MEMBRANE DEFORMATIONS OF TRANSLATIONAL SHELLS BY FINITE DIFFERENCES

By

SAMIR ALBERT LAWRENCE

Bachelor of Science

Al-Hikma University of Baghdad

Baghdad, Iraq

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Name: Samir Albert Lawrence Date of Degree: May 26, 1963

Institution: Oklahoma State University

Location: Stillwater, Oklahoma

Pages in Study: 43 Candidate for the degree of Master of Science

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Scope of Study: A systematic finite difference procedure for the solution of the differential equations of the membrane deformations of translational shells is presented.

The solution for the vertical deformation w is obtained in terms of the stress resultant $\overline{N}_y$. In this solution some simplifications are made in evaluating the term R. These simplifications are accomplished by the use of power series and the neglect of fourth order powers of the slopes in comparison with unity.

The solutions for the horizontal deformations u and v are obtained in terms of w.

Findings and Conclusions: The systematic finite difference procedure makes the solution for the membrane deformations of translational shells a simple and straight forward process.

The simplifications made in the term R eliminate complicated numerical differentiation and permit the evaluation of R in terms of the $\overline{N}_y$ membrane forces only.

ADVISER'S APPROVAL ______________________________

MEMBRANE DEFORMATIONS OF TRANSLATIONAL

SHELLS BY FINITE DIFFERENCES

Report Approved:

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Report Adviser

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Dean of the Graduate School

PREFACE

The writer wishes to express his sincere appreciation to the following individuals:

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S. A. L.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

NOMENCLATURE

h	Thickness of the Shell.
h_x, h_y	Total Rise of the Shell on the x and y Axes, Respectively.
p_o	Intensity of Uniformly Distributed Load.
p_z	Intensity of Vertical Load on Projected Area.
u, v, w	Deformations of the Shell in the x, y, and z Directions, Respectively.
w^*	Starting Value, Representing the Deformation in the z Direction at a Point.
x, y, z	Coordinates of a Point on the Middle Surface of the Shell.
z_x, z_y, z_{xx}, z_{yy}, z_{xy}	Partial Derivative of z with Respect to x or y as Indicated by the Subscript.
L_x, L_y	One-half the Total Lengths of the Shell in the x and y Directions, Respectively.
N_x, N_y, N_{xy}	Normal and Shearing Forces on an Element of the Shell.
$\overline{N}_x$, $\overline{N}_y$, $\overline{N}_{xy}$	Projected Normal and Shearing Forces in the x-y Plane.
α, β	The Angles Between the Middle Surface of the Shell and the Projected Plane when Measured along the x and y Axes, Respectively.
λ	Carried-Over Starting Value at a Point.
ν	Poisson's Ratio.
Δ_x, Δ_y	Interval of the Finite Network in the x and y Directions, Respectively.
$(\ )_{x,y}$	Partial Derivative of a Quantity with Respect to x or y as Indicated by the Subscript.

CHAPTER I

INTRODUCTION

1-1 Historical Study

In 1931 Pücher(1), in his dissertation, presented the equilibrium equations for the general shell of double curvature in terms of a stress function and projected forces. He also presented a series solution for stresses in an elliptical paraboloid.

Flügge(2), in 1950, discussed the analysis of translational shells by finite differences, using relaxation to solve the simultaneous equations.

In 1953 Shizuo Ban(3) extended the use of the membrane forces in shells of double curvature to the determination of membrane deformations of hyperbolic-paraboloid shells. At the same time and independently of Ban, F. T. Geyling(4) developed a general theory of deformations of membrane shells, and discussed the deformations of the elliptical paraboloid using finite differences. In 1955 Eric Reissner(5) presented a paper in which he discussed the membrane deformations and displacements of a hyperbolic shell due to its own weight according to shallow membrane theory.

In 1956 Parme(6) gave a detailed account of the partial differential equation for determination of shell stresses. In 1957 W. Flügge and F. T. Geyling(7, 8) presented two papers on the General Theory of Deformations of Membrane Shells based on the earlier work of Geyling(4).

The solution of finite difference equations by infinite geometric series for two dimensional, second order problems was presented by Tuma, Havner, and French[9] in 1958. The idea of extending the Algebraic Carry-Over method to the solution of translational shells was proposed by Havner[10] in shell lectures delivered in 1959. D. I. Tilden[11] carried out the extension of this method in 1961.

1-2 Scope of Study

A systematic finite difference procedure for the solution of membrane deformations of translational shells is presented. The partial differential equation in finite difference form is developed in terms of the vertical deformations, w, and the stress resultant $\overline{N}_y$. The latter values are directly obtained from the membrane force solution.

In solving the differential equations for w some simplifications are made in evaluating the term R. These simplifications are accomplished by the use of power series and the neglect of fourth order powers of the slopes as compared with unity.

A numerical example is included.

1-3 Membrane Equations of Equilibrium

An element of a shell, projected into the x-y plane is shown in Fig. 1-1. For simplification in the expressions of the equilibrium equations, the internal forces on the shell element have been transferred to the projected element. Internal forces are shown in the positive sense.

The shell is considered to be in the extensional (membrane) state. Thus, the forces on the element are membrane forces.

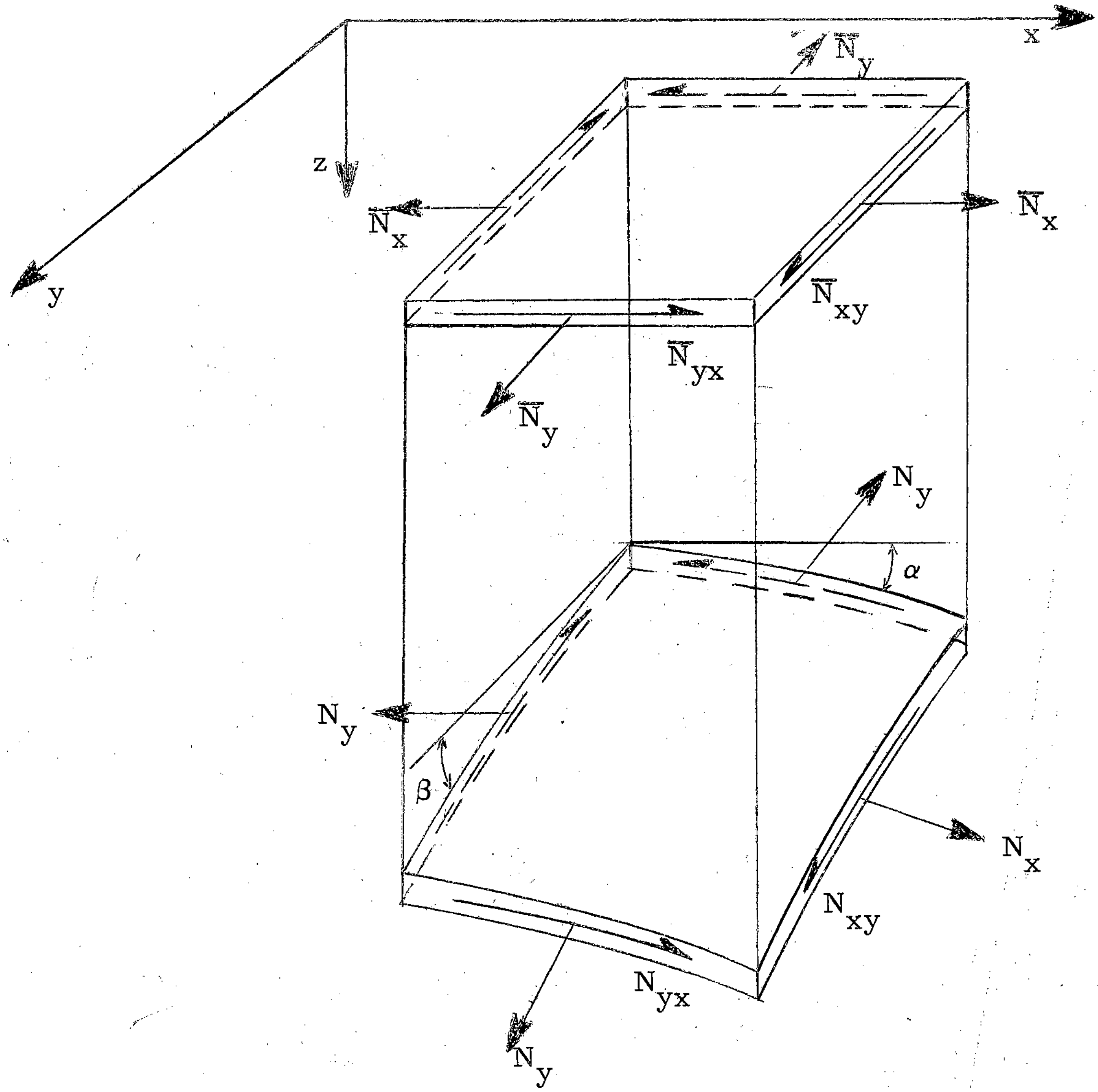

Figure 1-1

Element of Shell Projected in x-y Plane

The relationship between the projected internal forces and actual internal forces may be determined by geometry. The final equations are:

$$\left.\begin{aligned}\overline{N}_y &= N_y \frac{\cos\beta}{\cos\alpha} \\ \overline{N}_x &= N_x \frac{\cos\alpha}{\cos\beta} \\ \overline{N}_{xy} &= N_{xy}\end{aligned}\right\} \quad (1\text{-}1)$$

Considering loading in the z direction only, the final three equations of equilibrium are[1]

$$\frac{\partial \overline{N}_x}{\partial x} + \frac{\partial \overline{N}_{yx}}{\partial y} = 0 \quad (1\text{-}2)$$

$$\frac{\partial \overline{N}_y}{\partial y} + \frac{\partial \overline{N}_{xy}}{\partial x} = 0 \quad (1\text{-}3)$$

$$\overline{N}_x z_{xx} + \overline{N}_y z_{yy} + 2\,\overline{N}_{xy} z_{xy} = -p_z \quad . \quad (1\text{-}4)$$

1-4 Shells of Translation

The equation of the surface for a general shell of translation is given as

$$z = f_1(x) + f_2(y) \quad . \quad (1\text{-}5)$$

Differentiating Equation (1-5) twice with respect to x yields

$$z_{xx} = f_1''(x) \quad . \quad (1\text{-}6)$$

Similarly, with respect to y

$$z_{yy} = f_2''(y) \quad (1\text{-}7)$$

with respect to y and then x

$$z_{xy} = 0 \quad . \quad (1\text{-}8)$$

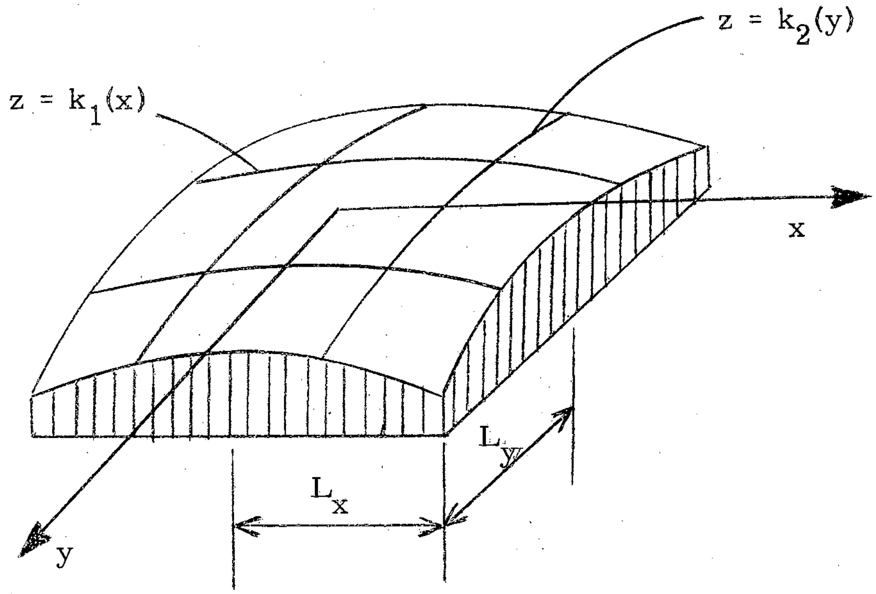

Figure 1-2

Shell of Positive Gaussian Curvature

1-5 Boundary Conditions

The shell is supported by vertical shear diaphragms, and it is assumed that no tangential sliding takes place between the shell and the diaphragms. The diaphragm is usually denied all rigidity transverse to its plane. Therefore, at the edges

$$x = \pm L_x \ , \qquad \bar{N}_x = 0$$

and at the edges

$$y = \pm L_y \ , \qquad \bar{N}_y = 0 \ .$$

CHAPTER II

MEMBRANE DEFORMATIONS OF TRANSLATIONAL SHELLS

2-1 Introduction

An element of the shell is shown in Figure 2-1 with the positive displacement components u, v, and w, and the geometric parameters α, β, and ω.

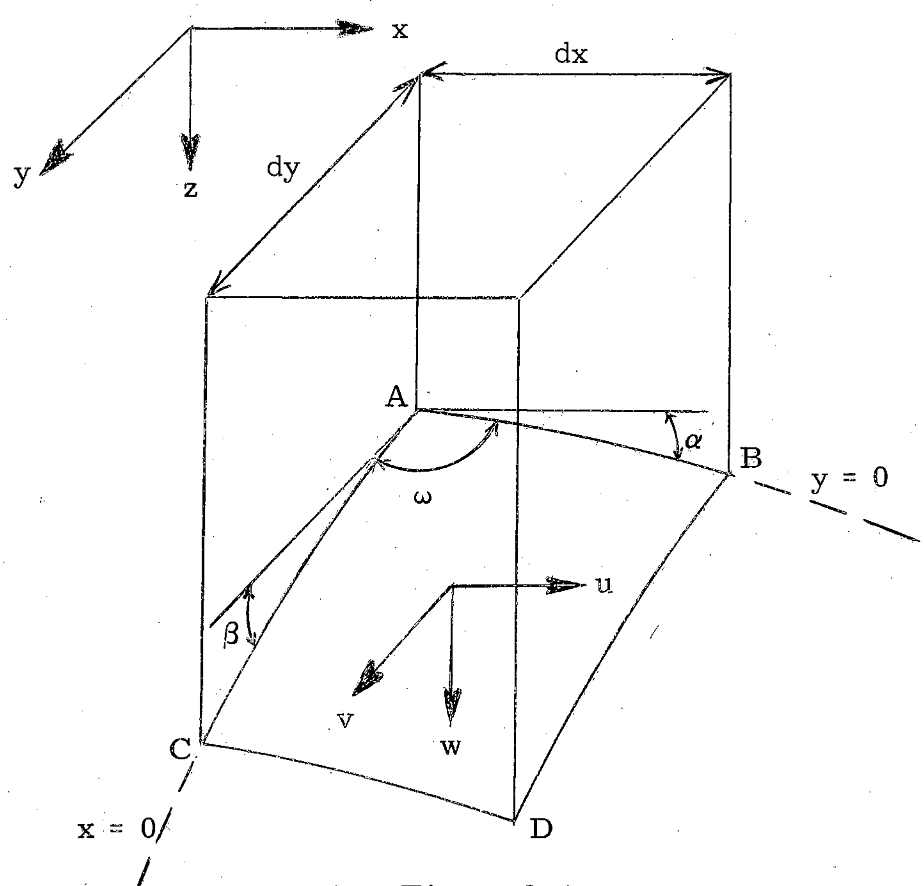

Figure 2-1

Element of Shell with Displacements Shown

2-2 Differential Equations for u, v, and w

The development of the equations of deformations appears elsewhere[4, 7, 8] and is not repeated here. The three equations of deformations are

$$u_x = - w_x z_x + f \qquad (2-1)$$

$$v_y = - w_y z_y + g \qquad (2-2)$$

$$u_y + v_x = - w_x z_y - w_y z_x + k \qquad (2-3)$$

where

$$f = \frac{1}{Eh(1 + z_x^2 + z_y^2)^{\frac{1}{2}}} \left\{ \bar{N}_x (1 + z_x^2)^2 + 2 \bar{N}_{xy} z_x z_y (1 + z_x^2) + \bar{N}_y \left[z_x^2 z_y^2 - \nu (1 + z_x^2 + z_y^2) \right] \right\}$$

$$g = \frac{1}{Eh(1 + z_x^2 + z_y^2)^{\frac{1}{2}}} \left\{ \bar{N}_y (1 + z_y^2) + 2 \bar{N}_{xy} z_x z_y (1 + z_y^2) + \bar{N}_x \left[z_x^2 z_y^2 - \nu (1 + z_x^2 + z_y^2) \right] \right\}$$

$$k = \frac{2}{Eh(1 + z_x^2 + z_y^2)^{\frac{1}{2}}} \left\{ \bar{N}_x z_x z_y (1 + z_x^2) + \bar{N}_{xy} \left[(1 + \nu)(1 + z_x^2 + z_y^2) + 2 z_x^2 z_y^2 \right] + \bar{N}_y z_x z_y (1 + z_y^2) \right\}$$

The final differential equation for the displacement w is:

$$w_{xx} z_{yy} - 2 w_{xy} z_{xy} + w_{yy} z_{xx} = k_{xy} - f_{yy} - g_{xx} = R \qquad (2-4)$$

Since $z_{xy} = 0$ for a translational shell, Equation (2-4) becomes

$$w_{xx}\left(\frac{z_{yy}}{z_{xx}}\right) + w_{yy} = \frac{1}{z_{xx}} R \quad . \tag{2-5}$$

2-3 Edge Conditions Imposed Upon Membrane Displacements by a Shear Diaphragm

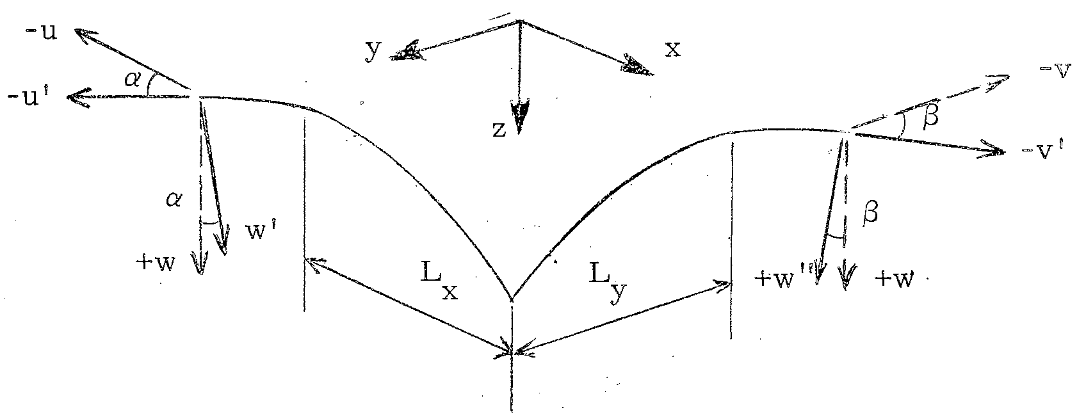

Figure 2-2

Edge Displacements

The shear diaphragm has no rigidity normal to its own plane. Thus, there will be no control imposed on displacement components u along the edges $x = \pm L_x$ nor on displacement components v along edges $y = \pm L_y$.

The boundary condition for w along $x = \pm L_x$ is:

$$w = \frac{1}{z_{yy}} \left[v' (1 + z_y^{\,2})^{\frac{1}{2}} \right]_y - \frac{g}{z_{yy}} \quad . \tag{2-6}$$

Similarly, along $y = \pm L_y$

$$w = \frac{1}{z_{xx}} \left[u' (1 + z_x^{\,2})^{\frac{1}{2}} \right]_x - \frac{f}{z_{xx}} \quad . \tag{2-7}$$

The boundary condition for v along the edges $x = \pm L_x$ is:

$$v = \frac{z_y}{z_{yy}} \left\{ g - z_y \left[\frac{v'}{z_y} (1 + z_y^{\,2})^{\frac{1}{2}} \right]_y \right\} . \tag{2-8}$$

Similarly, along $y = \pm L_y$

$$u = \frac{z_x}{z_{xx}} \left\{ f - z_x \left[\frac{u'}{z_x} (1 - z_x^{\,2})^{\frac{1}{2}} \right]_x \right\} \tag{2-9}$$

where u', v', w', and w'' are deformations tangential and normal, respectively, to the shear diaphragms (Figure 2-2).

The deformations of the edge members due to the shear forces from the shell are neglected, that is $u' = v' = 0$. Equations (2-6, 7, 8, 9) then become

$$w = - \frac{g}{z_{yy}} \tag{2-6a}$$

$$w = - \frac{f}{z_{xx}} \tag{2-7a}$$

$$v = \frac{z_y}{z_{yy}} \, g \tag{2-8a}$$

$$u = \frac{z_x}{z_{xx}} f \quad . \tag{2-9a}$$

CHAPTER III

FINITE DIFFERENCE EQUATIONS AND THEIR SOLUTION

3-1 Introduction

For most shells of double curvature, even for such a simple case as a translational shell, an algebraic solution becomes extremely involved. In such cases, the conversion of the various differential equations into finite-differences equations is more practical. The finite difference equations necessary for the solution of the deformation differential equation of any translational shell are developed in this chapter.

3-2 The Finite Difference Equation for the Vertical Deformation w

The differential equation for the vertical deformation, (Equation 2-5) is:

$$w_{xx}\left(\frac{z_{yy}}{z_{xx}}\right) + w_{yy} = \frac{1}{z_{xx}} R \quad . \tag{2-5a}$$

The finite difference approximation of $\frac{\partial^2 w}{\partial x^2}$ at the point i, j (Figure 3-1) is:

$$\frac{\partial^2 w}{\partial x^2} = \frac{w_{i+1,j} - 2w_{i,j} + w_{i-1,j}}{\Delta x^2} \quad . \tag{3-1}$$

Similarly in the y-direction

$$\frac{\partial^2 w}{\partial y^2} = \frac{w_{i,j+1} - 2w_{i,j} + w_{i,j-1}}{\Delta y^2} \quad . \tag{3-2}$$

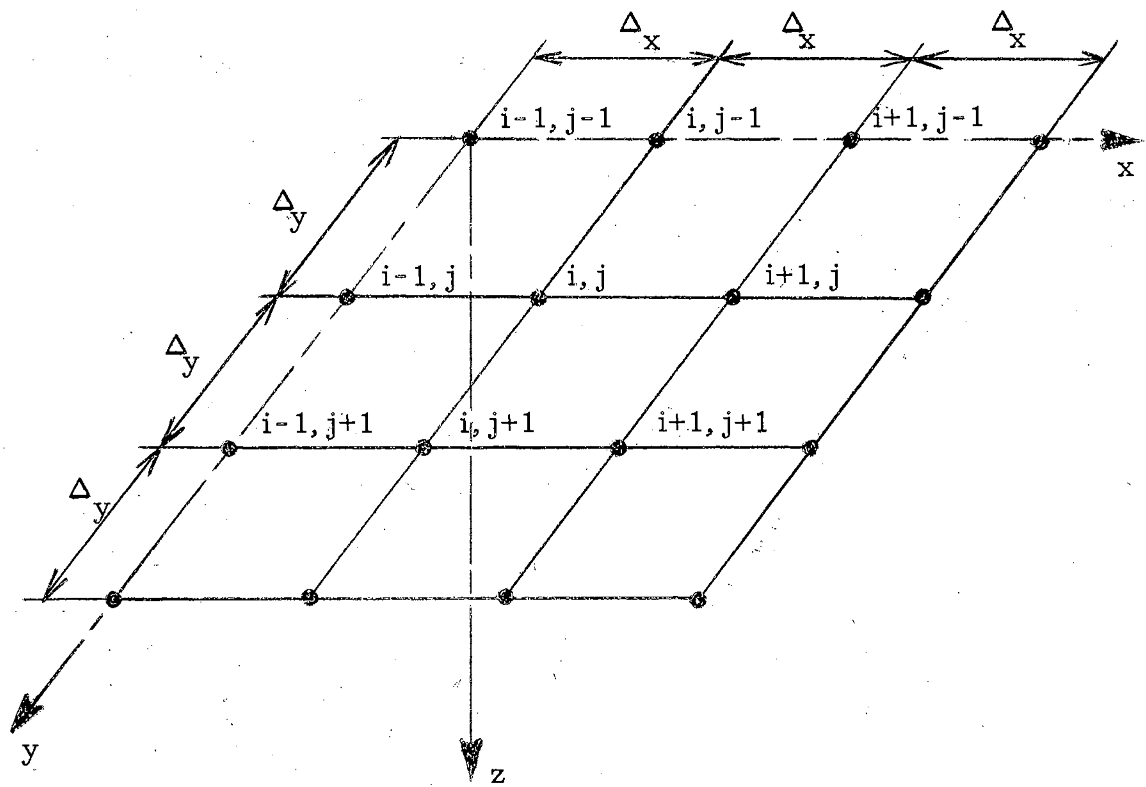

Figure 3-1

Finite Difference Network

Substituting Equations (3-1, 2) into Equation (2-5) results in

$$\frac{w_{i+1,j} - 2w_{i,j} + w_{i-1,j}}{\Delta x^2}\left(\frac{z_{yy}}{z_{xx}}\right) + \frac{w_{i,j+1} - 2w_{i,j} + w_{i,j-1}}{\Delta y^2} = \frac{1}{z_{xx}} R_{ij}$$

(3-3)

In a form suitable for iteration Equation (3-3) becomes

$$w_{i,j} = a_{i+1,ij}\, w_{i+1,j} + a_{i-1,ij}\, w_{i-1,j} + b_{ij}(w_{i,j+1} + w_{i,j-1}) + w^*_{ij} \qquad (3\text{-}4)$$

where

$$t = \frac{\Delta_x}{\Delta_y}$$

$$a_{i+1,ij} = \frac{(z_{yy}/z_{xx})_{ij}}{2(z_{yy}/z_{xx} + t^2)}$$

$$a_{i-1,ij} = \frac{(z_{yy}/z_{xx})_{ij}}{2(z_{yy}/z_{xx} + t^2)}$$

$$b_{ij} = \frac{1}{2(1/t^2 \;\; z_{yy}/z_{xx} + 1)}$$

$$w^*_{ij} = \frac{1}{2(z_{yy}/\Delta_x^{\,2} + z_{xx}/\Delta_y^{\,2})} R_{ij}$$

The value of the deformation w in the z-direction may now be computed at each pivotal point through a system of simultaneous equations of the type (3-4).

3-3 Evaluation of the R Value

According to Equation (2-4), the R value is:

$$R = k_{xy} - f_{yy} - g_{xx}$$

For simplification of the terms k, f, and g the following procedure is followed.

1. The k, f, and g are expanded by power series:

$$(1+x)^k = 1 + kx + \frac{k(k-1)}{2!} x^2 + \cdots$$

2. All fourth order terms in the slopes (z_x^4, z_y^4, $z_x^2 z_y^2$) are neglected in comparison with unity. This assumption, requiring only that z_x and z_y be less than one, was first made by Reissner[(5)].

3. The $\overline{N}_x$ value is replaced by its equivalent value in terms of $\overline{N}_y$ according to Equation (1-4).

4. The load is considered to be uniform ($p_z = p_o$) .

The algebraic quantities k, f, and g then simplify to

$$k = \frac{2}{Eh}\left\{p_o\left[-\frac{z_x z_y}{z_{xx}}\right] + \overline{N}_y\left[-z_x z_y\left(\frac{z_{yy}}{z_{xx}}\right) + z_x z_y\right] + \overline{N}_{xy}\left[1 + \frac{1}{2}z_x^2 + \frac{1}{2}z_y^2 + \nu + \frac{1}{2}\nu z_y^2 + \frac{1}{2}\nu z_x^2\right]\right\} \tag{3-5}$$

$$f = \frac{1}{Eh}\left\{p_o\left[-\frac{1}{z_{xx}} - \frac{3}{2}\frac{z_x^2}{z_{xx}} + \frac{1}{2}\frac{z_y^2}{z_{xx}}\right] + \overline{N}_y\left[-\frac{z_{yy}}{z_{xx}} - \frac{3}{2}z_x^2\left(\frac{z_{yy}}{z_{xx}}\right) + \frac{1}{2}z_y^2\left(\frac{z_{yy}}{z_{xx}}\right) - \nu - \frac{1}{2}\nu z_x^2 - \frac{1}{2}\nu z_y^2\right] + \overline{N}_{xy}\left[2 z_x z_y\right]\right\} \tag{3-6}$$

$$g = \frac{1}{Eh} \left\{ p_o \left[\frac{\nu}{z_{xx}} - \frac{3}{2} \nu \frac{z_x^2}{z_{xx}} - \frac{3}{2} \nu \frac{z_y^2}{z_{xx}} \right] + \overline{N}_{xy} \left[2 z_x z_y \right] \right.$$

$$+ \overline{N}_y \left[\nu \left(\frac{z_{yy}}{z_{xx}} \right) - \frac{3}{2} \nu z_x^2 \left(\frac{z_{yy}}{z_{xx}} \right) - \frac{3}{2} \nu z_y^2 + 1 - \frac{1}{2} z_x^2 \right.$$

$$\left. \left. + \frac{3}{2} z_y^2 \right] + \overline{N}_{xy} \left[2 z_x z_y \right] \right\} . \qquad (3\text{-}7)$$

These quantities must now be differentiated for substitution into the equation for R. Differentiating k with respect to x, then with respect to y, solving Equations (1-2), (1-3), and (1-4) for $\overline{N}_x$ and $\overline{N}_{xy}$ in terms of N_y, and substituting into the differentiated term of k, the following equation is obtained:

$$k_{xy} = \frac{2}{Eh} \left\{ p_o \left[- z_{yy} \right] + \overline{N}_y \left[- z_{yy}^2 + z_{xx} z_{yy} \right] \right.$$

$$+ \frac{\partial \overline{N}_y}{\partial x} \left[- z_x \left(\frac{z_{yy}^2}{z_{xx}} \right) - \nu z_x z_{yy} \right]$$

$$+ \frac{\partial \overline{N}_y}{\partial y} \left[- 2 z_y z_{yy} + z_{xx} z_y - \frac{1}{2} \nu z_y z_{yy} \right]$$

$$+ \frac{\partial^2 \overline{N}_y}{\partial x \partial y} \left[- z_x z_y \left(\frac{z_{yy}}{z_{xx}} \right) + z_x z_y \right]$$

$$\left. + \frac{\partial^2 \overline{N}_y}{\partial y^2} \left[- 1 - \nu - \frac{1}{2} z_x^2 (1 + \nu) - \frac{1}{2} z_y^2 (1 + \nu) \right] \right\}$$

(3-8)

Similarly,

$$f_{yy} = \frac{1}{Eh}\left\{p_o\left[\frac{z_{yy}^2}{z_{xx}}\right] + \overline{N}_y\left[\frac{z_{yy}^3}{z_{xx}} - \nu z_{yy}^2\right]\right.$$

$$+ \frac{\partial \overline{N}_y}{\partial y}\left[2 z_y \left(\frac{z_{yy}^2}{z_{xx}}\right) - 2 \nu z_y z_{yy}\right]$$

$$+ \frac{\partial \overline{N}_y}{\partial x}\left[4 z_x z_{yy}\left(\frac{z_{yy}}{z_{xx}}\right)\right] + \frac{\partial^2 \overline{N}_y}{\partial x \partial y}\left[2 z_x z_y \frac{z_{yy}}{z_{xx}}\right]$$

$$+ \frac{\partial^2 \overline{N}_y}{\partial y^2}\left[- \frac{z_{yy}}{z_{xx}} - \frac{3}{2} z_x^2 \left(\frac{z_{yy}}{z_{xx}}\right) + \frac{1}{2} z_y^2 \left(\frac{z_{yy}}{z_{xx}}\right)\right.$$

$$\left.\left. - \nu - \frac{1}{2} \nu z_x^2 - \frac{1}{2} \nu z_y^2\right]\right\} \qquad (3\text{-}9)$$

and

$$g_{xx} = \frac{1}{Eh}\left\{p_o\left[- 3 \nu z_{xx}\right] + \overline{N}_y\left[- 3 \nu z_{xx} z_{yy} - z_{xx}^2\right]\right.$$

$$+ \frac{\partial \overline{N}_y}{\partial y}\left[- 4 z_{xx} z_y\right]$$

$$+ \frac{\partial \overline{N}_y}{\partial x}\left[- 6 \nu z_x z_{yy} - 2 z_x z_{xx}\right]$$

$$+ \frac{\partial^2 \overline{N}_y}{\partial x \partial y}\left[- 2 z_x z_y\right]$$

$$+ \frac{\partial^2 \overline{N}_y}{\partial x^2}\left[1 + \nu\left(\frac{z_{yy}}{z_{xx}}\right) - \frac{3}{2} z_x^2 \left(\frac{z_{yy}}{z_{xx}}\right)\right.$$

$$\left.\left. - \frac{3}{2} \nu z_y^2 \left(\frac{z_{yy}}{z_{xx}}\right) - \frac{1}{2} z_x^2 + \frac{3}{2} z_y^2\right]\right\} \qquad (3\text{-}10)$$

Equations (3-8, 9, 10), when substituted into Equation (2-4), yield a general expression for the function R in terms of the membrane force $\overline{N}_y$ and its derivatives. Using Equations (3-8, 9, 10) R becomes:

$$R = \frac{1}{Eh}\left\{ p_o \tau_1 + \overline{N}_y \tau_2 + \frac{\partial \overline{N}_y}{\partial x}\tau_3 + \frac{\partial \overline{N}_y}{\partial y}\tau_4 + \frac{\partial^2 \overline{N}_y}{\partial x \partial y}\tau_5 + \frac{\partial^2 \overline{N}_y}{\partial x^2}\tau_6 + \frac{\partial^2 \overline{N}_y}{\partial y^2}\tau_7 \right\} \qquad (3\text{-}11)$$

where the τ values are given in Table (3-1).

3-4. The Finite Difference Scheme for Determining the Value R in Terms of the Internal Forces $\overline{N}_y$

The finite difference approximations for derivatives of the internal force $\overline{N}_y$ at point i, j (Figure 4-1) are:

$$\frac{\partial \overline{N}_y}{\partial x} = \frac{N_{i+1,j} - N_{i-1,j}}{2\Delta_x} \qquad (3\text{-}11)$$

$$\frac{\partial \overline{N}_y}{\partial y} = \frac{N_{i,j+1} - N_{i,j-1}}{2\Delta_y} \qquad (3\text{-}12)$$

$$\frac{\partial^2 \overline{N}_y}{\partial x \partial y} = \frac{N_{i+1,j+1} - N_{i-1,j+1} - N_{i+1,j-1} + N_{i-1,j-1}}{4\Delta_x \Delta_y} \qquad (3\text{-}13)$$

$$\frac{\partial^2 \overline{N}_y}{\partial x^2} = \frac{N_{i+1,j} - 2N_{i,j} + N_{i-1,j}}{\Delta_x^2} \qquad (3\text{-}14)$$

$$\frac{\partial^2 \overline{N}_y}{\partial y^2} = \frac{N_{i,j+1} - 2N_{i,j} + N_{i,j-1}}{\Delta_y^2} \qquad (3\text{-}15)$$

TABLE 3-1 ALGEBRAIC τ VALUES

τ	Algebraic Equivalent
τ_1	$-\ 2\ z_{yy}\ -\ \frac{z_{yy}^2}{z_{xx}}\ +\ 3\ \nu\ z_{xx}$
τ_2	$z_{xx}^2\ +\ z_{xx}\ z_{yy}\ (2\ +\ 3\ \nu)\ +\ z_{yy}^2\ (\nu\ -\ 2)\ -\ \frac{z_{yy}^3}{z_{xx}}$
τ_3	$2\ z_x\ z_{xx}\ +\ 4\ \nu\ z_x\ z_{yy}\ -\ 6\ z_x\ \left(\frac{z_{yy}^2}{z_{xx}}\right)$
τ_4	$6\ z_{xx}\ z_y\ -\ 2\ z_y\ \left(\frac{z_{yy}^2}{z_{xx}}\right)\ +\ z_y\ z_{yy}\ (\nu\ -\ 4)$
τ_5	$4\ z_x\ z_y\quad 1\ -\left(\frac{z_{yy}}{z_{xx}}\right)$
τ_6	$1\ +\ \frac{z_{yy}}{z_{xx}}\ (\frac{3}{2}\ \nu\ z_x^2\ +\ \frac{3}{2}\ \nu\ z_y^2\ -\ \nu)\ +\ \frac{1}{2}\ z_x^2\ -\ \frac{3}{2}\ z_y^2$
τ_7	$-\ 2\ -\ \nu\ +\ \frac{z_{yy}}{z_{xx}}\ +\ z_x^2\ \left(-\ 1\ -\ \frac{1}{2}\ \nu\ +\ \frac{3}{2}\ \frac{z_{yy}}{z_{xx}}\right)\ +\ z_y^2\ \left(-\ 1\ -\ \frac{1}{2}\ \nu\ -\ \frac{1}{2}\ \frac{z_{yy}}{z_{xx}}\right)$

where for convenience the symbol $\overline{N}_y$ has been replaced by the symbol N.

Substituting Equations (3-11, 12, 14, 15) into Equation (3-11) and making some simplifications will result in

$$R_{ij} = \frac{1}{Eh} \left\{ p_o \phi_1^{ij} + N_{ij} \phi_2^{ij} + N_{i+1,j} \phi_3^{ij} + N_{i-1,j} \phi_4^{ij} + N_{i,j+1} \phi_5^{ij} + N_{i,j-1} \phi_6^{ij} + \left[N_{i+1,j+1} - N_{i-1,j+1} - N_{i+1,j-1} + N_{i-1,j-1} \right] \phi_7^{ij} \right\} \qquad (3\text{-}12)$$

where the ϕ values are given in Table (3-2).

The R value at each pivotal point may now be computed by substituting the ϕ and the N values at the pivotal points into Equation (3-12). The N values are either computed by the Pucher[1] solution of the stresses or by the finite difference method of solution of the forces as was done by Tilden[12].

3-5 The Finite Difference Equations for the Horizontal Deformations u and v

Knowing w, the horizontal deformations u and v can be obtained from equations (2-1) and(2-2) as follows.

Considering the case of a symmetrically loaded, symmetrical translational shell, it is evident that the horizontal deformation, u, equals zero on the y-axis as an axis of symmetry for u. In a similar manner the horizontal deformation v is zero on the x-axis.

Equation (2-1) is now written in finite difference form for the point, i + 1, j + 1

Table (3-2) ALGEBRAIC ϕ VALUES

ϕ^{ij}	Algebraic Equivalent
ϕ_1^{ij}	τ_1^{ij}
ϕ_2^{ij}	$\tau_2^{ij} - \frac{2\tau_7^{ij}}{\Delta_y^2} - \frac{2\tau_6^{ij}}{\Delta_x^2}$
ϕ_3^{ij}	$\frac{\tau_3^{ij}}{2\Delta_x} + \frac{\tau_6^{ij}}{\Delta_x^2}$
ϕ_4^{ij}	$-\frac{\tau_3^{ij}}{2\Delta_x} + \frac{\tau_6^{ij}}{\Delta_x^2}$
ϕ_5^{ij}	$\frac{\tau_4^{ij}}{2\Delta_y} + \frac{\tau_7^{ij}}{\Delta_y^2}$
ϕ_6^{ij}	$-\frac{\tau_4^{ij}}{2\Delta_y} + \frac{\tau_7^{ij}}{\Delta_y^2}$
ϕ_7^{ij}	$\frac{\tau_5^{ij}}{4\Delta_x\Delta_y}$

$$\frac{u_{i,j+1} - u_{i+2,j+1}}{2\Delta_x} = \frac{w_{i,j+1} - w_{i+2,j+1}}{2\Delta_x}(-z_x)_{i+1,j+1} + f_{i+1,j+1} \tag{3-13}$$

Considering the values of w known, the value of u at i+2, j+1 can be computed from Equation (4-11) as u at i, j+1 is equal to zero.

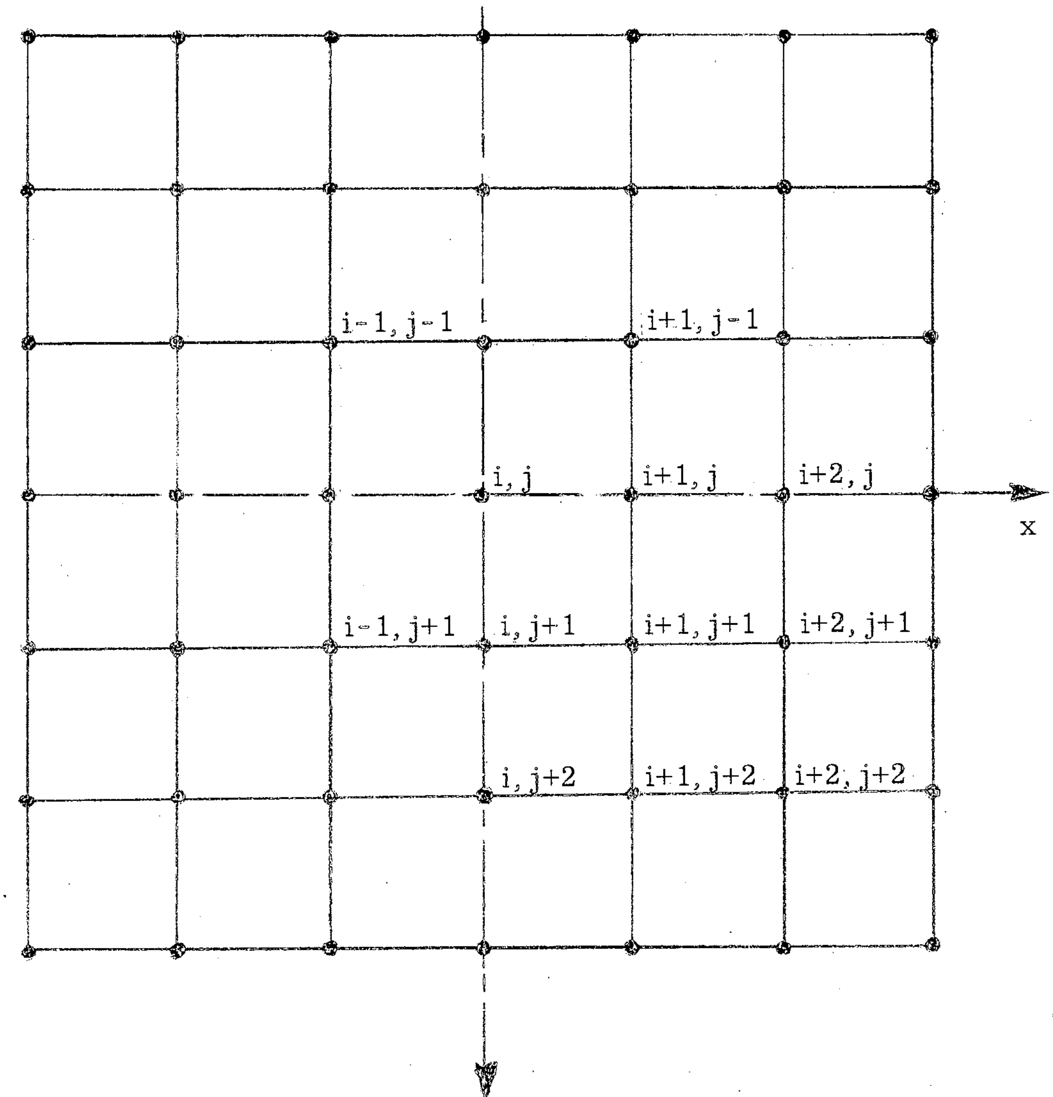

Figure 3-2

Symmetrical Finite Difference Network

Equation (2-1) is now written in finite difference form for the point i, j+1 as follows:

$$\frac{u_{i+1, j+1} - u_{i-1, j+1}}{2\Delta_x} = \frac{w_{i+1, j+1} - w_{i-1, j+1}}{2\Delta_x} (-z_x)_{i, j+1} + f_{i, j+1} \tag{3-14}$$

The right hand side of the above equation is known and the left hand side may be reduced to one unknown by symmetry, that is

$$u_{i+1,\,j+1} = -u_{i-1,\,j+1} \tag{3-15}$$

In a similar manner values of u at all points of the shell may be determined.

Similarly, values of v can be determined by using Equation (2-2) and following the same procedure as for u, resulting in:

At point i+1, j+1

$$\frac{v_{i+1,\,j+2} - v_{i+1,\,j}}{2\Delta_y} = \frac{w_{i+1,\,j+2} - w_{i+1,\,j}}{2\Delta_y}(-z_y)_{i+1,\,j+1} + g_{i+1,\,j+1} \tag{3-16}$$

At point i+1, j

$$\frac{v_{i+1,\,j+1} - v_{i+1,\,j-1}}{2\Delta_y} = \frac{w_{i+1,\,j+1} - w_{i+1,\,j-1}}{2\Delta_y}(-z_y)_{i+1,\,j} + g_{i+1,\,j} \tag{3-17}$$

When the shell is subjected to an unsymmetrical load, the horizontal deformations u, and v are no longer zero on the axis of symmetry. There are no starting points on the shell that can be used for the elemination of unknowns. Sufficient equations of the type (3-13, 16) are available to obtain a solution if forward or backward differences are used.

3-6 Solution of the Finite Difference Equations

A. Methods of Solution -

For a discussion on the methods of solution the reader is referred to Tilden[(11)]. His discussion on the solution of the internal forces is equally applicable to the deformations.

B. The Network -

Considering the case of the symmetrically loaded, symmetrical translational shell, the carry-over factors and deformations are symmetrical to both the x and y axes. By taking advantage of this symmetry, a twenty five point network (Figure 3-3) covering only one quadrant of the shell has been chosen for the analysis.

Carry-over factors that contribute to final values on the axis of symmetry are modified. As an example the point 1 (Figure 3-3) receives contributions of $2a_{2,1}\ w_2$ due to the fact that w_2 and $a_{2,1}$ have equivalent values, respectively, in the opposite symmetrical quadrant.

C. Reduction of the Network from Boundary Conditions -

From Equations (2-6a) and (2-7a), it is noted that w is a prescribed value at points 5, 15, 20, 21, 22, 23 and 24:

$$w = -\frac{g}{z_{yy}} \qquad \text{at } x = +L_y \qquad (2\text{-}6a)$$

$$w = -\frac{f}{z_{xx}} \qquad \text{at } y = +L_x \qquad (2\text{-}7a)$$

Since the value is prescribed, iteration can in no way effect w at points 5, 15, 20, 21, 22, 23 and 24. Therefore it follows that all carry-over factors to these points must equal to zero. That is

$$a_{4,5} = a_{9,10} = a_{14,15} = a_{19,20} = 2b_5 = b_{10} = b_{15} = b_{20} = 0 \qquad (3\text{-}18)$$

$$b_{21} = b_{22} = b_{23} = b_{24} = 2a_{22,21} = a_{23,22} = a_{24,23} = a_{25,24} = 0 \qquad (3\text{-}19)$$

From Equation (3-4), the expression for w_4 may be written

$$w_4 = a_{5,4}\ w_5 + a_{3,4}\ w_3 + 2b_4\ w_9 + w^*_4 \qquad (3\text{-}4a)$$

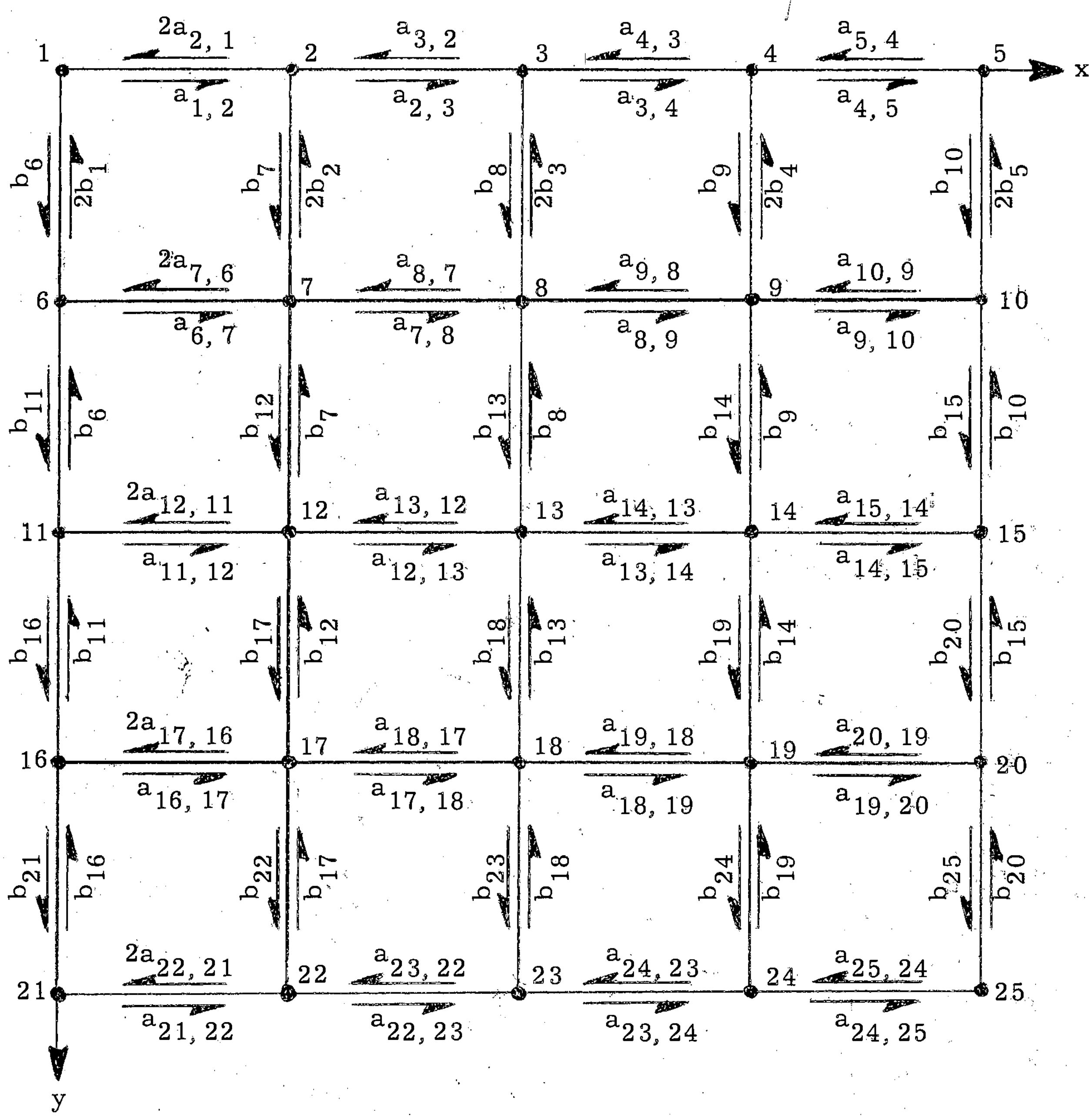

Figure 3-3

Twenty-Five Point Network

where $a_{5,4}\, w_5$ is a known value similar in form to the starting value, w^*_4. Denoting the carried-over starting value as λ, the values at points 4, 9, 14, 16. 17, 18 and 19 become

$$\begin{aligned} \lambda_4 &= a_{5,4}\, w_5 \\ \lambda_9 &= a_{10,9}\, w_{10} \\ \lambda_{14} &= a_{15,14}\, w_{15} \\ \lambda_{19} &= a_{20,19}\, w_{20} \end{aligned} \qquad (3\text{-}20)$$

$$\begin{aligned} \lambda_{16} &= b_{16}\, w_{21} \\ \lambda_{17} &= b_{17}\, w_{22} \\ \lambda_{18} &= b_{18}\, w_{23} \\ \lambda_{19} &= b_{19}\, w_{24} \end{aligned} \qquad (3\text{-}21)$$

The twenty five-point network has been reduced to a sixteen-point network as shown in Figure (3-4).

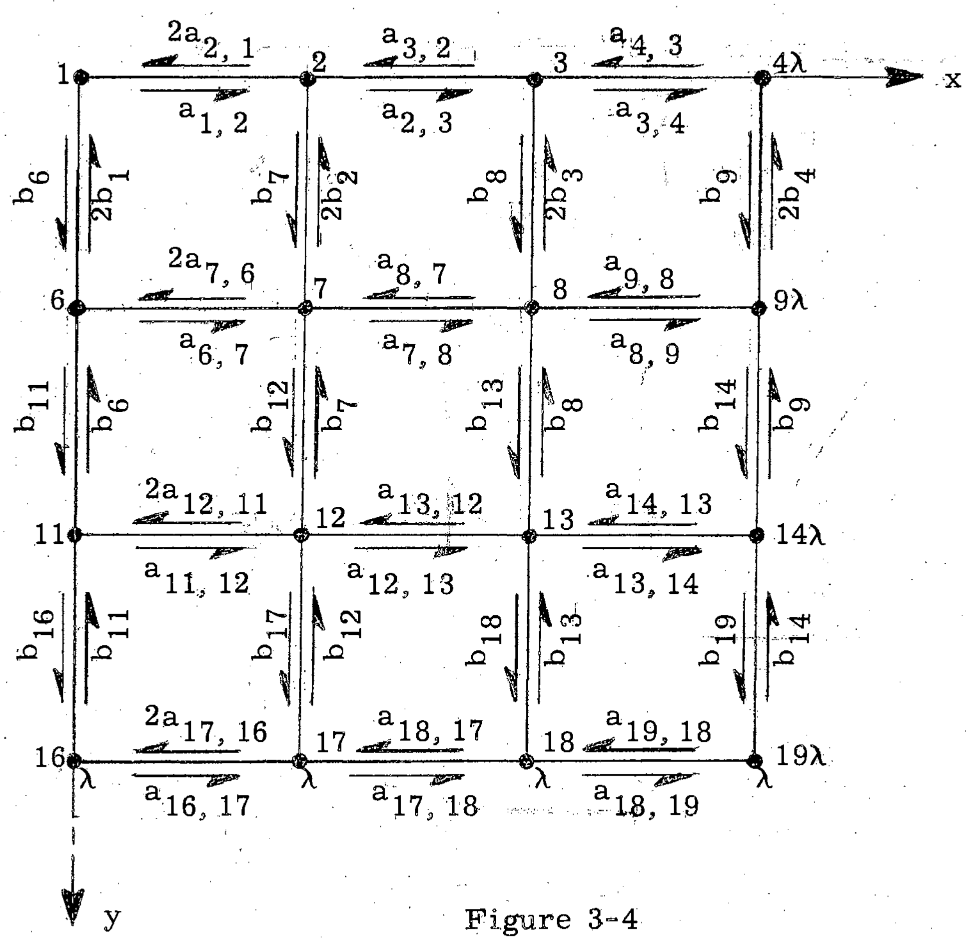

Figure 3-4

Sixteen-Point Network

CHAPTER IV

NUMERICAL EXAMPLE

It is required to compute the deformations w, u, and v for the elliptical paraboloid shell shown in Figure (4-1). The shell is subjected to a uniform load p_o. The edges of the shell are supported by shear diaphragms that are denied all rigidity transverse to their planes. The Poisson's ratio is considered to be zero. The values of the internal forces $\overline{N}_y$, $\overline{N}_x$, and $\overline{N}_{xy}$ have been computed by the use of tables and equations presented by Parme[6]. Thus,

$$\overline{N}_x = - 4 p_o L \text{ (Coeff.)}$$

$$\overline{N}_y = - 4 p_o L \text{ (Coeff.)}$$

$$\overline{N}_{xy} = - 4 p_o L \text{ (Coeff.)}$$

where the coefficients are as given in Reference 6.

The equation for the elliptical paraboloid shown in Figure (4-1) is:

$$z = \frac{h_x}{L_x^2} x^2 + \frac{h_y}{L_y^2} y^2$$

Substituting the known quantities, the slopes of the shell in the x and y directions are:

$$z_x = \frac{1}{2L} x \qquad z_y = \frac{1}{2L} y$$

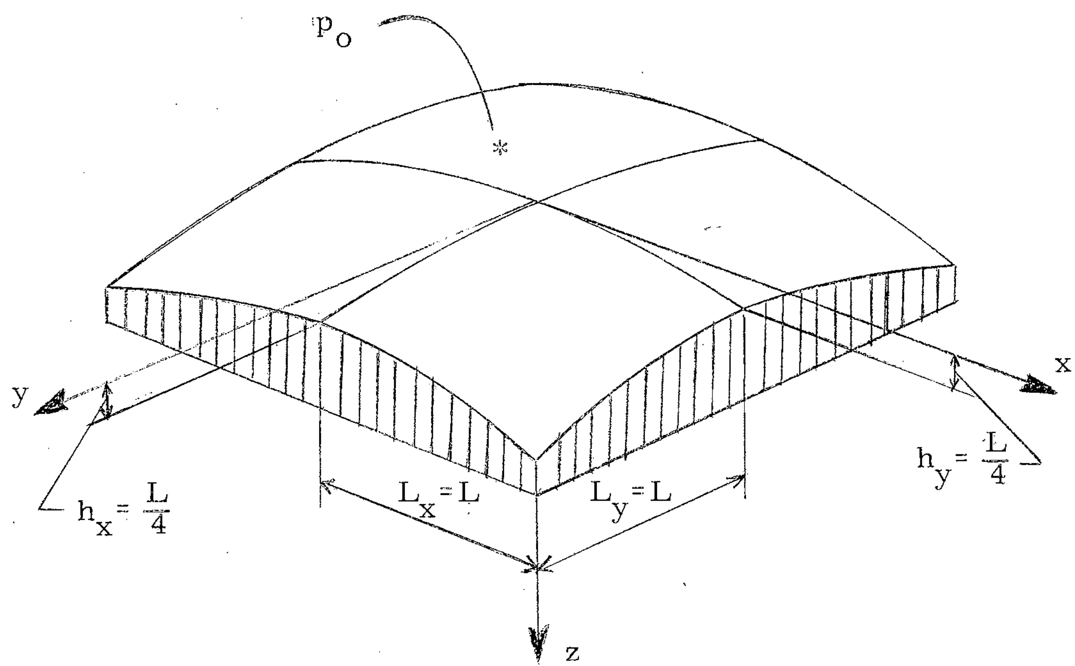

Figure 4-1

Elliptical Paraboloid

From Equations (1-6) and (1-7)

$$z_{xx} = \frac{1}{2L} \qquad z_{yy} = \frac{1}{2L}$$

From Equation (3-4)

$$a = \frac{(1/2\,L)}{(1/2\,L)} \div 2 \; \frac{1/2\,L}{1/2\,L} + t^2$$

$$b = 1 \div 2 \; \frac{1}{t^2} \; \frac{1/2\,L}{1/2\,L} + 1$$

Choosing a 25 pivotal point network

$$\Delta_x = \frac{L}{4}, \qquad \Delta_y = \frac{L}{4}, \qquad t = 1$$

then

$$a = 1 \div 2(1 + 1) = 0.2500$$

$$b = 1 \div 2(1 + 1) = 0.2500$$

It is noted that for the elliptical paraboloid the carry-over factors, a and b, are constant over the domain of the shell.

In order to determine the starting value w^*_{ij}, and the carry-over starting value λ, the following procedure is followed.

Since the elliptical paraboloid is a square, the vertical deformations are symmetrical about the diagonal of the shell. Therefore the vertical deformations are calculated for one octant of the shell only.

Using Equation (3-6) and substituting the known quantities at the points 1, 2, 3, 4, 6, 7, 8, 9, 11, 12, 13, 14, 16, 17, 18, and 19 will result in the f values given in Table (4-1).

Using Equation (3-7) and substituting the known quantities at the points 5, 10, 15, and 20 will result in the g values given in Table (4-2).

Using Table (3-1) and substituting the known quantities at the points 1, 2, 3, 4, 7, 8, 9, 13, 14, and 19 will result in the τ values given in Table (4-3).

Using Table (3-2) and substituting the known quantities at points 1, 2, 3, 4, 7, 8, 9, 13, 14 and 19 will result in the ϕ values given in Table (4-4).

Table 4-1 NUMERICAL f VALUES

Pivotal Point	f
1	$-1.0000 \frac{p_o L}{Eh}$
2	$-0.9541 \frac{p_o L}{Eh}$
3	$-0.7963 \frac{p_o L}{Eh}$
4	$-0.4893 \frac{p_o L}{Eh}$
6	$-1.0596 \frac{p_o L}{Eh}$
7	$-1.0193 \frac{p_o L}{Eh}$
8	$-0.8817 \frac{p_o L}{Eh}$
9	$-0.5703 \frac{p_o L}{Eh}$
11	$-1.2323 \frac{p_o L}{Eh}$
12	$-1.2117 \frac{p_o L}{Eh}$
13	$-1.1325 \frac{p_o L}{Eh}$
14	$-0.9091 \frac{p_o L}{Eh}$
16	$-1.4838 \frac{p_o L}{Eh}$
17	$-1.5191 \frac{p_o L}{Eh}$
18	$-1.5340 \frac{p_o L}{Eh}$
19	$-1.5410 \frac{p_o L}{Eh}$

TABLE 4-2 NUMERICAL g VALUES	
Pivotal Point	g
5	$-1.7850 \frac{p_o L}{Eh}$
10	$-1.8510 \frac{p_o L}{Eh}$
15	$-2.1800 \frac{p_o L}{Eh}$
20	$-2.8690 \frac{p_o L}{Eh}$

TABLE 4-3 NUMERICAL τ VALUES

Pivotal Point	τ_1	τ_2	τ_3	τ_4	τ_5	τ_6	τ_7
1	$-\frac{1.5000}{L}$	0	0	0	-1.0000	-1.0000	0
2	$-\frac{1.5000}{L}$	0	$-\frac{0.2500}{L}$	0	-0.9922	-0.9922	0
3	$-\frac{1.5000}{L}$	0	$-\frac{0.5000}{L}$	0	-0.9688	-0.9688	0
4	$-\frac{1.5000}{L}$	0	$-\frac{0.7500}{L}$	0	-0.9297	-0.9297	0
7	$-\frac{1.5000}{L}$	0	$-\frac{0.2500}{L}$	$+\frac{0.0625}{L}$	-1.0156	-1.0156	0
8	$-\frac{1.5000}{L}$	0	$-\frac{0.5000}{L}$	$+\frac{0.0625}{L}$	-0.9922	-0.9922	0
9	$-\frac{1.5000}{L}$	0	$-\frac{0.7500}{L}$	$+\frac{0.0625}{L}$	-0.9531	-0.9531	0
13	$-\frac{1.5000}{L}$	0	$-\frac{0.5000}{L}$	$+\frac{0.1250}{L}$	-1.0625	-1.0625	0
14	$-\frac{1.5000}{L}$	0	$-\frac{0.7500}{L}$	$+\frac{0.1250}{L}$	-1.0234	-1.0234	0
19	$-\frac{1.5000}{L}$	0	$-\frac{0.7500}{L}$	$+\frac{0.1875}{L}$	-1.1406	-1.1406	0

TABLE 4-4 ϕ VALUES

Pivotal Point	ϕ_1	ϕ_2	ϕ_3	ϕ_4	ϕ_5	ϕ_6	ϕ_7
1	$-\frac{1.5000}{L}$	$+\frac{64.000}{L^2}$	$-\frac{16.000}{L^2}$	$-\frac{16.000}{L^2}$	$-\frac{16.000}{L^2}$	$-\frac{16.000}{L^2}$	0
2	$-\frac{1.5000}{L}$	$+\frac{63.500}{L^2}$	$-\frac{16.375}{L^2}$	$-\frac{15.375}{L^2}$	$-\frac{15.875}{L^2}$	$-\frac{15.875}{L^2}$	0
3	$-\frac{1.5000}{L}$	$+\frac{62.000}{L^2}$	$-\frac{16.500}{L^2}$	$-\frac{14.500}{L^2}$	$-\frac{15.500}{L^2}$	$-\frac{15.500}{L^2}$	0
4	$-\frac{1.5000}{L}$	$+\frac{59.500}{L^2}$	$-\frac{16.375}{L^2}$	$-\frac{13.375}{L^2}$	$-\frac{14.875}{L^2}$	$-\frac{14.875}{L^2}$	0
7	$-\frac{1.5000}{L}$	$+\frac{65.000}{L^2}$	$-\frac{16.750}{L^2}$	$-\frac{15.750}{L^2}$	$-\frac{16.125}{L^2}$	$-\frac{16.375}{L^2}$	0
8	$-\frac{1.5000}{L}$	$+\frac{63.500}{L^2}$	$-\frac{16.875}{L^2}$	$-\frac{14.875}{L^2}$	$-\frac{15.750}{L^2}$	$-\frac{16.000}{L^2}$	0
9	$-\frac{1.5000}{L}$	$+\frac{61.000}{L^2}$	$-\frac{16.750}{L^2}$	$-\frac{13.750}{L^2}$	$-\frac{15.125}{L^2}$	$-\frac{15.375}{L^2}$	0
13	$-\frac{1.5000}{L}$	$+\frac{68.000}{L^2}$	$-\frac{18.000}{L^2}$	$-\frac{16.000}{L^2}$	$-\frac{16.750}{L^2}$	$-\frac{17.250}{L^2}$	0
14	$-\frac{1.5000}{L}$	$+\frac{65.500}{L^2}$	$-\frac{17.875}{L^2}$	$-\frac{14.875}{L^2}$	$-\frac{16.125}{L^2}$	$-\frac{16.625}{L^2}$	0
19	$-\frac{1.5000}{L}$	$+\frac{73.000}{L^2}$	$-\frac{19.750}{L^2}$	$-\frac{16.750}{L^2}$	$-\frac{17.875}{L^2}$	$-\frac{18.625}{L^2}$	0

Using Equation (3-12) and substituting the known quantities at points 1, 2, 3, 4, 7, 8, 9, 13, 14, and 19 will result in the R values given in Table (4-5).

TABLE 4-5 NUMERICAL R VALUES

Pivotal Point	R
1	$-1.5000 \frac{p_o}{EhL}$
2	$-1.3648 \frac{p_o}{EhL}$
3	$-1.2200 \frac{p_o}{EhL}$
4	$-0.4088 \frac{p_o}{EhL}$
7	$-5.9900 \frac{p_o}{EhL}$
8	$-0.7195 \frac{p_o}{EhL}$
9	$-0.6482 \frac{p_o}{EhL}$
13	$-0.7450 \frac{p_o}{EhL}$
14	$-0.5815 \frac{p_o}{EhL}$
19	$+1.1250 \frac{p_o}{EhL}$

The starting value w^*_{ij} is computed from Equation (3-4) by substituting the known quantities of R at each pivotal point according to Table (4-5). The w^*_{ij} values for the points 1, 2, 3, 4, 7, 8, 9, 13, 14, and 19 are given in Table (4-6).

TABLE 4-6 NUMERICAL w^*_{ij} VALUES

Pivotal Point	w^*_{ij}
1	$-\ 0.0469\ \frac{p_o L^2}{Eh}$
2	$-\ 0.0427\ \frac{p_o L^2}{Eh}$
3	$-\ 0.0381\ \frac{p_o L^2}{Eh}$
4	$-\ 0.0128\ \frac{p_o L^2}{Eh}$
7	$-\ 0.1872\ \frac{p_o L^2}{Eh}$
8	$-\ 0.0225\ \frac{p_o L^2}{Eh}$
9	$-\ 0.0203\ \frac{p_o L^2}{Eh}$
13	$-\ 0.0233\ \frac{p_o L^2}{Eh}$
14	$-\ 0.0182\ \frac{p_o L^2}{Eh}$
19	$+\ 0.0352\ \frac{p_o L^2}{Eh}$

The boundary condition for w along the line $x = +L$ is: according to Equation (2-6a),

$$w = -\frac{g}{z_{yy}} .$$

Therefore, at points 5, 10, 15, and 20, w is as given in Table (4-7).

TABLE 4-7 NUMERICAL w VALUES ALONG THE EDGE $x = +L$

Pivotal Point	w
5	$+ 3.5000 \frac{p_o L^2}{Eh}$
10	$+ 3.7020 \frac{p_o L^2}{Eh}$
15	$+ 4.3600 \frac{p_o L^2}{Eh}$
20	$+ 5.7380 \frac{p_o L^2}{Eh}$

Using Equations (3-20, 21) and substituting the quantities of w at points 4, 9, 14, and 19 will result in the starting values (λ) given in Table 4-8.

Note that 19 has $2\lambda_{19}$ due to the fact that it is a corner point and $w_{20} = w_{24}$.

Writing the carry-over Equations of the type (3-4) for the points 1, 2, 3, 4, 7, 8, 9, 13, 14 and 19 by using the known quantities found in Tables (4, 6, 8) will result in the matrix equation (4-1).

Solving this matrix equation for w, the values corresponding to one octant of the shell are obtained:

$$w = (\text{Coeff.}) \frac{p_o L^2}{Eh}$$

$$
\begin{bmatrix}
-1 & +1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
+0.2500 & -1 & +0.2500 & 0 & +0.5000 & 0 & 0 & 0 & 0 & 0 \\
0 & +0.2500 & -1 & +0.2500 & 0 & +0.5000 & 0 & 0 & 0 & 0 \\
0 & 0 & +0.2500 & -1 & 0 & 0 & +0.5000 & 0 & 0 & 0 \\
0 & +0.5000 & 0 & 0 & -1 & +0.5000 & 0 & 0 & 0 & 0 \\
0 & 0 & +0.2500 & 0 & +0.2500 & -1 & +0.2500 & +0.2500 & 0 & 0 \\
0 & 0 & 0 & +0.2500 & 0 & +0.2500 & -1 & 0 & +0.2500 & 0 \\
0 & 0 & 0 & 0 & 0 & +0.5000 & 0 & -1 & +0.5000 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & +0.2500 & +0.2500 & -1 & +0.2500 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & +0.5000 & -1
\end{bmatrix}
\begin{bmatrix}
w_1 \\ w_2 \\ w_3 \\ w_4 \\ w_7 \\ w_8 \\ w_9 \\ w_{13} \\ w_{14} \\ w_{19}
\end{bmatrix}
=
\begin{bmatrix}
+0.0469 \dfrac{p_o L^2}{Eh} \\
+0.0427 \dfrac{p_o L^2}{Eh} \\
+0.0381 \dfrac{p_o L^2}{Eh} \\
-0.8622 \dfrac{p_o L^2}{Eh} \\
+0.1872 \dfrac{p_o L^2}{Eh} \\
+0.0225 \dfrac{p_o L^2}{Eh} \\
-0.9053 \dfrac{p_o L^2}{Eh} \\
+0.0233 \dfrac{p_o L^2}{Eh} \\
-1.0718 \dfrac{p_o L^2}{Eh} \\
-2.9042 \dfrac{p_o L^2}{Eh}
\end{bmatrix}
\qquad (4\text{-}1)
$$

where the coefficients are given in Table 4-9. These values are shown graphically in Figure 4-1.

TABLE 4-8 NUMERICAL λ VALUES

Pivotal Point	λ
4	$+ 0.8750 \frac{p_o L^2}{Eh}$
9	$+ 0.9255 \frac{p_o L^2}{Eh}$
14	$+ 1.0900 \frac{p_o L^2}{Eh}$
19	$+ 2.8690 \frac{p_o L^2}{Eh}$

TABLE 4-9 COEFFICIENT VALUES OF w IN ONE OCTANT OF THE SHELL

	x/L				
y/L	0	0.25	0.50	0.75	1.00
0	2.7195	2.7664	3.0578	3.4927	3.5000
0.25		2.7289	3.0635	3.6372	3.7020
0.50			3.8393	4.2177	4.3600
0.75				5.0131	5.7380
1.00					infinite

The general results can be compared with those from the uniformly loaded cylindrical shell. In each case the membrane solution is not adequate to develop the desired boundary conditions of zero displacements w along the edges.

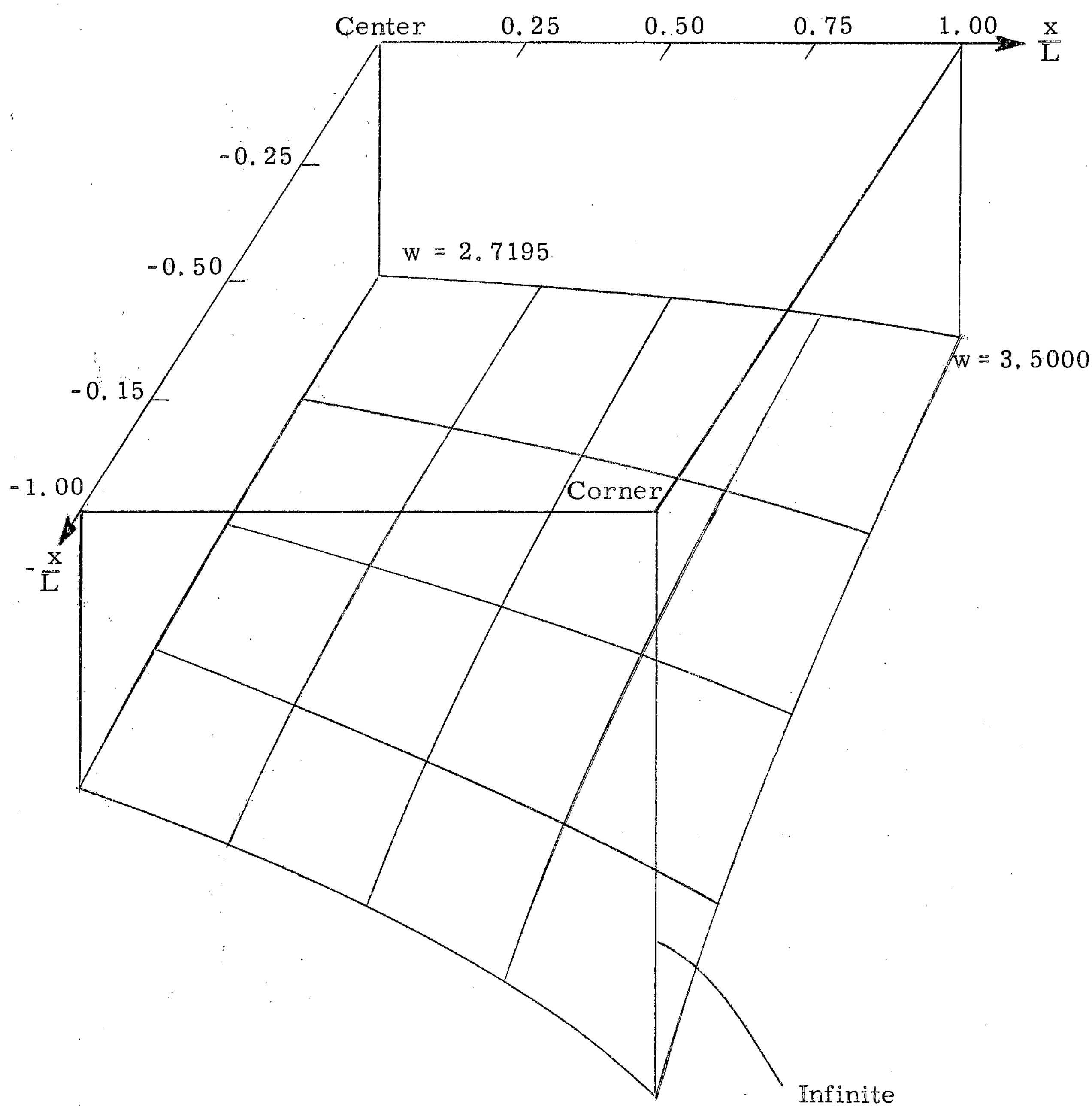

Figure 4-2

Deflections w in One Quadrant of the Shell

Having the values for w at all points, the displacements u and v must now be computed. u is determined according to the following procedure:

1. At points 1, 6, 11, 16, and 21 u is zero, and at points 21, 22, 23, and 24, the boundary conditions dictate the values of u (Equation 2-9a).
2. Using Equations of the type (3-14), and substituting the known values of w and f, u is determined for the points 2, 7, 12, and 17.
3. In a similar manner and by the use of Equation (3-13), u is determined for points 3, 5, 8, 9, 10, 13, 14, 15, 18, 19, and 20.

Table (4-10) gives the u values over the entire domain of the first quadrant.

TABLE 4-10 COEFFICIENT VALUES OF u IN ONE QUADRANT OF THE SHELL

	x/L				
y/L	0	0.25	0.50	0.75	1.00
0	0	-0.2500	-0.5193	-0.8298	-0.9298
0.25	0	-0.2649	-0.5468	-0.9329	-1.0714
0.50	0	-0.3081	-0.7036	-1.1630	-1.3535
0.75	0	-0.3710	-0.8502	-1.4820	-2.1908
1.00	0	-0.4628	-1.0905	-2.1518	infinite

Since u and v correspond to each other by symmetry, a table for v is obtained by transposing Table (4-10) about the diagonal of the shell.

Figure (4-2) shows the variation of the values for u over the domain of the first quadrant.

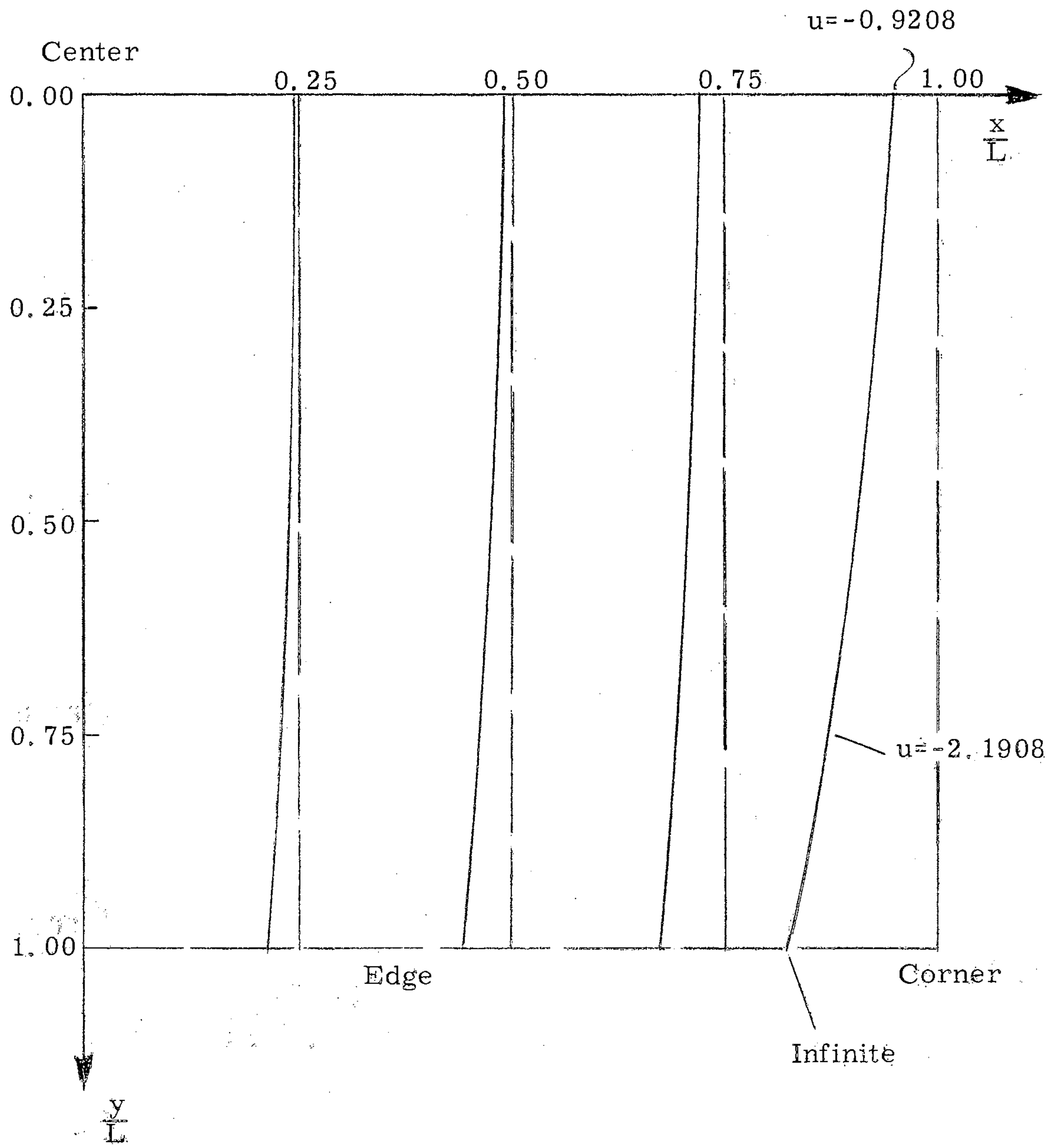

Figure 4-3

Deflections u in One Quadrant of the Shell

CHAPTER V

SUMMARY AND CONCLUSIONS

5-1 Summary

A systematic finite difference procedure for the solution of membrane deformations of translational shells is presented. The solution is accomplished through the following steps.

1. The finite difference equation for the vertical deformation w of the general shell of translation is formulated in terms of the value R.
2. The term R is simplified through the following procedure:
   a. The algebraic terms k, f, and g of R are expanded by power series, and all fourth order powers of the slopes are neglected in comparison with unity.
   b. The final value of R is expressed in terms of the membrane forces $\overline{N}_y$ only.
   c. The difference scheme of R is obtained for its evaluation at any point on the shell.
3. The finite difference equations for the horizontal deformations u and v in terms of the vertical deformations w are obtained.
4. A basic difference network of twenty-five pivotal points for one quadrant is chosen.
5. The twenty-five points are reduced to sixteen utilizing boundary

conditions and carry-over methods. The numerical solution is performed by matrix inversion.

6. The membrane deformations w, u, and v are found for an elliptical paraboloid of specific dimensions.

5-2 Conclusions

The systematic finite difference procedure outlined in this report makes the solution of the membrane deformations of translational shells a simple and straightforward process.

The simplification of the term R eliminates polynomial or higher order difference approximations in the numerical differtiations of the algebraic terms k, f, and g. Rather, the differentiation of these terms is carried out analytically. Moreover, R is evaluated through the solution of the $\overline{N}_y$ membrane forces only.

The application of the method presented in this report to a specific elliptical paraboloid resulted in a straight forward solution without any complex mathmatical operations.

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VITA

Samir Albert Lawrence

Candidate for the Degree of

Master of Science

Report: MEMBRANE DEFORMATIONS OF TRANSLATIONAL SHELLS BY FINITE DIFFERENCES

Major Field: Civil Engineering

Biographical:

Personal Data: Born October 24, 1938, in Baghdad, Iraq, the son of Albert and Esparance Lawrence

Education: Graduated from Baghdad College High School, Baghdad, Iraq, in June, 1956. Received the degree of Bachelor of Science in Engineering Physics from Al-Hikma University of Baghdad, June, 1960. Completed the requirements for the Master of Science degree in May, 1963.