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ROBUST DECISION-MAKING AND DYNAMIC RESILIENCE ESTIMATION FOR INTERDEPENDENT RISK ANALYSIS

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ROBUST DECISION-MAKING AND DYNAMIC RESILIENCE ESTIMATION
FOR INTERDEPENDENT RISK ANALYSIS

A DISSERTATION APPROVED FOR THE
SCHOOL OF INDUSTRIAL AND SYSTEMS ENGINEERING

BY

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Abstract

When systems and subsystems are put under external shocks and duress, they suffer physical and economic collapse. The ability of the system components to recover and operate at new stable production levels characterizes *resilience*. This research addresses the problem of estimating, quantifying and planning for resilience in interdependent systems, where interconnectedness adds to problem complexity. Interdependence drives the behavior of sectors before and after disruptions. Among other approaches this study concentrates on economic interdependence because it provides insights into other levels of interdependence. For sectors the normalized losses in economic outputs and demands are suitable metrics for measuring interdependent risk. As such the inoperability input-output model enterprise is employed and expanded in this study to provide a useful tool for measuring the cascading effects of disruptions across large-scale interdependent infrastructure systems. This research defines economic resilience for interdependent infrastructures as an "ability exhibited by such systems that allows them to recover productivity after a disruptive event in a desired time and/or with an acceptable cost". Through the dynamic interdependent risk model resilience for a disrupted infrastructure is quantified in terms of its *average system functionality*, *maximum loss in functionality* and the *time to recovery*, which make up a resilience estimation decision-space. Estimating such a decision-space through the dynamic model depends upon the estimation of the rate parameter in the model. This research proposes a new approach, based on dynamic data assimilation methods, for estimating the rate parameter and strengthening post-disaster resilience of economic systems. The solution to the data assimilation problem generates estimates for the rate of resilient recovery that reflects planning considerations interpreted as commodity substitutions, inventory management and incorporating redundancies. The research also presents a robust optimization based risk management approach for strengthening interdependent static resilience estimation. There is a paucity of research dealing with quantification and assessment of uncertainties in interdependency models. The focus here is more on the extreme bounds of event and data uncertainties. The deterministic optimization becomes a robust optimization problem when extremes of uncertainties are considered. Computationally tractable robust counterparts to nominal problems are presented here. Also presented in this research is a discrete event simulation based queuing model for studying multi-modal transportation systems with particular focus on inland waterway ports. Such models are used for impact analysis studies of inland port disruptions. They can be integrated with the resilience planning methodologies to develop a framework for large-scale interdependent risk and recovery analysis.

Chapter 1

Introduction and Motivation

1.1 Overview

Many large-scale systems and critical industries such as transportation, telecommunications, power, and banking share significant resources, and the flow of goods and information constantly takes place among these different industry sectors. Realizing the interdependent nature of US infrastructure and industry sectors, the Department of Homeland Security (DHS) stresses the urgency and need to protect infrastructures (DHS., 2009):

> Attacks on Critical Infrastructure and Key Resources (CIKR) could significantly disrupt the functioning of government and business alike and produce cascading effects far beyond the targeted sector and physical location of the incident. Direct terrorist attacks and natural, manmade, or technological hazards could produce catastrophic losses in terms of human casualties, property destruction, and economic effects, as well as profound damage to public morale and confidence.

Hence, increased connectivity of today's infrastructure systems (Pederson et al., 2006) has meant that direct impacts of disruptions lead to cascading indirect impacts called multiplier effects (Santos, 2006). For the economic health and security of a region, 'lifeline' infrastructures like transportation systems, emergency services and information and communications technology (ICT) need to be at full or almost full functionality during disruptive times.

With focus on security and global threats the approach towards infrastructure protection has lead to changes in viewpoints on the importance of infrastructures for a functioning society. The definition of ‘infrastructures’ has been evolving over the years due to changing threat perceptions. For considerable time US public policy makers considered infrastructures as economic facilities with “the common characteristics of capital intensiveness and high public investment at all levels of government” (CBO., 1983). The Clinton Executive Order (Clinton, 1996) redefined infrastructures in a security context as:

> The framework of interdependent networks and systems comprising identifiable industries, institutions (including people and procedures), and distribution capabilities that provide a reliable flow of products and services essential to the defense and economic security of the United States, the smooth functioning of government at all levels, and society as a whole.

Making ‘protection’ the central theme of national infrastructure security the Critical Infrastructure Protection (CIP) was a Presidential Directive (Clinton, 1998) that reflected the prevailing mandate at the time.

In recent years some extreme impacts of local and global significance have altered the thought process incorporated in the CIP approach. The September 11, 2001 terrorist attack on the World Trade Center (Kendra & Wachtendorf, 2003) highlighted the fact that protection and prevention against man-made disruptions is not always possible. Moreover monitoring and safeguarding against global human threats arising from different sources likes pandemics (H1N1, H5N1) and bio-chemical weapons is almost impossible. Also extreme weather events like hurricanes Katrina and Ike (Blake et al., 2007) have shown that some events are just too big and extreme to protect against. As such there has been an increased understanding that it is not possible to protect every potential target against every conceivable attack and eliminate all vulnerabilities (CITK., 2006).

While little can be done to prevent the occurrences of all extreme events, preventive measures help in lessening the impact of resulting disruptions. Interest lies in predicting the adverse impacts of disruptive events in an interdependent economy and evaluating risk management efforts to lessen these impacts. Since infrastructure interconnectedness leads to improved efficiency during normal operations there is interest in preserving interdependence during disruptions because it can be utilized for speedy recoveries. Due to the practicality of such an approach there has been a paradigm shift in infrastructure security, where the emphasis on 'preparedness and response' has been added to the 'protection and prevention' approach.

The Critical Infrastructure Resilience (CIR) (CITK., 2006) highlights the new take in policy making by introducing *resilience* as the overarching objective for safeguarding against risk. In its broadest definition and scope, as specified by the Infrastructure Security Partnership (TISP) (2011), a resilient sector would "prepare for, prevent, protect against, respond or mitigate any anticipated or unexpected significant threat or event", and "rapidly recover and reconstitute critical assets, operations, and services with minimum damage and disruption". Resilience has been incorporated into the security lexicon and the DHS National Infrastructure Protection Plan (NIPP) stresses the importance of building a resilient society (DHS., 2009):

> Build a safer, more secure, and more resilient America by preventing, deterring, neutralizing, or mitigating the effects of deliberate efforts by terrorists to destroy, incapacitate, or exploit elements of our Nation's CIKR, and to strengthen national preparedness, timely response, and rapid recovery of CIKR in the event of an attack, natural disaster, or other emergency.

To help realize the goal of improving interdependent infrastructure resilience to disruptions a concentrated research effort is required to understand the very nature of the problem at hand. Some relevant questions that need to be answered are: (i) What factors characterize interdependent infrastructure behaviors? (ii) How do we quantify interdependence and measure the system performances in terms of such interdependence? (iii) How is the interdependent performance eroded due to disruptive events and how can this erosion be quantified? (iv) Are there indicators in the interdependent sectors behaviors that exhibit properties of resilience and recovery? (v) How do we quantify the resilience through the properties that are inherent in the system and then enhance such resilience by strengthening such properties? (vi) What quantifiable planning strategies can be constructed to improve resilience and what aspects of the system do these strategies concentrate on? and (vii) Can an overall resilience estimation and planning framework be formalized and implemented practically?

1.2 Defining resilience and its domains

With current emphasis on resilience estimation and system preparedness, developing a quantifiable resilience estimation methodology presents an interesting research challenge. To this end, the research presented here aims to construct a resilience framework for interdependent infrastructure systems. The questions posed above have been answered in detail in the subsequent development of this work, to come up with a framework that is capable of quantifying system resilience to impacts on interdependent infrastructures that are of homeland security and national interest.

The primary research interest lies in analyzing large-scale infrastructure systems, like industry sectors, that are of socio-economic importance. For such systems, interdependence exists across many layers and over time increases, leading to physical, cyber, geographical, and logical interdependencies across sectors (Rinaldi et al., 2001). A broad analysis of interdependence would entail capturing all structural details explaining interactive system behaviors. But such analysis becomes system specific and is too complex to solve beyond a certain point. The problem is simplified if we concentrate on one aspect of interdependence that is good enough to explain most of interactive system behaviors.

Economic interactions among sectors provide suitable measures around which interdependence can be estimated and expanded to a generalized framework. The level of economic interdependence between infrastructure systems can be used as an indicator of other levels of interdependence to some degree. For large-scale industry sectors economic consequences of impacts are major drivers during recovery planning and decision-making. Understanding interdependent impacts in terms of business economic interruptions has been considered to be a useful tool for analyzing the capabilities of such systems to withstand disruptions (Tierney, 1997; Rose & Liao, 2005). Measures that quantify losses due to economic interruptions provide suitable metrics around which resilience can be expressed and planning can be considered.

With emphasis on economic aspects of interdependent behavior, this research concentrates on developing a quantifiable *economic resilience* estimation framework. Such a framework is applicable to studies of large-scale infrastructure recovery behaviors from extreme weather related events to man-made impacts. Before going into further details the definition of economic resilience that has been proposed in this study is presented here.

> Economic resilience for interdependent infrastructures describes an ability exhibited by such systems that allows them to recover productivity after a disruptive event in a desired time and/or with an acceptable cost, noting that resilience is planned for in advance of a disruptive event through preparedness policies and investments. Economic resilience planning leads systems towards targeted stable levels of productivity which indicate their recovery from disruptions.

Some of the aspects of system resilience defined above come from the properties of the infrastructure systems that are being analyzed. Large-scale infrastructures have similar properties to macro-economic systems that are spread over vast geographic and economic domains. As such most disruptive impacts can be absorbed by such systems without complete loss of functionality because there are several mechanisms in place to cope with the disruptive events. The recent Japanese earthquake and tsunami (Fackler, 2011) caused unprecedented loss of life and economic productivity, while an off-season snowstorm along the East-Coast of the US caused widespread electricity blackouts leading to business disruptions (Allen, 2011). But in the end economies have recovered from such adverse impacts. Therefore there exists an *inherent resilience* (Rose, 2004a) property that is present due to measures that are already part of the system. There is a general understanding that in macro-economic systems such resilience is realized through either the availability of resources (inventories) that are in place or the market prices that drive resource allocations towards necessary consumers and thus satisfy incomplete demands (Rose, 2007). The business coping behavior or community response properties (Tierney, 1997) are thus contributors towards inherent resilience properties. For economic systems inherent or inbuilt resilience can be quantified through the amount of economic losses they are able to avoid immediately after a disruptive event.

Having established that there would be some capability inherently present in the systems that allows for recovery, the research emphasis here is to improve on such capabilities. As highlighted through the definition above, improving resilience requires a planning mechanism that leads towards better system performance. Hence this research provides decision-making methods that help in resilience planning and estimation for disruptive recovery of interdependent economic systems. Disruptions in economic systems at the macro level result in losses of market supplies and demands that materialize themselves as interdependent risks. Such risks are instrumental in establishing the resilience planning objectives in two types of domains

1. *Static resilience* planning domain - In the framework presented here, when interest lies in improving the long term recovery behavior of the infrastructures after disruptions, then the resilience planning is said to be static resilience planning. Primarily the focus is to quantify the disruptive system response that is time independent and only concerned with the long term capability of the system to rearrange itself. The path taken towards recovery is irrelevant and the only driving factor for system resilience is its capability to withstand the initial disruptive impacts. In terms of estimating and quantifying such resilience, the

demand side risks of the macroeconomy provide suitable measures.

2. *Dynamic resilience* planning domain - When the short-term recovery behavior of interdependent systems needs to be improved then the planning involves understanding the path the system takes from the onset of disruption up until any given time. Such behavior shows properties that can indicate dynamic resilience planning. The quantification of the dynamic system responses helps establish the dynamic resilience planning measures. Both demand and supply side driven economic risks drive the dynamic interdependent system responses and are fundamental in establishing the measures for such resilience planning.

In establishing an improved dynamic response leading to strengthened system resilience there is also scope for incorporating new measures that help the system withstand further impacts and update its response to future disruptions. As such, it can be said that the planning introduces an *adaptive resilience* (Rose, 2004a) capability into the system behavior. Adaptive response comes from extra effort and ingenuity (Rose, 2007) that leads to system enhancement, and for interdependent economic systems it can be realized through adaptive responses in organizational responses in the public and private domains (Comfort, 1999).

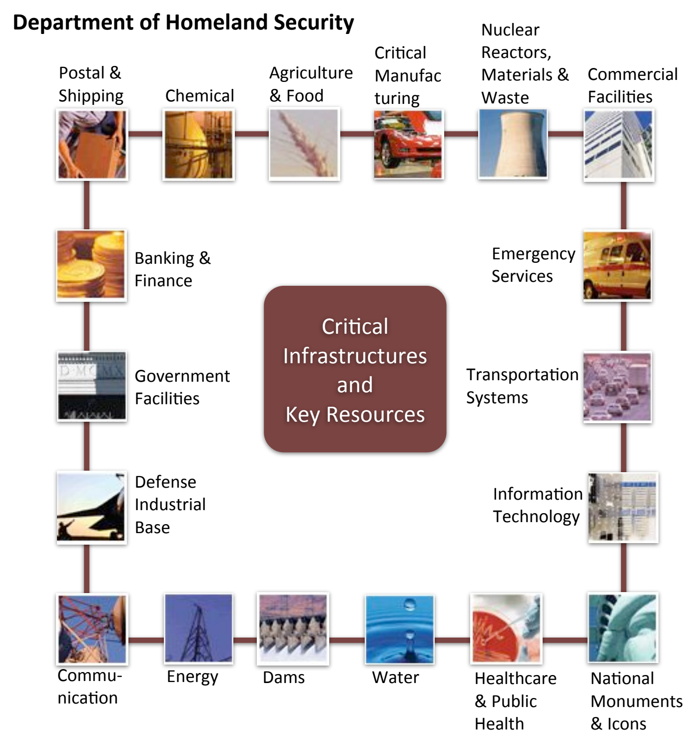
1.3 Components of a resilience framework and research approaches

Having established a broad definition for economic resilience and the primary types and domains of resilience of interest in this research, there is a need to establish a resilience framework. An understanding needs to be developed about the primary elements that motivate the formation of the framework in the first place. We look at the research approaches that have been undertaken and highlight the motivation for using our approach. Figure 1.1 shows the two principle elements of a resilience framework that need to be understood in a quantitative fashion. The resilience framework is built around these components, which are respectively (i) the interdependence of infrastructures, and (ii) the nature of the response and recovery of each infrastructure.

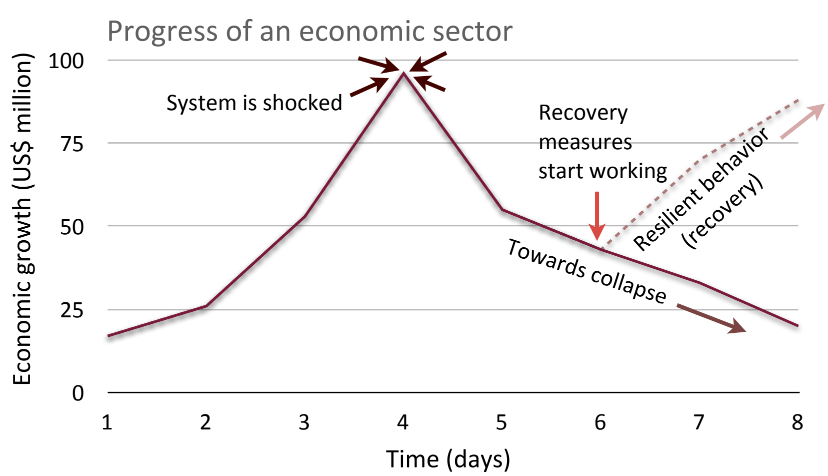
1.3.1 Modeling interdependence

Due to underlying interdependence, many industry and infrastructure sectors face direct and indirect risks due to disruptive events. The potentially wide-ranging indirect losses due to the cascading effects of disruptions in such interconnected systems (Rinaldi et al., 2001) are often greater than the direct impacts (Jiang & Haimes, 2004) of such disruptions. Hence, from a risk analysis perspective, an understanding of the impacts of a man-made attack, accident, or natural disaster must account for the interdependencies among industry and infrastructure sectors (Heal et al., 2006). Studies on potential disruptions due to earthquakes and floods (Cavallo et al., 2010), terrorist attacks (Gordon et al., 2007; Rose, 2009b), and power outages (Cavdaroglu et al., 2010), among others (Noy, 2009), have focused on interdependence of systems.

The different research approaches suggested for studying infrastructure interdependence include agent-based models (Bonabeau, 2002; Outkin et al., 2008), network models (Zhang et al., 2005; Lee et al., 2009), survey and expert judgement based models (Markowsky, 2009), among others (Pederson et al., 2006). Even though there are several merits of such schemes their scope in analyzing large-scale infrastructures is limited either by computational complexities and logistic elements (network models) or they have been designed for system specific interdependence estimations. As such the most widely used class of models for interdependent infrastructure analysis come from the Leontief based economic input-output family of models (Leontief, 1966). Explained in brief, the economic input-output models represent the equilib-



(a) Key interdependent infrastructures of interest



(b) Infrastructure response to shock and economic recovery behavior

Figure 1.1: Establishing the two main elements of the resilience framework.

rium balance in demand and supply for a system of many interconnected industries and thus are natural indicators for interdependence. These models are supported by vast data resources across the world (Atkinson et al., 1995; BEA., 2011). A few interesting applications that have incorporated the economic input-output models in large-scale interdependent system analysis include environmental life-cycle assessment (Hendrickson et al., 1998), hurricane damage assessment (Hallegatte, 2007, 2008) and earthquake impact analysis (Brookshire et al., 1997), terrorist attack impacts on ports (Gordon et al., 2005), analysis of electricity lifeline disruptions (Rose et al., 1997). Input-output model analysis has also been incorporated into broader modeling frameworks such as transportation network analysis models for spatial and temporal analysis of lifeline structures (Cho et al., 2001; Okuyama et al., 2004).

A risk-based extension to the economic input-output model is the inoperability input-output model (IIM). Introduced by (Haimes & Jiang, 2001), in the IIM study sector-wise economic risk is measured in terms of: (i) inoperability, or the fractional loss of industry economic output relative to its pre-disruption as-planned output level, and (ii) demand perturbation, or the fractional loss of sector final demand relative to its pre-disaster as-planned output level. Extensions of the model to dynamic (Lian & Haimes, 2006) and multi-regional (Crowther, 2007; Crowther & Haimes, 2010) analysis frameworks have been made. The static intra- and multi-regional IIMs are in fact normalized notions of the economic input-output model and thus bear structural resemblance to the economic input-output models. The rational behind using these models is that they provide metrics which allow for a comparative scale to measure the degrees of risk among interacting sectors. While economic sector outputs and demands might vary across a wide range depending upon the volume of commerce of sectors, the normalized IIM evaluates risk on the same scale allowing for comparison of disruption impacts on sectors. It might be of decision-making interest to evaluate economic impacts in terms of the fraction of damage instead of the amount of damage so that the decision-making is not biased towards the bigger economic systems. Though the dynamic IIMs are also built from normalizing the dynamic economic input-output models their interpretation of the dynamic rate terms are different. Insights into dynamic impact and recovery, which are not captured by the dynamic economic input-output models, can be provided through the dynamic risk models.

The IIM enterprise, including the dynamic and also multi-regional models, has been applied in numerous risk modeling applications, including: malevolent attacks (Andrijcic & Horowitz, 2006; Haimes et al., 2005b), supply chain disruptions (Wei et al., 2010; Barker & Santos, 2010a), workforce availability (Barker & Santos, 2010b; Orsi & Santos, 2010), transportation disruptions (Pant et al., 2011; MacKenzie et al., 2012a), and resource sustainability (Santos et al., 2008).

1.3.2 Resilience estimation

The importance of resilience in infrastructures has already been stressed so now its important to understand the meaning of resilience for different users. Different viewpoints about resilience exist within the research community (Rose, 2007), but there is consensus that resilience estimation is vital for risk decision-making (Klein et al., 2003). The main difference of opinion in defining and understanding resilience arises between the engineering approach that resilient recovery occurs by moving towards the previous stable state (Bruneau et al., 2003), and the ecological approach that resilience is developed to move towards a different system state (Handmer & Dovers, 1996). The original definition of resilience is attributed to Holling (1973) who stated that for ecological systems resilience is "a measure of the persistence of systems and of their ability to absorb change and disturbance and still maintain the same relationships between populations or state variables". Since then there have been several definitions of resilience across different disciplines, which has led some to question the relevance of the term in research (Klein et al., 2003; Rose, 2007, 2009a). Table 1.1 highlights some interpretations of resilience that have been misunderstood with other concepts used in disaster management.

Table 1.1: Different resilience definitions and conflicts in research.

Author	Definition	Comment
Holling (1973)	resilience implies ability to bounce back to original stability	Resilience can still exist after fluctuations lead to another state not necessarily stable
Mileti (1999)	"...a resilient community...takes mitigation actions consistent with achieving that level of protection."	Mitigation is implemented before a disruption and resilience after.
Timmerman (1981); Pelling (2003)	in context to hazard vulnerability "...resilience to natural hazards is the ability of an individual to cope with or adapt to hazard stress"	Resilience is post-disaster condition and vulnerability is pre-disaster.
Bruneau et al. (2003)	resilience results in reduced probability of failure and reduced consequences of failure.	Failure probability is reduced through mitigation and not resilience.
Godschalk (2004)	"future mitigation programs must also focus on teaching the city's social communities and institutions to reduce hazard risk and respond effectively to disasters, because they will be the ones most responsible for building ultimate urban resilience."	Resilience and mitigation are unrelated.

Primarily resilience has been defined in context to the speed of systems to go towards equilibrium (Adger, 2000), capability to cope and bounce back (Wildavsky, 1988), ability to adapt to new situations (Comfort, 1999), be inherently strong and flexible and adaptive (Tierney & Bruneau, 2007), ability to withstand external impacts and recover with least outside interferences (Mileti, 1999).

Economic resilience has been defined as the "inherent ability and adaptive response that enables firms and regions to avoid maximum potential losses" (Rose & Liao, 2005). Mainly economic resilience has been studied in context to seismic response and recovery (Tierney, 1997; Bruneau et al., 2003), community behavior (Chang & Shinozuka, 2004) and disaster hazard analysis (Rose, 2004b), among others (Rose, 2009a).

Even though there are different opinions in defining resilience there is some consensus in the measurement of system resilience. Generally resilience in measured in terms of the amount by which the system is able to avoid maximum impact (static resilience (Rose, 2004a)/robustness (McDaniels et al., 2008)) and the speed at which the system recovers from a disruption (dynamic resilience (Rose, 2004a)/ rapidity (Zobel, 2010)). In recent work Vugrin et al. (2010) have developed an economic resilience framework for measuring the targeted economic response of infrastructures.

1.4 Building resilience for large-scale infrastructures - approach and contributions

Several research components are considered here in building the resilience estimation framework for interdependent economic systems.

1.4.1 Interdependence modeling

This research study addresses the problem of estimating, quantifying and planning for resilience in interdependent systems, where interconnectedness adds to problem complexity. Understanding interdependence is critical for the development of the framework proposed here. The ultimate usefulness of understanding interdependent impacts, particularly from the standpoint of a preparedness decision maker, is not just a descriptor of property damage, but of business economic interruption (Tierney, 1997; Rose & Liao, 2005). That is, at the heart of understanding and planning for disruptive events is the quantification of (i) dollars of losses, and (ii) extent of and duration of system inoperability. The study of physical models of interdependency provides little benefit unless they are ultimately translated into those quantities. As such, this work focusses on modeling economic resilience in interdependent infrastructures and industries.

The input-output based models (Leontief, 1941; Haimes & Jiang, 2001) are utilized here for understanding and building an interdependent resilience framework. In particular the inoperability input-output enterprise provides a suitable framework on which interdependent infrastructure resilience concepts can be built. As outlined before, interdependence modeling is not a problem in these models due to the availability of economic input-output data. Moreover, economic interactions between infrastructures can be integrated with physical attributes to strengthen resilience estimations. During disruptions the inability of economic sectors to supply products or the loss of demand for commodities is an indicative of possible physical damages to infrastructure systems.

Rose (2007) points out that there is a similarity of the inoperability input-output model with static resilience estimation. Also the dynamic inoperability input-output model (DIIM), based on the economic input-output model, captures the interconnectedness of infrastructures and models recovery from disruptions. Hence, it is a useful resilience construct that captures dynamic aspects of resilient recovery. This research interprets and extends the inoperability input-output model capabilities of resilience estimation.

One way to develop resilience to economic losses would be by maintaining product inventories which reflect physical actions. In its present formulation the DIIM assumes that resilience comes from within a sector (Lian & Haimes, 2006). Such a treatment of resilience does not account for the interdependent effects of product substitution and inventory management. This research expands the capabilities of the DIIM in capturing such effects.

1.4.2 Risk management and system performance planning

In order to improve interdependent infrastructure resilience, this work provides a structured decision-making approach. Emphasis has been given to devising strategies to reduce infrastructure losses (Grabowski & Roberts, 1997), and this study seeks to provide an approach to quantify the efficacy of such strategies that prepare for timely post-disaster infrastructure functionality and performance, with emphasis on the interdependent relationships among infrastructures.

In the estimation of a static resilience planning the IIM can measure the efficacy of preparedness strategies in interdependent infrastructures by quantifying measures of interdependent inoperability and economic loss that may result from a disruptive event. Developing and choosing such a strategy requires quantitative trade-off analyses among different metrics such as cost, benefit, and risk, where interdependent sector risk is measured with the IIM. While the IIM can be used in a descriptive manner to model inoperability and economic loss resulting from a disruptive event, the ultimate usefulness of the model comes from its prescriptive ability to quantify how the implementation of risk management can lessen the interdependent impacts of a disruptive event. There are numerous ways to plan for risk management of interdependent systems using the IIM metrics (Jiang & Haimes, 2004; Anderson et al., 2007; Crowther, 2008). The risk management approach suggested here assumes that there exist planning policies that lead to reduction the demand losses, which translates to reduced economic losses. Planning economic loss reduction policy distributions based on the availability of budgets or deciding the budgets required for allowable economic losses provide two perspectives of the risk management decision-making. Such an approach contributes a simple scheme that adds to the capabilities of the IIM-based interdependent risk evaluation. Presenting the problem as a scheme to strengthen static resilience strengthening scheme is a new research approach.

Dynamic resilience explained through a model extension of the DIIM is also based on a planning decision. Such planning is aimed at making the model confirm to targeted behaviors that are representative of system resilience. The meaning and interpretation of dynamic resilience comes from certain properties that systems exhibit. In this research, dynamic resilience of a disrupted system is quantified in terms of three metrics: (i) average level of system operability/functionality (ii) maximum inoperability/loss of functionality, and (iii) time to recovery. These three metrics can be utilized to generate performance criteria for the dynamic behavior of interdependent recoveries. Specifically for large-scale interdependent systems that are being studied here the resilience quantification from the performance target-based planning approach is a new concept introduced here. Most of the models that discuss infrastructure resilience are limited in their treatment of quantifying the interdependent nature of resilience. Similar metrics exist in engineering resilience methods (Bruneau et al., 2003) but have been used it in a qualitative manner to discuss the collective resilience of systems. Quantitative treatment has been limited to individual systems (Zobel, 2011). Due to the wide scope of the input-output model the dynamic resilience estimation methods are applicable to many systems in unison. Network-based models have been used to quantify combined resilience for transportation (Dueñas-Osorio et al., 2007) or ICT systems (Ulieru, 2007), but they have to be infrastructure and network specific.

The dynamic resilience planning methods add more meaning to the resilience interpretation of the previous dynamic risk input-output schemes. It is shown, that in particular for resilient recovery, it is desirable for a sector to maintain stock of other sectors for utilization during recovery. Maintaining stock inventory would help a sector be prepared for the disruption in advance, thus providing an *inherent* interdependent resilience. Also *adaptive* resilience, which for interdependent sectors is achieved through changes in production and modified flows of resources when disruptions occur, can be quantified through the methods presented here. Previous DIIM research has ignored such effects and assumed that the interdependency structure remains invariant for entire analysis. Many researchers have pointed out that use of input-output models for disruption modeling should account for such changes (Kujawski, 2006; Hallegatte, 2008). Our methods account for updated interdependency structures between infrastructures reflecting new market situations as a result of disruptions.

In most resilience estimation studies, the notion of equilibrium is central to system recovery. The engineering resilience view (Bruneau et al., 2003; McDaniels et al., 2008; Zobel, 2011), shared by TISP, associates resilience with the ability to return to previous levels of stability. Rose & Liao (2005) have argued that for some disruptions economic systems cannot return to original stability levels. Nonetheless their economic resilience approaches are based on the notion that resilience leads to attaining different levels of equilibrium. Regional economies do not remain at equilibrium and hence resilience is truly adaptive if it impacts economic evolution (Simmie & Martin, 2010). Associating resilience with constant change is becoming a popular notion due to evolutionary nature of economic systems (Pendall et al., 2010). Though not discussed explicitly here, the approach presented in this research is capable of extending the "constant change" notion to interdependent resilience.

1.4.3 Mathematical concepts developed

Evaluation of model feasibility

It is important is establish the feasibility of the models being developed. The input data and output metrics in the interdependency models are constructed to confirm to real-world economic properties. Hence there are certain mathematical rules that the models should confirm to. In the static models the existence of the matrix inverses are critical for model solutions, while in dynamic models the stability of the system is important for the existence of feasible solutions. Since there are matrices involved here, eigenvalue analysis (Hu et al., 1998; Meyer, 2000; Lewis et al., 2006) is utilized to understand system behaviors. In particular the Greshgorins Circle Theorem (Moon & Stirling, 2000) has been utilized here to show the possible ranges of values of the dynamic system rate matrix elements that are required for stable solutions. Such mathematical treatment of these models has been missing from literature.

1.4.4 Robust optimization in static resilience planning

Risk studies of infrastructures requires quantification of the disruptive events and the underlying interdependent complexities of the systems. A deterministic approach for such problems has limited scope (Rose, 2004b), making uncertainty-based analysis a logical alternative. Sources of epistemic and aleatory uncertainties (Paté-Cornell, 1996; Haimes, 2009) arising due to unreliable data and limitations in disaster predictions need to be included in the analysis approach. In interdependent systems the exact relationships between the elements at the physical or economic level may be unknown though estimable, thereby creating scope for analyses that incorporate approaches such as structural fragility (Kim et al., 2007), covariance structure estimates (Hays & Kachi, 2008) and auto-regressive estimates (Bessler & Yang, 2003). Hence, many approaches have incorporated uncertainty in the modeling of interdependent sectors, including agent-based models (Lewis, 2006), discrete simulations and dynamic models (Brown et al., 2004; Min et al., 2007), among others.

Incorporating uncertainties in systems analyses increase the scope for decision making in planning and management for extreme events. Risk-based decision making involves finding optimal solutions to single or multiple objectives that provide guidelines to minimize risk while optimizing any number of other often competing objectives. Since seminal works by Dantzig (1955) and Charnes & Cooper (1959), stochastic optimization methods for decision making under uncertainty have been widely developed (Infanger, 1994; Kall & Mayer, 2010). These methods assume that underlying probability distributions for uncertain model parameters are know from historical data, and therefore future predictions are governed by past distributions. Decisions made from probability-driven optimal solutions are only as good as those underlying probability distributions (Huber, 2010). Some of these inaccuracies are addressed by the robust optimization framework (Soyster, 1973), that provides decision making solutions that are feasible and distribution independent. And since often is the case in risk-based decision making that we seek not the best option but rather to avoid making a bad decision, robust decision making can reflect worst case scenarios. Due to their ability to account for uncertainty in underlying parameters, robust optimization has seen several recent theoretical and methodological developments (Ben-Tal & Nemirovski, 2000; Ben-Tal et al., 2006; Atamturk & Zhang, 2007; Ben-Tal et al., 2009). While robust decision making has been applied in analyses of individual systems, e.g., inventory control (Bertsimas & Thiele, 2006; Bienstock & Özbay, 2008), its application to interdependent systems is limited.

Data assimilation in dynamic resilience planning

The dynamic resilience planning is a problem of estimating the rate parameter of the dynamic risk input-output model. Having set planned targets for the resilience metrics the dynamic resilience behavior is established by calibrating the model to meet the prescribed target values. Model calibration and prediction is a very widely used approach based on linear or non-linear least-square fit methods (Draper & Smith, 1998). If the model is dynamic, as is the case here, then the parameter estimation problem is a data assimilation scheme where data are combined with results from a predictive model to produce an estimate of the current state of the system (Lewis et al., 2006). The data assimilation problem developed here is solved using a forward sensitivity approach, in which parameter sensitivities are critical for the problem solution (Lakshmivarahan & Lewis, 2010). Data assimilation is typically used in forecasting, particularly in weather forecasting (Kanamitsu, 1989; Kalnay, 2003), hydrology (Reichle et al., 2002), among others (Lewis et al., 2006). No literature is available on the use of data assimilation to describe dynamic recovery of disrupted infrastructure and economic systems, considering its great potential for use in interdependent resilience planning.

1.5 Inland waterway applications

The application of the methods developed in this research can be made on multi-modal transportation systems and in particular inland waterway ports and network systems. Multi-modal transportation systems, identified by DHS to be among the critical US infrastructures (DHS., 2009), play a significant role in maintaining commodity flows across industries, and preserving the functionality of a multi-regional interdependent economy. A disruptive event that causes inoperability of the multi-modal transportation network is propagated to industry demand and supply, thereby causing production losses. For example, in 2002, Oklahoma witnessed the collapse of an I-40 bridge spanning the Arkansas River, due to a barge collision with a bridge pylon. The resulting daily detouring of 22,000 vehicles caused congestion, secondary road infrastructure accelerated wear and other economic losses that persisted for nearly two months, as the bridge was repaired (Schmitt et al., 2010). Similarly, the I-35W bridge collapse over the Mississippi river in Minneapolis, Minnesota caused the daily rerouting of 140,000 vehicles (Zhu et al., 2010) with a significant adverse economic impact. Events like these make risk analysis of freight disruptions an important research topic. Furthermore, the risks of larger-scale disruptive events, such as earthquakes and malevolent man-made attacks, could result in the protracted closure of key transportation facilities such as rail yards, cargo terminals, airports, seaports, or inland ports. Multi-modal risk assessment studies of various sorts have appeared recently (Sohn et al., 2004; Ham et al., 2005a,b; Tatano & Tsuchiya, 2008; Ishfaq & Sox, 2010); though risk studies of inland port disruptions have been particularly sparse in number. Transfer facilities, such as inland ports, are the locations that are particularly susceptible to disruptions in commodity flows that can cause losses of demand and supply to certain industries, which then propagate among other interdependent intra- and inter-regional industries. Inland port operations that are susceptible to disruptions include commodity arrivals at port, storage at unloading yards, transfer to docks by cranes, loading onto vessels, and departure to destinations. Inland waterways, although prominent in North America, are even more common in the European economy (Rodrigue et al., 2010).

The risk-based interdependency model quantifies the propagation of inoperability, or the extent to which industry output will not be produced, through a set of interconnected industry sectors. Operations at inland ports can be modeled through simulations as queueing systems capable of quantifying the number of commodities at each point of operation. By comparing the normal port operations with the disrupted port operations, the difference in number of arrivals and departures can be obtained to measure the losses for commodities/industries that use the port. Quantifying these losses respectively as loss in demand for the exporting region and loss in supply and demand for the importing region provides parameters for the risk input-output models. The interdependent nature of these models then cascades these losses to other industries locally and across regions, thereby providing an estimate for large-scale economic impacts of disruptions at an inland port.

The work presented here differs from other transportation disruption studies in that it does not use the traditional network analysis approach. Most of the existing studies in the literature build an optimized transportation network of roads and railways that minimize travel distance across regions and use the economic data to maintain the constraint that the supply is equal to the demand (Ham et al., 2005a,b). Such network approaches are computationally intensive; hence, our analysis of the inflow or outflow of goods through a region using queuing concepts at particular components in a network can reduce the computational burden and help build an efficient means to quantify multi-regional inoperability and economic losses due to disruptive events. In addition, existing studies assume that during a disruption, supply finds alternate paths on the network to meet demand. In practice, this might not be true for short time duration and across all industries. In particular, for port disruptions there are bulk products that are sitting at the port or are off shore for which alternative transportation arrangements are costly or impractical. In light of this perception, a company decision maker may prefer to wait for some time for the port to reopen.

1.6 Structure of the dissertation

The research framework developed in this study is shown in Figure 1.2. The discussion of the framework is organized in rest of the dissertation Chapters that follow.

Chapter 2 reviews the static and dynamic interdependent economic and risk input-output models. It presents mathematical proofs for the existence of feasible solutions to the static and dynamic input-output models. The discussion of the static economic input-output model includes model development, existence of model solution, and the data support on which the model is built. The static risk input-output model is presented with the risk metrics and the mathematical properties of the risk interdependency matrix are also discussed. The dynamic economic input-output model is presented and shown to be unstable. The dynamic risk input-output model is presented and its stability criteria are established.

In Chapter 3 the resilience framework using economic input-output data based risk models is constructed. The static risk input-output model metrics are shown to be suitable for resilience estimation. A static resilience problem is presented as a resource allocation and planning scheme. Subsequently a risk management optimization problem is formulated and solved to show resilience planning. The dynamic risk input-output model is developed as a resilience estimation construct. To develop the resilience methodology the dynamic model behavior to external shocks is analyzed and resilience is quantified through appropriate metrics. These metrics, called the *time averaged level of operability*, *maximum loss of functionality*, *time to recovery*, respectively denote the systems' ability to maintain functionality throughout the post-disruption response, withstand the maximum disruptive impact and still recover, and progress towards recovery with some speed. The functional relationship between the metrics is developed to generate a decision support space. Adaptive resilience behavior through the dynamic risk input-output model is also presented.

Chapter 4 discusses the nature of uncertainties in the static resilience planning problem developed in Chapter 3. Uncertainties change the problem objective due to which the static resilience risk management questions posed previously have to be reformulated as a problem of estimating the required amounts of resource allocations when economic losses need to be kept below certain thresholds. The general guidelines that need to be followed to make the problem robust are presented, which depends upon the uncertainty sets that are vital to the solution of the robust problem. The robust formulations to the nominal problems are built from the given uncertainty sets. Robust formulations, which increase the problem dimensions but preserve the nominal problem structure, are the result of the given data and event uncertainty sets. An example problem highlights the usefulness of the robust schemes by showing that small data uncertainties have great affect on the planning solutions, which are accounted for in the robust solution.

Chapter 5 proposes the scheme for solving the parameter estimation problem from dynamic resilience metrics. The inverse problem and the parameter estimation scheme is subsequently explained. The forward sensitivity scheme is presented and it is shown that control estimates are obtained from sensitivity functions. The computations of sensitivity matrices and the forward sensitivity scheme's computational tractability in solving the inverse problem are also discussed. An example problem is solved using the sensitivity algorithm and the different dynamic model and resilience concepts generated throughout Chapters 2 to 5 are discussed. The usefulness of the method in generating the model parameter essentially provides a useful tool to understand and interpret the interdependence of dynamic systems.

Chapter 6 is the application of the methodologies of concepts developed here on an inland port system. It discusses the relationship between the input-output risk metrics and transportation hubs (e.g., ports). A queueing-based simulation model is presented that describes normal port export and import operations and the adjustments that can be made to incorporate effects of disruptive events. A case study of the impacts of exports and imports disruptions through the Port of Catoosa in Oklahoma and the inland waterway network of the Mississippi River system is shown.

Chapter 7 concludes this study and discusses future research avenues for this work.

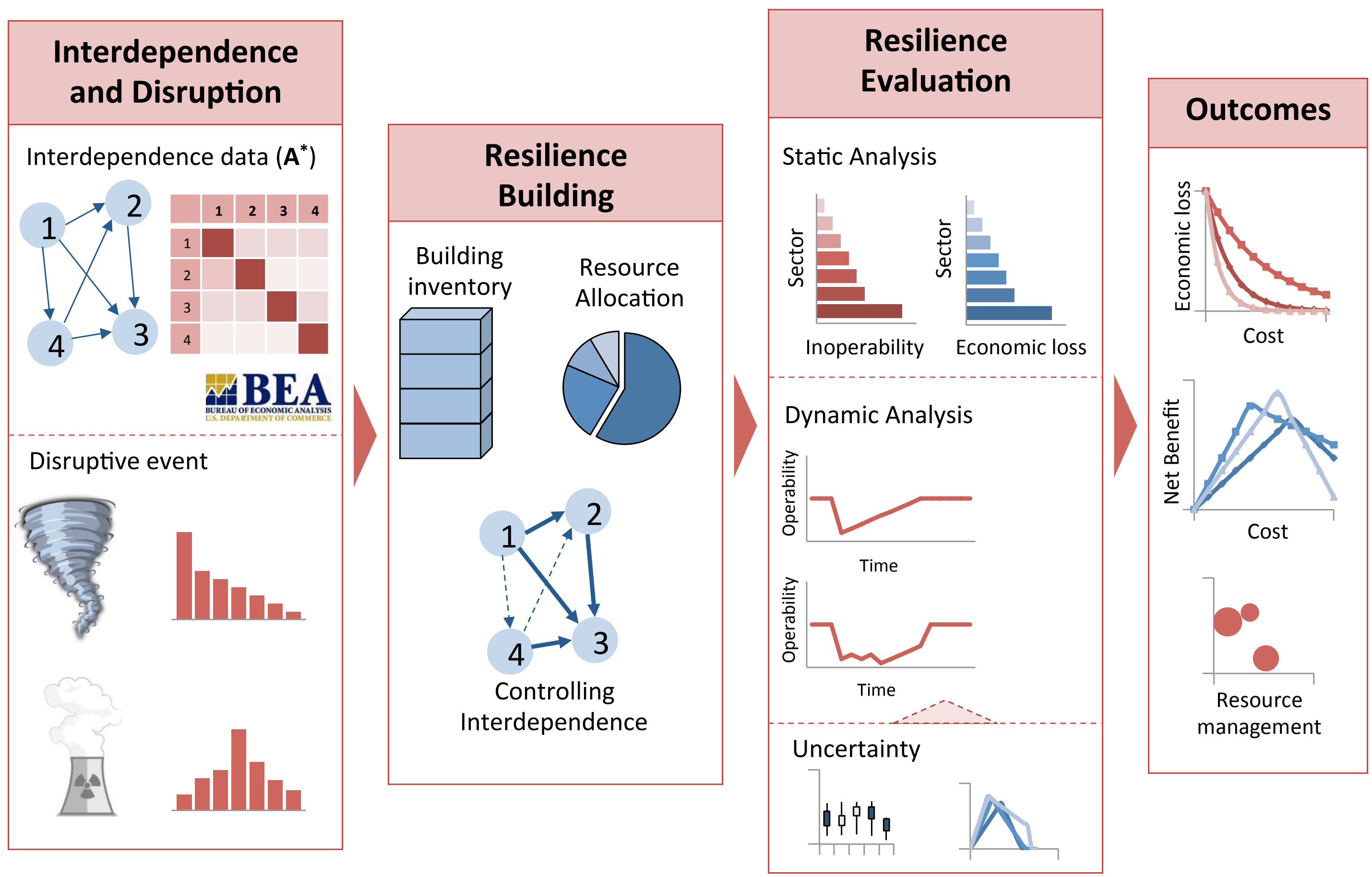


Figure 1.2: The research framework.

Chapter 2

Economic Input-Output Based Risk Interdependency Models

2.1 Introduction

This Chapter generates a discussion that includes a review of static and dynamic interdependent economic and risk input-output models, and analysis of the mathematical conditions required for these models to be feasible. Such analysis is crucial when the model metrics are used to describe economic risk to infrastructure systems, because it is desired that the model result is reflective of a quantifiable real world situation. While the mathematical analysis generates some already known results about the properties of the interdependency matrix in the economic input-output model, the presentation of such analysis for inoperability input-output models is an extension to research. The discussion in this Chapter is geared towards the development of the dynamic risk input-output model that is presented in the Section 2.5 with a different approach than previous research.

The primary research question here is:

> What are the models used for quantifying interdependence and do these models give feasible solutions?

The static interdependency models are systems of linear equations relating input and output vectors of the same dimension by a matrix operator that captures interdependence. Hence, the solution of a static model exists and is unique if the matrix operator is invertible. The dynamic interdependency models are systems of first-order linear differential equations that are temporal extensions of the static models. In dynamic behavior the stability of the model is important for the existence of feasible convergent solutions across all times, and it depends upon the growth/decay rate parameter in the model. We address these issues as we progress through the Chapter.

In Section 2.2 the discussion of the static economic input-output model includes model development (Section 2.2.1), the proof that the model solution exists due to the properties of the interdependency matrix arising from the problem structure (Section 2.2.2), and the data based approach for generating the model (Section 2.2.3). The static risk input-output model developed in Section 2.3.1 presents the risk metrics with the range of values they take, while in Section 2.3.2 the properties of the risk interdependency matrix that lead to an invertible matrix are presented. Sections 2.4.1 and 2.4.2 respectively present the dynamic economic input-output model and the conditions that make the model unstable. The Section 2.5 development of the dynamic risk input-output model addresses, in Section 2.5.2, the stability criteria necessary for convergent solutions. Section 2.5.3 establishes the possible values the model parameters can take for satisfy the stability criteria. This completes the development of the new risk input-output model presented in the research. Section 2.6 provides a closing remark to the findings of the Chapter.

2.2 Economic input-output model

The economic input-output model (Leontief, 1936, 1941, 1951, 1986), for which Wassily Leontief won a Nobel prize, has been widely accepted as a useful tool for analyzing the interdependent connections between industry sectors. Such interdependence makes for convenient usage in macroeconomic impact analysis studies. Leontief's model establishes a common comparative framework between sectors ranging from agriculture, energy, manufacturing to banking, communications, information technology, among others. Over the years the economic input-output model has been studied and developed to interpret intra-regional and multi-regional static and dynamic interactions among infrastructures (Isard et al., 1998; Lahr & Dietzenbacher, 2001; Leontief et al., 2004; Ten Raa, 2005; Miller & Blair, 2009). The wide use and popularity of the model is due to the fact that it is built for, supported, and verified by vast data resources (Polenske, 1980; Horowitz & Planting, 2006).

2.2.1 Leontief's economic model

The basic principle behind the economic input-output model is that it explains the supply and demand balance in an economy consisting of interacting sectors. It can be interpreted as a macroeconomic supply chain in which firms produce goods to satisfy demands of other firms and households and in return use resources from the other firms and households to make more of those goods. The value of transactions between industries in the group provides an indication of the supply and demand requirements in the economy. When supply and demand are balanced the economy is said to be at equilibrium. Equation (2.1) expresses this supply and demand balance mathematically for a group of n interacting industries. The total output of the i^{th} industry sector, measured in dollars, is distributed to all industries and also satisfies external demand. If x_i is the dollar value for industry i total output, z_{ij} is the dollar amount of industry i output purchased by industry j, and c_i is the dollar value of final demand for industry i output, then the input-output balance is expressed as

$$x_i = \sum_{j=1}^{n} z_{ij} + c_i \tag{2.1}$$

Equation (2.1) is called the demand side input-output model because it accounts for the amount of intermediary and final demands for industry output, thus giving an indication of the output required to balance such demands. A supply side input-output model also exists (Ghosh, 1958), and measures the balance of industry j output with the total amount of commodities it purchases from all other industries and the value added v_j expenditures it requires to generate output. Hence, in the supply side model the purchases required by an industry are a measure of supply amount needed to produce its output. Equation (2.2) expresses the supply side input-output balance discussed so far.

$$x_j = \sum_{i=1}^{n} z_{ij} + v_j \tag{2.2}$$

The demand and supply balances through the input-output model are depicted in Table 2.1, which shows the equilibrium economic accounting in terms of balances between selling and purchasing sectors. Such tables, called input-output transactions (flow) tables, have been adopted across number of countries for understanding and accounting of their economic structures. The final demand column in these transaction tables is made up of consumer/household purchases, private investment purchases, federal, state and local government purchases, and exports sales of industry products. The value added rows of the tables consist of payments made by the sector in labor expenses, government taxes, capital interest, rental and other payments, and import purchases by industries to make their product. The input-output balance between economic sectors typically depicts a long-run behavior of interindustry flows. The numbers in the transaction tables would generally show annual amounts of selling and purchasing balances between industries.

A key assumption in the input-output model is that z_{ij} depends upon x_j, which implies that the amount of product an industry purchases depends upon its output production. In the input-output model this relationship is assumed to be linear and is expressed as

$$z_{ij} = a_{ij} x_j \tag{2.3}$$

where a_{ij}, called the technical coefficient, is explained as the value of product industry j purchases from industry i for producing \$1 of its own output. Under the

Table 2.1: Economic Input-Output transactions(flow) table for an n sector economy

		Buying Sector					Final Demand	Total Output
		1	...	j	...	n		
Selling sector	1	z_{11}	...	z_{1j}	...	z_{1n}	c_1	x_1
	$\vdots$	$\vdots$		$\vdots$		$\vdots$	$\vdots$	$\vdots$
	i	z_{i1}	...	z_{ij}	...	z_{in}	c_i	x_i
	$\vdots$	$\vdots$		$\vdots$		$\vdots$	$\vdots$	$\vdots$
	n	z_{n1}	...	z_{nj}	...	z_{nn}	c_n	x_n
Value Added		v_1	...	v_j	...	v_n		
Total Output		x_1	...	x_j	...	x_n		

Equation (2.3) assumption the input-output Equations (2.1) and (2.2) become

$$x_i = \sum_{j=1}^{n} a_{ij}x_j + c_i \tag{2.4}$$

$$x_j = \sum_{i=1}^{n} a_{ij}x_j + v_j \tag{2.5}$$

Research and application has primarily been focused on the development of the demand side input-output model, which was as intended through Leontief's formulation and model usage. The supply side model has been questioned because it depicts economic behavior for a constant supply distribution, which has raised concerns in research (Oosterhaven, 1988; Rose & Allison, 1989).

2.2.2 Existence of solution to input-output system

The Leontief input-output model expression of Equation (2.4) leads to the n sector matrix Equation (2.6) where $\mathbf{x}$ is an $n \times 1$ vector of industry production outputs, $\mathbf{A}$ is an $n \times n$ industry-by-industry matrix of technical coefficients, $\mathbf{c}$ is an $n \times 1$ vector of final demands, and $\mathbf{I}$ is an $n \times n$ identity matrix. The model shows that total production is made up of industry-to-industry intermediate production ($\mathbf{Ax}$)

and production to satisfy final demands ($\mathbf{c}$).

$$\mathbf{x} = \mathbf{A}\mathbf{x} + \mathbf{c} \implies \big(\mathbf{I} - \mathbf{A}\big)\mathbf{x} = \mathbf{c} \tag{2.6}$$

Equation (2.6) shows that the primary usage of the input-output model involves finding the industry outputs for given final demands, which depends upon the proposition that the matrix $\mathbf{I} - \mathbf{A}$ is invertible. We investigate and prove that this is always true through the following two properties of the elements of the $\mathbf{A}$ matrix.

1. In the economic system under consideration each industry purchases something from all the other industries. Hence the connectivity between two sectors is finite which means that $a_{ij} \geq 0, \forall 1 \leq i, j \leq n$. This leads to an irreducible non-negative technical coefficient matrix expressing a strong connected relationship across industries.

2. If both sides of Equation (2.5) are divided by x_j we get the following

$$1 = \sum_{i=1}^{n} a_{ij} + v_j/x_j \implies \sum_{i=1}^{n} a_{ij} = 1 - v_j/x_j < 1 \tag{2.7}$$

   which shows that the sum of elements along each column of the $\mathbf{A}$ matrix is at most equal to 1.

We now show that the above two conditions lead to an invertible $\mathbf{I} - \mathbf{A}$ matrix. For this we first state a property of the $\mathbf{A}$ matrix, which is essential for the existence of the inverse.

Proposition 1. *If a given non-negative irreducible matrix* $\mathbf{A} = \big\{a_{ij} : a_{ij} \geq 0, \forall 1 \leq i, j \leq n\big\}$ *has column sums less than 1 then the spectral radius of* $\mathbf{A}$*, given as* $\rho(\mathbf{A})$ *is always less than 1.*

Proof. Since $\mathbf{A}$ is an irreducible non-negative matrix and is also an $n \times n$ square matrix, from the Perron-Frobenius theorem (Meyer, 2000), the maximum eigenvalue λ of $\mathbf{A}^\intercal$, and hence $\mathbf{A}$, is a real number that satisfies the condition

$$0 \leq \min_j \sum_i a_{ij} \leq \lambda \leq \max_j \sum_i a_{ij} < 1 \tag{2.8}$$

where the upper bound of one exists because column sums of $\mathbf{A}$ are less than 1, while the lower bound is due to non-negative elements of $\mathbf{A}$. Hence, it can be concluded that the matrix $\mathbf{A}$ has a spectral radius less than 1, i.e., $\rho(\mathbf{A}) < 1$. □

The above Proposition means that the geometric series $\left\{\mathbf{A}, \mathbf{A}^2, ..., \mathbf{A}^k\right\}$ of powers of the $\mathbf{A}$ matrix converges towards 0, which is expressed as

$$\lim_{k \to \infty} \mathbf{A}^k = \mathbf{0} \tag{2.9}$$

Hence Equation (2.10) shows that the inverse of $\mathbf{I} - \mathbf{A}$, which by definition can be written as the infinite sum of the powers of $\mathbf{A}$ stops growing after some k and hence is less than infinity. This shows that the matrix $\mathbf{I} - \mathbf{A}$ is always invertible.

$$(\mathbf{I} - \mathbf{A})^{-1} = \sum_{k=0}^{\infty} \mathbf{A}^k < \infty \tag{2.10}$$

In the input-output literature $(\mathbf{I} - \mathbf{A})^{-1}$ is referred to as the Leontief inverse and its existence means that the industry output can be measured in terms of final demands as

$$\mathbf{x} = \left(\mathbf{I} - \mathbf{A}\right)^{-1} \mathbf{c} \tag{2.11}$$

The primary usage of Equation (2.11) is for economic impact analysis wherein we measure the change in output due to the change in demand resulting from growth or disruption to the economy.

2.2.3 Make-Use data tables for model development

The linear production function in the input-output model may appear to be an overly simple assumption, but nonetheless the model has been found to be applicable in vast amount of research efforts. Such research efforts have made sure that the validity of the model is attested for by the consistency of its results. The general consensus has been that input-output models can be useful tools in macroeconomic analysis when long run economic behavior is studied. In addition, there are detailed data sets available to support analysis with the model, including the commodity flow data published annually by the US Bureau of Economic Analysis (BEA., 2011) and the worldwide data maintained by the Organization for Economic Co-Operation and Development (OECD., 2011). The data sets used for constructing the input-output $\mathbf{A}$ matrix constitute what is known as the commodity-by-industry framework. Since the inception of this framework (Stone, 1961; Stone et al., 1963), it has been adopted by several countries around the world and ratified by the United Nations (UN., 2009) as the standard the input-output data gathering method.

The commodity-by-industry framework is based on the premise that industries make and use commodities. Every industry is associated with making one primary commodity in addition to several secondary commodities. The dollar value of commodities made by industries are collected in a *Make table*, which is an industry-by-commodity table. Most commodities are required by almost all industries for making products, which means every commodity is used by almost all industries. The *Use Table* collects information on the dollar commodity-by-industry usage in the economy.

In the commodity-by-industry framework generally the number of industries and commodities is not the same. If it is assumed that there are n industries producing m commodities, then these m commodities will be used by n industries. Hence, the make table translates into an $n \times m$ matrix $\mathbf{V} = \{v_{ik}, \forall 1 \leq i \leq n, 1 \leq k \leq m\}$, while the use table becomes an $m \times n$ matrix $\mathbf{U} = \{u_{kj}, \forall 1 \leq k \leq m, 1 \leq j \leq n\}$. Table 2.2 shows that input-output arrangement of the economy based on the make-use matrices. It can be seen from the table that the row sums of the make matrix are equal to the sector outputs ($\mathbf{x}$), while the column sums of the make matrix are equal to the commodity outputs or total industry inputs ($\mathbf{y}$).

Table 2.2: Commodity-by-industry input-output structure of the economy

		Commodities			Industries			Final Demand	Total Output
		1	...	m	1	...	n		
Commodities	1				u_{11}	...	u_{1n}	e_1	y_1
	⋮				⋮	⋱	⋮	⋮	⋮
	m				u_{m1}	...	u_{mn}	e_m	y_m
Industries	1	v_{11}	...	v_{1m}					x_1
	⋮	⋮	⋱	⋮					⋮
	n	v_{n1}	...	v_{nm}					x_n
Value Added					v_1	...	v_n		
Total Inputs		y_1	...	y_m	x_1	...	x_n		

While different forms of input-output $\mathbf{A}$ matrices can be obtained from the make-use data, the most relevant one has been found to be the industry-by-industry matrix. This matrix translates the make-use data into an n industry selling and purchasing structure similar to the Table 2.1 construct. The $\mathbf{A}$ matrix is obtained from normalized make ($\hat{\mathbf{V}}$) and normalized use ($\hat{\mathbf{U}}$) matrices, by establishing the input-output relationships through these matrices. The relationship between the matrices is shown in Table 2.3, which highlights the mathematical derivation of the $\mathbf{A}$ matrix through input-output relationships

Input-output accounting through the make-use tables has provided a data based credibility to the Leontief economic construct. Due to the the worldwide availability of such data and its organization into make-use tables the input-output method is also considered to represent a global supply chain characterizing inter-industry and

Table 2.3: Derivation for the input-output $\mathbf{A}$ matrix from make-use tables

Normalize matrices	$\hat{v}_{ik} = v_{ik}/y_k \implies \hat{\mathbf{V}} = \mathbf{V}[\text{diag}(\mathbf{y})]^{-1}$ $\hat{u}_{kj} = u_{kj}/x_j \implies \hat{\mathbf{U}} = \mathbf{U}[\text{diag}(\mathbf{x})]^{-1}$
Output-Input equation	$x_i = \sum_{k=1}^{n} v_{ik} = \sum_{k=1}^{n} \hat{v}_{ik} y_k$ $\implies \mathbf{x} = \hat{\mathbf{V}}\mathbf{y}$
Input-Output equation	$y_k = \sum_{j=1}^{m} u_{kj} + e_k = \sum_{j=1}^{m} \hat{u}_{kj} x_j$ $\implies \mathbf{y} = \hat{\mathbf{U}}\mathbf{x} + \mathbf{e}$
Derivation	$\hat{\mathbf{V}}\mathbf{y} = \hat{\mathbf{V}}\hat{\mathbf{U}}\mathbf{x} + \hat{\mathbf{V}}\mathbf{e}$ $\implies \mathbf{x} = (\hat{\mathbf{V}}\hat{\mathbf{U}})\mathbf{x} + \hat{\mathbf{V}}\mathbf{e}$ $\implies \mathbf{x} = \mathbf{A}\mathbf{x} + \mathbf{c}$
Result	$\mathbf{A} = \hat{\mathbf{V}}\hat{\mathbf{U}}$, $\mathbf{c} = \hat{\mathbf{V}}\mathbf{e}$

commodity flows. In the US this supply chain is defined and compiled for industries named according to the North American Industry Classification System (NAICS) (Kelton et al., 2008). From BEA data for NAICS industries input-output accounts can be generated at the national, regional and local level. Every 5 years the BEA gives a list of 65 NAICS industries for which input-output accounts are maintained, and also annually produces a list of 15 aggregated industries from combining the 65 sectors.

2.3 Inoperability input-output model

2.3.1 Model formulation

An extension of the economic input-output model of interest to this work is the Inoperability Input-Output Model (IIM) (Santos & Haimes, 2004; Santos, 2006). Instead of describing the connections between the interdependent industry sectors in terms of commodity flow dollars, the IIM illustrates how normalized production losses propagate through all interconnected industries.

Central to the development of the inoperability input-output analysis is the assumption that the interdependency structure of the economy, measured by the $\mathbf{A}$ matrix does not change when demands and resulting outputs change. If in the n sector economic system $\hat{\mathbf{x}}$ and $\hat{\mathbf{c}}$ represent equilibrium as-planned output and final demand levels respectively, while $\tilde{\mathbf{x}}$ and $\tilde{\mathbf{c}}$ represent disrupted/perturbed equilibrium levels for output and final demands respectively, then

$$\hat{\mathbf{x}} = \mathbf{A}\hat{\mathbf{x}} + \hat{\mathbf{c}} \tag{2.12}$$

$$\tilde{\mathbf{x}} = \mathbf{A}\tilde{\mathbf{x}} + \tilde{\mathbf{c}} \tag{2.13}$$

Here it is assumed that the production and demand levels at the perturbed conditions are lower than as-planned levels. The key premise of Equations (2.12) and (2.13) is that when the as-planned interdependent economy is disrupted there are reductions in demands for products, which results in reduced equilibrium industry outputs. The IIM approach builds on this notion of equilibrium shift by providing two metrics that translate degraded outputs and demands into risk measures. These metrics called inoperability and demand perturbations are explained below.

Inoperability for industry i, q_i, refers to the inability of the industry to perform its intended functions. In the context of economic loss analysis for measuring failure in industry sectors, q_i is the measure of the loss of production in industry i as a proportion of its original production level, as shown in Equation (2.14). The inoperability of a system lies between 0 and 1, where $q_i = 0(\tilde{x}_i = \hat{x}_i)$ is a measure of a perfectly operable industry i, and $q_i = 1(\tilde{x} = 0)$ is a measure of complete failure of industry i.

$$q_i = \frac{\text{As-planned Output}(\hat{x}_i) - \text{Perturbed Output}(\tilde{x}_i)}{\text{As-planned Output}(\hat{x}_i)} \tag{2.14}$$

For the n sector economy the inoperability vector $\mathbf{q}$ of size $n \times 1$ is a vector of industry inoperabilities expressed in terms of the changed outputs normalized by the as-planned outputs.

$$q_i = \big(\hat{x}_i - \tilde{x}_i\big)/\hat{x}_i \implies \mathbf{q} = \big[\mathrm{diag}(\hat{\mathbf{x}})\big]^{-1}\big(\hat{\mathbf{x}} - \tilde{\mathbf{x}}\big) \tag{2.15}$$

Demand perturbation for industry i, c_i^*, refers of the change in final demand for industry i output due to disruptive events. For economic systems it is the measure of the change in demand as a proportion of the original production level in industry i, as shown in Equation (2.16). Demand perturbation can occur due to the inability of the producing sector to meet the demands of the final consumers when there is a failure in the system. The values of c_i^* lie between 0 for no economic failure and loss of demand, and $c_i/\hat{x}_i < 1$ for total economic failure and loss in demand.

$$c_i^* = \frac{\text{As-planned Demand}(\hat{c}_i) - \text{Perturbed Demand}(\tilde{c}_i)}{\text{As-planned Output}(\hat{x}_i)} \tag{2.16}$$

The demand perturbation vector $\mathbf{c}^*$ of size $n \times 1$ for the n sector economy is thus a vector of industry demand perturbations expressed in terms of the changed final demands normalized by the as-planned outputs.

$$c_i^* = \big(\hat{c}_i - \tilde{c}_i\big)/\hat{x}_i \implies \mathbf{c}^* = \big[\mathrm{diag}(\hat{\mathbf{x}})\big]^{-1}\big(\hat{\mathbf{c}} - \tilde{\mathbf{c}}\big) \tag{2.17}$$

From the concepts developed in Equations (2.12) through (2.17) the IIM provided in Equation (2.18) is derived by subtracting Equation (2.13) from Equation (2.12) and normalizing with $\hat{\mathbf{x}}$, thus essentially maintaining a form similar to the Leontief economic input-output model.

$$\begin{aligned} \big[\mathrm{diag}(\hat{\mathbf{x}})\big]^{-1}(\hat{\mathbf{x}} - \tilde{\mathbf{x}}) &= \Big(\big[\mathrm{diag}(\hat{\mathbf{x}})\big]^{-1}\mathbf{A}\big[\mathrm{diag}(\hat{\mathbf{x}})\big]\Big)\big[\mathrm{diag}(\hat{\mathbf{x}})\big]^{-1}(\hat{\mathbf{x}} - \tilde{\mathbf{x}}) \\ &+ \big[\mathrm{diag}(\hat{\mathbf{x}})\big]^{-1}(\hat{\mathbf{c}} - \tilde{\mathbf{c}}) \\ \implies \mathbf{q} &= \mathbf{A}^*\mathbf{q} + \mathbf{c}^* \end{aligned} \tag{2.18}$$

Normalized interdependency matrix $\mathbf{A}^*$ of size $n \times n$ is a modified version of the original $\mathbf{A}$ matrix describing the extent of economic interdependence among a set of infrastructure and industry sectors. Shown in Equation (2.19), the row elements of $\mathbf{A}^*$ indicate the proportions of additional inoperability that are contributed by a column sector to the row sector.

$$\mathbf{A}^* = \big[\text{diag}(\hat{\mathbf{x}})\big]^{-1}\mathbf{A}\big[\text{diag}(\hat{\mathbf{x}})\big] \iff a^*_{ij} = a_{ij}\big(\hat{x}_j/\hat{x}_i\big) \tag{2.19}$$

2.3.2 Existence of solution to the inoperability input-output model

Similar to the input-output approach the IIM measures the inoperability in terms of the demand perturbations, which means that the solution of Equation (2.18) depends upon the existence of the inverse of the matrix $\mathbf{I} - \mathbf{A}^*$. We can prove that this inverse always exists due to the properties of the $\mathbf{A}$ and $\mathbf{A}^*$ matrices.

1. Since the elements of $\mathbf{A}^*$ are derived from $\mathbf{A}$ it is easy to see that $\mathbf{A}^*$ would also be an irreducible, non-negative matrix, i.e., $a^*_{ij} \geq 0, \forall 1 \leq i, j \leq n$.

2. If both sides of Equation (2.5) are divided by x_i we get

$$1 = \sum_{j=1}^{n} a_{ij}\frac{x_j}{x_i} + \frac{c_i}{x_i} \implies \sum_{j=1}^{n} a^*_{ij} = 1 - \frac{c_i}{x_i} \leq 1 \tag{2.20}$$

   Hence we can see that the sum of elements on each row of $\mathbf{A}^*$ is less than or equal to 1.

Arguing as before, we can show that the spectral radius of $\mathbf{A}^*$ is less than 1, which leads to an invertible $\mathbf{I} - \mathbf{A}^*$ matrix structure. This is shown to be true in two different ways. Proposition 2 uses the fact that $\mathbf{A}^*$ and $\mathbf{A}$ are similar matrices and thus have the same eigenvalues and spectral radius. Proposition 3 proves the same result for a general $\mathbf{A}^*$ matrix with the properties highlighted above.

Proposition 2. *The interdependency matrices* $\mathbf{A}^*$ *and* $\mathbf{A}$ *have the same eigenvalues.*

Proof. Let $\bar{\lambda}^*$ be an eigenvalue of $\mathbf{A}^*$. Therefore $\bar{\lambda}^*$ satisfies the characteristic equation of $\mathbf{A}^*$, which leads to the following

$$\begin{aligned}\det(\mathbf{A}^* - \bar{\lambda}^*\mathbf{I}) &= \det(\left[\text{diag}(\hat{\mathbf{x}})\right]^{-1}\mathbf{A}\left[\text{diag}(\hat{\mathbf{x}})\right] - \bar{\lambda}^*\mathbf{I}) \\ &= \left[\text{diag}(\hat{\mathbf{x}})\right]^{-1}\det(\mathbf{A} - \bar{\lambda}^*\mathbf{I})\left[\text{diag}(\hat{\mathbf{x}})\right] \\ &= \det(\mathbf{A} - \bar{\lambda}^*\mathbf{I}) = 0 \end{aligned} \tag{2.21}$$

$\bar{\lambda}^*$ also satisfies the characteristic equation of $\mathbf{A}$, which means $\mathbf{A}^*$ and $\mathbf{A}$ have the same eigenvalues. Hence, if $\mathbf{A}$ has a spectral radius less than 1, then so does $\mathbf{A}^*$. □

Proposition 3. *If a given non-negative irreducible matrix* $\mathbf{A}^* = \left\{a^*_{ij} : a^*_{ij} \geq 0, \forall 1 \leq i, j \leq n\right\}$ *has row sums less than 1 then the spectral radius of* $\mathbf{A}^*$*, given as* $\rho(\mathbf{A}^*)$ *is always less than 1.*

Proof. Since $\mathbf{A}^*$ is an irreducible non-negative matrix and is also an $n \times n$ square matrix, from the Perron-Frobenius theorem (Meyer, 2000), the largest eigenvalue λ^* of $\mathbf{A}^*$ is a real number that satisfies the condition

$$0 \leq \min_i \sum_j a^*_{ij} \leq \lambda^* \leq \max_i \sum_j a^*_{ij} \leq 1 \tag{2.22}$$

where the upper bound of one exists because row sums of $\mathbf{A}^*$ are less than 1, while the lower bound is due to non-negative elements of $\mathbf{A}^*$. Hence, it can be concluded that the matrix $\mathbf{A}^*$ has a spectral radius less than 1, i.e., $\rho(\mathbf{A}^*) \leq 1$. □

From either Proposition 2 or 3 we conclude that the geometric series $\left\{\mathbf{A}^*, (\mathbf{A}^*)^2, ...\right\}$ of powers of the $\mathbf{A}^*$ matrix converges towards 0, which is expressed as

$$\lim_{k\to\infty} (\mathbf{A}^*)^k = \mathbf{0} \tag{2.23}$$

Hence Equation (2.24) shows that the inverse of $\mathbf{I} - \mathbf{A}^*$, which by definition can be written as the infinite sum of the powers of $\mathbf{A}^*$ stops growing after some k and hence is less than infinity. This shows that the matrix $\mathbf{I} - \mathbf{A}^*$ is always invertible.

$$(\mathbf{I} - \mathbf{A}^*)^{-1} = \sum_{k=0}^{\infty} (\mathbf{A}^*)^k < \infty \tag{2.24}$$

Similar to the economic input-output model, the interdependent risk impact analysis has been concentrated on solving the system

$$\mathbf{q} = \left(\mathbf{I} - \mathbf{A}^*\right)^{-1} \mathbf{c}^* \tag{2.25}$$

2.4 Dynamic economic input-output model

2.4.1 Model development

While the static equilibrium analysis of Equation (2.11) allows for study of long-term economic behavior, interest lies in studying the short-term dynamic aspects of changes in interdependent economic systems. Research has focussed on developing different dynamic models using the input-output concepts (Solow, 1956; Duchin & Szyld, 1985; Steenge & Thissen, 2005). The Leontief (1986) dynamic input-output model is one such construct that builds a simple notion of a dynamically growing economy given by a first order differential equation. In a dynamic interdependent economy it is assumed that industry outputs $\mathbf{x}(t)$ and exogenous demands $\mathbf{c}(t)$ are now time-dependent and they evolve according to the dynamic model

$$\mathbf{x}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{c}(t) + \mathbf{B}\dot{\mathbf{x}}(t) \tag{2.26}$$

where $\mathbf{B}$ is an $n \times n$ matrix called the capital coefficient matrix, whose element $b_{ij}(\in [0, 1])$ represents the capital stock of industry i maintained by industry j per unit of its output (Leontief, 1986). Equation (2.26) shows that the time-dependent industry outputs are still interdependent on other sector outputs through the matrix $\mathbf{A}$ while being driven by the time-dependent exogenous demands. The additional dynamic term signifies, through the capital coefficient matrix, the ability of the sectors to invest in the capital resources of other sectors such as inventory, land, machines with their rate of change ($\dot{\mathbf{x}}(t)$) of output. For solving the dynamic input-output model the continuous or discrete-form solution of the differential equation given in Equation (2.27) is sought

$$\dot{\mathbf{x}}(t) = -\mathbf{B}^{-1}\big[\mathbf{A}\mathbf{x}(t) + \mathbf{c}(t) - \mathbf{x}(t)\big] \tag{2.27}$$

2.4.2 Model issues

The solutions of the dynamic economic system depend upon the existence of $\mathbf{B}^{-1}$, and as such, it is assumed that $\mathbf{B}^{-1}$ exists at all times. One of the problems with the dynamic economic input-output model is the unavailability of data for the $\mathbf{B}$ matrix. Data for the $\mathbf{B}$ matrix can be obtained through the capital flow tables provided by the BEA (BEA., 2011), which at present contain data updated till 1997. Santos (2006) outlines the procedure for generating the $\mathbf{B}$ matrix from the BEA capital flow data. Since, the elements of $\mathbf{B}$ are estimates of the investment in technology, equipment, etc. by sectors they are expected to vary substantially over time. Hence, using old estimates does not lead to a reliable model for estimating the temporal changes in sector outputs.

The bigger issue with the solution of the Equation (2.27) system is that there is no guarantee that the matrix $\mathbf{B}$ is invertible (Luenberger & Arbel, 1977). In reality not every sector needs to maintain capital stocks of other sectors, which means that some of the rows or columns of the $\mathbf{B}$ matrix would contain all zero elements. Moreover the amounts of capital stock requirements of some sectors might be identical leading to two rows of columns being the same, which also leads to a singular $\mathbf{B}$. Leontief (1986) suggested solving the discrete-time dynamic model recursively backward in time, with the assumption that the matrix $\mathbf{I} - \mathbf{A} + \mathbf{B}$ is always invertible. Assuming discrete time steps $k = 0, 1, 2, ..., N$ this solution is given as

$$\mathbf{x}(k) = \big(\mathbf{I} - \mathbf{A} + \mathbf{B}\big)^{-1}\big[\mathbf{B}\mathbf{x}(k+1) - \mathbf{c}(k)\big] \tag{2.28}$$

Although the above formulation gives a feasible solution due to the existence of the matrix inverse the problem with using it arises due to the fact that the system behavior at time-step N needs to be known. This is not practical because generally the initial behavior at time-step $k = 0$ is given as we estimate the future $\mathbf{x}(N)$ behavior from it and not the reverse.

2.5 Dynamic risk input-output model

2.5.1 Model development

The need for a dynamic risk input-output model arises because the static equilibrium approach of the IIM lacks in modeling temporal interdependent inoperability propagation. Economic impact analysis requires modeling the dynamic interactive effects between an engineering and economic perspective, and representing the different time scales over which the actual disruptive events take place and direct and multiplier economic effects are felt (Okuyama et al., 2004). Modeling dynamic interdependent inoperabilities from the onset of a disruption till its dissipation over time determines how long it takes sectors to recover from a disruption. This added model feature makes for a dynamic risk input-output process that can also be used as a risk management model because recovery can be improved through efficient resilience metrics.

A dynamic inoperability input-output model was proposed (Lian & Haimes, 2006) along similar lines to the Leontief dynamic input-output model. The dynamic risk input-output model we intend to discuss here is along similar lines to the Lian & Haimes (2006) model, with a subtle difference that will be discussed as the model is developed.

It is assumed that as-planned interdependent economic system is time invariant and obeys the Leontief input-output equilibrium construct at all times. Restating Equation (2.12) as-planned economic behavior as a dynamic model with rate of change $\dot{\hat{\mathbf{x}}} = 0$

$$\hat{\mathbf{x}} = \mathbf{A}\hat{\mathbf{x}} + \hat{\mathbf{c}} + \mathbf{B}\dot{\hat{\mathbf{x}}} \tag{2.29}$$

When a disruption occurs at time $t = 0$ it is assumed that industry outputs reduce to levels $\tilde{\mathbf{x}}(0)$ and exogenous demands reduce to $\tilde{\mathbf{c}}(0)$. The equilibrium between demand and supply no longer exists and for subsequent times t the industry output evolution is assumed to be governed by the dynamic input-output model

$$\tilde{\mathbf{x}}(t) = \mathbf{A}\tilde{\mathbf{x}}(t) + \tilde{\mathbf{c}}(t) + \mathbf{B}\dot{\tilde{\mathbf{x}}}(t), \forall t > 0 \tag{2.30}$$

Since, the perturbed output levels are lower than the as-planned levels the aim of the economic system is to move from the perturbed state towards the as-planned levels. If we subtract Equation (2.30) from Equation (2.29) we get

$$\hat{\mathbf{x}} - \tilde{\mathbf{x}}(t) = \mathbf{A}\big[\hat{\mathbf{x}} - \tilde{\mathbf{x}}(t)\big] + \hat{\mathbf{c}} - \tilde{\mathbf{c}}(t) + \mathbf{B}\big[\dot{\hat{\mathbf{x}}} - \dot{\tilde{\mathbf{x}}}(t)\big] \tag{2.31}$$

Utilizing the previous definitions and constructions a dynamic inoperability risk input-output model is constructed by normalizing Equation (2.31) with the diagonal matrix of as-planned output levels, diag($\hat{\mathbf{x}}$), to obtain

$$\mathbf{q}(t) = \mathbf{A}^*\mathbf{q}(t) + \mathbf{c}^*(t) + \mathbf{B}^*\dot{\mathbf{q}}(t) \tag{2.32}$$

where $\mathbf{q}(t) = [\text{diag}(\hat{\mathbf{x}})]^{-1}[\hat{\mathbf{x}} - \tilde{\mathbf{x}}(t)]$ is now a time-dependent inoperability vector that represents the loss of output as a ratio of as-planned output at all times, $\mathbf{c}^*(t) = [\text{diag}(\hat{\mathbf{x}})]^{-1}[\hat{\mathbf{c}} - \tilde{\mathbf{c}}(t)]$ is the time-dependent demand perturbation vector that represents the loss of demand as a ratio of as-planned output at all times, $\mathbf{A}^* = [\text{diag}(\hat{\mathbf{x}})]^{-1}\mathbf{A}[\text{diag}(\hat{\mathbf{x}})]$ is the risk interdependency matrix as before, and $\mathbf{B}^*$ is obtained from the $\mathbf{B}$ matrix as $\mathbf{B}^* = [\text{diag}(\hat{\mathbf{x}})]^{-1}\mathbf{B}[\text{diag}(\hat{\mathbf{x}})]$.

Equation (2.32) can be reformulated as Equation (2.33) to give a state-space presentation of the dynamic model. Here we introduce the matrix $\mathbf{K}^* = -(\mathbf{B}^*)^{-1}$ to separate the differential term from the rest of the equation.

$$\dot{\mathbf{q}}(t) = -\mathbf{K}^*(\mathbf{I} - \mathbf{A}^*)\mathbf{q}(t) + \mathbf{K}^*\mathbf{c}^*(t), \forall t \tag{2.33}$$

$\mathbf{K}^*$ matrix introduced in Equation (2.33) is an $n \times n$ matrix that controls the rate of change of industry output in relation to its inability to establish equilibrium between demand and supply. Although Equation (2.33) is similar in structure to the dynamic input-output model of Equation (2.27) it represents recovery instead of growth. It is assumed that $\mathbf{K}^*$ exists and even though we mathematically derived it from the $\mathbf{B}$ in reality it is not governed by the existence of the $\mathbf{B}^{-1}$ matrix. Understanding the meaning and role of the $\mathbf{K}^*$ is central to the development of the dynamic risk input-output model and is discussed is the next section.

Assuming the model evolution is studied in discrete time-steps given by $0, 1, 2, ...,$ with unit time interval, the continuous time model of Equation (2.34) can be discretized as

$$\mathbf{q}(k+1) = \big[\mathbf{I} - \mathbf{K}^*(\mathbf{I} - \mathbf{A}^*)\big]\mathbf{q}(k) + \mathbf{K}^*\mathbf{c}^*(k) \tag{2.34}$$

2.5.2 Dynamic risk input-output model stability conditions

A closed form analytic solution to the differential Equation (2.33) exists and is expressed as

$$\mathbf{q}(t) = e^{-\mathbf{K}^*(\mathbf{I}-\mathbf{A}^*)t}\mathbf{q}(0) + \int_0^t e^{-\mathbf{K}^*(\mathbf{I}-\mathbf{A}^*)(t-z)}\mathbf{K}^*\mathbf{c}^*(z)dz \tag{2.35}$$

which shows that the evolution of inoperability at any time t depends upon the initial inoperability $\mathbf{q}(0)$, the demand perturbations $\mathbf{c}^*(t)$ at all times, the matrices $\mathbf{A}^*$ and $\mathbf{K}^*$. These parameters are referred to as system controls.

The dynamic risk input-output model is primarily used to model recovery from disruptions. Hence it is important that the Equation (2.35) solution leads towards an inoperability that signifies dissipation of the disruptive effects resulting from initial inoperabilities and demand perturbations. These considerations govern the values that can be taken by the elements of the $\mathbf{K}^*$ matrix. Since, it is required that the model provide stable solutions to inoperability evolution at all times the right hand side of Equation (2.35) cannot diverge or grow over the long run. Hence, requirement for model stability leads to the necessary condition that

$$\lim_{t\to\infty} e^{-\mathbf{K}^*(\mathbf{I}-\mathbf{A}^*)t} \to 0 \tag{2.36}$$

Establishing the Equation (2.36) necessary stability condition also guarantees that the dynamic risk input-output model is bounded at at times. As seen in Equation (2.35) the first term on the right hand side signifies free evolution of the system with $\mathbf{0} \leq \mathbf{q}(0) \leq \mathbf{1}$ by definition. Hence this term asymptotically converges towards zero.

The second term contains the forcing function $\mathbf{c}^*(t)$, which is also bounded because $\mathbf{0} \leq \mathbf{c}^*(t) \leq \hat{\mathbf{c}}/\hat{\mathbf{x}}$.

$$\begin{aligned}
\int_0^t \mathbf{K}^* e^{-\mathbf{K}^*(\mathbf{I}-\mathbf{A}^*)(t-z)} \mathbf{c}^*(z) dz &\leq \int_0^t \mathbf{K}^* e^{-\mathbf{K}^*(\mathbf{I}-\mathbf{A}^*)(t-z)} (\hat{\mathbf{c}}/\hat{\mathbf{x}}) dz \\
&= [1 - e^{-\mathbf{K}^*(\mathbf{I}-\mathbf{A}^*)t}](\mathbf{I}-\mathbf{A}^*)^{-1}(\hat{\mathbf{c}}/\hat{\mathbf{x}}) \\
&\leq (\mathbf{I}-\mathbf{A}^*)^{-1}(\hat{\mathbf{c}}/\hat{\mathbf{x}}) \leq 1
\end{aligned} \tag{2.37}$$

Having established the necessary system stability criteria puts restrictions on the values that can be taken by the elements of the $\mathbf{K}^*$ matrix, due to the fact that $\mathbf{A}^*$ is mainly data driven and thus exists *a priori*. Therefore an exercise in establishing system stability requires finding the range of possible $\mathbf{K}^*$ values that lead to system convergence. Two results in matrix theory that establish the stability requirements of state-space systems based on properties of the eigenvalues of the state matrix ($-\mathbf{K}^*(\mathbf{I}-\mathbf{A}^*)$ in our case) are stated in Proposition 4 (Lewis et al., 2006). With these results we can decide on the possible acceptable values of the matrix $\mathbf{K}^*$.

Proposition 4. *For bounded initial conditions, the continuos state-space linear system* $\dot{\mathbf{y}} = \mathbf{M}\mathbf{y}$ *converges when every eigenvalue,* $\lambda_{\mathbf{M}}$*, of* $\mathbf{M}$ *has a negative real part* $Re(\lambda_{\mathbf{M}}) < 0$*. Moreover the discretized counterpart,* $\mathbf{y}(k+1) = (\mathbf{I} + \Delta t\mathbf{M})\mathbf{y}(k)$*, of the state-space system converges only when the spectral radius* $\rho(\mathbf{I} + \Delta t\mathbf{M}) < 1$*.*

From Proposition 4 we can conclude that the dynamic risk input-output model has a convergent solution if the matrix $\mathbf{K}^*(\mathbf{I}-\mathbf{A}^*)$ has positive real valued eigenvalues. We use this information and combine it with the practical interpretations of $\mathbf{K}^*$ to establish a possible range of values.

2.5.3 Establishing parameter values for stable system behavior

Equation (2.33), called the Dynamic inoperability input-output model (DIIM) when $\mathbf{K}^*$ is a diagonal matrix, is explained in detail in the works of Haimes et al. (2005b), Lian & Haimes (2006) among others. Lian & Haimes (2006) argue that the $\mathbf{K}^*$ matrix represents a short-term behavior of sectors to invest to adjustments in changing outputs. They consider $\mathbf{K}^*$ is a diagonal matrix, whose diagonal elements, $0 \leq k^*_{ii} \leq 1 (\forall i = \{1, 2, ..., n\})$, represent the ability of each sector to attain stable responses to changes in its production outputs and demands. We explore the system stability when such an assumption holds. Proposition 5 and 6 show that system stability is guaranteed.

Proposition 5. *Given non-negative irreducible matrix* $\mathbf{A}^* = \left\{a^*_{ij} : a^*_{ij} \geq 0, \forall 1 \leq i, j \leq n\right\}$ *which has row sums less than 1 there exists an* $n \times n$ *positive diagonal matrix* $\mathbf{K}^*$*, such that the matrix* $\mathbf{K}^*(\mathbf{I} - \mathbf{A}^*)$ *has positive eigenvalues.*

Proof. Since the sum of each row of $\mathbf{A}^*$ is less than 1, we can show that

$$\sum_{j=1}^{n} a^*_{ij} \leq 1 \implies \sum_{j \neq i} |a^*_{ij}| \leq 1 - a^*_{ii} \tag{2.38}$$

Hence, the matrix $\mathbf{I} - \mathbf{A}^*$ is a diagonally dominant matrix. From Greshgorins Circle Theorem (Moon & Stirling, 2000) every eigenvalue λ^*_i of $\mathbf{I} - \mathbf{A}^*$ satisfies the condition

$$|\lambda^*_i - (1 - a^*_{ii})| \leq \sum_{j \neq i} |a^*_{ij}|, \forall i = \{1, 2, ..., n\} \tag{2.39}$$

From Equation (2.39) the lower bound on the real part of the eigenvalues of $\mathbf{I} - \mathbf{A}^*$ is given as

$$\mathrm{Re}(\lambda^*_i) \geq 1 - \sum_{j=1} a^*_{ij} > 0 \tag{2.40}$$

Assume the matrix $\mathbf{K}^*$ is given as $\mathbf{K}^* = \{\mathrm{diag}(k_{ii}), k_{ii} > 0, \forall 1 \leq i \leq n\}$. The i^{th} row of the product matrix $\mathbf{K}^*(\mathbf{I} - \mathbf{A}^*)$ is given as $k^*_{ii}[1 - a^*_{i1}, -a^*_{i2}, ..., -a^*_{in}]$, which is the multiplication of the the i^{th} row of $\mathbf{I} - \mathbf{A}^*$ by k_{ii}. Hence, the lower bounds on every eigenvalue κ_i^* of the matrix $\mathbf{K}^*(\mathbf{I}-\mathbf{A}^*)$ can be expressed as

$$\mathrm{Re}(\kappa_i^*) \geq k_{ii}^*\left[1-\sum_{j=1} a_{ij}^*\right] > 0 \tag{2.41}$$

which proves that the eigenvalues of $\mathbf{K}^*(\mathbf{I}-\mathbf{A}^*)$ are positive. □

Proposition 6. *Given a non-negative irreducible matrix* $\mathbf{A}^* = \left\{a_{ij}^* : a_{ij}^* \geq 0, \forall 1 \leq i,j \leq n\right\}$ *which has row sums less than 1 bounds on the elements of the diagonal matrix* $\mathbf{K}^*$ *for which* $\mathbf{I}-\mathbf{K}^*(\mathbf{I}-\mathbf{A}^*)$ *has a spectral radius less than 1 are given by* $\left\{diag(k_{ii}), 0 < k_{ii} < 2/(1-a_{ii}^* + \sum_{j\neq i} a_{ij}^*), \forall 1 \leq i,j \leq n\right\}$.

Proof. The i^{th} row of the product matrix $\mathbf{I}-\mathbf{K}^*(\mathbf{I}-\mathbf{A}^*)$ is given as

$$\begin{bmatrix} k_{ii}^*a_{i1}^*, & k_{ii}^*a_{i2}^*, & \ldots, & 1-k_{ii}^*(1-a_{ii}^*), & \ldots, & k_{ii}^*a_{in}^* \end{bmatrix}$$

From Greshgorins Circle Theorem (Moon & Stirling, 2000) every eigenvalue $\bar{\kappa}_i^*$ of $\mathbf{I}-\mathbf{K}^*(\mathbf{I}-\mathbf{A}^*)$ satisfies the condition

$$|1-k_{ii}^*(1-a_{ii}^*)-\bar{\kappa}_i^*| \leq \sum_{j\neq i}^{n} |k_{ii}^*a_{ij}^*| \tag{2.42}$$

Hence the maximal bounds on the real value of the i^{th} eigenvalue are established as

$$1-k_{ii}^*\left(1-a_{ii}^*+\sum_{j\neq i}^{n} a_{ij}^*\right) \leq \bar{\kappa}_i^* \leq 1-k_{ii}^*\left(1-\sum_{j=1}^{n} k_{ii}^*a_{ij}^*\right) \tag{2.43}$$

Since the spectral radius of the $\mathbf{I}-\mathbf{K}^*(\mathbf{I}-\mathbf{A}^*)$ is less than 1, $|\bar{\kappa}_i^*| < 1, \forall 1 \leq i \leq n$. This establishes a maximal bound on the inequality (2.43), which is leads to the two inequalities

$$-1 < 1 - k_{ii}^*\left(1 - a_{ii}^* + \sum_{j\neq i}^{n} a_{ij}^*\right) \quad\Longrightarrow\quad k_{ii} < \frac{2}{\left(1 - a_{ii}^* + \sum_{j\neq i}^{n} a_{ij}^*\right)} \tag{2.44}$$

$$1 - k_{ii}^*\left(1 - \sum_{j=1}^{n} k_{ii}^* a_{ij}^*\right) < 1 \quad\Longrightarrow\quad k_{ii} > 0 \tag{2.45}$$

Hence (2.44) and (2.45) provide the limits for the range of $k_{ii}, \forall 1 \leq i \leq n$, which are as stated in the proposition. □

We have being able to show range of values that the elements of the $\mathbf{K}^*$ matrix can take if it is strictly a diagonal matrix. One caveat with choosing $\mathbf{K}^*$ from this range is that the upper bound values are greater than 1. Since in the dynamic risk input-output model we have the term $\mathbf{K}^*\mathbf{c}^*(t)$ which contributes towards the inoperability, there is a chance that the value of $\mathbf{q}(t)$ might overshoot 1 which violates its very definition. Hence, if a choice of $\mathbf{K}^*$ has to be made from the above defined range then care should be taken that the product $\mathbf{K}^*\mathbf{c}^*(t)$ does not exceed 1. For simplicity sake and to confirm to previously established research we recommend that for a diagonal $\mathbf{K}^*$ the elements should be kept between 0 and 1.

For a $\mathbf{K}^*$ matrix with non zero off-diagonal elements it is difficult to establish a range for the values of the matrix. Based on the Proposition 4 we want to establish a dense $\mathbf{K}^*$ for which the joint spectral radius of the term $\mathbf{I} - \mathbf{K}^*(\mathbf{I} - \mathbf{A}^*)$ is less than 1. Establishing this is not simple for any matrix, but here we can make an assumption about the $\mathbf{K}^*$ matrix that comes from its practical interpretation. The values of $\mathbf{K}^*$ in general represents the degree of resilience investment that a sector makes in safeguarding itself against a disruption. Such investments come by strengthening its own resources and productions or by investing in inventories of other sector products to substitute for its own. There is a general incentive of a sector to concentrate on safeguarding its own interests, which translates mathematically to a $\mathbf{K}^*$ that is a diagonally dominant matrix and in which the off-diagonals are small values. It can be assumed that an dense $\mathbf{K}^*$ is represented as $\mathbf{K}^* \rightarrow \text{diag}(\mathbf{K}^*) + \Delta\mathbf{K}^*$, where $\Delta\mathbf{K}^*$ is introduces a perturbation in the diagonal $\text{diag}(\mathbf{K}^*)$ matrix to give the $\mathbf{K}^*$.

2.6 Discussion and summary

This development of the economic and risk input-output models presented here shows that these models possess mathematical properties that makes them always produce feasible solutions. Moreover there is a credible and dedicated data support system in place for these models, which means that they can be readily used for large-scale infrastructure risk analysis. Although the linear structure of the static models is quite simplistic it is nonetheless a useful construct for a basic analysis of interdependent system behaviors and risk properties. Having established the existence of solutions for the static models the development of the dynamic models is done by introducing a rate term to the static models, which makes the dynamic models first-order linear differential equations. The stability criteria that is established for a convergent feasible solution of the dynamic models is important for obtaining solutions from such models. Also the mathematical rules that are developed here have been related to the real world importance of the terms in these models, which helps establish certain guidelines for estimating or choosing the possible values of model parameters for practical and mathematical convenience. Overall we have been able to derive certain results here that show the existence of matrix inverses in the static systems and the criteria for the model stability in dynamic systems. Such properties are useful because they provide a prescription for choosing possible values for data or parameters where data is sparse or there is no prior knowledge about the possible parameter values and hence a design or expert-based judgement is required.

Chapter 3

Resilience Estimation and Planning Through the Risk Input-Output Framework

3.1 Introduction

The primary focus of this Chapter is to develop a resilience framework using economic input-output data based risk models. Resilience denotes the ability of the system to "spring back, rebound, return to the original form, recover readily or adjust" (Oxford English Dictionary, (OED., 2012)) from an external shock. The static and dynamic risk input-output models that we discussed in Chapter 2 are suitable tools for quantifying and planning for improved system resilience, and we show how these models can be applied towards achieving such goals.

Our static risk input-output model (Haimes & Jiang, 2001; Santos, 2006) quantifies the ability of the system of interdependent infrastructures to move from one equilibrium state to another after disruption. This means that the model is able to capture the system's capability to rearrange itself, which shows a resilience quality. Much of this quality depends on the ability of the system to resist the immediate impact of the external perturbation it is subjected to. In the context of the static model the relevant resilience planning question, in its broadest context, is

> How can the shock impact be reduced so that the system of interdependent infrastructures shifts from one equilibrium to another with minimum losses given the constraints of the planning environment?

In Section 3.2 we expand on the above question by posing the static resilience problem as a resource allocation and planning scheme. Such an approach lends itself towards a risk management optimization problem, which is explained in Section 3.2.1, while Section 3.2.2 provides some numerical insights into the planning problem with an example.

For a dynamic system resilience is indicative of its ability to resist initial impact effects and also return to adequate functionality within a desirable time-frame. The dynamic risk input-output model (Lian & Haimes, 2006) quantifies the trajectory of inoperability from the time of the disruptive event up until the system has made recovery. It is a suitable indicator of properties that define resilience. Resilience estimation through the dynamic model requires answering the following set of questions

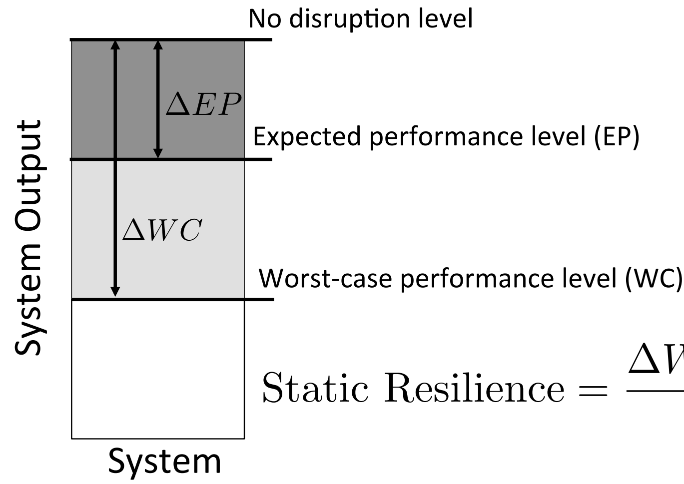
> What properties in the dynamic response to disruptions for the system of interdependent infrastructures imply resilience? How can we use these properties to estimate measures for resilience estimation?

These questions are handled in detail in Section 3.3, where we explain the dynamic risk input-output model as a resilience estimation construct. An understanding needs to be first developed on why the model is being used for resilience estimation. This comes from the type of disruptive events and their modeled behaviors, which are reflective of real world large-scale infrastructure responses to system shocks. It is understood that large-scale economic systems inherently possess resilient properties that lead towards recovery from disruptions (Klein et al., 2003). To this end Section 3.3.1 quantifies the type of external shocks and the model responses we are interested in. Once the system responses are understood resilience is quantified through appropriate metrics, explained in Section 3.3.2, which answers the first of our questions about dynamic resilience. These metrics, called the *time averaged level of operability*, *maximum loss of functionality*, *time to recovery*, respectively denote the systems' ability to maintain functionality throughout the post-disruption response, withstand the maximum disruptive impact and still recover, and progress towards recovery with some speed. The time averaged level of operability metric can be expressed as a function of the other two metrics and improving system speed to recovery can come at the cost of incurring more maximum impact. These complex behavior are better understood if the metrics can be used to generate a decision space that allows a user to choose appropriate levels of values and thus affect system resilience. Section 3.3.4 explains this construct.

Most resilience constructed in the static and dynamic model domains is in response to either an instantaneous or a continuos and short lived disruption. As such the resilient recovery is implied to come from pre-disruption preparedness and lends itself to be an inherent system property. In general a system could be subjected to multiple shocks that occur in phases. As such building the properties that allow a system to adapt its resilient behavior and rearrange itself every time it is shocked is required. Section 3.4 discusses this adaptive resilience behavior through the dynamic risk input-output model. Finally, Section 3.5 summarizes the topics discussed in this Chapter.

3.2 Static Resilience estimation

Static resilience is defined as "the ability of the system to maintain functionality when shocked" (Rose, 2007). Mathematically static economic resilience is measured in terms of the maximum potential drop in system performance and the estimated performance drop (Rose, 2004a). Figure 3.1 shows the graphical representation of static economic resilience and its mathematical formulation. The static economic resilience definition suggested is quite similar to the static inoperability metric defined in Equation (2.14), which suggests that the inoperability input-output model Equation (2.18) can be used as a resilience measuring construct. In essence if solution of the equation $\mathbf{q} = \mathbf{A}^{*}\mathbf{q} + \mathbf{c}^{*}$ gives us $\mathbf{q} < \mathbf{1}$, then the values $\mathbf{1} - \mathbf{q}$ tells us how much functionality the system is able to preserve after the disruptive impact. While simply



$$\text{Static Resilience} = \frac{\Delta WC - \Delta EP}{\Delta WC}$$

Figure 3.1: Static economic resilience visualization and mathematical definition (Rose, 2004a).

measuring the inoperabilities as a result of disruptive impacts is a good enough indication of the cascading collapse that is still able to preserve some levels of functionality, it is useful to improve the system performance such that cascading failures can be optimized. As such, minimizing the sector inoperabilities in some form leads to better system resilience. Such improvements are brought through efficient resource allocations during disruption recovery. Resource allocation requires developing strategies that reduce demand perturbations effectively leading to economic resilience. This demand driven approach is consistent with the idea that static economic resilience is a consequence of efficient utilization of resources and not system repair (Rose, 2007).

3.2.1 Strengthening static resilience through risk management

In order to improve static economic resilience a risk management approach needs to be adopted. A general risk management approach can be based on three questions (Haimes, 1991): (i) What can be done and what options are available? (ii) What are the associated trade-offs in terms of costs, risks and benefits? (iii) What are the impacts of current management decisions on future options? This triplet of questions motivates the development of a framework for interdependent risk decision-making formulation, which is explained below.

Consider a system consisting of n interdependent economic sectors related through the inoperability input-output model. Assuming a disruptive event results in initial demand perturbations for $m \leq n$ of the sectors. If such demand perturbations are given by $c_l^*(0), l = \{1, 2, ..., m\}$, then the risk management is concerned with reducing these $c_l^*(0)$'s through appropriate measures. If r_l signifies a risk management strategy adopted to reduce the initial sector l demand perturbation impact, then the effectiveness of r_l is measured in terms of the new demand perturbation resulting from implementing r_l on $c_l^*(0)$. This can be represented in functional form as

$$c_l^* = f_l(c_l^*(0), r_l) \tag{3.1}$$

Assuming a numerically higher value for r_l signifies a better management strategy, the graphical relationship between c_l^* and r_l is chosen as one of the possible forms shown in Figure 3.2. The upper bound $c_l^*(0)$ shows the inability of the strategy to reduce initial impacts, which is interpreted through the lowest value for r_l. The Figure 3.2

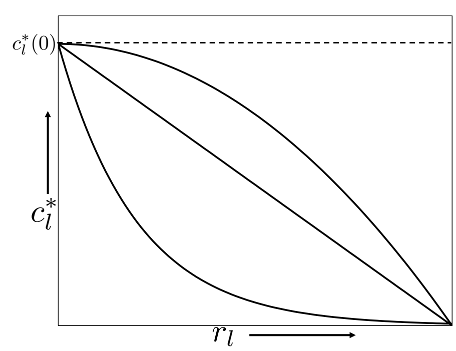


Figure 3.2: Possible functional relationships between c_l^* and r_l showing that the greater effect of the risk management strategy results in lesser demand perturbations

interpretation of the relationship between demand perturbations and management strategy suggests that an as high as possible value for r_l should be conveniently adopted. But since implementing risk management comes at a cost there is a finite budget that governs the maximum possible values taken by r_l. If $g_l(r_l)$ expresses the cost of implementing the strategy r_l then this budget as an upper bound. For an interdependent system planning the consideration of an overall budget would give a better planning option because it influences the distribution of resources for implementing the management strategies across all affected sectors. Thus, for the entire economy if at most budget b is available, then the Equation (3.2) constraint shows allocations that are decided on a fixed budget.

$$\sum_{l=1}^{m} g_l(r_l) \leq b \tag{3.2}$$

For an interdependent economic system being analyzed at the macro level overall system behavior needs to be quantified through a suitable metric. A metric for measurement of the overall impact of inoperabilities and demand perturbations for interdependent systems is the total economic loss, Q, experienced across all n sectors. Equation (3.3) shows how Q can be measured in terms of the as-planned outputs, x_i's, and the inoperabilities q_i's across all sectors. Since risk planning considerations entail the minimization of the total economic loss resulting from a disruptive event, minimizing Q helps achieve such an objective and answers the first of the questions posed above.

$$Q = \sum_{i=1}^{n} x_i q_i = \mathbf{x}^\mathsf{T}\mathbf{q} \tag{3.3}$$

As established already that the static inoperability input-output model is a demand driven model, Q is expressed in terms of the demand perturbations using the inoperability model. Thus expressing $Q = \mathbf{x}^{\intercal}[\mathbf{I} - \mathbf{A}^{*}]^{-1}\mathbf{c}^{*}$ shows that the interdependent nature of the economic system is reflected in the decision-making framework. Here the vector $\mathbf{c}^{*}$ is the $n \times 1$ demand perturbation vector for all n sectors and is given as

$$c_i^* = \begin{cases} c_l^* & \text{if } i \in l \\ 0 & \text{otherwise} \end{cases} \tag{3.4}$$

Combining the above equations into a risk management problem provides the statement for the improvement of static resilience of an interdependent economic system. Equation (3.4) shows the optimization formulation, which is understood through the following statement:

> For given initial impact on sectors in the form of demand perturbations if risk management strategies exist to reduce effects of such demand perturbations, then given a finite budget for implementation how do we allocate the strategies such that the overall interdependent economic losses are minimized?

$$\begin{aligned} \text{min} \quad & Q = \mathbf{x}^{\intercal}\mathbf{q} = \mathbf{x}^{\intercal}[\mathbf{I} - \mathbf{A}^{*}]^{-1}\mathbf{c}^{*} \\ \text{subject to} \quad & \\ & c_l^* = f_l(c_l^*(0), r_l), \forall l = \{1, 2, ..., m\} \\ & \textstyle\sum_{l=1}^{m} g_l(r_l) \leq b \\ & r_l \geq 0, \forall l = \{1, 2, ..., m\} \end{aligned} \tag{3.5}$$

MacKenzie et al. (2012b) have developed a similar risk management framework for accessing the budget allocations required for efficient recovery in the aftermath of the Deepwater Horizon oil spill (Cleveland, 2010). In their model f_l is exponential with a quadratic r_0 term in the exponent for allocating budget to all sectors.

3.2.2 Example planning problem insights

The type of the functional forms f_l and g_l govern the solution to the resilience planning problem. For macro level planning of the input-output kind of systems the r_l value might denote the amount of capital that can be invested to purchase and substitute for the lost demand denoted by c_l^*. With increased capital investment the demand perturbations are expected to decrease. Assuming there exists an exponential function that represents the decrease in demand perturbation from the initial level $c_l^*(0)$ as investments are made, the function f_l is represented as given in Equation (3.6)

$$c_l^* = c_l^*(0)e^{-\alpha_l r_l} \tag{3.6}$$

where $\alpha_l > 0$ can be regarded as the measure for the effectiveness of investment r_l, which also shows the return for substituting for lost demand for sector l. Using this formulation the optimization problem of Equation (3.5) becomes

$$\begin{aligned} \min \quad & Q = \mathbf{x}^\intercal[\mathbf{I} - \mathbf{A}^*]^{-1}\mathbf{c}^* \\ \text{subject to} \quad & \\ & c_l^* = c_l^*(0)e^{-\alpha_l r_l}, \forall l = \{1, 2, ..., m\} \\ & \textstyle\sum_{l=1}^m r_l \leq b \\ & r_l \geq 0, \forall l = \{1, 2, ..., m\} \end{aligned} \tag{3.7}$$

Assuming the matrix $\mathbf{D}^* = [\mathbf{I} - \mathbf{A}^*]^{-1} = [d_{ij}^*]$ denotes the interdependent propagation, the solution of the Equation (3.7) optimization problem is obtained by constructing the Lagrangian function

$$\mathcal{L} = \sum_{l=1}^{m} \left(\sum_{i=1}^{n} x_i d_{il}^* \right) c_l^*(0)e^{-\alpha_l r_l} + \lambda_0 \left(\sum_{l=1}^{m} r_l - b \right) - \sum_{l=1}^{m} \lambda_l r_l \tag{3.8}$$

where $\lambda_l, l = \{0, 1, 2, ..., m\}$ are the Lagrangian multipliers. Assuming

$$W_l = \left(\sum_{i=1}^{n} x_i d_{il}^*\right) c_l^*(0)$$

the optimal resource allocations r_l are obtained by solving the Karush-Kuhn-Tucker conditions (Boyd & Vandenberghe, 2004) given by

$$\frac{\partial \mathcal{L}}{\partial r_l} = 0 \implies -W_l \alpha_l e^{-\alpha_l r_l} + \lambda_0 - \lambda_l = 0, \forall l = \{1, 2, ...m\} \quad (3.9)$$

$$\sum_{l=1}^{m} r_l \leq b \quad (3.10)$$

$$r_l, \lambda_l \geq 0, \forall l = \{1, 2, ...m\} \quad (3.11)$$

$$\lambda_0 \left(\sum_{l=1}^{m} r_l - b\right) = 0 \quad (3.12)$$

$$\lambda_l r_l = 0, \forall l = \{1, 2, ...m\} \quad (3.13)$$

Setting $\lambda_0 > 0$ and $\lambda_l = 0, l = \{1, 2, ..., m\}$ leads to the optimal resource allocation values obtained by solving the system of Equations (3.14) and (3.15)

$$-W_l \alpha_l e^{-\alpha_l r_l} + \lambda_0 = 0, \forall l = \{1, 2, ...m\} \quad (3.14)$$

$$\sum_{l=1}^{m} r_l = b \quad (3.15)$$

An example problem illustrates the effectiveness of planning resource substitution for a decreasing demand perturbations and enhancing the static system resilience. Table 3.2 shows a three industry economy with given input-output transactions table. From the table the interdependency matrix $\mathbf{A}^*$ and the maximum possible demand loss vector $c^*(0)$ can be calculated, as shown in Equation (3.16). Also the substitution measures for the three sectors given by the $\boldsymbol{\alpha}$ vector are shown in Equation (3.16). These values come from expert elicitation or previous recovery planning data based on the properties of the system.

Table 3.1: Two industry input-output transaction data in million of dollars

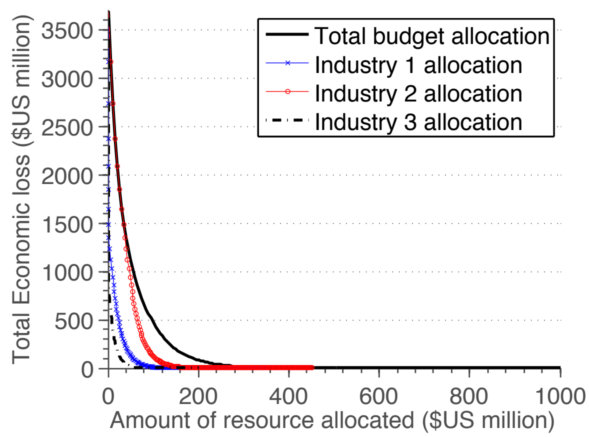
Industry	1	2	3	External Demand ($\mathbf{c}$) (\$ US million)	Total Output ($\mathbf{x}$) (\$ US million)
1	266	378	230	126	1000
2	267	110	224	899	1500
3	340	340	468	52	1200
Value added	127	672	278		
Total Output($\mathbf{x}^T$)	1000	1500	1200		

$$\mathbf{A}^* = \begin{bmatrix} 0.27 & 0.38 & 0.23 \\ 0.18 & 0.073 & 0.15 \\ 0.28 & 0.28 & 0.39 \end{bmatrix}, \quad \mathbf{c}^*(0) = \begin{bmatrix} 0.13 \\ 0.59 \\ 0.04 \end{bmatrix}, \quad \boldsymbol{\alpha} = \begin{bmatrix} 0.05 \\ 0.04 \\ 0.08 \end{bmatrix} \tag{3.16}$$

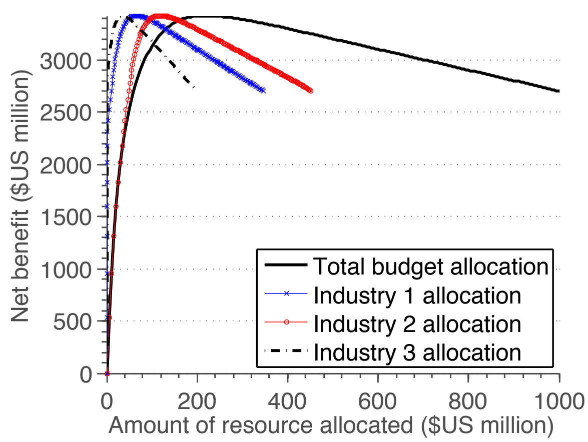
Two results which can be obtained from the resource allocation problem are shown in Figures 3.3(a) and 3.3(b). The decrease in the total economic losses for the entire economy with increasing budget allocation provides a estimate of the impact of budget allocation of the ability to improve system performance. Further, as shown in Figure 3.3(a) the sector-wise allocation of budget shows the optimal distribution of resource for most improved performances. In this example Industry 2 gets most of the budget, which can be explained from the fact that it is most impacted by the disruption. For low budget allocation almost all of the allocation goes to Industry 2. Figure 3.3(b) highlights the returns from the resource investments in the individual sectors and the entire economy. The net benefit of budget allocation can be estimated as (Anderson et al., 2007)

$$\begin{aligned} \text{Net Benefit} &= \text{Loss when no allocation} - \text{Loss due to allocation} - \text{Allocation Cost} \\ &= \text{Net loss avoided} - \text{Investment made} \end{aligned} \tag{3.17}$$

This metric when plotted against the amount of resource allocation shows that beyond a certain point there is a diminishing return of the investment. Hence, the maximum limit till which budget/resource allocation makes gives increasing returns can be accessed to help decision-making. As seen in Figure 3.3(b) investing resources to Industry 3 beyond approximately \$40 million does not provide any increasing returns for the investments. Hence, excess budgets can be directed towards the other two sectors for improving their performances. The above analysis is a basic frame-



(a) Economic losses vs Allocated budgets



(b) Net benefit vs Allocated budgets

Figure 3.3: Trade-offs between investments in losses.

work for a static resilience improvement and estimation problem, where the decision variable vector $\mathbf{r}$ can have different meanings depending upon the nature of the problem.

3.3 Dynamic Resilience estimation

Dynamic resilience behavior adds the time element to recovery properties desired from a system rebounding from disruptions. For economic systems the "speed of the system towards achieving a desired state" (Rose, 2007) becomes a factor in determining how dynamically resilient it is. In the dynamic risk input-output model construct most of the system behavior is indicative of a progression towards a stable state, and also the system inoperability responses are bounded. Hence, the dynamic model is a suitable candidate for analyzing and characterizing resilience in interdependent systems. We explore the dynamic risk input-output model behavior and build resilience interpretations through it.

3.3.1 Dynamic behavior

The n sector dynamic risk input-output model, which was discussed previously in Section 2.5.1, is restated here. This model, used for quantifying interdependent risk recovery for economic sectors, is given by the first order different equation

$$\dot{\mathbf{q}}(t) = -\mathbf{K}^*(\mathbf{I} - \mathbf{A}^*)\mathbf{q}(t) + \mathbf{K}^*\mathbf{c}^*(t), \quad \forall t > 0 \tag{3.18}$$

The analytic solution of the differential Equation (3.18) is given as

$$\mathbf{q}(t) = e^{-\mathbf{K}^*(\mathbf{I}-\mathbf{A}^*)t}\mathbf{q}(0) + \int_0^t e^{-\mathbf{K}^*(\mathbf{I}-\mathbf{A}^*)(t-z)}\mathbf{K}^*\mathbf{c}^*(z)dz \tag{3.19}$$

From Equation (3.19) it is evident that the temporal evolution of sector inoperability depends on the initial inoperability vector $\mathbf{q}(0)$, demand perturbation vectors $\mathbf{c}^*(t)$ at all times, model parameters $\mathbf{K}^*$ and $\mathbf{A}^*$ that effect the rate of recovery of the system.

From a system dynamics point of view the first term represents the motion that is inherently driven by system properties, and the second term represents system dynamics due to forced external loading. Some model properties, previously explained, are restated as follows:

1. The inoperability, $q_i(t)$ at any time t of sector i will lie between 0 and 1, where a value of 0 means that there is no loss of output and a value of 1 shows total loss of output. The $\mathbf{q}(t)$ vector can be treated as a measure of likelihood of damage to interdependent sectors and hence it is a risk metric.

2. The demand perturbation, $c^*(t)$ at any time t for sector i will lie between 0 for no loss of demand and $c_i/x_i < 1$ for complete loss in demand of sector i output. $\mathbf{c}^*(t)$ is a metric for measuring risk to interdependent sectors due to external disturbances.

3. $\mathbf{A}^*$ is a data-driven matrix derived from the Bureau of Economic Analysis data (BEA., 2011). In Section 2.3.2 it was shown to be an irreducible non-negative matrix with row sums less than 1. $\mathbf{A}^*$ measures the degree of interdependence between sectors, which provides an indication of the cascading effect of a disruption.

4. The $\mathbf{K}^*$ matrix represents a short-term behavior of sectors to invest to adjustments in changing outputs. In Section 2.5.3 we showed that when $\mathbf{K}^*$ is a diagonal matrix its elements are given as

$$\mathbf{K}^* = \left\{ \text{diag}(k_{ii}), 0 \leq k_{ii}^* \leq 1 (\forall i = \{1, 2, ..., n\}) \right\} \tag{3.20}$$

   A $\mathbf{K}^*$ with non-negative off-diagonal elements is also constructed in Section 2.5.3 and is an almost diagonal matrix. $\mathbf{K}^*$ elements represent the speed with which sectors can attain stable responses to changes in its production outputs and demands.

5. The bounds that are established for the elements of the $\mathbf{K}^*$ are determined by the consideration that the matrix $\mathbf{K}^*(\mathbf{I} - \mathbf{A}^*)$ has positive eigenvalues. This guarantees that the solution of Equation (3.19) does not diverge, which is an important consideration for modeling recovery through the model.

From a resilient recovery perspective we need to investigate possible functional forms of Equation (3.19) that suggest the system approaches towards an equilibrium state. Three possible cases are considered for analysis.

No demand perturbations

This condition arises when the disruption only affects the supply of economic commodities due to a direct impact on industry facilities. Such impacts would have localized effects leading to partial failure of economic sector productivity, but would not be widespread enough to affect demand losses. Mathematically it means $\mathbf{c}^* = 0, \forall t$ in Equation (3.19), which eliminates the second term to give

$$\mathbf{q}(t) = e^{-\mathbf{K}^*(\mathbf{I}-\mathbf{A}^*)t}\mathbf{q}(0) \tag{3.21}$$

The above solution always leads to the stable state $\mathbf{q}(t) \to \mathbf{0}, t \to \infty$, due to the exponential decay term. This implies that economic sectors are able to recover from supply only disruptions as long as they have some resilience. Such resilience can be built through component redundancies.

Constant demand perturbations

When the final demand perturbations are stationary during the entire time of analysis, the disruption is said to have lead to a constant demand perturbation. This condition implies that the economy has never fully recovered from the impact and there are always some residual demand perturbations, which might arise either due to inability of the economic sectors to meet final demands or decrease in demand from final consumers. In Equation (3.19) if we substitute $\mathbf{c}^*(t) = \mathbf{c}^*, \forall t$, the time-dependent inoperability becomes

$$\mathbf{q}(t) = (\mathbf{I} - \mathbf{A}^*)^{-1}\mathbf{c}^* + e^{-\mathbf{K}^*(\mathbf{I}-\mathbf{A}^*)t}\big[\mathbf{q}(0) - (\mathbf{I} - \mathbf{A}^*)^{-1}\mathbf{c}^*\big] \tag{3.22}$$

The stable state solution of the above Equation is $\mathbf{q}(t \to \infty) = (\mathbf{I} - \mathbf{A}^*)^{-1}\mathbf{c}^*$, because of the exponential decay term approaches zero over time. This stable state condition solution shows that at equilibrium the dynamic model reduces to the static inoperability input-output model. Such a condition is desirable for the input-output based economy because it shows the natural tendency of the system towards balancing the demand and supply shortages. For resilient economic recovery $\mathbf{c}^*$ should be such that $\mathbf{q}(t \to \infty) \leq \mathbf{q}(0)$ because it is desirable to move towards an equilibrium condition that is better than the initial system state. Hence, $\mathbf{c}^*$ should be a low value indicating only some residual shortages that exist in the economy.

Exponentially decreasing demand perturbations

A generalized recovery behavior is represented by demand perturbations that dissipate after an initial impact. There are entire families of curves that can be used to plan or represent the recovery of the demand perturbations over time. Exponentially decaying demand perturbations cover a wide variety of complex planning and behavior patterns, making them natural fits for quantifying system recoveries. If the demand perturbation is given as $\mathbf{c}^*(t) = e^{-\mathbf{P}t}\mathbf{c}^*(0)$, where $\mathbf{P}$ is a matrix for the rate of decay of demand perturbations. $\mathbf{P} = \text{diag}(p_i), 0 \leq p_i \leq 1$ would represent a good choice for the rate of recovery planning or behavior for each sector. Substituting the value of

$\mathbf{c}^*(t)$ into Equation (3.19) the time-dependent inoperability becomes.

$$\mathbf{q}(t) = e^{-\mathbf{P}t}\mathbf{Z}^{-1}\mathbf{K}^*\mathbf{c}^*(0) + e^{-\mathbf{K}^*(\mathbf{I}-\mathbf{A}^*)t}\big[\mathbf{q}(0) - \mathbf{Z}^{-1}\mathbf{K}^*\mathbf{c}^*(0)\big] \tag{3.23}$$

where $\mathbf{Z} = [\mathbf{K}^*(\mathbf{I} - \mathbf{A}^*) - \mathbf{P}]$. For mathematical design purposes, the choice of matrix $\mathbf{Z}$ is made such that it is invertible. The stable state condition for the Equation (3.23) inoperability is $\mathbf{q}(t \to \infty) = \mathbf{0}$, which is achieved quicker with larger values of p_i.

An example problem is provided to give a visual presentation of the model dynamics generated from the above three disruption scenarios. A two-sector economy is considered here with an input-output transaction as shown in Table 3.2. From the data the interdependency matrix $\mathbf{A}^*$ is calculated and shown in Equation (3.25). The $\mathbf{K}^*$ matrix is assumed to be a diagonal matrix with diagonal elements equal to 0.5 for both sectors. It is also assumed that a disruptive event occurs and causes an initial inoperability in sector 2, while sector 1 does not have any initial inoperability. Demand perturbation considered for the three scenarios are also shown in Equation (3.25). Figure 3.4 shows the operability (1 - $q(t)$) profiles for the sector recoveries for the three scenarios presented here.

Table 3.2: Two industry input-output transaction data in million of dollars

Industry	1	2	External Demand ($\mathbf{c}$)	Total Output ($\mathbf{x}$)
1	250	400	350	1000
2	200	100	1700	2000
Value added	550	1500		
Total Output($\mathbf{x}^T$)	1000	2000		

$$\mathbf{A}^* = \begin{bmatrix} 0.25 & 0.40 \\ 0.10 & 0.05 \end{bmatrix}, \quad \mathbf{K}^* = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}, \quad \mathbf{q}(0) = \begin{bmatrix} 0 \\ 0.5 \end{bmatrix},$$

$$\text{Case 1: } \mathbf{c}^*(t) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad \text{Case 2: } \mathbf{c}^*(t) = \begin{bmatrix} 0 \\ 0.2 \end{bmatrix}, \qquad (3.24)$$

$$\text{Case 3: } \mathbf{c}^*(t) = \begin{bmatrix} 0 \\ 0.3e^{-0.05t} \end{bmatrix}$$

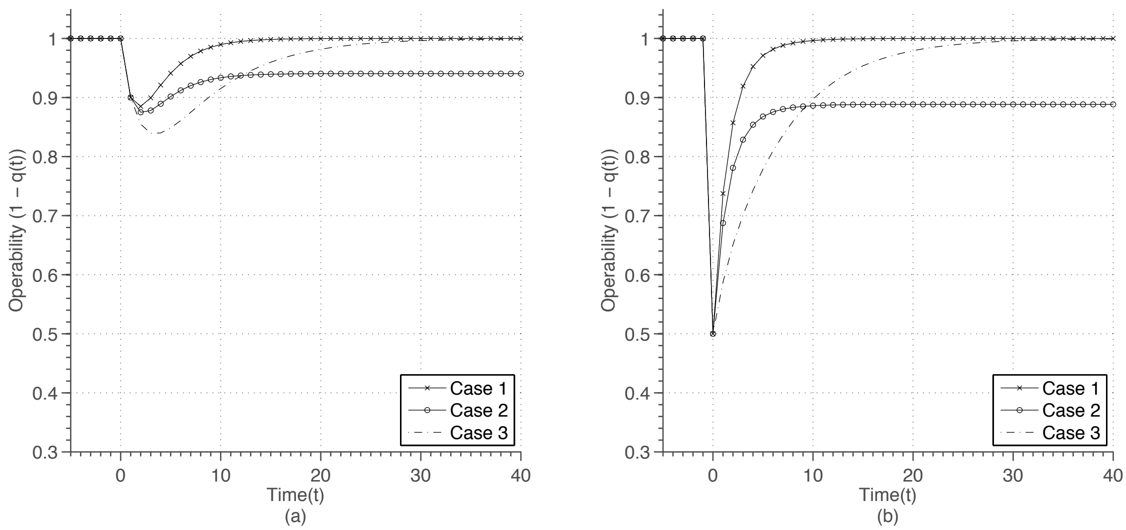


Figure 3.4: Dynamic profiles for sector operability vs time signifying recoveries from three scenarios of disruptions for a two sector economy. (a) Sector 1 risk profiles, and (b) Sector 2 risk profiles, for the three disruption scenarios characterized respectively by zero, constant and exponentially decaying demand perturbations. All scenarios show some form of recovery after initial impact.

3.3.2 Metrics for resilience using the dynamic risk input-output model

Having established that the dynamic risk input-output model is capable of modeling sector recoveries we need to quantify resilience through this model. For this the methodologies developed in the engineering resilience framework are explored and related to the model. Bruneau et al. (2003) developed a framework for seismic resilience estimation that captured technical, organizational, social and economic aspects of community behavior. The quantitative concepts of this framework apply to the dynamic economic resilience estimation framework we intend to discuss. Seismic resilience is estimated by measuring the expected degradation of the system from the time immediately after an earthquake till it makes suitable recovery. Figure 3.5 shows the graphical representation and the formulation for measuring system resilience based on the quality of the system at each time. From this approach, called the *resilience triangle* approach, the loss of resilience is quantified by the metric R which gives the shaded area showing the loss of quality during recovery. Based on this approach resilience is contended to have four dimensions applicable to any general system behavior characteristics.

1. Robustness - The measure of the sector's ability to resist the initial impact.
2. Rapidity - The measure for the time it takes a sector to attain recovery.
3. Redundancy - Ability of the sector to substitute for lost product through inventories and other means.
4. Resourcefulness - Capacity enhancing capabilities of a sector to improve its performance.

As shown in the Figure 3.5, the resilience triangle concentrates on quantifying system resilience in terms of the robustness and rapidity, which are understood to be achieved through incorporating redundancy and resourcefulness into the system (Bruneau et al., 2003). Research on the resilience triangle concept has been extended to include better methods to measure the resilience through multiple triangles showing more redundancies (Bruneau & Reinhorn, 2007), finding the area beneath the curve which actually indicates resilience (Cimellaro et al., 2010; Zobel, 2010, 2011). Probabilistic approaches have also been incorporated into the resilience triangle approach

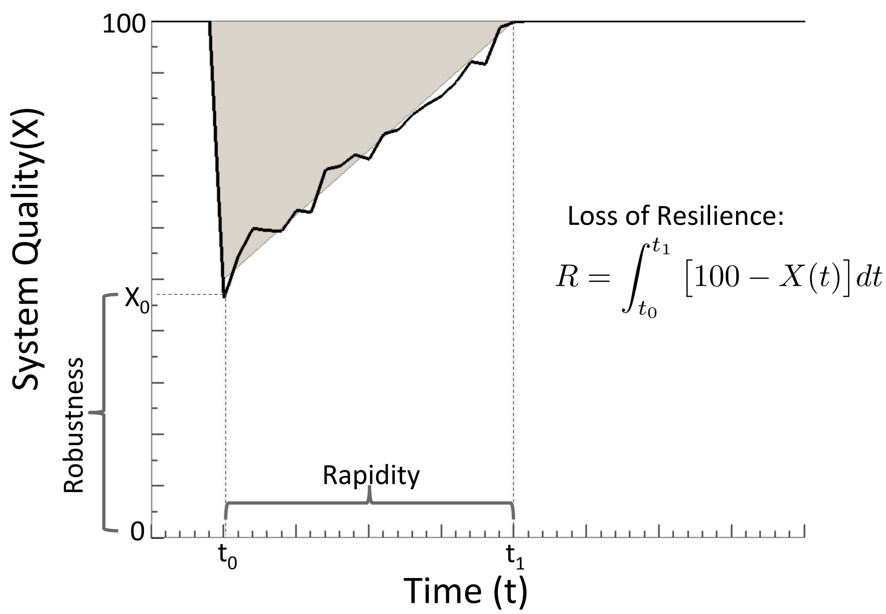


Figure 3.5: Resilience triangle concept showing the measures of robustness and rapidity and the calculation of the loss of resilience which is the shaded area (Bruneau et al., 2003).

(Chang & Shinozuka, 2004; Bruneau & Reinhorn, 2007).

Although resilience quantified through the above approach is based on four system characteristics it ultimately measures one quantity encapsulating entire system behavior. Zobel (2010, 2011) contend that a multi-dimensional representation of resilience is necessary because a single metric could be misleading. Mathematically the areas within two triangles can be the same indicating the same amount of resilience, but one behavior could be due to high robustness and low rapidity while the other might be the opposite. Figure 3.6 highlights this point. Such considerations require multiple metrics in the resilience estimation framework. Vugrin et al. (2010) discuss the importance of and develop metrics for building an infrastructure and economic resilience framework.

By its very definition and equation structure, the dynamic risk input-output model can be used to measure and quantify resilience. The four dimensions of infrastructure

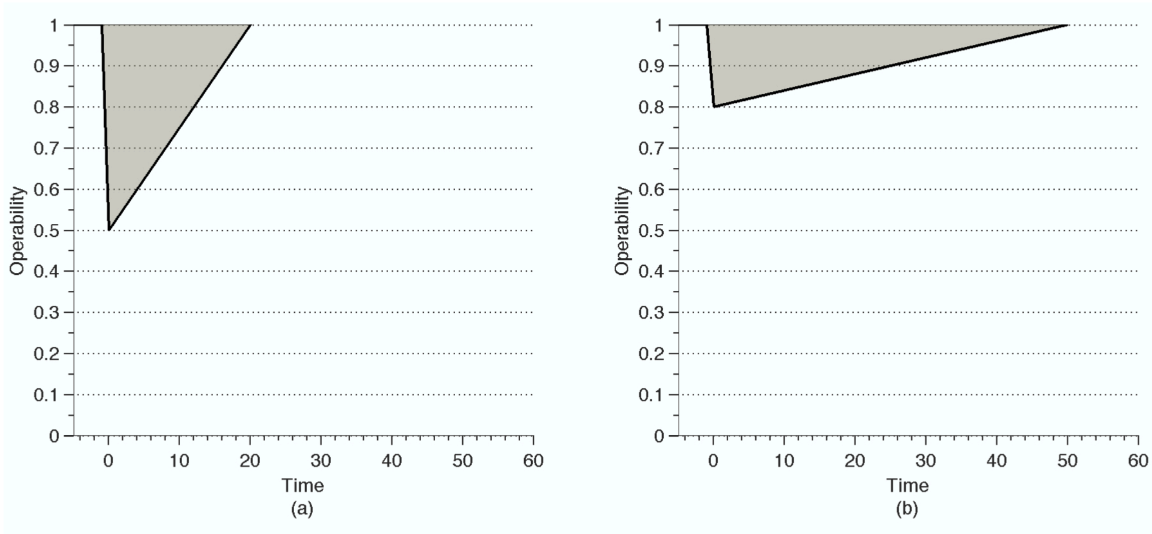


Figure 3.6: Resilience triangles having same shaded areas but different robustness and rapidity conditions. (a) Higher impacts but faster recovery, and (b) Lower impact but slower recovery.

resilience can be explained through the model to develop a multiple metric framework for building economic resilience frameworks. In previous dynamic inoperability input-output model (DIIM) analysis resilience is understood only in terms of the $\mathbf{K}^*$ matrix. Haimes et al. (2005b); Lian & Haimes (2006) call the coefficients of $\mathbf{K}^*$ industry resilience coefficients that measure the efficacy of a sector's risk managements options. For the diagonal $\mathbf{K}^*$, with diagonal elements between 0 and 1, higher resilience is indicated by closeness of elements towards 1 and lower resilience means diagonal elements tend towards zeros. Treating the $\mathbf{K}^*$ matrix as the only indicator of resilience is an incomplete and inaccurate analysis approach. In fact $\mathbf{K}^*$ is a factor that contributes towards the interdependent system resilience, and the actual resilience is built on such contributions. Figure 3.7 shows the caveat in treating $\mathbf{K}^*$ as the only resilience metric. Using the two sector example data of Table 3.2 we consider responses to same initial inoperability and demand perturbation conditions for two different $\mathbf{K}^*$ matrices. Using same $\mathbf{q}(0)$ as in Equation (3.25) and $\mathbf{c}^* = 0, \forall t$ we find responses for a lower $\mathbf{K}_1$ and a higher $\mathbf{K}_2$ resilience matrices given in Equation (3.25).

$$\begin{aligned}
\text{Given } \mathbf{c}^* &= 0, \forall t \text{ and} \\
\mathbf{A}^* &= \begin{bmatrix} 0.25 & 0.40 \\ 0.10 & 0.05 \end{bmatrix}, \quad \mathbf{q}(0) = \begin{bmatrix} 0 \\ 0.5 \end{bmatrix}, \\
\mathbf{K_1^*} &= \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}, \quad \mathbf{K_2^*} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}
\end{aligned} \tag{3.25}$$

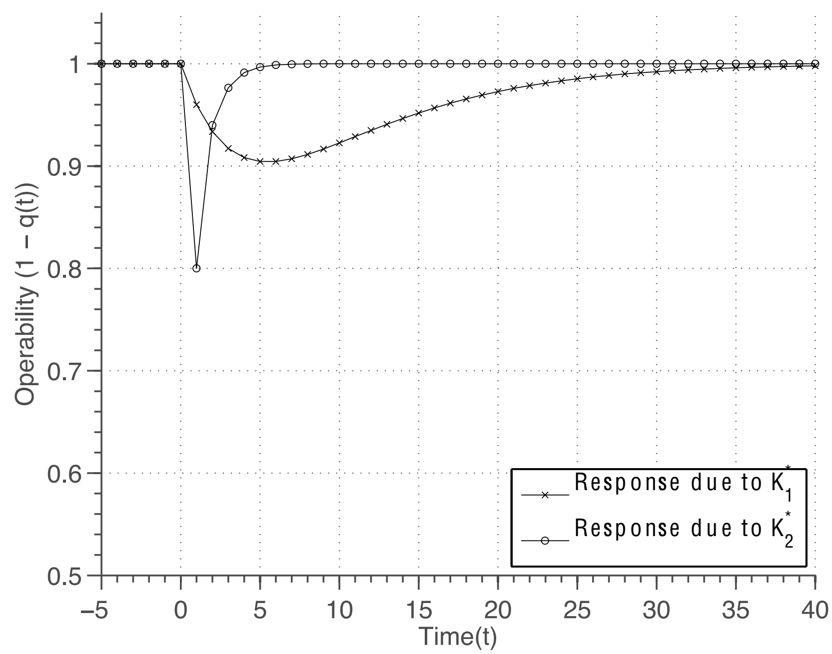


Figure 3.7: Sector 1 dynamic response profiles showing sector operability vs time from two different $\mathbf{K}^*$ matrices. The plot shows the trade-off between higher immediate impacts and recovery times for the two different $\mathbf{K}^*$ choices.

The analysis shows that for industry Sector 1 having a higher values for diagonal $\mathbf{K}^*$ leads to faster time to recovery but comes at a cost of increasing the initial impacts of disruptions. Hence, there is a trade off between achieving faster recovery at the cost of higher cascading impacts. Due to the coupling effects from interdependency the choice of higher $\mathbf{K}^*$ can lead to adverse non-desirable effects.

We need multiple metrics for a holistic quantification of resilience and measuring such trade-offs between the metrics. The three metrics proposed here to explain resilience through the dynamic risk input-output model are related to the resilience triangle approach and in essence explain the four resilience dimensions of the Bruneau et al. (2003) framework.

Time averaged level of operability for a sector

The overall level of functionality maintained by a system, starting from the time of initiation of disruptive effect till any time horizon, provides a good indication of its resilience to the disruptive event. From the dynamic risk input-output model the measure $1 - q_i(t)$ is the level of operability of sector i at each instance of time and can be used to quantify the overall sector functionality level. Starting at $t = 0$, if the system is being monitored till a long time $t = T$ then overall system performance can be quantified in terms of the *time averaged level of operability* over the analysis period. This metric, which we call F_i, is given in Equation (3.26) as

$$F_i = \frac{1}{T} \int_{t=0}^{t=T} \big[1 - q_i(t)\big] dt = 1 - \frac{1}{T} \int_{t=0}^{t=T} q_i(t) dt \tag{3.26}$$

From Equation (3.26) we observe that $0 \leq F_i \leq 1$, with $F_i = 0$ occurring for a totally inoperable system ($q_i(t) = 1, \forall t \in [0, T]$) and $F_i = 1$ corresponding to a fully functional system ($q_i(t) = 0, \forall t \in [0, T]$) during the entire time of analysis. Typically a system would prefer to have F_i closer to 1 because it indicates high levels of operability at each time step. A certain level of consideration needs to be made in deciding the time T over which the system functionality is being measured. In most analysis the dynamic risk input-output being used provides an exponentially decaying solution for sector inoperability. After a certain time the model values converge towards almost zero values. If T is large integral term involving $q_i(t)$ in Equation (3.26) will converge towards zero ($\lim_{T\to\infty} \frac{1}{T}\int_{t=0}^{t=T} q_i(t)dt = 0$) and the measure $F_i(T) \to 1$. A case-based study for the possible values for T and F_i is required based on the particular trajectory of $q_i(t)$, which depends upon the external forcing $c_i^*(t)\forall i$. The vector representation $\mathbf{F}$ of the Equation (3.26) metric for the entire n sector economy is given in Equation (3.27). Here $\mathbf{1}$ in a $n \times 1$ vector of ones.

$$\mathbf{F} = \mathbf{1} - \int_{t=0}^{t=T} \mathbf{q}(t)dt \qquad (3.27)$$

Zobel (2010, 2011) have used a similar approach to define a normalized resilience metric from the area beneath the resilience triangle, which they call the predicted resilience of a system. F_i can also be related to the economic loss metric Q_i, which is widely used in inoperability loss estimation studies of the infrastructure systems (Barker, 2008; Barker & Santos, 2010a). For a dynamic inoperability response Q_i is defined as

$$Q_i = x_i \int_{t=0}^{T} q_i(t)dt \qquad (3.28)$$

Hence, its functional relationship with F_i is as expressed as

$$F_i = 1 - \frac{Q_i}{x_i T} = \frac{x_i T - Q_i}{x_i T} \qquad (3.29)$$

Equation (3.29) suggests that if $x_i T$ represents the maximum loss the system could have incurred then F_i is a measure of the amount of loss the system avoided as fraction of potential maximum damage. This is consistent with the static economic resilience definition (Rose, 2007), which is often used as an overall performance measure even for dynamic system analysis.

Maximum loss of sector functionality

The immediate impact of a disruptive event on an infrastructure system is felt through the degradation of its output producing capacity. In addition, if the external demands for output are perturbed system operability is further eroded. A preferred resilient system is one that is capable of maintaining a high level of initial operability and continues being as much operable as possible during disruptions. In reality there are events that a capable of causing systems to lose most of their functionality, which makes it difficult for them to operate at high levels of productivity. As such there is interest in understanding the worst effect a disruption can have on the infrastructures before they can recover to better levels of performance. Since resilience is associated with the capability of a system to bounce back or recoil from disruptions, getting a perspective of the lowest productivity levels during recovery is needed to develop an understanding for system resilience. In the dynamic risk input-output model it is assumed that any infrastructure is capable of recovering from any level of inoperability below one to an equilibrium condition. Hence, in the model, inoperability reaches a maximum value before the sector rebounds towards recovery. This maximum sector i inoperability, called *maximum loss of functionality*, is quantified as

$$q_i^m = \max_{t \geq 0}[q_i(t)] \tag{3.30}$$

It is clear from Equation (3.30) that $0 \leq q_i^m \leq 1$ since it is directly measured in terms of inoperability which lies between zero and one. Like the previous resilience metric a case-based study is required for quantifying possible ranges in which q_i^m values lie depending upon the trajectory taken by the time-dependent inoperability. The n sector vector for maximum loss of functionality is expressed as

$$\mathbf{q}^m = \max_{t \geq 0}[\mathbf{q}(t)] \tag{3.31}$$

$1 - q_i^m$ is referred to the robustness measure in the resilience triangle framework, although there is a slight difference between its interpretation in the two approaches. In the engineering based interpretation robustness implies the ability to resist the direct impact and avoid immediate damages (Bruneau et al., 2003; McDaniels et al., 2008), which would be measured in terms $1 - q_i(0)$. q_i^m can manifest itself at a later time as seen in Figure 3.7, because it comes from a coupled system response and highlights the reaction of one sector to shocks in others. In the end strengthening robustness or reducing q_i^m require similar planning strategies that enhance sector and overall system performances.

Time to recovery

Dynamic resilience is best understood in context to the speed of recovery of systems. The faster a system is able to recover from a disruptive event the more resilient it is supposed to be. The notion of system recovery is important when trying to understand the time it takes to recover. For an ideal resilient system recovery implies return to pre-disruption levels of productivity and thereafter the capability to maintain functionality at the same levels. In reality systems might not be able to reorganize and recover to pre-disruption output levels due to the existence of permanent losses or different evaluation standards. Hence, recovery is best understood in terms of the capability of the system to achieve a stable condition where productivity levels are higher than they were immediately after the disruptive impacts. In the dynamic risk input-output model analysis we showed that the infrastructure systems approach equilibrium conditions from different initial inoperability and demand perturbation forcing conditions. Hence, we can define a suitable resilience metric, called the *time to recover* for indicating sector recovery through the dynamic model. As the dynamic model analysis suggests, the equilibrium levels of sector inoperability depending upon different dynamic conditions are known. Thus, time of recovery is measured in terms of said equilibrium level inoperabilities. If for sector i the equilibrium inoperability is given as q_i^e then its time to recovery τ_i is defined as the time when its inoperability $q_i(t)$ is within an $\epsilon(<< 1)$ neighborhood of q_i^e. This is stated in Equation (3.32) as

$$\tau_i = \left\{t : t > 0, |q_i(t) - q_i^e| \leq \epsilon\right\} \tag{3.32}$$

Similar to the above two metrics, the time to recoveries of all the sectors can be collected in an $n \times n$ matrix defined as

$$\boldsymbol{\tau} = \text{diag}\left[\tau_1, \tau_2, ..., \tau_n\right] \tag{3.33}$$

Time to recovery is synonymous with the notion of rapidity in all dynamic resilience frameworks (Bruneau et al., 2003; Rose, 2007), with slight difference in its role here. Rapidity is always associated with an improved dynamic resilience, because it is understood that recovery brings the system from its worst initial condition to a desired state and the increased speed implies more resilience. As shown in Figure 3.7 there is a tradeoff between the time to recover and maximum inoperability, which shows that increased speed does not always lead to a better performance throughout. Again system interdependence brings about such behaviors.

The graphical representation of the three resilience metrics generated from a general sector response to disruption is shown in Figure 3.8. Parallels can be drawn with the Figure 3.5 resilience triangle.

3.3.3 Relationship between the resilience metrics

Since the three resilience metrics defined above come from the dynamic risk input-output model, a functional relationship exists between them. Investigating such a function that relates the three metrics gives us a sense of bounds associated with the each metric based on its dependence on the other two. Moreover we can find

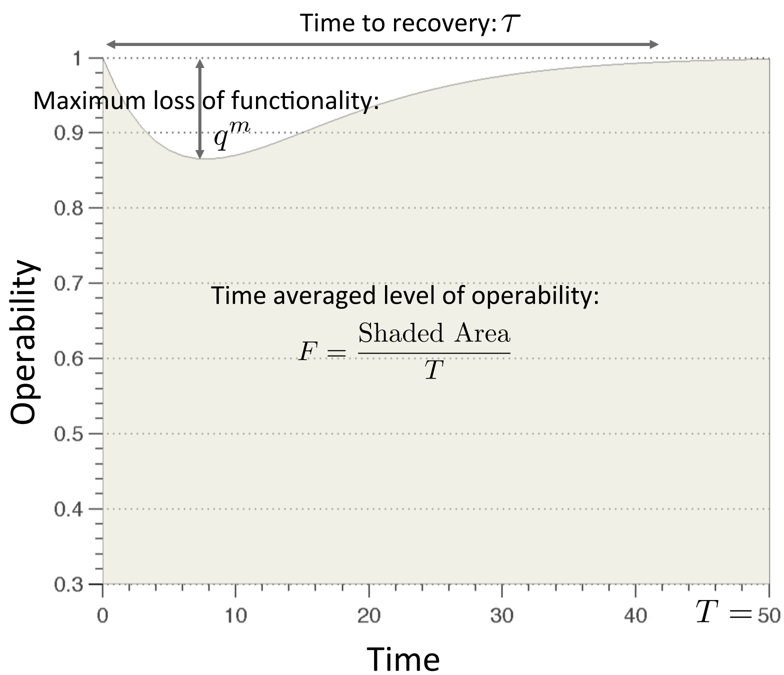


Figure 3.8: Resilience metrics as given by the dynamic risk input-output model.

the values for two of the metrics and use them to make an educated guess about the possible value of the third metric. From the definitions of the resilience metrics it is convenient to represent the time averaged level of operability in terms of the maximum loss of functionality and the time to recovery. Equation (3.34) gives the general functional form that expresses the relationship between the resilience metrics

$$F_i = g_i(T, q_i^m, \tau_i) \iff \mathbf{F} = \mathbf{g}(T, \mathbf{q}^m, \boldsymbol{\tau}) \tag{3.34}$$

The functional form of $\mathbf{g}$ can be developed by looking at the definitions developed from Equations (3.26) to (3.33). F_i given in Equation (3.27) can be estimated as

$$F_i = 1 - \frac{1}{T}\int_{t=0}^{t=\tau_i} q_i(t)dt - \frac{1}{T}\int_{t=\tau_i}^{t=T} q_i(t)dt \tag{3.35}$$

Since, by definition τ_i represents a time to attain stability the level of sector inoperability can be considered time invariant beyond τ_i. Substituting the equilibrium inoperability estimation q_i^e of Equation (3.32) in Equation (3.35) we get

$$\begin{aligned} F_i &= 1 - \frac{1}{T}\int_{t=0}^{t=\tau_i} q_i(t)dt - \frac{1}{T}\int_{t=\tau_i}^{t=T} q_i^e dt \\ &= 1 - \big(1 - \frac{\tau}{T}\big)q_i^e - \frac{1}{T}\int_{t=0}^{t=\tau_i} q_i(t)dt \end{aligned} \tag{3.36}$$

This relationship can be generalized to the entire interdependent economy to get the average operability vector $\mathbf{F}$ as

$$\mathbf{F} = \mathbf{1} - \big(\mathbf{I} - \frac{\boldsymbol{\tau}}{T}\big)\mathbf{q}^e - \frac{1}{T}\int_{t=0}^{t=\boldsymbol{\tau}} \mathbf{q}(t)dt \tag{3.37}$$

The particular form of the functional relationship in Equation (3.37) is further explained when we look at specific functions for the time-dependent sector inoperability. Therefore the expressions for inoperabilities derived previously will be used here to explain the exact functional relationships.

No demand disruptions

The interdependent inoperability vector for this case is an exponential decaying function and was calculated in Equation (3.21). It can be substituted into the Equation (3.37) expression for the time averaged level of operability.

$$\begin{aligned} \mathbf{F} &= \mathbf{1} - \big(\mathbf{I} - \frac{\boldsymbol{\tau}}{T}\big)\mathbf{q}^e - \frac{1}{T}\int_{t=0}^{t=\boldsymbol{\tau}} e^{-\mathbf{K}^*(\mathbf{I}-\mathbf{A}^*)t}\mathbf{q}(0)dt \\ &= \mathbf{1} - \frac{1}{T}[1 - e^{-\mathbf{K}^*(\mathbf{I}-\mathbf{A}^*)\boldsymbol{\tau}}][\mathbf{K}^*(\mathbf{I} - \mathbf{A}^*)]^{-1}\mathbf{q}(0) \end{aligned} \tag{3.38}$$

As shown before the equilibrium inoperability is given as $\mathbf{q}^e = \mathbf{0}$, which is achieved at $\boldsymbol{\tau} \rightarrow \infty$. For numerical and practical purposes we can assume that each τ_i represents a time beyond which there is no significant sector inoperability and numerically $q_i(t) \approx$

$0, \forall t > \tau_i$. Also, it is clear here that sector inoperabilities are obtained through the product of an exponentially decaying matrix function with a vector. Hence, for simplicity it is assumed that the maximum sector inoperabilities occur initially, which means $\mathbf{q}^m = \mathbf{q}(0)$. This might not be the general case as some inoperability grow from their initial levels due to system couplings. Substituting for $\mathbf{q}^m$ into the Equation (3.38) the functional relationship between the resilience metrics is described through Equation (3.39)

$$\mathbf{F} = \mathbf{1} - \frac{1}{T}[1 - e^{-\mathbf{K}^*(\mathbf{I}-\mathbf{A}^*)\boldsymbol{\tau}}][\mathbf{K}^*(\mathbf{I} - \mathbf{A}^*)]^{-1}\mathbf{q}^m \quad (3.39)$$

Constant demand perturbations

Similar to the case-based analysis for the no demand perturbation scenario we can find a relationship between the resilience metrics for a constant demand perturbation scenario. The time-dependent inoperability for this case was also calculated previously in Equation (3.22) and needs to be plugged into Equation (3.37). Here the equilibrium inoperability is $\mathbf{q}^e = [\mathbf{I} - \mathbf{A}^*]^{-1}\mathbf{c}^*$ and again it is assumed that $\mathbf{q}^m = \mathbf{q}(0)$.

$$\begin{aligned}\mathbf{F} &= \mathbf{1} - (\mathbf{I} - \mathbf{A}^*)^{-1}\mathbf{c}^* - \frac{1}{T}[1 - e^{-\mathbf{K}^*(\mathbf{I}-\mathbf{A}^*)\boldsymbol{\tau}}][\mathbf{K}^*(\mathbf{I} - \mathbf{A}^*)]^{-1}[\mathbf{q}^m \\ &- (\mathbf{I} - \mathbf{A}^*)^{-1}\mathbf{c}^*]\end{aligned} \quad (3.40)$$

Exponential decaying demand perturbations

The Equation (3.23) expression for the time-dependent inoperability is used to calculate the desired functional relationship in this case. As shown before, since this is an exponential decaying response $\mathbf{q}^e = 0$. Similar to previous assumptions $\mathbf{q}^m = \mathbf{q}(0)$ for this case also. Equation (3.41) provides the required expression relating $\mathbf{F}$ to T,

$\boldsymbol{\tau}$ and $\mathbf{q}^m$.

$$\begin{aligned}\mathbf{F} &= \mathbf{1} - \frac{1}{T}[1 - e^{-\mathbf{P}\boldsymbol{\tau}}]\mathbf{P}^{-1}\mathbf{Z}^{-1}\mathbf{K}^*\mathbf{c}^*(0) \\ &- \frac{1}{T}[1 - e^{-\mathbf{K}^*(\mathbf{I}-\mathbf{A}^*)\boldsymbol{\tau}}][\mathbf{K}^*(\mathbf{I} - \mathbf{A}^*)]^{-1}\big[\mathbf{q}^m - \mathbf{Z}^{-1}\mathbf{K}^*\mathbf{c}^*(0)\big] \qquad (3.41)\end{aligned}$$

3.3.4 Decision space generated by resilience metrics

Evaluating system performances through the resilience metrics allows for better understanding of the system, which leads to better decision-making. By capturing the overall resilience through the F metric and also through the characteristics that signify system robustness and rapidity allows for comparison between different disruption scenarios (Zobel, 2011). Since, we have been able to obtain a functional relationship between the metrics it is possible to use two metrics and estimate the third. This provides an overall picture for system performance objectives. Equations (3.39) to (3.41) are complex matrix functions that can become difficult to solve because they require measures for recovery times and maximum inoperabilities for all the sectors. Hence, getting a decision support through these constructs can becomes quite challenging. Nevertheless these equations provide valuable information that allows us to develop a visualization between the resilience metrics. We propose utilizing the functional forms of the derived functions for building a visualization tool for specific sectors. From example the Equation (3.39) suggests that for the no demand perturbation case the sector specific relationship between the resilience metrics is of the form

$$F_i = 1 - \frac{1}{\alpha_i T}(1 - e^{-\alpha_i \tau_i})q_i^m \qquad (3.42)$$

where α_i denotes some measure of interdependence suggested through $\mathbf{K}^*(\mathbf{I} - \mathbf{A}^*)$ matrix. Oliva et al. (2010) suggest that the term $\sum_{j=1}^{n} a_{ij}^*$ can be treated as quick global evaluation for sector resilience. It is called a dependency index because it reflects the infrastructures capabilities to maintain functionality under maximum interdependent inoperabilities. Deriving from the matrix form of Equation (3.39) the term α_i can take the values given by Equation (3.43), which is obtained by assuming a diagonal $\mathbf{K}^*$ matrix.

$$\alpha_i = k^*_{ii}\big(1 - \sum_{j=1}^{n} a^*_{ij}\big) \tag{3.43}$$

The above value for α_i can be utilized to reflect a decision-maker's control over the system resilience by controlling the possible values for k^*_{ii}. Combining expressions (3.42) and (3.43) provides a functional form for quick visualization of the system resilience metric relationships, that can be controlled through changing the degree of interdependence. A contour plot showing isolines for F_i based on all combination for τ_i and q_i^m can be constructed to generate a decision space that reflects recovery and maximum impacts for same levels of overall functionalities. The two-sector example problem previously discussed in Table 3.2 is considered here to show such contour plots. From the data we get the following relationships between the resilience metrics

$$\begin{aligned} F_1 &= 1 - \frac{1}{0.35k^*_{11}T}(1 - e^{-0.35k^*_{11}\tau_1})q_1^m \\ F_2 &= 1 - \frac{1}{0.85k^*_{22}T}(1 - e^{-0.85k^*_{22}\tau_2})q_2^m \end{aligned} \tag{3.44}$$

From Equation (3.44) Figures 3.9 and 3.10 show, respectively for Sector 1 and 2, the contour lines for F_i as τ_i and q_i^m vary for different choices of k_{ii}. Similar analysis is also presented in generating a multi-dimensional decision-space using the resilience triangle construct (Zobel, 2010, 2011; Zobel & Khansa, 2011). Such plots generate a decision-space which can be used to estimate the outcome of system performance when choosing recovery strategies reflected through $\mathbf{K}^*$. The values on the contour isolines, which denote the F_i values, can indicate to the decision-maker the outcome of different k_{ii}^* choices. A comparison is made in Figure 3.9 between two outcomes, shown as dots on the plots, that result in same recovery time and overall operability, but vary in the maximum inoperability values. Hence the choice of $k_{11}^* = 0.8$ is better in this case because it results in lower maximum losses while maintaining same values for other performance metrics. Similar tradeoffs can be accessed through the two decision-spaces for Sector 2. In general it is seen that higher k_{ii}^* leads to faster recovery and higher average operabilities for same levels of maximum inoperabilities. Another way of utilizing the resilience metrics is done through planning for the amount of investment required in rebuilding or substitution to recover from a disruption. This is quantified through the value of $\mathbf{K}^*$ elements that could indicate the desired tradeoff between the metrics. An example problem of such planning is presented in Equation (3.45). For the given interdependency matrix $\mathbf{A}^*$ it is known that there are not demand perturbations, and there is an initial inoperability $\mathbf{q}(0)$ that shows sector 2 suffers more disruption than sector 1. For sector 1 the decision is to determine the effect of its k_{11}^* planning on its resilience metrics, given that $k_{22}^* = 1$. Figure 3.11 shows both the change of maximum sector inoperability (q^m) and average inoperability ($1-F$) with the change in recovery time as k_{11}^* is varied from 1 to 0. As k_{11}^* is increased from 0 to 1 the time to recovery decreases along with the average inoperability, but the maximum inoperability increases. This shows a tradeoff analysis between the metrics that can be useful for resilience planning.

$$\begin{aligned} \text{Given } \mathbf{c}^* &= 0, \forall t \text{ and} \\ \mathbf{A}^* &= \begin{bmatrix} 0.25 & 0.40 \\ 0.10 & 0.05 \end{bmatrix}, \quad \mathbf{q}(0) = \begin{bmatrix} 0.1 \\ 0.5 \end{bmatrix}, \quad \mathbf{K}^* = \begin{bmatrix} k_{11}^* & 0 \\ 0 & 1 \end{bmatrix} \end{aligned} \tag{3.45}$$

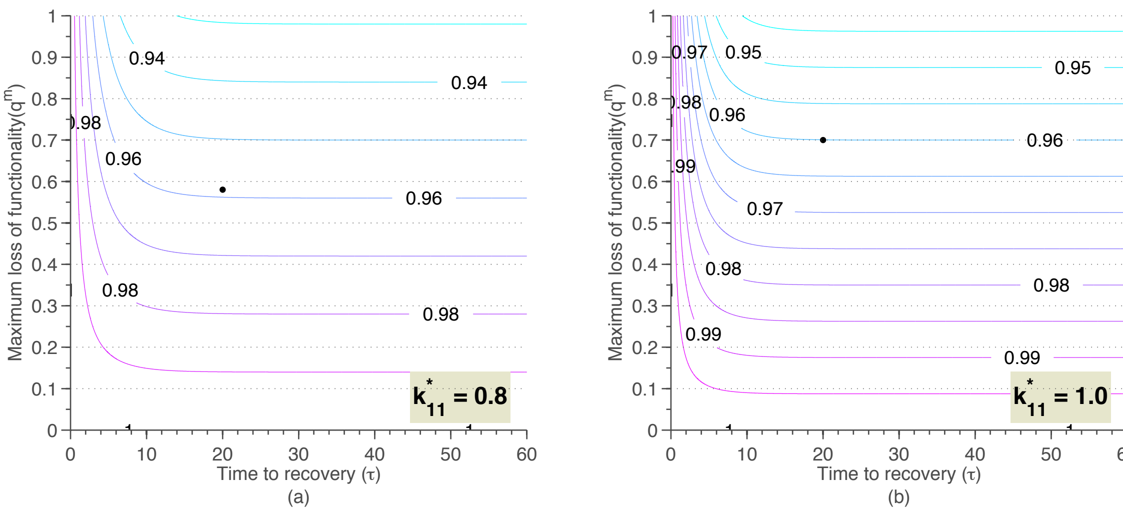


Figure 3.9: Sector 1 contour lines of F_1 for varying τ_1 and q_1^m for two type of responses generated as a result of choosing different k_{11}^* values. Decisions can be made between the two options based on where the sector performance lies on the plots.

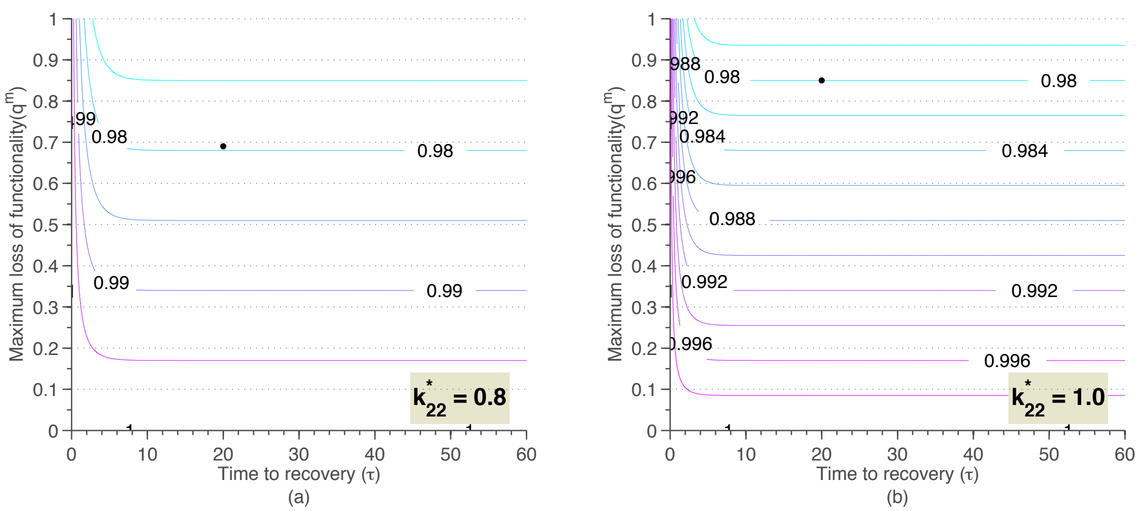


Figure 3.10: Sector 2 contour lines of F_2 for varying τ_2 and q_2^m for two type of responses generated as a result of choosing different k_{22}^* values. Decisions can be made between the two options based on where the sector performance lies on the plots.

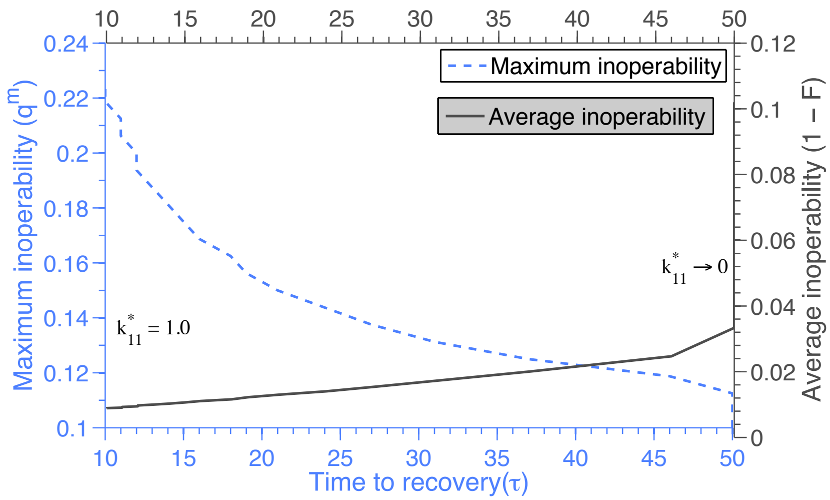


Figure 3.11: Sector 1 relationships between maximum inoperability (q_1^m) and average inoperability $(1 - F_1)$ vs recovery time (τ) for varying k_{11}^* values. As k_{11}^* is increased from 0 to 1 the time to recovery decreases along with the average inoperability, but the maximum inoperability increases. This shows a tradeoff analysis between the metrics that can be useful for resilience planning.

3.4 Adaptive resilience planning

The static and dynamic resilience estimation and planning schemes developed so far are based on the assumption that disruptive impacts are felt one time or follow a fixed trajectory during recovery. In the dynamic domain it is realistic to assume that disruption occurs in phases or there are multiple disruptions that affect the system. As such the system needs to readjust its recovery behavior and reevaluate its resilience. This means that the $\mathbf{K}^*$ matrix needs to be updated in order to accommodate such readjustment, which leads towards an an adaptive resilience estimation model.

In effect updating the $\mathbf{K}^*$ matrix shows the systems capability to "maintain function on the basis of ingenuity or extra effort" (Rose, 2009a). Such resilience has an emergent property that allows the system to reorganize against the changing risk (Haimes et al., 2008). The discussion here suggests a scheme of modeling dynamic resilient responses due to the changing risks.

3.4.1 An adaptive dynamic risk input-output model

The dynamic risk input-output model developed so far can also be used for estimating adaptive resilience for sector recoveries. Resilience is developed against disruptions in the supply, which are treated as instantaneous effects similar to the onset of the initial inoperability $\mathbf{q}(0)$. Also, resilience is developed for changing demand perturbations, which can be continuos in time. Given the dynamic risk input-output model of Equation (3.18) it is assumed that new disruptions occur at times $0, t_1, t_2, ..., t_n$ where $0 < t_1 < t_2 < ..... < t_s$. Also, it is assumed that the disruptions in supply/inoperability are quantified through the series of vectors $\mathbf{d}(0), \mathbf{d}(t_1), ..., \mathbf{d}(t_s)$. These disruptions add onto the already existing inoperability in the system, which implies that the inoperabilities are effected as

$$\begin{aligned} \mathbf{q}(0) &= \mathbf{d}(0) \\ \mathbf{q}(t_1) &\rightarrow \mathbf{q}(t_1) + \mathbf{d}(t_1) \\ &\vdots \\ \mathbf{q}(t_n) &\rightarrow \mathbf{q}(t_n) + \mathbf{d}(t_n) \end{aligned} \tag{3.46}$$

Disruptions can also have an effect on the demand perturbations, which means $\mathbf{c}^*(t)$ can be divided into different time intervals to reflect the changes in the demand side

$$\mathbf{c}^*(t) = \begin{cases} c^*_{0-t_1}(t) & 0 \leq t < t_1 \\ c^*_{t_1-t_2}(t) & t_1 \leq t < t_2 \\ \vdots & \vdots \\ c^*_{\geq t_s}(t) & t_s \leq t < \infty \end{cases} \tag{3.47}$$

Using Equations (3.46) and (3.47) we can divide the time evolution of inoperability into intervals that divide the different shocks and their effects. Equation (3.48) provides the formulation for the evolving inoperability

$$\mathbf{q}(t) = \begin{cases} e^{-\mathbf{K_1^*}(\mathbf{I}-\mathbf{A}^*)t}\mathbf{q}(0) + \int_0^t e^{-\mathbf{K_1^*}(\mathbf{I}-\mathbf{A}^*)(t-z)}\mathbf{K_1^*}\mathbf{c}^*(z)dz & 0 \le t < t_1 \\ e^{-\mathbf{K_2^*}(\mathbf{I}-\mathbf{A}^*)(t-t_1)}\mathbf{q}(t_1) + \int_{t_1}^{t_2} e^{-\mathbf{K_2^*}(\mathbf{I}-\mathbf{A}^*)(t_2-z)}\mathbf{K_2^*}\mathbf{c}^*(z)dz & t_1 \le t < t_2 \\ \vdots & \vdots \\ e^{-\mathbf{K_{s+1}^*}(\mathbf{I}-\mathbf{A}^*)(t-t_s)}\mathbf{q}(t_s) + \int_{t_s}^{t} e^{-\mathbf{K_{s+1}^*}(\mathbf{I}-\mathbf{A}^*)(t-z)}\mathbf{K_{s+1}^*}\mathbf{c}^*(z)dz & t_s \le t < \infty \end{cases} \tag{3.48}$$

As shown in the above equation, for recovery from a new impact of the disruptive event on the system would require different $\mathbf{K}^*$ estimates. Hence $\mathbf{K}^*$ is adapting to the requirements of the new system behavior. From Equation (3.48) the inoperability in a time interval $t_k \le t < t_{k+1}$ can be transformed into a time interval $0 \le t' < (t_{k+1} - t_k)$ where the model evolution now becomes

$$\begin{aligned} \mathbf{q}'(t') &= e^{-\mathbf{K_{k+1}^*}(\mathbf{I}-\mathbf{A}^*)t'}\mathbf{q}'(0) \\ &+ \int_0^{t'} e^{-\mathbf{K_{k+1}^*}(\mathbf{I}-\mathbf{A}^*)(t'-z)}\mathbf{K_{k+1}^*}\mathbf{c}'^*(z)dz, 0 \le t' < (t_{k+1} - t_k) \end{aligned} \tag{3.49}$$

where $\mathbf{q}'(t') = \mathbf{q}(t - t_k)$, $\mathbf{q}'(0) = \mathbf{q}(t_k)$ and $\mathbf{c}'^*(t') = \mathbf{c}^*(t - t_k)$. Hence we have a series of dynamic risk input-output models for each time interval that can be used for analysis of an adaptive response to disruptions.

An example problem shows how the inoperability response evolves when multiple disruptions affect the system. We consider the two sector economy system given in Table 3.2 whose interdependency matrix is restated here in Equation (3.50). Assuming that there are no demand perturbations in the economy, but three supply disruptions given in Equation (3.50) occur at times 0, 10 and 25. Different values of

$\mathbf{K}^*$ are used in the recovery planning as given in Equation (3.51).

$$\begin{aligned} \mathbf{A}^* &= \begin{bmatrix} 0.25 & 0.40 \\ 0.10 & 0.05 \end{bmatrix}, \quad \mathbf{d}(0) = \begin{bmatrix} 0 \\ 0.5 \end{bmatrix}, \\ \mathbf{d}(10) &= \begin{bmatrix} 0.2 \\ 0 \end{bmatrix}, \quad \mathbf{d}(25) = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix} \end{aligned} \tag{3.50}$$

$$\mathbf{K}^*_{0-9} = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}, \quad \mathbf{K}^*_{10-24} = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}, \quad \mathbf{K}^*_{\geq 25} = \begin{bmatrix} 0.8 & 0 \\ 0 & 0.8 \end{bmatrix} \tag{3.51}$$

Figure 3.12 shows the evolution of the sector operabilities due to the above values of $\mathbf{K}^*$ in response to the multiple disruptions. The resilience metrics that were developed in Section 3.3.2 can be applied to the adaptive analysis to also understand the system performance due to different resilience decisions.

3.5 Summary and discussion

This Chapter presented resilience estimation models for static and dynamic systems. It was argued that the static risk input-output model is in itself a resilience estimation scheme and thus can can be used to improve infrastructure resilience through risk management. The risk management problem is in essence a method to reduce the demand perturbation effects for the interdependent systems, which results in lowering the direct and indirect cascading inoperability effects of the disruptions. In general there are several decision-making options available, which are geared towards reducing the demand perturbations. It is assumed, and there is plausibility in such an assumption, that the decision-maker has a mathematical model that signifies the effectiveness of the risk policies in decreasing the demand risks. Hence the primary

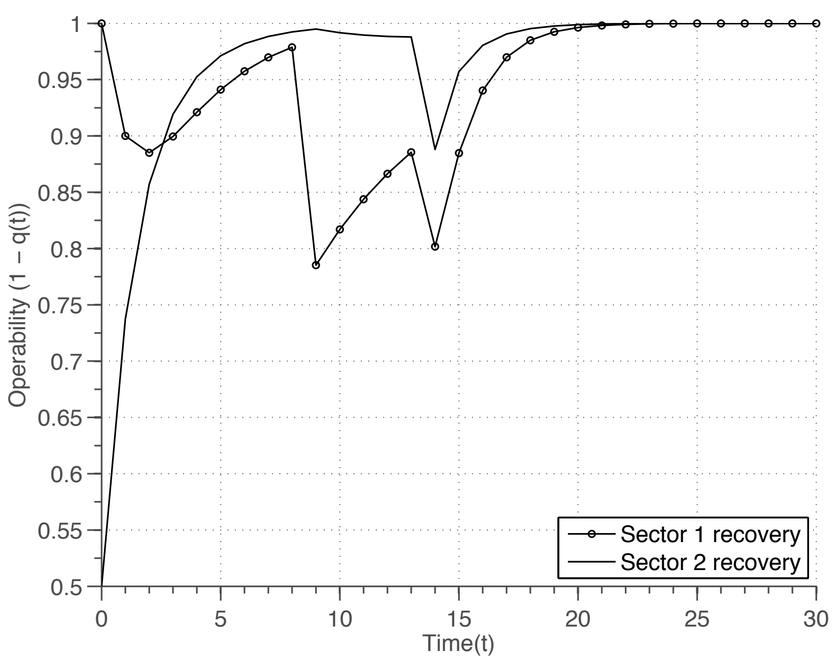


Figure 3.12: Operability profiles showing sector recoveries due to different $\mathbf{K}^*$ matrices to adapt to multiple disruptions of the interdependent system.

objective of the static resilience scheme is to come up with a decision to allocate the policies across interdependent sectors subjected to budget constraints for implementing said policies. The guiding objective of the planning decision is the preference to lower the total economic losses for the entire economy. For a particular budget resource allocation problem the solution of the risk management problem shows the effectiveness of budget allocation, and the limit beyond which there is no need to allocate more budget. Overall the scheme is a useful optimization based methodology that can provide planners with simple metrics and a prescription for strengthening static resilience in an interdependent economy.

Resilience is also constructed in the domain of the dynamic risk input-output model. The lack of a proper resilience estimation scheme in the dynamic model domain motivates the formulation of metrics that describe resilience. Even though the model has been capable of describing interdependent infrastructure resilience, its interpretation of resilience has been flawed. This is corrected here and improved to provide a holistic resilience construct through the dynamic model. Three characteristics in dynamic resilience behavior namely the average sector level of functionality/operability, the maximum inoperability/loss of functionality and the time to recovery are described. These provide a complete picture for resilience estimation in dynamic systems because they reflect the notion that resilience should indicate the ability to maintain functionality and posses a speed to recovery.

The usefulness of generating a trio of resilience metrics for the dynamic system behavior is shown through the conceptual and mathematical decision-making scheme that is prescribed here. The resilience metrics can be related to each other by a function, which typically expresses the average sector level of functionality in terms of the maximum inoperability and the time to recovery. As such a decision space can be generated to give the tradeoffs between the choice of resilience planning options, which are reflected through the values taken by the metrics.

Generating an adaptive scheme for resilience estimation and dynamic model behavior is a natural extension of the dynamic risk input-output model. Such an extension has been missing from previous research. An adaptive model is a better representative of actual recovery behavior because there are multiple shocks and changing resilience properties that are exhibited by the system. The resilience metric schemes can be also applied to the adaptive model behavior.

Chapter 4

Robust Static Resilience Planning Under an Uncertainty Framework

4.1 Introduction

The risk input-output model is a data-driven model. It contains uncertainties due to inaccuracies in data collection, including source data and assumptions inherent in input-output analysis. Some input-output assumptions that create uncertainty are the linearity or proportionality assumption, the allocation of resources distributed across sectors, and aggregation of multiple sectors into bigger sectors. In addition, the risk-based parameters such as demand perturbations and initial inoperabilities used in decision-making depend upon events that are often extreme in nature and difficult to understand *a priori*.

This Chapter discusses the nature of uncertainties in the problem at hand and provides planning solutions to the decision-makers for the worst-case uncertainties. The relevant research questions here is:

> What is the nature of the uncertainties present in the optimization scheme and how do they affect the problem formulations and solutions? What decision-making formulations can be constructed to consider the extreme realizations of the uncertainties in decision planning? Can we guarantee that the planning solutions are robust to every uncertainty specified within a prescribed limit?

Section 4.2 discusses the solution approach that answers the questions posed above. Uncertainties in the available data change the problem objective due to which the static resilience risk management questions posed in Section 3.2.1 has to be reformulated. While previously the planning objective was to minimize the overall economic losses with given budgets, due to uncertainty there is no certainty as the how much loss reduction would occur for a given budget allocation. Since planners would like to have estimates for their budgets the problem objective changes to estimating the required amounts of resource allocations when economic losses need to be kept below certain thresholds. Section 4.2.1 elaborates this further.

Since there is interest in analyzing the extreme uncertainty effects on the planning solutions the optimization problem is formulated as a robust optimization scheme. Section 4.2.2 gives the general guidelines that need to be followed to make the robust formulations. The nature of the uncertainties are fundamental to understanding solutions to planning objectives. Section 4.2.3 constructs the uncertainty sets that are vital to the solution of the robust problem. These uncertainty sets are constructed to reflect the real world interpretations of uncertainties in the input-output scheme and the model physics itself. The robust formulations to the nominal problems are built from the given uncertainty sets in Section 4.2.4. Robust formulations increase the problem dimensions considerably but here they preserve the nominal problem structure and hence are useful. The Section 4.3 example problem highlights the usefulness of the robust schemes by showing that small data uncertainties have great affect on the planning solutions, which are accounted for in the robust solution. Section 4.4 concludes the work.

4.2 Robust optimization in static resilience

4.2.1 Reformulating the risk management problem due to uncertainty

In Section 3.2.1 we presented the framework for building and strengthening static resilience through the inoperability input-output model. The framework was based on the notion that static economic resilience can be improved through optimal allocation of risk management options that contribute towards decreasing the demand perturbations that drive direct and indirect disruption propagation effects (Santos, 2006). For an n sector economic system we formulated a resource allocation based optimization framework whose objective is to minimize the total economic loss Q subjective to budget constraints. Also it is assumed that there are functional relationships between demand perturbations ($\mathbf{c}^*$) and the effectiveness of the risk management options ($\mathbf{r}$) that suggest decrease in demand perturbations as the risk management efficacy is increased. For $m \leq n$ sectors impacted by initial demand perturbations $\mathbf{c}^*(0)$ the static resilience planning problem we formulated is restated below.

$$
\begin{aligned}
\min \quad & Q = \mathbf{x}^\intercal[\mathbf{I} - \mathbf{A}^*]^{-1}\mathbf{c}^* \\
\text{subject to} \quad & \\
& c_l^* = f_l(c_l^*(0), r_l), \forall l = \{1, 2, ..., m\} \\
& \textstyle\sum_{l=1}^{m} g_l(r_l) \leq b \\
& r_l \geq 0, \forall l = \{1, 2, ..., m\}
\end{aligned}
\tag{4.1}
$$

where $\mathbf{x}$ is the $n \times 1$ vector of sector outputs, $\mathbf{I}$ is an $n \times n$ identity matrix, $\mathbf{A}^*$ is the $n \times n$ interdependency matrix, f_l is the functional relationship between the sector l initial demand perturbation $c_l^*(0)$ and the risk management option r_l, g_l is the cost function for implementing the risk management option r_l and b is the budget limit for implements risk management in the entire economy.

In Section 3.2.1 we discussed the solution of the optimization problem when $\mathbf{r}$ denote budget allocations, which are related to the demand perturbations through an exponentially function. Equation (4.2) is a particular case of the Equation (4.1), where $\alpha_l > 0$ is the measure of the effectiveness of the resource allocation.

$$
\begin{aligned}
\min \quad & Q = \mathbf{x}^{\intercal}[\mathbf{I} - \mathbf{A}^{*}]^{-1}\mathbf{c}^{*} \\
\text{subject to} \quad & \\
& c_l^{*} = c_l^{*}(0)e^{-\alpha_l r_l}, \forall l = \{1, 2, ..., m\} \\
& \textstyle\sum_{l=1}^{m} r_l \leq b \\
& r_l \geq 0, \forall l = \{1, 2, ..., m\}
\end{aligned} \tag{4.2}
$$

The above decision-making framework is a convex optimization problem, for which the solution was obtained through the Karush-Kuhn-Tucker (Boyd & Vandenberghe, 2004) optimality conditions. Even though it is a convenient framework for resilience strengthening through risk management planning there are some important system considerations that are omitted in the analysis. Primarily the framework is built on the assumption that the system properties and behavior are deterministic, which gives point estimates for the risk planning options. Such analysis is incomplete and in fact inaccurate because risk is uncertain by nature and in its extreme realization can have severe effects on the system behavior. If planning options do not consider extreme risk then they are rendered ineffective leading to severe systems failures.

The general optimization problem formulated in Equation (4.1) contains: (i) modeling or epistemic data uncertainties in estimation of the $\mathbf{A}^{*}$ matrix and magnitude of $\mathbf{x}$ vector and (ii) statistical or aleatory uncertainties in estimating probability distributions for $\mathbf{c}^{*}(\mathbf{0})$ vectors. Due to the presence of such uncertainties there arises a need to reformulate the problem statement. Previously our risk management problem was motivated by the question

> Given a finite budget for implementation how do we allocate the strategies such that the overall interdependent economic losses are minimized?

which showed that we were certain our planning would lead to a certain minimized economic loss estimate. Since, the objective Q is uncertain now the finite budget allocation approach does not apply because we are not sure if the allocated budget is sufficient to begin with. As a risk planner the interest still lies in minimizing the overall risk at an acceptable cost. Hence, due to the new planing paradigms the risk management problem is reformulated through the question

> For given initial impact on sectors in the form of demand perturbations if risk management strategies exist to reduce effects of such demand perturbations, then what is the minimum budget required for implementation of allocation of strategies such that the overall interdependent economic losses are below a certain acceptable threshold?

The mathematical statement of the above translates into the following optimization problem

$$\begin{aligned} \min \quad & b = \textstyle\sum_{l=1}^{m} g_l(r_l) \\ \text{subject to} \quad & \\ & \mathbf{x}^{\intercal}[\mathbf{I} - \mathbf{A}^*]^{-1}\mathbf{c}^* \leq Q \\ & c_l^* = f_l(c_l^*(0), r_l), \forall l = \{1, 2, ..., m\} \\ & r_l \geq 0, \forall l = \{1, 2, ..., m\} \end{aligned} \tag{4.3}$$

Applying this general framework to the specific problem of resource allocation we discussed for static resilience planning in Equation (4.2), the resource allocation problem is reformulated as

$$\begin{aligned} \min \quad & \textstyle\sum_{l=1}^{m} r_l \\ \text{subject to} \quad & \\ & \mathbf{x}^{\intercal}[\mathbf{I} - \mathbf{A}^*]^{-1}\mathbf{c}^* \leq Q \\ & c_l^* = c_l^*(0)e^{-\alpha_l r_l}, \forall l = \{1, 2, ..., m\} \\ & r_l \geq 0, \forall l = \{1, 2, ..., m\} \end{aligned} \tag{4.4}$$

Due to uncertainty there are certain modifications that need to be incorporated into the optimization frameworks of Equations (4.3) and (4.4). For this we investigate the components of the optimization problem that contain uncertainties and the effect said uncertainties have on the problem. Primarily we are concerned with knowing whether the functional forms of the objective function and constraints are preserved or changed when data and event uncertainties are incorporated into the framework.

Objective function

The general objective function of the resource allocation problem is to find the optimal budget that needs to be allocated to the risk management options. For this the planner would like to have a certain functional form of the objective function, which means that the parameters in the function g_l are certain. This does not mean that the budget allocated would be a certain budget. It essentially implies that there is certainty that the functional form of the objective function does not alter. We say that the optimization problem has a certain objective. Hence, in the resource allocation Equation (4.4) the objective is always $\sum_{l=1}^{m} r_l$, in which the coefficients associated with the r_l variables will always be 1.

Constraints

As mentioned previously, the estimates of the industry outputs ($\mathbf{x}$) and the interdependency structures in $\mathbf{A}^*$ are not known with certainty. This can result in the different sets of constraints for bounding the total economic loss. Hence, the constraint $\mathbf{x}^\intercal[\mathbf{I} - \mathbf{A}^*]^{-1}\mathbf{c}^* \leq Q$ is no longer a single constraint but belongs to a family of constraints given as

$$\mathbf{x}^\intercal[\mathbf{I} - \mathbf{A}^*]^{-1}\mathbf{c}^* \leq Q, \forall \mathbf{x} \in \mathcal{U}_x, \mathbf{A}^* \in \mathcal{U}_{A^*} \tag{4.5}$$

where $\mathcal{U}_x$ is a set that contains all possible realizations for the output vectors and $\mathcal{U}_{A^*}$ contains all possible realizations for the $\mathbf{A}^*$ matrices. Detailed descriptions of the uncertainty sets will be provided in the next section.

The functional relationship $c_l^* = f_l(c_l^*(0), r_l)$ between the demand perturbations and the risk management options also contains the initial demand perturbation estimates that initiate the onset of disruptions in the system. As is the nature of disruptions and risks, the estimates for $\mathbf{c}^*(0)$ cannot be known with certainty. As such the equality in the relationship cannot be maintained and from a planning perspective the best that can be done is to try to achieve a relationship that holds true within a bounded neighborhood. This implies that the equality constraint is transformed as

$$|c_l^* - f_l(c_l^*(0), r_l)| \leq \epsilon_l, \forall c_l^*(0) \in \mathcal{U}_{c^*(0)} \tag{4.6}$$

where $\epsilon << 1$ shows how close the risk management option translates into the desired demand perturbation, $\mathcal{U}_{c^*(0)}$ is the set that contains all possible realizations for the event that generates $c_l^*(0)$. Hence, the corresponding constraint in the resource allocation problem of Equation (4.4) becomes

$$|c_l^* - c_l^*(0)e^{-\alpha_l r_l}| \leq \epsilon_l, \forall c_l^*(0) \in \mathcal{U}_{c^*(0)} \tag{4.7}$$

4.2.2 Uncertain optimization and general robust formulation

The new formulation of the risk management problem (4.4) due to uncertainty becomes

$$
\begin{aligned}
\min \quad & \textstyle\sum_{l=1}^{m} r_l \\
\text{subject to} \quad & \\
& \mathbf{x}^{\intercal}[\mathbf{I}-\mathbf{A}^*]^{-1}\mathbf{c}^* \leq Q, \forall \mathbf{x} \in \mathcal{U}_x, \mathbf{A}^* \in \mathcal{U}_{A^*} \\
& |c_l^* - c_l^*(0)e^{-\alpha_l r_l}| \leq \epsilon_l, \forall c_l^*(0) \in \mathcal{U}_{c^*(0)}, \forall l = \{1,2,...,m\} \\
& r_l \geq 0, \forall l = \{1,2,...,m\}
\end{aligned}
\tag{4.8}
$$

Equation (4.8) is a collection of uncertain optimization problems of which Equation (4.4) is an instance. Since, now we are dealing with a collection of problems, instead of a single optimization problem, the concepts of optimality and feasibility lose their relevance. As decision-makers we are interested in obtaining a deterministic solution to an uncertain optimization, and thus are required to define paradigms for decision-making. Robust optimization principles for uncertain optimization problems provide such paradigms which are stated through the main proposition of robust optimization:

> When we solve the uncertain optimization problem we should obtain a deterministic value for the decision variable. This value should hold as long as the data is within the uncertainty sets defined for the problem and it never violates any of the constraints of the optimization.

The above proposition leads to the concept of a *robust feasible* solution set consisting on the decision variables that satisfies all the realizations of the constraints coming from the uncertainty set. In our risk management framework the decision variables are the $\mathbf{c}^*$ and $\mathbf{r}$ vectors, for which the robust feasible solution is one that belongs to the set

$$S(\mathcal{U}_x, \mathcal{U}_{A^*}, \mathcal{U}_{c^*(0)}) = \left\{ (\mathbf{c}^*, \mathbf{r}) : \begin{array}{ll} \mathbf{x}^\intercal [\mathbf{I} - \mathbf{A}^*]^{-1} \mathbf{c}^* \leq Q, & \forall \mathbf{x} \in \mathcal{U}_x, \mathbf{A}^* \in \mathcal{U}_{A^*} \\ |c_l^* - c_l^*(0) e^{-\alpha_l r_l}| \leq \epsilon_l, & \forall c_l^*(0) \in \mathcal{U}_{c^*(0)} \end{array} \right\} \quad (4.9)$$

It is very clear from the information so far that the ability to obtain the robust feasible set of solutions to the uncertain optimization problem hinges on the uncertainty sets. The theory of robust optimization is mainly concerned with constructing rules for the uncertainty sets that lead to feasible solutions. Since, new sets of rules are being created in addition to the ones defined by the problem statement, robust optimization leads to increased computational complexity of the uncertain optimization problem. Computational tractability is thus the governing issue when robust optimization formulations are made. Most of the recent theoretical research has focused on identifying problems for which tractable robust solutions can be obtained. It can be stated here that robust optimization is different from sensitivity analysis because the aim of robust optimization is to seek feasible solutions that do not depend upon the data uncertainties, but instead on the larger set which bounds them, and are constructed *a priori*. Sensitivity analysis, on the other hand, looks at a trajectory of solutions obtained from data variations and hence are data and perturbations sensitive.

We now provide a further rule that is required to make a start in solving the uncertain optimization problem of Equation (4.8).

1. **The uncertainty sets are closed, convex sets**. If a general non-convex set $\bar{\mathcal{U}}$ is chosen for the uncertainty then we can always take its Convex Hull (Conv($\bar{\mathcal{U}}$)) to construct $\mathcal{U}$. For the class of convex problems this means that the convexity structure of the problem is still preserved. Since, every uncertainty set is convex and the constraints in the problem Equation (4.8) are also convex, the robust feasible solution will hold for all uncertainties if it satisfies the worst-case of the

uncertainty. Hence, robust optimization problem is reduced to finding robust feasible $\mathbf{c}^*$ and $\mathbf{r}$ that satisfy

$$S(\mathcal{U}_x,\mathcal{U}_{A^*},\mathcal{U}_{c^*(0)}) = \left\{ (\mathbf{c}^*,\mathbf{r}) : \begin{array}{l} \max_{\mathbf{x},\mathbf{A}^*} \mathbf{x}^\intercal[\mathbf{I}-\mathbf{A}^*]^{-1}\mathbf{c}^* \leq Q \\ \max_{c_l^*(0)} |c_l^* - c_l^*(0)e^{-\alpha_l r_l}| \leq \epsilon_l \end{array} \right\} \tag{4.10}$$

This simplifies our analysis from an uncertain constraint problem to a certain constraint problem. This is one of the central principles of robust optimization and introduces the concept of the *robust counterpart* of an uncertain optimization problem. The robust counterpart of Equation (4.8) can now be stated as

$$\begin{array}{ll} \min & \sum_{l=1}^{m} r_l \\ \text{subject to} & \\ & \max_{\mathbf{x},\mathbf{A}^*} \mathbf{x}^\intercal[\mathbf{I}-\mathbf{A}^*]^{-1}\mathbf{c}^* \leq Q, \forall \mathbf{x} \in \mathcal{U}_x, \mathbf{A}^* \in \mathcal{U}_{A^*} \\ & \max_{c_l^*(0)} |c_l^* - c_l^*(0)e^{-\alpha_l r_l}| \leq \epsilon_l, \forall c_l^*(0) \in \mathcal{U}_{c^*(0)}, \forall l = \{1,2,...,m\} \\ & r_l \geq 0, \forall l = \{1,2,...,m\} \end{array} \tag{4.11}$$

4.2.3 Constructing uncertainty sets

The notion computational tractability of the robust optimization problem hinges on the structure of the uncertainty sets. Here we discuss the development of the uncertainties for which we seek solutions to the robust problem.

Data uncertainties

The coefficients of the $\mathbf{A}^*$ matrix, derived from the technical coefficient matrix $\mathbf{A}$, are subject to uncertainties arising from the inter-industry data collection efforts by the Bureau of Economic Analysis (BEA) (Horowitz & Planting, 2006). The BEA collects annual input-output records for a group of 15 aggregated industries and more detailed records for 65 industries every five years. Hence, there exists data uncertainty in the industry aggregations and in the temporal nature of inter-industry transactions. Furthermore, structural changes might occur in the $\mathbf{A}^*$ matrix due to occurrences of disruptive events (Percoco, 2006), which manifest themselves as forced substitutions in $\mathbf{A}^*$ whether by strategy *a priori* or by necessity *a posteriori*. Uncertainty in the interdependency matrix has been analyzed in a number of studies (Quandt, 1958, 1959; Bullard & Sebald, 1977; Percoco et al., 2006) and specifically for $\mathbf{A}^*$ (Barker & Haimes, 2009). The $\mathbf{x}$ vector is obtained from the same BEA data and hence, has uncertainties similar to the $\mathbf{A}^*$ matrix. Most of these studies concluded that the sample mean estimates coefficients of the $\mathbf{A}$ matrix, $\mathbf{x}$ vector or the $\mathbf{I} - \mathbf{A}^*$ matrix are bounded within a small interval of the published values (Bullard & Sebald, 1977).

Due to the assumption of having bounded intervals for the data uncertainty, we construct the uncertainty sets along similar lines to the theories of budgeted uncertainty sets (Bertsimas & Sim, 2004). We assume that each data value available to us is known to lie within an interval instead to having a point estimate. Hence, the actual output x_i of sector i lies somewhere in the interval $[\bar{x}_i - \hat{x}_i, \bar{x}_i + \hat{x}_i]$, where $\bar{x}_i$ is known as the nominal value that is free of any uncertainty, while $\hat{x}_i$ is the uncertain value that signifies the perturbation resulting in deviation from the nominal value. Generally it is expected that the perturbation is small or at least less than the nominal value. Since, x_i is known to occur within the specified interval, a scaled parameter ν_i is defined for quantifying the fractional deviation of x_i from $\bar{x}_i$ relative to the maximum allowed deviation $\hat{x}_i$.

$$\nu_i = \frac{x_i - \bar{x}_i}{\hat{x}_i} \in [-1, 1] \tag{4.12}$$

This means that x_i can be represented as

$$x_i = \bar{x}_i + \nu_i \hat{x}_i, \quad \nu_i \in [-1, 1] \tag{4.13}$$

While in a robust sense each ν_i can either be equal to -1 or 1 when we consider the maximum bounds of the uncertainty set. In reality every x_i will lie somewhere within the chosen interval and it would be a decision-makers preference to build a robust solution for the extreme interval as it might be too conservative. Hence, in order to control the amount of uncertain deviation the total uncertainty is limited by a *budget* that is given as

$$\sum_{i=1}^{n} |\nu_i| \leq \Lambda \tag{4.14}$$

where Λ lies in the interval $[0, n]$. By controlling the value to Λ the amount of uncertainty that is distributed to the sector outputs is regulated. It also shows the decision-makers preference in assigning uncertainty to his/her available nominal estimates for the output vales. Using the Equations (4.13) and (4.14) the uncertainty set $\mathcal{U}_x$ is given as follows

$$\mathcal{U}_x = \left\{ x_i : x_i = \bar{x}_i + \nu_i \hat{x}_i, |\nu_i| \leq 1, \sum_{i=1}^{n} |\nu_i| \leq \Lambda, i = \{1, 2, .., n\} \right\} \tag{4.15}$$

A similar approach can be adopted in constructing the uncertainty sets for the elements of the $n \times n$ $\mathbf{A}^*$ matrix. Assuming for each element a^*_{ij} the interval $[\bar{a}^*_{ij} - \hat{a}^*_{ij}, \bar{a}^*_{ij} + \hat{a}^*_{ij}]$ specifies the range within which the actual realization of the element lies. A scaling parameter η_{ij} is defined as

$$\eta_{ij} = \frac{a^*_{ij} - \bar{a}^*_{ij}}{\hat{a}^*_{ij}} \in [-1, 1] \tag{4.16}$$

This means that a^*_{ij} can be represented as

$$a^*_{ij} = \bar{a}^*_{ij} + \eta_{ij} \hat{a}^*_{ij}, \quad \eta_{ij} \in [-1, 1] \tag{4.17}$$

While incorporating uncertainties into the $\mathbf{A}^*$ matrix certain rules have to be followed that come from the definition of the matrix itself. It was established in Section 2.3.2 that each element of the interdependency matrix is greater than 0 and the sum of elements along a row are less than 1. This means that the uncertain $\mathbf{A}^*$ will have to belong to the set

$$\mathbf{A}^* = \left\{a_{ij}^*, a_{ij}^* \geq 0, \sum_{j=1}^{n} a_{ij}^* \leq 1\right\} \tag{4.18}$$

It is assumed here that the perturbed values $\hat{a}_{ij}^*$ are always less than the nominal values $\bar{a}_{ij}^*$, which implies that the condition that the elements in the uncertain matrix are greater than of equal to zero is always met. Guaranteeing the bound on the row sums of the matrix would lead to a budget over the amount of uncertainty that can be associated with the η_{ij} values. This is constructed as follows

$$\begin{aligned}
\sum_{j=1}^{n} a_{ij}^* &\leq 1 \\
\implies \sum_{j=1}^{n} \left(\bar{a}_{ij}^* + \eta_{ij}\hat{a}_{ij}^*\right) &\leq 1 \\
\implies \sum_{j=1}^{n} \eta_{ij}\hat{a}_{ij}^* &\leq 1 - \sum_{j=1}^{n} \bar{a}_{ij}^* \\
\implies \left|\sum_{j=1}^{n} \eta_{ij}\hat{a}_{ij}^*\right| &\leq \left|1 - \sum_{j=1}^{n} \bar{a}_{ij}^*\right| = \Gamma_i
\end{aligned} \tag{4.19}$$

Here Γ_i is associated with the allowed budget in the uncertainty, which has similar meaning to the budget Λ. The constrain (4.19) can be further modified to accommodate for the largest possible uncertainties associated with the scaling of the η_{ij} values for the upper bounds established for the budgets. Hence, the inequality (4.19) should be true for the largest value of the left hand side term. This is established as follows

$$\max \left| \sum_{j=1}^{n} \eta_{ij} \hat{a}^*_{ij} \right| \leq \Gamma_i \tag{4.20}$$

$$\implies \sum_{j=1}^{n} \hat{a}^*_{ij} |\eta_{ij}| \leq \Gamma_i \tag{4.21}$$

Using the Equation (4.15) and (4.20) the budgeted uncertainty set $\mathcal{U}_{A^*}$ for the $\mathbf{A}^*$ matrix is established as

$$\mathcal{U}_{A^*} = \left\{ a^*_{ij} : a^*_{ij} = \bar{a}^*_{ij} + \eta_{ij}\hat{a}^*_{ij}, |\eta_{ij}| \leq 1, \sum_{j=1}^{n} \hat{a}^*_{ij}|\eta_{ij}| \leq \Gamma_i, i = \{1, 2, .., n\} \right\} \tag{4.22}$$

Event uncertainties

Uncertainties associated with $c^*(0)$ are classified as event uncertainties because they depend upon the occurrence of a disruptive event. Instead of an interval bound on the possible realizations of the event uncertainties, it is preferred to associate such uncertainties with probability distributions. In reality we would consider the chances of a disruptive event occurring and causing an expected amount of damage, which results in an average value for the $c^*(0)$. Average estimates are not good enough for decision-making because there is also the possibility of the disruptive event resulting in a $c^*(0)$ that deviates substantially from the expected value. Hence, multiple realizations of $c^*(0)$ are required for a complete decision-making analysis. Instead of the actual probability distribution if it is assumed that the expected value $\mathbb{E}[c^*(0)]$ and the variance $Var[c^*(0)]$ of $c^*(0)$ are known, then we can construct an uncertainty set. Having the first two moments are good enough for our purposes of constructing an uncertainty set, and in general we might not have enough data to construct an actual distribution but can extract some estimates of the fist two moments. Any realization of $c^*(0)$ can be said to be within the interval $[\bar{c}^*(0) -$

$\kappa\hat{c}^*(0), \bar{c}^*(0) + \kappa\hat{c}^*(0)]$,where $\kappa \geq 0$ signifies the spread of the interval about its expected value. The uncertainty set $\mathcal{U}_{c^*(0)}$ is thus constructed as

$$\mathcal{U}_{c^*(0)} = \left\{ c^*(0) : \begin{array}{l} c^*(0) \in [\bar{c}^*(0) \pm \kappa\hat{c}^*(0)], \\ \mathbb{E}[c^*(0)] = \bar{c}^*(0), Var[c^*(0)] = \hat{c}^{*2}(0), \kappa \geq 0 \end{array} \right\} \tag{4.23}$$

4.2.4 Robust problem formulation

With the uncertainty sets that have been constructed, we need to see the corresponding robust formulations of the constraints in the optimization problem. As mentioned previously the robust formulation transforms the original problem into a higher dimension problem, which satisfies the constraints for the extreme uncertainty bounds imposed by the budgets. We look at the two types constraints in our resource allocation problem and construct robust counterparts for them.

Robust constraint due to data uncertainty

In the risk management problem we formulated in Equation (4.11) the robust formulation for data uncertainties requires us to formulate a tractable formulation for the constraint

$$\max_{\mathbf{x},\mathbf{A}^*} \mathbf{x}^\intercal [\mathbf{I} - \mathbf{A}^*]^{-1} \mathbf{c}^* \leq Q, \forall \mathbf{x} \in \mathcal{U}_x, \mathbf{A}^* \in \mathcal{U}_{A^*} \tag{4.24}$$

Due to the presence of the term $(\mathbf{I} - \mathbf{A}^*)^{-1}$ the uncertainty from $\mathbf{A}^*$ is not linearly transformed into the constraint unlike the $\mathbf{x}$ vector. We can construct an approximation of the $(\mathbf{I} - \mathbf{A}^*)^{-1}$ matrix that allows for a linear transformation of the uncertainties. If an uncertain $\mathbf{A}^*$ matrix is given as $\bar{\mathbf{A}}^* + \Delta\bar{\mathbf{A}}^* \in \mathcal{U}_{A^*}$ then assuming the terms of $\Delta\bar{\mathbf{A}}^*$ are small we get the following transformation of the inverse matrix

$[\mathbf{I} - (\bar{\mathbf{A}}^* + \Delta\bar{\mathbf{A}}^*)]^{-1}$.

$$\begin{aligned}
[\mathbf{I} - (\bar{\mathbf{A}}^* + \Delta\bar{\mathbf{A}}^*)]^{-1} &= \mathbf{I} + (\bar{\mathbf{A}}^* + \Delta\bar{\mathbf{A}}^*) + (\bar{\mathbf{A}}^* + \Delta\bar{\mathbf{A}}^*)^2 + ... \\
&\approx \mathbf{I} + \bar{\mathbf{A}}^* + (\bar{\mathbf{A}}^*)^2 + ... + \Delta\bar{\mathbf{A}}^*[\mathbf{I} + 2\bar{\mathbf{A}}^* + 3(\bar{\mathbf{A}}^*)^2 + ...] \\
&= (\mathbf{I} - \bar{\mathbf{A}}^*)^{-1} + \Delta\bar{\mathbf{A}}^*(\mathbf{I} - \bar{\mathbf{A}}^*)^{-2} \qquad (4.25)
\end{aligned}$$

Before building the robust constraint some notation is introduced here as follows

$$\begin{aligned}
\bar{\mathbf{A}}^* &= [\bar{a}^*_{ij}] \in \mathbb{R}^{n\times n} \\
\boldsymbol{\Delta}\bar{\mathbf{A}}^* &= [\eta_{ij}\hat{a}^*_{ij}] \in \mathbb{R}^{n\times n} \\
\bar{\mathbf{D}}^* &= [\bar{d}^*_{ij}] = (\mathbf{I} - \bar{\mathbf{A}}^*)^{-1} \in \mathbb{R}^{n\times n} \\
\bar{\mathbf{P}}^* &= [\bar{p}^*_{ij}] = (\mathbf{I} - \bar{\mathbf{A}}^*)^{-2} \in \mathbb{R}^{n\times n} \\
\boldsymbol{\Delta}\bar{\mathbf{D}}^* &= \Delta\bar{\mathbf{A}}^*\bar{\mathbf{P}}^* \in \mathbb{R}^{n\times n} \\
\bar{\mathbf{x}} &= [\bar{x}_i] \in \mathbb{R}^{n\times 1} \\
\boldsymbol{\Delta}\bar{\mathbf{x}} &= [\nu_i\hat{x}_i] \in \mathbb{R}^{n\times 1}
\end{aligned}$$

Due to uncertainty the left hand side of the Equation (4.24) is transformed as

$$\begin{aligned}
\mathbf{x}^\intercal[\mathbf{I} - \mathbf{A}^*]^{-1}\mathbf{c}^* &= (\bar{\mathbf{x}} + \boldsymbol{\Delta}\bar{\mathbf{x}})^\intercal[\mathbf{I} - (\bar{\mathbf{A}}^* + \boldsymbol{\Delta}\bar{\mathbf{A}}^*)]^{-1}\mathbf{c}^* \\
&= (\bar{\mathbf{x}} + \boldsymbol{\Delta}\bar{\mathbf{x}})^\intercal[\bar{\mathbf{D}}^* + \boldsymbol{\Delta}\bar{\mathbf{D}}^*]\mathbf{c}^* \\
&= \bar{\mathbf{x}}^\intercal\bar{\mathbf{D}}^*\mathbf{c}^* + \bar{\mathbf{x}}^\intercal\boldsymbol{\Delta}\bar{\mathbf{D}}^*\mathbf{c}^* + \boldsymbol{\Delta}\bar{\mathbf{x}}^\intercal\bar{\mathbf{D}}^*\mathbf{c}^* + \boldsymbol{\Delta}\bar{\mathbf{x}}^\intercal\boldsymbol{\Delta}\bar{\mathbf{D}}^*\mathbf{c}^* \\
&\approx \bar{\mathbf{x}}^\intercal\bar{\mathbf{D}}^*\mathbf{c}^* + \bar{\mathbf{x}}^\intercal\boldsymbol{\Delta}\bar{\mathbf{D}}^*\mathbf{c}^* + \boldsymbol{\Delta}\bar{\mathbf{x}}^\intercal\bar{\mathbf{D}}^*\mathbf{c}^* \qquad (4.26)
\end{aligned}$$

By separating the nominal data and the uncertain data the robust constraint of Equation (4.24) is interpreted as solving for the maximum of the uncertain parts of the inequality. Hence, the robust constraint is the one that satisfies

$$\bar{\mathbf{x}}^\intercal \bar{\mathbf{D}}^* \mathbf{c}^* + \max_{\mathcal{U}_{A^*}} \bar{\mathbf{x}}^\intercal \boldsymbol{\Delta} \bar{\mathbf{D}}^* \mathbf{c}^* + \max_{\mathcal{U}_x} \boldsymbol{\Delta} \bar{\mathbf{x}}^\intercal \bar{\mathbf{D}}^* \mathbf{c}^* \leq Q \tag{4.27}$$

We look at the element wise expansion of each component of the above formulation to construct the robust constraint.

$$\begin{aligned}
\max_{\mathcal{U}_{A^*}} \bar{\mathbf{x}}^\intercal \boldsymbol{\Delta} \bar{\mathbf{D}}^* \mathbf{c}^* &= \max_{\mathcal{U}_{A^*}} \sum_{j=1}^{n} \left(\sum_{i=1}^{n} \bar{x}_i \left(\sum_{k=1}^{n} \eta_{ik} \hat{a}_{ik}^* \bar{p}_{kj} \right) \right) |c_j^*| \\
&= \max_{\mathcal{U}_{A^*}} \sum_{i=1}^{n} \left(\sum_{k=1}^{n} \bar{x}_i \left(\sum_{j=1}^{n} \bar{p}_{kj} |c_j^*| \right) \hat{a}_{ik}^* \eta_{ik} \right) \\
&= \sum_{i=1}^{n} \max_{\eta_{ik} \in \mathcal{U}_{A^*}} \sum_{k=1}^{n} s_{ik} \hat{a}_{ik}^* \eta_{ik} \qquad (4.28)
\end{aligned}$$

where $s_{ik} = \bar{x}_i \left(\sum_{j=1}^{n} \bar{p}_{kj} |c_j^*| \right)$.

Finding the solution of the Equation (4.28) is equivalent to solving the n problems the i^{th} of which is given as

$$\begin{aligned}
\max_{\eta_{ik}} \quad & \textstyle\sum_{k=1}^{n} s_{ik} \hat{a}_{ik}^* \eta_{ik} \\
\text{subject to} \quad & \\
& \textstyle\sum_{k=1}^{n} \hat{a}_{ik}^* \eta_{ik} \leq \Gamma_i \\
& 0 \leq \eta_{ik} \leq 1
\end{aligned} \tag{4.29}$$

Since this is a linear optimization problem its optimal solution is the same as its dual. The dual of the Equation (4.29) optimization problem is calculated as

$$\begin{aligned}
\min \quad & \Gamma_i w_i + \textstyle\sum_{k=1}^{n} z_{ik} \\
\text{subject to} \quad & \\
& \hat{a}_{ik}^* w_i + z_{ik} \geq \hat{a}_{ik}^* \bar{x}_i \left(\textstyle\sum_{j=1}^{n} \bar{p}_{kj} |c_j^*| \right) \\
& w_i, z_{ik} \geq 0
\end{aligned} \tag{4.30}$$

Here w_i and z_{ij} are dual variables. There are n such problems each coming from constructing the dual for each term of the i^{th} sum in the final term of Equation (4.28). We can construct a similar formulation for the term $\max_{\mathcal{U}_x} \boldsymbol{\Delta}\bar{\mathbf{x}}^\intercal \bar{\mathbf{D}}^* \mathbf{c}^*$ which is given as

$$\begin{aligned} \max_{\mathcal{U}_x} \boldsymbol{\Delta}\bar{\mathbf{x}}^\intercal \bar{\mathbf{D}}^* \mathbf{c}^* &= \max_{\nu_i \in \mathcal{U}_x} \sum_{i=1}^{n} \left(\sum_{j=1}^{n} \bar{d}^*_{ij} |c^*_j| \right) \hat{x}_i \nu_i \\ &= \max_{\nu_i \in \mathcal{U}_x} \sum_{i=1}^{n} r_i \hat{x}_i \nu_i \end{aligned} \tag{4.31}$$

where $r_i = \sum_{j=1}^{n} \bar{d}^*_{ij} |c^*_j|$ The solution of the above problem is equivalent to solving the problem

$$\begin{aligned} \max_{v_i} \quad & \textstyle\sum_{i=1}^{n} r_i \hat{x}_i \nu_i \\ \text{subject to} \quad & \\ & \textstyle\sum_{i=1}^{n} \nu_i \leq \Lambda \\ & 0 \leq \nu_i \leq 1 \end{aligned} \tag{4.32}$$

The optimal solution of the above is the same as its dual, which is given as

$$\begin{aligned} \min \quad & \Lambda y + \textstyle\sum_{i=1}^{n} t_i \\ \text{subject to} \quad & \\ & y + t_i \geq \hat{x}_i \textstyle\sum_{j=1}^{n} \bar{d}^*_{ij} |c^*_j| \\ & y, t_i \geq 0 \end{aligned} \tag{4.33}$$

From the Equations (4.27), (4.30) and (4.33) the following theorem is proposed

Theorem 1. *The robust constraint*

$$\bar{\mathbf{x}}^\intercal \bar{\mathbf{D}}^* \mathbf{c}^* + \max_{\mathcal{U}_{A^*}} \bar{\mathbf{x}}^\intercal \boldsymbol{\Delta}\bar{\mathbf{D}}^* \mathbf{c}^* + \max_{\mathcal{U}_x} \boldsymbol{\Delta}\bar{\mathbf{x}}^\intercal \bar{\mathbf{D}}^* \mathbf{c}^* \leq Q$$

can be solved by the linear programming problem

$$
\begin{aligned}
\min \quad & \textstyle\sum_{i=1}^n \sum_{j=1}^n \bar{d}_{ij}^* c_j^* \bar{x}_i + \sum_{i=1}^n \Gamma_i w_i + \sum_{i=1}^n \sum_{k=1}^n z_{ik} + \Lambda y + \sum_{i=1}^n t_i \le Q \\
\textit{subject to} \quad & \\
& \hat{a}_{ik}^* w_i + z_{ik} \ge \hat{a}_{ik}^* \bar{x}_i \Big(\textstyle\sum_{j=1}^n \bar{p}_{kj} f_j^* \Big), \forall i, k \\
& y + t_i \ge \hat{x}_i \textstyle\sum_{j=1}^n \bar{d}_{ij}^* |c_j^*| \\
& -f_j^* \le c_j^* \le f_j^*, \forall j \\
& w_i, z_{ik} \ge 0, \forall i, k \\
& y, t_i \ge 0
\end{aligned} \tag{4.34}
$$

Proof. See Equations (4.27), (4.30) and (4.33) □

Robust constraint due to event uncertainty

As discussed previously the event uncertainty is given by a probability distribution for which the first two moments are known to us. The constraint in the Equation (4.11) for which we seek a robust formulation due to the event uncertainty shows the planning function that relates the effectiveness of the risk management option in reducing the effect of the initial disruption. We further make an assumption that due to the uncertainty in $c^*(0)$ we are not 100% certain that the inequality will be true to begin with. This implies that most of the times we are sure that our planning function and the targeted demand perturbation are within the bound ϵ_l, but there is a chance that the inequality is violated sometimes. This might happen due to fact that the planning function has been designed for a particular range of possible values for $c^*(0)$, but does not hold when $c^*(0)$ lies outside this range. Thus our inequality for which we seek a robust formulation is now a chance constraint given as

$$\mathbb{P}\big[|c_l^* - c_l^*(0)e^{-\alpha_l r_l}| > \epsilon_l\big] \le \gamma_l \tag{4.35}$$

where $\gamma_l << 1$ shows the small chance that the constraint is violated. The robust feasible counterpart to the above chance constraint is the one that satisfies the constraint for the maximum limit of the uncertainty set and is given by

$$\max_{c^*(0)\in\mathcal{U}_{c^*(0)}} \mathbb{P}\left[|c_l^* - c_l^*(0)e^{-\alpha_l r_l}| > \epsilon_l\right] \leq \gamma_l \tag{4.36}$$

Before we derive the robust formulation we state the following result from Chebychev (Stewart, 2009)

Proposition 7. *Let X be a random variable with finite expectation $\mathbb{E}[X]$ and nonzero variance $Var(X)$ then for some $t > 0$*

$$\mathbb{P}[|X - \mathbb{E}[X]| \geq t] \leq \frac{Var(X)}{t^2} \tag{4.37}$$

Proof. For the random number X, we have

$$t\mathbb{P}[|X| \geq t] = t\mathbb{E}[I_{|X|\geq t}] = \mathbb{E}[tI_{|X|\geq t}] \leq \mathbb{E}[|X|] \tag{4.38}$$

where $I_{|X|>t}$ is an indicator function which is 1 if $|X| \geq t$ and 0 if $|X| < t$, which means $tI_{|X|\geq t} \leq |X|$. From the constraint (4.38) we can see that

$$\mathbb{P}[|X| \geq t] \leq \frac{\mathbb{E}[|X|]}{t} \tag{4.39}$$

The above result is in fact known as the Markov inequality (Stewart, 2009). In (4.39) if we replace X by $X - \mathbb{E}[X]$ then we can get

$$\begin{aligned} \mathbb{P}[|X - \mathbb{E}[X]| \geq t] \implies \mathbb{P}[(X - \mathbb{E}[X])^2 \geq t^2] &\leq \frac{\mathbb{E}[(X - \mathbb{E}[X])^2]}{t^2} \\ &= \frac{Var(X)}{t^2} \end{aligned} \tag{4.40}$$

which proves the Chebychev's Inequality. □

From the above condition we can build a robust formulation for the chance constraint. Inequality (4.36) can have two variants given as

$$\mathbb{P}\Big[c_l^*(0)e^{-\alpha_l r_l} - c_l^* > \epsilon_l\Big] \leq \gamma_l \tag{4.41}$$

$$\mathbb{P}\Big[c_l^*(0)e^{-\alpha_l r_l} - c_l^* < -\epsilon_l\Big] \leq \gamma_l \tag{4.42}$$

. Rearranging the constraint (4.41) we get the following

$$\begin{aligned}\mathbb{P}\Big[c_l^*(0)e^{-\alpha_l r_l} - c_l^* > \epsilon_l\Big] &= \mathbb{P}\Big[c_l^*(0)e^{-\alpha_l r_l} > \epsilon_l + c_l^*\Big] \\ &= \mathbb{P}\left[\begin{array}{l}|c_l^*(0)e^{-\alpha_l r_l} - \bar{c}_l^*(0)e^{-\alpha_l r_l}| \\ > |\epsilon_l + c_l^* - \bar{c}_l^*(0)e^{-\alpha_l r_l}|\end{array}\right]\end{aligned} \tag{4.43}$$

Equation (4.43) is of the form of Equation (4.37), because $\mathbb{E}[c_l^*(0)e^{-\alpha_l r_l}] = \bar{c}_l^*(0)e^{-\alpha_l r_l}$. Hence we get

$$\mathbb{P}\Big[c_l^*(0)e^{-\alpha_l r_l} - c_l^* > \epsilon_l\Big] \leq \frac{\hat{c}^{*2}(0)e^{-2\alpha_l r_l}}{(\epsilon_l + c_l^* - \bar{c}_l^*(0)e^{-\alpha_l r_l})^2} \tag{4.44}$$

From Equation (4.44) we can assume that

$$\begin{aligned}\max_{c^*(0)\in\mathcal{U}_{c^*(0)}} \quad & \mathbb{P}\Big[|c_l^* - c_l^*(0)e^{-\alpha_l r_l}| > \epsilon_l\Big] \leq \gamma_l \\ \Longrightarrow \quad & \tfrac{\hat{c}^{*2}(0)e^{-2\alpha_l r_l}}{(\epsilon_l + c_l^* - \bar{c}_l^*(0)e^{-\alpha_l r_l})^2} \leq \gamma_i \\ \Longrightarrow \quad & |\epsilon_l + c_l^* - \bar{c}_l^*(0)e^{-\alpha_l r_l}| \geq \tfrac{1}{\sqrt{\gamma_i}}\hat{c}^*(0)e^{-\alpha_l r_l}\end{aligned} \tag{4.45}$$

The above outlined method generates a loose robust bound on the chance constraint. The factor γ_l can be related to the number of deviations κ of a random $c^*(0)$ from its mean. Specifically from the definition of the Chebychev's inequality in Equation (4.37) we can see that if $t = \kappa\sqrt{Var(X)}$ then we get $1/\kappa^2$ as the upper limit on the inequality. Thus $\gamma_l = 1/\kappa^2$ can be set as a bound on the chance constraint. This is not a very robust constraint bound because it means that the probability that the chance constraint is violated has a very high upper bound. For example if $\kappa = 2$, then $\gamma_l = 0.25$ which is a very loose upper bound for the probabilistic constraint. Nevertheless there is merit in such a bound because it helps us develop an estimate for the event uncertainty measure even though they are conservative.

Final robust formulation

Collecting the results from the development of the robust constraints the final robust counterpart of our original resource allocation problem is summarized as follows

Theorem 2. *Given the budget allocation problem for static resilience estimation*

$$\begin{aligned} \min \quad & \textstyle\sum_{l=1}^{n} r_l \\ \textit{subject to} \quad & \\ & \mathbf{x}^\intercal[\mathbf{I} - \mathbf{A}^*]^{-1}\mathbf{c}^* \leq Q \\ & c_l^* = c_l^*(0)e^{-\alpha_l r_l}, \forall l = \{1, 2, ..., n\} \\ & r_l \geq 0, \forall l = \{1, 2, ..., n\} \end{aligned} \tag{4.46}$$

If the uncertainties in the data are defined through budgeted sets $\mathcal{U}_x$ *and* $\mathcal{U}_{A^*}$ *defined in Equations* (4.15) *and* (4.23) *respectively and the event uncertainties are specified by the set* $\mathcal{U}_{c^*(0)}$ *defined in Equation* (4.24) *and the chance constraint in Equation* (4.34)*,* *then the robust counterpart of the problem is given as*

$$
\begin{array}{ll}
\min & \sum_{l=1}^{n} r_l \\
\textit{subject to} & \\
& \sum_{i=1}^{n}\sum_{j=1}^{n} \bar{d}_{ij}^{*} c_j^{*} \bar{x}_i + \sum_{i=1}^{n} \Gamma_i w_i + \sum_{i=1}^{n}\sum_{k=1}^{n} z_{ik} + \Lambda y + \sum_{i=1}^{n} t_i \leq Q \\
& \hat{a}_{ik}^{*} w_i + z_{ik} \geq \hat{a}_{ik}^{*} \bar{x}_i \left(\sum_{j=1}^{n} \bar{p}_{kj} f_j^{*} \right), \forall i, k \\
& y + t_i \geq \hat{x}_i \sum_{j=1}^{n} \bar{d}_{ij}^{*} f_j^{*}, \forall i, j \\
& -f_j^{*} \leq c_j^{*} \leq f_j^{*}, \forall j \\
& w_i, z_{ik} \geq 0, \forall i, k \\
& y, t_i \geq 0, \forall i \\
& \epsilon_l + c_l^{*} - \bar{c}_l^{*}(0) e^{-\alpha_l r_l} \geq \frac{1}{\sqrt{\gamma_i}} \hat{c}^{*}(0) e^{-\alpha_l r_l}, \forall l \\
& \epsilon_l + c_l^{*} - \bar{c}_l^{*}(0) e^{-\alpha_l r_l} \leq -\frac{1}{\sqrt{\gamma_i}} \hat{c}^{*}(0) e^{-\alpha_l r_l}, \forall l \\
& r_l \geq 0, \forall l
\end{array}
\tag{4.47}
$$

4.3 Example Problem

In order to show the effectiveness of the robust optimization scheme we illustrate the effect of uncertainties in the framework and the look at the nominal and robust solutions. The static resilience estimation problem of Section 3.2.2 is again examined with uncertainties introduced into the framework. Table 4.1 shows the transaction flow data for the economic system from which the interdependency matrix $\mathbf{A}^*$, and maximum demand perturbations $\mathbf{c}^*$ are generated. As previously mentioned the parameters α_l are available to the decision-maker.

$$
\mathbf{A}^* = \begin{bmatrix} 0.27 & 0.38 & 0.23 \\ 0.18 & 0.073 & 0.15 \\ 0.28 & 0.28 & 0.39 \end{bmatrix}, \quad \mathbf{c}^*(0) = \begin{bmatrix} 0.13 \\ 0.59 \\ 0.04 \end{bmatrix}, \quad \boldsymbol{\alpha} = \begin{bmatrix} 0.05 \\ 0.04 \\ 0.08 \end{bmatrix} \tag{4.48}
$$

Table 4.1: Two industry input-output transaction data in million of dollars

Industry	1	2	3	External Demand ($\mathbf{c}$) ($ US million)	Total Output ($\mathbf{x}$) ($ US million)
1	266	378	230	126	1000
2	267	110	224	899	1500
3	340	340	468	52	1200
Value added	127	672	278		
Total Output($\mathbf{x}^T$)	1000	1500	1200		

4.3.1 Data uncertainty effects

As a first stage analysis it is considered that there are no unknown events in the framework and the planner is certain about the policy functions that relate the demand perturbations to the allocated budgets. The only uncertainties that arise in the framework exist due to unreliable data estimates. Assuming a 5% uncertainty in both the output ($\mathbf{x}$) and the interdependency matrix coefficients the corresponding robust formulation is constructed. From previous formulations that data uncertainty sets take the following values

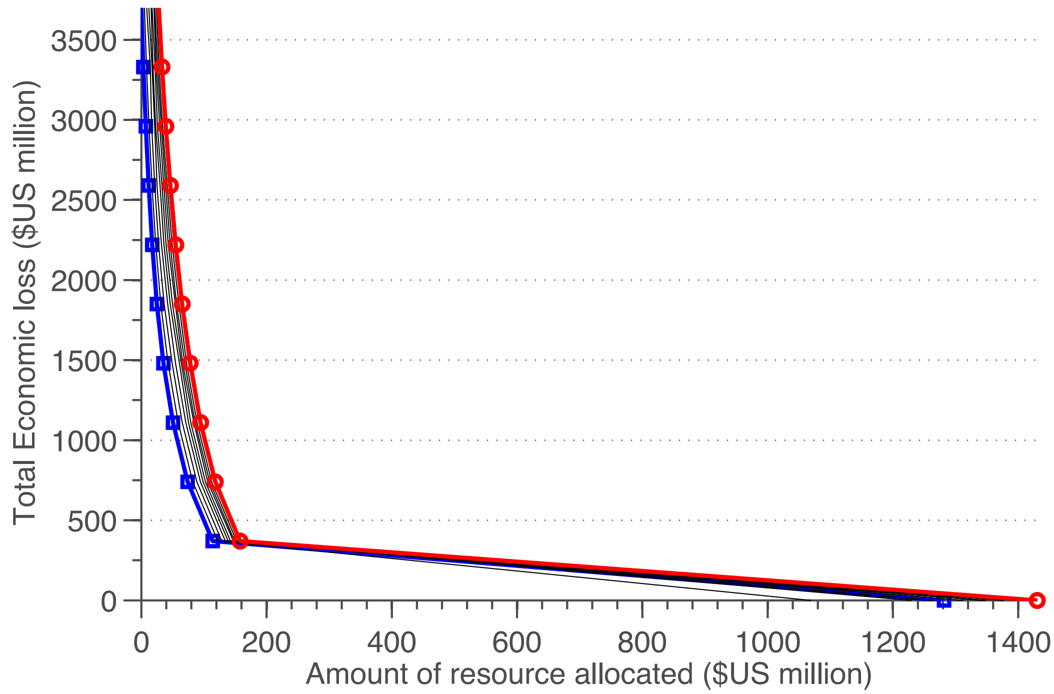
$$\mathcal{U}_x = \left\{x_i : x_i = \bar{x}_i + 0.05\nu_i\bar{x}_i, |\nu_i| \le 1, \sum_{i=1}^{3} |\nu_i| \le 3, i = \{1,2,3\}\right\} \quad (4.49)$$

$$\mathcal{U}_{A^*} = \left\{a^*_{ij} : \begin{array}{l} a^*_{ij} = \bar{a}^*_{ij} + 0.05\eta_{ij}\bar{a}^*_{ij}, |\eta_{ij}| \le 1, \sum_{j=1}^{3} \hat{a}^*_{ij}|\eta_{ij}| \le \Gamma_i, \\ \Gamma_i = \{0.1260, 0.5993, 0.0433\}, i = \{1,2,3\}, \end{array}\right\} \quad (4.50)$$

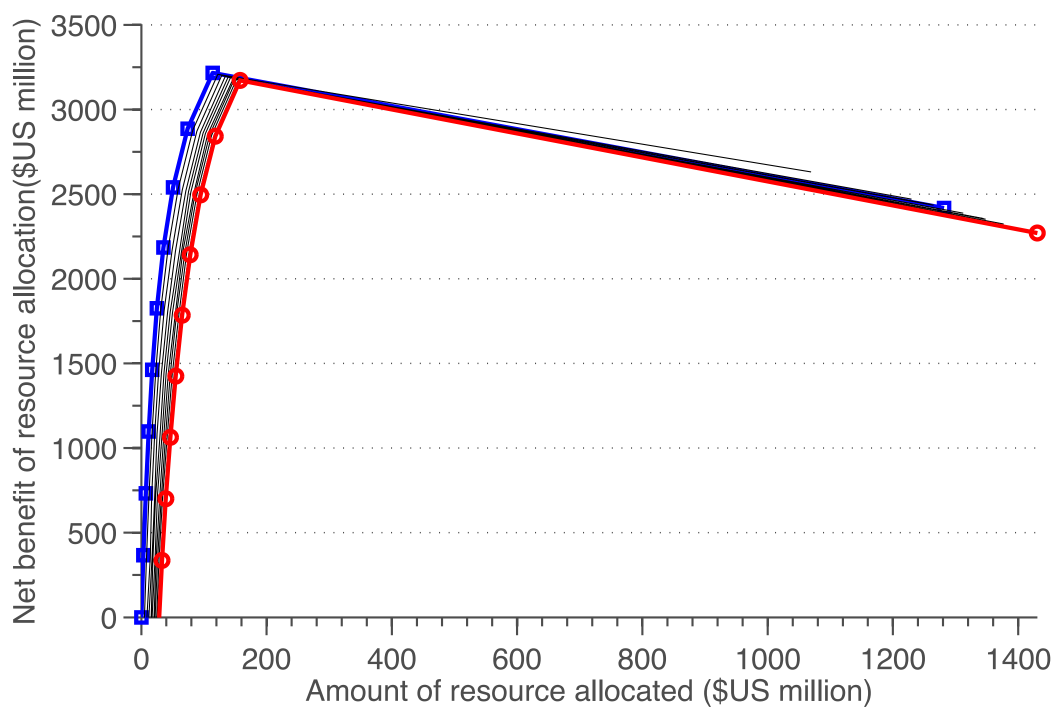
The values above are the maximum allowable uncertainty budgets that can be introduced into the formulations. As a planning exercise we examine the effects of varying the uncertainty budgets from 0 to their maximum values. There are significant differences in amounts of resources that need to be allocated to keep the total economic losses to targeted levels. Figure 4.1 shows that as the allowable uncertainties are increased by moving the realized values for the allocated budgets to their maximum values there is a considerable deviation from the nominal planning scenarios. For this problem the robust uncertainty solution shows that for just a 5% uncertainty in data estimates the required monetary values of resource allocations are considerably higher than their nominal counterparts. In particular the maximal robust solutions shown in the Figures 4.1(a) and 4.1(b) dominate all the other solutions, which show that they reflect the notion of the decision-making solution being robust to all realizations of the uncertainties within its limits. To further highlight the effect of uncertainties and the robust considerations the amounts of resource allocations to each sector for given total economic loss planning are shown in the Table 4.2. The nominal value results are denoted by r^N, while the robust planning results are denoted by r^R. The numbers show that there is a considerable difference in the requirements for each sectors when the worst-case uncertainties are incorporated into the planning. Also it is evident that the onset of resource allocation required for each sector is realized earlier than the nominal planning case. Such an analysis highlights the importance of the robust scheme developed and its requirement for the resource planning.

Table 4.2: Comparisons of the required resource allocations between the nominal planning and the robust planning. All numbers are in $US millions.

Q	Sector 1		Sector 2		Sector 3	
	r^N	r^R	r^N	r^R	r^N	r^R
3700	0	0	0	0	0	0
3330	0	0	3.3694	33.0776	0	0
2960	0	1.4269	7.2646	37.5941	0	0
2590	0	4.7042	11.8809	41.6833	0	0
2220	0	8.5048	17.5470	46.4342	0	0
1850	0	12.4287	24.8857	51.3458	0	1.4136
1480	0	16.9622	35.3182	57.0059	0	4.2523
1110	6.5781	22.7159	44.2229	64.1979	0	7.8483
740	15.4108	30.8252	55.2636	74.3347	3.2904	12.9163
370	29.2737	44.6179	72.5923	91.5825	11.9547	21.5303
0	431.0008	543.3183	571.0274	614.0301	262.2573	273.1499



(a) Economic losses vs Allocated budgets



(b) Net benefit vs Allocated budgets

Figure 4.1: Trade-offs between investments in losses for different levels of budgeted data uncertainties. The bold line with circles is the maximal robust solution under the available budgets

4.3.2 Event uncertainties

In addition to introducing data uncertainties in the analysis event uncertainties are also introduced. It is assumed that available data estimates are still known within ±5% accuracy. The uncertainty in the values of the $\mathbf{c}^*$ is assumed to be of ±2% deviation from the mean. This means the uncertainty set for the events, which is the set for $\mathbf{c}^*$ is defined as

$$\mathcal{U}_{c^*(0)} = \left\{ c_i^*(0) : \begin{array}{l} c_i^*(0) \in [\bar{c}_i^*(0) \pm 0.02\kappa\bar{c}_i^*(0)], i = \{1,2,3\} \\ \mathbb{E}[c_i^*(0)] = \bar{c}_i^*(0), Var[c_i^*(0)] = 0.0004\bar{c}_i^{*2}(0), \kappa \geq 0 \end{array} \right\} \quad (4.51)$$

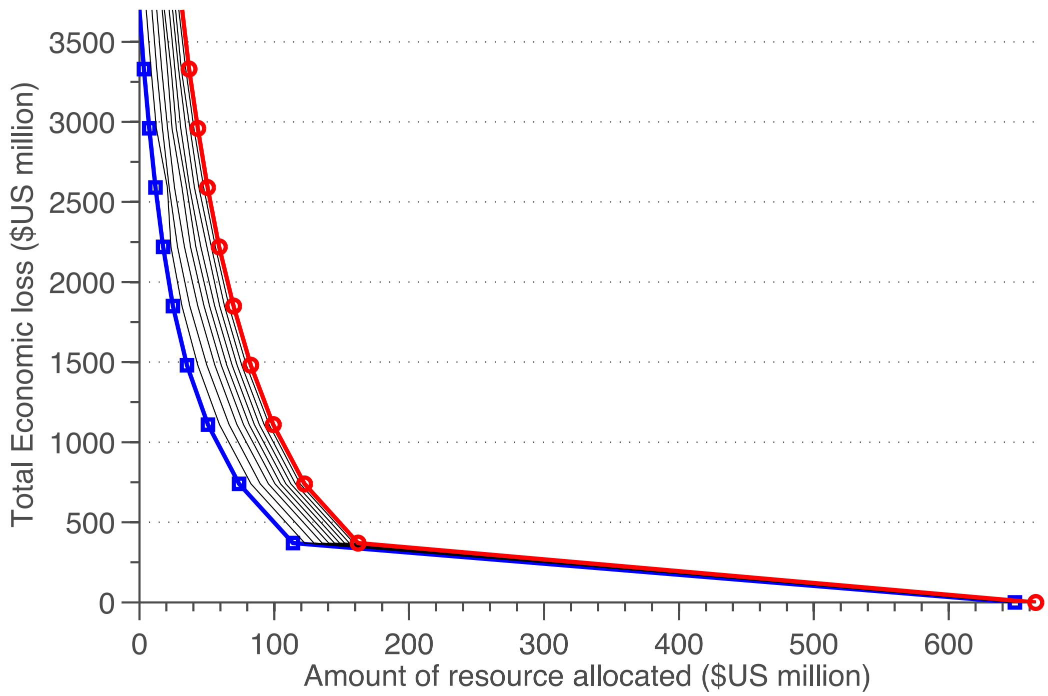
We choose $\kappa = 3$ here which means that the upper limit of the violation of the chance probabilistic chance constraint is $\gamma = 0.11$. The values above are the maximum allowable uncertainty budgets that can be introduced into the formulations. As done previously we examine the effects of varying the uncertainties in data and event budgets from 0 to the maximal limits that have been set for them. The amounts of resources that need to be allocated to keep the total economic losses to targeted levels are further increased when event uncertainties are added to the previously introduced data uncertainties. Figure 4.2 shows that as the allowable uncertainties are increased by moving the realized values for the allocated budgets to their maximum values there is a considerable deviation from the nominal planning scenarios. There is a greater variation in the budget allocation for just the additional 2% deviation in the event estimates along with the 5% data uncertainties. Now the required monetary values of resource allocations are considerably higher than their nominal counterparts. As shown in the previous case the maximal robust solutions shown in the Figures 4.2(a) and 4.2(b) dominate all the other solutions. The amounts of resource allocations to each sector for given total economic loss planning are shown in the Table 4.3. The nominal value results are denoted by r^N, while the robust planning results are denoted by r^R. The numbers show that due to event uncertainties there is further difference in the requirements for each sectors when the worst-case uncertainties are incorporated into the planning. The onset of resource allocation required for each sector is realized earlier than the nominal planning case, and great amounts of budgets are required for each level of economic loss. The only inconsistent result here is the value for resource allocations when $Q = 0$. The value for the robust solution is considerably less than the nominal value. This a due to the probabilistic limits we have placed over the chance constraint. The robust solutions obtained a violated with probability $1 - \gamma = 0.89$, which is a weak bound and needs to be improved. For other cases the robust results are as expected.

Table 4.3: Comparisons of the required resource allocations between the nominal planning and the robust planning. All numbers are in $US millions.

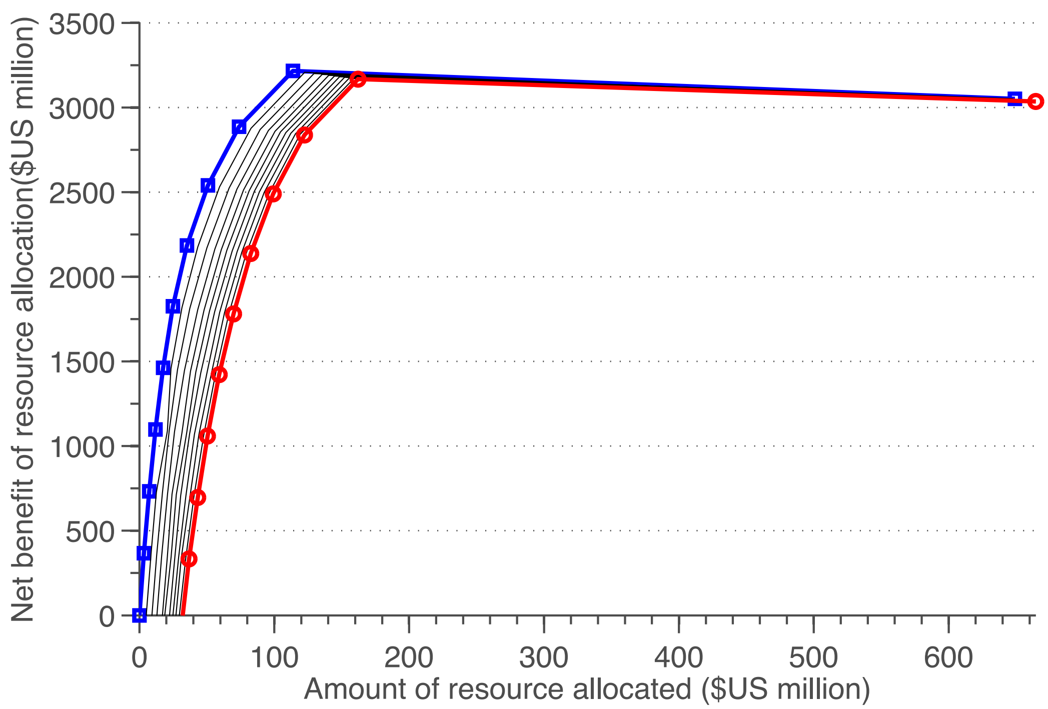
Q	Sector 1		Sector 2		Sector 3	
	r^N	r^R	r^N	r^R	r^N	r^R
3700	0	0	0	0	0	0
3330	0	0.4670	3.3694	36.3938	0	0
2960	0	3.3408	7.2646	39.9791	0	0
2590	0	6.5839	11.8809	44.0330	0	0
2220	0	10.3211	17.5470	48.7118	0	0.0949
1850	0	14.0377	24.8857	53.3502	0	2.4245
1480	0	18.5003	35.3182	58.9287	0	5.2135
1110	6.5781	24.2535	44.2229	66.1203	0	8.8093
740	15.4108	32.3621	55.2636	76.2559	3.2904	13.8773
370	29.2737	46.1529	72.5923	93.5013	11.9547	22.4895
0	431.0008	231.7977	571.0274	287.9179	262.2573	144.8736

4.4 Summary and discussion

This Chapter addresses the issue of uncertainties in the interdependent input-output framework and their effect on risk management. There is a need to consider such uncertainties because they will produce varying planning risks some of which would



(a) Economic losses vs Allocated budgets



(b) Net benefit vs Allocated budgets

Figure 4.2: Trade-offs between investments in losses for different levels of budgeted data uncertainties and also event uncertainties. The bold line with circles is the maximal robust solution under the available budgets

not be captured through the nominal planning strategies. Uncertainties manifest themselves in several ways and can be analyzed through various approaches. Since the planning objective is to guarantee results that consider the maximal effects of the uncertainties the solutions we seek to the risk management problem should be representative of the worst-case/best-case scenarios in the planning. This leads to the robust optimization approach, which is a useful construct for extreme uncertainty analysis.

In order to construct robust solutions to the static resilience strengthening and estimation problem the nature of the uncertainties present in the system need to be considered. In particular the mathematical properties of the uncertainty sets are important to us because they influence the structure of robust formulation. We have divided the uncertainties into data and event uncertainties to construct the uncertainty sets. Data is assumed to be estimated within an interval of accuracy, which means that the data uncertainty sets are bounded sets. The actual realization of the data values is controlled within the bounded sets by imposing budget limits that that control the amount of uncertainty within the prescribed interval. The budgets are also controlled through the considerations of the feasibility of the problem as is the case with the interdependency structures because there is a limit to which the interdependency matrix remains viable. For event uncertainties the introduction of chance constraints means that due to the uncertainty of the events the feasibility of the planning constraints is not known with complete certainty. The limits which give the bound the probabilities that the constraints are feasible or are violated shows the decision-makers preference and conservatism in handling the event uncertainty. Here we have introduced a bit conservative bound on the event chance constraint.

The robust formulation that is constructed here increases the size of the nominal problem significantly but the structure of the robust counterpart is the same as the nominal problem. The constraints containing the data were linear in the original problem and that linearity is preserved in the robust formulation. Also the chance constraints for event uncertainties have similar structures to the nominal problem, although that happens due to simplified nature of the planning functions. The overall robust formulation is not too complex to solve.

The static resilience planning results show the effect of small uncertainties in the solutions. In general the robust problem results show that small uncertainties in the data and event estimates make huge differences in the amounts of required resource allocations for keeping the total economic losses below accepted thresholds. As such the usefulness of the robust schemes is validated through the severity of the impact of small uncertainties in the problem. The robust solution guarantee that maximal worst-case scenarios are generated and for planning this is an important consideration.

Chapter 5

Forward Sensitivity Based Parameter Estimation in the Dynamic Risk Input-Output Model

5.1 Introduction

The dynamic risk input-output model presented in Chapters 2 and 3 was shown to be a useful tool in risk evaluation and in particular resilient recovery estimation. Resilience through the model has been explicitly expressed through a matrix, which we call $\mathbf{K}^*$, for which data or estimates do not exist. This Chapter proposes a scheme for obtaining the $\mathbf{K}^*$ by setting targets for the dynamic recovery that provide a feedback to estimate the model parameters.

In current research $\mathbf{K}^*$ has been modeled as a diagonal matrix in which each diagonal element is obtained from a recovery decision made for the particular sector it represents (Lian & Haimes, 2006). The general interpretation associated with the resilience matrix is that it represents a recovery rate during direct supply disruptions to sectors (Haimes et al., 2005b) or a substitution rate during demand disruptions (Haimes et al., 2005a). We argued in Chapter 3 that using the diagonal matrix alone as a resilience metric is an incomplete and inaccurate analysis through the model. Also, having a diagonal matrix that is derived based on the individual sector planning only does not reflect the interdependent effects of resilience planning in the economy. Interdependent systems are coupled so any parameter in the model should account for such coupling. It has been argued that the current dynamic risk input-output model does not account for resilience satisfactorily (Rose, 2007).

The three resilience metrics we proposed in Section 3.3.2 can be used provide a more complete picture of the resilient recovery through the dynamic risk input-output model. Since these metrics were derived from the model, we need to have all the model parameters, which brings us back to the problem that we do not know the $\mathbf{K}^*$ elements from any data or event *a prior*. The resilience metrics, called the average level of system operability (F), time to recovery (τ) and maximum inoperability (q^m), can be treated as performance metrics for evaluating the model performance. Decision-makers' interest is to make sure that systems perform within specific desirables. An example of desirables for evaluating the performance of sector i from initial impact ($q(t = 0) = 0.3$) till long run recovery ($T = 60$ days) could be: (1) $0.95 \leq F_i \leq 1$ - On an average the sector maintains at least 95% functionality in the long run; (2) $\tau_i \approx 30$ - The sector is able to almost attain stable productivity after 30 days; (3) $q_i^m \leq 0.4$ - At the most the sector loses 40% productivity before making a recovery.

Setting targets for performance metrics provides a benchmarking for the model. Such information can be utilized as a feedback to the model for setting values for the $\mathbf{K}^*$ elements. We desire sectors to have certain levels of resilience quantified through the metrics we defined. The relevant resilience performance questions for the estimation of $\mathbf{K}^*$ are:

> What should our estimates of $\mathbf{K}^*$ elements be so that we can take the system to a targeted level of recovery for a specified time? What should our estimates of $\mathbf{K}^*$ elements be so that we make sure that the system has a desired average level of operability performance target during the entire time of recovery?

Resilience has been expressed as "the ability to efficiently reduce both the magnitude and duration of the deviation from targeted system performance levels" (Vugrin et al., 2010). This is consistent with the approach we adopt here because in the absence of any prior knowledge of our system parameter $\mathbf{K}^*$, we estimate it through targeted knowledge of the physical and mathematical properties of the system. This approach is called the *inverse problem* and is used widely in feedback control systems research (Franklin et al., 1994; Lewis et al., 2006; Lakshmivarahan & Lewis, 2010).

The inverse problem and the parameter estimation scheme is subsequently explained in the Chapter. Section 5.2 outlines the mathematical statement of two inverse problems, one based on setting targets for the recovery metric τ, while the other based on setting targets for the performance metric F. It is a more complete metric for performance evaluation because it includes other metrics also. Section 5.3.1 explains and derives the formulation and algorithmic scheme, called the forward sensitivity method (Lakshmivarahan & Lewis, 2010), for solving the inverse problem. The algorithm is a least square fit of the model to the targeted data in which sensitivity functions lead to corrections in control estimates. The computations of sensitivity matrices required to solve the inverse problem are explained in Section 5.3.2. These computations also highlight the forward sensitivity scheme's computational tractability in solving the inverse problem. Section 5.3.3 examines the significance of the method and computational issues that limit the solution scheme. Section 5.4 suggests a scheme based on system planning for the generation the the performance metrics that help evaluate the model parameter. Section 5.5 explains the concepts developed through a numerical example. A summary and discussion of the topics presented in this Chapter is provided in Section 5.6.

5.2 Mathematical statement of the inverse problem

The n sector dynamic risk input-output model of Equation (2.33) describes a system of first-order differential equations. We restate the model here to note certain properties necessary for solving the inverse problem.

$$\dot{\mathbf{q}}(t) = -\mathbf{K}^*(\mathbf{I}_n - \mathbf{A}^*)\mathbf{q}(t) + \mathbf{K}^*\mathbf{c}^*(t), \forall t \tag{5.1}$$

where $\mathbf{q}$ is the $n \times 1$ inoperability vector, $\mathbf{c}^*(k)$ are $n \times 1$ vectors for demand perturbations, $\mathbf{A}^*$ is an $n \times n$ interdependency matrix, $\mathbf{K}^*$ is the $n \times n$ rate parameter matrix, and $\mathbf{I}_n$ is an $n \times n$ identity matrix. It is assumed that the given model produces a deterministic forecast for $\mathbf{q}(t)$. The formulation suggests that a solution $\mathbf{q}(t)$ of the model exists and in unique for a given set of initial conditions $\mathbf{q}(0)$, external forcing $\mathbf{c}^*(t)$ and parameters $\mathbf{K}^*, \mathbf{A}^*$. Also, we see that the first partial derivatives of the $\mathbf{q}(t)$ with respect to any of the variables exists. The dynamic discrete-time version of the model from Equation (2.34) is also restated and would henceforth be used for developing the inverse problem.

$$\mathbf{q}(k+1) = \left[\mathbf{I}_n - \mathbf{K}^*(\mathbf{I}_n - \mathbf{A}^*)\right]\mathbf{q}(k) + \mathbf{K}^*\mathbf{c}^*(k), \quad \forall k = 0, 1, 2, ... \tag{5.2}$$

As established previously the initial condition $\mathbf{q}(0)$ is given and is bounded between 0 and 1. Equation (5.2) can also be expressed explicitly in terms of the initial conditions as (Barker, 2008)

$$\begin{aligned} \mathbf{q}(k+1) &= \left[\mathbf{I}_n - \mathbf{K}^*(\mathbf{I}_n - \mathbf{A}^*)\right]^{k+1}\mathbf{q}(0) \\ &+ \sum_{r=0}^{k} \left[\mathbf{I}_n - \mathbf{K}^*(\mathbf{I}_n - \mathbf{A}^*)\right]^{r}\mathbf{K}^*\mathbf{c}^*(k-r) \quad \forall k = 0, 1, 2, ... \end{aligned} \tag{5.3}$$

Equation (5.3) shows that the evolution of inoperability at any time-step k depends upon the initial inoperability $\mathbf{q}(0)$, the demand perturbations $\mathbf{c}^*(r)$ at all previous time-steps, the matrices $\mathbf{A}^*$ and $\mathbf{K}^*$. These parameters are referred to as system controls.

As is evident from Equation (5.3), $\mathbf{q}(0)$ and $\mathbf{c}^*(r)$ are external control inputs, to which the system responds. The elements of $\mathbf{q}(0)$ represent measures of initial impacts on sectors due to disruptions. If disruptive events have a direct impact on economic sectors then we can obtain data-based estimates of the initial inoperabilities. For economic sector i, the value of $q_i(0)$ can be controlled through inventory management strategies (Barker & Haimes, 2009; Barker & Santos, 2010a). Similarly, demand perturbations are exogenous terms that depend upon responses to external stimuli. For a disruptive event, values of demand perturbations can be obtained from data of the impact or could be based on expert elicitation. In this study, we assume that $\mathbf{q}(0)$ and $\mathbf{c}^*(r)$ estimates come from data or models that need to be combined with the Equation (5.2) model.

From Equation (5.3) we also see that $\mathbf{K}^*$ and $\mathbf{A}^*$ reflect internal controls that determine the trajectory of inoperability. In Chapter 3 we established that these controls can be associated with redundancies and resourcefulness of sector resilience behavior. As mentioned previously, $\mathbf{A}^*$ can be obtained through the data (BEA., 2011). Hence, any form of system control would be incorporated through estimating $\mathbf{K}^*$, which is not known to us.

Two metrics derived from the dynamic risk input-output model are the time to recovery and average level of operability, which for a particular sector i are respectively given as

$$\tau_i = \left\{k : k > 0, |q_i(k) - q_i^e| \leq \epsilon\right\} \iff \tau = \left\{k : k > 0, |\mathbf{q}(k) - \mathbf{q}^e| \leq \boldsymbol{\epsilon}\right\} \tag{5.4}$$

$$F_i = 1 - \frac{1}{T}\sum_{k=0}^{T} q_i(k) \iff \mathbf{F} = \mathbf{1} - \frac{1}{T}\sum_{k=0}^{T} \mathbf{q}(k) \tag{5.5}$$

τ_i indicates the time it takes the disrupted system to recover to acceptable levels of functionality given by q_i^e, and F_i represents the average level of functionality that is maintained by the sector during the time frame T when recovery is analyzed. τ can be considered to indicate an overall recovery for the entire n sector system, while $\mathbf{F}$ is the $n \times 1$ vector quantifying average operability of all n sectors. These can be considered to represent performance metrics that indicate a resilience properties of the system.

As we do not know the values in the $\mathbf{K}^*$ matrix, we cannot readily estimate the above performance metrics. On the other hand there are physical meanings attached to these metrics, which can help us develop an estimate for them. For example saying that the disrupted sector recovers in 30 days means we know that $\tau_i = 30$ with q_i^e having value indicative of such recovery. Similarly saying that during recovery the sector is able to function at 90% capacity on an average means $F_i = 0.9$. Assuming that such estimates can be inferred they can be used to compute the model parameters. This is the inverse problem or a feedback problem. The estimated values for the performance metrics can be called 'observations' that help us calibrate the model.

If, for a sector, the value for one of the chosen performance metrics is decided then it is known that this target value can be achieved through the model with suitable values assigned to the parameters. If, for the chosen performance metric for sector i, z_i^{tar} denotes the observation value and z_i^{mod} denotes the true value given by the model then it can be assumed that

$$z_i^{tar} = z_i^{mod} + \nu_i \tag{5.6}$$

where $\nu_i \sim N(0, \sigma_i^2)$ is a random Gaussian unavoidable error that exists due to imprecise information in estimating the observation. This also shows some uncertainty that always exists in estimating the targeted value for the performance metric.

The inverse problem is an parameter estimation problem where it is desired that the model and observation are as close to each other as possible. The least squares criterion, which finds the best fit between modeled and observed values, is the obvious choice for solving the inverse problem. Equation (5.7) expresses the least squares problem, which is a mathematical statement of the inverse problem.

$$\min \left\{ J = \sum_{i=1}^{n} \frac{1}{\sigma_i^2} \left(z_i^{tar} - z_i^{mod} \right)^2 = \left(\mathbf{z}^{tar} - \mathbf{z}^{mod} \right)^{\mathsf{T}} \mathbf{W} \left(\mathbf{z}^{tar} - \mathbf{z}^{mod} \right) \right\} \tag{5.7}$$

where the $n \times n$ matrix $\mathbf{W} = \text{diag}(1/\sigma_i^2)$ in general represents the weights given to each target, $\mathbf{z}^{tar}$ is an $n \times 1$ vector of target values for all sectors and $\mathbf{z}^{mod}$ is an $n \times 1$ vector of all sector performance metric values given by the model.

Since z_i^{mod} is obtained from the model it is a function of the state variable $q_i(k)$, which can be represented as $z_i^{mod} = h_i(q_i(k))$ or $\mathbf{z}^{mod} = \mathbf{h}(\mathbf{q}(k))$. Using $z_i = z_i^{tar}$ and $\mathbf{z} = \mathbf{z}^{tar}$ for further notation the Equation (5.6) observation-model relationship is expressed as

$$z_i = h_i(q_i(k)) + \nu_i \tag{5.8}$$

As established $q_i(k)$ is a function of $\mathbf{K}^*$ among other parameters. Since the goal is to find the parameter $\mathbf{K}^*$ that provides the best model fit for the given observation Equation (5.7) is written as

$$\min_{\mathbf{K}^*} \left\{ J(\mathbf{K}^*) = \sum_{i=1}^{n} \frac{1}{\sigma_i^2} \big[z_i - h_i(q_i(k))\big]^2 = \big[\mathbf{z} - \mathbf{h}(\mathbf{q}(k))\big]^\mathsf{T} \mathbf{W} \big[\mathbf{z} - \mathbf{h}(\mathbf{q}(k))\big] \right\} \tag{5.9}$$

Here it is noted that the cost function J is an implicit function of $\mathbf{K}^*$ since the choice of $\mathbf{K}^*$ determines how well a fit is achieved.

The general formulation developed applies to any of the resilience metrics shown in Equations (5.4) and (5.5). If the time to recovery metric, τ is used as a performance metric then we are planning for the time when the economic sectors have inoperabilities which indicate stability. A planning decision can be made about deciding the time at which such stability should exist. Setting a target value for τ means a decision is made to attain stability from τ onwards. Stability means the system is able to reach an equilibrium from initial impact. The value that denotes stability depends upon the type of external stimulus to the system. Hence if it is assumed that we know the targeted equilibrium inoperability $\mathbf{z}_\tau$ at $\boldsymbol{\tau}$ then, $\mathbf{h}_\tau(\mathbf{q}(t)) = \mathbf{q}(\tau)$, and the inverse problem is given by the following set of equations

$$\begin{array}{ll} \text{Model space:} & \mathbf{q}(k+1) = \big[\mathbf{I}_n - \mathbf{K}^*(\mathbf{I}_n - \mathbf{A}^*)\big]\mathbf{q}(k) + \mathbf{K}^*\mathbf{c}^*(k) \\ \text{Observation space:} & \mathbf{z}_\tau = \mathbf{q}(\boldsymbol{\tau}) + \boldsymbol{\nu}_\tau \\ \text{Inverse problem:} & \min_{\mathbf{K}^*}\left\{J(\mathbf{K}^*) = \big[\mathbf{z}_\tau - \mathbf{q}(\boldsymbol{\tau})\big]^\intercal \mathbf{W}\big[\mathbf{z}_\tau - \mathbf{q}(\boldsymbol{\tau})\big]\right\} \end{array} \tag{5.10}$$

Similarly the overall average level of operability metric, $\mathbf{F}$ can also be used for performance evaluation. $\mathbf{F}$ is a more complete metric for performance that can be used to find $\mathbf{K}^*$ because it captures the most likely system behavior, which is averaged over every time-step. The Equation (3.37) representation for $\mathbf{F}$ is restated here in discrete form to show that quantifying $\mathbf{F}$ leads to estimating time to recovery and equilibrium inoperability, which gives a more complete picture of system performance.

$$\mathbf{F} = \mathbf{1} - \big(1 - \frac{\tau}{T}\big)\mathbf{q}^e - \frac{1}{T}\sum_{k=0}^{k=\boldsymbol{\tau}} \mathbf{q}(k)dt \tag{5.11}$$

Instead of $\mathbf{F}$ we use $\bar{\mathbf{F}}$ given by Equation (5.12) for setting a observational space model for which the target value is set at $\mathbf{z}_{\bar{F}}$. $\bar{\mathbf{F}}$ in fact represents an average loss of operability measure.

$$\bar{\mathbf{F}} = \mathbf{1} - \big(1 - \frac{\tau}{T}\big)\mathbf{q}^e - \mathbf{F} \tag{5.12}$$

The observational space model, which comes from Equation (5.11), is obtained as

$$\mathbf{h}_{\bar{F}}(\mathbf{q}(k)) = \frac{1}{T}\sum_{k=0}^{\tau} \mathbf{q}(k) \tag{5.13}$$

Hence, the inverse problem in this case is represented as

$$\begin{array}{ll} \text{Model space:} & \mathbf{q}(k+1) = \big[\mathbf{I}_n - \mathbf{K}^*(\mathbf{I}_n - \mathbf{A}^*)\big]\mathbf{q}(k) + \mathbf{K}^*\mathbf{c}^*(k) \\ \text{Observation space:} & \mathbf{z}_{\bar{F}} = \frac{1}{T}\sum_{k=0}^{\tau}\mathbf{q}(k) + \boldsymbol{\nu}_{\bar{F}} \\ \text{Inverse problem:} & \min_{\mathbf{K}^*}\left\{J(\mathbf{K}^*) = \big[\mathbf{z}_{\bar{F}} - \mathbf{h}(\mathbf{q}(k))\big]^{\intercal}\mathbf{W}\big[\mathbf{z}_{\bar{F}} - \mathbf{h}(\mathbf{q}(k))\big]\right\} \end{array} \tag{5.14}$$

Having set up the inverse problem our goal is to find the optimal $\mathbf{K}^*$ that solves systems in Equations (5.10) or (5.14). In the next sections we outline the general solution scheme that estimates such an optimal $\mathbf{K}^*$.

5.3 Solution scheme for the inverse problem

5.3.1 First-order forward sensitivity method

In order to solve the minimization problem in Equation (5.9) the computation of the gradient of J with respect to $\mathbf{K}^*$ is calculated and equated to zero. This gradient, denoted by $\nabla_{\mathbf{K}^*}J$ is a non-linear function of $\mathbf{K}^*$, which makes it difficult to get a value for $\mathbf{K}^*$ by simply calculating the gradient and equating it to zero. A method based on calculating the first-order directional derivative with respect to $\mathbf{K}^*$ is employed here to approach the optimal $\mathbf{K}^*$ through the $\delta\mathbf{K}^*$ incremental improvements following an initial estimate. In the subsequent development of a solution for the problem we will transform the $n \times n$ matrix $\mathbf{K}^*$ into an $n^2 \times 1$ vector $\mathbf{k}^*$, which is obtained by stacking the columns of $[\mathbf{K}^*]^{\intercal}$ into a column vector.

Equation (5.2) is a state space equation in which $\mathbf{q}(k)$ denotes the state of the system at time-step k. An initial guess can be made about the elements of $\mathbf{k}^*$ to obtain a value of $\mathbf{q}(k)$ that would satisfy Equation (5.8). But this value of sector inoperability calculated through the model is different from the true value. As such there will be an error in prediction of the observations. This error is called the forecast error (Lewis et al., 2006; Lakshmivarahan & Lewis, 2010), because it measures the amount by which the model is unable to match the observations. The forecast error $\mathbf{e}$ at time-step k can be defined as

$$\mathbf{e} \equiv \mathbf{z} - \mathbf{h}(\mathbf{q}(k)) = \mathbf{f}(\mathbf{k}^*) + \boldsymbol{\nu} \tag{5.15}$$

where $\mathbf{f}(\mathbf{k}^*)$ is a deterministic error induced by the incorrect control variables. We are interested in finding corrections $\delta\mathbf{k}^*$ such that the model forecast errors $\mathbf{e}$ updated to $\mathbf{e}_N = \mathbf{e} + \delta\mathbf{e}$ are purely random, that is $\mathbb{E}[\mathbf{e}_N] = 0$.

Using the first-order variational analysis approach (Lakshmivarahan & Lewis, 2010), the problem of improving the model forecast is tackled by improving the parameter via small increments and looking at the first variation effects of such increments on the model and observation functions. Figure 5.2 shows the schematic of the solution approach being adopted here. A $\delta\mathbf{k}^*$ change in the controls will induce a change

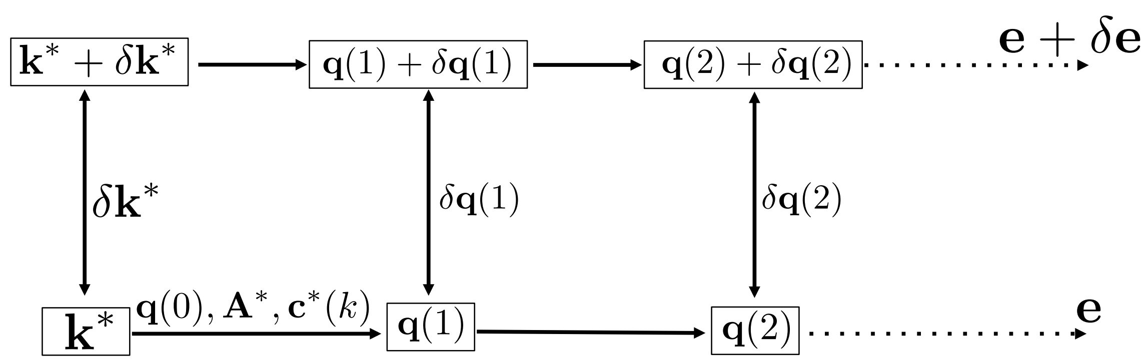


Figure 5.1: First-order variational analysis showing the increment of the parameter from a base case to a perturbed case, which results in the increment of the model values and therefore the error estimates

$\delta\mathbf{q}(k)$ in the model, which gives

$$\mathbf{e}_N = \mathbf{e} + \delta\mathbf{e} = \mathbf{z} - \mathbf{h}(\mathbf{q}(k) + \delta\mathbf{q}(k)) \tag{5.16}$$

From the first-order Taylor's expansion we can obtain

$$\mathbf{e}_N = \mathbf{e} - \mathbf{D}_{\mathbf{q}(k)}(\mathbf{h})\delta\mathbf{q}(k)) \tag{5.17}$$

where $\mathbf{D}_{\mathbf{q}(k)}(\mathbf{h}) = [\partial h_i/\partial q_j(k)]$ denotes the $n \times n$ Jacobian of $\mathbf{h}$ with respect to $\mathbf{q}(k)$. Further the change $\delta\mathbf{q}(k)$ is due to the perturbation of $\mathbf{k}^*$ by $\delta\mathbf{k}^*$. Using the Taylor's series expansion the first variation of this induced change can be expressed as

$$\delta\mathbf{q}(k) = \mathbf{D}_{\mathbf{k}^*}(\mathbf{q}(k))\delta\mathbf{k}^* \tag{5.18}$$

where $\mathbf{D}_{\mathbf{k}^*}(\mathbf{q}(k)) = [\partial q_i(k)/\partial k^{*'}_{ij}]$ is the $n \times n^2$ matrix for the Jacobian of $\mathbf{q}(k)$ with respect to $\mathbf{k}^*$ and represents the first-order sensitivity of the state with respect to the $\mathbf{k}^*$ vector. Combining Equation (5.17) and (5.18) we get

$$\begin{aligned} \mathbf{e}_N &= \mathbf{e} - \mathbf{D}_{\mathbf{q}(k)}(\mathbf{h})\mathbf{D}_{\mathbf{k}^*}(\mathbf{q}(k))\delta\mathbf{k}^* \\ &= \mathbf{e} - \mathbf{H}\delta\mathbf{k}^* \end{aligned} \tag{5.19}$$

where $\mathbf{H} = \mathbf{D}_{\mathbf{q}(k)}(\mathbf{h})\mathbf{D}_{\mathbf{k}^*}(\mathbf{q}(k))$. The perturbation that removes the systematic error from the forecast error satisfies

$$\mathbb{E}[\mathbf{e}_N] = 0 \implies \mathbf{e} = \mathbf{H}\delta\mathbf{k}^* \tag{5.20}$$

The above system is a linear least-squares problem, in which $\mathbf{e}$ is an $n \times 1$ vector, $\mathbf{H}$ is an $n \times n^2$ matrix and $\delta\mathbf{k}^*$ is an $n^2 \times 1$ vector. It is an under-determined system where the number of unknowns are more than the number of equations. Hence, the solution to this problem is obtained by minimizing the Tikhonov regularization function (Tikhonov et al., 1977; Groetsch, 1984)

$$\min_{\delta\mathbf{k}^*}\left\{\bar{J}(\delta\mathbf{k}^*) = \frac{1}{2}(\mathbf{e}-\mathbf{H}\delta\mathbf{k}^*)^\intercal\mathbf{W}(\mathbf{e}-\mathbf{H}\delta\mathbf{k}^*) + \frac{\mu}{2}(\delta\mathbf{k}^*)^\intercal(\delta\mathbf{k}^*)\right\} \tag{5.21}$$

where μ is a regularization constant. The solution of this linear least-squares problem can be found by directly equating the Jacobian $\nabla_{\delta\mathbf{k}^*}\bar{J} = 0$, which is now linear in $\delta\mathbf{k}^*$.

$$\begin{aligned}\nabla_{\delta\mathbf{k}^*}\bar{J} &= -\mathbf{H}^\intercal\mathbf{W}\mathbf{e} + (\mathbf{H}^\intercal\mathbf{W}\mathbf{H} + \mu\mathbf{I}_{n^2})\delta\mathbf{k}^* = 0 \\ \Longrightarrow \delta\mathbf{k}^* &= (\mathbf{H}^\intercal\mathbf{W}\mathbf{H} + \mu\mathbf{I}_{n^2})^{-1}\mathbf{H}^\intercal\mathbf{W}\mathbf{e}\end{aligned} \tag{5.22}$$

where $\mathbf{I}_{n^2}$ is an $n^2 \times n^2$ identity matrix. With the above calculated value for $\delta\mathbf{k}^*$ the updated error estimate of Equation (5.19) can be obtained to see how much improvement has been made in estimating the discrepancy between the model and the observation. If this improvement is satisfactory then the new value $\mathbf{k}^* + \delta\mathbf{k}^*$ indicates the value for the $\mathbf{K}^*$ that satisfies the targeted system behavior. Otherwise we can further improve the estimate by again calculating the next perturbation $\delta\mathbf{k}^*$ that is solved through the process outlined from Equation (5.15) to (5.22) above. Table 5.1 shows the iterative algorithm through which the optimal increment leading to the optimal $\mathbf{K}^*$ is obtained.

The analysis approach, called the forward sensitivity method (FSM) (Lakshmivarahan & Lewis, 2010), is useful for obtaining better estimates to the system and we know the sensitivity evolution of the state space in terms of the control parameters.

5.3.2 Computing the sensitivity functions

The solution of the forward sensitivity methods relies on finding the matrix $\mathbf{H}$ from Equation (5.19). For the two inverse problems defined in Equations (5.10) and (5.14)

Table 5.1: First-order sensitivity algorithm for solving the inverse problem

Given:	$\mathbf{z}, \mathbf{h}, \mathbf{W}, \mathbf{A}^*, \mathbf{q}(0), \mathbf{c}^*(k), \forall k, \mu$
Set:	$tol << 1$
Step 1:	Initial guess for $\mathbf{K}^*$
Step 2:	Find $\mathbf{e} = \mathbf{z} - \mathbf{h}(\mathbf{q}(k))$
Step 3:	If $\mathbf{e} \leq tol$; STOP
	else
Step 4:	Find $\mathbf{H} = \mathbf{D}_{\mathbf{q}(k)}(\mathbf{h})\mathbf{D}_{\mathbf{k}^*}(\mathbf{q}(k))$
Step 5	Solve $\delta\mathbf{k}^* = (\mathbf{H}^\intercal\mathbf{W}\mathbf{H} + \mu\mathbf{I}_{n^2})^{-1}\mathbf{H}^\intercal\mathbf{W}\mathbf{e}$
Step 6:	Update:
	$\mathbf{k}^* \to \mathbf{k}^* + \delta\mathbf{k}^*$
	$\mathbf{e}_N = \mathbf{e} - \mathbf{H}\delta\mathbf{k}^*$
Step 7:	If $\mathbf{e}_N \leq tol$; STOP
	else
Step 8:	$\mathbf{e} = \mathbf{e}_N$; GOTO Step 4

the specific expressions for $\mathbf{H}$ provides more insight into the forms of the sensitivity functions that generate the forward sensitivity calculations. Hence, we are interested in finding the two matrices $\mathbf{H}_\tau$ and $\mathbf{H}_{\bar{F}}$ given as

$$\mathbf{H}_\tau = \mathbf{D}_{\mathbf{q}(\tau)}(\mathbf{q}(\tau))\mathbf{D}_{\mathbf{k}^*}(\mathbf{q}(\tau)) \quad (5.23)$$

$$\mathbf{H}_{\bar{F}} = \mathbf{D}_{\mathbf{q}(k)}(\mathbf{h}_{\bar{F}})\mathbf{D}_{\mathbf{k}^*}(\mathbf{q}(k)) \quad (5.24)$$

In this section we provide the expressions from calculating the required Jacobian matrices, which reflect the forward sensitivities with respect to model parameters. Before proceeding we express the sector-wise temporal inoperability, which is needed for sensitivity calculations. From Equation (5.2), the sector i inoperability at time-step $k+1$ becomes

$$\begin{aligned} q_i(k+1) &= q_i(k) - \sum_{j=1}^{n} k_{ij}^{*'} q_j(k) + \sum_{j=1}^{n}\left(\sum_{r=1}^{n} k_{ir}^{*'} a_{rj}^*\right) q_j(k) + \sum_{j=1}^{n} k_{ij}^{*'} c_j^*(k) \\ &= G_i(\mathbf{K}^*, \mathbf{A}^*, \mathbf{q}(k), \mathbf{c}^*(k)) \end{aligned} \quad (5.25)$$

Calculations for $\mathbf{D}_{\mathbf{q}(k)}(\mathbf{h})$ depends upon the functional form of $\mathbf{h}$. When the observation space is defined by the Equation (5.10) then

$$\mathbf{D}_{\mathbf{q}(k)}(\mathbf{h}) = \mathbf{D}_{\mathbf{q}(\tau)}(\mathbf{q}(\tau)) = \mathbf{I}_n \tag{5.26}$$

For the observational space given by Equation (5.13) the expression for $\mathbf{D}_{\mathbf{q}(k)}(\mathbf{h})$ is

$$\begin{aligned}
\mathbf{D}_{\mathbf{q}(k)}(\mathbf{h}) &= \mathbf{D}_{\mathbf{q}(k)}\left(\mathbf{1} - \frac{1}{T}\sum_{k=0}^{T}\mathbf{q}(k)\right) \\
&= -\frac{1}{T}\sum_{k=0}^{T}\mathbf{D}_{\mathbf{q}(k)}(\mathbf{q}(k)) \\
&= -\frac{1}{T}\sum_{k=0}^{T}\mathbf{I}_n
\end{aligned} \tag{5.27}$$

$\mathbf{D}_{\mathbf{k}^*}(\mathbf{q}(k))$ are obtained from a recursive computation of the forward sensitivities of $\mathbf{q}(k+1)$ in dynamic risk input-output model Equation (5.2). At any time step we are interested in the following forward sensitivity matrices

$$\mathbf{D}_{\mathbf{k}^*}(\mathbf{q}(k+1)) = \mathbf{U}(k+1) = \left[u_{ij}(k+1)\right] = \begin{bmatrix} \frac{\partial q_1(k+1)}{\partial k^*_{11}} & \cdots & \frac{\partial q_1(k+1)}{\partial k^*_{nn}} \\ \vdots & \ddots & \vdots \\ \frac{\partial q_n(k+1)}{\partial k^*_{11}} & \cdots & \frac{\partial q_n(k+1)}{\partial k^*_{nn}} \end{bmatrix} \tag{5.28}$$

For calculating the forward sensitivities, we will need the Jacobian of G_i from Equation (5.25) with respect to $q_l(k)(l \in \{1, 2, ..., n\})$ and $k^{*'}_{pj}(p, j \in \{1, 2, ..., n\})$

$$\begin{aligned}
\frac{\partial G_i(k)}{\partial q_l(k)} &= \frac{\partial\big[q_i(k) - \sum_{j=1}^{n} k^{*'}_{ij} q_j(k) + \sum_{j=1}^{n}\Big(\sum_{r=1}^{n} k^{*'}_{ir} a^*_{rj}\Big) q_j(k) + \sum_{j=1}^{n} k^{*'}_{ij} c^*_j(k)\big]}{\partial q_l(k)} \\
&= \Big[\delta_{il} - k'^*_{il} + \sum_{r=1}^{n} k'^*_{ir} a^*_{rl}\Big] \qquad (5.29) \\
\frac{\partial G_i(k)}{\partial k^{*'}_{pj}} &= \frac{\partial\big[q_i(k) - \sum_{s=1}^{n} k^{*'}_{is} q_s(k) + \sum_{s=1}^{n}\Big(\sum_{r=1}^{n} k^{*'}_{ir} a^*_{rs}\Big) q_s(k) + \sum_{s=1}^{n} k^{*'}_{is} c^*_s(k)\big]}{\partial k^{*'}_{pj}} \\
&= 1_{p((p-1)\times n+j)}\Big[- q_j(k) + \sum_{s=1}^{n} a^*_{js} q_s(k) + c^*_j(k)\Big] \qquad (5.30)
\end{aligned}$$

where 1_{pg} is an indicator function that is equal to 1 at the (p, g) element of the matrix and 0 otherwise. Using Equation (5.28) and Equation (5.30) the calculations of the forward sensitivities are as follows:

$$
\begin{aligned}
\frac{\partial q_i(k+1)}{\partial k_{pj}^{*'}} &= \sum_{l=1}^{n} \frac{\partial q_i(k+1)}{\partial q_l(k)} \frac{\partial q_l(k)}{\partial k_{pj}^{*'}} + \frac{\partial G_i(k)}{\partial k_{pj}^{*'}} \\
&= \sum_{l=1}^{n} \frac{\partial G_i(k)}{\partial q_l(k)} \frac{\partial q_l(k)}{\partial k_{pj}^{*'}} + \frac{\partial G_i(k)}{\partial k_{pj}^{*'}} \\
&= \sum_{l=1}^{n} \Big[\delta_{il} - k_{il}^{'*} + \sum_{r=1}^{n} k_{ir}^{'*} a_{rl}^{*}\Big] \frac{\partial q_l(k)}{\partial k_{pj}^{*'}} \\
&+ 1_{p((p-1)\times n+j)} \Big[- q_j(k) + \sum_{s=1}^{n} a_{js}^{*} q_s(k) + c_j^{*}(k) \Big] \\
\Longrightarrow \begin{bmatrix} & \vdots & \\ \cdots & u_{ij}(k+1) & \cdots \\ & \vdots & \end{bmatrix} &= \begin{bmatrix} \frac{\partial G_i(k)}{\partial q_1(k)} & \cdots & \frac{\partial G_i(k)}{\partial q_l(k)} & \cdots \frac{\partial G_i(k)}{\partial q_n(k)} \end{bmatrix} \begin{bmatrix} u_{1j}(k) \\ \vdots \\ u_{lj}(k) \\ \vdots \\ u_{nj}(k) \end{bmatrix} \\
&+ 1_{p((p-1)\times n+j)} \Big[- q_j(k) + \sum_{s=1}^{n} a_{js}^{*} q_s(k) + c_j^{*}(k) \Big] \\
\Longrightarrow \mathbf{U}(k+1) &= \big[\mathbf{I}_n - \mathbf{K}^{*}(\mathbf{I}_n - \mathbf{A}^{*})\big]\mathbf{U}(k) \\
&+ \mathbf{T}(k), \quad \mathbf{U}(0) = \mathbf{0} \qquad (5.31)
\end{aligned}
$$

where $\mathbf{T}(k)$ is the $n \times n^2$ matrix of the $\partial G_i(k)/\partial k_{pj}^{*'}$ terms defined in the equations above. From the above expressions spanning Equations (5.23) to (5.31) the expressions for the required $\mathbf{H}$ matrices are given as

$$
\mathbf{H}_{\tau} = \mathbf{I}_n \mathbf{U}(\tau) \qquad (5.32)
$$

$$
\mathbf{H}_{\bar{F}} = -\frac{1}{T} \sum_{k=0}^{T} \mathbf{I}_n \mathbf{U}(k) \qquad (5.33)
$$

5.3.3 Importance of and issues with forward sensitivities

In general using the the inverse problem for parameter estimation is a useful scheme because it shows the fidelity of the model to actual data/observations. The approach is an online method in which the parameter estimate can be improved and updated as more data is available to us. Hence, the method leads to reliable estimates of model parameters.

Equations (5.32) and (5.33) suggest that the calculations of the matrices $\mathbf{H}_\tau$ and $\mathbf{H}_{\bar{F}}$ are based on obtaining $\mathbf{D}_{\mathbf{k}^*}(\mathbf{q}(k))$, which is the state variable sensitivity with respect to the control parameter. This further shows that the gradient of the function $\bar{J}$ in Equation (5.22) can be directly interpreted in terms of the sensitivities. Also the updated forecast error $\mathbf{e}_N$ is shown to be dependent on the forward sensitivities in Equation (5.19), which means that the structure of the forecast errors can be interpreted in terms of sensitivity calculations. Hence, the direction of the gradient and the error updates can be understood in terms of the sensitivity calculations using the method we presented here. Such analysis insights make the FSM a useful method in solving the inverse problem.

Substituting $\mathbf{M} = \big[\mathbf{I_n} - \mathbf{K}^*(\mathbf{I_n} - \mathbf{A}^*)\big]$, the expression for the forward sensitivity of Equation (5.31) can be expanded as a series of $\mathbf{M}$ and $\mathbf{T}(k), \forall k$ as shown in Equation (5.34)

$$
\begin{aligned}
\mathbf{U}(k+1) &= \mathbf{M}\mathbf{U}(k) + \mathbf{T}(k) \\
&= \mathbf{M}\big[\mathbf{M}\mathbf{U}(k-1) + \mathbf{T}(k-1)\big] + \mathbf{T}(k) \\
&= \mathbf{M}^2\big[\mathbf{M}\mathbf{U}(k-2) + \mathbf{T}(k-2)\big] + \mathbf{M}\mathbf{T}(k-1) + \mathbf{T}(k) \\
&\vdots \\
&= \mathbf{M}^k\mathbf{T}(0) + \mathbf{M}^{k-1}\mathbf{T}(1) + \cdots + \mathbf{M}\mathbf{T}(k-1) + \mathbf{T}(k)
\end{aligned} \tag{5.34}
$$

$\mathbf{M}$ is the state matrix of the dynamic risk input-output model and it was established in Section 2.5.3 that for the model to have a stable solution the spectral radius of $\mathbf{M}$ is less than 1. Therefore the powers of $\mathbf{M}$ progressively contain smaller terms and after some $k, \mathbf{M}^k \rightarrow 0$. Also, the elements of $\mathbf{T}(k)$ come from the vector $\mathbf{c}^*(k) - (\mathbf{I}_n - \mathbf{A}^*)\mathbf{q}(k)$, which also approaches 0 as the system moves towards stability. Therefore, it can be seen that beyond some value of $k, \mathbf{U}(k+1) \rightarrow 0$. Hence there is a region in the iterative scheme where the forward sensitivities are said to be saturated and do not change. This results in the matrix $\mathbf{H}$ being ill-conditioned. As such if the observations lie in this region then the FSM is not able to estimate the parameter. Such considerations a required while using the performance metrics to set targets for the system to generate the inverse problem. Lakshmivarahan & Lewis (2010) suggest that if the condition number of the matrix $\mathbf{H}^\intercal\mathbf{H}$ should be less than 10^4 for favorable estimates. Section 5.5 expands on the above discussion through a numerical example.

5.4 Setting values for observation space

A key issue in the development and solution of the inverse problem is the availability of the observation space or target values that will be used for parameter estimation. Knowledge of the system through past recovery data can provide us with estimates for the recovery time or average inoperability values, which can be used for solving the inverse problem. For example estimates are available for disaster impact and recovery from Hurricane Katrina (Hallegatte, 2008), and these can be used for future recovery planning. One caveat of using already available data is that it will not replicate itself in reality, which means planning decisions made from one disruptive event are unique to that problem. Also, in general there is little or no data for recovery planning of most macro level systems. The problem we are dealing with could be referred to as an 'online' parameter estimation problem, where the estimates depend upon the type of disruptive event characterized through the initial inoperability and demand perturbations. The observational space generated should represent a desired behavior we want to ascribe to the system that is reflective of the response to the given disruptive event given through the model space. Such an approach has been employed in economic planning and design, where the planner wants his/her system behavior to 'emerge' from the model itself. This planning philosophy for feedback control design for economic systems is elucidated by Archibald (2005) in his work 'Information, incentives and the economics of control'

> There is, however, a crucial distinction between the familiar use of feed-back systems for physical control and their potential use for economic control: in the former use, the target must be set, i.e. pre-selected; in the latter use, the target must somehow "emerge" as the process goes on. If this were not the case, we should at the most have a system for the implementation of a plan, not a substitute for the planning process itself.

Given the situation that there is no available data in our problem we use particular scenarios of the dynamic risk input-output model itself to generate the observation space. Based on the previous discussion on the dynamic risk input-output model in Chapter 2 setting $\mathbf{K}^* = \mathbf{I}_n$ gives some insights into the system recovery behavior. In Section 2.5.3 we established that since $\mathbf{K}^*$ is modeled to be diagonal or close to diagonal with its elements $k^*_{ij} \in [0, 1]$, having $\mathbf{K}^* = \mathbf{I}_n$ means that the interdependent recovery is fastest compared to other responses most of the times. Moreover. $\mathbf{K}^* = \mathbf{I}_n$ signifies that the system interdependence given through $\mathbf{A}^*$ is preserved and controls the response. Hence, we use the model given by Equation (5.35) to generate a benchmark for the observation data.

$$\mathbf{q}^{\mathbf{1}}(t) = e^{-(\mathbf{I}_n - \mathbf{A}^*)t}\mathbf{q}(0) + \int_0^t e^{-(\mathbf{I}_n - \mathbf{A}^*)(t-z)}\mathbf{c}^*(z)dz \tag{5.35}$$

Using the above model we can calculate for all the sector time to recovery $\boldsymbol{\tau}^1$, equilibrium inoperabilities $\mathbf{q}^{1e}$ and $\bar{\mathbf{F}}^1 = \frac{1}{T}\int_{t=0}^{T} \mathbf{q}^1(t)$ among other parameters. The $\mathbf{z}_\tau$

observational data can be conveniently generated as

$$\mathbf{z}_\tau = \mathbf{q}^{1e} + \boldsymbol{\nu}_\tau, \quad \boldsymbol{\nu}_\tau \sim (0, \text{diag}(\boldsymbol{\sigma}_\tau^2)) \tag{5.36}$$

In the inverse problem of Equation (5.13) setting $\boldsymbol{\tau} \geq \boldsymbol{\tau}^1$ would imply that the equilibrium has been shifted to a later time and hence the $\mathbf{K}^*$ estimation problem is posed as

> For times $\boldsymbol{\tau} \geq \boldsymbol{\tau}^1$ we ask ourselves the question: What is $\mathbf{K}^*$ such that
>
> $$\mathbf{z}_\tau = \mathbf{q}(\boldsymbol{\tau}) + \boldsymbol{\nu}_\tau?$$
>
> We are mainly shifting the equilibrium towards a later time and finding the $\mathbf{K}^*$ for the sectors for this new situation.

Similarly the $\mathbf{z}_{\bar{F}}$ observational data can be generated and used to estimate system performance

$$\mathbf{z}_{\bar{F}} = \bar{\mathbf{F}}^1 + \boldsymbol{\nu}_{\bar{F}}, \quad \boldsymbol{\nu}_{\bar{F}} \sim (0, \text{diag}(\boldsymbol{\sigma}_{\bar{F}}^2)) \tag{5.37}$$

where the Gaussian error terms $\boldsymbol{\nu}_{\bar{F}}$ arise because the assumptions that the targets cannot be set with complete certainty.

In the inverse problem of Equation (5.14) setting $\boldsymbol{\tau} \geq \boldsymbol{\tau}^1$ would imply that the equilibrium has been shifted to a later time and hence the $\mathbf{K}^*$ estimation problem is posed as

> For times $\boldsymbol{\tau} \geq \boldsymbol{\tau}^1$ we ask ourselves the question: What is $\mathbf{K}^*$ such that
>
> $$\mathbf{z}_{\bar{F}} = \frac{1}{T}\sum_{k=0}^{\tau} \mathbf{q}(k) + \boldsymbol{\nu}_\tau?$$
>
> We are mainly shifting the equilibrium towards a later time, while maintaining the same overall level of functionality, and finding the $\mathbf{K}^*$ for the sectors for this new situation.

Using $\mathbf{K}^* = \mathbf{I}_n$ for benchmarking and setting $\boldsymbol{\tau} \geq \boldsymbol{\tau}^1$ for performance evaluation of the system is justified due to the opposing tradeoff between the maximum inoperability and the time of recovery arising from coupled system behaviors. Figure 5.2 shows that faster recovery might come at the cost of greater maximum inoperability for the same level of overall performance. Same areas under curves means same overall level of operabilities and lower maximum impact at the cost of later recovery might be a preferred option. Table 5.2 summarizes the observational data generation approach.

Table 5.2: Algorithmic procedure to generate the observation data for the inverse problems

Given:	$\mathbf{A}^*, \mathbf{q}(0), \mathbf{c}^*(k), \forall k, \mu$
Set:	$\mathbf{K}^* = \mathbf{I}$
Calculate:	$\mathbf{q}^{1e}, \boldsymbol{\tau}^1, \bar{\mathbf{F}}^1$
Set:	$\mathbf{z}_\tau = \mathbf{q}^{1e} + \boldsymbol{\nu}_\tau, \quad \boldsymbol{\nu}_\tau \sim (0, \text{diag}(\boldsymbol{\sigma}_\tau^2))$
Set:	$\mathbf{z}_{\bar{F}} = \bar{\mathbf{F}}^1 + \boldsymbol{\nu}_{\bar{F}}, \quad \boldsymbol{\nu}_{\bar{F}} \sim (0, \text{diag}(\boldsymbol{\sigma}_{\bar{F}}^2))$

5.5 Example problem

The practical application of the above methods is highlighted through a 15 sector economy that shows the interdependence between economic sectors in the state of Oklahoma. This data is obtained from the BEA (BEA., 2011). Table 5.3 lists the economic sectors with annual output levels specified. Assuming that there is a disruptive event that results in initial inoperabilities for the mining and the manufacturing sectors only, the values in the initial inoperability vector are also given in Table 5.3. If there are no demand perturbations for this system at all times ($\mathbf{c}^*(k) = \mathbf{0}, \forall k$), then the problem of finding the $\mathbf{K}^*$ matrix is a recovery rate estimation exercise. We investigate the problem in context of the inverse problem given in Equation (5.10) where the $\boldsymbol{\tau}$ metric generates the performance criteria and also reflects the recov-

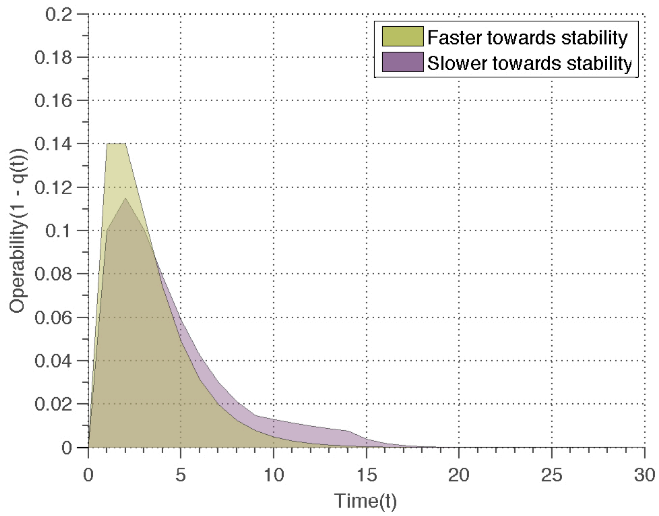


Figure 5.2: Two sector recoveries with same areas under the curves showing same levels of functionality. It can be seen that there is a tradeoff between the recovery time and the maximum inoperability which needs to be considered in setting recovery targets.

ery behaviors. From the Table 5.2 approach for estimating the observational data the recovery times τ^1 for all sectors are about the same ($\tau_i^1 = 6, \forall i \in [1, 15]$). The observational vectors $\mathbf{q}^{1e}$ and $\bar{\mathbf{F}}^1$ are also given in Table 5.3.

Setting different recovery times for the attainment of stable states is analogous to setting different planning horizons for recovery and monitors the consequences of such decisions. Figure 5.3 shows the trajectories of four major sectors as their recovery times vary. In the plots it is visible that the delayed recoveries result in later attainment of stabilities but there are lower maximum losses for the sectors. Figure 5.4 highlights such tradeoffs due to different recovery planning efforts. Shown in the figures 5.4(a) to 5.4(d) on the left side are the maximum economic/output losses incurred by the sectors for different recovery planning horizons and on the right side are the average economic/output losses the sectors incur due to such planning. Using these metrics instead of the previously defined q^m and F metrics gives a practical interpretation to the metrics the signify resilience. For a planner measures of economic losses help in understanding the implications of the planning decisions. As seen in the Figure 5.4 for each sector delayed recovery results in a higher average loss of functionality but there is a maximum impact effect due to the interdependence with other affected sectors, which results in initial cascading of the disruption. Such results highlight the justifications in using the proposed triplet of resilience metrics instead of a singular measure like total economic loss, which is generally done for such studies.

From the computational aspects of the forward sensitivity scheme interest lies in examining the evolution of the forward sensitivity operators because these are indicative of the capability of the model to provide a solution to the inverse problem. As discussed before the sensitivity functions $\frac{\partial q_i(k+1)}{\partial k^*_{jk}}$ approach towards zeros after some time which means that they not sensitive to the placement of the data/observation beyond a certain time. Figure 5.5 shows the sensitivity functions for the two affected sectors (mining and manufacturing) because these drive the responses for most of the interdependent economy. In this problem the size of the matrix $\mathbf{D}_{\mathbf{k}^*}(\mathbf{q}(k))$ is 15×225, which means there are 225 sensitivity operators for each sector. As seen in the figures beyond time $t = 25$ most of the sensitivity operators approach towards zero, and having data/observations beyond this point would not yield any solutions from the computational scheme. Also, another advantage of these sensitivity operators is that they can tell how much each $q_i(k)$ is sensitive to which k^*_{ij} elements and in what direction does that sensitivity progress. From a planning perspective we can interpret this as a indicator of the effectiveness of the sector investments in substitutions for certain products that would help in better recovery. Figure 5.6 shows the values for the $k^*_{ij}, j = [1, 15]$ in the form of a bubble plot for the mining sector as the recovery planning horizon is shifted for 6 days to 20 days. Here the sizes of the bubbles reflect the values of the k^*_{ij} elements. Greater the size of the bubble greater is the value of the parameter. Such a plot is indicative of two things: (i) The interdependent structure of the resilience parameters that shows the amount of substitution/inventory the mining sector needs to maintain to attain a targeted planning recovery, (ii) The affect of the substitutions on the overall performance metrics. As expected the mining sector needs to increase its self reliance to achieve faster recovery and also increasing most of its interdependent resilience leads towards the faster recovery. Since, we have been highlighting the fact that faster recovery comes at a cost of increased maximum impacts there is an interest to regulate the amount of interdependence due to substitution/inventory. In particular there is an indicator here that the mining sector can increase its k^*_{ij} value with respect to other sectors by more substitution and decrease its own k^*_{ii} by maintaing lesser redundancy to decrease the maximum economic loss impacts and achieve comparable recovery times.

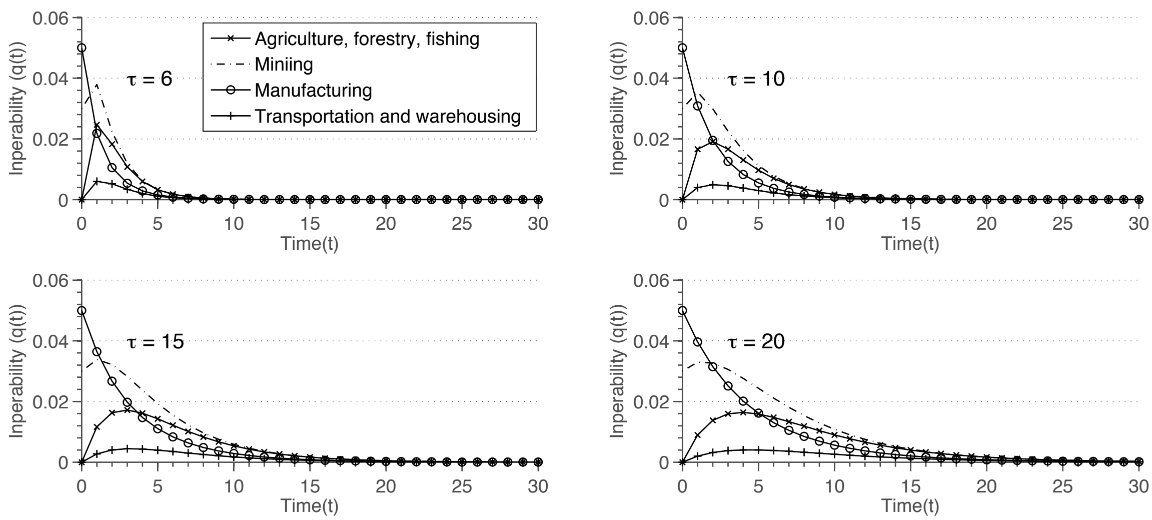
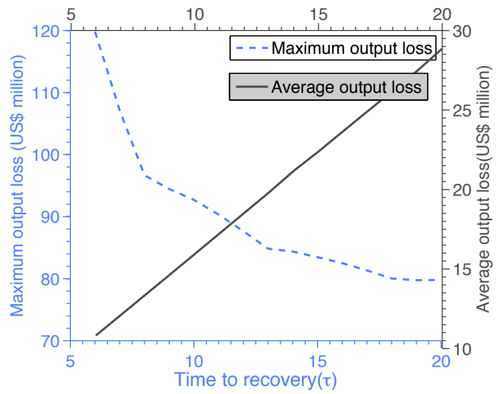
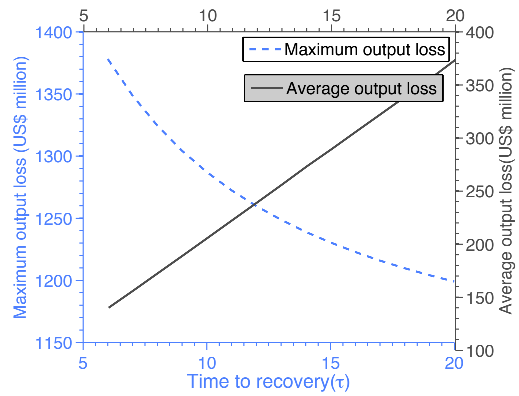


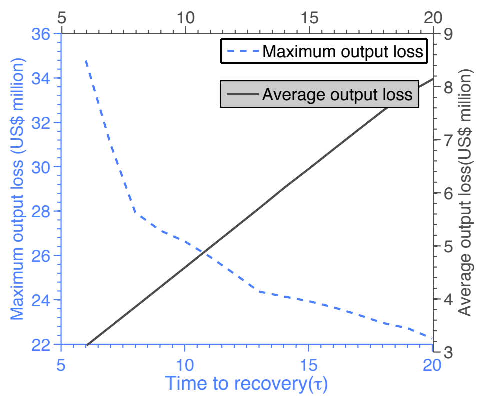
Figure 5.3: Recovery trajectories for four sectors as time to recovery is extended.



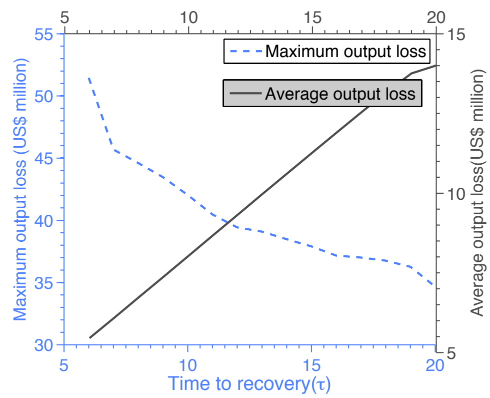
(a) Agriculture, forestry, fishing



(b) Mining

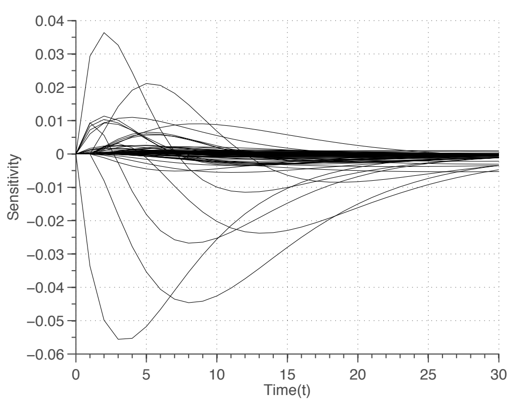


(c) Utilities

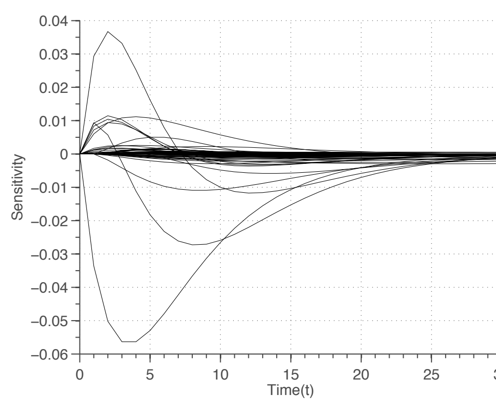


(d) Transportation and warehousing

Figure 5.4: Trade-offs between maximum output losses and average output losses as recovery is delayed.



(a) Mining



(b) Manufacturing

Figure 5.5: Evolution of the forward sensitivities for the inoperabilities of the two affected sectors.

Table 5.3: 15 sector economy with sector names, annual output levels, initial inoperabilities, observational data $\mathbf{q}^{1e}$ and $\bar{\mathbf{F}}^1$

Economic Sector	Output levels $\mathbf{x}$(\$ billion)	$\mathbf{q}(0)$	$\mathbf{q}^{1e}$	$\bar{\mathbf{F}}^1$
1. Agriculture, forestry, fishing	4.861	0	0.0010	0.0532
2. Mining	36.376	0.03	0.0010	0.0937
3. Utilities	4.998	0	0.0003	0.0147
4. Construction	8.380	0	0.0001	0.0023
5. Manufacturing	43.655	0.05	0.0004	0.0749
6. Wholesale trade	10.179	0	0.0002	0.0133
7. Retail trade	12.970	0	0.0000	0.0007
8. Transportation and warehousing	8.570	0	0.0003	0.0147
9. Information	8.112	0	0.0001	0.0030
10. Finance, insurance, real estate	29.672	0	0.0001	0.0023
11. Professional and business services	19.603	0	0.0002	0.0091
12. Educational services, health care	18.106	0	0.0000	0.0000
13. Arts, entertainment, recreation, accommodation	8.350	0	0.0001	0.0017
14. Other services, except government	6.606	0	0.0001	0.0028
15. Government	41.850	0	0.0000	0.0010

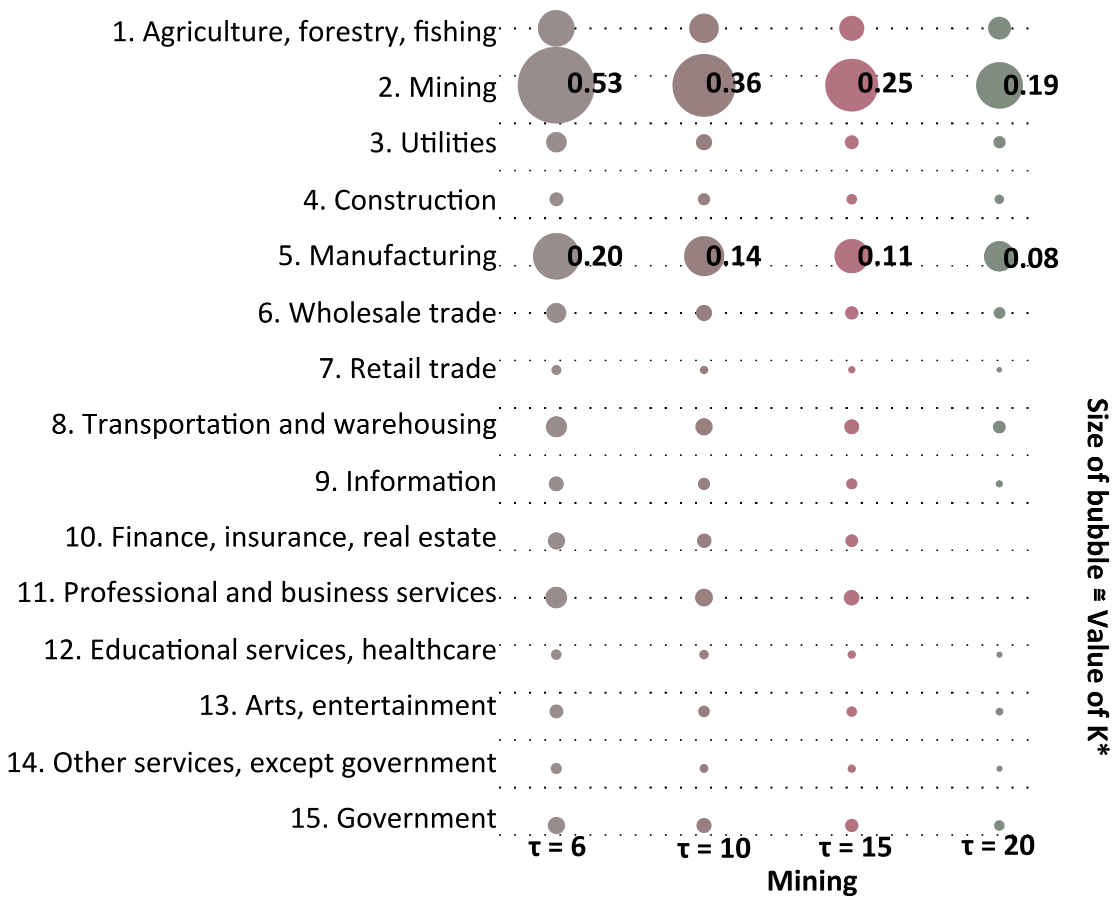


Figure 5.6: Values of the $k^*_{ij}, j = [1, 15]$ elements for the mining sector for different recovery times.

5.6 Summary and discussion

This Chapter presents the computational scheme for solving the parameter estimation problem of the dynamic risk input-output model. The approach adopted here comes from theories in control system designs and dynamic data assimilation methodologies. Primarily the parameter estimation problem is an inverse problem that is solved through the availability of data that the model is supposed to predict. A functional relationship exists between the model and the observation and the unknown parameter is estimated to satisfy this functional relationship with as much precision as possible. Though such methods are available in other research domains their usage in the parameter estimation for the dynamic risk input-output model is a novel approach.

Unlike other domains of application where the data is readily available to calibrate the model here the big issue is the lack of data which shows the recovery behavior that can be explained through the model. As such the problem is really a planning issue where the model is being used to agree to a specified decision preference. As the requirements of the model are that it should be a suitable construct for resilience estimation, the data is realized as performance targets set to meet specific resilience goals. Such assumptions are not too implausible, because in risk planning for future behaviors and for unknown scenarios there is a need to create manufactured scenarios.

The solution scheme adopted here is called the forward sensitivity scheme and its is based on calculating the sensitivity of the model state to the parameter being estimated. This approach has computational and practical relevance. Computationally sensitivity shows the effect of perturbation of the parameter on the model state, which translates to the magnitude of the effect on model state due to the parameter. More sensitivity indicates the parameter is significant in predicting the model states. Time evolving sensitivity, which is being used here, also shows the domain of analysis where the model does not respond to fluctuations in the parameter values. For the inverse problem this domain is important because it shows that the model parameter would not be able to predict the model state properly within the domain, hence the effect of observations on the model are negligent. These issues are discussed here in relation to the dynamic risk input-output model.

Having developed the solution scheme, the example problem brings together the resilience concepts and the forward sensitivity approach develop over the previous chapters and this one. Primarily the goal of developing the forward sensitivity approach here is the seek solutions to the possible values the $\mathbf{K}^*$ matrix takes during recovery and how these translate towards resilience indicators. Through the example problem the resilience conceptualization and model frameworks have been presented here. In the end we have a convenient tool for predicting dynamic resilience in interdependent systems.

Chapter 6

Inland Waterway Disruption Analysis

6.1 Introduction

This Chapter presents a study on multi-modal transportation risk assessment, with particular focus on inland ports. The purpose of this Chapter is to develop a methodology for applying the input-output risk models, so that the concepts that were developed in the previous chapters can be used for real world applications. Inland waterway systems are CIKR of homeland security importance and hence need to be studied for risk assessment and planning. The Chapter is organized in the following. Section 6.2 motivates the importance of studying inland waterway ports from a economic and security point of view. Section 6.3 presents a multi-regional framework for applying the input-output models that were discussed previously. The economic multi-regional model is based on the input-output principles and its risk-based extension is also constructed by following previously defined steps for model development. The models presented here are static, but they can be conveniently extended to the dynamic domains. In Section 6.4 the link between the commodity flows across inland waterway port hubs and input-output models is shown. This leads to the development of the risk metrics that express the losses in commodity flows as inoperabilities and demand perturbations. Hence, the unified framework that combines transportation hub commerce with input-output risk models is established. In order to estimate losses in the transportation hub we need to develop a model that as-planned and disrupted states of the system being analyzed. Hence, in Section 6.5 a port queuing model is presented for representing the flow of goods across the. The queuing model divides the commodity flows into four main components: *arrivals*, *yard storage*, *crane operations* and *departures*, which is useful in monitoring the flows across the different stages of the transport. Therefore, disruptions to each of these components can be separately modeled and their effect on the rest of the queue can be conveniently quantified. Such schemes are suggested for building different disruption scenarios that affect port commerce, and generate risk metrics. Section 6.6 presents a case study for the Port of Catoosa in Tulsa Oklahoma, where the methods developed are applied and their risk assessment application is shown. The work presented here shows the scope of the models, which can be applied to any general transportation hub and network. The importance of the work and its applicability to the risk management and resilience method is highlighted in Section 6.7.

6.2 Motivation

The prevention of, and recovery from, disruptions to large-scale economies often involves decision making by parties interested in the normal operation of more than one industry. Government policy makers and industry decision makers both have a need for distributing resources where they will be most useful in ensuring normal production levels, whether for the economy as a whole or for individual supply chains. This resource allocation depends on accurate information regarding the potential effects of preparedness and mitigation strategies that take into account the interconnected nature of the economy.

Such planning is particularly important risk-based policy and decision making for transportation infrastructure, listed among the US critical infrastructure and key resources (DHS., 2009). Roads and highways have been the primary mode for freight transport, moving an estimated 69% freight by weight and 65% by value in 2007

(Schmitt et al., 2010). With US domestic freight projected to grow annually by 1.6% from 2010 till 2040 (Schmitt et al., 2010) along with greater increase in general pedestrian traffic, there is concern about the future of the aging US transportation system (Hasley, 2010). Estimates suggest that in 2009, congestion in 423 metropolitan areas caused urban Americans to travel 4.8 billion hours more, and to purchase an extra 3.9 billion gallons of fuel for a congestion cost of $115 billion (Schrank et al., 2010). Congestion also exists on rail transport and is projected to increase along Class I railway lines and terminals along coasts (Association of American Railroads, 2007). A viable freight alternative is needed for sharing the burden with road and rail for future sustenance of the US multi-modal transportation infrastructure.

The US Maritime Administration, a division of the US Department of Transportation, has identified such a freight alternative, calling for an investment in inland waterways for general freight movement (Marine Administration, 2011). The increased use of the 25,000 miles of commercially navigable waterways for freight transport will likely lead to reduced congestions on US highways, as well as reduced risk of accidents relative to highway and rail transport and reduced air pollution emissions (US Department of Energy, Energy Information Administration, 2010). Transportation by barge is often cheaper than the alternatives of rail and truck, and there are many products that are too large for other methods of transport. Around 38 states depend on inland waterways to move as much as 630 million tons annually in the last decade (US Army Corps of Engineers, 2009), currently a distant third behind roadway and rail.

6.3 Multi-Regional Economic Impact Framework

Isard et al. (1998) proposed a multi-regional input-output (MRIO) framework for multi-regional trade supply and demand balance. If we assume that the n economic sectors trade across p regions then using the MRIO we can express the equilibrium between sector outputs supplies and intermediary and final product demands across all regions. Equation (6.1) expresses this trade balance as

$$x_i^r = \sum_{s=1}^{p} \sum_{j=1}^{n} t_i^{rs} a_{ij}^s x_j^s + \sum_{s=1}^{p} t_i^{rs} c_i^s \tag{6.1}$$

where x_i^r is the output of industry i in region r, t_i^{rs} is the proportion of industry i output that flows from region r to region s, a_{ij}^s is the proportion of industry i output used by industry j after it comes from all sources in region s, and c_i^s is the final exogenous demand for industry output i in region s. Expressed in matrix form Equation (6.1) leads to Equation (6.2) with $np \times 1$ vectors and $np \times np$ matrix structures. Each $n \times n$ sub-matrix $\mathbf{T}^{rs} = \left[\text{diag}(t_i^{rs})\right] \forall i = \{1, 2, ..., n\}$ is called the trade coefficient matrix for trade flow interdependence between region r and s. Also each sub-matrix $\mathbf{A}^s = \left[a_{ij}^s\right] \forall i, j = \{1, 2, ..., n\}$ is an $n \times n$ regional industry-to-industry interdependency matrix for region s. Each $n \times 1$ vector of regional industry outputs, $\mathbf{x}^r (= \left[x_i^r\right] \forall i = 1, 2, ..., n)$ and $n \times 1$ vector of regional exogenous demand, $\mathbf{c}^r (= \left[c_i^r\right] \forall i = 1, 2, ..., n)$ make up the multi-regional supply and demand vectors of the MRIO.

$$\begin{bmatrix} \mathbf{x}^1 \\ \vdots \\ \mathbf{x}^p \end{bmatrix} = \begin{bmatrix} \mathbf{T}^{11} & \cdots & \mathbf{T}^{1p} \\ \vdots & \ddots & \vdots \\ \mathbf{T}^{p1} & \cdots & \mathbf{T}^{pp} \end{bmatrix} \begin{bmatrix} \mathbf{A}^1 & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \mathbf{A}^p \end{bmatrix} \begin{bmatrix} \mathbf{x}^1 \\ \vdots \\ \mathbf{x}^p \end{bmatrix} + \begin{bmatrix} \mathbf{T}^{11} & \cdots & \mathbf{T}^{1p} \\ \vdots & \ddots & \vdots \\ \mathbf{T}^{p1} & \cdots & \mathbf{T}^{pp} \end{bmatrix} \begin{bmatrix} \mathbf{c}^1 \\ \vdots \\ \mathbf{c}^p \end{bmatrix} \tag{6.2}$$

The MRIO is also supported and validated through publicly available data sources. The Bureau of Transportation Statistics, maintains commodity flow data for entire US domestic and international trade (BTS., 2011), which is used in constructing the trade coefficient matrices. Also, the $\mathbf{A}$ matrix data available at the national level is converted to a regional $\mathbf{A}^r$ matrix through different regional multipliers methods

(Miller & Blair, 2009). The location quotient multipliers used here takes into account the regional contribution of an industry compared to its contribution to national outputs. As outlined by the BEA (BEA., 1997) the location quotient l_i^r for industry i in region r is the proportion between its regional output contribution to the entire regional economic output x_{to}^r and its national output contribution to entire national output x_{to}.

$$l_i^r = \frac{x_i^r / x_{to}^r}{x_i / x_{to}} \tag{6.3}$$

The regional input-output industry-by-industry interdependence matrix obtained by combining the two BEA data sources is expressed in Equation (6.4). As l_i^r approaches 1 industry i's contribution to regional demands approaches its national trade capacity, which means the national interdependency has a one-to-one correspondence to regional interdependency.

$$a_{ij}^r = \begin{cases} l_i^r a_{ij} & l_i^r < 1 \\ a_{ij} & l_i^r \geq 1 \end{cases} \tag{6.4}$$

The MRIO extension to the inoperability framework leads to a multi-regional inoperability input-output model (MRIIM) (Crowther & Haimes, 2010). In the MRIIM Equation (6.5), derived from (6.2), $\mathbf{q}^r$ and $\mathbf{c}^{*r}$ are respectively the $n \times 1$ vectors for inoperabilities and demand perturbations at the region r level. $\mathbf{A}^{*s}$ is now the $n \times n$ regional industry-to-industry inoperability propagation matrix for region s, and $\mathbf{T}^{*rs} = \left[\text{diag}(\mathbf{x}^r)\right]^{-1} \mathbf{T}^{rs} \left[\text{diag}(\mathbf{x}^r)\right]$ represents the $n \times n$ matrix for the inter-regional inoperability flow between regions r and s.

$$\begin{bmatrix} \mathbf{q}^1 \\ \vdots \\ \mathbf{q}^p \end{bmatrix} = \begin{bmatrix} \mathbf{T}^{*11} & \cdots & \mathbf{T}^{*1p} \\ \vdots & \ddots & \vdots \\ \mathbf{T}^{*p1} & \cdots & \mathbf{T}^{*pp} \end{bmatrix} \begin{bmatrix} \mathbf{A}^{*1} & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \mathbf{A}^{*p} \end{bmatrix} \begin{bmatrix} \mathbf{q}^1 \\ \vdots \\ \mathbf{q}^p \end{bmatrix} + \begin{bmatrix} \mathbf{T}^{*11} & \cdots & \mathbf{T}^{*1p} \\ \vdots & \ddots & \vdots \\ \mathbf{T}^{*p1} & \cdots & \mathbf{T}^{*pp} \end{bmatrix} \begin{bmatrix} \mathbf{c}^{*1} \\ \vdots \\ \mathbf{c}^{*p} \end{bmatrix} \tag{6.5}$$

6.4 Applications to Transportation facility disruptions

Transportation facilities, such as ports, are outlets for commodity flows across regions. Since the multi-regional input-output model quantifies the equilibrium of the imports and exports between regions, port facilities are suitable geographic locations where this equilibrium can be studied. For a port located in region $r(r \in \{1, 2, ..., p\})$ exporting to region $s(s \in \{1, 2, ..., p\}, s \neq r)$, the amount of commodity i that arrives at the port is the amount of final demand for that commodity for region r. Hence, Equation (6.6) shows how the total final demand, c_i^r, for commodity i in region r is divided, where $(c_i^r)_{re}$ is the amount of commodity i that is consumed internally or exported out of other locations except the port, and $(c_i^r)_{pe}$ is the amount of export out of the port into region s.

$$c_i^r = (c_i^r)_{re} + \sum_{s=1, s\neq r}^{p} (c_i^r)_{pe} \tag{6.6}$$

The amount of commodity i that is shipped to region s is then used by industries in s for their production and for final consumption. Hence, in region s the amount of import, $D_i^s = \sum_{r=1, r\neq s}^{p} D_i^{rs}$, contributes towards the total output, x_i^s, of the industry i and the final demand, c_i^s, for the commodity i. Value $(x_i^{rs})_{pi}$ is the amount of industry i output that comes through the port from region r, $(x_i^s)_{re}$ is the amount of industry i output in region s coming for sources other than the port import, $(c_i^{rs})_{pi}$ is the amount of final demand for commodity i coming from the port into region s, $(c_i^s)_{re}$ is the amount of final demand for commodity i through sources other that the port import.

$$\sum_{r \neq s} D_i^{rs} = \sum_{r \neq s} (x_i^{rs})_{pi} + \sum_{r \neq s} (c_i^{rs})_{pi} \tag{6.7}$$

$$x_i^s = (x_i^s)_{re} + \sum_{r \neq s} (x_i^{rs})_{pi} \tag{6.8}$$

$$c_i^s = (c_i^s)_{re} + \sum_{r \neq s} (c_i^{rs})_{pi} \tag{6.9}$$

When disruptions result in change in the amount of arrivals and departures of commodities at the port, they affect the exports and imports of the regions having commerce through the port. It is assumed that the disruptions cause losses in commodity flows only through the port while the rest of the flows are not affected. Hence, Equation (6.10) shows that for the entire economy of region r, port disruptions result in a demand perturbation for commodity i, c_i^{*r}, given by the loss of exports, $\sum_{s \neq r} (\Delta c_i^{rs})_{pe}$, as a proportion of the total output of commodity i in region, x_i^r.

$$c_i^{*r} = \frac{\sum_{s \neq r} (\Delta c_i^{rs})_{pe}}{x_i^r} \tag{6.10}$$

For the importing region s, the amount of import loss, $\sum_{r \neq s} \Delta D_i^{rs}$, in Equation (6.11) results in the loss of output, $\sum_{r \neq s} \Delta (x_i^{rs})_{pi}$, and final demand, $\sum_{r \neq s} \Delta (c_i^{rs})_{pi}$, for commodity i in region s.

$$\sum_{r \neq s} D_i^{rs} = \sum_{r \neq s} \Delta (x_i^{rs})_{pi} + \sum_{r \neq s} \Delta (c_i^{rs})_{pi} \tag{6.11}$$

Thus, for the entire economy of region s, the loss of imports causes an inoperability, $\hat{q}_i^s$, and demand perturbation, c_i^{*s}. The demand perturbation in industry i can be calculated.

$$c_i^{*s} = \frac{\sum_{r \neq s} \Delta(c_i^{rs})_{pi}}{x_i^s} \tag{6.12}$$

While export contributions toward input-output model metrics are straightforward to understand and model, import substitutions are more complicated. Typically in input-output tables imports are value-added contributions towards sector outputs and thus the inoperability due to them is expressed as

$$\hat{q}_i^s = \frac{\sum_{r \neq s} \Delta(x_i^{rs})_{pi}}{x_i^s} \tag{6.13}$$

Since the MRIIM also provides sector inoperability (denoted by $\tilde{q}_i^s$ here), we need to compare Equation (6.5) and (6.13) inoperabilities, and the maximum of the two values provides actual sector inoperability.

$$q_i^s = \max\left\{\tilde{q}_i^s, \hat{q}_i^s\right\} \tag{6.14}$$

The MRIIM interdependency equation uses the information from Equations (6.13) and (6.14) as inputs for calculating the inoperabilities and demand perturbations for interconnected industries across regions. If $m \in 1, 2, ..., n$ different commodities are transported through the port from region r to s, then in the event of a disruption there is a demand perturbation, given by Equation (6.13), only for those commodities, while the rest of the commodities experience no demand perturbation. Hence, the demand perturbation vector for region r is found with Equation (6.15).

$$c_j^{*r} = \begin{cases} \frac{\sum_{s \neq r} \Delta(c_j^{rs})_{pe}}{x_j^s}, & j \in \{1, 2, ..., m\} \\ 0, & \text{otherwise} \end{cases} \tag{6.15}$$

Similarly, for the importing region s, there is a demand perturbation for only those commodities imported through the port, while the rest of the commodities experience no perturbation in demand. The demand perturbation vector for the importing region s is calculated in Equation (6.17)

$$c_j^{*s} = \begin{cases} \frac{\sum_{r \neq s} \Delta(D_i^{rs})_{pe}}{x_j^s} - q_j^s, & j \in \{1, 2, ..., m\} \\ 0, & \text{otherwise} \end{cases} \tag{6.16}$$

Equations (6.15) and (6.17) combined with the MRIIM form a complete solvable system that quantifies the inoperability and demand perturbations for the entire regional economies for interconnected industries. The equations for demand perturbations developed above assume only exports from region r through the port, whereas in actual situations commodities are also imported into r through the port. Therefore, the total demand perturbation for region r is given in Equation (6.18).

$$c_j^{*r} = \begin{cases} \frac{\sum_{s \neq r} \Delta(c_j^{rs})_{pe}}{x_j^r} + \frac{\sum_{s \neq r} \Delta(D_i^{sr})_{pe}}{x_j^r} - q_j^r, & j \in \{1, 2, ..., m\} \\ 0, & \text{otherwise} \end{cases} \tag{6.17}$$

While there are several kinds of disruptions, any of which can be incorporated into transportation flow analyses, we are primarily interested in modeling impacts due to export-import losses. Such an approach is practical from the perspective of estimating economic losses because we are interested in quantifying lost commerce due to disruptions.

6.5 Port simulation model

A simulation model that provides estimates of the commodity arrivals and departures through the port is a useful tool for estimating the parameters for the MRIIM. Figures 6.1 and 6.2 depict a supply chain model for inland port operations, which can be extended to the entire waterway network. For the inter-regional commodity flow analysis we consider a queueing system for freight transfer through the supply chain. Supply chain modeling approaches have been applied in the transportation studies for different types of transfer facilities (Simão & Powell, 1992; Lee et al., 2003) and have been used in analyzing transportation disruptions (Wilson, 2007). In this study, the components of the different port operations are defined and explained as follows.

1. Delivery/Receipt - These operations include the arrival of commodities for exports out of the region and the departure of commodities for imports into the region.

2. Yard operations - These are storage operations for the temporary storage of commodities at the port where they are kept for further transport.

3. Crane operations - Cranes are used at the port to transfer commodities to and from the port docks.

4. Shipment - Freight shipment operations include the departure of commodities for exports and the arrival for imports.

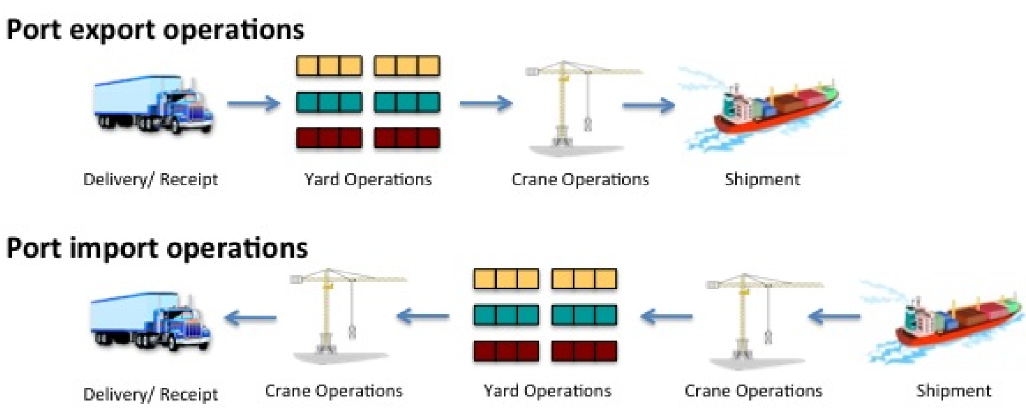


Figure 6.1: Port export-import model

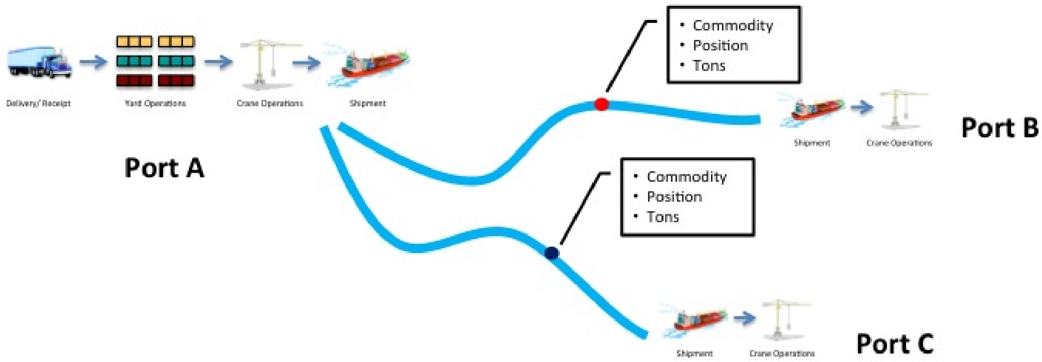


Figure 6.2: Overall inland waterway transportation model

A discrete time model, based on concepts developed by Simão & Powell (1992), can be built for simulating the above port operations. Due to different order of the operations, the simulation models for exports and imports will be separate. It is assumed that commodities arrive independent of each other at the port, and each commodity is transported through the port operations separately. Hence, for m commodities arriving at the port there are m parallel queueing systems in operation. Considering a time increment of Δt, the discrete time model can capture the evolution of queue model at all times $t = 0, \Delta t, 2\Delta t, ...$ Before developing the iterative equations for the queueing system, some random variables are defined for quantifying different elements of normal port operations:

1. $Y_i(t)$ = The number of units of commodity i arriving at the terminal in the time interval $(t - \Delta t, t]$;

2. $N_i(t)$ = Number of units of commodity i in yard storage at time t after commodities have arrived in the interval $(t - \Delta t, t]$;

3. $V_i(t)$ = The maximum units of commodity i that can be transferred by the cranes to the docks in the time interval $(t, t + \Delta t]$;

4. $W_i(t)$ = The maximum number of imported units of commodity i that can be

loaded from the yard to trucks or trains in the time interval $(t, t + \Delta t]$;

5. $U_i(t)$ = The number of units of commodity i that are transferred to the dock for shipment in the time interval $(t, t + \Delta t]$;

6. $D_i(t)$ = The number of units of commodity i departing in the interval $(t, t+\Delta t]$.

In simulating the commodity arrival process, service capabilities of the transfer cranes and import loading process are known from data describing port annual exports and imports and daily crane operations, respectively. Hence, assuming that $Y_i(t)$, $V_i(t)$ and $W_i(t)$ are known, the other variables are calculated by adding and subtracting random variables, as described in the following sections. The same variables are used for formulating the export and import operations as they have the same meaning for both operations.

6.5.1 Port export operations

When commodities arrive at the port, they are stocked at the yard. At time $t + \Delta t$, the number of units of commodity i at the yard is the sum of the units remaining to be carried by the transfer cranes and the units that arrive.

$$N_i(t + \Delta t) = \max[0, N_i(t) - V_i(t)] + Y_i(t + \Delta t) \tag{6.18}$$

The number of units of commodity i transferred by cranes is the minimum of the number of units at the yard and the crane capacities.

$$U_i(t) = \min[N_i(t), V_i(t)] \tag{6.19}$$

For normal port operations, the number of units of commodity i exported from the port is equal to the number of units transferred to the docks.

$$D_i(t) = U_i(t) \tag{6.20}$$

6.5.2 Port import operations

For imports the commodities are now arriving at the docks and transferred from the cranes to the yards. The number of units of commodity i transferred by the cranes becomes

$$U_i(t) = Y_i(t + \Delta t) \tag{6.21}$$

Equation (6.22) calculates the number of units of commodity i at the yard as the sum of the units remaining and the units transferred by the cranes.

$$N_i(t + \Delta t) = \max[0, N_i(t) - W_i(t)] + U_i(t) \tag{6.22}$$

Under normal port operations, the number of units of commodity i departing the port are equal to the units transferred by the crane, as shown

$$D_i(t) = \min[N_i(t), W_i(t)] \tag{6.23}$$

From the simulation equations it can be seen that the crane operations and departure processes lag the arrival and yard storage operations by Δt time. The above formulations for exports and imports involve the simple additions, subtractions and splitting of random variables. As mentioned before, if the distributions for $Y_i(t)$, $V_i(t)$ and $W_i(t)$ are known, then the rest of the distributions of the random variables can be obtained by convolutions of the probability mass or density functions. This allows for estimates of the random variables in the queueing model. Arrivals of the commodities can be modeled as independent non-stationary Poisson processes (with rate $\lambda_i(t)$ for commodity i). Similarly, the rate of service for the crane operations at the terminal could be modeled as a Poisson process with time-dependent rates ($\mu_i(t)$ for commodity i).

6.5.3 Modeling disruptive events

A disruptive event such as a man-made attack, an accident, or a natural disaster can cause damage to components of the transportation network. Different scenarios of incorporating disruptive events in the supply chain model can be explored to quantify the amount of loss incurred due to damages. In this study we investigate how disruptions can affect the parameters in the queueing system simulation model. Some situations considered are as follows.

1. Terminal Closure - A disruptive event, such as a storm, tornado or attack, may cause the closure of the terminal for some time ΔT. In this case there are no arrivals over the period of the storm, but normal service is resumed once the event subsides. In the simulation algorithm this is modeled as.

$$Y_i(i) = 0, \forall t \in [t, t + \Delta T] \tag{6.24}$$

   Such an event is a special case of the scenario where there is a partial disruption in the arrival of commodities due to a disruptive event. Using the assumption of time-dependent Poisson arrival rate of commodities, Equation (6.25) quantifies how the rates of arrival for commodity i change in the simulations, where $\lambda_i^*(t)$ are the the disrupted arrival rates and $\lambda_i(t)$ are the arrival rates under normal port operations.

$$\tilde{\lambda}_i(t) = \begin{cases} \lambda_i^*(t), & \forall t \in [t, t + \Delta T] \\ \lambda_i(t) & \text{otherwise} \end{cases} \tag{6.25}$$

2. Crane Outage - Disruptive events and normal wear and tear that damage some of the cranes may limit the number of commodities that are transferred to and away from the docks. Hence, if the disruption lasts for a time ΔT, in the simulation model the condition imposed is governed by Equation (6.26), where $\tilde{U}_i$ is the capacity limit on the amount of units of commodity i that can be transported by the cranes.

$$U_i(t) = \tilde{U}, \forall t \in (t, t + \Delta T] \tag{6.26}$$

A generalized simulation modeling scenario for such events could be the change in the time-dependent service rates of crane operations, as shown in Equation (6.27), where $\mu_i^*(t)$ are the the disrupted crane service rates and $\mu_i(t)$ are the service rates under normal port operations.

$$\tilde{\mu}_i(t) = \begin{cases} \mu_i^*(t), & \forall t \in [t, t + \Delta T] \\ \mu_i(t) & \text{otherwise} \end{cases} \tag{6.27}$$

3. Departure Stoppage - Similar to arrival disruptions, hazards such as floods in the river or barge accidents can cause disruptions in the departure of commodities for exports and imports. For commodity i, such disruptions over time ΔT change the number of units departed with Equation (6.28), where $\theta(t) \in [0, 1), \forall t \in (t, t + \Delta T]$ is the factor representing the reduction in depart-

ing commodity.

$$\tilde{D}_i(t) = \begin{cases} \theta(t)D_i(t), & \forall t \in [t, t+\Delta T] \\ D_i(t) & \text{otherwise} \end{cases} \tag{6.28}$$

The above three modeling formulations for disruptive events provide different scenarios for calculating the losses over the period of analysis. Some or all of these scenarios can occur in an actual port disaster. Each scenario can be incorporated easily into the queueing model while preserving the simple arithmetic of addition, subtraction, and splitting on the random variables.

6.6 Case Study: Inland Waterway Port Dock and Channel Disruptions

We demonstrate, through a case study of commerce disruption at the Port of Catoosa and along the Mississippi River System, the application of the network model with the MRIIM.

6.6.1 Port of Catoosa Overview

Connecting to the Mississippi River System, considered the most important commercial navigation corridor in the US, is the McClellan-Kerr Arkansas River Navigation System (MKARNS). Along the MKARNS is the Port of Catoosa located near Tulsa, Oklahoma, the subject of the case study that illustrates the dock-specific discrete-event queueing models for commodity flows and disruptions.

The port is an important transportation hub for the Midwest, as it is the farthest north inland port that remains unfrozen all year long. Commodities of all types, including grains, fertilizers, metal products, and chemicals, are shipped both in and out of the port. The port provides services for at least ten states, including Alabama, Arkansas, Illinois, Iowa, Kentucky, Louisiana, Mississippi, Ohio, Oklahoma, and Texas.

The Port of Catoosa has four main docks, each of which deals with a specific commodity type. The Dry Cargo dock handles large items, primarily steel, iron, and machinery. The Dry Bulk dock handles a variety of loose commodities that are moved by conveyer, such as sand, gravel, and fertilizers. The Grains dock moves agricultural products such as corn, wheat, and soybeans. Finally, the Liquid Bulk dock moves liquid products including chemicals, liquid fertilizers, and even molasses. If any of these docks were to become inoperable, it would stop the flow of the specific type of commodity handled by that dock.

Figure 6.3 lists the the combined estimates for the annual exports and imports (in US$ million) for major industries among the states that do the most commerce using the port. These estimates are obtained from the integration of different databases (BTS., 2011; Catoosa., 2011; USAC., 2011). These industries/commodities are the inputs for the port and network simulation model, with each commodity having its separate queue from arrival until departure.

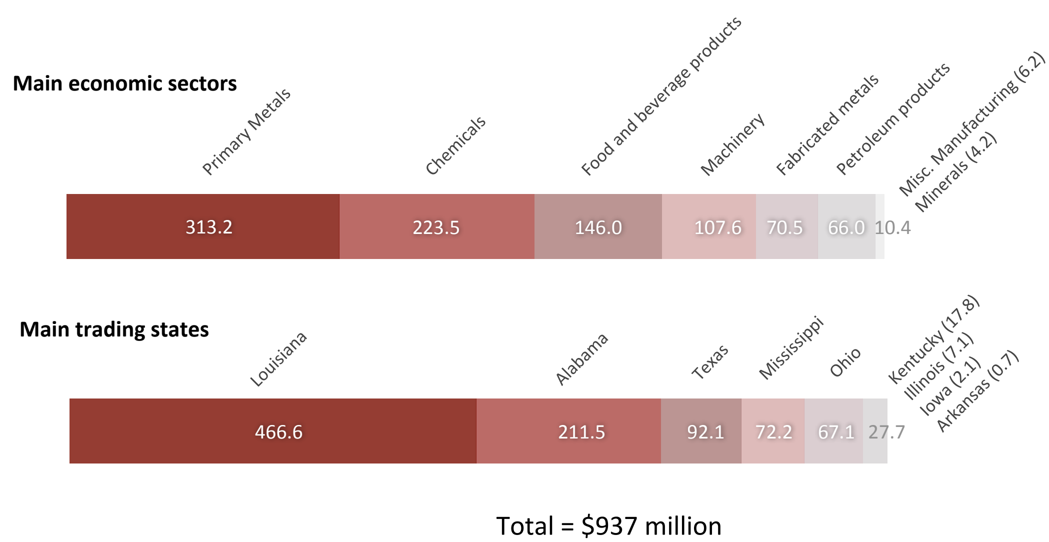


Figure 6.3: Estimates for the 2007 annual export-import commerce through Catoosa, in US $ million

6.6.2 Port Disruptions

We consider a port closure for a duration of two weeks and model the local and multi-regional effects of such a disruption. There are a number of natural and man-made events that could cause disruptions in the movement of commodities through the Port of Catoosa. In May of 2002, an Interstate 40 bridge spanning the Arkansas River collapsed, shutting down barge traffic for over two weeks. Heavy rains in the summer of 2008 led to flooding of the waterway, which greatly reduced the number of barges that could be moved through. A fire at a fertilizer company in early 2009 led to chemical run-off into the port, which required clean up before it spread. Events such as the bridge collapse and flooding affect the entire port's ability to move product, whereas a fire and the subsequent cleanup affect one or more docks.

We choose a day of the year when a disruptive event occurs and stop overall port and the local waterway commerce for a two week duration. In the port model, the disruptions manifest with the stopping of the arrival and departure queues in the port supply chain, which results in commodities not entering the network flow. Hence, export-import loss metrics are generated for the duration of disruption by estimating the amount of lost flow relative to a "no disruption" scenario. We also observe that, for the magnitude of the event we discuss: (1) disruptions at the port or waterway do not directly disrupt industry supply chains/ infrastructures, and (2) the values of demand perturbations or supply inoperabilities due to such a localized disruption are small fractions compared to entire state-wide industry outputs.

Figure 6.4 shows the cumulative loss of export-imports from the onset of disruption around day 25 of the year extended till the rest of the year. The individual effects of such a disruption felt across each of the docks in shown in Figure 6.5. The Dry Cargo and Liquid Bulk docks experience highest export-import losses due to volume and value of commerce through these docks. Overall, more losses are incurred as import losses than export with total losses amounting to an estimated \$45.0 million for such a disruptive event.

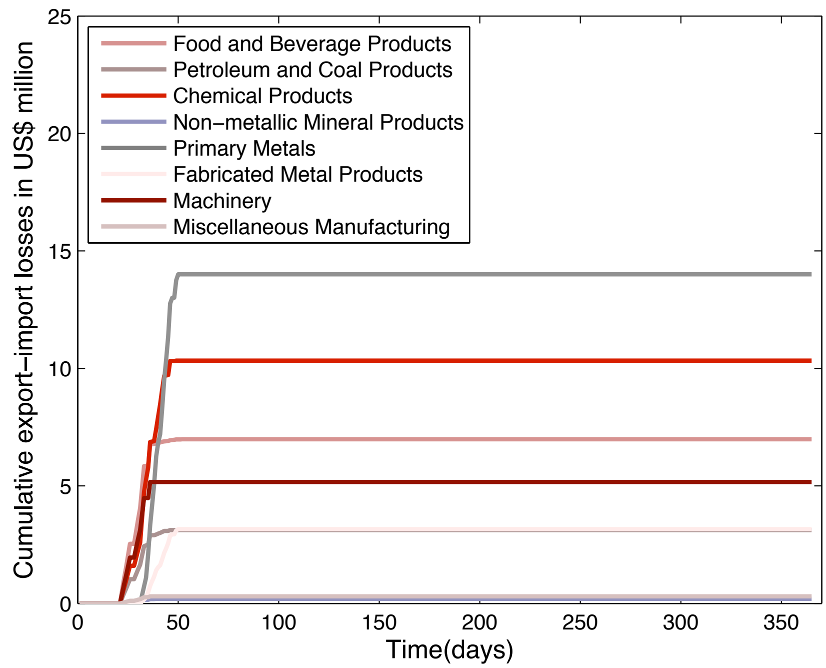


Figure 6.4: Sector-wise accumulation of export-import losses of the year due to 2-week port closure

The overall impacts of the disruption are propagated due to interdependencies cascading to regional economies of the states doing commerce with Oklahoma through the port. Out of many possible scenarios, we present here the effects on Oklahoma industries due to total and partial port closure. Figure 6.6 ranks the top 10 industries by the total amount of losses as a result of the port closure, while Figure 6.7 shows such ranking due to closure of only the Dry Cargo dock. Such analysis highlights the importance of using the dynamic MRIIM for studying macroeconomic risks. As shown in the figures, there are several industries which are indirectly affected by the port closure and incur substantial losses due to their interdependence with port industries.

Another aspect of port disruptions worth highlighting is the economic impact it

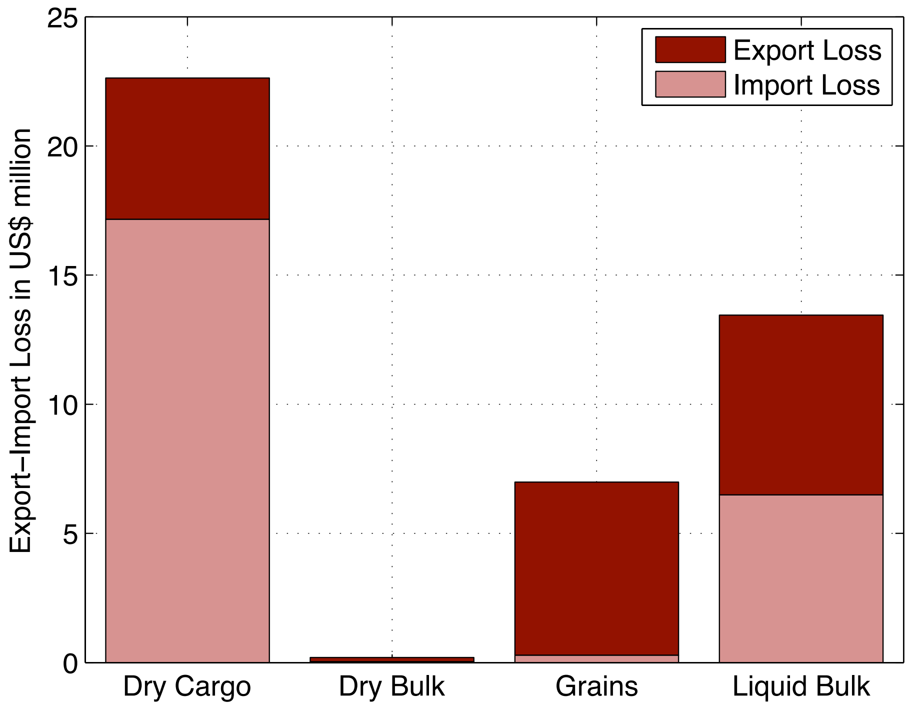


Figure 6.5: Dock specific export-import losses due to port closure

has across multi-regional economies. Figure 6.8 shows the direct, indirect, and total losses for regional economics of the 10 states using the port for commerce. In input-output modeling terms, direct losses come from the demand driven disruptions and are calculated as the sum of the $\mathbf{T}^{*}\mathbf{c}^{*}$ vector in the multi-regional models. These contribute towards backward linkage effects of disruption propagation that result in indirect losses. As can be seen from the Figure 6.8 total multi-regional direct losses of \$111.8 million due to two week port closure are accompanied by total indirect losses of \$72.9 million across all states. Hence, we have shown through data the severity of an localized transportation disruption on multi-regional commerce.

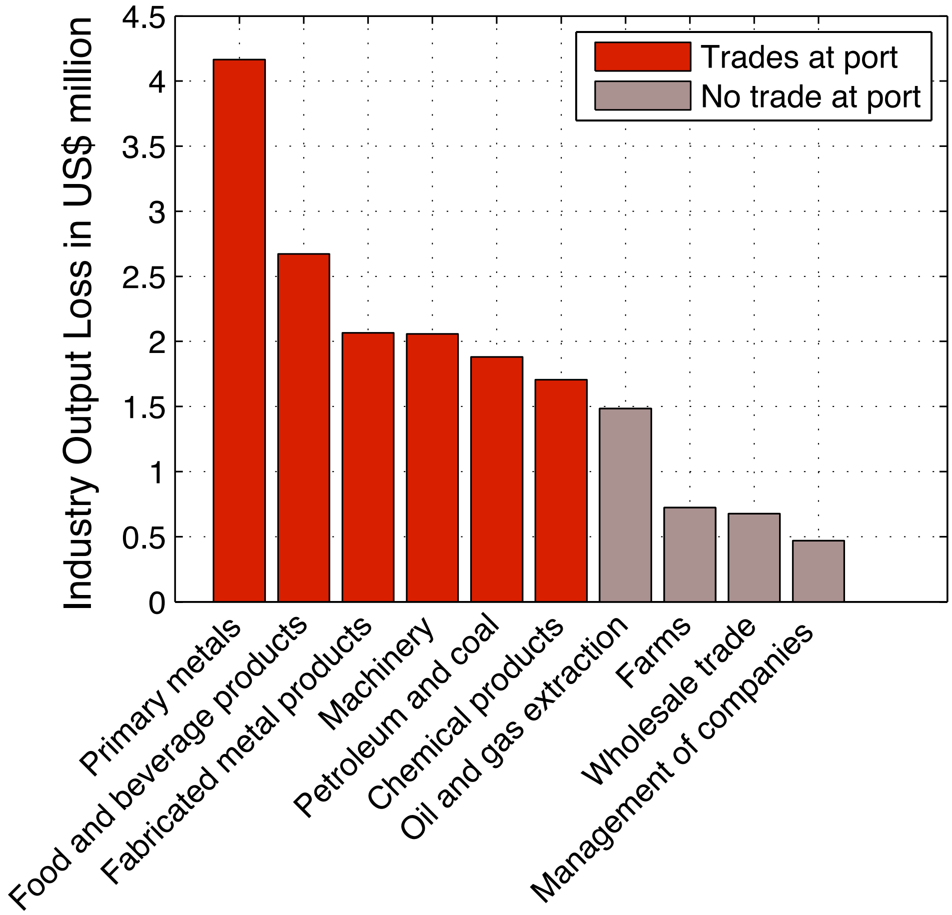


Figure 6.6: Output losses for Oklahoma industries due to total port shutdown

6.7 Summary and discussion

Multi-modal transportation systems are vital to the shipment of commodities among interdependent industries and across multiple regions, and freight disruptions at a number of nodes along the transportation system can have adverse impacts on the flow of commodities. One such node is an inland waterway port, whose risk studies in the literature have been sparse. This study provides a novel approach for modeling the adverse impact across interdependent industries and across multiple regions, resulting from a disruption in the operations of an inland port. The risk-based approach integrates the Multi-Regional Inoperability Input-Output Model (MRIIM)

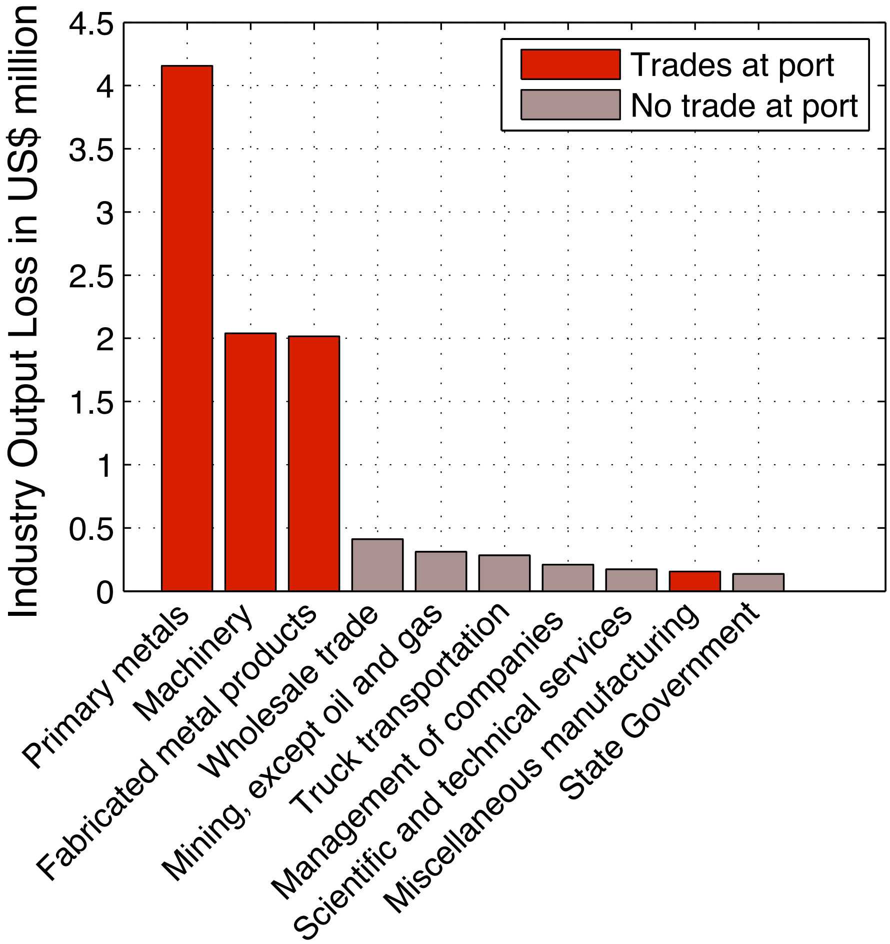


Figure 6.7: Output losses for Oklahoma industries due to Dry cargo dock shutdown

with a simulation model of inland port operations to quantify the impact of real disruptions in inter-regional commodity flow connected by a single terminal of usage in a multi-modal transportation system. The study considers three different disruption scenarios (terminal closure, crane outage, and departure stoppage) and quantifies interdependent impact in terms of inoperability (extent to which output is not being shipped and produced), and economic losses (dollar value of port inoperability).

The modeling approach depends upon existing and estimated data sources. The MRIIM is parameterized from commodity flow databases from the Bureau of Economic Analysis and the Bureau of Transportation Statistics, and the simulation model is parameterized by commodity flow data describing the operations of the inland port.

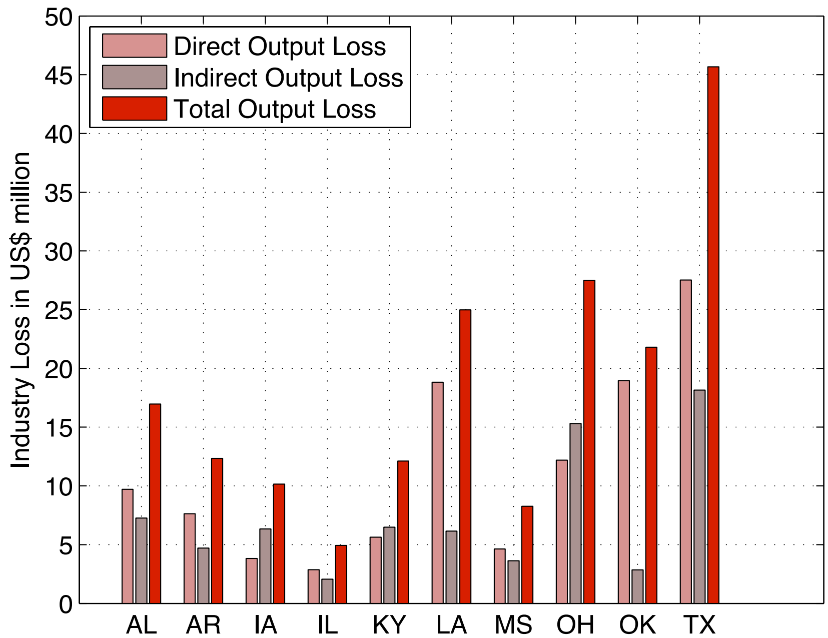


Figure 6.8: Estimates for industry direct and indirect losses across 10 states using Catoosa for commerce

The two-week disruption scenario for the inland Port of Catoosa in Oklahoma that we explore in this study shows that $45 million export-import losses in port industries result in $111.8 direct and $72.9 indirect losses for industry sector across 10 states. The ranking of sectors with most losses further highlights the fact that interdependence results in substantial losses of sectors that do not directly operate at the port. The dock specific losses we present here give an indication of the impacts on the port and industries if disruptions were more localized, identifying key industries or key docks facilitates investments to protect against and prepare for disruptive events.

The contributions of this paper are several. First, we broaden the scope of the MRIIM scheme (Crowther & Haimes, 2010) with the novel integration of the approach with multi-modal transportation systems for analyzing the interdependent adverse impacts of an inland port disruption. Second, our queuing model provides a much simpler analysis approach of tracking freight movement through the port compared to other discrete models (Simão & Powell, 1992). We also show how the components of our queuing model are altered due to disruptions, often ignored in much of the literature. Finally, the ultimate usefulness of our risk-based approach lies in its ability to measure the efficacy of risk management options. That is, investments in port protection (e.g., security, system hardening) may result in reduced initial effects of a disruption, and investments in preparedness (e.g., contingency routing options) may result in reduced downtime of a port. The approach described here is also useful to measure the interdependent and inter-regional benefit of implementing these risk management investments. Although this case study has been descriptive in nature, prescriptive uses of the model may be more useful and powerful.

Several opportunities for further research will be explored, including relaxing the equilibrium assumption of the MRIIM to include a dynamic analysis of inoperability and economic losses, and exploring more complex scenarios of port disruptions that highlight the realistic nature of man-made attacks, accidents, and natural disasters.

Chapter 7

Concluding Remarks

7.1 Summary and Conclusion

The overriding theme of this dissertation was to develop a static and dynamic resilience estimation scheme for interdependent sectors of national importance (DHS., 2009). To this end a methodology was constructed using the interdependent risk input-output models (Haimes & Jiang, 2001; Santos, 2006), which have been derived from economic input-output data and models (Leontief, 1966; Miller & Blair, 2009). The summary and outtakes from the research presented here are as follows:

1. An analysis of the interdependency models used in the study, with emphasis on the mathematical feasibility of these models, was presented in Chapter 2. Using eigenvalue analysis and convergence criteria rules, the stability criteria was developed for a convergent feasible solution of the dynamic models. The real world importance of the terms in the interdependency models were related to mathematical constructs, which helps establish certain guidelines for estimating or choosing the possible values of model parameters for practical and mathematical convenience. Such properties are for providing a prescription for choosing possible values for data or parameters, where data is sparse or there is no prior knowledge about the possible parameter values, and hence a design or expert-based judgement is required.

2. Chapter 3 presented resilience estimation models for static and dynamic systems. It was proposed that static resilience strengthening can be planned

through a risk management problem to reduce the demand perturbation effects for the interdependent systems, thereby lowering the inoperability effects of the disruptions. A general risk policy planning framework geared towards reducing the demand perturbations was presented. The guiding objective of the planning decision was the preference to lower the total economic losses for the entire economy. The solution of the risk management problem showed the effectiveness of budget allocation and the limit beyond which there is no need to allocate more budget. Hence a useful optimization based methodology was suggested for providing planners with simple metrics, and a prescription for strengthening static resilience in an interdependent economy.

3. Also in Chapter 3 dynamic resilience concepts were discussed. The lack of interdependent dynamic resilience estimation methods in the dynamic model domain motivated the formulation of metrics that describe resilience and provide a holistic resilience construct through the dynamic model. Resilience was constructed on the basis of the metrics (i) average sector level of functionality/operability, (ii) maximum inoperability/loss of functionality, and (iii) time to recovery from disruptions. Therefore, a complete picture for resilience estimation for dynamic systems was generated based on the notion that resilience should indicate the ability to maintain functionality and a speed to recovery.

4. The usefulness of generating a trio of resilience metrics for the dynamic system behavior helped in developing a decision-space that had both descriptive and prescriptive uses in resilience decision-making. The tradeoffs between the choice of resilience planning options which are reflected through the values taken by the metrics provides valuable insights into interdependent resilience planning.

5. An adaptive scheme for resilience estimation and dynamic model behavior was also provided in Chapter 3. This is a natural extension of the dynamic risk

input-output model, which has been missing from previous research. An adaptive model is a better representative of actual recovery behavior because there are multiple shocks and changing resilience properties that are exhibited by the system. The resilience metric schemes can be also applied to the adaptive model behavior.

6. Chapter 4 expanded on the static resilience planning framework of Chapter 3 with emphasis on developing optimization methods that account for uncertainties in the interdependent input-output framework. Such uncertainties produce varying planning risks some of which would not be captured through the nominal planning strategies. Given that the motivation of the planning strategies was to guarantee results that consider the maximal effects of the uncertainties, the solutions should be representative of the worst-case/best-case scenarios in the planning.

7. The robust formulation highlighted the nature of the uncertainties in data and event present in the problem. Each data was assumed to be realized within an interval, which gave rectangle bounded sets. For event uncertainties the introduction of chance constraints meant that due to the uncertainty of the events the feasibility of the planning constraints was not known with complete certainty. The limits, which give the bound the probabilities that the constraints are feasible or are violated, show the decision-makers preference and conservatism in handling the event uncertainty. The robust solutions guaranteed that maximal worst-case scenarios were generated and for planning this is an important consideration. Overall the robust method is a useful tool for the interdependency model analysis as the uncertainty sets constructed here can be applied to an uncertainty analysis of these models.

8. In Chapter 5 the rate parameter of the dynamic risk input-output model was es-

timated using concepts from dynamic data assimilation methodologies. Though such methods are available in other research domains their usage in the parameter estimation for the dynamic risk input-output model is a novel approach. Data generation for model calibration was achieved by interpreting the model as a resilience planning tool. The data was realized by setting performance targets to meet specific resilience goals. This is a new conceptual approach discussed here.

9. The parameter estimation problem was solved using a forward sensitivity scheme, which was based on calculating the sensitivity of the model state to the parameter being estimated. This approach has computational and practical relevance. Computationally sensitivity shows the effect of perturbation of the parameter on the model state, which translates to the magnitude of the effect on model state due to the parameter. More sensitivity indicates the parameter is significant in predicting the model states. Time evolving sensitivity shows the domain of analysis where the model does not respond to fluctuations in the parameter values. For the inverse problem this domain is important because it shows that the model parameter would not be able to predict the model state properly within the domain, hence the effect of observations on the model are negligent. Such consideration can be applied in the planning scheme because it is an indicator of the domain in which the performance targets need to be specified. In the end a convenient tool for predicting dynamic resilience in interdependent systems was developed.

10. The port study in Chapter 6 described the interdependent adverse effects of disruptive events on inter-regional commodity flows resulting from disruptions at an inland port terminal. This was done by integrating a risk-based Multi-Regional Inoperability Input-Output Model with models that simulate port

operations such as commodity arrival, unloading, sorting, and distributing. The study made several contributions to the homeland security literature in risk-based decision making for the little-explored, though vital, components of multi-modal commodity flows. There has been very limited research on interdependency risk analysis, particularly applied to inland waterway disruptions. Integrating these two research domains helped produce a framework that provides suitable metrics for risk-based decision making.

7.2 Future directions

There are several avenues of using and expanding the methods developed in this dissertation. Some of them are discussed below.

1. To begin with the static and dynamic resilience estimations framework needs to be integrated with the port and waterway network models to provide a real world application of the concepts presented here. Developing the resilience models for the port studies would help in testing and refining these models.

2. The robust static resilience optimization formulation can be improved by developing a more robust formulation for the chance constraint. This can be improved by developing a better uncertainty guarantee in the chance constraint.

3. The dynamic resilience parameter estimation problem considered here does not account for uncertainties in the model. There are several developments to the method if model uncertainties are incorporated. Robust concepts can be developed in similar fashion to the methodology presented for the static problem.

4. The port study can be further extended to include other network- and facility-specific details into the model. Uncertainties in analysis have not been discussed here and need a detailed investigation for a robust risk assessment and

planning. Overall, this work serves as an initial blueprint for further complex multi-regional interdependent risk analysis along inland waterways.

Bibliography

AAR. (2007). *National rail freight infrastructure capacity and investment study*. Technical Report Association of American Railroads.

Adger, W. (2000). Social and ecological resilience: are they related? *Progress in human geography*, *24*, 347–364.

Allen, N. (2011). US East Coast hit by record breaking snowstorms. The Telegraph. Accessed, October 2011.

Anderson, C., Santos, J., & Haimes, Y. (2007). A risk-based input–output methodology for measuring the effects of the august 2003 northeast blackout. *Economic Systems Research*, *19*, 183–204.

Andrijcic, E., & Horowitz, B. (2006). A macro-economic framework for evaluation of cyber security risks related to protection of intellectual property. *Risk analysis*, *26*, 907–923.

Archibald, G. (2005). *Information, incentives and the economics of control*. Cambridge University Press.

Atamturk, A., & Zhang, M. (2007). Two-stage robust network flow and design under demand uncertainty. *Operations Research*, *55*, 662–673.

Atkinson, A., Rainwater, L., & Smeeding, T. (1995). *Income distribution in OECD countries: evidence from the Luxembourg Income Study*. Organization for Economic Co-operation and Development Paris.

Barker, K. (2008). *Extensions of Inoperability Input-Output Modeling for preparedness decisionmaking: Uncertainty and inventory*. Ph.D. thesis University of Virginia.

Barker, K., & Haimes, Y. (2009). Uncertainty analysis of interdependencies in dynamic infrastructure recovery: Applications in risk-based decision making. *Journal of Infrastructure Systems*, *15*, 394.

Barker, K., & Santos, J. (2010a). Measuring the efficacy of inventory with a dynamic input-output model. *International Journal of Production Economics*, *126*, 130–143.

Barker, K., & Santos, J. (2010b). A risk-based approach for identifying key economic and infrastructure systems. *Risk Analysis*, *30*, 962–974.

BEA. (1997). Regional multipliers: A user handbook for the regional input-output modeling system (RIMS II). Bureau of Economic Analysis, U.S. Department of Commerce, Washington DC.

BEA. (2011). Interactive access to input-output accounts data. Bureau of Economic Analysis, U.S. Department of Commerce, Washington DC.

Ben-Tal, A., Boyd, S., & Nemirovski, A. (2006). Extending scope of robust optimization: Comprehensive robust counterparts of uncertain problems. *Mathematical Programming*, *107*, 63–89.

Ben-Tal, A., El Ghaoui, L., & Nemirovski, A. (2009). *Robust optimization*. Princeton University Press.

Ben-Tal, A., & Nemirovski, A. (2000). Robust solutions of linear programming problems contaminated with uncertain data. *Mathematical Programming*, *88*, 411–424.

Bertsimas, D., & Sim, M. (2004). The price of robustness. *Operations research*, *52*, 35–53.

Bertsimas, D., & Thiele, A. (2006). A robust optimization approach to inventory theory. *Operations Research*, *54*, 150–168.

Bessler, D., & Yang, J. (2003). The structure of interdependence in international stock markets. *Journal of International Money and Finance*, *22*, 261–287.

Bienstock, D., & Özbay, N. (2008). Computing robust basestock levels. *Discrete Optimization*, *5*, 389–414.

Blake, E., Rappaport, E., Landsea, C., & Center, T. (2007). *The deadliest, costliest, and most intense United States tropical cyclones from 1851 to 2006 (and other frequently requested hurricane facts)*. NOAA/National Weather Service, National Centers for Environmental Prediction, National Hurricane Center.

Bonabeau, E. (2002). Agent-based modeling: Methods and techniques for simulating human systems. *Proceedings of the National Academy of Sciences of the United States of America*, *99*, 7280–7287.

Boyd, S., & Vandenberghe, L. (2004). *Convex optimization*. Cambridge University Press.

Brookshire, D., Cochrane, H., & Steenson, J. (1997). Direct and indirect economic losses from earthquake damage. *Earthquake Spectra*, *13*, 683–701.

Brown, T., Beyeler, W., & Barton, D. (2004). Assessing infrastructure interdependencies: the challenge of risk analysis for complex adaptive systems. *International Journal of Critical Infrastructures*, *1*, 108–117.

Bruneau, M., Chang, S., Eguchi, R., Lee, G., ORourke, T., Reinhorn, A., Shinozuka, M., Tierney, K., Wallace, W., & von Winterfeldt, D. (2003). A framework to quantitatively assess and enhance the seismic resilience of communities. *Earthquake Spectra*, *19*, 733–752.

Bruneau, M., & Reinhorn, A. (2007). Exploring the concept of seismic resilience for acute care facilities. *Earthquake Spectra*, *23*, 41–62.

BTS. (2011). 2007 commodity flow survey overview and methodology. Bureau of Transportation Statistics.

Bullard, C., & Sebald, A. (1977). Effects of parametric uncertainty and technological change on input-output models. *The Review of Economics and Statistics*, *59*, 75–81.

Catoosa. (2011). Interactive access to website. Tulsa Port of Catoosa. Accessed October 2011.

Cavallo, E., Powell, A., & Becerra, O. (2010). Estimating the Direct Economic Damages of the Earthquake in Haiti*. *The Economic Journal*, *120*, 298–312.

Cavdaroglu, B., Hammel, E., Mitchell, J., Sharkey, T., & Wallace, W. (2010). Integrating restoration and scheduling decisions for disrupted interdependent infrastructure systems. *Annals of Operations Research*, (pp. 1–16).

CBO. (1983). *Public works infrastructure: policy considerations for the 1980s*. Technical Report Congress of the U.S., Congressional Budget Office.

Chang, S., & Shinozuka, M. (2004). Measuring improvements in the disaster resilience of communities. *Earthquake Spectra*, *20*, 739–755.

Charnes, A., & Cooper, W. (1959). Chance-constrained programming. *Management science*, *6*, 73–79.

Cho, S., Gordon, P., Moore II, J., Richardson, H., Shinozuka, M., & Chang, S. (2001). Integrating transportation network and regional economic models to estimate the costs of a large urban earthquake. *Journal of Regional Science*, *41*, 39–65.

Cimellaro, G., Reinhorn, A., & Bruneau, M. (2010). Seismic resilience of a hospital system. *Structure and Infrastructure Engineering*, *6*, 127–144.

CITK. (2006). *Report of the Critical Infrastructure Task Force*. Technical Report U.S. Homeland Security Advisory Council.

Cleveland, C. (2010). Deepwater horizon oil spill. *The Encyclopedia of Earth*, .

Clinton, W. (1996). *Executive Order 13010, establishing the president's commission on critical infrastructure protection (PCCIP)*. Technical Report U.S. Government Printing Office, Washington D.C.

Clinton, W. (1998). *63 (PDD-63), Protecting America's Critical Infrastructures*. Technical Report U.S. Government Printing Office, Washington D.C.

Comfort, L. (1999). *Shared risk: Complex systems in seismic response*. Emerald Group Publishing.

Crowther, K. (2007). *Development of a multiregional framework and demonstration of its feasibility for strategic preparedness of interdependent regions*. Ph.D. thesis University of Virginia.

Crowther, K. (2008). Decentralized risk management for strategic preparedness of critical infrastructure through decomposition of the inoperability input-output model. *International Journal on Critical Infrastructure Protection*, *1*, 53–67.

Crowther, K., & Haimes, Y. (2010). Development and deployment of the multiregional inoperability input-output model for strategic preparedness submitted to. *Systems Engineering*, *13*, 28–46.

Dantzig, G. (1955). Linear programming under uncertainty. *Management Science*, *1*, 197–206.

DHS. (2009). *National Infrastructure Protection Plan - partnering to enhance protection and resiliency 2009*. Technical Report U.S. Department of Homeland Security.

Draper, N., & Smith, H. (1998). *Applied regression analysis*. John Wiley & Sons.

Duchin, F., & Szyld, D. (1985). A dynamic input-output model with assured positive output (*). *Metroeconomica*, *37*, 269–282.

Dueñas-Osorio, L., Craig, J., & Goodno, B. (2007). Seismic response of critical interdependent networks. *Earthquake engineering & structural dynamics*, *36*, 285–306.

EIA. (2010). *Annual Energy Outlook 2010 With Projections to 2035*. Technical Report U.S. Department of Energy, Energy Information Administration.

Fackler, M. (2011). Powerful Quake and Tsunami Devastate Northern Japan. The New York Times. Accessed March 2011.

Franklin, G., Powell, J., Emami-Naeini, A., & Powell, J. (1994). *Feedback control of dynamic systems* volume 2. Addison-Wesley Reading, MA.

Ghosh, A. (1958). Input-output approach in an allocation system. *Economica*, *25*, 58–64.

Godschalk, D. (2004). Land use planning challenges: coping with conflicts in visions of sustainable development and livable communities. *Journal of the American Planning Association*, *70*, 5–13.

Gordon, P., Moore II, J., Park, J., & Richardson, H. (2007). The economic impacts of a terrorist attack on the us commercial aviation system. *Risk Analysis*, *27*, 505–512.

Gordon, P., Moore II, J., Richardson, H., & Pan, Q. (2005). The economic impact of a terrorist attack on the twin ports of los angeles-long beach. In P. Gordon, H. Richardson, & J. Moore II (Eds.), *The economic impacts of terrorist attacks* (pp. 262–289). Edward Elgar Publishing.

Grabowski, M., & Roberts, K. (1997). Risk mitigation in large-scale systems: Lessons from high reliability organizations. *California Management Review*, *39*, 152–162.

Groetsch, C. (1984). *The theory of Tikhonov regularization for Fredholm equations of the first kind* volume 105. Pitman Boston.

Haimes, Y. (1991). Total risk management. *Risk Analysis*, *11*, 169–171.

Haimes, Y. (2009). *Risk modeling, assessment, and management*. Wiley.

Haimes, Y., Crowther, K., & Horowitz, B. (2008). Homeland security preparedness: Balancing protection with resilience in emergent systems. *Systems Engineering*, *11*, 287–308.

Haimes, Y., Horowitz, B., Lambert, J., Santos, J., Crowther, K., & Lian, C. (2005a). Inoperability input-output model for interdependent infrastructure sectors. II: Case studies. *Journal of Infrastructure Systems*, *11*, 80–92.

Haimes, Y., Horowitz, B., Lambert, J., Santos, J., Lian, C., & Crowther, K. (2005b). Inoperability input-output model for interdependent infrastructure sectors. I: Theory and methodology. *Journal of Infrastructure Systems*, *11*, 67–79.

Haimes, Y., & Jiang, P. (2001). Leontief-based model of risk in complex interconnected infrastructures. *Journal of Infrastructure Systems*, *7*, 1–12.

Hallegatte, S. (2007). The economics of natural disasters. In *AGU Spring Meeting Abstracts* (p. 02). volume 1.

Hallegatte, S. (2008). An adaptive regional input-output model and its application to the assessment of the economic cost of katrina. *Risk analysis*, *28*, 779–799.

Ham, H., Kim, T., & Boyce, D. (2005a). Assessment of economic impacts from unexpected events with an interregional commodity flow and multimodal transportation network model. *Transportation Research Part A: Policy and Practice*, *39*, 849–860.

Ham, H., Kim, T., & Boyce, D. (2005b). Implementation and estimation of a combined model of interregional, multimodal commodity shipments and transportation network flows. *Transportation Research Part B: Methodological*, *39*, 65–79.

Handmer, J., & Dovers, S. (1996). A typology of resilience: Rethinking institutions for sustainable development. *Organization & Environment*, *9*, 482–511.

Hasley, A. (2010). Failing U.S. transportation system will imperil prosperity, report finds. The Washington Post.

Hays, J., & Kachi, A. (2008). Estimating interdependent duration models with an application to government formation and survival. In *Plenary Session Paper given at the Society for Political Methodologys 25th Annual Summer Conference, University of Michigan, Ann Arbor*.

Heal, G., Kearns, M., Kleindorfer, P., & Kunreuther, H. (2006). Interdependent security in interconnected networks. In P. Auerswald, L. Branscomb, T. LaPorte, & E. Michel-Kerjan (Eds.), *Seeds of Disaster, Roots of Response: How Private Action Can Reduce Public Vulnerability*. Cambridge University Press, New York.

Hendrickson, C., Horvath, A., Joshi, S., & Lave, L. (1998). Economic input–output models for environmental life-cycle assessment. *Environmental Science & Technology*, *32*, 184–191.

Holling, C. (1973). Resilience and stability of ecological systems. *Annual review of ecology and systematics*, *4*, 1–23.

Horowitz, K., & Planting, M. (2006). *Concepts and methods of the US input-output accounts*. Technical Report Bureau of Economic Analysis.

Hu, G., Yang, J., & Liu, W. (1998). Instability and controllability of linearly coupled oscillators: Eigenvalue analysis. *Physical Review E*, *58*, 4440–4453.

Huber, W. (2010). Ignorance is not probability. *Risk Analysis*, *30*, 371–376.

Infanger, G. (1994). *Planning under uncertainty: solving large-scale stochastic linear programs*. Boyd & Fraser San Francisco, CA.

Isard, W., Azis, I., Drennan, M., Miller, R., Saltzman, S., & Thorbecke, E. (1998). *Methods of interregional and regional analysis*. Ashgate Aldershot.

Ishfaq, R., & Sox, C. (2010). Intermodal logistics: The interplay of financial, operational and service issues. *Transportation Research Part E: Logistics and Transportation Review*, *46*, 926–949.

Jiang, P., & Haimes, Y. (2004). Risk management for Leontief-Based Interdependent Systems. *Risk Analysis*, *24*, 1215–1229.

Kall, P., & Mayer, J. (2010). *Stochastic linear programming: models, theory, and computation* volume 156. Springer Verlag.

Kalnay, E. (2003). *Atmospheric modeling, data assimilation, and predictability*. Cambridge University Press.

Kanamitsu, M. (1989). Description of the NMC global data assimilation and forecast system. *Weather and Forecasting*, *4*, 335–342.

Kelton, C., Pasquale, M., & Rebelein, R. (2008). Using the North American Industry Classification System (NAICS) to identify national industry cluster templates for applied regional analysis. *Regional Studies*, *42*, 305–321.

Kendra, J., & Wachtendorf, T. (2003). Elements of resilience after the World Trade Center disaster: Reconstituting New York City's emergency operations centre. *Disasters*, *27*, 37–53.

Kim, Y., Spencer Jr, B., Song, J., Elnashai, A., & Stokes, T. (2007). *Seismic performance assessment of interdependent lifeline systems*. Technical Report Mid-America Earthquake Center, University of Illinois at Urbana-Champaign.

Klein, R., Nicholls, R., & Thomalla, F. (2003). Resilience to natural hazards: How useful is this concept? *Global Environmental Change Part B: Environmental Hazards*, *5*, 35–45.

Kujawski, E. (2006). Multi-period model for disruptive events in interdependent systems. *Systems engineering*, *9*, 281–295.

Lahr, M., & Dietzenbacher, E. (2001). *Input-output analysis: frontiers and extensions*. Palgrave.

Lakshmivarahan, S., & Lewis, J. (2010). Forward sensitivity approach to dynamic data assimilation. *Advances in Meteorology*, *2010*, 1–13.

Lee, E., Mitchell, J., & Wallace, W. (2009). Network flow approaches for analyzing and managing disruptions to interdependent infrastructure systems. *Wiley Handbook of Science and Technology for Homeland Security*, *2*, 1419–1428.

Lee, T., Park, N., & Lee, D. (2003). A simulation study for the logistics planning of a container terminal in view of scm. *Maritime Policy & Management*, *30*, 243–254.

Leontief, W. (1936). Quantitative input and output relations in the economic systems of the united states. *The Review of Economics and Statistics*, *18*, 105–125.

Leontief, W. (1941). *The structure of American economy, 1919-1929: An empirical application of equilibrium analysis*. Harvard University Press.

Leontief, W. (1951). *The Structure of American Economy, 1919-1939: An Empirical Application of Equilibrium Analysis-Enlarged*. Oxford University Press.

Leontief, W. (1966). *Essays in economics: Theories, theorizing, facts, and policies*. Transaction Publishers.

Leontief, W. (1986). *Input-output economics*. Oxford University Press.

Leontief, W., Dietzenbacher, E., & Lahr, M. (2004). *Wassily Leontief and input-output economics*. Cambridge Univ Pr.

Lewis, J., Lakshmivarahan, S., & Dhall, S. (2006). *Dynamic data assimilation*. Cambridge University Press.

Lewis, T. (2006). *Critical infrastructure protection in homeland security: Defending a networked nation*. John Wiley and Sons.

Lian, C., & Haimes, Y. (2006). Managing the risk of terrorism to interdependent infrastructure systems through the dynamic inoperability input–output model. *Systems engineering*, *9*, 241–258.

Luenberger, D., & Arbel, A. (1977). Singular dynamic Leontief systems. *Econometrica: Journal of the Econometric Society*, *45*, 991–995.

MA. (2011). *Maritime Administration America's Marine Highway Report to Congress, April 2011*. Technical Report U.S. Department of Transportation, Marine Administration.

MacKenzie, C., Barker, K., & Grant, F. (2012a). Evaluating the consequences of an inland waterway port closure with a dynamic multiregional interdependence model. *Systems, Man and Cybernetics, Part A: Systems and Humans, IEEE Transactions on*, *42*, 359–370.

MacKenzie, C., Baroud, H., & Barker, K. (2012b). Optimal resource allocation for recovery of interdependent systems: Case study of the deepwater horizon oil spill. In *2012 Industrial and Systems Engineering Research Conference*. To appear.

Markowsky, G. (2009). Comparing apples and oranges how to select the most probable targets. In *Technologies for Homeland Security, 2009. HST'09. IEEE Conference on* (pp. 615–620). IEEE.

McDaniels, T., Chang, S., Cole, D., Mikawoz, J., & Longstaff, H. (2008). Fostering resilience to extreme events within infrastructure systems: characterizing decision contexts for mitigation and adaptation. *Global Environmental Change*, *18*, 310–318.

Meyer, C. (2000). *Matrix analysis and applied linear algebra: solutions manual* volume 2. Society for Industrial and Applied Mathematics.

Mileti, D. (1999). *Disasters by design: A reassessment of natural hazards in the United States*. National Academy Press.

Miller, R., & Blair, P. (2009). *Input-output analysis*. Cambridge University Press.

Min, H., Beyeler, W., Brown, T., Son, Y., & Jones, A. (2007). Toward modeling and simulation of critical national infrastructure interdependencies. *IIE Transactions*, *39*, 57–71.

Moon, T., & Stirling, W. (2000). *Mathematical methods and algorithms for signal processing* volume 204. Prentice hall.

Noy, I. (2009). The macroeconomic consequences of disasters. *Journal of Development Economics*, *88*, 221–231.

OECD. (2011). Interactive access to input-output accounts. Organization for Economic Co-Operation and Development.

OED. (2012). Resilience, n. Oxford English Dictionary.

Okuyama, Y., Hewings, G., & Sonis, M. (2004). Measuring economic impacts of disasters: interregional input-output analysis using sequential interindustry model. In Y. Okuyama, & S. Chang (Eds.), *Modeling Spatial and Economic Impacts of Disasters* (pp. 77–101). Springer Verlag.

Oliva, G., Panzieri, S., & Setola, R. (2010). Agent-based input-output interdependency model. *International Journal of Critical Infrastructure Protection*, *3*, 76–82.

Oosterhaven, J. (1988). On the plausibility of the supply-driven input-output model. *Journal of Regional Science*, *28*, 203–217.

Orsi, M., & Santos, J. (2010). Probabilistic modeling of workforce-based disruptions and input–output analysis of interdependent ripple effects. *Economic Systems Research*, *22*, 3–18.

Outkin, A., Flaim, S., Seirp, A., & Gavrilov, J. (2008). FinSim: A framework for modeling financial system interdependencies. In Y. Shan, & A. Yang (Eds.), *Applications of Complex Adaptive Systems* (pp. 257–277). IGI Global.

Pant, R., Barker, K., Grant, F., & Landers, T. (2011). Interdependent impacts of inoperability at multi-modal transportation container terminals. *Transportation Research Part E: Logistics and Transportation Review*, *47*, 722–737.

Paté-Cornell, M. (1996). Uncertainties in risk analysis: Six levels of treatment. *Reliability Engineering & System Safety*, *54*, 95–111.

Pederson, P., Dudenhoeffer, D., Hartley, S., & Permann, M. (2006). *Critical infrastructure interdependency modeling: a survey of US and international research*. Technical Report Idaho National Laboratory.

Pelling, M. (2003). *The vulnerability of cities: natural disasters and social resilience*. Earthscan/James & James.

Pendall, R., Foster, K., & Cowell, M. (2010). Resilience and regions: building understanding of the metaphor. *Cambridge Journal of Regions, Economy and Society*, *3*, 71–84.

Percoco, M. (2006). A note on the inoperability input-output model. *Risk analysis*, *26*, 589–594.

Percoco, M., Hewings, G., & Senn, L. (2006). Structural change decomposition through a global sensitivity analysis of input–output models. *Economic Systems Research*, *18*, 115–131.

Polenske, K. (1980). *US multiregional input-output accounts and model*. DC Heath and Company, Lexington, MA.

Quandt, R. (1958). Probabilistic errors in the leontief system. *Naval Research Logistics Quarterly*, *5*, 155–170.

Quandt, R. (1959). On the solution of probabilistic leontief systems. *Naval Research Logistics Quarterly*, *6*, 295–305.

Reichle, R., McLaughlin, D., & Entekhabi, D. (2002). Hydrologic data assimilation with the ensemble Kalman filter. *Monthly Weather Review*, *130*, 103–114.

Rinaldi, S., Peerenboom, J., & Kelly, T. (2001). Identifying, understanding, and analyzing critical infrastructure interdependencies. *Control Systems, IEEE*, *21*, 11–25.

Rodrigue, J., Debrie, J., Fremont, A., & Gouvernal, E. (2010). Functions and actors of inland ports: European and north american dynamics. *Journal of Transport Geography*, *18*, 519–529.

Rose, A. (2004a). Defining and measuring economic resilience to disasters. *Disaster Prevention and Management*, *13*, 307–314.

Rose, A. (2004b). Economic principles, issues, and research priorities in hazard loss estimation. In Y. Okuyama, & S. Chang (Eds.), *Modeling Spatial and Economic Impacts of Disasters* (pp. 13–36). Springer Verlag.

Rose, A. (2007). Economic resilience to natural and man-made disasters: Multidisciplinary origins and contextual dimensions. *Environmental Hazards*, *7*, 383–398.

Rose, A. (2009a). *Economic resilience to disasters, CARRI Research Report 8*. Center for Risk and Economic Analysis of Terrorism Events University of Southern California. Final Report to Community and Regional Resilience Institute.

Rose, A. (2009b). A framework for analyzing the total economic impacts of terrorist attacks and natural disasters. *Journal of Homeland Security and Emergency Management*, *6*, 1–27.

Rose, A., & Allison, T. (1989). On the plausibility of the supply-driven input-output model: Empirical evidence on joint stability*. *Journal of regional science*, *29*, 451–458.

Rose, A., Benavides, J., Chang, S., Szczesniak, P., & Lim, D. (1997). The regional economic impact of an earthquake: Direct and indirect effects of electricity lifeline disruptions. *Journal of Regional Science*, *37*, 437–458.

Rose, A., & Liao, S. (2005). Modeling regional economic resilience to disasters: A computable general equilibrium analysis of water service disruptions. *Journal of Regional Science*, *45*, 75–112.

Santos, J. (2006). Inoperability input-output modeling of disruptions to interdependent economic systems. *Systems engineering*, *9*, 20–34.

Santos, J., Barker, K., Paul, J., & Iv, Z. (2008). Sequential decision-making in interdependent sectors with multiobjective inoperability decision trees: application to biofuel subsidy analysis. *Economic Systems Research*, *20*, 29–56.

Santos, J., & Haimes, Y. (2004). Modeling the demand reduction input-output (I-O) inoperability due to terrorism of interconnected infrastructures*. *Risk Analysis*, *24*, 1437–1451.

Schmitt, R., Sprung, M., Rick, C., & Sedor, J. (2010). *Freight Facts and Figures 2010*. Technical Report U.S. Department of Transportation, Federal Highway Administration.

Schrank, D., Lomax, T., & Turner, S. (2010). *TTI's 2010 Urban Mobility Report*. Technical Report Texas Transportation Institute.

Simão, H., & Powell, W. (1992). Numerical methods for simulating transient, stochastic queueing networks-ii: Experimental design. *Transportation Science*, *26*, 312–329.

Simmie, J., & Martin, R. (2010). The economic resilience of regions: towards an evolutionary approach. *Cambridge Journal of Regions, Economy and Society*, *3*, 27–43.

Sohn, J., Hewings, G., Kim, T., Lee, J., & Jang, S. (2004). Analysis of economic impacts of an earthquake on transportation network. In Y. Okuyama, & S. Chang (Eds.), *Modeling Spatial and Economic Impacts of Disasters* (pp. 233–256). Springer Verlag.

Solow, R. (1956). A contribution to the theory of economic growth. *The Quarterly Journal of Economics*, *70*, 65–94.

Soyster, A. (1973). Convex programming with set-inclusive constraints and applications to inexact linear programming. *Operations research*, *21*, 1154–1157.

Steenge, A., & Thissen, M. (2005). A new matrix theorem: interpretation in terms of internal trade structure and implications for dynamic systems. *Journal of Economics*, *84*, 71–94.

Stewart, W. (2009). *Probability, Markov chains, queues, and simulation: the mathematical basis of performance modeling*. Princeton University Press.

Stone, R. (1961). *Input-output and national accounts*. Organization for Economic Co-Operation and Development.

Stone, R., Bacharach, M., & Bates, J. (1963). *Input-Output Relationships 1954-1966*. Chapman and Hall.

Tatano, H., & Tsuchiya, S. (2008). A framework for economic loss estimation due to seismic transportation network disruption: A spatial computable general equilibrium approach. *Natural Hazards*, *44*, 253–265.

Ten Raa, T. (2005). *The economics of input-output analysis*. Cambridge University Press.

Tierney, K. (1997). Business impacts of the northridge earthquake. *Journal of Contingencies and Crisis Management*, *5*, 87–97.

Tierney, K., & Bruneau, M. (2007). Conceptualizing and measuring resilience: a key to disaster loss reduction. *TR news*, *250*, 14–17.

Tikhonov, A., Arsenin, V., & John, F. (1977). *Solutions of ill-posed problems*. Winston Washington, DC.

Timmerman, P. (1981). *Vulnerability, resilience, and the collapse of society*. Environmental Monograph/Institute for Environmental Studies University of Toronto.

TISP. (2011). *Regional Disaster resilience Guide for Developing an Action Plan*. Technical Report The Infrastructure Security Partnership, American Society of Civil Engineers.

Ulieru, M. (2007). Design for resilience of networked critical infrastructures. In *Digital EcoSystems and Technologies Conference, 2007. DEST'07. Inaugural IEEE-IES* (pp. 540–545). IEEE.

UN. (2009). *System of National Accounts 2008*. Technical Report United Nations.

USAC. (2009). *Waterborne Commerce of the United States: Calender Year 2009*. Technical Report U.S. Army Corps.

USAC. (2011). Interactive access to the website. U.S. Army Corps. Accessed October 2011.

Vugrin, E., Warren, D., Ehlen, M., & Camphouse, R. (2010). *A Framework for Assessing the Resilience of Infrastructure and Economic Systems*. Springer-Verlag, Inc., forthcoming.

Wei, H., Dong, M., & Sun, S. (2010). Inoperability input-output modeling (iim) of disruptions to supply chain networks. *Systems Engineering*, *13*, 324–339.

Wildavsky, A. B. (1988). *Searching for safety* volume 10. Transaction publishers.

Wilson, M. (2007). The impact of transportation disruptions on supply chain performance. *Transportation Research Part E: Logistics and Transportation Review*, *43*, 295–320.

Zhang, P., Peeta, S., & Friesz, T. (2005). Dynamic game theoretic model of multi-layer infrastructure networks. *Networks and Spatial Economics*, *5*, 147–178.

Zhu, S., Levinson, D., Liu, H., & Harder, K. (2010). The traffic and behavioral effects of the i-35w mississippi river bridge collapse. *Transportation Research Part A: Policy and Practice*, *44*, 771–784.

Zobel, C. (2010). Comparative visualization of predicted disaster resilience. In *Proceedings of the 7th International ISCRAM Conference*.

Zobel, C. (2011). Representing perceived tradeoffs in defining disaster resilience. *Decision Support Systems*, *50*, 394–403.

Zobel, C., & Khansa, L. (2011). Characterizing multi-event disaster resilience. Computers & Operations Research. Forthcoming.