Name: Kenneth Arthur Smith Date of Degree: August 5, 1961

Institution: Oklahoma State University Location: Stillwater, Oklahoma

Title of Study: A COMPARISON OF MODERN AND TRADITIONAL FIRST YEAR ALGEBRA TEXTS

Pages in Study: 52 Candidate for Degree of Master of Science

Major Field: Natural Science

Scope of Study: The mathematics curriculum is in the midst of a revolution in which many changes are being made. These changes are concerned with new approaches to traditional subject matter although some new material is being introduced. The purpose of this report is to compare some of the more significant changes from what has been taught traditionally in first year algebra. This comparison involves the Ball State, the School Mathematics Study Group, and the University of Illinois Committee on School Mathematics projects and the traditional approach of First Course in Algebra, a revised text by Mallory, Meserve, and Skeen.

Findings and Conclusions: The study indicates that it is possible to take a more meaningful and understandable approach to first year algebra than has been utilized traditionally. In order to do this it is necessary to re-orient objectives and make some radical changes in course content and teaching methods. The Ball State, SMSG, and UICSM texts are fine examples of what can be accomplished with a unified, "modern" attack on the central themes of algebra.

ADVISER'S APPROVAL James H. Zant

A COMPARISON OF MODERN AND TRADITIONAL

FIRST YEAR ALGEBRA TEXTS

By

KENNETH ARTHUR SMITH

Bachelor of Science

Southern State Teachers College

Springfield, South Dakota

1956

Submitted to the Faculty of the Graduate School of
the Oklahoma State University
in partial fulfillment of the requirements
for the degree of
MASTER OF SCIENCE
August, 1961

A COMPARISON OF MODERN AND TRADITIONAL

FIRST YEAR ALGEBRA TEXTS

Report Approved:

Report Adviser

Dean of the Graduate School

PREFACE

The mathematics curriculum is in the midst of a revolution in which many changes are being made. These changes are concerned, in most instances, with new approaches to traditional subject matter although some new material is being introduced. The purpose of this report is to compare some of the more significant changes from what has been traditionally taught in first year algebra. This comparison involves the Ball State, the School Mathematics Study Group, and the University of Illinois Committee on School Mathematics programs and the traditional approach of Mallory, Meserve, and Skeen's text, _First Course in Algebra_.

A special thanks is extended to Dr. James H. Zant for his assistance and counsel in the preparation of this report and to the National Science Foundation, without which, this year of study would not have been possible.

TABLE OF CONTENTS

CHAPTER I

INTRODUCTION

At the present time, the high school mathematics curriculum is in a state of upheaval. The mathematics taught in the past is under heavy fire from many critics. The current trend of thought is that high school mathematics has not kept pace with twentieth century scientific and industrial applications of the subject.

There appear to be two definite camps to which those involved tend to congregate. One group believes changes are desirable but any new approaches should be centered around what has been taught traditionally. The other group, which has gained much support for its program, proclaims that the mathematics curriculum needs a complete "overhaul."

The philosophy of the latter group, frequently called the "modern" school, is concerned with radical changes from what has been taught traditionally. Objectives must be reviewed and, where necessary, be re-oriented. Some mathematics in the traditional program has no present day application and should be replaced by new material more appropriate to present day needs. Modern approaches to presenting much of the "old" mathematics must be developed. There is also a definite trend toward being more concerned with the development of the potential of the college capable students, the group from which the largest percentage of the future leaders of this country will come.

Several projects involving this "modern" approach to the teaching of high school mathematics have been developed. These programs, although differing in many respects, have as a prime objective the teaching of mathematics for understanding; manipulative skill is a secondary factor.

The purpose of this report is to compare first year algebra texts of several of the major modern programs and a revised edition of a traditional algebra text. In this way one can determine the main points of difference between the individual modern day approaches and a traditional method of teaching the same algebra. One modern text used was Algebra I by Brumfiel, Eicholz, and Shanks (1), published by Addison-Wesley. This text was developed as the result of a study centered at Ball State Teachers College, Muncie, Indiana. This report will frequently refer to the material included in the above text as the Ball State program.

Another set of texts being considered is probably the most popular of the modern day approaches. This is the School Mathematics Study Group (SMSG) text, First Course in Algebra (2).

The University of Illinois Committee on School Mathematics has produced a different type of text which is reviewed in this report. The UICSM First Course (3) will be referred to as the Illinois program.

The traditional text chosen is a 1961 revision of Mallory, Meserve, and Skeen's First Course in Algebra (4), published by the L.W. Singer Company. "Mallory" will be the term most frequently utilized in discussing this text.

In a report of this type, it is obviously not possible to compare all phases of algebra. Accordingly, only some of the more basic concepts and related material will be discussed. This involves approximately ten different major areas, a coverage which should acquaint the reader with basic similarities and differences of the texts under consideration. Ball State will generally be the first program to be reviewed in each area. SMSG, Illinois, and Mallory follow in that order.

CHAPTER II

INDIVIDUAL PREFACES

Although what is included in the preface of a text is usually not regarded as of major importance, what is said can have a large influence on how the student is to react toward material subsequently presented. For that reason, a review of the preface of each text is provided in this report.

Ball State takes an optimistic but strictly factual approach. The pupil is told that the purpose of the text is to present algebra in such a way that the reasons for algebraic processes become evident. The teacher is reminded that for best results, students must be treated as young adults.

In a foreword to students, algebra is described as more than a special technique that simplifies problem-solving. It is principally a study of number systems and the arithmetic operations possible in these systems. Skill is important but understanding the rules of why algebra works is the main objective. The discussion is closed by telling the pupil that a deeper understanding of numbers is enjoyable.

The approach used by SMSG is similar to that of Ball State. The teacher's manual suggests that the objective of the SMSG text is to help the student develop an understanding and appreciation of algebraic structure exhibited by the real number system and use these structures as a basis for techniques of algebra. In the preface of the student text, the pupil is told that he will be given "an opportunity which few high school students have had up to this time . . . Study of algebra in this course will be based upon an exploration of the behavior of numbers."(2, p. vii) It is added that excitement and fun can accompany study of the foundations of algebra. Manipulation with symbols will be made more clear and understandable.

There is no preface to the Illinois text. However, the teacher's commentary states that the objectives, or principles of the course are:

1. The learning of mathematics should be a delightful experience for youngsters.
2. Mathematics can be interesting without being watered down.
3. The teacher is an extremely important element in the classroom. (3, p. Int. A)

To best describe the Mallory preface, the following is quoted directly from the text.

> The authors of First Course in Algebra have taught high school algebra for many years. They have fully considered the Report of the Commission on Mathematics (5), the work of the School Mathematics Study Group, and the literature of the many current experimental programs. . . A natural transition from arithmetic to algebra has been provided . . . This book contains all the features which made earlier books in the series popular. It is modern in content and presentation. . . The review and maintenance program is a strong feature. . . Careful attention has been given to readability.(4, p. iv)

CHAPTER III

THE CONCEPT OF NUMBER

What is a number? This is a question that has never been answered clearly in the minds of many students of mathematics. Traditionally, little effort has been made toward clarifying this concept at the high school level. However, the modern texts were written with the philosophy that pupils must understand numbers, numerals, and symbols.

The second chapter of the Ball State text consists of a discussion of symbols in arithmetic and algebra. From this, a student should be able to understand the difference between symbols and numbers.

"A symbol is anything which represents, or stands for something else."(1, p. 17) Diplomas, report cards, letter sweaters, and words are mentioned as examples of symbols. "A symbol used to represent a number is called a numeral."(1, p. 6) A clear distinction is made between numerals and numbers. The marks "0", "1", "2", etc. are only numerals. "Numbers are ideas that are represented by these numerals." (1, p. 18)

Statements such as the following are used as illustrations.

> Anyone can see that there are two "9"'s in "99," but of course there are eleven 9's in 99, not two. When you said there are no "2"'s in "10", you were right. But can't you see there are five 2's in 10?"(1, p. 19)

It is interesting to note that the above quotation appears to be lifted directly from the Illinois text.(3, p. 1-3)

SMSG evidently feels that this concept is very important and the earlier it is introduced, the easier it will be to understand the principle involved. Accordingly, discussion of the topic begins on the first page of the text.

Mention is made of the fact that whole numbers like 3 may have other names such as "3/1", "6/2", "9/3", etc. It is added that, technically speaking, "3" is not a number; it is only a symbol for something involving three objects. The idea of "threeness" is described by the symbol "3". The number itself is an idea whose name is "3". The name given a number is called a numeral. After it is felt that the concept has been explained adequately, the statement is made that symbols such as 3 will be used to mean either the number or the numeral representing it.

In the Illinois text, the concept of number is introduced by a play on words. The procedure is concerned with a hypothetical situation in which a boy is trying to teach his friend arithmetic by mail. The fact is illustrated that symbolism must be precise if confusion is to be avoided in mathematics. The previously mentioned paragraph from the Ball State text is an example used by UICSM to illustrate the confusion that results if the difference between numbers and symbols is not clear.

"The marks we write on paper are only symbols for numbers. The marks are not the numbers themselves."(3, p. 1-5) After seven pages of preparatory material, it is added that "a symbol for a number is called a numeral."(3, p. 1-7) This comprehensive discussion is followed by exercises which pave the way for treatment of letter symbols in Unit 2. In explaining these exercises, stress is laid on the fact that in substitution, one symbol is replaced by another.

Mallory's discussion of the number concept is quite limited. In fact, it is confined to only one page. If a student does not know what the difference between a symbol, a number, and a numeral is before reading this material, there is a question whether the explanation will enlighten him any.

The following quote is all that is devoted to discussing the concept of number.

> The symbols 1, 2, 3, and so on are correctly called numerals. We use them to represent numbers. We also use Roman numerals I, II, III, and so on to represent numbers. In this book, we follow common usage and speak of these symbols as numbers. (4, p. 3)

Although there is some discussion related to different types of numbers, no exercises are concerned with the number concept.

CHAPTER IV

THE NOTION OF SETS

"So much of mathematics is permeated with set concepts and for that matter, can actually be made to rest upon set theory as a foundation . . ."(6, p. 282) Sets play an important role in modern mathematics. Therefore, it is appropriate that set theory be considered in this report.

Throughout the Ball State text, liberal use is made of the notion of sets. Sets are first mentioned in Chapter 1 in an exercise concerned with counting in different bases.

The next topic, "Numbers and Sets," consists of a full scale introduction to the concept. The student is reminded that his first experiences with numbers involved counting sets of objects such as cookies, apples, and playmates. As one continues the study of mathematics there is a tendency to think less about the sets of objects with which numbers are associated and more about the numbers themselves. This is called abstraction. As one becomes more skilled in mathematics, abstraction is used more and more. However, the sets of objects must not be forgotten; obvious mistakes can be made if the sets are ignored.

Deciding, without counting, whether two sets have the same number of objects, is discussed. This involves pairing of the objects in each set. Since this is usually not convenient, counting is generally accomplished by comparing the objects in a set with those of a pre-determined set containing a specified number of objects. This pre-determined set represents a certain number.

Chapter 4, "Addition and Multiplication," offers an example of the frequent use of sets by Brumfiel. The chapter begins with further discussion of the topic. Here, the subset is first introduced and defined. "A subset A of a given set B is a set of only certain elements of B." (1, p. 42) The null, or empty set is "the set which contains no objects . . . consider the set consisting of all wild elephants in your schoolroom."(1, p. 43) Parentheses with nothing between them, ( ), are used to indicate the empty set. Further, it is added that every set is a subset of itself and the null set is a subset of every set.

Other situations where sets are used include: "Variables are symbols we shall use in discussing numbers in a certain set."(1, p. 44). The natural and counting numbers are defined in terms of sets. The definition of a function involves sets. "The graph of a polynomial is the set of all ordered pairs (x,p(x))."(1, p. 201)

These examples are only samplings of situations where sets are utilized but they tend to substantiate the earlier contention that liberal use is made of the notion of sets throughout the Ball State text.

The concept of a set is one of the most important in the SMSG program. Early in the text, extensive treatment is given this area. Four units of over fifteen pages are devoted to sets.

After illustrating sets in several ways, a set is defined informally as a collection of elements. "In this course these elements will usually be numbers."(2, p. 9) Braces are used to enclose elements of a set. The symbol $\in$ is a shorthand notation for "is an element of." $\notin$ is used to indicate that an element does not belong to a set. The null, or empty set is described as "the set which contains no elements."(2, p. 11) The usual null set notation, $\emptyset$, is used by the text.

"If the members of a set can be counted, with the counting coming to an end, the set is a finite set. Otherwise, it is an infinite set."(2, p. 12) In the discussion of infinite sets, a proper subset is defined. The text then adds that an infinite set has the surprising property that "all its members can be paired off, one-to-one, with a proper subset of itself."(1, p. 17)

Several pages are devoted to an introduction to operations on members of sets and closure of a set under an operation.

Analysis of the complete course will reveal that extensive use is made of this concept. In fact, one may say without fear of contradiction that one of the main building blocks of the SMSG program is that of sets.

Use of sets by UICSM is quite limited in comparison with SMSG and Ball State. The first observed use of the concept is in the final unit of the text in an exercise which involves selecting ordered pairs from a set of numbers.

Intersection and union are defined, topics which neither of the two previous programs cover.

> The intersection of two sets of points is the set of all points which belong to both of the two given sets . . . The union of two sets of points is the set of all points which belong to either or both of the two given sets."(3, p. 4-13)

Some of the exercises involve the use of sets. Other than the instances just mentioned, little use is made of sets. However, the teacher's commentary suggests that if an instructor desires more information he should consult a modern text on the subject.

Mallory (4, p. 2) introduces sets as "a collection of objects or numbers. . . Each object or member of a set is called an element of the set." During the brief introduction to the concept, well-defined and not well-defined sets are discussed. Sets of numbers are given one paragraph of coverage. Other than this, the notion of sets is utilized sparingly throughout the text. In some situations, such as inequalities, sets are mentioned. However, such cases are limited in number.

This appears to be a topic that was included in the text to satisfy the modern trend. Little effort seems to have been made to "capitalize" and use the set material that is introduced.

CHAPTER V

THE VARIABLE

The variable concept is one which a student must understand thoroughly if algebra is to be meaningful. Accordingly, it is desirable that a textbook develop the idea carefully. The contrast between the various developments considered in this report is rather interesting.

Brumfiel, Eicholz, and Shanks evidently feel that the variable must be introduced fairly early in the study of algebra. However, there are certain prerequisites which must be covered if the concept is to be understood thoroughly. Therefore, careful consideration is given to preparatory material.

Chapter 1, "Sets and Counting," includes the development of various systems of marking used to represent numbers and a brief discussion of the relation between numbers and sets.

Chapter 2, "Symbols in Arithmetic and Algebra," prepares the student for variables by discussing symbols and numerals. Emphasis is placed on the fact that in mathematics, symbols stand for precise, clear-cut ideas. The discussion hovers near, but doesn't touch the variable concept.

Chapter 3 is concerned with logic. The Greek symbols α, β, and γ are introduced as abbreviations for statements involving logical principles.

After forty four pages of preparatory material, the authors believe that the necessary foundation has been laid for the introduction of variables. The concept is first presented in Chapter 4, "Addition and Multiplication."

The chapter begins with a discussion of sets. In the exercises following this introduction the student is asked to describe different sets. In the next subtopic, "Variables," the text suggests that it would be helpful to invent a simpler way to construct sentences such as: "Select a subset of S so that five more than twice each number in the subset is in S."(1, p. 43), [S = (1, 2, 3, 4)]

The student is then asked to select the subset that consists of the even numbers in S. It is added that another way to refer to the numbers in a set is to use a variable. "Variables are symbols we use in discussing numbers in a certain set. In algebra we usually use small letters of the alphabet to indicate variables."(1, p. 44)

SMSG approaches the variable concept with practically the same amount of preparatory material as Ball State. However, the material preceeding the variable concept differs slightly in each case.

Chapters 1 and 2 of SMSG are titled "The Real Number Line and Sets" and "Phrases and Sentences," respectively. The variable is first encountered in the latter chapter.

The concept is introduced by describing a class game with arithmetic. The instructions given the class are to "choose a number from set S, [S = 1, 2, 3, . . ., 30] add 3, multiply by 2, and subtract twice the number chosen."(2, p. 39) One person did the calculations mentally while John worked the problem completely out with numerals.

Rudy wanted to follow John's calculations but didn't hear what number he had chosen. Rudy then just used the symbol "John's number" and calculated with it. Don watched Rudy and caught the drift of his procedure but decided "John's number" was too complicated a phrase to write over and over, so he decided to abbreviate and use "n".

The calculations of each individual are compared in an illustration. Since each person had a final answer of 6, each method is correct. However, the one utilizing "n" involved less work and took less time to complete. The pupil is then asked to notice that the letter "n" has been used to represent a particular number, but this number may be any one of the set S. Don could just as well have used "t," "a," or "k" to represent the number he was calculating with.

The main points of the discussion are concluded by saying that "a letter such as 'n', when used as Don used it, is called a variable. The numbers that n may be are called its values." (2, p. 40)

The development of the variable concept is probably one of the most distinctive features of the UICSM program. The careful introduction of this concept is preceeded by a unit composed of seventy five pages of preparatory material.

Proponents of the UICSM program feel that directed numbers should be the first topic to be covered in a first course in algebra. The four arithmetic operations, the commutative, associative, and distributive laws, and the principles of 0 and 1 are included in Unit 1. These areas are followed by inequalities, development of the number line, and absolute value.

The teacher's commentary pinpoints several important ideas of Unit 2. The commentary suggests that the variable has always been a rather controversial idea in algebra. The name implies that it is something that changes. Because of the confusion arising from the word "variable", UICSM has invented the word "pronumeral" to precisely describe the concept.

The approach utilized in the student's text involves a mythical classroom situation. Miss Adams gives her class a different type of true-false test. She mimeographs the examination then punches holes in some of the numerals. Students are given this test and a second sheet which they slide under the first. They are then told to put numbers in the holes so it can be said precisely whether the mathematical statement is true or not.

After this introduction, the concept is further developed by using sentences with blanks. Another mythical situation is used to introduce symbols. Sammy's big brother told him that algebra was a lot different than arithmetic because "in algebra you add letters instead of numbers."(3, p. 2-7) As a class project, Sammy made out a true-false mathematics test and put letters instead of numbers in the blanks. This presented a problem to all, including Fred, the "brain" of the class. For the first time, Fred couldn't answer a mathematics problem presented him.

It is at this point that the student is introduced to the pronumeral. Here, instead of using letters, $\bigcirc$, $\square$, and $\hexagon$ are employed. Working with pronumerals is illustrated by such activities as $\square + \square + \square = 3 \times \square$.

The concept is developed further by giving the student exercises such as the following, and telling them to put the name of the same student in both blanks of a statement.

1. $\square$ is a boy and sits in the front row.
2. ⬡ is a girl and ⬡ has a brother in this school.(3, p. 2-18)

It is then carefully explained to the student that there are pronouns in mathematics as well as in English. In English, a pronoun stands in place of a name (or noun). "The pronouns in mathematics stand in place of names of numbers."(3, p. 2-20) An example of this would be $3 + \square = 8$. A noun is compared to a numeral and a pronoun with a pronumeral. "Since pronouns in mathematics stand in place of numerals, . . . , we shall call the symbols which act as pronouns, pronumerals."(3, p. 2-21)

Finally, on page 43 of Unit 2, one hundred eighteen pages from the beginning of the text, it is felt that sufficient background has been presented so that the student is ready to begin using letter symbols to replace numerals. One should note the difference between this and the preparatory material in traditional texts.

After pupils can see that $\square$, $\bigcirc$, and ⬡ serve only as frames and it doesn't matter whether these frames are kept after numerals are placed inside them, it is suggested that the task at hand will be easier if letter symbols are used as pronumerals. Exercises involving the use of letters follow immediately. Throughout the remaining portions of the text, letters are used exclusively.

Mallory's <u>First Course in Algebra</u> seemingly plunges "headfirst" into discussion involving the variable concept. The first chapter, "Numbers and Formulas," begins with a discussion of addition and multiplication in arithmetic. The arithmetic laws are introduced by numerical illustration. This is followed by subtraction and division in arithmetic and the properties of 0 and 1.

After fifteen pages of introductory material, the subtopic, "Use of Literal Numbers," is encountered. Here, the formula for the area of a rectangle is used to introduce literal numbers, the term used to describe variables in this text. Numbers, then letters are used to illustrate the use of the variable in this very short explanation. The equations $A = bh$ and $P = 2b + 2h$ are called general expressions. If number values are assigned to these formulas, they are than called particular rectangles. "Letters used to stand for numbers are called literal numbers."(4, p. 16)

How to add, subtract, multiply, and divide literal numbers is illustrated. More emphasis seems to be placed on defining various terms than in explaining the operations. After a comparison between arithmetic and algebraic computation, the student meets formulas somewhat abruptly. Such an approach and development may tend to leave many pupils with the question, "This is how we do it, but what are we doing?"

Finally, in the summary at the end of the chapter, some concrete procedures are presented. Among these are the commutative, associative, and distributive laws, expressed for the first time algebraically.

In Chapter 2, "Equations and Inequalities," an unknown is defined. "The X in the equation [$8X = 48$] is called the unknown; it stands for an unknown number. Since X stands for any set of values, X is called a variable."(4, p. 50) This appears to be the conclusion of a rather cloudy explanation of the concept under consideration.

CHAPTER VI

NUMBER SYSTEMS

The current trend of thought in mathematics is that a clear cut and mathematically sound picture of the structure of algebra gives the student a thorough introduction to some of the more important fundamentals of algebra. Since the various number systems are vital elements of algebraic structure, a review of the coverage given each system by the texts under consideration is warranted.

Before any operations can be performed there must be something with which to work. Therefore, in Chapter 4, "Addition and Multiplication," Ball State begins development of the number system.

First, the natural numbers are defined as "the set of numbers obtained by starting with one and successively adding one."(1, p. 46) Immediately thereafter, the counting numbers are introduced as the number zero and the set of natural numbers.

Brumfiel works exclusively with the counting numbers through the next two chapters, which deal with addition, multiplication, and subtraction. However, in subtraction equations such as $x + 4 = 0$ are encountered; equations of this type can't be solved until a new system of numbers is developed. These new numbers are created for the sole purpose of permitting solutions for such equations.

Important postulates and definitions include:

Postulate: For each natural number a there exists a new number called the negative of a, designated by the symbol '-a.' These new numbers, together with the counting numbers, form a set for which sums and products are defined. In particular $a + (-a) = 0$.

Definition: We call the set of numbers consisting of all these new numbers, and the counting numbers, the integers. The new numbers are called negative integers, and the natural numbers are called positive integers."(1, p. 79-80)

A postulate then follows which, in general, says that the integers obey the same laws for addition and multiplication as do the counting numbers for these two operations.

Throughout Chapter 8, only the integers are discussed. However, in the latter unit, "Division," it is clearly seen that since division of integers is not always possible, more numbers are needed. Hence, throughout the chapter, some of the properties that these new numbers must have are studied.

Just as the number -4 was invented in order to say that the equation $x + 4 = 0$ has a solution, a rational number $1/3$ is created so that the equation $3x = 1$ may have a solution. The following postulates and definitions are important in the development of this new system.

Postulate: For each pair of integers a and b with $b \neq 0$, there exists a new number, called "a over b," designated by the symbol "a/b." These new numbers together with the integers form a set of numbers for which sums and products are defined. In particular $b \cdot a/b = a$.

Definition: We call the set of numbers created by our postulate the set of rational numbers. We call the symbol "a/b" a fraction. The integer a is called the numerator of the fraction, and the integer b the denominator of the fraction.

Postulate: The same laws hold for the addition and multiplication of rational numbers as for the addition and multiplication of integers. (1, p. 128)

It is appropriate to mention here that these new numbers are very similar to the numbers studied in arithmetic called fractions. However, a distinction is made between the two terms. The text calls the new numbers rational numbers, not fractions. The word "fraction" will be used only as a symbol or name for rational numbers.

The student who has come to believe that he has become acquainted with the complete number system is shown differently in Chapter 14. "There is no rational number whose square root is two,"(1, p. 253) is presented as a theorem and proved by reductio ad absurdum.

The following postulate expands the rationals.

> We assume that there is a set consisting of
>
> (i) the rational numbers,
>
> (ii) a number, not necessarily rational, whose square root is two, which is denoted by $\sqrt{2}$, and
>
> (iii) all numbers which arise by addition and multiplication involving the numbers of (i) and (ii).
>
> Furthermore, we assume that all our old basic postulates hold for this new set of numbers.(1, p. 256)

Following this postulate is the definition:

> The new number $\sqrt{2}$ shall be called the square root of two. We also say that $\sqrt{2}$ is an irrational number. The set consisting of all the numbers $a + b\sqrt{2}$, with a and b rational, will be designated by "$Q(\sqrt{2})$" which we read as "Q of the square root of 2.(1, p. 258)

This system is not expanded beyond the rationals combined with 2 until the next chapter, "Real Numbers." Theorems are used to present the fact that the decimal representation of a rational number repeats.

An informed observer will note that the transition from rationals to reals is accomplished with the use of the Dedekind cut. The text does not mention the concept by name but the properties described are typical of the Dedekind cut.

A definition states:

> Each infinite decimal, positive, negative, or zero, repeating or not, represents a real number. Those real numbers which are represented by non-repeating decimals are called irrational numbers.(1, p. 277)

Since the rationals are embedded in the reals, it is mentioned that the basic laws for reals are practically the same as for the rationals. The basic postulates for real numbers are then given. This completes the main points of the Ball State development of the various number systems.

The first chapter of the SMSG text is titled "The Real Number Line and Sets." Accordingly, one must expect a quite different approach to the development of the number systems than that of Ball State. A student who compares both texts will readily see that Ball State's development is the more thorough of the two. It is possible that some authorities may consider Brumfiel to be rather meticulous in this particular area.

SMSG does not define the various number systems as explicitly. After an informal discussion and definition, one can see that the counting numbers and zero make up the set of whole numbers. Counting numbers are said to be sometimes called natural numbers.

It is appropriate to mention that Ball State defines counting numbers as those numbers consisting of zero and the natural numbers. Note the divergent definitions. If Brumfiel ever mentions "whole numbers," it is only in an insignificant position.

During discussion of the number line, fractions are mentioned and informally defined as zero and quotients of the counting numbers. The positive integers are introduced during development of the number line. The method used will be covered in a subsequent chapter.

Fractions are introduced by dividing intervals between positive integers. There are an infinite number of points between any two whole numbers since it is possible to find the midpoint of an interval, then find the midpoint of this half interval. If kept up endlessly, it would always be possible to find a point between any two points.

Still in the first subtopic on the number line, the student is asked, "Do you think we can label every point on the number line with a fraction?"(2, p. 6) The answer is no; a proof is given later in the text.

This is all the coverage given number system development until one hundred pages later in Chapter 5, "The Real Numbers." Here the student is reminded that lathough a line extends without end from left to right, so far, only all points to the right of zero have been labeled. These points and the numbers associated with them are developed in much the same way as those on the right. However, these points to the left are labeled $^{-}1$, $^{-}2$, etc. and are called negative one, negative two, and so on.

The text then states that "the complete set of points on this extended number line are called real numbers, those to the right being positive real numbers and those to the left, negative real numbers." (2, p. 120) It is mentioned that the integers, the set consisting of all the whole numbers and their negatives, are of particular importance.

In Chapter 8, "Factors, Exponents, and Radicals," the words "rational" and "irrational" are first mentioned. A theorem, $\sqrt{2}$ is irrational, is proved by reductio ad absurdum. The fact that all whole numbers are rationals completes discussion of number systems.

The UICSM First Course does very little with number systems. In fact, the words "integer," "rational," "irrational," and "real number" are mentioned sparingly, if at all. The teacher's commentary implies that work in this area will come in a later course.

Unit 1 of First Course is titled "Directed Numbers." The integers are introduced in this unit but are called positive and negative directed numbers. The positive and negative rationals are developed in much the same way. Positive rationals are mentioned two or three times in the text but neither they nor their negative counterparts are identified frequently as such. The irrational concept receives little consideration. Natural number and whole number concepts are included in the reading material but have different names.

An attempt is made to present the material in such a way that the student will see that an isomorphism exists between the positive directed numbers and the numbers of arithmetic under the operations of addition and multiplication.

Most critical observers will probably agree that Mallory's text has very little system in its approach to any of the number systems. About the only organization to this area is that almost all the definitions of number systems used in the text are included under the same heading, "Sets of Numbers in Arithmetic." Here, the natural numbers, (sometimes called the counting numbers) are defined as "the set of numbers you used most in arithmetic. This includes the whole numbers, starting with 1. Zero is not a natural number."(4, p. 3)

An attempt is made to describe the natural numbers by modifying several of Peano's postulates. This is certainly a step in the right direction and is something few modern texts attempt at this level. Most authors apparently feel that unless a creditable job can be done with Peano's postulates, they just as well be left alone. Mallory apparently doesn't adhere to this philosophy. The material in question is as follows:

From your study of arithmetic, you may observe many facts about the set of natural numbers. Some of these are:

1. It has a first element, 1, but no last element.
2. Each element is a whole number; that is, an integer.
3. It does not contain 0.
4. Each number after 1 is greater by one than the number before it. (4, p. 3)

The above mentioned quotation is one of the three places in the text where the word "integer" can be found. In one of the last chapters, there is a three paragraph discussion of rational numbers. The definition of rational numbers is precise, a statement which cannot be said of the irrationals. The latter topic is well done for the time alloted but it will probably be difficult for most students to see a clear relationship between the various number systems, especially since the real numbers are not mentioned. The fact that there are definitions of nine different types of numbers in the short topic that introduces natural numbers will undoubtedly tend to cloud the issue for many students.

CHAPTER VII

BASIC OPERATIONS AND LAWS

The development of the basic operations, addition, subtraction, multiplication, and division, will be reviewed in this chapter. Coverage will also be given to the commutative, associative, and distributive laws plus the properties of zero and one.

The Ball State development of basic operations is rather lengthy. In a report such as this, a resume of the topic must be brief. Accordingly, this analysis will begin with the end product of the development and review how the same was attained.

Over three-fourths of the Brumfiel text is untilized in developing this topic. The result is a very thorough foundation for algebra, based on the following basic postulates for the real numbers.

Each pair of real numbers, x and y, may be added to give a unique sum, $x + y$, and multiplied to give a unique product, xy. Addition and multiplication satisfy the following:

1. $x + y = y + x$
2. $xy = yx$ (commutative laws)
3. $x + (y + z) = (x + y) + z$
4. $x(yz) = (xy)z$ (associative laws)
5. $x(y + z) = xy + xz$ (distributive law)
6. Given x and y, there is a unique z such that $x = y + z$. We designate z by "$x - y$." (The subtraction postulate)
7. Given x and $y \neq 0$, there is a unique z such that $x = yz$. We designate z by "$x \div y$" or "x/y." (The division postulate)

9. $xy = 0$ implies either $x = 0$ or $y = 0$.

10. The number one has the property that $x.1 = x$ for all x. (1, p. 279)

Previous to this final climax, the integers are expanded to the rationals in order that division will be possible. (Excluding division by zero)

Important topics included in the discussion of rationals not introduced previously include the following theorems:

$a/b = c/d \Leftrightarrow ad = bc$

$a/b \cdot c/d = ac/bd$

$a/b + c/d = ad + bc/bd$ (1, p. 144)

Because numbers such as $\sqrt{2}$ are not rational, the rational system is expanded to include such numbers. All the basic postulates hold for this new system, the set of real numbers.

The predecessors of the rationals, the integers, result from expansion of the natural numbers. The latter are expanded because not all equations have solutions under the operation of subtraction. Topics introduced for the first time include:

Postulate. For each natural number a there exists a new number called the negative of a, designated by the symbol "$-a$." These new numbers, together with the counting numbers, form a set for which sums and products are defined. In particular $a + (-a) = 0$.

Theorem. For every integer x it is true that $-x = 0 - x$.

Theorem. For every integer x it is true that $-(-x) = x$.

Theorem. The difference, $x - y$, of any two integers x and y is the integer $x + (-y)$.

Theorem. For every pair of integers a and b it is true that $-(a + b) = (-a) + (-b)$. (1, p. 87-89)

The multiplication of integers with different signs is introduced by illustrations. "For example, to compute $4(-3)$, let us consider that $4(3 + (-3)) = 0$. However, $4(3 + (-3)) = 12 + 4(-3) = 0$." (1, p. 91)

The product of two negative numbers is illustrated by a similar method. The ideas developed by the examples are summarized by saying: "The product of two negative integers is positive, and the product of a positive and negative integer is negative."(1, p. 92) The following theorems are then introduced:

If $x.y = 0$ then either $x = 0$, or $y = 0$, or both are 0.

For every integer x, $(-1)x = -x$.

For every pair of integers a and b, $(-a).(-b) = (ab)$.

For any three integers a, b, and c, $a(b-c) = ab - ac$. (1, p. 92)

Important theorems included in the introduction to division are:

If x can be divided by a, then the quotient is unique. That is a division problem involving integers has either no solution or exactly one solution.

If $(x \div a) = (y \div a)$, then $x = y$. (1, p. 115)

Since the integers evolved from the natural numbers, the latter system must necessarily be developed first. While the natural numbers are being developed, the operations of addition and multiplication, along with the fundamental postulates, are introduced.

Important material presented at this point includes:

Theorems: If x is any counting number, then $x + x = 2x$.

For every counting number x, $x.0 = 0$.

Definition: If $x + b = a$ (or $b + x = a$), then the number x is called the difference of a and b, or a minus b. We write x as "$a - b$."

Theorems: If $x - a = b - a$ then $x = b$. If $r - x = r - s$ then $x = s$. (Cancellation law for subtraction)

For all counting numbers a, b, and c, $a - (b - c) = (a - b) + c$.

For all counting numbers a, b, and c, $a - (b + c) = (a - b) - c$.

For all counting numbers a, b, and c, $a + (b - c) = (a + b) - c$. (1, p. 65-75)

Summarizing the Ball State approach to basic operations and laws, first the natural numbers are developed and the basic laws postulated. The naturals are expanded to the integers, the integers to the rationals, and finally, the rationals to the reals. Generally, after the basic operations are postulated, they are re-introduced in next system as theorems. Only those properties not applicable to a preceeding system are defined or postulated. The fundamental properties are used throughout the text and are mentioned by name frequently.

The SMSG introduction to properties of operations on numbers somewhat resembles that of Ball State. However, SMSG's explanation may be described as being "more compact."

SMSG devotes one chapter of over thirty pages to properties of operations on numbers. Here, the various laws receive a more lengthy introduction than in Ball State. However, the two end products, the laws themselves, are stated almost word-for-word.

The laws and properties are first applied only to the numbers of arithmetic. These are the commutative, associative, and distributive laws, the additive and multiplicative properties of zero, and the multiplicative property of one. Later, properties are postulated for the real numbers. There is one additional inclusion, the addition property of opposites. "For every real number a, $a + (-a) = 0$." (2, p. 155) Opposites are discussed earlier.

Multiplication receives practically the same treatment in SMSG as in Ball State. The approaches to signed numbers under this operation are similar. However, SMSG defines the product of two real numbers a and b as follows:

If a and b are both negative or both non-negative, then $ab = |a||b|$.

If one of the numbers a and b is positive or zero and the other negative, then $ab = -(|a||b|)$. (2, p. 165)

Some of the more important theorems and definitions from the SMSG text which differ from those of Ball State includes:

Definition. To subtract a real number b from the real number a, add the opposite of b to a.

Definition. If c and d are real numbers such that $cd = 1$, then d is the multiplicative inverse of c.

Theorem. For any non-zero real numbers a and b, $1/a . 1/b = 1/ab$. (2, p. 196-223)

Important definitions and theorems related to division include:

Definition. For all real numbers a and b $(b \neq 0)$, a divided by b means a multiplied by the reciprocal of b.

Theorem. If $b \neq 0$, then $a = cb$ if and only if $a/b = c$.

Theorem. For any non-zero real number a, $a/a = 1$. (2, p. 223-238)

As is frequently the case with UICSM, basic operations are introduced with the motivating philosophy that if students "discover" a principle, their understanding will be more thorough. Accordingly, this development is somewhat concealed and cannot be followed with the ease possible in Ball State and SMSG.

The discussion of operations begins with the assumption that no authority such as a postulate or definition is needed to perform an operation. Since students have always added, subtracted, multiplied, and divided, the only question asked is whether these operations are possible with directed numbers.

Addition is first introduced by illustrating the operation with directed numbers. A short time later it is shown that an isomorphism exists between the numbers of arithmetic and directed numbers under addition. Scattered throughout the text are various principles related to this operation. After assuming students know they can add numbers of arithmetic, the same is assumed of directed numbers and later pronumerals. Concrete principles such as the following are then stated:

For any x, $-x$ is the opposite of x.

For any x, $+x = x$.

For any x, $x + (-x) = 0$.

For any x and y,
(a) if $x - y$ is a positive number, then $x > y$;
(b) if $x - y$ is a negative number, then $x < y$;
(c) if $x - y$ is 0, then $x = y$.

For any x and y, if $x < 0$ and $y > 0$, then $x + y = -(|x| + |y|)$.

For any x and y, if $x < 0$ and $y > 0$,
(a) and if $|x| < |y|$, then $x + y = |y| - |x|$;
(b) but if $|x| > |y|$, then $x + y = -(|x| - |y|)$.
(3, p. 2-48 - 2-54)

Multiplication receives a similar treatment. The student's knowledge of addition with arithmetic numbers is extended to directed numbers by illustration. It is then shown that an isomorphism exists between these two systems under multiplication.

The introduction to the laws of multiplication for signed numbers involves a novel exercise from which a student again "discovers rules" for this particular operation. The pupil is told that a motion picture, which can be run backward or forward, shows a pump filling a tank. Directed numbers correspond to the various changes as follows:

gal. increases in volume of water - positive numbers.
gal. decreases in volume of water - negative numbers.
gal. per minute, filling the tank - positive numbers.
gal. per minute, emptying the tank - negative numbers.
(3, p. 1-26)

To introduce multiplication of a negative number by a positive number the situation is presented where the pump is filling the tank four gallons per minute. The motion picture is running backward so the volume of water in the tank will appear to be decreased by eight gallons. This corresponds to $4 \times (-2) = 8$.

Multiplication of a negative number by a negative number involves somewhat the same procedure. In this situation, the pump is emptying the tank at a rate of four gallons per minute and the film is running backward for two minutes. This corresponds to $(-4) \times (-2) = +8$.

Pronumeral multiplication is introduced as follows:

> In Unit I, you learned how to multiply directed numbers. Probably you invented a rule which can be stated somewhat as follows: The absolute value of the product of two directed numbers is the product of their absolute values. If both numbers are negative, their product is positive. If one number is positive and the other negative, their product is negative.(3, p. 2-56)

This is followed by examples of multiplication with directed numbers. After each example a general rule is given. These rules include:

For any x and y, if $x < 0$ and $y < 0$, then $xy = |x| \cdot |y|$.

For any x and y, if $x > 0$ and $y < 0$, then $xy = -(|x| \cdot |y|)$.

For any x, $-x = (-1)x$.(3, p. 2-57)

To introduce subtraction, the student is asked to recall how he checked problems involving this operation in grade school; that was by adding the difference to the subtrahend. If the answer was the minuend, the problem was probably correct. It is then mentioned that solving a subtraction problem by stating it as one in addition illustrates a concept called the principle of inverse operations.

The statement, subtraction is the inverse of addition, is then illustrated and amplified by numerous examples and exercises.

Before going into pronumeral subtraction, opposites are introduced by illustrations involving directed numbers. The addition property of opposites and the subtraction property for pronumerals are introduced.

Division of directed numbers is introduced in much the same way as in traditional texts. The student is asked to study such examples as "$(+18) \div (-3) = -6$ because $(-6 \times (-3)) = +18$."(3, p. 1-53) In the chapter dealing with pronumerals, the statement is made that "the rules for dividing directed numbers are easily obtained by using the fact that division and multiplication are inverse operations." (3, p. 2-62)

The division rules for pronumerals, which follow the above statement, include:

For any x and y, if $x < 0$ and $y < 0$, then $x/y = |x|/|y|$.

For any x and y, if $x > 0$ and $y < 0$, then $x/y = -|x|/|y|$.

For any x and y, if $x < 0$ and $y > 0$, then $x/y \quad -|x|/|y|$.(3, p. 2-63)

In the first chapter of the Illinois text, no specific rules are stated which relate to the commutative, associative, and distributive laws. However, there are many illustrations which involve these concepts. These illustrations include the principles of zero and one, the commutative law for addition and multiplication, the associative law for addition and multiplication, and the distributive laws.

In the chapter on pronumerals, these principles (commutative, associative, and distribute laws) are stated using boxes and circles as pronumerals. When letters are introduced as pronumerals, the student is asked to write these laws, using letters for pronumerals.

The cancellation laws are not introduced until Unit 3 where they are called transformation principles.

Mallory's coverage of basic operations and laws is one which could be reorganized to a good advantage. This would not only tend to make a more compact and readable text but would eliminate many needless pages. At times, an operation or principle is introduced in several different situations throughout the text. A comprehensive introduction, applicable in general situations, if presented early in the text, would enable pupils to apply these principles and eliminate the need for extensive amplification each time a new situation were encountered.

The approach utilized in introducting operations may appear to be somewhat disorganized. As an example, consider addition. The text appears to use the philosophy that since the pupil has added since elementary school, why bore him with postulates giving one authority for such an operation in the natural numbers. Since literal numbers are merely letters used in place of numbers, little explanation is given to addition of this form. The summary at the end of the first chapter states, "in addition, we find the combined count of two or more sets."(4, p. 40)

The axioms of traditional algebra are used to solve equations. Axiom 4 is related to addition. "Adding the same number to both sides of an equation produces an equation with the same solution as the first equation."(4, p. 61) Although addition is the first operation discussed, Axiom 4 follows those for division, multiplication, and subtraction in that order.

The rules for addition of signed numbers utilize the absolute value concept.

Rule I: To add two numbers with like signs, add the absolute values of the addends. The sum has the sign of the addends.

Rule II: To add two numbers with unlike signs, subtract their absolute values. The result has the sign of the addend with the greater absolute value. (4, p. 98)

The following statement introduces subtraction:

Subtraction is the opposite (inverse) of addition. We may think of subtraction as separating a number into two parts.

We can also think of subtraction this way: $5 + ? = 9$. So, we say that in subtraction we look for a missing addend. (4, p. 10)

The student is evidently supposed to see that subtracting literal numbers is similar to subtracting arithmetic numbers since there is no explanation of the operation for literals. Two examples illustrate the operation for literal numbers.

Axiom 3 permits subtraction in the solution of equations. "Subtracting the same number from both sides of an equation produces an equation with the same solution as the first equation." (4, p. 58)

The rule for subtracting signed numbers is especially disheartening to those who advocate teaching mathematics for understanding and not manipulative skill alone. "To subtract signed numbers, change the sign of the subtrahend, then add." (4, p. 103) Later, in an exercise, another suggestion is made.

Another way of stating the subtraction rule is: To subtract a number, add its negative. The negative of a positive number is a negative number with the same absolute value. The negative of a negative number is a positive number with the same absolute value. (4, p. 104)

Multiplication is introduced by stating that this particular operation with natural numbers is a special form of addition. The operation is extended to literal numbers with the following statement:

"You have studied some important rules for multiplying in arithmetic. These rules must hold for algebra too, since algebra treats general facts as arithmetic treats particular facts."(4, p. 22)

Axiom 2 is concerned with solving equations by multiplication. "Multiplying both members of an equation by the same number, except zero, produces an equation with the same solution as the first equation."(4, p. 55) No mention is made of the fact that this is not true if the multiplier is a variable.

Multiplication of signed numbers is presented in much the same way as in Ball State. After going through several examples, the following statements are emphasized:

> The product of two negative numbers is a positive number . . .
>
> Rules I. If two numbers have like signs, their product is positive.
>
> II. If two numbers have unlike signs, their product is negative. (4, p. 106)

The following quote, which is used to introduce division, is an example of the sketchy explanation given this operation.

> Division is the opposite (inverse) of multiplication . . . In multiplication we know two numbers; we seek the product. In division we know one number and the product; we seek the other number . . . We can multiply natural numbers by repeated addition. We can divide by repeated subtraction.(4, p. 10)

There is a very good illustration as to why division by zero is impossible.

Only examples are used in making the transition from division of natural numbers to division of literal numbers. First, $18 is divided by 3; the $ is then replaced by "d" and the corresponding operation is completed.

Axiom 1 allows division of numbers in the solution of equations. "Dividing both members of an equation by the same number, except zero, produces an equation with the same solution as the first equation." (4, p. 23)

The introduction to division of signed numbers follows essentially the same procedure as used by Illinois. The examples lead to the rules for division:

I. If the dividend and the divisor have like signs, the quotient is positive.

II. If the dividend and divisor have unlike signs, the quotient is negative.(4, p. 110)

It is mentioned that division of fractions involves reciprocals. "When the product of two numbers is 1, either number is the reciprocal of the other,"(4, p. 327) defines a reciprocal.

Although there is some discussion of the properties of the various number systems, the coverage given this area is not as extensive as in Ball State, SMSG, and UICSM.

There is some early discussion concerned with the commutative, associative, and distributive laws and the properties of zero and one. Aside from Chapter 1, little mention is made of the commutative, associative, and distributive properties. This is in contrast with the frequent inclusion of the multiplicative and additive properties of zero and one throughout the text.

CHAPTER VIII

THE NUMBER LINE

The student who has a well-rounded understanding of the real number line is on a solid foundation in an important area of mathematics. An interesting feature of the texts under consideration is the difference in coverage given the evolution of the number line.

Ball State's introduction to the number line is contained in Chapter 7, "Some Applications of the Integers." The approach utilized is systematic and precise. The student is asked to consider a straight line.

> Choose any point on this line and associate the number 0 with the point. Choose any other point and name this point 1. We have placed the point 1 on the right side of the point 0 but this is unimportant. Choose a third point just as far to the left of the point 0 as the point 1 is to the right. Call this point -1. (1, p. 102)

A diagram illustrates how to continue. Later material suggests that a straight line may be thought of as endless and there is a point on the line for every integer. The integers are called coordinates of the points and each point is a graph of an integer. Mention is made that there are many more points on this line than have been marked by integers in the illustration. Also, there are other numbers besides integers. It is implied that coordinates can be given to the points between integers.

Later in the text, as each new system is encountered, the new numbers are incorporated into the number line.

After rationals are discussed, an illustration shows that there are certain numbers such as $\sqrt{2}$ for which no point can be found on the rational number line. In other words, the rational line has "holes" in it.

This last statement is clarified by separating the numbers of the rational line into two sets, set (a), all those rationals x such that $x^2 < 2$ and set (b), all those rationals y such that $y^2 > 2$.

> The two sets have the following properties: (1) Every number of the first set is less than every number of the second set. (2) The two sets together contain all rational numbers. (3) There is neither a greatest number in the first set nor a least number in the second set. In this sense the rational line has a hole in it where 2 should be.(1, p. 277)

One familiar with the contributions of Richard Dedekind to mathematics will readily see that the above properties describe the Dedekind cut. Ball State does not mention the name given the properties listed.

The pupil is told that when the non-rational, or irrational numbers, are considered along with the rational numbers, the "holes" in the rational line disappear. This, then, is the set of real numbers.

There is some discussion of an additional property of real numbers which is related to the fact that the reals "plug the holes" on the rational line. The end result of this property, that of completeness, is that there is a graph, called the real number line, which has no "holes" in it. The completeness postulate is stated as follows:

> Suppose S is any set of real numbers, which contains at least one number, and suppose there is a number A such that for every number x in S it is true that $x \leqq A$. Then there is a smallest number B with this property.(1, p. 285)

Probably the only significant difference between the SMSG and Ball State approaches to the real number line is where the individual developments commence. SMSG begins in Chapter 1 while Ball State waits until Chapter 7.

To inaugurate the SMSG presentation, a line is drawn, two distinct points are chosen, the point to the left is labeled 0, and the point to the right is labeled 1. The interval between these two points is used as a unit of measure to locate points equally spaced along the line to the right. The points are then labeled, each point being labeled with the successor of the number to the left of it.

Shortly thereafter, the intervals are divided and the points labeled with fractions. The student is shown, by finding midpoints of intervals, that there are infinitely many points between two whole numbers, and hence on the number line.

In Chapter 5, the points to the left of 0 are determined much the same as those to the right. These new points are labeled with negative numbers. The points corresponding to fractions are then determined and labeled accordingly.

The pupil is told that there are many points on the real number line whose coordinates do not correspond to fractions. One of these is $\sqrt{2}$; this will be proved in a later chapter.

The Illinois treatment of the number line is not as extensive as the two previous programs. Discussion of the concept begins with the following statement:

> You may be accustomed to thinking of the numbers of arithmetic as arranged along a line in order . . . The numbers themselves are not points on the line but we think of the numbers as corresponding to points on the line.(3, p. 1-62)

Graphs, coordinates, and the number line are defined and discussed. Near the middle of the text the number line is used extensively in describing situations such as the set of x such that $x > 3$.

Mallory's use of the number line concept is quite limited. The number line is introduced as the number scale, discussed, applied in material covering less than ten pages, then apparently discarded for the rest of the text.

Introduction of the arithmetic scale involves comparison with a yardstick. The pupil is asked to note that on both a yardstick and the arithmetic scale only distance can be measured. The discussion is terminated after mentioning that if signed numbers are used, an algebraic scale can be constructed which shows not only distance but direction.

CHAPTER IX

UNIQUE TOPICS OF INDIVIDUAL PROGRAMS

As the title suggests, this chapter will discuss significant topics which have been given major emphasis in only one or two of the texts under consideration.

A complete chapter devoted to logic is probably one of Ball State's most distinguishing features. In the fourteen pages of Chapter 3, "Logic," a brief introduction to logical concepts which are important in mathematics is encountered.

A statement is defined as "A sentence which is either true or false, but not both."(1, p. 27) The Greek letters α and β are used as abbreviations for statements. Ways complicated sentences can be constructed from simpler ones by using the words "and" and "or" are discussed. The student learns how to use truth tables in determining the truth or falsity of statements.

Further work involves formation of rules for determining the truth value of " α and β " and " α or β " statements. Negative, if then, converse, equivalent, and contrapositive statements are introduced as well as the corresponding symbollism.

Relations and sentences are topics emphasized much more by Ball State than any of the other programs with the exception of SMSG. Relations are pairs of elements in a certain set such as a set of numbers or people. These relations can be described by using a sentence, sometimes a mathematical sentence.

The Dedekind cut, completeness postulate, unique factorization theorem, and $Q(\sqrt{2})$ number system are topics included in Ball State but not covered by SMSG, UICSM, or Mallory. These areas have already been reviewed in this report so a repetition of such would be needless.

The final chapter of the Brumfiel text is "Similar Triangles and Trigonometry." A few geometric facts are covered which will be studied in detail in later courses. The main objective appears to be the introduction of trigometric functions. Previous to this properties of right and similar triangles are investigated.

Following the final chapter of the text is Appendix I which consists of a collection of postulates for the various number systems considered during the course. Also included are important definitions, theorems, and brief remarks concerned with each system. Appendix II is composed of supplementary exercises designed to improve manipulative skill and deepen insight. The exercises are grouped by chapters to facilitate use. "The collection includes exercises for the first thirteen chapters, which constitutes a minimum course."(1, p. 337)

Chapter 2 of SMSG, titled "Phrases and Sentences," contains much material similar to that included in Ball State's chapter on logic. However, the approach to the subject is not the same and some different areas are covered.

SMSG explains that algebra is composed, to a great extent, of sentences about numbers such as $6 + 2 = 7$. This particular sentence is used as an example to illustrate the point that even if a statement is false, it is still a sentence. The symbol "$=$" represents the verb "is" and "$\neq$" represents the verb "is not." Another important fact presented is if a sentence involves numbers, it is true or false, but not both.

Compound sentences are also introduced. An example of such a sentence is $4 + 1 = 5$ and $6 + 2 = 7$. "And" is described as meaning "both" so if one clause is false, the sentence is false. Another type of compound sentence discussed involves the connective "or." If one or more of the clauses is true, the compound sentence is true; otherwise it is false.

Phrases are said to be incomplete sentences such as $3x + 12$. A phrase containing a variable is said to be open since the decision has been left open as to what number to specify for the variable. It follows from this that a sentence is open in the sense that it contains at least one variable and the question is left open as to the truth or falsity of the statement until it is known what number the variable represents.

A truth set of an open sentence containing one variable is defined as "the set of all those numbers which make the sentence true. . ."(2. p. 49) An open sentence can be thought of as expressing a condition on numbers; the truth set is then just the set of all those numbers which fulfill this condition. Graphs of open sentences and truth sets are given ample coverage.

Probably the major distinguishing feature of the Illinois program is that of presenting the material with the objective in mind of letting the student "discover" principles. This is in contrast with "showing," the method generally used. If the student is to "discover," principles must be camouflaged to some extent. Therefore, at times it is difficult to determine precisely what terminology is used to describe concepts under consideration. Often one has difficulty finding the exact location of specific material covering certain concepts.

The pronumeral, another distinguishing feature of UICSM, has been reported earlier.

An area of special interest included in this program is that of probability and statistics. This material is introduced when working with plane lattices (graphs) in order to add interest. The text does not contain a large amount of material related to the subject but the teacher's section contains ample help if student interest is reasonably high and the teacher deems more thorough coverage feasible. If this is desired, the manual suggests the teacher lead the way into the informal discovery of such topics as independent and mutually exclusive events.

Mallory's "revised" text contains several topics which are generally included in traditional programs. However, most contemporary mathematics educators feel that the mathematical value of these areas is not commensurable with the time spent on them.

The topics in question include problems involving mixtures, money, age, levers, and pulleys and the previously mentioned axioms of traditional algebra. If the basic operations and the use of parenthesis are clearly understood, antiquated rules such as the following do not need to be mentioned.

> I. When a quantity in parenthesis is preceeded by a plus sign (+), the ( ) may be removed without changing the sign of any term within the parenthesis.
>
> II. When the quantity in ( ) is preceeded by a minus sign (-1), the parenthesis may be removed if the sign of every term within the ( ) is changed. (4, p. 121)

Frequently, the attempts to utilize modern concepts and approaches in this text have resulted in undesirable situations. Material has been placed in an area containing little related information. After the material is discussed, little use of it is made later in the text.

A specific example of a case such as this involves the introduction of statistics. The inclusion of statistics in the high school mathematics program is recommended by the CEEB Report.(5, p. 36) It appears that in endeavoring to follow this recommendation, a search was made to determine the best section to place the concept. Since graphs and statistics are somewhat related they are placed together in a chapter titled, "Statistics and Graphs."

Chapter 10, "Indirect Measurement With Trigonometry," gives a general introduction to trigonometry. Chapter 13 is "An Introduction to Geometry." One may question why trigonometry is covered before geometry when normally the opposite is true.

If Mallory is to be commended for presenting some topic by a unique method it may be for introducing the limit concept. This idea is usually difficult even for college students so, to present it in such a way that first year algebra students may realize what is involved requires much ingenuity. The following is how the limit is introduced:

> An old story is that of the jumping point frog, which is a frog the size of a point - that is, no size at all. The point frog starts from one end of a board and jumps toward the other end. The length of the board is x feet. The point frog jumps one-half of his distance from the end at each jump. Thus, in feet, he jumps distances of 1/2 x, 1/4 x, 1/8 x, and so on. . . The sum of his jumps is (1/2 x + 1/4 x + 1/8 x + 1/16 x + . . .) feet. You see that the frog may jump forever and never make the distance x feet.(4, p. 489)

CHAPTER X

SUMMARY

It is important to note that in the three modern texts considered, Ball State, SMSG, and Illinois, radical changes have not been made in content. The modern concept is concerned mainly with new approaches to traditional material and emphasizing areas that in the past have not been regarded as being so important. A careful observer may also note that more consideration is given to challenging and developing the potentialities of the more able student.

In the well organized Ball State text, a somewhat rigorous postulate approach is used to introduce new concepts. It is a rare occassion if one cannot determine what the text is "driving at." Ample exercises are provided to help the student understand this algebra based on the development of the real number system. Brumfiel tends to use a minimum of words; what is said is short and to the point.

Occassional full page pictures of important mathematicians and a few sentences describing their contributions are included in order to stimulate student interest. Generally, each chapter closes with a helpful summary of the more important topics just discussed. Page make-up and printing are conducive to easier reading than in many other texts.

Any criticisms would probably be related to the fact that the text may be too rigorous for the average first year algebra student.

Examples have already been mentioned which illustrate this point. This includes such topics as the Dedekind cut, completeness postulate, and the unique factorization theorem. Students are asked to prove many theorems. Experienced teachers say that attaining proficiency in this area is difficult for students who are no older than those who normally take first year algebra.

SMSG and Ball State are very much the same. However, as can be gathered from the preceeding pages of this report, there are minor differences.

One of the points of difference is that, in many instances, Ball State will cover a particular concept in a shorter time than will SMSG. The latter will tend to go into more detail in these areas. In general, the two programs include practically the same material.

Because SMSG texts are mimeographed, they seem somewhat harder to read than might normally be expected. The paperback construction may tend to make the texts less interesting to the student. No index is provided, which is a definite handicap. Summaries at the end of each chapter are usually not supplied.

A criticism of SMSG is that it is also possibly too rigorous. In comparison with Ball State, it appears that SMSG would be the program that could be handled by a larger percentage of students.

Of the three modern programs, Ball State, SMSG, and Illinois, the latter frequently takes the most "untraditional" approach. The method used to accent the difference between numbers and numerals, the introduction of pronumerals, and the development of plane lattices can be classified as being distinctly unique.

Of the three programs, UICSM will probably be more interesting to students in many instances. This is because concepts are frequently developed by using hypothetical situations from the world of everyday living.

The method of introduction used so much which involves "discovery of principles" by the student has definite advantages. However, one familiar with the situation often finds that many high school students are prone to be hesitant in approaching areas which require searching for facts. In the past this may be due to poor motivational procedures and corresponding approaches by the teacher and the text.

When summarizing and reviewing, it is difficult to pinpoint important material in the UICSM text. This is due to the fact that principles must be "hidden" in order that students can "discover" them.

The Illinois text used for reference in this report was mimeographed. Accordingly, it has some of the disadvantages possessed by SMSG which are due to the same condition.

UICSM takes more time to develop many concepts. This will lead to better pupil understanding of these areas but, to do this, time is required that normally could be used in developing more advanced topics. For that reason, the Illinois First Course does not contain much of the material covered in the latter part of the Ball State, SMSG, and even the more traditional texts.

The revised Mallory text appears to be inappropriate if one is looking for a text that contains modern mathematical concepts. Analysis indicates that in an attempt to be modern, topics have been placed throughout the text without enough careful thought as to overall effect. Much of the modern material appears to be isolated and is not closely related to that surrounding it.

Probably the greatest criticism of the Mallory text is that it is disorganized. A coordinated attack on the central themes of algebra is lacking. In many instances, topics are introduced, as mentioned earlier, in isolated situations. This could be avoided in case after case if more general concepts were developed early in the text. Then, when different applications of these concepts were encountered, extensive explanation would not be necessary.

The complete text seems to be composed of much piecemeal information. The chapters are divided into many more subtopics than the chapters of the modern texts and headings are seemingly repeated without sound reasons. An example of this is in Chapter 6, Topic 8, "Products of Two Binomials in Equations."

It is often difficult to determine which of the material is to be regarded as important although emphasized sentences are underlined in blue. At times there are so many blue lined sentences that one may overlook the significance of the more important points. Less promiscuous use of the blue lines would undoubtedly result in a better text.

The printing on individual pages is often crowded so that the pupil will have more difficulty than necessary in reading.

The language generally used cannot be classified as precise by mathematical standards. This handicaps both the student and the teacher.

The text contains, as has been previously mentioned, many topics and problems of traditional algebra that modern mathematicians do not feel are important enough to take time to teach. The book is rather long and eliminating such material would be a definite advantage in a modern day curriculum. Summaries are included at the end of each chapter but contain much irrelevant information.

BIBLIOGRAPHY

1. Brumfiel, C.R., Eicholz, R.E., and Shanks, M.E. Algebra I. Reading, Massachusetts: Addison-Wesley, 1961.

2. School Mathematics Study Group. Mathematics for High School. First Course in Algebra. Ann Arbor, Michigan: Cushing-Malloy, 1959.

3. University of Illinois Committee on School Mathematics. First Course. Urbana, Illinois: University of Illinois, 1955.

4. Mallory, V.S., Skeen, K.C., and Meserve, B.E. First Course in Algebra.

5. Commission on Mathematics, College Entrance Examination Board. Report of the Commission on Mathematics. Program for College Preparatory Mathematics. New York: College Entrance Examination Board, 1959.

6. Eves, Howard and Newsom, Carroll V. An Introduction to the Foundations and Fundamental Concepts of Mathematics. New York: Rinehart, 1960.

VITA

Kenneth Arthur Smith

Candidate for the Degree of

Master of Science

Title of Study: A COMPARISON OF MODERN AND TRADITIONAL FIRST YEAR ALGEBRA TEXTS

Major Field: Natural Science

Biographical:

Personal data: Born in Todd County, South Dakota, December 9, 1933, the son of Bennett and Marie Smith.

Education: Attended grade school in Witten and Winner, South Dakota, and rural Boyd County, Nebraska; graduated from Butte, Nebraska, High School in 1952; received the Bachelor of Science in Education degree from the Southern State Teachers College, Springfield, South Dakota, in 1956, with majors in mathematics and music; attended the South Dakota School of Mines and Technology and the Kansas State University during the summers of 1959 and 1960, respectively.

Professional experience: Served in the United States Army from June, 1959 until May, 1958; taught in the high schools of Geddes and Gregory, South Dakota, from 1958 until 1960.