of the two others is parallel to the others' intersection, then its intersection with the remaining surface will be parallel to the other intersections, and that of these three lines each pair of lines will be in a plane, but the remaining

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[416] See pp. 72-3 above.

[417] Appendix E., lines 472-513.

[418] It is tempting to use this attribution to make an argument that Campanus wrote or at least influenced this work, but the two texts of the proofs for these are not similar enough to argue that there is a real connection besides that they merely are proofs of the same thing. The letters, terminology, and especially the way that the two authors speak of how the three parallel lines are known to be parallel are rather different.

line will be outside that plane. This is not proved by the commentator. The third is that a ratio is composed of itself and a ratio of equality. The main proof then proceeds conceptually as Thabit's,[419] but once again with different terminology and with letters as in Ptolemy's proof of the sector figure. The author also points out that this conclusion applies to either of the principal arcs of the sector figure, i.e. there is a statement of composition about *ge* to *ga* and one about the sines of *bd* and *da*.[420] The commentator moves on to prove the conjoined sector figure (as we have seen, he already proved one specific case) in a way very similar to Thabit's proof—with only one, universal case.[421]

While many of these proofs are almost surely derived from Thabit, the closeness of the connection is not clear, because there are enough differences that there could be one or more missing links between Thabit and this commentator. The author of the Erfurt Commentary was clearly interested in expanding the knowledge of compound ratios and of the Menelaus Theorem, but that he proved the same propositions in different manners suggests that this commentary does not reflect a thoroughly thought-out understanding of the material, but rather it was written as the commentator learned about these topics.

[419] See p. 67 above. [crosreference]

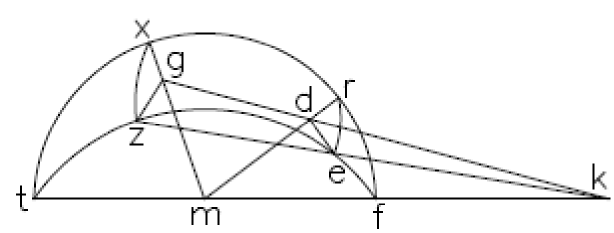
[420] Appendix E, lines 515-551.

[421] Ibid., lines 553-568.

Alternatives to the Sector Figure

Although he goes into great detail about the sector figure and compound ratios, the commentator proves and uses extensively a proposition that can take the place of the sector figure and does not require compound ratios. This theorem shows that in a spherical right triangle made of arcs of great circles with another great circle cutting the hypotenuse and one of the legs parallel to the other, the sine of the uncut leg is to the sine of the cutting arc as the sine of the hypotenuse is to the sine of the part of the hypotenuse below the section.[422]

This opaque enunciation is made clearer in the construction. The spherical right triangle is *fxz* with right angle *z*, and *er* is the arc of a great circle also making right angles with arc *fez* at point *e*. This theorem proves that the sine of *zx* is to the sine of *er* as the sine of *zf* is to the sine of *ef*. While this is put in different terms, this is conceptually similar to the first portion of Gebir's I.12, but the proof is different in some respects, and also the



commentator puts the proposition in astronomical terms. Radii *rm* and *xm* are drawn (unlike in Gebir's similar proof, the lines through points *d* and *g* meet at the center of the circle), and perpendiculars *ed* and *zg* are dropped on them. *Ed* is the sine of *er* and *zg* is the sine of *zx*. *Zg* and *ed* are parallel. The intersection of their plane with the equinoctial plane is line *dg*, and its intersection with the ecliptical

[422] Ibid., lines 585-9.

plane is line *ez*. The commentator states that if a plane cuts two other planes that intersect in such a way that its intersections with those planes are not parallel to the section of those two planes, then its intersections with the other planes will meet their section at one point. This is called point *k*. Right triangle *gkz* has now been formed with line *ed* cutting it parallel to base *zg*. Therefore, *zg* is to *ed* as *zk* is to *ek*. But from the fifth lemma to the sector figure, the whole line *zek* is to the portion of it outside the circle, that is line *ek*, as the sine of *zf* is to the sine of *ef*. Therefore, *zg*, the sine of *zx*, is to *ed*, the sine of *er*, as the sine of *zf* is to the sine of *ef*, which is what was to be proved.[423] Gebir's I.12 is more universal than this theorem. While this proposition is put in terms of right triangles cut by arcs parallel to the base, Gebir phrases his in terms of circles cutting each other. Gebir's proof applies in cases of two triangles with a reflected angle, while this commentator's proof requires one of the two triangles to be in the other.

The commentator proves another proposition that could be used as an alternative to the sector figure. It is that the chord of the arc subtending the right angle in a right angled spherical triangle can be found by adding the squares of the sines of the other sides of the triangles to the square of the difference between these two sides' verse sines, and then by taking the square root of this sum.[424] Given spherical triangle *abh* with right angle *b*, the author draws diameter *bm* and line *nhk*

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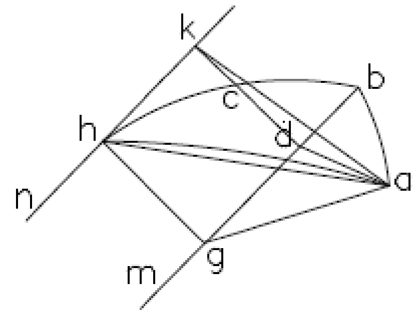
[423] The commentator does not consider the case where the intersections *fm*, *gd* and *ez* all are parallel, but in that case it is fairly obvious that *ed* will equal *zg* and that the sine of arcs *fe* and *fz* will be equal.

[424] In modern terms, this could be expressed as $\text{crd.}ah^2 = \sin bh^2 + \sin ab^2 + (\text{v.sin}bh - \text{v.sin}ab)^2$ where *abh* is a spherical triangle with right angle *b*.

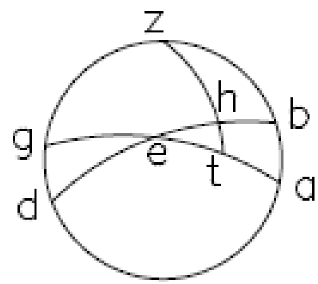
parallel to it. He then cuts off arc *bc* equal to *ba*, which is here taken to be the smaller of *ba* and *bh*. Through point *c*, he draws line *kcd* perpendiclar to *bm*. Perpendiculars *ad* and *hg* are also dropped to the diameter *bm*, and lines *ka* and *ah* are joined. By the Pythagorean Theorem, line *ah* is the square root of the sum of the squares of *ak* and *hk*, but *ak* is the square root of the squares of *kd* and *ad*, which are the sines of arcs *bh* and *ba*, and *hk* is the difference between *bg* and *bd*, which are the verse sines of arcs *bh* and *ba*. Therefore, the proposition is true.[425] The author writes that this conclusion is "of no little use" and that it can be used for several calculations such as finding the distance between two locations on the earth if their longitude and latitude are known. Although this proof of how to find the hypotenuse of a spherical right triangle from the legs of a right triangle uses fewer arcs in a more natural and simpler configuration than the sector figure, the corollary (the alternative to the sector figure just described above), or any of Gebir's alternatives, the commentator does not use it extensively. Since it only is useful for right triangles, it is not applicable in as many situations as the sector figure or other alternatives. Also, it requires verse sines, and tables of verse sines were not as common as tables of chords and of sines.



[425] Unfortunately, I have not been able to find a source for this proof.

Normal Applications of the Sector Figure and Alternative

The Erfurt Commentary uses the sector figure to show how to find all the quantities that Ptolemy finds with it in Book I and II of the *Almagest*. He follows Ptolemy's order, gives basically the same proofs, and also uses the same letters as the Gerard of Cremona edition. There are, however, some significant differences. One is that the commentator does not simply run through examples as Ptolemy usually does. Instead he proceeds more universally, dividing each book not into chapters as Ptolemy does, but into propositions divided into parts as the *Almagestum parvum* does. The proofs are generally given in a universal manner, i.e. he does not proceed by working through particular examples. However, perhaps struggling to leave Ptolemy's values completely behind, he occasionally gives particular values for some of the quantities. For example, when finding the declination of arcs of the ecliptic, he points out that the arc of the ecliptic, *eh*, is 30 degrees, but he does not give any other values in the proof.[426] The theorems and problems are divided into enunciations, constructions, proofs, and conclusions in a way similar to that of the *Almagestum parvum*. Like that work, this commentary has words and phrases marking the parts of the proofs (although not the same ones that are found in the *Almagestum parvum*).[427] As in the *Almagestum parvum*, the *Erfurt Commentary* contains rules for calculating unknown values. While most of the proofs follow the arguments of Ptolemy,



[426] Appendix E, line 736.

[427] See Appendix H.

several differ slightly and there are a few significant variations from Ptolemy's procedures.

The first of the major differences is that the commentator deviates from Ptolemy in the way of finding the unknown quantity from the statement of composition found from the sector figure. Unlike Ptolemy, he usually goes through the order of multiplications and divisions of known terms that are needed to find the unknown term or he refers the reader to the directions he has given for subtracting a ratio.[428]

In the applications the commentator shows more flexibility in treating compound ratios than Ptolemy does. In II.C.12, which shows how to find the angle between the horizon and the ecliptic, Ptolemy uses an altered version of the conjoined sector figure, presumably in order to avoid having the unknown term in the composed ratio and to avoid compounding ratios; with the altered statement of composition, Ptolemy could subtract ratios as he does in all other cases. After giving Ptolemy's method, the commentator notes that the unknown term can be had directly from the regular statement of composition by adding the two known composing ratios to find the unknown composed ratio.[429]

Often, the commentator paraphrases Ptolemy's proof and then adds a very simple proof through the "rule in four proportional quantities," which is the

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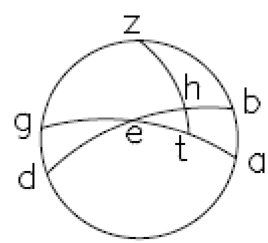
[428] E.g., Appendix E, lines 736-750.

[429] Ibid., lines 1848-1856.

corollary to the Menelaus Theorem that he proved after the proofs of the sector figure.[430] Unlike the alternative theorems that Gebir used to replace all of Ptolemy's applications of the Menelaus Theorem, this alternative is only used to supplement, not replace, five of Ptolemy's twelve applications in Books I and II. Although he also knows easier proofs and could have completely redone the spherical geometry like Gebir did, this commentator gives and explains Ptolemy's sector figure applications.

The author also shows a use of the second alternative to the sector figure. He observes that if an arc of the equator and the declination at that point are known, then the sine of the corresponding arc of the ecliptic can be easily found without tables through the spherical version of the Pythagorean theorem that he proved in I.16. In triangle *eht*, the arcs *et* and *ht* containing right angle *t* are known, so the sine of the hypotenuse *eh* can be found through that theorem.[431]



We see here the great creativity of the commentator to improve and modify the proofs of Ptolemy and to formulate and apply new proofs; however, the desire to explain Ptolemy leads the commentator to retain Ptolemy's original proofs (although sometimes slightly modified) and to only add his dramatically modified proofs or wholly new ones after those.

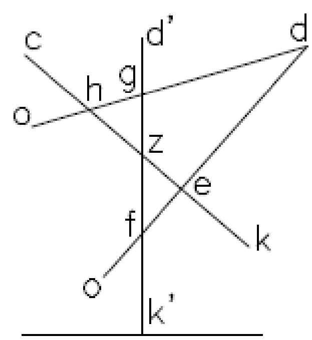
[430] E.g., ibid., lines 752-760.

[431] Ibid., lines 808-814.

New Applications

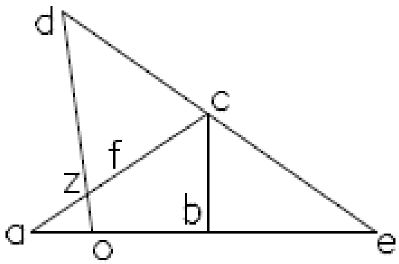
The commentator's creativity is seen even more clearly in his application of the Menelaus Theorem and its alternatives to non-astronomical matters. These new problems involve two topics. The first set of problems concerns the calculation of the distances to an object through instruments, and the second is on geographical problems about the distances and directions between locations on the earth.

The commentator uses the plane sector figure to show two methods of calculating the distances of objects. The first of these uses an instrument constructed by crossing two sticks, *he* and *gf*. First the observer places his or her eye *o* near the bottom of the instrument, observes the object *d*, and marks where the line of vision crosses the two sticks at *f* and *e*. The eye is then placed higher so that the object is seen above *z*, and the object *d* is sighted and marks made at *h* and *g*. The sticks and the lines of vision create a sector figure. After the figure has been set up, the commentator compounds the ratios of *gf* to *gz* and *hz* to *he* by multiplying *he* and *gz*, and then dividing the product by *hz* to get a quantity *l*. *Fg* to *l* is then composed of *fg* to *gz* and *hz* to *he*. Although this is a valid way of compounding, the commentator does not explain how it follows from the definition of compounding. From the conjoined sector figure, the ratio of *fd* to *ed* is composed from the same ratios, so *fg* is to *l* as *fd* is to *ed*. *Divisim*, the excess of *fg* over *l* is to *l* as *fe*, which is the excess of *fd* over *ed*, is to *ed*. Since all the values for the parts of the two sticks can be



measured, the length of *ed* can be had. Add *ef*, and the total distance to the object from point *f* is obtained.[432]

The second method of using the sector figure to calculate distance uses a tower. The observer stands on the ground and lines up the top of the tower *c* with the object *d* and marks the eye's location at point *e* of the diagram. Then, he stands on the other side of the tower and at point *a*, inserts a stick *af* pointing directly at *c*.



Then putting his eye on the ground a few feet closer to the tower at point *o*, he sights object *d*, and marks point *z* on the stick *af*. This makes a sector figure, but the commentator misapplies the conjoined sector figure. Uncorrected, the text has as its statement of composition that the ratio of *ae* to *oe* is composed of *az* to *zc* and *dz* to *zo*. The *az* should be *ac*, as the scribe of the Dijon manuscript has corrected above the line, and *zo* should be *do*. This passage is even more confusing in all but the Dijon manuscript because point *e* is designated *o*, which makes it difficult to tell which of the two points they mean when they write "*o*," and Erfurt, Fol. 393 is lacking several crucial words in the statement of composition. That the author originally had the statement of composition incorrectly is indicated by the way that he uses the ratio of *dz* to *zo* to find *do. Zo* is known so once the ratio of *dz* to *zo* is known, *dz* can be easily found. *Oz* is added to find the total length *do*. If he had realized that the ratio found through the sector figure was really *dz* to *do*, a slightly more

[432] Ibid., lines 118-138.

complicated process would have been required to find *do* from the measurable length *oz*.[433] None of the manuscripts, however, show any sign of this lengthier argument, so it seems to have never existed.[434]

A question that remains is why the commentator decided to deviate from the astronomical matters. These methods depend upon the two lines of sight in each being measurable inclined towards each other, but the lines of sight to celestial objects are sensibly parallel (at least with the difference in the viewer's location changing only a matter of feet). [435] The first method may have been of some actual utility but only for relatively short distances. The second was probably not a useful method in any circumstance. Since the object is sighted with a tower, it must be in the sky or on the top of a mountain, but as just shown, it could not be a celestial object. Even if there were a suitable object to observe, whoever tried to use this method would have difficulty ensuring that points *o*, *b*, *e*, and *a* are all on the same straight line. The implausibility of using this second method suggests that these two methods were primarily given as examples of the sector figure to show its versatility, to familiarize readers with it, or to highlight the mathematical skill of the commentator, but it was not to provide practical ways of measuring.

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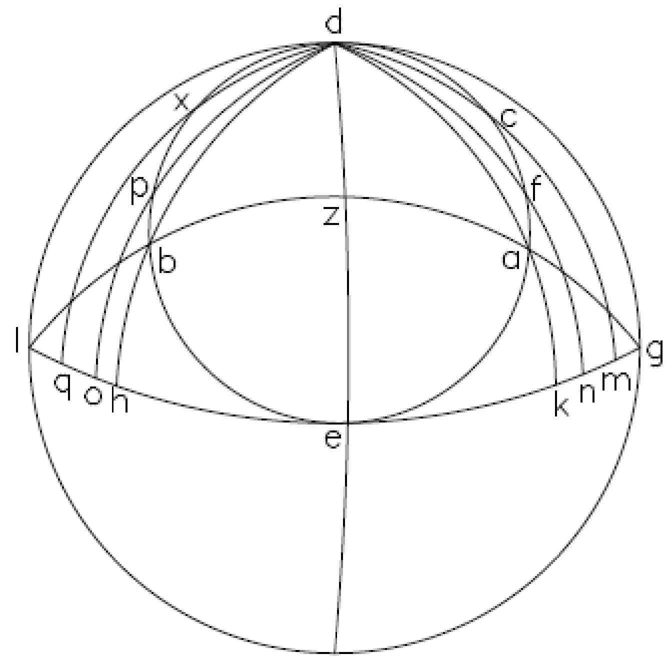
[433] Once the ratio of *dz* to *do* is known, the ratio of *oz* to *do* is the ratio of the difference of the antecedent and consequent of the ratio of *dz* to *do* to the consequent. E.g., if the ratio of *dz* to *do* were 9:10, the ratio of *oz* to *do* would be 1:10. *Oz*'s length is measurable, so *do*'s length can then be easily calculated.

[434] Appendix E, lines 139-153.

[435] E.g., in the second method, if the lines *ecd* and *ozd* are sensibly parallel, then the two triangles *eca* and *oza* are similar and the ratio of *ae* to *oe* is the same as of *ac to zc.* Since one of the composing ratios is the same as the composed ratio, the other composing ratio, *dz* to *do* must be one of equality.

The second set of additional applications of the sector figure and the commentator's alternative theorems is found in a very lengthy section on the sphere of the earth. This treatise added at the start of Book II begins with a discussion of the shape, size, and location of the dry portion of the earth.[436]

The commentator first uses the corollary to the sector figure to show how to find the portion of the equator that lies between the meridians of the two points that are farthest apart east and west (or in other words, the two points at the end of the circle's diameter that is parallel to the equator). These two points are *a* and *b* in the diagram, and arc *azb*



is the width of the land. The commentator states that since arc *azb* is a quarter circle, the angle it subtends at the center of the earth is a right angle and that

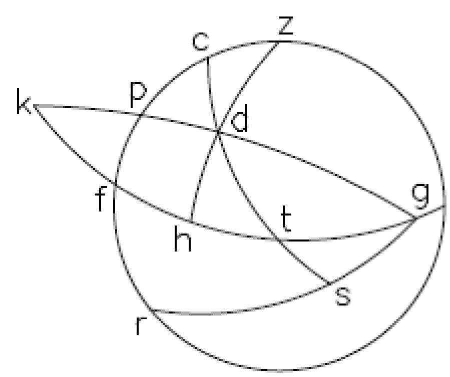
[436] Interestingly, the commentator thinks of the spheres of water and earth as being roughly the same size but with slightly different centers. He shows that this means that the dry portion of the earth is circular. He assumes that the circle of dry land extends from the North Pole to the equator, and therefore he argues that all shadows on land point north at the equinoxes, that the chord joining opposite points on the land is the side of the square inscribed within a great circle of the earth's surface, and that the land is only about a seventh of the total surface of the earth. Perhaps following Ptolemy or Alfraganus, the commentator divides the dry portion of the earth into seven climes. While for these earlier authors and many others, the climes ranged from the latitude with a longest day of 13 hours to one with a longest day of 16 hours, the commentator here may have a less exact division into seven parts. See Romeo Campani, ed., *Alfragano (al-Fargānī) Il 'libro dell'aggregazione delle stelle,'* (Città di Castello: S. Lapi, 1910), Ch. 8.

therefore the arc between their zeniths is a quarter of the great circle also passing through points *g*, *z*, and *l*. The meridians *dak*, *dze*, and *dbh* are drawn from the North Pole *d*. The proof relies upon two applications of the corollary to the sector figure from I.15. Angles *zek* and *eka* are right angles, so through the corollary to the sector figure, the sine of arc *ze* is to the sine of arc *ak* as the sine of arc *zag* is to the sine of arc *ag*. Arcs *ze* and *ag* are known to be eighths of circles, and arc *zag* is a quarter circle, so the unknown arc *ak* can be found. Its complement, arc *ad*, will also be known. Once again applying that same corollary to right triangle *edk* with arc *az* cutting arc *dze* at right angles, the sine of arc *ke* is to the sine of arc *az* as the sine of arc *kd* is to the sine of arc *ad*. Through the three known arcs in this proportion, the sine of arc *ke* is found. Arc *ke* is therefore known, as is its double, arc *kh*, which is the sought equatorial distance between the meridians of the two original points. He points out that the difference between the meridians of points *a* and *b* is more than six equal hours since arc *ke* is greater than arc *az*, which is an eighth of a great circle. Although he states that we could calculate the exact time, he does not do so. While this is a problem with only one obvious application, the world, the commentator chooses to remain on the general level.[437]

While this problem was solved with an alternative to the sector figure, the commentator does not completely separate himself from the sector figure. He outlines how the same arc *kh* can be calculated using the conjoined sector figure. The statement of composition that he is using here (although he does not state it

[437] Appendix E, lines 856- 877.

explicitly) is that the ratio of the sine of arc *de* to the sine of arc *ez* is composed from the ratio of the sine of arc *dk* to the sine of arc *ka* and from the ratio of the sine of *ga* to the sine of arc *gz*. Because all of the quantities are known except for the sine of arc *ka*, the ratio of the sine of arc *dk* to the sine of arc *ka* is found by subtracting the known composing ratio from the known composed ratio. Then the unknown quantity, the arc *ka*, can be found. With arc *ka* known, arc *ke,* which is half of arc *kh*, can be found through the disjoined sector figure.[438]



After a respite from applications of the sector figure and its alternative, in which he discusses the equatorial differences between the meridians of extremes of parallel circles such as of *cx* and *pf,*[439] and the methods of determining the diameter of

[438] Ibid., lines 893-898. He does not give any of these steps, but the statement of composition would be that the ratio of the sine of arc *dz* to the sine of arc *ez* is composed of the ratios of the sine of arc *da* to the sine of arc *ak* and of the sine of arc *gk* to the sine of arc *ge*. Arc *gk* can then be found from the other five quantities. Its complement is arc *ke*, so the sought arc *kh* can be found.

[439] He shows that as some parallel is taken nearer the pole, the greater is the equatorial difference between the meridians of the endpoints of that parallel. In the diagram, the two parallels he considers are *fp* and c*x*. He states that it is clear that the meridians *dcm* and *dxq* cut off a larger portion of the equator than meridians *dfn* and *dpo* do (Appendix E, lines 900-913). He points out that curiously as the extreme points of the parallels get closer together, the equatorial difference between their meridians grows. So, there can be two points only one league apart from each other on a parallel near the pole that have a greater difference in time between their meridians than do two points hundreds of leagues apart on a parallel much further south (lines 914-920). He then discusses that the maximum difference between meridians on a parallel is always less than twelve equal hours but that a parallel can always be taken further north with a greater difference of meridians (lines 922-941). In other words, twelve equal hours is the longitudinal limit that can be approached by taking points on parallels further and further north.

the earth,[440] the commentator arrives at the next matter connected with the Menelaus Theorem—the problem of how the distance between two cities can be found from the equatorial difference between their meridians and from their latitudes. Points *c* and *d* are the zeniths of two cities, and meridians *zcf* and *zdh* are taken from pole *z* to the equator, which is arc *kfhg*. The commentator writes that he can use the corollary to the sector figure if the two cities have the same latitude (i.e. when *c* and *d* are on the same parallel), but since the parallel circle joining points of equal latitude is not a great circle, this theorem does not apply. Besides this mistake, the commentator also errs in mixing in some chords of arcs where he should have sines or chords of double arcs—the proportion that should follow from the corollary is that the ratio of the sine of arc *fh* is to the sine of arc *cd* (again, arc *cd* is mistakenly taken as an arc of a great circle) as the sine of arc *hz* is to the sine of arc *dz*. Arc *cd* can be found from that proportion and the table of chords. To convert this quantity of arc into distance, the commentator assumes 700 stades for each degree. [441]

When the two cities are at different latitudes, the proof is more complicated. If *d* is assumed to be the southern city, a great circle is passed through it cutting the other city's meridian perpendicularly at point *p*. Because arc *zpf* cuts the two semicircles *gpk* and *gfk* at right angles, arcs *pg* and *fg* must be quarter circles. That same corollary that was used when the cities had the same latitude can be applied to

[440] Ibid., lines 943-957.

[441] Ibid., lines 959-978.

right triangle *zfh*, and thus the ratio of the sine of arc *hf* to the sine of arc *dp* is as the ratio of the sine of arc *hz* to the sine of arc *dz*. From the three known quantities in that proportion, the sine of arc *dp* can be found. With arc *dp* known, its complement, arc *dg*, is also known. Applying that same corollary to right triangle *pfg*, the sine of arc *fp* is to the sine of arc *hd* as the sine of arc *pg* is to the sine of arc *dg*. From this, arc *fp* can be found, and its complement *pz* will also be known. Subtracting known arc *cz* from arc *pz*, arc *pc* will be known.[442] In triangle *cpd*, angle *p* is right and the arcs *pc* and *dp* are known, so arc *cd* will also be known through the spherical Pythagorean theorem in I.16.[443]

Next the commentator shows how to find the longitude between two cities given the distance between them and the direction of one city to the other of which the latitude is known. He proves the case in which city *d*'s latitude and the direction from *c* to *d* is known. Arc *rs* is drawn as the horizon of point *c*. The direction towards city *d* is marked on the horizon, and therefore angle *scr* is known. Because arc *dp* is perpendicular upon arc *cr* in right triangle *rsc*, the corollary can be applied. The sine of arc *rs* is to the sine of arc *dp* as the sine of quarter circle *sc* is to the sine of arc *dc*. From the three known quantities in this proportion, the unknown arc *dp* is found. From the same corollary applied to triangle *fhz*, the sine

[442] Note that arc *pc* cannot be found merely by taking the difference of the two cities' latitudes since arc *dp* is not a parallel; it is an inclined great circle.

[443] Ibid., lines 979-1005.

of arc *dp* is to the sine of arc *fh* as the sine of arc *zd* is to the sine of arc *zh*.[444] The unknown sine of arc *fh* is then able to be found, and that arc is the sought longitude.[445]

He next shows how to find the longitude between the two cities when the distance between them and their latitudes are known. This demonstration involves three uses of the corollary and one of the conjoined sector figure. From using the corollary in right triangle *cft*, the sine of arc *cf* is to the sine of arc *dh* as the sine of arc *ct* is to the sine of arc *dt*. Only the first two terms in this proportion are known, but this is enough to determine the second ratio. Since arc *cd*, the distance between the cities, is also known, he is able to find arc *dt* and arc *ct*. He then uses newfound arc *ct* in an application of the sector figure. Arc *rs* must meet the equator at point *g* since arc *rfp* cuts all three circles *rs*, *kpg*, and *kfg* at right angles, so a sector figure is formed by arcs *cr*, *gr*, *cs*, and *gf*. From the conjoined catha, the ratio of the sine of arc *cr* to the sine of arc *rf* is composed from the ratio of the sine of arc *cs* to the sine of arc *ts* and from the ratio of the sine of arc *gt* to the sine of arc *gf*. Through subtraction of the known composing ratio from the known composed ratio, the ratio of the sine of arc *gt* to the sine of arc *gf* is found. Arc *gf* is known, so the arc *gt* and its complement, arc *tf*, can be found. Through the corollary applied to right triangle *tfc*, the sine of arc *tf*, which was just found, is to the sine of *dp* as the sine of *ct* is to

[444] The last term is missing in all four manuscripts, but after so many uses of the commentator's version of the rule of four quantities, presumably a reader would have been able to fill in what is missing.

[445] Ibid., lines 1007-1022.

the sine of *dc*. From this proportion, arc *dp* is found. Through the corollary again, the sine of arc *dp* is the sine of arc *fh* as the sine of arc *zd* is to the sine of arc *zh*, and from this, arc *fh*, the sought difference in longitude, is found.[446]

In these added sections, the commentator ambitiously attempts to apply the plane and spherical sector figures and his alternatives to a variety of non-astronomical matters. He does make some mistakes—although skilled enough to formulate new proofs, he has not completely mastered the intricacies of spherical geometry. While he once proves the same thing twice, once by the alternatives and once by the sector figure, for another new application, he only gives a proof using the alternatives, and for another he gives a proof that requires the use of the alternatives and the sector figure. He has not completely left behind the sector figure in the new applications, but he does not feel the necessity of giving two sets of proofs for each proposition.

The author of the Erfurt Commentary's systematizing of the material of the *Almagest* echoes what we have seen in the *Almagestum parvum* and in some of the marginal notes on the *Almagest*. The theorems are universalized and divided into parts similar to those of Euclid's *Elements*. Unlike the *Almagest*, this commentary goes into compound ratios in great detail. Denominations provide the basis for his understanding of them. While the commentator universalizes much of Ptolemy's

[446] Ibid., lines 1024-1053.

material, he retains a connection to the practical by including rules for finding the unknown terms in statements of composition and for calculating astronomical values. While a fairly advanced work, this commentary does show signs of the learning process of the author. He makes mistakes and sometimes gives a second way of doing something that he has already shown (e.g., he gives a proof of one case of the spherical conjoined sector figure but then later gives the universal proof of Thabit). Lastly, this work shows that some medieval mathematicians were sufficiently comfortable with the sector figure and its alternatives to formulate new proofs not only for problems that Ptolemy solves by the Menelaus Theorem, but also for new problems.

Chapter 6: The Vatican Commentary

A commentary on the entirety of the *Almagest* is found in two manuscripts in the Biblioteca Apostolica Vaticana. This work's incipit is "Quod perpendicularis cadens ex extremitate arcus super diametrum...," and its explicit is "... que est in capitulo quarto libri tertiidecimi." It covers all thirteen books in order and then has a few short notes on difficult passages throughout the work. Generally it is not organized into theorems as most of the *Almagest* commentaries, and much of it consists of notes linked to specific passages in Gerard of Cremona's translation of the *Almagest*. This commentary is also remarkable for the amount of detail it spends on the Menelaus Theorem; the section on I.12 of the *Almagest* runs to almost 11,000 words. In fact, this section of the commentary contains a full commentary on Thabit's *De figura sectore*. The commentator, who initially makes some errors, shows himself to be a knowledgeable and skilled mathematician who has, as will be clear, a fascination with completeness.

The Manuscripts

Città del Vaticano, Biblioteca Apostolica Vaticana, Vat. lat. 6795 (MS A).[447] This manuscript from the fourteenth century is made of 98 folios of parchment. The text is in one hand and in two columns. The manuscript is made up of quires usually of ten folios. The sixth and seventh of the original quires, which contained the

[447] This manuscript is described very briefly in of the *Inventarium Librorum Latinorum MSS Bib. Vat.,* tome VIII, p. 238.

commentary on the ending of Book V, Book VI, and the beginning of Book VII, are missing. After the end of the long commentary on 97r, there are several short notes on the *Almagest* in the same hand. These start with "De angulis. Duo anguli dicuntur esse ..." and ends on 97v with "... fecerunt sicut dictum est in notulis in libro." These *notulae* may have been marginalia from a manuscript of the *Almagest* that the scribe was reading as he copied out this commentary. This manuscript has some marginalia written in another hand.

Città del Vaticano, Biblioteca Apostolica Vaticana, Vat. lat. 3100 (MS B).[448] This manuscript, which consists of 120 sheets of parchment, appears to be from the early fourteenth century. On f. 1r there is a note saying that it was bought in 1318. The manuscript, which is made up of 14 quires, is written in a number of different hands. The first 7 are all in the same hand, the ninth has several hands in it, the tenth has a different hand for its outermost sheet of parchment, and the thirteenth and fourteenth are in the same hand. The commentary on the *Almagest* begins on f. 1r and continues to 109r. Two works follow the commentary. The first of these, which is found on 109r-110v, is Abū ʿAbd Allāh Muḥammad ibn Muʿādh al-Jayyānī's *De crepusculis*, which has the incipit "Liber Abomadhi malfagir idest crespulo matutino..." and explicit "... nulla est utilitas ideo pretermisi ea."[449] On 112ra-112v (there is no folio numbered 111) there is Alfraganus' *Liber de*

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[448] It is briefly described in the *Inventarium Librorum MSS Bib. Lat.*, tome IV, (Città del Vaticano, Biblioteca Apostolica Vaticana, Sala. Cons. Mss.304), p. 275.

[449] A. Mark Smith, "The Latin Version of Ibn Muʿādh's Treatise 'On Twilight and the Rising of Clouds'," *Arabic Sciences and Philosophy* 2 (1992): 83-132. Smith does not list this manuscript among those containing this work.

aggregationibus scientiae stellarum, which begins with “Numerus mensium anni ...” and ends with “... in hac arte intelligenti. Amen. Explicit liber Alfragani. Deo gratias. Amen.” Neither manuscript contains many diagrams. The author or a subsequent copyist must have thought the sources’ diagrams were sufficient although this commentary does require some alterations or wholly new figures occasionally.

Compound Ratios and Denominations

The commentator’s treatment of compounding is unsurprisingly filled with the use of denominations and the multiplication and division of ratios. However, he defines compound ratio without denominations; he writes, “A composed ratio is one which is between two quantities between which there are other quantities proportional to them.”[450] Although this definition does not explicitly mention that the composing ratios are continuous, the insertion of a “middle” or “middles” between two quantities amounts to the same thing. He later explains this further, writing, “And when ‘middles are placed’ is said, it must be understood that the ratio of the first of those between which are those middles [to the second] is composed of the ratio of that first to that which is after it and the ratio of that to another and thus in order until it comes to the last. And this is to place a quantity or quantities as middles in ratio between others.”[451]

[450] Appendix F, lines 9-10.

[451] Ibid., lines 273-7.

Despite the commentator's theoretical definition of continuous ratio, the rest of his use of compound ratios makes almost no use of it. In fact, immediately afterward, he explains how one works with compound ratios by giving an example in which he multiplies the denominations of ratios and even blurs the distinction between fractions and ratios; he writes, "The ratio of two to twelve is composed from the ratio of two to four, which is a half, and the ratio of four to twelve, which is a third. For a half is multiplied by a third and a sixth results. And it is the ratio of two to [twelve]."[452] Although this may be shorthand, he is saying that a fraction *is* a ratio. He does use and explain the insertion of middles in his commentary on Thabit's proofs of the modes, but he almost never uses middles or continuous ratios when not directly following Ptolemy or Thabit.

In fact, in the first application of the Menelaus Theorem and the first use of a statement of composition with actual astronomical values (I.13), the commentator explains how to find a ratio by dividing the antecedent by the consequent.[453] The number that is the quotient of the division *is* the ratio according to this commentator. He gives another very detailed account of this in his commentary on I.14 too. In his explanations of the processes of finding an unknown term when a ratio and the other term are known, both when the unknown is the antecedent and when it is the consequent, he treats the quotient of the division as the ratio. Perhaps

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[452] Ibid., lines 11-5.

[453] Ibid., lines 919-923.

because his focus in these passages is on practice and calculation instead of theory, he is not concerned about breaking the typical categorical boundaries.

Since he only pays initial lip service to the continuity conception of compound ratio, he feels that he needs to give an explanation of Ptolemy's recurring use of the subtraction of a known composing ratio from a known composed ratio to find the other, unknown composing ratio. He writes, "And finally it must be known that to subtract a ratio from a ratio is nothing other than to divide one ratio by the other and to take that which results, and thus Ptolemy understood [it] in this chapter."[454] That he defines subtraction of ratios this way harmonizes with his common ways of treating compound ratios although a definition by continuous ratios would correspond better to his definition of compound ratio, which used continuous ratios. He instructs the reader how to perform the multiplications and divisions necessary for the subtraction of ratios in his commentary on I.13 of the *Almagest* and he also returns to this issue of "what it may be to subtract a ratio from a ratio" near the end of Book II.[455] In the latter passage, he lays out the problem in general terms: if there are six quantities and the ratio of the first to the second is composed of the ratios of the third to the fourth and the fifth to the sixth, and one of the last four terms is unknown, how do you find the ratio containing the unknown quantity? He then gives the order of multiplications and divisions needed to find the unknown value: first, divide the

[454] Ibid., 968-970.

[455] Ibid., lines 919-37, 2000-2023.

first quantity by the second; second, divide the antecedent of the known composing ratio by its consequent; third, divide the first quotient by the second to reach the value of the unknown ratio. His justifications in Book I and in this later note are essentially the same; in I.13 he writes, "And this [is] thus because the second ratio is multiplied into this [the third ratio] and the first results… Therefore when the first ratio is divided by the second, the third results."[456] Although the word "denomination" is used in neither place, this explanation of how to deal with statements of composition relies just as much and probably more upon the concept of denominations than any other found in works related to the Menelaus Theorem. What he essentially is doing here is finding the denominations of the two ratios and then dividing the composed ratio's denomination by the composing ratio's denomination. This last step makes sense because composing ratios are multiplied together to reach a composed ratio and because multiplication and division are inverse operations (or in more Euclidean terms, because if the product of two quantities is divided by one of them, the other will result).

The commentator's identification of ratios with quantities is seen also in his proof of the validity of one of the modes, in which he explains that the ratio composed of the ratios of *e* to *d* and *d* to *e* is the number one. Uncharacteristically, he translates this into a generalized statement to be proved: "If any quantity is divided by another quantity, and the result is saved, and then the dividing quantity is divided by the divided quantity, and the result is saved, and then the first saved

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[456] Ibid., lines 931-4.

quantity is led into the second saved quantity, nothing results except one."[457] This restatement of the proposition he wants to prove does not even mention ratios or compound ratios—all the uses of ratio and compounding are replaced by talk of quantities, multiplication, and division. The ratios of *e* to *d* and *d* to *e* are now understood as the "results" of divisions, and these quantities are multiplied together to produce the number one, which is the composed ratio. He gives a proof using different quantities. He takes two quantities *a* and *b*, and *g* and *d* are the two saved quotients. He wants to prove that *g* times *d* is one. Since *g* times *b* is *a*, and one times *a* is *a*, he formulates the proportion that *g* is to one as *a* is to *b*. Similarly, because *d* times *a* and one times *b* are both *b*, *a* is to *b* as one is to *d*. From the two proportions, *g* is to one as one is to *d*. Therefore, *g* times *d* is one.[458]

An extra note that deals yet again with the subtraction of ratios, probably by a second commentator, is found at the end of both manuscripts. This second commentator feels that the topic "was not explained well."[459] After pointing out that "proici" and "minui" are synonyms,[460] he gives a new set of rules for subtracting a ratio from another. He orders the quantities such that the ratio of the first to the second is the ratio from which the ratio of the third to fourth is subtracted. Then he multiplies the first by the fourth and the second by the third. The first of these products is divided by the other, and the quotient is the ratio that

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[457] Ibid., lines 287-290.

[458] Ibid., lines 290-309.

[459] Ibid., lines 2144-5.

[460] Ibid., lines 2151-2.

results from the subtraction.[461] While this set of directions makes no appeals to either conception of compound ratio, it can be justified by either. While it seems that perhaps the second commentator disliked the first commentator's way of subtracting ratios because of its close connection to the denominative conception of compounding, this was probably not the reason for revisiting this topic since he follows this with a rule for how to find the unknown quantity given a known ratio between an unknown and a known quantity, in which he equates ratios with denominations.[462]

The commentator's treatment of compound ratios mixes the two traditions. Although he defines compounding according to continuous ratios and sometimes follows his sources in reasoning from continuity, when left to his own devices, he usually treats ratios as numbers and compounding as multiplication.

The Modes

As noted previously, the commentator includes a commentary on Thabit inside of his commentary on I.12 of the *Almagest*. Apparently, he realized the relevance of Thabit to the Menelaus Theorem and read and commented upon Thabit's *De figura sectore* before continuing his reading of the *Almagest*. Since Thabit includes much on the valid and invalid modes of a statement of composition, it is not surprising that this commentator has a lengthy section on

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[461] Ibid., lines 2156-2163.

[462] Ibid., lines 2163-7.

them. While Thabit jumps right into proofs of the eighteen valid modes, this commentator first enumerates the possible combinations and the relatively small number of combinations that are actually true given one statement of composition. Thabit had scattered his treatment of the number of combinations throughout his discussion of the modes, but the commentator collects this information in one location. He explains that there are fifteen different possible pairs of the first pair of the six quantities, and that only nine of these fifteen have valid modes.[463] After stating that for each pair there are twelve modes of the remaining four quantities, only two of which are valid, he lists the twelve different arrangements without saying which are valid.[464] He explains that since there are nine possible pairs and each has twelve modes, there are 108 possible modes, but since only two of the modes of the last four terms are valid, only eighteen modes are valid and the other ninety are invalid.[465] Interestingly, he does not mention the total number of logically possible combinations of the six terms.

The commentator treats Thabit's proofs of the eighteen valid modes. Noting that the first mode is assumed, he immediately moves on to the second. He relies heavily on Thabit; his proofs follow the lettering and structure of Thabit and sometimes even take phrases and sentences directly from Thabit. It is clear that the commentator expects his reader to have Thabit accessible because for many of the

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[463] Ibid., lines 236-244.

[464] Ibid., lines 249-262.

[465] Ibid., lines 263-7.

modes, he does not prove their validity but merely states that Thabit's text is clear enough. Also in his explanation of one of Thabit's proofs (that there are no valid modes with the composed ratio of *a* to *d*), he gives the proof but neglects to point out a crucial assumption (that lines *a* and *z* are equal).[466] Without this, the proof makes little sense, but this omission is understandable if he expects his readers to have Thabit's work open in front of them.

Despite having defined compound ratio according to the continuous conception and despite following Thabit in using the insertion of middles to get a statement of composition, the commentator uses the multiplication of ratios to explain certain steps of Thabit. For example, in the proof of the second mode, Thabit says that the ratio composed from the ratios of *g* to *d*, *d* to *e*, and *e* to *d* is the same as the ratio of *g* to *d*, which is immediately clear given the continuous understanding of compounding, but the commentator explains that this is so because the ratio composed of *d* to *e* and *e* to *d* is one, and when the ratio of *g* to *d* is multiplied by one, the result is *g* to *d*.[467]

The commentary on the section of Thabit's work in which he proves that there are only eighteen modes is difficult, and at parts is detailed while at other parts it adds little to Thabit's text, which is understandable since some of Thabit's proofs are much more difficult than others. The commentator devotes much more space and energy for the more complex arguments, such as Thabit's arguments for

[466] Ibid., lines 361-396.

[467] Ibid., lines 281-6.

the invalidity of any mode with the first and the fourth in the first two spots, while he quickly covers some passages that he feels are already clear enough in Thabit's text, such as the proofs of the invalidity of the other five impossible combinations of first two terms. He does, however, devote much attention to the invalidity of ten of the twelve possible arrangements of the last four terms. Unlike Thabit, the commentator goes through almost all of them. He treats the possible permutations in alphabetical order (skipping the last possibility), unlike Thabit who groups them into three main categories, of which he treats only one group, the one with ratios between *g* and *d* and between *e* and *u*, in detail. He treats another of the groups, those with ratios between *g* and *e* and between *d* and *u*, with one brief explanation, and he gives no proof of the invalidity of the group with ratios between *g* and *u* and between *d* and *e*. His proofs are generally similar to Thabit's, but Thabit classifies the possibilities by the pairs of terms in a ratio no matter which is the antecedent or the consequent, while the commentator is always specific. Thabit's treatment is much shorter and concise. For example, he is able to disprove four possible arrangements in one brief argument,[468] while the commentator argues these cases separately and differently (and with some difficulty).[469]

[468] If *a* to *b* were composed of a ratio between the terms *g* and *e* and a ratio between the terms *d* and *u*, then there would be a valid combination for the first two positions with either the terms *g* and *e* or *d* and *u*; however, he has shown that these are two of the six impossible first pairs in the sixth terms of a statement of composition.

[469] In his fifth mode he shows that the ratios *g* to *e* and *d* to *u* are not a valid set of the last four terms in a mode. His proof is a *reductio ad absurdum*, and he begins by assuming that it is true that *a* to *b* is composed of the ratios of *g* to *e* and of *d* to *u*. Taking that in tandem with the first mode, then the ratio compounded of *g* to *e* and *d* to *u* is the same as the ratio compounded of *g* to *d* and *e* to *u*. By inserting *d* and *e* as middles between the first two ratios, he expands it to the statement that the ratio compounded of *g* to *d*, *d* to *e*, *d* to *e*, and *e* to *u* is equal to the ratio compounded of *g* to *d* and *e* to *u*.

While a full treatment of the enumeration of possible arrangements of six terms and the determination of the valid and invalid statements of composition is not needed for the use of compound ratios in the *Almagest,* this commentator follows Thabit in treating the subject in great detail. Apparently, once he had decided to use Thabit's work to understand compound ratios, he wanted to understand them fully. As in the Erfurt Commentary's sections on range-finding and geographical problems, an understanding of astronomy is not the only goal of this commentary.

The Many Cases of the Menelaus Theorem

As with compound ratios, the author of the Vatican Commentary treats the different cases of the Menelaus Theorem in great depth. To explain Ptolemy's proof of the proposition, he turns to Thabit's *On the Sector Figure*, which leads

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That means that the ratio compounded of the ratios of *g* to *d* and *e* to *u* is equal to the ratio compounded from the very same ratios *and* the ratio twice of *d* to *e*. The commentator concludes: "Therefore, to multiply that [the ratio composed of *g* to *d* and *e* to *u*] into it [*d* to *e*] twice is equivalent to multiplying it into one, and because to multiply it into one does not make it different because nothing other results, then that multiplication is superfluous. And because it is superfluous, then it is not true that the ratio of *a* to *b* is composed of the ratio of *g* to *e* and of the ratio of *d* to *u*." (Ibid., lines 472-500).

The commentator could have reached his conclusion more clearly by stating that since the ratio of *d* to *e* is a ratio of equality, then *d* is equal to *e*, which is against what was assumed, but he makes this unclear jump from superfluity to falsity. What he seems to have in mind is that since one thing (the ratio composed of *g* to *d* and *e* to *u*) is the same as the product of itself and another ratio (the ratio composed of *d* to *e* and *d* to *e*), then the ratio of *d* to *e* must be one of equality although it was assumed that each of the six terms are different. The language of superfluity, however, makes this rather unclear and perhaps the commentator had trouble seeing exactly what the absurdity was to which his reduction led. The confusion of the commentator is also evident in a numerical example that he gives to ostensibly make the proof more apparent but that is a complicated string of multiplications that does not work as an example of the proof or any of its steps (lines489-493). Understandably, a reader had difficulty with this passage and tried to explain through another numerical example that "the superfluity was true" (line 488 apparatus).

him to a lengthy examination of the many different cases of the proofs for the conjoined sector figure.

The commentary on the Menelaus Theorem begins by following the *Almagest* fairly closely. The explanations of the lemmas are brief; for example, the commentator merely says for the third, "What is in the first circle follows because of the similitude of the triangles, and this because of the line falling across parallel lines."[470] He does add explanations when he believes that Ptolemy's steps are not clear or precise enough. His concern for precision is seen in the second lemma, in which he explains that to get from the proportions one would obtain by using similar triangles to the proportions that Ptolemy uses, one also has to alter those original proportions by taking the ratios *dividendo, componendo,* and *convertendo*.[471] In the fourth lemma, he explains how one can use Ptolemy's table of chords to find the angles of a right triangle when the lengths of its sides are known.[472]

The commentator then addresses the proof of the spherical sector figure and at this point his writing becomes a commentary on Thabit's *De figura sectore* within this commentary on the *Almagest*.[473] While he does not give the proof that

[470] Ibid., lines 29-30.

[471] Ibid., lines 25-8. Euclid explains the use of these terms in definitions 14, 15, 16 of Book V of the *Elements*.

[472] Appendix F, lines 33-41.

[473] Although the jump to Thabit's work is not marked explicitly, he does begin to follow Thabit's text closely and refers to Thabit. Also, the notes are introduced by phrases from *De figura sectore* marking the section that the note is explaining.

Ptolemy does, he carefully explains several aspects of the theorem and the different cases of it. He points out why the arcs in the figure must all be less than semicircles, explains why the figure is called the "sector figure," and notes that there are two propositions of the sector figure, one *compositionis* and one *divisionis*. These terms for the conjoined and disjoined propositions in the genitive forms of nouns are reminiscent of Thabit's terms, *compositionis* and *dissolutionis*.[474] He explains how each of these two *modi* of the sector figure are divided into three different cases depending on whether lines *ad* and *bh* meet as they do in Ptolemy's proof, they meet on the other side of the diagram, or they are parallel. The commentator makes this division very logically: "For in the figure they [lines *ad* and *bh*] either are parallel or they are not parallel. And when they are not parallel, they meet either on the left side as in the figure which Ptolemy provides or on the right side."[475] As we will see, however, this division for the conjoined spherical sector figure is not the important way of dividing the theorem into cases. The commentator assumes that since the disjoined sector figure is divided into three cases according to the ways that lines *ad* and *hb* meet, then also the conjoined sector figure can be divided the same way. Of course it is possible to make this division into cases, but for effective proofs, the division into cases will have to be done in another way for the conjoined sector figure. The commentator at this point in the text had apparently not yet tackled the conjoined sector figure in depth.

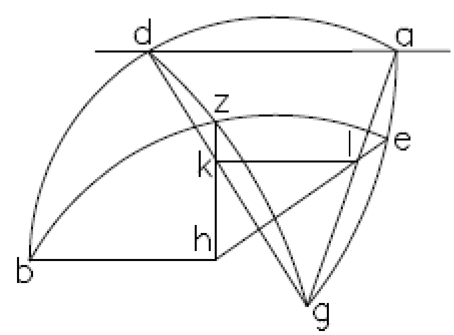
[474] Björnbo, *Thabits Werk*, p. 8, line 14.

[475] Appendix F, lines 57-60.

Since Ptolemy's proof is sufficient for the first case, he moves on to the case where they meet on the right side of the diagram.[476] He follows the proof of Thabit here although he keeps the letters from Ptolemy's diagram and has point *m* where Thabit has point *k*.[477] This version of the proof also differs from Thabit's in that it uses sines while Thabit's proof uses chords of double arcs. Although the author seems to not have been well versed yet in the nuances of the conjoined sector figure, he did know enough about the modes to point out that this proof of Thabit uses the seventh mode.

For the case where lines *ad* and *hb* are parallel, the commentator continues to follow Thabit's proof but with the letters from the Gerard of Cremona *Almagest*.[478] While Thabit's reasoning for why the lines *ht* and *bd* are parallel is rather unclear, the commentator explains more clearly why lines *kl* and *ad*, which correspond to Thabit's *ht* and *bd*, are parallel:

> It will be proved first that line *kl* is parallel to line *ad* in this way. It [line *kl*] is with it [line *ad*] in the same plane because they are in the plane of triangle *gad*. Therefore, if it is not parallel to it, then it would meet with it when extended. But because line *kl* is in the plane of circle *bze* and line *ad* is in the plane of circle *adb* which planes cut each other, they are able to meet in no way other than in their common section. But, the common section of the planes of these two



[476] Ibid., lines 60-78.

[477] Ptolemy does not have this point since it is not needed for the case that he proves.

[478] Appendix F, lines 80-111.

> circles is line *hb*. Therefore, the two lines *kl* and *ad* would meet upon it. This, however, is impossible because line *ad*, as was supposed, is parallel to line *hb*. Therefore line *ad* is parallel to line *kl*.[479]

The next steps are essentially Thabit's. Because *kl* and *ad* are parallel in triangle *gad*, line *gl* is to *la* as *gk* is to *kd*. Using the lemmas to the sector figure, the commentator is able to move from this proportion to a proportion about sines: that the sine of arc *ge* is to the sine of arc *ea* as the sine of arc *gz* is to the sine of arc *zd*.[480] He then seems to make a mistake, stating, "And because between two arcs *ga* and *ba*, two arcs *be* and *gd* intersect as was posited before, then the ratio of the sine of arc *ge* to the sine of arc *ea* is composed of the ratio of the sine of arc *gz* to the sine of arc *zd* and of the ratio of the sine of arc *db* to the sine of arc *ba*."[481] In doing so, he is calling upon the conclusion that he is trying to reach. He then explains that the ratio composed of the two composing ratios is the same as the ratio of the sine of arc *gz* to the sine of arc *zd* because the other ratio is a ratio of equality. He explains, "The ratio of one to the other is one, and when one is multiplied into any number, it is not changed. For this reason when the ratio of the sine of arc *db* to the sine of arc *ba* is multiplied into the ratio of the sine of arc *gz* to the sine of arc *zd*, nothing emerges except the same ratio."[482] Note that he identifies a ratio of equality with the unit and that he multiplies ratios. Although compounding has been defined by continuous ratios, the commentator here, as in

[479] Ibid., lines 80-9.

[480] Since the ratios of sines of arcs are the same as the chords of the doubles of the same arcs, the lemmas can be easily modified to be about sines.

[481] Appendix F, lines 95-8.

[482] Ibid., lines 103-7.

other places, uses ideas that are more in line with the denominative method of compounding.

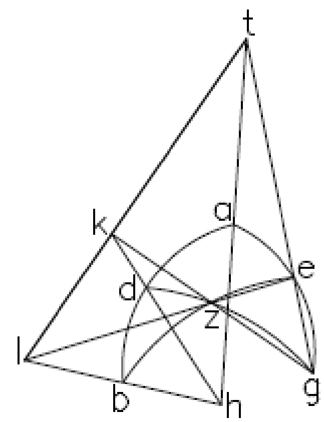
For the conjoined sector figure, the commentator first gives what is essentially Thabit's proof albeit with sines and with the initial points lettered as in the *Almagest*.[483] Although this proof is universal, succinct, and clear, he decides to prove it in another more complicated manner. As Thabit had argued, Ptolemy did not intend the reader to prove the conjoined case as Thabit did, so the commentator adds the proofs that he thinks Ptolemy did intend, ones that rely upon the plane conjoined sector figure. He initially attempts to do this in proofs of three cases, but he makes a crucial mistake. Unlike earlier, he now realizes that dividing the cases by how lines *ad* and *hb* meet or do not meet is not useful—how lines *ha* and *ge* meet or do not meet is the important criterion for dividing the proof into cases. He is partially correct, but there are subcases for each of these three cases.[484]

His proofs of the conjoined sector figure through the plane sector figure are similar to the one we have seen in the *Almagestum parvum*, but the author of the Vatican Commentary is more thorough than the author of the *Almagestum parvum*, who simply gave an outline of the proof. In the first proof, after he sets up the initial figure as Ptolemy does with arcs *aeg*, *adb*, *bze*, and *gzd* forming a sector figure, he draws radii *ha*, *hd*, and *hb* from point *h*, the center of the sphere, and he points out that these three lines are in one plane. The three lines *ge*, *gz*, and *ez* are

[483] Ibid., lines 112-132.

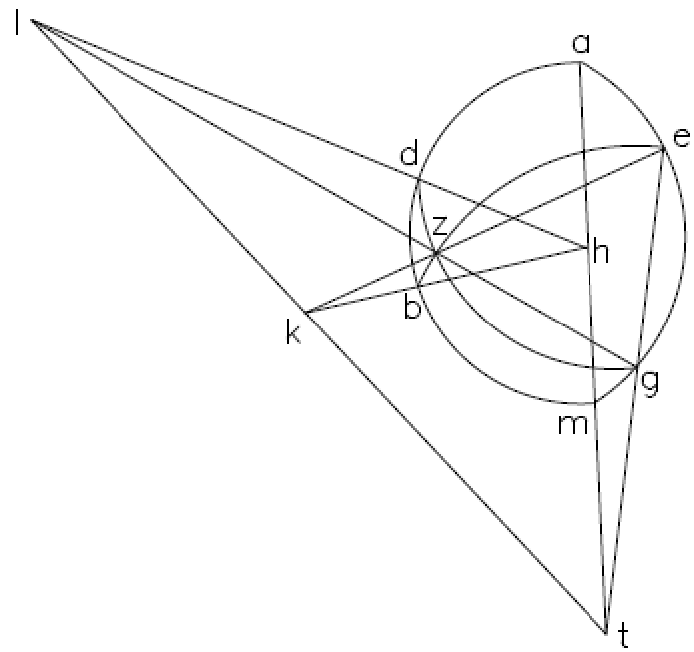
[484] Ibid., lines 133-220.

then produced. According to the commentator at this point in the text, the three cases are determined by how lines *ha* and *ge* meet or do not meet. In the case where they meet at the top of the diagram, they are extended until they meet at point *t*. The commentator next states that when extended, the lines *hb* and *hd* will meet *ez* and *gz* respectively at points *l* and *k*; however, depending on the sizes of arcs, it is possible for either line *gz* or *ez* to not meet *hd* or *ha* respectively or for them to meet on the "other side", i.e. lines *ez* could meet line *hb* on the side of point *e* instead of point *z*. This is clearly a mistake on the commentator's part and not merely a conscious oversimplification to save space, because he later returns to the division of the cases of the conjoined sector figure and redivides and proves all the cases correctly. After this initial error, his reasoning has no problems. He argues that points *t*, *k*, and *l* must be on a straight line because lines *ha*, *hd*, and *hb* are in one plane and the lines *ge*, *gz*, and *ez* are in one plane, and the intersection of these two planes must be a line. A plane sector figure is then formed by lines *tl* and *tg* meeting at point *t* and by lines *gk* and *le*, which intersect between them. However, since points *k* and *l* do not necessarily exist, and if they do, they are they not necessarily on the side of the diagram that the commentator assumes they are, it is not necessary that this plane sector figure exists or that any sector figure is formed. Using the plane conjoined sector figure, the ratio of line *gt* to *te* is composed of the ratio of line *gk* to *kz* and of the ratio of *zl* to *le*. The commentator uses the fifth



lemma to show that the ratio of the sine of arc *ga* to the sine of arc *ae* is composed of the ratio of the sine of arc *gd* to the sine of arc *dz* and the ratio of the sine of *bz* to the sine of *be*.[485]

After he considers the next case, in which lines *ah* and *ge* meet opposite point *a*, without any new major difficulties or errors (besides the commentator's initial assumption that once lines *ha* and *ge*'s arrangement is given, the arrangement of *gz* and *ez* with *hd* and *hb* respectively are given as well),[486] he proceeds to the third case, in which lines *ha* and *ge* are parallel.[487] In this he has trouble arguing that lines *gz* and *hd* must be parallel and that lines *ez* and *hb* must also be parallel, which is not surprising since it is not true. He states that it is so "... because since the first two lines *ge* and *ha* are parallel,



[485] Ibid., lines 152-174.

[486] Ibid., lines 175-201. Arcs *ag* and *ab* are extended downwards until they meet at point *m*, forming semicircles *agm* and *abm*. The diameter *ahm* is extended until it meets line *eg* at point *t*. Lines *ez* and *gz* meet lines *hb* and *hd* at points *k* and *l*. Again, there is a plane sector figure formed by lines *te*, *tl*, *gl*, and *ke*. Therefore, the ratio of line *et* to *tg* is composed of the ratios of *ek* to *kz* and *lz* to *gl*. Using the proportions from the other preliminary theorems and the equality of sines of supplementary arcs, the commentator reaches the statement that the ratio of the sine of arc *ae* to the sine of arc *ga* is composed of the ratios of the sine of arc *eb* to the sine of arc *bz* and of the sine of arc *dz* to the sine of arc *gd*. By inverting this, the sought conclusion is reached.

[487] Ibid., lines 202-220.

then the different planes in which they lie are parallel, and all the lines that are in those different planes are likewise parallel. Therefore, all the aforesaid lines are parallel, although they are not able to be parallel unless any two of them are in the same surface."[488] Of course, if there are two parallel lines, the planes in which these lines lie are not necessarily parallel.[489] As we will see later, the commentator came to realize that he had not imagined all the possible cases and that his reasoning was flawed in these first three attempts to prove the conjoined sector figure via the plane version. At this point in his treatment of the sector figure, however, the commentator is confused and he even makes another logical mistake. He points out the sector figure *abg*, and then states that therefore what he is trying to prove is true.[490]

The commentator realizes his mistake about the number of possible cases of the conjoined sector figure after his long treatment on the modes when he returns to Thabit's claim that there are thirty different cases of the sector figure and that sixteen of these are true and fourteen are false.[491] Three of these cases are for the disjoined sector figure, and have already been shown. The commentator then lays

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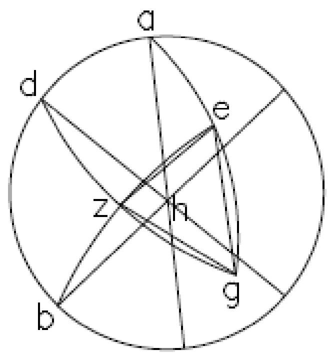
[488] Ibid., lines 203-8.

[489] That the conclusion that the commentator wants to reach, which is that if *ha* and *ge* are parallel, then *gz* and *hd* are parallel, and *ez* and *bh* are parallel, is not true can be easily seen. Imagine that points *a*, *b*, *g*, and *e* are all fixed on the surface of a sphere. By choosing different locations for point *d*, we will have new point *z*'s where the great circles through *g* and *d* and through *b* and *e* meet. Line *gz* can thus be made to meet *dh* anywhere along it.

[490] Appendix F, lines 209-213.

[491] Bjornbo, p. 8-9, section 3.

out the twenty-seven other cases for the conjoined sector figure.[492] He takes Ptolemy's diagram of the sector figure, and completes circle *adb* and the semicircles of arcs *aeg*, *gzd*, and *bze*. He then draws and extends lines *ge*, *gz*, and



ez. The twenty-seven different cases are determined by the way that each of these three lines in triangle *egz* meets or does not meet its corresponding diameter in the plane of circle *adb*. Since each of these three lines can meet the diameter on either side or be parallel to its diameter, there are twenty-seven different logical possibilities; however, only thirteen of these cases are geometrically possible, and the remaining fourteen are not possible.

The manner in which the commentator describes these different cases uses what appears to be a technical vocabulary of "sides", but it is sometimes ambiguous. For example, the first case is when the three lines all meet "on the side of *a*." Since *ge* is in the plane of circle *aeg*, it is clear which side of it is the side of *a*, but it is not obvious whether line *ez* is considered to meet its diameter on the side of point *a* when it meets the diameter on the side of *e* or the side of *z*. Likewise, it is not clear whether line *ze* meets its diameter "on the side of *a*" when it meets the diameter on the side of *e* or on the side of *z*. It is only by reading the proofs or disproofs that each case can be clearly distinguished.

[492] Appendix F, lines 564-600.

The commentator divides the cases into three main groups depending on whether the line *eg* meets its diameter on the side of *e* or *g* or is parallel to its diameter. Each of these groups is divided into three subgroups depending on how line *gz* meets or does not meet its diameter and these subgroups are further divided into three cases depending on whether line *ez* meets *hb* on the side of *e*, *ez* meets it on the side of *z*, or *ez* is parallel to it. The following list gives the enumeration of cases by the relation of the three lines *eg*, *gz*, and *ez* to their respective diameters, the validity of each case, and the location of the proof or disproof of the validity of each case in the critical edition.

	EG	*GZ*	*EZ*	Validity	Line #
1.	side *e*	side of *z*	side of *e*	true	601-624
2.	side *e*	side of *z*	side of *z*	true	625-634
3.	side *e*	side of *z*	parallel	true	635-651
4.	side *e*	parallel	side of *e*	true	652-672
5.	side *e*	parallel	side of *z*	false	673-677
6.	side *e*	parallel	parallel	false	677-682
7.	side *e*	side of *g*	side of *e*	true	683-712
8.	side *e*	side of *g*	side of *z*	false	713-718
9.	side *e*	side of *g*	parallel	false	718-720
10.	side *g*	side of *z*	side of *z*	true	725-742
11.	side *g*	side of *z*	side of *e*	false	743-746
12.	side *g*	side of *z*	parallel	false	746-750

13.	side *g*	side of *g*	side of *e*	true	752-772
14.	side *g*	side of *g*	side of *z*	true	773-793
15.	side *g*	side of *g*	parallel	true	794-814
16.	side *g*	parallel	side of *e*	false	815-821
17.	side *g*	parallel	parallel	false	821-822
18.	side *g*	parallel	side of z	true	822-835
19.	parallel	side of *z*	side of *e*	false	835-845
20.	parallel	side of *z*	parallel	false	845-847
21.	parallel	side of *z*	side of *z*	true	847-862
22.	parallel	side of *g*	side of *b*	false	863-867
23.	parallel	side of *g*	parallel	false	867-869
24.	parallel	side of *g*	side of *e*	true	869-883
25.	parallel	parallel	side of *e*	false	884-889
26.	parallel	parallel	side of *z*	false	884-889
27.	parallel	parallel	parallel	true	890-900

With the exception of Cases 25 and 26, of which the invalidity is shown together, each case is given its own treatment. The proofs require a number of different approaches. In many of the valid proofs (Cases 1, 2, 7, 10, 13, and 14), a plane sector figure is formed in the plane of triangle *gez* by the extended lines of the triangle and the line of intersection between the planes of triangle *gez* and circle *adb*. Using the plane conjoined sector figure, a statement of composition is reached between six lines. Each ratio of lines in this statement of composition is then changed to a ratio of sines of arcs through Ptolemy's fifth lemma. In many cases, the sines in the statement are of arcs that are supplements of the desired arcs, so the commentator converts these. In Cases 1, 7, and 13, he uses the eighth mode of Thabit to reach the sought conclusion from a statement of composition with the desired terms but not in the desired order.

In many of the valid cases, a plane sector figure cannot be formed. In all of these except Case 27, there is either a triangle cut by a line parallel to the base (Cases 3, 18, and 24) or two lines crossing between two parallel lines (Cases 4, 15, and 21), so the commentator is able to reach a proportion. In Cases 3, 4, 15, and 18, one side of the proportion is compounded with a ratio of equality to reach a statement of composition. As an example, in the third case, commentator writes:

> And the ratio of the sine of arc *gd* to the sine of arc *dz* is as the ratio of line *gt* to line *tz*. And the ratio of the sine of arc *bz* to the sine of arc *be* is one because each sine is equal to the other because they are between parallel lines. If therefore the ratio of the sine of arc *gd* to the sine of arc *dz* is multiplied into that [ratio of the sine of arc *bz* to the sine of arc *be*], nothing results except the ratio of the sine of arc *gd* to the sine of arc *dz*. For that reason the ratio of the sine of arc *ga* to the sine of arc *ae* is composed of the ratio of the sine of arc *gd* to the sine of arc *dz* and of the ratio of the sine of arc *bz* to the sine of arc *be*.[493]

Note that he starts with a proportion and a proportion of equality, which he identifies with the unit one, and then he multiplies one ratio in the proportion by the ratio of equality. Multiplication of ratios without the mediation of denominations leads directly to composition of ratios.

[493] Ibid., lines 643-650.

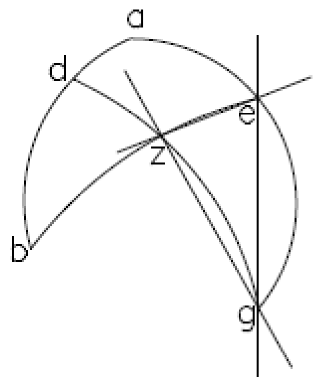
In Cases 21 and 24, another technique is used to reach the desired conclusion because the proportion is not conveniently turned into the desired statement of composition by merely the composition of one side of the proportion with a ratio of equality. In the proof of Case 21, the commentator argues:

> The ratio of the sine of arc *dz* to the sine of arc *gd* will be as the ratio of the sine of arc *bz* to the sine of arc *be*. But, the ratio of the sine of arc *dz* to the sine of arc *gd* and the ratio of the sine of arc *gd* to the sine of arc *dz* is one. Therefore the ratio of the sine of the arc *bz* to the sine of arc *be* and the ratio of the sine of arc *gd* to the sine of arc *dz* is one. And the ratio of the sine of arc *ga* to the sine of arc *ae* is one because the sines are equal. Therefore the ratio of the sine of arc *ga* to the sine of arc *ae* is composed of the ratio of the sine of arc *gd* to the sine of arc *dz* and of the ratio of the sine of arc *bz* to the sine of arc *be*.[494]

In the second and third sentences of this passage, he is making statements about compound ratios although the language is unusual. He is saying that the ratio of the sine of arc *dz* to the sine of arc *gd* compounded with the inverse of that ratio is a ratio of equality. He merely uses the word "and" to signify compounding and he uses "one" as equivalent to "a ratio of equality." He then can substitute a ratio from the proportion in the first sentence and a ratio of equality into the statement of composition to reach the conclusion. The use of "and" for compounding is interesting here since it seems to show that the commentator does retain some idea of compounding as akin to addition while he is multiplying ratios to compound them.

[494] Ibid., lines 853-862.

The last case has a unique proof. Since all three of the lines of triangle *gez* are parallel to their corresponding diameters in circle *adb*, the relevant ratios are all



ratios of equality. The sine of arc *ga* equals the sine of arc *ae*, the sine of arc *gd* equals the sine of arc *dz*, and the sine of arc *bz* equals the sine of arc *be*. The commentator immediately reaches the conclusion since it is clear to him that a ratio of equality is composed from ratios of equality, or, perhaps in terms in which he was more likely to think, a ratio of one times a ratio of one is a ratio of one. Earlier in the work, the commentator attempted this proof but made several mistakes.

While we have been focusing on the proofs of the valid cases, the invalid cases also deserve attention. The matter of importance here is the way that a plane can be inclined towards another. Given information about how two lines in that plane meet or do not meet the other plane, the commentator shows that some inclinations of a third line in that plane to the corresponding line in another plane are not possible. Although all but the last two get their own proof, the invalidity is shown in two ways. In the first, which is used for Cases 5, 8, 11, 12, 16, 19, 22, the basic principle is that if in a plane there is a line parallel to another plane, then the first plane can only meet the second plane on one side of this line. The wording is once again difficult. For example, in the fifth case, the proof simply reads, "Because when the plane of triangle *gez* meets the lower plane on the side of point *a* with line *gh*, then it would meet the same plane on the other side with line *ze*, which is impossible."[495] Although the obscure language of sides of points is used, other formulations of the same proof make it clearer. The proof eventually gets reformulated in terms of lines between others, and by the last of these proofs, it has been reduced to "... because the first line would be a middle that is parallel [to its diameter]."[496] What he means by "sides" is even more difficult to see in disproofs of cases in which none of the lines of triangle *gez* are parallel to the lower plane. For example, for the eighth case he merely states that line *ez* cannot meet its respective diameter on the side of *z* because if it did, "then the plane of the triangle would meet the lower plane at two opposite sides, which is impossible."[497]

The other cases, 6, 9, 17, 20, 23, 25, and 26, are proved with another argument. It is explained most clearly in the sixth case: "Again it [line *ez*] is not able to be parallel because the plane in which it would be parallel with line *gz*, would be parallel to the lower plane, but that meets it through line *gh*. Therefore it would be parallel to it and meet it, which is impossible."[498] In other words, since two of the lines in the plane of triangle *gez* are parallel to the plane of circle *adb*, the planes are parallel and no line in them can meet the other. Although this argument immediately disqualifies all these cases that have two lines parallel to their diameters and one meeting its diameter, the argument is given, albeit

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[495] Ibid., lines 675-7.

[496] Ibid., lines 866-7.

[497] Ibid., lines 716-8.

[498] Ibid., lines 677-681.

condensed, for each case except the last two of these, which are proved together in one sentence.

Applications of the Sector Figure and Mathematical Style of the Commentary

After the incredibly lengthy section on the sector figure, which ends with these proofs of the validity and invalidity of the twenty-seven cases of the conjoined spherical sector figure, the remainder of the commentary is on the astronomical content of the *Almagest*. The commentator does not give the entire proofs of Ptolemy, but he generally summarizes each chapter and explains some of the more difficult parts of Ptolemy's proofs in general terms, not in terms of the letters that Ptolemy assigns to each quantity in his figure. Since the sector figure has been treated so thoroughly, he does not feel any great need to explain the applications of it, and of some applications he merely says, "This all is easy."[499] The commentator sometimes explains how to get from a statement of composition to the knowledge of the unknown term in it; his process is to divide the known composed ratio by the known composing ratio to reach the other composing ratio. For example, in the section on oblique ascensions, he writes:

> And because the ratio crd. arc 2*th* to crd. arc 2*hz* is known, then because it is composed of the ratio of crd. arc 2*lk* to crd. arc 2*kz*, which also is known, and of the ratio of crd. arc 2*te* to crd. arc 2*el*, which is unknown, if the ratio

[499] E.g., while discussing some of the applications in the *Almagest* VIII.5-6, he twice writes, "Istud totum facile est." (MS A, fol. 55v, 56v).

> of crd. arc 2*th* to crd. arc 2*hz* be divided by the ratio of crd. arc 2*lk* to crd. arc 2*kz*, what results will be the ratio of crd. arc 2*te* to crd. arc 2*el*.[500]

As in many other places in this commentary, the subtraction of ratios is understood by the commentator as the division of ratios.

While the commentator sometimes gives series of directions to find the sought value, his *modus operandi* differs from that of other commentators. His rules, when they are stated, are not given in universal terms. In general, his work is not made up of a series of theorems and problems, but instead is a series of notes explaining passages of the *Almagest*. It treats the particular values that Ptolemy uses instead of generalizing. Interestingly, the commentator does sometimes use Gebir's alternative theorems, but he does not use them to replace Ptolemy's use of the Menelaus Theorem. For example, he uses Gebir II.12 in his proof of the last problem of *Almagest* II.3, which Ptolemy does not prove through the sector figure.[501]

This commentary's treatment of the sector figure is noteworthy in many ways. Much of the commentary on *Almagest* I.12 is actually more of a commentary on *De figura sectore* than the *Almagest*, but this commentator shows the most concern with covering the sector figure not only fully, but as Ptolemy intended it to be treated. While Thabit proves the disjoined and conjoined sector

[500] Appendix F, lines 1385-1392.

[501] Ibid., lines 1126-1130.

figure universally, the commentator goes painstakingly through all the sixteen valid cases, and similarly to the way in which Thabit shows that given all the logically possible arrangements of the six terms in a statement of composition, there are only eighteen valid modes, the commentator shows that of the twenty-seven logically possible arrangements of lines in triangle *gez*, thirteen and only thirteen are actually possible. The treatment of all these cases also leads to a great deal of attention being paid to the ways that lines in a plane can be inclined toward another plane. While this topic comes up sporadically in other treatments of the sector figure, the issues of three-dimensionality become subjects of interest in this commentary.

This commentary's treatment of compound ratios is also remarkable for its thoroughness. It does, however, show some inconsistencies. The author usually uses denominations or the operations of multiplication and division to understand compound ratios, but he retains some traces of his sources' understandings that connect compounding to continuity of ratios. As a final note, this commentary deserves to be examined more closely since it contains not only the finalized thoughts of its author, but his learning process; the division of the cases of the Menelaus Theorem and providing universal proofs initially proved difficult for the author of the Vatican Commentary even though he was a relatively skilled mathematician.

Chapter 7: Arzachel's Canons on the Toledan Tables and Related Works

While most of the works that we have been examining involve fairly complex mathematics and astronomy, it was possible for medieval scholars to do a good deal of astronomy without worrying themselves with the Menelaus Theorem and compound ratio. Using tables and canons, most astronomers were able to find the values derived from the Menelaus Theorem or its alternative theorems. Chief among these tools were the Toledan Tables and the Alfonsine Tables, which superseded the earlier tables, or tables derived from these tables.

Along with the Toledan Tables, which he constructed, al- Zarqālī, known in Latin as Arzachel, included a set of canons, which are instructions for how to use tables.[502] While nowhere in it does Arzachel describe the use of the Menelaus Theorem, the rules show signs of being derived from the Menelaus Theorem. For example, in the passage on how to calculate the right ascension of an arc of the ecliptic, the rule is to multiply the sine of the declination of the given arc of the ecliptic by the sine of the complement of the maximum declination, to divide the product by the sine of the maximum declination, then to multiply by the sine of a quarter circle, and finally to divide the quotient by the sine of the complement of the declination of the given arc of the ecliptic.[503] This results in the sine of the coascension of the given arc. This process requires the declination of any point on

[502] I am examining one of the three Latin versions of the canons that existed in the middle ages. Editions and translations of all three can be found in Fritz Pedersen, *The Toledan Tables*, Vol. 1-2, (Copenhagen: Reitzel, 2002).

[503] Ibid., pp. 410-2 (Cb72-Cb 77)

the ecliptic, but this can be had from a table of declinations. This rule for finding right ascensions is basically the same as the rule given in the *Almagestum parvum*. The same five quantities are used to find the sought one, but the order of the multiplications and divisions is different. That five quantities are needed to find the unknown shows that the latent principle behind this process is the Menelaus Theorem. While the focus is on the practical, even this rule is not necessary for his readers since as Arzachel points out this information can be obtained directly from tables of right ascension.[504]

Arzachel again shows the influence of the Menelaus Theorem although it is not needed on a practical level in his rule for finding the ascension in the declined sphere, which is again essentially the same rule as in the *Almagestum parvum*.[505] Because one could not expect to find tables of ascensions made for one's own latitude, Arzachel tells how to construct one's own table. This second method is much less labor-intensive than the first, which has a closer connection to the Menelaus Theorem. This easier rule is to multiply the "shadow" of the degree of the ecliptic for which the ascension is sought by the value given in the "table of ascensions of the entire world." That Arzarchel gives the rules derived from the Menelaus Theorem even when easier methods are also given reveals that while the emphasis remains practical, there is still some desire to retain vestiges of the theoretical.

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[504] Ibid., p. 412, (Cb78a).

[505] Ibid., pp. 412-4 (Cb79-Cb84).

Although Arzachel's canons focus on the practical rules for obtaining values and only have traces of their theoretical bases, some of commentaries included the theoretical justifications. Among these are two works that seem to be related to the canons of Arzachel.[506]

The first is a commentary on Arzachel's canons attributed merely to "Marsiliensis," who is thought to be Guillelmus Anglicus.[507] This work tries to provide an understanding of the Menelaus Theorem in order to justify the rules derived from it. First, the author sets up the geometric layout of the sector figure and describes what the different arcs are astronomically. Before getting into a proof of the Menelaus Theorem, he discusses compound ratio, beginning with a rule for finding the unknown sixth quantity in a statement of compounding. Then he explains what it means for a ratio to be compounded from others: "I say moreover that some ratio is composed from two others when with the two ratios multiplied into each other, that third is created."[508] He then gives numerical examples to illustrate this definition and the rule about finding the sixth unknown

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[506] Transcriptions of these are provided in Maximilian Curtze, "Urkunden zur Geschichte der Trigonometrie im christlichen Mittelalter." *Bibliotheca Mathematica* 3, 1 (1900): 321-416; here pp. 347-353, 353-372.

[507] This work has the incipit "Incipit compositio tabule que saphea dicitur sive astrolabium Arzachelis… Siderei motus et effectus motuum speculator…" Guillelmus Anglicus, who was an early thirteenth-century astronomer and doctor, lived in Marseilles and was referred to as "Marsiliensis." The text may be the result of collaboration between him and Profatius Judaeus. See Olga Weijers, *Le travail intellectuel à la Faculté des arts de Paris: textes et maîtres (ca. 1200-1500), 3, Répertoire des noms commençant par G*, (Turnhout: Brepols, 1998), pp. 99-100. See also Danielle Jacquart, "English, William," in *The Oxford Dictionary of National Biography*, vol. 18, 458-9, (Oxford: Oxford University Press, 2004). The work is found in Erfurt, Universitätsbibliothek Erfurt, Dep. Erf. CA 2º 394, and in Paris, Bibliothèque nationale de France, lat. 16652.

[508] Curtze, "Urkunden," p. 350. My translation. "Dico autem aliquam proportionem ex duabus aliis componi, quando multiplicatis duabus proportionibus inter se illa tertia procreatur."

quantity. This scholar is one of the few medieval scholars we have seen who multiply ratios directly and not through denominations. He states, "If moreover triple be multiplied by double, sextuple will emerge."[509] He then outlines the proof of the plane conjoined sector figure. He points out that it requires the assumption "If between any two quantities some quantity of any size be placed, the ratio of the first line to the last will be made from the ratio of the first to the middle multiplied by the ratio of the middle to the last."[510] The poor mathematical understanding of the commentator becomes apparent as he applies the plane conjoined sector figure to the spherical astronomical situation. The figure uses lines instead of curves, and the commentator jumps from calling the lines arcs to calling them sines of arcs without any explanation. From the statement of compound ratio, he then derives the rule for finding the unknown quantity in terms of the astronomical application.[511]

There is a similar but more detailed commentary found in four different manuscripts.[512] It was probably written in the mid- to late thirteenth century.[513]

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[509] Ibid., p. 350. My translation. "Si autem multiplicabitur triplum per duplum exibit sexcuplum."

[510] Ibid., p. 351. My translation. "[S]i inter duas quaslibet quantitates aliqua quantalibet ponatur media, proportio prime linee ad ultimam fit ex proportione prime ad mediam ducta in proportionem medie ad ultimam."

[511] Ibid., pp. 349-352.

[512] Vatican, Biblioteca Apostolica Vaticana, Reg. lat. 1904 (only contains the first portion of the text); Oxford, Bodleian Library, Laud. misc. 644; Erfurt, Universitätsbibliothek Erfurt, Dep. Erf. CA 2^{o} 394 (which also holds the other commentary on the canons); Munich, Bayerische Staatsbibliothek, Clm. 234. Curtze's transcription of portions of this are contained on pp. 353-372.

[513] Oxford, Bodleian Library, Laud. Misc. 644 was written ca. 1273. See Andrew Watson, Catalogue of Dated and Datable Manuscripts in Oxford Libraries, vol. 1, (Oxford, Oxford University Press, 1984), p. 102.

The incipit is "Kardaga est portio circuli constans ex 15 gradus..."[514] After defining terms such as "kardaga," "sinus rectus," "sinus versus," etc., the author states that a ratio of extremes is made from the multiplication of the ratio of the first to the middle and the ratio of the middle to the last. This is in fact given in the same words as in the other commentary; however, the explanation is in different wording and the explicit definition of compounding is not given. The concept of compounding is clearly the denominational one, and the commentator explains it with a numerical example. Unlike the other commentary, the denominations of ratios are multiplied, not the ratios themselves. He then gives a proof of the statement, setting out three quantities *a*, *b*, *c*, and the ratio (or its denomination) is called *h*. The ratio of *a* to *d* is *e* and the ratio of *b* to *c* is *f*. He multiplies *f* by *e* to make *p*. He wants to show that *p* is the ratio of *a* to *c*. Since *f* multiplied into *e* and *c* makes respectively *p* and *b*, *e* is to *c* as *p* is to *b*. Also, *e* times *b* makes *a*, and so does *c* times *h*. Therefore, *e* is to *c* as *h* is to *b*. But already *e* is to *c* as *p* is to *b*, so *h* is to *b* as *p* is to *b*. Therefore *p* is the same as *h*, the ratio of *a* to *c*.[515]

He then proves the plane conjoined sector figure generally following Ptolemy's proof but with more explanation. He follows this with a proof of the inverse of the statement that the ratio of *ba* to *da* is composed of the ratio of *bf* to *fe* and the ratio of *ce* to *ca*, which is that the ratio of *da* to *ab* is composed of the ratios

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[514] Also spelled "gardaga" and "cardaga."

[515] Curtze, "Urkunden," p. 356.

of *fe* to *bf* and of *ca* to *ce*.[516] After some theorems on finding the chords of arcs, the commentator goes very deliberately through the applications of the sector figure for finding the right and oblique ascensions of any arc on the ecliptic.[517] He lays out the situation clearly and explains how a sector figure is formed—which lines are the primary lines meeting at a point and which are the reflected lines. Like "Marsiliensis" this commentator applies the plane sector figure directly to a spherical situation, thus blurring the distinction between the lines of the plane figure, the arcs of the spherical, and the sines of those arcs. After setting up the sector figure to find the right ascension and giving the statement of composition to which this leads, the commentator very methodically shows that the rule of multiplying and dividing known quantities to reach the sought quantity is valid. The statement of compounding is that the ratio of *de* to *ea* is composed from the ratio of *cf* to *fa* multiplied by the ratio of *db* to *bc*. He then states that as *de* is to *ea*, so is *cf* to some quantity, which he calls *p*. By the rules for finding a fourth proportional, *p* is the result of multiplying *ea* by *cf* and dividing by *de*. Because of the original statement of compounding, the ratio of *cf* to *p* will be composed of those same composing ratios. If *fa* is placed as a middle between *cf* and *p*, then the ratio of *cf* to *p* will also be composed of *cf* to *fa* and of *fa* to *p*. He then shows that *fa* is to *p* as *db* is to *bc* by using the multiplication concept of compounding. The ratio *cf* to *p* is both the product of the ratio *cf* to *fa* times the ratio *db* to *bc* and the

[516] Although in Curtze and in Laud. misc. 644 and Erfurt fol. 394, the first ratio is given incorrectly, it is clear that this is what the proof should be proving. Curtze, "Urkunden," pp. 356-7.

[517] Curtze, "Urkunden," pp. 360-4.

product of the ratio of *cf* to *fa* times the ratio of *fa* to *p*, and if one quantity multiplies two others to reach equal products, then the two multiplied quantities must be equal. Therefore, *fa* is to *p* as *db* is to *bc*. The first three are known, so *bc* can be reached by multiplying *p* by *db* and dividing by *fa*. By putting this together with the steps used to determine *p*, the rule is reached and can be restated in the terms of its astronomical application.[518] The commentator proceeds in the same fashion for finding the oblique ascensions. He carefully explains how the sector figure applies, how the rule is derived from the Menelaus Theorem, and how the rule is understood in astronomical terms. He then gives some examples and explains how to work in other quadrants of the ecliptic. Although the sector figure can be applied to any part of the ecliptic with only small changes, the commentator here says that it only applies as he has done it for half the ecliptic, and he shows how to find the declinations in the other parts of the ecliptic through symmetry around the equinoxes and the solstices.[519]

These three works show that even the practical side of astronomy retained traces of its derivation from the sector figure and that medieval scholars were interested in understanding the theory behind the utilitarian side of astronomy. Although the commentators do not give thorough accounts of the Menelaus Theorem and misapply the plane sector figure to spherical problems, they do strive

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[518] This is closely related to Richard of Wallingford's method in *Quadripartitum* IV.7, which will be addressed in the following chapter.

[519] Curtze, "Urkunden,", pp. 360-7.

to understand it, and they fare better in their explanation of compound ratio and the method of finding an unknown in a statement of composition.

Chapter 8: Richard of Wallingford's *Quadripartitum*

Richard of Wallingford's *Quadripartitum*, which North dates to before 1326,[520] is another work that deals extensively with compound ratios, the Menelaus Theorem, and its astronomical applications. Richard was born in 1291-2 and after being educated at Oxford (ca. 1308-14 and ca. 1317-27), was an abbot of the Benedictine monastery of St. Albans.[521] For this type of technical astronomical work, the *Quadripartitum* was fairly popular and influential (it exists in nine manuscripts); Richard's interest in the Menelaus Theorem may have influenced others such as Simon Bredon and the author(s) of some short English works on the numbered sector figure.[522] Although he was very interested in mathematics and wrote several works on astronomical canons, astronomical instruments, and astrology, he was not an extremely innovative or creative mathematician; John North reports, "In mathematics his merit was to assemble existing knowledge, to organize it in a way which brought out the best of the scholastic method, and to make it accessible to the universities in the form of convenient treatises."[523] As we will see, Richard's characteristic collecting of existing knowledge is very much at work in the *Quadripartitum*.

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[520] North, *Richard of Wallingford*, Vol. 2, p. 23. Vol. 1 contains North's edition of the *Quadripartitum* and Vol. 2 contains notes upon it.

[521] See North, *Richard of Wallingford*, Vol. 2, pp. 1-16 for a more complete account of Richard's life.

[522] These works are found in Cambridge, Univ. Lib. MS. Ee. III. 61 (1017); Cambridge, University Library Gg. VI. 3 (1572); Cambridge, Gonville and Caius 141/191; and Oxford, Bodleian Library, Bodley 300. The short work explains the plane sector figure through numerical examples.

[523] North, Vol. 2, p. 15.

Richard's Use of Sources

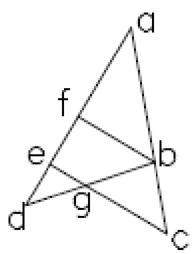
As its name suggests, the *Quadripartitum* is divided into four parts. Part I covers chords, right sines, and verse sines of arcs of circles, Part II is on the valid modes of compound ratios, Part III is made up of geometrical proofs of the 18 modes of the conjoined and disjoined plane sector figures, and Part IV consists of a treatment of the Menelaus Theorem and several astronomical applications of it. Much of Parts II-IV is derived from works written by others.

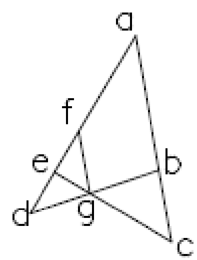
Parts II and III are copied albeit loosely from Campanus' treatise on compound ratios and from Part III of Ametus' *Epistola* respectively. North noticed that the definitions and the first five propositions were taken almost word for word from Campanus and that some of the rest shows a close conceptual connection,[524] but a closer look reveals that Richard must have written all of Part II with Campanus' work open before him. Almost every sentence has at least a phrase that is taken directly from Campanus. Richard was most of all interested in the concepts, not an exact copy of Campanus' treatise, so he freely wrote parts of it in his own words, added short phrases and sentences, and omitted some words, phrases, and sentences that he felt were unnecessary. The biggest change in the

[524] North states, "After II. 5, correspondence between the treatises appears to stop, but although the phraseology of II.6 diverges from that of the earlier text, the substance has some unusual properties in common (see comm. II. 6)" (Vol. 2, p. 54). The rest of Part II, however, matches up very closely to Busard's edition of Campanus' treatise in "Die Traktate." Perhaps the manuscript of Campanus' treatise that North used (Oxford, Corpus Christi College 41) has a different text after the first five propositions than the one on which Busard based his edition (Wien, Österreichische Nationalbibliothek 5277). While Busard published his article on the ps. Jordanus and the Campanus treatises in 1971, apparently North had not read it before his book on Richard was published in 1976.

text is the omission of the last paragraph of Campanus' treatise, which restates each of the eighteen modes.

Likewise Part III is taken almost wholly from the *Epistola* although North did not notice that it had any influence upon Richard.[525] As in Part II, Richard feels





free to paraphrase, add, and omit, but in some proofs he follows his source word for word, and in general he stays much closer to the language of his source than he does in Part II. For the conjoined sector figure, he also changes the letters standing for points in the

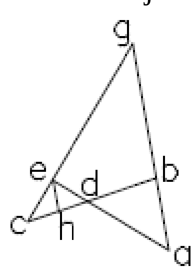
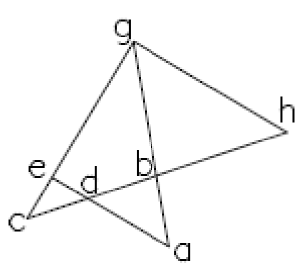


diagram. Where Ametus has his sector figure formed by lines *ag* and *gu* meeting at *g* with lines *ae* and *bu* crossing at *z*, Richard has lines *ac* and *ad* meeting at *a* with lines *ce* and *bd* crossing at



g. For the disjoined, his letters match Ametus' except that he has c where his source has *z*. Other slight modifications of Ametus' text on the geometrical proofs of the modes are that Richard also adds a short introduction and that he makes frequent references to Part II and to Euclid. Including these two different texts of Campanus and Ametus on the modes seems

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525 All but the first paragraph is taken from the *Epistola* from III A 1 (following Schrader's numbering) through III J 2 (Schrader, pp. 191-236). North omits several proofs because they are so repetitious, but I have checked these in Oxford, Bodleian Library, Digby 178 against Schrader's edition of the *Epistola*. North must not have known of Schrader's edition of Ametus' *Epistola*, so he relied on what must have been a cursory inspection of Oxford, Bodleian Library, Ashmole 357, which is difficult to read. North writes, "If this work influenced *Quadripartitum* indirectly, there are no obvious signs of the fact" (Vol. 2, p. 58).

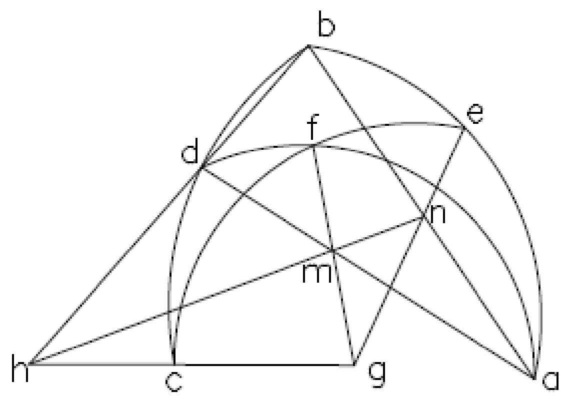
excessively repetitive since each mode is proven three times—once in a general manner, once for the conjoined sector figure, and once for the disjoined sector figure. It also causes a slight discrepancy in that Modes 9 and 10 of Campanus match respectively Modes 10 and 9 of Ametus.

Part IV of the *Quadripartitum* is also highly derivative. Richard begins by proving four propositions. The first and fourth are from Campanus' *De figura sectore* and the second and third are Richard's proofs of the third and fifth lemmas for the sector figure in the *Almagest*. While Richard provides the proof of the first of these four in his own words, he takes the enunciation of the first and the fourth directly from Campanus. The proof of the fourth is also taken from Campanus with only slight variations.[526]

Much of IV.5 and IV.6, in which the various cases of the spherical sector figure are proved, is also taken from Campanus. As before, Richard does not merely copy his source word for word, but he still remains extremely close to his source, using many of the identical words, phrases, and sentences but making small additions, omissions, and changes as well. Among the changes that he makes, he writes that Hipparchus, Gebir, and "others" used the sector figure. Since Gebir, as we have seen, does not use the sector figure, his inclusion by Richard supports North's argument that Richard had not read Gebir's *Correction* when he wrote the

[526] North, Vol. II, pp. 65-6 realized that these first four propositions were largely taken from Campanus.

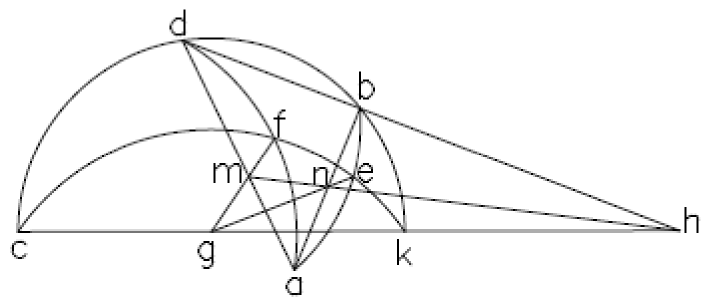
Quadripartitum.[527] He also adds short explanations and reminders that the arcs considered are all less than semicircles and that they are arcs of great circles. He



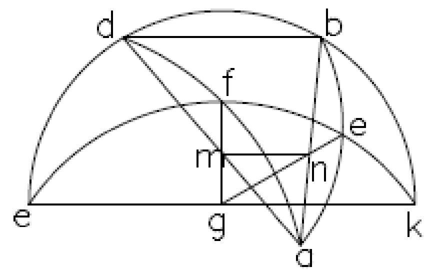
also is clearer than Campanus about the division of the sector figure into cases. Richard departs from Campanus in IV.5 to add a paraphrase of Ptolemy's proof of the the first case of the disjoined sector figure, which is found in the *Almagest*. Because he inserts this in the middle of his version of Campanus' *De figura sectore*, he uses the letters of Campanus' diagram (however with points *m* and *n* switched and with the lines *bd* and *cg* meeting at point *h* on the left side of the diagram). His proof is essentially that of Ptolemy, but it is much more detailed in its explanations, and Richard apparently was enough of a master of the sector figure that he was able to give it all in his own words.

His proofs of the second and third cases are taken (loosely, not always word for word) from Campanus although he supplies some additional explanations and references. He also has a few differences in lettering (*k* and *h* are switched, as are *m* and *n*). In the proof of the second case he uses the eleventh mode as does

[527] North, Vol. 2, pp. 23, 66. It is still unclear why Richard thought that Gebir had used the sector figure. Perhaps he confused the *Almagestum parvum* with Gebir's work. It is also possible that Richard did not know much about Gebir except that he was a distinguished astronomer, so he assumed that he used the sector figure and added his name to add some authority to his use of the sector figure.



Campanus.[528] At the end of this proof he adds that the third mode of the plane sector figure could be applied to the same plane sector figure to reach another conclusion as well. In the third case, the one with parallel lines, we find one of the more significant departures from Campanus. Richard does not appear to reach the sought statement of composition. He does not take the last step of moving from a statement of proportionality between two ratios to a statement that one of those ratios is composed from the other and a ratio of equality. His reason is that there cannot be a ratio between equals.[529] Strangely, Richard does not reach the conclusion, so he should not be able to assert that Ptolemy's conclusion applies universally, but he moves on as if he had proved it universally. After claiming that we can take eighteen modes of the disjoined statement of composition, he gives Campanus' text proving the conjoined sector figure, after which he adds a brief discussion of how one could prove all 18 modes of the



[528] North's edition and some of the *Quadripartitum* manuscripts say that it is the second mode, but this mistake is clearly a result of either a scribal or editorial error confusion over the Arabic numeral "11" and the Roman "ii."

[529] Perhaps what Richard means is that a ratio of equality is a ratio that adds nothing, but if that is his meaning, his language is strange. He writes, "Inter cordam dupli arcus CD et cordam dupli arcus CB nulla est proporcio, quia sunt idem. Relinquitur ergo quod proporcio corde dupli arcus AE ad cordam dupli arcus EB componitur et est omnino sicut corda dupli arcus AF ad cordam dupli arcus FD tantummodo." (Vol. 1, p. 108)

conjoined sector figure geometrically using IV.1-4 and III.C1-18 and D.1-18, and he gives as an example a proof of the second mode.

Most of the rest of the fourth book of the *Quadripartitum* consists of astronomical applications of the sector figure. While there are some passages that may have been Richard's own work, much of the remainder of Book IV is taken from the *Almagestum parvum* and the Arzachel's *Canons* commentary that begins with "Kardaga est …"[530]

In IV.7-8 there are found very detailed accounts of the construction of sector figures, and detailed explanations of each step, especially of the way to find the unknown term in a statement of composition for two applications of the sector figure—finding right and oblique ascensions. These chapters are taken from the commentary on Arzachel, much of it word for word and without major changes.[531] While IV.9, 10 and 11 are not to be found in Curtze's transcription of the "Kardaga est" text, I suspect that they may come from a part of the work that was not transcribed, from an unknown version of that text, or from a source common to both the *Quadripartitum* and the "Kardaga est" text that had more applications of

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[530] See pp. 237-240 above.

[531] IV.7 comes from Curtze, "Urkunden," pp. 360-2, and IV.8 from pp. 362-7. While most of the wording is altered, the letters are the same and some phrasing is the same. To give an idea of the similarity, the commentary on Arzachel has the sentences "Item line *be* sit quarta zodiaci, que est a principio Arietis usque in finem Geminorum. Sit ergo nobis propositum invenire elevationem totius Arietis ad circulum directum, et ducatur colurus per illum gradum, cuius volumus invenire elevationem, id est per ultimum gradum Arietis, et sit quarta illius coluri *ac*…" (p. 360) while the *Quadripartitum* has "Item, BE sit quarta zodiaci, a principio Arietis usque in finem Geminorum in ecliptica. Sit ergo nobis propositum invenire elevaciones tocius Arietis ad circulum directum; et ideo a puncto A ducatur colurus per illum gradum cuius volumus invenire elevacionem. Et sit quarta coluri illius AC …" (North, Vol. 2, p. 112).

the sector figure. My main reasons for this suspicion are: first, that almost all of the *Quadripartitum* has been shown to be taken from other sources so these few remaining chapters are probably also not Richard's original work; and secondly, that the manner of solving for an unknown quantity through an added quantity *p* in IV.9, 10, and 11 is so similar to the way this is done in IV.7-8, which is taken from the "Kardaga est" text. Phrases such as "eadem est proporcio … ad aliquid aliud" and "multiplica medium in medium" found in IV.10 are fairly unique and are found in the "Kardaga est" text.[532]

From IV.16 to 24, the bulk of what Richard does consists of quotations from and explanations of the rules from the *Almagestum parvum*. North knew about this work and surprisingly was confused that Richard refers to it as his source and not the *Almagest*.[533] IV.16 is taken directly from *Almagestum parvum* II.35, IV.18 from I.16, IV.19 from I.17, IV.20-23 from II.1-4, IV.24 from II.16. IV.17 does not come from the *Almagestum parvum* but directly from the *Almagest* VIII.5.

[532] E.g., North, Vol. 1, p. 130; and Curtze, "Urkunden," 361. Another explanation is that Richard used the concepts and methods that he learned while copying IV.7-8 and applied them to new problems in IV.10-11. North argues that IV.12 may be derived from this very "Kardaga est" text, but he apparently did not see the much more obvious connections that IV.7-8 had to it (Vol. 2, p. 75).

[533] I acknowledge the difficulty of finding matching sources in works that are organized differently, but North's treatment of Richard's sources is somewhat surprising given what what we have discovered about Richard's use of his sources. He was aware that the *Almagestum parvum* was a source that Richard used. He notes on Vol. 2, p. 77 that after IV.15 Richard gets his material from the *Almagest* or an "abbreviated version," and on Vol. 2, p. 140 he discusses it as a source for another of Richard's works. However, North fails to note which passages are taken almost directly from the *Almagestum parvum* (e.g., on Vol. 2, p. 78 he says, "The original source was *Almagest* I. 14, and it is surprising to find the rule ascribed to the commentator." The passage in question is taken almost word for word from the *Almagestum parvum*.)

Richard's Own Work on the Sector Figure and Compound Ratios?

Although in many ways, the *Quadripartitum* is a disharmonious collection of excerpts from works that do not always agree and perhaps all of it will be found to be derivative, we may have a few glimpses of Richard's own work in the chapters for which a source has not been found. While I suspect that the whole work is derivative, we must retain the possibility that these passages are Richard's own until sources are found.

We see an example of cleverness in IV.9, in which he finds the declination of a point on the ecliptic. He uses Mode 15 of the conjoined sector figure, which results in one of the composing ratios being a ratio of equality. The composed ratio is therefore the same as the other composing ratio. Although this procedure may have been taken from another source, Richard was at least able to comprehend it. If this is Richard's own work, it shows that he did learn from the texts on the sector figure and compound ratios that he gathered.

Richard also has applications of the sector figure that are not found elsewhere, although some of these new applications are similar to ones discussed by Ptolemy. For example, in IV.11, he uses the sector figure to find the arc of the equator which has risen while the sun or a star rises to a given altitude and how to find the azimuth of this star which is at a given altitude. Although these are similar to tasks performed by Ptolemy by application of the sector figure in *Almagest* II.7 and VIII.5-6, Richard's proofs show no close connection to Ptolemy's, and the problem requires that the altitude of the star be taken, presumably with instruments.

While in *Almagest* II.7, Ptolemy works from a given arc of the ecliptic to the arc of the equator that coascends with the sun, here Richard works from a given or observed altitude of the sun. While the problems are very similar, Ptolemy's is more useful for making a table of oblique ascensions while Richard's is more useful for someone who is actually observing the altitude of the sun. In IV.15, he finds the diurnal arc of a star, which is similar but not identical to finding the diurnal arc of the sun or the coascension of a star. IV.25-31 contain a string of proofs and rules for finding the declination and the mediations of stars. While these are essentially the conversions from ecliptical latitude and longitude to equatorial latitude and longtitude, which he has treated in IV.10 and that Ptolemy treats in VIII.5 of the *Almagest*, he divides the two problems into several, depending upon the star's location in relation to the ecliptic and equator.

In addition to these sections on astronomy that may be Richard's own work, one section of the *Quadripartitum* may show Richard's own work on compound ratios. IV.32 he returns to topic of how to find the unknown term in a statement of composition from the known five terms. He had explained this in terms of particular problems several times earlier, but here he gives general rules for the different locations of the unknown term in the statement of composition. [Table 2] Curiously, these rules do not match his solutions given earlier in Book IV. Richard's rules involve the following order of operations:[534]

[534] North, Vol. 1, pp. 164-5. Finding this section repetitious, North only transcribed the first two of these rules. I have transcribed the rest from Oxford, Bodleian Library, Digby 178, f. 37v:

Table 2

Unknown Term	Rule to Find Unknown Term
6^{th}	a) $1^{st} \times 4^{th} \div 3^{rd}$, save quotient, $2^{nd} \times 5^{th} \div$ saved value b) $2^{nd} \times 3^{rd} \div 4^{th} \times 5^{th} \div 1^{st}$
5^{th}	a) $1^{st} \times 4^{th} \div 3^{rd} \times 6^{th} \div 2^{nd}$ b) $2^{nd} \times 3^{rd} \div 4^{th}$, save quotient, $1^{st} \times 6^{th} \div$ by saved value
4^{th}	a) $1^{st} \times 6^{th} \div 5^{th}$, save quotient, $2^{nd} \times 3^{rd}$, $\div$ saved value b) $2^{nd} \times 5^{th} \div 6^{th} \times 3^{rd} \div$ by 1^{st}
3^{rd}	a) $1^{st} \times 6^{th} \div 5^{th} \times 4^{th} \div 2^{nd}$ b) $2^{nd} \times 5^{th} \div 6^{th}$, save quotient, $1^{st} \times 4^{th} \div$ saved value
2^{nd}	a) $1^{st} \times 4^{th} \div 3^{rd} \times 6^{th} \div$ by 5^{th} b) $1^{st} \times 6^{th} \div 5^{th} \times 4^{th} \div 3^{rd}$
1^{st}	$2^{nd} \times 3^{rd} \div 4^{th} \times 5^{th} \div 6^{th}$

“Et si quarta sit ignota, multiplica primum in sextum, et productum divide per quintum. Deinde multiplica secundum in tertium, et productum divide per illud quod in prima divisione exivit, et exibit quarta. Vel sic: duc secundum in quintum, et productum divide per sextum, et quod exit multiplica in tertium, et divide productum per primum, et exibit quartum.

“Et si tertia sit ignota, duc primum in sextum, et divide per quintum, et quod exit multiplica per quartam, et productum divide per secundum, et exibit tertium. Vel sic: duc secundum in quintum, et productum divide per sextum, et quod exit serva. Deinde duc primum in quartum et productum divide per servatum, et exit tertium.

“Et si secunda sit ignota, duc primum in quartum, et productum divide per tertium, et quod exit multiplica in sextum, et productum divide per quintum, et exibit secundum. Vel sic: duc primum in sextum, et productum divide per quintum et quod exit multiplica in quartum, et divide productum per tertium, et exibit secundum.

“Et si primum sit ignota, duc secundum in tertium et divide per quartum, et quod exit duc in quintum, et productum divide per sextum et exibit primum. Haec est simplex operatio.”

After giving rules for finding the unknown term in a statement of composition near the end of the *Quadripartitum*, Richard gives two methods of justifying these rules. The first does not use the word "denominations" but relies heavily on their use and identifies them with their ratios. After laying out a statement of composition, Richard writes:

> It must be noted that if you divide A by B, their ratio results, which let us call G. Likewise, divide C by D and their ratio results which we let be called H. Again, if you divide G by H, their ratio results, which let be K. I say, therefore, that the ratio which is between E and F is K, but E is known and likewise K, therefore F will necessarily be known. By multiplying E by K, F results necessarily if F is greater than E, or by dividing E in K, comes F if it is less. But now I prove that if G is divided by H, K results, which is the ratio of E to F. For because by dividing G by H comes K, it is determined that G is composed by the multiplication of K in H. But because G is the ratio of A to B and H the ratio of C to D, G is composed from H and the ratio of E to F. Because therefore it [G] is composed from H [multiplied] into K, K will be from the ratio of E to F, which we wanted to demonstrate, and thus we come to the knowledge of the unknown through division only.[535]

While this proof relies upon finding the ratios or their values by dividing their antecedents by their consequents, a procedure that makes ratios essentially numbers, Richard generally does not do this elsewhere in the *Quadripartitum*. Even the connection to the rules that he has given seems negligible. Given that this proof finds the value of the unknown sixth term through division, one would expect that a rule derived immediately from it would require one to divide the first by the second, to divide this quotient by the quotient of the division of the third by the

[535] North, Vol. 1, p. 166. My translation.

fourth, which would result in the value of the ratio with the unknown term, and then to divide the fifth by that quotient.

While the incongruity this proof seems to confirm that once again there is a mixture of Richard's own thoughts and several sources here, the following proof that Richard gives is more coherent with the rules given in IV.32. It reads:

> Again the same is able to be proved by multiplication thus as if A the first is multiplied by D the fourth, P results. Again let that be divided by C the third and H' results. Therefore, H' multiplied by C produces P. Let therefore A be the first, H' the second, C the third, D the fourth. Therefore, what results from the multiplication of A the first by D the fourth is equal to that which comes from the multiplication of H' the second by C the third. Therefore, the ratio of A the first to H' the second is that of C the third to D the fourth. But from the hypothesis the ratio of A to B comes from C to D and E to F, therefore A to B comes from A to H' and E to F. But A to B comes from A to H' and H' to B, so H' to B is as E to F. But the ratio of H' to B is known because each is known, and E is known, therefore F also because if the ratio is known and one of the terms is known, the other extreme will be known.[536]

This justification matches the first rule he gave in IV.32 about how to find the sixth term. The proof is also similar to what Richard does in many of the earlier proofs in Part IV. He finds a fourth proportional using one of his known ratios and a term from the other known ratio, then he uses two statements of composition that share two ratios to establish a proportion with the unknown term. While similar to how Richard usually finds his unknown term, the rule derived from this justification does not match what he does in the earlier sections of Book IV.

[536] North, Vol. 1, p. 166, 168. My translation.

Richard's Achievement

North praises Richard for his compilation of various sources and his rigorous treatment of trigonometry.[537] Once the truly derivative nature of the *Quadripartitum* is seen, however, Richard's achievement does not seem as praiseworthy. In general, his work is marked by nonuniformity that results from his style of gathering disparate material from his sources without fully integrating them into one consistent whole. For example, Book II has Campanus' abstract method of dealing with the modes, while Book III has Ametus' geometrical and particular method of dealing with them.

Book IV in particular lacks a clear order, although it covers most of Ptolemy's applications in *Almagest* I-II, VIII, and XIII. The content and ordering of the astronomical applications seems to not have been the product of careful deliberation, but of happenstance. Richard seems to have included proofs or rules in whatever order he happened to learn them. The astronomical applications (or rules apparently derived from sector figure applications) do not cover identical ground with the *Almagest* or any other work, nor do the applications that are found in other works strictly follow the same order. Richard may not have been quite as repetitious as North believed.[538] but at several points he has chapters on the same

[537] Vol. 2, p. 32.

[538] Vol. 2, p. 82. North believed that Richard repeated himself in many places, but a closer look shows that in several of these repetitions Richard does offer something new or at least a clearer rule for an operation that he had proved valid earlier but had not stated succinctly. For example, IV.17 and 24 are very similar to IV.8, but in the earlier section Richard shows how to find the oblique ascension of a point on the ecliptic, in IV.17 he shows how to find the oblique ascension of a star off

topic separated by other chapters on different topics. For example, IV.15 gives a rule for finding a value that he demonstrated how to find in IV.11 while IV.12-14 are on different applications. Richard's unique ordering immediately breaks the systematic, Euclid-like model of building upon axioms and previously derived truths when Richard shows how to find the right and oblique ascensions of arcs of the ecliptic before showing how to find the declination of points on the ecliptic, which is required in their proofs. There are also conspicuous absences; Richard does not treat any of the angles that Ptolemy finds near the end of Book II of the *Almagest*. Since these angles are used primarily for determining parallax and eclipses, this suggests that Richard was not interested in these subjects (at least when he compiled the *Quadripartitum*).

Unsurprisingly, the mixture of material from different sources makes it difficult for Richard to remain consistent in his manner of treating similar problems throughout the *Quadripartitum*. In astronomical applications Richard often goes through every step of finding the unknown term in a statement of composition.[539] Although he only gives the steps in terms of particular figures for finding the unknown sixth term, we can generalize these particular rules to find the order of operations to perform on the known five terms to find the unknown sixth term. We

of the ecliptic of which the ecliptical coordinates are known, and in IV.24 he gives a direct quotation of the general rule for finding the oblique ascension of a point on the ecliptic from the *Almagestum parvum* II.16. Likewise, IV.18 and 19 give the rules from the *Almagestum parvum* I.16 and 17 for finding the declination of a point on the ecliptic and for finding right ascensions, so they add to II.9 and 7 respectively. IV.25-7 also give general rules for procedures shown in particular terms in IV.10.

[539] An exception is the first proof of IV.10.

find, however, that Richard's rules do not match general rules that he later gives in IV.32. For example, if he were to state a general rule from the order of operations given in IV.7, it would be to multiply the third by the second, divide by the first, multiply by the fifth, and divide by the fourth, while his rule for finding the unknown term when it is the sixth in a statement of composition has two other orders, as will be shown below. In one case (in IV.11), the rule extracted from Richard's application does match the general rules given in IV.32, but it is unclear whether this is fortuitous or the result of these two chapters being the original work of Richard. Also, the astronomical rules or canons that can be extracted from Richard's early applications do not match the rules for finding the same quantities that Richard later quotes from the *Almagestum parvum*. For example, in IV.7, the rule to find the right ascension of a given arc of the ecliptic would be to multiply the sine of the declination of its final point by the sine of the complement of the maximum declination of the ecliptic, to divide by the sine of the maximum declination, to multiply by the radius, and to divide by the sine of the complement of the declination of the given arc of the ecliptic.[540] However, this rule is inconsistent with the rule for finding the right ascension that he quotes in IV.19 from the *Almagestum parvum*.[541]

[540] North, Vol. 1, pp. 112-5.

[541] Ibid., pp. 152-3.

De Sectore, Richard's Revision of the *Quadripartitum*

Sometime after he wrote the *Quadripartitum*, Richard read Gebir's *Correction of the Almagest*, which he had not read when he wrote the *Quadripartitum*, and in 1335 he wrote a revision of the *Quadripartitum*, which North calls "*Tracatus de sectore*."[542] The work is divided into four parts but not in the same way as the *Quadripartitum*. There are some changes in content from the *Quadripartitum*, and Richard rewrites almost all of the material covered in the earlier work. Most of the changes in content and in wording do not significantly affect the treatment of the sector figure and compound ratio much.[543] Nevertheless, there are still several changes related to compound ratios and the sector figure that deserve attention.

The first of these changes is the introduction, in which Richard explains what the sector figure is and why it is important. He refers to the *Almagest* and to Gebir and a "Commentarius,"[544] who used another method using only four proportional quantitities. In Book II there are more substantial changes. He omits the third and fourth definitions but adds four new ones, two of which concern compounding:

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[542] North, Vol. 2, p. 23.

[543] Because the work's material is fairly close to the *Quadripartitum* and because the manuscript is faded and difficult to read in several parts, neither North nor I provide a complete transcription.

[544] This probably refers to the author of the *Almagestum parvum*, whom Richard had called "the commentator' in the *Quadripartitum*, North, Vol. I, p. 152.

> 3. For a ratio to be composed of ratios is for the denomination to be created of the denominations by rising through multiplication or by declining through division.
>
> 4. When the quantities of whose ratios the ratio of extremes is composed continually grow or decrease from the first to the last, again the ratio of the extremes is composed of the multiplication of the denominations of the ratios of the intermediate terms one by one...[545]

While the third definition is relatively clear, the fourth is not. North understands it to be "a rule for cancelling a mean in a product of three ratios, the example (not printed above) being $\frac{2}{4}\cdot\frac{4}{12}\cdot\frac{12}{36}=\frac{2}{12}\cdot\frac{12}{36}=\frac{2}{36}=\frac{1}{18}$."[546] Through this numerical example, Richard is showing that continuity of ratios leads to a statement of composition, but compounding is still being understood by denominations, not continuity. The only other major difference in Part II is that Richard adds a proof that given three quantities, the product of the second by the third divided by the first has the same ratio to the third as the second does to the first.[547] Also, while most of the *De sectore* is a shorter paraphrase of the *Quadripartitum*, the later work has a more detailed treatment of the proofs that certain combinations of two of the six terms from a valid statement of composition do not have any valid modes.[548]

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[545] North, vol. 1, p. 173. My translation. North does not give the remainder of this passage. It is found on Gg.VI. 3, f. 60v; he gives a numerical example of showing that with numbers 2, 4, 12, and 36, the ratio of the extremes is the product of 2:4 "ducta in" 4:12 "ducta in" 12:36. He follows this with another example (2, 12, 6) in which the middle is not intermediate in size.

[546] Vol. 2, p. 59.

[547] Vol. 1, p. 174.

[548] Gg. VI. 3, ff.. 63v-64v.

Part III has content that corresponds to the *Quadripartitum*'s passages derived from Ametus' *Epistola*; however, Part III is divided into subsections differently. A portion is derived from the part of Campanus' *De sectore figura* on the sector figure (not the material on compound ratio and the modes, which is in Part II). There is no attribution to Campanus, and in fact Richard says that he is giving conclusions from the first book of the *Almagest*. Richard makes this section of the text more consistent by using the same diagram throughout, while the *Quadripartitum* and the *Epistola* have different diagrams for the conjoined and disjoined sector figures. This allows Richard to add a table of the seventy-two valid combinations of terms for the conjoined and disjoined plane sector figures (the eighteen modes and their inverses for each) and explains how the table is organized.[549] He follows this table with the rules (this time without any proofs) for finding unknown terms in statements of composition that were in IV.32 of the *Quadripartitum*. This section fits better here in a discussion of the modes than it did in the *Quadripartitum*, where it was placed after all the astronomical applications of the sector figure.

After proving the Menelaus Theorem with no major departures from the text of the *Quadripartitum*, the biggest difference from the earlier work is found. Richard provides several theorems from Gebir's *Correction*, some preliminary ones and then a "proposition of Gebir wonderfully succinct and short," by which he

[549] Gg. VI. 3, f. 68r.

means the alternatives to the sector figure.[550] He first gives paraphrases of Gebir I.11-15 and a proof from Gebir I.13 that if two sides and an angle or two angles and one side of a spherical triangle are known, then the other sides and angles will be known. He also gives Gebir I.24-25, which are similar proofs but about rectilinear triangles. Richard's paraphrases of Gebir's proofs remains so close to the original text that he clearly was writing as he read it.[551]

In the Part IV, Richard proves several of the astronomical applications that are in the *Quadripartitum*. The section on astronomy, which is mainly applications of the sector figure, is much better organized than the corresponding passages in the *Quadripartitum*. Instead of twenty-five different chapters, the material is condensed to 13. These chapters follow a more logical order; e.g. he shows how to find declinations before finding right and oblique ascensions. This revised treatment does omit a few of the proofs and rules, such as how to find the altitude of the pole using the sector figure, but it has practically every sector figure application from Books I, II, and VIII of the *Almagest*. Unlike the *Quadripartitum*, *De sectore* treats the angles formed by the ecliptic and other circles (the subject of the last few chapters of Book II of the *Almagest*). Also, unlike the *Quadripartitum*, *De sectore* includes the rules for finding each astronomical value in the same chapter in which the value is found.

[550] Vol. 1, p. 175.

[551] North only provides the enunciations, so one must turn to Gg. VI. 3, ff. 72r-76r for the text of the proofs.

The greatest difference in the astronomical applications is that Richard uses his new-found knowledge of Gebir's alternatives in *De sectore* to provide easier proofs for finding sought quantities. In a short introduction, he states, "It will be shown alternately how it is to be done through the six proportional quantities as Ptolemy relates . . . and how it is to be done through four proportional quantities in right spherical triangles in the way that Gebir teaches…"[552] As we have seen with the Erfurt Commentary, the alternatives to the sector figure are used to give additional proofs, not to replace the Menelaus Theorem.

We have seen that Richard was the author of only a very small portion of the *Quadripartitum*. Of the three parts of the work that deal with the sector figure and compound ratio, almost all of it was copied from Ametus' *Epistola*, Campanus's *De figura sectore*, the *Almagestum parvum*, the "Kardaga est" text, and the *Almagest*. Very likely Part I and the approximately ten chapters of Part IV for which I have not found a source are also copied. While these passages do show a thorough explanation of how to find unknown terms in statements of composition and a few clever uses of the eighteen modes to simplify proofs, they may prove to not be Richard's writing. Some credit has to be given to Richard for compiling material on the sector figure and compound ratio, but in light of the lack of originality, Richard's place in the history of trigonometry and astronomy should be

[552] Vol. 1, p. 177.

reconsidered. On the other hand, the history of mathematics and astronomy is not merely a history of innovation. Richard's choice of source material and his reorganization of it reveal how he considered the Menelaus Theorem and compound ratio as topics worth studying at length. The *Quadripartitum* reveals that Richard found the proving of cases beyond what Ptolemy proved, the proof that the insertion of middles produces a statement of composition, a full treatment of the modes, and rules for finding missing terms in a statement of composition to be important. His astronomical portions of the *Quadripartitum* show that he continued his predecessors' interest in general proofs concerning astronomy and general rules for various astronomical problems. *De sectore*, Richard's revision, similarly reveals a more unified whole, but like the *Quadripartitum* is a highly derivative work. In addition to showing many of the same qualities of the *Quadripartitum*, *De sectore* is another example of Gebir's alternatives being studied and used to supplement the sector figure, not to replace it.

Chapter 9: Simon of Bredon's Commentary on the *Almagest*

While Richard of Wallingford shows himself to be mainly an aggregator of texts on the Menelaus Theorem and related matters, another Oxford scholar, Simon of Bredon, shows more originality in his commentary on the first three books of the *Almagest*. He was probably born in the first decade of the fourteenth century and he lived until 1372. He arrived in Oxford in the late 1320s and may have been a member of Balliol College. Throughout the 1330s until 1341, he was a fellow at Merton College, where he was a colleague of some other prestigious mathematicians, physicists, and astronomers, including William Heytesbury, William Rede, and surely Thomas Bradwardine, who remained at Merton College until the mid-1330s. During much of the 1340s, he studied medicine at Oxford.[553] While other works are attributed to him, there are six mathematical, astronomical, astrological, and medical works that were in fact by him. As one would expect, his quadrivial and astrological work was probably done while he was at Merton College and his medical work was written while he was studying medicine.[554] His first work was probably his *Expositio arsmetrice Boicii*, a rudimentary overview of Boethius' *Arithmetica*, which does treat ratio and topics related to compound ratio.[555] Merton College was likely the best place in Europe at the time to study

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[553] For an overview of his life, see Keith Snedegar, "Simon Bredon, a Fourteenth-Century Astronomer and Physician," in *Between Demonstration and Imagination*, ed. by Lodi Nauta and Arjo Vanderjagt, 285-309, (Leiden: Brill, 1999), here pp. 285-9.

[554] Ibid., p. 291.

[555] Ibid., p. 292.

proportion theory—Bradwardine wrote his famous work on ratios right before Simon arrived and many scholars at Merton used ratios extensively.

General Description and Manuscripts

Simon's commentary on the *Almagest* only covers the first three books of the *Almagest*, but it shows the work of an expert mathematician who was able to not only comprehend the details of Ptolemy but also to incorporate material from Euclid, Menelaus, Theodosius, Thabit, Gebir, and Richard of Wallingford.[556] Despite Snedegar's claim of finding a fourth manuscript, this work exists in only three manuscripts, none of which contain the whole work.[557] Despite the apparently quite limited circulation of this text, it is revealing for its deep treatment of the sector figure, its alternatives, and compound ratio, as well as for its connections to both earlier and later mathematicians.

The manuscripts themselves have interesting and illuminating histories. Two, Oxford, Bodleian Library, Digby 168 and 178, have some text that seems to be in Simon's own hand. Digby 168 is a compilation of folios written by different scribes in the thirteenth and fourteenth centuries. The *Quadripartitum* and Simon's

[556] For a short description of the work see Snedegar, pp. 295-7.

[557] Snedegar, p. 296 claims that Paris, Bibliothèque nationale de France, lat. 7292 has I.8-13 of the work on folios 334r-345v; however, I find in these folios a short set of notes or commentary on the *Almagest* that mentions I.11 but then treats topics of Book II.9-13 of the *Almagest* as well as lunar parallax, which Simon does not treat in his commentary. This commentary at times is very close to direct quotations from the Gerard of Cremona translation of the *Almagest*, and I see no connections to Simon's commentary.

commentary appear to be written in his own hand, and the manuscript contains excerpts and notes on several works that are known to have been Simon's sources such as Richard's *Quadripartitum*, Thabit's *De figura sectore*, Ametus' *Epistola*, Menelaus' *Sphaerica*, and Gebir's *Correction*.[558] It also includes other short notes on compound ratio.[559] Surprisingly, while Simon's commentary in this manuscript is in his own hand, it is rather incomplete. It begins only in the middle of I.20 in the middle of the proof of the disjoined sector figure in which lines *ad* and *hb* meet on the side of point *g*, not *b* as in the case Ptolemy proves. It continues through Book III and ends with "... anni Nabuzodonosor qui sunt per quos intrabis in hunc librum."

Parts of the second manuscript, Digby 178, are also in Simon's own hand. It contains roughly the same parts of the commentary that Digby 168 does—it begins slightly earlier with I.13 ("Nunc superest ostendere quanta sit maxima declinatio...") and it ends at the same point in Book III. Like Digby 168, it has relevant works in it including the *Quadripartitum* and excerpts from Gebir and Menelaus.[560]

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[558] See the reprint of William D. Macray, *Catalogi Codicum Manuscriptorum Bibliothecae Bodleianae* IX, vol. 1, (1883), pp. 172-7; and IX, vol. 2, pp. 75-6. Available at http://databank.ora.ox.ac.uk/misccoll/datasets/QuartoDigby/Digby.pdf (accessed April 17, 2013). The manuscript has six main parts, but all the sector figure-related works are in the same part which was apparently owned by Simon and partly in his hand. Also, see North, Richard, Vol. 2, pp. 35-6.

[559] Transcriptions of these notes is found in Lorch, *Thabit*, pp. 423-5.

[560] Reprint of Macray, pp. 190-2.

The third manuscript, Cambridge, University Library, Ee.03.61, is yet another manuscript that is connected to Merton College. Simon's commentary does not cover much of the sector figure material since it ends in I.20 in the middle of the case of the disjoined sector figure in which lines *ad* and *bh* are parallel. It is, however, the only manuscript that has the portion of the commentary on the early chapters of the *Almagest* that treat plane trigonometry.[561]

Like the author of the *Almagestum parvum*, Simon follows a format similar to that of Euclid's *Elements*. He separates the mathematics into propositions or "conclusiones", each of which has its own general enunciations, rules (if the proposition is about how to find a quantity), and proofs. In fact, some of the enunciations are clearly related to those in the *Almagestum parvum*.[562] Interestingly, sometimes Simon tells in regular speech what it is that he is doing before he gives the formal proposition that will be proved or shown. For example, he introduces the section on the sector figure and its lemmas by saying:

> And because after this we will demonstrate how great the declination may be of any degree of the ecliptic, that is how great the colure arc passing through the world's poles between the equator and any grade of the ecliptic, therefore it is necessary that we first set out some conclusions that are

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[561] Charles Hardwick and Henry R. Luard, *Catalogue of the Manuscripts preserved in the Library of the University of Cambridge* II, (Cambridge: Cambridge University Press, 1857), pp. 114-20.

[562] While some of the enunciations show no similarities in wording with those in the *Almagestum parvum*, others show a close similarity. An example is in the third lemma for the Menelaus Theorem, which is I.9 in the *Almagestum parvum* and I.16 of Simon's commentary. While the first parts of the enunciations differs, they both share "... secabit secundum proportionem cordae dupli arcus unius ad cordam dupli arcus alterius."

> useful for this. Let there be this conclusion: with two straight lines drawn…"[563]

And the work is full of short phrases such as "This is another conclusion: …" These less formal lead-ins are followed by generalized enunciations, and the proofs are given in universal terms. The argument of the proof is sometimes set off by a phrase such as "for example,"[564] and the conclusions are often marked by phrases such as "Therefore what was proposed is clear" or "…which was what was proposed." For problems concerning finding astronomical values, the enunciation is usually followed by a corollary or rule for finding the sought value, introduced by the word "whence," but the corollary is not always placed immediately after the enunciation. As in some other commentaries, even non-mathematical sections are given universal enunciations.[565] The tone is sometimes more conversational as the commentator explains why proofs are done as they are done and how the proofs can be modified or how alternate proofs could be used.[566]

Treatment of the Menelaus Theorem

Simon treats the Menelaus Theorem similarly to the way that Ptolemy does, but he proves more of the cases. After the lemmas, which follow the basic steps of Ptolemy with only slight variations, he proves the case of the spherical disjoined

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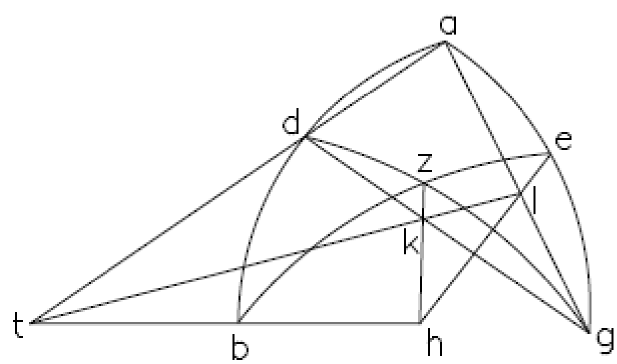
[563] Appendix G, lines 64-9.

[564] E.g., "verbi gratia" (ibid., line 95) and "gratia exempli" (ibid., line 440).

[565] E.g., ibid., lines 8-9.

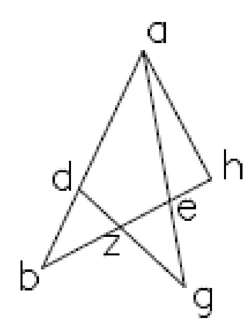
[566] E.g., ibid., lines 194-203.

sector figure that Ptolemy proves and follows Ptolemy's proof closely although with different words (throughout the commentary, the letters for the diagrams closely match those of the *Almagest*). The first major divergence comes after the



proof of the spherical sector figure, where Simon remarks that Ptolemy's proof is not universal because it only considers the case where *ad* and *hb* meet on point *b*'s side of the diagram while these lines could meet on the other side of the diagram or could be parallel.[567]

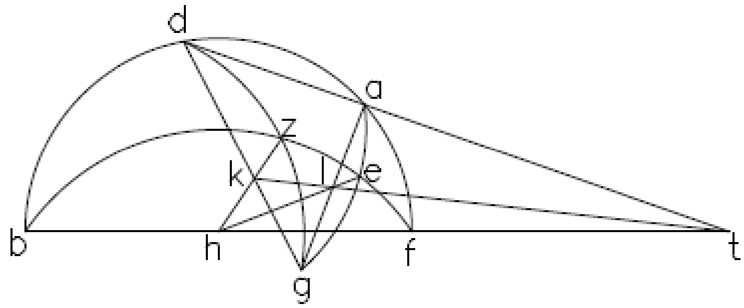
Unlike Thabit, Simon proves the case where lines *ad* and *hb* meet on the other side of the diagram by reducing the figure to a plane sector figure and not by merely applying the case that Ptolemy does prove. He first proves another mode of the plane disjoined sector figure—that *gz* to *zd* is composed of the ratio of *ge* to *ea* and *ba* to *bd* (in terms of the original Lemma 2, this is that the ratio of the third to fourth is composed of the ratios of the first to second and the sixth to fifth).[568] He sets up the four arcs such that arc *adb*'s supplement is less than arc *bd*, and he then continues the arcs *bda* and *bze* until they make semicircles at *f*. From *h*, the center of the sphere, he draws a line through *f* which is



[567] Ibid.

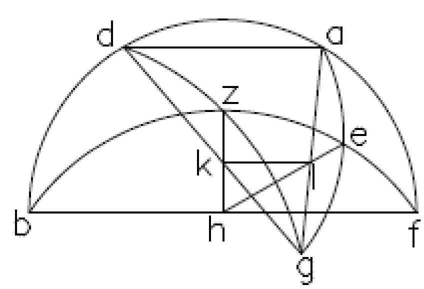
[568] Ibid., lines 204-213.

extended until it meets line *ad* at *t*. Also lines *he* and *hz* are made meeting chords *ga* and *gd* at points *l* and *k*. The points *t*, *l*, and *k* must be in a straight line because they are the plane of circle *bef* and in the plane of triangle *dkt*. The four lines *dt*, *dg*, *tk*, and *ga* make a plane sector figure, so the ratio of *gl* to *la* is composed of the ratios



of *gk* to *kd* and *dt* to *ta*. As in the case that Ptolemy proves, he uses the lemmas to move from this statement of composition of ratios of lines to one about the ratios of chords of arcs: the ratio of crd. arc 2*ge* to crd. arc 2*ea* is composed of the ratio of crd. arc 2*gz* to crd. arc 2*zd* and the ratio of crd. arc 2*df* to crd. arc 2*fa*. Because arc *bdf* is a semicircle, crd. arc 2*bd* is equal to crd. arc 2*df* and crd. arc 2*ba* is equal to crd. arc 2*af*. Therefore, the ratio of crd. arc 2*ge* to crd. arc 2*ea* is composed of the ratio of crd. arc 2*gz* to crd. arc 2*zd* and the ratio of crd. arc 2*bd* to crd. arc 2*ba*.[569]

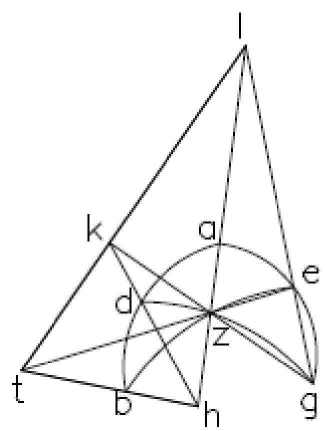
Simon then proves the case of the disjoined sector figure that has lines *ad* and *hb* parallel, which happens when arc *ab*'s supplement is equal to arc *bd*. The proof is basically the proof of Thabit, but its letters match Ptolemy's sector figure, not Thabit's.[570]



[569] Ibid., lines 214-238.

[570] Ibid., lines 238-274. See pp. 73-4 above for Thabit's proof.

Simon moves on to the conjoined sector figure, which he also proves by returning to the plane sector figure, unlike Thabit had done. Seeing Ptolemy's omission of a proof of this proposition in the best light possible, he explains that Ptolemy did not prove it perhaps because he thought that its proof was sufficiently obvious from the lemmas and the proof of the disjoined sector figure. The



construction, however, is not the same as for the disjoined sector figure. The chords of arcs *ge*, *gz*, and *ez* are extended until they meet the extensions of their corresponding radii of circle *bda*, i.e. *ha*, *hd*, and *hb*, at points *l*, *k*, and *t*. A plane sector figure is formed by lines *lg*, *lt*, *gk*, and *te*. The conjoined proposition is derived, and the sought statement of composition is reached by applying the fifth lemma. As we have seen in the section on the "Vatican" Commentary,[571] there are thirteen different cases of this proof depending on inclination of triangle *gze* to circle *bda*. Simon, however, only proves one. This is not an indication of misunderstanding the variety of cases as it appears to be in the *Almagestum parvum*. Perhaps with Geber's complaints about the pointless and confusing enumeration of cases of the sector figure in mind, Simon explains that he only proved all three cases of the disjoined sector figure because he wanted to show that Ptolemy's statement that the proposition holds for any arcs less than semicircles is true, but that he only proves one case of the conjoined sector figure

[571] See pp. 243-4 above.

in order to not "busy the reader tediously without fruit."[572] Simon also points out that Ptolemy only needed to prove the case of the disjoined sector figure where *ad* and *hb* meet on the side of points *d* and *b* since he only uses the Menelaus Theorem for applications in which the main arcs of the diagram are all less than quarter circles.[573]

Compound Ratio

Simon does not define compound ratio, but he does spend time describing how to work with compound ratios. His treatment shows the influence of different ways of thinking of compounding. While he sometimes treats compounding through geometrical proofs, at other times he treats it abstractly. On the continuity side, he does not think that Ptolemy's practice of inserting a quantity between two others to reach a statement of composition needs explanation, and in fact, he uses this method in his own proofs.[574] He also uses the language associated with thinking of ratios as quantities that are added in compounding. For example, he

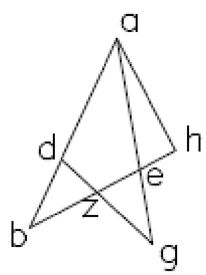
[572] Appendix G, lines 307-313.

[573] Ibid., lines 313-8. While Simon says that Ptolemy does not use the two propositions of the Menelaus Theorem for arcs greater than quarter circles, this is not strictly true since the arcs of *Almagest* II.11 can be larger than quarter circles. Simon redeems himself partially by pointing out that even if the arcs are greater than quarter circles, one can extend these arcs to semicircles and use the original arcs' supplements to form a sector figure with arcs less than quarter circles (lines 1300-1). Thabit does just this in some of his proofs.

[574] E.g., Appendix G, lines 208-9.

states that a ratio of equality "consequently neither increases nor decreases a composition."[575] At other times, he uses denominations.

While Ptolemy and some commentators, such as the author of *Almagestum parvum*, felt free to move from a statement of compounding to the statement that the ratio of one of the original composing ratios is composed of the original composed ratio and the inverse of the other composed ratio, Simon demonstrates one form of this that he needs for a case of the spherical disjoined sector figure.[576]



He proves this geometrically with what is essentially one of Ametus and Richard of Wallingford's proofs although the language and letters are different.[577] Unlike many of the commentators who addressed this issue of altering the two statements of composition given by Ptolemy, Simon declines to give a complete treatment of all the modes and instead merely offers what is needed to understand Ptolemy's use of modes in the *Almagest*.

While this treatment of a mode is geometrical and is in line with the continuous ratio idea of compounding, a more abstract, arithmetical approach to compound ratios is seen in Simon's proofs that unknown terms in a statement of composition can be found. He begins by showing that with the first five terms in a

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575 Ibid., line 270.

576 Ibid., 204-213.

577 The corresponding proof in the *Epistola* is labeled H.1 in Schrader's numbering, Schrader, pp. 226-7. The corresponding proof in the *Quadripartitum* is what is labeled D.11 in North's edition (Vol. 2, p. 95).

statement of compounding, the sixth can be found. He starts with: *a* to *b* composed of *c* to *d* and *e* to *f* with the first five terms known and *f* unknown. He divides *a* by *b*, and *c* by *d*, and he says that the quotients *g* and *h* are the ratios of *a* to *b* and *c* to *d*. He divides *g* by *h*, and the quotient *k* is the ratio of the ratio *g* to ratio *h*. He claims that *k* is also the ratio of *e* to *f*. Because *k* was found by dividing *g* by *h*, *k* times *h* equals *g*. However, the ratio of *e* to *f* times *h* also equals *g* because we assumed that *g* is composed of the ratio *h* and the ratio of *e* to *f*. Therefore, *k* is the same ratio as *e* to *f*. Because *k* and *e* are known, *f* will be known. If *f* is greater than *e*, multiply *e* by *k* to find *f* because "*k* is the quotient number denominating the ratio of *e* to *f*."[578] If *f* is less than *e*, divide *e* by *k* to find *f*. He explains why this is true: if *e* is greater than *f*, *f* cannot be divided by *e*, but *e* can be divided in *f*, which results in *k*, their ratio. Therefore, *k* times *f* produces *e*, so *e* divided by *k* is *f*. Note that it is assumed that the quotient of the antecedent divided by the consequent is their ratio. If the fifth term *e* is unknown and the other five are known, multiply *f* by *k* or divide *f* by *k* depending on whether *e* or *f* is greater. He continues to give the resulting operation to be followed if *a*, *b*, *c*, or *d* is the unknown quantity. Note that he is identifying ratios with quantities in this proof, and that his proof relies heavily upon denominations of ratios. Also, Simon's discussion of this matter is given when Simon encounters Ptolemy's method of subtracting ratios to find the

[578] Appendix G, lines 450-1.

unknown term; instead of explaining what it means to subtract a ratio, Simon gives these ways of finding unknown terms through denominations.[579]

Although he has given these rules for finding unknown terms in statements of composition, Simon also offers a set of simpler rules for finding the missing terms. [580] He provides these new rules because they only involve operations with the original six terms, not the denominations; avoiding using the denominations is worthwhile because "the relation of quantities is more well-known to us than the relation of ratios, and also quantities are more well-known to us than the ratio between them."[581] He begins by saying that there are four ways to proceed if the fifth or sixth is the unknown. Unfortunately the manuscripts with this section of the text are missing folios so we do not have the full set of rules.[582] Before the lacuna, he is able to lay out the first way of proceeding, which luckily is the one that he uses in many of the sector figure propositions of Book II. This way is to multiply the first by the fourth, divide by the second, which results in a quantity which is to the third as the fifth is to the sixth. While the proof is cut short, enough of it exists to see that it is conceptually similar to one of the rules that Campanus gives in his set of rules for finding an unknown term in a statement of composition. A further connection between the two is suggested by the fact that both Simon and

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[579] Ibid., lines 435-465.

[580] Ibid., lines 566-579.

[581] Ibid., lines 466-8.

[582] Ibid., line 479. From the foliation in Digby 168, the lacuna is one folio long.

Campanus give one way of finding the unknown and then give another set of four related ways of finding that same unknown term.[583]

Applications of the Menelaus Theorem and Gebir's Alternatives

In the astronomical applications, Simon usually gives a paraphrase of Ptolemy's proofs, but his reading of the *Almagest* is heavily influenced by Gebir. For many of the applications, he gives alternate proofs of Gebir, and after most of the propositions he notes the number of Gebir's corresponding proposition. He also switches from using chords of double arcs to using sines, and he explains why this is justified and cites the definition of sine given in Gebir's first book. As in some of the other commentaries, he gives rules or corollaries for the propositions that involve finding a value. For many of the propositions he gives more than one rule—a rule from Ptolemy's method of proof, a rule from an improved version of Ptolemy's proof, and a rule derived from a proof that uses Gebir's alternatives to the sector figure.

While Simon gives his proofs mostly in general terms partially out of a desire to follow the model of theoretical mathematics, at least part of his reason for doing this is that the values that Ptolemy uses are almost of no use for someone living in Britain. In II.17, the proposition about finding oblique ascensions, he

[583] See pp. 157-8 above.

states that he omits Ptolemy's example of Rhodes because finding values for such a latitude is of only "a modicum of usefulness."[584]

While Simon's commentary follows the same general pattern of the *Almagest*, its order at the beginning of Book II is closer but not identical to that of the *Almagestum parvum*. There are no proofs corresponding to Ptolemy's first two applications. Either Simon felt that these were sufficiently clear or more likely he thought that they were unnecessary. Ptolemy did give two ways of finding the arc of the horizon between the rising point of a point on the ecliptic and the equator, so Simon omits the first of these. He also omits the way of calculating the height of the pole at a certain latitude of which the longest day is known because Ptolemy has already shown another method for finding this quantity.

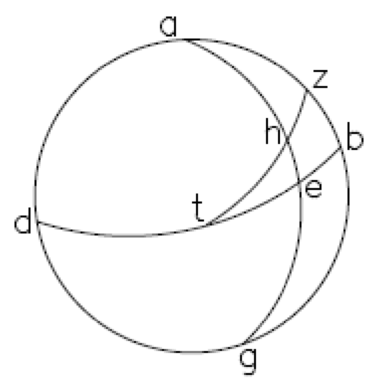
Simon's commentary shows some skill in treating the Menelaus Theorem, compound ratios, and Gebir's alternatives. Not only are proofs of Ptolemy and Gebir summarized, but they are also modified and proofs not found in Ptolemy or Gebir are provided. Some of the credit is probably due to Simon's sources. For example, while Ptolemy almost invariably subtracts ratios once he has reached a statement of composition, Simon shows when the statement of composition can be easily reduced to a proportion;[585] however, this method is used in the *Almagestum parvum*, albeit without a complete explanation, so Simon likely did not arrive at it

[584] Appendix G, lines 824-7.

[585] E.g., I.22, ibid., lines 352-360.

completely on his own.[586] After giving Ptolemy's second way of finding oblique ascensions, Simon also gives the clever proof of the *Almagestum parvum* that reduces the method of finding the oblique ascension from relying upon a statement of composition to just a proportion.[587] Again, the author of the *Almagestum parvum*, not Simon, appears to be responsible for this creative proof.

Simon's own expertise in working with ratios is seen more clearly in II.39.[588] After giving Ptolemy's proof for finding the distance from the zenith to any point on the ecliptic, Simon then observes that there is an easier way to find this arc since two of the arcs, *ab* and *ea*, are equal in the statement of composition, which is that the sine of *ab* to the sine of *bz* is composed of the sine of *ae* to the sine of *eh* and the sine of *th* to the sine of *tz*. The equal terms are not means of the composing ratios, so he cannot proceed as he does in several other cases. Instead he first substitutes *ab* for *ae* to obtain that the sine of *ab* to the sine of *bz* is composed of the sine of *ab* to the sine of *eh* and the sine of *th* to the sine of *tz*, but the same ratio of the sine of *ab* to the sine of *bz* is composed of the sine of *ab* to the sine of *eh* and the sine of *eh* to the sine of *bz* because the sine of *eh* is a middle between extremes, the sine of *ab* and the sine of *bz*. Therefore the sine of *th* is to the sine of *tz* as the



[586] See p. 179 above.

[587] Appendix G, lines 828-843.

[588] Ibid., lines 1480-1510.

sine of *eh* is to the sine of *bz*. Through Euclid VII.19, the unknown sine of *eh* will be found, and arc *eh* is the complement of the sought arc. While this is basically the proof in the *Almagestum parvum*, there the proof is only given in skeleton form and no explanation is given of how to get from the statement of composition to the unknown term or the rule; Simon shows that he is capable of filling in the details.

Since Simon has given rules for finding the unknown quantity in a statement of composition, he often just refers to these in his applications instead of going through every step each time. For example in II.1, he writes, "From the first rule of operating, the rule (*corelarium*) of this conclusion is clear, from which the conclusion is evident enough."[589] This first rule of operation is the rule that Simon gives near the end of the first book, the proof of which is cut short by the lacuna. In some of the applications, however, Simon uses Euclid VII.19 to move from proportions found through Gebir's rules to the rule of multiplication and division that results in the unknown quantity.[590]

Simon also shows that he can apply the sector figure in new ways. For example, after giving Ptolemy's first way of finding oblique ascensions, he supposes that one more quantity is known and then applies a different sector figure than the one used by Ptolemy to reach a statement of composition that he then resolves to a proportion through a common term among the composing ratios.[591]

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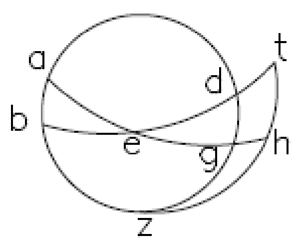
[589] Ibid., lines 525-6.

[590] Ibid., lines 531-9.

[591] II.19, ibid., lines 828-848.

He proceeds similarly in II.40, which is on finding the angle formed by the circle of altitude and the ecliptic at any given point taken on the ecliptic. Because he supposes that a quantity is known that Ptolemy does not assume or show how to find, Simon then gives another new application of the sector figure to show how to find this quantity.[592]

He shows some caution about applying the Menelaus Theorem in the proof for finding the angle between the horizon and the ecliptic. He warns the reader to set up the figure such that none of the arcs used will be over quarter circles. The reason for this might be that he uses the conjoined sector figure which he had not proved universally in I.21, or his concern might merely spring from an attempt to



set up the figure in a way that only one case needs to be proved. If arc *eg* were taken as more than a quarter circle, then the arc *zht* would cut arcs *eg* and *ed*, changing the figure. In such cases where *eg* is more than a quadrant, setting up the figure so that arc *ae*, which is then shorter than a quarter circle, is considered instead of arc *eg*, allows us to have the mirror image of the figure and everything proceeds in the same manner.[593]

Simon also shows his facility in working with Gebir's alternatives and applying them to new situations. For example, in his first two propositions of Book II, which have no corresponding proofs in Gebir's *Correction*, Simon applies one

[592] Ibid., lines 1550-1562.

[593] Ibid., lines 1298-1300.

of the alternatives (Gebir's I.15) to two different problems.[594] Simon proves two propositions about where the declination changes the most that are similar to I.17 and 18 of Gebir.[595] These are not the same as Gebir's, but clearly Simon is following his lead. Simon also has two related propositions saying that of two equal arcs of the ecliptic, the one nearer the tropic point will have a greater right ascension.[596] These propositions are similar to the ones derived from Gebir about the location of the greatest differences between declinations. Once again Simon justifies steps by referring to the "third preamble", which is Simon's missing proof of Geber I.12.

Given Simon's heavy use of Gebir and especially the Menelaus Theorem alternatives, it is somewhat surprising that he does not give the proofs or even state these propositions. His references, however, make it clear that he did prove at least one somewhere in a now-missing portion of his commentary. He refers many times to the "preambles" of his work to justify steps in his proofs, so the content of these preambles can be determined from context. What he calls the first and second have to do with greater and smaller arcs and angles in spherical triangles.[597] What he calls the third is Gebir I.12;[598] and the sixth is that two spherical triangles are equal if two angles and a side of one are equal to the respective quantities of the

[594] Ibid., lines 527-541, 555-560.

[595] Ibid., lines 378-419. See Apian-Gebir, pp. 14-6.

[596] Appendix G, lines 481-511.

[597] Ibid., lines 395-7.

[598] Ibid., lines 391-3.

other.[599] Since he does not refer to the other preambles, it is unclear what they are and even how many there were. He does use Gebir I.15, but he refers to "the fifteenth conclusion of his first book"[600] which suggests that Simon did not include it in his own preambles.

Simon's commentary shows a deep understanding of the Menelaus Theorem, its alternatives, and compound ratio, and it is a work organized in the manner of a Euclidean work; however, it is not meant to be a complete, systematic treatment that covers these topics entirely. Simon assumes that the definition and basic propositions about compound ratios are known and feels free to refer to Gebir instead of providing all of his alternatives. Although he realized that Ptolemy's treatment of the Menelaus Theorem was incomplete, Simon did not feel the need to prove every case of it. Simon's references to Gebir suggest that the *Correction* had become a canonical text at least among a small group. While many of the other commentators we have seen feel free to give references (as opposed to proving each proposition used in each proof) only when the proposition used is found in a well-known and authoritative text such as Euclid's *Elements*, Theodosius' *Sphaerica*, and Ametus' *Epistola*, Simon's references to Gebir show that the *Correction* was assumed by Simon to be both well-known to his readers and authoritative. It is hard to know who Simon's audience was, but even if it was only

[599] Ibid., lines 393-5.

[600] Ibid., lines 558-560.

a very small group of skilled mathematicians in Oxford, at least they were familiar with Gebir.

CONCLUSION

Thematic Conclusion

The texts on the Menelaus Theorem reveal much about medieval astronomy and mathematics. While the same theorem that Ptolemy, Menelaus, and Thabit proved continued to be a mainstay of medieval astronomy, the proofs of the different cases, the applications of the theorem, the formulation and style of the proofs, and the understanding of ratio were all modified.

Much of the work of this dissertation has been in examining works that have received little or no scholarly attention. Understandably, since not even the text of the medieval versions of the *Almagest* has yet been edited, historians have not examined the marginalia contained in *Almagest* manuscripts. The notations include a variety of types of notes. The *divisiones textus* contained in some manuscripts suggest that their sets of notes may have been preparations for lectures or taken during lectures; although the *Almagest* was too difficult of a text to be a required text in university curricula, it was taught occasionally. Many of the commentators' notes systematize the contents of the *Almagest* by adding references to the proofs of logically prior propositions, by justifying missing steps, by giving generalized enunciations, or in a few instances by giving universalized proofs. Many commentators attempt to define compounding of ratios, which Ptolemy had used without explanation, and both views of compounding are found. Surprisingly, while alternatives to the Menelaus Theorem are used in several of the full-length works (the Erfurt Commentary, the Vatican Commentary, Richard of Wallingford's

De sectore, and Simon Bredon's commentary), no traces of them are found in these notes.

While Campanus' *De figura sectore* and his treatise on ratios have been studied and transcribed, I have added evidence that these are indeed his writings and that they were written as parts of one work. I also have confirmed that Campanus wrote another work or set of notes on the *Almagest*. While many of the same ideas of compound ratio and the Menelaus Theorem are found in both this work and in the works by Campanus that have been examined, the *Almagest* commentary adds material on the algorismic aspects of compound ratio, i.e. how one performs the operations associated with compound ratios. While of the Menelaus Theorem related works translated into Latin, only Ametus' *Epistola* had some material on this topic, the algorism of ratios frequently accompanied the sector figure in the middle ages.

The notes and additions to Menelaus' *Sphaerica* show the ability of commentary to change the reader's experience of a text. Although Menelaus does not explain compounding and seems to accept without proof that the interposition of middles leads to a statement of composition, both the translated fragments from Arabic and the additions of Campanus rely upon the denominative idea of compounding, and readers of the *Sphaerica* in manuscripts with these additions would have understood compounding in the same way.

The *Almagestum parvum* does not dramatically alter the understandings of compounding or of the Menelaus Theorem that are found in the *Almagest*. The author does not define compounding, and his proofs of the lemmas and the disjoined spherical sector figure are conceptually similar to those of Ptolemy; however, the *Almagestum parvum* follows the model of an axiomatic, systematic mathematical work such as Euclid's *Elements*. Although this has been pointed out by Richard Lorch,[601] he does not define the degree to which the commentator follows his model of an ideal science. As in the *Elements*, principles are listed at the beginning of books, which are divided into propositions and proofs instead of chapters. Further, a typical proof includes a general enunciation, a setting out of the given situation and of any construction in terms of a lettered diagram, a statement of what is being proved in terms of this diagram, an argument in terms of the diagram, and a conclusion. Parts of the proof are marked by "key words" such as "exempli gratia" or "quod erat propositum." The systematization also affects the content; e.g, unlike Ptolemy who states the second part of the Menelaus Theorem without a proof, the commentator has to provide a proof for every enunciation, so he proves the conjoined spherical sector figure. The inclusion of generalized rules or corollaries describing the operations needed to find certain values cause the *Almagestum parvum* to be similar to a set of astronomical canons. The *Almagestum parvum*'s restructuring appears to have been a major catalyst for the widespread trend of axiomatizing astronomy.

[601] Lorch, "Some Remarks," 407-8.

The Erfurt Commentary is a work that has only been vaguely described until now as a version of the *Almagestum parvum*.[602] I have shown, however, that while it shares some enunciations with the *Almagestum parvum*, the Erfurt Commentary is its own text. Like the *Almagestum parvum*, the Erfurt Commentary is structured systematically and includes corollaries. The author offers a detailed explanation of compounding ratios according to the denominative concept and explains how Ptolemy's method of compounding can be justified from this idea of compounding. He proves the spherical Menelaus Theorem universally, and he also demonstrates alternatives to the Menelaus Theorem. He applies the Menelaus Theorem and its alternatives to new problems in the fields of astronomy, geography, and practical geometry. The commentator generally uses the sector figure where Ptolemy uses it, but he sometimes supplements these proofs with more streamlined applications of the Menelaus Theorem or with his alternatives to it.

While the Vatican Commentary, a work that has received no scholarly attention, breaks the trend of dramatic restructuring in the mold of an axiomatic, deductive discipline, it still shows some subdued changes in line with this general movement. The commentator shows a great desire for universality and for explaining Ptolemy's thinking; while he follows Thabit in proving the Menelaus Theorem universally with a small number of proofs of different cases, he then proves the conjoined spherical sector figure with the plane conjoined sector figure,

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[602] Weijers, *Le travail intellectual.2,C-F*, p. 33. Lorch, "Some Remarks," pp. 421-2

which is what he and Thabit believe is how Ptolemy intended it to be proved. This involves enumerating all twenty-seven possible arrangements of the figure and proving the validity or invalidity of each. The treatment of the modes is also thorough and complete, but this may be more the result of following Thabit than the result of the commentator's own desire for meticulousness. The commentary also contains some "key words" marking parts of proofs. Since the bulk of this work is a set of notes on passages of the *Almagest* and not a standalone text, the commentator usually remains on the same level of particulars as his source does. Like so many of these commentaries, this work deals with the algorismic issue of how to perform the operations involved in compounding and subtracting ratios as well as in finding an unknown term in a statement of composition. This commentator treats compounding inconsistently; while he initially defines compounding by continuity, he relies upon the idea of compounding as multiplication of denominations for the bulk of the commentary, and he blurs the distinction between ratios and the numbers that denominate them.

The canons on the Toledan Tables and the two works on them have been edited previously, but their methods have not been examined in the wider context of Menelaus Theorem related texts. The canons retained a connection to their theoretical justification, which was the Menelaus Theorem. The two works on the canons both use the Menelaus Theorem and discuss compounding. They use the denominative concept of compounding, and the text by "Marsiliensis" gives rules for finding unknown terms in statements of composition. Surprisingly, both of them discuss spherical sector figures as if they were rectilinear. While the other works discussed in this dissertation do not do this, Simon's commentary shows a similar conflation of curved and straight lines by improperly justifying arguments concerning spherical triangles with propositions from Euclid's *Elements* about plane triangles.[603]

The works of Richard of Wallingford have been edited by John North, who has also examined many aspects of his treatment of the Menelaus Theorem and compound ratios.[604] I have shown that North realized neither the extent to which Richard copied his sources nor the inconsistent and repetitious nature of the *Quadripartitum* due to Richard's use of disparate sources. For example, the rules that Richard gives for finding unknown terms in a statement of composition do not match his practices in the astronomical applications of the sector figure, and Richard proves each of the modes three times. Richard's *Quadripartitum* and *De sectore* have formats that bear some semblance to the structure of axiomatic mathematical works, but their disorganization and inconsistencies detract from their systematic character. For example, in the *Quadripartitum* the rule or corollary for finding a value is often not placed near the corresponding proof. In these two works, the Menelaus Theorem is proved universally, and compounding is accomplished through the multiplication of denominations. The modes are treated in a rather repetitious manner, and rules are given for finding unknown terms in

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[603] E.g., Appendix G, line 783.

[604] North, *Richard of Wallingford*.

statements of composition. While many of the astronomical applications are taken almost directly from Richard's sources, a few may be his own work. In *De sectore,* Richard summarizes and uses the proofs of Gebir's alternatives. As in most of the other medieval works that use alternatives to the sector figure, the alternate proofs do not take the place of proofs that use the sector figure.

In his commentary on the *Almagest*, which has only been described very succinctly by historians,[605] Simon Bredon uses the systematic format that has been shown to be characteristic of medieval astronomical commentaries. He tries to follow Ptolemy's intentions by proving cases of the spherical sector figure through the plane sector figures, but he does not go to the extreme of proving or disproving all twenty-seven logically possible cases as the author of the Vatican Commentary did. Simon proves the three cases of the disjoined, but only one of the conjoined. He realizes that he is not proving the theorem universally, but he balances the satisfaction of having universal knowledge against the usefulness of the various cases and the benefits of brevity. While the Vatican Commentary shows unwavering commitment to the ideal of completeness, Simon's commentary has comprehensiveness as one of several factors that determine what is proved. Like Richard, Simon shows some inconsistencies that may be the result of incorporating material from sources that do not completely agree with each other. While Simon does not define compounding, his treatment of compounding sometimes accords with the continuous idea and sometimes with the denominative. Like so many of

[605] Snedegar, "Simon Bredon."

the other texts, Simon gives rules for finding unknown terms in a statement of composition. Unlike in the other works which rarely cite Gebir, the *Correction* is treated as a canonical text in Simon's commentary. Simon correlates his propositions with those of Gebir. Although the part of the text in which he proves the alternatives of Gebir has been lost, many of the applications of the sector figure are supplemented with proofs that use the alternatives, and Simon often gives more than one generalized corollary for finding a certain astronomical value. Simon shows his expertise with both the Menelaus Theorem and the alternatives to it by utilizing them in ways found in the works of neither Ptolemy nor Gebir.

By looking at only a few of the more accessible medieval astronomical works such as *De sphaera* or the *Theorica planetarum*, one might conclude that medieval astronomy does not match the trend of axiomatization and systematizing that is found in other branches of mathematics (e.g. in Jordanus' *Arithmetica*), as well as in non-mathematical disciplines such as theology, law, and natural philosophy. The *Almagestum parvum* may appear to be one of only a few exceptions, but by examining a wider swath of astronomical works related to the Menelaus Theorem, it is clear that there was a significant program of axiomatization in medieval astronomy. While there are several ways in which to organize a topic systematically, medieval astronomers generally had a specific concept of a systematic format that differs from those generally used in theology or natural philosophy; they modeled astronomy after the systematic style of Euclid's

Elements. Not only did they generally divide their work into numbered propositions, but they also divided each proof into parts as in the *Elements* and marked these sections with “key words.” The *Almagestum parvum* goes so far as to have principles listed at the beginning of each book. With further research into the terminology used in each of these works and the different versions of the *Elements*, it may become apparent which versions of the *Elements* served as exemplars for which astronomical works.

Even this group of medieval mathematicians differed in their ideas of systematic astronomy. The *Almagestum parvum* never deviates from its systematic format, only giving principles and propositions, while Simon’s commentary has occasional statements about the principles and propositions (e.g. he points out the correspondence or lack of correspondence between his propositions and Gebir’s). Also, while all of the medieval scholars understood completeness as an aspect of systematizing, they did not all give it the same weight; while Campanus’ *De sectore figura*, the Erfurt commentary, the Vatican Commentary, and Richard’s works prove the Menelaus Theorem universally, the *Almagestum parvum* proves only two cases of the Menelaus Theorem and Simon only proves four of the sixteen cases.

As part of their systematizing of astronomy, medieval scholars attempted to universalize it. There are obvious benefits in giving enunciations and proofs in general terms instead of relying upon examples as Ptolemy did. Most importantly this manner accords with the Aristotelian position that scientific knowledge must be universal, but it also ensures that the methods given apply to all cases and permits the author to give only one proof for some related problems. On the other hand, the discipline of astronomy consists of both theoretical and practical parts. Universal proofs can detract from the practical aspects of astronomy, such as the calculation of specific values, and a synthetic approach may also obscure the way in which specific values were actually found. Perhaps as an attempt to counteract this turn and to retain a connection to the practice of astronomy, medieval scholars did not merely demonstrate that a quantity is known if others are, but they proved the set of operations necessary to calculate the sought value of that quantity.[606] A topic to be examined further is how practical aspects of astronomy were fitted (or not) into a structure taken from wholly theoretical subjects. We have noted in passing that the *Almagestum parvum*, the *Erfurt Commentary*, and others provided enunciations for practical tasks such as making instruments or taking observations, but a more complete investigation of which practical matters were retained, which modified, and which omitted, as well as why they were retained, modified, or omitted, will give a more complete picture of the unique characteristics of the systematic structure of medieval astronomy.

In these commentaries we have seen that compounding ratios was a subject of great interest to medieval astronomers. The two traditions of compounding are

[606] The inclusion of these sorts of rules may have been introduced to Latin scholars through translations of Arabic works related to the use of tables such as al-Battānī's *De motu stellarum.*

found in these works, and as Murdoch and Sylla have claimed,[607] the denominative concept is very common in them. Although the *Epistola* is the only one of the works containing the sector figure that were translated into Latin that defines compounding, these works tend to rely upon the continuous idea of compounding. Unlike their sources, most of the medieval works understand compounding according to the denominative method. Campanus' definition of compounding as the multiplication of denominations and his subsequent proofs that the interposition of a quantity or quantities between others leads to a statement of composition were especially influential; it was copied by the author of the Erfurt Commentary and by Richard of Wallingford. The text by "Marsiliensis" and the Kardaga text use the denominative idea of compounding. As Sylla has observed generally in medieval scholarship, a mixture of the two traditions is often found. The Vatican Commentary generally relies upon the multiplication of denominations for compound ratios, but it does include a definition of compounding according to continuity. Also, Simon's commentary contains some uses of compound ratio that accord with both views of compounding.

Besides the two ideas of compounding, the commentators usually address the issue of how one deals with compound ratio. Contrary to what Grant has stated, Nicole Oresme was not the first to write an algorism of ratios.[608] Some aspects of the algorism of ratio are present in Ametus' *Epistola*, and medieval astronomers

[607] See p. 24 above.

[608] Grant, "Part I," p. 341.

often added further directions. In his notes on the *Almagest*, Campanus gives directions for multiplying terms to produce a compound ratio and also to find the remainder of a subtraction of ratios. He also explains how to find the unknown term in a statement of composition. The Erfurt Commentary, the Vatican Commentary, the *Quadripartitum*, *De sectore*, and Simon's commentary all include rules for compounding and subtracting ratios or for finding the unknown term in a statement of composition. Because astronomy is not wholly theoretical, astronomers gave these sets of rules, which were used to justify the more specific rules for finding certain astronomical values. The practical focus of these algorismic rules stands in contrast to the rules of Oresme, whose algorism is closely connected to his theory of rational and irrational ratios. While Oresme focuses on the addition and subtraction of various types of ratios, the rules in astronomical texts focus on finding unknown terms in statements of composition.

Many medieval astronomers in both camps on the definition of compound ratio, treat ratios as if they were quantities, and the terminology used to denote compounding usually implies that ratios are being treated as quantities. The most common words for compounding are forms of the verbs "componere," "producere," and "aggregare." "Componere" and "aggregare" both imply that parts are placed together to make up a whole. Being placed together implies continuity. Speaking of ratios as components and composites leads naturally to treating them as things that can be added together or subtracted from each other. Again, ratios are whole and parts only analogously to the way in which quantities are wholes and parts;

ratios cannot be literally *placed together* as quantities can, but they are "placed together" through a common term. The other common word for compounding, "producitur," is a word that is often used in medieval works to signify the act of being the result of a multiplication. The related phrase "ducere in" means "to multiply by," and accordingly forms of "producere" and "ducere" are more closely associated with the denominative method of compounding. Medieval works use many other words for compounding that have connotations of parts and whole or of multiplication and division. For example, in the *Almagest* marginalia, the verbs and phrases "valere," "subtrahere," and "addere" are used; the last two are obviously connected to seeing compounding as addition and its opposing operation as subtraction. "Valere," which has several meanings, often means "to be worth," and in mathematics it means "to equal," which has implications of quantification since equality most properly applies to the Aristotelian category of quantity.[609] A note in an *Almagest* manuscript treats ratios in a quantitative manner by speaking of ratios as exceeding or falling short of other ratios.[610]

Unsurprisingly, in the works that stand in the confused area between the conceptions of compounding, the terminology is confused. The Vatican Commentary uses the phrase "proportio multiplicatur," which blurs any distinction between a ratio and its denomination; however, the work more often uses the forms of "componitur" and it talks about the subtraction of ratios ("ratio sublata") and

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[609] Aristotle, *Categories*, 6a.

[610] Appendix B, S edition, lines 22-3.

about ratios as remainders (“ratio remanet”). More surprisingly, even works that consistently treat compounding as the multiplication of denominations still retain terminology with connotations of wholes and parts. For example, Campanus uses the verbs “componere,” “constare,” and “abicere” in addition to verbs that correspond more closely to his idea of compounding such as “producere” and “dividere”.

Many of the works also quantify ratios not by treating them as wholes and parts, but by treating them as identical to the numbers that denominate them. While the Erfurt Commentary carefully applies words of multiplication such as “producere,” “multiplicare,” and “ducere” only to denominations and uses words denoting parts and wholes such as “componere,” “subtrahere,” and “demere” for ratios; however, several of the other works talk about directly multiplying ratios. Richard of Wallingford multiplies ratios by ratios, and he treats quotients of the division of antecedents by consequents as ratios.[611] “Marsiliensis” and Simon also speak of multiplying ratios, and Simon similarly identifies a ratio with the quotient of the antecedent of that ratio divided by its consequent. Multiplication in its most basic meaning involves numbers, so to talk of multiplying ratios requires the obscuring of the difference between ratios and numbers or fractions. Even more explicitly identifying ratios with numbers, the Vatican Commentary states that a ratio *is* the number one.[612]

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[611] E.g., North, Vol. 1, p. 62, 166.

[612] Appendix F, lines 283-6.

Thomas Bradwardine, Nicole Oresme, and other fourteenth-century scholars are more explicit about ratios having the quantity-like feature of being parts and wholes. Bradwardine speaks of ratios being double, triple, quadruple, etc. of other ratios.[613] If ratios can be added together, then it follows that multiples of ratios can be taken by adding a ratio to itself however many number of times. Nicole Oresme actually speaks of ratios as “partes,” and he not only talks about adding, subtracting, and taking multiples of ratios, but he also treats ratios of ratios.[614] If ratios are treated as quantities, then the relationships between them are ratios. While they treat ratios as quantities, Bradwardine and Oresme are clear that ratios are not numbers, and they consciously differentiate themselves from the trend found in the sector figure texts of identifying ratios with their denominations. Bradwardine takes statements about the nature of compounding ratios from Campanus’ treatise on compounding; but while Campanus proves these statements, Bradwardine takes them as statements that need no proofs.[615] In other words, he was aware of the denominative method of compounding, but he chose to define compounding by the interposition of middles. In his *Algorismus proportionum*, Oresme tells how to add and subtract various types of ratios. He includes a caution that “one ratio cannot be multiplied (multiplicatur) or divided (dividatur) by

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[613] Crosby, *Thomas of Bradwardine,* p. 78.

[614] Grant, *Nicole Oresme*.

[615] Crosby, p. 76.

another except improperly."[616] His reasoning is that because multiplication implies taking something a certain number of times, one of the terms involved in multiplication must be a number. This argument assumes that a ratio is not a number; if it were, then a ratio could multiply a ratio. Oresme is replying to the practice of multiplying ratios that some of the scholars who wrote on the sector figure had. While he and Bradwardine go further in developing the theory of ratios as quasi-quantities that can be whole and parts than the authors treated in this dissertation do, these two natural philosophers reject a form of quantification that is found in the sector figure works, i.e., treating ratios as numbers.

While many medieval mathematicians treated ratios as if they were quantities, it is unclear whether any went so far as to think that ratios actually were quantities. While the authorities generally define "proportio" as a type of relation, they are sometimes unclear. Boethius' *Institutiones de arithmetica* and several versions of Euclid's *Elements* (including Campanus') define "proportio" as a "habitudo," which was often understood as a relationship, but in other versions of the *Elements*, the wording is less clear. Adelard Versions I and II define ratio as a "certitudo," and Gerard's translation defines it as a "certitudo mensurationis."[617]

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[616] Grant, "Part I," p. 340. Grant explains this passage by discussing the addition and multiplication of exponents, so he misses the author's point. Oresme explains that you cannot multiply two doubles by two doubles to obtain four doubles by saying, "But this is nothing other than a multiplication of numbers, since the multiplication of two doubles by two triples comes to nothing, just as does the multiplication of a man by an ass." He means that multiplication properly involves numbers. When one tries to multiply two doubles by two doubles, one really multiplies the numbers (perhaps what we could call the coefficients). So, it is allowable to multiply a ratio by a number since what that means is to take the ratio a certain number of times, but multiplying a ratio by a ratio or a square by a square or a man by a man or a man by an ass is nonsensical.

[617] Knorr, pp. 7-16.

The exact meaning of “certitudo” is unclear here, and a reader could possible interpret “certitude of measure” as a denomination. Among other influential authors who define ratio, Jordanus defines it as a “relatio” and Bradwardine defines ratio as a habitudo.”[618] In the medieval sector figure works, Campanus, who defines ratio as a “habitudo,” provides the lone definition of ratio. Among the other authors, it is possible that some did not think that ratios were relations, but since the definition of ratio as a type of relationship was common enough that almost any serious mathematician would have at least known of it. Given the confusion to be found in sources, however, it is conceivable that some medieval mathematicians were unsure to which category ratios belong.

Although historians have not framed the issue in this manner, the core of the history of compounding is at its core a matter of quantification. The sector figure texts contain two main ways in which ratios were treated as if they were quantities. Ratios were seen as having parts and whole and being able to be added or subtracted, but ratios were also treated as numbers. We see then that the quantification movement in fourteenth-century natural philosophy was not an isolated program; from the second half of the twelfth century through the fourteenth, astronomers were treating ratios as if they were quantities. In other branches of mathematics, similar but different movements of quantifying ratios, which merit comparative study, were occurring. While fourteenth-century natural philosophers accepted the astronomer’s treatment of ratios as having parts and

[618] Crosby, *Thomas of Bradwardine*, p. 66. Busard, *Jordanus de Nemore*, p. 74.

whole as quantities do, they rejected the astronomers' treatment of ratios as numbers. Their selection accords with their conception of latitudes of forms, which as the term "latitude" suggests, were understood by analogy to continuous magnitudes such as lines rather than to numbers.

Determining the place of the Menelaus Theorem texts in the history of medieval mathematics opens up new avenues for future research. There are other works that treat the Menelaus Theorem that I was not able to examine.[619] For many of the works that I have examined, I have focused on the Menelaus Theorem, so the full import of these works in the history of astronomy remains to be investigated. The way in which medieval scholars accepted the solar, lunar, and planetary theories of Ptolemy and incorporated their own ideas as well as ones from other sources will not only permit us to better understand medieval astronomy but also to understand changes that were made in the fifteenth and sixteenth centuries. Closer comparison of the texts with the sector figure to other astronomical and mathematical texts will reveal how the practices of astronomical commentaries vary from those of other astronomical texts. Bernard of Verdun's *Astronomia*, for example, does not show the same degree of systematization, and unlike the sector

[619] These include two commentaries on the *Almagest* found in Paris, Bibliothèque nationale de France, lat. 7292; and Paris, Bibliothèque nationale de France, lat 7266. The latter may have ties to the Vatican Commentary. There are other *Almagest* manuscripts with marginalia such as Kraków, Biblioteka Jagiellońska, 589. Two short works that treat the plane sector figures through numerical examples are found in Cambridge, University Library Ee. III, 61 (1017); Cambridge, University Library Gg. VI. 3 (1572); Cambridge, Gonville and Caius 141/191; and Oxford, Bodleian Library, Bodley 300.

figure texts, it finds some astronomical values with a proof using Gebir's alternatives, without a corresponding proof through the sector figure.[620]

Closer comparison between the sector figure texts and other works that treat compounding will also be fruitful. Boethius and Euclid have their own approaches to compounding, as do the works of medieval mathematicians. For example, in Chapter 9 of the *Liber abbaci,* Leonardo of Pisa, also known as Fibonacci, discusses the modes and how to find an unknown term in a statement of composition, but his approach is more algebraic than any of the authors of the works on which this dissertation focuses.[621] While each discipline had its own ways of treating compound ratios, there was influence across the mathematical disciplines. The astronomical works that we have examined include references to Euclid's Elements, Boethius *Institutiones de arithmetica*, Jordanus *Arithmetica*, and Oresme's *Algorismus proportionum*. In Roger Bacon's *Communia mathematica*, which treats ratios at length since proportionality applies to both discrete and continuous quantity, the treatment of compounding and of the modes is based on Campanus' treatise on compounding.[622]

Now that many more texts involving the Menelaus Theorem and compounding are available, a comparison of methods, concepts, and terminology

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[620] Bernard de Verdun, *Tractatus super totam astrologiam*, Polykarp Hartmann, ed., (Werl: Westf, 1961).

[621] Leonardo of Pisa, *Scritti*, Vol. 1, esp. pp. 118-9 and 131-4. Leonardo also addresses the plane sector figure and compounding in his *Practica geometriae*. (Ibid., vol. 2, pp. 51-6)

[622] Bacon, pp. 103-4 and 129-139.

will do much to determine the interdisciplinary connections across the medieval mathematical landscape, the various ways that medieval scholars quantified things that belong in other categories, and how the very ideals of what a scientific work looks like varied in different disciplines.

Historical Epilogue: The Sector Figure in the 15th Century

Because authoritative mathematicians used the sector figure, the sector figure was studied as long as texts such as the *Almagest* and the *Sphaerica* continued to be studied.[623] But, while many of the themes that we have seen still appear at the end of the middle ages, there were new approaches and emphases in the treatment of the sector figure and compound ratios, as can be seen in the writings of four fifteenth-century astronomers, Iohannes Blanchinus, George of Trebizond, Georg Peurbach, and Iohannes Regiomontanus. In fact, by the late fifteenth century, the study of spherical triangles obviated the need for the Menelaus Theorem.

Iohannes Blanchinus

Iohannes Blanchinus, also known as Giovanni Bianchini (b. early 15th century, d. post 1469), was one of the foremost astronomers of his time.[624] Among various astronomical works, he wrote a summary of the first six books of the *Almagest* called the *Flores Almagesti*, which like so many of the earlier sector

[623] Oxford, Bodleian Library, Savile 9, which dates from the seventeenth century, contains notes on the sector figure and on Thabit's treatment of the modes although it also contains trigonometric material from Rheticus and Regiomontanus. In the sixteenth century, Maurolycus made a translation of Menelaus' *Sphaerica*, but in the field of astronomy he wrote the *Brevissima epitome totius almagesti* (found in Paris, Bibliothèque Nationale de France, lat. 7471), in which he uses Gebir's alternatives instead of the sector figure.

[624] See José Chabás and Bernard R. Goldstein, *The Astronomical Tables of Giovanni Bianchini*, (Brill, Leiden & Boston 2009) for a good summary of what is known of his life.

figure works is arranged systematically.[625] Of the nine treatises into which the *Flores Almagesti* is divided,[626] Treatises I-IV are on mathematics to be used in astronomy, and the fourth of these is divided into three books. In the first he provides definitions of terms relating to geometry and proportions, in the second he gives several proofs, and in the third he treats the sector figure.

Blanchinus has an understanding of compound ratio based upon denominations. His definition of ratio accords with this: "A ratio is a habitude of two quantities of the same kind towards each other through the third definition of the fifth book of Euclid, for a habitude is a common measure."[627] While this definition is taken from the *Elements*, Blanchinus understands this definition in a new light by taking "habitude" not as "relation" but as "common measure." While this concept of ratio is unusual, it fits well with the use of denominations, and the next few definitions deal with compounding in a way that makes it clear that by "common measure," Blanchinus means "denomination." He writes:

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[625] Chabás and Goldstein, p. 19; Grażyna Rosińska, "The 'Italian Algebra' in Latin and How It Spread to Central Europe: Giovanni Bianchini's 'De Algebra' (ca. 1440)," *Organon* 26-7 (1997-8): 131-145. There is some material on the parts of the *Almagest* after Bk. VI, but the theory of the planets and fixed stars is mostly absent. The work exists in seven manuscripts: Kraków, Biblioteka Jagiellońska 558 (has notes in Regiomontanus' hand); Perugia 10004; Città del Vaticano, Biblioteca Apostolica Vaticana, Reg. lat. 1904; Città del Vaticano, Biblioteca Apostolica Vaticana, Vat. lat. 2228; Paris, Bibliothèque nationale de France, lat. 10253; Bologna, Biblioteca Universitaria 198 (293); and Kraków, Biblioteka Jagiellońska 601 (has only the section on algebra).

[626] In different manuscripts the work is divided into 8, 9, or 10 treatises, and even the order of these parts varies from manuscript to manuscript. Fortunately, we are able to have an idea of what order and division the author intended since he describes the work in detail in his *Canones primi mobilis*, which follows the *Flores Almagesti* in Vat. lat. 2228. There he divides the work into nine treatises, some of which are divided further into books.

[627] Città del Vaticano, Biblioteca Apostolica Vaticana, Reg. lat. 1904, fol. 1r. "Proportio est ad invicem habitudo duarum eiusdem generis quantitatum per diffinitionem [tertiam] quinti Euclidis, habitudo enim est communis mensura."

> 3. To add or to unite a ratio to a ratio means to multiply a ratio or its common measure by the common measure [of the other].
> 4. To subtract a ratio from a ratio is done in the opposing way, because it means to divide common measure by common measure.
> 5. To double a ratio is to square the common measure, that is to multiply it by itself.
> 6. To triple a ratio, moreover, means to cube it, i.e. to multiply the common measure multiplied by itself again by itself.
> 7. To quadruple is to square a square, and thus it is able to be advanced through infinity. From which it is concluded that to add a ratio is the same as to multiply, but to take away is the same as to divide.[628]

These definitions show that he takes compound ratios to be the product of the operation of multiplication of denominations. While earlier astronomers blurred the lines between ratios and numbers, they may have merely been treating ratios as if they were numbers although they understood that technically they were relations; here, however, there is no doubt that Blanchinus understands ratios as being identical to common measures, which are numbers. Some of the earlier medieval sector figure authors, such as Simon Bredon and the authors of the Vatican commentary and the texts on the canons of Arzachel, may have understood ratio in a similar manner. Although much of his treatment of compounding is based on understanding it as the multiplication of ratios or denominations, traces of the other concept of compounding remain. Blanchinus provides two propositions that are reformulations of the proposition that continuous ratios lead to statements of

[628] Ibid., fol. 1r. "3. Addere proportionem [proportioni] seu continuare intelligitur proportionem seu communem mensuram per communem mensuram multiplicare. 4. Subtrahere proportionem a proportione [contrario] modo operatur quia intelligitur communem mensuram dividere per communem mensuram. 5. Duplare proportionem est communem mensuram quadrare idest in se multiplicare. 6. Triplicare autem proportionem intelligitur ipsam cubicare idest communem mensuram in se multiplicatam iterum productum multiplicare in ipsam. 7. Quadruplare enim est quadratum quadrare et sic per infinitum potest procedi. Ex quibus concluditur quod addere proportionem est idem quantum multiplicare, demere autem quantum dividere."

compositions, but his proof ultimately is circular because it calls upon a prior proof that implicitly relies upon the continuity concept of compounding. In trying to prove that the continuous concept relies upon the denominative, he tacitly relies upon the former.[629]

Like many of his medieval predecessors, Blanchinus attaches a treatment of the modes to his treatment of the sector figure. Unlike the thorough treatments of the modes that are given by the author of the Vatican Commentary or Richard, Blanchinus only lists eight of the eighteen modes in the text, and he only provides one general proof illustrating that given that a ratio is composed of two others, each of the composing ratios is composed of the original composed ratio and the inverse of the other original composing ratios.[630] While this may seem to show his disinterest in the modes, he merely has another approach to them. Without much of a concern for proof of the validity and invalidity of the possible combinations, he compiles tables that first illustrate the modes of the plane conjoined and disjoined sector figures with numerical examples, and then list the eighteen modes and their inverses for each of the four statements of the plane sector figure (two for the conjoined and two for the disjoined, because each one can have the composed ratio being of the parts of the line on the right or left side of the diagram).[631] This second part of the tables consists of 144 valid statements of composition arising

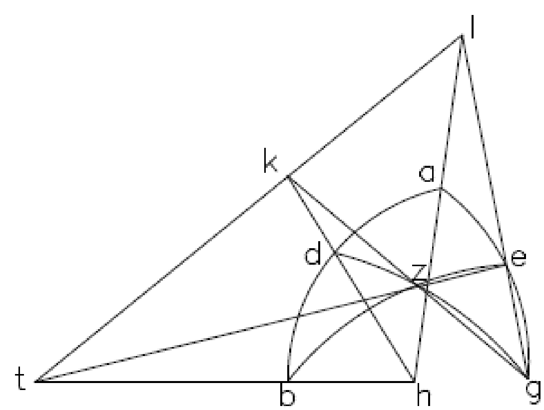
[629] Ibid., fol. 14v-15v.

[630] Ibid., fol. 22r-v.

[631] Ibid., fol. 23r- 25r.

from one diagram of the plane sector figure. In his *Canones primi mobilis*, Blanchinus refers often to these tables.[632]

Like many of the other writers on the sector figure, Blanchinus shows a great concern for the practice of astronomy, so he explains how the numbers of the ratios to be multiplied or divided should be arranged on a sheet of writing material and how to reach the correct answer by multiplying in the form of an X.   He also gives series of rules without proofs for how to find the missing quantity in a statement of composition, as well for how things simplify when two of the quantities in a statement of equality are equal, as is often the case in the *Almagest* and when working with sector figures that have two or more of their arcs quarter circles. He also offers some rules about how to simplify a statement of composition when four of the terms are proportional, which he follows with directions for finding unknown quantities in proportions.[633]



Blanchinus' treatment of the Menelaus Theorem is fairly typical of medieval sector figure works. He stays close to Ptolemy's proofs of the lemmas and the disjoined spherical sector figure, even using the same diagram letters, but he adds explanations and justifications for many of

[632] E.g., Città del Vaticano, Biblioteca Apostolica Vaticana, Vat. lat. 2228, fol. 57v.

[633] Città del Vaticano, Biblioteca Apostolica Vaticana, Reg. lat. 1904, fol. 13r-14v.

the steps. He points out that Ptolemy gives the conclusion from the conjoined sector figure but does not prove it since he wanted to be succinct and skip the proof and he assumed that "learned men" could fill it in if necessary. Blanchinus does not attempt to complete the proof fully, but he states that he will prove a sufficient number of the cases for the most useful applications.[634] He then proves the case of the spherical conjoined sector figure that has chords *ge*, *ez*, and *gz* meet their respective diameters of circle *adb* on the sides of points *e*, *z*, and *z* again respectively.[635]   Blanchinus informs his readers that this is only one of many possible cases of the spherical conjoined sector figure since the chords do not have to meet their respective diameters as in this case, but he does not prove any of the other cases.[636]

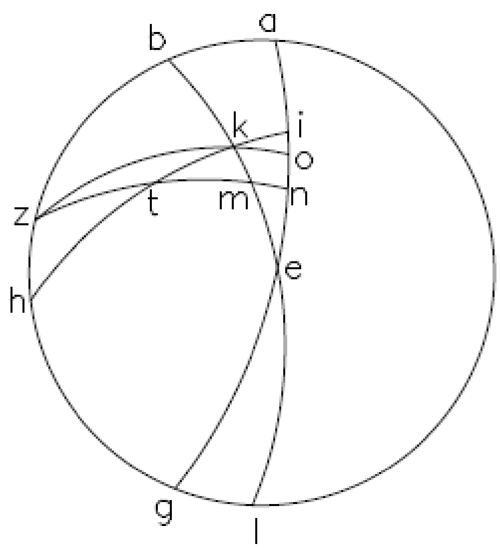
Blanchinus' application of the Menelaus Theorem to astronomical problems differs from that of Ptolemy. Trying to simplify matters, Blanchinus makes a "universal diagram" for several propositions relating to the right sphere, which he follows with another "universal diagram" for the oblique sphere. He refers constantly to his table of ratios to find the relevant statement of composition for each problem, and he uses the rules for simplifying statements of composition and for finding the unknown term. Unlike Ptolemy, Blanchinus groups propositions dealing with right or oblique spheres regardless of whether they involve arcs of the

[634] Ibid., fol. 21v.

[635] Ibid.

[636] Ibid., fol 22r.

ecliptic, planets, or fixed stars. For example, after showing how to find declinations of the ecliptic and right ascensions, he shows how to find the equatorial longitude of a star through its given arc of latitude. For some of the proofs, the relevant sector figure is not lettered as the one for the table of ratios, so



to find the corresponding statement of composition, he has to place the standard letters "in the place" of one in the particular example. For instance, if he has a sector figure formed by arcs *bkm*, *bzh*, with *mtz* and *htk* crossing between them, he has to put *g* in place of *m*, *e* in place of *k*, *a* in place of *b*, etc. Then with the sector figure relettered to match the table of ratios, the appropriate statement of composition can be found.[637]

George of Trebizond

In 1451, George of Trebizond (1395-1486), known as Trapezuntius, who came from Crete to Italy as a young man and who was a humanist renowned for his masterful style of writing, produced a new translation of the *Almagest* accompanied by an extensive commentary, in which he discusses the Menelaus Theorem and

[637] This example is found on Città del Vaticano, Biblioteca Apostolica Vaticana, Reg. lat. 1904, fol. 30v in his proof of how "to find the arc of the ecliptic corresponding to a star having a north latitude in the middle heaven."

issues of compound ratio at great length. George attempted to dedicate this work to his patron Pope Nicholas V, but early in 1452 he was involved in a dispute with Poggio Brachiolini, and he lost the favor of both Nicholas and Cardinal Bessarion, who had been a patron.[638] As we will see, his falling out with Bessarion would play a part in Peurbach and Regiomontanus' work. His translation and commentary were criticized harshly, but the translation was printed twice in the first half of the sixteenth century.[639] As a humanist, George structures his commentary differently than the scholastic astronomers that we have seen. He returns to the style of Ptolemy and organizes his work into chapters, not propositions. While his translation of the *Almagest* shows no major differences from the treatment of the sector figure and compound ratio found in Gerard's translation, George has much to say on these topics in his commentary.

George's commentary shows him struggling to understand compound ratios. Sometimes he speaks of a composed ratio being the whole that results from having component ratios made continuous, while at other times he speaks of a composed ratio being one whose denomination is the product of the multiplication of the denominations of other ratios. George writes, "For with two numbers written and any third added, the ratio of the extremes becomes composed, as with 2, 4, and

[638] John Monfasani, *George of Trebizond: A Biography and a Study of His Rhetoric and Logic*, (New York: Columbia University Press, 1976).

[639] *Claudii Ptolemaei Pheludiensis Alexandrini Almagestum, seu Magnae constructionis mathematicae opus plane divinum*. (Venice: calcographica Lucantonii Iunta officina aere proprio ac typis excussa, 1528). *Claudij Ptolemaei Pelusiensis Alexandrini Omnia quae extant opera, praeter Geographiam*, (Basel: in officina Henrichi Petri, 1551).

7, the ratio of 2 to 7 is composed by the ratios of 2 to 4 and of 4 to 7."[640] But, soon after, he writes, "[If] any two numbers multiplied by each other make a third, the ratio denominated by the third is composed of the ratios that are denominated by the numbers multiplied into each other."[641] He gives a special way of compounding for cases in which the ratios to be compounded together are not of the same "habitudo," by which he seems to mean one is a ratio of greater inequality while the other is a ratio of lesser inequality.[642] For such ratios, he states, "Because contraries cannot be multiplied by each other, the ratio of the denominations is the ratio of the extremes."[643] He returns to the continuous ratio concept, writing, "For a ratio of extremes is not composed properly except by the ratios that the extremes have to a single middle, for a composition of a magnitude with a magnitude and of a number with a number is as addition."[644] George treats ratios as numbers more explicitly than most of the medieval sector figure authors, but the inconsistencies

[640] Città del Vaticano, Biblioteca Apostolica Vaticana, Vat. lat. 2058, f. 12r: "Duobus enim numeris conscriptis tertioque addito quovis extremorum, proportio composita fit, ut 2 4 7, proportio 2 ad 7 componitur a proportionibus 2 ad 4 et 4 ad 7."

[641] Ibid. "[Si] quicumque duo numeri inter se multiplicati tertium faciunt, proportio a tertio denominata componitur ex proportionibus que a numeris in seipsos multiplicatis denominantur."

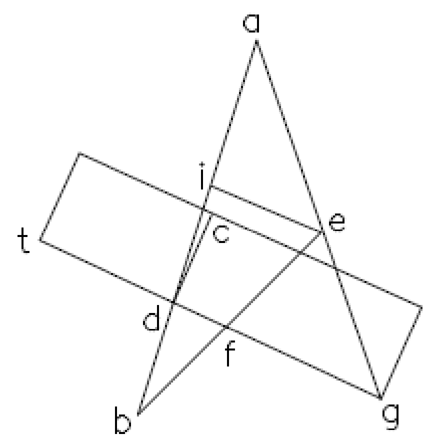
[642] The word "habitudo" had a variety of meanings. While in the context of mathematics, it often meant "relation," it sometimes meant "condition" more generally.

[643] Ibid., f. 12v. "Si autem non est eadem habitudo componentium tunc quoniam multiplicari interse contraria non possunt, proportio denominationum est proportio extremorum." What George has in mind is more complicated than he lets on in his brief statement. He gives the example of the numbers 2, 6, and 4. It is unclear in the first place why he thinks that it is invalid to multiply the denominations of 2:6 and 6:4. If we take the ratio of the denominations of these two ratios, we have the ratio of one third to three halves, which is not the ratio of 2 to 4. If, however, we take the ratio of the first denomination (1/3) to the reciprocal of the second denomination (2/3), then we have the ratio of 2 to 4.

[644] Ibid. f. 12v. "Nam proportio extremorum non componitur proprie nisi a proportionibus quas ad unicum medium habent extrema, compositio enim magnitudinis cum magnitudine numerique cum numero quasi additio est."

persist. For example, he says that a sesquialterate ratio (3:2) exceeds a sesquitertia ratio (4:3) by a sixth. Although this contradicts rules he gives elsewhere for the subtraction of ratios, it is true if ratios are numbers.[645]

Like Blanchinus, George's treatment of compounding is at times circular. Ptolemy introduces a statement of composition by inserting a middle between two quantities, and George attempts to prove that this is true despite having given the continuity of ratios as one of the explanations of compound ratio in the chapter on compound ratios. His proof adds the construction of rectangles and relies upon Euclid's *Elements* VI.23,[646] which states that the ratio of equiangular parallelograms is composed of the ratios of their sides.[647] However, *Elements* VI.23 relies upon deriving a statement of composition from continuous ratios, so George's proof is circular.



Another difference between George and many earlier commentators is that George does not deal extensively with the modes. He merely proves that when a

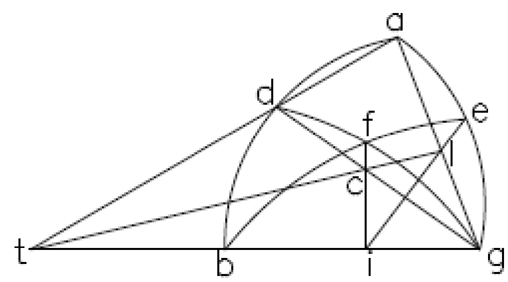
[645] Ibid. fol. 14v. "Sesqualtera excedit sequitertiam per sextam partem quoniam bis tria [est] sex. . . Hoc inde quoque affirmatur quia medietas tertiam partem per sextam excedit."

[646] VI.24 in some medieval numberings.

[647] Città del Vaticano, Biblioteca Apostolica Vaticana, Vat. lat. 2058, fol. 30r-v.

ratio is composed of two others, then one of the composing ratios is composed of the composed ratio and the inverse of the other.[648]

George runs into more difficulty while explaining the spherical sector figure. Although his proof of the disjoined sector figure is mostly a summary of Ptolemy's, he still makes a mistake. While Ptolemy says to take the center of the sphere without showing how, George tries to explain how it can be found. In this



explanation, he argues that *bi* and *ig* are in a straight line since they are both sections of the same circles.[649] However, they actually cannot be in a straight line![650] The source of George's mistake seems to have been an overreliance on his diagram, which (at least in Vat. lat. 2058) portrays lines *bi* and *ig* in a straight line. Ironically, George warns his reader that although the diagram makes it appear that all the lines and circles are in one plane although they are really in a variety of different planes, but then he immediately falls prey to exactly this problem.[651] His

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648 Ibid. fol. 29v.

649 Ibid. fol. 33r-v.

650 Because *bi* is in the plane of circle *bfe* and *ig* is in the plane of circle *dfg*, and these circles' intersection is line *fi*.

651 Città del Vaticano, Biblioteca Apostolica Vaticana, Vat. lat. 2058, fol. 34r. "Est autem *tcl* linea dictarum superficierum trianguli *adg* et circuli *bfe* protractarum terminus communis quo uniuntur se ipsos secantes, quamvis in plana descriptione trianguli et omnium circulorum esse videatur quod esse nequaquam potest, una enim et eadem superficies se ipsam secare non potest. . . Difficile igitur est intelligere cur *bfe* et non alterius circuli superficies dicti trianguli *adg* superficiem per *tcl* lineam secet cum in plana ut diximus superficie commune hoc omnium circulorum esse videatur quod impossibile est."

lack of mastery of the subject is even more apparent in his attempt to prove a single case of the spherical conjoined sector figure.[652] He misapplies the third and fifth lemmas to the sector figure, and he makes other crucial mistakes including not differentiating between arcs and the chords of their doubles.

Peurbach and Regiomontanus

In 1260 or possibly early 1261, Cardinal Bessarion encouraged Georg Peurbach to write a commentary on the *Almagest* to take the place of George of Trebizond's commentary. The first six books of this work, entitled the *Epitome Almagesti* were completed by Peurbach, who died in April, 1461, and the remainder was completed by his colleague Johannes Regiomontanus. Regiomontanus is known to have added a translation from the Greek of the first chapters of the *Almagest* to the *Epitome*, and the degree to which he edited and revised Peurbach's six books is not clear.[653] The *Epitome*, which bears a great resemblance to the *Almagestum parvum* in style and content,[654] is divided into propositions, which each generally have their own universal enunciations,

[652] Ibid. fol. 35r-v.

[653] Venezia, Biblioteca Nazionale Marciana, F.a. 329 has the earliest version of the work, so a comparison of it with later versions may shed some light on this issue. C. Doris Hellman and Noel M. Swerdlow, "Puerbach (or Peuerbach), Georg," *Complete Dictionary of Scientific Biography*, vol. 15, 473-9, (Detroit: Charles Scribner's Sons, 2008), here p. 476.

[654] Hellman and Swerdlow, p. 477, state, "[O]ne may legimately ask to what extent the present state of the first six books is really the result of Regiomontanus' revision of what Peurbach may have left as little more than a close parapharase of the *Almagestum minor*." That the connection is not coincidental is seen to be probable from similarities such as both works placing the explanation of how to find the greatest declination of the ecliptic after the Menelaus Theorem instead of before it as Ptolemy does and both works proving the same case of the spherical conjoined sector figure.

particular restatements in terms of the figure, explanations of the construction of the figure, proofs, conclusions, and often corollaries either giving the mathematical operations needed for finding sought quantities or giving proportions or statements of composition.

In the *Epitome*, the sector figure and its preliminaries are proved without any significant variations from the *Almagest*. Peurbach does prove both spherical sector figures, but only one case of each. In the first application of the sector figure, Peurbach deals with the statement of composition by subtracting a ratio from a ratio, as Ptolemy does. He later gives rules for the addition and subtraction of ratios as well as for finding unknown terms in statements of composition. His proofs of the correctness of these rules rely upon continuous ratios leading to statements of composition.[655]

In the applications of the sector, Puerbach and Regiomontanus use the methods of simplification that we have seen elsewhere, such as reducing a statement of composition to a proportion if one of the antecedents of one of the composing ratios is equal to the consequent of the other. In the *Epitome*, we see many original ways of finding sought values. For example, Peurbach or Regiomontanus shows how to calculate the difference between twelve hours and the shortest day of any latitude using only three quantities.[656] He does this by deriving a statement of composition, and then he effectively performs the part of

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[655] Georg Peurbach and Johannes Regiomontanus, *Epitome in Cl. Ptolemaei magnam compositionem*, (Basil: Henrichus Petrus, 1543), pp. 22-4.

[656] Ibid., pp. 27-8.

the solution involving the three terms that are not dependent upon the latitude. This gives him a new quantity that is in a proportion with the two quantities that are known when the particular latitude and the unknown difference between the mean and shortest days are given. This method of solving constant parts of the problem in advance is similar to the technique Ptolemy uses in his second way of finding oblique ascensions.[657] The *Epitome* also gives a new way of calculating the oblique ascensions at any given latitude, which the author says is the easiest for the production of tables.[658] He also uses Gebir's alternatives to show where on the ecliptic the greatest changes in declinations and ascensions occur. In these proofs, he refers to one of Gebir's alternatives to the Menelaus Theorem and he also uses Menelaus' *Sphaerica*.[659]

Later in the *Epitome*, Regiomontanus refers several times to the "science of spherical triangles." He explains how in several of the places where Ptolemy had used plane approximations for spherical problems related to the theory of the moon, fixed stars, and planets, one can use sector figures or the "scientia triangulorum

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[657] *Almagest* II.7.

[658] *Epitome in Cl. Ptolemaei magnam compositionem* II.25-27, pp. 34-6. This method involves finding the difference between right and oblique ascensions for the latitude of, and then showing that for any given latitude the ratio of the sine of the complement of the altitude of the pole to the sine of the altitude is as the ratio of the difference between a part of the ecliptic's right and oblique ascensions at the latitude of 45^{o} to the difference between the same part of the ecliptic's right and oblique ascensions at this new latitude. Both of these results come from the sector figure. He states that this is the easiest way to calculate the values for a table of oblique ascensions for a given latitude. If the difference between ascensions is divided by the single quantity that represents the first ratio of the proportion, the sought quantity results. In this way, after initial calculations have been performed, each of the at least 18 values that need to be calculated can be done by a simple act of division.

[659] Ibid., III.25-6, pp. 64-5.

sphaeralium" to find a true answer instead of the plane approximation.[660] Regiomontanus follows Ptolemy in using plane approximations some of the time, but he points out that the answer can be had more accurately through the "science of spherical triangles."[661] While Regiomontanus generally summarizes Ptolemy's applications of the sector figure in the later books, he often adds other proofs that use alternative theorems about spherical triangles.[662] Although he does not prove the validity of the eighteen modes, like Blanchinus he proves that one of the composing ratios in a statement of composition is composed of the original composed ratio and the inverse of the other composing ratio,[663] and he shows a knowledge of the modes by referring to this as "the eleventh way of alternation."[664] That he calls it the eleventh suggests that he has his own way of ordering the modes since this is the eleventh mode in neither Thabit nor Ametus, but he does not list the modes anywhere in this work.

While Regiomontanus only makes passing references to the science of triangles in the *Epitome Almagesti*, it is the subject of his *De trianguli omnimodis libri quinque*.[665] In this work, he proves not only the alternatives to the sector

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[660] E.g., Ibid., V.26, p. 108-9; V.31-2, pp. 111-2; VI.15, p. 125; VII.8-10, pp. 145-8.

[661] Ibid., VI.24, 28, pp. 132, 136.

[662] E.g., Ibid., VIII, 6, 12, pp. 154-5, 159-160.

[663] Ibid., VIII.7, pp. 155.

[664] Ibid., VIII.12, p. 160.

[665] Joannes Regiomontanus, *Doctissimi viri et mathematicarum disciplinarum eximii professoris Ioannis de Regio Monte De triangvlis omnímodis libri qvinqve: quibus explicantur res necessariæ cognitu, uolentibus ad scientiarum astronomicarum perfectionem deuenire : quæ cum nusquā alibi hoc tempore expositæ habeantur, frustra sine harum instructione ad illam quisquam aspirarit,*

figure that Gebir had given, but many more. Although he does not prove the Menelaus Theorem in this work, Regiomontanus does treat compound ratios at length and he bases his treatment of ratios heavily upon denominations.[666] He proves propositions that are equivalent to Gebir's alternatives to the sector figure,[667] and in a series of propositions, he provides all the possible solutions of spherical triangles—i.e. given certain sides and/or angles, that the remaining sides and angles will be known.[668] Although these propositions replace the sector figure almost completely, he still uses the composition of ratios a few times (in which he understands compounding through continuous ratios).[669] Surprisingly, in the penultimate proposition of the whole work, he gives a statement of composition without explaining why it is true, but it is true because of the conjoined sector figure.[670] On the whole, however, *De triangulis* marks a movement from general

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(Nuremberg: In ædibus Io. Petrei, 1533); available at http://fondosdigitales.us.es/fondos/libros/1164/1/doctissimi-ioannis-de-regio-monte-de-triangulis-omnimodis-libri-quinque-accesserunt-huic-in-calce-pleraque-d-nicolai-cusani-de-quadratura-circuli-deque-recti-ac-curui-commensuratione-itemque-io-de-monte-regio-eadem-de-re-elenktika/ (accessed May 2013). A facsimile of this edition accompanied by a translation is found in , Barnabas Hughes, ed., *Regiomontanus: On Triangles* (Madison: University of Wisconsin Press, 1967).

[666] At points Regiomontanus is essentially discussing compound ratios when it is not obvious at first glance that he is. For example, I.12 (pp. 15-6) shows that if the ratios of two quantities to a third are known, then the ratio between those two quantities is known. Although he does not use the ideas of compound ratio and subtraction, this would have been seen as showing that the subtraction of a known ratio from an unknown ratio results in a known ratio. Interestingly, his proof of this relies upon the nature of denominations, but also depends on *ex aequali*, which is closely connected with compounding by continuity.

[667] Regiomontanus, *De triangulis*, IV.15-19, pp. 101-8.

[668] Ibid., IV.25-34, pp. 114-125.

[669] Ibid., V.12, pp. 135-6.

[670] Ibid., V.14, pp. 136-7.

spherics to spherical triangles that would completely obviate the need for the sector figure.

In the *Defensio Theonis contra Trapezuntium*, Regiomontanus' thorough and harsh critique of George of Trebizond's commentary on the *Almagest*, George's treatment of compound ratios and the sector figure is criticized at length.[671] While most of Regiomontanus' complaints about George's errors in these matters are the ones that I have mentioned previously, Regiomontanus' critique also shows his own understanding of compound ratios. He defends Thabit, Gebir, and Ametus from George's claims that the Arabs did not understand Ptolemy's proof of the Menelaus Theorem; while George had not read any of these authors, Regiomontanus had clearly read them all.[672] His view on the definition of compound ratio is made clearer in his critique of George's attempt to prove that the insertion of a middle leads to a statement of composition. He observes that Euclid takes the statement that George tries to prove as a principle. That Regiomontanus interprets Euclid in this way suggests very strongly that that is how he himself viewed compound ratios.[673]

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[671] This work is the subject of an excellent website: http://regio.dartmouth.edu/index.html. This project headed by Richard Kremer and Michael Shank includes scans of each folio and a transcription of the text from the one surviving autograph manuscript of the work, Archive of the Russian Academy of Sciences, St. Petersburg Branch, IV-1-935. The following citations come from the website's transcriptions.

[672] Regiomontanus, *Defensio*, fol. 3r-v.

[673] Ibid., fol. 11v-12v.

Although my examination of the sector figure in the fifteenth century is cursory and preliminary, it is clear that the Menelaus Theorem remained a topic of interest.[674] Important mathematicians and humanists such as Blanchinus, Trapezuntius, Peurbach, and Regiomontanus were interested in it. Perhaps consciously trying to avoid the most pedantic aspects of medieval scholarship, they were not as systematic as many of the scholastic commentators; they did not attempt to prove all the modes or to prove all cases of the sector figure. On the other hand, many aspects of the earlier works were echoed in these later works. They gave proofs of the conjoined sector figure, which Ptolemy had not proved; they gave some proofs related to the modes; they used both the denominative and continuity ideas of compound ratio; they use terminology and concepts of quantity in their treatment of compound ratio; and they often use both the Menelaus Theorem and an alternative in astronomical applications. Regiomontanus' work, however, contains an advanced spherical geometry that would be used by some to replace spherical geometry based upon the sector figure. In 1543 Nicholas Copernicus' *De Revolutionibus Orbium Coelestium* was published, and not only can it be seen as the beginning of the end for Ptolemaic astronomy, but also for the

[674] Even the scholars I have addressed have more to say on the subject. Blanchinus and Regiomontanus corresponded with each other on a variety of astronomical issues, many of which involved applications of the Menelaus Theorem and its alternatives. See Ernst L. W. M. Curtze, "Der Briefweschsel Regiomontan's mit Giovanni Bianchini, Jacob von Speier und Christian Roder," in *Urkunden zur Geschichte der Mathematik im Mittelalter und der Renaissance*, 185-336, (Leipzig, 1902); and Armin Gerl, *Trigonometrisch-astronomisches Rechnen kurz vor Copernicus: der Briefwechsel Regiomontanus-Bianchini*, (Stuttgart: Steiner Verlag, 1972).

Menelaus Theorem; its spherical astronomy relies upon the solutions of spherical triangles, and no traces of the sector figure are to be found.[675]

[675] Whether Copernicus was directly influenced by Regiomontanus is debated (See Ernst Zinner, *Regiomontanus, His Life and Work*, (Amsterdam: North-Holland, 1990), pp. 183-4), but it is fairly clear that Regiomontanus' focus on the solution of triangles was influential and at least indirectly influenced Copernicus. See Mary C. Zeller, "The Development of Trigonometry from Regiomontanus to Pitiscus," (PhD diss., University of Michigan, 1944), pp. 53-6; Nicolaus Copernicus, *On the Revolutions*, Edward Rosen, ed., (Baltimore: Johns Hopkins University Press, 1992), p. 367; and Noel M Swerdlow, and Otto Neugebauer, *Mathematical Astronomy in Copernicus's De Revolutionibus*, (New York: Springer-Verlag, 1984), Part I, 103-4.

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Paris, Bibliothèque nationale de France, lat. 7255

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Paris, Bibliothèque nationale de France, lat. 7257

Paris, Bibliothèque nationale de France, lat. 7258

Paris, Bibliothèque nationale de France, lat. 7259

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Venezia, Biblioteca Nazionale Marciana, Marc. lat. XIV 291 (4631)

Victoria, State Library of Victoria, Australia, Sinclair 224

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Wien, Österreichische Nationalbibliothek 5273

Wien, Österreichische Nationalbibliothek, 5277

Wien, Österreichische Nationalbibliothek 5292

Wolfenbüttel, Herzog August Bibliothek, Co. Guelf. 24 Aug. quart.

APPENDICES

In the following appendices, the most commonly used abbreviations are:

a. m. = alia manu

add. = additur

adnot. = adnotatur

ant. = ante

del. = deletur

exp. = expunction [deletion by dot(s) under word(s)]

inv. = invertitur

mg. = in margine

om. = omittitur

pos. = postponitur

sup. lin. = supra lineam

In a list of the sigla of more than one manuscripts, information given in parantheses describes the preceding manuscript.

Appendix A: The Menelaus Theorem in Ptolemy's *Almagest*

My collation is taken from the following manuscripts:

Città del Vaticano, Biblioteca Apostolica Vaticana, Barb. lat. 182 = C

San Marino (CA), Huntington Library, HM 65 = E

Città del Vaticano, Biblioteca Apostolica Vaticana, Vat. lat 6788 = F

Paris, Bibliothèque nationale de France, lat. 16200 = H

Paris, Bibliothèque nationale de France, lat. 17864 = I

Paris, Bibliothèque nationale de France, lat. 14738 = J

Kraków, Biblioteka Jagiellońska 619 = K

Città del Vaticano, Biblioteca Apostolica Vaticana, Vat. lat. 2057 = L

Città del Vaticano, Biblioteca Apostolica Vaticana, Barb. lat. 173 = M

Città del Vaticano, Biblioteca Apostolica Vaticana, Barb. lat. 336 = N

Kraków, Biblioteka Jagiellońska 590 = R

Città del Vaticano, Biblioteca Apostolica Vaticana, Pal. lat. 1365 = S

Paris, Bibliothèque nationale de France, lat. 7260 = T

Paris, Bibliothèque nationale de France, lat. 7259 = U

Paris, Bibliothèque nationale de France, lat. 7258 = V

Paris, Bibliothèque nationale de France, lat. 7257 = W

Paris, Bibliothèque nationale de France, lat. 7256 = X

Paris, Bibliothèque nationale de France, lat. 7255 = Y

Paris, Bibliothèque nationale de France, lat. 7254 = Z

Venice 1515 Edition = P

Gerard of Cremona’s Translation of the Almagest, Selection from I.12

Et quoniam sequitur post hoc ut demonstrem numerum partium quantitatum arcuum qui sunt orbium maiorum descriptorum supra duos polos equationis diei et sunt arcus qui sunt inter lineam equationis diei et lineam medii orbis signorum. Oportet ut premittam capitula pauca utilitatem afferentia quibus possimus demonstrare plurimum scientie | I 8r demonstrationum sphericarum quam levius possibile est et sapientius.

[Lemma 1]

Describam ergo duas lineas | M 15vb *ab ag* et protraham in eo quod inter duas illas lineas est *be gd* sese supra *z* secantes. [Figura 1] Dico ergo quod proportio *ga* | S 13r ad *ea* aggregatur ex duabus proportionibus: ex proportione *gd* ad *dz* et ex proportione *zb* ad *be*. Quod sic probatur. Protraham ab *e* lineam *eh* equidistantem linee *gd*, | Y 10r et quia *eh* et *gd* sunt equidistantes, sit proportio *ga* ad *ae* sicut proportio *gd* ad *eh*. Ponam

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4 Et] Non est capitulum *adnot.* E | sequitur post] *pos*. demonstrem CHJPSTUV per hoc *pos*. demonstrem K | demonstrem] id est describam omnes arcus alios a predictis *adnot.* (*sup. lin. a. m. in codicibus et sic deinceps nisi aliter notetur*) H **5** orbium maiorum] *inv.* I | supra] super EFMNPRTVWX **6** polos] *sup. lin. a. m.* J orbis *add.* FP | et[1]] sunt *add.* IL | sunt[1]] *om.* HV | sunt arcus] *inv.* CJKTU | arcus] declinationis *adnot. supr. lin. a. m.* M | qui sunt] alii libri habent hic sunt *adnot.* H **7** lineam] id est vocatur linea mel ecliptica *adnot.* L | premittam] (check S) **8** quibus] qui U | possimus] possumus ISV | plurimum] multum MLRW | scientie demonstrationum] *inv. sup. lin*. N **9** possibile] *sup. lin.* L | possibile est] *supr. lin*. R | est] *add. supr. lin. a. m.* X **12** ab] et *add.* P | ag] *as* I | in…quod] *om.* LMW | in…inter] inter eas FMRW | quod] est *add.* S | inter] eas L eas *add.* ENX **13** duas illas] *inv.* CJKPUYZ | illas] *om.* EFLNVWX | est] *ant.* inter EINYZ *om.* FLMRXW | be] et *add.* P | sese…15 gd] *marg. a. m.* R | z] *s* P (s in diagrams instead of z) **14** proportio] portio T | ea] que est pars eius *adnot.* H | proportionibus] scilicet *add.* FR (*sup. lin.*) H **15** dz] *ds* P | ex] *om.* I | zb] *sb* P probatur] per 31 1[i] Euclidis *adnot*. *sup. lin. m. a.* L **16** Protraham] enim *add.* EFMNRWX ab] ad F | ab e] a *b e* V | eh[1]] per 31 1[i] (Euclidis *add*. R) *adnot*. FR | eh equidistantem] *inv*. V linee] lineam I | gd[1]] *dg* PT ex 31 primi *adnot*. *mg. a. m.* Y | quia] linee *add. sed exp.* N eh[2]…gd[2]] *inv*. V **17** equidistantes] per 29 1[i] Euclidem *adnot.* H per 31 1[i] Euclidis *adnot. sup. lin. a. m.* L | sit] fit ENT | ae] per 2[am] 6[i] et 4 eiusdem *adnot.* K | eh] per 4 6[i] Euclidem(*om*. W) *adnot.* H(*mg. a. m.* W)W per primam animi conceptionem sexti geometrie et quarta 6i ... et per quartam propostium sexti et per xiiii quinti *adnot. supr. lin. a. m.* Z ex 4 6i *adnot. mg. a. m.* Y propter triangules similes per 4am et primam sexti *adnot. supr. lin. a. m.* S | Ponam…18 eh] *om. (hom.)* V et *dz* posito in inter [sic] *gd* et *zd adnot. marg. a. m.* V

autem *zd* mediam inter *gd* | et *eh*. Manifestum est igitur quod proportio H 12vb
gd ad *eh* aggregatur ex duabus proportionibus: ex proportione *gd* ad *dz* et
ex proportione *dz* ad *eh*. Quapropter proportio *ga* | ad *ae* aggregatur ex | N 16va |
eisdem. Proportio vero *dz* ad *eh* est sicut proportio *bz* ad *be*. Et quoniam J 14v
due linee *eh zd* sunt equidistantes, ergo proportio *ag* ad *ae* aggregatur ex
proportione *gd* ad *dz* et ex proportione *bz* ad *be*. Et hoc est quod
proposuimus. | U 14va R 14rb

[Lemma 2]

Similiter declarabitur secundum modum dividendi quod proportio *ge* ad *ea* aggregatur ex duabus proportionibus: ex proportione *gz* ad *zd* et ex proportione *db* ad *ba*. [Figura 2] Quod sic probatur. Producam *ah* equidistantem *ez* et protraham *gd* ad *h*, et quia due linee *ah ez* sunt

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18 zd] *sd* P | mediam] scilicet rationis vel argumenti proportionibus et non mediam quantitatis *adnot.* K *add. supr. lin. a. m.* U | eh] ex 3a de de ... proportionibus *adnot. mg. a. m.* W **19** eh] per illud principium quinti et omnium trium pertinentium(?) L *eb* Z | aggregatur…20 eh] *add.* mg. X | proportionibus] per secundum cata *adnot. supr. lin. a. m.* Z | ex proportione] *om.* Z gd^2] *dg* I | dz] *ds* P **20** dz] *ds* P | eh] quoniam proportio extremorum componitur ex proportionibus intermediorum per 19 definitionem 7^i Euclidis *adnot.* K | Quapropter] qua *sed. corr. supr. lin. a. m.* W | ad^2] *om.* N | ae] *ea* Z | aggregatur] per 1 5^i Euclidis *adnot.* L **21** eisdem] proportione *gd* ad *dz* et ex proportione *dz*(*ds* P) ad *eh add.* ELMNPRWX proportionibus *add.* Y | dz] *ds* P | ad^1] *add. supr. lin. a. m.* Z | eh] *he* V | proportio] proportione N | bz] *zb* EFLMNW *bs* P | be] per 29 1^i Euclidis et per 4 6^i eiusdem *adnot.* H eadem est demonstratio que prius vero dictum sit proportio *ga* ad *ae* sicut *gd* ad *eh adnot. supr. lin. a. m.* Z ex 4^a 6i *adnot. mg. a. m.* W per 2^{am} et 3^{am} sexti *adnot. supr. lin. a. m.* S | Et] *exp.* HY *om.* EFLMNRSTWX | Et…23 be] *om.* (*hom.*) F **22** eh] et *add.* P | zd] *sd* P | ag] *ga* EFLMNRWX | ae] *ea* N *sup. lin.* K *om.* L etiam *add.* EFLMNWX | aggregatur] per primi quinti Euclidis *adnot.* LR ex duabus proportionibus (scilicet *add.* R) *add.* EFLMNPRWX **23** dz] *ds* P | ex] *om.* I | ex…bz] *mg.* K | bz] *zb* EFLMNRWX *bs* P | est] *add. supr. lin. a. m.* X *om.* EV **24** proposuimus] oportuit nos declarare EFLMNRWX sequitur de catha disiunctas *add. a. m.* K **27** Similiter] quoque *add.* FMPRW ergo *add.* L Similiter nec istud est capitulum ullum *adnot.* E | declarabitur] nobis *add.* N | secundum] hunc *add. sed exp.* L quod] quia I | proportio] gd *add. sed. exp.* S **28** proportionibus] scilicet *add. sup. lin.* HR gz…zd] *gs* ad *sd* P **29** db] *bd* N | ad] *mg. a. m.* R | ba] et *add.* I | probatur] per 11 1^i Euclidis *adnot.* L | Producam] enim *add.* EFMNPRWX **30** ez^1] *es* P | h] equidistantem *ez* et protraham *gd* ad *h add. sed del. signis va-cat* LR | ah] et *add.* P | ez^2] *es* P | sunt] super S(check S) equidistantes, | fit proportio *ge* ad *ea* sicut proportio | *gz* ad *zh*. Ponam autem *dz* mediam inter *gz* et *zh*. Manifestum est quod proportio *gz* ad *zh* aggregatur ex duabus proportionibus: ex proportione *gz* ad *zd* et ex proportione *zd* ad *zh*. Verum proportio *zd* ad *zh* est proportio *db* ad *ba*, quoniam due linee *ba zh* cadunt super duas *ah be* equidistantes. Ergo proportio *gz* ad *zh* aggregatur ex duabus proportionibus: ex proportione *gz* ad *zd* et ex proportione *bd* ad *ba*. Proportio autem *ge* | ad *ea* est sicut proportio *gz* ad *zh*, ergo proportio *ge* ad *ea* aggregatur ex duabus | proportionibus *gz* ad *zd* et *db* ad *ba*, quod voluimus ostendere. | | |

E 14v | X 9vb V 16r C 9va N 16vb | T 12r | W 10r M 16ra

[Lemma 3]

Describam etiam circulum supra quem sint | *a b g* supra centrum *d*, [Figura 3] et dividam ex circulo duos arcus *ab bg* et ponam

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31 equidistantes] per 31 1^{i} Euclidis *adnot.* LR | proportio^{1}] per penultimam(primam R) partem 2 6^{i} Euclidis *adnot.* LR | gz…zh] *gs* ad *sh* P | zh] *zd* I per 2 6^{i} Euclidis(*om.* K) *adnot.* HK per 1 iiii Geometrie sexti *adnot. supr. lin. a. m.* Z *zd add. sed. exp.* Y *zh add. supr. lin. a. m.* Y ex 2a 6i *adnot. supr. lin. a. m.* Y **32** dz] *ds* P | gz^{1}] *dz* R | gz^{1}…zh^{1}] *gs* et *sh* P | zh^{1}] per istud principium 5^{ti} 'et omnium trium quantitatum etc.' *adnot*. R | est] per propositionem positam in fine columne precedentis in margine *adnot.* H igitur *add.* EFLMNPSWX | quod] *om.* I | proportio] *sup. lin. a. m.* K | gz^{2}…zh^{2}] *gs* ad *sh* P **33** aggregatur] per illud partem id est ... communium trium ... *adnot. sup. lin. a. m.* L | proportionibus] scilicet *sup. lin.* H per 1 cata *adnot. supr. lin. a. m.* Z | gz…zd] *gc* ad *cd* C *gs* ad *sd* P **34** zd^{1}] *supr. lin*. Z | zd^{1}…zh^{1}] *sd* ad *sh* P | zh^{1}] *ch* H tamquam proportio extremorum ex proportionibus intermediorum *adnot.* K | Verum] unde Z | zd^{2}…zh^{2}] *sd* ad *sh* P | est] sicut *add.* P | ad^{3}] ex quarta sexti vere et xviii Geometrie eiusdem *adnot. supr. lin. a. m.* Z | ba] per 29 1^{i} et per 4 6^{i} et per 13 definitionem 5^{i} Euclidis *adnot.* H et *add. sed. exp.* Y **35** quoniam] id est propter triangulos similes *adnot. supr. lin. a. m.* Y | zh] et *sh* P | super] supra U | duas] lineas *add.* FLMPRW unde anguli coalterni sunt equales et similiter anguli contra se positi *adnot. mg. a. m.* S | ah] et *add.* P **36** proportio] *bz add. sed exp.* I *db* ad *ba add. sed. del*. U *om*. S | gz…zh] *gs* ad *sh* P zh] per 11 5*ti* (Euclidis *add*. R) *adnot*. LR | proportionibus] scilicet *sup. lin.* H scilicet *add.* P **37** gz…zd] *gs* ad *sd* P | zd] *zh* I ergo proportio *gz zd add.* I | Proportio] per ... sexti *adnot. supr. lin. a. m.* S | autem] *om.* V *gd add. sed. del*. S | est…38 ea] *om. (hom.)* V **38** gz…zh] *gs* ad *sh* P | ad ea] *marg. a. m.* R **39** proportionibus] scilicet (*om*. N) ex proportione *add.* EFMNPRWX ex *add.* L | gz…zd] *gs* ad *sd* P | gz…et] *mg. a. m.* F | zd] *dz* R | et] ex *add.* L ex proportione *add.* EMNPRWX | quod voluimus] et hoc est quod voluimus P quod…ostendere] et hoc est quod demonstrare voluimus(volumus E) EFMNRWX | voluimus] volumus HISZ | voluimus ostendere] demonstrare voluimus L | ostendere] sequitur ultra *add. a. m.* K **42** Describam] item hec est alia demonstratio *praem*. E | sint] fiunt H | supra^{2}] super FMR **43** dividam] separabo FLMRW | ex] supr. lin. W a *add. sed. exp*. W | ab] et *add.* P

unumquemque eorum semicirculo minorem et similiter dividentur omnes
arcus | in sequentibus. Hanc ergo exceptionem memorie commendemus. F 12r
Et protraham duas lineas *ag deb*. Dico ergo quod proportio *ae* ad *eg* est
sicut proportio corde dupli arcus *ab* ad cordam dupli arcus *bg*. Quod sic
probatur. Protraham duas lineas perpendiculares a duobus punctis *a g* ad
lineam *db* que sint *az gh*. | Et quia *az gh* equidistant et cadit super eas K 25v
linea *aeg*, erit proportio *az* ad *gh* sicut proportio *ae* ad *eg*. Proportio vero
az ad *gh* est sicut proportio corde dupli arcus *ab* ad cordam dupli arcus *bg*
quoniam unaqueque est medietas corde sui dupli.| Ergo proportio *ae* ad L 15v
eg| est sicut proportio corde dupli arcus | *ab* ad cordam dupli arcus *bg*. Et H 13ra |
hoc quod demonstrare voluimus. U 14vb

[Lemma 4]

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44 unumquemque] quemque N unumquodque V | minorem] minor *sed. corr. supr. lin. a. m.* V quoniam corda maior semicirculo non habet cordam eo quod talis corda correspondet minori portioni circuli et non maiori *adnot. mg. a. m.* S | similiter] ita scilicet quod sit minor semicirculo *adnot. sup. lin. a. m.* N | similiter dividentur] *inv.* C | dividentur] dividuntur Y dividentur omnes] dividetur omnis ENX | dividentur…45 arcus] omnis arcus quem separabo FLMRW **45** sequentibus] id est hoc sit generale quod omnis arcus dicantur linea minor semicirculo in quocumque circulo *adnot. mg. a. m.* H ita scilicet quod sit minor semicirculo *adnot. supr. lin. a. m.* X | ergo exceptionem] *inv.* H | exceptionem] expositionem *sed. corr. supr. lin. a. m. ad.* extrapositionem U **46** deb] *db* F *des* I **47** corde] *supr. lin. a. m.* S corde dupli] *inv.* V | ab…arcus^{2}] *om. (hom.)* I **48** Protraham] enim *add.* CEFJKMNPRSTUVWXYZ hic *add.* L per(ex Y) 12 1^{i} Euclidis *adnot.* LRY(*a. m.*) | lineas] *om.* MRW | a^{2}] et *add.* FP **49** sint] sunt INX | az^{1}] *as* et P | az^{1}…50 proportio^{1}] *om. (hom.)* C Et…gh^{2}] *om.* F | az^{2}] et *add. supr. lin.* H ad *del.* N et *add.* ENTX *as* et P | equidistant] per 12 1^{i} Euclidis *adnot.* L ex 27(?) primi *adnot.* W(*mg. a. m.*)Y(*supr. lin. a. m.*) **50** az] *as* P | gh] scilicet duarum equidistantium *adnot. mg. a. m.* M | eg] per 29 1^{i} et per 4 6^{i} Euclidis (*om.* F) *adnot.* FH per 4^{am} 6^{i} Geometrie(*om.* S) *adnot. supr. lin. a. m.* SZ ex 4a 6i *adnot. supr. lin. a. m.* Y | vero] *az* ad *gh* sicut proportio *ae* ad *eg* proportio *add.* S **51** az] *as* P | gh] que est enim proportio totius ad totum eadem est proportio medietas ad medietatem *adnot. mg. a. m.* S proportio] *om.* I | corde] *om.* H | ab] id est arcus qui est duplus ad arcum *ab* qui est subduplum eius *adnot.* H multiplicium et submultiplicium proportio est una per 15 quinti Euclidis *adnot.* R | ab…arcus^{2}] *om.* F | bg] *gb* F per 6^{i} 4^{am} Geometrie *adnot. supr. lin. a. m.* Z *dbg* V **52** quoniam] scilicet *az* ad *gh adnot.* L | corde sui] *inv.* I | sui dupli] supli *sed. corr. supr. lin. a. m.* Y ex 3a 3i *adnot. supr. lin. a. m.* Y **53** est] *sup. lin.* KN | proportio] per 11 5^{i} Euclidis *adnot.* LR | corde] *pos.* arcus K *om.* I | bg] et *be add. sed del.* K per 15 5i *adnot. supr. lin. a. m.* Y **54** hoc] illud est ELMNRWX est *add.* PSTUVZ | quod demonstrare] *inv.* Y demonstrare voluimus] voluimus demonstrare I *inv.* S | voluimus] volumus EFHIM ut sequitur ultra *add. a. m.* K

Hoc autem superest quod cum arcus *ag* totus fuerit notus et
proportio corde dupli arcus *ab* ad cordam dupli arcus *bg*, erit unusquisque
duorum arcuum *ab bg* notus. [Figura 4] Verbi gratia, reiterabo enim
figuram et protraham lineam *ad* et producam perpendicularem a *d* ad
lineam *aeg* que sit *dz*. Et quoniam cum fuerit arcus *ag* notus erit angulus
adz cuius basis est medietas arcus notus, | et erit totus triangulus *adz* | N 17ra |
notus. | Et manifestum est quod cum fuerit tota corda *ag* nota et iam R 14vr
firmum est quod proportio *ae* ad *eg* est sicut proportio corde dupli arcus S 12v
ab ad cordam dupli arcus *bg*, erit linea *ae* nota. Et post hoc sciemus *ze*. Et
propter hoc quod *dz* est nota, sciemus ex hoc angulum *edz* trianguli *edz*

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57 Hoc] Item alia demonstratio eiusdem capituli *adnot.* E | superest] super HKUVYZ est *add. sup. lin.* HK sequitur FLMRW *iter. sed del.* L scilicet quod ostensum est in hec(libra L) figura(huius L) 10 *adnot.* LR | quod cum] quodcumque *sed. corr.* Y | cum] est *sed corr. sup. lin.* K **58** corde] *sup. lin.* H | ab] id est qui contineret duos sinus *adnot.* H | bg] nota *add.* EFK(*sup. lin. a. m.*)LMNPRWX(*sup. lin. a. m.* Y)YZ **59** ab] et *add.* P | bg] *b sed corr. sup. lin.* K | reiterabo] retentabo(?) H id est resumam figuram supra immediate posita *adnot.* H enim] *om.* CHIJKSTUVYZ **60** lineam] *om.* H | ad^{1}] *ab* V | producam] ad *add.* F perpendicularem] *pos. d* FLMW | d] id est centro *adnot.* H puncto *adnot.* NX(*supr. lin. a. m.*) **61** lineam] *sup. lin.* K | aeg] *ag* F per 3 3^{i} *adnot.* F | dz] *ds* P per 12 primi Euclidis *adnot.* R notus] ex hypothesi scilicet *adnot.* H | angulus] notus *add.* S **62** adz^{1}] *ads* P | basis] arcus F arcus] *ag add. sup. lin. a. m.* LNX | notus] per 3 3i *adnot.* F quia per 30 3^{i} Euclidis ipsa sit medietas corde *adnot.* H ex 22^{a} 6^{i} *adnot. supr. lin. a. m.* W *om.* S | et] *om.* MZ etiam T | et erit] *inv.* P | adz^{2}] angulus *sed corr.* H *ads* P cuius basis est medietas arcus *add. sed. del. signis va--cat* R **63** notus] per 1^{am} 1^{i} et 30 3^{i} quia orthogonius *adnot.* F et cum totus triangulus *adz* notus *add. sed. del. signis va-cat* R | tota] nota R | nota] *del.* R quia si arcus nota et corda nota et econverso per 5am primas figuras[sic] huius *adnot. mg. a. m.* M | et] scilicet per arcum scilicet ex tabulis vel ex hypothesi *adnot.* H **64** firmum] *mut.* in firmatum *sup. lin. a. m.* K | firmum est] per precedens capitulum *adnot.* H | proportio^{1}] linee add. E ae] *e sed. corr. supr. lin. a. m.* Y **65** ad] *iter. sed. exp.* T | arcus] *om.* H | bg] et hec est nota ex hypothesi *adnot.* H que proportio per hypothesim est nota *adnot.* K per 10 figuram huius dictionis *adnot.* R per premissam *adnot. supr. lin. a. m.* X per primam figuram ante *adnot. supr. lin. a. m.* Z | linea] *e add. sed. exp.* L | nota] per 30 3^{i} *adnot.* F per secundam in Geometrie *adnot. supr. lin. a. m.* Z | post] *mut.* in per *sup. lin. a. m.* K | sciemus] quia si a noto subtrahatur(subtraham R) notum residuum erit notum *adnot.* LR | ze] per 30 3^{i} *adnot.* F que est residua *az* note *adnot.* H quia *az* est nota *adnot.* K se P quia si scimus totam *ag* et *az* et *eg* per premissa scimus *ze adnot. mg. a. m.* M **66** dz] *zd* Y *ds* P | nota] per penultimam 1^{i} Euclidis *adnot.* H ut iam patet *adnot. supr. lin. a. m.* L subtracto quadrato *az* de quadrato *ad adnot. sup. lin. a. m.* NX | angulum] augmentum *sed. corr. supr. lin.* Y | edz^{1}] *eds* P trianguli] anguli *sed corr. sup. lin.* I | edz^{2}] *eds* P

orthogonii, quoniam omnis trianguli orthogonii notorum laterum reliqui
eius | anguli sunt noti per id quod premisimus in tabulis loci portionis E 15r
cuiusque corde arcus. Sciemus ergo totum angulum *adb* et propter hoc
sciemus arcum *ab* et sciemus arcum *bg* qui est residuum arcus *ag*. Et hoc
est quod oportuit nos declarare.

[Lemma 5]

Describam etiam circulum supra quem sint *a b g* supra centrum | *d*. X 10ra
[Figura 5] Et sit quisque duorum arcuum *ab ag* minor semicirculo, et
similiter quisque arcus dividetur in | sequentibus existens minor Z 5r
semicirculo. Et | protraham duas lineas *ad gb* | et producam eas donec J 15r | M
concurrant super *e*. Dico ergo quod proportio *ge* ad *eb* est sicut proportio 16rb
| corde dupli arcus *ag* ad cordam dupli arcus *ab*. Huius autem P 10r
demonstratio prime similis est. | Protraham enim ad lineam | *da* duas U 15ra |
perpendiculares a *b* et a *g* que sint *bz gh*. Et quia ipse sunt equidistantes, V 16v
erit proportio *ge* ad *eb* sicut proportio *gh* ad *bz*. Et propter hoc erit

67 orthogonii[1]] per 30 3^{i} *adnot.* F | quoniam…orthogonii[2]] *om. (hom.)* N **68** noti] per 30 3^{i} *adnot.* F | id] illud P | portionis] proportionis Y **69** cuiusque] cuius W | arcus] per tabulas arcuum et cordarum *adnot.* F id est per tabulas precedentes ubi docetur quos arcus cui corde respondet et per primam partem 30 3^{i} Euclidis *adnot.* H | Sciemus] quia si notum noto addas totum erit notum *adnot. (supr. lin. a. m.* L)LR | totum] *exp.* K | adb] per sectionem et per 32 6^{i} cum ultima 6^{i} *adnot.* F dempto angulo *edz* nunc noto de angulo *adz* prius noto *adnot.* H propter] per F **70** sciemus[1]] per ultimam 6^{i} Euclidis (*adnot. supr. lin. a. m.* L)LR per tertiam ... *adnot. supr. lin. a. m.* Z | ab…arcum[2]] *om.* (*hom.*) F | sciemus[2]] quia a noto subtrahatur notus etc. *adnot.* R | est] *add. supr. lin.* X | residuum] residuus Z **71** oportuit nos] *inv.* F declarare] demonstrare FLMW **74** Describam] Item et similiter alia demonstratio eiusdem capituli *adnot.* E | supra[1]] super FH | supra[2]] super FLMW | centrum d] *inv.* F **75** quisque] unusquisque F sive sumpti *adnot. supr. lin. a. m.* U | ab] et *add.* P | et…77 semicirculo] *om. (hom.)* NX *add. mg.* E **76** dividetur] quem separabo FLMRW **77** semicirculo] id est arcus dividetur ... qui est minor semicirculo *adnot.* H | Et] *om.* I | ad] *hda* et P *ah* U | ad gb] *add. mg.* Y | gb] *g sed corr. sup. lin.* K | et] *om.* H **78** super] supra EHNRWX | ergo] *om.* CHJKTUVYZ **79** corde] *mg.* K *om.* F **80** est] id est tertie precedenti in illo circulo 'describam' *adnot.* H | Protraham] per 12 1^{i} Euclidis *adnot.* LR | enim] *om.* H | da] *ea* F *hda* P **81** a[1]] prepositio *adnot. supr. lin. a. m.* X | b] puncto scilicet (*om.* N) *adnot.* HN | a[2]] *om.* Z prepositio *adnot. supr. lin. a. m.* X | g] puncto *adnot.* HN | que…gh] *mg.* K | sint] sunt FI | bz] *bs* et P | ipse] per 12 et 28 1^{i} Euclidis *adnot.* LR | equidistantes] per definitiones 3^{i} *adnot.* F **82** ge…83 proportio[1]] *iter. (hom.)* N | bz] *hz* C *az* I per 2 6^{i} per 2 6^{i} *adnot.* F per 29 1^{i} Euclidis et per 4 6^{i} eiusdem *adnot.* H per 2am 6^{i} et 16am 5^{i} *adnot.* K eadem probatio est in secunda figura *adnot. supr. lin. a. m.* Z *bs* P quoniam unaqueque est medietas sui dupli *adnot. mg. a. m.* M | Et] per 11 5^{i} Euclidis *adnot.* L proportio | *ge* ad *eb* sicut proportio corde dupli arcus *ga* ad cordam dupli arcus *ab*. Et illud voluimus demonstrare. N 17rb

[Lemma 6]

Sequitur vero hoc quoniam cum fuerit hic arcus *gb* solum notus et fuerit proportio corde dupli arcus *ag* ad cordam dupli arcus *ab* nota, scietur arcus *ab*. [Figura 6] Quod sic probatur. Protraham in figura simili huic forme etiam a puncto *d* perpendicularem ad cordam *bg* que sit *dz*. Angulus ergo *bdz* cuius basis est medietas arcus *bg* erit notus. Quapropter totus triangulus *bdz* orthogonius est notus et quia proportio *ge* | ad *eb* est H 13rb nota, et corda *gb* est nota. Sciemus ex hoc *eb* et sciemus per hoc totam lineam *ebz*. Et quoniam *dz* est nota, erit angulus *edz* trianguli *edz*

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83 eb] *gh* C sicut proportio *ghe* ad *eb add. sed. del.* Z | sicut] sic W | corde] *om.* FU | ga] *ba sed. corr. supr. lin. a. m.* Y **84** ab] *ag sed. corr. supr. lin. a. m.* Y | illud] est quod *add.* LMNRW | voluimus] volumus HIKS quod volumus E | voluimus demonstrare] *inv.* F **87** Sequitur] Item aliud theorema eiusdem capituli scilicet duodecim *adnot.* E | vero] ergo F hoc] *add. supr. lin. a. m.* Y scilicet quod ostensum est in hac presenti figura *adnot.* R quoniam] *om.* F | hic] *om.* CEFLMNRWY | gb] *bg* N | solum notus] *ant.* arcus I | notus] ita quod non corda eius nec alii arcus *adnot.* H **88** corde] *post.* arcus V | ad…ab] *om.* U cordam] corde *add.* V **89** scietur] conclusio *adnot.* F | ab] nota scietur arcus *ab add.* U Protraham] namque *add.* EFMNPRWX scilicet aliam figuram *adnot.* H per 12 primi Euclidis *adnot.* R | figura simili] *inv.* U **90** etiam] et F | d] semidiametrum *db* et *add.* P | ad] id est super *adnot.* H | bg] per 30 3^{i} Euclidis *adnot.* H | dz] *ds* P **91** ergo] vero E | bdz] *bds* P arcus] *om.* C | erit] per ultimam 6^{i} Euclidis *adnot.* LR | notus] per 3 3^{i} *adnot.* F scilicet quia totus arcus notus ex hypothesi *adnot.* H **92** totus triangulus] *inv.* F | triangulus] angulus I bdz] *bds* P | notus] per 1^{am} 1^{i} vel 30 3^{i} *adnot.* F | ad eb] *mg.* K | est^{2}] cum *sed. corr. supr. lin.* R **93** nota^{1}] per precedentem *adnot.* F scilicet per figuram precedentem et ex hypothesi... expositum est superius *adnot.* H per secundam figuram huius *adnot.* L precedentem figuram *adnot. supr. lin. a. m.* Z | et^{1}] *om.* I | nota^{2}] per hypothesim *adnot.* FZ(*supr. lin. a. m.*) Sciemus] per 4 dictionem(?) *adnot. supr. lin. a. m.* Z | hoc^{1}] *om.* V | sciemus] *om.* P | per] ex *sed. corr. supr. lin. a. m.* C | hoc^{2}] consequens P **94** ebz] *ebs* P per 30 3^{i} *adnot.* F | ebz…95 angulum] *ebz* ideo totum angulum *cdz* erat autem an angulus *bdz* notus notius erit K | dz] *bz sed corr. mg.* H *bz* TV *ds* P | angulus] triangulus F | edz^{1}] *eds* P | edz^{2}] *eds* P

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84 demonstrare] *Hic ponitur adnotatio in codice* Z: Nota quod a duobus punctis circumferentie ... duorum maiorum orbium in sphera, ducuntur perpendiculares in reliquum eis que de eisdem punctis sunt perpendiculares in communem amborum diametrum aut eedem sunt aut proportionales.

orthogonii notus. Et sciemus angulum *edb* residuum. Fiet ergo arcus *ab*
notus. | W 10rb

[The Menelaus Theorem]

Et postquam premisimus hec antecedentia describam in superficie
spherica arcus orbium maiorum qui sint arcus *ab ag* et arcus *be gd* sese
secantes supra *z*, et sit quisque arcuum minor | semicirculo. [Figura 7] K 26r
Hanc vero exceptionem commendabimus memorie in omnibus formis.
Dico igitur | quod proportio corde dupli arcus *ge* ad cordam dupli | arcus R 14vb |
ea | aggregatur ex proportionibus duabus: ex proportione corde dupli T 12v
arcus *gz* ad cordam dupli arcus *zd* et ex proportione corde dupli arcus *db* N 17va
ad cordam dupli arcus *ba*. Quod sic probatur. Ponam enim centrum
sphere *h* et protraham a centro ad puncta *b z e* ubi se secant circuli lineas
hb hz he, et protraham cordam *ad* et producam *hb* | que est medietas C 9vb
diametri, donec concurrant supra punctum *t*. Et | protraham duas lineas | M 16va |
ga gd secantes duas lineas *hz he* supra duo puncta *k l*. | Fiunt ergo in linea U 15rb
una recta tria puncta que sunt *t k l* quoniam ipsa simul sunt in superficie E 15v

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95 notus] per 30 3[i] et tabulas arcuum et cordarum *adnot.* F anguli *edz* et *bda* anguli est notius eo qui arcus in quem cadit est notus *adnot.* H | Fiet ergo] cuius quantitas est P *inv.* R | ab] *ad* F **96** notus] quem querebamus P per ultimam 6i *add. a. m.* M **99** Et] Item *adnot.* E | Et postquam] post quam postquam N | antecedentia] accidentia R **100** maiorum] qui non sint duo arcus sed duo superficies *adnot.* F | sint] sunt EFMNX | ab] *ad* F et *add.* P | ag] sese secantes in puncto *a add. marg. manu secundi commentatoris* R | arcus[3]] residui *add. sup. lin. a. m.* F | be] et *add.* P **101** z] communis sectio *add. sup. lin. a. m.* F *s* P | arcuum] *ab ag add. sup. lin. a. m.* F | minor] maior N | semicirculo] circulo *sed corr. sup. lin.* HY(*a. m.* Y) **102** exceptionem] scilicet quod arcus vocatur semper qui est minor semicirculo ita dixit supra tertio circulo *adnot.* H **103** Dico] concludit *adnot.* F hic ostendit catham disiunctam *adnot.* H | dupli[1]] pli *sed. corr. supr. lin.* K | ad cordam] *iter.* Z | cordam] *om.* F **104** ex[1]] duabus *add. sed. exp.* N | proportionibus duabus] *inv.* FLRW **105** gz] *gs* P | zd] *sd* P | zd…106 arcus] *add. supr. lin.* Z | corde] *om.* CI *sup. lin.* LR *supr. lin. a. m.* U **106** ba] *be* N | enim] *om.* H centrum] omnium *add. supr. lin. a. m.* X **107** sphere] *om.* NX *add. mg. a. m.* E | b z] *b* et *s* et P | ubi se] visu *sed. corr. supr. lin. a. m.* Y | se] *om.* F *add. supr. lin. a. m.* M **108** hz] et *hs* et P | ad] et (*om.* J) faciam enim(*om.* CFJMUV eam RW) penetrare eam (*om.* CJLMRUVW) *add.* CFLR *mg. a. m.* JMUVW | medietas] circuli *add.* F **109** diametri] scilicet sphere *adnot.* H | concurrant] ad *zhb adnot.* H scilicet cum *ad adnot.* K scilicet *ad* et *hb adnot.* LR currant Y | t] extra spheram *adnot.* H **110** ga] *ag* U et *add.* P | ga gd] *inv.* F | hz] *hs* et P | duo puncta] *inv.* I | k] et *add.* P | k l] *b e l* H *d l* I | Fiunt] vel quod tota linea a puncto *l* per punctum *k* concurrat cum diametro in puncto *t adnot.* F | in] *add. supr. lin.* W | linea…111 una] *inv.* V **111** recta] *om.* F duo *add. sed del.* H | tria puncta] *inv.* H | sunt[1]] *om.* I | k] et *k* et P | simul sunt] *inv.* F trianguli *agd* et superficie circuli *bze*. He ergo due superficies se secant in linea *lkt*. Cum ergo he linee protrahentur, due linee *tl gd* se secabunt inter duas lineas *ta* | *ga*, et erit earum sectio supra *k*. Manifestum est ergo quod [F 12v] proportio *gl* ad *al* aggregatur ex duabus proportionibus: ex proportione *gk* ad *kd* et ex proportione *dt* ad *ta*. Proportio autem *gl* ad *la* est sicut proportio corde dupli arcus *ge* ad cordam dupli | arcus *ea*, | sicut [L 16r | X 10rb] ostensum est in primo quatuor circulorum precedentium | hanc figuram. Et proportio *gk* ad *kd* est sicut proportio corde dupli arcus *gz* ad cordam [N 17vb] dupli arcus *zd*, et proportio *dt* ad *ta* est sicut proportio corde dupli arcus *db* ad cordam dupli arcus | *ba*, quemadmodum ostensum est | in tertio [I 8v | V 17r] quatuor circulorum precedentium hanc figuram. Ergo proportio corde dupli arcus *ge* ad cordam dupli arcus *ea* aggregatur ex duabus proportionibus: ex proportione corde dupli arcus *db* ad cordam dupli arcus *ba* et ex proportione corde dupli arcus | *gz* ad cordam dupli arcus [Y 10v] *zd*. Ex eo autem quod demonstratum est ex proportionibus linearum in

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112 et] in I in *add.* P | bze] *bse* P | ergo] per 3 11^{i} *adnot.* K | due superficies] *inv*. E superficies] *agd* et *bze adnot.* F scilicet triangulus *agd* et circulus *bze adnot*. R **113** he…protrahentur] hec linea protrahetur(protrahatur K) KPW | he…linee^{2}] hec linea protrahatur F | protrahentur] protrahetur LMR | tl] et *add.* P | se] sese EFLMNRWX **114** ta] et *add.* P | ga] *praem.* et F | earum] eorum *sed. corr*. *supr. lin*. W | sectio] secatio *sed corr.* K supra] super K | Manifestum est] scilicet per illum capitulum pagine precedentis 'similiter' *adnot.* H | est] *om.* C **115** proportio] per secundam huius *adnot.* F | aggregatur] per 9 figuram huius dictionis *adnot.* LR | gk] *k sed. corr*. *supr. lin. a. m.* U **116** kd] *kg* C | dt] *gd sed. corr.* C | ta] per 9 huius *adnot.* K *th* V | autem] vero I **117** proportio] per 10^{am} figuram huius dictionis *adnot.* LR | corde] *om.* F | ge…arcus^{2}] *add. marg*. Z | ea] eam *sed corr.* I *da sed. corr.* N **118** primo] id est per tertiam huius *adnot.* F | quatuor] minor T | circulorum] scilicet per 9 5 H | figuram] per 10 vel 3 huius libri *adnot.* F | figuram…120 et] ergo I **119** Et] ergo CEHJNTUVXZ *sed corr. mg.* H | proportio^{1}] per eandem *adnot.* K | gz] *gs* P gz…120 arcus^{1}] *mg.* R | ad^{2}…120 zd] ad *zd* per eandem 3^{i} huius *mg. a. m.* F **120** zd] similiter per 9 6 H *sd* P per 10 figura huius dictionis *adnot*. R **121** db…arcus] *mg.* K | ad cordam] *iter*. U | arcus] *gz* ad cordam arcus *td* et proportio *dt* ad *ta* est sicut proportio corde [I 8v] dupli *db* ad cordam arcus *add.* I | ostensum] *om.* I | est] *om.* Y | tertio] id est 12 vel 5 huius *adnot*. F per 12 huius dictionis *adnot*. R **122** quatuor] minor T | Ergo] concludit *adnot.* F | corde] *om.* R **123** ea] *ee sed. corr*. *supr. lin*. T | aggregatur] per 11 5^{i} Euclidis pluries sumptam *adnot*. LR **124** corde] *add. sup. lin. a. m.* C *om*. R | db] *gs* P **125** ba] *sd* P | ex] *om.* EFLMNRWX *add. supr. lin. a. m*. Y | corde] *om*. R | arcus^{2}] *sup. lin.* H *om.* CEJKNUVXZ *iter*. Y | gz] *db* P arcus^{3}] *om*. ENXZ **126** zd] *ba* P | Ex] hic est catham coniunctam *adnot.* H | eo] videlicet ex 8 huius *adnot.* K | demonstratum est] sic in pagina precedenti in illo capitulo 'et quoniam' *adnot.* H forma superficiali | precedente, declaratur quod proportio corde dupli arcus *ga* | ad cordam dupli arcus *ae* aggregatur ex proportionibus duabus: ex proportione corde dupli arcus *gd* ad cordam dupli arcus *dz* et ex proportione corde dupli arcus *zb* ad cordam dupli arcus *be*. Et hoc intendimus probare. H 13va J 15v

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127 superficiali] triangulus *add. sup. lin. a. m.* F in figura octava *adnot. supr. lin*. N id est in figura octava *adnot. supr. lin. a. m.* X | corde] *add. mg.* M **128** ae] *db sed corr.* I aggregatur] aggregantur I | proportionibus duabus] *inv.* EFLMPRW | duabus] scilicet *add. supr. lin. a. m.* X **129** ex[1]] *om.* I | gd] *ga* C | dz] *ds* P *de* T **130** corde] *add. a. m. marg.* R zb] *sb* P | arcus[2]] *om.* FHIJKSUZ | hoc…131 probare] illud est quod demonstrare voluimus (volumus E) EFLNRWX illud est quod voluimus demonstrare M

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130 be] *Hic ponitur adnotatio in codice* Y: Tria puncta exterioris linee probantur esse in eadem linea recta quia sunt in superficie trianguli *gez* et circuli *bda* secantibus se. **131** probare] *Adnotatio in codice* W: Aliqua proportio ex duabus componatur. Si ipsa composita fuerit nota itemque altera componentium, secunda erit nota. Sicut enim patet in principiis de proportionibus datis proportionem componi ex proportionibus non est nisi denominationem proportionis composite produci ex ductu denominationis unius componentium in denominationem alterius. Denominatio autem proportionis est quod exit ex divisione unius proportionalium per reliquum. Si autem unum per reliquum ita quod nec remaneat dividi non possit ut in superparticulari et superpartienti proportione, contingit ex dividendo fiant [sic] minutia que dividantur per alterum proportionalium et nichil autem remanebit et relinquentur minuta scilicet denominatio illius proportionis. Verbi gratia. Dupla proportio componitur ex sexquialtera et sexquitertia ut 3 et 2 in sexquialtera 4 vero ad 3 in sexquitertia se habet proportione quia igitur 3/2 per 4/3 dividi non potest nisi aliquid remaneat ex tribus, fiant minuta. Et erunt 180 minuta que si dividantur per 2 exient 90 minuta que sunt denominatio sexquialtere proportionis. Item quia 4 per 3 partium pariter dividi non possunt, fiant ex 4 minuta et erunt 240 minuta que si divideantur per 3 exient 80 minuta scilicet denominatio sexquitertie proportionis. Si ergo ducantur 90 minuta in 80 minuta, fient 7200 secunda scilicet denominatio proportionis duple. Si enim ex hiis (regulis *add. sed. del.*) secundis faciamus integra, exient tamen 2 quod est duple proportionis denominatio. Data ergo proportione ex duabus composita ut si sit dupla, denominatio eius dividatur per denominationem alterius componentium ipsam, et exibit denominatio proportionis tertie quesite. Proportione autem data et uno proportionalium dato alterum proportionalis erit datum. Sumatur enim numeri minimi in proportione data ut sit *a* ad *b* data et *a* sit datum, sint etiam numeri minimi in proportione data *c d*. Quia sicut que est proportio *a* ad *b* eadem est *c* ad *d*, si *a* notum ducatur in *d* quartum notum et productum dividatur per tertium notum, exiet *b* quantitas secunda nota. Facilius tamen fient omnes he operationes per regulas *Minoris Almagesti*.

Ptolemy's *Almagest* Figures

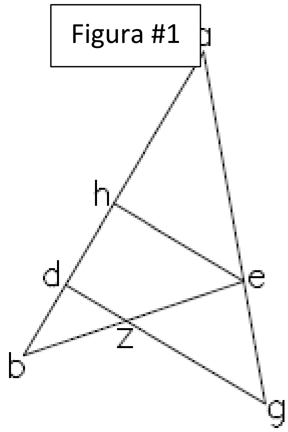

Figura #1

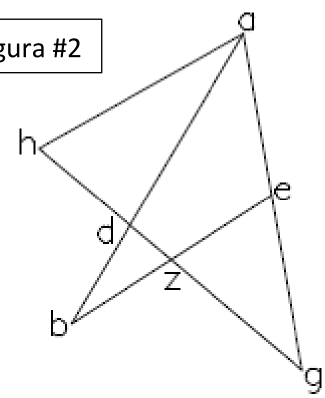

Figura #2

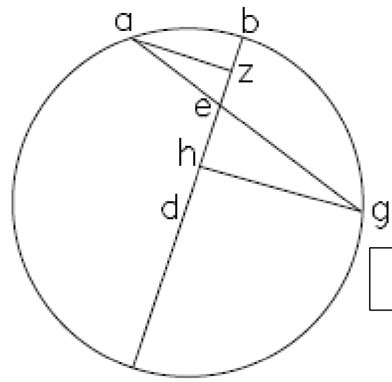

Figura #3

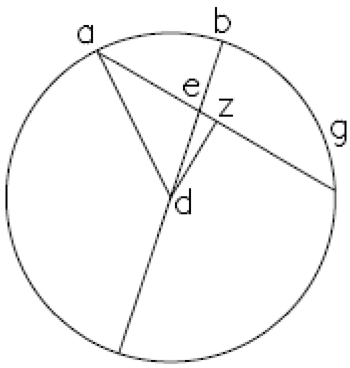

Figura #4

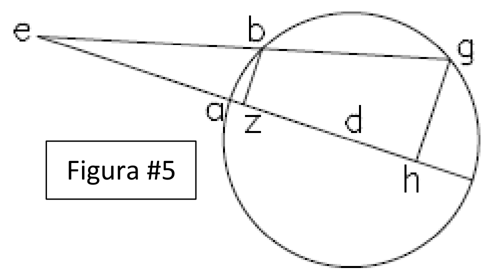

Figura #5

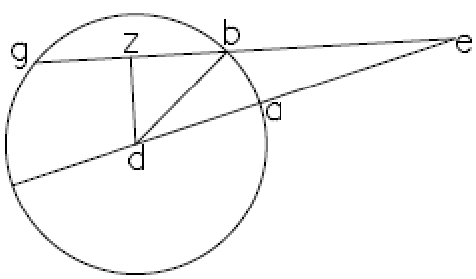

Figura #6

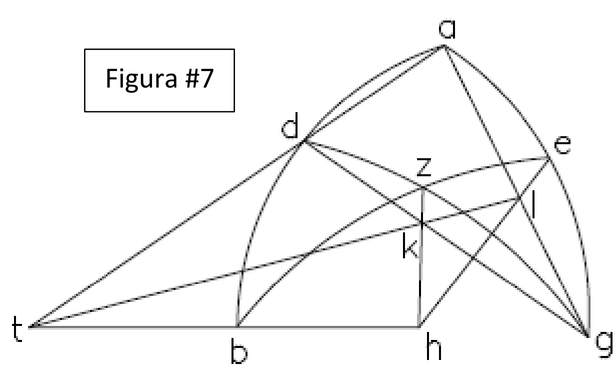

Figura #7

Appendix B: Marginalia from *Almagest* MSS F, HK, LMR, NX, & S

Città del Vaticano, Biblioteca Apostolica Vaticana, Vat. lat 6788 = F

Paris, Bibliothèque nationale de France, lat. 16200 = H

Kraków, Biblioteka Jagiellońska 619 = K

Città del Vaticano, Biblioteca Apostolica Vaticana, Vat. lat. 2057 = L

Città del Vaticano, Biblioteca Apostolica Vaticana, Barb. lat. 173 = M

Kraków, Biblioteka Jagiellońska 590 = R

Città del Vaticano, Biblioteca Apostolica Vaticana, Pal. lat. 1365 = S

Città del Vaticano, Biblioteca Apostolica Vaticana, Barb. lat. 336 = N

Paris, Bibliothèque nationale de France, lat. 7256 = X

Città del Vaticano, Biblioteca Apostolica Vaticana, Vat. lat. 6788 (MS F)

[Notes on Lemma 1]

| Nota pro triangulo coniuncto brevem probationem ex quo dicto quod linea *eh* secat latera *ag* et [*ad*] proportionaliter per 4^{m} 6^{i} libri Euclidis. Ergo qualis est proportio *ga* ad *ea* talis est *gd* ad *eh*. Posita inter *gd* et *eh zd* media, sequitur secundum descriptionem 7^{i} libri quod idem est *gd* ad *zd* et *zd* ad *eh* sicut *gd* ad *eh.* Quia primi ad ultimum proportio est composita ex omnibus intermediis. Sed *zd* ad *eh* est sicut *bz* ad *be* per 2^{m} 6^{i}, ergo patet. F 11v

Per 4^{am} 6^{i} *eh* equistans basi *gd* secat latera proportionaliter.

Posita *dz* in medio inter *gd eh* per descriptionem 7*i* libri Euclidis(?) per 7^{m} descriptionem libri ubi dicitur quod primi ad ultimum est etc.

Probatur ista per 2^{am} 6^{i} ubi dicitur latera triangulorum equidistantium sunt proportionalia.

In ista 8^{a} probatur in sensu composito quod proportio *ga* ad *ea* valent proportionum *gd* ad *dz* et *zb* ad *be*. Posita *dz* in medio inter *gd* ad *eh* per descriptionem 7^{i} Euclidis que talis est descriptio cum fuerint eedem vel diverse continuate proportiones, dicetur proportio primi ad ultimum ex aliis composita.

[Notes on Lemma 2]

Per 2^{am} 6^{i} *ze* secat trianguli *gha* latera *ga gh* proportionaliter cum sit equidistans basi *ah*. Igitur sicut *ge* ad *ea* ita *gz* ad *zh.* Posita *zd* in medio per descriptionem 7^{i} libri Euclidis, patet quoniam *ge* ad *ea* est composita ex *gz* ad *zd* et *zd* ad *zh* que est sicut *db* ad *ba,* et quod queritur igitur.

In 9^{a} probatur in sensu diviso quod proportio *ge* ad *ea* valet *gz* ad *zd* et *db* ad *ba*. Posita in medio *dz* inter *gz* et *zh*, ut sit *gz* ad *zd* et *zd* ad *zh* que valet *db* ad *ba* que queritur. Quoniam angulus *had* et *db*z sunt eque anguli cum *ah* linea sit equidistans ad *eb* ex hypothesi, ergo anguli coalterni per 29^{am} primi. Et per 15^{am} primi anguli *d* contra se positi [equales]. Ergo sunt ambo trianguli *bdz hda* equianguli. Ergo latera proportionalia per 4^{am} 6^{i} Euclidis.

6 ad] *ab* F **38** equales] equalis F

Nota quod ubi dicitur Ptolomeus quod ponatur *zd* mediam inter *gd* et *eh,* est propter hoc quoniam quod semper proportio primi ad ultimum est composita ex proportionibus intermediis, sicut dyapason est composita ex dyapente et dyatessaron. Etiam modo postquam nos demonstramus(?) generaliter(?): proportione *gd* ad *eh,* si ponas *dz* in medio, est sic scilicet quod in dicto compositione *gd* ad *dz* et *dz* ad *eh* ita quod proportio *gd* ad *eh,* scilicet primi ad tertium est composita ex proportione *gd* primi ad *dz* secundum et *dz* secundi ad *eh* tertium. Et hoc est ponere *dz* in medio, in quo dicto hypothesi ego laurentius episcopus aggregavi multitudinem dubitantium donec cogitando multitudinem respiciendo geminas considerando et ad memoriam reducendo descriptionem 7i Euclidis dicentis sic [cum] continuatis fuerint eedem vel diverse proportiones, dicetur proportio primi ad ultimum ex omnibus composita ut universaliter vidimus clareet cetera.

Nota per 2^{am} quod ex quo per 2^{am} 6^{i} latera sunt divisa proportionaliter per lineam *ze,* qualis est proportio *ge* ad *ea* talis est *gz* ad *zh*. Ponitur hic *zd* in medio per descriptionem 7^{i} libri. Sequitur quod etiam est idem *gz* ad *zh* sunt *gz* ad *zd* et *zd* ad *zh*. Sed *zd* ad *zh* est sicut *db* ad *ba*. Patet quoniam trianguli *bzd* et *dha* sunt equianguli quia *ah* et *zb* sunt equidistantes. Igitur per 29^{am} primi anguli coalterni *had* et *dbz* sunt equalis. Et angulus *d* ex utrisque partis contra se positus equaliter per 15^{am} primi, ergo equianguli. Ergo qualis est proportio *bd* ad *dz* talis est *da* ad *dh*. Ergo permutatim equalis est *bd* ad *da* talis est *zd* ad *dh*. Ergo coniunctim qualis erit *ba* ad *bd* talis erit *zh* ad *dh.* Ergo divisim qualis *bd* ad *ba* talis erit *zd* ad *zh,* quod queritur. Quoniam *ge* ad *ea* est sicut *gz* ad [*zh*], et *zd* ad *zh* est sicut *bd* ad *ba,* ergo *ge* ad *ea* est sicut *gz* ad *zd* et *db* ad *ba,* quod est intentum. Et sic patet proportio disiuncta et per ista probatur catha disiunctam reliquam.

[Notes on Lemma 3]

| Ista 10^{a} est talis quod captis duobus arcubus minoribus semicirculo F 12r
ut *ab bg,* a communi termino *b* ducatur dyametrus *bd* et coniungatur extremitates arcus per lineam *ag* secando dyametrum, erit proportio *ae* ad *eg* sicut est *az* ad *gh*. Patet ductis *gh* et *az* super dyametrum perpendiculariter cum trianguli *ghe aez* sint equianguli, latera proportionalia per 6^{am} 6^{i}.

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43 est[2]] ex F **52** cum] *illeg.* F **67** zh[1]] *zd* F **73** bg] in *add.* F

Per 4^{am} 6^{i} patet quod ex quo isti duo trianguli *aez* et *egh* sunt equianguli quod latera sunt ad invicem proportionalia. Hic probatur quod captis arcubus *ag ab* et ducta linea *ag* a termino primo ad terminum ultimum *g*, et ab alio puncto scilicet *b* linea ducta ad centrum secat *ag* lineam in proportione corde dupli arcus *ab* que est *az* medietas ad cordam dupli arcus *gb* que est *gh* etiam medietas. Quoniam sicut totum ad totum ita medium ad medium.

[Notes on Lemma 4]

Hic probatur quod scito toto arcu *ag* in eius corda *aeg* scientur arcus *ab bg*. Ex quo triangulus *ezd* est orthogonius per 30^{am} 3^{i}. Facto semicirculo *dze*, et corda *dz* est scita per ... primi. Scietur ergo corda *ze* per 30^{am} 3^{i} et sic per tabulas scietur arcus *ze*. Ergo angulus *zde* est scitus, ergo totus angulus *ade*, ergo arcus *ab* per ultimam 6^{i} quod queritur.

Et quod triangulus *adz* sit notus patet nam arcus *ag* est notus et per consequens(?) corda *ag* nota. Item cum *dz* cadat perpendiculariter super *ag*, dividit per 2 media et sic *az* est medietas. Ergo *dz* erit nota, per ix(?) primi Euclidis.

11^{a} propositio. Demonstrandum(?) quia nunc(?) video(?) istam conclusionem(?) quod si ex quo linea *ag* tota est nota, et suam quod sit proportio *ae* ad *eg* sicut proportio corde dupli arcus *ab* ad cordam dupli arcus *bg* quod propterea sit linea *ae* nota nunc video istud.

[Notes on Lemma 5]

Per 2^{am} 6^{i} Euclidis hic probatur solum datis duobus arcubus maioribus semicirculo, scilicet *ab ag* cum ductis lineis *ade gbe* donec concurrant in *e*, [Figura 1] quod proportio *ge* ad *be* est sicut proportio linee *gh* corde dupli arcus *ag* ad cordam *bz* dupli arcus *ab*. Patet per 2^{am} propositione 5^{i} Euclidis ex quo sunt trianguli equianguli.

[*The following note is rather illegible but I include it for its rare glimpses of an author and date for a set of marginalia.*]

Demonstrandum ex concursu undicime est necesse quod concurrant in 12^{a} propositione ...5 ..ditur(bene dicitur?) ex 5^{o} quia *ag* arcus non potest esse equalis *bz* residuum, ipsa proportio esset equilitatis corde dupli arcus *ag* ad cordam dupli arcus *ab* quia esset *bz* quoniam si arcus sunt equales, item corde etiam. Et sic videtur se habere dubii quod

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90 corda^{1}] est *add. sed. del.* F **109** propositione] 6^{i} *add. sed. exp.* F demonstrare habent ... quia sunt etiam inequales. Laurentius episcopus ... anno domini 1427, vidi hoc per me ipsum Brithonis(?) 24 martii(?) quando erat terribilis terremotus ... iam a prima martii semper per ... Cathalonge non destructione castrorum ... quam plurimarum in ...

[Notes on Lemma 6]

Hic probatur quod scito arcu et corda *gb* ... scietur *ga*. Patet hic sic per precedentem 6^{i} ... proportionem *ge* ad *be.* Ergo divisim *gb* ad *be*. Et quia *gb* est nota, scitur *be*. Ergo totus triangulus quia orthogonius *dze,* scitur angulus *edz* per 30^{am} $3^{i,}$ et quia erat notus iam ex ypothesi angulus *bdz,* residuus *edb* angulus erit notus, ergo arcus usque ...

Sic probantur duo. Primum quod captis duobus arcubus *ab* et *ag* quolibet semper semicirculo minori, facta superiori dispositione concludit quod talis est proportio *ge* ad *be* sicut corda *gh* ad *bz,* quod est primum. Secundo iterum probat per eandem figuram quod scito arcu *gb* et nota proportione corde *gh* ad *bz,* scietur totus arcus *ab*. Probat hoc ibi ubi etiam ... et facit ad ... figuram ... que est iam probata per 3^{am} 3^{i}.

[Note on Menelaus Theorem]

| Nota quod per eandem dispositionem per consequentia(?) probatur F 12v catha disiuncta. Per eandem potest probari catha coniuncta. Solum erat in textu una difficultas in dispositione de linea *lkt* que describitur concurre cum linea *ad* et *hb* in puncto *t,* sed universaliter melius videatur inferius notari dispositionem in tali signo [*mark given*]. [*At this mark, however, we only find:*] Itaque nota quod in nostro cistino papiri est clara probatio et etiam in almagesti minoris.

Marginal Notation from Paris, Bibliothèque nationale de France, lat 16200 (MS H) and Krakɵw, Biblioteka Jagiellońska 619 (MS K)

[Notes on Lemma 1]

H 12v | [In H] “Ponam [autem *zd*]...” Scilicet per secundam propositionem libri de proportionibus qui conditus fuit super has demonstrationes, que questio erit talis: prime quantitatis ad tertium quocumque medio interposito proportio producitur ex proportione prime ad mediam et medie ad tertium. Et hec propositio est exposita in notula quarte columne pagine folis sequentis, et nota quod hec propositio est eadem cum 36ª Iordani de numeris datis.

K 26r [In HK] | 8. Duabus rectis lineis ab angulo uno descendentibus aliisque duabus sese secantibus a descendentium terminis reliquis in easdem reflexis, utralibet reflexarum alterius conterminalem sic figet ut proportio ipsius fixe ad eam sui partem que supra fixionem est producatur ex duabus proportionibus; ex una quam habet sibi conterminalis reflexa ad eam sui partem que sectioni interiacet et fixioni et ex ea proportione quam habet alterius reflexe inferior sub sectione portio ad eam totam cuius pars est linea.

[In H] Hec est septima et est exemplum, proportio linee *ga* ad *ea* producitur ex proportione linee *gd* ad lineam *zd* et ex proportione linee *bz* ad lineam *be.* Et sit *eh* equidistans *gd*. Quare proportio *ga* ad *ea* est tamquam proportio *gd* ad *eh* inter quas *zd* linea media statuatur cuius proportio est ad *he* sicut *bz* ad *be*.

[H marks the diagram with “Hec est catha coniuncta in lineis rectis.” K labels it “katha coniuncta.”]

[In K] Nota Ptholomeus lineam *zd* mediam inter *gd* et *eh* et non videtur verum quia est minima inter illas ut faciliter probari potest. Dico quod non ponit eam medium quantitatum sed locale proportionali. Verbi gratia sint 3 linee, a ut 6, *b* ut 3, c ut 4. Igitur dico quod *b* illorum est

15 8] *om.* H **16** a] ab earum K **17** terminis reliquis] *inv.* K **19** ipsius] *om.* H **20** una] dico proportio *add.* K | sibi] *om.* H **22** fixioni] fixionem H | ex ea] alia K **23** portio] proportio *sed. corr.* H media et non quantitate sed loco. Et proportio *a* ad *c* colligitur ex duabus proportionibus scilicet ex *a* ad *b* et ex *b* ad *c* per 19^{am} diffinitionem 7^{i} Euclidis. Verbi gratia *a* ad *b* est dupla et *b* ad *c* est subsesquitertia. Igitur subtraham subsesquitertiam a dupla quia ... sub[sic] ut producitur sesquialtera que est proportio primi ad tertium et hoc faciam sic 6/3 4/3 et ducam unum in aliud per modum crucis ut docet algorismus proportionum, et proveniet 18 et 12 quorum proportio est sicut primi scilicet a quod fuit ut 6 ad tertium scilicet *c* quod fuit ut 4 que est utrique sesquialtera.

[In H] Angulus *h* trianguli *ahe* est equalis angulo *d* trianguli *adg* eo quod sunt inter equidistantes que sunt *he dg* per 29^{am} primi Euclidis. Ergo latera illos eque angulos respiciuntur. Item proportionalia per 4^{am} 6^{i} latera illa, item *ae ag gd* proportionalia. Itemque *dg he* respiciunt angulum communem videlicet *a* trianguli *dag*. Ergo latera *dg he* sunt proportionalia ea proportione qua *ae ag* per 2^{am} 6^{i}.

[In H] *Zd* minor est *gd* et minor *he* quomodo(?) et media inter hos.

[In H only] *Bz be* respiciunt equales angulos, ergo proportionales eadem proportione per 2^{am} 6^{i}. *He dz* respiciunt equales angulos, ergo proportionales eadem proportione per 2^{am} 6^{i}. Quare proportio *be bz* cum proportione *gd dz* faciunt proportionem *ga ae*.

[Notes on Lemma 2]

[In HK] 9. Duabus lineis rectis ab uno angulo descendentibus et aliis duabus se secantibus a descendentium reliquis terminis in easdem reflexis, utralibet reflexarum alterius conterminalem sic figet ut proportio portionis fixe inferioris, dico, partis ad superiorem producatur ex duabus proportionibus; ex una quam habet sibi conterminalis reflexe inferior sub sectione portio ad reliquam partem que sectioni interiacet et fixioni et ex alia proportione quam habet relique descendentis inferior sub fictione portio ad eam totam cuius est pars lineam.

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71 9] *om.* H | lineis rectis] *inv.* K | uno angulo] *inv.* K **72** et aliis] aliisque K | se] sese K | a] ab earum K **74** portionis] proportionum H **76** una] in quam proportione K **77** portio] proportio *sed. corr.* H **78** ex alia] aliaque K | habet…79 descendentis] relique descendentis habet K **79** fictione] sectione H **80** est pars] *inv.* K

[In H only] Hec est 8ª questio. Patet. Proportio *ge* ad [*ae*] producetur ex proportione *gz* ad *zd* et proportione *bd* ad *ba*. Protrahatur a puncto *a* linea equidistans *be* donec concurrat cum linea *hd*. Quare proportio *ge* ad *ea* sicut proportio *gz* ad *zh*. ... sit medium *zd* cuius proportio erit ad *dh* tamquam *bd* ad *da*. Ergo coniunctim(?) *zd* ad *zh* sicut *bd* ad *ba*.

[In H] Hec est catha disiuncta et dicitur catha secundum quosdam ... sagita unde catha postea et secundum Arabes catha sector et utreque propter figuram.

[In K] Cite quoniam ponebatur per hunc modum in numeris scilicet ageundo in katha coniuncta ... proportio surgit in disiuncta vero proportio equalitatis ut 4 ad 4 ut patet in hac suprascripta figura. [Figura 1]

[In K only] Si vero *eg* fuerit minor *ea* ut si sit *eg* ut 2 et *ea* ut 10 scilicet subquintupla, tunc sequentes proportiones debent sibi invicem addi et nam ast subtrahi, et *gz* esset 3 et *zd* 5 et *db* 4 et *ba* 12, tunc agendo disiunctim ex additione, provenit subquintupla. Et hoc in numeris habet verbi exempli ... proportio linearum fuerit nota et cum volueris numeros divideris. Si autem volueris uteris semper additionem, sed tunc ponatur subquintupla sic 2/10 ...

[In K] Utendo semper additione ponantur termini 4 3 6, volo igitur proportionem 4 ad 6. Ponam in terminos in ordine sesquitertiam, fac 4/3 et subduplam sic 3/6, et addam et provenient 12/18 inter quos est subsesquitertia. Si vero volueris e converso videlicet 6 ad 4 habere proportionem, ponas terminos sic 6/3 3/4, et adde et proveniet 18/12 videlicet 3/2.

[In K] Hec omnia probantur per secundam libri de 36 modis proportionum scilicet de aggregatione proportionum.

[In K] De aggregatione proportionum de qua hic [agit] Ptholomeus ut facilius a novellis geometrie concipiatur, potest in numeris exemplificari sic. Et quia iste modus arguendi seu probandi dicti Ptholomei a quibusdam catha variori consuetus, etiam est autem katha dupla, scilicet katha

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104 nota] *al add.* K **119** agit] agis HK coniuncta et katha disiuncta. Unde primo in coniuncta exemplificatur sic. Esto linea *ag* ut 12 et similiter linea *ab*, *ge* ut 8, *ea* ut 4 et similiter linea *bd* ut 8, et *da* ut 4, et *gz* et *bz* uterque singillatim ut 6, et *zd* et *ze* uterque ut duo. Manifestum igitur cum in coniuncta kata quod *ga* scilicet 12 ad *ae* scilicet 4 est proportio tripla que componitur ex *gd* que est 8 ad *dz* que est 2 scilicet quadruplo et *zb* scilicet 6 ad *bg* scilicet 8 que est proportio subsequitertia. Igitur ista subsesquitertia quia est sub scilicet *zb* ad *be* subtrahenda est a quadrupla scilicet a *gd* ad *dz* ut docet algorismus proportionum ut patet ducendo 2 in 8 et 6 in 8. Et provenient 16 et 48 inter que est proportio subtripla que creat *ga* ad *ae*. Sed in katha disiuncta linea *ge* scilicet 8 ad *ea* scilicet 4 est tripla que componitur ex *gz* ut 6 ad *zd* ut 2 que est tripla et ex *db* ut 8 ad *ba* ut 12 que est proportio subsesquialtera. Igitur ista sunt 3/2 quia est sub subtrahendum est de *gz* ad *zd* scilicet a tripla, scilicet ducendo per modum crucis, 6 in 8 et 48, et 2 in 12 et provenient 24, inter que est proportio dupla sicut creat *ge* ad *ea*, et hoc.

[Notes on Lemma 3]

[In HK] Si in circulo continui arcus sumantur et uterque minor semicirculo diameter producta a communi eorum termino lineam rectam reliquos eorumdem terminos continuantem secabit secundum proportionem corde dupli arcus unius ad cordam dupli arcus alterius.

[In H only] Hec est 9ª. Sit *gh* perpendicularis super diametrum *bd* et sit medietas corde duplantis [*ab*] et sit *az* perpendicularis ad eadem *bd,* et habet medietas corde arcus duplantis arcus *ab*. Quare fient trianguli *geh aez* similes.

[Notes on Lemma 4]

H 13r, K 26v | [In HK] 11ª. Si unus arcus notus in duo dividatur fuitque nota proportio corde dupli arcus unius ad cordam dupli arcus alterius, ambo illi erunt noti.

[In H] Hec est decima. Sit *dz* perpendicularis ad cordam arcus noti *ag*, et ergo totus triangulus *zda* in lineis et angulis erit notus. Ergo proportio *ge* ad *ea* per hypothesim et

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145 arcus sumantur] *inv*. K | et] *om*. H **146** minor semicirculo] *inv*. K diameter] diametros K **147** reliquos] *om*. K **157** 11a] *om*. H | notus] *supr. lin*. H | duo] duos K **159** alterius] eorum ad cordam dupli arcus alterius *add. sed. del*. H premissam erit nota, ergo proportio coniuncta *ga* ad *ea* nota, universaliter(?) demonstratio proportionis disiuncte sit nota. Ergo *ae* nota, ergo *ze zd ed* linee note respectu circuli et anguli. Ergo omnes anguli trianguli orthogonii *ezd* noti erunt per circulum circumscriptum respectu duorum rectorum, ergo et respectu quartorum. Dempto ergo angulo *zde* nunc noto ab angulo *zda* prius noto, relinquitur angulus *eda* notus, quare arcus *ab* notus, ergo *gb* notus.

[In H] ... linea *dz* secat lineam *ag* [per] medium per 30^{am} 3^{i} Euclidis, ergo et arcum *ag* si protrahatur ad ipsum. Ergo basis anguli *adz* est medietas arcus *ag*. Sed totus arcus *ag* est notus ex ypothesi, ergo et eius medietas nota, ergo et angulus *adz*, qui est qui(?) in eum cadit et sic habet eum pro basi est notus quia que est proportio arcus qui est minor quarta circulum ad quartam circuli sic et angulus primi arcus ad angulum rectum qui cadit in quartam circuli idest cui subtenditur quarta circuli.

[In K] Totus triangulus *adz* est notus quia *ad* latus est semidyameter, *az* est medietas arcus *abg* noti. Igitur perpendicularis primi(?) latus *dz* est notum. Angulus vero *adz* est notus quia arcus ipsius suscipiens, videlicet medietas arcus *ag* noti, est nota, et sit verbi gratia 60 gradus. Angulus vero *azd* quia rectus respondet 90 gradibus, igitur angulus *daz* id quod ad complendum 180 sufficit pertinebit videlicet 30.

Circumscribatur enim circulus triangulo *edz* notorum laterum. Sed quia latera sunt nota, igitur arcus quibus subtenditur propter missa sunt noti. Sed quia arcus sunt noti, etiam anguli quos idem arcus suscipit sunt noti. Igitur etc.

[Notes on Lemma 5]

[In HK] 12^{a}. Si ab uno termino arcus semicirculo minoris linea ipsum secans arcum educatur donec cum dyametro per reliquum eiusdem arcus terminum extracto concurrat, fiet proportio linee preter centrum transeuntis ad partem sui extrinsecam sicut proportio corde dupli arcus de quo est sermo ad cordam dupli arcus illius quem educte line includunt.

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197 12a] *om.* H **198** secans arcum] *inv.* H | cum] *om.* K **199** extracto] extracta H **201** corde] *post.* arcus H

[In H] Hec est undecima, et probat catham coniunctam. Esto *gh* sinus arcus *ga* cui equidistat *bz* sinus arcus *ba* in duabus lineis concurrentibus, quare linea *gbe* preter circulum(?) transitus(?) arcum *ga* secat altera *hae* diameter etiam extracta. Sit ergo triangulus *geh* totalis similis triangulo partiali *bez*.

[Notes on Lemma 6]

[In HK] 13ª. Si arcus dicto modo divisi lineis ut prescriptum est donec concurrant eductis, maior portio nota fuerit et proportio corde dupli arcus ipsius divisi ad cordam dupli arcus lineis eductis inclusi constiterit, ipse arcus inclusus notus erit.

[In H] Hec est duodecima. Esto *zb* medietas corde arcus noti *gb* nota. Item *db* nota quare totus triangulus *dzb* ortogonius notus in lineis et angulis. Item proportio *ge* ad *be* nota per primam inquisitionem. Quare per penultimam 3^{i} Euclidis, *ea* nota, ergo cuius triangulus orthogonius *ezd* notus, a quo dempto angulo *bdz* noto relinquetur angulus *adb* notus. Ergo et arcus [*ab*] notus.

[In H] "Sciemus ex hoc etc.” Notandum est est hic ad hoc probandum per numeros et cum proportio alicuius numeri totius ad partem sui erit nota sicut octanarii ad senarium que erit sesquitertia, et quantitas partis alterius erit tantum(?) nota sicut binarius, totus numerus etiam notus et utraque pars nota quia totus erit 8 et partes. Item 6 vel senarius sit nota pars et idem erunt ... Sic ostenditur. Subtrahatur numerus partis ignote scilicet 6 et 2 non curo quis a numero nominante totum scilicet 8. Et tunc que proportio erit illius numeri subtracti scilicet idest 6 ad residuum totius scilicet 2. Quia tripla erit, eadem erit partis ignote scilicet 6 ad partem notum scilicet 2. 6 item triplicat duo. Sic se habet in omni quantitate tam continua quam discreta observata(?) proportione.

[Notes on the Menelaus Theorem]

[In HK] 14ª. In superficie sphere duobus arcubus magnorum orbium semicirculo divisim minoribus ab uno communi termino descendentibus aliisque duobus non

212 Lemma] noti *add. sed. del.* H **213** 13a] *om.* H **225** ab] *aeb* H
238 scilicet[1]] ad *add.* H **243** 14a] *om.* H **245** duobus] *om.* H

minorum orbium ab illorum relictis terminis in eosdem se secando reflexis, utrius reflexorum alterius conterminalem arcum sic figet ut proportio corde arcus duplicantis inferiorem portionem arcus fixi ad cordam arcus duplicantis superiorem eiusdem fixi portionem producatur ex gemina proportione, scilicet ex ea quam habet corda arcus duplicantis inferiorem arcus reflexi portionem que ipsi fixo conterminalis est ad cordam arcus duplicantis reliquam eiusdem reflexi portionem et ex ea proportione quam habet corda arcus duplicantis inferiorem alterius descendentis arcus partem ad cordam duplicantis arcum ipsum cuius est pars totalem.

[In H] Hec est tertiadecima et patet ex kata disiuncta et 9^a huius et 11^a semel sumpta et primo et tertio circulo de 4 circulis precedentibus hanc figuram et per secundam et nonam libri 5^i Euclidis. ... arcus duorum magnorum circulorum describimus in superficie sphere *ab ag* inter quos sint alii duo *be gd* sese secantes in *z*. Dico ergo quia proportio corde duplantis *ge* ad cordam arcus *ea* dupli componitur ex gemina proportione ut in ... scilicet ex proportione quam habet corda arcus *gz* dupli ad cordam arcus *zd* duplantis ... et ex proportione que erit corde arcus *db* qui est duplus ad cordam arcus qui est duplus ad arcum *ba*. ... etiam centro .... et ab ipso scilicet *h* ad notas *b* z *e* circulorum sectiones linee ducantur. Recte linee *ad hb* transeuntes communicant ad notam *t*. Sed quia linee *ga gd* ... autem sunt *hz he* sectio..(?) ad *k l* ducantur sic(?) in una linea recta et sunt 3 note *t k l* quia sunt in superficie trianguli *agd* quamlibet extensa et in superficie circuli *bze* ... superficiem communis sectio linea, hac igitur linea protracta, restat ex ....dis.

K 27r [In HK] | In superficie sphere 4 arcubus supradicto modo depictis, fiet ut proportio corde arcus duplicantis unum descendentium totalem ad cordam arcus duplicantis superiorem ipsius descendentis portionem componatur ex gemina proportione scilicet ex ea quam habet corda duplantis arcum totum eiusdem descendentis [a] termino reflexum ad

246 relictis] reliquis K | se] sese K **247** utrius] alterius K **251** scilicet] videlicet *post.* ea K **252** que] qui H **255** arcus duplicantis] *inv.* K **256** est pars] *inv.* K **279** supradicto] predicto K **280** duplicantis] duplantis K **281** duplicantis] duplantis K **283** scilicet] videlicet *post.* ea K **284** totum] ab *add. sed. exp.* K cordam duplantem illam illius reflexi portionem que sectioni interiacet et fixioni et ex alia proportione quam habet corda arcus duplantis inferiorem sub sectione alterius reflexi portionem ad cordam arcus duplicis ad eundem reflexum cuius pars est totum.

[In H] Hec est 14^{a}, et patet per kata iunctam et 10^{am} et per 6^{am} undecimi libri. [Figura 2] Et non est hec satis bene facta et in aliquis libris non erit linea *tdze* que est hic nigra, et tantum debet esse infra, patet ex demonstratione. Ducantur ergo a *h* quod est centrum sphere linee *hal hdk*. Deinde ducatur *kzg* donec concurrat cum *hd* in *k*, et *ge* ducatur donec concurrat cum *ha* in *l*. Deinde ducatur *ez* que est hic linea nigra donec concurrat cum *hb* in puncto *t*. Et erunt tria puncta *t k l* in linea recta quia sunt in superficie trianguli *gkl* et in superficie circuli *ab*. Et illa, circulus ille *ba*, et triangulus ille *gkl* sunt in eadem superficie quantumlibet illa linea protrahatur. Ducatur ergo linea *lkt*, et tunc argue expletionem circulo quia proportio corde arcus duplantis arcus *ga* ad cordam duplantis *ea* est ex duplici proportione.

[Another Note]

H 13v [In H] | "Cum proicerimus." Ad sciendum quomodo proiciatur proportio ex proportione ... et cum fuerit proportio ex duabus proportionibus composita et voluerimus a tota proicere unam componentium ut relique que remanebit cognoscamur. Sic operabimur. Sit proportio *a* ad *b* duodecupla verbi gratia [*a*] erit 240 [et *b* erit 20] qui erit pars eius. Et sit composita ex duabus proportionibus, scilicet ex proportione *g* qui erit 120 ad *d* qui erit 40 que proportio erit tripla et et ex proportione *e* qui ad *f* que erit 30 que proportio erit quadrupla. Que erit igitur proportio *g* ad *d* scilicet tripla sicut *a* ad aliquem alium numerum. Quod sic invenitur. Ducatur [*d*] in *a* idest 40 in 240 et exeunt 9600, qui dividatur per *g* idest per 120. Et exit 80 qui vocetur *h* ad quem se habet *a* id est 240 sicut se habet *g* idest 120 ad *d* idest 40 idest in tripla proportione ex 18am 7i Euclidis libri. Et ergo proportio *h* qui erit 80 ad *b* qui erit 20 sicut *e* qui [erit 120] ad *f* qui erit 30 idest quadrupla, quia [cum] inter *a* et *b* ponitur *h* medium sicut 80, erit medium inter 240 ad quem erit subtriplum et 20

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285 duplantem] duplam K | illius] ipsius K **286** fixioni] fixionem H | ex] *om.* K que erit quadruplus. Et noto medium non quolibet numero sed medium proportionale. Ergo patet per libros(?) de proportionibus quos habes prima pars sole tabulo. Et ... proportio *a* ad *b* que erit duodecupla ut habitum erit composita ex proportione *a* ad *h* que erit tripla cum *h* ad *b* que erit quadruplam. Sed proportio *a* ad *b* producebatur ex proportione *g* ad *d* que erit tripla et proportione *e* ad *f* que erit quadrupla, quia illa tota est duodecupla ... ... 4(?), et e converso sunt 12^{a}(?), vel que erit *g* ad *d* eadem erit *a* ad *h* idest tripla quod patet per iam inventam propositionem. Restat ergo quod *h* ad *b* est sicut *e* ad *f* idest quadrupla. Si ergo quantitas *f* fuerit ignota, ducatur *b* in *e* et exibit 2400 et [dividatur per *h*], et exibit *f* qui erit 30 per 18am 7i Euclidis. Et si *e* fuerit ignotus, duc *h* in *f* idest 80 in 30 et exibit 2400 quem [dividatur per] *b* et exibit 120 qui erit *e* per eandem propositionem(?). [The note then explains how to proceed if *g* or *d* is unknown. Much of the remainder of the note is hidden in the gutter.]

[The notes continue to give enunciations, rules, and proofs from the *Almagestum parvum*.]

Marginalia in Città del Vaticano, Biblioteca Apostolica Vaticana, Vat. lat. 2057 (MS L); Città del Vaticano, Biblioteca Apostolica Vaticana, Barb. lat. 173 (MS M); and Krakɵw, Biblioteka Jagiellońska 590 (MS R)

[Notes on Lemma 1]

| [In MSS LMR] [“Quapropter proportio *ga* ad *ae*”] Nam hic sunt duo trianguli scilicet *agd* et *aeh*. Et angulus *agd* primi trianguli est equalis angulo *aeh* secundi trianguli per secundam partem 29 primi Euclidis et angulus qui sit in puncto *a* est communis utroque [tri]angulo. Ergo tertius remanet equalis tertio vel etiam tertius est equalis tertio per 29 primi Euclidis secundam partem. Ergo per 4^{am} 6^{i} Euclidis latera sunt proportionalia, ergo proportio *ga* ad *ea* etc. L 15r, M 15v, R 14r

[In MSS LMR] [“Proportio vero *dz* ad *eh*”] Nam hic sunt duo trianguli scilicet *beh* et *bzd*. Et angulus *beh* primi trianguli est equalis *bzd* secundi trianguli per secundam partem 29^{e} primi Euclidis. Et angulus qui fit in puncto *b* est communis in utroque angulo. Ergo tertius est equalis tertio vel etiam tertius est equalis tertio per secundam partem 29^{e} primi Euclidis. Ergo per quartam sexti Euclidis latera illorum triangulorum sunt proportionalia.

[In MS R] Illud quod fuerint pronotum in demonstratione huius 8^{e} figure scilicet quod proportio primi ad tertium aggregatur ex proportionibus primi ad secundum et secundi ad tertium ubi primus terminus est maior tertio et secundus minor ipso tertio, est potius ut credo segregatio quam aggregatio quia facta subtractione proportionis secundi ad tertium de proportione primi ad secundum, remanet proportio primi ad tertium, et in hiis terminis sic dispositis non habet veritatem illa regula scilicet multiplica proportionem primi ad secundum in proportionem secundi ad tertium.

[In MS R] Et ego credo quod velit dicere hic magis prescisimus quod multiplicando denominatorem proportionis prime ad secundam per denominatorem secunde ad tertiam exibit denominatio proportionis prime ad tertiam quod sic est.

[In MS R] In demonstratione huius uti debes 2^{a} 6^{ti} bis et 4^{a} eiusdem bis et 18 5^{ti} bis et regula data in commento super 24 6^{i} que talis omnium

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10 communis] in *add.* R **17** angulus] triangulus R trium quantitatum est proportio prime ad tertiam proveniens ex ductu proportionis prime ad secundam in proportionem secunde ad tertiam, verbi gratia sint 1 2 3 proportionis 1 ad 3 est dimidium duorum tertiarum. Et ideo dicit, “Ponam autem *zd* mediam etc.”

[Notes on Lemma 2]

[In MSS LR] Nam in precedenti figura quando dicebam “proportio *ga* ad *ea*,” istum erat coniunctim, sed tamen quando dico “proportio *ge* ad *ea* etc.,” istum est divisim respectu illius.

[In MSS LMR] [Verum proportio *zd* ad *zh*”] Nam hic sunt duo trianguli scilicet *dbz* et *dha* et angulus *dbz* primi trianguli est equalis angulo *dah* secundi trianguli per secundam partem 29^{e} primi Euclidis. Et similiter per eandem angulus *dzb* primi trianguli est equalis angulo *dha* secundi trianguli. Ergo tertius est equalis tertio vel est equalis tertio per 15^{am} primi Euclidis. Ergo per 4^{am} 6^{i} Euclidis latera sunt proportionalia. Ergo proportio *zd* ad *dh* est sicut proportio *bd* ad *da* et coniunctim et postea divisim.

[Notes on Lemma 3]

| [In MSS LMR] [“erit proportio *az* ad *gh*”] Nam hic sunt duo M 16r
trianguli scilicet *aze* et *hge* et anguli unius sunt equales angulis sicut potest ostendi per primam partem 29^{e} primi Euclidis quoad angulos fiunt in punctis *a* et *g* et per 12^{am} primi Euclidis quoad angulos *z* et *h.* Et per 4^{am} petitionem primi Euclidis etiam et per 15^{am} primi Euclidis quoad angulos qui fiunt in puncto *e* vel quia si a noto subtrahatur notum, residuum erit notum. Ergo latera sunt proportionalia per 4^{am} 6^{i} Euclidis.

[Notes on Lemma 4]

[In MSS LM] Nam si arcus *ag* est notus, ergo et eius medietas erit nota. Ergo angulus *adz* est notus per ultimam 6i Euclidis quia ei subtenditur medietas arcus *ag* que est nota ut iam patet. Et quod ei subtendatur medietas arcus *ag* patet quia *ag* est divisa in duo media linea *dz* per 2^{am} 3^{i} Euclidis. Ergo protrahamus *dz* donec occurat arcui *ag* et producamus cordas arcus *gz* et *za*. Dico quod per 4^{am} primi Euclidis ille corde erunt equales. Ergo per 27^{am} 3^{i} Euclidis arcus erunt equales.

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47 tamen] modo R **48** respectu] respectiva L **54** vel] etiam tertius *add.* LR **61** angulis] *om.* R **62** angulos] qui *add.* LR **63** Et] *om.* M **73** 2am] 3^{am} M **75** equales^{2}] Et per consequens totus triangulus *adz* erit notus sicut patet per glossam suprapositam ibi 'Nam trianguli *adz*' etc."*add. a. m.* M

| [In MSS LMR] [“erit linea *ae* nota”] Quia si aliqua quantitas est nota et proportio unius duarum partium in quas dividitur ad aliam fuerit nota, tunc erit unaqueque illarum partium nota. L 15v, R 14v

[In MSS LMR] [“*dz* est nota”] Nam trianguli *adz* angulus qui sit in puncto *z* est notus quia est rectus ut patet iam. Et angulus qui sit in puncto *d* est notus quia arcus cui subtenditur est notus. Ergo tertius erit notus quia si a noto subtrahatur notum, residuum erit notum. Ergo omnes anguli illius trianguli sunt noti. Etiam latera sunt nota. Describam enim circulum circa triangulum illum cuius diameter erit linea *ad* que est nota ex hypothesi quia ponitur 120 et *az* est notus quia angulus *zda* fuit notus. Ergo per ultimam 6^{i} Euclidis arcus cui subtenditur *az* est notus. Ergo per tabulas arcuum et cordarum iam factas corda *az* erit nota. Et similiter *dz* erit nota quia angulus *daz* fuit notus. Ergo per ultimam 6^{i} Euclidis arcus cui subtenditur *dz* est notus. Ergo per tabulas arcuum et cordarum iam factas *dz* erit nota.

[In MSS LMR] [“quoniam omnis trianguli orthogonii”] Nam describam circulum circa triangulum orthogonium *dze* cuius diameter sit *de*. Et quia *ze* est nota ut iam patet, ergo arcus cui subtenditur erit notus per tabulas arcuum et cordarum. Ergo per ultimam 6^{i} Euclidis angulus *edz* erit notus. Iterum cum linea *dz* sit nota ut iam patet, ergo per tabulas iam factas arcus cui *dz* subtenditur erit notus. Ergo per ultimam 6^{i} Euclidis angulus *dez* erit notus. Et angulus *z* est rectus, ergo omnes anguli predicti trianguli sunt noti et latera eius sunt nota. Et quia *dz* est notus ut iam patet et *ze* similiter est notus ut patet etiam iam, et *de* erit notum quia quadratum eius est equale quadratis *dz* et *ze* simul iunctis. Ergo quadratum *de* est notum, ergo *de* est notum, quia cuius quadratum est notum et ipsum est notum.

[Notes on Lemma 5]

[In MSS LMR] [“erit proportio *ge* ad *eb”* (first time)] Quia trianguli *egh* angulus *ehg* est equalis angulo *ezb* trianguli *ebz* quia uterque est rectus ut iam patet. Et angulus qui sit in puncto *e* est communis in utroque triangulo. Ergo tertius remanet equalis tertio. Ergo per 4^{am} 6^{i} Euclidis latera sunt proportionalia.

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102 similiter] *post*. notus M **110** Et…112 proportionalia] *om*. M

[IN MSS LMR] [“erit proportio *ge* ad *eb”* (second time)] Quia proportio *gh* ad *bz* est sicut proportio corde dupli arcus *ga* ad cordam dupli arcus *ba* quoniam *gh* est medietas corde sui dupli, et similiter *bz*. Et multiplicium et submultiplicium proportio est una ut patet per 15^{am} 5^{i} Euclidis.

[Notes on Lemma 6]

[In MSS LMR] [“cuius basis est”] Ut potest ostendi per 4 primi Euclidis protrahendo lineam *dz* primo donec occurat arcui *bg* et protrahendo postea cordas a punctis *b g* ad punctum *z* in circumferentia.

[In MSS LMR] [“Quapropter totus triangulus”] Quia cum duo anguli eius sint noti ut iam patet ergo tertius erit notus ergo latera erunt nota describendo circulum circa triangulum *bdz* et concludendo per ultimam 6^{i} Euclidis et per tabulas arcuum et cordarum iam factas.

[IN MSS LR] [“Sciemus ex hoc”] Quia si fuerit aliqua quantitas que dividatur in duas partes et fuerit proportio unius partis ad aliam nota et una illarum partium fuerit similiter nota, reliqua erit nota.

[In MS R] [“et sciemus per hoc”] Quia *bz* fuit iam nota et si notum noto addas, totum fiet notum.

[IN MS LR] [“*dz* est nota”] ut iam patet quia totus triangulus scilicet *edz* fuit notus.

[In MSS LR] [“erit angulus *edz*”] Quia totus angulus *edz* est notus ut iam patet angulus *zdh* fuit notus, ergo angulus *edb* erit notus quia si a noto subtrahatur etc.

[Note on I.13]

| [In MSS LR] Quia si aliquod aggregatur ex duobus et ab eo subtrahatur unum, remanet reliquum. L 16r, R ?

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123 circumferentia] circumferentiam LM **132** nota^{2}] Mihi non iret ad propositum ista expositio vel testus est corruptus *adnot. a. m.* R **137** edz] *dz* R

Marginalia in Città del Vaticano, Biblioteca Apostolica Vaticana, Pal. lat. 1365

[Notes on Lemma 1]

| Si volueris hanc figuram ponere in numeris, pone *ag* totam 36, *ge* 24, *ea* 12, item totum *gd* 24, *gz* 18, *zd* 6, item totam *be* 28, *bz* 21, *ze* 7, item *eh* 8. S 11v

[*There are numerical examples written giving lengths for the line segments of the plane sector figure.*]

ga	*ea*	*gd*	*dz*	*zb*	*be*	*gd*	*eh*	*gd*	*dz*	*dz*	*eh*
36	12	24	6	21	28	24	8	24	6	6	8

Sicut enim vult Iordanus in *Arismetica*, proportio extremorum ex proportionibus mediis est composita, verbi gratia, sicut patet in his tribus numeris 18 12 6. Proportio extremorum est tripla et illa composita est ex proportione sexquialtera scilicet 18 ad 12 et ex dupla scilicet 12 ad 6. Et ipse sunt intermedie. Similiter proportio dupla componitur ex duabus mediis scilicet sexquialtera et sexquitertia ut oportet in his numeris 12 9 6. Similiter si transponantur numeri sic 18 6 12, adhuc proportio veritatem habet. Sexquialtera enim potest dici esse composita ex tripla et subdupla nec minori, quantum enim tripla auget proportionem super sexquialteram tantum subdupla diminuit. Eodem modo ex dupla et subdupla componitur equalitas ut 4 2 4. Hac enim proportione utitur hic Ptholomeus statuendo *zd* mediam in proportione.

Dicit penultimo(?) quod inter omnes numeros cubicos est ponere duos numeros medios qui eadem proportione se habent ad extrema, ergo inter quelibet duo cubica erit ponere duo media que in eadem proportione se habent ad extrema. Unde primus numerus cubicus est bis duo bis, hoc est 8, secundus est ter tria tertio, hoc est 27. Et inter istos duos numeros est ponere duos medios qui in eadem proportione se habent ad extrema ut 12 et 18, isti enim omnes numeri se habent in sexquialtera proportio.

| [*Numbers given for the rectilinear disjoined sector figure*] S 12r

dz	*zh*	*dz*	*zd*	*zd*	*zh*	*ge*	*ea*	*gz*	*zd*	*db*	*ba*
18	9	18	6	6	9	24	12	18	6	28	42

29 duo[2]] cubica *add.* S

Sint due linee *abc* et *de*. [Figura 1] Dico quod proportio linee *ac* ad *de* constat ex proportione *ac* ad suam partem scilicet ad *bc* et illius partis *bc* ad lineam *de*, et hoc generaliter verum est in omnibus quantitatibus quod proportio alicuius quantitatis ad alteram constat ex proportione illius quantitatis ad suam partem et illius partis ad alteram quantitatem. Et hoc potest probari quia sunt tres proportiones scilicet 2 componentes et una composita, et si una componentium [p]roiciatur, manet altera. Et hoc experiri potes in numeris sicut patebit cum in glossa quadam post figura sectore posita super illum passum “cum ergo proiecerimus.”

[Notes on Lemma 4]

| Nota quod angulus hic dicitur esse notus quando latus ei oppositum S 13r est notum. Similiter angulus in circulo dicitur vel plus vel minus secundum quod latus ei oppositum plus vel minus de circulo continet.

“Et erit totus triangulus *adz* notus.” [Figura 2] Verum est si circumscribatur circulus vel intelligatur circumscribi huic angulo *d* quod sic patet. Omnis angulus stans in circumferentia in duplo maioris potentie est quam si staret in centro. Si ergo angulo *d* intelligatur circulus circumscribi in duplo plus valebit quam prius. Cum ergo centrum illius circuli circumscripti huic angulo sit in linea *ad*, erit ipsa linea *ad* dyameter talis circuli, et apponitur iste dyameter angulo *z* orthogonio qui angulus continet medietatem circuli, sic ergo in tali circulo notus est angulus *z* et latus ei oppositum scilicet dyameter *ad*. Cum ergo alii duo anguli preter orthogonium contineant medietatem circuli residuam, cum angulus *d* prius notus fuerit in circulo priori et cum modo in duplo plus valeat sive in duplo plus de arcu contineat, illa ergo portionem quam prius continuit duplatam. Subtrahe a semicirculo et medietas eius quod remanserit erit angulus *a* cum latere *dz* sibi opposito. Preterea nota quod cum in isto circulo circumscripto et incluso, angulus *z* orthogonius contineat semicirculum et integrum dyametrum, et prius in circulo maiori tamen semidyametrum in duplo plus valet nunc quamvis licet ipse prius non steterit in centro. Et cum alii duo scilicet *a* et *d* reliquam medietatem circuli contineant, tantum valent illam duo quam *z*. Cum ergo *z* in duplo plus valeat quam prius, ipsi etiam in duplo plus valent quam prius. Unde si noti sunt quantum ad parvum circulum, noti etiam erunt respectu magni.

“Et manifestum est quantum.” Nota corda *ag* et nota proportione partis ad partem ipsius *ag* ex ipsa proportione nota et toto noto, partes inscriptent per portionem coniunctam. Quandocumque enim totum notum

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40 ad bc] *iter*. S

est et proportiones partium sunt note, quantitas tamen partium ignorata, accipe aliquas quantitates notas in eadem proportione. Ubi totum sic notum et partes et sic proportio partium talis qualis est hic, accipe omnes partes et coniungas eas in unum, et videas que sit proportio totius coniuncti ad partem minorem vel maiorem. Eadem erit proportio illius totius ad suam partem maiorem vel minorem sicut et in priori. Hoc facto statuas totum notum pro primo et partem suam pro secundo et totum propositum cuius partes queris pro tertio, et tunc quere quartum ducendo secundum in tertium, et divide per primum, et exibit quartum. Et deinde subtrahe partes amborum eorum similes in proportione, et habebis propositum. Verbi gratia, tu scis ternarium et binarium et eorum proportionem et cognosces hunc numerum 30 et scis quod habet partes similes prioribus numeris in proportione et nescis illas partes. Coniunge ergo partes primas dicendo(?) sic que est proportio [coniunctionis] 3 et 2 idest 5 ad duo, eadem est proportio 30 ad aliqua sui partem, et si illam nescis, multiplica 30 per 2. Fuerit 60. Hoc divide per 5, exibunt 12 quod est unum proportionabilium quesitorum. Deinde subtrahe partes a totis ut 2 a 5 et 12 a 30, et remanebit ex prima parte trinarius, ex alia parte 18. Dico ergo quod 18 et 12 sunt partes quas quesivisti. Similiter in proposito si nota sic est linea *ag* et proportio partium eius sic est nota, non quantitas partium, accipe aliquos numeros in eadem proportione se habentes et per regulam prius datam invenies partes huius linee. Habens partibus inventis scilicet linea *ae* et *eg* cum *gz* prius nota tibi fuerit eo quod est medietas *ag* linee note, subtrahe *gz* de *ge* et remanebit *ez*. Illa habita intelligas omnem circumscriptum circulum cuius centrum sit linea *ed*. Tunc ergo *ez* in duplo plus valet quam prius, et erit corda integra arcus. Cum illa ergo corda inter tabulas arcuum [intra], et accipe eius arcum. Deinde applica illum ad istum magnum circulum propositum. Et tunc valeat dimidium eius et prius valuit. Per hoc ergo scies quantum arcum in hoc circulo contineat *ez* sed prius novisti totum arcum *ag*, unde et eius medietatem sue. Ergo medietati adde arcum inventum quem continet *ez* et habebis arcum *ag* et ex consequenti *ab*, et hoc est propositum.

[Notes on Lemma 6]

| Duplus arcus *ga* est arcus *gam* et corda huius dupli arcus sit linea *gm* S 12v
si protrahetur linea a puncto *g* ad *m*. *M* autem non est de linea Ptholomei.

"Sequitur [vero] hoc etc." "Sciemus ex hoc *eb...*" Apparatio talis est: nota proportione corde dupli arcus *ag* ad cordam dupli arcus *ab* ex

115 vero] ut S

consequenti, nota est proportio *ge* ad *be* quia est eadem proportio utrobique. [Figura 3] Si ergo non nescis quantitatem linee *ge*, illam investiga. Pone cordam dupli arcus *ag* et cordam dupli arcus *ab* in numeris aliquibus qui sunt eiusdem proportionis cuius sunt linee opposita. Deinde subtrahe minorem lineam a maiori integrando maiorem scilicet cordam dupli arcus *ag* resecari in puncto *t* licet Ptholomeus huius non faciat mentionem. Integras iterum a puncto *b* protrahi lineam equidistantem linee *ed* ad punctum *t*. Fiet ergo triangulus similis triangulo magno totali, ex quo ergo similes sunt que est proportio [medietatis] corde dupli arcus *ag* ad totam longam lineam *ge* eadem est proportio partis resecte in *t* scilicet linee *gt* ad lineam *gb*. Deinde pone *gt* in numero totali secundum quem exigit numerus ille in quo posuisti cordas duplorum arcuum.

Hoc autem de facili scire potes in quo numero *ge* ponere debes, *gt* enim est residuum de *gk* subtracto *bh* de *gk* ut patet in triangulo quem hic depictamus. Diximus enim quod deberes subtrahere minorem numerum de maiori scilicet lineam *bh* de [*gk*]. Unde *gk* sic secta est in *t* quod *gt* est residuum de *gk* subtracto a *gk* linea vel numero *bh*. Statue ergo tunc *gt* pro primo et *gb* quam Ptholomeus in linea(?) ex ypothesi ponit et nota pro secundo, et illam pone in numero sibi proprio et non aliunde accepto, et cordam dupli arcus *ag* scilicet *gk* pro tertio et lineam *ge* pro quarto. Postea duc secundum in tertium, divide per primum, et exibit quartum. Sic ergo erit nota tota linea *gb* ab illa. Ergo reseca *zg* deinde toti residuo circumscribe circulum. Et per hoc erit notus totus triangulus *odz* lineis et angulis. Unde linea *zd* etiam est nota. Linea *bz* est nota quia est medietas *bg* prius note. Has duas quadra ut habes *bd*. Huic pono 120. In eadem proportione *zd* prius inventa se habet ad cordam que deberetur triangulo *zdb*. Duc secundum etc. capiendo fac de *zb[d]* lineas ergo *zb* et *zd* prout sunt corde. Sciuntur anguli quibus subtenduntur scilicet *zbd zdb*. Deinde subtrahe [*zdb*] ab *edz* prius invento, et remanebit *bda*, verum est respectu duorum rectorum. Accipe ergo eius medietatem ut habeas uno 4 rectorum hic angulo habito. Habes cordam *ba*, quod est propositum.

[Notes on I.13

1	2	3	4	5	6	S 13v
za	*ba*	*zt*	*ht*	*eh*	*eb*	
120	48	120	24	60	120	[partes]
0	31		15			[minuta]

118 Figura 3] [The diagram in MS S is corrupt, so this is my reconstruction.] **132** gk[1]] *gh* S
145 zdb] *zbt* S

0 55 57 [secunda]

| "Cum ergo nos proiecerimus..." Ex maxima declinatione solis, S 14r invenies declinationem puncti terminalis 30 graduum acceptorum ab equinoctiali utpote finis arietis et hec declinatio est *ht* per hanc regulam: quandocumque 6 proportionalia habemus que sic se habeant quod ex proportione mediorum proportio extremorum sit composita, si omnia sint nota preter sextum, illud sic invenies. Duc primum in quartum, divide per tertium, et exibit quiddam quod sic se habet ad secundum sicut quintum ad sextum. Dimissis ergo omnibus aliis, capies 4 proportionalia ex hiis, primum scilicet illud quoddam quod quesivisti quod exivit per divisionem postea, secundum quod et prius fuit secundum ista se habent sicut quintum ad sextum. Illud ergo quod prius fuit quintum sit nunc tertium, et prius sextum nunc quartum. Hac facta duc secundum in tertium, divide per primum, exibit quartum.

Cum ergo proportio tripla composita est ex dupla et sexquialtera, ergo hoc probabis per hos numeros. Sit 12 primus, quaternarius secundus, horum proportio que est tripla constat ex dupla in qua sit 10 et 5, sit ergo 10 tertius, quinarius quartus, et ex sexquialtera que sit in nonario et senario, sit ergo 9 quintus, et sextus. In hiis si omnia sint nota preter senarium, illum invenies per predicatam regulam. Econverso etiam si omnia sint nota preter quartum, duc primum in sextum, divide per quintum, et exibit, quiddam quod sic se habet ad secundum senarium sicut tertium ad quartum, sicut patet in hiis eisdem numeris secundum hoc ergo cum ista proportionalia scilicet *za ba* etc. sic se habeant quod proportio extremorum ex proportione mediorum est composita sicut Ptholomeus inceptu dicit, duc *za* primum in *eb* quod est sextum, et divide per quintum scilicet per *eh*, et exibit quintum quod sic se habet ad *ba* quod est secundum sicut *zt* ad *ht*. Hoc facto abiectis residuis statue illud quiddam quod exivit primum, *ba* secundum, *zt* tertium, *ht* quartum. Tunc ergo duc secundum etc., et exibit quartum *ht*, que est declinatio quam queris, et hoc est subtrahere proportionem de proportione. In omnibus autem hiis operationibus non ducas arcum in arcum sed per arcubus sume cordas earum duplas.

Sed auctor minoris almagesti faciliorem ponit operationem et sunt hec verba omnes. Si sinus inchoate portionis ab equinoctiali cuius finalis puncti declinatio queritur ducatur in sinum maxime declinationis, productum dividatur per sinum quarte [circuli], exibit sinus quesite declinationis. Sinum autem vocas dimidia corda qua habita habetur integra et ex

170 sexquialtera] in hac *add.* S **181** et…182 eh] *add. mg.* S **191** productum] quod *add.* S constanti duplus arcus, et loquitur secundum modum Arzachelis qui utitur sinibus per quos hic dimidias arcus et ex consequenti integros. Utitur inter hic tantum 4 proportionalibus quia reducit 6 ad 4.

Ducatur primum in sextum, *za eb* idest 120 in 120, et dividatur per quintum *eh* idest per 60, et exibit numerus quidam qui se habet ad secundum sic tertium ad quartum idest *zt* ad *ht*. Deinde numerum qui exivit pone primo loco et *ba* secundo, *zt* tertio loco, deinde multiplica secundum per tertium et divide per primum, et proveniens ostendit lineam *ht* sex numerus notum ... inter has cordas... [Note continues but becomes illegible.]

[Notes on I.14]

| “Cum ergo nos proiecerimus...” Duc *zb* quod est primum in *ht* quod S 14v
est quartum, divide per tertium scilicet per *zh*, et exibit quiddam quod sic se habet ad secundum scilicet *ba* sicut quintum scilicet *et* ad sextum *ea*. Ergo per proportionem econverso sumpta secundum se habebit ad illud quiddam sicut sextum ad quintum. Pone tunc secundum per primo, et illud quiddam prius quesitum pro secundo, sextum pro tertio, quintum pro quarto, et et duc secundum in tertium etc.

1	2	3	4	5	6
zb	*ba*	*zh*	*ht*	*te*	*ea*

[Followed by their values and the chords of double arcs in seconds. S 15v has similar notes.]

[Notes on II.2]

| “Sectiones enim orbium...” Confinatio est eius quod dixit *h* et *t* S16r
simul provenit ad meridianum [*gd*] in quocumque paralello sic sol est illi paralelli in eadem proportione omnis secentur a meridiano puncta ergo ... simul ad meridianum provenient.

“Secundum ergo quod iam [precessit]...” Hic arguit Ptholomeus in figura sectoris per catam coniunctam, transponit autem proportiones secundum ordinem cate coniuncte. Deberet istorum ordo proportio *ea* ad *ta* aggregatur ex *eb* ad *hb* et *zh* ad *zt*. Ipse transponit sicut patet in littera, et hoc facit ideo ut in istis proportionibus *be* sit sextum et ultimum. Sit ergo si ordinem suum quem ponit *at* est primum, *ae* secundum, *tz* tertium, *hz*

196 proportionalibus] proportionibus S **223** gd] *qd* S quartum, *hb* quintum, *be* sextum. Primum itaque scilicet *ae* notum est per inventionem, invenit enim Ptholomeus per instrumenta quod arcus minimi diei in Rodo tantus est(?). Item *ae* secundum est notum quia quarta, similiter tertium scilicet *tz* est notum cum enim quarta coluri solsticiorum. Item quartum scilicet *hz* est notum cum omni *h* hic signat initium capricorni eo quod Ptholomeus docet hic invenire amplitidinem orientalem respectu minimi diei. Cum itaque secundum hoc *ht* sit maxima declinatio, residuum scilicet *hz* erit notum per subtractionem maxime declinationis a quarta. Hiis ergo proportionalibus notis duc primum in quartum, divide per tertium et exibit quiddam quod sic se habet ad secundum sicut quintum ad sextum. Ergo et econverso secundum habet se ad illud quiddam sicut sextum ad quintum. Hoc facto statue secundum pro primo, et illud inventum pro secundo, sextum pro tertio, quintum pro quarto. Et etiam ducas secundum in tertium [et divide per primum], et exibit quartum scilicet *hb*. Illo autem noto et subtracto a quarta, remanet *he* quod est propositum.

Nota quod quandocumque Ptholomeus investigat aliquas cordas per catas, si corde exteriores sint in sectione, et arguit per disiuncta, sed autem investigare volueris interiores in sectione figure, arguit per coniunctam, cuius ratio est quia cata coniuncta arguit per interiores et disiuncta per exteriores sicut patet inspicienti demonstrationes catharum superius positas. Item nota quod gradus equinoctialis vocat tempora quia ascensiones signorum mensurantur per ascensiones graduum equinoctialis.

[Note on II.7]

| “Quapropter cum nos proiecerimus...” Intentio Ptholomei est docere S 23r
invenire ascensiones signorum in sphera obliqua. Sit ergo secundum formam sue argumentatationis *ht* primum, *hz* secundum, *lk* tertium, *kz* quartum, *et* quintum, scilicet arcus excessus maximi diei super equalem, *el* differentia elevationis alicuius decane in sphera obliqua supra spheram rectam. Hoc inquam sit sextum. Duc primum scilicet *ht* in quartum scilicet *kz*, divide per tertium scilicet *lk*, exibit quiddam quod sic se habet ad *zh* quod est secundum sicut *et* quintum ad *el* sextum. Hoc facto statue quattuor proportionalia scilicet primum illud quiddam quesitum, *zh* secundum, *et* tertium, *el* quartum. In hiis quattuor proportionibus duo sunt nota scilicet illud quiddam quesitum, insuper secundum semper est notum scilicet *hz*. Tertium autem scilicet *et* quod est duplum excessus maximi diei in aliqua sphera obliqua super equalem, illum excessum in generali ratio

246 ducas] primum *add.* S

contentum(?) ad hanc spheram vel illam, pone 60 et tunc si pro prima decana operaberis et ducendo secundum in tertium, dividendo per primum, exibit duplum *el* 9 vel 18 partes, et(?) sicut parat in libra et hoc est duplum superflui elevationis decane prime in sphera recta super obliquam. [The remainder of this notes describes how to find the value of arc *el* for different decans at different latitudes.]

Notes in Città del Vaticano, Biblioteca Apostolica Vaticana, Barb. lat. 336 (MS N) and Paris, Bibliothèque nationale de France, lat. 7256 (MS X)

[There are a multitude of notes in the margins of MSS N and X and I have not attempted to include all the marginalia. The marginal notations are in different hands than the main text of the *Almagest.*]

| "Summa vero eius [quod narrando] etc." Hic primo ponit prohemium ad totum quod antecedit partem illam "Post hec que prediximus vere oportet ut ex summa etc.' Deinde exequitur de propositis, ibi "Primum quod intellexerunt [antiqui] etc." N 3r, X 2r

In hoc tertio capitulo ostendit quod motus celi sic sphericus et quod etiam ipsum celum sit sphericum. Et primo ponit intentionem, postea prosequitur, ibi "Premittam autem [paucos sermones]." Et quando prosequitur, primo ostendit hoc per rationes mathematicas, secundo per ratione naturales, ibi "Nunc quoque ergo dicemus." Prima adhuc in duos quia primo hic ostendit per rationes sumptas a considerationibus rerum absolute, secundo per rationem sumptam a considerationibus rerum in comparatione ad instrumenta, ibi "Demonstrat [etiam esse affirmandum] etc." Prima adhuc in duos quia primo hoc ostendit per rationes ostensivas, postea per rationes ducentes ad impossibile, ibi "Post hec vero [reliqua indicantia]." Prima adhuc in duos quia primo ostendit per considerationes motus solis et lune et aliarum stellarum que oriuntur et occidunt, secundo per considerationes earum que numquam oriuntur neque occidunt, ibi "Plurimum vero [quod perduxit]." Campanus.

[*There are several other Campanus notes dividing the chapters and portions of the text as this one does, but I omit most of them because they have little bearing on the mathematics of the text.*]

"Cum mensuratione alternata..." Non quod eadem stella hanc moram super terram et sub terra cum mensuratione alternata nisi ipsa fuerit in equatore sed quod quarumlibet duarum stellarum equatori utrinque equidistantes habent moras suas sub terra et supra terram alternatas cum equalitate mensurationis idest mora unius sub terra adequatur more alterius supra terram et econverso. Campanus.

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9 eius] *om.* N **26** Campanus] *om.* N **33** moram] motum N

“Necessario igitur oportet ut [punctum illud] etc.” Quoniam omnis punctus signatus in sphera et ad sphere motum motus movetur supra polum sphere, stella autem fixa est huius in octava sphera et ad eius motum mota, “Necessario igitur oportet...” Campanus.

| Quanto aliqua stella plus appropinquat polo, movetur in circulo N 3v, X 2v breviori, et quanto plus elongatur, movetur in circulo maiori. Et crescit ista maioratio circulorum donec proveniatur ad eas que occultantur. Campanus.

“Ut quecumque [earum singulis diebus] etc.” Quecumque stella oritur ab ortu cuiusque. Hec distributio est sic intelligenda quod unaqueque oritur ab ortu suo. Hec potest valere ad illud sophisma “Omnis homo est omnis homo.” Campanus.

| “Demonstrat etiam esse affirmandum etc.” Convenientia N 4r considerationum non probat celem esse sphericum sed motum eius esse sphericum. Posito enim celo quadrato vel cuiuslibet alterius forme et quod moveatur motu spherico adhuc remanebit convenientia considerationum. Campanus.

[In X only] Si enim stella a moveatur motu quadrato, quia in una et eadem revolutione erit in angulo et in latere apparebit in una et eadem revolutione diversarum magnitudinum. Campanus.

| 1. Data circuli dyametro ex ipsa latus exagoni, decagoni, N 8v, X 5r pentagoni, quadrati, trigoni equilaterorum circulo inscriptorum elicere.

| 2. Data corda arcus semicirculo minoris eam que subtenditur N 9v residuo semicirculo invenire.

3. Omnis quadrilateri circulo inscripti quod sub duabus eius dyametris continetur, equum est aggregato duarum superficierum a duobus lateribus oppositis contentarum.

| 4. Cognitis duabus cordis duorum arcuum inequalium in X 5v semicirculo, cordam superflui inter eas invenire.

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39 ut…etc] *om.* X **42** oportet] etc. *add.* X **49** Ut…etc] *om.* X

| 5. Data corda alicuius arcus semicirculo minoris noti, cordam medietatis illius invenire. N 10r

Hic est alius modus inveniendi cordam medietatis arcus noti cuius corda est nota quem ponit Geber. [Figura 1] Et est ut sit arcus *ab* notus cordam habens notam, et querimus inventionem corde medietatis arcus *ab* qui sit arcus *ag*. Et ad hoc producam a puncto *g* lineam in centrum circuli quod sit *e*, que dividat cordam *ab* in puncto *d*. Manifestum est igitur quod *ab* divisa est per equalia et orthogonaliter. Et quia tota *ab* est nota, erit eius medietas que est *ad* nota. Et cum eius quadratum minuitur ex quadrato *ae* noto, remanet quadratum *ed* notum et linea *ed* nota. Que si minuatur ex *eg* nota, remanebit *gd* nota. Quia ergo quadratum *ag* equatur duobus quadratis duarum linearum *ad* et *dg* quarum utraque est nota, erit linea *ag* nota, quod est propositum. Et hic modus est verior alio et videtur Ptolomeus fuisse operatus secundum hunc modum eo quod ponit cordam partis et medietatis partem unam et 34 minuta et 15 secunda, cum tamen secundum modum quem ponit non inveniatur nisi 14 secunda. Et similiter quia ponit cordam trium quartarum 47 minuta et 8 secunda cum tamen secundum modum quem ponit non inveniantur nisi 7 secunda, at secundum istum modum inveniuntur in prima 15 secunda et in alia 8.

4. Si in semicirculo corde arcuum inequalium note fuerint, corda quoque arcus quo maior superat minorem erit nota.

5. Si in semicirculo alicuius arcus corda nota fuerit, corda quoque que eius medietati subtenditur erit nota.

6. Si in semicirculo due corde duorum arcuum fuerint note, corda quoque que arcui ex ambobus composito subtenditur erit nota.

| 6. Datis duabus cordis conterminalibus duorum notorum arcuum semicirculo minorum quorum unus non sit pars alterius, cordam arcus ex eis compositi invenire. N 10v

Hic est alius modus compositionis quem ponit Geber. [Figura 2] Et est ut in circulo *abg* descripto supra centrum *d*, sint duo arcus *ab bg* noti cordas habentes notas, et inquiramus cordam totius arcus *ag*. Ad hoc a puncto *b* producam dyametrum *bde*, et protraham cordas *ae ge*. Quia igitur corda *ab* est nota, erit *ae* que subtenditur residuo semicirculi nota. Et similiter erit nota *ge* propter hoc quod posuimus notam *bg*. Et quia

81 arcus] *om.* N **88** gd] *dg* X **98** 4] 5 N **99** superat minorem] *inv.* X **100** 5] *om.* N **102** 6] *om.* N **105** notorum] *om.* N **109** est] etiam *add.* X **114** hoc] id X quadrilaterum *abge* est inscriptum circulo, erit quod fit ex *ab* in *ge* cum eo quod fit ex *bg* in *ae* quorum utrumque notum propter id quod 4 latera quadrilateri sunt nota, equum ei quod fit ex *be* in *ag*. Si igitur ipsum aggregatum dividatur per *be* notam, exibit *ag* nota, quod est propositum. Et hic modus est simplicior et facilior eo quem ponit Ptolomeus.

| 7. Si duobus arcubus inequalibus due corde subtendantur, minor N 11r, X 6r erit proportio longioris earum ad breviorem quam arcus longioris corde ad arcum brevioris.

"Et erit proportio dupli *az*." Cum fuerint 6 quantitates quarum sit prima ad secundum ut quarta ad quintam et secunda ad tertiam minor quam quinta ad sextam, erit prima ad tertiam minor quam quarta ad sextam. Sint 6 quantitates *ga az ae* et *gda adz eda*. Sitque *ga* ad *az* ut *gda* ad *adz*, et *az* ad *ae* minor quam *adz* ad *eda*. Erit ergo minor *ga* ad *ae* quam *gda* ad *eda*. Ipsa enim componitur ex duabus quarum altera equalis et altera minor. Aliter sit *az* ad *p* ut *adz* ad *eda*. Erit ergo minor *az* ad *ae* quam ad *p*. Ergo *p* minor *ae*, et erit *ga* ad *ae* minor quam ad *p*. Et quia *ga* ad *p* ut *gda* ad *eda*, erit *ga* ad *ae* minor quam *gda* ad *eda*, quod est propositum. Per quod patet quod dicit "Et cum diviserimus etc." Hic patet per id quod expositum est supra ubi "Cum ergo composuerimus."

| Sit circulus *abg* in quo sumantur duo arcus videlicet *ab* et *ag*. N 12r, X 6v [Figura 3] Sitque *ab* arcus trium medietatum et *ag* trium partium. Quia ergo omnes corde arcuum superfluentium tribus medietatibus note sunt, erunt due corde *ab* et *ag* note veraciter. Dividatur itaque arcus *bg* qui est trium medietatum in illas tres medietates, sintque *bd de eg*. Et protrahantur corde *ad* et *ae* quarum corda *ad* erit duarum partium et corda *ae* duarum partium et semis. Et iste sunt due que remanent ignote inter duas notas que sunt *ab* et *ag*. Istarum igitur duarum perscrutabitur inventio per cordam medietatis partis et duas circumstantes notas, prime earum per compositionem et secunde per superfluum augmenti. Subtendam enim arcui *bd* et arcui *eg* duas cordas quarum utraque erit corda medietatis partis et sit nota. Quia igitur corda *ab* est nota et *bd* etiam nota, erit per capitulum compositionis corda *ad* nota. Per capitulum vero superflui invenietur *ae*. Cum enim sit corda *ag* nota itemque corda *ge*, erit corda *ae* que est corda superflui nota. Sicque invenientur omnes

121 inequalibus] semicirculoque minoribus *add. sed. exp.* X **125** az] etc. *add.* X **132** et] quia *add. sed. exp*. N **135** ubi] dicitur *add.* X **138** Sitque] arcus *add.* N **140** veraciter] *om.* N due inter alias duas, prima per compositionem semis unius partis cum precedente duarum nota et secunda per superfluum sequentis note super cordam medietatis partis. Et est notandum quod licet corda *ae* posset haberi per compositionem medietatis partis et duarum partium idest corde *de* et *ad*, non tamen est sic inquirenda. Neutra enim cordarum *ad* et *de* est veraciter nota sed altera duarum *ag* et *ge* est veraciter nota videlicet *ag*, et ideo per illam est investiganda. Nulla enim corda est investiganda est per duas nisi altera earum vel ambe fuerit nota veraciter. Et hoc est quod dicit in fine precedentis capitulis, et "per hoc complebitur residuum reliquarum cordarum quas prediximus que sunt inter duas cordas notas." Campanus.

| 8. Maximam declinationem per instrumenti artificium et considerationem reperire. N 15r, X 9r

"Preparatio [vero primum horum duorum modorum] etc." Hii duo modi sunt erigere armillam orthogonaliter supra superficiem orizontis et facere eam equidistare orbi meridiei. Campanus.

Campanus. Omnis arcus cui aliquis angulus in circumferentia subtenditur omni arcui cui equalis angulus in centro subtenditur proportionaliter duplus existit. [Figura 4] Sit circulus *abg* supra centrum *d* et sit eius arcus *bg* super angulum *bag* qui est in circumferentia eius. Et fiat circulus alius *ehz* supra centrum *a* ut angulus *bag* sit in centro eius. Et producantur linee *ab ag* usque ad eius circumferentiam et occurrant circumferentie super duo puncta *e h*. Dico ergo quod arcus *bg* qui subtenditur angulo *bag* existenti in eius circumferentia est proportionaliter duplus ad arcum *eh* qui subtenditur eidem angulo in centro suo. Quod sic probatur. A centro circuli *abg* quod est punctum *d*, producam duas lineas *db dg*. Erit ergo angulus *bdg* duplus ad angulum *bag*. Eo igitur diviso in duo media per lineam *dm*, erit angulus *bdm* equalis angulo *bag*. Quare arcus *eh* erit similis arcui *bm.* Sed arcus *bg* est duplus ad arcum *bm*, ergo arcus *bg* est proportionaliter duplus ad arcum *eh* quod voluimus demonstrare. Campanus.

| "Et ponam medium extremitatis etc." Paxillus inferior non ponitur ibi nisi ut per ipsum et superiorem possit erigi orthogonaliter superficies N 15v, X 9r

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159 nota veraciter] *inv.* X **161** duas] *om.* X | notas] etc. *add.* X **174** bg] supra N **175** eius] *om.* N **182** bag] *a. m.* X | Eo] *bdt a.m. supr. lin.* X | diviso] scilicet *bdg add.* N **183** bag] *a. m.* X **187** Et…etc] *om.* X illa supra superficiem orizontis. Superior autem ponitur ibi propter hoc quod et propter hoc [sic] quod umbra in circumferentia quarte circuli ostendet altitudinem solis in meridie. Campanus.

Campanus. Omnis trianguli orthogonii notorum laterum reliqui anguli eius sunt noti. [Figura 5] Sint latera trianguli *abg* orthogonii nota et angulus *a* sit rectus. Dico ergo quod angulus *abg* et angulus *bga* sunt noti. Circumscripto enim semicirculo circa eum, patebit propositum quia enim latera eius sunt nota. Ergo proportio *bg* ad *ga* est nota. Ergo *ga* est nota prout *bg* est 120 partes. Similiter autem et *ab* est nota, prout *bg* est 120 partes cum sit earum proportio nota. Quare corde *ga* et *ab* sunt ambe note. Ergo arcus *ag* et *ab* sunt noti. Ergo duo anguli *abg* et *bga* sunt noti prout duo recti subtenduntur toti circulo, ergo prout 4 recti subtenduntur toti circulo. Rectus enim in circumferentia continet semicirculum, in centro autem quartam. Quare medietas anguli in circumferentia erit angulus in centro. Campanus.

| [“Per has ergo considerationes...”] Hic narrat quantitatem maxime N 16r
declinationis secundum quod eam per predicta instrumenti invenit, et dividitur in duas quia primo narrat quantitatem illius declinationis, secundo docet per illam considerationem invenire latitudinem regionis, ibi “hiis autem considerationibus.” Prima adhuc in duas quia primo eam narrat, secundo eam probat per considerationem alterius, ibi “et hec quidem etc.”

| "Et quoniam sequitur etc.” In hac parte comperta maxima X 10r
declinatione per considerationem, docet invenire via demonstrativa declinationem cuiusque puncti orbis signorum ab equatore, et dividitur in duas partes. In prima ponit quedam antecedentia ad inventionem illarum declinationum. In secunda ex illis antecedentibus demonstrat propositum, ibi "et postquam premisimus hoc capitulum."

Prima [dividitur] in duas: in prohemium et tractatum, secunda ibi "describam ergo." Hec secunda [dividitur] in duas. In prima ponit quedam antecedentia communia sive remota sive mediata. In secunda ponit alia propria sive propinqua sive immediata que demonstrantur ex prioribus, ibi "et postquam premisimus hec antecedentia." Prima [dividitur] in duas. In prima ponit quedam antecedentia generalia, in secunda quedam magis specialia, ibi "describam etiam." Prima in duas. In

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196 et[2]] *supr. lin*. N **215** In] post hoc *add.* X | hac] *om.* N **221** in[1]…222 dividitur] *mg*. X prima ponit unum antecedens penes coniunctam proportionalitatem. In secunda aliud penes disiunctam, ibi "similiter declarabitur." Pars illa "describam etiam" in qua ponit quedam antecedentia magis specialia similiter in duas. In prima ponit unum antecedens penes proportionalitatem disiunctam, in secunda aliud penes coniunctam, ibi "describam etiam." Utraque istarum in duas quia in prima utriusque demonstrat veritatem proportionis a parte rei, in secunda noticiam extremorum a parte nostra. Secunda prime ibi "hoc autem superest," secunda secunde ibi "sequitur vero hoc." Pars illa "et postquam premisimus hec antecedentia" in qua ponit antecedentia propria et immediata, dividitur in duas. In prima ponit unum antecedens penes disiunctam proportionalitatem. In secunda unum penes coniunctam, ibi "ex eo autem quod demonstratum est etc.”

Pars illa "et postquam premisimus hoc capitulum" in qua ex antecedentibus premissis demonstrat propositum dividitur in prohemium et tractatum, secunda ibi "describam ergo." Et hec secunda in duas. In prima demonstrat quantitatem declinationum particularium. In secunda tangit bene modum compositionis tabularum, ibi "et similiter referemus." Prima in duas secundum duas declarationes quas exempli gratia demonstrat. Et est prima declaratio ultimi puncti arietis, secunda ultimi tauri. Secunda istarum partium incipit ibi "ponam etiam" etc.

| 9. Duabus lineis ab angulo uno descendentibus si ab earum X 9v
terminis due linee inter eas sese secantes super eas reflectantur, erit utriuslibet descendentium ad eam sui partem que est inter punctum reflexionis et angulum proportio ex duplici proportione composita, ex ea videlicet que est sue conterminalis reflexe ad eam sui partem superiorem et ea que est inferioris partis alterius reflexarum ad totam.

"Ponam autem *zd* etc.” Supponit hanc propositionem hic et infra in multis locis que est quasi quedam conceptio. Si inter quaslibet duas quantitates eiusdem generis, alia quantitas quantalibet eiusdem generis ponatur media, erit proportio prime ad ultimam composita ex proportione prime ad secundam et secunde ad tertiam. [Figura 6] Sint due quantitates eiusdem generis *a* et *b* inter quas ponatur *c* eiusdem generis. Dico quod proportio *a* ad *b* componitur ex proportione *a* ad *c* et *c* ad *b* . Quod sic probatur. Sit *d* denominatio proportionis *a* ad *c*, et *e* denominatio

230 magis] *om.* X **236** Pars] *illeg.* N **239** secunda] proportionem N | penes] *om.* N coniunctam] unam *add.* N **244** declinationum] ostendit N **247** declaratio] *om.* N **259** generis[1]] alia quantitas *add. sed. del*. X proportionis *c* ad *b*, et *f a* ad [*b*]. Quia ergo ex *b* in *f* fit *a*, et ex *b* in *e* fit *c*, erit *a* ad *c* ut *f* ad *e*. Igitur cum *d* sit denominatio *a* ad *c*, erit denominatio *f* ad *e*. Quare ex *e* in *d* fit *f*, quod est propositum. Campanus.

| 10. Duabus lineis ab angulo uno descendentibus si ab earum N 16v
terminis due linee inter eas sese secantes super eas reflectantur, erit proportio partium utriuslibet descendentium composita ex proportione partium sue conterminalis reflexe et ea que est inter totam reliquam descendentem ab angulo et sui partem inferiorem sumptis eodem ordine harum trium proportionum extremis.

10. Si duobus arcubus quorum quisque sit semicirculo minor coniunctim corda subtendatur et a communi eorum puncto per centrum dyameter protrahatur, dividet cordam subtensam composito secundum proportionem cordarum arcuum duplicantium arcus propositos. Hec propositio patet per probationem quam ponit sive compositus ex illis duobus arcubus sit semicirculus sive eo maior sive eo minor. Dicit autem quod uterque sit semicirculo minor si enim uterque esset semicirculo equalis coniuncti perficerent circulum cui nulla corda subtenditur; quod si alter esset semicirculus et alter eo minor, dyameter a communi eorum puncto protracta cordam subtensam composito non secaret; quod si uterque esset semicirculo maior, cum quilibet eorum duplicaretur excederet circulum. Idem esset si unus maior semicirculo et alter minor vel semicirculus. Campanus.

"Hoc autem superest etc.” [Figura 4 of *Almagest* diagrams] Dato arcu *ag* et proportione corde dupli arcus *ab* ad cordam dupli arcus *bg*, erit uterque arcuum *ab bg* datus. Eo enim quod datus est arcus *ag*, data est corda *ag*. Eo vero quod data est proportio corde dupli arcus *ab* ad cordam dupli arcus *bg*, data est proportio *ae* ad *eg*. Et quia totius *ag* note data est proportio partium que sunt *ae* et *eg*, erit utraque earum data. Quare *ze* erit data que est superfluum *ae* super dimidium *ag* quod est *az*. Et quia *az* est nota et *ad* cum sit semidiameter nota et angulus *azd* est rectus, erit *zd* nota. Subtracto enim quadrato *az* note de quadrato *ad* note, remanebit quadratum *dz* quod, quia cum quadrato *ez* note equatur quadrato *de*, erit *de* nota. Latera ergo trianguli *dez* orthogonii sunt omnia nota. Ergo nota est proportio linee *de* ad lineam *ez*. Ergo *ze* est nota de partibus de quibus *de* est 120. Ergo est nota prout est corda arcus *ze* circuli descripti circa

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265 ergo] *d* N *om.* X **276** duobus] *om.* N **277** subtendatur] duos arcubus dico *adnot. supr. lin.* X **291** arcus[2]] *iter.* N **299** cum] quadratum *add. supr. lin.* X triangulum *dez* orthogonium. Ergo suus arcus erit notus cui subtenditur in circumferentia angulus *edz*. Ergo angulus *edz* est notus in circumferentia. Et quia eius medietas erit ipse idem angulus super centrum, ipse est notus in centro. Sed etiam angulus *adz* est notus in centro quia subtenditur medietati arcus *ag* noti. Quare totus angulus *eda* est notus super centrum. Ergo arcus *ab* erit notus cui ipse subtenditur in centro. Similiter quoque et arcus *bg* qui est residuum arcus *ag* noti. Campanus.

11. Si aliquis arcus notus dividatur in duos arcus quorum quisque sit semicirculo minor fueritque proportio corde dupli unius ad cordam dupli alterius nota, uterque eorum erit notus.

| "Et manifestum est" etc. Supponit hanc propositionem. Si aliquod totum notum dividatur in partes quarum proportio sit nota, erit utraque partium nota. [Figura 7] Sit totum *ag* notum divisum in partes *ae* et *eg* quarum sit proportio nota. Dico quod he partes erunt note. Quoniam cum proportio *ae* ad *eg* sit nota, sumam duos numeros in proportione illa et convenientius est ut minimos qui sint *h k*. Quia ergo *ae* ad *eg* ut *h* ad *k* erit coniunctim *ag* ad *eg* ut *hk* ad *k*. Quia igitur *ag* notum ad *eg* ut *hk* notum ad *k* notum, erit *eg* notum, quare et *ae*, quod est propositum. Campanus. N 17r

12. Si a termino alicuius arcus semicirculo minoris linea aliqua abscindens ex eo arcum aliquem minorem residuo semicirculi, protrahatur donec cum dyametro per reliquum terminum arcus protracta concurrat, erit proportio totius linee arcum secantis ad sui partem extrinsecam sicut corde dupli totius arcus predicti ad cordam dupli arcus abscisi.

13. Si alicuius arcus predicto modo divisi, cuius totius corde duplicati ad cordam dupli partis abscise sit data proportio, fuerit nota pars cuius intrinseca pars secantis linee tamquam corda subtenditur arcus, etiam reliquus qui abscisus est notus erit.

| "Et quia proportio" etc. [Figura 8] Sit enim numerus *hk* [i.e. *h* et *k*] ad *k* ut *ge* ad *eb* sive minimi sive non. Ergo disiunctim *gb* ad *be* ut *h* ad *k*. Quia ergo *gb* et *h* et *k* sunt nota, erit *eb* notum quod est propositum. Campanus. X 10r

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333 duplicati] ad sui partem abscisam sit de *add. sed. del.* X **338** ad[4]] erit X

14. Si in superficie spherica duo arcus ex orbibus magnis quorum quisque sit semicirculo minor ab uno communi descendant termino et ab eorum extremitatibus alii duo arcus similium orbium sese invicem intra eos secantes super eos reflectantur, erit proportio cordarum arcuum duplicantium partes utriuslibet descendentis producta ex proportione cordarum arcuum duplicantium partes sui conterminalis reflexi et ex proportione que est inter cordas arcuum duplicantium totum reliquum descendentem et eius inferiorem partem, sumptis eodem ordine harum trium proportionum extremis.

| "Et producam *hb* etc." [Figure 7 of *Almagest* diagrams] Linea *hb* N 17v
potest esse equidistans linee *ad* et potest etiam cum *ea* concurrere, quod si concurrant, possunt concurrere ex parte puncti *b* vel ex parte puncti *g*. Ptholomeus vero supponit hic quod concurrant et quod hic sit ex parte puncti *b*, et secundum hoc probat quod proportio corde dupli *ge* ad cordam dupli *ea* componitur ex proportione corde dupli *gz* ad cordam dupli *zd* et ex proportione corde dupli *bd* ad cordam dupli *ba*. Coniunctam autem non probat, que est ut proportio corde dupli *ga* ad cordam dupli *ea* componitur ex duplice proportione scilicet ex proportione corde dupli *gd* ad cordam dupli *dz* et ex proportione corde dupli *bz* ad cordam dupli *be*. Sed eius probationem latenter innuit ibi "ex eo autem quod demonstratum est etc." Nec etiam probat disiunctam aut coniunctam quando linee *bh* et *da* sunt equidistantes, nec etiam quando concurrunt ex parte puncti *g*. Et ideo Thebit fecit tractatum unum qui intitulatur Thebit de figura sectore in quo hec omnia probat. Ego etiam feci tractatum alterum de eodem planiorem ut puto et manifestiorem. Campanus.

| 15. Existentibus 4 arcubus in superficie spherica secundum N 17r
dispositionem premissam, producetur proportio corde duplicantis alterutrum descendentium ad cordam duplicantis eius superiorem partem ex proportione corde duplicantis totum suum conterminalem reflexum ad cordam duplicantis eiusdem reflexi partem superiorem et ex proportione corde duplicantis alterius reflexi partem inferiorem ad cordam que duplicat ipsum totum.

| 16. Dato puncto orbis signorum declinationem eius ab equinoctiali N 17v
circulo invenire. Unde manifestum est quod si sinus arcus orbis signorum qui intercipitur inter equatorem et punctum datum ducatur in sinum

358 corde] *iter.* X **360** scilicet] duplici X **364** bh] quoniam N | equidistantes] *de* N maxime declinationis, productum dividatur per sinum quarte, exibit sinus declinationis puncti dati.

[N has in the diagram for this: “Inquirit per katam coniunctam arcum *th* qui est declinatio arcus *eh*. Descendit autem kata ab *a* in *z e*.”]

| Proportionem produci ex proportionibus est denominationem N 18r, X 10v produci ex denominationibus. Proportionem componi vel aggregari ex proportionibus est ipsam produci ex componentibus. Proportionem dividi per proportionem est denominationem dividi per denominationem. Proportionem abici ex proportione est proportionem dividi per proportionem. Proportio aggregata ex quotlibet proportionibus est inter productum ex antecedentibus omnium ad productum ex consequentibus. Verbi gratia. Primo de composita ex duabus, sit proportio *a* ad *b* composita ex proportione *c* ad *d* et *e* ad *f*. Ducaturque *c* in *e* et fiat *g*, et *d* in *f* et fiat *h*. Dico quod proportio *a* ad *b* est inter *g* et *h*. Ducatur enim *d* in *e* et fiat *l* . Quia igitur ex *c* et *d* in *e* fiunt *g* et *l*, erit *g* ad *l* ut *c* ad *d*. Et quia item ex *d* in *e* et *f* fiunt *l* et *h*, erit *l* ad *h* ut *e* ad *f*. Quare *g* ad *h* componitur ex *c* ad *d* et *e* ad *f*, sed etiam *a* ad *b* ut propositum est. Ergo *a* ad *b* ut *g* ad *h*, quod est propositum. Quod si *a* ad *b* componatur ex pluribus quam ex duabus ut ex predictis et ea que est *m* ad *n*, remanentibus ceteris in habitudine prima, multiplicetur *c* in *e* et productum in *m*, et fiat *p*, itemque *d* in *f* et productum in *n* et fiat *q*. Eritque *p* productum ex *g* in *m*, et *q* ex *h* in *n*. Producebatur enim *g* ex *c* in *e* et *h* ex *d* in *f*. Et quia *a* ad *b* componitur ex *c* ad *d* et *e* ad *f* et *m* ad *n*, ac *g* ad *h* ex *c* ad *d* et *e* ad *f*, componetur *a* ad *b* ex *g* ad *h* et *m* ad *n*. Et quia *p* producitur ex *g* in *m* et *q* ex *h* in *n*, patet de tribus per duas quod *a* ad *b* ut *p* ad *q* sicque patebit de 4 per 3, et deinceps quod est propositum. Campanus." [The diagrams for this proof and the following notes consist of lines labeled with each of the letters used in the proof.]

Nota quod quandocumque investigatur aliquid alii duo [qui] se intersecant, procedendum est per katam disiunctam, cum autem aliquid ex hiis qui se intersecant inter eos, per coniunctam.

| Proportio que remanet abiectis quotlibet proportionibus ex una est N 18v inter productum ex antecedente eius a qua debent abici in consequentia omnium abiciendarum et productum inter consequens eius a qua debent abici et antecedentia omnium abiciendarum. Verbi gratia. Primo cum

388 produci] *iter*. X **397** g] *om.* X **398** c] componetur X **416** debent] *om.* N abicitur una ex una, ex proportione *a* ad *b* sit abicienda ea que est *c* ad *d*. Ducatur ergo *a* in *d* et proveniat *e*, et *b* in *c* et proveniat *f*. Dico quod *e* ad *f* est residua. Ducatur enim *b* in *d* et proveniant *g*. Quia ergo ex *a* et *b* in *d* proveniunt *e* et *g*, erit *e* ad *g* ut *a* ad *b*. Et quia ex *b* in *c* et in *d* fiunt *f* et *g*, erit *f* ad *g* ut *c* ad *d* . Quare *e* ad *g* constabit ex *e* ad *f* et *c* ad *d*, igitur et *a* ad *b* que est eadem illi. Patet igitur *e* ad *f* esse residuam. Quod si plures fuerint abiciende ex una ut *c* ad *d* et *m* ad *n* de *a* ad *b*, aggregentur abiciende inter *p* et *q* que sint producta ex *c* in *m* et *d* in *n*. Quia ergo inter producta ex *a* in *q* et *b* in *p* est proportio residua, at *a* in *q* quantum *a* in *d* et producti in *n* sicque *b* in *p* quantum in *c* et producti in *m*, patet propositum. Campanus.

Si qua proportio nota componatur ex duabus quarum una nota, erit relique nota. Sit *a* ad *b* nota composita ex *c* ad *d* nota et *e* ad *f* ignota. Ducaturque *a* in *d* et fiat *g*, et *b* in *c* et fiat *h*. Eritque proportio *g* ad *h* nota. Et quia ipsa est ut *e* ad *f*, cum ex ipsa et *c* ad [*d*] componatur eadem que ex *e* ad *f* et [*c*] ad *d*, erit *e* ad *f* nota, quod voluimus. Campanus.

Si quorumlibet duorum quorum proportio nota fuerit unum notum, reliquum erit notum. Sit inter *a* et *b* proportio nota sitque *a* notum. Et sint minimi numeri in proportione *a* ad *b* *c* et *d*. Quia ergo *a* ad *b* ut *c* ad *d*, quod fit ex *a* in *d* quorum utrumque notum equatur ei quod ex *c* in *b* quorum *c* notum. Eo ergo diviso per *c* exibit *b*. Campanus.

Cum aliqua proportio nota componitur ex una nota et alia ignota cuius ignote unum extremum fuerit notum, eam ignotam et eius extremum ignotum elicere. Sit proportio *a* ad *b* composita ex *c* ad *d* et *e* ad *f*. Sintque due prime et quantitas *e* note. Et sit propositum invenire proportionem *e* ad *f* et quantitatem *f*. Hoc autem fiet duobus modis. Productis enim *g* et *h* ex *a* in *d* et *b* in *c*, erit *g* ad *h* ut *e* ad *f*. Quare multiplicato *e* per *h* et producto diviso per *g*, exit *f*, sicque constat propositum. Alius modus est ut productum ex *b* in *c* quod est *h* dividatur per *a* et exeat *l*. Quia ergo ex *a* primo in *l* quartum fit *h*, itemque ex *b* secundum in *c* tertium fit idem, erit *a* ad *b* ut *c* ad *l*. Quare *c* ad *l* constabit ex *c* ad *d* et *e* ad *f*. At ipsa constat ex *c* ad *d* et *d* ad *l*. Ergo *d* ad *l* ut *e* ad *f*.

419] *supr*. *lin*. X **423** e^{2}] *iter*. N **433** h^{1}] etiam N **439** b^{2}] *om.* N **444** et] scilicet *add. supr. lin*. X | eius] *add. supr. lin*. NX **449** sicque] exibit X **450** ex] ut inveniatur. Illa proportio ignota que est *e* ad *f* inter unum extremitas[sic] duarum primarum qui sunt *a b c d* et aliquem alium quemadmodum si *add.* N

Multiplicato ergo *e* in *l* et producto diviso per *d*, exibit *f*, quod est propositum.

Sed diligenter oportet attendere quod isto secundo modo continget quadrupliciter operari. Primo enim modo ut iam premissum est, ponemus ut proportio *a* primi ad *b* secundum sit sicut *c* tertii ad aliquid aliud et ipsum sit *l*. Patetque quod *d* ad *l* erit ut *e* ad *f*. Secundo autem modo ut proportio *a* primi ad *b* secundum sit sicut alicuius ad *d* quartum et ipsum sit *m*. Quia igitur *m* ad *d* ut *a* ad *b*, constabit *m* ad *d* ex *c* ad *d* et *e* ad *f*. At ipsa constat ex *m* ad *c* et *c* ad *d*. Quare *m* ad *c* ut *e* ad *f*. Tertio autem modo ponemus ut proportio *c* tertii ad *d* quartum sit tamquam *a* primi ad aliquid aliud et ipsum sit *n*. Eritque *n* ad *b* ut *e* ad *f*. Quarto quoque modo ponemus ut proportio *c* tertii ad *d* quartum sit sicut alicuius ad *b* secundum. Sitque ipsum *n*. Eritque *a* ad *n* sicut *e* ad *f*. Sic igitur proportionem quinti *e* ad sextum *f*, possumus primo invenire inter quartum *d* et quoddam aliud *l*. Secundo inter quoddam aliud *m* et tertium *c*. Tertio inter quoddam aliud *n* et secundum *b*. Quarto inter primum *a* et quoddam aliud *n*. Campanus.

Commoditas autem quod accidit ex 30^{a} parte superflui tabule cordarum et arcuum est ista. Quia cum proportio *ed* note ad *zg* notam sit sicut arcus *bd* noti ad arcum *gb* ignotum, multiplicabimus *bd* tertium qui est 30 in *zg* secundum et dividemus per *ed* primum et exibit *bg* quartum. Sed ille idem *bg* exit dividendo *zg* per 30^{am} *ed* que est *eh*. Probatio quoniam ex ductu *bd* que est 30 in *zg* producitur aliquid et ipsum sit *k*, quod si dividatur per *ed* exibit *bg*. Quia igitur ex 30 in *eh* fit *ed* et iterum ex 30 in *zg* fit *k*, erit *eh* ad *zg* sicut *k* ad *ed*. Diviso ergo *k* per *ed* et *zg* per *eh*, idem exibit. Si etiam diviserimus superfluum *ed* super *zg* per *eh*, exiret arcus *gd*, quo detracto de *ad*, remaneret *ag* quoniam cum sit *db* ad *bg* ut *ed* ad *zg*, erit eversim *bd* ad *dg* ut *ed* ad superfluum *ed* super *zg*. Quare 30^{e} *db* ad *dg* ut 30^{e} *ed* ad superfluum *ed* supra *zg*.

| Cum alicuius arcus cuius corda non ponitur in tabulis voluerimus N 19r cordam investigare, sumemus cordam in tabulis quam inveniemus arcus proximo minoris cum 30^{a} parte superflui posita in directo eius. Et illam 30^{am} partem superflui multiplicabimus per differentiam arcuum, et productum adiungemus corde invente. Et aggregatum erit corda arcus propositi. Si autem per cordam que non invenitur in tabulis voluerimus

462 f] *c* N **464** modo] tertio X **469** inter^{2}] *d* N **480** ed] dividerimus X **486** tabulis] ponatur X invenire arcum, sumemus cordam sibi proximo minorem cum 30^{a} parte superflui que est in directo eius, et differentiam earum dividemus per 30^{am} partem superflui, et numerum exeuntem aggregabimus arcui corde suscepto. Et compositum erit arcus corde proposite.

Huius gratia sit circulus *abgd* cuius due corde *ab ad* sint immedietate in tabulis, et sit inter eas corda *ag* cuius arcus sit notus. [Figura 9] Ipsa tamen ignota quam volumus invenire. Quia igitur due corde *ab* et *ad* sunt note, erit excessus *ad* super *ab* et ipsum sit *de* notus. Quare eius 30^{a} que sit *eh* et ipsa erit corda unius minuti in toto arcu *bd* fere. Unde crescente corda *ab* per totum arcum *bd* secundum quantitatem linee *eh*, crescet arcus *ab* uno minuto, et ea crescente duplo *eh*, crescet arcus *ab* duobus minutis. Et sit usquequo ipsa crescente per trigintuplum *eh* quod est *ed* ut ipsa videlicet fiat *ad*; crescat arcus *ab* per 30 minuta, applicata enim corda equali *eh* ad punctum *b*, erit corda ducta ab *a* ad eius terminum equalis ei et *ab* secundum sensum. Unde duo latera trianguli erunt equa tertio. Quare proportio excessus corde *ad* super cordam *ab* ad excessum corde *ag* super cordam *ab* et ipse sit *zg* erit sicut proportio excessus arcus *ad* super arcum *ab* ad excessum arcus *ag* super arcum *ab*. Hoc est dicta quod proportio linee *ed* ad lineam *zg* est sicut arcus *db* ad arcum *bg*. Ergo proportio 30^{e} partis linee *ed* que est *eh* ad lineam *gz* est sicut 30^{e} partis arcus *bd* ad arcum *bg*. Tricesima vero pars arcus *db* est minutum unum, et *eh* est 30^{a} pars superflui duarum cordarum *ab* et *ad*. Ergo que est proportio unius minuti ad minuta arcus *bg* que sunt nota eadem est linee *eh* note ad *zg* ignotam. Multiplicamus ergo *bg* secundum quod est notum in *eh* tertium quod est notum, et dividimus per unum, et exit linea *zg*, que adiuncta ad *az* que est equalis *ab*, perficit *ag*. Et quia ex divisione per unum exit quod dividitur, non expedit nisi multiplicare *bg* in *eh* idest minuta arcus in 30^{am} partem superflui, et aggregare productum corde *ab*.

Convertamus iterum hoc et ponamus cordam *ag* mediam inter *ad* et *ab* immediatas in tabulis notam, et investigemus eius arcum qui est *ag*. Et quia ut prius patuit remanentibus ceteris in habitudine sua, est proportio *eh* ad *zg* sicut unius minuti ad arcum *bg*, si nos multiplicaverimus *zg* per unum minutum quod est secundum et diviserimus per *eh* quod est primum quorum unumquodque notum, exibit arcus *bg*. Comparato ad arcum *ab*, fiet notus arcus *ag*. Et quia ex multiplicatione *zg* per unum minutum non producitur nisi *zg*, dividimus *zg* per *eh*, et exit arcus *bg*.

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499 ab et[1]] *om.* N **502** eh[2]] duobus minutis *add. sed. exp.* N **503** duobus] *om.* N **518** quia] *a* N **527** arcum] quo composito X | ex] at X

Et est caute notandum quod cum proportio minutorum arcus *db* ad minuta arcus *bg* sit sicut linee *ed* ad lineam *zg*, si multiplicaverimus minuta arcus *bg* in lineam *ed*, productum eius generis cuius est *ed*. Minuta enim hoc loco naturam habent integri eo quod proportio *ed* ad *zg* sicut est inter minuta arcus *bd* ad minuta arcus *bg*. Sit inter tota integra quot minuta sunt in arcu *bd* ad tot integra quod minuta sunt in arcu *bg*, quare non salvatur ibi proportio tamquam inter minuta sed tamquam inter integra. Secunda vero, que quandoque erunt ultra minuta in arcu *bg*, ipsa habebunt naturam minutorum quantum ad hanc proportionem. Minutis enim factis integris, secunda sunt minuta eo quod integra minuta secunda, et deinceps sunt continue proportionalia. Campanus.

| 17. Dato arcu orbis signorum arcum equatoris qui cum eo oritur ad N 20r, X
situm sphere recte invenire. Unde manifestum est quod si sinus 11v
complementi maxime declinationis ducatur in sinum declinationis ultimi puncti arcus dati orbis signorum, productumque dividatur per sinum maxime declinationis, itemque quod exierit ducatur in sinum quarte, et productum dividatur per sinum complementi declinationis ultimi puncti arcus dati, quod tunc exibit erit sinus arcus equatoris qui oritur cum arcus orbis signorum qui est inter equatorem et ultimum punctum arcus dati.

"Et ipsa est proportio etc." Ipse inquirit aliquam quantitatem que referatur ad tertium tertii scilicet ad cordam dupli arcus *zh* in proportione qua prima proportio que est corde dupli *zb* ad cordam dupli *ba* superfluit secundam que est corde dupli *zh* ad cordam dupli *ht*. [Figura 10] Et illam sic invenit. Que est proportio secundi ad primum idest corde dupli arcus *ab* ad cordam dupli *bz* eadem est quarti idest corde dupli *th* ad aliquid aliud, et ipsum erit quantitas illa et est 54 partes et 52 minuta et 26 secunda.

Verbi gratia, sint sex quantitates *a b c d e f* que sint omnes note preter *f*. Et componatur proportio *a* ad *b* ex ea que est *c* ad *d* et ea que est *e* ad *f*. Volo inquirere *f*. Dico igitur quod proportio *b* secundi ad *a* primum est sicut *d* quarti ad aliquid aliud et ipsum sit *g*. Quia igitur *b* ad *a* ut *d* ad g, erit econverso *a* ad *b* ut *g* ad *d*. Quare *g* ad *d* constabit ex *c* ad *d* et *e* ad *f*, ex hiis enim constabit ea que est *a* ad *b*. Sed *g* ad *d* constat etiam ex *g* ad *c* et *c* ad *d*. Quare *g* ad *c* ut *e* ad *f*. Que igitur est *g* ad *c* eadem est *e* ad aliquid aliud, quod investigetur et ipsum erit *f*. Sic procedit Ptolomeus in hoc loco. [Diagram consists of lines for each quantity.]

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551 cordam…proportione] *mg.* X

| “Et propter hoc manifestum est quod relique quarte...” Intendit demonstrare quod cognitis elevationibus in una quarta orbis signorum ad spheram rectam, cognite sunt in aliis quartis cuius demonstrationis solum tangit medium, ibi “Ideo.” Hic autem potest patere per demonstrationem huius theorematis: Omnes duo arcus orbis signorum equales et equaliter distantes ab alterutro punctorum equalitatis elevantur ad spheram rectam cum arcubus equatoris equalibus. Hoc autem demonstrabo primo quando duo arcus orbis signorum equales equedistant ab uno puncto equalitatis, [Figura 11] et describam medietatem orbis magni transeuntis per polos equatoris et per puncta equalitatis supra quam sint *a b g*, sintque puncta *a* et *g* poli eqatoris, *b* vero punctum equalitatis vernale. Et describam arcum orbis signorum *hbz*, sitque *hb* equalis *bz*, et arcum equatoris supra quem sint *dbe*. Et protraham a polis duas quartas orbium maiorum supra quas sint *aze ghd*. Erit arcus *be* equatoris qui oritur cum *bz*, et *db* qui oritur cum *bh*. Dico igitur eos esse equales. Cum enim duo trianguli ex arcubus orbium magnorum qui sunt *abz gbh* sint equilateri, erunt equiangulo. Quare angulus *a* equatur angulo *g*. Et duo arcus *ab ae* equantur duobus *gb gd*, ergo *be* basis equatur *bd*, quod est propositum.

N 21r, X 17r

Si autem fuerint duo arcus orbis signorum equales quorum unus tantum distet ab uno puncto equalitatis quantum reliquus ex eadem parte a reliquo, fiat talis figura qualis est secunda supra quam addidimus ad tabulas declinatonis et maneant eedem hypotheses. [Figura 12] Quia ergo duo trianguli ex arcubus orbium maiorum qui sunt *abh ade* sunt equilateri, erunt equianguli. Ergo angulus *a* unius est equalis angulo *a* alterius. Sed duo arcus etiam *ab az* equantur duobus *ad ak* et angulus *a* angulo *a*, ergo basis *bz* equatur basi *dk*, quod est propositum.

Si autem sint duo arcus orbis signorum equales quorum unus tantum distet ab uno puncto equalitatis ex una parte equatoris, quantum reliquus a reliquo ex altera. Protrahatur *dm* arcus orbis signorum sicut in figura superiori. Protrahebat *ah*. Et erit per primum modum huius demonstrationis elevatio arcus *dm* equalis elevationi arcus *de*. Et per secundum elevatio arcus *de* est equalis elevationi arcus *bh*, ergo elevatio arcus *dm* est equalis elevationi arcus *bh.* Quod est propositum. Campanus.

[For Book II, I have only transcribed MS N]

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568 quod…569 demonstrare] *om.* N **578** equalitatis] utrum N **582** equales] eis N **586** fuerint…588 reliquo] *om.* X **588** qualis est] *inv.* X **594** sint duo] *om.* N **597** Et] protrahebatur X | erit] *om.* X **600** arcus^{1}…est^{2}] *iter.* N

| 1. Cognito excessu diei longioris super diem equalem et equalis N 22r super breviorem in quovis climate arcum orizontis interceptum inter ortum equatoris et ortum alterutrius tropicorum invenire. Ex quo patet quod si sinus medietatis arcus minimi ducatur in sinum complementi maxime declinationis, productumque dividatur per sinum quarte, exibit sinus complementi arcus orizontis qui intercipitur inter equatorem et tropicum. Simili quoque ratione sciri potest distantia ortus cuiusque punctorum orbis signorum ab ortu equatoris cognito arcu qui est differentia diei equalis et diei puncti illius. Patetque quod si ducatur sinus medietatis arcus diei puncti illius in sinum complementi declinationis eiusdem, productumque dividatur per sinum quarte, exibit sinus complementi arcus orizontis intercepti inter ortum equatoris et illius puncti.

[In diagram of for Prop. 1: “Inquirit *eh* arcum orizontis interceptum inter ortum capitis arietis et capricorni per conversionem kate coniuncte.” The diagram is repeated “bene et optime facta” and again in it is “Inquirit arcum *eh* per conversionem kate coniuncte descendentis ab *a* in *e* et in *z*.”]

| [2.] Latitudinem poli cuiusque regionis per excessum diei equalis N 22v et brevioris et per arcum orizontis interceptum inter ortum equatoris et tropicos reperire. Unde patet quod si sinus medietatis excessus dierum equalis et minimi ducatur in sinum complementi arcus orizontis intercepti inter ortum equatoris et tropici, productumque dividatur per sinum medietatis arcus diurni, itemque quod exit multiplicetur in sinum quadrantis, et inde productum dividatur per sinum arcus orizontis qui intercipitur inter ortum equatoris et tropici, exit sinus arcus altitudinis poli.

| 3. Data poli altitudine excessum diei equalis super breviorem N 23r invenire. Unde patet quod si sinus latitudinis regionis ducatur in sinum maxime declinationis, productumque dividatur per sinum complementi maxime declinationis, itemque quod exierit ducatur in sinum quarte, et productum dividatur per sinum complementi latitudinis regionis, exibit sinus medietatis excessus dierum equalis et minimi.

4. Data poli altitudine arcum orizontis interceptum inter occasum equatoris et tropici invenire. Unde patet quod si sinum maxime decliantionis ducas in sinum quarte, et productum dividatur per sinum complementi latitudinis regionis, exibit sinus arcus orizontis quesiti.

[Written in the diagram: "Demonstrat quod *ke* est equalis *eh* per 4am primi Milei."]

| 5. Dato puncto orbis signorum arcum orizontis interceptum inter ortum eius et ortu equatoris in regione cuius latitudo sit data investigare. Unde manifestum est quod cognito loco solis scietur differentia diei illius et diei equalis. Patet iterum quod si sinus latitudinis regionis ducatur in sinum declinationis orbis signorum puncti dati, et productum dividatur per sinum complementi declinationis eiusdem, itemque quod exierit ducatur in sinum quarte, et productum dividatur per sinum complementi declinationis eiusdem latitudinis regionis, exibit sinus medietatis excessus dierum equalis et minimi illius. Adhuc quoque manifestum est quod si sinum declinationis puncti eiusdem ducas in sinum quarte et productum dividas per sinum complementi latitudinis regionis, exibit sinus arcus orizontis intercepti inter ortum puncti illius et equatoris. N 23v

"Manifestum est igitur etc." Quodcumque punctum orbis signorum ponatur punctum *h*. Si detur altitudo poli et scientur semper duo arcus [*et*] et *eh,* per tabulam enim declinationis, semper erit notus arcus *ht*, quare et *hz* qui est residuum quarte. Scietur ergo arcus *et* per katam disiunctam et arcus *eh* per katam coniunctam.

6. Quilibet duo paralleli equinoctiali quorum ab ipso vel a duobus tropicis est equalis distantia secant ex orizonte arcus equales a duabus partibus equatoris et sit alternatim nox unius equalis diei alterius.

| Infinitas orizontum rectorum est propter longitudinem sphere, infinitas autem obliquorum propter latitudinem. Orbis equationis intersecat omnes orizontes rectum et obliquos, et ipsi se invicem [intersecant] cum eorum fuerit idem orbis meridiei in eodem puncto. Cum tantum orbis meridiei transeat per polos omnium et ipsi sint de maioribus in sphera transibunt et econverso omnes ipsi per polos eius. Quare in polis eius fiet omnium intersectio. Campanus. N 24r

7. Cognita solis altitudine proportionem instrumentorum ad umbras [solis] invenire. Unde patet et econverso. Unde patet quod si sinum complementi altitudinis solis ducatur in partes instrumenti, productumque dividatur per sinum altitudinis solis, exibunt partes umbre. Itemque econverso patet quod si radix duarum quadratorum instrumenti et umbre

681 Unde[1]] solas N

simul iunctorum ducatur in semidyametrum, productumque dividatur aut per partes instrumenti, exibit sinus altitudinis solis, aut per partes umbre, exibit sinus complementi altitudinis eiusdem.

| 8. Trium proportionum instrumentorum ad umbras que sunt in medietatibus dierum maximi, equalis, et minimi quibusque duabus cognitis, altitudinem poli et maximam solis declinationem inquirere. N 24v

9. Sub linea equinoctiali omnes dies sunt equales noctibus et sibi invicem omnesque stelle ibi oriuntur et occidunt. Umbre quoque facte in medietatibus dierum quandoque declinant ad septentrionalem, quandoque ad meridiem, quandoque vero nusquam.

10. Sub omni linea equidistante sibi fit dies equalis nocti tantum bis in anno et dies estivi prolixiores sunt hyemalibus, noctes autem econtrario. Quedam quoque stelle sunt semper apparentes et quedam semper occulte. Et distantia zenith ab equatore equalis est altitudini poli supra orizontem in orbe meridiei.

11. Sub remotiori linea equidistanti equatori ab equatore, maior est dierum et noctium inequalitas et maior celi pars semper apparens, et maior semper occulta.

12. Sub omni equidistante cuius minor est distantia ab equatore maxima declinatione, umbre facte in medietatibus dierum ad utramque partem declinant et bis in anno declinationem non habent.

13. Sub equidistante cuius ab equatore distantia est equalis maxime declinationi, umbre facte in medietatibus dierum numquam declinanant ad partem meridiem et tantum semel in anno declinationem non habent.

| 14. Sub equedistante quam describat polus zodiaci circa polum equatoris, in aliquo die reflectitur umbra ad omnes partem orizontis et est spacium diei illius 24 horarum sive nocte et ex opposito illius nox 24 horarum sine die, et quanto alicuius linee equidistantis maior est elongatio ab equatore, maius tempus abit sine nocte et ex opposito nox sive die. N 25r

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695 quandoque[1]] meridietatibus N **701** distantia] at vera *add.* N **705** apparens] celi *add.* N

15. Sub polo medietas celi semper est apparens et medietas occulta, et anni spacium dies una cum nocte sua.

| “Gnomones vero etc.” Cum voluerimus scire in qua parte orbis N 26r
signorum sol obumbret supra capita aliquorum quorum nota est distantia zenith ab equinoctiali, intrabimus cum hac distantia que est zenith ab equinoctiali in tabulam secundam tabule declinationis et sumemus quod in directo eius fuerit in tabula prima, et secundum distantiam eius quod repertum fuerit a principio arietis et libre versus tropicum eius estivalem erit obumbratio solis supra capita ipsorum. Eo igitur quod repertum est detracto de 90, residuum ostendet in quanta distantia a tropico estivali ad utramque duarum partium, erit solis obumbratio supra capita eorum. Campanus.

| “O quam bene quidem scietur...” Vult dicere quod ex tabula N 29v
declinationis sciemus cognitis partibus orbis signorum semper apparentibus quanta sit altitudo poli. Et etiam cognita altitudine poli sciemus quot sunt partes signorum semper apparentes circa tropicum estivalem, autem semper occulte circa tropicum hymalem. Verbi gratia, sint partes semper apparentes circa tropicum estivum vel semper occulte circa tropicum hyemalem 15 partes ab utraque parte. 15 itaque subtraham de 90 et cum eo quod remanebit intrabo in tabulam primam declinationis, et sumam declinationem quam in eius directo inveniam. Et ipsa erit distantia linee equidistantis contingentis orizonte ab equatore. Qua subtracta de 90 residuum erit altitudo poli. Et si altitudo poli sit cognita, subtraham ipsam de 90 et remanebit distantia equidistantis que contingit orizontem ab equatore. Cum hac intrabo in tabulam secundam declinationis et sumam quod erit ei oppositum in tabula prima. Et ipsam subtraham de 90. Residuum autem erit quantitas eius quod semper apparet ab utraque parte tropici estivi et eius quod semper occultatur ab utraque parte tropici hyemalis. Et hoc est quod vult dicere “O quam bene” sed littera valde male hoc signat quia mala.

| 16. Quilibet duo arcus orbis signorum equales et equaliter distantes N 30v
a puncto equinoctii elevantur in sphera declivi cum arcubus equatoris equalibus.

17. Quilibet duo arcus orbis signorum equales et equidistanter ab alterutro tropicorum habent suas ascensiones coniunctim ad spheram rectam et obliquum equales. Unde ex hoc et ex premissa patet quod si

note fuerint ascensiones et descensiones unius quarte in sphera obliqua, erint omnium reliquarum.

| 18. Dato arcu orbis signorum eius ascensionem in sphera declivi N 31r regionis cuius latitudo sit nota investigare. Unde patet quod si sinum latitudinis regionis ducatur in sinum declinationis ultimi puncti alicuius dati, productum vero dividatur per sinum complementi declinationis illius, quodque exierit ducatur in sinum quarte, et productum dividatur per sinum complementi latitudinis, exibit sinus differentie ascensionum ad spheram rectam et obliquam arcus orbis signorum inchoati ab equatore usque ad ultimam punctum arcus dati.

[In the diagram is written: “Inquirit arcum *em* per katam disiunctam descendentem a *g* in *k e*.”]

| 19. Differentiam ascensionum in sphera et obliqua eiusdem arcus N 32r orbis signorum per arcum circuli magni a polo venientis distinguere.

| 20. Cuiuslibet portionis orbis signorum elevationem in sphera N 32v declivi via rationis investigare. Unde patet quod si sinum medietatis differentie equalis diei et minimi ducatur in sinum elevationis portionis eiusdem in sphera recta, productumque dividatur per sinum quadratis, exibit sinus differentie sumpte portionis in elevatione sua ad spheram rectam et obliquam.

Sit orbis meridiei circulus *abgd*, [Figura 13] et medietas orizontis declivis orientalis *bed*, medietas quoque equatoris *aeg*, et medietas orbis signorum *meh*. Sitque punctum *e* orbis signorum quod est in orizonte punctum equationis vernale, eritque punctum *m* punctum tropici hyemalis. Sumam autem arcum *eq* orbis signorum cuius investigabo elevationem ad orizontem *bed*. Describam igitur equidistantes equinoctiali qui transeunt per duo puncta *m* et *q*, sintque *mn qp*. Et signabo punctum *z* polum meridianum. Et describam tres quartas orbium maiorum transeuntes per polum *z*, et per tria puncta *p n q* que sint *zpl znk zqt*. Eritque *ek* medietas differentie diei equalis et minimi. At vero *et* erit quod elevatur ad spheram rectam cum arcu *eq* orbis signorum sumpta. Et *el* erit differentia elevationis arcus *eq* ad spheram rectam et ad orizontem declinem *bed*. Hanc autem differentiam inquirimus. Quia igitur in duos arcus orbium maiorum *ke zk* intersecant se duo arcus magnorum orbium [*en*] *zl* supra punctum *p*, erit proportio sinus *zn* ad sinum *kn*, que est nota quoniam *nk* est maxima declinatio, composita ex proportione sinus *zp* ad

800 supra] *zn* N

sinum *pl*, que est etiam nota quoniam *pl* est declinatio puncti *q* noti, et ex proportione sinus *el* ad sinum *ek*. Et quia arcus *ek* est notus quoniam est medietas superflui diei equalis et minimi, erit arcus *el* notus, quod est propositum.

Correlarium autem sic patet. Proportio sinus [*zm*] ad sinum *ma* est sicut sinus *zn* ad sinum *nk*, et proportio sinus *zq* ad sinum *qt* est sicut sinus *zp* ad sinum *pl*. Quia igitur proportio *zn* ad *nk*, intelligantur semper loco arcuum sinus eorum, componatur ex proportione *zp* ad *pl* et ex proportione *el* ad *ek*, componetur proportio *zm* ad *ma* ex proportione *zq* ad *qt* et ex proportione *el* ad *ek*. Sed ipsa etiam proportio *zm* ad *ma* componitur ex *zq* ad *qt* et ex *et* ad *ea*. Ergo proportio *et* ad *ea* sicut *el* ad *ek* sumptis sinubus pro arcubus. Sitque patet correlarium. Nam *et* est elevatio *eq* in sphera recta et *ek* est medietas excessus diei equalis et minimi et *el* est differentia elevationis *eq* in sphera recta et declivi cuius orizon est *bed*.

| "Quapropter [cum nos prohicerimus] etc." Nota quod non dicit "et N 33v
in arcu cuius elongatio est 90 partes" quoniam arcus de quibus loquitur sunt *ek* qui est minor *eh*, et *eh* est 90. Campanus.

"Que est proportio lx partium..." Quoniam arcus *te* variatur in diversis regionibus et similiter arcus *el*, proportio tamen unius ad alium in omnibus regionibus est una. Ideo ponit arcum *te* 60 partes ut cognoscatur proportio eius ad arcum *el* in omni regione et per omnes partes quarte orbis signorum. Hac enim proportione nota et noto arcu *te* qui est in omni regione medietas superflui diei equalis super breviorem diem illius regionis, cognoscetur arcus *el*. Campanus.

"Et sciverimus proportionem eius etc." Que scilicet proportio est illa remanet abiecta proportione corde dupli arcus *lk* ad cordam dupli arcus *kz* de proportione corde dupli arcus *th* ad cordam dupli arcus *hz* que utraque est nota in omni regione et in omnibus partibus quarte orbis signorum. Campanus.

"Et quia proportio 60 ad 38 etc." Ostensum est supra quod proportio corde dupli arcus *te* ad cordam dupli arcus *el* in omni regione est, cum fuerit elongatio puncti orbis signorum qui oritur in puncto *k* a puncto *e* 10 partes, sicut 60 ad 9 partes et 33 minuta. *El* arcu cuius elongatio eadem fuerit 20 partes est illa proportio sicut 60 ad 18 partes et 57 minuta, et sic

806 sinum] *zn* N

de ceteris. Quia ergo volumus investigare arcum *el* in linea equidistante que est supra Rodum, cognito arcu et corda dupli cuius est 38 partes et 34 minuta vicinius, dicemus quod proportio 60 ad 9 partes et 33 minuta est sicut 38 partium et 34 minutorum ad aliquid aliud quod erit corda dupli *el* supra Rodum cum elongatio puncto quod ortus in *k* ab *e* est 10 partes. Quare permutatim proportio 60 ad 38 partes et 34 minuta est sicut 9 partium et 33 minutorum ad illud idem quod est corda dupli arcus *el* etc. Et ipse est 6 partes.

[N 35r contains lists of Hebrew, Egyptian, Macedonian, and Roman months.]

| "Accipimus [etiam quantitatem hore temporalis] etc." Ad sciendum horam temporalem diei accipe quod est in tabula tui climatis, illud quod est in directo partis solis in tabula aggregationum. Et similiter accipe id quod est in directo eiusdem partis scilicet in qua est sol in tabula aggregationis circuli equinoctialis. Et horis duorum vide differentiam cuius accipe sextam partem. Et adde eam super 15 tempora si sol est in medietate orbis signorum septentrionali vel minue a 15 si est in medietate meridiei. Et illud quod post diminutionem vel augmentum habebis erit numerus hore temporalis illius diei. Idem fac ad habendum horas noctis cum parte que opponitur partis solis. N 37v

Dare horas temporales est dare numerum earum et partes equatoris que oriuntur in una earum. Et quia in qualibet hora equalis oriuntur 15 partes equatoris, propter hoc dare horas equales est solum dare numerum earum. Sunt enim semper note partes equatoris que oriuntur in una earum.

21. Loco solis cognito arcum diei aut noctis per notas signorum elevationes invenire.

22. Numerum horarum equalium et tempora inequalium ex loco solis et notis ascensionibus elicere.

23. Datas horas temporales ad equales et datas equales ad temporales reducere.

24. Partem ascendentem et partem medii celi inquirere.

| "Quod si nos voluerimus etc." Gradum medii celi ita invenies. N 38r
Cognitis horis horas temporales preteritas a meridie precedenti reduc in gradus equatori scilicet multiplicando eas que sunt noctis in numerum partium hore temporalis nocturne et eas que sunt diei in numerum partium hore temporalis diurne, et quod ex eis aggregatur serva et quare

partem in qua est sol in elevationibus sphere recte. Et postea computa tot partes equatoris post illam que respondet parti in qua est sol, quot sunt ille quas aggregatas servaveras. Et ubi terminabitur considera partem orbis signorum que in eius directo ponitur, et ipsa est in medio celi super terram.

Aliter idem. Cognita parte orbis signorum oriente vide quid sit in directo eius in tabula aggregationum in elevationibus illius climatis, et ex eo si potes deme 90, si autem adde totum circulum ut ab aggregato demas 90, residuum vero quere in tabula aggregationum in elevationibus sphere recte. Et gradus orbis signorum qui ei opponitur est in medio celi super terram.

| "Nos nominamus [angulum quem continent] etc." Angulus rectus N 38v
spheralis est cui contento ab arcubus orbium maiorum subtenditur quarta omnis circuli in sphera cuius ipse polus. Obtusus autem cui talis circuli subtenditur magis quarta, acutus vero cui subtenditur minus quarta.

25. Proportio spheralis anguli supra polum alicuius circuli consistentis ad 4 angulos spherales rectos est sicut arcus eiusdem circuli qui ei subtenditur ad totam suam circumferentiam. Hanc propositionem non videtur Ptolomeus hoc modo proponere secundum quod proportio anguli rectilinei supra centrum illius circuli descripti cui arcus ille subtenditur ad quattuor angulos rectilineos est tanquam illius arcus ad circumferentiam suam. Angulus enim quem continet declinatio duarum superficierum illorum orbium de quibus loquitur est equalis angulo consistenti supra centrum eius circuli cui arcus ille subtenditur, et tamen ea quam proposuimus indigebit cum ipse investigabit quantitatem angulorum spheralium per quantitatem arcuum subtensorum, utraque tamen vera est.

Ea autem quam proposuimus hoc modo probatur sicut ultima 6^{ti} Euclidis. Sumantur duo circuli equales et equedistantes in sphera ut sunt duo circuli descripti a polis orbis signorum circa polum mundi, et sumantur ex eis duo equales arcus eruntque anguli super polos facti ab arcubus orbium maiorum transeuntium per finalia puncta illorum arcuum equales per secundam partem quarte primi Milei, quoniam tres arcus unius sunt equales tribus arcubus alterius. Quia igitur si ex circulis equalibus in sphera abscindantur arcus equales, anguli contenti ab orbibus magnis super polos eorum quibus illi arcus subtenduntur sunt equales. Demonstrabimus eo modo quo in ultima sexti Euclidis quod si inequalibus circulis sphere fiant anguli ab orbibus magnis super polos eorum, erit angulorum proportio sicut arcuum illorum circulorum illis angulis subtensorum. Quare etiam in uno circulo si fiant anguli ab orbibus magnis super polos eius, erit arcus ad arcum ut angulus ad angulum. Procedendo ergo coniunctim patebit propositum. Nam autem quam videtur Ptolomeus proponere patet ex ultima 6^{ti} Euclidis et coniuncta proportionalitate.

| 26. Omnes duo anguli provenientes ex sectione orbis meridiei et N 39r orbis signorum in duobus punctis equedistantibus uni punctorum equalitatis ex eadem parte sumpti extrinsecus intrinsecus sunt equales.

27. Omnes duo anguli provenientes ex sectione orbis meridiei et orbis signorum in duobus punctis equedistantibus uni tropicorum ex eadem parte sumpti extrinsecus cum inctrinseco sunt equales duobus angulis spheralibus rectis.

| 28. Angulus proveniens ex sectione orbis meridiei et orbis N 39v signorum apud punctum tropicum rectus esse necessario comprobatur.

"Et quia orbis [meridiei quod est *abgd*] etc." Cum duo circuli *aeg* et *bed* sint de maioribus in sphera, divident se per equalia per 12 primi Theodosius. Et quia circulus *abgd* transit per polos eorum, secabit portiones eorum in duo media per 9 secundi eiusdem. Quare *de* est quarta circuli.

29. Nota maxima declinatione angulum qui provenit ex sectione meridiani et orbis signorum apud utrumlibet punctum equinoctii invenire. Unde patet quod maxima declinatione addita super quartam vel ab ea diminuta, provenit angulus quesitus.

"Propter hoc ergo [quod circulus *abgd* est descriptus] etc." Eodem modo arguendum est sicut in premissa. Cum enim circuli *aeg bed* sint de maioribus secabunt se per equalia per 12 primi Theodosii. Et quia circulus *abgd* transit per polos amborum, dividet portiones eorum in duo media per 9 secundi eiusdem. Quare *de* erit quarta circuli. Quod autem *az* sit quarta circuli patet eadem ratione quia cum *aeg azg* sint duo circuli de maioribus in sphera, secabunt se per equalia per 12 primi Theodosii. Et quia *bzed* transit polos amborum, dividet portiones eorum in duo media per 9 secundi eiusdem. Quare *az* erit quarta circuli. ... alia ratione est arcus *az* quarta circuli. Quia cum [polo] *a* sit descriptus circa *bzed*

961 sit] polum N

secundum spacium lateris quadrati, erit unusquique orbis magnis descendens ab *a* polo usque(?) ad circumferentiam circuli *bzed* quarta circuli.

30. Angulum provenientem ex sectione orbis meridiei et orbis signorum apud punctum datum orbis signorum invenire. Unde patet quod si sinum declinationis puncti dati ducatur in sinum complementi portionis sumpte ab equinoctiali, productum vero dividatur per sinum portionis sumpte ab equinoctiali, quodque exierit ducatur in sinum quarte, et productum dividatur per sinum complementi declinationis illius puncti, exibit sinus differentie anguli recti et quesiti. Differentiam ergo illam si addideris angulo recto vel subtraxeris ab eodem, habebis angulum quesitum.

[In diagram is written: "Inquerit arcum et per katam disiuncta descendentem ab *h* in *b e*."]

| "Et quia circulus orbis meridiei etc." *Bh* et *bt* sunt quarte quoniam N 40r
sunt arcus orbium maiorum descendentes a polo orbis magni *hek* usque ad ipsum at vero hec est quarta. Quare duo circuli *hek aeg* sunt de maioribus et ideo dividunt se per equalia per 12 primi Theodosii. Et quoniam *abgd* orbis magnis transit per polos amborum, dividet portiones eorum per equalia per 9 secundi eiusdem. Quare *he* est quarta. Campanus.

| Vult dicere quod per istam demonstrationem iam dictam possumus N 40v
invenire quantitatem angulorum provenientium ex orbe meridiei et signorum ad quodlibet augmentum arcuum orbis signorum quod etiam sit minus uno signo, sed sufficit hic invenire in augmento signi et signi. Campanus.

"Post ista demonstrabo [qualiter operteat] etc." In hoc 11 capitulo docet invenire quantitatem angulorum provenientium ex orbe signorum et orizonte declivi, et superius docuit invenire quantitatem provenientium ex orbe signorum et orizonte recto, meridianus enim et orizon rectus idem sunt. Campanus.

31. Omnes duo anguli provenientes ex concursa orbis signorum in duobus punctis equedistantibus uni punctorum equalitatis et orizontis obliqui ab eadem parte intrinsecus cum extrinseco sunt equales.

| [“Propter ea quorum iam precessit...”] Per 6 huius et per 16, per 6 enim huius est arcus *el* equalis arcui *eh,* et per 16 huius est arcus *ek* equalis arcui *ez,* et arcus *kl* equalis arcui *zh* per hypothesis. Campanus. N 41r

32. Omnes duo anguli provenientes ex concursu orizontis declivis et orbis signorum in duobus punctis oppositis orientis et occidentis extrinsecus cum intrinseco ex eadem parte sumpti duobus rectis angulis sunt equales.

[“Angulus vero *zad* est equalis angulo *zgd*”] Eo quod si transeat circulus maior per polos amborum scilicet orizontis et orbis signorum, idem arcus qui erit *dz* subtendetur angulo *daz* et angulo *dgz*. Quare proportio uniuscuiusque duorum angulorum *daz dgz* ad quattuor rectos erit sicut arcus *dz* ad suam circumferentiam. Quia igitur illorum ad quattuor rectos est una proportio, ipse erunt equales. Campanus.

Correlaria 32^{e}. Unde et ex premissa manifestum est quod duo anguli eorumdem orbium equalis longitudinis ab uno punctorum tropicorum orientalis cum occidentali et extrinsecus cum intrinseco ex eadem parte sumpti duobus rectis angulis sunt equales. Quapropter notis angulis orientalibus qui sunt in una medietate orbis signorum que est ab ariete in libram, noti erunt orientales qui sunt in medietate altera et etiam occidentales qui sunt in ambabus medietatibus.

Correlarium 32^{e} sic patet. Sit orizon declivis circulus *abg*, et medietas orbis signorum supra terram *adzeb*. [Figura 13] Sitque punctum *a* principium piscium, *d* principium arietis, *e* principium cancri, *b* principium virginis. Sitque arcus *ze* equalis arcui *eb*. Quia igitur *ab* est medietas orbis signorum et arcus *de* est quarta, erunt duo arcus *ad eb* similiter equales quarte que est *ed.* Quare detracto *eb* ex *de* quod residuum erit, et ipsum est *zd* eo quod *eb* posuimus equale *ez*, erit equale *ad.* Quia igitur duo puncta *a z* sunt equalis distantie a puncto *d*, quod est punctum equinoctii, erunt per 31 huius anguli ab orizonte et orbe signorum in eis equales. Quia igitur per 32 huius angulus *b* orientalis extrinsecus cum angulo *a* occidentali intrinseco equatur duobus rectis, erit angulus *b* orientalis extrinsecus cum angulo *z* occidentali intrinseco equalis duobus rectis. Et hoc est prima pars correllarii.

Secunda pars sic patet. Angulus qui est ad finem arietis equatur angulo ad principium piscium per 31 huius. Et angulus ad finem tauri equatur angulo ad principium aquarii per eandem, et sic de ceteris. Quare uno nota alius erit notus. Cumque noti fuerint omnes orientales, detrahatur quilibet de duobus rectis, et remanebit angulus orientalis puncti oppositi. Campanus.

33. Per poli altitudinem et maximam solis declinationem angulum provenientem ex concursu orbis signorum et orizontis declivis apud utrumque punctum equinoctii invenire. Unde patet quod si differentia maxime declinationis ad latitudinem regionis cum latitudo maior fuerit detrahatur de quarta circuli aut supra quartam addatur cum fuerit minor, proveniet angulus sub capite libre a quo si dematur distantia que est inter duos tropicos, relinquitur angulus sub capite libre.

| 34. Angulum provenientem ex concursu orizontis declivis et orbis N 41v
signorum apud quodlibet punctum per partem medii celi et partis eiusdem declinationem restat invenire. Unde patet si sinum altitudinis gradus medii celi sub terra vel supra terram multiplicetur in sinum quarte, productum vero dividatur per sinum portionis intercepte inter orizontem et orbem signorum supra terram vel sub terra secundum quod contingit eam esse minorem quarta circuli, exibit sinus arcus subtensi angulo quesito.

Istud correlarium sic patet. Cum unusquisque arcuum *dgz thz egh* sit quarta circuli, erunt omnes equales. Et quia proportio *dg* ad *dz* et intelligatur de cordis duplorum arcuum componitur ex proportione *ge* ad *eh* et *ht* ad *tz*, erit eadem composita ex *ge* ad *dz* et ex *ht* ad *dz*. Quia igitur *gd* ad *dz* componitur ex *ge* ad *dz* et *ht* ad *dz*, et ipsa etiam composita ex *gd* ad *ge* et *ge* ad *dz*, erit *gd* ad *ge* sicut *ht* ad *dz*. Multiplica igitur *gd* primum in *dz* quartum et divide per *ge* secundum, et exibit *ht* tertium. Et intellige quidquid dictum est de arcubus de cordis duplorum arcuum. Et illud est quod correlarium proponebat.

Duo arcus *dz* et *tz* sunt quarta circulo, quia cum ambo sint arcus orbium maiorum, intersecabunt se illi orbes per equalia per 12 primi Theodosius. Et quia circulus orizon qui est *bed* transit per polos

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1050 detrahatur] maxima declinatione *add. supr. lin.* N **1051** angulus] latitudo maxima declinatione *add. supr. lin*. N | capite] orientalis *add. supr. lin.* N **1052** capite] orientalis *add. supr. lin.* N **1058** intercepte] orbis signorum *add. sed. del.* N **1070** proponebat] Campanus *add.* N [This note clearly belongs to the Almagestum parvum-like set of notes, and is not Campanus'. The style does not match the other notes marked as his, and he would be repeating himself in the the next note if this were his.]

amborum, dividet portiones per equalia per 9 secundi eiusdem. Quare unusquisque arcuum *dgz thz* erit quarta circuli. Campanus.

Nota in hac figura quod nullum punctum orbis signorum in suo ortu ad quemlibet orizonta dividit medietatem orizontis orientalem cuius termini sunt ad orbem meridiei in duo equalia preter duo puncta in quibus equator et circulus signorum sese intersecant. Cum enim equator dividat illam medietatem in duo equalia eo quod circulus meridiei qui transit per polos amborum dividit portiones amborum que sunt medietates circulorum in duo equalia per 9 secundi Theodosii, non dividet eam sic aliquis alius punctus orbis signorum. Ymo omnia puncta que sunt ab ariete in libram sic divident illam medietatem quod minor pars eius erit a loco sectionis ad septentrionalem partem orbis meridiei, maior vero a sectione usque ad partem australem orbis meridiei. In reliqua vero medietate orbis signorum que est a libra usque in finem piscium, erit econverso, nam minor pars erit a sectione usque ad partem australem orbis meridiei, maior vero a sectione usque partem eiusdem orbis septentrionalem.

Inde evenit quod per katam istam quam Ptolomeus ordinat sub orizonte non poterimus invenire angulos factos ab orizonte et orbe signorum nisi in medietatem que est ab ariete in libram. Secundum prius dicta per 31 huius, invenitur omnes alterius medietatis.

| "Et postquam iam scivisti etc." Quia inter duos arcus *te tz* sese N 42r
intersecant alii duo *eh zd* supra punctum g, erit per 15 primi proportio corde dupli arcus zt ad cordam dupli arcus *th* aggregata ex duplici proportione scilicet ex proportione corde dupli arcus *zd* ad cordam dupli arcus *dg* et ex proportione corde dupli arcus *eg* ad cordam dupli *eh*. Igitur erit econverso proportio *th* ad *tz,* et intelligatur de cordis duplorum arcuum, aggregata ex proportione *dg* ad *dz* et *eh* ad *eg.* Sed si proportio primi ad secundum componitur ex proportione tertii ad quartum et quinti ad sextum, erit proportio tertii ad quartum composita ex proportione primi ad secundum et sexti ad quintum per librum de proportione et combinationibus proportionuum quem composuimus. Erit ergo proportio *dg* tertii ad *z*[*d*] quartum composita ex proportione *th* primi ad *tz* secundum et *eg* sexti ad *eh* quintum.

Hoc autem modo arguit Ptolomeus quoniam proportio primi ad secundum que est inter *tz* et *ht* est ignota eo quod arcus *ht* est ignotus. Proportio vero que est inter tertium et quartum et inter quintum et sextum sive sextum et quintum est nota, posset tamen si vellet illas duas notas aggregari, et haberet eam que inter *tz* et *ht* notam cuius cum unum extremum scilicet *tz* sit notum esset reliquum notum. Sed hac via difficulius et prolixius esset opus. Campanus.

35. Omnes duos arcus duorum orbium altitudinis a polo orizontis in duo puncta orbis signorum equalis ab uno duorum tropicorum distantie descendentes, cum eadem puncta in que descendunt ab orbe meridiei ante et post secundum equa distiterint tempora equales sibi invicem esse necesse est. Anguli quoque duo ex eadem parte sumpti intrinsecus cum extrinseco duobus rectis angulis necessario sunt equales.

| "Erit angulus [*bgd* equalis angulo *bgz*] etc." Cum enim duo puncta orbis signorum que sunt *d z* equaliter distent a puncto tropico per hypothesis, equedistans equatori transiens per unum punctum transibit per alium. Sit itaque ille equidistans *dlz*, et sit punctum *l* in quo ille secat orbem meridiei. Quia igitur per hypothesis duo puncta *d* z secundum equalia tempora distant ab orbe meridiei ante et post, erunt duo arcus ex illo equedistante qui sunt *dl lz* equales. Quia iterum duorum punctorum *d* z equales sunt declinationes ab equatore quia equaliter distant a tropico, erunt duo arcus *gd* et *gz* equales. At rursus quoniam *dl* et *lz* sunt equales, erunt arcus equatoris intercepti inter eosdem orbes *gd ga gz* equales. Ergo duo trianguli ex istis tribus arcubus usque ad equatorem protensis et ex duobus arcubus equatoris eis subtensis, erit ex equalibus arcubus orbium maiorum. Quare per secundam partem quarte primi Milei erunt equianguli. Ergo anguli duo *bgz bgd* erunt equalis. Quia igitur duo arcus *gz gd* sunt equales, comitato arcu *gb* erunt duo arcus *gb gz* equales duobus *gb gd*. Et angulus *bgz* est equalis angulo *bgd*. Ergo per primum partem eiusdem erit basis *bz* equalis basi *bd*, quod est unum propositorum. Campanus. N 42v

"Qui est apud punctum unum orbis meridiei etc." Idest qui est apud illud unum punctum orbis signorum cum venerit ad orbem meridiei vel qui est apud unum illorum duorum punctorum situs ex illa puncto uno orbis signorum et orbe meridiei. Nam illi duo sunt equales eo quod quilibet punctus orbis signorum cum omnibus orbibus magnis transeuntibus per polos equatoris facit angulum extrinsecum equalem intrinseco ex eadem parte in quocumque situ fuerit punctus ille. Campanus.

Nota quod non apponit tertium membrum scilicet quod unum punctorum mediantium celum in illis duabus horis sit ad partem septentrionalem a puncto summitatis capitum et alterum ad partem meridianam ab eodem. Et licet istud sit possibile quia tunc non sequeretur illa conclusio quantum ad secundam sui partem, sed solum quantum ad primam. Duo enim anguli quos facerent circuli altitudinis ad illud punctum orbis signorum maiores essent duplo anguli facti in eodem puncto orbis signorum ab orbe meridiei in duobus angulis rectis vel etiam minores in duobus angulis rectis ut etiam demonstrat auctor infra. Campanus.

| 36. Omnes duo arcus duorum orbium altitudinis a polo orizontis in N 43r
unum et idem punctum orbis signorum cum ab orbe meridiei ante meridiem et post secundum equa distiterit tempora descendentes ad invicem sunt equales. Et si ambo puncta celum mediantia fuerint ad partem meridiei a polo orizontis aut ambo ad partem septentrionis ab eodem, facient duos angulos simul equales duplo anguli facti ad idem punctum ex concidentia orbis signorum et orbis meridiei. Quod si punctum medians celum eo existente ante meridiem fuerit ad partem meridiei et medians celum eo existente post meridiem fuerit ad partem septentrionis a polo orizontis, facient duos angulos simul maiores duplo anguli orbis signorum et meridiei in duobus angulis rectis. Quod si fuerit secundum huius conversionem, facient duos angulos minores duplo eiusdem anguli in duobus angulis rectis.

Unde manifestum est quod si noti fuerint anguli et arcus qui sunt a principio cancri usque ad principium capricorni in omni que est ante meridiem, noti erunt etiam anguli et arcus eorumdem signorum in omni declinatione post meridiem. Et cum hoc etiam anguli et arcus alterius medietatis qui sunt in omni declinatione ante meridiem et post.

"Puncti *e* et puncti *h* [ab orbe meridiei in utrisque partibus.]" Que duo puncta *e* et *h* non sunt duo puncta orbis signorum sed unum et idem in numero, et pono quod utrumque sit principium leonis. Sed sunt duo puncta solum secundum situm. Sunt duo puncti circuli cuiusdam fixi transeuntis per punctum illud orbis signorum equedistanter equatori. Campanus.

"Propterea ergo [que iam declarata sunt] etc." Hic incipit arguere et resumit istam hypothesim quod duo puncta situs scilicet *e h* distant secundum arcus equales equedistantis descripte supra punctum orbis signorum *h* sive *e* quia idem est ab orbe meridiei. Et per talem argumentem quale adducimus ad figuram premissam, probabis *gdh* equari angulo *gde*, et arcum *gh* arcui *ge.*

Et nota quod cum dicit “propterea ergo,” illud ergo non tenetur illatine quia quod ex eo videtur concludere cum dicit “erunt duo arcus eius” primo suppositum est. Sed illud et quod postea sequitur cum dicit “et erunt duo trianguli etc.” tenetur illatine quasi dicens “quia ergo,” ita est sicut supposuimus quod illi duo arcus illius equidistantis a duobus lateribus orbis meridiei sunt equales, et pro conclusio “erunt duo trianguli etc.” Campanus.

| 37. Angulum ex concidentia orbis signorum cum circulo altitudinis N 43v
in puncto dato exeunte in orbe meridiei vel in orizonte, arcum quoque continuans inter illud punctum ad predictos situs et summitatem capitum restat inquirere.

| 38. Arcum circuli altitudinis descendentem a puncto summitatis N 44r
capitum in quodlibet punctum orbis signorum a celi medio declinans restat investigare.

[By diagram is written: “Inquirit arcum *ah* circuli altitudinis per conversionem kate coniuncte descendentis a puncto *b* in *t a*.”]

| 39. Angulum ex concidentia circuli altitudinis cum orbe signorum N 44v
ad quodlibet punctum a celi medio declinans perscrutari.

Commentary in MS F

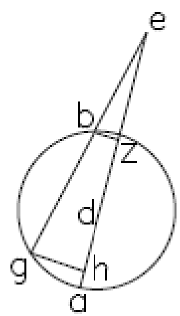


Figura #1

Commentary in MSS HK

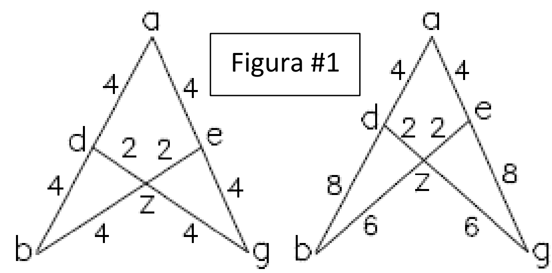


Figura #1

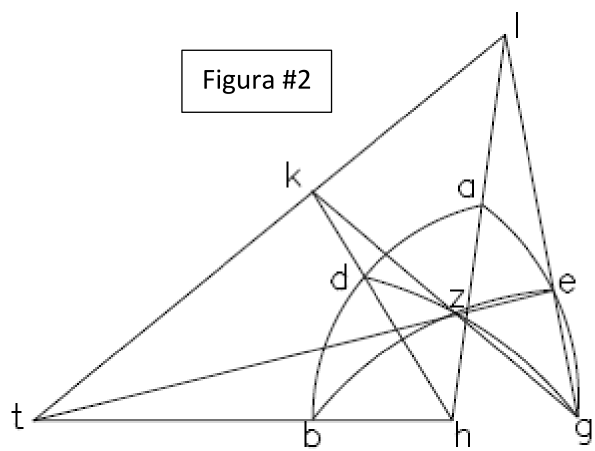


Figura #2

Commentary in MSS NX

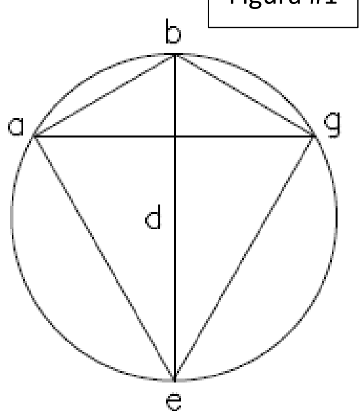
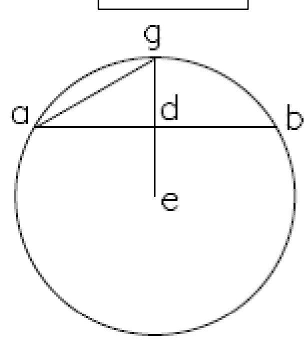


Figura #1



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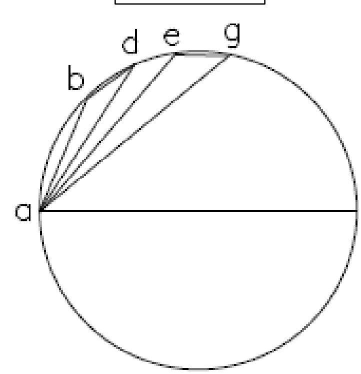

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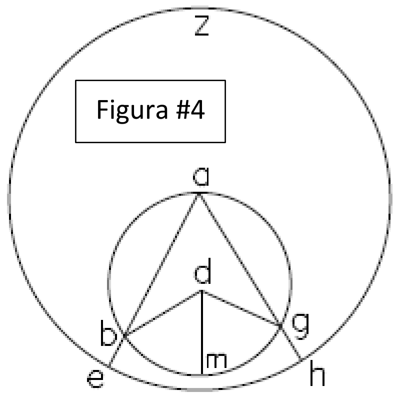

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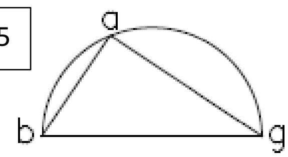

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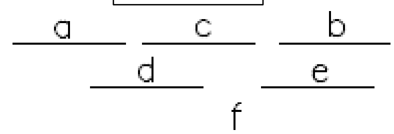

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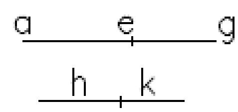


Figura #7

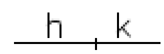


Figura #8

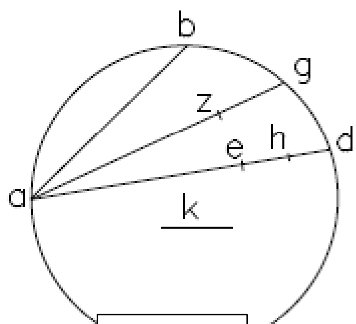


Figura #9

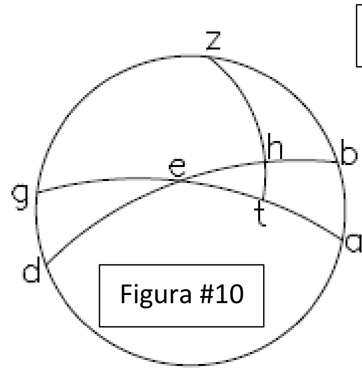


Figura #10

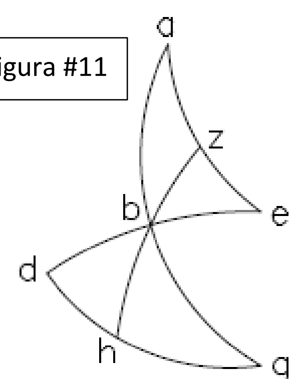


Figura #11

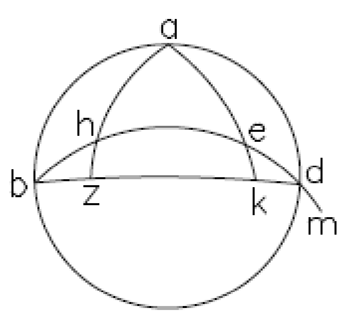


Figura #12

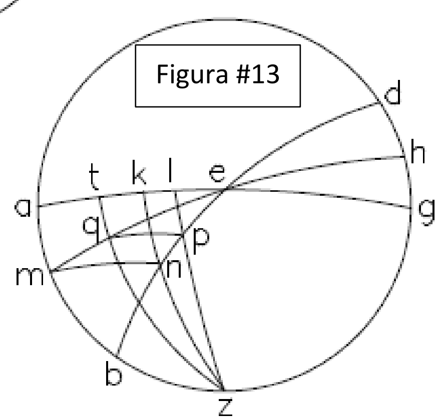


Figura #13

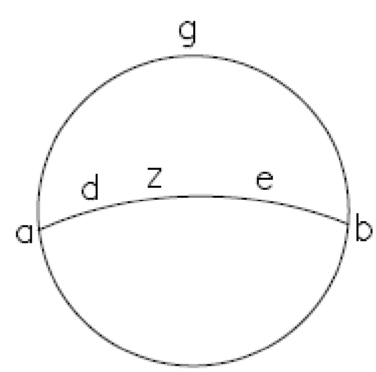


Figura #14

Commentary in MS S

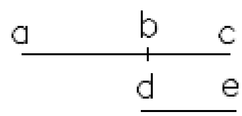
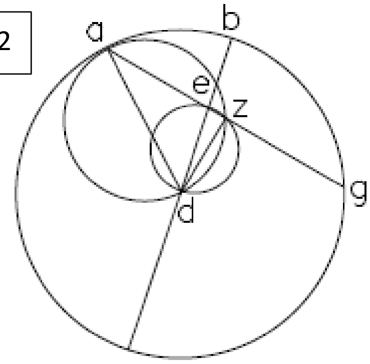


Figura #1

Figura #2



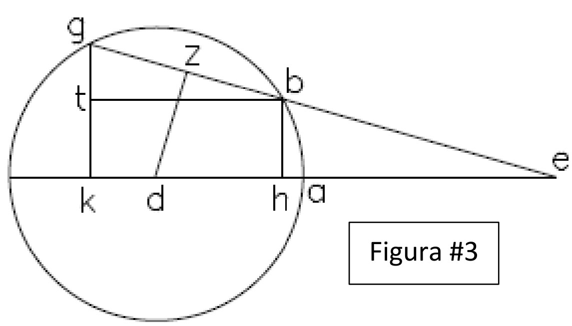


Figura #3

Appendix C: Menelaus' *Sphaerica* III.1

The following mansucripts containing Gerard of Cremona's translation of the *Sphaerica* were used in the following collation:

Schweinfurt, Stadtbibliothek, H 81*† = L

Venezia, Biblioteca Nazionale Marciana, Marc. lat. VIII 32*† = M[676]

Wolfenbüttel, Herzog August Bibliothek, Co. Guelf. 24 Aug. quart. = N

Milan, Biblioteca Ambrosiana, Q 69 sup.* = O

Città del Vaticano, Biblioteca Apostolica Vaticana, Vat. lat. 3380 = P

Città del Vaticano, Biblioteca Apostolica Vaticana, Ottob. lat. 2234 = Q

Città del Vaticano, Biblioteca Apostolica Vaticana, Reg. lat. 1261† = R

Città del Vaticano, Biblioteca Apostolica Vaticana, Reg. lat. 1268 = S

Paris, Bibliothèque nationale de France, lat. 9335 = T

Venezia, Biblioteca Nazionale Marciana, F.a. 328 = V

Wien, Österreichische Nationalbibliothek, 5277* = W

Città del Vaticano, Biblioteca Apostolica Vaticana, Pal. lat. 1351* = X

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[676] Björnbo, "Studien," pp. 151-2 lists this among the manuscripts with Campanus' commentary in the text, but parts of the commentary are found in the margins, not the text.

Città del Vaticano, Biblioteca Apostolica Vaticana, Vat. lat. 4571† = Y

Paris, Bibliothèque de l'Arsenal, 1035 = Z

† *Contains Campanus' commentary incorporated into the text.*

* *Contains Campanus' commentary in the margins.*

The following manuscripts also contain the *Sphaerica*:

Oxford, Bodleian Library, Digby 168 (contains Book I and excerpts of Book III)

Oxford, Bodleian Library, Digby 178 (most of the proofs are removed and sometimes replaced by notes)

Firenze, Biblioteca Medicea Laurenziana, S. Marci Florent. 184.

Firenze, Biblioteca Nazionale Centrale, Conv. soppr. J.V.30

Glasgow, Hunterian Museum, V.2.14

London, British Library, Harley 13 (only portions of Book I)

Paris, Bibliothèque nationale de France, lat. 7399 (incorrectly titled as a work of Gebir)

Madrid, Biblioteca Nacional, 10010

Paris, Bibliothèque nationale de France, lat. 7399 (only excerpts)

Utrecht, Universiteitsbibliotheek, 725 (text is mixed with unique commentary)

Venezia, Biblioteca Nazionale Marciana, F.a. 329.

Wolfenbüttel, Herzog August Bibliothek, Co. Guelf. 24 Aug. quart.

A commentary on the *Sphaerica* is found in Paris, Bibliothèque nationale de France, lat. 7377B, ff. 45v-60r.

Bjornbo listed Paris, Bibliothèque nationale de France, lat. 7251 as a work containing the Gerard of Cremona translation, but it does not have it. It contains Maurolycus' version of the *Sphaerica*.

Menelaus’ *Sphaerica*, III.1: The Menelaus Theorem

Sint in sphere superficie duo arcus duorum circulorum magnorum super quos sint *ne ln*. [Figura 1] Et protraham inter eos duos arcus *eta ltm* et secent se super punctum *t*. Dico ergo | quod proportio nadir arcus *an* ad R 248r nadir arcus *al* est composita | ex proportione nadir arcus *ne* ad nadir arcus Z 97vb *me* et ex proportione nadir arcus *mt* ad nadir arcus *tl*. Et ego quidem non significo cum dico ‘nadir arcus’ nisi lineam que subtenditur duplo illius arcus secundum quod est ille arcus minor semicirculo. Cuius hec est demonstratio. Ponam centrum sphere punctum *b* et protraham | lineas *nl* O 26r *nm lm tb eb asb sd*.

[Posito scilicet in puncto sectionis communis ubi linea ducta a centro sphere quod est *b* secat lineam rectam *nl* que est corda arcus *nal*. Et posito *d* in puncto communis sectionis ubi linea ducta a centro *b* secat lineam rectam *ml* que est corda arcus *mtl*.]

Et concurrant | inprimis due linee *nm* | *sd* cum protrahuntur super S 231r | punctum *c* secundum quod est in prima forma, et protraham lineam *ec*. X 273v Ergo erit punctum *c* |in unaquaque duarum superficierum duorum arcuum Q 51rb *ate nme*.

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2 Sint] sit in superficie sphere *praem*. sint L sunt P demonstrat figuram sectoris *adnot. mg*. L sphere superficie] *inv.* NOPQSTVZ | duorum] *om.* LMRY | duorum…magnorum] *om.* P **3** super] supra NTV | sint] sunt LP | ne] *lt* P | protraham] protrahuntur M | eta] *cte* P | ltm] *lm* M **4** secent] secat N | an] *na* L **5** arcus^{1}] *iter.* P *al* est sicut proportio composita ex proportione nadir arcus *ne* ad nadir arcus *em* et ex proportione nadir arcus *emt add. sed del. signis va-cat* Q | al] *em* QST *em add*. P est *add*. (*mg. a. m.* S)SZ (hic est melior *add*. N) in alio *al* (*el* T) est sicut proportio composita ex proportione (composita *add*. S) nadir arcus *ne* ad nadir arcus *em* et ex proportione nadir arcus *mt add. mg. a. m*. NST | est] *om.* W sicut proportio Z | proportione] composita *add. mg. a. m.* S | nadir^{2}…6 me] *ne* ad *em* QST | arcus^{2}] *om.* V | arcus^{2}…6 me] *ne* ad nadir *em* V **6** me] *em* QSVZ *mg. a. m.* S ex proportio nadir arcus *mt em* est composito ex proportione *ne* ad *em add*. Z | et] *om.* Z | arcus^{1}] *sup. lin. a. m.* Q *tm* ad nadir arcus *tl ne* ad *tm* et ex proportione nadir arcus *add*. P | mt] *et* P *sl* QSTZ | tl] *lt* W **7** arcus] *st add. sed exp.* Q | que] qui WX **8** secundum] sed V | est^{1}] sit NOQSTVZ *om.* P Cuius] inquam *add.* P **9** Ponam] *iter.* Y | nl] *na* Z **10** nm lm] *inv*. M | eb] *ab* Y | sd] *sa* N **12** Posito…15 mtl] *add*. LMO(*mg. a. m.*)RW(*mg. a. m.*)X(*mg.a.m.*)Y[The descriptions in the parentheses apply to the MS whose sigla they follow.] | sectionis communis] *inv*. O **13** sphere…est^{1}] *om.* M | nl] *ml* M *n* O **14** Et…15 mtl] *om.* M | d] *l* M **15** mtl] *ml* LX **17** Et…sd] *om.* L | concurrant] linee protracte *add.* MRY **18** est] *om.* Z | in prima] *om.* P prima] *om.* NOQSTVZ | lineam] *mg. a. m.* M | ec] *eec* Z **19** c] *t* M | duarum superficierum] *inv*. O | superficierum] *iter. sed del.* P

[Scholium. Hoc patet ex prima 11^{i} quia linea *sdc* sita est in superficie circuli *ate* cum punctum *s* sit in ipsa cum linea *asb* sit communis differentia superficiei circuli *nal* et superficiei circuli *ate* et punctum *d* similiter est in eadem superficie. Residua patent ex prima 11^{i}.]

Ac unumquodque duorum punctorum *e b* iterum est in istis duabus superficiebus, ergo | est *ceb* linea una recta. Et cum hec forma sit ita, tunc T 49ra proportio *ns* ad *sl* est sicut proportio composita ex proportione *nc* ad *cm* et ex proportione *md* ad *dl*.

[Patet sic: ducatur linea *me* equidistans linee *nsl*. [Figura 2] Patet igitur tibi quod trianguli *mde* et *sdl* sunt equianguli, anguli enim qui sunt ad *d* sunt equales quia contra se positi et angulus *med* est equalis angulo *dsl* quia unus est coalternus alteri, quare tertius est equalis tertio. Quare per 4^{am} 6^{i} proportio *me* ad *sl* est sicut *md* ad *dl.* Cum autem *me* sit equidistans *ns,* probabis de facili quod trianguli *cme cns* sunt similes. Quare per 4^{am} 6^{i} proportio *nc* ad *cm* est sicut *ns* ad *me*, sed proportio *ns* ad *sl* est composita ex proportione *ns* ad *me* et *me* ad *sl,* quare proportio *ns* ad *sl* erit composita ex proportione *nc*

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22 Scholium] *om*. MORWXY | Scholium…26 11i] *add*. L(*mg. a.m.*)MO(*mg. a.m.*)RW(*mg. a. m.*)X(*mg. a. m.*)Y | Hoc patet] *inv. pos.* 11^{i} LOWX **24** differentia] superiori L | nal…circuli^{2}] *om*. WX | ate] *atc* W *ace* X **25** similiter est] sit etiam L | patent] clarerent Y **28** Ac] *at* W unumquodque] utrumque NV | e] *a* P *b* Y | b] *d* Y | iterum] *om*. M *pos.* istis LRWXY | est] *pos.* istis M **29** est ceb] *inv.* RY | ceb] *deb* M *cbe* QY | Et…tunc] quod R *od* Y | tunc] quod L **30** est…proportio^{2}] sit LRY | proportio^{2}] proposita *add.* WX | ex] *sup. lin. a. m.* M proportione] *om.* P portione Q | cm] *mc* LRY **31** et] *om.* Y | ex] *om.* MRXW | dl] *ld* L **33** Patet…43 sl] *add.* LM(*mg. a. m.*)O(*mg. a. m.*)RW(*mg. a. m.*)X(*mg. a. m.*)Y | ducatur] educatur Y **34** tibi] si non dormis *add.* LOWX | et] *om*. L | anguli] *om*. MRWXY | enim] ei X unde Y **36** dsl] *sdl* O | quia] quoniam L **38** me] *mde* M **39** proportio] *na add. sed del*. X ns…40 proportio] *om*. WX **40** sl] *esl* Y | est] erit X | ns^{2}] *nc* LMR *ne* W *mc* Y | me^{2}] *mc* LMRY **41** et…42 mc] *om*. LMRY

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33 me] Hoc punctum *e* est aliud a puncto *e* linee *ceb* et est in linea *sdc,* non in arcu eta *adnot. supr. lin.* R **41** sl^{1}] Hoc pro tanto(?) dicitur quia proportio *nc* ad *mc* est sicut *ns* ad [*me*] et proportio *md* ad *dl* sicut *me* ad *sl* ut hic probatur. Item proportio *ns* ad *sl* sicut proportio duarum perpendicularium ductarum a duobus punctis *n* et *l* ad linea *ab* ut probatur in fine huius pagine. Ergo sunt hic 6 proportiones [sic] quarum bine et bine faciunt eandem proportionem. Unitas ... in quodcumque ducatur nichil producit nisi illud in quod ducebatur que(?) proportio duarum ultimarum dicitur esse(?) composita ex ... secundum modum positum ad *mc* que est sicut *ns* ad *me*, et ex proportione *md* ad *dl* que est sicut proportio *me* ad *sl*.]

Verum proportio *nc* ad *cm* est sicut proportio perpendicularis cadentis ex puncto | *n* super *ceb* ad perpendicularem cadentem ex puncto P 308v *m* super lineam *ceb* iterum. Sed perpendicularis cadens ex puncto *n* super lineam *bec* est medietas corde | dupli arcus *ne*, et perpendicularis cadens L?v ex puncto *m* super illam lineam est medietas corde dupli arcus *em*. Ergo proportio *nc* ad *cm* est sicut proportio nadir arcus *ne* ad nadir arcus *me*.

[Perpendiculares ductas a duobus punctis *n* et *l* ad lineam *ab* que transit per centrum ex eadem parte linee *nsl,* scilicet vel versus punctum *b* vel versus punctum *a* cadere est impossibile. [Figura 3] Si enim sit possibile, sint due linee *lc nd* perpendiculares ad lineam *ab.* Quare erit angulus *nds* equalis angulo *lcs*, quia uterque | rectus. Sed Y 14ra angulus *nse* est maior angulo *nds* quia extrinsecus ad ipsum. Quare angulus *nse* est maior recto. Quare et angulus *csl* qui ei contraponitur

42 dl] *ad* Y **43** sl] et (*et--ita *om.* L) cum hec forma sit ita, tunc proportio *ns* ad *sl* est sicut proportio composita ex proportione *nc* ad *cm* et proportione *md* ad *dl. add.* LR **45** nc] *me* Q ad *sl add. sed del.* P | sicut] *om.* Y **46** ceb] *ecb* T *ce* Y **47** super[1]…cadens] *sup. lin.* P perpendicularis] perpendiculariter X | super[2]…48 bec] *om.* P **48** bec] hec WX | et…49 medietas] *om.* X **49** super…lineam] *om.* P | corde] de X | dupli] arcus *ne* de dupli *add.* W **50** nc…ne] *om.* (*hom.*) W | ad[1]…ne] *mg. a. m.* W | nadir arcus[1]] *om.* X | ad[2]…me] *om.* (*hom.*) V | nadir[2]…me] *cm* est sicut proportio nadir arcus *ne* ad nadir arcus *me* X **52** Perpendiculares…68 ibi] *add.* (*post.* declaratur)LM(*mg. a. m.*)O(*mg. a. m.*)RW(*mg. a. m.*)X(*mg. a. m.*)Y | a] ex O **53** ex] et WX | vel] illis W **54** a] *om.* Y **58** csl] *cdl* W *scl* est rectus Y

in fine commenti huius pagine *adnot. mg. sed. del. signis 'va--cat'* R Quod proportio *ns* ad *sl* est composita ex proportione *nc* ad *mc* et ex proportione *md* ad *dl*. Hoc patet sic. Nam ut hic probat. Proportio *ns* ad *me* est sicut proportio *nc* ad *cm* et proportio *me* ad *sl* est sicut *md* ad *dl*. Ergo proportio *ns* ad *sl* est composita ex proportione nc ad *mc* et *md* ad dl. Quod patet in numeris. Sint enim tres numeri ex una parte ut 8 4 2, et quatuor in alio ordine ut 40 20 10 5. Et sit ita et sicut se habet in primo ordine primus ad secundum sic in secundo ordine primus ad secundum ut sicut 8 ad 4 sic 40 ad 20. Et sicut in primo ordine secundus se habet ad tertium sic in secundo ordine tercius ad quartum ut sicut 4 ad 2 sic 10 ad 5. Tunc ergo proportio primi ad tertium in primo ordine scilicet 8 ad 2 est composita ex proportione primi ad secundum et ex proportione tertii ad quartum in secundo ordine proportio. Enim(?) 8 ad 2 est composita ex proportione 40 ad 20 et ex proportione 10 ad 5, hoc est ex duabus proportionibus duplici. Dupla enim in se multiplicata per modum quadrati producit quadruplam ut habere(?) sunt 9 diffinitionem quinti geometrie. Proportio autem 8 ad 12 est quadrupla *adnot. mg.* R similiter est maior recto. Sed angulus *scl* est rectus, quare in uno triangulo duo anguli pariter accepti sunt maiores duobus rectis quod est impossibile. Similiter duceretur ad impossibile si darentur quod caderent ambe perpendiculares ex eadem parte linee *nsl* versus punctum *a*. Quare una cadet ex una parte ut *lc* ex puncto *l,* et alia ex alia parte ut *ne* ex puncto *n*. Patet ergo tibi cum angulus *csl* sit equalis angulo *nse* | quia contraposito, et angulus *lcs* sit equalis angulo *nes* R 248v quia uterque rectus, quod triangulus *nse* est equiangulus triangulo *lcs*. Quare per 4^{am} 6^{i} Euclidis sunt laterum proportionalium. Ex hoc patet quod dicit auctor ibi.]

Et similiter etiam declaratur quod proportio *ns* ad *sl* est sicut proportio nadir arcus *na* ad | nadir arcus *al*, et quod proportio *md* ad *dl* M 71v est sicut proportio nadir arcus *mt* ad nadir arcus *tl*. | Ergo proportio nadir N 30v arcus *na* ad nadir arcus *al* est | sicut proportio composita ex proportione L?r nadir arcus *ne* ad nadir arcus *me* et ex proportione nadir arcus *mt* ad nadir arcus *tl*. | Z 98ra

Et iterum nos ponemus lineam *sd* equidistantem linee *nm* et complebimus duas medietates duorum circulorum *etc enc* secundum quod est in forma secunda. [Figura 4] Et quoniam in duabus superficiebus *enc etc* sunt due linee equidistantes que sunt *sd mn,* erit sectio communis istis duabus superficiebus que est linea *ec* equidistans duabus lineis *sd mn.*

[Hoc patet, quia si non concurrent cum ipsa: aut ergo super unum et idem punctum, et tunc linee equidistantes concurrent quod est impossibile; aut super diversa puncta, et tunc patet per 7 11^{i} quod

59 similiter] *om.* L **61** duceretur] ducetur M | darentur] daretur LWX **65** contraposito] contraposita WX **66** rectus] sequitur *add.* L **67** Quare] *iter.* Y | proportionalium] et *add.* O **68** ibi] *supr. lin.* R “similiter etiam declaratur” *add.* LMWX **70** etiam] *om.* PXW | ns] *om.* P **71** arcus^1] *al add.* P | na] *ne* Y | al] est *add.* M | ad^2] *om.* P **72** proportio^1] composita ex proportione *add.* M | Ergo…75 tl] et ex proportione nadir arcus [N 30v] *en* ad nadir arcus *me* N **73** arcus^2] *om.* W | sicut proportio] *om.* V **74** arcus^1] *om.* V | ne] *en* P *mt* OQSTVZ | me] *tl* OQSTVZ *al* M est *add.* M | et] quod proportio *md* ad *dl* est sicut proportio composita *add.* M | nadir arcus^3] *inv.* L | mt] *en* OQSTVZ *me* P **75** tl] *me* OQSTVZ **76** ponemus] preponemus P | linee] dilinee *sed corr.* S | nm] *mn* V **77** etc] *enc* Q | enc] *emc* L **78** quod] sed P | forma] figura *add. sed del.* Z **79** superficiebus] similiter *add. sed del.* P | sunt^1] sint P sunt^2] due linee *add.* NV | erit] *mg.* Z **80** ec] *ca* P **81** lineis] his L | mn] *ma* N **83** Hoc…90 superficiebus] *add.* L(*praem.* que)MO(*mg. a.m.*)RX(*mg. a. m.*)Y | super] *om.* O linea recta que continuat duo puncta concursus que est pars linee *ebc* protracte. Et ideo tota linea *ebc* quantumcumque protrahatur, erit in eadem superficie in qua site sunt due linee *nm* et *sd*, quod falsum est, cum quelibet due sint in eadem superficie, omnes autem tres in diversis superficiebus.]

Et quoniam perpendicularis cadens ex puncto *n* super lineam *cbe* est medietas corde dupli arcus *cn*, et est iterum medietas corde dupli arcus *en*, [(quia eadem corda subtenditur duplo arcus *cn* et duplo arcus *en*)] erit nadir arcus *en* equalis nadir arcus *em*.

[Hoc patet sic. Cum enim nadir arcus *en* sit equalis nadir arcus *nc* ut probatum est. Nadir vero arcus *nc* sit equalis nadir arcus *me* ut probabo. Erit nadir arcus *en* equalis nadir arcus *me*, quod assumptum sit. Verum patet cum linea *nm* sit equidistans linee *ec* et si a duobus punctis | *m n* ducantur due perpendiculares ad lineam *ec*, sint L ?v similiter ille perpendiculares equidistantes quia site sunt in eadem superficie, videlicet in superficie circuli *emnc* et una linea cadens super eas, videlicet linea *ec* facit duos angulos intrinsecos equales duobus rectis. Quare patet quod sunt equidistantes. Quare due perpendiculares cum linea *mn* et cum parte linee *ec* continebunt superficiem equidistantium laterum. Quare latera opposita in illa superficie erunt equalia, quare perpendiculares erunt equales quare et dupla eorum equalia. Sed duplum perpendicularis que descendit ex puncto *m* super lineam *ec* est nadir arcus *me*. Duplum vero illius perpendicularis que descendit ex puncto *n* super lineam *ec* est nadir arcus *nc*. Quare patet quod nadir arcus *me* est equalis nadir arcus *nc*.]

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86 continuat] contineat L **87** quantumcumque] quomodocumque L quamcumque X **88** falsum] non L **89** sint] sunt L **92** cadens] ad *add. sed exp.* WX | n] *m* Y | cbe] *cbt* Z **93** arcus[1]] acus *sed corr. sup. lin.* X | cn] *en* M | et…94 en[1]] *om.* Q | est] *om.* LMVWX iterum medietas] *inv.* O **94** quia…en[2]] *add.* L(*mg. a. m.*)MRW(*sup. lin. a. m.*)X(*mg. a. m.*)Y | en[2]] equidistans etiam lateri *mn* indiviso quare patet quod dicit 2[a] 6[i] *add.* X **95** equalis] equale P | em] *en* V **97** Hoc…112 nc[2]] *add.* LMORX(*mg. a. m.*)Y | nadir arcus[2]] *mg.* M **98** nc[1]] *nd* Y | ut[1]…est] *om.* X **99** en] a *add.* M **100** sit[1]] sic *pos.* patet X | sit[1]…patet] verum sit patet sic L | patet] sic *add.* O | nm] *em* L **101** due] *om.* L | ad…102 perpendiculares] *om.* Y | ec] *et* X | ec sint] *om.* M | sint] sunt L **102** equidistantes] arcus O | site] *om.* L **103** in] *om.* M **104** linea ec] *inv.* O **105** sunt] sint Y **108** erunt[1]] sunt O | equalia] equales *sed corr.* M | quare[1]] et add. O | quare[1]…109 equalia] *om.* LMRY **110** super] punctum *add.* M | ec] *ce* Y **111** ec] *et* MX

Et quoniam linea *mn* est equidistans linee *ds*, erit proportio *ns* ad *sl* que est sicut proportio nadir arcus *na* ad nadir arcus *al* sicut proportio *md* ad *dl*, que est sicut | proportio nadir | arcus *mt* ad nadir arcus *tl*. [Quia linea recta *sd* secat duo latera trianguli *mnl* equidistanter lateri *mn* indiviso. Quare patet quod dicit per 2^{am} 6^{i} Geometrie.] Ergo proportio | nadir | arcus *na* ad nadir arcus *al* est sicut proportio composita ex proportione nadir arcus *mt* ad nadir arcus *tl* et ex proportione nadir arcus *ne* ad nadir arcus *em* cum sit *ae* equalis. [Id est cum nadir arcus *en* sit equalis nadir arcus *me*.]

M 72r | Y 14rb O 26v X 274r

Et per huiusmodi viam iterum declarantur reliqua que accidunt de hac specie proportionis [(ut patebit in 2^{a} huius que sequitur immediate)] in nadir horum arcuum. Et sciemus | illud ex dispositione linearum que iam secuerunt se in superficie quam diximus [scilicet trianguli ad modum dispositionis figure quinte primi Geometrie.]

T 49rb

Et declarantur | relique species huius | descriptionis | sicut nos declaravimus in hac forma quoniam proportio nadir arcus *al* iterum ad nadir arcus *an* est sicut proportio composita ex proportione nadir arcus *lt* ad nadir arcus *tm* et ex proportione nadir arcus *me* ad nadir arcus *en*. Et illud est quoniam iam nuper ostendimus quod proportio nadir arcus *na* ad nadir arcus *al* est sicut proportio composita ex proportione nadir arcus *mt* ad nadir arcus *tl* et ex proportione nadir arcus *ne* ad nadir arcus *me*. Cum

R 249r | W 370v | V 148

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115 sicut proportio[1]] *inv*. MO | sicut[2]…119 al] *om. (hom.)* N | md…119 proportio] *om.* (*hom.*) V **116** arcus[2]] *om.* P | tl] per 2 sexti Geometrie *add*. OL | Quia…118 Geometrie] *add*. LMO(*mg. a. m.*)RY **117** linea] *inv*. LO | sd] *om*. O | mnl] *mns* M **118** per] *om*. L Geometrie] *om*. LO **119** na…arcus[2]] *om.* (*hom.*) XW | al] [This is where L has the final note of Campanus.] **120** nadir[1]] *om*. L **121** arcus[1]] *me add.* M | ae] ei MN *a* Y | Id…122 me] *add*. LMO(*mg. a. m.*)RW(*mg. a. m.*)X(*sup. lin. a. m.*)Y **124** huiusmodi] huius L | viam] unam W | reliqua] aliqua Q relique Z **125** proportionis] dispositionis P | ut…126 immediate] *add*. RW(*mg. a. m.*)X(*sup. lin. a. m.*)Y: | patebit] patet WX **126** in] et MRY | horum arcuum] *inv.* P | Et] specialiter(?) *add.* P | illud] istud T | ex] e Z **127** quam] qua Z | scilicet] *om*. MRXY | scilicet…128 Geometrie] *add*. M(*mg. a. m.*)ORX(*sup. lin. a. m.*)Y **128** primi] prime *sed corr.* R **130** declarantur relique] *iter. sed del*. X | huius] propter *add*. O descriptionis] demonstrationis LMRWXY **131** declaravimus] declaramus QSTV | hac] *om.* NV | al] *ah* W | iterum…132 arcus[1]] *om.* WX **132** an] *en* Y | est…133 en] *om.* (*hom.*) Y | lt] *ht* W **133** arcus[1]] *an add. sed exp.* T | arcus[2]] *om.* MRWX | en] *em sed corr.* X **134** illud] id W | est] *om*. LMNQRWY | quoniam] quod X | iam] *om*. L | proportio] portio Q | arcus] nadir *add. sed del*. P | na] *a* P | na…135 arcus[1]] *mg.* M *om.* WX **135** al] est *add.* WX | est] *om.* QWX | mt…136 arcus[2]] *mg.* Q **136** me…137 arcus] *mg. a. m.* W ergo converterimus | proportionem, erit proportio nadir arcus *al* ad nadir L ?v arcus *an* sicut proportio composita ex proportione nadir arcus *lt* ad nadir arcus *tm*, et ex proportione nadir arcus *me* ad nadir arcus *en*.| Z 98rb

[Ubi dicit “ergo proportio nadir etc.,” cum ducere unam proportionem in aliam nihil aliud sit quam ducere illud quod denominat unam illarum proportionum in illud quod denominat aliam secundum quod definit Iordanus in commento 8 propositionis 9 libri Arithmetice sue. Et nadir arcus *ne* sit equalis nadir arcus *me*; quare proportionem nadir arcus *ne* ad nadir arcus *me* denominat unitas. Unitas autem in quodcumque ducatur, nihil producit nisi illud in quod ducebatur. Quare si ducatur illud quod denominat proportionem | nadir arcus *ne* ad nadir arcus *me* in illud quod L ?r denominat proportionem nadir arcus *mt* ad nadir arcus *tl*, nihil aliud producetur quam denominator proportionis nadir arcus *mt* ad nadir arcus *tl*. Quare et producetur denominator proportionis nadir arcus *na* ad nadir arcus *al*. Quare proportio nadir arcus *na* ad nadir arcus *al* composita est ex proportione nadir arcus *ne* ad nadir arcus *me* et ex proportione nadir arcus *mt* ad nadir arcus *tl*.]

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137 converterimus] convertimus MRXY coniunxerimus P | al] *ah* W **138** nadir[1]] *om.* Y | lt] *ht* W **139** proportione] propositione P | ad nadir] *om.* M | en] *em sed corr.* X *tn* Y et hoc etc. *add.* S **141** Ubi…etc] *om.* LO | Ubi…155 tl] *add.* LMO(*mg. a. m.*)RX(*mg. a. m.*)Y [In L this note is placed earlier in the text.] **142** illud] *om.* O **144** propositionis] proportionis M **147** in quodcumque] quocumque modo L **150** nihil] nec M **151** quam denominator] nisi quantum denominatio L **152** Quare] quia LMR | nadir] *om.* Y **154** arcus[2]] *om.* X

Menelaus' *Sphaerica* Figures

Figura #1

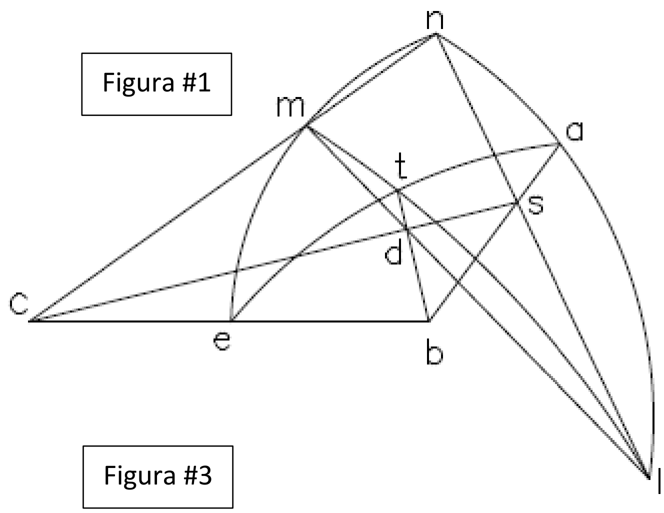


Figura #2

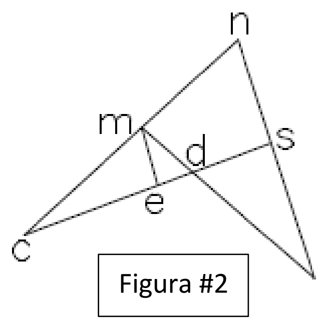


Figura #3

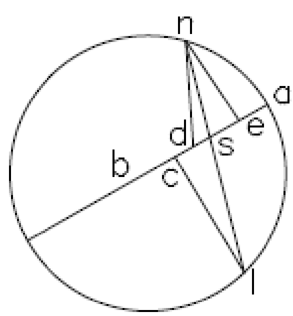
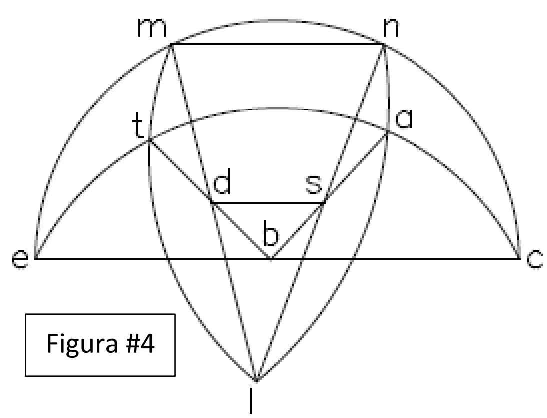


Figura #4



Appendix D: *Almagestum parvum,* Books I-II

I consulted the following manuscripts for my edition of Books I-II:

Paris, Bibliothèque nationale de France, lat. 7399 = O

Città del Vaticano, Biblioteca Apostolica Vaticana, Reg. lat. 1261 = MS F

For my edition of Book I, I also collated the following manuscripts:

Wien, Österreichische Nationalbibliothek 5292 = J

Utrecht, Bibliotheek der Rijksuniversiteit, 725 = L

Wien, Österreichische Nationalbibliothek 5273 = M

Paris, Bibliothèque nationale de France, lat. 16657 = N

Wien, Österreichische Nationalbibliothek 5266 = S

Leipszig, Universitätsbibliothek 1475 = MS T

The following have only been used in the collation of the introduction and the addition to I.6 if they contain it:

London, British Library, Harley 625 = U (has addition to I.6)

Città del Vaticano, Biblioteca Apostolica Vaticana, Reg lat 1012 = V (has addition to I.6)

Erfurt, Universitätsbibliothek Erfurt, Dep. Erf. CA 4º 356 = P

Erfurt, Universitätsbibliothek Erfurt, Dep. Erf. CA 2º, 383 = Q

Dresden, Sächsische Landesbibliothek, Db 87 = R

Basel, Universitätsbibliothek, F.II.33 = W

München, Bayerische Staatsbibliothek, Lat. 56 = X

Berlin, Staatsbibliothek, Lat. qu. 510 = Y

Toledo, Biblioteca de la Santa Iglesia Catedral, 98-22 = Z

The following manuscripts contain the *Almagestum parvum* but were not collated:

Firenze, Biblioteca Riccardiana, 885

Firenze, Biblioteca Laurenziana, Conv. soppr. 414

Kraków, Biblioteka Jagiellońska 1924

Memmingen, Stadtbibliothek, 2.33 fol. (F. 33)

Nürnberg, Stadtbibliothek, Cent. VI.12

Prag, Universitätsbibliothek, V A 11 (802)

Venezia, Biblioteca Nazionale Marciana, Marc. lat. XIV 291 (4631)

Almagestum Parvum

Omnium recte philosophantium non solum verisimilibus et credibilibus argumentis sed et firmissimis rationibus deprehensum est formam celi sphericam esse motumque ipsius orbicularem circa terram undique secus globosam in medio imoque defixam, que quidem etsi omnium cadentium tam gravitate corporis quam quantitate ponderis sit maxima ideoque immobilis. Ipsius tamen crassitudo comparatione infinitatis applani respectuque distantie fixorum luminum insensibilis et vicem centri obtinere physica indagatione comperta est. Ad hec duos principales et sibimet invicem contrarios motus superiorum sane animadverti etiam fides oculata comprobavit quorum alter semper ab oriente in occidentem pari et eadem concitatione per circulos et inter se et ad eum qui omnium spatiosissimus equinoctialem paralellos totum

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3 Omnium] incipit liber almagesti Ptholomei abbreviatus prefatio sex continens conclusiones *praem.* L incipit almagestum demonstratum de sex primis libris Ptolomei *praem.* R | non solum] *om.* JMPQUVWYZR | verisimilibus et] coniecturis MW | et] coniecturis OPSXYZ *om.* QV | et…4 credibilibus] credibilibusque JSUXR **4** argumentis] augurisque W firmissimis] verissimis L fortissimus W | deprehensum] deprensum M comprehensum W | est] *om.* J **5** sphericam] sphericum S | sphericam esse] *inv.* OY | esse] *add. mg.* M *om.* P motumque] motum quoque T **6** undique] volui terram *add. mg. a. m.* N | secus] sicut SX globosam] et *add.* M | medio] celi *add. supr. lin. a. m.* N | defixam] fixam *sed. corr. a. m.* S que] siti *add. supr. lin. a. m.* T **7** cadentium] entium P | gravitate] quantitate MOPQYZ corporis] *om.* P | quantitate] gravitate PQZ | ponderis] *mg.* W | sit…8 maxima] *inv.* MPQYZ maximaque sit O **8** tamen…comparatione] sicude(?) operatione Q **9** infinitatis] infinitas *sed. corr. supr. lin.* L | applani] applavit M applanes Q ad plani SX applanis W **10** obtinere] *om.* S optime T | physica] philosophica M philosophica *add. supr. lin. a. m.* N | indagatione] ratione T | comperta] compertum *sed. corr. supr. lin.* M compertum SWX comparata *sed. corr. exp.* R | Ad hec] adhuc Q | Ad…duos] duos insuper adhuc L | hec] hoc J **11** principales] motus *add.* W | sibimet] adversos *add.* W | contrarios] contrariosi J adversos MPQZ diversos OY | contrarios motus] *om.* W | motus superiorum] *inv.* MOPQYZ | sane] sani P *om.* U **12** animadverti] animadvertenti M | etiam] et J | fides] *om.* V | fides oculata] *inv.* MOPQZ occulta fides WY | oculata] ocultata *sed. corr. supr. lin.* T occulta UR comprobavit] approbavit T probavit W | semper] *pos.* oriente P semel T **13** in] ad M | pari] pare V | pari et] paratque P | et[1]] etiam J equali *add.* L atque MOQWYZ | concitatione] contentione JUVR concitione O **14** eum] illum OPY | qui] quidem P | omnium] *add. supr. lin.* M est *add.* SWX | spatiosissimus] spatiosimus L spatiorum U est *add.* OPQY | paralellos] parallelum P perambulans V mundane machine corpus movet et agitat cuius circumvolutio circa celestis sphere polos indefesse consistit. Alter e contrario solem et lunam et quinque erraticas circa alios diversosque polos circumducit et torquet. | V 1v His firme adeo fides conciliata est ut si quis iniuste calumnians obviet, aut cavillator verum scienter inficians | X 3v aut mente captus non indigne estimetur. Que cum ita sint superest ut propositum aggrediamur. | N 83r

(1.) Data circuli diametri latera decagoni pentagoni hexagoni tetragoni atque trianguli | S 176rb omni ab eodem circulo circumscriptorum reperire. Corollarium. Unde manifestum est quod si nota fuerit circuli diameter et prenominata latera erunt nota, corde quoque que residuis semicirculi arcubus subtenduntur note erunt.

Lineetur enim super *ag* diametrum semicirculus *abg*. [Figura 1] Sitque *db* a centro perpendiculariter erecta *h* medius punctus *dg zh* equalis *bh* | O 16r subtense angulo recto. Dico quia *zd* est latus decagoni et *zb* latus pentagoni. Ratio. Per sextam secundi Euclidis et penultimam primi

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15 mundane] meridiane O meridianale P meridiane *sed. corr. exp.* Y | mundane machine] *inv.* T | machine corpus] *inv.* MOPQYZ | corpus] etiam *add.* L | et] atque S | et agitat] atque exagitat MOPQWYZ | cuius] eius J quorum U | circumvolutio] revolutio MPYZ **16** indefesse…17 polos] *om. (hom.)* P | Alter] vero *add.* MQSWXZ aliter U | e contrario] vero OY **17** quinque] stellas *add.* M alios *add.* OY | erraticas] stellas *add.* L erraticos YZ diversosque polos] *inv.* W **18** firme] vero W | firme adeo] *inv.* SX | adeo fides] *inv.* OY fides] oculata et *add.* L | est] *om.* J | quis] quid Q | iniuste] etiam (*om.* M) iuste FLMNTV obviet] obiiciet L vel potius deviet *add.* PQSXZ **19** aut[1]] autem FJ potius deviet aut *add.* MOWY | verum] in *add.* V | inficians] inficiens JST in huiusmodi [X 3v] disciplina parum exercitatus *add.* QSX | aut[2]] autem FJNS | aut[2]…captus] *supr. lin.* M in huius disciplina parum exercitat *add. sed. exp.* M *om.* Q | mente captus] in huiusmodi diciplina parum (parium O) exercitatus *add.* OPQYZ | captus] est *add.* W **20** estimetur] existemetur M | Que] quod PV quare W | ita] ista P | sint] consistunt M(*mg.*)W constent OPQYZ | ut…aggrediamur] aggredi propositum LZ **21** Data] linea *add.* J propositio prima (*inv*. O) *add. ant.* data OS diametri] diametro MNOST | pentagoni hexagoni] *inv.* L **22** tetragoni] *del. signis* '*x*' J omni] omnium NO | omni…eodem] yssopleuros eidem L | circumscriptorum] inscriptorum L circumscriptibilium O **23** Corollarium] huius prime *add.* L corrumpere *adnot. mg.* L *om.* MST | Unde] inde T | est] *om.* J **24** diameter] eius *add.* J eius *add. mg.* M diametres M **25** semicirculi] partibus seu *add.* L circuli T | subtenduntur] intenduntur FNT | note erunt] *inv.* MOT | erunt] sunt J quia cum illis continent angulum semicirculi *adnot. supr. lin.* NO(*a. m.* O) **26** Lineetur…31 primi] quoniam imo quanticumque arcus corda nota arcusque residui de semicirculo semper nota erit L | semicirculus] semicirculum J et sit *add.* J **27** medius punctus] *inv.* ST | dg] *hz add. supr. lin. a. m.* O | zh] et *h* T **28** recto] et *gt* equalis semidiametro *add*. J | quia] quod MS | et] *om.* M **29** secundi] libri *add.* O | et] per *add.* S et nonam tertiidecimi. Patent cetera per tricesimam tertii et penultimam primi.

(2.) Si quadrilaterum intra circulum describatur rectangulum quod continetur sub duabus eius diametris est equale duobus rectangulis pariter acceptis que sub utrisque eius lateribus oppositis continentur.

Esto enim quadrilaterum cuius duo diametri *ag* et *bd* intra circulum descriptum fiatque angulus *abe* equalis angulo *dbg*. [Figura 2] Erit igitur *abd* angulus equalis *ebg* angulo communiter adiecto *ebd*. Sed etiam *adb* et *egb* anguli sunt equales quia super arcum eundem consistunt. Propter similitudinem ergo triangulorum, unde accidit proportionalitas laterum, quod sit ex ductu *ad* in *bg* equum est ei quod continetur sub *bd* et *ge*. Priori causa quod continetur sub *bd* et *be* equatur ei quod sit ex *ab* in *gb*. Restat per primam ergo secundi Euclidis argumentari. | M 36r

| (3.) Si in semicirculo corde arcuum inequalium certe fuerint, corda L 1v quoque arcus quo maior | minorem superat erit nota. S 176va

Sint enim *ab* et *ag* note, [Figura 3] ergo *db* | et *gd* quia subtenduntur J 1v residuis arcubus in semicirculo quia quadratam *da* videlicet duo quadrata

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30 nonam] etiam decimam *add.* J aut decimam decimi *add.* M et 10^{am} *add.* S | tertiidecimi] Euclidis *add.* S | Patent] quia (quoniam O) intelliges conversam ad probandum *zb* esse latus pentagoni necesse X XIII Euclidis *adnot. mg.* NO(*a. m.*) | Patent cetera] *inv.* JMOS **31** primi] Euclidis *add.* S **32** Si] propositio secunda *add. ant.* si S | intra] infra LNOT | describatur rectangulum] *inv.* T | rectangulum] rectangulus J **33** duabus] duobus FNO **34** utrisque] utriusque F | lateribus oppositis] *inv.* JMOS **35** Esto…42 argumentari] *om.* L | enim] *om.* T duo] *om.* M *add. supr. lin. a. m.* T | intra] infra NOT **36** descriptum] descripti T | angulus abe] *inv.* MO | angulo dbg] *inv.* MO **37** ebg angulo] *inv.* O | adb] *abd* S | adb…38 egb] *abd* et *ebg* M **38** anguli] *om.* T | super] sicut *sed. corr. mg.* J | arcum eundem] *inv.* JMO | eundem] *supr. lin.* M | consistunt] consistent S per 20^{am} tertii *add.* ST(*supr. lin. a. m.*) **39** ergo] qui sunt *abd* et *ebg add. supr. lin. a. m.* F *praem.* propter MOS | triangulorum] per 15^{am} 6^{i} *adnot. mg. a. m.* T | accidit] ex iiii vi Euclidis *adnot. supr. lin.* NO(*a. m.*) **40** sit] fit NOS | ductu] *om.* JO | bg] *gb* S | equum] de iii vi *adnot. supr. lin.* N ex xv vi *adnot. supr. lin. a. m.* O | est] *om.* JS | continetur sub] fit ex J | bd] *bgd sed. corr.* O | et] in J **41** Priori] consimili de MOS Priori causa] consimili ratione FNT | be] *ae* JMNOST | equatur] equum est J | sit] fit NOS gb] *dg* JMOS *gd* N **42** Restat] *pos.* Euclidis FNT | primam] ratiocinatus *add. sed exp.* M ergo] *praem.* per JMOS | secundi] libri *add.* O | argumentari] argumentum JS sit enim *ab* et *bd* nota ergo et *bd* et *gd* quia subtenduntur residuis arcubus *add.* M **43** Si] *om.* M | certe] note JS note *add. supr. lin.* M | fuerint] fuerunt O **44** erit nota] *inv.* L **45** Sint] sit M | Sint…48 nota] *om.* L | note] nota MOS | ergo] et *add.* JMO | db] *bd* nota JS *bd* MO | gd quia] que JS subtenduntur] per correlarium prime huius (*om.* O) *adnot. supr. lin.* NO(*a. m.*) subtendunt T **46** semicirculo] nota est *add.* FMNO nota fuerit *add.* T | quia…47 circumferentiam] *om.* FMNOT reliquorum laterum propter angulum rectum ad circumferentiam. Et quia diameter semicirculi notus, per proximam argue quod *bg* est nota. | N 83v

(4.) Si in semicirculo alicuius arcus corda nota fuerit corda quoque que eius medietati subtenditur erit nota.

Ex hypothesi *bg* est nota cuius arcus medius punctus *d*. [Figura 4] Ergo *ab* nota cui sit equalis *ae*. Ergo *ad* facta communi, erit *ed* equalis tam *bd* quam *dg*. Unde anguli *dab dag* sunt equales quia super equali circuli portione et latera *ab ad* sunt equalia lateribus *ad ae*. Quare demissa perpendiculari *dz* erit *gz* equalis *ez* et *gz* est nota quia *ae* nota que equalis est *ab*. Et ita *eg* nota cum diameter sit nota. Diameter nota quoque nota inter quas *dg* est proportionalis per sextum Euclidis, ergo etiam ipsa nota est. | T 2v

(5.) Si due corde duorum arcuum in semicirculo fuerint note, corda quoque que toti | subtenditur arcui composito ex illis duobus arcubus nota erit. F 1v

Ex hypothesi et *ab* et *bg* nota, [Figura 5] facta ergo tam *azd* quam *bzh* circuli diametro erit tam *bd* quam *gh* nota. Et quia *ab* nota est et *dh*,

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48 diameter] diametros M | diameter semicirculi] *inv.* FNT | notus] nota MOS | argue] *add. mg.* F argumentare M | quod…nota] *om.* FMNOT **49** Si] propositio quarta *add. ant.* si S corda[1]] *praem.* alicuius JMOS | nota fuerit] *inv.* JMOS **50** eius] eiusdem JMOS | erit nota] *inv.* L **51** Ex…58 est] *om.* L | est] *om.* FNOT | cuius] eius J | arcus] *mg.* M | punctus] scilicet *add.* JS **52** nota cui] cum nota est J | erit] *supr. lin.* N **53** dg] per 4[am] primi quia anguli sunt equales cum sint in portionibus equalibus *adnot. supr. lin.* NO(*a. m.*) *gd* T | Unde] quia JS dab] et *add.* S | dab…sunt] super N *e* et *g add. mg.* N super *e* et *g* OT | dab…54 ae] super *ez* (*e* et *g*) equales per heleufugam FM ["eleufuga" is a name given to *Elements* I.5] | quia…54 ae] *om.* NOT per heleufugam *add.* NT perelvetur [sic] *add.* O **54** ab] et *add.* S | Quare] quia M **55** demissa] dimissa *sed. corr.* O dimissa T | dz] *de* JS | equalis] *iter.* M | est] *om.* NO est…58 est] (et *add.* T) nota quoque (*om.*T) diametros quoque nota inter quas *bg* proportio(proportionalis T) quare et ipsa nota FT | nota[1]] cum triangulus *adg* et *zd*... sunt similes *adnot. mg.* N | quia…57 nota] diametrosque note MN diametrusque quoque nota O **56** Diameter] diametros S | nota[3]] *om.* S **57** nota] *zg* est nota quia duplum *eb* et *eg* quia tota diameter et reliqua pars *ae adnot. mg.* N | inter] cum trianguli *adg* et *zdg* sint similes *adnot. mg.* N | dg] *bg* S | est] *om.* NO | per…Euclidis] *om.* MO | per…ergo] *om.* N | sextum] sextam S | ergo…58 etiam] quare et S **58** etiam] et MO | nota est] *inv.* N | est] *om.* MO **59** Si] propositio quinta *add. ant.* si S **60** composito…arcubus] *om.* J | illis] *pos.* arcubus M duobus arcubus] *om.* O | nota] certa M | nota…61 erit] *inv.* JOST **62** Ex…67 propositum] *om.* L | et[1]] *om.* MT | facta] tertia JT factum M **63** diametro] diametris M | erit] per correlarium prime *adnot. supr. lin.* NO(*a. m.*) | tam] causa J | ab] *hb* J *ahb* S | nota[2]] *iter*. NS cum sint equales per 4[am] primi Euclidis *adnot. mg.* NO | est] *add. supr. lin. a. m.* J etiam S dh] nota est *add.* M

ergo | cum sit *bgdh* quadrilaterum circulo inscriptum cuius duo diametri O 16v
noti et tria latera nota, per tria erit quartum notum scilicet *dg*. Ergo et corda residui arcus de semicirculo *ag* videlicet nota est. Quod erat propositum.

(6.) Due linee inequales in circulo si protrahantur maioris ad minorem quam arcus longioris ad arcum brevioris minor est | proportio. S 176vb

Primo angulum *abg* linea *abd* per medium partiatur lineas deinceps *ag* et *ad* et *dg* protractis. [Figura 6] Quia ergo angulus *abg* per medium divisus est, lineas *ad* et *gd* constat esse equales. Linea etiam *ge* longiore existente quam linea *ea* in lineam *eg* perpendicularem *dz* protrahimus. Quia ergo *ad* quam *de* et *de* quam *dz* longiores sunt, circulus ad centrum *d* et ad distantiam *de* circumductus lineam *ad* procul dubio secabit linea, etiam *dz* altius protracta. Ipsum circulum *het* signabunt quia ergo sector portio *det* triangulo *dez* maior est sed etiam triangulum *dea* eo sectore qui est *deh* constat fieri maiorem. Erit per 8^{am} 5^{i} Euclidis trianguli *dez* ad triangulum *dea* proportio minor ea que est sectoris *det* ad sectorem *deh*. Sed sectoris ad sectorem que sui anguli ad suum angulum, ergo per primam sexti minor est proportio *ez* linee ad *ea* quam anguli *zde* ad *edh*.

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64 quadrilaterum] quadrata *sed. corr. supr. lin.* N | cuius] eius J **65** noti] note M | tria^{1}] duo M | tria^{2}] secundum et cetera J secundum M secundam O 3^{am} S | erit] et J et *add.* MOS | Ergo] *supr. lin.* N **66** semicirculo] scilicet *add.* T | videlicet] *om.* T erit *add.* T | nota est] *inv.* MO est] *om.* ST | erat] *om.* O est T | erat…67 propositum] proponebatur JMS **68** Due] si due corde vel L sexta *add. ant.* due S | Due…85 argumentari] *om.* L Si due corde vel linee inequales duobus arcubus in semicirculo subtendantur erit maioris corde ad minorem cordam proportio minor quam arcus longioris ad arcum brevioris *add.* L | linee inequales] *inv.* JMO **69** brevioris] breviorem JMS | est] erit O **70** linea abd] *om.* S | abd] *bd* MNOT | lineas] lineis NOT **71** dg] *gd* MO | abg] *om.* J **72** ad] *ab* J | ad…gd] *ab* et *dg* S | gd] *dg sed. corr. supr. lin.* F *dg* J | constat] *pos.* equales M per xxv tertii Euclidis *adnot.* N(*mg.*)O(*supr. lin. a. m.*) constat…equales] *add. mg.* M | esse] fieri FNOT **73** linea] *om.* FMOT | eg] *geg* J perpendicularem] perpendiculariter FN perpendiculare T | perpendicularem dz] *inv.* M | dz] *add. supr. lin.* F *ez* T | protrahimus] *de add.* JS **74** Quia] *om.* J | ergo] tam *add.* N(*supr. lin.*)OT | quam^{2}] *om.* J | circulus] circulo T per 25^{am} *adnot. mg. a. m.* T **75** lineam] *de add.* J **76** etiam…altius] et debet ulterius J | altius] ulterius S | het] *hez* FNST | signabunt] signabit M significabunt O | sector] sectio T **77** portio] *om.* JMO | det] *dez* FMNO | triangulo] angulo *sed. corr. supr. lin. a. m.* M | maior est] *inv.* O | sectore qui] sectione que T **78** deh] *dhe* J 8am] vanius *add. supr. lin.* N | Euclidis] *om.* T igitur per hoc quod est minus minore est minor maior *adnot. supr. lin.* F | dez] *hez* M **79** ea] *om.* J | sectoris] sector J | det] *exp. et. dez add. supr. lin.* F **80** sectoris] sector J | angulum] ex ultimam 6^{i} *adnot. supr. lin. a. m.* O **81** sexti] scilicet 8^{am} 5^{i} est(*om.* S) JS | ad^{1}] lineam *add.* T | ea] lineam *add.* JS | edh] angulum *add.* S

Ergo coniunctim ergo duple scilicet *ga* proportio ad eandem *ea* minor quam dupli anguli scilicet *gda* ad eundem *eda* angulum. Proportio est igitur disiunctim. Restat ergo per tertiam sexti et ultimam eiusdem argumentari.

Nunc quorsum hec tendant declarabimus. Interest presentis negotiationis cuiuslibet arcus noti respectu 360 graduum que est communis omnium circulorum partitio invenire cordam notam respectu 120 partium diametri ad quem numerum omnis | diameter secta J 2r intelligitur, cuius rei agnitio non minus utilis quam difficilis. Igitur ex prime speculationis ratione arcum 36 graduum habere cordam partium 37 punctorum sive minutorum | 4 secundorum 55 sollers practicus | inveniet, N 84r | S 177ra est enim ea corda latus decagoni, cordam vero pentagonicam que arcui 72 graduum subtenditur componi ex partibus 70 punctis 32 et secundis fere tribus. Sed et latus hexagoni supra quod arcus 60 graduum curvatur 60 itidem partibus terminari. Ad eundem quoque modum quia latus tetragoni existens arcus 90 partium corde quadratum medie diametros | duplat M 36v potentialiter latus trigonale existens 120 graduum corda medie diametros

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82 coniunctim] coniuncter F coniuctam ad eius correlarium *adnot. mg.* T | duple] dupli J | ea] *gda* M | minor] est *add.* S **83** eda] *gd* S *gda* T | Proportio est] *om.* M | est] *om.* JOS **84** disiunctim] disiuncti N | sexti] *om.* M **85** argumentari] declaratio propositionis sexte precedentis *add.* S **86** tendant] intendant *sed. corr.* O | declarabimus] declaremus M Interest] intus est J cuiuslibet argumentatos *add. sed. exp.* M in *add.* M **87** negotiationis] negotii LST negotii scilicet M | cuiuslibet] cuiusque L | que] qui L **88** communis] universalis JMOS | omnium] *supr. lin.* M | invenire] eorumdem *add.* S | cordam] eorumdem *sed. corr. supr. lin.* F eorumdem N | notam] cordam *add. mg.* N cordam *add.* T **89** quem] quam J numerum] universaliter *add.* JS | omnis] refertur *add.* JS | diameter] diametros FLNT generaliter *add.* MO diametri S **90** intelligitur] *om.* JS est L | agnitio] cognitio JMOT | utilis] est et laudabilis *add.* L erint M | Igitur…141 40] *om.* L videatur deinceps demonstratio 6ª primi Epythomatis *add.* L **91** prime] proprie N | speculationis] id est ex prime huius primi *adnot. supr. lin.* F id est propositionis *add. mg.* NO(*a. m.*) | arcum] qui est *add.* ST | graduum] qui est decima pars circuli *adnot. mg.* NO **92** 4] 9 N | sollers] sollerus J **93** enim] autem T cordam] corda J | vero pentagonicam] *inv.* T **94** et] *om.* JS | fere] *add. supr. lin. a. m.* J tertia *add. sed del.* J **95** Sed] *om.* JS | supra] super T | quod] quam J | arcus] *pos.* graduum T **96** terminari] per penultimi primi Euclidis *adnot. mg. a. m.* T | quoque] ergo O | quia] *om.* T latus] scilicet quadrate *adnot. supr. lin. a. m.* O | tetragoni] *om.* O **97** arcus] *add. supr. lin.* F *om.* JNOST | partium] circumferentie *adnot. supr. lin. a. m.* O | corde] corda N | quadratum] *pos.* diametros JS | duplat] per penultimam primi Euclidis *adnot. supr. lin.* F | duplat…98 potentialiter] *inv.* JMOS **98** latus] circa *add.* J vero *add.* M item *add.* NOS | trigonale] item T | existens] 15 *add.* F arcus *add. supr. lin.* F ens NO | 120] arcus supra ... quantitas est hoc(hec NO) *adnot.* L(*mg.*)NO(*supr. lin. a. m.*) | graduum] *om.* J partium O arcus *add.* T quadratum potentialiter triplat illud quod partibus 84 | O 17r punctis 15 secundis 10 fere concludi. Illud autem partibus 100 et tribus punctis 55 secundis 23 equarii diligens examinator compariet manente, dico, predicta diametri in 120 equas divisiones sectione. Ad hec ex eodem theoreumate cum sit corda arcui 36 graduum subtensa ex partibus 37 punctis 4 secundis 55 cordam que residuo arcui de semicirculo scilicet arcui 144 graduum subtenditur partibus 124 punctis 7 secundis 37 fere terminandam esse sobrius investigator agnoscet. Amplius ex sequentium demonstratione constat ad ceterorum arcuum differentias cordas multas posse invenire. Qua quidem ratione corda arcus 12 graduum reperienda est. His inquam qui sunt arcuum 60 atque 72 | T 3r cordis precognitis.

Deinceps plurimas diversorum arcuum cordas invenire inventas secundum arcum mediare sciemus utpote arcus 12 graduum cordam. Et deinde arcus 6 partium nec minus quoque trium eius, tunc qui habet

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99 quadratum] 6^{m} T | illud] scilicet latus tetragoni *adnot. supr. lin.* FO id J | illud quod] istud latus tetragoni quidem T | quod] quidem MOS | punctis] idest minutis *adnot. supr. lin. a. m.* O | 15] 51 JMOS 3 T **100** concludi] scilicet prima huius *adnot. mg. a. m.* T | Illud] scilicet latus trigoni *da* (*om.* F) *adnot. supr. lin.* FO(*a. m.*) istud JOST | 100…tribus] 103 JO | et tribus] 23 M **101** 23] 34 JS 34 *add. supr. lin.* M 33 *supr. lin.* N 33 OT | equarii] equari FNOST equali M | compariet] comparet J | dico] duo J *om.* M **102** predicta diametri] *inv.* M diametri] diameter JS | equas divisiones] portiones J equales portiones M | equas…sectione] portiones sectore S | divisiones] dicciones *sed. corr. supr. lin.* F dicciones *sed. del.* N portiones *add. mg.* N portiones O graduum *add.* T | Ad hec] adhuc M | hec] illud (istud N) respondet secunde parti corellarii prime propositionis *adnot.* (*supr. lin. a. m.*) NO hoc ST eodem…103 theoreumate] eadem proportione prima JS **103** theoreumate] scilicet ex prima primi huius *adnot. supr. lin.* F proportione prima *add. supr. lin.* M | graduum] *om.* FN qui est 10^{a} pars circuli *adnot. supr. lin. a. m.* O | partibus] graduum *add.* FN(*supr. lin.*) **104** 55…105 secundis] *om.* (*hom.*) S | cordam] corda M | de] ex M **105** graduum subtenditur] *om.* F graduum…124] *mg.* N | subtenditur] *om.* NOT | 7] scilicet *add.* J | 37 fere] *inv.* O **106** sobrius] subtilis JS | agnoscet] noscet J cognoscit *corr. ad.* cognoscet O | Amplius] ad *add.* T | ex sequentium] exequentium NT nisi extracta *adnot. supr. lin.* NO(*a. m.* O) **107** demonstratione] demonstrationem T | ad] *exp.* N ex *add. supr. lin.* N ex T | ceterorum] certorum MO | differentias] in successum sunt *adnot. supr. lin. a. m.* O differentiis T **108** posse] *om.* M | invenire] inveniri FT | 12] 14 *sed. corr. supr. lin.* N | graduum] *om.* J **109** est] *om.* O | est His] trahis J | qui] que MO *om.* S | qui sunt] que est J | sunt] corde *add.* T 60] graduum *add. supr. lin. a. m.* O | 72] 32 M 73 *sed. corr. mg.* N partium *add.* ST precognitis] quia diffinet tertii *adnot. supr. lin. a. m.* O **110** plurimas] *add. mg.* M | inventas] per 4^{am} propositio *adnot. supr. lin. a. m.* O **111** secundum] scilicet J | utpote] ut J | 12] 72 *sed. corr. supr. lin.* N | graduum] partium FMNOST **112** deinde] deinceps JS | nec] ne T trium] item *add.* JS | eius] eadem *add. supr. lin.* M | tunc] *om.* JS ratione M

partem et dimidiam. Et deinde qui ex media et quarta constabit. Docet autem hec observatio unius partis et medie cordam invenire ex parte una punctis 34 secundis 15 constare. | Retenta dico dicta diametro divisione et F 2r
ad eundem denique modum arcus medie partis et quarte cordam puncta 47 habere secundas 8. Amplius. Ex sequenti appodixi ratum est secundum arcum unius partis et medie et eius cordam quamlibet cordam multiplicis arcus posse inveniri. Nam eo arcu duplicato vel triplicato et deinceps omnes corde note occurrent. Verum cordam unius | gradus sub S 177rb
certa veritate nulla deprehendit. Ratio. Quamvis enim ad arcum unius gradus et medii corda constiterit eius tertie partis, corda sub numeri compoto nullatenus scibilis est. Eius tamen rei notitia presenti intentioni neccesaria est. Summo igitur studio et industria quamvis non verissime tamen omnis sensibilis erroris periculo depulso unius gradus corda per cordam unius gradus et medii. Sed etiam per medii et quarte in hunc modum reperta est. [Figura 7] Protrahimus in circulo cordam *ab* unius partis, *ag* vero unius gradus et medii. Quemadmodum ergo supradictum est quia *ag* ad *ab* quam arcus maioris ad arcum minoris minor est proportio *ag* autem arcus ad *ab* arcum sesquialter est linea ergo *ag* ad *ab* necessario quam sesquialtera minor erit. | Constat autem corda *ag* gradum J 2v

113 deinde qui] deinceps que J | media] dimidia JS **114** hec] hic M | invenire] *om.* FNOT parte] *add. supr. lin. pos.* una M | una] *om.* J **115** dico] itam J inquam MO itaque S | dicta] predicta O | diametro] diametri MNOST | et] *om.* FNS **116** denique] *om.* JS | cordam] corda N | puncta] punctorum M **117** secundas] secundorum M secunda NT | Ex] a M | sequenti] scilicet 5 *adnot. supr. lin.* NO(*a. m.*) | appodixi] id est probatione *adnot. supr. lin.* F ratum *add. sed. exp.* O **118** secundum] scilicet J | medie] partis *add.* sed *del.* J dimidie M | eius cordam] *inv.* O | cordam[2]] certam J **119** multiplicis] multipliciter J minoris M | duplicato] duplato M | triplicato] triplato M **120** occurrent] occurrunt NOS **121** deprehendit] deprehensio J | Ratio] *om.* J | ad arcum] *add. supr. lin.* M | arcum] cordam O **122** corda[1]] cordam T | partis] id est dimidium gradus *adnot. supr. lin. a. m.* O | numeri] numero M **123** scibilis] sensibilis *add. supr. lin.* F | Eius] cuius *add. sed. exp.* M | Eius tamen] *add. supr. lin.* M | notitia] in *add.* M **124** non] nisi O **125** omnis] omni M | erroris periculo] errore J **126** gradus] *om.* JM | medii[1]] dimidii JS | per] *add. supr. lin.* M **127** ab] *om.* JS **128** partis] scilicet *ab add.* JS | ag] alicuius J aliam S | gradus] *om.* M | medii] dimidii scilicet *ag* JS medie M | ergo] *om.* J **129** est[1]] 6ª huius primi *adnot. supr. lin.* F id est in 6ª propositione *adnot. supr. lin. a. m.* O | maioris] maior J | minoris] minorem J | est[2]] erit O **130** autem] aut J | ab[1]] ad T | sesquialter] sex qualiter J sesquialtera MS | ergo] *om.* JMO **131** necessario] *pos.* minor JS | minor] *praem.* quam O | corda] cordam OT | corda…gradum] cordam *ag* gradus M

unum puncta 34 secunda 15 habere. Unde corda *ab* maior quam gradus et puncti 2 secunda 50 profecto constabit unus namque gradus cum 34 punctis et secundis 15 gradum unum puncta 2 secundas 50 integraliter sesquialterat. Rursus *ab* lineam arcus medii gradus et quarte ipsam, vero *ag* ad unum gradum cordam statuimus. Igitur arcus *ag* ad *ab* sesquitertius est. Sed palam ex supradictis cordam *ab* punctis 47 secundis 8 concludi sed ad | hunc numerum scilicet puncta 47 secunda 8 sexquitertius O 17v
numerus est hic pars una puncta 2 secunda 50 tertia 40. Ergo corda unius gradus maior est quam pars una puncta 2 secunda 50 et minor quam pars una puncta 2 secunda 50 tertia 40. Non est ergo incongruum cordam unius gradus ponere partem unam puncta 2 secunda 50 quia minus quam in duabus tertiis unius tertii error erit. Quare multo minus quam in uno secundo sed inquisitione cordarum quam minus quam secundum fuerit postponitur. Unde manifestum est quoniam arcus dimidii gradus corda punctis 31 secundis 25 fere concluditur ad cuius quantitatis exemplar reliquas que inter duas certas cordas binatim cadunt possumus sine

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132 unum] unius *sed. corr. supr. lin.* M | puncta] punctum M | 34] puncta *add.* M | secunda] secundas J O | secunda 15] *inv.* M | Unde] cum *add.* JS | ab] sit *add.* S | quam] unius *add.* M gradus] puncta duo *add. sed. exp.* O **133** et] *om.* O | puncti] puncta JMNOT | 2] *supr. lin.* N et *sed. corr. supr. lin. a. m.* S | secunda] secunde J | constabit] haberi *add.* M | cum] est J **134** 2] et *add.* M | secundas] secunda FMOS **135** ipsam] ipsum JM | vero] *om.* FLN **136** cordam] cordas JMO | statuimus] constituimus JS | ab] *bg* ST **137** palam] patet J **138** sed…145 postponitur] *om.* JS Unde corda *ag* unum gradum (*inv.* S) puncta 2 secundas (secunda S) 50 minime complebis(completur J) que quidem summa ad 47 puncta et secundas 8 fere sesquitertia est (quia *de* est tertia pars 2^{arum} secundarum *cm adnot. mg. a. m.* J). Quia (ergo *add.* S) nec (nunc S) maior nec (nunc S) minor unius eiusdem gradus corda alio respectu consistit, optime visum est huius cordam partis unius punctorum 2 secundarum 50 minuto reputari. *add.* JS | scilicet] *om.* M **139** numerus] *om.* T | numerus est] *inv.* MO | tertia…140 50] *om.* (*hom.*) T | unius] arcus *add. sed. exp.* O **140** gradus] *supr. lin. a. m.* O | maior est] erit maior M | una…pars^{2}] *add. mg.* N | et…141 50] *om.* (*hom.*) FJ **141** 2] 3 T | ergo] *supr. lin. a. m.* O | incongruum] inconveniens L | cordam] *pos.* gradus M **142** quia] *om.* O quam T quia…145 postponitur] *om.* L **143** unius] minus N | error erit] *inv.* M | multo minus] *inv.* M **144** sed] in *add.* MOT | quam^{1}] quod NOT | quam minus] *om.* M **145** postponitur] propositum M eo quod ex hic non est sensibilis error *adnot. supr. lin. a. m.* O | manifestum] scilicet per 4^{am} *adnot. supr. lin. a. m.* O | est] *om.* FMN | quoniam] quod L quomodo *add. sed. corr. supr. lin.* M | gradus corda] *inv.* L | corda] *praem.* arcus JS **146** 25] 15 JOST | fere concluditur] *inv. pos.* punctis L | concluditur] terminatur JS | ad…153 agnitio] *om.* L **147** duas] ut id cordam arcus unius gradus et dimidii et cordam arcus trium graduum que alie sunt ... cadunt due corde que sunt corda arcus 2 graduum et corda arcus 2 graduum et dimidii *adnot. supr. lin.* F | binatim] bimediatum MN | sine…148 errore] *om.* JMS

sensibili errore deprehendere. Namque duorum | graduum cordam eius M 37r
que est dimidii ad unius et dimidii facit | cognosci adiectio. | Duorum S 177va |
tunc graduum atque dimidii corde poterit deprehendi si ab arcu trium N 84v
partium arcum medie partis differentiam sequestremus et ad hunc modum de ceteris facilis est. Ergo secundum premissorum tenorem cordarum ad arcus suos agnitio.

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148 Namque] cordam supra *add. supr. lin. a. m.* O | cordam] *om.* O **149** ad] minius *add. sed. exp.* F | cognosci] interrosti J | adiectio] adiunctio *sed. corr. in. mg. a. m.* J per 6^{am} *adnot. supr. lin.* N per 5^{am} *adnot. supr. lin. a. m.* O **150** tunc] circa J item MOST *del.* N item *add. supr. lin.* N | atque] et MST | corde] *om.* J corda MOT | deprehendi] per 3^{am} propositionem *adnot. supr. lin. a. m.* O | si] sed M **151** arcum] ad *add. sed. exp.* F *om.* J ad *add.* M *del.* N ad O | medie] dimidie ST | sequestremus] sesquitertius *add. sed. corr. mg.* M sequestramus T **152** ceteris] nunc *add.* S | est Ergo] *inv.* OT | premissorum] premissarum M | tenorem] *iter*. J **153** arcus suos] *inv.* T | agnitio] cognitio JMO sunt autem et ad hoc tabule composite *add.* M 7a *add.* T

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153 agnitio] *Additio in codicibus* JSUV: Quia tamen earum numerus et quantitas facilius ex oculo in subiecta figura deprehenditur, et scitu valde necessaria (inv. U) est in tabulis per ordinem disponantur(disponentur V) ita ut (non U quod V) unaqueque linea quatuor contineat quia(qui V) hucusque satis congrua est extensio. In prima itaque tabula partes arcuum et earum numerus subdimidii gradus augmento deorsum describuntur. In secunda vero partes cordarum non sine punctis et secundis ad prescriptos arcus pertinentium sub certo numero disponuntur (disponitur J deponuntur UV). In tertia quidem (vero U que V) partes trecesime ipsius differentie que inter quaslibet duas occurrit (occurrat U) cordas collocantur. Numero vero punctorum que ad unum numerum (mininum UV) attinent sub certa veritate et ad oculum deprehenso ab uno usque ad 30 singulas (singulos UV) singulorum que inter duas consistunt cordas particulas. Ob hoc (*om*. V) et rursus oportuna videtur (ut V) talis dispositio esse ut (*om*. U) dum (unde U) per hanc si quod (quidam U) erroris de numero vel quantitate (quantitas V) cordarum tabula ipsa contineat, agnoscatur. Et dictorum ratione verisimiliter (visibiliter U) corrigatur ut videlicet (dicunt precognita *add. sed. del*. J) cordam arcus duplicati et (*om*. V) certam cordam habentis prius cognoveris aut saltem differentia qua (quarti U *om*. V) certi (*om*. U) arcus certas (*om*. S) cordas habentes differunt (differint V) precognita (precogni V), vel si quemlibet arcum qui ad perfectionem semicirculi deest per arcum certum et (*om*. V) certe corde presciveris. Et ad (*om*. V) hunc quidem modum tabule ordinentur.

(7.) Duabus rectis lineis ab angulo uno descendentibus aliisque duabus sese secantibus ab earum descendentium reliquis terminis in easdem reflexis utralibet | J 3r reflexarum alterius conterminalem sic figet ut proportio ipsius fixe ad eam sui partem que supra fixionem est producatur | S 177vb ex duabus proportionibus ex una dico proportione quam habet sibi conterminalis reflexa ad eam sui partem que sectioni interiacet et fixioni et alia proportione quam habet alterius reflexe inferior sub sectione | T 3v portio ad eam totam cuius pars est lineam.

Exempli gratia. [Figura 8] Proportio linee *ga* ad *ea* producitur ex proportione linee *gd* ad lineam *zd* et proportione linee *bz* ad lineam *be*. Sit enim *eh* equidistans *gd*, quare proportio *ga* ad *ea* tamquam proportio

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154 Duabus] propositio septima *add. ant.* duabus S | rectis lineis] *inv.* O | lineis] ut *ag* et *ab adnot. supr. lin. a. m.* T | angulo uno] *inv.* JS **155** duabus] ut *gd* et *be adnot. supr. lin. a. m.* T | terminis] *supr. lin.* M **156** alterius] relique *add. supr. lin. a. m.* O **158** dico] videlicet L sibi] igitur J **159** eam] illam O | que] supra sectionem est producatur *add. sed. del.* S **160** alia] ex J ea S | inferior] inferiorum FN inferioris T **161** lineam] linea JS **162** Exempli…166 be] *om.* L **164** eh] *bh* S | proportio[1]] ea quod trianguli *azd* et *aeh* sunt similes *adnot. mg.* N | tamquam] eo quod sunt trianguli similes a*gd* et *aeh adnot. supr. lin. a. m.* O

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154 Duabus] *Adnotatio a. m. mg. in codice* T: Exemplum: a puncto a descendunt due linee scilicet *ag* et *ab*, et reflectantur ut *be* et *gd* interseant se in puncto *z*. Equedistantes linee in prima figura *gd eh*, in 2^a *be ha*.

Adnotatio a. m. in mg. in codice T: Pro 7^a: angulus ... trianguli *ahe* est equalis angulo *d* triangulo *adg* quia sunt inter equidistantes *he dg* per 29^{am} primi. Ergo latera illos equos ... respicientia sunt proportionalia per 4^{am} 6^i. Latera isti sunt *ae ag*, ergo sunt proportionalia. Itemque *dg he* respiciunt angulum eundem scilicet *a*. Ergo latera *dg he* sunt proportionalia ex proportione qua *ae ag* per 2^{am} 6^i. *Bz be* respiciunt equales angulos, ergo proportionales eadem propositione per quartam 6^i *he dz* respiciunt equales angulos. Ergo proportionales eadem proportione per 2^{am} 6^i. Quare proportio *he bz* cum proportione *gd dz* faciunt proportionem *ga ae*, quod sic probatur. Protraham ab *e* lineam *eh* equidistantem linee *gd*. Et quia *eh* et *gd* sunt equidistantes, sit proportio *ga* ad (*ed add. sed. del.*) *ae* sicut proportio *gd* ad *eh* per 29^{am} primi et 4^{am} 6^i. Ponam ergo autem *zd* mediam qualitercumque *dg* ad *eh* ...

gd ad *eh* inter quas *zd* linea statuatur media cuius proportio est ad *he* tamquam *bz* ad *be*.

(8.) Duabus rectis lineis ab angulo uno descendentibus aliisque duabus sese secantibus ab earum descendentium reliquis terminis in easdem reflexis | utralibet reflexarum alterius conterminalem sic figet ut L 5r proportio portionum fixe inferioris | dico partis ad superiorem producatur F 2v ex duabus proportionibus ex una inquam proportione quam habet sibi conterminalis reflexe inferior sub sectione portio ad reliquam partem que sectioni interiacet et fixioni et alia proportione quam habet relique descendentis | inferior sub fixione portio ad eam totam cuius pars est O 18r lineam.

Exempli gratia. [Figura 9] Proportio *ge* ad *ea* producitur ex proportione *gz* ad *zd* et proportione *bd* ad *ba* lineam. Protrahatur enim a puncto *a* linea equidistans *be* donec concurrat cum linea *gd*. Quare proportio *ge* ad *ea* tamquam proportio *gz* | ad *zh* inter que statuatur N 85r

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165 linea statuatur] *inv.* T | statuatur] *add. mg.* M situatur *add. sed. exp.* M | statuatur media] *inv.* S | media] *pos.* linea J | cuius] eius FN *zd add. supr. lin. a. m.* O **166** tamquam] propter trianguli similes sicut prius *adnot. supr. lin. a. m.* O | be] per similitudinem triangulorum *heb* et *dzb* probatur per 4^{am} 6^{i} Euclidis ut prius *add.* JMS **167** rectis lineis] *inv.* O | lineis] id est *ab* et *ag adnot. supr. lin. a. m.* T | angulo uno] *inv.* T **169** alterius] reflexe *add. supr. lin. a. m.* O **170** portionum] portionis JM | fixe] descendentis *add.* L **173** et alia] aliaque T | habet] *om.* T | relique…174 descendentis] reliqua descendentes J | relique…174 inferior] reliqua inferior descendens O **174** descendentis] descendentes S | fixione] sectione J | cuius…181 propositum] *om.* L **175** lineam] linea JMS *lm add.* M **176** Exempli] vel cause *add. supr. lin.* N | gratia] causa JOM | Proportio] que *add.* J | ea] *eha* O **177** zd] *dz* T | et] ex *add.* S **178** gd] *bdh* F *gdh* MNO *gh* S **179** proportio^{2}] *om.* F *add. infr. lin.* N | zh] ex 2^{a} 6^{i} Euclidis *adnot. supr. lin.* NO | inter que] *om.* N

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165 eh] *Adnotatio a. m. mg. in codice* V: Proportio linee *ga* ad *ea* est tamquam proportio linee *gd* ad *eh*. Sed proportio linee *gd* ad *eh* aggregatur sive producitur ex duabus proportionibus ex proportione sciicet *gd* ad *dz* et ex proportione *dz* et *eh*. At proportio *dz* ad *eh* est tamquam proportio *bz* ad *be*, ergo proportio *gd* ad *eh* aggregatur ex proportione *gd* ad *dz* et ex proportione *bz* ad *be*. Sed quia iam ostensum est proportionem *ga* ad *ea* esse tamquam proportionem *gd* ad *eh*, ergo proportio *ga* ad *ea* aggregatur ex proportionibus *gd* ad *dz* et *bz* ad *be*, quod est propositum.] **166** be] *Adnotatio mg. in codice* NO(*a. m.*): Media non dicitur per hoc medio loco, proportionalem sed secundam inter primam et tertiam ut patet in regula que supponitur super (supra O) 24^{a} 6^{i} Euclidis.

medium *zd* cuius proportio est ad *dh* tamquam *bd* ad *da*. Quare coniunctim *zd* ad *zh* sicut *bd* ad *ba*. Unde habemus propositum.

(9.) Si in circulo continui arcus sumantur et uterque minor semicirculo diametrus producta a communi eorum termino lineam rectam reliquos | eorumdem terminos continuantem secabit secundum S 178ra
proportionem corde dupli arcus unius ad cordam dupli arcus alterius.

[Figura 10] Fiat ergo *gh* linea perpendicularis super semidiametrum *bd* et sit medietas corde arcus duplicantis arcum *gb*. Item sit *az* perpendicularis super eandem diametrum et sit sinus arcus *ab*. Quare fient trianguli *geh* et *aez* similes.

(10.) Si unus arcus notus in duos dividatur fueritque nota proportio corde dupli arcus unius ad cordam dupli arcus alterius, ambo ipsi erunt noti.

[Figura 11] Sit *dz* perpendicularis ad cordam arcus *ag* noti, quare totus triangulus *zda* lineis et angulis notus. Item proportio *ge* ad *ea* per premissam et hypothesim nota est. Ergo proportio coniuncta *ga* ad *ea* addita unitate denominationi proportionis disiuncte fiet nota. | Ergo *ae* M 37v
nota ergo *ez* et *dz* et *ed* linee note respectu diametri circuli magni. Ergo

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180 medium] media M | zd] secundum regulam predictam *adnot. supr. lin.* NO | proportio] eo quod partialis (*om.* N) trianguli similes sunt scilicet *adh zdb* etiam (sunt O) enim equianguli ergo hanc proportio per 4^{am} 6^{i} *adnot. supr. lin.* NO | da] *dh* FN *dh sed. corr.* O | Quare] quia M **181** ba] *bh sed. corr.* O | habemus] habes JM | propositum] propositio nona *add.* S nona *add.* T **182** arcus sumantur] *inv.* MT | sumantur] sumatur S **183** diametrus] diametro J diametros MST **184** reliquos] reliquis M *om.* T **186** Fiat…189 similes] *om.* L | ergo] enim JMO | perpendicularis] perpendiculariter FJ | semidiametrum] diametrum O **187** arcus duplicantis] *inv.* O | duplicantis] duplantis J | arcum] medietas corde dupli arcum *add. mg. a. m.* T | gb] *gh* M | az] *ad* JOS **188** super] ad J | eandem] *om.* M | diametrum] semidiametrum T | sinus] medietas corde arcus duplantis J | sinus arcus] medietas arcus duplicantis arcum S **189** fient] sicut S | aez] *aed* JS | similes] et ex hoc habebitur (habebis OT) propositum cum adiutori 11(15 OT) (prime partis 29^{e} primi et 4^{e} *add.* T) 6^{i} (decima *add.* T) *add.* N(*supr. lin*)O(*supr. lin.*)T ex quibus collige propositum *add.* M **191** unius] *add. supr. lin. a. m.* FN eorum *add.* M | ipsi] illi JS **192** noti] exempli gratia *add.* O **193** Sit…201 notus[3]] *om.* L dz] *add. supr. lin.* J | arcus] *add. mg.* M | ag] et sic dividitur *ag* in duo mediis *adnot. supr. lin.* N | noti] nota M et sic dividitur *ag* in duo media *add. supr. lin. a. m.* O | quare] qualiter FN quare *iter. sed. del.* M **194** zda] et *add.* J *zad* M | notus] notis S | proportio] *om.* O **195** nota est] *inv.* JMOS **196** unitate] multe *add. mg.* M | denominationi] *add. mg.* M | proportionis disiuncte] *inv.* T | disiuncte] denominationi *add.* M **197** nota] quia totum *ag* notum *adnot. supr. lin.* NO(*a. m.*) | ez] *ed* est nota quia *ez* et *zd* sunt note *adnot. supr. lin. a. m.* O | ez…dz] *ze* et *zd* JS | dz] *zd* M | et ed] *supr. lin.* MO(*a. m.*) *om.* S | ed] *ed* est nota quia *ez* et *zd* sunt note *adnot. mg.* N | diametri] *om.* J omnes anguli trianguli orthogonii *ezd* noti sunt per circulum ei circumscriptum respectu duorum rectorum ergo respectu quattuor. Dempto ergo angulo *zd*e nunc noto | ab angulo *zda* prius noto relinquitur J 3v angulus *eda* notus quare arcus *ab* notus ergo et reliquus *gb* notus.

(11.) Si ab uno termino arcus semicirculo minoris linea ipsum arcum secans educatur donec cum diametro per reliquum eiusdem arcus terminum extracta concurrat, fiet proportio linee preter centrum transeuntis ad partem sui extrinsecam sicut proportio corde dupli arcus | T 4r de quo sermo est ad cordam dupli arcus illius que educte linee includunt.

[Figura 12] Esto igitur *gh* sinus arcus *ga* cui equidistat *bz* sinus arcus *ba* interclusi | O 18v lineis concurrentibus, quarum altera *gbe* preter centrum transiens arcum *ga* secat, altera *hae* secundum diametrum extracta fiet ergo triangulus *geh* totalis similis triangulo *hez* partiali. | S 178rb

(12.) Si arcus dicto modo divisi lineis ut prescriptum est donec concurrant eductis maior portio nota fuerit et | N 25v proportio corde dupli arcus ipsius divisi ad cordam dupli arcus lineis eductis inclusi constiterit ipse arcus inclusus notus erit.

[Figura 13] Esto *zb* medietas corde arcus *gb* noti nota. Item *db* nota quare totus triangulus *dzb* orthogonius notus est lineis et angulis. Item

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198 orthogonii] orthogonaliter FN orthogoni S | ezd] *edz* T | noti sunt] *inv.* O **201** notus^{2}] per ultimam 6^{i} *adnot. supr. lin. a. m.* O | et] *om.* J | gb] *gd* JS | notus^{3}] *om.* M propositio (*om.* T) undecima *add.* ST **203** arcum secans] *inv.* T | cum] *om.* T | eiusdem] eundem FN eisdem J arcus] *om.* O **204** fiet] fiat T **205** transeuntis] educte L | sui] suam L *om.* M **206** sermo est] *inv.* ST | que] quam J quem L **207** Esto…210 partiali] *om.* L | igitur] *om.* JMO | ga…208 arcus] *om.* (*hom.*) S | equidistat] equidistet M **208** interclusi] inclusi O | quarum] quare J gbe] *bge* FT | preter] inter T **209** hae] *hag* T **210** geh] *geb* N | triangulo…partiali] triangulus partiali *hez* | hez] *bez* MOT *om.* S | partiali] *bez* propositio duodecima *add.* S eriguitur [sic] ergo ultra per 2^{am} et 4^{am} 6^{i} *adnot. mg. a. m.* T **211** dicto] predicto MO **212** concurrant] occurrant M | eductis] eductus N | portio] *add. supr. lin.* FN | arcus…213 ipsius] *inv.* M **213** cordam] lineam L | constiterit] constituerit *sed. corr. supr. lin.* F constituerit N **215** Esto…220 notus^{2}] *om.* L | noti] non *add. supr. lin.* M | db] *ab* F *bd* J **216** quare] quia M | totus triangulus] *inv.* M | orthogonius] portione FN *om.* T | est] et *add.* S | angulis] angulo F

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201 notus^{3}] *Adnotatio mg. in codice* NO(*a. m.*): Addita: Hic generale est, in quacumque proportione duorum disiunctim si coniungantur (corrigantur O), erit denominatio proportionis coniuncte aucta supra denominationem proportionis disiuncte unitate. Verbi gratia. Denominatio proportionis 6 ad 3 est duo, denominatio vero (*om.* N) proportionis 9 ad 3 est trinarius, et sic de omnibus (omni alia N) proportionibus (proportione N).

proportio *ge* ad *be* nota per proximam et hypothesim. Quare per penultimam tertii Euclidis *ea* est nota. Ergo angulus trianguli orthogonii qui angulus est *ezd* notus a quo dempto angulo *bdz* noto relinquitur angulus *adb* notus ergo et arcus *ah* notus.

(13.) In superficie sphere duobus arcubus magnorum orbium semicirculo divisim minoribus | ab uno communi termino descendentibus F 3r aliisque duobus non minorum orbium ab illorum reliquis terminis in eosdem sese secando reflexis, utrius reflexorum alterius conterminalem arcum sic figet ut proportio corde arcus duplicantis inferiorem portionem arcus fixi ad cordam arcus duplicantis superiorem eiusdem fixi portionem producatur ex gemina proportione ex ea videlicet quam habet corda arcus duplicantis inferiorem arcus reflexi portionem qui ipsi fixo conterminalis est ad cordam arcus duplicantis reliquam eiusdem reflexi portionem et ex ea proportione quam habet corda arcus duplicantis inferiorem alterius descendentis arcus partem ad cordam duplicantis arcum ipsum cuius pars est totalem. | L 5v

Evidentia gratia. [Figura 14] Arcus magnorum orbium *ab* et *ag* in superficie sphere describimus inter quos alii duo *be* et *gd* sese intersecant apud *z*. Dico ergo quod proportio corde duplicantis *ge* ad cordam ipsius arcus *ea* dupli ex gemina proportione componitur sicut in kata disiuncta

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217 proportio] proposito S | nota] *supr. lin. a. m.* O **218** ea] *ed add. supr. lin. a. m.* O | est] *om.* FMNT | orthogonii] orthogonaliter F orthogonalii J orthogoni S **219** est ezd] *inv.* J | ezd] *exp.* O *edz add. supr. lin.* O *edz* M est *add.* S *zde* T | bdz] *bzd* S | relinquitur…220 angulus] *inv.* O **220** ergo] *om.* M | ergo…notus[2]] *om.* S 13^{a} *add.* S | ah] *ab* FMNO | notus[2]] tredicima *add.* T hoc modicum valet ad propositum *adnot. mg. a. m.* T **222** communi] in *add.* M **223** orbium] *supr. lin.* O **224** eosdem] eodem M | utrius] uterque M ulterius JS **225** arcus] *om.* L | portionem] portioni JS **227** ea] eadem O **228** qui…229 portionem] *om.* (*hom.*) S **229** eiusdem…230 quam] *mg.* F *om.* N | ex] *om.* JMO **230** quam] *om.* T | habet] *iter.* T arcus duplicantis] *inv.* FNT **231** arcus] *add. supr. lin.* M | pars…232 est] *inv.* O **233** ag] sint *add.* M **234** sphere describimus] descripti M | inter] in S | alii] ab T | intersecant] intersecent MO **235** apud] punctum *add.* O | quod] quia J | ad] totalem *add.* M | ipsius…236 arcus] *inv.*JOS **236** arcus] *om.* LM | in kata] *add. mg. a. m.* S | disiuncta] *iter. sed. del.* S

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217 Quare] *Adnotatio a. m. mg. in codice* O: Ergo disiunctim proportio *gb* ad *be* nota. Sed *bg* nota, ergo *be* nota. Ergo hec composita ex his scilicet *ge* nota. Ergo superficies contenta *ge* et *be* nota. Ergo per penultimam tertii quadratum linee contingentis circulum ab *e* puncto ducte est notum. Et quadratum semidiametri ad contactum ducti est notum. Ergo quadratum *de* est notum, ergo linea *de* nota. Ergo angulus *edz* notus cum centra latera sint nota. Cetera patent. **221** In] *Adnotatio a. m. mg. in codice* T: Notando posset subtilius et melius in hac figura, ponendo *h* ubi in rei veritate ... centrum sphere.

ex ea videlicet quam habet corda arcus ad *gz* dupli ad cordam arcus ipsum *zd* duplantis et ex ea que est corde arcus qui est duplus ad *db* ad cordam arcus ad arcum *ba* duplicis. Ratio. Centro sphere *h* posito | ab (S 178va) ipso ad notis *b z e* circulorum, dico sectiones linee ducantur linee rursum *ad* et *hb* descendentes ad notam *t* conveniant. | Sed etiam due *ga* et | *gd* (M 38r | O 19r) linee eas que sunt *hz* et *he* ad *k* et *l* puncta secantes protrahantur. Sic ergo in una recta linea sunt note tres scilicet *t k l*. Nam sunt et in superficie trianguli *agd* quantumlibet protensa et in | superficie circuli | relicti *bze*. (N 26r | J 4r) Quare superficierum communis sectio linea. Hac igitur linea protracta restat ex kata disiuncta et nona bis et 11ª semel assumpta propositum colligere.

(14.) In superficie sphere quattuor arcubus predicto modo depictis fiet ut proportio corde arcus duplicantis unum descendentium totalem ad

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237 videlicet] scilicet L | ad^{1}] *om.* O **238** duplantis] duplicantis MST | db] *bd* LM **239** ad] *gz* dupli ad cordam arcus ipsum *add. sed. del.* J | ad arcum] *ae* et unde N | arcum] ipsum JMOS | duplicis] duplicantis MT | Ratio] *add. supr. lin.* M ideo *add.* M | Centro] *h add. sed. del.* M | ab…240 ipso] a quo L **240** notis] puncta L notos N notas MOT | b…e] *del.* L *b e z* O *b* et *d* et *e* M | linee^{2}] *om.* M **241** notam] punctum L | t] tunc S | due] *supr. lin.* O **242** puncta] *praem. k* JMOS **243** tres] *supr. lin.* M *z add. sed. exp.* M | scilicet] sit M | et] *om.* JL etiam M **244** agd] per 3^{am} 11^{i} *adnot. supr. lin. a. m.* O | quantumlibet] quantum *l* et FN quamlibet T | protensa] extensa JMOS protracta L **245** Quare] quarum MNOST | linea^{1}] lineam T | igitur] recta *add.* M **246** 11a] 10ª M 5ª T | assumpta] *add. mg.* M | propositum] proportionem JL **247** colligere] collige M propositio 14ª *add.* S decima 4^{ti} *add.* T **248** predicto] supradicto JOS | depictis] de punctis J

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241 conveniant] *Adnotatio a. m. mg. in codice* O: Unde dicit "conveniant," possunt enim non convenire ut si fuerit *ab* maior quarta, ... enim protraheretur *hz* usque ad *a* posset contingere quod anguli *zhb* et *zad* valerent duos rectos. [Point *z* is here mistakenly taken to be on line *ha*.] **243** tres] *Adnotatio a. m. mg. in codice* O: De *l* et *k*: Patet sunt enim in duobus lateribus trianguli. Si vero tertium latus ad in continuum et directum protrahatur, deveniet usque ad *t*. Similiter sunt in superficie circuli *ezb* nam *l* et *k* sunt in suis diametris. Et si superficie extenderetur in infinitum *hd* semidiameter extenderetur usque ad *t* et ita centra *t* ita superficie circuli. **247** colligere] *Adnotatio mg. in codice* V: Proportio recte *gl* ad rectam *la* est sicut proportio corde arcus dupli ad *ge* ad cordam arcus dupli ad *ea* per 10^{am} huius et per eandem proportio recte *gk* ad rectam *kd* sicut proportio corde arcus dupli ad *gz* ad cordam arcus dupli ad *zd*. Item per 13^{am} huius proportio recte *td* ad *da* sicut proportio corde arcus dupli ad *bd* ad cordam arcus dupli ad *ba*. Et cum proportio recte *gl* ad rectam *la* producatur ex proportione recte *gk* ad rectam *kd* et ex proportione recte *td* ad rectam *ta* per kata disiunctam, propter hoc proportio corde arcus dupli ad *ge* [ad] cordam arcus dupli ad *ea* producetur ex proportione corde arcus dupli ad *gz* ad cordam arcus dupli *zd* et ex proportione corde arcus dupli ad *db* ad cordam arcus dupli ad *ba*.

cordam arcus duplicantis superiorem ipsius descendentis | portionem componatur ex gemina proportione ex ea videlicet quam habet corda duplantis arcum totum ab eiusdem descendentis termino reflexum ad cordam duplantem illam ipsius reflexi portionem que sectioni interiacet et fixioni et alia proportione quam habet corda arcus duplantis inferiorem sub sectione alterius reflexi portionem ad cordam arcus duplicis ad eundem reflexum cuius pars est totum. T 4v

Evidentie gratia. [Figura 15] Proportio corde arcus duplicantis arcum *ga* ad cordam arcus duplicantis arcum *ea* componitur ex gemina proportione scilicet ex proportione corde arcus duplicantis arcum *gi* ad cordam arcus duplicantis arcum *zi* et ex proportione corde dupli arcus *bz* ad cordam dupli arcus *be*. Ratio. A sphere centro *h* linee per sectiones circulorum *a b i* educantur donec singule cum singulis preter centrum transeuntibus ad notas *o d t* conveniant quas tres notas in eadem esse linea conveniet. Nam sunt et in superficie trianguli *gze* indefinita et in superficie circuli relicti *ba*, superficie, dico, quamlibet extensa. Hac igitur linea protracta *odt* per kata coniunctam et 11[am] argue quod proponitur. | S 178vb

(15.) Maximam declinationem per instrumenti artificium et considerationem reperire.

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250 duplicantis] totalem *add. sed. exp.* M | portionem] proportionem J **251** componatur] componitur J | ex ea] *pos.* videlicet M | corda] cordam J **252** duplantis] duplicantis MT reflexum] reflexu N **253** cordam duplantem] duplam M | duplantem] duplam *sed. corr. supr. lin.* F duplam N duplicantem T | duplantem illam] *inv.* O | illam] *om.* JS | ipsius] illius JS reflexi portionem] *inv.* L | sectioni] intersectioni S **254** duplantis] duplicantis MOST **255** duplicis] duplantis J duplicantis M **257** Evidentie] evidens J | Proportio] *supr. lin. a. m.* O **258** arcum[1]] *om.* L | arcus] *om.* J | arcus…ea] dupli arcus *ae* L | gemina…259 proportione[1]] *inv.* L **259** corde] *supr. lin.* N *add. mg. a. m.* M | duplicantis] duplantis O arcum] *om.* M | gi] *gd* MST **260** duplicantis] duplantis O | arcum] post *zi* M *om.* S | zi] *zl* M *zd* ST | dupli] duplicantis M | arcus[2]] *om.* M | bz] *be* ST | bz…261 arcus] *om.* (*hom.*) J **261** dupli arcus] duplicantis M | be] *bz* FST | Ratio] *add. mg. a. m.* M ideo *add.* M | sphere centro] *inv.* T | per] *om.* S | sectiones] sectionis S **262** circulorum] et *add.* T | educantur] et *ag* et *e add.* O | cum] *mg.* J **263** transeuntibus] transeuntes M | o] *e* ST | conveniant] perveniant *sed. corr. supr. lin. a. m.* M **264** linea conveniet] linee conveniunt S | conveniet] convenient J conveniat O quod eodem modo patet sicut in priori *adnot. supr. lin. a. m.* O | indefinita] infinita MO **265** ba] *ha* F | dico] ostendo istud sicut prius *adnot. supr. lin. a. m.* O quamlibet] quantumlibet MNS **266** odt] *edc* S *adt* T | kata] katam M | coniunctam] *add. mg. a. m.* M | 11am] primam S 5[am] T | proponitur] propositio quindecima *add.* S decimaquinta *add.* T **267** Maximam] solis *add.* T | instrumenti] instrumentum J

Paratur itaque lamina quadrate forme cubitalis vel eo amplius mensure ad unguem polita et planissima in cuius una superficie circulus ut modicum extra labrum relinquatur | (F 3v) describitur ipsumque labrum in circuitu in 360 partes equissime linea in centro semper posita dividitur. Et queque pars in minuta quot capere poterit subdistinguitur. Deinde ad circuli descriptionem cavatur et cavata aptissime planatur. Post hec minoris quantitatis et forme orbicularis nec minus plana quare lamina ad spissitudinem | (O 19v) labri in alia relicti spissa | (L 6r) ut cum ei super centrum inserta fuerit in una cum labro fiat in superficie et in huius minoris duobus punctis per diametrum oppositis due erigantur equales et per omnia sibi similes pinne sic ut linea secans utramque per medium pinnulam erecta sit super diametrum et a duobus | (N 26v) terminis diametri due in directam promineant lingule in extremitate sua gracillime quarum erit officium ut cum minor lamina infra maiorem super centrum rotata fuerit lingule sectiones partium in labro diametraliter oppositas numerent et indicent. Bis ergo ita paratis et minore maiori ut in ea volvi possit centraliter inserta quotiens opus erit per eas operari latus lamine quadrate super lineam meridianam in plano protractam erectum constituemus. Superficie minoris incluse ad meridiem conversa sicque aptabimus et firmabimus ut latus supremum horizonti equidistet et superficies erecta a meridiano non

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269 itaque] igitur T | quadrate] contracte J **270** mensure] *add. mg. a. m.* M | polita] posita T et] *add. supr. lin. a. m.* M | circulus] circulis J **271** modicum] modice J | describitur] describere J describatur S | labrum[2]] *pos.* circuitu M | in…272 circuitu] *om.* (*hom.*) L **272** equissime…posita] *om.* L | centro] centris *sed. corr. supr. lin. a. m.* M | semper posita] linea supposita M *inv. sed. corr.* T | dividitur] dividatur L dividatur *sed. corr. supr. lin. a. m.* M **273** queque] quaque pars | quot] quotquot M | poterit] possit J potest L | subdistinguitur] subdistinguatur LO **274** aptissime planatur] *inv.* L | hec] hoc JLST **275** minoris] minor J minoris quantitatis] *inv.* M | forme] fore J | forme orbicularis] *inv.* L | orbicularis] circulorum M | nec] non MO | quare] quam LS queritur O que M **276** super] supra M **277** in[1]] illud O in[2]] *om.* JMLOST **278** per diametrum] *pos.* oppositis L | oppositis] oppositus O | erigantur] eriguntur FNT | sibi] igitur J *om.* M **279** similes] prime *add. sed. exp.* FJ | pinne] *add. mg. a. m.* J | sic] si M | medium] mediam M **280** duobus] duabus FN | directam] directum JMT **281** promineant] premineant M | lingule] singule T | in] *add. supr. lin.* J ut *add. sed. exp.* J *om.* S | sua] *supr. lin.* N | gracillime] gratissime T | quarum] quorum J | erit] *add. mg.* J et sit *add. sed. exp.* J **282** super] supra M | lingule] singule T **283** partium] *om.* L | oppositas] oppresso J positis L appositionis T **284** Bis] eis T | maiori] maiorum F | ea] *om.* L | centraliter…285 inserta] circulariter iniecta *sed. corr. supr. lin. a. m.* M **285** erit] et sic J | erit…eas] fuerit poterimus M | operari] sic *add.* M **286** plano] planam T **287** minoris] minor J | ad meridiem] *om.* T | meridiem conversa] orientem versa JS | conversa] oblise(?) F obversa MNOT | firmabimus] confirmabimus M **288** supremum] suppositum JMN declinet quorum primum arte linelli efficies secundum experientiam perpendiculi. Solis ergo umbram circa | utramque solsticium in omni J 4v meridie observans tam diu volves interiorem rotulam donec superior pinna totam inferiorem obumbret et per hoc duorum tropicorum distantiam cuius medietas est maxima declinatio necnon et distantiam | S 179ra puncti in summitate capitum ab equinoctiali deprehendes.

Paratur et aliud commodius et facilius instrumentum. Laterem scilicet ligneum vel lapideum vel eneum quadratum | quere cubitalis M 38v latitudinis et apte altitudinis ut super latus sine tortuositate et inclinatione erigi possit sitque una superficierum levissima et equalis. Positoque centro in uno angulorum super ipsum quartam circuli describe et ab eo centro duas lineas rectas angulum rectum continentes et quartam circuli includentes protrahe et quartam circuli in 90 partes et unamquamque partium in minuta quot poteris partire. Deinde duas pinnulas tornatiles pyramidales equales longitudine et grossitudine quere et unam in centro orthogonaliter infige et alteram extremitati linee | a centro descendentis T 6r quo completo erige instrumentum super latus suum duabus pinnis ad orientem conversis. Et ea que in centro est superior et alia deorsum inferior sitque superficies in qua fixe sunt obversa orienti. Tunc

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289 linelli] libelli JMS armissis *sed. corr. supr. lin. ad.* libelli *a. m.* M | efficies] efficiet L efficiet *sed. corr. supr. lin. a. m.* M | secundum] per *add.* M | experientiam] experientia O **290** utramque] utrum J utrumque LOT **291** meridie] die M **292** pinna] *add. mg. a. m.* J prima *add. sed. exp.* J | totam inferiorem] *inv.* T **294** capitum] capita *sed. corr.* J capitis S deprehendes] propositio *add. sed. del.* S **295** et[1]] autem L etiam MT | commodius…facilius] utilis et brevius L **296** scilicet] *add. mg. a. m.* J vel *add. sed. exp.* J vel S *om.* T | vel[1]] *om.* T quadratum] scilicet quartam partem circuli id est potest sciri per officium quadrantis *adnot. mg. a. m.* T | quere] figure L quarte O **297** ut] et T | super] supra S **299** ipsum] *add. mg. a. m.* M **300** rectas] erectas *sed. corr.* T lineas *add.* T | continentes] continens L | et] *om.* O **301** includentes] concludentes L | partes] divide *add.* L | unamquamque] quamlibet L **302** minuta…poteris] quotlibet poteris minuta L | quot] *add. mg. a. m.* J | partire] *praem.* quot M **303** pyramidales] pyramides *sed. corr. supr. lin. a. m.* M | equales] in *add.* S | et[1]] *om.* T et grossitudine] grossiore FLNT | grossitudine] grossotie [sic] S | quere] *add. mg. a. m.* J quare *add. sed. exp.* J quem T | in] *om.* JOS **305** completo] expleto JMOS | pinnis] *add. mg. a. m.* J primis *add. sed. exp.* J **306** conversis] versis L *om.* T | Et] sit *add.* JS | superior] superiorum F superiore L superiori NOT superiori *sed. corr. supr. lin. a. m.* M | alia] altera J **307** inferior] inferiorum FLT inferiori MNO | Tunc] *om.* J tum *add. mg. a. m.* J nec *add. sed. exp.* J tuncque L est vero N cum O

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304 centro] [Folio 5 of T is just a small piece of parchment with a diagram.] **307** Tunc…310 attende] [This sentence does not make grammatical sense.]

perpendiculo a superiori pinna in inferiorem demisso ad meridiani superficiem et horizontis equidistantiam adapta umbramque pinnule in centro existentis quorsum in meridie cadat diligenter attende. Et per hoc sicut superius distantiam tropicorum et remotionem summitatis capitum ab equinoctiali contemplare. Notandum autem quod diversitas aliqua in maxima declinatione reperta est a diversis consideratoribus | in suis O 20r temporibus. Nam Indei invenerunt eam esse 24 graduum, Ptholomeus 23 graduum et 51 minutorum et 20 secundorum, Albategni vero 23 graduum et 35 minutorum, Arzachel quoque 23 graduum et 33 minutorum et 30 secundorum. Ideo sollerter adhuc est inspiciendum et magis visui quam auditui credendum.

(16.) Cuiuslibet puncti in circulo declivi cuius discessus ab equinoctiali est notus declinationem invenire. Unde manifesta est hec regula: si sinus portionis ab equinoctiali inchoate cuius finalis | puncti S 179rb declinatio queritur ducatur in sinum maxime declinationis productumque dividatur per sinum quadrantis, exibit sinus quesite declinationis. | L 6v |S 178*

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308 in] *om.* S | in inferiorem] *om.* T | meridiani] meridiam O meridianum M **309** superficiem] (could be superiorem L) | equidistantiam] equidistantem O | adapta] adaptata instrumentum JS adaptata instrumentum *add. mg.* M | umbramque] umbram J **310** in meridie] *pos.* cadit T **311** sicut superius] ut prius L | distantiam tropicorum] *inv.* L remotionem…capitum] zenith L | summitatis] summitatem S | capitum] capita *sed. corr.* J capitis S **312** Notandum…314 temporibus] *om.* L **313** reperta] comperta ST **314** temporibus] partibus S | Nam…esse] primum quidem ab Indis declinatio maxima inventa est L | Indei] Indi ST | eam] *om.* J | Ptholomeus] posterius invenit eam *add.* L **315** et[1]] *om.* LS | 51] *supr. lin. a. m.* N 52 T | et[2]] *om.* L | et[2]…316 minutorum[1]] *add. mg.* M | secundorum] secundarum FN | vero] *om.* L **316** et[1]] *om.* LS | quoque] aut invenit eam L vero O | 23] 33 O et[2]] *om.* LMO | et[3]] *om.* L | 30] 3 T **317** secundorum] posteriores vero 23 graduum 28 minutorum *add.* L | Ideo] *om.* L | sollerter] igitur *add.* L | adhuc] ad hoc JS | adhuc est] est ad hoc T | magis] magne J **318** credendum] et cetera *add.* L propositio 16[a] *add.* S sequitur 16[a] *add.* T **319** in] *add. supr. lin.* F *om.* N **320** equinoctiali] equinoctialia O | est[1]] *om.* L | Unde] et *add.* O | manifesta] manifestum O | hec] hac O **321** sinus] alicuius *add.* ST | portionis] *pos.* inchoate M circuli declivi *add. supr. lin.* O | cuius…322 queritur] *om.* L **322** queritur] quare N declinatio *add. sed. del.* S **323** quadrantis] id est quarta circuli *adnot. supr. lin.* F | quesite] *om.* L | declinationis] puncti finalis portionis proposite *add.* L

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318 credendum] *Adnotatio a. m. mg. in codice* T: Maxima declinatio secundum Indos 24, Ptolomeus 23 gradus 52 minuta 20 secundorum. Albategni 23 gradus et 35 minuta, Arzacel 23 gradus 33 minuta 23 secundorum.

[Figura 16] Describo circulum per polos circuli equinoctialis et etiam declivis transeuntem *abg* | F 4r infra quem equinoctialis medietas *aeg* et medietas circuli declivis *bed* ad notam *e* se intersecantes locentur. Et nota *e* vernale designet equinoctium punctus vero *d* hyemale solsticium et nota *b* estivale. | N 87r Polus equinoctialis circuli nota *z*. Arcus *eh* a declivi abscisus 20 partes contineat. Deinde arcum *zht* magni circuli circumduco. Est ergo propositum arcus *ht* quantus sit. Significatur. Cum ergo in huiusmodi figura duo arcus *az* et *ae* a communi termino descendant inter quos duo alii *zt* et *eb* ad notam *h* intersecantur et *zt* quadrans sit equalis *eb* quadranti, per katam coniunctam facto ergo sinu arcus *be* medio inter sinum *he* et sinum *ht* arcus, erit proportio corde dupli arcus *he* ad cordam dupli arcus *ht* que est | J 5r corde dupli arcus *az* ad cordam dupli arcus *ab*. Unde manifestum si sinus *he* ducatur in sinum *ab* productumque dividatur per sinum arcus *az*, exibit sinus arcus *ht*. Sinum voco medietatem corde dupli arcus. Posito igitur arcu *ab* duplicante ex partibus 47 punctis 42 secundis 40 secundum quod Ptholomeus distantiam inter

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324 Describo…349 elevationis] [In S, this section was skipped but then added in the same hand on a small leaf bound between folios 178 and 179.] | per polos] *add. supr. lin. a. m.* M circuli] *om.* LM per *add. supr. lin. a. m.* M | circuli equinoctialis] *inv.* O | equinoctialis] equinoctialem M **325** declivis] per polos *add. sed. exp.* M | transeuntem] scilicet *add.* M equinoctialis medietas] *inv.* L | aeg] *abg sed. corr. supr. lin.* F *abg* N *age* T **326** bed] *bde* JS ad notam] in puncto vernali L | Et] que JMO | Et…327 equinoctium] *om.* L **327** designet] designat MT | equinoctium] *add. mg. a. m.* J equinoctialem *add. sed. exp.* J | vero] autem L | et nota] *om.* L **328** equinoctialis] polus *z add.* T | equinoctialis…330 Significatur] mundi est per quem et punctum *h* per notam distantiam *ab* est elongatum transeat arcus circlui magni qui sit *zht* querimus arcum *h* qui est declinatio puncti *h* L | nota z] *om.* T **329** 20] 30 O per hypothesim *adnot. supr. lin. a. m.* O | contineat] contineant T | arcum] arcu *sed. corr. supr. lin. a. m.* M | magni] maximi O | circumduco] circumducto *sed. corr. supr. lin. a. m.* M circumducto S **330** propositum] propositi O | sit] fit O | Significatur] agnoscam JNS inquirere M agnoscere O cognoscam T | in…331 figura] *pos.* arcus M | huiusmodi] huius JST **331** termino] *a add.* L | descendant] descendunt L | duo^2…332 alii] *inv.* M **332** ad] super L intersecantur] se intersecant JMO | sit equalis] *inv.* M **333** katam] kata FNOT | facto ergo] factoque O | ergo] *om.* J | arcus] *om.* T **334** sinum^2] *om.* M | arcus^1] sui *add. sed. exp.* M et sic *add. mg.* M | erit] et sit J | he^2…335 arcus^1] *om.* (*hom.*) S **335** ht] *kt* M | que] qui FJ corde] M **336** manifestum] est *add.* LM **337** Sinum…342 declivi] *om.* L **338** arcu ab] *inv.* M | ex] *om.* M

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335 ab] *Adnotatio a. m. mg. in codice* O: Sed istarum 4 quantitatum proportionalium 3 sunt note. Ergo quarta nota. **336** manifestum] *Adnotatio a. m. mg. in codice* O: Scilicet per xvi vi quoniam si productum primi in quartum est equale producto secundi in tertio, si productum secundi in tertium dividatur per primum, exibit quartum.

duos tropicos invenit invenies ipsum *th* arcum ex partibus 11 punctis 40 fere componi. Ad hunc modum invenies cuiuslibet gradus finalis puncti declinationem in circulo declivi.

(17.) Cuiuslibet portionis circuli declivis elevationem in sphera recta invenire. Unde patet regula: si sinus perfectionis maxime declinationis ducatur in sinum declinationis portionis inchoate ab equinoctiali linea cuius portionis queritur elevatio productumque dividatur per sinum perfectionis declinationis illius portionis et quod exierit ducatur itidem in sinum elevationis unius quadrantis, productumque dividatur per sinum maxime declinationis, exibit sinus quesite elevationis. | O 20v *S returns to 179rb*

Elevatio portionis circuli declivis est arcus equinoctialis qui cum ipsa portione incipit et definit oriri. Ad huius rei expositionem supradicta figura in exemplum denuo assumatur. Est enim propositum quantus sit | T 6v arcus *et* et agnoscere qui est elevatio arcus *eh*. Cum ergo in huiusmodi figura *az* et *ae* arcus duo a communi termino descendant inter quos *zt* et *eb* alii duo se intersecant ad punctum *h*, quare per katam disiunctam proportio sinus *zb* ad *ba* constat ex proportionibus *zh* ad *ht* et *et* ad *ea.* De

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340 invenit] *praem.* distantiam M | arcum] *add. mg. a. m.* J **341** Ad] et M | invenies] *om.* FNOST *pos.* declivi M | gradus…puncti] puncti finalis gradus M **342** declivi] poteris invenire propositio (*om.* T) 17ª *add.* ST **343** circuli declivis] ecliptice L | in…recta] *om.* J recta] *om.* M **344** patet] hec *add.* M | perfectionis] complementi L **345** portionis] arcus L linea…346 elevatio] puncto L **347** perfectionis] complementi L | illius portionis] arcus eiusdem L | exierit] exibit J **348** sinum²] et *add.* FJM **349** sinus] maxime *add. sed. exp.* J **350** Elevatio…351 Ad] in sphera recta pro L **351** definit] desinit MNST | huius] cuius JS rei] regule T | expositionem] expositione L | supradicta] supraposita JMS **352** in exemplum] *om.* T | denuo] *om.* L | assumatur] *pos.* figura L | Est…sit] queritur autem quantitas L **353** et¹] *add. supr. lin.* F *om.* N c *add. supr. lin. a. m.* M | et²] *om.* MOS | et²…arcus²] qui est ascensio recta arcus ecliptice scilicet L | agnoscere] cognoscere MO | qui] que J quanta *sed. corr. ad*. que *mg. a. m.* M | est] sit T | huiusmodi] huius JT | huiusmodi…354 figura] *inv.* L **354** az…a] duo arcus *az* et *ae* ab uno L **355** alii…se] arcus sese L | intersecant] intersecent *sed. corr.* O | ad punctum] puncto L | punctum h] *inv.* T | katam] kata FO **356** sinus] *om.* T proportionibus] duo que sunt *add.* T | ea] *ae* J

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342 declivi] A*dnotatio a. m. mg. in codice* N: Nota a quod modus arguendi per catham coniunctam positus in isto commento multitudinem fecit musare donec percepi quod quando capit proportiones componentes que debet esse *he* ad *eb* et *zt* ad *ht* que component *az* ad *ab*-- de sinubus loquitur. Quia per 3ªᵐ de proportionalitate proportio extremorum ex omnibus intermediis componitur ... medium scilicet *eb*. Et hoc potest facere quia primus terminus secunde proportionis componentis *zt* est equalis secundo termino proportionis prime componentis et *eb* quia uterque arcus est quarta circuli. Et si sic non, enim modus simplicius et arguendi non valent sed propter causam dictam est valde bonus et brevis.

sinibus eorum arcuum loquor. Quare sinus *zb* si ducatur in sinum *ht* primum scilicet in quartum et productum dividatur per sinum *zh*, tertium exibit linea cuius proportio ad sinum arcus *ba* secundi sicut sinus *et* ad sinum *ea* quinti scilicet ad sextum. Ergo si linea illa ducatur in sinum *ea* qui est elevatio unius quadrantis et dividatur per sinum *ab* qui est maxima declinatio, exibit sinus *et* quesite elevationis. Posito ergo arcu *eh* 30 graduum invenies arcum *te* partibus 27 punctis 50 terminari. Quod si arcum *eh* ponas esse partium 60 reperies arcum *te* ex partibus 57 punctis 44. Ex his ergo constans est quod prima zodiaci pars 12ª ortus sui sive ascensionis tempus partibus 37 punctis 50. Linee, dico, equinoctialis terminat secunda 29 partibus punctis 54. Unde palam quod tertie ipsius duodecime elevationi relinquuntur de equinoctiali linee partes 32 puncta 16. Nam ascensus cuiuslibet zodiaci quarte cuilibet de recto circulo adequatur quod ex circulo per polos equinoctialis transeunte poterit deprehendi. Et vide quod uni quarte accidit alteri accidere necesse est dum circulus equinoctialis | N 87v horizonti recte sphere orthogonaliter insistat. Sufficit ergo inquisitio elevationum unius quarte | S 179va ad habendum omnes. Evidenter igitur ex his deprehenditur quot horis rectis pars zodiaci certa

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357 eorum…loquor] loquor arcuum ipsorum L | sinum] *zt add. sed. exp.* F *zt add.* N **358** primum scilicet] *inv.* L | et productum] productumque L **359** arcus] arcum N *ar add. sed. exp.* N | et] scilicet quinti *add.* L **360** quinti] *om.* L | illa] *hs* L ista T **361** qui[1]] que M elevatio] ascensio recta L **362** et] qui est sinus composite ascensionis recte *add.* L quesite…376 transierit] *om.* L **363** arcum] *supr. lin. a. m.* N | 27] partibus *add.* M **364** arcum[1]] arcus J | esse partium] *inv.* M | arcum te] *om.* M | te] *de sed. corr.* O **365** constans est] constat JS | zodiaci pars] *inv.* O | sui] *om.* M **366** 37] 27 MST **367** terminat] terminatur JM | 29] 30 *sed. corr.* N | palam] est *add.* M | tertie] tertius O ipsius…368 duodecime] *inv.* T **368** elevationi] levationi T | relinquuntur] relinquitur M linee] linea MS | puncta] puncti J **369** ascensus] ascensiones M | cuiuslibet] quarte *add.* S zodiaci quarte] *inv.* M | cuilibet] circulum J quarte *add.* M cuiuslibet T **370** quod] *add. mg. a. m.* J qui *add. sed. exp.* J | equinoctialis] equinoctiales J **371** vide] *exp.* J deinde M | uni] unicumque M | accidit alteri] *om.* N | accidere] poterit deprehendi *add.* M **372** recte sphere] *inv.* M | insistat] insubstat J **373** inquisitio] *pos.* quarte M **374** Evidenter] *add. mg. a. m.* J eunter *add. sed. exp.* J | deprehenditur] *add. mg. a. m.* J reprehendi *add. sed. exp.* J | certa] *add. mg. a. m.* J circa *add. sed. exp.* J *exp.* O circa *add. supr. lin. a. m.* O

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357 Quare] *Adnotatio a. m. mg. in codice* O: Sicut se habet *zh* ad *ht* sic se habet *zb* ad aliquam aliam lineam. Sit illa *m*. Ergo proportio *m* ad *ba* est proportio *et* ad *ea*. Sunt ergo iiii quantitates proportionales *zb* et *m* et *zh ht* et sic patet quod dicit "quare sinus et cetera." *adnot. mg. a. m.* O meridianum circulum ubique locorum et ab horizonte recte sphere transierit.

Book II

[From this point, my edition relies upon MSS F & O]

Orizon declivis est cui polus elevatur. Sphera declivis est hiis qui orizonte declivi utuntur. Cenith capitis est punctum summitatis capitis et est polus orizontis. Longitudo regionis est distantia eius ab orientis vel occidentis principio et est arcus paralelli ad equinoctialem inter zenith capitis et eum circulum qui est super amphitritis circuitum in celo est dispositus. Latitudo regionis est distantia cenith capitis ab equinoctiali et est arcus meridiani inter cenith capitis et circulum equinoctialem | O 21r
interceptus. Locus notus dicitur cuius longitudo et latitudo nota. Speralis angulus dicitur angulus ex duobus arcubus in superficie sphere provenient. Speralis angulus rectus dicitur cui sub duobus arcubus maiorum orbium contento quarta circuli supra cuius polum ipse angulus consistit subtenditur.

(1.) Arcum diei minimi vel maximi in quovis climate per notam poli altitudinem cognoscere. Unde manifestum quod si sinus altitudinis poli ducatur in sinum maxime declinationis et productum dividatur per sinum perfectionis maxime declinationis et quod proveniat ducatur in semidiametrum, productum dividatur per sinum perfectionis altitudinis poli, exibit differentia mediata minimi diei ad equinoctialem diem.

Sit ergo meridiei circulus *abgd* infra quem orientalis medietas orizontis *bed*. Sed etiam equinoctialis *aeg* designentur australem polum nota *z* hyemale solsticium ascendens in orizonte nota *h* notat. Deinde circuli per utrumque polum transeuntis quarta *zht* deducatur. Quia ergo *h* et *t* note motu suo parallelos in spera describunt circulos et spere revolucio super polos utriusque circumducitur, constat notas *h* et *t* ad arcum *ab* meridiani circuli uno et eodem tempore pariter devenire propter similes paralellorum circulorum porciones. Tempus autem quo nota *h* ad medium celum ab ortu suo conscendit est quantitas arcus *ta* de linea equinoctiali. Tempus autem a medio sub terra celo ad oriens est quantitas arcus *gt*. Quod inde apparet quia ipsius diei tempus est quantitas arcus ad

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375 ubique] *add. mg. a. m.* J verum *add. sed. exp.* J **380** est[2]] vel obliqua *add.* F **381** capitis[1]] capitum F | capitis[2]] capitum F **382** Longitudo…385 dispositus] *pos.* interceptus F **384** capitis] capitum F | est[1]] *om.* F | amphitritis] amphitrias O **385** capitis] capitum F **386** capitis] capitum F **392** diei] circulum F **395** proveniat] provenerit F **400** notat] notet O **405** similes] scilicet *add.* O **407** est…408 tempus] *add. in. mg.* O *ta* duplicis. Noctis vero tempus est quantitas arcus qui ad *gt* duplus est. Est ergo arcus *te* differentia equinoctialis et minime diei cum *e* sit medius punctus arcus *ag* ad quem punctum oritur aries vel libra. Hiis ita se habentibus vide quod inter duos arcus *az* et *ae* due quarte circulorum se intersecant scilicet *eb* et *tz*. Quare per kata disiunctam proportio sinus *zb* ad *ba* producitur ex proporcione sinus *zh* ad *ht* et sinus *et* ad *ea*. Sed primum est notum et secundum propter altitudinem poli notam et tercium propter maximam declinacionem notam esse et quartum similiter. Sextum vero quia est quarta circuli. Quapropter et quintum notum erit.

(2.) Arcum orizontis in quovis climate qui est inter ortum | tropici et O 21v equinoctialem per assignatum minimi diei arcum investigare. Unde patebit quod si ducatur sinus dimidii arcus diei minime in sinum perfectionis maxime declinationis productumque dividatur per sinum quadrantis exibit sinus perfectionis arcus orizontis qui est inter ortum utriuslibet tropicorum et circulum equinoctialem; similique ratione inveniri potest distantia ortus cuiuslibet signi vel gradus ab equinoctiali.

Premissa dispositione sicuti est manente arcu *ht* querimus. | Quare F 5r per kata coniunctam conversis proporcionibus proportio *at* ad *ae* de sinibus loquor producitur ex proportione sinus *bh* ad sinum *be* et eiusdem *be* sinus proportione ad sinum *hz*. Sed ex eisdem proportionibus constat proportio sinus *bh* ad *hz*. Ergo proportio sinus *at* ad sinum *ae* est sicut proportio sinus *hb* ad sinum *hz*. Ergo si primum ducas in quartum etc. Sed primum notum ex ypothesi quod arcus *ta* medietatis diei minime est tempus et quartum notum quia maxima declinatio nota et secundum notum quia est quarta circuli. Ergo tercium notum. Ergo eius arcus scilicet *hb* notus. Ergo reliquus de quarta scilicet *he* arcus notus est, quod proponebatur. Posito ergo quod dies longissima 14 horas rectis et media terminetur ut est in rodos insula invenies arcum *eh* partes 30 de ccclx continere.

(3.) Altitudinem poli per arcum diei minimi notum presto indagare. Regula: si sinum differentie medie diei minimi ad equinoctialem diem ducas in sinum perfectionis quarte orizontis, productumque dividatur per sinum arcus orizontis qui est inter ortum tropici et equinoctialem; atque quod exierit ducatur in sinum quadrantis productumque dividatur per sinum arcus medii minimi diei, exibit sinus altitudinis poli.

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409 Noctis vero] *inv.* F | duplus est] *inv.* O **423** similique ratione] simili quoque F **425** arcu ht] arcuum *he* O **426** coniunctam] et *add.* O **427** sinibus] similibus F | eiusdem] eundem F **429** bh] *hb* O | sicut] sinus *sed. exp.* O **431** notum] *om.* F | quod] quia O **434** reliquus] reliquum O **436** ccclx] 360 O **443** minimi] *add. supr. lin. a. m.* O

Supposita figura denuo assumpta quantitatem arcus *zb* que est altitudo poli. Querimus igitur per kata disiunctam proportio sinus *et* arcus ad sinum arcus *at* componitur ex proportione sinus *eh* ad *hb* sinum et proportione sinus *zb* ad sinum *za*. Quare si ducas primum in quartum et productum dividas per tertium, exibit quiddam quod sic se habebit ad secundum sicut quintum ad sextum. Sed tria nota. Duo enim per ypothesim, tercium quia est quarta circuli, ergo quartum notum est, quod intendebamus. | Posito ergo arcum diei minimi habere horas rectas 9 et dimidiam invenies altitudinem poli esse fere 36 graduum. O 22r

(4.) Arcum orizontis qui est inter ortum tropici et equinoctialem per altitudinem poli notam reperire. Unde patet regula. Si sinum maxime declinationis ducas in semidiametrum et productum dividas per sinum perfectionis altitudinis, exibit sinus arcus orizontis qui inter tropicum et equinoctialem deprehenditur.

Resumatur eadem figura nota quantitate arcus *zb*, querimus arcum orizontis *eh.* Igitur per kata coniunctam conversis proporcionibus et propter arcus *eb* et *zt* equales esse constat sinum *ab* ad sinum *az* eandem proporcionem habere quam sinus *th* ad sinum *eh.* Sed primum notum est quia est sinus perfectionis altitudinis poli note, et secundum quia est semidiametrum circuli, sed etiam tertium quia est sinum arcus maxime declinationis, quare quartum notum. Simili modo est cognoscere quemlibet arcum orizontis inter quemcumque gradum circuli declinis et equinoctialem deprehensum eo quod cuilibet gradus declinatio ex premissis est nota.

(5.) Quilibet duo circuli paralelli circulo equinoctiali eiusdem longitudinis a duobus tropicis sive ab ipso equinoctiali equales arcus orizontis ex utraque parte equinoctialis resecant, et sit alternatim nox unius diei alterius equalis.

Repetita itidem eadem figura [Figura 17] in ipsa duos circulos *hl* et *km* paralellos equinoctiali describimus et notam *q* quasi polum septentrionalem et ab eo per notam *k* quartam circuli magni *qks*. Quia ergo circuli *km* et *hl* eiusdem longitudinis sunt ab equinoctiali, eos equales esse constat et orizontem quia circulus magnus est equales arcus ab eis abscindere. Item *sg* equalis est arcui | *ta* quia similes eorum equales sunt. Relinquitur ergo arcus *se* equalis arcui *ta*, sed et arcus *ht* arcui *ks* propter declinationes equales esse. Sed et angulis *kse* angulo *hte* eo quod F 5v

444 Supposita] supraposita F **447** sinum] proportionem O | ducas] dividas F **455** dividas] *om.* F **459** et] *om.* F **460** zt] *et* F | esse] *om.* O **461** sinus] sinum O | est] *om.* O **464** est] *iter.* O **470** ex…equinoctialis] *pos.* resecant F **472** Repetita] recepta O **473** q quasi] *om.* F **475** ab equinoctiali] ad equinoctialem F **478** ht] *at* F uterque circulus erectus est super equinoctialem. Quare basis basi equalis scilicet arcus *ek* arcui *eh*, quod proposuimus.

(6.) Nota solis altitudine proporcionem umbre iacentis ad gnomonem | O 22v erectam vel umbre verse ad gnomonem iacentem invenire; et conversim, nota proporcione umbre ad gnomonem altitudinem solis indagare. Regula: Si sinum perfectionis altitudinis ducas in partes gnomonis quantaslibet et productum dividas per sinum altitudinis, exibunt partes quantitatis umbre similes partium gnomonis. Et econverso si radicem duorum quadratorum gnomonis et umbre cum nota sint extrahas et per eam id quod ex ductu gnomonis in semidametrum provenit dividas, exibit sinus quesite altitudinis.

[Figura 18] Sit ergo circulus altitudinis *adg* supra centrum *e* et *aeg* linea a summitate capitis perpendiculariter demissa supra lineam *gz* que linea orizontis intelligitur et est quidem super terram locata propter insensibilem tamen terre quantitatem ad celum centrum constituitur. Et sit *eg* gnomo erectus et *d* altitudo solis ab *f* quasi orizonte. Erit ergo radius solis per summitatem gnomonis *dez* et longitudo umbre *gz*. Propter similitudinem ergo triangulorum proporcio *et* ad *dt* eadem que *eg* ad *gz*. Cum ergo *et* sinus altitudinis notus et *dt* sinus perfectionis alter notus et quantitas gnomonis nota, erit quartum scilicet umbra nota. Pari ratione si *eb* sit gnomo iacens et *bc* umbra versa ponatur. Rursum si *ge* et *gz* sint nota, ergo *ez* basis que subtenditur angulo recto nota cuius ad *ed* semidiametrum est proporcio ut *ge* ad *et*. Simili modo *hf* arcus potest innocescere per umbram *gp*. Si ergo *h* sit maxima solis in meridie altitudo *zd*, minima erit *dh* distantia duorum tropicorum et eius medietas maxima declinacio circuli declinis.

(7.) Sub linea equinoctiali omnes dies sunt equales noctibus et sibi invicem et omnes stelle ortum habent et occasum et umbre quandoque meridiane quandoque ad meridiem quandoque ad septentrionem quandoque nusquam declinant... | O 23r

(8.) Sub omni alia linea equidistante linee equinoctiali bis tantum dies sit equalis nocti in anno et dies estivi hibernis prolixiores noctes vero breviores, et quanto ab equinoctio distantiores dies estivi productiores hyberni vero correptiores | F 6r et quedam stelle apparentes semper quedam occulte semper, et distiantia cenith ab equinoctiali equalis altitudini poli...

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483 erectam] erectum F **485** Si] *om.* F **486** sinum] cordam F cordam *sed. corr. supr. lin.* O **489** et] *om.* O **497** proporcio] *om.* F | eadem que] eademque *sed. exp.* F **503** umbram] umbras O **507** quandoque] *om.* O

(9.) Sub remotiori linea ab equinoctiali maior est inequalitas | O 23v dierum et noctium et maior pars celi apparens semper et maior pars celi occulta semper...

(10.) Sub omni linea cuius distantia minor ab equinoctiali maxima declinatione umbre meridiei ad utramque partem alternatim declinant et bis in anno declinatione carent...

(11.) Sub linea cuius descessus equalis est maxime declinationi umbra semel in anno declinatione caret et umbra meridiana numquam declinat ad meridiem...

(12.) Sub linea cuius discessus est ut poli zodiaci ab equinoctiali umbra in aliquo die ad omnem partem orizontis flectitur et sit spatium 24 horarum dies sine nocte et ex opposito nox sine die et quanto discessus ab hac linea maior maius tempus abit sine nocte et ex opposito maius tempus sine die... | O 24r F 6v

(13.) Sub polo medietas celi est apparens semper et medietas occulta semper et anni spatium dies una cum nocte sua...

(14.) In sphera declivi quilibet duo arcus equales circuli declivis et equaliter a puncto equinoctii distantes equales habent ascensiones.

[Figura 19] Sit ergo circulus meridianus *abgd* infra quem orizontis orientalis medietas *bed*, sed equinoctialis *aeg*, sitque *hz* arcus circuli declinis inchoata a puncto equinoctii et sit si placet signum piscium et est *z* punctum sectionis equinoctialis et circuli declinis sinus piscium et principium arietis. Palam ergo quod arcus *hz* oritur cum arcu *ez* quia *h* et *e* puncta pariter veniunt ad orizonte. Dico quod cum arcu equinoctiali equalis arcui *ez* oritur signum arietis et *t* idem punctum equinoctii communis sectio, palam ergo quod arcus *tk* oritur cum arcu equinoctialis *et*. Dico ergo quod arcus *ez* equalis est arcui *et*. Sint itaque note *m* et *l* duo poli et ab eis arcus magnorum circulorum *mh me mz lt le lk*. Quia ergo triangulus *mhz* equalis est triangulo *ltk*. Cum propter quartas magnorum circulorum, cum propter equales declinaciones principii piscium et sinis arietis, cum ex ypothesi, sunt ergo anguli *hmz* et *tlk* equales. Sed et arcus *he* equatur arcui *ek* ex quinta huius libri, est ergo angulus *hme* angulo *elk* equus. Relinquitur ergo angulus *emz* equalis angulo *elt* et latera continentia hos angulos sunt equalia. Ergo arcus *ez* equus est arcui *et*, quod intendebatur. Pari modo duo quilibet arcus maiores vel minores propositis inchoati a puncto equinoctii. Si equales sunt, equos habent ortus, et quia si ab equalibus equalia demantur, et est palam quod omnes

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524 discessus] discessio O **527** linea] *om.* F **534** orientalis] *inv.* O | sed] et *add.* O | hz] *haz* F **535** sit si] si sic F **536** z] *om.* F **539** equalis] *om.* O **542** lk] *lr* f **543** equalis] equilaterus F **549** intendebatur] intendebamus O | duo quilibet] *inv.* O equales et equaliter distantes a puncto equinoctiali equales habent ascensiones, quod proponitur.

(15.) Quilibet duo arcus circuli declivis equales et equaliter | ab O 24v alterutro punctorum tropicorum distantes habent in spera obliqua ascensiones coniunctas equas eis ascensionibus quas idem arcus habent in spera recta coniunctis. Ex quo et premissa proporcione [sic] manifestum est quod si note fuerint ascensiones unius quarte in spera obliqua note erunt ascensiones omnium.

[Figura 20] Describemus ad hoc circulum meridiei in duobus locis *abgd* infra quem orizontis medietas *bed* et medietas circuli equinoctiali *aeg*, et sit *t* punctum vernale *z* punctum autumpnale. Notandum autem quod cum orizon rectus per polos spere transeat et orizon declinus ipsum ad puncta equinoctialia secat necessario cum polus septentrionalis elevetur super eum inclinatur ab orizonte recto ad septentrionalem et elevatur super eum ad austrum. Unde sit ut arcus zodiaci a vernali puncto inchoatus et citra initium libre terminatus quantuscumque sit minorem moram faciat oriendo in orizonte declini quam oriendo in orizonte recto. Simul enim hic et ibi incipit, sed hic tardius oriri desinit. Econverso quilibet arcus ab autumpnali puncto inceptus et citra principium arietis finitus maiore moram facit ascendendo in spera declini quam ascendendo in spera recta. Simul enim incipit hic et ibi, sed hic prius oriri desinit. Differentias ergo ascensionum equalium arcuum | hinc inde sumptorum F 7r equales esse ostendemus. Et quia quilibet duo arcus equales ad punctum equinoctialem conterminales equales habent in quacumque spera eadem ascensiones, sit *th* arcus quantumlibet circuli declinis ad vernale punctum *t* finitus, et sit si placet signum piscium et *zh* equalis arcus signum libre et *khl* quarta orizontis recti a polo *k* australi venientis. Oritur itaque arcus *ht* in spera declini cum arcu *et* et in spera recta cum arcu *tl*. Est ergo differentia arcus *el*. Rursum arcus *zh* oritur infra declini cum arcu *ze* et in spera recta cum arcu *zl*. Est ergo differentia arcus *le*. Dico quod hee differentie equales sunt. Nam duo arcus *hl* et *hl* sunt equales propter eandem declinationem sinis libre et principii piscium et arcus ab orizonte recisi *he* et *he*. Cum sint hiidem equales et angulus *hle* utrobique rectus, ergo arcus *el* arcui *el* est equalis. Hoc enim similiter accidit in curvilineis maiorum orbium triangulis sicut in rectilineis cum angulus qui est ad *h* super polum equinoctialem non consistat et angulus qui est ad *l* sit rectus

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556 idem] hiidem O **558** est] *om.* F **562** z…autumpnale] *mg.* O | Notandum] nota O **570** principium] inicium O **573** hinc] hic O **580** el] *le* O | Rursum…581 le] *mg.* O **582** equales sunt] *inv.* O **584** sint hiidem] sit idem F **585** curvilineis] curvas lineas F **586** triangulis] trianglus F vel recto maior. Eodem modo constare potest de quibuslibet maioribus vel minoribus hiis arcubus sibi invicem equalibus. Palam ergo quod si note fuerint | ascensciones unius quarte, note erunt ascensiones omnium. O 25r Quia ascensiones a principio arietis usque ad initium cancri si note sunt, erunt note ascensiones ab initio capricorni usque ad principium arietis. Propter ascensiones equales esse et note erunt ascensiones ab initio cancri usque initium libre sive ab initio libre ad initium capricorni. Quia cum has ascensiones notas in spera declini quodlibet partium minueris ab ascensionibus earumdem partium in spera recta duplicatis prius notis, relinquuntur ascensiones quesite sumptarum partium, et hoc est quod volebamus.

(16.) Cuiuslibet portionis circuli declivis ascensionem in spera declivi invenire. Regula operationis. Si sinum altitudinis poli duxeris in sinum declinationis portionis inchoate ab equinoctiali puncto et productum dividas per sinum perfectionis declinationis et quod exierit itidem ducas in semidiametrum et productum dividas per sinum perfectionis altitudinis, exibit sinus differentie elevationum sumpte partis in spera recta et spera declivi.

Resumpta superiori figura arcum *el* querimus qui est differentia elevationum in spera recta et declini attinens arcui zodiaci *th*. Vides ergo quod in hac figura duo arcus *ak* et *ae* a communi termino *a* descendunt inter quos duo alii *kl* et *eb* se invicem secant ad punctum *h*. Per kata igitur disiunctam cum hec quinque sint nota, *kb* altitudo poli primum et *ba* secundum perfectio altitudinis et *kh* tercium perfectio declinationis et *hl* quartum declinatio sumpte partis et *ea* sextum quarta equinoctialis, erit quintum *el* notum. Quod si dempseris a *tl* noto quia est elevatio in spera recta, relinquitur et notum, quod est elevatio quesita arcus *ht* in spera declina. Est alia via et faciliori idem deprehendere.

(17.) Differentiam ascensionum in spera recta et spera declivi eiusdem portionis per arcum circuli magni a polo venientis determinare.

[Figura 21] Ponam circulum meridianum *abgd* et medietatem orizontis *bed*, sed et equinoctialis *aeg* et medietatem circuli signorum *hez* et sit *e* punctum vernale communis sectio trium circulorum in situ et nota *l* polus. Sumam ergo porcionem a puncto vernali *e* iam exortam quantam voluero et sit *et*, et describam super *l* polum et super *t* quartam magni | O 25v

590 fuerint ascensciones] fuerit ascendens F **592** ad] *om.* O **593** note erunt] *inv.* O **594** ab…libre[2]] *supr. lin.* O **595** has ascensiones] *inv.* O | quodlibet] quot O | minueris] minimus F **596** earumdem] eorum O **603** et productum] productumque F **606** arcum] arcuum F | qui] que F **608** a[1]] et F | descendunt] descendant F **610** nota] *om.* F **622** super[1]…t] *om.* F orbis *ltm*. Palam ergo quod portio | *et* oritur in spera recta cum arcu F 7v equinoctiale *em*. Determinabo per quartam magni circuli cum quo arcu oritur in spera declini. Describo ergo a puncto *t* arcum circuli equidistantis circulo equinoctialis donec secet arcum orizontis ad punctum *k* et sit *tk* et super polum et punctum *k* quartam magni orbis *lkn*. Dico quod cum arcu *mn* oritur portio *et* in spera declini et enim oritur cum arcu equidistantis *tk* simili arcui *mn*. At cum eadem portione oriuntur similes equidistantium arcus in omni loco et omni tempore. Est ergo *en* differentia ascensionum determinata per quartam magni circuli *lkn* transeuntem semper per commune punctum orizontis et equidistantis cuius distancia ab equinoctiali est ut declinatio porcionis sumpte. Unde et arcus *kn* equalis est arcui *tm*.

(18.) Cuiuslibet portionis elevationem in spera obliqua alia via rationis invenire. Unde manifestum erit quod si sinus differentie equalis diei ad minimum ducatur in sinum elevationis sumpte portionis in spera recta et quod exierit dividatur per sinum quadrantis, exibit sinus quesite differentie.

[Figura 22] Reponam ergo forma circuli meridiani et dimidii orizontis et dimidii equinoctiali et poli meridiani qui sit *z* et sit *e* punctum vernale et sit *zht* determinans differentiam elevationum tocius quarte ab initio capricorni ad finem piscium transiens per punctum commune orizontis et equidistantis tropici *h*. Est ergo *et* tota differentia et palam quod idem arcus *et* est differentia dimidia diei equalis ad minimum. Sit iterum quarta magni circuli *zkl* determinans differentiam elevationum portionis minoris quamcumque voluero, et sit piscium transiens per punctum *k* commune orizontis et illius equidistantis cuius distantia ab equinoctiali ut declinatio principii piscium vel alterius portionis sumpte. Est ergo arcus *el* differentia. Vides itaque arcus duorum magnorum orbium *et* et *tz* a communi puncto *t* venientium inter quos alii duo *eh* et *zl* se invicem secant super punctum *k*. Ergo per kata disiunctam proporcionem *zh* ad *ht* componunt proporcio *zk* ad *kl* et proporcio *el* ad *et*. De sinibus loquor. Sed eandem proporcionem componunt ut per ultimam prioris libri constat proporcio *zk* ad *kl* et proporcio sinus elevationum sumpte portionis scilicet piscium in spera recta ad sinum tocius quarte. Ergo proporcio sinus *te* totalis differentie ad sinum | O 26r differentie *el* equalis est proporcioni semidiametri ad sinum ascensionis

640 forma] scema O **642** determinans differentiam] *inv.* F **645** diei equalis] *inv.* O **650** duorum] duos O **653** proporcio[1]] *om.* O | ad[3]…654 et] *supr. lin.* O **655** sinus] tocius quarte ad sinum *add.* F **656** portionis] partis O | piscium] piscis O | ad…657 quarte] *om.* F piscium in spera recta. Ex quatuor ergo proporcionalibus tria sunt nota, primum propter arcum minimi diei notum esse et tercium quia semidiameter est, et quartum propter ascensiones omnes in spera recta notas esse. Collectis ergo de gradu in gradum huiusmodi differentiis usque ad completionem unius quarte subtrahantur gradatim ab ascensionibus quarte in spera recta illius que est ab initio arietis ad principium cancri vel illius que est a capite capriconi ad caput arietis. Addantur vero ascensiones in spera recta illius quarte que est ab initio cancri ad capite libre vel illius que est a capite libre ad principium capricorni, et sic invenientur omnes elevationes partium circuli declinis in spera obliqua, quod erat propositum.

(19.) Per notas ascensiones et locum solis notum quantitatem arcus diei et quantitatem arcus noctis et numerum equalium horarum diei vel noctis et tempora inequalium ascendensque et medium celi in omni hora reperire.

Quia enim magni circuli sunt circulus signorum et orizon necessario semper per equalia se secant. Unde necessario ab ortu solis ad occasum sex signa feruntur super terram et ab occasu ad ortum sex signa sub terra. Quare in spera cuius diem querimus ascensiones medietatis zodiaci | late F 8r super terram illa die sunt quantitas arcus diurni, quam cum minuerimus a toto circulo remanet quantitas arcus noctis eo quod in nocte et die completur una revolutio. Cum ergo acceperimus ascensiones a loco solis in oppositum sit quantitas diei, et cum acceperimus ab opposito solis ad partem solis, sit quantitas noctis. Et quia equalis hora est ascensio 15 graduum idest equinoctialium, si quantitatem arcus diurni notam diviseris per 15 vel nocturni, similiter exibit numerus equalium horarum diei vel noctis quam quesieris. Et si numerum equalium horarum diei dempseris de 24, remanet numerus horarum noctis vel econverso quia dies cum nocte 24 horas equales continet propter revolucionem 360 graduum. Et quia inequalis hora 12 pars diei dicitur quantacumque dies sit, tempus vero hore ascensio gradus equalis, palam quod si arcum diei in 12 diviserimus, exibunt tempora que sunt quantitas hore inequalis diei, et de horis noctis similiter. Autem si volueris considerare secundum ascensiones quid intersit inter arcum diei in spera obliqua et arcum eiusdem diei | in spera recta, dimidie differentie sextam vel totius O 26v duodecimam accipe. Et si locus solis septentrionalis fuerit, ad 15 adde, et si meridionalis, de 15 deme, et fient tempora hore inequalis. Et ex

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659 piscium] *om.* O | sunt] *om.* O **666** vero] ad *add.* O **677** cuius] circa *sed. corr. supr. lin.* O **678** minuerimus] minuimus O **682** hora est] *inv.* F **683** graduum] equalium *add.* O diurni] diei O **693** recta] et *add.* O premissis patens est, et si quantitatem hore diurne de 30 dempseris, remanebit quantitas hore nocturne. Hora enim nocturna et hora diurna semper complent 30 gradus propter revolutionem 360 graduum in die et nocte. Quod si volueris partem ascendentem in hora data accipe horas ab ortu solis in die vel ab occasu solis in nocte et in suos gradus per multiplicationem redige et exibit arcus equinoctialis circuli qui ab ortu vel occasu solis sursum emersit. Unde ergo quanta portio zodiaci a loco solis inchoata vel successionem signorum cum hoc arcu exorta sit et pars ad quam calculando perveneris ipsa est pars oriens. Et si volueris partem medii celi, sume horas a proximo meridie ad horam datam preteritas et eas in suos gradus redige, et fiet arcus equinoctialis qui a proximo meridie meridanum transiit. Quere ergo in spera recta cuius porcionis a loco solis sit illa elevatio et pars ad quam numerando perveneris est pars medii celi. Pars vero opposita orienti est occidens et que opponitur medio celi super terram est pars medii celi sub terra. Autem si velis per partem ascendentem scire partem medii celi sub terra, quere ascensiones in spera declivi porcionis ab initio arietis usque ad partem orientem et habebis gradum equinoctialis circuli qui cum parte ascendente venit ad ortum. Et quia semper ab orizonte ad medium celi est quarta equinoctialis circuli, deme ab illis ascensionibus 90 si fieri potest. Si minus adde super id quod inveneris 360 idest revolutionem unam et ex toto subtrahe xc, et relinquitur arcus equinoctialis qui ab initio arietis meridianum sub terra transiit in ortu dato. Quere ergo in spera recta cuius porcionis sit illa elevatio et invenies partem mediantem celum sub terra et vice versa. Si per medium celi super terram cognitum scire velis partem orientem, ab elevationibus in spera recta aufer xc et quere in spera declini cuius portionis residuum sit elevatio. Ecce ad quid utile est ascensiones circuli declinis noscere.

(20.) Datas horas temporales ad equales vertere et datas equales ad inequales reducere.

Datas nempe horas temporales multiplicando gradus effice et ex gradibus dividendo in 15 horas equales quotquot | poteris resitue. Item O 27r datas equales in suos gradus ducito et per tempora hora inequalis dividendo ad inequales reducito. Ratio in ianuis excubat.

696 hore…697 quantitas] *mg.* F **697** nocturna…diurna] diurna et hora nocturna O **700** die…in^2] *om.* F **702** Unde] vide O | quanta] quota O | portio] proportio F **703** vel] secundum O **711** sub terra] *om.* O **713** gradum] gradus F **716** xc] 90 O **717** sub terra] *om.* O **719** sub terra] *om.* O **721** xc] 90 O

| (21.) proporcio speralis anguli supra polum alicuius circuli O 27r consistens ad quatuor rectos | est sicut arcus eiusdem circuli qui F 8v subtenditur ad totam circumferentiam.

Hoc ex eque submultiplicibus primi et tercii et item secundi et quarti sicut in sexto euclidis de angulis planis facile comprobatur.

(22.) Omnes duo anguli ex duobus meridianis cum circulo signorum ad eandem distantiam a puncto equinoctiali provenientes quorum alter extrinsecus alter intrinsecus ex eadem parte sibi oppositus sunt equales.

[Figura 23] Ponam ergo arcum equinoctialis circuli *abg* et arcum circuli signorum *dbe* et punctum *b* equinoctiale a quo duo arcus equales *bh* et *ht*. Et describam duos arcus meridianos super polum *z* qui sint *zk*h et *ztl*. Dico quod angulus *zhb* equalis est angulo *zte*. Triangulus enim *khb* equilaterus est triangulo *tlb* cum propter ypothesim cum propter eandem declinationem cum propter equales ascensiones, ergo angulus *khb* est equalis angulo *ltb* qui equatur angulo *zte* quia sunt anguli centra se posita.

(23.) Omnes duo anguli ex duobus meridanis cum circulo signorum ad eandem distantiam a puncto tropico provenientes quorum alter extrinsecus alter vero intrinsecus ex eadem parte sibi oppositis equantur duobus rectis.

[Figura 24] Sit iterum orbis signorum arcus supra quem abg ex quo duo arcus equales a puncto tropico *b db* et *eb*, et sint duo arcus meridiani supra polum *z zd* et *ze*. Dico quod angulus *zdb* equus est angulo *zeb*. Quoniam duo latera trianguli *zde* propter eandem declinationem sunt equalia. Quare anguli ad basim *de* sunt equales quorum unus scilicet *zed* cum angulo *zeg* equatur duobus rectis.

(24.) Angulus ex circulo meridano cum circulo signorum | apud O 27v punctum tropicum proveniens rectus esse neccessario comprobatur.

[Figura 25] Sit denuo circulus meridianus *abgd* et medietas circuli signorum *aeg*. Et sit *a* punctum tropicum hyemale et describam super polum *a* secundum spacium lateris quadrati medietatem circuli *bed*. Quia ergo circulus meridianus *abgd* est descriptus super utriusque circuli *aeg bed* polos, erit arcus *ed* quarta circuli, quare angulus *dae* est rectus. Et propter idem est angulus qui est apud tropicum estivum rectus, et hoc est quod oportuit demonstrari.

(25.) Maxima declinatione nota angulum ex meridano et circulo signorum apud punctum equinoctii provenientem notum esse oportet.

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732 subtenditur] ei *add.* F **737** alter intrinsecus] *mg.* O **739** equales] *om.* O **743** est…744 equalis] *inv.* O **749** arcus] *om.* O | quem] commune F **751** zeb] *zeg* F **753** zed] *zde sed. corr. supr. lin.* O **758** a punctum] *inv.* O **761** polos] secundum theodosinum de speris *add.* O **762** est[2]] *om.* F **763** oportuit] oppositum est F

Unde patet quod si maximam declinationem addas super quartam vel ab ea subtrahas exibit angulus quesitus.

[Figura 26] Sit ergo ut solet circulus meridianus *abgd* et infra eum medietas circuli equinoctialis *aeg* et medietas circuli signorum *azg*. Et sit *a* punctum autumpnale, et describam supra polum *a* secundum spacium lateris quadrati semicirculum *bzed*. Propter hoc ergo quod circulus *abgd* est descriptus super polos orbium *aeg bed*, erit uterque istorum arcuum *az ed* quarta circuli. Est ergo *ze* maxima declinatio nota, ergo totus arcus *zd* notus, quare angulus *daz* notus respectu 4 rectorum. Reliquus ergo *baz* notus, quod opertuit demonstrari. Posito ergo quod maxima declinatio sit 23 partes et 51 minitum, erit angulus *baz* 66 partium et 9 minutum sicut in almagesti constitutum est.

(26.) Quantitatem cuiuslibet anguli ex meridiano cum circulo signorum apud quodlibet punctum provenientis per notam puncti declinationem inveniere. Unde liquet quod | si declinationis puncti cuius F 9r angulus queritur sinum ducas in sinum perfectionis sumpte portionis a puncto equinoctiali et productum dividas per sinum ipsius portionis et productum iterum multiplices in semidiametrum atque quod exierit dividas per sinum perfectionis declinationis, exibit sinus differentie duorum angulorum apud punctum | propositum valentium duos rectos O 28r quam si recto addideris vel subraxeris, habebis utrumque.

Rationis causa sit circulus meridianus agbd et medietas equinoctialis *aeg* et medietas circuli signorum *bzd*. [Figura 27] Et sit *z* punctum autumpnale et arcus *bz* pro libra sit signum virginis, et describam super polum secundum spacium lateris quadrati semicirculi *htek*. Quero ergo quantitatem *kbt*. Quoniam autem circulus *abgd* est descriptus super polos *aeg* et super polos *hek*, erit quilibet istorum arcuum *bh bt eh* quarta circuli. Et propter hanc formam proporcio *ba* ad *ha* per kata disiunctam ex geminis ducitur proporcionibus una *bz* ad *zt* et alia *te* ad *eh*, de sinibus intelligo. Sed quinque nota sunt: *ba* propter declinationem principii virginis, et *ah* propter perfectionem quarte, et *bz* propter signum virginis, et *zt* quia est perfectio quarte, et *eh* quarta. Relinquitur ergo *et* notum. Quare et totus *tk* arcus et angulus cui subtenditur *kbt* notus. Ergo secundum ptholomei inventam declinationem erit angulus qui apud caput virginis cxi partes et qui apud caput scorpii similiter propter equalem

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773 ed] propter hoc ergo quod circulus *add. sed. exp.* O | declinatio] et est *add.* F **774** 4] iiii O **776** 23] 33 F **787** sit] *om.* F **792** bh bt] *bht* F **793** ad] *om.* F **794** proporcionibus] portionibus F **796** virginis[1]] virginum O | ah] *ab* F | virginis[2]] virginum O **798** angulus] arcus *sed. corr. supr. lin.* O **800** cxi] iii O distantiam a puncto equinoctii. Et quam apud caput tauri vel piscium cum a duobus rectis illam quantitatem dempseris partes 69 ex antepremissa. Pari modo si ponas punctum *b* principium leonis lineis manentibus secundum suam habitudinem invenies angulum in capite leonis 102 partium et 30 minutorum, et eum qui in capite sagittarii similiter. Et cum a duobus rectis illum dempseris, occurrit angulus qui in capite geminorum vel in capite aquarii partes 77 et partis medietas ad hunc modum in singulis sectionibus angulos unius quarte et per eos angulos aliarum trium poteris comprehendere. Atque hec est noticia angulorum omnium in orizonte recto et circulo signorum provenientium.

(27.) Omnes duo anguli ex uno orizonte declivi cum circulo signorum ad eandem distantiam a puncto equinoctii provenientes quorum unus intrinsecus alter vero extrinsecus ex eadem parte sibi oppositus sunt equales.

[Figura 28] Propter hoc describo circulum meridianum *abgd* et dimidium equatoris diei *aeg* et orizontis *bed*. Et scribo duas porciones orbis signorum *zht* et *klm*, sitque utrumque *zk* punctum autumpnale, | et O 28v arcus *zh* equalis arcui *kl*. Dico quod angulus *eht* equalis est angulo *dlk*, latera namque trianguli *ehz* sunt equalia lateribus trianguli *ekl*, cum propter ypothesim cum propter ascensiones equales cum propter ascensiones orizontis equales. Ergo *ehz* equalis est angulo *elk*. Quare angulus *eht* residuus de duobus rectis equatur angulo *dlk* residuo.

(28.) Omnes duo anguli ex uno orizonte declivi cum circulo signorum apud puncta opposita orientis et occidentis provenientis extrinsecus cum intrinseco equantur duobus rectis. Unde colligitur quod duo quoque ad eandem distantiam a puncto tropico duobus rectis sunt equales. Quapropter notis angulis orientalibus unius medietatis ab ariete in libram noti erunt anguli orientales alterius medietatis et una anguli occidentales in ambabus partibus.

[Figura 29] Pono itaque circulum orizontis *abgd* et circulum signorum *aegz* et puncta sectionum *ag*. Palam quod anguli | *zad* et *dae* F 9v equales sunt duobus rectis. Angulus vero *zad* equatur angulo *dgz* quia arcus maxime declinacionis eorum circulorum *dz* secat utriusque medietatem per equalia. Quapropter angulus *dgz* et angulus *dae* simul valent duos rectos. Et quia anguli ad eandem distanciam a puncto

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801 equinoctii] equinoctiali F | quam] qui F qui *sed. corr. supr. lin.* O **802** antepremissa] *exp. et.* 23 *add. supr. lin.* O **803** lineis manentibus] *inv.* O **804** 102] cii F **806** occurrit] occuret O **808** singulis] singulus O **810** circulo signorum] signorum circulorum F **812** equinoctii] equinoctiali F **818** equalis est] *inv.* O **823** ex] in F **824** provenientis] *om.* F **832** zad] *zda* F **834** dgz] *dez* F equinoctiali sunt equales accidit ut anguli quoque duo eiusdem a puncto tropico distante orientalis duo et occidentalis duobus rectis sunt equales. Propter hoc ergo et premissam cognitis angulis orientalibus ab ariete in libram et orientales et occidentales in ambabus partibus erunt noti, et hoc est quod proponitur.

(29.) Nota poli altitudine et tropicorum distantia angulum ex concursu orientis orizontis declivis et signorum circuli apud utrumque punctum equinoctii notum esse neccesse est. Unde constat quod si differentiam que est inter regionis latitudinem et maximam declinationem cum latitudo maior fuerit a quarta circuli diminuas vel cum minor fuerit adicias, relinquetur angulus sub capite libre a quo si quantitatem distantie inter duos tropicos abiectis, residuum erit angulus sub capite arietis.

[Figura 30] Sit *abgd* meridianus circulus infra quem orientalis medietas | orizontis *aed* et quarta equatoris diei *ez* et due quarte orbis O 29r signorum *eb eg*, et sit punctum scilicet quod est quarte *eb* punctum autumpnale et quod est quarte *eg* punctum vernale et punctum *b* tropicum hyemale sub terra et punctum *g* tropicum estivum. Est ergo arcus *gb* tropicorum distantia notus et eius medietas arcus *bz* notus, sitque latitudo regionis *tz* maior sive *kz* minor nota. Quare propter *dt* vel *dk* esse quartam circuli erit uterque arcuum *bd* et *gd* notus. Et quia punctum *e* est polus meridiani, erit uterque angulus scilicet *bed* qui est sub capite libre et *ged* qui est sub capite arietis quia sunt cum dictis arcubus eiusdem quantitatis.

(30.) Quantitatem anguli ex concidentia orizontis et zodiaci apud quodlibet punctum per notum celi medium et eius declinationem notam investigare. Regula. Si semidiametrum multiplices in sinum altitudinis gradus celi medii sub terra vel super terram et productum dividas per sinum portionis que est inter orizontem et celi medium sub terra vel super terram prout contingerit eam portionem minorem esse quarta, exibit sinus et quesiti arcus et quesiti anguli.

[Figura 31] Pingo circulum meridianum *abgd* et infra eum medietatem orizontis orientalem *bed* et medietatem circuli signorum *aeg* et sit pro libito punctum *e* caput tauri ad ortum venientis et *g* celi medium sub terra, quod per ascensiones notas erit notum. Estque necessario secundum dictam positionem portio *eg* minor quarta. Describam autem super polum *e* secundum spacium lateris quadrati porcionem orbis maioris *zht* et complebo duas quartas *egh edt* et erit uterque duorum arcuum *zgd zht* quarta circuli eo quod orizon *bet* est descriptus supra

842 orientis] *om.* F | utrumque] *om.* O **843** esse] eius F **849** diei] *om.* O **853** arcus bz] *inv.* O **856** capite libre] *inv.* O **860** semidiametrum] diametrum F **867** et[1]] vel F **868** erit notum] *inv.* O **871** et[1]…872 zht] *mg.* O polum *zgd* meridiani et supra polum *zht* orbis magni. Vides ergo a puncto *t* duos arcus *te* et *tz* magnorum orbium descendentes inter quos alii duo se secant super punctum *g*. Igitur per kata coniunctam conversis proporcionibus erit proporcio sinus *th* ad sinum *tz* sicut sinus *gd* ad sinum *ge*. Sed tria nota sunt, *tz* propter esse quartam circuli gd propter declinationem gradus medii celi et latitudinem regionis esse notam. Nam cum *z* sit polus orizontis erit eius distancia in arcu meridiano *zgd* ab equinoctiali nota. Et cum *g* | sit celi medium, erit eius quoque distancia in F 10r eodem arcu ab equinoctiali nota, et propter hoc arcus *gz* notus. Quare perfectio quarte scilicet *gd* nota, et ipsa est altitudo partis medii celi ab orizonte *eg*. Vero propter notam esse porcionem | inter orizontem et celi O 29v medium, igitur primum notum *ht* cuius arcus quantitas est anguli quesiti quantitas. Eia, age ad hunc modum in ceteris sectionibus.

(31.) Omnes bini arcus binorum orbium altitudinis a polo orizontis egressi ad duo puncta circuli signorum eiusdem a puncto tropico distantie cum ipsa etiam a circulo medii diei ante et post secundum equalia tempora distiterint sint equales et faciunt angulos cum circulo signorum extrinsecum et intrinsecum ex eadem parte sibi oppositum equales duobus rectis.

[Figura 32] Describam itaque orbem meridiei supra quem sint *abg* et sit punctum *b* polus orizontis et *g* polus equinoctialis. Et ponam duas porciones orbis signorum *ade* et *azh*, et sint puncta *z* et *d* eiusdem longitudinis a puncto tropico et secundum equalia tempora distent a linea medii diei *abg* ante et post hoc est secundum equales arcus equidistantis equinoctiali. Et post hoc protraham duos arcus orbium altitudinis a puncto *b bz* et *bd*. Et dico quod ipsi sunt equales et quod angulus *bde* cum angulo *bza* equantur duobus rectis. Propter hoc etiam describo duos arcus meridianorum *gz* et *gd*. Quia ergo angulo *zbg* et angulo *bgd* equales arcus pro paralello resecti subtenduntur ipsi anguli quoque sunt equales. Quare *bg* linea facta communi duobus triangulis *zgb* et *gdb* cum duo latera duobus sint equalia, erit basis *bz* basi *bd* equalis, quod est unum ex propositis, et angulus *bzg* equalis angulo *bdg*. Sed ex 23ª presentis libri angulus *gza* et angulus *gde* equantur duobus rectis, ergo angulus *bza* cum angulo *bde* pariter equantur duobus rectis.

(32.) Omnes bini arcus binorum orbium altitudinis a cenith capitum egressi usque ad unum punctum circuli signorum cum ipsum a linea

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878 celi] circuli O **879** eius] *om.* F **880** Et…881 nota] *om.* O **882** medii celi] *inv.* O **883** Vero] *om.* O | esse] eius O **893** et^2] *om.* O **895** distent] post linea F **897** Et] *om.* O **898** b] *om.* F **899** etiam describo] *inv.* O **900** Quia ergo] *inv.* O | zbg] *zbg* F **901** pro] ex O anguli quoque] *inv.* O **902** facta] linea *add.* F **904** bzg] *b* et *g* F | 23a] 22ª F meridiei ante et post secundum equalia tempora distiterit sive zenith capitum a punctis celum mediantibus septentrionale fuerit sive meridanum sunt equales et faciunt angulos duos ad idem punctum duplo maiores pariter angulo ex concidentia meridiani et circulo signorum ad idem punctum proveniente. | O 30r

[Figura 33] Esto orbis meridiei *abgd* et summitas capitis punctus *g* primo ex parte septentrionis et *d* polus equatoris diei. Et sint due porciones orbis signorum *hb* et *ae* sitque *h* idem punctum quod *e* continuans duas porciones et secundum equalia tempora distans ante et post a linea meridiei. Et sint duo arcus orbium altitudinis *gh* et *ge*. Dico quod hii arcus sunt equales, et cum producta fuerint arcus meridianorum *dh* et *de*, erunt anguli *ghb* et *gez* duplo maiores angulo *dez* sive angulo *dhb*. Quia ergo puncta *h z e* secundum equalia tempora distant a linea medii diei sunt anguli *gdh* et *gde* equales. Facta ergo linea *gd* duobus triangulis communi erit linea *ge* equalis linee *gh* et erit angulus *ged* equalis angulo *ghd*. Sed et angulus *dhb* equalis est angulo *dez*, ergo ambo pariter *ged* et *ghb* sunt equales angulo *dez*. Quapropter ambo anguli *ghb* et *gez* totus equantur duplo anguli *dez*, quod intendimus.

Sit item cenit *g* meridianus a punctis celum mediantibus *a* et *b*. [Figura 34] Dico ergo quod similiter accidit scilicet quod duo anguli *kez* et *lhb* equantur duplo anguli *dez*. | Angulus enim *dez* equalis est angulo F 10v *dhb* imo idem. Sed et angulus *dek* equatur angulo *dhl*. Ergo totus angulus *lhb* equatur duobus angulis simul *dez* et *dek*. Quapropter anguli *lhb* et *kez* equales sunt duplo anguli *dez*.

(33.) Quod si unum punctorum celum mediantium sive orientalis portionis sive occidentalis meridianum fuerit a zenith capitum et alterum septentrionale anguli qui proveniunt ad punctum dictum superant duplum anguli ex arcu meridano ad idem punctum facti quantitatem duorum rectorum. Ex quibus omnibus colligitur quod si noti fuerint anguli ante meridiani et arcus in omni declinatione a principio cancri usque ad principium capricorni, noti erunt et arcus et anguli eorumdem signorum post meridiam et una [sic] anguli reliquorum signorum et arcus ante et post meridanam lineam.

[Figura 35] Describam formam predicte similem et sit punctum *a* porcionis orientalis in parte septentrionali a puncto *g* in linea medii celi et *b* punctum porcionis occidentalis in parte meridiana. Dico ergo quod duo anguli *kez* et *ghb* simul superant | duplum anguli *dez* quantitate duorum O 30v

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914 Esto] enim *add.* O | capitis] capitum F **916** porciones] proporciones F | ae] *be* F **923** gh] *ge* F **924** equalis[1]] equus F | dez] *deh* F **928** kez] *hez* F **932** duplo anguli] *inv.* F **937** quibus] omni F **940** et[3]] *om.* F **945** superant] superantur O rectorum. Ideo siquidem quod duo anguli *kez* et *ghb* simul superant a duobus angulis *dez* et *dhb* vel a duplo unius eorum quantitate duorum angulorum *dek* et *dhg*. Sed hii duo anguli equantur duobus rectis, et ille qui est ex *deg* equatur ei qui est ex *dhg*.

Sit rursum punctum *a* porcionis orientalis in medio celi in parte meridiana a puncto *g* et punctum *b* porcionis orientalis in parte septentrionali. [Figura 36] Dico quod similiter accidit. Angulus namque *dhg* equatur angulo *deg*. Duo vero anguli *dhg* et *dhl* equantur duobus rectis angulis, angulus autem *dez* est equalis angulo *dhb*. Quapropter erunt duo anguli *gez* et *lhb* superantes duos angulos *dez* et *dhb*. Autem duplum unius eorum quantitate duorum angulorum *deg* et *dhl* qui sunt equales duobus rectis, quod oportuit demonstrari. Palam ergo quod cum noti fuerint quilibet anguli ante meridiani ad quodlibet punctum, noti erunt post meridiani ad idem. Et ex 30 [huius] cum noti fuerint secundum quamlibet longitudinem anguli a tropico ex quacumque parte meridiei, noti erunt anguli secundum eandem longitudinem ex parte altera, et hoc est [quod intendimus].

(34.) Quemlibet angulum ex concidentia circuli altitudinis cum circulo signorum apud punctum medians celum vel apud punctum orizontis et arcum quoque a summitate capitum ad utrumlibet notum essse oportet.

[Figura 37] Ponam circulum meridianum *abgd* et infra eum medietatem orizontis *bed* et medietatem orbis signorum *zeh* qualitercumque ymaginemur itaque circulum altitudinis descriptum super *a* quod est summitas capitum et transeuntem per medium celi supra punctum *z*. Dico quod arcus *az* est notus ideo scilicet quod arcus *ez* notus est per 19^{am} huius et declinacio puncti *z* per 15 primi libri, et elongacio puncti *a* ab equatore diei quia est latitudo regionis. Et dico quod angulus *aze* cum circulus altitudinis hic sit meridianus est etiam notus ex 26 presentis. Rursus ymaginemur circulum altitudinis descriptum supra punctum *a* et transeuntem per *e* quod est punctum orientis scilicet *aeg*. Manifestum ergo quod arcus *ae* semper erit quarta circuli eo quod punctum *a* sit polus orizontis *bed*. Et propter has causas erit angulus *aed* rectus semper, sed et angulus *deh* qui est ex orbe signorum et orbe orizontis semper notus ex 30 presentis. Quare erit totus angulus *aeh* notus, et hoc est quod oportuit declarari. | F 11r O 31r

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946 superant] superantur O **948** rectis] eo quod duo anguli *dek* et *deg* equantur duobus rectis *add.* O **950** punctum] *om.* F **954** rectis angulis] *inv.* O **955** dez] *deg* F **958** noti^2…959 erunt] *inv.* O **961** parte altera] alia parte O **967** Ponam] pono F **968** zeh] *zth* F **971** punctum] *om.* O | ideo] id F **972** 19am] 18 O **974** cum] est *sed. corr.* O

(35.) Quantitatem arcus circuli altitudinis a summitate capitum ad quodlibet punctum circuli signorum invenire.

[Figura 38] Conscribimus itaque orbem meridiei *abgd* et infra eum medietatem orizontis *bed* et medietatem orbis signorum *zht*. Et sit punctum *h* caput cancri secundum quodlibet tempus distans a linea meridiana et exempli causa sit distans secundum unam horam, et punctum *z* medians celum et punctum *t* orientis per 18 [huius] notum. Faciam ergo super summitatem capitis *a* et super caput cancri *h* transire porcionem circuli altitudinis *aheg*. Scrutabor ergo quantitatem arcus *ah*. Est itaque sicut premisimus arcus *zt* notus, et arcus *ht* notus cum *h* sit principium cancri, et arcus *az* propter declinacionem puncti *z* et altitudinem poli notas notus, et arcus *zb* quia est complentum quare notus. Hiis ergo cognitis vides quod proporcio *bz* ad *bz* aggregatur ex duabus una scilicet que est *eh* ad *ea* quartam et alia que est *tz* ad *th*, de sinibus arcuum loquor. Cum ergo ceteri noti sunt, erit et arcus *eh* notus, ergo et reliquus *ah* notus.

Regula operationis. Si sinum arcus meridani deprehensi inter celi medium et orizontem multiplices in sinum arcus circuli signorum deprehensi inter orizontem et punctum circuli signorum ad quod circulus altitudinis deducitur et productum dividas per sinum arcus circuli signorum intercepta inter orizontem et celi medium, exibit sinus perfectionis arcus quesiti quam si a quarte dempseris relinquitur arcus circuli altitudinis a summitate capitum ad punctum circuli signorum destinant.

(36.) Quantitatem anguli ex concidentia circuli altitudinis cum circulo signorum ad quodlibet punctum a celi medio declinans perscrutari.

Resumamus positam figuram secundam habitudinem suam et describamus super polum puncti *h* secundum spacium lateris quadrati porcionem magni circuli *klm*. Quia ergo orbis *ahe* est descriptus supra duos polos etm et *klm*, erit uterque duorum arcuum *em km* quarta circuli. Propter hanc ergo formam per kata disiunctam proporcio sinus *eh* ad sinum *ek* componitur ex proporcione sinus *ht* ad sinum *lt* et proporcione sinus *lm* ad sinum *mk*. Sed quinque horum nota sunt. Relinquitur ergo *lm* notum, ergo et *kl* notum residuum. Quarte ergo angulus | *lhk* cui O 31v
subtenditur notus. Quapropter et angulus *aht* complementum duorum rectorum notus, quod volumus ostendere.

986 h] *b* F | secundum] sed F **989** capitis] capitum O **990** aheg] *ahet* F **998** celi] celum F **1011** orbis] *azb* O

Opus. Longitudinem puncti destinati ab ascendente vel ab occidente de 90 minue, et sinum residui in sinum altitudinis puncti destinati ducito. Quodque exierit per sinum longitudinis puncti destinati ab ascendente divide, et quod fuerit in diametri dimidium multiplica. Indeque collectum per sinum longitudinis puncti destinati a cenit capitum partire. Et quod exierit arcuabis et arcum de 90 minues et residuum de 180, et erit quantitas quesiti anguli. Ad hunc modum in ceteris punctis et arcus et angulos invenies. Atque hec est noticia omnium angulorum ex circulo altitudinis et orbe signorum quorum scientia necessaria est ad sciendum diversitatem aspectus lune sine cuius noticia solares eclipses sciri est impossibile.

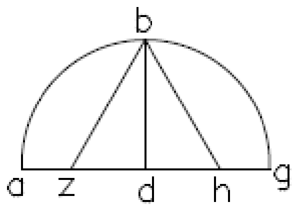
[I am including a proof from much later in the work. Only MS O was consulted.]

| Diversitatem aspectus lune in longitudine et in latitudine cum luna O 70v
latitudinem ab orbe signorum non habuit colligere.

Sit enim medietas circuli signorum *aeg* et medietas circuli altitudinis *bed* sese intersecantes ad punctum *e*. [Figura 39] Et sit circulus descriptus super polos utriusque *abgd* et polus circuli signorum nota *z* et cenith capitum punctum *p* et luna sit in puncto circuli signorum *e*. Et diversitas aspectus in circulo altituidnis arcus *he.* Duco itaque a polo *z* arcum circuli magni *zht*. Est ergo diversitas aspectus in latitudine arcus *ht* cum *h* sit visus locus lune, et arcus *et* diversitas aspectus in longitudine. Palam ergo ex positis quemlibet istorum arcuum *za zt eb ea* esse quartam circuli quid super polos suos invicem transeunt. Patet etiam ex ultima secundi libri quod angulus *bea* est notus ad iiii rectos. Ergo arcus *ba* similis scilicet quantitas est nota eo quod angulus super polum huius circuli consistat apud *e* ita quod cum a puncto *a* duo arcus magnorum circulorum descendant manifestum per kata coniunctam quod sinus arcus *ab* ad sinum *az* est sicut sinus *ht* ad sinum *he.* Sed tria nota sunt, ergo quartum notum scilicet sinus *ht*. Et ita arcus *ht* qui est diversitas aspectus in latitudinis notus. Rursum super polum *h* ad distantiam quarte *hk* vel *hn* lineo circulum magnum *knm*. Dico quod *mt* est quarta circuli quia enim *ztk* transit super polos circuli signorum *aeg* et circulus *aeg* necessario transit super polos circuli *ztk*. Quare in arcu *aeg* cum sit medietas circuli est polus circuli *ztk*. Item | *ztk* transit super polos *knm*, ergo et ille mutuo O 71r
transit super polos *ztk* est ergo punctus m polus circuli *ztk*. Quare mt est quarta circuli. Et dico quod arcus *kn* qui subtenditur the angulo

1019 ab[1]…vel] *om. (hom.)* F **1020** 90] xc F **1024** 90] xc F longitudinis est notus. Nam per kata disiunctam proporcio sinus *ht* ad *tk* componitur ex duabus una scilicet *he* ad *en* et alia *mn* ad *mk*. Cum reliqua sint nota, erit arcus *mn* notus, ergo et arcus *nk* qui deest ad perfectionem quarte est notus. Et nota quod si dempseris arcum *ba* sive angulum *bea* de quantitate unius recti, invenies reliquum fere equale arcui *kn* sive angulo *khn*. Cum ergo a puncto *k* duo arcus magnorum orbium descendant per kata coniunctam proporcio sinus *nk* ad sinum *mk* est sicut sinus et ad sinum *eh.* Cum ergo reliqua tria sint nota, erit arcus et notus et ipse est diversitas aspectus lune in longitudinis.

Operationis modus est ut ex opere ultime secundi libri vel ex tabulis ad hoc ... scias angulum ex cursu circuli altiduinis et orbis signorum et ex antepremissa vel ex tabulis ad hoc scias diversitatem aspectus in circulo altitudinis et addiscas cordam eius et cordam dicti anguli qui est angulus latitudinis et cordam mediatam eius quod deest ei ad completionem xc. Deinde multiplices sinum anguli latitudinis in sinum arcus altitudinis, et productum dividas per lx, et quod exit arcues. Nam iste arcus est diversitas aspectus in latitudine. Sinum vero anguli longitudinis multiplices similiter in sinum arcus altitudinis, et productum dividas per lx, nam arcus illius sinus qui exit est diversitas aspectus in longitudinis.



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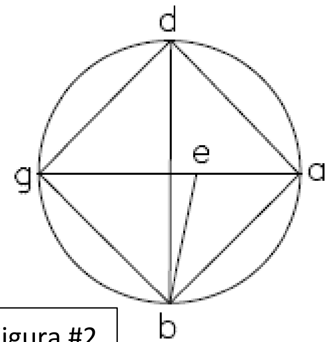

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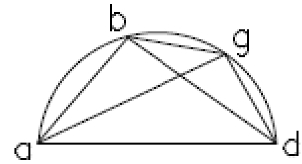

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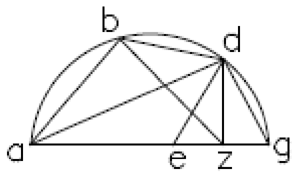

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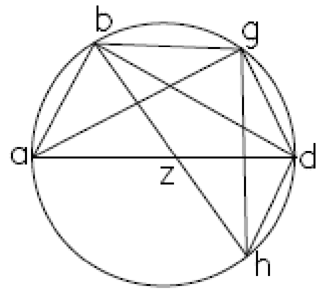

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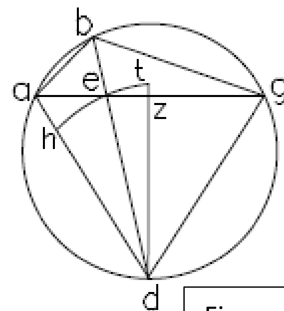

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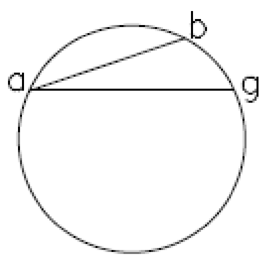

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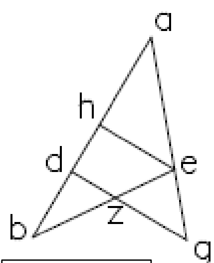

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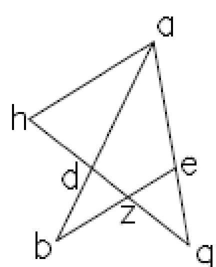

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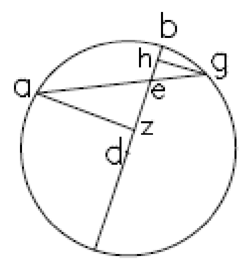

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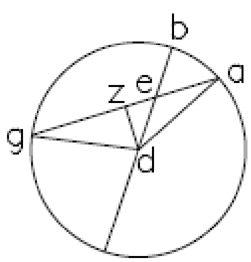

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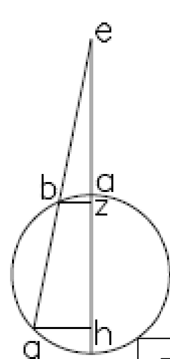

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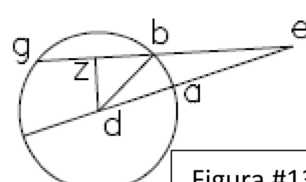

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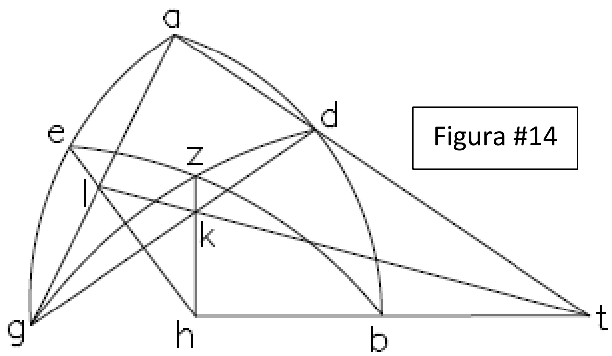

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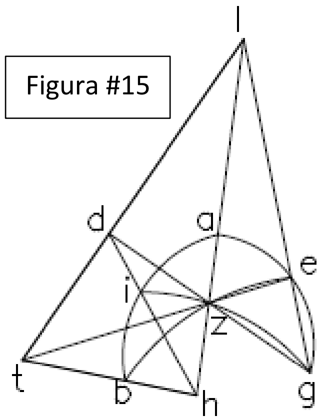

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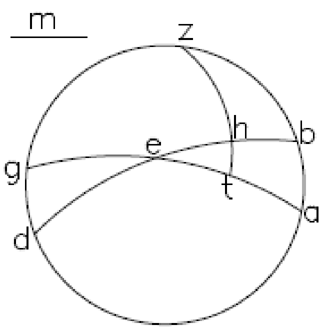

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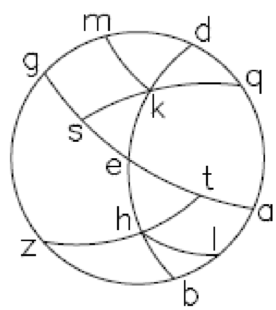

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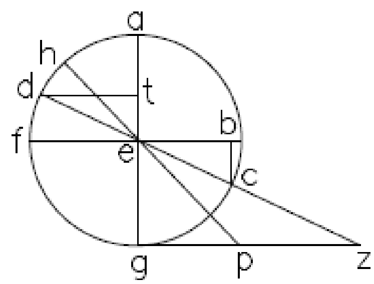

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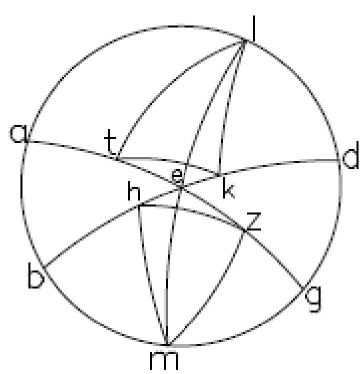

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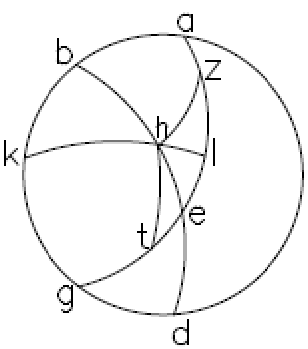

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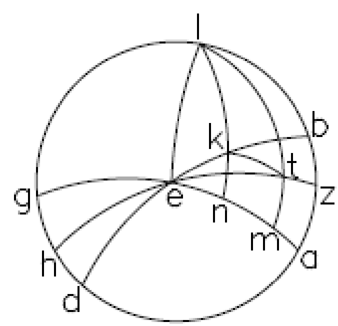

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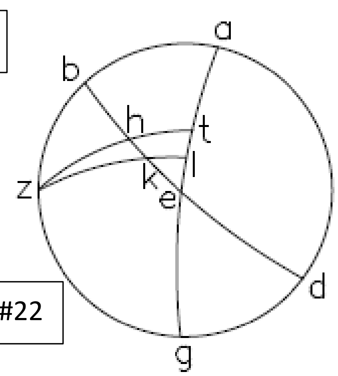

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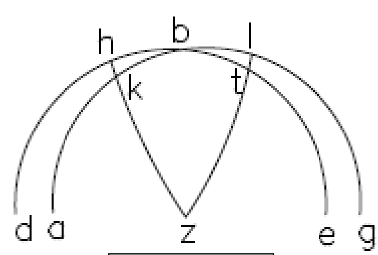

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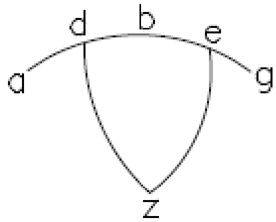

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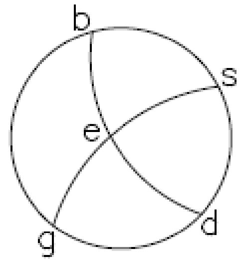

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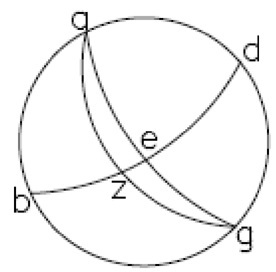

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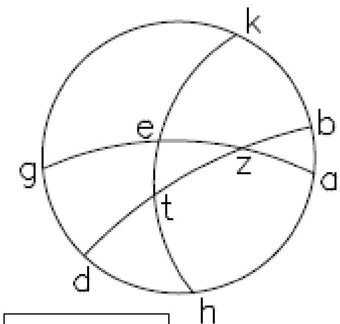

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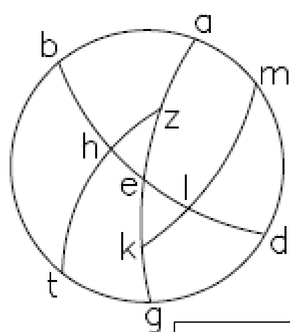

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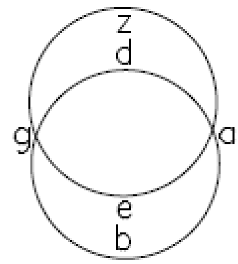

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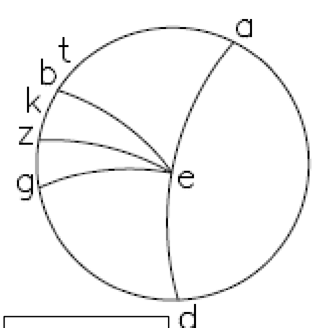

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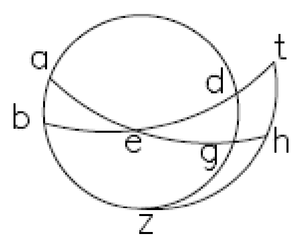

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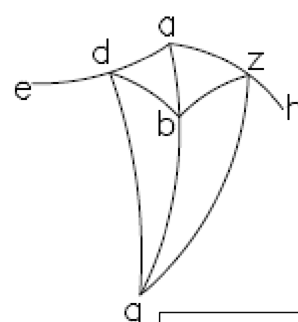

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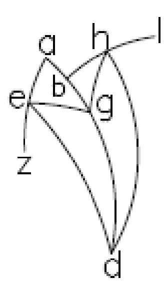

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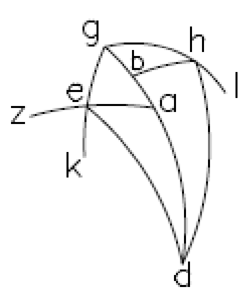

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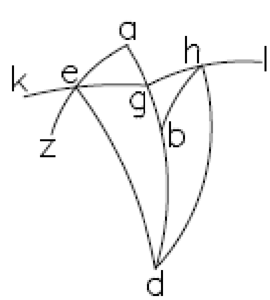

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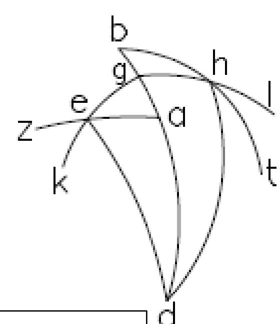
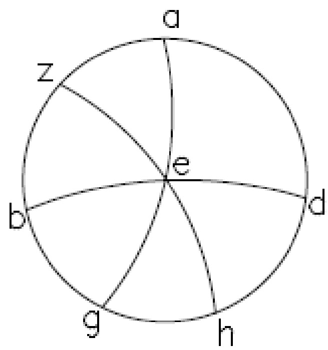


Figura #36

Figura #37



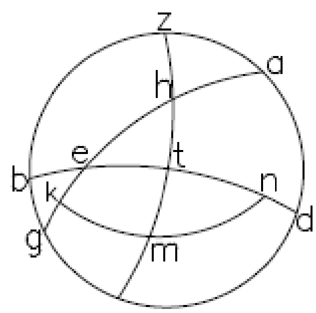
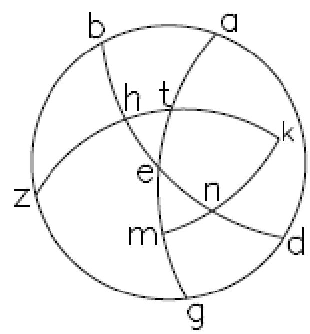


Figura #38

Figura #39



Appendix E: The Erfurt Commentary

My edition is based on the following manuscripts:

Dijon, Bibliothèque municipale, 441 = D

Erfurt, Universitätsbibliothek Erfurt, Dep. Erf. CA 2^{o} 375 = G

Erfurt, Universitätsbibliothek Erfurt, Dep. Erf. CA 2^{o} 393 = H

Città del Vaticano, Biblioteca Apostolica Vaticana, Pal. lat 1380= P

In the middle of the text in D, on fol. 220r-221r, we find a summary of and notations on the early portions of the commentary. The text of these summaries and notes are marked with the siglum “DD.”

I have omitted the early propositions on plane trigonometry.

[I.7]

Duabus rectis lineis ab uno angulo descendentibus aliisque duabus se secantibus a descendentium terminis reliquis in easdem reflexis, utralibet reflexarum alterius conterminalem sic figet ut proporcio fixe ad eam sui partem que supra fixionem est producatur ex duabus proporcionibus ex una quam habet conterminalis reflexa ad eam sui partem que sectioni interiacet et fixioni et ex ea proporcione quam habet alterius reflexe in inferiori sub sectione porcio ad eam totam cuius pars est linea.

Pro declaratione demonstrationis eius et generaliter figure sectoris, primo premittitur quod si denominatio proporcionis cuiuscumque primi ad aliquod secundum ducatur in secundum, semper producitur primum. Patet, quia denominatio proporcionis est numerus vel fractio vel fractiones vel numerus cum fractione vel fractionibus denotans quotiens maius contineat minus vel quota eius fractio vel fractiones sit minus ipsius maioris quo ad denominationes proporcionis inequalitatis minoris. Igitur necessario sive primum fuerit maius sive minus secundo ducta denominatione proporcionis eorum secundum artem algorismorum in secundum, proveniet primum ut si denominatio proporcionis duorum ad tria que est 2/3 multiplicetur in tria quod est secundum, provenient 2. Et e

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4 Duabus] 9ª de catha coniuncta *adnot. mg. a. m.* D 7ª *adnot. mg.* G 9ª *adnot. mg.* DD Duabus…11 linea] *om.* P | rectis] *om.* DDDH | angulo descendentibus] *inv.* DDH descendentibus…11 linea] et cetera D | aliisque…5 reliquis] a quarum terminis due linee DDH **5** reflexis…6 utralibet] reflectantur utraque DDH **6** alterius] sibi DDH | fixe…11 linea] proporcionis que infra infixionem est ad totam fixam componatur ex proporcionibus reflexe sibi conterminalis ad partem eius ultra sectionem eius (*inv.* H) et (*om.* DD) ex proporcione partis alterius reflexe infra sectionem ad partem eiusdem que supra sectionem est (tamen est dicere quod proporcio ga ad *ae* componitur ex duabus proporcionibus *gd* ad *dz* et *bz* ad *be add.* DD) DDH **12** Pro] prima suppositio *adnot. mg.* G incipit tercius tractatus *adnot. mg.* P | Pro…95 probandum] *om.* DD **16** vel[2]] cum *add.* HP **17** eius] *om.* GH *add. supr. lin.* P | sit] fuit P **18** denominationes] denominationem D | proporcionis] proporcionum HP | inequalitatis minoris] *inv.* P **19** necessario sive] necessarium si P **20** algorismorum] algoristicam G **21** proveniet] provenit D

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3 I7] [The numbering is not consistent in the manuscripts. I have attempted to stay as close to them as possible, especially internal references although this requires ignoring the numbers labelling sections and grouping together sections of text that perhaps more logically should be numbered separately.]

converso si denominatio proporcionis trium ad duo que est unitas et una medietas multiplicetur in binarium, proveniet trinarius.

Secundo supponitur quod quotienscumque duo numeri vel due fractiones vel duo numeri cum fractionibus multiplicantur in aliquod tertium, proporcio duorum productorum est sicut proporcio quantitatum multiplicantium. Patet ex 7º Euclidis propositione ut si tria et quatuor ducantur in 5 divisim provenient 15 et 20 sic se habentes sicut tria ad quatuor.

Tertio supponitur hic quod proporcionem componi ex proporcionibus est denominationem eius ex denominationibus earum invicem multiplicatis produci. Hec est auctoris de proporcione et proporcionalitate Euclidis et aliorum. Ut diapason in musica dicitur componi ex diatesseron et diapente ex eo | D 216v quod denominatio proporcionis sesquitertie que est unitas cum una tertia multiplicata in | H 68ra denominationem proporcionis sesquialtere que est unitas et una medietas producit binarium que est denominatio diapason vel proporcionis duple. Ex his sequitur una conclusio talis necessaria ad propositum.

Quibuscumque duobus extremis habentibus aliquam proporcionem ad invicem medium eiusdem rationis interponatur qualitercumque se habens ad ea, proporcio primi ad tertium componetur ex proporcione primi ad illud medium | P 126r et proporcione ipsius ad tertium.

[*Figura #1*] Nam ponitur inter *a* et *b, c* medium cuiuscumque fuerit quantitatis finite, et sit *d* denominatio proporcionis *a* ad *c* et *e* denominatio proporcionis *c* ad *b*, et denominatio proporcionis *a* ad *b* sit *f*. Cum igitur *e* denominet proporcionem *c* ad *b*, ergo per primam suppositionem *e* ductum in *b* producit *c*. Similiter quia *f* est denominatio proporcionis *a* ad *b*, ergo per eandem ex *f* in *b* fit *a*. Ex ductu ergo duarum quantitatum *f* et *e* in tertiam scilicet *b*, producuntur *a* et *c*. Ergo per secundam suppositionem proporcio *a* ad *c* est sicut *f* ad *e*. Tunc sic proporcio *a* ad *c* est sicut proporcio *f* ad *e*. Ergo cum *d* sit denominatio proporcionis *a* ad *c*, etiam *d* erit denominatio proporcionis *f* ad *e*,

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23 proporcionis] *add. supr. lin.* P | et] unitas *add. sed. del.* P **24** in] per P **25** Secundo] 2ª *adnot. mg.* G **27** quantitatum…28 multiplicantium] *inv.* D **28** propositione] *om.* D **29** provenient] proveniunt D **31** Tertio] 3ª suppositio *adnot. mg.* G | hic] *om.* D | ex] aliquibus *add.* D **32** proporcionibus] aliquibus *add.* HP | denominationibus] duabus D **33** auctoris] auctoritatis D **39** conclusio talis] *inv.* HP | propositum] valde videlicet ista *add.* H **40** Quibuscumque] 9ª *adnot. mg.* G **42** ad ea] *illeg.* P **43** et] ex D | proporcione] et proporcione *add. sed.del.* G | ipsius] secundi D **44** ponitur] ponatur HP *a* et *add. sed. exp.* H **47** igitur e] *inv.* P | denominet] denominat P **49** proporcionis] *om.* P **50** et[1]] *d add. sed. exp.* G | e] *d* P **52** proporcio[2]] *om.* GHP | Ergo cum] *inv.* H equalium enim proporcionum sunt equales denominationes. *D* ergo est denominatio proporcionis *f* ad *e*; ergo per predictam suppositionem *d* multiplicatum in *e* producit *f*. Denominatio ergo *d* ducta in denominationem *e* facit denominationem *f*. Ergo per tertiam suppositionem proporcio *a* ad *b* componitur ex proporcionibus *a* ad *c* et *c* ad *b*.

Similiter patet de quibuslibet mediis interpositis duobus quibuslibet extremis, proporcionem extremorum componi ex omnibus proporcionibus intermediis. [*Figura #2*] Patet, nam positis *c* et *b* mediis inter *a* et *g*, patet ex priori deductione quod denominatio *f* ducta in denominationem *h* producit denominationem *m* proporcionis *a* ad *g*. Similiter probaretur ulterius ex denominationibus *m* et *n* duarum proporcionum *a* ad *g* et *g* ad *k* produci denominationem *l* proporcionis *a* ad *k*. Ergo per tertiam suppositionem proporcio primi ad ultimum erit composita ex proporcionibus *a* ad *c* et *c* ad *b* et *b* ad *g* et *g* ad *k*. Verbi gratia in numeris ponantur inter 3 et 12 tres numeri sicut contingit videlicet 5 2 14 et manifestum erit quod denominationibus intermediarum proporcionum continue invicem multiplicatis, producetur denominatio proporcionis subquadruple videlicet trium ad 12. Ducantur enim tres quinte que est denominatio proporcionis prime in binarium | et unam (G 116v) medietatem que est denominatio secunda. Et iterum productum multiplicetur | in unam septimam que est denominatio proporcionis (H 68rb) duorum ad 14. Et iterum productum in denominationem 14 ad duodecim que est unitas et una sexta. Proveniet precise una quarta que est denominatio proporcionis primi ad ultimum. Et ita in aliis speciebus proporcionum eveniet quotcumque fuerint etiam inter aliqua duo extrema.

His premissis conclusio est satis manifesta. [*Figura #3*] Nam protracta linea *eh* equisdanter *zd* erunt duo trianguli *agd* et *aeh* similes.

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55 predictam] dictam P **58** a^1] *add. supr. lin.* P | ad^2] *d add. sed. del.* P | c^1] *b sed.corr. sup. lin.* G **59** b] *d* DP **60** Similiter] nota bene *adnot. mg.* G | quibuslibet^1] quotlibet GH | duobus quibuslibet] *inv.* D **63** et] *om.* H | g] mediis *add.* D **64** proporcionis…g] *om.* P **65** ulterius] ultius D **66** l] *om.* H **67** proporcio] *add. supr. lin.* P | erit…68 composita] componetur P **68** et^1…b^1] *mg.* G *om. (hom.)* H | c^2…et^2] *om. (hom.)* P | et^3…ad^4] *om.* P **69** in numeris] *om.* D | numeri] videlicet *add.* D **70** 2] et *sed. corr. supr. lin.* D et *add.* P **72** proporcionis] *add. supr. lin.* P | enim] *om.* G **73** et] in D **74** medietatem] unitatem D medietas P **75** denominatio] duorum *add. sed.del.* G | proporcionis] 1 *add. sed. del.* D **79** quotcumque] quibuscumque D | aliqua duo] *inv.* P **81** His] vide figuram post ad 4^{or} folia de catha coniuncta *adnot. mg. a. m.* D **82** equisdanter] linee *add.* D | zd] *gd sed. del. et corr.* D similes] equianguli D

Ergo per 4^{am} 6^{i} proporcio *ga* ad *gd* est sicut *ea* ad *eh*. Ergo permutatim sicut *ga* ad *ea* ita *gd* ad *eh*. Si igitur fuerimus hic imaginati tres quantitates scilicet *gd* prima *zd* secunda vel media et *eh* tertia, erit ex premissis proporcio prime ad tertiam composita ex proporcionibus prime ad mediam et medie ad tertiam. proporcio ergo linee *gd* ad *eh* est composita ex duabus proporcionibus, totius *gd* ad *zd* et ipsius *zd* ad *eh*. Sed *zd* ad *eh* est sicut *bz* ad totam *be* arguendo ut prius ex quarta sexti. Igitur etiam proporcio *gd* ad *eh* erit composita ex duabus proporcionibus *gd* ad *zd* et *bz* ad *be*, quia ex equalibus cum eadem constituitur equale. Cum ergo proporcio *gd* ad *eh* ex iam dictis duabus proporcionibus composita sit equalis ut iam probatum est proporcioni totius *ga* ad *ea*, ipsa etiam erit composita ex eisdem, videlicet *gd* ad *zd* et *bz* ad *be*. Quod fuit probandum. | D 217r

[I.8]

Duabus rectis lineis ab uno angulo descendentibus si ab earum termino due linee se secantes super eas reflectantur, erit utriusque descendentium ad eam sui partem que est inter punctum reflectionis et angulum proporcio ex duplici proporcione composita ex ea videlicet que est sue conterminalis reflexe ad sui partem superiorem et ea que est inferioris partis alterius reflexarum ad totam.

[*Figura #4*] Consequentia huius similiter manifesta est ex eisdem quoniam tracta linea *ah* equidistante *ez* erit propter similitudinem duorum triangulorum *ecz* et *ach* proporcio *ec* ad *cz* sicut *ca* ad *ch*; igitur

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83 proporcio] *om.* P | est sicut] *iter*. D *eh* ad *add. sed.del.* D **84** ita] igitur P | hic] huius DG **88** totius] videlicet G **91** eadem] eodem H **92** duabus…93 composita] *om.* D **93** composita] componatur *sed. corr.* P | iam] *om*. HP **94** erit] *om.* HP **97** Duabus] 8^{a} *adnot. mg.* G 10^{a} de catha disiuncta *adnot. mg. a. m.* D vide figuram huius cathe disiuncte ad 4or folia post *adnot. mg. a. m.* D 10^{a} DD | rectis] *om.* DDHP | ab^{1}…102 totam] et cetera G | uno] angulo *add.* P | angulo descendentibus] *inv.* DD | descendentibus…102 totam] et cetera D **98** linee] inter eas *add.* P inter *add.* H **99** reflectionis] reflexorum DD **100** ea videlicet] alia ut patet DD **102** inferioris] inferior DD | totam] sicut in 7^{a} sic figet ut proporcio porcionum fixe inferioris partis ad superiorem producatur ex duabus proporcionibus ex una quam habet sibi conterminalis reflexe inferior sub sectione porcio ad reliquam partem que sectioni interiacet et fixioni et ex alia proporcione quam habet relique descendentis inferior sub fixione porcio ad eam totam cuius pars est linea *add.* G **103** Consequentia] conclusio D nona P Consequentia…119 poterimus] *om.* DD **105** ecz…ach] *egz* et *agh* D | ca…ch] *aag* ad *gh* D

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102 totam] tantum est quod proporcio *ge* ad *ea* aggregatur ex duabus proporcionibus *gz* ad *zd* et *bd* ad totam *ba* patet quia ex quo *ah* est equidistans *ez* sicut *ge* ad *ga* ita *gz* ad *zh* per 2^{am} 6^{i} Euclidis *add.* DD permutatim erit *ce* ad *ca* sicut *cz* ad *ch*; ergo etiam divisim erit *ce* ad *ea* sicut *cz* ad *za*. Deinde positis duabus lineis videlicet *cz* et *zh* tamquam extremis interposita | *zd* media, erit ex premissis proporcio *cz* ad *zh* [H 68va] composita ex duabus proporcionibus linearum [*cz*] ad *zd* et *zd* ad *zh*. Sed proporcio *zd* ad *zh* est equalis proporcioni | *bd* ad totam *ba* quia duo [P 126v] trianguli *bdz* et *adh* sunt similes; ergo proporcio *bd* ad *dz* est sicut *ad* ad *dh*. Ergo permutatim *bd* se habet ad *da* sicut *zd* ad *da*; ergo coniunctim etiam *bd* se habet ad *ba* sicut *zd* ad *zh*. Quare etiam proporcio *cz* ad *zh* erit composita ex duabus proporcionibus linearum *cz* ad *zd* et *bd* ad *ba*. Cum proporcio *ce* ad *ea* sit sicut *cz* ad *zh* ut probatum est, ergo erit etiam proporcio *ce* ad *ea* composita ex eisdem, scilicet *cz* ad *zd* et *bd* ad *ba* quod fuit probandum.

Ex his conclusionibus maxime ex coniuncta medietate ostendo incidentaliter quomodo poterimus cuiuslibet rei non excessive distantis ab oculo elongationem mensurare. [*Figura #5*] Nam sit *d* talis res. Erigam *dk* virgam note quantitatis super terram perpendiculariter vel oblique sicut contingit et exigit differentia positionis rei distantis cui

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106 permutatim] permutando DHP | ce[1]…ca] *ge* ad *ga* D | cz…ch] *gz* ad *gh* D | ch] *zh* H | ce[2]] *ge* D **107** cz[1]…za] *gz* ad *zh* D | za] *zh* HP | cz[2]] *gz* D *gz sed. corr. supr. lin*. H **108** premissis] premissa D | cz] *gz* D **109** cz] *gz* D *az* GHP | zh] *za* G **110** zh] *za* G **111** similes] ex hypothesi *add.* DHP **112** dh] *gh sed. corr. supr. lin*. H | permutatim] vel coniunctim *add. supr. lin. a. m.* D | da[1]…113 ad[1]] *om.* D | da[2]] *dh* HP **113** cz] *gz* D **114** cz] *gd* D | Cum] igitur *add.* D **115** ce] *ge* D | cz] *gz* D | probatum] propositum H | ergo] cum D **116** ce] *ge* D | ea] *del. et. add. eh* G | cz] *gz* D **118** Ex] additio *adnot. mg.* G | coniuncta medietate] 5ª D accidentaliter *add. sed. del.* D | medietate] *mte* P *om.* H | ostendo…119 incidentaliter] *inv.* HP **119** incidentaliter] *om.* G | distantis] *post.* oculo DD **120** mensurare] secundum alium modum quam postea *d* reperies *add.* D | Nam] *om.* DD | Nam…138 propositum] *om.* D | talis] *om.* DD **121** Erigam] *om.* DD | dk virgam] *bk* virga DD | super] supra P virgam *add. sed. exp.* H **122** sicut] sed quod DD | et…positionis] elevare et secundum posicionis differentiam DD | rei…153 quesita] *om.* DD

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122 positionis] *ck* virga alia posicio note quantitatis predicte tractis(?) ... infixe ut angulus *gze* obtusus vel rectus sit versus rem *d.* Respiciatur igitur *d* linea visuali infra et supra sectionem et notentur puncta *f e h g.* (cum *add. sed.del.*) Igitur quatuor erunt nota et etiam tria igitur etiam linee *he* et *fg* et partes eius scilicet *zg* et *hz*. proporcio igitur *fg* ad *zg* et similiter *hz* ad *he* componantur simul, et hoc sic ducatur *he* in *zg* et productum dividatur per *hz,* et ad illud quod exibit se habebit *zg* sicut *hz* ad *he* ex 15ª 6^{i}. Sit igitur illud *l* et erit per catham coniunctam proporcio *fg* ad *l* sicut *fd* ad *ed,* ergo etiam divisim sicut se habet excessus ipsius *fg* super *l* ad *l* ita *fe* notum *ad* ad *od. Ed* ignotum multiplicetur ergo *l* per *fe* et productum dividatur per

virge affigatur quedam aliqua simili note quantitatis ut est *ck* per modum crucis ita ut angulus *gze* obtusus vel rectus ponitur versus *d*. Deinde oculo *o* posito infra sectionem virgarum aspiciatur *d* secundum lineam visualem *od* cuius intersectiones notentur cum duabus virgis in punctis *f* et *e* et similiter posito oculo supra sectionem virgarum et viso iterum *d* notentur similiter sectiones radii visualis cum ambabus virgis, et sint note *h* et *g*. Cum ergo quinque puncta *h g f e* et *z* sint data in illis virgis, notis erunt *fg* et *he* linee note et partes earum videlicet *zg* et *hz*. proporcio ergo *fg* ad *zg* est nota et similiter ipsius *hz* ad *he* que componantur simul. Et hoc sic ducatur *he* in *zg* et productum dividatur per *hz*, et ad illud quod exibit se habebit *zg* sicut *hz* ad *he* ex 15^{am} 6^{i}. Sit ergo illud *l* et erit per catham coniunctam proporcio *fg* ad *l* sicut *fd* ad *ed*. Ergo etiam divisim sicut se habet excessus ipsius *fg* super *l* ad *l* ita *fe* notum ad *ed* ignotum. Multiplicetur ergo *l* per *fe* et productum dividatur per dictum excessum et exibit *ed* nota. | Cui si addatur *fe* etiam nota, erit tota linea *fd* nota quod H 68vb
fuit propositum.

[*Figura #6*] Vel aliter aspiciatur *d* per summitatem alicuius turris que sit *cb* stantis super planum *ae* et sit oculus *e*. Deinde sumantur *ba* una nota distantia quantaque fuerit ab ipso *b* in eodem plano ex opposito oculi. Tunc quia *cb* est nota et similiter *ba* et angulus *cba* est rectus, erit per penultimam primi linea *ac* nota. Erigatur ergo super punctum *a* quedam virga verbi gratia *fa* que deferet visum oculi positi in *a* ad summitatem turris que est *c*. Qua sic fixa ponatur oculus ad distantiam duorum vel trium pedum ab *a* que sit *o*, et aspiciatur *d* per lineam visualem *od* cuius intersectio cum virga *af* notetur et sit *z*. Erit ergo

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123 aliqua simili] alia similiter GH autem similiter P **124** ponitur] ponatur HP **125** o] *d* H **126** punctis] puncto H | et^{2}] *om.* P **127** supra] super H | et] visum *add.* H **128** visualis] ex his conclusionibus maxime ex coniuncta incidentaliter ostenditur quomodo *add.* G **130** he] *be* P | partes] omnes *add.* H **132** exibit] exivit H **133** hz] *az* HP | catham] alkatam HP **134** ed] *gd* H **135** ad^{1}] ipsum *add.* PH **137** linea] *om.* H | fd] etiam *add.* H **139** Vel aliter] *om.* D **140** ae] *ao* GHP | e] *o* GHP | sumantur] sumatur HP **141** quantaque] qualitercumque D quantacumque H **142** cb] *cd* H | angulus] scilicet *add.* G | cba] *bca* P **144** verbi gratia] ut DP | fa] *af* D **146** a] sub ipsa *add.* DHP | que] *om.* P

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predictum excessum. Et exibit *ed* nota cui si addatur *fe* etiam nota erit tota linea *fd* nota quod fuit propositum. *add.* DD

140 ae] [These three MSS assign the same letter *o* (for oculus) to the two different locations of the eye. This confusing practice may explain why they all apply the conjunct sector figure incorrectly.]

proporcio *ac* et *zc* notarum linearum nota que subtrahatur a proporcione etiam notarum quantitatum scilicet *ae* ad *oe* et remanebit ex eadem catha coniuncta proporcio *dz* ignoti ad [*do*] notum nota. Multiplica ergo antecedens illius proporcionis in *zo*, et productum divide per eius consequens, et exibit *zd* nota cui addatur *oz* et erit linea *od* nota que est distantia quesita. | P 127r

[I.9] G 117r

| Investigationem maximarum declinationum solis via D 215v instrumentorum a mathematica prescisione propter tria vel quattuor DD 220v deficere. Primum est quod centra instrumentorum non sunt centrum orbis H 67ra meridiani propter quod maxima elongatio solis a summitate capitum in divisione instrumenti repariter maior quam est in firmamento. [*Figura #7*] Quoniam posito *o* centro instrumenti et *oc* horizonte eius et *gn* horizonte centri terre *gd*, vero sit axis orizontis et scilicet sol elevatus in hemispherio. Tunc quia due linee horizontales *co* et *ng* equidistant, erit angulus *cos* equalis angulo *nko*. Sed angulus *ngs* est maior angulo *k* per 16^{am} primi quia est extrinsecus, ergo etiam angulo *cos*. Ergo per ultimam 6^{ti} arcus *ns* quem capit maior illorum erit plurium graduum quam arcus *rs* qui capitur ab angulo *cos* super centro instrumenti. Quare subtracto arcu *rs* a 90 remanet elongatio solis a zenith maior quam in rei veritate sit quod est propositum. Quod etiam levius patet quoniam angulus *sod* extrinsecus est maior angulo *sgd*. Ergo etiam arcus maior arcu.

Secundum est fractio radiorum propter quam radius solis cum linea horizontali facit maiorem angulum super centro instrumenti. Verbi gratia.

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148 ac] *az sed. corr. supr. lin.* D *az* GHP | notarum linearum] *om.* P | notarum…149 ad] et H **149** ae] *ao* GP | oe] *oo* GHP **150** do] *zo* DGHP **151** in] *supr. lin.* D *om.* G | divide] *om.* G **152** oz] *zo* D **153** quesita] et cetera *add.* H **155** Investigationem] 8^{a} *adnot. mg.* DD via…157 deficere] et cetera D **157** est] si *add.* D | quod] quia DD postquam dicitur quia *add.* D **158** propter…169 arcu] *om.* D et *add.* D **160** Quoniam] ut patet in figura DD | et^{1}] *g* centro terre et *gd* axi orizontis *add.* DD | eius] instrumenti DD **161** gd…169 arcu] *om.* DD **164** 16am] 26^{am} H | etiam] angulus *add. sed.exp.* G **165** ns] *ng* P | rs] *rg* P **166** subtracto] *rs add. sed.del.* P **167** remanet] remanebit P | a^{2}] ad H **168** sod] *god* P **170** est] quod *add.* H propter…175 signatur] *om.* D quia supposita figura secunda *add.* D | linea…171 horizontali] *inv.* DD **171** angulum] cum *add. sed.del.* DD | centro] centrum DD | Verbi…182 goc] *om.* DD

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150 Multiplica…153 quesita] [Because the author has the wrong statement of composition, this argument is faulty. The ratio he should find is *dz*:*do*. Given that the difference of these lines, which is *zo*, is known, line *do* can be found.]

[*Figura #8*] In hemispherio *abf* posito *o* centro instrumenti et *oc* horizonte eius et posita *edk* circumferentia medii interstitii aeris vel alibi ubi est confinium mediorum maxime differentium in raritate et densitate, manifestum erit quod sol qui signatur *s* non radiabit super *o* secundum lineam rectam *sro*. Ymo | in occursu puncti *r* frangetur inferius versus *h* H 67rb centrum terre. Ergo continget necessario aliquem radium incidentem in aliquem punctum supra *r* qui sit *d* refractum versus perpendicularem *sh* cadere in *o* centrum instrumenti. Radius ergo refractus *do* facit angulum *doc* maiorem angulo *soc*, quod fuit propositum. Est ergo altitudo solis visa in hoc situ maior altitudine eius vera in hemispherio in arcu *sg* qui est excessus duorum angulorum *soc* et *goc*.

Tertium est quod error in angulo insensibili nobis super centro instrumenti propter magnitudinem distantie circumferentie celi a terra facit errorem magne quantitatis in basi. Error vero iste non posset a nobis caveri si etiam centrum instrumenti esset simul cum centro mundi quia nihilominus bene contingeret lineam fiducie vel medium umbre a verissimo situ deviare | per aliquem angulum manentem omnino in sensu P 127v eadem apparentia de situ regule. Nam non a qualibet deminutione quantitatis pedalis obiecte visui variaretur iudicium pedalitatis veritatum licet huiusmodi insensibili angulo corresponderet magnus arcus celi propter nimietatem distantie scilicet 10 vel 100 miliarium. Tamen angulus sub quo huiusmodi quantitas arcus celi innumerum visum esset imperceptibilis visui. Igitur cum iudicium quantitatis visibilis fiat penes quantitatem anguli visionis per Euclidem de visibus, huiusmodi arcus quantitas apud visum non experietur. Verbi gratia. [*Figura #9*] Si due linee *ba* et *ca* apparerent mihi contigue circa punctum *a* quamvis facerent angulum certe quantitatis si etiam in infinitum protraherentur angulo

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172 #8] [The d**iagrams don't all match the text I have here in this section on imprecision!--I need to figure this** out!!!][DELETE] | oc] *ec* H **175** s] *g* DP **176** sro] *ro* D h] quod est *add.* D **178** supra] super H | sh] scilicet *h* D **180** fuit] est D **181** visa] viso P sg] scilicet *g* D **184** circumferentie celi] *inv.* D **185** posset] potest DDD | a] *om.* P **186** simul…200 basi] *om.* DD centrum signorum *add.* DD **187** nihilominus] eque *add.* P **188** manentem] manente DP | omnino] *om.* P **189** non] *om.* D | a…deminutione] ad quamlibet diminutionem DGH **190** pedalis] *om.* D | veritatum] (really doesn't look like veritatum in DGH but I have no better guess, I'll have to go to Erfurt I guess to figure that out. P looks like veriitaii with lines over both sets of minims, check Capelli)[DELETE] **191** licet] *om.* D videlicet P **192** Tamen] *illeg.* G quia *add.* H **193** esset] [there's a verb missing here, figure out what it should be and add.][DELETE] **194** visui] visu HP **195** quantitatem] quantitates H **197** contigue] continue D contingere H | facerent] facerem P eorum cadente super centrum oculi, numquam fieret apparentia alicuius quantitatis pro basi.

Propter illud ergo tertium non evenit quoquomodo error sensibilis in dicta observatione declinationum solis nec etiam propter primum pari ratione. [*Figura #10*] Cum duo anguli *sgn* et *soc* insensibiliter se excedant eo quod semidiameter terre *og* comparata ad circumferentiam celi se habet tamquam punctus et sub angulo insensibili videretur si esset tantum distans.

Error vero ratione secundi possibilis accidere quodammodo minuitur ratione primi ut apparuit satis. Et etiam si sensibilis fuerit, poterit caveri nam differentia altitudinis solis meridiane vise et eius que invenitur per almutantharat per notum gradum solis | faciliter erit nota. Et D 216r patet ergo ex his quod cum preciso astrologice observationis sufficiens sit | si secundum eam numquam vel saltem in longissimo tempore aliquis H 67va error sensibilis accidat observationum Ptholomei via instrumentorum, distantia tropicorum invenienda in presenti capitulo sufficere merito in hac arte.

Quartum vero potest esse motus accessus et recessus octave sphere qui nondum tempore Ptholomei astrologis apparuit ratione cuius contingit maximas declinationes solis quandoque fuisse minores quam modo sint et e converso. [*Figura #11*] Nam ex quo ponitur in huiusmodi ecliptica octave sphere que sit *che* semper inseparaliter adherere puncto solsticii estivalis none sphere qui sit *h*. Manifestum est quod ipsa intersecabit medietatem equinoctialis videlicet *ad* ad maiores angulos spherales qui sunt *hkl* et *hbl* quam ecliptica none sphere que est *hd* faciat. Quod sic

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199 eorum] etiam *a* D | super] supra P **201** illud ergo] *inv.* H | ergo] *om.* DD | non] nec P quoquomodo] quocumque modo D **202** dicta] *om.* H | etiam…204 excedant] pari vel propter primum DD **204** og] *eg* H | og comparata] *om.* DD | circumferentiam…205 celi] *inv.* DD **205** et…206 distans] *om.* DD | insensibili] sensibili D **206** tantum] tamen D **207** ratione] rationis G | quodammodo…208 minuitur] *inv.* DD **208** minuitur] minuetur G | ut…215 arte] *om.* DD | fuerit] fieret D **209** nam] *om.* H | meridiane vise] *om.* D | que] quam P **210** almutantharat] almatantharat GP almitantarat H | Et] *om.* DHP **211** astrologice] astronomice D **212** longissimo] longo H **213** observationum] observationem D instrumentorum] de *add.* HP **216** vero…esse] est DD | potest] probatum H | et] ex *add. sed. exp.* H **218** declinationes solis] *inv.* D | minores] maiores DD | modo] nunc(?) H | sint] *om.* DD | et…219 converso] *add. supr. lin.* P **219** Nam…246 sectoris] *om*. DD | huiusmodi] motu *add.* H **220** che] *chae* H | puncto solsticii] *inv.* D | solsticii] solsticiali H **221** estivalis] estivali DH **222** videlicet] scilicet P | spherales] *om.* D **223** ecliptica] eclipticam G | hd] *ahd* H | faciat] faciet P patet nam ex quo arcus *kl* equinoxialis est minor quarta cum *dl* sit quarta. Ponatur ergo punctus *k* polus et describam super ipsum circumferentia una de maioribus distans ab eo per quartam equinoctialis *km* et erit circumferentia *pnmc* transiens per duos polos mundi. Erit ergo propter hoc uterque arcuum *kn* et *km* quarta circuli. Ergo arcus *nm* est quantitas anguli | *hkl*. Arcus vero *hl* que est anguli *hdl* est minor quam arcus *nm* eo [P 128r] quod proporcio sinus arcus *nm* ad sinum arcus *hl* est sicut sinus totius *nk* ad *hk* ut inferius circa compositionem tabule declinationis solis videbitur. Ergo cum contingat quandoque maximam declinationem solis esse arcum *nm* et quandoque *hl* et etiam intermedius arcus ut patet intelligenti dictum motum derelinquitur propositum. Ex ita etiam demonstratione patet quod quotiens equatio argumenti octave sphere fuerit maxima, erit etiam maxima quantitas maximarum declinationum solis que contingere potest. Quotiens vero argumentum eius vel motus accessus et recessus quod idem est erit 156 graduum ut est arcus *fcg* vel 336 graduum ut est arcus *fcgx*. Tunc erunt declinationes minime eo quod ecliptice duarum spherarum in illis sitibus sibi invicem superponuntur, et intersecant equinoxialem none sphere super eisdem punctis scilicet capitibus Arietis et | Libre, que ponuntur centra parvorum circulorum in quibus capita [H 67vb] Arietis et Libre octave sphere revolvuntur secundum imaginationem. Quomodo autem inveniri debeat maxima diversitas illarum declinationum solis | videlicet maxime super minimam habet in compositione tabule [G 117v] equationis motus octave sphere videri que sit per figuram sectoris.

[I.10]

| Si in circulo aliquo continui arcus sumantur et uterque minor [D 217r H] semicirculo, diameter producta a communi eorum termino lineam rectam [68r]

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224 cum] quam P | sit] *om.* P **225** Ponatur] ponitur P | k] *l* P | describam] describatur H **226** km] *kmt* D | et…227 pnmc] *om.* D **227** circumferentia] *pnmt add. sed.del.* P **228** arcuum] rectus *sed. corr.* D | kn] *kl* D *kl sed. corr.* P **229** hl] *supr. lin.* P *hbg* D | que] qui P | est[1]] quantitas *add.* DH | nm] *mn* H **230** arcus[2]] arcum H | hl] *lh* D | totius] arcus D **231** inferius] *add. supr. lin.* G | compositionem tabule] tabulam P | declinationis] declinationum D **232** contingat quandoque] *inv.* D | declinationem solis] *inv.* D **233** et[2]…234 propositum] *om.* P **234** motum derelinquitur] notum relinquitur H | ita] ista D demonstratione] *om.* P **236** que…potest] *mg. a. m.* P **237** vel] *exp.* P quod *add. sed. del.* P et] vel P | quod…238 fcg] erit 170 P **238** graduum[2]] *om.* P | ut[2]…arcus[2]] *iter.* P **239** fcgx] *fgcx* P **240** intersecant] intersecent H **241** scilicet] *om.* H **242** capita] capitibus H **243** secundum…244 debeat] *om.* P | imaginationem] Thebit *add.* H **248** Si] 10ª *adnot. mg.* DDD 9ª *adnot. mg.* G | Si…251 alterius] *om.* P | aliquo] *om.* G | et…251 alterius] et cetera D minor…249 semicirculo] *inv.* DD semicirculo minor tunc H reliquos eorumdem terminos continuantem secabit secundum proporcionem corde dupli arcus unius ad cordam dupli arcus alterius.

Probatur quoniam manifestum est quod medietas corde cuiuslibet dupli arcus semicirculi minoris est perpendicularis super diametrum dividentem ipsum per medium. Ergo sinus rectus cuiuslibet arcus est super diametrum terminatam ad alterum extremum eius perpendiculariter quia nihil aliud est sinus rectus alicuius arcus propositi quam medietas corde dupli arcus. Sinus versus est porcio dyametri cadens inter extrema arcus dati et sinus eius recti. proporcio ergo sinuum rectorum quorumlibet arcuum erit sicut proporcio cordarum duplorum arcuum ad ipsos quoniam semper proporcio totorum est sicut partium equisubmultiplicium ex 5º Euclidis.

[*Figura #12*] Tractis ergo duabus lineis *gh* et *az* perpendiculariter super diametrum transeuntem ad punctum *b* coniunctionis duorum arcuum *ab* et *bg*, constat eas esse medietates cordarum duplorum arcuum ad illos arcus partiales et ipsam diametrum intersecare cordam *ag* aggregati arcus in duas partes secundum proporcionem *az* ad *hg*. Nam ex quo duo trianguli *zae* et *hge* propter hypothesim sunt equianguli, erit ex | H 69ra
4^{ta} 6^{i} proporcio *za* ad *ae* sicut *hg* ad *ge*. Ergo permutatim erit *za* ad *hg* sicut *ae* ad *eg*, quod fuit probandum.

[I.11]

| Si unus notus arcus in duos dividatur fueritque nota proporcio DD 221r
corde dupli arcus unius ad cordam dupli arcus alterius, ambo illi erunt noti.

| Pro declaratione propositionis huius et sequentium multarum etiam P 128v
in dictionibus 3ª et 4ª, aliqua oportet hic premitti, quorum primum est quod scire quantitatem arcus anguli est scire quantitatem arcus circumferentie super quam vel super cuius centrum ipse constituitur. Eidem correspondeat quia proporcio angulorum supra circumferentiam

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250 eorumdem] corde DD | continuantem] continentem DD **252** Probatur…269 probandum] *om.* DD | quoniam] *om.* P **253** dupli] *om.* DP | semicirculi] semicirculo H **255** extremum] igitur *add.* P | perpendiculariter] perpendicularis DP **256** alicuius…propositi] *om.* D **257** dupli arcus] *inv.* GHP | porcio] proporcio H **259** sicut] ut GH | proporcio] arcuum *add. sed. del.* H **261** ex] *om.* G **262** duabus lineis] *inv.* D **264** esse] *om.* P | duplorum arcuum] *inv.* D **269** fuit] erat G **271** Si] 11ª *adnot. mg.* DDD 10ª *adnot. mg.* G | Si…273 noti] *om.* P in duos] *post.* dividatur D | fueritque…273 noti] *om.* D **272** corde] *om.* DDH **274** Pro…377 propositum] *om.* DD | propositionis huius] *inv.* DHP | sequentium multarum] *inv.* DP **276** arcus[1]] alicuius *add.* H **277** ipse] *om.* DP | constituitur] et *add.* H **278** supra…279 centrum] sive super centrum sive super circumferentiam D sive centrum constitutorum est sicut proporcio arcuum inter eorum latera cadentium ex ultima 6^{i} Euclidis.

Secundum est quod | semper idem angulus vel equalis super D 217v circumferentiam cuiuscumque circuli cadens capit arcum in duplo plurium partium circumferentie illius quam si supra centrum circuli alterius cuiuscumque ceciderit. [*Figura #13*] Patet nam posito *d* puncto conali anguli *fdg* cadentis super circumferentiam minoris circuli centro circuli *kb* cuiuscumque quantitatis et tractis lineis *dk* et *db* et *eg* et *ef* a centris circulorum, erit ex ultima 6^{i} numerus partium arcus *kb* ad 360 sicut angulus *kdb* ad quatuor rectos. Similiter per eandem numerus graduum vel partium arcus *gf* ad 360 sicut angulus *feg* ad quatuor rectos. Ergo necessario proporcio plus capiet arcus *gf* de 360 arcu *kb* sicut angulus *feg* plus capit de quatuor rectis angulo *bdk* quia si fuerint duo ordines quantitatum verbi gratia a prima, *b* secunda, *c* tertia, *d* quarta, et iterum quatuor videlicet *e* prima, *f* secunda, *g* tertia, *h* quarta. Tunc si proporcio prime ad secundam in primo ordine fuerit sicut tercie ad quartam in secundo ordine et iterum tercia ad quartam primi ordinis sicut prime ad secundam secundi ordinis, ex propositione una 5^{i} Euclidis *a* ad *c* sicut *e* ad *g*. Sed angulus *feg* in duplo plus capit de quatuor rectis quam angulus *bdk* cum sit duplus ad ipsum ex propositione 3^{i} Euclidis. Ergo arcus *gf* est in duplo plurium partium de 360 partibus sui totius quam arcus *kb* sui, quod fuit propositum.

Ex quibus manifestum est quod cognito numero partium alicuius arcus qui capitur ab aliquo angulo super circumferentiam circuli constituto, medietatem illarum idem angulus capiet de circumferentia quacumque super cuius centrum ceciderit. Et ergo quando alius angulus est notus super circumferentiam circuli, tunc etiam erit notus super centrum | quia semper subduplum arcum capit super centrum. Et e H 69rb converso cognito alio angulo super centro circuli, erit etiam ex hoc eius quantitas nota super circumferentiam cum sit dupli illius.

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279 sive] vel P supra *add.* H **282** cuiuscumque] cuiuslibet D **283** supra] super D **284** alterius] *om.* H **285** super] supra P **286** kb] *bl* P | eg] *dg sed. corr.* G **287** arcus kb] *inv.* P **290** proporcio] proporcionaliter G **292** c tertia] *inv.* P | d quarta] *inv.* P **293** videlicet] scilicet D | prima] et *add.* H **295** tercia] et *add. sed. del.* D **296** prime] prima H | una] *om.* GP 20ª H **297** sicut] *g* ad *add. sed. del.* D **298** cum] tamen D | propositione] *om.* D | 3i] 3º D **299** plurium partium] *om.* D **300** sui] suo H | fuit] est D | propositum] probandum H **302** super] supra HP | circumferentiam] alicuius *add.* D **303** constituto] constitutam H **304** alius] aliquis P | alius angulus] *inv.* H **305** circuli] arcus P | notus²] notius H **306** quia…centrum²] *om.* D | capit] accipit P **307** cognito…313 constituti] *om.* D | alio] aliquo HP arcu *add. sed.exp.* P | angulo] *supr. lin.* P | circuli] certum P **308** dupli] duplum P

Si igitur angulus rectus super centrum circuli capiat semper quartam partem circuli, oportet quod rectus angulus super circumferentiam capiat semicirculum, quod etiam patet ex 30^{am} 3^{i}. Et per consequens duo recti super circumferentiam capiunt 360 gradus circumferentiales quemadmodum quattuor faciunt super centrum circuli constituti. Manifestum est etiam ex his equales angulos super centris quorumlibet circulorum semper equaliter partes circumferentie eorum capere. Et ita etiam est de equalibus angulis super circumferentiam terminatis quantumlibet inequales videlicet quod semper capiant arcus proporcionales ad circumferentias totas. Nam proporcio arcuum ab eis captorum ad totas circumferentias est sicut proporcio cuiuslibet angulorum equalium super circumferentiam consistentium ad duos rectos et super centrum consistentium ad quattuor rectos. Cum ergo proporcio quorumlibet equalium angulorum ad quattuor vel duos rectos semper sit una, erit necessario etiam proporcio arcuum eis correspondentium ad totas circumferentias una quia omnes proporciones equalibus equales inter se sunt equales.

Tertio declarare oportet quomodo cuiuslibet orthogonii trianguli notorum laterum anguli sunt noti tam super circumferentiam quam super centrum cuiuscumque circuli. [*Figura #14*] Sit enim orthogonius triangulus *abc*, et posito *ab* latere eius quod opponitur recto angulo diametro super quam semicirculus descriptus | per 30^{am} 3^{i} | transit per *c* P 129r |
conum anguli recti, sint ergo gratia exempli illa tria latera nota secundum G 118r
quantitatem pedalem ita quod *ab* sit 20 pedum *ac* 12 et *cb* 8. proporcio ergo *ab* ad *ac* est nota quia est ut 20 ad 12. Sicut ergo et ex quo *ab* est diameter, ergo ipsa est 120 partium. Queratur ergo quantitas ad quam 120 se habeant sicut 20 ad 12, et ipsa erit linea *ac* secundum quantitatem qua *ab* vel diameter est 120 partium. Quod sic fit. Multiplica 120 in 12 et productum divide per 20, et exibit *ac* nota secundum gradus cordales. Corda ergo *ac* est nota. Ergo per tabulas cordarum eius arcus scilicet *ac*

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309 igitur] *om.* H | semper] *om.* P **312** 360] 160 H | gradus] *om.* P **315** equaliter] quartas *add. sed.del.* G equent P eque totis H | ita…316 est] idem D **316** equalibus] aliquibus P terminatis] constitutis P **317** quantumlibet…318 Nam] et D | capiant] capiunt H **321** et…rectos] *om.* D **322** equalium angulorum] *inv.* D **323** erit…etiam] etiam necessario erit D **326** Tertio] nota bene *adnot. mg.* G **327** super[1]] supra H | super[2]] supra H **328** cuiuscumque] cuiuslibet D **329** posito ab] *inv.* D | eius…opponitur] opposito D | recto angulo] *inv.* D **330** c…331 conum] *inv.* P **332** 20] 10 H | pedum] et *add.* D | 12] pedum *add.* H **334** ergo[1]] *om.* DH **335** habeant] habent G | 20] se habent *add.* G **336** vel] *om.* H | est] *om.* H | fit] sit P fuit *sed. corr.* H **338** ac[2]] *a* et *c* H erit notus, qui secundum predictam est quantitas anguli *cba* super circumferentiam constituti. Qua quantitate subtracta de semicirculo id est de 180, remanebit arcus *cb* notus qui est quantitas alterius anguli *cab*. Ergo sunt tres arcus noti quos capiunt tres anguli dati orthogonii super circumferentiam circuli constituti. Ergo etiam ex predictis erunt tres arcus noti quos caperent super centra quorumlibet circulorum. Et hoc est angulos illos esse notos, ergo et cetera.

Similiter patet quod noto uno angulo alicuius orthogonii preter rectum et uno quocumque alio latere, alia latera et anguli eiusdem | erunt H 69va noti. Nam da quod angulus *b* sit notus et latus *cb* notum gratia exempli 8 pedum. Igitur quia angulus *cba* est, notus erit arcus *ac* notus. Ergo et arcus *cb* notus et erunt due linee *ac* et *cb* note secundum quantitatem qua *ab* est 120 partium. Sit ergo *cb* 16 partium. proporcio ergo *cb* ad *ab* est nota quia ipsa est sicut 16 ad 120. Ergo posita *cb* 8 pedum erit secundum modum predictum etiam *ab* nota secundum quantitatem qua *cb* est 8 pedum. Et hoc sic. | Multiplica 8 in 120 et productum dividatur per 16, et D 218r exibit *ab* nota secundum divisionem pedalitatis. Et simili modo erit *ac* nota secundum eandem quantitatem. Quod est propositum.

His ergo premissis retenta figura et hypothesi Ptholomei, [*Figura # 15*] erit proporcio *ge* ad *ea* nota quia ipsa est per propositionem precedentem sicut proporcio sinuum duarum partium *ab* et *bg* arcus *ag* dati. Sumantur ergo aliqui numeri vel quantitates proporcionales secundum illam proporcionem, et sint gratia exempli tria et quatuor. Sicut ergo se habent tria ad quatuor ita *ge* ad *ea*. Ergo coniunctim sicut tria et quatuor ad quatuor vel sicut 7 ad quatuor ita tota corda *ga* que est nota ex tabula cordarum ex quo eius arcus habetur notus ad partem eius *ea* ignotam. Multiplica ergo 4 in *ga* et productum divide per 7, et exibit *ea* nota. Deinde quia *dz* vadit a centro perpendiculariter super cordam *ag* notam, ergo *az* eius medietas est etiam nota. Qua subtracta ab *ae* iam nota, relinquitur *ze* nota. Quia ergo angulus *azd* est rectus subtrahatur

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339 qui] igitur D *om.* H | predictam] predicta P | anguli] *bca add. sed.del.* D **340** Qua] quia H | id est] *om.* G **342** Ergo sunt] *inv.* DH | dati] *org add. sed.del.* G dicti P **343** Ergo etiam] et P **344** centra] centris DHP **345** illos] illorum D | ergo…cetera] *om.* H | et cetera] *om.* G **346** Similiter] nota bene *adnot. mg.* G **347** quocumque alio] *om.* H | quocumque…latere] latere quocumque DP **348** cb] *bc* H **349** Igitur] *add. mg.* G **350** arcus] *ab add. sed.del.* D cb[1]] *bc* P **351** cb[1]] *c* H | partium[2]] talium *add.* D **352** ipsa] *om.* P **353** etiam] *om.* H | nota] notum D **354** productum] pro H **355** pedalitatis] pedalis P **356** est] fuit H **358** proporcio] *om.* D | ge] *eg* P | propositionem] proporcionem P **362** ergo] *om.* P | ge] *eg* P | Ergo] ita P **364** habetur] supponitur D **365** divide] dividatur P **366** centro] perpendicularis *add. sed.del.* D **368** ze] *ez* D | angulus] *om.* D quadratum linee *az* notum a quadrato *ad* etiam note que est semidiameter, et remanet quadratum *zd* notum et erit *zd* radix eius nota. Similiter iunctis duobus quadratis notarum linearum *zd* et *ze,* erit quadratum linee *ad* notum, et sit linea *ed* nota. Sunt ergo tria latera trianguli orthogonii *dze* nota. Ergo ex premissis erunt tres anguli eius noti tam super centrum quam super circuli circumferentiam. Arcus ergo quem capit angulus *zde* super centrum circuli est notus. Quo addito super medietatem arcus *ag* a principio noti, erit unus arcuum partialium videlicet *ab* notus. Qui dematur de toto *ag*, et erit etiam *bg* alter notus, quod est propositum.

[I.12A]

Si ab uno termino arcus semicirculo minoris linea ipsum arcum secans educatur donec cum diametro per reliquum | eiusdem arcus H 69vb terminum extracta concurrat, fiet proporcio linee preter centrum transeuntis ad partem sui extrinsecam sicut proporcio corde dupli arcus de quo sermo est ad cordam dupli arcus illius quem educte linee includunt.

[*Figura #16*] Rescindat ergo corda *gb* de arcu *ga* minore semicirculo aliquem arcum ita quod arcus *ba* | residuus sit minor arcu in P 129v quo semicirculus excedit arcum totalem *gba* alias enim non oporteret cordam *gb* alibi extra circulum cum diametro exeunte per punctum *a* concurrere. Tractis ergo duabus perpendicularibus *gh* et *bz,* ipse erunt sinus vel medietates cordarum duorum arcuum videlicet totius *ga* et partis eius *ba*. Et erunt duo trianguli *hge* et *zbe* equianguli. Ergo ex 4^{a} 6^{i} *hg* se habebit ad *ge* sicut *zb* ad *be*. Ergo permutando erit proporcio totius

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369 notum] notis DG | quadrato] linee *add.* P | note] noto D **370** remanet] remanebit P **371** notarum linearum] *inv.* D **372** tria latera] *om.* D | trianguli] *om.* H **374** circuli] *om.* P circuli circumferentiam] *inv.* D **375** medietatem] subtracto a medietate *adnot. supr. lin.* D **379** Si] 11^{a} *adnot. mg.* DDG | Si…384 includunt] *om.* P | semicirculo…384 includunt] et cetera D **380** secans] secando DD | diametro] per *add.* DDH **381** concurrat] concurrerit DDH **382** transeuntis] *om.* DD | corde] *om.* DDH **383** sermo est] *inv.* DDH **385** Rescindat] rescindatur H | Rescindat…394 intentum] *om.* DD | minore] minori P **388** alibi] *om.* DH alii(?) P **390** duorum] duplorum P | ga] videlicet totius *add. sed. del.* H **391** partis] partes H **392** Ergo] *om.* D

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384 includunt] Illa proposicio vult dicere quod dato aliquo arcu principia *ga* qui sit semicirculo minor et ab una eius extremitate trahatur linea secans illum arcum principia *gbe* tunc si trahatur diameter per aliam extremitatem principia *dac*, tunc proporcio totius *ge* ad *be* est sicut corde dupli arcus *ga* ad cordam dupli arcus *ba* patet quia *gq* est corda arcus *gaq* qui arcus duplus ad arcum *ga* et sic patet propositum per 2^{am} 6^{i} Euclidis vel etiam per 4^{am}. *adnot.* DD *ge* ad partem eius que extra circulum cadit *be* sicut *gh* ad *bz,* quod est intentum.

[I.12B]

Si arcus dicto modo divisi lineis maior porcio nota fuerit et proporcio corde dupli arcus ipsius divisi ad cordam dupli arcus lineis eductis inclusi constiterit ipse arcus inclusus notus erit. [*Figura #17*] Sit enim in eadem hypothesi solum arcus *gb* notus. Et cum hoc proporcio sinus totius *ga* ad sinum arcus *ba* nota que sit gratia exempli inter *g* et *f*. Erit etiam arcus *ba* residuus notus. Nam ex precedenti proporcio sinus totius arcus *ga* ad sinum partis eius *ba* est sicut totius linee *ge* ad *be*. Ergo etiam proporcio *ge* ad *be* est sicut *g* ad *f* propter quod erit etiam disiunctim proporcio excessus ipsius *g* super *f* que sit *k* ad *f* sicut proporcio *gb* corde note eo quod arcus eius ponebatur notus ad lineam *be* ignotam. Ducatur ergo *f* quantitas nota in cordam *gb* et productum dividatur per *k,* et exibit *be* nota. Cui cum addatur *zb* medietas *gb* corde note, erit tota linea *ze* nota. Deinde subtracto quadrato linee *zb* a quadrato semidiametri *bd,* relinquitur ex penultima primi quadratum linee *zd* notum. | Quare et ipsa *zd* eius radix erit nota. Iunctis ergo simul duobus H 70ra
quadratis | notarum linearum *zd* et *ze,* erit similiter quadratum *dz* notum. G 118v
Et sic erit orthogonius triangulus *dze* notorum laterum. Ergo ex commento 11^{e} huius erunt eius anguli noti. Ergo arcus circumferentie quem capit angulus *zde* super centrum circuli constitutus est notus. A quo

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396 Si] 12^{a} *adnot. mg.* D DDG | Si…398 erit] *om.* P | lineis] ut prescriptum est donec concurrant eductis *add.* DDH | maior…398 erit] et cetera D | porcio] proporcio H **398** Sit…416 querebatur] *om.* DD **399** hoc] *om.* P **401** sinus] *add. supr. lin.* P **402** ga] *g* ad *a* P **405** lineam] *om.* H | be] *eb* P **407** be] *eb* P | Cui] *b add.* D **410** notum] et *add.* P | et] *om.* P | eius radix] *inv.* D **411** notarum] dica(?) *praem.* notarum G | ze] *ez* D **413** 11e] 9^{e} D noti] recti D **414** A] de D

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398 notus] Illa proposicio vult dicere quod cum fuerit arcus *ga* sic divisus ut precedens proponit et arcus *gb* sit solum notus, proporcio tamen sinus *ga* ad sinum *ba* sit nota. Tunc dicit proposicio quod etiam arcus *ab* erit notus et per ... totus arcus *ga* quia angulus *bdz* est notus quia medietas arcus *gb* noti qua propter totus triangulus *bdz* orthogonius notus. Et quia proporcio *ge* ad *eb* et corda *gb* est nota, sciemus ex hoc *eb* et per consequens etiam *ebz,* et ex consequenti totus triangulus *edz*. Et subtracto angulo *zdb* ab angulo totali, *d* remanebit angulus *edb* notus ergo et cetera. *adnot.* DD **412** ex…413 11e] [I.11 paragraph 6, one of the proofs used to prove Lemma 4, lines 326-345.]

si dematur angulus *zdb* notus qui capit medietatem arcus *gb* noti, relinquitur arcus *ba* notus qui querebatur.

[I.13A]

In superficie sphere duobus arcubus magnorum orbium semicirculo divisim minoribus ab uno communi termino descendentibus aliisque non minorum orbium ab illorum relictis terminis in eosdem se secando reflexis, utrius reflexorum alterius conterminalem arcum sic figet ut proporcio corde arcus duplicantis inferiorem porcionem arcus fixi ad cordam arcus duplicantis superiorem eiusdem fixi porcionem producatur ex gemina proporcione scilicet ex ea quam habet corda arcus duplicantis inferiorem arcus reflexi porcionem que ipso fixo conterminalis ad cordam arcus duplicantis reliquam eiusdem reflexi porcionem et ex ea proporcione quam habet corda arcus duplicantis inferiorem alterius descendentis arcus ad cordam arcus duplicantis arcum ipsum cuius est pars totalis.

[*Figura #18*] Probatur retenta figura et hypothesi Ptholomei quia *gl* ad *la* per 10^{am} huius est sicut proporcio sinus arcus *ge* ad sinum arcus *ea* ex quo *h* ponitur centrum omnium arcuum figure. Et per eandem proporcio *gk* ad *kd* est sicut sinus arcus *gz* ad sinum arcus *zd*. Et per 12^{am} huius proporcio totius *at* ad *dt* partem eius extrinsicam est sicut proporcio sinus totius arcus *adb* ad sinum arcus *db* partis eius. Ergo etiam conversim erit ut sicut *td* se habet ad totam *tda* ita etiam se habeat sinus arcus *bd* ad sinum totius arcus *bda*. Tunc ex quo duo puncta *l k* videlicet sectionum semidiametrorum *he* et *hz* cum cordis totorum arcuum *gea* et

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415 zdb] etiam *add.* H | qui] quoniam D **418** In] 13^{a} *adnot. mg.* DG | In…429 totalis] *om.* P arcubus] *om.* H | arcubus magnorum] magnorum circulorum orbibus D **419** divisim] divisi G **420** relictis] reliquis DH **421** utrius] utroque DH utrius G | arcum] *om.* DH **422** corde] *om.* H **424** gemina] se DH **425** porcionem…429 totalis] et cetera G | que] quoniam D que…426 porcionem] *mg.* D | ipso] ipse D **426** reliquam] reliquum D **429** totalis] totalem H **433** 12am] 11^{am} D **434** sicut] sic G **435** arcus^{1}] ad sinum arcus *add.* P | sinum] sinus D eius] etiam D **437** l k] *om.* P **438** semidiametrorum] diametrorum *sed. corr. supr. lin.* P gea] *ged* P

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416 querebatur] “In superficie sphere duobus magnorum circulorum et cetera” vide inferius per duo folia *add.* D

[Here D has the summary of the text up to this point. That text, which is on fol. 220r-221r, has been marked by “DD.”] **418** In] Ipsa proposicio non vocatur(?) probare aliud nisi catham disiunctam in arcubus ut quod sinus *ge* ad sinum *ea* integratur ex proporcione *gz* ad *zd* et *bz* ad *be* totam, intendo de sinubus vel de cordis duplicium arcuum *adnot. mg.* D **431** per…huius] [Lemma 3] **433** 12am…434 huius] [Lemma 5.]

gld | cadunt sive signantur in una recta linea videlicet *lkt* que est sectio H 70rb communis duarum superficierum scilicet trianguli *agd* que spheram imaginatione exiens eam transversaliter secat et superficiei circuli *ezb* que super axem *tbh* secat | spheram. Que ergo necessario secatur a dicta P 130r superficie trianguli. Quarum sectio signatur linea *tkl*. Facta est ergo catha quedam rectilinea cuius conus est *a* et linee reflexe *gkd* et *tkl* in una superficie plana dicti trianguli. Cum ergo per 9^{am} huius proporcio linee *gl* ad *la* componatur ex proporcionibus *gk* ad *kd* linearum et *td* ad *tda* lineam totam, sequitur quod etiam equalis proporcio est sinus arcus *ge* ad sinum arcus *ea* ipsi proporcioni *gl* ad *la* componatur ex equalibus duabus proporcionibus componentibus ipsius que sunt proporcio sinus *gz* ad sinum *zd* et proporcio sinus *bd* ad sinum totius arcus *bda* ut patuit ex 10^{a} et [12^{a}] huius quia equalia ex equalibus numero et quantitate | habent D 221v componi. Habetur ergo proporcionem sinus *ge* ad sinum arcus *ea* esse compositam ex duabus proporcionibus etiam sinuum arcuum *gz* et *zd* et *bd* et *bda*, quod est propositum.

[I.14A**]**

[*Figura #19*] Ex his coniunctam catham in eadem hypothesi lineis ultra punctum *b* concurrentibus restat demonstrare. Nam per 12^{am} proporcio *gl* ad *el* est sicut proporcio sinus totius arcus *ga* ad sinum arcus partialis *ea*. Et per eandem linea *gzk* se habet ad *zk* sicut sinus totius arcus *gd* ad sinum arcus *zd*, et per eandem 12^{am} linea *tz* se habet ad lineam totam *tze* sicut sinus *bz* arcus ad sinum arcus totius *bze*. Sed quia similiter hic facta est quedam catha rectilinea in superficie una cuius angulus conalis est *l* et linee reflexe *gzk* et *tze*. Propter hoc quod due linee recte *tze* et *tkl* sunt due sectiones superficiei trianguli *gze* cum duabus superficiebus circulorum *be* et *ba* intersecantibus se super axem sphere *tbh*. Ergo proporcio totius linee *gl* ad *el* est composita ex proporcionibus

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439 gld] *gzd* D **440** agd] *agb* G **441** imaginatione] imaginare transiens sive P | eam transversaliter] equitransversaliter D | secat] *om.* D | superficiei] superficie D **443** tkl] *ckh* P est] *om.* H | est ergo] *inv.* D **444** quedam] que H | linee reflexe] linea reflexa H **445** 9am] 10^{am} D **446** ad^{2}] et G **447** proporcio] scilicet *add.* D est *add. sed. exp.* G **448** arcus] *iter.* P **450** bd] *ba* P **451** 12a] 11^{a} DGHP **456** Ex] 14^{a} *adnot. mg.* DG | Ex…457 demonstrare] *om.* P **457** 12am] 11^{am} D **458** el] *dl* GHP | totius] *om.* D **459** habet] sicut *add.* P | totius] ad *add. sed.exp.* G **460** 12am] 11^{am} D **461** arcus totius] *inv.* P | similiter…462 hic] super hoc P **462** hic] similiter *add. sed.del.* G **463** reflexe] *dzk add. sed.del.* D **464** trianguli] circuli D **465** se] *om.* P

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445 9am huius] [Lemma 2.] **450** 10a…451 huius] [Lemmas 3 and 5.] **457** 12am] [Lemma 5] **460** 12am] [Lemma 5.]

gk ad *zk* et *tz* ad totam *tze*. Ergo etiam equalis proporcio sinus arcus *ga* ad sinum arcus *ea* componetur ex proporcionibus | H 70va equalibus illis componentibus videlicet sinuum arcuum *gd* ad *zd* et *bz* ad *be*, quod fuit probandum.

[I.13B]

Linea *da* concurrente cum *bh* versus punctum *g* nihilominus catham disiunctam demonstrare. Notum est enim cum *ba* sit semper minor semicirculo secundum hypothesim Ptholomei, si arcus residuus ad complendum cum *ba* semicirculum fuerit minor arcu *bd*, quod tunc corda arcus *ad* concurrit cum linea *hb* versus *g* si maior concurrit cum eadem versus *b*, si equalis equidistabit ipsi *hb*. Ptholomeus autem solum demonstravit stante secunda hypothesi. Sunt ergo alie due partes ex precedentibus faciliter demonstrande divisim et coniunctim.

[*Figura #20*] Pro quo premitto duas suppositiones quarum prima est quod quorumlibet arcuum constituentium semicirculum est equalis sinus ut linea *bz* est sinus utriusque arcus *ab* et similiter *cb* ex quibus aggregatur semicirculus *cb*. Secunda est quod si proporcio primi ad secundum componitur ex proporcionibus tertii ad quartum et quinti ad sextum, tunc proporcio etiam tertii ad quartum erit composita ex proporcione primi ad secundum et sexti ad quintum. Patet ex | P 130v libello de proporcione et proporcionalitate. Verbi gratia: proporcio 4 ad 2 componitur ex proporcionibus 6 ad 2 et 2 ad 3. Ergo etiam proporcio 6 ad duo componitur ex proporcionibus 4 ad 2 et 3 ad 2 videlicet primi ad secundum et sexti ad quintum. Quia si ducatur denominatio proporcionis primi ad secundum que est binarius in denominatione sexti ad quintum que est unitas cum una medietate, exibit trinarius qui est denominatio proporcionis tertii ad quartum scilicet 6 ad 2.

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467 equalis] *om.* D | ad^{3}…468 arcus] *iter*. D **469** ad^{1}] et GHP | ad^{2}] et GHP **472** Linea] additio *adnot. mg.* DG | da] *ad* P **473** disiunctam] demonstratum(?) D | est enim] *inv.* DHP semper] *om.* G **476** ad] *bd* H | hb] *bh* D *ahb* G | cum eadem] *post. b* P **477** ipsi] *om.* P autem…478 demonstravit] demonstravit solum P **478** secunda] hypothesim *add. sed.del.* P ergo] alique *add. sed.exp.* P **482** arcus] scilicet *add.* P | similiter] *om.* P | ex…483 cb] *om. (hom.)* P **483** cb] alias *ca add.* D | quod] *om.* GHP **484** quinti] coniunctim H **488** ex] proporcione seu *add.* P **490** et] quinti ad sextum *add. sed.exp.* P **491** denominatione] denominationem HP **492** una medietate] 1/2 P | medietate] et *add.* G | denominatio…493 proporcionis] *inv.* G **493** scilicet] *om.* P

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486 libello…487 proporcionalitate] [The *Epistola* of Ametus.]

Retenta ergo figura Ptholomei, sit *da* concurrens cum *bh* ex parte *g* et serventur omnes linee in locis suis nisi quod perficientur semicirculi arcuum *ba* et *be*, et concursus eorum versus *g* sit *f*. Tunc capta partiali catha cuius conus est *d* et arcus reflexi *fez* et *gea*, tunc manifestum est per argumentum et modum arguendi Ptholomei quod erit proporcio sinus *gz* arcus primi ad sinum *dz* secundi composita ex proporcione sinus arcus *ge* tertii ad sinum *ea* quarti et ex proporcione sinus *fa* quinti ad sinum *fd* sexti. Hoc patet posito *h* omnium circulorum centro et tracta linea *lkt*. Facta erit catha rectilinea *gdt* in una superficie trianguli *gda* per similem imaginationem ut prima hypothesis. | H 70vb Igitur per secundam suppositionem proporcio sinus *ge* tertii ad sinum *ea* quarti est composita ex proporcione sinus *gz* primi | G 119r ad sinum *dz* secundi et ex proporcione sinus *fd* sexti ad sinum *fa* quinti. Sed proporcio sinus *fd* ad sinum *fa* est sicut proporcio sinus *bd* ad sinum *ba* quia per primam suppositionem sinus arcuum *fd* et *bd* est idem et similiter arcuum *fa* et *ba* est sinus idem et proporcio equalium ad idem est eadem. Ergo proporcio sinus *ge* arcus ad sinum arcus *ea* erit etiam composita ex proporcionibus scilicet sinus *gz* ad sinum *zd* et proporcione sinus *bd* ad sinum | D 222r arcus *bda* ex quo ipsa est equalis proporcioni *fd* ad *fa*, et hoc stante dicta hypothesi quod corda *da* concurrat cum linea *bh* versus *g*, quod erat probandum.

[I.13B]

[*Figura #21*] Linea *dh* equidistante *bh* idem propositum declarare. Pro quo pono tres suppositiones. Prima est quod sumpto arcu in semicirculo cuius corda diametro equidistet, erit sinus arcus intercepti equidistantibus equalis sinui arcus aggregati ex arcu intercepto et arcu sumpto ut sinus arcus *bd* est equalis sinui *bda* vel *fad* si *da* corda equidistet diametro *bf*. Patet de se. Secunda suppositio est si una superficies secet alteram duarum secantium se, equidistanter earum communi sectioni secabit et reliquam equidistanter sectioni eidem vel dum trium linearum quelibet due fuerint in superficie una et non omnes in una, si due earum equidistent omnes sibi invicem equidistabunt. Tertia

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494 parte] *add. supr. lin.* P **495** locis suis] *inv.* D | perficientur] perficiantur P **496** ba] *ab* H **498** quod] *om.* DHP **499** dz] *zd* DHP **501** centro] *praem.* omnium D | lkt] *klt* D **502** Facta erit] patebit G | gdt] *om.* D | trianguli] *om.* P | similem] consimilem H **506** sinum^{2}] *om.* P **507** per] *om.* G **508** bd] *ba sed. corr.* P | ba] *b* H **510** etiam] *om.* P **512** fd…fa] *om.* D **515** Linea] alia additio Campani *adnot. mg.* D additio *adnot. mg.* G **516** sumpto] supposito H | in] *om.* D **517** equidistet] equidistat H | arcus intercepti] *inv.* P **518** sinui] sinus H **519** bd est] *bdz* G **520** est] *om.* G **522** eidem] est idem H **524** equidistabunt] distabunt H suppositio est quod omnis proporcio componitur ex seipsa et proporcione equalitatis. Patet quia denominatio proporcionis equalitatis ducta in denominationem cuiuscumque proporcionis producit ipsius denominationem, ergo et cetera.

[*Figura #22*] Retenta ergo eadem catha *bag* cum lineis suis nisi quod *da* ponatur equidistare *bhf*. Planum est ergo quod due linee *da* et *bf* in superficie una scilicet circuli *baf* et equidistantes | ex hypothesi. P 131r Similiter due linee *da* et *lk* sunt in superficie una trianguli *gda* que superficies trianguli secat superficies duas duorum circulorum *bzf* et *bdf* secantium se super *bf* sectionem communem super duabus lineis *da* et *lk*. Ergo per secundam suppositionem *lk* equidistat *bf*. Ergo *lk* et *da* equidistant | eidem linee tertie scilicet *bf* ut patet ex secunda parte H 71ra secunde supposicionis.

Quia ergo linea *lk* est in superficie una cum *bf* scilicet in superficie circuli *bzf*, ergo per 2^{am} 6^{i} proporcio *gk* ad *ka* est sicut *gl* ad *ld*. Ergo per 10^{am} huius proporcio sinus arcus *ge* ad sinum *ea* est sicut proporcio sinus *gz* ad sinum *zd*. Sed proporcio sinus *bd* ad sinum arcus *bda* est proporcio equalitatis ex prima suppositione. Ergo per 3^{am} supposicionem proporcio sinus *ge* ad sinum *ea* est composita ex eadem proporcione que est inter sinum *gz* ad sinum *zd* et proporcione equalitatis que est sinus *bd* ad sinum arcus *bda*, quod est propositum secundum assumptam hypothesim.

Si autem vis in tertia positione probare compositionem proporcionis *bd* ad *da*, tunc trahe cordam arcus *bda*, et erit corda arcus *ae* similiter se habens ad diametrum semicirculi arcus *gea* sicut *da* se habuit in semicirculo *bda*. Et linea *lk* cadet inferius versus *g*. Vel ponam arcum *ge* ad *ea* sicut *bd* ad *da*. Et arguo sicut dictum est. Patet ergo utilitas figure sectoris quantumcumque etiam fuerit arcus *ba* minor semicirculo.

[I.14B]

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525 suppositio] *om.* H | suppositio…quod] *om.* G | proporcio] composita *add.* G proporcione] proporcioni P **529** eadem] arcualis *add.* D **530** ergo] *om.* P | da^2] *bda sed. corr.* D *ba* P **532** lk] *kl* D | gda] *dga* G **534** sectionem communem] *inv.* D sectione communi H | lk] *kl* D **539** ka] *kd* lineam D | est] *om.* D | ld] *la* D **542** supposicionem] *om.* D **543** eadem proporcione] *inv.* H **544** et] ex H **545** assumptam] adsuptam D **546** tertia positione] disposicione H | compositionem proporcionis] proporcionis composiciones H **548** ad] *om.* D | da] *post.* habuit P | habuit] habent D habet P **549** lk] *lz*(?) D *lkt* H **550** arguo] ergo P **551** sectoris…semicirculo] *om.* D | etiam fuerit] *inv.* P

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540 10am huius] [Lemma 3.]

[*Figura #20*] Coniunctam catham in dictis hypothesibus universaliter demonstrare. Retenta ergo dispositione tertia, patet ex eo quod proporcio sinus arcus *bd* ad sinum *da* est composita ex proporcionibus sinus *bz* ad sinum *ze* et proporcione sinus *ge* ad sinum *ga*. Tunc sumpta quadam alia catha versus *f* videlicet *gdf*, manifestum est ex precedenti suppositione tertie hypothesis premisse quod proporcio sinus *fd* coniunctim ad sinum *ad* est sicut proporcio sinus *bd* ad sinum eiusdem arcus *da* quia per illam suppositionem idem est sinus *fd* et *bd* arcuum cum *bdf* ponatur semicirculus. Ergo similiter proporcio sinus *fd* ad *ad* erit composita ex eisdem proporcionibus scilicet sinus *bz* ad sinum *ze* et sinus *ge* ad sinum *gea* quia equales proporciones ex eisdem componuntur. Sed per dictam primam suppositionem proporcio sinus *fz* ad *ez* est sicut sinus *bz* ad sinum eiusdem arcus *ez* quia arcuum *fz* et *bz* sunt equales sinus. Ergo in catha *gdf* coniunctim proporcio sinus *fd* ad sinum *ad* | H 71rb componitur ex proporcione sinus *fz* ad *ez* et sinus *ge* ad sinum *gea*, quod est propositum.

Cuiuscumque etiam quantitatis fuerint duo arcus *gd* et *fd* citra quantitatem semicirculi. Si vero volueris compositionem *gd* ad *dz*, ponatur tunc *gzd* loco *fad*. Ymo de compositione proporcionis *ba* ad *da* arguitur per compositionem *fa* ad *ad* eodem modo sicut prius. Ergo universaliter patet propositum.

[*Figura #22*] In quarta vero dispositione patet coniuncta quia in tali dispositione necessario *bz* erit equalis arcui *ef*. Ergo illorum arcuum per primam suppositionem eiusdem dispositionis erit idem sinus. Et patet quod proporcio | D 222v | sinuum eorumdem arcuum | P 131v est proporcio equalitatis. Ergo patet propositum arguendo sicut ibi.

[I.15]

Ex principiis etiam huius figure sectoris maxime ex tercia figura circulorum que est proposicio 12ª huius dictionis, sequitur unum correlarium per quod longe facilius invenitur calculando in quattuor quantitatibus totum quod per applicationem figure sectoris a Ptholomeo in sex quantitatibus calculatur. Et est hoc.

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553 in] ex G **554** eo] ea GHP **555** quod] ex precedentibus *add.* D **556** proporcione] proporcionibus H **557** quadam alia] *om.* D **559** coniunctim] coniunctum D | sinum[1]] *om.* P **560** per] illam supposicionem primam *add. sed. del.* P **561** cum] nam D | sinus] *om.* D **563** eisdem] eis D **569** arcus] *om.* P **570** semicirculi] circuli H **574** quarta] tercia D | quia] quare D **580** maxime] vero *add.* P **581** 12a] 11ª P) **583** totum] totis D

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581 12a…dictionis] [Lemma 5.]

In omni orthogonio qui fit ex concursu magnorum circulorum etiam in convexo sphere, proporcio sinus unius duorum arcuum rectum angulum continentium ad sinum arcus intercepti secantis reliquum latus ad angulum rectum spheralem est sicut proporcio sinus lateris oppositi angulo recto ad sinum partis eius que infra sectionem est.

[*Figura #23*] Pro quo probando supponitur quod si aliqua superficies duas secantes se secuerit non equidistanter earum communi sectioni, due sectiones eius cum earum communi sectione super punctum unum necessario concurrunt. Patet faciliter imaginanti. Sit ergo gratia exempli *xfz* triangulus orthogonius spheralis qui sit ex primis quartis zodiaci et equinoxialis et maxima solis declinatione que sit arcus *zx*. Et quarta equinoctialis sit *xf* et quarta zodiaci *zf*. Et erit *z* caput cancri et sit arcus *er* declinatio principii geminorum que secat quartam equinoxialis ad angulum rectum sicut facit etiam maxima declinatio solis. Dico ergo quod proporcio sinus arcus *zx* qui sinus sit *zg* ad sinum arcus intercepti *er* qui sinus sit *ed* est sicut proporcio sinus totius lateris *zef* ad sinum partis eius scilicet *ef*. Sit *m* centrum commune omnium circulorum ex quo omnes magni circuli in sphera habent centrum unum. | Et sint *xm* et *rm* G 119v, semidiametri equinoxialis quas duo sinus ducti secant orthogonaliter ut H 71va patet intuenti in solido manifeste licet in plano non appareat. Anguli enim *edr* et *zgx* erunt recti et erunt *xg* et *ed* super superficiem equinoxialis perpendiculares ex quo superficies arcuum *zx* et *er* secant superficiem equinoxialis ex hypothesi orthogonorum. Ergo *zg* et *ed* erunt equidistantes, et per consequens in una superficie perpendiculariter secant superficiem equinoxialis super linea *gdyn*, et similiter superficiem zodiaci super lineam rectam *ze*. Ergo cum linea *kfmt* sit sectio communis duarum superficierum semicirculorum *fzt* et *fxt*, patet manifeste ex dispositione

585 In] correlarium ex 12^{a} huius *adnot. mg.* G | circulorum] *om.* G | etiam] *om.* D cum P **586** unius] *om.* P **588** angulum…spheralem] angulos rectos spherales P | est] *om.* D **591** se] *om.* P **592** due] duas DGH | earum] *post.* sectione D | earum communi] *inv.* HP **595** zx] *xz* P **596** equinoctialis] que *add.* D **597** er] *ey* D | principii] *om.* P | que] qui H **598** etiam] et GH *om.* P | declinatio solis] *inv.* DHP **599** er] *ey* D **600** sinus^{2}] *om.* P **601** scilicet] *om.* P m] enim D | ex quo] eo quod D **602** habent] *praem.* in G | sint] sunt D | rm] *ym* D **603** ducti] *om.* P **604** solido] solidum H | manifeste] *om.* D **605** edr] *edy* D | erunt^{1}] essent G | erunt^{2}] essent G | xg] *zg* H | super superficiem] super superficie D superficies G equinoxialis] equalis H **606** perpendiculares] perpendicularis G | er] *ey* D **607** equinoxialis] equalis H | ex…609 equinoxialis] *om. (hom.)* D | orthogonorum] *om.* P **608** consequens] consequentiam P | secant] secat H **609** linea] lineam P | gdyn] *gd* D *gdyk* HP | similiter] super P **610** lineam rectam] linea recta GH **611** dispositione] supposicione H premissa duas lineas *gy* et *ze* in directum protractas super uno puncto dicte communis dicte sectionis concurrere qui punctus sit *k*. Factus est ergo triangulus rectilineus *gkz* in quo *ed* equidistat basi *zg* ut patuit. Ergo per 2^{am} 6^{i} Euclidis proporcio *zg* ad *ed* est sicut proporcio totius linee *zk* ad *ek*. Ex 12^{a} huius proporcio totius linee *zk* ad partem | eius extra circulum P 132r scilicet *ek* est sicut proporcio sinus totius arcus *zf* ad sinum partis eius scilicet *ef*. Ergo proporcio sinus *zg* ad sinum *ed* est sicut sinus arcus *zf* oppositi angulo recto ad sinum arcus *ef* quia quecumque proporciones uni proporcioni sunt equales inter se sunt equales. Patet ergo propositum probatum.

[I.16]

Si sinus utriusque arcus rectum angulum spheralem continentium in orthogonio spherali duceretur in se quadrate et producti queratur radix que in se ducta addatur quadrato excessus sinuum versorum eorumdem arcuum, et illius producti queratur radix, habebitur corda arcus subtensa angulo recto in illo orthogonio.

Patet in figura presenti imaginando quod due superficies papyri intersecent se perpendiculariter super lineam *bm* ad modum duorum parietum. [*Figura #24*] Et erit angulus *abh* contentus arcubus magnorum circulorum rectus spheralis. Et tracta linea *khn* equidistante *bm* erit *kd* equalis sinui *hg* arcus maioris *hcb*. Erint ergo due linee *ad* et *kd*

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612 gy] *gd* D | directum] directo G **613** dicte communis] *inv.* G | qui] que G | est] *om.* DH **614** triangulus] orthogonius *add.* G | ed] *de* P *gd* G **618** ef] *zg* G | zg] *ez* G **623** Si] additis *adnot. mg.* G | arcus] arcuum P **624** duceretur] ducatur P | producti…625 ducta] productum D | radix] habebitur corda arcus subtensi angulo recto in illo orthogonio *add. sed.exp.* P **626** producti] anguli D aggregati *add. supr. lin.* D | radix] habet *add. sed.del.* G | subtensa] subtensi P in illo *add. sed. exp.* P **628** presenti] precedenti P **631** spheralis] *om.* P | Et] *om.* H | linea] *om.* G | khn] *khm* G *kh* in H | equidistante] equidistanti GH **632** hcb] *hob* P | Erint] erunt DH | ad] *hb* G *hb sed. corr. supr. lin.* D

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616 12a huius] [Lemma 5.] **622** I16] [This enunciation is particularly opaque, because the author takes the square root of a quantity (the sum of the squares of the two legs of the triangle) and then immediately squares it. The marginal adnotation in D states the proposition in a more comprehensible manner.] **623** Si] Notetur dicere propositio quod si in orthogonio spherali coniunguntur duo quadrata sinuum rectorum arcuum continentium angulum rectum et productum addatur quadrato differentie sinuum versorum eorumdem arcuum, tunc radix aggregati est corda arcus oppositi angulo recto. Si autem contingeret quod arcus continentes angulum rectum spheralem esset equales, tunc radix aggregati ex duobus quadratis(quartis?) suorum sinuum rectorum corda lateris oppositi. *adnot. mg.* D continentes angulum rectum note quia equales sunt sinubus dictorum arcuum. Sed quia linea | H 71vb *nk* est perpendicularis super superficiem orthogonii *kda*, ergo radix quadratorum duarum linearum *ad* et *dk* facit rectum angulum cum *kn*. Imaginata ergo linea recta inter *a* et *k* que est illa radix, ipsa faciet angulum rectum cum *kh* que est equalis *dg* excessui sinus versi arcus *hb* super sinum versum arcus *ba*. Erit ergo triangulus *akh* orthogonius. Cum ergo radix *ak* sit nota et similiter linea *hk*, radix quadratorum earum erit linea *ah* que subtenditur arcui magni circuli opposito angulo recto spherali *abh* nota. Ergo per tabulam cordarum ille arcus est notus.

Ista etiam propositio est non parve utilitatis nam per eam invenitur | D 223r distantia civitatum quarumcumque in terra quarum latitudo et longitudo fuerint note. Similiter si regionum equalis latitudinis distantia in terra fuerit nota, quot hore fuerint inter meridianos earum ea innante(?) erit notum. Similiter nota elevatione poli in duabus civitatibus note distantie in terra, longitudo earum in equinoxiali per eam cum figura sectoris vel correlario precedentis erit nota. Quomodo autem hoc habeat fieri circa principium 2^{e} dictionis forte videbitur.

[I.17]

Dato puncto orbis signorum declinationem eius ab equinoxiali circulo invenire. | P 132v

Pro declaratione istius propositionis et generaliter propositionum dictionis secunde, oportet hic plura premitti per que figura sectoris applicari habet ad calculationem tabularum ascensionum et aliarum. Quorum primum est si unum duorum extremorum quorum proporcio est nota fuerit notum, reliquum erit notum. Patet sumendo duos numeros secundum illam notam proporcionem quos pono primum et secundum. Et illud extremum notum pono tertium vel quartum secundum quod requirit

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633 equales sunt] *inv.* P | sunt] *om.* G | dictorum…634 arcuum] *inv.* H **634** nk] *mk* G | super] *add. supr. lin.* H | superficiem] superficie G **635** kda] *kdha sed. corr.* G | ad] *da* H | et] *db add. sed.del.* G **636** rectum angulum] *inv.* D | et] *b add. sed.del.* G **637** dg] *bg* G **638** versi] versus D | arcus^{1}] *om.* G **639** akh] *add. supr. lin.* P *abh* G | linea] *om.* G | hk] *kh* HP **643** est] *om.* G | est non] *inv.* P **644** distantia] *pos.* terra G **646** fuerint] fiant P | earum] eorum D | ea innante] [I don't get these] | innante] mediantis *add. sed.del.* P mediante P **647** Similiter] etiam *add.* G **648** longitudo] longitudine G | in equinoxiali] inequali DP | per] *om.* DGP eam] simul *add.* H **649** precedentis] precedente DP **650** forte] *om.* GP **652** Dato…653 invenire] *om.* P **654** istius] huius H **656** ascensionum] *om.* P | aliarum] aliorum D **657** Quorum] *om.* D quarum H | unum] *om.* H **658** fuerit notum] *om.* D | Patet] hoc *add.* H **659** notam] *om.* D ordo proporcionum in quod multiplica secundum numerum vel primum et productum divide per primum vel secundum, et exibit illud extremum ignotum.

Secundo premittendum est quod quibuslibet tribus quantitatibus propositis notis aliqua quarta erit nota ad quam se habet tertia sicut prima ad secundam. Patet quia ducatur secunda in tertiam et dividatur productum per primam et exibit huiusmodi quarta.

Tertium principium est si una duarum proporcionum componentium aliquam notam fuerit nota per illius subtractionem residua erit nota. Patet quia illa residua est inter productum antecedentis composite in consequens subtrahende proporcionis note et productum consequentis eiusdem composite in antecedens subtrahende, | quod probatur. [*Figura* H 72ra *#25*] Et sit proporcio *a* ad *b* nota composita ex proporcione *c* ad *d* nota et quadam alia ignota quam volo dicto modo invenire. Et duco *a* in *d* et producatur *e* et ex *b* in *c* fiat *f*. Tunc ex *b* in *d* fiat *g*. Ergo cum ex *a* et *b* divisim in *d* fiant *e g* erit *e* ad *g* ut *a* ad *b* multiplicium *d*. Sed etiam ex *b* in *c* et *d* fiant *f* et *g*. Ergo *f* ad *g* est sicut *c* ad *d* ex 7º Euclidis. Ergo cum *f* sit medium oportet proporcionem *e* ad *f* esse residuam que cum *c* ad *d* componit *a* ad *b*. Ergo cum *e* et *f* sint duo producta predicto modo ex antecedente in consequens et ex consequente in antecedens componentis, relinquitur propositum. Et eadem regula est si essent plures proporciones subtrahende duceretur enim antecedens composite in omnia consequentia componentium notarum et consequens in omnia antecedentia, et provenirent duo producta inter que esset proporcio residua.

Quartum principium est si due proporciones componentes fuerint note, composita erit nota. Nam ducantur | omnia antecedentia earum in G 120r invicem et similiter consequentia, et inter producta erit illa proporcio ignota composita, quod probatur ut prius. Et sit proporcio composita *a* ad *b* ignota, proporciones vero componentes *c* ad *d* et *e* ad *f* note. Ducatur *c* antecedens in *e* antecedens et productum sit *g*, et ex *c* in *f* sit *l*, et

661 quod] quot P tertium *add.* D | numerum] *om.* D **664** Secundo] secundum D **667** quarta] et cetera H **668** principium] premittendum D **669** illius subtractionem] *inv.* H **671** productum] *om.* GH | productum consequentis] consequenter P **675** Tunc] et D | ex[2]] eodem *add.* D | in[2]] *c* fiat *f* et ex eodem *b* in *add.* D **676** fiant] fiat H fiunt P | g[1]] *f* GHP | g[2]] *f* GHP | d[2]] *om.* H | etiam] cum D **677** fiant] fiunt HP **678** residuam] residuum D **679** a] *adb add. sed.del.* D **684** provenirent] provenient D provenerent P | residua] residui H **685** principium] premittendum D | fuerint] sunt D **686** antecedentia earum] *inv.* P | in] ad HP **687** consequentia] invicem *add.* D | inter] illa duo *add.* P | illa proporcio] *inv.* P **688** composita[2]] *om.* DH **689** proporciones…componentes] proporcio H **690** f] *m* H productum *d* consequentis in *f* consequens sit *h*. Cum ergo ex *c* et *d* in *f* fiat, *l* et *h* erit *l* ad *h* ut *c* ad *d*. Et ex quo ex *c* in *e* et *f* fiunt *g* et *l*, erit *g* ad *l* ut *e* ad *f*. Ergo cum proporcio *g* ad *h* extremorum componatur ex duabus proporcionibus equalibus proporcionibus *c* ad *d* et *e* ad *f*, ipsa erit equalis proporcioni *a* et *b*. Et per consequens proporcio ignota proposita erit inter nota extrema *g* et *h*. Et sic etiam probaretur medietatibus duabus de tribus componentibus et consequenter in quotcumque.

Aliter etiam ex secundo premisso si proporcio primi ad secundum nota componitur ex proporcione tertii ad quartum etiam nota et ex proporcione quinti ad sextum ignota, ipsa erit nota. Et tunc si una extremitas | illius fuerit nota ex prima suppositione, reliqua erit nota. P 133r Nam per secundum premissorum eadem proporcio componens ignota potest quattuor vel sex modis ex aliis duabus proporcionibus notis extrahi, scilicet ducendo secundum in tertium dividendo per primum et exibit aliquid ad quod se habet quartum in proporcione illa ignota | H 72rb scilicet quinti ad sextum. | D 223v Secundo modo ducendo primum in quartum et dividendo per secundum et exibit aliquid ad quod est tertium secundum proporcionem quesitam. Tertio modo e converso comparando. Verbi gratia: sicut se habet *a* ad *b* ita *c* tertii ad aliquod quartum ad quod se habebit *d* sicut *e* ad *f*. Secundo modo sicut *b* ad *a* ita *d* se habet ad aliquod quartum ad quod tunc se habebit *c* tertium sicut *e* ad *f*. Tertio modo sicut *c* ad *d* ita *a* ad aliquod quartum quod tunc se habebit ad *b* sicut *e* ad *f*. Quarto modo iterum sicut *c* ad *d* ita *b* ad aliquod quartum inter quod et *a* iterum erit proporcio *e* ad *f*. Quinto modo sicut *d* ad *c* ita *b* ad aliquod quartum. Sexto modo sicut *d* ad *c* ita *a* ad aliquod quartum et semper illud quartum erit ex secundo principio premisso.

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692 ex[2]] *om.* H | erit[2]] erunt GHP | ad[3]] et GHP **693** componatur] *iter.* H **696** Et] *om.* D etiam] *om.* H | probaretur] probaberetur P | medietatibus] mediantibus D | duabus] duobus et D | de] *add. mg.* G aut P **697** consequenter] consimiliter D | quotcumque] quodcumque D quocumque H **698** etiam] *om.* H **701** fuerit nota] *inv.* P | suppositione] et *add.* D **703** duabus proporcionibus] *inv.* D | notis] *add. supr. lin.* D **704** scilicet] *om.* G | tertium] et *add.* P **705** quod] quo D | se…707 quod] *om. (hom.)* P **706** sextum] **709** c] *g* GP se habet *add.* H | tertii] tertium H | aliquod] *d* H **710** b] *a add. sed.del.* D | a] *d* GP **711** tunc] *post*. habebit H | c] *add. supr. lin.* D **712** a] *d* GP **713** iterum] *om.* D | inter] in GH **714** e] *f add.* H | aliquod] *om.* H **715** semper] *e add. sed. exp.* G

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700 nota] Si consideraveris 20[am] 7[i] Euclidis et quod proporcio extremorum componitur ex proporcione intermediorum, credo(?) quod facitur intelliges illos 6 numeros. *adnot. mg.* D

Et proporcionali modo multipliciter ex duobus componentibus notis erit composita nota ex eodem secundo premisso. Nam primo modo sicut *e* ad *f* ita *d* ad aliquod quartum ad quod necessario se habebit *c* in proporcione *a* ad *b* composita ignota. Similiter imaginendo quod sicut se habet *c* ad *d* ita *f* ad aliquod quartum ad quod tunc se habebit *e* secundum proporcionem *a* ad *b,* et ita de aliis comparationibus fieret. Et semper illud quartum erit notum per principium secundum. Habemus ergo ex istis omnibus premissis quomodo per duas vias proporcio potest subtrahi ab aliqua proporcione, autem etiam alicui addi sine quibus omnino usus figure sectoris in calculationibus haberi non poterit.

[*Figura #26*] Sit ergo gratia exempli *h* principium Tauri vel alius punctus zodiaci et eb prima quarta eius videlicet vernalis et *e* principium Arietis. Et trahatur a polo arctico quarta magni circuli *zht* cuius porcio *ht* erit declinatio puncti *h* ab equinoxiali *aegd*. Posito ergo circulo *abg* meridiano vel coluro solsticiali, consurget in convexo spherali catha *zae* ex quartis magnorum circulorum. Quare arguendo per coniunctam erit proporcio corde dupli arcus *za* vel sinus quod est eius medietas ad sinum arcus *ba* qui est maxima declinatio solis nota composita ex proporcione sinus *zt* quarte ad sinum arcus *ht* qui queritur et ex proporcione sinus *eh* arcus verbi gratia 30 gradus ad sinum *eb* quarte. Ducatur ergo sinus *za* qui est antecedens proporcionis composite in sinum *eh* qui est consequens note proporcionis subtrahende, et vocetur numerus inde proveniens *f* gratia exempli. Deinde ducatur sinus *ba* qui est consequens composite in | sinum arcus *eb* qui est antecedens proporcionis H 72va
subtrahende, et numerus inde proveniens vel productus vocetur *g*. Inter quos numeros per 3^{am} suppositionem est proporcio residua. Sed sinus *zt* arcus noti ad sinum *ht* arcus ignoti, ergo per primam suppositionem multiplicetur *g* in sinum quarte *zt,* et productum dividatur per *f,* et exibit sinus arcus *ht* declinationis puncti dati in distantia 30 graduum ab *e* capite Arietis. Et ita in aliis punctis secundum quantitatem note latitudinis | P 133v

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717 duobus] duabus DH | componentibus] proporcionibus H **718** nota] *om.* D | secundo premisso] *om.* H | modo] *om.* D **719** e] *d sed. corr. supr. lin. a. m.* D | c] *om.* P **720** quod] *om.* P **722** et] *om.* G **723** erit] fieret H | principium secundum] *inv.* P **724** istis omnibus] *inv.* P | omnibus] *om.* H **725** autem] aut P vel H **727** ergo] *om.* DH | gratia exempli] *inv.* D **728** punctus] eius *add. sed. exp.* P | eius] *om.* H **729** arctico] tercia *add.* DGHP | zht] *znt* H porcio] proporcio H **730** aegd] *aeg* DH **732** coniunctam] 5^{am} D **733** corde] *add. supr. lin.* P **734** declinatio solis] *inv.* P | nota] *om.* H **735** zt] *ze* G | quarte] *om.* P | et] *om.* H | sinus^{2}] *om.* G **736** gradus] graduum H **737** eh] *eb* D **739** gratia exempli] *inv.* P **740** composite] composito H | eb] *eh* D **741** vel productus] *om.* P **743** suppositionem] proposicionem H **745** 30] *iter.* D **746** secundum…747 maxime] *om.* DH maxime procederetur secundum istam viam subtractionis proporcionis unius ab alia. Possemus etiam procedere per secundam viam multis modis ut patuit, et communiter canones tabularum primi mobilis procedunt secundum ipsam ut apparebit magis inferius. Et similiter latitudines planetarum invenirentur secundum quantitatem note latitudinis maxime. Sed ut dictum est eedem declinationes inveniuntur facilius per regulam in quattuor quantitatibus proporcionalibus. Nam ex correlario premisso proporcio sinus maxime declinationis solis scilicet *ba* ad sinum arcus *ht* qui queritur est sicut proporcio sinus arcus *be* noti ad sinum arcus *he* qui etiam ponitur notus. Multiplicetur ergo sinus *ab* notus per tabulam cordarum in sinum arcus *he* etiam notum per easdem scilicet primum in quartum. Et productum dividatur per sinum quarte videlicet per tertium, et exeat sinus arcus *ht* ignoti notus qui ad arcum reducatur per eandem tabulam. Et erit arcus *ht* notus qui querebatur.

[I.18]

Cuiuslibet arcus zodiaci ascensionem in sphera recta invenire.

Pro quo supponatur quod proporcio ignota que cum alia nota componit notam potest multis modis extrahi per secundam viam ut dictum est. Ex quorum primo operabor hic: Verbi gratia sit proporcio *a* ad *b* composita ex *c* ad *d* et *e* ad *f*, et sit *c* ad *d* nota, erit etiam | proporcio D 224r
e ad *f* nota. Primo modo sic: ducatur primum sex quantitatum in *d* quartam, et productum dividatur per *b* secundam, et exibit quantitas que sit *g* ad quam se habet *d* per conversam 14^{e} 6^{i} sicut *b* ad *a*. Ergo e converso *a* est ad *b* sicut *g* ad *d*. Ergo cum *c* sit medium positum, erit necessario *g* ad *c* proporcio que cum proporcione *c* ad *d* componit proporcionem *a* ad *b*. Si ergo *g* notum se habet ad *c* notum sicut *e* ignotum ad *f* notum, tunc ducatur primum scilicet *g* in quartum scilicet *f*, et productum dividatur per *c*, et exibit *e* tertium quod fuit ignotum. Volo ergo tali modo ascensionem *eh* arcus 30 gradus gratia exempli | invenire. H 72vb
Et retenta priori figuratione manifestum est ex catha divisa quod proporcio sinus supplementi *zb* ad sinum *ba* componitur ex proporcione sinus arcus *zh* ad sinum arcus *ht* iam notum | ex precedenti et ex G 120v

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748 per] aliam *add. sed.del.* D | secundam viam] *inv.* D **749** ut patuit] *om.* D **750** ut] et H **751** latitudines] *iter*. G **752** inveniuntur] invenientur P **754** ba] *ab* H **756** sinus] *om.* H **757** easdem] eas P **759** exeat] exhibit DH **760** eandem tabulam] easdem tabulas DP **763** supponatur] supponitur DP **764** secundam] *om.* H **765** a…766 b] *ab* D **767** modo] *om.* P **770** cum] tunc H **771** necessario] *c* ad *add. sed. exp.* P **772** g] *om.* H | se habet] *inv.* P **773** g] ad secundum *add.* H **775** ascensionem] arcus *add.* H | gradus] graduum DH | gratia exempli] *inv.* P **776** est] quod *add.* H **778** arcus^{1}] *om.* D | ex^{2}] *om.* P proporcione ignota sinus arcus equinoxialis et ignoti qui est inveniendus ad sinum *ea* notum. Ducatur ergo in ordine istarum 6 quantitatum quarum quinta est ignota et invenienda conformiter ad exemplum iam predictum prima in 4am, et quod producitur dividatur per 2^{am} scilicet sinum *ba*, et exibit quantitas ex premissis se habens ad sinum *zh* supplementi sicut sinus *et* ad sinum arcus *ea*. Ducatur ergo illa quantitas quam imaginor primam in ordine quatuor quantitatum iam dictarum in 4^{am} scilicet in sinum arcus *ea*, et productum dividatur per sinum arcus *zh* que ponitur quantitas 2^{a}. Et exibit sinus arcus *et* 3^{e} notus que fuit 5^{m} ignotum in ordine 6 quantitatum primo propositarum. Et sic hic arguitur ex primo dictorum modorum ita | posset ex aliis conformiter procedi secundum P 134r exigentiam illorum modorum aliam vel alias comparationes faciendo. Etiam possemus procedere hic per primam viam subtractionis proporcionis sicut fiebat inquirendo declinationes solis que minus intricata est et forte expeditioris calculationis quia habet duas multiplicationes ubi alia habet unam multiplicationem et unam divisionem.

Aliter et facilius arcus *et* equinoxialis qui est ascensio arcus noti *eh* in zodiaco per correlarium predictum ratiocinatur. Nam secundum ipsam imaginando triangulum orthogonium *azt* cuius angulus *a* est rectus cum etiam angulus *b* sit rectus, erit proporcio sinus arcus *tz* ad sinum *zh* etiam notum quia arcus *ht* est notus cum sit declinatio dati gradus zodiaci sicut proporcio sinus arcus equinoxialis *at* ignoti ad sinum arcus *bh* notum quia continet quartam zodiaci cum dato arcu *eh*. Ducatur ergo sinus arcus *hz* in sinum arcus *bh* scilicet secundum in tercium, et productum dividatur per sinum quadrantis vel arcus *tz* et exibit sinus arcus equinoxialis *at* qui arcuetur per tabulas cordarum. Et erit arcus *at* notus, qui subtrahatur de una quarta scilicet de 90 gradibus, et relinquitur *te* notus qui querebatur.

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779 et] *om.* H **780** sinum] *om.* H | notum…781 invenienda] *om.* D **781** iam] *om.* H **782** et[1]] *g* D **784** arcus] *om.* DH **785** in[1]] illa *add.* H | in 4am] *om.* D **786** et] *d add. sed. exp.* H **787** sinus] scilicet *add.* H | et] *zt sed. corr.* P **788** primo[1]] prius D | sic] sicut D **790** comparationes] proporciones D | faciendo] et *add.* G **791** procedere hic] *inv.* P **792** proporcionis] proporcionum DP | inquirendo] inquirendi G inquerendo H **796** qui] que P **797** ratiocinatur] ratiocinetur P **798** a] *az sed. corr.* G **799** erit] et D | etiam[2]] est D **800** notum] nota P | cum] igitur *add. sed. del.* H | dati] *om.* D **801** at] *ht sed. corr.* G ignoti…804 at] *om. (hom.)* P **802** continet] complet DH | dato arcu] *inv.* D | hz] *tz* D **803** secundum…tercium] primum in quartum D **804** quadrantis…tz] arcus *hz* D **805** tabulas] tabulam G **806** relinquitur] derelinquitur D | te] *et* P

Ex istis duabus propositionibus componuntur due tabule scilicet declinationis solis et tabula ascencionum signorum in sphera recta. Per propositionem vero additam precedentem eas immedietate, quelibet data ascensio in sphera recta | poterit ad arcum zodiaci ei correspondentem H 73ra
sine tabula reduci. Nam in orthogonio *hte* duo arcus angulum rectum scilicet *t* continentes essent noti. Ergo per ipsam corda arcus *eh* est nota et per consequens arcus zodiaci *eh*. Et sic est finis commenti prime dictionis Almagesti.

Explicit super primam dictionem.

[Book II]

[II.A.1]

Cum terra sit sperica ut Ptholomeus in prologo dictionis prime et consequenter auctores probant, necesse est totam porcionem aridam superficiei eius rotundam esse nam terminus eius est sectio communis duarum superficierum speralium scilicet convexitatis spere terre et spere aque. Igitur terminus ille necessario erit circumferencia quia omnis superficies plana vel speralis secans aliquam speram secat convexitatem eius super unam circumferentiam. Et patet eciam exemplariter. Nam si aliquod siccum spericum aque successive immergatur, manifestum est quod omnis porcio eius superficialis sicca remanens rotunda est, ergo etc. Ex quo clarum est porcionem terrre aridam non esse magis extensam versus orientalis et occidentalis quam versus meridiem et septentrionem, licet forte maius eius spacium inhabitetur in longitudine quam in latitudine, | propter hoc quod ex utraque parte eius latitudinis inhabitabilis D 224v

807 Ex] nota *adnot. mg.* G | propositionibus] proporcionibus DH **808** ascencionum] ascensionis D **809** eas] *om.* D | eas immedietate] *inv.* P **810** ei] eis D **811** hte] *het* H angulum rectum scilicet *t* arcus continentes *add. sed. exp.* G | duo arcus] *inv.* P **812** scilicet] *om.* G | arcus] *ht* et *te add.* H | corda arcus] *inv.* G **813** Et] *ez* D | Et…814 Almagesti] et cetera H | est finis] finiunt P | est…815 dictionem] finitur prima dictio D | commenti] demonstrationes P | prime…814 dictionis] *inv.* P **815** Explicit] in dei nomine amen *add.* P Explicit…dictionem] *om.* P expliciunt demonstrationes dictionis prime Almagesti cum principiis et regulis ad intellectum eius et totius libri necessariis H **821** consequenter] communiter DGP | probant] prout P **822** superficiei] superficiem H | eius[2]] illius P **823** convexitatis] convexarum D **826** Nam] *om.* G **827** aque successive] *inv.* P immergatur] immergitur G **828** remanens] manens P **829** esse magis] *inv.* P **830** orientalis…occidentalis] oriens et occidens DP **832** ex] ab D | eius] *om.* DHP redditur propter excessum calitatis in meridie et frigitatis in septentrione. | Versus oriens vero et occidens in omnibus regionibus super unam P 134v equidistantem equinoxiali situatis est equalis temperies vel intemperies frigitatis vel calitatis. Ergo quantumlibet in longitudine procedendo nihil prohibet terram inhabitari donec litus maris occeani occurrat in oriente et occidente.

Secundo infertur quod si ariditas terre solum se extendat ab equinoxiali usque ad sub polum articum ut Ptholomeus quoddammodo probare videtur in capitulo primo huius dictionis 2^{e} per hoc quod umbre meridiane in omnibus regionibus tempore equinoxiorum flectuntur ad septentrionem, tunc necessario linea que cordat arcum superficiei terre aridum erit latus quadrati inscripti uni magnorum circulorum superficiei terre. Ex quo ulterius sequitur totam porcionem | aridam esse vix H 73rb septimam partem tocius speralis convexi terre. Patet quia sex superficiales porciones circulares descriptibiles sunt in quolibet convexo sperali quarum cuiuslibet dyameter est latus ei inscripti quadrati. Nihilominus preter has remanentibus scilicet triangularibus superficiebus que simul iuncte plus faciunt quam una illarum porcionum 6 ut clare patet facienti figuram in aliquo globo sperico. Quarto exemplum infertur medium clima esse longissimum, primum vero et ultimum brevissima eo quod porcio arida terre est sperica et terminatur secundum predicta sub equinoxiali et polo artico.

[II.A.2]

Stante eciam eadem ypothesi poterit demonstrari per quantum arcum equinoxialis distent meridiani duorum punctorum terre aride maxime distancium versus oriens et occidens.

[*Figura #27*] Nam signatis illis punctis gratia exempli *a* et *b* in litore occeani in oriente et occidente, manifestum est ex dictis quod ipsa cadunt sub medio quarte celi que est inter equinoxialem et articum polum. Et quia arcus *azb* maxime longitudinis | terre aride sit quarta circuli, erit G 121r

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833 excessum calitatis] excessivam calitatem D | frigitatis] frigitatem D **834** oriens vero] *inv.* P **835** equidistantem] distanciam D | equinoxiali] equinoxialis H **837** maris] et *add.* D occeani] *om.* P | occurrat] concurrat D **839** ariditas] arida P | terre] *om.* P **840** sub] *om.* D **841** huius dictionis] *inv.* P **843** superficiei] superficierum G **844** superficiei] superficierum G **845** ulterius] *om.* P | esse] versus 2^{am} *add. sed. exp.* G | esse vix] *inv.* DP **850** plus faciunt] *inv.* P **851** figuram] figuracionem DH | globo] plano *add.* H | exemplum] exinde P **852** ultimum] esse *add.* P **853** arida] terra *add. sed. del.* H | arida terre] *inv.* DP | secundum predicta] *om.* P | predicta] secundum *add.* D **857** duorum] predictorum *add. sed. del.* H **858** distancium] distancia D **859** gratia exempli] verbi gratia P propter hoc angulus super centrum terre comprehendens arcum illum rectus. Ergo et distancia cenith duorum punctorum *a* et *b* in convexo celi erit una quarta magni circuli transeuntis per 3ª cenith scilicet duorum *a* et *b* et per cenith *z* puncti medii terre usque ad verum oriens et occidens que signo super *g* et *l*. Tunc tractis duobus meridianis videlicet *dak* et *dbh*, et sit *d* polus articus et circulus *gel* equinoxialis. Et protracto *dze* meridiano puncti medii tocius terre aride, clarum est quod in triangulo sperali *zge* duo anguli *zek* et *eka* sunt recti. Ergo per correlarium figure sectoris erit proporcio sinus arcus *ze* noti cum sit medietas quarta circuli ex hypthesi ad sinum arcus *ak* ignoti sicut proporcio sinus tocius quarte *zag* ad sinum arcus *ag* noti cum ipse sit medietas unius quarte quia ex ypothesi arcus *za* est quarta. Ducatur ergo sinus arcus *ze* in sinum | arcus *ag* et productum (P 135r) dividatur per sinum arcus *zg* videlicet per tercium. Et exibit sinus arcus *ak* notus. Ergo per tabulas sinuum erit ipse arcus *ak* notus, et ex hoc erit eciam arcus *ad* residuus de quarta notus. Cum ergo eciam angulus | *aze* (H 73va) sit rectus, erit per idem corellarium in triangulo orthogonio sperali *kde* proporcio sinus arcus equinoxialis *ke* ignoti qui est medietas arcus equinoxialis cadentis inter meridianos dictorum punctorum ad sinum arcus *az* noti sicut proporcio sinus quarte meridiani *kd* ad sinum arcus *ad* noti ex iam precedenti deductione. Si igitur ducatur sinus arcus *za* in sinum arcus *kd* et productum dividatur per sinum arcus *ad*, proveniet sinus arcus *ek* notus. Ergo et arcus *kh* ad ipsum duplus erit notus, et iste est arcus equinoxialis cadens inter meridanos dictorum punctorum *a* et *b* que ponebantur maxime distare secundum longitudinem terre aride, quod fuit probandum.

Ex quibus immedietate patet quod meridies duarum civitatum vel regionum maxime distancium secundum longitudinem plus distant quam per 6 horas equales quia arcus *ke* est maior quam arcus *az* qui est dimidium quarte ex ypothesi. Et similiter ex isto nota est quantitas temporis prescisa inter meridies earum.

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864 et[1]] eciam DP **865** 3a] 2 DP | et] *om.* D **867** super] puncta *add.* DP | l] *n* P | videlicet] *dah* et *add. sed. exp.* G **868** circulus] triangulis H | dze] *dez* D **870** eka] *akh* D **871** medietas] *add. supr. lin.* DP *om.* GH | ex hypthesi] *om.* H **873** za] *ba* D **876** erit[1]] *post. ak* G | erit[2]…877 eciam[1]] *inv.* H **878** per…corellarium] *om.* P | idem] *om.* D | sperali] per corellarium *add.* P **879** ke…880 equinoxialis] *om. (hom.)* D **880** cadentis] cadentes D **884** arcus ek] *inv. sed. corr. signis* P | kh] *hk* P | erit notus] *inv.* DP **890** per] *om.* P **891** Et] *om.* P | isto] istis P

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870 correlarium…sectoris] [I.15.] **878** idem corellarium] [I.15.]

Idem posset calculari in | 6 quantitabus per catham coniunctam que D 225r esset *deg* subtrahendo proporcionem notam sinus arcus *ga* ad sinum arcus *gz* a proporcione composita nota, videlicet sinus arcus *de* ad sinum arcus *ze*. Et remanet proporcio nota sinus noti arcus *dk* ad sinum ignotum arcus *ak*. Et sic arcus *ak* erit notus. Deinde arguendo per disiunctam catham erit ex hoc arcus *ke* notus qui querebatur.

[II.A.3]

Potest eciam elici ex hiis quod quanto aliquis paralellus terre aride fuerit propinquior polo artico tanto regiones vel civitates secundum eum maxime distantes habent meridies magis diversos sive meridianorum in equinoxiali maiorem distantiam.

Nam descriptis paralellis *fp* et *cx* manifestum est quod duo puncta *f p* secundum primum paralellum maxime distant, et *c x* per 2m. Protractis ergo meridianis per cenith istorum punctorum manifestum erit duos meridianos *dcm* et *dxq* maiorem arcum equinoxialis secundum *mq* intercipere quam duo | meridiani *dfn* et *dpo*, et ita de aliis paralellis H 73vb propinquioribus polo accidit. Ex hiis apparet quod non sequitur iste due regiones plus distant abinvicem ab oriente in occidentem, ergo meridies istarum vel meridiani in equinoxiali plus distant, quia maior est differencia meridierum regionum *e* et *x* quam *a* et *b* maxime tamen distantium secundum longitudinem terre aride, ergo etc.

Ymo quanto due regiones vel civitates super extremam circumferenciam | porcionis aride situate equedistanter polo artico P 135v propinquiores adinvicem fuerint tanto maiorem differenciam habent in meridiebus. Patet ex ymaginacione iam dicta, et sic possibile est duas civitates distantes ab oriente in occidentem solum per unam leucam habere maiorem differenciam meridierum aliis duabus distantibus per 100 leucas.

[II.A.4]

Patet eciam quod si porcio arida terre dictomodo se habet quod impossibile est aliquarum duarum regionum meridies distare integre per 12 horas equales, hoc est per medietatem circuli equinoxialis.

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894 deg] *edg sed. corr.* P **896** dk] *ak sed. corr. supr. lin.* P **902** maxime distantes] *inv.* D meridies] meridianos P | diversos] diversas GH **904** f…905 secundum] *iter*. D **906** ergo] *om.* D | istorum] illorum DH | erit] est D **907** dcm…dxq] *dc* et *dx* D | arcum] angulum D secundum] scilicet P **908** dpo] *dto* GHP **909** hiis] isto P **911** istarum] illarum D *om.* P **913** etc] *om.* G **915** circumferenciam] *post.* aride H **919** meridierum] meridianorum P

Patet quia ymaginendo duo puncta ex utraque parte quamlibet propinqua ipsi *d* puncto, quod supponitur polo artico, in terra semper meridiani transeuntes per cenith illorum intercipient minorem equinoxialis eius medietate que signatur per *gl* in figura, licet omnem arcum minorem medietate equinoxialis inter aliquas civitates possibile sit cadere ut evidenter poterit haberi ex precedenti ymaginacione. Patet ergo quod decipiuntur credentes concludere ex quantitate distancie meridierum vel horarum unius eclipsis lunaris in diversis regionibus quantitatem longitudinis terre aride ab oriente in occidentem. Nam non sequitur. Quantumcumque tempus infra 12 horis inter meridianos aliquorum punctorum terre aride ceciderit, maius cadat inter meridies aliquorum aliorum. Ergo quolibet arcu superficiei terre aride minore semicirculo datur arcus maior aridus ut potest colligi ex antedictis. Eciam quia angulus contingencie *cdg* est quolibet angulo meridiani cum recto orizonte *dg* minor, ergo dato meridiano alicuius puncti quantumlibet propinqui ipsi *d*, datur meridianus alicuius puncti ei propinquioris magis | H 74ra
appropinquas ad verum oriens *g* in equinoxiali circulo ergo etc.

[II.A.5]

Ex eo eciam quod dicebatur sperale convexum aque secare convexam superficiem terre, manifestum est centrum magnitudinis terre non esse centrum spere aque et per consequens nec centrum mundi. [*Figura #28*] Ex quo ulterius faciliter demonstrari poterit maximam spissitudinem aque adiacentis extrinsecus superficiei terre duplam vel minorem esse quam dupla ad distanciam centrorum magnitudinis terre et celi ut alibi demonstravi. Ex quo clarum est quod per modum communem inveniendi dyametrum terre quem Alfraganus et auctor spere ponunt, non invenitur vera quantitas dyametri magnitudinis terre sed dyametri circumferencie longe maioris convexitate spere terre ymo maioris quam | D 225v
convexum aque, si ariditas terre fuerit elevacior ad centrum mundi

925 quamlibet] quantumlibet DP **926** d] a *sed. corr. supr. lin*. P | quod] qui P **928** equinoxialis] porcionem D | in figura] *om.* D **933** aride] *om.* P **934** horis] horas H **939** quantumlibet…940 puncti] *om. (hom.)* P **940** ipsi] *g add. sed. del.* H **941** appropinquas] propinquans P **943** eo] quo H **944** est] *om.* D **945** mundi] *om.* D **948** esse] *om.* GHP dupla] duplam est P | distanciam] magnitudinis *add. sed. del.* D **949** alibi] aliter P **951** dyametri[2]] dyameter H **952** longe] longo H | convexitate] convexitatis DH **953** ad centrum] a centro P

949 demonstravi] Linea *ea* excedit lineam *eb* per duplum *de* que est differencia centrorum. Modo *eg* est equalis *ea*, ergo etiam excedit eandem per duplum *de*. Et tamen prescise excedit per *bg* que est spissitudo aque. *adnot. mg.* D superficie convexa spere aque. Et ergo si vera quantitas dyametri debet haberi, oportet maximam spissitudinem aque investigare et eam subtrahi a quantitate dyametri secundum communem modum inventum. Et quod relinquitur erit quantitas dyametri terre propinquissima veritati.

[II.A.6]

Hiis igitur ita se habentibus volo ostendere quomodo | duarum G 121v civitatum distancia in superficie terre ex nota differencia meridierum earum simul cum | distancia nota cenith earum ab equinoxiali habeat P 136r investigari, et e converso quomodo si distancia duarum civitatum in uno magnorum circulorum superficiei terre fuerit nota simul et elevacio poli super orizontem earum, habeatur differencia meridierum earum in equinoxiali que vocatur civitatum longitudo.

[*Figura #29*] Ponam ergo duo puncta scilicet *c* et *d* cenith duarum civitatum quarum longitudo et latitudines sint note, et sint earum meridiani *zcf* et *zdh*. Equinoxialis vero sit *kfhg* et polus articus *z*. Tunc si latitudines habuerint equales vel si cenith earum equaliter disteterint ab equinoxiali quod idem est, manifestum erit ex correllario allegato quod proporcio tocius corde arcus equinoxialis *fh* qui ponitur illarum civitatum longitudo ad totam cordam arcus magni circuli transeuntis per earum cenith qui signetur cd est sicut proporcio sinus tocius quarte *hz* ad sinum noti arcus *dz*. Ergo cum illarum 4 quantitatum proporcionalium tres sint note ex ypothesi, erit secundum dictum modum operandi | corda arcus *cd* H 74rb nota, et consequenter ipse arcus per tabulas cordarum erit notus. Si ergo cuilibet gradui eius dentur 700 stadia terre que uni gradui celi correspondent, habebitur propositum.

954 convexa] convexe P | spere] *om.* P **955** oportet] (unclear, could be omnem but looks more like oportet in H) **956** inventum] inventam H invente P (check P) **957** propinquissima] propinquissime G | veritati] veritate P **961** earum[1]] *om.* P | earum[2]] *om.* P **962** quomodo] sit *add. sed. del.* H | uno] una G[I think ‘una’ makes more sense because it can modify ‘superficiei’] **963** nota] *om.* D **964** orizontem] orizontes D | habeatur] habeat P **966** scilicet] *ced add. sed. del.* G **967** et[1]] per P | sint[1]] sunt D **968** zcf…zdh] *inv.* D | vero] namque D nam H **971** ponitur] cenith *add.* D | illarum] ipsarum P **973** signetur] assignatur D | cd] *pd* D | est] *om.* DGH | sinus] *add. supr. lin.* D | quarte] corde G **975** cd] *pd* D **977** dentur] dantur et P | que] qui D

965 longitudo] Nota quod modus sui inveniendi et practicandi civitatum longitudines est per ignes successos(?) in distancia 30 vel 40 leucarum vel etiam alio modo per circulos paralellos vel per modos quos hic reperies. *adnot. mg. a. m.* D **970** correllario allegato] [I.15.]

Si vero latitudines habuerint inequales sit gratia exempli cenith *d* propinquius equinoxiali. Deinde describam unam magnam circumferentiam in convexo spere transeuntem per cenith *d* ita quod secet meridianum alterius civitatis ad angulos rectos sperales. Et sit sectio illa *p*. Tunc quia meridianus *zpf* intersecat utrumque duorum semicirculorum *gpk* et *gfk* ad rectos angulos super punctis *p* et *f*, erunt necessario arcus *pg* et *fg* quarte magnorum circulorum. Et quia eciam arcus *dp* erit perpendicularis super meridianum *zf*, erit propter hoc ex corollario proporcio sinus longitudinis civitatum *hf* note ad sinum arcus *dp* ignoti sicud proporcio sinus quarte *hz* ad sinum notum arcus *dz*. Ducatur ergo sinus arcus *hf* in sinum arcus *dz*, et productum per sinum quadrantis dividatur. Et exibit sinus arcus *dp*. Est ergo hoc modo arcus *dp* notus. Ergo cum arcus *pg* sit quarta, erit arcus *dg* residuus notus. Cum ergo in triangulo orthogonio *pfg* angulus *f* sit rectus et arcus *dh* sit perpendicularis super latus eius videlicet *fg* eorum que sunt circa angulum rectum, erit iterum ex corollario eodem proporcio sinus arcus *fp* ignoti ad sinum noti arcus *hd* ut proporcio sinus quarte *pg* ad sinum arcus *dg*. Ducatur ergo sinus arcus *hd* in sinum quadrantis, et productum dividatur per sinum arcus *dg*. Et exibit sinus arcus *fp* notus. Noto ergo arcu *fp* ipse dematur de quarta *fz*, remanet arcus *pz* notus. | A quo P 136v subtracto arcu noto *cz* qui est inter cenith civitatis maioris latitudinis et polum, erit arcus *pc* notus. Deinde quia in triangulo *cpd* angulus *p* est rectus et duo arcus *cp* et *dp* sunt noti, erit per propositionem additam de orthogonio sperali in fine prime dictionis corda arcus *cd* nota. Quare distancia cenith civitatum dicto modo se habencium secundum proporcionem magni circuli erit nota dando | igitur cuilibet gradui eius H 74va 700 stadia.

[II.A.7]

Ex hoc econverso ostendo si duarum civitatum inequalium latitudinum distancia in terra secundum unum magnum circulum fuerit nota et cum hoc notum fuerit ad quem ventum vel ad quam differenciam

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979 latitudines habuerint] *inv.* P **980** describam] scribam P | unam] *om.* P **981** transeuntem] transeuntis H **982** illa] illud H **983** semicirculorum] circulorum G **984** punctis] *om.* D **990** hoc] *om.* D **992** f] *om.* H | f sit] *inv.* G | sit[1]] est D | et] *om.* GHP (check P) **993** eorum] *om.* D **994** erit] et D **997** arcus[1]] *om.* D **999** noto] *om.* D **1000** cpd] *zpd* H | p] *om.* D **1001** cp…dp] *inv.* P **1007** Ex] et *sed. corr. mg.* G | hoc] ex *add.* G **1009** ad[1]…vel] *om.* H

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986 corollario] [I.15.] **994** corollario eodem] [I.15.]

positionis iacuerit civitas note latitudinis meridionalior respectu septentrionalioris, quod eciam longitudo earum in equinoxiali erit nota. | D 226r

Describo super cenith septentrionalius videlicet c orizontem *rs*. Si igitur differencia positionis ad quam iacet respectu *c* sit data, erit propter hoc angulus *scr* notus qui est arcus *rs* orizontis. Cum igitur eciam arcus ignotus sit perpendicularis super quartam *cr*, erit ex corollario proporcio sinus arcus *rs* ad sinum arcus *dp* sicut proporcio sinus quarte *sc* ad sinum arcus noti *dc* qui ponitur distancia duorum cenith civitatum datarum. Ergo calculando ut supra erit arcus *dp* notus cuius sinus ex eodem corellario se habet ad sinum longitudinis *fh* civitatum sicut sinus arcus *zd* noti cum ponatur latitudo civitatis *d* nota [ad sinum arcus *zh* noti]. Ergo calculando in quatuor quantitatibus ut prius erit arcus *fh* notus videlicet longitudo civitatum propositarum.

[II.A.8]

Et in eadem figuracione ostenditur eciam quod si latitudines duarum civitatum fuerint note et similiter distancia earum in terra, erit iterum earum longitudo in equinoxiali circulo nota.

Sint ergo duarum civitatum que sint sub duobus cenith videlicet *c* et *d* latitudines *dh* et *cf* note. Et cum hoc sit arcus *cd* notus eo quod earum distancia in terra ponitur nota. Et erit in triangulo orthogonio *cft* ex corollario proporcio sinus arcus *cf* videlicet latitudinis maioris ad sinum arcus *dh* latitudinis minoris sicut proporcio sinus tocius arcus *ct* ad sinum arcus *dt*. Proporcio ergo sinus tocius arcus *ct* minoris quarta ad partem eius *dt* est nota que cadit inter lineam extractam et dyametrum. Ergo per 13^{am} dictionis prime cum arcus *cd* sit notus ex ypothesi, erit eciam arcus *dt* notus, et consequenter totus arcus *ct* notus est. Deinde descripto orizonte *rs* super cenith *c*, ipse neccessario intersecabit equinoxialem super puncto *g* eo quod meridianus *cfr* intersecat quemlibet trium circulorum | *kpg* et *kfg* et *rsg* ad rectos angulos sperales. Facta est ergo H 74vb

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1011 erit] fuerit P | nota] et *add. supr. lin.* H **1012** Describo super] *inv.* D | videlicet c] *vzt* DG *bzt* H **1013** positionis] porcionis D **1014** eciam] *om.* D **1020** cum ponatur] componatur H | ad…noti[2]] *om.* DGHP **1026** circulo] *om.* D **1027** sint] sunt H | et] *om.* H **1032** ct] *d add. sed. exp.* P | minoris] maioris *sed. corr. exp.* D **1033** eius] *om.* P **1034** cd] *ct* G | sit notus] *inv.* P **1035** est] *om.* P **1036** super] orizontis *add. sed. del.* H **1037** puncto] puncta H | trium…1038 circulorum] *inv.* P **1038** circulorum] triangulorum D | rectos angulos] *inv.* D | sperales] et *add. sed. exp.* D

1015 corollario] [I.15.] **1018** eodem…1019 corellario] [I.15.] **1030** corollario] [I.15.] **1034** 13am…prime] [The author seems to have in mind the first supposition of I.17] hic una catha ex quartis magnorum circulorum cuius angulus communis est *r* et arcus reflexi sunt quarte *cs* et *gf*. Quare proporcio sinus quarte *cr* ad sinum eciam noti arcus *fr* componitur ex proporcionibus sinus quarte *cs* | [P 137r] ad sinum eciam arcus noti *ts* et proporcione sinus ignoti arcus *gt* ad sinum quarte *gf*. Ergo per subtractionem proporcionis componentis note a proporcione composita iam dicta, relinquitur inter aliquas duas quantitates notas proporcio sinus arcus ignoti *gt* ad sinum quarte *gf*. Quare *gt* erit notus, et per consequens eciam arcus *tf*. Cuius sinus per correlarium se habet ad sinum arcus ignoti *dp* sicut sinus tocius noti arcus *tc* ad sinum arcus dati *dc*. Ergo calculando ut prius in 4 quantitabus erit arcus *dp* notus. Cuius sinus iterum | [G 122r] ex eodem correlario se habet ad sinum arcus *fh* qui ponitur longtitudo civitatum inquirenda sicut se habet sinus noti arcus *zd* ad sinum quarte *zh*. Ergo cum tres illarum quattuorum quantitatum proporcionalium fuerint note, erit arcus *fh* notus quod fuit probandum.

Longitudo eciam civitatum communiter invenitur per observacionem distanciarum in horis eiusdem eclipsis lunaris a meridiebus diversarum regionum. Quia hore in quibus distancie ille differunt inter meridies earum cadunt et si cuilibet hore dentur 15 gradus equinoxialis eciam erit arcus equinoxialis notus cadens inter meridianos. Idem poterimus habere per duas consideraciones simul factas in duabus civitatibus alicuius coniunctionis lune cum aliqua stella fixa quia huiusmodi coniunctio distat per minus tempus a meridie precedentis diei civitatis occidentalioris quam orientalioris in tempore quod est inter earum meridianos.

Postquam dictum est de longitudinibus regionum de quibus Ptholomeus modicum tetigit in primo capitulo huius dictionis, restat tractacus de invesigacione latitudinis earum vel elevationis poli artici super orizontem in quavis earum et pro hoc ponit propositionem hanc. | | [D 226v | H 75ra]

[II.B.1]

1040 gf] *fg* H **1042** et] ex H **1043** a] ex D **1049** iterum] eciam DP **1051** illarum] istarum P **1052** fuerint] fiunt H sint P **1054** communiter invenitur] *inv.* D **1057** 15] 5 D 25 H **1058** eciam erit] *inv.* P **1060** alicuius…lune] *om.* D **1063** meridianos] et sic est finis tractatus *add.* D **1066** latitudinis earum] *inv.* H | elevationis] elevatione G **1067** in] *om.* P pro] propter H | propositionem hanc] *inv.* G

1047 correlarium] [I.15.] **1049** correlario] [I.15.]

Quantum vero dies equinoxii excedant idest in quot horis diem minimum vel hyemalem per horologia et plura instrumentorum ingenia poterit inveniri absque eciam eo quod latitudo regionis nota fuerit. Nam si in aliquo die equinoxii situaretur transversaliter super lineam meridianam protractam prescise in aliqua superficie equidistante orizonti quedam lamina rotunda regularis spissitudinis non tamen perpendiculariter sed inclinata versus solem sic quod lumen solis utrique superficiei eius scilicet septentrionali et meridionali per totum illum diem adhereat vel saltem sic quod toto die latitudo umbre eius sit equalis spissitudini ipsius lamine, tunc enim superficies eius meridionalis erit precise equidistans equinoxiali. Si ergo stilus erigatur | a centro eius P 137v perpendiculariter versus solem, umbra ipsius movebitur uniformiter cum firmamento in superficie eius meridionali per medietatem anni, et umbra oppositi stili in opposita superficie circuiet eadem uniformiter videlicet in superficie lamine septentrionali per aliam medietatem anni. Observetur ergo in una die solsticii hiemalis vel estivalis ortus vel occasus solis. Et cum aliqua porcio solis incipit emergi, notetur medium latitudinis umbre cum instrumento. Et per quot gradus circumferentie instrumenti illa nota disteterit a dyametro eius equidistante orizonti ipsi erunt dimidium excessus arcus diurni diei equinoxii et diei solsticialis, et illa medietas excessus semper est arcus equinoxialis cadens inter duos orizontes scilicet rectum et obliquum simul intersecantes se super quovis punctorum solsticialium.

Simili eciam modo ymaginanda est medietas differencie arcus diurni diei cuiuscumque gradus zodiaci et arcus diurni diei equinoxii. Hoc patet ex hoc quod sectio cuiuslibet orizontis recti cum equinoxiali transeuntis super aliquem gradum zodiaci iam orientem super aliquem obliquum orizontem simul venit cum illo gradu ad lineam meridianam. Ergo necessario arcus equinoxialis interceptus inter illam sectionem et meridianum erit medietas arcus diurni diei artificialis illius gradus. Cum

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1069 Quantum] quarum D | excedant] *om.* P **1070** et] vel H | plura] plana P **1073** meridianam] meridiani P | aliqua…equidistante] aliquam superficiem equidistantem D **1075** sic] ita P **1076** totum illum] totam illam D **1077** toto] illa *add.* P **1080** ipsius] eius D **1082** oppositi] opposita GH | opposita] altera H | circuiet] circuibit G circuet HP in *add.* P uniformiter] uniformitate GHP | in^{2}…1083 lamine] *om.* H **1084** ergo] autem P hiemalis…estivalis] *inv.* DGP **1085** medium latitudinis] deinde laminis H **1087** disteterit] distaverit G **1088** illa] ista P **1090** simul] similiter H | quovis] quavis H **1091** solsticialium] equinoxialium H **1092** Simili] similiter H | modo] *om.* H | differencie] differencia P **1094** Hoc] et H **1095** orientem] orizentem *sed. corr. supr. lin.* H **1098** illius gradus] *inv.* DH igitur semper a meridiano ad orizontem obliquum sit una quarta | H 75rb equinoxialis, sequitur manifeste arcum equinoxialis iacentem inter duas sectiones duorum orizontum cum ipso scilicet recti et obliqui intersecantium se super gradu zodiaci qui oritur esse dimidium differencie arcuum diurnorum videlicet diei equinoxii et diei gradus illius.

[II.B.2]

[*Figura #30*] Hoc premisso sit *abg* meridanus et *aeg* medietas equinoxialis. *bed* vero sit medietas orientalis orizontis obliqui. Et sit *z* polus antarticus et ponatur *h* punctus obliqui orizontis super quem gratia exempli oritur in illa regione caput capricorni quod ymaginemur iam esse super ipsum *h*. Deinde si protrahatur eciam unus rectus orizon a polo *z* per idem *h* qui sit *zht*, erit ex immedietate premissis arcus equinoxialis *te* dimidium superflui vel excessus diei equinoxii super diem minimum vel diei maximi illius regionis super diem equalitatis quia excessus sunt equales ut patebit inferius. Et erit secundum hoc arcus equinoxialis *ta* medietas arcus diurni diei minimi, et erit consequenter *tg* arcus residuus de medietate equinoxialis | P 138r dimidium arcus nocturni noctis maxime eo quod in revolucione firmamenti duo puncta *h* et *t* simul veniunt ad meridianum in utroque hemisperio. Et ergo arcus equinoxialis *tg* revolvitur super lineam medie noctis interim quod gradus *h* zodiaci revolvitur ab eadem ad orizontem obliquum assignatum. Similiter et revolucio arcus *ta* super meridianum mensurat revolucionem eiusdem gradus *h* usque ad meridiem. Manifestum est ergo *at* esse medietatem arcus diurni diei minimi et *tg* medietatem arcus noctis maxime qui est equalis medietati arcus diurni diei maximi ut ostendetur inferius. Et quia meridianus transit per duos polos tam orizontis quam equinoxialis, erit arcus equinoxialis *ae* quarta circuli. Ergo ipse est medietas arcus diurni diei equalitatis. | G 122v Factus est ergo ex hiis protractionibus sector *zae* ex concursu | D 227r quartarum circulorum magnorum in convexo spere quod erit convertendo proporciones coniuncte cathe proporcio sinus arcus *at* ad sinum arcus tocius *ae* composita ex proporcionibus videlicet sinus *tz* ad sinum *hz* eciam noti, eo quod arcus *ht* est notus cum sit maxima declinacio solis ab | H 75va equinoxiali si *h* ponitur aliquis punctorum

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1102 gradu] gradus D **1107** bed] *bea* G **1109** iam] *om.* P **1110** super] supra D | eciam] post. deinde P **1113** quia] qui DG **1114** ta] *td* P **1116** dimidium] dimidii D **1117** et] *om.* D **1121** mensurat] mensuratur D | revolucionem] *om.* D **1122** esse medietatem] *inv.* P **1123** tg] *illeg.* G **1124** quia] *om.* D **1125** erit…1126 equinoxialis] *om.* D **1126** Ergo ipse] que D | ipse] ipsa P **1127** hiis] istis P **1128** quod erit] et tunc D **1130** videlicet] *pos.* composita P **1131** ht est] *htz* G **1132** declinacio solis] *inv.* P solsticialium, si vero alter gradus zodiaci adhuc erit notus per tabulam declinacionis, et ex proporcione sinus ignoti arcus orizontis *bh* ad sinum quarte orizontis *be*. Si ergo proporcio sinus quarte *tz* ad sinum arcus *hz* subtrahatur secundum aliquem modorum dictorum circa finem dictionis prime a proporcione illa composita nota eo quod arcus meridianus *at* diei minimi est datus, remanebit proporcio alia componens inter duas quantitates notas, que ponantur primum et secundum in ordine quattuor quantitatum. Et sinus quarte *be* ponitur quartum. Ducatur ergo primum in quartum, et productum dividatur per secundum, et exibit tercium scilicet sinus arcus *bh* notus. Quare eciam arcus *bh* est notus, quo dempto a quarta orizontis *be* nota, remanebit arcus *he* eiusdem orizontis notus qui cadit inter ortum capitis capricorni et equinoxialem qui fuit inquirendus.

Quantitatem arcuum circuli orizontis qui sunt inter orbem equacionis diei et ortum cuiuslibet signi vel gradus zodiaci per quantitatem datam diei maximi vel minimi invenire.

Patet nam retenta eadem figuracione ponatur punctus *h* quicumque gradus medietatis zodiaci meridionalis et sit gratia exempli principium sagitarii. Procedatur ergo in sectore *zae* omnino sicud in precedenti. Idem poterit per correlarium sectoris inveniri et facilius. Nam secundum ipsum in triangulo orthogonio *azt* proporcio sinus mediurni | *at* diei cuiusvis P 138v gradus zodiaci *h* ad sinum arcus orizontis ignoti *bh* sicud proporcio sinus tocius quarte *tz* ad sinum arcus noti *hz*. Ducatur ergo in illis 4 quantitatibus proporcionalibus prima in quartam et fiat divisio per secundam scilicet per sinum quadrantis et exibit sinus arcus *bh* notus. Quare iterum arcus *he* erit notus. Si vero *h* ponatur arcus capitis cancri vel alterius septentrionalis medietatis zodiaci, tunc cadet *t* punctus sub orizonte, et *z* erit polus articus et proveniet sector | similis ex parte illa P finis sicud ex ista ergo et cetera.

[II.B.3]

1136 modorum] modum H | modorum dictorum] predictorum modorum DP **1137** illa] ista P **1139** in ordine] inter G **1140** ponitur] ponatur D | quartum] dividatur *add. sed. del.* D **1141** productum] *om.* DGP **1143** qui] quia G **1144** equinoxialem] et *add.* D | qui] que P inquirendus] inquirenda P **1145** Quantitatem] quantitatis H proposicio *adnot. mg.* P **1148** retenta] recitata H **1150** ergo] *inv.* D | sectore] *zhe add. sed. exp.* G **1152** orthogonio] *om.* G | azt] *zat* G **1153** arcus] gradus G | proporcio] tocius *add. sed. del.* G **1156** sinum] signum D **1158** sub…1159 orizonte] super orizontem H **1160** ex] de D | ergo…cetera] *om.* DG

1136 aliquem…1137 prime] [I.17 paragraph 4 and 6, ca. lines 668-684 and 698-716.] **1151** correlarium sectoris] [I.15.]

Altitudinem poli per arcum diei minime presto indagare.

Nam | H 75vb sit in figuracione precedenti medietas diurni arcus diei minime ingenio instrumentorum nota. Erit propter hoc ex precedenti arcus orizontis *he* notus qui est inter ortum tropici hyemalis et equinoxialem. Erit etiam arcus et notus quia est dimidium excessus diei equalitatis super diem minimum. Erit igitur in sectore *eaz* proporcio sinus arcus *et* ad sinum arcus *ta* composita ex proporcionibus videlicet sinus arcus orizontis *eh* noti ad sinum eciam noti arcus *hb* residui et ex proporcione arcus ignoti *zb* qui est depressio antartici poli sub orizonte ad sinum quarte *za*. Ergo demendo primam proporcionem componentem que est nota ab illa composita nota relinquitur secunda proporcio componens nota inter aliqua extrema nota. Erit ergo ut prius altitudo sinus arcus *zb* notus et per consequens ipse arcus *zb* qui querebatur.

Aliter per correlarium idem levius invenitur nam in triangulo orthogonio *cab* proporcio sinuum duorum arcuum *ht* et ba quorum primus est notus ex ypothesi est sicud proporcio sinuum arcuum duorum notorum *eh* et *eb*. Ergo secundum modum calculandi in quattuor quantitatibus erit arcus *ab* notus, quo dempto de quarta relinquitur *bz* notus.

[II.B. 4]

Arcum minimi vel maximi diei in quovis climate per notam poli altitudinem invenire.

Nam sit in figuracione precedenti altitudo poli *zb* nota. Erit propter hoc in sectore *zae* proporcio sinuum arcuum duorum *zb* et *ba* notorum ex duabus proporcionibus videlicet sinuum duorum eciam notorum arcuum *zh* et *ht* et proporcione sinus arcus ignoti et qui ex premissis est dimidium differencie diei equalitatis et minime ad sinum quarte *ea*. Subtrahatur ergo prima duarum proporcionum componentium que est nota ex declaratis de ipsa composita | D 227v eciam nota. Relinquitur altera componencium nota. Ergo cum unum extremum illius scilicet sinus arcus *ea* sit notum, erit similiter alterum extremorum eius scilicet sinus arcus *et* notus. Ergo et ipse arcus *et* notus, quo dempto de quarta equinoxialis *ea* relinquitur *ta* medietas arcus diurni diei minime notus. Et addito | H 76ra arcu *et*

1163 diurni arcus] *inv.* D **1166** Erit] et D | notus] *om.* D **1167** Erit igitur] *inv.* G **1170** depressio] depressi GH **1185** zae] *zea sed. corr.* G **1186** sinuum] sinum H **1192** extremorum] extremum G | sinus] *add. mg.* G **1193** ea] *iter. sed. exp.* D **1194** Et] *om.* H

1175 correlarium idem] [I.15.]

super quartam equinoctialis *eg*, erit arcus *tg* notus videlicet medietas arcus maxime noctis qui est equalis arcui mediurno diei maximi ut patebit inferius, ergo et cetera.

Arcum orizontis qui est inter ortum tropici et equinoctialem per altitudinem poli notam invenire. Patet arguendo per coniunctam catham in sectore *zae* quia si arcus *zb* ponitur notus, erit similiter arcus *ba* notus. Ergo proporcio sinus tocius quarte *za* ad sinum arcus *ba* erit nota, a qua subtrahatur proporcio nota sinus arcus *zt* ad sinum arcus eciam noti *ht*, et relinquitur eciam nota proporcio sinus arcus *eh* orizontis qui queritur ad sinum quarte *eb*. Erit ergo calculando ut supra sinus arcus *eh* notus qui querebatur. Idem potest haberi in quattuor quantitatibus per correllarium comparando in triangulo orthogonio *bae* arcum *ba* ad *ht* et quartam *be* ad arcum *he* qui queritur.

[II.B. 5]

Quilibet duo circuli paralleli circulo equinoxiali eiusdem latitudinis a duobus tropicis sive ab ipso equinoxiali equales arcus orizontis resecant ex utraque parte equinoxialis, et fit(sit) alternatim nox unius diei alterius equalis.

Pro quo premittitur primo quod arcus similes dicuntur quorum est proporcio una | ad suas circumferentias totas. Secundo supponitur quod G 123r
latera cuiuslibet anguli speralis ad aliquem polum terminati de omnibus circumferenciis tam minoribus quam maioribus super polo eodem descriptis resecant arcus similes ut latera anguli recti de qualibet earum unam quartam separant, et medietati recti cuiuslibet earum una octava correspondet et sic. [*Figura #31*] Sint ergo *mkr* et *phl* huiusmodi 2 paralelli equinoxiali *aeg*, et obliquus orizon sit *bed*. Et signentur sectiones eius cum illis paralellis duobus punctis *k* et *h*. Protracta ergo quarta magni circuli a *q* polo septentrionali per sectionem *k* ad punctum *s* in equinoxiali, erit ex secundo iam premisso porcio *km* paralelli septentrionalis que est ab orizonte usque ad meridianum sub terra similis | H 76rb

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1197 ergo…cetera] *om.* DG **1198** ortum] arcum G | equinoctialem] equinoxialis D **1200** notus[1]] *om.* H **1203** eciam] *om.* DG **1204** qui] quod DH **1205** Idem] ergo GH correllarium] in *add. sed. exp.* H **1206** in triangulo] *om.* D | ad ht] et *dht* GH **1207** he] *hez* G **1211** ex] quia H **1213** dicuntur] *illeg.* G **1214** suas] *om.* D | Secundo supponitur] *inv.* G **1218** separant] separantur D separamus G | et] *om.* D **1219** correspondet] et sic *add.* H | Sint ergo] tunc ergo sint D | phl] *vhl* G | 2] *om.* H **1221** sectiones] sectores H | duobus] *om.* D **1222** septentrionali] *om.* D | punctum] *om.* D | s] *dg* G

1205 correllarium] [I.15.]

et per consequens est equitot graduum porcioni equinoxialis *sg* que ex premissis est medietas arcus nocturni illius gradus zodiaci qui revolucione firmamenti describit datum paralellum. Et arcus *sa* residuus de medietate erit medietas diurni arcus diei gradus eiusdem. Et per idem secundo premissis protracta a polo antartico *z* quarta circuli magni *zht*, erit porcio *hl* paralleli meridionalis eciam similis arcui equinoxialis *ta* qui est dimidium diurni arcus diei qui sit sole describente ipsum paralellum [*lhp*]. Cum ergo ille due porciones paralellorum scilicet *km* et *lh* sint equales eo quod eque et equaliter distant a basibus duorum triangulorum omnino equalium videlicet *edg* et *abe*, erunt propter hoc eciam duo arcus equinoctialis *at* et *as* equales quod fuit secunda pars propositionis. Subtractis eciam duobus arcubus equalibus *sg* et *at* a medietate equinoxialis *aeg*, erunt similiter duo arcus *sa* et *tg* equales quorum primus est arcus mediurnus diei gradus zodiaci orientis super *k* et duas medietas arcus noctis gradus describentis paralellum ex alia parte equinoxialis in equali distancia. Deinde eciam manifestum est duos arcus orizontis *ke* et *he* esse equales. Nam demptis dictis arcubus equalibus videlicet *sg* et *ta* de duabus quartis equinoxalis *ae* et *eg*, remanet duo arcus *et* et *es* equales. Et eciam duo arcus *ht* et *ks* sunt equales ex ypothesi cum sint distancie paralellorum. Cum igitur angulus *hte* sit equalis angulo *esk* eo quod uterque est rectus, erit propter hoc *he* equalis basi *ek* quod fuit probandum.

Ex qua demonstracione clare patet excessus diei cuiuscumque septentrionalis paralelli super diem equalitatis esse equalem excessui diei equalitatis | super diem paralelli meridionalis equaliter distantis ab D 228r equinoxiali. Similiter manifestum est obliquum orizontem quemlibet paralellum dividere in porciones duas similes duobus arcubus equinoxialis scilicet diurno et nocturno noctis vel diei paralelli eiusdem. Cum ergo ad depressionem orizontis | versus septentrionem sequatur H 76va minoracio arcuum septentrionalium paralellorum relictorum sub orizonte, sequitur necessario in septentrionalioribus noctium et dierum maiorem inequalitatem fore ex quo huiusmodi arcus parvorum circulorum sint

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1225 est] *om.* D | est equitot] *illeg.* G | graduum] gradus G propter *add. sed. del.* H | porcioni] porcionum D **1227** sa] scilicet *a* GH **1228** medietate] dimidietate G **1229** premissis] premissum D **1230** porcio] proporcio H **1232** lhp] *bhp* DH *bht* G | lh] *hl* D **1234** omnino] *om.* G | erunt propter] quapropter erunt D | hoc] *om.* DH **1235** as] *ag* GH **1236** Subtractis eciam] subtractisque D | arcubus] *om.* D | equalibus] scilicet *add.* D | a] de G **1238** duas] eius H **1239** noctis] nocturni G **1242** de] a DH **1245** he] *eh* H **1246** ek] equa H **1247** patet] quod *add. sed. del.* H | excessus] excessum D | cuiuscumque] a *add. sed. del.* H **1255** septentrionalioribus] septentrionalibus D | et] vel H similes ex precedentibus arcubus equinoxialis penes quorum revoluciones nox et dies artificiales quantificantur.

[II.B.6]

Nota solis altitudine proporcionem umbre iacentis ad gnomonem rectum invenire.

Umbra iacens vel recta est que causatur a re perpendiculariter erecta ad superficiem orizontis vel ei equidistantem que res vocatur gnomo. Umbra vero versa causatur per gnomonem perpendiculariter extractum a gnomone umbre recte versus solem. Et crescente una earum decrescit alia et econverso secundum talem proporcionem quod semper gnomonica(?) quantitas que ponitur communiter 12 punctorum, licet Ptholomeus ponat omnem gnomonem 60 parcium, est medium proporcionale inter umbram rectam et versam ut ostendetur inferius. Secundo advertendum est quod summitates gnomonum quorumcumque ymaginande sunt terminari ad centrum mundi. Quia semidyameter terre insensibilis quantitatis est respectu distancie ad solem, quare eedem proveniunt proporciones umbrarum ad gnomones erectos ad superficiem orizontis naturalis sicut artificialis que per centrum mundi transit et sic superficiem ad quam proiciuntur umbre recte ymaginamur equidistantem sub orizonte artificiali ad distanciam longitudinis gnomonis. Superficiem vero ad quam cadunt umbre verse ymaginamur equidistantis a latere dixi orizontis secundum longitudinem versi gnomonis.

[*Figura #32*] Ymaginor igitur *e* centrum mundi et lineam *eg* descendentem quemdam gnomonem et describam super eam sub meridiano vel alio azimuth circulum *abg*, ponamque *a* cenith. Et erit linea *gzn* ad quam umbre iaciuntur contingens circulum super puncto *g*. Sint gratia exempli *h* et *l* puncta sectionum tropicorum cum meridiano et *b* sectio equinoxialis cum eodem. Si ergo elevacio poli fuerit nota, erit arcus *ab* | notus scilicet distancia cenith ab equinoxiali, et *bl* est maxima H 76vb
solis declinacio. Ergo *al* maxima solis declinacio a cenith in illa regione versus meridiem est nota. Ergo angulus *lea* super centrum est notus. Ergo

1257 revoluciones] revolucionem H **1258** artificiales] equefit *add. sed. del.* H **1260** solis altitudine] *iter.* H **1264** vero] *om.* D | extractum] erectum D **1267** ponat] componat D **1268** omnem] *om.* D **1269** est] hoc *add.* H **1270** ymaginande] ymaginate D **1271** semidyameter] dyameter D **1273** umbrarum] gnomonum D **1274** superficiem] superficies D **1275** ymaginamur] ymaginantur D | equidistantem] equidistantes D **1276** Superficiem] superficies D **1277** ymaginamur] ymaginantur D | equidistantis] dici *add.* G | dixi] dicti D *om.* G **1281** alio] alia H | alio azimuth] a cenith D **1282** quam] *om.* H | g] *gk* H **1283** h…l] *om.* G **1285** arcus] *pos.* notus D | scilicet] *om.* G | bl est] *bh* D eciam angulus *gen* ei contrapositus super centrum est notus, ergo et super circumferentiam circuli. Orthogonii *egn* tres anguli super circumferenciam circuli cuius dyameter est linea *en* inscribentis orthogonium sunt noti. Ergo ex dictis in commento propositionis 11^{e} dictionis prime eciam tria eius latera erunt nota secundum quamcumque quantitatem qua unum eorum fuerit notum. Erit ergo linea *gn* nota secundum quantitatem qua gnomo *eg* ponitur esse 12 punctorum vel 60, et ipsa est meridiana umbra recta diei minimi hyemalis. Et eodem modo omnino proporciones umbrarum ad gnomones in meridiebus diei equalitatis et maximi invenire ymo in omni hora diei cuiuscumque cum nota fuerit altitudo solis vel elongacio a cenith cum instrumento.

Aliter idem et levius invenitur nam trahatur linea *lt* perpendiculariter super orizontem *rp*. Ipsa erit sinus rectus altitudinis solis exeuntis super *l*, et erit triangulus *lte* equiangulus triangulo *egn*. Quare ex 4^{a} 6^{ti} erit proporcio *lt* linee note secundum quantitatem qua re est 60 parcium ad ipsam *te* eciam notam secundum eandem quantitatem eo quod sinus versus *lt* altitudinis solis que ponitur nota per tabulas cordarum est notus sicud proporcio gnomonis *eg* qui ponitur gratia exempli 12 | ad lineam *gn* G 123v
que est eius umbra ignota secundum illam quantitatem. Ducatur ergo *te* in *eg* sive in 12, et productum dividatur per sinum altitudinis solis scilicet *lt*. Et exibit *gn* nota secundum quantitatem qua gnomo *eg* est 12 parcium. Et ita proceditur de omni altitudine nota inveniendo proporciones umbrarum rectarum ad suos gnomones. | D 228v

[II.B.7]

Proporciones gnomonum et umbrarum versarum respectu omnis altitudinis invenire.

Ymaginemur enim lineam *ep* gnomonem versum et superficiem ad quam iacitur eius umbra contingere [circulum] *abg* super puncto *p* que sit linea *px*. Et erit umbra meridiana versa diei minimi hyemalis *ps*, et diei equalitatis *pv*, diei vero maximi *po*. Arguatur ergo de triangulis

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1288 eciam…1289 circuli] insuper D **1291** propositionis 11e] *inv.* G **1292** dictionis prime] *inv.* G **1294** 12 punctorum] *inv.* D **1295** ipsa…meridiana] ipso meridiano H | est] *om.* G **1296** meridiebus] medietatibus D **1300** rectus] arcus H **1302** lt] *l* et G **1304** versus] *om.* D lt] *rt* G | ponitur] ponatur G **1307** eg] *e* H | sive…12] *om.* D | dividatur] dividitur DG scilicet] *om.* D **1308** eg] *egz* H **1309** proceditur] procedatur D **1313** altitudinis] latitudines D **1315** circulum] angulum DGH | abg] *agb* G **1317** Arguatur] arguitur D

1291 commento…1292 prime] [Presumably, he is referring to I.11 paragraph 6, ca. lines 326-345.]

orthogonalis *eps* et *dei* equalitatis *pve* et *epo* secundum duos modos respectu omnis altitudinis solis sicud prius et sic patet proposicio. | H 77ra

Per notam proporcionem cuiusvis umbre ad suum gnomonem altitudinem solis invenire.

Nam si proporcio huiusmodi est nota, erit eciam nota umbra secundum quantitatem qua gnomo ponitur 12 vel 60 parcium inveniatur, ergo radix duorum quartorum gnomonis et umbre simul iunctorum. Et erunt tria latera orthogonii trianguli cuius rectum angulum continent umbra et gnomo nota, et tres anguli eiusdem orthogonii erunt noti. Sed angulus ipsius quem facit radius supra summitatem gnomonis ut est angulus *gen* capit arcum elongacionis solis a cenith capitis si umbra fuerit recta vel si fuerit versa altitudinem solis seu arcum altitudinis solis ab orizonte. Ergo huiusmodi arcus erunt noti.

Vel aliter per secundam viam quia propter similitudinem triangulorum *neg* et *elt* erit proporcio dicte radicis note que est linea *ne* ad gnomonem *eg* notum sicut *el* que est 60 cum sit semidyameter ad sinum *lt* altitudinis solis ignote. Ducatur ergo gnomo *eg* in [*el*], et productum dividatur per radicem iam dictam scilicet per *ne*. Et exibit *lt* sinus altitudinis solis notus qui arcuetur per tabulas sinuum, et exibit arcus altitudinis solis *lr* notus. Et ita proporcionali modo habet fieri si umbra versa fuerit data respectu sui gnomonis.

[II.B.8]

Altitudinem poli in quovis climate per meridianam umbram alicuius trium dierum predictorum invenire.

Patet quia si umbra *ng* minimi diei hiemalis fuerit cognita secundum quantitatem qua gnomo *eg* est 12 vel 60 parcium, erunt ex hoc tria latera orthogonii trianguli *egn* nota. Ergo angulus *neg* notus super centrum circuli, ergo et angulus *ael* equalis qui capit angulum *al*, erit ergo arcus *al* notus. A quo dematur maxima solis declinacio *bl*, et relinquitur arcus *ba* notus videlicet distancia cenith ab equinoxiali que est elevacio poli super

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1318 orthogonalis] orthogoniis D | pve] *pv* GH **1319** et…proposicio] *om.* DG **1322** proporcio huiusmodi] *inv.* D **1324** iunctorum] coniunctorum D **1325** rectum angulum] *inv.* D **1327** supra] super D **1329** altitudinem…seu] *om.* DH **1331** secundam] illam H **1333** eg] *zeg* H **1334** gnomo] *om.* D | el] *eb* DGH **1335** iam] *om.* DH **1337** lr] *om.* D proporcionali modo] proporcionaliter G **1338** gnomonis] *om.* H **1342** cognita] nota G **1343** eg] *om.* G | eg est] *om.* D **1344** neg] est *add.* D **1345** circuli] circulus H | ael] *el* GH equalis] *gel add.* GH | al^1] *gl* GH | al^2] *gl* GH **1347** cenith] solis D orizontem. Et conformiter de umbris diei equalitatis et maximi argueretur. Ergo eciam tertius tamen ex precedentibus idem invenitur.

[II.B.9]

Gnomonitam quantitatem inter umbram rectam et versam in omni hora medium proporcionale existere.

Probatur in figuracione presenti. Nam sole elevato per altitudinem *lr* ymaginatis duabus | lineis *ep* et *eg* gnomonibus, erit versa umbra H 77rb altitudinis illius *ps*, recta vero *gn*. Cum ergo due linee *ep* et *gn* equidistent, erunt duo trianguli *eps* et *egn* equianguli. Ergo per 4^{am} 6^{ti} proporcio umbre *ng* ad gnomonem *eg* vel ad 12 erit sicud gnomonis *ep* ad umbram versam *ps*. Erit ergo 12 vel quicumque numerus secundum quem ponitur gnomo dividi medium proporcionale inter umbram rectam *ng* et *ps*. Et consimiliter omnino de aliis quibuslibet gnomonibus et solis altitudine arguetur ergo propositum. Ex qua demonstracione patet quod si una duarum umbrarum fuerit nota, reliqua respectu eiusdem altitudinis solis erit nota. Nam per notam earum dividatur quadratum gnomonite quantitatis idest 144 si gnomo dividatur in 12 partes vel 3600 si gnomo supponatur 60 parcium, et exibit umbra reliqua secundum eandem quantitatem nota, quod fuit declarandum.

[II.B.10]

Sub linea equinoxiali omnes dies sunt equales noctibus et sibi invicem, et omnes stelle ortum habent et occasum, et umbre quandoque sunt meridiane, quandoque(?) septentrionales, et quandoque nusquam declinantes.

Prima pars patet quia ibidem orizon neccesario transibit per polos mundi ex quo cenith quod est polus eius cadit in equinoxiali. Ergo dividit omnes circumferentias super eisdem polis descriptas in medietates, et ideo omnes proporciones | parallelorum relicte super orizontem sunt D 229r similes inter se cum quilibet sit sui tocius medietas. Ergo cum quinta dictionis huius porciones huiusmodi sunt eciam similes arcubus diurnis,

1353 presenti] precedente D presente G | lr] *rl* H **1355** recta] rectas H | due] alie H **1357** ng] *zg* GH | erit] *om.* D | sicud] proporcio *add.* D | ep] et k *add.* G **1359** ponitur] *om.* G | gnomo] dicitur *add.* G | ng] *gn* D **1361** arguetur] argueretur GH **1365** supponatur] supponitur DH | 60] 600 D **1366** fuit declarandum] declarandum erat H **1368** linea] *om.* G **1369** habent et] *iter. sed. del.* D **1370** quandoque[1]] quando H | et] *om.* DH **1372** ibidem] illud H **1373** equinoxiali] equinoxialem H **1375** omnes] *om.* D **1376** se] omnino *add.* D quilibet] quelibet D **1377** sunt] sint H

1376 quinta…1377 huius] [II.B.5.]

patet secunda pars. Tercia vero pars patet ut prima ex eo quod ibi contingit solem declinare a cenith versus utrumque polum et quandoque transire per ipsum cenith.

[II.B.11]

Sub omni alia linea equidistante equinoxiali bis tantum dies sit equalis nocti in anno, et dies estivi fiunt prolixiores et noctes breviores, et quanto dies estivi distanciores | G 124r sunt ab equinoxio, productiores sunt, hyemales vero correptiores, et quedam stelle apparentes semper quedam semper occulte, et distancia cenith ab equinoxiali est equalis altitudini poli.

Patet quia impossibile est describi duas circumferentias magnas in convexo spere quin dividant alternatim | H 77va se in medietates. Ergo omnis orizon cuius poli cadunt extra polos mundi dividit necessario equinoxialem per medietatem. Ex quo eciam secunda pars et tercia patent simul ex commento propositionis 5^{e} huius. Sed quarta eciam manifesta est quoniam omnes stelle minus distantes ab orizonte elevacione poli artici super eundem sempiterne apparicionis sunt. Ultimam eciam patet ex hoc quod poli altitudo et distancia cenith ab equinoxiali relinquuntur subtracta distancia cenith ad articum polum a duabus quartis equalibus, ergo etc.

[II.B.12]

Sub remotiori linea ab equinoxiali maior est inequalitas dierum et noctium, et continue maior pars septentrionalis celi apparens et minor meridionalis. Patet quantum ad primum similiter ex hiis que dicta sunt in probacione 5^{te} huius, quoad secundum vero clarum est ex accessu orizontis ad equinoxialem versus meridiem et recessu ipsius ab ipso versus septentrionem.

[II.B.13]

1378 vero] *om.* D **1379** a] ad G | utrumque] *om.* D **1382** linea] *om.* G | equidistante] equidistanti H | dies] sunt *add. sed. exp.* H **1384** estivi] *om.* D | sunt[1]] fuerint D | ab] *om.* G **1385** correptiores] corruptiores H | stelle] sunt *add.* G | apparentes semper] *inv.* G **1386** est equalis] *inv.* G **1390** poli cadunt] polus cadit *sed. corr. mg.* G | necessario] quod *add.* H **1391** medietatem] medium D | eciam] hic D **1392** simul] et *add.* DH | propositionis 5e] *inv.* G | Sed] *om.* D **1393** quoniam] quando D **1394** apparicionis] *illeg.* H **1395** ab equinoxiali] *om.* D | relinquuntur] relinquatur G **1396** articum polum] *inv.* H **1400** celi] *pos.* continue G **1401** quantum ad] *om.* G

1392 commento…huius] [II.B.5.] **1402** 5te huius] [II.B.5.]

Sub omni linea cuius distancia minor fuerit ab equinoxiali maxima declinacione solis, umbre meridiei ad utramque partem alternatim declinant et bis in anno declinacione carent.

Patet quia omnis huiusmodi linea intersecabit zodiacum super duobus gradibus medietatis eius septentrionalis in quibus sole exeunte nullo erunt umbre meridiei. Ipso vero peragrante arcum inter eos septentrionalem erunt umbre in meridiebus ad austrum, et septentrionem in peragracione arcus residui.

[II.B.14]

Sub linea cuius decessus ab equinoxiali equalis fuerit maxime declinacioni solis umbre semel in anno declinacione carent, et umbra meridiana numquam declinat ad meridiem.

Patet quia huiusmodi linea continget eclipticam solum super uno puncto scilicet capite cancri. Ergo cum fuerit sol in ipso, flectetur umbra meridiei in se. Cum ergo in omnibus aliis meridiebus anni sol distet a cenith ad austrum, necessario iacientur umbre in omnibus aliis meridiebus dierum ad septentrionem.

[II.B.15]

Sub linea cuius decessus ab equinoxiali est ut poli zodiaci ab eodem, umbra in aliquo die ad omnem partem orizontis flectitur, et erit spacium 24 horarum dies sine nocte et ex opposito nox sine die, et quanto decessus ab hac linea maior est, tanto maius ipsius abit sine nocte et ex opposito maius sine die.

Patet | quia sub ea polus orizontis per articum polum computando H 77vb
distat a tropico cancri per unam quartam meridiani. Ergo ille tropicus solum continget illud orizontem sub polo et ita totus erit supra orizontem. Et secundum quod polus orizontis ab hac linea magis versus polum discedit, necesse est orizontem in tanto maiori distancia a tropico cancri arcum maxime declinacionis intersecare. Per quam intersectionem si paralellus equinoxiali circumducatur, ipse intersecabit arcum zodiaci in punctis equaliter distantibus a capite cancri maiorem vel minorem

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1406 minor] maior *sed. corr. mg.* D **1407** declinacione solis] *inv.* G | meridiei] meridiane D **1409** intersecabit] intersecat D **1411** meridiei] meridiani D **1413** peragracione] peragracionem G **1415** decessus] discessus G | equalis] *om.* H **1416** declinacioni solis] *inv.* G **1418** super] sub H **1419** cum…sol] sole exeunte G | ipso] puncto H **1420** sol] solis G **1424** decessus] discessus G | poli] polus G **1425** flectitur] flectetur D **1426** et²…1428 die] *om. (hom.)* D **1427** decessus] discessus G | tanto] *om.* H | ex] ab H **1429** sub ea] *add. supr. lin.* D **1430** unam] et *add.* D **1431** supra] super G **1435** intersecabit] resecabit H secundum quod huiusmodi intersectio plus destiterit ab estivo tropico. Ex quo manifestum est quod talis arcus sic reflexus semper super orizontem manebit. Et ergo tantum tempus erit sine nocte in quanto sol talem arcum poterit proprio motu pertransire, et in opposito idem erit nox sine die et sic. Quia contingit solem in aliquo die ad omnem differenciam positionis a cenith declinare, quare similiter umbram ad omnem partem orizontis continget flecti. | D 229v

[II.B.16]

Sub polo medietas celi septentrionalis semper est apparens et meridionalis semper occulta, et anni spacium dies una cum nocte sua.

Patet eo quod ibi polus articus et polus orizontis sunt idem. Ergo cum sint duo de circulis maioribus, necessario erit idem circulus orizon et equinoxialis. Ergo semper medietas zodiaci septentrionalis revolvetur super orizontem. Ergo quam diu sol in ea manserit, erit dies sine capite.

[II.B.17]

In spera declivi quilibet duo arcus equales circuli declivis et equaliter a puncto equinoxii distantes equales habent ascensiones.

Sit ergo utrumque punctorum *z* et *t* punctum vernale in medietate equinoxialis *aeg* et sint duo arcus *tk* et *zh* equales se habentes ex utraque parte equinoxialis. Sit gratia exempli aries et pisces. Et protracto orizonte obliquo *bed* super quem ymaginor unum illorum arcuum esse iam ortum et alium oriendum. Manifestum erit arcum equinoxialis *te* esse ascensionem arcus *tk* et arcum *ez* esse ascensionem arcus *zh*. Que ascensiones probantur esse equales positis duobus punctis *l* et *m* polis equinoxialis et protractis quartis magnorum orbium ab eis videlicet *lt* et *le* et ex alia parte *mz* et *me*. Nam arcus *lk* | G 124v inter polum et finem piscium est equalis arcui *mh* qui cadit | H 78ra inter alterum polum et finem arietis. Quare duo trianguli *klt* et *hmz* erunt equilateri, ergo eciam equianguli. Erit ergo angulus *l* totalis equalis angulo *m* totali. Cum duo puncta *h* et *k* ponantur

1438 super] supra H **1439** quanto] quantum D **1440** proprio motu] *inv.* G | idem] eundem(?) GH **1445** Sub…1450 capite] *om.* D **1447** polus[2]] *om.* H **1448** sint] fuerit H **1450** in ea] sol est in capite et H | erit dies] *inv.* H | capite] nocte G **1452** quilibet] *om.* D | equales] *pos.* declivis G **1453** puncto] punctis G | equinoxii] equinoxiali D | distantes] distantis D **1454** punctorum] punctum D | et] *om.* D | punctum] *om.* DG **1456** Sit] ut DG **1457** quem] quod H | esse] *om.* H **1458** esse] et GH **1461** orbium] circulorum G | videlicet] sed G **1462** ex] *om.* G | polum] g *add.* G gradum *add.* H **1463** arietis] prima *add. sed. del.* H **1464** klt] *lkt* GH | et] *om.* H | eciam] *om.* D | Erit] et D **1465** equalis…totali] angulo totali *m* equalis D | Cum] eciam *add.* D equaliter distare ab equinoxiali, erunt ex 5ª huius duo arcus orizontis obliqui ex utraque parte equinoxialis videlicet *ke* et *he* equales. Quare similiter duo trianguli *elk* et *emh* erunt equilateri, ergo et equianguli. Et erit angulus per consequens *elk* equalis *emh*, quibus demptis a duobus angulis totalibus eciam equalibus scilicet *l* et *m*, remanebunt duo anguli residui *elt* et *emz* equales. Et cum hoc duo latera *el* et *tl* unius equalia sunt duobus lateribus *em* et *zm* alterius quia sunt quarte magnorum circulorum, ergo per quartam primi euclidis que similiter in libro Milii monstrata est in triangulis curvilineis, erit basis *ez* equalis basi et, quod fuit probandum.

[II.B.18]

Quilibet duo arcus circuli declivis equales et equaliter distantes ab alterutro punctorum tropicorum habent in spera obliqua ascensiones coniunctas equales ascensionibus quas eidem arcus habent in spera recta coniunctis. Ex qua et premissa manifestum est quod si note fuerint ascensiones unius quarte in spera obliqua, note erunt ascensiones omnium.

[*Figura #33*] Verbi gratia sint duo arcus *zh* et *th* aries et virgo qui equaliter distant a puncto tropici et *aeg* equinoxialis. Et ymaginor unum illorum arcuum ortum et alium oriendum respectu utriusque orizontis scilicet obliqui *bed* et recti *khl*. Et *t z* sint puncta equinoxiorum. Erit ergo arcus equinoxialis *tl* ascensio *th* in spera recta et *lz* ascensio arcus *hz* in eadem. Ascensiones ergo coniuncte illorum arcuum in recta spera equantur toti arcui *zt*. Et similiter due ascensiones *te* et ez eorumdem arcuum respectu obliqui orizontis *bed* equantur eidem arcui *zt*, ergo propositum. Et manifestum est eciam ex hoc omnium duorum arcuum dicto modo se habencium differencias ascensionum earum in utraque spera equales esse. Nam idem arcus *el* differencia talis est duorum arcuum propositorum et ita de aliis contingeret.

1466 equaliter] erunt *add. sed. del.* H | distare] distantes H | ex] *om.* D **1467** Quare] quasi H **1468** equilateri] equales DG | ergo…equianguli] *om.* D **1469** equalis] angulo *add.* D **1470** totalibus] et *add.* D | equalibus] totalibus H **1474** monstrata] demonstrata H | monstrata est] demonstratur D **1475** fuit] est D **1477** equales et] *om.* H **1479** eidem] idem D | habent] haberet D | recta] *om.* H **1480** premissa] premissis H **1483** th] *ht* D **1484** equaliter] equalo D | aeg equinoxialis] ab equinoxiali H **1485** oriendum] scilicet *add.* D **1486** khl] *ahl* H sint] *om.* H **1487** ascensio[2]] *om.* H **1488** recta spera] *inv.* D **1489** ez] *z* H **1494** de] in DG contingeret] ergo et cetera *add.* H

1466 5a huius] [II.B.5.] **1473** libro Milii] [Menelaus' *Sphaerica*.]

[II.B.19]

Cuiuslibet porcionis circuli declivis | ascensionem in spera declivi H 78rb per notam poli elevacionem invenire.

[*Figura #34*] Pro quo sit meridianus *abgd* et elevacio *k* poli artici super orizontem *bed* nota que elevacio est arcus *kd*. Et equinoxialis sit *aeg* et zodiacus *thz* exeunte *h* puncto vernali. Et ponam gratia exempli arcum zodiaci *hl* signum arietis quem ymaginemur iam elevatum versus meridianum super orizontem *bed*. Deinde protracto orizonte recto a polo *k* septentrionali videlicet *klm*, erit arcus *mh* ascensio arietis in spera recta et *eh* ascensio eiusdem in declivi. Quare *em* erit differencia ascensionum eius in duabus speris que sic inquiritur. Nam due quarte *ed* et *km* intersecant se super *l* communi sectione duorum orizontum et zodiaci infra duas quartas *ge* et *gk* ab angulo *g* descendentes. Quare ex catha disiuncta erit proporcio sinuum duorum arcuum *kd* et *dg* notorum ex ypothesi composita ex duabus proporcionibus | sinuum videlicet arcus *kl* D 230r et *lm* et arcus em ignoti qui queritur ad sinum quarte *eg*. Sed proporcio prima componens est nota eo quod arcus *kl* est notus, cum sit residuum declinacionis dati gradus zodiaci qui ponitur oriri super *l*. Ergo per subtractionem secundum regulas antedictas in dictione prima erit residua proporcio nota. Ergo cum unum extremum eius videlicet sinus quarte *eg* sit notum, erit ex eisdem et reliquum notum quod est sinus arcus equinoxialis *em*, ergo et ipse arcus notus qui est differencia ascensionum quesita. Que subtrahatur ab *mh* ascensione arietis in spera recta que nota habetur ex tabula ascensionum signorum in circulo recto cuius composicio in prima dictione precessit vel respectu puncti equalitatis autumpnalis huiusmodi differencia addatur et relinquitur ascensio in spera declivi. Quam si subtraxeris ab ascensionibus coniunctis arietis et virginis in circulo recto que note sunt, relinquitur ex precedenti ascensio virginis in data spera declivi. Et ita ex premissis duabus propositionibus, manifestum est quod cognitis ascensionibus unius quarte zodiaci erunt

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1496 circuli] dati *add.* H **1498** Pro quo] probatur D | elevacio] *add. mg.* H **1500** Et] *om.* D ponam] pono G **1501** arietis] zodiaci G | ymaginemur] ymaginor D **1502** protracto] tracto G | orizonte recto] *inv.* D **1503** ascensio] *om.* H **1506** sectione] sectore H **1507** descendentes] descendentis H **1508** et] ad D **1509** videlicet] scilicet G **1510** proporcio…1511 prima] *inv.* D **1511** kl] *l* H **1513** erit] et *add.* G | residua…1514 proporcio] *inv.* G **1514** Ergo cum] *inv.* H | eius] *om.* H | quarte] quarto H **1515** notum[2]] videlicet *add.* H **1519** prima dictione] *inv.* D **1520** relinquitur] *h add.* D relinquetur H **1522** precedenti] precedentibus DG **1524** quod] *om.* H

1513 regulas…prima] [I.17 paragraphs 4 and 6, lines 668-684 and 698-716.]

ascensiones trium aliarum quartarum note et similiter arcuum intermediorum.

[II.B.20]

Differenciam ascensionum in spera recta et declivi eiusdem porcionis aliter per arcum circuli magni a polo venientis | assignare. H 78va

[*Figura #35*] Nam ymaginemur arcum zodiaci *et* sumptum ab *e* puncto vernali versus capricornum iam esse ortum super orizontem *bed* versus meridianum *abg*. Et sit *tk* porcio paralelli equinoxialis transeuntis per finem illius arcus cadens inter *t* et sectionem *k* eiusdem paralleli equinoxialis cum orizonte. Duo igitur arcus *tk* et *te* simul ascenderunt super orizontem *bed*. Ergo cum arcus *tk* sit equedistans equinoxiali, ipse erit similis ascensioni arcus zodiaci et super orizontem declivem. Protractis arcubus magnorum circulorum a polo equinoxialis *l* per *k* et *t*, erit arcus equinoxialis *mn* similis arcui *tk* ex quo ab eodem angulo *nlm* super polos eorum descripto capiuntur. Ergo arcus equinoxialis *nm* erit equalis ascensioni arcus et in declivi orizonte. Et quia *ltm* est rectus orizon, erit *me* ascensio eiusdem arcus in spera recta. Erit ergo *ne* differencia ascensionum arcus zodiaci et assignanda quod fuit propositum.

[II.B.21]

Cuiuslibet porcionis circuli declivis elevacionem in obliqua spera per notum excessum diei equalitatis super minimum invenire.

[*Figura #36*] Pro quo ponam *bed* orizontem sicud ante et *ae* quartam equinoxialis inter ipsum et meridianum *abg*. Et sit *h* gratia exempli sectio paralelli hyemalis cum ipso orizonte. | Quare protracta a G 125r
polo meridionali *z* quarta *zht*, erit ex dictis circa principium huius dictionis arcus equinoxialis *te* medietas differencie diei equalitatis et minimi que ex noticia altitudinis poli vel aliunde ut ibidem ostendebatur habetur nota. Deinde sit *k* finis cuiusvis arcus zodiaci sumpti gratia exempli a puncto vernali ut est signum piscium cuius ascensio queritur.

1525 quartarum] *om.* DH | arcuum…1526 intermediorum] *inv.* DG **1531** ortum] arcum H **1532** equinoxialis] *om.* DH **1534** equinoxialis] *om.* G | ascenderunt] ascendunt D **1536** declivem] et tunc istis stantibus *add.* D **1537** polo] meridionali *add.* D | per…t] *lkn lt* D | et] *om.* G **1539** polos] polis D | descripto] descripti D **1541** me] *met* D **1543** propositum] ostendendum D **1547** ante] (an with line, check Cappelli!!) **1549** protracta] proiecta H **1550** quarta] est *add. sed. del.* H | huius…1551 dictionis] *inv.* D **1553** finis…1554 signum] intersectio paralelli equinoxialis transeuntis per principium D **1554** signum] *om.* H

Et ymaginemur *k* iam in orizonte, et trahatur quarta *zkl*. Tunc manifestum est ex dictis ubi supra quod arcus equinoxialis *le* est differencia ascensionum signi piscium in duabus speris cuius noticia sic habebitur. Nam in sectore *zte* cuius arcus reflexi sunt *eh* et *zl* intersecantes se super *k*, convertendo proporciones cathe disiuncte erit proporcio | sinuum H 78vb duorum arcuum notorum *th* qui est maxima solis declinacio et *hz* composita ex duabus proporcionibus videlicet sinus arcus *te* qui est excessus notus dierum ad sinum arcus *le* ignoti qui queritur et ex proporcione sinuum duorum notorum arcuum videlicet *lk* qui est declinacio finis piscium ab equinoxiali et *kz* residui de quarta. Per subtractionem secunde proporcionis note ab illa composita nota, erit etiam altera proporcio componens nota. Cum ergo unum extremum eius scilicet sinus arcus *te* sit notus, erit ex regulis ubi supra | alterum D 230v extremum notum scilicet sinus arcus *le*. Quare et ipse arcus *le* notus qui si dematur de notis ascensionibus signi piscium in orizonte recto, relinquetur eius ascensio nota in spera declivi.

[II.B.22]

Per notas ascensiones et per locum solis notum quantitatem arcus diei et quantitatem arcus noctis invenire, et preterea numerum horarum equalium diei vel noctis et tempora inequalium ascendensque et medium celi in omni hora reperire.

Prima pars patet quia si ascensiones arcus zodiaci ab ariete usque ad gradum solis in spera tua subtraxeris ab ascensionibus arcus eiusdem qui protenditur usque ad eius nadyr que ascensiones note habentur ex tabulis ascensionum, quarum composicio in capitulis precedentibus dicebatur, remanebit arcus diei dati gradus solis. Si vero ascensiones nadir solis subtrahantur ab ascensionibus gradus, remanebit arcus noctis. Quo diviso per 15 patebit quot horas equales habeat nox et similiter sit de arcu diei. Si vero aliquis istorum arcuum per 12 dividatur, exibit quantitas unius

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1555 Et…orizonte] cum orizonte obliquo D **1557** duabus] duobus H **1558** sectore] kathe D sectorem H | zte] *te* H | eh] *ez* H **1559** proporciones] proporcionem D | disiuncte] deconiuncte D **1561** composita] componitur D **1562** et] *om.* H **1564** residui] residuum D **1566** etiam] et D | proporcio] *om.* D **1567** alterum] ultimum H **1570** declivi] quod est propositum *add.* D **1572** solis notum] *inv.* G **1573** arcus] *om.* D **1574** diei…inequalium] *om. (hom.)* H **1575** in] *om.* G **1577** solis] ab orizonte *add.* H | eiusdem] *om.* D **1578** usque…1579 ascensionum] *om.* H **1579** capitulis] tabulis G | dicebatur] docebatur D **1580** nadir] gradus D **1581** gradus] eius *add.* GH | noctis] notus D **1582** per] patet posicionibus *add. sed. del.* H | arcu] ortu D **1583** vero] *om.* H

1567 regulis…supra] [I.17 paragraph 2, ca. lines 557-663.]

hore inequalis diei vel noctis idest quot gradus equinoxialis in ea oriuntur. Invenitur eciam illa quantitas aliter scilicet addendo vel subtrahendo duodecimam partem differencie diei gradus solis et equalitatis respectu 15 graduum qui sunt quantitas inequalis hore diei equalitatis. Quarta pars patet nam si fuerit in die per numerum horarum que sunt inter ortum solis et horam tuam, multiplica quantitatem unius hore inequalis si fuerint hore inequales vel 15 si equales. Et quod provenit adde ascensionibus gradus solis in spera tua, et totum illum arcum aggregatum reduc in tabulis ascensionum regionis eiusdem ad gradum equalem zodiaci. Et patebit quotus gradus quoti signi in illa hora ascendat. Si vero fuerit in nocte, tunc adde arcum qui | provenit ex H 79ra multiplicacione horarum quibus distat hora tua ab occasu solis asccensionibus radii solis, et aggregatum arcum reduc ad gradum equalem et patebit ascendens in illa hora noctis. Ultima vero pars similiter manifesta est quia addito arcu qui provenit ex multiplicatione horarum quibus distat hora tua a meridie diei precedentis ascensionibus gradus solis in spera recta aggregatos arcus reduc ad gradum equalem in tabula ascensionum eiusdem spere scilicet recte. Et patebit gradus medii celi in illa hora. Per gradum eciam notum medii celi, gradus ascendens est notus et econverso ut clarum est ex calculo nono dictionis huius quod comprehendit proposicio presens.

[II.C.1]

Proporcio speralis anguli super polum alicuius circuli consistentis ad quattuor rectos est sicud proporcio arcus eiusdem circuli qui ei subtenditur ad totam circumferenciam. Probatur conformiter sicud ultima sexti euclidis quia sumptis equimultiplicibus ad primum et tercium et secundum et quartum. Tunc si multiplex primi scilicet arcus addit super multiplex secundi quod eciam ponitur arcus eciam multiplex tercii quod ponitur unus angulorum addit super multiplex quarti quod est alter angulus. Et si diminuit minuit, et si equat equat eo quod maiori

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1584 vel noctis] *om.* H | gradus…1585 Invenitur] *om.* H **1585** quantitas] hore inequalis *add.* D **1588** equalitatis] ultima pars patet *add. sed. del.* G | fuerit] fuerint G | die] diei H **1591** adde] de G **1592** arcum] numerum D | eiusdem…1604 presens] et cetera secundum modum communem G **1594** in] de D **1599** meridie] meridiei *sed. corr. exp.* H **1603** nono] *om.* D **1606** Proporcio] tractatus prima *adnot. mg.* D **1607** ad] a H **1609** equimultiplicibus] eque multiplicibus D eque multiplicantibus H | et[1]] ad D **1610** scilicet] in 6 D | arcus addit] ad dicit H | super] supra D **1611** eciam[2]] *om.* H **1612** super] supra D **1613** angulus] angulorum G | diminuit] diminuat D | et] *om.* H

1603 nono…huius] [II.B.19(?).] **1608** ultima…1609 euclidis] [*Elements* VI.33.]

huiusmodi angulorum subtenditur arcus maior et minori minor et equali equalis. Ergo ex diffinicionibus 5^{ti} euclidis sequitur quod proporcio primi ad secundum est sicud tercii ad quartum. Ex quo conformiter concludi potest propositum.

[II.C.2]

Omnes duos circulos magnos qualitercumque in convexo sperali descriptos alternatim in medietates se dividere.

Patet quoniam ipsi necessario se intersecabunt ex quo centrum spere est centrum cuiuslibet eorum. Protracta ergo linea recta inter eorum sectiones que si per centrum spere transeat habetur propositum. Si vero preter centrum transeat, ergo cum ipsa sit communis sectio rectarum superficierum eorum et cum recte superficies se intersecantes solum communicent in punctis cadentibus in eorum sectione communi, ergo sequitur speram habere duo centra eo quod non concurrent ille superficies super suis centris quorum cum quodlibet oportet esse centrum spere cum sint circulorum magnorum.

[II.C.3]

Quaslibet duas circumferentias de maioribus super quarum polis circumferencia consimilis describitur ad quartas intersecari.

Probatur nam ex precedenti ipse | necessario intersecabunt sese ad H 79rb
medietates. Ergo puncta intersectionum | earum erunt opposita secundum D 231r
dyametrum in utraque. Cum igitur de ratione poli sit cadere in medio equidistanter inter quelibet duo puncta dyametraliter opposita circumferencie eius, sequitur earum polos necessario distare per quartas magnorum circulorum a duabus earum sectionibus. Quare sectiones ille erunt duo poli circumferencie descripte super earum polis, et per consequens ipsa intersecabit utriusque earum semicirculum cadentem inter earum sectiones in duas medietates quod est propositum.

[II.C.4]

Omnes duo anguli ex duobus meridianis cum circulo signorum ad eandem distanciam a puncto equinoxii provenientes quorum alter extrinsecus alter intrinsecus ex eadem parte sibi oppositus sunt equales.

1614 huiusmodi] modi *add.* G | angulorum] circulorum D **1619** Omnes] 2^a *adnot. mg.* D duos] *om.* DG **1620** dividere] dividunt D **1622** eorum^1] horum D **1627** concurrent] concurrerent DH | ille] due *add.* G **1628** quodlibet] quotlibet D **1631** Quaslibet] 3^a *adnot. mg.* D | quarum] quorum D **1632** consimilis] similis D **1636** equidistanter] equaliter D **1637** sequitur] super D **1643** Omnes] 4^a *adnot. mg.* D **1644** equinoxii] equinoxiali D **1645** sibi] *om.* D soli H

[*Figura #37*] Verbi gratia sint duo puncta zodiaci *t* et *h* equaliter distancia a puncto *b* vernali. Et posito *z* polo equinoxialis *abg* describantur duo meridiani ab eo videlicet *zkh* et *ztl*. Et erunt propter hoc duo trianguli *hbk* et *lbt* equianguli eo quod sunt equilateri | nam duo arcus G 125v *hb* et *bt* sunt equales ex ypothesi et similiter *kh* et *cl* quia sunt declinaciones punctorum equaliter distancium a puncto equalitatis. Similiter arcus equinoxialis *kb* est equalis *bl* eo quod sunt ascensiones in circulo recto arcuum equalium et equaliter distancium ab equinoxio. Cum ergo duo anguli *hbk* et *lbt* sunt equales propter contrapositionem et similiter duo anguli *hkb* et *blt* quia uterque est rectus relinquitur necessario angulum *khb* esse equalem *ltb*. Sed *ltb* est equalis *zte* ratione contraposicionis, ergo eidem equalis erit angulus *khb*. Erunt igitur duo anguli *zte* extrinsecus et *zhb* intrinsecus ex eadem parte sibi invicem equales, quod fuit probandum.

[II.C.5]

Omnes duo anguli ex duobus meridianis cum circulo signorum ad eandem distanciam a puncto tropici provenientes quorum alter extrinsecus alter intrinsecus ex eadem sibi oppositi equantur duobus rectis.

[*Figura #38*] Sint igitur *d* et *e* duo puncta equaliter distancia verbi gratia a puncto *b* tropici estivalis. Tunc protractis duobus meridianis a polo septentrionali *z* videlicet *zd* et *ze*, erunt duo arcus eorum *zd* et *ze* equales eo quod declinaciones punctorum *d* et *e* ab equinoxiali sunt equales. Triangulus ergo *dze* est duorum equalium | laterum, ergo duo H 79va anguli *zde* et *zed* super basim sunt equales. Sed angulus extrinsecus *zeg* cum angulo *zed* valet duos rectos sperales, ergo eciam cum angulo *zde* intrinseco ex eadem parte sibi opposito valebit tantum quod fuit propositum.

[II.C.6]

Angulus circuli meridiani cum circulo signorum apud punctum tropicum proveniens rectus esse necessario comprobatur.

1646 sint] si fuerint DH | equaliter] equidistantes *sed. corr. ad*. equidistanter H **1648** duo meridiani] *pos*. eo G | videlicet] scilicet G **1650** quia] *om.* D **1658** sibi invicem] sumpti D **1661** Omnes] 5ª *adnot. mg.* D **1662** tropici] *om.* H **1663** extrinsecus…intrinsecus] *inv.* D eadem] parte *add.* G **1665** Sint] sunt D **1667** et[1]] *om.* D **1668** e] *t* D | ab] *iter.* D **1669** dze] *zed* D *ze* H **1670** extrinsecus] est *add. sed. del.* H **1671** zde] *corr. supr. lin.* H valet duos rectos *add. sed. del.* H **1672** valebit] valebunt D | quod] quantum D **1675** Angulus] 6ª *adnot. mg.* D | Angulus…1676 comprobatur] [sic] | apud] ad DH **1676** rectus] rectum D

[*Figura #39*] Transeat enim meridianus *abg* per *a* punctum tropicum hiemalem, et sit *aeg* una medietas ecliptice. Deinde ponatur *g* polus, et describatur *bed* quedam circumferencia distans ab ipso per quartam circuli. Cum ergo meridianus *abg* transeat per polos utriusque duorum circulorum *bed* et *aeg*, ipsi per 3^am^ tractatus huius necessario dividentur in quartas. Ergo arcus *be* erit quarta et similiter *ed*. Et ipsi capiuntur a duobus angulis *bae* et *dae* consistentibus super polum *a*. Ergo per primam tractatus huius uterque eorum est rectus.

[II.C.7]

Maxima declinacione solis nota angulum ex meridano et circulo signorum apud punctum equinoxii provenientem notum esse oportet.

Hoc manifestum est quoniam si maxima solis declinacio subtrahatur de 90 gradibus vel una quarta, remanebit unus illorum angulorum qui ex eadem parte proveniret, et si addatur uni quarte proveniet alter. [*Figura #40*] Verbi gratia ponatur a punctus autumpnalis polus, et describatur circulus *bed* secundum distanciam quarte ab ipso. Et sit *z* punctum tropicum hyemale in medietate zodiaci *azg* et *aeg* medietas equinoxialis. Tunc quia poli duorum circulorum *aeg* et *bed* cadunt in circulo uno scilicet meridiano *abgd*, ergo ex 3ª tractatus huius uterque arcuum *ed* et *eb* erit quarta. Ergo uterque angulorum *ead* et *eab* rectus. Cum ergo arcus *ze* sit maxima solis declinacio, manifestum est quod ipsa cum quarta *ed* constituit arcum quem capit angulus zad. Et similiter quod ipsa dempta de quarta *eb* relinquitur arcus *zb* quem capit alter illorum angulorum videlicet *zab*, quod fuit propositum. | D 231v

[II.C.8]

Quantitatem cuiuslibet anguli ex meridiano cum circulo signorum apud quodlibet | punctum eius provenientis per notam illius puncti declinacionem invenire. H 79vb

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1677 abg] *abgd* D **1678** ponatur] *a* vel *add.* D **1679** ipso] *a add.* D **1680** duorum…1681 circulorum] *inv.* D **1681** ipsi] ipsa DH | dividentur] dividetur DH **1684** huius] istius G rectus] ergo et cetera *add.* H **1686** Maxima] 7ª *adnot. mg.* D | declinacione solis] *inv.* G **1687** equinoxii] tropici G **1689** vel] de *add.* D **1690** proveniret] provenencium GH **1692** circulus] semicirculus D **1695** meridiano] minimo D | huius] *om.* D **1697** declinacio] necesse est *add.* D **1699** zb] *zdb* H | capit] *post. zab* D **1700** videlicet] *om.* D | propositum] probandum G **1702** Quantitatem] 8ª *adnot. mg.* D | ex] cum G | cum] et G **1703** eius] tropicis G | provenientis] provenientem G | per…1704 invenire] notum esse oportet G

1681 3am…huius] [II.C.3.] **1684** primam…huius] [II.C.1.] **1695** 3a…huius] [II.C.3.]

[*Figura #41*] Sit ergo gratia exempli *b* principium virginis nunc in meridiano *abgd*, et *aeg* medietas equinoxialis, *btd* vero medietas zodiaci. Et *z* sit punctus autumpnalis. Inquirendo ergo quantitatem anguli *kbt*. Ponatur *b* punctus zodiaci datus polus, et describatur circulus *htek* distans ab eo per quartam. Transibit ergo meridianus *agb* per polos duorum circulorum *aeg* et *hek*. Erit ergo ex premissis uterque trium arcuum *bh* et *eh* et *bt* quarta. In sectore ergo *bhe* cuius angulus communis cadit versus meridiem erit ex kata disiuncta proporcio sinus arcus *ba* noti qui est declinacio puncti dati ad sinum arcus *ah* residui de quarta composita ex proporcionibus duabus videlicet sinuum arcuum duorum notorum *bz* et *zt* et *et* ad *eh*. Subtracta ergo prima proporcione componente que est nota eo quod arcus *bz* ponitur notus scilicet 30 graduum quo dempto a tota quarta *bt* erit eciam arcus *zt* notus, remanebit proporcio altera nota. Cuius cum unum extremum scilicet sinus quarte *eh* sit notum, erit ex regulis in dictione prima premissis similiter alterum notum scilicet sinus arcus ignoti *et*. Et sic erit arcus *et* notus qui dematur de quarta *eh*, et remanebit arcus *th* notus qui est quantitas unius angulorum zodiaci cum meridiano super *b* puncto ex parte eadem proveniencium. Dempto ergo a duobus rectis speralibus idest a 180 gradus, erit alter videlicet angulus *kbt* notus. Vel addatur arcus *et* super quartam *ek*, et provenit arcus *tk* notus qui capitur ab illo angulo super polo consistente. Cum ergo similiter respectu cuiuslibet alterius puncti zodiaci possit argui super quem anguli non sunt noti ex precedentibus, patet propositum.

Posset eciam istud facilius ex corellario sectoris inveniri. Nam secundum ipsum proporcio sinus arcus *bz* dati ad sinum quarte *bt* est sicud proporcio sinus arcus *za* qui est | ascensio arcus dati in spera recta G 126r
ad sinum ignotum arcus *th*. Ducetur ergo secundum in tercium et

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1705 gratia exempli] *post.* virginis DH **1706** btd] omnino *add.* H | vero] *om.* D *bd* H **1707** Et] *om.* D **1708** circulus] *htb add. sed. del.* D | distans…1709 agb] *om.* D **1710** aeg] *heg sed. corr. supr. lin.* D | hek] transit predictus meridianus *add.* D | et[2]] *om.* D **1711** eh] *ek* D **1714** sinuum] sinum H | arcuum duorum] *inv.* G | duorum] arcuum *add.* D | notorum] arcuum *add.* H **1717** cum] *om.* D **1719** dictione] demonstracione H | alterum] uterque D **1720** et[1]] *te add.* D **1722** parte eadem] *inv.* G **1723** notus…1725 capitur] *om.* H notus…1725 angulo] videlicet D **1725** angulo] videlicet H | Cum ergo] *inv.* G **1726** argui] arcui H **1730** qui est] *iter.* G **1731** Ducetur] duceretur G | Ducetur…1732 th] *om. (hom.)* D

1718 regulis…1719 prima] [I.17, paragraph 2, ca. lines 657-663.] **1728** corellario sectoris] [I.15.]

dividatur per primum, et exibit sinus ignotus arcus *th*, quod fuit propositum.

[II.C.9]

Omnes duo anguli ex uno orizonte declivi cum circulo signorum ad eandem distanciam a puncto equinoxii provenientes quorum unus intrinsecus alter extrinsecus ex | eadem parte sibi oppositus sunt equales. H 80ra

Probatur. [*Figura #42*] Et stet meridianus et equinoxialis ut prius, et describatur medietas declivis orizontis *bed*. Et ymaginemur utrumque duorum punctorum *z k* punctum gratia exempli autumpnale, et sint *zh* et *kl* duo arcus sumpti ab eis in zodiaco equales unus versus septentrionem alter versus meridiem ut virgo et libra. Et patebit duos angulos *dlk* et *eht* esse equales, nam ex 5ª dictionis huius duo arcus orizontis *eh* et *el* sunt equales. Similiter duo arcus equinoxialis *ez* et *ek* cum sint ascensiones dictorum arcuum in spera declivi sunt equales ex predemonstratis. Ergo quia eciam dati *zh* et *kl* sint equales, erunt duo trianguli *ehz* et *elk* omnino equilateri, ergo et equianguli. Sed duo anguli *hez* et *hz*e unius sunt equales duobus angulis *kel* et *lke* alterius. Ergo erunt eciam duo anguli residui videlicet *ehz* et *elk* equales. Ergo propter hoc quod unus eorum cum angulo *eht* valet duos rectos et tantum valet alter cum angulo *dlk*, eciam ipsi erunt equales. Et ita eciam manifestum est duos angulos ex alia parte orizontis scilicet extrinsecus intrinseco ut *dlm* et *ehz* equales esse, quod querebatur.

[II.C.10]

Omnes duo anguli ab uno orizonte declivi cum circulo signorum apud opposita orientis et occidentis extrinsecus cum intrinseco equantur duobus rectis. Unde colligitur correlarium quod eciam duo anguli ad eandem distanciam a puncto tropico orientalis eorum cum occidentali duobus rectis sunt equales. Quapropter notis angulis orientialibus unius medietatis ab ariete in libram, erunt noti orientales anguli alterius medietatis. Et similiter occidentales utriusque.

1732 dividatur] divideretur G | primum] 4m G | fuit] est G **1735** Omnes] 9a *adnot. mg.* D circulo] signo D circulorum H **1736** equinoxii] equinoxiali H **1738** Et] sic *add. sed. del.* H ut] sicut G **1742** eht] *ekt* G **1743** et el] *iter.* H **1745** dictorum] duorum H **1746** eciam] arcus *add.* D | elk] *ek* D | omnino] sunt *add.* H **1747** et[1]] *om.* DG **1748** erunt] *om.* H **1751** ita eciam] *om.* D **1755** Omnes] 10ª *adnot. mg.* D | ab] ex G **1756** equantur] equatur H **1757** colligitur] correlarium *add.* G | correlarium] *om.* DH | duo] *om.* D **1759** notis] noctis G

1743 5a…huius] [II.B.5.]

[*Figura #43*] Prima pars patet nam duobus circulis equalibus se secantibus quorum *abg* sit orizon et *aeg* zodiacus super duobus punctis scilicet *a* et *g*, manifestum est duos angulos *dae* et *zad* simul equari duobus rectis. Ergo cum angulus occidentalis *zgd* sit equalis angulo *zad*, equantur similiter duo anguli *zgd* et *dae* duobus rectis, quod est intentum. Secunda pars et tercia sequuntur ex ista propositione | precedente. H 80rb [*Figura #44*] Nam sit iterato arcus *lka* porcio zodiaci incipiens a fine | D 232r libre videlicet puncto *l* protensa super terram, et oriatur idem *l* supra medietatem septentrionalem *deb* obliqui orizontis. Deinde ymaginemur [caput] virginis videlicet punctum *h* in eodem vel simili orizonte scilicet [*b'e'hd*]. Et sit caput cancri *t*, et *p* polus articus et meridianus *gtp* et equinoxialis *age* et *a* caput arietis et *k* caput libre. Tunc manifestum est quod duo trianguli *elk* et *e'hk* vel *e'hz* sunt equianguli quia ex precedenti angulus *elk* vel *z* intrinsecus orientalis septentrionalis super orizontem qui vocetur *x* est equalis angulo [*dht*]. Super duobus punctis oppositis in oriente et occidente videlicet super *l* et *f*, ergo ex declaracione prime partis propositionis presentis, erunt duo anguli *dlt* et *dft* equales, ergo uterque angulorum *x* est equalis angulo *o* qui est angulus *dft*. Sed angulus *dhk* extrinsecus orientalis cum angulo *x* valet duos rectos, ergo idem cum angulo *o* intrinsico occidentali orizontis et zodiaci super *f*, quod est principium tauri cum *l* sit finis libre. Per subtractionem ergo angulorum notorum a duobus rectis speralibus, patet tercia pars.

[II.C.11]

Nota poli altitudine et tropicorum distancia angulum ex concursu orizontis declivis et signorum circuli apud utrumque punctum equinoxii notum esse. Unde constat quod si differenciam que est inter latitudinem regionis et maximam declinacionem cum maior latitudo fuerit declinacione a quarta circuli subtrahas, vel cum minor fuerit, addas, relinquitur angulus cum capite arietis. A quo si distancia inter duos tropicos abieceris, residuum erit angulus super capite libre.

1762 se] sese H **1763** aeg] *zaeg* H **1765** rectis] et *add. sed. del.* D **1768** fine] v *add. sed. del.* H **1769** idem] iam H **1771** caput] finem DGH **1772** behd] *bdehd* DH *dehd* G **1774** trianguli] anguli G | et ehk] *om.* G **1775** elk…orizontem] *om.* D | super] supra H **1776** dht] *dhtl* DGH **1780** dhk] *dak* D **1785** Nota] 11ª *adnot. mg.* D **1787** differenciam] differencia H | latitudinem] altitudinem H **1788** regionis] solis D | maior] *add. mg.* H *pos.* declinacione G | latitudo] altitudo H **1791** abieceris] abicietis H

1771 caput] [The MSS have 'finem,' but then point arcs *hk* and *kl* would not be equal and neither would arcs *ht* and *tf*.] **1772** behd] [The arcs *bed* and *b'e'd* are the same horizon at different times.] **1774** precedenti] [II.C.9.]

[*Figura #45*] Ut posito meridiano *abg* et medietate orientali orizontis *dea*, et sit *ez* quarta equinoxialis inter ipsum et meridanum et *eb* quarta incipiens a libra, et *ge* una quarta zodiaci incipiencium a capite cancri quod sit *g*. Et sit cenith gratia exempli *c*. Tunc quia altitudo est nota, erit distancia cenith ab equinoxiali nota, videlicet *cz*. A qua dematur *zg* maxima declinacio solis, et remanebit *gc* differencia nota. Que si subtrahatur a quarta *cd*, remanebit *gd* notus qui est quantitas anguli *ged* | H 80va ex quo est punctus ortus equinoxii est polus meridiani. A quo dematur distancia *gb* distancia tropicorum, et remanebit arcus *bd* notus qui est quantitas *bed*. Et propter hoc erit eciam quilibet aliorum angulorum notus orizontis et zodiaci super duobus punctis equaliter proveniencium.

[II.C.12]

Quantitatem anguli orientalis apud quemlibet punctum circuli declivis cum orizonte per notum gradum medii celi et latitudinem regionis notam presto invenire.

Hic propter modum demonstrandi ptholomei hanc ex katha coniuncta est advertendum quod quocienscumque proporcio primi ad secundum componitur ex proporcionibus tercii ad quartum et quinti ad sextum, eciam proporcio quarti ad tercium componitur ex proporcione quinti ad sextum et secundi ad primum. Verbi gratia proporcio 8 ad 4 componitur ex duabus | proporcionibus 4 ad 3 et trium ad duo. Patet G 126v ymaginando 6 esse medium inter 8 et 4. Et constat quod si denominacio secundi ad primum que est 1/2 ducatur in denominacionem quinti ad sextum que est unum et 1/2, provenient 3/4 que est denominacio proporcionis quarti ad tercium, videlicet trium ad 4. Istud eciam et consimiles conbinaciones plures habentur ex libro de proporcione et proporcionalitate.

[*Figura #46*] Sit ergo meridianus *abg* et medietas orizontis orientalis *bed* quam ymaginemur modo intersecari a medietate zodiaci super puncto *e* qui gratia exempli sit caput tauri quod ascendit. Erit ex

1792 Ut] verbi gratia H | Ut posito] verbi gratia proposito D **1794** quarta²] quartarum H **1796** videlicet] scilicet G | cz] *tcz* D **1797** gc] *gcd* GH idest *add.* H **1800** gb] *egb* H **1801** erit eciam] *inv.* G **1804** Quantitatem] 12ª *adnot. mg.* D **1811** quinti…sextum] sexti ad quintum GH **1812** proporcionibus] *om.* G **1814** 1/2] una medietas G **1815** et 1/2] 2 1/3 G 1/2] 1/3 H | 3/4] 3 D **1816** proporcionis] tercii ad *add. sed. exp.* D | quarti] primi GH **1819** abg] *abgd* D | et] *om.* D **1820** bed] *aeg* GH **1821** caput tauri] tauri principium D quod] quo H | ascendit] ascendat et D ascensio H

1817 libro…1818 proporcionalitate] [Ametus' *Epistola*.]

predictis in tabulis ascensionum gradus zodiaci pro eodem tunc existens in linea meridiana sub terra notus qui secundum elevacionem poli in insula rodi est xviii cancri. Est ergo *g* 18 cancri existens in linea medie noctis ascendente e principio tauri. Deinde ymaginor e polum super quem ad distanciam unius quarte zodiaci scilicet *eh* revolvitur circulus quidam qui sit *thz*. Tunc quia tam poli ipsius quam meridiani cadunt in circulo uno scilicet in orizonte *bedt*, erunt ex predictis in principio huius tractatus duo arcus *zd* et *zt* quarte. Posito ergo *p* polo artico et *k* puncto sectionis equinoxialis cum linea medie noctis, erit arcus *kp* quarta. A qua dempto arcu *gk* qui est declinacio nota gradus zodiaci qui ponitur in medio celi sub terra, remanet | arcus gp notus a quo dematur arcus pd | scilicet D 232v | elevacio poli super orizontem. Et erit arcus *gd* notus, ergo per H 80vb subtraccionem eius a quarta *dz*, erit eciam arcus *gz* notus. Eciam quia duo puncta zodiaci scilicet *e* et *g* sunt data, ergo arcus *eg* est notus qui secundum latitudinem Alexandrie est inter principium tauri et 18 cancri. Quare est minor una quarta, quo ergo dempto a quarta *eh* remanet *gh* notus. Capto ergo sectore *zte* cuius arcus reflexi sunt *zd* et *eh*, erit per katam coniunctam proporcio sinus quarte *zt* ad sinum arcus ignoti *ht* qui est quantitas anguli *teh* qui queritur composita ex duabus proporcionibus sinuum arcuum *zd* et *gd* et *eg* et *eh*. Ergo propter hoc quod prenotabatur erit proporcio sinuum duorum arcuum *dg* et *dz* scilicet quarti et tercii composita ex duabus proporcionibus sinuum duorum arcuum *eg* et *eh* scilicet quinti et sexti et duorum arcuum *th* et *tz* qui sunt secundum et primum. Subtracta ergo proporcione *eg* ad *eh* ex proporcione *gd* ad *dz*, remanebit proporcio *th* ad *tz* cuius unus terminus *tz* est notus, et illo modo proceditur Ptolomeus in littera.

Sed alio modo potest procedi sicud iam tactum est in catha coniuncta scilicet ut addantur due proporciones componentes que sunt note ex declaratis adinvicem secundum modos predictos in fine dictionis prime, resultabit proporcio composita nota. Cum ergo ipsius secundum extremum scilicet sinus quarte *tz* sit notum, erit sinus arcus ignoti *th* eciam notus, et ipse arcus notus. Et ipse est quem capit angulus *teh*

1823 in[1]] libra *add. sed. del.* H **1824** g] gradus D **1825** e[1]] in H **1826** ad distanciam] a distancia G **1827** circulo] *illeg.* D **1829** puncto] polo GH **1830** A qua] *om.* D **1835** e…g] *inv.* D **1838** zte] *zet* G | reflexi sunt] sunt reflexi super G **1840** qui] que G **1841** prenotabatur] proponebatur G **1842** erit] eciam DH | et tercii] ad tercium D **1843** et] ad D **1844** scilicet] sed H | et sexti] ad sextum D **1845** Subtracta…1849 ut] *om.* GH **1852** notum] notus D **1853** teh] *zeh* H

1828 predictis…tractatus] [II.C.3]

orientalis sub terra orizontis zodiaci super capite tauri. Qui si dematur de duobus rectis, erit angulus *dea* notus orientalis super orizontem, quod est propositum. | GH finis

[II.C.13]

Cum fuerint duo puncta orbis signorum equalis elongacionis ab uno et eodem tropico fueritque eorum longitudo a circulo meridiei ad orientem et occidentem cum temporibus equalibus, arcus euntes per ea et per summitatem capitum sunt equales et eciam anguli quos continent hii arcus cum orbem signorum intrinsecus et extrinsecus ei oppositus sunt equales duobus rectis.

[*Figura #47*] Verbi gratia sit polus articus *g*, et sint *z* et *d* principium geminorum et finis cancri et *a* principium cancri et *b* cenith capitum. Dico ergo quod arcus *bd* equatur arcui *bz* et angulus *bde* cum angulo *bza* equatur duobus angulis rectis. Prima pars patet quia duorum triangulorum *bdg* et *bzg* duo anguli super *g* sunt equales quia includunt ascensiones equales duarum porcionum equalium *ad az*. Eciam predictorum triangulorum duo latera *gd gz* sunt equalia quia illa puncta habet equales declinaciones ab equinoxiali, et latus *bg* est eis commune. Ergo bases erunt equales quod est primum propositum. Sequitur eciam quod angulus *bzg* erit equalis angulo *bdg*. Secunda pars patet quia ex 5^{a} huius 2^{i} duo anguli *gde* et *gza* valent duos rectos. Addendo ergo primo scilicet angulo *gde* angulum *bdg* et removendo a secundo angulum equalem scilicet *bzg*, adhuc remanencia valebunt duos angulos rectos quod est secunda pars.

[II.C.14]

Quando unius puncti orbis signorum elongacio ab utroque latere orbis meridiei ad orientem et occidentem cum temporibus equalibus, tunc arcus transeuntes per illud et per cenith capitum sunt equales, et duo anguli quos hii arcus continent cum circulo signorum aggregati sunt duplum quem super idem punctum faciunt meridianus et zodiacus. Et hoc si fuerint duo puncta cum quibus zodiacus secat meridianum [declinabunt] aut ad septentrionem aut ad meridiem.

Sit igitur prima ut illa puncta sint versus meridiem a cenith. [*Figura #48*] Pro figuracione illius oportet ymaginari quod unus punctus zodiaci primo sit versus orientem et distet a meridiano per 15 gradus verbi gratia, et tunc vocetur *e*, et gradus qui est tunc in meridiano sit *a*. Deinde oportet

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1854 orizontis] scilicet *add.* D **1855** est] fuit H **1856** propositum] [GH END HERE!]
1884 declinabunt] *illeg.* D

1873 5a…2i] [II.C.5.]

postea eundem punctum zodiaci ymaginari versus occidentem eiusdem distare a meridiano ut prius scilicet per 15 gradus, et tunc vocetur ille punctus *h* et gradus qui pro tempore(?) est in medio celi scilicet in meridiano sit *b*. Et tunc sit polus *d* articus, *g* zenith capitum. Tunc dico quod duo arcus *gh* et *ge* equales sunt quod probatur eodem modo sicut in precedenti propositione. Probatur quod [*de* et *dh*] erunt equales ex quo eciam patet | quod anguli *ged* et *ghd* sunt equales. Et sic patet prima pars H 233r propositionis. Sed dico secundo quod duo anguli simul sumpti *gez* et *ghb* equantur duplo anguli *dez*. Quod patet quia angulus *dez* et *dhb* sunt equales ymo successive sunt unus et idem. Si igitur tu addas secundo illorum scilicet angulo *dez* angulum *ged*, et a secundo scilicet *dhb* removeas *ghd* equalem illi qui addebatur, tunc remanencia valebunt tantumdem sicut duo primi. Et sic duo anguli *gez* et *ghb* valent duplum anguli *dez*, quod est 2a pars.

[*Figura #49*] Describam quoque illas porciones prout duo puncta *a* et *b* que sunt in medio celi sunt versa septentrionem a puncto *g* sicut apparet in figura. Dico igitur quod illud similiter accidit scilicet quod duo anguli simul qui sunt *kez* et *lhd* equantur duplo anguli *dez*. Angulus enim *dez* est equalis angulo *dhb*, sed angulus *dek* equatur angulo *dhl* quia sunt duo residui duorum angulorum intrinsicorum equalium. Ergo totus angulus *lhb* equatur duobus angulis simul qui sunt *dez* et *dek*. Quapropter erunt duo anguli qui sunt *lhb kez* equales duplo anguli *dez*.

[*Figura #50*] Describam quoque similem huius forme et hoc prout *g* summitas capitum sit inter a et b, et hoc potest contingere dupliciter. Unde sit primo *a* punctum porcionis orientalis in medio celi in parte meridiana a puncto *g*, et sit *b* porcionis occidentalis punctus qui est in medio celi a parte septentrionali cum *g*. Dico ergo quod duo anguli *gez* et *lhb* sunt maiores duplo anguli *dez* secundum duos angulos rectos. Patet quia angulus *dhg* equatur angulo *deg* ut patet sicut prius ex prima figura angulorum. Duo vero anguli *dh*g et *dhl* equantur duobus rectis. Angulus autem *dez* est equalis angulo *dhb*, ymo est unus et idem angulus qui ymaginatur esse successive in duobus locis equidistantibus a meridiano. Quapropter erunt anguli duo *gez* et *lhb* maiores duobus angulis *dez dhb* scilicet maiores duplo anguli *dez* secundum duos angulos *deg* et *dhl* qui tamen sunt equales duobus rectis.

1894 de…dh] *bz* et *be* D **1898** ymo successive] ymo successive ymaginatur idem angulus *adnot. mg.* D **1907** quia…1908 equalium] *adnot. mg.* D **1909** sunt] ex *add.* D **1921** angulis] *gez add. sed. exp.* D

1917 prima…1918 angulorum] [II.C.13.]

[*Figura #51*] Sit autem punctum *a* porcionis orientalis in linea medii celi in parte septentrionali a puncto *g*, et sit punctum *b* porcionis occidentalis in linea medii celi videlicet in parte meridiana a puncto *g*. Dico ergo quod duo anguli *kez* et *ghb* simul sumpti sunt minores duplo anguli *dez* secundum duos angulos rectos. Qui duo anguli *kez* et *ghb* sunt minores duobus angulis *dez dhb* scilicet minores duplo anguli *dez* secundum duos angulos simul qui sunt ex *dek* et *dhg*. Licet hii duo anguli equantur duobus anguli rectis eo quod ambo anguli qui sunt ex *dek* et *deg* eequantur duobus angulis rectis. Modo deg est equalis *dhg*, et sic patet clare propositum. Unde anguli *dez* et *dhb* sunt equales, ymo ymaginantur successive idem angulus a quibus duobus equalibus demptis duobus angulis *kez ghb*, remanent duo anguli *dek* et *dh*g qui valent duos rectos.

[II.C.15]

Si fuerit datum zodiaci punctum notum in circulo meridiano celum mediante vel in linea orizontis, notum esse oportet tam arcum magni circuli inter ipsum et cenith capitum notum civitatum quam angulum in eodem puncto ab eodem arcu et zodiaco contentum.

[*Figura #52*] Sit punctum capitum *a*, quod necesse est esse in meridiano superius emisperam mediante, quod sit *abgd*. Sit zodiacus *zeh*, orizon notus *bed*, notum zodiaci punctum *z* in meridiano. Dico quod notus est arcus *az* cadens inter cenith et *z* datum punctum zodiaci. Nota est enim arcus ab *a* veniens ad equinoxialem cum sit *a* punctum notum, hoc est noti orizontis polus, sed nota est declinacio *z* puncti noti ex precedentibus, ergo arcus *az*. Dico eciam notum esse angulum *aze*. Ipse enim in hoc situ est idem ei quem facit cum zodiaco meridianus. Sit modo datum punctum zodiaci *e* in orizonte *bed* positum. Dico notum esse arcum *ae* nec mirum cum sit quarta circuli, est enim summitas capitis polus orizontis. Dico eciam notum esse angulum *aez*. Notus est enim angulus *aeb* quia rectus, eciam notus *bez* per 23^{am} de Gebri. Ergo eciam notus est angulus *aez* quod probandum erat.

Et manifestum ait Gebri quod cum nos sciverimus quantitates arcuum et angulorum qui eveniunt ab arcu transeunte per cenith capitum in medietate orbis signorum qui est ab inicio | cancri usque ad inicium D 233v

1930 et] *deg add. sed. exp.* D

1952 23am…Gebri] [Gebir, p. 34-5. This refers to the series of propositions that Simon of Bredon refers to as II.23-6.] **1954** ait Gebri] [Gebir, p. 35-38. Simon of Bredon numbers these propositions that are used to find these arcs and angles as II.27-34.] || Gebri] [Gebir's *Critique*] capricorni in declinacione posita idest data duo nota que sunt, sciemus ex eis per id cuius delinacio precessit quantitates arcuum et angulorum que eveniunt illis signis post orbem meridiei et sciemus cum hoc iterum arcus et angulos qui eveniunt medietati secunde orbis signorum ante meridiem et post.

[II.C.16]

Dato quolibet puncto zodiaci cuius elongacio a puncto meridiei sit cum tempore noto, arcum circuli magni cadente inter ipsum et polum orizontis dati et noti quantus sit inquirendi. Anguli eciam quantitatem que apud idem punctum ab eodem arcu et zodiaco continentur invenire.

Istud declaratur verbi gratia in orizonte cuius altitudo poli 36 gradus, et sit verbi gratia primus punctus cancri distans a meridiano versus orientem per unam horam equalem. [*Figura #53*] Et sic tunc in medio celi erit fere 16 gradus geminorum quod sit *z*, et ascendens erit circiter 18 gradus virginis qui sit *t*. Sitque *h* principium cancri, et sit *btd* medietas orizontis obliqui. Deinde trahatur circulus magnus *aheg*. Isto premisso prius perscrutabor quantitatem arcus *ah*. Manifestum enim est quod illi arcus sunt noti videlicet *az zb zh zt*, et tunc habetur catha cuius angulus *b*. Arguendo ergo per conversam cathe coniuncte erit proporcio corde dupli arcus *zb* ad cordam dupli arcus *ba* aggregata ex duabus proporcionibus ex proporcione corde dupli arcus *zt* ad *th* et ex proporcione corde dupli arcus *he* ad *ea*. Cum igitur tota composita sit nota et una componencium et unus terminus alterius componentis principiam *ae* cum sit quarta. Patet tunc prima pars propositionis.

Sed secunda pars scilicet invencio anguli *aht* patet quia faciemus punctum *h* polum et describemus secundum longitudinem lateris quadrati porcionem orbis magni supra quam sint *k l m*. Et quia orbis *ahe* descriptus est super duos polos *etm* et *lkm* erit propter hoc quilibet duorum arcuum em *km* quarta circuli. Et sic habemus catham cuius angulus *k*. Et tunc arguendo per catham disiunctam proporcio sinus *he* ad *ek* aggregata ex duabus proporcionibus videlicet *ht* ad *tl* et ex proporcione *lm* ad *mk*. Modo composita est nota et prima componentium et unus terminus secunde componentis. Est enim *ah* arcus notus per primam partem, quare et *he* cum sit complementum quarte, quare et *ek* cum sit complementum quarte *hk*. Proporcionaliter eciam *ht* et *tl* arcus sunt noti, et *km* notus est nec mirum quia quarta subtrahendo ergo primam proporcionem componentem notam a composita eciam nota, remanebit proporcio *ml* ad quartam eciam nota. Quare eciam arcus *ml* notus, quare eciam et arcus *lk* qui est quantitas anguli *eht*. Quare eciam et notus erit angulus *aht* residuus de duobus rectis, et hoc est quod voluimus declare.

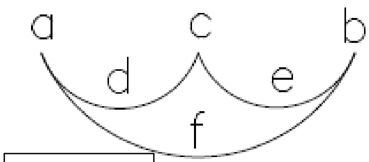
Consimiliter autem sciemus arcus et angulos alios cum principium cancri distabit per duas horas vel *z* a meridiano versus orientem et non solum de cancri principio sed eciam quocumque alio gradu zodiaci ut principio leonis virginis etc.

Deinde Ptolomeus ponit tabulas de quantitatibus arcuum qui sunt inter cenith et inter inicia signorum. Verbi gratia de inicio cancri exeunte in meridiano, ponit arcum qui est inter cenith climatis respectu cuius facta est tabula et inter illum principium cancri. Et ponit eciam quantitatem anguli cancri ex meridiano et zodiaco versa orientem. Postea elongat principio cancri a meridiano versus orientem per unam horam. Ponit quantitatem arcus exeuntis inter ipsum principium cancri et cenith et quantitatem anguli qui causatur ab eodem arcu cum zodiaco super principium eiusdem cancri, et hoc versus orientem semper et ita semper si fiat elongacio per duas horas vel 3 vel 4 etc. usque ad orizontem scilicet ponendo quantitatem anguli et arcus. Eodem modo ponit angulos qui fiunt post meridiem sed ponit intrinsecus scilicet illos qui sunt versus meridianum ex parte tamen septentrionali. De quibus prius rursum est quod cum suis orientalibus extrinsecis valent duos rectos. Proporcionaliter intelligendum est de aliis signis in quolibet climate etc.

Et sic est expleta dictio secunda Almagesti.

1999 ut] principia *add.* D

Erfurt Commentary Figures



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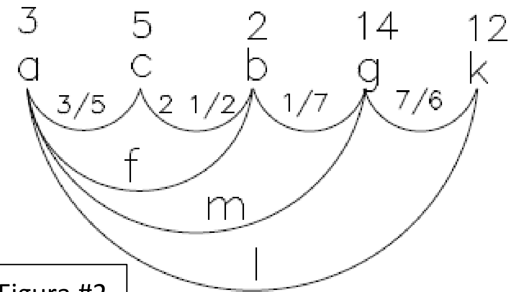

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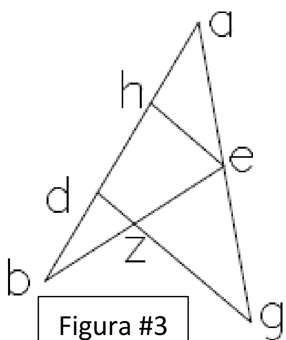

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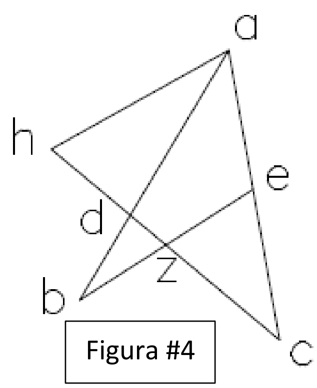

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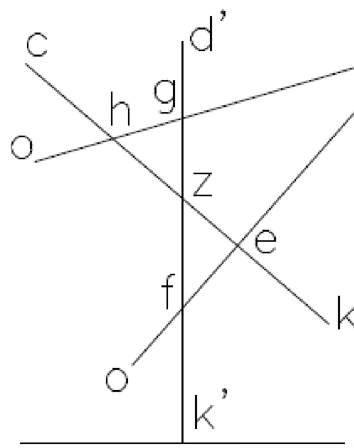

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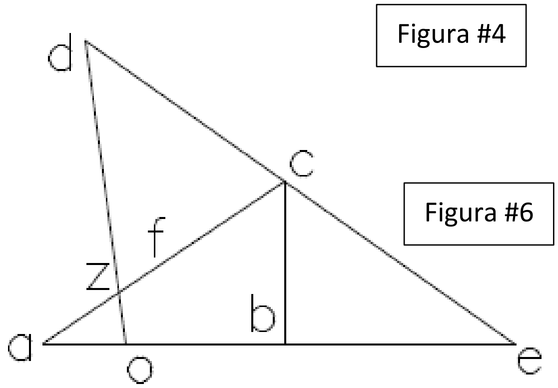

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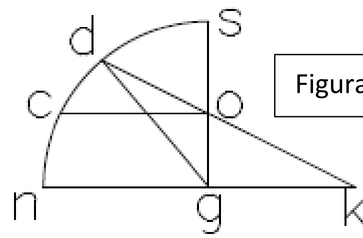

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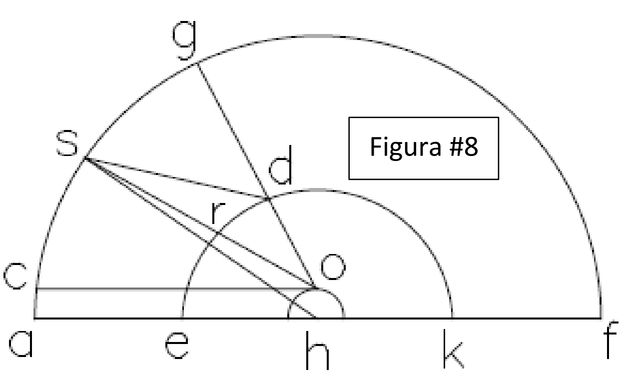

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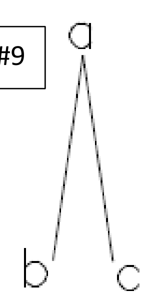

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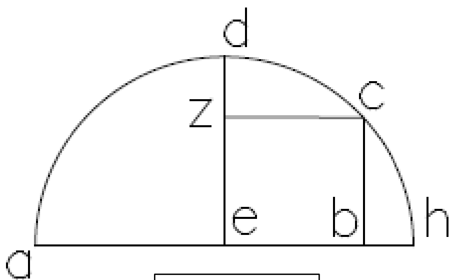

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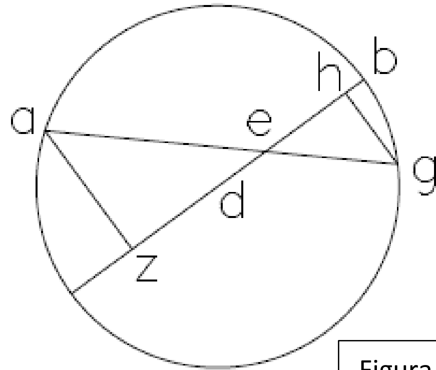

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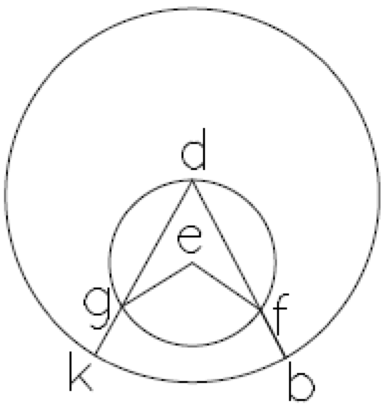

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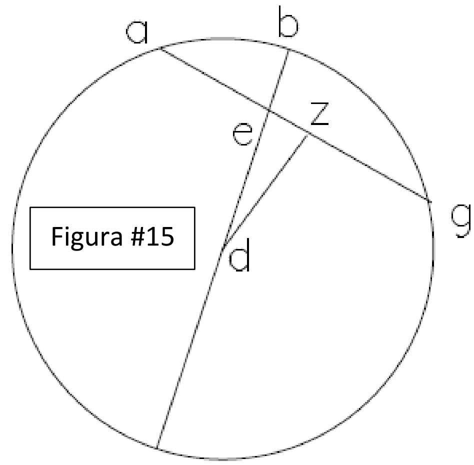

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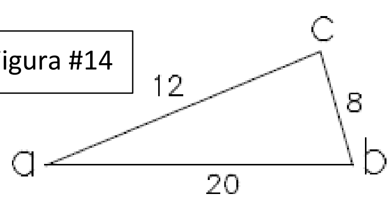
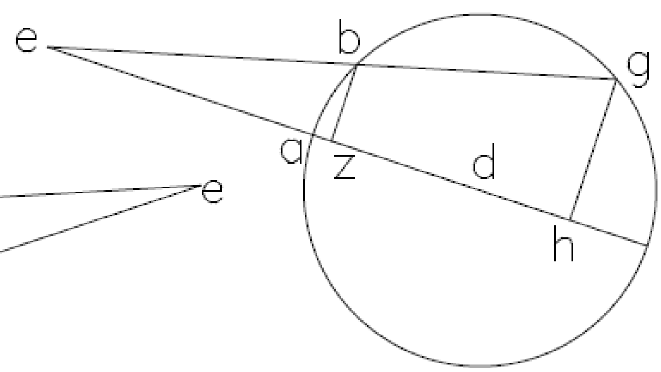
Figura #14





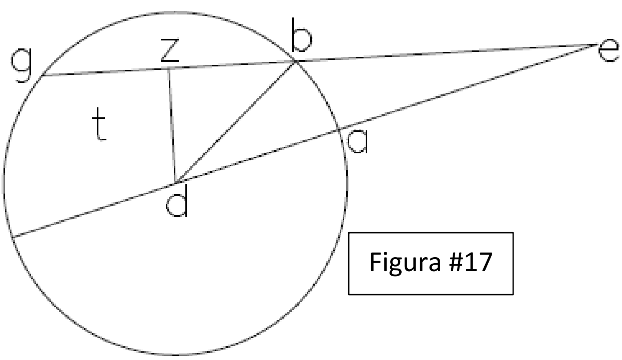


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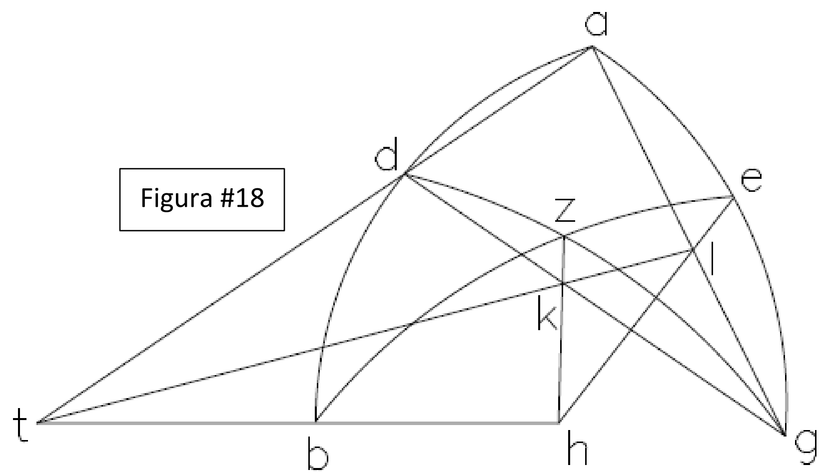
Figura #16



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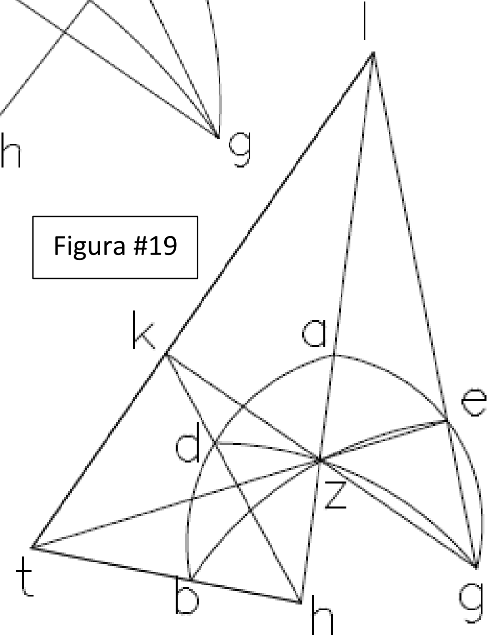

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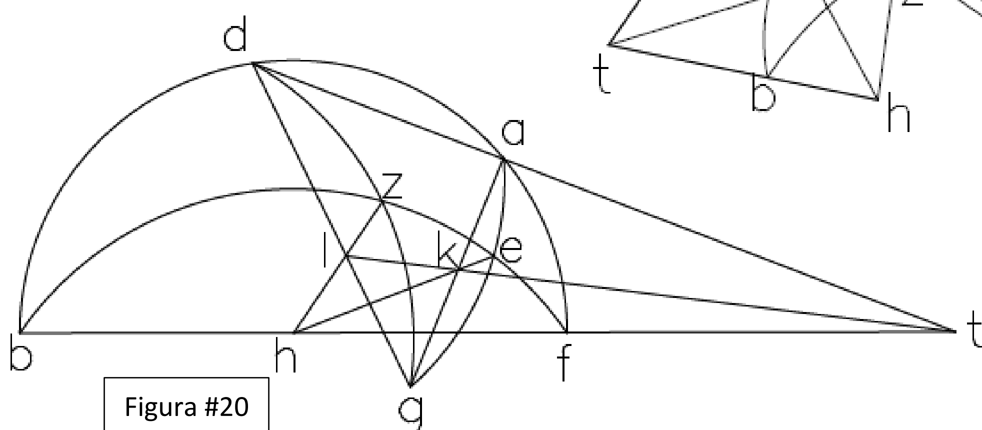

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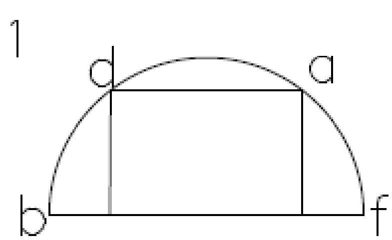

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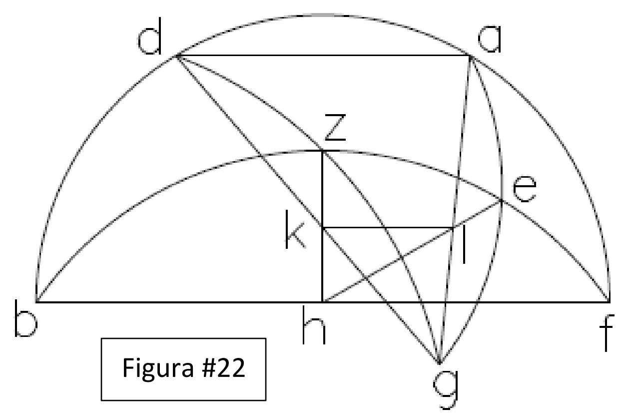

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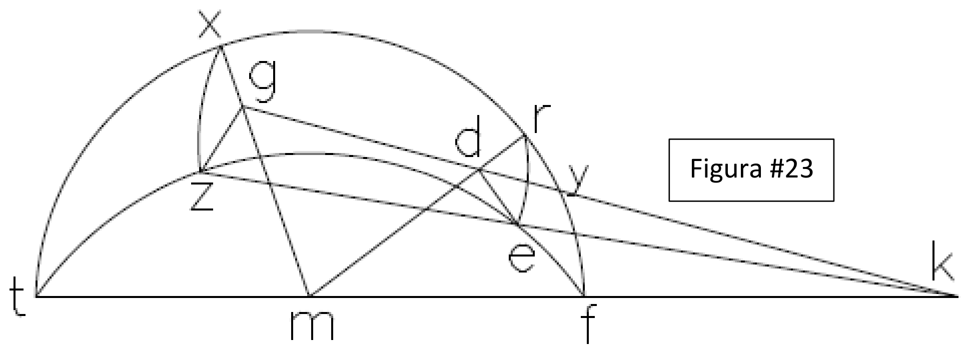

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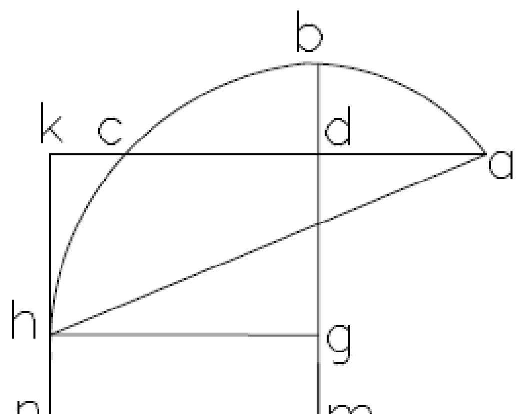

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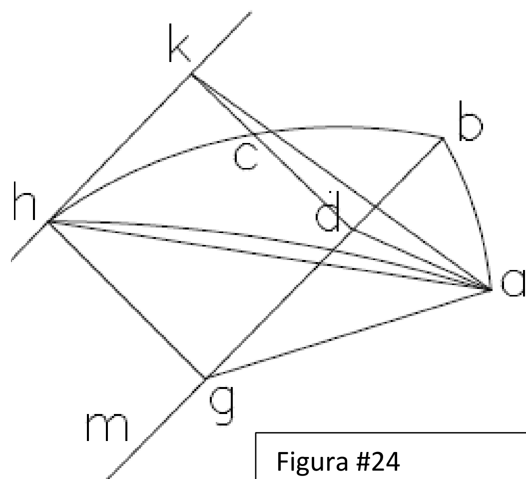
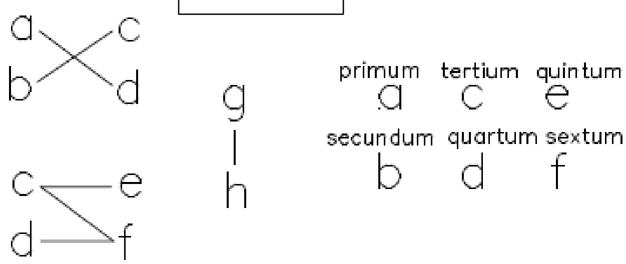
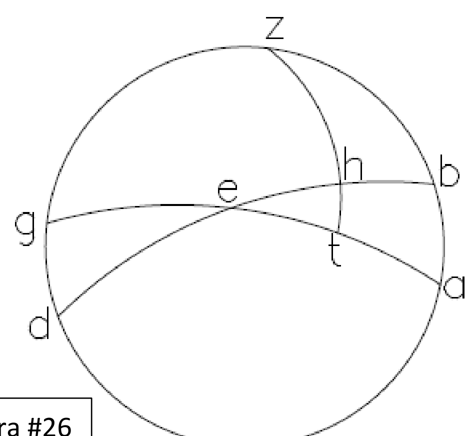


Figura #24
(from another angle)

Figura #25





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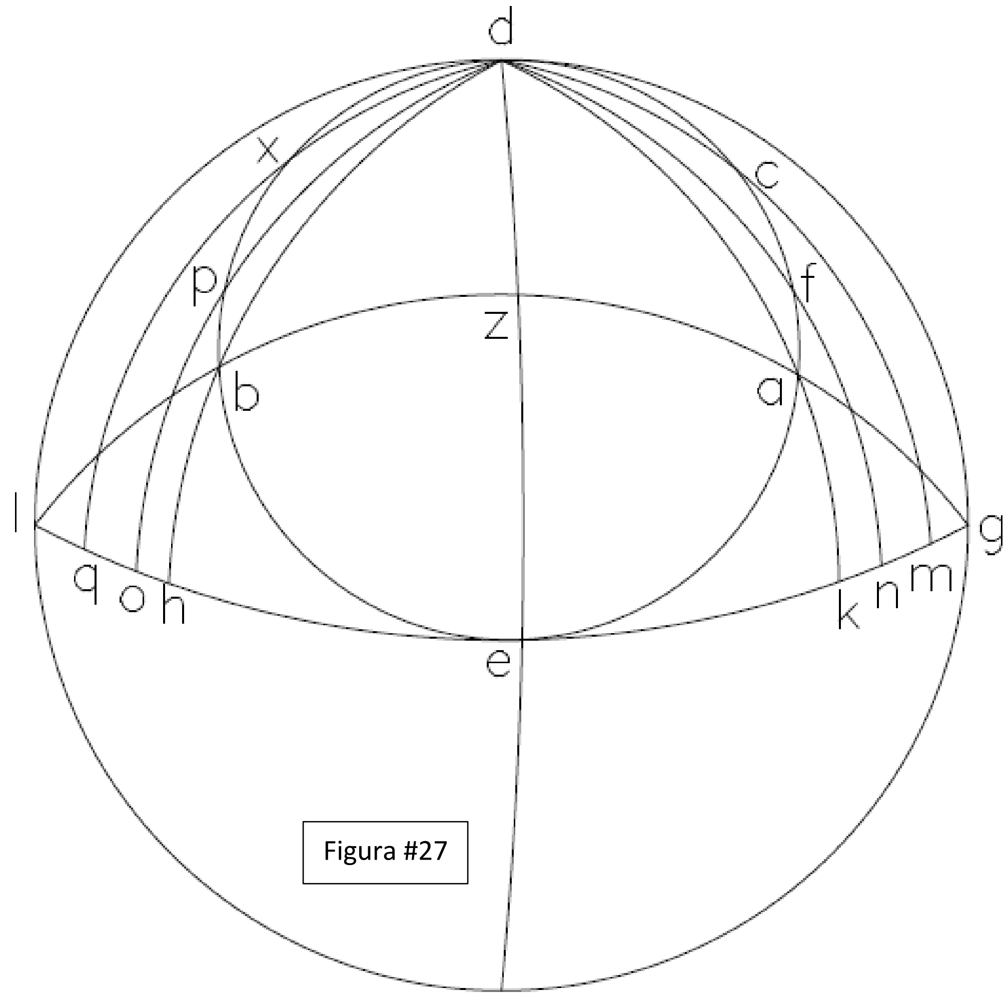

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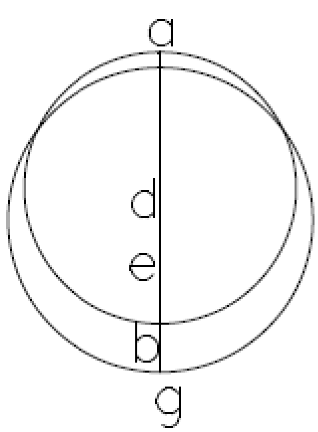

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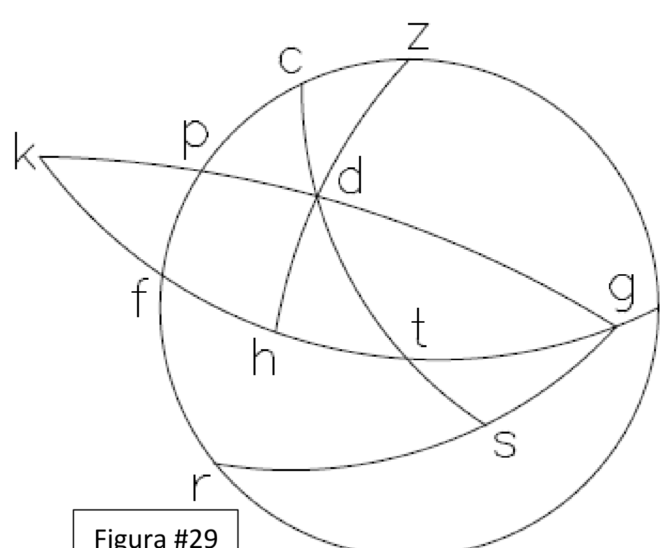

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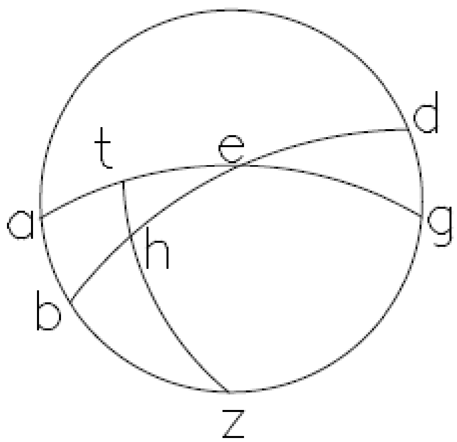

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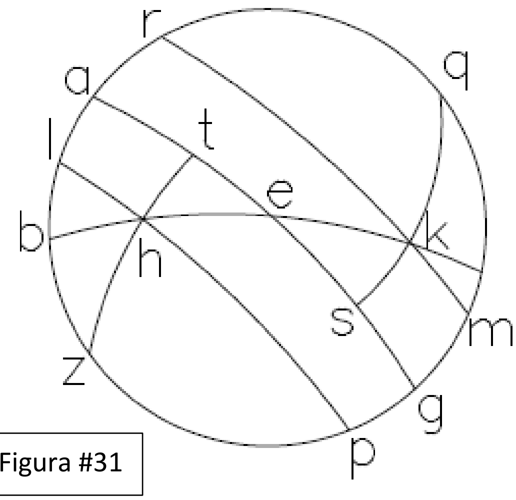

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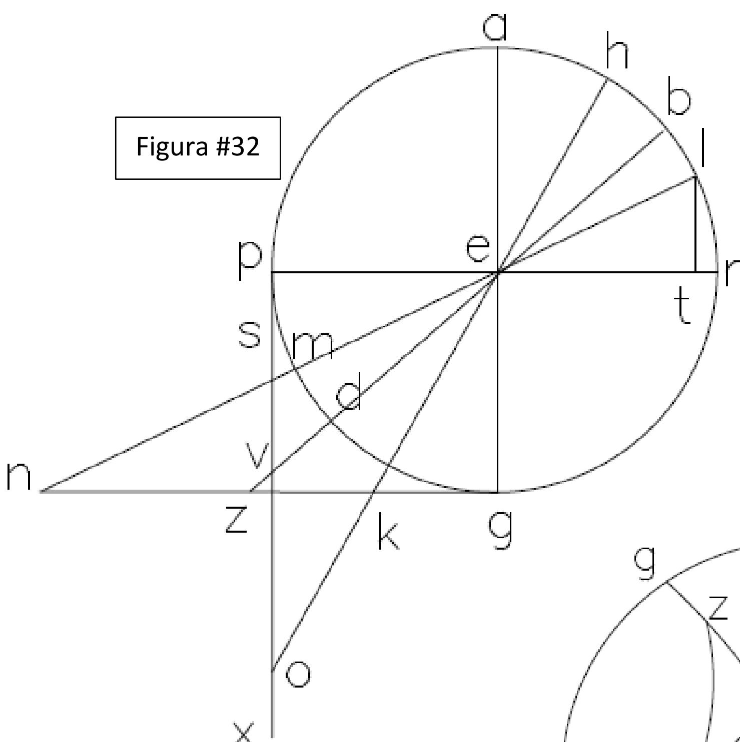

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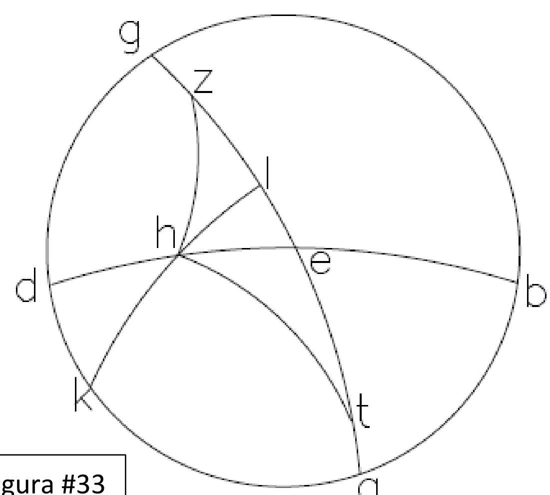

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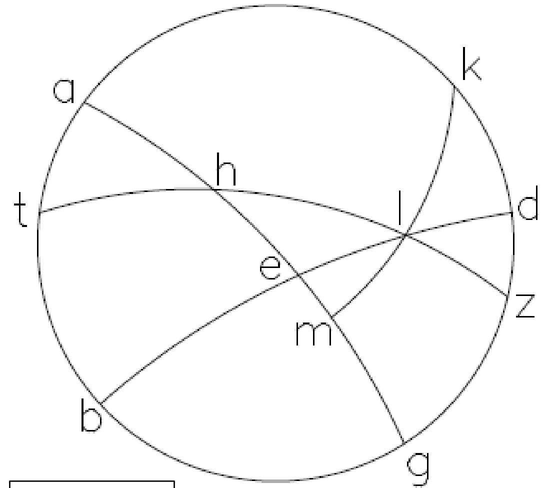

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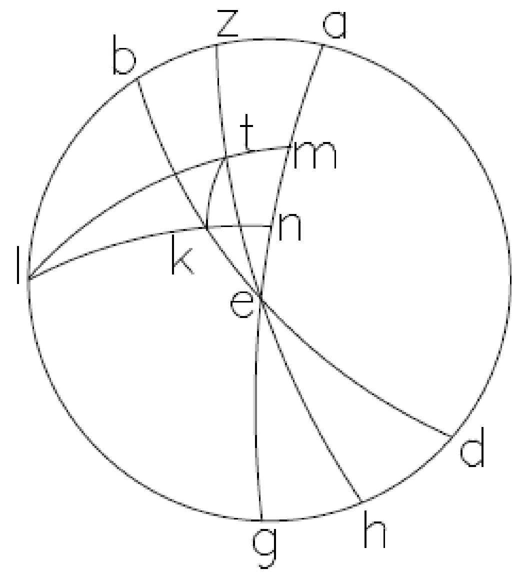

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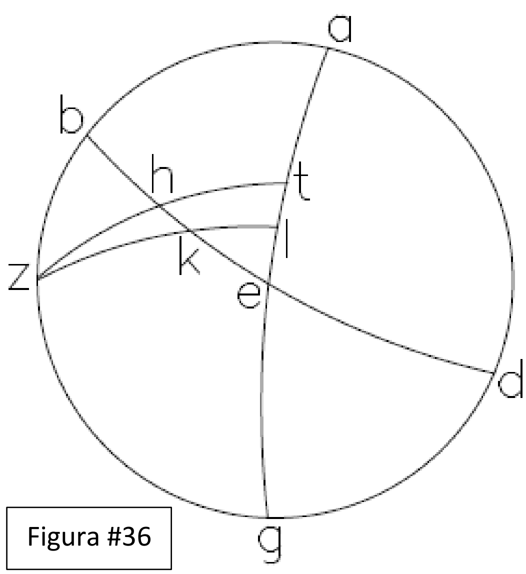

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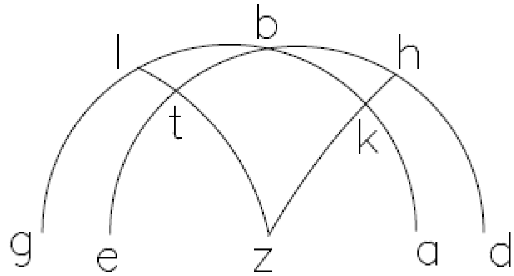

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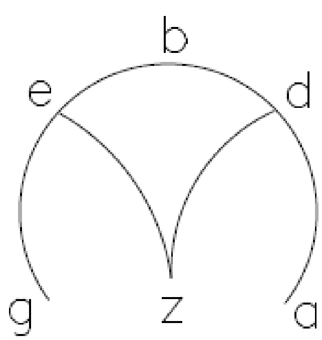

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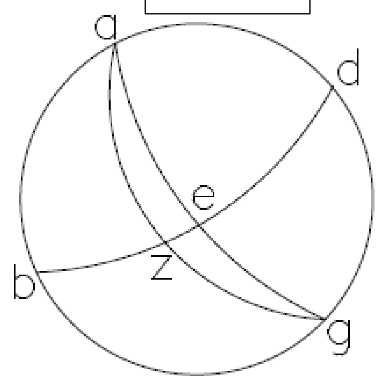
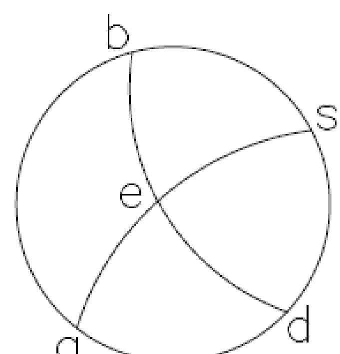
Figura #39





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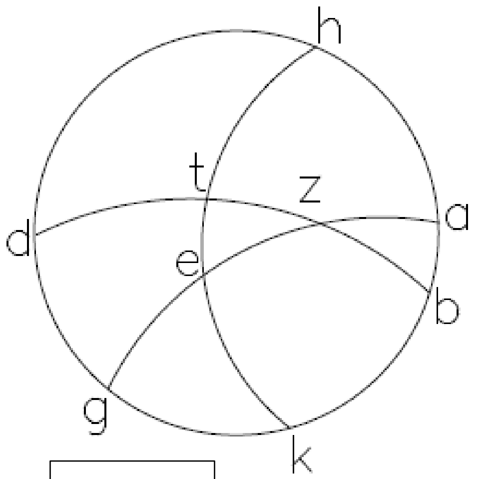

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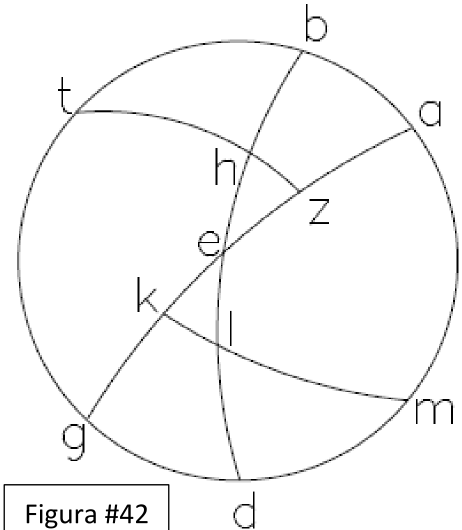

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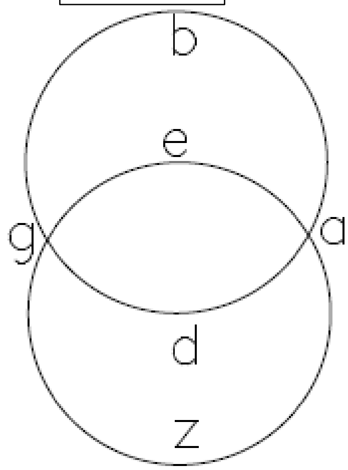

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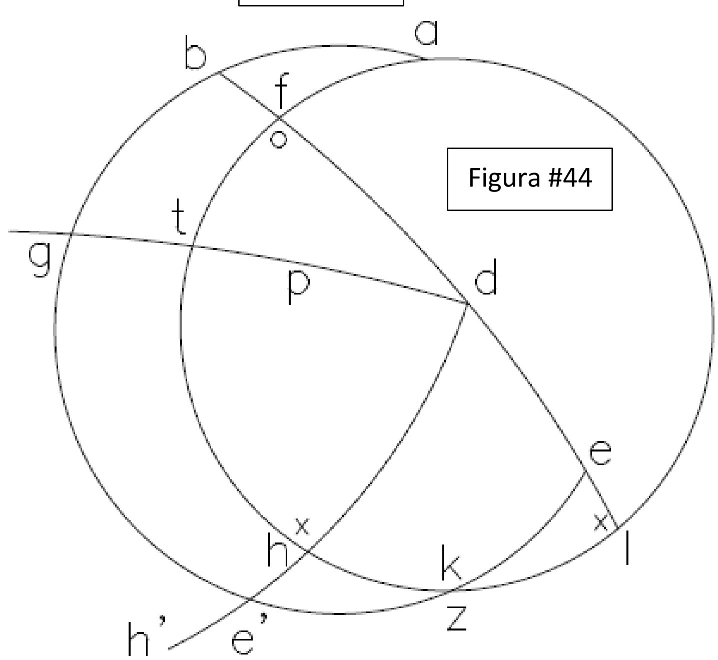

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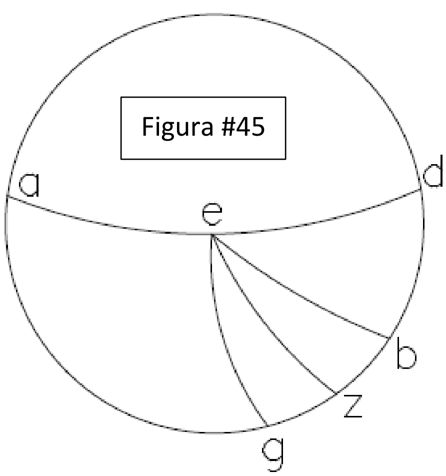

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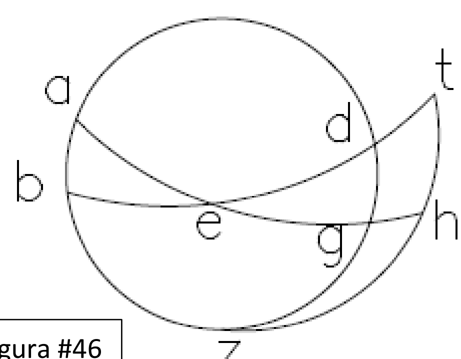

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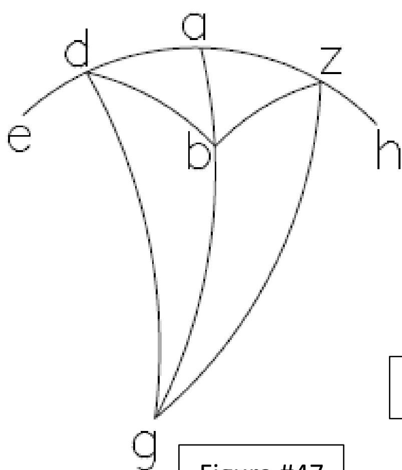

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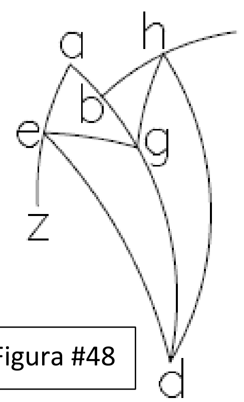

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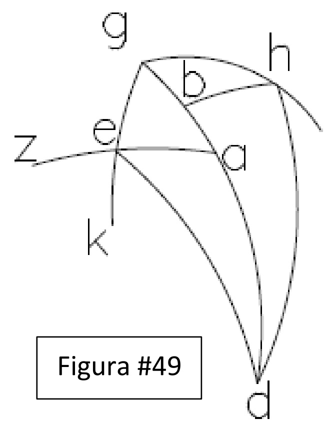

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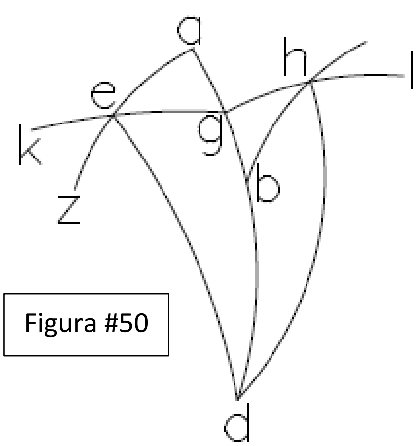

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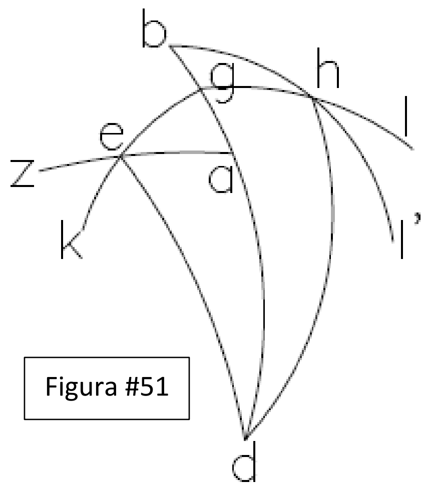

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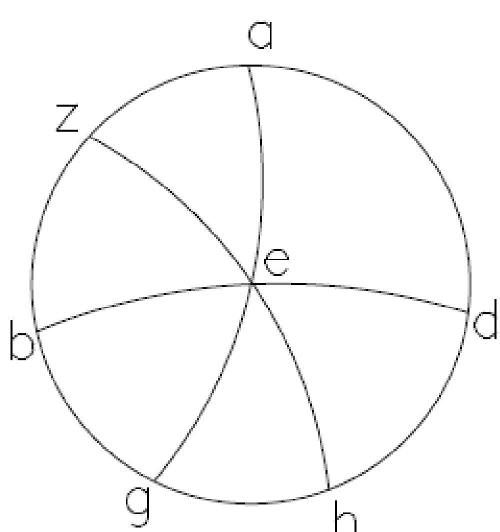

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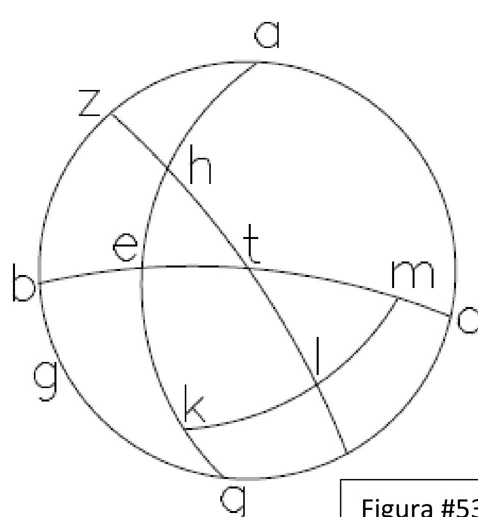

Figura #53

Appendix F: The Vatican Commentary

This work is found in two mansucripts:

Città del Vaticano, Biblioteca Apostolica Vaticana, Vat. lat. 6795 = A

Città del Vaticano, Biblioteca Apostolica Vaticana, Vat. lat. 3100 = B

I am starting my collation where the text begins to address the Menelaus Theorem.

[Notes on I.12: Lemmas]

| 'Et quoniam sequitur etc.' Hic intendit docere qualiter possit sciri quantitas cuiusque arcus qui est inter equatorem diei et orbem signorum. Postquam demonstravit arcum circuli meridiei qui est inter eos. A 5rb B 3vb

'Describam duas lineas etc.' Quia Ptolomeus facit hic mentionem proportionis composite, ideo videndum que sit composita proportio et qualiter fiat. Composita proportio est que est inter duas quantitates inter quas alie sunt quantitates proportionales eis. Et sit ex proportionibus earum ut duo quattuor duodecim. Proportio duorum ad duodecim componitur ex proportione binarii ad quaternarium que est medietas et proportione quaternarii ad duodenarium que est tertia. Ducitur enim medietas in tertiam et provenit sexta. Et ipsa est proportio binarii ad [duodecim]. Et ita sit omnis composita proportio ex proportionibus omnium quantitatum que inter primam et ultimam existunt. 'Et quia *he* et *gd* sunt equidistantes' etc. Hoc sequitur cum probatum fuerit quod omnes anguli illorum duorum triangulorum sunt equales et quod illi duo trianguli sunt similes *aeh* et *adg*. Similiter sequitur quod proportio *dz* ad *eh* sit sicut proportio *zb* ad *be* cum constet quod anguli duorum triangulorum *bhe* et *bdz* sint equales et quod ipsi sint similes. Et hoc totum propter lineas cadentes super lineas equidistantes.

In secunda quoque figura contingit illud idem propter similitudinem triangulorum et hoc propter lineas cadentes super lineas equidistantes. Verum hic prius dividendum est, ut sit proportio linee *hd* ad *dz* sicut proportio *ad* ad *db*, deinde componendum ut sit proportio *hz* ad *zd* sicut proportio *ab* ad *bd*, deinde convertendum ut sit proportio *zd* ad *zh* sicut proportio *db* ad *ba*. Et post sequitur illud quod est in libro.

Quod est in primo circulo sequitur propter similitudinem triangulorum et hoc propter lineam cadentem super lineas equidistantes.

'Hoc autem sequitur etc.' Id est post hoc sequitur illud quod hic dicitur, 'Quoniam omnis trianguli orthogonii notorum laterum reliqui eius anguli a recto sunt noti per id etc.' Id est cum sciemus quantum de arcu debeatur cuilibet portioni corde, quod tamen superius non ostendit in omni corda. Hoc autem sciemus cum minuemus cordam sequentem de precedente in tabulis, et unum minutorum eius aut duo et deinceps

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1 Vat[1]…3100] Vat lat 6795=A; Vat lat 3100=B **15** duodecim] senarium AB | proportionibus] proportionalibus A **16** que] est *add. sed. exp.* A **17** omnes] omnis B **19** similes] scilicet *add.* B | dz] *ad add. sed. exp.* A **26** deinde…27 bd] *om. (hom.)* B **29** Quod] autem *add. sed. del.* A multiplicaverimus in triginta. Et quod provenerit dividemus per cordam illam cuius minutum vel minuta multiplicavimus scilicet sciemus quantum de arcu debeatur uni minuto corde vel duobus et sic de aliis. Et est conversio eius quod fecit Ptolomeus cum fecit tabulas in quibus ostendit quantum proveniat unicuique triginta minutorum de corda.

Tercii autem circuli figura probatur sicut prima per lineas equidistantes et per similitudinem triangulorum.

Quarti similiter figura probatur per easdem proportiones et per noticiam laterum et angulorum. | A 5va

[Notes on Spherical Sector Figure]

'Et postquam premisimus' etc. In hoc loco describit Ptolomeus figuram que sector vocatur et eam probat. In qua etiam ut in precedentibus repetit 'quod componitur' et sit unusquisque arcuum minor semicirculo, quod ideo totiens repetiit quia intendit hic agere de proportione sinuum arcuum et nullus arcus habet sinum nisi minor semicirculo, quia neque semicirculus nec arcus maior semicirculo duplari non possit. De figura vero sectore sciendum est quod vocatur 'sector' propter multas arcuum sectiones que in ea fiunt. Et est notandum quod ipsa probatur secundum duos modos quorum unus est divisionis et alter compositionis quorum unusquisque tribus potest diversificari modis que diversitas contingit secundum situm duarum linearum *ad hb*. Ipse enim in figura aut sunt equidistantes aut non equidistantes. Et cum sunt nonequidistantes aut concurrunt a parte sinistra ut in figura quam ponit Ptolomeus aut a parte dextra. Cum autem a parte sinistra, tunc probatur secundum divisionem probatione quam ponit Ptolomeus que facilis est satis. [*Figura #1*] Cum autem a parte dextra, tunc [producuntur] arcus *ba* et arcus *be* usquequo concurrant supra punctum *m*, et producuntur due linee *da bh* usquequo similiter concurrant. Et tunc secant se duo arcus *zm* et *ga* supra punctum *e*. Quare tunc proportio sinus arcus *gz* ad sinum arcus *ze* componitur ex proportione sinus arcus *ge* ad sinum arcus *ga* et ex proportione sinus arcus *ma* ad sinum arcus *ad*. Cum autem proportio prime ad secundam componitur ex proportione tertie ad quartam et quinte ad sextam, tunc secundum septimum modum compositionis proportionum sex quantitatum, componitur proportio tertie ad quartam ex proportione prime ad secundam et ex proportione sexte ad quintam. Ergo proportio sinus *ge* tertie ad sinum *ea* quartam componitur ex proportione sinus *gz* prime ad sinum *zd* secunde et ex proportione sinus *dm* sexte ad

37 provenerit] divi *add.* B **49** 'quod componitur'] *om.* A **52** semicirculo[2]] cum *add.* AB **56** quorum] sunt *add.* A **59** nonequidistantes] equidistantes B **62** producuntur] producitur AB **72** sinus] *add. supr. lin. a. m.* A sinum *am* quinte. At vero sinus arcus *dm* est sinus arcus *db* et sinus arcus *am* est sinus arcus *ab*. Ergo proportio sinus arcus *dm* ad *am* est proportio sinus arcus *db* ad sinum arcus *ba*. Ergo proportio sinus arcus *ge* ad sinum arcus *ea* componitur ex proportione sinus *gz* ad sinum arcus *zd* et ex proportione sinus arcus *db* ad sinum arcus *ba*. Ecce iam rediit probatio | B 4ra hec ad probationem Ptolomei.

Quod si linea *ad* fuerit equidistans linee *hb*, probabitur prius quod linea *kl* est equidistans linee *ad* hoc modo. [*Figura #2*] Ipsa est cum ea in eadem superficie quia in superficie trianguli *gad*. Ergo si non est ei equidistans, tunc concurret cum ea cum protrahetur. Sed cum linea *kl* sit in superficie circuli *bze* et linea *ad* sit in superficie circuli *adb* que se secant, nullo modo concurrere possunt nisi in communi sectione earum. Communis autem sectio superficierum illorum duorum circulorum est linea *hb*, ergo super eam concurrent due linee *kl* et *ad*. Hoc autem est impossibile cum linea *ad* ut positum est sit equidistans linee *hb*. Ergo linea *ad* linee *kl* equidistat. Quare et linea *kl* iterum equidistat linee *bh*. Et quoniam linea *kl* equidistat linee *ad*, tunc proportio linee *gl* ad lineam *la* est sicut proportio linee *gk* ad lineam *kd*. Proportio vero sinus arcus *ge* ad sinum arcus *ea* est sicut proportio linee *gl* ad lineam *la*. Et proportio sinus arcus *gz* ad sinum arcus *zd* est sicut proportio linee *gk* ad lineam *kd*. Ergo proportio sinus arcus *ge* ad sinum arcus *ea* est sicut proportio sinus arcus *gz* ad sinum arcus *zd*. Et quoniam inter duos arcus *ga* et *ba* se secuerunt duo arcus *be* et *gd* ut prius positum est, tunc proportio sinus arcus *ge* ad sinum arcus *ea* componitur ex proportione sinus arcus *gz* ad sinum arcus *zd* et ex proportione sinus arcus *db* ad sinum arcus *ba*. At vero proportio composita ex proportione sinus arcus *gz* ad sinum arcus *zd* et ex proportione sinus arcus *db* ad sinum arcus *ba* est proportio sinus arcus *gz* ad sinum arcus *zd*, quoniam sinus arcus *db* et sinus arcus *ba* est idem eo quod ipsi sint perpendiculares super lineam *bh* et sint inter duas equidistantes lineas *bh* et *ad*. Quare et proportio unius ad alterum est unum. Et cum | A 5vb unum multiplicatur in aliquem numerum non mutatur, quamobrem cum multiplicatur proportio sinus arcus *db* ad sinum arcus *ba* in proportionem sinus arcus *gz* ad sinum arcus *zd*, nihil provenit nisi eadem proportio. Cum igitur proportio sinus arcus *ge* ad sinus arcus *ea* sit composita ex proportione sinus arcus *gz* ad sinum arcus *zd* et ex proportione sinus arcus *db* ad sinum arcus *ba*, tunc est composita ex sola

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80 si] *add. supr. lin. a. m.* A **84** que] superficies *add.* B scilicet superficies *add. mg. a. m.* A **85** modo] *om.* B **87** eam] ipsam B **98** ex] *iter.* B **101** est] unus *add. sed. del.* A **103** et[2]] *om.* B **106** nihil] non A **109** arcus[1]] *om.* A proportione *gz* ad proportionem arcus *zd*. Hoc itaque modo probatur figura cum predicte linee sunt equidistantes.

Restat ergo nunc ut ostendamus qualiter secundum compositionem probetur. Manente itaque eadem dispositione, dico quod proportio sinus arcus *ga* ad sinum arcus *ae* componitur ex proportione sinus arcus *gd* ad sinum arcus *dz* et ex proportione *zb* ad sinum arcus *be*. Quod sic probatur. [*Figura #3*] Producam duos arcus *ga* et *gd* quousque conveniant supra punctum aliquod quod verbi gratia sit punctum *m*. Quare erit unusquisque eorum semicirculus scilicet arcus *gam* et arcus *gdm*. Convertetur ergo figura quoniam inter duos *eam* et *ezb* secabunt se duo arcus *zdm* et *bda* supra punctum *d*. Ergo erit proportio sinus arcus *ma* ad sinum arcus *ae* composita ex proportione sinus arcus *md* ad sinum arcus *dz* et ex proportione sinus arcus *zb* ad sinum arcus *be*. Verum sinus arcus *ma* est sinus arcus *ga,* et sinus arcus *md* est sinus arcus *dg* eo quod unusquisque duorum arcuum *gam* et *gdm* sit semicirculus. Proportio ergo sinus arcus *ma* ad sinum arcus *ae* est proportio sinus arcus *ga* ad sinum arcus *ae*. Et proportio sinus arcus *md* ad sinum arcus *dz* est proportio sinus *gd* ad sinum arcus *dz*. Ergo quia proportio sinus arcus *ma* ad sinum arcus *ae* est composita ex proportione sinus arcus *md* ad sinum arcus *dz* et ex proportione sinus arcus *zb* ad sinum arcus *be*, tunc proportio sinus arcus *ga* ad sinum arcus *ge* est composita ex proportione sinus arcus *gd* ad sinum arcus *dz* et ex proportione sinus arcus *zb* ad sinum arcus *be*. Et illud quod declarandum premisimus.

Predicte probationes non sunt secundum Ptolomeum sed secundum Thebit qui de hac figura libellum composuit. Sed ne nichil estimetur Ptolomeus dixisse cum dixit, 'ex eo autem quod demonstratum est ex proportionibus linearum etc.,’ et ne pro nichilo videatur premisisse primum antecedens quod est de proportione linearum in superficiali figura secundum compositionem, ostendamus qualiter per lineas rectas possit probari hec figura secundum compositionem tribus modis, scilicet quando linee non sunt equidistantes, quod duobus sit modis ut ostendemus et quando equidistant. Sint itaque duo arcus circulorum maiorum qui sunt in sphera illi quos ponit Ptolomeus scilicet *gea* et *bda* sese supra punctum *a* secantes et inter eos similiter secent se duo alii arcus *bze gzd* supra punctum *z*. [*Figura #4*] Dico ergo quod proportio sinus arcus *ga* ad sinum arcus *ae* componitur ex proportione sinus *gd* ad sinum arcus *dz* et ex proportione sinus arcus *zb* ad sinum arcus *be*. Quod sic probatur. Ponam sicut est in figura Ptolomei centrum sphere punctum

110 itaque] ita B **120** Ergo erit] *inv.* B **127** quia] *om.* B **131** arcus^2] *om.* A **134** ne] *add. supr. lin.* B **141** Sint] sintque *sed. corr. exp.* A *h* a quo producam lineam rectam ad punctum *a* et aliam rectam ad punctum *d* et aliam ad punctum *b* que omnes erunt in una superficie. Deinde protraham a puncto *g* lineam rectam ad punctum *e* et aliam rectam ab eodem ad punctum *z* et aliam rectam a puncto *e* iterum ad punctum *z* que omnes erunt in superficie una. Et ponam inprimis ut linea *ha* et linea *ge* non sint equidistantes et concurrant a parte *a*. Cum ergo protrahentur, concurrent supra punctum unum. Et similiter alie due que sunt in eadem superficie cum linea *ha* et incipiunt ab *h* concurrent cum duabus lineis *gz* et *ez* supra duo puncta quecumque cum sua relativa. Et dico quod illa tria puncta que fiunt ex concursu earum sunt super lineam quoniam in communi sectione duarum superficierum in una quarum sunt tres linee que producuntur a centro et in alia tres alie. Cum enim superficies due se secant, earum communis sectio est linea. Ergo in illa communi sectione illarum duarum sectionum sunt illa tria puncta in quibus concurrunt predicte linee. In hac | igitur figura coniunguntur due A 6ra linee in puncto uno quod sit *t* scilicet illa que est communis differentia in qua sunt predicta puncta tria et linea *ge*, et inter eas secant se due linee *gzk* et *ezl* ut scilicet sint tria puncta signata tribus litteris *t k l*. Est ergo ex primo antecedente Ptolomei proportio linee *gt* ad lineam *te* composita ex proportione linee *gk* ad lineam *kz* et ex proportione linee *zl* ad lineam *le*. At vero proportio linee *gt* ad lineam *te* est sicut proportio sinus arcus *ga* ad sinum arcus *ae*. | Et proportio linee *gk* ad lineam *kz* est sicut proportio B 4rb sinus arcus *gd* ad sinum arcus *dz*. Et proportio linee *zl* ad lineam *le* est sicut proportio sinus arcus *zb* ad sinum arcus *be*. Ergo proportio sinus arcus *ga* ad sinum arcus *ae* est composita ex proportione sinus arcus *gd* ad sinum arcus *dz* et ex proportione sinus arcus *zb* ad sinum arcus *be*. Et hoc est illud quod fuit declarandum.

Et dico quod si due linee scilicet *ha* et *ge* non concurrerint a parte *a* sed ab alia parte, similiter probabitur illud idem. [Figura #5] Sed erit necessarium ut arcus *ag* et arcus *ab* producantur ad partem *g* et partem *b* quousque coniungantur super punctum aliquod quod sit *m*. Erit ergo arcus *aegm* semicirculus et arcus *adem* semicirculus. Deinde diameter producta a puncto *a* transiens per centrum *h* et proveniens ad punctum *m* protrahatur usquequo concurrat cum linea *eg* supra punctum *t*. Et linea *ez* concurrat cum linea *hb* supra punctum *k* et linea *gz* cum linea *hd* supra punctum *l*. Erit ergo in figura hoc mutatum quod in loco *t* erit *l* et in loco *l* erit *t*. Quapropter sic erit in principio argumentandum inter duas lineas

162 hac] fi- *add.* A **164** predicta] a *add. supr. lin. a. m.* A | puncta tria] *inv.* B | linee] scilicet *add. supr. lin. a. m.* A **165** signata] figura B **179** et…semicirculus[2]] *add. mg. a. m.* A **184** argumentandum] augmentandum B *egt* et *tkl* secant se due linee *gzl* et *ezk*. Ergo proportio linee *et* ad lineam *tg* est composita ex proportione linee *ek* ad lineam *kz* et ex proportione linee *lz* ad lineam *lg*. Proportio vero linee *et* ad lineam *tg* est sicut proportio sinus arcus *em* ad sinum arcus *mg*. Et proportio linee *ek* ad lineam *kz* est sicut proportio sinus arcus *eb* ad sinum arcus *bz*. Et proportio linee *lz* ad lineam *lg* est sicut proportio sinus arcus *zd* ad sinum arcus *dg*. Ergo proportio sinus arcus *em* ad sinum arcus *mg* est composita ex proportione sinus arcus *eb* ad sinum arcus *bz* et ex proportione sinus arcus *zd* ad sinum arcus *dg*. Sinus vero arcus *em* est sinus arcus *ae* cum arcus *aem* sit semicirculus. Et sinus arcus *mg* est sinus arcus *ga*. Ergo proportio sinus arcus *ae* prime ad sinum arcus *ga* secunde est composita ex proportione sinus arcus *eb* tertie ad sinum arcus *bz* quarte et ex proportione sinus arcus *zd* quinte ad sinum arcus *dg* sexte. Cum ergo converterimus, erit proportio secunde que est sinus arcus *ga* ad sinum arcus *ae* prime composita ex proportione sinus arcus *zb* quarte ad sinum arcus *be* tertie et ex proportione sinus arcus *gd* sexte ad sinum arcus *dz* quinte. Et illud est quod demonstrare voluimus.

Et ponam ut linea *ha* equidistet linee *ge*. [*Figura #6*] Dico ergo quod similiter equidistat linea *gz* linee *hd* et linea *ez* linee *hb*, quod ideo est quoniam cum prime due linee *ge* et *ha* sint equidistantes, tunc diverse superficies in quibus ipse sunt equidistant et omnes linee que sunt in illis diversis superficiebus similiter equidistant. Quare omnes predicte linee equidistant, licet equidistare non possint nisi sint in eadem superficie quecumque illarum duarum, ut *ha* et *eg* in superficie circuli *aeg*, et similiter *gz* et *hd* in superficie circuli *gzd*, et sic de aliis. Et inter duos quidem arcus *gea* et *bda* secant se duo arcus *be* et *gd* supra punctum *z*. Ergo proportio sinus arcus *ga* ad sinum arcus *ae* componitur ex proportione sinus arcus *gd* ad sinum arcus *dz* et ex proportione sinus arcus *zb* ad sinum arcus *be.* Proportio vero sinus arcus *gd* ad sinum arcus *dz* est unum. Et proportio sinus arcus *zb* ad sinum arcus *be* similiter est unum eo quod omnes sinus sint inter equidistantes lineas. Quare sinus unius arcus est sinus alterius. Et similiter sinus arcus *ga* et sinus arcus *ae* est idem propter eandem rationem. Quapropter verum est quod premisimus scilicet quod proportio sinus | arcus *ga* ad sinum arcus *ae* est A 6rb composita ex illis aliis proportionibus quia ex omnibus eadem provenit proportio, et illud est quod demonstrare voluimus.

Hic vero notandum est quod ubicumque posui proportionem sinus arcus ad sinum arcus, debet esse proportio corde dupli arcus ad cordam

201 est] *om.* A **204** prime due] *inv.* B **213** vero] ergo B **215** inter] *add. mg. a. m.* A dupli arcus. His enim proportionibus utitur Ptolomeus et Thebit in figura sectore. Ego vero ideo apposui illud quia eadem est proportio sinus arcus ad sinum arcus, que corde dupli arcus ad cordam dupli arcus cum corda dupli arcus sit dupla sinus arcus. Dupli namque ad duplum eadem est proportio que medii ad medium. Et hoc facile probari potest. Item notandum est quod illud quod diximus de repetitione huius sermonis ‘et sint arcus minores semicirculo,’ alia de causa dictum intelligi potest, scilicet ut minor portio minori comparetur et non maiori, nec maior minori. Et est sciendum quod sicut minores arcus se secant inter duos arcus ita, et maiores arcus qui supersunt et cum illis parvis complent circulos, secant se ab alia parte inter illos duos arcus. Sed quia duplari non possunt, ideo non sunt necesarii ad figuram sectorem.

Nunc vero de variatione proportionis sex quantitatum agendum est. Et est sciendum quod ex combinatione sex quantitatum que omnes sint diverse in processione quindecim proveniunt combinationes, videlicet ex prima cum reliquis quinque et ex secunda cum reliquis iiii et ex tertia cum reliquis tres et ex quarta cum reliquis due et ex quinta cum ultima, id est sexta una. Et ita omnes fiunt quindecim. Et totidem proveniunt in conversione. Et cum dico ‘cum reliquis,’ intelligo quantitates sequentes illam a qua incipio, ut si a secunda intelligo tertiam et quartam et quintam et sextam et sic in aliis. Et est sciendum quod istarum quindecim sex probantur esse impossibiles, et nonem possibiles. Item cuiusque combinationis proportio componitur duodecim modis de quibus duo tantum veri sunt et decem falsi.

Quod ut manifestum fiat, ponamus sex quantitates quas ponit Thebit supra quas sint *a b g d e u* et sit prima *a* secunda *b* tertia *g* quarta *d* quinta *e* sexta *u*. Dico ergo quod proportio prime ad secundam componitur ex proportionibus reliquarum quattuor duodecim modis quia componitur ex proportione tertie ad quartam et proportione quinte ad sextam; ecce unus. Et ex proportione tertie ad quartam et sexte ad quintam; ecce duo. Et ex proportione quarte ad tertiam et quinte ad sextam; ecce tres. Et ex proportione quarte ad tertiam et sexte ad quintam; ecce quattuor. Et iterum ex proportione tertie ad quintam et quarte ad sextam; ecce quinque. Et proportione tertie ad quintam | et sexte ad quartam; ecce sex. B 4va
Et proportione quinte ad tertiam et quarte ad sextam; ecce septem. Et proportione quinte ad tertiam et sexte ad quartam; ecce octo. Et iterum ex proportione tertie ad sextam et quarte ad quintam; ecce nonem. Et proportione tertie ad sextam et quinte ad quartam; ecce decem. Et ex

223 Thebit] *illeg.* B **229** potest] ponitur B **230** minor] maior B | comparetur] comparatur B **238** ex[1]] *om.* B **240** fiunt] faciunt B **248** e] *add. mg. a. m.* A **256** tertie] *om.* B proportione sexte ad tertiam et quarte ad quintam; ecce undecim. Et proportione sexte ad tertiam et quinte ad quartam; ecce duodecim.

Et licet sint duodecim modi, duo tantum sunt veri. Unde cum de quindecim combinationibus nonem tantum sunt possibiles ut diximus et sex impossibiles et de nonem possibilibus proveniant 108 modi cum ex unaquaque proveniant duodecim. Decem et octo tantum sunt veri, reliqui vero scilicet nonaginta falsi.

Illos vero decem et octo ponit Thebit et probat, quorum primus non probatur quia manifestus est et est radix omnium. Secundus vero probatur hoc modo. Proportio prime ad secundam, id est *a* ad *b*, componitur ex proportione tertie ad sextam, id est *g* ad *u*, et ex proportione quinte ad quartam, id est *e* ad *d*. Sed ad hoc probandum, ponende sunt *d* et *e* medie inter *g* et *u*. Et cum dicitur 'ponuntur medie' intelligendum est ut proportio prime illarum inter quas ille existunt medie sit composita ex proportione illius prime ad illam que est post ipsam et illius ad aliam et sic deinceps in ordine usquoque perveniatur ad ultimam. Et hoc est ponere quantitatem vel quantitates medias in proportione inter alias. Proportio igitur *g* ad *u* est composita ex proportione *g* ad *d* et ex proportione | *d* ad *e* et proportione *e* ad *u*. Ergo proportio *g* ad *u* et A 6va
proportio *e* ad *d* est proportio composita ex proportione *g* ad *d* et proportione *d* ad *e* et proportione *e* ad *u* et proportione *e* ad *d*. Sed proportio composita ex proportione *g* ad *d* et ex proportione *d* ad *e* et ex proportione *e* ad *d* est proportio *g* ad *d*. Quod ideo est quoniam proportio *d* ad *e* et proportio *e* ad *d* est unum. Ergo cum multiplicetur proportio *g* ad *d* in proportione *d* ad *e* et *e* ad *d* que est unum, non provenit nisi proportio *g* ad *d*. Quod autem proportio *d* ad *e* et *e* ad *d* sit una sic probatur. Si aliqua quantitas per quantitatem aliam dividatur et quod provenit servetur et postea dividens quantitas per divisam dividatur et quod inde provenit servetur, deinde primum servatum in servatum secundum ducatur, non proveniet nisi unum. Verbi gratia, sint due quantitates *a* et *b* et dividatur *a* per *b* et proveniat *g*. Deinde dividatur *b* per *a* et proveniat *d*. Dico ergo quod si multiplicetur *g* in *d*, non proveniet nisi unum. Quod sic probatur. Si enim *g* multiplicetur in *b*, provenit *a*. Et si unum multiplicetur in *a*, provenit *a*. Sunt igitur quattuor quantitates prima *g*, secunda unum, tertia *a* et quarta *b*. Et quod fit ex prima id est *g* in quartam id est *b* equum est ei quod fit ex secunda id est unum in tertiam id est *a*. Ergo proportio prime ad secundam est sicut proportio

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263 sint duodecim] *inv.* B | modi] *add. marg. a. m.* A | duo] tamen *add.* B **264** sunt] sint B **269** manifestus] manifestius *sed. corr.* A **273** est] *om.* A **277** proportione] portione B **279** et[2]] erit B **294** provenit a] *add. mg. a. m.* A tertie ad quartam. Item si *d* multiplicetur in *a*, provenit *b*, et si unum multiplicetur in *b*, provenit *b*. Sunt igitur quantitates prima *d* et quarta *a* et secunda unum et tertiam *b*, et quod fit ex prima in quartam equum est ei quod fit ex secunda in tertiam. Ergo proportio *d* prime ad secundam que est unum est sicut proportio *b* tertie ad quartam *a*. Si ergo convertatur proportio, erit proportio quarte *a* ad tertiam *b* sicut proportio secunde que est unum ad primam *d*. Et iam fuit proportio *g* ad unum sicut *a* ad *b*. Sublatis ergo mediis *a* et *b* remanet proportio *g* prime ad secundam que est unum sicut proportio tertie que est unum ad quartam que est *d*. Ergo quod fit ex ductu prime in quartam equum est ei quod fit ex ductu secunde in tertiam. Sed ex ductu secunde in tertiam non fit nisi unum, ergo ex ductu prime in quartam non fit nisi unum. Ergo si aliqua quantitas per aliquam dividatur et dividens per divisam et quod provenit ex prima divisione in id quod provenit ex secunda multiplicetur, non proveniet nisi unum. Et illud est quod demonstrare voluimus. Ergo proportio composita ex proportione *g* ad *u* et ex proportione *e* ad *d* est proportio composita ex proportione *g* ad *d* et ex proportione *e* ad *u*. Proportio vero *a* ad *b* iam fuit composita ex proportione *g* ad *d* et ex proportione *e* ad *u*, ergo proportio *a* ad *b* ex proportione *g* ad *u* et ex proportione *e* ad *d*.

Modus vero tertius et modus quartus faciles sunt et ita probantur ut sunt in libro. In modo autem quinto in probatione, dicit proportionem *a* ad *b* compositam esse ex proportione *e* ad *u* et ex proportione *g* ad *d* preponendo *e* ad *u*. Quod ideo fecit quia necesarium erat ei ad probandum illud quod volebat.

In sexto vero modo nichil est dicendum nisi quod est in libro et est sciendum quod in modo quinto cum in quarto egisset de proportione prime ad tertiam pretermissa proportione prime ad quartam eo quod sit impossibile ut ipse ostendet. In sequentibus transivit ad proportionem prime ad quintam.

In modo item septimo interrupit ordinem cum enim in precedentibus modis tractaverit de proportione prime ad eas ad quas possibilis est eius proportio. Pretermissa proportione secunde ad alias, transivit ad proportionem tertie ad alias. Quod ideo fecit quia necessarium fuit ei ad probationem sequentium modorum ut ipsemet dicit. Et in septimo quidem ubi dicit, 'proportio autem *g* ad *d* est composita ex proportione *g* ad *z* et ex proportione *z* ad *d*,' intelligendum est quod sicut posuit proportionem *d* ad *z* ut proportionem *e* ad *u*, ita e converso est proportio *z* ad *d* sicut

329 est] erit B

333 'proportio…334 d'] Björnbo, p. 17, lines 11-3.

proportio *u* ad *e*. Quare cum ipse iam | probaverit quod proportio *g* ad *z* A 6vb sit proportio composita ex proportione *g* ad *d* et proportione *e* ad *u* que est proportio *a* ad *b*, sequitur tunc posita *z* media inter *g* et *d* quod proportio *g* ad *d* sit composita ex proportione *g* ad *z* que est proportio *a* ad *b* et ex proportione *z* ad *d* que est proportio *u* ad *e*.

In modo similiter octavo interrupit ordinem transiendo ad proportionem quinte ad sextam propter eandem causam quam in septimo diximus. Et ut in quinto ita hic dicit quod proportio *a* ad *b* est etiam composita ex proportione *e* ad *u* et ex proportione *g* ad *d*. Quod ideo facit quia necessarium fuit ei ad ordinandas quantitates in proportione ad hoc ut probet quod vult. Et in eo nichil aliud est dicendum nisi quod est in libro.

In modo autem nono redit ad ordinem ut ipsemet dicit tractando de proportione secunde ad eas ad quas est possibilis. Probatio vero ipsius est sicut est in libro. In decimo vero modo nichil est dicendum nisi quod est in libro. In undecimo itidem et 12^{o} et 13^{o} nichil est dicendum nisi quod est in libro, et similiter in quartodecimo et similiter in 15^{o}.

In sextodecimo | vero cum dicit quod proportio *d* ad *e* componitur B 4vb ex proportione *d* ad *g* et ex proportione *g* ad *u* et ex proportione *u* ad *e*, intelligendum est quod *g* et *u* sint medie inter *d* et *e*. Proportio vero composita ex proportione *d* ad *g* et ex proportione *u* ad *e* est sicut proportio *b* ad *a*. Hoc secundum conversionem. Sequitur ergo propositum.

In modo vero septimodecimo et in modo 18 nichil est dicendum nisi quod est in libro.

His expletis [Thabit] ponit quedam que facilia sunt de tabulis quas fecit et aliis que facilia sunt usque ad locum in quo probat quod non est necesse ut proportio prime *a* ad quartam *d* sit composita ex duabus proportionibus que sunt inter reliquas quattuor quantitates quocumque modo sumantur, directe vel conversim. Quod quidem probatur ex 25^{a} theoremate sexti libri Euclidis in quo date superficiei similem aliique proposite superficiem equalem designare iubet. Quapropter cum dicit 'ponam lineam *z* etc.' et 'ponam superficiem orthogoniam equalem ei quod est ex *a* in *d* etc.,' intellexit ut sit superficies ex *a* in *d* proposita et fiat ei superficies equalis scilicet illa quam ponit scilicet *ht* que sit similis superficiei que fit ex *z* in *d*. Probatio autem quam ponit ista est. Proportio

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338 posita] composita B **351** itidem] idem B | 13o] et *add.* B **352** similiter[2]] *om.* B **360** est] *om.* B **362** fecit] facit B **364** quocumque] vel *add. sed. exp.* A **369** in d[2]] inde B **370** similis] simul B

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368 'ponam[1]…z] Björnbo, p. 19, line 14. || 'ponam[2]…369 etc] Björnbo, p. 19, lines 14-5.

prime *a* sex quantitatum predictarum ad quartam earum *d* est composita ex duabus proportionibus reliquarum quattuor. Et quod fit ex *a* in [*d*] equum est ei quod fit ex *h* in *t*. Posita igitur *h* prima et *t* quarta, tunc proportio *h* ad *a* est sicut proportio *d* ad *t*. Proportio vero *a* ad *b* componitur ex proportione *g* ad *d* et ex proportione *e* ad *u*. Ergo proportio composita ex proportione *h* ad *a* et ex proportione *a* ad *b* que est sicut proportio *h* ad *b*, est equalis proportioni composite ex proportione *d* ad *t* que est proportio *h* ad *a* et ex proportione *g* ad *d* et ex proportione *e* ad *u*. Est ergo proportio *h* ad *b* equalis proportioni composite ex proportione *g* ad *t*, que est proportio composita ex proportione *g* ad *d* et ex proportione *d* ad *t*, et ex proportione *e* ad *u*. Quattuor igitur harum sex quantitatum scilicet *h b g t e u* scilicet *b g e u* sunt quattuor sex primarum scilicet *a b g d e u* et non differunt nisi in primis et quartis. Si ergo esset modus aliquis necessitatis ut proportio prime omnium sex quantitatum ad quartam earum esset composita ex eisdem duabus proportionibus reliquarum quattuor, et ille quattuor relique sex primarum essent quattuor relique sex postremarum. Tunc hic esset proportio [prime] sex primarum que est *a* ad quartam earum que est *d* sicut proportio prime sex postremarum que est *h* ad quartam earum que est *t*. Sed positum est quod proportio *h* ad *t* est sicut proportio *z* ad *d*. Ergo proportio *a* ad *d* est sicut proportio *z* ad *d*. Ergo *z* est equalis *a*. Secundum positionem vero una earum fuit longior altera, quod quidem est contrarium et impossibile. Non est igitur hic modus necessitatis aliquis per quem | sit necesse ut sit A 7ra
proportio prime ad quartam composita ex eisdem duabus proportionibus inter reliquas quatuor.

'Et dico quod non sunt hic modi diversi per quos fiat proportio prime ad quartam aliquando et in quibusdam quantitatibus composita ex duabus proportionibus inter reliquas quattuor et in quibusdam earum ex aliis duabus proportionibus earum.' Quod ideo dicit quia aliquis posset dicere, 'non est necesse ut una earum sit equalis alii sicut diximus de *z* et *a*, quoniam proportio unius prime ad quartam componitur uno modo ex duabus proportionibus inter reliquas quattuor et proportio alterius prime ad quartam componitur alio modo ex duabus proportionibus inter reliquas quattuor.' Quod ipse removet dicens, 'manifestum est etc.'

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372 predictarum] predicatur B **373** d] *b* AB **378** proportioni] proportionis A **380** proportioni] proportionem *sed. corr.* A **384** et[2]] *om.* B **391** Ergo…392 d] *om. (hom.)* B **401** ut] *om.* B

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397 Et…400 earum] Björnbo, p. 19 line 37 to p. 20 line 3. **405** 'manifestum…etc] Björnbo, p. 20, lines 3-4.

‘Numerum diffinitum comprehensum,’ id est duodenarium sicut prius diximus quod proportio inter quantitates duodecim componitur modis de quibus duo tantum sunt veri et decem falsi et impossibiles ut in proximo patebit.

'De lineis vero etc.’ Hoc sic intelligendum est: numerus quo componitur proportio inter quattuor quantitates non est nisi duodenarius, sed numerus quo possumus ponere lineas proportionales ad quartam ut *z* posuimus ad *d* diversas a prima ut ab *a* potest esse maior quam duodecim quia possumus ponere tredecim lineas ut posuimus lineam *z*. Et possumus ostendere quod proportio uniuscuiusque illarum ad *d* est composita ex duabus proportionibus reliquarum quattuor. Et quoniam invenimus lineas que taliter possunt proportionari ad *d* ut *z* plures numero quam sit numerus modorum qui aggregantur ex duabus proportionibus quattuor quantitatum, tunc necesse est ut sit proportio alicuius vel aliquarum de illis lineis que proportionantur ad *d* sicut *z* ad *d* sicut proportio alterius linee de eis ad lineam *d*, quoniam ipsa componitur ex eisdem duabus proportionibus iteratis, id est ex proportionibus quantitatum quattuor. Quod ideo contingit quoniam cum non sint nisi duodecim modi compositionis, tunc necesse est ut proportio que est tertiedecime linee ad *d* cum componatur ex duabus proportionibus illarum quattuor sit una illarum duodecim. Ergo oportebit ut illa tertiadecima linea et una de duodecim precedentibus sint equales cum ambarum proportio ad *d* sit una. Sed secundum positionem omnes fuerunt diverse. Hoc autem contrarium est.

'Non igitur oportet etc.’ Et sicut ostendimus in hac combinatione ita possumus ostendere in reliquis quinque combinationibus per id quod simile est isti, id est faciendo figuram quadratam ut fecimus et ponendo sex lineas et aliam ut *z* proportionalem illi cuius combinationem cum prima id est *a* inprobare volumus ut *u* et sic de ceteris, et ostendendo omnia que ostendimus in *a d*, aut quia ostensum est illud in prima et quarta, id est ponemus unam illarum quartam et aliam primam. Et probabimus in eis sicut probavimus in *a d* mutando solummodo litteras. Et hoc intendit Thebit cum dicit 'quoniam si esset neccessarium in *a d* etc.’ Erunt ergo tunc *a d* loco prime et sexte. Ecce per exempla ostendit se intendisse illud quod diximus scilicet ut una manente prima et alia

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417 numero] *add. mg. a. m.* A **423** cum] *add. supr. lin. a. m.* A **428** positionem] dispositionem *sed. corr.* A **432** faciendo] facientibus B | fecimus] facimus B **434** id…a] *add. mg. a. m.* A

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406 Numerum…comprehensum] Björnbo, p. 20, line 6. **410** 'De…etc] Björnbo, p. 20, line 6. **430** 'Non…etc] Björnbo, p. 20, line 7. **438** 'quoniam…439 etc] Björnbo, p. 21, lines 16-7.

illarum que cum ea combinari non possunt, quarta ponatur *d* in loco eius ut *u*, ponatur quarta et *d* sexta. Et ita in secunda et tertia ponendo *b* primam et *a* secundam et *g* quartam et *d* tertiam. Et similiter in aliis.

'Nos autem iam ostendimus etc.' In hoc loco intendit Thebit demonstrare quod dixi superius scilicet quod licet proportio alicuius combinationis proveniat ex compositione quattuor quantitatum reliquarum duodecim modis, tamen non sunt possibiles in unaquaque combinatione possibilium que nonem sunt, tantum ut predictum est nisi duo modi. Quod ut manifestius fiat ostendamus hic in combinatione prima que constat ex *a b* cuius proportio componitur ex proportione tertie id est *g* ad quartam id est *d* et ex proportione | quinte id est *e* ad sextam id B 5ra est *u*. Dico autem quod non componitur tertie *g* ad quartam *d* et ex proportione sexte [*u*] ad quintam *e*. Quod sic probatur. Cum enim proportio *a* ad *b* sit composita ex proportione *g* ad *d* et ex proportione *e* ad *u*, tunc si componeretur ex proportione *g* | ad *d* et ex proportione *u* ad A 7rb *e*, esset proportio *u* ad *e* et proportio *e* ad *u* eadem. Quare *e* et *u* essent equales, quod esset inconveniens posito prius quod omnes ille sex quantitates *a b g d e u* sint diverse ut Thebit ponit.

Et dico quod non componitur ex proportione quarte *d* ad tertiam *g* et ex proportione quinte *e* ad sextam *u* quoniam si componeretur ex his duabus proportionibus cum iam composita sit ex proportione *g* ad *d* et *e* ad *u*, sequeretur quod proportio *g* ad *d* et proportio *d* ad *g* esset eadem. Quare essent *g* et *d* equales quod esset contrarium et inconveniens cum sint diverse.

Et dico iterum quod non componitur ex proportione quarte *d* ad tertiam *g* et ex proportione sexte *u* ad quintam *e* quoniam cum proportio *b* ad *a* que est conversa proportionis *a* ad *b* componatur ex proportione *d* ad *g* et ex proportione *u* ad *e*, si proportio *a* ad *b* componeretur ex eisdem proportionibus, esset proportio *a* ad *b* et proportio *b* ad *a* eadem. Quamobrem *a* et *b* essent equales, quod esset contrarium et impossibile cum sint diverse.

Et dico quod nullo modo componitur proportio *a* ad *b* ex proportione tertie *g* ad quintam *e* et ex proportione quarte *d* ad sextam *u*. Quoniam si componeretur ex eis, sequeretur quod proportio composita ex proportione *g* ad *d* et ex proportione *e* ad *u* esset equalis proportioni composite ex proportione *g* ad *e* et ex proportione *d* ad *u*. Proportio vero

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441 cum] *iter.* B **442** d] terciam et similiter in aliis *add. sed. del.* B **453** u] *d* AB **456** esset…e[2]] *add. supr. lin. a. m.* A **461** iam] sint *add.* B | sit] *om.* B **468** eisdem] isdem B **473** quarte] *om.* B

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444 'Nos…etc] Björnbo, p. 22, line 1.

composita ex proportione *g* ad *e* et ex proportione *d* ad *u* est composita ex proportione *g* ad *d* et ex proportione *d* ad *e* bis et ex proportione *e* ad *u*. Ergo proportio composita ex duabus proportionibus tantum que sunt *g* ad *d* et *e* ad *u* est equalis proportioni composite ex eisdem duabus et ex proportione *d* ad *e* bis, quod tantum valet quantum si proportio composita ex proportione *g* ad *d* et ex proportione *e* ad *u* componatur ex proportione *d* ad *e* bis vel multiplicetur in eam bis, verum ipsa bis accepta est eadem. Quare tantum valet multiplicare illam in eam bis quantum multiplicare eam in unum. Et quia multiplicare ipsam in unum non est diversificare ipsam quia non provenit aliud, tunc multiplicatio illa superflua est. Et quia superflua est, tunc non est verum quod proportio *a* ad *b* componatur ex proportione *g* ad *e* et ex proportione *d* ad *u*. Quod autem istud ita sit videri potest ex numeris. Ponamus quattuor numeros 2 3 4 6. Si ergo multiplicemus 2 in 3, provenient 6, et 4 in 6, 24. Si ergo multiplicemus 6 qui provenit ex multiplicatione 2 in 3 in 24 qui provenit ex multiplicatione 4 in 6, provenient 144 quod est idem cum eo quod provenit ex multiplicatione 3 in 4 id est 12 in se. Similiter ergo multiplicare proportionem *g* ad *d* in proportione *e* ad *u* debet esse idem quod multiplicare proportionem *d* ad *e* in proportione *d* ad *e*. Sed ex hac multiplicatione non provenit nisi unum, ergo neque ex illa multiplicatione provenit nisi unum quod est inconveniens vel ut superius ostendimus est superflua, compositio proportionis *d* ad *e* bis cum ex multiplicatione proportionis composite ex proportione *g* ad *d* et ex proportione *e* ad *u* non proveniat alia proportio nisi ipsa.

Et dico quod non componitur ex proportione *g* ad *e* et ex proportione *u* ad *d* quoniam componeretur ex proportionibus ex quibus iste due componuntur scilicet ex proportione *g* ad *d* et ex proportione *d* ad *e* et ex proportione *u* ad *e* et ex proportione *e* ad *d*. Et sic proportio composita ex proportione *g* ad *d* et ex proportione *e* ad *u* esset equalis proportioni composite ex proportione *g* ad *d* et ex proportione *d* ad *e* et ex proportione *u* ad *e* et ex proportione *e* ad *d*. Sed proportio *d* ad *e* et proportio *e* ad *d* est unum. Ergo sublata ea remanet proportio composita ex proportione *g* ad *d* et ex proportione *e* ad *u* equalis proportioni

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478 ex[1]] *om.* B **480** eisdem] isdem B **486** multiplicatio] est *add. sed. del.* B **488** u] Totum istud superfluum est verum quod ita sit potest ostendi ex proportione istorum numerorum 2 3 3 6, quoniam sicut apparet proportio composita ex proportione 2 ad 3 et ex proportione 3 ad 6. Si multiplicetur in proportionem 3 ad 3, non provenit alia proportio quia equales sunt 3 3. Ita oporteret ut predicte quantitates *g* et *d* essent equales ad hoc ut illud esset quod dicitur. *adnot. mg. a. m.* A **491** ex[2]…493 provenit] *add. mg. a. m.* A **497** ut] *add. supr. lin. a. m.* A **503** g…504 proportione[2]] *add. mg. a. m.* A composite ex proportione *g* ad *d* et ex proportione *u* ad *e*. Ergo proportio *e* ad *u* et proportio *u* ad *e* est una. Ergo *u* et *e* sunt equales quod est impossibile.

Et similiter dico quod non componitur ex proportione *e* ad *g* et ex proportione *d* ad *u* quoniam componeretur ex proportione *e* ad *d* et proportione *d* ad *g* et ex proportione *d* ad *e* et ex proportione *e* ad *u*. Et sublata proportione *e* ad *d* et proportione *d* ad *e* que est unum, remaneret | A 7va proportio composita ex proportione *g* ad *d* et proportione *e* ad *u* equalis proportioni composite ex proportione *d* ad *g* et ex proportione *e* ad *u*. Quare esset proportio *g* ad *d* et proportio *d* ad *g* eadem. Quare *g* et *d* essent equales quod esset impossibile.

Et similiter dico quod non componitur ex proportione *e* ad *g* et ex proportione *u* ad *d*. Quod sic probatur. Si componitur ex eis, tunc componitur ex proportionibus ex quibus ille componuntur scilicet ex proportione *e* ad *d* et ex proportione *d* ad *g* et ex proportione *u* ad *e* et ex proportione *e* ad *d*. Et ipsa componitur ex proportione *g* ad *d* et proportione *e* ad *u*. Ergo proportio composita ex istis duabus est equalis proportioni composite ex illis quatuor. Sed proportio composita ex proportione *u* ad *e* et proportione *e* ad *d* et proportione *d* ad *g* est equalis proportioni *u* ad *g*. Ergo proportio composita ex proportione *g* ad *d* et proportione *e* ad *u* est equalis proportioni *u* ad *g* et proportioni *e* ad *d*. Ponam autem proportionem *d* ad *e* communem cum utrisque. Proportio igitur composita ex proportione *g* ad *d* et proportione *d* ad *e* et proportione *e* ad *u* est equalis proportioni composite ex proportione *u* ad *g* et proportione *e* ad *d* et proportione *d* ad *e*. Sed proportio composita ex proportione *g* ad *d* et proportione *d* ad *e* et proportione *e* ad *u* est equalis proportioni *g* ad *u*. Sublata igitur proportione *d* ad *e* et proportione *e* ad *d* que est unum, remanet proportio *u* ad *g* equalis proportioni *g* ad *u*. Quare *g* et *u* sunt equales quod est contrarium et inconveniens cum ipse sint diverse. Non igitur proportio *a* ad *b* componitur ex proportione *e* ad *g* et proportione *u* ad *d*.

Dico autem quod ipsa componitur ex proportione *g* ad *u* et proportione *e* ad *d*, et hoc superius probatum est, verum non componitur ex proportione *g* ad *u* et proportione *d* ad *e* quoniam si hoc esset, tunc proportio *e* ad *d* et proportio *d* ad *e* esset eadem. Et sic *d* et *e* essent equales quod esset contrarium et impossibile.

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510 ex[2]] *om.* A **515** ex[2]] *add. mg. a. m.* A *om.* B **517** equalis…518 u] *add. mg. a. m.* A **518** composite] composito A | et ex] *om.* B **524** ex[3]] *om.* A **531** proportionem] portionem B **536** Sublata] Non aufertur proportio *d* ad *e* ideo quod communis sit sed quia proportio *d* ad *e* et proportio *e* ad *d* est unum ut dictum est *adnot. mg. a. m.* A

Et similiter non componitur ex proportione *u* ad *g* et proportione *d* ad *e* quoniam cum hec proportio composita sit conversa proportionis composite ex proportione *g* ad *u* et proportione *e* ad *d* et sit illa ex qua componitur proportio *b* ad *a.* Tunc si proportio *a* ad *b* componeretur ex | B 5rb eadem, esset proportio *a* ad *b* et proportio *b* ad *a* eadem, et ita essent *a* et *b* equales, quod esset contrarium et impossibile. Manifestum est igitur quod huius combinationis, proportio licet duodecim modis componatur ex proportionibus reliquarum quattuor, non sunt tamen possibiles et veri nisi duo. Et hoc totum innuit Thebit, sed implicite ut in libro habetur.

His ita se habentibus nunc redeundum est ad illud quod dicit Thebit in principio sui libri scilicet quod divisiones huius figure que proveniunt secundum duos modos divisionis et compositionis sunt xxx de quibus sunt vere xvi tantum et impossibiles et false xiiii. Sciendum est itaque quod secundum modum divisionis, proveniunt tres divisiones, tantum quarum unam posuit Ptolomeus et probavit. Et Thebit probavit illam eandem et alias duas que omnes vere sunt et necessarie. Oportet ergo ut demonstremus qualiter secundum modum compositionis proveniant divisiones 27 et quod earum 14 sunt impossibiles et 13 vere.

Compleatur ergo circulus *adb* et erigatur super ipsum semicirculus cuius pars est arcus *gea* et arcus *gd* et arcus *be.* [*Figura #7*] Et secet arcus *gea* circulum *adb* supra punctum *a* ut est in figura Ptolomei, et ab alia parte super aliud punctum cum factus sit semicirculus, et arcus *gd* supra punctum *d* et arcus *be* supra punctum *b* ab una parte ut est in figura et ab alia parte super aliud punctum cum sit semicirculus. Deinde producantur diametri horum trium semicirculorum scilicet *gea* et *gd* et *be* in superficie circuli *adb,* et producantur ille tres diametri utrinque extra circumferentiam circuli *adb* in infinitum. Deinde protrahantur a puncto *g* due linee recte quarum una perveniat ad punctum *e* et alia ad punctum *z* et alia | a puncto *z* ad punctum *e.* Dico ergo quod cum iste linee A 7vb protrahentur in infinitum aut coniungentur omnes cum suis diametris aut erunt eis equidistantes aut quedam erunt equidistantes et quedam coniungentur et iterum aut coniungentur ab una parte aut a diversis.

548 sit] sic B | ex[2]] est B **549** ex] eorum B **550** proportio[1]] *b* ad *a add. sed. del.* B **555** ita] itaque B | illud] aliud B **557** modos] scilicet *add.* B **558** itaque] itemque B **561** et[2]] *om.* B **564** super] supra B **567** super] cum B **568** in figura] nisi *ga* B **570** gea] *ga* B **575** coniungentur] protrahentur B

551 impossibile] He neglects to treat the twelfth mode. It could be disproved as most of the others are. Assume that it is true, i.e. that *a* to *b* is composed of *u* to *g* and *e* to *d*. From the tenth mode, which is valid, *a* to *b* must also be composed of *g* to *u* and *e* to *d*, so *g* to *u* is as *u* to *g*. Therefore, *u* equals *g*, which is absurd.

Ponamus ergo lineam [*ge*] primam et lineam *gz* secundam et lineam *ze* tertiam. Dico ergo quod linea [*ge*] coniuncta sue diametro a parte *a*, proveniuntur divisiones 9 quoniam potest esse ut ea coniuncta sic, sit linea *gz* coniuncta ab eadem parte sue diametro et sit linea *ze* coniuncta sue diametro ab eadem parte et ab alia parte et equidistans. Et sic sunt tres divisiones. Et eadem prima coniuncta ab eadem parte sit linea *gz* equidistans et linea *ze* coniungatur ab eadem parte et ab alia et equidistans sicut prius. Et erunt tres alie divisiones. Et iterum eadem prima ab eadem parte coniuncta sit coniuncta linea *gz* sue diametro ab alia parte et linea *ze* coniungatur utrinque et sit equidistans. Et provenient tres alie divisiones. Quare linea prima coniuncta a parte *a* sue diametro, proveniunt divisiones nonem. Eadem quoque equidistante sue diametro, provenient nonem alie quia poterit coniungi linea *gz* a parte *a*, et provenient tres divisiones eo quod linea *ze* poterit coniungi utrinque et esse equidistans, et poterit coniungi linea *gz* ab alia parte, et linea *ze* coniungetur tribus sicut prius, et provenient tres alie. Et poterit esse equidistans linea *gz* et provenient tres alie linea *ze* tribus modis se habente ut in precedentibus. Et ita linea [*ge*] equistante provenient nonem divisiones alie. Similiter quoque linea [*ge*] coniuncta sue diametro ab alia parte, provenient nonem divisiones alie eo quod linea *gz* potest coniungi utrinque et esse equidistans linea *ze* se habente tribus modis ut in precedentibus. Ecce nunc patet quod secundum modum compositionis proveniunt divisiones 27.

Videramus ergo quot earum vere sint et quot false. Prima igitur que est cum omnes tres coniunguntur suis diametris a parte *a* vera est. Et sic probatur. Sit itaque dispositio figure ut positum est et sit linea in qua coniunguntur linee diametris linea *thk*. [*Figura #8*] Et coniungatur linea *gz* diametro super punctum *t* et linea [*ge*] diametro super punctum *h* et linea *ze* diametro super punctum *k*. Inter duas igitur lineas *gzt* et *thk* secant se due linee *geh* et *zek* supra punctum *e*. Ergo proportio *gzt* ad *tz* componitur ex proportione linee *geh* ad lineam *he* et ex proportione *ke* ad *kez*. At proportio sinus arcus *gzd* ad sinum arcus *dz* est sicut proportio linee *gzt* ad lineam *tz*. Et proportio sinus arcus *gea* ad sinum arcus *ae* est sicut proportio linee *geh* ad lineam *he*. Et proportio sinus arcus *le* ad sinum arcus *lez* est sicut proportio linee *ke* ad lineam *kez*. Ergo proportio sinus arcus *gzd* ad sinum arcus *dz* componitur ex proportione sinus arcus

578 ge] *ga* AB *add. mg. a. m.* A **579** ge] *ga* AB **582** sue] suo B **585** tres alie] *inv.* B **588** a[1]] *mg. a. m.* A **595** ge] *ga* AB *ge add. mg. a. m.* A **596** ge] *ga* AB **605** ge] *ga* AB *ge add. mg. a. m.* A **606** thk] *ghk* B **609** sinum] *add. mg. a. m.* A **613** dz] *gz* B *gea* ad sinum arcus *ae* et ex proportione sinus arcus *le* ad sinum arcus *lez*. Sed sinus arcus *le* est sinus arcus *ezb*. Et sinus arcus *lez* est sinus arcus *zb*. Ergo proportio sinus arcus *gd* ad sinum arcus [*dz*] componitur ex proportione sinus arcus *be* ad sinum arcus *bz* et ex proportione sinus arcus *ga* ad sinum arcus *ae*. Posito igitur quantitate prima sinu arcus *gd* et secunda sinu arcus *dz* et tertia sinu arcus *bz* et quarta sinu arcus *be* et quinta sinu arcus *ga* et sexta sinu arcus *ae*, erit secundum octavum modum proportio quinte que est sinus arcus *ga* ad sextam que est sinus arcus *ae* composita ex proportione prime que est sinus arcus *gd* ad secundam que est sinus arcus *dz* et ex proportione tertie que est sinus arcus *bz* ad quartam que est sinus arcus *be.*

Secunda divisio est ut linea *ze* iungatur diametro sue ab alia parte. [*Figura #9*] Et tunc inter duas lineas *kth* et *geh* secant se due linee *gzt* | et A 8ra *kze*. Ergo proportio linee *gh* ad *he* est composita ex proportione linee *gt* ad lineam *tz* et ex proportione linee *kz* ad lineam *ke*. At proportio linee *gh* ad lineam *he* est sicut proportio sinus arcus *ga* ad sinum arcus *ae*. Et proportio linee *gt* ad lineam *tz* est sicut proportio sinus arcus *gd* ad sinum arcus *dz*. Et proportio linee *kz* ad lineam *ke* est sicut proportio sinus arcus *bz* ad sinum arcus *be.* Ergo proportio sinus arcus *ga* ad sinum arcus *ae* est composita ex proportione sinus arcus *gd* ad sinum arcus *dz* et ex proportione sinus arcus *bz* ad sinum arcus *be.*

Tertia divisio est ut prima linea que est *ge* et secunda que est *gz* suis diametris sint iuncte a parte *a* et tertia que est *ze* sit equidistans sue diametro. [*Figura #10*] Quare erit equidistans linee *ht* que equidistat diametro. Cum ergo trianguli *ght* duo latera secet linea *ze* equidistans basi *ht*, tunc latera eius sunt proportionalia. Ergo proportio linee | *ge* ad B 5va lineam *eh* est sicut proportio linee *gz* ad lineam *zt*. Cum ergo composuerimus, erit proportio linee *gh* ad lineam *he* sicut proportio linee *gt* ad lineam *tz*. Sed proportio sinus arcus *ga* ad sinum arcus *ae* est sicut proportio linee *gh* ad lineam *he*. Et proportio sinus arcus *gd* ad sinum arcus *dz* est sicut proportio linee *gt* ad lineam *tz*. Et proportio sinus arcus *bz* ad sinum arcus *be* est unum eo quod uterque sinus est equalis alii cum sint inter lineas equidistantes. Si ergo multiplicetur in ipsam proportio sinus arcus *gd* ad sinum arcus *dz*, non proveniet nisi proportio sinus arcus *gd* ad sinum arcus *dz*. Quamobrem proportio sinus arcus *ga* ad sinum

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614 gea…arcus[2]] *om. (hom.)* B **615** zb] ergo proportio sinus arcus *zb add. sed. del.* A **616** dz] *de* AB **619** bz] *dz* A **625** iungatur] iungantur A **642** arcus[1]] *add. mg. a. m.* A *om.* B

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619 tertia…be] Strangely, the commentator does not order these in the order they are in the statement of composition, in which the sine of arc *be* is the third term and the sine of arc *dz* is the fourth.

arcus *ae* est composita ex proportione sinus arcus *gd* ad sinum arcus *dz* et ex proportione sinus arcus *bz* ad sinum arcus *be.* Ecce iam habemus tres divisiones veras prima linea et secunda fixis a parte *a.*

Nunc consequens est ut linea prima coniuncta sue diametro, secunda que est *gz* sit sue diametro equidistans, et tertia que est *ze* sit coniuncta sue diametro aut a parte *a* aut a parte *b* aut equidistans. [*Figura #11*] Coniungatur autem a parte *a* et sit linea *gz* equidistans. Dico ergo quod proportio sinus arcus *ga* ad sinum arcus *ae* componitur ex proportione sinus arcus *gd* ad sinum arcus *dz* et ex proportione sinus *bz* ad sinum arcus *be.* Quod sic probatur. Linea *gz* est equidistans linee *hk* quoniam ipsa *gz* est equidistans sue diametro que est equidistans linee *hk*. Et inter eas secant se due linee *gh* et *zk* super punctum *e,* ergo proportio linee *gh* ad lineam *he* est sicut proportio linee *zk* ad lineam *ke*. Quod verum est ex libro Euclidis propter similitudinem triangulorum. At proportio sinus arcus *ga* ad sinum arcus *ae* est sicut proportio linee *gh* ad lineam *he*. Et proportio sinus arcus [*zl*] ad sinum arcus *le* est sicut proportio linee [*zk*] ad lineam *ke*. Et sinus arcus *zl* est sinus arcus *bz*. Et sinus arcus *le* est sinus arcus *be.* Ergo proportio sinus arcus *ga* ad sinum arcus *ae* est sicut proportio sinus arcus *bz* ad sinum arcus *be.* Sed proportio sinus arcus *bz* ad sinum arcus *be* multiplicata in proportionem sinus arcus *gd* ad sinum arcus *dz* est eadem proportio cum proportio sinus arcus *gd* ad sinum arcus *dz* sit unitas. Quamobrem sequitur quod proportio sinus arcus *ga* ad sinum arcus *ae* est composita ex proportione sinus arcus *bz* ad sinum arcus *be* et ex proportione sinus arcus *gd* ad sinum arcus *dz*.

Duo vero alii modi, [*Figura #12-13*] scilicet quod linea *ze* coniungatur sue diametro a parte *b* et quod sit equidistans aliis duabus sic dispositis sunt impossibiles, quoniam cum superficies trianguli *gze* coniungatur a parte *a* superficiei inferiori per lineam *gh*, coniungeretur ab alia parte eidem superficiei per lineam *ze* quod est impossibile. Item equidistans esse non potest quoniam superficies in qua esset cum linea *gz* equidistante esset equidistans inferiori superficiei et ipsa est ei coniuncta per lineam *gh*. Quare esset | ei equidistans et coniuncta quod A 8rb
est impossibile. Et ita una tantum de his tribus divisionibus tantum est vera.

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651 veras] *om.* B **656** arcus[1]] *om.* A **658** quoniam…659 hk] *om.* B **661** Quod] quidem *add.* B **664** zl] *gl* AB *zl add. mg. a. m.* A | zk] *gk* AB *zk add. supr. lin. a. m.* A **665** ke] arcus *be* factus semicirculus secat circulum *adb* supra punctum *l* a parte *a adnot. mg. a. m.* A **666** arcus[1]] *om.* B **667** bz[1]] *dz* B **670** sit unitas] *mg. a. m.* A *om.* B **675** sunt impossibiles] *mg. a. m.* A *om.* B **681** his] istis B

Item prima linea coniuncta sue diametro a parte *a* potest coniungi secunda ab alia parte et proveniunt inde tres divisiones secundum tres modos dispositionis linee tertie que est *ze,* de quibus una tantum vera est et due false. Sit ergo figura talis. [*Figura #14*] Linea prima que est *ge* sic coniuncta supra punctum *h* ut fuit in precedentibus et linea tertia supra punctum *k* et linea secunda ab alia parte super punctum *m*. Et secet arcus *gd* factus semicirculus ab alia parte circulum *adb* supra punctum *n*. Inter duas igitur lineas *zgm* et *mkh* secant se due linee *gh* et *zk* super punctum *e*. Ergo proportio linee *zm* ad lineam *mg* composita est ex proportione linee *zk* ad lineam *ke* et ex proportione linee *he* ad lineam *hg*. Proportio vero linee *zm* ad lineam *mg* est sicut proportio sinus arcus *zn* ad sinum arcus *ng*. Et proportio linee *zk* ad lineam *ke* est sicut proportio sinus arcus *zl* ad sinum arcus *le*. Et proportio linee *he* ad lineam *hg* est sicut proportio sinus arcus *ae* ad sinum arcus *ag*. Ergo proportio sinus arcus *zn* ad sinum arcus *ng* componitur ex proportione sinus arcus *zl* ad sinum arcus *le* et ex proportione sinus arcus *ae* ad sinum arcus *ag*. Sinus vero arcus *zn* est sinus arcus *dz*, et sinus arcus *ng* est sinus arcus *gd*. Et sinus arcus *zl* est sinus arcus *bz*, et sinus arcus *le* est sinus arcus *be*. Ergo proportio prime que est sinus arcus *dz* ad secundam que est sinus arcus *dg* componitur ex proportione tertie que est sinus arcus *bz* ad quartam que est sinus arcus *be* et ex proportione quinte que est sinus arcus *ae* ad sextam que est sinus arcus *ag*. Cum ergo converterimus, erit proportio sinus arcus *gd* ad sinum arcus *dz* composita ex proportione sinus arcus *be* ad sinum arcus *bz* et ex proportione sinus arcus *ga* ad sinum arcus *ae*. Secundum igitur modum octavum posita prima sinu arcus *gd* et secunda sinu arcus *dz* et tertia sinu arcus *be* et quarta sinu arcus *bz* et quinta sinu arcus *ga* et sexta sinu arcus *ae,* erit proportio quinte que est sinus arcus *ga* ad sextam que est sinus arcus *ae* composita ex proportione prime que est sinus arcus *gd* ad secundam que est sinus arcus *dz* et ex proportione quarte que est sinus arcus *bz* ad tertiam que est sinus arcus *be*.

Alii vero duo modi, linea secunda sic coniuncta ab alia parte et prima disposita ut in omnibus predictis divisionibus, sunt impossibiles, [*Figura #15*] scilicet ut linea tertia coniungatur ab alia parte id est a parte *b* et ut sit equidistans. Quoniam si coniungeretur ab alia parte, tunc superficies trianguli coniungeretur inferiori superficiei a duabus partibus contrariis quod est impossibile. Et si esset equidistans, [*Figura #16*] tunc superficies in qua est esset equidistans inferiori superficiei et ipsa est coniuncta. Et hoc similiter est impossibile. Constat igitur quod linea

687 coniuncta] coniunctam B | supra[1]] super B | supra[2]] super B **688** super] erit B **692** zk] *zke* B **693** ad[2]] *iter*. B **707** posita] composita B prima que est *ge* manente coniuncta sue diametro a parte *a*, proveniunt 9 divisiones quarum 5 sunt vere et quattuor false et impossibiles.

Et est sciendum ut diximus quod linea *ge* prima coniuncta sue diametro ab alia parte, proveniunt 9 divisiones. Videramus ergo que illarum sint vere et que impossibiles. [*Figura #17*] Sit ergo linea *ge* prima coniuncta sue diametro ab alia parte super punctum *h* et linea secunda *gz* a parte in qua fuit inprimis supra punctum *t* et linea tertia que est *ez* a parte [*b*] supra punctum *k*. Et secet arcus *ga* factus semicirculus | ab alia B 5vb parte circulum *adb* super punctum *c*. In hac figura igitur inter duas lineas *egh* et *hkt* secant se due linee *gt* et *ek* supra punctum *z*. Ergo proportio linee *eh* ad lineam *hg* componitur ex proportione linee *ek* ad lineam *kz* et ex proportione linee *tz* ad lineam *tg*. Sed proportio linee *eh* ad lineam *hg* est sicut proportio sinus arcus *ec* ad sinum arcus *cg*. Et proportio linee *ek* ad lineam *kz* est sicut proportio sinus arcus *eb* ad sinum arcus *bz*. Et proportio linee *tz* ad lineam *tg* est sicut proportio sinus arcus *dz* ad sinum arcus | *dg*. Sed sinus arcus *ec* est sinus arcus *ae*, et sinus arcus *cg* est A 8va sinus arcus *ga*. Ergo proportio sinus arcus *ae* ad sinum arcus *ag* componitur ex proportione sinus arcus *eb* ad sinum arcus *bz* et ex proportione sinus arcus *dz* ad sinum arcus *dg*. Cum ergo converterimus, erit proportio sinus arcus *ga* ad sinum arcus *ae* composita ex proportione sinus arcus *gd* ad sinum arcus *dz* et ex proportione sinus *bz* ad sinum arcus *be*. Ecce divisio ista vera est.

Sed prima et secunda in hac dispositione manentibus, tertia non potest coniungi sue diametro ab alia parte quoniam sic coniungeretur superficies trianguli inferiori superficiei a duabus partibus contrariis, quod est impossibile. [*Figura #18-19*] Neque potest esse equidistans quoniam est in medio. Sic enim superficies eadem coniungeretur superficiei et esset ei equidistans, quod item est impossibile. Et non est possibile ut secunda sic manens sit media et tertia sit extrema ab illa parte quoniam arcus *be* non secaret arcum *gd* ab illa parte. Constat ergo quod harum trium divisionum una tantum est vera et due false.

Et sit prima coniuncta sue diametro sicut fuit nunc, et sit secunda coniuncta sue diametro ab alia parte super punctum *t*, et tertia post ipsam sue diametro ab illa parte scilicet que est inter *c* et *a* et non a parte *b*. [*Figura # 20*] Et sit locus in quo arcus *dg* factus semicirculus secat circulum *adb* ab illa parte ille super quem est *l*. Et ille in quo ipsum secat arcus *be* factus semicirculus, ille super quem est *m*. Inter duas igitur lineas *zek* et *kth* secant se due linee *zt* et *eh* supra punctum *g*. Ergo

722 divisiones] et *add. sed. exp.* A **728** b] *d* AB **729** figura igitur] *inv.* B **742** est] *om.* B
744 sic] si B **754** et[2]] *om.* A **757** quem] *add. mg. a. m.* A

proportio linee *zk* ad lineam *ke* componitur ex proportione linee *zt* ad lineam *tg* et ex proportione *hg* ad lineam *he*. At proportio linee *zk* ad lineam *ke* est sicut proportio sinus arcus *zm* ad sinum arcus *me*. Et proportio linee *zt* ad lineam *tg* est sicut proportio sinus arcus *zl* ad sinum arcus *lg*. Et proportio linee *hg* ad lineam *he* est sicut proportio sinus arcus *cg* ad sinum arcus *ce*. Sed sinus arcus *zm* est sinus arcus *bz*, et sinus arcus *me* est sinus arcus *be*. Et sinus arcus *zl* est sinus arcus *dz*, et sinus arcus *lg* est sinus arcus *gd*. Et sinus arcus *cg* est sinus arcus *ga*, et sinus arcus [*ce*] est sinus arcus *ae.* Ergo proportio sinus arcus *bz* ad sinum arcus *be* est composita ex proportione sinus arcus *dz* ad sinum arcus *dg* et ex sinu arcus *ga* ad sinum arcum *ae.* Secundum igitur octavum modum erit proportio sinus arcus *ga* ad sinum arcus *ae* et composita ex proportione sinus arcus *bz* ad sinum arcus *be* et ex proportione sinus arcus *gd* ad sinum arcus *dz*.

Et maneant prima et secunda sic coniuncte ut in hac precedenti fuerunt, et sit tertia coniuncta sue diametro a parte *b* ita quod sit prima media. [*Figura #21*] Inter duas igitur lineas *ezk* et *kht* secant se due linee *zt* et *he* supra punctum *g*. Ergo proportio linee *ek* ad lineam *kz* componitur ex proportione linee *eh* ad lineam *hg* et ex porportione linee *tg* ad lineam *tz*. At proportio linee *ek* ad lineam *kz* est sicut proportio sinus arcus *eb* ad sinum arcus *bz*. Et proportio linee *eh* ad lineam *hg* est sicut proportio sinus arcus *ec* ad sinum arcus *cg*. Et proportio linee *tg* ad lineam *tz* est sicut proportio sinus arcus *lg* ad sinum arcus *lz*. Sed sinus arcus *lg* est sinus arcus *gd*. Et sinus arcus *zl* est sinus arcus *dz*. Et sinus arcus *ec* est sinus arcus *ae*. Et sinus arcus *cg* est sinus arcus *ga*. Ergo proportio sinus arcus *eb* ad sinum arcus *bz* componitur ex proportione sinus arcus *gd* ad sinum arcus *dz* et ex proportione sinus arcus *ae* ad sinum arcus *ga*. Cum ergo converterimus, erit proportio sinus arcus *bz* ad sinum arcus *be* composita ex proportione sinus arcus *dz* ad sinum arcus *gd* et ex proportione sinus arcus *ga* ad sinum arcus *ae*. Positis igitur prima sinu arcus *bz* et secunda sinu arcus *be* et tertia sinu arcus *dz* et quarta sinu arcus *gd* et quinta sinu arcus *ga* et sexta sinu arcus *ae,* erit proportio sinus arcus *ga* ad sinum arcus *ae* secundum octavum | modum A 8vb composita ex proportione sinus arcus *bz* ad sinum arcus *be* et ex proportione sinus arcus [*gd*] ad sinum arcus *dz*.

Et iterum prima et secunda sic coniunctis ut in duabus precedentibus, potest tertia esse equidistans. [*Figura #22*] Sit itaque linea *ze* que est tertia equidistans sue diametro reliquis duabus manentibus ut in

759 zk] *tk* A **764** ce] *cg* B **766** ce] *ge* AB **770** et] *om.* B **777** eh] *bh sed. corr.* B **785** ae] *ge* B **791** ae] *gae* B **793** gd] *gb* B duabus precedentibus. Ipsa ergo est equidistans linee *ht.* Inter duas igitur equidistantes lineas secant se due linee supra punctum *g*. Quare propter similitudinem duorum triangulorum est proportio linee *zt* ad lineam *tg* sicut proportio linee *eh* ad lineam *hg.* Sed proportio linee *eh* ad lineam *hg* est sicut proportio sinus arcus *ec* ad sinum arcus *cg*. Et proportio linee *zt* ad lineam *tg* est sicut proportio sinus arcus *zl* ad sinum arcus *lg*. Et sinus arcus *ec* est sinus arcus *ae,* et sinus arcus *cg* est sinus arcus [*ga*]. Ergo proportio sinus arcus *ae* ad sinum arcus *ga* est sicut proportio sinus arcus *dz* ad sinum arcus *gd*. Cum ergo converterimus, erit proportio sinus arcus *ga* ad sinum arcus *ae* sicut proportio sinus arcus *gd* ad sinum arcus *dz*. Sed proportio sinus arcus *bz* ad sinum arcus *be* est unum eo quod uterque sinus est unus. Proportio igitur sinus arcus *gd* ad sinum arcus *dz* multiplicata in proportionem sinus arcus *be* ad sinum arcus *bz* est ipsa eadem quoniam est multiplicata in unum. Ergo proportio sinus arcus *ga* ad sinum arcus *ae* componitur ex proportione sinus arcus *gd* ad sinum arcus *dz* et ex proportione sinus arcus *bz* ad sinum arcus *be.* Et ita habemus tres modos quos facit tertia cum | secunda coniuncta est ab ista veros. B 6ra

Nunc superest ut ostendimus de tribus modis quos facit linea tertia an sint veri linea prima coniuncta sue diametro a parte *c* ut in precedentibus et linea secunda equidistante sue diametro. Dico igitur quod unus illorum tantum est verus qui est cum coniungitur linea *ez* sue diametro a parte *b.* Quoniam si ab alia iungeretur parte, esset inter illam primam et tertiam linea secunda equidistans quod est impossibile propter illud quod dictum est. Neque potest esse equidistans quoniam essent due linee equidistantes et tertia coniuncta, quod est impossibile. Sit itaque dipositio figure ut diximus. [*Figura #23*] Cum ergo linea *zg* sit equidistans linee *hk,* tunc proportio linee *eh* ad lineam *hg* que est sicut proportio sinus arcus *ec* ad sinum arcus *cg* est sicut proportio linee *ek* ad lineam *kz* que est sicut proportio sinus *eb* ad sinum arcus *bz.* Sed sinus arcus *ec* est sinus arcus *ae,* et sinus arcus *cg* est sinus arcus *ga*. Ergo proportio sinus arcus *ae* ad sinum arcus *ga* est sicut proportio sinus arcus *eb* ad sinum arcus *bz*. Cum ergo converterimus, erit proportio sinus arcus *ga* ad sinum arcus *ae* sicut proportio sinus arcus *bz* ad sinum arcus *be.* Sed cum proportio sinus arcus *gd* ad sinum arcus *dz* sit unum eo quod utriusque sinus [est idem], tunc proportio sinus arcus *bz* ad sinum arcus *be* multiplicata in ipsam est eadem. Ergo proportio sinus arcus *ga* ad

797 ht] *he* B **803** ga] *gd* AB **809** in…810 multiplicata] *add. mg. a. m.* A | be] *bz* B **813** quos] quas A **819** illam] lineam A **827** ec] ad sinum arcus *cg add.* AB **832** sinus[1]] *iter.* A sinum arcus *ae* est composita ex proportione sinus arcus *gd* ad sinum arcus *dz* et ex proportione sinus arcus *bz* ad sinum arcus *be*.

Ostenso quot divisiones vere sint et quot false cum linea *ge* coniungitur sue diametro a parte *a* et quot sint vere et quot sint false cum ei coniungitur ab alia parte, demonstrandum est quot vere divisiones proveniant et quot false cum ipsa equidistat sue diametro. Sit ergo equidistans linea prima que est *ge* et sit linea secunda que est *gz* coniuncta sue diametro a parte *d*. Dico ergo quod de tribus modis quos facit linea tertia que est *ze* non est possibilis nisi unus scilicet ut ipsa coniungatur diametro a parte *b*. [*Figura #24*] Non enim potest coniungi ab alia parte quoniam linea prima que est equidistans esset in medio scilicet inter ipsam et secundam, quod est impossibile. Neque potest esse equidistans quia essent due linee unius superficiei equidistantes et una coniuncta, similiter quod est impossibile. Qualiter ergo figura illa probetur ostendamus. Linea *ge* equidistat linee *kt* quoniam equidistat sue diametro que equidistat linee *kt*. Ergo proportio linee *gt* ad lineam *tz* est sicut proportio linee *ek* ad lineam *kz*, et hoc propter similitudinem triangulorum. At proportio linee *gt* ad lineam | *tz* est sicut proportio sinus A 9ra arcus *gd* ad sinum arcus *dz*. Et proportio linee *ek* ad lineam *kz* est sicut proportio sinus arcus *eb* ad sinum arcus *bz*. Ergo proportio sinus arcus *gd* ad sinum arcus *dz* est sicut sinus arcus *eb* ad sinum arcus *bz*. Et cum converterimus, erit proportio sinus arcus *dz* ad sinum arcus *gd* sicut proportio sinus arcus *bz* ad sinum arcus *be*. At proportio sinus arcus *dz* ad sinum arcus *gd* et proportio sinus arcus *gd* ad sinum arcus *dz* est unum. Ergo proportio sinus arcus *bz* ad sinum arcus *be* et proportio sinus arcus *gd* ad sinum arcus *dz* est unum. Et proportio sinus arcus *ga* ad sinum arcus *ae* est unum eo quod utrique sinus sint equales. Ergo proportio sinus arcus *ga* ad sinum arcus *ae* componitur ex proportione sinus arcus *gd* ad sinum arcus *dz* et ex proportione sinus arcus *bz* ad sinum arcus *be*.

Et dico quod linea prima equidistante et secunda coniuncta ab alia parte, non est possibilis de tribus modis quos tertia facit nisi unus scilicet ut ipsa sit iuncta a parte *a* qua coniungitur linea secunda. [*Figura # 25*] Non enim potest coniungi a parte *b* quoniam linea prima esset media que est equidistans, quod est impossibile. Neque potest esse equidistans quoniam essent due linee equidistantes unius superficiei et una coniuncta, quod est impossibile. Ostendamus ergo quomodo figura in qua ita disposite sunt linee probetur. Linea *ge* equidistat linee *tk*, ergo proportio linee *zt* ad lineam *tg* est sicut proportio linee *zk* ad lineam *ke*. Sed

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837 sint[2]] *om.* A **841** d] *add. mg. a. m.* A | de] *om.* B **847** similiter quod] *inv.* B **860** utrique] utrisque *sed. corr.* A **870** tk] *tg* B proportio linee *zt* ad lineam *tg* est sicut proportio sinus arcus *zl* ad sinum arcus *lg*. Et proportio linee *zk* ad lineam *ke* est sicut proportio sinus arcus *zm* ad sinum arcus *me*. At sinus arcus *zl* est sinus arcus *dz*, et sinus arcus *lg* est sinus arcus *gd*. Et sinus arcus *zm* est sinus arcus *bz*, et sinus arcus *me* est sinus arcus *be*. Ergo proportio sinus arcus *dz* ad sinum arcus *gd* est sicut proportio sinus arcus *bz* ad sinum arcus *be*. At proportio sinus arcus *dz* ad sinum arcus *gd* et proportio sinus arcus *gd* ad sinum arcus *dz* est unum. Ergo proportio sinus arcus *bz* ad sinum arcus *be* et proportio sinus arcus *gd* ad sinum arcus *dz* est unum. Et proportio sinus arcus *ga* ad sinum arcus *ae* est unum, ut ostensum est. Ergo proportio sinus arcus *ga* ad sinum arcus *ae* est composita ex proportione sinus arcus *gd* ad sinum arcus *dz* et ex proportione sinus arcus *bz* ad sinum arcus *be*.

Nunc videamus linea prima equidistante et secunda similiter equidistante quot modi de tribus quos facit linea tertia sint veri. [*Figura #26*] Dico ergo quod illis duabus equidistantibus impossibile est ut tertia sit coniuncta quoniam sic esset superficies una equidistans uni superficie et coniuncta eidem. Quare necesse est ut illis duabus equidistantibus tertia sit equidistans. Et sic de illis tribus modis unus tantum est verus. Sint ergo equidistantes suis diametris ille tres linee. Dico ergo quod proportio sinus arcus *ga* ad sinum arcus *ae* est composita ex proportione sinus arcus *gd* ad sinum arcus *dz* et ex proportione sinus arcus *bz* ad sinum arcus *be*. Proportio namque sinus arcus *ga* ad sinum arcus *ae* est unum eo quod uterque sinus est equalis propter equidistantiam linearum. Et similiter proportio sinus arcus *gd* ad sinum arcus *dz* est unum propter eandem causam, et similiter proportio sinus arcus *bz* ad sinum arcus *be* [est unum] eadem de causa. Ergo proportio sinus arcus *ga* ad sinum arcus *ae* est composita ex proportione sinus arcus *gd* ad sinum arcus *dz* et ex proportione sinus arcus *bz* ad sinum arcus *be*. Et illud est quod demonstrare voluimus. Ecce, habemus quod linea prima coniuncta diametro a parte *a* proveniunt quinque divisiones vere et quattuor false. Et similiter ea coniuncta ab alia parte proveniunt quinque vere et quatuor false. Et ea equidistante, | proveniunt tres vere et sex false. Sunt ergo B 6rb omnes vigintiseptem quarum tredecim sunt vere et quatuordecim false sicut dixit Thebit in principio sui libri.

[Notes on I.13]

‘Capitulum 13 de scientia quantitatum etc.’ Intendit Ptholomeus in hoc capitulo docere qualiter inveniantur declinationes omnium graduum

877 be…878 arcus[1]] *add. mg. a. m.* A | arcus[3]] *om.* B **897** est unum] *om.* AB | de] *supr. lin.* A orbis signorum qui sunt arcus circulorum maiorum transeuntium per duos polos orbis equatoris | diei existentes inter orbem signorum et orbem A 9rb equationis diei. Et hoc per proportiones sectionum arcuum circulorum inter duos arcus secundum quod ostensum est in figura sectore. Et totam declinationem iam invenit per instrumentum. Et ideo ea manifesta. Docet in duobus exemplis qualiter inveniatur scilicet cum ponitur arcus orbis signorum 30 partes et cum ponitur arcus eius 60 partes. Et ita sit manifesta declinatio 30 graduum et 60 et 90 qui omnes sunt tota quarta. Et declinatio unius quarte est declinatio aliarum trium quartarum.

Declinatio autem 30 graduum sic reperitur. Invenitur proportio corde dupli arcus qui est ab equatore diei usque ad polum ad cordam dupli arcus totius declinationis. Et hoc sit dividendo maiorem cordam per minorem et producendo utramque ad secunda, et postea in divisionem perveniendo usque ad tertia. Deinde invenitur proportio corde dupli arcus 30 graduum ad cordam dupli arcus 90 graduum, que invenitur dividendo minorem cordam per maiorem, per quam inventam dividitur prima proportio inventa scilicet que est corde dupli arcus qui est corde ab equatore diei usque ad polum ad cordam dupli arcus totius declinationis. Et quod provenit est proportio corde dupli arcus protensi a polo equatoris diei transeuntis per finem 30 gradus orbis signorum ad equatorem diei ad cordam dupli arcus qui est inter 30 gradum orbis signorum et equatorem diei. Et hoc ideo quoniam proportio secunda multiplicatur in hanc et provenit prima scilicet proportio corde dupli arcus qui est ab equatore diei ad polum ad cordam dupli arcus totius declinationis. Ergo cum proportio prima dividitur per secundam, provenit tertia. Cum ergo per eam dividitur corda dupli arcus protensi a polo ad equatorem diei et transeuntis per 30 gradum orbis signorum, provenit corda dupli arcus qui est inter 30 gradum orbis signorum et equatorem diei. Deinde inveniatur illius corde arcus et accipiatur eius medietas, et ipsa est declinatio 30 graduum orbis signorum. Et ita faciendum est ad invendiendas omnes alias declinationes et maiores et minores. Et observandum est semper ut in dividendo nichil pretermittatur sed perveniatur usque ad tertia.

'Dico quod cum nos nominamus etc.' Hec littera sic est in arabico. Volumus intelligere per partes arcuum nisi illas que sunt 360 partes in quas dividitur maior circulus et volumus per partes cordarum intelligere nisi illas que sunt 120 in quas dividitur diametrus.

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932 qui est] *add. mg. a. m.* A **936** provenit…937 signorum] *add. mg. a. m.* A *iter.* B **940** declinationes] declinationis A

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942 Hec…arabico] This note is strange because the commentator was clearly using the Gerard of Cremona version of the *Almagest*, which does not contain any Arabic words at this point.

Numeri communes. In tabulis ideo dicuntur numeri communes quia a loco in quo se secant duo circuli orbis signorum et equator diei usque ad finem gradus cuiuslibet declinatio est termini declinatio residui circuli qui est ab alia parte sectionis usque ad eundem finem, licet secet alium circulum in alio puncto.

Et est sciendum quod cum invenitur corda per arcum et sunt in arcu minuta plura quam sint in libro, tunc minuta illa superflua multiplicanda sunt in numerum qui est in directo illius linee in qua intrasti tabule que inscribitur 'pars 30' etc. Et si sunt sola minuta que multiplicas in illum numerum, non mutatur illud quod provenit ex illa multiplicatione ab ultimis fractionibus in quas multiplicas id est tertiis. Et hoc ideo quia minuta non sunt ibi nisi numerus. Si autem sunt cum minutis secunda, quoniam ipsa minuta descendunt una differentia, descendit et illud quod provenit ex multiplicatione una differentia videlicet ad quarta. Item ad inveniendum arcum per cordam, illud quod minus est in linea cum qua intras minuitur de corda maiore quam habes. Et quod remanet multiplicatur in 30 eo quod tabula crescit per 30. Et minuitur linea cum qua intrasti de linea que sequitur ipsam, et per illud quod remanet dividitur illud quod provenit ex multiplicatione primi residui in triginta, et quod provenit additur primo arcui. Vel si vis, divide 30 per residuum quod provenit diminuta prima linea ex secunda, et quod provenit ex divisione multiplica in residuum primum. Et habebis illud idem.

Et ad ultimum sciendum est quod minuere proportionem ex proportione non est nisi dividere unam proportionem per aliam et accipere illud quod provenit et ita intellexit Ptolomeus | in hoc capitulo. A 9va

[Notes on I.14]

Capitulum 14. Intendit Ptolomeus in ipso docere quot gradus equinoctialis circuli eleventur super orizonta cum aliquibus gradibus circuli signorum datis, et hoc per figuram sectorem ut in precedenti fecit. Verum ibi secundum compositionem, hic vero secundum divisionem quoniam ponit quod proportio corde dupli arcus qui est ab orbe signorum ad polum ad cordam dupli arcus totius declinationis componitur ex proportione corde dupli arcus qui est usque ad aliquem gradum orbis signorum a polo ad cordam dupli arcus qui est ab illo gradu orbis signorum usque ad orbem equatoris diei et ex proportione corde dupli arcus qui est a loco communis sectionis orbis signorum et orbis equatoris diei et est arcus de equatore diei usque ad locum per quem transit

949 ab] *iter.* A **958** ipsa] secunda *add.* B **965** divide] per *add. sed. del.* B **979** gradum] graduum B

predictus arcus veniens a polo ad orbem signorum et orbem equatoris diei ad duplum corde arcus qui est ab eadem sectione usque ad locum in quo circulus meridiei secat orbem equatoris diei. Et ipse arcus est equatoris diei. Et quia proportio prime ad secundam componitur ex proportione secunde ad tertiam et proportione quinte ad sextam secundum ordinem quem premisimus, tunc cum dividitur proportio prime ad secundam per proportionem tertie ad quartam, remanet proportio quinte ad sextam. Et quoniam sexta nota est et quinta ignota et proportio quinte ignote ad sextam notam nota que scilicet provenit ex predicta divisione proportionum, debet multiplicari ipsa proportio in sextam et proveniet quinta que erat ignota. Quod quidem diversum est ab eo quod in superiore factum est capitulo. Ibi enim divisimus quintam que nota erat per proportionem que provenit ex divisione | proportionum et provenit B 6va
sexta que erat ignota. Hic vero multiplicamus proportionem in sextam et provenit quinta. Quod ideo sit, quoniam cum dicitur proportio esse quantitatis alicuius ad aliam quantitatem, ad hoc ut proportio earum nota sit, dividenda est illa cuius proportio dicitur ad aliam per aliam sive sit maior sive minor. Et ideo si illa que dividitur fuerit nota et dividatur per proportionem que nota est, proveniet illa alia per quam ipsa primo fuit divisa cum provenit proportio. Si vero illa que dividitur ignota fuerit et illa per quam dividitur nota, tunc si proportio que nota est multiplicetur in eam, proveniet illa que divisa fuit per eam. Quod ut manifestius fiat exemplis ostendamus. Duodenarius numerus dividitur per quaternarium et provenit trinarius. Et si ignotus fuerit quaternarius, dividatur duodenarius per ternarium et proveniet quaternarius. Quod si duodenarius fuerit ignotus, multiplicetur trinarius in quaternarium et proveniet duodenarius.

'Et est etiam proportio 54 partium et minutorum 52' etc. Hoc fere verum est quoniam in minutis non est diversitas. 'Ergo corda dupli arcus *te* est 57 partes.' Istud in actione falsum est quoniam non est nisi 56 partes et minutum unum et 52 secunda. 'Que est sicut proportio nonaginta quinque partium et duorum minutorum et 41 secundi' etc. Istud iterum fere verum est quoniam in minutis non est diversitas. '101 pars et 28 minuta et viginta.' Istud verum est. 'Cum fuerit elevatio orbium

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989 cum] *om.* A | per] *add. supr. lin. a. m.* A **994** erat] erit B **1001** per] *add. supr. lin. a. m.* A **1002** fuit] fuerit B **1003** provenit] propuerat [sic] B **1008** duodenarius[1]] trinarius B ternarium] quaternarium B **1011** 54] 4 A

1012 'Ergo…1013 partes'] This mistake in the text of the *Almagest* manuscript that this user was using may help us determine the manuscript or family of manuscripts that the commentator was using.

descriptorum.' Istud verum est in loco super quem transit orbis equatoris diei quoniam omnes orbes qui elevantur ibi super orizonta qui distinguunt predictas elevationes sunt descripti super duos polos orbis equatoris diei.

[Book II]

Tractatus secundum libri Almagesti. Hic queritur quare Ptolomeus hic distinctionem fecerit tractatus, cuius rei duplex est causa quarum una est quia volebat tractare de orbe declivi et secunda quia volebat tractare de illis que contingunt in terra secundum motum circuli declivis. Et est sciendum quod ea que fiunt in celo alia considerantur secundum celum tantum, ut declinatio et sectio circulorum de quibus in priori tractatu egit Ptolomeus tractando de declinatione et figura sectore. Alia considerantur secundum terram in eclipses, et similiter ea que contingunt in terra quandocumque considerantur secundum celum ut prolixitas diei et noctis. Et est iterum sciendum quod ea que fiunt in terra cum secundum celum attenduntur, non consideratur quantitas terre ut prolixitas diei et noctis. Et cum ea que fiunt in celo attenduntur secundum terram, consideratur magnitudo terre. Ptolemeus | itaque ut dictum est in primo tractatu A 9vb tractavit de illis que in celo fiunt quantum ad celum tantum, quia de declinatione et sectione circulorum et de elevatione etiam partium orbis equationis diei cum partibus orbis declivis, quod ideo fecit ut esset antecedens ad probandum illud de quo in hoc secundo tractatu agit. Non autem in hoc secundo tractatu agit de circulo declivi idest orbe signorum et de illis que contingunt in terra secundum motum eius.

[II.1]

‘Primum capitulum de scientia locorum habitabilium terre,’ idest ad sciendum universaliter que loca terre inhabitentur non particulariter idest ad sciendum an pars terre sit orientalis aut occidentalis aut septentrionalis aut meridionalis. Non autem hic ... vel hoc.

‘Capitulum quintum etc. in medietatibus diei.’ Hic diem intelligit spacium duodecim horarum quando sol est super terram.

‘De forma tocius et communitate eius que est in ea sicut principia et antecedentia.’ Vel eorum que sunt que ipse in ea tractat de forma tocius et de aliis que sunt communia ad multa scilicet ad ea que secuntur et sunt quasi principia et antecedentia et quid est necessarium.

‘Et quod una duarum quartarum etc.’ Sciendum quod sicut in celo ita et in terra intelliguntur duo circuli qui dividunt ipsam in quattuor quartas, circulus aquationis diei et circulus orizon transiens per duos polos equationis diei. Et una illarum quartarum quas continet medietas

1046 Non…hoc] The meaning is unclear.

orbis equationis diei et medietas sub orizontis, a parte septentrionis terra inhabitatur.

‘Illud vero etc.’ Duobus modis demonstrantur quod illa quarta tantum inhabitatur propter latitudinem que est spacium a meridie ad septentrionem. Hic debuit dicere ‘ab equinoctiali ad polum septentrionalem,’ et dixit ‘a meridie ad septentrionem,’ quod ideo secat quoniam illum spacium sive sit ad septentrionem sive ad meridiem vocatur latitudo.

‘Declinatio erit semper ad septentrionem etc.’ Videtur in sequentibus sibi adversari cum dicit quod loca quedam inhabitantur inter equinoctialem et caput cancri. Unde ab viscera dicit vel quod ipse prius nescivit illud scilicet quod ibi esset habitacio vel quod alii qui sciverunt illum apposuerunt postea in libro eius.

‘Alter propter longitudinem etc.,’ scilicet quia non invenimus in scriptis aliquorum quod differentia que est inter illos qui vident eclipsius in orient et illos qui vident eam in occidente non excedit quantitatem 12 horarum, idest 180 gradus. Sic non invenitur in scriptis aliquorum quod umbre instrumentorum vadant ad meridiem. Et hee due rationes non sunt necessarie sed verisimiles ad probandum quod quarta septentrionalis tantum inhabitetur.

[II.2]

Capitulum secundum. Intendit in hoc capitulo ostendere qualiter possit sciri quantitas arcus orizontis qui est inter ortum alicuius gradus circuli signorum et circulum equinoctialem. Et hoc non sit nisi ponendo longiorem diem illius regionis in qua hoc invenitur. Et scitur hoc propterea quod circulus medii diei secat in omni regione illum quod est de circulis equidistantibus equatori diei et ipso in duo media, et illud quod de eisdem est sub terra similiter in duo media.

‘Ergo proportio corde dupli etc.’ Hic intulit a conversione propositionis. Prius enim debet sic inferri quoniam inter duos arcus orbium maiorum qui sunt *ae az* secant se duo arcus *ehb zht* super *h*. Ergo proportio corde dupli arcus *ea* ad cordam dupli arcus *ta* componitur ex proportione corde dupli arcus *eb* ad cordam dupli arcus *hb* et ex proportione corde dupli arcus *zh* ad cordam dupli arcus *zt.* Deinde convertendo hanc proportione sequitur illud quod Ptolomeus conclusit. Et hoc fecit causa abbreviandi. Cetera omnia plana sunt.

[II.3]

Capitulum tercium. ‘Propter hos arcus,’ scilicet quos probavit in precedenti capitulo, et est notandum quod superius ostendit qualiter

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1078 qualiter] *mg.* A   **1088** ea…arcus[2]] *mg.* A scietur arcus orizontis qui est ab ortu alicuius gradus circuli signorum propter diem longiorem datum. Nunc autem vult ostendere qualiter per hunc arcum orizontis datum scietur arcus altitudinis poli, et qualiter scietur per arcum superfluitatis diei prolixioris datum, et qualiter isti duo arcus scientur per illum datum, et qualiter [scietur] arcus orizontis per arcum superfluitatis diei quod iam ostensum est, et qualiter per ipsum sciatur arcus | superfluitatis diei. Istud vero non facit nisi converse A 10ra quoniam cum deberet istud facere, ostendit prius qualiter sciatur arcus altitudinis poli per arcum orizontis. Deinde dicit se ostensurum conversionem illius quod non facit, immo ostendit qualiter per arcum altitudinis poli datum sciatur arcus superfluitatis diei prolixioris. Et postea innuit in sequentibus alia que diximus.

‘Manifestum est igitur etc.’ Quasi dicat totum istum scietur propter arcum declinationis qui notus est quoniam quicumque arcus a polo ducatur transiens per orbem signorum ad equatorem diei scietur pars eius que est ab orbis signorum ad equatorem diei que est declinatio.

‘Premisimus tabula etc. et illum est relatum etc.’ Idest sicut *ht* est hic declinatio in hac figura ita erit in qualibet figura arcus ab orbe signorum ad equatorem diei declinatio nota propter tabulas premissas.

‘Sequitur autem illum vero orbes equidistantes etc.’ Istum quod prius ponit est ad ostendum quod dies diei sit equalis et nox nocti. Illum vero quod sequitur quod dies nocti sit equalis et nox diei, et ideo hic dicitur equalitas ibi alternitas.

‘Et hoc est quod si nos signa[verimus] etc.’ Figura hec sic probatur ex positione arcus *hl* equalis est arcui *km*, et arcus *hl* similis est arcui *ta*. Ergo arcus *ta* similis est arcui *km*, ideo quod arcus *hl* est similis arcui *km* eo quod est ei equalis, quoniam omnes arcus equales sunt similes sed non convertitur. Et arcus noster *km* est similis arcui *sg*, ergo arcus *ta* est similis arcui *sg*. Ipsi vero sunt unius circuli, ergo sunt equales ex Euclide. Arcus autem *ea* est equalis arcui *eg* eo quod quisque eorum est quarta circuli. Quare remanet arcus *et* equalis arcui *es*. Quod autem arcus *ks* sit equalis arcui *ht* probatur per theorema illud in quo dicitur si secuerint se duo circuli magni et signata fuerint super unum eorum duo puncta etc. Ex hoc theoremato sequitur quod proportio sinus arcus *ks* ad sinum arcus *se* est sicut proportio sinus arcus *ht* ad sinum arcus *te*. Et similiter cum permutaverimus, erit proportio sinus arcus *ks* ad sinum arcus *ht* sicut proportio sinus arcus *se* ad sinum arcus *et*. Sinus vero arcus *se* est equalis sinui arcus *et*, ergo sinus arcus *ks* est equalis sinui arcus *ht*. Sed

1100 scietur] *om.* A **1123** ergo…1124 sg] *iter. sed. del.* A **1129** ks…1130 arcus[1]] *mg.* A

1127 theorema…1128 etc] This is a reference to Gebir, I.12.

unusquisque eorum est minor quarta circuli, ergo ipsi sunt equales. Hoc probato quod in figura est, facile sequitur.

[II.4]

Capitulum quartum. Quicquid in ipso continetur facile est et exponere non eget. Illum inde quod ibi dicitur ‘ut intremus numerum partium etc.,’ sic intelligendum est omnium locorum latitudo que sunt a capite cancri usque ad equinoctialem circulum, est declinatio graduum orbis signorum ab equinoctiali. Cum scitur ergo latitudo loci, scitur declinatio duorum graduum orbis signorum scilicet unius ab ariete ad cancrum et alterius a libra ad cancrum qui sunt equalis longitudinis a capite cancri. Cum numero ergo latitudinis tractandum est in tabulam declinationis que inscribitur gradus declinationis et dicitur secunda eo quod ante ipsum est alia tabula que inscribitur “numeri communes,” et numerus graduum quarte qui sunt in illa prima tabula que est ante secundam que dicitur declinationis indicat quando sol obumbrabit super capita habitancium in loco illo. Est transibit ab ariete ad cancrum semel et cum a cancro ad libram iterum cum fuerit in simili numero graduum a libra in quo fuit ab ariete cum transivit super capita.

[II.5]

Capitulum quintum. ‘Propter quedam que prediximus cum fuerint data.’ Ea que predixit que ad hoc valent sunt arcus orizontis qui est ab ortu alicuius puncti usque ad equinoctialem et arcus altitudinis poli et arcus superfluitatis diei longioris, de quibus omnibus superius ostendit qualiter unus per alium inveniatur. Non tamen hic ad ostendendas propositiones gnomonum ad umbras suas ponit alium de predictis arcubus notum nisi arcum altitudinis poli cum tamen ita bene possit inveniri per quemlibet aliorum datum.

‘Postquam iam scivimus arcum qui est inter etc.’ Arcus qui est inter duos tropicos est arcus declinationis duplatus et arcus qui est inter duos polos | et orizontem qui similiter est arcus elevationis poli ab orizonte A 10rb duplatus eo quod quantum unus polus elevatur ab orizonte tantum alter deprimitur ab orizonte.

‘Manifestum est igitur quod hec line etc.’ Hic notandum est quod Ptolomeus, quoniam cum ea que contingunt in terra considerantur secundum celum, terra nullius sensibilis quantitatis invenitur ut dictum est, posuit figuram suam acsi esset centrum et transiret radius solis per eam usque ad aliam partem circumferentia circuli meridiei et posuit medietatem diametri circuli meridiei gnomonem et in superficie circuli meridiei inferius produxit lineam *gn* super quam cadit umbra. Quodquod in actu non est. Sed quia constans est ut ipse dicit quod hec linea sic producta equidistat linee que transit per centrum et per duo puncta super que circulus orizon secat circulum meridiei, ideo quicquid verum est de hoc quod sit in has figura super hanc lineam quam iste ponit, verum est si fiat super illam lineam equidistantem. Et quia inter centrum terre et superficiem eius et caput gnomonis positi super eam non est diversitas sensibilis quantum ad magnitudinem spere solis ut ipse dicit, tunc si [super] superficiem terre ponatur gnomo sexaginta cubitos habens, omnes probationes quas ipse vere invenientur.

Omnia plana sunt usque ad locum ut dicitur, ‘Et quia arcus *gd* equatur altitudini poli.’ Hoc autem sic probatur. Arcus *ab* est latitudo regionis eo quod punctum *a* sit supra summitatem capitum et punctum *b* sit circuli equinoctialis. Arcus vero *ab* est equalis arcui elevationis poli ab orizonte. Ergo arcus *gd* est equalis arcui elevationis poli ab orizonte quoniam arcus *gd* est equalis arcui *ab*. Quod ideo est quoniam angulus *aeb* est equalis angulo *deg*. Quapropter arcus subtensus uni eorum est equalis subtenso alteri.

‘Quapropter anguli qui sunt sub eis secundum quantitatem qua quattuor anguli etc.’ Hic notandum est quod illud quod prius dixit attendendum est cum vult aliquis scire quota pars angulus aliquis in circulo factus de quattuor angulis rectis. Illum autem quod secundo dixit scilicet ‘secundum quantitatem que duo anguli recti etc.’ tunc attendendum est cum vult aliquis scire quota pars sit angulus *keg* de duobus rectis cum duo recti positi fuerint 360 partes. Quod ideo fit ut sciantur corde scilicet quanta sit linea *gk* et relique linee trianguli *gke*. Cum enim unus angulorum eius sit rectus scilicet *g*, tunc scimus quantum ipse sit et scimus quantum sit angulus *keg* de alio recto. Cum ipse et angulus *k* sit unus rectus, sciemus arcum qui erit super lineam *gk* et ita sciemus lineam *gk* que est corda eius. Et sciemus angulum *k* et sciemus arcum cuius corda erit *eg* et sciemus cordam *eg*. Erit enim angulus *keg* cum considerabitur quota pars sit de duobus rectis positis trecentis sexaginta partibus, duplum eius quod fuit cum considerabatur quota pars esset de quattuor rectis cum ponuntur 360 partes. Et ad ultimum sciendum est quod quattuor anguli ponuntur 360 partes ad sciendum arcum et duo anguli recti ponuntur trecente et sexaginta partes ad sciendum cordam.

‘Arcus ergo qui sunt porciones etc.’ Sectis angulis secundum quod duo anguli recti sunt trecente et sexaginta partes, sciunt arcus qui sunt super lineas subtensas illis angulis. Et scitis arcubus sciuntur corde illorum arcuum. Et quantitatem [sic] qua augetur una linea de illis tribus

1188 Quapropter…1189 alteri] *iter. sed. del.* va--cat A **1198** scimus] sciamus *sed. corr. exp.* A que sunt umbra, minuitur linea *eg*, cum ipsa cum *ea* contineat rectum angulum qui est in semicirculo.

‘Secundum illam ergo qua instrumentum *ge* etc.’ Hic sciendum est ut diximus quod debemus ponere gnomonem in terra erectum super lineam meridiei ut dictum est superius qui sit 60 cubitorum vel palmorum et facere omnia que dicuntur in predicta figura. Et provenient anguli qui erunt equales predictis angulis quiscumque videlicet suo relativo ut autem proveniat numerus quem auctor in hoc loco ponit, scilicet ut cum gnomo *ge* est 60 partes, sit umbra | *gk* estivalis 12 partes... A 10va

[I skip ahead to Chapter 7, which has the next use of the Menelaus Theorem.]

[II.7]

| Capitulum septimum etc. “Proprietates linearum equidistancium.” A 11rb
Proprietates ut diximus sunt ea que ita propria sunt tocius linee quod non unius partis linee tantum.

‘Et summam.’ Per summam, illum quod contingit communiter in illis.

‘Eveniunt.’ Sicut de elevatione poli que non aparet.

'Aparent.’ Sicut umbra et prolixitas diei et brevitas eius.

‘Numeri temporum.’ Tempora vocat gradus equinoctialis per quorum scientiam sciemus divisiones eorum que sunt preter hoc ut horarum dierum et noctium et partes scilicet horarum et aliorum et nominabimus partes duodecim orbis signorum.

‘Que est a puncto equalitatis ad id quod sequitur,’ idest que incipit a capite arietis ut se secant circulus signorum et equator diei.

‘Ad id quod sequitur,’ scilicet ad taurum etc.

‘Unusquisque eorum etc.,’ idest unus cum uno et alius cum alio quorum unusquisque est equalis alteri.

‘*Zh* equatur *tk*’ exponere. Et *lk* equatur *mh* eo quod unusquisque eorum producitur a polo ad gradum orbis signorum quorum utriusque declinatio est equalis. Et *ek* equatur *eh* eo quod sunt equales duabus declinationibus illorum duorum graduum. Et *lt* equatur *mz* propterea quod unusquisque eorum est quarta circuli eo quod provenit unusquisque a polo ad equatorem diei, et anguli equantur angulis propterea quod anguli qui supponuntur eis sunt equales. Et est sciendum quod hec figura aliter deberet fieri scilicet ut triangulus *klt* esset super orizonta et triangulus *zmh* sub orizonte et tunc esset secundum modum quo elevatur aries.

‘Erunt equalium elevationum.’ Hec littera mala est et impropria, sed ita debet intelligi idest erunt equalibus duobus arcubus cum quibus

1221 12] 212 *sed. corr. exp.* A

elevantur isti duo arcus orbis signorum coniunctis, idest isti duo arcus equatoris diei coniunctis sunt equales illis duobus coniunctis.

‘Cum his duobus arcubus,’ idest orbis signorum.

‘Punctum *h c*ommune elevationibus eorum et orizonti,’ idest commune orizonti scilicet ut ibi communicent elevationes eorum et orizon.

‘Equalem in potencia,’ hic non proprie ponitur ‘in potencia,’ sed ita debet intelligi “in potencia” idest sicut per quartam circuli productam a polo ad equatorem diei inveniuntur elevationes horum duorum arcuum ita [reperitur] hoc in probatione huius figure per arcum *khl*. Hic notandum est Ptolomeum non posuisse figuram convenientem omnibus modis | A 11va quibus istud accidit. Abstulit enim illud quod est in medio et posuit quasi semper coniungeret unus arcus illos duos arcus elevationum et quasi illi duo arcus orbis signorum essent coniuncti semper. Sed si quis vult probare illud ita ut conveniat omnibus et continuis et disiunctis, ducat arcus duos a polo ad unumquemque duorum graduum orbis signorum terminancium illos duos arcus equales, et producat eos usque ad equatorem diei. Et erunt duo arcus qui sunt a gradibus ad equatorem diei equales eo quod sunt declinationes graduum duarum equidistancium a tropico. Et similiter ducat duos arcus ad eosdem gradus a loco in quo orizon secat circulum meridiei, et producat eos usque ad equatorem diei. Et erunt similiter duo arcus qui sunt inter orbem equatoris diei et illos gradus equales quia sunt declinaciones. Deinde argumentetur ostendendo quod proportio sinus complementi lateris subtensi recto qui sit ab arcu producto a polo ad complementum sinus alterius continentis rectum qui est ei continuus est sicut proportio complementi sinus tercii lateris ad sinum complementi quarte circuli. Et ita fiat in alio triangulo, deinde fiat permutatio proportionum. Et concludetur in ultimo quod proportio sinus complementi illius arcus qui est inter duas notas in equatore ad complementum sinus quarte circuli est sicut proportio sinus complementi alterius arcus eiusdem orbis equatoris diei ad sinum complementi quarte circuli. Et ipsi sunt minus quarta circuli, ergo sunt equales. Ergo si dupletur arcus elevacionis in circulo directo unius eorum, erit quod proveniet equale elevationibus amborum in eodem circulo. Et erit arcus parvus que prediximus equalis alii, et remanebit alius arcus equalis alii. Quod ut manifestius fiat, ponamus duos arcus se secantes supra quem sunt *agdb* et alius equatoris diei supra quem sunt *aezhtb*. [*Figura #27*] Et

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1261 reperitur] reperiuntur A **1262** posuisse] possuisse *sed. corr. exp.* A **1271** in quo] *iter.* A

1275 quod…1278 circuli] Gebir, I.15.

sit linea *ge* pars arcus qui producitur a polo transiens per punctum *g* usque ad orbem signorum. Et similiter arcus *gz* ille qui producitur a loco in quo orizon secat circulum meridiei transiens per idem punctum *g* usque ad orbem signorum. Et sit arcus *dh* a polo et *dt* a sectione orizontis et circuli meridiei. Et erit arcus *ge* equalis *dh* et arcus *gz* equalis arcui *dt* quoniam sunt arcus equalium spaciorum. Deinde modo per dicto argumentando probetur quod arcus *ez* arcui *ht*. Deinde dupletur arcus *ae*, et erit quod proveniet equale arcui *ae* et arcui *hb* simul quoniam sunt equales. Si ergo minuatur ex arcu addito super arcum *ae* ex quo provenit duplum eius qui sit *ek* arcus *ez* equalis arcui *ht*, remanebit arcus *zk* equalis arcui *tb*. Et illud est quod voluimus declarare.

‘Iam ergo declaratum est etc. in omni declinacione,’ idest orizonte declivi. Hec figura probatur per figuram sectorem et numeri qui in libro correpti sunt. Non sunt convenientes ad hoc ut illud quod ponit in fine et non proveniat, immo desunt duo minuta. Sed si opus fiat per numeros qui exponuntur in margine et erant prius in libro, proveniet illud. Unde credo tabulas cordarum non fore veraces.

'Ex duplo temporum etc.,’ idest cum elevatio arietis in orbe recto duplatur et minuitur elevatio inde eius in hoc orizonte, quod remanet est illum quo elevatur libra et virgo in hoc orizonte propter illud quod dictum est superius et probatum.

Deinde omnia plana sunt usque ad hanc litteras, ‘Ergo manifestum est que est a cancro ad finem sagitarii etc.,’ quod sic est intelligendum cum sol est in capite cancri, oritur sol capite cancri in orizonte orientali et capite capricorni in occidentali. Cum ergo iam occidit, iam elevatum est super orizonta quicquid elevatur de equatore diei cum omnibus signis que sunt a capite cancri usque ad caput capricorni. Et quia dies maior est quartuordecim horarum equalium et semis, et ipsa est maior dies in illo climate et quecumque hora equalis constat ex 15 gradibus equatoris diei qui elevantur super orizonta, tunc manifestum est quod medietas illa que est a capite cancri | usque ad finem sagitarii elevatur cum 217 temporibus A 11vb
idest gradibus et medio tempore que proveniunt ex multiplicatione

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1294 Deinde…1295 ht] Applying Gebir I.15 to the right triangles *gez* and *dht*, we find that the sine of the complement of *gz* is to the sine of the complement of *ge* as the sine of the complement of *ez* is to the sine of a quarter circle, and that the sine of the complement of *dt* is to the sine of the complement of *dh* as the sine of the complement of ht is to the sine of a quarter circle. The first ratio in the first proportion is the same as the first of the second proportion, therefore, the other ratios are equal. Since the sine of the complement of *ez* has the same ratio to the sine of a quarter circle as the sine of the complement of *ht* does, arc *ez* equals arc *ht*.

quattuordecim [et semis] in 15. Et similiter intelligendum de alia medietate que est a capite capricorni usque in fine geminorum. Et ideo dixit “Ergo manifestum est quod medietas orbis etc.”

Alia omnia plana sunt quia postquam manifestum est cum quot temporibus unaqueque medietas satis patet cum quot elevetur unaqueque quarta. Et similiter postquam notum est cum quot temporibus elevetur unumquodque duorum signorum quarte, satis constat cum quot elevetur tercium quoniam tota cum quarta cum tota quarta elevatur. Et postquam sciuntur elevationes quarte unius, sciuntur elevationes aliarum quartarum et signorum quartarum ut ostensum est per diminutionem elevationum signorum de duplo elevationum eorum in orbe recto. Et [modo] eodem quo docuit invenire elevationes signorum possunt inveniri elevationes partium signorum et partium partium.

‘Has quoque elevationes etc.’ Hic notandum est quod in orizontibus declinatis contingit diversitas elevationum duabus de causis, scilicet ex orizonte ipso et ex partibus signorum quoniam alie sunt elevationes orizontis regionis cuius latitudo est 20 graduum et alie cuius est 24. Et similiter alie sunt trium partium, alie quattuor. Et ipse iam ostendit qualiter inveniri possint elevationes signorum in regione una per illud quod premisit. Et quoniam totum illum faciendum est ad hoc ut in qualibet regione inveniantur et est valde laboriosum, subtiliatus est et invenit ingenium alium ut per unam solam figuram inveniri possint elevationes signorum in omni climate simul, et non fiat permutatio proportionum set quantitatum solum. Et ad illud premisit figuram istam qua docet qualiter sciatur illud quod elevationes signorum in orbe recto addunt super elevationes eorum in orbe obliquo.

‘Ergo ex hoc manifestum est quod [portio] *et* etc.’ Hic ideo verum est quoniam cum elevatur *tk* elevatur *te*, et cum elevatur *te em* elevatur, et hic in orbe recto. Et similiter in orbe obliquo cum elevatur *et* elevatur *tk*, et cum elevatur *tk* elevatur *nm* propter similitudinem arcuum. Ergo cum elevatur *te* elevatur *nm*.

‘Arcus vero similes etc.’ Hoc quasi per se notum ponit. Inde patet ex hoc esse verum quia cum sint orbium equidistancium et similes, simul pertranseunt terminum aliquem ita quod cum unus eorum incipit pertransire terminum et alius similiter et cum unus pertransit totum terminum et alius similiter, arcus *lkn* determinat portionem *en*, idest distinguit que portio *en* est superfluum quod est inter elevationes *et* in spera declivi et inter elevationes eius in spera recta arcuum, idest que *et*

1338 similiter] simile *sed. corr. exp.* A **1347** portio] proportio A **1355** et^2…1356 similiter] *mg. illeg.* A [taken from B] est de arcubus orbis signorum quem idest arcum *et* determinat punctum *e* et equidistans descripta supra *t*.

'Postquam prescivimus ista etc.' Hic facit figuram in qua ponit orbem meridiei et medietatem orbis orizontis et medietatem orbis equationis diei. Sed medietatem orbis signorum non posuit quia non fuit ei necessarium. Et est sciendum quod ipse posuit hic conversionem proportionis. Cum enim inter duos arcus *zt et* secent se duo arcus *zkl* et *ekh*, sequitur primum ut proportio corde dupli arcus *zh* ad cordam dupli arcus *ht* agregetur ex proportione corde dupli arcus *zk* ad cordam dupli arcus *kl* et ex proportione corde dupli arcus *el* ad cordam dupli arcus *et*. Cum ergo convertitur hec proportio, sequitur ut sit proportio corde dupli arcus *th* ad cordam dupli arcus *hz* composita ex proportione dupli arcus *lk* ad cordam dupli arcus *kz* et ex proportione corde dupli arcus *te* ad cordam dupli arcus *el*. Et hoc est illud quod posuit Ptolomeus. Et est notandum quod figura hec magne est utilitatis, cum per eam in omnibus climatibus facile possint reperiri elevationes. Et hoc ideo quoniam proportio corde dupli arcus *th* ad cordam dupli arcus *hz* in omnibus climatibus est una eo quod est proportio corde arcus dupli declinationis tocius ad cordam dupli arcus residui qui est usque ad polum. Et iterum | proportio corde dupli A 12ra arcus *kl* ad cordam dupli arcus *kz* in omni climate est una quoniam qualis est proportio que est inter eas cum arcus *zkl* transit super decenam primam in uno climate talis est in omni climate. Et similiter quando transit super decenam secundam talis est in omnibus climatibus qualis est in uno quod contingit propter declinationem que non variatur que est arcus *kl*. Arcus vero *et* diversificatur secundum diversificatem climatum eo quod est arcus additionis medietatis diei maioris super equalem vel equalis super minorem. Et hoc posuit Ptolomeus. Et quoniam proportio corde dupli arcus *th* ad cordam dupli arcus *hz* est nota, tunc cum componatur ex proportione corde dupli arcus *lk* ad cordam dupli arcus *kz* que item nota est et ex proportione corde dupli arcus *te* ad cordam dupli arcus *el* que est ignota, si dividatur proportio corde dupli arcus *th* ad cordam dupli arcus [*hz*] per proportionem corde dupli arcus *lk* ad cordam dupli arcus *kz*, quod proveniet erit proportio corde dupli arcus *te* ad cordam dupli arcus *el*. Et quoniam *el* est nota semper, erit *el* nota que est corda dupli arcus eius quod ascensiones orbis recti addunt super elevationes orbis obliqui. Cum ergo invenerimus illius corde arcum et

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1360 descripta] *mg.* A **1370** th] *tkh sed. corr. exp.* A | arcus[2]] *lk* ad cordam *add. sed. del.* A **1389** proportio] d *add. sed. exp.* A **1390** hz] *ht* A

1361 prescivimus] 1515 *Almagest* edition reads 'premisimus.'

acceperimus eius medietatem, erit notus arcus additionis quem minuemus de elevationibus orbis recti, et remanebunt elevationes orbis obliqui.

‘In omnibus locis declinationionis.’ Hic et ubicumque dicit ‘locis declinationis,’ intelliga obliquitatem orizontis.

‘Postquam igitur hec sunt quemadmodum narravimus etc.’ Quid velit hic agere videramus. Prius ponit numerum dupli arcus *th* et numerum corde ipsius et numerum dupli arcus *hz* et numerum corde eius. Deinde ostendit numerum dupli arcus *lk* et eius cordam et dupli arcus *kz* et eius cordam cum est super primam decenam. Deinde numerum amborum et cordas eorum cum est super decenam secundam idest 20 gradus. Deinde cum est super terciam idest 30. Postea cum est super quartam idest 40. Deinde cum super 50. Postea cum est super 60. Deinde cum est super 70. Postea cum est super 80 idest octavam decenam. Super nonam vero decenam transit arcus *thz* de quo prius locuti fuimus. Post hoc videndum est que sit proportio corde dupli arcus *lk* ad cordam dupli arcus *kz* secundum unamquamque decenam quarum nulla in nullo climato variatur, et hoc propter declinationem graduum orbis signorum que non diversificatur. Et per illam proportionem secundum unamquamque decenam dividenda est proportio corde dupli arcus *th* ad cordam dupli arcus *hz,* et quod proveniet ex divisione servandum est. Et quia sunt octo decene, ex divisione provenient octo proportiones. Cum per unamquamque illarum dividatur proportio corde dupli arcus *th* ad cordam dupli arcus *hz*, et unaqueque illarum est proportio corde dupli arcus *te* ad cordam dupli arcus *el* secundum suam decenam. Et duplum arcus *te* cum sit superfluum diei equalis super breviorem et illum superfluum secundum diversa climata sit diversum, sciendus est eius numerus in unoquoque numero, qui scitur per horas que superfluunt super diem equalem. Ut si superfluunt due hore, erit arcus *te* 30 gradus, si tres 45, et sic de aliis. Deinde dividendus est numerus ille per illas proportiones, et numeri inde provenientes erunt numeri corde dupli arcus *el* secundum illas octo decenas. Deinde inveniantur arcus illius corde secundum omnes numeros illos, et accipiatur medietas cuiuscumque arcus que erit *el* secundum decenas predictas quisque videlicet secundum suam decenam. Et hoc innuit Ptolomeus cum posuit lineam *te* 60 et ostendit que esset proportio 60 in arcu prime decene et secunde et sic deinceps ad numeros quos ipse posuit secundum unamquamque decenam. Quod solius exempli gratia posuit quoniam similiter ponere quemlibet alium numerum quemadmodum ipse in sequentibus ostendit cum ponit lineam equidistantem que est super Rodum pro exemplo.

1400 ponit] *mg.* A **1407** 80] postea cum est *add.* A **1413** th…1414 arcus] *mg.* A

‘Ergo ex hoc manifestum est nobis etc. dupli arcus *te* in omni declinatione.’ | Idest orizonte obliquo. Hic *te* accipitur pro arcu hec A 12rb figure. Cum vero dicit, ‘tunc sciemus elevationes arcus *te* dati,’ tunc intelligit arcum *te* orbis signorum quem posuit in precedenti figura.

‘Et quia proportio 60 ad 38 partes.’ Notandum est quod ipse utitur proportione permutata sicut est cum 60 dividitur per proportiones quas prediximus, proveniunt inde numeri ad quos est eius proportio quos etiam ipse posuit superius ita cum 38 dividuntur per easdem proportiones, et proveniunt inde numeri ad quos 38 proportionalis existat. Quare proportio ad illos numeros est sicut proportio 38 ad istos numeros. Et cum permutatur, est proportio 60 ad 38 sicut proportio illorum numerorum adinvicem et sunt numeri eius quod prius erat ignotum. Et hoc est illud quod ponit Ptolomeus. ‘Erit corda dupli arcus *el.’* In hac littera est defectus, et est supplemendum in ea hoc scilicet ille partes quas predixi.

‘Et eius medietas que est *el* in decena prima erit due partes.’ Hic de arcu loquitur, quia prius per predictas cordas inveniuntur arcus, deinde accipiuntur medietates arcuum et proveniunt numeri illi quos ipse ponit hic scilicet ut sit medietas eius idest arcus corde predicte in decena prima due partes et 56 minuta, et sic de aliis. Et est notandum quod numeri quos ipse posuit superius cum posuit arcum *te* 60 partes et hic cum posuit ipsum 37 partes et 30 minute in decenis sunt agregati, scilicet qui est in decena secunda continet illum qui est prime et secunde. Qui est in tercia continet illum qui est prime et secunde et tercie, et sicut de aliis. Unde cum aliquis vult scire numerum alicuius decene, et de numero eius minuat numerum precedentis decene et qui remanserit erit numerus illius decene, ut si voluerit scire numerum quarte decene, numerum tercie minuat quarte et qui remanserit erit quarte. Et similiter est in arcubus cum dicit eius medietas que est *el* in decena prima est due partes et 56 minute et in decena secunda 5 partes et 50 minute et sic de ceteris, scilicet secunda decena continet numerum prime, et tercia prime et secunde, et sic de ceteris. Quare cum aliquis vult scire numerum qui est alicuius decene, minuat numerum precedentis decene de numero sequentis et qui remanet est sequentis. Et manifestum est quod in decena none est 18 partes et 45 minuta. Hoc namque manifestum est ideo quia cum dies equalis superet in hac regione diem minorem duabus horis equalibus et semisse quod est triginta septem gradus et 30 minute cuius medietas est 18 gradus et 45 minute, tunc medietas diei equalis superat diei minoris

1456 Qui…1457 secunde] *mg.* A

1463 50] 1515 *Almagest* edition has 4 minutes, not 50.

medietatem hoc numero. Ergo cum decena octava superat 18 partibus, nona superat eo quod deest de tota et medietate illa scilicet 21 minuto quod cum predicto numero octave decene fit 18 partes et quadragintaquinque minute.

‘Et quemadmodum precessit etc.’ Reiterat hic quod superius dixerat de elevationibus decenarum in spera recta quas ponit secundum agregationem usquoque pervenit ad nonaginta. Et hoc ideo facit ut ostendat quare ostenderit qualiter inveniendum esset illud quo decena quelibet spere recte superet se ipsam in spera obliqua, scilicet quia si illum minuatur de elevationibus eius in spera recta, quod remanebit eius elevatio in spera obliqua erit. Et si illum quo superat se ipsam in spera obliqua additum fuerit elevationibus in spera recta, quod perveniet erit elevatio decene illi opposite in spera obliqua. Quod ideo contingit quoniam quantum dies equalis addit super diem minorem, tantum dies maior addit super diem equalem. Et ideo quantum ascensiones que sunt a capite capricorni usque ad finem geminorum minuuntur de 180 tantum addunt elevationes que sunt a capite cancri usque ad finem sagitarii.

[II.8]

‘Capitulum 8. De modo positionis,’ idest faciendi tabulas.

‘Ex eis quorum iam declarata est scientia etc.,’ idest per ea que demonstrata sunt de elevationibus unius quarte.

‘Sciemus que secuntur de elevationibus trium quartarum reliquarum.’ Quoniam cum scierimus elevationes quarte que est | a A 12va
capricorno ad arietem que sciuntur per diminutionem superflui de elevationibus eius in spera recta, sciemus elevationes quarte que est ab ariete ad finem geminorum, cum sint equales elevationes unius elevationibus alterius. Et oppositas harum duarum sciemus per additionem superflui que similiter sunt equales ut sciamus cum necesse est que sunt preter illud sicut horas diei et noctis et ascendens et domos duodecim et arcum diei et noctis etc.

‘Et describam in ea nomina signorum,’ scilicet in prima tabula et in secunda partes signorum 36, non quod partes signorum sint 36 tantum. Si sic intelligendum est cum sint duodecim arcus secundum duodecim signa, et in unaquaque arcu sint tres linee numerorum partium signorum, constat quod sunt 36 cum ter 12 36 faciat. Et hoc est quod dicit 36 partes signorum que superfluunt decem gradibus cum ter x sint 30.

‘Consequenter post unumquodque,’ idest post signum in prima tabula positum, ponuntur partes sue. Et in tercia tempora equatoris diei

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1492 demonstrata] est *add. sed. exp.* A **1503** 36[2]] modo *add. sed. exp.* A **1507** x] de *add. sed. exp.* A idest gradus qui elevantur cum omnibus decem partibus orbis signorum et minute temporum. Et in tabula quarta summas idest agregationes temporum etc.

[II.9]

Capitulum nonum. “De divisione eorum que secuntur,” idest que consequenter sunt scienda post scientiam elevationum.

‘Postquam narravimus etc. tunc omnia reliqua que sunt necessaria in hoc modo.’ Idest que scienda sunt necessario ex hoc modo quoniam per illud quod premissum est de elevationibus omnia ista que ipse ponit in hoc capitulo que quidem sunt pauca et ad sciendum levia.

‘Non indigentibus etc.,’ idest non oportet nos uti lineis etc. cum per ea que dicta sunt totum declaretur.

'Longitudinis diei.’ Longitudinem diei vocat spacium quod est ab ortu solis usque ad eius occasum et longitudinem noctis quod est ab occasu solis usque ad ipsius ortum.

'Partem quintamdecimam,’ idest cum diviserimus illa tempora per 15, numerus proveniens ex divisione erit numerus hora 12 equalium.

‘Partem duodecimam.’ idest cum diviserimus per 12.

‘Accipiemus etiam quantitatem hore temporalis facilius.’ [*Figura #28*] Quod hic dicit Ptolomeus sine demonstratione potuit, quod tamen hoc modo demonstrari potest. Sint duo circuli sese secantes *abgd* et *aegz*, et sit circulus *abgd* circulus signorum et circulus *aegz* circulus equatoris diei. Et signetur pars aliqua orbis signorum que sit *ah* et pars ei opposita et equalis que sit *gk*. Et quod elevatur cum *ah* in orbe recto sit *al* de equatore diei, et quod elevatur cum *gk* sit *gm* de equatore diei. Et quod elevatur cum *ah* in orbe declivi sit *an* et quod elevatur cum *gk* in orbe declivi sit *gc*. Et signetur inter *a* et *l* gradus oppositus *c* sitque *p*. Dico ergo quod duplum *cm* est *pn* et quod est superflui duplum inter elevationes *ah* in circulo directo et elevationes ipsius in circulo obliquo. Quod sic probatur. *Gm* est quod elevatur de equatore diei cum arcu *gk* in orbe recto. Ergo duplum *gm* est equale ei quod elevatur de equatore diei in orbe recto cum duobus arcubus orbis signorum *gk ah* eo quod ipsi sunt equales. Quod autem elevatur cum eis in orbe declivi est equale ei quod elevatur in orbe recto cum ipsis, ergo est equale duplo *gm*. Sed quod elevatur cum eis in orbe declivi est arcus *an* et arcus *gc*, ergo *an* et *gc* simul est equale duplo *gm*. Verum quod est equale duplo *gm* est equale duplo *gc* et duplo *cm*. Ergo cum *ap* et *gc* sint equales, quod probari potest ex hoc quod *pac* est semicirculus et *azg* est semicirculus et sunt unius circuli, quare sunt equales et sic ductis duabus diametris se supra centrum

1511 Et…1512 etc] *iter*. A **1539** elevatur de] *iter*. A secantibus duo anguli oppositi erunt equales, tunc simul ipsi duo sunt duplum *gc*. Relinquitur ergo ut duplum *cm* sit *pn*. Sed cum *cm* sit equalis *pl* quod demonstrari potest per semicirculos ut in precedentibus, tunc *pn* est duplum *pl*. Quare est duplum *ln* | Et quia *pac* est semicirculus, tunc A 12vb divisus per 15 continet horas equales 12. Si ergo *pn* qui superfluus est supra semicirculum *pac* dividatur per 12, habebimus quantum addendum sit de eo unicuique 12 horarum scilicet duodecimam eius. Sed quia idem est accipere duodecimam alicuius numeri et sextam medietatis eius, tunc si acceperimus sextam *ln* qui est medietas *pn*, proveniet illud idem quod provenit ex duodecima *pn*. Et *ln* est superfluum quod est inter elevationes *ah* in circulo recto et elevationes ipsius in orbe obliquo. Et ideo dicitur ‘cum acceperimus ex illo sextam etc.’

Et hec sexta additur super 15 cum pars est septentrionalis eo quod dies longior, et cum meridiana minuitur propterea quod dies est brevior. Post hoc probari facilius ita ut non sit in figura, idest hoc modo duo arcus *ah* et *gk* orbis signorum sunt equales et oppositi, ergo elevationes eorum in circulo directo sunt equales. Duplum igitur elevationum unius eorum est equale elevationibus eorum in circulo directo. Sed elevationes *gk* in circulo directo sunt arcus *gm*. Ergo duplum arcus *gm* est equale elevationibus eorum in circulo directo. Sed elevationum eorum duplum in circulo directo est equale elevationibus eorum in circulo obliquo simul. Sunt duo arcus *an* et *gc*, ergo duplum arcus *gm* est equale duobus arcubus *an* et *gc*. Et duplum arcus *gm* est equale duplo *gc* et duplo *cm*. Et duplum *gc* est *gc* et *ap* eo quod sint equales quod patet propter equalitatem semicirculorum. Reliquitur ergo ut *pn* sit duplum *cm*, et illum est quod voluimus declarare.

‘Inter summam consequentem partem solis que est diei.’ Hec littera sic intelligendi est. Considerandum est in qua parte idest gradu signorum sit sol ea die, et in die consideranda est in tabulis elevationum in orbe recto et in orbe obliquo summa elevationum, idest agregatio earum que est in directo partis solis et in nocte in directo partis opposite parti solis. Et ita intelligenda est summa que est diei, idest que in die accipitur a parte solis. Et similiter intelligitur que est noctis.

Totum quod sequitur usque ‘Et postquam equaverimus etc.’ planum est. Est pars ad quam pervenit numerus illius signi idest pars illius signi in quo est sol in die vel oppositi eius in nocte ad quam pervenit numerus.

1571 duplo[1]] gm *add. sed. del.* A

1573 Reliquitur] He has shown that *ap* and double *cm* equals *an*. The result follows by subtracting *ap* from these equals.

‘Quod si nos voluerimus invenire partem medii celi etc.’ Hec littera plana est. Sed ut quod hic dicitur planius fiat, geometrice hoc demonstremus. [*Figura #29*] Sint itaque duo circuli se secantes *abgd aegz*, et sit circulus *abgd* circulus signorum et circulus *aegz* [sit circulus equatoris diei]. Et sit pars orbis signorum oriens super quam est *n* que oriatur cum parte equatoris diei super quam est *h*. Et sit pars orbis signorum que est in circulo meridiei super quam est *t*, et pars equatoris diei in eodem circulo meridiei illa super quam est *k*. Dico ergo quod pars super quam est *t* mediat celum. Quod sic probavit cum toto arcu *an* elevatur totus arcus *ah*, sed arcus *kh* est quarta circuli, propterea quod *h* est in orizonte et *k* in circulo meridiei quod idem est quoniam omnis orizon secat equatorem diei in duo media semper et similiter omnis circulus meridiei. Et cum ipso scilicet arcu *kh* elevatur arcus [*tn*], diminuto arcu *kh* qui est quarta circuli, remanet arcus *ka* qui in circulo directo elevatur cum arcu *at* eo quod circulus meridiei omnis est orizon in circulo directo. Pars igitur *t* mediat celum cum pars *n* est oriens. Et ideo dicitur quarta circuli. Et quia multociens est inter partem orientem et arietis principium minus quarta circuli, ideo addendus est ortus circulus super partem orientem, et deinde minuenda est quarta circuli, et cum residuo intrandum est in tabulas circuli directi. Et pars orbis signorum que invenitur in directo illius est pars medians celum. Quod ut in predicta figura manifestum fiat. Ponamus partem orientem super quam est *t* cum parte super quam est *k* equatoris die. Et sit pars equatoris diei in circulo meridiei super quam est *l*, et sit pars orbis signorum que est in circulo meridiei cum ea super quam est *m*. Dico ergo quod pars orbis signorum | A 13ra
super quam est *ca* est medians celum. Ponam prius ut punctum *a* sit principium arietis. Et quoniam *ak* est minus quarta circuli, tunc ad hoc ut quartam circuli minuere possim, oportet me addere super *ak* totum circulum de quo dempta quarta una. Remanebunt *ak* et tres quarte que sint *azge*. Ostenso ergo quod arcus *kazge* sit equalis arcui *aegzl*, patebit propositum. Hoc autem sic demonstratur. Arcus *kl* est semii quarta circuli propter rationem predictam quia est ab orizonte ad circulum meridiei. Et arcus *ae* quarta circuli. Et sunt unius circuli, ergo sunt equales, ergo arcus *al* est equalis arcui *ke*. Dempto arcu *ak* communi, ergo arcus *kegl* est equalis arcui *alge*. Addito ergo arcu *ak* communi utricumque, erit arcus *kalge* equalis arcui *aegl*. Cum eo igitur fiat ingressum in tabulas circuli directi et invenietur in directo eius pars orbis signorum super quam est *m*. Et illud est quod demonstratur voluimus.

1586 dicitur] dicetur *sed. corr. exp.* A **1597** kh…1598 arcu] *mg.* A | tn] *illeg.* A

Item probari potest geometrice illud quod dixit prius scilicet ‘si voluerimus invenire partem medii celi accipiemus semper etc.’ hoc modo. Sint duo circuli se secantes *abgd aegz*. Et sit orbis signorum *abgd* et equator diei *aegz*. Et sit pars solis illa super quam est *h*. Et sit pars equatoris diei in medio celi super terram illa super quam est *t*. Et illa que est in orizonte de eo illa super quem est *l*. Et illa cum qua oritur in declini super quam est *m*. Et sit quarta circuli *mn*, ergo arcus *mn* est equalis arcui *kt* cum unusquisque eorum sit quarta circuli. Sublato ergo communi *km* remanet arcus *kn* equalis arcui *mt*. Ergo arcus *mtan* est equalis arcui *mnat*. Sed arcus *mtan* est arcus qui a meridie diei preteriti est elevatus cum parte solis. Ergo si egressio fiat in tabulas spere recte cum arcu *mnat* invenietur illud quod dixit per partem solis.

‘Manifestum est autem quod eis qui sunt sub uno etc.’ Idest per quot horas equales differt sol a linea meridei super terram aliquibus, per tot differt omnibus, et similiter sub terra. Et illis qui non sunt sub uno meridie, numerus diversitatis meridiei secundum partes equalitatis idest horarum equalium est equalis numero partium que sunt inter duos orbes scilicet meridiei, idest quot horis equalibus aut partibus hore equalis differt unus circulus meridiei ab alio, tot partes sunt inter illos duos orbes.

[II.10]

Capitulum 10. ‘De scientia angulorum etc. Postquam remansit de complemento.’ Idest ad complendum ea que narravimus etc.

‘Premittam propositum.’ Propositum vero quod ipse premittit est istud.

'Nos nominamus angulum etc.’ Istud sic intelligendum est, angulus rectus est ‘quem continent due portiones orbium maiorum cum communis locus sectionis eorum ponitur polus, et super ipsum polum describitur circulus secundum quodlibet spacium,’ idest vel magnum vel parvum, ‘et est arcus illius circuli’ qui describitur ‘quem comprehendunt due predicte portiones continentes angulum quarta circuli’ sive magnus sit circulus ille sive parvus.

‘Et universaliter dico quod proportio huius arcus ad circulum suum de quo ipse est secundum modum quem prediximus,’ idest secundum magnum spacium aut parvum, “est sicut proportio anguli quem continet declinatio duarum superficierum,” idest obliquatio unius earum ab alia incipiens a loco sectionis earum “ad quattuor angulos rectos.” Hoc dixit

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1628 est[2]] *k add. sed. exp.* A **1635** Idest] *supr. lin.* A **1655** prediximus] diximus *sed.* pre- *add. mg.* A

1631 Ergo…1632 mnat] It is unclear how this step follows since it assumes that *tm* equals *mn*, which has not been shown. Perhaps *‘mtan’* is a mistake by the author or scribe for *‘kmtan*.’

ut suppleret quod minus dixerat superius, dixerat autem superius de recto tantum. Hic autem innuit quod si fuerit arcus ille quarta circuli, angulus erit rectus. Et si minor quarta, angulus erit minor recto. Et si maior quarta, angulus erit maior recto. Et hoc innuit per proportionem.

'Erit quantitas partium arcus ad circulum suum,' idest de circulo suo, 'ad quattuor angulos rectos,' idest de quattuor angulus rectis.

'Angulorum autem etc.' Iam tractavit de angulis ex sectione orbis signorum et equatoris diei et sunt illi qui fiunt in duabus equalitatibus. Nunc autem intendit tractare de angulis qui proveniunt ex sectione orbis declivis | idest signorum et orbis meridiei, et ex sectione orbis orizontis et A 13rb
ipsius orbis signorum, et ex sectione orbis declivis et orbis magni descripti super duos polos orizontis idest transeuntis super sont capitum ab oriente ad occidentem et est in regione qualibet loco equatoris diei illis qui sunt sub equalitate.

'Et cum scientia,' idest per scientiam 'horum angulorum sciemus arcus huius orbis quos terminat,' idest qui sunt inter locum sectionis ambarum circulorum et polum orizontis idest sont capitum.

'Cum enim declarata,' idest omnia ista que prediximus manifeste scita fuerint. 'Erit locus eius,' idest scientie eorum, 'in hac scientia,' idest astrologia, 'et in eo' similiter erit magnus 'in quo est eius necessitas,' idest ipsa est necessaria, 'ad sciendam diversitatem que est inter locum lune secundum' quod videtur et 'considerationem.' 'Considerationem' vocat illum quod invenitur per regulas et tabulas et alia. 'Visum' [vocat] quod ipso visu percipitur 'et locum eius verum.' 'Scire [tamen] eam,' idest diversitatem predictam.

'Et quia anguli etc.' Tractaturus de angulis qui fiunt ex sectione orbis signorum et alicuius illorum trium orbium quos predixit. Cum ex omni sectione eorum fiant quattuor anguli, vult ostendere quod vero de omnibus illis tractare intendit, sed de uno tantum, et de quo uno insinuat, scilicet de illo qui est septentrionalis ab orbe signorum et orientalis. Ratio vero quare de uno tantum intendat hec est: quoniam scito illo uno cum ipse et alius qui cum eo est meridianus sint equales duobus rectis, diminuto eo de duobus rectis, restat ut ille alius sit notus cum ipse sit illud quod remanet de duobus rectis. Similiter ille cum suo socio septentrionali et occidentali equatur duobus rectis. Quare scito ipso scitur alius. Et illo alio scito, scitur compar eius meridianus occidentalis. Et ita uno scito sciuntur reliqui tres. Et ideo noluit tractare nisi de uno.

1670 sont capitum] This is the term he uses for 'zenith.'

‘Et quoniam declaratio etc.’ Continuat se ostendens de quibus angulis velit prius tractare et reddit causam quoniam facilius possunt sciri.

‘Et ostendemus prius quod puncta orbis signorum que sunt equalis intervalli,’ idest que habent equales declinationes faciunt hos angulos equales. Quod ideo est quoniam arcus qui proveniunt a polo ad ea sunt equales eo quod arcus illi a polo ad equatorem diei omnes sint quarte circuli. Tunc diminutis declinationibus equalibus, remanent illi qui sunt a polo ad illa puncta equales. Et cum sint, erunt similiter arcus qui subtenduntur angulis illis equales. Quare et anguli erunt equales.

'*Hb* equale *bt*’ ex positione, ‘*hk* equale *tl*’ quoniam sunt equales declinationes, ‘*bk* equale *bl’* quoniam punctum *b* est medium inter duo puncta *h* et *t* que equedistant ab equatore diei. Ecce in hac figura probat quod angulus *khb* [meridionalis] orientalis est equalis angulo *zte* meridionali orientali. Et hoc est illud quod ipse proposuit.

'Ostendam etiam quod etc.’ In hac figura vult demonstrare quod unus angulus septentrionalis orientalis cum alio angulo orientali septentrionali qui fiunt super duo puncta orbis signorum elongationis equalis ab uno puncto tropico sunt equales duobus rectis.

‘Erit arcus *dz* equalis arcui *ze’* propter rationem predictam, scilicet quod cum eorum declinationes sint equales, eis sublatis remanent isti duo arcus equales.

‘Angulus *zdb* est equalis angulo *zeb*’ propter eo quod equales arcus eis subtenduntur.

‘Et post scientiam eorum.’ In hac figura vult probare quod omnis angulus qui sit apud quodlibet duorum tropicorum cum circulo meridiei est rectus. Quod patet ex exemplo quod ponit et ex eo quod ipse proposuit de angulo recto scilicet cum subponitur quarta circuli angulo. Et hoc propterea quod punctum tropici ponitur polus, et transit circulus meridiei per polos illorum duorum circulorum.

‘Describam etiam circulum etc.’ Intendit demonstrare in hac figura quantus sit angulus qui sit aput punctum equalitatis autumpnalis idest caput libre, et probat quod est plus recto tantum quantum est declinatio tota. Et cum iste sit plus recto, et sint iste et ille qui sit in alio puncto equalitatis equales duobus rectis, tunc manifestum est quod ille alius tantum est minus recto quantum iste est plus. Et hoc est quod dicit ‘complementum eorum etc.’

1702 quod] est *add.* A **1705** subtenduntur] subtenditur *sed. corr. exp.* A **1709** meridionalis] septentrionalis A

‘Et | describam etiam circulum orbis etc.’ In hac figura intendit demonstrare quantitatem anguli provenientis ex sectione orbis meridiei et principio alicuius signi. Et ponit prius angulum qui sit apud principium virginis, et facta dispositione probat illum per figuram sectorem que probatio satis facilis est scienti figuram sectorem. A 13va

‘Ergo unusquisque duorum scilicet arcus *tek* et angulus *kbt* etc.’ Quod ideo est quoniam arcus *ke* est nonaginta partes cum sit quarta circuli et arcus *te* est vigintiuna pars.

'Et angulus qui est apud caput scorpii etc.’ Propterea quod eque distant ab equatore diei.

‘Et unusquisque duorum qui sunt apud caput tauri et caput piscis.’ Quod propterea est quoniam caput virginis et caput tauri equalis elongationis sunt a puncto tropici estivi. Quare angulus unius cum angulo alterius equatur duobus rectis ut probatur superius. Unde cum iste sit maior recto, ille est minor tanto quanto ille est maior. Et ita de aliis duobus qui fiunt ab alia parte.

‘In hac quacumque forma ponam arcum *zb* etc.’ Sicut ostendit quantitatem anguli qui sit ex sectione orbis meridiei et orbis signorum super punctum distans a puncto equalitatis quantitate unius signi, ita ostendit nunc quantitatem eius cum punctum distat ab equalitate quantitate duorum signorum. Et non est diversitas nisi in numeris tantum.

‘Iam vero manifestum est etc.’ Et est sensus quod sicut inveniuntur quantitates angulorum plurium partium ita et pauciorum quoniam sicut scimus quantitatem anguli unius signi ita scimus quantitatem unius partis et duarum et trium et sic de ceteris.

‘Sed in opere unius signi et signi,’ idest cuiuscumque signi est sufficientia quoniam non est diversitas in opere. Et est sciendum quod in numeris superius posito est defectus et ideo dixit “fere.” Et illa 20 secunda superflua sunt et fuerunt inventa in translatione Ysahac.

[II.11]

Capitulum 11. In hoc capitulo intendit tractare de angulis qui proveniunt ex sectione orbis signorum et orbis orizontis.

‘In climate dato.’ Hoc ideo dicit quoniam illi qui fiunt ex sectione orbis signorum et orizontis circuli recti sunt illi qui fiunt ex sectione orbis signorum et orbis meridiei. Et hoc ipse subiunxit dicens, ‘Manifestum est autem quod anguli etc.’ Et quare hoc eciam dixerit in sequentibus patebit quoniam necessarium erit ei hoc ‘ut autem sciamus invenire angulos in spera declivi etc.’ Sicut in superiore capitulo ita et hic premittit dicens

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1760 20] The Ptolemy text has 23 seconds **1761** in…Ysahac] The commentator mentions this source more than once later in the following books.

quod puncta orbis signorum equalis elongationis a puncto equalitatis faciunt angulos apud unum orizontem equales.

‘Unum’ dicit quoniam apud diversos orizontes non equales sed diversos faciunt angulos.

‘Et describam propter hoc etc.’ In hac figura ponit ambo puncta *z* et *k* pro uno scilicet capite libre, et hoc ideo quoniam aliter non poterat ostendere quod volebat.

‘*Zh* equale *kl*’ expositione.

‘Et *he* orizontis equale *el’* similiter orizontis propter equales declinationes duorum punctorum *h* et *l*. Et *ez* oriens equale *ek* similiter orienti quoniam *ze* elevatur cum *zh* et *ek* cum *kl*, et quoniam *zh* et *lk* sunt equales et elongationis equalis a puncto equalitatis, sunt eorum elevationes equales.

'Ergo angulus *ehz* etc.’ Hic ideo verum est quoniam cum angulus *ehz* et angulus *eht* sint equales duobus rectis et similiter angulus *elk* et angulus *kld* equales duobus rectis. Quare illi duo istis duobus sunt equales, tunc cum unus eorum sit equalis alii scilicet *ehz* angulo *elk*, reliquitur ut alius alii sit equalis.

‘Et dico quod duo anguli qui sunt apud duo puncta etc.’ ‘Puncta opposita’ vocat oppositorum signorum in eodem numero ut scorpionis et tauri in eodem numero quoniam primum unius primo alterius, secundum secundo, et sic de ceteris. Intendit in hac figura probare quod duo anguli qui fiunt apud duo opposita puncta ut dictum est quorum unus est orientalis et alter occidentalis sunt equales duobus rectis.

‘Nos namque si de-[scripserimus] etc.’ In fine huius littere est defectus ut dicitur ‘secantes’ et est addendum ‘patebit istud.’

‘Nam duo anguli etc.’ | quod hic dicit satis patet. A 13vb

‘Angulus vero *zad* etc.’ Hoc ex premissa figura patet.

‘Quapropter ambo etc.’ Hoc nunc satis manifestum est.

‘Et quia iam ostensum est etc.’ Hoc sic intelligendum. Nos ostendimus iam quod anguli qui fiunt in punctis orbis signorum equalis longitudinis a puncto equalitatis sunt equales in uno [orizonte], et nunc similiter ostendimus quod duo anguli qui fiunt in duo punctis oppositis quorum unus est orientalis et alter occidentalis sunt equales duobus rectis. Et ex istis duobus sequitur illud quod ipse ponit, scilicet ut anguli equalis longitudinis a puncto tropici orientalis cum occidentali sunt equales duobus rectis quod ideo est verum cum duo oppositi sint equales duobus rectis, orientalis cum occidentali, et alius qui est equalis longitudinis ab

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1802 orizonte] orizontum A **1807** quod…1808 rectis] *mg.* A

1797 Nam] 1515 edition has ‘tunc.’

equalitate ab alia parte sit ei equalis, et tantum distet a tropico quantum. Et primus patet verum esse quod ipse dicit.

'Tunc iam sequitur illud,' idest post illum vel propter illud.

'Quapropter cum scierimus.' Istum totum planum est.

'Angulo vero qui sunt ex puncto orbis signorum quod est equatoris diei.' Dicitur punctum orbis signorum et equatoris diei idem, propterea quod secant se super unum punctum quod est commune utrique.

'Et describam propter hoc circulum orbis etc.' Intendit in hac figura ostendere quantitatem angulorum duorum provenientium ex sectore duorum punctorum orbis signorum que sunt apud duas equalitates et orizontis. Et intelligendum est in hac figura quod arcus *be* est ille qui est a capite libre ad capricornum, et qui est arcus *ge* est ille qui est a capite arietis ad cancrum. Et arcus orbis meridei *dgz* est ille qui est sub terra cum caput arietis est in orizonte orientali et ideo non est nisi quod 54 partes, cum polus sit elevatus ab orizonte 36 partibus, et est arcus meridiei orbis qui est ab orizonte ad equatorem diei super terram 126 partes. Et idem est angulus *dez* 54 partes. Et arcus *dgb* orbis meridiei est ille qui est sub terra cum caput libre est in orizonte orientali. Et cum arcus *dz* sit 54 partes et arcus *bz* 23 partes et 51 minutum qui simul iuncti sunt 77 partes et 51 minutum, quare angulus qui sit a capite libre et orizonte est 77 partes et 51 minutum. Et ille qui sit a capite arietis et orizonte est 30 partes et 9 minutum cum arcus *dg* sit tantundem. Et hoc est illud quod dicit Ptolomeus in hac figura.

'Sed ut sit acceptio nostra etc.' Ostendit qualiter sciantur anguli qui fiunt a capite arietis et capite libre cum orizonte dato, nunc intendit demonstrare qualiter sciantur anguli orientalis qui fiunt ex capitibus signorum cum orizonte, verum etiam ex quibuslibet gradibus et partibus graduum cum eodem orizonte. Et sicut inveniuntur anguli in illo orizonte ita inveniuntur et in quolibet orizonte.

'Decem et septem partes et quadra- etc.' Sic invenit per elevationes huius climatis.

'Et erit unusquisque duorum arcuum *dgz zht* quarta circuli.' Hoc ideo verum est quoniam unusquisque eorum est productus a polo ad circumferentiam sui circuli scilicet orizontis quoniam poli orizontis sunt super circulum meridiei et super circulum *zht*.

'Et etiam quia parcium cancri etc.' Hoc ideo verum est quoniam punctum *g* orbis signorum cum distet ab equatore diei tot gradibus quot

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1809 tantum…quantum] **1823** est] *supr. lin.* A **1834** orientalis] *mg.* A

1813 Angulo…1814 diei[1]] The corresponding text in the 1515 edition reads, "Anguli vero qui proveniunt ex duobus punctis equalitatis orbis signorum..."

ipse ponit, et equator diei distet a puncto *z* qui est polus orizontis versus meridiem 36 gradibus. Tunc verum est quod linea *gz* est tot partes quot ipse ponit.

'Proportio corde dupli arcus *gd* ad cordam dupli arcus *dz*.' Hic Ptolomeus multa pretermisit prius, enim debuit istud premittere, scilicet proportio corde dupli arcus *zt* ad cordam dupli arcus *th* agregatur ex proportione corde dupli arcus *zd* ad cordam dupli arcus *gd* et ex proportione corde dupli arcus *eg* ad cordam dupli arcus *eh*. Deinde debuit convertere ita, tunc proportio corde dupli arcus *ht* ad | cordam dupli arcus A 14ra *tz* agregatur ex proportione corde dupli arcus *gd* ad cordam dupli arcus *dz* et ex proportione corde dupli arcus *eh* ad cordam dupli arcus *eg*. Hoc autem premisso, sequitur postea ut secundum septimum modum ut sit proportio corde dupli arcus *gd* tercie ad cordam dupli arcus *dz* quartam composita ex proportione corde dupli arcus *ht* prime ad cordam dupli arcus *tz* secundam et ex proportione corde dupli arcus *eg* sexte ad cordam dupli arcus *eh* quintam.

[II.12]

Capitum 12. 'Postquam iam restat etc.' In hoc capitulo intendit ostendere qualiter sciantur anguli qui fiunt ex orbe signorum et orbe descripto super duos polos orizontis ab oriente in occidente.

'Ex quorum scientia sciemus in omni etc.' Hoc sic intelligendum est punctum quod est super summitatem capitum est unus de polis orizontis. Et cum scimus angulos de quibus ipse loquitur hic, scimus arcum illius orbis qui est ab illo puncto usque ad sectionem illam ut sit angulus.

'Tunc ponam etiam etc.' Premittit hoc 'quod duo puncta orbis signorum equalis longitudinis a puncto tropico' et sunt elevationes temporum ipsorum 'ab utrisque lateribus orbis meridiei equales,' idest in orbe recto quoniam orbis meridiei et orizon circuli recti ut sepe dictum est est idem, 'quorum unum est ad orientem et alterum ad occidentem, faciunt arcus etc.' a duobus punctis tropici idest a duobus tropicis quoniam ex quo equidistant ab uno et equidistant ab alio.

'Angulorum quoque etc.' Equales duobus rectis secundum modum quem prediximus scilicet quod de duobus orientalibus et duobus occidentalibus debemus accipere orientalem de orientalibus et orientalem de occidentalibus quoniam ex sectione duorum circulorum fiunt quattuor anguli quorum duo sunt orientales et duo occidentales. Et hoc sit in puncto orientali et in puncto occidentali, et de illis omnibus debemus

1863 12] 'In omni declinatione,' idest in omni latitudine. 'Et in omni loco,' scilicet hora. *add. mg. a. m.* A accipere unum orientalis in puncto orientali et unum orientalem in puncto occidentali. Et iste est modus quem predixit.

‘Describam itaque etc.’ Hoc planum est usque ut dicitur ‘et sunt duo arcus qui secantur etc.’ Istud dicit affirmando non ponendo qui secantur a linea meridiei.

‘Duos arcus super unumquodque duorum positorum,’ idest quattuor. Etiam hoc innuit cum a *g* duos et a *b* duos.

‘Erit angulus *bgd* equalis angulo *bgz*.’ Hoc sit propterea quod duo arcus equidistanti subtensi eis sunt equales. Et sic intelligendum est in reliquis quattuor figuris que secuntur.

‘*Gd* equale *gz*,’ propterea quod a polo equatoris diei ad puncta orbis signorum equalis longitudinis a puncto tropico quoniam declinationes eorum sunt equales. Totum quod sequitur est planum.

'Et quia iam ostensum fuit etc.’ Hoc in secunda figura de angulis totum quod sequitur planum est.

‘Ostendam quoque quod cum elongatio unius puncti etc.’ Incepit tractare de angulis qui fiunt ex sectione orbis signorum et orbis descripti super duos polos orizontis, et probavit in prima figura illud quod proposuit de duobus punctis orbis signorum equalis elongationis a puncto tropico quorum elevationes temporum sunt equales, scilicet quod anguli qui fiunt in illis duobus punctis ex sectione predictarum circulorum orientalis occidentalis cum orientali orientali equatur duobus angulis rectis. Nunc autem intendit ostendere quod cum fuerint duo puncta orbis signorum ab utrisque partibus orbis meridiei que habeant tempora equalia, idest que pertranseant orbem meridiei in temporibus equalibus sive sint portiones illorum punctorum orbis signorum equales sive non (et ob hoc est illum quod dicit hic communius quam illum quod dixit prius), erunt tunc arcus orbium productorum a puncto summitatis capitum ad duo predicta puncta orbis signorum equales, et duo anguli qui fiunt apud illa duo puncta quorum unum est ad orientem et altera ad occidentem ex sectione predictorum orbium [erunt] equales duplo anguli qui est apud unum punctum orbis meridiei, idest qui sit ex circulo meridiei. Qui scilicet est de illis qui considerantur secundum situm equinoctialis qui omnes sunt orizontes | et circuli meridiei. Et hoc cum fuerit A 14rb
unumquodque duorum punctorum orbis signorum mediantium celum, idest que sunt in medio celi idest in circulo meridiei, ita quod unum erit principium portionis orbis signorum que est ad orientem et alterum eius que est ad occidentem.

1883 accipere] *mg.* A **1891** equidistanti] equidistans *sed. corr. supr. lin.* A **1904** orientali[2]] *mg.* A

‘Aut ad partem meridianam a puncto summitatis capitis,’ cum [sont] capitum est septentrionale ab orbe signorum, ‘aut ad partem septentrionalem ab eo,’ cum sont capitum est meridianum ab orbe signorum.

‘Sint ergo.’ Prius ponit quod sint ad partem meridianam. Et totum quod sequitur usque ad finem illius figure est planum.

‘Describam quoque.’ Hec iterum figura facilis est. Arcus qui est a *g* scilicet *ghl* et alius *gek* sunt illi qui sunt a summitate capitum, et arcus *bh* et alius *aez* sunt illi qui sunt orbis signorum. Et reliqua omnia que sunt usque ad finem huius figure plana sunt.

‘Describam quoque similem huius forme.’ In hac forma ponit ut punctum *a* portionis orientalis orbis signorum in medio celi sit in parte meridiana a puncto *g* quod est sont capitum, et punctum *b* portionis occidentalis orbis signorum in medio celi in parte septentrionali a puncto *g*. Totum vero quod sequitur distinctis arcubus orbis signorum et arcubus qui protenduntur a puncto sont capitum et angulis factis ab eis patet, et ita est ut in libro habetur.

‘Describam quoque ad id quod residuum est de hoc capitulo.’ In hac forma ponit duo puncta a quibus incipiunt due portiones orbis signorum scilicet *a b* e converso eius quod fuit in precedenti ipsam, scilicet ut *b* sit meridianum a puncto *g* et *a* septentrionale. Reliqua vero plana sunt usque ad finem distinctis arcubus et angulis ut in precedentibus, scilicet qui sint orbis signorum, qui orbis descripti super duos polos orizontis, qui orbis meridiei. Et similiter de angulis qui a quorum sectione fiant. Et est notandum quod ideo posuit portiones orbis signorum in figuris istis ita segregatas, quia aliter non poterat ostendere varietatem angulorum qui fiunt in figuris illis nisi hoc modo. Et est iterum sciendum quod portiones equidistantis que sunt ab utroque latere orbis meridiei dicuntur equales, propter quas etiam et duo anguli qui sunt in puncto *d* sunt equales, vel quia ipse sunt equalis longitudinis, vel propter similitudinem arcuum quoniam arcus minor qui est ab una parte est similis arcui maiori qui est ab eadem parte, et facit angulum equalem angulo cui subtenditur ab alia parte arcus ei equalis et eiusdem circuli. Et tota pars est minor arcus de suo circulo quota est maior arcus sibi similis de suo circulo. Et ideo facit angulum equalem angulo illius, imo eundem.

‘Et ex hoc declarabitur nobis etc.’ Hec littera sic est intelligenda. Levius possunt [inveniri] anguli provenientes ‘in orbe meridiei et orbe orizontis’ et arcus qui ‘sunt ex orbe declivi et orbe magno descripto’ super sont capitum ‘secundum modum quem prediximus,’ idest

1921 sont] sunt A

accipiendo septentrionalis orientalis. Et vult docere quod quando arcus protensus a polo orizontis ad orbem declivem est in circulo meridiei, tunc sciendum est angulus quem facit ille arcus [cum] orbe declivi, qui sciri potest per ea que dicta sunt de scientia angulorum qui fiunt ex sectione orbis meridiei et orbis declivis, et hoc est quod dicit 'in orbe meridei.' Et iterum [vult docere] quod sciendus est angulus qui sit ex sectione orbis transeuntis super sont capitum et orbe declivi in loco orizontis, qui similiter sciri ponit propter ea quod ille quem facit cum orizonte est rectus ideo quia transit per polum eius, et illi duo quos facit cum orbe declivi sunt equales duobus rectis. Cum ergo ille rectus quem facit cum orizonte sit pars maioris quem facit cum orbe declivi, scito minore et diminuto de duobus rectis, scitur quantus sit maior cuius pars est rectus, et hoc est quod dicit "et orbe orizontis." Et postquam isti duo anguli fuerint sciti quorum unus sit in circulo meridiei et alter in circulo orizontis a sectione illorum duorum circulorum et arcus duo quorum unus protensus est a sont capitum per circulum meridiei ita quod ipse est pars circuli meridiei ad orbem declivis et alter ab eodem | sont capitum ad A 14va orizontem in loco, ut cum orbis declivis cum quo facit angulum fuerint sciti, tunc omnes anguli et arcus qui fient in quarta illa leviter scientur. Et hoc intendit ostendere in hac littera: 'Nos enim cum de- etc.' Ponit hic figuram in qua ostendit illud quod diximus omnia plana sunt.

'Descripserimus super ea *aeg*,' idest orbem transeuntem per sont capitum. Totum planum est usque ad finem figure.

'Manifestum est autem quod cum nos scierimus etc. in omni declinatione,' idest latitudine qui sunt ad orbem meridiei idest qui fiunt ab oriente ad occidentem usque ad circulum meridiei, 'sciemus angulos et arcus qui sunt post orbem meridiei,' idest a circulo meridiei usque ad occidentem eo quod omnes duo quorum unus est orientalis et alter occidentalis sunt equales duobus rectis. Et per illud sciemus 'angulos et arcus reliquorum signorum qui sunt ante meridiem,' idest pre orbem meridiei, quod ita intelligatur ut est predictum. Intendit hic Ptolomeus docere qualiter sciantur anguli et arcus de quibus incepit loqui. Ponit exemplum unum in una linea equidistante cuius latitudo est 26 partes, et per figura sectore probat qualiter sciatur angulus qui provenit ex sectione orbis signorum et orbis transeuntis super sont capitum. Et ponit illum causa exempli ut sicut sit hic in uno signo et una linea equidistante, ita fiat in omnibus signis et in omnibus lineis equidistantibus.

1962 cum] est A **1970** minore] vel aliter et ille qui sit ab orizonte (ad *add. sed. exp.*) et ab orbe declivi qui est recto facit illum totalem angulum sit notus per ea que dicta sunt *adnot. mg. a. m.* A **1992** 26] 27 *sed. corr. supr. lin.* A

Omnia autem plana sunt usque ad locum ut dicitur ‘Ergo proiecerimus etc.’ Et hic prius tractat de scientia arcus, postea tractabit de scientia anguli.

Et est sciendum hic et ubicumque quod cum sunt sex quantitates [et] proportio duarum quarum componitur ex proportione quatuor reliquarum, tunc cum voluerimus invenire aliquam illarum quattuor ignotam, debemus prohicere ex proportione prime ad secundam proportionem duarum reliquarum, ut inveniamus ignotam. Ideo videndum est quid sit prohicere proportionem ex proportione. Cum ergo sunt sex quantitates et volumus hoc facere, debemus prius invenire proportionem prime ad secundam quod fit dividendo primam per secundam. Deinde debemus invenire proportionem tertie ad quartam quod itidem fit dividendo tertiam per quartam. Hoc facto dividenda est proportio prime ad secundam per proportionem tertie ad quartam, et quod provenit est proportio quinte ad sextam. Quod ideo contingit quoniam proportio prime ad secundam provenit ex multiplicatione proportionis tertie ad quartam in proportionem quinte ad sextam. Ergo cum una earum scienda est, si dividatur proportio prime ad secundam per aliam, proveniet altera, ut si dividatur per proportionem tertie ad quartam, proveniet proportio quinte ad sextam. Et similiter si dividatur per proportionem quinte ad sextam, proveniet proportio tertie ad quartam, que postea multiplicanda est in quantitatem notam et proveniet ignota vel per ipsam dividenda est. Et hoc est prohicere proportionem ex proportione. Et est sciendum quod si proveniunt tantum fractiones et numerus in quem multiplicande sunt fuerit integer, tunc integer descendet ad illas fractiones, et dividendo postea reducendus est ad integros per 60.

Et est notandum quod ipse facit hic tres figuras. Prima est ut doceat qualiter sciantur angulus qui fit ab arcu producto a sont capitum per circulum meridiei cuius etiam ipse est pars tunc et ab orbe signorum in circulo meridiei, et angulus qui fit ab arcu producto ab eodem puncto scilicet sont capitum et ab orbe signorum apud orizontem. Et scitur primus per illud quod ductum est de scientia anguli qui fit a circulo meridiei et ab orbe signorum. Et secundus per angulum qui fit ab orizonte et orbe signorum qui similiter scitur per ea que dicta sunt, et per angulum qui fit ab orizonte et ab arcu producto a suo polo idest sont capitum qui rectus est cuius utriusque quantitatem continet secundus angulus quem prediximus.

Secunda vero figura facta est ad sciendum arcum qui fit ab arcu producto a sont capitum ad aliquam partem orbis signorum. Et est causa exempli aposita | ut sicut hic invenitur in una linea equidistante et in una A 14vb
parte signorum et cum una hora, ita inveniatur in qualibet linea

equidistante et qualibet parte signorum et qualibet hora arcus predictus qui est inter arcum meridiei prime figure et arcum productum a sont capitum ad orbem signorum in loco orizontis similiter prime figure. Et ita possunt inveniri omnes arcus producti a sont capitum ad orbem signorum qui sunt in medio inter illos duos arcus in quarta una.

Et tercia figura facta est ad inveniendos angulos qui fiunt in eadem quarta inter illos duos angulos de quibus loquitur in prima figura. Et hoc per horas equales que sunt ante meridiem. Et ideo per horas equales quia sunt elevationes circuli directi eo quod circuli meridiei sunt orizontes in circulo directo.

Et per istas duas figuras sunt facte tabule que secuntur et per istas demonstrationes et proportiones que sunt in ipsis. Et ipse non fecit tabulas angulorum aliorum de quibus ipse superius tractavit nisi de illis qui fiunt ab arcu producto a sont capitum ad orbem signorum et ab ipso orbe signorum qui fiunt inter duos angulos prime figure et ante meridiem. Et ponit in eis.

Tabule vero ita facte sunt. Prius ponuntur arcus et anguli quos faciunt capita signorum cum circulo meridiei. Et ipse per primas figuras docuit invenire illos angulos cum proposuit et probavit quod anguli qui fiunt in duobus punctis equalis elongationis ab uno puncto equalitatis sunt equales. Et hic secundum latitudinem. Et postea similiter secundum longitudinem quod anguli qui fiunt in duobus punctis elongationis equalis ab uno tropicorum vel a duobus sunt equales duobus rectis. Et ostendit quod anguli qui fiunt a capitibus duorum tropicorum in circulo meridiei sunt recti, et illi qui fiunt a duobus punctis equalitatis sunt equales duobus rectis, et quod quantum unus eorum est maior recto tanto alter est minor recto. Et ostendit quantus sit unusquisque. Deinde per figuram sectorem ostendit quanti sint anguli qui fiunt a capite virginis et capite leonis in circulo meridiei. Et postquam isti anguli sciti sunt, sciuntur anguli qui fiunt a capitibus duorum signorum qui ab alia parte circuli que sunt elongationis equalis ab uno puncto equalitatis eo quod sint ipsis equales ut ipse premisit. Et sunt illi qui fiunt a capite scorpionis et capite sagitarii. Et similiter per eosdem sciuntur anguli a duobus capitibus duorum signorum que sunt equalis elongationis ab uno puncto tropici scilicet geminorum et tauri eo quod unus istorum cum uno illorum qui sibi est compar equatur duobus rectis. Quare si ille qui scitur minuatur ex duobus rectis, remanet ille alter. Et similiter istis duobus satis sciuntur duo alii qui fiunt a duobus capitibus duorum signorum alterius partis circuli que sunt equalis elongationis a puncto equalitatis scilicet pisci et aquarii.

2038 hora] *add. mg. a. m.* A

Quamobrem non fuit ei necessarium invenire nisi illos duo angulos quos faciunt caput virginis et leonis in circulo meridiei quoniam illis duobus satis sciuntur anguli sex reliquorum. Et anguli qui fiunt a capitibus omnium signorum in circulo meridiei sicut invenit et probavit ita positi sunt in omnibus tabulis climatum. Et sunt idem in omnibus tabulis quoniam illi qui sunt in tabulis primi climatis sunt in tabulis secundi et aliorum omnium.

Arcus vero sciuntur ex latitudine climatum sicut arcus primi climatis propterea quod ipsum est inter caput cancri et equinoctialem minuitur ex declinatione tota, et quod remanet est arcus cum caput cancri est in circulo meridei. Similiter sit in omnibus capitibus cum sunt in circulo meridiei. Et hoc in omnibus climatibus. Inventis his angulis et arcubus, inveniendi sunt anguli et arcus qui fiunt a capitibus signorum in orizonte cum arcu producto a sont capitum. Qui hoc modo inveniuntur. Scitur angulus qui fit a capite signi ab orizonte et minuitur de duobus rectis, et ille qui remanet est equalis suo opposito. Quare oppositus ille addatur | A 15ra
super unum rectum aut minuatur, et remanet angulus ille qui fit a capite signi et arcu predicto in orizonte eo quod angulus qui fit ab illo arcu et orizonte est rectus. Nunc ostendendum est qualiter anguli qui sunt in medio et arcus inveniantur. Ipsi vero inveniuntur per duas figuras quas causa exempli ponit in quibus per figuram sectorem operatur et per regulas alias quas premittit et probat. In illis vero regulis dicit quot duo anguli qui fiunt a duobus arcubus productis a sont capitum ad duo puncta orbis signorum quorum elevationes sunt equalis elongationis a puncto tropico equantur duobus rectis. Et probat illum, deinde dicit quod illi duo anguli sunt equales duplo anguli qui fit in circulo meridiei ab illo puncto cum fuerint duo puncta duarum portionum orbis signorum in circulo meridiei aut ad partem meridiei aut ad partem septentrionis. Et si punctum portionis orientalis fuerit meridianum a sont capitum et punctum portionis occidentalis fuerit septentrionale ab eo, addunt super duplum predicti anguli duos angulos rectos. Et si fuerint converso minuunt de duplo eius angulos duos rectos. Viderimus ergo qualiter iste regule observentur in tabulis istis.

Tabule cancri et capricorni sunt per se ita quod queque est per se. In tabula enim cancri duo anguli eius sunt equales duobus rectis orientalis scilicet et occidentalis, non ob alium nisi quia sunt equales duplo anguli qui fit a capite cancri in circulo meridiei. Ergo cum unus minuitur ex duplo illius et ille sit rectus unde duplum illius est duo recti, remanet. Et

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2088 hoc] *supr. lin.* A **2092** addatur] [the split word] **2094** orizonte] eo quod angulus qui fit a capite signi et arcu predicto in orizonte *add. sed. del.* va--cat A ob hoc illi duo sunt equales duobus rectis. Et similiter sit in tabula capricorni. In tabulis vero aliorum invenitur angulus orientalis et minuitur de duobus rectis, et remanet angulus occidentalis sui relativi, ut in leone et geminis. Ut vero orientalis sui relativi inveniatur, tali arte est procedendum. Minuendus est angulus orientalis inventus utputa leonis de duplo anguli qui fit a capite signi in circulo meridiei, et remanebit angulus eius occidentalis, quo diminuto de duobus angulis rectis remanebit angulus orientalis sui comparis. Et tamen attendendum in hoc aut anguli illi addunt super duplum illius duos angulos rectos aut diminuant. Unde postquam additi sunt duo anguli recti super duplum ipsius aut diminuti, postea diminuendus est predictus angulus. Si vero scire vult aliquis aut puncta illa sint septentrionalia aut meridiana a puncto summitatis capitum aut unum meridianum et alterum septentrionale, considere declinationem signi et ita poterit hoc scire. Et ita fit ut scitis angulis orientalibus leonis, virginis, et libre, sciantur anguli geminorum et tauri et arietis, et scitis angulis orientalibus scorpionis et sagitarii, sciantur anguli aquarii et piscium occidentales. Et sciuntur iterum anguli ipsorum occidentales modo quem prediximus et aliorum anguli orientales. Et cancri per se et capricorni per se.

'Et postquam etc. unaquaque spera,' idest uno climate. Ipse enim superius tractavit de climatibus non singulariter ponens aliquem locum nisi causa exempli. Nunc vero dicit se facturum librum in quo tractabit de locis qui inhabitantur in unoquoque climatum in longitudine et latitudine secundum considerationem eorum que aparent et accidunt in eis sicut de horis et elevationibus et reliquis. Quod vero sequitur totum planum est.

'Alexandriam,' quoniam secundum comperationem eius constituit horas aliarum et alia. Totum vero quicquid est usque ad finem planum est.

[Note at End of Commentary]

| 'Sciendum est quod in secundo libro est quiddam de proportionibus [A 96v] quod non est ibi bene enucleatum. Dicitur enim ibi in multis locis, 'Si [B 108v] ergo proiecerimus ex proportione alicuius quantitatis ad aliam quantitatem que sit composita ex proportionibus duarum quantitatum ad alias duas quantitates, proportionem unius earum ad aliam, proportio alterius ad alteram remanebit.' Quod ut bene appareat et manifestum fiat,

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2117 rectis] Et est sciendum quod quando invenitur in tabulis in loco angulorum sifre aut [180](illeg. A), tunc non est ibi angulus quoniam arcus orbus signorum est arcus productus a sont capitis adnot *mg. a. m.* A **2130** et arietis] *mg. a. m.* A **2137** inhabitantur] inhabitabantur *sed. corr.* A **2146** quantitatis] quantitas B **2148** proportionem] proportione B | proportio] proportionem B unum de exemplis ibi positis ponamus. Et qualiter proportio ex proportione prohiciatur ostendamus vel qualiter ex ea minuatur. Idem enim est proportionem ex proportione minui et ipsam ex ea prohici. Sit autem exemplum quod ponimus illud quod ipse ponit in fine undecimi capituli secunde dictionis libri Almagesti ubi dicit, 'Cum ergo nos prohicerimus ex proportione sexaginta duarum partium etc.” Hic itaque sic operandum est. Ponenda est prima sexaginta due partes et viginti et quatuor minuta, et secunda centum et viginti partis, et tertia centum et decem et septem partes et quatuordecim minuta, et quarta centum et viginti partes. Deinde multiplicanda est prima in quartam et secunda in tertiam. Deinde dividenda multiplicatio duarum per multiplicationem | A 96vb aliarum duarum, et quod egreditur de divisione est proportio sexaginta trium partium et quinquaginta duorum minutorum ad centum et viginti partes. Illa ergo proportio, idest ille numerus qui provenit ex divisione, si minor est notus et maior ignotus, multiplicanda est in minorem, et proveniet maior numerus. Et si maior numerus est notus et minor ignotus, per illam proportionem dividendus est maior numerus, et quod proveniet erit minor numerus. Et hec est regula universalis que observanda est in proiectione proportionis ex proportione et diminutione proportionis ex proportione. Et hoc modo agere est proportionem ex proportione eicere et minuere proportionem ex proportione. Et est sciendum quod cum dixi, “Si minor est notus et maior ignotus,” intellexi de duobus numeris qui supersunt ex sex numeris ex proportionibus duorum quorum ad alios duos composita est proportio primi ad secundum.

2150 Et] *om.* B **2155** sexaginta] *om.* B **2156** et[2]] *om.* A **2161** et…2162 duorum] *om.* B **2165** proveniet] provenit B | est] *om.* B **2167** universalis] universaliter B **2169** eicere…2170 proportione] *mg. a. m.* A **2170** dixi…2171 Si] dixisti B

Vatican Commentary Figures

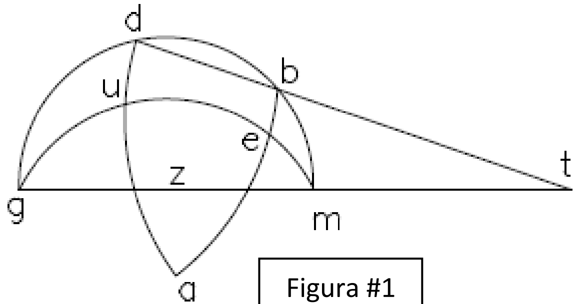
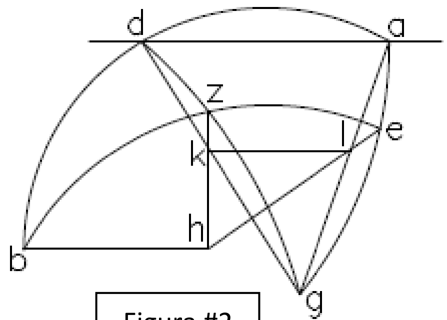


Figura #1



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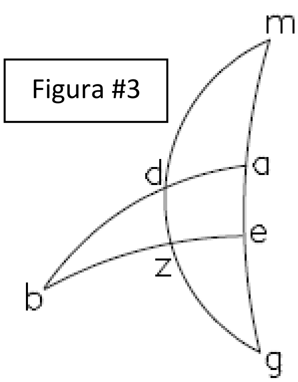

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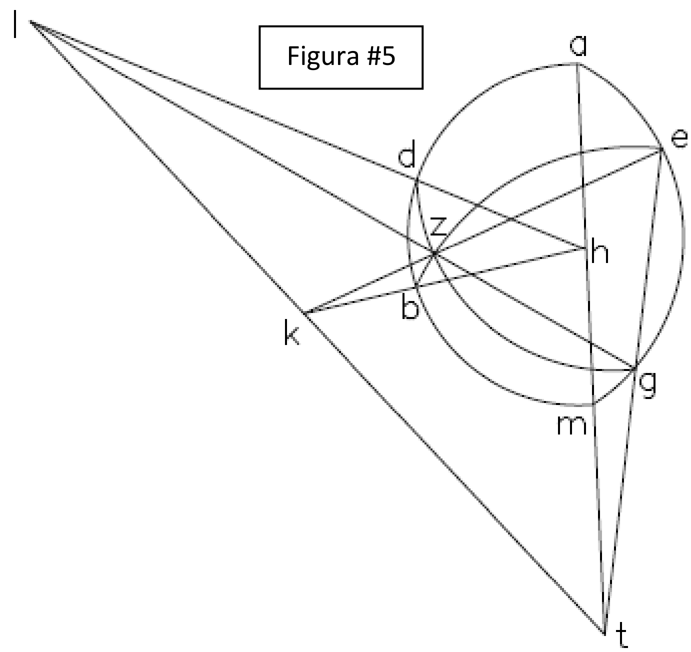

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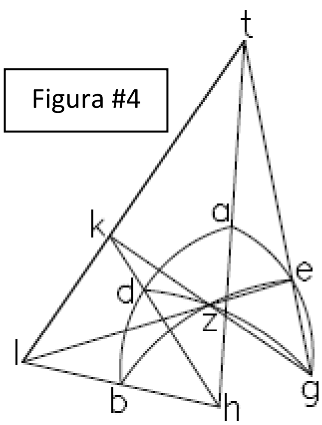

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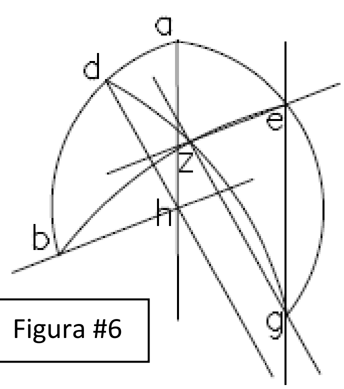

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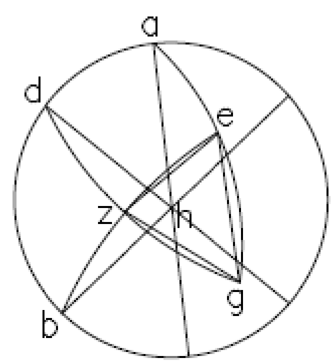

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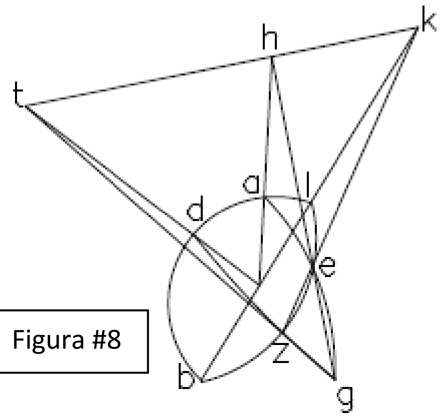

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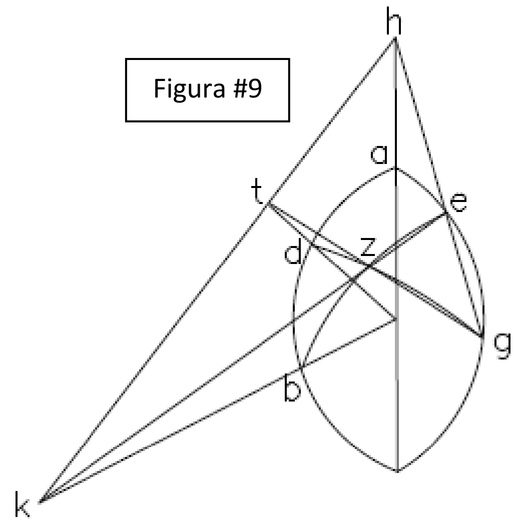
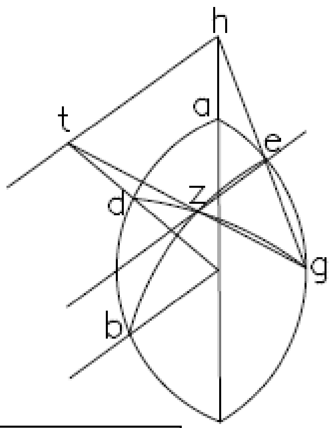


Figura #9



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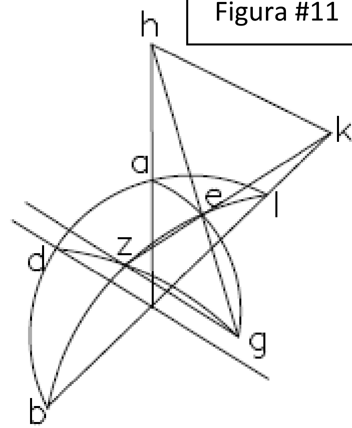

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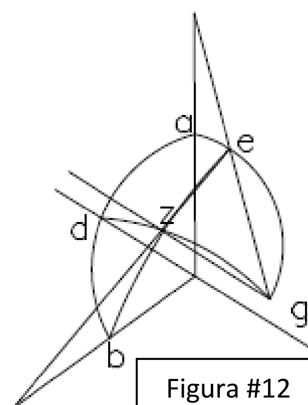

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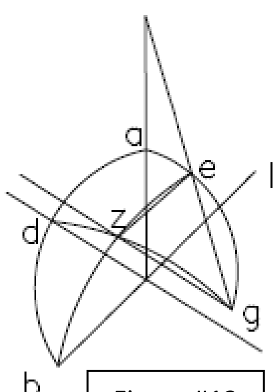

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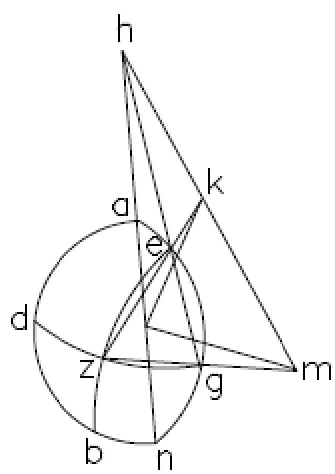

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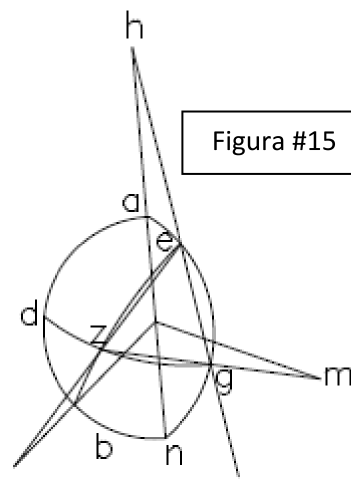

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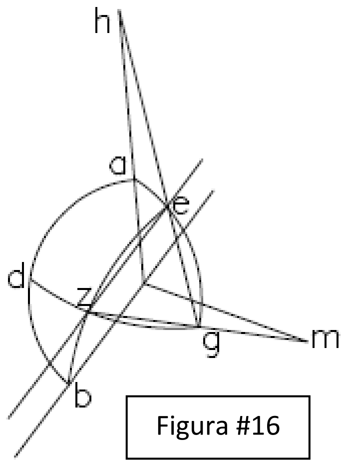

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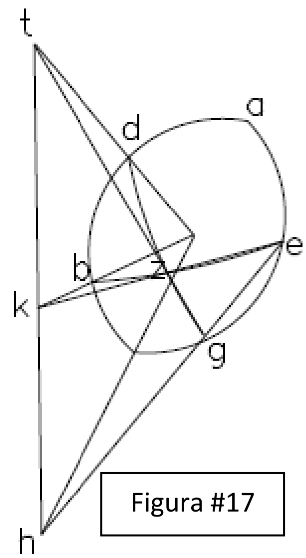

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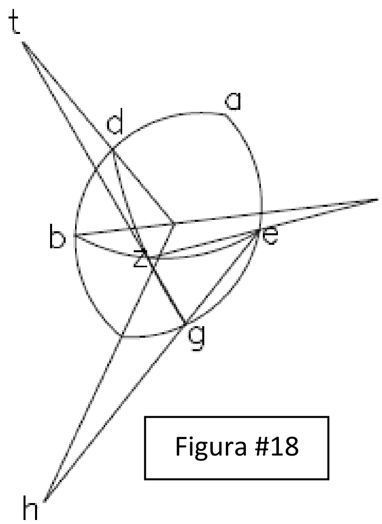
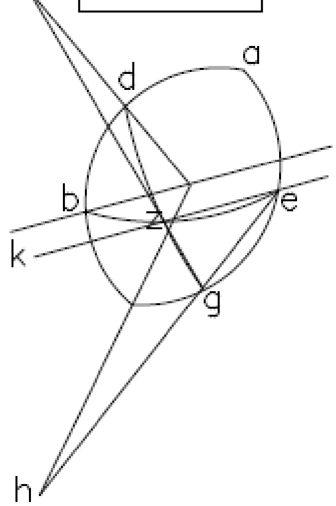


Figura #18

Figura #19



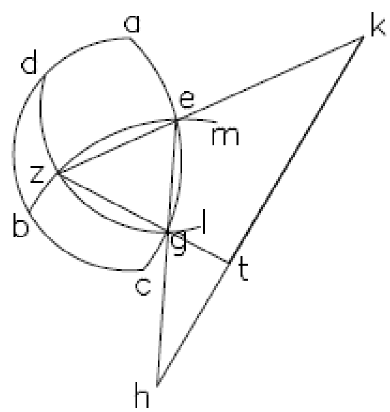
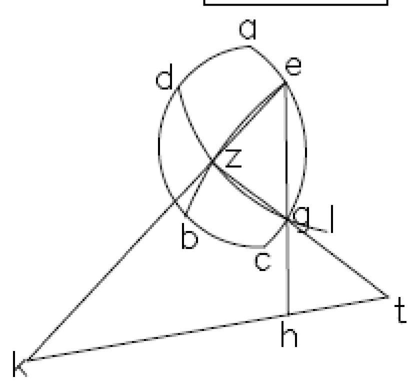
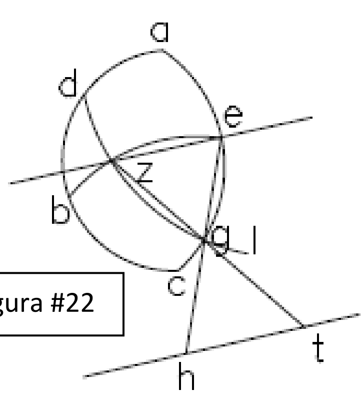


Figura #20

Figura #21





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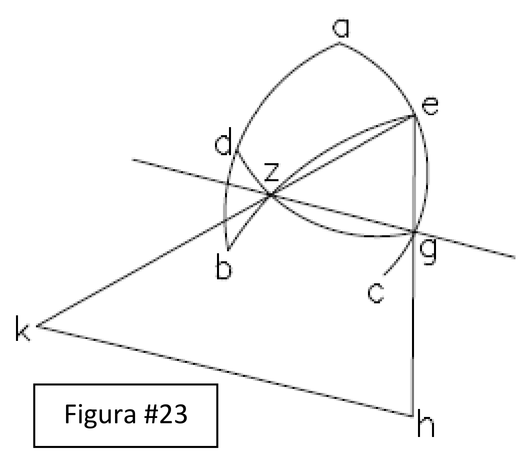

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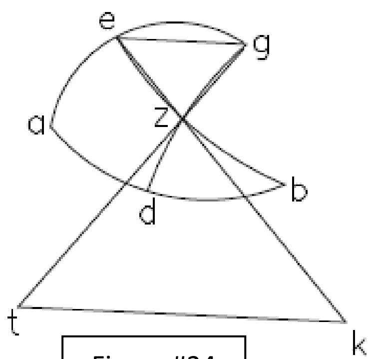

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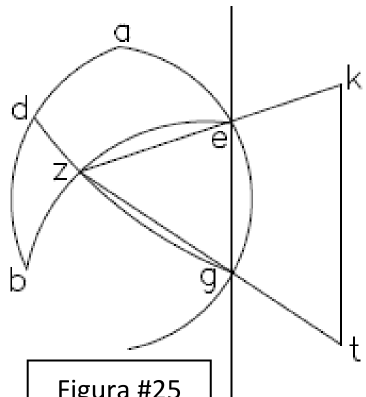

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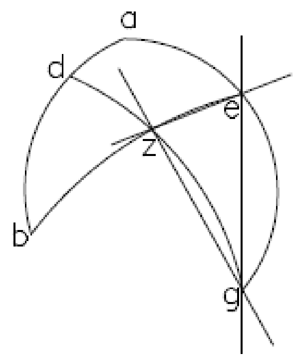
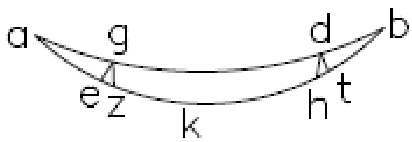


Figura #26



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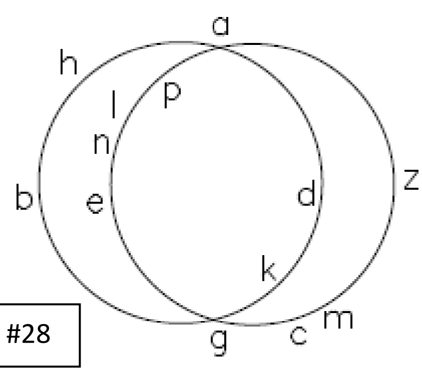

Figura #28

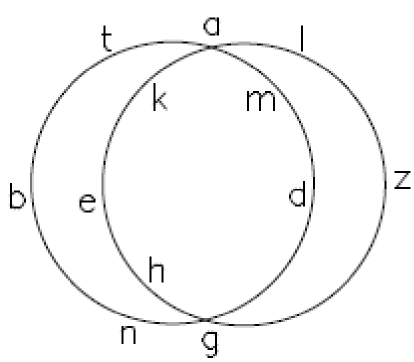

Figura #29

Appendix G: Simon of Bredon's Commentary, Books I-II

The following manuscripts contain this work:

Cambridge, University Library, Ee. III. 61 = P

Oxford, Bodleian Library, Digby 178 = R

Oxford, Bodleian Library, Digby 168 = S

For Book II, I have only transcribed material from R.

Commentum Magistri Symonis Bredon Super Aliquas Demonstrationes *Almagesti*

[I.13]

| Nunc superest ostendere quanta sit maxima declinatio ecliptice ab P 43r
equinoctiali et per consequens quantum distat polus zodiaci a polo mundi. R 42r
Hoc autem sciri potest per quoddam instrumentum. Sit igitur hec conclusio 13ª.

Instrumentum componere per quod maxima declinatio ecliptice ab equinoctiali certitudinaliter poterit mensurari.

Huius instrumenti compositio est hec: capiatur una lamina enea rotunda tante spissitudinis quod poterit concavari instar matris astrolabii pro receptione unius alterius laminis. Qua modo dicto concavata dividatur eius limbus in 360 partes equales et quelibet pars in quotquot poteris fractiones. Deinde in extremitatibus diametri minoris lamine erigantur due regule parve equalis altitudinis et latitudinis a quarum mediis punctis in radicibus earum exeant due lingule cum extremitatibus gracilibus tangentes divisiones in limbo lamine maioris in punctis oppositis. Et circumrotetur minor lamina circa centrum lamine maioris ita quod lamina minor et limbus maioris appareant in superficie una. Deinde cum voluerimus per hoc operari, erigatur aliquod perpendiculum fixum in loco non moto in quo pendeat instrumentum cum armilla in capite eius defixa et equetur cum filo alicuius perpendiculi quousque habemus filum transiens per punctum capitis a quo incipiunt divisiones. Transiat etiam per punctum oppositum in parte inferiori, et sit illa superficies que est facies instrumenti equedistans linee meridianali quod equari potest. Sed sub instrumento sic pendente sit superficies equedistans horizonti fixa in qua protrahitur linea meridianalis. Si tunc filum perpendiculi motum super faciem instrumenti aliquando ex parte meridiei aliquando ex parte septentrionalis continue descendat super illam lineam meridiei signatam in superficie equedistante horizonti, tunc quantum ad hoc verissima erit equatio instrumenti. In diebus igitur solsticiorum sole exeunte in meridie elevetur vel deprimatur lamina inferior quousque obumbretur tota regula inferior per umbram regule superioris. Et quotquot fuerint partes inter superiorem lingulam et caput instrumenti tanta est distantia solis a zenith capitis.

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4 Nunc…223 tria] *om.* S **7** 13a] 9 *add.* R **20** voluerimus] volueris P **21** pendeat] pendat *sed. corr. supr. lin. a. m.* P **22** habemus] huiusmodi P **25** equari] possunt *add. sed. exp.* P **29** continue] continuo P **30** equedistante] equidistanter P

Item compositio alterius instrumenti quod levius et vicinius operatur ad ostendendum effectum est hoc. Capio laterem lapideum vel ligneum quadratum competentis magnitudinis tanteque spissitudinis ut supra sue spissitudinis superficiem erigi possit absque declinatione aut tortuositate. Tunc iuxta secundo eius latera conterminalia in superficie eius quadrata, protrahantur due linee recte predictis lateribus equidistantes in quarum concursu posito centro describatur quarta circuli quam predicte linee necessario terminabunt. Hec autem quarta in 90 gradus equales dividatur cum divisione graduum in fractiones alias si sit possibile. Deinde tornatis duabus paxillis parvis piramidalibus equalibus in magnitudine et grossitie, figatur alter illorum in centro qui sit in angulo superiori meridiano et reliquus figatur in altero extremo eiusdem linee descendentis. Stetque instrumentum super superficiem equidistantem horizonti ita quod eius facies in qua describitur dicta quarta sit equidistans linee meridiei, et libretur linea inter paxillos per filum perpendiculi. Deinde sole exeunte in meridie videatur ubi cadit umbra paxilli superioris in circumferentia ut per umbre medium solis elevatio indicetur. Per has igitur considerationes quas in tropicis estivalibus et hyemalibus frequenter fecimus in pluribus | revolutionibus annorum, R 42v invenimus illas | longitudines easdem esse totaliter non mutatas ita quod P 43v invenimus semper distantiam inter tropicos 47 gradus et plus quam 40 minuta, minus tamen quam 45 minuta. Et hec quidem consideratio convenit considerationi quam consideravit Archusianus philosophus quam Abrachim operatus est. Illud enim quod est inter duos tropicos est fere 11 partes secundum quantitatem qua linea orbis meridiei est 83. Huius autem distantie medietas est arcus inter duos polos equinoctialis et ecliptice interceptus.

[I.14]

Et quoniam post hoc demonstrabimus quanta sit declinatio cuiuscumque gradus zodiaci ab equinoctiali quantus scilicet sit arcus coluri transiunctis per polos mundi interiacens equinoctiali et gradui ecliptice cuicumque, ideo oportet ut conclusiones aliquas ad hoc utiles premittamus.

Sit hec conclusio: protractis duabus rectis lineis angulum quemcumque causantibus si ab earum terminis non coniunctis alie due recte linee sese intersecantes in principales lineas reflectantur, utralibet reflexarum alterius conterminabilem sic dividet ut proportio totius linee

36 Item] Aliud instrumentum *adnot. mg. a. m.* R | vicinius] vicinus P **37** ostendendum] eundem P | hoc] hec P **46** figatur] figantur R **59** duos] duo P **69** conclusio] vide plura infra eversis 24 foliis *adnot. mg. a. m.* P sic divise ad eam sui partem que cum alia linea principali coniungitur ex duabus proportionibus componetur scilicet ex proportione totius reflexe sibi conterminalis ad eiusdem reflexe partem sibi non conterminalem et etiam ex proportione quam ad reliquam reflexam habet illa eius pars que conterminalis est cum reliqua linea principali.

Ut verbi gratia. [*Figura #1*] Proportio *ga* ad *ae* componitur ex proportione *gd* ad *dz* et ex proportione *bz* ad *be*. Protrahatur enim *eh* equidistanter *gd* linee. Tunc per 4^{am} 6^{i} proportio *ga* ad *ae* est sicut proportio *gd* ad *he*. Posita ergo *dz* media inter *gd* et *he,* erit proportio *gd* ad *he* composita ex proportione *gd* ad *dz* et *dz* ad *he.* Sed *dz* ad *he* est sicut *bz* ad *be* per 4^{am} 6^{i}. Ergo proportio *ga* ad *ae* componitur ex proportione *gd* ad *dz* et *bz* ad *be*, quod fuit propositum. Et hec vocatur kata coniuncta.

[I.15]

Alia conclusio est hec: descendentibus duabus rectis lineis ab aliquo angulo sicut prius et reflexis ab earum terminis aliis duabus sese secantibus usque ad alterutras principales protractis, utralibet reflexarum conterminalem alterius sic secabit ut proportio partis inferioris cuiuslibet linee principalis ad partem superiorem eiusdem ex duabus proportionibus componetur scilicet ex proportione partis inferioris linee reflexe sibi conterminalis ad partem eius superiorem et ex proportione partis inferioris alterius linee principalis ad eandem totam lineam principalem.

Verbi gratia. [*Figura #2*] Proportio *ge* ad *ea* componitur ex proportione *gz* ad *zd* et ex proportione *bd* ad *ba*. Ducatur enim *ah* equidistanter line *eb* ad quam protrahatur linea *gd*. Erit ergo proportio *ge* ad *ea* sicut *gz* ad *zh*. Posita ergo *dz* media inter *gz* et *zh* erit proportio *gz* ad *zh* composita ex proportionibus *gz* ad *zd* et *zd* ad *zh.* Sed proportio *zd* ad *zh* est sicut proportio *bd* ad *ba* propter similitudinem triangulorum *dbz dha*. Ergo proportio *ge* ad *ea* componitur ex proportionibus *gz* ad *zd* et *bd* ad *ba*. Et hoc est quod volumus demonstrare et hec est kata disiuncta.

[I.16]

Alia conclusio est hec. Continuatis in circulo duobus arcubus quorum uterque semicirculo extet minor, si ab eorum communi termino producatur diameter, ipsa cordam inter reliquos arcuum terminos protractam secabit secundum proportionem corde dupli arcus unius ad cordam dupli arcus alterius.

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73 partem] partemque R **81** Posita…82 he^{2}] *om. (hom.)* R **90** cuiuslibet…91 linee] *add. supr. lin. a. m.* P civis sit littere *add. sed. del.* P **96** proportione^{1}] partis inferioris *add. sed. del.* P **98** et…99 zd^{1}] *add. mg.* R **102** disiuncta] a b a *add.* PR

Sint enim duo arcus *ab bg* a quorum termino *b* communi. [*Figura #3*] Ducatur *bd* diameter dividens *ag* cordam in puncto *e*. Deinde protrahantur *az gh* perpendiculares super diametrum quia ergo *az gh* | P 44r sunt linee equidistanter inter quas cadit linea *aeg*. Erit propter similitudinem triangulorum *aez geh* proportio *az* ad *gh* ut *ae* ad *eg*. Ergo et proportio dupli *az* ad duplum *gh* ut *ae* ad *eg*. Sed duplum *az* est corda dupli arcus *ab* et duplum *gh* est corda dupli arcus *bg*. Ergo liquet propositum.

[I.17]

Alia conclusio est hec. | R 43r Quocumque arcu noto in duos arcus diviso si proportio corde dupli arcus unius ad cordam dupli arcus alterius nota fuerit, uterque partialium arcuum notus erit.

Quilibet arcus de quo loquimur aut loquemur in huiusmodi conclusionibus intelligitur esse minor semicirculo, et ideo hanc rem memorie commendemus. [*Figura #4*] Sit ergo *abg* arcus notus et proportio *eg* ad *ae* nota per premissam. Et ducatur a *d* centro *dz* perpendicularis super *ag*. Cum ergo *ag* corda sit nota, erit angulus ei oppositus in centro notus respectu quatuorum rectorum. Ergo et eius medietas qui est angulus *adz* erit nota. Ergo totus [tri]angulus orthogonius *adz* tam in lateribus quam in angulis erit notus. Sed cum *ag* sit nota et per precedentem proportio *ae* ad *eg* est nota, ergo *ae* est nota et tota *az* nota. Ergo *ez* erit nota. Sed et *zd* est nota et angulus *z* rectus, ergo triangulus *ezd* est totus notus per dulkum et per tabulas de corda et arcu imaginando circulum describi circa *d* ad quantitatem de angulus. Ergo *edz* est notus. Sed etiam totus angulus *adz* prius fuit notus. Remanet ergo angulus *adb* notus. Est ergo *ab* arcus notus, et per consequens *bg* arcus residuus est notus cum totum compositum fuit notum.

[I.18]

Alia conclusio est hec. Si ab altero termino alicuius arcus semicirculo minoris linea educta arcum illum secaverit fueritque protracta ulterius quousque diameter per reliquum eiusdem arcus terminum transiens cum illa occurrerit, erit proportio totius linee sic arcum secantis ad partem eius extra circulum sicut proportio corde dupli arcus totius resecti ad cordam dupli arcus inter dictas lineas intercepti.

[*Figura #5*] Ductis enim perpendicularibus *bz gh* super diametrum *ead*, erit proportio *gb* ad *eb* ut *gh* ad *bz*, ergo ut duplum *gh* ad duplum *bz*.

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110 Ducatur] duc *add. sed. exp.* P **131** et[1]] *add. supr. lin. a. m.* P ductis *add. sed. del.* P **133** etiam] et P

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131 dulkum] I am not aware of what this term means.

Sed duplum *gh* est corda dupli arcus *gba* et duplum *bz* est corda dupli arcus *ba*. Ergo patet propositum.

[I.19]

Alia conclusio est hec. Diviso arcu per lineam occurrentem diametro sicut prius si nota fuerit maior portio dicti arcus fueritque nota proportio corde dupli arcus totius divisi ad cordam dupli arcus inter lineas eductas inclusi, ille arcus inclusus inter lineas notus erit.

[*Figura #6*] Protrahatur *dz* perpendicularis super *bg.* Quia ergo arcus *bg* est notus, erit angulus *bdz* qui est medietas anguli *bdg* notus. Sed etiam *bz bd* latera sunt nota. Ergo totus triangulus orthogonius erit notus. Et quia *gb* est nota et proportio *ge* ad *eb* nota, erit propter hoc *eb* nota per disiunctionem proportionis, quia in proportione illa nota auferendo minorem terminum de maiori, residuum termini maioris se habebit ad terminum minorem sicut *gb* ad *be*. Cum ergo *zb* sit nota erit tota *ze* nota. Sed etiam *zd* est nota, ergo totus orthogonius *edz* in lateribus et in angulis erit notus. Ergo notus est angulus *edz*. Sed prius fuit notus angulus *dbz*. Ergo notus est angulus residuus scilicet *adb*. Erit ergo arcus *ba* notus, quod fuit propositum.

[I.20]

Egredientibus ab uno communi termino in superficie sphere duobus arcubus duorum circulorum magnorum quorum uterque semicirculo extet minor, si ab eorum disiunctis terminis alii duo arcus circulorum magnorum sese secando in principales arcus reflectantur, uterque reflexorum reliquum reflexum et arcum principalem in quem reflectitur sic secabit ut proportio corde dupli arcus portionis illius arcus alterius principalis que est a termino communi remotior ad cordam dupli arcus portionis residue eiusdem principalis ex duabus proportionibus componetur scilicet ex proportione corde dupli arcus portionis reflexi sibi conterminalis ad cordam dupli arcus portionis residue eiusdem reflexi et ex | proportione corde dupli arcus portionis illius alterius arcus principalis P 44v que a termino communi est remotior ad cordam dupli arcus totius principalis eiusdem.

[*Figura #7*] Id est proportio corde dupli arcus *ge* ad cordam dupli arcus *ea* componitur ex duabus proportionibus scilicet ex proportione corde dupli arcus *gz* ad cordam dupli arcus *zd* et ex proportione corde dupli arcus *bd* ad cordam dupli arcus *ba*. Sit enim *h* centrum sphere a quo

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149 portio] proportio R **150** divisi] *add. supr. lin. a. m.* P **154** etiam] et P **155** et] etiam *add.* R **156** in] ex *add.* sed *del.* P et *iter. supr. lin. a. m.* P **159** etiam] et P **160** dbz] *bdz* R **171** principalis…173 eiusdem] *add. mg. a. m.* R **173** residue] residui R **177** Id est] idem P **179** dupli[1]…180 cordam] *add. mg.* P **180** dupli arcus[1]] *om.* P ad puncta *b z e* ubi se secant circuli protraham tres lineas *hb hz he.* Et producam lineam *ad* ulterius quousque concurrat cum | *hb* semidiametro R 43v protracta ulterius in *t* puncto. Et protraham lineas *ga gd* secantes lineas *hz he* in punctis *k l*. Erunt ergo puncta *t k l* in eadem linea recta per 3^{am} 11^{i} propter hoc quod sunt tam in superficie trianguli *agd* in infinitum extensa quam in superficie circuli *bze* in infinitum extensa. Ille enim due superficies secant se super lineam unam rectam per 3^{am} 11^{i} que quidem linea est *lkt*. Arguendo ergo in lineis rectis cum kata disiuncta, erit proportio *gl* ad *la* composita ex proportionibus *gk* ad *kd* et *td* ad *ta*. Sed per 16^{am} huius *gl* ad *la* est ut corda dupli arcus *ge* ad cordam dupli arcus *ea*. Et per eandem *gk* ad *kd* est ut corda dupli arcus *gz* ad cordam dupli arcus *zd*. Et per 18^{am} huius convertendo sicut *td* ad *ta* ita proportio corde dupli arcus *bd* ad cordam dupli arcus *ba*, quo liquet propositum.

Hic notare oportet quod hec probatio Ptholomei non est universalis eo quod supponit lineas *ad* et *hb* concurrere ex parte punctorum *d b*. Cum tamen contingere potest *ad* et *bh* esse lineas equidistanter in casu scilicet quo arcui *bda* qui supponitur esse minor semicirculo deficit ad complendum semicirculum arcus prescise equalis arcui *bd.* Contingit etiam *da* et *bh* lineas concurrere ex parte punctorum *h* et *a* in casu scilicet quo arcui *bda* deficit ad complendum semicirculum arcus minor quam *db*. Ne igitur huius conclusionis probatio insufficiens reputetur quod Ptholomeus omisit, expedit hic supplere pro quo premittam hanc conclusionem:

[*Figura #8*] Si ab *a* puncto descendant *ab ag* linee recte a quarum terminis *g b* reflectantur due linee *gd be* secantes se in puncto *z*, erit proportio *gz* ad *zd* composita ex proportionibus *ge* ad *ea* et *ba* ad *bd*.

Probatio huius. Protrahatur *ah* equidistanter *dg* quousque cum ea concurrat linea *be.* Posita ergo *ha* media inter *gz* et *zd* erit proportio *gz* ad *zd* composita ex proportione *gz* ad *ha* et *ha* ad *zd*. Sed proportio *gz* ad *ha* est sicut *ge* ad *ea* propter similitudinem triangulorum *ezg eha*, et proportio *ha* ad *zd* est sicut proportio *ab* ad *db* per 2^{am} 6^{i}. Ergo proportio *gz* ad *zd* componitur ex proportionibus *ge* ad *ea* et *ba* ad *bd*, quod fuit propositum conclusionis qua premissa.

[*Figura #9*] Egrediantur ab *a* puncto in superficie sphere duo arcus duorum circulorum magnorum qui sint *adb aeg* a quorum terminis reflectantur alii duo arcus magni qui sint *bze* et *gzd*. Et sit ita quod arcui *adb* deficiat ad complendum semicirculum minor arcus quam *db*. Et

185 tam] *add. supr. lin.* P **186** quam…extensa] *add. mg. a. m.* P **195** quod] *add. supr. lin.* R **208** zd] *ze* R **210** ea] et *add.* P *ba* ad *bd* quod fuit propositum conclusionis qua premissa egrediantur ab *a* puncto in superficie sphere duo arcus *add. sed. exp.* P protrahantur arcus *bda* et *bze* uterque ulterius usque ad completionem semicirculum ubi secabunt se per 12^{am} primi Theodosii. Sit igitur ibi *f* punctum, et ab *h* centro sphere protrahatur linea pro *f* quousque concurret cum linea *da* in puncto *t*, quod necessario continget eo quod *fa* arcus minor est arcu *db*. Deinde protractis lineis *he hz* concurrentibus cum lineis *ga gd* in punctis *l k*, oportet quod tria | puncta *t l k* sint in eadem linea recta. [S | begins] Sunt enim tam in superficie circuli *bef* quam in superficie trianguli *dkt* utraque in infinitum extensa. Procedant ergo a *d* termino communi due linee *dt dg* a quarum | terminis *t* et *g* reflectuntur due linee [R 44r] *tk* et *ga* secantes se in puncto *l*. Ergo per hanc conclusionem quam premisi proportio *gl* ad *la* componitur ex proportionibus *gk* ad *kd* et etiam *td* ad *ta*. Sed per 16^{am} huius *gl* ad *la* est ut corda dupli arcus *ge* ad cordam dupli arcus *ea*. Et per eandem *gk* ad *kd* est ut corda dupli arcus *gz* ad cordam dupli arcus *zd*, et per 18^{am} huius sicut *td* ad *ta* ita corda dupli arcus *df* ad cordam dupli arcus *fa*. Cum igitur [semicirculi] *bdf* eadem sit corda | dupli arcus *bd* et dupli arcus *df* residui et similiter eadem corda [P 45r] dupli arcus *ba* et dupli arcus *af* residui. Ergo sicut *td* ad *ta* ita corda dupli arcus *bd* ad cordam dupli arcus *ba*. Ergo a primo proportio corde dupli arcus *ge* ad cordam dupli arcus *ea* componitur ex proportione corde dupli arcus *gz* ad cordam dupli arcus *zd* et proportione corde dupli arcus *bd* ad cordam dupli arcus *ba*, quod fuit propositum.

Item sint *ad hb* linee equidistantes iuxta tertium modum variationis figure prout contingit in casu quo arcui *ba* deficit ad completionem semicirculi arcus equalis arcui *bd*. [*Figura #10*] Tunc compleantur arcus *ba be* usque ad complementum semicirculorum qui intersecabunt se in puncto *f*. Et protractis cordis *ga gd* et lineis *hz he* productis a centro sphere ad sectiones circulorum in concursu linearum *hz he* cum cordis *gd ga*, signentur puncta *k l*. Probabitur ergo primo quod linea *kl* sit equidistans tam linee *da* quam linee *hf*. Est enim in eadem superficie cum linea *da* propter hoc quod utraque linea est in triangulo *gda*, et eadem *kl* est in superficie eadem cum *hf* propter hoc quod utraque est in superficie circuli *bef*. Est etiam *da* in eadem superficie cum *hf* quia utraque est in superficie *baf*. Tamen ille tres linee *da kl hf* non fuerint in eadem superficie propter hoc quod triangulus *gda* in cuius superficie sunt *da kl*

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219 secabunt] *add. supr. lin. a. m.* R | Theodosii] thedosi *sed. corr. supr. lin. a. m.* P **220** concurret] concurrat R **226** a] *add. supr. lin.* P **229** la] componitur ex proportionibus *add. sed. del.* P **231** td] *ed* P | ta] *iter. sed. del.* P | ita] *om.* P **232** cordam] *om.* S semicirculi] semidiameter P semidiametri RS **236** arcus^{1}] *ba add.* P **243** he] erit cordis *gd ga* signentur puncta *k l add. sed. del.* P **247** da] et *add.* R **250** kl hf] *kh hl* P | fuerint] sunt RS | in] *del.* R non transit per centrum sphere. Illarum ergo trium linearum *da kl hf* quelibet duo sunt in superficie una et ille tres non sunt in superficie una. Cum ergo due illarum scilicet *da hf* sunt equidistanter utraque earum erit tertia equidistans scilicet *kl*. Probatio. Nam si alteri earum sit equidistans, ergo per 9^{am} 11^{i} erit equidistans utrique. Si neutri fuerit equidistans et cum utraque est in superficie una, igitur cum utraque concurret, et si sic vel ergo in eodem puncto vel in diversis punctis. Si in eodem puncto ergo et alie due concurrent | in eodem puncto, et per consequens alie due non (P ends) sunt equidistantes quod est contra hypothesim. Si *kl* concurreret cum aliis duabus in diversis punctis, ergo per 7^{am} 11^{i} omnes ille tres scilicet *da kl hf* sunt in superficie una, [quod] | contra prius probatum. Relinquitur ergo (R 44v) *kl* et *da* esse equidistantes ex quo argumento propositum. Cum enim sint in triangulo *gda*, ideo per 2^{am} 6^{i} sicut *gl* ad *la* ita *gk* ad *kd*. Ergo proportio corde dupli arcus *ge* ad cordam dupli arcus *ea* est eadem cum proportione corde dupli arcus *gz* ad cordam dupli arcus *zd* per 16^{am} huius. Cum ergo corda dupli arcus *ba* sit eadem cum corda dupli arcus *af* residui de semicirculo, et per consequens est equalis corde dupli arcus *bd*, erit proportio corde dupli arcus *bd* ad cordam dupli arcus *ba* proportio equalitatis et per consequens nec maiorat compositionem nec minorat. Proportio igitur corde dupli arcus *ge* ad cordam dupli arcus *ea* componitur ex proportione corde dupli arcus *gz* ad cordam dupli arcus *zd* et proportione corde dupli arcus *bd* ad cordam dupli arcus *ba*, quod fuit probandum sicut ergo in omni casu probatur conclusio Ptholomei.

[I.21]

Alia conclusio eiusdem est hec. Egredientibus ab uno communi termino duobus arcubus sicut prius et aliis duobus a primorum terminis in eosdem arcus ut prius reflexis, uterque reflexorum reliquum reflexum et arcum principalem in quem reflectitur se secabit ut proportio corde dupli arcus alterius principalis arcus ad cordam dupli arcus portionis eius que est termino communi propinquior ex duabus proportionibus componetur scilicet ex proportione corde dupli arcus totius reflexi sibi conterminalis ad cordam dupli arcus portionis eiusdem reflexi que in reliquum principalem arcum incidit et ex proportione corde dupli arcus portionis reliqui reflexi que conterminatur alteri principali ad cordam dupli arcus illius totius reflexi.

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252 hf] *gf* S **253** duo] due S **254** equidistanter] equidistantes S | utraque] utrique S **256** neutri] neutre S **257** si sic] sicut P | sic] *add. supr. lin.* R **260** concurreret] concurret S **262** quod] contra RS **263** argumento] arguetur S **267** de] *om.* R **274** probandum] *om.* S sicut] sic S **277** primorum] priorum S **279** se] sic S **280** eius] eiusdem *sed. corr.* R

Ut verbi gratia in figuris prioribus proportio corde dupli arcus *ga* ad cordam dupli arcus *ae* componitur ex proportionibus corde scilicet dupli arcus *gd* ad cordam dupli arcus *dz* et corde dupli arcus *zb* ad cordam dupli arcus *be*. [*Figura #11*] Hanc consequens ponit Ptholomeus sed non probat eam quia fortasse videtur sibi satis nota tamquam sequens ex premissis et ex kata coniuncta. Supponendo tamen quod nullus arcuum predictorum sit maior quarta circuli prout sufficit ad propositum Ptholomei, ut patebit inferius, hec conclusio poterit sic probari. Protrahatur linea *ge* donec concurrat cum *ha* producta ab *h* centro sphere, et protrahatur linea *gz* donec concurrat cum *hd* producta ulterius. Et protrahatur *ez* donec concurrat cum *hb* producta ulterius. Sint ergo illi concursus in punctis *l k t*. Oportet enim eas sic concurrere in casu quo nullus arcus est maior quarta circuli. Dico ergo quod tria puncta *t k l* sunt in eadem linea recta. Sunt enim tam in superficie trianguli *gze* extensa in infinitum quam in superficie circuli *adb* extensa in infinitum. Ergo per katam coniunctam proportio *gl* ad *le* componitur ex proportionibus *gk* ad *kz* et *tz* ad *te*. Ergo per 18^{am} huius patet intentum.

Et notandum quod sicut hec figura probat conclusionem supponendo quod nullus arcuum excedat quartam circuli, ita supponendo illud idem in figura Ptholomei conclusionis precendentis hanc, figura eius probat intentum conclusionis. Et quia ubi operabitur cum istis conclusionibus non utetur arcubus maioribus | quarta circuli. Forte igitur R 45r
ex illa causa non variavit figuram sed supposuit simpliciter quod *ad hb* linee concurrerent ex parte punctorum *d b*. Quia tamen textus suus ampliat conclusionem ad omnem arcum minorem semicirculo ideo iuxta diversos casus variavi figuram in conclusione premissa. Hic tamen illud omitto ne legentem tediose occupem sine fructu. Hoc enim cuicumque debet sufficere quod ex quo tenet probatio Ptholemei in omni casu quo quilibet arcus est minor quarta circuli cum cuiuscumque talis arcus duplati corda sit eadem cum corda residui de semicirculo duplati. Et per consequens equivalens est probatio cum arcuum cordis sive quartis sive semicirculis sint minores.

De hoc igitur contenti antequam ad propositum rediamus, notandum est quod cuilibet conclusioni sequenti pertinenti ad practicam cuius operatio dependet in katis, addam unum correlarium in quo consistet operatio pertinens ad conclusionis intentum. Et ulterius est notandum quod cum eadem sit proportio multiplicium et submultiplicium et per consequens omnis proportio que tenet in cordis arcuum duplatorum tenet

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291 fortasse] forte S | videtur] videbatur S **304** sicut] *add. supr. lin.* R **307** ubi] *add. supr. lin.* R **311** conclusionem] suam *add.* S **317** cum] *supr. lin. a. m.* R | cordis] *add. sup. lin.* R etiam in dictarum cordarum dimidiis, ideo que Ptholomeus arguit de cordis duplatorum arcuum, arguam ego de medietatibus illarum cordarum et illas lineas medietas voco sinus arcuum mediatorum ita quod iuxta diffinitionem Gebir in libro suo primo. Sinus alicuius arcus est medietas corde dupli arcus et est etiam perpendicularis cadens ab altera extremitate arcus super diametrum exeuntem a reliqua extremitate eiusdem arcus. His premissis ad Ptholomei propositum revertamur.

[I.22]

Sequitur igitur alia conclusio que est hec. Cuiuscumque gradus vel puncti noti ecliptice declinationem ab equinoctiali ostendere. Unde si sinus arcus ecliptice incipientis ab equinoctiali et terminantis in punctum cuius declinatio queritur in | sinum ducatur maxime declinationis et S 22v productum dividatur per sinum quarte circuli, exibit sinus declinationis quesite.

[*Figura #12*] Sit *abg* colurus transiens per puncta solsticialia et sit *aeg* medietas equinoctialis et *bed* medietas ecliptice. Tunc a polo *z* septentrionali protrahatur arcus circuli magni per *h* punctum cuius queritur declinatio usque ad equinoctialem. Arcus ergo *ht* est quem querimus. Cum igitur ab *a* termino procedant duo arcus *az ae* a quorum terminis reflectuntur alii duo *eb zt* intersecantes se in puncto *h,* erit proportio sinuum *za* ad *ba* composita ex proportionibus sinuum *zt* ad *th* et *eh* ad *eb*. Cum igitur arcus *az* sit notus quia quarta circuli et *ab* notus quia maxima declinatio et *eb zt* noti quia quarte circulorum et *eh* notus quia est arcus cuius queritur declinatio, remanet tamen *ht* arcus ignotus. Subtracta igitur proportione sinuum *eh* ad *eb* ex proportione sinuum *za* ad *ab*, remanet proportio sinuum *zt* ad *th*. Cum ergo *zt* sit notus, erit etiam *th* notus. Hec est probatio Ptholomei tenens per 14^{am} huius.

Sed quia non videtur facile proportionem a proportione subtrahere, ostendam hanc conclusionem aliter in quattuor quantitatibus. Nam convertendo | proportiones in kata pretacta, erit proportio sinus *ab* ad R 45v sinum *az* composita ex proportionibus sinuum *th* ad *tz* et *be* ad *he.* Sed *tz* et *be* sunt equales cum uterque sit quarta circuli. Ergo proportio sinuum *ab* ad *az* est composita ex proportionibus sinuum *th* ad *tz* et *tz* ad *eh*. Ex eisdem autem componitur proportio sinuum *th* ad *eh* propter *tz* medium interceptum. Ergo sicut sinus *ab* ad sinum *az* quarte circuli ita sinus *th* ad sinum *eh*. Sed *ab* est maxima declinatio et *th* est arcus quesitus et *he* arcus cuius queritur declinatio. Cum quilibet istorum sit notus preter tertium, multiplicando primum per quartum et productum dividendo per

335 terminantis] terminati R **346** arcus] *om.* R **362** multiplicando] multitudo R secundum, resultabit tertium per 19^{am} 7^{i} Euclidis. Sic ergo patet correlarium conclusionis ex quo conclusio nota erit. Hanc conclusionem secundum proportionem harum quatuor quantitatum probat Gebyr. Est enim quarta conclusio 2^{i} libri eius.

Hic autem ponit Ptholomeus quod arcus inter tropicos secundum proportionem xi ad 83 est 47 partes 42 minuta et 40 secunda cuius corda est 48 partes 31 minuta 55 secunda. Est [declinatio 30 graduum] xi partes 40 minuta vicinius. Item duplum declinationis tauri est 41 partes 0 minuta 18 secunda et eius corda 42 partes 1 minutum et 48 secunda, et est declinatio tauri 20 partes 30 minuta 9 secunda. Et notandum quod universaliter quando nominamus partes arcuum intellegimus partes de quibus circumferentia continet 360. Et per partes cordarum intelligimus partes de quibus diameter continet 120. Sic igitur inventis declinationibus quorumlibet graduum quarte circuli, ponende sunt singule declinationes in directo suorum graduum prout patet in tabula que hic sola in textu.

Et est hic notandum quod differentie declinationum arcuum equalium maiores sunt apud puncta equinoctialia quam apud puncta tropica ita quod declinatio duorum graduum a principio arietis plus excedit declinationem unius gradus quam excedat declinatio trium graduum declinationem duorum, et sic deinceps captis arcubus equalibus quibuscumque.

Hoc autem ostendam sic. [*Figura #13*] Sit equinoctialis *ac* cuius polus *z* et sit zodiacus *ab* ita quod *a* sit principium arietis vel libre et *b* principium capricorni vel cancri. Deinde sumantur arcus equales zodiaci qui sint *df fh* per quorum terminos protendantur arcus circulorum magnorum a polo *z* qui sint *zde zfg zhl*. Et sint arcus *dp hk* perpendiculares super arcum *zg*. Quia ergo duo arcus *ab zg* secant se super punctum *f* et signata sunt duo puncta *d* et *h* in altero arcu a quibus exeunt due perpendiculares super reliquum arcum. Ergo per 3^{am} preambuli huius libri eadem est proportio sinuum *fh* ad *hk* sicut sinuum *fd* ad *dp*. Sed *fh dp* sunt equales. Ergo *hk dp* erunt equales. Quare propter

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363 Euclidis] *add. supr. lin.* S **369** Est…370 vicinius] item duplum declinationis arietis est 23 partes et 19 minuta et 59 secunda et eius corda est 48 partes 15 minuta et 57 secunda S **372** quod] *add. supr. lin.* R **376** singule] singulis *sed. corr.* R **377** que] sequentur *add.* S sola] *om.* S **378** differentie…419 Ptholomei] *om. sed. add. f. 23r* S **384** sic] *supr. lin.* S **389** Quia] quod S **390** et^{2}] *om.* S **393** dp^{2}] *fd* S

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366 quarta…eius] Gebir, p. 36. **391** 3am…392 libri] The preambles that Simon refers to do not appear anywhere in his commentary, so they must be derived from context. What he calls the third is Gebir I.12 (Gebir, p. 10-11).

hoc et propter angulos *p k* rectos, [erunt] per 6^{am} preambuli huius et *kf* et *fp* equales. Et quia per primam preambuli huius arcus *zd* maior est arcu *zp* propter hoc quod angulus *p* est maximus angulus | *zpd* trianguli per R 46r secundum preambuli huius. Ideo arcus *pg* maior est arcu *de* propter hoc quod arcus *ze zg* sunt equales. Arcus ergo *fg* excedit arcum *de* per plus quam per *fp*. Item cum per idem arcus *zh* sit maior arcu *zk*, erit arcus *hl* minor arcu *kg*. Ergo arcus *hl* excedit arcum *fg* per minus quam per *kf*, ergo per minus quam per *pf* sibi equale. Sed iam patuit quod *fg* excedit *de* per maius quam per *fp*. Ergo maior est excessus *fg* supra *de* quam sit excessus *hl* supra *fg*, quod fuit probandum.

Consimiliter arguendum est cum arcubus equalibus non continuatis adinvicem ut si arcus *df* ponatur equalis arcui *hm* et protendatur circulus magnus *zmn* et perpendicularis *mq* supra *zl*. Tunc per 3^{am} preambuli huius proportio sinuum *fd* ad *dp* est sicut *fz* ad *zb*, et proportio sinuum *hm* ad *mq* est sicut *hz* ad *zb*. Sed maior est proportio sinuum *fz* ad *zb* quam *hz* ad *zb* cum *fz* sit maior *hz*. Ergo maior est proportio sinuum *fd* ad *dp* quam *hm* ad *mq*. Ergo propter equalitatem arcuum *fd* et *hm*, erit per 8^{am} 5^{i} Euclidis arcus *mq* maior quam arcus *dp*. Ergo complementum arcus *mq* minor est complemento arcus *pd*. Quare propter equalitatem arcuum *mh fd*, maior erit proportio sinus complementi *mh* ad sinum complementi *mq* quam sinus complementi *fd* ad sinum complementi *dp*. Et per consequens per 6^{am} preambuli huius proportio sinus complementi arcus *qh* ad sinum quarte circuli maior est proportione sinus complementi arcus *fp* ad sinum quarte circuli. Quare complementum arcus *qh* maius est complemento arcus *fp*. Ergo arcus *qh* minor est arcu *fp*. Patet ergo residuum per omnia sicut prius. Igitur redeundum est ad propositum Ptholomei.

[I.23]

Post hec demonstrabimus ascencionem cuiuscumque arcus ecliptice in sphera recta. Sit igitur hec conclusio 23^{a}. Cuiuslibet arcus ecliptice ascencionem in directo circulo demonstrare. Unde si in sinum quarte circuli ducatur sinus complementi illius arcus cuius ascensio queritur et

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394 et propter] *add. supr. lin.* R | erunt] erit R **398** zg] *za* S **401** pf] *fp* S **409** fz] *illeg.* S **414** fd…415 complementi] *om. (hom.)* R **417** circuli] quarte *add. sed. exp.* R **421** Post] et *praem.* post S **422** Cuiuslibet] Gebir libro 2^{o} capitulo 8^{o} in figure *adnot. mg.* R | arcus] arcubus R

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394 6am…huius] The sixth preamble must refer to the angle-side-angle principle of congruence in spherical triangles. **406** 3am…huius] Again, this preamble is not found in Simon’s commentary, but he must mean Gebir I.12 (Gebir, p. 10-11). **415** 6am…huius] The angle-side-angle principle of congruence of spherical triangles.

productum dividatur per sinum complementi declinationis arcus eiusdem, proveniet sinus complementi ascensionis quesite.

Manente eadem figura ut prius et eisdem arcubus circulorum, [*Figura # 12*] patet per conversam kate disiuncte quod proportio sinus *ab* ad sinum *bz* componitur ex duabus proportionibus scilicet ex proportione sinus *th* ad sinum *hz* et sinus *ae* ad sinum *te*. Sed istorum sinuum omnes noti sunt preter sinum arcus *te* quem inquiris cuius arcus est ascencio arcus *he.* Subtracta ergo proportione sinuum *th* ad | *hz* ex proportione R46v sinuum *ab* ad *bz,* remanet proportio sinuum *ae* ad *te*. Sed *ae* arcus est notus quia est quarta circuli, ergo *te* erit notus.

Pro subtractione autem proportionis a proportione qua utitur Ptholomeus, oportet ut dicam aliqua hic notanda. Unde si fuerint sex quantitates quarum proportio prime que sit *a* ad secundam que sit *b* componitur ex proportionibus *c* tertie ad *d* quartam et *e* quinte ad *f* sextam, quibuscumque quinque earum notis sexta nota erit. Sit enim gratia exempli ultimum ignotum et reliqua quinque nota, et arguitur sic. Si dividatur *a* per *b*, resultat eorum proportio que sit *g*. Similiter diviso *c* per *d,* resultat eorum proportio que sit *h*. Item diviso *g* per *h*, resultat eorum proportio que sit *k*. Dico ergo quod *k* est proportio *e* ad *f*. Probatio. Nam ex ductu *k* in *h* sit *g* eo quod ex divisione *g* per *h* resultabat *k*. Sed ex multiplicatione numeri quotiens in divisorem, resultat numerus divisus. Ergo ex *k* in *h* fit *g.* Sed ex proportione *e* [ad] *f* in *h* fit *g* per hypothesim, eo quod *g* proportio componitur ex *h* et ex proportione *e* in *f*. Ergo proportio *e* ad *f* eadem est cum proportio *k*, quod fuit probandum. Cum igitur *k* sit nota et *e* nota, erit *f* nota. Nam si *f* sit maius quam *e,* ex

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427 ut] que S **428** per] *om.* R **436** dicam] dictam R **443** Probatio] probo R

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432 ad] *adnotatio mg. in codice* R: Nota secundum Geber libro 2º capitulo 8º in fine [This is a paraphrase of Gebir's proof, pp. 36-7. The numbering given in this note does not accord with the numbering that Simon gives in his references to Gebir.]. Hic potest 4 numeros proportionales secundum viam Geber quorum 3 sunt noti, et quartus ignotus. Et sic habetur per quatuor proportionalia hoc quod Ptholomeus invenit per 6 quantitates proportionales per figuram sectoris. Et per hanc viam invenitur sinus complementi arcus equinoctialis qualiter quod arcuabis et sinum arci deveniens de quarta circuli, et remanebit portio equinoctialis elevata cum arcu zodiaci dato. Primus in proportione est sinus complementi arcus zodiaci dati cuius ascensio queritur et notus. Secundus sinus est sinus complementi arcus equinoctialis elevati cum arcu zodiaci et ignotus. Tertius sinus est sinus complementi declinationis arcus zodiaci cuius ascensio queritur et notus resultans. Quartus sinus est sinus quarte circuli et similiter notus. Ducatur primus in quartum vel ignota et dividatur per tertium et exibit cuius queris circuli portionem. Et illam deme de 90 et remanebit arcus equinoctialis elevatus. *adnot. marg.* R ductu *e* in *k* resultabit *f* cum *k* sit numerus quotiens denominans proportionem *e* ad *f*. Si vero *f* sit minus quam *e*, ergo dividendo *e* per *k* resultabit *f*. Si enim *e* sit maius *f*, non potest *f* dividi per *e*, sed e conversa *e* per *f* et resultabit *k* cum sit eorum proportio. Ergo multiplicando *k* quod est quotiens per *f* divisorem resultabit *e*. Ergo e conversa dividendo *e* per *k* resultabit *f*, quod fuit propositum. Consimiliter si *e* sit ignotum et reliqua quinque nota, multiplicando *f* per *k* si *f* sit minus *e*, vel dividendo si sit maius resultabit *e*.

Item si *c* vel *d* sint ignota, dividatur *g* per *k* et resultabit *h*. Per *h* ergo que est proportio *c* ad *d*, multiplica illud eorum quod est notum si sit minus ignoto vel divide si sit maius, et resultabit ignotum. Item ad hunc si *a* vel *b* sint ignota cum proportio *a* ad *b* sit composita ex *h k* proportionibus, ducatur *h* in *k* et resultabit *g* que est proportio *a* ad *b*. Per *g* ergo multiplica notum eorum si sit minus ignoto vel divide si sit maius, et resultabit ignotum. Sic ergo patent ea que sunt necessaria ad modum operandi Ptholomei qui per subtractionem proportionis a proportione cum quinque quantitatibus notis devenit in ultimam ignotam. Verum quia relatione proportionum est nobis notior relatio quantitatum ac etiam sunt nobis notiores quantitates quam proportio inter illas, ideo ad habendum sextam quantitatem ignotam per quinque notas ubi proportio primi ad secundum componitur | ex proportionibus tertii ad quartum et quinti ad sextum, regule faciliores prioribus possunt dari. Sit enim ut prius proportio *a* ad *b* composita ex proportionibus *c* ad *d* et *e* ad *f*. Si ergo quintum vel sextum ignotum fuerit et cetera quinque nota, quadrupliciter per illa nota devenire poterimus ad ignotum.

R missing folios

Si enim *a* primum ducamus in *d* quartum et productum dividamus per *b* secundum, resultabit quiddam quod sit *z*. Illa ergo *z* se habebit ad *c* tertium sicut *e* quintum ad *f* sextum. Probatio. Nam per 19^{am} 5^{i} sicut *a* ad *b* ita *z* ad *d*, et ideo proportio *z* ad *d* componitur ex proportione *c* ad *d* et *e* ad *f*. Sed eadem proportio *z* ad *d* componitur... |

S missing folios

[I.24]

S 24v | R 47r

| | Ascensiones equalium portionum zodiaci maiores sunt apud puncta tropica quam apud puncta equinoctialia ita quod ascensio secunde medietatis arietis maior est prime medietatis ascensione eiusdem, et ascensio prime medietatis tauri maior est ascensione secunde medietatis

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452 f^{1}] *k* R cum *k* sit eorum proportio ergo multiplicando *add. sed. exp.* R | Si] et S **459** ergo] *g* R **460** ad hunc] adhuc S **466** ultimam] sextam S **468** notiores] *praem.* sunt S **469** ad…470 secundum] *add. supr. lin.* R **470** ex…479 componitur] *om.* R **482** secunde] notiores sunt nobis S **484** $medietatis^{2}$] *praem.* ascensione S

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479 componitur] The next portion of the text is missing.

arietis, et sic deinceps captis portionibus equalibus quibuscumque. Hoc autem ostendam sic. [*Figura #14*] Sit equinoctialis *ac* cuius polus *z* et sit zodiacus *ab* ita quod *a* sit principium libre vel arietis et *b* principium capricorni vel cancri. Deinde captis de zodiaco portionibus equalibus que sunt *ad df* et *fh* quarum ascensiones sint arcus *ae eg* et *gl* protractis circulis magnis *zde zfg* et *zhl*. Dico quod arcus *ge* est maior arcu *ea* et arcus *lg* maior est arcu *ge.* Producta enim perpendiculari *fk* super arcum *zde,* erit per 3^{am} preambuli huius eadem proportio sinuum *df* ad *fk* et *da* ad *ae*. Quare cum *df* et *da* sunt equales, erunt *fk* et *ae* equales. Ergo eadem est proportio *ge* ad *fk* et ad *ea*. Sed proportio sinuum *ge* ad *fk* est sicut *gz* ad *fz*, et *gz* est maior quam *fz*. Ergo *ge* est maior quam *ea*. Item protracta *dm* perpendiculari super *fg* et *hn* perpendiculari super *zf*, erunt *dm* et *hn* equales per 3^{am} preambuli huius. Ergo per eadem proportio sinuum *lg* ad *md* est sicut *bz* ad *hz*, ergo maior quam *lz* ad *dz*, et per consequens maior quam *ez* ad *dz*. Sed eadem est proportio sinuum *ez* ad *dz* et *ge* ad *md,* ergo maior est proportio sinuum *lg* ad *md* quam *ge* ad *md*. Ergo per 8^{am} 5^{i} Euclidis *lg* maior est quam *ge,* quod fuit probandum.

Consimiliter arguendum est in arcubus equalibus non continuatis adinvicem ut si arcus *hp* ponatur equalis arcui *fd* cuius ascensio sit *lq*, protracto arcu *zpq* et ducta perpendiculari *pr* super arcum *zh*. Nam per 3^{am} preambuli huius eadem est proportio sinuum *hp* ad *pr* et *hz* ad *zb*. Et etiam est eadem proportio sinuum *fd* ad *dm* et *fz* ad *zb*. Sed maior est proportio sinuum *fz* ad *zb* quam *hz* ad *zb*. Ergo maior est proportio sinuum *fd* ad *dm* quam *hp* ad *pr*. Quare propter equalitatem arcuum *fd* et *hp* erit *md* minor quam *pr*. Ergo maior est proportio sinuum *ql* ad *md* quam ad *pr*, ergo maior quam *qz* ad *pz*, ergo a fortiori maior quam *ez* ad *dz,* et per consequens maior quam *ge* ad *md*. Ergo *ql* est maior quam *ge.* Patet ergo totum quod intendebam ultra propositum Ptholomei. Et terminatur hic sententia primi libri.

[II.1]

Arcum diei maximi seu minimi per notam poli altitudinem reperire. Unde si sinus altitudinis poli in sinum maxime declinationis ducatur, productumque dividatur per sinum complementi altitudinis poli, resultabit quiddam quod si ducatur in sinum quarte circuli et productum | R 47v

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487 et…488 cancri] *add. mg.* S **489** sunt] sint S **493** sunt] sint S **500** et ge] *illeg.* S **502** in] cum S **505** Et…506 etiam] *illeg.* S **506** est eadem] *inv.* S **510** ergo^{1}] item S **511** dz] *illeg.* S **512** totum] *illeg.* S

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492 3am…huius] Gebir I.12 (pp. 10-11). **497** 3am…huius] Gebir I.12 (pp. 10-11). **505** 3am…huius] Gebir I.12 (pp. 10-11). **514** II1] I have only used MS R in my transcription of Book II.

dividatur per sinum complementi maxime declinationis, resultabit sinus differentie que inter quartam circuli et medietatem arcus diei maximi seu minimi.

Manente enim dispositione priori [Figura #12] erit per katam disiunctam proportio sinus arcus *zb* ad sinum arcus *ba* composita ex proportionibus sinus arcus *zh* ad sinum arcus *ht* et sinus arcus *te* ad sinum arcus *ea*. Ergo ex prima regula operandi patet corelarium huius conclusionis ex quo conclusio satis liquet.

Unde notandum est quod quia ita conclusio non potest demonstrari per quattuor quantitates solummodo nisi supponendo notum esse azimuth ortus cancri, ideo ne plura supponam esse nota quam Ptholomeus, posui corelarium cum sex quantitatibus iuxta modum operacionis Ptholomei. Supposita tamen noticia azimuth ortus capriconi vel cancri qui gratia exempli sit arcus *he*, erit proportio sinus *at* ad sinum *ae* sicut sinus *bh* ad sinum *zh* prout in conclusione prima huius ostendi. Sed arcus *zh* est complementum maxime declinationis eo quod *he* est azimuth tropicorum. Ergo per 19^{am} 7*i* Euclidis erit in quattuor quantitatibus corelarium istud verum, scilicet: Si sinus complementi azimuth ortus cancri ducatur in sinum quarte circuli et productum per sinum complementi declinationis maxime dividatur, quod inde resultaverit medietatis arcus diei minime sinus erit. Hanc conclusionem non probat Gebyr neque de ea facit mencionem. Istud tamen ultimum corelarium quod adiunxi per libri sui primi conclusionem 15^{am} satis liquet.

[II.4]

Cuiuslibet gradus orientis azimuth per notam poli altitudinem indagari. Unde si in sinum quarte circuli ducatur sinus declinationis gradus dati dividaturque productum per sinum complementi altitudinis poli, resultabit sinus azimuth gradus dati.

Pro probatione Ptholomei manente dispositione priori ita quod hoc sit indifferenter quiscumque punctus zodiaci volueris. [*Figura #12*] Per katam coniunctam quod proportio sinus *za* ad sinum *ab* componitur ex proportionibus sinus *zt* ad sinum *th* et sinus *eh* ad sinum *eb*. Ergo ex

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539 erit] nota renductionem sex quantitatum proportionalium ad quattuor et de Gebro *adnot. mg.* R

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540 libri…541 15am] Simon does not explain his steps here in detail. He is applying I.15 of Gebir (Gebir, p. 13) to right triangle *eth*. **542** II4] I have struggled to number the propositions of Book II. Although this is the second proposition of the book, it refers to the second conclusion as if it were another and the following proposition is referred to as the fifth proposition of the book by several internal references. Therefore, I have chosen to number this II.4, and to have no II.2 or II.3.

prima regula operandi si ducatur sinus *za* primi in sinum *th* quarta et productum dividatur per sinum *ab* secundum, resultabit quiddam quod si ducatur in sinum *eb* sexti et productum dividatur per sinum *zt* tercii, resultabit sinus *eh* quinti qui est azimuth gradus dati. Et hec est probatio Ptholomei. Aliter tamen per quatuor quantitates deveniam ad intentum. Nam proportio sinus *ab* ad sinum *az* quarte circuli est sicut proportio sinus *th* ad sinum *eh*, prout in secunda conclusione huius probavi. Ergo per 19^{am} 7^{i} Euclidis tam conclusio quam corellarium satis patent. Hanc conclusionem omittit Gebyr preter tamen sicut et precedens per primi libri sui conclusionem 15^{am} demonstrari.

[II.5]

Quilibet duo circuli equinoctiali circulo paralelli per equalem longitudinem a duobus tropicis vel ab ipso equinoctiali distantes | equales R 48r
arcus orizontis ex utraque parte equinoctialis resecant et sit alternatim nox unius equalis diei alterius et e contra.

Retenta figura priori discribantur [*Figura #15*] duo paralleli *lh* et et *km* equaliter distantes ab equinoctiali, *lh* ex parte meridiei et *km* ex parte septentrionis, et transiat quarta circuli magni per polum septentrionalem et per punctum *k* usque ad equinoctialem que sit arcus *qks*. Est ergo totalis circulus *lh* equalis totali circulo *km* per 6^{am} primi Theodosii, quos non secet circulus magnus *bed* qui est orizon non transiens per polos eorum. Ergo sic dividit eos quod portiones eorum coalterne erunt equales per ultimam partem 18^{e} 2^{i} Theodosii. Ergo pars unius supra orizontem erit equalis parti alterius sub orizonte, quare dies unius equalis nocti alterius et e conversa. Item arcus *at* equalis est arcui *sg* per 10^{am} 2^{i} Theodosii eo quod arcus *lh* et *km* similes illis sunt equales propter hoc quod unus est medietas arcus diei puncti *h* et alter est medietas arcus noctis puncti *k*. Remanent ergo *te* et [*es*] arcus equales, sed et *th* et *sk* arcus sunt equales propter equales declinationes circulorum equaliter ab equinoctiali distancium. Ergo triangulorum *eth* et *esk* duo latera unius *ht te* sunt equalia duobus lateribus alterius *sk ke*, et angulus *t* inter duo latera unius contentus equalis angulo *s* inter duo latera alterius contento cum uterque sit rectus, et sunt trianguli circulorum magnorum. Ergo basis unius que est *he* equalis est basi alterius que est *ek*. Et hoc est quod volumus demonstrare. Patet eciam ita conclusio quam ad primam eius

555 intentum] hic per quattuor quantitates *adnot. mg.* R **572** quod] *supr. lin.* R **578** es] *fs* R

557 prout…probavi] As in the second proof of II.1, he reaches a proportion through Gebir I.15 (Gebir, p. 13). **559** primi…560 15am] Gebir I.15, p. 13.

partem per 17^{am} 2^{i} Theodosii et quam ad secundam eius partem per 18am eiusdem, est eciam 8^{a} conclusio 2^{i} libri Gebyr.

[II.6]

Qui sunt illi super quorum capitum summitatem sol transiat et quando et quociens illud accidet declarare.

Manifestum est enim quod sol numquam transit supra cenith capitum eorum qui manent sub parallelum plus distantibus ab equinoctiali quam ab eodem distiterit alteruter tropicorum, que quidem distancia est 23 gradus 51 minuta et 20 secunda. Habitantibus autem sub alterutro tropicorum transit sol semel in anno supra earum capitum summitatem. Habitantibus eciam inter duos tropicos transit, eciam sol supra eorum capita bis in anno. Unde ut in omni loco inter tropicos sciamus quando hoc accidet.

Videamus primo quanta sit distantia cenith capitis ibidem habitantium ab equinoctiali, et illam distantiam queramus in secunda linea tabule declinationis in directo, cuius in linea prima eiusdem tabule inveniemus gradus ad quos cum sol devenerit(?), accidet quod vellemus. Est eciam hec 9^{a} conclusio 2^{i} libri Gebyr.

[II.7]

Per distanciam inter tropicos et per altitudinem poli notas, proporcionem umbre solis meridiane ad rem erectam cuius est umbra sole in equinoctiali circulo vel in tropicis exeunte patefacere et e conversa, vel universalius per altitudinem solis notam in cuiuscumque diei parte | R 48v
qualibet ostendere illud idem. Unde si sinus complementi altitudinis solis ducatur in altitudinem rei cuius est umbra et productum per sinum solaris altitudinis dividatur, resultabunt partes umbre similes partibus obumbratis. Eciam econtra si per radicem aggregati ex duobus quadratis umbre scilicet et umbrosi dividatur illud quod ex ductu eiusdem umbrosi in semidiametrum circuli provenit exibit sinus altitudinis solis quesite.

[*Figura #16*] Sit *abg* circulus meridiei cuius centrum *e* et sit *a* cenith capitis. Et protrahatur diameter *aeg* a cuius termino *g*, protrahatur *gn* othogonaliter super eam. Erit igitur *gn* linea equedistans orizonti et quia tocius terre quantitas respectu quantitatis orbis solis est sicut punctum vel centrum. Ponatur *e* centrum summitas rei erecte ita quod res erecta causans umbram sit *eg* et sit *gn* linea super quam cadit extremitas umbre in medietatibus dierum. Et sit radius solis sole exeunte in tropico estivali in medietate diei line *hetk*, et in equinoctiali *bedz*, et in tropico

587 8a…Gebyr] Gebir, p. 29-30 (incorrectly numbered as 37-8). **603** 9a…Gebyr] Gebir, p. 30 (incorrectly numbered as 38).

yemali *lemn*. Cum igitur distancia cenith ab equinoctiali sit equalis altitudini poli, erit arcus *ab* et per consequens arcus *gd* sibi equalis notus. Sed et arcus *dm* et *dt* erunt noti cum uterque sit equalis maxime declinationi, quare et arcus *tg* erit notus nisi(?) non et totus arcus *gm* eciam erit notus. Ergo secundum quantitatem qua quattuor anguli recti sunt 360 noti, erunt anguli *neg zeg keg*. Nondum(?) eciam erunt noti secundum quantitatem qua duo anguli recti sunt 360. Sed et angulus *g* cuiuslibet triangulorum est rectus. Ergo arcus qui sunt porciones circuli descripti super tres angulos *neg zeg keg* erunt noti. Quare et earum corde erunt note. Umbra ergo *ng* que causatur ab umbroso *ge* in meridie diei brevioris erit nota secundum proporcionem quantitatis *ge*. Eciam umbra *zg* que est umbra meridiana diei equinoctialis et umbra *kg* que est umbra meridiana diei longioris secundum eandem proporcionem erunt note, quod fuit propositum. Patet eciam huius conversa ita quod quibuscumque duabus istarum umbrarum notis in comperacione ad umbrosum, tam altitudo poli quam distancia inter tropicos erunt note, nam scitis quibuscumque duobus angulorum *meg deg teg,* tercius erit notus propter hoc quod duo arcus *td dm* sunt equales. Ergo residua satis patent.

Tamen distancia inter tropicos verius scietur per 13^{am} primi huius et altitudo poli per 2^{am} huius certius quam per hunc modum habebitur. Tum propter hoc quod extremitates capitum umbrarum yemalium | pre sui R 49r
longitudine difficulter apprehenduntur. Tum eciam propter hoc quod tempus umbre equalitatis in se non est discretum eo quod in meridie diei equinoctialis frequentitudine contingit solem vel nondum ad equinoctialem circulum pervenisse vel ab eo per aliquid recessisse. Hoc est totum quod tangit Gebyr in 10^{a} conclusione 2^{i} libri sui et totum eciam quod in isto capitulo tangit Ptholomeus preter hoc quod ponit quantitatem predictarum umbrarum in Rodo insula. Et quia ista pars conclusionis proposite est universalior parte prima et eam includit ideo ostendam eam sic. [*Figura #17*] Sit circulus altitudinis qui est circulus transiens per cenith et per locum solis *abc* et sit *a* cenith et *b* locus solis *fd* orizon et *aec* diameter circuli altitudinis [perpendicularis] orizonti. Et sit *ce* umbrosum erectum quod vocitur gnomo a Ptholomeo. Erit ergo *bd* sinus altitudinis solis et *bg* sinus complementi eiusdem altitudinis. Cum ergo

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623 distancia] *iter.* R **642** 2am] Nota quod distancia inter tropicos et altitudo poli melius scietur per conclusiones premissas scilicet 13^{am} et 2^{am} huius. *adnot. mg.* R **654** perpendicularis] equedistans R

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642 2am…certius] Simon does not tell how to find the altitude of the pole in II.2 or anywhere else. Presumably, II.2-3 are missing. **648** 10a…libri] Gebir, p. 30 (incorrectly numbered 38).

trianguli *bge* et *hce* sunt similes eo quod eorum anguli sunt equales per 29^{am} et 30^{am} primi Euclidis, erit per 4^{am} 6^{i} Euclidis proportio *bg* ad *ge* sicut *hc* ad *ce*. Quare per 19^{am} 7^{i} Euclidis corelarium primum liquet. Item pro conversa eiusdem cum quadratum *he* valet duo quadrata *hc ce,* si coniungantur duo quadrata *hc ce*, radix aggregati erit linea *he,* ad quam per 4^{am} 6^{i} se habebit *ce* sicut *db* se habet ad *be* semidiametrum circuli. Quare per 19^{am} 7^{i}, patet corelarium secundum. Item est notandum quod consimiliter devenire poterimus ad umbram gnomonis iacentis per solis altitudinem et e conversa. Nam sit *fe* gnomo iacens, *fk* eius umbra, tunc propter similitudinem triangulorum *ekf* et *ech* sicut *ef* ad *fk* ita *hc* ad *ce.* Et equaliter per consequens sicut gnomo iacens ad eius umbram ita e conversa umbra gnomonis erecti ad gnomonem erectum, ex quo cetera patent.

[II.8]

Sub equinoctiali circulo omnes dies suis noctibus et sibi invicem sunt equales et omnes stelle oriuntur et occidunt umbre eciam meridiane quandoque ad meridiem quandoque ad septentrionem et quandoque nusquam declinant...

[II.9]

| Sub omni circulo qui equinoctiali fuerit equedistans, erit dies nocti R 49v
equalis tanto modo bis in anno et erunt dies estivales noctibus suis et diebus yemalibus longiores dies vero yemales econtra erit eciam quedam pars celi semper apparens et alia sibi equalis perpetuo occultata...

[II.10]

| In omni orizonte obliquo quantum polus ab orizonte distiterit R 50r
tantum ab equinoctiali circulo distat cenith, et ubi distancia illa fit longior ibi dierum et noctium inequalitas erit maior, eritque pars celi semper obiecta visui maior ibi quam in orizonte quo poli elevacio extat minor...

[II.11]

| Sub omni linea inter equinoctialem et alterum tropicorum, umbra R 50v
meridiana quandoque ad meridiem quandoque ad septentrionalem declinat. Ipsius eciam bis in anno declinatio nusquam erit. Unde sub omnibus huiusmodi lineis a se invicem per quartam hore distantibus quante sint umbre meridiane in solsticiis et equinoctiis exponemus...

[II.12]

| Sub linea cuius distancia ab equinoctiali est equalis declinatione R 51r
solis maxime ab eodem, umbra meridiana semel in anno declinatione

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674 declinant] Since this proposition and the following do not concern the Menelaus Theorem, I have not transcribed them in their entirety.

carebit et in omni alia parte anni ad septentrionem et numquam ad meridiem declinabit. ....

[II.13]

Sub omni linea que est inter tropicum estivalem et circulum quem describit polus septentrionalis zodiaci, umbra meridiana numquam declinacione carebit. Sed in omni parte anni ad septentrionalem et numquam ad meridiem flectetur...

[II.14]

| Sub linea quam circa polum mundi septentrionalem describit, polus R 52v zodiaci flectetur umbra dum sol tropicum estivalem descripserit ad partem quamlibet orizontis, et erit spacium 24 horarum dies continuus sine nocte, dum vero sol tropicum descripserit yemalem erit econtra spacium 24 horarum nox continua sine die et in intermediis anni diebus erit crementum et decrementum dierum et noctium alternatim...

[II.15]

| In omni orizonte cuius cenith inter circulum articum et polum R 53r mundi septentrionalem extiterit, eo per maius tempus erit dies continuus sine nocte et nox continua sine die, quo fuerit cenith orizontis propinquior polo mundi...

[II.16]

| In orizonte cuius cenith fuerit polus mundi, erit una medietas anni R 53v dies continuus sine nocte et alia medietas nox continua sine die....

[II.17]

In omni orizonte obliquo quilibet duo arcus equales zodiaci et equaliter ab altero puncto equinocti utrimque distantes cum equalibus arcubus equinoctialis ascenderit.

[*Figura #12*] Sit circulus meridianus *abg* in quo describatur orientalis medietas orizontis que sit *bed*, et equinoctialis *aeg* sitque *tk* arcus zodiaci incipiens in *t* puncto ab equinoctio vernali ita quod *tk* sit gratia exempli signum arietis. Liquet ergo quod arcus *tk* ascendit cum arcu *te* de equinoctiali eo quod *k* et *e* simul tangunt orizontem et punctus *t* est inicium utriusque arcus. Dico ergo quod cum arcu equinoctialis equali | arcui *te* ascendet signum piscium. Sit enim propter commoditatem R 54r figure arcus *zh* signum piscium ita quod tam *z* quam *t* ymaginentur esse idem punctum equinoctii vernalis, *t* scilicet quando finis arietis incipit ascendere supra orizontem, et *z* quando principium piscium incipit ascendere supra eundem. Palam est ergo quod arcus *zh* ascendit cum arcu *ze*. Dico igitur quod arcus *ze* equalis est arcui *te*. Sint enim *l* et *m* poli mundi a quibus ad terminos arietis et piscium. Protrahantur arcus circulorum magnorum qui sint *lt lk mz mh*, et protrahatur colurus *mel*. Quia igitur *k* finis arietis et *h* finis piscium equaliter distant utrinque a puncto equinoctii vernalis, ideo per 22^{am} primi huius eorum declinationes erunt equales, quibus abscisis a quarta circuli, remanet *mh* equalis *kl*. Sed et *mz tl* sunt equales cum utraque sit quarta circuli, et basis *zh* equalis est basi *tk* scilicet piscis arieti. Ergo *zmh* et *tlk* sunt trianguli equilateri. Quare per secundam partem quarte primi Euclidis angulus *hmz* equalis est angulo *klt*. Sed et per eandem angulus *hme* equalis est angulo *kle* cum *eh* [*ek*] arcus sint equales per 5^{am} huius et reliqua latera equalia, relinquitur ergo angulus *zme* equalis angulo [*tle*]. Sed et latera illos angulos continencia sunt equalis. Ergo per 7^{am} primi Euclidis basis *ez* equalis est basi *et*, quod fuit probandum. Et pari racione quilibet alii duo arcus equales incoati a puncto equinoctiali ex utraque parte sumpti habebunt ascensiones equales. Et quia si ab equalibus equalia demantur relinquentur equalia, palam est quod omnes arcus zodiaci equales et equaliter distantes a puncto equinoctii ascensiones habebunt equales. Sic ergo patet conclusio et est 11^{a} conclusio 2^{i} libri Gebyr.

[II.18]

Quilibet duo arcus zodiaci equales et equaliter ab alterutro punctorum tropicorum distantes habent in orizonte obliquo ascensiones coniunctas equales illis ascencionibus quas in orizonte recto coniunctim obtinent iidem arcus. Unde ex illa et premissa conclusione liquebit quod si note fuerint ascensiones unius quarte in orizonte obliquo omni aliarum quartarum ascensiones eciam note erunt.

[*Figura #19*] Sit circulus meridianus *abgd* et medietas orientalis orizontis obliqui *bed* et sit equinoctialis *aeg*. Et describantur duo arcus zodiaci equales et equalis elongacionis a tropico yemali qui sint *zh th*. Et sit *zh* gratia exempli signum libre et *th* signum piscium et *t* punctum vernale et *z* punctum autumnale, ita quod *h* intelligatur quasi esset duo puncta principium scilicet piscium et finis libre eo quod utrumque | R 54v
illorum tangit idem circulus equedistans equinoctiali propter equales eorum distancias a punctis equinoctialibus propter eorum declinationes equales ab equinoctiali. Et per consequens principium piscium et finis libre in eodem puncto tangent orizontem scilicet in puncto *h* ita ut intelligatur speram verti sicut in figura precedente, ut finis libre et nunc principium piscium ponatur super orizontem. Deinde a polo mundi meridiano qui sit *k* transiat quarta circuli magni per punctum *h* usque ad *b* in equinoctiali. Palam est ergo quod arcus *zh* ascendit in orizonte obliquo cum arcu *ze* et in orizonte recto cum arcu *zl*. Item arcus *ht* ascendit in orizonte obliquo cum arcu *te* et in orizonte recto cum arcu *tb*. Igitur

741 ek] *tk* R **742** tle] *kle* R

749 11a…Gebyr] Gebir, p.30 (incorrectly numbered 38) to 31.

ascenciones arcuum *zh ht* coniunctim sunt arcus *zt* in orizonte recto, et ascensiones eorumdem coniunctim sunt arcus *zt* in orizonte obliquo. Ergo liquet propositum. Nec potest dici quod arcus *le* qui est differentia ascensionum tam libre quam piscium in orizonte recto et obliquo sit maior in una ascensione quam in alia eo quod declinatio finis libre eadem est vel equalis declinationi principii piscium. Utraque ergo signari potest per arcum *hl,* et arcus *he* orizontis est idem cum utraque ascensione eo quod per 5^{am} huius tam finis libre quam principium piscium secat orizontem in *h* puncto. Ergo trianguli *lhe* duo latera *lh* et *he* in ascensione libre sunt equalia *lh* et *he* lateribus in ascensione piscium et angulus *l* est rectus in utroque et angulus *e* non rectus. Ergo per 13^{am} primi Euclidis basis *le* tanta est in uno sicut in alio. Ergo differentia ascensionum libre in orizonte recto et obliquo equalis est differentie ascensionum piscium in orizonte recto et obliquo, et ita de quibuslibet partibus zodiaci equalibus et equaliter distantibus ab altero tropicorum. Sine calumnia ergo patet conclusio et est 12^{a} 2^{i} Gebyr.

Unde cum note fuerint ascensiones quarte zodiaci a principio arietis usque ad principium cancri, note erunt eciam ascensiones quarte a principio capriconi usque ad principium arietis per premissam propter ascensiones eorum equales. Sed et note erunt eciam ascensiones quarte ab inicio cancri usque inicium libre sive ab inicio libre usque ad inicium capricorni, quia si ascensiones singularum partium prime quarte zodiaci in orizonte obliquo habueris et illas a duplo ascensionum earumdem partium in orizonte recto subtraxeris, remanebunt per conclusionem presentem ascensiones partium secunde quarte et tercie que eis secundum equalitatem et equalem distanciam a tropico correspondent.

[II.19]

Cuiuslibet arcus zodiaci ascensionem in obliquo circulo reperire. Unde si in sinum declinationis arcus zodiaci incoati a puncto equinoctii ducatur sinus altitudinis poli et productum per | sinum complementi R 55r
eiusdem altitudinis dividatur, resultabit quiddam quod si ducatur in sinum quarte circuli et productum dividatur per sinum complementi declinationis arcus dati, resultabit sinus differencie ascensionum in recto circulo et obliquo.

[*Figura #20*] Sit meridianus *abg* et orizon *bed* equinoctialis *aeg* et zodiacus *thlz*. Et sit arcus *hl* supra orizontem qui gratia exempli sit signum arietis. Est ergo eius ascensio in orizonte obliquo arcus *he*. Sit *h*

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781 lh] *supr. lin.* R

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783 13am…Euclidis] [Note the use of a proposition about plane triangles to justify an argument about spherical triangles.] **788** 12a…Gebyr] Gebir, p. 31.

punctum equinoctii vernalis, et transiat *klm* quarta circuli magni a polo mundi septentrionali qui sit *k*. Est ergo *hm* arcus ascensio arietis in spera recta que nota est per 23^{am} primi huius, et arcus *em* est differentia illius ascensionis supra ascensionem arietis in spera declivi. Illam ergo querimus, ut per eam subtractam ab ascensione arietis in circulo directo, remanet nobis *he* que est ascensio eiusdem in circulo obliquo. Cum igitur a puncto *g* procedant duo arcus *ge gk* a quorum terminis *k* et *e* reflectuntur alii duo arcus *eld klm*. Ergo per katam disiunctam proportio sinuum *kd* ad *gd* componitur ex proporcionibus sinuum *kl* ad *lm* et *em* ad *eg* quartam circuli. Ergo ex prima regula operandi data in fine primi huius, notus erit arcus *em*, et per consequens *he* que est ascensio arietis in orizonte obliquo dato. Habitata autem ascensione arietis consimiliter inquiratur ascensio arietis et tauri simul, a qua quidem ascensione subtrahatur ascensio arietis, et remanebit ascensio tauri. In aliis eciam similibus, similiter est agendum. Ptholomeus autem ponit hic consequenter quantitates omni arcuum predictorum et eciam ascensiones signorum pro Rodo insula ubi longior dies est 14 horarum dimidii et altitudo poli 36 gradus, quas quia modicum utilitatis obtinent hic omitto.

Est eciam hic notandum quod .... conclusio non potest demonstrari per quattuor quantitates solomodo nisi supponantur alii arcus esse noti quam hic supponit Ptolomeus. Ideo posui corellarium cum sex quantitatibus iuxta modum operationis Ptholomei. Supposita tamen notitia azimuth finis arietis dum in ortu fuerit seu cuiuscumque alterius porcionis quod idem azimuth in presenti figura signatur per arcum *el*. Patet per conversam kate coniunctis quod proportio *gm* ad *ge* quartam circuli componitur ex proporcionibus *dl* ad *de* et *km* ad *kl*. Sed *de* et *km* sunt equales quia uterque est quarta circuli. Ergo proporcio *gm* ad *ge* componitur ex proporcionibus *dl* ad *de* et *de* ad *kl*. Cum igitur ex eisdem componatur proporcio *dl* ad *kl* propter *de* medium interceptum, ergo sicut *gm* ad *ge* ita *dl* ad *kl*, intendo totum in sinibus. Sed *gm* est complementum differencie ascensionum arietis in spera recta et obliqua, et *dl* est complementum azimuth *l* finis arietis et *kl* est complementum declinationis eiusdem *l* puncti. Ergo per 19^{am} 7^{i} Euclidis erit corellarium, illud verum quod scilicet:

| Si in sinum quarte circuli ducatur sinus complementi azimuth R 55v
puncti terminantis arcum cuius ascensio queritur in puncto equinoctii incoatum, et productum dividatur per sinum complementi declinationis

815 remanet] remaneat *add. supr. lin.* R puncti eiusdem, resultabit sinus complementi differentie ascensionum quesite et hoc ostendi 13^{a} conclusio 2^{i} libri Gebyr.

[II.20]

Ad differenciam ascensionum cuiuscumque portionis zodiaci in obliquo circulo et directo, per modum alium facilius et compendiosius devenire. Unde si in sinum ascensionis porcionis date zodiaci in spera recta ducatur sinus differencie que est inter medietates arcuum diei equalis et minime, et productum per sinum quarte circuli dividatur, resultabit sinus differencie ascensionum proposite porcionis.

[*Figura #21*] Sit meridianus *abg*, orizon *bed*, equinoctialis *aeg*, et zodiacus *zeh*, et sit *e* punctum sectionis punctum vernale. Et ponatur arcus *et* quantum voluerimus a cuius termino *t*. Protrahatur *tk* equedistans equinoctiali quousque secet orizontem in puncto *k*, et a polo mundi qui sit *l* protrahantur portiones circulorum magnorum que sint *ltm lkn* et *le*. Palam igitur quod arcus *et* ascendit in spera recta cum arcu *em.* Ascendit eciam in orizonte *bed* cum arcu equalis arcui *mn*, quod sic patet. Nam per 10^{am} 2^{i} Theodosii arcus *mn* et *tk* sunt similes et arcus similes orbium equedistancium ascendunt in temporibus equalibus in omni orizonte ut dici potest ex 13^{am} 2^{i} Theodosii propter hoc quod orizon quilibet obliquus continget continue unum circulum semperiterne apparicionis ex parte septronali et alium sibi equalem sempiterne occultacionis ex parte meridionali quorum uterque est equedistans equinoctiali. Circulos vero equedistantes inter predictos circulos interceptos dividit orizon predictus equinoctialem scilicet in partes equales et reliquos singulos in partes inequales ut patuit in nona huius. Cum igitur propter motum celi contingat orizon circulum sempiterne apparicionis continue in diverso puncto et diversa, et consimiliter dividit reliquos circulos in diversis punctis. Et diversis captis punctis contactuum in principio et in fine alicuius temporis, et captis eciam punctis sectionum ex parte orientali seu occidentali in principio et fine eiusdem temporis, erunt arcus predictorum circulorum intercepti inter predicta puncta similes per 13^{am} 2^{i} Theodosii. Et illi arcus simul ascendunt, ergo modi orizonte arcus similes equedistancium circulorum in temporibus equalibus ascendunt. Arcus igitur *te* ascendit in orizonte *bed* cum arcu equali arcui *mn* propter hoc quod | ascendit cum arcu *tk* simili. Arcus igitur *ne* est differencia R 56r ascensionum *te* arcus in circulo recto et obliquo.

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848 Gebyr] hoc est prima regula Albategni capitulo 13 de eodem in principio capituli ... *adnot. mg. a. m.* R **863** et^{2}…$similes^{2}$] *add. mg. a. m.* R **867** occultacionis] *supr. lin. a. m.* R

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848 13a…Gebyr] Gebir, p. 31-2.

Ad quantitatem vero huius differencie devenit Ptholomeus per hunc modum. [*Figura #22*] Sit orizon *bed*, equinoctialis *aeg*, et meridianus *abg*. Et sit *h* punctus in orizonte per quem transit tropicus yemalis in ortu suo, et sit *k* punctus orizontis per quem transit principium piscis vel alterius cuiuscumque partis illius quarte zodiaci. Deinde a polo antartico qui sit *z* transiant arcus circulorum magnorum per puncta predicta qui sint arcus *zht zkl*. Cum igitur ab arcubus *tz te* reflectantur *zkl ekh*, erit per katam disiunctam proportio *th* ad *hz* composita ex proporcionibus *lk* ad *kz* et *te* ad *le*, intelligendo scilicet de sinibus illorum arcuum. Sed *th* est idem in omni orizonte cum sit maxima declinatio, et per consequens *hz* est idem in omni orizonte cum sit complementum maxime declinationis. Eciam *lk* et *kz* sunt idem in omni orizonte cum *lk* sit declinatio puncti zodiaci ut principii piscis et [*kz*] eius complementum. Illi ergo quattuor arcus erunt noti in omni orizonte. Sed et arcus *te* erit notus in orizonte noto cum sit differentia medietatis arcus diei minime ad diem equalem in illo orizonte. Remanebit ergo arcus *le* notus. Ex prima enim regula operandi data in fine primi huius, patet istud corellarium videlicet quod:

Si in sinum declinationis maxime ducatur sinus complementi declinationis arcus cuius ascensio quem et productum per sinum complementi declinationis maxime dividatur, resultabit quiddam per quod si dividatur illud quod sit ex ductu sinus declinationis arcus dati in sinum differencie quem est inter medietates diei minime et equalis, resultabit sinus differencie ascensionum quesite. Et hec est probatio Ptholomei.

Sed probabo intentum facilius per quattuor quantitates nam ut prius proportio *th* ad *hz* componitur per katam disiunctam ex proporcionibus *lk* ad *kz* et *te* ad *le*. Sed ut patuit per ultimam primi huius eadem proporcio *th* ad *hz* scilicet proporcio maxime declinationis ad eius complementum componitur ex proporcionibus *lk* ad *kz* et tocius quarte ad ascensionem arcus dati in spera recta, intelligendo de sinibus. Ergo eadem est proportio *te* ad *le* et quarte circuli ad ascensionem arcus dati in spera recta. Cum igitur arcus *te* sit differencia ascensionis tocius quarte a principio capricorni usque in finem piscium ut patuit superius, et idem arcus *te* est differencia medietatum diei equalis et minime. Arcus eciam *le* est differencia ascensionum piscium vel cuiuslibet alterius porcionis illius quarte, ergo sicut sinus quarte circuli ad sinum ascensionis piscium vel alterius proposite porcionis in spera | recta ita sinus differencie R 56v
medietatum arcuum diei equalis et minimi ad sinum differencie

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895 ut] principia *add.* R | kz] *k* R **907** quantitates] idem per quattuor quantitates *adnot. mg.* R

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899 fine…huius] I.23, ca. lines 504-556.

ascensionum proposite porcionis. Ex 19^{am} igitur 7^{i} Euclidis patet corellarium conclusioni annexum hic.

Ulterius in illo capitulo ponit Ptholomeus quantitate dupli arcus *th* qui secundum eum est 47 gradus 42 minuta 40 secunda et corda eius 28 partis 3 minuta et 55 secunda et dupli arcus *hz* residui qui est 132 gradus 17 minuta et 20 secunda et corda eius 109 partes 44 minuta et 53 secunda. Ponit eciam quantitatem dupli arcus *kl* et dupli *kz* residui in distancia puncti *k* a puncto vernali per 10 gradus et 10 gradus usque in finem quarte circuli quod quia per tabulam declinationum haberi potest satis faciliter hic omisi inserere. Deinde per subtractionem distanciarum predictarum, concludit Ptholemeus quod remanebit proportio sinuum *te* ad *le* que in distancia 10 graduum a puncto vernali erit ut 60 partium ad 9 partes 33 minuta. Et in distancia 20 graduum ut 60 ad 18 partes 57 minuta et in distancia 30 graduum ut 60 ad 28 partes et 1 minuta et in distancia 40 graduum ut 60 ad 36 partes 33 minuta et in distancia 50 graduum ut 60 ad 44 partes 12 minuta et in distancia 60 graduum ut 60 ad 50 partes 44 minuta, et in distancia 70 graduum 60 ad 58 partes 55 minuta, ulterius exemplicando ponit Ptholomeus quantitatem. Deinde ponit ascensiones eorumdem arcuum in spera recta, a quibus subtrahens predictas differencias, illud quod remanet ponit pro ascensionibus illorum arcuum in Rodo insula. Universaliter autem collectis huiusmodi differencionis de gradu in gradum vel de 10 gradibus in 10 gradus usque ad complementum unius quarte zodiaci, si huiusmodi differencie subtrahantur ab ascensionibus illorum arcuum in spera recta in quarta que est ab inicio arietis usque ad principium cancri et in quarta ab inicio capricorni usque ad principium arietis, sique addantur eedem differencie super ascensiones suorum arcuum in libre ad inicium capricorni, habebuntur ascensiones arcuum predictorum in spera obliqua.

Et ut huiusmodi ascensiones in diversis orizontibus promptius(?) habeamus, faciemus duo tabulas, quarum prima ostendet ascensiones singularum decimarum in orizonte recto cuius quilibet dies 12 est horarum. Secunda tabula in orizonte cuius longior dies continet 12 horas. Et dicitur tercia | tabula in orizonte cuius longior dies continet 13 horas, R 57r et sic continue variando orizontas per medietatem unius hore quousque devenerimus ad orizontem cuius longior dies 17 horas continet. Unde in prima cellula prime linee tabularum ponentur 10 gradus, et in secunda cellula eiusdem prime linee descendendo 20, et in tertia 30, et in quarta 40, et sic crescendo continue per 10 gradus usque ad 360. Et sic erit longitudo tabule 36 cellularum. Deinde in directo trium primarum cellularum ponatur aries in parte sinistra, et in directo trium duarum cellularum ponatur tauris, et sic deinceps, et sic habemus duas lineas descendentes. Deinde in tertia linea ponemus correspondent ascensiones singularum decimarum zodiaci in gradibus et minutis ut contra primos 10 gradus arietis ponantur ascensiones illarum 10 graduum in orizonte, pro quod deservit illa tabula. Et contra 20 ponantur ascensiones secundorum 10 graduum arietis scilicet a decimo gradu ad 21^{m} gradum et sic continue singillatim. In linea quarta ponatur summa ascensionum predictarum ut contra 10 ponatur ascensio 10 graduum, et contra 20 ponatur ascensio 20 graduum, et sic deinceps summatim. Et hic sequitur ulterius inscriptio tabularum.

[II.21]

Per notas ascensiones quantitatem arcus diei cuiuscumque gradus zodiaci et quantitatem eciam arcus noctis numerumque horarum equalium tam noctium quam dierum nec(?) termino(?) et quantitatem horarum inequalium eorumdem infallibiliter reperire.

Cum enim propter 12^{am} primi Theodosii ecliptica et orizon in equalia se secent, necessario orientur sex signa de die et eciam sex de noctis. Capta ergo medietate zodiaci a gradu solis ad gradum oppositum secundum successionem signorum, pars equinoctialis que cum illa medietate zodiaci ascenderit, erit quantitas arcus diei illius gradus quem sol occupaverit illo die. Item capta ascensione alterius medietatis zodiaci scilicet ab opposito loci solis ad locum eius secundum successionem signorum, resultabit quantitas arcus noctis. Vel brevius subtracta quantitate arcus diei ab una revolucionis, remanet illud idem eo quod in die et nocte coniunctim revolucio una integra compleatur. Item cum quantitas arcus diei seu noctis fuerit divisa per 15, resultabit numerus horarum equalium diei vel noctis eo quod hora equalis sit ascensio 15 graduum propter hoc quod 15 gradus in una revolucione integra 24 vicibus continetur. Et ideo eciam si numerum equalium horarum diei de 24 minuamus, remanebit numerus horarum noctis equalium et econtra. Item si arcum diei seu noctis per duodecimam dividamus, exibit quantitas hore inequalis seu temporalis quod idem est diei vel noctis eo quod hora inequalis diei vel noctis sit duodecima pars diei vel noctis. Et ideo eciam si quantitatem hore diurne de 30 | dempserimus, remanebit quantitas hore R 57v
noctis eo quod hora diurna et hora nocturna coniunctim 30 gradus complebunt, qui quidem numerus in una revolucione duodecies continetur. Aliter eciam possumus devenire ad quantitatem hore in equalis diei per differenciam ascensionum medietatis zodiaci incoate a gradu solis et in gradum oppositum terminate, et eciam ad quantitatem hore noctis per differenciam ascensionum alterius medietatis zodiaci

966 continue] singularum *add. sed. del.* R scilicet ab opposito solis ad solem. Nam si illius differencie ascensionum in spera recta et obliqua capiatur pars duodecima vel sue medietate pars sexta, et addatur illud ad 15 gradus si locus solis fuerint vernalis, et si fuerit meridionalis a 15 gradibus minuatur, resultabit quantitas hore in equalis diei, et si econtra seceris, resultabit quantitas hore noctis. Et hoc est 14^{a} conclusio 2^{i} libri Gebyr.

[II.22]

Datas horas inequales ad equales reducere et econtra.

Hore enim inequales sive diurne sive nocturne per multiplicationem redigantur in gradus, productoque diviso per 15, resultabit numerus horarum equalium. Item si horas equales in suos gradus multitudo per 15 resolvamus, productumque diviserimus per numerum graduum hore inequalis sive nocturne sive diurne, quod inde exierit horarum inequalium erit summa. Et hec est 15^{a} conclusio 2^{i} libri Gebyr.

[II.23]

Per notas ascensiones et horas preteritas ascendens et celi medium invenire.

Accipiemus enim horas ab ortu solis in die vel ab eius occasu in nocte, quibus per multiplicationem reductis in gradus exibit arcus equinoctialis qui ab ortu solis si fuerit de die vel ab eius occasu si de nocte fuerit est ex ortus. Videamus igitur quanta pars zodiaci incoata a loco solis de die et ab eius opposito de nocte secundum successionem signorum ascenderit in spera obliqua cum arcu equinoctialis iam habito, nam punctus terminans illam partem zodiaci est ascendens. Et si medium celi habere voluerimus, accipiemus horas a proximo meridie precedente, quibus per premissam in gradus reductis, exibit arcus equinoctialis qui a meridie proxima meridianum pertransiit. Videamus igitur quanta porcionis zodiaci a loco solis incoate sit tanta ascensio in spera recta. Et finis calculi celi medium intimabit. Punctus vero oppositus ascendenti est occidens, et punctus oppositus medio celi supra terram est medium celi sub terra, qui et angulus terre aliter nuncupatur.

Item si per gradum ascendentem habere voluerimus medium celi vel angulum terre, queramus in circulo obliquo ascensiones a principio arietis ad gradum ascendentem secundum successionem signorum. Et habebimus gradum equinoctialis qui simul est in ortu cum gradu zodiaci ascendente. Cum igitur semper inter punctum equinoctialis orientem et eius punctum tam in medio celi quam in angulo terre sint 90 gradus precise, si ad punctum equinoctialis | orientem addamus 90 gradus et R 58r
videamus quis punctus zodiaci cum fine calculi in spera recta ascenderit.

1006 14a…Gebyr] Gebir, p. 32. **1014** 15a…Gebyr] Gebir, p. 32.

Habebimus angulum terre si eciam de puncto equinoctialis oriente subtrahamus 90 gradus si poterimus vel si non poterimus addamus unam revolucionem et subtrahamus postea 90. Punctus zodiaci qui in spera recta ascenderit cum puncto equinoctialis ad quem per huius calculum devenimus, erit in celi medio super terram.

Item eciam e contrario si per celi medium super terram devenire voluerimus ad ascendens, ascensioni medii celi in spera recta addamus 90. Et capto puncto in quem terminatur addicio, videamus in spera declivi quis punctus zodiaci cum illo puncto ascenderit, nam ille procul dubio est ascendens. Et notandum est quod omnibus eandem lineam meridionalem habentibus erit longitudo solis a linea meridionali sive supra terram sive sub terra secundum eundem numerum horarum equalium. Sed quibus illa meridionalis linea est diversa, erit diversitas meridiei secundum quantitatem arcus equinoctialis inter meridionales lineas. Et hoc tota docet conclusio 16ª 17ª 18ª 2^{i} libri Gebyr.

[II.24]

Cuiuslibet anguli speralis supra polum alicuius circuli consistentis ad quatuor rectos proporcio est sicut arcus eiusdem circuli qui angulo predicto subtenditur ad circumferenciam eius totam.

Istud per sumta equemultiplicia angulorum et arcuum probari poterit sicut in ultima 6^{ti} Euclidis in planis angulis est probatum. Unde ex hoc patet quod in omni angulo sperali posito polo et circumducto secundum quamcumque longitudinem circulo, ubi arcus illius circuli inter latera predictum angulum continencia interceptus sit quarta circuli, ille angulus est rectus. Et si ille arcus sit maior quarta circuli, est angulus obtusus vel expansus. Et si minor, est acutus. Et est notandum quod omnis angulus speralis hic diffinitus est ex concursu circulorum maiorum qui fuerit in spera.

[II.25]

Omnes duo anguli ex duobus meridianis equaliter ab altero equinoctii puncto distantibus et ex circulo signorum causati quorum uterque sit septentrionalis vel uterque meridionalis et uterque eciam orientalis vel uterque occidentalis, neccessario sunt equales.

[*Figura #23*] Sit arcus equinoctialis *abg* et arcus zodiaci *dbe*, et sit *b* alter punctorum equinoctialium a quo sumantur duo arcus *bh bt* equales. Et a polo septentrionali qui sit *z* transiant duo arcus per puncta *t h* qui sint *zhk* et *zlt*. Dico ergo quod angulus septentrionalis orientalis qui est ad *h* est equalis angulo septentrionali orientali qui est ad *t* scilicet angulus *zhd*

1053 lineas] intercepi *add*. R **1064** obtusus] *supr. lin. a. m.* R

1054 16a…Gebyr] Gebir, p. 32-3.

est equalis angulo *ztb*. Et idem est arguendum de aliis angulis sumptis secundum easdem differencias ad puncta *t* et *h* terminatis(?) propter hoc quod anguli contra se positi | sunt equales. Probatur autem hec conclusio R 59v sit triangulus *bkh* equilaterus est et equiangulus triangulo *blt*. Nam *bt bh* sunt equales ex ypotesi, et *bk bl* sunt equales propter equales ascensiones arcuum *bt bh* equalium, et *kh lt* sunt equales propter equales declinationes arcuum *tb bh*. Ergo per 4am primi Euclidis angulus *btl* est equalis angulo *bhk*, quare et angulo *zhd* sibi equali cum sint anguli contra se positi. Sic ergo patet propositum. Cum igitur duo anguli *zhd* et *zhb* simul sumpti valeant duos rectos et similiter duo anguli *ltb* et *lte* valeant duos rectos et anguli *zhd* et *ltb* sunt equales, ergo relinquuntur anguli *zhb* et *lte* equales. Et sic omnes anguli terminati ad puncta *t h* secundum easdem differencias posicionis sumpti sunt equales. Et ideo sufficiet nobis in conclusionibus sequentibus cum angulis septentrionalibus orientalibus propositum declare. Est autem hec 19a conclusio secundi Gebyr.

[II.26]

Omnem duo anguli ex duobus meridianis equaliter ab altero puncto tropicorum distantibus et ex circulo signorum causati quorum uterque sit septentrionalis vel uterque meridionalis et uterque eciam orientalis vel uterque occidentalis duobus rectis angulis sunt equales.

[*Figura #24*] Sit arcus orbis signorum *abg* cuius punctus *b* sit alter tropicorum a quo sumantur *bd be* arcus equales. Et ad puncta *d e* protrahantur duo meridiani *zd ze* a polo septentrionali qui sit *z*. Dico ergo quod duo anguli *adz* et *bez* coniunctim sunt equales duobus rectis, nam in triangulo *zde* duo latera *zd* et *ze* sunt equalia propter equalem declinationem punctorum *d* et *e*. Ergo per 2am primi Euclidis anguli super basim sunt equales. Sed anguli *adz* et *bdz* sunt equales duobus rectis, ergo et anguli *adz* et *bez* erunt equales duobus rectis. Et hoc ostendit 20a 2i Gebyr.

[II.27]

Angulus super punctum tropicum ex meridiano et orbe signorum causatus neccesario erit rectus.

[*Figura #25*] Sit meridianus *abgd* et medietas zodiaci *aeg* ita quod *a* sit tropicum hyemale et super polum *a* secundum spacium lateris quadrati describatur semicirculus *bed*. Quia ergo meridianus *abgd* transit super polos utriusque circulorum *aeg bed*, erit arcus *ed* quarta circuli eo quod oportet *d* et *b* puncta esse polos circuli *aeg*, propter hoc quod arcus *ad* est quarta circuli per 17am primi Theodosius cum *a* sit polus circuli *deb* ex

1092 19a…Gebyr] Gebir, p. 33. **1103** 2am…Euclidis] This should refer to *Elements* I.5.
1105 20a…1106 Gebyr] Gebir, p. 33-4.

ypotesi. Et quia *de* arcus est quarta circuli, erit per 24^{am} huius angulus *a* cui subtenditur rectus.

[II.28]

Maxima declinatione nota notus erit angulus ex meridiano et circulo signorum in puncto equinoctii designatus. Unde si declinationem maximam a quarta subtraxeris vel eam addideris supra quartam, | R 59r resultabit angulus ille quesitus.

[*Figura #26*] Sit meridanus *abgd* et medietas equinoctialis *aeg* medietas, vero zodiaci *azg*, et sit *a* punctum equinoctii autumnalis. Ubi polo signato discribatur super eum secundum spacium lateris quadrati semicirculus *bzed*, erit ergo per 17^{am} Theodosii tam *az* quam *ed* arcus quarta circuli propter hoc quod circulus meridianus *abgd* transit per polos utriusque circulorum *aeg bed* ut patuit in premissa. Est ergo *z* punctus tropici estivalis et arcus *ze* maxima declinatio est eciam nota cum sit 23 gradus 51 minuta. Erit ergo totus arcus *zed* notus cum sit 113 gradus 51 minuta. Quare angulus *daz* est notus cum sit se habeat ad quattuor rectos sicut arcus *zd* ad totam circumferenciam. Relinquitur ergo angulus *baz* notus cum sit residuum de duobus rectis, a quibus demitur angulus *daz*. Erit ergo angulus *baz* 66 gradus 9 minuta, et hoc est quod querere intendebam. Et hoc ostendit 21^{a} conclusio 2^{i} Gebyr.

[II.29]

Cuiuslibet anguli quantitatem ex meridiano et circulo signorum in quocumque puncto noto causati certitudine reperire. Unde si in sinum quarte ducatur sinus ascensionis in spera recta cuiuscumque arcus zodiaci in puncto equinoctii incoati et terminati in puncto proposito, dividaturque productum per sinum eiusdem arcus zodiaci, resultabit sinus anguli iam quesitis.

Ad huius anguli quantitatem devenit Ptholomeus per notam declinationem illius puncti in quo angulus propositus designatur, et hoc sic. [*Figura #27*] Sit meridianus *abgd*, et equator *aeg*, zodiacus *bztd*. Et sit *z* punctum autumnalem, et sit *zb* gratia exempli signum virginis. Erit ergo arcus *ba* declinatio virginis. Posito ergo polo in *b* puncto describatur secundum distanciam lateris quadrati semicirculus *keth*. Quia igitur circulus *abgd* transit per polos utriusque circulorum *aeg* et *hek*, erit per 17^{am} primi Theodosii quilibet istorum arcuum *bh bt eh* quarta circuli. Intellecto ergo quomodo ab *h* puncto descendunt duo arcus *he hb* a quibus reflectuntur alii duo *bzt* et *eza*, patet per katam disiunctam, quod intelligendo de sinibus, proporcio *ba* ad *ha* componitur ex proporcionibus *bz* ad *zt* et *et* ad *eh* quartam. Sed *ba* est notus cum sit declinatio virginis,

1135 21a…Gebyr] Gebir, p. 34.

et per consequens *ha* residuum erit notum. Eciam *zb* quod est signum virginis est notum, et *zt* residuum per consequens erit notum cum sit duo signa. Quarta eciam *eh* est nota, ergo *te* arcus neccessario erit notus, quare et totus arcus *tek*. Ergo per 24^{am} huius notus erit angulus *tbk* cuius quantitatem quesivimus. Unde pro eius noticia habenda per primam regulam operandi datam in fine primi huius patet istud corelarium, videlicet quod:

Si in sinum declinationis puncti cuius queritur angulus ducatur sinus complementi porcionis sumpte ab equinoctio et in punctum propositum terminate, dividaturque productum per sinum complementi declinationis predicte, resultabit quiddam quod si in sinum quarte ducatur et productum per sinum porcionis proposite | dividatur, exibit sinus R 59v
differencie anguli recti ad angulum quem inquiris. Quam si recto addideris vel ab eo subtraxeris, non deficies a quesito. Id est si dictam differenciam angulo recto addideris, habebis angulum *tbk* maiorem duorum angulorum ex meridiano et zodiaco in *b* puncto proposito causatorum duos rectos valencium, et si differenciam illam subtraxeris, habebis angulum *hbt* qui est minor eorum. Et hec est operacio Ptholomei.

Sed quia ascensiones in spera recta propter tabulas ascensionum sunt eque note cum declinationibus arcuum quorumcumque, ideo ubi Ptholomeus supponit declinationem arcus propositi, ego supponam ascensionem eiusdem et deveniam per quattuor sinus ad quesiti anguli quantitatem. Nam per conversam kate coniuncte proporcio *ht* ad *he* componitur ex proporcionibus *az* ad *ae* et *tb* ad *zb*. Sed *ae* est quarta cum *e* sit polus meridiani, et *tb* est quarta ut ostensum est supra, et per consequens *ae* et *tb* sunt equales. Ergo proporcio *th* ad *he* componitur ex proporcionibus *az* ad *ae* et *ae* ad *zb*. Sed ex iisdem componitur proporcio *az* ad *zb* propter *ae* medium interpositum, ergo sicut *th* qui est arcus anguli *hbt* ad *he* quartam circuli ita *az* qui est ascensio porcionis proposite ad *zb* propositam porcionem. Intelligendo omnia argumenta in sinibus. Patet ergo corellarium conclusioni annexum, et idem ostendit 22^{a} conclusio 2^{i} Gebyr.

Et est notandum quod per investigacionem Ptholomei in hoc capitulo erit angulus *kbt* qui est angulus capitis virginis 111 graduum, et angulus capitis scorpionis tot graduum per 25^{am} huius propter equalem distanciam a puncto equinoctii. Anguli eciam qui sunt apud caput tauri et caput piscium, uterque erit 69 graduum per 26^{am} huius, quia illud est residuum de duobus rectis. Consimiliter si ponatur punctum *b* principium leonis, erunt anguli principii leonis et principii sagittarii quilibet 102

1160 fine…huius] Ca. lines 504-556. **1185** 22a…1186 Gebyr] Gebir, p. 34.

graduum et 30 minuta, et uterque angulorum apud principium geminorum et principium aquarii 77 graduum et 30 minuta. Et sic per angulos in singulis sectionibus unius quarte zodiaci, possumus ad angulos aliorum quartarum facillime devenire.

[II.30]

Omnes duo anguli ex uno orizonte declivi et circulo signorum in duobus punctis secundum equalem distanciam ab equinoctiali puncto signati, quorum uterque sit septentrionalis vel uterque meridionalis et eciam uterque sub orizonte vel uterque supra orizontem, neccesario sunt equales.

Anguli causati ex orbe signorum et orizonte recto sunt iidem cum angulis ex zodiaco et meridiano causatis, et ideo de illis angulis non competit plus tractare. [*Figura #28*] Sed ad propositum sit meridianus *abgd,* et orizon obliquus *bed,* equinoctialis *aeg.* Et sint due porciones orbis signorum *zht* et *klm,* sitque tam *z* quam *k* punctum autumnale, et arcus *zh* equalis arcui *kl*. Dico ergo quod angulus *eht* equalis est angulo *dlk,* et ita de aliis angulis | correspondentibus alia puncta *l* et *h* signatis. In R 60r triangulis enim *zhe kle* duo latera *zh* et *kl* sunt equalia ex ypotesi et eorum azimuth *he* et *le* sunt equales. Eorum eciam ascensiones *ze* et *ek* sunt equales, ergo per 4^{am} primi Euclidis angulus *ehz* equales est angulo *elk*. Quare angulus *eht* residuus duorum rectorum equalis est angulo *dlk* residuo, quod erat probandum. Idem docet 23^{a} conclusio 2^{i} Gebir.

[II.31]

Omnes duo anguli ex uno orizonte declivi et circulo signorum simul et semel in oppositis punctis causati quorum uterque sit septentrionalis vel uterque meridionalis et alter supra orizontem vel uterque sub orizonte et alter septentrionalis et reliquus meridionalis et reliquus sub orizonte aut quorum vice versa sit uterque supra orizontem, omnes tales duo anguli duobus rectis angulis sunt equales.

[*Figura #29*] Sit orizon *abgd* et zodiacus *aegz.* Angulus *zad* equalis est angulo *zgd* per 4^{am} primi Euclidis, eo quod arcus *zd* qui est arcus maxime declinationis illorum circulorum secat utriusque medietatem per equalia. Cum igitur anguli *zad* et *dae* sunt equales duobus rectis, erunt duo anguli *dae* et *zgd* equales duobus rectis. Quare et anguli *zab* et *egb* qui cum prioribus sunt anguli contra se positi sunt eciam equales duobus rectis. Ergo et anguli eis equales scilicet *dge* [*zad*]. Et similiter *zgb* et *eab*

1229 zad] *zae* R

1215 23a…Gebir] Gebir, p. 34. **1224** 4am…Euclidis] Note again the incorrect use of plane geometry to justify spherical geometry.

sunt equales duobus rectis, quod fuit probandum. Et hec est 24^{a} conclusio 2^{i} Gebyr.

[II.32]

Omnis angulus ex orizonte declivi et zodiaco in puncto preter tropicum ex parte orientis signatus equalis est angulo qui in parte occidentis secundum equalem ab eodem tropico distanciam et secundum easdem sub orizonte vel supra easdemque septentrionis vel meridiei differencias est descriptus. Quapropter notis angulis septentrionalibus supra orizontem ex parte orientis in altera medietate zodiaci ab ariete scilicet usque in libram, omnes utriusque medietatis anguli tam ex occidentis parte quam orientis et tam sub orizonte quam supra neccesario noti erunt.

Ista conclusio statim patet nam per antepremissam angulus capitis scorpionis est equalis angulo capitis virginis ubi uterque est ex parte orientis et uterque septentrionalis et supra orizontem. Sed angulus capitis scorpionis sic signatus est equalis angulo capitis tauri septentrionali et supra orizontem ex parte occidentis signato eo quod simul tangunt orizontem, alter ex parte orientis et reliquus ex parte occidentis. Et per consequens sunt sectiones medietatis zodiaci et orizontis quarum angulos per 4^{am} primi Euclidis oportet esse equales eo quod arcus maxime declinationis illarum medietatum secat utramque medietatem per equalia. Ergo angulus capitis virginis septentrionalis et supra orizontem ex parte orientis signatus equalis est angulo capitis tauri | septentrionali et supra R 60v orizontem ex parte occidentis signato cum igitur illi duo anguli sint in punctis equaliter distantibus a tropico. Et sicut arguitur de illis, arguitur de singulis aliis secundum equidistanciam signatis. Ergo patet conclusio. Cum igitur per antepremissam noti erunt anguli orientales medietatis zodiaci a libra in arietem propter noticiam angulorum orientalium alterius medietatis ab ariete in libram, et per illam conclusionem noti erunt anguli occidentales propter noticiam angulorum orientalium, ergo notis angulis orientalibus ab ariete in libram, noti erunt omnes anguli tam orientales quam occidentales tocius zodiaci.

[II.33]

Nota latitudine regionis et tropicorum distancia, angulus ex illo orizonte declivi et circulo signorum causatus apud utrumque punctum equinoctii notus erit. Unde si ab altitudine capitis arietis vel quod idem est, a complemento latitudinis regionis, maximam declinationem subtraxeris, resultabit sub orizonte angulus orientalis arietis, cui si distanciam tropicorum addideris, quod inde resultaverit angulus erit libre.

1230 24a…1231 Gebyr] Gebir, p. 34-5.

[*Figura #30*] Discripto meridiano *abgd* sit orientalis medietas orizontis *aed* et quarta equinoctialis *ez*. Et sint duo quarte zodiaci *eb eg* ita quod *b* sit punctus tropici yemalis et *g* punctus tropici estivalis. Et sit *e* in quarta *eb* punctum autumnale, in quarta vero *eg* sit *e* punctum vernale. Et sint ille tres quarte sub orizonte. Cum igitur punctus oppositus cenith distet ab orizontis per quartam circuli et per 10^{am} huius, quanta est latitudo regionis tantum distat cenith ab equinoctiali et per consequens tantum distat punctum sibi oppositum. Ergo arcus *zd* est complementum latitudinis regionis. Arcus ergo *zd* erit notus per subtractionem latitudinis regionis a quarta circuli, a quo si arcum *zg* subtraxeris qui est maxima declinatio, remanet arcus *gd* notus. Cui si addatur arcus *gb* qui est distancia inter tropicos, erit arcus *db* notus. Cum igitur punctus *e* sit polus meridiani, tam angulus *bed* quam angulus *ged* per 24^{am} huius notus erit. Sed angulus *bed* est angulus orientalis capitis libre sub orizonte ex parte septentrionis, et angulus *geb* consimiliter est angulus arietis. Ergo liquet conclusio, et hanc docet 25^{a} 2^{i} libri Gebyr.

[II.34]

Ad cuiuslibet anguli quantitatem ex orizonte declivi et circulo signorum in quocumque zodiaci puncto noto causati infallibiliter devenire. Unde capto puncto medii celi in zodiaco supra terram vel sub terra, prout porcionem zodiaci inter punctum illud et orientalem partem | R 61r
orizontis quarta circuli minorem esse contigerit, si in sinum quarte circuli ducatur sinus profunditatis seu altitudinis puncti capti dividaturque productum per sinum pretacte porcionis zodiaci, resultabit sinus anguli quem inquiris.

[*Figura #31*] Sit meridianus *abgd*, et orientalis medietas orizontis *bed*, medietas vero zodiaci *aeg*. Et sit gratia exempli punctum *e* caput tauri et *g* celi medium sub terra quod propter notas ascensiones in orizonte dato erit notum per 23^{am} huius. Quia ergo punctum *e* est in quarta zodiaci inter principium arietis et principium cancri, erit neccessario arcus *ge* minor quarta circuli ut liquere potest per tabulas ascensionum. Operabimur igitur per arcum *ge*, qui si foret maior quarta circuli, tunc operaremur per arcum *ea*. Describatur igitur super polum *e* secundum quantitatem lateris quadrato porcio circuli maioris qui sit *zht* concurrens cum meridiano in puncto *z*, et compleantur duo quarte *edt* et *egh*. Erit ergo per 17^{am} primi Theodosii polus orizontis *bed* in circulo *zht*, et per eandem 17^{am} id polus orizontis erit in meridiana *zgd*. Ergo oportet ut punctus *z* sit polus orizontis *bedt*, et per consequens uterque arcuum

1281 erit] *supr. lin. a. m.* R

1284 25a…Gebyr] Gebir, p. 35.

zgd et *zht* est quarta circuli. Cum igitur a puncto *t* descendant duo arcus *te tz* a quorum terminis *z* et *e* reflectuntur alii duo arcus *zd* et *eh* secantes se in puncto *g*, erit per conversam kate coniuncte proporcio *th* ad *tz* composita ex proporcionibus *dg* ad *dz* et *he* ad *ge*, intelligendo totum argumentum in sinibus. Ergo per subtractionem proporcionis a proporcione liquet propositum Ptholomei.

Sed ut operationem abbreviem, arguam sic. Proporcio *th* ad *tz* componitur ex proporcionibus *gd* ad *dz* et *he* ad *ge*. Sed *dz* et *he* sunt equales cum uterque sit quarta circuli. Ergo proporcio *th* ad *tz* componitur ex proporcionibus *gd* ad *he* et *he* ad *ge*. Cum igitur proporcio *gd* ad *ge* componitur ex iisdem propter medium *he* sumptum inter extrema, ergo eadem est proporcio *th* ad *tz* et *gd* ad *ge*. Quia ergo *tz* secundum est notum cum sit quarta circuli, et *gd* tertium est distancia puncti medii celi in zodiaco ab orizonto, quam oportet esse notam eo quod distancia *z* poli orizontis ab equinoctiali in circulo meridiano est nota, et declinacio *g* medii puncti celi ab equinoctiali in eodem meridiano erit nota, quare oportet arcum *gz* esse notum. Ergo et *gd* residuum de quarta oportet esse notum. *Ge* eciam quartum est notum propter ascensiones notas per 23^{am} huius, ergo per 19^{am} 7^{i} Euclidis arcus primus qui est arcus *th* neccessario erit notus. Sed ille est arcus anguli *ged* eo quod *e* est polus circuli *zht*. Ergo patet conclusio.

Ad huiusmodi angulos aliter devenit Gebyr et est 26^{a} conclusio 2^{i} Gebir. [*Figura #32*] Sit *gdz* meridianus et *gbha* orientalis medietas orizontis. Sit *zb* equinoctialis et *ehd* zodiacus ita quod *e* sit punctus equinoctii vernalis et *eh* arcus sit quantuscumque volueris dummodo non sit maior quarta circuli. Volo ergo scire quantitatem anguli *ehb*. Nam cum arcus *eh* sit notus, arcus *eb* qui est eius ascensio in orizonte dato erit notus per | 19^{am} huius. Et quia triangulus *ebh* est ex [porcionibus] R 61v
circulorum magnorum, erit proporcio sinus lateris *he* oppositi angulo *b* ad sinum lateris *eb* oppositi angulo *h* sicut proporcio sinus arcus anguli *b* ad sinum arcus anguli *h* ut patet 13^{a} conclusione primi Gebyr, que est hec: omnis trianguli ex arcubus circulorum magnorum est proporcio sinus cuiuslibet laterum ad sinum arcus anguli cui latus illud subtenditur proporcio una. Cum igitur sinus arcus anguli *b* sit sinus complementi altitudinis poli scilicet sinus arcus *gz* qui est idem cum sinu arcus *za*, per 19^{am} 7^{i} Euclidis patet corellarium istud Gebyr scilicet quod:

1316 ad ge^{1}] *add. mg. a. m.* R **1333** eius] *supr. lin.* R **1334** Et quia] *supr. lin.* R porcionibus] proporcionibus R

1328 26a…2i] Gebir, p. 35. **1337** 13a…Gebyr] Gebir, pp. 11-2.

Si in sinum complementi altitudinis poli ducatur sinus ascensionis in circulo obliquo arcus zodiaci non maioris quarta circuli in orizonte proposito elevati, et productum dividatur per sinum eiusdem arcus zodiaci, proveniet angulus ex arcu predicto et orizonte causatus. Istud correlarium inferni ut per diversos modos operandi devenire poterimus ad intentum. Nam Gebyr hic utitur aliis arcubus quam utitur Ptholomeus. Quomodo autem per angulos habitos unius quarte habere poterimus angulos aliarum quartarum ex 30ª huius et duabus eam sequentibus satis liquet.

[II.35]

Situato in linea meridionali puncto tropico orbis signorum et signatis in zodiaco quibuscumque duobus punctis a dicto puncto tropico equaliter utrimque distantibus, duo arcus circulorum magnorum qui a cenith capitum ad duo signata puncta descenderint sibi invicem erunt equales. Necnon et duo anguli coniunctim quos cum orbe signorum, alterum extrinsecum et reliquum inrinsecum, ad eandem differenciam posicionis causaverint(?), duobus rectis angulis equabuntur.

[*Figura #33*] Sit meridianus *gba* ita quod *b* sit cenith capitum qui est polus orizontis, et sit *g* polus equinoctialis. Et sit *hae* zodiacus ita quod *a* sit punctus alterius tropici in linea meridionali, et sint *d* et *z* duo puncta zodiaci equaliter distancia ab *a* puncto tropico. Et descendant a *b* cenith ad duo puncta *d* et *z* duo arcus circulorum magnorum, et a puncto *g* ad eadem puncta descendant alii duo arcus circulorum magnorum. Dico ergo quod arcus *bd* equabitur arcui *bz* et duo anguli *bza* et *bde* simul sumpti sunt equales duobus rectis. Nam in triangulis *gad* et *gaz* duo latera *ga* et *ad* unius sunt equalis duobus lateribus *ga* et *az* alterius, sed et anguli ab illis lateribus contenti scilicet angulus *a* et angulus *a* alterius sunt equales cum uterque per 27^{am} huius sit rectus. Ergo per 4^{am} primi Euclidis latera *gd* et *gz* sunt equalia. Quare per 8^{am} primi Euclidis angulus [*dga*] equalis est angulo *zga*. Et quia sic cum in triangulis *bdg* et *bzg* duo latera *bg gd* unius sunt equalia duobus lateribus | *bg gz* alterius, et anguli ab illis R 62r
lateribus contenti scilicet *g* unius et alterius equales, ergo per 4^{am} primi Euclidis basis *bd* equalis est basi *bz*, quod fuit primum. Et angulus *bdg* equalis est angulo *bzg* per eandem. Quia eciam per 26^{am} huius duo anguli *gza* et *gde* simul sumpti equantur duobus rectis, si de angulo *gza* subtrahatur angulus *gzb*, et angulo *gde* addatur angulus [*bdg*] equalis angulo prius subtracto, erunt duo anguli *bza* et *bde* simul sumpti equales

1371 dga] *gda* R **1372** in] *supr. lin.* R **1378** bdg] *bgd* R

1371 8am…Euclidis] Yet another use of a proposition in plane geometry to justify an argument in spherical geometry.

duobus rectis, quod fuit secundum probandum. Et hec est 27^{a} conclusio 2^{i} Gebyr.

[II.36]

Signato ex orientali parte meridiani aliquo puncto zodiaci et postea eodem in puncto in occidentali parte meridani secundum equalem a meridiano distanciam situato, si cenith captitis septentrionalis vel meridionalis fuerit ab utroque punctorum zodiaci mediancium celum in casibus antedictis, tunc duo arcus circulorum magnorum a dicto cenith ad punctum illud in predictis sitibus descendentes sibi invicem sunt equales. Necnon et duo anguli qui ex predictis arcubus et circulo signorum aput punctum predictum in dictis sitibus versus eandem positionis differenciam sunt causati, dupli sunt coniunctim ad angulum qui ad idem punctum ex meridano et zodiaco procreatur.

[*Figura #34*] Sit ergo primo cenith capitis septentrionalis respectu medii celi in utroque situ et sit meridianus *abgd*. Sitque cenith punctum *g*, et sit *d* polus equinoctialis, et sint duo porciones zodiaci *aez* et *bht*, et sit *h* idem punctum cum *e*. Sintque *h* et *e* equaliter a linea meridiei ex utraque parte distancia ita quod arcus paralelli inter *e* et meridianum interceptus sit equalis arcui paralelli inter *h* et meridianum intercepto. Et transeant duo arcus circulorum magnorum a *d* polo mundi ad puncti *e h*, et alii duo a cenith *g* ad eadem puncta *e* et *h*. Dico ergo quod duo arcus *gh ge* sunt equales et quod duo anguli *ghb* et *gez* simul sumpti dupli sunt ad angulum *dez* seu ad angulum *dhb* sibi equalem. Quia enim *h* et *e* per equales arcus eiusdem paralelli distant a meridiano, erit per 4^{am} primi Euclidis angulus *gdh* equalis angulo *gde*. Sed eciam latera illos equos angulos continencia sunt equalia, ergo per eandem Euclidis *gh* et *ge* bases sunt equales, quod fuit primum, et erit angulus *ged* equalis angulo *ghd*. Cum eciam angulus *dhb* sit equalis angulo *dez*, erunt duo anguli *gez* et *ghb* simul sumpti equales angulo *dez*. Ergo ambo anguli *gez* et *ghb* simul sumpti dupli sunt ad angulum *dez,* quod fuit secundum.

[*Figura #35*] Sit item cenith meridionalis | respectu medii celi in R 62v
utroque situ, a quo ad puncta *h* et *e* protrahantur arcus *ghl* et *gek* circulorum magnorum. Dico ergo quod duo anguli *lhb* et [*kez*] simul sumpti sunt dupli ad angulum *dez*. Angulus enim *dez* equalis est angulo *dhb* immo idem. Sed et angulus *dek* equalis est angulo *dhl* eo quod per 4^{am} primi Euclidis anguli *deg* et *dhg* sunt equales. Totus ergo angulus *lhb* equalis est duobus angulis *dek* et *dez* coniunctim. Quare duo anguli *lhb* et

1412 kez] *keg* R

1380 27a…1381 Gebyr] Gebir, pp. 35-6. **1403** 4am…1404 Euclidis] Yet another use of a proposition in plane geometry to justify an argument in spherical geometry.

kez simul sumpti dupli sunt ad angulum *dez*, quod duo fuit probandum. Hec eadem probant 28^a et 29^a 2^i Gebyr.

[II.37]

Situato eodem puncto zodiaci in diversis sitibus per omnia sicut prius, si a puncto medii celi in porcione orientali fuerit cenith septentrionalis et meridionalis ab altero, tunc predicti duo anguli versus orientem et septentrionem signati ultra duplum anguli tercii coniunctim continent duos rectos. Si vero e conversa fuerit cenith meridionalis a puncto medii celi in porcione orientali et septentrionalis ab altero, tunc predicti duo anguli per duos rectos deficiunt a duplo tercii anguli memorati. Unde notis angulis et arcubus altitudinum ex orientali parte meridiani in medietate zodiaci a principio cancri ad principium capricorni tam arcus quam anguli ex occidentali parte meridiani in eadem medietate zodiaci necnon et arcus et anguli alterius medietatis ex utraque parte meridiani certitudine noti erunt.

[*Figura #36*] Stante similitudine figure prioris sit cenith *g* septentrionalis ab *a* puncto medii celi in porcione orientali, et sit idem *g* cenith meridionalis a *b* puncto medii celi in porcione occidentali. Dico ergo quod duo anguli *gez lhb* sunt maiores duplo anguli *dez* per quantitatem duorum rectorum. Nam per 4^{am} primi Euclidis duo anguli *deg dhg* sunt equales, sed et duo anguli *dhg dhl* sunt equales duobus rectis. Angulus eciam *dez* equalis est angulo *dhb* vel pocius idem angulus, ergo duo anguli *lhb ged* superant duos equos angulos *dhb dez*, et per consequens duplum anguli *dez*, per duos angulos *lhd* et *ged*, quos probabimus equales esse duobus rectis. Et hoc est primum quod volumus.

[*Figura #37*] Sit iterum cenith *g* meridionalis ab *a* puncto medii celi in porcione orientali, et sit septentrionalis a *b* puncto medii celi in porcione occidentali. Dico ergo quod duo anguli *kez* et *ghb*, qui sunt septentrionales orientales respectu zodiaci, coniunctim sunt minores duplo anguli *dez* per quantitatem duorum rectorum. Nam duo anguli *kez ghb* sunt minores | duobus equis angulis *dez* et *dhb*, et per consequens R 63r
duplo anguli *dez*, per angulos *dek* et *dhg*. Sed duo anguli *dek* et *dhg* sunt equales duobus rectis propter hoc quod duo anguli [*deg*] et *dhg* per 4^{am} primi Euclidis sunt equales et duo anguli *deg* et *dek* sunt equales duobus rectis. Ergo duo anguli *kez* et *ghb* sunt minores duplo anguli *dez* per duos rectos, quod fuit probandum. Hec eadem probant 30^a et 31^a 2^i Gebyr.

Ex his igitur manifestum est quod scitis angulis et arcubus altitudinum ex orientali parte meridiani in medietate zodiaci a principio

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1449 deg] *dg* R

1418 28a…Gebyr] Gebir, p. 36. **1452** 30a…Gebyr] Gebir, pp. 36-7.

cancri ad principium capricorni scientur per 35^{am} huius tam arcus quam anguli alterius medietatis zodiaci in occidentali parte meridiani secundum equalem a meridano distanciam. Et scito quocumque angulo tali ex orientali parte meridani, scietur angulus eiusdem puncti in equali distancia ex parte occidentali, et e conversa ut patet hanc conclusionem et per premissam, ergo corellarium liquet.

[II.38]

Cuiuslibet arcus altitudinis et anguli eciam ex circulo altitudinis et zodiaco in medio celi vel in orizonte signati quantitatem cognoscere.

[*Figura #38*] Sit meridianus *abgd* et orizon *bed*, zodiacus *zhe* quomodocumque contingat. Et sit cenith capitis *a* a quo descendat circulus altitudinis per punctum *z* qui quidem circulus erit idem cum meridiano eo quod punctus *z* est in medio celi. Dico ergo quod arcus *az* est notus. Nam distancia cenith ab equinoctiali in orizonte dato est nota per 10^{am} huius et declinatio puncti *z* ab equinoctiali est nota per 22^{am} primi huius. Ergo arcus *az* erit notus. Item angulus *aze* etiam erit notus per 29^{am} huius eo quod ibidem est meridianus et circulus altitudinis. Iterum ab *a* cenith descendat *aeg* circulus altitudinis ad *e* punctum zodiaci existens in orizonte. Dico ergo quod arcus *ae* est notus. Quia est quarta circuli eo quod *d* est polus in orizontis, quare et angulus *aed* neccesario erit rectus. Sed et angulus *deh* notus est per 34^{am} huius eo quod *e* punctum zodiaci per 23^{am} huius oportet esse notum. Ergo totus angulus *aeh* est notus, et hoc est quod volumus. Idem patet 32^{a} et 33^{a} 2^{i} Gebyr.

[II.39]

Quantitatem arcus circuli altitudinis a cenith capitis ad quodcumque notum punctum zodiaci in quacumque nota distancia a meridiano circulo reperire. Unde si in sinum arcus zodiaci inter orizontem et punctum zodiaci ad quem producitur arcus altitudinis intercepti ducatur sinus arcus meridiani inter orizontem et punctum zodiaci in celi medio deprehensi, et productum per sinum arcus zodiaci qui est inter orizontem et celi medium dividatur, resultabit sinus | altitudinis puncti propositi scilicet sinus R 63v complementi arcus quesiti.

[Figura #39] Sit meridianus *abgd*, et medietas orizontis *betd*, medietas zodiaci *zht*. Et sit gratia exempli punctum *h* principium cancri distans secundum quodcumque tempus voluerimus a circulo meridiei, et distet gratia exempli per unam horam. Et sit punctum *z* medians celum et

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1456 parte] per *sed. corr. supr. lin.* R **1465** a[1]] *supr. lin.* R **1472** ab] *supr. lin.* R

1469 10am huius] This seems to be a mistaken reference to II.7. **1477** 32a…1478 Gebyr] Gebir, pp. 37-8.

punctum *t* in ortu. Igitur per 23^{am} huius tam *z* quam *t* puncta erunt nota. Descendat ergo circulus altitudinis qui sit *aheg* ab *a* cenith per *h* principium cancri. Inquirimus igitur quantitatem arcus *ha*. Cum ergo a puncto *b* descendant duo arcus *bt ba* a quibus reflectuntur alii duo *tz ae*, erit per conversam kate coniuncte, intelligendo totum in sinibus, proporcio *bz* ad *ba* composita ex proporcionibus *tz* ad *th* et *eh* ad *ea*. Subtracta ergo proporcione sinuum *tz* ad *th* a proporcione sinuum *zb* ad *ba*, remanet proporcio sinuum *eh* ad *ea* que est quarta circuli.

Et hec est operatio Ptholomei, sed pro faciliori practica arguam sic. Proporcio sinuum *ab* ad *bz* componitur per katam coniunctam ex proporcionibus sinuum *ea* ad *eh* et *th* ad *tz*. Ergo propter equalitatem quartarum *ab* et *ea* descendencium a cenith ad orizontem, erit proporcio sinuum *ab* ad *bz* composita ex proportionibus sinuum *ab* ad *eh* et *th* ad *tz*. Sed eadem proportio sinuum *ab* ad *bz* est composita ex proportionibus sinuum *ab* ad *eh* et *eh* ad *bz* propter *eh* medium interceptum. Ergo eadem est proporcio sinuum *eh* ad *bz* et sinuum *th* ad *tz*. Sed arcus *bz* est notus propter *za* notum per premissam, et duo arcus *th tz* sunt noti per 23^{am} huius. Ergo per 19^{am} 7^{i} Euclidis arcus *eh* erit notus. Quare tam conclusio quam eius corellarium satis liquent.

[II.40]

Quantitatem anguli ex circulo altitudinis et zodiaco ad quodcumque zodiaci punctum causati in quacumque distancia a meridiano circulo perscrutari. Unde si in sinum complementi arcus zodiaci inter orizontem et punctum propositum intercepti ducatur sinus altitudinis puncti eiusdem, et productum per sinum complementi eiusdem altitudinis dividatur, resultabit quiddam quod si ducatur in sinum quarte circuli et dividatur productum per sinum arcus zodiaci inter orizontem et punctum propositum deprehensi, resultabit sinus cuiusdam anguli, quem si a duobus rectis subtraxeris, remanebit angulus quem inquiris.

Fiat figura similis figure priori. Et posito polo in puncto *h* describatur secundum quantitatem lateris quadrati porcio circuli maioris qui sit *klm*. Quia ergo polus circuli *ahe* est tam in circulo *etm* quam in circulo *klm* per 14^{am} primi Theodosii, erit per 17^{am} eiusdem quilibet arcuum *em km kh* quarta circuli. | Cum igitur a *k* puncto descendant duo R 64r
arcus *kh km* a quibus reflectuntur arcus *hl me*, erit per katam disiunctam proporcio sinuum *he* ad *ek* composita ex proporcionibus sinuum *ht* ad *tl* et *ml* ad *mk*. Sed *he* est notus per premissam et per consequens *ek* residuum erit notum. Item *th* est notus per 23^{am} huius et per ypotesim, et per consequens *tl* residuum erit notum. Sed et *km* est quarta circuli, ergo

1509 19am] huius *add*. R

per primam regulam operandi datam in fine primi huius arcus *lm* est notus. Quare et *kl* residuum erit notum, et per consequens angulus *khl* erit notus, quem si de duobus rectis subtraxeris, remanet angulus *aht* notus. Et ille est angulus quem inquiris.

Hec autem est probatio Pthomei in sex quantitatibus, sed supposita noticia arcus *te* ad quam devinire poterimus per modum qui patebit infra, erit per conversam kate coniuncte proporcio sinuum *kl* ad *km* composita ex proporcionibus sinuum *et* ad *em* et *lh* ad *th*. Sed *lh* et *em* sunt equales cum utraque sit quarta circuli. Ergo proporcio sinuum *kl* ad *km* composita est ex proporcionibus sinuum *et* ad *em* et *em* ad *th*. Ex iisdem autem componitur proporcio sinuum *et* ad *th* propter *em* medium interceptum. Ergo eadem est proporcio sinuum *kl* ad *km* et *et* ad *th*. Ergo per 19^{am} 7^{i} Euclidis erit in quattuor quantitatibus corelarium istud verum quod:

Si in sinum quarte circuli ducatur sinus arcus orizontis deprehensi inter gradum zodiaci exeuntem in orizonte et circulum altitudinis qui per propositum punctum descenderit, et dividatur productum per sinum arcus zodiaci inter orizontem et punctum propositum intercepti, resultabit sinus arcus cuiusdam anguli, quo a duobus rectis subtracto remanebit angulus ille quesitus.

Istud correlarium non adiunxi conclusioni quia presupponit noticiam arcus *te* quam non supponit Ptholomeus. Deveniam ad eam tamen sic. Proporcio sinuum *ke* ad *kh* erit composita per conversam kate coniuncte ex propocionibus sinuum *lt* ad *lh*, et *em* ad *tm*, ergo componitur ex proporcionibus sinuum *lt* ad *lh* et *lh* ad *tm* propter equalitatem quartarum *lh* et *em*. Sed ex iisdem componitur proporcio sinuum *lt* ad *tm* propter *lh* medium interceptum, ergo sicut sinus *ke* ad *kh* ita sinus *lt* ad *tm*. Ergo per 19^{am} 7^{i} Euclidis, si in sinum quarte circuli ducatur sinus complementi arcus zodiaci inter orizontem et punctum propositum deprehensi, et productum dividatur per sinum complementi altitudinis eiusdem puncti propositi, resultabit sinus complementi arcus orizontis inter gradum zodiaci exeuntem in orizonte et predictum circulum altitudinis intercepti. Id est resultabit sinus arcus *te* quem inquisivimus. Hanc conclusionem et premissam probat Gebyr in | ultima conclusione 2^{i} libri sui. Et quia ipse R 64v
per alios arcus operatur in conclusione ita quam nos facimus, ponam probationem suam. [*Figura #41*] Descendat in figura priori ab *a* cenith ad *t* punctum zodiaci in orizonte arcus circuli magni qui sit arcus *at*. Quia igitur angulus *eth* est notus per 34^{am} huius et angulus *eta* est rectus, remanebit angulus *hta* notus. Ergo in triangulo *aht* qui est ex arcubus

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1545 per] *supr. lin.* R **1554** et lh] *add. mg.* R **1565** suam] demonstratio Gebir *adnot. mg.* R

1531 primam…primi] I.?? **1563** ultima…sui] Gebir, II.34, p. 38.

circulorum magnorum, erit per 13^{am} primi Gebir quam in 34^{a} huius inferni, proporcio sinus *at* ad sinum *ah* sicut proporcio sinus arcus anguli *aht* ad sinum arcus anguli *hta*. Ergo per 19^{am} 7^{i} Euclidis patebit corellarium istud Gebyr quod:

Si in sinum quarte circuli ducatur sinus arcus anguli per quem angulus ex zodiaco et orizonte causatus angulum rectum excedit, et dividatur productum per sinum arcus circuli altitudinis inter cenith capitis et quodcumque punctum zodiaci deprehensi, resultabit sinus arcus anguli ex zodiaco et circulo altitudinis in puncto illo proposito designati. Id est resultabit in figura proposita sinus arcus anguli *tha*. Et hec est operatio Gebyr.

Revertar igitur ad propositum Ptholomei. Unde habito modo inveniendi huiusmodi angulos ut eos cum eis indignerimus [sic] facilius et promptius habeamus, faciemus ad hoc tabulas incipiendo primo a climate cuius longior dies est 13 horarum equalium, et procedendo cum augmento dimidie hore continue quousque devenerimus ad clima cuius longior dies est 16 horarum equalium. Et inferemus in tabulis angulos resultantes ex circulo altitudinis et zodiaco in iniciis signorum quorumlibet procedendo in distanciis a circulo meridiano ex parte orientis et occidentis pro augmentum unius hore equalis. Et unius ita quod in prima linea cuiuscumque tabule ponetur numerus horarum equalium que sunt longitudo meridiani ad orientem vel ad occidentem. In secunda linea ponentur quantitates arcuum circuli altitudinis a cenith capitum ad inicia signorum quorumlibet incipiendo a cancro in distanciis horarum equalium singularum. In tertia linea ponentur quantitates angulorum huiusmodi ex orientali parte meridiani circuli causatorum. In quarta linea ponentur quantitates occidentalium angulorum. Et recordemur quod solummodo loquimur de angulis qui sunt orientales respectu circuli altitudinis et septentrionales respectu zodiaci in quacumque parte meridiani circuli situentur. Et ostendemus quantitates angulorum predictorum secundum quantitatem qua angulus rectus continet 90, et hic sequitur insercio tabularum.

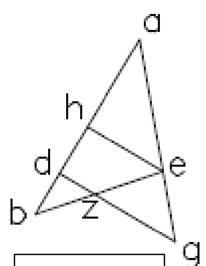
Et postquam exsecuti sinus scientiam angulorum, sine quorum scientiam diversitas aspectus lune non poterit comprehendi, restat inquirere de longitudinibus et latitudinibus civitatum. Sed ad hoc faciam librum alium singularem | in quo sequar vestigia antiquorum habencium R 65r
studium et scientiam de huiusmodi, et ponam longitudinem et latitudinem singularum civitatum in comperacione ad Alexandriam quocienscumque. Ergo voluerimus scire in hora nobis nota apud unum locorum que sit hora

1569 13am…Gebir] Gebir, pp. 11-2.

in loco alio. Oportebit scire longitudinem inter illa duo loca et quis sit orientalior et quis occidentalior. Et addemus vel minuemus secundum quantitatem temporis correspondentis illi longitudini, et fiet addicio pro hora habenda in loco orientaliore, et diminucio pro hora habenda in loco occidentaliore.

Expleta est secunda dictio Almagesti in qua notatur quod arcus orbis signorum equales et equaliter ab alterutro punctorum equinoctialium distantes equales in obliquo circulo habent ascensiones, et arcus equales etiam et equaliter ab alterutro punctorum tropicorum distantes habent in circulo obliquo suas ascensiones coniunctim equales suis ascensionibus in spera recta. Signorum autem que sunt a principio capricorni usque ad principium cancri elevantur arcus semper cum paucioribus partibus in obliquo circulo quam in recto, et a principio cancri ad principium capricorni secundum plures. A longiori longitudine solis usque ad principium cancri, cursus solis medius 25 et 29, et usque ad principium libre 100 16 40, et ad principium capricorni 200 12 31, et ad principium arietis 200 92 20.

1614 equaliter] qualiter *sed. corr. supr. lin.* R **1623** 16] 15 *add. supr. lin.* R



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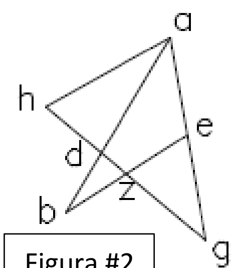

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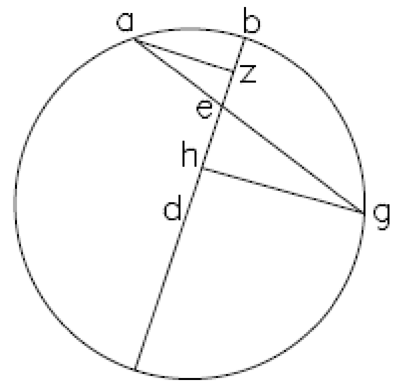

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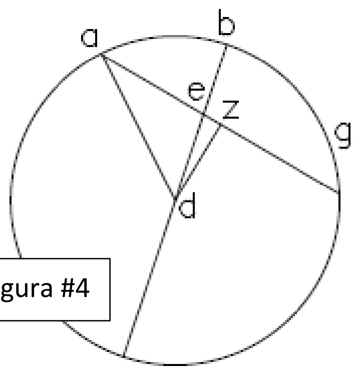

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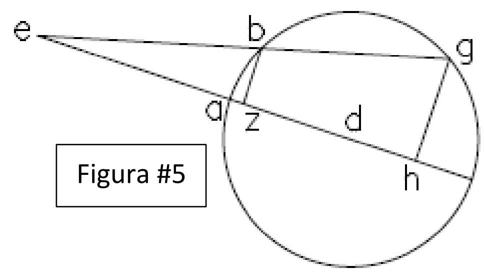

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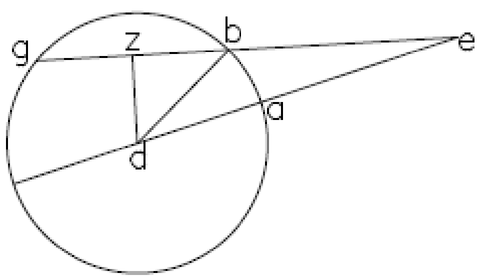

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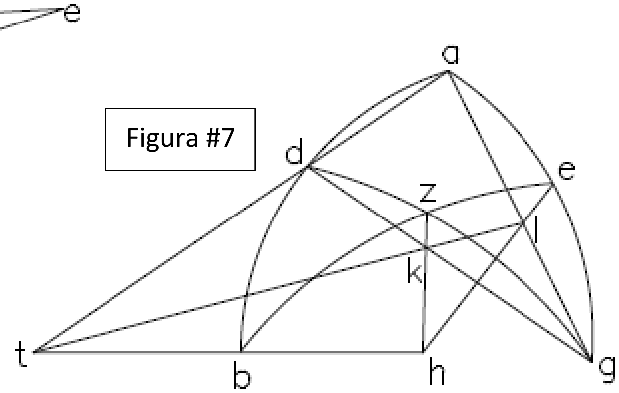

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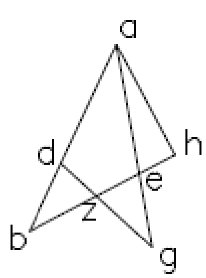

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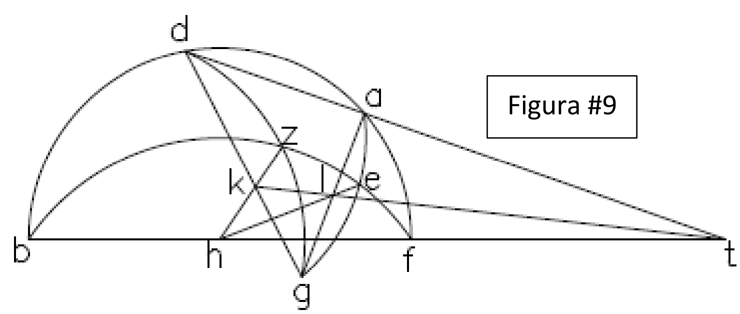

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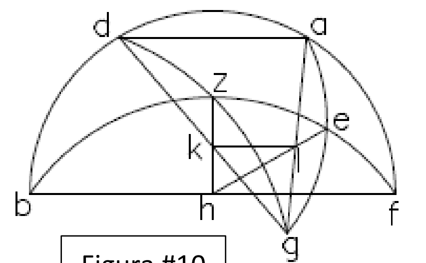

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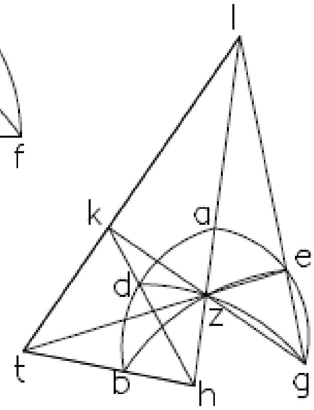

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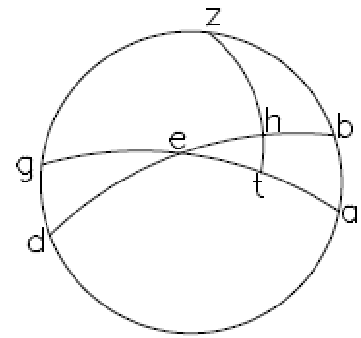

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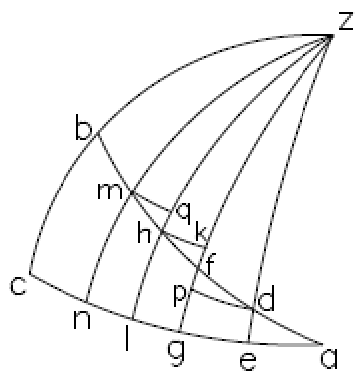

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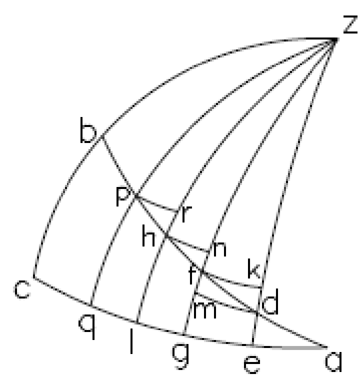

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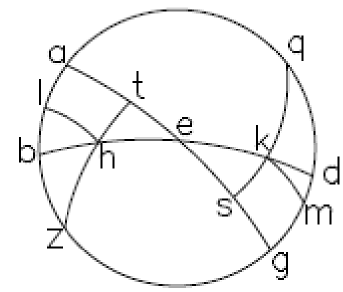

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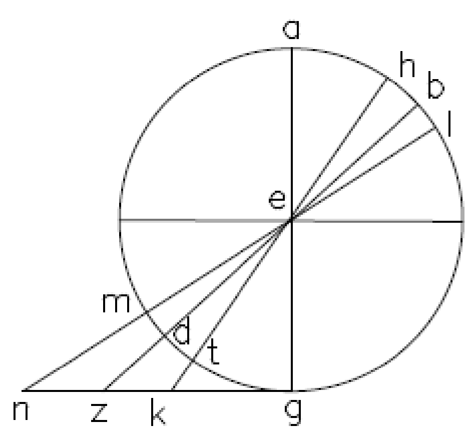

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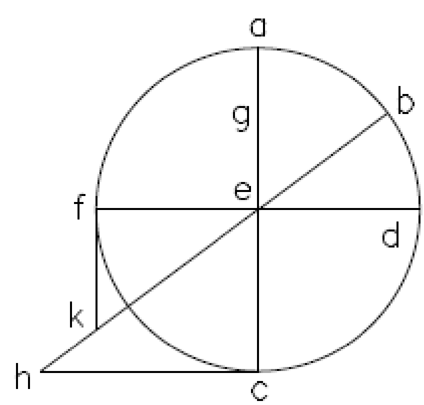

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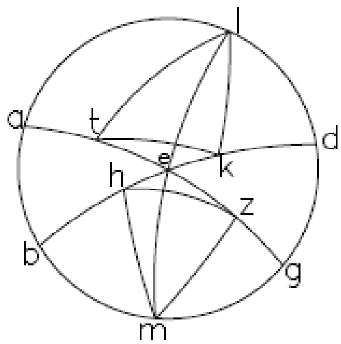

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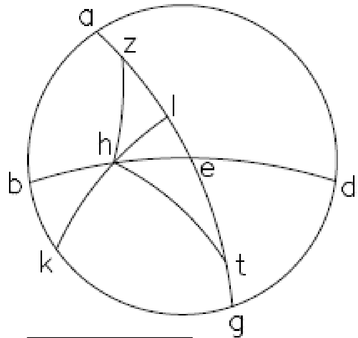

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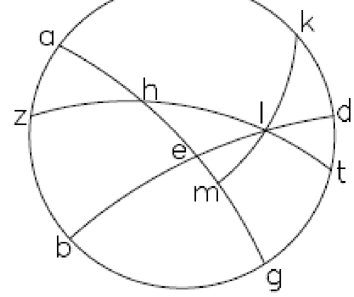
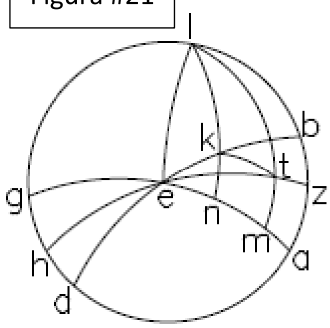


Figura #20

Figura #21



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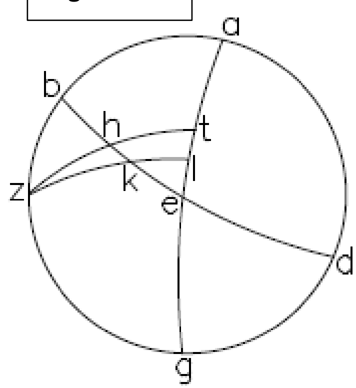

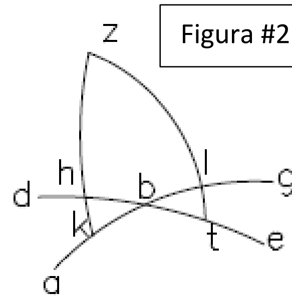

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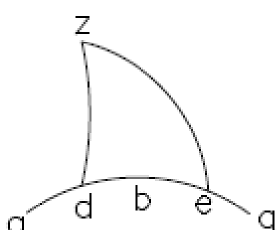

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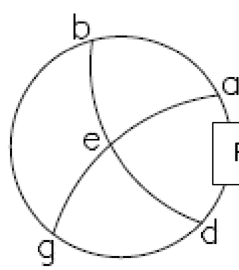

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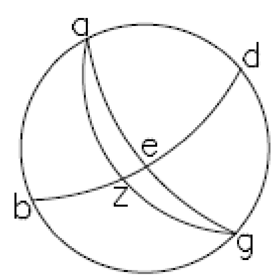


Figura #26

Figura #27

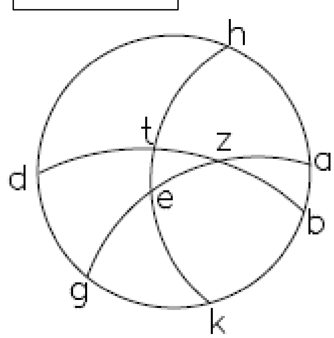
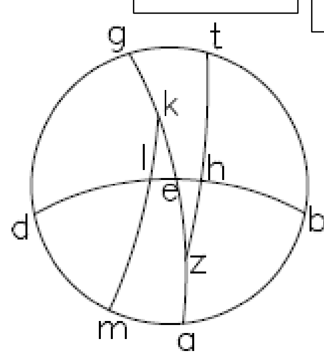


Figura #28



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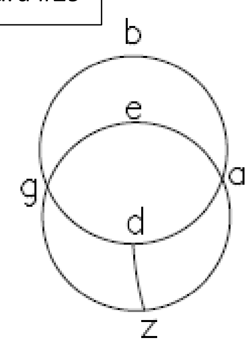

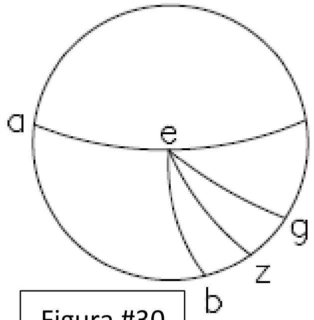

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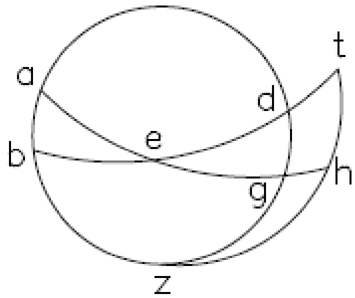

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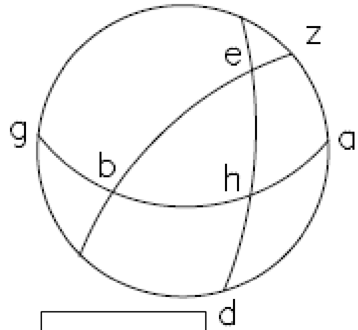
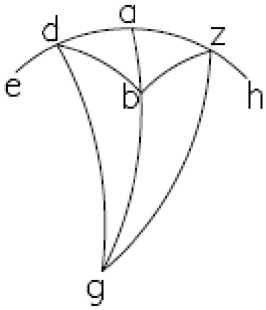


Figura #32

Figura #33



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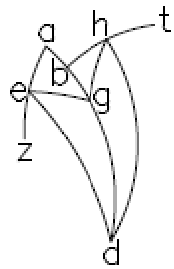

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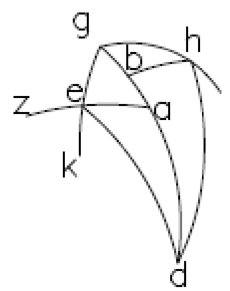

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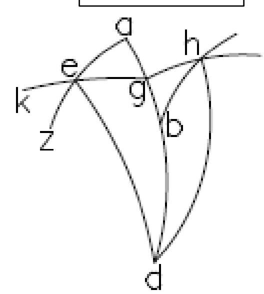

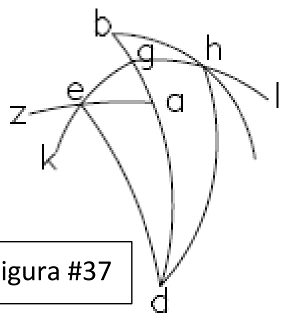

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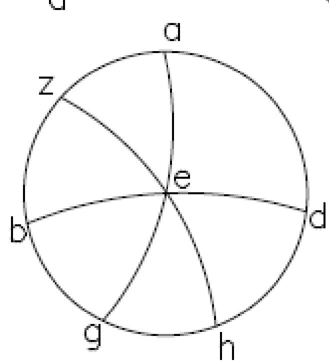

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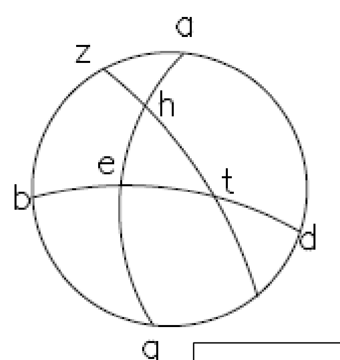

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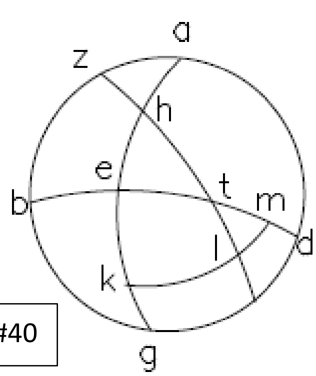

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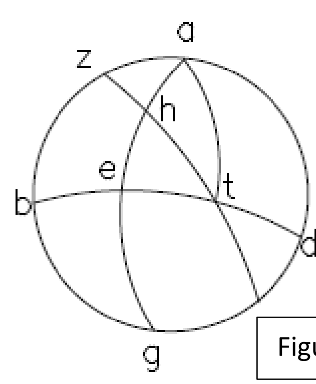

Figura #41

Appendix H: Comparison of Enunciations

	Particulars	Argument	Conclusion	Corollary
Elements from Greek	n/a	n/a	"quod oportet ostendere"	"porisma" (after theorem)
Elements Adelard	"exempli gratia"	"rationis causa"	"et hoc est quod in hac figura demonstrare intendimus"	n/a (after theorem)
Elements Gerard	"verbi gratia"/ "exempli gratia"	"huius probatio est"	"et hoc est quod demonstrare voluimus"	"corollarium" (after theorem)
Elements Hermann	n/a	n/a	n/a	n/a
Elements Campanus	n/a	n/a	"quod est propositum"	"correlarium" (after enunciation)
Almagestum parvum	"exempli gratia"/ "evidentiae gratia"	"ratio" (for particulars and argument)	"quod erat propositum"	"corollarium" (after enunciation)
Erfurt Com.	"gratia exempli" / "verbi gratia"	n/a	"qui quaerebatur" / "quod fuit propositum/ probandum" / "quod est propositum"	"unde colligitur corollarium" (after enunciation, rarely used)
Vatican Com.	n/a	"sic probatur"	"et illud est quod demonstrare voluimus" / "et illud cuius voluimus declarationem"/ (many different similar varations)	n/a
Simon of Bredon	"ut verbi gratia" / "verbi gratia"	"probatio huius" / "probatio"	"et hoc est quod volumus demonstrare" / "ergo liquet/patet propositum" / "quod fuit propositum"	"unde" (after enunciation, he uses "corollarium" but not as a signal word)
Campanus Notes	"verbi gratia"	n/a	"quod est propositum" / "quod voluimus"	n/a