SIMULATION OF DISPATCH RELIABILITY FOR A FLEET OF LARGE COMMERCIAL AIRCRAFT

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PREFACE

This study is concerned with the analysis and prediction of performance for the American Airlines fleet of DC-10 commercial jet aircraft. The purpose is to develop a means to predict the performance of the DC-10 fleet whereby airlines management may determine whether its goals will be attained. The analysis includes the application of computer programs to determine the statistical properties of aircraft delay times and times-between-delays. Monte Carlo simulations based on analysis of (1) both delay times and times-between-delays, and (2) delay times only, provide statistical estimates of historical performance. Similar techniques are then used to predict the future performance of the fleet.

Accordingly, Chapter I defines the subject area and scope and introduces the performance measures of interest: observed availability and dispatch reliability. Chapter II sketches the development of theories of reliability, maintainability, availability, and the development of Monte Carlo simulation techniques. Chapter III describes the delay times and times-between-delays used in the study. Chapters IV and V detail the results of fitting the data to appropriate families of distributions; the fitting procedure, the estimated parameters obtained, and the results of goodness-of-fit analysis are discussed. Chapter VI deals with a "new" distribution derived by the author to handle a set of fitting problems. Chapter VII demonstrates how the results of the statistical analysis are used to simulate past and future performance; the results of the simulations are discussed. Chapter VIII summarizes the procedure and presents the conclusions reached.

I should like to express my appreciation to my major adviser, Dr. Mitchell O. Locks, for his inestimable guidance, assistance, and encouragement throughout this study. Appreciation is also given to my other committee members, Dr. J. Leroy Folks, Dr. Wayne A. Meinhart, Dr. Donald R. Williams, and Professor Fred Black, for their support and suggestions during the development of this study. I am also indebted to Dean Richard L. Williamson, College of Business Administration, Loyola University of Los Angeles, for his continued support and encouragement during the final phases of research.

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CHAPTER I

INTRODUCTION

Subject Area

In August, 1971, American Airlines introduced a fleet of DC-10 wide-bodied commercial jet aircraft into revenue service. The subject area of this study is the prediction of fleet performance based on data generated during revenue operation and comparison of those predictions to expected performance. In 1968, when American Airlines contracted for the acquisition of its fleet of 25 DC-10 aircraft, certain performance goals were specified as part of the purchase agreement with the manufacturer, McDonnell Douglas Corporation. These goals, formulated during the design stage, are expected to be realized in revenue operation. One of the goals was expressed in terms of "dispatch reliability" (DR), a performance measure calculated as the ratio of departures within a stated time of scheduled departure to total departures.

Purpose

The purpose of this study was to establish from data accumulated during the first 30 months (August, 1971, through January, 1974) of revenue operation of the DC-10 fleet a means to predict DR and to determine whether management's objectives for dispatch reliability would be attained. The DR goals were established as 99% DR for delays over

15 minutes by the end of the third year of revenue operation - that is, no more than a one percent probability of a delay over 15 minutes - and 99.75% DR for delays over 60 minutes. Although DR is monitored for each month of operation, management does not have a method for predicting DR from the current trends of DR. Preliminary results of this study demonstrated through simulation of DR to the end of the third year of revenue operation that management's objectives for July, 1974, would not be met.* In the present study, DR values are predicted to the end of the fourth year of revenue operation (July, 1975) based on the analysis of performance data from the first 30 months.

Scope

This study treats primarily the analysis of data to yield the subsequent predictions. To a lesser extent, certain analyses were undertaken to establish the comparability, and thus the usefulness, of observed availability (A_o) to DR.** Problems associated with component level performance are not considered, nor are specific maintenance procedures or policies. The emphasis is rather on the application of certain statistical concepts and procedures which ultimately yield a straightforward means of obtaining predictions of DR.

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*These results, based on the analysis of data from the first 18 months of revenue operation, were presented at the 1974 Reliability and Maintainability Symposium [1].

**A_o, as used in this study, is calculated for a single availability cycle as the ratio of time-between-delays to time-between-delays plus delay time.

Methodology

The study proceeded in several stages. The initial step was to compile the data. Delay times (DT) for each of the 30 months were collected. Times-between-delays (TBD), necessary for the assessment of A_o, were determined for the first 18 months. The first phase in analyzing the data consisted of determining the characteristics of the delay times by fitting the data from each of the 30 months to several different candidate families of distributions. In general, good fits were obtained to the lognormal family of distributions; however, in some cases it was necessary to use a mixture of distributions in order to obtain goodness-of-fit. When this occurred, a derived distribution, which could be called "log-uniform," was fitted to the tails of the otherwise lognormally distributed data. By applying fitting and testing techniques to the TBD data from the first 18 months, good fits were obtained to the Weibull family of distributions.

The findings from the analysis of delay times and times-between-delays allowed for Monte Carlo simulations of A_o, using data from the first 18 months. Resulting assessments of A_o did not compare closely enough to the DR values as calculated by American Airlines to be fully useful for the prediction of DR. Monte Carlo simulation of DR using the results of the analysis of delay times, however, yielded values quite close to those computed by American. The same Monte Carlo simulation technique was then used to predict DR for future months of operation of the DC-10 fleet.

This methodology resulted in the development of a procedure which provides a means to predict performance levels for any specified period and is capable of providing airlines management with timely predictions based on actual performance levels. Such predictions can be used for comparison with future goals as part of a comprehensive performance evaluation program.

This study describes in detail the methods used to fit the DT and TBD data to the appropriate distributions, the tests used to determine the goodness-of-fit to those distributions, the development and application of the "log-uniform" distribution, and the Monte Carlo simulation used to assess and predict DR.

CHAPTER II

SURVEY OF LITERATURE

Overview

This chapter presents a representative survey of the literature associated with reliability, maintainability and availability.* Since the body of literature has become quite large in the last two decades, this survey traces the main lines of development with special emphasis on aspects relevant to this study.** Overlapping areas also considered are estimation of parameters, fitting data to distributions and Monte Carlo techniques.

Reliability

The field of reliability is generally traced to the experience in World War II with complex military systems: Barlow and Proschan [2] provide a general historical sketch. Shooman [3] points out that the fields of communication and transportation had gained rapidly in

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*Relevant definitions are (1) Reliability: the probability that a device will operate according to specification in a given environment; (2) Maintainability: the probability that a device will be restored to specification within a given time; and (3) Availability: the probability that a device will be operational at a required time.

**Studies of general usefulness peripheral to this thesis are listed in the Bibliography.

complexity when reliability engineering became identified as a separate discipline in the late 1940's and early 1950's. This development may be viewed as an outgrowth from the field of quality control since certain aspects such as "life testing" may be shown to be special applications of quality control procedures according to Duncan [4]. Some of the earliest procedures in life testing and the use of the exponential distribution were developed by Epstein and Sobel [5]. An influential series of papers followed ([6]-[11]), which, in conjunction with a paper by Davis [12], presented much evidence for the use of the exponential distribution with failure data. This influence is present in the reports of the nine task groups of the Advisory Group on Reliability of Electronic Equipment (AGREE) [13], which is significant since many current reliability practices can be traced to their reports.

Studies extending the applicability of the Weibull distribution to reliability have also become important. This distribution was first proposed by Weibull [14] and received greater notice due to the influence of Kao, who treats estimation of its parameters in [15] and [16]. In [17], Kao discusses a mixture of Weibull distributions. Zelen and Dannemiller [18] contributed further to the use of the Weibull distribution by questioning the use of the exponential distribution for life testing. The work of Nancy R. Mann has been a factor in the usefulness of the Weibull distribution. Her contributions include the development of the following items: (1) linear techniques for goodness-of-fit (with Fertig and Scheur [19]); (2) a series of tables for weighted estimates of parameters [20]; and (3) confidence and tolerance bounds (with Fertig [21]).

Of special interest in this study is the lognormal distribution. While it has found application to failure times (for example, see Epstein [22] and Freudenthal [23]) its greatest value seems to be involved with maintainability, e.g., repair times. This and other distributions and procedures relevant to reliability are treated in studies of general interest in the Bibliography.*

Maintainability

The study of maintainability grew out of the recognition that for repairable systems or components, the measure of reliability is only part of the total problem in actual operation. Like reliability, maintainability as a measure of system effectiveness is based on applied probability. Many of the same procedures, therefore, apply.

Since much of the work in maintainability is conducted in the design stage, the literature concerns phases of the maintenance operation and distributions of down times. Studies representative of this approach include Aeronautical Radio Corporation's (ARINC) [27], Pieruschka [28], Bazovsky [29], and Retterer [30]. In the effort to assess and predict maintainability, Bovaird and Zagor [31], drawing on the work of Howard, Howard and Hadden [32], proposed the distribution of down times as a suitable tool. They showed that the lognormal distribution provides for meaningful parameters of down times. More recently, Locks [33] has shown how to assess maintainability when repair times follow either the exponential or lognormal distribution.

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*Special bibliographies of interest are found in Weiss's article in Zelen [24], in Balaban [25], and in Aitchison and Brown [26].

Availability

Like maintainability, availability studies show a variety of approaches and definitions. Thus, Hosford [34] uses measures he calls "pointwise availability" and "interval availability"; Pieruschka [28] defines availability as the ratio of the number of units ready for use to the total number; Barlow and Proschan [2] follow Hosford's terminology with the addition of "limiting interval availability"; Sandler [35] refers to the same measures as "instantaneous availability," "average up-time," and "steady state availability." Measures relevant to the design phase include "intrinsic availability" and "operational availability" [27]. In general, these measures have in common the combined analysis of times-between-failures (TBF) and times-to-repair (TTR). There are also a variety of distributions used. Assuming exponential TBF and lognormal TTR, Gray and Lewis [36] tabulate the distribution of the ratio, where availability is given by A = TBF/(TBF + TTR). Gray and Schucany [37], assuming the same distributions, establish lower confidence limits for the availability ratio. Locks [33] uses measures of "inherent availability" (with an example using exponential TBF and TTR) and "observed availability (A_o)" (with examples of both exponential TBF and TTR and Weibull TBF with lognormal TTR). Also shown is a Monte Carlo technique which yields confidence levels for the various estimates of A_o.

Monte Carlo

In order to analyze and predict dispatch reliability for the DC-10 fleet, a simulation model was constructed which uses Monte Carlo techniques.* The development of Monte Carlo techniques has a lengthy history. Teichroew [39] suggests that simulation is an extension of distribution sampling practiced by statisticians since the turn of the century and provides an extensive bibliography of early studies. Investigation of Monte Carlo techniques thus preceded, by quite a while, the origin of the term.** Current development is attributed to the work of von Neumann and Ulam during World War II on neutron diffusion. The paper by Metropolis and Ulam [40] introduced the term "Monte Carlo" and is considered to be historically significant. Their approach, still an application of Monte Carlo, was essentially a statistical one applied to integrals and differential equations. The development of Monte Carlo techniques has been enhanced by the concurrent development of computers so that it is now relatively simple to apply to a wide range of problems.***

Monte Carlo simulation, as applied in this study, consists generally of transforming random variables to variates of selected density functions based on observed data. General discussions are in Amstadter [42] and Brown [43], with more detailed treatments in Chorafas [44], Fabrycky [45], and Buslenko et al. [46].

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*There is some disagreement regarding appropriate terminology. Harling [38], for example, suggests that "simulation" is to be preferred to "Monte Carlo" since the latter term suggests limitation to statistical sampling experiments and the former implies a more inclusive stochastic model. General practice, however, does not tend to make this distinction.

**Buffon's needle problem and Lord Rayleigh's "random walks" are examples.

***A bibliography with a sampling of procedures and applications is given by Malcolm [41]

Monte Carlo is also described in much of the literature of operations management. Chase and Aquilano [47], Bierman, Bonini, and Hausman [48], King [49], and Buffa [50] give methodologies and sample applications, especially to queuing problems. In reliability studies, Thoman, Bain, and Antle [51] and Nancy R. Mann [52], have used Monte Carlo for work with the Weibull distribution. Complex systems are treated by Curtin [53] and Gilmore [54].

Since Monte Carlo techniques require a source of random numbers, the problem of their generation appears frequently in the literature. Three methods have found favor. The first, and earliest to develop, is tables of random numbers which have been subjected to statistical tests for randomness. The Rand Corporation, for example, in 1947 generated 10^6 random digits from a physical source. The use of tables, however, is generally unsuited for use with computers. Von Neuman and Metropolis proposed an alternate means of generating random numbers, which is described by Haugen [55] and Chambers [56]. This method, however, has faults also and has been superseded by methods which are more rapid and economical for computer use [57]. This study employs the method used by IBM for their subroutine package RANDU, described by Schmidt and Taylor [58], pp. 225-229. Although there is concern with the uniformity of distribution of randomly generated sequences ([46], [57] [58] [3]), this method was considered sufficiently accurate for this study. Once a random number is generated, however, it is then necessary to transform it to a variate based on the distribution being considered. General discussions of transformations are in [44] [47] [55] [58] [59] and [60]. A detailed treatment is given by Kahn [61].

Kamins [62] developed a method for transformation using the lognormal distribution, which has been refined by Locks [33].

CHAPTER III

DATA BASE

Reliability Program for the DC-10

The development of reliability into a separate discipline and the increasing cost of maintenance of newer, larger, and more complex aircraft induced commercial operators to increase requirements for manufacturers. When American Airlines was contracting for its DC-10 fleet, the respective roles for both the operator and the designer were established in order to provide for a reliable, maintainable aircraft [63]. One important aspect from the contract negotiations was the specification of goals for dispatch reliability (DR). As indicated previously, DR is calculated by American Airlines as the ratio of aircraft departures within a stated time of scheduled departure to total departures. For example, in January, 1973, there were 2752 total departures, of which 2531 departed within five minutes of schedule; therefore, DR for that case was 91.97%. More accurately, an estimate, $\widehat{DR}$ of DR, has been obtained which is an estimate of the probability that an aircraft will depart within a stated time of scheduled departure. Its complement, $1 - \widehat{DR}$, is therefore an estimate of the probability of delay.* Values

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*Mosteller, Rourke, and Thomas [64] present a clear, basic discussion regarding this type of estimate.

of DR as calculated by American Airlines are denoted as DR, while the estimates obtained in this study are shown as $\widehat{DR}$.

Delays of 15 minutes and under are not considered to have a significant impact on revenue service. Therefore, contractual DR goals are expressed in terms of delays over 15 minutes and delays over 60 minutes. For delays over 15 minutes, the contractual agreement specifies a DR goal of 99% at the end of the third year of revenue service; for delays over 60 minutes, the specified DR goal is 99.75%. For each of these categories, the associated DR is the ratio of departures within the stated time of scheduled departure to total departures. For January, 1973, for instance, there were 168 delays over 15 minutes and 65 delays over 60 minutes. Therefore, the DR for delays over 15 minutes, denoted by DR_{15}, was 93.90%; for over 60 minutes, DR_{60} was 97.64%.

Delay Times Data

Values for dispatch reliability are determined each month by American Airlines using their delay time reports for that month. Accordingly, the reports of delay times for the DC-10 fleet for the 30 months from the inauguration of revenue service, August, 1971, through January, 1974, provided the basic data for this study. Delays are reported only when certain safety-related equipment or certain passenger convenience items do not meet requirements for scheduled dispatch. It is only when the corrective maintainance causes a delay from scheduled departure of the aircraft for over five minutes that the delay time is reported.* For example, if failure of a given dispatch-related item is

*See Appendix A for the "Dispatch Inoperative List" determining the necessity of unscheduled repair or maintenance.

not corrected within five minutes of a scheduled departure, a delay from scheduled departure is reported. Since delay times are reported in whole minutes, the smallest delay time possible in the reporting system is six minutes.

Times-Between-Delays

A portion of this study was devoted to comparing dispatch reliability to an alternative measure of performance discussed by Locks [33], observed availability (A_o).* Since A_o, as applied here, is the ratio of time-between delays to time-between-delays plus delay time, an additional data set, times-between-delays (TBD) was extracted from the American Airlines reporting system. Since American does not assess A_o, TBD's are not monitored. This data was obtained by correlating information contained in aircraft log books, routing charts, and delay reports. It was collected for the 18 months, August, 1971, through January, 1973. The extraction process yielded monthly TBD sets expressed in hours of actual operating time between reported delays.

*Examples using this measure are contained in Appendix B.

CHAPTER IV

CHARACTERISTICS OF THE DATA

Overview

Analysis of the delay times (DT) and times-between-delays (TBD) was undertaken as the initial step in developing predictions of performance for the DC-10 fleet. In order to determine the nature of the DT and TBD, both random variables, they were fitted by month to appropriate families of distributions. Estimated parameters from these distributions were then used for Monte Carlo simulations of performance.

Distributions of Delay Times

Delay times from the first 30 months of revenue operation were fitted to several different candidate families of distributions, including normal, lognormal, exponential, and Weibull. The application of probability plotting,* and subsequent goodness-of-fit tests (Chapter V) to a wide range of data determined that the best fits were obtained to lognormal distributions.** Since only delays of six minutes or over

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*Nelson [65] describes methods of probability plotting for several different distributions applicable to this study.

**Aitchison and Brown, whose monograph on the lognormal distribution [26] is quite useful, note a variety of applications of this distribution. Bovaird and Zagor [31], Howard, Howard, and Hadden [32], and Goldman and Slattery [66] all present evidence that down times are lognormally distributed. Pieruschka [28], p. 165, suggests that when time are reported for data collection by American Airlines, the fitting procedure begins with the subtraction of 5.5 minutes from all delay times in order to provide a fit from a point closer to zero; that is, 0.5 minutes. This means that the difference between the actual delay time and 5.5 minutes is lognormally distributed. The 5.5-minute value is practically the same as the cut-off used by American Airlines for reporting a delay time.

Next, delay times were fitted, by month, to lognormal distributions using a least squares technique. Let DT denote the delay time, in minutes, and let $t = DT - 5.5$. Also, let μ and σ represent the parameters of the lognormal distribution (the mean and standard deviation, respectively, of the normally distributed logarithms of the values). The lognormal distribution has the probability density function (pdf),

$$f(t) = (2\pi)^{-1/2}(\sigma t)^{-1}\exp\{-1/2(\ell n\ t - \mu)^2/\sigma^2\}, \quad t \geq 0. \tag{1}$$

Least-Squares was used to fit the delay times to Equation (1). Let

$$\ell n\ t = \sigma z + \mu, \tag{2}$$

where z is the standard normal deviate with mean zero and standard deviation one. A given set of delay time data consists of n order statistics $\ell n\ t_1 \leq \ell n\ t_2 \leq \cdots \leq \ell n\ t_n$. Corresponding to each order

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**(Continued) spent in fault isolation predominates, down times tend to be exponentially distributed; otherwise down times are appropriately described by lognormal distributions. Since delay times exhibit characteristics similar to repair times and down times (when repair times are predominant), such as positive skewness, the lognormal family of distributions was also intuitively appealing as a choice for describing delay times.

statistic $\ell n\ t_i$ there is an estimated plotting value $\hat{F}_i$ of the normal distribution function given by,

$$\hat{F}_i = (i - ½)/n, \tag{3}$$

and an associated $\hat{Z}_i$ which is a function of $\hat{F}_i$ (when successive equal values of $\ell n\ t_i$ are encountered, an average value of i is used to obtain the value of $\hat{F}_i$).

The estimates $\hat{\mu}$ of μ and $\hat{\sigma}$ of σ were obtained by a two-parameter least-squares fit of the n observed data points in Equation (2) using a computer program which was especially prepared for this analysis (Appendix F). Results from the first 30 months of revenue operation yielded estimated values of μ ranging from 2.34, corresponding to a median delay of approximately 16 minutes, to 3.73, corresponding to a median delay of approximately 47 minutes. The estimated scale parameter σ is approximately 1.28. The results are shown in Table I, including the corresponding estimated median delay times.

By viewing the values displayed in Table I over the period from August, 1971, to January, 1974, it is clear that the delay times display an increasing tendency while the values obtained for $\hat{\sigma}$, the estimated standard deviation, exhibit a relative stability about their average value of 1.28. Since $\hat{\mu}$ and $\hat{\sigma}$ are in terms of the logarithms of the delay times, this means that the dispersion in terms of minutes is increasing. For example, suppose that three successive values of μ were 2.97, 3.10, and 3.23 with a σ of 1.28. Then the percentage increase in the dispersion of minutes for the range $\mu \pm \sigma$ between successive distributions is a constant of approximately 13.9%. Thus, with respect to the

TABLE I

ESTIMATED PARAMETERS FOR DELAY TIMES FROM THE FIRST 30 MONTHS OF REVENUE OPERATIONS

Month	$\hat{\mu}$	$\exp(\hat{\mu}) + 5.5$ (Median DT)	$\hat{\sigma}$
August, 1971	3.085	27.4	1.310
September	2.818	22.2	0.623
October	2.490	17.6	1.373
November	2.545	18.2	1.529
December	2.340	15.9	1.431
January, 1972	2.683	20.1	1.373
February	2.949	24.6	1.186
March	3.188	29.7	1.431
April	2.791	21.8	1.317
May	2.817	22.2	1.299
June	3.088	27.4	1.250
July	2.833	22.5	1.416
August	3.030	26.2	1.243
September	3.341	33.8	1.305
October	3.250	31.3	1.330
November	3.296	32.5	1.207
December	3.533	39.7	1.229
January, 1973	3.233	30.9	1.329
February	3.242	31.1	1.288
March	3.079	27.2	1.322
April	3.167	29.2	1.220

TABLE I (Continued)

Month	$\hat{\mu}$	$\exp(\hat{\mu}) + 5.5$ (Median DT)	$\hat{\sigma}$
May, 1973	2.914	23.9	1.249
June	3.283	32.1	1.355
July	3.085	27.4	1.396
August	3.196	29.9	1.538
September	3.048	26.6	1.135
October	2.921	24.0	1.121
November	3.264	31.6	1.217
December	3.730	47.2	1.386
January, 1974	3.424	36.2	1.220

delay times, a constant value of σ with increasing values of μ means that the spread of the delay times is increasing.*

Distributions of Times-Between-Delays

Times-between-delays (TBD) from the first 18 months of revenue operation were also fitted to several different candidate families of distributions. Using a combination of probability plotting, a goodness-of-fit test by Mann, Fertig, and Scheur [19] and the chi-square test (Chapter V), the best fits were obtained to the Weibull family of distributions.

The computer program developed for this analysis uses a least-squares calculation to obtain estimated parameters of the Weibull distributions for each month (Appendix E supplies a listing of the program used for analysis of TBD). Since delays can occur as soon as an aircraft begins operation, a location parameter was not used and estimation of δ and β (the "characteristic TBD" and "shape" parameters) was accomplished by using the two-parameter Weibull distribution. Using t to denote TBD, the pdf is given by

$$f(t|\delta,\beta) = (\beta t^{\beta-1}/\delta^{\beta}) \exp\{-(t/\delta)^{\beta}\}. \quad (4)$$

Let R denote the probability that delay occurs after t; that is,

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*Aitchison and Brown [26] show the effect of a variety of values for μ and σ by displaying graphs of lognormal probability density functions. For a given σ, increasing values of μ mean decreasing values of f(t) obtained by Equation (1) when the value of t is taken as the mode of the lognormal distribution, $\exp\{\mu-\sigma^2\}$. In terms of this study, this represents a "flattening" of the shape of the lognormal distributions of the delay times. Additionally, this also means a decrease in the density of the median ($\exp\{\mu\}$).

$$R(t) = 1 - F(t) = \exp\{-(t/\delta)^{\beta}\}. \quad (5)$$

From Equation (5), the straight line function corresponding to $y = \beta x + c$ is

$$\ln(-\ln R) = \beta \ln t - \beta \ln \delta, \quad (6)$$

so that when $\ln(-\ln R) = 0$, β is the slope and δ is obtained by $\delta = \exp(-c/\beta)$.

For a sample of n observations, let the order statistics be given by $\ln t_1 \leq \ln t_2 \leq \cdots \leq \ln t_n$, and let $\ln(-\ln R)$ be the corresponding function with a corresponding plotting value $\hat{F}_i$ as given by Equation (3).

The estimates $\hat{\beta}$ of β and $\hat{\delta}$ of δ are obtained by a two-parameter least-squares fit of the n observed data points to Equation (6). Estimated parameters are shown in Table II. Results show that while the values of $\hat{\delta}$, the "characteristic TBD" appear to be increasing slightly, the value of $\hat{\beta}$ is approximately 1.0.*

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*In Equation (4), setting $\beta = 1.0$ yields the exponential pdf, $f(t) = \lambda \exp\{-\lambda t\}$, $\lambda = 1/\delta$.

TABLE II

ESTIMATED PARAMETERS FOR TIMES-BETWEEN-DELAY FROM THE FIRST 18 MONTHS OF REVENUE OPERATION

Month	$\hat{\delta}$	$\hat{\beta}$
August, 1971	14.66	1.139
September	5.31	0.856
October	19.34	1.342
November	7.53	0.978
December	12.70	1.131
January, 1972	21.54	1.130
February	25.40	1.092
March	34.48	0.984
April	29.55	1.054
May	39.21	1.014
June	25.21	1.108
July	25.82	1.055
August	21.69	1.032
September	29.68	0.990
October	30.18	0.970
November	36.47	1.133
December	31.92	1.049
January, 1973	24.35	1.011

CHAPTER V

TESTS OF THE DATA

Overview

After fitting the delay times and times-between-delays to the lognormal and Weibull distributions, respectively, different goodness-of-fit tests were considered in order to establish the usefulness of the fitted distributions for the simulations of performance and the predictions of dispatch reliability.

Although different tests were used in early experimentation and analysis,* chi-square was adopted as the primary test for use in this study. The chi-square test, originated by Pearson in 1900, is well documented in many textbooks and manuals.**

Goodness-of-Fit for the Delay Times

The lognormal distributions based upon the values of $\hat{\mu}$ and $\hat{\sigma}$ fitted to the monthly delay times were tested against the data by means of chi-square goodness-of-fit tests. Let k be the number of segments over the

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*Such as the Lilliefors versions of the K-S test (for lognormal distributions) [67] and the Mann-Fertig-Scheuer test (for Weibull distributions) [19].

**Good reviews are provided by Cochran [68] and Watson [69]. Choice of intervals for the test are discussed by Williams [70], Mann and Wald [71], Gumbel [72], and Pieruschka [28].

range of values of the delay times. For each segment i, let $\hat{f}_i$ denote the number of observations in that segment based upon the distribution with parameters $\hat{\mu}$ and $\hat{\sigma}$, and let f_{io} be the number observed from the data; then,

$$\chi^2 = \sum_{i=1}^{k} (\hat{f}_i - f_{io})^2/\hat{f}_i. \tag{7}$$

The computer program prepared for the analysis of delay times computes the number of segments, k, according to a rule that there be at least five observations in each segment and that each segment should contain a specified percentage of the total number. In general, a percentage specification of .05 or .10 provided a satisfactory division of the data. (The computer program listed in Appendix F shows the detailed Fortran steps used to perform the analysis.)

The χ^2 values were computed by Equation (7). Since two parameters, $\hat{\mu}$ and $\hat{\sigma}$, are estimated for fitting the data to the lognormal distribution, the number of degrees of freedom for χ^2 is $\nu = k - 1 - r$, where $r = 2$, unless portions of the data are truncated, in which case additional restrictions are imposed which are discussed in Chapter VI for mixed distributions. The results, shown in Table III, reveal that on the whole, the fits were good. Analysis of the first three months (August, 1971, through October, 1971) of the first 30 months of delay times are not shown in Table III because of a small number of data points. The chi-square values at the .05 level were obtained from Harter's tables [73].

TABLE III

SUMMARY OF CHI-SQUARE GOODNESS-OF-FIT ANALYSIS
FOR DELAY TIMES

Month	Delays	k	ν	χ^2	χ^2 at .05 level
November, 1971	33	4	1	1.39	3.84
December	42	6	3	3.00	7.81
January, 1972	49	8	5	10.78	11.07
February	58	9	6	9.40	12.59
March	49	8	5	0.90	11.07
April	75	12	9	10.00	16.92
May	61	10	7	4.00	14.07
June	99	10	7	6.35	14.07
July	129	10	7	6.43	14.07
August	160	18	15	21.72	25.00
September	140	10	7	6.43	14.07
October	134	10	7	6.60	14.07
November	130	18	15	25.77*	25.00
December	156	10	7	7.59	14.07
January, 1973	221	19	16	24.08	26.30
February	231	20	17	18.44	27.59
March	265	20	17	13.87	27.59
April	196	18	15	25.34*	25.00
May	206	10	7	9.63	14.07
June	223	19	16	24.76	26.30
July	222	19	16	16.74	26.30

TABLE III (Continued)

Month	Delays	k	ν	χ^2	χ^2 at .05 level
August	250	10	5	10.15	11.07
September	177	18	15	22.55	25.00
October	134	16	12	14.25	21.03
November	100	14	11	14.90	19.68
December	99	11	7	12.90	14.07
January, 1974	115	10	7	3.70	14.07

*Significant at 5% level.

Goodness-of-Fit of the Times-Between-Delays

A chi-square test was also included in the computer program used for analysis of the TBD (Appendix E). The number of classes is established in a manner identical to that used for DT analysis; class boundaries, however, are given by

$$BD = \ell n \{-\ell n\ (1 - BK)\} - C/\beta, \qquad (8)$$

where BD denotes the boundary and BK denotes the percentage rule used to determine expected frequency for each class. The results, shown in Table IV, demonstrate that on the whole, the fits are good.

An additional test also showed that, for this data, there is no significant difference between a Weibull and an exponential distribution. A test statistic from Epstein [9] was used to test the hypothesis that $\hat{\beta}$ is not different from 1.0. Application of the statistic is also discussed in detail by Fercho and Ringer [74].

TABLE IV

SUMMARY OF CHI-SQUARE GOODNESS-OF-FIT ANALYSIS
FOR TIMES-BETWEEN-DELAY

Month	TBD	κ	ν	χ^2	χ^2 at .05 level
November, 1971	33	5	2	8.01*	5.99
December	42	6	3	4.46	7.81
January, 1972	49	6	3	1.92	7.81
February	57	9	6	5.54	12.59
March	49	6	3	0.90	7.81
April	75	8	5	4.47	11.07
May	61	8	5	9.90	11.07
June	98	10	7	5.88	14.07
July	126	10	7	7.49	14.07
August	157	10	7	8.67	14.07
September	139	10	7	9.99	14.07
October	131	10	7	7.24	14.07
November	127	9	6	13.08*	12.59
December	152	10	7	7.21	14.07
January, 1973	219	10	7	10.45	14.07

*Significant at 5% level.

CHAPTER VI

A MIXTURE OF DISTRIBUTIONS

Detection of the Mix

Aggregations of the monthly delay times showed lack of fit to the lognormal distribution because of the nature of the data in the tails of the distribution. Analytical procedures were developed to handle this problem when encountered with monthly delay times. Of the 30 months, three - August, October, and December, 1973 - showed lack of fit to the lognormal distribution upon application of the chi-square test using the .05 level of significance ($\chi^2_{.05}$). A procedure was adopted by which the tails of the ordered data, fitted to the lognormal family of distributions for each of these months, were truncated. These truncated portions were then fitted to a derived distribution which may be termed "log-uniform."

The Derived Distribution

Enough delay times were removed from the tail(s) of the observed distribution of monthly delay times until goodness-of-fit to the lognormal distribution was obtained for the remaining portion. The portions represented by the truncated tails were then fitted and tested against alternative distributions. Since only a few data points were involved, the uniform distribution was found to be satisfactory for the upper and/or lower portions of the problem months; however, even better fits were obtained by using the logarithms of the delay times. Thus, by using the $\ell n\ t_i$ as had been done in the center portion, goodness-of-fit was established for the lower and/or upper portions.

A uniform distribution with parameters a and b is defined by the pdf,

$$f(x) = 1/(b - a), \qquad a \leq x \leq b. \tag{9}$$

For $x = \ell n\ t$, a transformation of variables results in the pdf of what may be termed a "log-uniform" distribution,

$$f(t) = 1/(b - a)t, \qquad \exp\{a\} \leq t \leq \exp\{b\}. \tag{10}$$

Although this derived distribution was a natural step in the research, since logarithms had been used for previous analysis, no reference to this particular form of distribution has been found in the literature.

Analysis

The analysis of the tails of the distribution begins with visual inspection of the plotted data fit to the lognormal distribution which is provided for by the initial computer analysis of the monthly delay times. By such inspection, an approximate percentage point for each truncation is determined. A chi-square value is then determined for each portion, resulting in a combined total χ^2 value which is tested against χ^2 at the .05 level of significance. Since trial and error is necessary to establish the estimate of the exact percentage breaking point, the computer program for analysis of delay times (Appendix F) is adapted to analyze up to 25 different combinations per run in order that the optimum mix can be chosen on the basis of the total chi-square statistics. For each χ^2 test statistic, one additional degree of freedom is subtracted to account for the additional restraints on the data imposed by each estimated percentage point on the cumulative distribution function at which a truncation occurs. This breaking point is given by the percentage point of the cumulative distribution function which corresponds to the value of z obtained by rearrangement of Equation (2):

$$z = (\ell n\ t_i - \hat{\mu})/\hat{\sigma}. \tag{11}$$

Figure 1 displays the estimated percentage points for the month of August, 1973, which separate the data into a mix of log-uniform and lognormal distributions. Thus, the log-uniform distribution applies from 0 to .0348 and from .8051 to 1.0, while a lognormal fit explains the center portion.

Selected results for the months of August, October, and December, 1973, which show both the initial fit to the lognormal distribution and the fit to the mixed situation are shown in Tables V and VI. Note, for example, that when the 250 delay times for the month of August, 1973, are fitted to and tested against the lognormal distribution, the χ^2 obtained is 18.24 (Table V). Since μ and σ are estimated, the degrees of freedom are $\nu = k - 1 - 2$, where $k = 10$, for a $\chi^2_{.05}$ test statistic of 14.07. Clearly, a significant difference between the observed data and the lognormal distribution is noted. The subsequent fit to a mixture of lognormal and log-uniform distributions, however, provided a total χ^2 of

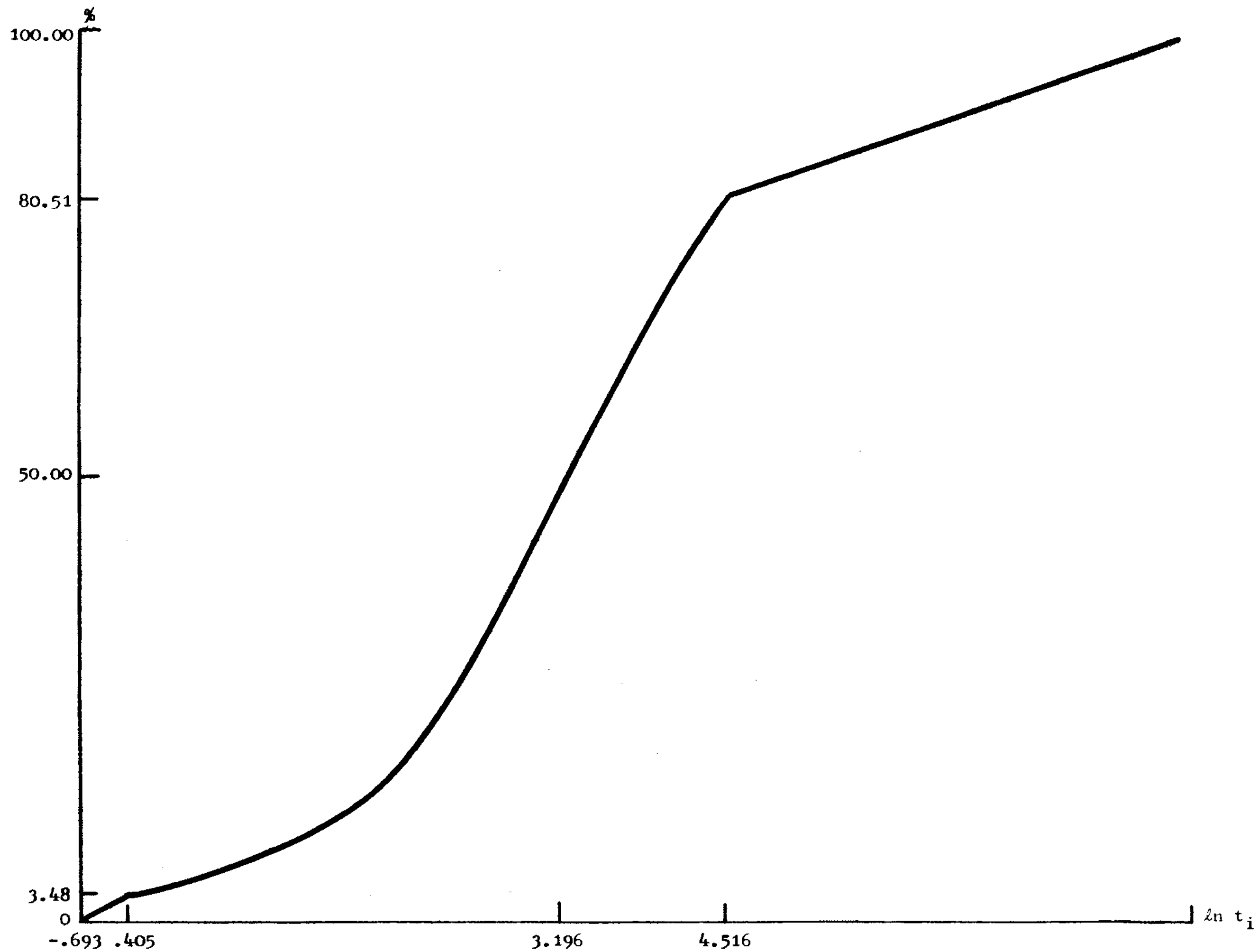


Figure 1. Cumulative Distribution Function for Delay Times From August, 1973, Fitted to a Mixture of the Lognormal and Log-Uniform Distribution

TABLE V

ANALYSIS OF SELECTED DELAY TIMES SHOWING FIT TO LOGNORMAL DISTRIBUTION ONLY

Month	Delays	χ^2 for all delays	k	ν	χ^2 at .05 level
August, 1973	250	18.24	10	7	14.07
October, 1973	134	26.45	15	12	21.03
December, 1973	99	23.00	10	7	14.07

TABLE VI

ANALYSIS OF SELECTED DELAY TIMES SHOWING FIT TO A MIXTURE OF LOG-UNIFORM AND LOGNORMAL DISTRIBUTIONS

Month	Selected Range of Delays	Estimated Boundaries of cdf (between 0 and 1)	Lower χ^2	Center χ^2	Upper χ^2	k	ν	Total χ^2	χ^2 at .05 Level
August, 1973	9-210	.0348, .8051	0.06	8.53	1.56	10	5	10.15	11.07
October, 1973	1-124	--- , .9032	----	13.49	0.68	16	12	14.17	21.03
December, 1973	7- 99	.0543, ---	0.07	9.07	----	11	7	9.15	14.07

10.15 (Table VI). Using $\nu = k - 1 - 4$, since two additional parameters were estimated, that is, the percentage points separating the distributions, the $\chi^2_{.05}$ test statistic is 11.07.

For October, 1973, an upper truncation was made at .9032 in order to establish goodness-of-fit while for December a lower truncation at .0543 was sufficient. In general, about five percent of the data in each tail of the monthly delay times was not well behaved even though, in most cases, good fits were obtained to lognormal distributions. When this was not the case, the analysis resulted in definitive mixtures such as explained above.

CHAPTER VII

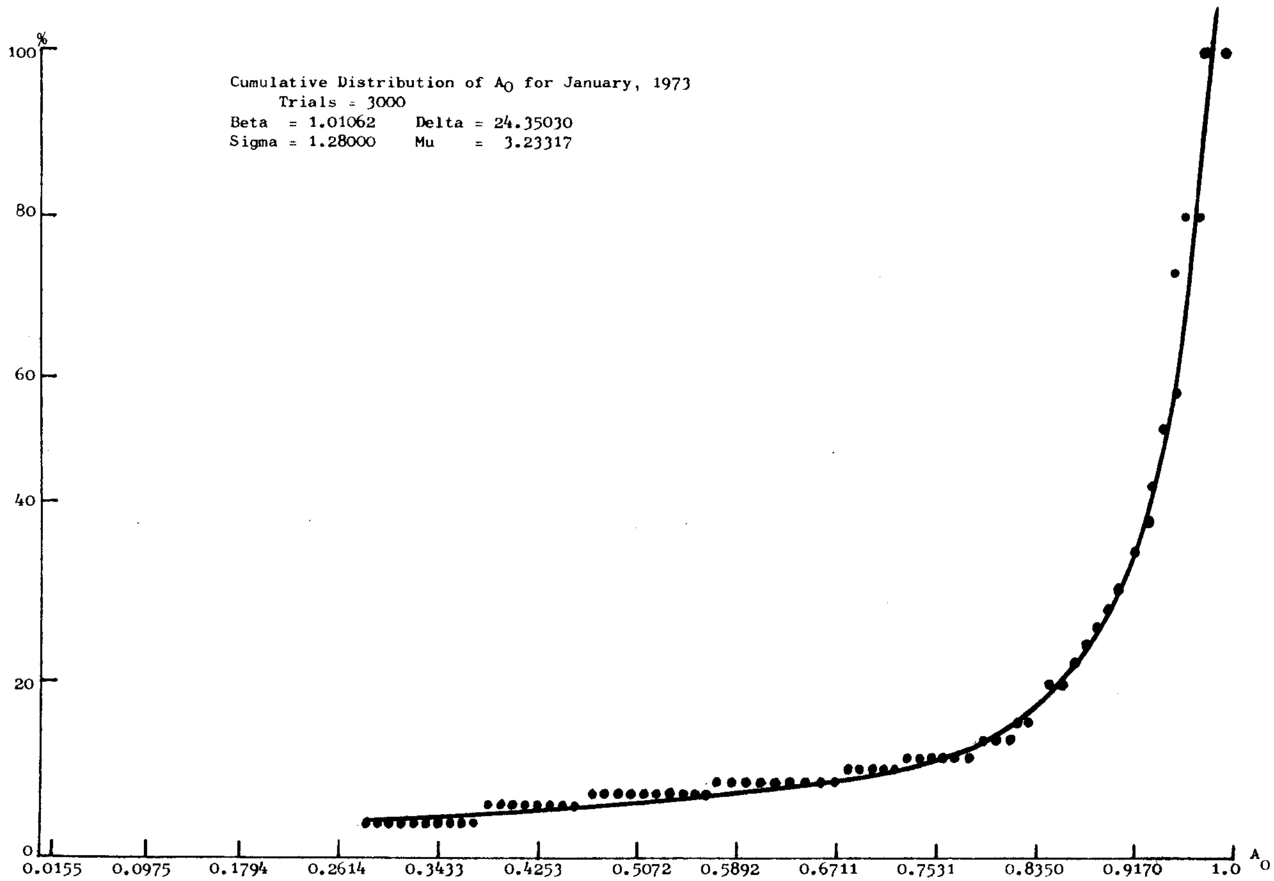
SIMULATIONS OF PERFORMANCE

Overview

The preceding analysis of the delay times (DT) and times-between-delays (TBD) provided estimated parameters of distributions for use in the Monte Carlo simulations of performance. Since the simulations of observed availability (A_o) did not compare closely to historical values of dispatch reliability, final predictions of dispatch reliability were accomplished based on a simulation technique which uses the analysis of the delay times.

Observed Availability

Results from the analysis of data from the first 18 months of revenue service of the DC-10 fleet were used to evaluate performance in terms of A_o. The value of A_o for a single cycle is obtained by a Monte Carlo selection of values based upon the distributions of the DT and TBD. A large number of Monte Carlo trials is employed to generate a distribution of A_o. Since the percentage points of the resulting simulated distribution of A_o are the confidence limits on A_o, assessments may be performed for any particular confidence level desired [33]. For example, data from the month of January, 1973, is shown as a graph of the cumulative distribution function of A_o in Figure 2. Using 60% as the level of



Confidence Level	90.0	80.0	70.0	60.0	50.0	40.0	30.0	20.0	10.0
A_O	0.69999	0.85880	0.91332	0.93831	0.95648	0.96801	0.97801	0.98454	0.99031

Figure 2. Distribution of Observed Availabilities With Lognormal Delay Times and Weibull Times-Between-Delays

confidence desired for assessment, the A_o value of 93.80% may be noted. Similar assessments were performed for each of the first 18 months of revenue service and compared to the DR values as calculated by American Airlines. Appendix D contains the computer program used for the simulation of A_o.*

Comparison of Observed Availability and Dispatch Reliability

Since the DR measures of interest were for delays over 15 minutes and delays over one hour, the assessments and predictions of A_o were made with regard to such delays. Comparison with DR values is shown in Table VII. In general, at a given confidence level applied to all periods to determine values of A_o, close comparison with the historical values of DR is not displayed. By using a range of confidence levels for A_o between 40 and 70 percent, values of DR are generally bracketed; however, the necessity of this procedure in order to establish comparability with DR values demonstrated the need for a more definitive procedure for predicting values of DR. For example, Table VII shows that the month of January, 1973, has a value of DR_{15} of .9390 which is close to the A_o value of .9383 at the 60% level of confidence, but for the month of November, 1972, with a DR_{15} of .9466, it is necessary to select a confidence level of 70% in order to obtain a closely related A_o value of .9480. Figure 3 also demonstrates this lack of comparability by showing A_o values at two different levels of confidence which bracket the DR_{15}

*This program represents an adaptation of methods explained by Locks [33].

TABLE VII

COMPARISON OF OBSERVED AVAILABILITY AND DISPATCH RELIABILITY

Month	Delays Over 15 Minutes			Delays Over 60 Minutes		
	A_o at .7 Confidence Level (γ = .7)	DR_{15}	A_o at .6 Confidence Level (γ = .6)	A_o at .5 Confidence Level (γ = .5)	DR_{60}	A_o at .4 Confidence Level (γ = .4)
August, 1971	.8885	.9091	.9189	.9546	.9545	.9674
September	.8056	.8644	.8618	.9948	1.0000	.9959
October	.9456	.9471	.9586	.9822	.9765	.9869
November	.8357	.8909	.8730	.9495	.9697	.9615
December	.9215	.9330	.9395	.9807	.9854	.9852
January, 1972	.9404	.9360	.9554	.9815	.9801	.9860
February	.9413	.9245	.9569	.9817	.9832	.9867
March	.9314	.9577	.9519	.9727	.9819	.9805
April	.9523	.9456	.9655	.9867	.9843	.9900
May	.9617	.9681	.9718	.9892	.9891	.9919
June	.9329	.9423	.9512	.9743	.9833	.9818
July	.9380	.9468	.9539	.9783	.9816	.9842
August	.9218	.9359	.9432	.9717	.9788	.9805
September	.9194	.9432	.9442	.9652	.9773	.9749
October	.9263	.9488	.9481	.9703	.9802	.9788
November	.9480	.9466	.9634	.9769	.9776	.9831
December	.9227	.9465	.9472	.9632	.9739	.9725
January, 1973	.9133	.9390	.9383	.9638	.9764	.9739

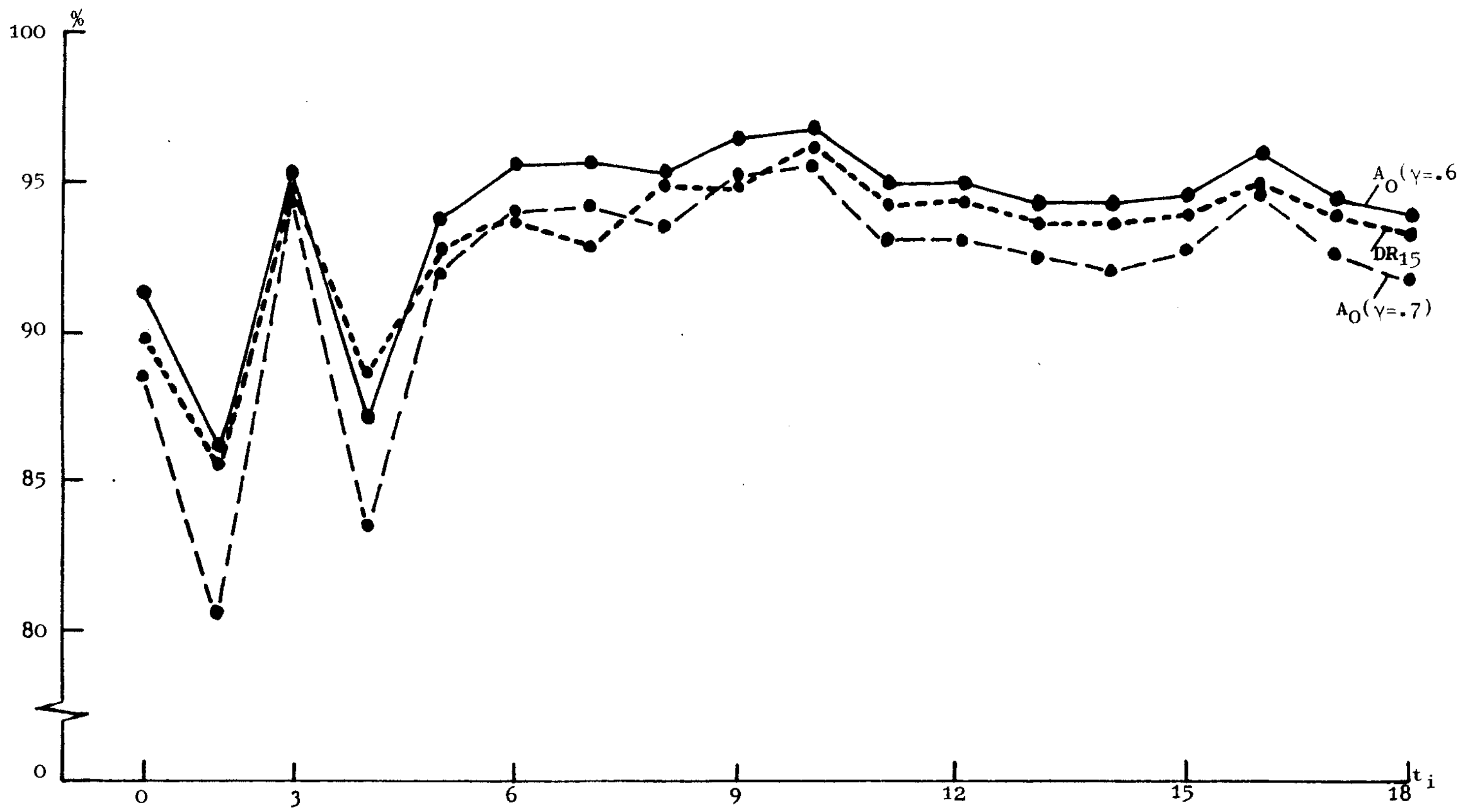


Figure 3. Comparison of Observed Availability and Dispatch Reliability

values. Note that the confidence levels of 60% and 70% only provide a general bracketing since some values of DR_{15}, such as April, 1972, are not contained in the range of A_o values between the two levels of confidence shown.

Simulations of Historical Dispatch Reliability

While the simulations of A_o represented a partial solution to the prediction problem by indicating the general trend of aircraft performance levels, simulations of historical DR values provided a more direct and accurate comparison to DR as calculated by American Airlines. Accordingly, this procedure was adopted for further analysis.

Each run of the computer program for simulating historical values of DR requires as input data: estimates of lognormal parameters μ and σ, estimated percentage points delineating log-uniform boundaries, the number of departures for each month, the number of delays of 5.5 minutes or more (that is, 6 minutes or more in the American Airlines maintenance reporting system), and a specified number of Monte Carlo trials to represent aircraft departures.* In initial analyses, up to 20,000 trials were used to determine an efficient number. Simulation of 3000 trials was adopted for final runs since a computer run could be obtained faster with this number of trials than by using a much larger number and the results from simulations using different numbers of trials were practically the same.

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*The computer program for simulation of dispatch reliability, which was especially prepared for this study, is shown in Appendix C.

The simulation model works as follows. For each trial, a random number is first generated to determine whether a delay of 5.5 minutes or more occurs by comparison with the ratio of departures with 5.5 minutes or more delay to total departures. If the random number is equal to or less than this ratio, then a delay time is determined by obtaining a second random number which is used to select a percentage point on either the lognormal distribution with estimated parameters μ and σ, or the log-uniform distribution, based on the input specifications. A value of the scale parameter of the lognormal distribution, $\sigma = 1.28$, was assumed as a good representative value as previously noted in discussion of Table I. By specifying a log-uniform distribution for the lower and upper 5% of the cumulative distribution function, better estimates of DR were obtained than otherwise. The percentage point obtained is then converted to a specific delay time for the trial based on the estimated parameters of the applicable distribution. When all trials are complete, values of DR are estimated for DR_{15} and DR_{60}.

Using a 90% confidence coefficient, lower confidence limits for estimated DR values are established by,

$$\widehat{DR} - Z\sqrt{\frac{\widehat{DR}(1 - \widehat{DR})}{n}}, \qquad Z = 1.645. \tag{12}$$

This type of calculation is discussed by Mosteller [64] as a suitable approach to confidence limits when the value of $\bar{p}$, expressed in the above as $\widehat{DR}$, is not close to 50% and n is large.*

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*Smith and Williams [75] also provide a good discussion of various methods used for estimating confidence limits.

The estimated value, $\widehat{DR}_{15}$, of the 15-minute dispatch reliability is the ratio of the number of trials with less than 15.5 minutes to the total number of trials; likewise, $\widehat{DR}_{60}$ is the ratio of the number of trials with less than 60.5 minutes delay to total trials. The results of the DR simulations shown in Table VIII display very close agreement between the estimates, $\widehat{DR}_{15}$ and $\widehat{DR}_{60}$, and the corresponding calculated values of DR_{15} and DR_{60}, for the first 30 months of revenue operations. Also shown in Table VIII are the lower confidence limits for $\widehat{DR}_{15}$ and $\widehat{DR}_{60}$. Although 3000 trials were used, the table shows the confidence limits calculated by using the number of actual departures for the value of n in Equation (12) in order to provide, in most cases, more conservative values. For example, for January, 1974, the lower confidence limit for $\widehat{DR}_{15}$ using 3000 as n would be .9364, a value slightly higher than that shown in the table as .9343, a value which results from using the number of departures as the value of n. In general, the differences between the DR values as calculated by American Airlines and the estimated values are in the neighborhood of .005. This closeness is a good indication that the methodology for simulating DR is suitable for prediction of future values of DR.

Predictions of Dispatch Reliability

Predictions of dispatch reliability are performed using the same Monte Carlo technique which yielded the simulated values of historical DR. The predicted values of DR are based on trends of $\hat{\mu}$ from the fitted distributions of the delay times for each of the first 30 months, the .05 allowance for the log-uniform in the tails, estimated delays of 5.5

TABLE VIII

ANALYSIS OF DISPATCH RELIABILITY FOR THE FIRST 30 MONTHS OF REVENUE OPERATIONS

Month	Departures	Delays Over 15 Minutes				Delays Over 60 Minutes			
		Delays	DR_{15}	$\widehat{DR}_{15}$	$\widehat{DR}_{15}$ for .9 Confidence Level ($\gamma = .9$)	Delays	DR_{60}	$\widehat{DR}_{60}$	$\widehat{DR}_{60}$ for .9 Confidence Level ($\gamma = .9$)
August, 1971	44	4	.9091	.9200	.8527	2	.9545	.9710	.9294
September	59	8	.8644	.9027	.8392	0	1.0000	.9747	.9410
October	170	9	.9471	.9403	.9104	4	.9765	.9863	.9717
November	165	18	.8909	.8807	.8392	5	.9697	.9737	.9532
December	343	23	.9329	.9357	.9139	5	.9854	.9873	.9774
January, 1972	453	29	.9360	.9327	.9133	9	.9801	.9850	.9756
February	596	45	.9245	.9333	.9165	10	.9832	.9770	.9669
March	827	35	.9577	.9547	.9428	15	.9819	.9837	.9764
April	956	52	.9456	.9437	.9314	15	.9843	.9883	.9826
May	1285	41	.9681	.9720	.9644	14	.9891	.9937	.9900
June	1317	76	.9423	.9367	.9256	22	.9833	.9823	.9764
July	1523	81	.9468	.9460	.9365	28	.9816	.9860	.9810
August	1794	115	.9359	.9350	.9254	38	.9788	.9800	.9746
September	1936	110	.9432	.9493	.9411	44	.9773	.9823	.9774
October	1974	101	.9488	.9440	.9355	39	.9802	.9790	.9737
November	2005	107	.9466	.9497	.9416	45	.9776	.9847	.9802

TABLE VIII (Continued)

Month	Departures	Delays Over 15 Minutes				Delays Over 60 Minutes			
		Delays	DR_{15}	$\widehat{DR}_{15}$	$\widehat{DR}_{15}$ for .9 Confidence Level (γ = .9)	Delays	DR_{60}	$\widehat{DR}_{60}$	$\widehat{DR}_{60}$ for .9 Confidence Level (γ = .9)
December, 1972	2372	127	.9465	.9497	.9423	62	.9739	.9790	.9742
January, 1973	2752	168	.9390	.9413	.9340	65	.9764	.9783	.9738
February	2531	172	.9320	.9373	.9294	72	.9716	.9763	.9714
March	2840	192	.9324	.9290	.9211	62	.9782	.9737	.9687
April	2877	147	.9489	.9460	.9391	51	.9823	.9807	.9764
May	2710	141	.9480	.9467	.9396	44	.9838	.9840	.9800
June	3156	167	.9471	.9457	.9390	74	.9766	.9800	.9759
July	3272	154	.9529	.9440	.9374	64	.9804	.9827	.9789
August	3336	186	.9442	.9530	.9470	69	.9793	.9837	.9801
September	3012	127	.9578	.9597	.9538	36	.9880	.9883	.9851
October	2863	95	.9668	.9527	.9461	26	.9909	.9750	.9702
November	2100	75	.9643	.9640	.9573	31	.9852	.9870	.9829
December	1984	81	.9592	.9583	.9509	46	.9768	.9817	.9767
January, 1974	1772	95	.9464	.9433	.9343	36	.9797	.9777	.9719

minutes or more, and estimated departures.

The critical input to the computer program for prediction of dispatch reliability is the projected values of $\hat{\mu}$. These were determined by regression of different structures of the estimated values of $\hat{\mu}$ obtained from the analysis of each of the first 30 months of revenue operation. Using a computer program called FUTURE [76], results from regression of the 30 values are shown in Figure 4. A relatively high positive slope of .022 may be noted for this regression. The next regression, shown in Figure 5, uses the last 24 months of the first 30, resulting in a slightly smaller positive slope of .014. In a variation of this regression, which also does not use the first six months of "start up" data, the four highest values were replaced by the original regression values shown in Figure 5. Because of the relatively conservative nature of these results (Figure 6), with a slope of .011, they were considered to be the most suitable for obtaining future values of $\hat{\mu}$ to be used in the predictions of DR.

Summary results showing selected regression equations obtained, by different structures of the 30 values, with their corresponding projected values of $\hat{\mu}$ for July, 1974, January, 1975, and July, 1975, are shown in Table IX. The regression equation used, as discussed above for Figure 6, $\hat{\mu}_i = 2.969 + .011\ t_{i-6}$, is noted by underlining. Comparison of this equation with the others also displays its appropriateness to the predictions. Table IX shows, for example, that projected values from this regression are slightly lower than those obtained by regression of the last 24 months and slightly higher than those from regression of only the last 18 months. Thus, projected values of $\hat{\mu}$ for the

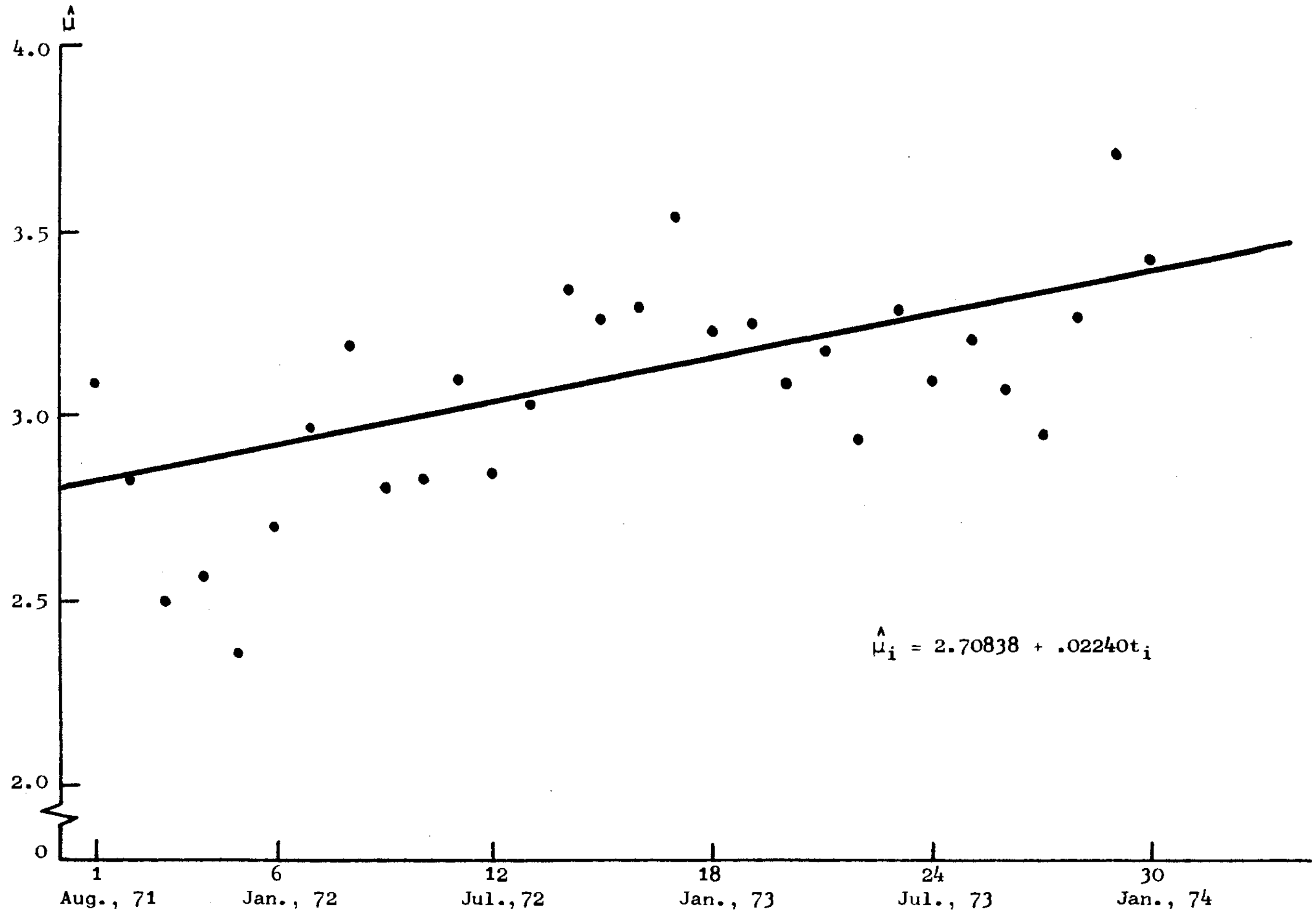

Figure 4. Trend of Values of $\hat{\mu}$ From August, 1971, Through January, 1974

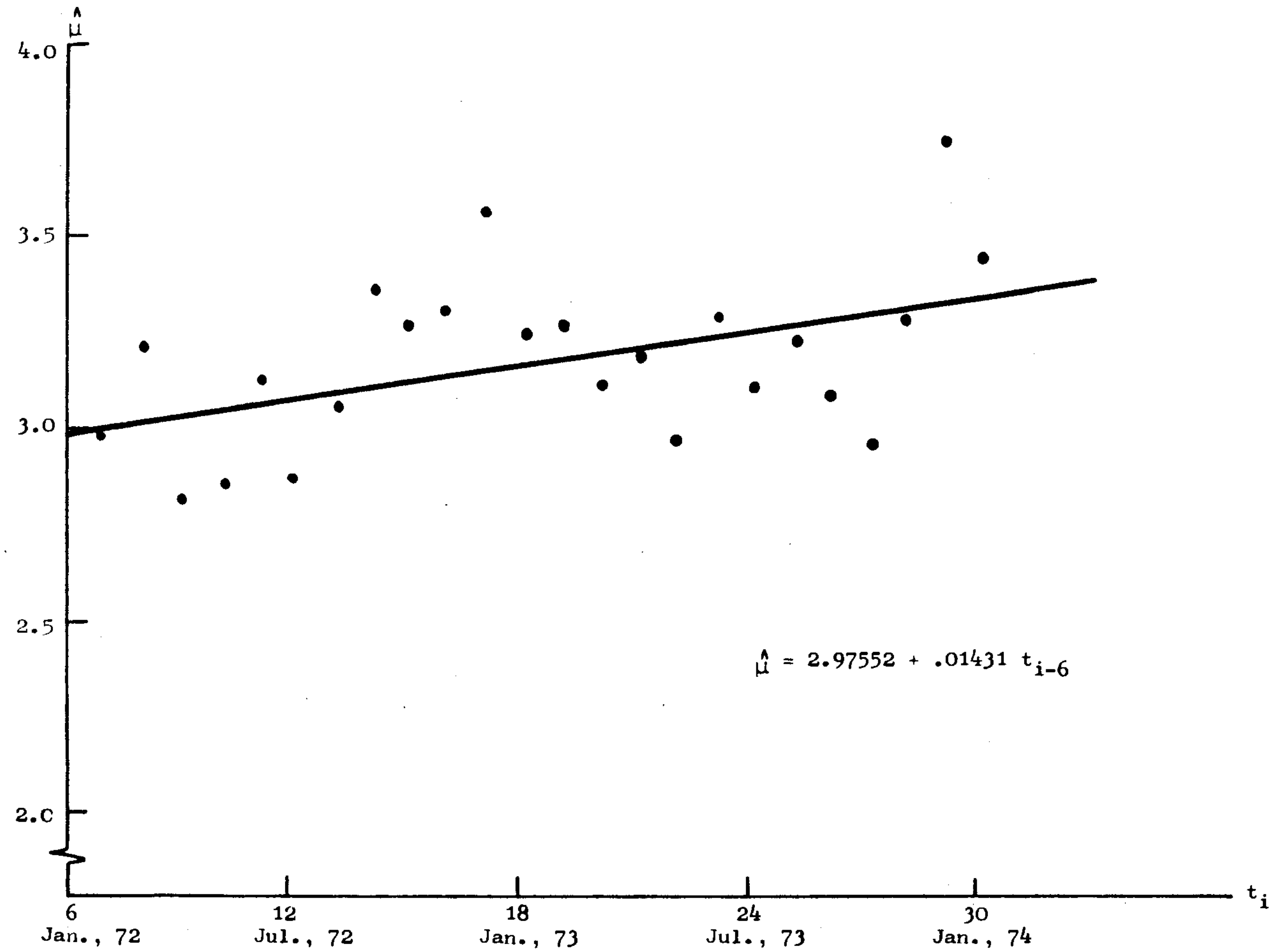

Figure 5. Trend of Values of $\hat{\mu}$ From February, 1972, Through January, 1974

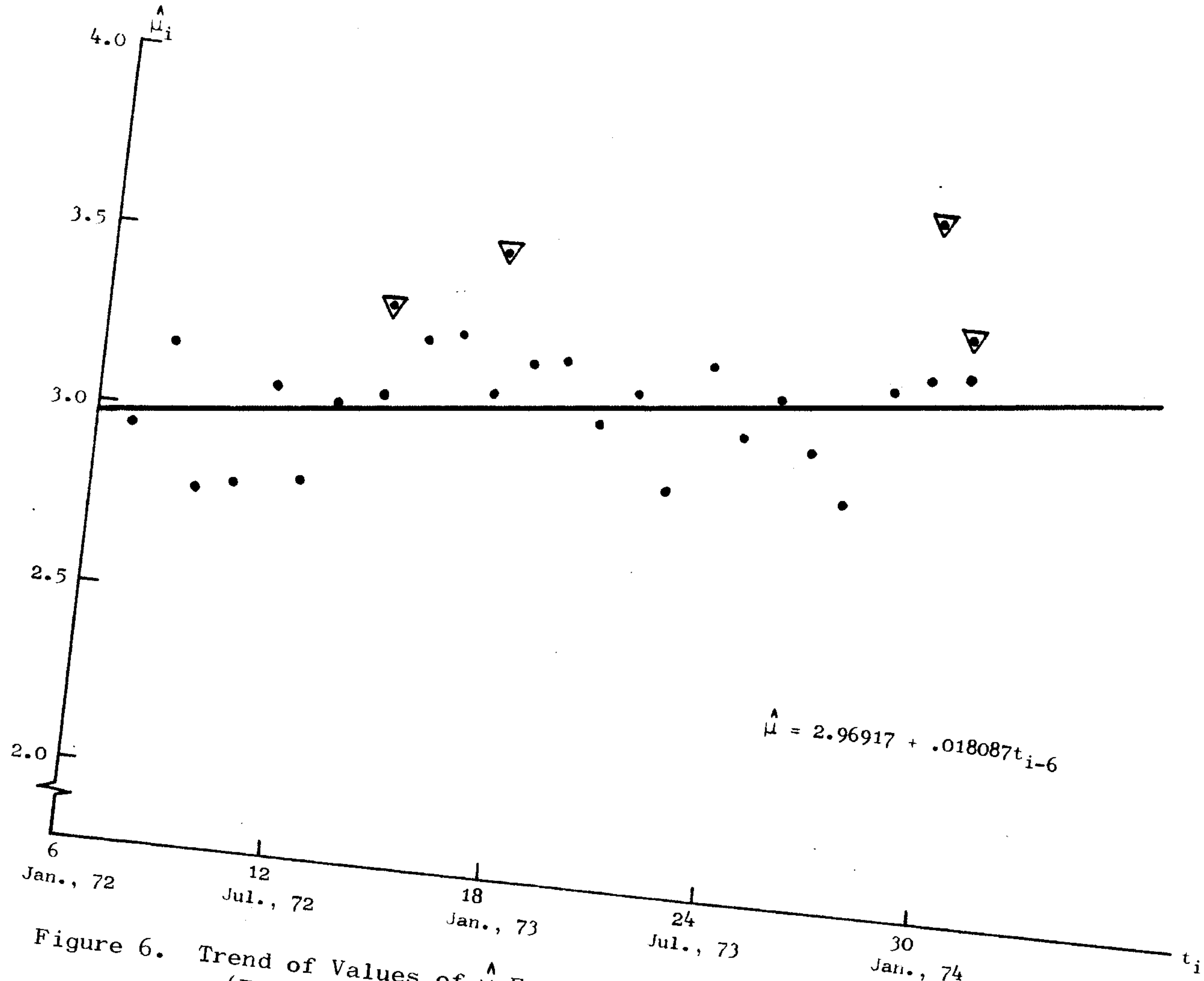

Figure 6. Trend of Values of $\hat{\mu}$ From February, 1972, Through January, 1974 (Four Values Replaced)

TABLE IX

REGRESSION EQUATIONS AND PROJECTED VALUES OF $\hat{\mu}$

Structure of Regression Values of $\hat{\mu}$ From:	Regression Equation for $\hat{\mu}_i$	Projected Values of $\hat{\mu}$ for Selected Months		
		July, 1974	January, 1975	July, 1975
All 30 months	$2.708 + .022\ t_i$*	3.515	3.649	3.783
The last 18 of the first 24 months	$2.984 + .014\ t_{i-6}$	3.406	3.490	3.574
The last 24 of the first 30 months	$2.976 + .014\ t_{i-6}$	3.405	3.490	3.576
The last 24 of the first 30 months with 4 RV**	$2.969 + .011\ t_{i-6}$	3.295	3.361	3.426
The last 24 of the first 30 months with 6 RV	$2.957 + .011\ t_{i-6}$	3.288	3.354	3.420
The last 18 of the first 30 months	$3.190 + .004\ t_{i-12}$	3.277	3.299	3.321
The last 12 of the first 30 months	$3.015 + .028\ t_{i-18}$	3.517	3.684	3.852

*t_i denotes time period, for i = 1, 2, 3, ···, 48, where month 1 is August, 1971.

**RV = Replacement values for the high values of $\hat{\mu}$.

36th month, July, 1974, the 42nd month, January, 1975, and the 48th month, July, 1975, were selected for the predictions of DR based on projections of $\hat{\mu}$ as shown by Figure 6. Results are shown in Table X, where it can be seen that the predicted values, $\widehat{DR}_{15}$ = .9780 and $\widehat{DR}_{60}$ = .9907 for the end of the fourth year of revenue operation.

TABLE X

PREDICTED VALUES OF DISPATCH RELIABILITY

Month	$\widehat{DR}_{15}$	.9 Lower Confidence Limit (γ = .90)	$\widehat{DR}_{60}$	.9 Lower Confidence Limit (γ = .90)
July, 1974	.9563	.9495	.9843	.9802
January, 1975	.9660	.9606	.9863	.9828
July, 1975	.9780	.9736	.9907	.9878

Significance of the Predictions

The significance of the predictions shown in Table X is that the predicted values of $\widehat{DR}_{15}$ and $\widehat{DR}_{60}$ for the end of fourth year of operation are both still less than the original management objectives established for achievement by the end of the third year of revenue operation.*

*In a recent check of data with American Airlines, it was learned that for the month of July, 1974, they reported a DR_{15} of .9743 and a DR_{60} of .9877. By reference to Table X, it can be seen that these values

The increasing trend in the values of $\hat{\mu}$, evidenced by analysis shown in Table IX and Figures 4, 5, and 6, indicate that longer and longer delay times are being experienced by DC-10 aircraft. This tendency could be a result of a trend toward less problems from the systems which have primarily caused less than 16-minute delays; that is, these problems are being overcome with relative success compared to problems with systems which primarily account for delays over 15 minutes. Thus, the implication is that, to achieve an improvement in future values of DR and to attain the DR goals specified in the contract between American Airlines and McDonnell Douglas, the values of μ to be attained in future months must be substantially reduced.

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*(Continued) are bounded from below by the .9 confidence limits on the estimates, $\widehat{DR}_{15}$ and $\widehat{DR}_{60}$. The variance between DR_{15} and $\widehat{DR}_{15}$ is .018 and between DR_{60} and $\widehat{DR}_{60}$ it is .003.

CHAPTER VIII

SUMMARY AND CONCLUSIONS

Summary

The dispatch reliability goals for American Airlines fleet of DC-10 aircraft, originally established for achievement by July, 1974, have not yet been attained; nor is it likely that they will be by July, 1975, based upon current trends of increasing delay times. Analysis of both delay times and times-between-delays was accomplished for use in assessment, by Monte Carlo simulation, of observed availability in an effort to establish comparability of this measure to that of dispatch reliability. The findings were that these simulations do not compare well enough to historical values of DR, as calculated by American Airlines, to provide a suitable basis for prediction of DR. Thus, the need for the development of different techniques to establish a more suitable foundation for the prediction of dispatch reliability was indicated. Detailed analysis of the delay times from the first 30 months of revenue operation of the fleet (August, 1971, through January, 1974) constituted basic input to the Monte Carlo simulations of historical values of DR. Goodness-of-fit analysis revealed that delay times for departures delayed six minutes or more tend to fit lognormal distributions. In certain cases, a mixture of the lognormal and log-uniform distributions was found to provide an even better explanation of the delay times. By allowance for a log-uniform distribution in the lower and upper five percent of the otherwise lognormally-distributed delay times, estimated DR values were obtained by Monte Carlo simulations in terms of delays over 15 minutes and delays over one hour for each of the first 30 months. The mixed distribution with the allowance for log-uniform portions in the tails had the additional virtue of yielding better values of $\widehat{DR}$ than obtained by using only the estimated parameters from the fits provided to lognormal distributions only. Consequently, findings of these simulations showed very close agreement with DR values as computed by American Airlines. By similar allowance for the log-uniform portions and by using projected values of $\hat{\mu}$, with a representative value of $\hat{\sigma}$, predictions of DR to the end of the fourth year of revenue operation were accomplished using the same simulation technique developed for the assessments of historical values of DR. For July, 1975, predictions are $\widehat{DR}_{15} = .9780$ and $\widehat{DR}_{60} = .9907$.

Conclusions

Since the projection of values of $\hat{\mu}$ used in the simulations of DR are based on a conservative evaluation of several alternative projections, all of which showed a positive slope value, the chances are that the DR_{15} and DR_{60} values to be attained in future months will be less than the established goals. Because of the effect of the increasing values of $\hat{\mu}$, as indicated by analysis of past data, the DR_{15} goal of .9900 and DR_{60} goal of .9975 are still higher than the predicted values for the end of the fourth year (July, 1975). Since these predictions are $\widehat{DR}_{15} = .9780$ and $\widehat{DR}_{60} = .9907$, it may be noted that they are short of the goals by .0120 and .0068, respectively. Additionally, because of the relatively stable value of $\hat{\sigma}$, accompanied by the increasing values of $\hat{\mu}$, the spread of the delay times is increasing. Statistically, this means that the median and mode parameters of the lognormally distributed delay times are also increasing; in terms of density, however, the occurrence of the mode and median values is decreasing. This might indicate, for instance, that solutions are being obtained to problems typified by delay times around these densities.

The procedures, integrated into specially prepared computer programs, developed by this study for the specific predictions constitute analytical tools for detailed analyses of delay times and their effect on dispatch reliability. By continual tracking and analysis of delay times, future findings from revenue operation can be used in a similar manner as input for the provision of continual predictions which serve to monitor progress toward goals and provide airlines management with an objective view of present and future performance in terms of dispatch reliability.

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APPENDIX A

DISPATCH INOPERATIVE LIST

DISPATCH INOPERATIVE LIST

Air Transport Association (ATA) is a classification scheme which reports data according to the following major system number and identifier:

ATA System Number	Identifier
21	Air Conditioning
22	Autopilot
23	Communications
23A	Entertainment
24	Electrical
25	Equipment and Furnishings
25A	Buffet
25B	Furnishings
26	Fire Protection
27	Flight Controls
28	Fuel
29	Hydraulic Power
30	Ice-Rain/Pneumatics
31	Instruments
32	Landing Gear
33	Lights
33A	Interior Lights
33B	Exterior Lights
34	Navigation
35	Oxygen
36	Pneumatic
38	Water/Waste
49	Auxiliary Power Unit (APU)
50	Structures
52	Exterior Doors
53	Fuselage
54	Pylons
55	Stabilizers
56	Windows
57	Wings
70	Power Plant
71	Cowling
72	Engines
73	Engine Fuel
74	Ignition
75	Engine Bleed

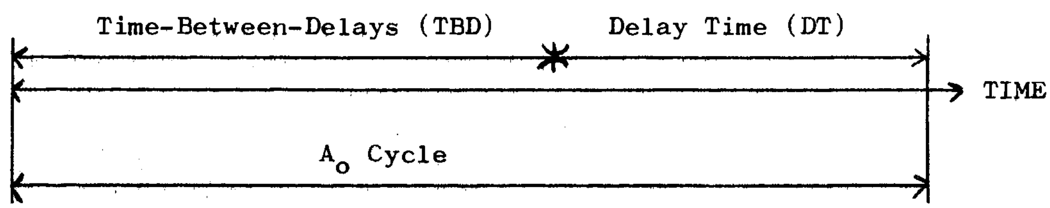
ATA System Number	Identifier
76	Engine Control
77	Engine Indicators
78	Exhaust
79	Engine Oil
80	Starting

APPENDIX B

MEANING AND EXAMPLES OF OBSERVED AVAILABILITY (A_O)

MEANING AND EXAMPLES OF OBSERVED AVAILABILITY (A_O)

A_o, as applied to a fleet system, is the probability that an individual aircraft, selected at random, will meet a scheduled departure within a stated time period. A_o is a random variable defined in terms of an availability cycle. The two consecutive periods of the cycle are (1) an operation time until delay, and (2) a delay time. A_o is the ratio of time in the first period to the time of the cycle:



For example, suppose a new aircraft begins service and makes several flights, adding up to 98 hours, prior to a delay. Suppose further that it experiences a two-hour delay prior to beginning its next A_o cycle. Measurement of A_o for the first cycle is thus:

$$A_o = \frac{\text{Operation Time Until Delay}}{\text{Time in Cycle}} = \frac{98}{100} = .98,$$

or

$$A_o = \frac{\text{TBD}}{\text{TBD} + \text{DT}} = \frac{98}{98 + 2} = .98.$$

On the basis of this information only, A_o would be assessed as 98 percent. With no other information, this aircraft's next cycle could be expected to yield an A_o of 98 percent. Another example, which would also yield an A_o of 98 percent could be a TBD of 9.8 hours and a DT of .2 hours:

$$A_o = \frac{TBD}{TBD + DT} = \frac{9.8}{9.8 + .2} = .98.$$

This illustrates that different combinations of TBD and DT can result in the same A_o. Thus, A_o may also be interpreted as the probability of being in service during a specified time period.

One approach to the assessment of A_o is to estimate the parameters of the family of distributions which govern a given collection of TBD and DT. Monte Carlo simulations of A_o can then be performed in order to obtain a distribution of A_o where the ordinate (0 to 1.0) may be used to determine confidence levels (using γ as the confidence level, $1 - \gamma$ is the corresponding point on the ordinate). The range of the abscissa is determined by the range of simulated values of A_o, for example, from .01 to 1.0.

Monthly collections of TBD and DT are used for assessment of A_o. Since several aircraft compose a fleet and each contributes to the TBD and DT collections, A_o may be viewed as the probability of take-off within a specified time after scheduled departure for an aircraft selected at random.

APPENDIX C

PROGRAM LIST FOR SIMULATION OF DISPATCH RELIABILITY

```
C       DISPATCH RELIABILITY PROGRAM
C       THIS PROGRAM SIMULATES AIRCRAFT FLEET PERFORMANCE ON A MONTHLY BASIS.
C   BY USING A LARGE NUMBER OF TRIALS(DEPARTURES).E.G., 3000, AND DETERMINING
C   FOR EACH DEPARTURE, IF A DELAY OCCURS.  A PROPORTIONALITY FACTOR(Q) WHICH IS
C   TOTAL  DELAYS OVER TOTAL DEPARTURES, FOR THAT MONTH, IS CHECKED AGAINST A
C   RANDOM NUMBER TO DETERMINE IF A DELAY OCCURS FOR THE DEPARTURE. I.E., TO
C   MONTE CARLO A DEPARTURE ON TIME:  A DEPARTURE ON TIME OCCURS IF THE RANDOM
C   NUMBER IS GREATER THAN Q.   IF IT DOES, THEN    THE          DELAY IS
C   DETERMINED FROM RANDOM SELECTION OF A TIME FROM THE DISTRIBUTION OF DELAY
C   TIMES FOR THAT  MONTH.  IN THE CASE OF FUTURE MONTHS, RANDOM SELECTION IS
C   MADE FROM THE DISTRIBUTION USING PROJECTED VALUES OF THE PARAMETERS.
C     THE  OUTPUT CONSISTS OF THE MEASURE DISPATCH RELIABILITY FOR ALL DELAYS,I.E
C   OVER  5 MINUTES, FOR DELAYS OVER 15 MINUTES AND FOR DELAYS OVER 60 MINUTES.
C   A CONFIDENCE LEVEL IS USED TO DETERMINE THE LOWER AND UPPER CONFIDENCE
C   LIMITS ON THE MEASURE OF DR.  FOR EXAMPLE FOR 90 PERCENT=100(1-ALPHA)PERCENT
C   ALPHA /2=.05 AND SINCE P(Z>1.645)=.05,Z(.05)=1.645.THEN FOR THE CONFIDENCE
C STATEMENT: P(F)(DRBAR-(Z*SIG/SQRT(N))<DR<DRBAR+(Z*SIG/SQRT(N))=90%.
      DIMENSION M(35),RD(35),ALOW(35),AD(35),DRO(35),R15(35),DRL(35),
     1D15(35),DRO15(35),R60(35),DR6L(35),D60(35),DRO60(35),XM(35),
     2SIG(35)
      INTEGER RNDSD
      IX=1
      DA=0.0
      DA15=0.0
      DA60=0.0
      NCARD = 1
      NPRNT = 3
C     READ PARAMETERS
      READ (NCARD, 101) ZE, CONF, ITRYS, RNDSD, NO, KO
101   FORMAT ( 2E10.2, 4I10 )
      WRITE(NPRNT,106)ITRYS,CCNF
 106  FORMAT(1H1,T60,'DISPATCH RELIABILITY',//,T55,'NUMBER OF TRIALS IS'
     7,I6,' (= N)',//,                         T30,'SIMULATED DR COLUMNS
     1 SHOW THE LOWER CONFIDENCE LIMIT USING THE',F6.2,' LEVEL OF CONFID
     2ENCE',///,T10,'MONTH  DPTRS  DELAY>5     DR       SIM DR     DR EST
     3 DELAY>15     DR       SIM DR     DR EST  DELAY>60    DR    SIM DR    DR
     4 EST')
C     READ MONTH DATA
1     READ (NCARD, 102) MO, DEL, DEP, XMU, SIGMA,DEL15,DEL60
102   FORMAT ( I10, 6E10.3 )
      M(IX)=MO
      XM(IX)=XMU
      SIG(IX)=SIGMA
C     IF THE BLANK CARD AFTER ALL MONTH CARDS IS DETECTED, FINAL RESULTS ARE
C     PRINTED AND PROGRAM ENDED.
      IF (MO) 2, 3, 2
  3   IX=IX-1
      JO=NO+1
      WRITE(NPRNT,200)
200   FORMAT(1H1,T53,'ANALYSIS OF DISPATCH RELIABILITY',//,T30,'SIMULATE
     6D DR COLUMNS SHOWS LOWER CONFIDENCE LIMIT USING NUMBER OF DEPARTUR
     7ES AS N'///,T4,'MONTH',T14,
     2'DR>5',T24,'SIM DR',T34,'DR EST',T44,'DR-DR EST',T54,
     3'DR>15',T64,'SIM DR',T74,'DR EST',T84,'DR-DR EST',T94,
     4'DR>60',T104,'SIM DR',T114,'DR EST',T123,'DR-DR EST')
      WRITE(NPRNT,300)(M(J),RD(J),ALOW(J),AD(J),DRO(J),R15(J),DRL(J),
     1D15(J),DRO15(J),R60(J),DR6L(J),D60(J),DRO60(J),J=1,NO)
300   FORMAT(1H0,5X,I3,3X,12F10.6)
      WRITE(NPRNT,320)(M(J),ALOW(J),AD(J),DRL(J),D15(J),DR6L(J),D60(J),
     1J=JO,KO)
 320  FORMAT(1H0,5X,I3,13X,2F10.6,20X,2F10.6,20X,2F10.6)
      WRITE(NPRNT,350)
350   FORMAT(1H1,T14,'ESTIMATED VALUES OF MU AND SIGMA',//,T19,'MO',T34,
     5'MU',T48,'SIGMA',/)
400   FORMAT(1H0,15X,I3,2F10.5)
      WRITE(NPRNT,400)(M(JI),XM(JI),SIG(JI),JI=1,KO)
      DO 500 J=1,NO
      DA=DRO(J)+DA
      DA15=DA15+DRO15(J)
      DA60=DA60+DRO60(J)
 500  CONTINUE
      WRITE(NPRNT,551)DA,CA15,CA60
 551  FORMAT(1H1,T5,'TOTAL DIFFERENCES OF DR AND DR ESTIMATES',//T10,3F1
     20.6)
      CALL EXIT
  2     CONTINUE
```

```
C      REINITIALIZE VALUES.
       IOVR5 = 0
       IOVR15 = 0
       IOVR60 = 0
       SIGN  = 1.0
       FACTOR = DEL / DEP
       RDR=1.0-FACTOR
       RD(IX)=RDR
C      COMPUTE ACTUAL OR VALUES THROUGH THE LAST MONTH OF DATA. THEN COMPUTE FOR
C      PREDICTED MONTHS.
       IF(MO-NO)321,321,322
321    CONTINUE
       RDR15=1.-(DEL15/DEP)
       R15(IX)=RDR15
       RDR60=1.-(DEL60/DEP)
       R60(IX)=RDR60
322    CONTINUE
C      COMPUTE DELAY DATA FOR A GIVEN NUMBER OF TRIALS.
       DO 10  I = 1, ITRYS
       JX = RNDSD
       CALL RANDU (JX, RNDSD, RNDNO)
       IF (RNDNO - FACTOR) 11, 11, 10
C  DETERMINE OF A DELAY OCCURS FOR THIS DEPARTURE(ITRY).
11     JX = RNDSD
       CALL RANDU (JX, RNDSD, XI )
C  ALLOWANCE FOR LOGUNIFORM PORTIONS.
       IF(XI-.05)544,545,545
 545   IF(XI-.95)546,546,548
 544   R=XI/.05
       DELTM=EXP((R*1.7)-.7)
       GO TO 550
 548   DELTM=EXP(XI*5.75)
       GO TO 550
 546   CONTINUE
       Q = XI
       IF (Q - 0.5) 12, 13, 13
13     Q = 1.0 - XI
C      HASTINGS APPROXIMATION OF NORMAL DISTRIBUTION.
       SIGN = -1.0
12     ETA = SQRT( -ALOG( Q*Q ))
       Z = 2.515517 + 0.802853*ETA + 0.010328*ETA*ETA
       Z = Z / (1.0 + 1.432788*ETA + 0.189269*ETA*ETA + 0.001308*ETA*ETA
      1      *ETA        )
       Z = SIGN*(ETA - Z)
       SIGN = 1.0
C      THE SIMULATED DELAY TIME IS INTERPRETED IN MINUTES.
       DELTM = EXP(SIGMA * Z + XMU )
C  IF ALL ORIGINAL DT SUBTRACTED 5.5 MINUTES PRIOR TO FITTING THEN THE
C  SIMULATED TIME MUST ADD BACK THIS 5.5 MINUTES.
 550   CONTINUE
       DELTM=DELTM+5.5
       IF (DELTM - 5.5) 11, 14, 14
14     IOVR5 = IOVR5 + 1
       IF (DELTM - 15.5) 16, 15, 15
15     IOVR15 = IOVR15 + 1
16     IF (DELTM - 60.5) 10, 17, 17
17     IOVR60 = IOVR60 + 1
10     CONTINUE
       RTRYS=FLOAT(ITRYS)
       IF(MO-NO)323,323,324
324    X15=FLOAT(IOVR15)
       DEL15=DEP*(X15/RTRYS)
       RDR15=1.-(DEL15/DEP)
       R15(IX)=RDR15
       X60=FLOAT(IOVR60)
       DEL60=DEP*(X60/RTRYS)
       RDR60=1.0-(DEL60/DEP)
       R60(IX)=RDR60
       X5=FLOAT(IOVR5)
       DEL=DEP*(X5/RTRYS)
       RD(IX)=1.0-(DEL/DEP)
323    CONTINUE
C      CALCULATE CONFIDENCE INTERVAL FOR THE THREE CATEGORIES.
       X5=FLOAT(IOVR5)
       ADR=1.0-(X5/RTRYS)
       AD(IX)=ADR
```

```
      HOLD=ZE*SQRT((ADR*(1.0-ADR))/RTRYS)
      ALOWDR = ADR - HOLD
      HOL =ZE*SQRT((ADR*(1.0-ADR))/DEP)
      ALOW(IX)=ADR-HOL
      AUPDR = ADR + HOLD
      DR15 = FLOAT(ITRYS - ICVR15) / RTRYS
      D15(IX)=DR15
      HOLD=ZE*SQRT((DR15*(1.0-DR15))/RTRYS)
      DR15LO = DR15 - HOLD
      HOL =ZE*SQRT((DR15*(1.C-DR15))/DEP)
      DRL(IX)=DR15-HOL
      DR15UP = CR15 + HOLD
      DR60 = FLCAT(ITRYS - IOVR60) / RTRYS
      D60(IX)=DR60
      HOLD=ZE*SQRT((DR60*(1.0-DR60))/RTRYS)
      DR60LO = DR 60 - HOLD
      DR60UP = DR60 + HOLD
      DR60UP = DR60 + HOLD
      DR60UP = DR60 + HOLD
      HOL =ZE*SQRT((DR60*(1.C-DR60))/DEP)
      DR6L(IX)=DR60-HOL
      IF(MO-NC) 301,301,312
C     SET DR EQUAL TO THE ESTIMATE WHEN PREDICTING SINCE THERE IS NO ACTUAL DR.
  312 AD(IX)=1.-(DEL/DEP)
      D15(IX)=1.-(DEL15/DEP)
      D60(IX)=1.-(DEL60/DEP)
  301 CONTINUE
C     FIND THE DIFFERENCE BETWEEN THE ACTUAL DR AND THE ESTIMATED DR IN EACH
C     CATEGORY,>5 MINUTES,>15 MINUTES AND >60 MINUTES.
      DRO(IX)=RDR-ADR
      DRO15(IX)=RDR15-DR15
      DRO60(IX)=RDR60-DR60
      IX=1+IX
C     WRITE RESULTS.
      IF(MO-NC)100,100,112
  100 WRITE(NPRNT,107)MO,DEP,DEL,RDR,ALOWDR,ADR,DEL15,RDR15,DR15LO,DR15,
     1DEL60,RDR60,DR60LO,DR60
  107 FORMAT(1H0,10X,I3,2F7.0,3F10.6,F9.0,3F10.6,F6.0,3F10.6)
      GO TO 1
  112 WRITE(NPRNT,113)MO,DEP,DEL,ALOWDR,ADR,DEL15,DR15LO,DR15,DEL60,
     1DR60LO,DR60
  113 FORMAT(1H0,10X,I3,2F7.C,10X,2F10.6,2X,F7.0,10X,2F10.6,F6.0,10X,2F1
     10.6)
      GO TO 1
      END
```

APPENDIX D

PROGRAM LIST FOR SIMULATION OF

OBSERVED AVAILABILITY

```
C     LOGNORMAL DT-WEIBULL TBD SIMULATION
C     AND CONFIDENCE ASSESSMENT OF AO:
C     PROGRAM REQUIRES TWO INPUT CARDS:
C
C     CARD 1
C       COL           DATA
C       1-10          INTEGER VALUE FOR RANDOM NUMBER
C                     GENERATOR SEED, NOT A POWER OF 2.
C
C     CARD 2
C       COL           DATA
C       1-4           INTEGER NUMBER OF TRIALS, L=5000
C       5-11          REAL VALUE OF MU, TYPE IN DECIMAL
C       12-18         REAL VALUE OF SIGMA, TYPE IN DECIMAL
C       19-25         REAL VALUE OF DELTA, TYPE IN DECIMAL
C       26-32         REAL VALUE OF BETA, TYPE IN DECIMAL
C       33-39         DELAY IN MINUTES, TYPE IN DECIMAL
C      45-52          BLOCK TIME FOR DR
C       67-72         MONTH AND YEAR OF DATA
C
C
      DIMENSION NA(5000),IN(122),PLT(121),P(10),A(13)
      DATA PT/'+'/
      DATA BLNK/' '/
    2 READ(1,15)IX
      READ(1,5)ITRILS,AMUHAT,SIGMA,DELTA,BETA,DELAY,BLOCK,MO,IYR
      NTTWO=0
      TTBD=0.0
      DELSPC=6./60.
      XPEC=16./60.
      GSPEC=61./60.
      IF(ITRILS.EQ.0)GO TO 846
C
C     IF AMUHAT IS INPUT IN TERMS OF MINUTES AND DELTA IS IN TERMS OF
C     HOURS THIS ROUTINE IS NECESSARY FOR COMPATIBILITY .
      CONVT=EXP(AMUHAT)/60.
      AMUHAT=ALOG(CONVT)
C
    5 FORMAT (I4,2F7.5,F7.4,F7.5,F7.0,5X,F8.6,14X,1A4,I2)
   15 FORMAT(I10)
C     BLANKS OUT PLOT LINE
      DO 1 I=1,121
      PLT(I)=BLNK
    1 CONTINUE
C     AO CALCULATIONS
      DO300 M=1,ITRILS
C     RANDU IS IBM SYSTEM RANDOM NUMBER GENERATOR
      CALL RANDU(IX,IY,XI)
      IX=IY
C     TONE IS DERIVED FROM WEIBULL DISTRIBUTION FUNCTION
      TONE=DELTA*((-ALOG(1.-XI))**(1./BETA))
   73 CONTINUE
      CALL RANDU(IX,IY,XI)
      IX=IY
      Q=XI
C     REQUIRES RANDOM NUMBER TO BE LESS THAN .5 IN
C     ORDER TO CALCULATE ONLY UPPER ONE-HALF OF
C     THE NORMAL DISTRIBUTION
      SIGN=1.
      IF(Q.LE.0.5) GO TO 150
      Q=1.-XI
C     SIGN UTILIZES THE SYMMETRY OF THE NORMAL DISTRIBUTION
      SIGN=-1.
  150 ETA=SQRT(-ALOG(Q*Q))
      Z=2.515517+0.802853*ETA+0.010328*ETA*ETA
      Z=Z/(1.+1.432788*ETA+0.189269*ETA*ETA+0.001308*ETA**3.)
      Z=SIGN*(ETA-Z)
      SIGN=1.
      TTWO=EXP(SIGMA*Z+AMUHAT)
       NTTWO=NTTWO+1
      IF(TTWO.LT.DELSPC) GO TO 73
      IF(DELAY.EQ.60.) GO TO 163
      IF(DELAY.EQ.15.) GO TO 164
      GO TO 169
  163 IF(TTWO.GE.GSPEC) GO TO 169
```

```
      CALL RANDU(IX,IY,XI)
      IX=IY
      TONE=TONE+(DELTA*((-ALCG(1.-XI))**(1./BETA)))
      GO TO 73
  164 IF(TTWO.GE.XPEC) GO TO 169
      CALL RANDU(IX,IY,XI)
      IX=IY
      TONE=TONE+(DELTA*((-ALCG(1.-XI))**(1./BETA)))
      GO TO 73
  169 CONTINUE
      IF(BLOCK.EQ.0.0) GO TO 170
      TTBD=TONE+TTBD
      XT=NTTWO
      TONE=TONE/BLOCK
      TTWO=1.0
  170 CONTINUE
      AOH=TONE/(TONE+TTWO)
C     INTEGER ARRAY UNTILIZED TC CONSERVE CORE
      NA(M)=IFIX(AOH*100000+.5)
  300 CONTINUE
C     SORT ROUTINE, ASCENDING
      IITRIL=ITRILS-1
      DO 500 I=1,IITRIL
      ISM=10000000
      II=I+1
      NO=1
      DO 400 J= I,ITRILS
      IF(ISM.LE.NA(J)) GO TO 400
      ISM=NA(J)
      NO=J
  400 CONTINUE
      IF(NO.EQ.1) GO TO 460
      DO 450 KL=II,NO
      K=NO+II-KL
      KK=K-1
      NA(K)=NA(KK)
  450 CONTINUE
  460 CONTINUE
      NA(I)=ISM
  500 CONTINUE
C     EVALUATES AO VALUES
C     SETS VALUES TO TOP AND BOTTOM OF RANGE
      ALG=NA(ITRILS)+.5
      ASM=NA(1)-.5
      AINCR=(ALG-ASM)/12.
      ISLT=IFIX(ASM)
      SUMT=0.
      APCT=FLOAT(ITRILS)/10.
C     ZEROS OUT PERCENTAGE ARRAY
      DO 520 I=1,122
  520 IN(I)=0
      K=0
      I=0
  530 K=K+1
  540 I=I+1
      IF(I.GT.ITRILS) GO TO 600
      IF(NA(I).GT.ISLT) GO TO 550
C     PUTS NUMBER CF OCCURRANCES IN ARRAY IN
      IN(K)=IN(K)+1
      GO TO 540
  550 CONTINUE
      SUMT=SUMT+IN(K)
C     THERE ARE 10 PLOTTING SLOTS WITHIN EACH INCREMENT
      ISLT=ISLT+AINCR/10.
  590 I=I-1
      GO TO 530
  600 CONTINUE
C     PUTS PERCENTAGES OF OCCURANCES IN ARRAY IN
      SUMT=SUMT+IN(K)
      CUM=0.
      DO 700 I=1,122
      CUM=CUM+FLOAT(IN(I))/FLOAT(ITRILS)
      IN(I)=IFIX((CUM+.005C)*100)
  700 CONTINUE
C     WRITES GRAPH HEADINGS
      IF(BLOCK.NE.0.0) GO TO 7C6
```

```
      WRITE(3,701)MO,IYR,ITRILS
      GO TO 707
 706  WRITE(3,708)MO,IYR,ITRILS
 708  FORMAT(1H1,40X,'CUMULATIVE DISTRIBUTION OF DR FOR',1X,1A4,I2,
     1/,47X,'TRIALS = ',I8)
 707  CONTINUE
 701  FORMAT(1H1,40X,'CUMULATIVE DISTRIBUTION OF AO FOR',1X,1A4,I2,
     1/,47X,'TRIALS = ',I8)
      WRITE(3,702)BETA,DELTA,SIGMA,AMUHAT
 702  FORMAT(1X,32X,'BETA = ',F10.5,'     DELTA = ',F10.5,
     1/,33X,'SIGMA = ',F10.5,'     MU= ',F10.5)
C     PLOTTING ROUTINE
      IDOT=122
      DO 800 I=1,50
      NUM=0
      IPCT=102-2*I
 705  IF(IN(IDOT).LT.IPCT) GO TO 710
      NDOT=IDOT-1
      PLT(NDOT)=PT
      IDOT=IDOT-1
      NUM=NUM+1
      GO TO 705
 710  INTGER = IFIX((FLOAT(I)-1.)/10.)*10
      II=I-1
      IF(INTGER.EQ.II) GOTO 730
      WRITE(3,720) PLT
 720  FORMAT(1X, 7X,'*',121A1)
      GOTO 750
 730  WRITE(3,725)IPCT,PLT
 725  FORMAT(2X,I4,'X -',121A1)
 750  IF(NUM.EQ.0) GOTO 800
      DO 760 K=1,NUM
      KDOT=IDOT+K-1
      PLT(KDOT)=BLNK
 760  CONTINUE
 800  CONTINUE
C     WRITES BOTTOM LINE OF GRAPH
      WRITE(3,805)
 805  FORMAT(4X,'0%     /',12('*********/'))
      DO 810 I=1,13
      A(I)=(ASM + AINCR*(I-1))/100000.
 810  CONTINUE
C     WRITES VALUES FOR GRAPH
      WRITE(3,820)(A(L),L=1,13)
 820  FORMAT(1X,5X,12(F6.4,4X),F5.1)
      DO 830 I=1,10
      P(I)=100.-I*10.
 830  CONTINUE
C     WRITES VALUES FOR CUMULATIVE PERCENTAGES
      WRITE(3,835)(P(L),L=1,10)
 835  FORMAT(///,1X,'CUM PCT ',10F10.1)
      DO 840 I=1,10
      IA=(FLOAT(ITRILS)/10.)*I
      A(I)=FLOAT(NA(IA))/100000.
 840  CONTINUE
      IF(BLOCK.NE.0.0) GO TO 841
      WRITE(3,845)(A(L),L=1,10)
      GO TO 842
 841  WRITE(3,847)(A(L),L=1,10)
 847  FORMAT(1X,'DR',6X,10F10.5)
 842  CONTINUE
 845  FORMAT(1X,'AO',6X,10F10.5)
      GO TO 2
 844  CONTINUE
 846  CONTINUE
      END
```

APPENDIX E

PROGRAM LIST FOR ANALYSIS OF

TIMES-BETWEEN-DELAYS

```
C    PROGRAM WEIBULL
C
C    PROGRAM INPUT IS A SET OF TIME BETWEEN DELAYS(TBD).  THE PROGRAM
C    NULL HYPOTHESIS(HO) IS THAT THE DATA WERE DRAWN FROM THE FAMILY
C    OF WEIBULL DISTRIBUTIONS.  THE WEIBULL DISTRIBUTION HAS PARAMETERS
C    DELTA(SCALE) AND BETA(SHAPE) AND MU(LOCATION). SETTING MU TO 0 THE
C    P.D.F. IS F(TBD|DELTA, BETA)=(BETA*TBD**BETA-1/DELTA**BETA)*
C    EXP(-(TBD/DELTA)**BETA).  ITS D.F. IS F(TBD)=1-EXP(-(TBD/DELTA)**BETA).
C    BY TAKING THE NATURAL LOGARITHMS OF BOTH SIDES -LNR=(TBD/DELTA)**BETA
C    IS OBTAINED.  TAKING NATURAL LOGARITHMS OF BOTH SIDES AGAIN THE FORM
C    Y=BX+C IS OBTAINED WHERE Y=LN(-LNR),X=LN(TBD) AND C=-BETA*LN(DELTA).
C    THUS LEAST SQUARES CALCULATION MAY BE USED TO ESTIMATE THE PARAMETERS
C    BETA AND DELTA.  DELTA=EXP(-C/BETA).
C    A PLOT IS OBTAINED BY USING A VERTICAL SCALE OF LN(-LNR) AND
C    LOGARITHMIC SCALE IN THE HORIZONTAL FOR LN(TBD). EACH TBD IS
C    PLOTTED AGAINST A CORRESPONDING EMPIRICAL PLOT POINT. FI=I-.5/N.
C     ALLOW THE PROGRAM TO READ SEVERAL SETS OF TBD,DO VARIOUS SELECTIONS
C      PER SET AND OUTPUT RESULTS.
C*********************************************************************************
      DIMENSION        TBD(1000),FI(1000),R(1000),ALNR(1000),TBDLN(1000
     1),XY(1000),SQX(1000),FY(1000)
      DIMENSION  Y(1000),ECP(100),XCHISQ(100),IOBX(100),CUMCHI(100)
      DIMENSION KLASES(100)
      DIMENSION CUQCHI(100)
      DIMENSION LOW(50),LP(50)
      DIMENSION DUM(1000)
      DIMENSION IDUM(1000)
      READ(5,6)NUMR
    6 FORMAT(I3)
      DIMENSION COPY1(1000),COPY2(1000),COPY3(1000)
C  READ THE PERCENTAGE FOR CLASS INTERVALS FOR A CHI-SQUARE CALCULATION.
      READ(5,5) XBYX
C     PROGRAM READS A BLANK AT END OF A TBD SET.THEN IF NUMR.GE.2 IT
C     LOOKS FOR ANOTHER SET.  ALL SETS ARE FOLLOWED BY A BLANK,A SELECTION
C     3 GRAPH CARDS, ETC. UNTIL A BLANK OR GRAPH CARD IS READ WHICH STOPS
C     THE SELECTION. IN THE CASE OF SUCCESSIVE TBD SETS, INSERT A BLANK.
C     THE TBD SET, ETC.
C*********************************************************************************
      DO 2000 JK=1,NUMR
    5 FORMAT(F8.5)
C  READ THE TBD
      DO 20 N=1,1000
      READ(5,10) IDUM(N),TBD(N)
   10 FORMAT(I5,F9.4)
      IF(TBD(N)) 20,30,20
   20 CONTINUE
   30 LOL=1
      N=N-1
C  ASSIGN A CORRESPONDING EMPIRICAL PLOT POINT TO EACH OBSERVATION OF TBD.
      DO 50 L=2,N
      IF(TBD(LOL).EQ.TBD(L)) GO TO 50
      LLL=L-1
      C=LOL
      CC=L-1
      XNO=N
      FA=((C-.5)+(CC-.5))/(2.*XNO)
      DO 60 IJK= LOL,LLL
   60 FI(IJK)=FA
      LOL=L
   50 CONTINUE
      LLL=L
      C=LOL
      CC=L
      FA=((C-.5)+(CC-.5))/(2.*XNO)
      DO 70 IJK=LOL,LLL
   70 FI(IJK)=FA
C  ASSIGN A CORRESPONDING NATURAL LOGARITHM FOR EACH TBD.
      DO 80 I=1,N
      R(I)=1.-FI(I)
      ALNR(I)=ALOG(-ALOG(R(I)))
      TBDLN(I)=ALOG(TBD(I))
   80 CONTINUE
C*********************************************************************************
C  A VARIABLE NUMBER OF TBD MAY BE READ INTO THE PROGRAM.  ALL ARE ARRAYED
C  PRIOR TO ANY CENSORING.
C*********************************************************************************
```

```
C  ARRANGEMENT OF THE DATA DECK: CARD 1 IS F8.5 FOR XBYX. NEXT IS
C    THE TBDS. ONE PER CARD, FOLLOWED BY A BLANK CARD.  THEN A VARIABLE
C    NUMBER OF SELECTICN CARDS CONTAINIG XLO AND UP IN 2I3.FOLLOWED
C    BY A BLANK. LASTLY ARE THE PARAMETER CARDS FOR THE PLOT.
C*****************************************************************************
C    THIS LOOP SELECTS A VARIABLE NUMBER OF TBD RANGES FOR ESTIMATION OF
C  BETA AND DELTA BY LEAST SQUARES AND FOR A CHI-SQUARE CALCULATION.
C*****************************************************************************
      DO 90 NP=1,25
      READ(5,40) LOW(NP),LP(NP)
      LLO=LOW(NP)
      IP=LP(NP)
 40   FORMAT(2I3)
      IF(LOW(NP).EQ.0)GO TO 55
      WRITE(6,63)LLO,IP
 63   FORMAT(1H1,T5,'TABULAR INFORMATION FOR SELECTION OF TIMES BETWEEN
     1DELAY FROM ORDER NUMBER ',I3,' TO ',I3/T5,'FOLLOWED BY A WEIBULL
     2PROBABILITY X LOGARITHMIC PLOT AND CHI-SQUARE DATA.')
      WRITE(6,65)
 65   FORMAT(1H0,T5,'ORDER',T18,'TBD',T26,'LN(TBD)',T39,'FI',T48,'R=1.-F
     1I',T57,'LN(-LN(R))',T70,'Y=BX+C',T79,'FY=1-EXP(-EXP(Y))')
C    CREATE AN ARRAY FOR X*Y WHERE X=LN(TBD) AND Y=LN(-LN(R)).
      SUMY=0.0
      SUMX=0.0
      SUMX2=0.0
      SUMXY=0.0
      SMTBD=0.0
 45   DO 110 I=LLO,IP
      XY(I)=TBDLN(I)*ALNR(I)
      SQX(I)=TBDLN(I)**2
C    SUM APPROPRIATE ARRAYS FOR LEAST SQUARES ESTIMATORS.
      SUMY=SUMY+ALNR(I)
      SUMX=SUMX+TBDLN(I)
      SUMX2=SUMX2+SQX(I)
      SUMXY=SUMXY+XY(I)
      SMTBD=SMTBD+TBD(I)
 110  CONTINUE
      DIF=IP-LLO+1
C THIS ROUTINE CALCULATES AN EPSTEIN STATISTIC TO TEST THE FOLLOWING HYPOTHESES:
C                HO: BETA.EQ.1
C                HA: BETA.NE.1
C DATA REQUIRED ARE N, SUM OF TDB, THE LN(TBD).AND SUM OF LN(TBD)
      A=ALOG(SMTBD/DIF)
      B=SUMX/DIF
      C=1.+((DIF+1.)/(6.*DIF))
      EPS=2.*DIF*(A-B)/C
      BETA=(DIF*SUMXY-SUMX*SUMY)/(DIF*SUMX2-SUMX**2)
      CEPT=(SUMX2*SUMY-SUMX*SUMXY)/(DIF*SUMX2-SUMX**2)
      DELTA=1./EXP(CEPT/BETA)
      DO 126 I=LLO,IP
      Y(I)=BETA*TBDLN(I)+CEPT
      FY(I)=1.-EXP(-EXP(Y(I)))
      WRITE(6,115)I,TBD(I),TBDLN(I),FI(I),R(I),ALNR(I),Y(I),FY(I)
 115  FORMAT(1H ,T5,I3,T12,F10.4,T23,F10.5,T34,F10.5,T45,F10.5,T56,
     1F10.5,T67,F10.5,T78,F10.5)
 126   CONTINUE
C    USING THE LEAST SQUARES ESTIMATORS A CHI-SQUARE GOODNESS-OF-FIT TEST
C    MAY BE PERFORMED BY COUNTING OBSERVATIONS WITHIN INTERVALS DIVIDED BY
C    NATURAL LOGARITHMS ALONG THE HORIZONTAL AXIS.  THE COUNT BEGINS
C    WITH TBDLN(XLO).  ITS CORRESPONDING POINT IS OBTAINED BY SOLVING FOR Y IN
C    Y=BX+C.FOR INCREMENTING, Y IS CONVERTED TO A PROBABILITY,
C    FY=1-EXP(-EXP(Y)).  A PERCENTAGE INTERVAL(XBYX) DETERMINES THE
C    SUBSEQUENT BOUNDARY POINTS.
      IF(LP(NP).EQ.N) FY(IP)=1.
      IF(LLO.EQ.1)FY(LLO)=0.0
      BK=FY(LLO)+XBYX
      BY=ALOG(-ALOG(1.-BK))
      XLNBD=(BY-CEPT)/BETA
      IXX=1
      NX=((FY(IP)-FY(LLO))/XBYX)+1.
C    CHECK TO SEE IF THE NUMBER OF OBSERVATIONS EXCEEDS THE NUMBER OF
C    ORIGINAL INTERVALS AS A FUNCTION OF XBYX.
      IF(NX.GT.N)GO TO 1201
      GO TO 1203
 1201 WRITE(6,1204)
 1204 FORMAT(1H0,30X,' NEED TO INCREASE THE XBYX')
 1203 CONTINUE
```

```
C    INITIALIZE THE ARRAYS TO ZERO.
      DO 120 I=1,NX
      ECP(I)=0.0
      XCHISQ(I)=0.0
      IOBX(I)=0
 120  CONTINUE
C  THIS LOOP COUNTS OBSERVATIONS OF TBDLN WITH LOG BOUNDARIES FOR NX CLASSES.
      DO 150 I=LLO,IP
 130  IF(TBDLN(I).LT.XLNBD) GO TO 140
      BK =BK+XBYX
      IF(BK.GE.1.)BK=.999999
      BY=ALOG(-ALOG(1.-BK))
      XLNBD=(BY-CEPT)/BETA
  134 IXX=IXX+1
      GO TO 130
 140  IOBX(IXX)=IOBX(IXX)+1
 150  CONTINUE
C    CALCULATE EXPECTED FREQUENCY IN THE INTERVALS.
      XNO=N
      IXX=1
      BK=FY(LLO)
      IF(LLO.NE.1)GO TO 151
      IF(LLO.EQ.1) BK=0.0
      IF(BK.EQ.0.0) BC=.01
      BY=ALOG(-ALOG(1.-BC))
      GO TO 152
 151  BY=ALOG(-ALOG(1.-BK))
 152  CONTINUE
      XLNBD=(BY-CEPT)/BETA
      XXBYX=0.0
      DO 160 IX=1,NX
      IF(BK+XBYX.GT.FY(IP))XXBYX=FY(IP)-BK
      IF(IP.EQ.N.AND.BK+XBYX.GT.1.)XXBYX=1.-BK
      ECP(IX)=XBYX*XNO
      IF(IX.EQ.NX)ECP(IX)=XXBYX*XNO
      BK=BK+XBYX
 160  CONTINUE
      TOTCHI=0.0
      BK=FY(LLO)
      BX=FY(LLO)
      IF(LLO.EQ.1)BK=0.0
      IF(LLO.EQ.1)BX=0.0
      IF(BK.NE.0.0)GO TO 161
      IF(BK.EQ.0.0) BC=.01
      BY=ALOG(-ALOG(1.-BC))
      GO TO 162
 161  BY=ALOG(-ALOG(1.-BK))
 162  CONTINUE
      XLNBD=(BY-CEPT)/BETA
C    WRITE NEW PAGE HEADING AND OTHER INFORMATION.
      WRITE(6,170)
 170  FORMAT(1H1,50X,'TBD ANALYSIS')
      WRITE(6,171)CEPT,BETA,DELTA
 171  FORMAT(1H0,T10,'LEAST SQUARES ESTIMATORS;  INTERCEPT = ',F10.5,
     1'    BETA = ',F10.5,'    AND DELTA = ',F10.5//)
      WRITE(6,169)
 169  FORMAT(1H0,T5,'CLASS INTERVAL',T32,'CLASS INTERVAL',T55,
     1'EXPECTED',T65,'OBSERVED',T75,'INDIVIDUAL',T88,'CUMULATIVE'/T5,
     2'IN PERCENT',T32,'IN LOGARITHMS',T55,'FREQUENCY',T65,
     3'FREQUENCY',T75,'CHI-SQUARE',T88,'CHI-SQUARE')
C    WRITE OUT CLASS INTERVAL IN PERCENT, INTERVAL IN LOGS, EXPECTED
C    FREQUENCY, OBSERVED FREQUENCY, INDIVIDUAL CHI-SQUARE AND
C    CUMULATIVE CHI-SQUARE, ASCERTAINING THAT EACH CLASS HAS
C    AT LEAST FIVE OBSERVATIONS.
      KLAS=0
      DO260 L=1,NX
      IF(IOBX(L).GE.5)GO TO 230
      IF(L.EQ.NX.AND.IOBX(L).LT.5) GO TO 6500
      GO TO 6600
 6500 OBSVD=IOBX(L-1)+IOBX(L)
      EXPD=ECP(L-1)+ECP(L)
      BX=BX+XBYX
      IF(BX.GE.1.)BX=.999999
      BY=ALOG(-ALOG(1.-BX))
      XLNBX=(BY-CEPT)/BETA
      GO TO 6700
 6600  CONTINUE
```

```
      IF(L.EQ.NX)GO TO 240
220   ECP(L+1)=ECP(L+1)+ECP(L)
      IOBX(L+1)=IOBX(L+1)+IOBX(L)
         BX=BX+XBYX
      BY=ALOG(-ALOG(1.-BX))
      XLNBX=(BY-CEPT)/BETA
      GO TO 260
230   IF(L.EQ.NX) GO TO 240
      IF(IOBX(L+1).EQ.0) GO TO 220
240   IF(ECP(L).EQ.0.0) GO TO 250
      OBSVD=IOBX(L)
      EXPD=ECP(L)
6700  CONTINUE
      XCHISQ(L)=((EXPD-OBSVD)**2)/EXPD
250   TOTCHI=TOTCHI+XCHISQ(L)
      CUMCHI(L)=TOTCHI
      BX=BX+XBYX
      IF(BX.GE.1.)BX=.99999999
      BY=ALOG(-ALOG(1.-BX))
      XLNBX=(BY-CEPT)/BETA
      IF(L.EQ.NX)BX=FY(IP)
      IF(IP.EQ.N.AND.L.EQ.NX)BX=1.0
      WRITE(6,270)BK,BX,XLNBD,XLNBX,ECP(L),IOBX(L),XCHISQ(L),CUMCHI(L)
      KLAS=KLAS+1
      IF(L.EQ.NX)KLASES(NP)=KLAS
      BK=BX
      IF(BK.GE.1.)BK=.99999999
      BY=ALOG(-ALOG(1.-BK))
      XLNBD=(BY-CEPT)/BETA
      IF(L.EQ.NX)CUQCHI(NP)=CUMCHI(NX)
265   CONTINUE
260    CONTINUE
270   FORMAT(1H0,F8.5,' TO ',F8.5,3X,F10.5,' TO ',F10.5,5X,F10.4,3X,I3,
     13X,F10.5,3X,F10.5)
      WRITE(6,180)LLO,IP,TBD(LLO),TBD(IP),TBDLN(LLO),TBDLN(IP),FY(LLO),
     1FY(IP),KLAS,XBYX,CUMCHI(NX)
180   FORMAT(1H0,T7,'RANGE OF',T28,'RANGE OF',T52,'RANGE OF',T72,
     1'PERCENT',T87,'NUMBER OF',T98,'CLASS',T107,'CHI-SQUARE'/T7,
     2'ORDER',T28,'TBD',T52,'LN(TBD)',T72,'COVERAGE',T87,'CLASSES',T98,
     3'SIZE'//T5,I3,' TO ',I3,T18,F9.4,' TO ',F9.4,T40,F10.5,' TO ',
     4F10.5,T66,F8.5,' TO ',F8.5,T89,I3,T95,F8.5,T105,F10.5)
      IDIF=DIF
      DO 1000 ICOPY=1,IDIF
      COPY1(ICOPY)=TBD(LLO +ICOPY-1)
      COPY2(ICOPY)=ALNR(LLO +ICOPY-1)
      COPY3(ICOPY)=DUM(LLO +ICOPY-1)
1000  CONTINUE
      CALL PLOT(COPY1,3,COPY2,0,COPY3,0,IDIF,1,1,3,2,1,1)
      WRITE(6,410)EPS,DIF
 410  FORMAT(1H1,T20,'THE EPSTEIN STATISTIC IS',F10.5,'WITH K =',F5.0//)
90    CONTINUE
55    CONTINUE
      WRITE(6,281)
281   FORMAT(1H0,45X,'SUMMARY FOR WEIBULL SELECTION'/)
      WRITE(6,280)
280   FORMAT(1H0,T7,'RANGE OF',T28,'RANGE OF',T52,'RANGE OF',T72,
     1'PERCENT',T87,'NUMBER OF',T98,'CLASS',T107,'CHI-SQUARE'/T7,
     2'ORDER',T28,'TBD',T52,'LN(TBD)',T72,'COVERAGE',T87,'CLASSES',T98,
     3'SIZE'//)
      NPI=NP-1
      DO 300 IW=1,NPI
      LLO=LOW(IW)
      IP=LP(IW)
      WRITE(6,380)LLO,IP,TBD(LLO),TBD(IP),TBDLN(LLO),TBDLN(IP),FY(LLO),
     1FY(IP),KLASES(IW),XBYX,CUQCHI(IW)
300   CONTINUE
2000  CONTINUE
380   FORMAT(1H0,T5,I3,' TO ',I3,T18,F9.4,' TO ',F9.4,T40,F10.5,' TO ',
     4F10.5,T66,F8.5,' TO ',F8.5,T89,I3,T95,F8.5,T105,F10.5)
      STOP
      END
```

APPENDIX F

PROGRAM LIST FOR ANALYSIS OF DELAY TIMES

```
C    PROGRAM LOG-NORMAL
C*******************************************************************************
C    THIS PROGRAM PROVIDES FOR BOTH A CHI-SQUARE GOODNESS OF FIT(GOF)
C    TEST AND A LILLIEFORS KOLMOGOROV-SMIRNOV (K-S) GOF TEST TO THE LOG NORMAL
C    FAMILY OF DISTRIBUTIONS.  IT USES THE ARITHMETIC MEAN (XBAR) FOR THE K-S
C    TEST AND USES LEAST SQUARES ESTIMATES OF MEAN(MU) AND STANDARD DEVIATION
C    (SIGMA) FOR THE CHI-SQUARE GOF TEST.  THE K-S TEST IS APPLICABLE
C    ONLY TO A COMPLETE DATA SET.  WHEN CENSORING IS USED THE CHI-SQUARE GOF TEST
C    APPLIES.  IN THIS CASE THE PROGRAM ALSO CALCULATES CHI-SQUARE FOR LOWER
C    AND UPPER TAILS FOR GOF TO THE LOG-UNIFORM FAMILY OF DISTRIBUTIONS.
C*******************************************************************************
C       K-S LILLEFORS TEST FOR AN UNSPECIFIED LOG-NORMAL DISTRIBUTION
C    ***************************************************************************
C    THIS PROGRAM ACCEPTS SETS OF DELAY TIMES, ESTIMATES THE PARAMETERS AND
C    PERFORMS A GOODNESS-OF-FIT TEST TO THE LOG NORMAL FAMILY OF DISTRIBUTIONS.
C    ***************************************************************************
C    THE OUTPUT PROVIDES A LISTING AND A PLOT OF THE DELAY TIMES AND THE
C    CORRESPONDING EMPIRICAL PLOT POINT. FHAT=I/N, PARAMETERS AND THE D STATISTIC
C    ARE ALSO SHOWN WITH A STATEMENT REGARDING THE GOODNESS-OF-FIT.
C    THE TEST IS BASED ON THE DIFFERENCE BETWEEN THE EMPIRICAL PLOT POINT(THE
C    HYPOTHESIZED CUMULATIVE DISTRIBUTION FUNCTION(C.D.F.)) AND THE C.D.F. FOR
C    THE SAMPLE DATA.
C    ***************************************************************************
C    IF THE TEST STATISTIC IS SMALL ENOUGH, THE NULL HYPOTHESIS IS ACCEPTED,
C    IMPLYING THAT THERE IS NO OBSERVED EVIDENCE OF A POOR FIT.  IF IT IS TOO
C    LARGE,I.E., EXEEDS THE LILLIEFORS TABULATED D STATISTIC, THIS IMPLIES A
C    POOR FIT.
C    ***************************************************************************
C    THE LILLIEFORS TEST STATISTIC D IS THE LARGEST ABSOLUTE DIFFERENCE BETWEEN
C    THE EMPIRICAL AND THE CALCULATED C.D.F. FOR ANY VALUE OF THE RANDOM
C    VARIABLE X:  D=MAX(X)|F(X)-FHAT(X)|.
C    ***************************************************************************
C   LET X BE AN R.V. WITH CUMULATIVE DISTRIBUTION FUNCTION
C   FOR X SUB-1 ,X SUB-2,...,X SUB-N OF SIZE N ORDERED X SUB-1 < OR =
C     X SUB-2 < OR =,...,< OR = X SUB-N,THEN EMPIRICAL DIST. FUNC. IS:0 FOR
C     X < X SUB-1.
C   F SUB-N(X) = I/N FOR X SUB-I < OR = X < OR = X SUB-(I+1)     I = 1,2,.
C    ....,N-1.
C         1 FOR X SUB-N < OR = X
C   F(X) = PROBABILITY (X< OR = X)
C
C
C   TABLES USED ARE FOR UNSPECIFIED   LN POPULATION, I.E., THE PARAMETERS
C    ARE ESTIMATED FROM THE SAMPLE, REF:JASA ARTICLE BY LILLIEFORS
C     VOL. 62 #318
C   THE D STATISTIC INTRODUCED BY KOLMOGOROV IN 1933 IS:
C      D  = LEAST UPPER BOUND|F(X) -F SUB-N(X)|
C   REF: ARTICLE BY BIRNBAUM IN JASA.  VOL 47,PP. 425-441
C
C   XBAR IS THE MEAN
C
C
C   VAR IS THE VARIANCE
      DIMENSION GVAL(1000),GFHAT(1000),DUM(1000),GD(134,8)
      DIMENSION GPLOT(1000)
      DIMENSION VAL(1000),T(1000),G(1000),PROB(1000),DAT1(19)
      DIMENSION EXPD(1000),FXSQ(1000),UXSQ(1000),IOBX(1000),LOW(100),
     1LP(100)
      DIMENSION ZP(1000)
      DIMENSION CUXSQ(100)
      DIMENSION XCHISQ(60),CUMCHI(60)
C  DIMENSION OF THE LEAST SQUARES CALCULATION,LN(DT)=ZPLOT*SIGMA+MU
      DIMENSION ZXLNDT(1000),SQZ(1000)                               LS
      DIMENSION TRCB(1000)
      DIMENSION OBX(100)
      DIMENSION IE(100)
      INTEGER XLO,UP
      EXTERNAL CNORM
      DATA BLANK/3HXXX/
      REAL MEAN,MU
C*******************************************************************************
C      INPUT CAPABILITY FOR NBR, XBYX, A VARIABLE SET OF DELAY TIMES
C      (DT), A VARIABLE SET OF SELECTIONS, A PROGRAM NBR LOOP TO READ
C      ANOTHER SET OF DT AND SELECTIONS.  IN THE CASE OF RUNNING SEVERAL
C      SETS OF DT SELECTIONS MIGHT BE FOR ALL DATA PER SET.
C*******************************************************************************
```

```
      RPIE=1./SQRT(6.283184)
      B=0.5
      J=1
      X=0.
C
C
C  CREATION OF THE OBSERVED [redacted] DIST. FUNCTION
C
    6  Y=0.005/6.*(RPIE*1./EXP(X**2/2.)+4.*(RPIE*1./EXP((X+0.0025)**2/2.
     $))+(RPIE*1./EXP((X+0.005)**2/2.)))
      B=B+Y
      PROB(J)=B
      IF(X.GT.4.1)GO TO 79
      J=J+1
      X=X+0.005
      GO TO 6
 79   CONTINUE
      READ(5,102)NBR
      READ(5,206)XBYX
 206  FORMAT(F8.5)
      DO 1000 JJ=1,NBR
      DO 202 JF=1,1000
      READ(5,2) VAL(JF)
      IF(VAL(JF).EQ.0.)GO TO 203
      VAL(JF)=VAL(JF)-5.5
 202  CONTINUE
      WRITE(6,5123)
 5123 FORMAT(1H1,'DECK OVER 1000')
      GO TO 1009
 203  N=JF-1
C THIS 'DO LOOP, DO 1000' ALLOWS THE PROGRAM TO BE EXECUTED 'NBR' TIMES
C INITIALIZE THE LS VALUES
C*****************************************************************************
C     NBR AND XBYX ARE READ, AFTER WHICH ALL DT ARE READ IN AND
C     ASSIGNED LOGARITHMS AFTER SORTING.
C*****************************************************************************
C  SORT INPUT - LOWEST TO HIGHEST
      I=N
 31   IF(I.EQ.N)GO TO 39
      IF(VAL(I).LE.VAL(I+1))GO TO 36
      K=I+1
   32 IF(K.EQ.1)GO TO 36
      IF(VAL(K).GE.VAL(K-1))GO TO 36
      VALSV=VAL(K)
      VAL(K)=VAL(K-1)
      VAL(K-1)=VALSV
      K=K-1
      GO TO 32
   36 I=I+1
      GO TO 31
 39   CONTINUE
C     ASSIGN LOGARITHM FOR EACH DT
      DO 3 K=1,N
       T(K)=ALOG(VAL(K))
 3    CONTINUE
C*********************************************************************GFHAT ROUTINE
C THE FOLLOWING STATEMENTS THROUGH STATEMENT NUMBER 360 ARE USED FOR
C  RESOLUTION OF EQUAL VALUES OF THE OBSERVATIONS.  A CORRESPONDING
C  EMPIRICAL PLOT POINT IS USED WHICH IS AN AVERAGE VALUE.
C     THIS LOOP WILL BE REPEATED FOR THE SELECTIONS READ IN.
C     READ VARIABLE NUMBER OF SELECTIONS OF CENSORED DATA.
      DO 209 NP=1,25
      READ(5,208)LOW(NP),LP(NP)
      IF(LOW(NP).EQ.0)GO TO 1000
 208  FORMAT(2I3)
      XLO=LOW(NP)
      UP=LP(NP)
      SUM=0.
      SUMSQ=0.
      DO 207 K=XLO,UP
      XNI=N
      CALL HISTO(5,T(K),1.0/XNI,6.0,1.7,30.0)
       SUM=SUM+T(K)
      SUMSQ=SUMSQ+T(K)**2
 207  CONTINUE
      Y=UP-XLO+1
```

```
      XBAR=SUM/Y
      VAR=(SUMSQ-(SUM**2/Y))/(Y-1)
      SD=SQRT(VAR)
C     ASSIGN EMPIRICAL FLCT VALUE
      LOL=XLO
      IXLC=XLC+1
      DO 350 L=IXLO,UP
      IF(VAL(LOL).EQ.VAL(L)) GC TC 350
      ICTR=L-LOL
      LLL=L-1
      C=LOL
      CC=L-1
      GFHT=(((C-.5)+(CC-.5))/2.)/XNI
      DO 340 IJK=LOL,LLL
 340  GFHAT(IJK)=GFHT
      LOL=L
 350  CONTINUE
      ICTR=L-LCL+1
      LLL=L
      C=LOL
      CC=L
      GFHT=(((C-.5)+(CC-.5))/2.)/XNI
      DO 360 IJK=LOL,LLL
 360  GFHAT(IJK)=GFHT
      SUMT=0.0
      SUMZ=0.0
       SUMZ2=0.0
        SUMZT=0.0                                                        LS VALS
C*****************************************************************GFHAT ROUTINE
      DO 4 L=XLO,UP                                                      DO LOOP
       C=L
C
C [redacted] DISTRIBUTION FUNCTION
      FHAT=GFHAT(L)
      ZPLOT=SINRT(0.,1.,FHAT,CNCRM,.01,400)
      LL=L-(XLC-1.)
      GPLOT(LL)=ZPLOT
      GVAL(LL)=VAL(L)
C CREATE A COLUMN FOR Z*LN(DT). USE Z FROM FHAT.
      ZXLNDT(L)=ZPLOT*T(L)
C  CREATE A COLUMN FO Z SQUARED.
      IF(ZPLOT.GT.-.001.ANC.ZPLOT.LT..001) GO TO 98
      SQZ(L)=ZPLOT**2
      GO TO 198
 98   SQZ(L)=0.0
 198  CONTINUE
C SUM COLUMNS OF ZPLOT, SQZ, T, AND ZXLNDT FOR THE CALCULATION OF MU & SIGMA.
      SUMT=SUMT+T(L)
      SUMZ=SUMZ+ZPLOT
      SUMZ2=SUMZ2+SQZ(L)
      SUMZT=SUMZT+ZXLNCT(L)
  999 CONTINUE
    4 CONTINUE
C  AFTER THE LOOP FIND THE ESTIMATES OF MU AND SIGMA BY LEAST SQUARES FORMULA.
      XNO=UP-XLO+1
      MU=((SUMZ2*SUMT)-(SUMZ*SUMZT))/((XNO*SUMZ2)-(SUMZ**2))
      SIGMA=((XNO*SUMZT)-(SUMZ*SUMT))/((XNO*SUMZ2)-(SUMZ**2))
      WRITE(6,62)XLO,UP
   62 FORMAT(1H1,T4,' TABULAR INFORMATION FOR SELECTION OF DELAY TIMES
     1FROM ORDER NUMBER ',I3,' TO ',I3/T5,' FOLLOWED BY A PROBABILITY X
     2LOGARITHMIC PLOT, A HISTOGRAM AND CHI-SQUARE DATA.')
      WRITE(6,59)
      DO 1999 L=XLO,UP                                                   DO LOOP
      LL=L-(XLC-1.)
      Z=(T(L)-MU)/SIGMA
      IF(Z.LT..05.AND.Z.GT.-.05)GO TO 789
      ZP(L)=Z
      EST=Z*200.
      IF(Z.LT.0.)GO TO 89
      JO=EST
      BO=JO
      IF(EST-BO.GT.0.5)JO=JO+1
      TROB(L)=PROB(JO)
      GO TO 99
   89 JO=(-EST)
      BO=JO
```

```
      EST=(-EST)
      IF(EST-BO.GT.0.5)JO=JO+1
      TROB(L)=1.-PRCB(JO)
      GO TO 99
  789 TROB(L)=CNORM(2)
C
C      THIS IS ONLY APPLICABLE FOR NONCENSORED DATA.
C  THIS IS THE D STATISTIC WHICH MEASURES ABSOLUTE DIFFERENCE BETWEEN
C    THE EMPIRICAL AND THE OBSERVED CUMULATIVE DISTRIBUTION FUCTION.
C     USE FI=I/N FOR THE D STATISTIC.  THUS CORRECT WITH +.5/N
 99   G(L)=ABS(GFHAT(L)-TROE(L))+.5/N
C
C
      WRITE(6,51)L,VAL(L),GFHAT(L),T(L),Z,TROB(L),G(L),GPLOT(LL)
C
       IF(L.EQ.1)GO TO 7
       IF(G(L).GT.BIGD)GO TO 8
      GO TO 1999
    7  BIGD=G(1)
      GO TO 1999
    8  BIGD=G(L)
 51   FORMAT(1H ,T5,I3,T10,F5.1,T20,F8.5,T32,F8.5,T42,F8.5,T51,F8.5,T62,
     1F8.5,T72,F8.5)
 1999 CONTINUE
      IDIFR=UP-XLO+1
      WRITE (6,1021)IDIFR
      WRITE(6,1997)SUMT,SUMZ,SUMZ2,SUMZT
 1997 FORMAT(1H0,'SUM OF LNS=',F13.5,'    SUM OF Z=',F13.5,'    SUM OF Z S
     1QUARED=',F13.5,'    SUM OF Z * LNS=',F13.5)
      M = (N/8) +1
      K = 1
      DO 1001 I = 1,M
      DO 1001 J = 1,8
      IF (K.GT.N) GO TO  1002
 1003 GD(I,J) = G(K)
      K = K+1
      GO TO  1001
 1002 IF ((M*8)-K) 1007,1005,1005
 1005 GD (I,J) = BLANK
 1001 CONTINUE
 1004 CALL PLOT(GVAL,3,GPLOT,0,DUM,0,1DIFR,1,1,3,2,1,1)
      WRITE(6,50)
C***************************************************************************
C      NOTE THE LOWER LIMIT
C***************************************************************************
      CALL HISTO(5,T,0.0,6.0,-.7,30,1)
      WRITE(6,50)
      WRITE(6,1100)
 1100 FORMAT(' THE FOLLOWING STATEMENTS REGARD A GOF TEST USING D STATIS
     1TIC'/'  DISREGARD WHEN CENSORING AND USE CHI-SQUARE'//)
      V=N
      S=SQRT(V)
      IF(BIGD.GT.(0.805/S))GO TO 9
      WRITE(6,10)
      GO TO 11
    9 IF(BIGD.GT.(0.886/S))GO TO 12
      WRITE(6,13)
      GO TO 11
   12 IF(BIGD.GT.(1.031/S))GO TO 14
      WRITE(6,15)
      GO TO 11
   14 WRITE(6,16)
  11  MEAN=EXP((VAR/2.)+XBAR)
      VARY=EXP((2.*XBAR)+(2.*VAR))-EXP((2.*XBAR)+VAR)
      STDY=SQRT(VARY)
      WRITE(6,60)XBAR,VAR,SD
      WRITE(6,50)
      WRITE(6,1111)
 1111 FORMAT(' THE FOLLOWING INFORMATION APPLIES TO A GOF TEST USING CHI-
     1-SQUARE'/' THE MU AND SIGMA ARE OBTAINED BY LEAST SQUARES USING-'/
     1' LN(DELAY TIME)=(FHAT Z * SIGMA)+ MU'//)
      WRITE(6,58)MU,SIGMA                                              LS WRITE
 58   FORMAT(1H ,'LEAST SQUARES MU IS',F9.5,' AND SIGMA IS',F9.5)
      WRITE(6,184) TROB(XLO),TROB(UP),T(XLO),T(UP),VAL(XLO),VAL(UP)
 184  FORMAT(1H0,'RANGE OF F IS ',F8.5,' TO ',F8.5,'    LOGARITHMS  ',
     1F8.5,' TO ',F8.5,' FOR DELAY TIMES ',F5.1,' TO ',F5.1,' MINUTES'/)
```

```
C  THE NEXT SEVERAL INSTRUCTIONS CALCULATE THE X SQUARE STATISTIC
C*************************************************************CHISQUARE*****
C  SET THE NUMBER OF CLASSES FOR THE CHI-SQUARE CALCULTIONS.
      IF(LCW(NP).EQ.1)TROB(XLO)=0.0
      IF(LP(NP).EQ.N)TROB(UP)=1.0
      NX=((TROB(UP)-TROB(XLO))/XBYX)+1.
      IF(NX.GT.N)GO TO 9700
      GO TO 9710
 9700 WRITE(6,9711)
 9711 FORMAT(1H0,' INCREASE XBYX')
      GO TO 209
 9710 CONTINUE
      XNO=N
C   SET THE BREAK POINT AS XBYX.E.G.,.05, IF STARTING WITH ORDER NUMBER
C  ONE OR IF DATA IS TRUNCATED.USE XBYX PLUS THE PERCENTAGE ASSOCIATED
C   WITH THE STARTING ORDER NUMBER
      BK=TROB(XLO)+XBYX
      IF(XLO.EQ.1)BK=XBYX
      ZIP=SINRT(0.,1.,BK,CNORM,.01,400)
      BKL=ZIP*SIGMA+MU
      IXX=1
C  INITIALIZE THE ARRAYS TO 0.
      DO 121 I=1,NX
      IE(I)=0
      XCHISQ(I)=0.0
  121 OBX(I)=0.0
C  THIS ROUTINE COUNTS THE EXPECTED AND OBSERVED OCCURRENCES IN EACH CLASS.
C  THIS LOOP COUNTS LN(DT) WITHIN LOGARITHMIC BOUNDS CORRESPONDING TO VALUES
C  OF Z OBTAINED FROM THE INCREMENTS OF F.
C***************************************************************************
      DO 151 I=XLO,UP
 150  IF(T(I).LE.BKL) GO TO 149
      BK=BK+XBYX
 7351 IF(BK.GE.1.)BK=.999999
      ZIP=SINRT(0.,1.,BK,CNORM,.01,400)
      BKL=ZIP*SIGMA+MU
      IXX=IXX+1
      GO TO 150
 149  IE(IXX)=IE(IXX)+1
 151  CONTINUE
C***************************************************************************
C   THE OBSERVATIONS ARE NOW IN NX ARRAYS OF IE.
      IXX=1
      BK=TROB(XLO)
      IF(XLO.EQ.1)BK=0.0
      XBCX=0.0
C   ARRAY THE EXPECTED FREQUENCIES IN NX CLASSES.
      DO 105 IX=1,NX
      XBCX=TROB(UP)-BK
      OBX(IX)=XBYX*XNO
      IF(IX.EQ.NX)OBX(IX)=XBCX*XNO
      BK=BK+XBYX
 105  CONTINUE
      TOTCHI=0.0
      BK=TROB(XLO)
      BX=TROB(XLO)
      WRITE(6,166)
      KLS=0
      DO 108 L=1,NX
      IF(IE(L).GE.5) GO TO 103
      IF(L.EQ.NX.AND.IE(L).LT.5)GO TO 6500
      GO TO 6600
 6500 EXZ=IE(L-1)+IE(L)
      OBV=OBX(L-1)+OBX(L)
      BX=BK+XBYX
      GO TO 701
 6600 CONTINUE
      IF(L.EQ.NX)GO TO 104
  109 OBX(L+1)=OBX(L+1)+OBX(L)
      IE(L+1)=IE(L+1)+IE(L)
      BX=BX+XBYX
      GO TO 108
  103 IF(L.EQ.NX)GO TO 104
      IF(IE(L+1).EQ.0) GO TO 109
 104  IF(OBX(L).EQ.0.0) GO TO 107
      EXZ=IE(L)
```

```
      OBV=OBX(L)
  701 XCHISQ(L)=((OBV-EXZ)**2)/OBV
  107 TOTCHI=TOTCHI+XCHISQ(L)
      CUMCHI(L)=TOTCHI
      BX=BX+XBYX
      IF(L.EQ.NX) BX=TROB(UP)
      IF(UP.EQ.N.AND.L.EQ.NX)BX=1.0
      WRITE(6,167)BK,BX,OBX(L),IE(L),XCHISQ(L),CUMCHI(L)
      KLS=KLS+1
      BK=BX
  108 CONTINUE
C*******SPACER*************************************************SPACER********************
      WRITE(6,19)
   19 FORMAT(1H0,//)
      WRITE(6,17) CUMCHI(NX),KLS,XBYX
   17 FORMAT('0',10X,'CHI SQUARE STATISTIC EQUALS',F10.5,' FOR ',I3,
     1'  INTERVALS OF ',F5.3,'  OR MULTIPLES.')
  166 FORMAT(1H0,'CLASS INTERVAL',12X,'EXPECTED',8X,'OBSERVED',
     16X,'INDIV CHISQ',9X,'CUM CHISQ')
  167 FORMAT(1H0,F8.5,' - ',F8.5,5X,F9.4,10X,I3,9X,F10.5,8X,F10.5)
C************************************************LOWER CHI-SQUARE**************
C     ONCE A CENSORED SELECTION HAS BEEN MADE A CHI-SQUARE IS
C     CALCULATED FOR THE CENSORED PORTIONS.
C     CALCULATE EXPECTED FREQUENCY FOR LOWER PART.
      IF(XLO.EQ.1.AND.UP.EQ.N)GO TO 209
      IF(XLO.EQ.1.AND.UP.NE.N)GO TO 3061
      INT=(VAL(XLO)-VAL(1)+1.)/2.
      XNT=INT
      IF(((VAL(XLO)-VAL(1)+1.)/XNT).NE.2.) GO TO 2030
      NXX=4
      GO TO 2035
 2030 NXX=3
 2035 KLASES=0
      TOXSQ=0.0
 2040 DO 2020 IK=1,NXX
      EXPD(IK)=0.0
      FXSQ(IK)=0.0
      UXSQ(IK)=0.0
      IOBX(IK)=0.0
 2020 CONTINUE
C     CALCULATE THE PERCENTAGE FOR EACH INTERVAL. THEN THE EXPECTED FREQUENCY
C     PER INTERVAL.
      XNXX=NXX
      PCT=TROB(XLO)/XNXX
 2045 DO 2050 IX=1,NXX
      EXPD(IX)=PCT*XNO
 2050 CONTINUE
C*******************************************************************************
C     CALCULATE OBSERVED FREQUENCIES IN EACH CLASS USING LN(DT)
C     BOUNDARIES.
      NIP=XLO-1
      XLN=(T(XLO)-T(1))/XNXX
      IXI=1
      XLNBD=XLN+T(1)
 2080 DO 2060 IO=1,NIP
 2083 IF(T(IO).LT.XLNBD) GO TO 2070
      XII=IXI
      XLNBD=XLNBD+XLN
      IXI=IXI+1
      GO TO 2083
 2070 IOBX(IXI)=IOBX(IXI)+1
 2060 CONTINUE
      XLNBD=T(1)
      PCT=0.0
      PCTBK=0.0
      XLNBK=T(1)
C   INSERT HEADING FOR CALCULATION OF THE CHI-SQUARE STATISTIC FOR THE LOWER
C   PART OF THE DISTRIBUTION.
      WRITE(6,3001)
 3001    FORMAT(1H1,'  THE CHI-SQUARE STATISTIC FOR THE LOWER PART OF TH
     1E DISTRIBUTION'///'  LOG BOUNDS',13X,'PERCENT BOUNDS',
     212X,'EXPECTED',3X,'OBSERVED',4X,'CHI-SQUARE'/T74,
     3'INDIV',6X,'CUM'//)
C*******************************************************************************
C     THE OBSERVED AND EXPECTED FREQUENCIES ARE NOW AVAILABLE IN
C     EACH CLASS FOR CALCULATION OF THE CHI-SQUARE STATISTIC.
      DO 3060 L=1,NXX
```

```
      IF(IOBX(L).GE.5) GO TO 3030
      IF(L.EQ.NXX) GO TO 3040
 3020 EXPD(L+1)=EXPD(L+1)+EXPD(L)
      IOBX(L+1)=IOBX(L+1)+IOBX(L)
      XLNBK=XLNBK+XLN
      PCTBK=PCTBK+TROB(XLO)/XNXX
      GO TO 3060
 3030 IF (L.EQ.NXX) GO TO 3040
      IF(IOBX(L+1).EQ.0) GO TO 3020
 3040 IF(EXPD(L).EQ.0.0) GO TO 3050
      OSVD=IOBX(L)
      ECP=EXPD(L)
      FXSQ(L)=((ECP-OSVD)**2)/ECP
 3050 TOXSQ=TOXSQ+FXSQ(L)
      CUXSQ(L)=TOXSQ
      XLNBK=XLNBK+XLN
      PCTBK=PCTBK+TROB(XLO)/XNXX
      WRITE(6,3070)XLNBD,XLNBK,PCT,PCTBK,EXPD(L),IOBX(L),FXSQ(L),
     1CUXSQ(L)
      KLASES=KLASES+1
      XLNBD=XLNBK
      PCT=PCTBK
 3060 CONTINUE
      GO TO 3062
 3061 NXX=1
      CUXSQ(NXX)=0.0
 3062 CONTINUE
      XDIST=0.0
      XDIST=CUXSQ(NXX)+CUMCHI(NX)
      WRITE(6,50)
C*************************************************UPPER CHI-SQUARE**************
C  NOW THAT THE CHI-SQUARE HAS BEEN CALCULATED FOR A LOWER AND/OR CENTER PORTION
C  OF THE DATA, A CHI-SQUARE IS CALCULATED FOR UPPER PART(IF ANY). THUS A TOTAL
C  CHI-SQUARE STATISTIC IS OBTAINED WHICH APPLIES TO THE TOTAL DISTRIBUTION.
      OBSU=0.0
      EXPU=0.0
      UCHI=0.0
      IF(UP.EQ.N)GO TO 3075
C  CALCULATE NUMBER OF OBSERVATIONS IN UPPER TAIL OF THE DISTRIBUTION.
      XNUB=N
      XUPR=UP
      OBSU=XNUB-XUPR
C  CALCULATE THE EXPECTED FREQUENCY
      EXPU=XNUB*(1.-TROB(UP))
C  CALCULATE THE CHI-SQUARE CONTRIBUTION OF THE UPPER TAIL.
      UCHI=((EXPU-OBSU)**2)/EXPU
 3075 CONTINUE
C  PRINT OUT THE TOTAL CHI-SQUARE STATISTIC
      IF(UP.EQ.N)UCHI=0.0
      IF(XLO.EQ.1)NXX=1
      IF(XLO.EQ.1)CUXSQ(NXX)=0.0
      WRITE(6,3093)OBSU,EXPU,UCHI
 3093 FORMAT(1H0,' OBSERVED FREQUENCY FOR THE UPPER TAIL IS ',F10.5,
     1' AND THE EXPECTED FREQUENCY IS ',F10.5,' FOR A CHI-SQUARE OF ',
     2F10.5//)
      WRITE(6,3077)CUXSQ(NXX),CUMCHI(NX),UCHI
 3077 FORMAT(1H0,'CHI-SQUARE FOR LOWER PART IS ',F10.5,' AND FOR THE CEN
     1TER PART IS ',F10.5,' FOR THE UPPER PART IT IS ',F10.5//)
      IF(UP.NE.N)XDIST=CUXSQ(NXX)+CUMCHI(NX)+UCHI
      WRITE(6,3071)XDIST
 209  CONTINUE
 1000 CONTINUE
 3070 FORMAT(1H0,F8.5,' TO ',F8.5,3X,F8.5,' TO ',F8.5,5X,F10.5,3X,I3,
     13X,F10.5,3X,F10.5)
 3071 FORMAT(1H0,' THE CHI-SQUARE STATISTIC FOR THE TOTAL DISTRIBUTION I
     1S ',F10.5)
 1030 FORMAT('0',25X,F10.5)
      WRITE(6,33)
    2 FORMAT(F3.0)
   10 FORMAT('0THERE IS NO EVIDENCE AT THE TEN PCT, FIVE PCT,'/' OR ONE
     $PERCENT LEVEL THAT THE DATA POORLY FIT A LOG NORMAL'/' DISTRIBUTIO
     $N.')
   13 FORMAT('THERE IS EVIDENCE AT THE 10 PCT LEVEL'/' THAT THE DATA PO
     $ORLY  FIT A LOG NORMAL DISTRIBUTION.')
   15 FORMAT('0THERE IS EVIDENCE AT THE FIVE PCT LEVEL'/' THAT THE DATA
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     $POORLY FIT A LCG NORMAL DISTRIBUTION.')
  16 FORMAT('OTHERE IS EVIDENCE AT THE ONE PCT LEVEL'/' THAT THE DATA
     $POORLY FIT A LCG NORMAL DISTRIBUTION.')
  18 FORMAT(1H0, 8F8.4)
  33 FORMAT( 1H1)
  50 FORMAT(1H0,/////)
 59  FORMAT(1H0,T5,'OBS',T10,'DELAY',T20,'FHAT',T32,'LN(DT)',T40,'Z=LN(
    1DT)-MU',T54,'CDF',T65,'D',T72,'Z FROM FHAT'/T43,'SIGMA'//)
  60 FORMAT(1H0,'  XEAR   =',F10.4,3X,'     VAR =',F10.4,3X,
     $'     STDEV =',F10.4)
  61 FORMAT(1H0,'POP. MEAN =',F10.4,'    POP VAR =',F10.4,3X,
     $'POP STDEV =',F10.4)
 102 FORMAT(I4)
1008 FORMAT (1H1,'DG IS TOO SMALL')
1020 FORMAT(10A4,11X,6A4)
1010 FORMAT (20A4)
1021 FORMAT ( 1H0, 'TOTAL DELAYS= ',I3)
     GC TO  1009
1007 WRITE (6,1008)
1009 STOP
     END
     SUBROUTINE HISTO(N1,A1,W,A2,A3,N2,N3)
     DIMENSION BIN(10,52),ERR(10,52),NEV(10,52),KOUNT(10)
     DIMENSICN NONUN(10),KH(104)
     DATA NHMAX/10/,NBMAX/5C/
     DATA MNLIN/20/
     DATA KHLIN/120/
     DATA MAXER/50/
     DATA KBL/' '/,KPL/'+'/,KMI/'-'/,KX/'X'/
     DATA INISH/0/,NERR/0/
     KW=6
     NHIST=N1
     A=A1
     WT=W
     AMAX=A2
     AMIN=A3
     NBINS=N2
     NSENS=N3
     IF(NHIST)150,150,10
  10 IF(NHIST-NHMAX)20,20,150
  20 IF(NBINS)150,150,30
  30 IF(NBINS-NBMAX)40,40,150
  40 IF(AMAX-AMIN)150,150,5C
  50 IF(INISH)80,60,80
  60 INISH=7
     DO 70 J=1,NHMAX
     KOUNT(J)=0
     NONUN(J)=0
     KTCP=NBMAX+2
     DO 70 K=1,KTOP
     BIN(J,K)=0.
     ERR(J,K)=0.
  70 NEV(J,K)=0
  80 KOVFL=NBINS+2
     IF(NSENS)180,90,180
  90 BINS=NBINS
     K=((A-AMIN)/(AMAX-AMIN))*BINS+2.
     IF(K)100,100,110
 100 K=1
 110 IF(K-KOVFL)130,130,120
 120 K=KOVFL
 130 BIN(NHIST,K)=BIN(NHIST,K)+WT
     ERR(NHIST,K)=ERR(NHIST,K)+WT**2
     NEV(NHIST,K)=NEV(NHIST,K)+1
     KOUNT(NHIST)=KOUNT(NHIST)+1
     IF(WT-1.)140,730,140
 140 NONUN(NHIST)=7
     GO TO 730
 150 IF(NERR-MAXER)160,160,730
 160 NERR=NERR+1
     WRITE(KW,170)NHIST,NBINS,AMAX,AMIN
 170 FORMAT(/34H ILLEGAL INPUT TO HISTO.  NHIST = I5,5X 8HNBINS = I5,  HISTW 99
    *    5X 7HAMAX = E12.5,5X 7HAMIN = E12.5)
     GC TC 730
 180 DO 190 K=1,KOVFL
 190 ERR(NHIST,K)=SQRT(ERR(NHIST,K))
     NBPU=NBINS+1
```

```
      NIN=0
      HMAX=0.
      DO 210 K=2,NBPU
      NIN=NIN+NEV(NHIST,K)
      H=BIN(NHIST,K)+ERR(NHIST,K)
      IF(H-HMAX)210,210,200
 200  HMAX=H
 210  CONTINUE
      LNBIN=MNLIN/NBINS+(MNLIN-(MNLIN/NBINS)*NBINS+NBINS-1)/NBINS
      JUMP=0
      IF(NONUN(NHIST))230,220,230
 220  JUMP=-1
 230  NBEFR=(LNBIN-1)/2
      NAFTR=LNBIN-1-NBEFR
      NCH=KHLIN-54
      IF(JUMP)240,250,240
 240  NCH=KHLIN-34
 250  ENCH=NCH
      SCALE=0.
      IF(HMAX)270,270,260
 260  SCALE=ENCH/HMAX
 270  WRITE(KW,280)NHIST,AMAX,AMIN,NBINS,SCALE
 280  FORMAT(//18H HISTOGRAM NUMBER I3,5X7HAMAX = E12.5,5X7HAMIN =
     *       E12.5,5X8HNBINS = I3,5X8HSCALE = E10.3)
      IF(JUMP)290,320,320
 290  WRITE(KW,300)
 300  FORMAT(///7X10HBIN LIMITS 5X6HEVENTS/6X12H------------7X            HI
     *    8H--------)
      WRITE(KW,310)NEV(NHIST,1)
 310  FORMAT(/7X9HUNDERFLOW 5XI7/1H )
      GO TO 350
 320  WRITE(KW,330)
 330  FORMAT(///7X10HBIN LIMITS 5X6HEVENTS 3X6HHEIGHT 4X5HERROR/
     *       6X12H------------7X8H--------1X8H--------2X7H-------)
      WRITE(KW,340)NEV(NHIST,1),BIN(NHIST,1),ERR(NHIST,1)
 340  FORMAT(/7X9HUNDERFLOW 5XI7,2E10.3/1H )
 350  ENBIN=NBINS
      DEL=(AMAX-AMIN)/ENBIN
      DO 660 K=2,NBPU
      DO 360 J=1,NCH
 360  KH(J)=KBL
      NX=SCALE*BIN(NHIST,K)+.5
      IF(NX)440,440,370
 370  IF(NX-NCH)390,390,380
 380  NX=NCH
 390  DO 400 J=1,NX
 400  KH(J)=KX
      NX=SCALE*(BIN(NHIST,K)-ERR(NHIST,K))+.5
      IF(NX)440,440,410
 410  IF(NX-NCH)430,430,420
 420  NX=NCH
 430  KH(NX)=KMI
 440  NX=SCALE*(BIN(NHIST,K)+ERR(NHIST,K))+.5
      IF(NX)450,450,460
 450  NX=1
      GO TO 490
 460  IF(NX-NCH)480,480,470
 470  NX=NCH
 480  KH(NX)=KPL
 490  IF(NBEFR)560,560,500
 500  DO 550 J=1,NBEFR
      IF(JUMP)510,530,510
 510  WRITE(KW,520)(KH(L),L=1,NX)
 520  FORMAT(33X1HI86A1)
      GO TO 550
 530  WRITE(KW,540)(KH(L),L=1,NX)
 540  FORMAT(53X1HI66A1)
 550  CONTINUE
 560  AK=K
      XL=AMIN+(AK-2.)*DEL
      XH=XL+DEL
      IF(JUMP)570,590,570
 570  WRITE(KW,580)XL,XH,NEV(NHIST,K),(KH(L),L=1,NX)
 580  FORMAT(1XE10.3,4H TO E10.3,I7,2H I 86A1)
      GO TO 610
 590  WRITE(KW,600)XL,XH,NEV(NHIST,K),BIN(NHIST,K),ERR(NHIST,K),
```

```
     *      (KH(L),L=1,NX)
  600 FORMAT(1XE10.3,4H TO E10.3,I7,2E10.3,2H I 66A1)
  610 IF(NAFTR)660,660,620
  620 DO 650 J=1,NAFTR
      IF(JUMP)630,640,630
  630 WRITE(KW,520)(KH(L),L=1,NX)
      GO TO 650
  640 WRITE(KW,540)(KH(L),L=1,NX)
  650 CONTINUE
  660 CONTINUE
      IF(JUMP)670,690,670
  670 WRITE(KW,680)NEV(NHIST,KOVFL)
  680 FORMAT(/7X8HOVERFLOW 10XI7,2E10.3)
      GO TO 700
  690 WRITE(KW,680)NEV(NHIST,KOVFL),BIN(NHIST,KOVFL),ERR(NHIST,KOVFL)    HISTW22
  700 WRITE(KW,710)NIN,KOUNT(NHIST)
  710 FORMAT(///42H NUMBER OF EVENTS BETWEEN XMIN AND XMAX = I7,5X
     *      26H TOTAL NUMBER OF EVENTS = I7    )
      KOUNT(NHIST)=0
      NONUN(NHIST)=0
      DO 720 K=1,KTOP
      BIN(NHIST,K)=0.
      ERR(NHIST,K)=0.
  720 NEV(NHIST,K)=0
  730 RETURN
      END
      DOUBLE PRECISION FUNCTION CNORM(YYY)
C COMPUTES CUMULATIVE NORMAL DISTRIBUTION FUNCTION FOR A STANDAWRD NORMAL
C WITH ERROR<1.5 X 10**(-7)
      Y=YYY
      IF(Y)2,1,1
    2 X=-Y
      CNORM=1.0-0.5*(1.0+X*(.0498673470+X*(.0211410061+X*(.0032776263+
     1X*(.0000380036+X*(.0000488906+X*.0000053830))))))**(-16.)
      CNORM=1.0-CNORM
      RETURN
    1 X=Y
      CNORM=1.0-0.5*(1.0+X*(.0498673470+X*(.0211410061+X*(.0032776263+
     1X*(.0000380036+X*(.0000488906+X*.0000053830))))))**(-16.)
      RETURN
      END
      FUNCTION SINRT(A,B,C,F,ERROX,ITER)
      IF(B-A)1,1,2
  1   WRITE(6,3)A,B
  3   FORMAT(1H 'SINRT WAS CALLED WITH LEFT ENPT',E14.7,'> THAN RIGHT
     2 ENDPT ',E14.7,' NO VALUE RETURNED')
      RETURN
  2   X1=A
      X2=B
      DO 50 I=1,ITER
      Y2=F(X2)-C
      Y1=F(X1)-C
      IF (Y2-Y1)10,15,10
  15  IF(Y2)16,17,16
  16  IF(Y1)18,19,18
  17  SINRT=X2
      RETURN
  19  SINRT=X1
      RETURN
  18  CONTINUE
C 18  WRITE(6,5)X1,X2,Y2
C 5   FORMAT(1H, ' SINRT ERROR X1=',E14.7,'X2= ',E14.7,' F(X1)-C=
C    2=F(X2)-C=',E14.7,'X2 RETURNED')
      SINRT=X2
      RETURN
  10  X3=X2-Y2*(X2-X1)/(Y2-Y1)
      X1=X2
  50  X2=X3
      SINRT=X3
      RETURN
      END
```

VITA

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Candidate for the Degree of

Doctor of Philosophy

Thesis: SIMULATION OF DISPATCH RELIABILITY FOR A FLEET OF LARGE COMMERCIAL AIRCRAFT

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