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Abstract

The need for spatial thinkers is evident in the growing concerns regarding the performance of U.S. students in mathematics and the lack of interest in spatially-driven fields such as science, technology, engineering, and mathematics. Although the focus on spatial research has fluctuated over decades of educational reform, a platform has been established through the support of national organizations such as the National Research Council (2006) and the National Council of Teachers of Mathematics (NCTM, 2000). Even with such powerful recognition, purposeful cultivation of spatial thinking is commonly overshadowed by other factors in the mathematics classroom, especially at the undergraduate level. According to NCTM, problem solving is an integral part of all mathematics learning. Further, research has linked spatial thinking to problem solving, indicating that spatial thinking is a necessary skill for success in solving problems in mathematics. This embedded case study examined how the inclusion of spatial tasks influenced problem-solving performance, spatial thinking ability, and beliefs of undergraduate mathematics students. Data were collected through quantitative instruments, such as the Purdue Spatial Visualization Test, the Mathematical Processing Instrument, and the Spatial Thinking Attitude Survey, as well as qualitative instruments, such as student-written journal responses, focus group interviews, and observations. The findings of this study suggest the inclusion of spatial thinking tasks has an influence on students' spatial visualization ability, problem-solving strategies, and beliefs about the relevance of spatial thinking. As long as problem solving remains a goal for learners of mathematics, spatial thinking must be fostered in students of mathematics as well as those who desire to teach mathematics.

Chapter I

INTRODUCTION

Meaningful mathematics learning is almost always based in spatial imagery. While some forms of mathematical reasoning do not require imagery, the majority of mathematical activities involve a spatial component (Wheatley & Abshire, 2002). Spatial thinking, therefore, plays an integral part in mathematical ability (Anderson, 2000) and is an essential skill for success in the STEM fields of science, technology, engineering, and mathematics. Proficiency in spatial ability can lead to a multitude of career choices. A quick Internet search will reveal sites that list over 100 different occupations that rely on adept spatial skills, many of which are currently in demand or are projected to be in demand in the next decade (Halpern, et al., 2007).

Spatial thinking is a skill used in everyday life, the workplace, and mathematics to solve problems using concepts of space, visualization, and reasoning. The inclusion of spatial ability in education is crucial for the future disciplines of science, engineering, architecture, medicine, geography, and mathematics, to name a few. With specific reference to mathematics, the research has shown a high correlation between spatial ability and success in life (Mohler, 2008), spatial ability and general mathematics achievement (Brating & Pejlare, 2008; Casey, et al., 2008; Pittalis & Christou, 2010; Unal, Jakubowski, & Corey, 2009), spatial ability and creativity in mathematical thinking (Clements, 1998; Lohman, 1993), and spatial ability and problem solving (Battista, Wheatley, & Talsma, 1989; Edens & Potter, 2007; Hegarty & Kozhevnikov, 1999; Lean &

Clements, 1981; Moses, 1977; National Research Council [NRC], 2006; Tartre, 1990; van Garderen, 2006).

The significance of spatial reasoning in the teaching and learning of mathematics is reinforced by the National Council of Teachers of Mathematics (NCTM) standards from kindergarten through grade 12. The geometry standard explicitly states that all instructional programs should enable students to "use visualization, spatial reasoning, and geometric modeling to solve problems" (NCTM, 2000, p. 41). Alongside the geometric standard, NCTM promotes a problem-solving standard that highlights four outcomes through enabling students to "build new mathematical knowledge through problem solving; solve problems that arise in mathematics and other contexts; apply and adapt a variety of appropriate strategies to solve problems; [and] monitor and reflect on the process of mathematical problem solving" (p. 41).

Additionally, the National Research Council (NRC, 2006) suggests that spatial reasoning is essential for progress in mathematical problem solving and states, "spatial thinking can be learned and it should be taught in all levels of the education system" (p. 3). Traditional problem-solving strategies encourage students to find a pattern, make a table, work backwards, guess and check, draw a picture, make a list, or write a number sentence. Wheatley and Reynolds (1999) stress the necessity of creating a mental image before using diagrams to help solve problems. They imply that in order for a student to construct a useful illustration for problem solving—one of the seven strategies previously listed—they must first create a useful mental image. Albert Einstein conveyed this same idea in a letter to his friend, Jacques Hadamard, in 1945 when Einstein explained that his thought processes began with a play of visual images (Hadamard, 1996). Van de Walle (2010) and Johnson (2008) agree that spatial skills are important and that students can develop spatial skills given appropriate design and implementation with spatial experiences over a period of time. Again, the NRC (2006) agrees that spatial ability can be developed through activities in education, and, further, that this skill will develop within each student according to each individual's proclivity. With spatial thinking as one thread in the foundational fabric of mathematical reasoning and ability, growth in this skill can have a tremendous impact on the overall development of a student's ability to mathematically problem solve.

Foundation of the Problem

For more than 100 years, leaders in government, industry, and education have decried the performance of U.S. students in mathematics. In 1892, concerned with the state of secondary education, the National Education Association (NEA) appointed the Committee of Ten to study core courses and provide a national force for standardizing secondary school curriculum. The committee examined nine areas of study and became concerned that the quality of education was being negatively affected because too much was being asked of students. The reactions to the report released by the Committee of Ten were generally positive. However, the impact the report had on schools was minimal, because the committee was appointed during a time of social unrest.

In 1940, with World War II at the center of attention, much of the decade was spent educating students for functional competence rather than increasing the rigor of mathematical content. Mathematics was considered an important component of success in the engineering and technical support needed by the war effort. The induction testing of military recruits, however, revealed that many youth and young adults were underprepared in mathematical content (Senk & Thompson, 2003). Researchers at universities agreed and began to voice their concerns for the lack of higher levels of mathematics offered in school programs (Fauvel & Van Maanen, 2000). The new technologies of World War II and the lack of qualified manpower needed to fill new positions in government, engineering, and other facets of industry, revealed the weaknesses of the Depression-era mathematics curriculum. Six years after the war ended, a content-based curriculum, which eventually became known as "New Math," was catapulted into acceptance with the successful launching of the Soviet Union's *Sputnik*, the first space satellite, on October 4th, 1957. The American press treated *Sputnik* as a major humiliation and called attention to the low quality of math instruction in schools. In response, U.S. legislators hurriedly passed the National Defense Education Act, pumping millions of dollars into science and mathematics education, desperate to stay competitive in the "space race" (U.S. Department of Education, n.d.).

Teaching for meaning became a goal during the 1960s as psychologists pushed for an emphasis in "discovery learning," a teaching method of inquiry-based instruction founded in the idea that it is best for learners to discover facts and relationships for themselves. Jerome Bruner and Jean Piaget were the lead psychologists in this movement. They recommended that school curricula emphasize the individual student by allowing them to problem solve and formulate solutions based on their own knowledge of the world around them (Coxford & Jones, 2002). The 1969 moon landing meant that the U.S. had won the "space race," and funding for curriculum development dwindled, as did the movements for "New Math" and discovery learning.

In the early 1980s, shortly after standardized testing had become the norm, there was a widespread recognition that the quality of mathematics education had been deteriorating. This recognition resulted in various reports and commissions calling for an investigation of K-12 education. Of these reports, two stand out: *An Agenda for Action* and *A Nation at Risk*. Both reports held strong opinions about the need for change in school curriculum. NCTM released *An Agenda for Action* in 1980, and it became a strong position statement, placing the organization at the head of mathematics education. The report called for new directions in mathematics curriculum implementation, which would later be codified in 1989 in the form of national standards. Specifically, this document emphasized the need for problem-solving skills, manipulatives, and the implementation of calculator use in the classroom.

Today, two primary influences shape mathematics instruction in the U.S.: NCTM's *Principles and Standards for School Mathematics* (2000), and the "No Child Left Behind Act" of 2001 (NCLB). However, reform is always on the horizon and by the 2014-2015 school year, a new document will be added to this list. Rooted in the NCTM Process Standards (2000), the Common Core State Standards for Mathematics (CCSSM) will be implemented across the nation and were "designed to be robust and relevant to the real world, reflecting the knowledge and skills that our young people need for success in college and careers" (CCSSI, 2010). For the time being, most students today are learning under the inquiry-based umbrella set forth by *Principles and Standards*, which emphasize both content and process standards, including problem solving and spatial reasoning. In contrast, the NCLB Act, former President George W. Bush's reauthorization of the Elementary and Secondary Education Act, required school districts to show adequate yearly progress for all students in mathematics, forcing many districts to focus heavily on preparation for the standardized tests used to measure this progress. Currently, the NCLB Act faces reform under President Obama, and promises to "win the future and prepare [U.S.] students to out-educate and out-compete the world" (U.S. Government, 2011, p. 1). Further, President Barack Obama indicates that his mission is to create students who are ready for college and, eventually, careers. While reform to strengthen NCLB will, most likely, be welcomed, it is unlikely the emphasis on accountability for student performance will diminish.

Whatever the relative merit of these efforts, the harsh truth is that in more than four decades of regular standardized testing, both international and internal assessments of our students' mathematics achievement reflect little, if any, improvement. Since 1959, the International Association for the Evaluation of Educational Achievement (IEA) has conducted numerous international comparative studies of the mathematics and science performance of students. The most recent report, the 2007 *Trends in International Mathematics and Science Study* (TIMSS), revealed disappointing news about the progress of U.S. students compared to other countries worldwide. While increases in achievement at different grade levels have been attained, the mathematics achievement of U.S. students continues to lag behind. The National Center for Education Statistics (2003) noted that the improvement is deceiving since U.S. students scored as much as 66 points below economic competitors like Japan. According to the 2007 TIMSS study, benchmark results show that fourth- and eighth-grade students did not perform well in the "knowing and reasoning domains in terms of comparisons with other countries" (Gonzales, Williams, Roey, Kastberg, & Brenwald, 2009, p. 13). Also, the number of students in both the fourth and eighth grades with advanced scores trailed behind students from seven other nations (Gonzales, et al., 2009).

Alongside the TIMSS report, other studies have confirmed similar distressing results. For instance, the Program for International Student Assessment (PISA) released highlights from its 2006 study on U.S. performance of science and mathematics literacy. Among the findings, this study concluded that, on average, the U.S. was below the international average on the mathematical literacy scale (Baldi, Jin, Skemer, Green, & Herget, 2007). Assessment results for college-bound students are of equal concern. According to the report released in August 2010 by ACT (2010), only 24% of all high school graduates met or surpassed all four of the ACT College Readiness Benchmarks. Only 43% of those tested in the graduating class of 2010 met the mathematics benchmark. This benchmark is defined as the minimum score to indicate a 75% chance of making a "C" or higher, or a 50% chance of obtaining a "B" or higher in a typical college algebra course. These results further verify the Rasmussen, et al. (2011) concern over the percentage increase of college students that are in need of remedial mathematics courses.

More specifically, the TIMSS report revealed an interesting gap in geometric knowledge, an area grounded in spatial ability. The lowest performance from U.S. eighth grade students was in the geometric and spatial strand, where they scored 20 points below the international average of the 41 participating TIMSS countries. However, the ranking for eighth grade students in the U.S. was near average in algebra, fractions, data representation, analysis, and probability. In summary of the report, the U.S. overall score in mathematics was in the top third, but in geometry and spatial ability, the U.S. was in the bottom half for eighth grade students. Within the geometry and spatial thinking strand, the number of correct problems is low, "…indicating that substantial room for improvement remains in this content area" (Sowder, Wearne, Martin, & Strutchens, 2004, p. 124). Clearly, when considering both national and international education in mathematics, classrooms in the U.S. do not promote the kind of reasoning and problem-solving skills desired.

Problem Statement

Despite decades of reform, U.S. mathematics education faces its greatest challenge yet: how to overcome our nation's lackluster performance and transform educational practices allowing students to develop as mathematical problem solvers. In *Adding It Up*, authors Kilpatrick, Swafford, and Findell note that although U.S. students "may not fare badly when asked to perform straightforward computational procedures, they tend to have a limited understanding of basic mathematics concepts. They are also notably deficient in their ability to apply mathematical skills to solve even simple problems" (2001, p. 4). Improving performance, however, is easier if there is evidence of what students know, what they do not know, and where progress is already being made; and one only needs to visit past research to help answer these questions in part.

Despite the pervasiveness of spatial thinking in research and recommendations, this skill is still largely unrecognized in the educational system (Casey, Andrews, Schindler, Kersh, Samper, & Copley, 2008; Clements & Sarama, 2002; National Research Council, 2006). In *A Visual Approach to Algebra*, Frances Van Dyke briefly discusses the advantage to teaching concepts visually or, at least, with a visual component. Developmentally, students' ability to think in images precedes the capacity to think in words, and understanding follows the same model (Van Dyke, 1998). This natural order of learning supports activities that involve imagery and is, therefore, beneficial to all students.

Spatial thinking can help students in mathematical problem solving (Lean & Clements, 1981; Moses, 1977; National Research Council, 2006; Sorby, 2009). A majority of research literature links the concept of spatial reasoning to mathematical ability, and more specifically, to problem solving. The National Research Council (2006) claims that spatial representations can help students in learning mathematics and in problem solving. Literature in this area suggests that mathematics educators should,

1) have students generate their own representations;

2) use spatial representations to provide multiple and, where possible, interlocking and complimentary representations of situations, especially where the phenomena are not readily available to direct sensory perception;
3) use a wide variety of spatial representations;
4) use spatial representations to convey different types of thinking (e.g. data about how something is structured now, how it could or should appear in the future or did appear in the past); and
5) learn where—and which types of—spatial representations can be useful. (National Research Council, 2006, p. 108)

Further, NCTM recommends that academic programs "enable all students to use visualization, spatial reasoning, and geometric modeling to solve problems" (NCTM, 2000, p. 41). These same suggestions should be taken seriously at the university level since students of this age are also capable of honing this important skill (Sorby, 2009).

With all recommendations considered, it is unfortunate that students today are still not getting adequate training in spatial skills (Sommer, 1978; Tall, 1991; Wheatley G. , 1991), including undergraduate candidates (Mohler, 2008; Sorby, 2009). This lack of opportunity to develop spatial thinking could reside in mathematics teachers' underdevelopment in their own spatial ability (Richardson & Stein, 2008). Regardless, the outcome of insufficient training in spatial thinking has created a *need* for an increase in spatial training for mathematics students and educators alike (Kotze, 2007). Luckily, a multitude of studies suggest that spatial reasoning can be improved through rich, mental and hands-on experiences that are appropriate for the age of the student (see Mohler, 2008; Van de Walle, Karp, & Bay-Williams, 2010).

Although numerous studies exist that relate spatial ability to problem solving, few, if any, have been conducted with university-age students with varying majors in a regular undergraduate mathematics course. Former research with undergraduate-age participants indicates that engineering students (Mohler & Miller, 2008), education majors (Battista, Wheatley, & Talsma, 1982), and students in the arts (Edens & Potter, 2007) are all beneficiaries from spatial instruction in their field of interest. The goal of exposing university students in mathematics to spatial tasks is to expand their ability to utilize spatial visualization to better understand the world around them, and enhance their overall problem-solving capability.

Purpose of the Study

Spatial visualization plays an important role in any productive mathematical endeavor; it is one of the processes by which mental representations are created. This qualitative research study explored the influential nature of spatial reasoning tasks on spatial and problem-solving abilities of undergraduate students in a low-level mathematics course. A combination of spatial activities were used to exercise this skill and ranged from drawing activities (see Van Dyke, 1998; Wheatley, 2007) and mental activities (Wheatley, 2007), to hands-on manipulations with three-dimensional (3D) materials (see Johnson, 2008; Winter, Lappan, Phillips, &

Fitzgerald, 1986). The tasks used varied and, when possible, correlated to the mathematical objective presented in class.

Focus of the Study

In consideration of the international reports, a question emerges: why do U.S. students' performance lag behind when compared to students' peers in other countries? The answer to this question could reside in a number of explanations. This study posits that U.S. students of mathematics are weak in spatial reasoning skills and are therefore not performing well on assessments that require mathematical problem solving. NCTM recognizes spatial ability as a foundation for learning mathematics and recommends that a variety of spatial representations be made available to students when learning how to represent and solve problems (NCTM, 2000). Additionally, the NRC urges for spatial thinking instruction to be "infused across and throughout the curriculum," and for mathematics instruction to "create skills that promote a lifelong interest in spatial thinking" (2006, p. 109).

In consideration of NCTM and NRC's recommendations for mental representations, the study that follows examined how spatial ability of university students affects their ability to problem solve in mathematics. The study was shaped by the following theoretical assumptions:

- Students construct their learning, individually and collectively, in relation to their experiences (Piaget, 1970; Vygotsky, 1978).
- Ideal learning environments include opportunities for students to construct meaning and engage in spatial tasks to further understanding (Bishop, 1980; Wheatley G. H., 1991).

- Spatial reasoning supports student learning and mathematical problem solving (Tartre, 1990; van Garderen, 2006).

These assumptions, if implemented with careful thought and consideration for student learning, will foster mathematical thinking, spatial reasoning, and, hopefully, problem solving in the classroom.

In consideration of the assumptions stated, the following questions will guide this study:

1. How does the integration of spatial activities in an undergraduate mathematics content course impact student spatial ability?
2. In what ways does the integration of spatial reasoning tasks into an undergraduate mathematics content course influence problem-solving strategies?
3. How does the integration of spatial reasoning tasks influence the beliefs on spatial thinking of pre-service elementary teachers?

These questions were formed through the influence of the pragmatics worldview of experientialism, the theory that personal experience is the basis of knowledge. Pragmatism derives, in part, from the work of Peirce, James, Mead and Dewey (Cherryholmes, 1992). Many forms of this philosophy exist, but for this study, pragmatism as a worldview arises out of actions, situations, and consequences. Pragmatism strives to understand consequences of actions, is problem-centered, pluralistic, and is real-world practice oriented (Creswell, 2009). This pragmatic study is concerned with what strategies work and possible solutions to the problems presented above (Patton, 1990). A pragmatic researcher is less concerned with methods, but rather, emphasizes the research questions and all approaches available to understand the problem (Creswell, 2009).

As earlier stated, there is presently a paucity of studies that examine the effects of spatial reasoning on problem solving for undergraduate students. Pragmatist studies are not confined to a limiting research method, meaning multiple forms of data can be collected. This study used an embedded case study design, which includes both quantitative and qualitative components, to adequately answer the aforementioned research questions. The results of this research study are important because they may provide a partial explanation for why the performance of U.S. students remains far behind the performance of economic competitors, and it will contribute to the body of literature regarding the preparation of students of mathematics at the undergraduate level to problem solve through sharpening spatial reasoning skills.

Organization of the Study

The contents of each of the five chapters describing this study are as follows: Chapter I consists of the introduction, foundation of the problem, problem statement, purpose of the study, and focus of the study of the study. Chapter II will be a review of the literature as it pertains to the study. Chapter III will include the methodology, which consists of the research design, definition of terms, assumptions and limitations, participant information, instruments, data collection procedures, and analysis of the data. Chapter IV will analyze the results, while Chapter V will discuss the implications of the results and make suggestions for future research.

Summary

Spatial reasoning is a foundational component for meaningful mathematical learning. It is a skill used in everyday life and a necessity to succeed in any career under the STEM umbrella. Academically, spatial aptitude can support general mathematical achievement, creativity in mathematical thinking, and mathematical problem solving. NCTM (2000) and NRC (2006) both recommend that spatial skills be taught in the classroom to encourage successful problem solving. This suggestion, among many others, is the result of decades of educational reform and standardized testing. From the launching of *Sputnik* to the "No Child Left Behind Act" of 2001, America's citizens and government have been at odds over the lack of progress concerning mathematics education. Results from the most recent TIMSS (2007) report, PISA (2006) assessment, and ACT (2010) release only further the debate. Clearly, U.S. students are not being given the opportunity to develop the reasoning and problem-solving skills needed to shine locally, not to mention in the global spotlight desired by U.S. leadership.

The education system in the U.S. is resilient and may overcome this detriment if instructional programs across the nation start pushing the inclusion of problem-solving skills in the classroom. One component research has linked to problem solving is spatial reasoning. Students' ability to think in images precedes their ability to think in words. Therefore, spatial thinking is essential in the learning of mathematics. Unfortunately, this skill is being overlooked in U.S. classrooms, a mistake that educational programs cannot afford to make if they wish to test competitively. The purpose of this study is to determine the influence of spatial tasks on the ability of undergraduate students to problem solve and think spatially. Further, students' beliefs concerning the importance of spatial ability will also be addressed. As with any qualitative study, this study has several factors limiting generalizations. Even with all things considered, the aim of this study is to shed light on the influence of spatial thinking and its ability to help students in mathematics. A review of the literature will help set the stage for which this study is built.

Chapter II

REVIEW OF THE LITERATURE

Of the many contributions that American psychologist Jerome Bruner made to the scientific study of education, perhaps none is more influential than his assertion that students must be challenged to learn and that educators must support their doing so. In 1959, Bruner reported that students learn about geography in one of two ways and offered the following example on how to foster thinking:

> One group learned geography as a set of rational acts of induction—that cities spring up where there is water, where there are natural resources, where there are things to be processed and shipped. The other group learned passively that there were arbitrary cities at arbitrary places by arbitrary bodies of water and arbitrary sources of supply. One learned geography as a form of activity. The other stored some names and positions as a passive form of registration. (Bruner, 1959, p. 188)

Bruner goes on to describe the work of the first group highlighting the type of environment created when rich thinking is promoted:

> We hit upon the happy idea of presenting this chunk of geography not as a set of knowns, but as a set of unknowns. One class was presented blank maps, containing only tracings of the rivers and lakes of the area as well as the natural resources. They were asked as a first exercise to indicate where the principle cities would be located, where the railroads, and where the main highways. Books and maps were not permitted and "looking up facts" was cast in a sinful light. Upon completing this exercise, a class discussion

> was begun in which children attempted to justify why the major city would be here, a large city there, a railroad on this line, etc.
>
> The discussion was a hot one. After an hour, and much pleading, permission was given to consult the rolled up wall map. I will never forget one student, as he pointed his finger at the foot of lake Michigan, shouting, "Yipee, Chicago is at the end of the pointing-down lake." And another replying, "Well, OK: but Chicago's no good for the rivers and it should be here where there is a big city (St. Louis)." These children were thinking, and learning was an instrument for checking and improving the process. To at least a half dozen children in the class it is not a matter of indifference that no big city is to be found at the junction of Lake Heron, Lake Michigan, and Lake Superior. They were slightly shaken up transportation theorists when the facts were in. (Bruner, 1959, pp. 187-188)

Clearly, this group of students was engaged in meaningful thinking, spatial thinking, and all it took was a simple map and well-written prompts. This example is just one of many that encompassed the need for spatial skills. Hidden behind many of the daily tasks of everyday life, the workplace, and science, spatial thinking is integral to the success of problem solving, and mathematics is no exception.

Teachers have a responsibility to create classrooms that meet the standards necessary in order to prepare students for the demands of the 21st century, which includes proficiency in spatial thinking. To meet this responsibility, teachers must expect more than passive learning and create opportunities for students to actively engage in activities that allow them to construct their own knowledge. The philosophy of learning from which this study was designed is known as the constructivist learning theory. The basis of the constructivist theory is the belief that learners construct their own understanding and meaning based on prior knowledge and information they acquire through experience (Noddings, 2006; Richardson, 2003). Constructivism recognizes that knowing is active, it is individual, and that it is based on previous knowledge (Ernest, 2010). The first principle of constructivism is based on the idea of construction as expressed by von Glassersfeld, "knowledge is not passively received but actively built up by the cognizing subject" (1989, p. 182). The constructivist theory originated with Dewey's pragmatism pedagogy (Reich, 2007). In the constructivist theory, activities that are grounded provide individuals with the opportunity to draw upon prior knowledge, learn through practice, problem solve, and reflect on the learning process and knowledge gained (Bruner, 1960; Richardson, 2003).

Problem-solving and decision-making activities are core components to the constructivist-learning model. Both Dewey (1933) and Vygotsky (1978) contend that each student in a learning environment has different prior knowledge, which affects how each responds to new information since prior knowledge serves as a foundation for decisions the student employs (Bruner, 1960). Further, in a constructivist approach, learners participate in activities that foster communication during and after the learning process (Ernest, 2010). Such activities should provide the opportunity for students to collaborate and share in the process of constructing their ideas with others (Lunenburg, 1998). This process allows students to develop a shared understanding of the topic. Engaging in communication after an activity allows students the opportunity to "develop a metawareness [*sic*] of their own understanding and learning process" (Richardson, 2003, p. 1626). Adhering to this learning process allows students to extend their knowledge to other contexts (Boddy, Watson, & Aubusson, 2003), within mathematics and otherwise.

The goal of this chapter is to review the research literature that is pertinent to the study of undergraduate mathematics students' spatial ability, their problem-solving skills, and their beliefs pertaining to spatial thinking. The research questions, which guide this study, are:

1. How does the integration of spatial activities in an undergraduate mathematics content course impact student spatial ability?
2. In what ways does the integration of spatial reasoning tasks into an undergraduate mathematics content course influence problem-solving strategies?
3. How does the integration of spatial reasoning tasks influence the beliefs on spatial thinking of pre-service elementary teachers?

The major areas of research relevant to the present study include:

1. Spatial Thinking: A historical background including the developmental research supporting spatial thinking, followed by connections regarding gender, aptitude, teaching, and research regarding the difficulties of measuring an introspective activity.

2. Problem Solving: Problem solving as a goal, followed by historical contributions, connections to teaching, teachers, and research regarding methods used to measure this goal.
3. Spatial Thinking and Problem Solving: Connections between spatial thinking and mathematics, Krutetskii's factors in mathematical performance, connections between spatial thinking and problem solving, and implications for teaching mathematics.
4. Beliefs regarding spatial thinking and problem solving.

Spatial Thinking

The Beginnings of the Research

The history of spatial thinking is rich in theory and research. While an exhaustive description of the history behind spatial thinking is not practical, an overview is appropriate to provide an introduction to the focal points of the research study. With implications for nearly every technical field, spatial ability has been an active thread of research for some time. As early as 1880, spatial abilities were under study when Sir Francis Galton began recording on his systematic psychological inquiry into mental imagery. Since that time, research in the field has continued and the chronology of spatial ability research can be broken into four major fields of activity (Mohler, 2008).

Starting with Galton, the first range of research occurred from 1880 until approximately 1940. While many credit Galton (1911) with being the initiator of the research, it was not until the 1920s that publications emerged with a special focus in spatial thinking. Contributions during this time acknowledged spatial ability and, for the first time, defined the ability as separate from general intelligence. More specifically, researchers classified intellect into two categories: the first being verbal/rational/logical and the second being visual-spatial/nonverbal/intuitive. Researchers still use these divisions today (Cooper, 2000).

The first published identification of spatial ability was a 1921 paper by Thorndike. He drew a defining distinction between classes of intelligence and argued that standard intelligence tests only measured abstract intelligence (1921). Thorndike's publication set the stage for all spatial ability research that would follow. Other researchers, such as McFarlane (1925) and Kelley (1928), also influenced some of the better-known researchers of this time. El Koussy (1935), a researcher who more clearly defined spatial thinking as the capacity "to obtain, manipulate, and utilize visual spatial imagery" (p. 86), compiled 28 tests of spatial intelligence, both his own and those of other researchers.

Thurstone (1938) first introduced the concept of kinesthetic imagery, which is the visual factor of spatial intelligence. His theory was that intelligence was made up of several primary mental abilities as opposed to a single, holistic factor. He further described three aspects of kinesthetic imagery as being able to recognize an object when viewed from different angles, being able to imagine the internal movement of parts within a configuration, and the ability to visualize one's body within the object for the purpose of viewing the object from every perspective possible. Thurstone recognized that the ability to think spatially was one from a set of abilities that was needed to be successful in mathematics (Bishop, 1980).

Between 1940 and 1960 there was an acknowledgement of multiple space factors and a surge of instruments developed to assess them. During this time, researchers focused their efforts on creating measures of assessment and definitions for spatial areas of ability. Which, as it turned out, was a somewhat futile effort. Some of these researchers denied the importance of the skill and deemed it unimportant for practical purposes (Mohler, 2008). This undervaluation created confusion among the scholarly community resulting in contradictory names and definitions for spatial factors that created complications for the instruments used to measure them. Nevertheless, spatial testing gained a strong foothold due to the large-scale assessments conducted by the U.S. military to help place and assign recruits (see Guilford & Zimmerman, 1947). By the end of this period, researchers agreed that spatial thinking was not unitary and multiple tests were available for use (Eliot & Smith, 1983).

Developmental Research

The goal of developmental research is to help answer questions related to how and when spatial ability develops. Research in this area began around the 1960s and continued through the 1980s. Perhaps the most influential to this body of knowledge is the work of Swiss developmental psychologist, Jean Piaget. With the belief that material and social conditions determined a child's development, Piaget (1970) posited that the development of space begins at infancy. Piaget and Inhelder (1967) are credited with much of what has been accepted with respect to children's construction of conceptual space. They defined two types of spatial ability, Perceptual Spatial Ability and Conceptual Spatial Ability, when a child interacts with his or her environment. Perceptual Spatial Ability is defined as the ability to perceive the spatial relationships between objects, while Conceptual Spatial Ability is the ability to build and manipulate a mental model of the environment (Piaget & Inhelder, 1967). Further, Piaget and Inhelder (1967) suggest that children progress through three stages in the development of their cognitive spatial ability: preoperational, concrete, and the formal operational stage.

Another theory that is sequential in nature is the van Hiele model of geometric thought. Developed in the 1950s by Dutch mathematics educators P.M. van Hiele and D. van Heile-Geldof, this theory suggested that all students progress through five levels of geometric reasoning, and must master one level before moving onto the next. Each level describes how a student thinks and to what level a student is able consider geometric ideas, not necessarily how much knowledge a student has. These levels, as arranged from lowest to highest, include: Level 1—Visualization, Level 2—Analysis, Level 3—Informal Deduction, Level 4—Deduction, and Level 5—Rigor (Van de Walle, Karp, & Bay-Williams, 2010). The van Hiele Level 1, Visualization, is a nonverbal level and is of most interest to this study. At this level, recognition is the primary focus as figures are recognized by appearance alone. Therefore, this level is heavily dependent upon visual processing (Van Hiele, 1999). As the van Hiele levels increase, there is a decrease in emphasis of visual processing skills and in increased emphasis on verbal knowledge (Clements & Battista, 1992).

There have been many studies that have focused on developmental issues. A more recent study confirmed that developmental stages concerning spatial thinking do occur in children, but that they may be different from what Piaget and Inhelder proposed. According to Huttenlocher, Newcombe, and Vasilyeva (1999), there exist more than three stages and that some children are able to think using some forms of advanced spatial thinking earlier than once thought. Other studies have focused on spatial ability differences with respect to age or how the ability changes over time, and can be found in works by Battista (1990); Salthouse, Babcock, Mitchell, Palmon, and Skovronek (1990); Tartre (1990a); and Coleman and Gotch (1998). While scientists agree that stages of development exist for spatial thinking and that the ability progresses with age, there is not a general consensus when these abilities develop. Furthermore, research indicates that age is not the sole indicator of spatial aptitude.

Differential Research

Is it is possible that sex and general intelligence are also factors in a child's ability to reason spatially? The research seems to indicate that a relationship does exist. Since the mid-1970s, research has investigated specifics regarding spatial thinking. Sex differences, modifications for high and low level learners, biological factors, and improvement on methods of measurement are among some of the branches research in spatial ability has explored.

When considering differential research, sex is easily the most recognized factor for deliberation. Literature consistently notes the discrepancies in spatial performance between males and females. Maccoby and Jacklin (1974) ignited a surge of interest within this area when they suggested four areas in which sex differences become apparent, most notably in spatial ability. When considering differences in spatial ability, three big questions usually emerge: Do sex differences exist? If so, how significant are they? What causes them—biological or environmental factors? The research that was conducted in response to these questions is expansive. Unfortunately, differential research in this area appears to be one of the most contested, resulting in an inconclusive consensus (La Pierre, 1993).

There is a plethora of research that supports the existence of male dominated sex differences in spatial thinking (Boakes, 2009; Cochran & Wheatley, 1982; McDaniel, 1976; Shepard & Metzler, 1971; Voyer, 1998). Eals and Silverman (1994) posit that male dominance in spatial ability exists and that the dominance holds across "regions, classes, ethnic groups, ages and virtually every other conceivable demographic variable" (p. 95). In a 1990 study, 145 geometry students, both male and female, were given paper-and-pencil tests that measured spatial visualization, logical reasoning, geometric achievement, geometric problem solving, use of drawing strategy, use of visualization without drawing, use of nonspatial strategy, correct drawings made, and discrepancy between the logical and spatial score (Battista). There was no evidence of difference in logical reasoning among the sexes, but males scored significantly higher than females on spatial visualization and geometric problem solving.

Another study, however, examined fourth and fifth-grade students' drawings to determine the relationship between spatial understanding and mathematical problem solving (Edens & Potter, 2007). Students were asked to draw a picture of themselves with their friends playing on the school playground.

In addition, they were asked to include their school building in the background and a dog in front of them in the picture. Interestingly, the analysis of the results indicated that a significant difference existed between boys and girls with regard to spatial understanding, but in the girls' favor. Since this study contained a verbal component, one possible explanation for this unique finding is the fact that females are more fluent in verbal communication and, therefore, had an advantage (Kimura, 1996). Other studies, however, would not find these results so surprising.

A seminal study by Linn and Peterson (1985) examined sex differences in spatial abilities by focusing on three distinct areas of spatial reasoning: spatial perception, mental rotation, and spatial visualization. The researchers suggested that females use less effective strategies than males, which influence performance on spatial tasks. For example, Linn and Peterson observed that females tend to be more cautious, double check their answers more frequently, take more time answering questions, and noted that females find spatial tasks more difficult than males (Yilmaz, 2009). The results of this study found that of the three areas of focus—spatial perception, mental rotation, and spatial visualization—there was little to no difference in performance on spatial perception and spatial visualization. A separate study confirmed that the only area where there was a significant difference in ability concerning sex was mental rotation (Casey et al., 2008). These studies suggest that bias among sex differences may only exist in certain areas of spatial thinking.

Researchers Fennema and Sherman (1977) took this suggestion one step further by asserting that they "do not support either the expectations that males are invariably superior in mathematics achievement and spatial visualization or the idea that differences between the sexes increase with age and/or mathematics difficulty" (p. 69). They argued that some studies might not control participants' backgrounds as they should, skewing differences among the sexes. Similar studies supported the idea that the differences in spatial ability among the sexes are either very small or nonexistent (Fennema & Tartre, 1985; Lord & Holland, 1997).

Perhaps to what extent sex differences exist is of less importance than what can be done about the gap to help improve discrepancies in spatial understanding. Along with examining if spatial differences subsist in spatial ability, many researchers also considered if or how the differences could be modified. Some research suggests that the learning gap between sexes with respect to spatial thinking can be reduced (Ben-Chaim, Lappan, & Houang, 1988; Fennema & Sherman, 1978; Spence, Yu, Feng, & Marshman, 2009; Stransky, Wilcox, & Dubrowski, 2010) while others furhter state that the gap can be eliminated (Lord, 1987; Sorby, 2009). In one study, a group of 116 first-grade children were split up and placed in either a control group or an experimental group and were administered a mental rotation test (Tzuriel & Egozi, 2010). The experimental group received instruction aimed at improving representation and transformation of spatial information while the control group received a substitute program. After three months, the two groups retested and the results revealed that initial differences among the sexes in spatial ability were eliminated in the experimental group but not the control group. This study, along with the others stated, indicated that training can improve spatial ability at any age using methods aimed at spatial improvement.

The results between sex differences and spatial ability have led researchers to investigate possible differences resulting from biological factors. Studies searching to discover gender differences include those focused on left- or right-handedness (Gilleta, 2007), brain activity (Jausovec & Jausovec, 2007), and the parietal lobe of the brain with reference to gray and white matter (Koscik, O'Leary, Moser, Andreasen, & Nopolous, 2009) to name a few. In the latter, researchers found that structural differences in the parietal lobe of the brain significantly corresponded to spatial ability when measured using the Mental Rotations test. Women were found to have more gray matter volume in their parietal lobe, a disadvantage with respect to spatial ability, while men were found to have a larger surface area of the parietal lobe, an advantage when measuring spatial rotations.

Psychologists and educational researchers are not the only scientists interested in this area of study. There exists a need for spatial thinking in the workplace as it pertains to fields in science, technology, engineering and mathematics, also known as STEM, which is currently neglected (Lubinski, 2010; Wai, Lubinski, & Benbow, 2009). Further, Wai, Lubinski, and Benbow (2009) noted that students who have an aptitude for spatial thinking go on to pursue STEM domains. Since both men and women are pursuing careers in STEM fields, efforts to deveolp spatial ability should not be isolated to a specific sex. But what about gifted or remedial students; should instruction in spatial reasoning differ?

Shepard (1988) compiled autobiographical accounts of notable figures in the arts, sciences, and literature who claimed to be influenced, at least in part, by spatial thinking in the creation of original ideas. For example, Einstein described developing the concept of spatial relativity in part through experiments in his mind where he imagined the properties of space and time. In a study involving sixth-grade students, Van Garderen (2002) classified three levels of learners—students with a learning disorder, average achieving students, and gifted students—and observed the types of images students were using to solve mathematical word problems. He found that differences in imagery use existed among the three groups, where the highest level of spatial reasoning was observed in the gifted group. Regardless of which types of students prefer to use spatial reasoning, Wai, Lubinski, and Benbow (2009) would argue that spatial ability assessment and training would benefit all students and that, “basic science indicates that students throughout the ability range could profit from spatial ability assessments and the provision of educational opportunities aimed at developing spatial ability” (p. 818). Therefore, it is crucial that spatial training be available for all learners and a priority for teachers in classroom instruction.

Spatial Thinking in the Classroom

Is spatial thinking really a key to science, technology, engineering, and mathematics—the so-called STEM disciplines? Several studies conducted over the last fifty years seem to indicate just that. One of the most robust studies, named Project Talent, started in the 1950s and tracked approximately 400,000 for people from their high school years until just recently (Wai, Lubinski, & Benbow, 2009).

The study found that people who tested high in spatial ability were much more likely to choose a career in a STEM field than those with lower scores. Further, these same students tended to have higher verbal and mathematical scores as well. Luckily, research has shown that spatial skills are beneficial for all students, even those who end up serving in a non-STEM related field (Mohler, 2008; National Research Council, 2006). Studies such as Project Talent support the National Council of Teachers of Mathematics (NCTM) demand for spatial thinking in the classroom (2000). The National Research Council (NRC) recognized the importance of spatial ability and calls for the skill to be emphasized beyond high school stating, "spatial thinking can be learned, *and* it can and should be taught at all levels in the education system" (2006).

Even with highest recommendations, spatial thinking is still overlooked as an important tool in education (Casey et al., 2008; Clements & Sarama, 2011; Wheatley, 1991). McArthur and Wellner (1996) acknowledged that the spatial ability of students is becoming poorer due to decreased focus of this skill in schools. This is problematic because studies have described spatial skills, such as mental rotation, as a "critical mediator" with performance on the mathematics portion of the SAT exam—an assessment used to grant students admission into undergraduate programs (Casey, Nuttall, & Pezaris, 1997, p. 676). Such results should not be a surprise since the recommendation to include imagery in education is far from new. Svensen (1948) noted that physical and non-physical things can be visualized and that with the "increase in accuracy of representation and visualization has come the life and growth of civilized man" (p. 20). Svensen continues, "Thus I repeat that the accuracy of visualization is the yardstick of education. The power of visualization has been and must continue to be developed as an essential factor in education" (p. 20). Svensen's challenge begs the question: Who is responsible for making sure today's students are given the opportunity to sharpen their spatial thinking? Rowe (1945) would argue that it is the "instructor's job to teach [students] to visualize and to think" (p. 3).

Since spatial ability is malleable, spatial thinking can and should be fostered with the right kind of instruction. It has been shown that spatial thinking improves more over the school year than over the summer months (Huttenlocher, Levine, & Vevea, 1998), so teachers should feel some sense of accomplishment. In one study, undergraduates were given extended, semester-long practice on mental rotation using the game of Tetris (Terlecki, Newcombe, & Little, 2008). Results showed that training effects were massive, lasted several months, and generalized to other spatial tasks such as constructing two- and three-dimensional images.

However strong the case is for teaching spatial thinking, there are some challenges with convincing pre- and in-service teachers to prioritize implementation of the skill. Because spatial thinking is not a subject, not something in which children are explicitly tested, it often gets lost among reading, mathematics, and the other content areas standardized testing demands attention in classroom instruction. Another challenge is with regards to teacher training. Studies have found that space and shape are problematic areas for teachers (Kotze, 2007) and, more troublesome, that "the majority of teachers have had minimal experiences with spatial tasks as part of their own K-12 mathematics curriculum;

thus, their spatial ability is underdeveloped" (Richardson & Stein, 2008, p. 107). Presmeg (1986) found that mathematics teachers who are visual are more inclined to teach holistically:

> It was found that teachers in the non-visual group were more inclined to adopt a lecturing style, and to teach formally, logically, rigorously, in a matter which could be called convergent…the visual group of teachers—and to some extent the middle group—used many other teaching aspects which are summed up in one characteristic principle, as follows. The visual teachers made connections between mathematics curriculum and many other areas of pupils' experience, including other subjects, other parts of the syllabus, mathematics learned in past years, and, above all, the real world. Visual teachers expressed in their teaching many traits commonly associated with creativity …Non-visual teaching had the effect of leading visualizers to believe that success in mathematics depends on rote memorization of rules and formulae. (p. 46)

To combat this discrepancy among teachers of mathematics, and to best prepare pre-service teachers for the challenges of the classroom, some researchers have advised that spatial ability training be a part of pre-service teacher programs (Lord & Holland, 1997; Unal, Jakubowski, & Corey, 2009). Unal, Jakubowski, and Corey (2009) point out that, "it was identified that instructional activities that afford opportunities for fostering spatial abilities must be included in pre-service programmes so that future teachers have a mathematical foundation from which to teach" (p. 997).

When a teacher creates an environment rich in spatial thinking, students are more likely to learn in a meaningful way. Explicitly, Wheatley and Abshire (2002) explain,

> Meaningful mathematics is usually (always?) image-based. While there may be certain forms of mathematical reasoning that do not use imagery, most mathematical activity has a spatial component. If school mathematics is only procedural, students may fail to develop their capacity to form necessary images of mathematical images of mathematical patterns and relationships. (p. 32)

To help students use spatial reasoning to their benefit, it is important to understand how students develop naturally. As stated earlier, many psychologists believe that students learn in stages. Bruner (1973), in particular, believes that a child explores new things first through action then through imagery before, finally, using language to describe and comprehend the world around them. A thorough description of one young lady, Elaine, going through this process can be found in a study conducted by Reynolds and Wheatley (1997). Elaine, a fifth-grade student, was recorded and given the following task: "A videotape can record two hours on short play and four hours on long play. After recording thirty minutes on short play, how many minutes can it record on long play?" (p. 101). After a brief discussion about the meaning of "short play" and "long play," Elaine thought about the problem and began drawing. When reviewing the video, the authors noted, "It was almost possible to turn off the sound and see the solution emerge through her drawings and hand movements" (p. 102). Elaine was successful in solving the stated task, along with others, by initially constructing images that eventually led her to a solution. Elaine's well-developed spatial thinking skills gave her mathematical power, which enabled her to construct, examine, and reconstruct complex relationships—a goal for all students.

Measuring Spatial Thinking

Measurement of spatial abilities has an extensive history and usually takes on four general types of tests: performance tests, paper-and-pencil tests, verbal tests, and film or dynamic computer-based tests (Lohman, 1993). How the tests are conducted, however, are of less importance than what area of spatial thinking they cover. In general, researchers suggest three categories of spatial ability: spatial perception, mental rotation, and spatial visualization (Clements & Battista, 1990; Linn & Peterson, 1985). Consequently, there exists an abundance of tests that can be used to measure each of these categories.

Debate over the best method to assess spatial thinking is evident in the literature. This should come as no surprise since measuring spatial thinking is much like grabbing smoke—the very act of reaching out to take hold of it disperses it. When considering measuring spatial ability, one must question how three-dimensional spatial reasoning can be tested thoroughly on paper, a two-dimensional medium. Yilmaz (2009) suggested focusing on spatial factors to measure spatial ability while others suggest administering tests to measure different stages of development (Sorby, 2009). Thus far, research has not provided a fail-proof method but it does offer several assessments that can be used with confidence; one example is the Purdue Spatial Visualization Test (PSVT).

The PSVT is comprised of three parts: Developments, Rotations, and Views. Each test is designed to measure one of the three categories of spatial ability described previously. The Developments section consists of 12 questions designed to measure an individual's spatial visualization through spatial structuring, the Rotations section consists of 12 questions designed to measure an individual's mental rotation ability, and the Views section consists of 12 questions designed to measure in individual's spatial perception. Other tests commonly used to measure spatial ability such as the Mental Rotations Test, the Paper Folding Test, the Card Rotations Test, and the Cube Comparison Test tend to only emphasize one of the three components of spatial ability. Since spatial thinking is found in multiple contexts, each unique, and each can be used to measure multiple areas of interest, researchers have also developed their own spatial thinking instruments to satisfy individual research requirements (Ganesh, Wilhelm, & Sherrod, 2009).

Attempting to understand and discuss spatial thinking, which is by definition intuitive and nonverbal, is a difficult task indeed. It could be argued that any attempt to verbalize the processes involved in spatial activity ceases to be spatial thinking. Any evidence about how the skill is manifested must be indirect since it is impossible to experience another's spatial thoughts. The resulting indirectness of the research in this area does set limits, but should not curtail it. If spatial thinking is important to meaningful learning, then researchers must find ways to identify, measure, and improve the skill.

Problem Solving in Mathematics

Goals of Problem Solving

Learning to solve problems is a principal focus in the study of mathematics. A primary goal of mathematics teaching and learning is to develop the ability to solve a wide variety of complex mathematics problems. To many mathematically knowledgeable people, mathematics is synonymous with solving problems; solving contextual problems, creating patterns, interpreting figures, proving theorems, etc. On the other hand, persons who are not well versed in mathematical reasoning often think that any activity involving numbers is problem solving. The sad truth is that most of what is happening in schools is simply procedural knowledge (Wilson, Fernandez, & Hadaway, 1993).

Problem solving is, by definition, engaging in a task for which the solution method is not obvious or known in advance. In other words, problem solving is what you do when you do not know what to do. NCTM (2000) believes that problem solving is an integral part of mathematics learning and asks that,

> Instructional programs from prekindergarten through grade 12 enable all students to build new mathematical knowledge through problem solving; solve problems that arise in mathematics and in other contexts; apply and adapt a variety of appropriate strategies to solve problems; [and] monitor and reflect on the process of mathematical problem solving. (p. 52)

By engaging in problem solving in mathematics, students should learn to develop new strategies of thinking, habits of questioning and curiosity, and confidence in dealing with a situation that causes perturbation. It would then seem intuitive that problem-solving activities would provide opportunities for students to expand their strategies, not stifle them. Unfortunately, with the expectation that the teaching and learning of mathematics is procedural, some attempts at incorporating problem solving have done more harm than good.

Historically, the term problem solving has had more than one meaning. Many times, problem solving is thought of as "solving highly structured word problems appearing in texts" (Wheatley & Abshire, 2002, p. 19). Attributing some relationship to Polya's problem-solving strategies, some textbooks use a four-step method to teach problem solving. Strategies such as Polya's have been taken to imply linear thinking when problem solving, an unfortunate misinterpretation. Polya's four-phase heuristic process guides students to understand the problem, devise a plan, carry out the plan, and then check the result (Polya, 1973). While the steps of Polya's plan are linear, it is unlikely they were written to be as rigid as most textbooks present them—with little room for students to think for themselves.

Cognitive Process of Problem Solving

It can be argued that in order to teach mathematical problem-solving skills effectively, an understanding between the demands of problem solving and cognitive processes involved should be reached. Wu (2004) identified two problem-solving cognitive processes: the factor-analytic approach and the information processing approach. The former approach is generally empirical in that the characteristics are identified through the use of exploratory factor analysis. Based on these factors, conclusions can be made about the nature of mathematical thinking. One sub factor in this area is visual perception. Information processing approach, on the other hand, is concerned with the sequential steps of cognitive demand involved when solving a mathematical problem. Polya's problem-solving strategies fall into this category. Another well-known information processing theorist refined Polya's steps and defined mathematical problem solving in five episodes: reading, analysis, exploration, planning/implementation, and verification (Schoenfeld, 1983). Polya's and Schoenfeld's stages of problem solving can serve as useful prompts for students to evaluate their own thought processes. Without these processes, early problem solving can be somewhat ad hoc and disorganized. Approaching problem solving in a systematic way can help students acquire the necessary skills to move past a set of steps and onto more creative strategies.

Implications for Teachers

Pre- and in-service teachers need to have well-developed problem-solving skills so they can aid students in learning mathematics *through* problem solving. While teaching problem solving to pre-service teachers, Krulik and Rudnck (1982) determined that before teachers can provide effective instruction in mathematical problem solving, they themselves must become adequate problem solvers. If teachers were more fluent problem solvers, they might be more apt to create space for problem-solving opportunities in their classrooms. Unfortunately, teachers are often hesitant to include problem solving in their everyday routine for a number of reasons: problem solving is too difficult, it takes up too much class time, school curriculum is already too heavy a load and there is no room for additional goals, since problem solving is not tested it is also not important, problem solving is not in the textbook, and lastly, the belief that basic facts must first be mastered before students can be expected to problem solve (Wilson, Fernandez, & Hadaway, 1993). These ideas short change the students and misrepresent the true meaning of learning mathematics.

As the emphasis on problem solving in mathematics increases, the need for evaluation of progress in problem solving becomes more pressing. While correct answers will always be desired, just knowing if an answer is correct or incorrect will no longer suffice. Schoenfeld (1988) warns:

> All too often we focus on a narrow collection of well-defined tasks and train students to execute those tasks in a routine, if not algorithmic fashion. Then we test the students on tasks that are very close to the ones they have been taught. If they succeed on those problems, we and they congratulate each other on the fact that they have learned some powerful mathematics techniques. In fact, they may be able to use such techniques mechanically while lacking some rudimentary thinking skills. To allow them, and ourselves, to believe that they "understand" the mathematics is deceptive and fraudulent. (p. 30)

Creating an atmosphere where problem solving is the norm and choosing problems wisely is a difficult, but necessary, part of teaching mathematics. Further, it is the teacher's job to embed opportunities for the students to use strategies of problem solving that will cross content areas (NCTM, 2000).

Measuring Problem Solving

The art of problem solving is the heart of mathematics. Thus, mathematics instruction should be designed so that students experience mathematics as problem solving. The challenge here is to choose tasks that help create this experience. Teachers and future educators will have several variables to juggle when selecting tasks, two such variables are: task variables and subject variables. Should a task be chosen based on the complexity of the problem or for the relevance to the subject at hand? A 1979 study considered these questions (Days, Klum, & Wheatley, 1979). Fifty-eight eighth-grade students were divided into concrete- and formal-operational groups and given a series of problems. The problems ranged from simple to complex and the students were interviewed about their problem-solving strategies. Analysis of the interviews revealed that formal-operational students used a greater variety of processes on the complicated problems than the simple problems. Also, problem structure had a greater effect on problem difficulty for the formal group than in the concrete group. In summary, the researchers concluded that both task variables and subject variables should be considered when teaching problem solving. This is a reasonable request for educators since the first person in the classroom that must become a problem solver is the teacher.

Spatial Thinking and Problem Solving

Effective problem solving in mathematics depends in part on spatial thinking. "Meaningful mathematics learning is usually (always?) image-based. While there may be certain forms of mathematical reasoning that do not use imagery, most mathematical activity has a spatial component" (Wheatley & Abshire, 2002, p. 32). While it is important to know algebraic symbols and productive procedures to get answers, it is also important to be able to think about a problem conceptually. Without the help of an illustration, consider the following problem:

> Imagine a large cube made up of twenty-seven smaller cubes, that is, three layers of nine cubes each. Imagine further that the entire outer surface of the large cube is painted red and ask yourself how many of the smaller cubes will be red on three sides, two sides, one side, and no side at all. (Arnheim, 1980, p. 491)

As long as you are thinking of the 27 cubes as a pile of blocks, attempting to count sides, your procedure is mostly mechanical. Now, think of the cubes as a well-structured centrally symmetrical structure, like a Rubik's cube, that you can rotate in your mind. With practice, this ability will allow you to make quick and accurate conjectures about the number of sides that are painted red. The thinking required by an exercise such as this is the substance of mathematical thinking.

Connections Between Spatial Thinking and Mathematics

In the example above, was it seeing or thinking that solved the problem? Obviously, the distinction between the two is absurd since each is dependent upon the other. It is well documented that spatial ability is positively related to the achievement and understanding of mathematics (Battista, 1980; Brating & Pejlare, 2008; Fennema & Sherman, 1977). Further, spatial skills help students with sense making in practical, every day areas as well. The National Council of Teachers of Mathematics (1989) recommends that spatial thinking be included in the mathematics classroom because "spatial understandings are necessary for interperting, understanding, and apreciating our inherently geometric world" (p. 48).

According to Clements and Battista (1992), high correlations were found between mathematics achievement and spatial ability at all grade levels. In a different study, undergraduate pre-service teachers showed mathematical growth after a period of spatial instruction (Unal, Jakubowski, & Corey, 2009). In 1985, Fennema and Tarte confirmed through research that a high correlation exists between spatial visualization and mathematics for both girls and boys.

Creativity in mathematics is also important for meaningful understanding, and spatial thinking is a tool that helps students do just that (Clements, 1998). Empirical evidence has indicated that spatial imagery reflects general intelligence as well as specific abilies that are highly related to solve mathematical problems, especially nonroutine problems (Wheatley, Brown, & Solano, 1994). The National Research Council (2006) recognizes that spatial representations can help students in learning and problem solving and suggests that educators:

> (1) have students generate their own spatial representations; (2) use spatial representations to provide multiple and, where possible, interlocking and complimentary representations of situations, especially when the phenomena are not readily available to direct sensory perception; (3) use a wide variety of spatial representations; (4) use spatial representations to convey a variety of kinds of thinking…; and (5) learn where—and which types of—spatial representations can be useful. (p. 108)

Whether working in a numerical or geometric setting, when students are engaged in learning mathematics meaningfully, as opposed to rote computation, it is quite likely some form of imagery is being used (Wheatley & Abshire, 2002). Clearly, spatial thinking can be powerful. However, using spatial representation is not a panacea.

Some resarchers do not believe spatial thinking is essential for success in mathematics. Krutetskii was one example (1976). Spatial abilities may be conceptualized on a continuum from concrete to abstract, implying that students may differ greatly when using imagery (Presmeg, 1992). Similarly, Krutetskii's research was also based on two factors. Krutetskii (1976) identified two factors in school mathematical performance. The first of these was a verbal/logical component of thinking which contributed to the level of mathematical ability, while the second was a preference for visual/nonvisual methods of problem solving which contributed to the form of mathematical thinking. While Krutetskii acknowledged that spatial ability could play a role in mathematical reasoning, he concluded that success in problem solving was related to a logical reasoning component of mathematical ability rather than the ability to form spatial images. Further, Lean and Clements (1981) found that students who preferred to use verbal/logical means to process mathematical information outperformed those who preferred visual methods on a mathematical and spatial test. It is arguable, however, that the methods used to measure problem-solving performance in these studies confounded the results (Fennema & Tartre, 1985).

Connections Between Spatial Thinking and Problem Solving

Spatial thinking can be useful in problem solving in many aspects of life. Everything from understanding directions and maps to shooting a basketball to rearranging furniture are spatial activities. In this research study, spatial thinking and problem solving were the two factors being investigated with respect to mathematics. As with the connection between spatial thinking and general mathematics, there is a profusion of research linking spatial thinking to mathematical problem solving (Battista, 1990; Edens & Potter, 2007; Hegarty & Waller, 2005; Moses, 1977; Reynolds & Wheatley, 1997). Fisher (2005) stated,

> Visual knowledge is valuable as an aid to thinking, not only for potential artists, scientists, architects, and engineers but for all children. Visualizing can help in the expression of information and ideas. Visual expression provides a means of formulating and solving problems. (p. 16)

A great example of this can be found in Marriott's (1978) study of two comparable sixth-grade classes learning about fractions. The children in the experimental class made their own sets of circular cutouts and then used the shapes to embody fractional concepts throughout the time they spent studying fractions. The other class received a thorough but traditional algorithmic treatment on how to work with fractions. When the two classes were tested, no statistically significant differences were found from the pre- and post-test gains. However, a qualitative analysis of the data showed that the children who had learned using the cutouts were much more likely to solve using diagrams, usually circular. Furthermore, this group of children tended to think about fraction questions by creating visual images, which involved circles and sectors of circles, while the other group relied solely on numerical algorithms.

The students in the experimental group were more likely to have learned about fractions in a way that was meaningful to them and, therefore, were more likely to be able to use the knowledge outside of a mathematics classroom. Battista (2007) explains that, “individuals reason about a situation by activiating mental models that enable them to simulate interactions within the situaton so that they can explore possible scenarios and solutions to problems” (p. 861). In an earlier example, a research study was described where a little girl, named Elaine, was able to construct powerful mental solutions to mathematical situations (Reynolds & Wheatley, 1997). Elaine was performing reasoning just as Battista (2007) described. It is clear that she was able to draw from a rich background of previous imaging experiences to construct a context to the mathematical tasks she was given.

In 2006, van Garderen studied sixth-grade students with varying mathematical abilities. Sixty-six students were divided into three groups based on ability: students with learning disabilities, average-achieving students, and gifted students. These groups were assessed on measures of mathematical problem solving, visual imagery representation, and spatial visualization ability and compared. The instrument used to measure mathematical problem solving was called the Mathematical Processing Instrument, the same used in this study, where significant and positive correlations between each of the spatial visualization measures and mathematical word problem-solving performance were found. Further, the results indicated that gifted students performed better on both of the spatial measures than the other two groups. In general, students who performed well on the mathematical problem-solving measure also scored well on the spatial measures, implying a correlation.

Measuring problem solving and spatial reasoning is common in mathematics education research and it is almost always performed in one way, by having students complete mathematical tasks that involve a spatial component. One of the better-known instruments that use this approach is the Mathematical Processing Instrument (MPI), created by Suwarsono (1982) to test seventh-grade students' preference for using imagery on nonroutine tasks. The instrument has been mildly modified and validated over the years and continues to be a reliable source for measuring spatial thinking and problem solving (Clements, 1981; Lean & Clements, 1981; Presmeg, 1986; Van Garderen, 2006). A more in-depth review of this instrument will be presented in the next chapter.

Overall, the variety of outcomes indicated by research would seem to infer that the relationship between spatial thinking and mathematical problem solving is complex, multi-dimensional, and sometimes difficult to measure. The variables are the types of spatial thinking being measured and the classification of the mathematics being used for problem solving. One goal of the research in this area was to discover how best to teach problem solving in mathematics because we know that a student who has "the ability to construct and transform mental images leads to flexibility and power. In doing mathematics, it is advantageous to know more than one way to solve a problem or complete a routine task" (Wheatley & Reynolds, 1999, p. 374).

Implications for Teachers

An embedded area of interest for this research study was to examine what relationships exist between spatial thinking and teaching. Wheatley and Abshire's (2002) book points out that students who reason using dynamic images also tend to be powerful mathematics students.

> When students are encouraged to develop mental images and to use those images in mathematics, they show suprising growth. Even though individual differences in imaging among children are striking, all students can learn to use mental imagery effectively. Thus, every mathematics teacher or parent should make improving spatial sense a priority. (p. 32)

Lord and Holland (1997) discovered that pre-service secondary mathematics and science teachers were higher in spatial ability than pre-service teachers in other disciplines. Research has also shown that teachers who are more confident in their own spatial abilities are more likely to use such strategies in their classroom (Battista, 1990; Presmeg, 1986). Explicitly, Presmeg (1986) points out that teachers "in the visual group used and encouraged visual methods" (p. 308) while teachers in the other groups did not.

If teacher educators expect pre-service teachers to use spatial strategies in future mathematics classrooms, spatial thinking needs to be a part of the pre-service teacher's curriculum. Since "teachers will tend to teach in ways that are consistent with how they learned mathematics" (Sundberg & Goodman, 2005, p. 29), including spatial thinking activities in teacher education programs is essential. Having pre-service teachers in a mathematics class can either be a phenomenal opportunity or an educational disappointment. Sommer (1978) posits that the education system is partly to blame for the poor reputation with regards to spatial thinking by claiming, "School more than any other institution is responsible for downgrading visual thinking. Most educators are not only disinterested in visualization, they are positively hostile to it" (p. 54). Much needs to be done to reverse this trend and perhaps a good place to start would be to identify and influence beliefs and attitudes about spatial thinking.

Beliefs Regarding Spatial Thinking and Problem Solving

In general, attitudes pertaining to mathematics are comprised of two elements: "feelings about mathematics and feelings about oneself as a learner of mathematics" (Reyes, 1980, p. 164). Comparably, this quote could be restated as attitudes pertaining to spatial thinking are similar in that there are two factors: feelings about spatial thinking and feelings about oneself as a learner of spatial thinking. These feelings about spatial thinking can be better described as beliefs. These convictions, either positive or negative, can alter the confidence of a person's ability to think spatially and/or problem solve mathematically.

Problem Solving

In 1985, Silver recognized that beliefs about mathematics should be studied to better understand how students learn problem solving. National assessment data indicate that a staggering 83% of seventh-grade and 81% of eleventh-grade students agree or strongly agree with the erroneous belief that there is *always* a rule to follow when solving mathematics (Dossey, Mullis, Lindquist, & Chambers, 1988). Once students come to believe that mathematics is constructed of memorization and sequential rule applying, it is difficult for them to respond meaningfully to tasks that require decision making. Wheatley and Abshire (2002) explain it this way:

> Many students are not confident problem solvers because they believe good problem solvers know what to do and will write down the solution in an orderly set of steps. Since they don't know what "the first step" is, they do nothing. Once they come to believe there is no one first step and that problem solving involves exploration, they are on their way to becoming effective problem solvers. (p. 22)

Schoenfeld (1988, 1989) reported results from a year-long study of detailed observations, analysis of recorded instruction, and follow-up questionnaire data from two tenth-grade geometry classes. The participants were chosen based on high performance on a state exam. The students reported beliefs that they could be creative in mathematics and that the subject helped them think clearly, yet, they also claimed that mathematics was best learned through memorization. These contradictory beliefs should cause concern for educators. Unfortunately, some of these beliefs have stemmed directly from teachers themselves.

There is a common misconception that boys are better at mathematics-related material than girls. When formed early, gender stereotyped expectations do not disappear with age (Casey, Nuttall, & Pezaris, 1997). Research has shown that, generally, these biases are incorrect. An interesting study by Moe and Pazzaglia (2006) divided high school students into three groups, both male and female, and assigned the Mental Rotation Test as a pre- and post-measure. After the first assessment, the first group was told that males were better at mental rotation, the second group was told that women were better at mental rotation, while the third group was not given any gender preference before taking the second test. The posttest revealed that females showed a significant decrease in the first group, an improvement in the second, and no difference in the third. Taken together, the data suggested that beliefs in gender superiority are able to affect performance on a spatial test. Armed with this knowledge, teachers should set the standard of beliefs in their classrooms. Polya (1973) said it best:

> The first rule about teaching is to know what you are supposed to teach. The second rule of teaching is to know a little more than what you are supposed to teach…Yet it should not be forgotten that a teacher of mathematics should know some mathematics, and that the teacher wishing to impart the right attitude of mind toward problems to his students should have acquired that attitude himself. (p. 173)

Spatial Thinking

Research has shown that teachers who are more confident in their own spatial abilities are more likely to incorporate spatial learning into their classrooms (Battista, 1990). One of the many positive aspects of spatial thinking is the power it has in mathematics, espeically problem solving. One easy way to incorporate spatial thinking into a mathematics classroom is through the use of drawing images or shapes to represent mathematical problems. Clements (1998) points out that drawing is a type of representation that demonstrates understanding of an idea or concept. Additionally, teachers need to avoid infusing students with anxiety about spatial tasks (Newcombe, 2010) and believe in all learners. Although there is an absence of literature associated with attitudes regarding spatial thinking, teachers' beliefs about the topic are of obvious importance. Teachers' attitudes toward spatial thinking, problem solving, and students' abilities to perform these tasks will impact student beliefs and motivation as well.

Beliefs cannot be changed easily. When problem solving is mistaken for answer getting and mathematics as a set of rules, beliefs about mathematics will be shaped accordingly. Many students are content with the way they view mathematics and their competence in learning the subject matter (Kloosterman & Stage, 1992). It is not uncommon for a student to request the "steps" for a problem in lieu of thinking creatively to craft an autonomous solution. This misrepresents the learning of mathematics. Students need to believe that they can do time-consuming problems and that time thinking about a problem is time well spent. They need to believe that word problems are important to the learning of mathematics, not the punishment at the end of a homework assignment. They need to believe that effort and spatial thinking will help them better understand the beauty and power of mathematics. Teachers can play a significant role in helping shape their students' beliefs in this way.

Summary

This literature review includes both theoretical perspectives and empirical studies supporting the following propositions, which constitute the conceptual framework of this study: (a) students construct their learning, individually and collectively, in relation to their experiences; (b) ideal learning environments include opportunities for students to construct meaning and engage in spatial tasks to further understanding; and (c) spatial reasoning supports student learning and mathematical problem solving. In response to this review, I have identified at least two significant gaps in the literature I hope to address in this study.

The first concerns the influence of spatial reasoning and problem-solving strategies among undergraduate students in a low-level mathematics course. Studies have shown that students with high spatial ability scores perform better on questions requiring problem-solving skills (Pribyl & Bodner, 1987; Small & Morton, 1983). Further, spatial ability has shown to be malleable across age groups including elementary age students (Edens & Potter, 2007), secondary students (Tartre, 1990a), and undergraduate students (Mohler & Miller, 2008). However, very few studies have included undergraduate students as participants and none have focused the research on students in low-level undergraduate mathematics courses. Since all students benefit from spatial thinking, this group of learners may benefit greatly from spatial tasks.

The examination of how the inclusion of spatial tasks influence pre-service teachers' beliefs about spatial thinking is a second critical gap in the literature. Some studies have identified general beliefs about spatial thinking among pre-service teachers, but none have considered the beliefs in the context of a low-level mathematics course. This is important since the mathematics classroom is where pre-service teachers will, in part, form their beliefs about mathematics teaching and learning.

This study provided an opportunity to consider spatial thinking as a tool for problem solving and space for pre-service teachers to reconsider their beliefs about spatial thinking in a mathematics classroom. Insight into these areas could provide support for future educational policy within the mathematics classroom and pre-service teacher programs. The introduction in chapter one and the review of the literature in this chapter establish the foundation for this study. Using the theory of constructivism as a starting point, three research questions and a method for studying these questions was designed. Chapter three provides a description of this design and the details of the research project itself.

Chapter III

METHODOLOGY

The purpose of this case study was to understand how spatial reasoning tasks influenced the development of spatial ability, problem-solving ability, and pre-service teachers' beliefs about spatial thinking in undergraduate students of mathematics. Both quantitative and qualitative data were collected as part of this embedded case study design. This chapter provides a description of the instruments used to collect data as well as methods performed to organize and analyze the data. Quantitative instruments such as the Purdue Spatial Visualization Test (PSVT), the Mathematical Processing Instrument (MPI), and the Spatial Thinking Attitude Survey (STAS), together with qualitative data garnered through student-written journal responses, focus group interviews, and observations were collected as data and analyzed to address the research questions in this study.

An amalgamation of quantitative and qualitative instruments provided a means in which to evaluate the influence spatial tasks had on students of mathematics. Specifically, the MPI and the PSVT were used as pre- and post-measures of students' problem-solving and spatial abilities, respectively. The STAS survey helped to identify pre-service teachers' perceptions of the importance of spatial thinking over the course of the study. Since the MPI and PSVT share some overlap, collecting both quantitative and qualitative data concerning spatial and problem-solving ability informed the researcher through interpretation of the results from two different perspectives. Analysis of the qualitative data partnered with the qualitative instruments assisted in answering the guiding questions of this study:

1. How does the integration of spatial activities in an undergraduate mathematics content course impact student spatial ability?
2. In what ways does the integration of spatial reasoning tasks into an undergraduate mathematics content course influence problem-solving strategies?
3. How does the integration of spatial reasoning tasks influence the beliefs of pre-service elementary teachers concerning spatial thinking?

According to Clark and Creswell (2007), "case study design is sensible because it will allow all methods available to be used to address the research questions" (p. 9). The single case studied was an undergraduate mathematics content course, and the units studied were the students enrolled in that course. A focus group was organized and consisted of pre-service elementary teachers. Pre- and post-assessments in spatial ability, pre- and post-tests in mathematical problem solving, journals, observations, and the focus group served as means for data collection. The quantitative data and subsequent analysis provided a general understanding of the research questions, while the qualitative data and analysis offered a rich description of the statistical results by exploring the views of participants more in depth (Creswell, 2009; Marshall & Rossman, 2011).

Rationale for Methodology

A controversy exists regarding the mixture of quantitative and qualitative methods of data collection, as case study allows. Yin (2009) argues that while case study is considered a form of qualitative research; it goes beyond that description "by using a mix of quantitative and qualitative evidence" (p. 19). However, some quantitative and qualitative purists believe that these approaches should not be mixed due to the fact that the theoretical perspectives that inform each of the designs are in opposition. These purists contend that multiple realities abound and that it is impossible for the knower and the known to be separated since the knower is the one source of reality (Guba, 1990).

Morse (1991) differentiates between the paradigms by explaining that a quantitative precedence is guided by a post-positivistic worldview, a qualitative precedence is guided by a naturalistic worldview, while the combination of the two, either equally or unequally divided, is driven by the pragmatic worldview. For the purpose of this study, pragmatism, as a philosophy, is an approach that assesses the truth of meaning in terms of practical application. Further, pragmatism asserts that research methods should be mixed in accordance with the best opportunities for answering important research questions (Creswell, 2009). Dewey was a pragmatist and was interested in examining practical consequences and empirical findings to help better understand real world phenomena. He stated that "in order to discover the meaning of the idea [we must] ask for its consequences" (Dewey, 1948, p. 132). Since the research questions in this study were "how" questions that are best answered using both quantitative and qualitative data, a pragmatist approach was fitting. In keeping with the nature of this study, by using the pragmatist approach, research became a problem-solving activity.

Qualitative Research

Case study design aligns with the methods of qualitative research. Qualitative inquiry – even when absolute conclusions or truths are not the result, or even the goal – supports literature by giving insight into the area of focus. Stake (1995) characterized qualitative study as holistic, naturalistic, interpretive, and empathetic. As defined by Marshall and Rossman (2011), qualitative research is typically "enacted in naturalistic settings, draws on multiple methods that respect the humanity of the participants in the study, focuses on content, is emergent and evolving, and is fundamentally interpretive" (p. 2). Maxwell (2005) discussed the research questions most appropriate for qualitative study, a good method when trying to understand the process by which events and actions take place. Denzin and Lincoln (2005) offer this definition:

> Qualitative research is a situated activity that locates the observer in the world. It consists of a set of interpretive, material practices that make the world visible. These practices transform the world…At this level, qualitative research involves an interpretive, naturalistic approach to the world. This means that qualitative researchers study things in their natural settings, attempting to make sense of, or interpret, phenomena in terms of the meanings people bring to them. (p. 3)

In summary, while qualitative research does adhere to guidelines to ensure trustworthiness, it does not attempt to separate the researcher from the research, the subjects from their setting, questions from context, or perception from reality. One method of qualitative research that incorporates these ideals is case study—a method that can opt to include quantitative measures, but is based in qualitative strategies (Creswell, 2009).

Case Study

Qualitative research can take many forms. Shutz, Chambless, and DeCuir (2004) state, "When we conceptualize research as a problem-solving activity, we also suggest that any method, within moral and ethical constraints, can be used" (p. 274). Case study, defined by Stake (1995), but cited by Creswell (2009) is,

> …a strategy of inquiry in which the researcher explores in depth a program, event, activity, process, or one or more individuals. Cases are bounded by time and activity, and researchers collect detailed information using a variety of data collection procedures over a sustained period of time. (p. 13)

In discussing the circumstances for designing a case study, Yin (2009) identified three conditions that make it an appropriate choice: a) the researcher's questions are "how" or "why" questions, explanatory in nature, which need to be observed over time; b) the investigator does not control behavioral events; and c) the focus is on contemporary, as opposed to historical, events.

An embedded case study is a special style of case study design in that it contains a sub-unit of analysis (Yin, 2009). An embedded case study methodology, like case study, lends itself a means of integrating both quantitative and qualitative methods into a single study. The sub-units, usually an individual or focus group, allows for a more detailed level of inquiry. For this study, the focus group helped determine whether or not spatial skills differ among undergraduate students based on their respective major areas of study and the influence these activities have on pre-service teachers' beliefs on spatial thinking. Yin (2009) suggests an embedded case study approach when the boundaries between the phenomenon of interest and context are not clearly evident.

One criticism of case study is that this particular methodology provides little room for generalization (Yin, 2009). However, Stake (1995) argues that particularization, not generalization, is the point of case study research when he states: "We take a particular case and come to know it well, not primarily as to how it is different from others but what it is, what it does" (p. 8). Further, Yin explained that case studies generalize to "theoretical propositions and not to populations and universes" (p. 15). The case study researcher, then, must not simply tell the story of the case, but rather interpret the data and develop conclusions that might be applicable beyond the case itself.

An embedded case study design was determined to be most appropriate since this study aimed to document and explore undergraduate students' spatial abilities and the influence this skill had on mathematical problem solving and beliefs about spatial reasoning. According to Yin (2009), a single case is appropriate when it is representative or typical of the phenomenon in study. For a single-case study, the objective is "to capture the circumstances and conditions of an everyday or commonplace situation" (p. 48). In this study, the single case, an undergraduate mathematics course, was divided into subunits for analysis. This subgroup was an embedded unit, called a focus group, and was used to collect in-depth information that helped analyze data collected from the entire class. Using more than one unit of analysis, as described by Yin (2009), utilizes embedded case study methods and made up the design used to answer the three focus questions in this study.

Definitions

Little consensus exists on the definition of spatial intelligence, and this is complicated by the use of a variety of terms to describe the phenomenon. For example, "spatial reasoning," "spatial skills," "spatial intuition," "spatial perception," "spatial thinking," "spatial ability," "spatial relations," "spatial orientation," "spatial insight," "spatial imagery," and "spatial visualization" are all terms that imply an interaction with a spatial environment through images and have been used interchangeably throughout the literature, and this list is not exhaustive. To complicate matters, there are some definitions with similar descriptions but different names, as well as identical names for different components of spatial ability (Yilmaz, 2009). For the purpose of flow in writing, "spatial reasoning," "spatial ability," "spatial skill," and "spatial thinking" were chosen to represent the key areas of study in this paper. Any variation in terms will be discussed in this section. The terms that are used throughout this research are defined below.

- *Spatial Ability*: the ability to effectively generate, retain, compare, retrieve, manipulate, and transform well-structured mental images (Lohman, 1993).
- *Spatial Reasoning*: the process and ability to go beyond information given and reason with spatial images.
- *Spatial Skill*: the mental skills involved when thinking and reasoning through the comparison, manipulation, and transformation of mental pictures (Casey et al., 2008).

- *Spatial Thinking*: thinking concerned with objects in space, their locations, their shapes, their relations to each other, and the paths they take when they move (Newcombe, 2010).
- *Problem Solving*: the cognitive process directed at achieving a goal when no solution method is obvious to the problem solver (Meyer, 1992).
- *Pictorial Imagery*: the construction of vivid and detailed visual images (Hegarty & Kozhevnikov, 1999).
- *Schematic Imagery*: the representation of spatial relationships between objects and imagining spatial transformations (Hegarty & Kozhevnikov, 1999).
- *Undergraduate or University Student*: refers to students whose major course of study requires completion of a low-level mathematics course, namely Elements of Mathematics I.
- *Elements of Mathematics*: a small, private, midwestern university's Survey of Mathematics course designed to give the liberal arts student a comprehensive overview of the applications of mathematics in today's society. Topics include set theory, logic, probability, systems of numeration and number theory.

Assumptions and Limitations

When human subjects are involved, assumptions and limitations are unavoidable, and this study is no exception. One assumption is that although the researcher is also the instructor of the course from which participants were selected, responses from the participants were authentic and uninfluenced by the researcher's dual role. One limitation is that this study only examined one mathematics course, which included a total of 35 students. The dynamic of this group of students is predominately white, middle to upper-middle class, and enrolled in Elements of Mathematics. Being a sample of convenience, the results may not be suitable for generalization to the general population of those taking a Survey of Mathematics course. Second, due to the fact that the researcher of this study has taught the Elements course every semester for five years, in addition to teaching teacher-preparation courses that focus on spatial skills, there is a possibility that the researcher had some preconceived ideas regarding problem solving, spatial thinking, and the abilities of students in both areas. Third, this study will be time-sensitive since it took place over the course of one semester. The data collected through this study will be time-intensive, so only a subset of the participants will be interviewed, which might limit the generality of this set of data. Lastly, in qualitative research, the researcher is also an instrument; therefore, all coding and interpretations of the formal and informal interviews, observations, journals, and student work will be through the researcher's lens, albeit grounded in the data (Yin, 2009).

The positionality of the researcher can also be viewed as a limitation. For mathematics education researchers, as with all researchers, lived experiences impact researcher positionality. This positionality ultimately informs the research questions asked, the data gathered, and the interpretations drawn from that data (Foote & Bartell, 2011). To begin describing positionality, Harvey (1996) calls for researchers to examine the similarities and differences between themselves and their research participants.

As will be further discussed in the third chapter of this paper, I have a dual role in this study since I am both teacher and researcher. Since I was the instructor of the course under study, the main differences between my students and myself were our roles in the classroom. Since I had the most authority in the classroom, I set the expectations and temperament. Because of this, I took intentional measures to create a learning environment where communication was accepted and valued between teacher-student and student-student relationships. The similarities between my students and myself were much more apparent. I am a Caucasian female with an education degree. A little more than half of the students in the course declared education as their field of study. Over 70% of the participants were female while 88.6% of the participants identified themselves as Caucasian. Another significant similarity between my students and myself was our religion. This study took place at a faith-based institution where the vast majority shared a common religious belief. This likeness in spirituality created an automatic camaraderie in the classroom.

My experience as an educator has influenced my position on spatial thinking and problem solving. I am of the opinion that problem-solving ability is enhanced by spatial ability, but not necessarily always dependent on it. I think many problems encountered in mathematics, at any level, involve a spatial component. Lastly, I believe that problem solving should be a priority in every mathematics classroom. Therefore, any aspect of thinking that can encourage effective problem solving should be considered a worthy endeavor. These positions were recognized and considered when making decisions about how to create and conduct the various aspects of this study.

Trustworthiness

Although qualitative research is not judged using statistical tests that measure validity and trustworthiness, Merriam (1998) offered six basic strategies that enhance the validity of qualitative research: triangulation, member checking, long-term observation, peer examination, participatory research, and bias declaration. This study included five of the six recommendations to enhance validity: triangulation, member checking, long-term observation, peer examination, and bias declaration. Specifically, sequential triangulation was utilized as a method of collecting data, and this strategy allowed the findings to be corroborated (Creswell, 2009). Triangulation of data was generated through the MPI and PSVT scores, focus group interviews, and observations; along with the researcher's notes, these measures helped strengthen the study. Patton (2002) noted the following:

> Understanding inconsistencies in findings across different kinds of data can be illuminative and important. Finding such inconsistencies ought not to be viewed as weakening the credibility of results, but rather as offering opportunities for deeper insight into the relationship between inquiry approach and the phenomena under study. (p. 556)

According to Merriam (2002), member checking requires taking the findings and final report to the participants who validate the accuracy of the information. Participants in this study were given the opportunity to verify data throughout the study through weekly discussions during class time. Long-term observation requires repeated observations over the course of a period of time. Observations for this study took place twice a week, for 75 minutes each time, over an eight-week period or time. Asking a third party, a colleague from the math department in this case, for verification of themes satisfied the peer examination recommendation. The cooperating colleague was asked to verify results throughout the study and at the final stage.

In addressing personal bias, the role of the researcher is a necessary consideration. It has been suggested that a good qualitative researcher be familiar with the phenomena, interested in contextual understanding, aware of personal bias, and competent in gathering data (Miles & Huberman, 1994; Yin, 2009). In addition to reading widely on the topics involved, tools and methods recommended by qualitative research were used to maintain an understanding and focus in examining this case study. It should be noted that the possibility of researcher influence exists on the interpretations of the interviews, observations, and artifacts, due to the fact that the researcher is also the instructor of the course being studied (Creswell, 2009).

However, there are positive aspects to being both the teacher and researcher. In response to the responsibilities of the teacher as researcher, Stake (1995) notes that the "intention of research is to inform, to sophisticate, to assist the increase of competence and maturity, to socialize, and to liberate. These also are the responsibilities of the teacher" (pp. 91-92). Considered this way, the teacher as researcher is a natural fit. Marshall and Rossman (2011) agree that researching one's own classroom has advantages since teachers have "relatively easy access to participants, reduced time expenditure for certain aspects of data collection, a feasible location for research, [and] the potential to build trusting relationships" (p. 101).

In response to the fact that the researcher is naturally a data collection instrument in qualitative research, a journal was kept and referred to throughout the study to aid in researcher reflexivity (Watt, 2007). Thoughts about the experience of conducting the study as well as details about decisions made were described in the journal for review during the study and final analysis. Further, an effort was made throughout the study to build strong teacher-student relationships so constant communication with the participants could be maintained to help avoid bias in the study.

Procedure

The focus of this study was a specific case, bounded by place and time, using multiple sources of data as required by case study design (Stake, 1995). The bounded system used for this study was a course at a small, private midwestern university over the course of 12 weeks, as shown in Figure 1. The research questions were answered through data collected from formal instruments such as the MPI, the PSVT, and the STAS. Informal measures such as group interviews, observations, and journals were also used to clarify any findings. Any willing student, who had declared elementary education as their major, formed the focus group (FG).

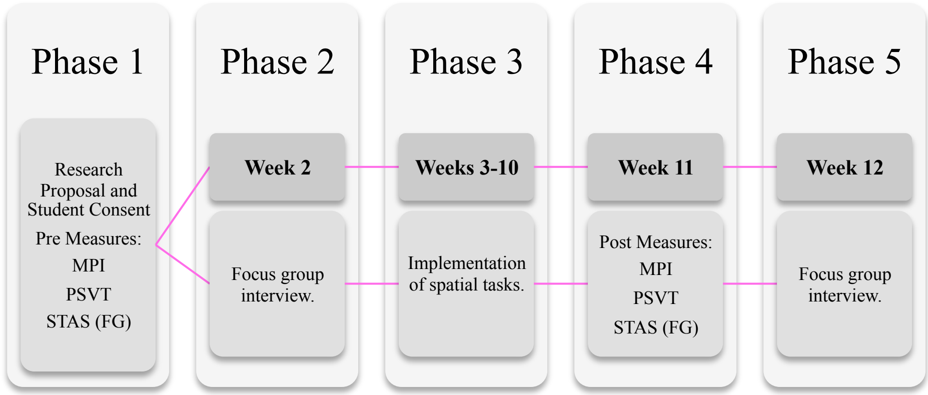

Figure 1. Synopsis of the embedded mixed methods case study design.

To explore the research questions, pre- and post-tests were administered, as well as eight weeks of spatial tasks in between. During the first phase, I, the researcher and instructor of the course, described the study to my students therefore satisfying the requirements set forth by the IRB concerning student consent (see Appendix A). A colleague of mine was responsible for passing out and collecting these forms in my absence to ensure student privacy. After obtaining consent, the participants took the MPI and the PSVT. In addition, the focus group responded to the STAS questionnaire.

During phase two, results from both the MPI and the PSVT were recorded and analyzed. Once the STAS results were combined, the focus group met to discuss general feedback on the MPI and the PSVT. Additionally, I asked the focus group members to expand on their responses to the STAS survey. The third phase devoted eight weeks to spatial tasks where participants had the opportunity to engage in spatial activities during every class period. These activities ranged from pencil and paper tasks to working with 3D materials, and were not necessarily in conjunction with the mathematical objective of the day. Class time was also used for observations, informal interviews with all participants, and student journals.

After eight weeks, at the start of phase four, students took the identical post-measures of the MPI and the PSVT. The focus group also re-took the STAS. Finally, during the fifth phase, I conducted interviews with the focus group after the data were analyzed. Participants were asked about any significant changes in ability or beliefs that surfaced in the findings.

Examples of Spatial Tasks

During phase three, participants engaged in a number of spatial tasks. Each task took between 10 and 20 minutes, depending on class discussion, and typically took place at the beginning of class. These tasks were specifically chosen to engage students in spatial thinking by encouraging students to create, manipulate, rotate, transform and/or recall mental images. It is through these activities that students were encouraged to think spatially before solving the task at hand. Six specific tasks are highlighted below. However, variations of these activities were also included and will be discussed in chapter four. The following represent the six types of activities participants were asked to perform.

Quick Draw is a student activity that encourages the transformation of self-constructed images. According to Wheatley (2007), "Quick Draw is designed to develop powerful imagery that will come in to play in both numerical and geometric settings and to encourage students to explore alternative ways to solve a problem" (p. 1). In Quick Draw, students are shown a unique figure for three seconds (see Figure 2), then asked to "draw what they saw" once the figure was concealed (p. 5). The image was projected using a document camera large enough for all students to see at one time, covered, then revealed.

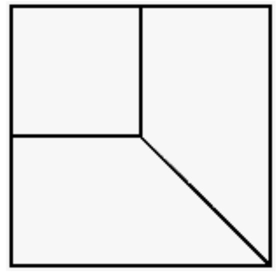

Figure 2. Example of a *Quick Draw* figure.

When needed, students were allowed a second or third three-second viewing of the image before engaging in whole-class discussion. Since the image was covered up before the students were allowed to sketch, they were forced to reconstruct the figure using the mental images they had created. Once all students completed their drawings, the class engaged in discussions about the image, and students were asked to share their strategies and consider the images created by their classmates. Through questions such as "What did you see?", "How did you decide to draw your figure?", "What did you draw first?", and "How would you explain your drawing?" the participants discussed how they conceptualized the shape during the viewing stage and then shared strategies on reconstructing the image. Typically, student interpretations ranged from simple two-dimensional (2D) explanations to complex 3D comparisons to real-life objects. Once students understood the method of this activity, a lack of discussion was rarely a problem.

Unit cubes are a powerful manipulative for spatial tasks. They can be used to create 3D models of 2D representations or can be used as a stand-alone figure that students must consider when drawing 2D illustrations. The fact is, "most students' mathematical experience with the three-dimensional world is obtained from two-dimensional pictures" (Winter, Lappan, Phillips, & Fitzgerald, 1986, p.

3). Therefore, it is necessary that students learn to deal with 2D representations of 3D objects if they are expected to perform in a mathematical setting. Further, as students work with unit cubes to construct 3D formations, they "use their spatial skills in more discriminating ways" and learn to "visually distinguish between a left-right or a front-back orientation in both two and three dimensions" (p. 3). As stated, there are many ways to use unit cubes in a spatial task. For example, I gave students a map of the top, front, and right side of a figure (see Figure 3) and asked them to create the 3D model with the cubes. Since the maps are not true representations of the top, front, or right side of a figure, the guess-and-check method was rarely effective, forcing students to think spatially about the figure.

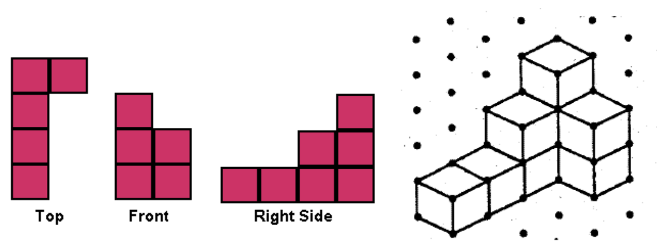

Figure 3. Map of three sides of a 3D figure and solution.

Participants took pride in this activity and requested that the 3D solution not be revealed until they had solved it first. This activity was also completed in reverse order by presenting a 3D figure, then asking the students to draw the 2D representations of at least three of its sides.

Drawing and sketching was often part of the in-class spatial activities. In the "Sketch" activities, students were asked to mentally rotate a 2D image of a 3D figure made from unit cubes then to sketch the new figure or mentally fold the net of a figure and sketch the image (Serra, 1992). See Figure 4 for an example. Dot or graph paper was provided to those who preferred to use it.

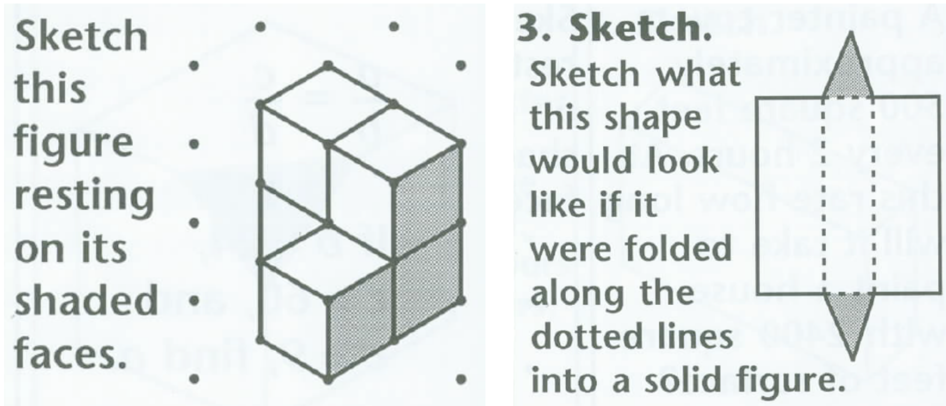

Figure 4. Examples of a sketching activity.

Once participants finished sketching the new image, I encouraged them to create the shape with unit cubes or sketch the image from different perspectives.

Another activity with unit cubes is one that I call Complete the Cube. For example, I showed students Figure 5 and asked them to create the 3D shape required to make the image on the left a solid cube like the image on the right.

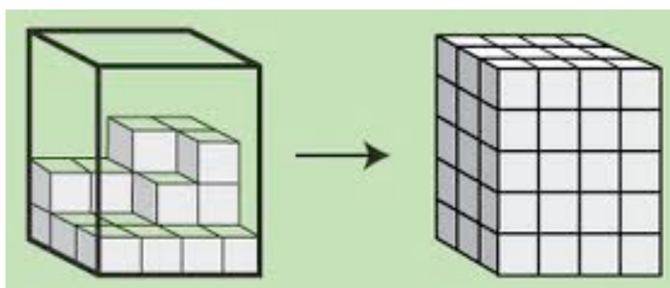

Figure 5. Complete the Cube activity.

To check students' solutions, I created the 3D figure on the left and let students connect the two.

One more way unit cubes were used to support spatial thinking was the "dunk" activity. For instance, I showed a 2D representation of a 3D figure, or created the 3D replica of a figure, and showed it to students (see Figure 6). The image or figure was then concealed and the students were asked to determine the number of cubes, or sides of cubes, that would get wet if the figure were dunked into a bucket of water.

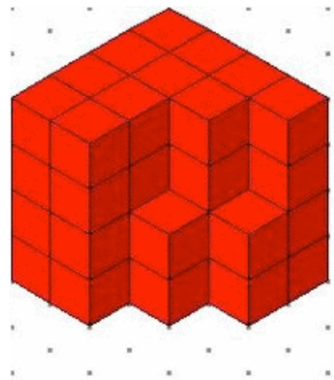

Figure 6. The Dunk task.

Once all students decided on an answer, they were encouraged to discuss and defend their answers before the image was once again revealed. These activities were great for encouraging dialogue since rich imagery and dialogue result from activities that incorporate unit cubes (Winter, Lappan, Phillips, & Fitzgerald, 1986).

Graphs and tables are very important for mathematics representation and communication (NCTM, 2000), and they are also a great tool when working towards spatial thinking. Van Dyke (1998)—who recognized that visualization needs to be fostered in the classroom to "help students understand mathematics concepts and strengthen their connection with mathematics" (p. v)—created the following activity. Given Figure 7, students were informed that each flask was filled by a steady drip of water and that the graphs described the height of the water as a function of volume. Then, students were instructed to match each flask to a graph. Discussion amongst participants was a common occurrence during this activity and the topic of conversation was almost always spatially driven. Once students had matched each of the graphs to a flask, they were asked to defend their answers.

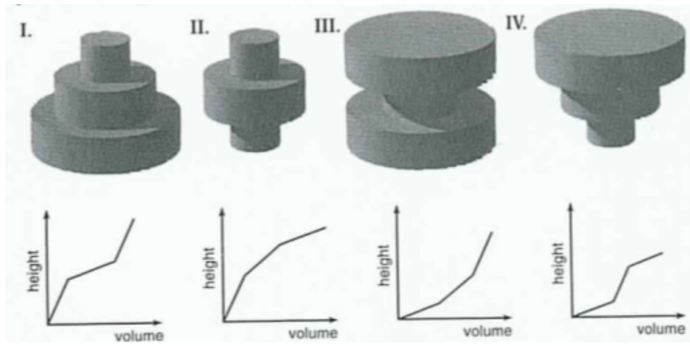

Figure 7. The Flask task.

Asking students to create a graph or sketch a flask of their own, then having another student produce the coordinating graph or flask, easily created an extension of this activity. The participants in this study appreciated this task as they enjoyed creating activities that their peers helped solve.

Participants and Setting

The participants who were asked to engage in the above activities consisted of 33 undergraduate students who were enrolled in the researcher's Elements of Mathematics, Fall 2011 course. Originally, 35 participants were expected to take part in the study. However, two of the participants were absent for more than 75% of the course and, therefore, all data collected from these two participants was omitted from the study, resulting in a total of 33 active participants. Further, due to the fact that quantitative and qualitative data were collected over the course of 12 weeks, the sample size of 33 fluctuated due to excused and unexcused absences of the participants.

The Elements of Mathematics course was a low-level mathematics course designed for liberal arts students. The class met twice a week for 75 minutes each. A comprehensive overview of relevant mathematics in today's society was covered during that time. Topics such as set theory, logic, probability, systems of numeration and number theory was considered. Spatial thinking was treated as a support topic to these areas and was addressed on a daily basis. The goal was to implement at least 15-20 minutes of intentional spatial thinking activities into each class meeting with additional tasks, such as journaling, completed outside of class time. The in-class activities were typically implemented at the beginning of class.

The physical classroom was located in the University's Mathematics and Engineering building on the top floor. The classroom could hold up to 45 students, so ample space was available when students needed to maneuver location. Students sat at large tables that could easily accommodate individual or group work. Large whiteboards surrounded the perimeter of the classroom. In the front right corner of the classroom was a large screen with a projector. A document camera, speakers, and laptop connections were also available for use. The document camera was used for projecting images and student solutions. When time allowed, students were encouraged to work out the daily spatial activity on one of the white boards or use the document camera to project their solution for whole-class discussion. All 33 of the participants presented solutions or led class discussion at some point in the semester.

Participants varied both physically and mentally. Any student with a math ACT score at or above 23, not majoring in a STEM field, was required to take the

Elements of Mathematics course. The participants, also students, in this study attended a small, private university situated in the midwestern United States. Although the sample was one of convenience, as described by Gall, Gall, and Borg (2007), Creswell (2009) and Merriam (2002) noted that in conducting qualitative research, the participants are purposely selected because they exhibit characteristics relevant for the investigation. These university students were partly chosen out of convenience, but they also satisfied purposeful selection because they were enrolled in a course that evaluated mathematical problem solving as a consideration of satisfactory completion. All participants were informed about the study and only those who volunteered, following IRB protocol, were included in the study. A demographic survey was obtained during the first two weeks of the course and results are documented below in Table 1.

Table 1

Demographic Information

N=33

Category	
Age	
Mean	19.61
Standard Deviation	4.37
Range	17-29
Sex	
Female	75.7%
Male	24.3%

Race	
Caucasian	90.9%
African American	0.0%
Hispanic	0.0%
Native American	0.0%
Asian	0.0%
Other	9.1%
College Major	
Education	51.5%
Graphic Design/Art	12.1%
English/Writing	9.1%
Family Studies	9.1%
Other	18.2%

There were 17 purposely sampled participants who served as the focus group and took part in pre- and post- group interviews as well as one group interview during phase three. These 17 students, all of which had declared education as their major area of focus, were comprised of 13 females and four males. Their average age was 19.12 and all 17 were Caucasian. Of these participants, 10 were freshmen, three were sophomores, three were juniors, and one was a senior. All members of the focus group had taken high school Algebra I, Geometry, and Algebra II with the exception of one participant who had taken Algebra I and Geometry, but not Algebra II. The fact as to whether the participants had taken three credits of high school mathematics was noted, but was determined to be of no importance for two reasons. First, all students in the focus group had taken high school geometry, the course most commonly noted as being heavily concerned with spatial thinking. Second, any changes to a student's spatial and problem-solving ability incurred throughout the semester were assumed to be with respect to the participants' initial level of spatial and problem-solving aptitude recorded at the beginning of the semester.

The Elements of Mathematics course was chosen because these particular students did not choose to major in an academic field of STEM, which may complicate the results since students who traditionally pursue STEM fields are naturally strong in spatial thinking (Newcombe, 2010; Wai, Lubinski, & Benbow, 2009). The intention of the study was to examine spatial reasoning with respect to problem solving, which was not a direct topic of study for the course. Apart from some diagrams and other visual aids, this course was void of topics that would specifically encourage spatial thinking.

Instrumentation

Data were collected throughout the 12-week study. There was a combination of qualitative and quantitative data collected with the expectation that both sets of data obtained would combine their strengths and result in a more rigorous analysis. The participants in this study were measured by a number of instruments to help gather these data. According to Creswell (2009), qualitative research involves multiple methods that are interactive, humanistic, and involve active participation. Qualitative research is "emergent" and not "tightly prescribed," which lends this method to gathering "multiple forms of data" (p.

175). Three instruments were used in this study: the Purdue Spatial Visualization Test, the Mathematical Processing Instrument, and the Spatial Thinking Attitude Survey, as well as observations, journals, and a focus group. Together, this collection of quantitative and qualitative data helped answer the research questions posed in this study.

Purdue Spatial Visualization Test

The Purdue Spatial Visualization Test (PSVT), developed by Guay (1980), is comprised of three parts: Developments, Rotations, and Views (see Appendix B). This instrument was chosen because of the various aspects of spatial thinking it measures. The Developments section (PSVT/DEV) measures spatial structuring, the Rotations section (PSVT/ROT) measures an individual's mental rotation ability, while the Views section (PSVT/VIEW) measures spatial perception. The three sections consist of 12 problems each. In this study, the three areas were each scored separately since each test measured a unique aspect of spatial thinking. Once separate scores were tallied, an overall score was assigned. To restrict analytical processing, Bodner and Guay (1997) used a time limit, where the minutes allowed were half the number of questions, or 30 seconds per question. Since there were a total of 36 questions, 18 minutes were given to complete the full PSVT.

Assessing the individual sections of the PSVT was completely objective and based on the answer key provided by the test makers. The correct number of answers for each section was recorded, as well as the number of problems considered incomplete or not attempted. The overall score was simply the sum of the three sections making the overall percentage of correct answers for each participant easily accessible. Example problems from each of the three sections are as follows:

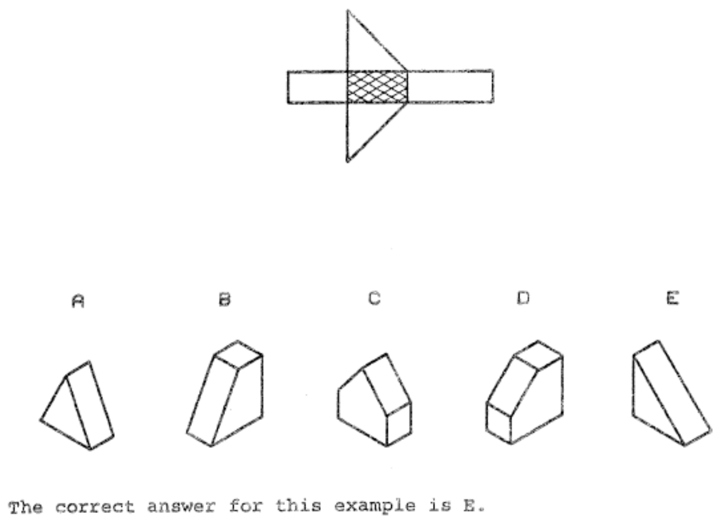

Figure 8. PSVT/DEV example problem.

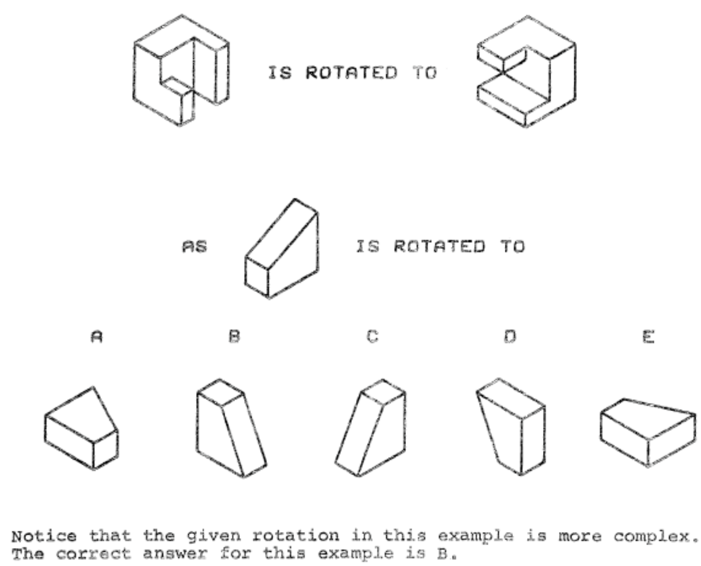

Figure 9. PSVT/ROT example problem.

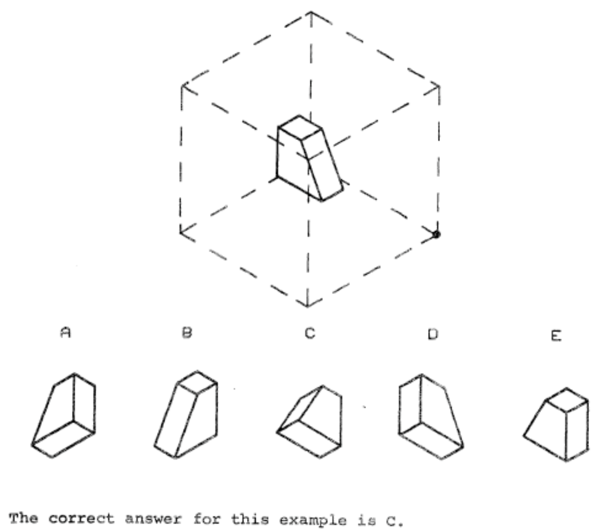

Figure 10. PSVT/VIEW example problem.

In the PSVT/DEV section, students were asked to choose the 3D object, from options A-E, which represented the development when folded. The shaded portion of the development was representative of the bottom of the object. For the PSVT/ROT portion, participants were asked to rotate an object in the same manner as the example above it and choose the correct answer from the options available. Finally, in the PSVT/ROT section of the test, students were asked to view the object from the viewpoint of the black dot and select what it would look like from the answers given.

The PSVT has been found to be a reliable instrument – with Kuder-Richardson-20 (KR-20) coefficients of internal consistency being reported as .87, .89, and, .92, respectively – for the Developments, Rotations, and Views section of the PSVT (Guay, 1980). Permission to use this instrument was granted and can be found in Appendix E.

Mathematical Processing Instrument

The Mathematical Processing Instrument (MPI) was developed by Suwarsono (1982) as an instrument to measure a student's performance in mathematical problem solving in either a visual or non-visual mode (see Appendix C). The MPI consists of 20 problems, either taken from previous studies (Hegarty & Kozhevnikov, 1999; Lean & Clements, 1981) or composed specifically for this study. The uniqueness of this instrument relies on the fact that each of the word problems includes a spatial component, lending the solver the opportunity to use imagery in the solving process. Validity for the MPI can be found in Suwarsono's dissertation and later confirmed in a study led by Lean and Clements (1981) and again by Presmeg (1986).

Scoring the MPI can be difficult since images play a role in the grading strategy. Hegarty and Kozhevnikov (1999) assist in this task by differentiating between two types of imagery: pictorial imagery and schematic imagery. Pictorial imagery is defined as detailed visual images, while schematic imagery is an individual's representation of the spatial relationship between objects and mental spatial images. Schematic imagery, not pictorial imagery, has been linked to spatial ability and will be considered when scoring the MPI.

The MPI was given during phases one and four of the study and was timed and scored as previously described. Three minutes for each question, or one hour total, was given to complete the test (Hegarty & Kozhevnikov, 1999). The original MPI developed by Suwarsono included 30 word problems, but this study only used

20 questions due to time constraints. Scores for each problem were given according to the following criteria:

+2 for a correct answer and reasoning was based on schematic imagery.

+1 for an incorrect answer and reasoning was based on schematic imagery.

0 points for no attempt or unclear method.

-1 for an incorrect answer and no use of schematic imagery attempted.

-2 for a correct answer and no use of schematic imagery attempted.

For example, the first question on the MPI asked,

> At each of the two ends of a straight path, a man planted a tree; then again every 5 meters along the path he planted another tree. The length of the path is 15 meters. How many trees are planted?

Figure 11 shows a solution where schematic imagery was used and the correct answer was concluded.

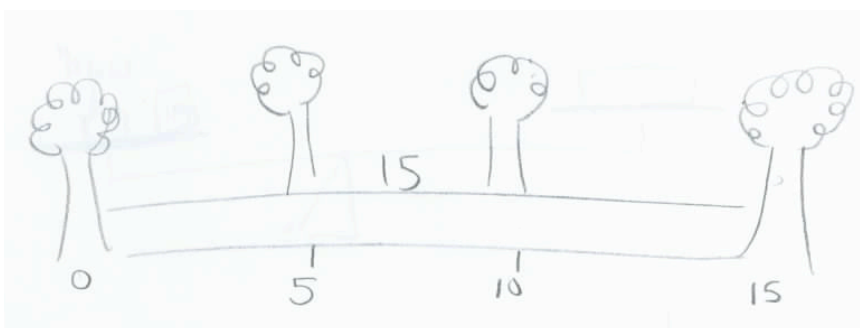

Figure 11. Example of schematic imagery.

Clearly, this student used an image of trees and lines to help create a solution. The next figure also includes trees, but in this example, the relation of the trees is either pictorial or unclearly related to the solution.

Figure 12. Example of pictorial imagery.

Figure 12 shows a solution where the correct answer was also obtained but the image was pictorial, not schematic. Even though both students gave the correct solution to the problem, the score given to each attempt was different based on the approach. The solution in Figure 11 earned a score of +2 while a score of -2 was given to the example in Figure 12. Thus, for this study, the 20 problems on the MPI could result in a possible individual score ranging from -40 to +40 on a purely analytical to visual scale.

This study took into consideration both the success in solving the given problem and the manner in which the problem was represented. To give a measure with which to compare the PSVT, standard grading, where correct responses were scored, was also performed. Therefore, two scores were obtained for both the pre- and post-measure of the MPI: the analytical/visual score and the grade. Permission to use this instrument could not be obtained. However, it may not be necessary, as many other studies have used the original or modified version without knowable consent (Hegarty & Kozhevnikov, 1999; Lean & Clements, 1981; Lowrie, 2001; Lowrie & Kay, 2001).

Spatial Thinking Attitude Survey

The Spatial Thinking Attitude Survey (STAS), developed by Hanlon (2009), was a 15 question, five-point, Likert-type survey (see Appendix D). The survey had two areas of focus, with one measuring beliefs regarding spatial thinking and the other dealing with confidence regarding the drawing of a 2D or 3D shape subsequent to the mental construction of such shape. The STAS was developed through a sequential exploratory mixed method study where reliability statistics show the STAS to have a coefficient alpha of 0.877. Focus group participants were asked to take this survey outside of class during phase one and phase four. The 15 questions were scaled as following: 1 = strongly disagree, 2 = disagree, 3 = neutral, 4 = agree, and 5 = strongly agree. The possible range for raw individual scores ranged from 15 to 75. Permission to use this instrument can be found in Appendix F.

Focus Group

According to Rea and Parker (2005), focus groups provide "semi-structured discussion among individuals deemed to have some knowledge of or interest in the issues associated with the research study" (p. 31). Further, research has shown that teachers who are more confident in their own spatial abilities are more likely to incorporate spatial thinking into learning situations in their own classrooms (Battista, 1990). The focus group for this study consisted of the 17 willing participants who had declared education as their major area of study. Their common interest in education gave the group a sense of community and helped keep discussion guided (Marshall & Rossman, 2011). To ensure a higher response rate and participation, the focus group met at times chosen by the members. This focus group was relevant to the study since these participants will be responsible for teaching spatial skills to their own future students. The focus group participated in three discussions outside of class, with one discussion taking place in each of the phases two, three, and five. The researcher used a guided interview method, as described by Marshall and Rossman (2011), to gain insight on the views and abilities in problem solving and spatial thinking of the participants. See Appendix H for guiding questions for each of the three interviews. This method was chosen since some of the questions were asked in a traditional interview style, where all participants were expected to answer, and others naturally became a part of group discussion as tangents to an individual's answer.

To ensure qualitative data trustworthiness, member checking and peer examination were utilized once the audio-recorded discussions had been transcribed (Merriam, 1998). Ideas or emerging themes based on the transcripts of the interviews were reviewed to corroborate the data obtained from other sources (Patton, 2002). In addition, the researcher was in bi-weekly contact with the focus group during class time where any concerns or questions were addressed verbally with the participants.

Observations

Like interviews, observations represent a powerful tool for the qualitative researcher. Bernard (2002) provided five reasons for participant observation: (1) it makes it possible for the researcher to gather multiple forms of data; (2) it reduces the problem of reactivity, which occurs when people change their behavior when they know they are being studied; (c) it helps the researcher to ask questions that are sensible to the participants; (d) it gives the researcher an intuitive sense of the happenings of a situation; and (e) observation is sometimes the only way to address a research problem adequately (pp. 333-335). Stake (1995) and Yin (2009) also advocate the use of multiple observations as a means of triangulating data.

For this study, bi-weekly observations were conducted, as the researcher was also the instructor for the course. During phase three, the researcher maintained a journal of in-class notes that assisted in identifying or confirming themes that surfaced in other data. Since these notes were used primarily to address questions and conversations between the researcher and the students, these data were not coded and themed. However, the content of these notes were discussed with the participants to help ensure validity.

Other Data Sources

Throughout the 12-week study, participants were occasionally asked to respond to journal questions. These prompts can be found in Appendix G. Marshall and Rossman (2011) contend that artifacts such as journals are potentially "rich in portraying the values and beliefs of participants in the setting" (p. 160). A total of 10 journal questions were given to the students throughout phases three, four, and five. The number of journal questions was determined by time availability and by an as-needed basis as questions and topics of discussion surfaced throughout the study. These prompts served as another insight into the participants' beliefs and ability to think spatially.

Data Analysis

The process of data analysis involves moving into a deeper understanding of the data by representing the data, and interpreting the deeper meaning of the data (Creswell, 2009). The quantitative data were analyzed using statistical methods by means of Microsoft Excel to determine whether the implementation of spatial tasks significantly influenced undergraduates' spatial ability and mathematical problem solving. Using a .05 significance level, a paired t-test performed on each of the three instruments, the MPI, PSVT, and STAS, was conducted to assess for overall differences in the pre- and post-results for each measure. Also of importance, this test calculated descriptive statistics such as mean, variance, and standard deviation for the MPI and PSVT.

Another statistical analysis focused on finding correlations between the two independent variable measures, the PSVT and the MPI, since these two instruments both contained spatial components. A linear correlation test was used to test for a relationship between the PSVT and grades from the MPI. This test was chosen for several reasons: the subjects were independent, the post-PSVT and post-MPI were measured independently, neither instrument was a controlled measure, and a clear alternative to testing for linear correlation was not apparent. The final analysis focused on changes in responses on the STAS for those in the focus group.

The qualitative data were analyzed to determine whether the implementation of spatial tasks influenced undergraduate students' spatial ability, problem solving, and beliefs about spatial thinking. All focus group discussions and written responses were coded and classified using the constant comparison method to create a framework for interpretation (Creswell, 2009). To do this, verbal and written responses were written on note cards and laid out. Note cards with similarities were grouped together. As codes became evident a new note card was made, in a different color, and considered as a possible theme. These possible themes were discussed with the students or posed as journal questions. The ongoing analysis allowed for the reduction of data into themes and emerging patterns (Gall, Gall, & Borg, 2007). At the end of the semester, all data were considered as a whole and final codes and themes were decided upon. All findings were corroborated with a colleague who reviewed the data. These themes will be described, interpreted, and presented as findings in the next chapter (Creswell, 2009).

Ethical Considerations

Following IRB protocol, all participants were informed of the study and given an assurance of confidentiality as part of the informed consent process (Appendix A). To ensure privacy, all participants' scores and responses were coded and pseudonyms were used. Additionally, considering the researcher was also the instructor of the course under study, a third party, a colleague of the researcher, collected and stored informed consent forms in the absence of the researcher. All data collected that could be used in future research, including transcripts, test scores, and written responses, are to be kept in a locked drawer and destroyed after three years.

Summary

Case study research is helpful in exploring an exemplary model to inform educators of effective strategies and practices implemented in the classroom. The case study methodology described in this chapter was the most effective way for conducting a study of this nature. Merriam (1998) stated, "A case study design is employed to gain an in-depth understanding of the situation and outcomes in context…insights gleaned from case studies can directly influence policy, practice, and future research" (p. 19). To reach this level of understanding, the three research questions posed are best answered through case study design, which is used in this study. Overall, data were collected through instruments such as the MPI, the PSVT, and the STAS, as well as through methods such as focus group discussions, classroom observations, and student-written journal responses. All data, quantitative and qualitative, was analyzed to provide in-depth answers to the guiding questions in consideration and is presented in the next chapter.

Chapter IV

RESULTS

Both quantitative and qualitative data from undergraduate mathematics students were collected as part of this embedded case study for the purpose of understanding the influence of spatial thinking activities on spatial visualization abilities, problem-solving strategies, and pre-service teachers' beliefs about spatial thinking. This chapter provides an accounting of the data acquired from the quantitative instruments, such as the Purdue Spatial Visualization Test (PSVT), the Mathematical Processing Instrument (MPI), and the Spatial Thinking Attitude Survey (STAS), as well as qualitative data garnered through student-written journal responses, focus group interviews, and observations. The pre- and post-measures, focus group interviews, and implementation of spatial tasks occurred over 12 weeks and were broken up into five phases.

During the first phase, IRB requirements were met and pre-measures of the MPI, the PSVT, and the STAS were given. The focus group met for the first time during phase two before implementation of the spatial tasks began in phase three. The focus group met for the second time during phase three as well. Post-measures of the MPI, the PSVT, and the STAS were given during phase four. During the final stage of the study, identified as phase five, the focus group met for the third and final time.

The embedded piece of this study, the focus group, was comprised of the subgroup of participants who had declared elementary education as their major area of study. As stated, this group met on three separate occasions: once during phases two, once during phase three, and once during phase five. The purpose of the focus group was to give deeper insight into the participants' experiences with the study and beliefs about spatial thinking.

Implementation began with a description of the study followed by satisfaction of IRB requirements (Appendix A). Pre-measures of the PSVT, the MPI and the STAS were also given to participants during the first phase. Once all three pre-measures were scored, the focus group met to discuss initial perceptions about the three pre-assessments and areas of interest concerning spatial thinking (Appendix H). During the following eight weeks, daily spatial thinking activities and reflective journal prompts were incorporated into classroom practices. Naturally, class discussions ensued which provided insightful classroom observations regarding these assignments. The second of three focus group interviews also took place during this phase. A feel for students' beliefs concerning spatial thinking during the eight weeks of implementation was the focus of this discussion. During the final two weeks of the study, post-measures of the PSVT, the MPI, and the STAS were executed and scored, as well as the final focus group discussion. These data were collected and fully analyzed after grades for the course had been finalized and posted for the semester.

Results from the quantitative data were used to determine if the integration of eight weeks of spatial activities resulted in significant differences in scores on the PSVT, the MPI, and individual statements on the STAS. Analysis of the qualitative data, responses to journal prompts, focus group interviews, and observations, was used to examine the influence of the spatial tasks on students' perceptions about spatial thinking as well as pre-service elementary teachers' beliefs about spatial thinking. Moreover, this same data were used to evaluate how pre-service teachers' viewed their own understanding of spatial thinking and its relevance in their daily lives and future classrooms. After the quantitative data were scored and tested, and the qualitative data were coded and themed, the data were analyzed in its entirety and conclusions were drawn. This section will present the findings for each of the three themes that emerged from the qualitative data gathered as well as results from the PSVT, the MPI, and the STAS which were chosen to help answer the three focus questions of this study.

The literature review provided both theoretical perspectives and empirical studies supporting the following propositions, which constitute the conceptual framework of this study: (a) students construct their learning, individually and collectively, in relation to their experiences; (b) ideal learning environments include opportunities for students to construct meaning and engage in spatial tasks to further understanding; and (c) spatial reasoning supports student learning and mathematical problem solving. Based on this conceptual framework, the following research questions guiding this study were:

1. How does the integration of spatial activities in an undergraduate mathematics content course impact student spatial ability?
2. In what ways does the integration of spatial reasoning tasks into an undergraduate mathematics content course influence problem-solving strategies?

3. How does the integration of spatial reasoning tasks influence the beliefs on spatial thinking of pre-service elementary teachers?

This chapter is presented in three key sections to address the guiding research questions. The following three sections reflect the themes found through qualitative analysis of focus group discussions, participants' written responses to journal questions, and relevant findings from observational notes obtained from class. The themes were drawn based on triangulated data that identified patterns and relationships. Quantitative analysis of the PSVT, the MPI, differences in the pre- and post-results of these measures, and any correlation between these two instruments are presented as findings. Data collected from the focus group, including results from the STAS and the views of the pre-service elementary teachers as they relate to spatial thinking and problem solving, help support these themes and the discussions surrounding them. Spatial activities performed in class are explained and discussed. Finally, a summary is provided to identify and clarify contributing factors that led to changes in spatial ability, problem-solving strategies, and beliefs about spatial thinking.

Improving Problem Solving and Spatial Thinking

Students' Perceptions and Beliefs

The first theme that emerged through the collected data was that problem solving and spatial thinking could improve with practice. During phase two of this study, just after the pre-PSVT and pre-MPI were completed, students commented on the difficulty of the two measures. The in-class conversation was light-hearted as students compared thoughts and strategies on the two instruments. Comments such as "It's been too long since I've thought about problems like on [the MPI]" and "I think I could do better on [the PSVT] if I could take it again now that I've thought about the shapes" surfaced in student-to-student conversations. In response, I asked the focus group to discuss the difficulty of the pre-MPI and pre-PSVT (Appendix H). Maggie, a 20-year-old female and focus group member, summed up the group responses when she said, "I felt like I knew the stuff on the [MPI] and [PSVT], but I just couldn't quite do it. At least I don't think I did. I know if I were to practice solving problems like [on the MPI] and practice working with shapes like [on the PSVT], I would do much better. I just need to practice." Further, the second and fifth journal prompt given to students asked them to discuss their ability to think spatially and the importance of problem solving (Appendix G). Grouped together, responses from class discussions, the focus group, and journal responses revealed the first theme. Below are a few examples of student responses:

- I don't like word problems, I never have, but I always do better with them after I practice awhile.
- Mrs. Prugh, are we going to do problems like [those on the MPI and PSVT] this semester? I hope so. I think it would help me do better if I practice.
- It's hard to think about those types of problems [on the PSVT] because it's tricky thinking. But once I can get the hang of it, I know I can do it.
- I think I would do better on [the MPI and PSVT] if we could work on similar problems in class.

Participants were given the opportunity to practice spatial thinking throughout phase three of this study. During the first class of phase three, students were given the following *Quick Draw* image (see Figure 13) and asked to recreate the image once it was concealed.

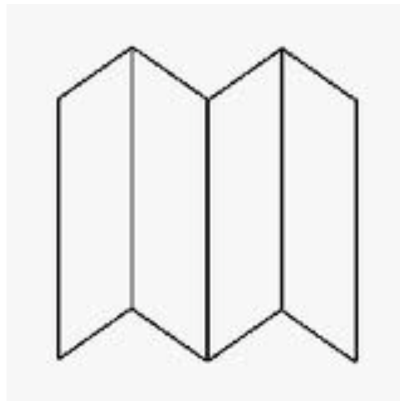

Figure 13. *Quick Draw* image.

An interesting conversation about perspective ensued when I asked the students to describe their strategies when recreating the image (see Figure 14).

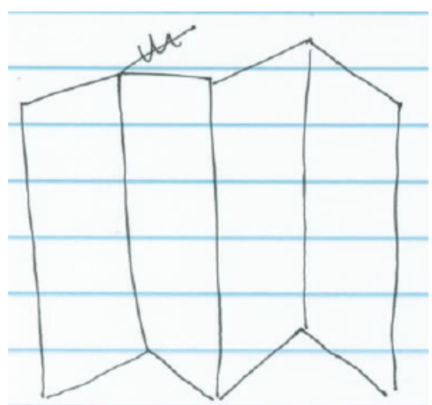

Figure 14. Student recreation of the *Quick Draw* image.

One student, a 19-year-old female named Erin, described the figure as "two books standing open on…maybe a table or something…with the texting facing towards me and the edges touching." Other members of the class considered this explanation and offered further insights. "I see it from a different perspective," followed Brett, a 19-year-old male who sat next to Erin. He continued, "I think the way you draw your picture all depends on your perspective. I see the two books, but for me they are opening away from me. I think that makes a difference." I asked the class to define perspective and how it pertained to the *Quick Draw* activity. Erin answered, "I think of perspective as how I look at something, my point of view, and it can affect how I see the [*Quick Draw*] picture before it's covered up." Another student, a 20-year-old female named Lexi, indicated that she first saw four parallelograms that reminded her of "two rooftops where I am in the air looking down." "I guess my perspective," Lexi explained, "was from the top. What's neat is how all of our pictures still look the same…or mostly the same." Other explanations were offered, but the conversation about perspective and point of view eventually ignited the first discussion about spatial thinking.

By the end of the first week of phase three, students had responded to the first and second journal prompts (see Appendix G) that asked students to describe a situation where spatial thinking was emphasized and describe their ability to think spatially. Over half the class stated that they used spatial reasoning in some form on a daily basis. With respect to ability, 13 students described themselves as having strong spatial ability, 13 students described themselves as having average or normal spatial ability, and seven students admitted to having poor spatial ability. The pre-PSVT revealed a discrepancy between the students' self assessment and actual ability.

Students' Initial Abilities

Participants in this study believed they could improve their spatial thinking. The PSVT, a quantitative instrument, was used in this study and found that students' spatial visualization skills did improve. The PVST (Guay, 1980) was made up of three sections, the Developments section (PSVT/DEV), the Rotations section (PSVT/ROT), and the Views section (PSVT/VIEW), each measuring a different aspect of spatial ability for a total of 36 questions. The scoring for each section of the PSVT consisted of a raw score between 0 and 12 before being added together for an overall score. Using Excel, the descriptive statistics of mean and range were obtained with N=33 for the pre-PSVT (see Table 2).

Table 2

Descriptive Statistics for the Pre-PSVT

	Mean	Range
PSVT/DEV	5.76	1-12
PSVT/ROT	4.73	1-12
PSVT/VIEWS	3.79	0-12
PSVT Total	14.27	6-36

Note. PSVT=Purdue Spatial Visualization Test, DEV=the Developments section, ROT=the Rotations section, and VIEW=the Views section.

Individual scores revealed that 72.7% of the participants answered over half of the 36 questions incorrectly. Also of interest was the number of questions not attempted on the pre-PSVT. While scoring the PSVT, when an answer was counted incorrect, a note was made as to whether the problem had been attempted or not. Of the total 1,188 questions given on the pre-PSVT, 146 (12.3%) of the questions were not attempted, with the majority of these questions coming from the Views section. Clearly, this portion of the instrument gave students the most trouble.

The Views portion of the PSVT was designed to measure how well someone could visualize a 3D object from various viewing positions (Figure 15).

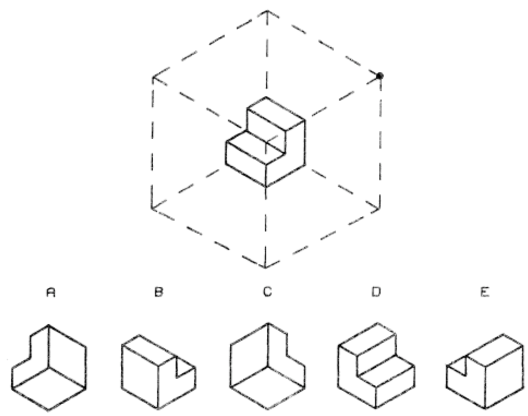

Figure 15. PSVT/VIEW sample problem.

The example above shows an object sitting in the center of a glass box with a black dot on the upper right vertex. The test-taker was asked to select the figure, from options A-E, which represented the figure when viewed from the position of the black dot. The answer was E. The focus group expressed their frustration with this portion of the instrument.

The focus group met for the first time during phase two of this study, just after the pre-PSVT was scored but before spatial exercises had begun. Michelle, a 19-year-old female who had been homeschooled during her junior high and high school years, was the first to bring up the Views section during the group discussion. In almost a whisper, Michelle stated that, "the [PSVT/VIEW] part of the spatial test we took was really difficult." She continued, "I used to make sculptures in high school for art projects so I thought it would be easy for me. I feel like I didn't do very well. Maybe it was because the shapes were so difficult." Mitch, an 18-year-old male and friend of Michelle's, followed this statement by adding, “Yeah, I agree. I think I would have done better if the test showed both sides of the shape. Like, the front and the back. Then I could have been able to see it better. I felt like I didn’t know what the whole shape looked like on a lot of the problems.” When I asked others in the focus group how they felt they performed on the PSVT (Appendix H), Katy responded that she felt she had “done okay except for the last problems on the [PSVT/VIEW] part.” The Views portion of the PSVT was not the only instrument where students’ confidence seemingly waned.

Surprisingly, several members of the focus group felt as though they did not perform well on the pre-MPI, even though the graded scores indicated a much higher success rate as measured by this instrument than the initial PSVT. The MPI, developed by Suwarsono (1982), is an instrument intended to measure a student’s performance in mathematical problem solving in either a visual or non-visual mode (Appendix C). This test was primarily scored on a +2 to -2, visual to non-visual scale, but was also graded for correct responses for discussion purposes. Without regard to *how* the problem was solved, the graded scores of the pre-MPI revealed that 75.8% of the students answered more than half of the twenty questions correctly, with 56% of this group scoring at or above the 70% mark. Participants had a much higher attempt rate on the pre-MPI as well. An individual score of “0” was given to problems where no clear attempt was made. Of the 660 total questions given to the class on the pre-MPI, only 11 (1.7%) of the questions were skipped over completely. Since the average score of the focus group was comparable to the average score of the class as a whole, 64.7% to 60.6%

respectively, the similarity of statements overheard from both groups was not surprising.

While discussing the MPI, focus group members were quick to voice their beliefs that they could “improve with practice” even though they felt disdain for “word problems.” Descriptive words such as “confusing,” “complicated,” and “hate” surfaced while the pre-MPI was the topic of discussion. One student, a 22-year-old male named Seth, felt like some of the questions might have been a trick. “Mrs. Prugh, were all of the questions [on the MPI] real?” he asked. I asked him to define “real” and he explained, “Well, I think that some of the questions on [the MPI] were a math joke. I tried working out every problem, and I thought I would be able to solve them, but when I tried to work them out there wasn’t enough information. I think that maybe ‘not possible’ was the solution to some of the problems.”

I gave the students a few seconds to think about Seth’s remark, but before I replied, the youngest student in the focus group, 17-year-old Ashlee, responded, “A lot of math problems have more than one solution, and [the MPI] probably had questions like that. You probably knew how to solve it, but maybe you were solving for the wrong solution. I know problem solving has always been hard for me, and [the MPI] reminded me of that.” I seized the opportunity to briefly discuss solution strategies with the focus group and examples where “more than one solution” might be appropriate. While I may not have convinced Seth that the MPI did not include trickery as a questioning strategy, I did discover that many of the students felt as thought there was hope when learning to problem solve.

As previously noted, the first theme that emerged from the data indicated that participants in this study believed spatial thinking ability and problem-solving ability could improve with practice. Regularly, statements such as “I knew this would get easier for me” and “I knew I could do this eventually” were verbally concluded at the end of in-class spatial activities. The quantitative results from the pre- and post-measures used in this study supported this belief. The MPI and the PSVT were given as measures to determine each participants’ preference for using spatial strategies when problem solving and spatial visualization abilities, respectively.

Dependent sample t-tests were used to evaluate the changes in the pre- and post-measures of these instruments. The null hypothesis, H_0: no significant difference between the pre- and post-intervention scores, and the alternative hypothesis, H_1: significant difference between the pre and post intervention scores were set with a significance level of .05. A two-tailed test was chosen since a difference in the pre- and post-data was desired and an increase or decrease was not guaranteed. This test was selected since the sample data consisted of matched pairs, or, more specifically, the students in this study. Also, the sample was a simple random sample for the population, which included those who would be required to take the Elements of Mathematics course. Lastly, since the sample population was greater than 30, normal distribution was assumed.

Students’ Abilities and the Mathematical Processing Instrument

The MPI was an instrument designed to measure a student’s preference for solving mathematical problems in a visual or non-visual manner. Assessing this instrument was challenging since there were two components to consider: a visual or non-visual attempt at the solution and a correct or incorrect answer. The greatest challenge was with scoring the latter, the visual component. Hegarty and Kozhevnikov (1999) offered great insight on how to grade visual-spatial representations when they differentiated between pictorial imagery and schematic imagery. Schematic imagery—imagery that is engaged when an individual represents the spatial relationship between objects and, therefore, imagines spatial images —was considered when scoring this instrument. For this study, the 20 problems on the MPI could have resulted in a possible individual score ranging from -40 to +40 on a purely analytical to visual scale. This scale was useful for tallying information to gain simple percentages before any other statistical analysis was conducted.

Once the pre- and post-measures of the MPI were scaled, differences in the sets of data were immediately observed and considered. Of the total 660 possible questions given to the 33 students on the MPI, 55.6% of the questions on the pre-MPI were attempted using a spatial approach. This percentage rose to 62.3% on the post-MPI. Strategy used on individual questions was noted as well. Number six on the MPI posed the following scenario:

> From a long stick of wood, a man cut 6 short sticks, each 2 feet long. He then found he had a piece 1 foot long left over. Find the length of the original stick.

Questions 1, 15, and 20 each dropped 3% on the number of participants who chose to use spatial imagery on the pre-MPI versus the post-MPI, while question six, above, dropped the most at 6.1%. Conversely, data revealed that 14 of the questions had a percentage gain with respect to those who chose to use schematic imagery as to the problem-solving strategy. Of the most significant, questions 2, 14, 16, and 19 increased by 12.1% while questions 9 and 12 increased by 18.2% and 21.2% respectively. The only two questions that showed no change were questions 7 and 10.

The preference for using a spatial strategy on the MPI was not the only initial change found in the pre- to post-results. The grades on the MPI changed as well, increasing from an average grade of 60.6% on the pre-measure to 67.7% on the post-measure. In addition, an astounding 90.9% of the participants were able to correctly answer at least half of the problems with 73.3% of these students performing at or above the 70% mark. With these changes in mind, a t-test was performed to test for a significant difference between the pre- and post-MPI results.

A t-test for paired samples was conducted to compare the preference of using a visual-spatial approach for problem solving before and after eight weeks of spatial tasks. There was a significant difference in the scores for the pre-MPI ($M=4.76$, $SD=14.16$) and the post-MPI ($M=8.76$, $SD=15.94$) conditions; $t(32)=2.42$, $p=0.02$. Therefore, the null hypothesis was rejected. These results suggest that the inclusion of spatial tasks had an effect on the participants' preference for using a spatial approach when solving problems on the MPI. Specifically, these results suggest that the inclusion of spatial activities for eight weeks increased the preference for using schematic drawings and, therefore, a spatial approach when solving mathematical problems, as well as the accuracy with which spatial thinking was used. One of the spatial activities used during this study that encouraged spatial thinking was building three-dimensional figures using unit cubes.

Spatial Activity: Unit Cubes

When unit cubes were brought into class for the first time, students expressed their excitement to "play with blocks." After explaining that the manipulatives were called "Unit Cubes," I passed out a set of 20 blocks to each student (see Figure 16).

Figure 16. Unit Cubes

First, I asked the students to describe their unit cubes. Initially, students gave superficial answers such as "I have three reds," "[the unit cubes] connect," and "there are 10 different colors." Soon after, Eric, an 18-year-old graphic arts student, pointed out that all the cubes had the same dimensions. I asked him to further explain. "Well," he demonstrated, "if you ignore the little piece that connects to another piece and set them side-by-side, they are all the same shape and are all the same height." I responded by asking the entire class if two figures with the same shape and height would always have identical dimensions. Eric quickly amended his explanation, "Oh, wait, what I meant to say is that if you set them side-by-side you can see that they have the same height, the same width, and the same…uh…depth! Right? Like, you can see they take up the same amount of space." Others nodded with approval and, after a short discussion about side lengths, agreed that each cube had the same dimension and were therefore identical. This conversation set the stage for future dialogue when only 2D figures were available for discussion concerning 3D objects. From here, I felt the class was ready to start the activity of building with Unit Cubes.

Given the following image, I asked the students to recreate the image using their unit cubes (see Figure 17).

Figure 17. 2D image of a 3D figure.

To no surprise, the students recreated the figure with ease. I then asked them to hold their figure under the table, where they could not see it, and asked them to imagine seeing their figure from afar. I specifically asked, "What would your figure look like if you set it on the ground, stood directly over it, and viewed it from above? Would it appear different?" About half the class nodded in agreement. I asked those students to draw what their figure would look like from an arial view. For the others, I asked them to physically stand over their figures, and then draw what they had seen. Once everyone had completed their sketch, I had them hold up their drawings. Figure 18 demonstrates the variation in student answers.

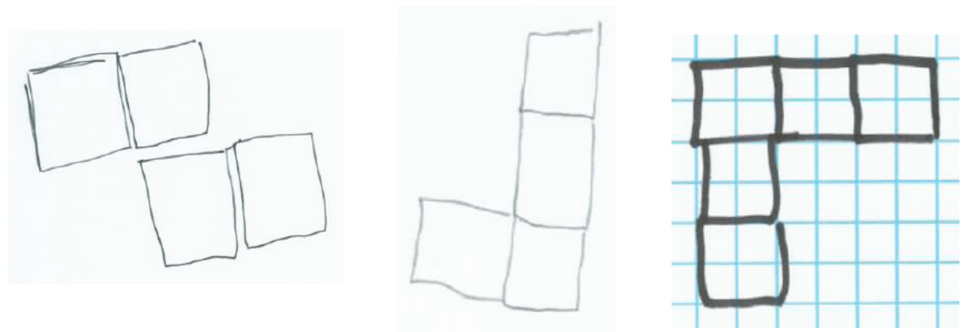

Figure 18. Student sketches of the “top” of Figure 17.

I asked the class to look around the room and consider the drawings. Emilia, a 19-year-old female, was puzzled. “Hey, why does your drawing [pointing to a classmate’s sketch] look like a ‘Z’?” she asked. I replied by asking Emilia why her sketch looked like an “L.” She initially thought she made a mistake. “Did I do something wrong?” she asked. By the time Emilia had asked the knee-jerk question and handed me her figure and drawing to critique, most of the class had realized the discrepancy in their drawings. “Emilia, your drawing isn’t wrong, you just drew [the figure] from a different angle! See, my picture is an ‘L,’ too!” realized Katy. Handing the figure and drawing back to Emilia, I explained to the class that they had just sketched maps of the figure they created with unit cubes. I then asked if they thought it possible for a stranger to recreate their 3D figure given the three different maps they had created and a set of unit cubes. The students decided it was possible, so for the next class, we did just that.

At the beginning of class, students were again given sets of unit cubes and presented with the map of three sides of a figure (see Figure 19). They were asked to create the figure, represented by the maps, using the cubes.

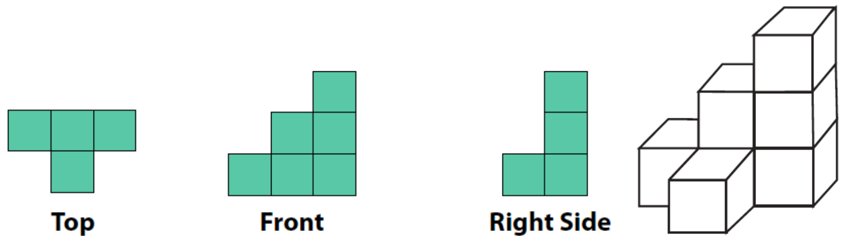

Figure 19. Map of three sides of a figure and solution.

This activity was considerably more challenging, but students persisted. After several successful rounds of this activity, Seth noted that he was getting better at creating the figures. "I can tell I'm getting faster [at constructing the figures] because it used to take me a long time. I just have to remember not to take the maps literally—like you said, they are just a representation." This activity was often modified by only giving maps of two sides of a figure or by completing the task in reverse. For example, I would present a 2D image (see Figure 20) of a 3D figure and ask the students to draw the matching top, front, and side maps.

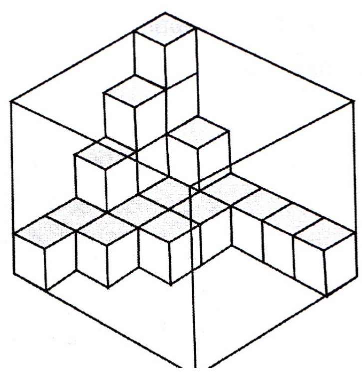

Figure 20. 2D image of a 3D figure.

The activities that involved unit cubes were always seemingly well received by the participants. The focus group mentioned that they would use these activities with their future students. 18-year-old Alison stated that she enjoyed the unit cube tasks because they taught her "how to think of figures in a different way." She believed that learning to think of figures in more than one way would be "really good for students since math has so many shapes in it." Undoubtedly, the ability to perceive "shapes" using multiple strategies is beneficial on instruments such as the PSVT.

Students' Abilities and the Purdue Spatial Visualization Test

The Purdue Spatial Visualization Test (PSVT), developed by Guay (1980), was comprised of three parts: Developments, Rotations, and Views (Appendix B). The Developments section (PSVT/DEV) measured spatial structuring, the Rotations section (PSVT/ROT) measured mental rotation ability, while the Views section (PSVT/VIEW) measured spatial perception. Students were given 18 minutes to complete the test and four scores were given: one for each of the three sections and an overall grade. Each section had 12 problems for a total of 36 questions.

Initial assessment of the data revealed an increase in the test scores and a decrease in the number of incomplete responses. For the PSVT/DEV section, there was a 27.9% increase on the number of correct responses from the pre- to post-PSVT. The PSVT/ROT portion showed a 33.3% increase while the PSVT/VIEW section increased by 60% in correct responses from the pre- to post-results. The individual increases resulted in an overall increase of 38.2% on the total scores of the pre- versus post-PSVT. One reason for the success was the ability of the students to complete the test. As noted earlier, of the 1,188 total questions given to the students on the pre-measure of the PSVT, 12.3% of those questions were left completely blank. On the post-measure, however, that number dropped considerably to .4%, a 96.6% drop. To evaluate whether these changes were significant, a t-test was used on various aspects of the data.

Paired samples t-tests were conducted to compare the spatial abilities of students in three areas—Developments, Rotations, and Views—before and after eight weeks of spatial tasks. A t-test was performed on each of the three pre- and post-results individually and subsequently on the overall scores. A significant difference was found in the scores for all areas tested. A summary of the relevant statistics can be found in Table 3.

Table 3

Paired Samples T-test Data for the PSVT

N=33

	M	SD	Range	t-value	p-value
PSVT/DEV					
Pre	5.76	3.2	0-12	3.98	0.0004*
Post	7.36	3.3	1-12		
PSVT/ROT					
Pre	4.73	2.7	1-12	3.9	0.0005*
Post	6.3	2.76	2-12		
PSVT/VIEW					
Pre	3.79	3.43	0-12	3.99	0.0004*
Post	6.06	3.01	1-12		

TOTAL SCORE					
Pre	14.27	6.71	6-36	6.2	0.0000006*
Post	19.73	7.64	7-36		

Note. PSVT=Purdue Spatial Visualization Test, DEV=the Developments section, ROT=the Rotations section, and VIEW=the Views section.

*Significant at the p<.05 level (2-tailed).

These results suggest that the inclusion of spatial tasks had an effect on the participants' spatial ability with regard to developments, rotations, and views when solving problems on the PSVT. Specifically, these results suggest that the inclusion of spatial activities for eight weeks increased the students' ability to think spatially. Responses from the journal questions and the focus group interview from phase three of the study highlighted some of the class activities that students felt had an impact on their ability to perform spatially.

Spatial Activity: Mental Rotations

The second focus group interview took place during week nine of the study and lasted for over two hours. The guiding questions I had prepared (see Appendix H) proved to be worthwhile in that they provided entry into discussion of other related topics of interest for the focus group participants. This helped create open dialogue in which everyone participated. When I asked the focus group if they could see themselves incorporating spatial thinking activities into their future curriculum, every member responded in the affirmative. Jill, an 18-year-old freshman, explained, "I want to use the [spatial] activities we do in class in my classroom because I have learned so much from them. I think it would be good for my students to be good at thinking this way so early on." Erin continued, "Yeah, I think we should all use [spatial] activities when we teach someday. Even if we don't teach math." Specifically, the participants mentioned two in-class activities they felt helped them improve their spatial thinking: rotating the *Quick Draw* figures and sketching 2D representations of 3D objects.

The third time participants engaged in a *Quick Draw* task, I changed the rules. Usually, students were shown an image for approximately three to five seconds and asked to draw the figure once it was covered. This day in class, however, I instructed the students to rotate the image 90° before drawing it on paper, without initially drawing the original (see Figure 21). Before the activity began, we discussed what a 90° rotation might look like for other objects in the classroom since the topic of rotations was not covered in the regular course material.

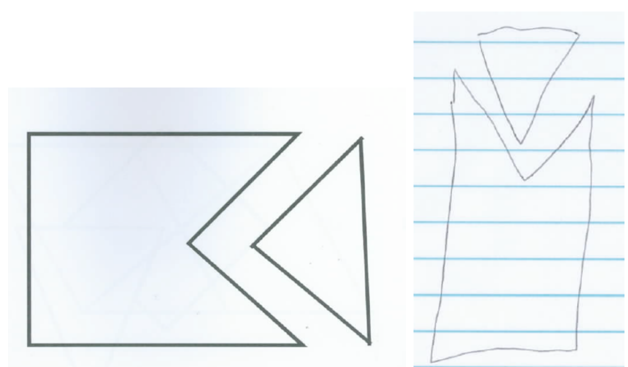

Figure 21. *Quick Draw* image and student solution of a 90° rotation.

Focus group participants were not the only students influenced by this activity. The seventh journal prompt asked students to reflect on an activity that had an impact on their spatial skills (Appendix G). In her response, Wendy described the modified *Quick Draw* activity as follows: "The thing that sticks out to me most would definitely be the rotating of a shape that we saw for just five seconds. I really had to think about the exact lines of the shape, as well as how it would appear if it was shown in the opposite directions. This influenced my spatial thinking by helping me to be able to manipulate items or shapes more willingly in my head." This was just one activity where students had to mentally manipulate images prior to drawing them. Another activity mentioned by the focus group participants was the "Sketch the Cubes" activity. Seth felt like this activity "really helped [his] spatial thinking because [he] had to think in 3D and then draw in 3D."

The sketching activity asked students to view an image made from unit cubes, rotate the image mentally, and then sketch the new image on paper (see Figure 22).

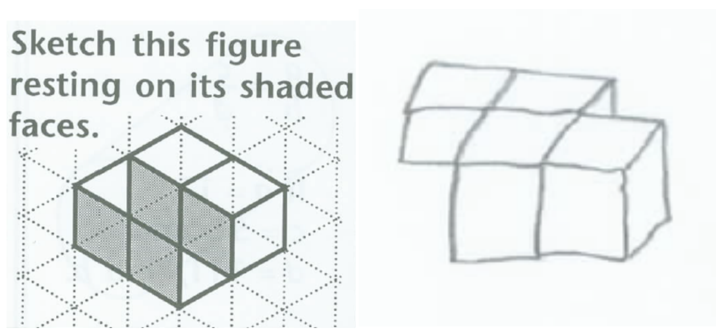

Figure 22. Sketch the Cubes activity and student solution.

When all the students had completed their sketches, I passed out sets of unit cubes and let the students create the figure. With the 3D model complete, I asked participants to compare and discuss their drawings. At first, students voiced frustrations with their inability to draw 3D images. Prompted by this response, I handed out grid paper to those who preferred it. Eventually, students became comfortable with this activity and, without instruction, sketched the image from several different viewpoints before creating the figure with unit cubes. Michelle, a member of the focus group, stated in the final interview session that the activities that involved drawing “helped [her] figure out solutions on the [MPI] and PSVT when [she] took them the second time.” She concluded, “I could tell I was much better at thinking spatially.” This admission led me to wonder if the two instruments, the MPI and the PSVT, had any correlation.

Correlation Test for the PSVT and MPI

A t-test for paired samples was computed to assess the relationship between the overall scores of the post-PSVT and the graded scores of the post-MPI. This test was chosen for several reasons: the subjects were independent, the post-PSVT and post-MPI were measured independently, neither instrument was a controlled measure, and a clear alternative to testing for linear correlation was not apparent. The test revealed a correlation between the overall scores of the post-PSVT and the graded scores of the post-MPI, r=0.467, p=0.006. A scatter plot summarizes these results (Figure 23).

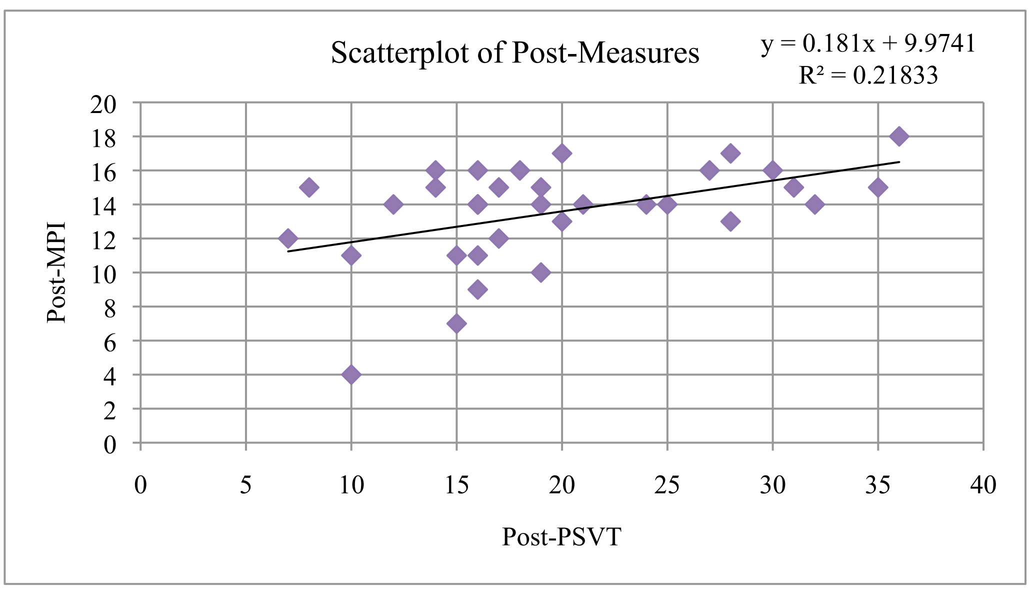

Figure 23. Scatter plot of the overall scores of the post-PSVT and graded scores of the post-MPI.

Overall, there was a positive correlation between the graded scores of the post-MPI and the total scores of the post-PSVT. Increases on the graded scale of the post-MPI correlated with increases on the overall scores of the post-PSVT. While not a statistical measure, qualitative data indicated that confidence in taking these tests also increased.

Students' Confidence Levels in the PSVT and MPI

After taking the post-PSVT and the post-MPI, participants were asked to write about their confidence levels while taking the pre- and post-measure for both instruments (Appendix G). One student wrote about how poorly she felt she performed on the tests but did not mention her confidence levels, four students discussed how they felt confident taking both the pre- and post-measures, while the other 28 participants reported gaining confidence in their answers from the pre- to post-tests. Among those last 28 answers, words and phrases such as "more comfortable," "easier," "more confident," and "better equipped" surfaced in the writings. Wendy explained, "The first time I took the test, I felt that I had done extremely poorly. I rushed through it because I really had no idea what I was doing. The second time, however, I felt much more confident. When [Mrs. Prugh] first handed out the test that second time, I was a little worried thinking I wouldn't have improved. As I took the test though, my confidence went up a great deal as I realized, 'Yes! I know how to do this!'"

Encouraged by the overwhelming response, I asked the focus group members to tell me how they felt they performed on the two measures during the third and final group interview (Appendix H). Erin enthusiastically responded that she felt more confident by stating, “I instantly saw an improvement while I was taking both tests at the end of the semester as opposed to the beginning. I was excited to see that I was having an easier time figuring things out and rationalizing through them. I even finished the spatial test the second time I took it! The first time, I think I only got halfway through before time was up. That right there is definitely a sign of improvement.” Wyatt, a 22-year-old male, attributed, in part, his improvement on these tests to the “Blind Cube” activity. He described the activity as “something that helped me think of how shapes are related.”

Spatial Activity: Blind Cube Task

The “Blind Cube” task was an extension of one of the many unit cube activities conducted during phase three of this study. After asking the class to create a figure with their set of unit cubes, I asked them to pair themselves with another student in class. To begin this exercise, both partners dismantled their figures. One partner closed their eyes while the other created a new figure using his or her set of unit cubes before handing it to the partner with their eyes closed—the “blind” partner. Without looking, the “blind” student would get to feel the figure for approximately 10 seconds before returning it. Once the figure was hidden from sight, the student opened their eyes and attempted to recreate the figure using his or her own set of unit cubes. Once complete, the original figure was revealed and compared to the recreation. To add more of a challenge, the second time the participants performed this task, the “blind” student was instructed to keep their eyes closed until they were finished recreating the figure. The class dialogue that followed this task indicated that students connected with this activity and the spatial thinking it promoted. Lori, a 19-year-old graphic arts major, enjoyed this task because "it took creative thinking to figure out how to build the figure. I think like this in my art classes, so I knew if I really focused I would be able to recreate [the original] figure." This was not the first mention of "creative thinking" when discussing spatial activities. This idea emerged throughout discussions and responses to journal questions and eventually became a theme.

The Uniqueness of Spatial Thinking

The idea that spatial thinking was a unique way of thinking was the most unexpected theme that surfaced through the analysis of the qualitative data. Throughout discussions and journal responses, phrases such as "creative thinking," "new way of thinking," "deep thinking," "artistic thinking," and "thinking outside the box" were common descriptions when discussing spatial activities and associated thought processes. When asked if the in-class spatial tasks were difficult, Ashlee responded, "It's not that the stuff we do in class is that hard, it's just different. It takes a different kind of thinking to work through those activities." Another student commented that she had to be "in the spatial mindset" to be able to do well on the tasks given in class. Clearly, students were not considering spatial thinking as a common mental practice.

The first journal prompt given to the participants during phase three of this study asked participants to reflect on experiences involving spatial thinking. It read,

> Thinking back on previous experiences, can you identify instances where you:
>
> a. Used spatial skills or spatial thinking?

b. Were taught spatial thinking?

c. Were in a situation where spatial thinking was emphasized?

There was significant crossover in the responses when describing real-world applications to spatial thinking. Many students discussed how spatial ability was important in certain aspects of art, such as sculptures, paintings, drawings, graphic design, and illustration. A sophomore graphic arts major named Jacob was confident in his spatial thinking skills and expressed that he used his ability often, writing, “My freshman year I was in a 3D design class, and if you had no spatial thinking, you couldn’t excel. You had to figure out how things would look in 3D and put them into the computer.” Other responses mentioned spatial thinking during “particular activities” such as driving and parking, the game of Tetris, geometry topics, and building blocks. Students regarded these activities as special since a “unique part of the brain” was used when engaged in the above activities. The second and third most mentioned topics included packing a car and rearranging furniture, respectively. Of the 11 students who mentioned one or both of these topics, 10 admitted to having just used the skill when they prepared to return to college for the school year. Interestingly, only 9 of the 33 participants wrote about, either directly or indirectly, using spatial thinking on a regular basis. The first interview with the focus group offered more depth to these responses.

Focus Group Beliefs About Spatial Thinking

Pre-STAS analysis showed that the average response to questions 4, 10, and 14, was “Disagree.” These questions can be found in Appendix D but are listed here for convenience:

4. Spatial thinking skills are useful in other areas besides mathematics.

10. I can see spatial thinking in many aspects of my daily life.

14. I struggle drawing two-dimensional shapes.

Students could select “Strongly Disagree,” “Disagree,” “Neutral,” “Agree,” or “Strongly Agree” to the questions on the Spatial Thinking Attitude Survey (STAS). Not a single focus group member selected “Strongly Agree” for any of the above statements while “Strongly Disagree” was selected five times on questions 4 and 14 and twice on question 10. Prompted by these responses, I asked the focus group three similar but not identical questions (Appendix H):

- Do you think there are any other areas besides mathematics where spatial thinking skills are useful? Explain.
- Does spatial thinking ever play a role in your daily life? Explain.
- Do you generally find drawing 2D shapes easy or difficult? Explain.

Jill was the first to respond to the opening question. “There might be, but I think math class is where we use them the most. I’ve probably done things that have involved spatial thinking all throughout my school years, especially in elementary school,” she explained. Michelle agreed, “This way of thinking,” referring to spatial thinking, “is useful when creative thinking is involved. We use it when we are solving math problems and doing art projects.” Rephrasing the question, I asked the focus group participants if the only time spatial thinking is used would be in a mathematics or art class. “No,” Mitch replied. “I think I probably use spatial thinking all the time. Like when I build stuff or sketch pictures or try to estimate the shortest route on a trip.” Seizing the opportunity to address the third question, I asked Mitch to explain how he uses spatial thinking when he sketches. "Well, I'm not an artist or anything, but I like to sketch cars and bikes and stuff. I have to imagine what the car I'm drawing would look like on paper before I draw it, or it looks all funny. And sometimes I will draw the same car from different sides. That way I can see what it would look like in real life," he described. Following this explanation, I asked the rest of the group if they ever sketched 2D or 3D shapes for recreation or academic purposes and whether or not they found it difficult.

Lisa, an 18-year-old freshman, expressed that she enjoyed drawing, especially for fun, and used images whenever possible to help solve problems. "I'm a very visual learner," she explained, "so being able to draw is really important to me. I think it helps me with a lot of things like rearranging a room and drawing maps. I struggle a little more when I have to draw in 3D though." Four other participants' responses mirrored Lisa's in that they felt being able to think in images was valuable, but that their 3D representations were lacking. Several of the in-class activities designed to enhance spatial thinking included a drawing component, so I decided to incorporate these tasks as soon as possible. Two tasks in particular were presented on days where 15 to 20 minutes were spent discussing 3D images: the "Folding Cubes" activity and the "Sketch the Figure" activity.

Spatial Activity: Mental Folding

The first of these, the "Folding Cubes" activity, began by showing students the net of a cube and its result when folded on the lines (see Figure 24).

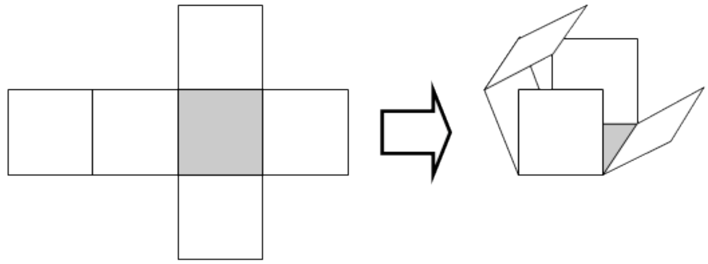

Figure 24. The net of a cube and solution.

After a discussion about nets and the properties of a cube, like the number of sides and vertices, a cluster of nets was shown to the students (see Figure 25). Participants were asked to decide which of the images would create a solid cube when folded on the lines.

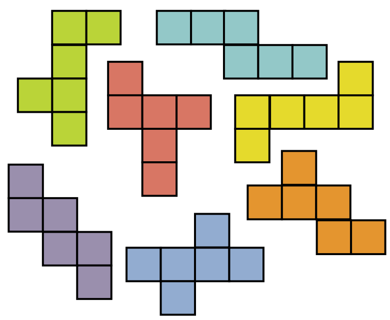

Figure 25. Example of the Folding Cubes activity.

The first time this activity was presented, the class could not reach an agreement on a single set of solutions. So, the class took a poll for each color before I prompted them to redraw the nets they felt could *not* form a cube, cut them out, and fold them to check their answers. After this, a re-vote was taken and it was unanimous that all the nets, at least for this example, would create a solid cube. This discovery astonished Alison: “I just couldn’t believe that the purple and orange net could create a cube! I still can’t make the purple cube in my mind like I can the other colors. But now that I can fold the [net] into a cube with my hands, I can see it!" Eric offered a strategy: "I also had a hard time with the orange net. But with the net that I made, I can see that it's easier if it's flipped upside down," he said as he demonstrated by turning his net in the air, "I think from now on I will rotate any net I don't feel can make a cube before I decide for sure. I think it's easier that way." This activity offered the students the chance to sketch in 2D but think in 3D.

Desiring to give the students the opportunity to think and sketch in 3D, the "Sketch the Figure" task was introduced once students were proficient with the "Folding Cubes" activity. In this exercise, students were given the nets of various 3D shapes and asked to sketch the shape when folded into a solid figure (see Figure 26). Blank paper and grid paper were always available for students to use if they chose.

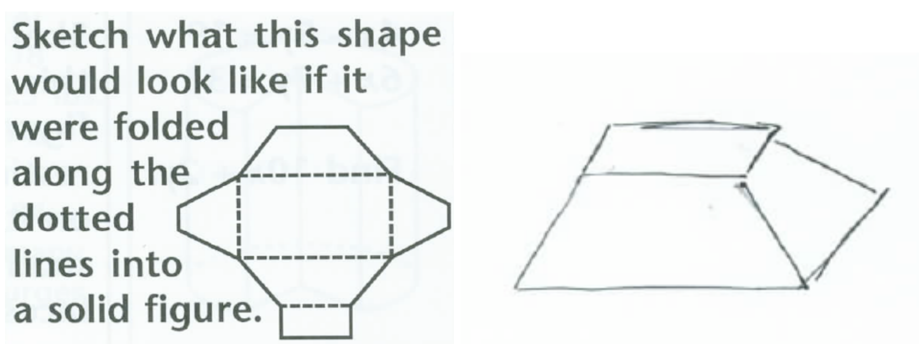

Figure 26. Example of a Sketch the Figure activity and student solution.

At first, students could draw the figure from any viewpoint of choice. The third time this task was performed, however, I instructed the students to draw the figure from two different viewpoints. For example, I might have asked students to draw Figure 26 from a top view and a side view. This encouraged participants to think about the figure from different angles. When asked to discuss the "Folding Cubes" and "Sketch the Figure" activities, the notion of spatial thinking as a unique way of thinking surfaced, reinforcing the theme. "This [Sketch the Figure] activity really stretches my brain," Jill commented, "so I know I must be thinking spatially."

At the end of each task, I inquired about the challenges and difficulties of creating the figures both mentally and on paper. Lexi admitted that these tasks were challenging. "Mrs. Prugh, this new way of thinking is hurting my brain. It's difficult," she began, "but I also think it's fun. I wish I would have learned to think this way sooner because it would have helped me in geometry." Emilia described the "Folding Cubes" activity as "tricky" but she enjoyed the activities as a whole. She claimed that activities like "Folding Cubes" and "Sketch the Figure" helped her brain "get ready to think about math." Kelly, a junior English major, summarized the general ideas of the class when she stated, "Spatial thinking is really deep thinking. These [activities] are challenging because we aren't used to thinking this way—but it's good that we do. They teach us how to think creatively, which we must know how to do if we expect to do well in a math class…or probably in any class." Several students nodded in unison at this remark. Kelly's thought on the importance of spatial thinking was not a solitary opinion.

Problem Solving and Spatial Thinking as Important Life Skills

The third theme that emerged from coding the focus group discussions and journal responses was that problem solving and spatial thinking are important life skills. No matter the topic, whether it be discussing an activity or responding to a journal prompt, participants in this study continually acknowledged the importance of problem solving and spatial thinking skills. Jill wrote that spatial skills "are very important. They are used in everyday life, all of the time." Other participant responses supported this belief:

- I think that being able to use spatial skills are very important. Everyday people face problems where they need to use their spatial thinking skills. For example, knowing how to look at things from a different perspective, using your mind to think and not just our intuition, is something that we need to do everyday.
- I think that spatial thinking skills are important for daily life. It helps with multiple things. If you are not able to think spatially then I believe that it is harder to do things sometimes.
- I think that spatial thinking skills are [a] very important quality to have in life. Spatial thinking skills are seen all around us. For example, being able to look at a building and know how many stories high it is, or even knowing how many objects can fit into another object. That is how general and often used spatial thinking is.
- I do think [spatial thinking skills] are important and relate to real life. I think spatial reasoning skills are important when trying to quickly decide where objects should be placed in relation to size and shape.

A common occurrence was for students to share stories of when and how they had used spatial thinking or problem-solving strategies in other areas of life outside of class. Robyn, a 19-year-old female, began a class discussion when she exclaimed, "I think these spatial thinking skills are more important than we realize! Did you know that I used my spatial reasoning and problem-solving skills twice already today? It's true. My communications class got cancelled so I got to eat lunch before coming to class. Since I was coming from the cafeteria, I had to visualize how far I needed to walk and then used my problem-solving skills to estimate how quickly I needed to eat so I wasn't late for class! I was really proud of myself." Other student-to-student conversations centered on the practicality of problem-solving strategies outside of mathematics.

Directly addressing this phenomenon, the fifth journal prompt given to the participants asked if they thought problem solving was important in everyday life (Appendix G). All 33 students felt that problem-solving skills were not only important but "very important" or, even, "extremely important" in everyday life. Students recognized that problem solving was not only an important skill needed to succeed in a mathematics course, but also in every other area of study. In his response to whether problem solving was important, Wyatt declared, "Yes! You may not realize sometimes when you're trying to resolve something or put something together, but you are using problem solving. Life is about putting things together and using your mind. Math makes you use that and brings it out to its fullest ability. Math is just practice for everyday problem solving." Ashlee agreed, "Problem solving is present in absolutely everything that we do. It is important to be able to think and work out things in your head so that you can make the best possible decision." One math-related task presented in class, which was designed to be a problem-solving activity through spatial thinking, was the "dunk" task.

Spatial Activity: Mental Cubes

The "dunk" task was designed to help students create strong mental images they could manipulate to answer questions. For example, I would ask students to imagine a 3x3x3 cube. If needed, I would show an example (see Figure 27).

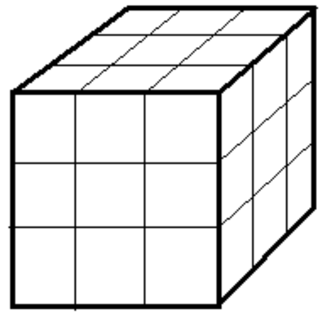

Figure 27. Example of a 3x3x3 cube.

Having concealed the example, I asked students to figure out how many individual cubes with one, two, or three sides would get wet if the figure was dunked into a bucket of water (see Figure 28).

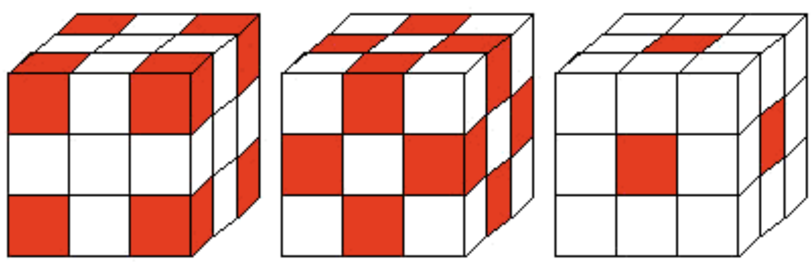

Figure 28. Example solution to a Dunk problem.

Similar to the "dunk" task, the "complete the cube" activity used unit cubes as a manipulative to help with spatial thinking. In this activity, students were given an image of an incomplete cube and asked to create the figure that would complete it (see Figure 29). After viewing the image for 5 to 10 seconds, students were asked to create the missing part of the cube with their set of unit cubes.

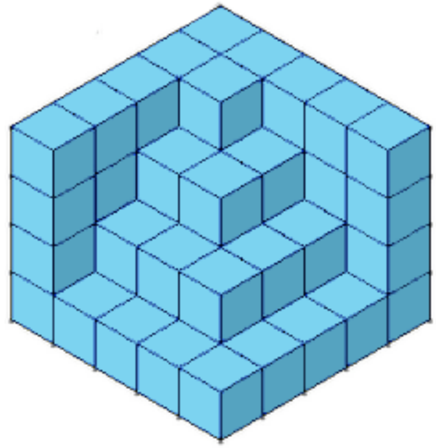

Figure 29. Example of a Complete the Cube activity.

To check students' solutions, I created the incomplete cube shown to the class and allowed students to "fit" their solutions to mine. If their attempt did not work, I permitted students to study the picture of the incomplete cube and try again. These two activities generated a healthy amount of student-to-student discourse. While helping a fellow classmate with a "complete the cube" activity, I overheard Ashlee encouraging a struggling friend stating, "Try it again. You need to think like a sculptor. Think, what would I have to chisel away to make that figure [pointing to the image I had put back up on the projector]? Now, make that piece." Over time, students were able to mentally manipulate cubes with more efficiency and their opinions of these tasks began to change. "When we first did a ["dunk"] activity, I didn't like it because I couldn't figure out how some cubes only had one side that got wet and another had two. I thought they all had the same. But, after the first time, I could see what was happening with the separate cubes and now I get it—I like it," explained Seth, a member of the focus group. Other focus group participants shared their opinions on spatial thinking as well.

Student Beliefs and the Spatial Thinking Attitude Survey

During phases one and four of this study, the focus group took the Spatial Thinking Attitude Survey (STAS). The STAS, developed by Hanlon (2009), was a 15-question, five-point, Likert-type survey (see Appendix D). The survey had two areas of focus: one area measured beliefs regarding spatial thinking and the other measured confidence regarding the drawing of a 2D or 3D shape. Initial assessment of the data revealed changes concerning the average answers for each of the 15 questions. Table 4 summarizes these results.

Table 4

Summary of the Changes in Average Responses on the Pre- and Post-STAS

n=17

Statement	Pre-STAS	Post-STAS	Difference
1	3.06	3.41	0.35
2	3.18	4.24	1.06
3	3.29	4.12	0.82
4	2.59	3.82	1.24
5	3.47	4.18	0.71
6	3.35	4.18	0.82
7	3.41	3.65	0.24
8	3.53	3.88	0.35
9	3.41	3.82	0.41
10	2.71	3.65	0.94
11	3.00	3.47	0.47

12	2.82	2.71	-0.12
13	3.06	3.12	0.06
14	2.12	2.06	-0.06
15	3.41	3.12	-0.29

Note. STAS=Spatial Thinking Attitude Survey. Also, these numbers represent the average response for each question.

Initial considerations of the data revealed the majority of responses on questions 12, 14, and 15 were either “Disagree” or “Strongly Disagree”; all other questions reflected an increase of change, indicating that more students agreed with the statement on the post-PSVT. To evaluate whether these changes were significant, a t-test was administered on each of the 15 statements.

A t-test for paired samples was conducted to compare students’ responses on each of the 15 statements on the pre-STAS and post-STAS taken before and after eight weeks of spatial tasks, respectively. A significant change occurred in seven of the responses. A summary of the p-values can be found in Table 5.

Table 5

Paired Samples T-test for the STAS

n=17

Statement	p-value
1	0.083
2	0.001*
3	0.011*
4	0.00008*

5	0.002*
6	0.0002*
7	0.260
8	0.111
9	0.130
10	0.002*
11	0.041*
12	0.608
13	0.718
14	0.718
15	0.136

Note. STAS=Spatial Thinking Attitude Survey.

*Significant at the p<.05 level (2-tailed).

These results suggest the inclusion of spatial tasks had an effect on the participants' beliefs regarding seven of the statements on the STAS. Specifically, these results suggest the inclusion of spatial activities significantly altered the beliefs in these seven areas:

- Spatial thinking skills are important for students to be successful at the elementary school level.
- I am sure that I can improve my spatial thinking abilities.
- Spatial thinking skills are useful in other areas besides mathematics.
- Spatial thinking skills can be developed.
- I can see spatial thinking in many aspects of my daily life.

- I am confident that I can draw geometric shapes accurately.
- I will incorporate spatial thinking activities into the classroom.

Data collected from the focus group interview from phase five of the study highlighted a class activity that members hoped to use in their future classroom: the "snowflake" task.

Spatial Activity: The Snowflake Task

The paper-folding activity, or the "snowflake" task as named by the participants, was a simple task that encouraged students to mentally manipulate a folded piece of paper (Wheatley & Reynolds, 1999). The students were given a blank 8.5x11 inch sheet of paper and a pencil and were instructed to watch as I folded and cut an identical 8.5x11 inch sheet of paper. To start the mental exercise, I twice folded my piece of paper while the students watched. Then, I used scissors to cut small shapes from the folded piece of paper (see Figure 30). At this point, I asked students to "shade in" all the areas on their piece of unfolded paper they thought were missing from my piece of paper.

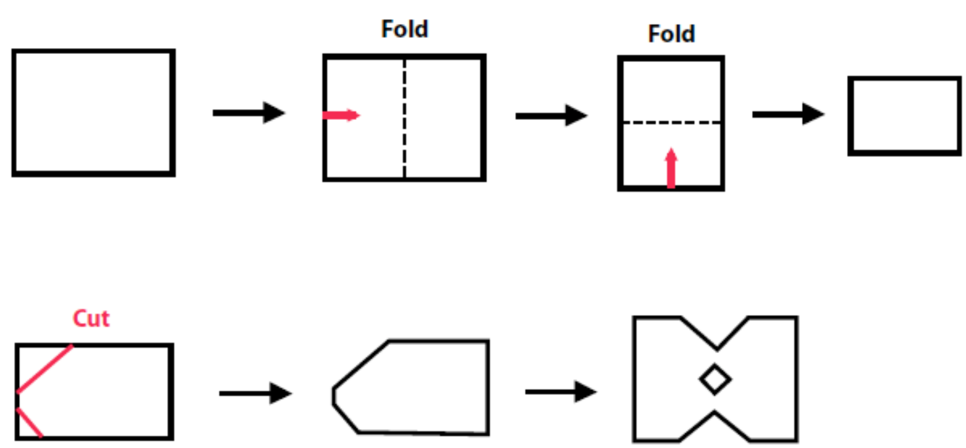

Figure 30. Example of the Snowflake activity.

On occasion, pursuant to students' requests, I identified the location of the folds on my piece of paper. Once everyone was finished, I revealed the "snowflake" by unfolding the paper in the reverse order in which I had folded it prior to cutting. I then asked students to discuss their thinking process while shading in the missing pieces. I asked: "How is your snowflake different than mine?" and, for example, "Why is this diamond shape in the middle of the snowflake?" To create more of a challenge, I added a fold, changed the folding pattern, or rotated the folded piece of paper before cutting it. These changes were made as the students watched every move. When asked why they would like to use this activity in their future classrooms, focus group members described the activity as one that would engage students in spatial thinking in a "fun way."

Pressing the topic, I asked the members if they felt similar activities that encouraged spatial thinking were important enough to incorporate throughout the year as more than just a "fun" activity (Appendix H). With the majority of the room nodding, Michelle vocalized the general consensus: "I think being able to use spatial skills are very important, so I think we need to be teaching it in every grade. Everyday, people face problems where they need to use their spatial thinking skills. Spatial skills help us think outside the box and help our minds grow." Wyatt followed, "Yeah, doing [spatial thinking] activities that you don't know how to do makes you think. That's what makes spatial learning fun. I plan to use [spatial] activities when I can." While the participants might not know exactly how often they will incorporate spatial activities into their future classrooms, they were all in agreement that spatial thinking is important and worthy of instruction time.

Conclusion

This embedded case study investigated the influential nature of spatial thinking activities on students' spatial visualization ability, problem-solving strategies, and beliefs about spatial thinking. Quantitative data were analyzed to examine the influence of eight weeks of spatial activities on these areas. Additionally, qualitative data were gathered through focus group discussions, written responses, and observations and were analyzed in conjunction with the quantitative data to answer the three research questions.

Both quantitative and qualitative data were analyzed to examine the impact of integration of spatial activities on student spatial visualization ability. Qualitative data revealed that students believed their spatial abilities could improve. The sixth journal prompt, given near the end of the study, asked students if they felt their spatial ability had improved. Of the 33 participants, 29, or 87.9% of the students, felt that their spatial thinking abilities had improved, an opinion supported by the results from the PSVT. The results of the quantitative data gathered from the pre- and post-PSVT revealed a significant difference on all three areas of the PSVT—the Developments, Rotations, and Views sections—which resulted in a significant difference on the overall scores as well. Completion rate of the instrument also improved, as there was a 96.6% drop on the number of questions left blank from the pre- to post-measure of the PSVT. Specifically, the results suggest the inclusion of spatial activities for eight weeks increased the students' ability to think spatially as measured by the PSVT.

Both quantitative and qualitative data were analyzed to investigate the influence of spatial reasoning tasks on students' mathematical problem-solving strategies. Qualitative analysis revealed that the majority of students felt as though spatial thinking was a unique way of thinking, stating that spatial thinking was "creative thinking," "a mindset," or a "new" way of thinking. Further, responses to journal prompts established the view that students felt that spatial thinking was important when problem solving, even if they did not or could not explain their reasoning. Regardless, this "new" way of thinking proved to be helpful when solving the 20 questions on the MPI. The results of the quantitative analysis on the pre- and post-MPI showed a significant difference in the scores from the pre- to post-MPI. This finding suggests that the inclusion of spatial tasks had an effect on the participants' preference for using a spatial approach when solving problems on the MPI. Specifically, these results suggest that the inclusion of spatial activities for eight weeks increased the preference for using schematic drawings and, therefore, a spatial approach when solving mathematical problems as measured by the MPI.

Together, quantitative and qualitative data were analyzed to explore the influence of spatial tasks on pre-service elementary teachers' beliefs about spatial thinking. Qualitative data analysis revealed that participants in this study thought that problem solving and spatial thinking were important life skills. Students specifically expressed why they thought spatial thinking skills were important through written responses to journal prompt number three (Appendix G). Focus group members, who were all pre-service elementary teachers, verbally expressed this belief through group discussions and further confirmed their thoughts through the STAS survey. The quantitative results showed a significant difference in opinion on seven statements on the STAS. Specifically, these results suggest the inclusion of spatial activities significantly altered the pre-service teachers' beliefs in these seven areas:

- Spatial thinking skills are important for students to be successful at the elementary school level.
- I am sure that I can improve my spatial thinking abilities.
- Spatial thinking skills are useful in other areas besides mathematics.
- Spatial thinking skills can be developed.
- I will incorporate spatial thinking activities into the classroom.
- I can see spatial thinking in many aspects of my daily life.
- I am confident that I can draw geometric shapes accurately.

Additionally, focus group members were able to discuss the in-class spatial activities and their preferences regarding which tasks they plan to use in their future classrooms.

In Chapter V, a summation of the findings, along with conclusions, are offered. Chapter V also presents a discussion of the consequences of the results with respect to implications regarding practices for mathematics education and pre-service teacher education programs in addition to a discussion concerning future directions for research in the area of spatial thinking.

Chapter V

CONCLUSION

Spatial thinking is not only necessary for success in many aspects of daily life, but it is also an essential skill for STEM fields from which many scientific discoveries and progress are made (National Research Council, 2006). The importance of spatial thinking throughout a child's kindergarten through grade-12 education is emphasized in the geometric standards set forth by the National Council of Teachers of Mathematics (NCTM, 2000). This recommendation is mirrored through the work of the National Research Council (NRC), which asserts that spatial thinking is a learnable skill that should be matriculated throughout a student's educational experience. Spatial activities are a worthwhile investment in the mathematics classroom, since the skill of spatial thinking has been repeatedly linked to problem solving (Battista, 1990; Edens & Potter, 2007; Hegarty & Waller, 2005; Moses, 1977; Reynolds & Wheatley, 1997).

Additionally, taking into consideration that future teachers of mathematics will most likely teach material in the same manner in which they learned it (Sundberg & Goodman, 2005) and are more likely to incorporate spatial activities in their own classrooms if they are confident in their own abilities (Battista, 1990), mathematics courses required for education majors have the opportunity to promote the learning of this skill in current as well as future learners of mathematics. The inclusion of spatial tasks in the classroom could lead to more effective problem-solving strategies and improved instructional strategies in the K-12 classroom. For these changes to be made, present and future students must be given the opportunity to engage in spatial thinking whenever possible, especially in the mathematics classroom.

Overview of the Study

The purpose of this embedded case study was to understand how the inclusion of spatial tasks influenced undergraduate students' spatial visualization ability, problem-solving strategies, and beliefs about spatial thinking. Despite decades of reform, the U.S. still trails economic competitors like Japan (National Center for Education Statistics, 2003). One missing piece to this puzzle could be the lackluster ability of U.S students to think spatially and problem solve with regard to mathematics (IEA, 2008). Despite the pervasiveness of spatial thinking research and recommendations, this skill is largely unrecognized in the educational system (National Research Council, 2006). As a result, this study examined how the inclusion of spatial tasks could influence problem-solving performance, spatial thinking ability, and beliefs of undergraduate mathematics students. The following section will provide insight into understanding undergraduate students' problem-solving ability, spatial ability, and beliefs about spatial thinking by addressing the three research questions that guided this study:

1. How does the integration of spatial activities in an undergraduate mathematics content course impact student spatial ability?
2. In what ways does the integration of spatial reasoning tasks into an undergraduate mathematics content course influence problem-solving strategies?

3. How does the integration of spatial reasoning tasks influence the beliefs on spatial thinking of pre-service elementary teachers?

The participants were 33 undergraduate students who were enrolled in the researcher's Elements of Mathematics, Fall 2011 course. The course was a low-level mathematics course designed for liberal arts students, so the participants' majors ranged in concentration. A little over half of the participants were education majors while the others had declared interest in areas such as art, English, and family studies. In addition to demographic information, quantitative and qualitative data were collected during the 12 weeks of study. Quantitative data were collected through the Purdue Spatial Visualization Test (PSVT), the Mathematical Processing Instrument (MPI), and the Spatial Thinking Attitude Survey (STAS). Qualitative data were garnered through student-written journal responses, focus group interviews, and observations. The focus group was comprised of the 17 participants who had declared elementary education as their area of study. This group met on three separate occasions: once during phases two, three, and five. The purpose of the focus group was to give deeper insight into the participants' experiences with the study and beliefs about spatial thinking.

Implementation began with a description of the study followed by satisfaction of IRB requirements (Appendix A). Pre-measures of the PSVT, the MPI and the STAS were also given to participants during the first phase. Once all three pre-measures were scored, the focus group met to discuss initial perceptions about the three pre-assessments and areas of interest concerning spatial thinking (Appendix H). During the following eight weeks, daily spatial thinking activities and reflective journal prompts were incorporated into classroom practices. Class discussions regarding these assignments ensued, allowing for insightful classroom observations. The second of three focus group interviews also took place during the first phase. A feel for students' beliefs concerning spatial thinking during the eight weeks of implementation was the focus of this discussion. During the final two weeks of the study, the final focus group discussion was held, and post-measures of the PSVT, the MPI, and the STAS were executed and scored. These data were collected and fully analyzed after grades for the course had been finalized and posted for the semester.

Results from the quantitative data were used to determine if the integration of eight weeks of spatial activities resulted in significant differences in scores on the PSVT, the MPI, and individual statements on the STAS. Analysis of the qualitative data—responses to journal prompts, focus group interviews, and observations—was used to examine the influence of the spatial tasks on the perceptions and beliefs about spatial thinking on students and pre-service elementary teachers. Moreover, this same data were used to evaluate how pre-service teachers viewed their own understanding of spatial thinking and its relevance in their daily lives and future classrooms. After the quantitative data had been scored and tested and the qualitative data had been coded and themed, the data were analyzed in its entirety and conclusions were drawn. This section presents the findings for each of the research questions, implications for undergraduate-level mathematics education, and recommendations for action and further study.

Summary of the Findings

Spatial Tasks and Spatial Ability

The first research question investigated the influence of spatial tasks on students' spatial visualization abilities in an undergraduate mathematics content course. Spatial visualization abilities are crucial to learning. Bruner (1973) believed children explore new things first through action then through imagery before, finally, using language to describe and comprehend the world around them. Through this reasoning, spatial thinking is a necessary step to learning.

To help investigate the first research question, both qualitative and quantitative data were collected and analyzed. Qualitative analysis on student-written responses and focus group discussions revealed that students believed their spatial thinking abilities could improve with practice. This was encouraging given the fact that 60.6% of the class described themselves as possessing average or below-average ability at best in response to journal prompt number two, which asked students to describe their ability to think spatially. With respect to the quantitative analysis, the PSVT was used as a pre- and post-measure to assess student spatial visualization ability at the beginning and end of the study. The PSVT, developed by Guay (1980), was comprised of three parts: Developments, Rotations, and Views (Appendix B). The Developments section (PSVT/DEV) measured spatial structuring; the Rotations section (PSVT/ROT) measured mental rotation ability; while the Views section (PSVT/VIEW) measured spatial perception. Initial assessment of the data revealed an increase in test scores and a decrease in the number of incomplete responses. For the PSVT/DEV section, there was a 27.9% increase on the number of correct responses from the pre- to post-PSVT. The PSVT/ROT portion showed a 33.3% increase in correct responses, while the PSVT/VIEW section demonstrated a 60% increase in correct responses from the pre- to post-results. The individual increases resulted in an overall increase of 38.2% on the total scores from the pre-PSVT to the post-PSVT. Paired samples t-tests were conducted to evaluate whether these changes were significant.

A t-test was performed on each of the three pre- and post-results individually and later on the overall scores. The results of the quantitative analysis revealed a difference in the scores for all areas tested. Notably, these changes were most evident in the overall pre-PSVT scores ($M=14.27$, $SD=6.71$) and the overall post-PSVT scores ($M=19.73$, $SD=7.64$), with $t(32)=6.2$, $p=0.0000006$. Specifically, these results suggest that inclusion of spatial activities for eight weeks increased the students' ability to think spatially, as measured by the PSVT. Van Garderen (2002) found that differences in imagery use existed among different levels of learners, where the highest level of spatial reasoning was observed in a gifted group. Regardless of the level of giftedness the participants would be assigned if tested, these results support the participants' predictions as well as the NRC's (2006) assertion that spatial thinking can be learned. This implies that spatial imagery is useful for all levels of learners, not just the gifted. This knowledge is powerful with respect to problem solving since the two have been repeatedly linked.

Spatial Tasks and Problem Solving

The second area of focus in this study involved spatial thinking and problem solving. Specifically, the second research question sought to identify ways for which the inclusion of spatial tasks influenced mathematical problem-solving strategies. Learning to solve problems is a principal reason for studying mathematics. Problem solving is engaging in a task for which the solution method is not obvious or known in advance, and the NCTM (2000) strongly believes this activity is an integral part of mathematics learning. Wu (2004) identified two problem-solving cognitive processes: the factor-analytic approach and the information-processing approach. The former approach is generally empirical, and one factor in this area is visual perception—the concept that spatial/visual aptitude, however strong, will play a role in mathematical problem solving. Several studies support this conjecture (Battista, 1990; Edens & Potter, 2007; van Garderen, 2006).

Analysis of the relevant qualitative data collected in this study exposed several themes that involved problem solving. Students felt their problem-solving skills could improve with practice and were important for everyday situations. Another interesting theme emerged in addition to the previous two themes. Participants in this study believed spatial thinking was unique way of thinking. Phrases such as “new way of thinking,” “creative thinking,” and “spatial mindset” were just a few of the descriptions students used when discussing spatial thinking. Students’ perceptions of this “unique” way of thinking did not hinder them from using the skill to aid in problem solving as measured by the MPI.

The MPI was used as a measure to identify the students' preference for solving problems using a visual or non-visual approach. Schematic imagery, as defined by Hegarty and Kozhevnikov (1999), was used when scoring this instrument. Of the 660 possible questions given to the 33 students on the MPI, 55.6% of the questions on the pre-MPI were attempted using a spatial approach. This percentage rose to 62.3% on the post-MPI. The average grade on the pre-MPI to post-MPI changed as well, increasing from 60.6% to 67.7%. As with the PSVT, a t-test for paired samples was used to compare the students' preference for using a visual-spatial approach for problem solving before and after eight weeks of spatial task implementation. A significant difference was revealed in the scores for the pre-MPI ($M=4.76$, $SD=14.16$) and the post-MPI ($M=8.76$, $SD=15.94$) conditions; $t(32)=2.42$, $p=0.021$. These results suggest that inclusion of spatial tasks had an effect on the participants' preference for using a spatial approach when solving problems on the MPI. Specifically, these results suggest that the inclusion of spatial activities increased the preference for using schematic drawings and, therefore, a spatial approach when solving mathematical problems.

This study showed a positive correlation between the PSVT and the MPI, and thereby strengthened the body of existing literature on the relationship between spatial thinking and problem solving (Battista, 1990; Edens & Potter, 2007; Hegarty & Waller, 2005; Moses, 1977; Reynolds & Wheatley, 1997). Improvement on one post-measure typically indicated improvement on the other. Through journal responses and discussions, participants stated they had "more confidence" when taking the MPI the second time. Fisher (2005) explained that “visual expression provides a means of formulating and solving problems” (p.16), so improvement on these two instruments makes sense. Based on these results, it is apparent that exercises in spatial thinking affect spatial ability as well as one’s preference for using a spatial approach when problem solving in mathematics. A change in students’ beliefs seems like a logical extension of the change in students’ confidence and ability.

Spatial Thinking and Pre-service Teachers’ Beliefs

In addition to examining the influence spatial tasks had on ability, this study explored the impact of spatial activities on beliefs of pre-service elementary teachers. The beliefs of pre-service teachers are an important component of spatial thinking and problem solving, since research has shown that teachers who are more confident in their own spatial abilities are more likely to use such strategies in their classrooms (Battista, 1990; Presmeg, 1986). Again, both qualitative and quantitative data were collected and analyzed to address this question.

Analysis of the qualitative data revealed that students felt the ability to think spatially was an important life skill. However, this was not a unanimous consensus at the beginning. During the first interview with the focus group, I asked participants if they felt spatial thinking was useful in any other area besides mathematics (Appendix H). One group member explained that she was not sure and felt that “math class is where we use [spatial thinking] the most.” Qualitative analysis on the STAS showed considerable change in teacher beliefs concerning the usefulness of spatial thinking outside of mathematics.

The STAS, developed by Hanlon (2009), was a 15-question, five-point, Likert-type survey that partially focused on measuring beliefs regarding spatial thinking (Appendix D). Question number four asked if "spatial thinking skills are useful in other areas besides mathematics." Seven of the 17 students answered "Disagree" or "Strongly Disagree" on the pre-STAS while on the post-STAS only two students answered "Disagree" or "Strongly Disagree". A t-test for paired samples was used to measure the significant change in responses on question number four of the STAS as well as the other 14 statements. A significant change was found for the following seven areas of spatial thinking and geometrical drawing:

- Spatial thinking skills are important for students to be successful at the elementary school level.
- I am sure that I can improve my spatial thinking abilities.
- Spatial thinking skills are useful in other areas besides mathematics.
- Spatial thinking skills can be developed.
- I will incorporate spatial thinking activities into the classroom.
- I can see spatial thinking in many aspects of my daily life.
- I am confident that I can draw geometric shapes accurately.

These results indicate that eight weeks of spatial tasks changed the beliefs of pre-service elementary teachers. Specifically, after the implementation of spatial activities, the participants were more likely to believe that spatial skills are malleable, useful outside the mathematics classroom, and worthy of inclusion in future curricula. This shift in beliefs can be tied back to Bruner's (1973) theory that learners explore concepts through a linear progression, starting with action, then imagery, and finally through language. The "hands-on" exercises students experienced, possibly for the first time, throughout the eight weeks of implementation promoted understanding of spatial concepts and allowed students the opportunity to identify other areas in everyday life where spatial skills are useful.

Implications

Results of this study contain implications for mathematics courses as well as courses required for pre-service elementary teachers. Ultimately, the responsibility falls upon mathematics educators to prioritize problem solving and sense making as opposed to rote memorization in the mathematics classroom. Spatial thinking is one key component to achieving this goal. Further, the utilization of spatial thinking skills in future mathematics classrooms is dependent upon an emphasis of fostering such skills in teacher education programs. In either scenario, inclusion of spatial thinking activities is vital to the success of the objectives of the course.

With respect to undergraduate mathematics education, this study revealed that undergraduate learners of low-level mathematics have the potential to increase their spatial thinking skills. This result has many implications for this particular population of students. The participants in this study indicated that they felt spatial thinking was an important life skill, and that it could improve with practice. Many focus group members also admitted to "not liking math" or not being "good at math" through group discussions. The possibility exists that the reason these students felt left behind in previous mathematics courses is because their spatial skills were subpar. Since spatial aptitude has been linked to problem-solving ability, inclusion of this skill could also result in more powerful problem solvers in the mathematics classroom—an overarching goal of any mathematics course. Therefore, the benefits of spatial thinking activities are two-fold: enhanced spatial thinking ability and more efficient problem-solving skills. If the goal is for every student of mathematics to think critically and problem solve, then inclusion of spatial thinking as a primary focus simply cannot be ignored. The inclusion of spatial thinking could also have a positive impact in the areas of STEM.

A current need exists for qualified persons in science, technology, engineering, and mathematics in the U.S. While each of these four areas is distinct, overlap exists with respect to the presence of problem solving and the need for practiced spatial thinkers. Inclusion of spatial activities in undergraduate mathematics classrooms can potentially further develop the spatial skills of those already interested in the STEM fields and encourage those who might not consider the possibility otherwise. This outcome will be best realized if spatial skills are fostered throughout a child's educational experience.

Spatial thinking and related activities should be included in the elementary classroom and in teacher education programs. This study showed that exposure to eight weeks of spatial tasks changed pre-service teachers' beliefs about the importance of spatial thinking. Specifically, participants' beliefs about the malleability of spatial thinking, the practical applications of spatial thinking, and the importance of spatial activities in the classroom changed during this study. As a result of this change, participants expressed their desire to use spatial activities in their future curricula. This is encouraging, because spatial thinking is recognized as an important component in the mathematics classroom.

Spatial thinking is emphasized throughout the standards set by NCTM (2000); consequently, it is a crucial component to teacher education programs. Further, pre-service teachers need to be comfortable not only with implementing spatial thinking activities in their own classrooms but also with learning through spatial thinking activities themselves. Since teachers are more likely to teach in a manner in which they were taught, incorporating space for spatial thinking activities into teacher education programs is vital. The responsibility to help prepare pre-service teachers lies within both education courses and content courses.

The inclusion of eight weeks of spatial tasks resulted in increased spatial visualization ability, effective problem-solving strategies, and positive beliefs about spatial thinking in this study. The findings and conclusions of this study could be used to redefine objectives to include or increase a spatial focus for courses in undergraduate mathematics as well as teacher education programs. Utilization of spatial thinking activities as a component in mathematics and mathematics education curricula could support the efforts of leaders in mathematics and mathematics education to maximize student ability in problem solving and effectiveness of pre-service teachers in future classrooms.

Recommendations for Future Research

This case study focused on how implementation of spatial tasks influenced spatial visualization ability, problem-solving strategies, and pre-service teachers' beliefs about spatial thinking. Additional qualitative or mixed methods studies could attempt to flesh out the three themes that emerged through this study: that problem solving and spatial thinking are important life skills, that spatial thinking is a unique way of thinking, and that both problem solving and spatial thinking can improve with practice. Additionally, future research with respect to pre-service teachers is needed to bring about the necessary updating of teacher education programs. Such studies might investigate the following:

1. A change in mathematical ability could greatly alter beliefs about spatial thinking. A similar study could be conducted in upper-level mathematics courses to investigate differences in student beliefs.
2. With further exploration into the belief that spatial thinking is a unique way of thinking, a future study could attempt to identify how mathematics students think they use spatial thinking in problem solving and other areas.
3. Longitudinal studies could be conducted following pre-service teachers through other mathematics courses and into in-service teaching. An investigation as to how spatial thinking evolved for these students or how it was utilized in later mathematics courses and classroom curriculum could be conducted.
4. This study disclosed a discrepancy between the participants' self assessment and actual ability on the pre-PSVT. Further mixed methods research could investigate student confidence and actual ability with another instrument designed to measure spatial ability.

5. This study could be repeated with careful consideration to specific spatial activities. Selective elimination could reveal which, if any, of the specific spatial tasks have a unique influence on spatial ability or beliefs.

Finally, more longitudinal studies are needed to address the issue of novelty. This study found changes in spatial ability and beliefs based on implementation of spatial tasks. Questions remain regarding whether participants will continue to use spatial strategies when problem solving and whether beliefs will revert back once there is an absence of weekly spatial activities. At this time, very little research is available exploring spatial thinking within the confines of lower-level mathematics courses at the collegiate level. Comparisons between this study and similar research could be investigated to determine if different implementation practices affect student performance. Future studies in spatial thinking are important if we wish to make the most of mathematics teaching and learning.

Concluding Comments

The need for practiced spatial thinkers is evident in the growing concern over performance of U.S. students in mathematics as well as lack of interest in spatially driven fields such as science, technology, engineering, and mathematics. In addition to this need, spatial thinking is a beneficial skill that reaches beyond the STEM fields, as good problem-solving techniques are valuable for everyday life. A participant in this study told of a stressful situation at his job at a local video rental store:

> Last week I was put in charge of five employees and instructed to box up all the old VHS tapes in the warehouse. The problem was that the boxes I was given to use were all different sizes so I couldn't just count out the number of tapes for each box. Instead, I used spatial thinking to estimate how many tapes could be packed into each box. Then I gave the box and the estimated number of tapes to a coworker and asked them to pack it. There were a lot of boxes. I felt good when I was done because I was never more than three tapes over or under the number needed. My spatial thinking saved me a lot of time that day.

Clearly, this student was using spatial thinking and problem solving well outside the confines of a mathematics classroom. This is expected and encouraging since one goal of education is to promote autonomous thinkers. Without critical thinking, human life would cease to make the necessary changes and adjustments necessary for survival. Since spatial thinking is related to problem solving, and problem solving is important in many facets of life, spatial thinking should be a skill that is fostered and encouraged within the classroom.

Focus on spatial research has fluctuated over decades of educational reform and has established a platform through the support of national organizations such as the National Research Council (2006) and the National Council of Teachers of Mathematics (2000). Even with such powerful recognition, purposeful cultivation of spatial thinking is commonly overshadowed by other factors in the mathematics classroom. More often than not, spatial thinking is a second-hand skill fostered through interaction with shapes and taught in courses with a geometric focus.

Currently, the message being sent to students of mathematics is that spatial thinking is unimportant or, worse, best utilized when no other strategies, such as procedural steps, are available. This approach to spatial teaching puts present as well as future students at a disadvantage.

According to NCTM, problem solving is an integral part of all mathematics learning. Further, research has linked spatial thinking to problem solving, indicating that spatial thinking is a necessary skill for success in solving problems in mathematics. Therefore, students must be given the opportunity to foster this skill from the beginning to the conclusion of their educational experience. Thankfully, research, including this study, has shown that this vital skill can be improved as late as post-secondary school. Spatial skills need to be intentionally nurtured if educators desire to give students a global competitive edge and help students develop an effective arsenal of strategies to problem solve. While this skill is not explicitly tested by state exams, the benefits of honing spatial skills will pay off long after the final bells of a classroom have rung. If the purpose of education is to create productive citizens to advance our way of life, then spatial thinking must be incorporated into the classroom. To do this, we must first equip our future teachers.

The role of research concentrating on pre-service teachers' spatial thinking and spatial ability needs to be a priority if change is desired. The spatial thinking and beliefs surrounding spatial thinking of pre-service educators is a critical component to the likelihood of this skill being fostered in future mathematics classrooms. The spotlight is now on teacher education programs, because pre-service teachers must first be proficient spatial thinkers before they are able to infuse this skill into their own teaching methods. Mathematics courses—especially those required for education majors—should be used as a fundamental piece to this design. In conclusion, for change to occur, inclusion of spatial thinking and spatial thinking activities must permeate the mathematics classrooms and teacher education programs of today and tomorrow.

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Appendix A

IRB Letters of Approval

The University of Oklahoma®

OFFICE OF HUMAN RESEARCH PARTICIPANT PROTECTION - IRB

IRB Number: **13562**
Approval Date: **September 16, 2011**

September 16, 2011

Lindsay Prugh
Instructional Leadership and Academic Curriculum
820 Van Fleet Oval, ECH 114
Norman, OK 73019

RE: Spatial Reasoning In Undergraduate Mathematics A Case Study

Dear Ms. Prugh:

On behalf of the Institutional Review Board (IRB), I have reviewed and granted expedited approval of the above-referenced research study. This study meets the criteria for expedited approval category 6, 7. It is my judgment as Chairperson of the IRB that the rights and welfare of individuals who may be asked to participate in this study will be respected; that the proposed research, including the process of obtaining informed consent, will be conducted in a manner consistent with the requirements of 45 CFR 46 as amended; and that the research involves no more than minimal risk to participants.

This letter documents approval to conduct the research as described:

Consent form - Subject Dated: September 12, 2011
Survey Instrument Dated: September 12, 2011 Interview Questions
Other Dated: September 12, 2011 Recruitment script
Protocol Dated: September 12, 2011
IRB Application Dated: September 12, 2011
Survey Instrument Dated: August 31, 2011 Purdue Spatial Visualization Test
Survey Instrument Dated: August 31, 2011 Mathematical Processing Instrument
Survey Instrument Dated: August 31, 2011 Spatial Thinking Attitude Survey
Letter Dated: August 18, 2011 OK Christian University-Review Committee Letter

As principal investigator of this protocol, it is your responsibility to make sure that this study is conducted as approved. Any modifications to the protocol or consent form, initiated by you or by the sponsor, will require prior approval, which you may request by completing a protocol modification form. All study records, including copies of signed consent forms, must be retained for three (3) years after termination of the study.

The approval granted expires on September 15, 2012. Should you wish to maintain this protocol in an active status beyond that date, you will need to provide the IRB with an IRB Application for Continuing Review (Progress Report) summarizing study results to date. The IRB will request an IRB Application for Continuing Review from you approximately two months before the anniversary date of your current approval.

If you have questions about these procedures, or need any additional assistance from the IRB, please call the IRB office at (405) 325-8110 or send an email to irb@ou.edu.

Cordially,

Todd T. Sandel

Todd Sandel, Ph.D.
Vice Chair, Institutional Review Board

1816 West Lindsey, Suite 150 Norman, Oklahoma 73069 PHONE: (405) 325-8110

Ltr_Prot_Fappv_Exp

OKLAHOMA CHRISTIAN UNIVERSITY

Memorandum

To: Dissertation Committee for Lindsay Prugh, IRB Committee Members, University of Oklahoma

CC: Lindsay Prugh

From: David N. Lowry, Ph.D, Dean of the College of Arts and Sciences, Chair, University Research Committee

Date: 8/18/2011

Re: Approval for Campus Research Study

Dear Committee Members,

We have received Lindsay Prugh's request to conduct her doctoral research project at Oklahoma Christian University. After a careful review of her research proposal, *Spatial Reasoning in Undergraduate Mathematics: A Case Study,* we have granted her permission to conduct this case study research project during the Fall Semester of 2011—contingent upon her receiving approval from the Institutional Review Board at the University of Oklahoma.

Respectfully,

Dr. David N. Lowry
Dean, College of Arts and Sciences

DAVID N. LOWRY, PH.D.
Dean, College of Arts and Sciences

ph 405.425.1940 fax 405.425.1945 email david.lowry@oc.edu BOX 11000 OKLAHOMA CITY, OK 73136-1100 www.oc.edu

Appendix B

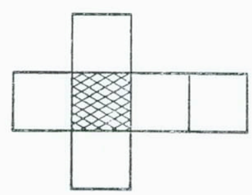
Sample Problems from the Purdue Spatial Visualization Test

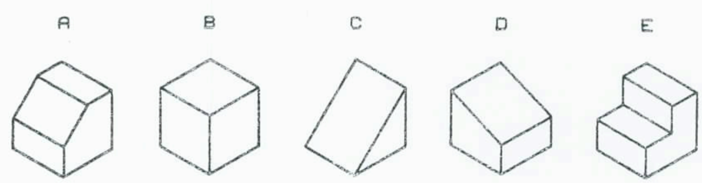
Do NOT make any marks in this booklet.
Mark your answers on the separate answer card.

SECTION 1: DEVELOPMENTS

Directions

The first section of this test consists of 12 questions designed to see how well you can visualize the folding of developments into three-dimensional objects. Shown below is an example of the type of question included in the first section of this test.





Presented is a development and five three-dimensional objects. The development shows the inside surfaces of a three-dimensional object. The shaded portion of the development indicates the bottom surface of the three-dimensional object. You are to:

1. picture in your mind what the development looks like when folded into a three-dimensional object;
2. select from among the five objects (A, B, C, D, or E) the one that looks like the folded development.

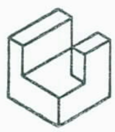
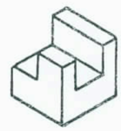
What is the correct answer to the example shown above?

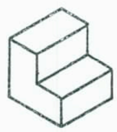
Do NOT make any marks in this booklet.
Mark your answers on the separate answer card.

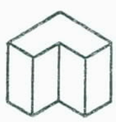
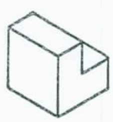
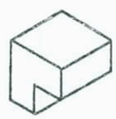
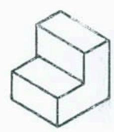
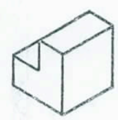
SECTION 2: ROTATIONS

Directions

The second section consists of 12 questions designed to see how well you can visualize the rotation of three-dimensional objects. Shown below is an example of the type of question included in the second section.

 IS ROTATED TO 

AS  IS ROTATED TO

A 
B 
C 
D 
E 

You are to:

1. study how the object in the top line of the question is rotated;
2. picture in your mind what the object shown in the middle line of the question looks like when rotated in exactly the same manner;
3. select from among the five drawings (A, B, C, D, or E) given in the bottom line of the question the one that looks like the object rotated in the correct position.

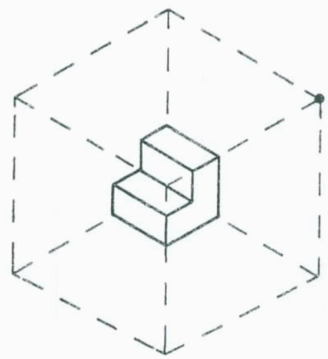
What is the correct answer to the example shown above?

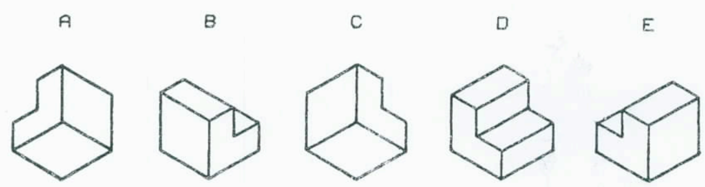
Do NOT make any marks in this booklet.
Mark your answers on the separate answer card.

SECTION 3: VIEWS

Directions

The third section consists of 12 questions designed to see how well you can visualize what three-dimensional objects look like from various viewing positions. Shown below is an example of the type of question included in the third section.





The example shows an object positioned in the middle of a "glass box" and five drawings representing what the same object looks like when seen from different viewing positions. The black dot in the top right corner of the "glass box" identifies the desired viewing position. You are to:

1. imagine yourself moving around the "glass box" until the black dot is located directly between you and the object;

Appendix C

Mathematical Processing Instrument

The Mathematical Processing Instrument (MPI)

1. At each of the two ends of a straight path, a man planted a tree; then again every 5 meters along the path he planted another tree. The length of the path is 15 meters. How many trees were planted?
2. On one side of a scale there is a 1kg weight and half a brick. On the other side there is one full brick. The scale is balanced. What is the weight of the brick?
3. A balloon first rose 200 meters from the ground, then moved 100 meters to the east, and finally dropped straight to the ground. How far was the balloon from its original starting point?
4. In an athletics race, Jim is four meters ahead of Tom, and Peter is three meters behind Jim. How far is Peter ahead of Tom?
5. A square (A) has an area of 1 square meter. Another square (B) has sides twice as long. What is the area of B?
6. From a long stick of wood, a man cut 6 short sticks, each 2 feet long. He then found he had a piece of 1 foot long left over. Find the length of the original stick.
7. The area of a rectangular field is 60 square meters. If its length is 10 meters, how far would you have traveled if you walked the whole way around the field?
8. Jack, Paul, and Brian all have birthdays in the 1st of January, but Jack is one year older than Paul and Jack is three years younger than Brian. If Brian is 10 years old, how old is Paul?
9. The diameter of a tin of peaches is 10 cm. How many tins will fit in a box 30 cm by 40 cm (one layer only)?
10. Four young trees were set out in a row 10 meters apart. A well was situated beside the last tree. A bucket of water is needed to water two trees. How far would a gardener have to walk altogether If he had to water the four trees using only one bucket?
11. A hitchhiker set out on a journey of 60 miles. He walked the first 5 miles and then got a lift from a lorry driver. When the driver dropped him off he still had half of his journey to travel. How far had he traveled in the lorry?
12. How many picture frames 6 cm long and 4 cm wide can be made from a piece of framing 200 cm long?
13. On one side of a scale there are three pots of jam and a 100 g weight. On the other side there are a 200 g and a 500 g weight. The scale is balanced. What is the weight of a pot of jam?
14. A ship was Northwest. It made a turn of 90 degrees to the right. An hour later it made a turn through 45 degrees to the left. In what direction was it then traveling?
15. There are 8 animals on a farm. Some of them are hens and some are rabbits. Between them they have 22 legs. How many hens and how many rabbits are on the farm?
16. A passenger who had traveled half his journey fell asleep. When he awoke, he still had to travel half the distance that he had traveled while sleeping. For what part of the entire journey had he been asleep?
17. Ten plums weigh as much as three apricots and one mango. Six plums and one apricot are equal in weight to a mango. How many plums balance the scales against one mango?
18. What time is it now if the time that has passed since noon constitutes a third of the time that remains until midnight?
19. One day Amy and Rea visit a library together. After that, Amy visits the library regularly every two days, at noon. Rea visits the library every three days, also at noon. If the library is open every day, how many days after the first visit will it be before Amy and Rea are, once again, in the library together?
20. A mother is six times as old as her daughter. The difference between their ages is 25 years. How old are they?

Appendix D

Spatial Thinking Attitude Survey

SPATIAL THINKING
ATTITUDE SURVEY

Spatial thinking is a combination of a person's intuition with respect to direction, distance, location, pattern and shape and the relationships among direction, distance, location, pattern and shape, as well as a person's ability to visualize and manipulate direction, distance, location, pattern and shape in space.

Please indicate the degree to which you agree or disagree with each statement below by circling the appropriate number to the right of the statement.

Strongly Disagree	Disagree	Neutral	Agree	Strongly Agree
1	**2**	**3**	**4**	**5**

1. Achievement in mathematics is directly related to spatial thinking ability.	1	2	3	4	5
2. Spatial thinking skills are important for students to be successful at the elementary school level.	1	2	3	4	5
3. I am sure that I can improve my spatial thinking abilities.	1	2	3	4	5
4. Spatial thinking skills are useful in other areas besides mathematics.	1	2	3	4	5
5. Spatial thinking skills can be developed.	1	2	3	4	5
6. I will incorporate spatial thinking activities into the classroom.	1	2	3	4	5
7. Spatial thinking skills are important in order for students to be successful in math at the high school level.	1	2	3	4	5
8. I believe that I will need to have good spatial thinking skills for my future.	1	2	3	4	5
9. There are some manipulatives that can encourage the development of spatial thinking.	1	2	3	4	5
10. I can see spatial thinking in many aspects of my daily life.	1	2	3	4	5

CONTINUED ON BACK PAGE

Please indicate the degree to which you agree or disagree with each statement below by circling the appropriate number to the right of the statement.

Strongly Disagree	Disagree	Neutral	Agree	Strongly Agree
1	**2**	**3**	**4**	**5**

11. I am confident that I can draw geometric shapes accurately.	1	2	3	4	5
12. When I am asked to picture a three-dimensional object, I have a hard time.	1	2	3	4	5
13. Manipulating shapes in my head is challenging.	1	2	3	4	5
14. I struggle drawing two-dimensional shapes.	1	2	3	4	5
15. I struggle drawing three-dimensional shapes.	1	2	3	4	5

Appendix E

Permission to Use the Purdue Spatial Visualization Test

Subject: RE: Purdue Spatial Visualization Test
Date: Monday, July 18, 2011 10:24 AM
From: George M. Bodner <gmbodner@purdue.edu>
To: Lindsay Prugh <Lindsay.Prugh@oc.edu>

Feel free to use it any way you wish.

The only information I have about the test is contained in the three PDF files I usually send people. One contains the test, as it was published. Another contains the paper on construct validity and the third is the scoring rubric.

What other information would you like?

-----Original Message-----
From: Lindsay Prugh [mailto:Lindsay.Prugh@oc.edu]
Sent: Saturday, July 16, 2011 12:13 AM
To: gmbodner@purdue.edu
Subject: Purdue Spatial Visualization Test

Hi Dr. Bodner,

My name is Lindsay Prugh and I am currently working on my doctorate at the University of Oklahoma in mathematics education. I am just beginning my dissertation and am interested in using your Purdue Spatial Visualization Test as a pre and post measure for my research in spatial thinking and problem solving. I am writing to ask your permission to use this instrument during this next school year in my Elements of Mathematics I course. Also, if you could give me some direction on where to locate your test in its entirety, I would greatly appreciate it!

Thank you so much for your time and I look forward to hearing back from you.

Respectfully,
Lindsay Prugh

Lindsay Prugh
Mathematics Instructor
Oklahoma Christian University
(405)425-5394

Appendix F

Permission to Use the Spatial Thinking Attitude Survey

Monday, July 25, 2011 8:54 AM

Subject: Re: Spatial Thinking Attitude Survey
Date: Sunday, July 24, 2011 9:01 PM
From: adele hanlon <adele@hanlons.org>
To: Lindsay Prugh <Lindsay.Prugh@oc.edu>

Hi Lindsay,

You are more than welcome to use all of or just part of theSpatial Thinking Attitude Survey. I have attached the survey. One thing I have learned about using these surveys is that students seem to underestimate/overestimate on the pre-survey and in turn this seems to skew the post-survey. Very interesting...I have found a new type of survey that I also will attach that I believe will help with this issue. Although I have not had a chance to analyze the data I collected using this survey just take a look at how this pre/post is designed and see if you like this better:)

Best of luck on your endeavors!

Adele

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Appendix G

Journal Prompts

Journal Prompts
Fall 2011

1. Thinking back on previous experiences, can you identify instances where you:
    a. Used spatial skills or spatial thinking?
    b. Were taught spatial skills?
    c. Were in a situation where spatial thinking was emphasized?

2. How would you describe your ability to think spatially?

3. Do you think spatial thinking skills are important in general? Why or why not?

4. Thinking back to previous experiences, can you identify instances where you:
    a. Were expected to problem solve?
    b. Were taught to problem solve?
    c. Were in a situation where problem solving helped you answer a question?

5. Do you think problem solving is important in everyday life? Why or why not?

6. Do you think your spatial skills have improved throughout this semester? Please explain.

7. Reflecting on the problems and activities we did throughout the semester, describe something we did in class that had an impact on your spatial reasoning skills. Further, how did it influence your spatial thinking ability?

8. Do you feel your ability to problem solve has improved throughout the semester? Please explain.

9. Reflecting on the problems and activities we did throughout the semester, describe something we did in class that had an impact on your problem solving skills. Further, how did it influence your problem solving abilities?

10. This semester you took a spatial test (PSVT) and a problem-solving test (MPI) two separate times. Discuss your confidence level taking those tests at the beginning and end of the semester.

Appendix H

Focus Group Guided Interview Questions

Focus Group Guiding Questions
Fall 2011

<u>Group Interview—Phase 2</u>

1. Ask participants for general thoughts on the PSVT and the MPI. Avoid discussing specific questions so post-measure will not be compromised.
    a. Were the questions difficult for you? Why or why not?
    b. How do you feel you performed on the PSVT?
    c. How do you feel you performed on the MPI?

2. Ask participants for general feedback on the STAS.

3. Due to the fact that the general group answer to questions four, ten, and fourteen on the STAS was "Disagree", ask the following:
    a. Do you think there are any other areas besides mathematics where spatial thinking skills are useful? Explain.
    b. Does spatial thinking ever play a role in your daily life? Explain.
    c. Do you generally find drawing 2D shapes easy or difficult? Explain.

<u>Group Interview—Phase 3</u>

1. How important is the development of spatial thinking and/or problem solving skills in your future students?
    a. Is this development your responsibility?
    b. Are these skills something you can help foster?

2. Discuss the in-class activities. Which activities do you like or dislike? Explain.
    a. Would you have benefitted from these activities, or more of these activities, in your elementary or secondary education? Explain.
    b. Do you think your future students would enjoy any of the mentioned activities?

3. Could you see yourself incorporating spatial thinking activities into your future classroom curriculum?
    a. If yes, when would you use them?
    b. If yes, how would you use them?
    c. If no, why not?

Group Interview—Phase 5

1. Ask participants for general thoughts on the second PSVT and MPI. Do NOT give students their results so that confidence levels are not altered. Specific questions may be discussed.
    a. Were the questions easier or more difficult for you the second time?
    b. How do you feel you performed on the PSVT compared with the first time you took it?
    c. How do you feel you performed on the MPI compared with the first time you took it?

2. Due to the fact that the average group response to questions two, three, four, five, six, and ten changed category, ask the following:
    a. Prompted by Q2 and Q6: Do you think spatial skills are important enough to incorporate into your future classroom? Explain.
    b. Prompted by Q3 and Q5: Do you think spatial thinking skills can be developed and/or improved? Explain.
    c. Prompted by Q4 and Q10: How and/or where are spatial skills useful in daily life? Explain.

3. Discuss general feedback on the STAS.