UNIVERSITY OF OKLAHOMA

GRADUATE COLLEGE

MATHEMATICS FOR CRITICAL NUMERACY: A CASE STUDY OF A SOCIAL JUSTICE MATHEMATICS COURSE FOR PRESERVICE ELEMENTARY TEACHERS

A DISSERTATION

SUBMITTED TO THE GRADUATE FACULTY

in partial fulfillment of the requirements for the

Degree of

DOCTOR OF PHILOSOPHY

By

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Norman, Oklahoma
2010

MATHEMATICS FOR CRITICAL NUMERACY: A CASE STUDY OF A SOCIAL JUSTICE MATHEMATICS COURSE FOR PRESERVICE ELEMENTARY TEACHERS

A DISSERTATION APPROVED FOR THE
DEPARTMENT OF INSTRUCTIONAL LEADERSHIP AND ACADEMIC CURRICULUM

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ACKNOWLEDGEMENTS

This has been a long and arduous journey. Just as I found many paradoxes in the evolution of the course in this study, I came across many paradoxes in the relationships that led to this dissertation. Developing my own body of work was only possible because of the interactions between me and the many integral people in my life. I have them to thank for nurturing my intellectual, psychological, and emotional growth throughout this process. I would first like to thank the educators in this department. I would like to express my gratitude to my advisor, Stacy Reeder, who expanded my notions about mathematics teaching and learning, gave me confidence in my ideas, and allowed me the freedom to follow my interest in mathematics for social justice. Without the endless hours she spent with me, I would not have been able to reach the autonomy necessary to create this project. I would like to state my appreciation to Neil Houser, who helped me view the world through new eyes and question many of my assumptions about myself and society. Without these questions, I could not have even conceived of such a thing as social justice mathematics. I would also like to thank my committee members for being my teachers and providing me with valuable feedback, as the ideas for this study emerged.

Second, I would like to communicate the importance of my family to this project. I would like to express my deep appreciation for my husband, Zeyad, who spent many nights listening to and discussing the ideas of this dissertation with me. I value all of his love, support, and open-mindedness. I would like to thank my parents, Radwan and Susan, who have played an important role in nurturing my intellectual curiosity from birth. I would like to express gratitude to my father for his encouragement of

mathematics in our home, his rejection of the absoluteness of the norm, and his love of diversity. He sparked many of the interests that drove my curiosity in mathematics and persuaded me to want to question what it means to teach and learn the subject. I would like to thank my mother for her creativity, her rejection of the absoluteness of the norm, and her love of diversity. She helped me develop my writing and spent many hours reading and editing my work. I would also like to express my appreciation for my siblings, Thamer, Abdul-Rahman, and Sarah, for lending a helping hand when I needed it. They stepped in on many occasions when I thought I would not make it and helped me take care of the many things that were entangled in my life throughout the past few years. Last but not least, I would like to thank my two beautiful boys, Yusef and Zade, who have brought much joy and purpose to my life. They were the love that pushed me to keep going, even when I thought I could not. This dissertation is for all of you.

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ABSTRACT

In spite of undeniable technological progress in our Western society, injustice continues to plague the twenty-first century. More than ever in human history, our civilization faces many obstacles that require our attention. Although some educators have contended that the primary goal of education should be to ensure social well-being, many still question whether all subject areas are inviting students to think about current world conditions and their transformation. For the most part, little focus has been placed on considering a critical mathematics curriculum that relates both content and delivery to considering transformative possibilities for society. Some mathematics educators have begun to question the role their subject plays in developing social well-being; however, not much research exists on how to formulate courses that develop mathematical as well as social knowledge and beliefs, particularly with preservice teachers. In light of these concerns, through my own teaching, I decided to explore the critical factors involved in the evolution of a social justice mathematics content course for elementary preservice teachers. Further, I sought to understand student perceptions about teaching and learning in such a course. My findings suggest that participating in this study increased enjoyment of learning mathematics for the participants, and it transformed their understandings of mathematics and social issues. However, resistance from a few students was encountered.

CHAPTER ONE

INTRODUCTION

Despite undeniable technological progress and endless educational reform efforts, human culture faces what may arguably be its most difficult time in history. Scientists continue to warn us of the devastating effects of our actions (e.g., Bender, Burns, Burns, & Guggenheim, 2006), and we continue to search for solutions, with what seems like little progress and even more damage—a result of what Capra (1996) has called a crisis of perception.

> The more we study the major problems of our time, the more we come to realize that they cannot be understood in isolation. They are systemic problems, which means that they are interconnected and interdependent. For example, stabilizing world populations will be possible only when poverty is reduced worldwide… Scarcities of resources and environmental degradation combine with rapidly expanding populations to lead to the breakdown of local communities and to the ethnic and tribal violence… Ultimately these problems must be seen as just different facets of one single crisis, which is largely a crisis of perception. (pp. 3-4)

Capra suggests that current cultural conditions are the result of an interconnected web of relationships that are often viewed as disjointed, isolated events in history. This misunderstanding is the result of "an outdated worldview [often referred to as modernity], a perception of reality inadequate for dealing with our overpopulated, globally interconnected world" (p. 3), which manifests itself in all cultural productions, including education.

Although some educators have contended that the primary goal of education should be to ensure social well-being (e.g., Freire, 1970), scholars such as D'Ambrosio (1985) question whether schools are inviting students to think about current world conditions and their transformation. Except for a few exceptions (e.g., Powell &Frankenstein, 1997), for the most part, little focus has been placed on considering a critical mathematics curriculum that relates both content and delivery to transforming society rather than maintaining it. Our education system continues to disregard social and economic disparities, while world markets collapse, global warming is at its worst, people are going hungry, and terror is at an all time high.

In light of the concerns I have expressed, I believe in order to begin contemplating a more just world and more relevant mathematics education, both curriculum and pedagogy must be explored from a social justice approach. Therefore, this study focuses on the development and implementation of a social justice mathematics curriculum in a college level mathematics course for elementary teachers.

Organization of the Dissertation

My dissertation is composed of five chapters. Chapter One explains the purpose of the study and provides the research questions addressed in it. Chapter Two gives a background for the research. Chapter Three describes the research methodology and the setting for the study. Chapters Four and Five provide the research findings and a summary of those findings. Finally, Chapter Six presents a research analysis and discussion along with the implications of this study.

Purpose of the Study

Education has traditionally played a key role in perpetuating world conditions. Modernity, the scientific mindset that has come to dominate Western culture, has driven the production model of schooling that currently exists in our culture and has produced a competitive mindset that tends to reproduce and enable inequality in many cases rather than transform it. "Modernist traditions and the need to find comfort in modernist paradigms drive the neoconservative movement and the desire for increased test scores, uniform curriculum, and successful international competition" (Fleener, 2000, p. 18). Mathematics education has often escaped any responsibility for these problems because of the false notion that mathematics is value-free and universal, hence, incapable of contributing to ideologies and philosophies that may result in unjust actions that enable global problems. However, academic mathematics, in its interactions with and extrapolation from culture and society, has influenced human action and been a brick in the foundation of modernity. A closer look into the history of academic mathematics gives us a clearer depiction of how this subject has helped maintain social and economic disparities that have aided in the formation of a crisis of perception.

Mathematics has existed for as long as humans have sought patterns to make sense of the world in which they live. "[I]t is clear that mathematics arose as a part of everyday life" (Boyer, 1968, p. 3). It began with ideas of contrast and likeness—"the difference between one wolf and many, the inequality in size of a minnow and a whale" and "one wolf, one sheep, and one tree have something in common" (p. 3), which illuminated an understanding of abstract ideas and brought about the concept of number. With time, different cultures began developing more sophisticated mathematics that reflected their social and political needs. For example, Egyptians developed devices for counting time and measuring land. Pre-Colombian societies formulated and utilized a base 20 number system--a positional vigesimal system. “It employs three symbols to write any whole number from zero to whatever quantity is desired” (Ortiz-Franco, 2006, p. 72). It was used in the creation of calendars and the study of astronomical sciences by the Zapotecs of Oaxaca and Mayas in pre-colonized Central America.

As the mathematics of different cultures and societies progressed, so did interaction between and among them. Humans began understanding and recording the mathematics of others and assimilating this new information into their own bodies of knowledge. The field grew and cultures learned from others and expanded their own mathematical ideas and the subject became essential for human achievement and advancement; a new worldview began to emerge. With mathematical inventions such as the mechanical clock, “[m]edieval mysticism and qualitative interpretations of time were replaced in the 16th and 17th centuries with quantitative, numerical notions of time” (Fleener, 2002, p. 39). Time became isolated and separated from life and was used to control workers’ behavior and quantify the universe—what Fleener has called a mathematization of reality.

> The invention of the mechanical clock and the acceptance of the perspective of mechanical time provided a necessary foundation for the modern scientific paradigm. The relationship between objects moving in space and time and the mathematics that captured that relationship, fundamentally changed how Western societies approached nature and conceived of the potential of science to understand the inner workings of nature…Mathematics, originally a way of

> modeling our ideas about nature, soon became mistaken for an exact representation of the inner workings of nature. (p. 43)

The mathematization of reality led to the separation of humans from nature. A shift in consciousness emerged in which people began believing that with enough scientific advancement, nature could be controlled, and the leading metaphor for the world became that of a machine—the idea being that like a machine, the world could be pulled apart, understood, and hence manipulated by its pieces. And inherent in this new metaphor was a logic of domination (Fleener, 2002), which aided in creating a crisis of perception and perpetuated social and economic disparities that have contributed to the problems we now face.

Mathematics and science became idolized among the studies, fragmenting thinking into categories with technicality as superior to emotion and thought. World structures began to and still do reflect this mode of thought which "touts competition over cooperation, individualization over community, and progress over process" (Fleener, 2002, p. 47). Humans began competing for advancement and resources. Modernity separated the world, and manifested itself in many ways, including the colonization efforts of Europe, which molded the structure of education, including mathematics education, maintained separation, and perpetuated a logic of dominance.

With the need for dominance and superiority, certain groups of people began to take credit for and distribute knowledge that would further their ascendancy in the world. For example, although current academic mathematical ideas can be attributed to the endeavors of people around the world, because of European colonization efforts, mathematics took-on a uni-cultural appearance and created the false impression that only

European males have held the key to unlocking its mysteries. This depiction has disregarded the efforts of other cultures and led many scholars to the conclusion that mathematics education in Western societies perpetuates modernity and Eurocentrism, maintaining economic and social disparities that allow the "elite to assume effective management of the productive sector" (D'Ambrosio, 1985, p. 16).

History shows that Egyptians, Chinese, and Indians contributed tremendously to the development of modern academic mathematics. However, credit for their undertakings has often been attributed to scholars such as Euclid and Pythagoras.

> Euclid – the so-called 'father' of plane geometry – spent 21 years studying and translating mathematic tracts in Egypt. Pythagoras also spent years studying philosophy and science in Egypt, and possibly journeyed East to India and/or Persia, where he 'discovered' the so-called Pythmatical documents (c. 800-500 B.C.). How could a theorem whose proof was recorded in Babylonian documents dating 1,000 years before he was born be attributed to Pythagoras? (Anderson, 2006, p. 44)

Today, these myths still manifest themselves in current Western mathematics texts—Euclid is still considered to have introduced plane geometry to the world, and Pythagoras is still the title holder for the first proof of the theorem that was named after him.

Moreover, as Europe colonized the world and brought with it the mathematics that proved useful to its expansion, mass education systems were put in place. They brought with them ideologies that imposed and rewarded Europe's ways of thinking on other societal groups. Students became expected to study the mathematics brought to them by European settlers, which disregarded their own mathematical ideas and contributions. The effect has been an educational marginalization along socio-economic, race, and gender lines (Powell & Frankenstein, 1997). For example, Ezeife notes that "[o]ne of the reasons advanced for the high dropout rate and poor performance in examinations by the few aboriginal students who enroll in mathematics or science is that mathematics and science taught in school is bereft of aboriginal cultural and environmental content" (2002, p. 177). And Davison has said, "American Indian students' capacity to learn mathematics is influenced by language, culture, and learning style. However, the methods by which mathematics is typically presented do not take into consideration these factors" (as cited by Ezeife, p. 177).

Although the effects of this modern mindset that fragments mathematics from other subjects and allows inaccessibility of academic mathematics for many students has not gone unnoticed (Dewey 1902), meaningful change in mathematics education has yet to materialize. During the past century, parents, educators, and psychologists have acknowledged that not all students are learning mathematics, sparking many debates, often referred to as math wars, over what and how mathematics should be taught. On the one hand, progressives have suggested that the child and the curriculum should be viewed in correlation with one another—that the child should construct mathematical knowledge about the world, but not in a manner that is detached from his/her life. On the other hand, essentialists have argued that the mathematics taught in schools should be the mathematics that maintains a competitive society—that students can only advance their positions in society if they know the type of mathematics that would put them ahead of others. This has been referred to as essentialist ideology.

Regardless of what progressives and essentialists originally meant, the shifts that have been called progressive and essentialist in mathematics education over the past century have brought about little to no meaningful change and continue to perpetuate a system that disregards world conditions and fails many. Both factions have discounted the connectivity of history and culture to social and economic injustice and hence disconnected students from the idea that the mathematics that brings success in one form helps carry on problems in another, which maintains dominance and illustrates Capra's (1996) suggestion that we face a crisis of perception.

Some scholars have begun reconsidering the role mathematics education should play; however, for the most part, mathematics education still consists of curriculum filled with formulas and theorems, often seen as disconnected from the outside world and meant only for memorization and regurgitation, and pedagogy that reinforces a teacher-centered approach to learning. Those who strive to re-envision mathematics are contemplating it as a means of uncovering the effects of modernity by connecting it to world conditions and empowering students to create change. Recent efforts to dissipate the destructive structures of society and encourage social action through mathematics have emerged in the form of a social justice approach to teaching mathematics (e.g., Gutstein & Peterson, 2006). Within this context, mathematics reveals the effects of domination in Western culture and turns mathematics into a "tool to understand and potentially change the world" (Gutstein & Peterson, 2006, p. 2). The idea is that understanding traditional mathematics as a means to "*reading* the world" can give power to marginalized students (Gutstein, 2003) while at the same time helping students from the dominant culture recognize injustice so that social and economic disparities may dissolve and world conditions may begin to improve. In this context, mathematics and problems in society become interconnected and mathematics curriculum becomes more holistic, combating a crisis of perception.

Educators have begun discovering the power of mathematics as a tool for understanding social issues. For example, Steele (2006) taught his high school accounting students, through traditional mathematics concepts, the effects of "sweatshops" on the economic and social well-being of the workers who manufacture our products. Andrew Brantlinger (2006) used the inequity present in certain geographical areas, such as South Central Los Angeles during the riots of 1992 to illuminate geometric concepts. Through mathematics, he helped his students, from Chicago's north side, understand that social conditions are often a result of culture, not individual deficiencies, as is sometimes portrayed by modernity. And Beatriz Font Strawhun, a teacher in a middle school in New York (Turner & Font Strawhun, 2006), used concepts such as linear and area measurement, ratio, operations with fractions, and mixed numbers to help her students uncover the injustice in the conditions of their overcrowded school when compared to those of a more affluent nearby school. "The opportunity to investigate real issues pushed students to construct and apply important mathematical concepts" (p. 86) and prompted them to take action to change the conditions of their school.

Other educators have attempted to combat dominance in mathematics education by legitimizing the voices of their students. Frankenstein (Powell & Frankenstein, 1997) used interviews and dialogue to help her students communicate their thoughts and empower their voices. Powell, Jeffries, and Shelby (1989) asked their students to write in journals so they could analyze and reflect on their own mathematics as well as what it means to teach and learn. Frankenstein, Powell, Jeffries, and Shelby asked their students to reconsider the roles of teaching and learning, thereby allowing them to question the assumption that the teacher is in control of the students. Students became empowered by the fact that their ideas and contributions were heard and valued, rather than disregarded and underestimated. The dichotomy between authoritative figure and subservient individual was replaced by mutual influence and diverse abilities. However, more research needs to be done. Few resources exist that portray what can happen when social justice is infused with mathematics education, particularly with preservice teachers.

As I consider the above points, I come to some questions. Preservice teachers today face many challenges as they prepare to enter the mathematics classroom. If they are expected to teach certain "value-free" concepts, can we help them reconsider the idea that mathematics is value-free? In addition, can we help them examine how the power of mathematics has the potential to help us work together worldwide by explicitly addressing social issues that maintain injustice in our society? If so, can they make sense of the mathematics that is intended by the strict guidelines prescribed by the traditional curriculum as well as the social concepts introduced by a teacher educator? What challenges arise in a social justice mathematics content course for elementary teachers? How is social justice mathematics defined in this type of classroom? How is preservice teacher understanding assessed? What is the role of the teacher educator in such a classroom? These are some of the questions that arise when I think of challenging the mathematics education methods of the last century. They lead me to the goal of my study, which hopes to add a piece to this body of knowledge.

Goal of the Study

Attempts have been made to challenge the academic traditions of mathematics educators over the past century. Some scholars have begun searching for answers to the above questions (e.g., Powell & Frankenstein, 1997; Gutstein, 2003) in order to begin contemplating what it means to teach mathematics during a time that is bombarded with social problems. They have done so while emphasizing the importance of students' voices in the classroom and questioning the traditional roles of teacher and learner. They created an atmosphere of cooperative learning, where students constructed their own ideas about mathematics concepts and taught their peers as well as instructors.

With the above questions in mind, and the efforts of other social justice mathematics educators before me, I come to the goal of my study. I hope to clearly define what a social justice curriculum might look like in a mathematics class for elementary teachers, where predefined mathematical curricular objectives have been imposed externally. I also hope to understand how preservice teachers perceive learning in such a class.

Focus Questions

This study focuses on the implementation of a social justice mathematics curriculum in a mathematics course for elementary teachers. In this class, issues of social and economic justice were integrated into the curriculum and the traditional roles of teaching and learning were reconsidered. This study specifically addresses the following questions:

1. What critical factors were involved in the evolution of a mathematics course that incorporated social justice?
2. What were students' perceptions about learning mathematics in a course that combined mathematics and social issues in the way they are presented here?
3. What were students' perceptions of their understanding of mathematics, social issues, and the relationship between mathematics and social issues when they were presented in this way?

Summary

Since the beginning of time, humans have developed and distributed mathematical knowledge. With colonization efforts, a certain kind of mathematics became favored and distributed worldwide. This mathematics now dominates our school systems. Unfortunately, it is a mathematics that does not address many social issues we face today. Poverty, hunger, world wealth, and war are some of the realities that constitute our present. They are interconnected with mathematics but have yet to find a place within its curriculum. Some scholars have attempted to weave such issues into the math class, yet very few such instances exist. Mathematics educators need more preparation and resources that describe what addressing issues of social justice might look like in a mathematics class. Therefore, this project focuses on what it entails to create a social justice mathematics class for elementary preservice teachers and what its effects are. Drawing on the ideas of those scholars before me, along with the theories of a crisis of perception, led by modernity, and perpetuated by dominance, I explore the effects of teaching social justice mathematics to students traditionally considered to be mathematically "deficient." I illuminate what happens to elementary preservice teachers' ideas about the mathematics class when social justice issues along with a reconsideration of the roles of teaching and learning are infused with mathematics in a content course for elementary teachers.

Chapter Two gives a historical background for this study and explores what it may mean to teach mathematics for social justice. It also provides a brief literature review of preservice teachers' mathematical and cultural understanding.

CHAPTER TWO

BACKGROUND

Leopold Kronecker once wrote, “God made the integers; all else is the work of man” (Bell, 1986, p. 477)—white, European man, at least according to any mathematics textbook that utilizes Western or “academic” mathematics. The names are well-known: Euclid, Descartes, Newton, Laplace, Gauss, and Reimann are a few players in this game of domination that has given credit to the European male for creating mathematics. God and Europe seem to be the only contributors to the field. Western society has adopted and marketed this attitude that now thrives in the world of academics. Through conquest and colonization of the world by the European empire, a uni-cultural view of the subject has given mathematics a universal appearance and created the false suggestion that European males have alone unlocked the mysteries that made mathematics a superpower among the sciences—this is what is often referred to as Eurocentrism. Moreover, Eurocentric mathematics has aided in the perpetuation and maintenance of a hierarchical social and economic structure. However, a deeper look into history exposes the myth of Europe as the only contributor to mathematics and reveals that academic mathematics has been the culmination of the contributions of many cultures throughout time and space. Moreover, it divulges that other forms of valid mathematics, practiced around the world existed and still exist today.

Although academic mathematics of Western culture has dominated, due in no small part to the triumph of European colonization efforts, different forms of mathematics have been practiced worldwide since the beginning of human civilization. Humans naturally developed ways to measure, organize, classify, and even form abstractions (Boyer, 1986). As the Agricultural Revolution commenced between 10,000 and 15,000 years ago, populations began to expand and cultures adopted ideas of hierarchy and new forms of structure (Houser, 2006), which necessitated more sophisticated forms of mathematics. Contact and communication between and among different groups of people grew, and humans embarked on journeys of studying and adopting mathematical ideas from others and assimilating them into their own cultural bodies of knowledge. The mathematics of each culture reflected the social, political, as well as economic needs of that society, the study of which birthed the concept of ethnomathematics.

Although ethnomathematics has existed since people set in motion the idea of learning mathematics from other cultural groups, ethnomathematics research did not become a formal field of study until the 1980s. Ethnomathematics is the study of "the mathematical ideas of peoples, manifested in written or non-written, oral or non-oral forms…" (Powell & Frankenstein, 1997, p. 9). An etymological look at the word ethnomathematics gives a better understanding of its meaning.

> It is a construct using the roots *ethno* (meaning the natural, social, cultural and imaginary environment) + *mathema* (meaning explaining, learning, knowing, coping with) + *tics* (a simplified form of *techne*`, meaning modes, styles, arts and techniques). Breaking the word would allow for saying that ethnomathematics is a theoretical reflection on the *tics* of *mathema* in distinct *ethnos.* (D'Ambrosio, 2006, p. 77)

The Program Ethnomathematics, or ethnomathematics as a research field developed as a way to combat the Eurocentric, universal view of mathematics. It has been described by D'Ambrosio (2006) as

> a research program which focuses on the ways, the styles, the arts, the techniques, generated by identifiable cultural groups to explain, to understand and to cope with the environment, particularly in the development of methods of comparing, classifying, quantifying, measuring, explaining, generalizing, inferring, and, in some way, evaluating. (p.79)

Ethnomathematics has attempted to fight the colonial strategy to disregard the mathematical contributions of the conquered. Therefore, ethnomathematics research can be understood as the study of the formation of, standardization of, and dispersion of mathematical ideas throughout history and culture for the purpose of combating dominant social and economic hierarchical structures that maintain marginalized groups in society.

This chapter provides a brief recorded historiography of the ethnomathematics that led to the academic mathematics that reflects the social, economic, and political demands of a Eurocentric view of the world. It explains the concerns and reform efforts that have stemmed from Euro-mathematics, and it leads to the development of ethnomathematical research as a formal field of study. Further, it includes a synthesis of the research shifts that have occurred in theoretical orientation and philosophical perspectives, and it describes how ethnomathematics, along with recent global concerns, have become catalysts for teaching a social justice approach to mathematics, which begins with the preparation of preservice teachers.

Ethnomathematics and a Eurocentric Mathematics

Since humans began intersecting with one another, interest in mathematics of different cultures has existed. However, in the Western world, records of these encounters are dominated by European accounts of their interactions with other cultures. In 440 BC, the Greek historian Herodotus of Halicarnassus published *The History of Herodotus*. This effort has been considered one of the first historical works of Western literature. *The Histories*, as it has commonly been referred to, consists of nine books that chronicle the Greco-Persian wars, as compiled by Herodotus from stories and interviews accumulated through his extensive travels around the ancient world. Among other ideas considered in this work, Herodotus records the Geometry of Egypt. The mathematics required by the Egyptians at the time included measuring land, calculating economic needs, and counting time--a reflection of the social, political, and economic system of that culture. Greek scholars were now gaining access to mathematical ideas that were unfamiliar to them, and therefore considered new. Works from several cultures began to enter the Greek culture, and intellectuals opened up the doors that began the infusion of mathematical ideas from other parts of the world into Western culture. This time became known as the Golden Age.

As Greece rose and fell and the Roman Empire took hold, mathematical ideas continued to prosper and transact with one another throughout the world. However, with the rise of Christianity, Rome gradually became dominated by this new religion, which developed into the official faith under the rule of Emperor Theodosius. Greek scholars abandoned their mathematical endeavors and focused their energies on issues of theology, since knowledge was now believed to come solely from the Bible. A period of scientific inactivity took hold of Europe and continued throughout the Dark Ages (Joseph, 2000). Although the stagnation of mathematics could only be attributed to Europe during this time, this period is often misrepresented as a time when mathematical endeavors were not being pursued anywhere because they were not thriving in Europe. "A variety of mathematical activity and exchange between a number of cultural areas went on while Europe was in a deep slumber" (Joseph, p. 9). India, Babylonia, China, Baghdad, Southern Spain, and Egypt, to name a few, all interacted with each other mathematically and continued the formulation and cultivation of the field. For example, the Arab Empire was responsible for synthesizing and refining scientific ideas that stemmed from India, China, Egypt, and Greece, and the numbers we currently use in our base ten system, 0 through 9, were developed in India at that time (Joseph, 1993). These major contributions, although milestones in the formation of academic mathematics, are often ignored in Western discussions of the origins of mathematics and attributed to European endeavors.

Although the vitality of mathematics studies thrived around the world, the Renaissance became known as the period of revitalization of many cultural undertakings, including mathematical work, since that was when Europe returned to its interest in the subject. As is well known, this movement began in northern Italy and stretched throughout the rest of Europe. During this time, trade with the Arabs helped nurture the advancements throughout Europe. Moreover, contributions came with the chronicles of European travelers throughout Asia, Africa, and the Americas—endeavors that could be considered ethnomathematical in nature. Some of the most remarkable contributions to the history of mathematics entered the West during this time, but were disregarded as efforts of non-Westerners and later attributed to Taylor, Newton, and Euler. For example, the deductive proof was present in India long before European scholars utilized this method of proof, and the Indian scholar Madhava of Sangamagramma is known to have moved from working with finite series to infinite ones, the cornerstone of modern classical analysis (Joseph, 2000). It was during this time that indigenous people in the Americas were practicing sophisticated mathematics, such as the ethnomathematics Juan Diaz Freyle describes in his book *El Sumario compendioso de las quentas de plata y oro que en los reinos del Pirú son necessarias a los mercaderes y todo genero de tratantes: Con algunas reglas tocantes al arithmética*, published in 1556, in Mexico city. "Pre-Columbian achievements in the New World have long eluded traditionalists. The Maya invented zero about the same time as the Indians, and practiced math and astronomy far beyond that of medieval Europe. Native Americans built pyramids and other structures in the American Midwest larger than anything then in Europe" (Teresi, 2002, p. 13). After the Europeans invaded the new world in the sixteenth century, they "began to apply commercial arithmetic [of indigenous people] to the purchase of citizens in North America from local chiefs and kings, and the later sale of those still alive, to entrepreneurs and landowners across to the Americas" (Grattan-Guinness, 1997, p. 112). However the conquerors "made little effort to conserve the culture of either their slaves or of the indigenous tribes" (p. 113). Although these contributions were recorded, they have still been underappreciated as having any significant involvement in the history of Western mathematics.

As Europe ascended in the eighteenth and nineteenth centuries, cultural interaction grew and capitalism expanded. The countries of this region became industrialized and continued to search for new supplies and cheap labor, in order to manufacture supplies at low wages. Although these interactions allowed for the expansion of mathematical ideas, the world began to change socially, as well as economically. A new modern worldview (modernity) began to capture the world, and the mathematics that became idolized was one that seemed most useful for prediction and control (Houser, 2006). Europeans were writing and distributing this mathematics through colonization and mass education (Joseph, 2000), spreading modernity throughout the world. With the introduction of this mass education system, a Eurocentric mathematics took over. The colonized form of mathematics that was a consequence of this time came to dominate the United States and the world. It brought with it the disregard of different cultural contributions to academic mathematics as well as other forms of mathematics, and it became the basis for the technologies that have developed many of the problems we face today. Students became marginalized educationally along race, gender, and socio-economic lines (Jacobson, 2000), resources began to be horded by some, killing with weapons of mass destruction became possible, and global warning emerged as a problem for all because of the possibilities that stemmed from this form of mathematics.

Reform of a Eurocentric Mathematics

As one form of mathematics came to dominate education across the United States, many educators, mathematicians, parents, and psychologists became concerned about student success rates under the system. Improving mathematics instruction developed into a major distress for many, and the twentieth century became filled with debates over what and how mathematics should be taught. While a series of what have been referred to as “math wars” have raged battles between whether pedagogy or curriculum should be reformed, little meaningful change has materialized, social and economic structures that perpetuate many of the problems industrialized cultures face have remained intact; and mathematics education has failed to address these issues.

In the late nineteenth century, Dewey began a progressive movement in education with a school he created in Chicago. The idea was that a disconnection existed between children and the curriculum they encountered in schools (Dewey, 1902). Dewey explained the world of the child before school, as whole and complete. As that child begins attending school, the subjects begin to “divide and fractionalize the world” (p. 6) for that child. “Facts are torn away from their original place in experience and rearranged with reference to some general principle” (p. 6). However, he believed that the child and the curriculum should be viewed in correlation with one another. The child should construct knowledge about the world, but not in a manner that is detached from life. The “child and the facts and truths of studies [should] define instruction” (p. 11), just as “two points define a straight line” (p. 11). That was the philosophy of his laboratory school.

Although Dewey’s ideas were well-received in the mathematics education realm, for the most part, little focus has been placed on considering a critical mathematics curriculum that relates both content and delivery to relevant issues in students’ lives. The result of the last hundred years has been a mathematics education system that

> maintains the economic and social structure, reminiscent of that given to the aristocracy when a good training in mathematics was essential for preparing the elite (as advocated by Plato), and at the same time allows this elite to assume effective management of the productive sector (D’Ambrosio, 1985, p. 16).

Very few mathematics educators, labeled as either progressive or essentialist, have addressed or greatly impacted the mathematics education community in a way that reconsiders mathematics education as Dewey had envisioned. An overview of the last hundred years demonstrates this point. It also leads to why I have chosen to do this study and why I think a need for a new kind of mathematics, namely social justice mathematics, exists.

In 1902, E. H. Moore, the president of the American Mathematical Society (AMS) "proposed an ambitious program of educational reform for secondary schools and colleges. He championed the 'laboratory method' of instruction and called for mathematicians to take a larger role in educational issues" (Roberts, 2001). He understood the abstract nature of mathematical subjects that seemed disconnected to students and proposed the introduction of manipulatives, technology, and a group learning approach to teaching mathematics (Lott, 2002). He also called for teacher preparation for more developed content knowledge.

Mathematics classrooms, however, continued to epitomize traditional ideas of memorization, drill and practice, and disregard for individual student needs, and by the beginning of World War I both mathematicians and math educators were expressing concern for the current state of affairs. Although both groups were apprehensive about the quality of mathematics education, conflicting views existed about what, how, and why mathematics should be taught. In response to a report published by the National Education Association (NEA) in 1920, summarizing the state of affairs of secondary education in the United States, the NEA compiled a committee of educators that was headed by William Heard Kilpatrick, an education student of Dewey's (Center for the

Study of Mathematics Curriculum [CSMC], 2004). The Kilpatrick report concluded that the higher level mathematics taught in schools was unnecessary to students and called for less mathematics for the majority of students. Even though Kilpatrick's report influenced the progressive movement, the mathematics education community became defensive about their stance that students needed more mathematics rather than less and criticized the fact that no mathematicians participated in Kilpatrick's committee. The community responded to what it considered an outrage.

Several organizations expressed their indignation towards Kilpatrick's efforts. In 1920, the National Council of Teachers of Mathematics (NCTM), based on essentialist ideas, formed to challenge the policies expressed by progressives (Winston & Royer, 2003). In that same year, the Mathematical Association of America (MAA) published a preliminary report defending the need for higher level mathematics in secondary school. This report was compiled by a committee consisting of mathematicians as well as representatives of secondary mathematics teachers. In 1923, the MAA committee published a 639-page volume of a collection of reports called *The Reorganization of Mathematics in Secondary Education*, or the 1923 Report (CSMC, 2004). "The goal of the committee was to investigate the whole field of mathematics education from secondary school through college. They would then make recommendations on the best way to reorganize mathematics courses and improve mathematics teaching" (p. 1). The main point essentialists tried to make was that higher-level mathematics courses, such as algebra, are important to every person.

Although essentialists tried to combat Kilpatrick's efforts, his report overshadowed these efforts, due in no small part to the fact that he chaired the NEA sponsored committee report entitled *The Problem of Mathematics in Secondary Education*, produced in 1920. However, as the 1920s and 1930s progressed, concern about the decline of mathematics in the schools emerged. Even though in theory, curriculum was formed according to the needs of children, as seen by their educators, fewer students were taking mathematics courses and failure rates were high—an outcome Kilpatrick had not anticipated (Winston & Royer, 2003). NCTM as well as the MAA decided to address what they viewed as neglected mathematics in public schools with a report they began working on in 1934 (CSMC, 2004). Although they questioned progressive ideas of teaching, they recognized that there is no one way to teach but nevertheless advocated more mathematics with a detailed 253-page set of guiding principles that outlined the direction that secondary mathematics education should take.

Nonetheless, concerns still echoed throughout the country by the 1940s. The need for a certain type of practical mathematical competency became necessary during World War II. Military personnel complained that recruits could not perform simple arithmetic and schools needed to teach basic skills (Winston & Royer, 2003). The public once again seemed to disagree with educational policies, and by the mid-1940s the Board of Directors of NCTM established a commission to plan a post-war mathematics program (CSMS, 2004). The commission put out three reports that guided the implementation of new mathematics courses and emphasized the importance of mathematics to life, equating it to literacy. Educational leaders implemented a program for the 60% of students who did not display the attributes perceived as necessary for college or skilled labor (Winston & Royer). The movement called for the implementation of mathematics courses that focused on practical concepts such as budgeting, taxation, and home buying.

The expectation was that these students would not go into professional fields, and so this type of mathematics was more useful to them than algebra or trigonometry.

Algebra and Geometry enrollment had been on the decline since the early 1900s, but by the 1950s science and math began gaining momentum due to developments of technologies, including the introduction of the atomic bomb (Jones & Coxford, 1970). Critics of the progressive movement began to push for a return to math literacy that would encourage an increase in the numbers of technically scientific workers. The New Math era was born, and lasted well into the sixties. The University of Illinois commissioned Max Beberman to head the Committee on School Mathematics, initially established to assess competencies a college-bound high school student needed in order to pursue science or mathematics studies. Later, the committee wrote and published a series of textbooks that contained the mathematics concepts they deemed fit for college studies. Although initially this push did not take off, when Sputnik was launched in 1957, Beberman's ideas dominated (Winston & Royer, 2003). The United States media portrayed a deficient picture of school mathematics and science, and embarrassed Americans by comparing them to the Russians and labeling them as less competent. A recession had hit the economy creating an increase in unemployment and decrease in incomes. Congress passed the 1958 National Defense Education Act to increase mathematics, science, and foreign language instruction in schools. The focus became on adding courses such as abstract algebra, topology, and set theory to mathematics curriculum, even at the elementary level. Although this era began a push towards conceptual rather than procedural understanding, soon enthusiasm for New Math began to waiver.

By the end of the sixties, the abstract nature of New Math brought about its own demise. Mathematicians as well as parents criticized the intangibility of courses to students, and teachers were "not well prepared to deal with the demanding content of the New Math curricula" (Klein, 2003, p. 133). A shift emerged—a push "back to the basics." Disapproval of New Math had brought with it a perceived need to return to a focus on basic math skills. In 1973, the National Institute of Education (NIE) was formed to investigate the question of what basic math skills are. The NIE, along with other organizations, such as the National Council of Supervisors of Mathematics compiled lists of what basic mathematical skills should be taught (CSMC, 2004). Many of the components on those lists are still considered relevant today (i.e., problem solving, estimation and approximation, alertness to the reasonableness of results, etc.).

Prior to the push "back to the basics" and essentialist ideas about mathematics education, progressive educators had begun gaining ground. A new book called Summerhill, by A. S. Neill (1960), depicted a progressive school in England and the success an open school can have, where students choose their own curriculum at their own pace (Winston & Royer, 2003). It spread throughout the country, and many schools attempted to create reformations necessary to become more like Neill's school. However, schools' attempts seemed not to last, and this new time of returning to basic skills brought with it more focus on test scores. Standardized tests became considered an efficient means of assessing student learning, promoting the ideas of behaviorist theorists, where the teacher was at the center. The demands on teachers sparked NCTM to create *An Agenda for Action: Recommendations for School Mathematics in the 1980s* (NCTM, 1980). This document placed emphasis on problem solving in the mathematics classroom and opened up discussions about broader ideas of what basic skills are. NCTM had become the thing it originally intended to fight. Its progressive ideas soon spread throughout the country, launching NCTM towards fame.

Although mathematics educators seemed exited by the change, in 1983, Ronald Reagan gave control back to essentialists. It had become apparent that other countries around the world were manufacturing higher quality products more inexpensively than the United States. Again, the U.S. education system was to blame. The idea was that American students were not keeping up with students abroad. *A Nation at Risk*, published by the National Commission on Excellence in Education, spoke of the state of affairs of education, including mathematics education, of the time (CSMC, 2004). It emphasized American schools as mediocre, and this idea spread throughout the nation. States began focusing on curriculum and the idea of advancing ahead of other nations. The document overshadowed the efforts of NCTM and progressivism and put essentialist ideas at the forefront.

As the 1980s moved forward, students continued to lag in performance on national and international scales of assessment. The American educational system was still looking for the magic set of prescriptions that would dictate how to "best" teach students mathematics and remedy this problem. Motivated by *An Agenda for Action* (NCTM, 1980) and other publications of the time, NCTM produced the *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989). The document, splitup into strands: K-4, 5-8, and 9-12, emphasized the notion of "big ideas" and focused on teaching based on constructivist notions that had become popular. The basic constructivist view was that the learner must make connections to form knowledge. It is not enough to just listen to a knower; the pupil must experience and make associations in order to internalize new ideas, which he/she will do in a different way than any other individual. The document hoped to emphasize the importance of creating mathematical literacy through non-traditional means of instruction. It contained progressive ideas of group work, the use of manipulatives, discovery learning, spotlighted the "beauty of mathematics," and accentuated the importance of technology to developing accessibility to mathematics that seemed previously incomprehensible without paper-and-pencil proficiency. Often referred to as the NCTM *Standards*, the document hoped to invite all students to participate in meaningful mathematics learning. As a result of the NCTM *Standards*, the National Science Foundation (NSF) funded the generation of several curricular materials such as *Everyday Mathematics* (University of Chicago School Mathematics Project), K-6; *Connected Mathematics* (Michigan State University Connected Math Project), Middle Grades; and *MATH Connections: A Secondary Mathematics Core Curriculum* (Connecticut Business and Industry Association), High School (CSMC, 2004).

During the 1990s, research continued to head in the direction the *Standards* had paved the way for. People became disenchanted with standardized tests. Researchers began seeing and revealing some of the harmful effects of testing, including their insistence on emphasizing basic facts versus deep understanding (Linn, 2000). States developed their own performance-based assessments as well as standards based on those in NCTM's document. Moreover, schools began adopting reform-based curricular materials; a shift from learning by listening and working in isolation to gaining understanding by constructing and reinforcing socially had hit the country and continued to grow throughout the decade. Radical constructivists such as von Glasersfeld (1995) grew in popularity, and encouraged researchers to draw heavily on the ideas of Piaget and Vygotsky.

However, with the emergence of a new decade along with a new president, the year 2000 brought with it a second Bush administration that called into question the idea of accountability. With the creation of the *Elementary and Secondary Act of 2001* (also known as *No Child Left Behind*), weight shifted from growing qualitative methods of research to large-scale quantitative forms of determining value. The swing again placed pressure on teachers to induce strong performance on high-stakes tests, reverted emphasis back to a focus on teaching basic skills, and placed a strain on researchers to sway back towards scientifically-based methods. The justification came from a perceived lack of organized, systematic methodology that constructivist theories of learning set in motion. Critics of NCTM's *Standards* and constructivism have disapproved of what they have viewed as ill defined constraints on what sound teaching should entail and have condemned what they perceive as unprepared teachers for such radical ideas about teaching mathematics (Cohen & Hill, 2001). However, the war still rages between what has been called reform (progressive) and accountability (essentialist), carrying with it vast implications about what good teaching and research entail in the field of mathematics education.

Although progressive as well as essentialist ideas have shifted somewhat throughout the past century, our education system has rarely changed in meaningful ways and continues to fail many. On the one hand, essentialists believe that certain "value-free" concepts should be taught to children in order to create workers that will maintain a competitive, hierarchical society. However, when students participate in a system that places the teacher as the knower and portrays mathematics as "somehow pure, abstract, and value-free" (Che, 2005, p. 26), the fact that academic mathematics is socially constructed becomes overshadowed. It becomes mysterious, reserved for certain people, and used "for quantification, reduction, and objectification" becoming a "weed-out or gatekeeper subject" (p. 26). Of course, this is most detrimental to marginalized individuals. When emphasis is placed on teaching this kind of mathematics, those with access to resources are more likely to succeed.

On the other hand, progressive mathematics educators encourage child-centered mathematics, which is intended to capitalize on a child's interest to study the mathematics he/she pleases or to concentrate on the use of manipulatives and group work to allow students to construct knowledge about mathematical concepts that are either abstract or connected only to superficial "real world" situations such as whale weights or pizza (considered "value-free" concepts). However, when progressive mathematics classrooms focus on whatever mathematics the students are interested in, they fail to address critical issues in the child's society and in turn tend to maintain economic structures that keep oppressive social constructions intact. For example, in the 1970s, a series of "open schools," based on Neill's (1960) book, opened across the country. They focused on the idea that the child should dictate the curriculum. It quickly became apparent that these schools were "devastating to children with limited resources" (Winston & Royer, 2003, p. 185). Although children from financially successful parts of society are not formally taught academic mathematics in an open school, "their parents make sure they get what they need" (Delpit as cited by Winston & Royer, 2003, p. 186).

Thus, the open school system gave children from higher up on the economic ladder an advantage to explore in depth the curriculum that would keep them in control, whereas this system hindered children who came from lower socioeconomic status, because they studied what they pleased, detached from a curriculum that could transform their social and economic positions. This system did not permit the resistance of injustice through an open school movement. The economic and social structure was maintained by the fact that curriculum cannot simply be chosen by students in school; culture affects these decisions.

Even when progressive educators shifted focus from a child-centered curriculum that was chosen by the student to a standardized mathematics curriculum that was taught in a child-centered way, mathematics education continued to disregard social problems. Even the reform efforts of NCTM in 1989, which were intended to appease those concerned with progressive ideas that allowed students to choose their own curriculum and those focused on a child-centered approach to learning, did not explicitly address social issues in the curriculum. Although NCTM had developed a set of so-called rigorous mathematical competencies within the context of a problem-centered approach to pedagogy that focused on the child, the curriculum still advocated a traditional, "value-free" mathematics. However, the traditional mathematics curriculum "ignores the culture and history of the oppressed. It operates to homogenize groups into a 'common' culture and does not invite critical inquiry of the social problems that threaten our democracy" (Crockett, 2008, p. 99). Although the focus reverted back to a more child-centered one, it did not address social injustice, violating the ideas of scholars such as Friere (1970) who advocate educating for social well-being.

Neither progressives nor essentialists in the mathematics realm have considered an alternative view to teaching mathematics. The effects of Eurocentrism in mathematics are still relevant today. As stated in Chapter One, Ezeife notes that "[o]ne of the reasons advanced for the high dropout rate and poor performance in examinations by the few aboriginal students who enrol in mathematics or science is that mathematics and science taught in school is bereft of aboriginal cultural and environmental content" (2002, p. 177). And Davison has said, "American Indian students' capacity to learn mathematics is influenced by language, culture, and learning style. However, the methods by which mathematics is typically presented do not take into consideration these factors" (as cited by Ezeife, p. 177). Academic mathematics continues to devalue diverse ways of practicing the subject, maintains its status as a universal subject, and still masks the fact that it was established and legitimized by one kind of people which ultimately sustains those people.

Ethnomathematics Research

As reform efforts in mathematics education continued to fail, scholars began noticing the effects of Eurocentrism in mathematics on people outside of the mainstream culture. They recognized that reform efforts did not address these concerns and did little to create meaningful change in mathematics teaching and learning. "Ethnomathematics emerged as a new conceptual category from the discourse on the interplay among mathematics education, culture, and politics" (Powell & Frankenstein, 1997, p. 5). It materialized when the mathematician Ubiratan D'Ambrosio began to view "much of the history and philosophy of mathematics, as well as of mathematical cognition, redundant and biased" (2002). He wanted "to look into different ways of doing mathematics, taking into account the appropriation of academic mathematics by different sectors of society and the way different cultures deal with mathematical ideas" (D'Ambrosio) in order to combat a Eurocentric system that maintains an unjust hierarchy both socially as well as economically. However, since his introduction of the word, two dominant definitions of the term have led research efforts in this field.

The first definition described ethnomathematics as "the study of the mathematical ideas of nonliterate peoples" (Ascher & Ascher, 1986, p. 125). This characterization intended "to challenge Erurocentric historical and anthropological notions about the locus of mathematical ideas, including pernicious statements in the mathematical literature concerning the value of the mathematical ideas of nonliterate, non-Western peoples" (Powell & Frankenstein, 1997, p. 6). Ascher and Ascher pointed out that "most statements about nonliterate peoples are usually (1) in preliminary chapters in histories of mathematics or in texts on the spirit of the subject, and (2) theoretically and factually flawed" (Powell & Frankenstein, p. 6). Nonliterate people are thought of as inferior to or not intellectually capable, often referred to as "uncivilized." However, Ascher and Ascher (1986) argued that the mathematics practiced by nonliterate people is as abstract and complex as current Western mathematics. The idea was to broaden historical considerations of mathematics and include multicultural contributions to the field (Powell & Frankenstein).

However, the second definition of ethnomathematics, and the one this section most heavily focuses on, encompasses an even broader perspective. D'Ambrosio, the so-called "father of ethnomathematics" pointed out

> that the belief in the universality of mathematics can limit one from considering and recognizing that different modes of thought or culture may lead to different forms of mathematics, radically different ways of counting, ordering, sorting, measuring, inferring, classifying, and modeling. That is, once we abandon notions of general universality, which often cover for Eurocentric particularities, we can acquire an anthropological awareness; different cultures can produce different mathematics and the mathematics of one culture can change over time, reflecting changes in the culture. (as cited by Powell & Frankenstein, 1997, p. 6)

More than the first definition which attempts to expand the history of mathematics by including the contributions of nonliterate people, this description calls for a reassessment of what is considered mathematics by understanding the mathematical practices of various societal groups, including people from non-Western as well as Western societies, who use a mathematics that is different from the traditional academic form found in the classroom. Academic mathematics that is produced and distributed as the only kind of mathematics should exist alongside discussions of other forms of mathematical practice. To talk about mathematics must include "the mathematics which is practiced among identifiable cultural groups, such as national-tribal societies, labor groups, children of a certain age bracket, professional classes" (D'Ambrosio, 1985, p. 45) and academic mathematicians. This designation enlarges the participants included in this field of study. Moreover, it expounds the notion that the role of culture should not be ignored when discussing the production of mathematical ideas. Just as music and art are cultural productions, mathematics stems from contexts entrenched in tradition and belief.

With the introduction of D'Ambrosio's definition of ethnomathematics came the need for research that reassesses what counts as mathematical knowledge. New questions arose about the nature of mathematics—questions such as:

> Are the mathematics found in different cultural processes and artifacts different mathematics or different manifestations of one universal mathematics?...Further, do we miss truly different mathematics because we examine different cultural traditions through the lens of academic mathematics? (Powell & Frankenstein, 1997, pp. 321-322)

Although scholars continue to investigate these questions, the pursuit of understanding ethnomathematics of cultural groups initiated an effort to bridge the gap that exists between academic mathematics and practical mathematics.

Ethnomathematical studies began by examining the categorization of mathematics into abstract versus practical. They found that labeling school mathematics as something different from the mathematics that is practiced outside of the classroom contributed to students' frustrations about mathematics and created the false sense that they do not know how to do mathematics when in their natural endeavors they may contemplate very abstract concepts (e.g., Spradbery, 1976; Haris, 1987). Spradbery conducted a study in England in which he worked with sixteen-year-old students who had not succeeded in elementary mathematics courses. After receiving remediation ineffectively, these students "left school 'hating everyfink what goes on in maffs'" (p. 237). Yet, Spradbery reports that in their free time these same students "kept and raced pigeons…Weighing, measuring, timing, using map scales, buying, selling, interpreting timetables, devising schedules, calculating probabilities and averages…were a natural part of their stock of commonsense knowledge" (p. 273). Although in the school setting these students could barely perform elementary arithmetic, in their natural settings they could do very complicated academic mathematics but viewed it as something distinctly different from the abstract concepts they were taught.

Similarly, Haris (1987) revealed how commonsense mathematical knowledge used traditionally by females is often unacknowledged as mathematical when in fact it may contain as much mathematical content as engineering, for example. Haris explained the similarity between knitting the heel of a sock and configuring a right-angled cylindrical pipe. Both are ethnomathematical, but traditionally one is thought of as knitting and having nothing to do with mathematics while the other is considered an engineering mathematics problem. Inherent in these findings is the idea that "sexism also underpins the dichotomy between 'school' mathematics and one's stock of commonsense knowledge and perverts what counts as mathematical knowledge" (Powell & Frankenstein, 1997, p. 194). The Eurocentric bias that exists in this dichotomy devalues the mathematical knowledge of cultures outside of the traditionally successful European-male one.

Ethnomathamticians, however, did not stop there. Shortly after researchers embarked on studying the ethnomathematics of cultural groups such as students who race pigeons or women who knit, scholars started to examine the implications of ethnomathematical curriculum for shattering the dominant hierarchical culture and empowering students in the classroom. Naturally, ethnomathemtical research shifted to school mathematics, and questions began to formulate about the role of ethnomathematics in education, since the lack of cultural sensitivity in traditional mathematics curriculum sparked a formal field for ethnomathematical research in the first place. Efforts set in motion a focus on student empowerment by understanding the mathematics developed by students through dialogue and writing (e.g., Powell & Lopez, 1989). Moreover, researchers began considering how to embrace the mathematics of various cultures in the classroom without undervaluing it and creating the false sense that other mathematics is an introduction to or subset of real mathematics (e.g., Zaslavsky, 1991). The goal became to create confidence in students through culturally relevant pedagogy that would ultimately lead to "action against oppression and domination" (Powell & Frankenstein, 1997, p. 327).

Students have traditionally had little influence in the classroom. In the late 1980s Ethnomathematicians began to question the traditional roles of teaching and learning and continue to seek ways to challenge these norms. For example, Frankenstein (Frankenstein & Powell, 1989) used interviews with her students as well as dialogue to encourage students to understand their own ethnomathematics. They were asked to explain their reasoning and communicate with other students as well as the instructor in order to break down the dichotomy between the role of teacher and learner and give power and authority to all participants in the classroom. Powell, Jeffries, and Shelby (1989) utilized writing through journaling. Similar to Frankenstein's ideas about communication, these writing activities prompted students to analyze their own mathematics as well as reflect critically on the methodologies of teaching and learning. Further, students critiqued the pedagogical techniques of the instructors and mutually studied the journals of their classmates. Students' voices became valued, heard, and considered.

Others empowered their students by creating curriculum that acknowledged significant mathematical contributions of cultures outside of the mainstream European one. In Mozambique, Gerdes (Powell and Frankenstein, 1997) used baskets, fish traps, and other traditional Mozambican artifacts to teach Geometry and illustrate the idea that mathematical understanding exists inherently in Mozambican culture. After his students discovered the Pythagorean Theorem, by studying the construction of a woven button used to fasten the top of a basket, they recognized that

> Had Pythagoras—or somebody else before him—not discovered this theorem, we would have discovered it!...Could our ancestors have discovered the 'Theorem of Pythagoras'? Did they?...Why don't we know it?...Slavery, colonialism… (as cited by Powell & Frankenstein, p. 253)

Gerdes opened up a space for his students that allowed them to gain an awareness of the mathematics that Mozambican culture has utilized. Moreover, they began to understand that Pythagoras may have taken credit for the theorem, but historical factors aided him. Students became aware of the fallacy that awards superiority to one culture over others.

Educators continue to contemplate and develop a role for ethnomathematics in the classroom. In this pursuit, their research has mostly utilized qualitative research, such as case study (Powell & Lopez, 1989) and ethnographic designs (Lave, 1988). Qualitative versus quantitative approaches have held more weight in this field because of the unknown nature of this unsystematic approach to curriculum—ethnomathematics in the classroom does not seem to lend itself to the formation of a null and alternative hypothesis. Researchers have sought description rather than prediction and refused modern scientific approaches. They have utilized observation, interview, and other non-traditional scientific forms of data collection. "To give authority to the voices of students and to incorporate their perspectives in transforming mathematics pedagogy" (Powell & Frankenstein, 1997, p. 325), mathematics education research that attempts to understand the nature of ethnomathematical pedagogy "must begin by listening to students and finding authentic ways to incorporate students' perspectives into" (p. 325) the research. Students themes have been organized and new themes emerge that hope to shatter "the commonly held myths about the structure of both society and knowledge and that interfere with critical consciousness" (p. 325). Experimental approaches seem incapable of giving the detail necessary to accomplish this feet. Nevertheless, world conditions illustrate that there is still a lack of critical awareness and social action in Western culture; therefore, ethnomathematicians continue to search for ways to conduct research and implement new ideas into mathematics curriculum that will create the change necessary to sustain a just world for all.

Social Justice Mathematics

Although ethnomathematical research aided in uncovering the mathematics of different cultural groups, formulated ideas for instruction of culturally relevant mathematics, and promoted ways to build self-confidence in students, "there is no confirmation that this knowledge results in action against oppression and domination" (Powell & Frankenstein, 1997, p. 327). More recent efforts to dissipate the destructive structures of society and encourage social action have emerged in the form of a social justice approach to teaching mathematics (e.g., Gutstein & Peterson, 2006). However, social justice pedagogy takes a slightly different path towards emancipation. Unlike its ethnomathematical counterpart, which has aided in the development of non-Eurocentric mathematics, social justice mathematics is a pedagogical approach that utilizes the idea of counter-hegemony to challenge the dominant culture.

Hegemony, coined by the philosopher Antonio Gramsci, can be understood "as the process by which dominant groups establish the legitimacy of their version of reality throughout society" (Che, 2005, p. 22). Traditional mathematics curriculum that maintains social and economic hierarchical structures is one way that hegemony manifests itself in Western culture. The mathematics that is now believed by many to be the one and only universal mathematics and its delivery to students has perpetuated the Eurocentric bias that aids in the legitimizing of domination. Ethnomathematicians have attempted to combat hegemony in mathematics by understanding and introducing other forms of mathematics to the history and curriculum of the field. However, a counter-hegemony approach to teaching mathematics is slightly different. Within this context, the traditional, universal curriculum reveals the agenda of the dominant group and instead turns Mathematics into a "tool to understand and potentially change the world" (Gutstein & Peterson, 2006, p. 2). Understanding traditional mathematics as a means to reading the world can give power to marginalized students and help students from the dominant culture recognize injustice so that social and economic disparities may dissolve. In other words, a social justice mathematics curriculum can be defined as one that:

- helps students develop positive dispositions towards mathematics
- promotes social awareness
- poses questions that help students address and understand social problems and the "forces and institutions that shape their world" (Gutstein, 2003, p. 40), which in turn influences students to "pose their own questions" (p.40),

- advocates "*writing* the world" or creating "a sense of *agency*," in which students see themselves "as people who can make a difference in the world, as ones who are makers of history" (p. 40),
- helps students "develop positive social and cultural identities by validating their language and culture" and helps "them understand their history" (p. 40),
- helps students develop positive dispositions towards others by validating other groups' cultures, contributions, and histories, and
- questions the role of teacher and learner.

Rather than struggle against the curriculum, some educators saw a social justice approach to teaching mathematics as a means by which school mathematics can reveal social and economic disparities. For example, Steele (2006) taught a high school accounting course in which sweatshops were examined. In a lesson entitled "Sweatshop Math," as the students began learning what the textbook called "the language of business," including terminology such as "profit," "net profit," and "bottom line," they investigated the human consequence of businesses' focus on the "bottom line" or "total sales minus total expenses" (p. 54). They were shocked to learn that for each $140 Nike shirt produced in El Salvador, workers who sewed the shirt got paid 29 cents. They examined a graph depicting all the costs that make-up a $100 Nike shoe: $50 to the retail store, $13.50 to the brand company, $11 for research, $8.50 for advertising and publicity, $8 for materials, $5 for transportation, $2 to the factory, $1.60 for production costs, and $0.40 for factory worker wages. The students also studied working and living conditions for the factory workers. The lesson initiated a conversation that included comments such as

> If somebody takes a job its their own choice. It must be better than what they were doing before… They could pay the factory workers twice as much and it would barely dent the shoe company's share… Reduce the retail store's costs… Your paycheck would be a part of the retail store's costs. How would you feel about taking the cut in salary? (p. 54)

Many of the students echoed messages they received from society. Nonetheless, they began to contemplate the injustice embedded in business tactics. Although Steele (2006) did not create a different curriculum than the one the text intended for the students, he instead used traditional mathematics as a tool to help reveal the effects of a system which conducts business in the traditional sense of the word.

Other educators taught school mathematics which revealed the inequity that brought about historical events and thereby empowered their marginalized students. Andrew Brantlinger (2006) taught a summer geometry course to a group of students, from Chicago's north side, who had failed the subject during the regular school year. Eighty-five percent of the students were classified as low-income and the majority where not European-American. The objective of one of the lessons was learning area of a circle, which included understanding radius. Brantlinger approached the topic with a discussion of Rodney King and the riots that took place in South Central Los Angeles (L.A.) in 1992. The students estimated and predicted the number of liquor stores, community centers (field houses in city parks, Boys and Girls Clubs, YMCAs, etc.), and movie theaters in South Central L.A. in 1992. They were told that South Central L.A. covered an area with a radius of three-miles. They had to compute square miles and square blocks for the area, without the use of a formula, and they were permitted to use data from a suburb of Chicago, Evanston, which was one-third the size of South Central L.A., to make their predictions. Evanston had nine or 10 movie theaters, 26 community centers, and 27 liquor stores. When students discovered that there were 640 liquor stores, no community centers, and no movie theaters in South Central L.A. in 1992, they responded with comments such as, "What?"… "All they want them to do is drink"… "That's why they be on the streets" (p. 99). Students began to understand that unequal conditions existed in their society, even though often times they were masked by prejudice, pinning acts of hostility on groups of individuals versus social conditions that might prompt people to rise up. Although the lesson utilized "traditional" mathematics concepts, it connected the ideas of area and radius to helping students understand that their social and economic discrepancies were not due to any deficiencies on their part.

Moreover, social justice mathematics also initiated social action by some educators and their students. "In an overcrowded New York middle school, students discovered that math was a path to investigating and working to change conditions at their school" (Turner & Font Strawhun, 2006). Beatriz Font Strawhun taught mathematics in a predominately African-American, Dominican, and Puerto Rican working class community. During one unit that focused on linear and area measurement, ratio, operations with fractions, and mixed numbers, Font Strawhun had her students brainstorm concerns they had about their school or community as an avenue for teaching mathematics that would be relevant to students' lives. After considerable thought, she decided to utilize the topic of overcrowding in the school. Font Strawhun developed several mini lessons that addressed the mathematics concepts such as finding areas of spaces with fractional dimensions or understanding ratios to compare hallway space at this school to hallway space at a more affluent school in the community. "As the class continued to analyze overcrowding at their school, they discovered disparities between their own space and that of other schools, and numerous instances where their school violated district building codes" (p. 83). However, Font Strawhun wanted her students to share their information rather than just understand the discrepancies they found. They distributed flyers, visited the school board, and created floor plans of the school to share with the district. After a district meeting in which students argued that their school either needed to be made bigger or have fewer students, one student remarked that in this class "we did something with it [mathematics]…Without the math, then, we wouldn't have the area of the school, and we wouldn't really know. And the [district] meeting wouldn't have been as powerful as it was" (p. 86). Mathematics became meaningful and necessary. In order to create a plan of action, students used traditional mathematics to attain the proof they needed to make their case. The following year, thirty fewer students were allowed to enroll in the school than initially planned for.

Eurocentric superiority, aided by the events of history, has created a hierarchical structure in which social and economic disparities exist. Mathematics, often thought of as universal and culture-free, has generally been excluded from contributing to domination in Western culture. However, a look at ethnomathematics throughout history reveals that this false notion of mathematics has assisted in the perpetuation of unequal social conditions. Ethnomathematicians have continued to find ways to uncover the significance of non-European cultures to the development of academic mathematics as well as other forms of the subject. They attempt to debunk the idea that European males have been the advanced scientific thinkers of our culture, and that all others should follow their path in order to attain success. In the classroom, they have sought student empowerment by legitimizing student voices and introducing multicultural approaches to mathematics as well as other types of mathematics. Although their focus has been on the marginalized, they paved the way for teaching mathematics for social justice and prompted educators to explore alternative avenues for emancipating socially and economically imprisoned individuals and educating privileged members of society about the ramifications of historical outcomes.

Preservice Teachers

Although teaching mathematics for social justice has opened up possibilities for liberation, few resources exist that can help mathematics teachers become aware of current social conditions and prepare them to deal with creating meaningful change through and for mathematics. Preservice elementary teachers, who often times bring misconceptions or limited understandings of academic mathematics into their college careers (Ball, 1990), are often invited to explore mathematics through problem-solving and within only superficial "real world" contexts (i.e., teaching proportional reasoning with pizzas or candy bars), however, rarely are these problems situated within meaningful social contexts. This is problematic because it is vital that teachers enter their profession prepared to "help all students acquire the knowledge, attitudes, and skills needed to participate in cross-cultural interactions and in personal, social, and civic action" for a more just world (Cochran-Smith, 2004, pp. vii-viii).

However, engaging preservice teachers in mathematics for social justice poses more than one obstacle for mathematics teacher educators. The obstacles they face include but are not limited to both facilitating the development of mathematical understanding and providing space for the extension of social understanding. These can be challenging, as preservice teachers often enter their college preparation courses with misunderstanding about and a limited amount of mathematical content and social issue knowledge (Ball, 1990; Sleeter, 2001).

Elementary preservice teachers' high anxiety levels and negative emotions towards academic mathematics have been well documented for some time now (Fisher, 1992). Many of them are apprehensive about teaching the subject and often hope to teach lower grade levels because of their self-perceived lack of mathematical content knowledge (Ball, 1988a, 1988b). Preservice teachers often view themselves as nonmathematical thinkers and incapable of doing mathematics (Powell & Frankenstein, 1997), a result of the traditional algorithmically focused training they encountered in school (Ball, 1990). As is portrayed by the Eurocentric formation of mathematics in Western culture, preservice teachers tend to view mathematics as absolute and dualistic, rule-bound and procedural, rather than logical and meaningful (Schiftner & Fosnot, 1993). They perceive mathematics differently than other subjects, viewing it as more rigid and containing either right or wrong answers and no ambiguity (Benbow, 1993). They often times enter college mathematics classrooms with the idea that in order to succeed in such a setting, they must search for prescribed algorithms and memorize formulas (Ball, 1990). Moreover, they often believe that people who succeed mathematically have a gift or innate ability to understand the subject (Frank, 1990). These attitudes and beliefs are deeply embedded within the culture and their schooling experiences. Overcoming these ideologies has been a major concern of teacher education reform in mathematics for several decades (Civil, 1990).

Further, preservice teachers often exhibit low levels of autonomy in the mathematics classroom. They rely heavily on external authority for validation, and have little confidence in themselves as mathematicians (Dupree, 1999). They search for one right method for finding one correct answer and view themselves as unable to verify the validity of methods they create and process. They often want to rely on the instructor to show them the correct method or provide them with the right answer (Chazan & Ball, 1991). They find it difficult and unnecessary to engage in mathematical discourse about different ways of solving problems, from multiple perspectives, when they believe that there is only one way. Therefore, they tend to not engage in the process of searching for various methods to verify their own answers (Ma, 1999).

However, preservice teachers not only come into their college training programs with preconceived notions about mathematics but also with already developed perceptions of society. Preservice teachers in the United States tend to be predominately European-American and often have little cross-cultural background or experience (Sleeter, 2001). In 2007, it was reported that 86% of elementary and secondary teachers were European-American (United States Department of Education). The majority of these White preservice and in-service teachers enter their classrooms with a view of society that stems from their cultural perspectives (Gay, 2002), culture being defined as "the sum total of ways of living built up by a group of human beings and transmitted from one generation to another" (Cordeiro, 2006, p. 93). Often times, the representations of society that many of them have developed include a propensity towards stereotypic beliefs about diversity and cultural issues that impact individuals outside of the mainstream (Carpenter, 2000). They include many preconceived notions about issues of diversity in society, including gender, religion, socioeconomic status, and politics. They tend to "hold preconceived notions of education and diversity that have developed from a variety of perspectives and values, shaped by past experiences and cultural environment" (Locke, 2005, p. 20).

Moreover, preservice teachers' beliefs tend to be "relatively stable and resistant to change" (Tatto, 1996, p. 157). Much of this resistance stems from their culturally developed idealistic attitude about the United States as a place where equality exists for all (Carpenter, 2000). Therefore, viewing their culture as less-than-perfect tends to create a great deal of dissonance for and resistance from preservice teachers. Research has contended that teacher education programs have limited capabilities when transforming sociocultural beliefs that have developed and become culturally embedded in preservice teachers' attitudes and ideologies about social issues (Haberman & Post, 1992).

Because of the lack of impact teacher education programs have had on meaningful social change and understanding in educational techniques, there is a need for every content area to promote critical citizenship. Although a plethora of research exists on preservice teachers' pedagogical knowledge and beliefs about mathematics, and many studies emphasize preservice teachers' knowledge of social issues, little research focuses on this populations' mathematical content knowledge and understanding (Ball, Lubienski, & Mewborn, 2001), particularly when it is embedded within significant global problems. Therefore, there is a need for educators to look for ways to weave social issues into all content areas of preservice education and explore how preservice teachers construct knowledge about mathematics and social issues when this is done.

Summary

Ethnomathematics has existed since the beginning of human existence. People studied the mathematics of others and assimilated it into their own from the time they began interacting with each other. However, eventually, Europe began to distribute a mathematics that became viewed as universal and the correct form of mathematics. This Eurocentric mathematics came to dominate Western school systems and began to marginalize students who did not come from the mainstream European culture and became the foundation for many of the problems current society faces.

In the early twentieth century, reform efforts began to emerge, and battles between progressives and essentialists ensued. Although some progressives, such as Dewey, recognized the lack of a critical education system that teaches students to become more conscious citizens, neither progressives nor essentialists were able to create meaningful change in a system that was created for and continues to benefit a few and marginalizes many. Maybe Dewey's ideas about linking the child and the curriculum were misinterpreted by progressives. Further, maybe essentialists have not considered the perspective that the mathematics that is most essential in this era is not a mathematics that places our students above others, but rather a mathematics that helps us work together to form a more just world. Regardless, maybe this new century should bring with it a reconsideration of what mathematics is essential for our students and how that mathematics can be effectively taught to them.

If teachers are to begin teaching a mathematics that considers understanding critical social issues for and through mathematics, then preservice teachers need to be prepared to teach this kind of mathematics. They must be exposed to social justice mathematics, and their perceptions of it must be studied. However, no research has addressed preservice teachers' perceptions of learning mathematics content in a social justice mathematics course. Therefore, this research study focuses on contributing to that body of knowledge with an attempt to develop and implement a social justice mathematics curriculum and pedagogy. My goal is to illuminate what factors were involved in the undertaking of such a project in a mathematics class for elementary teachers, with externally imposed mathematical objectives. I offer an analysis of the process assumed by this project. Chapter Three offers the methodology for this analysis.

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CHAPTER THREE

METHODOLOGY

This study had two purposes. They were to (a) illuminate the factors involved in creating a social justice curriculum (i.e., what criteria were used to define issues of social concern, how were social issues incorporated with mathematics so that the mathematical as well as social justice objectives were achieved, what limitations existed when trying to create a social justice mathematics curriculum) and (b) understand students' perception of learning in this way (i.e., what did students think of this approach to learning mathematics, how did they understand mathematics and social issues when they were taught in this way). That is, I give an in-depth illustration of how I, the instructor, went about creating the activities and learning situations for this classroom so that readers may conceptualize how this was done. Further, I re-create the implementation and effects of these activities and situations on the students and myself by providing as much detail as possible so that readers may understand the participants' reactions to and interactions with this particular curriculum.

However, as I engaged in the process of fulfilling the purposes of the study, I wondered how what I do as a mathematics teacher enables inequity and whether or not I may be able to find ways to challenge what I may have contributed to perpetuating. I sought answers by investigating my own environment as a mathematics teacher. As I investigated this particular class, I did so in recognition of the fact that my search was clouded by my own history and judgments.

Therefore, in this chapter, I discuss why I chose to utilize a practitioner research study embedded within a case study methodology, and I include how data collection occurred as well as how I analyzed the data. Further, I provide a brief description of the curricular routines of the class, and I conclude with a discussion of the methodology, procedures, and limitations of this study.

Practitioner-Research

In choosing a research design, one must decide first what the research questions are and second, which design best leads to answering those questions. I chose practitioner-research because in this design, the researcher is the practitioner, as I was in this study. Practitioner-research studies, such as this one, stem from the reflections of the practitioner's experiences from the unique position of an insider, immersed in the reality of the study (Anderson, Herr, & Nihlen, 1994). They examine teaching and learning from the experiences of teachers and students by teachers and students. They allow teachers, who constantly reevaluate and transform their instructional sequences and activities, to reflect on, improve, and voice those practices.

Despite its growing popularity and acceptance as legitimate research, practitioner-research has been greatly criticized. It has been viewed by some as an inadequate form of research.

> The first criticism is that teachers are not properly trained to conduct research and that the research they have conducted has not been up to an acceptable standard…The second criticism, which is based on a positivist view of external validity, is that practitioner research is of questionable value because many studies do not involve the investigation of groups that are representative of larger populations…Third, and apart from concerns about teachers' qualifications to conduct research, were concerns that the demands of teachers' jobs make it

> difficult for them to find time to do research and that, when they do so, their attention is drawn away from their main task of educating students (Zeichner & Noffke, 2001, p. 2).

The primary concern these critics display stems from the following question: Is there a difference between academic research and action research? For them, the answer is yes, academic research is legitimate and action research is questionable in integrity. Despite such views, "there has been growing support for its knowledge-generating potential" (Zeichner & Noffke, 2001, p. 4). In fact, some scholars have argued that there are many advantages to being an "insider" and that "researchers [should] justify themselves to practitioners, not practitioners to researchers" (Stenhouse in Cochran-Smith & Lytle, 2003, p. 19). Cochran-Smith & Lytle have characterized teacher research as "systematic and intentional inquiry carried out by teachers" (p. 7). They believe that through action-research, teachers can significantly improve and strengthen their practices. When teachers engage in practitioner-research, they engage in a systematic, reflective process that requires substantial evidence to support claims. It is a way for teachers to explore some particular action or actions and investigate the results of doing so.

Case Study

I chose a case study approach because it provided the in-depth description and perspectives that could help shed light on what it meant to teach mathematics and social justice in a college level mathematics class for elementary teachers. A case study is one of several ways of conducting research. It focuses on in-depth descriptions of an instance or event from multiple sources of information. Case studies have utilized both quantitative and qualitative approaches; however, this project focuses on the qualitative case study. Case studies are well-known to social scientists because of their popularity across various fields (e.g., psychology, medicine, education, etc.). Cresswell (2007) described this type of research as "a qualitative approach in which the investigator explores a bounded system (a *case*) or multiple bounded systems (cases) over time, through detailed, in-depth data collection involving ***multiple sources of information***" (p. 73). And an intrinsic case study is one in which the case has been preselected because of its unusualness or particular relevance to the researcher (Stake, 1995). This study was an intrinsic case study because of the unique social justice aspect of the class created and investigated.

Although one universal procedure for conducting a case study does not exist, I draw on Creswell's approach for mine.

- Researchers next need to identify their case or cases. These cases may involve an individual, several individuals, a program, an event, or an activity.
- The data collection in case study research is typically extensive, drawing on multiple sources of information, such as observations, interviews, documents, and audiovisual materials.
- The type of analysis of these data can be a ***holistic analysis*** of the entire case or an ***embedded analysis*** of a specific aspect of the case.
- In the final interpretive phase, the researcher reports the meaning of the case, whether that meaning comes from learning about the issue of the case (an instrumental case) or learning about an unusual situation (an intrinsic case). (pp. 74-75)

This methodology allowed me to give an in-depth, longitudinal account of the course, which gave a sharpened image of why instances occurred as they did. It allowed me to investigate the development of, implementation of, and reactions to instructional activities. The idea of the intrinsic case study was not to ensure that others could replicate or reenact classroom events in precisely the same manner. Rather, it was to illustrate one case in-depth so that others may understand what one social justice mathematics classroom emerged as and what the implications of the outcomes of the class might be for other mathematics classroom settings.

My Role

Engaging in practitioner-research required a reconsideration of traditional roles, particularly my own. In the action-research process the teacher and the students are considered learners, collaborators, and researchers who mutually engage in creating and learning from the process. Because of the emergent nature of this type of research, the researcher must continuously engage in the process of hypothesizing, testing, and re-hypothesizing.

The process of forming hypotheses, executing plans of action, and reforming them, was embedded within some fundamental beliefs I had about teaching and learning. From my perspective, students construct knowledge by constantly negotiating and renegotiating new knowledge in relation to past experiences (Piaget, 1972). Much of this negotiation process is affected by social interaction (von Glasersfeld, 1995), an important component in my classroom. The idea is that students bring with them social and cultural histories and backgrounds that interact with the social and cultural histories and backgrounds of those around them. This process allows students to propose new ideas, engage in dialogue about those ideas, and make sense of them. It was from this perspective that I created my identity as a teacher. I viewed my role as teacher not as that of someone who hoped to impose certain knowledge on her students regardless of who they were. Rather, I saw myself as someone who hoped to facilitate the development of problem solving and logical thinking by taking into consideration the socio-political and socio-historical backgrounds of my students. My aim in this class was to allow my students to engage in a transactional (Houser, 2006) relationship with the curriculum, their peers, and the instructor.

From the time I began teaching, it seemed natural to reflect on and refine my own teaching practices, including my own role with students. With the progression of time, I began to take a more deliberate role in creating and executing learning experiences for my students and myself. In this study, I had to become even more aware of my role as teacher, learner, and researcher. I believe it was this consciousness of my role that helped me develop a strong relationship with my students in this class and created an environment that nurtured student comfort and engagement, allowing me to be viewed as someone who was truly interested in their perspectives and who was merely observing what emerged. Although I was still considered the instructor, I believe I was able to form trusting relationships with my students which created a safe and caring environment where they felt free to present themselves naturally rather than in a way that was meant to please me.

Assuming the role of teacher as researcher, however, presented some challenges, some of which could be anticipated and some of which could not. For example, upon beginning the research and teaching process, I knew I would be faced with my own biases and judgments when documenting and analyzing the teaching and learning processes. However, I did not anticipate the amount of reassessing, regrouping, and reworking I encountered while trying to answer my research questions. Taking on the role of creator of classroom curricular opportunities as well as researcher was more difficult than I anticipated. I found myself under much more difficult time constraints than I hoped. Although I felt I had a firm grasp of the mathematical content I was teaching, and I thought I had developed my ideas about integrating mathematics and relevant social issues, I found it very time consuming to create lessons that fulfilled the objectives of the research and the class and to reflect on them during the duration of the semester in which I taught. I knew that the reflection process was necessary, and it existed in my teaching previously; however, in this class, making sure I was addressing particular research questions necessitated a deliberate decision to formulate, reformulate, and learn new ways by which I worked.

Setting

The mathematical, verbal, and cognitive transactions documented in this research project provide valuable insight into what can happen when mathematics is taught in conjunction with social issues. They illustrate what happened to one teacher and her students when they collaborated and transformed their thinking about the learning of mathematics. This class was chosen because of the participants, as preservice elementary teachers, who for the most part have been portrayed by many research studies as students who tend to have misunderstandings/misconceptions or a limited amount of mathematical knowledge (Ball, 1990). The underlying research questions of this study were based on elementary preservice teachers' mathematical knowledge formation of new meanings and shared understandings when mathematics was presented in the way it is described here. The research questions discussed here were:

1. What critical factors were involved in the evolution of a mathematics course that incorporates social justice?
2. What were students' perceptions about learning mathematics in a course that combines mathematics and social issues in the way they are presented here?
3. What were students' perceptions of their understanding of mathematics, social issues, and the relationship between mathematics and social issues when they were presented in this way?

In order to better address these questions, the rest of this chapter first provides a contextual background of the physical setting for the study. It then offers an account of the data collection procedures and analysis, including a brief overview of the course. Finally, it concludes with discussion of the methodology and a summary of the chapter.

The Community College and the Classroom

Situated on the south side of a city in the southwestern region of the United States, the community college in which this study took place opened its doors on September 25, 1972. Initiated by the city's chamber of commerce, the junior college began with an enrollment of a little over 1,000 students and grew to its current size with over 19,000 students on its roster. The college now offers 36 Associate in Arts and Associate in Science degree programs, 24 Associate in Applied Science degree programs, and 18 Certificate of Mastery programs. However, it is not only these programs that draw students to the campus; a variety of reasons bring students here. Although some students are interested in the programs offered, others transfer courses to universities and some obtain skill training in various fields. Attracting students from many cultures, backgrounds, and occupational tracks, this junior college has become a vital part of the city's community.

Within the buildings of the college, classroom spaces vary. Our classroom (see Figure 1) was very spacious, and the layout was fairly inviting: well lit; high ceilings; long tables (rather than desks); and well maintained. The room was also well equipped, containing two large cabinets full of manipulatives, a computer (with internet access), and a *Smartboard* projector (images from the projector or computer were displayed in the center of the dry erase board). The manipulatives in the cabinets consisted of base-ten blocks, fraction pieces, cuisinaire rods, color counters, etc. The space was set up in a traditional way--the tables and chairs were placed in rows, with one table perpendicular to all the others along one wall, and with the instructor's table and chair at the front. The classroom contained 32 student seats. Although in the first few weeks of class, we attempted to rearrange the desks and groupings of students in the class (see Figure 2), after a while, students settled on keeping the desks as they were and rearranging their chairs instead (see Figure 3). (The seats students occupied are black in the figures 2 and 3 below.) In the final figure, students had formed groups that they maintained for the majority of the rest of the semester; although, a few students rotated groups periodically. In particular, the students who spoke Spanish as their primary language always worked together.

Figure 1. Original Classroom Layout

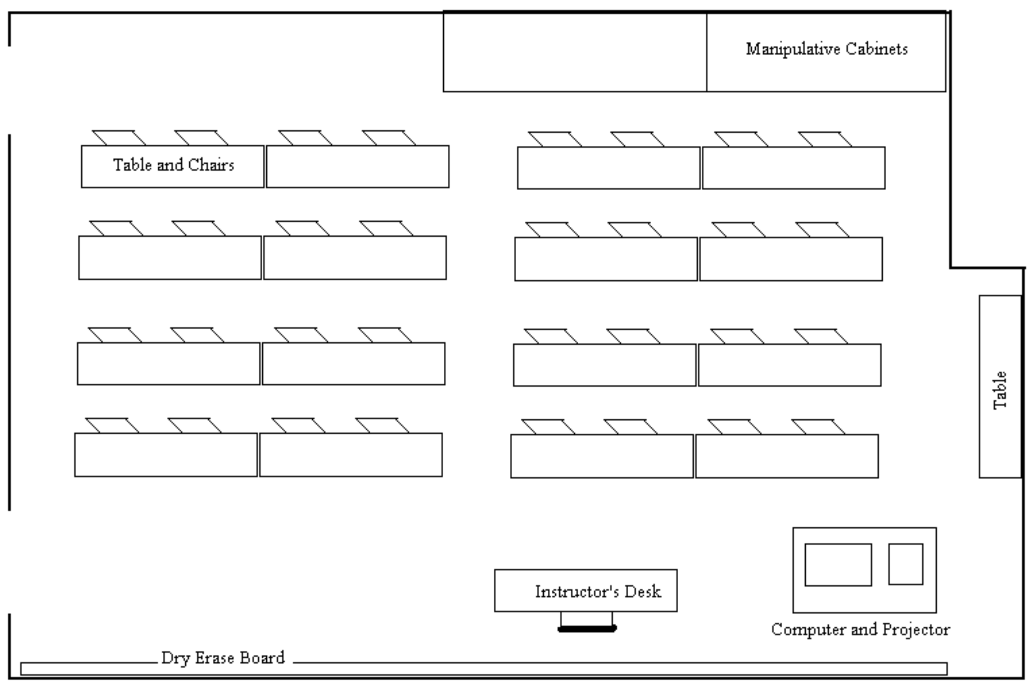


Figure 2. Developing Classroom Layout

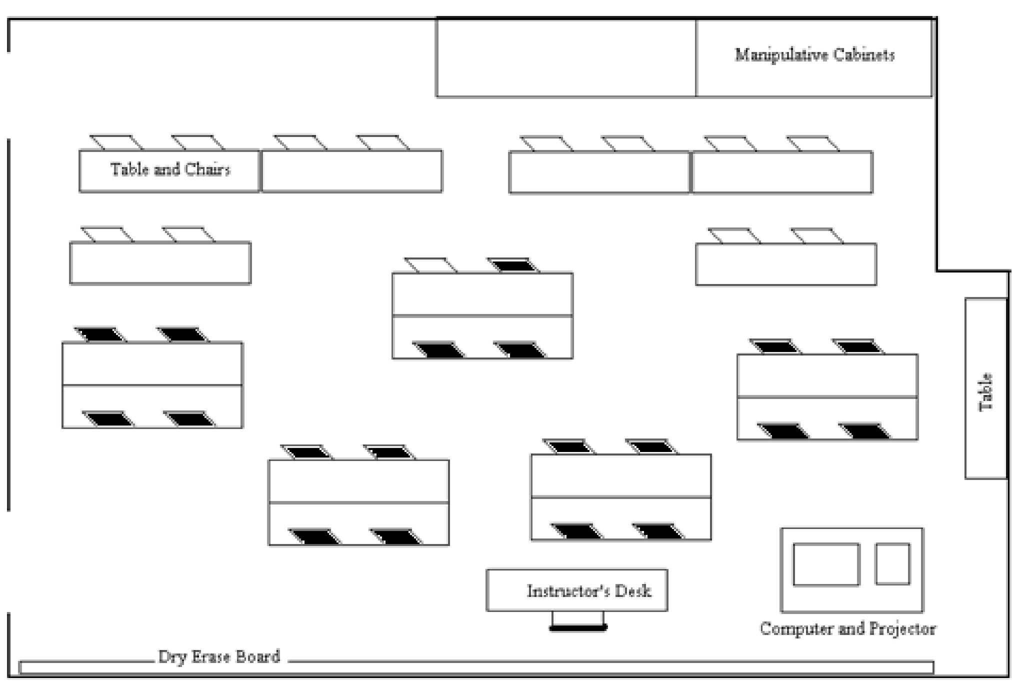
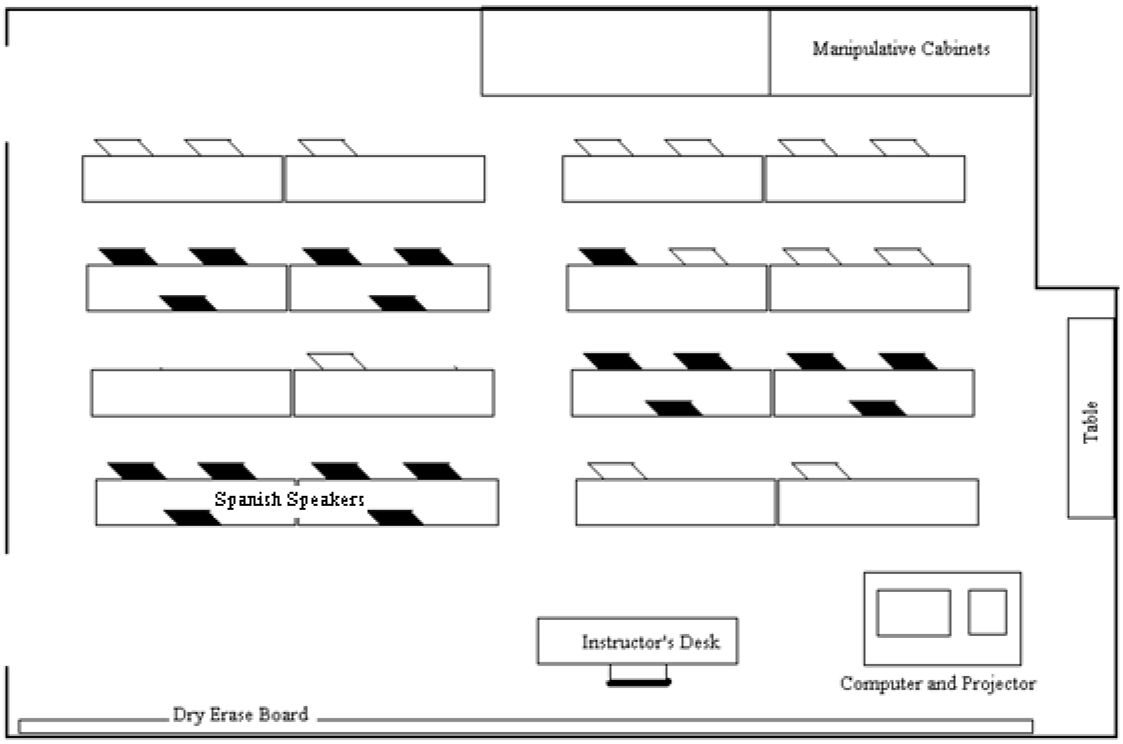


Figure 3. Final Classroom Layout



Participants

All nineteen students in the course were female, several had children (12 with children and one pregnant with her first child), and many worked outside of the home (no exact figure available—I only knew this from conversations with and between student). They ranged in age from the youngest at eighteen years to the oldest at forty three years (see Table 1). Many of the students were non-traditional students, returning to school after several years of being away. However, all of the students had at least one pre-requisite college mathematics course or had passed a placement exam prior to entering the course. About half of the students had taken at least one mathematics class within the last two years; for the rest, it had been a number of years since their last mathematics course (see Table 2). Seven of the nineteen students were English language learners—all but one spoke Spanish as their native language; the other spoke Korean.

Table 1: Age Range

Age Group	Below 20	20-29	30-39	40-49
Number of Students	2	7	8	2

Table 2: Last Mathematics Course

Number of years	< 1 year	1-2 years	3-5 years	6-10 years	No response
Number of Students	7	4	4	2	2

As the practitioner-researcher, I too was a participant in the study. Upon beginning the study, I had been teaching for six years. All my teaching experience was at the college/university level. This was my first semester to teach at this community college; however, I had taught several mathematics courses at two universities in the state. I had also taught an elementary mathematics methods course at one of those universities. Most of my teaching experience was as a graduate student.

I completed my Bachelor's of Science in Mathematics in 2003 and immediately proceeded to graduate school. Upon completing my Master's of Science in Mathematics in 2006, I began my Ph. D. in Mathematics Education. I was twenty-eight years old, and younger than most of my students, when I began teaching this course.

Being bilingual, female, a student, and a working mother myself, I felt I shared many commonalities with my students. These connections formed a strong bond between us. Although I did not share neither Hispanic nor Korean roots, and I was aware that sociocultural, educational, economic, and historical background differences existed between my students and myself, I did not feel this prevented me from empathizing with their experiences. Further, I believe that being in the role of practioner-researcher allowed me to interact with the students in ways that gave me deeper access to their lives than might have otherwise been the case.

Data Collection

This study aimed at assessing the effects of using social issues and mathematics in conjunction with one another in an undergraduate mathematics course. It included the students enrolled in the class as well as myself. Data were gathered using qualitative methodologies throughout the semester. Collecting the data, documenting it, and analyzing it all interacted and overlapped as an ongoing and evolving process. This study being a practitioner-research study embedded within a case study designed allowed for the emergence of various forms of data collection and analysis. The study took shape and changed throughout the process as new discursive and interactive patterns formed. I used case study (Stake, 1995) techniques to collect data for this study. Along with my own field notes, my reflective journal, student journals, and online discussions, I used videotapes and audiotapes to systematically record classroom sessions and conversations during several class sessions and *mid-term conferences* (discussed further later in the chapter). I identified the "case" as the mathematics classroom, which included all the participants, and its response to a social justice approach to mathematics curriculum and pedagogy. It was a bounded system, bounded by time (the duration of the semester) and place (situated in a single classroom). However, the curriculum and conversation that took place within was unbounded and evolving.

Beginning in late August of 2009, I began to record observations of interactions between and among students, the curriculum, and myself in a reflective journal. I structured formal time to write notes during and after the discussions and intermingling that took place during our classroom routines and non-routines. (In the next section, I discuss classroom routines further.)   I used several sources of information in data collection to provide a detailed description of the classroom response to social justice mathematics.

Along with reflections on my part, I focused heavily on journal entries collected from the students, as well as online discussions students took part in outside of class time. Students maintained a journal throughout the semester. In it, they included all mathematical work they completed for the class along with reflections about the course, all of which I would direct them to do periodically throughout the semester. Mathematical journal entries including solving problems provided in the *Journal Activities* for the week, which are described in detail in the next chapter. Reflective journal entries included responses to questions such as: How did you feel about mathematics when you entered this course? Describe some of your previous mathematical experiences? Have you ever used mathematics outside of a classroom setting? If so, how? What did you learn from this assignment? Have you learned anything in this class so far? If so, what did you learn, and what helped you learn it? What did you think of the lessons that incorporated social issues? Please be specific. How do you feel about your interactions with the other students in the class? What about the instructor? Students also engaged in discussions about lessons online, through *DesireToLearn*, the school's online course companion. Discussion forums were created for students each week. They could login, view posts, and write their own comments. These were used as data, as well.

I also videotaped several class periods. Although video recordings of various class sessions were taken, I focused most heavily on documenting those class periods that explicitly addressed social justice issues in the curricular assignments. Each class (about two and a half hours) that included the discussion of social issues was video recorded. Periodically, I was only interested in parts of a particular class period, so recordings ranged in length from ten minutes to the full length of the class period. Video tapes were used for accuracy and a comprehensive account of interactions between and among students, the curriculum, and the teacher. In all the class sessions, social issues were embedded within the interactions of the students and myself; therefore, I decided to not only record the classes where we discussed social issues but also some other meetings because I felt the *hidden curriculum*—the social issues embedded within our interactions (Anyon, 1979)—would emerge during those times, as well.

Moreover, at about the midway point of the semester, I scheduled *mid-term conferences* with all the students, most of whom met with me in groups, to discuss the course up to that point. I met with students in small groups to converse about the curriculum, the social structure, their interactions with the two, and their progress in the course. I asked students questions such as, "What do you think of this course so far? Have you learned anything in this course? If so, what? Are there any particular aspects of the course you would like to discuss?" Although most of the conferences began with some focused questions, most of them turned into discussions that developed and emerged naturally rather than from a particular question. These were audio-recorded and later transcribed to be included as part of the data.

Data Analysis

Data for this study were analyzed using an interpretive and descriptive framework. Analysis of this data came heavily from journal entries from students as well as instructor, online posts, and video/audio tapes. I searched for common groupings and tendencies as I sifted through the data. Throughout the semester, as I collected these data, I continuously reflected on it and mentally analyzed what was happening so that the results could begin to take shape for me, and so that I could cycle through and look back at what had taken place, what was taking place, and what was to come.

The more formal phases of analysis began after the semester concluded as the videotapes, audiotapes, online posts, student and instructor journals, and field notes were examined for the establishment of patterns. I first did an exploratory read of student as well as instructor reflective journals. I wanted to get an overall picture of what student and instructor perceptions of the course were. I made anecdotal notes, and followed up with several subsequent readings. As I recursively read through journal entries, I began to develop categories and trends. At the same time, I began to view and listen to the video/audio tapes for the semester. A secondary set of field notes was established directly from them. The recorded video and audio tapes were transcribed. Documentation of my observations was recorded in a log using anecdotal records. I recorded the different patterns of interactions that occurred when social issues and mathematics were intertwined. I also recorded my interpretations of these observations in my reflective journal. Each student was given a psudename preceding this process, for identification proposes throughout the process. These pseudonames were used to form individual data records that were annotated and placed in the corresponding category of analysis. They allowed me to create a data record that could easily be associated with each student, in case I needed to refer back to original data.

As I recursively cycled back through the data, after the initial explorations and theme creations described above, I began to identify answers to my research questions. I followed up with organizing a summary of the data and checking to verify that all the research questions had been answered. I then cycled through the data one more time for a final review and analysis. The idea was to verify previously constructed themes and to extract examples to be used in the description of the findings.

The first theme of analysis, *Paradox and the Formation of a Social Justice Mathematics Classroom*, addressed the first research question: What critical factors were involved in the evolution of a social justice mathematics course for elementary teachers? This theme emerged from my reflective journal, online posts, recorded video and audio, student work, and field notes that captured the interactions and classroom incidences that prompted me and challenged me to create the curricular materials and learning situations I chose to engage my class and myself in.

The second theme of analysis, *Enjoyment of Learning Mathematics for Social Justice*, addressed the second research question: What were students' perceptions of learning mathematics in a classroom that combines mathematics and social issues in the way they are presented here? This theme emerged from video recordings of classroom discourse, audio recordings of *mid-term conferences*, student journal reflections, and my own journal reflections. All of these sources addressed students' discussions and thoughts about participating in this class.

The third and fourth themes of analysis, *A Critical Perception of Mathematics* and *A Critical Perception of Social Issues and Their Connection to Mathematics* addressed the third research question: What were students' perceptions of their understanding of mathematics, social issues, and the relationship between mathematics and social issues when they are presented in this way? For this theme, I analyzed student reflective journals, my reflective journal, video recordings of the class, and audio recordings of *mid-term conferences*. I examined those materials that depicted students' attitudes and comments about their interactions with and perceptions of mathematics and social issues.

The four themes presented here centered around the transactions that occurred in a social justice mathematics course for elementary teachers. These events emerged from the role of social justice mathematics and the curricular routines students engaged in. In the following two sections, I first examine the role of social justice mathematics in this class, and then I describe the curricular routines of the class.

The Role of Social Justice Mathematics

Treated as a gatekeeper subject, mathematics has often prevented students from overcoming educational, economic, and social barriers. At one point in history, reading literacy was viewed in the same light and became seen as a civil right—a tool for understanding and changing the world. Mathematics literacy can also be viewed as a civil right (Osler, 2007). Mathematics literacy, as a civil right, includes "[l]essons and activities that increase students': math literacy; problem solving, reasoning and critical thinking abilities; ability to apply knowledge and skills; sense of themselves as mathematicians; knowledge of the math in their own culture; ownership of learning process; preparedness for math-based college majors and careers; etc." (Oslar, p. 3).

In light of this view of mathematics literacy as a civil right, social justice mathematics manifested itself in two ways in this class. I (a) incorporated issues of social and economic justice in the mathematics curriculum, and (b) reconsidered the traditional social structure of the mathematics classroom, as power relationships in society are a major social justice issue. Social issues such as poverty, healthcare, consumerism and sweatshops were incorporated into *Journal Activities* (described below). Moreover, students chose many of the social issues we investigated for and through mathematics. They took on the roles of teachers and researchers, not just learners, while I engaged in the role of learner, not just teacher and researcher; power relationships were constantly rearranged (see description of small-group and whole-class discussions below).

Although students controlled many of the social issues that were incorporated into lessons, and much of the mathematics that emerged was driven by them, I faced strict curricular guidelines as the instructor of the course. Another instructor in the mathematics department had developed the course in the semester before I taught it. Aware of the fact that I wanted to incorporate ideas of social justice into the curriculum, she gave me freedom to do so; however, under fairly strict guidelines (see Appendix A). I was given a list of mathematical concepts that I had to address each week. I was informed that in this course student would be expected to emerge with a solid understanding of the real number system, operations, properties, patterns, sequences, and functions. Therefore, I faced an added challenge of trying to incorporate the social interests of students' within the mathematical restrictions that were externally imposed on us.

Curriculum Routines

The participants in this study brought to the classroom already formulated understandings and conceptions about subjects, including mathematics, social issues, and their interaction with the world around them. These understandings then interacted with the perceptions of others in the situational and conversational setting of the classroom (von Glasersfeld, 1995). In this context, new meanings and understandings about mathematics and other topics were regenerated and recreated. This construction of new knowledge occurred within the curricular routines of the class. The class met once a week (on Thursday evenings) from 5:30 p.m. to 8:00 p.m., for fifteen weeks. During that time, classroom routines generally (but not always) followed the pattern described below:

- Closed-Book Journal
- Snacks
- Small-Group Discussion
- Whole-Class Discussion
- Introduction of New Journal Activities

These curricular routines emerged as vital components to the construction of meanings and knowledge in the class. They are discussed in more detail in the sections that follow.

Closed-Book Journal

These one-page reflections of daily assignments included answering one or more questions about the topics discussed in class or in homework assignments. Administered at the beginning of class, students completed and handed them in within ten minutes of class time. I read and reflected on them, returning them the next class period. In the comments I included I would ask questions I wanted students to elaborate on. Students kept all of these in their *Journals* and return them with any necessary corrections at the end of the semester. I informed students that if they answered all the questions I asked them to and returned all *closed-book journals* to me at the end of the semester they could theoretically obtain a "perfect" score for their *closed-book journals*.

At the end of the semester, I asked students to complete an extended *closed-book journal* entry that consisted of reflecting on mathematics content questions from the various sources of material encountered throughout the semester. It also included several questions about the way the course was conducted: How do you feel about the way this class was conducted? What were some specific strong points or moments of the course for you? What were some specific weak points or moments of the course for you? Do you think you learned anything in this course? If so, what did you learn and what helped you learn it?

Snacks

In an attempt to create a relaxed and trusting community atmosphere, I asked students to sign-up to bring snacks to class each week. Two students signed-up for each week. I brought snacks three times during the semester. I wanted to encourage everyone to enjoy their time during class and to become less focused on traditional formalities they might have encountered in previous mathematics classes. Students usually got their food upon completing their *closed-book journal* entries. I wanted it to be a way to relieve some of the stress associated with mathematics and the *closed-book journal*. One student summed-up what I hoped the snacks would achieve with the following comment:

> I like the atmosphere…and I like the snack thing, too...It kind of makes it feel like a get together instead of going to class. I just really like it.

Small-Group Discussion

Every week students were expected to complete what we referred to as *Journal Activities*. These were assigned at the end of each class session and discussed at the beginning of the next. They included a series of questions or assignments about the mathematical objective for the week. These were designed specifically for the purpose of asking students to complete tasks using an inquiry-based approach, rather than doing the problems in the text, which tended to be more procedural and not conceptually based. Their construction was based on student interest in particular social issues (see group discussions below) and the mathematical objectives of the course. At the end of each class session, I would distribute these questions to the students. Class time was reserved for discussing the problems in these *Journal Activities* and introducing topics in innovative ways. If time allotted, students might discuss the next week's set of *Journal Activities* at the end of class. These will be discussed in more detail in the next chapter of this dissertation.

Students completed these problems at home and returned to class ready to discuss them. I placed students in groups to do so. Initially, I assigned groups randomly, by giving each student a number from 1 to 5 and placing all individuals with the number 1 together, all those with the number 2 together, and so on. After a few weeks of doing this, I noticed that some students worked more efficiently and diligently when they were placed with certain other students. For example, the students who spoke Spanish as their first language tended to contribute more within groups when they could speak to other

Spanish speakers. In the fourth week of class, I allowed students to choose their own groups. The process was natural and the students had become acquainted with one another enough that they gravitated towards certain people. Although for the major part of the rest of the semester they maintained the same groups, with some students rotating at times, every so often I rearranged them so they could hear what others had to say.

During the time they spent with their groups, students discussed and answered one another's questions, as they pertained to the week's *Journal Activities* or *Closed-Book Journal*. They were only given three guidelines: (1) stay on task, (2) do not move to a discussion of a new problem or assignment until everyone in the group feels comfortable with the solution because I will ask students randomly to discuss any problem, and (3) after an adequate amount of time, if a problem or discussion cannot be resolved within the group, record it so that it may be addressed to the whole-class later. As students worked, I walked around, observed, took notes, and asked questions as I probed and learned about students' mathematical interactions and thoughts. I never searched for right or wrong answers; rather, I was investigating the various strategies students used to approach problems. I intended to find students' different methods of solving problems and where students might have faced difficulties so that I could orchestrate the whole-class discussion afterwards. I also took note of the discussions about social issues that emerged. I used students' interests to develop future *Journal Activities* (discussed further in the next chapter).

As the instructor, and a participant in this process, my role became that of monitor, facilitator, and persuader of student participation. As a monitor, I listened to students as they grappled with ideas and made sense of them. I encouraged them to reiterate and reflect on solutions so that others could make sense of what they had done. As a facilitator, I posed questions and initiated dialogue, encouraging students to approach problems from different angles. I wanted to encourage students to work cooperatively and listen to each other. My intention was to promote an atmosphere in which students constructed and developed ideas about mathematics from various perspectives and in ways that made sense to them (Wheatley & Reynolds, 1999).

Whole-Class Discussion

Unlike what may happen in a traditional mathematics setting where the instructor lectures and students wait for ideas and answers to be handed to them, in this classroom, the students and I engaged in open discussions where everyone was expected to listen, respond, and interact with the curriculum and one another in a respectful manner. I aimed at an exchange of ideas from different points of view. In fact, during the first day of class, students were told that professional participation would be a heavily weighed component of their grade for the course. I indicated that course grades would be determined in the following manner.

1.	Class preparation and participation	25%
2.	Journal Activities	55%
4.	Closed-Book Journal	10%
5.	Mid-term Conferences	10%

As an explanation for what I expected in terms of class preparation and participation, the syllabus included the following statement:

> Students are expected to attend and be prepared for **every** class. Poor attendance, punctuality or preparation habits will result in the unsatisfactory completion of

> this class. Your grade for professionalism will be evaluated on the following (to name a few): cooperation/participation in group work; participation in class discussions; attendance; thoughtful reflection; and communication with your peers and instructor.

Students began the semester aware of these guidelines. As they participated in whole-class discussions, they did so in a respectful manner and with focus on the topics of discussion. As students presented problems to the class or discussed ideas with everyone, I took note of what interested students. I asked questions such as, "Did anyone solve this problem in a different way? Do you agree or disagree with this? Why or why not? Does anyone have anything else to add or share? What other ideas would you be interested in investigating?" I hoped to encourage students to question one another and think about various topics, mathematical or otherwise, from diverse methods and perspectives. I wanted them to deepen not only their mathematical understanding but also to learn to question and justify their own ideas about themselves and society, as a social justice component. The social issues that emerged in the discussions were used as the basis for the formation of future *Journal Activities*.

Introduction of New Journal Activities

Upon completion of the whole-class discussion, I introduced the *Journal Activities* for the next week. Aware of the fact that students construct knowledge based on their previous experiences, in these introductions, I connected new concepts with previously encountered material. Sometimes, this was a previously discussed mathematical concept, sometimes it was a previously encountered social issue, and sometimes it was a mathematical or social topic I thought all the students would have some knowledge of. I introduced mathematics through children's stories, newspaper articles, current issues, and pure mathematics concepts. These prefaces set up the activities that emphasized tasks students participated in. They never gave student specific procedures for finding solutions. At times, I asked students to do activities before ending class. I created these activities to help students become interested in and begin tackling the tasks for the week ahead.

Discussion

A practitioner-research study is an emergent one; therefore, replication of data is irrelevant in this case because the theoretical nature of the teaching experiment is that it evolves as the process takes place, and each teaching experiment may lead to different results. However, what is desirable is the analysis of episodes, interactions, and perceptions of participants when teachers engage in the evolution of and implementation of innovative ideas in their particular and unique circumstances. This design was not intended to suggest generalizations that would apply to the preservice teacher population. There were several limitations to this study that would hinder making broad sweeping statements about the findings. First, a large proportion of the preservice teachers in the course were not European-American students. Second, many of the students were non-traditional and had children. Third, all the participants in the course were female. This population did not represent the "typical" population found in many elementary preservice education classrooms. Fourth, as the instructor of the course, I came into the semester very excited about teaching a social justice mathematics course. Fifth, the sample size was small, and the study was only conducted in one classroom and not compared to a similar classroom using "traditional" teaching techniques. Therefore, the findings for this study were viewed from the perspectives of the participants, rather than that of a traditional one. Many of the interactions and transactions that ensued were greatly influenced by these limitations; however, exploring this course was a way to provide insight that may help other preservice teacher educators form their own ideas about teaching and learning in their unique circumstances (Cobb, 2000)

The factor that became relevant, though, was the trustworthiness of the analysis found in this study. Credibility in this research was based on the longitudinal engagement with and observation of the student participants of the study. I still maintain contact with several of my students. I have continued discussing the course with them and confirming the results to verify the dependability of my conclusions. Further, I confirmed my results with the theories and results of other authors who wrote about and conducted similar studies.

Summary

This chapter attended to the components involved in the methodological approach, data collection and analysis of the mathematical and social interactions between and among students, a social justice curriculum, and the instructor. It also described the teacher's role and framed it within a methodology that was naturalistic and utilized case study techniques to present a descriptive and interpretive analysis. Further, this chapter provided a sense of the community college, the classroom, the participants in the study, and the four themes of analysis that were examined within the curriculum routines of the class. In the next chapter, Chapter Four, I explore the first question of the research with an explanation of the first theme of analysis, *Paradox and the Formation of a Social Justice Mathematics Classroom.*

CHAPTER FOUR

THE EVOLUTION OF A SOCIAL JUSTICE MATHEMATICS COURSE

Overall Results

The first question this study sought to answer was, "What critical factors were involved in the evolution of a social justice mathematics course for elementary teachers?" In considering this question, I explored the possibilities that emerged from within the curricular routines described in Chapter Three. These routines opened up space for students to engage in problem-solving opportunities that allowed them to stretch their ideas about mathematics and social issues. They allowed students to listen to each other, negotiate meaning, explore multiple representations, create curricular contexts, and reconsider time and space for mathematics. They encouraged students to communicate with all course participants about mathematics and social issues, and they pushed students to continuously revisit problems and questions throughout the semester.

The curricular routines offered in this course were a new and innovative way of participating in a mathematics classroom for most of the students. *Journal Activities* were at the heart of this newness. Activities were assigned to students each week for the duration of the fifteen week semester. Although some introduction to the problems would sometimes take place during class, students were expected to grapple with these on their own time and at their own pace for one week before they were discussed in class. Each week, the journal activities from the previous week would be discussed in groups and with the entire class. Students recorded all journal activity work in a journal for the course. They were allowed to revisit and redo problems at anytime during the semester, even though journals were graded twice before the end of the semester. They submitted completed journals during the final class period. All revised work was regarded.

Each week's journal activities were aligned with exposure to and experience with a particular mathematical concept in mind. Often times the topics were embedded within a social context that addressed a critical social issue. Although the activities, problems, and scenarios included in the journals were created to address particular mathematical ideas, they provided opportunities for exploring multiple concepts, procedures, and solutions of mathematics and social issues. For example, one activity asked students to explore the concept of exponential growth. This was done through a children's story, *One Grain of Rice* (Demi, 1997), that explores the social issues of greed, power, and wealth. During the exploration, students wrote about and discussed exponents, factors, multiples, patterns, summation notation, and variables. Although the primary focus of the activity was to engage students in understanding exponential growth, students worked with multiple concepts and ideas all at once, considering social problems as well as mathematics to understand much more than exponential growth.

These journal activities were embedded within a classroom community that revolved around curricular routines and provided students with the opportunity to engage in thinking about mathematics and society. The overall results of this process were high student attendance, participation, and interaction. Grade records and reflective journals depicted the success of this type of engagement. I begin this chapter with a brief overview of some of these results and then address the three themes of this chapter, namely the three major paradoxes that led to the evolution of a social justice mathematics course with these outcomes.

Presence

Retention and attendance rates in this course were extremely high. Zero percent of the students dropped the course after the first week of class. Although during the first week of class five out of the initially enrolled twenty-five students dropped the course—one of whom never attended the class, the drop rate for the course was extremely low. I did not have contact with any of the twenty percent of students who did not return to class after the first meeting, therefore, I cannot make any predictions about why they decided not take part in the class. However, the fact that zero percent of the remaining students dropped during the rest of the semester indicates that this class resulted in an extremely high student retention rate.

Because class participation was such a major component of the course, I enforced a strict attendance policy. On the first day of class, I informed students that attendance would be recorded on a regular basis. I also implemented a make-up paper policy that was included in the syllabus. It read:

> You will be expected to write a make-up paper for any class session you may miss. The paper should include a summary of any materials that were discussed and worked-on for that class period. It will be due at the beginning of class the week you return.

Students were informed that it would be their responsibility to contact at least one classmate to discuss what occurred during the time missed in order to successfully complete their make-up paper. In these papers, students included closed-book journal activities, journal activities, and any other topic we discussed during the class they missed. They turned these in upon returning the week after the absence. My intention was to make sure that everyone could participate fully during class time. Since many of the concepts we discussed emerged from previous discussions, I found this system the closest method of ensuring students knew what the class engaged in during the time they were absent.

The devout enforcement of this attendance policy might have been one reason why attendance in this class was so regular. I consistently reminded students about the policy, particularly after an absence. Every student who missed a class contacted me within a day of the missed class, and I responded to e-mails or telephone calls from these students, reminding them about the make-up paper. It became well-known in the class that this assignment would be a consequence of an absence. On more than one occasion students commented that they did not want to miss class, so they would not have to write a make-up paper. The worst attendance by a single student was two class periods missed.

However, other factors also influenced the high turn-out. They included but were not limited to the enjoyment students exhibited, the rigorous mathematics they engaged in, and the passionate discussions of mathematics and social issues students took part in. I discuss those aspects further in the remainder of this chapter and in Chapter Five.

Concepts Covered

I taught this course during its second semester at the community college after it had been developed by a full-time faculty member in the Mathematics Department. Before the semester began, the original course coordinator provided me with a set of curricular materials and instructions on how to conduct the course, including a week-by-week mathematical concept agenda. For example, I was instructed to “teach” *Problem Solving* in week 1 (Sections 1.1 and 1.2 of the text), *Sets and Venn Diagrams* in week 2

(Section 2.1 of the text), *Numerations Systems* in week 3 (Section 3.1 of the text), and so on (see Appendix A for a complete list of concepts). The fifteen week semester was mapped-out in detail, including suggested homework problems from the book.

Although I was given a rigorous schedule of events for the semester, I was allowed the autonomy to teach using my own pedagogic approaches, which created space for the formation of a class in which students, rather than the instructor, decided whether or not mathematical concepts were "covered". During my interview for the job, I explained to the interviewer, the chair of the mathematics department, that I wanted to teach a mathematics course through social issues as part of my dissertation study. She informed me that I could teach the course any way I liked, as long as I "covered all the material". Knowing that I had some freedom in how I included curricular materials gave me the liberty to create the curricular situations I chose, including discussions, Journal Activities, and reflective journal entries, among other things. I was not limited to illustrating examples on the board and assigning homework problems directly from the text. This liberty allowed me to invite students to think about mathematics in a less linear and traditional way. In other words, teaching problem solving did not occur only in the first week, as the schedule indicated it should, it was an important component of the entire semester. Unlike in a traditional mathematics class where student might be told, "Today we are going to learn problem solving," the entire semester focused on problem solving and students were never told what they were going to learn, rather they were consistently asked what they thought they had learned. This process revealed what students felt they had gained from the course in a meaningful way. Rather than my saying I "covered" all the mathematics I was supposed to, students' work, discussions, and journal reflections revealed that they had made sense of the mathematical concepts I hoped they would. They became the assessors of whether or not mathematics was "covered" in this class. I provide specific examples of these student judgments in the next chapter.

Incomplete/Missing Assignments

Students were given more than one opportunity to complete assignments and closed-book journals. Every week, students were expected to discuss homework assignments and closed-book journals. They were expected to be prepared for class in order to do so, and they were held accountable for their preparation, as I would ask them to answer questions in closed-book journal entries and discussions. Students were also informed that they could re-do any journal activity or closed-book journal for a new grade at the end of the semester. Upon grading assignments and closed-book journal entries, I would return them to students with comments, questions, and suggestions for students to consider for a satisfactory completion of the assignment. Several students discussed appreciating this system during *mid-term conferences*. By the end of the semester, every student but one turned-in every assignment and closed-book journal entry completed. Although not every one of these resulted in a "perfect" score, they indicated that students were willing to complete and revisit all their work, if given another chance.

Final Results

Although students were informed they would be given an opportunity to re-do this final journal entry after it was graded, students' initial average on this assessment of comprehension was a B, and not one student chose to re-do her work. I decided to give the students a comprehensive final, although that was not what we called it, so I could see how students would perform on a traditional form of assessment when they engage in a non-traditional social justice mathematics course. Most of the questions on the exam were aligned with previous journal activities or closed-book journal questions; however, the problems were written in a way that made them stand alone, rather than relate to a context as they might have in the journal activities from which they were taken. For example, one question appeared as the following on the final:

> Illustrate how you could find 221 divided by 17 without using the traditional division algorithm.

However, in the journal activities, it appeared as the following:

> The local Union is planning a rally at the local community center in order to fight for better wages for home health care workers. The union believes that it is unfair for these workers to only make minimum wage and are upset that many of the union members working long hours are denied overtime pay. There is a coalition (made up of community groups, unions, activists, and various political parties) that has planned for the rally to happen in two weeks and the union is responsible for bringing people from their organization to attend and show support for this important cause. Two-hundred twenty-one people have confirmed their attendance at the event. The room reserved for the occasion holds 17 seats in each row. How many rows are needed for everyone to sit? Perform the necessary calculation with illustrations or using manipulatives. Do not use the traditional algorithm.

The idea was to see if students could translate contextual problems into abstract ones on a traditional form of assessment, and the result was that they did so successfully with a B average.

After averaging all grades and considering attendance, the final results for the course were eighteen students achieved an A average and one obtained a B. The one with a B was also the one who had the missing assignment. She also had the lowest grade on the comprehensive final. Although she did not receive an A for the course, at the end of the semester, she expressed satisfaction with her performance in the course.

Paradox and This Chapter

What factors were involved in the formation of a class that yielded these results? Three major paradoxes that emerged in the creation and execution of the class answer this question.

I chose to discuss the formation of the class in terms of paradoxes because they seemed to capture the essence of the transactional relationships that materialized between and among the instructor, students, and the curriculum throughout the semester (Houser, 2006). These relationships questioned notions of "reality" and "truth" as absolute and knowable. They began to reconsider notions of time and space as linear and separable. Teaching and learning were not simply seen as a verbal transmissions of a prescribed set of facts from a more knowing active authority to less knowing passive novices all during a linear span of time. As transactional theorists have suggested,

> [L]earners are cognitively active agents who interpret the environment and make personal decisions regarding subsequent encounters and experiences…[The] organism-environment relationship is reciprocal and context specific and…at least

> where humans are concerned this relationship is mediated by dialogical communications…(Houser, 2006, p. 18).

These dynamic relationships that connect the self to society are what led the construction of mathematical and social understanding in this classroom, and they only seemed explicable by the idea of paradox.

A *paradox* is a statement that appears to be a contradiction but in fact is or may be true. All three paradoxes described here embodied the essence of the transactional relationships between and among participants, mathematics, and social issues within this course. They overlapped and interacted in the evolution of the course; however, each one contained unique characteristics that contributed to the success of the course. The three statements were:

- less traditional teaching resulted in more meaningful learning,
- learning from students yielded more effective teaching, and
- releasing control of time and space created more time and space.

Less Traditional Teaching

The first paradox that drove the evolution of this course was *less traditional teaching resulted in more meaningful learning*. To give a clearer understanding of how this paradox materialized, I begin with a description of how the roles of *communicating* and *perturbation* became catalysts for the formation of a community atmosphere, where a reconsideration of traditional roles between students and teacher emerged. Unlike a traditional classroom that includes lecture, individual problem-solving, and an emphasis on right or wrong answers, I consider how providing a space for listening to one another and negotiating meaning prompted students' to stretch themselves as problem-solvers in this class. Developing a community in the classroom began on the first day. I knew that an important element to providing a caring environment in the course would be authentic *communication* if I expected my students to engage actively in the teaching and learning process I hoped would emerge (Davis, 1997).

Communication became a key hallmark of the class. An important component to the community atmosphere that developed in the classroom was the facilitation of an environment where students would listen, be listened to, and discuss with one another. Therefore, a primary focus of the first class meeting was getting students to communicate with me and one another. I describe how communication emerged on the first day of class.

After discussing the syllabus and telling the students a little about myself, I asked students to engage in dialogue where they would have to interact with and listen to each other. I began by asking each student to tell the class a little about herself. I wanted students to begin the process of speaking with each other by talking about something familiar, rather than beginning with a discussion about mathematics, which I anticipated might be intimidating and could create resistance towards participation. Although I wrote in my reflective journal that I sensed that students were "tense about and nervous with the idea of talking on the first day," I knew I wanted to break the ice somehow. Further, I wanted to model the communication process. I did this by reiterating many of the statements students made and connecting them to the accounts of others. I wanted to make a concerted effort to use students' names, as a way to emphasize their value as individuals rather than random students. (Students placed name cards in front of them for the first several weeks of class.) I provide an example from my reflective journal:

> Two students, Erica and Wanda told the class they were pregnant, and another, Alex, humorously said she had recently gotten married and was trying to get pregnant. I joked and said, "Maybe you should sit next to Erica and Wanda. It might be contagious." Several students laughed at this statement. I wanted students to see that I was listening to them and interacting with them. I also wanted them to see that I recognized commonalities between them.

After talking about ourselves for a short while, as a way to engage students in practicing listening to and communicating with one another, I randomly placed students in groups of four or five individuals and asked them to discuss an introductory mathematical task I provided for them. This initial group work was a way to engage the students in a negotiation of the social norms for the class. I wanted students to begin accepting each other as having equal value. The goal of this assignment was to introduce students to the idea that they would all be active participants who are respected and heard in this course. According to Noddings (1992) "[s]tudents will not succeed academically if they are not cared about…Dialogue taking place while learning in communion connects children [or adults, in this case] to each other, and it provides knowledge about each other that forms a foundation for caring" (p. 23). Setting the stage in this way would carry out through the remainder of the semester. This was the mathematical task I gave them:

> Find the next three terms in the following sequence: 7, 11, 15, 19, 23,…(This is called an arithmetic sequence.)
>
> Find the next three terms in the following sequence: 3, 6, 12, 24, 48,…(This is called a geometric sequence.)

What do you think is the difference between an arithmetic sequence and a geometric sequence?

After they discussed, I invited each group to share what they found during their time together. As students spoke, I used their names to address them. I also asked students whether or not they agreed with other groups' ideas, as they emerged. I illustrate how this transpired with a reflection I wrote on a whole-class discussion of the task above.

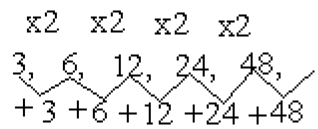
Mary volunteered to show the class how her group solved the first question. She wrote on the board:

7, 11, 15, 19, 23, 27, 31

She explained that her group added four to generate the next three terms of the sequence. I reiterated Mary's statement, and asked the class if anyone had done the problem any differently. No one interceded. Then, I asked Kyran if she minded illustrating to the class how her group solved the second problem. Although she seemed hesitant, she came to the board and wrote:

x2 x2 x2 x2
3, 6, 12, 24, 48,

She explained that her group noticed that each term was multiplied by two to generate the next term of the sequence. After Kyran explained that the next three terms would be 96, 192, 384, I asked the class if anyone had anything different. Megan interjected that her group "added the double." I asked her to show the class what she meant. Megan wrote on the board:



She explained that her group generated the numbers in the sequence by doubling the number they were adding each time. They began with three and added it's double, six, the next time, repeating this pattern until they generated three new terms. Megan's method surprised me and prompted me to ask the students what they thought the difference between an arithmetic and geometric sequence is. Megan said her group had decided that "an arithmetic sequence is generated by addition and a geometric is generated by multiplication." Kyran, on the other hand, said her group had written "an arithmetic is generated by adding or subtracting and a geometric is generated by multiplying or dividing." I asked the class if there was a difference. Another student explained there was not and wrote the following on the board:

$3 - 2 = 3 + (-2)$ $\qquad\qquad$ $4 \div 2 = 4 \times \frac{1}{2}$

Although some students seemed perturbed by these statements, the conversation took another direction. Angel asked a question:

Angel: Megan added up there, so can a geometric sequence be arithmetic? [Angel was referring to Megan's assertion that "you add for a arithmetic and you multiply for a geometric"]

Angel's question made sense to me. She was confused by the other students' definition of a geometric sequence dealing with multiplication and division alone when Megan's method for generating new terms included addition.

Several students began participating in the conversation at this point. One student suggested that both sequences could be arithmetic. Then another noticed that in the first sequence the same number is added and in the second, a different number is added. Andrea told the class that an "arithmetic is adding the same number over and a geometric is multiplying the same number over and over, even if you are adding those numbers." Kim called this number a "constant." This led Angel to conclude that the two sequences are different. I asked April to tell the class what she thought the difference was between an arithmetic and a geometric sequence. After she spoke, in my own words, I retold the incident and asked the class to help me write the definitions for the two sequences on the board. Students spoke as I wrote. After a couple of revisions, these were the agreed upon definitions:

> An arithmetic sequence is a sequence of numbers in which each term is generated by adding a constant number to the previous term in the sequence.
>
> A geometric sequence is a sequence of numbers in which each term is generated by multiplying a constant number to the previous term in the sequence.

Although I could have easily surpassed all the questioning and debating that occurred and could have simply told my students what arithmetic and geometric sequences are and how to generate terms for each, the discourse that came to light was much more than just reaching conclusions about how to generate terms for arithmetic and geometric sequences. In this discussion, students had to listen to classmates to make sense of what other students had constructed. Angel heard Megan say that an arithmetic sequence uses addition and a geometric uses multiplication, but she saw her add in the geometric sequence. This led her to propose a question about whether or not a geometric sequence could be the same as an arithmetic sequence. However, by listening to Mary, Megan, Andrea, and Kim discuss, Angel understood that adding could be one way to generate terms of a geometric sequence but that did not mean it could be characterized as an arithmetic sequence. Through their discussion, they communicated with one another and discovered the fact that adding by a "constant" number is what differentiates the arithmetic from the geometric sequence. By communication with one another, they prompted ideas in each other and eventually recognized that a geometric sequence, even when numbers are added, is generated by multiplying a constant number.

The whole-class discussion that emerged during our first meeting engaged students in open participation, collaborative work, and mutual respect for one another's ideas. Students began to articulate their ideas and share them with others. Unlike the traditional mathematics classroom situation where communication rarely takes a form other than that of the teacher lecturing, listening for correct answers, and quickly correcting incorrect ones, communication in this classroom began to occur between and among students and instructor (Davis, 1997). Not just listening to a lecture or listening for correct answers, students and teacher listened to and commented on all the statements made by classroom participants. The discussion was focused on students developing an equal voice in the classroom and displaying that knowledge could be constructed within this process. It was a way to illustrate that I was not an authoritarian with the correct answers and methods but a participant in the process of forming connections.

This introduction to the class was important because it set a foundation for the community of the class. Creating a comfortable environment where students could discuss, make decisions, express themselves, and listen to one another enhanced mathematical discourse as the semester progressed because of the safety and care that was developed for students (Davis, 1997). One student seemed to sum up it up when, after several classroom meetings, she wrote,

> I think group work is excellent. It really helps people understand with every type of thinking. I also think it is great to have interaction with the teacher. It shows the teacher really cares and it helps with the learning process.

The first day communication that occurred in this class provided opportunities for students to be heard, where they could explicate their thinking through interactions with the teacher and other students. It was a time for personal sharing, giving, and learning, and it carried through for the remainder of the semester. It was the students' first experience as active participants and listeners in this class.

However, not only was *communication* a significant hallmark of the course, but also *perturbation* emerged as an important characteristic of the class. The tasks and assignments provided by the journal activities offered perturbation for many students. This conflict was a necessary component of the class that became a precursor to learning to negotiate meaning. I provide an example of this process with a problem that caused concern for students.

As the semester progressed, students began to realize that I, the “teacher,” would not provide them with “correct” answers to problems they encountered—that it would be their responsibility to decipher what was intended by and how to reach solutions to the tasks and problems they were assigned. On the third day of class, students returned to a problem that had been assigned to them on the first day. They had discussed it during the second class session and online between the first week and the third; however, they still had not reached a consensus. I provide the problem below and illustrate how students negotiated meaning through their online posts and in-class interactions. The problem led students to discover the Fibonacci sequence.

> Suppose that a pair of baby rabbits is too young to produce more rabbits until they are 2 months old. However in every month thereafter, they produce a pair and the same happens for each pair they produce. How many pairs of rabbits are there in each of the first six months? What do you notice about the number of pairs of rabbits in month four as compared to months two and three? What about month five as compared to months three and four?

Online Posts (Some comments were omitted):

Andrea: DOES ANYONE HAVE A SOLUTION FOR THE STINKIN' BUNNY PROBLEM? IT IS DRIVING ME CRAZY!

Sally: I have to agree about wanting to find the solution to the bunny problem! The more I read the more the words start to run together and more and more bunnies start to appear. AHH..

Megan: I was also confused by this problem. The wording is a little tricky because it says they arent *[sic.]* mature enough to have babies until they are 2 months old, and then every month thereafter they have a set. When I read it that sounded to me, like the additional sets didn't actually arrive until the third month (the month thereafter). This would allow a gestation time I suppose. Anyways, I decided I was overthinking *[sic.]* the problem and went with the additional sets coming in the 2nd month, either way I am looking forward to finding out the answer.

Laney: I was also confused as to when to start counting. For example, do you go ahead and use 2 months of waiting time for the first pair or do we start with that first pair having a pair? Once I decided to just start on the pair having a pair it seemed easier but I really have no idea if it's right or wrong. I am very interested and look forward to getting the correct answer and seeing how others worked the question. I personally had to do a chart.

Brandy: I figured out when I got home that our group was reading the question wrong. It was asking how many are there total in EACH of the six months. We were thinking how many total at the end of six months. So when I read the question like this, it made it much easier to figure out and to see the pattern forming from month to month.

it helped me to visualize the problem by making a chart. I had six columns representing the six months then just labeled the different bunnies with letters. IE...my original bunny pair was A1, their first offspring born in month two was B1, born in month three was B2, etc...it is hard to explain w/o seeing it.

So now I THINK I have the answer...it will be interesting to see what the answer really is and if I am even close at all!

Angel: I think I may actually be in Brandy's group, because I was reading the problem wrong too! Tonight I got out 6 pieces of paper, labeled them for each month, and took out the color tile manipulatives we got with our book. I then created bunny "families" on each paper. This way I was able to count out each month's bunnies. After month 3 I began to sequence to the order of bunny reproduction. I may be totally off on this, but I believe that the reproduction sequence is a geometric sequence. It will be interesting to find out the true answer. I'm really ready to get these bunnies out off my mind!

Kyran: For me I had to create a chart that was 1-6 for the months. We started off with 2 bunnies in month 1. I didn't add the original bunnies babies until the third month. So I had 2 bunnies in the first month...then 2 more bunnies in the third month. After the third month the original pair had babies every month, and the third month baby bunnies didn't have babies until they were 2 months old which would be in the fifth month and so on. Wow thats really confusing but I hope it's right and helps someone out.

Sandy: That sounds like the same way I worked.

Brandy: So here is my attempted explanation via message...

I made A1 my original pair of bunnies. They had a set of babies(B1, B2, B3, B4, and B5) each in months 2, 3, 4, 5 and 6. B1 started having babies(C1, C2 and C3) in months 4, 5 and 6. B2, born in month three had babies (D1 and D2) born in months 5 and 6. B3 and C1 both born in month 4, had babies (E1 and F1) in month 6.

So, now that you are thoroughly confused:

Here are the totals for each month:

month 1 = 1 pair

month 2 = 2 pairs

month 3 = 3 pairs

month 4 = 5 pairs

month 5 = 8 pairs

month 6 = 13 pairs

with 32 pairs total.

The number of pairs of rabbits in months two PLUS the pairs in months three EQUALS the number of pairs of rabbits in month four.

> The number of pairs in month three PLUS the number of pairs in month four EQUALS the number of pairs in month five.

On the one hand, several students seemed to be confused about which months to begin adding new rabbits. Some students understood the problem as a new pair is born during the second month and therefore would add a new pair every two months after each new pair was born. One student, Brandy, incorrectly gave the original pair of rabbits a pair of offspring in the second month, but did not repeat this logic with all new offspring, beginning the problem incorrectly but completing all other months correctly. Kyran, on the other hand, understood that no rabbits would be born until the third month of each new pairs' lives.

Although most students came to class the next week believing they had correct solutions, many had misunderstood the problem. In my reflective journal, I noted the following scenario:

> As students worked in groups and discussed their "bunny problem" solutions, Kyran told her group that she thought the bunnies would not reproduce until the third month because they don't reproduce "until they are 2 months old," which she explained would mean they would have completed two months of age. I heard her statement and asked her to share it with the whole class. This comment made several students recognize the mistake they had made and led to several more minutes of group discussions. Although I had noticed this miscomprehension in the online posts, I refrained from telling the students that they had misread the problem. Instead, I allowed them to negotiate the meaning of the wording in the problem on their own. Students then volunteered to present

their groups' solutions on the board. Kyran was first. She drew the following diagram (see Figure 4) on the board.

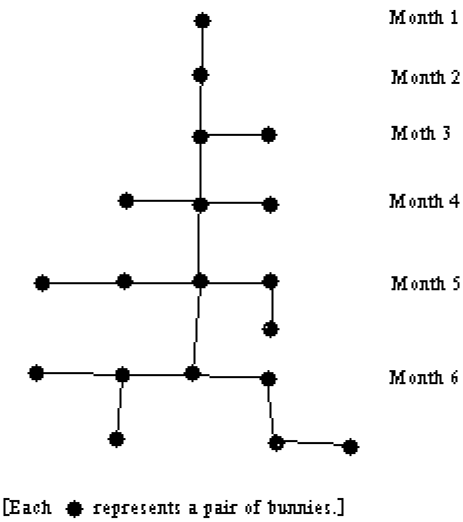


Figure 4. Rabbit One

She wrote on the board, "Month 6 had 7 pairs," and I asked the students what they thought. Sandy exclaimed that she disagreed. I asked her to show us why, and she drew the following diagram (see Figure 5) on the board.

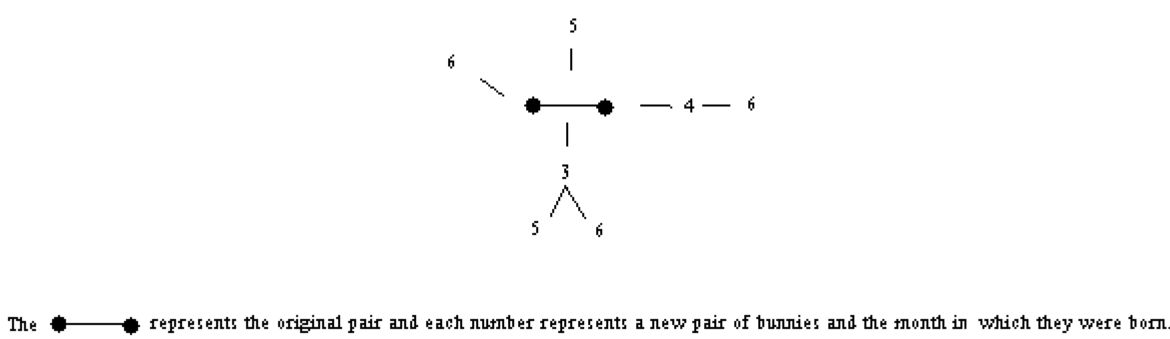


Figure 5. Rabbit Two

Sandy wrote, "Month 6 has 8 pairs." I asked the class which figure they thought was correct, and students began discussing this within their groups. One group noticed that Kyran "forgot to add a pair in Month 6 for the original pair." Kyran agreed, and all the other groups decided the answer should be "Month 6 has 8 pairs." One student then added that the number of pairs in each month is the sum of the two previous months' totals. Another student then explained to the class that she had discovered, from the text, that this is called a Fibonacci sequence. She also explained to the class the common appearance of this sequence in nature.

The investigation of this problem perturbed students which prompted them to negotiate and enforced the idea that they can and must work with one another to solve their misunderstandings, rather than look to an authoritative figure for a right or wrong answer. They were beginning to support their thoughts and claims with logical arguments and started asking their classmates for affirmation. In fact, during the class session that followed this scenario, one group encountered a perturbing problem and decided to refrain from asking me for help. Two students from the group looked in my direction and I heard one say to the other, "Don't even ask her, she's not going to tell you the answer." In my reflective journal I wrote, "It's become a recurring joke in the class that I will never tell students how to do a problem."

The process described here was unlike the traditional mathematics classroom scenario in which the teacher shows students how to "read" word problems, look for "key words," and solve them, by illustrating examples of solutions to similar problems. Most traditional mathematics classroom events begin with an instructional lecture, proceed with students solving problems individually to obtain correct answers, and end with teachers grading student work. However, this pattern has restricted mathematical activity, prompting students to respond to teachers' questions and expectations, rather than giving them the opportunity to think, negotiate meaning, and develop ideas with others. Often times, teachers in traditional classrooms assign homework problems to give their students the opportunities to practice problems, expecting them to become experts at them by doing so, but never allowing them the chance to engage in mathematical discourse with other members of the class (Wood, Cobb, & Yackel, 1991). This restrictive unidirectional communication between teacher and student only imposes on students the idea that they must accept a mathematics that has been developed by others and viewed as the “right” or “important” mathematics.

Engaging students in mathematical discourse though perturbing problems stimulates students’ thinking and encourages them to negotiate and construct meaning. Throughout the semester, *Journal Activities* continued to provide complex tasks for students to grapple with and construct mathematical meaning through. Rather than focusing on specific procedures, the focus was on problem-solving and the process of negotiating and constructing meaning socially. The emphasis in this mathematics class no longer lay in the solutions; rather, it was in the reflective thinking and communication that occurred.

More Student Decision-Making

The second paradox that drove the evolution of this course was *learning from students yielded more effective teaching*. Although the episodes above illuminated the listening and communication that occurred during class time, they did not, however, illustrate these ideas through problems embedded within a social context. Mathematics entrenched in social issues was a major component of the class. However, the process of utilizing social issues to discuss mathematics materialized gradually. Aware that a delicate balance exists between dissonance and affective safety, I wanted to approach the introduction of social issues in educative rather than "miseducative" ways (Dewey, 1938). Dewey described the need for disequilibrium for the attainment of new knowledge; however, he also expressed that too much dissonance is dangerous and can cause what he referred to as "miseducative experiences," or a hindrance of the creation of new ideas. For students to engage in meaningful learning entails both maximizing intellectual conflict and affective safety. Therefore, a difficulty I encountered was negotiating where that balance lay when discussing critical social issues with mathematics. I decided to utilize student interest as the driving force for the social issue lessons I included in *Journal Activities*.

In deciding what social issues I should include in the course, I had to listen to my students and try to understand what interested them. The initial social issue was chosen by me; however, it became a catalyst for discussing society in the class and prompted students to share concerns they had about society. I used these interest and concerns to develop the lessons that included social issues. In the first set of journal activities, along with the "bunny" task, I included four tasks that addressed poverty, one of which asked students to create a monthly budget for a family of three living in poverty, according to the federal government's definition of poverty. Students had to research government aid programs in order to figure out how to spend such a limited amount of money over a thirty day period. I then asked students to reflect on this process. We discussed students' budgets during the third week of class. For some students, completing this task was not an abstract assignment. They did not have to imagine such a life, since they lived under similar circumstances. I provide an example of one students' budget (see Figure 6) and explain how student reflections and discussing this assignment opened up space for including other social issues in journal activities.

Figure 6. Poverty

Monthly Budget for a Family of 3 with a Monthly Income of $1525.83

Expense	Estimated Monthly Cost	Comments:
Rent	$650	Family lives in a 2 bedroom apartment in NW [City Name]
Food	$30	Family has qualified for State Nutrition Assistance Plan which provides $526 in food per month. Additionally, the child qualifies for the free school breakfast and lunch program.
Car Payment	$350	Family shares 1 car, a 2002 Nissan Sentra
Auto Insurance	$65	Family carries full coverage on the car because they have a loan on the vehicle.
Gas for car	$40	Mom and Dad try to only travel within a 20 mile radius of their home
Health Insurance	0	Child qualifies for Sooner Care, parents currently do not have health benefits
Clothing	$50	Mom goes to the local thrift store each week looking for good deals. All other clothing such as underwear and socks are brought at Wal-Mart or Target when on clearance
Utilities	$125	The family lives in an apartment in which the gas and water are paid. Family is responsible for their electric bill and telephone.
Childcare	0	Mom only works part-time when her child is in school, because the family cannot afford to pay for childcare.
Toiletries / Cleaning Supplies	$30	Toiletries include: feminine products, deodorant, soap, shampoo, conditioner, toilet paper Cleaning Supplies include: paper towels, dish soap, bathroom and kitchen cleaner.
Haircuts	0	Mom has learned how to cut everyone's hair from a book she checked out from the library.
Charitable Giving (Tithe)	$153	This family has strong beliefs and is very involved in their local church. They feel it is important to tithe 10% of their income each month.
Entertainment	$30	Because the family is on such a strict budget, entertainment is limited to activities such as renting a DVD, going to the dollar movie, or going to McDonalds. The family also goes to free festivals, parades, museums on their "get in free day", way back Wednesdays at the zoo where admission is only 50 cents, and the park.
Total Expenses	$1,523	
Total amount left over at the end of the month	$2.17	Financially speaking, this family is living in a very dangerous place. They have almost no money left over at the end of the month to save. Additionally, they do not have an emergency fund. If Mom or Dad was to get sick and could not work, this family would be in a tragic situation.

For some students it was difficult to imagine such a financial situation and for others it was not; however, similar to this students' concern about the family's financial situation, every student wrote about how a family in this financial situation is "living in a very dangerous place." One student wrote:

> I had some difficulty because most of these expenses I do not have myself so it was hard for me to make sure all the basics were covered and I'm sure many necessities were left out or underestimated which is bothersome because there was already many areas such as insurance and rent that may have already been estimated a little lower than possible. To me, [this] puts quite a strain on day to day living...

Student concerns such as this one's emerged during the whole-class discussion of this assignment. Several of them discussed many of their personal experiences with living under strict financial guidelines. In my reflective journal, I wrote about one student who told the class that she could not afford healthcare because she battled diabetes, and insurance companies would not accept her with a "preexisting condition." She explained that she was fortunate enough to come from Native American roots and could therefore utilize the services of a tribal clinic in the state. However, she had to drive two hours to reach the clinic. She told a story about a time when she became very ill and was unable to make the two hour drive and knew she could not afford an emergency room visit, so she decided to stay home and "hope for the best." Fortunately, she recovered; however, she explained that she believed that "this is not right." This story instigated a discussion about whether healthcare reform was necessary or not and students began expressing their opinions about the issue.

As I reflected on students' comments and concerns, I decided that an appropriate social issue to include in the next set of *Journal Activities* would be healthcare reform. I created the following assignment—The Healthcare Debate (see Appendix B).

A major mathematical objective of the course was proportional reasoning. In an attempt to create a connection between proportional reasoning (fractions, decimals, and percentages) and understanding major current issues, I asked the students to watch President Obama's Healthcare speech (In Full, 2009) and the Republican response (Republican Response, 2009). This preceded a mathematical debate over the healthcare reform bill (which included the "Public Option") of the time (see Appendix B). The debate took place during Week 6 of the course. Students were asked to research the mathematics used for or against the bill. They were asked to use and explain the mathematics they found (with citations) to convince their classmates of their points of view. The mathematics students brought in included mostly decimals and percentages (e.g., one student found that the "United States had not contained costs as effectively as nations with broader public coverage. As the *OECD Observer* notes: 'U.S. health expenditure grew 2.3 times faster than GDP, rising from 13% in 1997 to 14.6% in 2002. Across other OECD countries, health expenditure outpaced economic growth by 1.7 times.'"—this student had to explain what these numbers meant when looking at GDP) and some fractions and graphs.

Students were divided into two groups, Republicans and Democrats; some students wanted to be placed in a certain political party group, and some had no preference. I intentionally placed students who were both for and against the political party they represented in each group, so that they would have to use mathematics to make any sort of argument and convince each other of the political party's perspective. The debate was recorded and students wrote reflections over it in their journals.

During the debate students discussed mathematics that I did not intend for them to. For example, one group brought in a set of graphs that depicted "Annual Small Business Profits Lost Due to Healthcare Costs" and "Estimated Family Income with and without Health Care Reform." They had to explain what the graphs meant and how they supported their argument.

Creating journal activities that were perturbing yet based on student interest in social issues seemed to engage students in the tasks and impact their understanding of both mathematics and social issues (discussed further in the next chapter). Every student described learning something from the lesson, and more than half wrote about enjoying engaging in this debate (see Chapter Five).

Engaging students in discussions about fractions, decimals, percentages, and graphs through a social issue they were concerned about opened up space for meaningful understanding of mathematics to surface. When they discussed what they learned, students connected mathematics to a relevant issue in their lives. Just as I expected students to negotiate meaning with the mathematical task described above, I expected them to negotiate the meaning of the mathematics they were using to make their arguments. This allowed them to make decisions about social issues, without my interference. I did not want to illustrate my perspective. I wanted the mathematics to drive their decisions about social issues.

Although I could have chosen the social issues to integrate with mathematics, utilizing the interests of students' proved to be very effective. Students had definite perspectives on the issue when the debate began and many of those changed during the assignment (as is discussed in Chapter Five). Learning what students were interested in helped me facilitate mathematical as well as social development through those interests.

I used this technique of revolving mathematics lessons around student interests throughout the semester. For example, after overhearing a conversation about the impact of commercials on their children, I developed an activity called *Commercial Math* (see Appendix B), in which students recorded how many and what types of commercials appeared during a two-hour period of children's shows. The data were combined and graphed, and this activity led to a discussion of consumerism in the United States and who makes all the products we use. I used that topic to engage students in an exploration of linear equations through sweatshops. This led to a discussion of issues of power and greed which prompted an exponential growth lesson that included these social concerns. They became consumers of information by understanding mathematics and mathematical thinkers by consuming social issues. Learning what social issues interested students allowed me to become a more effective teacher.

More Time and Space

The third paradox that drove the evolution of this course was *releasing control of time and space created more time and space* for mathematics. Time and space for mathematics in this classroom was not viewed linearly and only as existent in the classroom setting. Although I had certain mathematical objectives for the semester, I did not abide by a rigid schedule and allowed students to discuss whatever mathematics might emerge beyond that of the objectives. Initially, I held some concerns about not taking charge of class time and discussing every mathematical concept during that time.

I was apprehensive about how much time it would take to discuss extra mathematical concepts and social issues in conjunction with mathematics, particularly since when I taught traditionally, I always managed to barely cover the minimum lessons I was expected to. However, as the semester progressed, I realized that the release of time constraints actually created more time for meaningful mathematical discourse to come forward.

When I taught in a traditional mathematics classroom setting, students tended to rely on me to show them how to solve problems. I never encouraged students to work on problems before they were "covered" in class. They waited and saw how I solved the problems and then mimicked this process in the homework. This forced the responsibility of finding time to solve every kind of problem on me. I viewed class time as a time for showing students how to do every possible kind of problem they might encounter when working in isolation. When students in this traditional situation encountered a problem they had not seen before, they often times quickly reverted to asking me for help and I quickly provided an answer. Therefore, students came to rely on me for supplying them with all the mathematics of the course. Time and space for mathematical concepts to emerge was limited to the time and space we had together.

When students were invited to engage in creating their own mathematical knowledge in this class, class time took on a different meaning. Time together did not focus on what I was doing to provide students with information, but rather, it centered around what students chose to engage in. In this class, students were expected to work on *Journal Activities* for one week in advance of discussing them in class. They could post comments online to one another and utilize any resources they found outside of class, including a mathematics tutoring lab located in the Science and Mathematics Department of the community college. Gradually, working on mathematics became a continuous part of students' weeks. Time to learn mathematics became versatile and individualized, and students spent different amounts of hours discussing and working on problems. They interacted with each other outside of class, and they began including mathematics in other aspects of their lives and discussing it with other individuals outside of class. Time during class was reserved for resolving concerns or questions students wanted to address communally. It was also a time to introduce new concepts through relevant topics such as social issues. The linearity of time dissolved and concepts and subjects overlapped, eventually leading to more time for mathematics.

An important component of the class was allowing students to grapple with ideas individually before they discussed them in class. This process encouraged students to "do" as much mathematics as they could before class time was utilized for the topic. Often times, it greatly reduced the amount of time spent discussing a problem in class. In the traditional mathematics class, students wait for the teacher to describe what it is they are supposed to be doing and how they should do it. That process consumes a significant amount of class time. When the students in this class were given questions to answer outside of class, it created time for them to find ways to do the mathematics without instruction and then allowed them to find weak points they wanted to discuss with others.

In one set of *Journal Activities*, for example, students were asked to find the solutions to several problems that dealt with exponential growth. Upon beginning the small-group discussions of the exercise, I rotated around the room and noticed that every student had completed the assignment and therefore had immediately moved to discussing the next problem in the set. Time to discuss something they could all do individually was eliminated and given to working on something they wanted to address in groups. Often times, in traditional mathematics classrooms, student abilities are underestimated and so teachers believe they must tell all students how to do everything. Had I taught this concept in a traditional way, I would have spent several minutes describing how to solve the problems. However, allowing students to work on problems for a week before discussing them gave students time to grapple with and discover solutions on their own. It eliminated my responsibility "to do it all" with and certainly, for them.

Not only did engaging in this class shift responsibility of doing problems from the teacher to individuals during their time away from the classroom, but also it encouraged them to interact with one another outside of our scheduled time together. The community atmosphere that had developed in the class shifted to the "real world," and students began turning to each other for mathematical understanding when we were not together. I noticed this phenomenon after online posts began to dwindle four weeks into the semester. In my reflective journal I wrote about asking students why they were not posting comments online anymore. One student explained that she did not need to anymore because she was meeting with her peers outside of class. I asked other students if this was the case for them and received several nods. Most of the students had become friends outside of class and therefore interacted with and asked each other questions about the material when they were not in the classroom. During midterm conferences, fifteen of the nineteen students said they were in contact with at least one other student from the class during the week. One student even explained how this class had developed a strong bond between her and other students. She said, “I know Brandy and I are going to be friends forever. I’m so glad we got to meet each other in this class.”

The non-traditional curricular materials and pedagogical techniques allowed students to involve people who were not enrolled in the class in mathematical discussions and contemplations, as well. In an online post, referring to the rabbit problem discussed previously, one student wrote, “I am getting stress with these bunnies my husband and I are steel *[sic.]* working on it.” Another student, during the healthcare debate, shared a story about meeting a woman at her job who was battling cancer and had accrued over $40,000 in medical expenses because of it. She explained that the woman’s mother had developed the same form of cancer in Canada and was debt free. She described how in her discussion with the women she discovered the mathematical benefits of a universal healthcare system. Another student described working on mathematics problems at her job and coworkers engaging in solving problems with her and describing how “fun” it was to do the problems. Yet another student told the class that many of the mathematical concepts we were discussing in the class overlapped with concepts her children were encountering in school. They would work on the problems from this class and those from her child’s school together, helping each other discover solutions. The problems we negotiated in class seemed to engage others who were not direct participants of the course. They allowed the students in the course to engage in mathematics outside of class with others and yet again eliminated much of the time that would have been spent explaining mathematics problems, allowing us to focus on other issues instead.

Further, time and space for mathematics was extended by my giving students the freedom to revisit problems at any time during the semester. Allowing students to re-do problems gave time for understanding problems and reconsidering them, even after in-class discussions ended. Working mathematics problems correctly was not restricted to certain time periods in the course. I encouraged students to return to *Journal Activities* and *Closed Book Journal*s and re-do them anytime during the semester. Students were not expected to get it "right" in one try. They had the entire semester to revisit and re-work problems when they felt they understood them. During her mid-term conference, one student said:

> I like the grading system where you have another chance to be able to get your grade, because me going through here trying to get the journals, I probably wouldn't be able to pass because I don't get it at first. I need the explanation and working it over again and time to kind of go through and figure it out.

Knowing that they could return to problems encouraged students to go back to concepts that had already come to a conclusion during class time but were still unresolved for them. In a traditional classroom setting, students often never know why they perform calculations incorrectly because they are given a grade for an assignment and never encouraged to revisit the "incorrect" problems. Students often give up on the concept and view themselves as incapable of understanding it.

Instead of giving up, when students were unable to resolve problems they talked to one another and included individuals in their lives to solve and discuss problems during time outside of class, and they revisited problems after they were discussed in class. As the semester progressed, because students were interacting with the curriculum individually, with each other, and with other individuals, much of the mathematics that I would have traditionally discussed in class was resolved and topics were either discussed in more depth or new concepts emerged when we were together. At the beginning of each class, after completing their closed book journal entries students would begin contemplating solutions to journal activities and closed-book journal entries. Because of their efficient use of time outside of class, many problems were discussed briefly or not at all during class and did not require a great deal of time. In her final reflection for the course, one student wrote:

> Home journals, closed book journals, and class discussions were very appropriate methods; we could save time to solve problems in class, it led us to participate actively in discussion and to share our ideas (sometimes, it was a good chance to think from others' point of view).

When time for whole-class discussion would begin, I would systematically ask random students from each group to provide the class with their group's answer to a question. I did this for each problem of the *Journal Activities* for that week. If all other groups agreed, I might ask a group or two to present a solution or discuss a problem from a different perspective. For the most part, if we reached consensus, we would move to the next problem or begin discussing a new topic. If there seemed to be discrepancies' in different groups' answers, we would discuss the problem and negotiate a resolution. This use of time for resolving issues or seeing multiple representations reserved class time for addressing difficulties or engaging in new ideas through social issues in a way that could only be possible in the social setting of the classroom community. It was time to introduce or revisit concepts from various perspectives, rather than illustrate something the students could do outside of class. Further, students could return to problems after they were discussed in class and resolve any issues that seemed unresolved before classroom discussions took place. One student commented on her appreciation for this process during her mid-term conference. She said:

> It's like light bulbs going on. Once we talk about it and talk about it and talk about it, it's like those moments like, oh yeah, that makes complete sense. Why didn't I think about that before? I was sitting at home looking and looking and not finding it and then, okay. You know?

Recognizing that students could rely on themselves and others for mathematical knowledge made me have to rethink my ideas about how time should be utilized in the mathematics classroom. I had to let go of the idea that I needed to "cover" EVERYTHING with them in the time allotted together. Once I was able to do this, I realized that by releasing control of time in this way, I was actually creating time for more meaningful discussions to ensue.

Summary

This chapter described the critical factors that were involved in the evolution of this course. It addressed the three paradoxes that emerged as the major themes of this process. Through the curricular routines of the classroom and the utilization of *Journal Activities* that engaged students in mathematical learning, the course evolved into a setting where students could reconsider their roles as teachers and learners, where they could contribute to the formation of classroom lesson ideas, and where time and space could be reconsidered. Curricular routines provided a context for listening to each other and engaging in discussions about mathematics. These conversations emerged from the challenging tasks provided by journal activities. Although these problems often frustrated students, they proved to be motivational rather than debilitating. Moreover, the perturbation and engagement allowed students to engage in mathematical discourse over long periods of time and space individually, and with classmates, friends, coworkers, and children. The classroom community and culture that transpired as well as the quality of journal activities provided a milieu for prolonged emersion and interest in mathematics, social issues, and the course in general.

Chapter Five describes how students perceived interacting with the curriculum, participants, and classroom setting. It addressed the second and third research questions for the study. Students' emotions towards mathematics changed after engaging in this class. Moreover, students came into the course with many assumptions and understandings about mathematics, social issues, and the relationship between the two. Many of these understandings changed throughout the semester. These shifts in emotion, assumption, and understanding are discussed in the next chapter.

CHAPTER FIVE

STUDENT PERCEPTIONS

The second and third questions this study sought to answer were, "What were students' perceptions about learning mathematics in a course that combined mathematics and social issues in the way they are presented here?" and "What were students' perceptions of their understanding of mathematics, social issues, and the relationship between mathematics and social issues when they were presented in this way?" Students made many references about their perceptions towards the course in classroom discussions, during mid-term conferences, and in reflective journal entries. In these references, they described the impact participating in this course had on their attitudes about and emotions towards learning mathematics, their assumptions about mathematics and social issues, and the relationship that began to exist for them between mathematics and social issues. The first section of this chapter addresses the themes than answer the first question, namely students' perceptions of learning mathematics in this social justice mathematics classroom.

Enjoyment of Learning Mathematics for Social Justice

Elementary preservice teachers often carry negative emotions towards learning mathematics. These include but are not limited to fear, anxiety, anger, and frustration. The students in this social justice mathematics course entered the classroom with a certain amount of emotional baggage towards learning the subject. These feelings tended to overwhelm them and were often connected with certain beliefs about mathematical knowledge and its attainment. Often times, carrying repressive emotions of this sort can restrict or hinder mathematical learning (Brubaker, 1994). This section describes the mathematical attitudes students came to this course with and explores how they changed after engaging in learning mathematics the way it was presented in this social justice mathematics class. I first provide a table (see Table 3) with a summary of the change in attitude towards learning mathematics and towards student perceptions of themselves as mathematical learners. I then explain the qualitative results that led to the formation of the table and were derived from my reflective journal, classroom discussions, mid-term conferences, and student reflective journals.

Mathematical Learning	**Number of Students Beginning of the Semester**	**Number of Students End of the Semester**
Positive	4	19
Negative	14	0
Indifferent	1	0
Self-Perception	**Beginning of the Semester**	**End of the Semester**
Positive	3	13
Negative	15	2
Indifferent	1	4

Table 3. Transformed Attitudes

Initial Attitudes

Most of the students in this course described having negative attitudes towards mathematics and learning mathematics when they entered this course. During the second week of class, I asked students to reflect on how they felt about learning mathematics when they entered this class. In their responses students used strong emotive words to describe their feelings and attitudes towards studying the subject. Many chose emotionally charged words such as hate, fear, boredom, anxiety, dread, and phobia to describe their dispositions towards learning the subject. One student summed up many students feelings towards taking another mathematics course when she wrote, “I was dredding *[sic.]* this math class." Other students wrote, "Before I started this class I had a fairly negative attitude about math in general" and "When we started this class I came expecting the same stress from my previous math classes." All but four students described having negative emotions towards mathematics and learning mathematics in their reflections.

A large number of students' negative attitudes were expressed by fear and anxiety towards learning mathematics. One student wrote:

> Before I started this class I was very scared. Like three weeks before I kept waking up in the middle of the night and could not go back to sleep because I was so stress *[sic.]*.

Much of the time, this was a fear of being wrong when doing mathematics. In her reflections, one student wrote, "Mathematics was overwhelming. I was extremely nervous and always fearfull *[sic.]* that I was not doing it properly."

Often times, during this course, fear manifested itself and prevented students from engaging in discussions and defending their thoughts. During one class session, a student discovered a pattern for summing exponential numbers. I wrote in my reflective journal that "her group became very excited and asked her to show the class what she had discovered." When I asked her to explain the pattern to the class, she said, "I don't know if it's right." She was apprehensive about sharing her ideas because of the fear of being wrong. Further, in their reflections or during classroom conversations, I recorded that eight of the nineteen students discussed reservations about presenting their ideas to the class because of their fears. One student wrote, "I tend to be shy around new people because I am not that confident with my math abilities."

One of the ways we dealt with this trepidation was by developing an open, caring discourse in the classroom community (as described in Chapter Four). On the first day of class, students read in the syllabus that an important component of the class would be participation—they were told that respectful consideration of others and discussion would be expected. Moreover, throughout the semester, I would stress to students that "[I]t's okay to be wrong. A good mathematician is wrong many times before she is right. That's what doing math is all about," and students were consistently asked whether they "agreed" or "disagreed" with one anothers' arguments. These techniques provided a way for students to respectfully confirm or disprove classmates' ideas in a way that addressed their fears and anxieties rather than reinforced them.

Moreover, most of the students perceived themselves as incompetent when it came to mathematics learning. All but four students described themselves as incapable in some form or another when it came to learning mathematics. In their reflective journals, students wrote:

> I am not that confident with my math abilities it was and can be uneasy for me…
>
> [M]ath has always been very hard for me…
>
> I've never had a good relationship with math. It has always been hard for me to understand…

However, a few students saw themselves as successful at learning mathematics. Three students described their feelings towards learning mathematics with strong positive emotions.

> I was excited to take this class…I think it should be really easy and fun…

> Math is my favorite subject…I love doing math problems [e]specially solving equations…I'm always excited when I find the answer…
>
> I love to work with numbers and for me math was my special subject…

One student seemed indifferent towards studying the subject. She wrote, "I am not very interested in math outside of practical applications…"

From the experiences I have had as a mathematics educator, students tend to view themselves in a variety of ways as mathematics learners, usually with most students not having very strong emotions for or against learning the subject. However, in this class, there seemed to be many students who felt negatively about themselves as mathematical learners. Except for a few exceptions, most students did not seem to possess a positive self perception of themselves related to their mathematical abilities.

The preservice teachers in this study brought a considerable amount of emotions towards learning mathematics. Most held negative feelings towards it, a few viewed the study of mathematics as "fun," "interesting," and "exciting," and one student illustrated indifference in her attitude towards engaging with the subject. However, as the semester progressed, many students' opinions changed.

Transformed Attitudes

As the semester progressed, many of the sentiments students expressed initially seemed to transform. Students began writing about changed attitudes towards learning mathematics, expressing less fear and anxiety. They began to exude more confidence and empowerment towards their attainment of the subject. However, this was not the case for everyone.

A couple of weeks after mid-term conferences, I asked students to reflect on their feelings towards learning mathematics, at this point. Every student who had expressed negativity towards the subject previously used positive words and phrases to describe their changes in emotions. Students wrote:

> Now that I have been in this class for several weeks I have begun to feel something strange…a <u>love</u> for math! It is so weird because this is actually my favorite class this semester…
>
> To me now, math is a fun subject…It seems to be less stressing…
>
> Now that I have been in this class for about a month I have found that math can be much more fun and enjoyable than I thought…

At the end of the semester, all the students described this course as a positive experience. One student wrote, "It was the most enjoyable math class I've ever taken." Others wrote:

> I can honestly say I will miss this class and all of the people in it. It is rare for me to feel so comfortable with everyone in a classroom setting. I am usually pretty quiet and tend not to talk much but the nature of the structure in the classroom was totally different. We were like a big group of friends! I loved it!...
>
> I feel that the way this class was conducted was close to perfect. Each Thursday I knew that I would be attending a class where I would be learning, be respected, and have fun. It was really nice to actually be looking forward to class each week…

Several students began accepting the idea of "being wrong" as a stepping stone, an often necessary part of the process, towards mathematical understanding. During the same class session, when I asked students to reflect on what they felt they had learned in the class thus far, they described learning that the process of being wrong was a necessary step towards reaching a correct answer and illustrated how this acceptance was helping them become more mathematical. One student wrote:

> I like to understand why I came up with an incorrect answer rather than just knowing that it is wrong. I have learned that [this] has helped me to feel more confident in my work.

During one mid-term conference two students discussed the impact of "being wrong" on their learning. When they were asked about their feelings towards the class, they responded as follows:

> Melissa: I like it also because, I mean, we're not always wrong on our answers. I mean, we could be wrong, but we try to fix it.
>
> Sandy: I think we learn more like that… I had one class before like that, where it wasn't pressure, pressure, pressure, and I learned an awful lot more from her than I did from other classes where it was like here's a test; you have a study guide; do the study guide, you know.

As students became more comfortable with the notion of "being wrong," they also became less fearful of speaking in front of one another. One student wrote, "I kind of got over my fear of coming up to the board and it got easier each time." In classroom conversations, students began sharing ideas with each other, even when they knew they were wrong, so that they could receive input from others and understand their mistakes. In my reflective journal, on several occasions, I wrote about discussions I overheard where students would say things like, "I know this is wrong, but I'm not sure where to go" or "What do you guys think of this?" They would explain how they solved problems and would ask for group input, even when they thought they might be wrong. They seemingly no longer feared discussing ideas with their peers. They welcomed other students' perspectives and engaged in trying to solve problems mutually rather than shying away from discussing mathematics if they were unsure of themselves. Their thinking changed from a focus on "right" or "wrong" to a focus on "why" and "how" as they began to understand their own ideas and work within the support of a community. One student wrote:

> In the approach in our class, I feel a lot of the pressure that I have felt in previous math classes to not be able to mess up, is not there. I really feel the freedom to ask questions, get help, and have fun!

Students even began competing over who would present problems on the board. I noted several incidences in my reflective journal where students would announce before whole-class discussions which problems they wanted to present. One student came into class one day and said, "I want to do number ten," and another student said, "Man, I wanted to do it." During mid-term conferences two students talked about "holding back" because they felt they were "talking too much" and wanted to give others a chance.

Many students began to perceive themselves as capable of learning mathematics. During a classroom discussion, one student shared with me that this class was helping her with a geometry course she was taking in conjunction with this course. She explained that she feels comfortable doing the geometry problems in her own way, "even if it's different from the teacher's way." She had become more confident in her own abilities as a mathematician, and did not feel the need to always revert to an external authority for solving problems. Other students wrote about perceiving themselves differently as mathematics learners in their reflections. They wrote:

> I definitely still get frustrated with it, but I have definitely thought more about how often I use it, and have caught myself using it without really thinking about it…
>
> I'm slowly but surely getting it…
>
> I like math more and I'm starting to understand it better…
>
> Yes, I am gaining confidence and an increase in familiarity with problem solving and hopefully logic and reasoning…

However, for one student, the change in feelings towards mathematics was initially not a transformation towards more enjoyment and confidence. During her reflection, after mid-term conferences, she expressed the opposite feelings. In the beginning of the semester, she described mathematics as her "special subject." She explained that math was always her "strong area." She reflected on her feelings towards mathematics after several weeks in the class.

> Before the first day of class, I t[h]ought it was going to be an easy class…Now I feel like I am clueless sometimes. For me a math problem was a piece of cake but in this class it has been very difficult. I was so used to just get numbers and solve the problems. Now I feel [it] is not like that I have to think more than once to solve a problem and sometimes it is so hard for me.

The traditional nature of mathematics classroom settings that encourage memorization and mimicking was comfortable for this student. She enjoyed mathematics when it meant that she could plug numbers into handed down formulas, and when mathematics became a subject that necessitated creativity and innovation, she began to view the subject in a negative way. It took much longer for her to appreciate the structure of a mathematics classroom that encouraged discourse and a social approach to learning. However, by the end of the semester, she wrote:

> This class was conducted in an excellent way. I really enjoyed it. It helped me to see math in a different way. Interacting with my classmates was a great idea. It helped me a lot during the entire course. I think everyone learned a lot from each other.

Students changes in attitudes seemed to stem from various aspects of the class they described as enjoyable. However, the two most popular aspects were learning mathematics with others and learning mathematics through meaningful contexts. For some, a cooperative learning environment was the most dominantly pleasurable experience of learning mathematics in this way. Students wrote:

> I loved the interaction with the other students. I have to admit that on the first night when we had to work with other students in small groups, I did cringe (just a little). I was not used to working with other people, especially people that I didn't know. It was uncomfortable. But, I'm so glad you made us do it. This was another great aspect of the class that made me step out of my box…
>
> I enjoyed working with other people and being social, therefore being able to communicate freely with the other students and the professor was one of my favorite parts…
>
> I felt that everyone got along really well…

> I felt really connected in this class. I like the age range of people and the different backgrounds of experience that we all came from. Most everyone participated well and made an effort to be a part of group projects…

Other students seemed more excited about learning mathematics when problems were embedded in meaningful contexts, such as social issues. At the end of the semester, students reflected on the course one final time. Every student commented on the significance of incorporating social issues into the course. They reported:

> I think it's awesome to be able to incorporate modern day issues into math, for me it made math more interesting...
>
> I was really impressed and so glad that you brought up such important issues in our class. By just assigning those types of projects, you help people to become aware of the details of major issues such as health care and poverty…
>
> Thank you for bringing social issues in the class: healthcare reform, Indian King, commercials in T.V, and sweat shops. Personally, these issues were also my long concerns toward our society. I believe that we can transform our society to better place to live by taking responsibility for it and reclaiming it as our own…

Summary

Most of the students in this class entered the course with negative emotions and attitudes towards mathematics. Many had encountered bad experiences in their previous mathematics courses and felt negatively about learning mathematics. A few students entered the course with a positive outlook on learning the subject. However, after participating in this class, every student described learning mathematics in this way as enjoyable. Even though this positive shift did not change every student's understandings and beliefs about mathematics and/or social issues (as is described in the next two sections of the chapter), it did illustrate that teaching mathematics and social issues in the way they were taught in this course can result in an enjoyable experience for the participants of such a course.

A Critical Perception of Mathematics

Students bring to a college mathematics course already constructed knowledge and beliefs about mathematics (Ball, 1991). These come from over 2,000 hours in an "apprenticeship of observation" (Lortie, 1975), extended over at least fourteen years of mathematics education. The understandings they have formed about mathematics emerge from within these experiences and their interactions with student knowledge, beliefs, and emotions (Ball, 1991; 1998), all constructed within a social view of mathematics. During the semester, many students revealed several assumptions about mathematics that dealt with issues of power, attitudes and beliefs, and autonomy. Reading through student journals, listening to their classroom discussions, and conversing with them during mid-term conferences revealed several common themes about students' understandings of mathematics. I begin by describing those initial commonalities and proceed with an illustration of how they altered after engaging in this course.

Initial Mathematical Knowledge and Beliefs

The mathematical topics explored in this course have already been "studied," at some point in their mathematics education history. They include understanding number, arithmetic operations, understanding fractions, decimals, percentages, performing operations with proportions, understanding sets, performing operations with integers, and so on. These concepts and procedures would generally be characterized as elementary, and are visited and revisited in most mathematics courses from elementary school to college. It is expected that students who enter this course have a certain amount of prerequisite knowledge of most of these concepts. However, I found that although all of my students had at least twelve years of mathematics courses and some had taken at least one college mathematics course, few students met this expectation. The mathematical knowledge they brought seemed to be fragmented, false, limited, and disconnected from other subject areas.

Many students seemed to recognize that they had encountered many of the concepts in other mathematics courses but could not recall them clearly. The concepts were faded or contained conceptual gaps. In my reflective journal, I recorded an incident in which the class was discussing exponents and one student said to her group, “Aren’t these called exponents? I know I did these before. Isn’t there an easy way to do this?” Students often recalled seeing topics in other classes, but they had difficulty describing concepts in meaningful ways. During one class period, as a discussion of fractions emerged, students began recalling formulas for dealing with fractions. Several students explained that they recognized the operations as something they had done previously; however, many of them could not recall “the rules” for performing the operations. One student explained that she always “skipped those problems” when she encountered them on exams or in homework and another student said, “I never learned fractions when we did them in school.” During one mid-term conference, a student said:

> Percentages are clueless to me. I wish I could learn them, because I go to the store and it will say "20% off" and I'm like "man, okay, how much is that?" I hate it when I can't figure it out.

In my reflective journal, on several occasions I wrote about students commenting that "they thought they understood concepts" but often their understandings were procedural rather than conceptual, and they faced difficulties applying them. During a discussion about multiplication of multiple-digit numbers with each other, one student said, "I thought I understood how to multiply a two-digit number with another two-digit number, but it never occurred to me why we scoot over on the second line." She was referring to the commonly used procedure of multiplying the one's digit of the second number by the two digits of the first number and then leaving a space in the next row when multiplying the ten's digit by the two digits of the first number. She had always used the procedure but never thought about the logic for performing the calculation in this way.

Moreover, many students saw mathematical concepts, topics, and procedures as unrelated to each other or other subjects (the connection between mathematics and other subjects will be discussed later in the chapter). Mathematics seemed to be compartmentalized and connections between and among mathematical concepts did not exist for many students. In my reflective journal, on several occasions, I wrote about students being surprised by connections between mathematical concepts. For example, during a discussion of multiplication of fractions, I asked the students to describe how they thought about multiplication of whole numbers. After some discussion of this topic, one student said, "I had no idea you could think of multiplication of fractions in the same way." Several students affirmed this statement. During another discussion, I asked students to find 20% of the United States' population. One student said she divided by five to do this. Another student commented that she had never considered the fact that dividing by a number could be the same as finding a percentage. Most of the students' conceptual knowledge was fragmented or faded, assumed to exist, but only retrievable on a superficial level, if at all.

Not only was mathematical knowledge faded and/or fragmented for many students, but also, for several, the mathematical knowledge they brought to the course was incorrect. They had constructed formulas and notions that were mathematically invalid. During one class discussion about closure of sets under certain operations, students discovered that integers are not closed under division. This led one student to ask, "Is there a set of numbers that is closed under division?" One student asked if fractions were; however, she initially called them "irrational numbers," even though they are actually "rational numbers." Further, as the discussion continued, I asked students if decimals could be grouped under rational numbers. One student answered, "Yes, but not repeating ones." I asked the class if they agreed with her statement, and no one objected. However, several were unsure. Her statement was incorrect, and after a week of investigation the issue was resolved; however, the incident revealed how often times students construct misconstrued ideas about mathematics and the terminology associated with it.

Several such instances occurred throughout the semester. During a discussion of operations with fractions, I asked students to tell me what they knew about multiplication of fractions. One student said, "Don't you need a common denominator?" She was using "the rule" for addition of fractions and applying it to multiplication of fractions.

Yet on another occasion, a student proposed that for addition of integers, “when the signs are different, your answer is negative and when the signs are the same, your answer is positive,” applying “the rule” for multiplication of integers to addition of integers. This illustration of false knowledge appeared on many occasions during the semester and demonstrated that many students had taken bits of knowledge, combined them, and created misconceptions about mathematics.

For several students, mathematical knowledge seemed to consist of memorized formulas and procedures that allowed them to conduct the motions of doing mathematics but limited their abilities to explain how or why they did mathematics in this way. Students began to recognize this tacit knowledge in themselves. One student wrote in her journal:

> I’ve never thought about the mathematical fundamentals which the formula derived from real examples, but just memorized formula, substitute some numbers for X or Y in it, and did the mechanical calculations.

Another student expressed anger when she realized that she had been taught in this way. In my reflective journal I noted her saying, “Why don’t they teach us in school where the rules come from?” Early on in the semester, many students would explain they could perform a calculation, but did not know why they could. One student said, “I know that that’s the rule, but I don’t know how to explain it.” It became a recurring joke in the class that students were not allowed to explain their reasoning for a solution by saying, “That’s the rule.”

This limited knowledge became evident as students began to grapple with mathematical concepts they were unfamiliar with from school. For example, in one set of journal activities students were asked to perform various calculations in bases other than base ten. Many students illustrated their tacit knowledge of place value. In base ten, often students add and subtract using procedures that include techniques such as lining numbers up one on top of the other and "carrying the one" when the numbers add to a sum larger than nine. However, this procedure does not work in other bases. For example, in one problem, students were asked to find the following sums:

$$304_{\text{five}} + 20_{\text{five}} + 120_{\text{five}} + 22_{\text{five}} =$$

$$201_{\text{three}} + 102_{\text{three}} =$$

Many students proposed the answers as:

$$304_{\text{five}} + 20_{\text{five}} + 120_{\text{five}} + 22_{\text{five}} = 466_{\text{five}} \text{ or } 121_{\text{five}}$$

$$201_{\text{three}} + 102_{\text{three}} = 303_{\text{three}}$$

Correct Answers:

$$304_{\text{five}} + 20_{\text{five}} + 120_{\text{five}} + 22_{\text{five}} = 1021_{\text{five}}$$

$$201_{\text{three}} + 102_{\text{three}} = 1010_{\text{three}}$$

They had difficulty recognizing when to switch from one digit's place value to the next, and even when they did, they often forgot to place a zero in the number if one digit's place value was not a number greater than zero (as is indicated by the second incorrect solution to the base-five problem).

The preservice teachers in this class often seemed to only have a limited understanding of mathematical concepts. However, many of these concepts, such as conducting mathematical operations on numbers in bases other than base-ten, they are expected to teach as elementary school instructors. Without a deeper understanding, how could they be expected to facilitate their own students' learning or recognize misconceptions or misunderstanding between and among them?

Not only did students in this class bring with them faded, fragmented, incorrect, and limited knowledge, but also they brought many beliefs about mathematics. Most of the students initially saw mathematics as dualistic, as containing only right and wrong answers. They also viewed mathematics as a subject void of creativity and contexts, outside of situations such as "budgeting"," recipes," and "construction." Two students described many of their classmates' attitudes when they wrote, "[b]efore I took this math class, I thought that math was pointless…" and "[a] lot of it I saw as something I'd never need." Further, when many students reflected on their previous experiences with mathematics, they often explained that for them mathematics meant performing mathematical operations—"doing math problems," "solving equations," "adding and subtracting problems," and "working with numbers"—that are handed down and verified by an external authority.

The students in this course at the beginning of the semester tended to view mathematics as void of creativity, context, and imagination and as consisting of "numbers and formulas." Early in the semester, in her reflective journal, one student wrote that mathematics in school is "teaching kids to always have the right answer or just teaching them to pass tests." Others wrote, "[t]here is a wrong way and a right way to do a math problem," "math was the way the teacher taught it" and "I thought math was a hard and boring subject that just plays with mysterious numbers and symbols." Moreover, viewing mathematics in this way separated it from other subjects. Many students described mathematics as being a subject that has "one right answer" or is done "one way." In their descriptions of their previous mathematical experiences, almost every student, including those who enjoyed mathematics, referred to the idea that there is only "one right way" to do mathematics. Doing mathematics "one right way" sparked resentment towards the subject for many of the students. In a classroom discussion, I noted one student describing being punished for not doing mathematics in a particular way. She said, "I used to have a teacher that would count off if we didn't do problems exactly like she wanted." Many students echoed this statement, saying they had similar experiences. Several students explained they were not encouraged to ask *why* in their mathematics classes. One student said her teachers always told her "that's the rule" or "you'll find out later," so she began to refrain from asking. Students viewed mathematics as a subject that is transmitted as whole and complete from an all knowing authority to the less knowing student. One student wrote that in her mathematics classes, "there was always someone showing the how to's, the right way per say *[sic]* of doing a problem."

Further, several students viewed mathematics as reserved for certain people. They often viewed themselves as "different" from those people. In my reflective journal, on several occasions, I noted one student saying, "I'm just not a math person." Students often wrote about not possessing special mathematical abilities. One student wrote, "I think my brain does not automatically think mathematically it is something I have to work at." Another student once said in class, "My brain just doesn't work that way."

These types of beliefs and attitudes about mathematics revealed how mathematics is often viewed in society. They illustrated the idea that mathematics is reserved for "special" people and that only some students have the ability of or the "gift" for doing mathematics. They reinforce the constructed, oppressive implication that there is one correct way to solve problems and do mathematics, and that only those who have the "gift" can dictate to all others what and how mathematics should be taught and learned. However, although many students described resenting the authority of the teacher or the lack of questioning they had previously experienced, one commonality that emerged in the first few weeks of class was the degree to which students sought external verification of their work.

As students interacted with one another and encountered different solutions and answers to problems, they showed an excessive urge to legalize their answers, even if they thought they were right. However, I expected them to discuss and resolve issues on their own. Just as in society, where there is no right or wrong answer to everything, and people must engage in developing ideas, sharing them with others, and negotiating their validity and/or transformation, students in this class were expected to justify their thoughts and decide whether or not they agreed with one another. I was instigating the social construction of knowledge rather than asking my students to find single answers in one way (von Glasersfeld, 1995).

After the first class session, when students were asked to work on several mathematical tasks in groups, I wrote about students' initial frustrations with this process. As I rotated from one group to another, various forms of one question kept emerging: "Is this right?" I kept giving the same answers: "I don't know. What do you think? Why?" Students were not always pleased with these inquiries as answers. They seemed frustrated when I refused to answer their questions, and they exuded little faith in themselves and their answers. Various questions and comments such as, "Why can't you just tell us?" or "I don't know. It just seems right" radiated at each table. I asked students to convince other group members, and I probed their thinking by asking, "What made you think to go about the problem that way? Is there a difference between what you did and what she did? Where?" During one class session, as the time for discussion came to an end and one task remained unresolved, I announced to the class that I would not give them the answer. Students seemed shocked. I noted one student saying, "What? How are we supposed to figure it out? What if we can't figure it out?" Several students expected me to announce the answer or give them a solution during the next class session; however, as one more week passed, without my doing so, students became even more frustrated. This set in motion the recognition by students that intellectual autonomy rested in their hands as they worked towards agreement (Kamii, 2000). After the first week, one student wrote in her journal, "I am now nervous because this class seems like it is going to be a lot." Many students used the word "different" when describing the way the class was being conducted. For the first several weeks, I wrote about the frustrations students depicted in the classroom as they were forced to determine solutions and validate ideas, without my affirmations.

The need for external validation from an authority seems to come from the misconception that mathematics is something complete and whole that is handed down from an expert to a novice. For several weeks in this class, students seemed uneasy about the notion of constructing ideas socially. They viewed themselves as unable to know the answer or verify solutions on their own. Ironically, this view of mathematics is contrary to the construction of the subject in our world. Mathematics has emerged and continues to be created from the questions that are answered and verified through the internal and social interactions of people. They are not verified externally, even though, that is often how they are transmitted in the mathematics classroom.

Transformed Mathematical Knowledge and Beliefs

As students interacted with one another, the instructor, the curriculum, and individuals in their lives outside of the classroom, they began to change many of their ideas and beliefs about mathematics. By the end of the semester, every student had written about how this course had helped her mathematical understanding. The most common theme was the emergence of a conceptual understanding of mathematics. Further, students began to ask *why*, they started to develop questions outside the scope of the mathematical objectives, they began to use and appreciate multiple representations for solving problems, and they began to make connections between mathematics and their worlds outside of the classrooms.

Many students wrote about how engaging in this course created a deeper conceptual understanding of mathematics for them. They could now explain formulas they knew, understand other students' mathematical ideas, and teach mathematics. They began to view mathematics as more than procedures and formulas. Several students wrote and commented about understanding the concepts "behind" the rote motions. At the end of the semester, two students wrote:

> I definately *[sic]* feel I learned alot *[sic]* from this class. Mostly, I have learned the concepts behind the math problems that I've been doing my whole life which I thought was very important, and I realized that the way I had been taught was kind of sad in a way! I was taught repetetive *[sic]* procedures and had no clue as to what was behind the concept…

> [T]his class introduced me a very new way of solving math problems which was different that I learned at school before. I've never thought about the mathematical fundamentals which the formula derived from real examples, but just memorized formula, substitute some numbers for X or Y in it, and did the mechanical calculations. Even though the math problems were all in elementary level, it was more important to understand the ground principles.

During a discussion about fractions, one student said, "I've been through elementary school, middle school, high school, and two college math classes, and I never understood fractions the way I understood them in this class." Another student, who confessed that she had a physics degree and was only taking the class to maintain a student visa, in one of her reflections wrote, "I feel comfortable to come to the class and I enjoy the class…Now I wonder if I can understand the equations in Quantum Mechanics which was a great distress throughout my college life."

As students began to develop deeper understandings of mathematical concepts, they began to justify and defend their solutions. After several weeks of enrollment in the course, students stopped asking me for verification of answers. They began to ask one another and trust themselves. Several students commented on the effect of allowing them "to figure it out" on their own. Students wrote:

> The main thing that impacted me was your teaching style. I really hope to be able to encourage my students to learn by letting them figure it out on their own like you did with us…

> I think the thing I like the most about it [the class] is being able to figure out the problems…A lot of times it takes me a little bit of extra work to figure things out. But that[‘s] okay.

Some even began to ask questions and explore mathematics beyond the expectations of the course. They became intellectually curious about mathematics.

Often, in traditional mathematics classrooms, students’ intellectual curiosity is limited to searching for correct answers to mathematics problems. Traditional teaching methods transmit procedures for solving problems and when students utilize these procedures and produce answers which are externally verified by an authoritative figure, the need for further thought or exploration is limited because the goal of producing the correct answer has been achieved. However, when students begin to ask questions and understand mathematics more deeply, creativity emerges, and students begin to form connections and ask further questions. This curiosity materialized on several occasions throughout the duration of the course.

On several occasions, students noticed patterns that caused them to pose questions. In my reflective journal, I noted one class period when a question about even and odd numbers turned into a conjecture and a proof. While studying sets and whole numbers, one student noticed that every time she would subtract two odd numbers, she would obtain an even number. She asked me if I thought that would always be the case. I redirected the question to the class and asked them what they thought and how they might find out. By the end of the class, a student illustrated a proof of the conjecture that “an odd number minus an odd number equals an even number.” In another class period, I observed a student noticing a pattern when working with exponential growth. She proposed the pattern to the class, and we worked together to verify it. It was an unexpected result that led the class to discuss summation notation and how finding a pattern can lead to writing a formula. Yet on another occasion, I wrote about a student asking if "repeating decimals" could be rational numbers. Again the question was redirected to the class, and students worked together to decide if "repeating decimals" could be fractions. I did not anticipate any of these questions or results.

The curricular routines of this class allowed students to explore multiple ways of representing mathematics. Students began drawing pictures, using manipulatives, and relating topics to previously studied concepts to solve mathematics problems. Several students wrote about the impact of using multiple representations on their mathematics learning. Students wrote:

> I've gained the knowledge of how to use manipulatives and visuals...
>
> All the hands on activities have helped me a lot…
>
> I learned that there isn't just one way to solve a math problem. You don't always have to remember the "rules". You can use base ten, fraction bars and other manipulative kits…
>
> I love that we learned the reasons why some math works and how to do it with different materials rather than just teaching rules…
>
> I definitely learned more about fractions! Thinking of it as multiplication and benchmarks helped…

Another student described the manipulatives as the main "tool" that impacted "learning or retraining my brain to do and think about math in new and different ways." On several occasions, I wrote in my reflective journal that students would comment on the effects manipulatives had on their mathematical understandings. During a discussion of decimals, one student said, "I never thought to use base-ten blocks when working with decimals, but they really help." Another student described the importance of color counters in her development of integer multiplication. Yet another student shared with the class her request for manipulatives for Christmas because of how much they had helped her. Students no longer limited working mathematics problems to paper and pencil calculations.

As the semester progressed, not only did students begin to transform their mathematical knowledge, but also, they began to express their change in beliefs about mathematics. Although many initially viewed mathematics as dualistic, right or wrong, and void of creativity and context, by the middle of the semester many students began viewing mathematics differently. Students began rethinking the nature of mathematics. They began to express their newfound ideas about the significance of mathematics and the process of learning mathematics. Engaging in mathematical learning became an active rather than a passive endeavor. In their reflections students wrote:

> While I attend this class and solve some math problems, I rethink the importance of math education. I realize that math is [a] significant mean[s] to facilitate the ability of reasoning and logical thinking…
>
> Now I feel like that I really use my brain…
>
> I learned how to see math in a different way. This will help me to teach math in a different way…

Many students commented on the variety of ways of doing mathematics. Students wrote:

> I have learned that math is a subject that causes you to think…I have realized that not everybody thinks the same way. There is always more than one right way… There are more than one way to get an answer and it's ok if its *[sic.]* not the way the teacher taught it…
>
> I really enjoyed this math class. It expanded my horizons and allowe[d] me to understand math a little bit better. I began to realize that there is not just one way to find an answer…

Further, students began to change their views of mathematics as reserved for certain "special" or "gifted" individuals. During one class session, I heard a student who had previously said that her brain did not think mathematically, say, "I can do this kind of mathematics." One student who wrote about her "bad experiences" with mathematics early on, in a later classroom discussion said:

> Letting us do math this way is also good for students who come from different countries because in their countries, the teacher tells them to do problems one way and when they come here the teacher says do it another way. Like this, they can do it how it makes sense for them.

Other students wrote in their journals, "I'm learning that there are certain parts of math that do click with me," and "I think I would have done much better in math if my teachers let me do it like this." They began to express learning about themselves and their capabilities as mathematics learners and teachers after engaging in this course.

> I learned a lot about my self in this class. This class gave me a boost of confidence mathematically and as a future teacher. I felt so bad about math when I began this class and now I know that I can learn math and I can teach it too!...

> I learned how to teach math and that it's okay to go against the way I was taught as a child…
>
> So far, I've learned the importance of having a flexible mind when working with math. I need it to be reminded that there are many, many ways to solving math problems…

Students also began to view mathematics as connected to their worlds. By the end of the semester, more than half of the students wrote about recognizing mathematics outside of the classroom. Students wrote:

> Math is around us every second. It has to do with everything that we do eventhough *[sic.]* we do not think about it…
>
> A lot of it I saw as something I'd never need; however I guess I'm kind of noticing how often I use it…
>
> We use math in everything we do…

On several occasions throughout the semester, I noted students making references to connections between mathematics and other areas. During one class session, a student described to me how she "had an epiphany" during her geology class. She explained that as she was learning about plate tectonics, she realized she was exploring Venn diagrams, something we had done in our class. Another student, during her mid-term conference discussed realizing in a social studies class that "the maps we were studying are math. I never thought about that before." Yet another student, during her mid-term conference, described "getting annoyed" because she kept "recognizing math" and realizing that she was recognizing it. One student even proposed that "we use math more than we use words."

Working in groups and realizing that people view mathematics in different ways seemed to contribute greatly to students beginning to see mathematics differently. In their final reflections, almost every student commented on the effect of seeing "more than one way" to perceive or solve problems. The welcoming atmosphere of the class that encouraged diverse perspectives expanded many students' notions of mathematics. They wrote:

> Working in groups and sharing other people experiences with math has started changing my point of view in many aspects of math. I'm actually starting to enjoy math!...
>
> It was very insightful for our peers to help us with our problems. It allowed for different techniques to be shown…
>
> It was also a pleasent *[sic]* surprise to be able to see that there are many ways to achieve the right answer and that it was alright for everyone to have different opinions and approaches to logic…the overall acceptance and understanding of view points was comforting…

As students were exposed to multiple methods and solutions to problems, they began to reconsider their ideas about the notion of there only being "one right way." They began to appreciate diversity in problem solving and gain confidence in their own mathematical thinking. One student wrote, "I have learned that working with others as a group has helped me to feel more confident in my work and I seem not to second guess myself as much." Providing students with the freedom and space to engage in mathematics in their own ways and through multiple perspectives and representations boosted their confidence and understanding.

Resistance

Although most of the students' understandings of and beliefs about mathematics changed, for some, this change was limited to the mathematics we engaged in as part of this class. They viewed this mathematics as different from other mathematics and continued to harbor many of their initial beliefs towards the subject when they wrote and spoke about other mathematics courses. Students wrote in their journals:

> I do not know if this approach would work for all math classes but I was grateful to be a part of it and I am looking forward to incorporating some of these ideas in to my classrooms if given the opportunity…
>
> Higher math did, and still does seem kind of pointless, but my opinion is changing.

During a classroom discussion, I noted one group of students discussing enjoying "the way we do math in this class." They went on to explain they did not believe this approach could be taken in an Algebra course or any other upper-level mathematics course.

Further, in their final reflections for the course, three students depicted that they still maintained some of their initial perceptions about mathematics and themselves as mathematical thinkers. Students wrote:

> I still struggle with math so there where *[sic.]* many different weak points for me. I always struggled with the homework…
>
> I learned that math is something that I really need to work on. I need to practice alot and really retrain myself in my ways of thinking about math…

It is difficult to determine why some students continued to see mathematics in the ways described above. It could be a result of the many years of experience in traditional mathematics classroom settings, or it may be other factors. Further research is needed to determine what exactly maintained these beliefs for some students in this course. However, while this was the case for some students, it was only so for a small minority of the class.

Summary

Indisputably, engagement in this class increased most of the students' understandings of mathematics and changed their beliefs about the subject. As students confronted their mathematical capabilities, they began to perceive themselves as capable of teaching and learning mathematics. They gained confidence in themselves as mathematicians, they began to see more than one way to understand mathematics, they began to appreciate diverse processes and perspectives, and they became autonomous learners and teachers of the subject.

A Critical Perception of Social Issues and Their Connection to Mathematics

Just as students bring to a college mathematics course already constructed knowledge and beliefs about mathematics, they also bring already constructed knowledge and beliefs about society. Preservice teachers enter their college careers with well established and tacit beliefs about culture and society. Many of these stem from their cultural identities that are composed of race, social class, gender, health, age, geographic region, socioeconomic status, sexuality, religion, language, disability, ethnicity (Cushner, 2003) and the interactions between and among these with student knowledge, beliefs, and emotions. Preservice teachers' perspectives and opinions about social issues have been constructed from within this framework.

During the semester, many students revealed several assumptions about social issues that dealt with perceived knowledge, beliefs, and autonomy. Reading through students' reflective journals, listening to their classroom discussions, and conversing with them revealed several common themes about students' understandings of society. I begin by describing those initial commonalities and proceed with an illustration of how some of them altered after engaging in this course.

Initial Social Issue Knowledge and Beliefs

Social issues knowledge in this study is taken to mean an understanding of critical concepts and problems that affect society. Most of the students in this study had encountered the concepts and issues we studied in this class in some form or another prior to their discussions in class. Knowledge of critical social issues and concepts was limited for many students. Some acknowledged themselves as having little knowledge, while others discovered their limited understandings as issues emerged in class, and others perceived themselves as knowledgeable but could not explain their thoughts when asked to discuss them in class.

In their reflective journals, students were asked to write about social issues, sometimes before an issue was discussed in class, and sometimes after. Students wrote about and discussed their lack of social issue understanding in many of these reflections. Some revealed that they "do not follow" or are confused by many issues. They wrote:

> Personally, I don't follow what President Obama is doing…
>
> I'm personally not involved in politics and I'm not particularly tied to any party…

> As an international student, I really had trouble to understand what is the current health insurance system in the United States. Besides, the idea of health care reform made me more puzzled…
>
> I do not follow politics very much…

Other students discovered their limited knowledge as issues and concepts were discussed in class. During a discussion of poverty in our state, students discovered that approximately sixteen percent of the population lives in poverty. I wrote in my reflective journal that students seemed shocked by the figures. One of the tasks of the journal activities for that week asked students to calculate how many cities with a population of 40,000 each it would take to hold all people living in poverty in our state. They discovered that it would take approximately thirteen of them. I noted several students making comments that indicated their surprise by this discovery. In another activity, while students learned about sweatshop worker wages and conditions, they realized that many sweatshops exist in the United States. Students made comments like, "I didn't know we have sweatshops here. I thought they were only in other countries." As students reflected on the lessons we did, many wrote about being surprised by the limited knowledge people have of critical social issues. One student wrote, "I learned how little most people truly know about very important subjects."

Although many students discovered their limited knowledge as the semester progressed, some initially thought they had strong understandings of issues, but when they described those understandings, they portrayed conceptual gaps in their depictions. For example, before the healthcare debate, one student wrote in her journal:

> Other countries have universal healthcare, but they pay for it through taxes…Those countries have high taxes and we don't want to pay higher taxes. We have always prided ourselves on self-sufficiency and freedom, not on government handouts and mandates.

This student believed that a universal healthcare system would mean higher taxes and therefore more money out of her pocket to pay for healthcare. However, in her assessment, she did not consider how much money Americans pay for private insurance. Although she understood that taxes would increase, she did not consider whether overall healthcare costs would increase or decrease per person. Her understanding was incomplete and did not consider all factors involved in individual cost. Similarly, in my reflective journal I noted one student objecting to health care reform "because doctors would make less money, and no one will want to be a doctor." When I asked her how much doctors would make, she answered, "I don't know, thirty or forty thousand." She believed she had a solid argument, but it was limited to a superficial statement, not a solid understanding.

Moreover, some students illustrated their limited understandings by using one or two example as "proof" of an argument. For example, one student wrote in her journal:

> Doctors don't make money through Medicaid or Medicare…when I was 18 years I used to work with a Dr and his wife used to put things in the forms that was not even done to the patients so they could get more money. By the time I retired and want to get Medicare, there is not going to be any…

This student's evidence of Medicaid and Medicare not working was based on one personal experience.

One reason why students' knowledge of social issues was shallow may be attributed to the fragmented nature of the information students receive from outlets such as the media. Most students had some exposure to the topics and issues that emerged in this class, but their knowledge was often limited and unsupported, often preventing broader understandings.

Likewise, students seemed to have inaccurate knowledge about social issues. Before the healthcare debate, one student wrote in her journal, "I don't find it right for me to pay for someone else's healthcare." She had explained that her opposition to healthcare reform stemmed from this reason. She seemed oblivious to the fact that she pays for uninsured citizens' healthcare under the current system. Further, several students agreed that they opposed healthcare reform because, as one student put it, "People with socialized medicine pay more for healthcare." However, during the healthcare debate, after conducting their own research, they discovered that Americans pay more for healthcare per capita than any other industrialized country in the world.

Many students based their knowledge on belief rather than evidence. They perceived viewpoints of social issues as connected with opinion, not substantiated support. When they spoke about social issues, they used their outlooks to describe problems in society and whether or not they believed issues should be of concern. Student wrote in their reflective journals:

> I know that for the past years the federal government has repeatedly try to extend health care coverage to all Americans and they have failed. Now, the Republicans cannot permit a health care bill to pass and the Democrats want to pass it to hold the majority in government. My very personal opinion is that the

government does not care about the people and that politicians are addicted to our money…

I think people are afraid to let the government 'have control' of their medical issues and are worried about long waiting periods for specialists…

Another student described poverty as a problem in our state because, as she put it, "I think people are just lazy and don't wanna' get a job." Several students echoed her sentiments, while one student objected to this statement and explained that she believed that a lot of those people "can't get a job" for one reason or another. However, she too, explained this as her opinion. On several occasions, I noted in my reflective journal that students would use phrases like "I think," "I believe," or "in my opinion" to describe their perspectives of issues we came across. They did not seem to find it necessary to verify their opinions with supportive data.

Some students adamantly expressed they understood concepts and issues but when probed could not explain why. For example, I wrote in my reflective journal about a discussion of socialism that emerged during one class period. One student said, "I know that socialism isn't good." Several students agreed with this statement. When I asked students to explain to me why they believed this, one student said, "We know it doesn't work. Look at the Soviet Union." As I probed further and asked her what she meant, she could not explain exactly why she believed the Soviet Union failed. One student tried to help out by saying, "I just have a bad feeling when I think of socialism. I imagine this scary, dark place, where everyone is working and no one is happy." No one could give evidence, other than mentioning the Soviet Union as an example, for why socialism is bad. Many simply said things like, "I just know" or "We've always known that." Further, students seemed surprised when I suggested that socialism exists in the United States. I asked them if they agreed with having a fire department, police department, public library, and public schools that served all citizens. Everyone agreed that these are "good," but seemed surprised by my terminology when I called this socialism. One student said, "Maybe sometimes socialism is good."

Students often compartmentalized their knowledge of social concepts and grouped certain ideas together, viewing them as dualistic and disconnected from each other. For example, many students used words such as socialism, oppression, and dictatorship together and words such as capitalism, freedom, and democracy together, and they saw these groups as disjointed and unrelated to each other. They had difficulty envisioning overlap, as was the case when we discussed socialism and the United States. They believed that the categories they had formed were correct but could not explain why. The isolation of this knowledge often prevents people from seeing interconnectivity between social concepts and prevents envisioning alternatives to society. When students encountered words such as socialism, democracy, oppression, and freedom, they often viewed them as good or bad, without being able to describe these concepts in meaningful ways. They viewed discussing these concepts as engaging in a moral debate about whether they were good or bad, and seemed to believe that when this is the debate, there can only be one answer. They had difficulty envisioning that there are instances when concepts such as socialism can be good.

Moreover, students in this class brought many beliefs about the nature of social issues. The majority saw social issues as dualistic, having two sides—a right side and a wrong side. Their positions were seemingly based on opinion and the verifications of others, and they often reverted to emotions for constructing ideas about social issues. Their journal reflections and comments in class revealed a belief that solutions to problems in society could be found quickly and easily and indicated a common held belief by many people regarding the nature of critical social problems, as simplistic and easy to solve.

Students in this course tended to view social issues as existing within two perspectives, a correct one and an incorrect one, often times based on their faith in others. Students used the words "right" and "wrong" when they discussed social issues. After being placed in the political party that she opposed for the healthcare debate, one student said, "I could do this project if I was on the right side." For many of them, there seemed to be no room for common view points from the two sides or an alternative perspective to the two.

Often times, students' beliefs about social issues seemed to be based on the verification of an absent external authority. In my reflective journal I wrote about one student saying she had received her information from the news, while another student explained that her family had discussed these issues. Many students' perceived right and wrong perspectives seemed to come from their affiliations with certain political parties or their connections to significant others in their lives. Many students categorized themselves as Democrats or Republicans. One student explained that although she does not always understand issues well, "I do know that when it comes down to it I am a republican." I noted several occasions when students discussed their spouses, parents, churches, and coworkers' when they described their own ideologies.

I would frequently try to perturb students' assertions, and they would often become frustrated and fall back on external authority or previous experience as their verification for their versions of the truth. Many students would revert to their respective political parties as having the correct answers to issues. The loyalty they felt towards their political parties and significant others was difficult for some to overcome if they began to view issues from other perspectives. They seemed to feel it was a betrayal if they opposed one of the stances those others perceived as correct. One student explained how difficult it was to rethink some of the issues we studied because of her husband's viewpoint. In class, she explained trying to discuss issues with her husband but encountering resistance when she did so. She said, "He's a staunch Republican and he just doesn't wanna' hear it." Another student, after a discussion about distribution of wealth and a change in opinion said, "I feel like I'm betraying my party if I agree with you," as if she was doing something "wrong" by considering an alternate perspective. They often resisted the perturbation I offered.

Many of students' opinions of social issues were seemingly tied to emotions rather than understanding, and the most common emotion students connected with their view points was fear. Students often used fear or worry to describe why they believed in certain ideas. Before the healthcare debate, several students discussed fearing "a government takeover of healthcare," "increase in taxes," "bureaucracy", long lines at doctors' offices, and so on. During a discussion about war, several students expressed that war was necessary because they feared for our national security. In a conversation about immigration, one student explained that we needed to take control of illegal immigration because she worried that "they'll take all our jobs." I wrote that several students did not like the idea of redistributing wealth because they feared the money they would work hard to make would be taken away from them. Often this fear contributed to their resistance of exploring other ideas and was an easy way to maintain a certain perspective.

The complicated nature of social issues can make it difficult to decide what to believe; therefore, many students accepted ideas they encountered in their surroundings that were based on the beliefs of people they trusted or political parties they were tied to. They perceived social issues as based on opinion and often swayed in the direction of what felt less threatening emotionally. However, as the semester progressed, many students began to refrain from relying on opinion and emotion to make decisions. New knowledge began to emerge as they investigated social issues for and through mathematics.

Transformed Social Issue Knowledge and Beliefs

As students reflected on the social issue lessons we did in class, everyone described learning something from them. Many started with phrases similar to "I learned a lot from this lesson." The majority described making sense of social issues they did not understand well previously. They were able to develop arguments to support claims they made, and many of their understandings of social issues changed once they investigated them through mathematics.

Many students came with definite opinions about social issues. For example, before the healthcare debate, many students opposed healthcare reform because of many of the reasons described above. However, after concluding the debate, most of the students in the class were surprised to discover that they favored healthcare reform. They began to defend this viewpoint with evidential support. As students reflected on the debate in their journals, they wrote:

> The U.S. spends far more than any other industrialized nation on healthcare. Yet, other nations insure everyone while America has 46 million uninsured, a number which will grow as health insurance costs rise…
>
> A fundamental problem in evaluating reform proposals is the difficulty of estimating their cost and potential impact. Based on the convincing financial data, U.S. government should persuade the people and get more support from them…

Further, by the end of the debate, many students began to not only support healthcare reform but also favor a universal healthcare system. One student who was completely opposed to healthcare reform at the beginning of the assignment later wrote:

> According to statistics from 2003, the United States spends $5,711 per capita per year for health care while Canada spends about half of that, $2,998 per capita per year (Kaiser Family Foundation, 2007). Cuba with their well socialized healthcare system spent only $251 per capita on healthcare in 2006. The reason socialized insurance is much cheaper is because single payer eliminates the health insurance racket with all of its waste in capitalist profits, paperwork, and overpaid CEOs. According to (United Nations Development Program, 2006) socialized healthcare does work.

Another student wrote:

> I learned a lot of great statistics from the debate. I learned that socialized healthcare would benefit more people than it would hurt. ..Socialized healthcare is

> established in many countries…This program is working perfectly fine in these countries.

They began to realize there is more to forming judgments about social issues than simply relying on emotion, opinion, or others. Their understanding of social issues became more explicit and they were able to make decisions based on a more precise understanding. Further, as students' understanding developed, they began to defend their new found ideas. During the classroom debate, one student from the Republican side gave a figure for the cost of healthcare reform on families in the United States. A student from the other side quickly referenced a different figure that showed a lowering of cost for families in the U.S. and she explained why this would be the case. Students even became more interested in social issues and began to ask questions about them, expanding their intellectual curiosity.

Students became more critical of their understandings of social issues. They began to develop interest in and questions about them. In their reflective journals, students wrote:

> I learned that I have many more questions economical, fiscal, financial and political...
>
> All of this leads to one question. Why are we not looking into what other countries are doing?...
>
> Why is the U.S. not following countries where socialism is working?...

They began to explain how incorporating social issues into this course sparked their curiosity and interest in understanding social issues. Students wrote:

> The lessons on social issues I think helped the class not only to incorporate and think about the math within the subject, but sparked further interest in the issue itself…
>
> I found that I would continue to think about these issues days and weeks after the lesson had concluded…
>
> It has become a subject I think about daily. Not only how it effects *[sic.]* me now, but how it will effect *[sic.]* my kids in 10-20 years…

They began to question one another's knowledge and ask for evidence to claims others would make. In my journal, I wrote that after students engaged in a couple of social issue lessons, they no longer took classmates' comments at face value. I wrote about one student making a comment regarding sweatshops and another saying, "Where did you get that information, because I found something different?" They began to view a meaningful understanding of social issues based on factual information rather than beliefs as important.

Students became more critical of their own understandings and began advocating teaching for social understanding in all subject areas. One student, wrote about herself, "To be honest I did not know a thing about this healthcare reform…This is bad for me because I should be informed." Other students wrote:

> Incorporating math into those everyday things is so important, especially for kids because we should be teaching them to become better PEOPLE, not just better STUDENTS!!
>
> We reclaim society from giving attention, rediscovering on many controversial social issues. Throughout this process we can find possible answers. Teachers are

> not people who hand down only scholastic knowledge to the next generation, but also help them to build desirable insight into our social problems…
>
> I believe that there are some social issues I think that we as educators have a responsibility to raise an awareness of and to urge a greater accountability for people in all areas…

Engaging students in mathematics for social justice deepened their understanding of social issues. It made it more explicit and gave them the ability to investigate issues and make decisions based on the evidence they accumulated. It also created an interest in the issues, encouraging them to continue to explore issues, even when we were not deliberately addressing them in class. They began to value understanding social issues and some started advocating educating for social well-being.

Moreover, after engaging in mathematics lessons that incorporate social issues and concepts, students began to change many of their beliefs about the nature of social issues. They began to reconsider their quick conclusions about which side is "right." Initially, students seemed to believe that they had to confer with one perspective or another when discussing issues of social concern. One student explained in class that she believed she had to "choose a side" when she encountered arguments for or against certain solutions to social issues, but after the healthcare debate, she expressed that she was "torn" and did not know which side was right. Other students seemed similarly confused by their inability to quickly choose a side when considering social issues and concepts. They began to view issues as not so black and white, and they began to revert to mathematics as the connection to their beliefs rather than emotion or an external authority.

Although most of the students in the course continued to view social issues as stemming from two perspectives, they began to have difficulty categorizing each side as right or wrong. Students no longer seemed to be able to quickly label others' points of view as simply correct or incorrect. Many began viewing positives and negatives from both sides. Students wrote in their journals:

> After researching I found that on this topic both sides have great points and I am torn…
>
> I learned to keep my mind open. There are many good points on each side of the healthcare debate…

Students began to question their reliance on significant others in their lives for understanding social issues. They began to trust their own abilities to investigate and understand problems in society. Students wrote:

> I learned how important it is to investigate issues for your self and not just believe what your friends tell you or what you hear on the news…
>
> We should not take our leaders information to heart, always question, always do your research…
>
> It was interesting to investigate both sides and then make my own choice…

Participating in this class opened up space for students to discover verification and construction of ideas from within themselves. Students began to author their own learning and understanding, validating and verifying their ideas without the reliance on an external authority.

The transformation of students' beliefs about the nature of social issues stemmed in large part from considering other perspectives through mathematical research and working with students' whose perspectives were different than their own. Many students, during mid-term conferences, explained having never thought about diverse outlooks of issues. In their reflections, several students wrote about being affected by other viewpoints. Students wrote:

> The political debate was really interesting, especially since I was on a side that I had never investigated before. Each of the social issue lessons pushed me to think about issues I was choosing to not think about…
>
> I also found this project to be eyeopening to others views…

One student explained that she was open to new ideas because of her interactions with group members. For her, the fact that we were all female and interested in education made a difference. In her journal, she wrote, "I also realized how grateful I was to be debating the subject with just women and teachers…We didn't have any irrational outburst because we were all understanding." In my reflective journal, I noted that the single gender of all the classroom participants might have been a factor in the development of an open, comfortable, community in this class. I wrote about a student commenting that one of the reasons she was open to listening and discussing in this class was this factor. During that class period, several students agreed with her. However, I did not systematically assess this factor; therefore, further research is needed to conclude more definitively whether or not the results of the course were affected by the fact that the class was comprised entirely of women.

Initially, students seemed to revert to emotions or an external authority for making decisions about social issues. At the end of the semester, almost every student commented on the effect of realizing that mathematics can shape beliefs about social issues.

> As a student of the math class, I realized that we are using "MATH" a lot in our real life, not only calculating for our receipt in a store but also reading what happens in our community. Statistic and many kinds of graph can convey a whole story…
>
> I loved the lessons that covered social issues. These lessons made me think outside of my personal box. Additionally, the lessons showed me just how important math is in our daily lives.
>
> I liked the lessons dealing with social issues. It brought another dimension to the class and it helped me see how math is used in other ways.

As the semester progressed, students no longer justified their beliefs with phrases such as "I worry" or "I fear;" rather, they began to use mathematical information to explain why they accepted or questioned one viewpoint over another. Connecting mathematics to social issues allowed students to become more autonomous decision makers. They began to expand their understandings of social issues and were able to extend their thinking about social issues beyond an emotional level.

However, not only did students discover that mathematics and social issues are connected, but also they began to question the role of mathematics in society. In my reflective journal, I wrote about students becoming confused during the healthcare debate when the two political parties would discuss the same issue and have different figures—numbers that were skewed to fit each party's agenda. During a class discussion of the debate, one student described how "math is used to make us believe things." Students began agreeing with her; one student said, “Politicians use numbers to convince us they’re right.” In her journal, another student wrote, “I learned that the numbers and statistics that politicians use can be inaccurate or inflated or deflated.”

Resistance

Just as was the case for students’ understandings and beliefs about mathematics, for some students, the change in understandings and beliefs about social issues seemed to be limited to the social issues we encountered in this class. They continued to harbor many of their initial beliefs towards issues when they wrote and spoke about them. At the end of the semester, students presented the mathematics they found about a social issue of their choice. One student chose abortion, and in her presentation, she used mathematical evidence to illustrate why she opposed abortion. However, in her explanation, she only used mathematical information from one side of the debate. Although she had stressed the importance of investigating both sides of an issue before making a decision in her healthcare debate journal reflection, she failed to investigate both sides of the abortion debate before making a decision about the issue. Even though understanding healthcare reform prompted her to change her opinion drastically from no healthcare reform to a universal healthcare system, which she attributed to researching more than one side of the issue, when she encountered another topic of social concern, she reverted to describing it from the perspective of her political affiliation.

Further, in their reflections on social issue lessons, a few students maintained some of their initial false beliefs about social issues. One student continued to believe that healthcare reform would be more costly than the current system, while another student wrote:

> I learned that many people have many different opinions about Universal Healthcare. ..This type of healthcare could benefit many people, but at the same time it could place a higher burden on the people who then must chip in to support everyone else, as well as the people in the medical field…I say no Universal Healthcare because I don't find it right for me to pay for someone else's healthcare if they're doing things that are harmful to their health, or if they are to lazy to go out and get a job.

She continued to believe that she would have to pay for those who are uninsured, even though several students presented data that illustrated how much we currently pay for uninsured individuals through our private healthcare system.

Again, it is difficult to determine why a few students continued to see social issues in the ways described here. Although these students were perturbed by the social issue mathematics journal activities, they did not seem able or willing to transform their initial beliefs and understandings. This may be the result of previous experiences, a dependence on others, or any number of unknown factors. Further research is needed to determine what exactly maintained these beliefs and understandings in some students in this course. Regardless, although this was the case for some students, it was only so for a small minority of the class.

Summary

Engaging in the curricular routines that incorporated social issues increased most of the students' understandings of social issues and changed their beliefs about the subject. As students confronted their assumptions about social issues through mathematics, they began to develop deeper understandings of and new beliefs about critical problems in society. They gained confidence in themselves as autonomous decision makers, they began to see more than one perspective of social issues, they started to discover the power of mathematics in understanding issues, and they even began to question the role of mathematics in society.

Summary

Most students brought with them previous experiences with mathematics that were negative, shallow, and disconnected from contexts outside of the classroom. In this course, I endeavored to provide a meaningful mathematics learning environment where mathematics connected to critical issues in students' lives, and I attempted to unveil and revisit students' previous mathematical understandings through a reconfiguration of traditional classroom roles of teacher and student. All of the students enjoyed participating in the course, and most were successful in deepening and expanding their ideas and beliefs about mathematics, social issues, and the relationship between the two. In Chapter Six, I discuss these findings and their implications for mathematics education.

CHAPTER SIX

RESEARCH ANALYSIS & DISCUSSION

With world conditions more dire than ever and the need for a re-envisioning of the mathematics education reform efforts of the past century, I ventured to explore teaching mathematics for social justice with elementary preservice teachers. The context of this study included nineteen female students engaged in a mathematical environment that incorporated issues of social and economic justice into the curriculum and did so through a problem-rich learning environment where traditional roles of teacher and learner were reconsidered. I set out to create a college classroom community in which communication would be encouraged, student interest in social issues would drive mathematical lessons, and time and space for mathematics would be reconsidered. The three questions of the study were:

1. What critical factors were involved in the evolution of a mathematics course that incorporated social justice?
2. What were students' perceptions about learning mathematics in a course that combined mathematics and social issues in the way they are presented here?
3. What were students' perceptions of their understanding of mathematics, social issues, and the relationship between mathematics and social issues when they were presented in this way?

In this chapter, I make sense of the findings through a discussion of complex systems, and I describe the implication of this study for mathematics education.

Making Sense through Complex Systems

A complex system is a system in which interconnected components work together to form properties that cannot be understood by examining each of the parts separately (Briggs & Peat, 1989). Such a system is difficult to understand, reduce, predict, or verify because it emerges from the interactions between and among the individual parts, rather than from the individual contributions of each part. A complex system is highly sensitive, evolving when it encounters perturbation or instability and unfolding over time. During each phase of evolution, it experiences three stages—equilibrium, disequilibrium, and re-equilibrium (Briggs & Peat).

The world we live in is now becoming understood as a series of complex systems and subsystems (Capra, 1996). Every field from biology to economics to social studies has found structures that are inexplicable when reduced to their individual parts. For example, biologists were "unable to explain the self-preservation of the animal organism by recourse to the physical laws governing the behavior of its atoms and molecules" (Laszlo, p. 8). Social scientists have begun to understand that often times, beings or large groups of things have their own personalities that can remain intact or shift very slowly, even when individual members change or are wiped out. For example, a football team is an entity which will replace members throughout the years but may continue to maintain its characteristics – "their tactics and techniques, their fighting spirit, and so on" (Laszlo, p. 5). Such structures cannot be reduced to the characteristics of the individual parts but nevertheless exhibit unique characteristics as wholes. Scholars (Briggs and Peat, 1989) have begun to understand that the interactions between the individual components in the whole form create the phenomenon, not the individuals themselves.

This emergent understanding of complex systems has begun to call into question all-encompassing theories that attempt to analyze and explain everything (Briggs & Peat, 1989). This contrast to the classical disciplines has major implications for a change in consciousness (Laszlo, 1996). The metaphor that previously attached the universe to the image of a machine is replaced with that of a natural, ever-changing, dynamic system. The individual is no longer perceived as separate from but a part of a human world. The view of things as measurable and knowable is replaced with the idea that some things are self-creative and unpredictable. The power-hungry survival of the fittest attitude becomes a cooperative spirit, with an emphasis on helping others versus beating them for material prosperity. The overuse of resources becomes the understanding of sustenance for all and hence sacrifice and flexibility. World problems and education become linked. Diversity is appreciated and the complexity of life is not underestimated.

Once mathematics becomes understood as a complex system of certain kind of mathematics that evolved from a particular worldview that has benefited a few and contributed to many of the problems of our current culture, we can begin to imagine the transformation of such a system. With this transformation comes the understanding that mathematics must be taught in systemic terms. Therefore, in this classroom, the success of the evolution of the course and participant perceptions are better understood when they are viewed as complex systems.

Evolution of the Course

Viewing the make-up of the universe as a series of systems carries many implication about finding solutions to the problems our culture has come to possess. Scholars, such as Capra, have begun to propose solutions that would have previously seemed incomprehensible and illogical. For example, he (Capra, 1996) explained that an interconnected, paradoxical view can eliminate problems such as overpopulation. He proposes that “stabilizing world population will be possible only when poverty is reduced worldwide” (p. 3). Rather than thinking in terms of hierarchy and separation, solutions may be found in horizontal and interdependent ways. Human beings begin to be understood as systems, living within a culture or larger system, part of a world or an even larger system, and so forth. Viewing the world as a series of systems within systems that interact with and react to one another requires us to understand that although there is value in dissecting parts of these structures, they must be seen in relation to one another and survival of each means sustenance for all.

A systemic approach to life carries with it implications for all aspects of the current world, including education--all education--even in the field of mathematics. From this perspective, the mathematics classroom and all its participants, students, subject, environment, and teacher alike, can be viewed as complex systems that interact to form a whole that cannot be separated and understood or analyzed by its parts but rather exists in the interactions between and among them. Because of the dynamic nature of this system, no prescription can be given to any two teachers and the same outcomes be expected. Therefore, the evolution of this social justice mathematics course can only be viewed in a systemic way, but it cannot be imposed on others for the purposes of creating the same results.

This course evolved from an interconnected, paradoxical view of teaching and learning mathematics, as a complex system. Three paradoxes became the catalysts for the emergence of a complex system that intertwined and interrelated with the complexities of critical world issues and the participants engaged in the course. Although I have categorized the formation of this course into three paradoxes, just as with any system, it is important to understand that the evolution of this course was emergent, and could only truly be in the interaction of these categories with each other and the participants of the course. The evolution of this course transpired through multiple iterations of the process of equilibrium, disequilibrium, and re-equilibrium. This system emerged from each of these phases as each of the subsystems of the class interacted with time, space, and a social justice mathematics curriculum.

The evolution of the course continuously fluctuated between states of equilibrium, disequilibrium, and re-equilibrium. It began with strict guidelines and initial conditions, placing it in a state of equilibrium. However, a complex system is open and unpredictable. Although some things were anticipated, others could not be and were attended to as they emerged. The traditional roles of teacher and learner began with the instructor as authoritarian and students as novices, but as the semester progressed, students were provided space to interact and grapple with complex mathematical and social problems in a supportive, collaborative environment. Through communication, students learned to work together and negotiate meaning, participating in an open, active learning environment with each other and the instructor. This shook traditional notions of teaching and learning and prompted the evolution of the course towards a reorganization of the social structure of the classroom. Students became active participants who prompted ideas in one another. Even seemingly insignificant comments and interactions initiated the formation of new tasks based on student interest, which shifted the balance of the course yet again. Tasks were carefully chosen based on student interest, and the evolution of the course was perturbed as students examined previously encountered mathematical and social issues and concepts in a way that allowed them to revisit, understand, and (re)construct ideas in more meaningful ways. Time and space took on new meaning as students grappled with problems individually and with others before, during, and after class time. As time and space for mathematics was reconsidered, the course continued to evolve and follow new paths and directions, taking us to a resting place at the end of the semester that could not have been predicted at the beginning.

Student Perceptions

Just as the evolution of this course could be viewed as a complex system, so could student perceptions of participating and learning in this social justice mathematics course. Students brought with them many initial attitudes about learning mathematics and themselves as mathematical thinkers. They entered the course with certain knowledge and beliefs about mathematics and social issues that seemed to be stagnant and in a state of equilibrium. Nonetheless, as they interacted with the curricular routines of the class, their ideas and thoughts became perturbed and perceptions reached a state of disequilibrium, only to be re-equilibrated once they reorganized many of their previous misunderstandings or misconceptions.

Students entered the course with certain perceptions about their knowledge of mathematics and social issues, their self-perceptions as mathematicians, and their emotions towards learning the subject. These initial discriminations affected their abilities to learn and interact with each other (Tosey, 2002). However, as students interacted with each other, the instructor, and the curriculum, their perceptions were perturbed and reached critical points that allowed them to restructure their ideas and reorganize them into new ones. The transition from previously held ideas to the creation of new ones emerged from within the interactions of student perceptions with each other and with the curriculum, which provided complex tasks that facilitated the perturbation process.

Engaging in the curricular routines of this classroom affected the negative attitudes students initially held about learning mathematics when they came into the course. They began to view learning mathematics as a positive endeavor. Their initial fears and anxieties seemed to dissolve, and they began to view themselves as capable of doing mathematics. They became more confident and empowered as they created their own ways of doing mathematics and engaged in a curriculum that often addressed their interests and concerns about society. They became authors of their own mathematical learning, and surprised themselves as they recognized their abilities as mathematicians. When students became liberated by utilizing their own mathematics and constructing ideas in a community, the restrictive nature of their previous mathematics courses no longer oppressed their abilities and allowed them to transform their attitudes about learning mathematics. This course perturbed students' perceptions about what it means to engage in mathematical teaching and learning. The level of comfort they felt in the classroom community allowed them to participate, risk being wrong, and explore other questions. It encouraged them to develop new perceptions that dealt with restructuring their ideas about engaging in a mathematics course, including learning to understand through methods that worked best for them. This course transformed their ideas about mathematics being a series of pointless formulas and it related it to a world outside of the classroom. The engagement students began to feel deepened their understandings of mathematics, social issues, and the relationship between the two, showing them that learning mathematics can be critical and enjoyable.

The curricular routines of the course changed students' perceptions of their understandings of mathematics and social issues. They entered the semester with fragmented and limited knowledge about many of the topics. As the semester progressed, students wrote about and discussed the deeper understandings they developed. They became more able to communicate and explain their thoughts about mathematics and social issues. They supported their arguments and defended their perspectives for and through mathematics. Some students even became intellectually curious about mathematics and society and began to ask and investigate questions beyond the scope of the course. They formulated conjectures and wrote proofs. They also started recognizing the importance of a meaningful understanding of mathematics and social issues. Further, this explicit knowledge affected their ideas about teaching mathematics. Many described the impact of this course on their future plans for teaching mathematics in non-traditional ways and through connections to important social issues. Becoming more informed about mathematics and society not only deepened student understanding but also affected student beliefs about the subjects.

The curricular routines of this course restructured student beliefs about the nature of mathematics and social issues. The experiences they encountered allowed them to become mathematical creators and encouraged them to investigate social issues through and for mathematics. Their sense of autonomy increased as they learned to develop their ideas by verifying them and validating them through the use of mathematics, without the reliance on opinion, emotion, or an external authority. Encountering mathematics and social issues with others in the class and contemplating ideas in groups, gave students insight to other ways of thinking about mathematics and social issues. This helped students see that there is more than one way to do mathematics and view social issues. It also sparked interest in the subjects and changed perceptions about the importance of understanding mathematics and society. Students wrote about the value of other's perspectives in shaping their ideas about teaching in the future.

Further, students who had previously perceived themselves as incapable of doing mathematics or believed it was reserved for others, began to envision themselves as successful at it. They became more confident in their abilities. Even students who enjoyed mathematics and did well in courses before this one, expressed the positive impact of this experience on their beliefs. They wrote about expanding their ideas beyond a technical level and developing deeper meanings of mathematics.

Journal Activities

At the heart of these complex systems that worked to form the outcomes of this course were carefully chosen *Journal Activities* that perturbed participants, initiated communication between them, and continued to challenge them before, during, and after classroom time and space. The facilitation of dissonance through carefully selected tasks was a key component in the evolution of the course and the alteration of perceptions in this course. They provided the disequilibrium that was necessary to propel the emergence of a social justice mathematics class where students' ideas of learning mathematics and social issues could be reconsidered. A certain amount of dissonance is necessary for intellectual growth (Piaget, 1972); however, too much mental disagreement can lead to miseducative experiences—experiences that hinder learning (Dewey, 1938). Motivation for understanding occurs in a space between being challenged and being paralyzed. The boundaries for too much dissonance may not be the same from one student to the next. Students must make errors, question assumptions, and confront confusion in order to construct meaningful ideas (Doll, 1993). However, the process of distributing discomfort must be well thought out and structured, not just randomly administered (Doll). Although the delicate balance between dissonance and affective safety is impossible to pinpoint, providing students with too little discomfort may limit learning and providing them with too much may halt the process. For the few students in this course who seemed unable to change some of their perceptions, the dissonance they encountered may have been too extreme and may have hindered a shift in perceptions. For the rest, the process of providing students with tasks that related to their interests contributed enough anxiety for them to transform their negative attitudes about mathematics and better understand mathematics, social issues, and the relationship therein. The tasks became the bifurcation points (Briggs & Peat, 1989) that transformed and fueled the evolution of the course and student perceptions.

> Human beings are autopoietic structures (Briggs & Peat, 1989). These structures: lie at the highly sophisticated end of nature's spectrum of 'open structures'…they are remarkable creatures of paradox…Each autopoietic structure has a unique history, but its history is tied to the history of the larger environment and other autopoietic structures: an interwovenness of times' arrows. (p. 154)

Autopoietic structures change and evolve, operating on many interconnected levels at once. They acts on and react to their surroundings. Their thoughts and feelings emerge from “a constant feedback and flow-through of the thoughts and feelings of others” who have influenced them. “Our individuality is decidedly a part of a collective movement. That movement has feedback at its roots” (Briggs & Peat, p. 154).

This interconnectedness of ourselves with and in our surroundings allows us to sense disequilibrium in our world and become motivated to act when given the chance. Human culture has reached a time of dissonance in which social structures are reaching bifurcation points where humanity must either form a new order or descend into chaos. When mathematics is taught in a manner that separates it from this reality, it becomes fragmented and disconnected from students, irrelevant to their surroundings. Viewing mathematics as connected to and created from culture, history, society and students brings mathematics to life as a complex system that correlates with the natural structures of students. The tasks for this course were created with these ideas in mind. They connected to students and allowed them to form a mathematics that related to them as individuals and addressed them as a collective part of a society. The tasks in this course were paradoxical in nature, just as complex systems are. They changed and evolved from one week to the next. In order to understand mathematics and critical issues in society, sometimes mathematics needed to be understood in the abstract, sometimes it needed to be understood through a critical context, sometimes it needed to be understood through a non-critical context, sometimes a social issue needed to be understood in the abstract, sometimes it needed to be understood through mathematics, and so on. The journal activities of this course engaged students in a way that motivated them to want to participate in learning mathematics.

Implications of the Study

Until our modernistic, anthropocentric ideals are revised and transformed, humans will continue to struggle with problems within the structures that they have constructed and have come to take for granted. Mathematics has become understood to be culture free, almost as though it were god-given. Eurocentrism and modernity have played a major hand in creating such an attitude and misconception. However, the results of this understanding of mathematics have become a major concern for educators and the world. Once mathematics and culture can be linked, not only as playing a hand in forming one-another, but also as means for understanding each other, systemic implications for the mathematics classroom can begin to emerge.

In the current educational system, there exist answers to questions of teaching and learning mathematics, with current mathematics curriculum taken for granted. Research that has sought to find these answers has been considered most valid when it had a quantitative component, a mathematical one. The idea is that if a perfect lesson plan or set of prescriptions could be created and measured, it could be distributed to teachers and mathematics learning would occur. However, the reform efforts of the past century have made it obvious that these questions remain unanswered. No quantitative research has given the prescriptions for successful teaching and learning to occur for all. "Given the broad and pervasive discussion on the figurative (read: qualitative) bases of scientific pursuits over the past half-century" (Davis & Sumara, 2005, p. 306) scholars such as Francis Bacon have "argued that analogy and other figurative devices are as important to scientific inquiry as measurement and replication" (Davis & Sumara, p. 306). This alternative perspective asks us to reconsider the questions of the past century. From a systemic view, there can be no answer to the question of how to best teach or what to teach, for that matter, there can only be examinations of classrooms where students find meaning in mathematics and begin to consider possibilities for reinventing society.

Therefore, the implications I provide in this study are only based on the descriptive experiences my students and I had in this mathematics course for preservice elementary teachers. First, the findings imply that the preservice teachers in this research responded well to a social justice approach to teaching mathematics because the curricular materials utilized were relevant to their lives. More meaningful learning occurred when topics were connected to life. The students described here discussed what they had learned with more passion when it was related to their world. They began to perceive their understandings of mathematics and society as deeper and more explicit, and their beliefs about the nature of mathematics and social issues transformed as they explored innovative ways of viewing mathematics. Therefore, I would recommend teaching mathematics to preservice elementary teachers using material that is applicable to their lives and relates to mathematics in the context of larger critical issues so that they may gain familiarity with a social justice mathematics classroom and transfer the experience to their own teaching.

Second, the findings of this study suggest that using *Journal Activities* that incorporate student concerns about significant social problems in a safe community aided students in reconsidering many of their previous experiences with mathematics and social issues and concepts. Communication, perturbation, and a reconsideration of time and space for mathematics were important components to supporting students' stretching and reconfiguring of notions. Without communication, knowledge may not be constructed in many instances such as those when students get stuck in isolation. Without perturbation, students may not become interested or motivated to understand a new idea. And without freedom of time and space for mathematics, students may give up on concepts and disengage in further learning. Therefore, I would recommend teaching mathematics to preservice elementary teachers in a way that provides space for discussion and negotiation, perturbation, and a reconstruction of traditional time and space restrictions.

If we are to advocate teaching preservice teachers mathematics for social justice, we must keep in mind that doing so is not just about supplementing traditional mathematics curriculum with a few lessons that incorporate social issues. It is about creating an environment that questions traditionally accepted assumptions about society and teaching and learning; it is about understanding how the mathematics we utilize in our schools was created and who it has traditionally benefitted; and it is about addressing issues of power and equity through the explicit curriculum as well as the “hidden” curriculum (Anyon, 1979). We must constantly remind ourselves that any attempt to universalize an idea or theory is a dangerous one (Houser, 2006). Humans have mountains to climb in terms of understanding life and human interaction and how these influence teaching and learning--not to mention what is of value when engaging in developing human beliefs.

Further Research

There is a need for more research in this area. One of the immediate concerns that comes to mind for me is what the long term effects of such a course are on preservice elementary teachers. I would like for my students to change the way mathematics is perceived and taught; however, I recognize that they have been participating in a traditional system for many years and they face many challenges in a public school system that stems from modernity. I believe that many of my students' perceptions of mathematics and social issues were changed by their engagement in this course, and I believe those changes were meaningful and have the potential to be long lasting. However, only time can tell whether or not this is truly the case.

Other questions that can be explored further include what happens when social justice mathematics is explored in other mathematics courses or other subject areas entirely, what happens when mathematics and social issues are explored with children, why was there some resistance from a few students, what happens when mathematics and social issues are explored with male students, or what is the effect of the reflection process on understanding and beliefs. Further I would be interested in conducting case study research of a few students' perceptions for a more in-depth understanding of what happens to their perceptions when they engage in a course such as this one. We should continue to question and search for answers as we explore the roles of teaching and learning within the context of a struggling modern world, as we are faced with a crisis of perception (Capra, 1996). Finally, we must always remind ourselves that teaching is not about the teacher, the curriculum, the student, or the world; rather, it is about all of them simultaneously.

REFERENCES

Anderson, S. (2006). Historical, cultural, and social implications of mathematics. In E. Gutstein & B. Peterson (Eds.), *Rethinking mathematics: teaching social justice by the number* (pp. 43-44). Milwaukee, WI: Rethinking Schools.

Anderson, G. L., Herr, K., & Nihlen, A. S. (1994). *Studying your own school: An educator's guide to qualitative research*. Thousand Oaks, CA: Cortin Press, Inc.

Anyon, J. (1979). Social class and the hidden curriculum of work. *Journal of Education, 162(1),* 67-92.

Ascher, M., & Ascher, R. (1986). Ethnomathematics. In A. B. Powell & M. Frankenstein (Eds.), *Ethnomathematics: Challenging Eurocentrism in Mathematics Education* (pp. 25-50). Albany, NY: State University of New York Press.

Ball, D. L. (1988a). Knowledge and reasoning in mathematical pedagogy: Examining what prospective teachers bring to teacher education. *Dissertation Abstracts International, 50*(02), 419.

Ball, D. L. (1988b). Unlearning to teach mathematics. *For the Learning of Mathematics, 8*(1), 40-48.

Ball, D. L. (1990). The mathematics understanding that prospective teachers bring to teacher education. *Elementary School Journa, 90*(4), 449-466.

Ball, D. L., Lubienski, S. T., & Mewborn, D. S. (2001). Research on teaching mathematics: The unsolved problem of teachers' mathematical knowledge. In V. Richardson (Ed.), *Handbook of Research on Teaching* (pp. 433-456). New York: Macmillan.

Bell, E. T. (1986). *Men of mathematics*. New York: Simon & Schuster.

Benbow, R. M. (1993, April). *Tracing mathematical beliefs of preservice teachers through integrated content-methods courses.* A paper presented at the Annual Meeting of the American Educational Research Association, Atlanta, GA.

Bender, L. (Producer), Burns, S. (Producer), Burns, S.Z. (Producer), & Guggenheim, D. (Director). (2006). *An inconvenient truth* [Motion Picture]. United States: Lawrence Bender Productions.

Boyer, C. (1968). *A history of mathematics*. New York: John Wiley & Sons, Inc.

Bratlinger, A. (2006). The geometry of inequality. In E. Gutstein & B. Peterson (Eds.), *Rethinking mathematics: teaching social justice by the number* (pp. 97-100). Milwaukee, WI: Rethinking Schools.

Briggs, J., & Peat, F. D. (1989). *Seven life lessons of chaos: Spiritual wisdom from the science of change*. New York: Harper Collins Publishers.

Burton, L. (2003). *Which way social justice in mathematics education?* Westport CT: Greenwood Press.

Capra, R. (1996). *The web of life*. New York: Anchor Books.

Carpenter, A. (2000, April). *An ethnographic study of preservice teachers' resistance to multiculturalism: Implications for teaching.* Paper presented at the Annual Meeting of the American Educational Research Association, New Orleans, LA.

Center for the Study of Mathematics Curriculum. (2004). *Curriculum resources*. Retrieved December 20, 2008 from http://mathcurriculumcenter.org/CCM/ccm_resources.php

Chazan, D., & Ball, D. (1991). *Beyond exhortations not to tell: The teacher's role in discussion-intensive mathematics classes* (Craft Paper 95-2). East Lansing: Michigan State University, National Center for Research on Teacher Education.

Che, S. M. (2005). *Cameroonian teachers' perceptions of culture, education, and mathematics*. Doctoral dissertation, University of Oklahoma, Oklahoma.

Cohen, D., & Hill, H. (2001). *Learning policy: When state education reform works.* New Haven, CT: Yale University Press.

Cochran-Smith (2004). *Walking the road: Race, diversity, and social justice in teacher education.* New York: Teachers College Press.

Cochran-Smith, M., & Lytle, S. L. (1993). *Inside/outside. Teacher researcher and knowledge.* New York: Teachers College Press, Columbia University.

Cordeiro, P., & Cunningham, N. (2006). *Introduction to educational administration: A Problem based approach.* Boston, MA: Allyn & Bacon.

Crockett, M. (2008). *Mathematics and teaching: Reflective teaching and the social conditions of schooling.* Florence, KY: Routledge.

Cresswell, J. W. (2007). *Qualitative inquiry and research design: Choosing among five approaches* (2nd ed.). Thousand Oaks, CA: Sage.

Cushner, K., McClelland, A., & Safford, P. (2003). *Human diversity in education: An integrative approach* (4th ed.). New York: McGraw-Hill.

D'Ambrosio, U. (2007). Peace, social justice and ethnomathematics [Monograph]. *The Montana Mathematics Enthusiast*, *1*, 25-34. Retrieved January 3, 2008, from http://www.math.umt.edu/TMME/Monograph1/D'Ambrosio_FINAL_pp25_34.pdf

D'Ambrosio, U. (2006). The program ethnomathematics and the challenges of globalization. *Circumscribere: International Journal for the History of Science, 1*(2006), 74-82. Retrieved January 1, 2009, from http://revistas.pucsp.br/index.php/circumhc/article/viewFile/552/1007

D'Ambrosio, U. (2002). Alustapasivistykselitys or the name ethnomathematics: My personal view. Retrieved January 2, 2009, from http://vello.sites.uol.com.br/alusta.htm

D'Ambrosio, U. (1985). Ethnomathematics and its place in the history and pedagogy of mathematics. In A. B. Powell & M. Frankenstein (Eds.), *Ethnomathematics: Challenging Eurocentrism in Mathematics Education.* (pp. 13-24). Albany, NY: State University of New York Press.

Davis, B. (1997). Listening for differences: An evolving conception of mathematics teaching. *Journal for Research in Mathematics Education*, *28*(3), 355-376.

Davis, B. & Sumara, D. J. (2005). Challenging images of knowing: complexity science and educational research. *International Journal of Qualitative Studies in Education*, *18*(3), 305-321.

Demi. (1997). *One grain of rice: A mathematical folktale*. New York: Scholastic Press.

Dewey, J. (1902). *The child & the curriculum*. Chicago: The University of Chicago Press.

Dewey, J. (1938). *Experience and education*. New York: Collier Books.

Doll, W. E. (1993). *A post-modern perspective on curriculum.* New York: Teachers College Press.

Dupree, G. N. (1999). Mathematical empowerment: A case study of relational classroom learning. *Dissertation Abstracts International, 60*(04), 1055.

Ezeife, A. (2002). Mathematics and culture nexus: The interactions of culture and mathematics in an aboriginal classroom. *International Education Journal, 3*(3), 176-187. Retrieved January 1, 2009, from http://ehlt.flinders.edu.au/education/iej/articles/v3n3/Ezeife/paper.pdf

Fisher, P. O. (1992). Beliefs about and attitudes towards mathematics and mathematics teaching held by prospective elementary teachers at the University of Nebraska-Lincoln. *Dissertation Abstracts International, 50*(02A), 0450.

Fleener, M. J. (2002). *Curriculum dynamics: Recreating heart*. New York: Peter Lang.

Frank, M. L. (1990). What myths about mathematics are held and conveyed by teachers? *Arithmetic Teacher, 37*(5), 10-12.

Frankenstein, M. & Powell, A. (1989). Empowering non-traditional college students: On social ideology and mathematics education. *Science and Nature, 9*(10), 100-112.

Freire, P. (1970). *Pedagogy of the oppressed* (M. B. Ramos, Trans.). New York: Continuum.

Gay, G. (2002). Preparing for culturally responsive teaching. *Journal of Teacher Education, 53*, 106-116.

Gay, G. & Kirkland, K. (2003). Developing cultural critical consciousness and self-reflection in preservice teacher education. *Theory into Practice. 42(3)*. 181-187.

Grattan-Guinness, I. (1997). *The Rainbow of Mathematics: A History of the Mathematical Sciences*. London: W. W. Norton and Company.

Gray, S. B., & Sandifer, E. C. (2001). The *Sumario Compendiso*: The New World's first mathematics book. *Mathematics Teacher*. *94*(2), 98-105.

Gutstein, E. (2003). Teaching and learning mathematics for social justice in an urban, latino school. *Journal for Research in Mathematics Education*, *34*(1), 37-73.

Gutstein, E., & Peterson, B. (Eds.). (2006). *Rethinking mathematics: teaching social justice by the numbers.* Milwaukee, WI: Rethinking Schools.

Haberman, M., & Post, L. (1992). Do direct experiences change education students' perceptions of low-income students? *Midwestern Educational Researcher, 5*, 29-31.

Haris, M. (1987). An example of traditional women's work as a mathematics resource. *For the Learning of Mamthematics*, *7*(3), 26-28.

Herodotus. *The History of Herodotus* (G. Rawlinson, Trans.). Retrieved January 1, 2009 from http://classics.mit.edu/Herodotus/history.html.

Houser, N. (2006). Worldviews and learning theories: A transactional reconsideration of teaching and learning. *Curriculum & Teaching*, *21*(1), 5-32.

In full: Obama health care address [Video]. (2009). Retrieved September 15, 2009, from http://www.youtube.com/watch?v=U1YNF9I25yU

Jacobson, K. (2000). Central tensions: A crtitical framework for examining high school mathematics education. Paper presented at the Annual Meeting of the American Educational Research Association, New Orleans, LA. (ERIC Document Reproduction Service No. ED 442 673)

Jones, P., & Coxford, A. (1970). Mathematics in the evolving schools. In NCTM (Ed.), *A history of mathematics education in the United States and Canada* (pp. 11-92). Washington, DC: National Council of Teachers of Mathematics.

Joseph, G. G. (2000). *The Crest of the Peacock, non-European roots of Mathematics*. Princeton and Oxford: Princeton University Press.

Joseph, G. G. (1993). A rationale for a multicultural approach to mathematics. In G. G. Joseph, D. Nelson, & J. Williams (Eds.), *Multicultural Mathematics: Teaching Mathematics from a Global Perspective* (pp. 1-24). Oxford: Oxford University Press.

Kamii, C. (1985/2000). *Young children reinvent arithmetic*. New York: Teachers College Press.

Klein, D. (2003). A brief history of American K-12 mathematics education in the 20th century. In J. Royer (Ed.), *Mathematical cognition.* Greenwich, CT: Information Age Publishing.

Laszlo, E. (1996). *The systems view of the world*. Cresskill, NJ: Hamton Press, Inc.

Lave, J. (1988). *Cognition in practice*. Cambridge, England: Cambridge.

Legaspi, A. & Rickard, A. (2004). A case study of multicultural education and problem-centered mathematics curricula. *National Forum of Multicultural Issues Journal, 1E*(1), 1-18.

Linn, R. (2000). Assessments and accountability. *Educational Researcher, 29*(2), 4-16.

Locke, S. (2005). Institutional, social, and cultural influences on the multicultural perspectives of preservice teachers. *Multicultural Persectives, 7*, 20-28.

Lott, J. W. (2002). A static mathematics curriculum is unhealthy! Retrieved December 20, 2008, from http://www.nctm.org/about/content.aspx?id=974

National Council of Teachers of Mathematics (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA: Author.

National Council of Teachers of Mathematics. (1980). *An Agenda for Action: Recommendations for school mathematics of the 1980s*. Reston, VA: Author.

Neill, A. (1960). *Summerhill*: *A radical approach to child rearing*. New York: Hart.

Noddings, N. (1992). *The challenge to care in schools: Alternative approach to education.* New York: Teachers College Press.

Novak, M. (2000). Defining Social Justice. *First Things*, *108*, 11-13.

Ortiz-Franco, L. (2006). Chicanos have math in their blood. In E. Gutstein & B. Peterson (Eds.), *Rethinking mathematics: teaching social justice by the number* (pp. 70-75). Milwaukee, WI: Rethinking Schools.

Oslar, J. (2007). A guide for integrating issues of social and economic justice into mathematics curriculum. Retreived January 7, 2010, from http://www.radicalmath.org/docs/SJMathGuide.pdf

Palmer, P. J. (1998). *The courage to teach*. San Francisco: Jossey-Bass.

Piaget, J. (1972). *The principles of genetic epistemology*. NY: Basic.

Powell, B. & Frankenstein, M. (Eds.) (1997). *Ethnomathematics: Challenging Eurocentrism in Mathematics Education.* Albany, NY: State University of New York Press.

Powell, A.; Jeffries, D.; & Shelby, A. (1989). An empowering, participatory research model for humanistic mathematics pedagogy. *Humanistic Mathematics Network Newsletter*, *4*, 29-38.

Powell, A., & Lopez, J. (1989). The role of writing in learning mathematics and science: A case study. In P. Connolly and T. Vilardi (Eds.), *The Role of Writing in Learning Mathmetics and Science* (pp. 157-177). New York: Teachers College.

Republican response to obama's speech on healthcare [Video]. (2009). Retrieved September 15, 2009, from http://www.youtube.com/watch?v=qJ9z4PFA1I4

Roberts, D. (2001). E. H. Moore's early twentieth-century program for reform in mathematics education. Retrieved December 20, 2008, from http://www.maa.org

Schiftner, D., & Fosnot, C. T. (1993). *Reconstructing mathematics education: Stories of teachers meeting the challenge of reform.* New York: Teachers College Press.

Sleeter, C. E. (2001). Restructuring schools for multicultural education. *Journal of Teacher Education, 43*(2), 141-149.

Spradbery, J. (1976). Conservative pupils? Pupil resistance to curriculum innovation in mathematics. In G. Witty and M. Young (Eds.), *Explorations in the Politics of School Knowledge* (pp. 236-243). Driffeld: Nafferton.

Stake, R. (1995). *The art of case research*. Newbury Park, CA: Sage Publications.

Steele, L. (2006). Sweatshop Accounting. In E. Gutstein & B. Peterson (Eds.), *Rethinking mathematics: teaching social justice by the number* (pp. 53-61). Milwaukee, WI: Rethinking Schools.

Tatto, M. (1996). Examining values and beliefs about teaching diverse students: Understanding the challenges of teacher education. *Educational Evaluation and Policy Analysis, 18*, 155-180.

Teresi, D. (2002). *Lost discoveries: The ancient roots of modern science – from the Babylonians to the Maya*. New York: Simon & Schuster.

Tossey, P. (2002). Teaching on the edge of chaos: Complexity theory, learning systems and the idea of enhancement. Retreived January 15, 2010 from the World Wide Web: http://epubs.surrey.ac.uk/cgi/viewcontent.cgi?article=1011&context=humresmgmt

Turner E. E. & Font Strawhun B. T. (2006). With math, it's like you have more defense. In E. Gutstein & B. Peterson (Eds.), *Rethinking mathematics: teaching social justice by the number* (pp. 81-87). Milwaukee, WI: Rethinking Schools.

von Glasersfeld, E. (1995). *Radical constructivism a way of knowing and learning*. New York: RoutledgFalmer.

United States Department of Education. (2007). Digest of Education Statistics: 2007. Retreived February 9, 2010 from http://nces.ed.gov/programs/digest/d07

Wood, T., Cobb, P., & Yackel, E. (1991). Change in teaching mathematics: A case study. *American Educational Research Journal*, *28*(3), 587-616.

Warrington, M., & Kamii, C. (1998). Multiplication of fractions: A piagetian, constructivist approach. *Mathematics Teaching in the Middle School, 3(5),* 339-343.

Wheatley, G. H., & Reynolds, A. M. (1999). *Coming to know number: A mathematics activity resource for elementary school teachers.* Tallahassee, FL: Mathematics Learning.

Winston, D. & Royer, J. (2003). *Mathematical Cognition*. Greenwich, CT: Information Age Publishing.

Zaslavsky, C. (1991). World cultures in the mathematics class. *For the Learning of Mathematics, 11*(2), 32-36.

APPENDIX A

Externally Imposed Weekly Schedule

Week	Course Material
Week 1 August 21	Course Information Section 1.1 and Section 1.2 Problem Solving
Week 2 August 28	Section 2.1 Sets and Venn Diagrams
Week 3 September 4	Section 3.1 Numeration Systems
Week 4 September 11	Start Section 3.2 Addition and Subtraction
Week 5 September 18	Finish Section 3.2 and Start Section 3.3 Addition, Subtraction, and Multiplication
Week 6 September 25	Section 3.3 Multiplication
Week 7 October 2	Section 3.4 Division and Exponents
Week 8 October 9	Section 4.1 and Section 4.2 Factors and Multiples GCF and LCM
Week 9 October 16	Section 5.1 Integers
Week 10 October 23	Start Section 5.2 Introduction to Fractions
Week 11 October 30	Finish Section 5.2 and Start Section 5.3 Fractions & Operations with Fractions
Week 12 November 6	Section 5.3 Operations with Fractions
Week 13 November 13	Section 6.1 Decimals and Rational Numbers
Week 14 November 20	Section 6.2 and Section 6.3 Operations with Decimals Ratio and Percents
Week 15 December 4	Section 6.4 Irrational and Real Numbers And Project Sharing
Week 16 December 11	Project

APPENDIX B

Sample Journal Activities

Healthcare Debate

Answer the following questions in your journal by September 17th. You will be given an opportunity to devise a plan of how you will make your argument with your group on that date. An in-class debate will take place on September 24th.

Note: Some places you may want to look for information are the internet (YouTube clips, newspapter articles, google, etc.), C-SPAN, newspaper articles, and the various news channels. Please indicate where you obtained your information when wrting up your responses.

1. Listen to President Obama's healthcare reform speech. What were the president's main points? Address these points when discussing your party's point of view.
2. What was the Republican response to the president's speech?
3. What does your party think about the effects of President Obama's plan on the deficit? Explain your point using mathematical data.
4. What does your party think about the effects of this plan on Medicare and Medicaid? Explain your points using mathematical data.
5. What does your party think about the effects of this plan on individual businesses? Explain your points using mathematical data.
6. Explain what the "public option" is and what your party thinks about it? Justify your viewpoint using mathematical data.
7. What are some other relevant ideas or questions for us to consider? Again, justify what you say with mathematical data.

Commercial Math

Watch two hours of kids' shows (preferably cartoons). In your journal, record the number of commercials you see during that time span, the amount of time dedicated to commercials, and the product types and brand names depicted in each commercial. You might find making some tables similar to the ones below helpful.

First Set of Commercials
Time Begin:
Time End:
Length of Commercial Break:

Commercial 1	Type: junk food; Brand: Fruit Roll Ups
Commercial 2	Type: ; Brand:
…	…
…	…

Second Set of Commercials
Time Begin:
Time End:
Length of Commercial Break:

Commercial 1	Type: junk food; Brand: Fruit Roll Ups
Commercial 2	Type: ; Brand:
…	…
…	…

In your groups, combine your data and create a visual representation of your findings. Record these findings in your journal.