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Major Field: Civil Engineering

Scope of Study: The objective of this report is to show the inherent advantages of plastic design over conventional elestic design. The application of plastic design is limited to continuous beams and single or multi span frames.

Findings and Conclusions: It is shown that for a two span frame, the plastic method of approach offers an easier solution and eliminates the solution of simultaneous equations which is a necessary step in conventional elastic methods of analysis. This would become a cumbersome task, as the indeterminacy of structure increases. The writer is in favour of the rational approach of plastic design for the analysis and design of indeterminate structures. Furthermore, it economises the material.

ADVISER'S APPROVAL ______________________________

With Sincere Wishes
to Dr. James. W. Gillespie.
Aug 8, 1963

COMPARATIVE DESIGNS BY ELASTIC AND PLASTIC METHODS FOR A TWO SPAN PIN-BASED GABLE FRAME

By

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I would be grateful to my parents for their assistance in carrying out my academic career up to this level. In last, I would appreciate the efforts of Judy Tilton for typing the manuscripts of this report.

S. M. A.

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NOMENCLATURE

α	=	Roof Rise Parameter
β	=	Column Height Parameter
E	=	Young's Modulus of Elasticity
F	=	Load Factor of Safety
f	=	Shape Factor $= \frac{MP}{My} = \frac{Z}{S}$
I	=	Moment of incrtia.
L	=	Span Length
M_s	=	Simple Span Moment
M_P	=	Plastic Moment
N	=	No. of Possible Plastic hinges -- Normal force
n	=	No. of Possible Indepent Mechanisms
P_u	=	Ultimate Load
r	=	Radius of Gyration or Thrust Induction Factor
S	=	Section Modulus I/C
V	=	Shear Force
W	=	Total Distributed Load
X	=	Number of Redundancies
Z	=	Plastic Modulus $Z = MP/\sigma y$.
σ_y	=	Yield Stress
σ_w	=	Working Stress

CHAPTER I

INTRODUCTION

1-1. GENERAL

The very existence of innumerable steel structures is sufficient proof that the design methods in practice are adequate enough to produce useful structures but when we study the failure of a structure we find that it may fail by reaching its limit of usefulness through instability, fatigue or excessive deflection. Alternatively if none of these modes of failure occur then the structure will continue to carry load beyond the elastic limit until it reaches its ultimate load, through plastic deformation and then collapses. Most indeterminate structural frames fall into this category. It means the outstanding property of structural steel that is "Ductility" was not taken into account in elastic methods of analysis while plastic methods make conscious use of this property and provide a rational approach to the analysis of indeterminate structures.

Plastic Design is not based on the allowable stress concept but on the maximum load carrying capacity of the structure. Thus it economizes material as compared with conventional elastic design.

The objective of this report is to show the influence of some variation in the geometric proportions of a certain type of gable frame upon the differences in the member sizes as determined by plastic and conventional elastic methods. Indeterminate frames designed by plastic methods are known to be lighter than if they are designed by conventional elastic methods. It is hoped that this report will indicate the magnitude of the differences.

1-2. FURTHER ASSUMPTIONS (PLASTIC ANALYSIS)

1. The material is ductile. It has the capacity of absorbing plastic deformation without the danger of fracture.

2. Each member has a maximum moment of resistance "M_p", a moment that is attained through plastic yield of the entire cross section.

3. Due to the ductility of steel, rotation at relatively constant moment will occur through a considerable angle resulting in the formation of plastic hinges.

4. Connections will transmit the desired "M_p" forming a theoretical continuity and provide for hinge action.

5. As a result of the formation of plastic hinges at connections and other points of maximum moment redistribution of moment will occur allowing the formation of plastic hinges at points that are otherwise less highly stressed.

6. The ultimate load may be computed with accuracy on the basis that a sufficient number of plastic hinges have formed to create a mechanism.

7. The effect of Axial force and the shear force is neglected.

See the illustration of above in Figure 1-1. from reference "3".

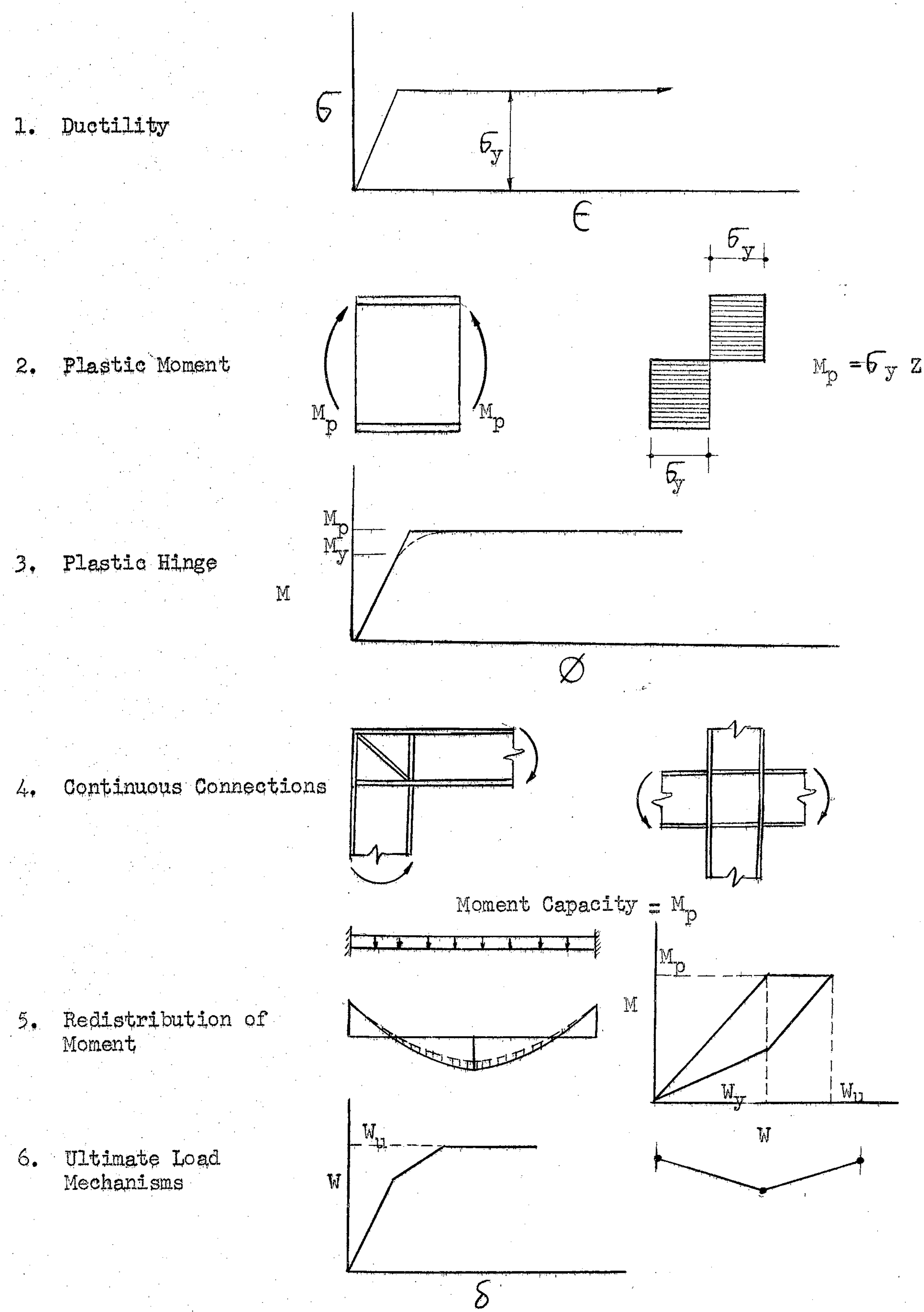


Fig. 1-1. Assumptions Illustration

1-3. REPORT DESCRIPTION (IN BRIEF).

The two-span pin-based "gable frame" was chosen to demonstrate the comparative analysis and design.

The elastic analysis is done by the method of moment distribution and the results of the three sets of parameters were tabulated in Chapter II. The plastic analysis was carried out in Chapter III along with the detailed example. In both analyses the gravity load governs due to higher load factor being provided in case of plastic design.

The analyses were carried out by assuming straight connections, although by introducing haunches the sections could be further reduced. However for the sake of comparison the connections were kept straight and the detailed procedure of design by both methods presented in Chapter IV.

In conclusion a summary of the comparative results by both methods is presented in tabular and chart form. The influence of the parameters is clearly indicated by the sections chosen and the resulting savings in weight.

CHAPTER II

ELASTIC ANALYSIS

2-1. "Data for Analysis" of "Two-Span Pin-Based Gable Frame":

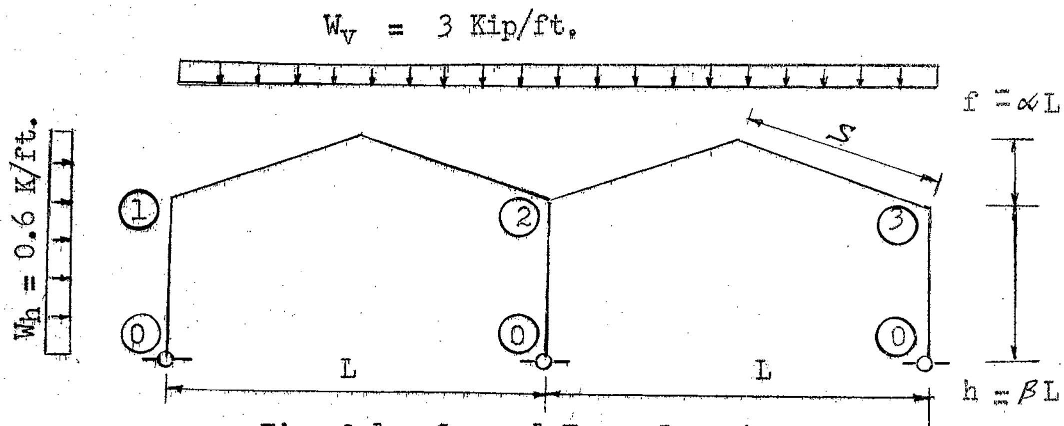


Fig. 2-1. General Frame Layout

1 PARAMETERS

a. Span length L = 60 ft.

b. Height & Roof Rise Coeff: with respect to span length.

Case	1		2		3	
α	0.1		0.2		0.3	
β	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{3}$

Table 2-1: Parameter Combinations

2 <u>FRAME SPACING</u> -------- 20 ft.

3 <u>LOADING</u>

W_v = Vertical Loading = 3 Kips per foot

W_h = Horizontal Loading = 0.6 Kips per foot

2-2. Analysis by Moment Distribution (Vide Ref. No. 9 of Bibliography)

Although limited essentially to the continuous beams and frames is so much faster than any other method that it has revolutionized structural analysis.

Thus for analyzing the end moments, shearing forces and normal forces at the given frame we will employ the method of "Moment Distribution". The procedure is outlined in detail.

GENERAL PROCEDURE

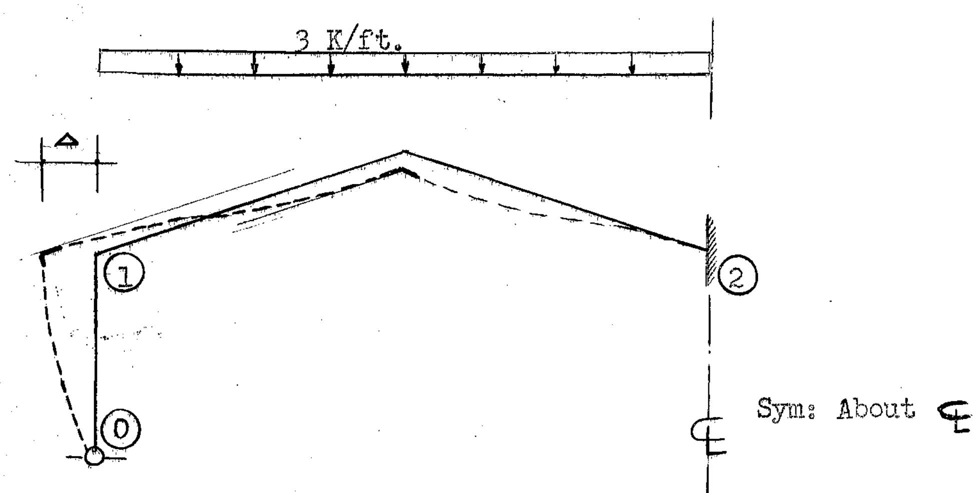


Fig. 2-2.

1 <u>DEFORMATIONS</u>

Due to symmetry of loading and structure we will assume that it is fixed at "2" and analysis will be done for half the structure.

$\theta_1 = \theta_3$; $\theta_5 = \theta_0$ $\theta_2 = 0$

$\Delta_{1x} = -\Delta_{3x}$; $\Delta_{2x} = 0$

2 STIFFNESS FACTORS

a. For bent members

$$K_{12} = K_{21} = K_{23} = K_{32} = \frac{7EI}{2S}$$

b. For columns

$$K'_{10} = K'_{20} = K'_{30} = \frac{3}{4}\left(\frac{4EI}{h}\right) = \frac{3EI}{h}$$

3 CARRY OVER STIFFNESS FACTOR

a. For bent members

$$C_{12} = C_{21} = C_{23} = C_{32} = -\frac{1}{7}$$

b. For column (Being pinned base)

$$C_{10} = C_{20} = C_{30} = 0$$

4 DISTRIBUTION FACTORS

$$D_{10} = \frac{K'_{10}}{\Sigma K_{10}} \qquad \therefore D_{12} = (1 - D_{10})$$

5 FIXED END MOMENTS

(i) Due to loads

$$FM^{L}_{12} = -FM^{L}_{21} = -\frac{wL^2}{48}$$

(ii) Due to Δ_x

a. For bent members

$$FM^{\Delta}_{12} = -FM^{\Delta}_{21} = -\frac{3EI}{sf}$$

b. For column (modified)

$$EM^{\Delta}_{10} = FM^{\Delta}_{10} - \frac{1}{2} FM^{\Delta}_{01} = \frac{1}{2} FM^{\Delta}_{10} = \frac{1}{2} \frac{6EI}{h^2} \Delta$$

6 FIXED END THRUSTS

(i) Due to load

$$FH^{L}_{12} = \frac{wL^2}{8f}$$

(ii) Due to $\triangle$ (for bent members)

$$FH_{12}^{\triangle} = -\frac{6\,EI}{s\,f^2}\triangle_x$$

7 THRUST INDUCTION FACTOR

$$r_{12} = \frac{3}{4f}$$

Definition (brief):

The coefficient, if applied to rotational moment, gives the corresponding rotational thrust. When $FH^L = 0 = FH^{\triangle}$.

Thus, in other words, thrust induced in a member such that it procuces a unit rotational moment.

8 MOMENT DISTRIBUTION PROCEDURES

(i) Due to load

(ii) Due to $\triangle$

9 EVALUATION OF THRUST

$$H_{12} = r_{12}\,(RM_{12} - RM_{21}) + FH_{12} + FH_{12}^{L}$$

10 SHEAR EQUATION (For Symmetrical Case)

$$\sum F_x = 0 \qquad H_{12} - V_{10} = 0$$

11 FINAL MOMENTS & THRUSTS

2-3 ILLUSTRATION

Case I: $\alpha = 0.1$ $\quad \beta = \frac{1}{2}$

$f = \alpha L = 0.1\,(60) = 6$ ft. ; $\quad s = \sqrt{6^2 + 30^2} = 30.6$ ft.

$h = \beta L = 0.5\,(60) = 30$ ft. ; $\quad e = \frac{f}{2} = \frac{6}{2} = 3$ ft.

Elastic Constants:

$$I_{xx} = \frac{sf^2}{6EI} = \frac{(30.6)\,(6)^2}{6EI} = \frac{183.8}{EI}$$

$$I_{yy} = \frac{sL^2}{6EI} = \frac{(30.6)\ (\overline{60}^2)}{6EI} = \frac{18380}{EI}$$

2 Stiffness Factor:

$$K_{12} = K_{21} = K_{23} = K_{32} = \frac{7EI}{2\ (30.6)} = 0.1144\ EI$$

$$K'_{10} = K'_{20} = K'_{30} = \frac{3EI}{h} = \frac{3EI}{30} = 0.100\ EI$$

3 Carry over Stiffness Factors:

$$C_{12} = C_{21} = C_{23} = C_{32} = -\frac{1}{7}$$

$$C_{10} = C_{20} = C_{30} = 0$$

4 Fixed End Moment:

(i) Due to loads

$$FM_{12}^{L} = -FM_{21}^{L} = -\frac{wL^2}{48} = -\frac{w\ (60)^2}{48} = -75\ w$$

(ii) Due to $\triangle_x$

$$FM_{12}^{\triangle} = FM_{21}^{\triangle} = -\frac{e}{I_x}\triangle_x = -\frac{3EI}{sf}\triangle_x = -\frac{3EI}{30.6x6}\left\{ 0 - \right.$$

$$= -\frac{EI}{61.2}\triangle_x = -0.0\ 1633\ EI\triangle_x = -490\ x$$

$$EM_{10}^{\triangle} = \frac{1}{2}FM_{10}^{\triangle} = \frac{1}{2}\ \frac{6EI}{h^2}\triangle_x = \frac{3EI}{(30)^2}\triangle_x$$

$$= 0.0033\ EI\triangle_x = 100\ x\ \text{(assumed)}$$

To facilitate working in the distribution table and thus eliminating errors of less than 1% in the distribution process.

5 Distribution factors:

$$D_{10} = \frac{K_{10}}{K_{10} + K_{12}} = \frac{0.100\ EI}{0.100\ EI + 0.1144\ EI} = \frac{0.100}{0.2144} = 0.467$$

$$D_{12} = (1 - 0.467) = 0.533$$

6 Fixed End Thrusts:

(i) Due to load

$$FH_{12}^{L} = \frac{wL^2}{8f} = \frac{W\,(60)^2}{8(6)} = 75\ w \qquad \text{Kips.}$$

(ii) Due to Δ_x

$$FH_{12}^{\Delta} = -\frac{1}{\bar{I}_x}\Delta_x = -\frac{6\,EI}{sf^2}\Delta_x$$

$$= -\frac{EI}{183.8}\Delta_x = -0.00\ 545\ EI\ {}_x = -163.5\ x$$

7 Thrust Induction Factor:

$$r_{12} = \frac{3}{4f} = \frac{3}{4 \times 6} = \frac{1}{8} = 0.125$$

8 Moment Distribution Procedure:

(i) Due to load

	10	12	21
D.F	0.467	0.533	- - -
C.F	- - -	-1/7	- - -
$F.M^L$	- - -	-75 w	+ 75 w
	+35.0 w	+40.0 w →	-5.72 w
R.M	+35.0 w	+40.0 w	-5.72 w
M'S	+35.0 w	-35.0 w	+69.28 w

(ii) Due to Δ_x

	10	12	21
D.F	-0.467	-0.533	- - -
C.F	- - -	- 1/7	- - -
$F.M^{\Delta}$	+100 x	-490 x	+490 x
	+182 x	+208 x →	-29.7 x
R.M	+182 x	+208 x	-29.7 x
M'S	+282 x	-282 x	+460.3 x

9 Thrust:

$$H_{12} = r_{12}\,(RM_{12} - RM_{21}) + FH^{\triangle}_{12} + FH^{L}_{12}$$

$$= 0.125 \quad (40.0\ w + 208\ x) - (-5.72\ w - 29.7\ x)$$

$$- 163.5\ x + 75\ w$$

$$= 0.125\ (45.72\ w + 237.7\ x) \ - 163.5\ x + 75\ w$$

$$= 80.7\ w - 133.9\ x$$

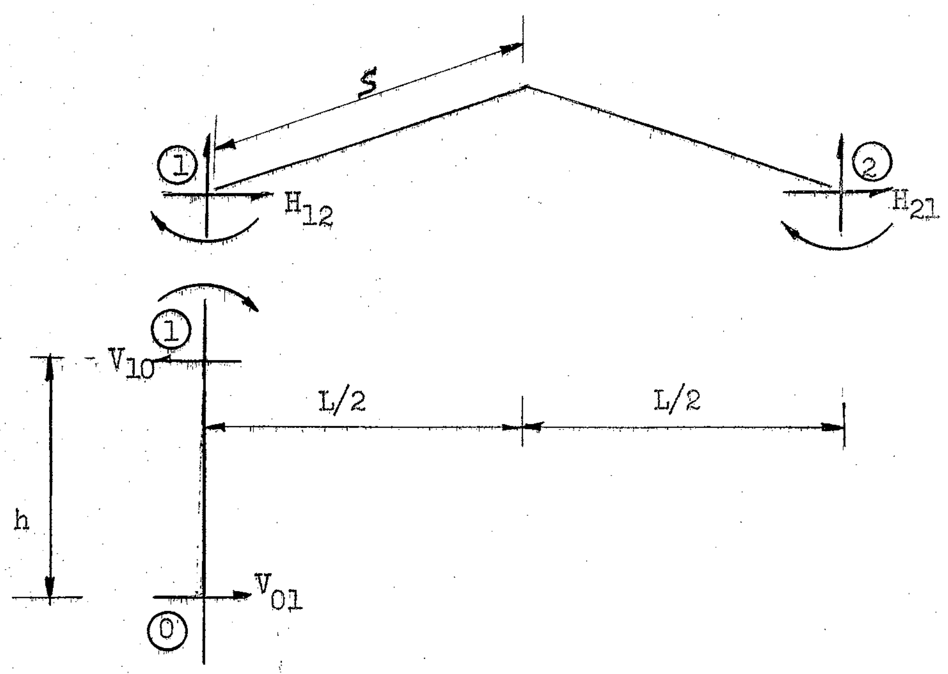


Fig. 2-3

10 Shear Equations:

$$\sum F_x = 0$$

$$-V_{10} + H_{12} = 0$$

$$\frac{M_{10}}{30} + (80.7\ w - 133.9\ x) = 0$$

$$-\frac{1}{30}\ (35.\ \ w + 282\ x) - (80.7\ w - 133.9\ x) = 0$$

$$143.3\ x = 79.53 \qquad \therefore \qquad x = 0.555\ w$$

11. Final Moments & Thrusts:

$M_{01} = M_{02} = M_{03} = M_{20} = 0$

$M_{10} = -M_{30} = 35.0\ w + 282\ (0.555\ w) = +191.5\ w$

$M_{12} = -M_{32} = -35.0\ w - 282\ (0.555\ w) = -191.5\ w$

$M_{21} = -M_{23} = +69.28 + 460.3\ (0.555\ w) = +325.3\ w$

$H_{12} = H_{23} = -H_{21} = -H_{32} = 80.7\ w - 133.9\ (0.555\ w) = +6.3\ w$

2-3(a) INVESTIGATION FOR WIND EFFECT.

For the cases I and II, the roof rise was within moderate limits, it was presumed that wind effect may not be critical and the analysis expedited for gravity loads only. For the case III roof rise was "18ft." Therefore, analysis run for with wind and without wind effect. The entire analysis would be the same as presented for case I only the final thrusts are presented here.

Case A: Vertical Loading Only (without wind)

The final thrust value deduced by routine analysis as

$$H_{12} = H_{23} = -H_{21} = -H_{23} = \boxed{7.51\ w}$$

Case B: Vertical and Horizontal Loading (with wind)

In this case again the final thrust value evaluated by the routine analysis as

$$H_{12} = H_{23} = -H_{21} = -H_{23} = \boxed{2.88\ w}$$

Thus, even in this extreme case the thrust values indicate that the case would be critical without considering the wind effect in the analysis.

2-3(b). FINAL RESULTS.

The final end moment and axial forces in the members of the two-span pin-based "Gable Frame" for three different cases of parameters and $w = 3$ k/ft. have been tabulated in Table 2-1.

TABLE 2-2

FOR END MOMENTS, AXIAL FORCES AND THRUST IN THE MEMBERS OF THE TWO-SPAN PIN-BASED GABLE FRAME ANALYZED BY MOMENT DISTRIBUTION METHOD

W = 3 K/ft.

Parameters		End Moments (Kip - ft)					Axial Forces (Kips)			Thrusts (Kips)		
α	β	M_{10} M_{32}	M_{12} M_{30}	— M_{21}	— M_{23}	$M_{01}M_{02}$ $M_{03}M_{20}$	N_{10} N_{01}	N_{20} N_{02}	N_{30} N_{03}	H_{12} H_{23}	H_{21} H_{32}	N_{12} N_{21} N_{23} N_{32}
0.1	$\frac{1}{2}$	+575.0	-575.0	+975.9	-975.9	0	-90	-180	-90	+18.9	-18.9	-36.15
	$\frac{1}{3}$	+652.0	-652.0	+875.0	-875.0	0	-90	-180	-90	+33.0	-33.0	-50.0
0.2	$\frac{1}{2}$	+583.0	-583.0	+854.0	-854.0	0	-90	-180	-90	+19.5	-19.5	-51.5
	$\frac{1}{3}$	+645.0	-645.0	+728.0	-728.0	0	-90	-180	-90	+32.50	-32.5	-63.60
0.3	$\frac{1}{2}$	+603.0	-603.0	+770.0	-770.0	0	-90	-180	-90	+20.0	-20.0	-63.3
	$\frac{1}{3}$	+716.0	-716.0	+698.0	-698.0	0	-90	-180	-90	+22.50	-22.5	-65.45

CHAPTER III

PLASTIC ANALYSIS

3-1. MECHANISM METHOD (Based on Upper Bound Theorem)

This method is a more powerful tool for solution of complicated structures.

Objective: Find a mechanism (independent or composite) such that

$$M \leqslant M_p$$

Procedure:

a. Determine location of possible plastic hinges (load points connections, point of zero shear)

b. Select possible "independent" and composite mechanism

c. Solve equilibrium equation (virtual displacement method) for the lowest load

d. Check to see $M \leqslant M_p$ at all sections.

Illustrative example:

Gable Frame Analysis (Pin-Based)

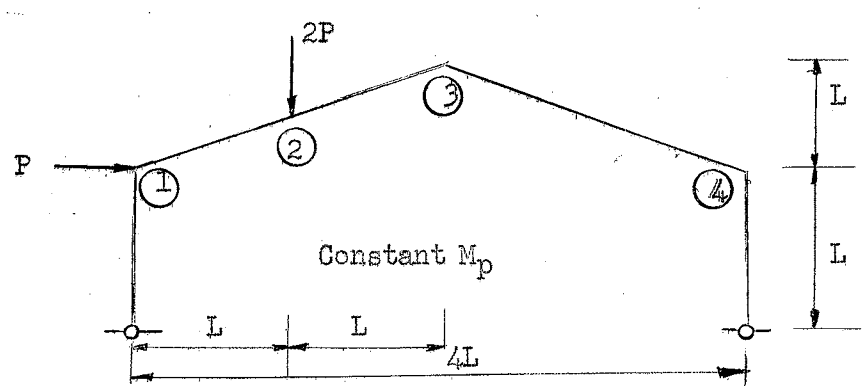


Fig. 3-1

It will be assumed that the structure is of uniform cross-section and the plastic moment equals M_p. Noting that maximum moments within a structure can occur only at junction, where the shear equals zero. There are four possible plastic hinges. It is also observed that structure is one time statically indeterminate. Therefore, the rule devised by Dr. Thurlimann. $(n - x) = N$.

Where "n"	Number of possible hinges	4
Where "x"	Number of redundants	1
Where "N"	Number of independent mechanisms	3

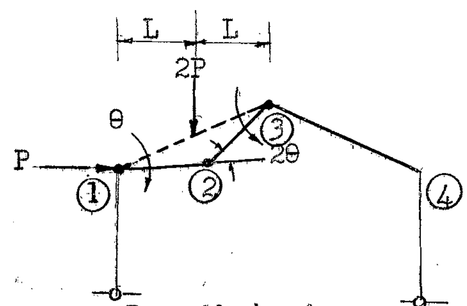
a. Beam Mechanisms:

Obviously the rafter 1 - 2 - 3 could fail as a beam. We will, therefore, choose this as one of our independent mechanisms. Assume hinges at top of column, ridge and under rafter loads. Equate internal work to external work for computing P_a or M_p.

External Work = Internal Work

$$2P(\theta L) = \underbrace{M_p\,\theta}_{\text{at } ①} + \underbrace{M_p(2\theta)}_{\text{at } ②} + \underbrace{M_p(\theta)}_{\text{at } ①}$$

$$P_a = \frac{2M_p}{L} \qquad \text{or} \qquad M_p = \frac{P_a L}{2}$$



Beam Mechanism
Fig. 3-2(a)

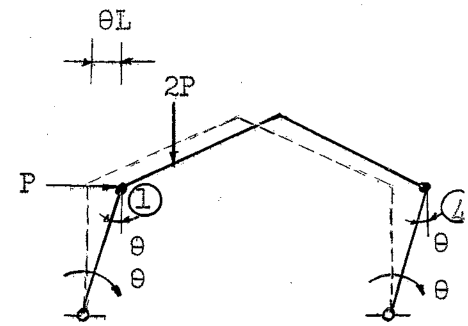
b. Panel Mechanism:

Likewise, the total "roof part" frame could deform in a panel type of configuration. Assume hinges at top of column and constitute virtual work relationship for computing P_b or M_p.

External Work = Internal Work

$$P\,(\theta L) = M_p(\theta) + M_p\,(\theta)$$

$$P_b = \frac{2M_p}{L} \quad \text{or} \quad M_p = \frac{P_b \cdot L}{2}$$



b. Panel Mechanism
fig. 3-2(b)

c. Gable Mechanism:

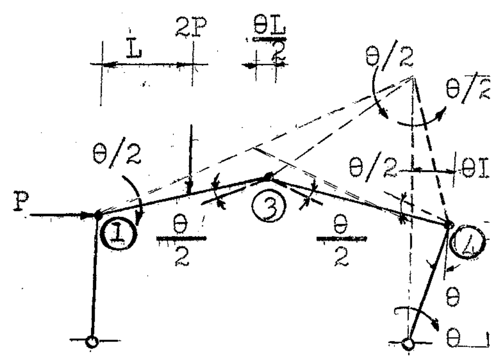
As a third independent mechanism the rafters 1 - 3 and 3 - 4 could remain rigid while point 1 remained fixed. This type of mechanism has been termed a Gable Mechanism, since it can occur only in gabled type of structures. Assuming hinges at top of column and at ridge established virtual work relationship.

External Work = Internal Work

$$2P\left(\frac{\theta L}{2}\right) = M_p\;\frac{\theta}{2} + \theta + \frac{3\theta}{2}$$

$$P\,(\theta l) = M_p\,(3\theta)$$

$$P_c = \frac{3M_p}{L} \quad \text{or} \quad M_p = \frac{P_c L}{3}$$



c. Gable Mechanism
fig. 3-2(c)

Not only we must consider these elementary mechanism forms to obtain a solution to the problem in question, but we must consider all possible combinations of them.

d. Composite Mechanism:

Shift hinge shown for panel mechanism from top of left column to load point in left rafter. Locate instantaneous center, I.c. by proportic al triangles we can evaluate the "virtual" rotations at each hinge and thus established the virtual work relationship.

External Work = Internal Work.

$$P\left(\frac{3}{5}\theta\right)L + 2P\left(\frac{3}{5}\theta L\right) = M_p\left[\underbrace{\frac{3}{5}\theta + \frac{\theta}{5}}_{\text{at (2)}} + \underbrace{\frac{\theta}{5} + \theta}_{\text{at (4)}}\right]$$

i.e. $P_d = \frac{10\ M_p}{9\ L}$ ——— (4)

or $M_p = \frac{9\ P_d\ L}{10}$

Based on the preceeding investigation, the ultimate load solution is given by equation (4)

i.e. $P_u = \frac{10}{9}\ \frac{M_p}{L}$

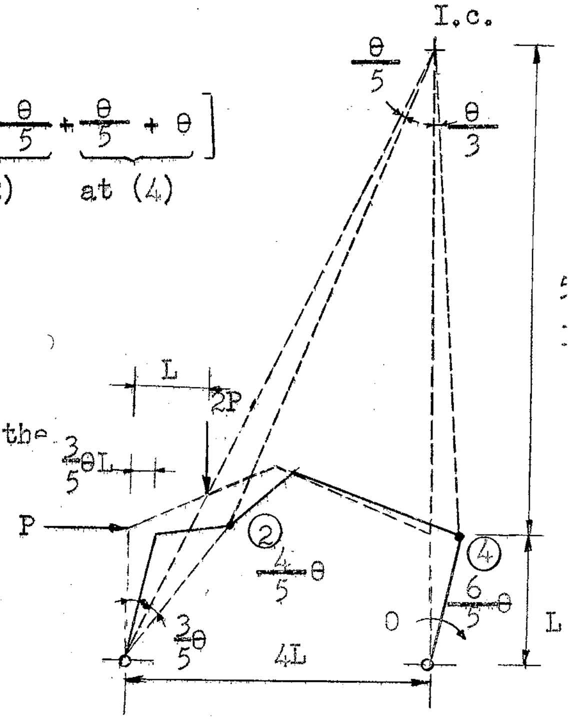
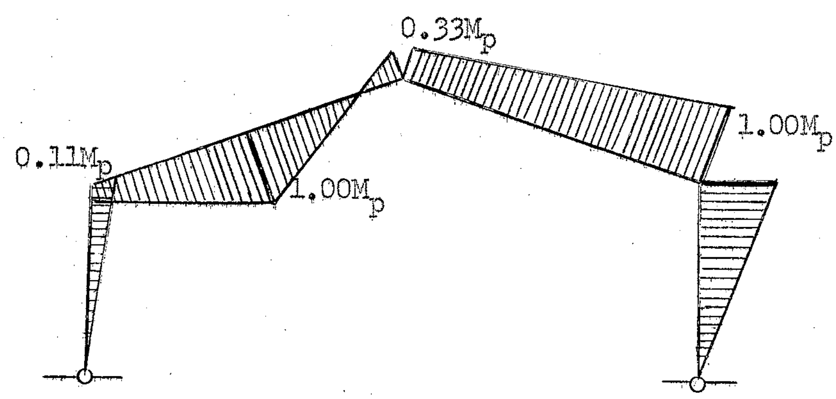


fig. 3-2 (d)

Moment Check:

Since the moment diagram is in equilibrium with the applied external loads, since there is a sufficient number and arrangement of hinges to produce a mechanism, and since nowhere does the moment value exceed the full plastic moment of the section, this is the ultimate load that structure can carry.



Moment Diagram

fig. 3-2 (e)

3-2. ANALYSIS BY MECHANISM METHOD.

1. The frame is symmetrical with individual spans of 60 ft. column height of 20 ft. and 30 ft. and roof rise varies from 0.1 to 0.3 L.

The loading is concentrated at the quarter points of the rafters and might be considered as a closer approximation to a uniformly distributed load of w Kip-ft., where w is 3 K/ft. the horizontal load is 0.6 K/ft.

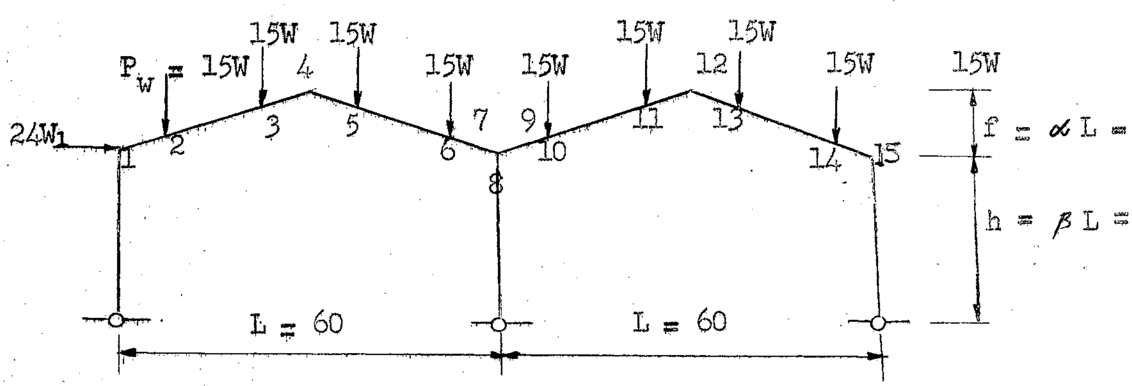


fig. 3-3

2. The most critical ultimate load condition for this given load is $P_u = F.P_W$ Where F = Load Factor.

Load Condition:

Case A: D.L. + L.L N (1.85)

Case B: D.L. + L.L + w.L (left) N (1.40)

So far Case I P_u = 15 w (1.85) = 27.75 w (without wind)

For Case B

(with wind) P_{u_v} = 15 w (1.4) = 21.0 w

3. Plastic Moment Ratio:

Try constant section throughout.

4. Mechanism Method.

Case I. Independent Mechanisms.

Location of possible plastic hinges (under load points, joints, point of zero shear)	N = 15
Redundants (remove mid support and H)	X = 3
No. of independent mechanisms	n = N - X = 12

The 12 possible independent mechanisms are shown in Fig.

Mechanism 1-4
Beam Mechanism

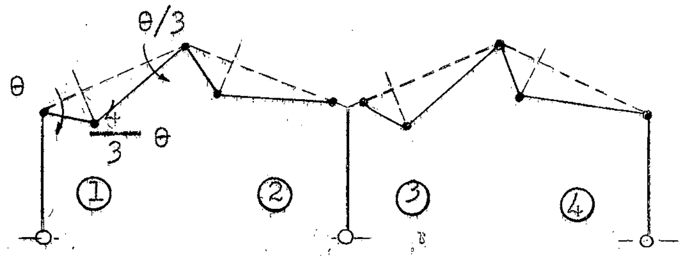
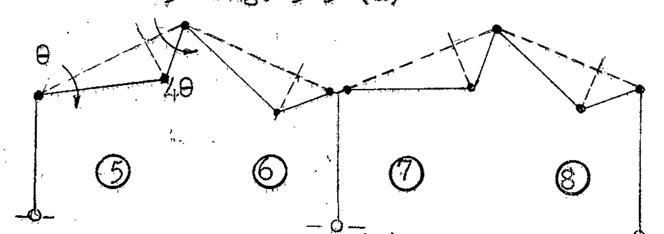


fig. 3-3 (a)

Mechanism 5-8
Beam Mechanism



fit. 3-3 (b)

Mechanism 9
Panel Mechanism

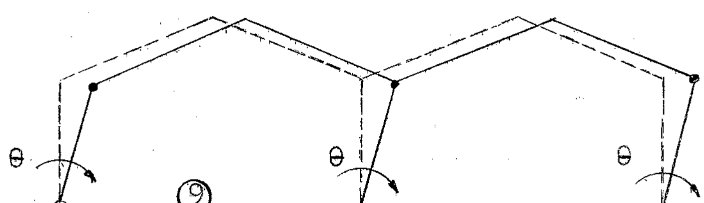


fig. 3-3 (c).

Mechanism 10-11
Case I $\frac{7}{5}\theta$
Case II $\frac{2}{5}\theta$
Case III $\frac{11}{5}\theta$

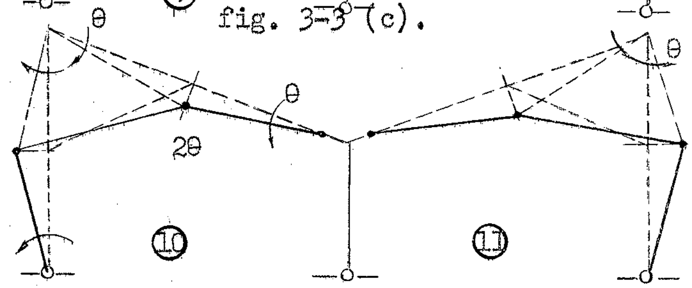


fig. 3-3 (d)

Mechanism 12 Case I $\frac{2}{5}\theta$

Joint Mechanism Case II $\frac{4}{5}\theta$

Case III $\frac{6}{5}\theta$

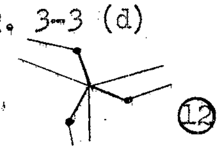


fig. 3-3 (e)

3-3. h = 30 ft.

Table 3-1.
Mechanism Analysis for Two Span Pin Based Gable Frame in Plastic Design (Case I $\alpha = 0.1$)

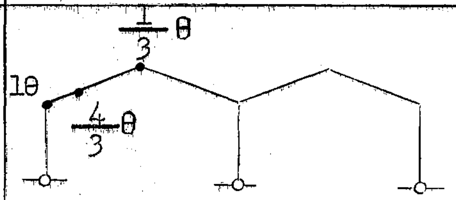
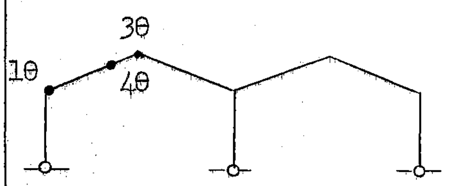
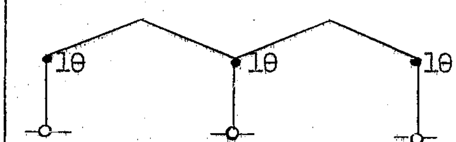
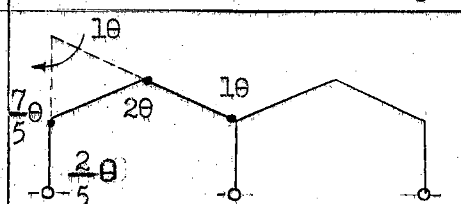
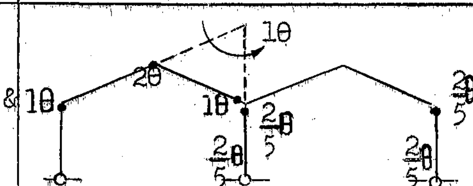
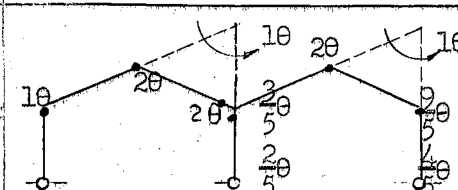
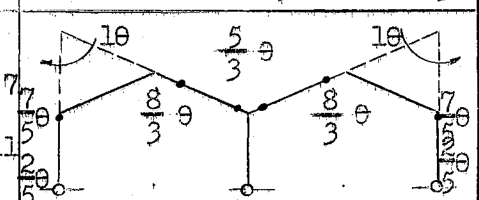
1	2	3	4	5
No:	Mechanism	Internal Work $W_I/M_p \cdot \theta$	External Work $W_E/P_u \cdot L\ \theta$	$\frac{M_p}{P_u L}$
1-4		$1+\frac{4}{3}+\frac{1}{3}=\frac{8}{3}$	$\frac{1}{8}+(\frac{1}{8}\times\frac{1}{3})=\frac{1}{6}$	$\frac{1}{16}$
5-8		$1+4+3=8$	$\frac{1}{8}+\frac{1}{8}(3)=\frac{1}{2}$	$\frac{1}{16}$
9		$1+1+1=3$	0	0
10 11		$\frac{7}{5}+2+1=\frac{22}{5}$	$\frac{1}{8}+\frac{3}{8}+\frac{3}{8}+\frac{1}{8}=1$	$\frac{5}{22}$
13 9 & 10		$1+2+1+\frac{2}{5}+\frac{2}{5}=\frac{24}{5}$	$\frac{1}{8}+\frac{3}{8}+\frac{3}{8}+\frac{1}{8}=1$	$\frac{5}{24}$
14 11, 12 & 13		$1+2+2+2+\frac{3}{5}+\frac{9}{5}=\frac{47}{5}$	$2(\frac{1}{8}+\frac{3}{8}+\frac{3}{8}+\frac{1}{8})=2$	$\frac{5}{23.5}$
15 2 + 7 + 10+11		$(\frac{7}{5}+\frac{8}{3}+\frac{5}{3})2=\frac{172}{15}$	$2(\frac{1}{8}+\frac{3}{8}+\frac{5}{8}+\frac{1}{8}\cdot\frac{5}{3})=\frac{8}{3}$	$\frac{5}{21.5}$

Table 3-2. Mechanism Analysis for Two-Span Pin Based Gable Frame in Plastic Design (for Case II $\alpha = 0.2$)

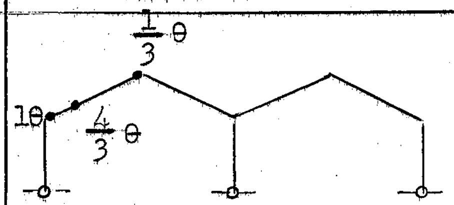
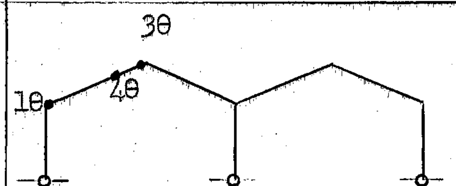
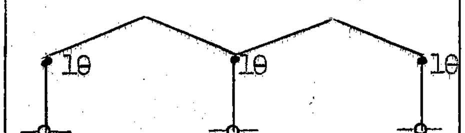
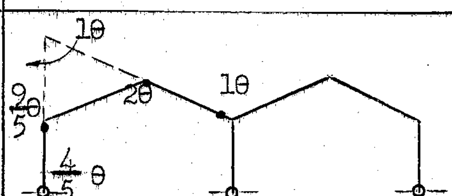
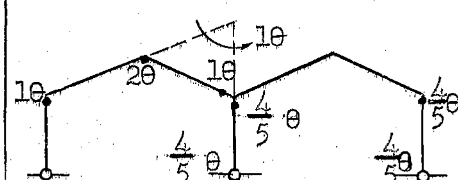
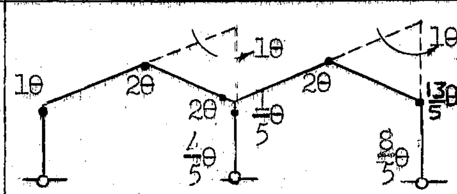
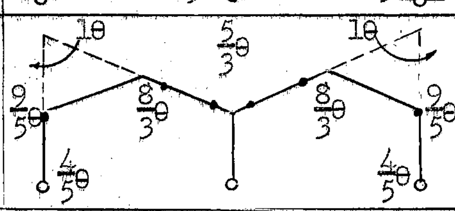
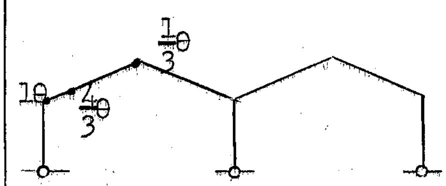
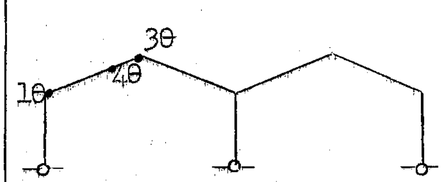
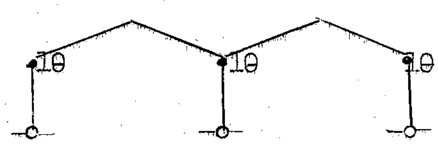
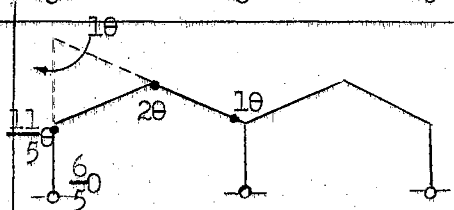
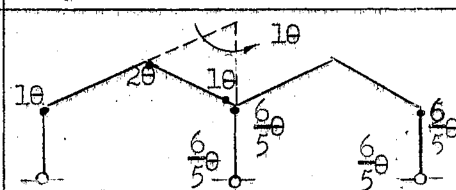
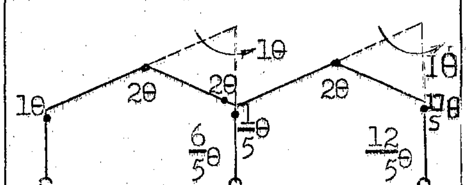
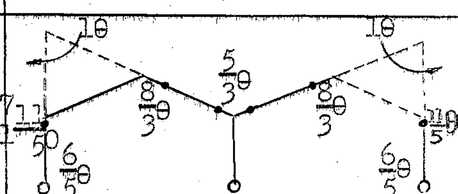
1	2	3	4	5
No.	Mechanism	Internal Work $W_I/M_p\,\theta$	Internal Work $W_E/P_u.L\theta$	$\frac{M_p}{P_u L}$
1-4		$1+\frac{4}{3}+\frac{1}{3}=\frac{8}{3}$	$\frac{1}{8}+(\frac{1}{8}\times\frac{1}{3})=\frac{1}{6}$	$\frac{1}{16}$
5-8		$1+4+3=8$	$\frac{1}{8}+\frac{1}{8}(3)=\frac{1}{2}$	$\frac{1}{16}$
9		$1+1+1=3$	0	0
10 11		$\frac{9}{5}+2+1=\frac{24}{5}$	$\frac{1}{8}+\frac{3}{8}+\frac{3}{8}+\frac{1}{8}=1$	$\frac{5}{24}$
13 9 & 10		$1+2+1+\frac{4}{5}+\frac{4}{5}=\frac{28}{5}$	$\frac{1}{8}+\frac{3}{8}+\frac{3}{8}+\frac{1}{8}=1$	$\frac{5}{28}$
14 11, 12 & 13		$1+2+2+2+\frac{1}{5}+\frac{13}{5}=\frac{49}{5}$	$2(\frac{1}{8}+\frac{3}{8}+\frac{3}{8}+\frac{1}{8})=2$	$\frac{5}{24.5}$
15 2 + 7 + 10+11		$(\frac{9}{5}+\frac{8}{3}+\frac{5}{3})2=\frac{184}{15}$	$2(\frac{1}{8}+\frac{3}{8}+\frac{5}{8}+\frac{1}{8}\cdot\frac{5}{3})=\frac{8}{3}$	$\frac{5}{23.0}$

Table 3-3. Mechanism Analysis for Two-Span Pin Based Gable Frame in Plastic Design (Case III $\alpha = 0.3$).

1	2	3	4	5
No.	Mechanism	Internal Work	External Work	$\frac{M_p}{P_u L}$
1-4		$1+\frac{4}{3}+\frac{1}{3}=\frac{8}{3}$	$\frac{3}{8}+(\frac{1}{8}\times\frac{1}{3})=\frac{1}{6}$	$\frac{1}{16}$
5-8		$1+4+3=8$	$\frac{1}{8}+\frac{1}{8}(3)=\frac{1}{2}$	$\frac{1}{16}$
9		$1+1+1=3$	0	0
10 11		$\frac{11}{5}+2+1=\frac{26}{5}$	$\frac{1}{8}+\frac{3}{8}+\frac{3}{8}+\frac{1}{8}=1$	$\frac{5}{26}$
13 9 & 10		$1+2+1+\frac{6}{5}+\frac{6}{5}=\frac{32}{5}$	$\frac{1}{8}+\frac{3}{8}+\frac{3}{8}+\frac{1}{8}=1$	$\frac{5}{32}$
14 11 + 12 + 13		$1+2+2+2+\frac{1}{5}-\frac{17}{5}=\frac{53}{5}$	$2(\frac{1}{8}+\frac{3}{8}+\frac{3}{8}+\frac{1}{8})=2$	$\frac{5}{26.5}$
15 2 + 7 10+11		$2(\frac{11}{5}+\frac{8}{3}+\frac{5}{3})=\frac{196}{15}$	$2(\frac{1}{8}+\frac{3}{8}+\frac{5}{8}+\frac{1}{8}\cdot\frac{5}{3})=\frac{8}{3}$	$\frac{5}{24.5}$

Mechanism Analysis.

To facilitate the calculations the analysis is done in tabular form (See Table 3-1 to 3). Column "1" bears the mechanism number column "2" contains a mechanism sketch. Since the deformed shape was drawn in fig. 3 and 4, this feature is not repeated in the small sketch. The internal work is computed in column "3" and column "4" contains the computation of external work, listing the work done by each load in the same sequence. The ratio of M_P to $P_{\ddot{u}}.L$ is given in column "5".

3-4. ILLUSTRATION FOR MECHANISM 10 CASE I $\alpha = 0.1$.

The mechanism angle at section 1, 4 and 7 are $\frac{7}{5}\theta$, 2θ, and θ respectively.

Thus the total internal work is = $\frac{22}{5} M_P\theta$ or $\frac{W_I}{M_P\theta} = \frac{22}{5}$ (See column 3)

Using the instantaneous center the load at 2 does the work equal to $P(\frac{L}{8})(\theta)$ at 3 the work equal to $P(\frac{3L}{8})(\theta)$

Segment 4-7 rotates about 7 through the angle θ and, therefore, the work done by the load at 5 equals $P(\frac{3L}{8})(\theta)$ and the work done by the load at 6 equals $P(\frac{L}{8})(\theta)$.

Then total external work $PL\theta(\frac{1}{8} + \frac{3}{8} + \frac{3}{8} + \frac{1}{8}) = 1\ PL\theta$

$W_E/PL\theta = 1$ shown in column 4

By the virtual work relation

Internal work = External work

$$\frac{22}{5} M_P \theta = 1\ P_u L \theta$$

or $\frac{M_P}{P_{\ddot{u}} L} = \frac{5}{22}$ Shown in column 5

Similarly, we can get for mechanism 10

for Case II $\qquad \alpha = 0.2 \qquad ; \qquad \frac{M_P}{P_u L} = \frac{5}{24}$

for Case III $\qquad \alpha = 0.3 \qquad ; \qquad \frac{M_P}{P_u L} = \frac{5}{26}$

Reviewing now the possible combinations these are made in such a way as to eliminate plastic hinges, because only by this means the ratio M_P/P_u L can be increased.

Mechanism 13 is formed by combining mechanism 11, 12 and 10 and mechanism 15 is formed by combining mechanisms 2, 7, 10 and 11.

3-5. MOMENT CHECK (for Mechanism 15 $h = 30$ ft.)

Referring the tables 3-3a, b, c the collapsed mechanism 15 is critical for all three cases.

which gives: $\quad M_P = \frac{5}{21.5} P_u L \quad$ for Case I

$M_P = \frac{5}{23.0} P_u L \quad$ for Case II

$M_P = \frac{5}{24.5} P_u L \quad$ for Case III

Case I: $\quad (\alpha = 0.1)$

The moment diagram for this case is shown in fig. 3-4.

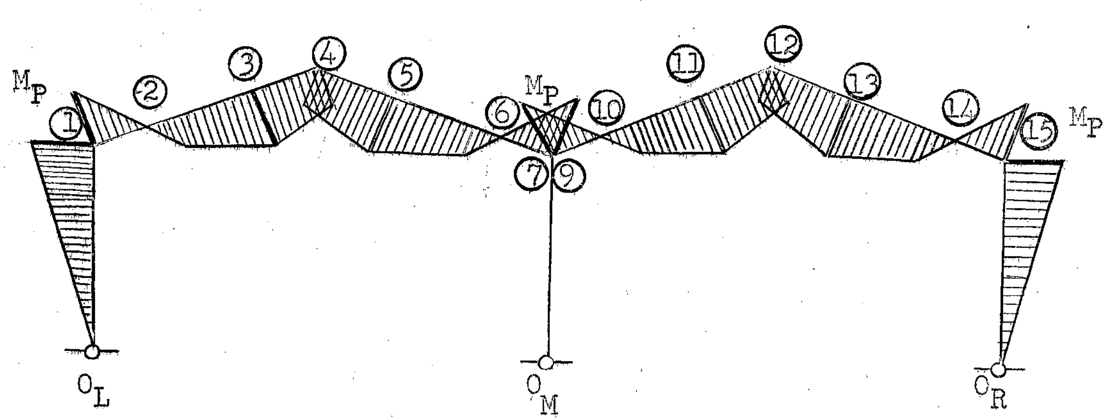


Fig. 3-4

Check for moment at each critical section.

(Rafter equilibrium)

Beam 1-4

$$M_3 = \frac{1}{4} M_1 - \frac{3}{4} M_4 + \frac{PL}{B} = -\frac{10}{40} M_P + \frac{28.50}{40} M_P + \frac{21.5}{40} M_P$$

$$= \frac{40}{40} M_P = M_P$$

Beam 4-7

$$M_5 = \frac{3}{4} M_4 - \frac{1}{4} M_7 + \frac{PL}{8}$$

$$M_4 = \frac{4}{3} M_5 + \frac{1}{3} M_7 - \frac{P_u L}{6}$$

$$= \frac{4}{3} M_P + \frac{1}{3} M_P - \frac{21.50}{30} M_P = \frac{28.5}{30} M_P$$

$$M_5 = \frac{3}{4} \left(\frac{28.50}{30} M_P\right) - \frac{10}{40} M_P + \frac{21.50}{40} = \frac{40}{40} M_P = M_P$$

b) M_P value:

Since all $M \leqslant M_P$ plastic moment condition fulfills

$$M_P = \frac{5 P_u L}{21.5} = \frac{5}{21.5} \left(27.75 w (60)\right) = \boxed{387 w}$$

For the cases II and III the same type of mechanism considered to be critical as we have seen in case I. Thus we can fairly say that moment for the cases II and III will also fulfill the plastic moment condition.

TABLE 3-4

RESULTS OF MECHANISM ANALYSIS IN COEFFICIENTS OF "$\frac{M_P}{P_u . L}$" DIMENSIONLESS RATIO

Parameters		Mechanism Referring to Table 3-1, 2, 3						
α	β \ Mech. No.	1 1+4	5 5+8	9 "9"	10 10+11	13 9+10	14 11+12+13	15 1+10+2+7
0.1	$\frac{1}{2}$	$\frac{1}{16}$	$\frac{1}{16}$	0	$\frac{5}{22}$	$\frac{5}{24}$	$\frac{5}{23.5}$	$\frac{5}{21.5}$
	$\frac{1}{3}$	$\frac{1}{16}$	$\frac{1}{16}$	0	$\frac{5}{23}$	$\frac{5}{26}$	$\frac{5}{24}$	$\frac{5}{22.25}$
0.2	$\frac{1}{2}$	$\frac{1}{16}$	$\frac{1}{16}$	0	$\frac{5}{24}$	$\frac{5}{28}$	$\frac{5}{24.5}$	$\frac{5}{23.0}$
	$\frac{1}{3}$	$\frac{1}{16}$	$\frac{1}{16}$	0	$\frac{5}{26}$	$\frac{5}{32}$	$\frac{5}{26.5}$	$\frac{5}{24.5}$
0.3	$\frac{1}{2}$	$\frac{1}{16}$	$\frac{1}{16}$	0	$\frac{5}{26}$	$\frac{5}{32}$	$\frac{5}{26.5}$	$\frac{5}{24.5}$
	$\frac{1}{3}$	$\frac{1}{16}$	$\frac{1}{16}$	0	$\frac{5}{29}$	$\frac{5}{38}$	$\frac{5}{31}$	$\frac{5}{26.7}$

NB: - The critical mechanism was No.10. The plastic moment was checked for this and it fulfilled the plastic moment condition. The lightest section selected on this critical "M_p" given by mechanism "10"

CHAPTER IV

DESIGN

4-1. GENERAL

The two-span pin-based gable frame was chosen for comparative analysis. Gable frames are becoming much more popular than column truss construction. These frames provide much room on one hand, and the other reason is that they are easy to erect. No doubt frame construction involves more steel than the column truss construction, but due to numerous advantages the material factor may be overlooked.

4-2. SPECIFICATIONS

1. Load factors adopted for continuous frames in analysis governed by AISC Nov: 1961.

Case A. When we consider dead load and live load only, the load factor should be 1.85.

Case B. Assuming the effect of wind in conjunction with the dead load and live load its value reduces to 1.4

2. ASTM A-7 structural steel was employed for the sections in both cases.

3. For proportioning the minimum thickness ratio and secondary design requirements governed by AISC specifications.

4-3(a) GENERAL PROCEDURE FOR PLASTIC DESIGN

The general procedure is outlined as follows:

1. Determine possible loading condition. There are two loading conditions. These are:

A. Live load and dead load.

B. Live load and dead load and wind load.

2. Compute ultimate load by multiplying working load by load factor, F.

$$P_u = F P_w$$

For Buildings:

D.L + L.L	F = 1.85
D.L + L.L + Wind	F = 1.40

For other forms of construction:

$$F = \frac{\sigma_y}{\sigma_w} f$$

Where σ_w = working stress allowed by AISC.

3. Estimate plastic bending moment ratio for various members.

A. Determine absolute plastic moment values for separate loading condition. (Assume all joints fixed against rotation, but frame free to sway).

a. Beams: Solve beam mechanism equation.

b. Columns: Solve panel mechanism equation.

B. Select plastic moment ratio using the following guides:

a. Beams: Use the value determined in step A above.

b. Columns: At corner connections

$$M_p \text{ (Column)} = M_p \text{ (Beam)}$$

c. Joints: Establish equilibrium.

C. Compute the maximum plastic moment (M_p).

D. Examine the frame for further economics as may be apparent from considerations of relative beam and sway moments.

4. Analyze each loading condition for maximum M_p.

5. Compute reactions for each loading condition.

6. Select section.

Plastic modulus Z is equal to section modulus S multiplies by shape factor "f" or $Z = f.S$

For wF section $1.10 < f < 1.23$

Plastic moment $M_p = \sigma_y . Z$ or $Z = 0.364 M_p$

(where $\sigma_y = 33000$ Psl for ASTM A-7 steel)

For cover plate $\Delta_z = A_p . d_p = A_p (d - t_p)$

Net Z for members with hole in flange or web

$$Z \text{ (net)} = Z - \frac{t (d-t)}{2} - t_w . y$$

where:

t = Flange thickness

d = Depth of section

t_w = Web thickness

A_p = Area of plates

d_p = Clear distance of plates

y = Distance from centroid to hole.

7. Check design to see that it satisfies all limitations.

8. Check deflection if necessary the analysis undermentioned, does not required to check the deflection.

4-3(b) GENERAL PROCEDURE FOR ELASTIC DESIGN

1. Determine possible loading condition.

a) Dead load and live load

b) Dead load and live load and wind load

The investigation in conjunction with the wind load revealed that it is not critical.

2. Analysis done by the method of "Moment Distribution" for determining the end moments, shearing forces thrusts, etc.

3. The critical design section was considered at the inside face of column and bottom of girder; it was proportioned on the value evaluated in step 2.

4. Allowable bending stress at these critical sections kept limited to 20 K S 1 modified where necessary according to the formula.

$$F_b = \frac{12000}{Ld/bt}$$

For allowable compressive stress to be limited to $F_a = 17000 - 0.485 \frac{L^2}{r^2}$ and that the max combined stress be limited by the provisions of section (12) (a) of the AISC specifications.

5. The max combined stress be determined by the conventional formula:

$$f = \frac{N}{A} + \frac{Mc}{I}$$

6. Check for stresses of the designed section.

$$\frac{f_a}{F_a} + \frac{f_b}{F_b} \leqslant 1$$

where f_a = Actual axial stress = $\frac{N}{A}$

F_a = Allowable compressive stress with respect to $(\frac{L}{r})$ ratio.

f_b = Flexural compressive stress = $\frac{Mc}{I}$

F_b = Allowable compressive stress 20 K S 1, specified by AISC.

4-4(a). SELECTION OF THE SECTION

The choice of section done with due emphasis to the following points as outlined here:

a. The section should be of the minimum weight which can be permitted by the Elastic Design procedure.

b. Selected section should be checked against the stress with,

$$\frac{f_a}{F_a} + \frac{f_b}{F_b} \leqslant 1$$

Thus with these two major consideration sections were selected for all the three cases as mentioned before and the results were tabulated in table 4-1. Illustrations are given below to show the design procedure in detail.

4-4(b). ILLUSTRATIVE EXAMPLE

Case I. $\alpha = 0.1$ $\beta = \frac{1}{3}$ $w = 3$ K/ft.

The critical section for this case is at the middle bent member, with a bending moment of +875 K-ft. and its corresponding axial force of -50 Kips using the factor of safety of 1.65.

Required section Modulus $S' = \frac{M(1.65)}{33}$

$$= \frac{875\ (12)\ (1.65)}{33} \doteq 525 \text{ in}^3$$

Try 36 WF 182

1. Check for stress at the critical section (middle bent member)

Assume purlin spacing 7.5 ft. or L = (7.5) (12) = 90 in

From AISC manual for 36 WF 182, $A = 53.54$ in^2, $d = 36.32$ in, $b = 12.072$, $S_x = 621.2$ in^3, $r_y = 2.47$ in

i) Allowable Stress:

For bending,

$$\frac{Ld}{bt} = \frac{90 \times 36.32}{12.072 \times 1.180} = 229 < 600 \qquad F_b = 20 \text{ K.S I}$$

For Axial force

$$\frac{L}{r} = \frac{90}{2.47} = 36.5 < 120 \qquad \text{From AISC tables } F_a = 16.35 \text{ K S}$$

ii) Actual Stress

For bending.

$$f_b = \frac{M}{S} = \frac{875\ (12)}{621.2} = 16.9 \text{ KS1}$$

For Axial

$$f_a = \frac{N}{A} = \frac{50.0}{53.54} = 0.932 \quad \text{KS1}$$

Thus,

$$\frac{f_a}{F_a} + \frac{f_b}{F_b} = \frac{0.932}{16.35} + \frac{16.9}{20} = 0.0570 - 0.840$$

$$= 0.8970 < 1 \text{ O.K}$$

2. Check for stress at exterior column:

From Table $M = 550.05$ K ft., $N = 90$ K

Assume girt spacing = 5 ft. or L = 5 (12) = 60 in.

i) Allowable Stress

For bending.

$$\frac{Ld}{bt} = \frac{60 \times 36.32}{12.072 \times 1.18} = 153 < 600 \qquad F_b = 20 \text{ K S 1}$$

For Axial.

$$\frac{L}{r} = \frac{60}{2.47} = 24.3 \quad \text{From table} \quad F_a = 16.71 \text{ K S 1}$$

ii) Actual Stress

For bending.

$$f_b = \frac{M}{S} = \frac{652.0\ (12)}{621.2} = 12.60 \quad \text{K S 1}$$

For Axial.

$$f_a = \frac{N}{A} = \frac{90}{53.54} = 1.69 \quad \text{K S 1}$$

Thus,

$$\frac{f_a}{F_a} + \frac{f_b}{F_b} = \frac{1.69}{16.71} + \frac{12.6}{20} = 0.101 + 0.63 = 0.731 < 1 \quad \text{O.K}$$

3. Check for stresses at interior column

From table 4-1 $M = 0$, $N = 180$ K

Assume no girt is needed $L = h = 20(12) = 240$ in.

i) Allowable Stress

For bending.

$$\frac{Ld}{bt} = \frac{240 \times 36.32}{12.072 \times 1.180} = 610 > 600 , \; F_b = \frac{12000}{\frac{Ld}{bt}}$$

$$= \frac{12000}{610} = 19.65 \text{ K S I}$$

For Axial.

$$\frac{L}{r} = \frac{240}{2.47} = 97.0 < 120 \quad \text{From table} \quad F_a = 12.44 \text{ K S I}$$

ii) Actual Stress

For bending.

$$f_b = \frac{M}{S} = 0$$

For Axial.

$$f_a = \frac{N}{A} = \frac{180}{53.54} = 3.36 \text{ K S I}$$

Thus,

$$\frac{f_a}{F_a} + \frac{f_b}{F_b} = \frac{3.36}{12.44} + 0 = 0.27 < 1 \quad \text{O K}$$

Use 36 wF 182

TABLE 4-1

LIGHTEST WF SECTIONS FOR THE TWO-SPAN PIN-BASED GABLE FRAME SELECTED BY ELASTIC DESIGN

Parameters		Critical Section Elements			Selected Sections	Allowable Stress		Actual Stress		Check for Stress
α	β	M (K ft)	N (Kips)	Location	WF Section	Axial F_a	Bending F_b	Axial $f_a = \frac{N}{A}$	Bending $f_b = \frac{M}{S}$	$\frac{f_a}{F_a} + \frac{f_b}{F_b} \leqslant 1$
0.1	$\frac{1}{2}$	+975.9	-36.15	Middle Bent Member	36 WF 194	16.35	20	0.635	17.7	0.919 < 1 O.K
	$\frac{1}{3}$	+875.0	-50.0	Middle Bent Member	36 WF 182	16.35	20	0.932	16.9	0.897 < 1 O.K
0.2	$\frac{1}{2}$	+854.0	-51.50	Middle Bent Member	36 WF 170	16.35	20	1.03	17.7	0.948 < 1 O.K
	$\frac{1}{3}$	+728.0	-63.60	Middle Bent Member	36 WF 150	16.30	20	1.44	17.40	0.959 < 1 O.K
0.3	$\frac{1}{2}$	+770.0	-63.3	Middle Bent Member	36 WF 160	16.34	20	1.34	17.1	0.930 < 1 O.K
	$\frac{1}{3}$	+716	-90.00	Exterior Column	36 WF 150	16.30	20	2.04	15.90	0.915 < 1 O.K

4-6. SELECTION OF SECTIONS (For Plastic Design)

The possible section selected in this case again is made with due consideration of

a. Min weight wF section

b. Plastic moment of the section should be sufficient enough to transmit the required M_P.

Thus on this basis, sections were selected for each of the three cases with w = 3 K/ft. and employing A - 7 steel.

a. Case I ($\alpha = 0.1$)

i)	ii)
h = 20 ft.	h = 30 ft.
$M_P = 374$ w	$M_P = 387$ w
$M_P = 1122$ K ft.	$M_P = 1161$ K ft.
Trh 33 wF 118 which gives (M_P = 1139)	Try 30 wF 132 which gives (M_P = 1201 K ft)

b. Case II ($\alpha = 0.2$)

i)	ii)
h = 20 ft.	h = 30 ft.
$M_P = 340$ w	$M_P = 362$ w
$M_P = 1020$ K ft.	$M_P = 1086$ K ft.
Try 30 wF 116 which gives (M_P = 1038)	Try 30 wF 124 which gives (M_P = 1120 K ft.)

c. Case III ($\alpha = 0.3$)

i)	ii)
h = 20 ft.	h = 30 ft.
$M_P = 311$ w	$M_P = 340$ w
i.e $M_P = 933$ K ft.	i.e $M_P = 1020$ K ft.
Try 30 wF 103 which gives (M_P = 950 K ft.)	Try 30 wF 116 which gives (M_P = 1038 K ft.)

4-7. SIMPLIFIED PROCEDURES (FRAME CHART 4-2) USED FOR THIS PLASTIC DESIGN HAS BEEN DERIVED AND PLOTTED.

a. Case I ($\alpha = 0.1$)

i) $h = 20$ ft.

$f = \alpha L = 0.1 \times 60 = 6$ ft.

$Q = \frac{f}{h} = \frac{6}{20} = 0.3$

$D = 0$

From frame chart we get:

$\frac{M_p}{WL^2} = 0.0545$

$M_p = 0.0545 \times 1.85 \times (60)^2 \times W = 363\ W$ K ft.

for $W = 3$ K/ft. $M_p = 1089$ K ft.

Try 33 wF 118 ($M_p = 1139$ k ft.)

ii) $h = 30$ ft.

$f = \alpha L = 0.1 \times 60 = 6$ ft.

$Q = \frac{f}{h} = \frac{6}{30} = 0.2$

$D = 0$

From frame chart we get:

$\frac{M_p}{WL^2} = 0.0569$

$M_p = 0.0569 \times 1.85\ (60)^2 \times W = 379\ W$

for $W = 3$ K/ft. $M_p = 1137$ K ft.

Try 30 wF 132 which gives ($M_p = 1201$ K ft.)

b. Case II ($\alpha = 0.2$)

i) $h = 20$ ft.

$f = \alpha L = 0.2 \times 60 = 12$ ft.

$Q = \frac{f}{h} = \frac{12}{20} = 0.6$

$D = 0$

From frame chart (fig 4-2) we get:

$$\frac{M_P}{WL^2} = 0.0487$$

$$M_P = 0.0487 \times 1.85 \times (60)^2 \times W = 324\ W$$

for W = 3 K/ft. M_P = 972 K ft.

Try 30 wF 116 (M_P = 1038 K ft.)

ii) h = 30 ft.

$$f = \alpha L = 0.2 \times 60 = 12 \text{ ft.}$$

$$Q = \frac{f}{h} = \frac{12}{30} = 0.4$$

$$D = 0$$

From frame chart (fig. 4-2) we get:

$$\frac{M_P}{WL^2} = 0.0525$$

$$M_P = 0.0525 \times 1.85 \times (60)^2 \times W = 350\ W$$

for W = 3 K/ft. M_P = 1050 K ft.

Try 30 wF 124 (M_P = 1120 K ft.)

c. Case III ($\alpha = 0.3$)

i) h = 20 ft.

$$f = \alpha L = 0.3 \times 60 = 18 \text{ ft.}$$

$$Q = \frac{f}{h} = \frac{18}{20} = 0.9$$

$$D = 0$$

From frame chart (fig. 4-2) we get:

$$\frac{M_P}{WL^2} = 0.444$$

$$M_P = 0.444 \times 1.85 \times (60)^2 \times W = 296\ W$$

for W = 3 K/ft. M_P = 888 K ft.

Try 30 wF 108 (M_P = 950 K ft.)

ii) $h = 30$ ft.

$f = L = 0.3 \times 60 = 18$ ft.

$Q = \frac{f}{h} = \frac{18}{30} = 0.6$

$D = 0$

From frame chart (fig. 4-2) we get:

$$\frac{M_P}{WL^2} = 0.0487$$

$$M_P = 0.0487 \times 1.85 \times (60)^2 \times W = 324\ W$$

for $W = 3$ K/ft. $\qquad M_P = 972$ K ft.

Try 30 wF 116 ($M_P = 1038$ K ft.)

4-8(a) FRAME CHART USED FOR THIS PLASTIC DESIGN.

a. Derivation.

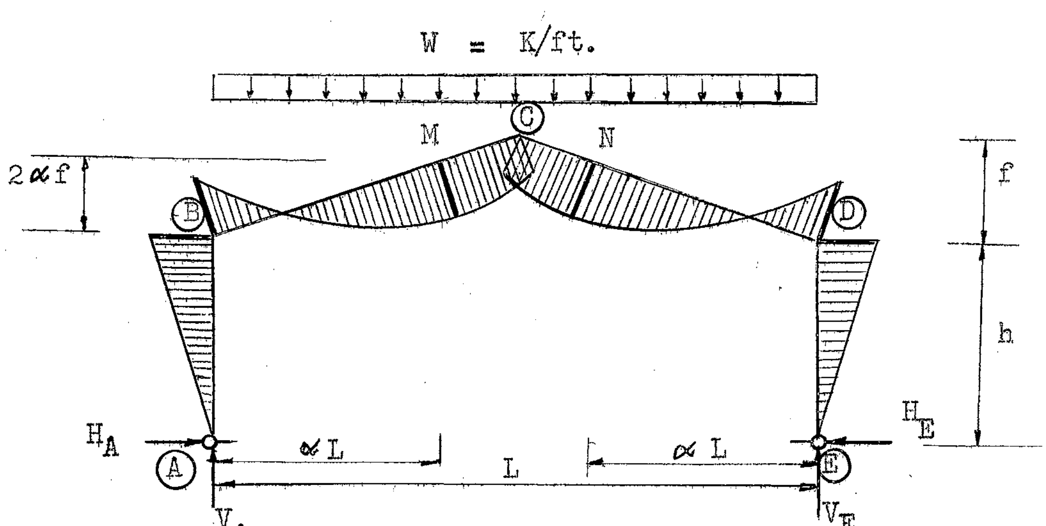


fig. 4-1(a)

Denoting $Q = \frac{f}{h}$

Assuming plastic hinges occurring at B, M, N and D as shown.

Take Column AB as free body

$$\sum M_B = 0 \qquad H_A = \frac{M_P}{h}$$

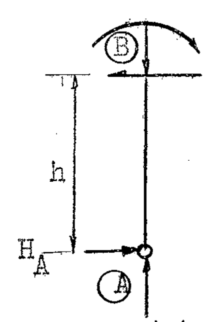


fig. 4-1(b)

Then take whole frame as free body

$$\sum F_x = 0 \qquad H_B + H_A = \frac{M_P}{h}$$

$$\sum F_y = 0 \qquad V_A = V_E = \frac{WL}{2}$$

Consider member ABM as free body

$$\sum M_M = 0$$

$$-\frac{1}{2}WL(\alpha L) + \frac{M_P}{h}(h + 2\alpha f) + M_P + \frac{W\alpha^2L^2}{2} = 0$$

$$M_P\left(1 + 1 + \frac{2\alpha f}{h}\right) = \frac{1}{2}\alpha WL^2 - \frac{1}{2}W\alpha^2L^2$$

$$M_P = \frac{1}{\left(2 + \frac{2\alpha f}{h}\right)}\frac{W}{2}\alpha L^2(1-\alpha)$$

$$= \frac{1}{4}WL^2\frac{\alpha - \alpha^2}{1 + \alpha Q} \qquad \text{—— I}$$

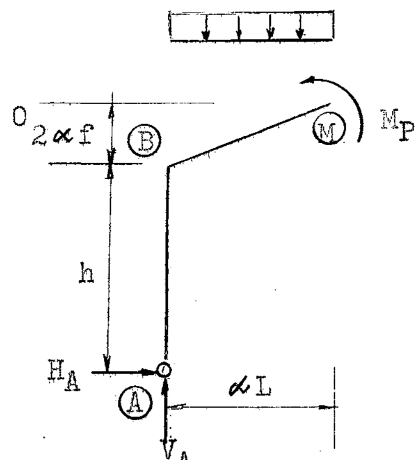


fig. 4-1(c)

Differentiating w.r.t α

$$\frac{dM_P}{d\alpha} = \frac{1}{4}WL^2\frac{(1-2\alpha)(1+\alpha Q) - (\alpha - \alpha^2)Q}{(1+\alpha Q)^2} = 0$$

$$= (1-2\alpha)(1+\alpha Q) - (\alpha - \alpha^2)Q = 0$$

$$1 + \alpha Q - 2\alpha - 2\alpha^2 Q - \alpha Q + \alpha^2 Q = 0$$

$$\alpha^2 Q + 2\alpha - 1 = 0 \quad \therefore \alpha = \frac{1}{Q}\left(\sqrt{1+Q} - 1\right)$$

Substituting the value of α into Eq. I

$$M_P = \frac{WL^2}{4}\frac{\alpha(1-\alpha)}{1 + Q\left(\frac{1}{Q}\sqrt{1+Q} - 1\right)} = \frac{WL^2}{4}\frac{\alpha(1-\alpha)}{\sqrt{1+Q}}$$

i.e. $$\boxed{\frac{M_P}{WL^2} = \frac{1}{4}\frac{\alpha(1-\alpha)}{\sqrt{1+Q}}}$$

b. Frame Chart: from the above derivation a curve can be plotted between $\frac{M_P}{WL^2}$ vs Q as shown in fig. 4-2.

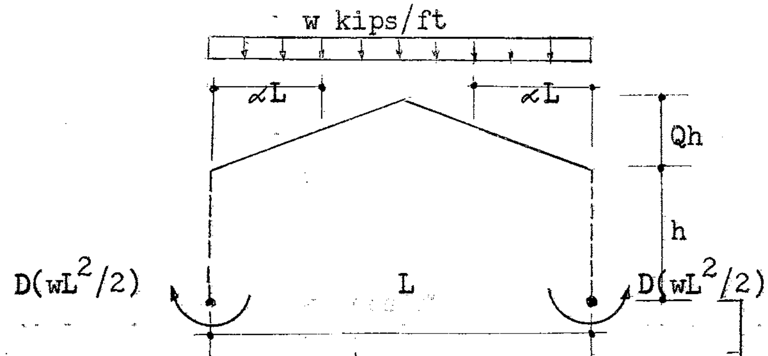
Formula derived for this frame chart:

1) When Q=0 (D=0)

$$\frac{M_p}{wL^2} = \frac{1}{16} \quad , \quad \alpha = \tfrac{1}{2}$$

2) When Q > 0 (D=0)

$$\frac{M_p}{wL^2} = \frac{1}{4}\frac{(\quad)}{\sqrt{1+Q}} \quad , \quad \alpha = \frac{1}{Q}\left(\sqrt{1+Q} - 1\right)$$



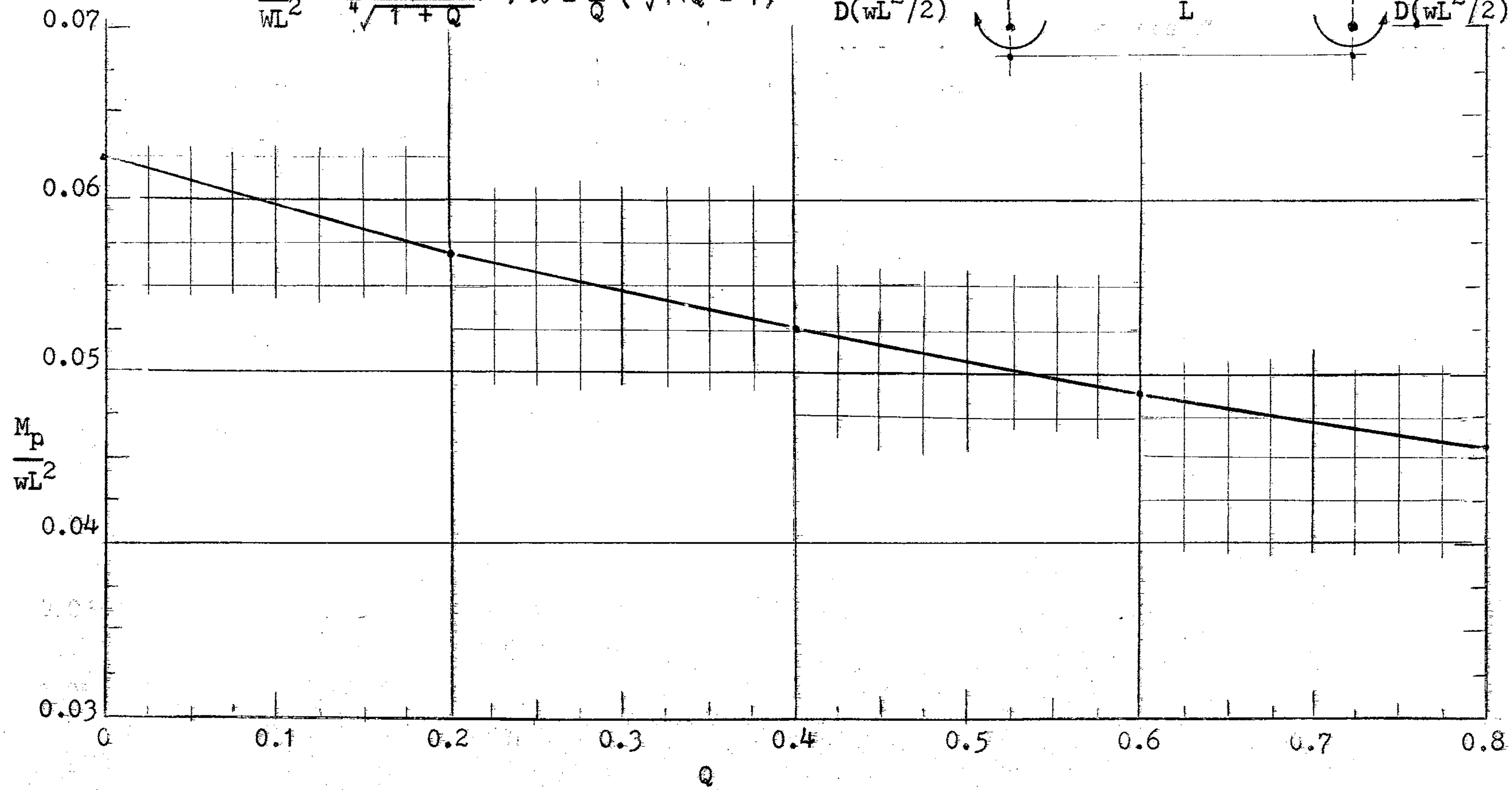


Fig. 4-2 : FRAME CHART

4-8(b) SELECTED SECTIONS

The possible lightest wF sections obtained by two different procedures for three different cases with $W = 3$ K/ft. have been tabulated in table 4-3.

Illustrative example: For the Case $\alpha = 0.1$

$h = 20$ ft $\qquad W = 3$ K/ft.

By employing the mechanism method we get:

$$M_P = 1122 \text{ K ft.}$$

By using simplified procedure we get:

$$M_P = 1089 \text{ K ft.}$$

From AISC (Rolled beams properties for Plastic Design) Try 33 wF 118

Reactions can be found easily:

$$V_1 = V_R = \frac{1}{4} W(2L) = \frac{1}{2} WL = \frac{1}{2} \times 3 \times 60 = 90 \text{ Kips}$$

$$V_C = 2V_L = 2 \times 90 = 180 \text{ Kips}$$

$$H_L = H_R = \frac{M_P}{A} = \frac{1122}{20} = 50.61 \text{ Kips}$$

Thus maximum axial force at middle column.

a. Check for axial force at middle column P_y for 33 wF 118 section can be found from tables $\frac{P}{P_y} = \frac{V_C}{P_y} = \frac{180}{1145} = 0.157 > 0.15$

33 wF 118 is inadequate.

Try 33 wF 130 From table, $P_y = 1263$ Kips

$$\frac{P}{P_y} = \frac{V_C}{P_y} = \frac{180}{1263} = 0.1425 < 0.15 \qquad \text{O.K.}$$

b. Check for shearing stress, Max shear $= V_u = 50.61$ K

$$\frac{V_u}{W_d} = \frac{50.61}{(0.554)(32.86)} = 2.78 < 18 \text{ K S I} \qquad \text{O. K} \qquad \text{Use 33 wF 130}$$

TABLE 4-3

LIGHTEST WF SECTIONS FOR THE TWO-SPAN PIN-BASED GABLE FRAME SELECTED BY PLASTIC DESIGN

$W = 3$ K/ft.

Parameters		Mechanism Method		Simplified Procedure		Reactions (Kips)			Check for Axial Force at Middle Column		
α	β	M_P (K ft.)	Selected Lightest Section	M_P (K ft.)	Selected Lightest Section	$H_L = H_R$	$V_L = V_R$	V_C	P_y (K)	$\frac{P}{P_y} = \frac{V_C}{P_y}$	Allowable $\frac{P}{P_y} \leq 0.15$
0.1	$\frac{1}{2}$	1161	30 WF 132	1137	30 WF 132	38.80 37.90	90	180	1281	0.1402	O.K
	$\frac{1}{3}$	1122	33 WF 130	1089	33 WF 130	50.61 50.45	90	180	1263	0.1425	O.K
0.2	$\frac{1}{2}$	1086	30 WF 124	1050	30 WF 124	36.20 35.00	90	180	1203	0.1495	O.K
	$\frac{1}{3}$	1020	33 WF 130	972	33 WF 130	51.00 48.60	90	180	1263	0.1425	O.K
0.3	$\frac{1}{2}$	1020	33 WF 130	972	33 WF 130	34.00 32.40	90	180	1263	0.1425	O.K
	$\frac{1}{3}$	933	33 WF 130	888	33 WF 130	46.65 44.40	90	180	1263	0.1425	O.K

CHAPTER V

SUMMARY AND CONCLUSIONS

5-1. SUMMARY

A two-span pin-based steel gable frame was analyzed both by elastic and plastic methods and briefly designed for its critical section assuming straight connection and constant section throughout.

In the case of elastic analysis end moments for two values of column height in conjunction with three different values of roof rise assuming constant vertical and wind loads. The elastic analysis was carried out by the method of moment distribution. The plastic analysis of the said frame with the mentioned conditions was expedited by employing mechanism methods and simplified procedures.

The final results obtained by both methods were tabulated in Table 5-1. At first glance, the benefits achieved by plastic design may be summarized as,

1. Time saving:

   The computations involved in the analysis devised by plastic methods are simple and brief compared with those by elastic methods.

2. Material saving:

   The comparative results of selected sections shows that an average of 20% material could be saved by employing plastic design.

3. Load factors:

   May be predicted with more accuracy than is possible from safe

TABLE 5-1

THE RESULTS OF THE COMPARATIVE DESIGNS FOR A TWO-SPAN PIN-BASED GABLE FRAME BY THE ELASTIC AND PLASTIC METHODS

Parameters		Elastic Moment M_y (K ft.)	Plastic Moment M_p (K ft.)		Shape Factor $\frac{Mp}{My}$		Sections Selected by			% Saving of wt. by Simplified Method	% Saving of wt. by Mechanism Method
α	β		Mechanism Method	Simplified Method	Mechanism Method	Simplified Method	Elastic Method	Mechanism Method	Simplified Method		
0.1	$\frac{1}{2}$	-967.0	1161.0	1137.0	1.190	1.180	36 WF 194	36 WF 132	30 WF 132	31.9%	31.9%
	$\frac{1}{3}$	-875.0	1122.0	1089.0	1.285	1.245	36 WF 182	33 WF 130	33 WF 130	28.6%	28.6%
0.2	$\frac{1}{2}$	-854.0	1086.0	1050.0	1.270	1.230	36 WF 170	30 WF 124	30 WF 124	27.1%	27.1%
	$\frac{1}{3}$	-728.0	1020.0	972.0	1.40	1.340	36 WF 150	33 WF 130	33 WF 130	13.4%	13.4%
0.3	$\frac{1}{2}$	-770	1020.0	972.0	1.320	1.260	36 WF 160	33 WF 130	33 WF 130	18.7%	18.7%
	$\frac{1}{3}$	-716	933.0	888.0	1.30	1.235	37 WF 150	33 WF 130	33 WF 130	13.4%	13.4%

Fig. 5-2
WEIGHT SAVING CHART
with respect to M_p / M_y

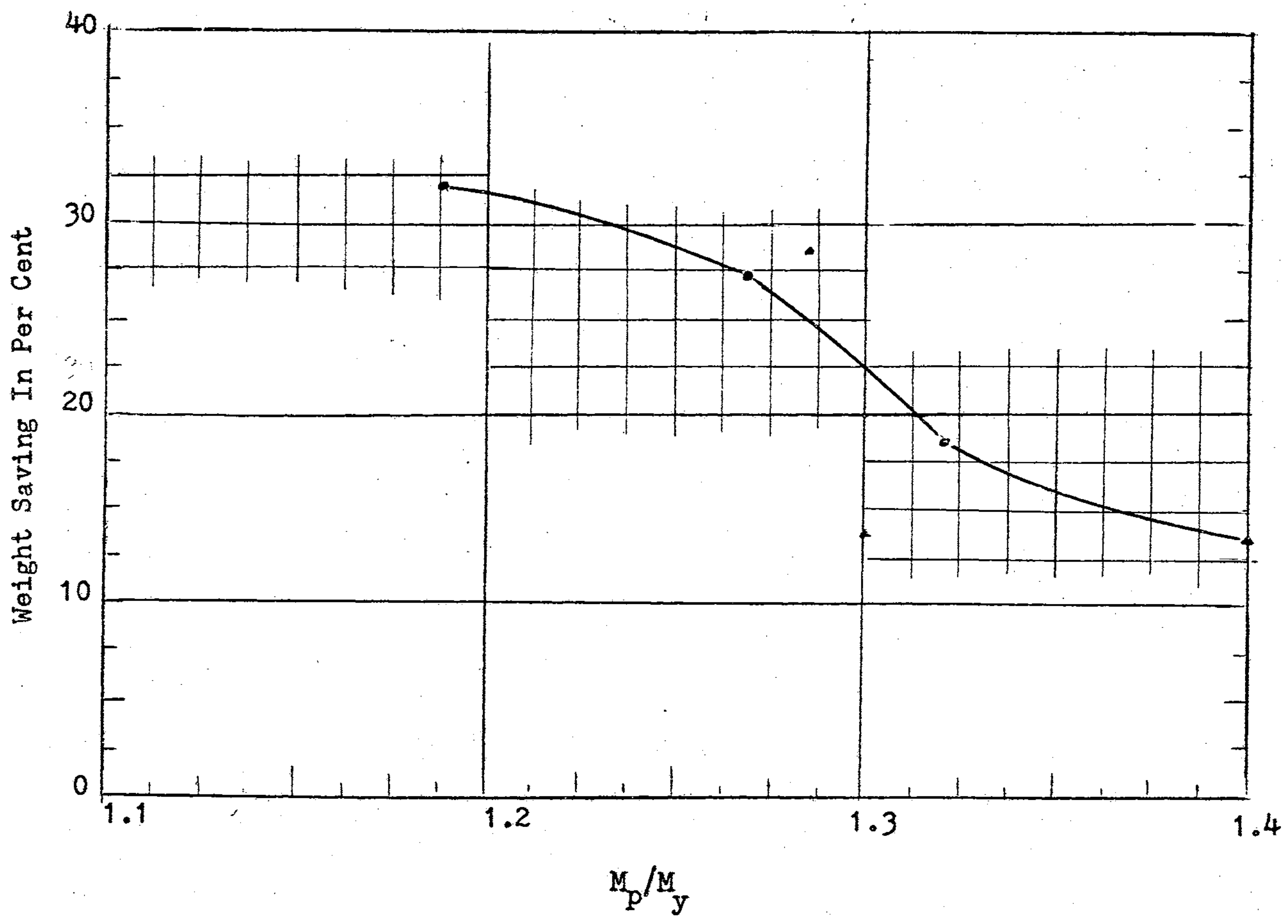
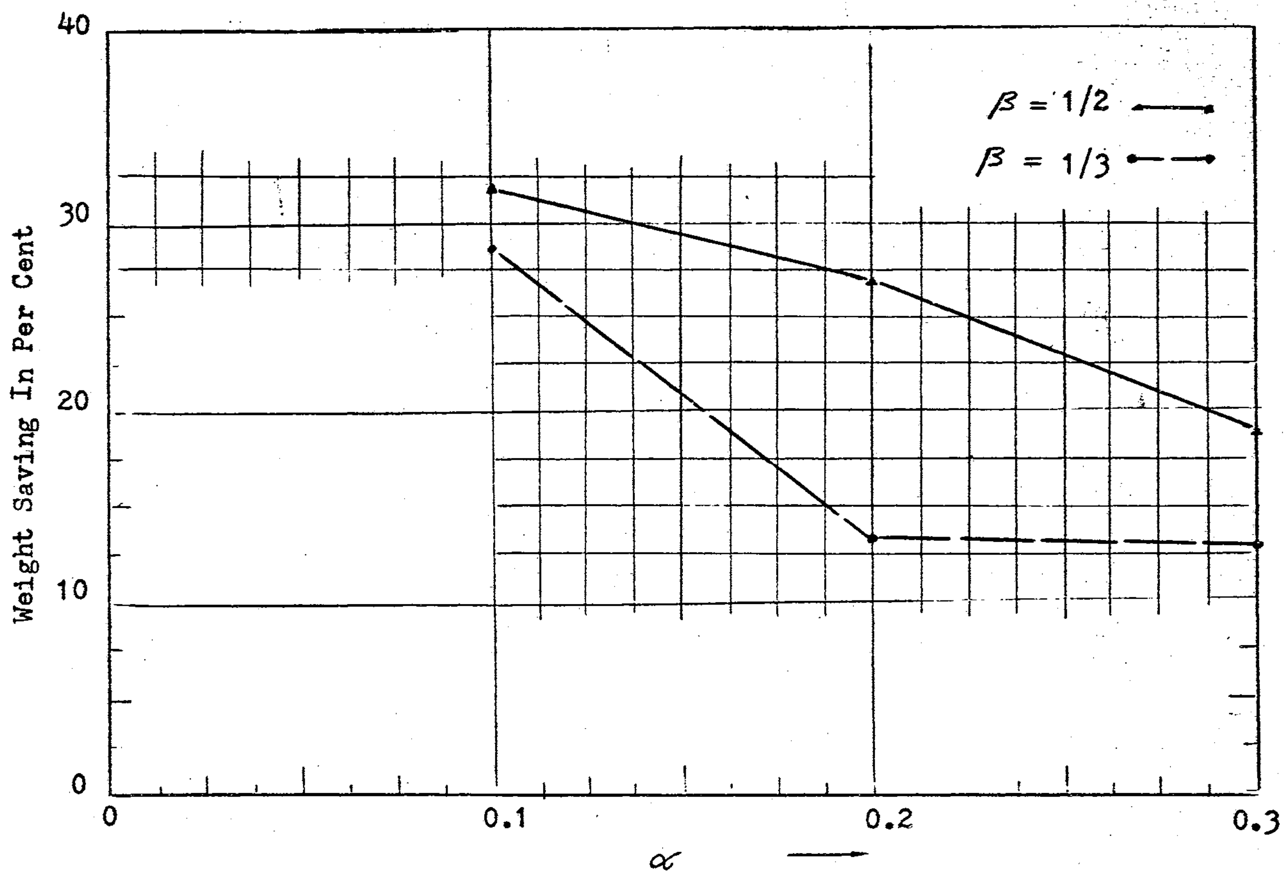


Fig. 5-3
WEIGHT SAVING CHART
with respect to parameters



unit stress at working load, and also a very uniform factor of safety will result.

5-3. INFLUENCE OF PARAMETERS

From the results obtained by two different methods of analysis it was shown that, plastic design is weight saving and time saving. Figure 5-3 clearly indicates that the per cent of weight saving varies with different combination of parameters.

Thus, the full benefits of plastic design in this particular case are greater when the " α" roof rise to span ratio is low, while they are lower when this ratio is large.

Furthermore, for the same roof rise, if the "β" column height to span ratio would give more saving when it is high as compared to a low order ratio.

These conclusions are based upon a collapse mechanism due to vertical load only. In brief we can say parameters play a vital role in weight saving.

5-4. CONCLUSION.

Yet this is not the conclusive proof that plastic design is feasible in all cases, because it is still in an experimental stage and has the following limitations:

1. Uncertainties of loading might result in fatigue failures.
2. Accuracy of design loads in representing present actual loads and possible future increase in magnitude and nature of loading.
3. The superior type of fabrication is required in the shop and the field so that the "ductility" which is considered in the plastic theory concepts may not be impair and with loss in strength.

4. Monumental structures, where a greater margin of safety is required.

5. Possibility of deterioration due to exposure to sea water or other corrosive inducing environments.

While in the elastic analysis it is assumed that no yielding takes place. It is desirable to check the following points that favor the plastic method.

1. Correctness; that is the suitable margins of safety for all parts of structure under critical conditions.

2. Simplicity; when a convenient method is required for analysis.

3. Adaptability; the flexibility in the extension and application of the method to unusual structures.

4. Elastically designed structures in ideal conditions should possess the uniform variation of x-section corresponding to the moment.

Further research in the area of plastic design will broaden its application to multi-story structures; trusses, etc. Though the efficiency of either method is entirely dependent upon the problem and the designer, however, the judgement evaluation in its applicability would yield better results.

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