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This book is dedicated to my great-grandmother, “Mamaw,” who always loved me unconditionally, believed in my abilities, and supported me until the end of her journey.

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Abstract

A continued and ongoing call for improvement and change in mathematics teaching and learning suggests a need for re-visioning the ways in which mathematics teachers are educated. Suggestions include incorporating reform teaching in university courses that integrate mathematics content with pedagogy and perturbate mathematical beliefs. The context of this study was a group of preservice teachers in a mathematics content course that incorporated meaning-making, dialogue, space and justification into classroom learning experiences. Further, the usual power dynamics between teacher and student were revisited and revised as part of the social norms established in the classroom. Due to the learning experiences in this non-traditional course, students reported plans for their future pedagogical practices as being conceptually oriented, gaining mathematical empowerment, a change in beliefs about the nature of mathematics, a new appreciation for mathematics in general, and enjoyment of group work, presentations, and the use of manipulatives (term used for the use of physical models).

Chapter 1: Introduction

> If you want to understand today, you have to search yesterday. –Pearl Buck

In a world that is rampant with change, mathematics, together with science, technology, and engineering are often seen as a key to dominance in the ever-competitive world market (Friedman, 2007). This idea has dominated mathematics education for many generations—as seen by the "new math" implemented after Sputnik (1957), the "back to basics" in the seventies, and the culture of crisis surrounding mathematics education in the eighties. This "crisis" mode in the eighties that has continued on a lesser scale today was catapulted by the publication of *A Nation at Risk* in 1983 by the National Commission on Excellence in Education and *Everybody Counts* in 1989 by The National Research Council which highlighted the United States' dwindling superiority as well as the woes surrounding the current curriculum (Hofmeister, 2004; Klein, 2002; Schoenfeld, 2004).

For more than a century, a dynamic relationship has existed between developments in research, changes in curriculum, and trends in practice in the United States educational system. To this relationship can be added a fourth element, social climate, upon which many transformations were influenced. Social climate is a general term describing a phenomena whose authority can be seen as given from a variety of sources: the political arena, educational research, the various news mediums, and 'the masses' (which encompass parents, educators, and the general public). Some deviations in mathematics education occurred with no clear indication as to the role of the elements of research or practice. Since the interactions between elements of change are fundamentally dynamic, one can only hypothesize as to the level of influence of any one element, but all major modifications can be described in terms of these essentials. Often, social climate seems to be central in the interaction between research, curriculum, and practice when there are alterations in education, and mathematics education specifically (Schoenfeld, 2004).

The trends in mathematics education and practice that we see currently in the United States are a reflection of the evolution of: beliefs about the goals of mathematics education; beliefs about mathematics teaching and learning; and educational decisions that have transpired over the past century. Therefore, I will first present an overview of the history of mathematics education—beliefs and values, curriculum and policy. I will then examine recent trends in mathematics practice. Finally, I will describe the consequent influence on mathematics teachers' preparation.

Historical Background and Context

Mathematical knowledge is often seen as a way in which to further social mobility and access on the smaller scale, and a foundation for economic and military global standing on the larger scale; subsequently concern with success in mathematics always seems to be at the forefront of our nation's educational worries. This is blatantly obvious when the social and political forces that have fashioned and shaped mathematics education over the last hundred years or so are examined. Major shifts in a curriculum (and as a consequence, practice) rooted in the traditional (computational) can be seen in the sixties, seventies, eighties, and nineties. These changes are best understood in the context of the social and political culture in which they occurred. Schoenfeld (2004) noted the differences between interpretations (and themes) of the controlling forces in mathematics education by looking at the work of an anthropologist, Rosen, and a historian, Stanic, and their respective views on the development of mathematics education in the United States.

According to the anthropologist Rosen (2000), there have been three major "master narratives" during the past century of education in the United States that have shaped the educational system: education for democratic equality, education for social efficiency, and education for social mobility. Stanic (1987) stated that there were four perspectives on mathematics fighting for dominance in the nineteenth century: humanists, developmentalists, social efficiency educators, and social meliorists.

Humanists value the reason, logic, and cultural achievements associated with mathematics; developmentalists focus on mental capabilities of children (e.g. Piaget); social efficiency educators (also one of Rosen's narratives) see schooling as preparing students to fit different positions in society, could be preordained; and social meliorists, considered to be in opposition to social efficiency, see schools and mathematics as having the potential to be a great equalizer, an opportunity for social mobility and access. To Stanic's perspectives, Schoenfeld added one more; that mathematics is seen as the foundation for military and economic superiority. These identified themes of Rosen, Stanic, and Schoenfeld provide an important way to consider the history of mathematics education and understand social forces, in particular the beliefs and values behind them, that shape education—policies, curriculum, and practice.

In the early years of the formal education system in the United States, “education for the masses” meant mainly elementary school since, in 1890, less than 7% of 14-year-olds were enrolled in high school. Therefore, the mathematics taught to the majority of students in the system was rather basic (Rippa, 1988; Saracho & Spodek, 2009; Schoenfeld, 2004). From that time until the beginning of World War II, the education system had a major growth in population which created tremendous pressure in the system with almost 75% of 14 to 17-year-olds attending high school; from 1910 until 1930 there was a 400 percent increase in high school enrollments (Rippa, 1988; Schoenfeld, 2004). Despite this influx of students, whom were largely unprepared and diverse compared with past students, and embarrassing complaints made by the army about potential officer candidates and navy candidates’ paucity of mathematical skills (Klein, 2002; Rippa, 1988; Schoenfeld, 2004), no immediate major changes were made to curriculum to address these facts (Schoenfeld, 2004).

One of the foremost changes to curriculum occurred in the early sixties after the launch of Sputnik in October of 1957 created worries that the United States was falling behind in mathematics and science (Klein, 2002; Hofmeister, 2004; Schoenfeld, 2004). Therefore, with support from the National Science Foundation (NSF), "new math" with "modern" content was created and implemented: the "new math" curriculum had fresh content embedded in the form of set theory, modular arithmetic, and symbolic logic (Becker & Jacob, 2000; Klein, 2002; Schoenfeld, 2004). Unfortunately, the public as a whole (teachers and parents included) were not well-educated about this change. Consequently, many teachers were uncomfortable with the new curriculum and many parents did not understand its usefulness. As a result, new math "died" by the early seventies (Klein, 2002; Schoenfeld, 2004). Thus, the "back to basics" curriculum shift of the seventies was in response to the excessive curriculum of the sixties (Klein, 2002; Schoenfeld, 2004).

Although the curriculum of the seventies was based on the original mathematics curriculum (focused on skills and procedures), by the eighties it was clear that this "back to basics" curriculum was unsuccessful as well (Klein, 2002; Schoenfeld, 2004). As a result, in 1980 the National Council of Teachers of Mathematics (NCTM) published *An Agenda for Action* which stated that exclusive teaching of back to basics was wrong and that a primary goal of mathematics teaching should be an emphasis on problem-solving.

The reason that the public's attention was drawn to mathematics education in the eighties was due to the downfall of the United States economy compared with Japanese and Asian economies. Furthermore, the publication of *A Nation at Risk* in 1983 by the National Commission on Excellence in Education and *Everybody Counts* in 1989 by The National Research Council which highlighted the presumed dwindling of the United States' superiority as well as the woes surrounding the current curriculum (Hofmeister, 2004; Klein, 2002; Schoenfeld, 2004) drew public attention. Additional contextual factors such as the cognitive revolution, paltry scores on the Second International Mathematics Study, and the fact of the two-part planned coordinated release of *Everybody Counts* by the National Research Council in the spring of 1989 followed by the *Curriculum and Evaluation Standards for School Mathematics* in the fall of 1989 by the National Council of Teachers of Mathematics (Hofmeister, 2004; Klein, 2002; Schoenfeld, 2004) contributed to the perceived need for a reform curriculum that would address the “crisis” in mathematics education so broadcasted during the eighties (Hofmeister, 2004; Rippa, 1988; Schoenfeld, 2004).

Publishers responded by making trifling problem-solving editions of their texts which consisted of embedding a problem-solving section at the end of each chapter (Schoenfeld, 2004). The reform curriculum implemented in the early nineties was based upon research on problem solving and constructivism that was mainly unfinished (although the basic theory was sound). As with the original “new math,” the public was largely uninformed as to the methodology or theory supporting the curriculum. The textbooks’ format were unfamiliar and inaccessible to parents thus dooming it to be labeled as impractical and “fuzzy” (Rosen, 2000; Schoenfeld, 2004). It was eventually ridiculed and called the “new-new math” in reference to the failure of reform texts in the sixties. The jump to produce new “reform” texts without allowing the full magnitude of research into problem-solving and constructivism to guide the natural progression of influencing texts and practice had disastrous consequences on the move to implement anything reform-oriented for the next time period. This seemed to begin a battle in mathematics education between traditionalists and reformists over curriculum (and consequently instructional practices) called the "Math Wars." This battle primarily raged in California and perhaps a large part of the heatedness can be explained by the social climate of the eighties surrounding mathematics but, it is still ongoing in many educational settings (Schoenfeld, 2004).

Traditionalists believe that the classroom should be run in a lecture-based teaching style with emphasis on skills and procedures, rather than conceptual understanding, whereas reformists believe that a holistic view of mathematics is important with a focus on conceptual understanding that is more than simply rote memorization of facts and practicing algorithms using computation (Schoenfeld, 2004; Wheatley & Abshire, 2002). Although some educators are stanch traditionalists, with new research into the ways in which people learn supporting the tenets of reform, most mathematics educators align with the reformists' views of teaching and learning mathematics. Thus, revising teaching practices often comes along with reform curriculum according to the 'normal' progression of mathematics education.

The developments during the seventies and eighties regarding research into understanding how learning occurs, along with the enactment of NCTM's *An Agenda for Action* in 1980 and the *Curriculum and Evaluation Standards for School Mathematics* in 1989, paved the way for alternative, or "reformed", ways of instruction (practice) that primarily involved problem-solving. The foundation of the reform movements in mathematics education can be viewed as a shift in orientation from a procedural to a conceptual orientation. *Procedural*, in this context, refers to computational methods and algorithms and the procedure for solving is typically shown to students by the teacher. Further, the expectation is that all students solve the problems in the same way along with the belief that there is one best way to solve certain problems. *Conceptual*, in this context, refers to a focus on mathematical concepts rather than computations; students are encouraged to construct meaning and their own methods for solving problems and sense making is the overall objective.

A classroom practice that has been shown to aid students in building conceptual understanding of mathematics is quite different from typical procedurally-driven classroom practice. Rather than focusing on memorization and algorithmic computation, teachers encourage students to focus on connections, sense-making, and problem-solving (Wheatley & Abshire, 2002; Wheatley & Reynolds, 1999; Reynolds, Fleener, Wheatley, & Robbins, 2004). While there have been many “reform” movements throughout our nation’s educational history, the current reform is based upon knowledge of the ways in which children learn (Schoenfeld, 2004).

The change in orientation and practices reflected in the current reform movement stems from a belief that “knowledge originates in a learner’s activity performed on mental constructs which are directly related to the action and experience of that learner” and “that learning occurs when an individual adapts his or her functioning schemes to cope with a problematic situation” (Lo & Wheatley, 1994, p.146) which is a conviction resting upon constructivism. This change of direction (from procedural to conceptual) resulted in our current trends in classroom practice that are based upon a belief in constructivism and that learning mathematics is best facilitated with a focus on problem solving.

Problem-centered learning, as it applies to mathematics, is a theory of learning centered on the belief that learning mathematics is best facilitated by solving problems rather than rote memorization of facts and procedures (Wheatley, 1991); the belief that "knowledge is not acquired but constructed by the individual as he or she solves problems" (Wheatley & Abshire, 2002, p.3). Wheatley and Abshire (2002) believe that the more traditional methods of instruction do not support or encourage students' building of inter-connected mathematical ideas based upon prior knowledge. Thus their thought processes concerning solving mathematical problems become debilitated over the long run. Problem-centered learning as a classroom model is applicable to a variety of subjects due to its methods of provocative questioning, highlighting a paradox, new perspectives, focus on incomplete information, or posing a dilemma as the "problem" around which instruction is centered (Adams & Burns, 1999; Dooley, 1997). In the practice of mathematics classrooms, problem solving is a significant trend supported by reformers, NCTM, and policy makers.

The numerous changes in curriculum and practice "in the spirit of reform" in the last two decades, point to a change in beliefs. Reform may represent a change in understanding about how people best learn mathematics as well as change in beliefs about the preeminent methods in which to facilitate the learning of mathematics.

Rationale

Beliefs are formed through the process of enculturation and are socially constructed; belief formation often occurs in formal mathematics education classes (Anderson & Piazza, 1996; Ball, 1988; Pajares, 1992; Philipp, 2000) in which beliefs

> evolve as individuals are exposed to the ideas and mores of their parents, peers, teachers, neighbors, and various significant others. They are acquired and fostered through schooling, through the informal observation of others, and through the folklore of a culture, and they usually persist, unmodified, unless intentionally or explicitly challenged. (Lasley, 1980, p. 38)

Because beliefs are socially and contextually constructed, many preservice teachers' views of teaching mathematics are consistent with the ways in which they experienced mathematics learning (Ball, 1990; Cooney, 1999). For many preservice teachers, the beliefs they bring with them are created from an "apprenticeship of observation" (Anderson & Piazza, 1996) during their many years of schooling

(Ball, 1988; Ball, 1996; Calderhead & Robson, 1991; Philipp, 2000). Numerous beliefs about the nature of mathematics held by preservice teachers have emerged from formal mathematics education experiences which, taken together with their 'apprenticeship' and early formation has the effect of resistance to change during teacher education which consequently influences practice (Calderhead & Robson, 1991; Chapman, 2002; Philipp et al., 2007; Stuart & Thurlow, 2000).

Views about the nature of mathematics form a basis for mental models of mathematics teaching and learning (Ernest, 1989) whereas beliefs about mathematics and mathematics teaching play a subtle but significant role in the shaping of behavior (Cooney, 1985). Raymond (1997) found that preservice teachers' practices are more consistent with beliefs about mathematics than with beliefs about teaching and learning. She suggested that "deeply held, traditional beliefs about the nature of mathematics have the potential to perpetuate mathematics teaching that is more traditional, even when teachers hold nontraditional beliefs about mathematics pedagogy" (p. 574).

Additionally, perturbations (pedagogical conflicts) played a significant role in changing beliefs, along with the desire to resolve the cognitive dissonance (Chapman, 2002; Gregoire, 2003; Middleton, 2002). Most new ideas are assimilated rather than accommodated because the lack of a challenge to beliefs in learning experiences, in which the new information is encountered, does not require reflection upon existing schema about conceptions (Lasley, 1980; Philipp, 2000).

The relationship between beliefs about mathematics, the context in which many beliefs are formed (mathematics educational experiences), and the need for challenging situations coupled with reflection in order for beliefs to change point to a new venue for future research. Cooney (1999) calls for the integration of content and pedagogy to ease curricular problems in mathematics teacher education in order to "influence teachers' ways of knowing so as to promote a more reflective orientation toward teaching" (p. 175). Raymond (1997) posits that preservice teachers' "early and continued reflection about mathematics beliefs and practices" (p. 574) in teacher education classes may be the key to change in teacher beliefs. Wilson and Cooney (2002) point to the need to rethink the separation of mathematical and pedagogical beliefs in research. Thompson (1984) elaborates the fact that research into what role preservice teachers conceptions of mathematics might play in their teaching practices has principally been ignored.

A report put out by the American Mathematics Society, *The Mathematical Education of Teachers* (CBMS, 2001), made recommendations for changes in the preparation of preservice teachers such as "College courses...should make connections between the mathematics being studied and mathematics prospective teachers will teach" (p. 7) with a call to rekindle preservice teachers mathematical thinking. They listed as first priority "classroom experiences in which *their* ideas for solving problems are elicited and taken seriously, their sound reasoning affirmed, and their missteps challenged in ways that help them make sense of their errors (p. 17). Additionally, Burnaford, Fischer, and Hobson (2001) point out that in the subject of educational research, few studies are of university teaching (compared with high school) and that "it also appears that university teachers are not known for using in their own teaching the practices they urge on teachers" (p. 109). Thus, although change is called for in teacher preparation programs and although “reform” is touted to future teachers as a model for classroom instructional experiences, little of either is seen in teacher preparation classes in the university setting.

In summary, research reveals the following: beliefs about mathematics and mathematics pedagogy are socially constructed through an apprenticeship of observation, perturbations during learning experiences coupled with reflection bring about the most change in beliefs (solely observing a new teaching practice will not bring change), preservice teachers’ practices are more consistent with beliefs about mathematics than with pedagogical beliefs, a call for change in preservice teacher preparation, and a lack of research into the role of mathematical beliefs in preservice teachers’ teaching practices.

Therefore, the need to consider the connections between mathematical beliefs and mathematical pedagogy in mathematics content classes in a university setting is important. Consequently, the following research project that encompassed challenging preservice teachers’ beliefs about mathematics and mathematics teaching and learning while asking them to reflect upon their beliefs, was proposed. The study took place with students in a mathematics classroom that integrated content and pedagogy as Cooney (1999) suggested, and whose structure was consistent with reform teaching practices as opposed to a traditional university mathematics classroom structure.

The design of the course incorporated a view of preservice teachers as social constructors of knowledge—as entering the teacher education program with preconceived beliefs (and knowledge) about mathematics and mathematics teaching and learning formed through an apprenticeship of observation during their formal education. Instruction was situated among a reform model based upon conceptual rather than a procedural orientation with a focus on meaning making, connections, patterns, justification, and dialogue. Because the relationship between reflection and perturbations are vital to change in teacher beliefs, I sought to design perplexing classroom experiences which evolved throughout the course of the study. My goal through this course was to provide an opportunity for a new kind of "apprenticeship of observation", to develop "teachers' ability and their desire to think seriously, deeply, and continuously about the purposes and consequences of what they do—about the ways in which their curriculum and teaching methods, classroom and school organization, testing and grading procedures, affect purpose and are affected by it" (Silberman, 1970, pg. 472) as well as reflect on their own belief systems. A concerted effort was made to establish social norms in the classroom that supported and encouraged discourse, investigation, and questioning. I sought to agitate the students' beliefs about mathematics and mathematics teaching and learning through the structure of the class (learning experiences, social norms, etc.). Additionally, I incorporated writing assignments that addressed beliefs (although not always explicitly stated).

Purpose

The purpose of this study was to describe the characteristics of a non-traditional mathematics content course for preservice teachers and to describe their perceptions about the impact of such a course on their beliefs about mathematics, beliefs about mathematics pedagogy, and mathematical empowerment as told from their perspective. This study also sought to intentionally explain the participants' experiences from their perspective as much as possible in the vein of their own words, oftentimes using their words in the descriptions.

Research Questions

The questions this study aimed to answer were as follows:

- What are the characteristics of a mathematics content course for preservice teachers taught from a non-traditional orientation?
- From the preservice teachers' perspective, what impact, if any, does participation in this course have on mathematical beliefs?
- What influence, if any, do preservice teachers believe the curriculum and structure of this class have on their empowerment?

The results of this study contribute to the literature on the preparation, mathematically and pedagogically, of preservice teachers. Additionally, it specifically adds to the literature on preservice teachers learning mathematics through classroom experiences based upon a non-traditional conceptual format based upon constructivist learning theory. Lastly, it adds to the current literature on preservice teachers' beliefs about mathematics and mathematics pedagogy.

Assumptions

1. Participant's responses during classroom experiences, writing assignments, and questionnaires were thoughtful, thorough, and complete and were not influenced by their perceptions of the instructors' beliefs.
2. The instructor (and investigator) had a positive attitude towards mathematics as well as had a high teaching self-efficacy towards mathematics.

Limitations

1. The sample of participants was a sample of convenience. The participants of this study were students from a small four-year college located in a community comprised of approximately seventeen-thousand people in the southern Midwest region of the United States; all were preservice teachers enrolled in a mathematics content course intended for early childhood and elementary majors. Thus, the findings may not be generalizable to the general population of all preservice teachers.
2. The participants were largely female and of Caucasian ethnicity.

3. As the principal investigator, I had previously taught this course and as a consequence brought with me to the study preconceived notions about the characteristics of the course as well as the types of students enrolled in the course. Additionally, although the development of the course changes along with my research study, I also had preconceived notions about the impact the study might have upon my participants.

Organization of the Study

This study, which comprises my dissertation, is organized into a five chapter format. Chapter one provides the history and background, rationale, questions, assumptions, limitations, and organization of my study. Likewise, it provides a general overview of my reasoning for my study as well as situates it in the literature. Chapter two presents a review of the relevant literature associated with my study. Chapter three describes the research methodology, participants, design, data collection procedures, and setting for the study. Lastly, chapters four and five examine and illustrate the findings of the research study as well as present conclusions and recommendations.

CHAPTER TWO: REVIEW OF THE LITERATURE

Throughout this study the organization, analysis, and results, numerous key ideas were at play. This study is situated in the literature related to the following topics: Constructivism, Problem Solving, Beliefs, and Empowerment. Empowerment came to the forefront of the study during the analysis of the findings stage. Therefore, it was added to the literature later in an effort to develop a complete and thorough background for the study in the literature.

Constructivism

Constructivism is a theory of learning based upon cognitive psychology, educational research, and neurological science (Adams & Burns, 1999) in which the learner is viewed as actively constructing knowledge in ways that seek coherence and organization (Mayer, 2004). According to constructivist theories of learning, learning is an adaptive activity situated in the context where it occurs and constructed by the learner (Boethel & Dimock, 1999). Furthermore, learning is internally controlled and mediated; knowledge is constructed in multiple ways through a variety of tools, resources, experiences, and contexts; learning is a process of accommodation, assimilation, or rejection to construct new conceptual structures, meaningful representations, or new mental models; learning is both an active and reflective process (Adams & Burns, 1999). In other words, knowledge is constructed (Adams & Burns, 1999; Boethel & Dimock, 1999; Davis, 2004; Mayer, 2004; Wheatley & Abshire, 2002). Under the wide umbrella of constructivism fall the categories of social constructivism or constructionism, trivial constructivism, and radical constructivism (Davis, 2004).

The nuances between the "different" types of constructivist theories of learning are minute thus it is challenging to distinguish absolutely the differentiating line separating them. Broadly, however, there are some general definitions. Radical constructivism tenants that there is no absolute "Truth" since truth is relative to each individual. It is a theory of knowing versus knowledge in which one's knowledge is never exactly the same as another individual, and because of this, no one has access to the true world of reality (Goldin, 1990). Social constructivism asserts that there is an element of the social in meaning making and constructing knowledge. Thus, the constructions of meanings are always socially constructed— nothing that is "known" is without a social element in constructing that knowledge. For example, a given classroom may have a shared understanding of a mathematical concept due to the construction together of the knowledge acquired.

Some of the major theorists of constructivism are Piaget, Vygotsky, Bruner, von Glasersfeld and Dewey (Davis, 2004; Dimitriadis & Kamberelis, 2006; Houser, 2006; Steffe & Kieren, 1994). It is a generally-held belief that Piaget was the first theorist to lay out the tenets of what we call constructivism today (Davis, 2004; Dimitriadis & Kamberelis, 2006; Houser, 2006; Steffe & Kieren, 1994) followed by Vygotsky's work which looked at how learners incorporate into the body politic which is some of the early workings of social constructivism or constructionism (Davis, 2004). Bruner's influences on the development of the theory of constructivism can be seen in his concept of readiness as well as discussions about knowing as doing (Bruner, 1996; Steffe & Kieren, 1994). Dewey's influence can be seen in his notion of reflective inquiry when discussing thinking as it relates to spectators, inquirers, and the traits of inquirers (Boisvert, 1998; Hiebert et al., 1996). Finally von

Glasersfeld is seen as being the main developer of trivial constructivism which evolved from Piaget's radical constructivism and encompasses ideas about the relationships between the learner and reality as well as the roles of action and reflection in knowledge acquisition (Steffe & Kieren, 1994).

Although constructivism is only a theory of learning, many educators mistakenly believe that it is also a classroom model (Adams & Burns, 1999). Educators often misconstrue constructivism's tenet of learning as active into meaning that in order to learn students must be active physically. Therefore, the idea that teaching methods must also be activity based (learner is behaviorally active) is a common belief among educators (Mayer, 2004). While this notion is not entirely correct, pedagogic practices can in fact try to implement the views of learning and knowledge acquisition purported by constructivism. Hence, after the publication of the *Curriculum and Evaluation Standards for School Mathematics* in 1989 by NCTM, many educators felt that the best way to teach mathematics was with problem solving. However, what problem solving entails as far as classroom practice (or classroom learning experiences) remains a matter of opinion among mathematics educators.

Problem Solving

Descriptive terms for classroom models based on problem solving are as follows: *problem-centered learning experiences, problem-centered classroom, problem-centered instruction, case-based instruction and problem-based instruction* as well as loosely "discovery" and "inquiry-based" teaching methods (Dooley, 1997; Hiebert et al., 1996; Kirschner, Sweller, & Clark, 2006; Mayer, 2004; Merrill & Gilbert, 2008; Wheatley & Abshire, 2002). Problem-centered learning can also be viewed as a model of instruction centered on ill-structured problems (Dooley, 1997) in which solving a problem is seen as key to the acquisition of knowledge. There is some debate as to the details of this type of classroom model as seen by varying descriptions found in academia and the literature. Problem-based learning as a classroom model is applicable to a variety of subjects due to its methods of provocative questioning, highlighting a paradox, new perspectives, focus on incomplete information, or posing a dilemma as the "problem" around which instruction is centered (Adams & Burns, 1999; Dooley, 1997). In the practice of mathematics classrooms, problem solving is a huge trend supported by reformers, NCTM, and policy makers. It is seen as a way to help students become more competent and our nation to become more competitive globally. Unfortunately, the term "problem solving" for classroom practice can be interpreted and utilized in ways that are procedurally focused as well as conceptually focused. Traditional teachers can implement more worksheets, problems of the day, and more end of the chapter problems—all done with little or no discussion, reflection, or sense-making—and call his classroom a "problem-centered" learning environment. Contrast this with the problem-centered learning model developed by Grayson Wheatley (1991) upon which is based the belief of learning found in constructivism in that it strives to keep the learner actively engaged—through tasks, collaboration, and presentation (Boethel & Dimock, 1999; Wheatley, 1991; Wheatley & Abshire, 2002).

Problem-centered learning, as it applies to mathematics, is a theory of learning centered on the belief that learning mathematics is best facilitated by solving problems instead of rote memorization of facts and procedures—the belief that "knowledge is not acquired but constructed by the individual as he or she solves problems" (Wheatley & Abshire, 2002, p.3). Wheatley (1991) believes that the more traditional methods of instruction can bypass students' building of inter-connected mathematical ideas based upon prior knowledge. Thus their thought processes concerning solving mathematical problems becoming debilitated in mathematics over the long run.

Failing to make connections between ideas in mathematics is crucial because accessing knowledge in a competent and well-organized manner leads to the solution of problems (of course ingenuity also helps). Lack of success in problem solving is most often due to the fact that students are not using their resources of time and past knowledge efficiently (Schoenfeld, 2004). This is because they have failed to create connections between concepts. Consequently, traditional classrooms only tend to give students a strong knowledge base without students also acquiring strategies, metacognition, or positive beliefs about the mathematical enterprise (Schoenfeld, 2004). Problem-centered learning theorists see mathematics as a set of patterns and relationships instead of just a set of rules as the traditionalists view mathematics. Therefore, because mathematics is about reasoning and reflection about relationships, solving problems can be seen as similar to finding one’s way around a park that has many trails. In other words, there are various paths to the same solution—some are more efficient in getting to a location and all of them are interconnected (Wheatley & Abshire, 2002, p.3). Therefore, supporters of problem-centered learning feel that in the process of solving problems, students will develop strategies as well as interconnections between relationships, and along the way will start seeing mathematics in a positive light.

Problem-centered learning can be broken up into two main categories based upon the amount of guidance involved—minimally guided versus guided. Problem-based instruction has minimal guidance whereas problem-centered instruction provides guidance in the form of carefully sequenced and well-thought out problems in which students are taught some component skills (Merrill & Gilbert, 2008; Wheatley & Abshire, 2002). This minimal guidance approach can be seen as the early version of the problem-centered learning classroom model and has many other names such as discovery learning, inquiry learning, and experiential learning (Kirschner, et al., 2006). The guided approach of problem-centered instruction is what has developed over the years as more research has been conducted on the cognitive architecture of the structures and functions of working memory, both short and long-term (Kirschner, et al., 2006). Also key to this change in classroom models is the transition of the “focus” of the mathematics classroom from an emphasis on problem solving as an end itself into problem solving as a way in which to learn mathematical content and processes as seen by NCTM’s *Curriculum and Evaluation Standards for School Mathematics* (Lubienski, 2000).

Even though it has been shown that some guidance is needed in a problem-centered learning instructional environment, the question remains how much guidance is appropriate? The goal is to provide students adequate information because too much guidance can impair later performance (do to the fact that they haven’t constructed information, merely memorized it) while too little can inhibit effective learning of strategies as well as linking of concepts (Kirschner, et al., 2006). The “guide on the side” view of the teacher in the classroom is what is currently accepted as being most conducive to meeting the *Curriculum and Evaluation Standards for School Mathematics* put forth by NCTM of having a constructivist methodology of teaching (White-Clark, DiCarlo & Gilchriest, 2008). Although this does not specifically address how much guidance to give, common sense dictates that "students need enough freedom to become cognitively active in the process of sense making, and students need enough guidance so that their cognitive activity results in the construction of useful knowledge" (Mayer, 2004, p.16). Thus, teaching in a problem-centered learning environment is difficult because the teacher must be mindful of students' past knowledge base and current understandings all the while keeping in mind curriculum goals.

Reform Movements

Reform, in mathematics education, is a general term that can be used to describe changes in curriculum, instruction, and policy (or standards). For the United States' educational system, there has existed a multi-faceted and complicated relationship between research developments, curriculum changes, trends in instructional practices, and social climate. Social climate can be seen as central to the interactions of the other three.

The launch of Sputnik in 1957 caused worries about the capabilities of the United States to compete with other nations; furthering mathematics and science was seen as the way in which to ensure domination. Curriculum changes were made (in the 60's) because of this social climate of fear, but implementation was limited because information about "what" and "why" was lacking consequently leading to teacher and parent uncomfortability with the "new math" (Schoenfeld, 2004). Consequently, practice was primarily unchanged. Research did not seem to play a huge role in either the development or implementation of the curriculum. A similar story was played out in the seventies.

Research in mathematics education during the late sixties and early seventies mainly focused on student learning and cognitive development. Major theories of learning, such as constructivism, were emerging as well as ways in which to best facilitate students' construction of knowledge. During the eighties (and early nineties), research primarily concentrated on sense-making, student ability, student understanding, as well as attributes, attitudes, and processes (Hoyles, 1992; Wilson & Cooney, 2002). Additionally, student beliefs were studied and the impact of those on performance. Research found that mathematical self-efficacy (belief in ones abilities for a specific area or topic) has a strong effect upon individuals' effort, choices, and perseverance (Bandura, 1986; Hackett, 1985; Hackett & Betz, 1989; Lent & Hackett, 1987) and on performance (Meece, Wigfield & Eccles, 1990; Lent & Hackett, 1987; Fennema & Sherman, 1977). Little emphasis in any of this research was given to the teacher—to the influence she might have on learning through classroom practices; she was primarily viewed as a facilitator—a dispenser of information, materials, strategies, and grades (Hoyles, 1992; Wilson & Cooney, 2002).

Cognitively-focused research, along with societal worries such as US competence globally after the Second International Mathematics Study showed low math scores compared with other countries, led to curriculum development that focused on problem solving and learning mathematics conceptually. This new-new math curriculum was implemented in the early nineties (Becker & Jacob, 2000; Klein, 2002; Schoenfeld, 2004). Implementation of the reform curriculum necessarily called for changes in teacher practice (Schoenfeld, 2004). The curriculum summoned a turn away from traditional teaching practices that focus on procedures in a “show and tell” manner to reform teaching practices that focus on meaning making and conceptual learning. Reform teaching practices are built upon the foundation of a constructivist view of learning in which learning is an adaptive activity situated in the context where it occurs and constructed by the learner (Boethel & Dimock, 1999).

Implementation of the majority of reform curriculums during the eighties and nineties did not go well; teaching practices did not change to support the new curriculum and parents were concerned about a curriculum that was inaccessible to them because they did not recognize nor understand it (Becker & Jacob, 2000; Klein, 2002; Rosen, 2000; Schoenfeld, 2004). Debate over the changes in mathematics education, in particular the potential effect on student learning, led to the “Math Wars” between traditionalists and reformists (Klein, 2002; Schoenfeld, 2004). Coinciding with this same time period, research in the late eighties and nineties transitioned from a focus solely on student knowledge, thinking, and learning—namely, cognition—to attention centered on the impact of the teacher on the learning process (Schoenfeld, 2004;

Hoyles, 1992; Wilson & Cooney, 2002). The context in which learning occurred was seen as important and influential to learning—the teacher's behavior was finally appreciated as an important factor in the learning process. Research then transitioned from a look at teacher behavior to teacher cognition with the realization that beliefs about mathematics and pedagogy were critical to student learning, reform teaching, and curriculum implementation (Hoyles, 1992; Lloyd, 1999; Philipp, 2000; Wilson & Cooney, 2002).

All of the developments in ways of understanding how learning occurs during the seventies and eighties, along with the enactment of NCTM's *An Agenda for Action* in 1980 and the *Curriculum and Evaluation Standards for School Mathematics* in 1989, paved the way for alternative, or "reformed," ways of instruction (practice) that primarily involve problem-solving. The reform movements' foundation for mathematics education can be viewed as a shift in orientation from a procedural to a conceptual orientation. *Procedural* refers to computational methods and algorithms in which the procedure for solving is typically shown to students by the teacher. The expectation is that all students solve the problems in the same way and the belief is that there is one best way to solve certain problems. *Conceptual* refers to a focus on mathematical concepts rather than computation; students are encouraged to construct meaning and their own methods for solving problems. Sense-making is the overall objective. This change of direction (from procedural to conceptual) resulted in our current trends in classroom practice that are based upon a belief in constructivism and that learning mathematics is best facilitated with a focus on problem solving.

A classroom practice that aids students in building conceptual understanding of mathematics is quite different from procedurally-driven classroom practice. Instead of focusing on memorization and algorithmic computation, teachers are encouraging students to focus on connections, sense-making, and problem-solving (Wheatley & Abshire, 2002; Wheatley & Reynolds, 1999; Reynolds, Fleener, Wheatley, & Robbins, 2004). While there have been many so-called "reform" movements throughout our nation's educational history, the current reform is based upon knowledge of the ways in which children learn (Schoenfeld, 2004). The change in orientation and practices stems from a belief that "knowledge originates in a learner's activity performed on mental constructs which are directly related to the action and experience of that learner" and "that learning occurs when an individual adapts his or her functioning schemes to cope with a problematic situation" (Lo & Wheatley, 1994, p.146). This belief system rests upon constructivism.

It remains unclear which was most powerful or which came first—problems with implementing reform curriculum or research into teachers' influence on the learning environment of students. Like most periods in education, the relationships between social context, research, curriculum, and practice remain intricate and with no known causality as far as change. What is most prominent to note is that teacher beliefs came to be seen as a major force in mathematics education, an added dimension to the already murky relationships affecting learning environments.

Beliefs

The topic of teacher beliefs related to mathematics encompasses beliefs about the nature of mathematics, the teaching and learning of mathematics, and the goals of mathematics education. These beliefs serve as a critical filter and influence perceptions—the way in which the world is interpreted (Philipp, et al., 2007; Pajares, 1992; Grant, Hiebert, and Wearne, 1998). They are formed through the process of enculturation and are socially constructed—many times in formal mathematics education classes (Anderson & Piazz, 1996; Ball, 1988; Parjares, 1992; Philipp, 2000) and evolve as individuals are exposed to the ideas and mores of their parents, peers, teachers, neighbors, and various significant others. They are acquired and fostered through schooling, through the informal observation of others, and through the folklore of a culture, and they usually persist, unmodified, unless intentionally or explicitly challenged (Lasley, 1980, p. 38).

The literature on the subject covers approximately the past thirty years and is quite extensive. The following review of the literature will include an examination of the various definitions of teacher beliefs, a discussion of the role of knowledge in relation to beliefs and a description of two conceptual frameworks for beliefs. Finally, some specific findings concerning teacher beliefs about mathematics, the role of context, and the influences of beliefs and context on practice will be examined.

Definition?

As with most topics found in any field, there is no general consensus on the definition of beliefs, conceptual framework for the structure of interactions, or levels of intensity (Cooney, 1999; Pajares 1992; Philipp, 2000; Torner, 2002; Wilson & Cooney, 2002). Some of the various names for beliefs or descriptors of belief structures are as follows: affect, emotions, attitudes, belief systems, conception, identity, knowledge, value, judgments, axioms, opinions, ideology, perceptions, conceptual systems, preconceptions, dispositions, implicit theories, explicit theories, personal theories, internal mental processes, action strategies, rules of practice, practical principles, perspectives, repertories of understanding, and social strategy (Pajares, 1992; Philipp, 2000). Authors use different names for descriptors of basically the same construct (with varying levels). Most commonly the terms beliefs, knowledge, and concept(tions) are used synonymously, with occasional distinctions between terminologies pointed out. In order to avoid confusion, the word beliefs will be used solely in place of the other terms found in the literature. However, perhaps the biggest differentiation made in the mathematical theoretical literature was the difference between knowledge and beliefs thus this distinction will be further examined.

Knowledge

Beliefs and knowledge are intricately related—on this much the research agrees—but the hierarchy and interrelations that characterize the complex relationship are not agreed upon. Even though there are no general definitions, the overall consensus of the literature supports that knowledge is considered set and beliefs are subject to change depending on the situation of the belief (primary, derivative) in the belief structure and the context of the situation (Chapman, 2002; Cooney, Shealy, & Arvold, 1998; Ernest, 1989; Hoyles, 1992; Pajares, 1992; Philipp, 2000; Raymond, 1997; Skott, 2001; Sztajn, 2003). In particular, the only contradiction I found was that Nespor (1987) characterized knowledge as most malleable in relation to change whereas beliefs were more inflexible (despite being disputable due to their non-evidential nature) and that when change does occur, it happens not because of reason but instead from a "conversion or gestalt shift" (p. 321). Although the transition between knowledge and beliefs is hard to distinguish, the literature does reveal some ways in which to differentiate them.

Beliefs are held with varying levels of intensity or conviction whereas knowledge is not typically thought of or expressed in that way (Green, 1971; Philipp, 2000). Knowledge is belief held with certainty; it is purer with a true-false component (Pajares, 1992; Philipp, 2000). Knowledge is also consensual (general agreement can usually be reached) and warranted (evidential in nature) whereas people hold varying beliefs that cannot be disproved because they are deeply personal in nature as well as non-evidentially based; consequently they are unaffected by attempts at persuasion to the contrary (Nespor, 1987; Pajares, 1992; Philipp, 2000). Philipp (2000) in his review of beliefs and affect noted that he found it helpful to think of a conception as *belief* when a person could respect a position to the contrary as reasonable and intelligent and *knowledge* when a person could not respect a position to the contrary as reasonable and intelligent. Since knowledge has a belief component in addition to a cognitive component, it is as vital to know *how* a person holds a conception as knowing *what* the person holds as a conception because this can further understanding about the capacity of change (Chapman, 2002; Philipp, 2000). Consequently, a structure for beliefs is imperative for understanding *how* a person holds a belief in relation to other conceptions in order to be able to adequately describe how changes in beliefs occur.

Belief System

Although there are various theoretical structures for teacher conceptions, those postulated by Green (1971) and Rokeach (1960, 1968) were most represented in the literature along with adding to them in the body of knowledge of relations among beliefs. Green (1971) described a conceptual framework for beliefs and their relationships to each other in which a belief system is composed of three dimensions: First there is the quasi-logical relation between beliefs. They are primary or derivative. Secondly, there are relations between beliefs having to do with their spatial order or their psychological strength. They are central or peripheral. But there is a third dimension. Beliefs are held in clusters, as it were, more or less in isolation from other clusters and protected from any relationship with other sets of beliefs. Each of these characteristics of belief systems has to do not with the content of our beliefs but with the way we hold them (pp. 47-48).

The quasi-logical relationship between beliefs is quite different from psychological strength (central or peripheral). For instance, a teacher can believe that mathematics should be taught with meaning making as a goal (primary belief) and therefore feel that students should be allowed to use manipulatives (derivative belief). But if the psychological strength of the belief is not strong (peripherally-held rather than central/strongly-felt), then when faced with the usual constraints of the classroom environment, such as lack of time, the commitment to using manipulatives can fade swiftly. Thus, the effect of beliefs held in clusters, isolated from each other, is the avoidance of conflict between belief structures; note that because of their isolation and consequent lack of confrontation, they can appear contradictory or inconsistent with each other (Philipp, 2000).

Also important to Green was the role of evidence in how a belief is held. He argued that a belief not founded on evidence was impervious to change (even when confronted with reason or evidence to the contrary) but a belief founded on evidence can change with reflection. Green (1971) described the difference between nonevidentially and evidentially held beliefs as follows:

> When beliefs are held without regard to evidence, or contrary to evidence, or apart from good reasons or the cannons for testing reasons and evidence, then I shall say they are held nonevidentially. If follows immediately those beliefs held nonevidentially cannot be modified by introducing evidence or reasons; they cannot be changed by rational criticism. The point is embodied in a familiar attitude: "Don't bother me with facts; I have made up my mind." When beliefs, however, are held on the basis of evidence or reasons, they can be rationally criticized and therefore can be modified in the light of further evidence or better reasons. I shall say that beliefs held in that way are held evidentially. (p. 48)

Thus, the foundation for the formation of beliefs is significant when considering how beliefs are modified (Cooney, et al., 1998).

Chapman (2002) confirmed and furthered this conceptual framework in his study of inservice high school mathematics teachers. He found that perturbations (pedagogical conflicts) between the teaching act, teacher's expectations or intentions, and the outcomes of the teaching act played a significant role in changing beliefs, along with the desire to resolve them. The additional findings that the structure and content of the mathematical beliefs seemed to influence both the conflicts and resolutions were believed to confirm Green's theory of primary and peripheral beliefs. Chapman added to the framework by making a schematic representation of the relationships that seemed important to the evolution of participants teaching practice. He posited that the psychological strength of primary beliefs about mathematics most likely influenced desire and persistence to have teaching practices reflect those beliefs but that this was not sufficient to implement change. The primary belief seemed to be a theoretical construct that required an inferential belief and attribute (similar to peripheral belief) to bridge the gap to action—a change in practice. An added barrier was the connection between the primary belief to the primary context from which it derived; the connection gave the impression of being the initial obstacle to implementation in a new context. Thus, Chapman added the ideas of inferential beliefs and attributes as well as a schematic diagram to further understandings.

Rokeach (1960) contributed the notion of an open mind versus a closed mind with reference to context as a justifying reason for holding beliefs. Cooney, et al. (1998) described it in the following manner: "The more open-minded person attends to context. In contrast, a closed-minded person sees no shades of gray because the world is seen from a perspective in which context is largely considered irrelevant" (p. 311). This stance is akin to Green's conception of clusters of beliefs. In 1968, Rokeach continued his theoretical constructs of beliefs by arguing that they have a cognitive, affective, and behavioral component (respectively—representing knowledge; capable of arousing emotion; and activated when action is required) (Pajares, 1992). He cautioned that understanding beliefs necessitates making assumptions about underlying states which is problematic because of unwillingness or inability to accurately represent beliefs (for many reasons) (Pajares, 1992). Rokeach's three assumptions were that "beliefs differ in intensity and power; beliefs vary along a central-peripheral dimension; and, the more central a belief, the more it will resist change" (Pajares, 1992, p. 318). He also stated that belief systems were organized but not always logical (Chapman, 2002; Cooney, et al., 1998; Pajares, 1992), which can be seen in further research even if not explicitly stated.

Cooney, et al. (1998), in a study of four preservice teachers found that Green's (1971) framework was a viable means of describing the ways in which participants held their beliefs. They suggested a partial scheme for conceptualizing teachers' professional development; these four characterizations were the naïve idealist, isolationist, naïve connectionist, and reflective connectionist. The naïve idealist absorbed others' beliefs without reflection due to seeking mutual consensus; the isolationist had clusters of beliefs which caused a rejection of others' beliefs and hence lack of accommodation; the naïve connectionist reflected on experiences but failed to resolve all conflicts between beliefs and practice; and the reflective connectionist not only saw connections but was able to reformulate core beliefs when perturbations occurred (Cooney, et al., 1998). One of their chief findings related to change was that reflection was key and its catalyst was perplexing situations (intentional or not). Hence, context is significant to the creation of knowledge (constructivism) and how these beliefs and knowledge are then implemented because behavior is adaptive (Cooney, et al., 1998).

Context

The role of context is an important consideration in seeking to understand the development of beliefs, implementation of beliefs, capability of change, and apparent inconsistencies noted in practice (Cooney, 1999; Cooney, et al., 1998; Ernest, 1989; Hoyles, 1992; Philipp, 2000; Raymond, 1997; Skott, 2001; Sztajn, 2003). Cooney (1999) felt that teachers contextual knowledge, shaped and framed by experiences, served as a "mediating factor in conceptualizing and acting out a course of action in the classroom" (p. 171). He also identified teaching as telling and caring as barriers to changing beliefs of which context played a huge role. For instance, even when a teacher believes that less guidance better facilitates student learning, in incidents when lack of guidance effects students quitting, teachers often give the (perceived) needed support due to beliefs about care (Cooney, 1999).

Raymond (1997), found that inconsistencies existed between a participant's beliefs about mathematics (traditional), beliefs about learning mathematics (non-traditional), beliefs about teaching mathematics (nontraditional), and her practice (primarily traditional with occasional innovation) and that these inconsistencies could in part be explained by looking at the context of the learning situation. Raymond noted that such factors as time constraints, scarcity of resources, concerns over standardized testing, and behavior of students as possible causes of discrepancies between beliefs and practice; for instance, traditional teaching practices require fewer resources and remain time efficient.

Skott (2001) studied the relationship between a novice teacher's image of school mathematics (which were strongly influenced by the current reform) and coping with the complexities of the mathematics classroom. He found that the teachers' actions in different visits seemed to be inconsistent in that one episode seemed to support the teachers' professed school mathematics image of reform whereas the second episode appeared inconsistent in that the teacher led the students through a series of computational steps when asked for help. After further investigation, Skott found that the apparent inconsistencies were explained by differing goals in various contexts; the beliefs, however, stayed the same. For instance, although the teacher believed in the importance of students' self-confidence and ability to solve tasks on their own, his goal of classroom management would preclude this belief on occasion. Thus, context produces new priorities and goals without necessarily representing a conflict in beliefs.

Hoyles (1992) cited Stigler and Perry (1987) as claiming that the happenings in the classroom were a reflection of the culture of the classroom as well as wider society and Moreira (1991) who found attitudes among English and Portuguese teachers to be dissimilar about mathematics and mathematics teaching; Moreira ascribed this to the differing systems and social contexts. After more research on this topic, Hoyles (1992) concluded that

> teacher decisions stem more from the social practices which frame teaching than the cognitive structures and beliefs of individual teachers. Yet, if we go too far along this road, there is a danger of viewing the teacher as "determined" by the constraints of the role and failing to acknowledge the diversity in both beliefs and practice. (p. 37)

Therefore, some of the inconsistency or disconnect between beliefs and practice can be linked to the context of the social atmosphere of education as Ernest (1989) noted arises from the expectations of students, parents, peers, and superiors in addition to "the institutionalized curriculum: the adopted text or curricular scheme, the system of assessment, and the overall national system of schooling" (p. 253).

Furthermore, Sztajn (2003) considered context a mediating factor in the relationship between beliefs and practice in her study of two teachers with similar beliefs teaching in diverse contexts. She felt that the notion of students' needs which encompasses beliefs about children, society, and education accounted for differences in instruction between the two teachers due to the variation in the schools view of children. The teacher who taught in the lower socioeconomic school felt that her students came from chaotic home environments and hence instruction focused on facts and procedures that could help prepare them for the workplace; she taught this way despite professing problem solving and higher order thinking skills as important in the mathematics classroom. In contrast, the classroom of the teacher (whose beliefs about mathematics were similar to the previous teacher) with students from the higher socioeconomic level was structured around problem solving and projects with an emphasis on sharing solution strategies. An interesting note is that this same teacher said that in the past, she had also taught with more drill and practice when teaching lower socioeconomic level students. It seems quite clear after noting this fact that context does indeed have quite a large influence on the practice of mathematics teaching when compared with the influence of beliefs.

Although Sztajn argued that these teachers are the "heroes" and are not trying to lessen students chances or hold them back socially because they are using their best judgment in assessing their particular students' needs, I cannot help but be reminded of Jean Anyon's (1980) article, "Social Class and the Hidden Curriculum of Work", in which she found that working class schools, middle class schools, affluent professional schools, and executive elite schools were preparing their respective students to fill their almost preordained roles in a social class system that is rarely noticed or looked at critically in the mainstream public. While I feel that these teachers are doing what they deem best, I have to criticize the way in which the research community sometimes tiptoes around issues related to classism and its effect in the mathematics classroom.

The contexts in which teachers find themselves, and consequent beliefs about children, society, and education in general, have an effect on practice that seems to mediate with beliefs about mathematics and mathematics teaching and learning. That beliefs about the nature of mathematics (which itself influence beliefs about teaching and learning and practice) are formed in context (socially constructed) helps to illuminate the trouble in understanding or defining the dynamics between context and beliefs about mathematics, teaching and learning, and practice.

Beliefs about Mathematics

Beliefs about the nature of mathematics (for all students—in particular preservice teachers) are formed through the context of culture as well as social constructions made by observations, interactions, and experiences during school mathematics settings (Ball, 1988; Hoyles, 1992; Pajares, 1992; Philipp, 2000). Some of these beliefs comprise the following: mathematics is computation (fixed set of rules and procedures), mathematics problems should be quickly solvable in just a few steps, the goal of doing mathematics is to obtain "right answers," the role of the mathematics student is to receive mathematical knowledge and to demonstrate that it has been received, and the role of the mathematics teacher is to transmit mathematical knowledge and to verify that students have received this knowledge (Ball, 1988; Ball, 1990; Frank, 1988; Kloosterman, 2002). Consequently, preservice teachers' views of teaching mathematics are consistent with the ways they experienced mathematics learning (Ball, 1990; Cooney, 1999). Additionally, their practice is more consistent with beliefs about mathematics than with beliefs about teaching and learning (Raymond, 1997), and these views about the nature of mathematics form a basis for mental models of mathematics teaching and learning (Ernest, 1989); thus, beliefs about mathematics influence the ways in which teachers teach mathematics (Skemp, 1978; Sullivan & Mousley, 2001). Further, these beliefs about mathematics and teaching play a subtle but significant role in the shaping of behavior (Cooney, 1985).

Stigler and Hiebert (1999) state that "the typical US lesson is consistent with the belief that school mathematics is a set of procedures" (p. 89) and Knoll, Earner, and Morgan (2004) point out the sharp contrast between pure mathematicians and school mathematics. Additionally, Hersh (1986, p.13) as cited by Thompson (1992), states that "One's conception of what mathematics *is* affects one's conception of how it should be represented. One's manner of presenting it is an indication of what one believes to be most essential in it… The issue, then, is not, What is the best way to teach? but, What is mathematics really all about?” Szydlik, Szydlik, and Benson (2003) found that the culture and sociomathematical norms established in the mathematics classroom attributed to preservice teachers’ change in beliefs about mathematics and supported the development of autonomous behavior. Thus, the culture of the classroom affect beliefs about mathematics and teachers’ beliefs about what mathematics *is* affect *how* they teach mathematics. Therefore, the context of classroom learning experiences for preservice teachers, as well as their beliefs about mathematics are key influences in their pedagogic practices.

In reference to teacher education, these findings are critical in that most teacher education programs focus on pedagogy and changing beliefs about teaching and learning mathematics whereas few classes aim at changing beliefs about mathematics. Yet it appears that conceptions about the nature of mathematics are as large a factor in teacher practice as conceptions about pedagogy.

Practice

The research on the influence of beliefs on practice has its roots in the history of reform curriculum as well as the cognitive revolution. The realization of a teachers' influence on learning (and curriculum implementation) was a long process partly because teacher knowledge was initially viewed simplistically as knowledge about mathematics.

Research on teacher effectiveness initially focused only on teachers' knowledge (or lack thereof) about content (Cooney, 1999; Thompson, 1984). Cooney (1999) cites Begle (1968) and Eisenberg (1977) as drawing attention to the fact that there is much more to effectual teaching than simply being mathematically competent. Bishop (1980) noted that there was "no doubt that the teacher was the key person in mathematics education" (p.343), and Fenstermacher (1979) felt that the focus of teacher effectiveness research would be studies of teacher beliefs. Looking at teacher effectiveness has naturally led into looking at teacher change and the role of beliefs in this process—through preservice education, professional development, or other factors (Wilson & Cooney, 2002; Carpenter, Fennema, Peterson, Chiang, & Loef, 1989; Lloyd, 2002).

Teacher change, growth, and development of teaching practices studies are valued due to the interest in reform recommendations presented by the National Council of Teachers of Mathematics (NCTM) in their publications of *Curriculum and Evaluation Standards for School Mathematics* in 1989 and *Principles and Standards for School Mathematics* in 2000 as well as the influence of teachers on implementation of these reforms and reform curriculums (Chapman, 2002; Ernest, 1989; Lloyd, 2002; Wilson & Cooney, 2002). In fact, most research into teacher beliefs has a reform focus (Lloyd & Wilson, 1998) on those studies even if not explicitly mentioned (Philipp, 2000). For example, Lloyd (2002) looked at the effect of professional development, in which teachers had experiences with innovative curriculum materials, upon their beliefs. She stated that

> professional development based upon curriculum has the potential to involve and impact teachers' beliefs about mathematics, student learning, and mathematics pedagogy, as well as their beliefs about mathematics curriculum…The distinctions between reform-oriented and traditional curricula provide immediate opportunities for teachers to explore, and possibly experience, multiple approaches to mathematical subject matter and mathematics pedagogy. (pp.156, 157)

Thus, beliefs about mathematics, mathematics teaching and learning, and curriculum are seen as influential to their practices, curriculum implementation, and learning potential of students.

A range of studies have found that teacher beliefs (views, preferences, conceptions, values, etc.) have an influence on instructional practices and that the relationship between teachers' conceptions and instructional decisions and behavior is extremely complex as beliefs are situated among context, activity, and culture (Ball, 1988; Chapman, 2002; Cooney, 1985; Ernest, 1989; Hoyles, 1992; Renzaglia, Hutchins, & Lee, 1997; Stuart & Thurlow, 2000; Thompson, 1984). Many preservice teachers bring to their teacher education programs beliefs about teaching and learning created from an "apprenticeship of observation" (Anderson & Piazz, 1996) during their many years of schooling (Ball, 1988; Calderhead & Robson, 1991; Philipp, 2000). These beliefs, values, and practices internalized by observations, interactions, and experiences are well-established but usually unarticulated, simplistic and may be implicitly held (Ernest, 1989; Pajares, 1992). Many beliefs about the nature of mathematics held by preservice teachers have emerged from formal mathematics education experiences which, taken together with their 'apprenticeship' and early formation has the effect of resistance to change during teacher education which consequently influences practice (Calderhead & Robson, 1991; Chapman, 2002; Philipp et al., 2007; Stuart & Thurlow, 2000).

Several studies, however, have pointed out that the level of intensity or psychological strength with which a belief is held affects the likelihood of change (Chapman, 2002; Cooney, et al., 1998). If a person has held a belief that is peripheral (felt less intensely) rather than primary, as Green's (1971) conceptual framework discusses, then it is more susceptible to change (Chapman, 2002; Cooney, et al., 1998). However, most beliefs will remain unchanged without a challenge or perturbation (Nespor, 1987; Chapman, 2002). Most new ideas are assimilated rather than accommodated because the lack of a challenge to beliefs in learning experiences, in which the new information is encountered, does not require reflection upon existing schema about conceptions (Lasley, 1980; Philipp, 2000).

Stuart and Thurlow (2000) found that when preservice teachers were asked to analyze both teacher observations and personal conceptions (about mathematics and pedagogy) and connections between beliefs and practice, it affected reflection. Reflection caused preservice teachers to question and challenge how perceptions influence their practice and student learning as well as make connections between prior mathematical experiences and pedagogical beliefs.

Grant, Hiebert, and Wearne (1998) found evidence that if preservice teachers' beliefs are at odds with an instructional approach, then solely observing the approach to classroom practice may not effect change and that these beliefs serve as a filter for what they see and internalize. They suggested that activities which bring reflection combined with observations of reform instructional strategies are more likely to lead to change. This highlights the importance of preservice teachers' ability to bring beliefs to a conscious level, and examine and articulate them (Lasley, 1980) in the evolution of change; hence reflection is vital to amending beliefs (Ernest, 1989; Renzaglia et al., 1997; Stuart & Thurlow, 2000). This reflection component to change can be seen in studies that focus on presenting student thinking about mathematics to preservice teachers.

Beliefs about Teaching and Learning versus Students' Mathematical Thinking

The role of knowledge about students' mathematical thinking plays prominently in influencing teacher beliefs and hence teaching practices. Studies find that this knowledge along with attention to content knowledge or pedagogy influenced beliefs about teaching and learning (Carpenter, et al., 1989; Philipp et al., 2007; Vacc & Bright, 1999). Philipp, et al. (2007) concluded that preservice teachers who studied children's mathematical thinking concurrently with learning mathematics changed beliefs more than those that did not and those beliefs were more sophisticated.

Vacc and Bright (1999) found that giving preservice teachers explicit information about research on children's mathematical understandings along with an emphasis on pedagogy may have influenced thinking about teaching and learning mathematics to a more constructivist approach. Carpenter et al. (1989) found that teachers involved with the Cognitively Guided Instruction (CGI) program that consequently learned about children's mathematical thinking taught problem-solving more, encouraged students' use of a variety of strategies, listened to students' descriptions of process more, and professed the belief that instruction should build on existing knowledge as compared with control teachers. Consequently, there is support for the view that learning about children's mathematical thinking can alter beliefs about teaching and learning mathematics as well as practice.

Empowerment

Empowerment is "the gaining of power in particular domains of activity by individuals or groups and the processes of giving power to them, or processes that foster and facilitate their taking of power" (Ernest, 2002). Consequently, mathematical empowerment concerns the goals and objectives of teaching and learning mathematics as well as the role and impact of mathematics on the life of the learner (Earnest, 2002). The word empowerment, in mathematics education literature, is often used to denote autonomy or efficacy; they are often used synonymously. In order to avoid confusion, the word empowerment will be used solely in place of the other two. There are three main domains of empowerment—mathematical, social, and epistemological; to these can be added a fourth domain of empowerment—the professional empowerment of the mathematical teacher (Ernest, 2002) which I will refer to as pedagogical empowerment.

Mathematical, Social, and Epistemological

Mathematical empowerment involves gaining power over the domain of school mathematics which entails using and applying the language, practices, and skills; likewise, it has cognitive and semiotic perspectives which are complementary (Ernest, 2002). The cognitive psychological perspective of mathematical empowerment involves the procurement of concepts, skills, facts, and general problem solving strategies whereas the semiotic perspective demands the development of power over the 'texts' of mathematics. These powers over the 'texts' of mathematics include the abilities to read and make sense of mathematical tasks, transform text into smaller tasks, pose problems and write questions, and make sense of text in computational form (Ernest, 2002).

Social empowerment encompasses the use of mathematics to increase a person's life chances and critical participation in work, study, and society (Ernest, 2002). In a utilitarian way, throughout history success in mathematics (often judged by performance on examinations) serves as a 'gatekeeper' or 'critical filter' controlling access into further education as well as occupations with greater pay (Ernest, 2002; Lemann, 1999; Oakes, 1985; Oakes, Ormseth, Bell, & Camp, 1990; Stanic, 1986; Standards, 1989). Moreover, researchers have long noted the perceived inequity in mathematics education for women and other minorities (Fennema & Sherman, 1977; Oakes, 1985; Oakes, Ormseth, Bell, & Camp, 1990; Sells, 1976; Walkerdine, 1997). The second facet of social empowerment deals with a 'critical mathematical citizenship' which involves empowering students to

> think mathematically, and be able to use their mathematical knowledge and skills in their lives to empower themselves both personally and as citizens, and through their broadened perspectives, to appreciate the role of mathematics in history, culture, and the contemporary world (Ernest, 2002, p. 4).

Thus, social empowerment includes not only access to upward mobility educationally and economically but also developing critical understanding and awareness of the uses and value of mathematics in society.

Epistemological empowerment concerns both one's confidence in the use of mathematics and a "personal sense of power over the creation and validation of knowledge" (Ernest, 2002, p. 8). It is in this category that the professional empowerment (or pedagogical empowerment) of the mathematics teacher falls. For many teachers and students, past experiences have led them to the belief that knowledge is created, legitimized, and exists outside of themselves. The stages of the epistemological empowerment of learners begins with 'silence' (passive acceptance of assertions) and gradually transitions through received knowledge: the voice of others (acceptance of assertions by authority but with the ability to repeat them), subjective knowledge: the inner voice (own subjective intuitive judgments are valued and responded to), procedural knowledge, separated or connected knowing, and constructed knowledge: integrating the voices in which the learner is active and "all knowledge is understood to be constructed by the knower herself, and the voices of intuition and reason are integrated" (Ernest, 2002, p. 9). It is with this conception of empowering the learner that empowering the teacher in the classroom can be seen as equally vital.

Pedagogical Empowerment

Pedagogical empowerment (or professional empowerment) refers to teachers developing into autonomous and reflective participants of the educational world. Empowered teachers contain the confidence to critically assess and construct mathematics teaching and learning (Ernest, 2002). Szydlik, Szydlik, and Benson (2003) found that the culture and sociomathematical norms of the classroom affected a change in preservice teachers' mathematical beliefs as well as served to further their autonomy. Sociomathematical norms established in the classroom are distinct from social norms in that they are unique only to mathematics classrooms (Yackel & Cobb, 1996). For example, adequate justification is a social norm in many subject areas but what constitutes as *relevant* and *elegant* for proof of a claim remains exclusive for mathematics. Additionally,

> what becomes mathematically normative in a classroom is constrained by the current goals, beliefs, suppositions, and assumptions of the classroom participants. At the same time these goals and largely implicit understandings are themselves influenced by what is legitimized as acceptable mathematical activity. (Yackel & Cobb, 1996, p. 460)

These sociomathematical norms established in the classroom studied by Szydlik et al. (2003) affected their participants autonomy as seen by students indicating that they were "now aware that mathematics is a human creation and they can be a part of making mathematics themselves (p. 272) in a culture that views mathematics as making sense. Mathematical autonomy refers to behavior involving sense-making instead of memorizing or appealing to authority. Additionally, Anderson and Piazza (1996) found that a classroom practice that eliminated lecture as the main form of instruction together with the use of physical models (manipulatives, pictures, diagrams) served to influence students in that they felt less anxiety about learning and teaching mathematics and felt a greater sense of confidence. Consequently, the culture of the classroom serves to empower preservice teachers mathematically and pedagogically.

Conclusions

Beliefs about mathematics, learning, and mathematics pedagogy all intertwine to form the foundational framework upon which teachers rely when planning learning experiences. Mathematical beliefs as to the nature of mathematics, along with prior mathematical learning experiences, influence the ways in which teachers teach mathematics (Skemp, 1978; Sullivan & Mousely, 2001). Perturbations and reflection on these beliefs through learning experiences that are in line with reform teaching relying on the notions of constructivism can serve to affect preservice teachers’ ideas and intentions for mathematics pedagogy. Additionally, university mathematics content courses are rarely taught in a manner aligned with this non-traditional format focused on conceptual rather than procedural reasoning, that encourages meaning making. Therefore, this study focused on contributing to the body of knowledge by attempting to develop, implement, and examine a non-traditional mathematics content course for preservice teachers. Chapter three outlines the methodology for this analysis.

CHAPTER THREE: METHODOLOGY

My mathematical and pedagogical experiences as a student, teacher and observer in mathematics and mathematics education courses served to intrigue me as to the effect of teaching in a way supportive of social constructivism. After further research on the topic of 'teacher beliefs' and the ways in which conceptions, values, emotions, attitudes, knowledge, and perception might influence teaching practices, the goals of this research study evolved due to the gap in the literature as well as my own curiosities on the subject. In particular, I wondered if or how learning mathematics in traditional, procedurally-driven classrooms might influence the ways in which preservice teachers view reform instructional practices (conceptual focus on meaning making and connections) along with alternative curriculum in a university mathematics content class. Additionally, I questioned the effect (if any) a mathematics content course taught conceptually, with a focus on meaning making, connections, dialogue, reflection, and patterns, might have on preservice teachers' beliefs about mathematics and on beliefs about mathematics teaching and learning. Consequently, this study had a dual purpose. The first was to describe what a “reform” mathematical content course might look like in a university setting; the creation and evolution of the course will be described as well as the limitations and demands of such a course. The second purpose of this study was to describe, from the preservice teachers’ perspective, the impact (if any) classroom learning experiences taught in this manner had upon their mathematical beliefs.

The design for this study incorporated a view of preservice teachers as social constructors of knowledge—as entering the program with preconceived beliefs (and knowledge) about mathematics and mathematics teaching and learning formed through an internship of observation during their formal mathematics schooling (Anderson & Piazz, 1996; Ball, 1988; Calderhead & Robson, 1991; Philipp, 2000). My instruction was situated among a reform model based upon conceptual rather than procedural orientations with a focus on meaning making, connections, patterns, justification, and dialogue. Because both reflection and perturbation (or challenges) are so vital to change in teacher beliefs (Ernest, 1989; Chapman, 2002; Cooney, et.al., 1998), I sought to design perplexing learning experiences for my students.

Thus, through providing an opportunity for a new kind of "apprenticeship of observation," my aim was

> developing teachers' ability and their desire to think seriously, deeply, and continuously about the purposes and consequences of what they do—about the ways in which their curriculum and teaching methods, classroom and school organization, testing and grading procedures, affect purpose and are affected by it(Silberman, 1970, pg. 472)

as well as reflect on their own belief systems. A concerted effort was made to establish social norms in the classroom that supported and encouraged discourse, investigation, and questioning. I sought to agitate their beliefs about mathematics and mathematics teaching and learning by the structure of the class (learning experiences, social norms, etc.) in addition to incorporating writing assignments that address beliefs (although not always explicitly stated). The questions this study explored were as follows:

- What are the characteristics of a mathematics content course for preservice teachers taught from a non-traditional orientation?
- From the preservice teachers' perspective, what impact, if any, does participation in this course have on mathematical beliefs?

- What influence, if any, do preservice teachers believe the curriculum and structure of this class have on their empowerment?

This research study was both timely and needed. Reform efforts in the classroom have yet to reap any major changes in mathematics teaching norms (Confrey, 2000). Ball (1988) called for prospective elementary teachers' ideas of what it means to learn to be challenged and extended and stated that "if teacher education is to become a more effective intervention in preparing elementary teachers to teach mathematics, we need to examine the influence of different kinds of teacher education" (Ball, 1988, p.16). Also, since preservice teachers adhere to an "apprenticeship of observation" (Anderson & Piazza, 1996) and because historically most faculty use a traditional lecture style of teaching, there is a call for university mathematics content courses to be taught in ways consistent with reform.

Throughout the course of this chapter, teacher action research as a methodology will be described as well as the data collection methods, and how the data were used in the action research cycle. Additionally, I will describe the participants, classroom setting, my role as instructor and researcher, and how data were analyzed.

Teacher Action Research

Teacher action research (also commonly referred to as "practitioner research" or "teacher research") was selected based upon careful reflection on the following concerns: the research topic, the fact that teacher action research supports an emergent design, applicability, and my personal beliefs. Throughout this section, the choice of teacher action research will be explained: why it is most appropriate for this study, the decision to use qualitative rather than quantitative or mixed methods, personal beliefs that influenced this choice, and finally an overview of teacher action research.

In order to better understand the topic of research for this study, beliefs about mathematics, I felt it would best be explored in a college mathematics classroom. Due to the gap in the literature concerning college mathematics content courses along with the availability of selecting participants from among my own students for practicality, practitioner research best facilitated answering the research questions. Practitioner research was selected because I would be working as both the researcher and practitioner: "whereas traditional research is undertaken by people who are essentially outside (external) to the phenomena under study, action research is undertaken by people who are part of the phenomena" (Cain & Milovic, 2010, p.19).

Teacher action research would be natural as the research methodology for this study because it "develops through a *self-reflective spiral* of planning, acting, observing, reflecting, and then replanning, further implementation, observing, and reflecting" (Burnaford, Fischer, & Hobson, 2001, p.43) as described by Kemmis and McTaggart (1988), accurately pronounces what I do as a college teacher each semester (although perhaps without the same level of rigor and formal documentation). This study was emergent and interpretivistic, and teacher action research is congruent with that point of view. The emergent nature of action research lies in the fact that the process of the research occurs simultaneously with action being taken to improve the situation which is differentiated from other research forms that study in retrospect (Foreman-Peck & Murray, 2008).

Perhaps the most appealing aspect of the teacher action research methodology is its potential applicability in the classroom. It has potential usefulness to other educators in teaching situations similar to mine. Many classroom teachers feel that academic research is not applicable to their particular classroom because they cannot see themselves or their students in the research. They cannot draw parallels (Rudduck, 1988), therefore my study has a local rather than global orientation (Feinberg & Soltis, 2004; Foreman-Peck & Murray, 2008) so that teachers may be better able to orient it within their own practice.

One of the criticisms of teacher research is that it lacks the possibility for generalization because it is considered too local (Foreman-Peck & Murray, 2008). This argument does not seem to have bearing within the educational sphere due to my belief—articulated by Stenhouse (1985)—that "research can be adequately applied to education only when it develops theory which can be tested by teachers in classrooms. Research guides action by generating action research (or at least the adoption of action as a systematic mode of enquiry)" (p. 29). Teacher research facilitates collaboration amongst universities, schools, and the community by extending their confines (Keiny & Orland-Barak, 2009). My hope is that practitioners can recognize in the findings of this study their own educational situation and be able to draw parallels between their classrooms and mine (Cochran-Smith & Lytle, 1993; Foreman-Peck & Murray, 2008).

Practitioners are often not taken seriously in research settings; they are not respected as researchers or seen as having specialized knowledge about teaching and learning not accessible to outsiders (Orland-Barak, 2009; Burnaford, Fischer, & Hobson, 2001; Cain & Milovic, 2010; Cochran-Smith & Lytle, 1993; Foreman-Peck & Murray, 2008). Stenhouse contrarily feels that teachers were in an ideal laboratory to not only test educational theories but study teaching. His students chose the following statement for his commemorative plaque "It is teachers who, in the end, will change the world of the classroom by understanding it" (Rudduck, 1988). Consequently, an element of the rationale to select teacher action research was a conscious personal decision to take a stance against the power roles that dominate most domains of research in education.

Although teacher action research does not have a particular method for research (Cochran-Smith & Lytle, 1993; Foreman-Peck & Murray, 2008; Orland-Barak, 2009; Noffke, 1997), I felt that my research topic and questions would be best facilitated by qualitative rather than quantitative data collection. Qualitative data provides a richness to the study and description that is often left bereft by quantitative data. Additionally, the goal was to describe the phenomena from my participants' perspective; I wanted to give a voice to my student-participants. A qualitative research method helped to ensure this aim.

Another major criticism of teacher research is that it is not considered very reliable as a whole (Cochran-Smith & Lytle, 1993; Foreman-Peck & Murray, 2008). This idea may stem from the fact that teachers choose their research topics based upon their interests and goals and consequently seem unable to distance themselves from their research, to remain objective so that their results have validity (Cochran-Smith & Lytle, 1993; Burnaford, Fischer, & Hobson, 2001; Cain & Milovic, 2010; Foreman-Peck & Murray, 2008). Teacher action research assumes that it is impossible to eliminate bias whereas traditional university-based research assumes that the researcher can become objective (Cain & Milovic, 2010; Cochran-Smith & Lytle, 1993). Teacher research purposively does not seek to distance the researcher from the research, the researcher is intricately involved, as Heron and Reason

(1997) articulated: “To experience anything is to participate in it, and to participate is both to mould and to encounter” (p. 278). Therefore, “teacher researchers do not strive for dispassionate objectivity; they work toward systematic and detailed data that teach them something about their professional world” (Burnaford, et al., 2001, p.56). Instead of being a fly-on-the-wall observing a natural phenomenon, teacher action researchers “study social reality by acting within it and studying the effects of their actions” (Anderson, Herr, & Nihlen, 2007, p.1); rather than *looking in* at phenomena, teacher action research examines *from the inside* (Ball, 2000). Additionally, its aim is the transformation of educational settings rather than simply observing, describing, and interpreting (Anderson, Herr, & Nihlen, 2007).

The teacher researcher is reflective both inter-personally (looking out) and intra-personally (looking in) in order to gain insight into self and students (Burnaford, et al., 2001). Despite being personally invested in the research, a teacher researcher can take steps to retain validity and be purposeful in the level of rigor involved with the use of systematic analysis of the data, peer examination and discussion, as well as triangulation (Bartlett & Burton, 2006; Foreman-Peck & Murray, 2008). For this study, a systematic analysis of the data involved coding and theming; peer examination and discussion entailed corroboration and examination of the findings and themes with a peer; triangulation necessitated examination of the themes from participants' responses, my reflective journal, and the peer discussions.

Teacher action research is an emergent research design, a methodology unlike other forms of practitioner research (generally referred to as teacher research) that study situations in retrospect (Burnaford, et al., 2001; Foreman-Peck & Murray, 2008). Action research involves process simultaneously occurring with action in a continual cycle (Foreman-Peck & Murray, 2008). This involves reflection-in-action and reflection-on-action in order to generate positive change in the recurring spiral of planning, implementing, monitoring (observing and evaluating), and reflecting (Burnaford, et al., 2001; Cain & Milovic, 2010; Elliott, 1991). Cochran-Smith and Lytle (1993) based their definition of teacher research as "systematic and intentional inquiry carried out by teachers" (p. 7) on the work of Lawrence Stenhouse.

Stenhouse (1985) wrote that "research guides action by generating action research, or at least the adoption of action as systematic mode of inquiry" (p. 29). Systematic refers to ordered ways of obtaining information and intentional clarifies that action is planned versus spontaneous (Cochran-Smith & Lytle, 1993). Inherent in this definition is the key role reflection plays in the course of action.

John Dewey (1929) encouraged teachers to reflect on their practice since "each day of teaching ought to enable a teacher to revise and better in some respects the objectives aimed at in previous work" (p. 74). Reflection helps to ensure that systematic and intentional research occurs. In the process of reflection, writing can be a valuable tool (Burnaford, et al., 2001). Writing in response to prompts such as "What decisions did I make during the lesson?; What responses and reactions from the students affected those decisions?; What was I thinking about and feeling during the action?" (Burnaford, et al., 2001, p. 9) can aid in the process of reflecting upon classroom experiences.

Teacher research does not have a set method or model (Cochran-Smith & Lytle, 1993; Feldman & Minstrell, 2000;

Foreman-Peck & Murray, 2008; Orland-Barak, 2009); since teacher action research is a methodology and not a model, there exists a vast array of data collection methods employable including journals, questionnaires, interviews, classroom observations, videotaping, and audiotaping (Feldman & Minstrell, 2000; Forman-Peck & Murray, 2008).

For this study, qualitative methods, specifically teacher action research, were a better option for relevance and usefulness to educators than quantitative research than other more quantitative approaches. Quantitative research often attempts to "make caricatures through some sort of oversimplification" versus looking at a phenomenon as "mysterious and multifaceted" (Dowling, 2005), which is the reality of any experience that brings about change. My goal was to situate this study, and myself as the researcher, among the existing body of literature describing, interpreting, and illuminating context-specific aspects of pre-service teacher change in beliefs rather than seeking to find all-encompassing explanations of change (Feinberg & Soltis, 2004). Therefore, the choice of teacher action research coincides with my ontological perspective.

Both practitioner research and teacher action research come in many shapes and forms. Orland-Barak (2009), elaborates on two main themes in practitioner inquiry: practitioner inquiry as a paradigm for change and practitioner inquiry as a practice of variety. Foreman, Peck, and Murray (2008), differentiate three conceptions of teacher action research and define each conceptions' knowledge characteristics: action research as professional learning, action research as a form of practical philosophy, action research as a form of critical social science, and action research in the service of policy implementation. This study is situated within the "paradigm for change" (Orland-Barak, 2009) since there exists a call for change in prospective teacher education.

Participants and Instructional Setting

The participants for this study were students from a small 4-year college located in a community comprised of approximately 17,000 people in the southern Midwest region of the United States. The college campus which is quite large and spread out (37 buildings and 135 acres) is nestled in a neighborhood occupied by two of the town's elementary schools. The university began as a normal school in 1907; the consequent six normal schools established over the years transitioned to teacher's colleges and then the university began offering bachelor's degrees in 1919. Currently, the university offers 50 bachelor's degrees with 71 different degree options and 4 master's degrees with 18 different degree options. The approximate student population is 5,000 and contains a mix of local and commuter population.

The mathematics building is one of the oldest buildings on campus. As such, it has beautiful architecture, spacious rooms, and a comfortable atmosphere. Various portions of the building have been remodeled and the building itself has been well-maintained so it has a lived-in feeling without feeling too old or dingy. At the center of the mathematics floor is a theater in which smaller plays are performed (the university recently built a larger fine arts building) and special smaller events take place.

The classroom in which the study took place is located on the southeast end of the building on the second floor. The room is quite large and spacious with windows lining the south and east sides of the room. The room is equipped with four rows, each composed of three large long tables, and a fifth row (front) that only has two long tables between which is the podium. Students would move chairs around to accommodate group work, but we rarely moved desks due to their bulkiness. Typically students stayed in the seat they originally chose and would work with the people surrounding them (beside, in-front, and behind). Students worked with the same local partners for the duration of the semester with the exception of my reassigning a student to a different group if their group members were absent or a student moving their location in the classroom. Two chalkboards, an overhead projector, and a Smartboard comprised the teaching tools available in the classroom. The mathematics department also had various manipulatives, children's literature books, and basic project supplies that I used for the classroom experiences. I typically had a cart filled with "stuff" that I would bring to our classroom. Also on the cart was a large plastic file tub with separating files for the students to turn in their homework journals.

All participants were preservice teachers enrolled in a mathematics content course intended for early childhood and elementary majors although there were a few participants taking the course that had other majors such as special education. The research study included two sections of the same course totaling 47 students of which 37 chose to be participants: 35 females (95%) and 2 males (5%). Most of the students were traditional, but several were non-traditional, at least two of which were coming back for additional degrees while the remaining had returned to school after a varied number of years absence from school.

The ages of the participants' ranged from 19 to 50. The majority of participants were Caucasian with lesser percentages in the other ethnicity categories. The variety of mathematics courses taken in high school included the following: Algebra I, Algebra II, Geometry, Trigonometry, and College Algebra. The prior college mathematics courses included the following: Intermediate Algebra, College Algebra, Survey of Mathematics, and Statistics. The college majors of the participants included the following: Elementary Education, Special Education, Early Childhood Education, and Physical Education (see Table 1).

Table 1: College Major

College Major	Elementary Educ.	Special Educ.	Early Childhood Educ.	Physical Educ.	Business Admin.
Number of Participants	15	5	15	1	1

The instructional setting for this course was a non-traditional design that incorporated meaning-making, dialogue, connections, reflection, and patterns. The focus on conceptual rather than procedural was intentional in an effort to help preservice teachers gain understanding of the mathematical topics covered as well as challenge their preconceived beliefs about mathematics and mathematics learning. Participation in the study was primarily incorporated into the normal framework of the course. The only extra items that participants were asked to do, more than participate in normal class activities and assignments, were to complete the background form and the final survey.

Ethical and Methodological Rigour

Marshall and Rossman (2011) point out that concern with trustworthiness criteria in qualitative research originated from quantitative research. Therefore, terms such as reliability, validity, and objectivity were the measure of soundness. However, with the onset of postmodernism and the variety of ways it is viewed, these criteria as well as the need for them are challenged and debated with no apparent consensus in sight. Principally vital to the trustworthiness of any study is ethics. Ethics, in research, encompasses respect for persons, beneficence, and justice—which goes beyond mere informed consent—from procedures to relationships with participants, peers, and discourse community (Marshall & Rossman, 2011). Davis and Dodd (2002) state

> ethics are an essential part of rigorous research. Ethics are more than a set of principles or abstract rules that sit as an overarching entity guiding our research…Ethics exist in our actions and in our ways of doing and practicing our research; we perceive ethics to be always in progress, never to be taken for granted, flexible, and responsive to change. (p. 281)

Ethics are interwoven throughout the research process—not simply addressed for the ethics committee during the approval process (Marshall & Rossman, 2011).

For purposes of this research project, approval was granted by the Institutional Review Board of both participating universities(primary and data collection university) through a process wherein the methods, procedures, and goal were scrutinized to ensure respect for persons (privacy, anonymity, right to participate—or not, not used as means to an end), beneficence (first, do no harm), and justice (who benefits from study—and who does not) were all present in the research process

(Marshall & Rossman, 2011). Therefore, I have sought to maintain the trustworthiness and integrity of this study.

Upon approval of the project by the Institutional Review Boards, participants were contacted by an announcement made by a colleague who distributed and explained the information sheet and consent forms during a small portion of one class period. In order to minimize undue influence and preserve the integrity of this study, my colleague maintained the signed consent forms until after the final grades had been submitted at the completion of the semester; thus, the identities of the participants were not revealed until the end of the semester after all grades for the course were submitted.

Data Collection

Data were collected over a period of time of about seven months. The majority of the data came from participants' writing, but other sources included a personal journal, student metaphors, and classroom conversations. The variety and amount of data compiled helped to advance a more complete and accurate sense of the participants' perspectives, experiences, and beliefs.

Personal Journal

At various times throughout the study, I would record my reflections about a particular class, the classes as a whole, or specific conversations that occurred. The length of the journal entries and time between journal entries depended upon factors such as noteworthy classroom conversations or experiences, whether it was a test week, or if data had been recently collected and reviewed. This was done so I could retain my experiences and also so I could take a step back from the data. These times were used to reflect upon classroom interactions, record anecdotal notes about student-to-student conversations, and plan new actions for perturbation that I then implemented in the classroom experiences. At times I felt that I was not getting enough information from the writing portion of the homework assignments, thus I would add a couple of “free” questions on the test about students’ opinions and beliefs in order to better gauge the students’ perceptions of the climate of my class. At this point in their experiences in the class, the participants had already received grades for opinion-related questions based solely upon whether their responses were “thoughtful and thorough,” and as a consequence, I felt that their responses were valid as a data source. The content of some of the responses supported this idea since the students did not always reflect positively on certain aspects of the course or their experiences in the course; they seemed comfortable to honestly share their feelings and beliefs.

Course Documents

Course documents included various writing assignments throughout the duration of the course. Some were included as part of homework assignments, others were questions on the exams (in-class as well as take-home portions of exams). Due to the fact that these data collections were built into the structure of the course, a complete data set for some participants was not obtained(for example, some students did not turn in completed homework assignments). Overall, the data collected for the majority of the participants were complete with regard to assignments and test questions.

Final Survey

Once final grades for the course were submitted, I learned the identities of my participants. At this point, I contacted each of them and asked them to respond to a final survey. They were asked to return the final survey in a couple of weeks. The participants did not receive the final survey until after the break between semesters which allowed them some time to reflect upon their experiences. Participants were asked to write in response to nine specific questions (see Appendix E) on the final survey. Responses were not asked to be a certain length and thus varied in length as well as depth of response. Additionally, many participants chose not to respond to the final survey.

All students' homework folders for the assignments that dealt with my research study were copied and kept in labeled folders in a secure location. Once final grades were submitted and the identity of the participants was known, data sets for those students who elected not to participate were removed. Those data were later shredded in order to maintain those students' privacy. Therefore, as my study progressed, I had the files of assignments to examine and appraise as part of the recursive process of data collection, documentation, analysis, and implementation of new content, processes, and experiences. Although this process sounds linear, it was in fact dynamic in that all of the pieces were interwoven and overlapping in many ways so as to become part of the intricately plaited developments that encompassed the overall study. This progression continued on into the phase in which data were analyzed, results found, and then findings summarized.

My Role

My role in this study was two-fold: instructor and researcher. My first role was that of instructor, which entailed negotiating social norms in the classroom to help establish a mathematical community through a recursive process (Wheatley & Abshire, 2002). The very first day, I initialized the social norm of discourse as well as sought to establish a community by asking everyone to state their name, major, where they are from, favorite subject, and an interesting fact about themselves. Likewise, I introduced myself in order to establish my role as co-learner in the learning experiences and to help the students create a level of comfort with me necessary for a safe and caring community of learners.

The second class meeting was likely the most interruptive to their pre-conceived notions of what a typical mathematics classroom looks like and helped to establish norms for our classroom experiences for the remainder of the semester. Rather than lecturing over a section from the textbook, as is typical in a traditional university mathematics course, I split the class into groups of two to three (or more) to work on problems that focused on patterns and relationships. These were atypical (or non-routine) problems in that they did not focus on a particular topic and could not be solved using algorithms or algebraic manipulations.

The collaborative working times were followed by whole-class discussions in which each group (and individual members) was encouraged to participate. I expected participation in a professional manner during all classroom experiences (which I explained on the first day of the semester) from my participants which included contributing to the group collaboration as well as respect for one another during sharing times through active listening.

Since social norms cannot be established by a teacher and are instead established by the community of learners which includes the teacher (Wheatley & Abshire, 2002), I tried to negotiate my students' preconceived conceptions about mathematics classroom learning experiences (Ball, 1988; Calderhead & Robson, 1991). For instance, students frequently asked me to look at their answers and judge their correctness, with little regard to the process or reasoning involved. Consequently, I declined to judge the precision of their answer and instead asked them questions about the process. Additionally, I asked the class as a whole how they reasoned about a problem and then let them "argue" about the solution. Many students seemed quite frustrated with this early in the semester but later transitioned into justifying their work first which naturally leads to the answer in most instances.

My second role was that of researcher; to be a recorder and facilitator of the experience. Entailed in this was listening, reflecting, journaling, copying, recording, formulating plans of action, testing hypothesis, and then repeating this multi-faceted cycle again and again. The emergent design of this study requires this iterative process and inherently positioned me as co-learner along with my students as participants. I believe that individuals are active constructors of knowledge, the majority of which occurs in social interactions. Further, learning is an active process of meaning making that happens socially and is a dynamic interaction between teacher-learner and learner-learner (Davis, 2004). Learning also involves reflection on experiences in order to be able to apply that knowledge (Bruner, 1996). Freire (1998) claims that learning is unfinished and that it is both horizontal and vertical (Gee, 2004). I believed that I could learn from my students as they learned from me and one another. Therefore, as my students were actively constructing knowledge about mathematics and developing beliefs, I was also constructing ideas about their beliefs during classroom experiences. My students and I were continually negotiating and renegotiating new information with past knowledge in order to form new knowledge (Piaget, 1972).

My combined role of teacher-researcher was certainly a challenging role for which I was both prepared and unprepared at the same time. The amount of time spent reflecting on and planning for each class and the research process was difficult to resolve within my time constraints. The time spent on these two tasks was much more than I had previously spent on teaching because I had the added task of writing my reflections as well as trying to ensure that students' writing assignments would help to answer my research questions. Likewise, the iterative process of regularly reviewing student reflections and determining how those reflections would or should impact subsequent classes was quite involved. Additionally, having adequate time during classroom learning experiences to facilitate dialogue, while at the same time attending to the concepts and material I was expected to cover, proved to be a constant source of tension. As with all worthwhile endeavors, it was challenging at times.

Data Analysis

The data collected in this research study was qualitative in nature and primarily collected throughout one semester in a mathematics content course. The data were analyzed through a series of iterative processes. I began by reading through each separate data source throughout the semester in order to gauge my participants' response to classroom learning experiences and beliefs looking for trends; I then used this for reflection to plan further data collection throughout the research process, learning experiences, homework assignments, and exams.

Once I determined which students were participants in the study following the completion of the semester I typed each separate data source for each assignment again looking for trends. I made anecdotal notes on the participants' papers as well as in a notebook as I worked my way through typing data sets.

The next portion of the iterative cycle involved me looking at each individual data source as a whole, making categories, coding the categories and placing each participant response into a category based upon the code beside the source. Many times as I coded the responses the first time, another theme emerged as either a sub-category of a code or as another category altogether. This would necessitate the re-coding of the data set. After developing categories, I would then somewhat linearly, in a step-by-step straightforward way, write about the results of that particular data set.

In the midst of the linear writing, larger themes began to emerge that in turn began to shape the organization of chapter four. The overarching organization of the results section was centered around the research questions. When the writing process was complete, I again examined each major section (with all of the smaller portions comprising it) of the results chapter and then as a whole to see if any further themes emerged. During this time, I also checked for congruence so that the results would be reliable and trustworthy for future reference. I then reorganized, edited, themed, and wrote as necessary. A general time frame or class schedule for the course is provided below to aid in understanding the descriptions of the results. This will allow the reader to see the evolution of the data and participants' beliefs.

Table 2:Class Schedule for the course

Intro day

Classes 2-9

- Class 2
- Class 6 & 7

Test #1 (Review in class day before exam)

- Questions about classroom environment

Classes 10-15

- Class 10, 11, 12

Test #2 (Review in class day before exam)

Classes 16-24

- Class 21

Test #3 (Review in class day before exam)

- “Math is…”

Classes 25-31

Test #4 (Review in class day before exam)

Work Day

Presentations (3 days)

Final

- Take home about class format & any change in beliefs

Final survey

CHAPTER FOUR: RESULTS

After the analysis of results, this study found that the structure of the course affected students in that they stated having positive classroom experiences, changed beliefs about mathematics, and gaining mathematical empowerment. Participants expressed that they enjoyed the format of learning experiences; the aspects most enjoyed were group work, presentations and manipulatives (term used to encompass the use of physical models). Additionally, the non-traditional format of the course had an effect on these preservice teachers' plans for their future teaching endeavors; participants cited a non-traditional classroom experience as their preferred choice of learning format. Furthermore, the course structure was influential to changing participant's mathematical beliefs. The beliefs with the most amount of change were those perturbated most often during classroom experiences which were "there is one right way to solve a problem" and "the goal of mathematics is to obtain 'right' answers." Lastly, the results revealed that after the course participants were mathematically empowered. They expressed: greater confidence in their abilities to learn, feeling more comfortable teaching mathematics, and finding a new appreciation for mathematics in general. The continued discussion of the results that follows are arranged in the order of the questions this study sought to answer.

A Non-Traditional Mathematics Content Course

The first question this study sought to answer was "What are the characteristics of a mathematics content course for preservice teachers taught from a non-traditional orientation?" In considering this question, I examined the social norms established in the classroom throughout the study and my beliefs which affected the negotiation of social norms as well as my definition of "classroom experiences taught from a non-traditional orientation" in which a focus on conceptual rather than procedural and meaning-making are central. I begin with a discussion of the role of the teacher and student followed by the importance of dialogue, decentralized control, curricular material (enabling constraints), space, and justification (or reasoning).

Instructor Reflexivity

The way in which social norms negotiation were approached during classroom experiences stems from my beliefs about the characteristics of a classroom based on conceptual rather than a procedural orientation—a non-traditional rather than traditional college mathematics classroom. Rather than viewing the classroom as a place where I must “transmit” information so that my students can in turn “know” what I know, I viewed the classroom as a place of mutual learning in which a conversation takes place between and among the students, teacher, and curriculum; dialogue is thus central to a classroom focused on meaning-making and a conceptual orientation. This classroom was one in which the teacher’s role is seen as being a facilitator and supporter of learning rather than a carrier of knowledge and one who “views the classroom as a community, the teacher as co-learner, and the curriculum as an ongoing conversation” (Reeder, 2005 p.253). Inherent is this view of a classroom learning experience is the belief that “the only learning which significantly influences behavior is self-discovered, self-appropriated learning… Such self-discovered learning, truth that has been personally appropriated and assimilated in experience, cannot be directly communicated to another” (Rogers, 1969, p.152). I believe that creating space in classroom learning experiences for dialogue and meaning-making helps to ensure that true understanding occurs.

Dialogue

My strongly-held belief in the role of dialogue is supportive of a transactional theory of education which blurs the borders of traditional teacher and student roles by insisting that the teacher-student environment be reciprocal and mediated by dialogue (Houser, 2006). Therefore, rather than teacher-centered or student-centered classrooms, transactional theory suggests that both teacher and student together complete the circle of learning in the classroom. Learning is not seen as "banking" with the teacher depositing knowledge into students who are passive "empty vessels" (Freire, 1998), but as an interaction of interested parties. My beliefs include a holistic view of the classroom and "envision active humans transacting within an equally active social environment" (Houser, 2006, p.21). Consequently, as an effect of my beliefs, I changed the way that I related to my students; instead of trying to control classroom interactions, I facilitated "back and forth" interactions in which I was an active participant in the conversation. I aimed to enable the learning process through the support of constant dialogue with my students. Complexity theory provides ideas for ways to facilitate this environment. Choosing to decentralize control of the classroom is a view often held by complexity theory educators (Bowsfield, et al., 2004; Davis & Sumara, 2005).

Decentralized Control

Decentralized control entails relinquishing control of classroom learning experiences which is not normally perceived as a good pedagogical practice (Bowsfield, et al., 2004; Davis & Sumara, 2005). In addition to going against perceived teaching norms, the idea of implementation was an intimidating task for me as teacher. However, it is important to note that decentralized control necessitates incorporating “space” for multiple possibilities in the classroom rather than being about resigning control over curriculum or goals and objectives for student learning (Bowsfield, et al., 2004). In contrast with traditional mathematics classrooms in which the teacher must retain the image of “expert” as well as the power and authority over classroom learning experiences, decentralized control allows the teacher to be a participant in the development of the classroom’s communal personality (Bowsfield, et al., 2004; Davis &

Sumara, 2005); my classroom learning experiences were centered around *being prepared* rather than *having plans*. My focus on being prepared enabled me to move with confidence from a lesson plan when appropriate and allowed students the needed "space" for active meaning-making with the material to occur.

Enabling Constraints

As I planned classroom experiences, I also kept the notion of enabling constraints in mind. *Enabling constraints*, in reference to curriculum, suggests that a task is neither "overly prescriptive" nor "anything goes" (Bowsfield, et al., 2004). My view of an activity that enables while also providing necessary constraints were ones that guided students toward a concept yet with boundaries that did not allow them to go too far down an unproductive path. Therefore, the notion of enabling constraints when applied to learning activities created opportunities for students to think and learn on their own yet have boundaries that enable constructive learning experiences to occur.

Dynamic Process

Teaching is a dynamic process, replete with cycles of interchange between teachers and students, teachers and the curriculum, the students and the curriculum, and students and students, rather than purely prescriptive. Teaching for learning is not a formula or recipe that can be followed or figured out by breaking classroom experiences into smaller parts, analyzing them separately, and then defining it as the "sum of the parts." Learning is dynamic because students are diverse in their backgrounds, interests, abilities, and personalities; consequently, each class as a whole has a culture all its own. I believe that learning experiences (and what our students learn) depend not upon the "plans" that I make but upon the multi-faceted social interactions that occur during class time; what my students learn depends on them and the collective as a whole. My belief in social constructivism (Ernest, 1998; Houser, 2006; Vygotsky, 1978) as the way in which students learn guides how I approach the planning of learning experiences such that they are set up to facilitate group interaction and knowledge construction of individuals as well as the group.

The overarching goal, in my planning of classroom experiences, was to encourage students to make sense of mathematics: to ask "why?" as much as possible. Implicit in this goal are my beliefs about mathematics and mathematics teaching and learning. I feel that mathematics is logical and makes sense therefore is learned best by students frequently asking “why?” and “how?” Furthermore, students are capable of answering these questions and of making sense of a problem or situation. In order to support learning wherein students ask questions and search for answers, collaboration and discussion need to be facilitated. Understanding mathematics was extremely important for learning in the non-traditional format of this particular mathematics content course.

Justification and Reasoning

Lastly, I feel that justification and reasoning that leads to a given result should be an integral part of mathematics learning. In particular, for a mathematics content course for preservice teachers, justification serves to facilitate better communication of mathematical concepts as well as promotes reflection upon said concepts and hence furthers understanding.

Philosophy in Action

Due to my pedagogical and mathematical beliefs, lecture was eliminated as the primary form of instruction during classroom learning experiences. Instead, group work and active learning using manipulatives, pictures, and diagrams were the emphasis during class time.

Group Work and Presentation

As each group worked on non-routine problems, I encouraged the students to talk to one another and to share their ideas and methods. After all groups had adequate time to work on their problems, I then used the interactive whiteboard to display the ways the groups worked various problems; as a group verbally described their thinking and solution to the class, I would write it on the whiteboard. I tried to ensure that at least two different ways of thinking about and solving each problem were represented, but in some cases up to six different ways of solving a particular problem were shown. A few times I asked that a representative member of a group share their work on the interactive whiteboard or overhead but most often I would ask them to verbally lead me through their work as I interpreted and wrote down their ideas and methods.

The semester began with groups presenting their work to the class but was modified shortly after the beginning of the course. My decision to interpret and write the group solutions on the interactive whiteboard in class as described rather than having my students come to the front of the room and present them was intentional and based on two reasons. The first was that it made my students feel more comfortable. Their responses in a written portion of the first exam as well as individual conversations with students and my observations during class learning experiences led me to take action and change this classroom practice for "presentation" portions of classroom experiences. This portion of classroom experiences evolved as part of what were many social norms established throughout the semester. The second reason was based on time management. Presenting took more class time when students presented their work rather than me writing it on the board while they explained their ideas and methods.

Something that I struggled with throughout the semester was the fine line between time spent working on problems in groups, discussion of the route to solution of the problems, and "lecture." I wondered continually how I could maintain "a practice that respects the integrity both of mathematics as a discipline *and* of children as mathematical thinkers" (Ball, 1993, p.376) except in my case the students were adults. This caused significant disequilibrium for me as a teacher. I found myself in a recurrent state of reflection constantly analyzing every upcoming classroom learning experience in light of my students' reactions to prior ones; I repeatedly looked for ways to make more time for discussion and working on problems with less time spent on "lecture." This characterizes the recursive cycle of teacher reflection, action, and reflection that occurred throughout the semester with my courses as a normal part of my teaching practice and as part of this teacher action research project.

"Dirty Papers"

Throughout this study many themes emerged as I observed, reflected, journaled, and analyzed classroom learning experiences. One theme was the students' desire to not "dirty" their papers. Students did not seem to want to write anything down on their papers unless they were sure that it was "right" so their papers would be neat and organized. It is my belief that this tendency was rooted in their beliefs about mathematics grounded in the notions that "There is one right way to solve a problem," "The goal of mathematics is to obtain 'right answers’,” and “The teacher and the textbook are the mathematical authority.”

Challenging “One Right Way”

Throughout the semester I worked to combat these seemingly well-established beliefs by giving up control of the problem-solving process and validating their processes and answers. I further worked to perturbate these beliefs by refusing to give students answers, giving help minimally time-wise (I would scan their work and either ask a leading question, encourage their work, or give a small hint), and almost never showing “my” way to complete a problem. On the rare occasion that I did demonstrate a way not represented by any of the groups, I would say “One way that I saw a previous student (or group) work this problem was…” so students would not perceive this as the correct solution or only solution but rather just one of many that were valid. This was only done if I felt seeing the other method was vital. The following excerpts from my reflection notes support these ideas:

- Still too many not wanting to “dirty” their papers
- (MWF class) I noticed that I actively gave up control of the problem solving process and didn’t give as many hints and went with their flow of thinking/ways of solving. For instance, in Class18,19,20 farmer’s market problem, I didn’t show the algebraic way but only talked about their methods. I did this to

validate their work. If I had shown another way, they would have thought my way was "better" because it was mine!

- (T/Th class) The class as a whole seems to be waiting for me to "solve" the problem. I need to back off and force them to figure it out. Own it. Validate them. Empower!
- The T/Th class almost refuses to talk. They seem to always wait for me. They are too scared to mar their papers. Too scared that they might have to erase. I am not sure what to do next (week, semester, yr. etc.) to curb this. Maybe I can get their groups to show their work to me. Maybe explain that this is part of the process [of classroom practice]. Or switch up groups more.

Student Perceptions of Learning Experiences

As part of their exams, participants were asked to provide written responses of varying lengths focused on certain aspects of the class learning experiences; they varied from one-word responses to three pages for the final. The responses to these questions were graded much like the homework written response in that they were given a grade based not on *opinion* but on the *thoughtfulness* and *thoroughness* of their response. I sought to establish a social norm of trust and openness early in the semester so students would feel comfortable expressing their thoughts and feelings about classroom experiences and their personal opinions and beliefs about mathematics and mathematics learning.

The written portion of the first exam consisted of three short response questions. Credit was given for any response, no matter the length, in order to build trust between me and the students. I also addressed their statements in writing on their tests when applicable as well as in class verbally in order to validate their opinions. The questions they were asked were as follows: During classroom learning time, what specifically do you enjoy about classroom interactions (if anything)?; During classroom learning times, what specifically causes you discomfort during classroom interactions (if anything)?; During classroom learning times, what specifically would you change about the way in which classroom interactions occur (if anything)? The first question analyzed was participants' responses dealing with the aspects of the learning experiences that they found enjoyable.

Enjoy About Classroom Interactions

The data from the responses to this particular question were analyzed by using a *Wordle*. A wordle is a tool that generates "word clouds" from a provided text. The "clouds" appear as a cluster of words created by using all of the words in the provided text. The sizes of the words in appearance are proportional to how many times they appear in the text in relationship to the other words (http://www.wordle.net/).

Figure 1: Wordle depicting what participants enjoyed about classroom interactions.

Analysis of the data displayed in the wordle revealed that some of the most prominently used words were enjoy, group(s), understand, problem(s), like, work(ing), different, able, get, and class(room). The words enjoy and class(room) were included in the phrasing of the question thus may have appeared as predominant words simply because students rephrased the question as part of their responses. The vast majority of students mentioned the structure of the classroom experiences as what they enjoyed the most; the three components mentioned most often in order of prominence were group work, presentations, and manipulatives (or hands-on). Note that the mentions of the first two components of the classroom learning experiences were often in conjunction thus they are discussed jointly rather than separately.

Group Work and Presentations

Students indicated that they enjoyed working in groups because they were able to see how other students figured out a problem—both as a student and a future teacher. For instance, one participant stated that [this course]

> has taught me how to teach my students that you can learn mathematics in fun ways, different procedures, and stay interactive. I never was a believer about group work until I came into this class. I thought group work was a way for people to cheat off of one another while there are only one or two people actually trying to solve the problem, but I really changed my mind on group work. What I learned from this class the most is how we can really learn different ways to solves problems. I've always tried one way to solve a problem, but I learned many new things from others while I was working in groups.

The second feature of the class that students frequently mentioned as enjoying was presentations. The presentations had varied descriptions; participants articulated seeing other ways to work the problem both from the group work and during whole-class discussions. Students in both instances explained that they enjoyed seeing the variety of ways to understand and solve a given mathematics problem and that they found it helpful, as a future teacher, to see how other students thought about mathematics. Group work was mentioned in 22 of the 32 responses from participants, followed by presentations, which was mentioned in 12 of the 32 short answer responses. The following are examples of these two types of responses:

- I enjoy when the class splits into small groups to work on a problem or set of problems and then comes back as a whole group to discuss the answers. Not only does it allow us individual time to familiarize ourselves with the assignment, but it also allows us a chance to see how our peers worked the same assignment.
- I like that we get to work in groups and I also like that we get to work on a problem and try to figure it out before we are told how or why.
- I enjoy working with others on problems. The help I get from the other students has probably benefited me more than anything in my college math classes. The discussions between us and the teacher really help me to understand concepts better.

Manipulatives and Physical Models

The third component of the classroom experiences mentioned when participants were asked what they specifically enjoyed about classroom interactions was using “manipulatives” or “being hands-on.” This is an interesting result due to the fact that at this point in the course, students had only used one manipulative, Base10 blocks, on one occasion. In fact, throughout the duration of the course only two manipulatives, Base10 blocks and two-colored chips, were used, but students expressed their feelings that this was a significant feature of the course. I believe that the students embraced a broad definition of the terms "hands-on" and "manipulative"; they seemed to include the meaning-making component of the classroom learning experiences which included the use of physical models (pictures, diagrams, and manipulatives) with being hands-on (or using manipulatives). When students were engaged in an attempt to understand problematic mathematical situations, I encouraged them to use pictures, tables, objects, mental manipulatives, and various other tactics to help them make meaning in a situation.

Students also indicated they did not struggle to understand as much as they had in the past due to the use of physical manipulatives (or being hands-on). Given that we did not use many manipulatives, this result may have been due to the sense-making focus of our activities which included the use of physical models such as graphs, pictures, diagrams, as well as manipulatives. They seemed to attribute their success in the course (unusual for some compared with past experiences) to the use of physical objects. Due to the number of responses that mentioned manipulatives or hands-on, the impact of the structure of the learning experiences is undoubtedly great; the newness of this experience in a mathematics content course seemed to leave these participants without the language to adequately define and describe their experience. Therefore, instead of recognizing this new breadth in understanding as relating to simply asking "why?" and/or trying to make sense of an activity, they point their success to the use of manipulatives. This type of learning experience was mentioned in 7 of the 32 responses as an enjoyable aspect of learning interactions. Some of their responses that supported this belief include:

- During classroom learning time, I specifically enjoy being able to interact hands-on. For example, it really helped me to better understand after being able to use building blocks. It helps for me to actually see the problem layed out in front of me.
- I like using manipulatives to teach math. It seems like it's easier to explain and understand. Students learn better when they can see why something is the way it is, rather than just being told that's how it is.

- I also enjoy using manipulatives. The hands-on experience not only lets me see it but helps me to remember it later.
- I am a very visual person so I like to see things worked out. I also like to be in groups and see how other people get different answers. I enjoy being able to draw things out and also using the blocks like we did in class.

Discomfort in Classroom Interactions

When participants were asked the question, "During classroom learning times, what specifically causes you discomfort during classroom interactions (if anything)?" 20 of 32 participants who responded to the question indicated either nothing caused them discomfort or that they had not felt discomfort to this point in the class. Their feelings were contrary to my reflections on the classroom learning experiences recorded in my research journal. Only four of the participants mentioned discomfort, citing that if they had to speak or show work in front of the class as a whole they would feel distress (for various reasons).

Upon noting (during class learning experiences as well as responses on the first exam) that many of my students lacked the confidence and perhaps mathematical empowerment necessary to feel comfortable sharing their ideas publically, I modified classroom learning experiences so students were rarely (if ever) asked to come to the front of the classroom (either as a group or singularly) to present their work. Typically I walked around the room observing and taking mental notes as the students worked in groups on a task. When adequate time had elapsed to ask three or more groups to share their findings, I interpreted what they were saying and wrote it on the board, trying my best to capture their problem solving process. My mental notes as well as knowledge of the problems aided me in this portion of the class activities. Additionally, when referencing a picture to solve a particular problem, I would say “Here is what I saw as I observed the groups working” and then draw the variety of pictures that I had observed with guidance from the groups for details. This was typically followed with the question “Did anyone do it differently?” in order to include those that I might have missed. Only in rare cases (I could not interpret their verbal descriptions) did I ask a person or group to come to the board and present their work.

This classroom practice seemed to have a positive effect in that by the first exam, most felt comfortable with classroom interactions. I worried that this allowed participants to remain reticent in sharing their work and/or ideas and put the responsibility on me to be the mathematical "authority" in the classroom. This was a problem for the duration of the semester for my Tuesday/Thursday class and caused me much frustration.

Out of the group of participants who mentioned discomfort, several mentioned moving on before they were ready or uneasiness with working in a group before they had had adequate time to think about the problem on their own. After reading their responses, I addressed the latter by announcing to the class before they started working in groups that they should feel free to take time to read, understand, and work on the problem(s) individually before working as a group if that was more comfortable for them. This helped to establish the classroom practice of a "think-partner-share" work atmosphere even though it was never specifically referred in that name. The two participants that mentioned being uncomfortable due to moving on too soon were students who seemed to struggle with many aspects of the class. They were not representative of the class as a whole. Thus, I addressed their concerns in a note, offering extra help outside of class time (few took advantage of this help).

Specifically what would you change?

Participant response to the question "During classroom learning times, what specifically would you change about the way in which classroom interactions occur (if anything)?" revealed that 26 of the 32 returned with an answer of "nothing" or its equivalent. Examples that espouse this finding are shown below:

- I enjoy for the class to be primarily small group time and the last portion of class to be discussion as a whole group. I like to hear other peoples reasoning!
- I wouldn't change anything about the way class works. I enjoy this class, and that is a big thing for me to say because I strongly dislike math. But this class is making it more enjoyable.
- I actually would not change anything. The way we work together in this class has taught me a lot. I enjoy seeing how the others learned as well as how I learned it.

Final survey

The overall consensus, by those who completed the final survey following the end of the semester in which data were gathered, was unanimous in that they all liked the structure of classroom learning experiences. Several mentioned the consequent effect on the ways in which they will approach teaching mathematics in their own classroom (this will be discussed in a later question the study sought to examine). Some examples supportive of this finding are:

- I like class discussion better than lectures. Personally, I learn better that way.
- I actually like discussions because it gets all of us involved and lecturing tends to bore the students or never gives them time to ask questions.
- Since I first experienced a discussion-type class, I have thoroughly enjoyed this type of format rather than a lecture-style seminar. I feel that interaction encourages learning and understanding better than merely listening to a professor talk about his or her preferred subject. I think that by utilizing this kind of classroom structure, an instructor encourages both teacher-student and student-student relationships which allows for more effective sharing of ideas.

In response to the question on the final survey, "If you were to tell someone else about your experiences in mathematical concepts, what would you tell them?" participants had a variety of responses ranging from indicating that they liked the class in general to that they learned a lot. The excerpts shown below relay participants' enjoyment of the format of this non-traditional mathematics content course.

- …I had worthwhile experiences in this class and I learned a lot of things that I will be able to bring into my own classroom someday.

- …it was good and it’s the only math class I ever really understood any part of.
- With the people I have already told about this class, I have expressed my surprise that this became my favorite class during that particular semester. Since math has never been my strong point, the fact that I was easily doing well with the unique work was a fact of slight pride in my collegiate abilities (even though the actual performing of math is most like not considered collegiate-level work). I think that I would encourage anyone who had the opportunity to take this/these courses if for no other reason than to experience a new and/or different way of doing mathematics.
- I would tell them that it is a very educating course. It is worth the time and money, especially if your major has to do with education. From being in this class, I have already learned so much about to teach my future students mathematics, and I couldn’t be more ready to get out there and start teaching.

Modifying Classroom Practices

Throughout the duration of this non-traditional mathematics content course, learning experiences were flexible and open to change. As I reflected upon daily interactions with the students as well as their writings (on homework assignments and tests), I modified aspects of the course to better suit the participants as well as goals of the research project. The students’ perceptions of learning experiences captured as part of writing response questions presented previously played an integral role in the continual cycle of reflection on action and reflection in action. For example, although most participants noted that "nothing" caused them discomfort during classroom interactions, my journal entries revealed, in addition to a few class features, my belief that the students felt unease with the idea of presenting findings in front of the class as a whole. The following reflections provide insight into this belief:

- Back-row girls still largely uncooperative. They seem defensive about the way in which we learn—almost implying with their language and manner that I am not doing my job and being a "good" teacher by helping them/spoon-feeding them during learning experiences. I get the sense that they are uncomfortable with mathematics and lack confidence.
- Still some anger and discomfort with drawing pictures; students slamming paper down and childishly exclaiming "I can't draw a pic." Many students ask "Why can't we do it the old way?" showing discomfort with the structure of the learning experiences since they is most likely a pronounced difference between them and what they have experienced in their past mathematics classes.
- I seem to have a lot of procedurally-driven students (future teachers!) that cannot seem to represent the situation with a picture. I am not sure as to why but I feel that it is because there is disconnect between the "real" world and the "math" world. Therefore, mathematics is simply steps and key words. It does not have to be used a lot in a logical way—"sometimes it makes no sense, period." I think that there is a link between actually being able to represent a mathematical situation and thinking that math is more than rule and procedures.

Beliefs

The second question this study sought to explore was "From the preservice teachers' perspective, what impact, if any, does participation in this course have on mathematical beliefs?" In considering this question, I looked at various responses throughout the semester in which my participants stated their beliefs in answer to questions stated both directly and indirectly about their mathematical beliefs. Additionally, the majority of the data used to answer this question were the responses to questions students were given as a portion of their take-home final.

Prior Beliefs

The first homework assignment was accompanied with many questions, misunderstandings, and negotiations of classroom practices related to format, length, and expectations. Many of the responses to the homework for Class 2 brought to light beliefs held by participants prior to this course. The assignment was turned in at the beginning of Class 4 which is important to note due to its proximity to the beginning of the semester. This potentially had an effect on responses in that "common beliefs about mathematics" were discussed on the first day of class and that perturbations and negotiating social norms dealing with the belief "there is one right way to solve a problem" had already occurred through classroom learning experiences involving group problem solving and presentations displaying numerous solutions to problems.

What is mathematics?

As part of their first homework assignment, participants were asked to respond to the question "What is mathematics?" in their homework journals. I asked them to define mathematics in their own way without using other methods of finding a definition such as a dictionary or Google. The depth and language used in their definitions of mathematics helped to illuminate their beliefs about mathematics, as well as whether they might have used other sources.

The majority of the responses, 13 out of 22 responses, mentioned mathematics as primarily being about solving problems. Also stated, somewhat implicitly in a few cases, were the beliefs that "the goal of mathematics is to obtain 'right answers'" and "elementary school mathematics is computation." Sample responses included:

- The manipulation of numbers in order to appropriate [*sic*] and solve numerical problems. Basic math is comprised of addition, subtraction, multiplication, and division.

- The study of numbers and patterns. It is being able to manipulate numbers to create and solve problems. There are many formulas in the study of mathematics.
- Problem solving (typically with numbers) it involves strategies and often times using memorization.
- A subject in school where kids learn how to add, subtract, multiply, and divide among other things. They learn how to do problems they never thought they could do with numbers and letters.

Some participants' responses revealed a deeper understanding of mathematics in their beliefs. For instance, one participant stated that mathematics is

> the use of numbers to solve problems. It can be used to find out how many, or how much; but can also be used to find values or relationships between two things. It is not simply methods and formulas however. Math is common sense problem solving, through the use of numbers and values.

Another stated that "Math to me is just another science. A science involving numbers, quantities and applying these numbers to real life situations in which they can be used. Math is critical thinking just with numbers." These two responses appear to indicate the belief that mathematics is thinking critically about real-world situations and has sense-making as a goal rather than a view of mathematics as simply "solving problems."

What is most vital to learn in mathematics?

Responses to the prompt "What is most vital to learn in mathematics? Why?" indicated that the belief that "elementary school mathematics is computation" was centrally-held in that it seemed personal and strongly held and can be seen by participant's use of the following descriptors: important, always, personally, and believe; these descriptors were used in 10 of the 19 responses. Additionally, other common mathematical beliefs revealed by participant responses were "mathematics is a set of rules and procedures" and "learning mathematics is predominantly about memorizing." The following participant responses support this idea:

- I think basic adding and subtracting because it can always and will always take longer but it is possible for all ages. First objective you learn.
- It is most vital to add and subtract. Because at least with this skill children as adults can manage a checkbook, pay bills, and care for the needs of their own children.
- I think the most vital is learning how to do the basics that we are taught when we are little, like adding, subtracting, etc. Why is because it's the foundation of math, without that you can't really do any of the other things later on.
- The most vital thing to learn for math is counting. In order to work an equation or to simply add numbers, a person must be able to count whether it be on their fingers, in their heads or with visual aids, I feel it is most important.

Interesting to note were the number of responses, 7 out of 19, indicating the belief that "there is one right way to solve a problem." The number of responses dealing with this belief could be due to the proximity of this assignment to the first day of class in which this belief was briefly mentioned. It could also be due to this belief having been addressed (although not explicitly) and perhaps perturbated during every classroom learning experience to this point in the semester through the structure of the classroom. For example, multiple groups were asked to share the way they approached the solution of the problem sets thus seeking to challenge the belief (if priorly held) that "there is one right way to solve a problem."

Do beliefs help or interfere with teaching children?

As part of the homework assignment for Class 2, ten students chose to respond to the question "How do beliefs help and/or interfere with the ability to teach children to problem solve?" As mentioned earlier, students seemed to focus on the belief "There is one right way to solve a problem" in their discussions most likely due to classroom experiences discreetly focused on perturbating this belief. Instead of addressing how they feel beliefs, in general, affect the ability to teach children to problem solve, participants instead focused on a specific belief that they perceived as problematic with a teacher adequately teaching children to problem solve in mathematics. Some responses can be seen below.

- By thinking there's one right way to do something. Every child is different in their way of thinking and processing information. So if a teacher has a belief that this is how something has to be done, but the child does not understand that method of thinking, then it would therefore interfere with the ability to teach that child problem solving.
- I think when teaching children math, you should first let them try to figure out a way to solve the problem. If you teach children one specific strategy, they won't ever want to think. They might believe that there is always only just one answer to questions in life or that there is just only one specific way to do something such as folding towels.
- While the belief that the goal of math is to obtain "right answers" provides something tangible to work towards it can overshadow the process if one is struggling to get that "right answer". Especially is this belief is paired with the belief that math problems should be solved quickly.

Only two participants responded to the question with a general statement about their opinion on how beliefs interfere with the ability to teach children to problem solve.

- If we as teachers hold certain beliefs, then our children (even though we aren't actively teaching those beliefs) might pick up on them and inhibit their natural curiosity and creativity. There is also the possibility that our beliefs may cause us to neglect something that may have better aided a student as they're trying to learn a new concept.

- Beliefs about math, if negative, can hurt a child's thinking about math. If you as a teacher believe math is hard, your students will as well because they feed off of your attitude. If you have a "can do" attitude about math, your class will be more willing to learn.

Should skills development be the focus in teaching children mathematics?

A little over half, 14 out of 23, of the responses to the question "What skills are we trying to develop in teaching children mathematics?" mentioned problem solving as a skill to develop while teaching children mathematics. Participants' phrasing in describing this trait included: problem solving skills, independent thinking, critical thinking skills, logic or reasoning, and complex thinking. Many mentioned the notion of everyday life as part of their response. They believed that understanding mathematics was helpful or necessary for success. Some of their responses supporting this finding include:

- I believe we are trying to teach children how to solve the problems instead of just memorizing answers.
- By teaching mathematics, we are trying to develop critical thinking and problem solving skills. We want children (and ourselves for that matter) to have good

reasoning skills in everyday life as well as in mathematics. All of these thinking skills can be obtained through our math classes if presented correctly.

- When teaching children math I think the goal is to not only provide them with the tools to do computation but to also build their logic, reasoning and problem solving skills in order to create independent thinkers.
- We are trying to teach them logic and how numbers work. Hopefully they can apply it to everyday life, as long as it’s not too abstract.
- By teaching children mathematics we are trying to give them skills to get through everyday life. Whether we realize it or not we use math every day. And if we don’t’ teach the children proper math then they may not be able to get throughout the day as easy as they could have otherwise.

Other notable mentions in regards to skills were confidence and empowerment. Participants felt that students need to develop confidence, and additionally mentioned empowerment implicitly by letting students do problems “their” way.

Beliefs about the Goals of Mathematics

The homework assignment given for Class 21 proved to be significant in that it served to illuminate participants’ prior and transitioning beliefs about mathematics. The first half of the assignment asked students to address specific questions to do with their beliefs about the goal of mathematics. The second half of the assignment asked students to choose a mascot to represent mathematics and then describe why they made that choice.

Is the goal of mathematics to find the "right" answer?

For the first half of the homework for Class 21, students were asked to respond to the following: In mathematics, we almost always find an answer to a given problem or problematic situation. So,……

- Is the goal of mathematics to find the right answer?
- In mathematics teaching, which is more important, the answer to a problem or the process of working on the problem? Which one and why?

Thirty-four participants chose to complete this portion of the homework assignment. Of those who responded to the first question, 15 answered yes, 9 answered no, 9 were neutral, and 1 answer proved indecipherable as to opinion. The following category boundaries were determined from the analysis of not only the base answer but also the descriptions that followed them.

Participant responses were placed in the category of "yes" for either simply responding "yes" along with some supporting statements or they responded "no" but their supporting statements actually maintained agreement with the belief that the goal of mathematics is to find the right answer. The following are examples of both types of responses:

- Yes, I think the goal of mathematics is to find the right answer but not in just one way.
- No, getting the process down will lead you to the correct answer through steps of manipulation.
- The goal of mathematics is to learn a process to find the right answer. In mathematics it is important to find a process that will help you get the correct answer.
- I think so. We need to know the process to solve the problems but the goal is to find the right answer.

Participant responses were placed in the category of "no" for either simply responding "no" or for stating that the answer was important but not the main goal. What was most interesting about the results of this category was that of the nine participants in this category, many had no supporting statements to support their belief. Additionally, some of the supporting statements given were neutral; their beliefs seemed not fully evolved or explicit at times. Participants mentioned phrasing such as knowing how, figuring out, and understanding. Whether this was due to the structure of the classroom or simply lack of reflection on their part remains unclear. The following shows answers supporting this category:

- It is important that the right answer be reached in math, but I would not say that is the ultimate goal.
- The goal of mathematics isn't to just find the right answer, but to learn different ways to solve problems and come up with your own ways of finding the answers.
- No, I don't think it is. I think it is [to] understand how to find the right answer and to understand why it works.

Participant responses were placed in the category of "neutral" if their answers indicated that they felt both the process and the answer were equally important. Again, there were some responses from participants that could be considered borderline for the no category. The following are examples from this category:

- The goal of mathematics to me, would be finding the correct answer and being able to explain how and why you got that one answer. If a student can explain the reasoning it means more than getting the answer right. Maybe a student is working the problem out correctly but isn't getting the correct answer but they understand what they are doing. If a student knows how and what they are doing and can explain how this happens would be the main goal in mathematics.
- Yes, the goal of mathematics is to find the right answer. However, the goal is also to understand how and who you found the right answer.
- Yes it is but that's not the only goal. It is important to know the process.
- The ultimate goal in mathematics is always to find the right answer, but the process of getting there is equally important, as is what you may learn by finding the wrong answer. So even though the goal is to find the right answer, actually learning from it is a goal too.

What is most important—the answer or the process?

The second question in which participants stated whether the answer or the process was more important produced interesting results. Of the 33 participants who answered, 4 answered "both" while 27 said the "process" and only 2 said the "answer" (and one of those was somewhat borderline "both"). This is quite interesting to note considering the fact that the answers to the first question were not quite as skewed. Since the majority of the participants felt that the process was the most important aspect of mathematics teaching, the responses to the first question were re-examined in light of the second with a conflict in beliefs between pedagogy and mathematics in mind. I found 11 examples of participants with differing beliefs about mathematics and mathematics teaching; for each of them, they answered the first question with "yes" and the second with "process." Below are a few of the supporting examples of this finding:

- Q1: I think in mathematics that the eventual goal is to get the correct answer. Of course it is important to understand the overall process it took to get that answer. However, when you are solving a mathematics problem, you don't just do it for the fun of it, you do it to get the answer.
  Q2: In the teaching of mathematics the process of working the problem is more important than the actual answer. This is because you are actually teaching

children how to do something, and what good will it do your students?

- Q1: Essentially yes! Mathematics equations generally have only one answer, but developing the "know-how" to achieve those problems and understand the processes is extremely important as well.
  Q2: I think that working the problem is the most important part in mathematics. If students do not understand the processed necessary for working certain problems, they will never get the correct answer.
- Q1: Yes, I believe the ultimate goal of mathematics is to find the right answer. Otherwise it wouldn't e the standard method for working with quantities and computations and all the other utilitarian purposes it fills.
  Q2: When teaching math the process is more important. While it is possible to memorize rules and arrive at the correct answer without understanding how or why it is a more difficult and less accurate approach.
- Q1: I don't think the goal of math is to find <u>one</u> distinct answer. In many cases there are many answers but whatever answers we find should be correct.
  Q2: I believe the process of working problems is most important because it helps students understand what is going on, and what/why they are doing what they are doing, getting answers sometimes is easier than knowing how you got them and being able to explain the process that you went through to get them, as teachers I think we should be more worried about processes and the pursuit of knowledge, however answers and tests help us gauge whether or not our lessons have had any effect on our students.

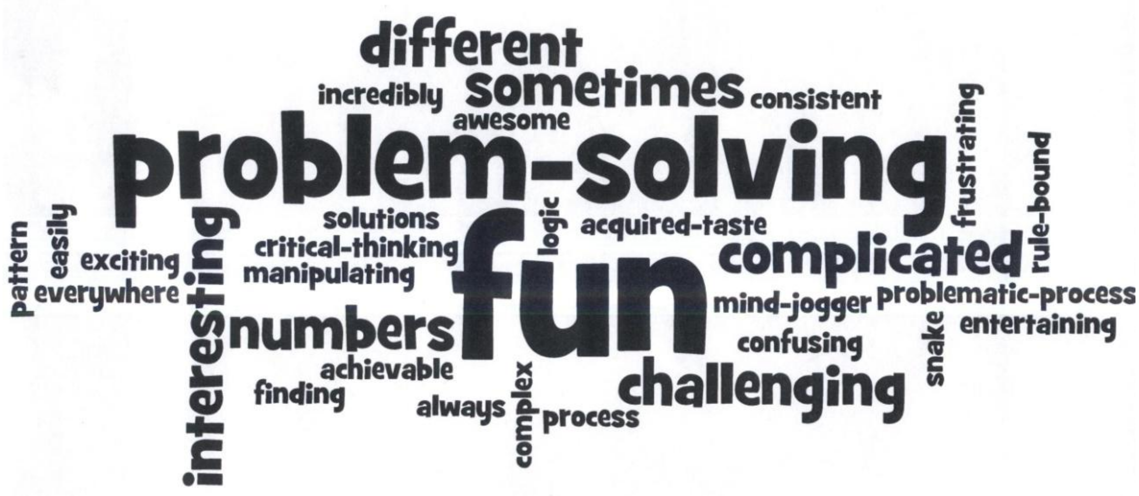
Analyzing the responses to these two questions was extremely difficult because like most aspects of education, the relationships between their mathematical and pedagogical beliefs were convoluted, dynamic, and conflicting yet made sense to the participant.

Mathematics is....

In order to maintain consistency and social norms for the testing portion of the course, I added a small written portion of the third exam in which students were given credit for giving a response regardless of the response. Again, this was to reinforce the social norm established on the first day that credit would be given not for opinions but for thoughtfulness and thoroughness *only*. While the instructions on this particular exam were to "Complete the sentence (Math is…) with a one/two word descriptor" which did not involve in-depth writing, it did involve an opinion thus I sought to further their trust in me and continue to establish and nurture this particular social norm. I did this knowing that this social norm needed to be firmly in place before the take-home final since it would be specifically requiring them to state their beliefs about mathematics in the context of the course.

Their answers to "Math is…" were analyzed and represented using a wordle. Of the 37 participants, 36 responded to the statement with 12 statements being one word descriptors and the rest two or more.

Figure 2: Wordle depicting participants' responses to "Math is..."



Examination of the data displayed in the Wordle revealed that some of the most prominently used words were fun, solving, problem(s), challenging, complicated, and interesting. The most interesting result was that the word "fun" was used more than any other word. This was surprising both because of students' defensive attitudes during classroom learning experiences and participant responses to earlier homework journal assignments. Perhaps this result can be attributed to the influence that I exerted as the teacher for the course. Despite the fact that I assured them multiple times and ways that credit would not be given for opinions, perhaps the fact that it was a one or two word response left them feeling vulnerable. It seemed that in writing long responses students felt freer to share their true opinions.

The next most prominently used word was "problem-solving" which was not particularly surprising given that mathematics is typically centered on a problematic situation or problem, but the context in which participants were referring could be different. Whether their view of mathematics was as being problematic or mathematics as merely solving problems could not be ascertained by this particular data.

Beliefs about Mathematics Revisited

In an effort to determine the impact of this course on students' beliefs about mathematics I provided a final opportunity for students to share their thinking. Thus, as a portion of their cumulative final, there was a written take-home exam which asked students to *thoughtfully* and *thoroughly* respond to four to seven of the listed mathematics beliefs in light of their experiences from the class. Additionally, I gave them some prompt questions such as "How did you feel about these beliefs at the beginning of…" to help in explaining the task. Except for the fourth exam, there had been a writing component of varying length and opinion-level on the exams; some were take-home and some were not. Therefore, due to the trust and confidence in sharing opinions established through prior homework and exams experiences, I felt comfortable with the reliability of the data I received. The mathematical beliefs are listed below along with the number of participants (out of 37) that addressed that particular belief in their take-home final.

- There is one right way to solve a problem (36)
- Mathematics is a set of rules and procedures (18)
- Learning mathematics is mostly memorizing (25)
- Elementary school mathematics is computation (6)
- Mathematics problems should be solved quickly (28)
- The goal of mathematics is to obtain "right answers" (29)
- The teacher and the textbook are the mathematical authority (22)

Although students were asked to "thoughtfully and thoroughly discuss your feelings about 4-7 of these beliefs in light of your experiences in this class and its consequent affect or non-affect as well as why," some participants did not explicitly state if the beliefs they professed in their writing had changed or remained the same as a consequence of the course. The discussion of the findings will illuminate this where necessary.

Due to the fact that students chose particular beliefs to respond to based upon their classroom experiences, it was significant that the belief addressed most often was "There is one right way to solve a problem" followed by "The goal of mathematics is to obtain 'right answers'" and "Mathematics problems should be solved quickly." Since the majority of classroom learning experiences confronted the first and third most chosen belief, the fact that these were chosen was not a surprise. The fact that students chose to write the goal of math is to obtain right answers follows since examining this belief was part of the second exam as well as a follow-up homework assignment. Therefore, these three beliefs were most prominently dealt with and articulated in the duration of this non-traditional mathematics content course.

One right way to solve a problem

Not one participant reflected that he or she felt that there is only one right way to solve a problem. The participants that wrote about this belief, 13 out of 36, stated that the course effected a change in their beliefs. Of the participants that held this belief before the class, many stated that their belief was stronger after the class. For instance, one participant stated "'There is one right way to solve a problem,' this I don't agree on for one because of all the "hundreds" of ways you had us to try and solve one problem. Also, I felt this way before this class but now I have a strong belief in this now, and probably always will." Another wrote

> Of all the beliefs that have been stated, "There is one right way to solve a problem" is probably the one that I am most confident in answering: I disagree. During the course of this class, we discovered and discussed numerous ways to work the same problem. The multiplicity held true for all of the topics we covered, and we covered all the cases. In mathematical problems, there is a starting point (the problem) and an ending point (the answer) but it doesn't matter which road you take from one to the other, as long as you get there in the end.

For the participants that prior to the course believed "there is one right way to solve a problem" due to past experiences in mathematics classes, the structure and format of course experiences proved fundamental in changing their perceptions about mathematics and the ways in which they believed mathematics should be taught. Examples supporting this result follow:

- There are beliefs about math that I used to live by and believe one hundred percent, the belief that "there is one right way to solve a problem" and I saw this as true. Now because of this I also thought that there was only one way of solving each problem and the right way was the way the teacher did it on the board. In fact if the problems weren't done verbatim we were docked points. But after taking this class I was shocked to see all the different ways that each problem can be solved. Problems that were difficult for me in the past became so clear and I was able to do them with ease.

- One of the things I had to think deeply about in this class was whether or not there is one right way to solve a problem. When I was in high school I had a math teacher who only taught me one way to solve a specific problem, and acted as if that was the only way to do it. In this class I have realized there is usually more than one way to solve a problem, and that not just one specific way is right. I think that teachers should teach their students the simplest method to solve the problem and then also encourage them to find a method of their own that works best for them.
- Before this class I thought that there was only one right way to solve a problem because growing up my teachers only showed me one way. Some teachers even required that if we did not do it their way we would get points taken off. This has even happened to me at this college, but now I think differently. In lectures I was able to see multiple ways students solved problems to get the same answer… Now I believe that there is more than one way to solve a problem and not one way is the right way. This will affect how I teach my classroom because I will teach the students multiple ways to solve the problem and whatever way they like best they can do. I will be open to new ways because they might find ways I would have not seen before. I want my students to succeed in my classroom and making them only doing *[sic]* it one way will not have them succeed because some students will not understand how to do it that certain way. So allowing them to do it the most comfortable way to them, they will increase their success.
- "_____, you cannot solve those problems that way, do it how I showed you on the board." I heard this as an

elementary student and did not know why I had to do it the same exact way as my teacher as long as I got the right answer that should have been enough. Because I was taught to do a problem out of the book so much as a youngster I felt at the beginning of this semester that you could only do math problems one way and one way alone. During the course of this class, I learned that as long as you get the right answer you can work the problem anyway you want.

Whether this belief changed or as an alternative was further strengthened as a result of the structure of classroom learning experiences, many participants made mention of the variety of ways of thinking about mathematics problems and that they planned to incorporate this new-found perspective into their own classrooms. Many of the mentions of pedagogy related back to differentiated instruction and the fact that everyone's brain works differently. I feel that these participants' pedagogical statements highlight the point that these preservice teachers were making connections between this course and their education courses. Some examples holding up this finding follow.

- I believe that teachers should teach more than one right way to solve a problem. Not every child that you teach is going to learn the same way. There's no such thing as a perfect class. As a future teacher, I may start with one way, but if I notice some students aren't getting it, then I will approach that group of children with a different way.

- I do not believe that any student should be limited to just one single right way to solve a problem. They should be free to find a way that works with them and they can remember how to do later on. When students are limited to only one way to solve the problem they may not fully understand how to do it and when it comes to the test they are left not knowing how to do it and being held accountable when they could've had their own way to solve the problem.
- When I first started this class, I thought that there was only one way to solve a math problem. As the week went on in class I found out that there was more than one way to solve a math problem… So I feel that there is no one specific right way to solve one problem there could be many different ways to solve one problem. With this knowledge now I can help my children learn that it is okay that you don't understand the way I showed you on the board. Maybe we need to figure a way that you can still get the same answer and understand how to work the math problems. It has showed me that not everyone is the same.
- I like that you can find a way to solve a problem according to how the numbers make sense to you… I believe it's important for teachers to teach a few different ways to solve problems so that the students can find the ways that work best for them. I don't think it's a good idea to put things in a box and label it the only correct way.

Mathematics as a set of rules and procedures

Of the 18 responses to the mathematical belief that "mathematics is a set of rules and procedures" none of the participants disagreed with the statement, and of those 18 only 5 showed a partial belief (only revealed minimal support in the belief). It is possible that this belief had little change associated with it due to the level of strength within which this belief is often supported and nurtured. Mathematics is traditionally taught with rules and procedures; despite the fact that all throughout the semester in this non-traditional course their classroom learning experiences focused on conceptual understanding rather than procedural understanding (which caused perturbation of this belief), this deeply ingrained belief from participants' "apprenticeship of observation" through their many years of formal schooling demonstrated little change. Nevertheless, the encouraging note is that this study indicated that some change did occur. Of the five that showed a partial belief, three explained their partial change in this belief due to the structure of the course (the other two were inconclusive based upon their statements as to how they felt before the class). Following are the three statements supporting this finding.

- In the beginning I would have said yes that is all mathematics is but throughout this class I have learned that it is to a point rules and procedures. Also math uses a lot of mnemonics like, Please Excuse My Dear Aunt Sally; we use this with the order of operations. But we do not have to follow a specific rule or procedure every single time we solve a problem. However, in a classroom environment, teachers need to be able to teach them because in math they are used and used throughout any future level of math.

- To a certain extent math is a set of rules and procedures. You have to do baby steps to learn math. First you learn numbers and what the numbers mean. Then second you learn how to add those numbers then you also learn how to take away the numbers… Through this class, I learned that there are many rules or should we say guidelines for math. I always thought that math was black and white now I know that there are many gray areas. I feel this way because you can bend some of the math rules to get the answers to a different mathematic equation.
- "Mathematics is a set of rules and procedures," is partially true. Those things are important, but I don't believe that's all there is to math. Math is about thinking about things. You have to think about how they work, why they work, and when they work. The rules and procedures help along the way, but they aren't the main idea. I think I came into this class thinking that math was only rules and procedures, but my mind has really been opened to the idea of thinking about problems deeper.

Additionally, it is important to note that a few of the participants' arguments for believing that mathematics is a set of rules and procedures are critical, well-thought out, and are logical with regard to some aspects of the nature of mathematics. For instance, one participant stated that

> Another strong belief I was caused to critically think about in this class was whether mathematics is

> just a set of rules and procedures. I actually agree with this statement. I felt that mathematics was a set of rules and procedures before this class, and still continue to after it. Mathematics is something I just do not enjoy doing so maybe I don't have another fun way to explain it, but to me this definition of mathematics makes sense. Every mathematics problem you solve has some sort of procedure you have to follow to get the answer or rule you have to follow. The procedures and rules may vary depending on the method the person uses to solve the problem, but still at the end of the day you have to follow some sort of procedure or rule to solve the problem.

While a person could develop a set of rules and procedures for every mathematics problem and a general set of rules and procedures for certain types of problems (which is where the "rules" in mathematics books came from), simply knowing rules will not necessarily help anyone be a mathematical problem solver. Furthermore, knowing rules only is problematic given that rules can be applied to the wrong scenario, as well as forgotten or misinterpreted. Thus, although this participant's reasoning is sound, it demonstrates only a surface level of understanding about the nature of mathematics and the way in which mathematics is "done" despite the example of the format of the course contradicting this traditional procedurally-driven belief.

Mathematics learning as mostly memorization

Of 37 participants, 25 chose to write about their perceptions on the mathematical belief that "learning mathematics is mostly memorizing." Of those 25, 8 agreed, 6 disagreed, 10 had a partial belief, and 1 was incomprehensible with regard to this belief. The label of "partial belief" was given for those whose proclamations demonstrated support for both sides. I looked at those participants that agreed with this belief compared with their statement about the belief "mathematics is a set of rules and procedures." I was interested to see if the beliefs go hand-in-hand for some participants. Of the five who fit this comparison, three believed both statements; further, their beliefs on both seemed firmly entrenched. The other two had partial beliefs about mathematics being rules and procedures.

Although there were only six participants that disagreed with this mathematical belief, this minimal result is not surprising. Similar to the belief that mathematics is a set of rules and procedures, this belief tends to have a firm foundation based upon prior learning experiences in which rules and memorizing those rules were the key to success in mathematics and how mathematics was “done” and taught in traditional classroom learning experiences. Of the six that disagreed, three explicitly stated that their experiences in a non-traditional mathematics classroom helped to change their perspectives. Two examples supporting this follow:

- Before this class, I thought math was mostly memorizing. But now that I think about it, I have not memorized anything for this class. I used to try to remember formulas for certain math problems. That is not the case for me anymore. I remember how to do things because I have been shown why they work. If I forget how to finish a math problem, I know now how to work it out from the beginning in a way that makes sense. The memorization comes naturally once the student understands why and how something works. I do not have to try to memorize rules, I just know them.
- “Learning mathematics is mostly memorizing” is a basic way for getting most students through tests. My friends and even some teachers would say do not stress over this test it’s all about memorizing. So I believed them and would try to memorize crazy equations and the answers. I did more work than necessary because instead of cramming a bunch of numbers and equations I could have found ways to make each problem more appropriate for me. For example we learned that to add 12 and 5, take 2 away from 12 and you get 10. 10 and 5 are easier to add than 12 and 5… You do not need to memorize to do this and that is why my thought process on this has changed.

Of those ten participants that had only a partial belief that learning mathematics is mostly memorizing, four plainly stated that this change occurred due to the course. One participant stated

> I used to strongly believe that mathematics was mostly memorizing before coming into math concepts one this semester. Throughout the course of this semester my belief on that issue has changed some. I do still believe that a big part of mathematics is being able to remember things and commit them to your memory, but I have now added the belief that you have to be able to understand the concept. I have added to my beliefs about mathematics that another big part of mathematics is knowing the why and how of things; such as why you have to memorize and certain formula or algorithm and how and why that formula or algorithm works the way it does.

Another explains,

> At the beginning of math concepts my beliefs about the concepts of math were completely different than they are now. I never stopped and thought about the process of math and how it is studied as well as taught. If learning mathematics were as simple as memorizing a set of formulas, every student would excel in this area. Most students' associate math with memorization by the way this subject is taught. When I was in high school I fell under this category. My teacher only taught one method of solving problems to the class. If you had a separate approach to each problem and received the same answer it was still wrong. This teaching style resulted in most students associating math with memorization.

In conclusion, the minimal change in beliefs of the participants in this category, 4 out of 25, most likely can be linked to prior mathematics classroom learning experiences that were traditional in nature and served to strengthen and deepen this belief. As a result, perturbation seemed to only affect a small amount of change in this strongly-held mathematical belief.

Elementary mathematics as computation

Although only six participants chose to write about this belief, statements from earlier data revealed strong participant support for the belief that elementary school mathematics is computation. The one participant agreeing with this belief stated, "I agree with this because in elementary school you are basically just learning the basics of math, nothing too great in depth." Four of the six participants did not agree with this belief; two participants showed direct backing of their change in belief based upon their experiences in the course. Their statements are shown below.

- I thought elementary school mathematics was just computation. But, now I do not think that elementary school mathematics is just about addition and subtraction. I think that they do a lot more complex problems other than addition and subtraction. This class showed me all the problems that an elementary math student would do. I thought that algebra was taught in high school, but it is taught earlier than I expected.
- Having only been taught the traditional algorithms, I did believe that elementary school math was all about computation. While it is of course a vital part of what a student should learn, it is not enough. I believe an elementary school student should be

> gaining a number sense that is not imparted when the focus is solely on computation. The number sense that I have been self-taught and self-constructed. It was not until this class that I even realized I could develop a stronger number sense. At 38 years old I have gained a new confidence in my ability to do math. Imagine what I could have accomplished if I had gained that in grade school!

Mathematics as a process of speed

When participants were asked to thoughtfully and thoroughly discuss their feelings in light of their experiences in this class about the belief that mathematics problems should be solved quickly, only one person professed minute support of this belief and was categorized as “partial belief.” Those that were placed into the category of partial belief (9 out of the 28 respondents) represented those who did not believe that the focus should be on solving mathematics problems quickly but still placed some level of importance on speed; many felt that there was a place for speed in mathematics mostly due to testing. Their beliefs seemed to be more peripheral in that they were not strongly held. Many of these participants did not explicitly state the influence of the course on this belief (only 5 of the participants stated explicitly that the course changed their belief). Examples of this result follow.

- Math problems do not have to be solved quickly to be solved correctly. Timed math tests can be very challenging for students, myself included. It was common knowledge that whoever finished these tests first in class was the smartest student. If you finished last, you were obviously the dumbest student in the class. But solving math problems quickly does not make you any better at math than the student who solves the same number of problems correctly in longer amount of time. These timed tests make children feel inferior to their peers. Because of this class, I will not allow my students to take timed tests in the way that I did in school. I will have my students keep their tests at their desks and turn them in all at the same time. It will still be timed, but no one will know who finished and who did not. That pressure is too great on young minds. Sometimes, it is important for math problems to be solved quickly. But this is not the case all the time. Speed is not what is important when it comes to math. Correctness and method should be placed above all other aspects with speed being one of the least important aspects.
- I don't think it is vital for students to be able to solve a math problem quickly, but I do think it can be beneficial for them. However, I don't believe the focus should be placed on speed. I think the biggest objective should be to help the student understand the process of *how* to solve the problem. Being able to solve a math problem quickly helps, but it won't matter if the student doesn't know how to find the right answer.
- I came into math concepts solving problems quickly and I just assumed that speed is something that should not necessarily be taught but worked on through practice. I still stand by this statement and my view on it because of standardized tests. My perspective on this statement is that students should be introduced to a concept and then taught all the strategies to solve problems that represent the

> concept. Then the students can explore which strategy works for them and being to use the strategy to work the problems. As the student practices a problem using the strategy the student will begin to understand the concept better and speed will become natural to the student. I think it is important for students to practice the strategies they are using because it allows them to not only gain speed but to become more efficient in solving the problem. As a teacher I feel that you should strive to have your students become consistent in speed and getting the correct answer because I feel that they need to be able to do this when they are taking timed standardized tests.

Five of the participants explicitly stated that this non-traditional class changed their feelings about the need for mathematics problems to be solved with speed. For instance, one stated that

> At the beginning of math concepts 1, I felt that math problems should be solved quickly to excel in mathematics. I feel completely different about the time frame used to solve each equation. Upon completing this class I feel more comfortable taking my time with each problem to make sure I get the correct answer. It is not fair to expect a whole class of students to complete a set of problems in a certain time frame. Every child learns at a different rate as well as excels at a different rate. Children should be set to a certain standard, but not all children should be set to the same standard.

Another pointed out the fact that those not able to solve mathematics problems quickly in a traditional format often feel stupid or that they are not good at “doing mathematics” by the following account

> I always thought I was bad at math because I was always the slowest to answer the problems and it made me feel stupid. But now I do not think solving the problem quickly matters. What I think matters is doing the problem correctly and making no errors and doing all the steps. So, in my classroom, I am not going to influence the students to think faster is better and smarter. I want to let the students know that it does not matter who finishes first, but what is important is to take your time and make sure you do everything correctly. One way to incorporate this in my class is to give a problem and say wait until everyone is done so that the answer is not yelled out and the slower students get discouraged.

The goal of mathematics is to obtain correct answers

Of the 29 statements of beliefs about whether the goal of mathematics is to obtain "right answers" a larger portion, 11 participants, stated that their beliefs changed. Of those that changed their belief, five disagreed with the belief and six partially agreed with the belief; those that were placed in the "partial belief" category expressed feelings that the process was more important than getting the "right answer" but that getting the right answer was a large goal in mathematics. Examples of those that changed their beliefs and now disagreed are presented first (3) followed by those that changed their beliefs and now only partially agree with the belief (2).

- I believe in this because in mathematics, there is usually only one right answer. However, after this class, my beliefs have changed. I now know the biggest and most important goal of mathematics is to understand *how* to obtain the right answer. It is great if a student obtains the right answer, but they must know how they did it. If they understand the various problem solving processes, then they will know how to solve the problem again in the future or in real-life situations.

- At the beginning of the semester, I believed that math is all about having the right answer, but now I believe that it's not the goal. The goal is that you want your students to understand the problem, understand what steps to take to get to the right answer(s). As a future teacher I would teach my class to show how they got their answer, if they get it wrong I can help them with that problem by showing them where the mistake was made. It also helps the teacher get to know how their students comprehend things and how they like to work things out, or if they have a different way of finding the answer than what you are teaching.
- After this course, I have begun to realize that math can be expressed in multiple areas and that it is much more than writing down the correct answer. The goal of math is not to "obtain right answers" but to understand why the answers are right. It is two totally different concepts. Obtaining right answers does not always mean that the student understands why the answers are right. When I was younger, if I did not understand how to complete a problem, I would just look in the back of the book and write down the answer. I then would make up the procedure I used or just say I did it in my head and show no work. If the objective of math is to obtain right answers, then I was successful. But that is not the purpose of math. I did not understand why my answers were correct. That has much more value than simply knowing the answer. It is in the process of knowing why that math becomes real to us.
- In this class we had to think deeply on the concept of if the goal of mathematics is to obtain the right answer or not. The belief that most people hold is

that the goal of mathematics is to obtain the right answer, but I am in between with this belief. If you would have asked me this before I took this class I would have said yes automatically that the goal of mathematics is to get the right answer to a problem. However, now I realize that it is also important in mathematics to know and understand how to do the problem as well. If a student randomly comes across the right answer but has no clue how they did it, then what good has that done them?

- I used to think that the goal of mathematics was to get the "right answer" because when I was growing up all my math teachers would only give points for right and not wrong answers. Now I think that getting the right answer is not the only thing that matters. What I think that matters is all the steps throughout the whole equations. Sometimes you can make a small math error and that could get you the wrong answer, but if you do all the steps that will prove that you know what you are doing.

Additionally, only one participant cited no change in her initial feelings of disagreement with this mathematical belief due to the experiences in this course.

The mathematical authority in the classroom is the teacher or the textbook

Although participants were asked to discuss their mathematical beliefs in light of their experiences in this course, on this belief, many simply stated their viewpoint with no dialogue about a connection to experiences in this course or prior beliefs. One might assume, since the instructions asked them to discuss their beliefs and the consequent affect or non-effect of the course, that many changed their beliefs simply by them mentioning their opinion, however, I choose to focus on those that unambiguously stated the change due to the classroom experiences. Four of the participants stated that they had changed their belief that the teacher and the textbook are the mathematical authority due to their experiences in this non-traditional mathematics content course. Of those four, three disagreed with the belief after the course and one stated only a partial agreement with the belief. Following are a sampling of those statements supporting this finding:

- The last belief I will talk about is that "The teacher and the textbook are the mathematical authority." Now this for any child is true because we are raised to respect and listen to our elders. Especially in school, what the teacher says goes. So if it was like this growing up it will be like that in college as well. But what Concepts of Math One has taught me was that the student has the authority. This is because we cannot see what goes on in their head and if they find a way that makes more sense to them then that is what they should do. Everyone is a different learner and teachers it does not mean we get to choose which learner they are. As educators we should be open minded to new ways to solve equations.
- One of the most important things that I have learned is that neither the teacher nor the math book has mathematical authority. When it comes to math you can come up with your very own way to do a problem if you

> wanted to. There are no “set” rules saying that you have to go exactly by what the book or the teacher tells you. I believe that if a child finds an easier way to do a math problem than in the book that he/she should do it that way.

Moreover, some participants explicitly stated that their experiences in the course did not change their beliefs. Of the four that made this claim, two agreed with the belief, one supported the belief, and one only partially supported this belief. It seems that the level of robust feeling with which this belief is held affected the amount of change able to be had. Examples of agreement and disagreement are shown below.

- In the beginning of Math Concepts, I believed that the teacher and the textbook are the mathematical authority. My belief still hasn’t changed during the semester. I still believe that the teacher and the textbook have the authority because the teacher has been taught to teach us how to learn and concept mathematics, also the textbook has many mathematicians that they base off of. They have all the right answers. If they were not the authority, there wouldn’t be a way they could teach students nor help them with math.

- I don’t believe in this concept. I never have and I probably never will. I don’t believe in this because I know students can solve mathematical problems differently than the teacher, and still get the right answer. Teachers need to understand that sometimes they can be wrong, and so can the textbooks. While I do believe teachers should have classroom authority and I support using the textbooks, I know that everyone makes mistakes, and that teachers should be willing to also learn from the students.

Mathematical Empowerment

A significant result of this study was that the structure of the classroom affected the mathematical empowerment felt by the participants. The third question this study sought to explore was "What influence, if any, do preservice teachers believe the curriculum and structure of this class had on their empowerment?" Analysis of the data related to this question revealed that participants' perspectives about the nature of mathematics as well as their-selves in relation to mathematics changed significantly for many students. It appears that their altered beliefs about mathematics and the culture of the classroom dynamically interacted to affect their mathematical autonomy. Students enjoyed learning mathematics and had greater confidence in their mathematical capabilities. One participant stated

> I think, in general, most of the experiences in this course have enhanced my confidence and enthusiasm for mathematics. Being encouraged to work with out-of-the-box algorithms has expanded my perceived horizons and opened up a new field of interest for me.

Participants' perceptions about the way in which mathematics is "done" as well as their feelings about mathematics changed over the course of a semester in this non-traditional mathematics content course. Many felt more comfortable with the mathematics that they would eventually teach and found a new appreciation for mathematics in general; a few even grew to like mathematics. Participants talked about this change in confidence levels extensively; some excerpts from their writings are as follows:

- After this class, math is still not my most liked subject, however it isn't my most disliked either. I do feel a lot more confident in teaching math to students now that I have had this class.
- I think that this course has made me more confident in learning math because it made me realize there was not just one way to find the "answers" to math problems… I am not sure that I will ever really enjoy math but I am not so afraid to take it on now.
- I feel that, had I been allowed to discover nontraditional algorithms during my early education, I would have enjoyed mathematics much more than I did. Being now able to see different ways to work a problem, I have encountered less frustration and horror when confronting numbers, even to the point, surprisingly, of enjoying applying new-to-me techniques to solve an equation.
- Before this semester began, I had never really thought much about math; I just did it. After going through this semester, though, I now think about questions like "What is math?" or "Why does this happen?" This course has made me think long and hard about different questions dealing with mathematics.
- My feelings about mathematics have changed a lot since I have been in the math concept classes. I find myself liking math a lot more now. I feel like I know more now, and I feel better prepared to teach my students than I did before.
- This class has changed my thoughts about math in many ways. Before this class I hated math and I struggled in all my other previous math courses. This class has showed me that math can be enjoyable and that I can do well in this course and not just squeeze by.

Furthermore, after taking a course structured in a non-traditional format that focused on conceptual understanding, meaning-making, answering “why,” and working in groups, participants’ reported feeling more confidence in their mathematical ability as well as their pedagogical skills to teach mathematics. They spoke about their mathematical and epistemological empowerment related to understanding the mathematics. Participants often linked mathematical empowerment with pedagogical empowerment; they described their newfound confidence to teach others the mathematics content they felt comfortable. For example, one student stated “I know I will be able to teach certain math well because I understand it,” and another said “Since taking this course, I have already begun to help my friends and younger siblings with their mathematical endeavors. I think that with more practice I will be an effective teacher with more than just my stronger subjects.”

Although I initially separated these perspective changes into the categories of “Perspectives on Mathematics” and “Perspectives about Self,” the intertwining of their beliefs about mathematics, teaching and learning mathematics, and their ability in mathematics form such a dynamic relationship that I did not want to reduce it by separating statements into these categories. This is reductionistic and would perhaps not capture the dynamic and complex relationship between the empowerment and confidence that comes for teaching when one understands with depth what they are teaching. Therefore, below are participant statements about changes in their viewpoints after participation in this non-traditional mathematics content course. Due to the nature of the importance of this finding, more than the usual amounts of examples were included.

- In the beginning of Mrs. Harper's class I thought she would be a teacher from the textbook like every other kind of math class I have took in the past. But with Mrs. Harper's class it was different. She not only took a little form the standard textbook but from her own ways. She makes us think outside the box… This class has opened my eyes to a new math world, a math world that I will gladly share with my students and colleagues over the years that will come.
- During the course of this past semester I have learned so much. I was apprehensive taking what I felt was a lower level math class again. Because I am not good at math and have never had good math instructors I felt that it would be like every other generic math class I have ever taken; the kind of class where the teacher stands in the front of the classroom and lectures and teaches only from the book and the examples come straight from the book and no further. However this class challenges its students to think outside the box and to get the answer by thinking in a non-traditional sense.

- I've always disliked math. Since my basic computation skills have been adequate to serve me in my everyday life, I have never cared to learn more or understand more about the subject than was absolutely necessary… Prior to this class I had every intention of getting my certification in Language Arts. Now I am planning to get my certification in math as well. Working with the student I mentioned previously as well as my 10-year-old son has reinforced my belief that gaining number sense is a vital part of the learning process. That non-traditional approaches to problem solving are just as important, if not more so, than the traditional algorithms. After all, does *anyone* actually use the traditional algorithm to do math in their head? I cannot wait to demystify the subject for my future students!
- Yes, I think my ability to teach mathematics has improved because I understand the concepts more fully and know how to present them in simpler ways.
- I am much more comfortable working these types of math problems and I am not as afraid of being asked to teach someone else how to do math.

CHAPTER FIVE: DISCUSSION, CONCLUSIONS, AND IMPLICATIONS

The implementation of the Common Core State Standards for Mathematics (CCSSM) in 2011 marks the most recent attempt to improve the quality of education and are

> designed to be robust and relevant to the real world, reflecting the knowledge and skills that our young people need for success in college and careers. With American students fully prepared for the future, our communities will be best positioned to compete successfully in the global economy (http://www.corestandards.org/),

With the adoption of the CCSSM the supposed glaring lack of effective mathematics education in the United States has again been brought to the forefront of the public's attention. The current high-stakes testing environment supports the idea that the purpose of education is economic and holds a singular view of the worth of teachers based upon performance (Rose, 2011). The continual and ongoing calls for change and improvement in mathematics teaching and learning suggests that there is a need for re-visioning the ways in which mathematics teachers are educated. Consequently, this study sought to explore the enactment of a mathematics content course for preservice teachers taught in a non-traditional format and its impact on the participants' beliefs about mathematics teaching and learning. This study included thirty-seven students participating in classroom learning experiences that focused on meaning-making, dialogue, space, and justification in which the usual power dynamics between teacher and student were revisited and revised as part of the social norms established in the classroom. The questions this study sought to examine were the following:

- What are the characteristics of a mathematics content course for preservice teachers taught from a non-traditional orientation?

- From the preservice teachers' perspective, what impact, if any, does participation in this course have on mathematical beliefs?

- What influence, if any, do preservice teachers believe the curriculum and structure of this class have on their empowerment?

Throughout this chapter, I will provide an overview of the study, discuss the findings, and finally present the implications of this study situated in the body of literature as well as possibilities for future research.

Preservice teachers come to their mathematics content education classes with beliefs about mathematics pedagogy formed by prior experiences and understandings throughout their many years of formal education (Ball, 1996). Further, they have rarely experienced mathematics learning experiences based on reform ideas about mathematics teaching and learning (Ball, 1996); particularly in college mathematics content courses, reform-minded teaching is preached but rarely practiced (Burnaford, Fischer, and Hobson, 2001). Therefore, since mathematics classroom learning experiences impact the ways in which preservice teachers approach teaching mathematics, this course aimed to perturbate and ultimately challenge beliefs about mathematics as well as mathematics pedagogy through the non-traditional format and structure of the classroom learning experiences.

Subsequently, this study employed qualitative teacher action research in a mathematics content course for preservice teachers in which the participants were primarily female. As part of the normal course, students completed homework journals as well as take-home and in-class written portions of exams and these items were used as data in this study. In addition, I kept a reflective journal and participants were asked to complete a final survey following the completion of the semester-long course. The results of the data were used to describe the characteristics of the course, the impact of the course on preservice teachers’ mathematical beliefs, and the influence of the course on the empowerment of the participants.

Experiencing a mathematics content course taught in a non-traditional manner that centered on conceptual rather than procedural understanding seemed to have a profound effect upon many participants. It transformed participants’ beliefs about mathematics, changed the ways in which they envisioned mathematics pedagogy, and empowered them mathematically, epistemologically, and pedagogically. The structure of the course changed their perceptions of themselves, mathematics, and their own teaching; their beliefs evolved throughout their experiences in a semester of a non-traditional mathematics content course for teachers. One participant captured this profound impact in the following statement about her experiences in the course:

> My views on math have certainly changed since the beginning of this semester. I used to think that math was all about memorizing rules, writing down the right answer, and giving the teacher what they wanted to hear. After this course, I have begun to realize that math can be expressed in multiple areas and that it is much more than writing down the correct answer. My perception of math has changed because of this class. I no longer see it as simply solving problems quickly and I now know multiple ways of figuring problems out. I feel prepared and confident about sharing these concepts I have learned with my future students.

Additional findings will first be briefly discussed below together with situating the study in the current body of research as well as implications for future studies and educating preservice teachers.

Characteristics of a Non-Traditional Mathematics Course

The first research question explored the characteristics of a non-traditional mathematics content course designed for prospective early childhood and elementary teachers and their perceptions about these characteristics. The course was non-traditional in that it did not adhere to the usual traditional format of "show and tell" (lecture followed by homework) but instead focused on meaning-making, dialogue, space, decentralized control, and justification among others; the course was conceptually rather than procedurally driven. This course aimed to develop and create dynamic classroom learning experiences wherein students felt comfortable making sense of and dialoguing about a problematic situation with substantial mathematics content. The instructor's (my) role in the course was as facilitator; planning learning experiences, supporting dialogue, listening and answering questions as the groups worked, supporting and interpreting for(when needed) groups when they presented their findings, and assisting in the negotiation of social norms established throughout the duration of the course.

In order to confront and perturbate beliefs (Chapman, 2002) formed from past mathematics classroom experiences such as the belief that "the teacher and the textbook are the mathematical authority," the instructor rarely lectured or gave definitions (students were asked to define terms based upon their real-world experiences and common sense as a class and only later were these compared to the textbook definition); the instructor also refused to give or evaluate answers seeking to decentralize control of the mathematical authority in the classroom. Instead, the instructor asked groups (or individuals) to reason through problems and then to justify their work to the class; as a consensus, the class then agreed on the answer. Additionally, the instructor viewed students as learners actively constructing understandings about mathematics content and pedagogy (Ball, 1988) and consequently supported the student's construction of conceptual understanding by helping them to connect new understandings with prior knowledge (Adams & Burns, 1999; Boethel & Dimock, 1999; Davis, 2004; Mayer, 2004; Wheatley & Abshire, 2002). Ball (1993) noted the difficulty in developing "a practice that respects the integrity both of mathematics as a discipline *and* of children as mathematical thinkers" (p. 376). While focused on adult learners, the classroom experiences as part of this study were developed with this Ball's 1993 aim in mind with the constant struggle to incorporate more time working on problem solving and discussion and less on traditional lecture.

Preservice teachers' perceptions about this non-traditional mathematics content courses' format for learning experiences was explored in the study. Analyzing the data provided a narrative of how the participants felt about the structure of the course. The majority of the preservice teachers enjoyed having ample time and space to work on problems, working in groups, seeing how others worked on the problems (seeing a variety of ways to solve) and the hands-on or manipulative component of classroom experiences. The classroom format centered around meaning making which included the use of physical models such as pictures, diagrams, and manipulatives affected participants in that they felt their performance was better and they enjoyed the course more than their past mathematics classes. When participants' mentioned the hands-on aspect of the classroom, they were referencing not only the use of manipulatives (which only occurred twice in the semester) but also the pictures, tables, objects, mental manipulatives, and various other tactics they were encouraged to use to help them understand and find meaning in a situation. Although some students stated that they did not like mathematics, they still expressed that they felt comfortable in the learning environment established and that it positively affected their success in mathematics. One student stated "I like the whole discussion of the day's topic. I learn math a lot better by being about to talk about things instead of just being told this is what it is and this is why, now do it.”

Students liked being able to discuss topics and wrestle with problems during classroom experiences (as contrasted with formal lectures), before they attempted the problems on the homework assignment. Many participants felt that this helped their understanding of the content. One student stated

> when teaching mathematics you can’t do the traditional lecture, because you have to be very descriptive of whatever topic you are going over. It’s not just sentences of information, but problems that you have to take the time to work out. I like that we take up a whole class period going over one topic because I feel like I understand it better when we take the time to go over it.

The course also changed the ways in which the preservice teachers viewed the teaching and learning of mathematics. Their learning experiences in this mathematics content course taught conceptually rather than procedurally altered their perceptions about the teaching of mathematics. Similar to Raymond (1997), this study found that preservice teachers often linked their future pedagogical practices with beliefs about mathematics. For instance, one student stated that

> I have learned so many different ways of looking at math during this class. While going through high school we were taught only one way to do certain problems and we definitely never asked why you have to do it that way. Every time you did ask, the answer was always the same "because that is the easiest way to do it." This class has helped me become a believer in the phrase "you don't have to be a genius to understand math." It has helped broaden my views on the way math can be taught. Before this class I just assumed that there was only one certain way math could be taught, but now I understand that there are many ways to teach the subject. I wish more than anything that I would have had the privilege of being taught that math is more than being fast, or that there is more than one way to solve a problem but I wasn't, so therefore I plan to teach my students the way I wish I could have been taught.

The notion that mathematics pedagogy is primarily "show and tell" was challenged through the format of the course. Consequently, many students stated that they altered their vision of their own mathematics pedagogy to include hands-on activities, creativity, explanation (of thinking), and exploration among other descriptors as being part of their mathematics pedagogy. For instance, one participant noted that "the thing that impacted me the most would have to be learning how many different teaching and learning styles there are. I like the fact that there isn't just one way to do everything."

Mathematical Beliefs

Similar to the findings of Chapman's (2002) study, examination of the data reflecting statements about beliefs revealed that the most changes due to classroom experiences, from the preservice teachers' perspective, seemed to be those that were perturbated the most in learning experiences (through the format of the class) or those that might have been only partially-held prior to class. For instance, the most changed were the beliefs that "there is one right way to solve a problem" and "the goal of mathematics is to obtain 'right answers'." Of these two, "there is one right way to solve a problem" was perturbated the most and "the goal of mathematics is to obtain 'right answers'" was a belief held by my participants prior to the course and was changed to either disagreement or only partial agreement after participation in the non-traditional format of the course.

Additionally, mathematical beliefs that were partially-held or maybe even subconscious, formed before the course through their prior mathematics classes in early school years (Anderson & Piazza, 1996; Ball, 1988; Ball, 1996; Calderhead & Robson, 1991; Philipp, 2000), were brought to consciousness due to the non-traditional format of the course that integrated content and pedagogy as called for by Cooney (1999). A contrast between statement of belief and the reinforcing statements to support a participants' belief can be seen by many of the members in the "partial belief" category. This is perhaps due to the fact that these participants failed to spend the time in "early and continued reflection about mathematical beliefs and practices" (Raymond, 1997, p. 574) that is necessary for a change in beliefs.

Due to the small number of participants affirming a change in beliefs and the large number of statements displaying partially-held beliefs after the course, this study found that preservice teachers' mathematical beliefs were resistant to change (Calderhead & Robson, 1991; Chapman, 2002; Philipp et al., 2007; Stuart & Thurlow, 2000). If beliefs prior to the course were central (held with strong conviction), then despite the fact that classroom learning situations perturbated these beliefs again and again, participants showed little change in belief structure. For instance, even though rules and procedures were not a focus of the course, participants still believe that mathematics is simply rules and procedures; the structure of learning and the social norms established were that groups solved problems with no prior stated “rules” yet participants still made statements like “I agree with this *[mathematics is a set of rules and procedures]* because there are rules that we follow to solve mathematical problems, and these rules always have to be followed.” This result for the beliefs strongly-held was most likely due to the “apprenticeship of observation” (Anderson & Piazza, 1996) that occurred in past mathematics classes with traditional “show and tell” classroom learning experiences structure. These resilient beliefs were affected minutely in that many participants were placed in the “partial belief” category but overall little change was stated explicitly although several stated change implicitly.

Empowerment

Perhaps one of the most noteworthy findings, after beliefs, is that the structure and format of classroom interactions served to empower participants both mathematically and pedagogically. The word *empowerment* encompasses feelings about capability as well as self-confidence (Ernest, 2002). This course eliminated traditional lecture (focused on procedural reasoning) as a primary source of instruction and instead focused on problem-based instruction, student-led solutions, and collaboration time (Gasser, 2011). This study found that the format of class practice affected students in that they reported feeling more confident in their mathematical prowess as well as their ability to teach the mathematical topics covered in the course which supports Anderson and Piazza's findings (1996). For instance, participants stated

- I think, in general, most of the experiences in this course have enhanced my confidence and enthusiasm for mathematics. Being encouraged to work with out-of-the-box algorithms has expanded my perceived horizons and opened up a new field of interest for me.
- I am much more comfortable working these types of math problems and I am not as afraid of being asked to teach someone else how to do math.
- I used to view all mathematics very negatively because I was never good at it. However, in here by using visual manipulatives and other methods I was able to better understand mathematics, therefore I can feel more confident about it….because this class gave me a better understanding of mathematics I am able to enjoy it more, instead of being stressed out by it.

Implications

The body of literature on teachers' beliefs is vast and focuses on both beliefs and pedagogic practice. Most of the recommendations for research focused on the ways in which teacher education and professional development influence teacher beliefs and practice—by looking at curriculum, student thinking and learning, context, and challenging mathematical and pedagogical beliefs (Carpenter, et al., 1989; Pajares, 1992). The suggestions in the literature for future teacher education classes that integrated mathematics and pedagogy (Cooney, 1999) in ways supporting constructivist learning propelled this research study. Thus, this study sought to fill the perceived hole of research examining the role preservice teachers' conception of mathematics might play in teaching practices (Thompson, 1984). The structure of this course aimed to challenge beliefs about mathematics and mathematics teaching and learning and was built on the notion that preservice teachers come to their teacher education programs with prior pedagogical knowledge. It was with these ideas in mind that this study was formed and conducted. Consequently, the findings of this study added to the body of the literature through the integration of mathematics content and pedagogy in classroom learning experiences that supported constructivist learning as well as challenged beliefs about mathematics and mathematics pedagogy. This study revealed that challenging preservice teachers' beliefs about mathematics and mathematics pedagogy through the culture of the classroom as well as sociomathematical norms, altered their perceptions about the teaching of mathematics and the nature of mathematics (Szydlik, Szydlik, & Benson, 2003). Preservice teachers' participatory experiences in a mathematics content course taught in a non-traditional manner that centered on conceptual rather than procedural understanding seemed to have a marked effect upon many aspects of participants' beliefs and views—about themselves and mathematics.

Further, the non-traditional format of the learning space of this mathematics content course served to empower participants both mathematically and pedagogically. While the results of this study are not intended to be generalized, it may be used to advance preservice teacher preparation programs as well as point to future directions to pursue in research on this topic. After investigating the results of this study, one area for future studies would be to examine how and why preservice teachers assimilate new ideas to fit existing beliefs rather than accommodate their existing beliefs to internalize new ideas. Moreover, since this study focused on perturbing a variety of mathematical beliefs and found that those perturbed most were seemingly impacted the most, future studies might focus on perturbing specific mathematical beliefs throughout a course to observe the effect on belief structure.

References

Adams, S. & Burns, M. (1999). *Connecting student learning & technology*. Austin, TX: Southwest Educational Developmental Laboratory. Available from http://www.bazzirk.com

Anderson, D., & Piazza, J. (1996). Changing beliefs: Teaching and learning mathematics in constructivist preservice classrooms. *Action in Teacher Education, 18(2)*, 51-62.

Anderson, G.L., Herr, K., & Nihlen, A.S. (2007). *Studying your own school: An educator's guide to practitioner action research.* Thousand Oaks, CA: Sage Publications Inc.

Anyon, J. (1980). Social class and the hidden curriculum of work. *Journal of Education, 162(1)*, 67-92.

Ball, D. B. (1988). Unlearning to teach mathematics. *For the Learning of Mathematics, 8(1)*, 40-48.

Ball, D. B. (1990). Breaking with experience in learning to teach mathematics: The role of a preservice methods course. *For the Learning of Mathematics, 10(2)*, 10-16.

Ball, D. B. (1996). Teacher learning and the mathematics reforms: What we think we know and what we need to learn. *Phi Delta Kappan, 77(7)*, 500-508.

Ball, D.B. (2000). Working on the inside: Using one's own practice as a site for studying teaching and learning. In A.E. Kelly and R.A. Lesh (Eds.), *Handbook of Research Design in Mathematics and Science Education* (pp.365-402). Mahwah, NJ: Lawrence Erlbaum Associates Inc.

Ball, D.L. (1993). With an eye on the mathematical horizon: Dilemmas of teaching elementary school mathematics. *Elementary School Journal, 93*, 373-397.

Bandura, A. (1986). *Social foundations of thought and action: A social cognitive theory.* Englewood Cliffs, NJ: Prentice-Hall.

Bartlett, S. & Burton, D. (2006). Practitioner research or descriptions of classroom practice? A discussion of teachers investigating their classrooms. *Educational Action Research, 14(3)*, 395-405.

Becker, J.P., & Jacob, B. (2000, March). The politics of California school mathematics: The anti-reform of 1997-99. *Phi Delta Kappan,* 529-537.

Begle, E. (1968). Curriculum Research in Mathematics. In H. Klausmeier and G. O'Hearn(Eds.), *Research and Development toward the Improvement of Education*(pp. 44-48). Madison, WI: Dembar Educational Research Services.

Bishop, A.J. (1980). Classroom conditions for learning mathematics, *Proceedings of PME4*, Berkeley, California. pp. 338-344.

Boethel, M. & Dimock, K.V. (1999). *Constructing knowledge with technology: A review of the literature*. Austin, TX: Southwest Educational Development Laboratory. Available from http://www.sedl.org/

Boisvert, R.D. (1998). *John Dewey: Rethinking our time.* Albany: State University of New York Press.

Bowsfield, S., Breckenridge, A., Davis, B., Hur, S.J., Ibrahim-Didi, K., Kim, M., et al. (2004). *Safe and caring schools in a complex world: A guide for teachers*. Edmonton Canada: The Society for Safe and Caring Schools and Communities.

Bruner, J. (1996). *The culture of education.* Cambridge, MA: Harvard University Press.

Burnaford, G., Fischer, J., & Hobson, D. (2001). *Teachers doing research: The power of action through inquiry*. Mahwah, NJ: Lawrence Erlbaum Associates Inc.

Cain, T., & Milovic, S. (2010, February). Action research as a tool of professional development of advisers and teachers in Croatia. *European Journal of Teacher Education, 33(1)*, 19-30.

Calderhead, J., & Robson, M. (1991). Images of teaching: Student teachers' early conceptions of classroom practice. *Teaching & Teacher Education, 7*, 1-8.

Carpenter, T. P., Fennema, E., Peterson, P. L., Chiang, C.–P., & Loef, M. (1989). Using knowledge of children's mathematics thinking in classroom teaching: An experimental study. *American Educational Research Journal, 26*, 499-531.

Chapman, O. (2002). Belief structure and inservice high school mathematics and teacher growth. In G.C. Leder, E. Pehkonen, & G. Torner (Eds.), *Beliefs: A hidden variable in mathematics education* (pp. 177-193). Dordrecht, The Netherlands: Kluwer.

Cockran-Smith, M. & Lytle, S.L. (1993). *Inside/Outside: Teacher research and knowledge*. New York: Teachers College Press, Columbia University.

Common Core Standards. http://www.corestandards.org/

Conference Board of the Mathematical Sciences. (2001). *The mathematical education of teachers.* Washington, DC: Mathematical Association of America.

Confrey, J. (2000). Improving research and systemic reform toward equity and quality. In A.E. Kelly and R.A. Lesh (Eds.), *Handbook of Research Design in Mathematics and Science Education* (pp.87-106). Mahwah, NJ: Lawrence Erlbaum Associates Inc.

Cooney, T. J. (1985). A beginning teacher's view of problem solving. *Journal for Research in Mathematics Education, 16(5)*, 324-336.

Cooney, T. J. (1999). Conceptualizing teacher's ways of knowing. *Educational Studies in Mathematics, 38*, 163-187.

Cooney, T. J., Shealy, B. E., & Arvold, B. (1998). Conceptualizing belief structures of preservice secondary mathematics teachers. *Journal for Research in Mathematics Education, 29(3)*, 306-333.

Davis, B. (2004). *Inventions of teaching: A genealogy.* New York: Lawrence Erlbaum Associates.

Davis, D., & Dodd, J. (2002). Qualitative Research and the question of rigor. *Qualitative Health Research, 12(2)*, 279-289.

Davis, B. & Sumara, D.J. (2005). Challenging images of knowing: Complexity science and educational research. *International Journal of Qualitative Studies in Education, 18(3)*, 305-321.

Dewey, J. (1929). *The quest for certainty.* New York: Minton, Balch, & Co.

Dimitriadis, G., & Kamberelis, G. (2006). *Theory for education.* New York: Routledge.

Dooley, C. (1997). Problem-centered learning experiences: Exploring past, present and future perspectives. *Roeper Review, 19(4)*, 192-195.

Dowling, M. (2005, May). From Husserl to van manen: A review of different phenomenological approaches. *International Journal of Nursing Studies, 44*, 131-142.

Eisenberg, T. (1977). Begle revisited: Teacher knowledge and student achievement in algebra. *Journal for Research in Mathematics Education, 8*, 216-222.

Elliott, J. (1991). *Action Research for Educational Change.* Philadelphia, PA: Keynes.

Ernest, P. (1989). The impact of beliefs on the teaching of mathematics. In C. Keitel, P. Damerow, A. Bishop, & P. Gerdes (Eds.), *Mathematics, education, and society* (pp. 99-101). Paris: UNESCO.

Ernest, P. (1998). *Social constructivism as a philosophy of mathematics.* Albany, NY: SUNY Press.

Ernest, P. (2002). Empowerment in mathematics education. *Philosophy of Mathematics Education Journal, 15.* Retrieved July 15, 2012 from http://people.exeter.ac.uk/PErnest/pome15/empowerment.htm

Feinberg, W. & Soltis, J.F. (2004). *School and society*. New York: Teachers College Press, Columbia University.

Feldman, A. & Minstrell, J. (2000). Action research as a research methodology for the study of the teaching and learning of science. In A.E. Kelly and R.A. Lesh (Eds.), *Handbook of Research Design in Mathematics and Science Education* (pp.429-455). Mahwah, NJ: Lawrence Erlbaum Associates Inc.

Fennema, E., & Sherman, J. (1977). Sex-related differences in mathematics achievement, spatial visualization, and affective factors. *American Educational Research Journal, 14*, 51-71.

Fenstermacher, G. D. (1979). A philosophical consideration of recent research on teacher effectiveness. In L.S. Shulman (Ed.), *Review of Research in education* (Vol. 6, pp. 157-185). Itasca, IL: Peacock.

Foreman-Peck, L. & Murray, J. (2008). Action research and policy. *Journal of Philosophy of Education, 42(S1)*, 145-163.

Frank, M. L. (1988). Problem solving and mathematical beliefs. *Arithmetic Teacher, 35(5)*, 32-34.

Freire, Paulo. (1998). *Pedagogy of freedom: Ethics, democracy, and civic courage.* New York: Rowman & Littlefield.

Gasser, K.W. (2011). Five ideas for 21st century math classrooms. *American Secondary Education, 39(3)*, 108-116.

Gee, J.P. (2004). *Situated language and learning: A critique of traditional schooling.* New York: Routledge.

Goldin, G.A. (1990). Epistemology, constructivism, and discovery learning in mathematics. In R. B. Davis, C.A. Maher, and N. Noddings (Eds.), *Constructivist Views on the Teaching and Learning of Mathematics* (pp. 31-47). Reston, VA: National Council of Teachers of Mathematics.

Grant, T.J., Hiebert, J., & Wearne, D. (1998). Observing and teaching reform-minded lessons: What do teachers see? *Journal of Mathematics Teacher Education, 1*, 217-236.

Green , T. (1971). *The activities of teaching.* New York: McGraw-Hill.

Gregoire, M. (2003). Is it a challenge or a threat? A dual process model of teachers' cognition and appraisal processes during conceptual change. *Educational Psychology Review, 15*, 147-179.

Hackett, G. (1985). Role of mathematics self-efficacy in the choice of math-related majors of college women and men: A path analysis. *Journal of counseling Psychology, 32(1)*, 47-56.

Hackett, G., & Betz, N.E. (1989). An exploration of the mathematics self-efficacy/mathematics performance correspondence. *Journal for Research in Mathematics Education, 20(3)*, 261-273.

Heron, J. & Reason, P. (1997). A participatory inquiry paradigm. *Qualitative Inquiry, 3(3)*, 274-294.

Hersh, R. (1986). Some proposals for revising the philosophy of mathematics. In T. Tymoczko (Ed.), *New directions in the philosophy of mathematics* (pp. 9-28). Boston: Birkhauser.

Hiebert, J., Carpenter, T.P., Fennema, E., Fuson, K., Human, P., Murray, H., Olivier, A., et al. (1996). Problem solving as a basis for reform in curriculum and instruction: The case of mathematics. *Educational Researcher, 25(4)*, 12-21.

Hofmeister, A.M. (2004). Education reform in mathematics: A history ignored?. *Journal of Direct Instruction, 4(1)*, 5-11.

Houser, N.O. (2006). Worldviews and learning theories: A transactional reconsideration of teaching and learning. *Curriculum and Teaching, 21(1)*, 5-31.

Hoyles, C. (1992). Mathematics teaching and mathematics teachers: A meta-case study. *For the Learning of Mathematics, 12(3)*, 32-44.

Keiny, S., & Orland-Barak, L. (2009). Educational action research as a paradigm for change. In B. Somekh & S. Nofke (Eds.), *Handbook of Educational Research* (pp. 166-177). Thousand Oaks, CA: Sage Publications Inc.

Kemmis, S., & McTaggart, R. (1988). *The action research planner.* Victoria, Australia: Deakin University Press.

Kirschner, P., Clark, R., & Sweller, J. (2006). Why minimal guidance during instruction does not work: An analysis of the failure of constructivist, discovery, problem-based, experiential, and inquiry-based teaching. *Educational Psychologist, 41(2)*, 75-86.

Klein, D. (2002). A brief history of American k-12 mathematics education in the 20th century. In James M. Royer (Ed.), *Mathematical Cognition: A Volume in Current Perspectives on Cognition, Learning, and Instruction* (pp. 175-225). CT: Information Age.

Kloosterman, P. (2002). Beliefs about mathematics and mathematics learning in the secondary school: Measurement and implications for motivation. In G.C. Leder, E. Pehkonen, & G. Torner (Eds.), *Beliefs: A hidden variable in mathematics education* (pp. 247-269). Dordrecht, The Netherlands: Kluwer.

Knoll, E., Earnest, Pl, & Morgan, S. (2004). Experiencing research practice in pure mathematics in a teacher training context. In M. J. Hoines & A. B. Fuglestad (Eds.), *Proceedings of the 28th annual conference of the International Group for the Psychology of Mathematics Education* (Vol. 3, pp. 161-168). Bergen: Bergen University College.

Lasley, T. J. (1980). Preservice teacher beliefs about teaching. *Journal of Teacher Education, 31(4)*, 38-41.

Lemann, N. (1999). *The big test: The secret history of the American meritocracy.* New York: Farrar Straus and Giroux.

Lent, R.W., & Hackett, G. (1987). Career self-efficacy: Empirical status and future directions. *Journal of Vocational Behavior, 30*, 347-382.

Lloyd, G.M. (1999). Two teachers' conceptions of a reform-oriented curriculum: Implications for mathematics teacher development. *Journal of Mathematics Teacher Education, 2,* 227-252.

Lloyd, G.M. (2002). Mathematics teachers' beliefs and experiences with innovative curriculum materials: The role of curriculum in teacher development. In G.C. Leder, E. Pehkonen, & G. Torner (Eds.), *Beliefs: A hidden variable in mathematics education* (pp. 149-159). Dordrecht, The Netherlands: Kluwer.

Lloyd, G.M., & Wilson, M.S. (1998). Supporting innovation: The impact of a teacher's conceptions of functions on his implementation of a reform curriculum. *Journal for Research in Mathematics Education, 29,* 248-274.

Lo, J.J., & Wheatley, G. (1994). Learning opportunities and negotiating social norms in mathematics class discussion. *Educational Studies in Mathematics, 27,* 145-164.

Lubienski, S.T. (2000). Problem solving as a means toward mathematics for all: An exploratory look though a class lens. *Journal for Research in Mathematics Education, 31(4),* 454-482. Retrieved February 27, 2009 from http://www.jstor.org/stable/749653

Marshall, C., & Rossman, G.B. (2011). *Designing Qualitative Research.* Thousand Oaks, CA: Sage Publications Inc.

Mayer, R. (2004). Should there be a three-strikes rule against pure discovery learning? *American Psychologist, 59(1),* 14-19.

Meece, J.L., Wigfield, A., & Eccles, J.S. (1990). Predictors of math anxiety and its influence on young adolescents' course enrollment intentions and performance in mathematics. *Journal of Educational Psychology, 82(1),* 60-70.

Merrill, M., & Gilbert, C. (2008). Effective peer interaction in a problem-centered instructional strategy. *Distance Education, 29(2)*, 199-207.

Middleton, Valerie. (2002). Increasing preservice teachers' diversity beliefs and commitment. *Urban Review, 34*, 343-361.

Moreira, C. (1991). Teacher attitudes towards mathematics and mathematics teaching: Perspectives across two countries. *Proceedings of PME 15*, Vol. II, 17-24. Assisi, Italy.

National Commision on Excellence in Education. (1983). *A nation at risk: The imperative for educational reform.* Washington, DC: Government Printing Office. Available from http://www.ed.gov/pubs/NatAtRisk/risk.html

National Council of Teachers of Mathematics. (1980). *An agenda for action: Recommendations for school mathematics of the 1980s.* Reston, VA: Author.

National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics.* Reston, VA: National Council of Teachers of Mathematics.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics.* Reston, VA: National Council of Teachers of Mathematics.

National Research Council. (1989). *Everybody counts: A report to the nation on the future of mathematics education.* Washington, DC: National Academy Press.

Nespor, J. (1987). The role of beliefs in the practice of teaching. *Journal of Curriculum Studies, 19*, 317-328.

Noffke,S.E. (1997). Professional, personal and political dimensions of action research. *Review of Research in Education, 2,* 305-343.

Oakes, J. (1985). *Keeping track: How schools structure inequality.* New Haven: Yale University Press.

Oakes, J., Ormseth, T., Bell, R., & Camp, P. (1990). *Multiplying inequalities: The effects of race, social class, and tracking on opportunities to learn mathematics and science.* Santa Monica, CA: Rand Corporation.

Orland-Barak, L. (2009). Unpacking variety in practitioner inquiry on teaching and teacher education. *Educational Action Research, 17(1)*, 111-119.

Pajares, M. F. (1992). Teachers' beliefs and educational research: Cleaning up a messy construct. *Review of Educational Research, 62,* 307-332.

Philipp, R.A. (2000). Mathematics teachers' beliefs and affect. In F. Lester (Eds.), *Second handbook of research on mathematics teaching and learning*. United States: Information Age Pub Inc.

Philipp, R.A., Ambrose, R., Lamb, L.C., Sowder, J.T., Schappelle, B.P., Sowder, L., et al. (2007). Effects of early field experiences on the mathematical content knowledge and beliefs of prospective elementary school teachers: An experimental study. *Journal for Research in Mathematics Education, 38(5)*, 438-476.

Piaget, J. (1972). *The principles of genetic epistemology*. New York: Basic.

Raymond, A. M. (1997). Inconsistency between a beginning elementary school teacher's mathematics beliefs and teaching practice. *Journal for Research in Mathematics Education, 28*, 550-576.

Reeder, S. (2005). Classroom dynamics and emergent curriculum. In W.E. Doll, M.J. Fleener, D. Trueit, & J.St. Julien (Eds.), *Chaos, complexity, curriculum, and culture: A conversation* (pp. 247-260). New York, NY: Peter Lang Publishing Inc.

Renzaglia, A., Hutchins, M., & Lee, S. (1997). The impact of teacher education on the beliefs, attitudes, and dispositions of preservice special educators. *Teacher Education and Special Education, 20(4)*, 360-377.

Reynolds, A., Fleener, M.J., Wheatley, G.H., & Robbins, R. (2004, June). The making of intelligence: Poststructural perspectives of learning. *International Journal of Learning, 11*.

Richardson, V. (1996). The role of attitudes and beliefs in learning to teach. In J.P. Sikula, T.J. Buttery, & E. Guyton (Eds), *Handbook of research on teacher education: A project of the Association of Teacher Educators* (2^{nd} ed., pp. 102-119). New York: Macmillan.

Rippa, S.A. (1988). *Education in a Free Society: An American History*. New York: Longman Incorporated.

Rogers, C. (1969). *Freedom to learn: A view of what education might become*. Columbus, OH: Merrill Publishing Co.

Rokeach, M. (1960). *The open and closed mind.* New York: Basic Books.

Rokeach, M. (1968). *Beliefs, attitudes, and values: A theory of organization and change.* San Francisco: Jossey-Bass.

Rosen, L. (2000). *Calculating concerns: The politics of representation in California's "math wars"* (Doctoral dissertation, University of California, San Diego, 2000).

Rudduck, J. (1988). Changing the world of the classroom by understanding it: A review of some aspects of the work of Lawrence Stenhouse. *Journal of Curriculum and Supervision, 4(1)*, 30-42.

Saracho, O.N., & Spodek, B. (2009). Educating the young mathematician: A historical perspective through the nineteenth century. *Early Childhood Education Journal, 36*, 297-303.

Schoenfeld, A.H. (2004). The math wars. *Educational Policy, 18(1)*, 253-286.

Sells, L. (1976). *The mathematics filter and the education of women and minorities*. Paper presented at the Annual Meeting of American Association for the Advancement of Science, Boston, MA.

Silberman, C.E. (1970). *Crisis in the Classroom: The Remaking of American Education*. New York: Random House.

Skemp, R. R. (1978). Relational understanding and instrumental understanding. *Arithmetic Teacher, 26(3)*, 9-15.

Skott, J. (2001). The emerging practices of a novice teacher: The roles of his school mathematics images. *Journal of Mathematics Teacher Education, 4,* 3-28.

Stanic, G.M.A. (1986). The growing crisis in mathematics education in the early twentieth century. *Journal for Research in Mathematics Education, 17(3),* 190-205.

Stanic, G.M.A. (1987). Mathematics education in the United States at the beginning of the twentieth century. In Thomas S. Popkewitz (Ed.), *The formation of school subjects: The struggle for creating an American institution* (pp. 147-183). New York: Falmer.

Steffe, L.P., & Kieren, T. (1994). Radical constructivism and mathematics education. *Journal for Research in Mathematics Education, 25,* 711-733.

Stenhouse, L.A. (1985). Applying Research to Education. In J. Rudduck & D. Hopkins (Eds.), *Research as a Basis for Teaching: Readings from the Work of Lawrence Stenhouse.* London: Heinemann Educational Books.

Stigler, J.W., & Hiebert, J. (1999). *The Teaching Gap: Best Ideas from the World's Teachers for Improving Education in the Classroom.* New York, NY: The Free Press.

Stigler, J., & Perry, M. (1987). Cross-cultural studies of mathematics teaching and learning: Recent findings and new directions. *Proceedings of PME 11,* Vol. III, 421-422. Montreal.

Stuart, C. & Thurlow, D. (2000). Making it their own: Preservice teachers' experiences, beliefs, and classroom practices. *Journal of Teacher Education, 51(2)*, 113-121.

Sulivan, P., & Mousley, J. (2001). Thinking teaching: Seeing mathematics teachers as active decision makers. In F.-L. Lin & T.J. Cooney (Eds.), *Making Sense of Mathematics Teacher Education* (pp. 147-163). Dordrecht: Kluwer Academic Publishers.

Sztajn, P. (2003). Adapting reform ideas in different mathematics classrooms: Beliefs beyond mathematics. *Journal of Mathematics Teacher Education, 6*, 53-75.

Szydlik, J.E., Szydlik, S.D., & Benson, S.R. (2003). Exploring changes in pre-service elementary teachers' mathematical beliefs. *Journal of Mathematics Teacher Education, 6,* 253-279.

Thompson, A.G. (1984). The relationship of teachers' conceptions of mathematics and mathematics teaching to instructional practice. *Educational Studies in Mathematics, 15*, 105-127.

Thompson, A.G. (1992). Teachers' beliefs and conceptions: A synthesis of the research. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 127-146). New York: Macmillan Publishing Company.

Torner, G. (2002). Mathematical beliefs—a search for a common ground: Some theoretical considerations on structuring beliefs, some research questions, and some phenomenological observations. In G.C. Leder, E. Pehkonen, & G. Torner (Eds.), *Beliefs: A hidden variable in mathematics education* (pp. 73-94). Dordrecht, The Netherlands: Kluwer.

Vacc, N. N., & Bright, G. W. (1999). Elementary preservice teachers' changing beliefs and instructional use of children's mathematical thinking. *Journal for Research in Mathematics Education, 30,* 89-110.

Vygotshy, L.S. (1978). *Mind in society.* Cambridge, MA: Harvard University Press.

Yackel, E., & Cobb, P. (1996). Sociomathematical norms, argumentation, and autonomy in mathematics. *Journal for Research in Mathematics Education, 27(4)*, 458-477.

Walkerdine, V. (1997). *Counting girls out.* London: Falmer Press.

Wheatley, G.H. (1991). Constructivist perspectives on science and mathematics learning. Science Education, 75(1), 9-21.

Wheatley, G.H., & Abshire, G.E. (2002). *Developing mathematical fluency.* Bethany Beach, DE: Mathematics Learning.

Wheatley, G., & Reynolds, A.M. (1999). "Image maker": Developing spatial sense. *Teaching Children Mathematics*, *5(6)*, 374-378.

White-Clark, R., DiCarlo M., & Gilchriest, N. (2008, April). Guide on the side: An instructional approach to meet mathematics standards. *The High School Journal*, *91(4)*, 40-44.

Wilson, M.S., & Cooney, T. (2002). Mathematics teacher change and development. In G.C. Leder, E. Pehkonen, & G. Torner (Eds.), *Beliefs: A hidden variable in mathematics education* (pp. 127-147). Dordrecht, The Netherlands: Kluwer.

Wordle definition. http://www.wordle.net/

Appendix A: Demographic Form

Name:____________________

Demographic Information

Gender: M or F (circle one)   Age:

______________years

Major:_________________________________

Ethnicity: ____Native American   ____Latino

____African-American

____Caucasian   ____Asian

____Other (please specify:_______________________)

Place an X beside each Mathematics course listed below that you took in High School.

____Algebra I   ____Pre-Calculus or Math Analysis

____Algebra II   ____Calculus

____Geometry   ____Statistics

____Algebra III   ____Other (please specify:_______________)

____Trigonometry

Place an X beside each Mathematics course listed below that you have taken in college.

____Intermediate Algebra ____College Algebra

____Trigonometry ____Statistics

____Calculus ____Survey of Mathematics

____Other (please specify:_______________________)

Appendix B: Final Survey

*Please remember that your identity will be kept anonymous. Please answer these as honestly and thoughtfully as possible(on a separate sheet(s) of paper)

1. Based upon your experiences in this course, describe how you think mathematics should be taught.
2. Describe your feelings about mathematics now based upon your experiences in this course.
3. From this course, what impacted you the most? Describe why:
4. What experiences in this course, if any, caused you the most discomfort? Explain.
5. Have the experiences in this course had any effect on your confidence to learn mathematics? Which ones, if any?
6. Has your feelings about your ability to teach mathematics been affected by your experiences in this course?
7. Has your enjoyment of mathematics been affected by your experiences in this course?
8. Most of the class time in this course is taken up by whole class discussion of the day's topic. This

replaces the more traditional lecture. Your thoughts and feelings about this are…

9. If you were to tell someone else about your experience in mathematical concepts, what would you tell them?