AN EXPERIMENTAL STUDY OF MODEL POLES UNDER LATERAL LOADS

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CHAPTER I

INTRODUCTION

Posts and poles have long been used in agricultural construction. Because treatments have been developed that greatly prolong the life of wood, poles may be used in important structures. For instance, plans have been developed by the Oklahoma State University Agricultural Experiment Station for a laminated arch building that will be used for a dairy barn. The arches will be anchored into the ground in the same manner as posts or poles are anchored.

Research at the Oklahoma State University Agricultural Experiment Station has revealed that there is a need for a method of predicting the behavior of laterally loaded poles. In farm structures these poles are usually embedded in the earth to a relatively shallow depth.

A rational analysis would be an ideal method of solving the problem. Several such attempts have been made. The results have been less than satisfactory, however, because of the simplifying assumptions that were necessary to render the problem solvable.

It has been suggested that an investigation of the use of physically similar models might be a fruitful line of study. The general laws of physical similarity have been published by E. Buckingham and others.

If model tests which give valid results could be performed, they would be valuable research tools. They might also be useful in field application.

If the principles outlined herein are valid for the soils that were used, it should be possible to adapt them to other soils.

CHAPTER II

OBJECTIVES

The primary objective of the study described in this thesis was to investigate the possibility of using small-scale models for studying the behavior of laterally loaded poles. It was felt that this could be done by attempting to predict the behavior of prototype poles with models. It appeared that a reasonable method of doing this would be to obtain by experimental methods a prediction equation describing the behavior of a laterally loaded model pole and then to use the same procedure to obtain a prediction equation for a prototype pole. A comparison of these two equations would give a measure of success of the experiments. To give some degree of generality to the results three different studies were made. One was made in dry sand with an internal friction angle of 29°; another was made in dry sand with an internal friction angle of 36°; the third study was made in a saturated sandy clay.

A secondary objective was to determine how changes in pole embedment depths affected the behavior of the poles in dry sand. Before the secondary objective could be accomplished, it was necessary for the studies mentioned in the primary objective to prove that the model studies gave results which were valid for prototypes.

Because these investigations were made with farm buildings in mind, the dimensions of the models were chosen so that the results would be applicable to poles 6 inches in diameter and embedded from $2\frac{1}{2}$ to $4\frac{1}{2}$ feet in the ground. This is the range of embedment depths often used for poles in farm construction. The application of the results is not limited to this range of sizes. They may be applied to any size which is consistent with the conditions of the pi terms.

CHAPTER III

A REVIEW OF PREVIOUS WORK

Several approaches have been made to the problem of predicting the behavior of a laterally loaded pole. No approach so far used gives consistently satisfactory results. A brief review of the various methods used follows:

Rational derivations. Generally, rational analysis of a problem requires less work than an experimental study. Often results of rational analysis have wider application than those of an experimental study. For these reasons a complete rational analysis of the behavior of a pole under lateral load is a desirable goal. This goal has not been achieved.

Czerniak (1) proposed a rational solution to the problem which made use of the assumptions that the pole was perfectly rigid and that the soil had a definite modulus that varied as the first power of the depth.

Palmer and Thompson (2) proposed an approximate solution for piles. The pile was considered to be a non-uniformly loaded beam. The soil modulus was allowed to vary with any power of the depth, and the pile was assumed to be flexible. The fundamental differential equations of the non-uniformly loaded beam were set up for these conditions. An approximate solution was obtained with difference equations.

Neither of these solutions was completely satisfactory. Although in both solutions the soil modulus was assumed to vary only with depth, in reality it was a function of the width of the pole, the magnitude of the applied load, the deflection, and the depth.

Nelson (3) derived an equation for the deflection of an elastic pole under lateral load when the deflection caused by anchorage yield was known. He recommended that this deflection be determined from field measurements.

Full-scale tests. The results of the full-scale studies are usually applicable only to the post-soil-load conditions which were used in the tests described.

Shilts and Graves (4) conducted tests for the Outdoor Advertisers Association. Anderson (5) studied the overturning resistance of utility poles. These tests were carried out, in general, with larger poles and deeper settings than are normally used in farm construction. The deflections allowable in utility poles and outdoor signs would cause great damage to a farm building.

Nelson (6) and his associates studied the effect of lateral loads on 6-inch diameter poles that were embedded $2\frac{1}{2}$ feet, $3\frac{1}{2}$ feet, and 5 feet deep. The poles projected 14 feet above grade. The tests were made for the purpose of getting results that would be applicable to farm structures. The results, however, are applicable only to poles of similar size and embedment depths in the soil types tested.

In experiments with full-scale piles McClelland and Focht (7) found that the soil modulus increased almost linearly with depth. They also found that it decreased as load increased. McNulty (8) concluded that the soil modulus decreased with deflection. His experiments were with piles also.

Full-scale tests are expensive and cumbersome whether used in research or field application.

Model studies. In soil mechanics several model studies have been reported. None of the reports, however, mentioned the classical theories of dimensional analysis and physical similarity.

Wen (9) made lateral loading studies on small-scale wood piles. For small deflections, load versus deflection was a straight line on semi-log paper. He did not report a dimensional analysis or mention the conditions necessary for physical similarity.

Tschebotarioff (10) described lateral load tests on tapered model piles. The piles were driven into 15 inches of sandy clay which was overlain by 14 inches of sand. The soil was contained in a concrete tank. Lateral loads were applied, and deflections were measured. Deflections ranged from zero to fifteen inches and loads from zero to forty-five pounds. Physical similarity was not mentioned in the paper.

In the oral discussion after the paper Cummings asked Tschebotarioff why the deflection obtained by models was so much greater than that from full-scale pile tests (10). In his answer the author did not mention similarity.

In the same discussion Gleser told of model piles tested in sand by the United States Army Engineers at St. Louis. He reported great variation between piles with the same treatment. Tschebotarioff suggested that the variation was caused by differences in sand density. These differences were caused by the method of compaction. He stated that the sand should be compacted uniformly over the entire area.

CHAPTER IV

A BRIEF REVIEW OF MODEL THEORY

Buckingham (11) offered the following theorem: "If an equation is dimensionally homogeneous, it can be reduced to a relationship among a complete set of dimensionless products."

The article which contained this theorem also contained a statement which in effect said that the number of dimensionless and independent quantities required to express the relationship among the variables in any phenomenon is equal to the number of variables minus the number of basic dimensions in which those variables may be measured. In equation form the statement is:

$$s = n - b$$

The number of dimensionless and independent quantities, usually called pi terms is represented by s. The number of variables is represented by n, and b is the number of basic dimensions.

Bridgman (12) pointed out that there were some exceptions to this rule. Langhaar (13) showed that, if the rule were restated to say that the number of pi terms is equal to the numbers of variables minus the rank of the dimensional matrix, there would be no exceptions.

The only restrictions on the pi terms are that they be dimensionless and independent.

If fourteen variables were required to describe a phenomenon and if the rank of the dimensional matrix were three, eleven pi terms would be required. The rank of a matrix is defined as the size of the largest non-zero determinant in that matrix. The relationship among the pi terms could be written as:

$$\pi_1 = f(\pi_2, \pi_3, \cdots\cdots \pi_{11})$$

When it is desired to produce a model that will yield results that will be valid for a prototype, the following conditions must be met:

$$\pi_{1p} = \pi_{1m}; \quad \pi_{2p} = \pi_{2m}, \cdots\cdots \pi_{11p} = \pi_{11m}$$

When these conditions exist, the model is known as a true model, and the following equation may be written:

$$F(\pi_{1p}, \pi_{2p}, \cdots\cdots \pi_{11p}) = F(\pi_{1m}, \pi_{2m}, \cdots\cdots \pi_{11m})$$

In words, this says that results obtained from a model are applicable to the prototype when the pi terms of the model are equal to those of the prototype.

Models are used extensively in studying fluid flow and to some extent in studying the behavior of structures. For instance, prior to its construction, two models were made of the Hoover dam to study its structural behavior. P. B. Bucky (14) used models in studying problems in the design of mine workings.

The most critical part of any study with models is the determination of the variables which affect the phenomena being investigated. If an important variable is omitted, the study will not be valid.

The way in which the variables are combined into pi terms is a matter of choice of the investigator. As mentioned previously, the only conditions that must be met is that the pi terms be dimensionless and independent. Of course, they should be arranged in a manner that will cause investigation to be of the greatest possible value.

CHAPTER V

THE MOVEMENT OF POLES UNDER LATERAL LOADS

A schematic diagram of a pole under lateral load is shown in Figure 1. When a load is applied at C, referring to Figure 1, the soil is put in compression at A and B. If the load is great enough, there will be movement. For this movement to occur the soil in contact with the post at A and B must move.

If the sand is in a loose state, some movement could be allowed by a decrease in volume caused by compaction of the sand in areas A and B. If the sand is in a dense state, there can be no significant decrease in volume. In fact, the volume of a dense sand increases when it is deformed.

In both cases the sand in regions A and B will be subjected to greater compressive stresses than sand at a greater distance from the pole. Shearing stresses will also be present. The compressive stresses and shearing stresses are resisted by the weight of the soil and the internal friction of the sand. For a particular sand the internal friction angle increases directly as the unit weight.

The amount of movement under a given lateral load will be determined by the internal friction angle, the unit weight, and specific gravity of the sand, the depth of embedment of the post, and the diameter of the post. The deformation of individual sand particles will be a factor also. The amount of movement resulting from this will

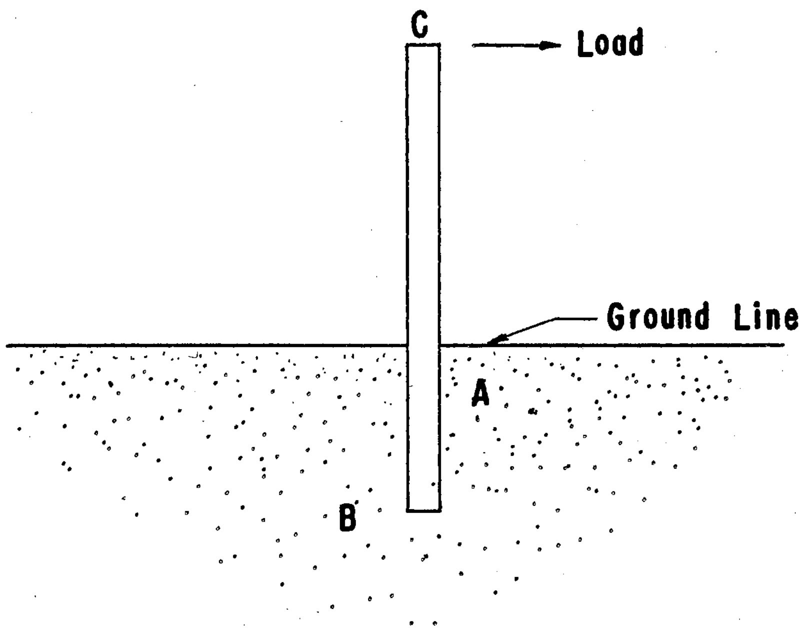

Figure 1. A Pole Under Lateral Load

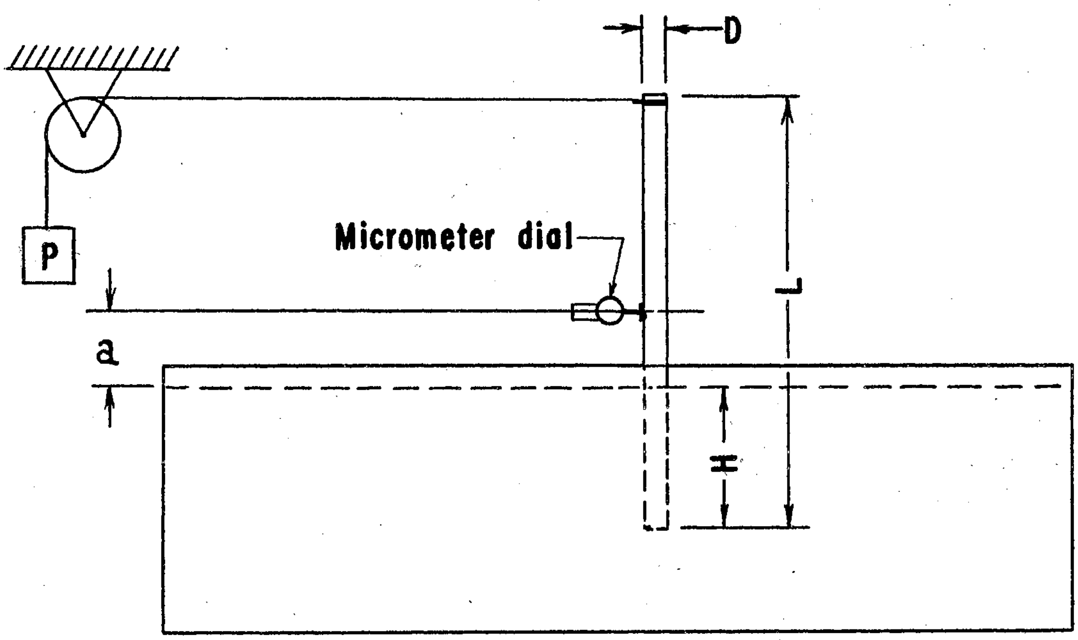

Figure 2. Symbols Involving Geometry of the Pole

be very small in relation to that resulting from displacement of the particles. For this reason particle deformation was omitted from the analysis.

Bending of the poles will also contribute to deflection. In these studies solid steel poles were used, and deflection was measured near the ground line. It was assumed that pole bending could be neglected under these conditions.

If the sand is saturated and is very fine, the rate of movement under a given load will be determined by permeability. For all dry sand and saturated coarse sand the time interval between application of the load and pole movement will be small.

In a saturated mixture of sand and clay the permeability will control the rate of movement.

CHAPTER VI

DIMENSIONAL ANALYSIS

The three soil conditions used were dry loose sand, dry compacted sand, and a saturated mixture of sand and clay.

Analysis for dry sand. The following variables were thought to be involved to an appreciable extent in the behavior of a pole under lateral loads in dry sand. They were selected on the basis of the analysis made in the previous chapter.

	Variable	Symbol	Dimensions
1.	Pole diameter	D	L^1
2.	Depth of embedment	H	L^1
3.	Weight of soil per unit volume	γ	FL^{-3}
4.	Internal friction angle	ϕ	-
5.	Load applied	P	F^1
6.	Point of load application	L	L^1
7.	Lateral movement	y	L^1
8.	Specific gravity of soil solids	G_s	-
9.	Point at which deflection is measured	a	L^1

The dimensional matrix for these variables is:

	D	H	γ	P	L	y	a
F	0	0	1	1	0	0	0
L	1	1	-3	0	1	1	1

The rank of this matrix is two. The number of variables is nine. Therefore, the number of pi terms required is seven. The pi terms chosen were:

$\pi_1 = H/D$ $\pi_4 = P/D^3\gamma$ $\pi_6 = L/H$

$\pi_2 = \phi$ $\pi_5 = y/D$ $\pi_9 = H/a$

$\pi_3 = G_s$.

The values and relationships of the variables concerning geometry are shown in Figure 2 and in Table I.

TABLE I

POLE DIMENSIONS IN INCHES FOR DIFFERENT VALUES OF π_1.
THE DIMENSION COLUMN REFERS TO FIGURE 1.

Dimension	Model			Prototype
	π_{15}	π_{17}	π_{19}	π_{17}
L	11 1/4	15 3/4	20 1/4	31 1/2
H	3 3/4	5 1/4	6 3/4	10 1/2
a	1 3/16	1 2/3	2 1/8	3 1/3
D	3/4	3/4	3/4	1 1/2

Analysis for saturated sandy clay. The variables thought to be involved to an important degree in the rate of movement of laterally loaded poles embedded in a saturated sandy clay were:

	Variable	Symbol	Dimensions
1.	Pole diameter	D	L^1
2.	Depth of embedment	H	L^1
3.	Weight of soil per unit volume	γ	FL^{-3}
4.	Load applied	P	F^1

5.	Point of load application	L	L^1
6.	Lateral movement	y	L^1
7.	Point where deflection is measured	a	L^1
8.	Water content	w	-
9.	Permeability	k	LT^{-1}
10.	Specific gravity of solids	G_s	-
11.	Time elapsed since loading	t	T^1

The rank of the dimensional matrix is three. The pi terms chosen were:

$$\pi_1 = H/D \qquad \pi_6 = L/H$$

$$\pi_3 = G_s \qquad \pi_7 = w$$

$$\pi_4 = P/H^3\gamma \qquad \pi_8 = kt/D$$

$$\pi_5 = y/D \qquad \pi_9 = H/a$$

CHAPTER VII

GENERAL PROCEDURE

Loose sand experiments. Since prediction equations were desired, it was necessary to hold all but two pi terms constant in any given test. One of these two pi terms was varied through a range of values. The effect of this variation on the second pi term was noted. In all cases the pi term containing deflection was used as the dependent variable.

In the sand experiments there were seven pi terms. Experiments to determine a complete prediction equation would be very time-consuming. The value of the results obtained probably would not justify the expense involved. This would also be unnecessary to achieve the primary objective of this study.

The pi terms containing load, deflection, and embedment depth were chosen as the ones to be varied. An outline of how the pi terms were treated in the investigation is given below:

1. π_2 and π_3, which are internal friction angle and specific gravity, were held constant by using the same sand in the same condition for all loose sand tests. π_6 was held constant at a value of 3. The value of π_9 in all tests was 3.15.
2. π_4 was varied by changing P. The effect of this variation on π_5 which contains the deflection term was noted.

3. Step two was carried out for three values of π_1. The values of π_1 were 5, 7, and 9. The notation for these values is π_{15}, π_{17}, and π_{19}, respectively.

The results of steps two and three were plotted on logarithmic paper. The graphs are shown in Figure 15. If all variables that substantially affect the system were considered, these curves are valid for any size post set at any depth as long as the pi terms other than π_1, π_4, and π_5 have the same value as the ones used in this study. π_1 and π_4 must fall in the range of values investigated.

A prediction equation was developed and is given in the analysis of results. It has the form $\pi_5 = f(\pi_4, \pi_1)$.

The model studies were validated by using prototype poles twice the diameter of the model. The procedure for arriving at the curve π_{17} was repeated, and prototype poles were used. The curves for π_{17} prototype and π_{17} model are shown in Figure 13.

Dense sand experiments. The procedure described above was used for dense sand tests. The use of dense sand caused the weight per unit volume and internal friction angle to change.

Saturated sandy clay. As in the previous experiments, prediction equations for the behavior of the model and prototype systems were desired. The addition of clay and water to the system made it necessary to add two new pi terms to the analysis. They were π_7, which is the water content of the soil, and π_8, which includes time and permeability variables. The pi terms were treated as follows:

1. For a constant value of π_4, the value of π_5 was noted for several values of π_8. The deflection of the pole was included in π_5. The time elapsed since loading was included in π_8.

2. The above procedure was repeated for five values of π_4, the load term.

3. An attempt was made to keep all pi terms related to geometry alone constant and at the same values for both model and prototype. As will be explained later, the attempt was not entirely successful, and corrections in the data were necessary.

The data from these experiments are tabulated in Tables X and XI in Appendix A.

CHAPTER VIII

MECHANICS OF THE STUDY

Loose Sand Experiments

All studies were conducted indoors. It was felt that the size of the model selected was the smallest that could be used and still obtain fairly precise results with available measuring and loading equipment.

The size of the prototype selected was smaller than would be found in farm construction. It was desirable for physical reasons to use this size since larger sizes would have required excessively large testing apparatus. It was felt that the size selected would give some degree of validation. The procedure and equipment used in carrying out the experiments are described below:

1. A box 6 feet long, 3 feet wide, and 2 feet deep was used to contain the soil material, which was Ottawa sand. The box, which is shown in Figure 3, was constructed of 2-inch Redwood staves.

2. The poles tested were rigidly suspended from cross-members above the box. They projected into the empty box by an amount equal to the depth of embedment plus approximately two inches. Sand was poured in 1-inch layers into the box until it came to the proper level on the poles. The suspension mechanism was then removed. Figure 4 gives a view of the suspended poles

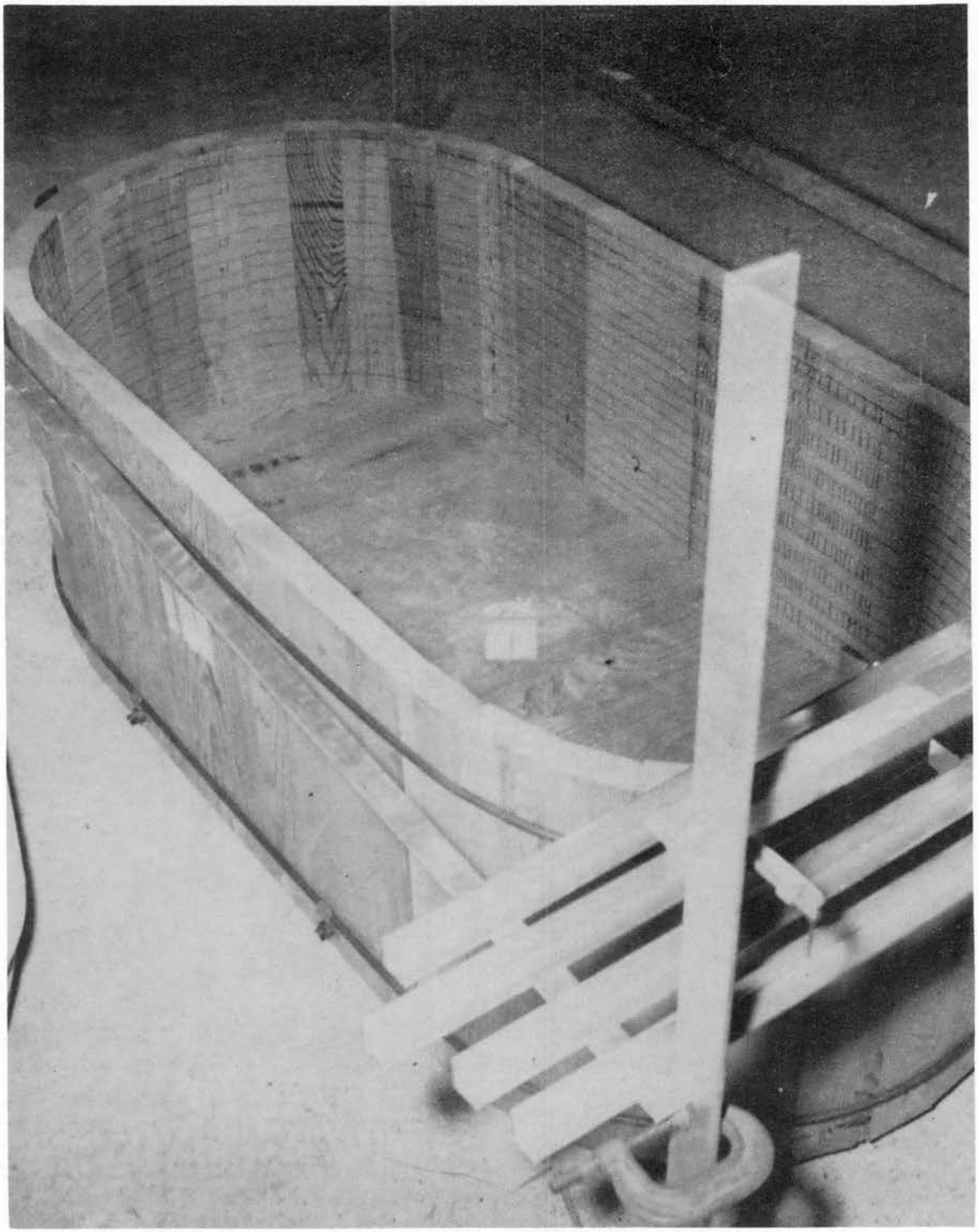

Figure 3. The Box Used To Contain the Soil Material

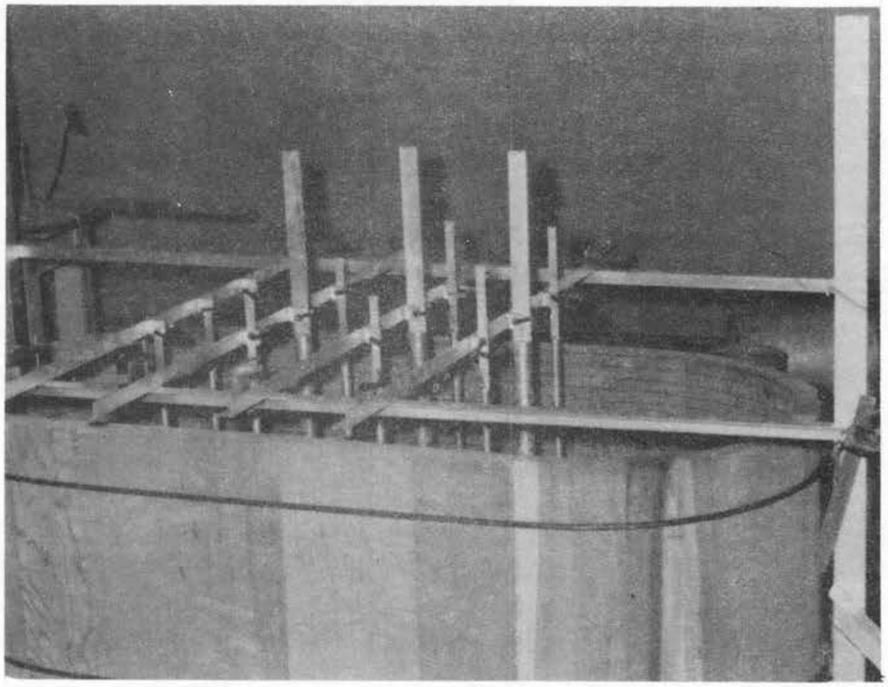

Figure 4. The Poles Suspended Into the Box

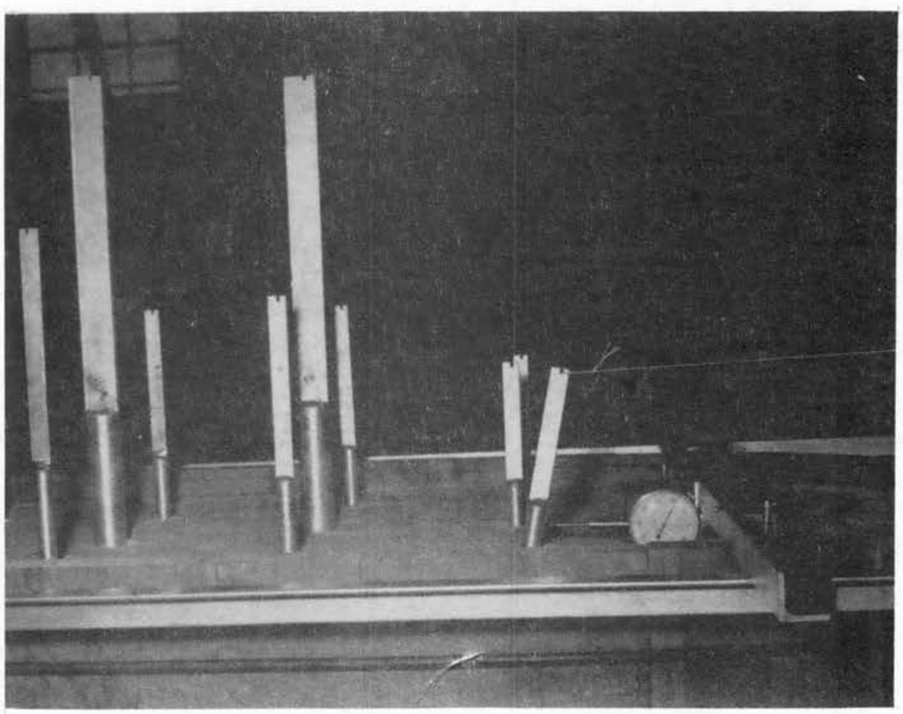

Figure 5. A Pole Under Load

before the sand was added.

3. The model poles were three-fourths of an inch in diameter. Three embedment depths were used. They were 3 3/4 inches, 5 1/4 inches, and 6 3/4 inches. These depths caused π_1, which is H/D, to have values of 5, 7, and 9. The overall lengths of these poles were 11 1/4 inches, 15 3/4 inches, and 20 1/4 inches respectively. The 1 1/2-inch diameter prototype poles were embedded 10 1/2 inches, and their overall length was 31 1/2 inches. Other pi terms concerning the geometry of the pole were constant.

4. The poles were made of low carbon steel. The lower portion of the poles was round. The upper part was a flat bar. The poles are shown in Figure 5.

5. The deflection of the pole was measured 1 3/16 inches above the soil surface for π_{15}. This caused π_9 to have a value of 3.15. π_9 was held constant for other values of π_1 by varying the height of the point at which deflection was measured. A micrometer dial was used to measure deflection. The springs were removed from the dial to prevent the dial from applying pressure to the pole. The point of the dial was fastened to the pole with an Alnico magnet as shown in Figure 6.

6. A string was fastened to the top of the pole and led horizontally through a pulley supported by a cross-member. A weight pan attached to the string was used for loading. A schematic diagram of the loading and deflection measuring mechanism is shown in Figure 2.

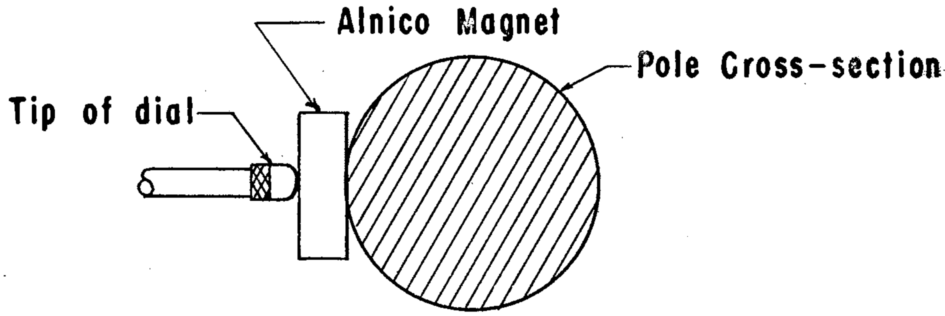

Figure 6. The Method of Fastening the Tip of the Dial to the Pole

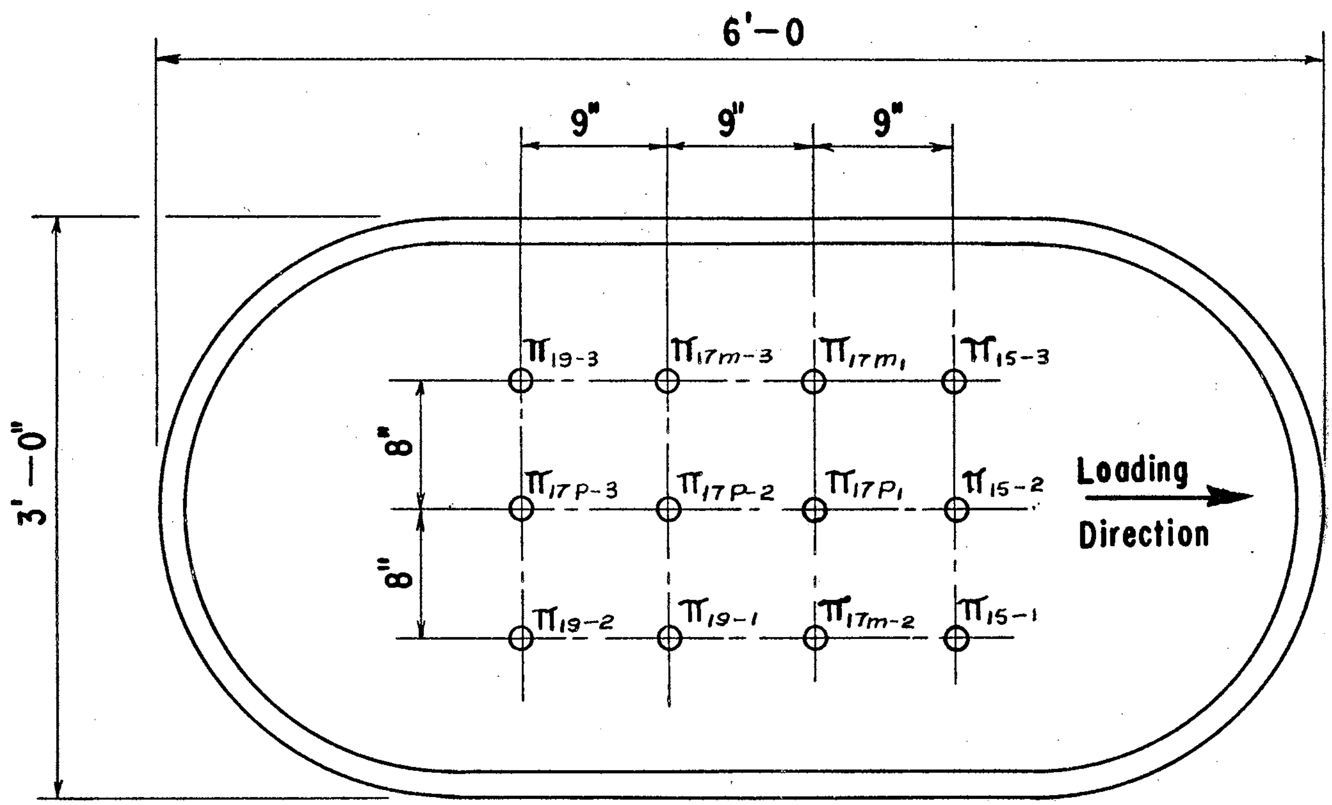

Figure 7. The Relative Location of Poles in Loose Sand Tests. The Second Subscript Number is the Value of π_1 for that Pole. The Subscripts m and p Refer to Model and Prototype Respectively. The Last Number is the Pole Replication Number.

7. Load was added in varying increments until the pole rotated through an angle of more than ten degrees. The micrometer dial reading was noted after each load increment was added.

8. Three replications were used for each depth. A total of twelve poles were used. The approximate location of the poles in the box is shown in Figure 7. Three of the poles were prototypes, and nine were models.

Dense Sand Experiments

Compacting sand increases the internal friction angle. It also increases the weight per unit volume. The latter variable is included in the pi term containing applied load.

Except as outlined below, the procedure for dense sand was the same as that for loose sand.

1. The poles were suspended as described above. The sand was placed in 2-inch increments and compacted. The equipment used for compaction was the modified vibratory sander shown in Figure 8. The sand was compacted in two steps. In the first step the sander was placed on the surface of the sand and vibrated for about three seconds. It was then moved to an adjoining area and again vibrated. This process was repeated until the entire surface of the increment was vibrated. This left the surface in a rough condition.

   In the second step a 1-inch by 6-inch by 14-inch board was placed on the surface. The sander was then placed on this board and vibrated for about three seconds. The board was moved to an adjacent area, and the process was repeated. The

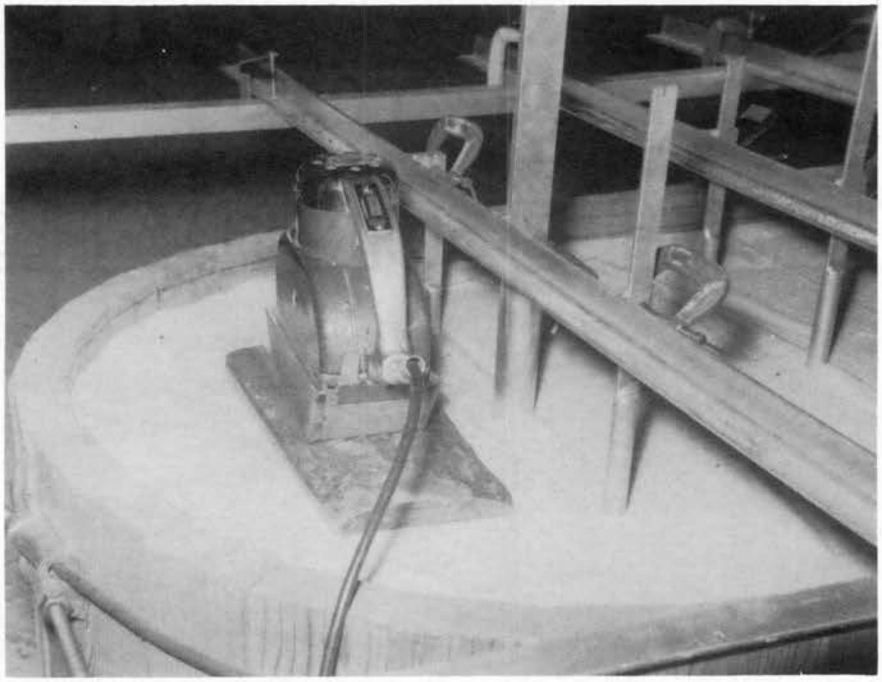

Figure 8. The Vibratory Sander Used for Compacting the Sand

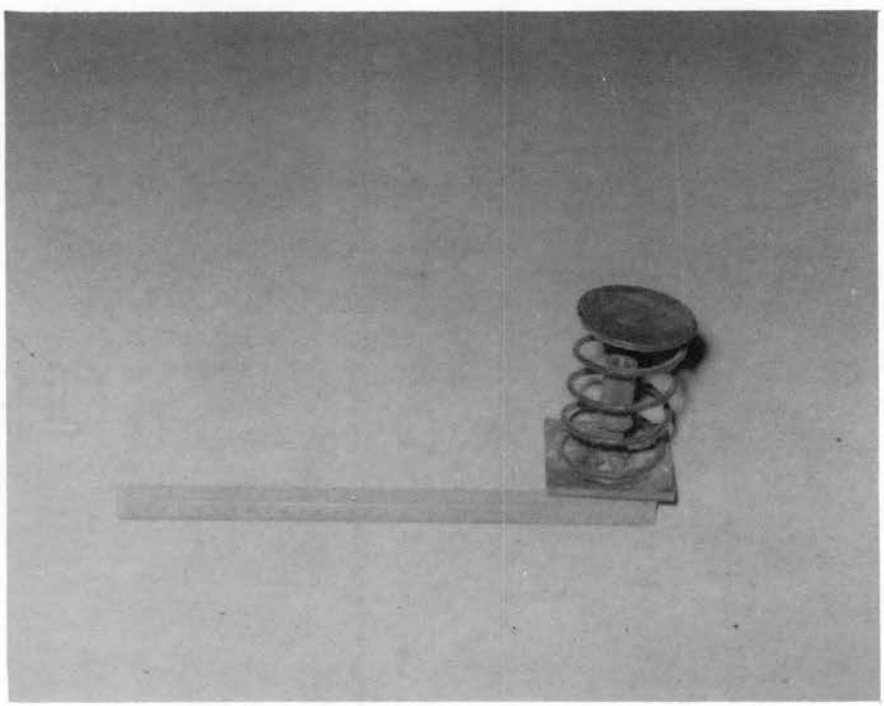

Figure 9. The Device Used for Packing the Sandy Clay

entire surface of the increment was treated in this manner.

The top increment was given the board treatment twice.

2. Load was applied until the pole failed. Pole movement was allowed to cease before reading the micrometer dial. When enough load was applied to cause failure, the pole moved rapidly until it was stopped by the end of the dial.

Saturated Sandy Clay

The experimental procedure for these tests was similar to that used in the ones described previously. The soil material was a mixture by weight of five parts of Ottawa sand to three and a half parts of a clay-type soil. The clay was material that had been removed from a cellar excavation. It was dried to a very low moisture content and then passed through a hammer mill, which pulverized it to a fine powder. The clay was then mixed with the sand in a small electrically driven mortar mixer.

As in the sand experiments, the model and prototype poles were suspended over the empty box. A layer of Ottawa sand 2 1/8 inches thick was placed in the box. After this, the mixture of dry sand and clay was placed and packed in approximately one-inch layers. The device shown in Figure 9 was used for packing. It was adjusted to give a pressure of 3 pounds per square inch. A cross section of the filled soil container is shown in Figure 10.

The model poles, which were of the same dimensions as those used in previous experiments when the value of π_1 was 7, were suspended over the box so that the depth of embedment was $5\frac{1}{4}$ inches. This depth was $10\frac{1}{2}$ inches for the prototype. π_1 was not varied in this series of experiments.

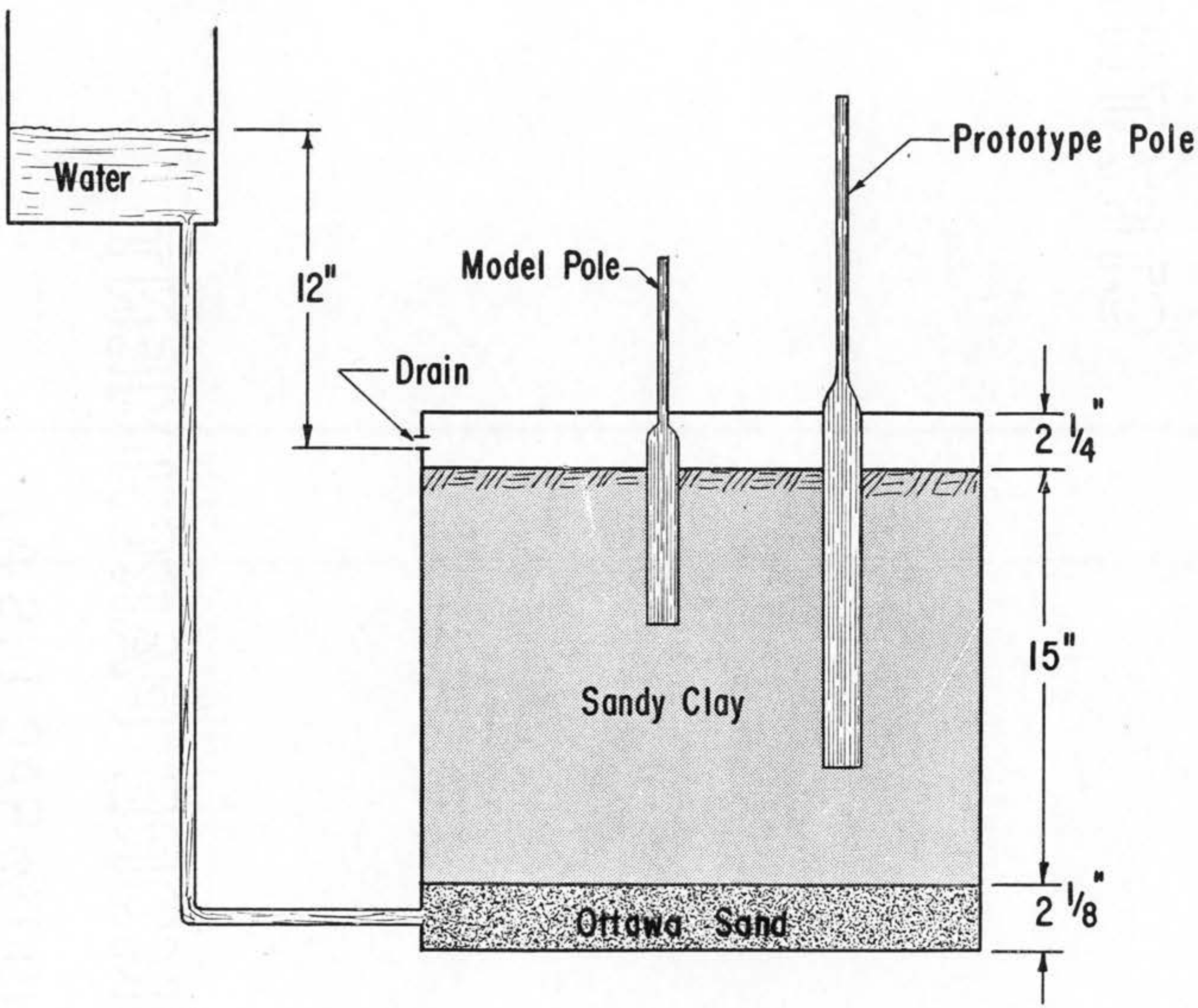

Figure 10. A Cross Section of the Apparatus Used for Saturating and Measuring the Permeability of Sandy Clay

The mixture was then saturated by introducing water into the sand in the bottom of the tank. A schematic diagram of the apparatus is shown in Figure 10. The time required for damp spots to appear on the surface was 24 hours. Water began to collect on the surface after about five days.

Loading method. Loads were applied through the same pulley apparatus as before. Five loads were used. The loads on the prototype were eight times as heavy as those on the model in each case. The loads used on the model were 100 grams, 150 grams, 200 grams, 250 grams, and 300 grams. The 100-gram load was applied to the model pole, and the deflection was recorded. The first deflection reading for the model was taken after 15 seconds; the second after 30 seconds; and the next after 60 seconds. This process of taking deflection readings as time doubled was continued until time reached 16 hours in the model. After this procedure was complete for the first model pole, loads were applied in 50-gram increments to this pole and left on for 1 hour. This was done to determine what loads should be used on the remaining poles.

The time increments used in the prototype were double those of the model. The last reading on the prototype pole with 800 grams load was taken at 32 hours.

The remaining model poles were tested for 24 hours, and the corresponding prototypes were kept under a single load for 48 hours. Additional load increments were used only on the pair of poles, one model, and one prototype described above. The load test data are given in Tables X and XI in Appendix A.

When the tests were complete, the poles were marked at the ground line and pulled out. Depth of embedment was again determined. The model poles were embedded 5 9/16 inches, and the prototypes were embedded 10 13/16 inches. This was five-sixteenths of an inch greater than the original depth setting in both cases. This increase in depth was probably caused by swelling of the soil while the poles were still fastened to the rigid supporting frame.

This change in depth made changes in some pi terms necessary. These changes are discussed in the analysis of data.

Determination of Soil Properties

Weight per unit volume. The weight per unit volume of sand was determined by measuring the volume of the box and weighing the sand that was placed in it.

The volume of the box was determined by weighing the water required to fill the box to within two inches of the top and then by dividing this weight by the unit weight of water, which is 62.4 pounds per cubic foot. The water was weighed on platform scales. The pail used to weigh the water held about twenty-four pounds. The total weight of water was 1204 pounds.

Sand was weighed on the same scales in approximately 100-pound increments. The unit weight for loose sand was 98.16 pounds per cubic foot. For convenience in calculating pi terms this was converted to 25.76 grams per cubic inch. For dense sand these figures were 108.3 and 28.44 respectively. The submerged unit weight of the sandy clay mixture was 14.4 grams per cubic inch.

Internal friction angle. The internal friction angle of a material is usually designated by the symbol ϕ. In this paper it is designated by π_2.

The properties of Ottawa sand in very loose and very dense states were known. The internal friction angle of sand varies directly as the unit weight. Since the unit weights used in these experiments were known, the problem of finding the internal friction angles was one of interpolation.

Permeability. The permeability of the sandy clay was 42.3×10^{-5} inches per minute. The permeability determination was made with the apparatus shown in Figure 11.

Water covered the upper surface of the soil when it was time to begin the permeability tests. It had not, however, reached the upper drain. Water was added to the surface until it reached the upper drain. One hour was allowed for the surface of the water to reach equilibrium before the tests were started. A head of 1 foot was maintained during the tests as shown schematically in Figure 10.

The water that flowed through the upper drain in $1\frac{1}{2}$ hours was collected and measured. The amount of evaporation during this period was determined by placing a container of water near the tank and measuring the change in height of its surface with a point gauge. The apparatus is shown in Figure 12. The evaporation amounted to approximately one-tenth of one percent of the 78 cubic inches of water that passed through the drain during the test. The layer of sandy clay was 17 inches thick. The water temperature was 18 degrees centigrade. The area of the soil was 2005 square inches. From these data the permeability was calculated.

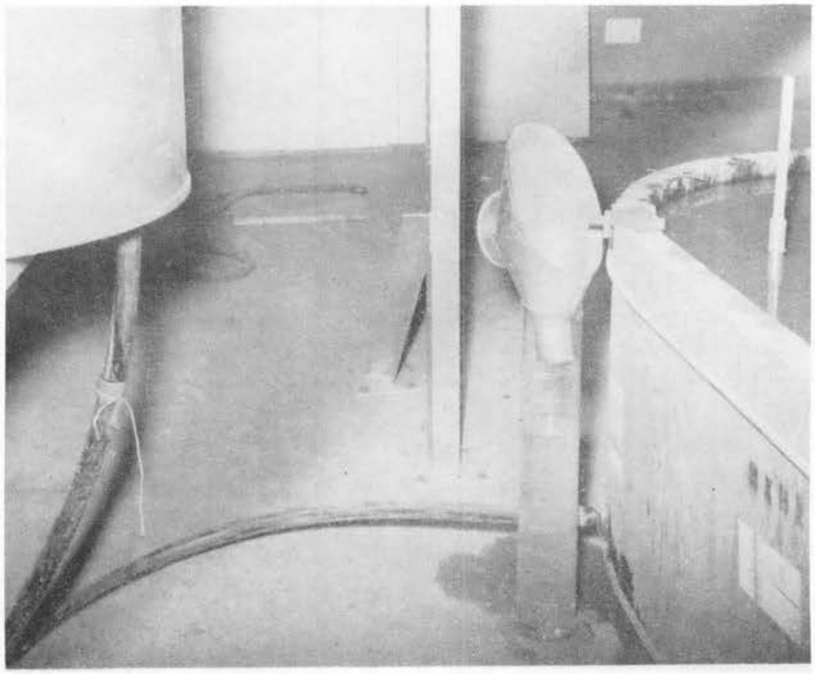

Figure 11. The Apparatus Used for Permeability Determination

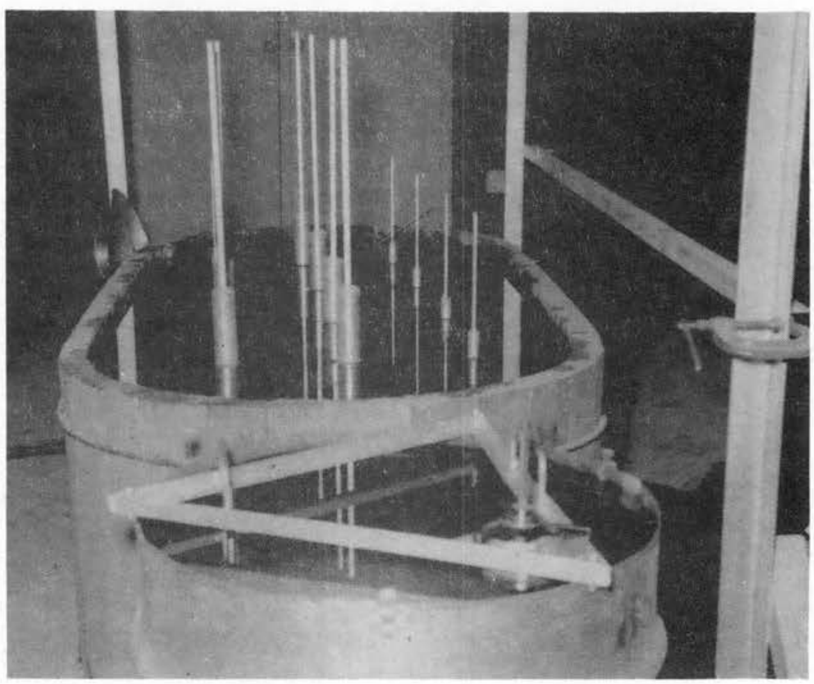

Figure 12. The Apparatus Used for Measuring Evaporation

Determination of the unit weight of sandy clay. The mixture was weighed as it was placed in the tank. Since the volume of the tank was known, determination of the dry unit weight was simple. The value was 23.28 grams per cubic inch. The specific gravity of solids was estimated to be 2.67, and the water content at saturation was calculated to be 33 percent. The soil unit weight submerged was then found to be 14.4 grams per cubic inch.

CHAPTER IX

STATISTICAL METHODS

Sand. The curves π_4 versus π_5 were fitted to the data by the method of least squares as outlined by Snedecor (15). The confidence intervals on the slopes were also calculated by methods outlined by Snedecor.

The confidence intervals on the difference between points on the model and prototype curves were calculated by methods developed by Dr. Franklin Graybill of the Oklahoma State University Department of Mathematics. Dr. Graybill also developed the method for setting confidence intervals on the difference between the slopes of the model and prototype curves. The developments are given in Appendix B.

The formula used for the confidence interval on the slopes was:

$$b_m - b_p - \frac{t\sqrt{E}}{n_m+n_p-4}\left[\frac{1}{\Sigma x_m^2}+\frac{1}{\Sigma x_p^2}\right]^{\frac{1}{2}} \le B_m - B_p \le b_m - b_p + \frac{t\sqrt{E}}{n_m+n_p+4}\left[\frac{1}{\Sigma x_m^2}+\frac{1}{\Sigma x_p^2}\right]^{\frac{1}{2}}$$

The confidence interval on $Y_m - Y_p$ for a given X_o was calculated by the following formula:

$$a_m + b_m X_o - a_p - bX_o - tC\sqrt{E} \leq Y_m - Y_p \leq a_m + b_m X_o - a_p - b_p X_o + tC\sqrt{E}$$

This gave a confidence interval of the form:

$$k_1 \leq Y_m - Y_p \leq k_2$$

This was converted to percentage differences in rectangular coordinates by:

$$10^{k_1} - 1 \leq \frac{\Pi 5 m_o - \Pi 5 p_o}{\Pi 5 m_o} \leq 10^{k_2} - 1$$

Saturated sandy clay. The data for these experiments seemed to form a plane in logarithmic space. A plane was fitted to the data by means of a multiple regression study as outlined by Snedecor (15). The partial regression coefficients were obtained by the Abbreviated Doolittle Procedure (16). The variances used in setting the confidence intervals were obtained with the aid of multipliers from the inverse of the Doolittle matrix.

Symbols. The definitions of symbols given here generally apply only to this chapter. They are symbols frequently used in statistical analysis.

Symbol	Definition
b	An estimate of the mean slope of the line.
B	The true mean slope of the line.
p	Subscript p designates the prototype quantity.
m	Subscript m designates the model quantity.
n	Number of observations.
X	Log π_4.
Y	Log π_5.
o	Subscript o indicates a specific value of the quantity.
a	The Y intercept of the curve in logarithmic space.
$\bar{\bar{x}}$	Mean of X.
x	Deviation from the mean $\bar{\bar{x}}$.
$\bar{\bar{y}}$	Mean of Y.
y	Deviation from the mean $\bar{\bar{y}}$.
t	The Student t value at the 95 percent confidence level for $n_m + n_p - 4$ degrees of freedom.
E	$\Sigma y_m - b_m \Sigma y_m x_m + \Sigma y_p - b_p \Sigma y_p x_p$
C	$\left\{ \frac{1}{n_m + n_p - 4} \left[\frac{1}{n_m} + \frac{1}{n_p} + \frac{(\bar{x}_m - X_o)^2}{\Sigma x_m^2} + \frac{(\bar{x}_p - X_o)^2}{\Sigma x_p^2} \right] \right\}^{\frac{1}{2}}$

<u>Confidence statements</u>. The confidence statement is a probability statement that the true value of a quantity lies within the range stated. For example, suppose that it can be said with 95 percent confidence that the true slope of a line lies within the range of 5 to 7.

This does not mean that, if the experiment were repeated many times, the slope would always be in this range. It does mean that, if the experiment were repeated many times and if a new range were selected each time, the true slope would lie in the range selected 95 percent of the time.

The confidence interval on the slope of a particular line is one indication of the precision of the experimental results. For example, a 95 percent confidence interval of from 8 to 10 would indicate greater precision than one of 6 to 12 at the 95 percent level.

The confidence interval on the difference between the slopes of two lines gives an estimate of the probable range of differences between the slopes.

The confidence interval on the difference in Y values from two curves at a particular value of X gives an estimate of the range of differences to be expected at that point. The difference in Y values at a given point is made up of two factors. One is the difference in the slopes of the lines, and the other is the difference in the Y intercepts of the two lines.

The 95 percent confidence interval has no particular significance. When another confidence level seems more appropriate, it should be used.

CHAPTER X

ANALYSIS OF RESULTS

Loose Sand

In these studies a group of model poles which were three-fourths of an inch in diameter were used in an attempt to predict the behavior of a group of 1 1/2 inch-diameter poles under lateral loads. Good agreement was obtained between the predictions of the model and the actual behavior of the prototype. Figure 13 gives the results of both model and prototype tests plotted on logarithmic paper. Figure 14 gives the same results on rectangular coordinates. A plot of the logarithms of π_4 versus the logarithms of π_5 on rectangular coordinate paper resulted in a straight line. The slopes were 3.4153 and 3.2644 for the model and prototype respectively. The data for these studies are given in Tables II through V in Appendix A.

Assumptions. The assumptions used in these experiments were:

1. The mineral particles making up the sand did not deform enough to affect the behavior of the pole.
2. The change in geometry caused by movement of the pole under load did not affect the results.
3. The effect of pole bending on the movement of the poles at the point where deflection was measured was negligible.

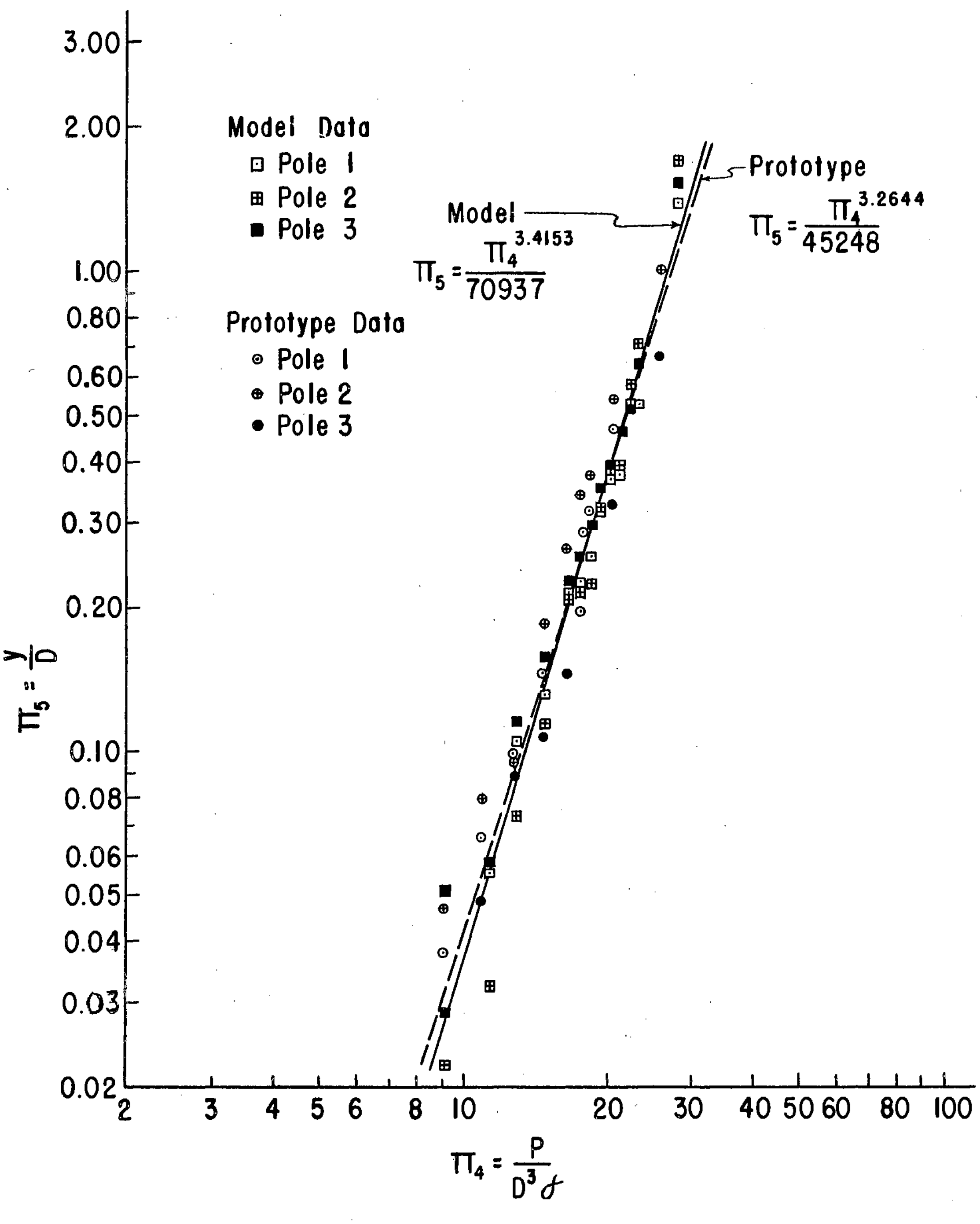

Figure 13. A Logarithmic Plot of Load and Deflection Pi Terms for the Model and Prototype in Loose Sand. The Value of Π_1 is 7 for Both Model and Prototype

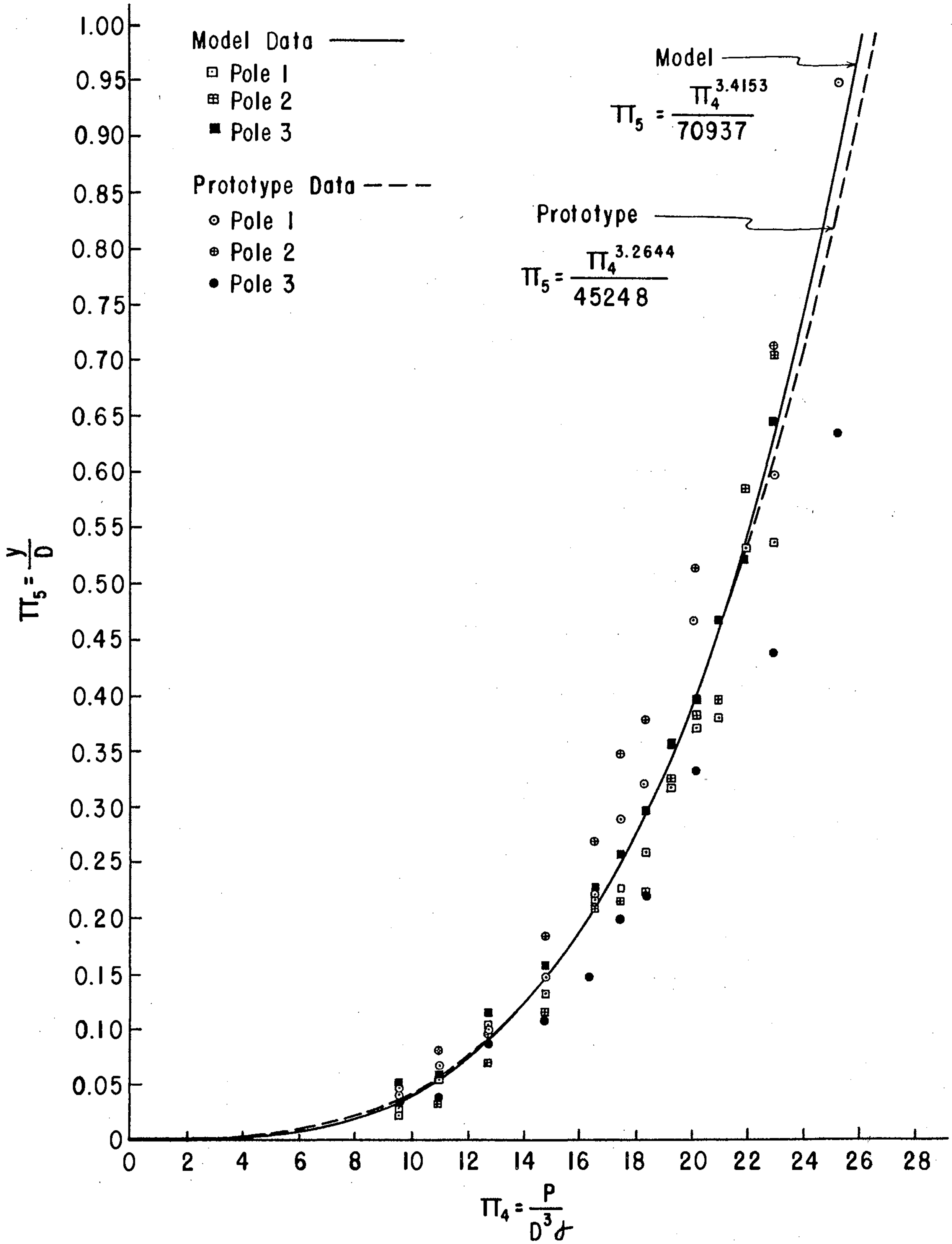

Figure 14. A Plot of Π_4 vs. Π_5 on Rectangular Coordinates for Model and Prototype in Loose Sand

<u>Confidence intervals</u>. As was mentioned earlier, the most probable slope of the logarithmic plot of π_4 versus π_5 for the model is 3.4153. The 95 percent confidence interval on this slope is from 3.2015 to 3.6291. The most probable slope for the prototype is 3.2644, and the confidence interval is from 2.9779 to 3.5509. It is possible to say with 95 percent confidence that the true difference between the slopes lies within the interval 0.0977 to 0.2039.

An analysis for the percentage confidence interval for the value of $\pi_4 = 9$ revealed that, with 95 percent confidence, it could be said that the value of π_5 for the prototype might be as low as 88 percent or as high as 143 percent of that predicted by the model. For a π_4 value of 25 this interval is from 76 percent to 123 percent. Since the curves cross between these two points, all differences between the points $\pi_4 = 9$ and $\pi_4 = 25$ would be less than the ones given above.

In the range of these experiments it can be said with 95 percent confidence that π_5 taken from the prototype curve will never exceed 1.43 π_5 taken from the model curve. In this case 1.43 might be defined as an appropriate safety factor to use with model studies.

The most probable equation for the behavior of the model is:

$$\pi_5 = \frac{\pi_4^{3.4153}}{70937}$$

The equation for the prototype is:

$$\pi_5 = \frac{\pi_4^{3.2644}}{45248}$$

These equations can be used with confidence only in the range of the experiments from which they were derived. Since in the loose sand tests the poles were not loaded to complete failure, it could be said that additional load would give additional deflection. The above equations, however, might not predict this additional deflection accurately.

Variant data. One set of readings was taken with π_4 equal to 4.6. The results of these readings were not used in deriving the above equations. For the model these points were far below the line through the rest of the data. For the prototype, however, these points fell nearly on the line.

One possible explanation of the discrepancy of the model could be inaccuracies of measurement of deflection. Small irregularities in seating the magnet on the pole could possibly have allowed it to move slightly in the beginning and thus have caused the readings to be too low.

Of course, the possibility that the readings are correct should not be overlooked.

Depth effect. As a secondary part of this project, the effect of depth on deflection was determined for a small range of depths. Depth is contained in the term π_1. The values of π_1 used were 5, 7, and 9. The deflection was measured at different loads for each of the values as has been described. Logarithmic plots of π_4 versus π_5 for the three values of π_1 are shown in Figure 15. From these graphs it appears that the slopes of the π_4 versus π_5 lines decrease as π_1 increases. This would be expected, for if the depth were zero, the line would be vertical. If the pole were embedded to an infinite depth, the π_4 versus π_5 line would be horizontal.

It appears that if the slopes of the π_4 versus π_5 curves were plotted against π_1, the resulting curve would be asymptotic to the horizontal and vertical axes in the first quadrant. π_1 versus slope, fitted to such a curve is shown in Figure 16.

The constants in the equations shown in Figure 15 do not differ greatly. These constants, which are the intercepts on the vertical axis, are affected by two factors. One is the slope of the π_4 versus π_5 curve, and the other is the maximum value of π_4 for which π_5 is 0. It appeared that up to a certain value of π_4 there was no measurable deflection. This was as expected since the internal resistance caused by friction of the sand had to be exceeded before movement could occur.

These two effects are compensating. Greater depths gave flatter slopes and therefore smaller y intercepts. On the other hand, greater embedment depths caused π_4 to be greater before any deflection occurred. For the three values of π_1 tested it appeared reasonable to assume that the y intercept was the same. In computing the following equation the mean of the three values was used.

<u>General equation</u>. The equation of any one of the lines shown in Figure 15 may be represented by:

$$\text{Log}\,\pi_5 = K_1\,\text{Log}\,\pi_4 + K_2$$

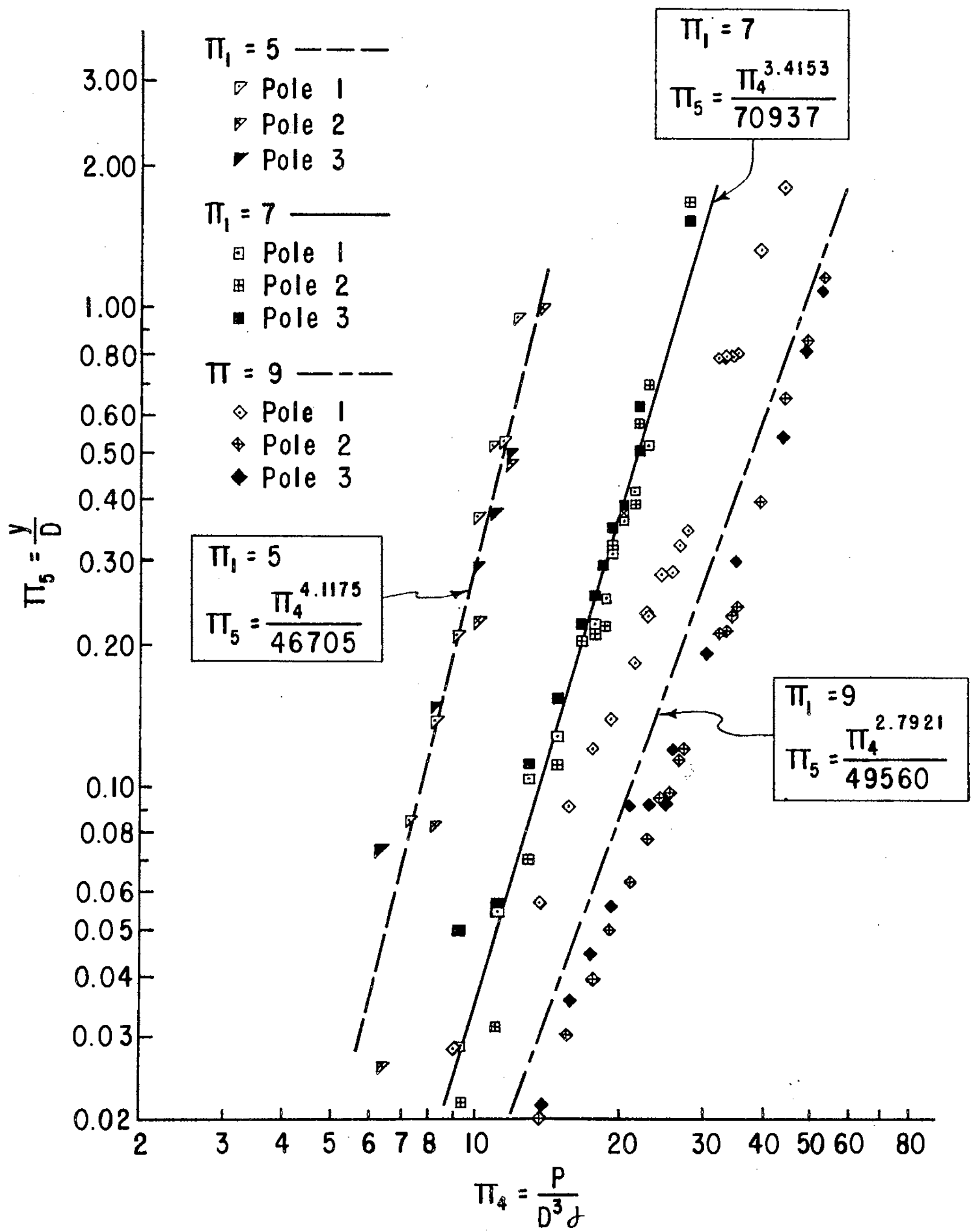

Figure 15. Load vs. Deflection Pi Terms for 3 Values of Π_1. All Curves are for Model Poles in Loose Sand

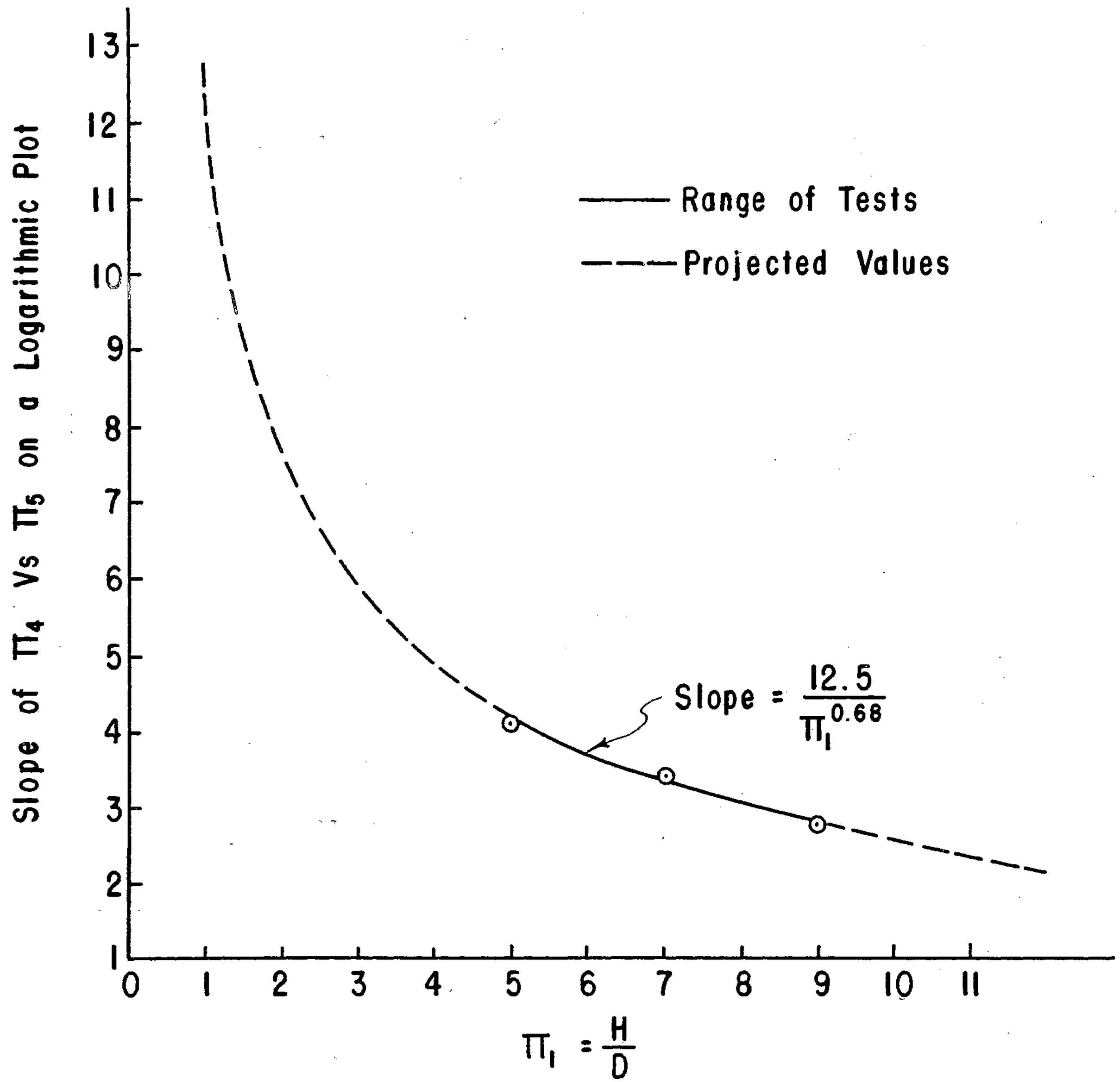

Figure 16. π_1 vs. the Slope of π_4 vs. π_5 for Loose Sand

As was explained above, K_1 and K_2 are functions of π_1. If this is true, the general equation describing the loose sand experiments may be written as:

$$\text{Log}\,\pi_5 = \text{Log}\,\pi_4\, f_1(\pi_1) + f_2(\pi_1)$$

By taking the $f_1(\pi_1)$ to be the function shown in Figure 16 and $f_2(\pi_1)$ to be the mean of the constants, the above equation may be written as:

$$\text{Log}\,\pi_5 = \left(\frac{12.5}{\pi_1^{0.68}}\right) \text{Log}\,\pi_4 - 4.7338$$

In rectangular coordinates this is:

$$\pi_5 = 1.824\,(\pi_4)^{\frac{12.5}{\pi_1^{0.68}}} \times 10^{-5}$$

Substituting the variables for the pi terms gives:

$$y = 1.824\,D\left(\frac{P}{D^3\gamma}\right)^{12.5\left(\frac{D}{H}\right)^{0.68}} \times 10^{-5}$$

Since the curve shown in Figure 16 does not fit the slopes exactly, the three equations given in Figure 15 represent the observed data better than the above equation. For example, when $\pi_1 = 5$ and $\pi_4 = 10$, the value of π_5 from the above equation is 0.283, but the value read from the curve in Figure 15 is 0.285. When $\pi_1 = 9$ and $\pi_4 = 30$, the calculated value of π_5 is 0.257, whereas the value read from the curve is 0.275.

Figure 17 is a plot of all observed values of deflection versus the calculated values from the general equation. The calculated values are measured along the horizontal axis. If perfect agreement existed between observed and calculated values, all the points on the graph would be on the line making a 45 degree angle with the horizontal and vertical axes.

Validity of the equation. It must be remembered that the validity of this equation has not been proved outside the range of the data given. It appears, however, that the general form should be valid for any value of π_1. The terms π_2, π_3, π_6, and π_{10} were constant in the tests described.

Dense Sand

The experiments in dense sand were performed in the same manner as those in loose sand. The difference was that in dense sand the internal friction angle was 36 degrees but that for loose sand was 29 degrees.

As with loose sand, good agreement was obtained between the predictions of the model and the actual behavior of the prototype. The data for the experiments are given in Tables VI through IX in Appendix A. The curves π_4 versus π_5 on semi-logarithmic paper for both the model and prototype are shown in Figure 18. These same curves are

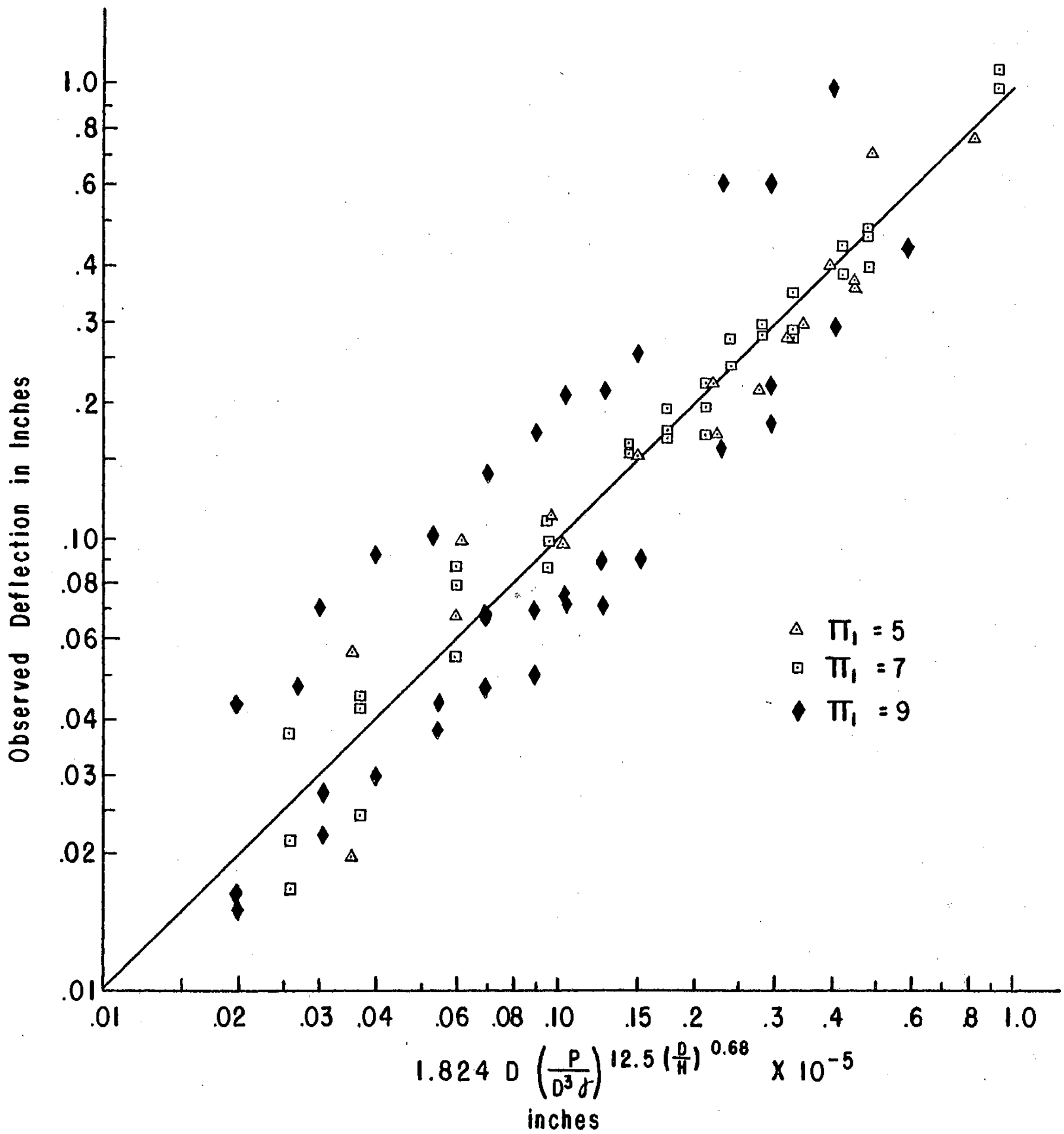

Figure 17. A Plot of Calculated Values of Deflection vs. Observed Values of Deflection for Loose Sand

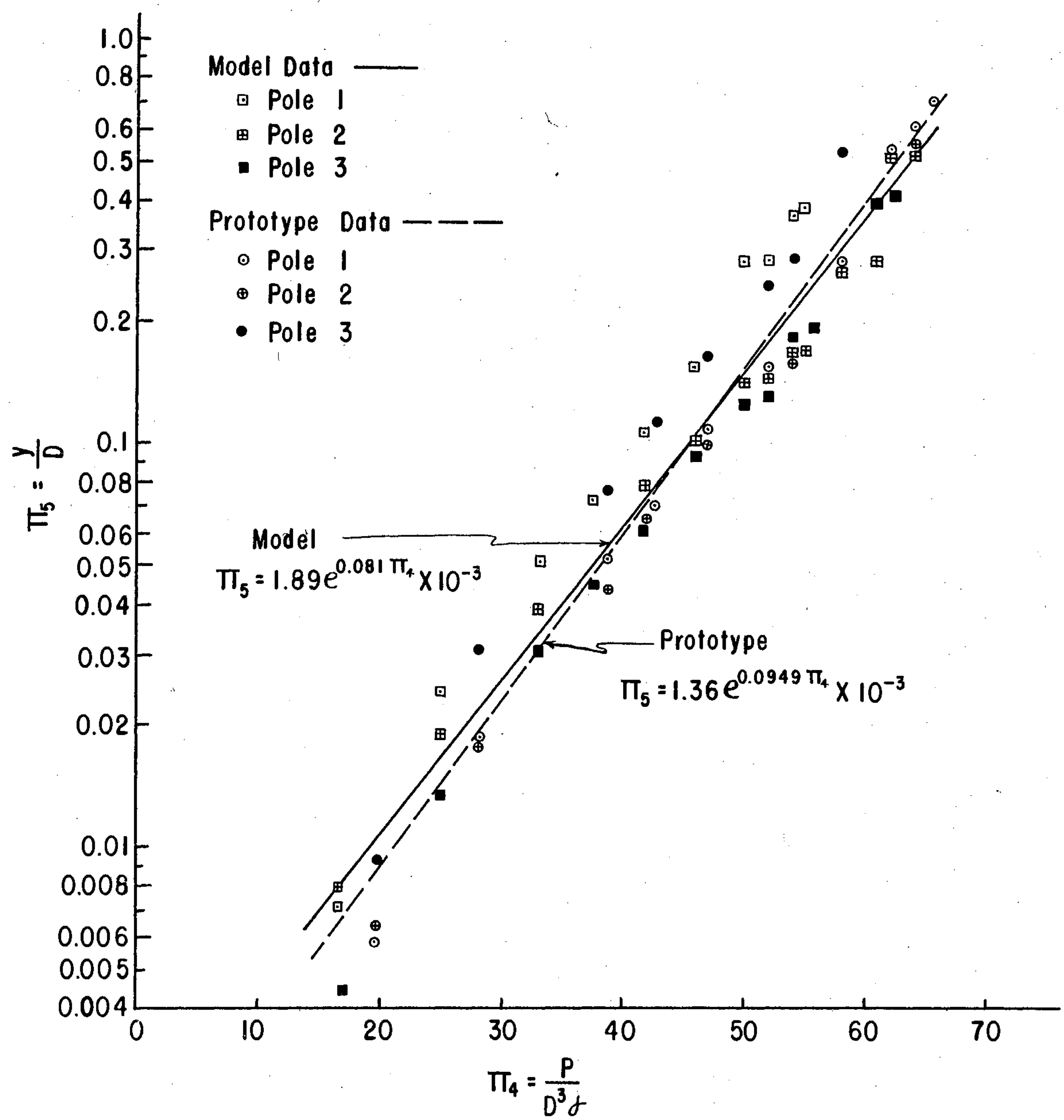

Figure 18. A Logarithmic Plot of π_4 vs. π_5 for Model and Prototype in Dense Sand

plotted on rectangular coordinate paper in Figure 19.

The equation for the model curve is:

$$\pi_5 = 1.89e^{0.0881\pi_4} \times 10^{-3}$$

The prototype equation is:

$$\pi_5 = 1.36e^{0.0949\pi_4} \times 10^{-3}$$

These curves cross when π_4 is 48.5.

Confidence intervals. The most probable slope of the model curve is 0.0881. The 95 percent confidence interval on the slope of this curve is from 0.0808 to 0.0954. This same interval for the prototype is from 0.0882 to 0.1016. The magnitude of the spread of the 95 percent confidence interval gives an indication of the experimental error.

It is possible to say with 95 percent confidence that the difference between the slopes of the model and the prototype curves lies within the interval of -0.0030 to 0.0166. This seems to indicate that the two curves are different. They are so close together, however, that the results of the model may be used to predict the behavior of the prototype without excessive error.

The greatest percentage differences between the model and prototype are at the ends of the curves. When π_4 is 15, it is possible to say with 95 percent confidence that the value of π_5 taken from the prototype curve might be as little as 25 percent of π_5 for the model or as great as 110 percent. When π_4 is 65, these values are 88 and 130.

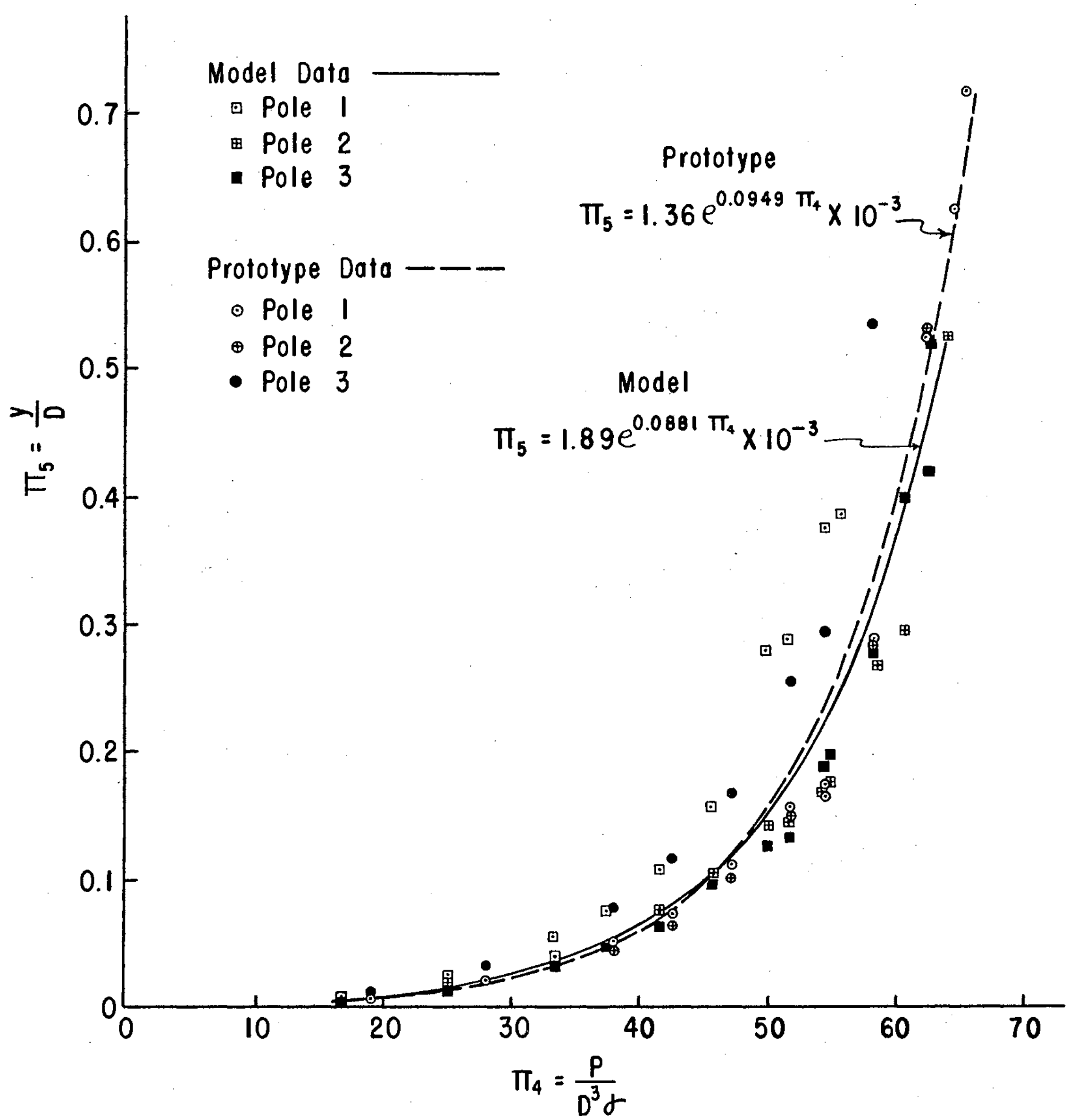

Figure 19. A Plot on Rectanguler Coordinates of π_4 vs. π_5 for Model and Prototype in Dense Sand

Although closer agreement would be desirable, these differences do not appear to be excessive. If the model value of π_5 were multiplied by 1.3, the prototype value of π_5 would be covered more than 95 percent of the time.

Depth effect. As for loose sand, studies were carried out for three values of π_1 to determine how depth affected deflection at different loads. The curves for the three values of π_1 are shown in Figure 20. As with loose sand, it appears that as π_1 decreases in value, the π_4 versus π_5 curve should approach a vertical line, but if π_1 increases, the curve should approach a horizontal line. A plot of π_1 versus the slope of π_4 versus π_5 should be asymptotic to the horizontal and vertical axes. π_1 versus slope is fitted to such a curve in Figure 21. The slopes are the exponents of e from the equations in Figure 20.

The constants in the single equations do not appear to vary in a predictable manner. The factors affecting the variation of these constants are the same as those discussed under loose sand.

General equation. The equation for a single value of π_1 may be represented by:

$$\operatorname{Log}_e \pi_5 = K_1 (\pi_4) + K_2$$

As was explained above, K_1 and K_2 are functions of π_1. If this is true, the general equation describing the loose sand experiments may be written as:

$$\operatorname{Log}_e \pi_5 = \pi_4 f_1 (\pi_1) + f_2 (\pi_1)$$

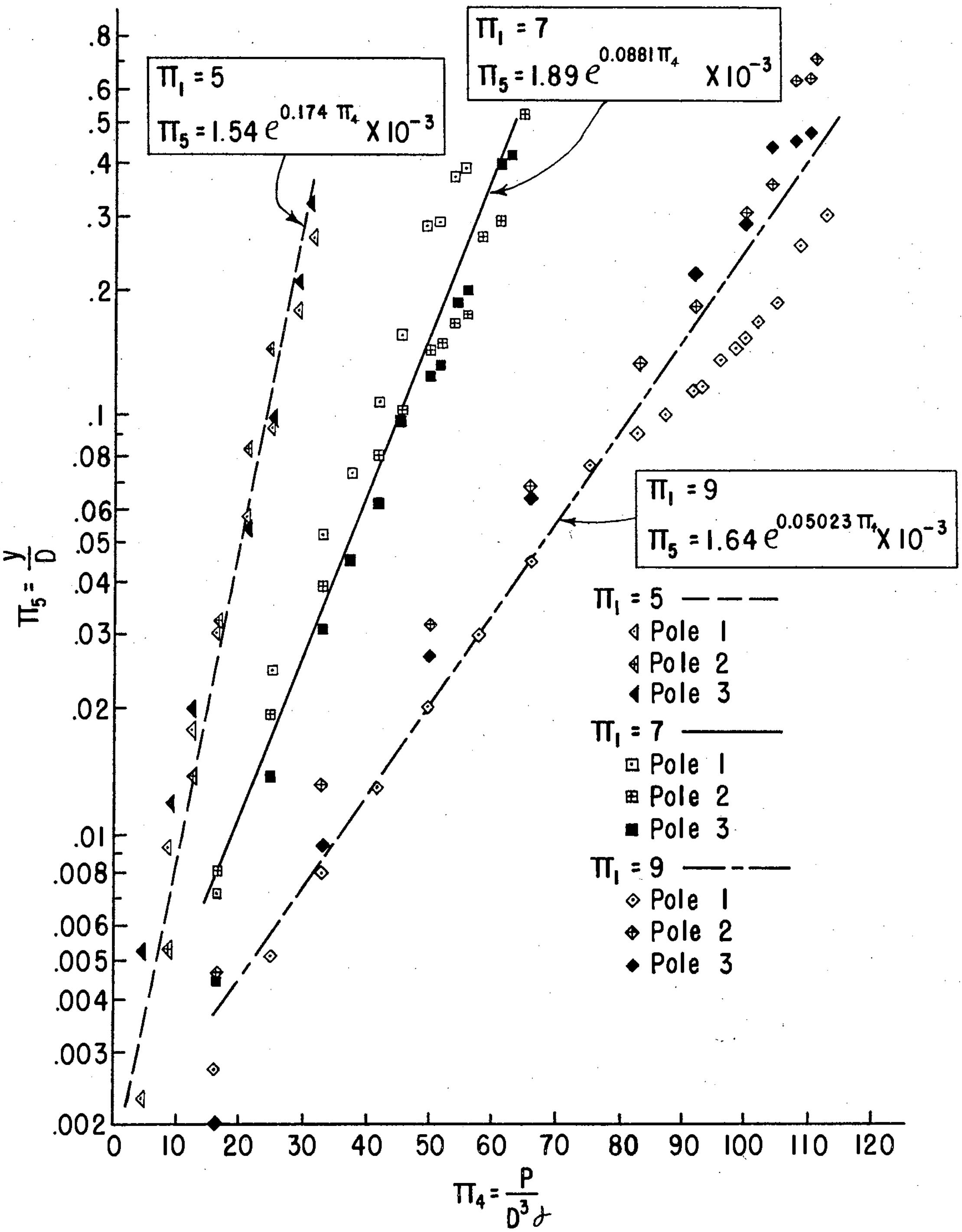

Figure 20. Load vs. Deflection Pi Terms for 3 Values of Π_1. All Curves are for Models in Dense Sand.

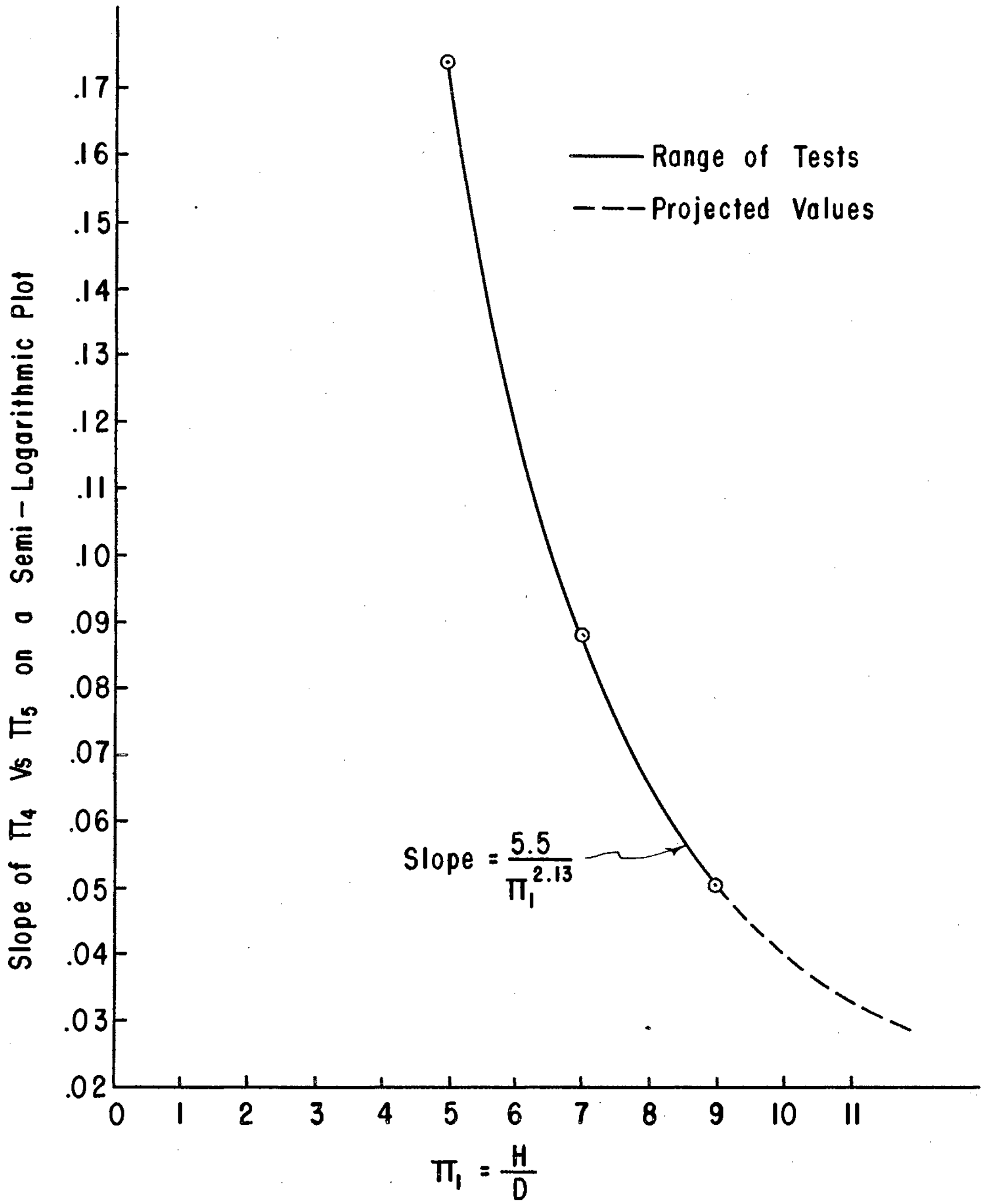

Figure 21. Π_1 vs. the Slope of Π_4 vs. Π_5 for Dense Sand

If $f_1(\pi_1)$ is the function shown in Figure 21 and $f_2(\pi_1)$ is the mean of the constants of the individual equations, the above equation may be written as:

$$\log_e \pi_5 = \frac{5.5\,\pi_4}{\pi_1^{2.13}} - 6.3893$$

In rectangular coordinates this is:

$$\pi_5 = 1.68 e^{\frac{5.5\,\pi_4}{\pi_1^{2.13}}} \times 10^{-3}$$

Substituting the variables for the pi terms gives:

$$y = 1.68 D e^{\frac{5.5 P}{D^{0.87} H^{2.13} \gamma}} \times 10^{-3}$$

Although the slopes of the three individual curves shown in Figure 20 fit the curve given in Figure 21 almost perfectly, the general equation does not fit the data as well as the individual curves. This is caused by using the average of the constants of the three individual curves.

The fit, however, is fairly good in spite of this. For example, when π_1 is 5 and π_4 is 10, the value of π_5 predicted by the general equation is 0.0097, but this value read from Figure 20 is 0.0088. When π_1 is 5 and π_4 is 30, these values are 0.292 and 0.285 respectively.

When π_1 is 7 and π_4 is 60, π_5 predicted by the general equation is 0.334, whereas, when read from the curve, this value is 0.380.

Figure 22 is a plot of all observed values of deflection versus the calculated values. The calculated values are measured along the horizontal axis. If perfect agreement existed between the observed and calculated values, all points would be on the line which makes a 45-degree angle with the horizontal and vertical axes.

It must be remembered that these equations are valid only within the range in which they were determined.

Failure loads. In the dense sand tests failure of the poles under lateral loads was abrupt and definite. The results in the dense sand tables show that when π_1 is 7 the largest values of π_4 for which failure did not occur were 55.8, 64.2 and 62.5 for model poles 1, 2, and 3 respectively. For the prototype poles these values were 65.6, 64.6, and 58.3. The results of the model tests are not directly comparable to the results of the prototype tests since the increments of π_4 used were not exactly the same. They, however, are obviously not greatly different.

Pole 1 in the model and poles 1 and 2 in the prototype did not fail exactly like the other poles. They deflected under load like the other poles up to a certain point and then began to move very rapidly as did the other poles. Unlike the majority of the poles, however, they stopped moving before the point of the micrometer dial had been moved to its limit. This action may have been caused by the pole's striking a pocket of very dense sand. This point where rapid movement began was considered the point of failure.

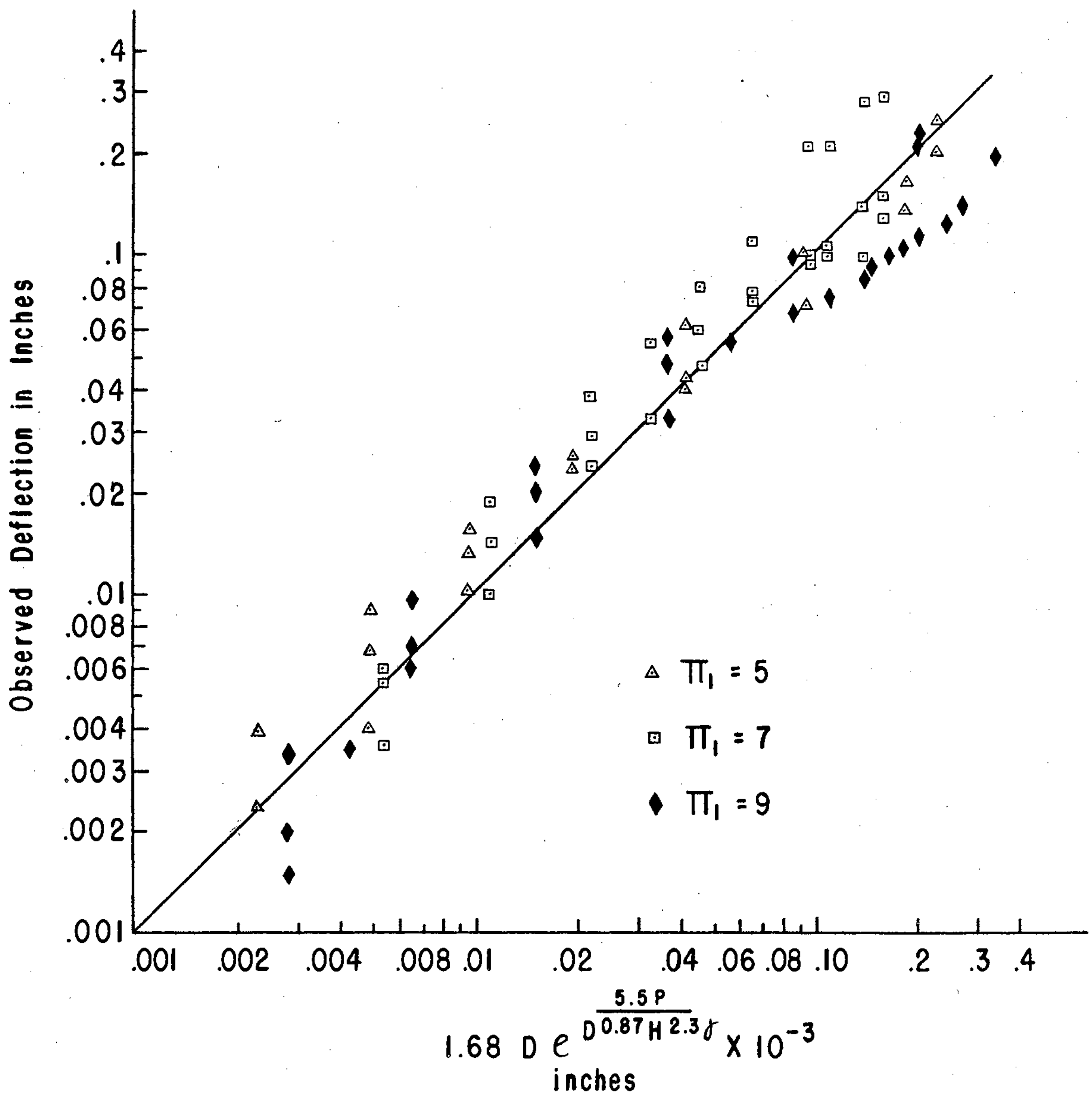

Figure 22. Calculated Values vs. Observed Values of Deflection for Dense Sand

When π_1 was 5, the values of π_4 at failure were 30.8, 25.0, and 30.8 for poles 1, 2, and 3 respectively. When π_1 was 9, these values were 113.3, 111.6, and 111.7 respectively.

The effect of internal friction angle. The only pi term that was changed in going from loose sand to dense sand was π_2, which is the internal friction angle. In the loose sand tests it was 29 degrees but in the dense sand tests it was 36 degrees.

This change made a drastic difference in the π_4 versus π_5 curves. For purposes of comparison the model curves for $\pi_1 = 7$ are shown in Figure 23. These curves indicate that the deflection for a given value of π_4 was much smaller for the larger internal friction angle.

Another difference that might be noted is that failure occurred for π_5 values of about 0.5 for dense sand but that failure did not occur at all in loose sand even though load was added until π_5 was greater than 1.

Saturated Sandy Clay

These experiments were in several ways different from those described previously. The soil material was a mixture of sand and clay that was saturated and ponded. In general, only one load was used on one pole. The poles did not fail as they did in dense sand. In dry sand the deflection occurred very quickly after the application of the load. In these studies deflection was still in progress when the load was removed twenty-four to forty-eight hours after application.

Pi term corrections. As mentioned in the procedure, the embedment depth of the poles at the time of testing was not $5\frac{1}{4}$ inches for the model and $10\frac{1}{2}$ inches for the prototype, as had been planned. The actual

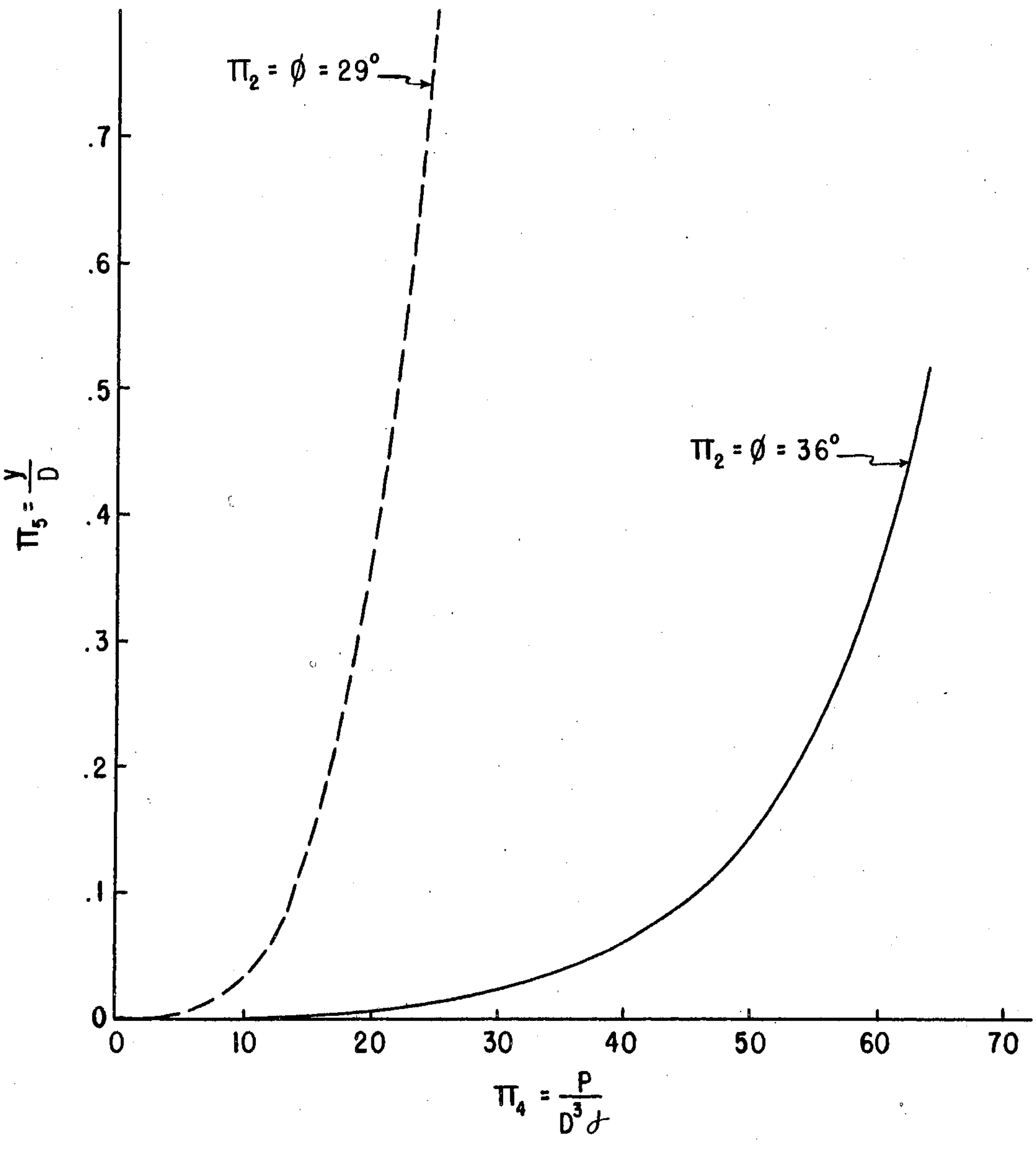

Figure 23. Π_4 vs. Π_5 When Π_1 is 7 for Different Values of Internal Friction Angle

depths were 5 9/16 inches and 10 13/16 inches for the model and prototype respectively. This made some adjustments in the pi terms necessary.

The ratio of the depth of embedment to the distance above the ground to where the deflection was measured was changed by the swelling of the soil. This ratio, which is π_9, was 3.15 before the change in depth occurred. After the change in depth, it was 4.12 for the model. The micrometer dial would have had to be lowered on the prototype for it to have had this same value. The only pi term affected by π_9 was π_5, the term containing deflection. It was possible to adjust π_5 in the prototype to the value it would have had if the dial had been at the proper location. This adjustment was made on the basis of two assumptions. The first was that the point about which the pole rotated was located at a point two-thirds of the depth of embedment below the soil surface. The second assumption was that the load applied to the pole caused no translation.

The reduction in π_5 for the prototype resulting from this correction was 4 percent. The deflection data given in Tables X and XI in Appendix A are the actual measurements made in the experiment. The pi terms are calculated from the corrected data.

The ratio of depth of embedment to length of pole, which is π_6, was also changed. It would have been necessary to increase the length of the model poles to make the two values of π_6 equal. This increase in length would have resulted in an increase in moment at the ground line in the model. This increase would have caused greater deflection in the model and therefore would have brought the model and prototype deflection closer together. The magnitude of this correction was found to be about one percent. Since this was so small, the correction was not made. π_1 was also changed. To have kept it constant would have required that the diameter of the model pole be increased by 0.026 of an inch. A change of this magnitude would probably have had little effect.

The change in depth made considerable difference in π_4. Because depth was the only length variable in this term, it was possible, with the aid of the multiple regression study mentioned under statistical analysis, to use the actual depths in this term.

Equation determination. Plots of π_8 versus π_5 for different values of π_4 are given in Figures 24 and 25 for the model and prototype respectively. These plots appear to be straight lines on logarithmic paper. The lines for the model and prototype cannot be compared directly, however, since the values of π_4, the term containing the load, were not the same. It was possible to make a multiple regression study of both cases and compare the resulting expressions. If the model had predicted the behavior of the prototype exactly, the two expressions would have been the same.

The expression for the model study in logarithmic form was found to be:

$$\text{Log}\,\pi_5 = 2.7602 + 3.2308\ \text{Log}\,\pi_4 + 0.0943\ \text{Log}\,\pi_8$$

Converted to rectangular coordinates it is:

$$\pi_5 = 576\ (\pi_4^{3.2308})(\pi_8^{0.0943})$$

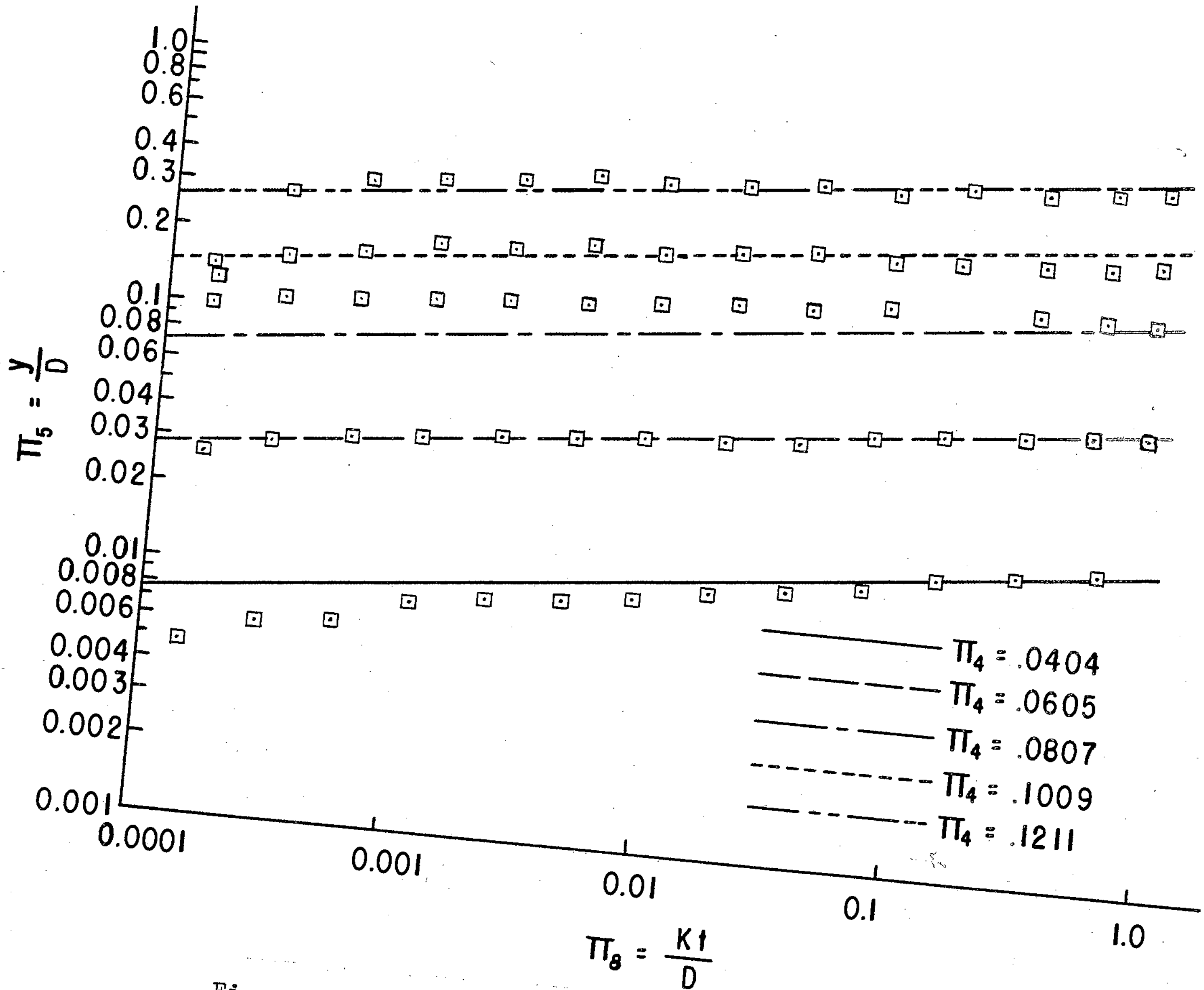

Figure 24. Π_8 vs. Π_5 for the Model at 5 Values of Π_4 for Saturated Sandy Clay

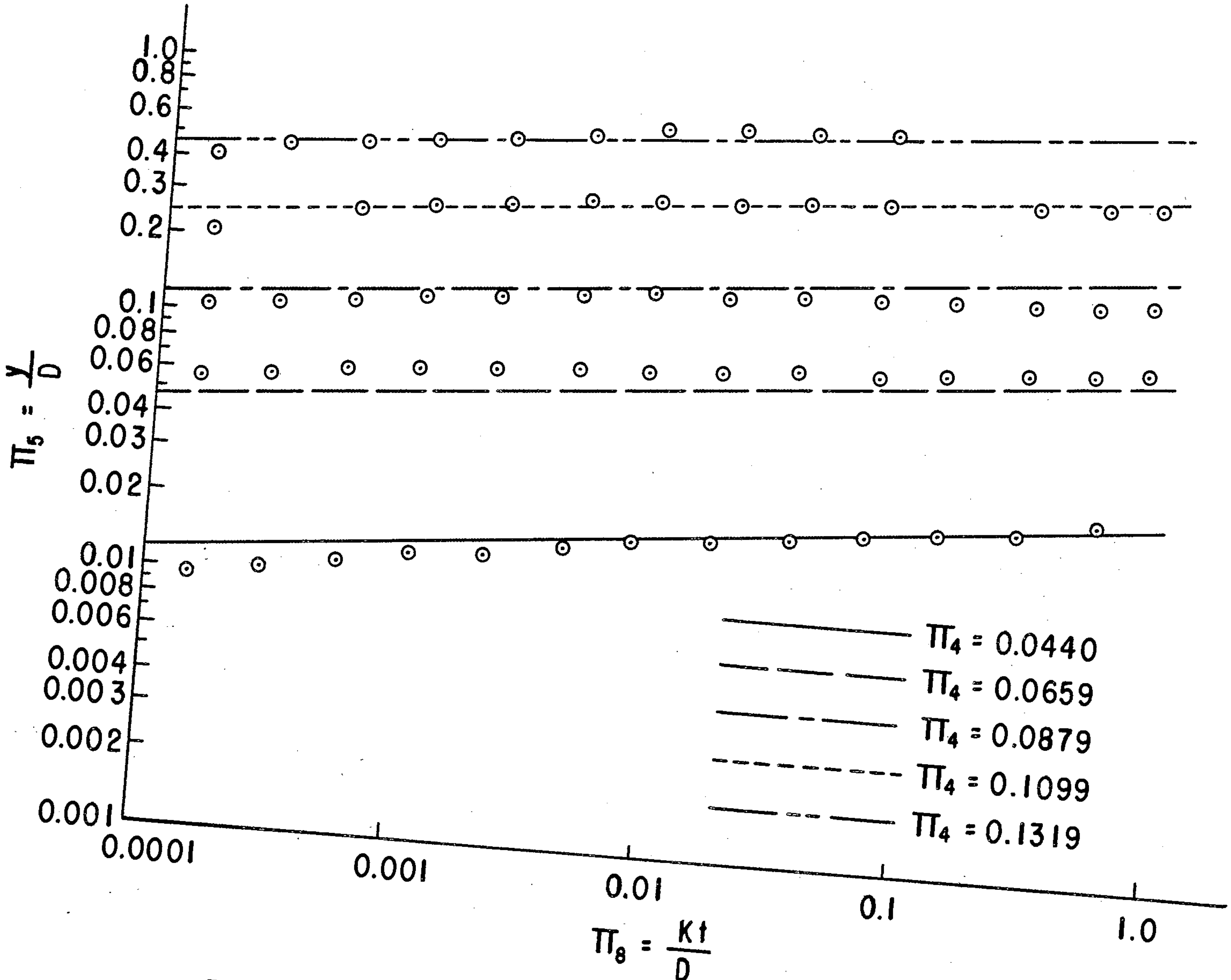

Figure 25. π_8 vs. π_5 for the Prototype at 5 Values of π_4 for Saturated Sandy Clay

Substituting the variables for the pi terms gives:

$$y = 576 D^{0.9057} \left(\frac{P}{H^3 \gamma}\right)^{3.2308} (kt)^{0.0943}$$

These equations for the prototype are:

$$\mathrm{Log}\,\pi_5 = 2.8119 + 3.2417\ \mathrm{Log}\,\pi_4 + 0.0828\ \mathrm{Log}\,\pi_8$$

$$\pi_5 = 648 (\pi_4^{3.2417}) (\pi_8^{0.0828})$$

$$y = 648 D^{0.4172} \left(\frac{P}{H^3 \gamma}\right)^{3.2417} (kt)^{0.0828}$$

The expressions for the model and prototype are nearly the same.

Confidence intervals. The exponent of π_4 in the model equation is 3.2308. In the prototype this exponent is 3.2417. It can be said with 95 percent confidence that the difference between these exponents is in the range from -0.2120 to 0.1793.

The exponents of π_8 are 0.0943 and 0.0828 for the model and prototype respectively. The 95 percent confidence intervals on the difference between the π_8 exponents is from -0.0403 to 0.0173. The prototype value was subtracted from the model value in each case.

The values for the logarithms of the constants are 2.7602 for the model and 2.8119 for the prototype. The 95 percent confidence interval on the difference between the logarithms of the constants in the model and prototype is from -0.2881 to 0.1339.

The small differences between the model and the prototype constants and exponents indicate that the prototype behavior was very close to that predicted by the model. A logarithmic plot of π_4 versus π_5 for a constant value of π_8 for both the model and the prototype is shown in Figure 26. The lines are very close together.

Combined equation. The data from both the model and the prototype tests were combined and the following equation obtained:

$$\pi_5 = 632 \left(\pi_4^{3.2546}\right) \left(\pi_8^{0.08009}\right)$$

Since the data from both the model and the prototype are for practical purposes from the same system, this equation should describe the system more accurately than either of the equations given before. A plot of the π_5 given by this equation versus π_5 from the observed data is shown in Figure 27. If there were perfect agreement between values of π_5 from the equation and values directly from the data, all the points in the figure would be on the line shown.

Comparison of Model and Prototype Results

The equations representing prototype behavior were not exactly the same as those representing model behavior in any of the three situations studied. When the value of π_4 was small in loose sand, π_5 for the prototype was greater than π_5 for the model. At relatively large π_4

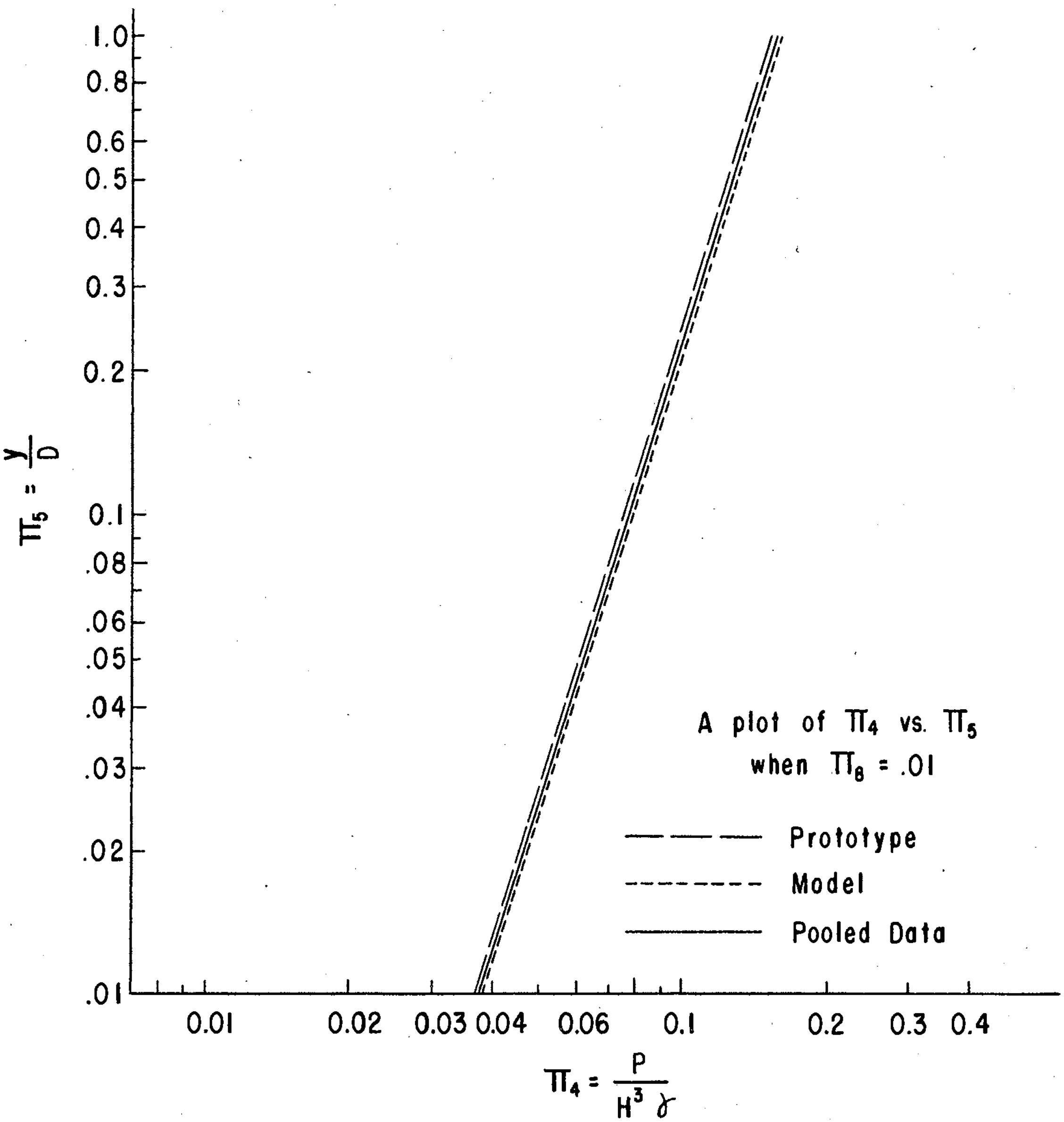

Figure 26. Π_4 vs. Π_5 When Π_8 is 0.01

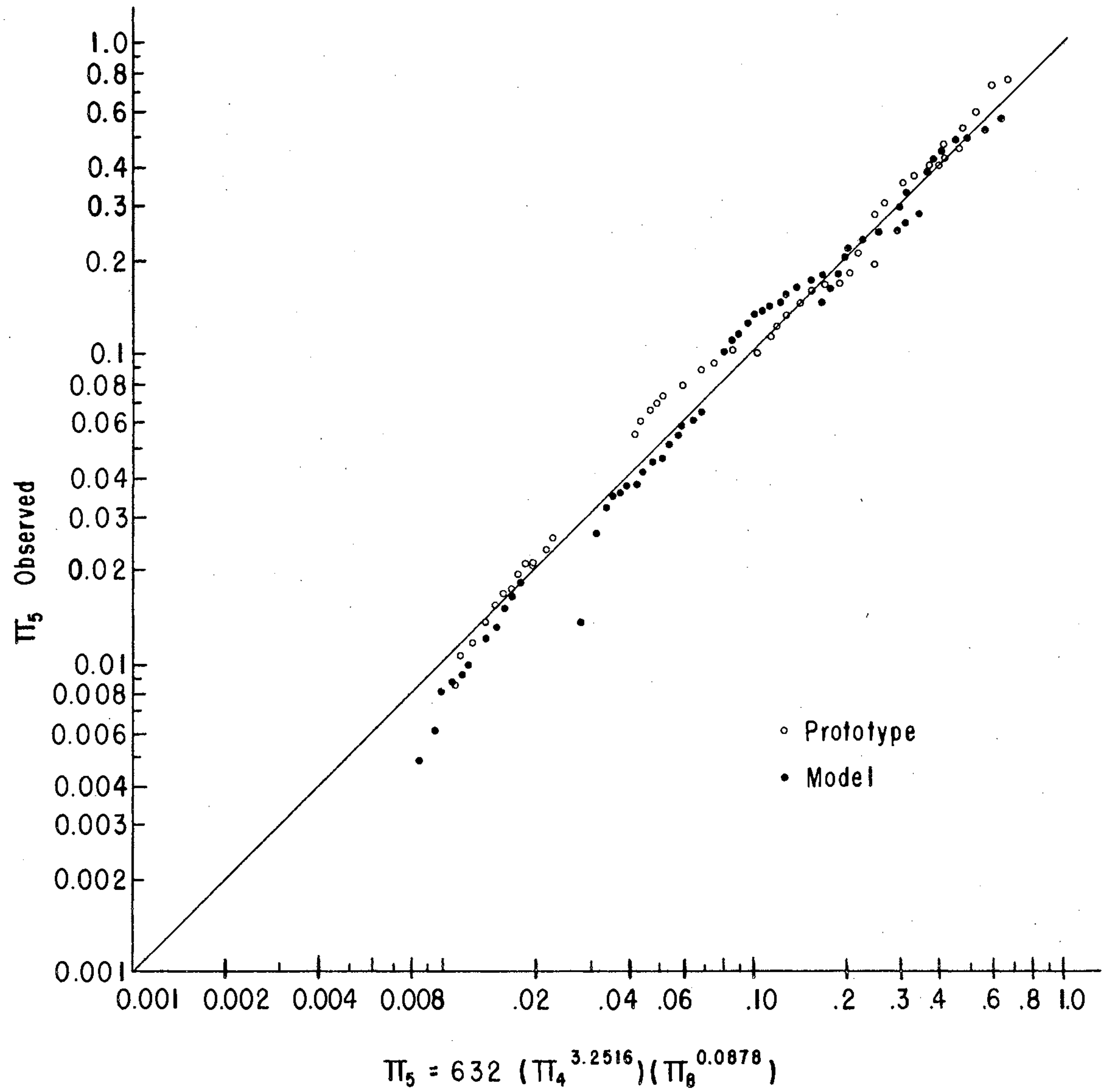

Figure 27. Π_5 Calculated vs. Π_5 Observed for Saturated Sandy Clay

values, π_5 for the model was greater than π_5 for the prototype. This situation was reversed in dense sand. In saturated sandy clay π_5 for the prototype was slightly greater than π_5 for the model at all values of π_4.

Examination of Figures 13, 18, and 26 reveals that these differences are relatively small. Confidence intervals given earlier in the paper give quantitative estimates of these differences.

The causes of the differences are not apparent. Many people intuitively feel that the size of the model affects the results of a model study. This may have been true in these experiments. No evidence, however, was found to support this view. Size quantities are only one of several types of quantities considered in the systems investigated. It appears equally valid to assume that the results of a study might change as the magnitude of the variables involving force change. The same statement might be made concerning variables containing time. There was no evidence to support such assumptions.

It is not out of the realm of possibility that these differences were the result of random variation. Repeated experimental errors could have caused the differences. Omission from the analysis of a variable that affected the system might be the explanation.

Philosophers say that no two events are ever exactly the same. The implication is that the variables producing events are ever-changing. Many statisticians accept this view and say that it is impossible to predict any event with absolute accuracy. According to those who hold this view, the objective of research should be to determine how much prediction error can be tolerated and then to strive to get within these limits with predictions.

The ultimate test of any prediction method is the comparison of predicted behavior with observed behavior. To settle the question of the effect of size on the results of the studies reported here more experiments would be necessary. Such experiments are suggested in Chapter XII.

CHAPTER XI

SUMMARY AND CONCLUSIONS

The possibility of using a model pole under lateral load to predict the behavior of a prototype subjected to a similar load was investigated. The experiments were designed by combining the factors involved in pole behavior into dimensionless parameters according to the theory of dimensional analysis.

The three soils used were loose sand, dense sand, and saturated sandy clay. Load and depth terms were independent variables in the sand studies. Independent variables in the saturated sandy clay experiments were depth and time terms. The dependent variable was the term containing deflection.

The deflection of the prototype was close to that predicted by the model in every case.

Deflection of the prototype in loose sand was about 112 percent of that predicted by the model at light loads. For loads near the maximum this value was 99 percent. These figures for dense sand were 70 percent and 112 percent. For saturated sandy clay these figures were 111 percent and 113 percent.

Statistical analyses were made, and confidence intervals were set on extreme differences between deflections predicted by the models and the deflections observed in the prototypes for sand studies. The statements which follow refer to curves and planes in logarithmic space.

Confidence intervals were set on the differences between the slopes of the model and the prototype curves in sand studies. They were also determined for the differences in the π_5 intercepts of the planes representing the deflection of the poles in saturated sandy clay. These intercepts were determined when π_4 and π_8 were 0. Confidence intervals were set on the differences between the slopes of the planes also.

When load had been applied to poles in dense sand until π_5 reached a certain value, which was approximately 0.5, they failed abruptly. The load required to produce an equal value of π_5 in loose sand was about one third the load required in dense sand. Deflections in saturated sandy clay for a specific load were less than those produced by the same load in loose sand but greater than those in dense sand.

Prediction equations were obtained for each soil condition. The equations follow:

For loose sand:

$$y = 1.824 D \left(\frac{P}{D^3 \gamma}\right)^{12.5 \left(\frac{D}{H}\right)^{0.68}} \times 10^{-5}$$

For dense sand:

$$y = 1.68 D e^{\frac{5.5 P}{D^{0.87} H^{2.13} \gamma}} \times 10^{-3}$$

For saturated sandy clay:

$$\Pi_5 = 632 \left(\Pi_4^{3.2546}\right)\left(\Pi_8^{0.08009}\right)$$

The equations above are directly applicable to any pole of any size that meets the conditions of the dimensional analysis. Any consistent system of units may be used in the equations since they are dimensionally homogenous.

It appears that model poles may be used to predict the behavior of prototypes under lateral loads for the three situations investigated. By using the methods of analysis used in this study it should be possible to design model experiments that will predict the behavior of poles in other conditions.

CHAPTER XII

SUGGESTIONS FOR FURTHER RESEARCH

New experiments. These studies were not made with direct practical application in mind. The results, however, could be applied directly to a practical pole in the event that the conditions of the dimensional analysis for one of the three situations investigated were met.

Although there was good agreement between the behavior of the model and the prototype in the studies reported here, the results cannot be regarded as absolutely conclusive, for the prototype poles were considerably smaller than those ordinarily used in farm construction.

The next logical step in the research would be to make model studies of poles under field conditions and to validate the models with poles of a size that might be used in farm construction. The easiest way to do this would be to find a soil that has uniform properties to a depth of about six feet. In this situation the model and prototype studies could be carried out on the same site.

It would be desirable to use wood poles in these studies since they are most frequently used in farm construction. In this case it would be necessary to consider the bending of the pole. This would make it necessary to add modulus of elasticity to the list of variables considered. A new pi term would be necessary. The most logical combination appears to be one of soil unit weight, pole diameter, and modulus of elasticity. With this combination it would be necessary for the moduli of elasticity to have the same ratio as that of pole diameters. For a one half size model the use of White cedar in the model and Southern Yellow pine in the prototype would meet this requirement.

The differences between the model and the prototype reported in this paper, although not great, might indicate that there was a size effect. To test this possibility, poles of several sizes should be studied. The most direct approach would be to use 1-inch diameter models and 2-inch diameter models in one or more of the soils that were used in this study. The value of π_1 should be held constant at 7 so that the results of the new studies can be compared with the results given in this paper. The apparatus described in this paper could be used.

An alternative procedure would be to use different pole sizes on the site proposed for full-scale tests.

Improvements in experimental procedure. If it is necessary to fasten the tip of the micrometer dial to the pole with a magnet in future studies, the magnet should be bonded to the pole with a strong cement to prevent movement.

Soil in the sandy clay experiments swelled when water was added. This caused a change in pole depth. In future studies this depth change should be prevented if possible. This might be done by estimating the amount of swelling and setting the poles so that embedment depths after swelling would be correct. Another possible procedure would be to scrape the soil off the surface until the poles were embedded to desired depths. This would be done after swelling had occurred.

It appears that the poles for models in this study were about as small as can be used to produce reliable results.

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APPENDIX A

TABLE II

DATA FOR MODEL TESTS CONDUCTED IN LOOSE SAND WHEN THE VALUE OF π_1 WAS 5

Pole Number	Load P Grams	Deflection y Inches	π_4 $\frac{P}{D^3\gamma}$	π_5 $\frac{y}{D}$
1	80	0.0637	7.36	0.0849
	90	0.1047	8.28	0.1396
	100	0.1580	9.20	0.2107
	110	0.2749	10.12	0.3665
	120	0.3927	11.04	0.5236
	125	0.3954	11.50	0.5272
	135	0.7238	12.24	0.9651
2	70	0.0194	6.44	0.0259
	90	0.0624	8.28	0.0832
	110	0.1676	10.12	0.2235
	130	0.3611	11.96	0.4815
	150	0.7737	13.80	1.1399
3	70	0.0559	6.44	0.0745
	90	0.1115	8.28	0.1487
	110	0.2205	10.12	0.2940
	120	0.2805	11.04	0.3740
	130	0.3755	11.96	0.5007
	150	0.7659	13.80	1.0212

TABLE III

DATA FOR MODEL TESTS CONDUCTED IN LOOSE SAND WHEN THE VALUE OF π_1 WAS 7

Pole Number	Load P Grams	Deflection y Inches	π_4 $\frac{P}{D^3\gamma}$	π_5 $\frac{y}{D}$
1	100	0.0212	9.20	0.0283
	120	0.0419	11.04	0.0559
	140	0.0788	12.88	0.1051
	160	0.0981	14.72	0.1308
	180	0.1620	16.56	0.2160
	190	0.1687	17.48	0.2249
	200	0.1931	18.40	0.2575
	210	0.2389	19.32	0.3185
	220	0.2790	20.24	0.3720
	230	0.2841	21.16	0.3788
	240	0.3982	22.08	0.5309
	250	0.4016	23.00	0.5355
	300	1.0607	27.60	1.4143
2	100	0.0166	9.20	0.0221
	120	0.0242	11.04	0.0323
	140	0.0539	12.88	0.0719
	160	0.0861	14.72	0.1148
	180	0.1570	16.56	0.2093
	190	0.1613	17.48	0.2151
	200	0.1691	18.40	0.2255
	210	0.2436	19.32	0.3248
	220	0.2874	20.24	0.3832
	230	0.2960	21.16	0.3946
	240	0.4354	22.08	0.5845
	250	0.4624	23.00	0.7056
	300	1.2939	27.60	1.7252
3	100	0.0384	9.20	0.0512
	120	0.0441	11.04	0.0588
	140	0.0865	12.88	0.1153
	160	0.1181	14.72	0.1574
	180	0.1710	16.56	0.2280
	190	0.1934	17.48	0.2579
	200	0.2234	18.40	0.2978
	210	0.2679	19.32	0.3572
	220	0.2969	20.24	0.3959
	230	0.3520	21.16	0.4693
	240	0.3919	22.08	0.5225
	250	0.4851	23.00	0.6486
	300	1.1632	27.60	1.5509

TABLE IV

DATA FOR MODEL TESTS CONDUCTED IN LOOSE SAND WHEN THE VALUE OF π_1 WAS 9

Pole Number	Load P Grams	Deflection y Inches	π_4 $\frac{P}{D^3\gamma}$	π_5 $\frac{y}{D}$
1	50	0.0035	4.60	0.0047
	100	0.0214	9.20	0.0285
	150	0.0437	13.80	0.0583
	170	0.0685	15.64	0.0913
	190	0.0914	17.48	0.1219
	210	0.1043	19.32	0.1391
	230	0.1378	21.16	0.1837
	250	0.1736	23.00	0.2315
	270	0.2115	24.84	0.2820
	280	0.2146	25.76	0.2861
	290	0.2417	26.68	0.3223
	300	0.2569	27.60	0.3425
	350	0.6097	32.20	0.8129
	360	0.6097	33.12	0.8129
	370	0.6110	34.04	0.8147
	380	0.6159	34.96	0.8212
	430	1.0050	39.56	1.3400
	480	1.3823	44.16	1.8431
2	50		4.60	
	100	0.0032	9.20	0.0043
	150	0.0151	13.80	0.0201
	170	0.0226	15.64	0.0301
	190	0.0298	17.48	0.0397
	210	0.0380	19.32	0.0507
	230	0.0480	21.16	0.0640
	250	0.0586	23.00	0.0781
	270	0.0718	24.84	0.0957
	280	0.0738	25.76	0.0984
	290	0.0864	26.68	0.1152
	300	0.0908	27.60	0.1211
	350	0.1599	32.20	0.2132
	360	0.1620	33.12	0.2160
	370	0.1728	34.04	0.2304
	380	0.1814	34.96	0.2419
	430	0.3000	39.56	0.4000
	480	0.5016	44.16	0.6688
	530	0.6593	48.76	0.8791
	580	0.8881	53.36	1.1841

TABLE IV (Continued)

Pole Number	Load P Grams	Deflection y Inches	π_4 $\frac{P}{D^3\gamma}$	π_5 $\frac{y}{D}$
3	50		4.60	
	100	0.0055	9.20	0.0073
	150	0.0164	13.80	0.0219
	170	0.0270	15.64	0.0360
	190	0.0338	17.48	0.0451
	210	0.0428	19.32	0.0571
	230	0.0681	21.16	0.0908
	250	0.0690	23.00	0.0920
	260	0.0724	23.92	0.0965
	270	0.0731	24.84	0.0975
	280	0.0900	25.76	0.1200
	330	0.1451	30.36	0.1935
	380	0.2250	34.96	0.3000
	430	0.3080	39.56	0.4107
	480	0.4097	44.16	0.5463
	530	0.5714	48.76	0.7619
	580	0.8348	53.36	1.1131

TABLE V

DATA FOR PROTOTYPE TESTS CONDUCTED IN LOOSE SAND WHEN THE VALUE OF π_1 WAS 7

Pole Number	Load P Grams	Deflection y Inches	π_4 $\frac{P}{D^3\gamma}$	π_5 $\frac{y}{D}$
1	800	0.0584	9.20	0.0389
	960	0.0997	11.04	0.0664
	1120	0.1498	12.88	0.0999
	1280	0.2197	14.72	0.1453
	1440	0.3253	16.56	0.2168
	1520	0.4296	17.48	0.2864
	1600	0.4800	18.40	0.3200
	1800	0.7020	20.70	0.4680
	2000	0.8985	23.00	0.5990
	2200	1.4296	25.30	0.9531
2	800	0.0704	9.20	0.0469
	960	0.1196	11.04	0.0797
	1120	0.1483	12.88	0.0989
	1280	0.2766	14.72	0.1844
	1440	0.3940	16.56	0.2627
	1520	0.4840	17.48	0.3466
	1600	0.5666	18.40	0.3777
	1800	0.8120	20.70	0.5413
	2000	1.0716	23.00	0.7144
	2200	1.4896	25.30	1.0193
3	800	0.0440	9.20	0.0293
	960	0.0729	11.04	0.0486
	1120	0.1328	12.88	0.0885
	1280	0.1620	14.72	0.1080
	1440	0.2188	16.56	0.1459
	1520	0.2991	17.48	0.1994
	1600	0.3334	18.40	0.2223
	1800	0.4966	20.70	0.3311
	2000	0.6555	23.00	0.4370
	2200	0.9538	25.30	0.6359

TABLE VI

DATA FOR MODEL TESTS CONDUCTED IN DENSE SAND WHEN THE VALUE OF π_1 WAS 5

Pole Number	Load P Grams	Deflection y Inches	π_4 $\frac{P}{D^3\gamma}$	π_5 $\frac{y}{D}$
1	50	0.0023	4.1673	0.0031
	100	0.0069	8.3347	0.0092
	150	0.0131	12.5020	0.0175
	200	0.0234	16.6693	0.0312
	250	0.0433	20.8366	0.0577
	300	0.0705	25.0040	0.0940
	350	0.1360	29.1712	0.1813
	370	0.1997	30.8382	0.2662
2	50	0.0004	4.1673	0.0005
	100	0.0040	8.3347	0.0053
	150	0.0105	12.5020	0.0140
	200	0.0253	16.6693	0.0337
	250	0.0622	20.8366	0.0829
	300	0.1084	25.0040	0.1445
3	50	0.0039	4.1673	0.0052
	100	0.0088	8.3347	0.0117
	150	0.0153	12.5020	0.0204
	200	0.0248	16.6693	0.0331
	250	0.0406	20.8366	0.0541
	300	0.0737	25.0040	0.0982
	350	0.1638	29.1712	0.2183
	370	0.2582	30.8382	0.3442

TABLE VII

DATA FOR MODEL TESTS CONDUCTED IN DENSE SAND WHEN THE VALUE OF π_1 WAS 7

Pole Number	Load P Grams	Deflection y Inches	π_4 $\frac{P}{D^3\gamma}$	π_5 $\frac{y}{D}$
1	100		8.3347	
	200	0.0054	16.6693	0.0072
	300	0.0184	25.0040	0.0245
	400	0.0390	33.3386	0.0520
	450	0.0550	37.5059	0.0733
	500	0.0818	41.6733	0.1090
	550	0.1170	45.8405	0.1560
	600	0.2154	50.0079	0.2871
	620	0.2161	51.6748	0.2881
	650	0.2792	54.1752	0.3722
	670	0.2904	55.8422	0.3871
2	100	0.0018	8.3347	0.0024
	200	0.0060	16.6693	0.0080
	300	0.0143	25.0040	0.0191
	400	0.0294	33.3386	0.0392
	500	0.0598	41.6733	0.0797
	550	0.0774	45.8405	0.1032
	600	0.1070	50.0079	0.1426
	620	0.1089	51.6748	0.1452
	650	0.1267	54.1752	0.1689
	670	0.1299	55.8422	0.1732
	700	0.2031	58.3426	0.2707
	730	0.2179	60.8429	0.2905
	750	0.3908	62.5099	0.5209
	770	0.3966	64.1768	0.5257
3	100	0.0000	8.3347	0.0000
	200	0.0033	16.6693	0.0044
	300	0.0101	25.0040	0.0135
	400	0.0240	33.3386	0.0320
	450	0.0339	37.5059	0.0452
	500	0.0466	41.6733	0.0621
	550	0.0739	45.8405	0.0985
	600	0.0941	50.0079	0.1254
	620	0.1001	51.6748	0.1334
	650	0.1405	54.1752	0.1803
	670	0.1499	55.8422	0.1998
	700	0.2076	58.3426	0.2767
	730	0.3011	60.8429	0.4014
	750	0.3170	62.5099	0.4226

TABLE VIII

DATA FOR MODEL TESTS CONDUCTED IN DENSE SAND WHEN THE VALUE OF π_1 WAS 9

Pole Number	Load P Grams	Deflection y Inches	π_4 $\frac{P}{D^3\gamma}$	π_5 $\frac{y}{D}$
1	100	0.0006	8.3347	0.0008
	200	0.0020	16.6693	0.0027
	300	0.0038	25.0040	0.0051
	400	0.0061	33.3386	0.0081
	500	0.0095	41.6733	0.0127
	600	0.0150	50.0079	0.0200
	700	0.0220	58.3426	0.0293
	800	0.0335	66.6772	0.0447
	900	0.0571	75.0119	0.0761
	1000	0.0685	83.3465	0.0913
	1050	0.0761	87.5138	0.1014
	1100	0.0861	91.6811	0.1148
	1120	0.0895	93.3481	0.1193
	1150	0.1011	95.8485	0.1348
	1180	0.1085	98.3489	0.1446
	1200	0.1140	100.0158	0.1520
	1230	0.1247	102.5162	0.1662
	1260	0.1420	105.0166	0.1893
	1310	0.1953	109.1839	0.2603
	1360	0.2307	113.3512	0.3075
2	200	0.0035	16.6693	0.0047
	400	0.0097	33.3386	0.0129
	600	0.0236	50.0079	0.0315
	800	0.0511	66.6772	0.0681
	1000	0.0992	83.3465	0.1322
	1100	0.1391	91.6811	0.1854
	1200	0.2332	100.0158	0.3109
	1250	0.2705	104.1832	0.3606
	1300	0.4809	108.3504	0.6410
	1320	0.4900	110.0174	0.6532
	1340	0.5395	111.6844	0.7192
3	200	0.0015	16.6693	0.0020
	400	0.0070	33.3386	0.0093
	600	0.4955	50.0079	0.0267
	800	0.5231	66.6772	0.0635
	1000	0.5775	83.3465	0.1360
	1100	0.6388	91.6811	0.2177
	1200	0.6891	100.0158	0.2847
	1250	0.8051	104.1832	0.4394
	1300	0.8202	108.3504	0.4595
	1320	0.8377	110.0174	0.4828

TABLE IX

DATA FOR PROTOTYPE TESTS CONDUCTED IN DENSE SAND WHEN THE VALUE OF π_1 WAS 7

Pole Number	Load P Grams	Deflection y Inches	π_4 $\frac{P}{D^3\gamma}$	π_5 $\frac{y}{D}$
1	908	0.0011	9.4598	0.0007
	1816	0.0089	18.9196	0.0059
	2724	0.0280	28.3794	0.0187
	3632	0.0789	37.8392	0.0526
	4086	0.1070	42.5692	0.0713
	4530	0.1664	47.1949	0.1109
	4984	0.2341	51.9248	0.1561
	5200	0.2572	54.1751	0.1715
	5600	0.4317	58.3424	0.2878
	6000	0.7836	62.5098	0.5224
	6200	0.9389	64.5934	0.6260
	6300	1.0763	65.6353	0.7176
2	908	0.0004	9.4598	0.0003
	1816	0.0096	18.9196	0.0064
	2724	0.0265	28.3794	0.0177
	3632	0.0665	37.8392	0.0443
	4086	0.0997	42.5692	0.0665
	4530	0.1520	47.1949	0.1013
	4984	0.2225	51.9248	0.1483
	5200	0.2486	54.1751	0.1657
	5600	0.4265	58.3424	0.2843
	6000	0.7978	62.5098	0.5319
	6200	0.8435	64.5934	0.5625
3	908	0.0016	9.4598	0.0011
	1816	0.0139	18.9196	0.0093
	2724	0.0480	28.3794	0.0320
	3632	0.1156	37.8392	0.0771
	4086	0.1723	42.5692	0.1149
	4530	0.2471	47.1949	0.1647
	4984	0.3799	51.9248	0.2533
	5200	0.4372	54.1751	0.2915
	5600	0.8020	58.3424	0.5347

TABLE X

DATA FOR LATERAL LOAD TESTS ON MODEL POLES IN SATURATED SANDY CLAY

Load P Grams	π_4 $\frac{P}{H^3\gamma}$	Time t Minutes	Deflection y Inches	π_8 $\frac{kt}{D}$	π_5 $\frac{y}{D}$
100	0.0404	0.25	0.0037	0.00014	0.0049
100	0.0404	0.50	0.0046	0.00028	0.0061
100	0.0404	1	0.0049	0.00056	0.0065
100	0.0404	2	0.0059	0.00113	0.0079
100	0.0404	4	0.0065	0.00226	0.0087
100	0.0404	8	0.0069	0.00451	0.0092
100	0.0404	15	0.0075	0.00856	0.0100
100	0.0404	30	0.0083	0.01692	0.0111
100	0.0404	60	0.0090	0.03384	0.0120
100	0.0404	120	0.0099	0.06768	0.0132
100	0.0404	240	0.0114	0.13536	0.0152
100	0.0404	480	0.0123	0.27072	0.0164
100	0.0404	960	0.0125	0.54144	0.0167
150	0.0605	0.25	0.0199	0.00014	0.0265
150	0.0605	0.50	0.0227	0.00028	0.0303
150	0.0605	1	0.0248	0.00056	0.0331
150	0.0605	2	0.0264	0.00113	0.0352
150	0.0605	4	0.0280	0.00226	0.0373
150	0.0605	8	0.0295	0.00451	0.0393
150	0.0605	15	0.0308	0.00856	0.0411
150	0.0605	30	0.0330	0.01692	0.0440
150	0.0605	60	0.0354	0.03384	0.0472
150	0.0605	120	0.0381	0.06768	0.0508
150	0.0605	240	0.0404	0.13536	0.0539
150	0.0605	480	0.0429	0.27072	0.0572
150	0.0605	900	0.0452	0.50760	0.0603
150	0.0605	1440	0.0469	0.81216	0.0625
200	0.0807	0.25	0.0775	0.00014	0.1033
200	0.0807	0.50	0.0835	0.00028	0.1113
200	0.0807	1	0.0883	0.00056	0.1177
200	0.0807	2	0.0925	0.00113	0.1233
200	0.0807	4	0.0964	0.00226	0.1285
200	0.0807	8	0.1000	0.00451	0.1333
200	0.0807	15	0.1045	0.00856	0.1393
200	0.0807	30	0.1105	0.01692	0.1473
200	0.0807	60	0.1155	0.03384	0.1540
200	0.0807	120	0.1203	0.06768	0.1604
200	0.0807	300	0.1235	0.17200	0.1633
200	0.0807	480	0.1263	0.27072	0.1684
200	0.0807	960	0.1321	0.54144	0.1761
200	0.0807	1440	0.1326	0.81216	0.1767

TABLE X (Continued)

Load P Grams	π_4 $\frac{P}{H^3\gamma}$	Time t Minutes	Deflection y Inches	π_8 $\frac{kt}{D}$	π_5 $\frac{y}{D}$
250	0.1009	0.25	0.1108	0.00014	0.1477
250	0.1009	0.50	0.1268	0.00028	0.1691
250	0.1009	1	0.1398	0.00056	0.1864
250	0.1009	2	0.1493	0.00113	0.1991
250	0.1009	4	0.1598	0.00226	0.2131
250	0.1009	8	0.1683	0.00451	0.2244
250	0.1009	15	0.1738	0.00856	0.2317
250	0.1009	30	0.1783	0.01692	0.2377
250	0.1009	60	0.1850	0.03384	0.2467
250	0.1009	120	0.1906	0.06768	0.2541
250	0.1009	240	0.1960	0.13536	0.2613
250	0.1009	480	0.2022	0.27072	0.2696
250	0.1009	900	0.2128	0.50760	0.2837
250	0.1009	1440	0.2221	0.81216	0.2961
300	0.1211	0.25	0.1010	0.00014	0.1347
300	0.1211	0.50	0.2228	0.00028	0.2971
300	0.1211	1	0.2470	0.00056	0.3293
300	0.1211	2	0.2720	0.00113	0.3627
300	0.1211	4	0.2880	0.00226	0.3840
300	0.1211	8	0.3020	0.00451	0.4027
300	0.1211	15	0.3121	0.00856	0.4161
300	0.1211	30	0.3280	0.01692	0.4373
300	0.1211	60	0.3405	0.03384	0.4540
300	0.1211	120	0.3555	0.06768	0.4740
300	0.1211	240	0.3723	0.13536	0.4964
300	0.1211	480	0.3910	0.27072	0.5213
300	0.1211	900	0.4127	0.50760	0.5503
300	0.1211	1440	0.4271	0.81216	0.5695

TABLE XI

DATA FOR LATERAL LOAD TESTS ON PROTOTYPE POLES IN SATURATED SANDY CLAY

Load P Grams	π_4 $\frac{P}{H^3\gamma}$	Time t Minutes	Deflection y Inches	π_8 $\frac{kt}{D}$	π_5 $\frac{y}{D}$
800	0.0440	0.50	0.0147	0.00014	0.0094
800	0.0440	1	0.0165	0.00028	0.0105
800	0.0440	2	0.0183	0.00056	0.0117
800	0.0440	4	0.0202	0.00113	0.0130
800	0.0440	8	0.0220	0.00226	0.0141
800	0.0440	16	0.0239	0.00451	0.0153
800	0.0440	30	0.0259	0.00856	0.0166
800	0.0440	60	0.0279	0.01692	0.0179
800	0.0440	120	0.0300	0.03384	0.0192
800	0.0440	240	0.0324	0.06768	0.0208
800	0.0440	480	0.0358	0.13536	0.0230
800	0.0440	960	0.0369	0.27072	0.0237
800	0.0440	1920	0.0404	0.54144	0.0259
1200	0.0659	0.50	0.0875	0.00014	0.0561
1200	0.0659	1	0.0965	0.00028	0.0619
1200	0.0659	2	0.1035	0.00056	0.0664
1200	0.0659	4	0.1095	0.00113	0.0702
1200	0.0659	8	0.1147	0.00226	0.0734
1200	0.0659	16	0.1200	0.00451	0.0770
1200	0.0659	30	0.1249	0.00856	0.0801
1200	0.0659	60	0.1299	0.01692	0.0833
1200	0.0659	120	0.1349	0.03384	0.0865
1200	0.0659	240	0.1405	0.06768	0.0901
1200	0.0659	480	0.1469	0.13536	0.0942
1200	0.0659	960	0.1532	0.27072	0.0982
1200	0.0659	1800	0.1627	0.50760	0.1044
1200	0.0659	2880	0.1675	0.81216	0.1075
1600	0.0879	0.50	0.1600	0.00014	0.1026
1600	0.0879	1	0.1750	0.00028	0.1123
1600	0.0879	2	0.1900	0.00056	0.1219
1600	0.0879	4	0.2040	0.00113	0.1308
1600	0.0879	8	0.2158	0.00226	0.1384
1600	0.0879	16	0.2310	0.00451	0.1481
1600	0.0879	30	0.2439	0.00856	0.1564
1600	0.0879	60	0.2542	0.01962	0.1631
1600	0.0879	120	0.2642	0.03384	0.1694
1600	0.0879	240	0.2724	0.06768	0.1747
1600	0.0879	480	0.2810	0.13536	0.1802
1600	0.0879	960	0.2900	0.27072	0.1860
1600	0.0879	1800	0.2986	0.50760	0.1915
1600	0.0879	2880	0.3056	0.82160	0.1960
1600	0.0879	4320	0.3220	1.21840	0.2065

TABLE XI (Continued)

Load P Grams	π_4 $\frac{P}{H^3\gamma}$	Time t Minutes	Deflection y Inches	π_8 $\frac{kt}{D}$	π_5 $\frac{y}{D}$
2000	0.1099	0.50	0.3320	0.00014	0.2129
2000	0.1099	1	0.3710	0.00028	0.2379
2000	0.1099	2	0.4062	0.00056	0.2605
2000	0.1099	4	0.4630	0.00113	0.2970
2000	0.1099	8	0.4840	0.00226	0.3104
2000	0.1099	16	0.5112	0.00451	0.3278
2000	0.1099	30	0.5320	0.00856	0.3412
2000	0.1099	60	0.5670	0.01692	0.3636
2000	0.1099	120	0.5820	0.03384	0.3733
2000	0.1099	240	0.6020	0.06768	0.3861
2000	0.1099	480	0.6191	0.13536	0.3971
2000	0.1099	960	0.6360	0.27072	0.4079
2000	0.1099	1800	0.6508	0.50760	0.4174
2000	0.1099	2880	0.6670	0.81216	0.4278
2000	0.1099	4320	0.7070	1.21840	0.4535
2400	0.1319	0.50	0.6510	0.00014	0.4175
2400	0.1319	1	0.7190	0.00028	0.4785
2400	0.1319	2	0.7860	0.00056	0.5041
2400	0.1319	4	0.8380	0.00113	0.5375
2400	0.1319	8	0.9015	0.00226	0.5782
2400	0.1319	16	0.9920	0.00451	0.6363
2400	0.1319	30	1.0610	0.00856	0.6805
2400	0.1319	60	1.1250	0.01692	0.7215
2400	0.1319	120	1.1790	0.03384	0.7561
2400	0.1319	240	1.2220	0.06768	0.7837

APPENDIX B

THE DEVELOPMENT OF EXPRESSIONS FOR SETTING CONFIDENCE INTERVALS ON THE DIFFERENCE BETWEEN:

1. THE SLOPES OF TWO LINES.
2. VALUES OF Y ON THE 2 LINES AT ONE VALUE OF X.

Given: Estimates of 2 Lines.

$$Y = a_1 + b_1 X$$

$$Y = a_2 + b_2 X$$

Problem: Estimate the difference $(B_1 - B_2)$.

$$b_1 \sim N\left(B_1, \frac{\sigma^2}{\Sigma x_1^2}\right) \; : \; b_2 \sim N\left(B_2, \frac{\sigma^2}{\Sigma x_2^2}\right)$$

$$b_1 - b_2 \sim N\left(B_1 - B_2, \; \sigma_2 \left[\frac{1}{\Sigma x_2^2} + \frac{1}{\Sigma x_2^2}\right]\right)$$

$$\frac{\text{Pooled error Sum of squares}}{\sigma^2} \sim x^2$$

$$\frac{E}{\sigma^2} \sim x^2 \; (n_1 + n_2 - 4)$$

$$\frac{b_1 - b_2 - (B_1 - B_2)}{\sigma \left[\frac{1}{\Sigma x_1^2} + \frac{1}{\Sigma x_2^2}\right]^{\frac{1}{2}}} \sim N(0,1)$$

So

$$\frac{\left[b_1 - b_2 - (B_1 - B_2)\right]\sqrt{n_1 + n_2 - 4}}{\sqrt{E}\left[\frac{1}{\Sigma x_1^2} + \frac{1}{\Sigma x_2^2}\right]^{\frac{1}{2}}} \sim t(n_1 + n_2 - 4)$$

$$E = \Sigma y_1^2 - b_1 \Sigma y x_1 + \Sigma y_2^2 - b_2 \Sigma y x_2$$

$$-t_1 \leq \frac{\left[b_1 - b_2 - (b_1 + b_2)\right]\sqrt{n_1 + n_2 - 4}}{\sqrt{E}\left[\frac{1}{\Sigma x_1^2} + \frac{1}{\Sigma x_2^2}\right]^{\frac{1}{2}}} \leq t_1$$

$$b_1 - b_2 - \frac{t_1\sqrt{E}}{\sqrt{n + n - 4}}\left[\frac{1}{\Sigma x_1^2} + \frac{1}{\Sigma x_2^2}\right]^{\frac{1}{2}} \leq B_1 - B_2 \leq b_1 - b_2 +$$

$$\frac{t\sqrt{E}}{\sqrt{n_1 + n_2 - 4}}\left[\frac{1}{\Sigma x_1^2} + \frac{1}{\Sigma x_2^2}\right]^{\frac{1}{2}}$$

Problem: Estimate the difference between 2 values of Y at one value of X.

$$a_1 + b_1 X_0 \sim N\left[A_1 + B_1 X_0,\ \sigma^2 \frac{\Sigma (X_1 - X_0)^2}{n_1 \Sigma x_1^2}\right]$$

$$a_2 + b_2 X_0 \sim N\left[A_2 + B_2 X_0,\ \sigma^2 \frac{\Sigma (X_2 - X_0)^2}{n_2 \Sigma x_2^2}\right]$$

$$a_1 + b_1 X_0 - a_2 - b_2 X_0 \sim N \left\{ A_1 + B_1 X_0 - (A_2 + B_2 X_0), \; \sigma^2 \left[\frac{\Sigma (X_1 - X_0)^2}{n_1 \Sigma x_1^2} + \frac{\Sigma (X_2 - X_0)^2}{n_2 \Sigma x_2^2} \right] \right\}$$

So

$$\frac{a_1 + b_1 X_0 - (a_2 + b_2 X_0) - \left[A_1 + B_1 X_0 - (A_2 + B_2 X_0) \right]}{\sigma \left[\frac{\Sigma (X_1 - X_0)^2}{n_1 \Sigma x_1^2} + \frac{\Sigma (X_2 - X_0)^2}{n_2 \Sigma x_2^2} \right]^{\frac{1}{2}}} \sim N(0,1)$$

Let

$$C^2 = \left[\frac{\Sigma (X_1 - X_0)^2}{n_1 \Sigma x_1^2} + \frac{\Sigma (X_2 - X_0)^2}{n_2 \Sigma x_2^2} \right] \frac{1}{n_1 + n_2 - 4}$$

Then

$$-t_1 \leq \frac{(a_1 + b_1 X_0) - (a_2 + b_2 X_0) - (Y_1 - Y_2)}{\sqrt{E} \quad C} \leq t_1$$

$$a_1 + b_1 X_0 - a_2 - b_2 X_0 - t_1 C \sqrt{E} \leq Y_1 - Y_2 \leq a_1 + b_1 X_0 - a_2 - b_2 X_0 + t_1 C \sqrt{E}$$

This will reduce to

$$k_1 \leq Y_1 - Y_2 \leq k_2$$

If $Y_1 = \text{Log } \pi_{51}$ and $Y_2 = \text{Log } \pi_{52}$

$$k_1 \leq \text{Log } \pi_{51} - \text{Log } \pi_{52} \leq k_2$$

$$k_1 \leq \text{Log } \frac{\pi_{51}}{\pi_{52}} \leq k_2$$

$$e^{k_1} \leq \frac{\pi_{51}}{\pi_{52}} \leq e^{k_2}$$

$$e^{k_1} - 1 \leq \frac{\pi_{51}}{\pi_{52}} - 1 \leq e^{k_2}$$

$$e^{k_1} - 1 \leq \frac{\pi_{51} - \pi_{52}}{\pi_{52}} \leq e^{k_1} - 1$$

Symbols :

Symbols not defined in chapter IX are those used by Snedecor (15).

VITA

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