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Title of Study: A COMPARISON OF TRADITIONAL AND SMSG 7th AND 8th GRADE MATHEMATICS TEXTS

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Scope of Study: Unfamiliarity is an inherent cause of hesitancy. Those administrators and teachers who fail to supplement their present mathematics programs or implement a new program involving the contempory concepts and approaches do so not so much because of malice or dishonesty but rather because of unfamiliarity with new subject materials. This report, in bulk, presents actual subject matter from both the traditional and modern mathematics textbooks in the 7th and 8th grades with statements of respective critique and summary. Overall introduction and summary state recent trends and criteria, and give implication of the responsibility for valid selection of text materials. In this way it is hoped that a reduction in unfamiliarity is effected and that a comparison of the respective text materials, herein, by those who select textbooks for their individual schools will result in a preparatory mathematics program adequate for today's secondary student to meet the demands of future instruction and vocation.

ADVISOR'S APPROVAL ______________________

A COMPARISON OF TRADITIONAL AND SMSG
7th AND 8th GRADE MATHEMATICS TEXTS

By

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A COMPARISON OF TRADITIONAL AND SMSG

7th AND 8th GRADE MATHEMATICS TEXTS

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Dean of the Graduate School

PREFACE

The cry of "modern mathematics" is being echoed in the ears of those responsible for selecting textbooks and planning curriculum schedules in our secondary schools, especially in Oklahoma. In many cases this is the superintendent or principal. The purpose of this report is to allow administrators the opportunity to hear what has been said about the modern approach to the mathematics curriculum and observe the actual content materials of the School Mathematics Study Group which are not as readily available as the traditional texts now on their shelves. It is not intended that the selected topics found in the content materials used in the comparison herein should be expressed in terminology and symbolism decipherable only by mathematics teachers and fellow mathematicians. On the contrary, the selection of the few pages representing the total 7th and 8th grade SMSG program was done in an effort to present topics which are somewhat self-explanatory or readily understandable to those who are interested in mathematics curriculum but restricted in contact with mathematics as a discipline.

A special thanks is extended to Dr. James H. Zant for his counsel and assistance in the preparation of this report and to the National Science Foundation who made this year of study possible.

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CHAPTER I

INTRODUCTION

Today there is tremendous pressure on the United States as the stronghold of the free world to maintain her position of scientific and productive leadership and keep pace with the proponents of communism in the quest for space. To do so we must fill the evergrowing demand for scientists, engineers, technicians, and mathematicians. An initial purpose of this report is to enable those who select the curriculum in our high schools to observe and consider the traditional and modern textbook approaches as preparations for just such persons.

Of course not every student will be a space scientist or computer statistician, but our secondary schools being instrumental in preparation of students for college capability should supply an opportunity for groundwork on which students could build toward these endeavors in future instruction. The nature and level of this opportunity will naturally depend on the local school situation and the attitudes and abilities of the administrators, teachers, students, and parents. With this in mind, the overall aim of this report is to show a comparative presentation of the traditional and modern 7th and 8th grade mathematics programs to be used as a basis of selection of the appropriate curriculum presentation for the individual school. The 7th and 8th grades are chosen because of their primary position in secondary study and a conviction that during the adolescent years basic concepts should be stressed.

To present this comparison four traditional texts, with respective Teachers Editions, from the state adoption list of Oklahoma, 1960, are selected as typical. Two of these, The New Thinking With Numbers and The New Knowing About Numbers, both by Leo J. Brueckner, Foster E. Grossnickle, and Elda L. Merton, John C. Winston Company, Dallas, 1956, cover 7th and 8th grades respectively. The other two are Exploring Arithmetic (7) and Exploring Arithmetic (8), by Jesse Osborn, Adeline Riefling, and Herbert F. Spitzer, Webster Publishing Company, Dallas, 1957. To discuss and display the modern or contemporary approach the texts of the School Mathematics Study Group are preferred.

> The School Mathematics Study Group (SMSG) represents the largest united effort for improvement in the history of mathematics education. It is national in scope. The director is Professor E. G. Begle, whose office was, until recently, at Yale University. In the fall of 1961, Professor Begle and SMSG headquarters moved to Stanford University. SMSG if financed by the National Science Foundation.
>
> The development of the SMSG material is unique in that it represents the combined thinking of many people — psychologists, testmakers, mathematicians from colleges and industry, biologists, and high school teachers. Approximately 100 mathematicians and 100 high school teachers did the writing, and in order to produce material that is both mathematically sound and teachable, each writing team had an equal number from each group.(1, p. 17)

The choice of SMSG texts was conditioned, by extent of local significance, in a report read on October 1, 1960, to the Annual Meeting of Oklahoma School Administrators at Norman, Oklahoma, by Dr. James H. Zant, National Science Foundation Director, Oklahoma State University, Stillwater, Oklahoma.

> There has been some individual experimentation going on in Oklahoma schools at all levels from Grades 1-12.
>
> ***
>
> ..., the bulk of the experimentation with new programs in mathematics has resulted from co-operative efforts

> between the Oklahoma State Committee for the Improvement of Mathematics Instruction and the School Mathematics Study Group. During 1959-60 SMSG assigned 7 of its 49 Centers of Teaching SMSG Textbooks to this state. These involved the use in regular classes of textbooks for Grades 9, 10 and 11. Twenty-three school systems were involved with 42 teachers, 84 classes, and approximately 2,500 students. Eight college staff members served as consultants.
>
> This program was considered very successful. Teachers reported much more interest and understanding on the part of these students than did those in classes taught from the traditional program. School administrators and parents were enthusiastic about the program and the results obtained. Students seemed to be as proficient in the standard skills of mathematics as those taking traditional courses, though no particular effort was made to make comparisons. Preliminary comparisons made by SMSG and the Minnesota National Mathematics Laboratory also indicate this is the case. Teachers reported that student ability in handling word or "story" problems was outstanding.
>
> Examination of actual orders sent to SMSG for revised copies of SMSG textbooks for use in Oklahoma schools during 1960-61 reveals that 52 school systems will use the books in some or all grades 7-12. The total number of texts ordered by Oklahoma schools for next year is 13,494. This exceeds five times the number of books used last year and again (as in the case of the Experimental Centers for 1959-60) Oklahoma's use of the books exceeds that of any state except California. The total number of orders in the entire nation was 110,921. Oklahoma's share was more than 12 per cent!
>
> ***
>
> In Oklahoma, then, we have placed most emphasis on the teaching material, Grades 4-12, developed by the School Mathematics Study Group. There have been several reasons for this. We have been able to take advantage of SMSG's program of experimentation and the fact that some subsidy has been available. Other considerations were: (1) SMSG has complete teaching material for all grade levels; (2) a program initiated in a particular school can be continued into the next year; (3) a program can be expanded to include other schools in the state. The success of this idea is indicated by the large number of SMSG books ordered in Oklahoma this year. We are convinced that the books present good mathematics which children can understand and enjoy.(2, p. 597-598)

The SMSG books to be used are <u>Mathematics for Junior High School</u>, Volume 1, Books 1, 2, 3, New Haven, Connecticut: SMSG, 1960. These for College Preparatory Mathematics was issued in the spring of 1959. (3, p. ix) The overriding objective of the Commision in this Report is "a curriculum suitable for students and oriented to the needs of mathematics, natural science, social science, business, technology, and industry in the second half of the twentieth century. ..... Whatever of the old has disappeared, whatever of the traditional yet remains, whatever of the new appears, is in or out of the curriculum solely to effect necessary modification and improvement."(3, p. 34)

CHAPTER II

THE TRADITIONAL PROGRAM

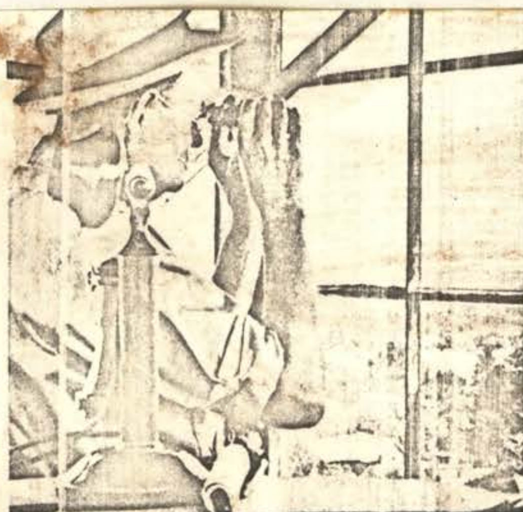
Overview

Here we shall take a look at the traditional program content in an effort to display in brief compilation the topics typical of the great majority of traditional textbooks. In accordance with the purpose of this report it is of importance to examine carefully the approaches in these topics.

In this chapter are areas concerning review of operations, percent, percent applications, measurement, and economic and business environments. The approach in the section on review of arithmetic operations is one of algorithms and "rule" method with hardly any explanation of "why". For example, the student is shown how to "cancel" and if the student wonders why this operation is legal he is misled by the implications that "canceling makes multiplication easier." Granted that this last statement is true, there is still no basis presented for the mathematical soundness of this operation. Percent and applications of percent are approached in their relationship to the business world. Knowledge of the business world is commendable, but terms such as collateral, list price, and net cost are of little interest to the junior high school student, and of no use in the structure of mathematical systems on which the concept of percent and operations with percent are based. Measurement is also approached from a social application viewpoint based, for example, on a need to know "what the volume of a home-use cord is when the sticks are 16 inches long" or "the number of gallons of disinfectant, with $\frac{1}{4}$ gallon per square yard of surface, needed to treat playgrounds having the following dimensions: 150' x 375'." Lastly, in the part on economic and business environments there is very little mathematical approach in reading gas and electric meters, finding a bank balance, or knowing what becomes of the Florida orange crop.

Review of Operations

> Traditional mathematics courses for grades 7 and 8 include a review of the operations with whole numbers, fractions, and decimals.(5, p. 5)



Ewing Galloway

Looking for Danger Spots

This test will show how well you are able to work the kinds of examples you have studied in previous grades.

If you make errors in any of the processes in whole numbers, take the diagnostic tests in those processes. Do the same for fractions and decimals. The directions on page 7 will show where to find the Study Helps and the diagnostic tests to use.

Whole Numbers | **Common Fractions** | **Decimal Fractions**

I. Addition:

Whole Numbers		Common Fractions		Decimal Fractions	
1. 736	2. $ 4.87	3. $6\frac{1}{4}$	4. $1\frac{1}{2}$	5. .47	6. 14.8
358	23.54	$4\frac{1}{2}$	$4\frac{2}{3}$	.58	9.6
407	9.75	$5\frac{7}{8}$	$2\frac{5}{6}$	.25	3.7
935	17.86	$1\frac{3}{4}$	$7\frac{5}{12}$	.68	11.4
689	4.98				
3125	$61.00	$18\frac{3}{8}$	$16\frac{5}{12}$	1.98	39.5

three paperback editions comprise the 7th grade portion. Also to be used are Mathematics For Junior High School, Volume 2, Books 1, 2, 3, New Haven, Connecticut: SMSG, 1960, comprising the 8th grade work. Each of these six books has a respective Commentary for Teachers making a total of twelve paperback texts.

To obtain a useful comparison in view of the overall purpose of this report -- selection of an appropriate curriculum for the individual school -- decidable contrasts in subject matter must be shown. In order to accomplish this the bulk of this report will consist of a compilation of content from, first, the traditional books; and, secondly, the SMSG texts. These selected materials will exemplify specifically prefaced statements of what can be found in the respective programs. A short critique after both chapters serves as a brief summary of approaches.

The topics from SMSG are chosen with respect to emphasis of the 1959 Report of Commission on Mathematics of the College Entrance Examination Board whose proposals for college capable students include:

1. Judicious use of unifying ideas -- Sets
2. Treatment of Inequalities along with Equations
3. Appreciation of Mathematical Structure Patterns
4. Strong preparation in Concepts and Skills
5. Understanding of the nature and role of Deductive Reasoning(3, p. 33)

This Commission was the result of appointment by the Mathematics Examiners of the College Entrance Examination Board, the distinguished school and college teachers who are responsible for the Board's entrance examinations in mathematics, who wanted to know what kind of mathematics should be studied by today's American youth capable of going on to college work.(3, p. xi)

Their work began in August, 1955, under the Chairmanship of Professor A. W. Tucker of Princeton University and the final report on a Program

II. Subtraction:

1. 7213	2. 10,302	3. $17\frac{3}{4}$	4. $11\frac{1}{2}$	5. .703	6. 4.75
4215	9,305	$9\frac{5}{8}$	$4\frac{2}{3}$	.518	3.98
2998	*997*	*$8\frac{1}{8}$*	*$6\frac{5}{6}$*	*.185*	*0.77*

III. Multiplication:

1. 78	2. 609	3. $7\frac{1}{2} \times 3\frac{1}{3} =$ *25*	5. 4.36	6. 3.25
47	507	4. $9 \times 8\frac{1}{2} =$ *$76\frac{1}{2}$*	9	1.2
3666	*308,763*		*39.24*	*3.9*

IV. Division:

1. 17)9524 *560 r 4*
2. 36)75,204 *2,089*
3. $36 \div \frac{4}{9} =$ *81*
4. $1\frac{1}{2} \div 3\frac{3}{4} =$ *$\frac{2}{5}$*
5. 15)1.2 *.08*
6. .8)16 *20*

(6, p. 6)

3. Study the examples below and show how each quotient figure was found. Then copy the examples and work them with your book closed.

a. 230)690 = 3	b. 420)956 = 2 r116	c. 320)1045 = 3 r85
690	840	960
	116	85

(6, p. 28)

To find the lowest common denominator of two or more unlike fractions,

a. Test whether the largest denominator contains each of the other denominators evenly. If it does, this is the lowest common denominator.

b. If the largest denominator is not the common denominator, multiply it first by 2, then by 3, then by 4, and so on, until the smallest number that contains each denominator evenly is found.

Find the lowest common denominator and do these subtractions.

21.	$\frac{2}{3} - \frac{1}{6}$	$\frac{3}{4} - \frac{1}{2}$	$\frac{9}{10} - \frac{4}{5}$	$\frac{1}{2} - \frac{3}{8}$
22.	$\frac{3}{5} - \frac{2}{10}$	$\frac{5}{6} - \frac{2}{3}$	$\frac{7}{8} - \frac{3}{4}$	$\frac{3}{4} - \frac{9}{16}$
23.	$\frac{2}{3} - \frac{2}{5}$	$\frac{1}{2} - \frac{1}{8}$	$\frac{1}{3} - \frac{1}{4}$	$\frac{3}{4} - \frac{1}{3}$
24.	$\frac{5}{6} - \frac{3}{8}$	$\frac{3}{4} - \frac{3}{16}$	$\frac{17}{18} - \frac{2}{6}$	$\frac{5}{8} - \frac{3}{7}$

(8, p. 31)

Study and explain this short way of changing a mixed number to an improper fraction.

$$4\frac{1}{2}=\frac{4\times 2+1}{2}=\frac{9}{2}$$

To change a mixed number to an improper fraction, multiply the denominator of the fraction by the whole number. Then add the numerator of the fraction. This will give the numerator of the improper fraction. Use the same denominator.

Explain this multiplication.

$$\frac{6}{1}\times\frac{9}{2}=\frac{54}{2}=27$$

Change to improper fractions.

14. $2\frac{1}{2}$ $1\frac{1}{2}$ $3\frac{1}{2}$ $4\frac{1}{2}$ $6\frac{1}{2}$
15. $3\frac{1}{3}$ $1\frac{2}{3}$ $6\frac{2}{3}$ $8\frac{1}{3}$ $4\frac{2}{3}$
16. $2\frac{1}{4}$ $6\frac{1}{4}$ $3\frac{3}{4}$ $4\frac{3}{4}$ $8\frac{1}{4}$
17. $2\frac{1}{6}$ $3\frac{5}{6}$ $3\frac{1}{6}$ $4\frac{5}{6}$ $5\frac{1}{6}$
18. $1\frac{1}{8}$ $2\frac{3}{8}$ $3\frac{5}{8}$ $1\frac{7}{8}$ $3\frac{7}{8}$

Multiply. Use a different way of multiplying as a check.

19. $8\times 2\frac{1}{2}$ $5\times 3\frac{1}{2}$ $4\frac{1}{2}\times 10$
20. $6\times 1\frac{2}{3}$ $6\frac{2}{3}\times 6$ $3\frac{1}{3}\times 12$
21. $8\times 2\frac{1}{4}$ $4\times 3\frac{3}{4}$ $1\frac{3}{4}\times 20$
22. $3\times 4\frac{1}{6}$ $8\times 2\frac{5}{6}$ $1\frac{5}{6}\times 15$
23. $4\times 3\frac{5}{8}$ $6\times 3\frac{7}{8}$ $4\frac{3}{8}\times 18$

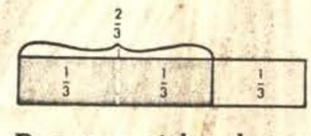
When multiplying small mixed numbers, change both mixed numbers to improper fractions.

Multiply.

24. $1\frac{3}{4}\times 2\frac{2}{3}$ $6\frac{2}{3}\times 1\frac{7}{8}$ $6\frac{1}{4}\times 5\frac{1}{3}$
25. $8\frac{1}{3}\times 3\frac{3}{5}$ $4\frac{3}{8}\times 3\frac{1}{7}$ $5\frac{5}{6}\times 1\frac{4}{5}$

26. Study this multiplication. Explain how the drawing proves it.

$$\frac{1}{2}\times\frac{2}{3}=\frac{2}{6}=\frac{1}{3}$$



Multiply. Prove with drawings.

27. $\frac{1}{2}\times\frac{3}{5}$ $\frac{1}{2}\times\frac{4}{5}$ $\frac{1}{3}\times\frac{3}{5}$
28. $\frac{1}{4}\times\frac{2}{3}$ $\frac{3}{4}\times\frac{2}{3}$ $\frac{2}{3}\times\frac{1}{2}$

29. The two multiplications below show two ways of finding the answer in lowest terms. Explain both examples. Dividing before multiplying, as in B, is called *canceling*. Why does canceling make multiplication of fractions easier?

A. $\frac{3}{4}\times\frac{8}{15}=\frac{\overset{2}{\cancel{24}}}{\underset{5}{\cancel{60}}}=\frac{2}{5}$

B. $\frac{\overset{1}{\cancel{3}}}{\underset{1}{\cancel{4}}}\times\frac{\overset{2}{\cancel{8}}}{\underset{5}{\cancel{15}}}=\frac{2}{5}$

In the multiplication of fractions, to cancel means to divide a numerator and a denominator by the same number.

Multiply. Be sure to cancel.

30. $\frac{2}{3}\times\frac{5}{8}$ $6\times\frac{3}{8}$ $2\frac{1}{4}\times 2\frac{2}{3}$
31. $\frac{3}{4}\times\frac{2}{9}$ $\frac{5}{6}\times 4$ $1\frac{7}{8}\times 2$
32. $\frac{7}{8}\times\frac{4}{5}$ $\frac{3}{4}\times 6$ $8\times 1\frac{3}{4}$

(8, p. 35)

Four Kinds of Examples in Division of Decimals

There are four kinds of examples in division of decimals.

A. Dividing a decimal by a whole number, as 2)1.2 (quotient .6)

B. Dividing two whole numbers and getting a decimal in the quotient, as 5)7 (quotient 1.4)

C. Dividing a whole number by a decimal, as .5)2 = 5)20 (quotient 4)

D. Dividing a decimal by a decimal, as .5).2 = 5)2.0 (quotient .4)

These sets of exercises illustrate the four kinds of examples in division of decimals. Solve and check.

Set A

	a	b	c	d	e
1.	6).48 (.08)	8).36 (.045)	4).1 (.025)	3)8.4 ([illegible]8)	8)52.4 (6.55)
2.	3).81 (.27)	9).72 (.08)	5).3 (.06)	24).72 (.03)	14)19.6 (1.4)

Set B

	a	b	c	d	e
1.	6)3 (.5)	25)4 (.16)	8)7 (.875)	20)9 (.45)	16)11 (.6875)
2.	50)1 (.02)	28)14 (.5)	36)90 (2.5)	75)6 (.08)	44)550 (12.5)

Set C

	a	b	c	d	e
1.	.3)9 (30)	.4)3 (7.5)	.5)8 (16)	.06)12 (200)	1.8)9 (5)
2.	7.5)3 (.4)	2.5)5 (2)	3.2)432 (135)	1.6)64 (40)	.36)27 (75)
3.	1.5)9 (6)	1.6)264 (165)	2.5)2 (.8)	.12)15 (125)	.08)1 (12.5)

Set D

	a	b	c	d	e
1.	.2)7.2 (36)	.4).172 (.43)	.8)5.84 (7[illegible])	.5).31 (.62)	.09)3.6 (40)
2.	.5)107.1 (214.2)	6.5).1625 (.025)	6.8).34 (.05)	4.5)10.35 (2.3)	3.6)93.6 (26)

MORE PRACTICE 378 379

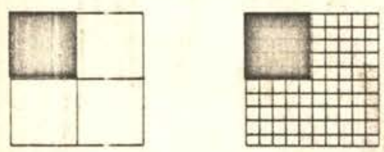
(7, p. 71)

Percent

Percent is introduced, usually in terms of the three cases of percents, each of which is treated separately after various manipulations with percents, including fractional and decimal equivalents of percents. (5, p. 5)

Fractions and Per Cents

1. How do these squares show that $\frac{1}{4}$ is equal to $\frac{25}{100}$ or 25%? What fraction of the large square at right is each small square? Each small square is what per cent?



2. Draw squares and color them to show these per cents:

50% 75% 10% 20% 98%

3. Jim's basketball team won 3 out of 5 games. What fraction did they win? Don's team won 64% of its games. Which team won the larger per cent of its games?

THINK: $\frac{3}{5} = .60 = ?\%$

4. Bob's team had won only $\frac{1}{8}$ of its games. What decimal is this? What per cent?

Study these examples.

$\frac{3}{5} = .60 = .60.\% = 60\%$

The decimal point at the end of a number may be omitted.

$\frac{1}{8} = .12\frac{1}{2} = .12\frac{1}{2}.\% = 12\frac{1}{2}\%$

A fraction does not occupy a place in our number system.

5. What happens to a decimal when you move the decimal point one place to the right? By what number are you multiplying when you move the decimal point two places to the right?

6. Multiply .75 by 100 by moving the decimal point two places to the right. Now we have ? wholes. When we give these wholes the name *per cent*, we see that .75 equals 75%. Explain.

To change a decimal to a per cent, move the decimal point two places to the right and add a per cent sign.

7. Change .375 to a per cent.

THINK: .375 = .37.5% = ? %

Change to per cents.

8.	.75	.35	.08	.40	.05
9.	.01	$.06\frac{1}{4}$	$.37\frac{1}{2}$	.18	$.33\frac{1}{3}$
10.	.125	.98	.875	$.66\frac{2}{3}$	.03

11. Explain how these per cents were changed to decimals.

$25\% = .25.\% = .25$

$87.5\% = .87.5\% = .875$

$4\% = .04.\% = .04$

$12\frac{1}{2}\% = .12\frac{1}{2}.\% = .12\frac{1}{2}$

12. Make up a rule for changing a per cent to a decimal.

Change to decimals.

13.	20%	32%	3%	$4\frac{1}{2}\%$	10%
14.	16%	18.5%	5%	$6\frac{1}{4}\%$	87.5%

(8, p. 98)

Saying Hundredths in Three Ways

You have seen that hundredths may be expressed as a

1. **Common fraction,** as $\frac{3}{100}$
2. **Decimal fraction,** as .03
3. **Per cent,** as 3%

You learned that 3% means 3 out of 100. This is the same as three hundredths, or .03.

Per cent means hundredths, or two decimal places.

Express the following as per cents:

	a	b	c	d	e	f
1.	$\frac{13}{100}$ 13%	$\frac{9}{100}$ 9%	$\frac{21}{100}$ 21%	$\frac{10}{100}$ 10%	$\frac{67}{100}$ 67%	$\frac{91}{100}$ 91%
2.	.12 12%	.04 4%	.15 15%	.25 25%	.45 45%	.76 76%
3.	$\frac{1}{2}$ 50%	$\frac{1}{4}$ 25%	$\frac{3}{10}$ 30%	$\frac{7}{20}$ 35%	$\frac{9}{25}$ 36%	$\frac{11}{50}$ 22%
4.	$\frac{23}{100}$ 23%	$\frac{4}{5}$ 80%	.06 6%	$\frac{9}{100}$ 9%	.54 54%	$\frac{7}{100}$ 7%
5.	$\frac{3}{4}$ 75%	$\frac{3}{20}$ 15%	$\frac{11}{25}$ 44%	$\frac{1}{5}$ 20%	$\frac{19}{100}$ 19%	.61 61%
6.	$\frac{2}{5}$ 40%	$\frac{9}{10}$ 90%	.02 2%	$\frac{24}{25}$ 96%	$\frac{19}{20}$ 95%	.05 5%

Express the following as hundredths:

	a	b	c	d	e
7.	15% .15	2% .02	25% .25	11% .11	40% .40
8.	50% .50	47% .47	8% .08	30% .30	95% .95
9.	100% 1.00	10% .10	$\frac{3}{10}$ .30	$\frac{7}{25}$ .28	$\frac{17}{20}$ .85
10.	$\frac{4}{5}$ .80	20% .20	$\frac{3}{20}$ .15	44% .44	1% .01

11. Write the following as hundredths in three different ways: $\frac{1}{2}$; $\frac{3}{5}$; $\frac{1}{20}$; $\frac{4}{25}$; $\frac{9}{10}$. *See teacher's column for answers*

12. Write the following as hundredths in two other ways: 14%; $\frac{7}{100}$; .30; .08; 5%. *See teacher's column for answers*

13. Write the following as common fractions in lowest terms: 50%; $\frac{1}{2}$ 40%; $\frac{1}{5}$ 75%; $\frac{3}{4}$ 20%; $\frac{1}{5}$ 15%. $\frac{3}{20}$

14. Write each of the following in two other ways: 60%; → .60, $\frac{60}{100}$, $\frac{3}{5}$ $\frac{7}{25}$; .09; 36%; 72%; $\frac{11}{20}$. → $\frac{55}{100}$, .55, 55%

.09 → $\frac{9}{100}$, 9%;

$\frac{7}{25}$ → $\frac{28}{100}$, .28, 28%; 36% → .36, $\frac{36}{100}$, $\frac{9}{25}$; 72% → .72, $\frac{72}{100}$, $\frac{18}{25}$;

(6, p. 137)

Percent Applications

The traditional courses also have rather extensive treatments of percent applications such as commission, simple interest, discount, and insurance.(5, p. 5)

More about Commission

1. Mr. Scott wanted to find the rate of commission offered in the advertisement at the right. He figured that the price of a new car was about $2500. At this price, how much would 20 cars bring in? What per cent would the commission be of the total sales?

AUTO SALESMAN WANTED
EXPERIENCED ONLY
If you are capable of delivering 20 cars or better per month, this job will pay upwards of $1500 per month. Apply in person. See sales manager.
ABC AUTO SALES
1800 S. Main St.

2. Where Mr. Scott now works he receives $250 per month plus a commission of 2% on all sales over $5000. Last month his sales were $26,000. What was his commission? What were his total earnings?

3. Mr. Robbins received $900 for selling $22,500 worth of cars. What was the rate of his commission?

4. The Parker Real Estate Company paid a salesman $348 for selling a home for $11,600. What rate of commission did the salesman receive?

5. Another salesman received $247.50 for selling a home costing $12,375. What rate of commission did he receive?

6. Because a house was difficult to sell, Mrs. Luden received a commission of 6%. She sold the house for $14,000. What was her commission?

7. What are the advantages of paying a salesman a commission rather than a salary? Why do some employers pay both?

8. Robert sells magazines at 15¢ each. What is the rate of his commission if he receives $1.50 for selling 25 magazines?

9. Jerry helps in his father's drug store on week ends. One week his sales amounted to $293.20 and he was paid $14.66. What was the rate of Jerry's commission?

10. The amount of money left after commission has been paid is called the *net proceeds.* What were the net proceeds for Mr. Black's company if he sold $8000 worth of goods and received a commission of $680? What was the rate of commission? What per cent were the net proceeds of the selling price?

11. What was the commission on a sale of $2400 if the rate was $7\frac{1}{2}$%? The net proceeds?

(8, p. 104)

Finding Interest

1. Find the interest on $450 at 4% for 1 year.

4% = .04 .04 × $450 = $18

The interest on $450 at 4% for 1 year is (?).

Find the interest for one year on the following:

	a	b	c
2.	$600 at 2% $12	$750 at 3% $22.50	$250 at 4% $10
3.	$860 at 1% $8.60	$960 at 5% $48	$475 at 1% $4.75
4.	$350 at 6% $21	$400 at 2% $8	$925 at 2% $18.50
5.	$275 at 4% $11	$720 at 1% $7.20	$175 at 6% $10.50
6.	$2100 at 1% $21	$1240 at 3% $37.20	$1850 at 2% $37

7. The Camp Fire Girls borrowed $450 for a year at 6%. They paid back at the end of the year the sum borrowed and the interest. What was the total payment? $477 The girls had to earn an average of how much a month to pay the loan plus the interest? $39.75

8. Mr. Wells borrowed $750 for a year at 6%. He is to repay the loan plus the interest at the end of the year. How much must he set aside on the average each month? $66.25

9. The following loans are for a year. Each loan plus the interest for a year is paid back at the end of the year. What are the average monthly savings needed?

a. $600 at 5% $52.50 c. $900 at 3% $77.25 e. $840 at 6% $74.20
b. $750 at 4% $65 d. $270 at 6% $23.85 f. $1200 at 5% $105

10. Miss Hicks borrowed $300 from a bank for a year at 6%. The bank subtracted the yearly interest from the amount she borrowed. How much did Miss Hicks receive? $282 She agreed to repay the full amount of the loan in 12 equal monthly payments. What was the amount of each payment? $25

★11. In problem 10, the loan is called a **personal loan.** A personal loan does not require **collateral.** Find out what is meant by collateral. Find when the interest is paid on a personal loan.

(6, p. 304)

Buying at a Discount

The coach of a football team orders the supplies for his team from a catalogue of a sporting goods store. He receives a **discount** on the **list price** given in the catalogue. The cost after the discount is subtracted is called the **net cost.**

1. The list price of a football is $26. The coach is given a 25% discount. What is the net cost of the football?

25% of $26 = ?	**25% = $\frac{1}{4}$**
.25 × $26 = $6.50	**$\frac{1}{4}$ × $26 = $6.50**
$26.00 − $6.50 = $19.50	**$26.00 − $6.50 = $19.50**

The net cost of the football is $19.50.

2. In problem 1, the net cost is what per cent of the list price? 75%

3. When a discount of 30% is given, the net cost is what per cent of the list price? 70%

4. Suppose only one discount is given. Prove that the per cent which is the net cost is equal to 100% of the list price less the per cent of discount.

Find the missing numbers in the table:

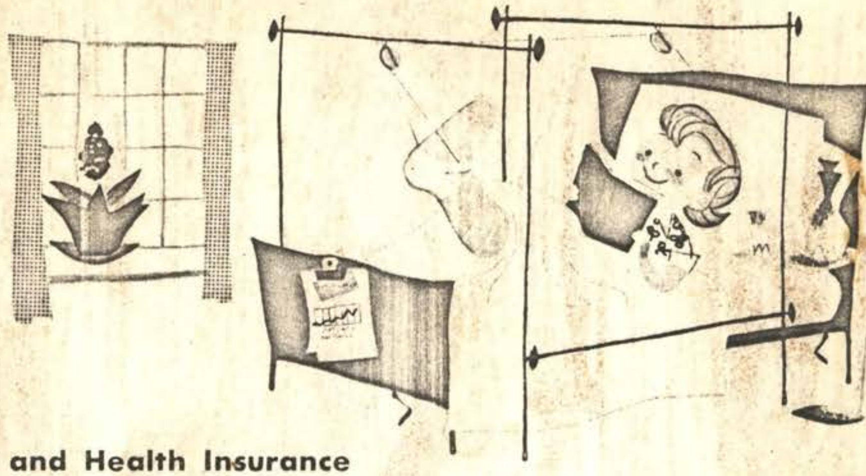
	List Price	Per Cent of Discount	Net Cost
5.	$60	30%	? $42
6.	$7.50	20%	? $6
7.	$10	? 25%	$7.50
8.	$25	? 22%	$19.50
★9.	$15 ?	20%	$12
★10.	$50 ?	30%	$35

11. A bookstore receives a discount of 25% on a book listed at $3. The store sells the book at $2.75. What is the margin? 50¢

12. The net cost of athletic equipment listed at $960 was $768. What was the per cent of discount? 20%

★ 13. John bought a football at a net cost of $16. This was 80% of the list price. What was the list price? $20

(7, p. 119)



Accident and Health Insurance

▶ Jane Connor had an accident which kept her in the hospital for two weeks. Her hospital and doctor bills amounted to $196. Because her father carried a health and accident policy on himself and his family, he had to pay only $28 of the total amount.

1. Jane's father pays $10 each month for this insurance protection. He had paid premiums for a year and three months when Jane had her accident. How much had Mr. Connor paid in premiums? How much had the insurance saved him?

▶ Some types of accident and health insurance protect against loss of income at a time of illness.

2. Miss Hill has an accident and health policy that pays her $50 a week while she is disabled. She has paid $70 each year for three years and has been in good health. She has just lost 7 weeks of work because of an accident. Was the insurance worthwhile? Why?

3. Mr. Jones paid $70 each year for 5 years for accident and health insurance. During this time he has not had an accident or illness. How much has he paid for this insurance? Should he drop his policy? Why?

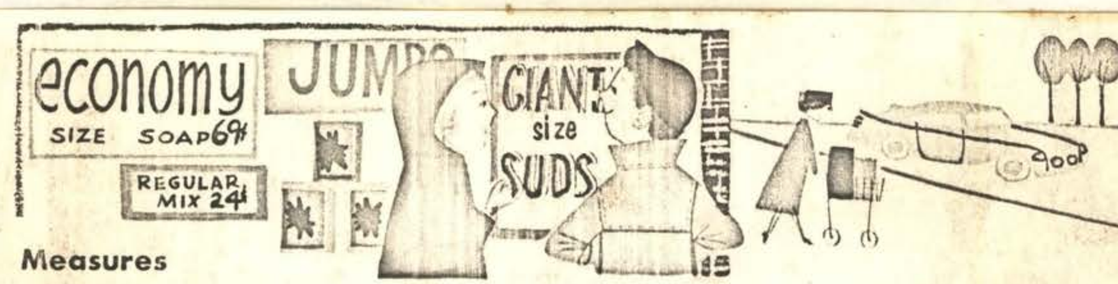
4. Mr. Kem pays $12 each year for surgical benefits. His company will pay a surgeon up to $200 for an operation. After 17 years, Mr. Kem had an operation that cost $375. Was Mr. Kem wise in having this insurance? Why?

5. Health and accident insurance usually protects the buyer in two ways. Explain.

(9, p. 106)

Measurement

A study of measurement has had an important place, but again much of this is a review of work done in earlier grades and little or none of it new from a mathematical point of view. (7, p. 5)



Measures

1. Mrs. Smith bought 5 dozen eggs for $3. Mrs. Fall paid only $2 for a basketful. Can you tell who got the better bargain? Why?

Explain why one measure in each of the following pairs tells the quantity more exactly than the other.

2. Bucketful of milk — 3 gallons of milk

3. Bag of potatoes — 10 pounds of potatoes

4. Pinch of salt — $\frac{1}{4}$ teaspoonful of salt

Find exact answers to as many of the following problems as you can. When you cannot find an exact answer, tell what is lacking in the problem.

5. A 5-ounce tube of toothpaste costs 65¢. A 3-ounce tube costs 45¢. What is the cost per ounce for each tube? Which is the better bargain?

6. A large tube of toothpaste costs 49¢ and a jumbo tube costs 69¢. Which is the better bargain?

7. A 12-ounce box of soap powder costs 24¢. A 32-ounce box costs 48¢. Which is the better bargain?

8. A large box of soap flakes costs 23¢, but a giant-size box can be bought for 49¢. Which is the better bargain?

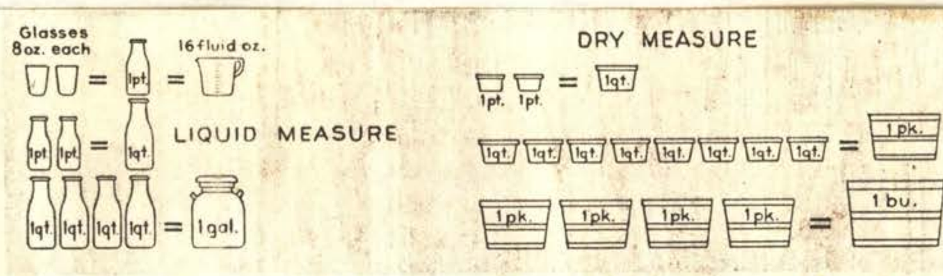
9. A 16-ounce box of cereal costs 32¢, but a 6-ounce box costs only 15¢. Which is the better bargain?

■ **Such measures as the ounce, pound, pint, quart, inch, foot, and mile are called standard units of measure.**

▶ Standard units of measure have exact meanings. Buyers should not be misled by such words as **jumbo** and **giant**.

10. Which is larger, a 2-pound or a 3-pound package of soap? Which is largest, a regular-size package, a jumbo package, an economy-size package, or a giant package?

(8, p. 49)



Making and Using Tables of Measures

1. Copy the following tables and fill in the missing numbers:

2 ? glasses = 1 pint (pt.)	2 ? pints = 1 quart
2 ? pints = 1 quart (qt.)	8 ? quarts = 1 peck (pk.)
4 ? quarts = 1 gallon (gal.)	4 ? pecks = 1 bushel (bu.)
1 quart = ? fluid ounces (fl. oz.) 32	

2. Ten gallons of milk will fill how many pint bottles? 80

3. A gallon of milk will fill how many 8-oz. glasses? 16

4. How many quarts are there in a bushel? 32

5. Change 8 ft. to inches. 6. Change 48 in. to feet.

8×12 in. = 96 in. $48 \div 12 = 4$, no. of feet.

In problem 5 we changed feet to inches. Since a foot is a larger measure than an inch, we multiplied.

In problem 6 we changed from inches to feet. Since an inch is a smaller measure than a foot, we divided.

7. Show that 4 gal. 1 qt. = 3 gal. 5 qt.

8. Show that 2 gal. 5 qt. = 3 gal. 1 qt.

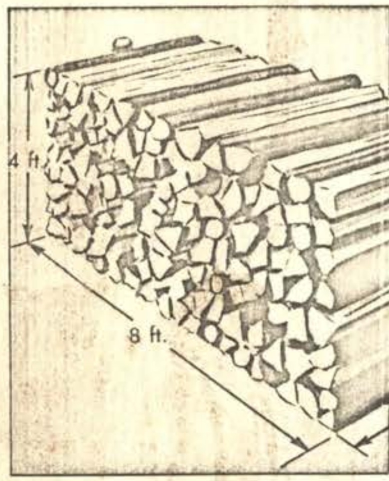
Copy the examples. Fill in the missing numbers. If you need help, use the tables above and on page 335.

9. 12 ft. = ? yd. 4	16. 5 hr. 72 min. = 6 hr. ? min. 12
10. 24 fl. oz. = ? pt. $1\frac{1}{2}$	17. 2 ft. 20 in. = ? ft. 8 in. 3
11. $4\frac{1}{2}$ gal. = ? qt. 18	18. 3 hr. 18 min. = 2 hr. ? min. 78
12. $1\frac{1}{4}$ mi. = ? ft. 6600	19. 3 gal. 2 qt. = ? qt. 14
13. $1\frac{1}{2}$ rd. = ? ft. $24\frac{3}{4}$	20. 3 lb. 14 oz. = ? lb. $3\frac{7}{8}$
14. $2\frac{1}{2}$ min. = ? sec. 150	21. 4 lb. 18 oz. = ? lb. $5\frac{1}{8}$
15. $2\frac{1}{4}$ yd. = ? in. 81	22. 2 hr. 66 min. = ? hr. $3\frac{1}{10}$

(6, p. 52)

Measures for Wood and Lumber

During Harlan's visit to the country, he helped his grandfather put 4-ft. sticks of wood into piles 8 ft. long and 4 ft. high. He learned that each pile was a legal cord.



1. A legal cord is a rectangular solid. What is its length? Its width? Its height? Its volume?

The 4-ft. sticks need to be cut into shorter lengths to fit into fireplaces and stoves. Some dealers call any pile 8 ft. long and 4 ft. high a cord, whatever the length of the sticks. We can call this a home-use cord.

2. Grandfather cut a legal cord of wood into sticks that were 2 ft. long. How many home-use cords did this make? What is the volume of a home-use cord when the sticks are 2 ft. long?

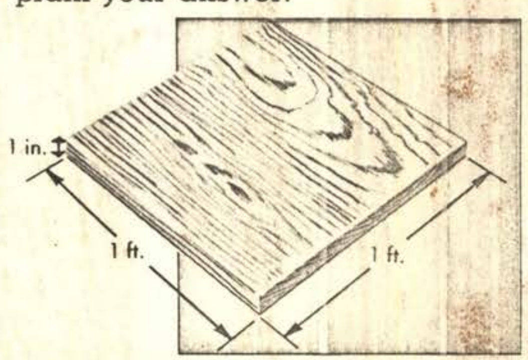
3. Grandfather cut a legal cord of wood into sticks 16 in. long. How many home-use cords did this make? What is the volume of a home-use cord when the sticks are 16 in. long?

4. A fuel dealer bought wood at $20 per legal cord. He cut it into 16-in. lengths. He sold the home-use cords that it made at $16 per cord. How many home-use cords did he have? Find the amount of his profit.

At a sawmill, Harlan's grandfather bought 1000 board feet of lumber.

Harlan learned that a board 1 ft. square and 1 in. thick is called a board foot. This is the unit of measure for lumber.

5. According to this measurement, how many board feet would be needed to make 1 cu. ft.? Explain your answer.



(9, p. 274)

Proper Lighting Is Essential

The total area of the windows of a classroom should be at least 20% of the area of its floor.

1. A classroom is 21 feet wide and 29 feet long. To the nearest square foot, what should be the least number of square feet of window space? 122 sq. ft.

2. The window space in the following classrooms is how much more or how much less than 20% of the floor area?

	Dimensions of Room	Number of Windows	Size of Windows	Answer
a.	21'×32'	5	3'×7'	29.4 sq. ft. less
b.	21'×35'	5	4'×7.5'	3 sq. ft. more
c.	19'×22.5'	4	3'×7.5'	4.5 sq. ft. more
d.	24'×38.5'	6	4'×7.5'	4.8 sq. ft. less

3. How many square feet are there in a square yard? If you forget, how can you figure it out? 9 sq. ft.

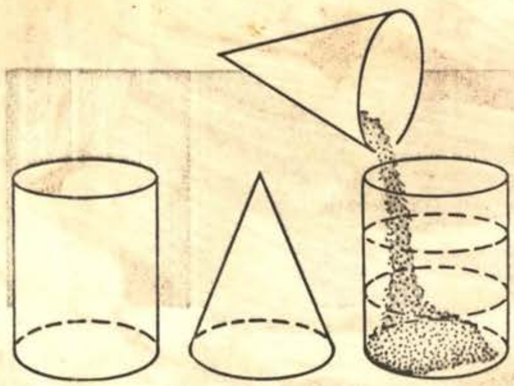
4. All playgrounds should be treated twice a year with $\frac{1}{4}$ gallon of disinfectant per square yard of surface. If treated twice a year, find the number of gallons needed to treat playgrounds having the following dimensions:

a. 150' × 375' b. 300' × 675' c. 450' × 725' d. 250' × 450'

3,125 gal. 11,250 gal. 18,125 gal 6,250 gal.

★5. The area of a certain playground is an acre (43,560 sq. ft.). How many gallons of disinfectant are needed if $\frac{1}{4}$ gallon is used per square yard twice a year? 2,420 gal.

(7, p. 131)



Volume of a Cone

The bases of a cylinder and of a cone are each equal to πr^2. Why?

The bases of the above cylinder and cone are equal. The heights of the cylinder and cone are each equal to h.

By pouring sand or salt from cone to cylinder, you can show that the volume of the cone is one third the volume of the cylinder.

▶ The volume of a cone is given by this formula:

$$V = \frac{1}{3}\pi r^2 h$$

1. The radius of the base of a cone is $3\frac{1}{2}$ in. The height of the cone is 6 in. Find the volume.

$$V = \frac{1}{3} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 6 = ?$$

2. A cylinder and a cone have equal bases and equal heights. The volume of the cylinder is 78 cu. in. What is the volume of the cone?

3. A cylinder and a cone have equal bases and equal heights. The volume of the cone is 36 cu. in. What is the volume of the cylinder?

4. Find the volume of a cylinder when $r = 7$ in. and $h = 12$ in.

5. Find the volume of a cone when $r = 7$ in. and $h = 12$ in.

6. A pile of sand is a cone. The diameter of the base is 21 ft. The height is 12 ft. Find the volume.

7. A paper drinking cup is a cone. The diameter of the top is $3\frac{1}{2}$ in. The height is $5\frac{1}{4}$ in. How much water will it hold?

8. A paper cone used at soda fountains is $4\frac{1}{2}$ in. deep and $3\frac{1}{2}$ in. across the top. What is its volume in cubic inches?

9. An Indian chief's tent is cone shaped. The diameter of the base is 14 ft. The height is 15 ft. How many cubic feet of air are in the tent?

10. A pile of wheat is a cone. The diameter of the base is 35 ft. The height is 15 ft. Find the volume in cubic feet.

11. A cone and cylinder have equal diameters and volumes. Compare the height of the cone with the height of the cylinder.

(9, p. 189)

Economic and Business Environments

Not to be overlooked in the mainstream of traditional content are the "daily living" exercises of economic and business environments.

Reading Gas and Electric Meters

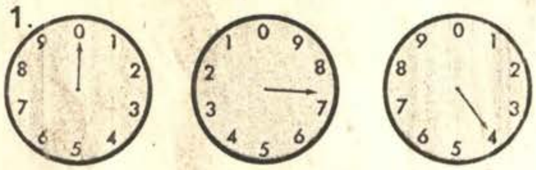
▶ Jack made a report to his class on gas meters and gave this information:

- A gas meter measures the cubic feet of gas used.
- A gas meter has three dials.
- The figures and hands on the end dials go clockwise.
- The figures and hands on the middle dial go counterclockwise.

Jack showed the class a picture of this meter. It has a reading of 42,800 cubic feet.

Read these gas meters.

1.



2.

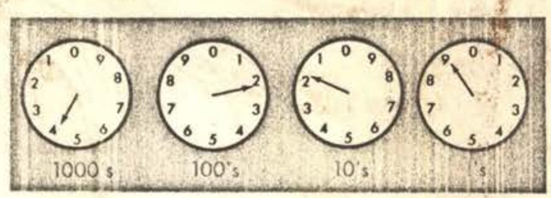
3.

4. Draw the dials of a gas meter and show these readings:
7800 23,400

▶ Helen's class report on electric meters gave this information:

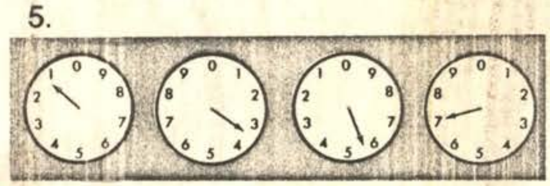
- An electric meter measures kilowatt hours.
- The abbreviation for kilowatt hours is KWH.
- An electric meter has four dials.

Helen showed the class this picture of an electric meter. It has a reading of 4219 KWH.

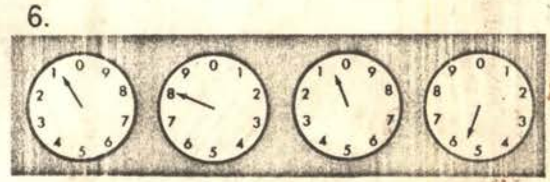


Read these electric meters.

5.



6.



7. Draw the dials of an electric meter and show these readings:
4038 7744

(8, p. 258)

Finding a Bank Balance

The First National Bank is a **commercial bank.** A commercial bank permits its depositors to write checks drawn on their accounts.

The illustration shows a form of a **check stub** for keeping a record of each check written. Joan's checkbook contains one sheet as shown for each three blank checks.

DEPOSITS	AMOUNT		CHECKS DRAWN	AMOUNT	
BALANCE FORW'D May 3	147	39	NO. 11 DATE May 12 195– ORDER OF Costume Designers FOR Rental of Costumes	25	85
May 12	14	75	NO. 12 DATE May 14 195– ORDER OF Gift Shop FOR Decorations for Hall	21	68
TOTAL DEPOSITS			NO. 13 DATE May 15 195– ORDER OF Fine Groceries FOR Fruit for punch at dance	9	45
LESS CHECKS					
BALANCE FORW'D			TOTAL CHECKS		

1. What was the amount brought forward on May 3? This is the amount of money in the account at the beginning of May. $147.39

2. How much was deposited on May 12? $14.75

3. What was the amount of the total deposits? $162.14

4. What was the amount of the three checks written? What were the numbers of the checks? 11, 12, 13 $56.98

5. Find the **balance** to be carried forward to the next record sheet in the checkbook. **A bank balance is the difference between deposits and withdrawals when deposits are greater than withdrawals.** $105.16

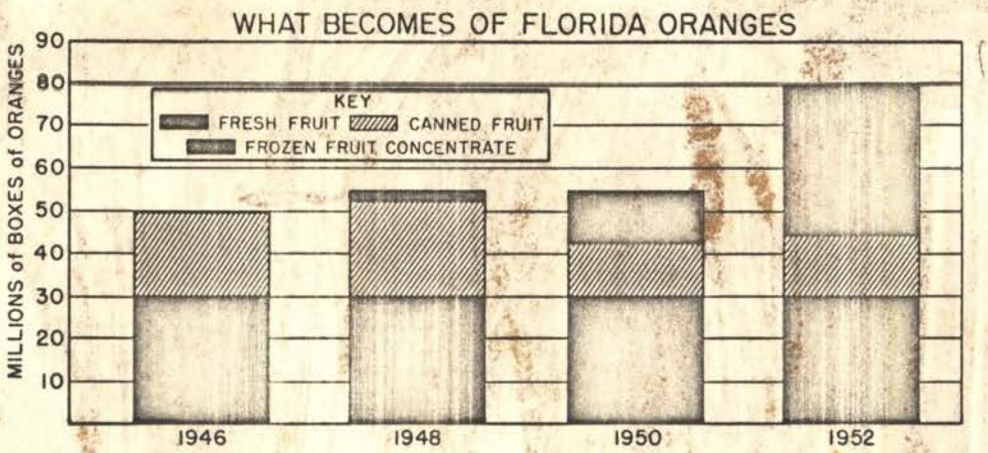
Find the bank balance, if any, for the following:

6. Brought forward, $38.62; deposited, $41.50, $19.35, $108.75. Checks drawn, $54.83, $18.45, $9.49, $23.59, $16.98. $84.88

7. Brought forward, $203.41; deposited, $11.56, $148.75, $62.00. Checks drawn, $17.24, $92.53, $1.89, $64.39, $127.91. $121.76

8. Brought forward, $39.58; deposited, $75.00, $41.85. Checks drawn, $112.50, $19.36, $10.45, $15.25. Since the amount of the checks exceeded the amount of the deposits, we say the account was **overdrawn.** By how much was the account overdrawn? $1.13

(6, p. 300)



The Florida Orange Crop

The graph shows how the orange crop of Florida has been used.

1. The number of oranges in a box depends upon the size of the oranges. How many dozen are there in a box which contains 180 oranges? 216 oranges? *18 dozen* *15 dozen*

2. Approximately how many boxes of oranges were used in 1946? in 1948? in 1950? in 1952? *80 million boxes* *50 million* *55 million* *55 million*

3. How many different uses are shown on the graph for 1946? for 1948? for 1950? for 1952? *3 uses* *2 uses* *3 uses* *3 uses*

4. For each year shown, approximately how many boxes of oranges were used as fresh fruit?
30 million boxes for each year shown.

5. In 1946, the number of boxes of oranges used as fresh fruit was what fractional part of the total number of boxes? Express the answer as a common fraction; as a decimal fraction. *.6* *3/5*

6. Approximately how many boxes of oranges were used as frozen fruit concentrate in 1948? in 1950? in 1952? *35 million boxes* *2 million* *10 million*

7. Use an average price of $3.25 a box. What was the value of all of the Florida oranges in 1946? in 1952? *$260,000,000* *$162,500,000*

8. Approximately what fractional part of the entire crop in 1952 was used as fresh fruit? for other purposes? *5/8* *3/8*

★9. What is meant by frozen fruit concentrate?

(7, p. 64)

Critique and Summary

It is hoped that the reader also notes that while there is much to do with numbers and various manipulations with numbers in various social and economic settings in the traditional content there is no mention of the real number system. Yet, use is made of π, an irrational number, fractions which are rational numbers, and decimals which can be rational or can be irrational. There is little evidence of the real number system as a progressing development. Also the geometry herein deals almost entirely with the measurement of geometric figures and emphasizes the use of such measurements in social situations. Non-metric or non-measured relations and common properties of geometric figures are not presented. It is the author's belief that basic concepts such as the real number system and geometric properties are fundamental in the preparation of students for the vocations mentioned in the introduction of this report.

> One of the major defects of seventh and eighth grade mathematics curricula is that pupils of all ability levels are required to give too much attention to so-called practical applications. Most junior high school pupils are not fascinated by such topics as taxation, banking, interest notes, installment buying and other applications of the three cases of percent. Emphasizing such topics not only fails to motivate the learning of mathematics but actually serves to reduce substantially the amount of time that can be devoted to the development of new mathematical concepts. As a result, seventh and eighth grade mathematics has become largely a review of the arithmetic learned earlier with little or no advance in the pupils' understanding of mathematics. (1, p. 62-63)

CHAPTER III

THE SMSG PROGRAM

Overview

It should be noted here after viewing the traditional program and before considering the SMSG program, that the 7th and 8th grade traditional texts are a culmination of arithmetic presentation started in the third grade and terminate a series of six grades while the 7th and 8th grade SMSG texts are primary in the SMSG program which terminates in the 12th grade.

> The School Mathematics Study Group believes it particularly important that greater substance and interest be given to the mathematics of grades 7 and 8. Our general point of view has been to think of grades 7 and 8 not as the end of elementary school mathematics, but rather as a foundation for the work of the senior high school. The curriculum for these grades should include a sound intuitive basis for algebra and geometry courses to follow. (4, p. 455)

Now we shall take a look at selected topics found in the SMSG program in an effort to cover those considerations emphasized in the introduction to this report. As was done in the preceding chapter, we must also consider the approaches found in the topics of this chapter. Much of the SMSG content is self-explanatory in approach and the use of the teacher's commentaries further facilitates our search.

In this chapter are topics, as mentioned earlier, that are chosen with emphasis on sets, inequalities, structure, concepts and skills, and deductive reasoning. This includes areas concerned with the definition and uses of set language, properties of structure patterns, number system concepts and skills, concept of percent, non-metric and metric geometry, and proof of geometric properties. The approaches inherent in all of these areas are explained thusly:

> Careful attention is paid to the appreciation of abstract concepts, the role of definition, development of precise vocabulary and thought, experimentation, and proof. Materials are chosen with the intent to capture the fascinating features of mathematics, creation and discovery, rather than just utility alone.(4, p. 455)

Sets and Their Uses

> Many high school pupils have seen mathematics as a series of separate and unrelated manipulative tricks. The improved programs, on the other hand, have tried to use as central themes the permeating ideas in mathematics. In some cases this has led to the introduction of words and ideas from college mathematics.
>
> Set theory, for example, is a unifying idea found in higher mathematics; however, it is simple enough in its beginnings to be taught in high school.
>
> DEFINITION: A set is any well-defined collection of distinguishable objects of our perception or thought.
>
> Once the notion of set is introduced, it may be used throughout the course. The student refers to solution sets of equations, truth sets, sets of ordered pairs, etc. He is given exercises to strengthen and clarify his idea of sets before any symbols are used. He will use these symbols and terminology in more advanced mathematics, and the ideas will be further developed as he takes additional courses.(1, p. 22-23)

A set of numbers: 5, 36, 7, 8

A set of marks: / / / / / / /

A set of stars in a diagram: * * * * * *

Other <u>examples</u> <u>of</u> <u>sets</u> are: the set of coins in your pocket, the set of vowels in our alphabet, a set of chessmen, a set of cattle (you might say a herd of cattle), the set of cities in the U.S.A. which have a population of more than one million.(10, p. 85)

The counting numbers form a set. Remember that the counting numbers are 1, 2, 3, 4, 5, 6, ... where the three dots are used to indicate that the set of numbers continues indefinitely. There is no last number. We are going to use N to represent the set of counting numbers and we will put the counting numbers within braces { } to indicate that they are the objects in the set which is designated by N. Hence, we may write

$$N = \{1, 2, 3, \ldots\}$$

and read it "N is the set of counting numbers."

We may choose any capital letter to represent the set. If we have the set S = {1, 2, 3, 4, 5, 6, 7} we may describe this by saying that S is the set of counting numbers from 1 to 7 inclusive, or S is the set consisting of the counting numbers less than 8.

A few more examples of sets and the abbreviated way of writing them will help make the concept clear. "V is the set of vowels in our alphabet" becomes "V = {a, e, i, o, u}." "M is the set of counting numbers which are greater than 20 and less than 25" becomes "M = {21, 22, 23, 24} ." "E is the set of states in the U. S. A. which are touched by Lake Erie" becomes "E = {Michigan, Ohio, Pennsylvania, New York} ." (10, p. 85)

1. The set of numerals {1, 2, 3, 4, 5, . . .} is the set of symbols for the counting numbers.
2. The set of numerals {0, 1, 2, 3, 4, 5, . . .} is the set of symbols for the whole numbers.
3. The commutative property for addition: $a + b = b + a$, where a and b represent any whole numbers.
4. The commutative property for multiplication: $a \cdot b = b \cdot a$, where a and b represent any whole numbers.

5. The associative property for addition: $a + (b + c) = (a + b) + c$ where a, b, c represent any whole numbers.

6. The associative property for multiplication: $a \cdot (b \cdot c) = (a \cdot b) \cdot c$ where a, b, c represent any whole numbers.

7. The distributive property: $a \cdot (b + c) = (a \cdot b) + (a \cdot c)$ and $(b + c) \cdot a = (b \cdot a) + (c \cdot a)$ where a, b, c are any whole numbers.

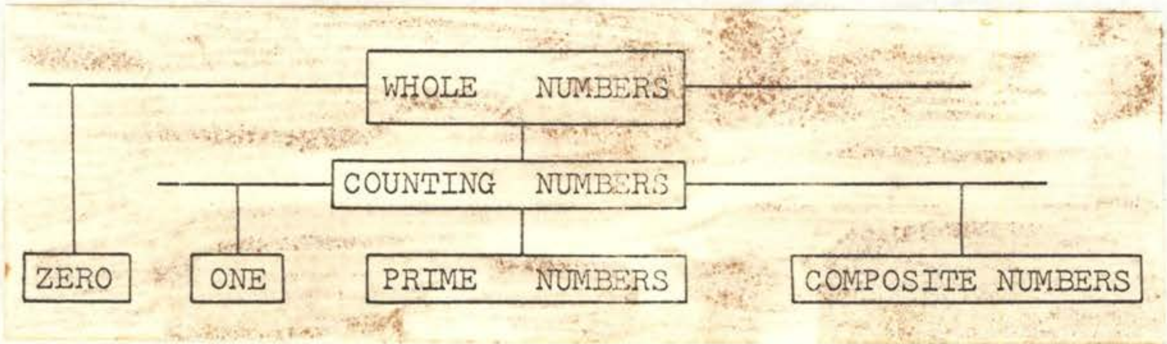
8. New symbols: { set of elements }; $>$ is greater than; $<$ is less than; $\neq$ is not equal to.

9. Set and closure. A set is closed under an operation if the combination of any two elements of the set gives an element of the set. The set of counting numbers is closed under addition and multiplication but not under division or subtraction.

10. Inverse operations. Subtraction is the inverse of addition, but subtraction is not always possible in the set of whole numbers. Division is the inverse of multiplication, but division is not always possible in the set of whole numbers; that is, division of one whole number by another whole number does not always yield a whole number.

11. The number line and betweenness. Each whole number is associated with a point on the number line. There is not always a whole number between two whole numbers.

12. Special numbers: 0 and 1. Zero is the identity for addition; 1 is the identity for multiplication; multiplication by 0 does not have an inverse; division by 0 is not possible.(10, p. 102-103)



Note that zero is a member of the set of whole numbers, but not a member of the set of counting numbers. The ONE, the PRIME NUMBERS, and the COMPOSITE NUMBERS are members of the set of COUNTING NUMBERS and also members of the set of WHOLE NUMBERS.

Every member of the set of counting numbers is a member of the set of whole numbers.(10, p. 184)

We think of <u>space</u> as being a <u>set of points</u>. There is an unlimited quantity of points in space. In a way, we think of the points of space as being described or determined by position -- whether they are in this room, in the world, or in the universe.

For us, a <u>line</u> is a <u>set of points</u> in space, not any set of points but a particular type of set of points. The term "line" means "straight line." All lines in our geometry are understood to be straight. A line is suggested by the edge of a ruler. It is suggested by the intersection of a side wall and the front wall of your classroom.

A geometric line extends without limit in each of two directions. It does not stop at a point. The intersection of a side wall and the front wall of your classroom stops at the floor and the ceiling. The line suggested by that <u>intersection</u> extends both up and down, indefinitely far.(10, p. 106)

The notion of "set" will be helpful in explaining what is meant by "equal" when applied this way. Let's see what facts can be ascertained about the situation described by the figure:

1. Plane ABC is a set of points extending beyond the book cover.
2. Plane ABE is a set of points extending beyond the book cover.
3. Points A, B, C, E, and others not indicated are in plane ABC and are also in plane ABE.

In other words, all elements of plane ABC, (a set of points) and elements of plane ABE (a set of points) seem to be contained in both sets (planes). We shall say, "Two sets are equal if and only if they contain the same elements." According to this, plane ABC = plane ABE. In other words, we say set M is equal to set N if M and N are two names for the same set. (10, p. 118)

Intersection of Sets

We now shall introduce some useful and important ideas about sets.

Let set A = { 1, 2, 3, 4, 5, 6, 7, 8, 9 }

Let set B = { 2, 4, 6, 8, 10, 12, 14, 16 }

Let set C be the set of those elements which are in set A and are also in set B. We can write set C = {2, 4, 6, 8}. We call C the intersection of set A and set B.

Let set R be the set of pupils with red hair.

Let set S be the set of pupils who can swim.

It might happen that an element of set R (a pupil with red hair) might be an element of set S (a pupil who can swim). In fact, there may be no such common elements or there may be several. In any case, the set of red-headed swimmers is the intersection of set S and set R.

A set with no elements in it is called the "empty set." Thus, if there are no red-headed swimmers, then the intersection of set S and set R is the empty set.

Let set H be the set of pupils in your classroom and let set K be the set of people under two years of age. Then the intersection of H and K is the empty set. There are no pupils in your classroom under two years of age!(10, p. 122)

Definition: Numbers which are multiples of more than one number are called common multiples of those numbers. "Common" means belonging to more than one. Thus 6 and 12 are common multiples of 2 and 3.

Let's try another example. List the common multiples of 3 and 4. First, we list the multiples of each:

Set of multiples of 3: { 0, 3, 6, 9, 12, 15, 18, 21, 24, ..., }

Set of multiples of 4: { 0, 4, 8, 12, 16, 20, 24, ...,}(10, p. 178)

The numbers that these sets have in common are the common multiples of 3 and 4. This set is written as follows:

{0, 12, 24, 36, 48, ..., }

This set is the intersection of the two previous sets.

Common multiples are very useful in arithmetic. For example, let us add $\frac{1}{2} + \frac{1}{3}$. We write $\frac{1}{2}$ as $\frac{3}{6}$ and $\frac{1}{3}$ as $\frac{2}{6}$. Then $\frac{3}{6} + \frac{2}{6} = \frac{3 + 2}{6} = \frac{5}{6}$. Here we use a common multiple of 2 and 3. In doing such problems you may have called the 6 a "common denominator." It is a common multiple of the denominators of the given fractions.(10, p. 179)

Not only may we talk about intersection of sets, but we also find it convenient to talk about the union of sets. The word "union" suggests uniting or combining two sets into a new set. The union of two sets consists of those objects which belong to at least one of the two sets. For example, in the figure above, the union of $\overline{AB}$ and $\overline{BC}$ consists of all points of $\overline{AB}$, together with all points of $\overline{BC}$, that is, the segment $\overline{AC}$.

We use the symbol $\cup$ to mean "union." That is, $X \cup Y$ means "the union of set X and set Y." Suppose that set X is the set of numbers $\{1, 2, 3, 4\}$ and set Y is the set of numbers $\{2, 4, 6, 8, 10\}$. Do you have any idea of what $X \cup Y$ is? Yes, it is $\{1, 2, 3, 4, 6, 8, 10\}$. In the union of two sets we do not think of an element which occurs in both sets as appearing twice in the union. Instead, it appears just once.

Again, let us think of the set of all pupils who have red hair and the set of all pupils who can swim. We may think:

Let set R be the set of pupils with red hair.

Let set S be the set of pupils who can swim.

Then $R \cup S$ is the set of all pupils who either have red hair (whether or not they can swim) or who can swim (whether or not they have red hair).

Exercises 4-6

1. Draw a horizontal line. Label four points on it P, Q, R, and S in that order from left to right. Name two segments:
   a. Whose intersection is a segment.
   b. Whose intersection is a point.
   c. Whose intersection is empty.
   d. Whose union is not a segment.
2. Draw a line. Label three points of the line A, B, and C with B between A and C.
   a. What is $\overline{AB} \cap \overline{BC}$?
   b. What is $\overline{AC} \cap \overline{BC}$?
   c. What is $\overline{AB} \cup \overline{BC}$?
   d. What is $\overline{AB} \cup \overline{AC}$?(10, p. 132-133)

Suppose a number sentence involves a symbol like "x" or "y." If the symbol can refer to anyone of many numbers the sentence is called an open sentence. It is not necessarily a true sentence. It is not necessarily a false sentence. It leaves the matter open for further consideration. Look at this open sentence:

$$x + 7 = \frac{10}{x - 2}.$$

It is composed of three parts: a verb, "=", and two open phrases, "x + 7" and "$\frac{10}{x - 2}$." The open sentence states that for a certain number x these two open phrases represent the same number. Can you discover such a number x? Can you find more than one? Try some numbers. After working for a while you might say, "The sentence is true if $x = 3$ or $x = {}^{-}8$, but it is false if x is any other number." The numbers 3 and $^{-}8$ are called solutions of the open sentence. The set 3, $^{-}8$ is called the set of solutions of the open sentence.

When we find the entire set of solutions of an open sentence, we say that we have solved the sentence. An equation is a particular kind of number sentence. It is a number sentence which involves the verb "=". Hence to solve an equation means to find its entire set of solutions. The set of solutions of an equation may contain one member or it may contain several members. It might even be the empty set. (i.e., contain no members)

Is this sentence an equation?

$$"x - 4 > 7."$$

What is the verb in the sentence above? Is it "="? Since the verb is not "=", the sentence is not an equation. We might say that the sentence indicates that the two phrases, x-4 and 7 are not equal. Such a sentence is called inequality.

Can you determine the set of solutions for the inequality $x - 4 > 7$? How large must the number x be in order for the inequality to be true? Is $5 - 4 > 7$? Is $7 - 4 > 7$? Is $12 - 4 > 7$? Do you see that "$x - 4 > 7$" is true if x is any number greater than 11? Also, "$x - 4 > 7$" is false for any other value of x. Thus the set of solutions of the inequality is the set of all numbers which are greater than 11.(13, p. 71-72)

A pair in which the objects are considered in a definite order is called an ordered pair.

The ordered pair (2,7) is the same as the ordered pair (x,y) if $x = 2$ and $y = 7$, and only then. This pair is different from the ordered pair (7,2).

The solution set of the above sentence

$$x + 1 = y$$

is a set of ordered pairs of numbers. For what number y is the ordered pair (2,y) in the solution set?

In order to picture the solution set on your graph paper, pick out two lines for the X-axis and the Y-axis and draw them in heavily with your pencil. Label the vertical and horizontal lines as shown. Mark off on your graph paper all the points (0,1), (1,2), etc., whose coordinates are in the solution set of $x + 1 = y$. What do you notice about them? They all lie on a simple geometric figure. To what set of points does the solution set correspond? In Chapter 1 you learned to call it the graph of the given number sentence, or equation.(13, p. 95-96)

Structure

Another area of emphasis common to all the improved programs is structure. It is reflected in the careful development of mathematics as a deductive system. Since

> the phrase structure of mathematics is used frequently in the improved programs, we should consider its meaning. The study of the structure of mathematics is the study of the basic principles or properties common to all systems of mathematics. These systems, in fact, may not even be concerned with numbers.(1, p. 23)

A set is <u>closed</u> under a binary operation if every two elements of the set can be combined by the operation and the result is always an element of the set.

An <u>identity element</u> for a binary operation defined on a set is an element of the set which does not change any element with which it is combined.

Two elements are <u>inverses</u> of each other under a certain binary operation if the result of this operation on the two elements is an identity element for that operation.

A binary operation is <u>commutative</u> if, for any two elements, the same result is obtained by combining them first in one order, and then in the other.

A binary operation is * <u>associative</u> if, for any three elements, the result of combining the first with the combination of the second and third is the same as the result of combining the combination of the first and second with the third.

$$a * (b * c) = (a * b) * c.$$

The binary operation * <u>distributes</u> over the binary operation o provided

$$a * (b \circ c) = (a * b) \circ (a * c)$$

for all elements a, b, c.

A set S is generated by an element b under the operation * if

$$S = \left\{ b,\ (b * b),\ (b * b) * b,\ \left[(b * b) * b \right] * b,\ \ldots \right\}$$ (12, p. 578)

What Is a Mathematical System?

The idea of a set has been a very convenient one in this book -- some use has been made of it in almost every chapter. But there is really not a great deal that can be done with just a set of elements. It is much more interesting if something can be done with the elements (for instance, if the elements are numbers, they can be added or multiplied). If we have a set and an operation defined on the set, it is interesting to find out how the operation behaves. Is it commutative? associative? Is there an identity element? Does each element have an inverse? The "behavior" of the arithmetic operations (addition, subtraction, multiplication and division) on numbers was discussed in Chapters 3 and 6. We have seen that different operations may "behave alike" in some ways (both commutative, for instance). This suggests that we study sets with operations defined on them to see what different possibilities there are. It is too hard for us to list all the possibilities, but some examples will be given in this section and the next. These are examples of mathematical systems.

Definition. A mathematical system is a set of elements together with one or more binary operations defined on the set.

The elements do not have to be numbers. They may be any objects whatever. Some of the examples below are concerned with letters or geometric figures instead of numbers.

Example 1: Let's look at egg-timer arithmetic -- arithmetic mod 3.

(a) There is a set of elements the set of numbers $\{0, 1, 2\}$.

(b) There is an operation + mod 3, defined on the set $\{0, 1, 2\}$.

Mod 3

+	1	2	0
1	2	0	1
2	0	1	2
0	1	2	0

Therefore, egg-timer arithmetic is a mathematical system. Does this system have any interesting <u>properties</u>?

(c) The operation, + mod 3, has the <u>commutative property</u>. Can you tell by the table? If so, how? We can check some special cases too. $1 + 2 \equiv 0 \pmod 3$ and $2 + 1 \equiv 0 \pmod 3$, so $1 + 2 \equiv 2 + 1 \pmod 3$.

(d) There is an <u>identity</u> for the operation + mod 3 (the number 0).

(e) Each element of the set has an <u>inverse</u> for the operation + mod 3.(12, p. 559-560)

Study the following tables.

(a)

○	A	B
A	A	B
B	A	B

(b)

*	P	Q	R	S
P	R	S	P	Q
Q	S	R	Q	P
R	P	Q	R	S
S	Q	P	S	R

(c)

~	△	□	○	\
△	△	□	○	\
□	□	○	\	△
○	○	\	△	□
\	\	△	□	○

Exercises 12-6

1. Which one, or ones, of the Tables (a), (b), (c) describes a mathematical system? Show that your answer is correct.

2. Use the tables above to complete the following statements correctly.

(a) B O A = ?	(e) Q * R = ?	(i) \ ~ □ = ?
(b) Δ ~ ○ = ?	(f) R * S = ?	(j) B O B = ?
(c) \ ~ \ = ?	(g) P * R = ?	(k) A O A = ?
(d) A O B = ?	(h) □ ~ ○ = ?	( ) S * S = ?

3. Which one, or ones, of the binary operations O, *, ~ is commutative? Show that your answer is correct.

4. Which one, or ones, of the binary operations O, *, ~ has an identity element? What is it in each case?

5. Use the tables above to complete the following statements correctly.

(a) P * (Q * R) = ?	(f) R * (P * S) = ?
(b) (P * Q) * R = ?	(g) Δ ~ (Δ ~ \) = ?
(c) P * (Q * S) = ?	(h) (Δ ~ Δ) ~ \ = ?
(d) (P * Q) * S = ?	(i) (○ ~ □) ~ Δ = ?
(e) (R * P) * S = ?	(j) ○ ~ (□ ~ Δ) = ?

6. Does either of the operations described by Table (b) or Table (c) seem to be associative? Shy? How could you prove your statement? What would another person have to do to prove you wrong?

7. (a) In Table (c) what set is generated by the element □?

   (b) In Table (b) what set is generated by the element P?

8. BRAINBUSTER. For each of the following tables, tell why it does not describe a mathematical system.

(a)

*	1	2
1	1	1
2	1	

(b)

*	1	2
1	the product of 3 and 6	the sum of 2 and 4
2	a number between 3 and 8	0

(c)

*	1	2
3	1	4
4	2	3

(12, p. 561-562)

Definition. Suppose we have a set and two binary operations, * and o, defined on the set. The operation * distributes over the operation o if a * (b o c) = (a * b) o (a * c) for any elements a, b, c, of the set. (And we can perform all these operations.)

In a mathematical system with two operations, there are the properties which we previously discussed for each of these operations separately. The only property which is concerned with both operations together is the distributive property.

Exercises 12-8

1. Consider the set of counting numbers:
   (a) Is the set closed under addition? under multiplication? Explain.
   (b) Do the commutative and associative properties hold for addition? for multiplication? Give an example of each.
   (c) What is the identity element for addition? for multiplication?
   (d) Is the set of counting numbers closed under subtraction? under division? Explain.

   The answers to (a), (b), and (c) tell us some of the properties of the mathematical system composed of the set of counting numbers and the operations of addition and multiplication.

2. Answer the questions of Problem 1 (a), (b) (c) for the set of whole numbers. Are your answers the same as for the counting numbers?

3. (a) For the system of whole numbers, write three number sentences illustrating that multiplication distributes over addition.

   (b) Does addition distribute over multiplication? Try some examples.(12, p. 571)

Properties of Rational Numbers

You have seen that the whole number 3 can be written, $\frac{3}{1}$, which shows that 3 is a rational number. In a similar way, you can show that each whole number is a rational number.

When you studied whole numbers you learned that the whole numbers had certain properties. Learning about rational numbers is made easier by knowing that the rational numbers have some of the same properties.

You remember that the sum of two whole numbers is always a whole number, and the product of two whole numbers is always a whole number. That is, if a and b are whole numbers, there is a whole number c for which $a + b = c$ and a whole number d for which $a \cdot b = d$. The set of whole numbers has the closure property for addition and multiplication.

The set of rational numbers also has the closure property for addition and multiplication. The sum of two rational numbers is a rational number. You know that $\frac{2}{5} + \frac{1}{5} = \frac{3}{5}$, $\frac{12}{4} + \frac{1}{4} = \frac{13}{4}$, $\frac{1}{2} + \frac{1}{3} = \frac{5}{6}$, $2 + \frac{1}{4} = \frac{9}{4}$. The product of two rational numbers is a rational number. Notice that $\frac{1}{3} \cdot \frac{1}{4} = \frac{1}{12}$, $\frac{2}{3} \cdot \frac{6}{7} = \frac{12}{21}$, $\frac{5}{1} \cdot \frac{1}{6} = \frac{5}{6}$. In precise language we state:

1) The set of rational numbers is closed with respect to the operations of addition and multiplication.

You know that $3 + 4 = 4 + 3$ and $3 \cdot 4 = 4 \cdot 3$ because, for the whole numbers, addition and multiplication have the commutative property. These operations also have the commutative property for the rational numbers. You know that $\frac{2}{5} + \frac{3}{5} = \frac{3}{5} + \frac{2}{5}$, and $\frac{2}{5} \cdot \frac{3}{5} = \frac{3}{5} \cdot \frac{2}{5}$. In precise language we state:

2) The operations of addition and multiplication for the rational numbers have the <u>commutative property</u>, that is:

$$a + b = b + a \text{ and } a \cdot b = b \cdot a$$

You also remember that $5 + (3 + 4) = (5 + 3) + 4$, and $5 \cdot (3 \cdot 4) = (5 \cdot 3) \cdot 4$, because addition and multiplication have the associative property for the whole numbers. For the rational numbers also, these operations have the associative property. You know that $\frac{5}{3} + (\frac{1}{3} + \frac{2}{3}) = (\frac{5}{3} + \frac{1}{3}) + \frac{2}{3}$, and that $\frac{3}{1} \cdot (\frac{1}{3} \cdot \frac{6}{3}) = (\frac{3}{1} \cdot \frac{1}{3}) \cdot \frac{6}{3}$. In precise language, we state:

3) The operations of addition and multiplication for the rational numbers have the <u>associative property</u>, that is:

$$a + (b + c) = (a + b) + c, \text{ and } a(bc) = (ab)c.$$

What is $5 \cdot (2 + 3)$? From the distributive property you know that you get the same result if you think of $5 \cdot 5 = 25$ as you do if you think of $5 \cdot 2 + 5 \cdot 3 = 10 + 15 = 25$. For the whole numbers, multiplication is distributive over addition. The distributive property also holds for the rational numbers. You have used this property for the rational numbers when you multiplied $4\frac{1}{5}$ by 5. In our symbols, $5 \cdot (4\frac{1}{5}) = 5 \cdot (4 + \frac{1}{5}) = 5 \cdot 4 + 5 \cdot \frac{1}{5} = 20 + 1 = 21$. In precise language, we state:

4) The operation of multiplication is <u>distributive</u> over addition for the rational numbers; that is:

$$a(b + c) = ab + ac$$

Among the whole numbers were two special numbers 1 and 0. These are also rational numbers.

5) Among the rational numbers are special numbers 0 and 1; 0 is the <u>identity for addition</u> and 1 is the <u>identity for multiplication</u>.

When we say that 0 is the identity for addition we mean, for instance, that $0 + 3 = 3 + 0 = 3$; that is, that adding zero to any number does not change it. This can be expressed in symbols as:

$$0 + a = a + 0 = a,$$

no matter what number a is. Similarly when we say that 1 is the identity for multiplication, we mean, for instance, that $1 \cdot 5 = 5 \cdot 1 = 5$; that is, multiplying any number by 1 does not change it. This can be expressed in symbols as:

$$1 \cdot a = a \cdot 1 = a.$$

You can see that 1 is a rational number by writing it as the fraction $\frac{1}{1}$. To see that 0 is a rational number you should remember that 0 divided by any counting number is 0. If $x = \frac{0}{1}$, $1 \cdot x = 0$ and x must equal zero. In defining a rational number, $\frac{a}{b}$, we said that b could not be zero. You can see the reason for this by seeing what happens to $\frac{5}{0}$.

If $x = \frac{5}{0}$, then $0 \cdot x = 5$. There is no number x for which $0 \cdot x = 5$ so there is no number $\frac{5}{0}$.

These five properties of the rational numbers let you see the reasons for some of the rules you state for fractions. Let us use these properties to show $\frac{3}{2} = \frac{15}{10}$. If $x = \frac{3}{2}$, then $2x = 3$. Since 2x and 3 are names for the same number,

$$5 \cdot (2x) = 5 \cdot 3.$$

By the associative property,

$$(5 \cdot 2)x = 5 \cdot 3$$

$$10\,x = 15$$

$$x = \frac{15}{10}.$$

But x is a name for $\frac{3}{2}$, so

$$\frac{3}{2} = \frac{15}{10}$$

If you write the last equation,

$$\frac{3}{2} = \frac{5 \cdot 3}{5 \cdot 2},$$

you see that you would have arrived at the same fraction if you had multiplied the numerator and denominator of $\frac{3}{2}$ by 5.

Generalizing, we get

Property 1. If the numerator and denominator of a fraction are multiplied by the same counting number, the number represented is not changed. If the numerator and denominator are divided by the same counting number, the number represented is not changed.

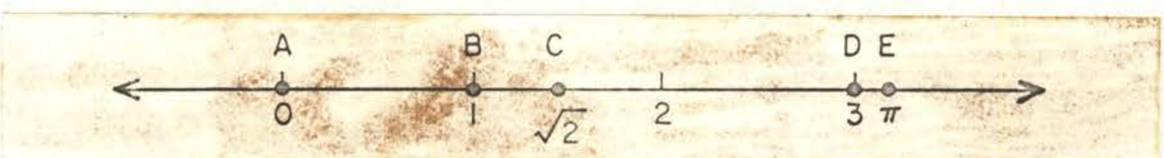
$$\frac{a}{b} = \frac{kc}{kd} = \frac{c}{d}.$$

(11, p. 196-198)

Geometric Properties of the Real Number Line

The one-to-one correspondence between the real numbers and the points of the number line gives us for the first time a satisfactory geometric representation. For this reason it is customary to refer to the number line as the real number line.

We know that there are no gaps or missing points in the real number line. We can speak of tracing the real number line continuously and know that the segment described at any stage has a length which is measured by a real number. Thus in the number line indicated below



we know that $\overline{BC}$ has a length of measure $\sqrt{2} - 1$, the length of $\overline{CD}$ has measure $3 - \sqrt{2}$, $\overline{BE}$ has measure $\pi - 1$, $\overline{CE}$ is measured by $\pi - \sqrt{2}$. (Note: π and $\sqrt{2}$ are irrational numbers; the rationals and the irrationals constitute the real number system.)

We can think of a point moving continuously from 0 to 1. At every location we may associate with it a real number.

Because of this continuous property of our real number system, we sometimes refer to it as the continuum of real numbers. (14, p. 262-263)

We list first the familiar properties which the real number system shares with the rational number system.

Property 1. Closure.

a) Closure under Addition. The real number system is closed under the operation of addition, i.e., if a and b are real numbers then $a + b$ is a real number.

b) Closure under Subtraction. The real number system is closed under the operation of subtraction (the inverse of addition), i.e., if a and b are real numbers then $a - b$ is a real number.

c) Closure under Multiplication. The real number system is closed under the operation of multiplication, i.e., if a and b are real numbers then $a \cdot b$ is a real number.

d) Closure under Division. The real number system is closed under the operation of division (the inverse of multiplication), i.e., if a and b are real numbers then $a \div b$ (when $b \neq 0$) is a real number.

The operations of addition, subtraction, multiplication, and division on real numbers display the properties which we have already observed for rationals. These may be summarized as follows:

Property 2. Commutativity.

a) If a and b are real numbers, then $a + b = b + a$.

b) If a and b are real numbers, then $a \cdot b = b \cdot a$.

Property 3. Associativity.

a) If a, b, and c are real numbers, then $a + (b + c) = (a + b) + c$.

b) If a, b, and c are real numbers, then $(a \cdot b) \cdot c = a \cdot (b \cdot c)$.

Property 4. Identities.

a) If a is a real number, then $a + 0 = a$, i.e., zero is the identity element for the operation of addition.

b) If a is a real number, then $a \cdot 1 = a$, i.e., one is the identity element for the operation of multiplication.

Property 5. Distributivity.

If a, b, and c are real numbers, then $a \cdot (b + c) = (a \cdot b) + (a \cdot c)$.

Property 6. Inverses.

a) If a is a real number, there is a real number (^{-}a), called the additive inverse of a such that $a + (^{-}a) = 0$.

b) If a is a real number and $a \neq 0$ there is a real number b, called the multiplicative inverse of a such that $a \cdot b = 1$.

Property 7. Order.

The real number system is ordered, i.e., if a and b are different real numbers then either $a < b$ or $a > b$.

Property 8. Density.

The real number system is dense, i.e., between any two distinct real numbers there is always another real number. Consequently, between any tao real numbers we find as many more real numbers as we wish. In fact we easily see that: 1) There is always a rational number between any two distinct real numbers, no matter how close. 2) There is always an irrational number between any two distinct real numbers, no matter how close.

The ninth property of the system of real numbers is one which is not shared by the rationals.

Property 9. Completeness.

The real number system is complete, i.e., to each point on the number line there corresponds a real number, and, conversely, to each real number there corresponds a point on the number line.(14, p. 256-258)

Review of the Fundamentals of Arithmetic

> While the new SMSG courses provide for review of the fundamentals of arithmetic, this review has been placed in a new setting with emphasis on number systems.(5, p. 5)

One of the best ways to delve into the reasons for the procedures for carrying out the addition and multiplication operations is to consider systems of number notations using bases other than ten. Since in using a new base, the pupil must necessarily look at the reasons for "carrying" and the other mechanical procedures in a new light, he should gain deeper insight into the decimal system.(16, p. 13)

An attempt has been made to use " number" and "numeral" with precise meaning in the text. For example, "numerals" are written, but "numbers" are added. A numeral is a written symbol. A number is a concept. Later in the text it may be cumbersome to the point of annoyance to speak of "adding the numbers represented by the numerals below." In such case the expression may be elided to "adding the numbers below."

At several points, numbers are represented by collections of x's. Exercises of this kind are important, because they show the role of the base in grouping the x's, as well as the significance of the digits (i.e., symbols) in the numeral for the number.(16, p. 14)

The decimal system uses the idea of place value to represent the size of a group. The size of the group represented by a symbol depends upon the position of the symbol or digit in a numeral. The symbol tells us how many of that group we have. In the numeral 123, the "1" represents one group of one hundred; the "2" represents two groups of ten, or twenty; and the "3" represents three ones, or three. This clever idea of place value makes the decimal system the most convenient system in the world.

Since we group by tens in the decimal system, we say its base is ten. Because of this, each successive (or next) place to the left represents a group ten times that of the preceding place. The first place tells us how many groups of one. The second place tells us how many groups of ten, or ten times one (10 × 1). The third place tells us how many groups of ten times ten (10 × 10), or one hundred; the next, ten times ten times ten (10 × 10 × 10), or one thousand, and so on. By using a base and the ideas of place value, it is possible to write any number in the decimal system using only the ten basic symbols. There is no limit to the size of numbers which can be represented by the decimal system.

To understand the meaning of the number represented by a numeral such as 123 we add the numbers represented by each symbol. Thus 123 means (1 × 100) + (2 × 10) + (3 × 1), or 100 + 20 + 3. The same number is represented by 100 + 20 + 3 and by 123. When we write a numeral such as 123 we are using number symbols, the idea of place value, and base ten.(10, p. 27)

Numerals in Base Seven

You have known and used decimal numerals for a long time, and you may think you understand all about them. Some of their characteristics, however, may have escaped your notice simply because the numerals are familiar to you. In this section you will study a system of notation with a different base. This will increase your understanding of decimal numerals.

Suppose we found people living on Mars with seven fingers. Instead of counting by tens, a Martian might count by sevens. Let us see how to write numerals in base seven notation. This time we plan to work with groups of seven. Look at the x's below and notice how they are grouped in sevens with some x's left over.

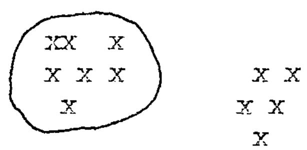


Figure 2-4-a

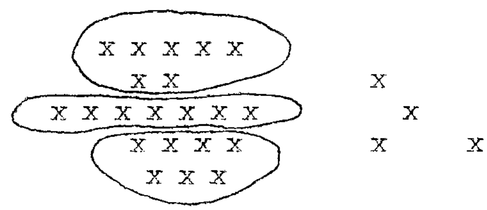


Figure 2-4-b

In figure 2-4-a we see one group of seven and five more. The numeral is written 15_{seven}. In this numeral, the 1 shows that there is one group of seven, and the 5 means that there are five ones.

In figure 2-4-b, how many groups of seven are there? How many x's are left outside the groups of seven? The numeral representing this number of x's is 34_{seven}. The 3 stands for three groups of seven, and the 4 represents four single x's or four ones. The "lowered" seven merely shows that we are working in base seven.

When we group in sevens the number of individual objects left can only be zero, one, two, three, four, five, or six. Symbols are needed to represent those numbers. Suppose we use the familiar 0, 1, 2, 3, 4, 5 and 6 for these, rather than invent new symbols. As you will discover, no other symbols are needed for the base seven system. (10, p. 33-34)

Place Values in Base Seven

$(\text{seven})^5$	$(\text{seven})^4$	$(\text{seven})^3$	$(\text{seven})^2$	$(\text{seven})^1$	(one)

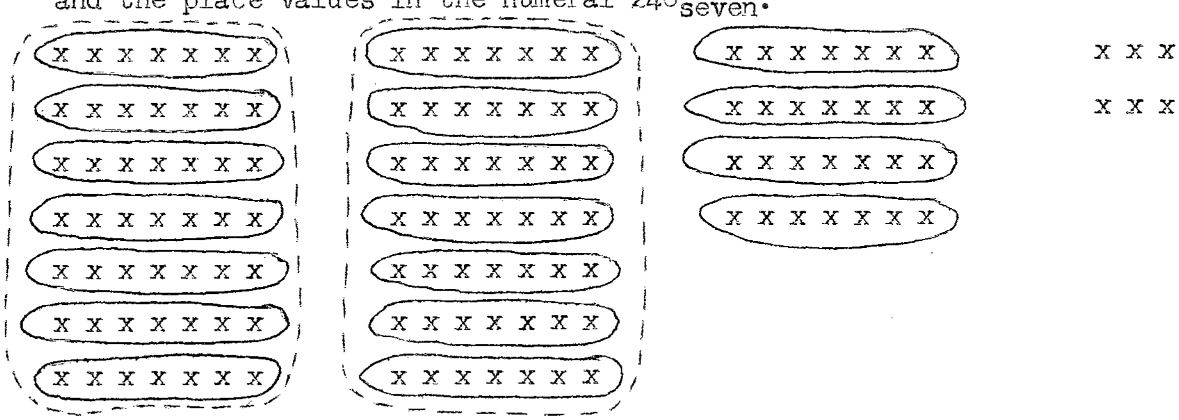
Notice that each place represents seven times the value of the next place to the right. The first place on the right is the one place in both the decimal and the seven systems. The value of the second place is the base times one. In this case what is it? The value in the third place from the right is (seven × seven), and in the next place (seven × seven × seven).

What is the decimal name for (seven × seven)? We need to use this (forty-nine) when we change from base seven to base ten. Show that the decimal numeral for $(\text{seven})^3$ is 343. What is the decimal numeral for $(\text{seven})^4$?

Using the chart above, we see that

$$246_{\text{seven}} = (2 \times \text{seven} \times \text{seven}) + (4 \times \text{seven}) + (6 \times \text{one}).$$

The diagram shows the actual grouping represented by the digits and the place values in the numeral 246_{seven}:



(2 × seven × seven) (4 × seven) (6 × one)

If we wish to write the number of x's above in the decimal system of notation we may write:

$$
\begin{aligned}
246_{seven} &= (2 \times 7 \times 7) + (4 \times 7) + (6 \times 1) \\
&= (2 \times 49) + (4 \times 7) + (6 \times 1) \\
&= 98 + 28 + 6 \\
&= 132_{ten}
\end{aligned}
$$

Regroup the x's above to show that there are 1 (ten × ten) group, 3 (ten) groups, and 2 more. This should help you understand that $246_{seven} = 132_{ten}$.(10, p. 35-36)

Subtraction

How did you learn to subtract in base ten? You probably used subtraction combinations such as 14 - 5 until you were thoroughly familiar with them. You know the answer to this problem but suppose, for the moment, that you did not. Could you get the answer from the addition table? You really want to ask the following question "What is the number which, when added to 5, yields 14?" Since the seventh row of the base ten addition table gives the results of adding various numbers to 5, we should look for 14 in that row. Where do you find the answer to 14 - 5? Did you answer "the last column"? Use the base ten addition table to find

9 - 2, 8 - 5, 12 - 7, 17 - 9.

The idea discussed above is used in every subtraction problem. One other idea is needed in many problems, the idea of "borrowing" or "regrouping." This last idea is illustrated below for base ten to find 761 - 283:

7 hundreds + 6 tens + 1 one	=	6 hundreds + 15 tens + 11 ones	=	761
2 hundreds + 8 tens + 3 ones	=	2 hundreds + 8 tens + 3 ones	=	283
		4 hundreds + 7 tens + 8 ones	=	478

Now let us try subtraction in base seven. How would you find $6_{seven} - 2_{seven}$? Find $13_{seven} - 6_{seven}$. How did you use the addition table for base seven? Find answers to the following subtraction examples:

15_{seven}	12_{seven}	11_{seven}	14_{seven}	13_{seven}
6_{seven}	4_{seven}	6_{seven}	5_{seven}	4_{seven}

The answers to these problems are 6_{seven}, 5_{seven}, 2_{seven}, 6_{seven}, and 6_{seven}.

Let us work a harder subtraction problem in base seven comparing the procedure with that used above:

43_{seven}	=	4 sevens + 3 ones	=	3 sevens + 13 ones	=	43_{seven}
16_{seven}	=	1 seven + 6 ones	=	1 seven + 6 ones	=	16_{seven}
				2 sevens + 4 ones	=	24_{seven} (10, p. 42)

The decimal system is so familiar that its structure and the ideas involved in its algorithms are easily overlooked. (16, p. 22)

Number Systems from an Algebraic Viewpoint

> Number systems are treated from an algebraic viewpoint not only to deepen the student's understanding of arithmetic but also to prepare him for the algebra which is to come. The work on fractions is introduced by defining a fraction as a numeral for the rational number such that $b(\frac{a}{b}) = a$, $b \neq o$. (5, p. 5)

A symbol " $\frac{a}{b}$ " where a and b are numbers, with b not zero, is called a <u>fraction</u>. If a and b are whole numbers, with b not zero, the number represented by the fraction, $\frac{a}{b}$, is called a <u>rational number</u>; any number which can be written in this form is called a <u>rational number</u>. For example, 0.5 represents a rational number because the same number can be written $\frac{1}{2}$. A fraction is a name for a rational number just as numeral is a name for a number. Different names for the same number are:

$$3, \text{III}, \frac{6}{2}, \frac{9}{3}, \frac{63}{21}.$$

The names

$$\frac{6}{2}, \frac{9}{3}, \frac{63}{21}$$

are fractions.

Sometimes $\frac{a}{b}$ is a whole number. This happens when b is a factor of a and only then.

Sometimes $\frac{a}{b}$ is not a whole number. Is there a whole number for which $3x = 4$? Is $\frac{4}{3}$ a whole number?

Two fractions which represent the same number are called <u>equivalent fractions</u>. But it will not often be necessary to use this term.(11, p. 193)

<u>Multiplication of Rational Numbers</u>

You recall that $3 \cdot \frac{1}{3} = 1$ and $3 \cdot \frac{2}{3} = 2$. In general if a and b are whole numbers, where b is not zero,

$$b \cdot \frac{1}{b} = 1 \text{ and } b \cdot \frac{a}{b} = a.$$

What does $\frac{5}{7} \cdot 2$ equal? You can find an answer if you can find a number x for which

$$x = \frac{5}{7} \cdot 2.$$

But then, x and $\frac{5}{7}$ are names for the same number, so that

$$7x = 7(\frac{5}{7} \cdot 2).$$

Using the <u>associative property</u> for multiplication

$$7x = (7 \cdot \frac{5}{7}) \cdot 2 = 5 \cdot 2 = 10$$

$$x = \frac{5 \cdot 2}{7} = \frac{10}{7}$$

You started with $x = \frac{5}{7} \cdot 2$, and you have shown that $x = \frac{5 \cdot 2}{7}$, so

$$\frac{5}{7} \cdot 2 = \frac{5 \cdot 2}{7} .$$

In general, if a, b, and c are whole numbers and b is not zero,

$$c \cdot \frac{a}{b} = \frac{c \cdot a}{b} .$$

Similarly, you can find a number x for which

$$x = \frac{2}{3} \cdot \frac{5}{7} .$$

Since x and $\frac{2}{3} \cdot \frac{5}{7}$ are names for the same number

$$3x = 3(\frac{2}{3} \cdot \frac{5}{7}).$$

Using the <u>associative</u> <u>property</u>,

$$3x = (3 \cdot \frac{2}{3}) \cdot \frac{5}{7}$$

$$3x = 2 \cdot \frac{5}{7}$$

$$3x = \frac{2 \cdot 5}{7}$$

But 3x and $\frac{2 \cdot 5}{7}$ are names for the same number. Thus

$$7 \cdot (3x) = 7(\frac{2 \cdot 5}{7})$$

$$7 \cdot (3x) = 2 \cdot 5$$

Using the associative property for multiplication,

$$(7 \cdot 3)x = 2 \cdot 5.$$

$$x = \frac{2 \cdot 5}{7 \cdot 3} = \frac{2 \cdot 5}{3 \cdot 7} = \frac{10}{21}$$

You started with $x = \frac{2}{3} \cdot \frac{5}{7}$, and you have shown that $x = \frac{2 \cdot 5}{3 \cdot 7}$, so

$$\frac{2}{3} \cdot \frac{5}{7} = \frac{2 \cdot 5}{3 \cdot 7}$$

This example shows that:

To <u>multiply</u> <u>two</u> <u>rational</u> <u>numbers</u> <u>written</u> <u>as</u> <u>fractions</u>, you find a fraction whose numerator is the product of the numerators and whose denominator is the product of the denominators.

In symbols, if $\frac{a}{b}$ and $\frac{c}{d}$ are rational numbers,

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d} = \frac{ac}{bd} .$$

You can use this property to reduce $\frac{12}{16}$ to simplest form, instead of Property 1, in this way

$$\frac{12}{16} = \frac{4 \cdot 3}{4 \cdot 4} = \frac{4}{4} \cdot \frac{3}{4} = 1 \cdot \frac{3}{4} = \frac{3}{4} .$$

Similarly,

$$\frac{21}{35} = \frac{7 \cdot 3}{7 \cdot 5} = \frac{7}{7} \cdot \frac{3}{5} = 1 \cdot \frac{3}{5} = \frac{3}{5} .$$

You can even show Property 1 for the rational number $\frac{a}{b}$ this way if k is not zero:

$$\frac{a}{b} = 1 \cdot \frac{a}{b} = \frac{k}{k} \cdot \frac{a}{b} = \frac{k \cdot a}{k \cdot b} = \frac{ka}{kb} .$$

(11, p. 212-213)

Addition and Subtraction of Rational Numbers

We have examined the elements of the set of rational numbers. We know that there are many names for the same rational number. We have used the operations of multiplication and division. Only two operations remain to be considered: addition and its inverse operation subtraction. Let us look at the addition operation first.

We are all familiar with the idea that $\frac{1}{3} + \frac{1}{3} = 2 \cdot \frac{1}{3} = \frac{2}{3}$; also $\frac{4}{3} = 4 \cdot \frac{1}{3} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3}$. Continuing with other rational numbers, let us find a single fraction for:

$$\frac{2}{3} + \frac{4}{3} .$$

Using what you already know let us write this as:

$$\frac{2}{3} = 2 \cdot \frac{1}{3} = \frac{1}{3} + \frac{1}{3}$$

and

$$\frac{4}{3} = 4 \cdot \frac{1}{3} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} .$$

Then

$$\frac{2}{3} + \frac{4}{3} = (\frac{1}{3} + \frac{1}{3}) + (\frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3}) = 6 \cdot \frac{1}{3} = \frac{6}{3}$$

or we may write

$$\frac{2}{3} + \frac{4}{3} = (2 \cdot \frac{1}{3}) + (4 \cdot \frac{1}{3})$$

$$= (2 + 4)\frac{1}{3} = 6 \cdot \frac{1}{3}$$

$$= \frac{6}{3} ,$$

using the distributive property to get the second line.

What is the result of adding $\frac{a}{b}$ to $\frac{c}{b}$, where a, b, and c are whole numbers and b is not 0? $\frac{a}{b} + \frac{c}{b}$ may be written $a \cdot \frac{1}{b} + c \cdot \frac{1}{b}$. By the distributive property this is equal to

$$(a + c) \cdot \frac{1}{b} = \frac{a + c}{b} .$$

If a, b, and c are whole numbers and b is not 0, then $\frac{a}{b} + \frac{c}{b} = \frac{a + c}{b}$. This may be read in words: The sum of two numbers whose fractions have the same denominator is the sum of the numerators divided by the common denominator.

Let us look at a more difficult addition example:

$$\frac{3}{4} + \frac{7}{10} .$$

The least common multiple of the denominators 4 and 10 is 20. We write each fraction with a denominator 20. You recall that

$$\frac{3}{4} = \frac{3 \cdot 5}{4 \cdot 5} = \frac{15}{20} \text{ and } \frac{7}{10} = \frac{7 \cdot 2}{10 \cdot 2} = \frac{14}{20} .$$

Then

$$\frac{3}{4} + \frac{7}{10} = \frac{15}{20} + \frac{14}{20} = \frac{15 + 14}{20} = \frac{29}{20} .$$

Also $$\frac{3}{10} + \frac{7}{15} = \frac{3 \cdot 3}{10 \cdot 3} + \frac{7 \cdot 2}{15 \cdot 2} = \frac{9}{30} + \frac{14}{30} = \frac{23}{30} .$$

The sum of any two rational numbers $\frac{a}{b}$ and $\frac{c}{d}$ may be found similarly. A common multiple of b and d is bd.

$$\frac{a}{b} = \frac{a \cdot d}{b \cdot d} = \frac{ad}{bd} \text{ and } \frac{c}{d} = \frac{b \cdot c}{b \cdot d} = \frac{bc}{bd}$$

Using what we know about adding rational numbers whose fractions have the same denominator we have:

$$\frac{a}{b} + \frac{c}{d} = \frac{ad}{bd} + \frac{bc}{bd}$$
$$= \frac{ad + bc}{bd} .$$

Thus we may say:

If a, b, c, and d are whole numbers and b and d are not zero, then

$$\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd} .$$

(11, p. 220-221)

Percent and Applications

> Percent applications have a place in the new courses, as do other social applications, for example through governmental statistics in the chapter on graphs and in probability. (5, p. 5)

Percent is used to express ratios of numerical quantities in everyday experience. It is important for you to understand the notation of percent, and also, to be accurate in computing with numbers written as percents. Let us look at some examples in the use of percent.

Example 1. Suppose that a family has an annual income of $4500 (after withholding tax). The family budget includes an item for food of $33\frac{1}{3}\%$ of the budget. How much money is allowed for food for the year? Can you answer this question without using a pencil and paper? If you can, do so, and check what you find with the result below. We work this example as we do, to show a method which we shall be using in more difficult examples later. We know that $33\frac{1}{3}\%$ is equal to $\frac{1}{3}$ and hence if we let x stand for the number of dollars allowed for food we have

$$\frac{x}{4500} = \frac{1}{3} .$$

To find x we use Property 1 of Section 9-1 which tells us that

$$\text{if } \frac{a}{b} = \frac{c}{d} \text{ then } ad = bc.$$

For example, this means that

$$\text{if } \frac{6}{10} = \frac{9}{15} \text{ then } 6 \cdot 15 = 9 \cdot 10 .$$

Then in our example,

$$\text{if } \frac{x}{4500} = \frac{1}{3} \text{ then } 3x = 4500 .$$

Hence $x = \frac{4500}{3} = 1500$, and 1500 will be the number of dollars used for food. (11, p. 382)

It is unfortunate that in most texts, percent is considered a separate topic and traditionally receives very special and separate treatment. This separation from the rest of the course is the reason for much of the difficulty encountered. One should not go on to the applications of percent until its relationships with ratio and decimals have been firmly established. (17, p. 275)

The meaning of percent is based on the idea that "a"% means $\frac{a}{100} = a \times \frac{1}{100}$. All three "cases" of percent are introduced informally in Section 2 with numbers that are easily handled. However, you will notice that the three "cases" of percent are not referred to in this textbook. (17, p. 262)

If the background is firm along the lines suggested, there should be little trouble with the applications since terminology is kept at a minimum. The purpose of the section on applications is to show how percentage works, rather than to make the student a facile doer of problems in discount, commission and the like.

Notice that at the beginning of this section, the form $\frac{a}{b} = \frac{c}{d}$ is used exclusively. The reason for this is to stress the essential unity of all problems in percentage, to strengthen the relationship:

$$\text{if } \frac{a}{b} = \frac{c}{d}, \quad \text{then } ad = bc, \quad (b \neq 0, \; d \neq 0)$$

and to prepare for the solution of equations later on. If the pupils discover valid short-cuts for themselves they should be encouraged to use them if they understand what they are doing. But the moment one starts to use special methods for special problems, one is led inexorably to the classification of problems by types, which, as every good teacher knows, must be avoided at all costs.

Of course, one must introduce the ideas of discount, commission and interest one at a time with a little practice on each, but they are mixed as much as feasible so that the pupil will acquire the idea that, though the terms are different, the mathematical processes are the same. (17, p. 276)

The principal difference in treatment in the SMSG and the traditional texts is that in SMSG materials these topics are more closely related to the properties of number systems from a mathematical point of view, and less time is given to a discussion of the social situations in which the various applications arise. (17, p. 258)

Table 13-2 shows the population in millions for every census from 1790 through 1950. The table shows that the population in 1790 was 3.9 millions. This means there were 3,900,000 (3.9 × 1,000,000) people in the U. S. at that time. Is this an exact or approximate number? The

> from the traditional. Geometric ideas are introduced, first of all, from a non-metric point of view and then, after a careful treatment of measurement, students are led gradually to a study of properties of triangles and other geometric figures, plane and solid, through an informal deductive approach.(5, p. 5-6)

Points, Lines, and Space

1. Understandings:

   (a) A point has no size.

   (b) A line is a certain set of points.

   (c) A line is unlimited in extent.

   (d) Through two points there is one and only one line.

   (e) Space is a set of points.

2. Teaching Suggestions:

Just as we use representations to develop the concept of the "counting numbers" (2 cars, 2 people, 2 hands, 2 balls, 2 chairs, etc., to develop the concept of twoness) similarly we must select representations for developing the concepts of point, line, plane, and space.

Point. Identify things which suggest the idea of a point keeping in mind that one suggestion by itself is not adequate for developing the idea of a point. One needs to use many illustrations in different situations. Suggestions: tip of pencil; needle; pointer; collection of boxes, putting one inside another and always being able to place one more inside of the last and the point indicated as being in all the boxes; pupil of the eye in intense brightness; progressive closure of shutter of camera; dot of light on some TV screens; particle of dust in the air.

Line. Identify two points using some of the situations as above, such as tips of two pencils, etc. Bring out the idea that given these two points there are many other points on the line that contain them. Some of these are between the two points, some are "beyond" the one, and some are "beyond" the other. Also, through two points there can be only one line. The line has no thickness and no width. It is considered to extend indefinitely. Use string held taut between two points to show representations of lines in positions that are horizontal, vertical, and slanting. Each student may represent lines by using a pencil between his fingertips. With each example talk about thinking of a line as unlimited in extent. Emphasize frequently that we use the word, "line" to mean straight line.(16, p. 78)

Planes

1. Understandings:

   (a) A plane is a set of points in space.

   (b) If a line contains two different points of a plane, it lies in the plane.

   (c) Many different planes contain a particular pair of points.

   (d) Three points not exactly in a straight line determine a unique plane.

2. Teaching Suggestions:

Identify surfaces in the room which suggest a plane -- walls, tops of desks, windows, floor, sheet of paper, piece of cardboard, chalkboard, shadow. Make use of Saran wrap, cellophane, and a wire frame to show further a representation of a plane since this more nearly approaches the mathematician's idea of a plane. With each example bring out the idea that a plane has no boundaries, that it is flat, and extends indefinitely. It is an "ideal" of a situation just as is a line and a point. We try to give this idea by suggesting things that represent a plane. It is important to suggest representations of planes in horizontal, vertical, and slanting positions. Note that if a line contains 2 points of a plane, it lies in the plane and that many planes may be on a particular pair of points as pages of a book, revolving door, etc.

Then using three fingers or sticks of different heights in sets of 3 (not in a straight line) as suggested by the sketch at the right, see what happens when a piece of cardboard is placed on them. Add a fourth finger or a fourth stick and observe what happens. (16, p. 80)

Intersections of Lines and Planes

1. Understandings:

   (a) Two lines may:

      (1) be in the same plane and intersect;

      (2) be in the same plane and not intersect (intersect in the empty set);

      (3) not be in the same plane and not intersect (intersect in the empty set).

   (b) A line and a plane may:

      (1) not intersect (intersect in the empty set);

      (2) intersect in one point;

      (3) intersect in a line.

   (c) Two different planes may

      (1) intersect and their intersection will be a line;

      (2) not intersect (have an empty intersection).

2. Teaching Suggestions:

Use models in order to explore the possible situations for two lines intersecting and not intersecting. (Let each student have materials, too.) Also, use a pencil or some other object to represent a line, and a card to represent a plane. Use two pieces of cardboard each cut to center with the two fitted together to represent the idea of two planes and their intersection, and, from these, state some generalizations that may be made.(16, p. 86)

Property 4: If the intersection of two different planes is not empty, then the intersection is a line.

If the intersection of two planes is the empty set, then the planes are said to be parallel. Several examples of pairs of parallel planes are represented by certain walls of a room or a stack of shelves. Can you think of others?

In discussing the intersection of two different planes we have considered two cases. Let M and N denote the two planes.

Case 1. $M \cap N$ is not empty. $M \cap N$ is a line.

Case 2. $M \cap N$ is empty. M and N are parallel.

Are there any other cases? Why?(10, p. 128)

Separations

In this section we shall consider a very important idea -- the idea of separation. We shall see this idea applied in three different cases.

Let M be the name of the plane of the front chalkboard. This plane separates space into two sets: (1) the set of points on your side of the plane of the chalkboard, and (2) the set of points on the far side (beyond the chalkboard from you). These two sets are called half-spaces. The plane M is not in either half-space.

Let A and B be any two points of space not in the plane M of the chalkboard. Then A and B are on the same side of the plane M if the intersection of $\overline{AB}$ and M is empty, that is, if $\overline{AB} \cap M$ is empty. Also, A and B are on opposite sides of the plane M if the intersection of $\overline{AB}$ and M is not empty; in other words, there is a point of M between A and B.

Any plane M separates space into two half-spaces.

If A and B are in the same half-space, $\overline{AB} \cap M$ is empty. If A and B are in different half-spaces, $\overline{AB} \cap M$ is not empty. We call M the boundary of each of the half-spaces. (10, p. 134)

Separations

1. Understandings:

   (a) A plane separates space into two half-spaces.

   (b) A line separates a plane into two half-planes.

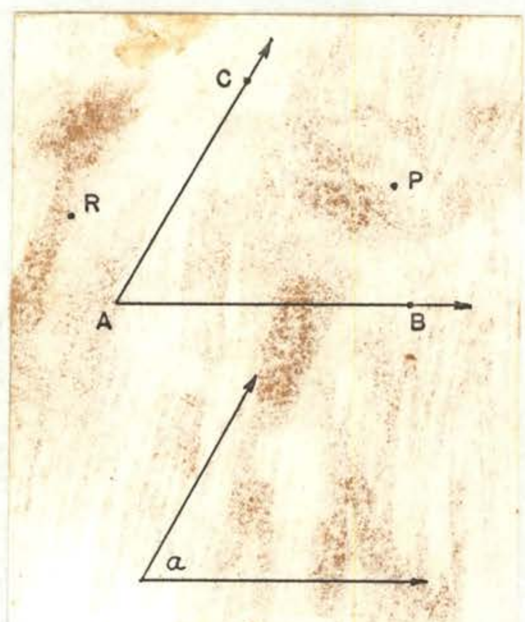
   (c) A point separates a line into two half-lines.

   (d) A ray is the union of a half-line and the point which determines the half-line. (16, p. 90)

Measurement of Angles

You have been studying ways of measuring line segments, plane closed regions, and solids. Now let us see how angles are measured.

Recall that an angle is the set of points on two rays with the same endpoint. In the drawing of the angle, the rays are $\overrightarrow{AB}$ and $\overrightarrow{AC}$. These rays are sides of angle BAC and point A is its vertex. Notice that the angle is named "angle BAC" or "angle CAB," with the vertex named second. Why should A be the middle letter? We can call it "angle A" if this name applies to only one angle. We also name angles by writing a small letter or numeral in the interior of the angle, near the vertex.



An angle determines three sets of points in the plane, the set of points in the interior of the angle, the set of points in the exterior of the angle, and the set of points on the angle itself. A point P is in the interior of angle BAC if it is on the same side of line $\overleftrightarrow{AB}$ as point C, and on the same side of line $\overleftrightarrow{AC}$ as point B. (See angle BAC above.) Any point in the plane which is not a point on the angle and not a point in the interior is a point in the exterior of the angle. Is point R a point in the interior or the exterior of angle BAC? Point P?

As you know, to measure anything you must use a unit of the same nature as the thing measured. To measure an angle, an angle is chosen as the unit of measure. Then an angle can be measured by drawing rays which subdivide its interior so that angles are formed which are exactly like the unit angle.(11, p. 287)

You learned in Chapter 7 how to measure angles. We shall frequently talk about the "measure of an angle" in this chapter. Therefore, it is convenient to have a symbol for this statement. To indicate the number of units in an angle, we will use the symbol "m" followed by the name of the angle enclosed in parentheses. For example, m(∠ABC) means the number of units in angle ABC.

As you learned, any angle can be used as a unit of measure, but in this chapter we will use the degree as the standard unit. Thus when we write m(∠ABC) = 40, we will understand angle ABC is a 40 degree (40°) angle. Note that since m(∠ABC) is a number, we write only m(∠ABC) = 40, not "m(∠ABC) = 40°."(12, p. 401)

To compare these segments, lay the edge of a piece of paper along $\overline{CD}$ and mark points C and D on this paper so that they match the points in the picture. Place the edge of the paper along $\overline{AB}$ with point C on point

A. Where does point D fall? If it is between A and B, $\overline{AB}$ is longer than $\overline{CD}$. If D falls on B, the segments are the same length. We say that segments of the same length are congruent. If B is between C and D, $\overline{CD}$ is longer than $\overline{AB}$.

We wish to consider the set of all points which either lie on a simple closed curve or are contained in the interior of the simple closed curve. Such a set of points will be called a closed region.

* * *

To compare the size of the closed region A with the size of the closed region B, cut out a copy of figure B and place it on the closed region A. Figure B separates A into the part covered by B and the part not covered by B. We say that the area of A is larger than the area of B.

When we use methods like this to compare the sizes of line segments or of closed regions, we assume these properties of geometric continuous quantities:

Motion Property. A geometric figure may be moved without changing its size or shape.

Comparison Property. Two continuous geometric figures or sets, of the same kind may be compared to determine whether they have the same size, or which one is the larger.(13, p. 245)

Matching Property. If two geometric continuous figures, or sets, are both made up of parts such that every part of one can be matched to a part of the same size in the other, then the two continuous figures, or sets have the same size.

Subdivision Property. A geometric continuous figure or set may be subdivided.(11, p. 249)

Construct a triangle as follows:

(a) Use $\overline{JK}$ for one side.

(b) Construct $\angle J$ at point J. Set the compass for the length of $\overline{JM}$ and with J as a center make an arc intersecting the second side of the angle.

(c) Label this intersection M.

(d) Draw side $\overline{MK}$ through points M and K.

Is your triangle the same size and shape as triangle JKM? Use your compass and ruler to check.

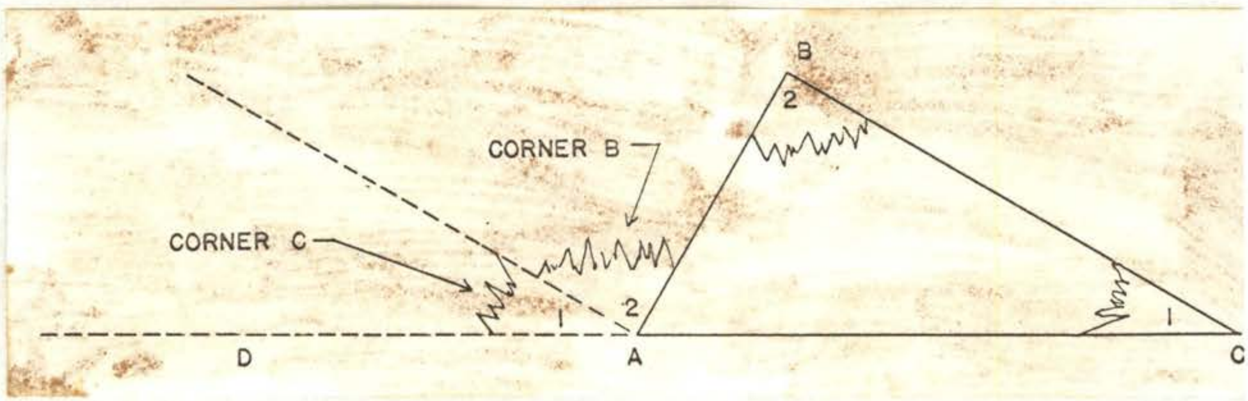
This triangle was constructed by using side $\overline{JM}$, $\angle$ J, and side $\overline{JK}$. What do you notice about the position of the angle in relation to the sides? This arrangement of two sides and an angle is called "two sides and the included angle." Another such group would be side $\overline{JK}$, $\angle$ K, and side $\overline{KM}$. Is there another such group?

Two triangles are congruent if two sides and the included angle of one triangle are congruent respectively to two sides and the included angle of the other triangle. We will refer to this as Property S. A. S. (Side, Angle, Side). (14, p. 190-191)

Inductive and Deductive Reasoning

Although there is no attempt to give a system of postulates for the geometry, properties are identified on an intuitive or inductive basis and then these properties are used to draw conclusions about, or to prove, other properties. (5, p. 6)

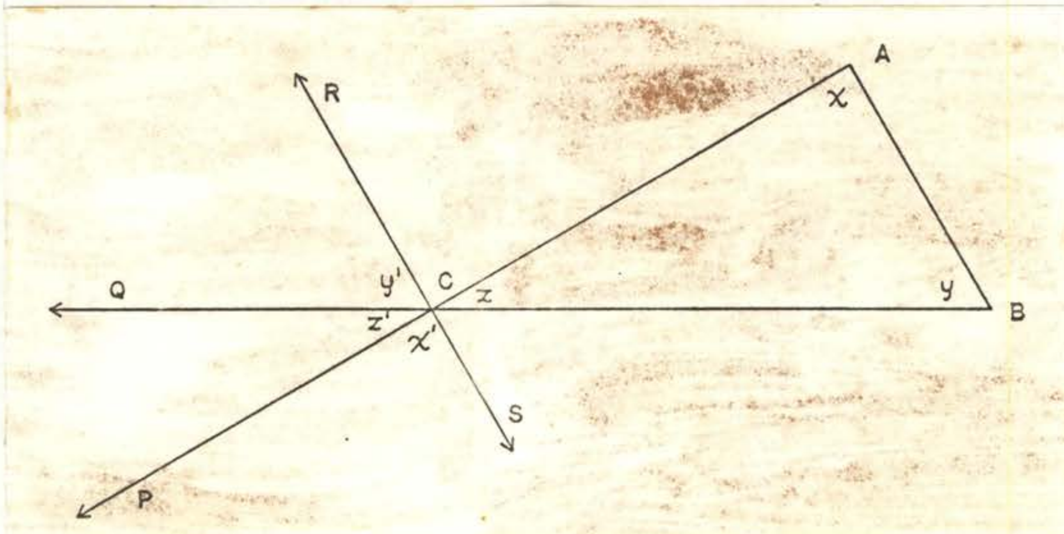
1. Draw a triangle, making each side about two or three inches in length. Cut out the triangular region by cutting along the sides of the triangle. Tear off two of the corners of this region and mount the whole figure on cardboard or a sheet of paper as shown in the figure below. Note: Corners B and C are placed around the vertex A. (The corners may be pasted or stapled in place.)



(a) Find the measures of the three angles with vertices at A. Find the sum of these three measures and compare your results with those of your classmates. In the drawing above, does it appear that $\overrightarrow{AD}$ and $\overrightarrow{AC}$ are on the same line? Does this appear to be true on the figure you made?

(b) What do you observe about angle 1, angle 2, and angle BAC in this new arrangement?(12, p. 425)

3. Consider the triangle ABC and the rays $\overrightarrow{AP}$ and $\overrightarrow{BA}$ shown below. Line RS is drawn through point C so that the measure of $\angle$ y and the measure of $\angle$ y' are equal. Here we are using a new notation, y'. y' is read "y prime." (In this problem we are using this notation in naming angles.)



Use a property to explain "why" for each of the following whenever you can:

(a) Is $\overleftrightarrow{RS}$ parallel to $\overline{AB}$? Why?

(b) What kind of angles are the pair of angles marked x and x'? Is $m(\angle x) = m(\angle x')$? Why?

(c) What kind of angles are the pair of angles marked z and z'? Is $m(\angle z) = m(\angle z')$? Why?

(d) $m(\angle y) = m(\angle y')$ Why?

(e) $m(\angle x) + m(\angle y) + m(\angle z) = m(\angle x') + m(\angle y') + m(\angle z')$ Why?

(f) $m(\angle x) + m(\angle y) + m(\angle z)$ is the sum of the measures of the angles of the triangle. Why?

(g) $m(\angle x') + m(\angle y') + m(\angle z') = 180$ Why?

(h) $m(\angle x) + m(\angle y) + m(\angle z) = 180$ Why?

(i) We conclude therefore that the sum of the measures of the angles of the triangle is 180. Why?

This is a <u>proof</u> of Property 4.

Property 4. The sum of the measures in degrees of the angles of any triangle is 180.(12, p. 427-428)

You obtained this property by using the inductive method of reasoning when you tore the corners from a triangular region and placed them as adjacent angles. You also learned to prove this property by the deductive method when you used previously proved properties to show that

$$m(\angle x) + m(\angle y) + m(\angle z) = 180$$

where $\angle x$, $\angle y$, and $\angle z$ represent the angles of a triangle.(12, p. 459)

Critique and Summary

In this piecework of SMSG course content the reader should note that the approach here is concerned with basic mathematical concepts as evidenced by the symbolism and terminology.

> In the new programs we find that familiar ideas often have a strange appearance because of new symbolism.
>
> ***
>
> They are not substitutes for thinking. They are not a set of symbols whose manipulation miraculously produces valid conclusions. To present them thus would merely replace one kind of meaningless symbol-pushing with another. Instead we must present them as a new language which facilitates thought by enabling us to express abstract ideas with greater clarity.(1, p. 85)
>
> ... judicious use of sets and of the language and concepts of "abstract" algebra may bring more coherence and unity into the high-school curriculum. Yet, the spirit of modern mathematics cannot be taught by merely repeating its terminology. Consistently with our principles, we wish that the introduction of new terms and concepts should be preceded by sufficient "concrete" preparation and followed by genuine challenging applications and not by thin and pointless material: one must motivate and apply a new concept if one wishes to convince an intelligent youngster that the concept warrants attention. (22, p. 193)

The emphases of the approaches of the SMSG program are summed up thusly:

Emphasized in the texts for junior high school are the following important ideas of junior high school mathematics: structure of arithmetic from an algebraic viewpoint; the real number system as a progressing development; and metric and nonmetric relations in geometry. These ideas are constantly associated with their applications. Time is given to the topics of measurement and elementary statistics.(4, p. 455)

CHAPTER IV

SUMMARY

In a brief effort to point out content of these two programs which total over 3,000 textbook pages combined, many of the topics of the total programs must be left out and a number of discrepancies and "holes" in the continuity are inevitable. The choice of materials is biased by the attitude of the author as to what would be useful to those who select curriculum texts for our junior high schools. This situation arises from the conviction that it is better to see the actual subject matter content to which this report refers and about which this report asserts opinions than to fight a battle of semantics (i.e., "modern" versus "traditional").

It is hoped that the reader, being more familiar with the traditional texts, will be stimulated enough to observe and read the actual SMSG texts for a more full account of what is found in them in order to make a fair appraisal for himself.

If there is serious interest on the part of any administrator, superintendent, principal, supervisor, or teacher who wishes to know about the progress in mathematics and its implications for the schools, the drive to improve school mathematics, and classroom experiences with the new mathematics programs, he is referred to The Revolution in School Mathematics; A Challenge for Administrators and Teachers as listed (1.) in the bibliography of this report. Herein will also be found a specific section of questions and answers concerning administrative problems, individual differences in ability to learn, teacher training, evaluation and comparison, availability of the new texts, etc.

Also included are the steps considered basic to the implementation of a new mathematics program in the school.

1. Recognition by school authorities of the need for a new mathematics program.
2. Adequate preparation of teachers in the mathematics now being taught for the first time in secondary schools.
3. Selection of a new program.
4. Selection of students for the program.
5. Informing parents about the new program.
6. Informing other members of the school system about the new program and its implications for the mathematics program K-12.
7. Continuation of teacher preparation for carrying the new program to higher and lower grades.
8. Provision for adequate time and compensation to carry on the new program year after year.(1, p. 38)

Each of these steps is discussed in detail, describing and evaluating the techniques involved. Concerning teacher preparation, for example, is a statement of a situation which must be resolved.

> In the past some administrators thought that mathematics was so easy to teach -- the same, year after year -- that anybody could teach it. Therefore, they assigned mathematics classes to some teachers who were not mathematically qualified. Administrators must give up the idea that mathematics is easy to teach. The day is gone when administrator can assign just anyone on his staff to teach mathematics and come up with an acceptable program. Mathematics must not be taught today with the same content and techniques that have been commonly used in the past. (1, p. 50)

As a final consideration, it is further hoped that efforts of this report have served to influence the attitude of those who plan the curriculum in our schools to take an objective look at their present mathematics programs and act in accordance with the individual school needs of their students in the rapidly approaching future.

The leading scholars in each field must take much of the responsibility for determining the nature of college-preparatory courses. Our mathematicians, until recently, have been too busy devising new mathematics to fulfill their obligations in this regard. Now some of them, like their counterparts in Europe, have produced expository materials that are useful in college-preparatory sequences. These materials, written with the aid and advice of classroom teachers, learning specialists, psychologists, and mathematics educators, delineate the patterns of reasoning that characterize mathematical thought. They spell out, in terms appropriate for each grade level, some of the implications of the explosive growth and manifold applications of twentieth century mathematics.

These new materials can be used to provide better preparation for the study of mathematics in college, thus relieving colleges of a heavy burden of remedial instruction and enabling students to learn more mathematics during their college years. For many students this means the attainment of the higher levels of mathematical literacy which are now necessary for the effective study of science, engineering, and many other subjects. For a few students it means earlier access to the frontiers of mathematical thought and longer careers as creative mathematicians. This is important because mathematicians are now in short supply, and mathematicians, like jet pilots, reach their maximum efficiency early in life.

The new programs in school mathematics will serve to increase the nation's supply of technicians, engineers, scientists, and mathematicians. They will also help, in some degree, to bridge the terrifying gap that now exists between mathematics instruction and the outrushing frontiers of mathematics and science. Indeed, they are typical of the programs that must be established in all fields (and are already being established in science) if the members of the next generation are to have the knowledge necessary to operate the complex civilization they inherit.

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The school administrator is the educational statesman who is called upon to supply the understanding and administrative guidance necessary for the success of the improved programs now available in several academic subjects. He is in a better position than the subject-matter specialist for a view of the entire picture.

Some school administrators see the drive to improve school mathematics as part of a nation-wide drive to achieve excellence in the classroom. Some view it as evidence that our schools have a new sense of mission which has its goals in high academic achievement. Many administrators are encouraged by the fact that the improved programs in mathematics, while designed primarily for the college-capable pupils, offer many ways to improve instruction for pupils who are mathematically less talented. They recognize the urgent need for improvement, but they do not regard the drive to improve school mathematics as a crash program whose sole justification is found in the menacing attitude of our potential enemies, and whose sole purpose is the production of mathematicians and scientists to serve, like nuclear weapons, as instruments of national survival. Instead, they regard it as a program for long-range improvement which aims to produce people who are not only more competent in science and technology but are also better able to meet the responsibilities of citizenship in a free society.(1, p. 84-86)

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