COMPARATIVE STUDY OF METHODS OF ANALYSIS OF PRESTRESSED CONCRETE CONTINUOUS BEAMS

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C. M. D.

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NOMENCLATURE

i, j, k . . . . Letters designating intermediate supports.

x, x' . . . . Coordinates of cross section.

cgc . . . . Center of gravity of concrete section.

cgs . . . . Center of gravity of steel.

e . . . . Excentricity of prestress.

m_j . . . . Starting moments.

r_{ij} . . . . Carry over factor

b, c, d, e . . . . Algebraic carry over factors.

w . . . . Intencity of uniform load.

t_1, t_2 . . . . Angular load function coefficients.

m . . . . Position coefficient.

m' . . . . Position coefficient from support to the position of moving load.

f'c . . . . Cylinder strength at 28 days.

$f^t_{H_iT}$, $f^b_{H_iT}$ . . . . . Stress at the top or bottom fiber of girder due to initial prestress force and all loads.

$f^t_{H_fT}$, $f^b_{H_fT}$ . . . . . Stress at the top or bottom fiber of girder due to final prestress force and all loads.

A, B, C, D . . . . . . Notation for unyielding supports .

L . . . . Length of span.

M_i, M_j . . . . Final moments at i and j respectively.

$M^{(B)}$. . . . . Moment coefficients due to unit starting moment at (B).

BM_x . . . . Bending moment at x due to loads.

H . . . . Prestressing force.

H_i . . . . Initial prestress.

H_f . . . . Final prestress.

F_{ij} . . . . Angular flexibility.

G_{ji} . . . . Angular carry over value.

E . . . . Modulus of elasticity.

I_x . . . . Moment of inertia with respect to x.

U . . . . Strain energy.

U_L . . . . External work due to loads.

U_R . . . . External work due to reactions.

C.O.F . . . . Carry over factors. (abbreviation)

S.M. . . . . Starting moment. (abbreviation)

C.O.M. . . . . Carry over moment. (abbreviation)

K . . . . Stiffness factor.

K' . . . . Modified stiffness factor.

CK . . . . Carry over stiffness factor.

D_{ji} . . . . Distribution factor.

CD_{ji} . . . . Carry over distribution factor.

$FM^{(L)}$. . . . . Fixed end moment due to loads.

$FM^{(H)}$. . . . . Fixed end moment due to prestress.

$FM^{(\Delta)}$. . . . . Fixed end moment due to displacement of supports.

(GL) . . . . Gravity load.

(UL) . . . . Uniform live load.

P . . . . Concentrated live load.

Q . . . . Moving load position designation.

C_t . . . . Distance from neutral axis to top fiber.

C_b . . . . Distance from neutral axis to bottom fiber.

Z_b, Z_t . . . . Section modulus of the girder about its centrotral axis with respect to top and bottom fiber.

A . . . . Area of girder concrete section. (steel not deduced)

M_G . . . . Bending moment due to dead load of girder.

M_L . . . . Bending moment due to live load plus impact.

$\tau_{ji}^{(L)}$ . . . . Angular load functions.

$\tau_{ij}^{(H)}$ . . . . Angular prestress functions.

Δ . . . . Vertical displacement.

θ_j, θ_i . . . . Angular rotations at j and i respectively.

Σ . . . . Summation.

N, γ, η, ψ. . . . . . Denoted constants.

CHAPTER I

HISTORICAL DEVELOPEMENT OF THE ANALYSIS OF STATICALLY INDETERMINATE PRESTRESSED CONCRETE BEAMS

An analysis of continuous concrete beams was first undertaken by G. Magnel of Belgium (1, 2). The analysis included continuous beams with equal spans worked out by virtual work method with reactions at supports selected as redundants.

E. Freyssinet (3) of France, contributed practical notes for designers on methods and applications to various continuous and discontinuous structures.

A. L. Parme and G. H. Paris (4) determined the redundant moments at the supports due to prestressing by one cycle distribution method.

D. W. Cracknell and W. A. Knight (5) suggested the method of "cut" in their paper. By this method the beam is assumed to be cut at the points of support so that the moment continuity is destroyed. The relative slope of the ends of members meeting at the cut are easily obtained by area moment propositions, and the method proceeds directly to the determination of the moments required to re-establish continuity.

E.G. Trimble (6) of England calculated the fixed end moments due to prestress and distributed these by Hardy Cross Method.

Y. Guyon (7) investigated the pressure curves and their relation to the line of gravity due to resulting conditions imposed on the cable, the effect of the superposition of external loading, and several methods of determining "concordant" cable lines satisfying given conditions.

R. B. B. Moorman (8) introduced "the equivalent load method" for the analysis. The effect of cable tension can be expressed as distributed or concentrated load from the shape of the moment diagram due to prestress. The analysis becomes a simple matter of applying any ordinary method.

A. E. Komendant (9) deserves special credit for discussing prestressed continuous truss girders.

E. I. Feisenheiser (10) made use of the advantage of prestressing combined with continuity in the determination of fixed end moment formulas for various conditions of prestressing, which helped to make combination feasible. The line of thrust and the kern boundary concepts are advocated for the use of design.

Newmark (11, 12) applied his numerical procedure in analysis of continuous prestressed beams. The rotation of the beams over the supports can be determined by this procedure.

Moorman's "equivalent load method" was applied to the moment distribution method by T. Y. Lin (13).

Kao (14) presented the example of slope deflection method to prestressed continuous beams.

The virtual work method with the support moments selected as redundants was used by the writer to present the example of Clapeyron's three moment equation to the extension of prestress.

Finally, the application of the Carry-Over concept originated by J. J. Tuma (15) was extended to prestressed concrete continuous beams by Munshi (16).

CHAPTER II

DERIVATIONS OF THE SELECTED METHODS

Selected Methods:

(II A) Virtual Work

(II B) Carry Over

(II C) Slope Deflection

(II D) Moment Distribution

Definition of Problem.

The continuous prestressed beam of variable cross section acted on by a general system of transverse loads, with the prestressing cable of any shape is considered: (Fig. 1)

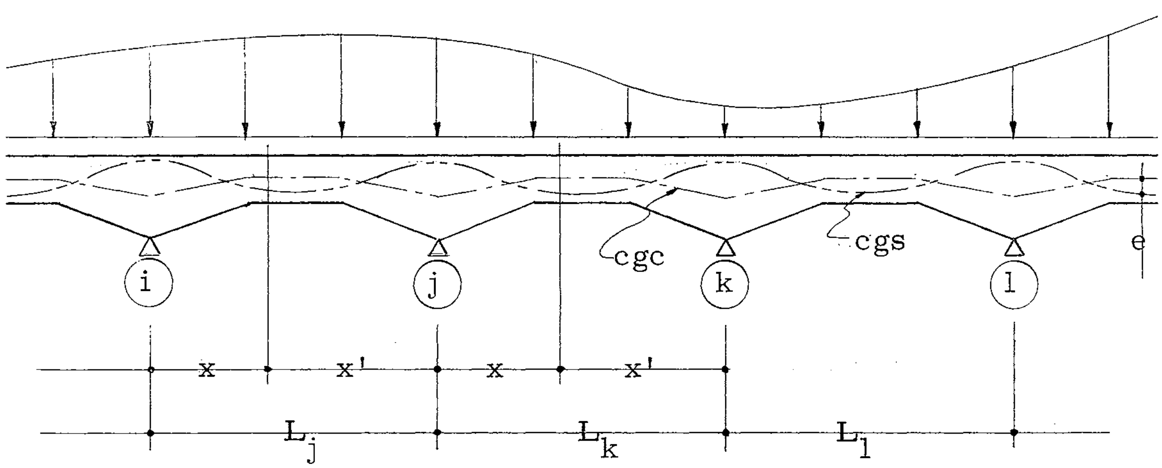


Fig. 1
GENERAL SKETCH

The bending moment on any arbitrarily selected section of the basic structure ij can be written as:

$$M_x^{(i)} = BM_x^{(i)} + M_i \frac{x'}{L_j} + M_j \frac{x}{L_j} + He \qquad (\text{Eq } 1\text{-a})$$
$$x = 0 \rightarrow L_j$$

Similarly:

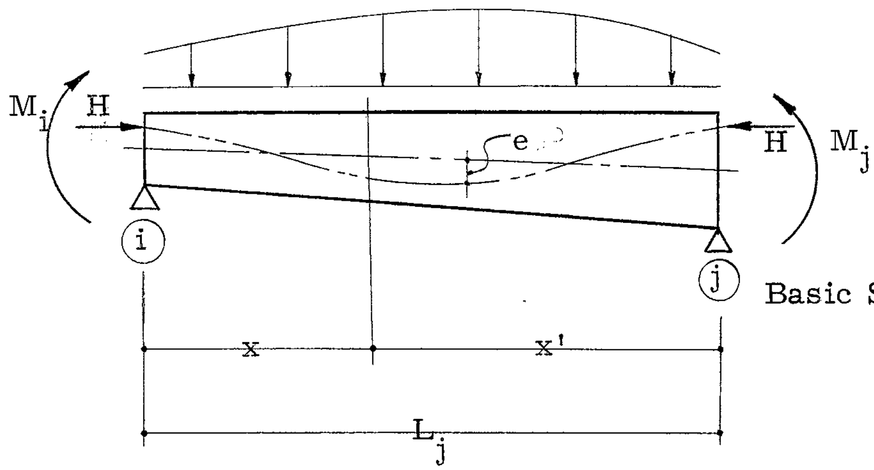
$$M_x^{(j)} = BM_x^{(j)} + M_j \frac{x'}{L_k} + M_i \frac{x}{L_k} + He \qquad (\text{Eq } 1\text{-b})$$
$$x = 0 \rightarrow L_k$$

The vertical reactions in terms of the end moments are:

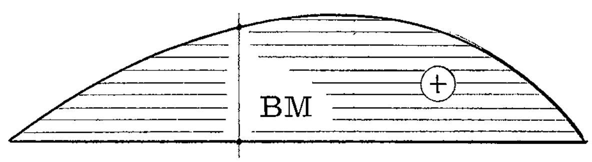
$$R_{iy} = \frac{M_j - M_i}{L_j} + BR_{iy} \qquad (\text{Eq } 2\text{-a})$$

$$R_{jy} = \frac{M_i - M_j}{L_j} + BR_{jy} \qquad (\text{Eq } 2\text{-b})$$

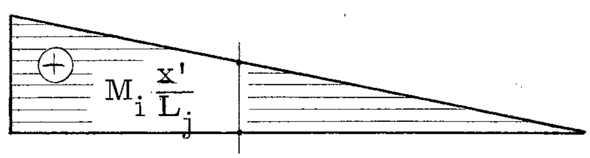
where BR_{iy} is the reaction of a simple beam at i due to the applied loads.



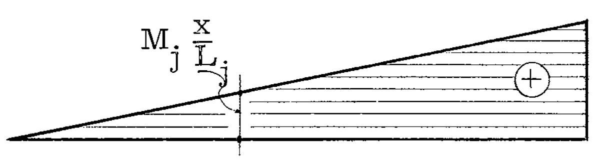
Basic Structure



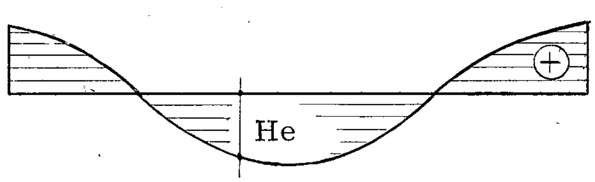
Bending Moment due to Loads



Bending Moment due to M_i



Bending Moment due to M_j



Bending Moment due to Prestressing Force (H)

(He diagram is plotted by measuring the cgs eccentricity from the cgc.)

Free Body Diagram of Span ij for Bending Moments. Fig. 2

BEAM FUNCTIONS TABLE I

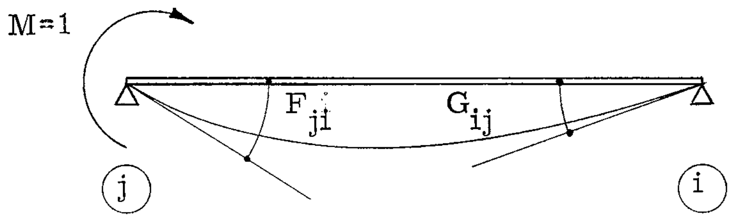


Fig. 3-a

$$\int_j^i \frac{x^2 dx}{L^2 EI_x} = F_{ji}$$

The Angular Flexibility (F_{ji}) is the end slope of the simple beam ij at j due to a unit moment applied at that end. Fig. 3-a

$$\int_j^i \frac{xx' \, dx}{L^2 EI_x} = G_{ij}$$

The Carry Over (G_{ij}) is the end slope of the simple beam ij at i due to unit moment applied at the far end j. Fig. 3-a

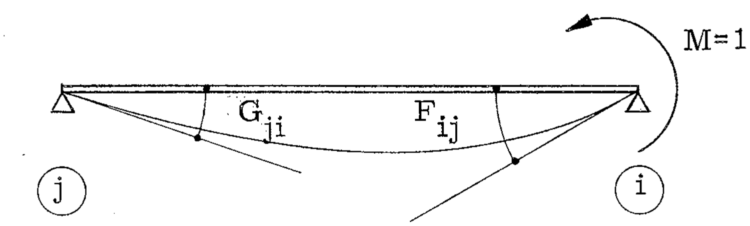


Fig. 3-b

$$\int_j^i \frac{x'^2 \, dx}{L^2 \, EI_x} = F_{ij}$$

Angular Flexibility (F_{ij}) is the end slope of the simple beam ij at j due to a unit moment applied at that end. Fig. 3-b

$$\int_j^i \frac{x'x \, dx}{L^2 \, EI_x} = G_{ji}$$

The Carry Over (G_{ji}) is the end slope of the simple beam ij at j due to a unit moment applied at the far end i. Fig. 3-b

TABLE I continued

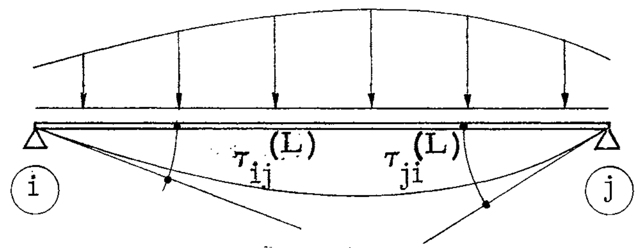


Fig. 3-c

$$\int_i^j \frac{BM_x\, x'\, dx}{LEI_x} = \tau_{ij}^{(L)}$$	$$\int_i^j \frac{BM_x\, x\, dx}{LEI_x} = \tau_{ji}^{(L)}$$
<u>Angular Load Function</u> ($\tau_{ij}^{(L)}$) is the end slope of the simple beam ij at i due to loads. Fig. 3-c	<u>Angular Load Function</u> ($\tau_{ji}^{(L)}$) is the end slope of the simple beam ij at j due to loads. Fig. 3-c

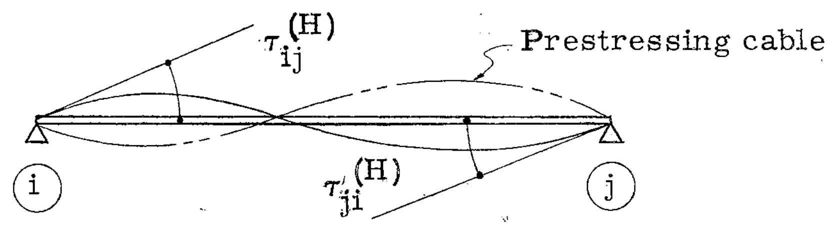


Fig. 3-d

$$\int_i^j \frac{He\, x'\, dx}{LEI_x} = \tau_{ij}^{(H)}$$	$$\int_i^j \frac{He\, x\, dx}{LEI_x} = \tau_{ji}^{(H)}$$
<u>Angular Prestress Function</u> ($\tau_{ij}^{(H)}$) is the end slope of the simple beam ij at i due to prestress. Fig. 3-d	<u>Angular Prestress Function</u> ($\tau_{ji}^{(H)}$) is the end slope of the simple beam ij at j due to prestress. Fig. 3-d

(II A) VIRTUAL WORK

Derivation of Three Moment Equation

The spans $\overline{ij}$ and $\overline{jk}$ of the continuous prestressed beam are considered.

Neglecting the strain energy of shear, normal force and volume change, the total strain energy will be equal to the sum of the energies absorbed by each of the spans, thus:

$$U_{ijk} = U_{ij} + U_{jk} \qquad \text{(EQ II A-1)}$$

where:

$$U_{ij} = \int_i^j \frac{\left(M_x{}^{(i)}\right)^2 \, dx}{2EI_x} \qquad \text{(EQ II A-2a)}$$

$$U_{jk} = \int_j^k \frac{\left(M_x{}^{(j)}\right)^2 \, dx}{2EI_x} \qquad \text{(EQ II A-2b)}$$

Selecting support moment M_j as a redundant, and applying Castigliano's First Theorem of minimum energy:

$$\frac{\partial U_{ijk}}{\partial M_j} = \frac{\partial U_{ij}}{\partial M_j} + \frac{\partial U_{jk}}{\partial M_j} = 0 \qquad \text{(EQ II A-3)}$$

$$\frac{\partial U_{ij}}{\partial M_j} = \int_i^j \frac{M_x{}^{(i)}\left(\dfrac{\partial M_x{}^{(i)}}{\partial M_j}\right) dx}{EI_x} \qquad \text{(EQ II A-4a)}$$

and

$$\frac{\partial U_{jk}}{\partial M_j} = \int_j^k \frac{M_x^{(j)} \left(\frac{\partial M_x^{(j)}}{\partial M_j} \right) dx}{EI_x} \qquad \text{(EQ II A-4b)}$$

The values $M_x^{(i)}$ and $M_x^{(j)}$ of EQ- 1a, 1b are differentiated partially with respect to M_j yielding the results:

$$\frac{\partial M_x^{(i)}}{\partial M_j} = \frac{x}{L_j} \text{ , and } \frac{\partial M_x^{(j)}}{\partial M_j} = \frac{x'}{L_k} \qquad \text{(EQ II A-5)}$$

Substituting the results of EQ II A-5 , and the expressions for the bending moments from EQ 1a, 1b into the EQ's II A-4a , II A-4b, and again substituting these results in EQ's II A-1, after expanding and rearranging:

$$\int_i^j \frac{BM_x^{(i)} x \, dx}{L_j EI_x} + M_i \int_i^j \frac{x \, x' \, dx}{L_j^2 \, EI_x} + M_j \int_i^j \frac{x^2 \, dx}{L_j^2 \, EI_x} +$$

$$\int_i^j \frac{He^{(i)} x \, dx}{L_j \, EI_x} +$$

$$\int_j^k \frac{BM_x^{(j)} x' \, dx}{L_k EI_x} + M_j \int_j^k \frac{x'^2 \, dx}{L_k^2 \, EI_x} + M_k \int_j^k \frac{x \, x' \, dx}{L_k^2 \, EI_x} +$$

$$\int_j^k \frac{H_e^{(j)} x' \, dx}{L_k \, EI_x} = 0 \qquad \text{(EQ II A-6)}$$

Replacing the integrals of the above equation by their nomenclature of beam functions from (Table I) to EQ II A-6, the equation can be written as:

$$M_i G_{ij} + M_j (F_{ji} + F_{jk}) + M_k G_{kj} + \tau_{ji}^{(L)} + \tau_{jk}^{(L)} + \tau_{ji}^{(H)} + \tau_{jk}^{(H)} = 0$$

(EQ II A-7)

Denoting:

$$\left(F_{ji} + F_{jk}\right) = \Sigma F_j \qquad \text{(EQ II A-7a)}$$

$$\left(\tau_{ji}^{(L)} + \tau_{jk}^{(L)}\right) = \Sigma \tau_j^{(L)} \qquad \text{(EQ II A-7b)}$$

and,

$$\left(\tau_{ji}^{(H)} + \tau_{jk}^{(H)}\right) = \Sigma \tau_j^{(H)} \qquad \text{(EQ II A-7c)}$$

the general Clapeyron's Three Moment Equation, with the introduction of the effect of prestress, can be written in the final form as:

$$M_i G_{ij} + M_j \Sigma F_j + M_k G_{kj} + \Sigma \tau_j^{(L)} + \Sigma \tau_j^{(H)} = 0$$

(EQ II A-8)

By considering every pair of adjacent spans, in a like manner, the required number of equations can always be written and solved simultaneously.

(II B) CARRY OVER

The three moment equation, EQ II A-8 is considered again:

$$M_i G_{ij} + M_j \Sigma F_j + M_k G_{kj} + \Sigma \tau_j^{(L)} + \Sigma \tau_j^{(H)} = 0$$

Dividing the equation through out by ΣF_j and rearranging:

$$M_j = -\frac{\tau_j^{(L)}}{\Sigma F_j} - \frac{\tau_j^{(H)}}{\Sigma F_j} - \frac{M_j G_{ij}}{\Sigma F_j} - \frac{M_k G_{kj}}{\Sigma F_j} \qquad \text{(EQ II B-1)}$$

denoting:

$$-\frac{\left(\tau_j^{(H)} + \tau_j^{(L)}\right)}{\Sigma F_j} = m_j \qquad \text{(EQ II B-1a)}$$

where m_j is the starting moment at j.

$$-\frac{G_{ij}}{\Sigma F_j} = r_{ij} \qquad \text{(EQ II B-1b)}$$

where r_{ij} is the carry over factor from i to j.

$$-\frac{G_{kj}}{\Sigma F_j} = r_{kj} \qquad \text{(EQ II B-1c)}$$

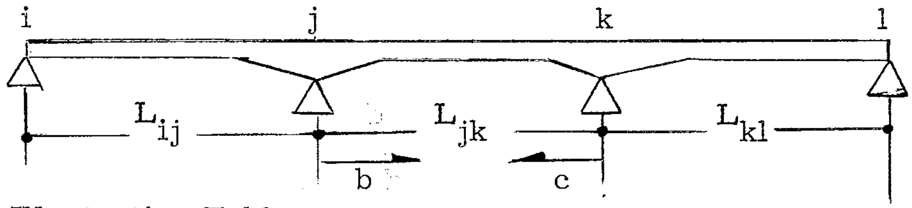
where r_{kj} is the carry over moment factor from k to j.

Replacing these values in EQ II B-1 the final equation of the redundant moment becomes:

$$M_j = m_j + M_i r_{ij} + M_k r_{kj} \qquad \text{(EQ II B-2)}$$

For a n-span continuous beam there are n-1 interior supports, and n-1 moment equations in the above form, may be written to solve the unknown support moments. By the algebraic carry over method each moment consist of an infinite series, convergent and geometric which is a basic series.

For a starting moment of m = 1, and with carry over factors of b and c the procedure is shown for a three span bridge by the following illustrative table.



Illustrative Table:

Carry Over Procedure		
Supports	1	2
C.O.F	b →	← c
S. M.	1	o
1st C.O.M.		b
2nd C.O.M.	bc	
3rd C.O.M.		b^2c
Final Mom.	Σ	Σ

The procedure of the algebraic carry over will be shown in greater detail in the numerical example.

(II - C) SLOPE DEFLECTION

Considering again the basic structure of span ij with the slope deflection - moment distribution, sign convention, (Fig. 4):

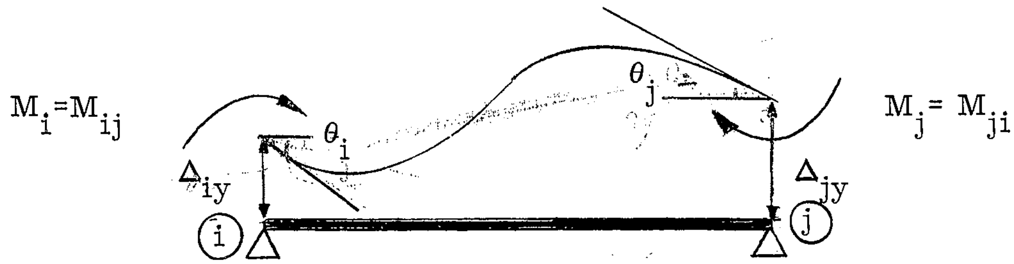


Fig. 4

External work = internal work, where the external work is due to the loads and reactions, and the internal work being the strain energy due to bending only:

$$U_L + U_R = U_S \qquad \text{(EQ II C-1)}$$

where:

U_L = External work due to loads.

U_R = External work due to reactions.

U_S = Strain energy due to bending only.

Selecting M_i, and M_j as the redundant moments and differentiating EQ II C-1 with respect to these redundant moments, deformation EQ's are obtained.

Thus:

$$0 + \frac{\partial U_R}{\partial M_i} = \frac{\partial U_S}{\partial M_i} \qquad \text{(EQ II C-1a)}$$

$$0 + \frac{\partial U_R}{\partial M_j} = \frac{\partial U_S}{\partial M_j} \qquad \text{(EQ II C-1b)}$$

Expressing,

$$U_S = \int_i^j \frac{M_x^{(i)^2}\, dx}{2EI_x}$$

where $M_x^{(i)}$ is given by EQ 1a.

$$U_R = R_{iy}\Delta_{iy} + M_i\,\theta_i$$
$$R_{jy}\,\Delta_{jy} + M_j\,\theta_j$$

where:

θ_i = angular rotation at i.

θ_j = angular rotation at j.

Δ_{iy} = vertical displacement at i.

Δ_{jy} = vertical displacement at j.

R_{iy} and R_{jy} are given by EQ's 2a, 2b.

After proper substitutions and rearranging, deformation equations follow:

$$-\frac{\Delta_{iy} + \Delta_{jy}}{L_j} + \theta_i = M_i \int_i^j \frac{x'^2\, dx}{L^2{}_j EI_x} - M_j \int_i^j \frac{x\, x'\, dx}{L^2{}_j EI_x}$$

$$+ \int_i^j \frac{He^{(i)}\, x'\, dx}{L_j EI_x} + \int_i^j \frac{BM_x^{(i)}\, x'\, dx}{L_j EI_x} \quad \text{(EQ II C-2a)}$$

and,

$$-\frac{\Delta_{iy}+\Delta_{jy}}{L_j}+\theta_j=-M_i\int_i^j\frac{x\,x'\,dx}{L_j^2EI_x}+M_j\int_i^j\frac{x^2\,dx}{L_j^2EI_x}$$

$$-\int_i^j\frac{He^{(i)}x\,dx}{L_mEI_x}-\int_i^j\frac{BM_x^{(i)}x\,dx}{L_jEI_x}\quad\text{(EQ II C-2b)}$$

Replacing again the above integrals by their values given in table I, the equations above can be written as:

$$-\frac{\Delta_{iy}+\Delta_{jy}}{L_j}+\theta_i=F_{ij}\,M_i-G_{ij}\,M_j+\tau_{ij}^{(L)}+\tau_{ij}^{(H)}$$

(EQ II C-3a)

$$-\frac{\Delta_{iy}+\Delta_{jy}}{L_j}+\theta_j=-G_{ij}\,M_i+F_{ji}\,M_j-\tau_{ji}^{(L)}-\tau_{ji}^{(H)}$$

(EQ II C-3b)

Solving these two equations simultaneously for end moments M_i and M_j, and denoting:

$$F_{ji}\,F_{ij}-G_{ij}\,G_{ij}=N\qquad\text{(EQ II C-4)}$$

$$\frac{-\Delta_{iy}+\Delta_{jy}}{L_j}=\frac{\Delta}{L}=\psi\qquad\text{(EQ II C-5)}$$

$$M_i = M_{ij} = \frac{F_{ji}}{N}\,\theta_i + \frac{G_{ij}}{N}\,\theta_j + \frac{F_{ji} + G_{ij}}{N}\,\psi + \frac{-F_{ji}\tau_{ij}^{(L)} + G_{ij}\tau_{ji}^{(L)}}{N}$$

$$+ \frac{-F_{ji}\tau_{ij}^{(H)} + G_{ij}\tau_{ji}^{(H)}}{N} \qquad \text{(EQ II C-6a)}$$

$$M_j = M_{ji} = \frac{F_{ij}}{N}\,\theta_j + \frac{G_{ij}}{N}\,\theta_i + \frac{G_{ij} + F_{ij}}{N}\,\psi + \frac{-G_{ij}\tau_{ij}^{(L)} + F_{ij}\tau_{ji}^{(L)}}{N}$$

$$+ \frac{-G_{ij}\tau_{ij}^{(H)} + F_{ij}\tau_{ji}^{(H)}}{N} \qquad \text{(EQ II C-6b)}$$

where each function is denoted as in the following table II.

Using the notation of:

$$FM_{ij} = FM_{ij}^{(L)} + FM_{ij}^{(H)} \qquad \text{(EQ II C-7a)}$$

$$FM_{ji} = FM_{ji}^{(L)} + FM_{ji}^{(H)} \qquad \text{(EQ II C-7b)}$$

and writing the equations II C-6a and II C-6b by the nomenclature of table II, the general slope deflection equation is presented:

$$M_{ij} = K_{ij}\theta_i + CK_{ji}\,\theta_j + FM_{ij} + FM_{ij}^{(\Delta)} \qquad \text{(EQ II C-8a)}$$

$$M_{ji} = K_{ji}\theta_j + CK_{ij}\theta_i + FM_{ji} + FM_{ji}^{(\Delta)} \qquad \text{(EQ II C-8b)}$$

The condition of continuity of the beam over the supports ($M_{ji}+M_{jk} = 0$) give one equation for each support. We, therefore always have enough number of equations to be solved simultaneously for the unknown rotations at each support. Substituting the values of these rotations in the above slope deflection equations the final end moments are obtained.

BEAM FUNCTIONS

TABLE II

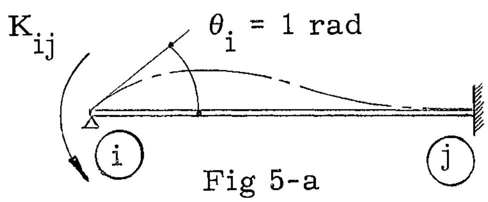
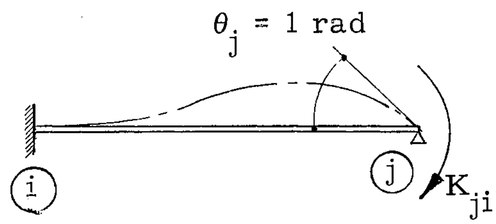
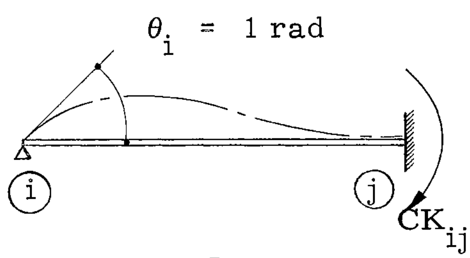
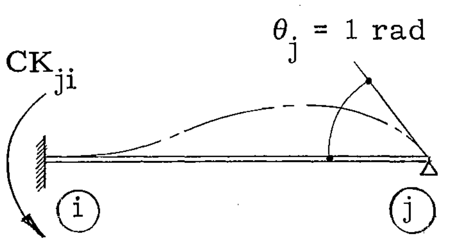
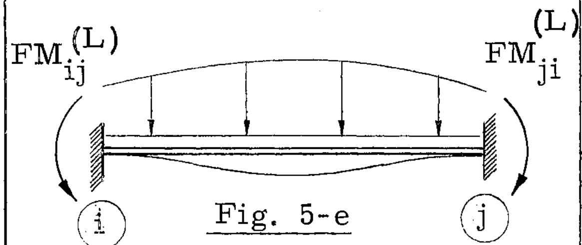
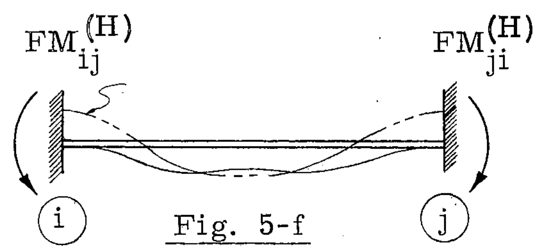
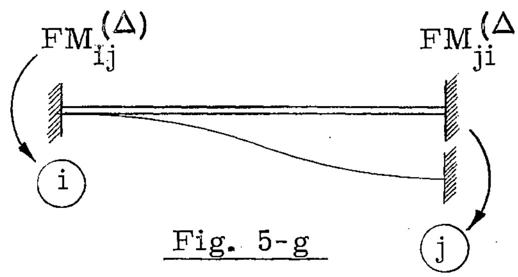
<table>
<tr><td rowspan="2">Fig 5-a</td><td><u>The stiffness factor</u> K_{ij} is the moment required at i to produce a unit rotation at i when end j is fixed. Fig. 5-a</td></tr>
<tr><td>$$K_{ij} = \frac{F_{ji}}{N}$$</td></tr>
<tr><td rowspan="2">Fig 5-b</td><td><u>The stiffness factor</u> K_{ji} is the moment required at j to produce unit rotation at j when end i is fixed. Fig. 5-b</td></tr>
<tr><td>$$K_{ji} = \frac{F_{ij}}{N}$$</td></tr>
<tr><td rowspan="2">Fig 5-c</td><td><u>The carry-over stiffness factor CK_{ij}</u> is the moment induced at j due to a unit rotation at i when j is fixed. Fig 5-c</td></tr>
<tr><td>$$CK_{ij} = \frac{G_{ij}}{N}$$</td></tr>
<tr><td rowspan="2">Fig 5-d</td><td><u>The carry-over stiffness factor CK_{ji}</u> is the moment induced at i due to a unit rotation at j when i is fixed. Fig. 5-d</td></tr>
<tr><td>$$CK_{ji} = \frac{G_{ij}}{N}$$</td></tr>
</table>

TABLE II continued

$FM_{ij}^{(L)} = \dfrac{-F_{ji}\tau_{ij}^{(L)} + G_{ij}\tau_{ji}^{(L)}}{N}$  Fig. 5-e	The fixed end moment $FM_{ij}^{(L)}$ of the beam at i due to external loads. Fig 5-e
$\dfrac{-G_{ij}\tau_{ij}^{(L)} + F_{ij}\tau_{ji}^{(L)}}{N} = FM_{ji}^{(L)}$	The fixed end moment $FM_{ji}^{(L)}$ of the beam at j due to external loads. Fig. 5-e
$FM_{ij}^{(H)} = \dfrac{-F_{ji}\tau_{ij}^{(H)} + G_{ij}\tau_{ji}^{(H)}}{N}$  Fig. 5-f	The fixed end moment $FM_{ij}^{(H)}$ of the beam at i due to prestress Fig. 5-f
$\dfrac{-G_{ij}\tau_{ij}^{(H)} + F_{ij}\tau_{ji}^{(H)}}{N} = FM_{ji}^{(H)}$	The fixed end moment $FM_{ji}^{(H)}$ of the beam at j due to prestress Fig. 5-f
$FM_{ij}^{(\Delta)} = \dfrac{F_{ji} + G_{ij}}{N}\,\psi$  Fig. 5-g	The fixed end moment $FM_{ij}^{(\Delta)}$ of the beam at i due to displacement of support at i. Fig. 5-g
$\dfrac{G_{ij} + F_{ji}}{N}\,\psi = FM_{ji}^{(\Delta)}$	The fixed end moment $FM_{ji}^{(\Delta)}$ of the beam at j due to displacement of support at j. Fig. 5-g

(II D) MOMENT DISTRIBUTION

Considering the equilibrium of the beam at the support j:

$$\Sigma M_j = M_{ji} + M_{jk} = 0 \qquad \text{(EQ II D-1)}$$

The above end moments in form of the slope deflection equations are:

$$M_{ji} = K_{ji}\theta_j + CK_{ij}\theta_i + FM_{ji} \qquad \text{(EQ II D-2a)}$$

$$M_{jk} = K_{jk}\theta_j + CK_{ij}\theta_k + FM_{jk} \qquad \text{(EQ II D-2b)}$$

The effect of rotation of the beam at j due to unequal fixed end moments on the two sides, when the beam is fixed against rotation at i and r can be represented by the following equations:

$$M_{ji} = K_{ji}\theta_j + FM_{ji} \qquad \text{(EQ II D-3a)}$$

$$M_{jk} = K_{jk}\theta_j + FM_{jk} \qquad \text{(EQ II D-3b)}$$

Substituting the above equations to EQ II D-1 for their end-moment values we have:

$$\theta_j (K_{ji} + K_{jk}) = -(FM_{ji} + FM_{jk}) \qquad \text{(EQ II D-4)}$$

denoting

$$K_{ji} + K_{jk} = \Sigma K_j$$

and,

$$FM_{ji} + FM_{jk} = \Sigma FM_j$$

and solving for θ_j:

$$\theta_j = - \frac{\Sigma FM_j}{\Sigma K_j} \qquad \text{(EQ II D-5a)}$$

By a similar approach the effect of rotation of beam at supports i and k, etc., when the beam is fixed against rotation at corresponding adjacent supports we can show that:

$$\theta_i = -\frac{\Sigma FM_i}{\Sigma K_i} \qquad \text{(EQ II D-5b)}$$

and,

$$\theta_k = -\frac{\Sigma FM_k}{\Sigma K_k} \qquad \text{(EQ II D-5c)}$$

then, substituting these values back in equations II D-2a, II D-2b:

$$M_{ji} = \frac{K_{ji}}{\Sigma K_j} \Sigma FM_j + \frac{CK_{ij}}{\Sigma K_i} \Sigma FM_i + FM_{ji}$$

(EQ II D-6a)

$$M_{jk} = \frac{K_{jk}}{\Sigma K_j} \Sigma FM_j + \frac{CK_{kj}}{\Sigma K_k} \Sigma FM_k + FM_{jk}$$

(EQ II D-6b)

where:

$-\frac{K_{ji}}{\Sigma K_j} = D_{ji}$; D_{ji} is the distribution factor.

(EQ II D-7)

$-\frac{CK_{ji}}{\Sigma K_j} = CD_{ji}$; CD_{ji} is the carry over distribution factor.

(EQ II D-8)

then, the equations above become:

$$M_{ji} = D_{ji} \Sigma FM_j + CD_{ji} \Sigma FM_j + FM_{ji} \qquad \text{(EQ II D-9a)}$$

$$M_{jk} = D_{ij} \Sigma FM_j + CD_{kj} \Sigma FM_k + FM_{jk} \qquad \text{(EQ II D-9b)}$$

where the fixed end moments are due to loads, settlement of supports, and prestressing, and they are shown by their equivalent expressions in Table II.

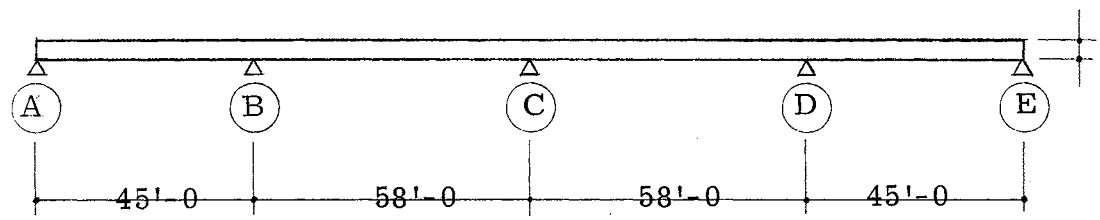
Starting with the fixed end moments as the end moments, numerical procedure of balancing each joint when other joints are treated as fixed, and carrying over their effects on adjacent joints, can be established which on repeated operations converges to the final end moments.

The procedure is fully illustrated in the numerical example.

CHAPTER III

ILLUSTRATIVE NUMERICAL EXAMPLE

The selected structure below will be analysed by the four methods presented previously.



General Layout

General Data:

"Del Valle St. Pedestrian Over Cross", built recently by California State Highway Department, a four span continuously prestressed pedestrian overcrossing with constant moment of inertia is considered as shown on the drawing sheet. The drawing sheet contains a detail drawing of typical cross sections, position of prestressing cable, computed eccentricities of steel, influence lines due to prestressing and live load.

Loads:

Gravity load (GL) = 1.463 k/ft.
Uniform live load (UL) = 0.45 k/ft.

Prestressing by "Roads Incorporated"

H_i = 3,369 k
H_f = 2,864 k

Stresses:

$fc' = 4500$ psi at 28 days

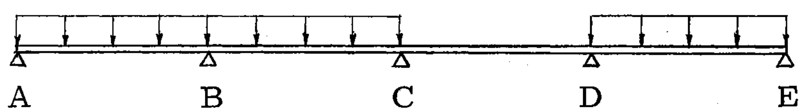
$fc^{t} = 0.40\ fc'$

$fc^{b} = 0$

Units:

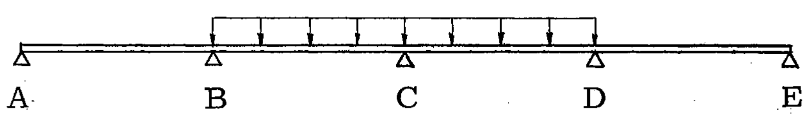
All moments, all shears, and all dimensions are in (k-ft), (k) and (ft) respectively, unless specified.

Loading Conditions Studied



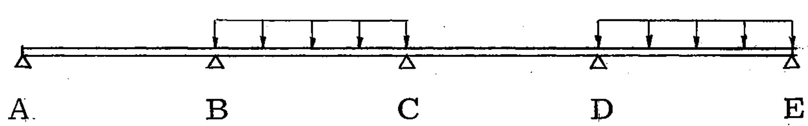
Condition I

for Maximum Moment at B



Condition II

for Maximum Moment at C



Condition III

for Positive Moment at Center of Spans DE or BA; BC or CD

Fig. 6

(III-A) VIRTUAL WORK METHOD

Calculation of Elastic Constants:

Angular Flexibilities (Table I)

For constant moment of inertia:

$$F_{BA} = F_{DE} = \frac{L}{3EI} = \frac{45}{3EI} = \frac{15}{EI}$$

$$F_{BC} = F_{CB} = F_{CD} = F_{DC} = \frac{58}{3EI} = \frac{19.33}{EI}$$

$$\Sigma F_B = \Sigma F_D = F_{BA} + F_{BC} = \frac{34.33}{EI}$$

$$\Sigma F_C = F_{BC} + F_{CD} = \frac{38.66}{EI}$$

Angular Carry Over Values (Table I)

For constant moment of inertia:

$$G_{BC} = G_{CB} = G_{CD} = G_{DC} = \frac{L}{6EI} = \frac{58}{6EI} = \frac{9.66}{EI}$$

Angular Load Functions (Table I)

For constant moment of inertia and due to uniformly distributed load of intensity $W = 1$ k/ft:

$$\tau_{BA}^{(1)} = \tau_{DE}^{(1)} = \frac{WL^3}{24EI} = \frac{(1)(45)^3}{24EI} = \frac{3796.88}{EI}$$

$$\tau_{BC}^{(1)} = \tau_{CB}^{(1)} = \tau_{DC}^{(1)} = \tau_{CD}^{(1)} = \frac{WL^3}{24EI} = \frac{(1)(58)^3}{24\ EI} =$$

$$= \frac{8129.66}{EI}$$

<u>Angular Prestress Functions</u> (Table I, and refer the drawing)

Due to H = 1 kip:

Applying the conjugate beam method;

$$\tau_{AB}^{(H)} = \frac{H}{LEI}\left[\text{Moment of He diagram on AB about B.}\right]$$

$$= \frac{1}{45EI}\left[(7.61)(6.18) - (0.986)(23.05) - (1.10)(31.39) + (0.805)(42.64)\right]$$

$$\tau_{AB}^{(H)} = \frac{(1)(24.10)}{45EI} = \frac{0.535}{EI} = \tau_{ED}^{(H)}$$

$$\tau_{BA}^{(H)} = \tau_{DE}^{(H)} = \frac{H}{LEI}\left[\text{Moment of He diagram on AB about A.}\right]$$

$$= \frac{1}{45EI}\left[(0.805)(2.36) - (1.10)(13.61) - (0.986)(21.94) + (7.61)(28.51)\right]$$

$$\tau_{BA}^{(H)} = \frac{4.05}{EI} = \tau_{DE}^{(H)}$$

$$\tau_{BC}^{(H)} = \tau_{CB}^{(H)} = \tau_{CD}^{(H)} = \tau_{DC}^{(H)} = \frac{H}{EI}\left[\text{Area of 1/2 He diagram on BC.}\right]$$

$$= \frac{3.19}{EI}$$

<u>Calculations for Different Combinations of Loading and Prestressing.</u>

<u>Angular Load Functions due to Gravity Load.</u>

$$\tau_{BA}^{(GL)} = \tau_{DE}^{(GL)} = 1.463 \times \frac{3796.88}{EI} = \frac{5555}{EI}$$

$$\tau_{BC}^{(GL)} = \tau_{CB}^{(GL)} = \tau_{DC}^{(GL)} = \tau_{CD}^{(GL)} = 1.463 \times \frac{8129.66}{EI} = \frac{11,894}{EI}$$

$$\Sigma\tau_B^{(GL)} = \tau_{BA}^{(GL)} + \tau_{BC}^{(GL)}$$

$$= \frac{5555}{EI} + \frac{11,894}{EI} = \frac{17,449}{EI} \qquad \text{(from EQ II A-7b)}$$

$$\Sigma\tau_C^{(GL)} = 2\tau_{CB} = \frac{2(11,894)}{EI} = \frac{23,788}{EI}$$

<u>Angular Prestress Functions due to Hi.</u>

$\tau_{AB}^{(H_i)}$ = not required in this case.

$$\tau_{BA}^{(H_i)} = \tau_{DE}^{(H_i)} = 3,369 \times \frac{4.05}{EI} = \frac{13,644}{EI}$$

$$\tau_{BC}^{(H_i)} = \tau_{CB}^{(H_i)} = \tau_{CB}^{(H_i)} = \tau_{DC}^{(H_i)} = 3,369 \times \frac{3.19}{EI} =$$

$$= \frac{10,747}{EI}$$

$$\Sigma\tau_B^{(H_i)} = \tau_{BA}^{(H_i)} + \tau_{BC}^{(H_i)} \qquad \text{(from EQ II A-7c)}$$

$$= \frac{13,644 + 10,747}{EI} = \frac{24,391}{EI}$$

$$\Sigma\tau_C^{(H_i)} = 2\tau_{CB}^{(H_i)} = \frac{2\,(10,747)}{EI} = \frac{21,494}{EI}$$

<u>Angular Load Functions due to Combinations of Gravity Load and Initial Prestress.</u>

$$\Sigma\tau_B^{(GL + H_i)} = \frac{17,449 + 24,391}{EI} = \frac{41,840}{EI}$$

$$\Sigma\tau_C^{(GL + H_i)} = \frac{23,788 + 21,494}{EI} = \frac{45,282}{EI}$$

Angular Prestress Functions due to H_f:

$$\tau_{AB}^{(H_f)} = \text{is not required in this case.}$$

$$\tau_{BA}^{(H_f)} = \tau_{DE}^{(H_f)} = 2,864 \times \frac{4.05}{EI} = \frac{11,599}{EI}$$

$$\tau_{BC}^{(H_f)} = \tau_{CB}^{(H_f)} = \tau_{DC}^{(H_f)} = \tau_{CD}^{(H_f)} = 2,864 \times \frac{3.19}{EI} =$$

$$= \frac{9,136}{EI}$$

$$\Sigma\tau_B^{(H_f)} = \tau_{BA}^{(H_f)} + \tau_{BC}^{(H_f)} \qquad \text{(from EQ II A-7c)}$$

$$= \frac{11,599 + 9,136}{EI} = \frac{20,735}{EI}$$

$$\Sigma\tau_C^{(H_f)} = 2\tau_{CB}^{(H_f)} = \frac{2(9,136)}{EI} = \frac{18,272}{EI}$$

Angular Load Functions due to Combinations of Gravity Load and Final Prestress.

$$\Sigma\tau_B^{(GL + H_f)} = \frac{17,449 + 20,735}{EI} = \frac{38,184}{EI}$$

$$\Sigma\tau_C^{(GL + H_f)} = \frac{23,788 + 18,272}{EI} = \frac{42,060}{EI}$$

Combination of Gravity Load and Initial Prestressing.

Substituting the numerical values in the simultaneous equations, EQ II A-8, and multiplying throughout by EI.

$$M_B\ (34.33) + M_C\ (9.66) + 41,840 = 0$$

$$M_B\ (19.32) + M_C\ (38.66) + 45,282 = 0$$

Solving the two unknown support moments by determinants we get:

$$\text{Det.} = \begin{bmatrix} 34.33 & 9.66 \\ 19.32 & 38.66 \end{bmatrix} = 1140.46$$

$$M_B = \frac{\begin{bmatrix} (-41,840) & 9.66 \\ (-45,282) & 38.66 \end{bmatrix}}{1140.46} = -1034.76$$

$$M_C = \frac{\begin{bmatrix} 34.33 & -41,840 \\ 19.32 & -45,282 \end{bmatrix}}{1140.46} = -654.28$$

Combination of Gravity Load and Final Prestressing.

Similar to the procedure above we have, for the case of H_f,

$$M_B\,(34.33) + M_C\,(9.66) + 38,364 = 0$$

$$M_B\,(19.32) + M_C\,(38.66) + 42,060 = 0$$

$$M_B = \frac{\begin{bmatrix} (-38,184) & 9.66 \\ -42,060 & 38.66 \end{bmatrix}}{1140.46} = -938.12$$

$$M_C = \frac{\begin{bmatrix} 34.33 & -38,184 \\ 19.32 & -42,060 \end{bmatrix}}{1140.46} = -619.23$$

Maximum Negative Moment at B Due to Uniform Live Load Only: Using Condition I, Fig. 6.

Angular Load Functions.

$$\tau_{BA}^{(UL)} = 3796.88 \times 0.45 = \frac{1708.59}{EI}$$

$$\tau_{BC}^{(UL)} = \tau_{CB}^{(UL)} = 8129.66 \times 0.45 = \frac{3658.35}{EI}$$

$$\tau_{CD}^{(UL)} = \tau_{DC}^{(UL)} = 0$$

$$\tau_{DE}^{(UL)} = 3796.88 \times 0.45 = \frac{1708.59}{EI}$$

$$\Sigma\tau_{B}^{(UL)} = \frac{5366.94}{EI}$$

Symultaneous EQ's:

Substituting the numerical values in the simultaneous equations, and solving these equations by Gauss's * Elimination Method final support moments are obtained.

$$M_B\ (34.33) + M_C\ (9.66) + 5366.94 = 0$$

$$M_B\ (9.66) + M_C\ (38.66) + M_D\ (9.66) + 3658.35 = 0$$

$$M_C\ (9.66) + M_D\ (34.33) + 1708.59 = 0$$

* See Salvadori and Baron, "Numerical Methods in Engineering", pp. 17-20, Prentince-Hall, Inc., N.J. 1959.

Representing the above EQ's in Matrix Form:

M_B	M_C	M_D	$_7(UL)$
34.33	9.66	0	-5366.94
9.66	38.66	9.66	-3658.35
0	9.66	34.33	-1708.59

we have:

1	0.281	0	-156.33
0	1	0.269	- 59.76
0	0	1	- 35.65

Reading the matrix:

$$M_D = -35.65$$

$$M_C = -59.76 - 0.269\ (-35.65) = -50.16$$

$$M_B = -156.33 - (0.281)(-50.16) = -142.24$$

Final Results for Maximum Negative Moment at B.

Combinations of: GL + H_i + UL for Condition I.

$$GL + H_i = -1034.76$$
$$UL = -142.24$$
$$= -1177.00$$

$$M_B = -1177.00$$

Combinations of: GL + H_f + UL for Condition I.

$$GL + H_f = -938.12$$
$$UL = -142.24$$
$$= -1080.36$$

$$M_B = -1080.36$$

Maximum Negative Moment at C due to uniform Live Load Only: Using Condition II, Fig. 6.

Angular Load Functions.

$$\Sigma\tau_B^{(UL)} = \frac{3658.35}{EI} = \Sigma\tau_D^{(UL)}$$

$$\Sigma\tau_C^{(UL)} = \frac{7316.70}{EI}$$

Symultaneous EQ's

The resulting three moment equations for this condition, written simultaneously, are:

$$M_B\,(34.33) + M_C\,(9.66) + 3658.35 = 0$$

$$M_B\,(9.66) + M_C\,(38.66) + M_D\,(9.66) + 7316.70 = 0$$

$$+ M_C\,(9.66) + M_D\,(34.33) + 3658.35 = 0$$

Representing these equations in matrix form:

M_B	M_C	M_D	$\tau^{(UL)}$
34.33	9.66	0	-3658.35
9.66	38.66	9.66	-7316.70
0	9.66	34.33	-3658.35

solving the above matrix by Gauss's eumination:

1	0.2814	0	-106.56
0	1	0.268	- 174.94
0	0	1	- 62.00

reading the above matrix for the end moments:

$$M_D = -62.00$$

$$M_C = -174.94 - (0.268)(-62.00)$$

$$M_C = -158.32$$

Final Results of Maximum Negative Moment at C

Combinations of: $GL + H_i + UL$ for Condition II.

$$GL + H_i = -654.28$$
$$UL = -158.32$$
$$-812.60$$

$$M_C = -812.60$$

Combinations of: $GL + H_f + UL$ for Condition II.

$$GL + H_f = -619.23$$
$$UL = -158.32$$
$$-777.55$$

$$M_C = -777.55$$

For Maximum Positive Moments at Center of Spans BA, or DE; BC, or CD Due to Uniform Live Load Only, Using Condition III, Fig. 6.

Angular Load Functions.

$$\tau_{AB}^{(UL)} = \tau_{BA}^{(UL)} = \tau_{CD}^{(UL)} = \tau_{DC}^{(UL)} = 0$$

$$\tau_{BC}^{(UL)} = \tau_{CB}^{(UL)} = \frac{3658.35}{EI}$$

$$\tau_{DE} = \frac{1708.59}{EI}$$

Symultaneous Equations .

$$M_B\,(34.33) + M_C\,(9.66) + 3658.35 = 0$$

$$M_B\,(9.66) + M_C\,(38.66) + M_D\,(9.66) + 3658.35 = 0$$

$$+ M_C\,(9.66) + M_D\,(34.33) + 1708.59 = 0$$

putting in matrix form:

M_B	M_C	M_D	$_7(UL)$
34.33	9.66	0.	-3658.35
9.66	38.66	9.66	-3658.35
0	9.66	34.33	-1708.59

Solution of the matrix by Gauss's elimination gives the support moments of the spans considered:

1	0.281	0	-106.56
0	1	0.2687	- 73.13
0	0	1	- 31.60

Reading the above matrix for:

$$M_D = -31.60$$

$$M_C = -73.13 - 0.2687\ (-31.60) = -64.64$$

$$M_B = -106.56 - 0.281\ (-64.64) = -88.40$$

<u>Final Moments at Center of Spans BA, or DE: BC, or CD.</u>

<u>Span DE is considered</u>:

<u>Combinations of GL + H_i + UL which will produce positive maximum moments at the center of span.</u>

$$M_D + M_D^{(UL)} = -1034.76 - 31.60 = -1066.36$$

Therefore, the moment at the center of the span due to M_D is -533.18.

The moment of a simple beam at the center due to uniformly distributed load is $\frac{wL^2}{8}$. Plugging the values in we get:

$$\frac{(1.913)\ (45)^2}{8} = +\ 484.22$$

The moment of a simple beam due to prestressing only is the He. Therefore:

$$H_i e = \frac{1.38}{12} \times (3,369) = -387.43$$

Superimposing these three cases, the final moment due to the above conditions is: - 436.39

<u>Combinations of GL + H_f + UL producing maximum positive moment at center of span.</u>

$$M_D + M_D^{(UL)} = -938.12 - 31.60 = -969.72$$

Therefore, at center of span the moment is -484.86.

The moment of a simple beam at the center due to the uniformly distributed load was + 484.22.

The moment of a simple beam due to final prestressing only:

$$H_f e = \frac{1.38}{12} \times (2,864) = -329.36$$

Adding these values together the final moment due to the condition above is: -329.36

<u>Spans BC or CB is considered:</u>

<u>Combinations of GL + H_i + UL which will produce positive maximum moments at the center of spans.</u>

End moments are:

$$M_B + M_B^{(UL)} = -1034.76 - 88.40 = -1123.16$$

$$M_C + M_C^{(UL)} = -654.28 - 64.64 = -718.92$$

Moment at center of span due to M_B is = -561.58

Moment at center of span due to M_C is = -718.92

The moment of the simple beam at the center of the span, loaded by distributed uniform load only:

$$\frac{1.913\,(58)^2}{8} = +\,804.42$$

The moment of the simple beam due to the initial prestress only:

$$H_i e = -\frac{3.69}{12} \times (3.369) = -1037.65$$

Adding these values together we obtain the final moment due to above conditions, the moment being equal to -1154.27.

<u>Combination of GL + H_f + UL producing maximum positive moment at center of spans.</u>

End moments are:

$$M_B + M_B^{(UL)} = -938.12 - 88.40 = -1026.52$$

$$M_C + M_C^{(UL)} = -619.23 - 64.64 = -683.87$$

Moment at center of span due to M_B is = -513.26

Moment at center of span due to M_C is = -341.94

The moment of the simple beam at the center of the span, loaded by distributed uniform load only: + 804.42

Moment of a simple beam due to final prestress only:

$$H_f e = -\frac{3.69}{12} \times (2,864) = -882.11$$

Adding these values together we obtain the final moment due to above conditions, the moment being equal to -932.89.

(III B) CARRY OVER METHOD

Calculation of Elastic Constants:

Angular Flexibilities (Table I)

The values are already calculated. See page 25.

Angular Carry Over Factors

The values are already calculated. See page 25.

Moment Carry Over Factors

(EQ's II B-1b, and II B-1c)

$$r_{BC} = r_{DC} = \frac{-G_{BC}}{\Sigma F_C} = \frac{-9.66/EI}{38.66/EI} = -0.2498$$

$$r_{CB} = r_{CD} = \frac{-G_{CB}}{\Sigma F_B} = \frac{-9.66/EI}{34.33/EI} = -0.2814$$

Algebraic Carry Over Tables

$m_B = m_D = 1$ TABLE IIIa

Supports	B	C	D
C.O.F.	b $\rightarrow$	$\leftarrow$ c d $\rightarrow$	$\leftarrow$ e
S. M	1	0	0
1st C.O.M.		b	
2nd C.O.M.	bc		be
3rd C.O.M.		$b(bc+e^2)$	
	$bc(bc+e^2)$		$bd(bc+e^2)$
		⋮	
Σ	$\frac{1-de}{Y_1}$	$\frac{b}{Y_1}$	$\frac{bd}{Y_1}$
Final Moment	$M_B^{(B)}$	$M_C^{(B)}$	$M_D^{(B)}$

The table demonstrates that the moments form infinite convergent geometric series.

$$Y_1 = 1 - bc - de$$

$$Y_1 = 1 - (0.2498)(0.2814) - (0.2814)(0.2498)$$

$$Y_1 = 0.8594$$

$$M_B^{(B)} = \frac{1 - (0.2498)(0.2814)}{0.8594} = 1.0818$$

$$M_D^{(B)} = \frac{(0.2498)(0.2814)}{0.8594} = 0.0818$$

$$M_C^{(B)} = \frac{-0.2498}{0.8594} = -0.291$$

$m_C = 1$ TABLE IIIb

Supports	B	C	D
C.O.F.	b	c d	e
S.M.	0	1	0
1st C.O.M.	c		d
2nd C.O.M.		bc + de	
3rd C.O.M.	c (bc + de)		d (bc + de)
4th C.O.M.		$(bc + de)^2$	
5th C.O.M.	$c\ (bc + de)^2$		$d\ (bc + de)^2$
	⋮	⋮	⋮
	$\frac{c}{Y_1}$	$\frac{1}{Y_1}$	$\frac{d}{Y_1}$
Final Moment	$M_B^{(C)}$	$M_C^{(C)}$	$M_D^{(C)}$

$$M_B^{(C)} = \frac{-0.2814}{0.8594} = -0.3274$$

$$M_C^{(C)} = \frac{1}{0.8594} = 1.164$$

$$M_D^{(C)} = \frac{-0.2814}{0.8594} = -0.3274$$

Angular Load Functions

The angular load functions for the uniformly distributed load, and for the different cases of loading are already computed. See Page .

Angular Prestress Functions

The angular prestress functions are already calculated See page .

Starting Moments (EQ II B-1a)

S. M. due to GL:

$$m_B^{(GL)} = m_D^{(GL)} = -\frac{\tau_{BA}^{(GL)} + \tau_{BC}^{(GL)}}{\Sigma F_B} = \frac{-17449}{34.33} =$$

$$= -508.27$$

$$m_C^{(GL)} = -\frac{\tau_{BC}^{(GL)} + \tau_{CD}^{(GL)}}{\Sigma F_C} = \frac{-23788}{38.66} = -615.31$$

S. M. due to H_i:

$$m_B^{(H_i)} = m_D^{(H_i)} = -\frac{\tau_{BA}^{(H_i)} + \tau_{BC}^{(H_i)}}{\Sigma F_B} = \frac{-24{,}391}{34.33} =$$

$$= -710.49$$

$$m_C^{(H_i)} = -\frac{\tau_{CB}^{(H_i)} + \tau_{CD}^{(H_i)}}{\Sigma F_C} = -\frac{21,494}{38.66} = -555.98$$

S. M. due to H_f

$$m_B^{(H_f)} = m_D^{(H_f)} = -\frac{\tau_{BA}^{(H_f)} + \tau_{BC}^{(H_f)}}{\Sigma F_B} = \frac{-20,735}{34.33} = -603.99$$

$$m_C^{(H_f)} = -\frac{\tau_{CB}^{(H_f)} + \tau_{CD}^{(H_f)}}{\Sigma F_C} = \frac{-18,272}{38.66} = -472.63$$

Final Support - Moment Equations

$$M_B = m_B M_B^{(B)} + m_C M_B^{(C)} + m_D M_B^{(D)}$$

$$M_C = m_B M_C^{(B)} + m_C M_C^{(C)} + m_D M_D^{(D)}$$

$$M_D = m_B M_D^{(B)} + m_C M_D^{(C)} + m_D M_D^{(D)}$$

Combination of GL + H_i

Starting Moments

$$m_B = m_D = -1218.76$$

$$m_C = -1171.29$$

Final Moments due to above conditions:

$$M_B = (-1218.76)(1.0818) + (-1171.29)(-0.3274) + (-1218.76)(0.0818)$$

$$M_B = -1318.45 + 383.48 - 99.695$$

$$M_B = M_D = -1034.67$$

M_C = (-1218.76) (-p.291) + (-1171.29) (1.164) + (-1218.76) (-0.291)

M_C = + 354.66 - 1363.38 + 354.66

M_C = -654.06

<u>Combination of GL and H_f</u>

<u>Starting Moments</u>

$m_B = m_D = -1112.26$

$m_C = -1087.94$

<u>Final Moments due to above conditions:</u>

M_B = (-112.26) (1.0818) + (-1087.94) (-0.3274) + (-1112.26) (0.0818)

M_B = -1203.24 + 356.19 - 90.98

M_B = -938.03 = M_D

M_C = (-1112.26) (-0.291) + (-1087.94) (1.164) + (-1112.26) (-0.291)

= 323.67 - 1266.36 + 323.67

M_C = -619.02

<u>Maximum Negative Moment at B Due to Uniform Live Load Only: Using Condition I, Fig. 6.</u>

<u>Angular Load Functions</u>

The angular load functions for this case are already worked out. See page .

<u>Starting Moments</u>

$$m_B^{(UL)} = - \frac{\tau_{BA}^{(UL)} + \tau_{BC}^{(UL)}}{\Sigma F_B} = \frac{-5366.94}{34.33} = -156.33$$

$$m_C^{(UL)} = \frac{-\tau_{CB}^{(UL)}}{\Sigma F_C} = \frac{-3658.35}{38.66} = -94.63$$

$$m_D^{(UL)} = \frac{-\tau_{DE}^{(UL)}}{\Sigma F_D} = \frac{-1708.59}{34.33} = -49.77$$

Moment equations due to above conditions:

$$M_B = (1.0818)(-156.33) + (-0.3274)(-94.63) + (0.0818)(-49.77)$$

$$M_B^{(UL)} = -142.21$$

Final results of maximum negative moment at B .

Combinations of GL + H_i + UL for Condition I.

$$M_B^{(GL + H_i)} + M_B^{(UL)} = -1034.67 - 142.21 = -1176.88$$

Combinations of GL + H_f + UL for Condition I.

$$M_B^{(GL + H_f)} + M_B^{(UL)} = -938.03 - 142.21 = -1080.24$$

Maximum negative moment at C due to uniform live load only:
Using Condition II, Fig. 6.

Angular Load Functions.

The angular load functions for this case are already calculated. See page .

Starting Moments

$$m_B^{(UL)} = -\frac{\tau_{BC}}{\Sigma F_B} = \frac{-3658.35}{34.33} = -106.56$$

$$m_C^{(UL)} = -\frac{\tau_{BC} + \tau_{CD}}{\Sigma F_C} = \frac{-7316.70}{38.66} = -189.26$$

$$m_D^{(UL)} = -\frac{\tau_{DC}}{\Sigma F_D} = -106.56$$

Moment equations due to above conditions:

$$M_C = (-106.56)\ (-0.291) + (-189.26)(1.164) + (-0.291)(-106.56)$$

$$M_C = 31.01 - 220.30 + 31.01$$

$$M_C = -158.26$$

Final results of maximum negative moment at C.

Combinations of GL + H_i + UL for Condition II.

$$M_C^{(GL + H_i)} + M_C^{(UL)} = -654.06 - 158.26 = -812.26$$

Combinations of GL + H_f + UL for Condition II.

$$M_C^{(GL + H_f)} + M_C^{(UL)} = -619.02 - 158.26 = -777.28$$

For maximum positive moments at center of spans AB or DE; BC, or CD Due to Uniform Live Load Only, Using Condition III, Fig. 6.

Angular Load Functions.

The angular load functions due to uniform live load for this case are computed already in page .

Starting Moments.

$$m_B^{(UL)} = -\frac{\tau_{BC}^{(UL)}}{\Sigma F_B} = \frac{-3658.35}{34.33} = -106.56$$

$$m_C^{(UL)} = -\frac{\tau_{CB}^{(UL)}}{\Sigma F_C} = \frac{-3658.35}{38.66} = -94.63$$

$$m_D^{(UL)} = -\frac{\tau_{DE}^{(UL)}}{\Sigma F_D} = \frac{-1708.59}{34.33} = -49.77$$

Moment equations due to above conditions:

$$M_B^{(UL)} = (-106.56)(1.0818) + (-94.63)(-0.3274) + (49.77)(0.0818)$$

$$= -115.28 + 30.98 - 4.071$$

$$M_B^{(UL)} = -88.37$$

$$M_C = (-106.56)(-0.291) + (-94.63)(1.164) + (-49.77)(-0.291)$$

$$= +31.01 - 110.15 + 14.48$$

$$M_C^{(UL)} = -64.66$$

$$M_D = (-106.56)(0.0818) + (-94.63)(-0.3274) + (-49.77)(1.0818)$$

$$= -8.72 + 30.98 - 53.84$$

$$M_D^{(UL)} = -31.58$$

Final Support Moments.

Combination of GL + H_i + UL.

$$M_B^{(GL + H_i)} + M_B^{(UL)} = -1034.67 - 88.37 = -1123.04$$

$$M_C^{(GL + H_i)} + M_C^{(UL)} = -654.06 - 64.66 = -718.72$$

$$M_D^{(GL + H_i)} + M_D^{(UL)} = -1034.67 - 31.58 = -1066.25$$

Combinations of GL + H_f + UL for condition III.

$$M_B^{(GL + H_f)} + M_B^{(UL)} = -938103 - 88.37 = -1026.40$$

$$M_C^{(GL + H_f)} + M_C^{(UL)} = -619.02 - 64.66 = -683.68$$

$$M_D^{(GL + H_f)} + M_D^{(UL)} = -938.03 - 31.58 = -969.61$$

Once the above support moments are calculated the procedure of superposition in finding the moments in the center of the spans is exactly the same as for previous methods, so that part of the procedure will not be illustrated again.

(III C) SLOPE DEFLECTION

Calculation of elastic constants.

Angular Flexibilities.

$$F_{BA} = F_{AB} = 15/EI$$

$$F_{CB} = F_{BC} = \frac{19.33}{EI}$$

See page 25.

Angular Carry Over Values.

$$G_{AB} = \frac{L}{6EI} = \frac{45}{6EI} = \frac{7.50}{EI}$$

See page25 .

$$G_{BC} = G_{CB} = \frac{9.66}{EI}$$

Evaluation of Constant N. (EQ II C-4)

$$N_{AB} = N_{BA} = F_{AB}\,F_{BA} - G_{AB}\,G_{AB}$$

$$= (15)^2 - (7.50)^2 = \frac{168.75}{EI^2}$$

$$N_{BC} = N_{CB} = F_{BC}\,F_{CB} - G_{BC}\,G_{BC}$$

$$= \frac{(19.33)^2 - (9.66)^2}{EI^2} = \frac{280.32}{EI^2}$$

Stiffness Factors(From Table II).

$$K_{AB} = K_{BA} = \frac{F_{BA}}{N_{AB}} = \frac{15}{168.75} = 0.888\ EI$$

$$K_{BC} = K_{CB} = \frac{19.33}{280.32} = 0.689 \text{ EI}$$

Carry Over Stiffness Factors (From Table II).

$$CK_{BA} = CK_{AB} = \frac{G_{AB}}{N_{AB}} = \frac{7.50}{168.75} = 0.444 \text{ EI}$$

$$CK_{CB} = CK_{BC} = \frac{G_{BC}}{N_{CB}} = \frac{9.66}{280.32} = 0.345 \text{ EI}$$

Fixed End Moments (From Table II).

Due to unit loading.

$$FM_{AB}^{(1)} = -\frac{F_{BA}\,\tau_{AB}^{(1)} + \tau_{BA}^{(1)}\,G_{AB}}{N_{AB}} = \tau_{AB} = BA\,\frac{G_{AB} - F_{AB}}{N_{AB}}$$

$$= 3796.88\ \frac{(7.50-15)}{168.75} = -168.74$$

$$FM_{BA}^{(1)} = -\frac{G_{BA}\tau_{AB}^{(1)} + F_{BA}\tau_{BA}^{(1)}}{N_{AB}} = +168.74$$

$$FM_{BC} = -\frac{F_{CB}\tau_{CB}^{(1)} + G_{BC}\tau_{CB}^{(1)}}{N_{BC}} =$$

$$= \tau_{CB} = BC\ \frac{(-F_{CB} + G_{BC})}{N_{BC}} =$$

$$= \frac{8129.66\,(-19.33 + 9.66)}{280.32} = -280.44$$

$$FM_{CB} = \frac{-G_{BC}\tau_{BC} + F_{CB}\tau_{CB}}{280.32} = +280.44$$

Due to GL.

$$FM_{AB} = -168.74 \times 1.463 = -246.86$$

$$FM_{BA} = +168.74 \times 1.463 = +246.86$$

$$FM_{BC} = -280.44 \times 1.463 = -410.28$$

$$FM_{CB} = +280.44 \times 1.463 = +410.28$$

Angular Prestress Functions.

Due to Unity.

$$\tau_{AB} = \tau_{ED} = \frac{0.535}{EI}$$

$$\tau_{BA} = \tau_{DE} = \frac{4.05}{EI}$$

$$\tau_{BC} = \tau_{CB} = \tau_{CD} = \tau_{DE} = \frac{3.19}{EI}$$

Due to Initial Prestressing.

$$\tau_{AB} = \tau_{ED} = 3,369\ (0.535) = \frac{1802.42}{EI}$$

$$\tau_{BA} = \tau_{DE} = \frac{13,644}{EI}$$

$$\tau_{BC} = \tau_{CB} = \tau_{CD} = \tau_{DC} = \frac{10,747}{EI}$$

Due to Final Prestressing.

$$\tau_{AB} = \tau_{ED} = 2,864\ (0.535) = \frac{1532.24}{EI}$$

Due to H_i.

$$FM_{AB}^{(H_i)} = -\frac{F_{AB}\,\tau_{AB}^{(H_i)} + G_{AB}\tau_{BA}^{(H_i)}}{N_{AB}}$$

$$FM_{AB}^{(H_i)} = -\frac{15\,(1802.42) + 7.50\,(13,644)}{168.75} = 446.18$$

$$FM_{BA}^{(H_i)} = -\frac{G_{BA}\tau_{AB}^{(H_i)} + F_{BA}\,\tau_{BA}^{(H_i)}}{N_{AB}}$$

$$= \frac{(-7.50)\,(1802.42) + (15)\,(13,644)}{168.75} = 1132.69$$

$$FM_{BC}^{(H_i)} = \frac{\tau_{CB}^{(H_i)}\,(-F_{CB} + G_{BC})}{N_{BC}}$$

$$FM_{BC}^{(H_i)} = \frac{9,136\,(-19.33 + 9.66)}{(280.32)} = -370.74$$

$$FM_{CB}^{(H_i)} = +370.74$$

Due to H_f.

$$FM_{AB}^{(H_f)} = -\frac{F_{BA}\tau_{AB}^{(H_f)} + G_{AB}\tau_{BA}^{(H_f)}}{N_{AB}}$$

$$= -\frac{15\,(1532.24) + 7.50\,(11,599.2)}{168.75} = +379.26$$

$$FM_{BA}^{(H_f)} = -\frac{G_{BA}\tau_{AB}^{(H_f)} + F_{BA}\,\tau_{BA}^{(H_f)}}{N_{AB}}$$

$$= \frac{(-7.50)\,(1532.24) + 15\,(11,599.2)}{168.75} = +963.84$$

$$FM_{BC}^{(H_f)} = \frac{\tau_{CB}^{(H_f)} (-F_{CB} + G_{BC})}{N_{BC}} = -315.16$$

$$FM_{CB} = +315.16$$

Due to combination of GL + H_i.

$$FM_{AB}^{(H_i)} + FM_{AB}^{(GL)} = +446.18 - 246.86 = +199.32$$

$$FM_{BA}^{(H_i)} + FM_{BA}^{(GL)} = +1132.69 + 246.86 = +1379.55$$

$$FM_{BC}^{(H_i)} + FM_{BC}^{(GL)} = -370.74 - 410.28 = -781.02$$

$$FM_{CB}^{(H_i)} + FM_{CB}^{(GL)} = +410.28 + 370.74 = +781.02$$

Slope Deflection Equations:

$$M_{AB} = 0.888\ EI\theta_A + 0.4444\ EI\theta_B + FM_{AB} = M_{ED}$$

$$M_{BA} = 0.888\ EI\theta_B + 0.4444\ EI\theta_A + FM_{BA} = M_{DE}$$

$$M_{BC} = 0.689\ EI\theta_B + 0.345\ EI\theta_C + FM_{BC} = M_{DC}$$

$$M_{CB} = 0.689\ EI\theta_C + 0.345\ EI\theta_B + FM_{CB} = M_{CD}$$

Final Support Moments due to GL + H_i.

From condition of support equilibrium;

$$M_{BA} + M_{BC} = 0 = 1.5770\ EI\theta_B + 0.4444\ EI\theta_A + (1379.55 - 781.02)$$

$$M_{AB} = 0 = 0.4444\ EI\theta_B + 0.8889\ EI\theta_A + 199.32$$

Solving the simultaneous equations for $EI\theta_B$ and $EI\theta_A$ respectively by determinants:

$$EI\theta_B = \frac{\begin{bmatrix} 598.53 & 0.4444 \\ 199.32 & 0.8889 \end{bmatrix}}{\begin{bmatrix} 1.5770 & 0.4444 \\ 0.4444 & 0.8889 \end{bmatrix}} = \frac{-443.45}{1.2043} = -368.22$$

$$EI\theta_A = \frac{\begin{bmatrix} 1.5770 & 598.53 \\ 0.4444 & 199.32 \end{bmatrix}}{1.2043} = \frac{-48.34}{1.2043} = -40.14$$

Substituting the values of $EI\theta_A$ and $EI\theta_B$ back to the original slope deflection equations, the final support moments due to above conditions are obtained:

$$M_{BA} = (0.8889)(-368.22) + (0.4444)(-40.181) + 1379.55$$

$$M_{BA} = + 1034.40$$

$$M_{CB} = 0.345\ EI\theta_B + 781.02$$

$$= 0.345\ (-368.53) + 781.02$$

$$M_{CB} = +653.98$$

Fixed-end moments due to combination of GL + H_f.

$$FM_{AB}^{(H_f)} + FM_{AB}^{(GL)} = +379.26 - 246.86 = +132.40$$

$$FM_{BA}^{(H_f)} + FM_{BA}^{(GL)} = +963.84 + 246.86 = +1210.70$$

$$FM_{BC}^{(H_f)} + FM_{BC}^{(GL)} = -315.16 - 410.28 = -725.44$$

$$FM_{CB}^{(H_f)} + FM_{CB}^{(GL)} = +315.16 + 370174 = +685.90$$

Final support moments due to combination of GL + H_f.

From condition of support equilibrium:

$$M_{BA} + M_{BC} = 0 = 1.5770\ EI\theta_B + 0.4444\ EI\theta_A + (1210.70 - 725.44)$$

$$M_{AB} = 0 = 0.4444\ EI\theta_B + 0.8889\ EI\theta_A + 132.40$$

Solving the equations for $EI\theta_A$ and $EI\theta_B$ simultaneously we have:

$$EI\theta_B = \frac{\begin{bmatrix} -485.26 & 0.4444 \\ -132.40 & 0.8889 \end{bmatrix}}{1.2043} = \frac{-372.51}{1.2043} = -309.32$$

$$EI\theta_A = \frac{\begin{bmatrix} 1.5770 & +485.26 \\ 0.4444 & +132.40 \end{bmatrix}}{1.2043} = \frac{+6.86}{1.2043} = +5.70$$

Substituting the values of $EI\theta_A$ and $EI\theta_B$ back to the original slope deflection equations, the final support moments due to above conditions are obtained.

$$M_{BA} = (0.8889)(-309.32) + (0.4444)(5.70) + 1210.70$$

$$M_{BA} = +938.28$$

$$M_{CB} = 0.345(-309.32) + 725.44$$

$$M_{CB} = 618.72$$

Maximum Negative Moment at B due to Uniform Live Load Only: Using Condition I, Fig. 6.

Fixed End Moments.

$$FM_{AB}^{(UL)} = \frac{-F_{BA}\tau_{AB}^{(UL)} + G_{AB}\,\tau_{BA}^{(UL)}}{N_{AB}}$$

$$= \frac{-15\,(1708.59) + 7.50\,(1708.59)}{168.75}$$

$$FM_{AB}^{(UL)} = -75.94 = FM_{DE}^{(UL)}$$

Similarly,

$$FM_{BA}^{(UL)} = +75.94$$

$$FM_{BC}^{(UL)} = \frac{-F_{CB}\tau_{BC}^{(UL)} + G_{BC}\tau_{CB}}{N_{BC}}$$

$$= \frac{3658.35\,(-19.33 + 9.66)}{280.32} = 126.20$$

$$FM_{CB}^{(UL)} = +126.20$$

$$FM_{CD}^{(UL)} = FM_{CD}^{(UL)} = 0$$

Slope Deflection Equations.

$M_{BA} + M_{BC} = 0$

$$0.4444\ EI\theta_A + 1.5779\ EI\theta_B + 0.345\ EI\theta_C - 50.26$$

$M_{CB} + M_{CD} = 0$

$$1.378\ EI\theta_C + 0.345\ EI\theta_B + 0.345\ EI\theta_D + 126.20$$

$M_{AB} = 0$

$$1.578\ EI\theta_D + 0.345\ EI\theta_C + 0.4444\ EI\theta_E - 75.94$$

$$M_{AB} = 0.8889\ EI\theta_A + 0.444\ EI\theta_B - 75.94 = 0$$

$$M_{ED} = 0.8889\ EI\theta_E + 0.4444\ EI\theta_D + 75.94 = 0$$

Expressing these equations in matrix form and solve for the unknown slopes by Gauss elimination:

$EI\theta_A$	$EI\theta_B$	$EI\theta_C$	$EI\theta_D$	$EI\theta_E$	FM's
0.8889	0.4444	0	0	0	+ 75.94
0.4444	1.578	0.345	0	0	+ 50.26
0	0.345	1.378	0.345	0	- 126.20
0	0	0.345	1.478	0.4444	+75.94
0	0	0	0.4444	0.8889	- 75.94

Representing the matrix in upper triangular matrix form:

$EI\theta_A$	$EI\theta_B$	$EI\theta_C$	$EI\theta_D$	$EI\theta_E$	FM's
1	0.4999	0	0	0	85.43
0	1	0.2544	0	0	9.06
0	0	1	0.267	0	-100.26
0	0	0	1	0.2991	+ 74.38
0	0	0	0	1	-144.15

and solving for $EI\theta$'s:

$$EI\theta_E = -144.15$$

$$EI\theta_D = (0.2991)(144.15) + 74.38 = +117.50$$

$$EI\theta_C = -100.26 + 0.267\ (117.50) = -131.63$$

$$EI\theta_B = 0.2544\ (-121.63) + 9.06 = +42.54$$

$$EI\theta_A = 0.4999\ (42.54) + 85.43 = +64.17$$

Substituting the values of $EI\theta$'s back to the original slope deflection equations the support moment BA due to above conditions becomes;

$$M_{BA} = 0.8889\ (42.54) + 0.4444\ (64.17) + 75.94$$

$$M_{BA} = +142.27$$

Final Results For Maximum Negative Moment at B.

Combinations of GL + H_i + UL for Condition I.

$$M_{BA}^{(GL + H_i)} + M_{BA}^{(UL)} = +1034.40 + 142.27 = +1176.67$$

Combinations of GL + H_f + UL for Condition I.

$$M_{BA}^{(GL + H_f)} + M_{BA}^{(UL)} = +938.28 + 142.27 = +1080.55$$

Maximum Negative Moment at Support C Due to Uniform Live Load Only: Using Condition II Fig. 6.

Fixed End Moments.

$$FM_{AB}^{(UL)} = FM_{BA}^{(UL)} = FM_{DE}^{(UL)} = FM_{ED}^{(UL)} = 0$$

$$FM_{BC}^{(UL)} = FM_{CD}^{(UL)} = -126.20$$

$$FM_{CB}^{(UL)} = FM_{DC}^{(UL)} = +126.20$$

Slope Deflection Equations.

$$M_{BA} + M_{BC} = 0$$

$$0.4444\, EI\theta_A + 1.578\, EI\theta_B - 126.20 = 0$$

and,

$$M_{AB} = 0.8889\, EI\theta_A + 0.4444\, EI\theta_B = 0$$

Solving the two symultaneous equations for $EI\theta_B$ by determinants:

$$EI\theta_B = \frac{\begin{bmatrix} 0.8889 & 0 \\ 0.4444 & 126.20 \end{bmatrix}}{1.2043} = \frac{112.17}{1.2043} = +93.14$$

substituting the value of $EI\theta_B$ to the original slope deflection equation for the support moment:

$$M_{CB}^{(UL)} = 0.345\,(93.14) + 126.20 = 0$$

$$M_{CB}^{(UL)} = +158.33$$

Final Results For Maximum Negative Moment At C.

Combinations of GL + H_i + UL for Condition II.

$$M_{CB}^{(H_i + GL)} + M_{CB}^{(UL)} = +653.98 + 158.33 = +812.31$$

For Maximum Positive Moments at Center of Spans BA or DE; BC or CD Due to Uniform Live Load only: Using Condition III, Fig. 6.

Fixed End Moments.

$$FM_{AB}^{(UL)} = 0 = FM_{BA}^{(UL)}$$

$$FM_{BC}^{(UL)} = -126.20$$

$$FM_{CB}^{(UL)} = +126.20$$

$$FM_{CD}^{(UL)} = 0 = FM_{DC}^{(UL)}$$

$$FM_{DE}^{(UL)} = -75.94$$

$$FM_{ED}^{(UL)} = +75.94$$

Slope Deflection Equations.

$$M_{AB} = 0.8889\ EI\theta_A + 0.4444\ EI\theta_B$$

$$M_{BA} + M_{BC} = 0$$

$$0.4444\ EI\theta_A + 1.5779\ EI\theta_B + 0.345\ EI\theta_C - 126.20$$

$$M_{CB} + M_{CD} = 0$$

$$1.378\ EI\theta_C + 0.345\ EI\theta_B + 0.345\ EI\theta_D + 126.20$$

$$1.578\ EI\theta_D + 0.345\ EI\theta_C + 0.4444\ EI\theta_E - 75.94$$

$$0.8889\ EI\theta_E + 0.4444\ EI\theta_D + 75.94$$

Expressing the above equations in matrix form and solve for the unknown $EI\theta$'s by Gauss elimination:

$EI\theta_A$	$EI\theta_B$	$EI\theta_C$	$EI\theta_D$	$EI\theta_E$	FM's
0.8889	0.4444	0	0	0	0
0.4444	1.5779	0.345	0	0	+126.20
0	0.345	1.378	0.345	0	-126.20
0	0	0.345	1.578	0.4444	+ 75.94
0	0	0	0.4444	0.8889	- 75.94

Putting the matrix in upper triangular matrix form:

$EI\theta_A$	$EI\theta_B$	$EI\theta_C$	$EI\theta_D$	$EI\theta_E$	FM's
1	0.4999	0	0	0	0
0	1	0.2544	0	0	+ 93.07
0	0	1	0.267	0	-122.63
0	0	0	1	0.2991	+ 79.58
0	0	0	0	1	- 147.28

Reading the matrix for the solution of $EI\theta$'s:

$$EI\theta_E = -147.28$$

$$EI\theta_D = 79.58 + 0.2991\,(147.28) \doteq 123.63$$

$$EI\theta_C = -122.63 - 0.267\,(123.63) = -155.64$$

$$EI\theta_B = 93.07 + 0.2544\,(155.64) = +132.66$$

$$EI\theta_A = -0.4999\,(132.66) = -66.32$$

Substituting the values of the $EI\theta$'s above, to the original sope deflection equations for the final support moments:

$$M_{BA}^{(UL)} = (0.4444)(-66.32) + 0.8889\,(132.66) = 0$$

$$M_{BA}^{(UL)} = +88.45$$

$$M_{CB}^{(UL)} = 0.689\,(-155.64) + 0.345\,(132.66) + 126.20$$

$$M_{CB}^{(UL)} = +64.73$$

$$M_{DE}^{(UL)} = 0.8889\,(123.63) + 0.4444\,(-147.28) - 75.94$$

$$M_{DE}^{(UL)} = -31.50$$

Final Support Moments.

Combination of GL + H_i + UL for Condition III.

$$M_{BA}^{(GL + H_i)} + M_{BA}^{(UL)} = 1034.40 + 88.45 = +1122.85$$

$$M_{CB}^{(GL + H_i)} + M_{CB}^{(UL)} = +653.98 + 64.73 = +718.71$$

$$M_{DC}^{(GL + H_i)} + M_{DC}^{(UL)} = +1034.40 + 31.50 = +1065.90$$

Combination of GL + H_f + UL for Condition III.

$$M_{BA}^{(GL + H_f)} + M_{BA}^{(UL)} = +938.28 + 88.45 = +1026.73$$

$$M_{CB}^{(GL + H_f)} + M_{CB}^{(UL)} = +618.72 + 64.73 = +683.45$$

$$M_{DC}^{(GL + H_f)} + M_{DC}^{(UL)} = +938.28 + 31.50 = +969.78$$

Once the above support moments are calculated the procedure of superposition in finding the moments in the center of the spans is exactly the same as for previous methods, so that part of the procedure will not be illustrated again.

(III-D) MOMENT DISTRIBUTION

Calculation of Elastic Constants.

Stiffness Factors.

For constant moment of inertia:

$$K_{AB} = K_{ED} = \frac{4EI}{L_{AB}} = \frac{4EI}{45} = 0.08889\ EI$$

$$K_{BC} = K_{CD} = \frac{4EI}{L_{AB}} = \frac{4\ EI}{58} = 0.06897\ EI$$

Modified Stiffness Factors.

$$K'_{AB} = K'_{ED} = \frac{3}{4} \times 0.08889 = 0.06667\ EI$$

Distribution Factors. (EQ II D-7)

$$D_{BA} = -\frac{0.06667}{0.06667 + 0.06897} = -0.4915$$

$$D_{BC} = -\frac{0.06897}{0.13564} = -0.5085$$

$$D_{CB} = -\frac{0.06897}{0.13794} = -0.5000$$

Carry Over Factors.

For constant moment of inertia:

$$C_{BC} = C_{CB} = C_{CD} = C_{DC} = \frac{2\ EI / L}{4\ EI / L} = 0.5000$$

Fixed End Moments.

Calculated already in the slope deflection method.

Moment Distribution Tables, (IV)

Due to GL.

Due to symmetry:

TABLE IVa

Spans	AB	BA	BC	CB
D		-0.4915	-0.5085	
C.O.F	0.500		0.500	
(GL) F.M	-246.86	+246.86	-410.28	+410.28
	+246.86			
	0	+123.43		
		370.29	-410.28	
		+19.66	+20.344	
				10.172
		+389.95	-389.94	+420.45

Due to H_i.

Due to symmetry:

TABLE IVb

Spans	AB	BA	BC	CB
D		-0.4915	-0.5085	
C.O.F.	0.500		0.500	
$FM^{(H_i)}$	+446.18	+1132.69	-370.74	+370.74
	-446.18	-223.09		
		-264.86	-273.99	
				-136.99
		+644.74	-644.73	+233.75

Due to H_f.

Due to symmetry:

TABLE IVc

Spans	AB	BA	BC	CB
		-0.4915	-0.5085	
	0.500		0.500	
$FM^{(H_f)}$	+379.26	+963.84	-315.16	+315.16
	-379.26	-189.63		
		-225.62	-233.43	
				-116.72
		+548.59	-548.59	+198.44

Final Results of Maximum Negative Moment at B.

Combinations of GL + H_i + UL for Condition I.

$$M_B^{(GL + H_i)} + M_B^{(UL)} = 1034.69 + 142.32 = -1177.01$$

Combinations of GL + H_f + UL for Condition I.

$$M_B^{(GL + H_f)} + M_B^{(UL)} = 938.54 + 142.32 = -1080.86$$

Maximum Negative Moment at C Due to Uniform Live Load Only:

Using Condition II Fig. 6.

Due to Symmetry:

TABLE IV d

Spans	BA	BC	CB
D	-0.4915	-0.5085	-0.500
C.O.F.		0.500	0.500
F.M.		-126.20	+126.20
	+62.03	+64.17	
			32.09
			158.29

Final Negative Moments At Support C:

Combinations of GL + H_i + UL for Condition II.

$$M_C^{(GL + H_i)} + M_C^{(UL)} = -654.20 + 158.29 = -812.49$$

Combinations of GL + H_f + UL for Condition II.

$$M_C^{(GL + H_f)} + M_C^{(UL)} = -618.89 - 158.29 = -777.18$$

Maximum Negative Moment at B due to uniform live load only; Using condition I, Fig. 6. TABLE IVe

Supports	B		C		D	
Spans	BA	BC	CB	CD	DC	DE
D	- 0.4915	- 0.500	- 0.500	- 5.00	- .5085	- 0.4915
C.O.		0.5	0.5	0.5	0.5	
	+113.91	-126.20	+126.20	0	0	-113.91
	+ 6.04	+ 6.25	- 63.10	- 63.10	+57.92	+ 55.99
		- 31.55	+ 3.13	+28.97	-31.55	
	+ 15.51	+ 16.04	- 16.03	-16.03	+16.04	+ 15.51
		- 8.15	+ 8.02	+ 8.02	- 8.15	
	+ 4.01	+ 4.14	- 8.02	- 8.02	+ 4.01	= 4.14
		- 4.01	+ 2.07	+ 2.07	- 4.01	
	+ 1.97	+ 2.04	- 2.07	- 2.07	+ 2.04	+ 1.97
		- 1.04	+ 1.04	+ 1.04	- 1.04	
	0.51	+ 0.53	- 1.04	- 1.04	+ 0.53	+ 0.51
		- 0.52	+ 0.27	+ 0.27	- 0.52	
	+ 0.26	+ 0.26	- 0.27	- 0.27	+ 0.26	+ 0.26
		- 0.14	+ 0.13	+ 0.13	- 0.14	
	+ 0.07	+ 0.07	- 0.13	- 0.13	+ 0.07	+ 0.07
		- 0.08	+ 0.04	+ 0.04	+ 0.08	
	+ 0.04	+ 0.04	- 0.04	- 0.04	- 0.04	- 0.04
		- 0.01	+ 0.02	+ 0.02	+ 0.01	
	+142.32	-142.32	+ 50.20	- 50.18	+ 35.51	- 35.50

For Maximum Positive Moments at center of spans BA or De; BC or CD due to UL only; Using Cond. III, Fig. 6. TABLE IVf

	Joint B		Joint C		Joint D	
	BA	BC	CB	CD	DC	DE
D	-0.4195	-0.5085	-0.500	-0.500	-0.5085	-0.4915
C.O.F		0.5	0.5	0.5	0.5	
FM(UL)		-126.20	+126.20	0	0	-113.91
	+62.03	+64.17	-63.10	-63.10	+57.92	+55.99
		-31.55	+32.09	+28.96	-31.55	
	+15.51	+16.04	-30.53	-30.53	+16.04	+15.51
		-15.27	+8.02	+8.02	-15.27	
	+7.51	+7.76	-8.02	-8.02	+7.76	+7.51
		-4.01	+3.88	+3.88	-4.01	
	1.97	+2.04	-3.88	-3.88	+2.39	+1.97
		-1.94	+1.02	+1.02	-1.94	
	+0.95	+0.97	-1.02	-1.02	+0.97	+0.95
		-0.51	+0.49	+0.49	-0.51	
	+0.25	+0.26	-0.49	-0.49	+0.26	+0.25
		-0.26	+0.13	+0.13	+0.26	
	+0.13	+0.13	-0.13	-0.13	+0.13	+0.13
		-0.08	+0.08	+0.08	-0.08	
	+0.04	+0.04	-0.08	-0.08	+0.04	+0.04
		-0.04	+0.02	+0.02	-0.04	
	+0.02	+0.02	-0.02	-0.02	+0.02	+0.02
	+88.41	-88.42	+64.66	-64.67	+31.54	-31.54

Final Support Moments.

Combination of GL + H_i + UL for Condition III.

$$M_B^{(GL + H_i)} + M_B^{(UL)} = -1034.69 - 88.41 = -1123.10$$

$$M_C^{(GL + H_i)} + M_C^{(UL)} = -654.20 + 64.66 = -718.86$$

$$M_D^{(GL + H_i)} + M_D^{(UL)} = -1034.69 - 31.54 = -1066.23$$

Combination of GL + H_f + UL for Condition III.

$$M_B^{(GL + H_f)} + M_B^{(UL)} = -938.54 - 88.41 = -1026.95$$

$$M_C^{(GL + H_f)} + M_C^{(UL)} = -618.89 - 64.66 = -683.55$$

$$M_D^{(GL + H_f)} + M_D^{(UL)} = -938.54 - 31.54 = -970.08$$

Again, once the above support moments are calculated the procedure of super position in finding the moments in the center of the spans, is exactly the same as for previous methods, so that part of the procedure will not be illustrated again.

(III-E) INFLUENCE LINES

The object of this part is to determine the safe concentrated live load (P) that can be put on the second span.

The influence lines for the moments at various points on the second span due to the concentrated unit live load on the second span will be determined as well as the moment influence lines of supports B and C.

The method for the computations of the influence lines is selected to be the carry over method.

Load in the First Span.

$$M_B = M_B^{(B)} - \frac{\tau_{BA}^{(P)}}{\Sigma F_B}$$

$$M_B = - \frac{1.0818}{34.33} \tau_{BA}^{(P)} = -0.0315 \quad \tau_{BA}^{(P)}$$

$$M_C = M_C^{(B)} - \frac{\tau_{BA}^{(P)}}{\Sigma F_B}$$

$$= - \frac{0.291}{34.33} \tau_{BA}^{(P)} = + 0.00848 \quad \tau_{BA}^{(P)}$$

$$M_D = M_D^{(B)} - \frac{\tau_{BA}^{(P)}}{\Sigma F_B}$$

$$= - \frac{0.0818}{34.33} \tau_{BA}^{(P)} = - 0.00238 \quad \tau_{BA}^{(P)}$$

Load in Second Span.

$$M_B = M_B^{(B)} - \frac{\tau_{BC}^{(P)}}{\Sigma F_B} + M_B^{(C)} - \frac{\tau_{CB}^{(P)}}{\Sigma F_C}$$

$$M_B = -0.0315\ \tau_{BC}^{(P)} + 0.00847\ \tau_{CB}^{(P)}$$

$$M_C = M_C^{(B)} - \frac{\tau_{BC}^{(P)}}{\Sigma F_B} + M_C^{(C)} - \frac{\tau_{CB}^{(P)}}{\Sigma F_C}$$

$$= +0.00858\ \tau_{BC}^{(P)} - 0.0301\ \tau_{CB}^{(P)}$$

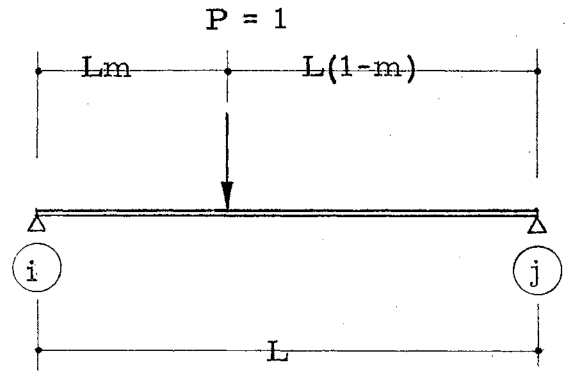
$$M_D = M_D^{(B)} - \frac{\tau_{BC}^{(P)}}{\Sigma F_B} + M_D^{(C)} - \frac{\tau_{CB}^{(P)}}{\Sigma F_C}$$

$$M_D = -0.00238\ \tau_{BC}^{(P)} + 0.00847\ \tau_{CB}^{(P)}$$

The bending moments over the supports due to unit load in the third and fourth spans are obtained by symmetry.

The final moments M_B, M_C, M_D may now be easily obtained by substituting the values τ from Table 5 into the formulas stated above as shown in Tables 6-a and 6-b.

END SLOPE COEFFICIENTS TABLE V



$$\tau_{ij} = \frac{PL^2 m\,(1-m)\,(2-m)}{6EI}, \text{ where } t_1 = m\,(1-m)(2-m)$$

$$\tau_{ij} = \frac{PL^2 m\,(1-m^2)}{6\,EI}, \text{ where } t_2 = m\,(1-m^2)$$

t_1 and t_2 are the angular load function coefficients.

m	t_1	t_2	τ_{BA}	τ_{BC}	τ_{CB}
0.1	0.171	.099	33.41	95.88	55.10
0.2	0.288	0.192	64.80	161.48	107.65
0.3	0.357	0.273	92.14	200.17	153.07
0.4	0.384	0.336	113.40	215.31	188.40
0.5	0.375	0.375	126.56	210.26	210.26
0.6	0.336	0.384	129.60	188.40	215.31
0.7	0.273	0.357	120.49	153.07	200.17
0.8	0.192	0.288	97.20	107.65	161.48
0.9	0.099	0.171	57.71	55.51	95.88

SUPPORT MOMENT INFLUENCE VALUES TABLE VIa

Load in 1st or 4th Span					
m	$\tau_{BA} = \frac{337.5}{EI} t^2$	M_B	M_C	M_D	
		$-0.0315\ \tau_{BA}$	$+0.00848\ \tau_{BA}$	$-0.00238\ \tau_{BA}$	
0.0	0	0	0	0	1.0
0.1	33.41	− 1.052	+ 0.2833	− 0.0795	0.9
0.2	64.80	− 2.041	+ 0.5495	− 0.1542	0.8
0.3	92.14	− 2.902	+ 0.7813	− 0.2193	0.7
0.4	113.40	− 3.572	+ 0.9616	− 0.2699	0.6
0.5	126.56	− 3.986	+ 1.0732	− 0.3012	0.5
0.6	129.60	− 4.082	+ 1.0990	− 0.3085	0.4
0.7	120.49	− 3.795	+ 1.0218	− 0.2868	0.3
0.8	97.20	− 3.062	+ 0.8243	− 0.2313	0.2
0.9	57.71	− 1.818	+ 0.4894	− 0.1373	0.1
1.0	0	0	0	0	0.0
		M_D	M_C	M_B	m

SUPPORT MOMENT INFLUENCE VALUES — TABLE VIb

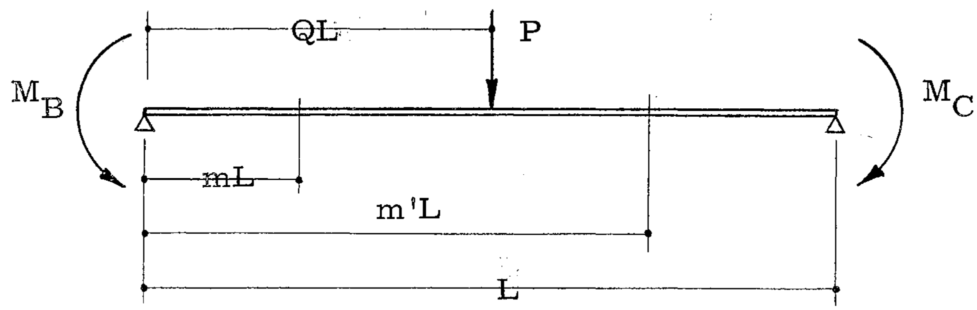
Load in 2nd or 3rd Span

m	τ_{BC} = 560.7 t_1	τ_{CB} = 560.7 t_2	M_B: -0.0315 τ_{BC}	M_B: +0.0085 τ_{CB}	M_B: Σ	M_C: +0.0085 τ_{BC}	M_C: -0.0301 τ_{CB}	M_C: Σ	M_D: -0.0024 τ_{BC}	M_D: +0.0085 τ_{CB}	M_D: Σ	
0.0	0	0	0	0	0	0	0	0	0	0	0	1.0
0.1	95.88	55.10	- 3.020	0.466	- 2.550	+ 0.813	- 1.659	- 0.846	- 0.228	+ 0.466	+ 0.238	0.9
0.2	161.48	107.65	- 5.086	0.912	- 4.174	+ 1.369	- 3.240	- 1.871	- 0.384	+ 0.912	+ 0.528	0.8
0.3	200.17	153.07	- 6.305	1.297	- 5.010	+ 1.697	- 4.607	- 2.910	- 0.476	+ 1.297	+ 0.821	0.7
0.4	215.31	188.40	- 6.780	1.596	- 5.184	+ 1.824	- 5.671	- 3.847	- 0.512	+ 1.596	+ 1.084	0.6
0.5	210.26	210.26	- 6.620	1.781	- 4.840	+ 1.783	- 6.328	- 4.545	- 0.500	+ 1.781	+ 1.281	0.5
0.6	188.40	215.31	- 5.930	1.824	- 4.106	+ 1.598	- 6.481	- 4.883	- 0.448	+ 1.824	+ 1.376	0.4
0.7	153.07	200.17	- 4.820	1.697	- 3.123	+ 1.298	- 6.025	- 4.727	- 0.364	+ 1.695	+ 1.331	0.3
0.8	107.65	161.48	- 3.390	1.367	- 2.020	+ 0.913	- 4.861	- 3.948	- 0.256	+ 1.367	+ 1.111	0.2
0.9	55.51	95.88	- 1.750	0.812	- 0.938	+ 0.471	- 2.885	- 2.414	- 0.132	+ 0.812	+ 0.680	0.1
1.0	0	0	0	0	0	0	0	0	0	0	0	0.0
			M_D			M_C			M_B			m

Bending Moment Influence Values for the Load on 2nd or 3rd Span

TABLE VII

Formulas used in Tables VII



m< Q

$$M_{mL} = M_B + (M_C - M_B)\ m + L\ (1 - Q)$$

Denoting the constants γ, η for tabulation:

$$M_{mL} = M_B + \gamma m + \eta m$$

m> Q

$$M_{m'L} = M_B + \gamma m + \eta m - L\ (m - Q)$$

Q = 0.1

TABLE VIIa

	M_B	γm	ηm	-L (m-Q)	Σ
0.0	- 2.5500	0	0		- 2.5500
0.1	- 2.5500	+ 0.1704	+ 5.2200		+ 2.8404
0.2	- 2.5500	+ 0.3408	+10.4400	- 5.8000	+ 2.4308
0.3	- 2.5500	+ 0.5112	+15.6600	-11.6000	+ 2.0212
0.4	- 2.5500	+ 0.6816	+20.8800	-17.4000	+ 1.6116
0.5	- 2.5500	+ 0.8520	+26.1000	-23.2000	+ 1.2020
0.6	- 2.5500	+ 1.0220	+31.3200	-29.0000	+ 0.7920
0.7	- 2.5500	+ 1.1928	+36.5400	-34.8000	+ 0.3828
0.8	- 2.5500	+ 1.3632	+41.7600	-40.6000	- 0.1027
0.9	- 2.5500	+ 1.5340	+46.9800	-46.4000	- 0.4360
1.0	- 2.5500	+ 1.7040	+52.2000	-52.2000	- 0.8460

TABLE VII b

Q = 0.2

	M_B	γm	ηm	-L (m-Q)	Σ
0.0	- 4.1740	0	0		- 4.1740
0.1	- 4.1740	+ 0.2303	+ 4.6400		+ 0.6963
0.2	- 4.1740	+ 0.4606	+ 9.2800		+ 5.5660
0.3	- 4.1740	+ 0.6909	+13.9200	- 5.8000	+ 4.6360
0.4	- 4.1740	+ 0.9212	+18.5600	-11.6000	+ 3.7072
0.5	- 4.1740	+ 1.1515	+23.2000	-17.4000	+ 2.7775
0.6	- 4.1740	+ 1.3818	+27.8400	-23.2000	+ 1.8478
0.7	- 4.1740	+ 1.6120	+32.4800	-29.0000	+ 0.9180
0.8	- 4.1740	+ 1.8424	+37.1200	-34.8000	- 0.0116
0.9	- 4.1740	+ 2.0730	+41.7600	-40.6000	- 0.9410
1.0	- 4.1740	+ 2.3030	+46.4000	-46.4000	- 1.8710

Q = 0.3

TABLE VII c

	M_B	γm	ηm	-L (m-Q)	Σ
0.0	- 5.0100	0	0		- 5.0100
0.1	- 5.0100	+ 0.2100	+ 4.0600		- 0.7400
0.2	- 5.0100	+ 0.4200	+ 8.1200		+ 3.5300
0.3	- 5.0100	+ 0.6300	+12.1800		+ 7.8000
0.4	- 5.0100	+ 0.8400	+16.2400	- 5.8000	+ 6.2700
0.5	- 5.0100	+ 1.0500	+20.3000	-11.6000	+ 4.7400
0.6	- 5.0100	+ 1.2600	+24.3600	-17.4000	+ 3.2100
0.7	- 5.0100	+ 1.4700	+28.4200	-23.2000	+ 1.6800
0.8	- 5.0100	+ 1.6800	+32.4800	-29.0000	+ 0.1500
0.9	- 5.0100	+ 1.8900	+36.5400	-34.8000	- 1.3800
1.0	- 5.0100	+ 2.1000	+40.6000	-40.6000	- 2.9100

TABLE VII d

Q = 0.4					
	M_B	γm	ηm	-L (m-Q)	Σ
0.0	- 5.1840	0	0		- 5.1840
0.1	- 5.1840	+ 0.1337	+ 3.4800		- 1.5700
0.2	- 5.1840	+ 0.2674	+ 6.9600		+ 2.0430
0.3	- 5.1840	+ 0.4011	+10.4400		+ 5.6570
0.4	- 5.1840	+ 0.5348	+13.9200		+ 9.2710
0.5	- 5.1840	+ 0.6685	+17.4000	- 5.8000	+ 7.0850
0.6	- 5.1840	+ 0.8022	+20.8800	-11.6000	+ 4.8980
0.7	- 5.1840	+ 0.9359	+24.3600	-17.4000	+ 2.7119
0.8	- 5.1840	+ 1.0696	+27.8400	-23.0000	+ 0.5256
0.9	- 5.1840	+ 1.2033	+31.3200	-29.0000	- 1.6607
1.0	- 5.1840	+ 1.3370	+34.8000	-34.8000	- 3.8470

TABLE VII e

Q = 0.5					
	M_B	γm	ηm	-L (m-Q)	Σ
0.0	- 4.8400	0	0		- 4.8400
0.1	- 4.8400	+ 0.0295	+ 2.9000		- 1.9105
0.2	- 4.8400	+ 0.0590	+ 5.8000		+ 1.0190
0.3	- 4.8400	+ 0.0885	+ 8.7000		+ 3.9485
0.4	- 4.8400	+ 0.1180	+11.6000		+ 6.8780
0.5	- 4.8400	+ 0.1475	+14.5000		+ 9.8075
0.6	- 4.8400	+ 0.1770	+17.4000	- 5.8000	+ 6.9370
0.7	- 4.8400	+ 0.2065	+20.3000	-11.6000	+ 4.0665
0.8	- 4.8400	+ 0.2360	+23.2000	-17.4000	+ 1.1960
0.9	- 4.8400	+ 0.2655	+26.1000	-23.2000	- 1.6745
1.0	- 4.8400	+ 0.2950	+29.0000	-29.0000	- 4.5450

TABLE VII f

Q = 0.6

	M_B	γm	ηm	-L (m-Q)	Σ
0.0	- 4.1060	0	0		- 4.1060
0.1	- 4.1060	- 0.0777	+ 2.3400		- 1.8437
0.2	- 4.1060	- 0.1554	+ 4.6800		+ 0.4186
0.3	- 4.1060	- 0.2331	+ 7.0200		+ 2.6809
0.4	- 4.1060	- 0.3108	+ 9.3600		+ 4.9432
0.5	- 4.1060	- 0.3885	+11.7000		+ 7.2055
0.6	- 4.1060	- 0.4662	+14.0400		+ 9.4678
0.7	- 4.1060	- 0.5439	+16.3800	- 5.8000	+ 5.9300
0.8	- 4.1060	- 0.6216	+18.7200	-11.6000	+ 2.3920
0.9	- 4.1060	- 0.6993	+21.0600	-17.4000	- 1.1453
1.0	- 4.1060	- 0.7770	+23.2000	-23.2000	- 4.8830

TABLE VII g

Q = 0.7

	M_B	γm	ηm	-L (m-Q)	Σ
0.0	- 3.1230	0	0		- 3.1230
0.1	- 3.1230	- 0.1604	+ 1.7400		- 1.5434
0.2	- 3.1230	- 0.3208	+ 3.4800		+ 0.0362
0.3	- 3.1230	- 0.4812	+ 5.2200		+ 1.6158
0.4	- 3.1230	- 0.6416	+ 6.9600		+ 3.1954
0.5	- 3.1230	- 0.8020	+ 8.7000		+ 4.7750
0.6	- 3.1230	- 0.9624	+10.4400		+ 6.3546
0.7	- 3.1230	- 1.1230	+12.1800		+ 7.9340
0.8	- 3.1230	- 1.2830	+13.9200	- 5.8000	+ 3.7140
0.9	- 3.1230	- 1.4436	+15.6600	-11.6000	- 0.5066
1.0	- 3.1230	- 1.6040	+17.4000	-17.4000	- 4.7270

TABLE VIIh

Q = 0.8

	M_B	γm	ηm	-L (m-Q)	Σ
0.0	- 2.0230	0	0		- 2.0230
0.1	- 2.0230	- 0.1925	+ 1.1600		- 1.0560
0.2	- 2.0230	- 0.3850	+ 2.3200		- 0.0880
0.3	- 2.0230	- 0.5775	+ 3.4800		+ 0.8795
0.4	- 2.0230	- 0.7700	+ 4.6400		+ 1.8470
0.5	- 2.0230	- 0.9625	+ 5.8000		+ 2.8145
0.6	- 2.0230	- 1.1550	+ 6.9600		+ 3.7020
0.7	- 2.0230	- 1.3480	+ 8.1200		+ 4.7490
0.8	- 2.0230	- 1.5400	+ 9.2800		+ 5.7170
0.9	- 2.0230	- 1.7330	+10.4400	- 5.8000	+ 0.8840
1.0	- 2.0230	- 1.9250	+11.6000	-11.6000	+ 3.9570

TABLE VIIi

Q = 0.9

	M_B	γm	ηm	-L (m-Q)	Σ
0.0	- 0.9380	0	0		- 0.9380
0.1	- 0.9380	- 0.1476	+ 0.5800		- 0.5056
0.2	- 0.9380	- 0.2952	+ 1.1600		- 0.0732
0.3	- 0.9380	- 0.4428	+ 1.7400		+ 0.3592
0.4	- 0.9380	- 0.5904	+ 2.3200		+ 0.7916
0.5	- 0.9380	- 0.7380	+ 2.9000		+ 1.2240
0.6	- 0.9380	- 0.8856	+ 3.4800		+ 1.6564
0.7	- 0.9380	- 1.0330	+ 4.0600		+ 2.0890
0.8	- 0.9380	- 1.1808	+ 4.6400		+ 2.5212
0.9	- 0.9380	- 1.3284	+ 5.2200		+ 2.9540
1.0	- 0.9380	- 1.4760	+ 5.8000		+ 3.3860

Determination of Safe Load P:

Allowable Stress Conditions.

$fc^t = 0.40\ f'c = 0.40 \times 4500 = 1800$ psi

fc^b 4 0

Section Data.

Area	I_x	C_t	C_b	Z_t	Z_b
1404 in.	23,933 in^4	5.308 in	6.692 in	4509 in^3	3,576 in^3

Girder Moment at 0.5 L.

Areas for the moment influence lines for M_B and M_C are calculated by the trapesoidal area approximation method, thus:

Area for M_C.

$4.5\ (7.0834) \times 2\ -5.8\ (29.91) \times 2 = -283.206\ FT^2$

Area for M_B.

$-4.5\ (26.31) - 5.8\ (40.387) + 5.8\ (8.45) - 4.5\ (1.988) =$

$= -312.575\ FT^2$

Thus multiplying the areas by the intensity of gravity load:

$M_C = 1.463 \times -283.206 = -414.330$

$M_B = 1.463 \times -312.576 = -457.299$

The moment due to simple beam at the center is $\frac{WL^2}{8} =$

$= +615.19$

By superposition the value at 0.5L is obtained to be:

$M_G = 179.376 \times 12 = 2,152,440$ # in.

Moment due to Live Load.

Form the maximum moment ordinate of the influence line due to concentrated load in the second span:

$$9.8075\ P \times 12 = 117.69\ P$$

Stress Equation:

The stress equation in a prestress concrete beam from the application of external loads and the prestressing force is given as:

$$f^t_{H_iT} = + \frac{H_i}{A} - \frac{H_ie}{Z_t} + \frac{M_G}{Z_t} + \frac{M_L}{Z_t} \qquad \text{(EQ F-1)}$$

$$f^b_{H_iT} = + \frac{H_i}{A} + \frac{H_ie}{Z_b} - \frac{M_G}{Z_b} - \frac{M_L}{Z_b} \qquad \text{(EQ F-2)}$$

$$f^t_{H_fT} = + \frac{H_f}{A} - \frac{H_fe}{Z_t} + \frac{M_G}{Z_t} + \frac{M_L}{Z_t} \qquad \text{(EQ F-3)}$$

$$f^b_{H_fT} = + \frac{H_f}{A} + \frac{H_fe}{Z_b} - \frac{M_G}{Z_b} - \frac{M_L}{Z_b} \qquad \text{(EQ F-4)}$$

representing each term with its numerical value and considering both cases of initial and final prestressing =

$$\frac{H_i}{A} = \frac{3,369,000}{1404} = 2,399.57 \text{ psi}$$

$$\frac{H_f}{A} = \frac{2,864,000}{1404} = 2,039.89 \text{ psi}$$

$$\frac{H_ie}{Z_t} = \frac{3,369,000\ (3.69)}{4509} = 2757.06 \text{ psi}$$

$$\frac{H_i e}{Z_b} = \frac{3,369,000\ (3.69)}{3576} = 3476.40 \text{ psi}$$

$$\frac{H_f e}{Z_t} = \frac{2,864,000\ (3.69)}{4509} = 2344.02 \text{ psi}$$

$$\frac{H_f e}{Z_b} = \frac{2,864,000\ (3.69)}{3576} = 2954.86 \text{ psi}$$

$$\frac{M_G}{Z_t} = \frac{2,152,440}{4509} = 477.365 \text{ psi}$$

$$\frac{M_G}{Z_b} = \frac{2,152,440}{3576} = 601.912 \text{ psi}$$

$$\frac{M_L}{Z_t} = \frac{117.69\ P}{4509} = 0.02610\ P$$

$$\frac{M_L}{Z_b} = \frac{117.69\ P}{3576} = 0.0329\ P$$

$$f^t_{H_iG} = 0.55 \times f'c = 0.55 \times 4500 = 2475 \text{ psi}$$

$$f^b_{H_iG} = 0 \text{ psi}$$

$$f^t_{H_fG} = 1800 \text{ psi}$$

$$f^b_{H_fG} = 0 \text{ psi}$$

Considering (+) for compression, and with the above values substituting in the stress equation the value for P can be determined:

Due to H_i for the Top Fiber Stresses.

$$2475 = 2,399.57 - 2757.06 + 477.37 + 0.02610\,P$$

$$2355.12 = 0.0261\,P$$

$$90.23 = P$$

Due to H_i for the Bottom Fiber Stresses.

$$0 = 2,399.57 + 3476.40 - 601.9 - 0.0329\,P$$

$$5274.06 = +0.0329P$$

$$160.30 = P$$

Due to H_f For the Top Fiber Stresses.

$$1800 = 2039.89 - 2344.02 + 477.365 + 0.02610\,P$$

$$1626.8 = 0.0261\,P$$

$$62.33 = P$$

Due to H_f For the Bottom Fiber Stresses.

$$0 = 2039.89 + 2954.86 - 601.912 - 0.0329\,P$$

$$4392.84 = 0.0329\,P$$

$$133.52 = P$$

The Safe Concentrated Live Load:

$$P = 62.33$$

CHAPTER IV

COMPARISON AND CONCLUSION

The general equations of the four selected methods of analysis, namely the

(1) Virtual Work

(2) Carry Over

(3) Slope-Deflection

(4) Moment Distribution

including the effect of prestressing, were derived in such a way that the angular functions F_{ij}, G_{ij}, $\tau^{(L)}$, $\tau^{(H)}$, etc., would remain common in all the expressions of these methods.

The derivations included expressions for starting moments, and fixed-end moments, in terms of τ's, the end slopes of a simple beam due to load or prestress, was easily evaluated by considering the conjugate beams of simple spans loaded with $\frac{He}{EI}$ diagrams. This is certainly more advantageous than finding equivalent loads due to prestress to evaluate these functions, since He diagram itself is completely known.

Tables with coefficients for the above mentioned angular functions for beams with variable moment of inertia, for dead load, uniform live load, and unit live load are prepared in Reference (15).

As the derivations of all the methods contained in this work are in terms of these angular functions the procedure of analysis by any of these methods is greatly simplified by use of these tables where needed.

Fundamentally the derivation for each method used the condition of consistent deformation and established continuity of elastic curve over the supports.

The method of Virtual Work results in a set of Three Moment Equations to be solved symultaneously. In general this method is simple and easily applicable for a reasonable number of continuous spans.

The slope Deflection Method is particularly useful when settlements of supports or given rotations of any ends are involved. This method has the advantage of good physical interest in as much as it leads to the iterative procedures like the Moment Distribution.

As said above the Method of Moment Distribution is a numerical method of balancing the support moments on either side of each support.

The Method of Carry Over Moments is also a numerical method of successive approximations, which again can be carried out to a desired degree of accuracy. This method differs from the method of Moment Distribution in relaxation technique as well as the concept of approach.

The method is obviously superior to that of Moment Distribution as it doesn't contain any distribution of moments.

The regular carry over tables reduce to almost half of the size of those in moment distribution, since the carry over factors are usually very small and the convergence is rapid. By the use of Algebraic Carry Over Tables the labor is more simplified. and yields a greater degree of accuracy.

This method is simple and especially handy for an increasing number of spans.

As illustrated in the numerical example, this method has easy means of calculations for influence lines, which is an important factor in considering continuous spans.

A SELECTED BIBLIOGRAPHY

1. Magnel, G., "Le Calcul des Poutres Continues a Fravecs e Gales en Beton Precontraint" - Annales des Travaux Publics de Belgique. Vol. 100 n3,June 1947 p. 285-320.

2. Magnel, G., "Prestressed Concrete" -McGraw - Hill Book Company, Inc., 1954, N.Y. Third Edition.

3. Frexssinet, E., "Prestressed Concrete" - Principles and Application. Inst. C.E. Journal, Vol. 33 n.4, Feb. 1950, p. 331-380.

4. A.L. Parme and G.H. Pans, "Designing for Continuity in Prestressed Concrete Structures" - Am. Conc. Inst. Journal Vol. 23,n1, Sept. 1951, p. 45-64.

5. Crackneil, D.W. and W.A. Knight, "The Analysis of Prestressed Concrete Statically Indeterminate Structures" - "A Symposium on Prestressed Concrete Statically Indetermined Structures" Sept., 1951. Published by Cement and Concrete Association, London, SWI. p. 24,25

6. Trimble, E. G., "Determination of Continuity Bending Moments in Prestressed Continuous Beams" - "A Symposium on Prestressed Concrete Statically Indeterminate Structures" Sept., 1951. Published by Cement and Concrete Association, London, SWI. p. 95-98.

7. Guyon, Y., "Quelques Problems des Constructions Hyperstatiques Precontraintes Par Cables" - Ingeniur, Vol. 64,n21,23 May 23, 1952 p. Bt 29-35, June 6, 1955 p. Bt 37-42.

8. Moorman, R.B.B., "Equivalent Load Method for Analysing Prestressed Concrete Structures" - Am.Conc. Inst. Journal Vol. 23, n5, Jan., 1952, p. 405-416.

9. Komendant, A.E., "Prestressed Concrete Structures" - McGraw-Hill Book Co., New York, Toronto, London, 1952.

10. Feisenheiser, E. I., "Rapid Design of Continuous Prestressed Members" - Am.Conc Inst. Journal, Vol. 25,n8, April, 1954. p. 669-676.

11. Newmark, N.M., "Newmark Numerical Procedure and Its Application in Analysis of Continuous Prestressed Beams" - Rein. Conc. Review, Vol. 3, n5, 1954, p. 303-312.

12. Newmark, N.M., ASCE Proc. May, 1942, Jan., 1943.

13. Lin, T.Y., "Design of Prestressed Concrete Structures" - New York, John Wiley and Sons, Inc., London, Chapman and Hall, Limited. Jan., 1958. (Third Edition).

14. Kao, M., "Slope Deflection Equations for Prestressed Concrete Beams" - M.S. Report, Oklahoma State University, Stillwater, 1956.

15. Tuma, J.J. "Analysis of Continuous Beam Bridges Using Carry-Over Procedure" - Publication No. III, Engineering Experiment Station, Oklahoma State University, Stillwater, March, 1960.

16. Munshi, R., "Analysis of Prestressed Concrete Continuous Beams by Carry-Over Procedure" - M.S. Thesis, Oklahoma State University, Stillwater. (in preperation).

17. Tuma, J.J.,"Structural Lectures and Seminar Notes" - Civil Engineering Department, Oklahoma State University, Stillwater, Spring, 1959.- Summer, 1960.

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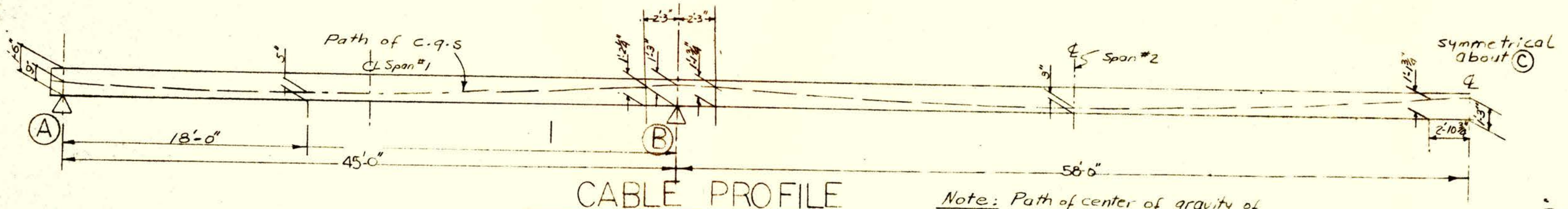
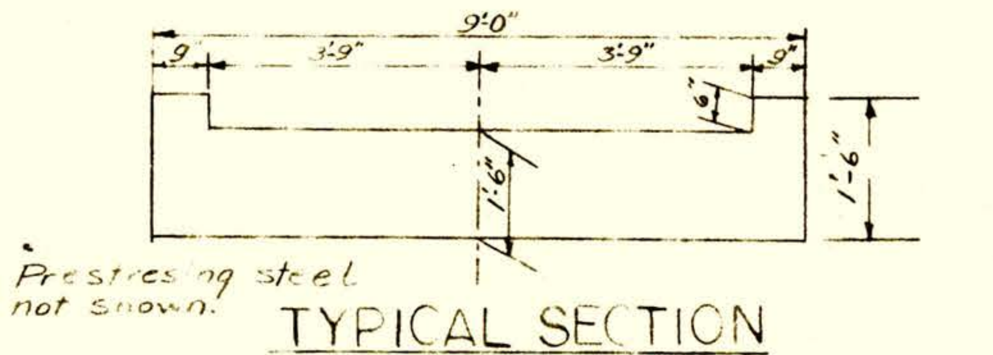


CABLE PROFILE

NO SCALE

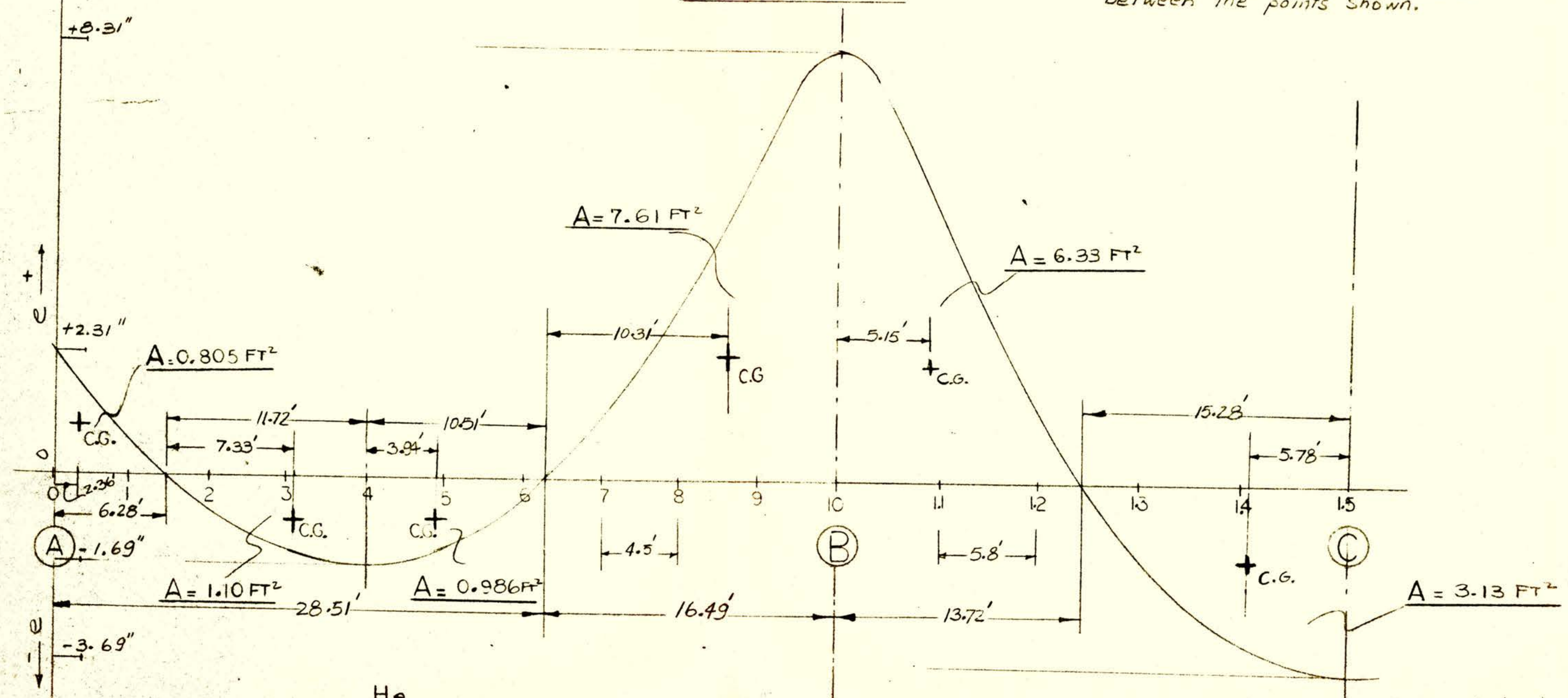
Note: Path of center of gravity of prestressing force is a series of second degree parabolas between the points shown.



TYPICAL SECTION

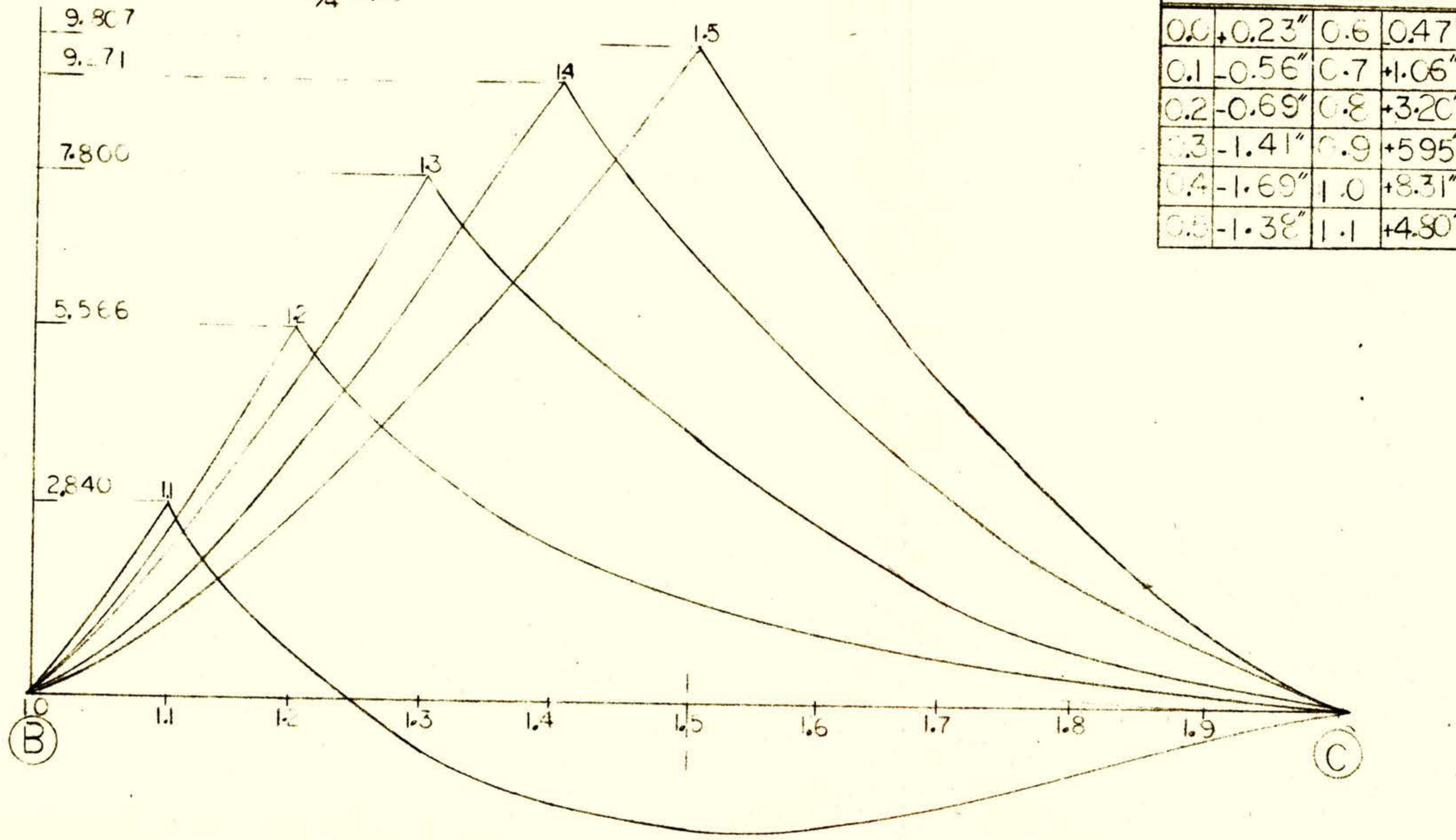
3/4" = 1'-0"

TABLE OF ECCENTRICITY OF STEEL					
0.0	+0.23"	0.6	0.47"	1.2	+1.09"
0.1	-0.56"	0.7	+1.06"	1.3	-1.57"
0.2	-0.69"	0.8	+3.20"	1.4	-3.16"
0.3	-1.41"	0.9	+5.95"	1.5	-3.69"
0.4	-1.69"	1.0	+8.31"		
0.5	-1.38"	1.1	+4.80"		



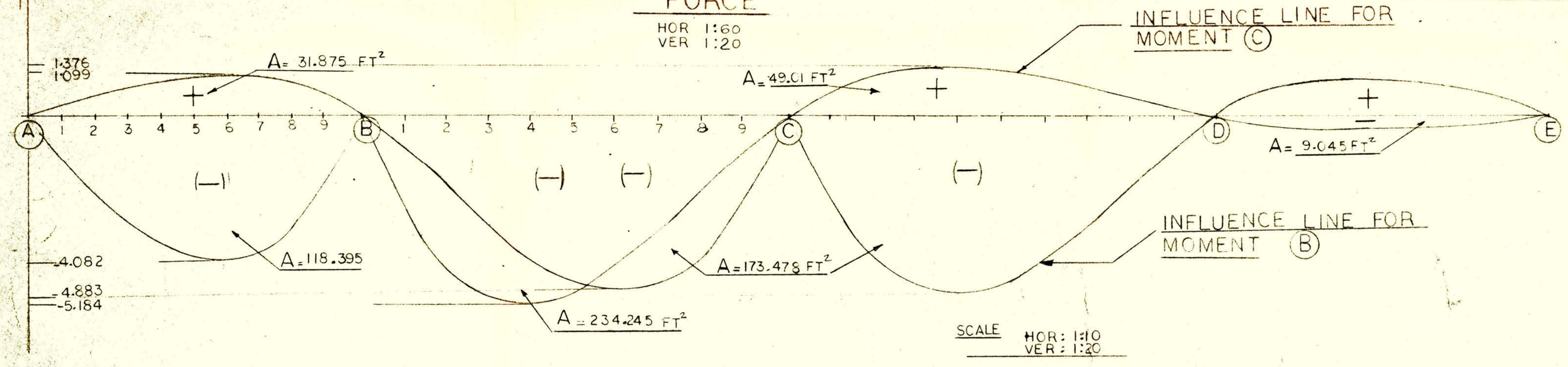
$\frac{He}{EI}$ - DIAGRAM or INFLUENCE LINE FOR PRESTRESSING FORCE

HOR 1:60
VER 1:20



INFLUENCE LINES FOR MOMENTS AT STATIONS FOR THE SECOND SPAN ONLY

SCALE HOR: 1:60
VER: 1:20



SCALE HOR: 1:10
VER: 1:20

INFLUENCE LINES FOR MOMENTS

OKLAHOMA STATE UNIVERSITY SCHOOL OF CIVIL ENGEERING
ANALYISIS OF FOUR SPAN CONTINUOUS PRESTRESSED GIRDER
DRAWN BY: M.C. DIRI
CHECKED BY:
DATE: SPRING 1961
SCALE: SPECIFIED