AN ANALYSIS AND BRIEF DESIGN OF TWO SPAN REINFORCED CONCRETE GABLE FRAME

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Scope of Study: The analysis and brief design of a two span reinforced concrete gable frame by the elastic and the concept of limit design approaches are presented in this report. The procedure of analysis for each of these methods is outlined and the design procedures for typical sections are presented. The elastic analysis is accomplished by using "Moment Coefficient Tables," and the "Plastic Hinge Theory for Reinforced Concrete Frames" is applied in the limit design approach. Finally, cross-sectional dimensions and the corresponding steel requirements for several critical sections are shown and compared.

Findings and Conclusions: It is shown that for a two span frame the plastic method of approach offers a simpler solution and eliminates the solution of simultaneous equations, which is a necessary step in the conventional elastic method of analysis, and would become a cumbersome and quite tedious task as the number of redundants of an indeterminate structure increases. The writer feels inclined to recommend the careful adoption of the limit design approach for the analysis of indeterminate reinforced concrete structures with a higher degree of indeterminacy.

ADVISER'S APPROVAL Roger L. Flanders

AN ANALYSIS AND BRIEF DESIGN OF TWO SPAN

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CHAPTER I

INTRODUCTION

The rigid frame structure is becoming more popular than the column-truss combination structure. The reason for this general trend of development is that they provide a greater free space under certain specified conditions and result in a more pleasing appearance and over-all economy. In some parts of the world where structural steel is either very expensive or not readily obtainable, and the locality offers plenty of aggregates and cheap labor, reinforced concrete rigid gable frame structures are more frequently adopted than that of steel, with advantages which result in greater economy. Realizing the practical importance of reinforced concrete gable frames in structural engineering, two different methods for the analysis of a two-span pinned base gable frame and brief designs of the same frame are presented by the writer.

The elastic method of analysis, using the moment coefficents prepared by Gillespie (1) and Hale (2), is described in Chapter II. The plastic method of analysis, based on the plastic hinge theory for reinforced concrete frames developed by Baker (3), is briefly introduced in Chapter III and with its application to the same structure shown in Chapter

IV. In Chapter V, brief designs of critical sections are presented and finally, the results of design are summarized and compared in Chapter VI.

In the analysis by the elastic or conventional method, the analysis is greatly facilitated by the use of moment coefficients for continuous gable frames. Since the method is based on Hooke's Law, a large portion of the structure is understressed, resulting in uneconomical use of material. This is particularly true for a statically indeterminate structure.

The analysis by plastic hinge theory of reinforced concrete is based upon the plastic behavior of both concrete and reinforcing steel bars after yield stresses of both materials are reached. The design is dependent on the ultimate load a structure will support. Besides those three necessary conditions for the plastic analysis of steel structure, i.e., (1) equilibrium condition, (2) collapse condition and (3) yield condition, the rotation capacity of the concrete should be investigated in detail in order to ensure the simultaneous formation of all the necessary hinges as required by a collapse mechanism and prevent any undesirable sudden failure of the structure.

Since the method of analysis by plastic theory recognizes the redistribution of stresses during the period of occurrence of first yield in a certain highly stressed portion of a structure and the formation of n + 1 plastic hinges for an n times statically indeterminate structure, more economical use of material can be achieved than by the conventional method of analysis. Also, for a structure with a large number of redundants, the approach by plastic theory seems to offer a simpler solution.

CHAPTER II

ELASTIC ANALYSIS

2-1. General

A two span pin based reinforced concrete gable frame is analyzed by the method of moment distribution and the adoption of "The Moment Coefficient Tables." The frame has three different values of the ratio β of the rise in gable to the span length. Spacing of frames is 20 ft. on centers and each span is 60 ft., the column height is 20 ft. Sections are considered constant through the analysis for all members. A set of three different gravity loads, including dead weight of members, is assumed. The wind load is 30 pounds per vertical square foot. A general sketch of the frame and its loading is shown below.

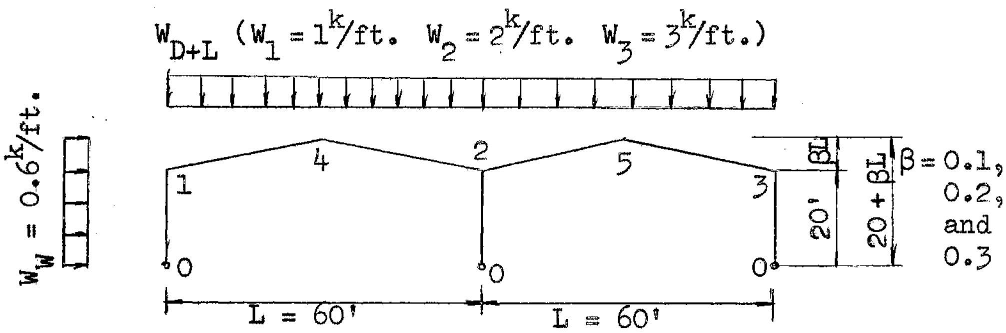


Figure 2-1. Two Span Gable Frame

2-2. Analysis by Moment Coefficient Tables

In this elastic analysis, the moment distribution method, with the aid of the moment coefficient tables, is followed. Two cases of loading are considered in the analysis, and by superposing the results of those two cases, the moments for the third case of loading are obtained. Those cases of loading are shown in the following:

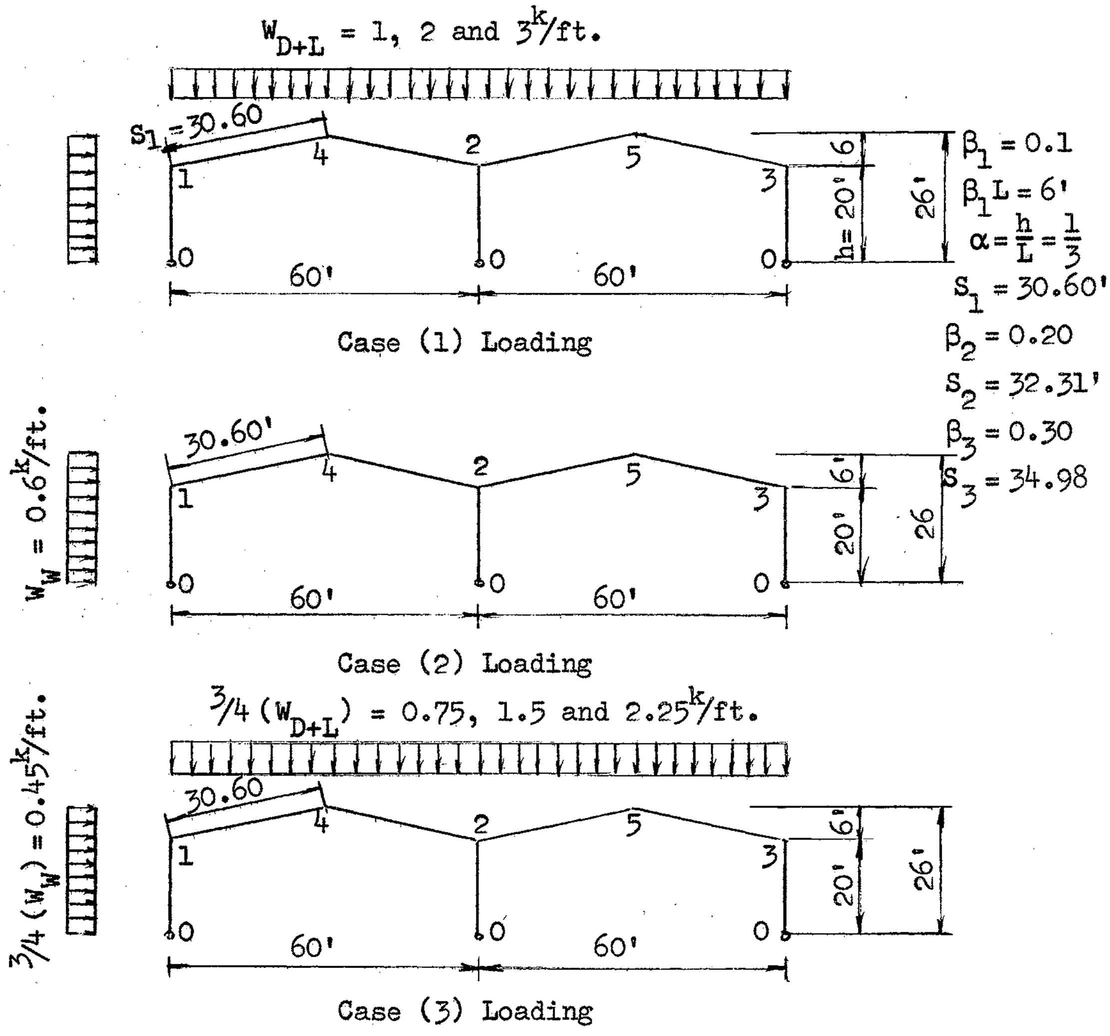


Figure 2-2. Cases of Loading

(A) Analysis of Case (1) Loading

1. For parameters $\alpha = 1/3 \quad \beta = \beta_1 = 0.10$

(a) Values of $(W_{D+L})_i L^2 \quad i = 1, 2, \text{ and } 3$

$W_1 = 1 \text{ k/ft.} \qquad W_1 L^2 = 3600 \text{ k-ft.}$

(b) End Moment Coefficients:

$$\begin{bmatrix} M_{12} \\ M_{21} \\ \\ M_{23} \\ M_{32} \end{bmatrix} = \begin{bmatrix} -Q_{12} \\ +Q_{21} \\ \\ -Q_{21} \\ +Q_{12} \end{bmatrix} (W_1 L^2) = \begin{bmatrix} -.060563 \\ +.080777 \\ \\ -.080777 \\ +.060563 \end{bmatrix} (3600)$$

(c) End Moments for $W_1 = 1 \text{ k/ft.}$

$M_{12} = -218.20^{k\text{-}ft} \quad M_{21} = +290.79^{k\text{-}ft} \quad M_{23} = -290.79^{k\text{-}ft}$

$M_{32} = +218.20^{k\text{-}ft}$

(d) End Moments for $W_2 = 2 \text{ k/ft.}$ and $W_3 = 3 \text{ k/ft.}$

For $W_2 = 2 \text{ k/ft.}$

$M_{12} = -436.40^{k\text{-}ft} \quad M_{21} = +581.58^{k\text{-}ft} \quad M_{23} = -581.58^{k\text{-}ft}$

$M_{32} = +436.40^{k\text{-}ft}$

For $W_3 = 3 \text{ k/ft.}$

$M_{12} = -654.60^{k\text{-}ft} \quad M_{21} = +872.37^{k\text{-}ft} \quad M_{23} = -872.37^{k\text{-}ft}$

$M_{32} = +654.60^{k\text{-}ft}$.

2. For parameters $\alpha = 1/3$ $\beta = B_2 = 0.20$ and

$\beta = \beta_3 = 0.30$.

End moments for $\beta = \beta_2 = 0.20$ and $\beta = \beta_3 = 0.30$ are found by the same method shown previously, and the results are tabulated in Table 2-1.

(B) Analysis of Case (2) Loading

1. Analysis for the case $\alpha = 1/3$ $\beta = \beta_1 = 0.10$

Consider first that all joints are fixed against translation, and end moments due to rotation only are computed by the method of moment distribution. In this analysis, the gable members are considered as single structural elements.

(a) Stiffness Factors

$$K'_{10} = K'_{20} = K'_{30} = \frac{3EI}{h} = 0.15EI$$

$$K_{12} = K_{21} = K_{23} = K_{32} = \frac{7EI}{2S_1} = 0.114EI$$

(b) Distribution Factors

$$D_{10} = D_{30} = 0.568$$

$$D_{12} = D_{32} = 0.432$$

$$D_{21} = D_{23} = 0.301$$

$$D_{20} = 0.398$$

(c) Carry-Over Factors

$$C_{10} = C_{20} = C_{30} = 0$$

$$C_{12} = C_{21} = C_{23} = C_{32} = -0.143.$$

TABLE 2-1

END MOMENTS FOR CASE (1) LOADING

End Moments	$\beta_1 = 0.10$			$\beta_2 = 0.20$			$\beta_3 = 0.30$		
	$W_1 = 1^k/ft.$	$W_2 = 2^k/ft.$	$W_3 = 3^k/ft.$	$W_1 = 1^k/ft.$	$W_2 = 2^k/ft.$	$W_3 = 3^k/ft.$	$W_1 = 1^k/ft.$	$W_2 = 2^k/ft.$	$W_3 = 3^k/ft.$
M_{12}	-218.20	-436.40	-654.60	-215.16	-430.32	-645.48	-205.71	-411.42	-617.13
M_{21}	+290.79	+581.58	+872.37	+243.60	+487.20	+730.80	+205.80	+411.60	+617.40
M_{23}	-290.79	-581.58	-872.37	-243.60	-487.20	-730.80	-205.80	-411.60	-617.40
M_{32}	+218.20	+436.40	+654.60	+215.16	+430.32	+645.48	+205.71	+411.42	+617.13

(d) Fixed End Moments

$$FM_{12} = -\frac{5wf^2}{48} = -2.25 \text{ k-ft.}$$

$$FM_{21} = -\frac{wf^2}{48} = -0.45 \text{ k-ft.}$$

$$EM_{10} = +\frac{wh^2}{8} = +30.00 \text{ k-ft.}$$

(e) Distribution Table

	(1)		(2)			(3)	
	10	12	21	20	23	32	30
- D's	- .568	- .432	- .301	- .398	- .301	- .432	- .568
C's		- .143	- .143		- .143	- .143	
FM's	+30.00	- 2.25	- .45				
	-15.76	-11.99	+ .14	+ .17	+ .14		
		- .02	+1.71			- .02	
	+ .01	+ .01	- .52	- .68	- .51	+ .01	+ .01
		+ .07	0		0	+ .07	
	- .04	- .03	0	0	0	- .03	- .04
RM's	-15.79	-11.96	+1.33	-0.51	-0.37	+0.03	-0.03
M's	+14.21	-14.21	+0.88	-0.51	-0.37	+0.03	-0.03

(f) Thrust Induction Factors

$$n_{12} = -n_{21} = n_{23} = -n_{32} = \frac{3}{4\beta L} = +0.125$$

(g) Fixed End Thrusts

$$FH_{12} = -\frac{3wf}{4} = -2.70^k$$

$$FH_{21} = -\frac{wf}{4} = -0.90^k$$

(h) Thrusts Due to Rotations

$$\begin{bmatrix} RH_{12} \\ RH_{21} \\ RH_{23} \\ RH_{32} \end{bmatrix} = \begin{bmatrix} RM_{12} & RM_{21} & 0 & 0 \\ RM_{21} & RM_{12} & 0 & 0 \\ 0 & 0 & RM_{23} & RM_{32} \\ 0 & 0 & RM_{32} & RM_{23} \end{bmatrix} \begin{bmatrix} n_{12} \\ n_{21} \\ n_{23} \\ n_{32} \end{bmatrix}$$

$$\begin{bmatrix} RH_{12} \\ RH_{21} \\ RH_{23} \\ RH_{32} \end{bmatrix} = \begin{bmatrix} -11.96 & +1.33 & 0 & 0 & +0.125 \\ +1.33 & -11.96 & 0 & 0 & -0.125 \\ 0 & 0 & -0.37 & +0.03 & +0.125 \\ 0 & 0 & +0.03 & -0.37 & -0.125 \end{bmatrix} = \begin{bmatrix} -1.66 \\ +1.66 \\ -0.05 \\ +0.05 \end{bmatrix}$$

(i) Total Horizontal Thrusts

$$\begin{bmatrix} H_{12} \\ H_{21} \\ H_{23} \\ H_{32} \end{bmatrix} = \begin{bmatrix} RH_{12} & & FH_{12} & & -4.36 \\ RH_{21} & & FH_{21} & & +0.76 \\ RH_{23} & + & FH_{23} & = & -0.05 \\ RH_{32} & & FH_{32} & & +0.05 \end{bmatrix}$$

(j) End Shears

$$\begin{bmatrix} V_{10} \\ V_{20} \\ V_{30} \end{bmatrix} = -\frac{1}{h} \begin{bmatrix} M_{10} + \frac{wh^2}{2} & -6.71 \\ M_{20} & +0.03 \\ M_{30} & 0 \end{bmatrix}$$

(k) Balancing Forces

Joint 1.

$$H_{12} + V_{10} = -4.36 - 6.71 = -11.07^k$$

Joint 2.

$$H_{21} + V_{20} + H_{23} = +0.76 + 0.03 - 0.05$$

$$= +0.74^k$$

Joint 3.

$$H_{32} + V_{30} = +0.05 + 0 = +0.05^k$$

Thus

$$P_1 = +\ 11.07^k \qquad P_2 = -\ 0.74^k$$

$$P_3 = -\ 0.05^k$$

$$P_1L = +664.20^{k\text{-}ft.} \qquad P_2L = -44.40^{k\text{-}ft.}$$

$$P_3L = -\ 3.00^{k\text{-}ft.}$$

(l) End Moments Due to Balancing Forces

$$\begin{bmatrix} M^{(P)}_{12} \\ M^{(P)}_{21} \\ M^{(P)}_{20} \\ M^{(P)}_{23} \\ M^{(P)}_{32} \end{bmatrix} = \begin{bmatrix} Q^{(1)}_{12} & Q^{(2)}_{12} & Q^{(3)}_{12} \\ Q^{(1)}_{21} & Q^{(2)}_{21} & Q^{(3)}_{21} \\ -Q^{(1)}_{20} & -Q^{(2)}_{20} & -Q^{(3)}_{20} \\ Q^{(1)}_{23} & Q^{(2)}_{23} & Q^{(3)}_{23} \\ Q^{(1)}_{32} & Q^{(2)}_{32} & Q^{(3)}_{32} \end{bmatrix} \begin{bmatrix} P_1L \\ \\ P_2L \\ \\ P_3L \end{bmatrix}$$

$$\begin{bmatrix} M^{(P)}_{12} \\ M^{(P)}_{21} \\ M^{(P)}_{20} \\ M^{(P)}_{23} \\ M^{(P)}_{32} \end{bmatrix} = \begin{bmatrix} .116819 & .079858 & .079063 \\ .045012 & .086809 & .092439 \\ -.137451 & -.173618 & -.137451 \\ .092439 & .086809 & .045012 \\ .079063 & .079858 & .116819 \end{bmatrix} \begin{bmatrix} +664.20 \\ \\ -\ 44.40 \\ \\ -\ 3.00 \end{bmatrix} = \begin{bmatrix} +73.81 \\ +25.76 \\ -83.17 \\ +57.41 \\ +48.62 \end{bmatrix}$$

(m) Final End Moments

$$\begin{bmatrix} M_{12} \\ M_{21} \\ M_{20} \\ M_{23} \\ M_{32} \end{bmatrix} = \begin{bmatrix} +59.60 \\ +26.64 \\ -83.68 \\ +57.04 \\ +48.65 \end{bmatrix}$$

2. Analysis For Cases $\alpha = 1/3$ $\beta = \beta_2 = 0.20$ and $\beta = \beta_3 = 0.30$

Analysis of these two cases was carried out by the same method and the results obtained are tabulated in the following.

TABLE 2-2

END MOMENTS FOR CASE (2) LOADING

Final Moments	$\beta_1 = 0.10$	$\beta_2 = 0.20$	$\beta_3 = 0.30$
M_{12}	+59.60	+ 98.03	+140.46
M_{21}	+26.64	+ 24.54	+ 21.90
M_{20}	-83.68	-109.17	-133.88
M_{23}	+57.04	+ 84.63	+111.98
M_{32}	+48.65	+ 56.74	- 61.43

(C) Final End Moments for Case (3) Loading

End moments for case (3) loading are obtained by superposing end moments for both case (1) and case (2) loading. The final moments obtained from this superposition are tabulated in Table 2-3. The superposition is carried out only for the case of combining $W_{D+L} = 1^{k}/ft.$ with $W_W = 0.6^{k}/ft.$

TABLE 2-3

END MOMENTS FOR CASE (3) LOADING

Final Moments	$\beta_1 = 0.10$	$\beta_2 = 0.20$	$\beta_3 = 0.30$
M_{12}	- 118.95	- 87.80	- 48.94
M_{21}	+ 238.07	+ 201.90	+ 170.78
M_{20}	- 62.76	- 81.80	- 100.41
M_{23}	- 175.31	- 119.10	- 70.37
M_{32}	+ 200.14	+ 204.00	+ 200.36

2-3. Tables of Moments, Shearing, and Normal Forces

From those end moments obtained in the foregoing analyses, shearing forces and normal forces for all members with different load intensity and parameter β are found by the conditions for static equilibrium. Finally, these end moments, shearing forces, and normal forces acting at various critical sections are tabulated in order to be used for brief designs of sections and their steel requirements in Chapter V. The tabulation follows in Tables 2-4, 2-5, and 2-6.

TABLE 2-4

ELASTIC ANALYSIS : END MOMENTS, SHEARING FORCES, AND NORMAL FORCES ($\alpha = 1/3 \; \beta = \beta_1 = 0.10$)

Member	End Moments					Shearing Forces					Normal Forces				
	M_{W_1}	M_{W_2}	M_{W_3}	M_{W_W}	0.75 $[M_{W_1}+M_{W_W}]$	V_{W_1}	V_{W_2}	V_{W_3}	V_{W_W}	0.75 $[V_{W_1}+V_{W_W}]$	N_{W_1}	N_{W_2}	N_{W_3}	N_{W_W}	0.75 $[N_{W_1}+N_{W_W}]$
1 - 0	+218.20	+436.40	+654.60	-59.60	+118.95	-10.91	-21.82	-32.73	-3.02	-10.45	-28.79	- 57.58	- 86.37	+1.62	-20.43
1 - 4	-218.20	-436.40	-654.60	+59.60	-118.95	+26.06	+52.12	+28.18	-2.18	+17.91	-16.34	- 32.68	- 49.02	-2.64	-14.24
4 - 1	-130.04	-260.09	-390.13	+17.80	- 84.18	- 3.34	- 6.68	-10.02	-2.89	- 4.67	-10.46	- 20.92	- 31.38	-6.16	-12.47
4 - 2	+130.04	+260.09	+390.13	-17.80	+ 84.18	+ 0.94	+18.80	+ 2.82	-0.29	+ 0.49	-10.94	- 21.88	- 32.82	-6.80	-13.31
2 - 4	+290.79	+581.58	+872.37	+26.64	+238.07	-28.46	-56.92	-85.35	+0.29	-21.13	-16.82	- 33.64	- 50.46	-6.80	-17.72
2 - 0	0	0	0	-83.62	- 62.76	0	0	0	+4.19	+ 3.14	-62.42	-124.84	-187.26	+0.14	-46.71
2 - 5	-290.79	-581.58	-872.37	+57.04	-175.31	+28.46	+56.92	+85.35	-2.20	+19.70	-16.82	- 33.64	- 50.46	-2.03	-14.14
5 - 2	-130.04	-260.09	-390.13	+10.40	- 89.73	- 0.94	-18.80	- 2.82	-2.20	- 2.36	-10.94	- 21.88	- 32.82	-2.03	- 9.73
5 - 3	+130.04	+260.09	+390.13	-10.40	+ 89.73	+ 3.34	+ 6.68	+10.02	-1.24	+ 1.58	-10.46	- 20.92	- 31.38	-2.73	- 9.89
3 - 5	+218.20	+436.40	+654.60	+48.65	+200.14	-26.06	-52.12	-78.18	-1.24	-20.48	-16.34	- 32.68	- 49.02	-2.73	-14.30
3 - 0	-218.20	-436.40	-654.60	-48.65	-200.14	+10.91	+21.82	+32.73	+2.43	+10.01	-28.79	- 57.58	- 86.37	-1.76	-22.91
0 - 1	0	0	0	0	0	-10.91	-21.82	-32.73	+8.98	- 1.45	-28.79	- 57.58	- 86.37	+1.62	-20.38
0 - 2	0	0	0	0	0	0	0	0	+4.19	+ 3.14	-62.42	-124.84	-187.26	+0.14	-46.71
0 - 3	0	0	0	0	0	+10.91	+21.82	+32.73	+2.43	+10.01	-28.79	-57.58	- 86.37	-1.76	-22.91

TABLE 2-5

ELASTIC ANALYSIS : END MOMENTS, SHEARING FORCES, AND NORMAL FORCES ($\alpha = 1/3$ $\beta = \beta_2 = 0.20$)

Member	End Moments					Shearing Forces					Normal Forces				
	M_{W_1}	M_{W_2}	M_{W_3}	M_{W_W}	0.75 $[M_{W_1}+M_{W_W}]$	V_{W_1}	V_{W_2}	V_{W_3}	V_{W_W}	0.75 $[V_{W_1}+V_{W_W}]$	N_{W_1}	N_{W_2}	N_{W_3}	N_{W_W}	0.75 $[N_{W_1}+N_{W_W}]$
1 - 0	+215.16	+430.32	+645.48	- 98.03	+ 87.80	-10.76	-21.52	-32.27	- 1.10	- 8.90	-29.53	- 59.06	- 88.59	+2.76	-20.08
1 - 4	-215.16	-430.32	-645.48	+ 98.03	- 87.80	+23.40	+46.80	+70.20	- 2.97	+15.32	-21.00	- 42.00	- 63.00	+0.01	-15.74
4 - 1	- 91.60	-183.20	-274.80	+ 41.17	- 37.80	- 4.44	- 8.88	-13.32	- 5.65	- 7.57	- 9.84	- 19.68	- 29.52	-6.67	-12.38
4 - 2	+ 91.60	+183.20	+274.80	- 41.17	+ 37.80	+ 3.54	+ 7.08	+10.62	+ 0.53	+ 3.05	-10.14	- 20.28	- 30.42	-8.73	-14.34
2 - 4	+243.60	+487.20	+730.80	+ 24.54	+201.90	-24.30	-48.60	-72.90	+ 0.53	-17.83	-21.33	- 42.66	- 63.99	-8.73	-22.55
2 - 0	0	0	0	-109.17	- 81.80	0	0	0	+ 5.46	+ 4.10	-60.94	-121.88	-182.82	-0.40	-46.01
2 - 5	-243.60	-487.20	-730.80	+ 84.63	-119.10	+24.30	+48.60	+72.90	- 3.25	+15.79	-21.33	- 42.66	- 63.99	-1.61	-17.21
5 - 2	- 91.60	-183.20	-274.80	+ 20.25	- 53.60	- 3.54	- 7.08	-10.62	- 3.25	-50.93	-10.14	- 20.28	- 30.42	-1.61	- 8.81
5 - 3	+ 91.60	+183.20	+274.80	- 20.25	+ 53.60	+ 4.44	+ 8.88	+13.32	- 1.13	+ 2.48	- 9.84	- 19.68	- 29.52	-3.25	- 9.82
3 - 5	+215.16	+430.32	+645.48	+ 56.74	+204.00	-23.40	-46.80	-70.20	- 1.13	-18.40	-21.00	- 42.00	- 63.00	-3.25	-18.19
3 - 0	-215.16	-430.32	-645.48	- 56.74	-204.00	+10.76	+21.52	+32.27	+ 2.84	+10.20	-29.53	- 59.06	- 88.59	-2.36	-23.92
0 - 1	0	0	0	0	0	-10.76	-21.52	-32.27	+10.90	+ 0.10	-29.53	- 59.06	- 88.59	+2.76	-20.07
0 - 2	0	0	0	0	0	0	0	0	+ 5.46	+ 4.10	-60.94	-121.88	-182.82	-0.40	-46.01
0 - 3	0	0	0	0	0	+10.76	+21.52	+32.27	+ 2.84	+10.20	-29.53	- 59.06	- 88.59	-2.36	-23.92

TABLE 2-6

ELASTIC ANALYSIS : END MOMENTS, SHEARING FORCES, AND NORMAL FORCES ($\alpha = 1/3$ $\beta = \beta_3 = 0.30$)

Member	End Moments					Shearing Forces					Normal Forces				
	M_{W_1}	M_{W_2}	M_{W_3}	M_{W_W}	$0.75 [M_{W_1}+M_{W_W}]$	V_{W_1}	V_{W_2}	V_{W_3}	V_{W_W}	$0.75 [V_{W_1}+V_{W_W}]$	N_{W_1}	N_{W_2}	N_{W_3}	N_{W_W}	$0.75 [N_{W_1}+N_{W_W}]$
1 - 0	+205.71	+411.42	+617.13	-140.46	+ 48.94	-10.29	-20.57	-30.86	+ 1.03	- 6.95	-30.00	- 60.00	- 90.00	+ 4.33	-19.25
1 - 4	-205.71	-411.42	-617.13	+140.46	- 48.94	+20.42	+40.84	+61.26	- 3.18	+12.93	-24.24	- 48.48	- 72.72	+ 3.11	-15.85
4 - 1	- 59.07	-118.14	-177.21	+ 68.10	+ 6.77	- 5.29	-10.58	-15.87	- 3.73	-10.52	- 8.82	- 17.64	- 26.46	- 6.14	-11.22
4 - 2	+ 59.07	+118.14	+177.21	- 68.10	- 6.77	+ 5.29	+10.58	+15.87	+ 1.31	+ 4.95	- 8.82	- 17.64	- 26.46	-10.60	-14.57
2 - 4	+205.80	+411.60	+617.40	+ 21.90	+170.78	-20.42	-40.84	-61.26	+ 1.31	-14.33	-24.24	- 48.48	- 72.72	-10.60	-26.13
2 - 0	0	0	0	-133.88	-100.41	0	0	0	+ 6.70	+ 5.03	-60.00	-120.00	-180.00	- 1.44	-46.08
2 - 5	-205.80	-411.60	-617.40	+111.98	- 70.37	+20.42	+40.84	+61.26	- 4.06	+12.27	-24.24	- 48.48	- 72.72	- 1.44	-19.26
5 - 2	- 59.07	-118.14	-177.21	+ 29.98	- 21.82	- 5.29	-10.58	-15.87	- 4.06	- 7.01	- 8.82	- 48.48	- 72.72	- 1.14	- 7.47
5 - 3	+ 59.07	+118.14	+177.21	- 29.98	+ 21.82	+ 5.29	+10.58	+15.87	- 0.90	+ 3.29	- 8.82	- 17.64	- 26.46	- 4.12	- 9.71
3 - 5	+205.71	+411.42	+617.13	+ 61.43	+200.36	-20.42	-40.84	-61.26	- 0.90	-15.99	-24.24	- 48.48	- 72.72	- 4.12	-21.27
3 - 0	-205.71	-411.42	-617.13	- 61.43	-200.36	+10.29	+20.57	+30.86	+ 3.07	+10.02	-30.00	- 60.00	- 90.00	- 2.89	-24.67
0 - 1	0	0	0	0	0	-10.29	-20.57	-30.86	+13.03	+ 2.06	-30.00	- 60.00	- 90.00	+ 4.33	-19.25
0 - 2	0	0	0	0	0	0	0	0	+ 6.70	+ 5.02	-60.00	-120.00	+180.00	- 1.44	-46.08
0 - 3	0	0	0	0	0	+10.29	+20.57	+30.86	+ 3.07	+10.02	-30.00	- 60.00	- 90.00	- 2.89	-24.67

CHAPTER III

THE PLASTIC HINGE THEORY OF REINFORCED CONCRETE FRAMES

3-1. General

The development of design methods based on inelastic behavior of redundant steel structures preceded that of similar methods for concrete structures. After World War II, engineers throughout the world concentrated much effort in investigating the behavior of steel frameworks at ultimate load and in the development of practical plastic design methods. Thus, a number of alternative methods of plastic design for steel frames have been developed. The approaches of these different methods differ, but they all recognize the following conditions as the requirements for collapse of an all-steel structure.

(A) Equilibrium Condition

Bending moment distribution must be in equilibrium with external loads.

(B) Collapse Mechanism Condition

A sufficient number of plastic hinges must exist to transform either the whole or part of the structure into a mechanism.

(C) The Yield Condition

Full plastic moment must nowhere be exceeded. A design is considered valid when all the three numerated conditions are satisfied at the final collapse stage.

In the case of designing reinforced concrete structures, not only the satisfaction of these conditions is necessary, but also two other important considerations related to a successful design should be investigated carefully.

(A) Rotation Capacity

In structural steel, little attention is paid to how much any one hinge section is strained, before all the other hinges are formed. Such considerations are usually not necessary for structural steel because of its high ductility. The ultimate strain for concrete in flexural compression is limited from 0.3% to 0.5%. Therefore, in limit design of structural concrete, rotation capacity of sections must be considered in greater detail than for structural steel.

Furthermore, to avoid excessive flexural cracking, it is desirable to limit hinge rotations for structural concrete even when considerable rotation capacity is present after extensive cracking.

(B) Distribution of Moment Resistance

By varying the amount and location of reinforcement, the positive and negative moment resistance of structural concrete members can easily be made different, and the moment capacity can be varied along the length of a prismatic member. It is therefore conveniently possible to reinforce a concrete structure in such a manner that all plastic hinges necessary to form a mechanism will then form at practically the same load, and thereby the hinge rotations required are small. Similarly, it is also possible to reinforce the structure in such a manner that the yield condition may be satisfied without causing yielding between the chosen plastic hinges.

An introductory explanation of "The Theory of Plastic Hinges for Reinforced Concrete Frames." developed by Professor A. L. L. Baker, will be shown in the following section.

3-2. The Plastic Hinge Theory

(A) Basic Concept

The classical elastic equations, developed by Müller-Breslau and others, may be applied to the "Idealized Frame" in order to check that the hinge positions chosen are at sections where plastic deformation will occur under ultimate load, other sections being reinforced to remain elastic, and to ensure that the rotation of the hinges is not excessive.

The principal aim, when designing a frame by the theory of plastic hinges, is to obtain uniformity of the cross-section of the various members, and an economical distribution of bending moments under the plastic conditions, which occur with over-loading prior to failure.

(B) Basic Assumptions

1. When a frame which is n times statically indeterminate, is increasingly loaded throughout, n plastic hinges form before failure occurs, and the structure becomes statically determinate.
2. The load applied when the n^{th} plastic hinge forms is the ultimate load.
3. The reinforcement in the members of the frame between the plastic hinges remains elastic and does not yield when ultimate load is applied.
4. The plastic hinges are concentrated at points.
5. Throughout the frame, under increasing load, the relation between load and moment of resistance follows a straight line portion OA, except at the hinge points where, after the plastic moment of resistance has developed, a horizontal line portion such as AB in Figure 3-1 is followed.

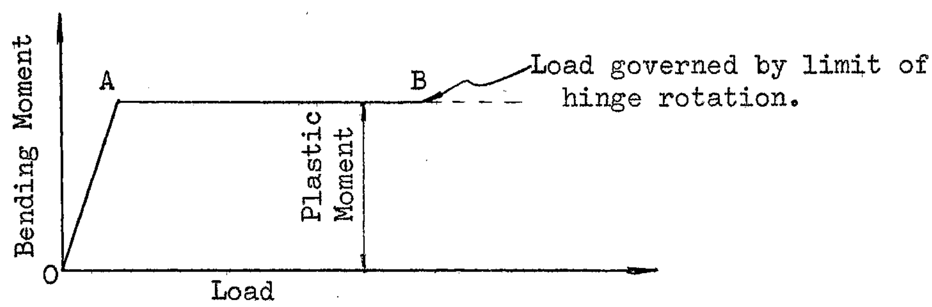


Figure 3-1. Idealized Bending Moment-Load Characteristics of a Plastic Hinge

(C) General Equations for Statically-Indeterminate Frames

In deriving the general equations for a frame n times statically indeterminate, n frictionless hinges are assumed to be inserted in the frame and n unknown equal and opposite bending moments $X_1 \ldots X_n$ are assumed to act on the members on either side of the hinges. For the elastic condition, the rotation at each hinge due to external load and all unknown moments acting is zero. Hence, for each of the hinges an equation is derived giving the following n equations from which the n unknowns may be found.

$$\delta_{i0} + \sum_{k=1}^{k=n} X_k \delta_{ik} = 0 \qquad i = 1, 2 \ldots n \qquad (3\text{-}1)$$

where

δ_{i0} = Rotation of hinge i due to external load only acting.

δ_{ik} = Rotation of hinge i due to unit bending moment acting at hinge k in direction X_k in a frame that has become statically determinate by the assumed insertion of sufficient number of hinges.

X_i = Unknown moment of resistance acting at hinge i when the section is elastic.

In a frame n times statically indeterminate, which has been loaded until n plastic hinges have formed, the rotations θ_1, θ_2 ... θ_n, are the sum of the rotations due to the external loads and the plastic moments acting at each hinge so that expressions of Equation (3-1) are then modified to

$$\delta_{i0} + \sum_{k=1}^{k=n} \overline{X}_k \delta_{ik} = -\theta_i \qquad i = 1, 2, \ldots n. \qquad (3\text{-}2)$$

Also, it can be shown that

$$\delta_{0k} = \int \frac{M_0 M_k}{EI} ds \qquad \delta_{ik} = \int \frac{M_i M_k}{EI} ds \qquad (3\text{-}3)$$

where,

$\overline{X}_i$ = Plastic moment of resistance for hinge section i.

M_0 = Free bending moments due to external load only acting on the frame being made statically determinate by the insertion of hinges.

M_i or M_k = Bending moments due to unit moment acting at hinge i or k in direction X_i or X_k on the frame made statically determinate.

θ_i = Resultant opening or rotation of hinge i in the direction opposite to X_i due to external load and all plastic hinge moments acting.

(D) Available Hinge Rotation

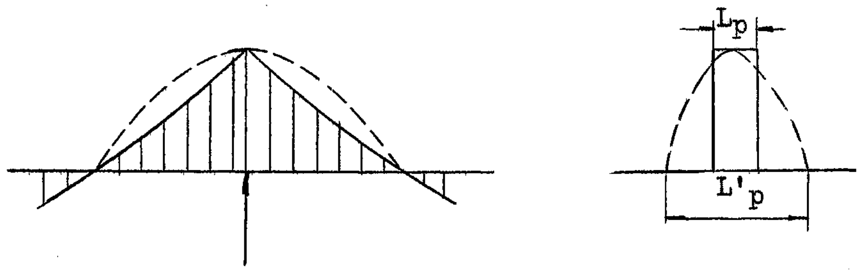
In the limit analysis of reinforced concrete frame, the amount of hinge rotation should be studied carefully and limited below a permissible value, so that to prevent the undesirable sudden failure of the structure, which might otherwise occur. The plastic deformation adjacent to any hinge section (Figures 3-2 and 3-3) equals

$$\int_0^{L'_p} \frac{S_p}{n_1 P_d} \, ds$$

for members in which tension develops, or

$$\int_0^{L'_p} \frac{S_d}{d} \, ds$$

for members such as columns in which no tension occurs.

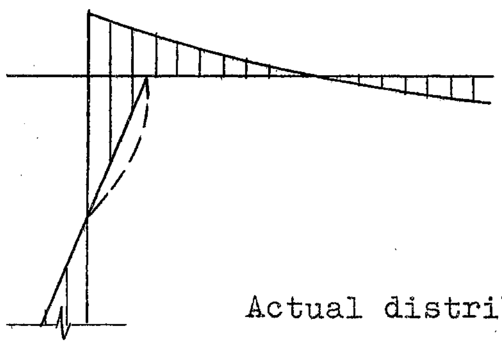


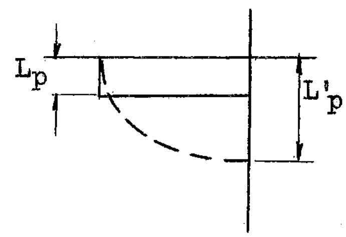
Actual distribution — — — — — —

Idealized distribution ————————

(a) Bending Stress (b) Plastic Strain

Typical Beam Support





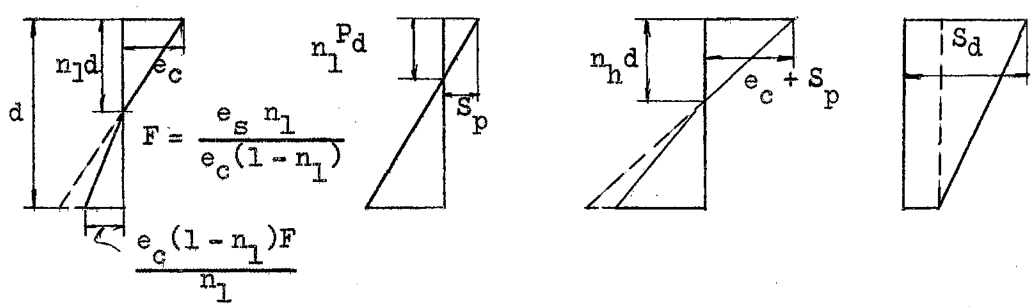
Actual distribution — — — — — —

Idealized distribution ————————

(c) Bending Stress (d) Plastic Strain

Typical Column

Figure 3-2. Actual and Idealized Distributions of Plastic Strain



(a) Elastic Strain (b) Plastic Strain (c) = (a) + (b) (d)

Beams Columns

Figure 3-3. Distribution of Strains at Hinge Section at Ultimate Load

The distribution of plasticity along a member is generally determined by:

1. The slope of the bending moment diagram.
2. The stress-strain curves of the concrete and steel.
3. The local behavior of the member in resisting bending.

It has been proved by tests that plasticity can spread over a length at least equal to the depth of the member. (3). Therefore, it is safe to assume an idealized uniform distribution of plastic strain, as shown in Figures 3-2 (b) and (d), over a length L_p equal to the depth of the member. The available plastic deformation then, before failure occurs, is (see Figures 3-2 and 3-3)

$$\int_0^{L_p} \frac{S_p}{n_1^P d}\, ds = \frac{S_p L_p}{n_1^P d} = \frac{S_p}{n_1^P} = \theta$$

for members in which tension develops. It is safe and generally convenient, when checking values of rotations, to assume $n_1^P = n_h$. And

$$\int_0^{L_p} \frac{S_d}{d}\, ds = \frac{S_d L_p}{d} = S_d = \theta$$

for members in which tension does not occur.

Safe limiting value of $S_d = 0.01$ for suitably bounded prismatic sections is given by Baker (3) based on test results.

(E) Safe Limiting Values of EI

The derivation of EI values from the basic stress-strain characteristics of the steel and concrete used has been discussed in various papers. The following results are reproduced from Baker's book (3).

TABLE 3-1

MOMENT OF INERTIA FOR CRACKED AND UNCRACKED SECTIONS

Generally E'c (elastic) = 500 Cu E'c (plastic) = 500 C' $I' = \frac{M' n_1 d}{C'}$

Condition	Section	E'c	I'	I'($n_1 \doteq 0.5$)
Elastic : Cracked	Rectangular	500 cu	$\frac{bd^3}{2}(n_1^3 - \gamma n_1^3)$**	0.120 bd^3
Plastic : Cracked	"	500 c'	$\frac{Cu}{C'}\frac{bd^3}{2}(n_1^2 - \gamma n_1^3)$**	0.120 bd^3
Elastic : Uncracked	"	500 cu	$\frac{bd^3}{12} + pbd^3(m-1)(1-n_1)^2$**	0.135 bd^3*

*Assuming $n_1 = 0.5$ $d = 0.9h$, $p = .01$ $m = \frac{E_s}{E_c}$

**Adopting British notations for ultimate strength design of concrete sections.

(F) Summary of Design Procedure

The procedure for designing a frame by The Plastic Hinge Theory may be summarized as follows:

1. Assume a general arrangement of the frame and concrete sections appropriate for the loads.

2. Assume sufficient hinges in the frame to make a statically determinate system. Also assume resisting moments acting at the hinges giving an economic distribution of bending moments. When necessary, assume different sets of hinges for different cases of load.
3. Check the positions of the hinges and value of rotation at ultimate load by applying general Equation (3-2), making adjustments until a satisfactory solution is obtained for each case of idealized frame.
4. Design a practical frame at least as strong in all parts as each case of idealized frame.

3-3. Design Criteria of Plastic Hinge Theory

A design can be considered valid, if for a set of assumed positions, plastic moments and rotations of hinges, the following conditions are satisfied:

1. The sum of the rotations at each hinge due to loads and all plastic-hinge moments, i.e.,

$$\delta_{i0} + \sum_{k=1}^{k=n} \bar{X}_k \delta_{ik}, \quad i = 1, 2 \ldots n$$

   is negative following the usual sign convention.
2. The resultant bending moments for ultimate load at all sections between the plastic hinges are within the elastic range of the main steel and the ultimate strength of the concrete.

3. The rotation at each hinge does not exceed a safe limiting value for that hinge, in order to avoid premature crushing of the concrete or fracture of the steel, if steel with considerably limited ductility is used.
4. At working load, elastic conditions obtained at all hinges and the strains are small enough to avoid wide cracks, large deflections or spalling of the concrete.

CHAPTER IV

ANALYSIS BY THE PLASTIC HINGE THEORY

4-1. General

The plastic hinge theory and the design procedure by the trial and adjustment method, developed by Baker (3) and briefly introduced in previous chapters, will now be applied in the limit analysis of the same structure which was analyzed in Chapter II by a conventional method based on elastic theory. The primary purpose of this chapter is to show the application of the theory to the limit analysis of a pin based reinforced concrete gable frame, having parameters $\beta = 0.1$, 0.2 and 0.3. A detailed analysis is carried out for the case with $\beta = 0.1$ and following cases of loading.

(A) Gravity load or dead plus live load: $W_{D+L} = 1^k/ft.$

(B) Wind load: $W_W = 0.6^k/ft.$

(C) $3/4\,(W_{D+L} + W_W)$.

For the cases with $\beta = 0.2$ and 0.3, only loadings (A) and (B) are considered in analyzing critical end moments and compared to the corresponding end moments obtained from the elastic analysis.

Other data necessary for the analysis are assumed as follows:

Load Factor = 2 $\qquad$ f'c = 3000 psi

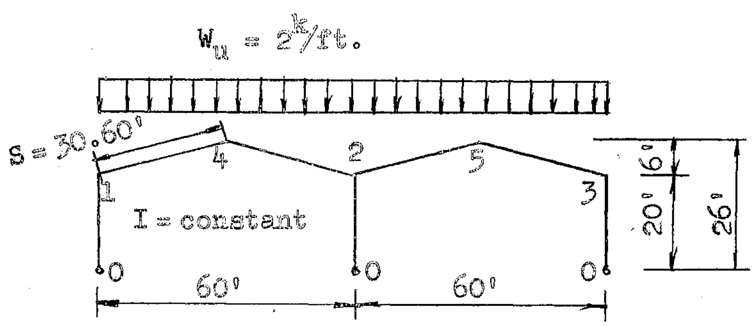
Cu ≐ 1.25 f'c = 3750 psi $\qquad$ C' = 0.85 Cu = 3200 psi

4-2. Analysis by The Plastic Hinge Theory

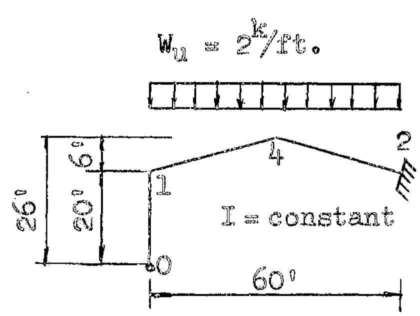
Each of the three cases of different loading is analyzed separately. The results obtained are tabulated.

(A) Analysis of Case (A) Loading

1. Sketch of Loaded Frame



(a). Frame With Case (A) Loading



(b). Reduced Frame With Case (A) Loading

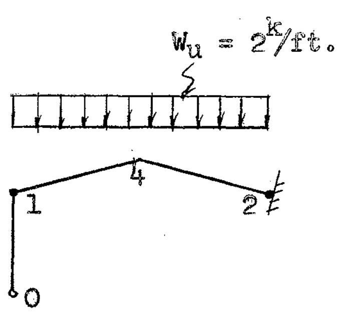
Figure 4-1. Sketch of Loaded Frame

The structure shown in Figure 4-1(a) is symmetrical about its center column and is also symmetrically loaded. Therefore, joint "2" can be considered as a fixed end support and, thus, reducing the frame to the one shown in Figure 4-1(b).

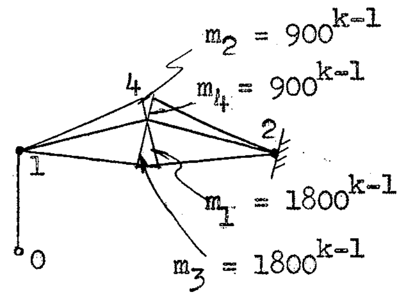
2. Assumption of Hinge Locations and Trial Sections

Since the reduced structure has two redundants, two plastic hinges are located at joint "1" and "2", thus making the structure a statically determinate one. A concrete section of 12" x 27" will be used in this trial and adjustment solution.

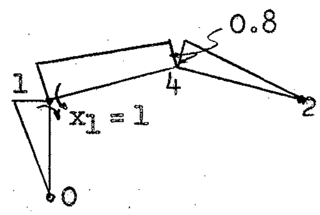
3. Moment Diagrams due to Loads and Plastic Moments



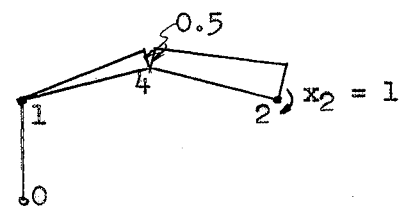
(a). Determinate Structure



(b). Moment Diag. due to Load



(c). Moment Diag. due to $\overline{X}_1 = 1$



(d). Moment Diag. due to $\overline{X}_2 = 1$

Figure 4-2. Moment Diagrams due to Load and Plastic Moments

4. Determination of E'_cI' Value

From Table 3-1, E'_cI' value for cracked section is given as follows:

$$E'_cI' = 500\ C' \times 0.120\ bd^3 .$$

For $f'c = 3000$ psi $Cu = 3750$ psi $C' = 0.85\ Cu$

$= 3200$ psi

$$E'_cI' = 500 \times 3200 \times 0.120 \times 12 \times 24^3 \times \frac{1}{144} = 221 \times 10^3 \text{ k-ft}^2$$

$$\frac{S}{E'_cI'} = \frac{30.60}{221 \times 10^3} \doteq 1.4 \times 10^{-4}$$

$$\frac{h}{E'_cI'} = \frac{20}{221 \times 10^3} = 0.9 \times 10^{-4} .$$

5. Solution by the Trial and Adjustment Method

The solution is carried out in tabular form, with properly assumed values of X_i's and the values of rotation coefficients of

$$\delta_{i0} = \int_0^L \frac{M_i M_0}{E'_c I'} \, ds \quad \text{and} \quad \delta_{ik} = \int_0^L \frac{M_i M_k}{E'_c I'} \, ds,$$

which are readily obtained by performing integration for all members concerned.

TABLE 4-1

NUMERICAL SOLUTION OF PLASTIC MOMENTS

				Plastic Hinge 1				Plastic Hinge 2			
	P	A_p	A_E	δ's*	P	A_P	A_E	δ's*	P	A_P	A_E
m_1	1800			- .607	-1091			-.233	-419		
m_2	900			+ .397	+ 357			+.175	+168		
m_3	1800			- .397	- 671			-.467	-838		
m_4	900			+ .280	+ 252			+.292	+263		
X_1	400	70	170	+1.364	+ 547	+96	+232	+.677	+271	+54	+115
X_2	400	70	150	+ .677	+ 271	+48	+102	+.700	+280	+49	+105
					-1155	+144	+334		-826	+103	+220
					+ 818		-337		+551		-275
1st Trial $-\theta_i$ rad.					-337×10^{-4}		$-3 \times 10^{-4} \doteq 0$		-275×10^{-4}		$-55 \times 10^{-4} \doteq 0$
					$+144 \times 10^{-4}$				$+103 \times 10^{-4}$		
2nd Trial $-\theta_i$ "					-193×10^{-4}				-172×10^{-4}		

$$*\delta_{i0} = \left[\int_0^L \frac{M_i M_0}{E'_c I'} \, ds\right] 10^{-4} \text{ rad.} \qquad *\delta_{ik} = \left[\int_0^L \frac{M_i M_k}{E'_c I'} \, ds\right] 10^{-4} \text{ rad.}$$

6. Investigation of Hinge Rotation

The permissible plastic hinge rotation for members in which tension develops is given by

$$\theta = \frac{S_p}{n_1^P} \quad \text{for} \quad S_p = 0.01,\ n_1^P = 0.5$$

$$\theta = 200 \times 10^{-4} \text{ radian .}$$

The calculated values of θ's at hinge 1 and 2 are

$$\theta_1 = 193 \times 10^{-4} \text{ rad.} < 200 \times 10^{-4} \text{ radian. O.K.}$$

$$\theta_2 = 172 \times 10^{-4} \text{ rad.} < 200 \times 10^{-4} \text{ radian. O.K.}$$

Since no excessive hinge rotation will occur, the analysis is considered satisfactory.

7. The Resultant Bending Moment Distribution

Applying the conditions of static equilibrium with the plastic moments $\overline{X}_1 = 470^{k-1}$, $\overline{X}_2 = 470^{k-1}$ the bending moment distribution under ultimate load is shown for the frame.

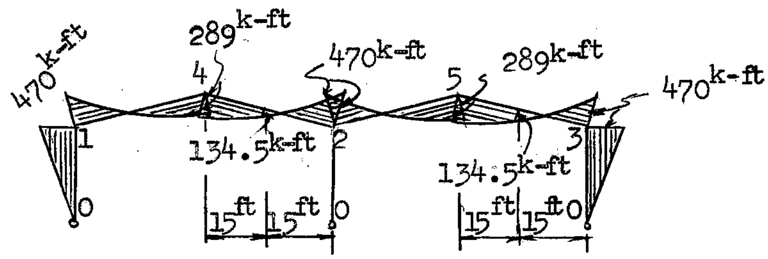


Figure 4-3. Bending Moment Distribution Under Ultimate Load

(B) Analysis of Case (B) Loading

1. Sketch of Loaded Frame

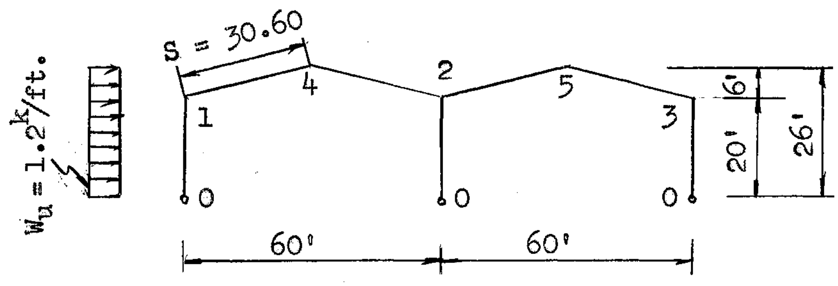
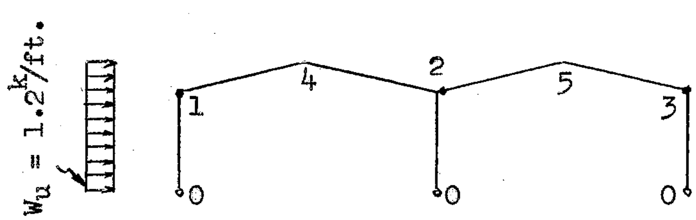


Figure 4-4. Frame With Case (B) Loading

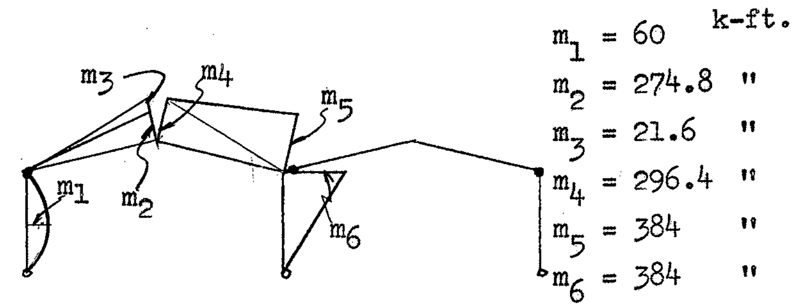
2. Assumption of Hinge Locations

The structure shown in Figure 4-4 is indeterminate to third degree, three hinges at joints 1, 2 and 3 are assumed.

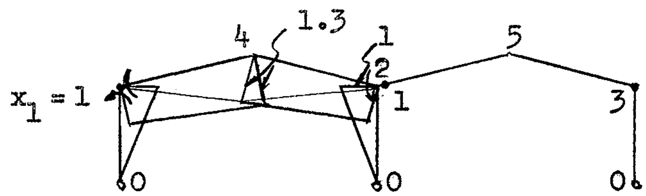
3. Moment Diagrams Due to Load and Plastic Moments



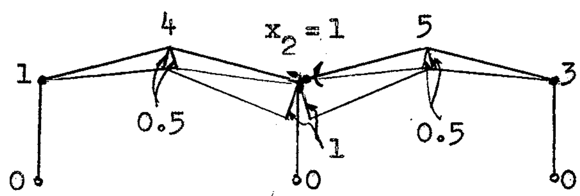
(a) Determinate Structure



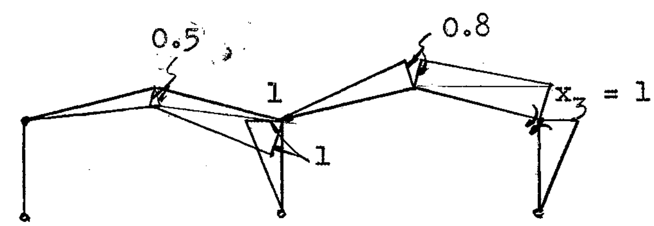
(b) Moment Diagram Due to Load



(c) Moment Diagram Due to Plastic Moment $\overline{X}_1 = 1$



(d) Moment Diagram Due to Plastic Moment $\overline{X}_2 = 1$



(e) Moment Diagram Due to Plastic Moment $\overline{X}_3 = 1$

Figure 4-5. Moment Diagrams Due to Load and Plastic Moments

4. Determination of $E'_c I'$ Value

$$E'_c I' = 221 \times 10^3 \quad k\text{-}ft^2$$

$$\frac{S}{E'_c I'} = 1.4 \times 10^{-4}$$

$$\frac{h}{E'_c I'} = 0.9 \times 10^{-4}$$

5. Solution by the Trial and Adjustment Method

The solution is carried out in tabular form as follows:

TABLE 4-2

NUMERICAL SOLUTION OF PLASTIC MOMENTS

	Moments			Plastic Hinge 1				Plastic Hinge 2				Plastic Hinge 3			
	P	A_P	A_E	δ'_S*	P	A_P	A_E	δ'_S*	P	A_P	A_E	δ'_S*	P	A_P	A_E
m_1	60			+ .300	+ 18			0	0			0	0		
m_2	274.8			- .840	-231			- .233	- 64			- .233	- 64		
m_3	21.6			- .572	- 12			- .175	- 4			- .175	- 4		
m_4	296.4			- .840	-249			- .467	-138			- .467	-138		
m_5	384			- .770	-296			- .583	-224			- .583	-224		
m_6	384			- .300	-115			0	0			0	0		
X_1	96	14	40	+3.110	+299	+44	+125	+1.610	+154	+23	+65	+1.910	+183	+27	+77
X_2	92	10	34	+1.610	+148	+16	+ 55	+1.400	+129	+14	+48	+ .257	+ 24	+ 3	+ 9
X_3	78	18	28	+1.910	+148	+35	+ 54	+ .257	+ 20	+ 5	+ 7	+2.365	+185	+43	+67
					-903	+95	+234		-430	+38	+120		-545	+70	+153
					+614		-289		+303		-127		+392		-153
1st. Trial - θ_i rad.					-289×10^{-4}		- 55		-127×10^{-4}		- 7		-153×10^{-4}		0
					$+ 95 \times 10^{-4}$		$\times 10^{-4}$		$+ 38 \times 10^{-4}$		$\times 10^{-4}$		$+ 70 \times 10^{-4}$		
2nd. Trial - θ_i rad.					-194×10^{-4}		$\doteq 0$		-107×10^{-4}		$\doteq 0$		-105×10^{-4}		

6. Investigation of Hinge Rotation

$\theta_1 = 194 \times 10^{-4}$ rad. $< 200 \times 10^{-4}$ rad. O.K.

$\theta_2 = 107 \times 10^{-4}$ rad. $< 200 \times 10^{-4}$ rad. O.K.

$\theta_3 = 105 \times 10^{-4}$ rad. $< 200 \times 10^{-4}$ rad. O.K.

The analysis, with the assumed positions of hinges and the plastic moments $\overline{X}_1 = 110^{k-1}$, $\overline{X}_2 = 102^{k-1}$, and $\overline{X}_3 = 96^{k-1}$ acting at joints 1, 2, and 3, respectively, can be considered satisfactory.

7. The Resultant Bending Moment Distribution

With those known plastic moments $\overline{X}_1$, $\overline{X}_2$, and $\overline{X}_3$, and applying the conditions for static equilibrium, the bending moment distribution under ultimate load is shown below.

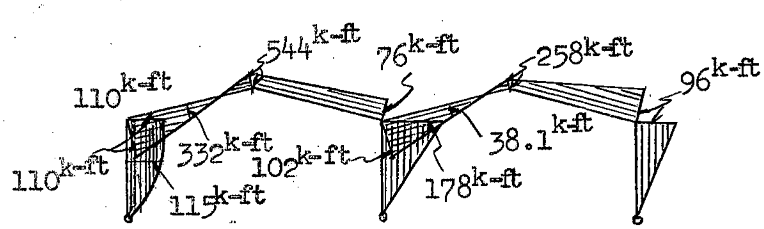


Figure 4-6. Bending Moment Distribution Under Ultimate Load

(C) Analysis of Case (C) Loading

1. Sketch of Loaded Frame

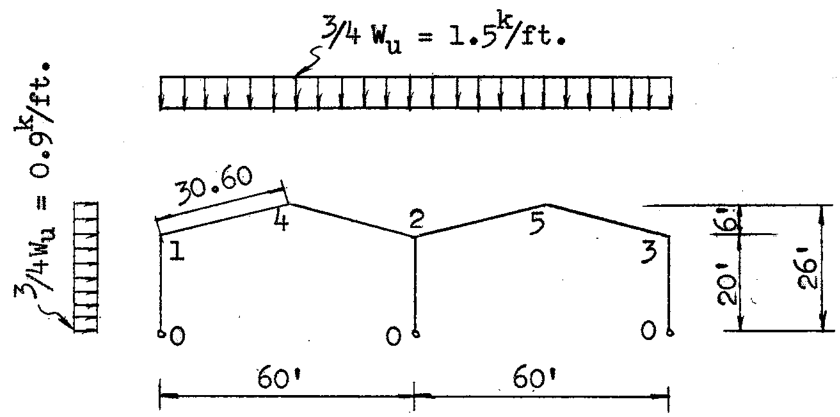
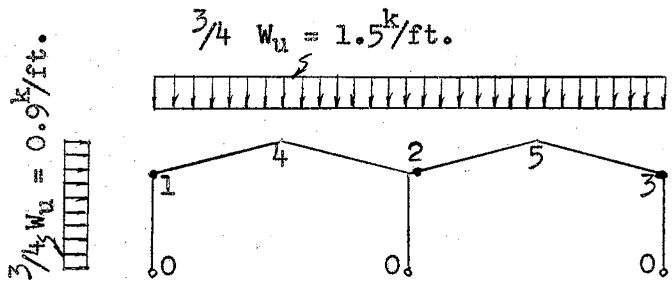


Figure 4-7. Sketch of Loaded Frame

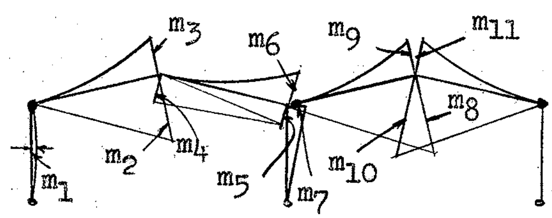
2. Assumption of Hinge Locations

As in the previous case, three hinges at joints 1, 2, and 3 are assumed.

3. Moment Diagrams due to Load and Plastic Moments

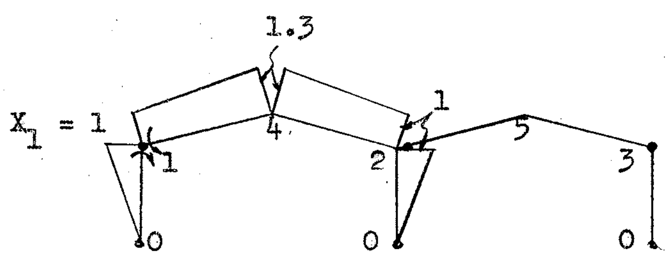


(a) Determinate Structure

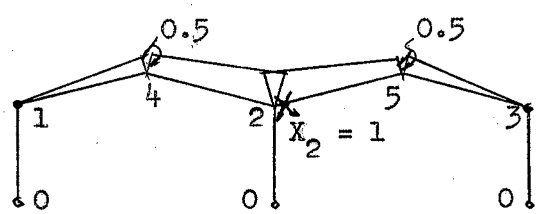


(b) Moment Diagram due to Load

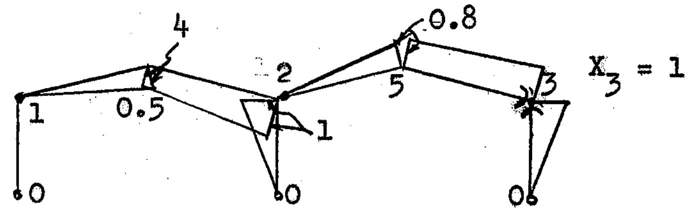
$m_1 = 40^{k\text{-}ft}$
$m_2 = 1143.9^{k\text{-}ft}$
$m_3 = 691.2^{k\text{-}ft}$
$m_4 = 452.7^{k\text{-}ft}$
$m_5 = 387 \ {k\text{-}ft}$
$m_6 = m_9 = m_{11}$
$= 675^{k\text{-}ft}$
$m_7 = 288^{k\text{-}ft}$
$m_8 = m_{10} = 1350^{k\text{-}ft}$



(c) Moment Diagram due to Plastic Moment $\overline{X}_1 = 1$



(d) Moment Diagram due to Plastic Moment $\overline{X}_2 = 1$



(e) Moment Diagram due to Plastic Moment $\overline{X}_3 = 1$

Figure 4-8. Moment Diagrams due to Load and Plastic Moments

4. Determination of E'_cI' Value

$$E'_cI' = 221 \times 10^3 \text{ k-ft}^2$$

$$\frac{S}{E'_cI'} = 1.4 \times 10^{-4}$$

$$\frac{h}{E'_cI'} = 0.9 \times 10^{-4}$$

5. Solution by the Trial and Adjustment Method

The solution is carried out in tabular form (Table 4-3) on the following page.

6. Investigation of Hinge Rotation

The θ_i values obtained at hinges 1, 2, and 3 are 179×10^{-4}, 76×10^{-4} and 200×10^{-4} radians, respectively. Since the permissible value of hinge rotation $\theta = 200 \times 10^{-4}$ radian (3) is not exceeded, the analysis can be considered satisfactory.

TABLE 4-3

NUMERICAL SOLUTION OF PLASTIC MOMENTS

	Moments			Plastic Hinge 1				Plastic Hinge 2				Plastic Hinge 3			
	P	A_P	A_E	δ'_S*	P	A_P	A_E	δ'_S*	P	A_P	A_E	δ'_S*	P	A_P	A_E
m_1	45			- .300	- 14			0	0			0	0		
m_2	1143.9			- .840	-960			- .233	-266			+ .233	+266		
m_3	691.2			+ .572	+395			+ .175	+121			- .175	-121		
m_4	452.7			- .840	-380			- .467	-211			+ .467	+211		
m_5	387			- .770	-298			- .583	-225			+ .583	+225		
m_6	675			+ .502	+338			+ .408	+275			- .408	-275		
m_7	288			+ .300	+ 86			0	0			- .300	- 86		
m_8	1350			0	0			- .467	-630			- .373	-503		
m_9	675			0	0			+ .292	+197			+ .280	+189		
m_{10}	1350			0	0			- .233	-314			- .607	-819		
m_{11}	675			0	0			+ .175	+118			+ .513	+346		
X_1	200	30	100	+3.110	+622	+ 93	+311	+1.610	+322	+ 48	+161	-1.910	-382	- 57	-191
X_2	300	120	180	+1.610	+483	+193	+290	+1.400	+420	+168	+252	- .257	- 77	- 31	- 46
X_3	300	86	200	-1.910	-573	-164	-382	- .257	- 77	- 22	- 52	+2.365	+710	+204	+473
					-2225	+122	+219		-1723	+194	+361		-2263	+116	+236
					+1924		-301		+1453		-270		+1947		-316
1st Trial - θ_i rad.					-301×10^{-4}		-82×10^{-4}		-270×10^{-4}		$+91\times10^{-4}$		-316×10^{-4}		-80×10^{-4}
					$+122\times10^{-4}$		≐ 0		$+194\times10^{-4}$		≐ 0		$+116\times10^{-4}$		≐ 0
2nd Trial - θ_i rad.					-179×10^{-4} rad.				-76×10^{-4} rad.				-200×10^{-4} rad.		

7. The Resultant Bending Moment Distribution

With those values of plastic moments, $\overline{X}_1 = 230^{k-ft}$, $\overline{X}_2 = 420^{k-ft}$, and $\overline{X}_3 = 386^{k-ft}$, and by applying the conditions for static equilibrium, the following bending moment distribution under ultimate load is obtained.

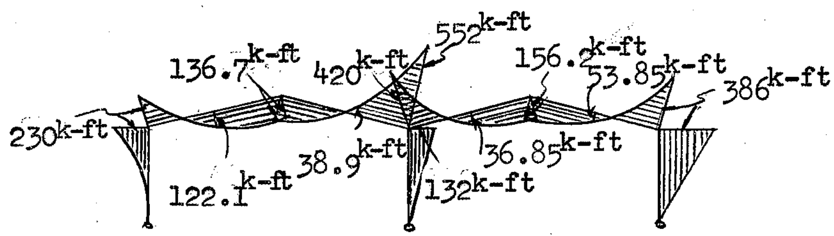


Figure 4-9. Bending Moment Distribution Under Ultimate Load

4-3. Table of Moments, Shearing and Normal Forces

Having found those ultimate moments at each critical section for different loadings, the corresponding shearing and normal forces are easily obtained by the conditions for static equilibrium (shown in Table 4-4).

TABLE 4-4

PLASTIC ANALYSIS : END MOMENTS, SHEARING FORCES, AND NORMAL FORCES ($\alpha = 1/3$ $\beta = \beta_1 = 0.10$)

Section	Moments				Shearing Forces				Normal Forces			
	Loading			Moments for Design	Loading			Shear for Design	Loading			Normal Force for Design
	(A)	(B)	(C)		(A)	(B)	(C)		(A)	(B)	(C)	
1 - 0	+470.0	-110.0	+230.0	+470.0 -110.0	-23.50	- 6.50	-20.50	-23.50	- 60.00	+ 3.46	-38.82	- 60.00
1 - 4	-470.0	+110.0	-230.0	-470.0 +110.0	+54.16	- 4.66	+33.98	+54.16	- 34.75	- 5.69	-27.71	- 34.75 - 5.69
4 - 1	-289.0	+ 54.4	-136.7	-289.0 + 54.4	- 4.60	- 6.07	-11.13	-11.13	- 23.00	-12.74	-24.14	- 23.00 - 12.74
4 - 2	+289.0	- 54.4	+136.7	+289.0 - 54.4	+ 4.60	- 0.71	- 0.99	+ 4.60	- 23.00	-14.10	-26.56	- 23.00 - 14.10
2 - 4	+470.0	+ 76.0	+552.0	+552.0	-54.16	- 0.71	+45.03	-54.16	- 34.75	-14.10	-35.35	- 35.35
2 - 0	0	-178.0	-132.0	0 -178.0	0	+ 8.90	+ 6.60	+ 8.90	-120.00	+ 1.34	-96.75	-120.00 + 1.34
2 - 5	-470.0	+102.0	-420.0	-470.0 +102.0	+54.16	- 4.17	+40.92	+54.16	- 34.75	- 4.05	-27.83	- 34.75 - 4.05
5 - 2	-289.0	+ 25.8	-156.2	-289.0 + 25.8	- 4.60	- 4.17	- 3.23	- 4.60	- 23.00	- 4.05	-19.01	- 23.00 - 4.05
5 - 3	+289.0	- 25.8	+156.2	+289.0 - 25.8	+ 4.60	- 2.29	+ 4.33	+ 4.60	- 23.00	- 5.35	-18.79	- 23.00 - 5.35
3 - 5	+470.0	+ 96.0	+386.0	+470.0	-54.16	- 2.29	-39.72	-54.16	- 34.75	- 5.35	-27.60	- 34.75
3 - 0	-470.0	- 96.0	-386.0	-470.0	+23.50	+ 4.80	+19.30	+23.50	- 60.00	- 3.30	-44.43	- 60.00
0 - 1	0	0	0	0	-23.50	+17.50	- 2.50	-23.90	- 60.00	+ 3.46	-38.82	- 60.00
0 - 2	0	0	0	0	0	+ 8.90	+ 6.60	+ 8.90	-120.00	+ 1.34	-96.75	-120.00
0 - 3	0	0	0	0	+23.50	+ 4.80	+19.30	+23.50	- 60.00	- 3.30	-44.43	- 60.00

4-4. Comparison of Critical Section Moments

The same structure is again analyzed for cases with $\beta = 0.20$ and $\beta = 0.30$ in a similar way for both cases of loading (A) and (B). The critical moments thus obtained from these analyses are tabulated and compared with the corresponding moments obtained in the previous elastic analysis, in Table 4-5.

From the "k" values of Table 4-5, it can be concluded that for an indeterminate structure, in general, portions of the structure which are less stressed as indicated by elastic analysis, carry greater ultimate moments as can be seen by values of "k" greater than 2.0 (load factor) while those portions of the structure which are highly stressed according to the same analysis, undergo reduction of moments, as shown by values of "k" less than 2.0, due to the redistribution of moments recognized in plastic or limit analysis.

In this particular study, it is observed, in the case of gravity loads that the degree of redistribution of moments among those critical sections reduces as the value of β increases (as β increases, the "k" value approaches load factor value = 2.0). However, no significant trend of redistribution in moments related to the value of β can be seen in the case of wind load. This is probably because of the resulting smaller values of moment due to wind load and the greater freedom allowed in limit design for a random selection of plastic moments within the limitation of rotation capacity requirements.

TABLE 4-5

COMPARATIVE TABLE OF END MOMENTS OBTAINED FROM BOTH THE ELASTIC AND PLASTIC ANALYSES

Sections	Elastic Analysis						Plastic Analysis						*k = $M_{plastic}/M_{elastic}$						Average "k"
	Case (A) Loading			Case (B) Loading			Case (A) Loading			Case (B) Loading			Case (A) Loading			Case (B) Loading			
	β=0.10	β=0.20	β=0.30	β=0.10	β=0.20	β=0.30	β=0.10	β=0.20	β=0.30	β=0.10	β=0.20	β=0.30	β=0.10	β=0.20	β=0.30	β=0.10	β=0.20	β=0.30	
1 - 0	+218.20	+215.16	+205.71	-59.60	- 98.03	-140.46	+470.00	+445.50	+410.86	-110.00	-210.00	-336.00	2.15	2.07	2.00	1.85	2.14	2.39	2.10
1 - 4	-218.20	-215.16	-205.71	+59.60	+ 98.03	+140.46	-470.00	-445.50	-410.86	+110.00	+210.00	+336.00	"	"	"	"	"	"	"
4 - 1	-130.04	- 91.60	- 59.07	+17.80	+ 41.17	+ 68.10	-289.00	-220.00	-130.00	+ 54.40	+ 42.60	+ 37.20	2.22	2.40	2.20	3.06	1.04	0.55	1.91
4 - 2	+130.04	+ 91.60	+ 59.07	-17.80	- 41.17	- 68.10	+289.00	+220.00	+130.00	- 54.40	- 42.60	- 37.20	"	"	"	"	"	"	"
2 - 4	+290.79	+243.60	+205.80	+26.64	+ 24.54	+ 21.90	+470.00	+380.00	+390.00	+ 76.00	0	0	1.62	1.56	1.90	2.85	-	-	1.98
2 - 0	0	0	0	-83.62	-109.17	-133.88	0	0	0	-178.00	-178.00	-220.00	-	-	-	2.13	1.63	1.65	1.80
2 - 5	-290.79	-243.60	-205.80	+57.04	+ 84.63	+111.98	-470.00	-380.00	-390.00	+102.00	+178.00	+220.00	1.62	1.56	1.90	1.79	2.10	1.97	1.82
5 - 2	-130.04	- 91.60	- 59.07	+10.40	+ 20.25	+ 29.98	-289.00	-220.00	-130.00	+ 25.80	+ 65.00	+ 52.40	2.22	2.40	2.20	2.48	3.21	1.75	2.38
5 - 3	+130.04	+ 91.60	+ 59.07	-10.40	- 20.25	- 29.98	+289.00	+220.00	+130.00	- 25.80	- 65.00	- 52.40	"	"	"	"	"	"	"
3 - 5	+218.20	+215.16	+205.71	+48.65	+ 56.74	+ 61.43	+470.00	+445.50	+410.86	+ 96.00	+140.00	+116.00	2.15	2.07	2.00	1.97	2.47	1.89	2.09
3 - 0	-218.20	-215.16	-205.71	-48.65	- 56.74	- 61.43	-470.00	-445.50	-410.86	- 96.00	-140.00	-116.00	"	"	"	"	"	"	"
							Range of Redistribution Ratios "k"						1.62 & 2.22	1.56 & 2.40	1.90 & 2.20	1.79 & 3.06	1.04 & 3.21	0.55 & 2.39	1.80 & 2.38

*Note: The load factor used in the plastic analysis is 2.0, therefore, a value of "k" greater than 2.0 indicates an increase in moment due to the redistribution of moments recognized in plastic analysis. Also, a value of "k" less than 2.0 indicates a decrease in moment due to the redistribution.

CHAPTER V

BRIEF DESIGN OF CRITICAL SECTIONS

5-1. General

A brief design of critical sections of the reinforced concrete frame is carried out in this chapter. The conventional design method for reinforced concrete members is applied to the resulting end moments under working load obtained in Chapter II, and the ultimate strength method of designing reinforced concrete members is applied to the resulting end moments under ultimate load obtained in Chapter IV. In the former case, the ACI Building Code of 1956, and ACI Reinforced Concrete Design Handbook are used, and for the latter case Guide for Ultimate Strength Design of Reinforced Concrete is followed. (7), (9), (10). Only those design procedures for a couple of typical sections are shown in detail, and no design for shearing and bond stress is considered.

The designed results of necessary cross-sectional dimensions and the corresponding steel requirements are tabulated for both methods of design, and compared in Table 5-1 (page 57).

5-2. Conventional Design

The following data are used in the design:

$$f_s = 20{,}000 \text{ psi} \qquad n = \frac{E_s}{E_c} = 10 \qquad f'_c = 3000 \text{ psi}$$

$$f_c = 0.45\, f'_c = 1350 \text{ psi} \qquad b = 12'' \quad t = 27''$$

$$d = 24'' \qquad d' = 3''$$

The design follows.

(A) Design of Girder (Section 2-4)

$$M = 290.79^{k\text{-}ft.} \quad N = -16.82^{k} \quad b = 12'' \quad d = 24''$$

$$d' = 3'' \qquad t = 27''$$

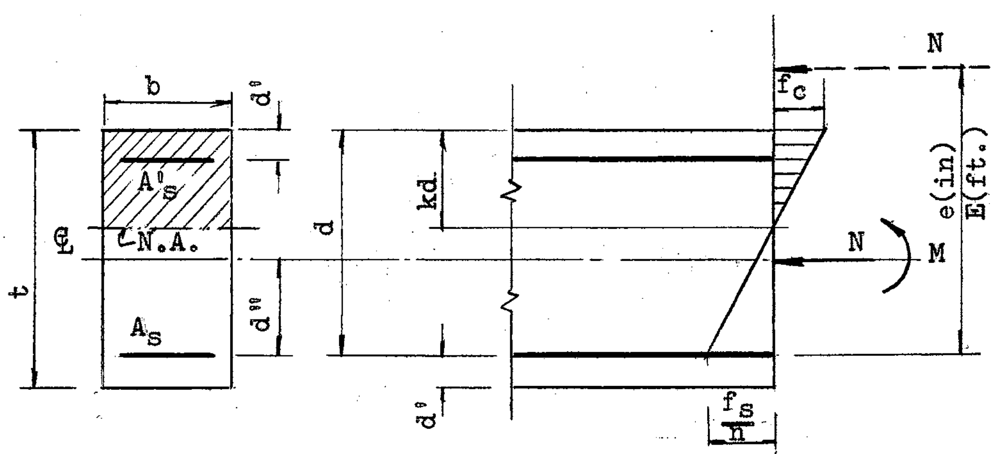


Figure 5-1. General Sketch of Sections

$$e = \frac{12 \times 290.79}{16.82} + 10.5 = 217.5'' \qquad E = \frac{e}{12} = \frac{217.5}{12} = 18.15$$

$$\frac{e}{d} = \frac{217.5}{24} = 9.07$$

1. From Table 1, for 20,000/10/1350 $\quad k = 236$

From Table 4, for bd = 12 x 24 $\quad F = 0.576$

then,

$$NE = 16.82 \times 18.15 = 305.4^{k\text{-}ft.}$$

$$kF = 236 \times 0.576 = 135.9^{k\text{-}ft.}$$

$$NE - kF = 169.5^{k\text{-}ft.}$$

Compressive reinforcement is required, since (NE - kF) is positive.

2. From Table 7b, for $\frac{d'}{d} = 0.125 \quad c = 1.29$;

therefore,

$$A's = \frac{NE - kF}{cd} = \frac{169.5}{1.29 \times 24} = 5.47 \text{ in}^2 .$$

3. From Table 10, for $\frac{e}{d} = 9.07 \quad j = 0.866 : i = 1.11$

From Table 1, for fs = 20,000 psi $\quad a = 1.44$;

therefore,

$$As = \frac{NE}{adi} = \frac{290.79}{1.44 \times 24 \times 1.11} = 7.58 \text{ in}^2 .$$

$$\Sigma As = As + A's = 7.58 + 5.47 = 13.05 \text{ in}^2 .$$

(B) Design of Column (Section 1 - 0) $\frac{e'}{t} > 1$

$M = 218.2^{k\text{-}ft} \quad P' = 28.79^{k} \quad b = 12'' \quad d = 24''$

$d' = 3'' \quad t = 27'' \quad h = 20 \text{ ft.}$

1. $\frac{h}{t}$ Ratio

$$\frac{h}{t} = \frac{20 \times 12}{12} = 20 > 10 \quad \text{Long Column}$$

According to ACI Code, Sec. 1107, the equivalent eccentrically applied load on a short column is given by

$$P = \frac{P'}{[1.3 - 0.03 \frac{h}{t}]} = \frac{28.79}{1.3 - 0.03 \times 20}$$

$$= 41.2^{k} \quad (10).$$

Also,

$$e = \frac{12 \times 218.2}{28.79} + 10.5 \doteq 101.5'' \quad E = 8.45''$$

$$\frac{e}{d} = \frac{101.5}{24} = 4.23 \; .$$

2. From Table 1, for 20,000/10/1350 $\quad k = 236$

From Table 4, for bd = 12 x 24 $\quad F = 0.576$.

Then

$$NE = 41.2 \times 8.45 = 348^{k\text{-}ft.}$$

$$kF = 236 \times 0.576 = 135.9^{k\text{-}ft.}$$

$$NE - kF = 212.1^{k\text{-}ft.}$$

Compressive reinforcement is required.

3. From Table 7b, for $\frac{d'}{d} = 0.125 \quad c = 1.29$

$$As' = \frac{NE - kF}{cd} = \frac{212.1}{1.29 \times 24} = 6.85 \text{ in.}^2 .$$

4. From Table 10, for $\frac{e}{d} = 4.23$ $j = 0.866$,

$i = 1.258$, also $a = 1.44$.

$$A_s = \frac{NE}{adi} = \frac{348}{1.44 \times 24 \times 1,258} = 8.02 \text{ in.}^2$$

$$\Sigma A_s = As + A's = 8.02 + 6.85 = 14.87 \text{ in.}^2$$

(C) Design of Column (Section 2-0) $\frac{e}{t} < 1$

$M = 83.68^{k-ft}$ $P' = 62.42^k$ $b = 12''$ $d = 15''$

$t = 20''$ $h = 20$ ft.

1. $\frac{h}{t}$ Ratio

$$\frac{h}{t} = \frac{20 \times 12}{12} = 20 > 10 \quad \text{Long Column}$$

therefore,

$$P = \frac{P'}{[1.3 - 0.03 \frac{h}{t}]} = \frac{62.42}{0.7} = 89.2^k .$$

2. For $g = 15/20 = 0.75$ from Table 27, part 1, the average value of $D = 5.25$, and from part 2, with 20,000/10/3,000 and an estimated $p = 0.030$, determined $C = 0.58$.

Compute

$$CD \frac{12M}{t} = 0.58 \times 5.25 \times \frac{12 \times 83.68}{20} = 153^k$$

Add $N = 89.2^k$

$P = 242.2^k$.

Equivalent eccentric load:

From Table 18, part 1, with

$A_g = 12 \times 20 = 240 \text{ in.}^2$

and $f'_c = 3000$ psi, load on concrete = $\underline{130^k}$

Balance to be carried by longitudinal bars = 112.2^k

From Table 18, part 2, select 6 No. 10 bars,

$A_s = 7.62 \text{ in}^2$

Actually $p = \frac{A_s}{A_g} = \frac{7.62}{240} = 0.031 \approx$ assumed $p = 0.030$

O.K.

(D) Design of Other Critical Sections

Other sections of the frame are also designed similarly for their cross-sectional dimensions and the corresponding steel requirements. The final results are tabulated in section 5-4.

5-3. Ultimate Strength Design (Whitney's Method)

The following data are used in the design:

$f_y = 40{,}000$ psi $\quad f'_c = 3000$ psi $\quad b = 12''$

$t = 27'' \quad d = 24'' \quad d' = 3''$

The design follows on page 53.

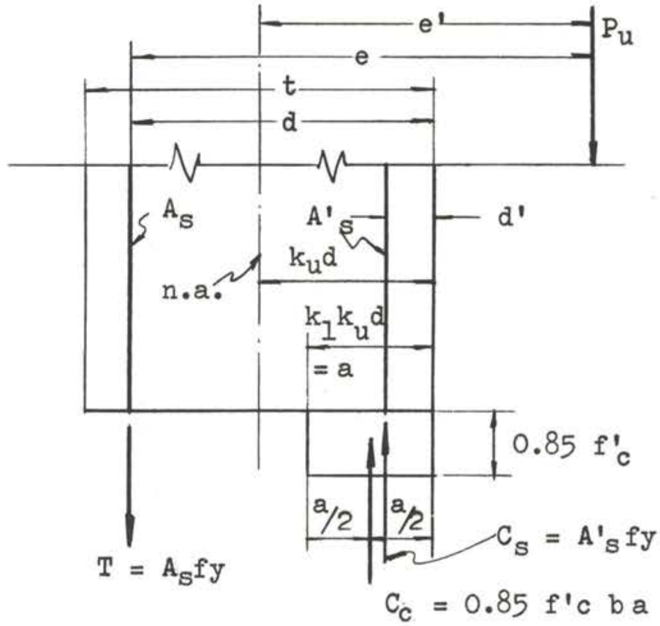


Figure 5-2. Ultimate Strength Notation With Rectangular Stress Block

(A) Design of Girder (Section 2-4)

$M_u = +552^{k-ft}$ and $Mu = -102^{k-ft}$ $\quad N = 35.35^k$

Since the normal force is relatively small as compared to the bending moment, only the bending moment Mu will be considered in the design.

As a rectangular beam without A's the maximum steel for A_{s1} is limited by $p_1 = 0.40 \frac{f'c}{fy}$, which provides

$$M_1 = 0.306 \ f'c \ bd^2 = 0.306 \times 3000 \times 12 \times 24^2 \times \frac{1}{12{,}000}$$

$$= 528^{k-ft}.$$

$$A_{s1} = 0.40\ f'c\ \frac{Ac}{fy} = 0.40 \times 3000 \times 12 \times 24/40{,}000$$

$$= 8.63\ \text{in.}^2$$

$$M_2 = M_u - M_1 = 552 - 528 = 24^{k\text{-}ft.}$$

$$T_2 = C_2 = M_2/(d - d') = \frac{24 \times 12{,}000}{24 - 3} = 13{,}700\ \text{lb.}$$

$$A_{s2} = \frac{T_2}{fy} = \frac{13{,}700}{40{,}000} = 0.34\ \text{in.}^2$$

$$\Sigma A_s = A_{s1} + A_{s2} = 8.63 + 0.34 = 8.97\ \text{in.}^2$$

Since $a = \frac{A_s f_s}{0.85\ f'c}$, the ultimate moment is expressed as:

$$Mu = A_s f_y\ (d - \frac{a}{2}) = A_s f_y (d - 1.308\ \frac{A_s}{2})$$

for $Mu = 102^{k\text{-}ft.}$, the steel requirement is found to be:

$$A_s = 1.35\ \text{in}^2 > A_{s2} = 0.34\ \text{in}^2\text{, use } A_{s2} = 1.35\ \text{in}^2$$

$$\Sigma A_s = 8.63 + 1.35 = 9.98\ \text{in}^2$$

(B) Design of Column (Section 1-0) $\frac{e'}{t} > 1$

$Mu = 470^{k\text{-}ft}$ $P'_u = 60^{k.}$ $b = 12''$ $d = 24''$

$d' = 3''$ $t = 27''$ $h = 20^{ft.}$

1. $\frac{h}{t}$ Ratio

$$\frac{h}{t} = \frac{20 \times 12}{12} = 20 > 15 \qquad \text{Long Column}$$

According to the recommendation given for the slender compression members in page 471 of the reference (7), the equivalent eccentrically applied load on a short column is given by:

$$P_u = \frac{P_{u'}}{[1.6 - 0.04 \frac{h}{t}]} = \frac{60}{0.8} = 75^k.$$

Since $\quad C_s(d - d') = P_u z \quad$ and $\quad e' = \frac{470 \times 12}{60} = 94''$

$$A'_s \, f_y(d - d') = P_u(e' - 0.5t + 0.5a).$$

For $\quad d' = 3'' \quad d - d' = t - 2d' = t - 6 \qquad$ assume $a = 2''$

then, $\quad 40,000 \, A'_s(t - 6) = 60,000(94 - 0.5t + 0.5 \times 2)$

For $\quad t = 27'' \qquad A'_s = 5.8 \text{ in}^2$

$$a = \frac{P'_u}{0.85 \, f'c \, b} = \frac{60,000}{0.85 \times 3000 \times 12} \doteq 1.96'' \doteq 2''$$

as assumed O.K.

$$P'_u z = 60,000 \, (94 - 13.5 + 0.98) \doteq 4,888,000^{\text{lb-in.}}$$

$$P_u z = 75,000 \times 81.48 = 6,111,000^{\text{lb-in.}}$$

$$A_s' f_y(d - d') = P_u z = 6,111,000^{\text{lb-in.}}$$

$$A'_s = \frac{6,111,000}{40,000 \times 21} \doteq 7.28 \text{ in}^2 = A_s$$

$$\Sigma A_s = 7.28 + 7.28 = 14.56 \text{ in.}^2$$

(C) Design of Column (Section 2-0)

1. $P'_u = 120$ k. $M_u = 0$ $b = 12''$ $t = 20''$

$$P_u = \frac{120}{0.8} = 150 \text{ k.} \qquad f_c = \frac{P_u}{bt} = \frac{150,000}{12 \times 27} = 463 \text{ psi}$$

$$< f'c = 3000 \text{ psi.}$$

No reinforcement is required.

2. $M_u = 178$ k-ft. $P'_u = 1.34 \text{ k} \doteq 0$ $b = 12''$

$t = 20''$ $d = 17.5''$ $d' = 2.5''$

In this case, the section can be considered as under the action of M_u only.

$$M_c = 0.306 f'c\, bd^2 = 0.306 \times 3000 \times 12 \times (20)^2$$

$$\doteq 441 \text{ k-ft.} > M_u = 178 \text{ k-ft.} \qquad \text{O.K.}$$

Also, $M_u = A_s f_y (d - \frac{a}{2})$ $a = 1.308\, A_s$

$$178,000 \times 12 = 40,000\, A_s(17 - 0.654\, A_s)$$

$$A_s \doteq 3.5 \text{ in.}^2 = A'_s$$

$$\Sigma A_s = 3.5 + 3.5 = 7.0 \text{ in.}^2$$

(D) Design of Other Critical Sections

Similarly, other critical sections of the frame are also designed for their cross-sectional dimensions and corresponding steel requirements. The final results are also tabulated in Table 5-1.

5-4. Table of Dimensions and Steel Requirements

TABLE 5-1

Section	Cross-Sectional Dimension	Steel Area Requirements					
		Elastic Analysis			Plastic Analysis		
		A_s	A'_s	$A_s+A'_s$	A_s	A'_s	$A_s+A'_s$
1 - 0 3 - 0	12" x 27" "	8.02	6.85	14.87	7.28	7.28	14.56
1 - 4 3 - 5	" "	5.93	3.14	9.07	7.35	1.45	8.80
4 - 1 4 - 2 5 - 2 5 - 3	" " " "	3.50	0.36	3.86	4.05	0.70	4.75
2 - 4 2 - 5	" "	7.58	5.47	13.05	8.63	1.35	9.98
2 - 0	12" x 20"	3.81	3.81	7.62	3.50	3.50	7.00
Total Steel Area				48.37 in²			45.09 in²

% saving of steel

$$\frac{\Sigma(A_s)_E - \Sigma(A_s)_P}{\Sigma(A_s)_P} \times 100 = \frac{48.37 - 45.09}{45.09} \times 100 \doteq 7.5\%$$

CHAPTER VI

SUMMARY AND CONCLUSIONS

6-1. Summary

A two-span hinged base reinforced concrete frame was analyzed both by elastic and plastic methods and briefly designed for its critical sections. The dimensions are the same, and the moment of inertia is assumed constant throughout the frame for both methods.

In the case of elastic analysis, end moments for three values of parameter β, with three different gravity loads and a constant wind load, were obtained by utilizing the moment coefficients. Analysis of the same frame, by the plastic approach, was done by the application of the plastic hinge theory for reinforced concrete frames, generally considered as a part of the theory of limit design.

A detailed analysis was carried out for the case with $\beta = 0.1$, $W_{D+L} = 1^k/ft.$, $W_W = .6^k/ft.$, and L.F. = 2.0, however, only brief analyses of the critical end moments for the cases with $\beta = 0.2$ and 0.3, were done for the purpose of comparison.

A brief design of cross-sectional dimensions and the corresponding steel requirements was carried out. For the conventional design, the 1956 ACI Building Code (10) was followed and, for the ultimate strength design, procedures recommended by Whitney (7) was followed. Finally, results obtained were tabulated and compared. In this particular study, it was revealed that approximately 7.5% of steel might be saved by the adoption of the plastic approach.

6-2. Conclusions

Two basically different methods of analysis were presented in this report. The conventional method of approach to the analysis of indeterminate structures is based on the theory of elastic deformation and is generally recognized as the most efficient and powerful tool in the solution of structural problems. The second method of approach differs from the first, in recognizing the readjustment in the relative magnitude of moments, and thereby the corresponding stresses at various sections, due to non-linear relationship between load and moment as ultimate load, is approached. Regarding the plastic method of approach, the following conclusions were made:

(A) The maximum load capacity of statically determinate structures with sections proportioned by ultimate strength design equals the capacity computed by equilibrium conditions alone. For indeterminate structures, however, the maximum moments at various sections as calculated by the theory of elastic displacements are due to different load combinations. Therefore, the maximum load capacity of an indeterminate structure as a whole may be considerably greater than that indicated by the ultimate strength of one section. By limit design, the moment redistribution involved is considered in design, and the maximum load capacity will equal the calculated capacity for an indeterminate structure.

(B) Limit design is simpler than the design by the theory of elastic displacements, especially for those structures with large number of redundants. The former permits an intelligent, arbitrary choice of redundant moments, while the latter requires solution of simultaneous equations.

(C) A reduction of negative support moments by limit design would avoid reinforcement congestion. This would be an advantage particularly in buildings where negative beam reinforcement in two directions intersects the column reinforcement. By reducing beam moments at these joints and, thereby, the corresponding amount of negative reinforcement required, better concrete placement and compaction would become possible, and an improved concrete structure would result.

This report is expected to serve as a guide to the future application of the concept of limit design to the analysis of indeterminate reinforced concrete framed structures.

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