OPTIMAL OPERATIONAL POLICIES FOR A SYSTEM OF SIX MULTI-PURPOSE RESERVOIRS BY DISCRETE DIFFERENTIAL DYNAMIC PROGRAMMING

By

ARTURO LE-BLANC PALMA

Ingeniero Agronomo
Universidad Catolica de Chile
Santiago, Chile
1968

Master of Science
Oklahoma State University
Stillwater, Oklahoma
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Thesis Approved:

Mitchell O. Locks

Lou Ineinhard

Richard H. Leftwich

P. Larry Claypool

Donald R. Williams

N N Durham
Dean of the Graduate College

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CHAPTER I

INTRODUCTION

It is frequently advisable to analyze a set of reservoirs which are in the same river basin as a system, because of the interrelationship between the reservoirs, in functions such as flood control, hydroelectric power, inland navigation, recreation, pollution abatement, irrigation, and fish and wildlife conservation.

In the Eastern Oklahoma watershed area, there is a system of seven multi-purpose reservoirs in the Arkansas River basin. These are: Keystone, at the confluence of the Cimarron and the Arkansas rivers; Fort Gibson, on the Grand River; Webbers Falls, on the Arkansas River; Tenkiller-Ferry, on the Illinois River; Eufaula, on the Canadian River; Robert S. Kerr, on the Arkansas River; and Oologah, on the Verdigris River. Six of these seven reservoirs are also capable of generating hydroelectric power, which is sold to local electric utilities, rural electrification cooperatives, and other major uses through the Southwestern Power Administration.

The ability to generate commercially salable power in this system is a substantial technological achievement, because of the nature of the topography of the area. Over this range the average water drop per mile of running river is relatively small, 1.96 feet per mile, with no mountains or large waterfalls. As a result, the system is what the power engineers would call a "low head project," that is, capable of generating relatively little power.

Another interesting feature of the system is the fact that high inflows have different effects upon storage and power generation capability depending on the nature of the reservoir. In this case, four of the six power-generating reservoirs, Keystone, Fort Gibson, Tenkiller-Ferry, and Eufaula, are storage reservoirs. Therefore, water can be stored in high flow periods and used to generate power at a later point of time. The other two reservoirs, Webbers Falls and Robert S. Kerr, are "run-of-river" reservoirs, which have storage capacity for at most a few hours and, therefore, cannot be used to generate hydroelectric power in high flows because all water has to be released to prevent flooding upstream.

An elevation pool as high as possible is required for hydroelectric power generation. However, because the reservoir has other purposes which require high downstream levels, it is not always possible to operate the reservoir so as to generate the maximum amount of power possible.

For flood control purposes, it is necessary to keep as much free storage space as possible behind the dam so that the excess runoff can be stored safely for the areas downstream. Hydroelectric power generation and recreation on the other hand, require an elevation pool as high as possible behind the dam.[1]

The purpose of this dissertation is to describe the application of a technique called differential dynamic programming for obtaining an optimal storage and release policy for a system of reservoirs, for a 12-month period based upon a fixed rainfall pattern predicted over the 12 months. This technique, which is a modification of Bellman's dynamic programming, is an iterative approximation method for arriving at an optimal policy.[2,3,4]

The releases from a given reservoir depend on the volume of water in storage and the releases from other reservoirs in the system in a given period of time. In this sense, the water allocation problem is a multi-stage decision process in which the decisions taken at each period in time are not independent. Besides we are interested in the entire span of time under study rather than in each period as an entity, this means that the releases from the reservoirs in the system must be optimal in view of the entire span of time under study.

As a mathematical technique, dynamic programming has many desirable characteristics. Unlike linear programming increasing the number of constraints makes the solution easier by limiting the policy space. It also eliminates the necessity for examining all the alternative options at one period of time by taking each stage as it comes and choosing the best decision, out of the alternative available at each time interval reducing in this way the size of the problem.

Dynamic programming also has its drawbacks. Unlike linear programming where the simplex algorithm is a fairly universal technique, dynamic programming does not provide a general purpose algorithm. Each problem has its own characteristics and the proper optimization technique must be found.

The most important disadvantage of dynamic programming is what Bellman calls "the curse of dimensionality," or the large amount of high speed computer memory required to implement the solution of the algorithm. Widespread use of dynamic programming in large and complex real life systems has been deterred because of this requirement for high-speed memory.

Several algorithms have been developed to solve this dimensionality problem of dynamic programming.[5] In general, these algorithms could be classified into two groups: (1) function space approximations[6] and (2) policy space approximations.[7]

In this study, a new and promising technique to alleviate the high speed memory disadvantage of dynamic programming is presented. This technique was developed by Mayne,[8] Jacobson,[9] Gerschwin and Jacobson,[10] and Jacobson and Mayne.[11] Jacobson named this technique "differential dynamic programming."

Differential dynamic programming is an iterative method which starts with a trial solution satisfying predetermined constraints. Applying increments to the trial solution values a new state space is created close to this trial solution.

Then the performance criterion is used to search for an improved solution among the values in the neighborhood of this trial solution. This improved solution is used as the trial solution of the next iteration. This iterative process continues until some convergence conditions are met.

Since the iterative procedure will always move in the direction of better solutions, the procedure will always move to at least a local maximum. This loss of global optimality assurance is the trade off for rather large reductions in computer memory requirements.

Review of the Literature

There have been several interesting applications of dynamic programming to water resources. The first big scale study of a complex water resource system that employed dynamic programming was the work done by Hall and Shephard et al. in which they obtained the optimal operational policy for a system of ten reservoirs, eight with power plants, in Northern California.[12] They combined linear and dynamic programming in the following way: first, they solved a dynamic linear programming model that maximizes the revenue from energy and water supply and combined all the reservoirs. This master program provided the water and energy commitments for each of the reservoirs of the system, and the shadow prices for energy and water. Then, for each individual reservoir they used dynamic programming to determine the optimal operational policy.

Larson and Keckler used the successive approximations method of dynamic programming to maximize the benefits from water for irrigation and power generation for a fictitious system of four reservoirs, which contained both series and parallel connections between reservoirs.[13]

Heidari, Chow, Kokotovic, and Meredith outlined the potentials of differential dynamic programming for optimizing the operation of complex water resources systems.[14] As an example, they applied this technique to the fictitious model of four reservoirs analyzed by Larson and Keckler using the method of successive approximations.[15]

Millham and Rusell employed dynamic programming on a simplified three reservoir system on the Snake and Columbia rivers in the states of Idaho, Oregon, and Washington, for studying the economic losses from power generation and pollution by diverting water from the Snake river to geographical areas other than the Pacific Northwest.[16] Later, Dutton and Millham expanded the previous model and included seven reservoirs, and studied the economic losses by diverting water from the Snake and Columbia rivers to other geographical areas.[17]

State incremental dynamic programming was used by Fults and Hancock to obtain the optimal operational policy for a system of five reservoirs in Northern California.[18]

But the objective was to maximize the firm energy production of two parallel reservoirs, Shasta Reservoir on the Sacramento River and Claire Engle on the Trinity River. Yeh and Trott used a combination of state incremental dynamic programming and the method of successive approximations to determine the optimum design parameters of six reservoirs on the Eel River and Cache Creek in Northern California.[19] First, they decomposed the problem by successive approximations, and they applied state incremental dynamic programming to optimize for each of the parameters under consideration for each of the reservoirs. The objective was to maximize the benefits from firm water supply from the six reservoirs.

Other mathematical techniques have also been used to study water resrouce systems. Dorfman used simulation to evaluate the operation of a fictitious system composed by two reservoirs and one power plant.[20] Fredrich likewise used simulation to analyze and evaluate a complex water resource system composed by the Arkansas, White, and Red rivers in the states of Arkansas, Missouri, Oklahoma, and Texas.[21] This sytem included 23 reservoirs, of which 19 had power plants.

Stephenson employed a network (transportation) model to minimize water conveyance costs in the Orange-Vaal basis in South Africa.[22] He considered a model with three reservoirs and eight rivers and twenty-two uses of the water.

Fitch, King, and Young proposed an algorithm based on maximizing the net returns from operating a multi-purpose system.[23] The net returns are represented by gain functions for each alternative use of the water, which are maximized using a recursive procedure rooted in dynamic programming. As an illustration they solved a fictitious problem consisting of three reservoirs and five treatment plants, and one gain function for recreation, power generation, and water supply maintenance.

O'Neill used a branch and bound procedure of mixed-integer programming to minimize the construction and operation costs of a proposed eight reservoirs system in South East England.[24]

Description of the Arkansas River Basin and the System of Reservoirs

The Arkansas River and its longest tributary, the Canadian River, have their sources in the Sangre de Cristo, Sawatch, and Front Ranges of the Southern Rocky Mountains in Colorado and New Mexico. The Arkansas River flows 1,450 miles southeasterly through Colorado, Kansas, Oklahoma, and Arkansas, and empties into the Mississippi River 575 miles above the Head of Passes, Louisiana. Moving eastward the High Plains become more dissected by stream valleys and give way to the Central Lowlands. Within this formation, drainage becomes more numerous and streamflow increases notably. In particular, the Verdigris and Grand (Neosho) Rivers contribute large flows to the main stem of the Arkansas River.[25]

The watershed covers 160,650 square miles and is about 870 miles long and averages 185 miles in width. The Arkansas River has a total fall of about 11,400 feet with a slope ranging from 110 feet per mile near the source to 0.4 feet per mile near the mouth.[26]

Climate in the Arkansas basin is semiarid to arid in the western part, subhumid in the central part and humid in the eastern part, and is characterized by long hot summers and short cold winters. Annual precipitation averages about 60 inches in the eastern part and decreases rather uniformly westward to about 12 inches in the Western Great Plains, then it increases to 32 inches in the mountains of Colorado and New Mexico. High wind velocities and high evaporation rates are associated with the dry climate of the western half of the region.

The population in the Arkansas basin above Fort Smith, Arkansas was about 3.3 million in 1970. Agriculture is the major economic activity with cattle, wheat, cotton, grain sorghums, and rice as the chief farm products.

Manufacturing represents a growing and increasingly important segment of the regional economy with food and lumber products, air-craft assembly, iron and steel milling, oil and other petrochemical refining and glass manufacturing as the most important industries.

Outdoor recreation and tourism industries have grown in the last 25 years and are expected to continue expanding. Millions of visitors come into the region every year, and their expenditures are the major source of revenue for the enterprises related to the travel and recreation industry.

There are many mineral resources in the region, petroleum and natural gas being the most important. Other significant minerals produced are zinc, lead, germanium, gold, silver, molybdenum, bauxite, copper, cadmium, mercury, tungsten, tin, iron, and manganese. There are important coal reserves, but coal mining is competitive only in localized areas. The region also has extensive deposits of cement, building stone, ceramic clays, sand, gravel, and salt.[27]

The McClellan-Kerr Arkansas River Navigation System is a major feature of the water resources development in the Arkansas Basin in Oklahoma, Arkansas, and Kansas. It extends from the Mississippi River to near Tulsa, Oklahoma.

In the Oklahoma portion of the Arkansas River Basin, the U. S. Army Corps of Engineers constructed and operates seven major upstream lakes: Keystone, Fort Gibson, Webbers Falls, Tenkiller-Ferry, Eufaula, Robert S. Kerr, and Oologah. The first six have power plants and only Oologah does not. A brief description of the main characteristics of the six reservoirs included in this study is as follows:

(1) Keystone Reservoir is located at mile 538.8 on the Arkansas River, in the Northwest corner of Tulsa County. The dam is about two miles downstream from the mouth of the Cimarron River and about 15 miles west of Tulsa, Oklahoma. Its purposes are flood control navigation,

hydroelectric power, fish and wildlife conservation, and water supply. The power plant has two generating units with an installed capacity of 70,000 kw.

The lake has a surface of 26,300 acres and a total storage capacity of 1,879,000 acre-feet, of which 1,216,000 acre-feet is available for the storage of flood waters. The conservation pool contains 351,000 acre-feet of dead storage.

(2) Fort Gibson Reservoir is located at mile 7.7 on the Grand (Neosho) River in Wagoner, Cherokee, and Mayer Counties, about five miles northeast of Fort Gibson, Oklahoma. Its purposes are flood control and hydroelectric power. The power plant has four generating units with an installed capacity of 45,000 kw.

Normally the lake has a surface area of 19,900 acres and retains 365,200 acre-feet of dead storage, and 53,900 acre-feet for power pondage.

(3) Webbers Falls Reservoir (Lock No. 16 in the Arkansas Navigation System) is located at mile 432.3 on the Arkansas River about five miles northwest of Webbers Falls in Muskogee County, Oklahoma. Its purposes are navigation, hydroelectric power, recreation, and fish and wildlife conservation. The power plant has three

generating units with an installed capacity of 66,000 kw.

The lake has a surface of about 10,900 acres and a storage capacity of 165,200 acre-feet, of which 135,000 acre-feet are dead storage and 30,000 acre-feet for power generation.

(4) Tenkiller-Ferry Reservoir is located at mile 12.8 on the Illinois River in Sequoyah County about 22 miles southeast of Muskogee, Oklahoma. Its uses are flood control and hydroelectric power. The power plant has two generating units with an installed capacity of 34,000 kw.

The lake has a surface area of 12,700 acres, with a power storage of 283,100 acre-feet, a dead pool for powerhead of 358,300 acre-feet, an additional capacity of 588,600 acre-feet is available for floodwaters storage.

(5) Eufaula Reservoir is located at mile 27 on the Canadian River, about 13 miles east of Eufaula in McIntosh County, Oklahoma. Its purposes are flood control, inland navigation, hydroelectric power, water supply, and fish and wildlife conservation.

It is the fifteenth largest man-made lake in the United States with a total storage capacity of 3,848,000 acre-feet. Of this capacity, 1,470,000 acre-feet are for flood control and 1,481,000 acre-feet are allocated for power, and 897,000 acre-feet of dead storage for powerhead and sedimentation. The power plant has three generating units with an installed capacity of 90,000 kw.

(6) Robert S. Kerr Reservoir (Lock No. 15 in the Arkansas Navigation System) is located at mile 395.4 on the Arkansas River, about eight miles south of Sallisaw, Oklahoma, in LaFlore and Sequoyah Counties. Its purposes are hydroelectric power, inland navigation, recreation, and fish and wildlife conservation.

The lake has a storage capacity of 493,600 acre-feet of which 79,500 acre-feet are for power pondage and 414,000 acre-feet for dead pool. The power plant has four generating units with an installed capacity of 110,000 kw.[28]

Analysis

The optimization procedure is performed using a deterministic discrete approach so that the future volumes of water in storage, the inflows, and the net evaporation rates are known quantities at each month for each of the six reservoirs. Three different situations were considered: a critical period or drought year, a year with average flows, and a high rainfall year. These are defined respectively as the sequence of twelve months in which the reservoirs displayed pronounced drought cycles in the hydrologic record, average and average plus one standard deviation.

In order to determine if the optimal solution corresponds to a global maximum for each of these three situations, the analysis is performed using two different trial or initial solutions or policies. In this study, a trial solution is the set of twelve storage and release values for each of the six power-generating reservoirs in the system. This string of values is also called a trajectory.

If the optimal release policies for the six reservoirs are identical for both trial trajectories, a global maximum has been attained. Otherwise the solution reached by each trajectory is a local maximum.

In each of the three situations considered, there was a substantial divergence between the optimal storage and release policies for the two different trajectories. Interestingly enough, in each case the optimal solution did not differ by very much in the value of the hydroelectric power generated. For example, in the high inflows period the difference in the value of the hydroelectric power was 0.10%; in the average period, it was only 0.0092%; and in the critical period, 0.37%.

This leads to the hypothesis, which we were not able to prove, that the value of the hydroelectric power generated will be approximately the same over a wide range of optimal policies.

Interpolation Using Spline Functions

At different stages of the optimization procedure, it is necessary to retrieve values of functions which have been stored as tables. This action requires the use of an interpolation procedure.

The interpolation technique used in this study is a cubic spline fitting which is a type of piecewise polynomial fitting. Using this technique, the data set is divided into a number of nonoverlapping intervals and the points in each interval are fitted by a polynomial.

Organization of this Study

Chapter II presents the discrete differential dynamic programming approach and the application of this technique to water resources systems analysis. In Chapter III, the problem is set up as a multi-stage sequential decision process which can be optimized using discrete differential dynamic programming. Chapter IV presents the optimal storage and release policies under the different hydrological conditions considered in the analysis. Chapter V contains the summary and conclusions of the study.

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CHAPTER II

THE THEORETICAL MODEL

This chapter presents the discrete differential dynamic programming approach for optimizing a multi-stage and multi-variable decision process.

Following, an example illustrates the application of this optimization technique to a water resources system.

The Discrete Differential Dynamic Programming Approach

Dynamic programming, developed by Richard E. Bellman, is a sequential technique for optimizing a multi-stage decision process.[1,2] The optimum policy is obtained by calculating the optimum solution at any given stage as a function of the optimum solution of the immediately preceding stage, for every possible state of the system at each stage.

Discrete differential dynamic programming is a variant of dynamic programming in which a nominal or trial trajectory is initially employed as an approximation of the optimal policy. Improvements are made iteratively, by imbedding the solution at any given iteration within the function defining the optimum for the next one. A perturbation technique with a relatively concise grid is employed to obtain the policy for the next iteration, where the grid has as many dimensions as there are decision or state variables.

The index which defines stage is the symbol k, that determines the order in which events occur in time. Since there are 12 months or time increments considered in the analysis $k = 1, 2, \ldots, 12$.

Let x denote the state variable indicating the level of storage in each of the six reservoirs, then a six dimensional state vector is employed in the analysis. At a given month x(k) is the state vector at the end of stage k.

The control or decision variable is denoted by u, and it refers to the volume of water released from the dam during a given month. Since in the system there are four storage reservoirs, Keystone, Fort Gibson, Tenkiller-Ferry, and Eufaula, on which decisions are to be made, the control variables are denoted by a four dimensional control vector u(k). In a time span of 12 months, 48 decisions need to be made for this sytem of reservoirs.

The domain of values for the state variables is limited to the sex X bounded by the maximum and minimum allowable storages for each of the six reservoirs. Also, the values of the control variables are delimited to the set U(k) bounded by the permitted minimum and maximum releases from each dam in a given month. Then,

$$\begin{aligned} x(k) &\in X \\ x(k) &\in U(k). \end{aligned} \tag{1}$$

Besides, in this study the six reservoirs start and finish the analysis at the same level of storage, imposing an initial and a final boundary condition on the values of the state vector as follows:

$$x(0) = x(12). \tag{2}$$

In discrete differential dynamic programming, it is assumed that the optimal control $u^o(k)$, $k = 0, 1, \ldots, 11$, is unknown, but a sequence of nonoptimal control variables $\overline{u}(k)$, $k = 0, 1, \ldots, 11$, which satisfies the control constraints in (1) is called a trial solution or policy; then using these values the state variables are calculated for the 12 stages under analysis. The sequence of values of the state vectors satisfying the state constraints in (1) and boundary conditions (2) is called a trial trajectory, and it is designated by $\overline{x}(k)$, $k = 0, 1, \ldots, 12$.

The system of equations which explains the dynamic behavior of the six reservoirs in the system is composed by six difference equations like the following:[3]

$$x_i(k) = x_i(k-1) + in_i(k-1) - u_i(k-1) - ev_i(k-1), \quad i = 1, 2, \ldots, 6. \tag{3}$$

where:

$x_i(k)$ = volume of water stored in the i^{th} reservoir in period k,

$x_i(k-1)$ = volume of water stored in the i^{th} reservoir in period k-1,

$in_i(k-1)$ = inflow into the i^{th} reservoir in period k-1,

$u_i(k-1)$ = volume of water released from the i^{th} reservoir in period k-1, and

$ev_i(k-1)$ = volume of water evaporated from the i^{th} reservoir in period k-1.

Equation (3) is known as the "principle of continuity" or "storage equation."[4] When the releases, inflows, and evaporation are defined properly and measured in standardized units, this equation is appropriate for storage accounting when the length of the period k in consideration is long enough so that the travel time through the reservoirs in the system is not significant. It should be noted that the definition of inflow implies that all diversions into the reservoirs are added to the natural inflow to obtain the inflow volume. The standardized unit of measurement employed in this study is a kilo-acre-feet (KAF), the volume of water contained in a surface of one thousand acres one foot deep.

In the form in which this problem is set up, we are searching for the best decision at stage k-1 in order to bring the system to a specific value of the state vector at stage k from a known value of the state vector at stage k-1.

The index h defines the iteration number during the iterative optimization procedure employed in this analysis.

Let $L[x(k-1), u(k-1)]$ represent the return or dollar value of the hydroelectric power generated in one time period as the result of decision u(k-1) made at stage k-1 with the system in state x(k-1). Also, let J represent the sum of returns from the system over a time horizon of 12 months, then

$$J = \sum_{k=1}^{12} L[x(k-1),\ u(k-1)]. \tag{4}$$

Let $J_h[x(k)]$ be the maximum total return from stage 0 to stage k when the system is x(k) and the analysis is at iteration h. If the optimization procedure is carried-out forward in time and the objective is maximize the return (4) over k stages, then applying Bellman's principle of optimality, the following variational performance equation is obtained:[5]

$$J_h[x(k)] = \max_{u(k-1) \in U(k-1)} \{L[x(k-1),\ u(k-1)] + J_h[x(k-1)]\}. \tag{5}$$

Now, if Equation (3) is solved for x(k-1), the value of the state vector at stage k-1 can be expressed as a function of the value of the state vector at stage k and the control variable values at stage k-1 in the following way:

$$x(k-1) = \theta[x(k),\ u(k-1)]. \tag{6}$$

Substituting (6) into (5), the following variational performance equation is obtained:

$$J_h[x(k)] = \max_{u(k-1) \in U(k-1)} \{ L[x(k),\ u(k-1)] + J_h[x(k-1)] \} \ . \tag{7}$$

Equation (7) may be solved for every x(k) as a function of u(k-1) only. The solution of (7) for a given value of the state vector in the sex X in (1) provides an optimal u(k-1), or the optimal decision that should be made for some state vector at stage k-1 to bring the system to a given value of the state vector at stage k.

Substituting the trial trajectory values $\overline{x}(k)$, k = 0, 1, ..., 12, and policy $\overline{u}(k)$, k = 0, 1, ..., 11, in Equation (4), it is obtained the total return associated with this trial trajectory and policy over a span of time of 12 months. This return is designated by $\overline{J}$ may not be the optimum return for the system.

$$\overline{J}_h = \sum_{k=1}^{12} L[\overline{x}(k-1),\ \overline{u}(k-1)] \tag{8}$$

where h indicates the iteration number.

Let $\delta x_{in}(k)$ represent a state increment value or perturbation associated with the n^{th} state variable, approximately equal to 10% of the smallest power storage capacity among the six reservoirs in the system. These perturbations form a six dimensional vector that when added to the trial trajectory define a subdomain for the state variables in the neighborhood, i.e., close to, the trial trajectory values. Let T be the total number of assumed increments for the state variable domain, and DX_t, $t = 1, 2, \ldots, T$, the actual increment value for the state variables, then any of the n^{th} components $\delta x_{in}(k)$ for $n = 1, 2, \ldots, 6$ and $k = 1, 2, \ldots, 12$, can take on any one value DX_t, $t = 1, 2, \ldots, T$, which is the t^{th} assumed increment for the state variable domain.

$L \Delta x_i(k)$ denote the number of increment vectors at stage k, then the total number of these vectors is given by T^n, or the total number of assumed state increments raised to a power equal to the number of state variables. In this study $T = 3(+DX, 0, -DX)$, and since there are four storage reservoirs on which decisions are to be made, the total number of increment vectors at stage k is 81 ($T^n = 3^4$). The perturbation vector is designated by

$$x_i(k) = \begin{bmatrix} x_{i1}(k) \\ x_{i2}(k) \\ \cdot \\ \cdot \\ \cdot \\ x_{i6}(k) \end{bmatrix} \qquad \begin{matrix} i = 1, 2, \ldots, 81 \\ k = 1, 2, \ldots, 12 \end{matrix} \tag{9}$$

When added to the trial trajectory at stage k these increment vectors form an n-dimensional subdomain that is designated as S(k).

Two examples of subdomains are presented in Figures 1 and 2 for two and three state variables.

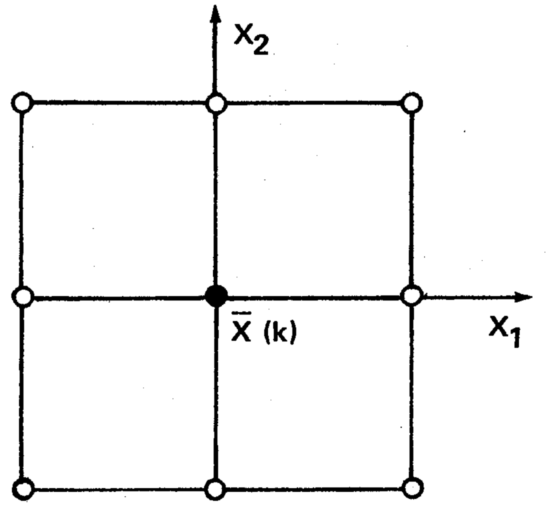

Figure 1. Subdomain S(k) Indicating the Nine Lattice Points Around the Trial Trajectory Value at Stage k, for a Problem of Two State Variables and Three Incremental Values (DX = +1, DX = 0, and DX = -1)

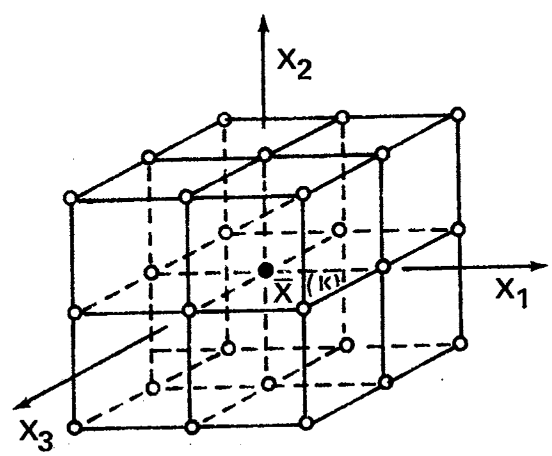

Figure 2. Subdomain S(k) Indicating the Twenty-Seven Lattice Points Around the Trial Trajectory Value at Stage k, for a Problem of Three State Variables and Three Incremental Values (DX = +1, DX = 0, and DX = -1)

Figure 1 shows the subdomain of nine lattice points around the trial trajectory value $\overline{x}(k)$ at stage k for a problem with two state variables and three incremental values. Figure 2 presents the subdomain of twenty-seven lattice points around the trial trajectory value $\overline{x}(k)$ at stage k, for a system with three state variables and three incremental values. Note in these figures that one of the incremental values has to be zero since the trial trajectory is always in the subdomain.

In differential dynamic programming, all subdomains S(K) for $k=1, \ldots, K$, considered together form a "corridor," that is, designated as C_h. In Figure 3 is hown the corridor C_h, at iteration h, in the neighborhood of a trial trajectory for a problem with one state variable, three incremental values, five stages, and an initial and final boundary condition. Using this approach, the corridor C_h is used as the set of admissible values for the state variables, and the optimization procedure constrained to these values.

When the optimum-searching procedure is constrained to certain values and the state variables do not vary continuously, a "lattice search" procedure is defined.[6,7] The only requirements of this procedure are that the number of points under consideration be finite and arrangeable in some order that will make the performance criterion unimodal.[8]

Let $x_n^*(k)$, $n=1, 2, \ldots, 6$, represent the value of the state vector composed by the trial trajectory values plus

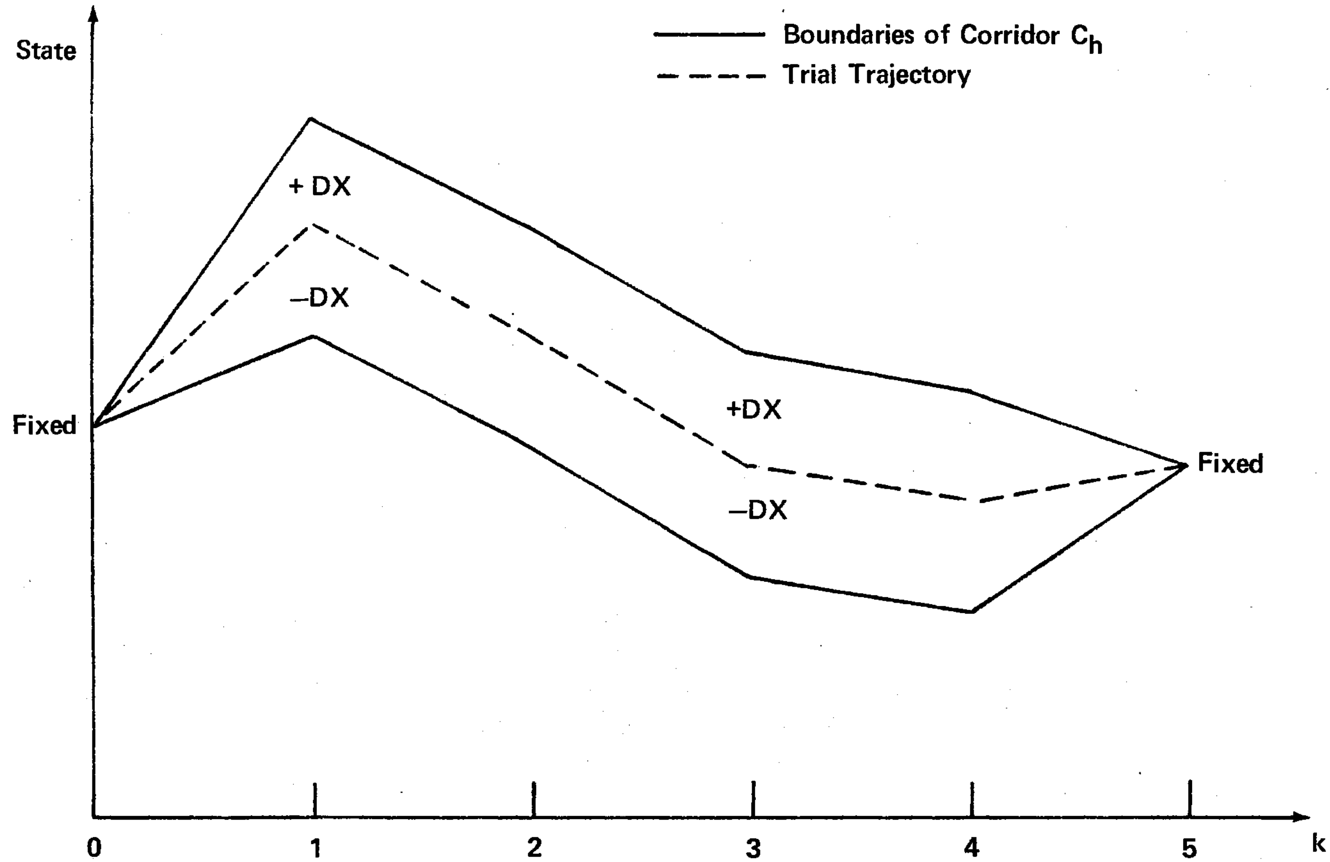

Figure 3. Trial Trajectory and the Boundaries of Corridor C_h in the Neighborhood of it, for a Problem with One State Variable, Three Incremental Values (DX = +1, DX = 0, DX = -1), Five Stages, and an Initial and Final Boundary Conditions

the perturbations, then the variational performance Equation (7) can be expressed in terms of the trial trajectory values and the increments by letting

$$x^*_n(k) = \overline{x}(k) + \delta x_{in}(k), \qquad \begin{matrix} k = 1,\ 2,\ \ldots,\ 12 \\ i = 1,\ 2,\ \ldots,\ 81 \end{matrix} \tag{10}$$

Substituting these values in Equation (7), the following variational function is obtained:

$$J^*_h[x(k)] = \max_{u(k-1) \in U(k-1)} \{[L\ \overline{x} + \delta x(k-1),\ u(k-1)] + J^*_h[\overline{x} + \delta x(k-1)]\} \tag{11}$$

where J^*_h represents the maximum total return at iteration h for the corricor C_h.

Using the total return value as a measure of convergence, the iterative analysis proceeds in the following way: if the return associated with the state variables with the increments (J^*) is greater than the total return associated with the trial trajectory ($\overline{J}$) by a certain predetermined convergence index value, the values of $x^*(k)$ and $u^*(k)$, $k = 1, 2, \ldots, 12$, are saved to define a new trajectory and repeat the process again until no improvements in total return can be achieved in relation to the convergence criterion. Given this characteristic of the discrete differential dynamic programming, it can be classified as an approximation in policy space algorithm.[9]

In the course of the iterative process, the size of the corridor may be changed gradually by choosing different

$\{DX_t\}_h$, $t = 1, 2, 3$. If the corridor size is kept constant at every iteration and no improvement is observed in the total return, it is suggested that $\{DX_t\}_h$, $t = 1, 2, 3$, be reduced starting at the $(h+1)^{th}$ iteration, and the process is continued with the new corridor size until the policy values, or release values coincide for two consecutive iterations. Then the corridor size is further reduced starting at the next iteration, and the process is repeated until a predetermined convergence condition is met.

In Figure 4 and following Jacobson, the overall computational algorithm for discrete differential programming is presented.[10]

Since the optimization search procedure of the algorithm is constrained to the quantized state variables values defined by the corridor C_h, the algorithm is designed to drastically reduce the number of grid points over which the optimization search procedure must search for the optimal trajectory. The rationale of the algorithm is based upon the concept that with a given starting trial solution, the search is carried out only on a certain constrained region of the state space around this trial trajectory. If a new and better solution is contained in this "corridor" of the state space surrounding the trial trajectory, this new solution is used as the basis for constructing a new "corridor" to be searched. This process is repeated until convergence is reached.

The differential dynamic programming algorithm differs

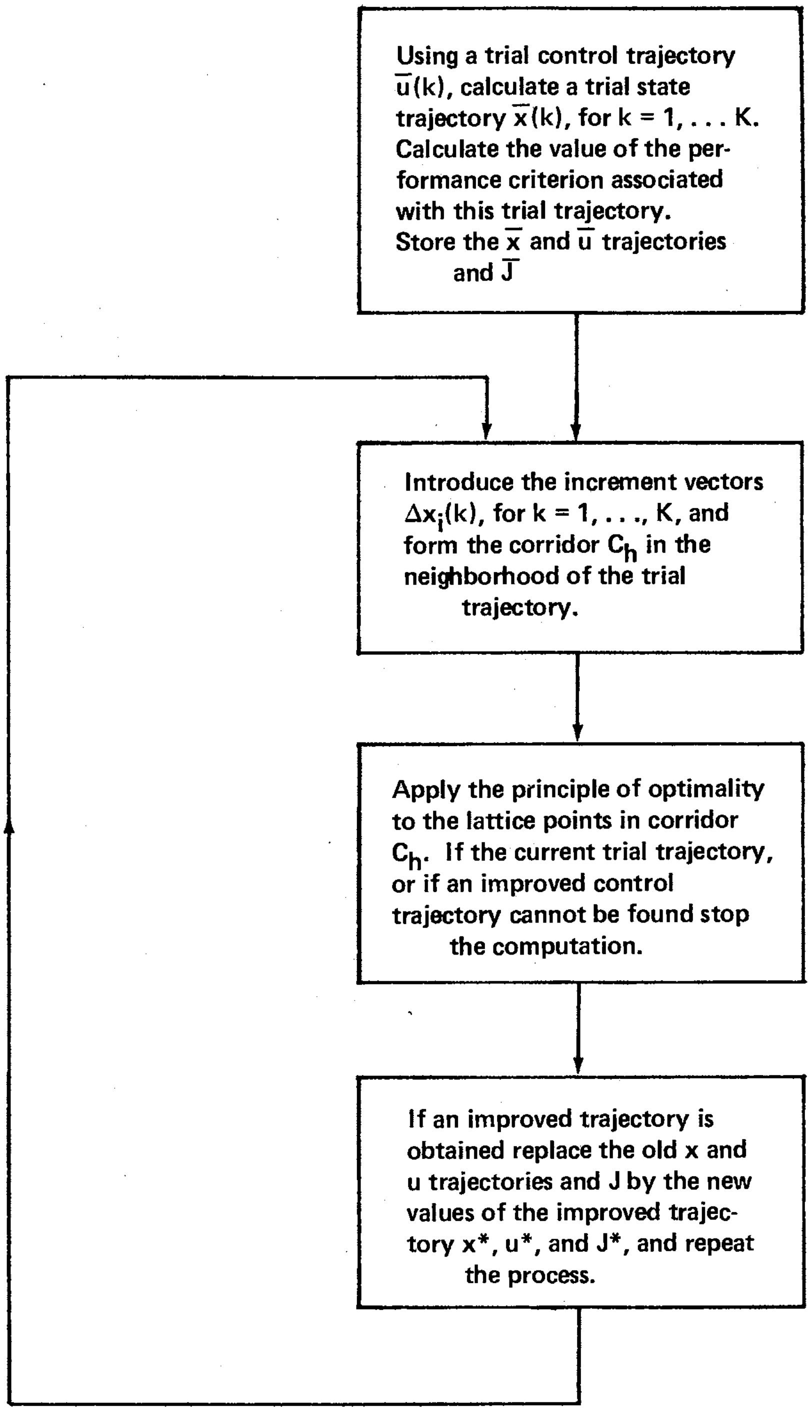

Figure 4. Overall Computational Algorithm for Discrete Differential Dynamic Programming

from classical dynamic programming in that, unlike the classical approach, it searches only in the immediate vicinity of a specified trial solution instead of all state space. The algorithm then uses the best solution found in the restricted space to form a new trial trajectory to iterate upon. Repeated iterations will find "a best" solution, unlike the "best solution" found by dynamic programming, that under the assumption of a sufficient grid fineness, is always a global maximum, differential dynamic programming may find only a local optimum solution.

The biggest obstacle which has prevented a widespread use of dynamic programming is the high speed memory requirement of the algorithm. This requirement refers to the number of locations in the high-speed access memory (core memory) which must be available during the computations. In addition to the locations needed for the program, the compiler, and other special functions, locations are required to store the values of the performance criterion values for all the feasible values of the state variables at a single stage. In general, this is done by storing one value of the performance criterion for every feasible quantized value of the state variable, and using an interpolation procedure for the non-quantized values. The minimum number of locations required by the classical dynamic programming is given by:

$$M = 2K \prod_{i=1}^{n} Q_i \tag{12}$$

where:

M = number of core locations,

K = number of stages,

Q_i = number of quantized values of the i^{th} variable, and

n = number of state variables.

In order to solve a problem with four state variables, ten stages and one hundred quantization levels for each variable ($Q_i = 100$, for $i = 1, 2, 3, 4.$), the number of locations required are:

$$M = 2(10)^9. \tag{13}$$

This number exceeds the total high-speed storage capacity of any existing computer.

Invertible Systems

If the control or decision variables can be expressed in terms of the values of the state vectors at stage k-1 and k the system Equation (3) can be written in the following form:

$$u(k-1) = \Psi[x(k),\ x(k-1)]. \tag{14}$$

A system of equations that can be expressed in this form was called an "invertible system" by Heidari, Chow, Kokotovic, and Meredith.[11]

An invertible system permits to calculate the optimal value of the control variable at stage k-1 in order to bring the system to a specific value of the state vector at stage k from a known value of the state vector at stage k-1. Figure 5 shows the possible decisions for a system of one state variable and three incremental values in order to take the system from the three points defined in the subdomain S(k-1) to a given value of the state variable in the subdomain S(k). The values of the control variables must be checked for feasibility in relation to the constraints at that stage.

In general, when the optimization is being performed in the states of the corridor C_h the use of the invertibility property provides with T^n possible values for the control variables, which when applied to the states in the subdomain S(k-1) will bring the system to x(k).

Furthermore, for an invertible system it is possible to assume first an admissible trial trajectory $\overline{x}(k)$, $k=0$, 1, ..., 12, and then use these values and the system Equation (12) to calculate the trial policy $\overline{u}(k)$, $k=0$, 1, ..., 11.

The Larson and Keckler four reservoirs system is presented here as an example of an invertible system.[12] The diagrammatic representation of this system is shown in Figure 6. The equations explaining the dynamic behavior of this system are the following:

$$x_1(k) = x_1(k-1) + in_1(k-1) - u_1(k-1) - ev_1(k-1)$$

$$x_2(k) = x_2(k-1) + in_2(k-1) - u_2(k-1) - ev_2(k-1)$$

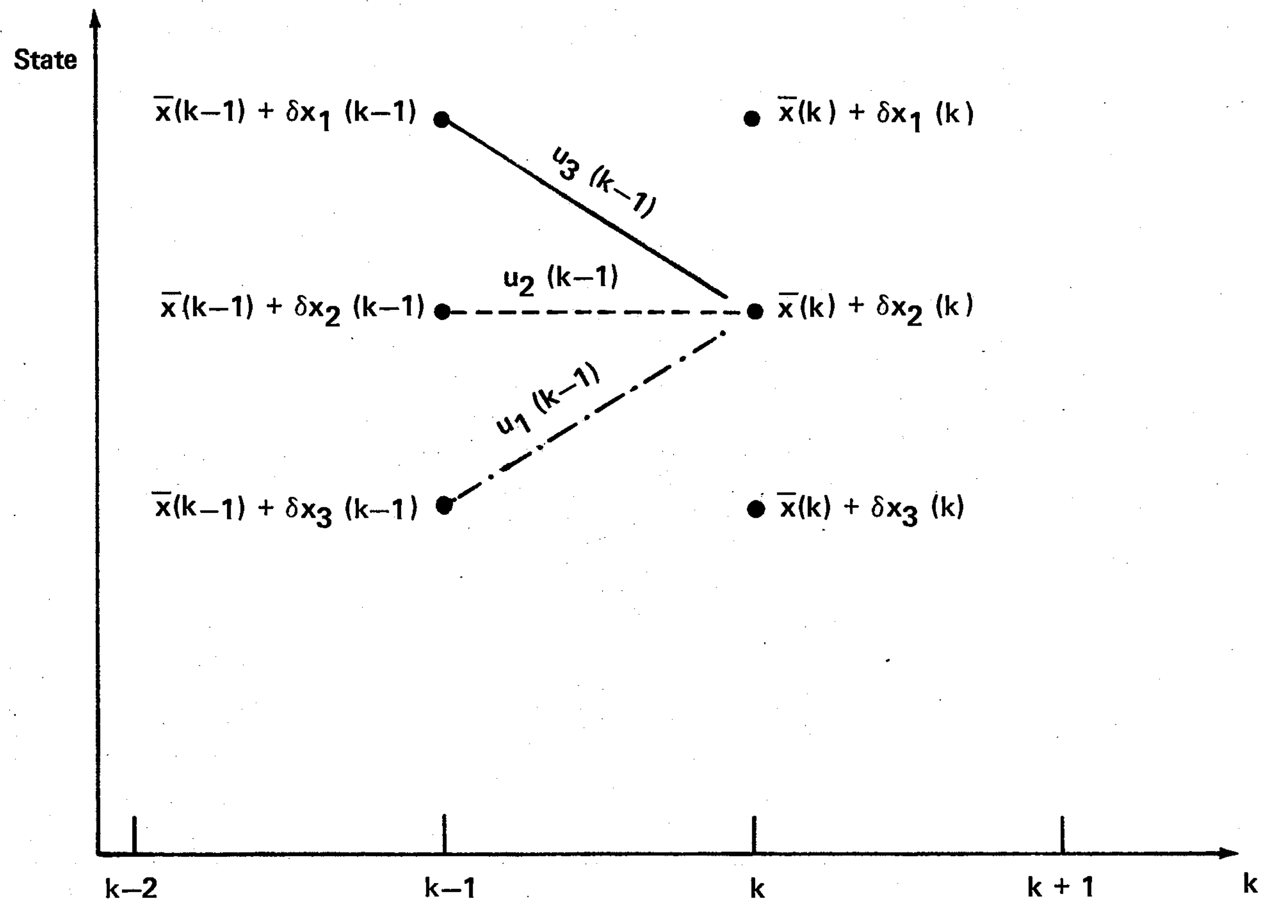

Figure 5. Possible Controls Leading to State $\overline{x}(k) + x_2(k)$ from Stage k-1, for a System with $n = 1$ and $T = 3$

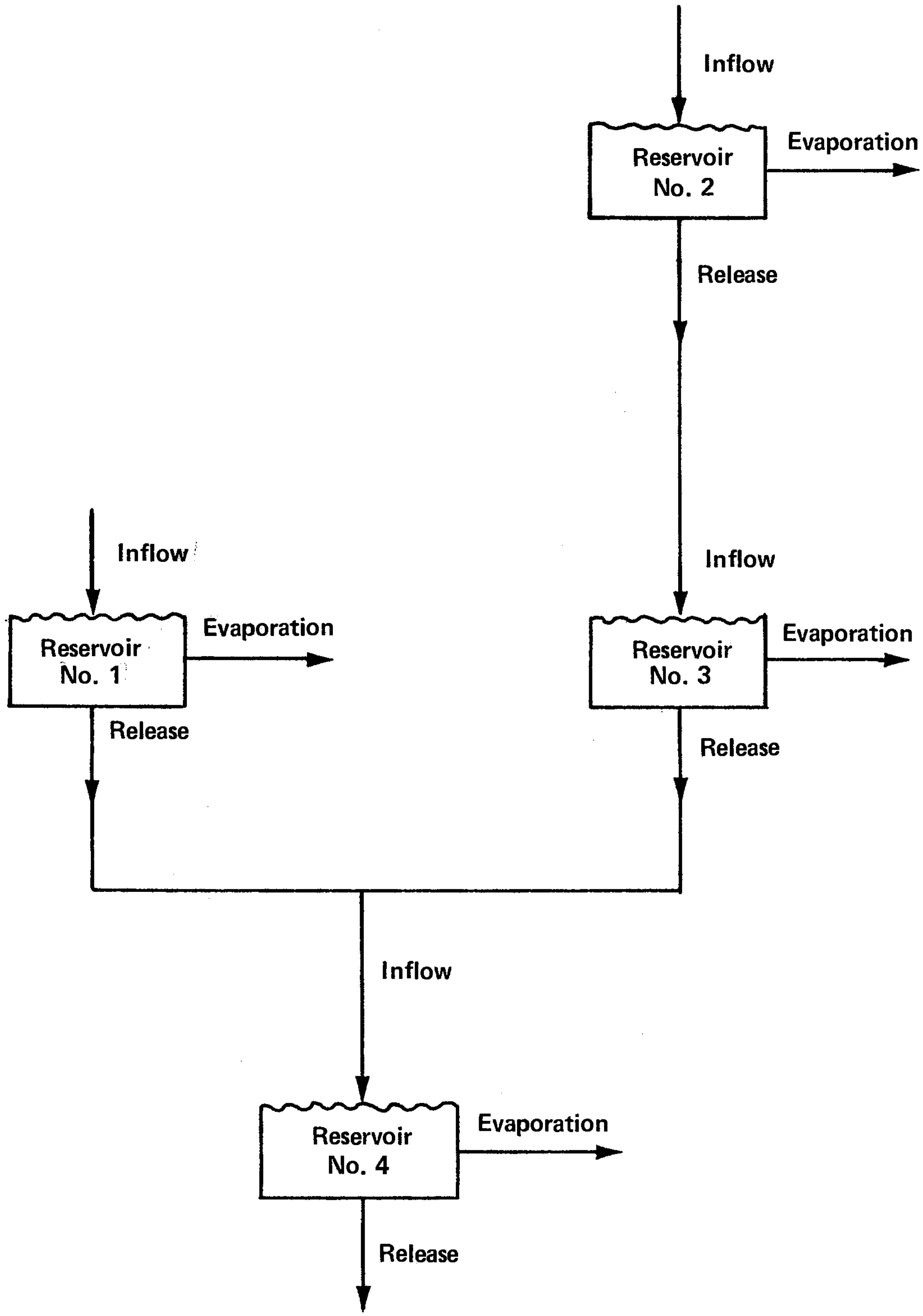

Figure 6. Diagrammatic Representation of the Larson and Keckler Four Reservoirs System

$$x_3(k) = x_3(k-1) + in_3(k-1) - u_3(k-1) - ev_3(k-1) + u_2(k-1)$$

$$x_4(k) = x_4(k-1) + in_4(k-1) - u_4(k-1) - ev_4(k-1) + u_1(k-1) + u_3(k-1). \tag{15}$$

Using a deterministic approach, the inflows and evaporation rates are known quantities at each stage; besides, if a forward algorithm is employed, the terms $x_i(k-1)$ $i = 1, 2, 3, 4$, are also known. Then, in matrix form, it is expressed as:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 1 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} + \begin{bmatrix} C \end{bmatrix}. \tag{16}$$

where $x_i = x_i(k)$ and $u_i = u_i(k-1)$, for $i = 1, 2, 3, 4$. Expressed in a more concise form:[13]

$$x = Bu + C. \tag{17}$$

We can solve Equation (17) in terms of u as proposed in Equation (14) under the assumption that matrix B is invertible, or nonsingular.

From matrix algebra we know the square matrix B, which is lower triangular ($b_{ij} = 0$ if $j > i$), is invertible if and only if the diagonal coefficients are different from zero ($b_{ij} \neq 0$, $i = j$).[14] The proof of this theorem is found in the fact that any matrix is nonsingular if the value of the determinant is different from zero. In this example, it is different from zero.

When the matrix B is invertible, we can express the control variables as a function of the state variables as follows:

$$u = B^{-1}x - B^{-1}C. \tag{18}$$

According to these findings, the Larson and Keckler system can be classified as an invertible system.

FOOTNOTES

[1]R. E. Bellman, Dynamic Programming (Princeton, New Jersey, 1957).

[2]R. E. Bellman and S. E. Dreyfus, Applied Dynamic Programming (Princeton, New Jersey, 1962).

[3]S. Goldberg, Introduction to Difference Equations (New York, New York, 1958), pp. 50-54.

[4]H. M. Morris and J. M. Wiggert, Applied Hydraulics in Engineering (2nd ed., New York, New York, 1972), p. 418.

[5]Bellman, Dynamic Programming, page 10. The principle of optimality states that:

> An optimal policy has the property that whatever the initial state and decision (i.e., control) are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision.

[6]J. Kiefer, "Optimum Sequential Search and Approximation Methods Under Minimum Regularity Assumptions," Journal of the Society for Industrial and Applied Mathematics, V (September, 1957), pp. 124, 125.

[7]D. J. Wilde, Optimum Seeking Methods (Englewood Cliffs, New Jersey, 1964), pp. 27, 28.

[8]Bellman and Dreyfus, pp. 152, 153. A function $f(x)$ is unimodal in an interval $[0,b]$ if there is a number x_o, $0 \leq x_o \leq b$, such that $f(x)$ is either strictly increasing for $x \leq x_o$, and strictly decreasing for $x \geq x_o$, or else strictly increasing for $x \leq x_o$, and strictly decreasing for $x \geq x_o$.

[9]H. M. Wagner, Principles of Operations Research Englewood Cliffs, New Jersey, 1969), pp. 383-385.

[10]D. H. Jacobson, "New Second Order and First Order Algorithms for Determining Optimal Control," Journal of Optimization Theory and Applications, II (November, 1968), p. 424.

[11]M. Heideri, V. T. Chow, P. V. Kokotovic, and D. D. Meredith, "Discrete Differential Dynamic Programming Approach to Water Resources Systems Organization," Water Resources Research, VII (1971), pp. 276, 277.

[12]R. E. Larson and W. B. Keckler, "Applications of Dynamic Programming to the Control of Water Resources Systems," Automatica, V (1969), pp. 19, 20.

[13]The matric C contains the known values for $x_i(k-1)$, $in_i(k-1)$, and $ev_i(k-1)$, for $i = 1, 2, 3, 4$.

[14]P. C. Shields, Elementary Linear Algebra (New York, New York, 1968), pp. 148, 165.

CHAPTER III

THE EMPIRICAL MODEL

In this chapter, the problem is set up as a multi-stage sequential decision process amenable to optimization by discrete differential dynamic programming. The system of equations is represented by six difference equations, which explain the dynamic behavior, and interrelations among the reservoirs according to the principle of continuity.

The monthly inflows data are presented for each reservoir. Three levels of monthly inflows are defined as "critical period inflows," "average inflows," and "high inflows," to be used later in the analysis of the behavior of the system under different hydrological conditions.

Next, the net evaporation rates data for each reservoir, and a method to determine the volume of water evaporated at each month are presented.

A methodology to determine the generation of hydroelectric energy, and its classification as on-peak and off-peak is developed for the two types of reservoirs included in the analysis.

Finally, the performance criterion and the storages and releases constraints for the system are presented and discussed.

The System of Equations

The system of six reservoirs to be analyzed in this study is composed by the Keystone, Fort Gibson, Webbers Falls, Tenkiller-Ferry, Eufaula, and Robert S. Kerr reservoirs on the Arkansas River basin in Eastern Oklahoma. The diagrammatic representation of the system is shown in Figure 7.

In order to use the continuity principle to analyze the storage behavior of the reservoirs, all the variables included in the analysis must be measured on a standard unit. In this study, the standardized unit of measurement is the kilo-acre feet or the volume of water contained in a surface of 1,000 acres one foot deep. This unit is abbreviated as KAF. All the variables related to water like storage, releases, inflows, and evaporation are measured with this unit.

Following the principle of continuity and the relations among the reservoirs shown in Figure 7, the system of difference equations describing the dynamic behavior of this system of six reservoirs is:

Keystone Reservoir (Reservoir No. 1)

$$x_1(k) = x_1(k-1) - u_1(k-1) + in_1(k-1) - ev_1(k-1) \tag{1}$$

Fort Gibson Reservoir (Reservoir No. 2)

$$x_2(k) = x_2(k-1) - u_2(k-1) - in_2(k-1) - ev_2(k-1) \tag{2}$$

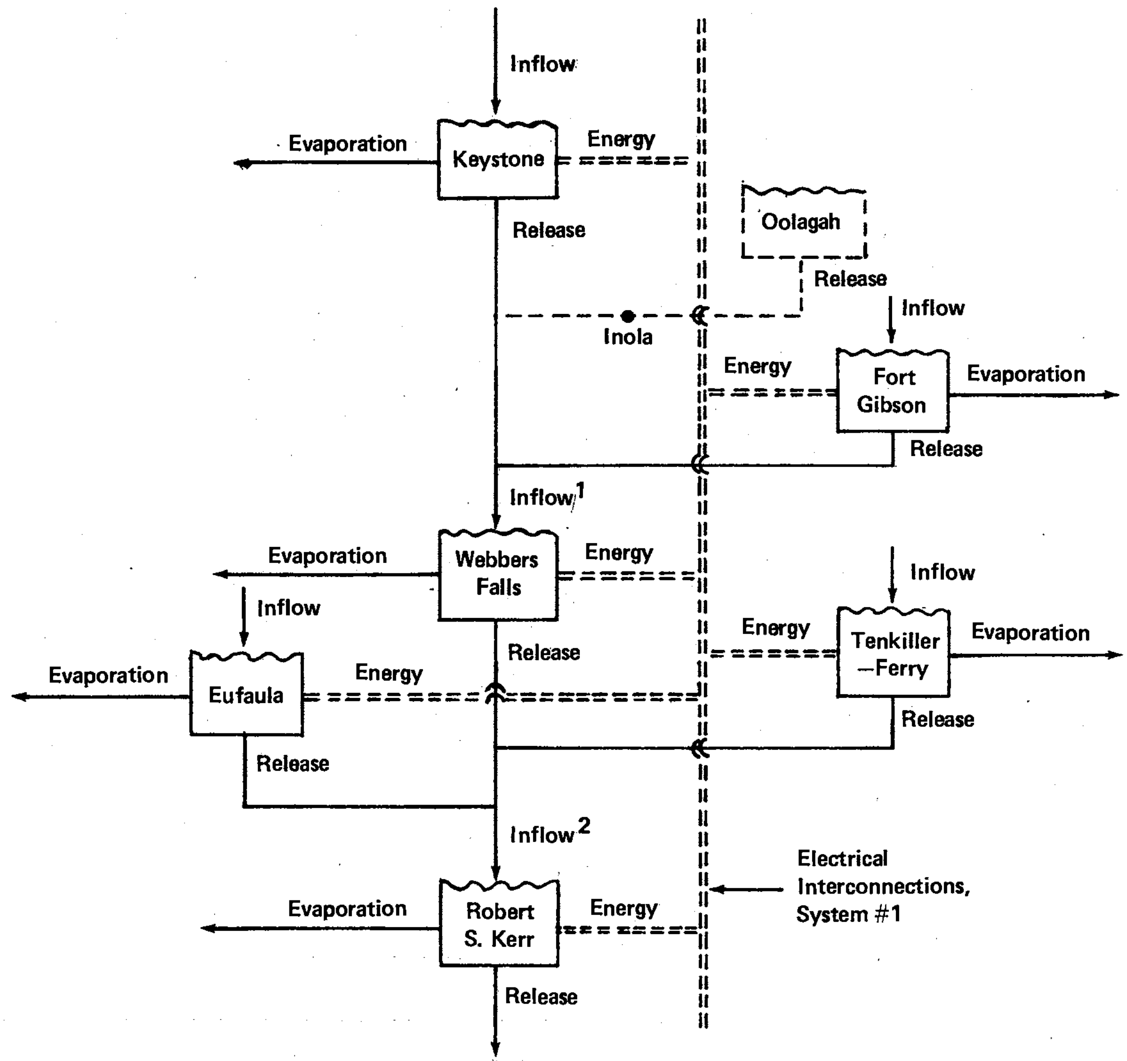

Figure 7. Diagrammatic Representation of the System

Tenkiller-Ferry Reservoir (Reservoir No. 3)

$$x_3(k) = x_3(k-1) - u_3(k-1) + in_3(k-1) - ev_3(k-1) \tag{3}$$

Eufaula Reservoir (Reservoir No. 4)

$$x_4(k) = x_4(k-1) - u_4(k-1) + in_4(k-1) - ev_4(k-1) \tag{4}$$

Webbers Falls Reservoir (Reservoir No. 5)

$$x_5(k) = x_5(k-1) - u_5(k-1) + in_5(k-1) - ev_5(k-1)$$
$$+ u_1(k-1) + u_2(k-1) + man(k-1)^1 \tag{5}$$

Robert S. Kerr Reservoir (Reservoir No. 6)

$$x_6(k) = x_6(k-1) - u_6(k-1) + in_6(k-1) - ev_6(k-1)$$
$$+ u_3(k-1) + u_4(k-1) + u_5(k-1). \tag{6}$$

In matrix notation, these equations look like as follows:

$$\begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \\ X_6 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 1 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \end{bmatrix} + \begin{bmatrix} C \end{bmatrix}$$

Since a deterministic approach is being used, the matrix C contains the known values for $x_i(k-1)$, $in_i(k-1)$, $ev_i(k-1)$, for $i = 1, 2, 3, 4, 5, 6$, and the Inola Gage flows.

In a more concise form

$$x = Bu + C.$$

The determinant of matrix B is different from θ, then this matrix is invertible. The equation can be expressed in terms of the control variables

$$u = B^{-1}x - B^{-1}C.$$

The six difference equations describing the system considered in this study form an invertible system.[2]

The Monthly Inflows

In order to analyze the sensibility of the optimal operation of the system under different hydrological conditions, the analysis is performed for three different levels of monthly inflows.

The first level corresponds to the "critical period" inflows, indicating the inflows during a sequence of 12 months in which the reservoirs displayed pronounced drought cycles in the hydrologic record.[3] The critical period is the year 1956 for Keystone, Fort Gibson, Webbers Falls, Eufaula, and Robert S. Kerr reservoirs, and the year 1964 for Tenkiller-Ferry reservoir. The critical period approach to the analysis is a conservative practice which assumes that if the energy and/or water commitments are met during a "worst period," they can also be satisfied during any other period. The monthly inflows corresponding to the critical period for the six reservoirs in the system are presented in Table I.

TABLE I

MONTHLY CRITICAL PERIOD INFLOWS FOR THE SIX RESERVOIRS
IN THOUSANDS OF ACRE-FEET

k	Month	Keystone	Fort Gibson	Webbers Falls	Tenkiller-Ferry	Eufaula	Robert S. Kerr
1	July	33.0	9.0	3.0	9.0	9.0	2.0
2	August	10.0	4.0	8.0	10.0	3.0	10.0
3	September	3.0	2.0	1.0	8.0	1.0	6.0
4	October	4.0	1.0	1.0	4.0	1.0	2.0
5	November	7.0	2.0	2.0	4.0	10.0	7.0
6	December	4.0	3.0	3.0	7.0	17.0	9.0
7	January	14.0	2.0	19.0	33.0	12.0	8.0
8	February	21.0	3.0	9.0	21.0	68.0	32.0
9	March	16.0	2.0	6.0	38.0	15.0	19.0
10	April	16.0	5.0	24.0	40.0	13.0	10.0
11	May	21.0	23.0	33.0	18.0	163.0	41.0
12	June	32.0	20.0	24.0	17.0	99.0	16.0

Source: U.S. Army Corps of Engineers, Southwestern Division, Basic Data, Vol. I of Arkansas, White, Red Rivers System Conservation Studies. 2 Vols., Dallas: Texas, January, 1970.

"Average inflows" are the second level of inflows considered, they are the average monthly inflows into the six reservoirs during the years 1923 to 1967.[4] The average inflows for the system are shown in Table II.

The third level of monthly inflows is the "high inflows," that are defined as the monthly average into each reservoir plus one monthly standard deviation. These values were obtained from the inflows record from 1923 to 1967.[5] The high inflows for the six reservoirs are presented in Table III.

In order to calculate the monthly average inflows and monthly standard deviations for the six reservoirs in the system, a computer program was developed with this particular purpose.

The inflows into Webbers Falls reservoir corresponding to the term man(k-1) are the natural flow above Inola Gage plus the releases from Oologah reservoir, when necessary. If the water passing at Inola Gage is below a certain minimum value, water is released from Oologah reservoir. For the purpose of this study, these inflows are set equal to the minimum flow at Inola Gage, and are going to remain constant along the analysis. These values are shown in Table IV.

Determination of the Volume of Water Evaporated

The net evaporation rates in inches for the six reservoirs are shown in Table V. The volume of water evaporating

TABLE II

MONTHLY AVERAGE INFLOWS FOR THE SIX RESERVOIRS
IN THOUSANDS OF ACRE-FEET

k	Month	Keystone	Fort Gibson	Webbers Falls	Tenkiller-Ferry	Eufaula	Robert S. Kerr
1	July	288.5	36.3	48.7	50.6	302.2	104.2
2	August	163.7	17.8	26.9	45.1	167.9	54.5
3	September	191.3	23.5	22.6	30.9	233.6	51.8
4	October	251.5	28.4	34.9	47.9	329.4	88.6
5	November	134.3	23.5	42.1	57.9	206.7	69.3
6	December	91.9	18.1	60.6	71.4	205.0	83.7
7	January	82.2	20.3	47.0	81.9	215.5	80.0
8	February	93.1	25.3	117.7	95.4	264.8	96.5
9	March	128.0	35.1	65.4	123.5	309.2	166.9
10	April	294.6	70.4	108.9	172.8	536.4	210.0
11	May	484.4	70.4	155.6	177.9	841.9	241.9
12	June	451.8	69.5	101.9	119.5	598.2	157.8

Source: U.S. Army Corps of Engineers, Southwestern Division, Basic Data, Vol. I of Arkansas, White, Red Rivers System Conservation Studies. 2 Vols., Dallas: Texas, January, 1970.

TABLE III

MONTHLY HIGH INFLOWS FOR THE SIX RESERVOIRS
IN THOUSANDS OF ACRE-FEET

k	Month	Keystone	Fort Gibson	Webbers Falls	Tenkiller-Ferry	Eufaula	Robert S. Kerr
1	July	653.4	95.4	107.7	112.0	642.1	237.5
2	August	376.4	42.1	77.0	124.4	413.2	148.7
3	September	376.9	55.1	59.0	64.5	488.6	177.5
4	October	695.7	69.3	85.7	119.3	952.8	278.3
5	November	313.7	57.4	116.9	136.2	515.2	192.8
6	December	182.4	36.0	199.0	147.4	469.2	184.0
7	January	158.6	41.4	108.8	159.9	560.2	179.1
8	February	230.5	49.4	494.4	186.5	629.2	232.3
9	March	247.0	66.7	171.8	249.1	672.3	464.0
10	April	663.7	152.7	293.1	360.1	1,159.5	472.8
11	May	1,022.3	149.4	380.1	367.5	1,599.0	478.5
12	June	930.1	159.4	220.1	264.0	1,221.4	388.4

Source: U.S. Army Corps of Engineers, Southwestern Division, Basic Data, Vol. I of Arkansas, White, Red Rivers System Conservation Studies. 2 Vols., Dallas: Texas, January, 1970.

TABLE IV

MINIMUM FLOW AT INOLA GAGE, INCLUDES FLOW AT INOLA PLUS RELEASES FROM OOLOGAH RESERVOIR (WHEN NECESSARY) IN THOUSANDS OF ACRE-FEET

k	Month	Flow
1	July	12.91
2	August	12.11
3	September	10.23
4	October	8.92
5	November	8.03
6	December	7.50
7	January	7.50
8	February	6.77
9	March	7.50
10	April	8.03
11	May	10.57
12	June	10.23

Source: U.S. Army Corps of Engineers, Tulsa District, Tulsa, Oklahoma.

TABLE V

MONTHLY NET EVAPORATION IN INCHES FOR THE SIX RESERVOIRS

k	Month	Keystone	Fort Gibson	Webbers Falls	Tenkiller-Ferry	Eufaula	Robert S. Kerr
1	July	5.23	5.16	3.78	4.61	4.44	3.78
2	August	4.41	4.95	3.27	4.14	4.08	3.27
3	September	3.24	3.41	1.59	2.53	1.79	1.59
4	October	2.01	2.16	-.07	1.09	1.49	-.07
5	November	1.12	1.21	-.16	.61	.16	-.16
6	December	.66	.56	-.14	.39	.29	-.14
7	January	.59	.47	-.03	.38	-.06	-.03
8	February	1.30	.72	.39	.78	.18	.39
9	March	2.48	1.84	.69	1.49	1.72	.69
10	April	2.85	2.39	.97	1.99	1.79	.97
11	May	2.25	2.68	1.05	2.10	1.33	1.05
12	June	3.27	3.91	2.13	3.04	2.75	2.13

Source: U.S. Army Corps of Engineers, Southwestern Division, <u>Basic Data</u>, Vol. I of <u>Arkansas</u>, <u>White</u>, <u>Red Rivers System Conservation Studies</u>. 2 Vols., Dallas: Texas, January, 1970.

from the reservoir depends on the area of the reservoir, at the same time the area is a function of the volume of water in storage.

In this study, the area of the reservoirs is a function of storage. Selected values of these functions are shown in Table VI for Keystone, in Table VII for Fort Gibson, in Table VIII for Tenkiller-Ferry, and in Table IX for Eufaula. As it is explained later in this chapter, the storages of Webbers Falls and Robert S. Kerr reservoirs remain constant for the period under study, and are set equal to:

Webbers Falls = 10,630 acres

Robert S. Kerr = 40,875 acres.

The volume of water evaporated from the reservoir is calculated using the following formula:

$$Evap_i(k) = S_i[x_i(k)] \cdot Evra_i(k) \tag{7}$$

where:

$Evap_i(k)$ = volume of water evaporated from the i^{th} reservoir in month k measured in KAF,

$S_i[x_i(k)]$ = surface of the i^{th} reservoir as a function of storage in month k, measured in thousands of acres, and

$Evra_i(k)$ = net evaporation rate in feet for the i^{th} reservoir in month k.

TABLE VI

AREA, ENERGY CAPACITY, ENERGY RATE, AND STORAGE FOR KEYSTONE RESERVOIR AND POWER PLANT

Area (Acres)	Energy Capacity (KW)	Energy Rate (KWH/KAF)	Storage (KAF)
14,490	60,200	0.0	287.5
15,400	62,250	5,050.4	317.6
16,290	64,250	9,624.8	349.2
17,240	66,250	13,790.5	382.8
18,520	68,250	17,646.9	418.6
20,150	70,000	21,302.0	457.0
22,000	70,000	24,807.5	498.6
23,590	70,000	28,124.6	543.9
25,240	70,000	31,262.6	592.4
26,020	70,000	32,760.5	618.0
26,940	70,000	34,221.0	644.4
28,540	70,000	37,007.6	700.1
29,070	70,000	37,942.8	720.4
30,090	70,000	39,645.5	758.9
31,740	70,000	42,157.6	820.8
33,280	70,000	44,570.3	885.5
34,950	70,000	46,913.7	952.6
36,250	70,000	49,051.9	1,025.2
38,090	70,000	51,180.7	1,099.3
39,570	70,000	53,233.0	1,176.3
41,470	70,000	55,231.6	1,256.4
43,540	70,000	57,086.5	1,342.7
45,630	70,000	58,955.2	1,431.9
47,900	70,000	60,767.4	1,526.0
50,320	70,000	62,574.8	1,624.2
52,810	70,000	64,382.6	1,726.6
55,320	70,000	66,074.0	1,836.5
58,080	70,000	67,767.9	1,951.1
60,890	70,000	69,536.4	2,068.6
63,830	70,000	71,224.4	2,193.7

Source: U.S. Department of the Interior, Southwestern Power Administration, Tulsa, Oklahoma.

TABLE VII

AREA, ENERGY CAPACITY, ENERGY RATE, AND STORAGE FOR FORT GIBSON RESERVOIR AND POWER PLANT

Area (Acres)	Energy Capacity (KW)	Energy Rate (KWH/KAF)	Storage (KAF)
17,000	50,000	0.0	311.3
17,600	50,000	2,617.9	328.5
19,000	50,000	2,617.9	365.2
20,600	50,000	11,913.5	404.5
22,100	50,000	15,991.0	447.0
23,700	50,000	19,748.3	492.6
25,500	50,000	23,326.7	541.6
27,400	50,000	26,503.4	594.3
29,400	50,000	29,574.4	650.9
31,700	50,000	32,477.9	711.9
34,000	50,000	35,239.9	777.5
36,800	50,000	37,893.3	847.9
39,500	50,000	40,454.6	923.8
42,300	50,000	42,298.3	1,005.4
45,200	50,000	45,249.4	1,092.7
48,100	50,000	47,517.1	1,185.7
51,000	50,000	49,704.9	1,184.4
53,800	50,000	51,812.3	1,388.8
56,600	50,000	53,850.2	1,498.6
59,600	50,000	55,851.5	1,613.3
62,800	50,000	57,769.5	1,735.0

Source: U.S. Department of the Interior, Southwestern Power Administration, Tulsa, Oklahoma.

TABLE VIII

AREA, ENERGY CAPACITY, ENERGY RATE, AND STORAGE FOR TENKILLER-FERRY RESERVOIR AND POWER PLANT

Area (Acres)	Energy Capacity (KW)	Energy Rate (KWH/KAF)	Storage (KAF)
7,500	31,200	0.0	283.1
7,530	32,000	3,596.2	294.2
7,760	33,000	8,133.1	309.6
7,992	34,000	12,453.9	325.2
8,230	35,000	16,557.4	341.6
8,490	36,000	20,494.1	358.2
8,730	37,000	24,256.8	375.4
9,020	38,000	27,873.3	393.1
9,298	39,000	31,356.3	411.4
9,590	39,000	34,694.5	430.5
9,890	39,000	37,941.7	449.9
10,180	39,000	41,054.9	470.2
10,500	39,000	44,106.4	490.7
10,820	39,000	47,043.5	512.1
11,180	39,000	49,926.9	533.9
11,520	39,000	52,699.3	556.8
11,840	39,000	55,429.3	580.0
12,190	39,000	58,066.5	604.1
12,355	39,000	59,360.8	616.4
12,520	39,000	60,653.7	628.7
12,700	39,000	61,948.5	641.0
12,880	39,000	63,167.7	654.1
13,040	39,000	64,388.5	667.2
13,200	39,000	65,610.7	680.3
13,570	39,000	68,023.7	706.9
13,940	39,000	70,343.0	734.7
14,309	39,000	72,684.6	762.5
14,667	39,000	74,897.1	791.9
15,025	39,000	77,131.4	821.3
15,383	39,000	79,266.4	852.0
15,741	39,000	81,381.3	883.2
16,099	39,000	83,414.1	915.6
16,499	39,000	85,389.9	949.0
16,899	39,000	87,355.0	983.0
17,299	39,000	89,203.0	1,018.8
17,759	39,000	91,104.7	1,054.6
18,219	39,000	92,910.6	1,092.2
18,739	39,000	94,733.7	1,130.4
19,319	39,000	96,588.2	1,169.2
19,899	39,000	98,349.8	1,210.2
21,200	39,000	100,286.1	1,251.2
21,700	39,000	102,204.1	1,294.4
22,200	39,000	104,127.2	1,338.2

Source: U.S. Department of the Interior, Southwestern Power Administration, Tulsa, Oklahoma.

TABLE IX

AREA, ENERGY CAPACITY, ENERGY RATE, AND STORAGE FOR EUFAULA RESERVOIR AND POWER PLANT

Area (Acres)	Energy Capacity (KW)	Energy Rate (KWH/KAF)	Storage (KAF)
46,910	60,000	0.0	864.8
49,350	61,400	3,083.6	913.2
54,500	64,100	8,892.2	1,015.5
59,680	66,700	14,426.6	1,113.5
64,900	69,300	19,090.7	1,254.9
69,790	72,300	23,557.2	1,389.3
75,200	74,600	27,654.6	1,534.1
80,920	77,800	31,449.9	1,689.7
86,670	80,400	34,953.4	1,857.9
92,700	83,200	38,243.5	2,036.5
99,100	86,200	41,310.7	2,228.5
102,200	87,500	42,766.9	2,329.7
105,400	88,800	44,187.6	2,433.6
111,800	91,600	46,938.8	2,649.3
118,700	94,500	49,516.0	2,880.1
125,600	97,250	51,981.1	3,124.0
132,700	100,050	54,319.1	3,382.9
140,000	102,712	56,584.3	3,654.6
143,750	103,500	57,648.4	3,798.3
147,300	103,500	58,712.6	3,944.3
155,000	103,500	60,836.2	4,243.9
162,200	103,500	62,832.4	4,562.2
169,800	103,500	64,736.2	4,897.4
177,500	103,500	66,645.4	5,243.2
185,200	103,500	68,454.4	5,608.2
193,100	103,500	70,202.3	5,990.2

Source: U.S. Department of the Interior, Southwestern Power Administration, Tulsa, Oklahoma.

Hydroelectric Energy Generation

As it was mentioned in the introduction, the only marketable use of the water with an associated monetary value is the production of hydroelectric energy. Hydroelectric energy is sold by the Southwestern Power Administration, a division of the U. S. Department of the Interior, wholesale under contract for "firm" or "on-peak" energy corresponding to the period during each month in which energy demands are high. "Non-firm" or "non-peak" energy is that one produced in excess of firm energy commitments which can be sold but at substantial lower prices than firm energy. The prices charged for these two types of energy by the Southwestern Power Administration, including the capacity charge, are for 1974:

On-peak energy: 8.99 mills/kilowatt-hour
Off-peak energy: 2.47 mills/kilowatt-hour.[6]

Reservoirs and their power plants are classified into two categories: run of water and storage reservoirs. A storage reservoir is one of sufficient capacity to permit carry-over storage from the high inflows season to the low inflows season. This characteristic allows a controlled firm flow above the minimum natural flow. A run-of-river reservoir has very limited storage capacity. Some run-of-river reservoirs have pondage, or storage volume which permits to store water during off-peak hours for its use during peak hours the same day.[7]

In this study, Keystone, Fort Gibson, Tenkiller-Ferry, and Eufaula are classified as storage reservoirs and Webbers Falls and Robert S. Kerr as run-of-river reservoirs.

Energy Generation at the Power Plant of a Storage Reservoir

For a storage reservoir, the energy generated depends on the "energy rate," that represents the energy stored as water in the reservoirs. In this study, it is measured as kilowatt-hours per thousands of acre-feet. We obtain the hydroelectric energy generated multiplying the energy rate times the volume of water flowing through the turbines during stage k.

The maximum capacity of hydroelectric energy production is given by the "energy capacity" that represents the maximum hydroelectric energy which can be produced given the technical characteristics of the turbines in the power plant.

Energy rate and energy capacity depend on the efficiency of the turbines, and on the head of water on the turbine during period k.[8] Assuming a constant turbine efficiency, the energy rate and energy capacity are only dependent on the head of water. The head in the reservoir depends primarily on the volume of water in storage at period k. This approach to energy generated had been used by Koopmans[9] and by Roefs and Bodin.[10]

Energy rate and energy capacity will be assumed to be functions of the volume of water in storage at period k. In mathematical terms,

$$\text{Energy rate} = ER[x_i(k)] = f[x_i(k)] \tag{8}$$

where

$ER[x_i(k)]$ = energy rate at storage x in month k, measured in kilowatt-hours per KAF,

$x_i(k)$ = volume of water stored in the i^{th} reservoir in stage k, measured in KAF.

$$\text{Energy capacity} = EC[x_i(k)] = f[x_i(k)] \tag{9}$$

where:

$EC[x_i(k)]$ = energy capacity at storage x in month k measured in kilowatt-hours,

$x_i(k)$ = volume of water stored in the i^{th} reservoir at stage k measured in KAF.

Selected values of the energy rate and energy capacity functions are shown in Table VI for Keystone, in Table VII for Fort Gibson, in Table VIII for Tenkiller-Ferry, and in Table IX for Eufaula.

To calculate the total hydroelectric energy generated by the power plant of a storage reservoir, the energy rate function is multiplied by the volume of water released through the turbines of the power plant in month k, as follows:

$$EN_i(k) = ER_i[x(k)] \cdot u_i(k) \tag{10}$$

where:

$EN_i(k)$ = total hydroelectric energy generated at the i^{th} power plant during month k, measured in kilowatt-hours,

$ER_i[x(k)]$ = energy rate in the i^{th} reservoir at storage x in month k, measured in kilowatt-hours per KAF,

$u_i(k)$ = volume of water released through the turbine at the i^{th} reservoir at stage k, measured in KAF.

Energy Generation at the Power Plant of a Run-of-River Reservoir

Energy generation at Webbers Falls and Robert S. Kerr reservoirs, the two run-of-river reservoirs included in this study, is explained by equations different from those for the storage reservoirs. The water at these reservoirs, that are also locks of the Arkansas River Navigation Project, can be used in energy generation as it comes, since there is no sufficient storage space to regulate the releases in a period of time longer than a day.

Given this characteristic, their storage capacities are going to be kept constant during the 12-month period under analysis. The constant storage for Webbers Falls is set to 160 KAF and for Robert S. Kerr at 473.7 KAF.

For these two reservoirs the energy generated is assumed as a function of the release through the power plant and the net head elevation, or the difference between the water elevation upstream of the dam minus the water elevation downstream of the dam, also called tailwater elevation.

Then the formula that provides the total hydroelectric energy generated at these two reservoirs, considering a turbine efficiency of 86.3 per cent, is the following:[11]

$$EN = 883.5248 \cdot \text{Release} \cdot [\text{Water Elevation} - \text{Tailwater Elevation}] \tag{11}$$

Since the volume in these reservoirs is going to be kept constant, the water elevation upstream of the dam is a constant. Thus, the energy-generation formula for the two reservoirs is

for Webbers Falls reservoir:

$$EN = 883.5248 \cdot u(k) \cdot [489.5 - T.E.] \tag{12}$$

where:

EN = hydroelectric energy generated in kilowatt-hours,

u(k) = release through the power plant at stage k in KAF,

489.5 = water elevation in feet equivalent to a storage of 160 KAF,

T.E. = tilwater elevation in feet, or height of the water downstream of the dam.

for Robert S. Kerr reservoir:

$$EN = 883.5248 \cdot u(k) \cdot [459.5 - T.E.] \tag{13}$$

where:

EN = hydroelectric energy generated in kilowatt-hours,

u(k) = release through the power plant at month k in KAF,

459.5 = water elevation in feet equivalent to a storage of 473.7 KAF,

T.E. = tailwater elevation in feet, or height of the water downstream the dam.

The tailwater elevation downstream of the dam is a function of the total release from the reservoir. Defining total release as the sum of all the releases for different purposes, the tailwater function is given by

$$T.E._i(tu) = f[tu_i(k)] \tag{14}$$

where:

$T.E._i(tu)$ = tailwater elevation at total release u for the i^{th} reservoir, measured in feet,

$tu_i(k)$ = total release from the i^{th} reservoir in month k, measured in KAF.

The energy capacity of the turbines in the power plant of a run-of-river reservoir is a function of the tailwater elevation, which is a function of the total release from the reservoir. In this study, the energy capacity of the run-of-river reservoirs is a function of the total release from the reservoir, as follows:

$$EC_i(tu) = f[tu_i(k)] \tag{15}$$

where:

$EC_i(tu)$ = energy capacity at a certain total release from the i^{th} reservoir measured in kilowatt,

$tu_i(k)$ = total release from the i^{th} reservoir measured in KAF.

Selected values of the tailwater-total release, and energy capacity-total release functions are shown in Table X for Webbers Falls reservoir, and in Table XI for Robert S. Kerr reservoir.

Determination of the On-peak and Off-peak Energies

Following the approach given by Hall and Shephard et al., the procedure to determine the on-peak and off-peak energies from the total energy produced at each power plant during a particular month.[12]

Before stating the procedure, the following terms need to be defined:

$h_{1i}(k)$ = number of peak-hours in month k for the i^{th} power plant,

$h_2(k)$ = total number of hours in month k.

The estimated number of peak-hours for the six reservoirs are shown in Table XII, and the total number of hours in each month are presented in Table XIII.

TABLE X

TAILWATER ELEVATION, TOTAL RELEASE (FLOW), AND ENERGY CAPACITY FOR WEBBERS FALLS RESERVOIR AND POWER PLANT

Tailwater Elevation (Feet)	Total Release (KAF)	Energy Capacity (KW)
460.0	0.0	69,000.0
461.0	1,487.6	69,000.0
462.0	2,082.7	69,000.0
463.0	2,796.7	69,000.0
464.0	3,421.5	69,000.0
465.0	4,076.1	69,000.0
466.0	4,849.6	69,000.0
468.0	6,277.8	60,000.0
470.0	7,884.4	49,500.0
472.0	9,550.5	39,000.0
474.0	11,454.7	0.0

Source: U.S. Department of the Interior, Southwestern Power Administration, Tulsa, Oklahoma.

TABLE XI

TAILWATER ELEVATION, TOTAL RELEASE (FLOW), AND ENERGY CAPACITY FOR ROBERT S. KERR RESERVOIR AND POWER PLANT

Tailwater Elevation (Feet)	Total Release (KAF)	Energy Capacity (KW)
413.4	0.0	126,500.0
415.1	297.5	126,500.0
416.5	595.0	126,500.0
417.7	892.6	126,500.0
418.7	1,190.0	126,500.0
419.6	1,487.6	126,500.0
420.4	1,785.1	126,500.0
421.2	2,082.7	126,500.0
422.0	2,380.2	126,500.0
422.8	2,677.7	126,500.0
423.6	2,975.2	126,500.0
427.0	4,165.3	113,000.0
431.6	5,950.5	94,000.0
436.5	8,925.7	73,000.0
439.8	11,901.0	59,000.0
443.0	15,768.8	0.0

Source: U.S. Department of the Interior, Southwestern Power Administration, Tulsa, Oklahoma.

TABLE XII

ESTIMATED MONTHLY ON-PEAK HOURS BY RESERVOIR, $h_1(k)$

k	Month	Keystone	Fort Gibson	Webbers Falls	Tenkiller-Ferry	Eufaula	Robert S. Kerr
1	July	432	502	180	374	339	548
2	August	324	376	135	280	254	411
3	September	216	251	90	187	169	274
4	October	135	157	56	117	106	171
5	November	135	157	56	117	106	171
6	December	216	251	90	187	169	274
7	January	216	251	90	187	169	274
8	February	162	188	67	140	127	205
9	March	162	188	67	140	127	205
10	April	162	188	67	140	127	205
11	May	162	188	67	140	127	205
12	June	378	439	158	327	296	480

Source: U.S. Department of the Interior, Southwestern Power Administration, Tulsa, Oklahoma.

TABLE XIII

TOTAL NUMBER OF HOURS IN EACH MONTH, $h_2(k)$

k	Month	Total Hours
1	July	744
2	August	744
3	September	720
4	October	744
5	November	720
6	December	744
7	January	744
8	February	672
9	March	744
10	April	720
11	May	744
12	June	720

Next, the maximum hydroelectric energy generation is defined as the product of the energy capacity times the total number of hours in month k.

Storage Reservoir: $EMAX_i(k) = EC_i[x_i(k)] \cdot h_2(k)$ (16)

where:

$EMAX_i(k)$ = maximum energy generated in month k at the i^{th} power plant in kilowatt-hours,

$EC_i[x_i(k)]$ = energy capacity at storage x(k) during month k at the i^{th} reservoir, in kilowatts,

$h_2(k)$ = total number of hours in month k.

Run-of-river reservoir: $EMAX_i(k) = EC_i[tu_i(k)] \cdot h_2(k)$ (17)

where:

$EMAX_i(k)$ = maximum energy generated in month k at the i^{th} power in kilowatt-hours,

$EC_i[tu_i(k)]$ = energy capacity at total release tu(k) during month k at the i^{th} reservoir in kilowatts,

$h_2(k)$ = total number of hours in month k.

The maximum on-peak energy generated is defined as the energy capacity for a given storage or total release times the number of peak-hours in month k.

Storage Reservoir: $EPEAK_i(k) = EC_i[x_i(k)] \cdot h_{1i}(k)$ (18)

where:

$EPEAK_i(k)$ = maximum on-peak energy generated during month k at the i^{th} power plant in kilowatt-hours,

$EC_i[x_i(k)]$ = energy capacity at storage x(k) during month k in the i^{th} reservoir in kilowatts,

$h_{1i}(k)$ = number of on-peak hours during month k and i^{th} power plant.

Run of river reservoir: $EPEAK_i(k) = EC_i[tu_i(k)] \cdot h_{1i}(k)$ (19)

where:

$EPEAK_i(k)$ = maximum on-peak energy generated in month k at the i^{th} power plant in kilowatt-hours,

$EC_i[tu_i(k)]$ = energy capacity at total release u(k) in the i^{th} reservoir during month k in kilowatts,

$h_{1i}(k)$ = number of on-peak hours for the i^{th} power plant in month k.

With these concepts already defined for the two types of reservoirs included in the system we can give a criterion to recognize when the energy generated will be on-peak or off-peak. Defining

EUF = on-peak energy generation in kilowatt-hours,

EVH = off-peak energy generation in kilowatt-hours,

According to this terminology, we set up the following inequalities:

if:

$$EN \leq EPEAK \quad \begin{cases} EUH = EN \\ EVH = 0 \end{cases}$$

if:

$$EPEAK < EN \leq EMAX \quad \begin{cases} EUH = EPEAK \\ EVH = EN - EPEAK \end{cases} \tag{20}$$

if:

$$EN > EMAX \quad \begin{cases} EUH = EPEAK \\ EVH = EMAX - EPEAK. \end{cases}$$

According to these inequalities we determine the on-peak and off-peak energies generated at each of the six power plants in the system.

The Performance Criterion

The performance criterion to be maximized is the sum of the returns due to the sale of energy generated at the six power plants during a period of twelve months.

$$J = \sum_{k=1}^{12} \sum_{i=1}^{6} L[PU(EUH_i) + PV(EVH_i)] \tag{21}$$

where:

J = total return from operating the system for a period of 12 months,

L = return from a single stage,

PU = price of the on-peak energy,

EUH_i = energy generated on-peak, at the i^{th} power plant,

PV = price of non-peak energy, and

EVH_i = energy generated off-peak, at the i^{th} power plant.

Constraints on the Operation of the System

Three types of constraints are imposed on the system: the first, is related to storages, i.e., the state variables; the second is related to releases and control variables; and the third to the initial and final boundary conditions.

For the storages we have

$$Stomin_i \leq x_i(k) \leq Stomax_i \tag{22}$$

where:

$x_i(k)$ = volume of water in storage in the i^{th} reservoir at month k, in KAF,

$Stomin_i$ = minimum storage allowable in the i^{th} reservoir, in KAF,

$Stomax_i$ = maximum storage allowable in the i^{th} reservoir, in KAF.

The values for Stomin and Stomax for the six reservoirs are presented in Table XIV.

The following constraints are imposed for the releases:

$$Relmin_i(k) \leq u_i(k) \leq Relmax_i(k) \tag{23}$$

TABLE XIV

MAXIMUM AND MINIMUM STORAGES FOR THE SIX RESERVOIRS IN THOUSANDS OF ACRE-FEET

Reservoir	STOMIN	STOMAX
Keystone	287.5	2, 193.70
Fort Gibson	311.3	1, 735.05
Webbers Falls	135.2	165.20
Tenkiller-Ferry	283.1	1, 338.20
Eufaula	864.8	5, 990.20
Robert S. Kerr	414.1	493.60

Source: U.S. Department of the Interior, Southwestern Power Administration, Tulsa, Oklahoma.

where:

$u_i(k)$ = release from the i^{th} reservoir in period k,

$Relmin_i(k)$ = minimum required release from the i^{th} reservoir to achieve all the uses of the water downstream of the dam in month k, in KAF,

$Relmax_i(k)$ = maximum allowed release from the i^{th} reservoir in month k without jeopardizing the areas downstream of the dam.

The values of Relmin and Relmax for each of the reservoirs and months are shown in Table XV. According to this data, the only reservoir that has minimum mandatory releases is Keystone reservoir.

The third type of constraints is represented by the initial and final boundary conditions specifying the initial and final storages of water required for the six reservoirs and the beginning and end of the period of 12 months under analysis.

The initial and final boundary conditions for the system of reservoirs are given by the following values:

Keystone reservoir	=	618.0 [KAF]	
Fort Gibson	=	365.2 [KAF]	
Tenkiller-Ferry reservoir	=	654.1 [KAF]	
Eufaula reservoir	=	2,329.7 [KAF]	(24)
Webbers Falls reservoir	=	160.0 [KAF]	
Robert S. Kerr reservoir	=	473.7 [KAF].	

TABLE XV

MONTHLY MINIMUM AND MAXIMUM RELEASES FOR THE SIX RESERVOIRS IN THOUSANDS OF ACRE-FEET

k	Month	Keystone		Fort Gibson		Webbers Falls		Tenkiller-Ferry		Eufaula		Robert S. Kerr	
		RELMIN	RELMAX	RELMIN	RELMAX	RELMIN	RELMAX	RELMIN	RELMAX	RELMIN	RELMAX	RELMIN	RELMAX
1	July	45.01	860.84	0	688.67	0	2,189.0	0	260.65	0	882.00	0	2,576.4
2	August	50.05	860.84	0	688.67	0	2,189.0	0	260.65	0	882.00	0	2,576.4
3	September	35.52	833.07	0	666.45	0	2,118.4	0	252.24	0	853.54	0	2,493.3
4	October	25.03	860.84	0	688.67	0	2,189.0	0	260.65	0	882.00	0	2,576.4
5	November	16.13	833.07	0	666.45	0	2,118.4	0	252.24	0	853.54	0	2,493.3
6	December	11.68	860.84	0	688.67	0	2,189.0	0	260.65	0	882.00	0	2,576.4
7	January	11.68	860.84	0	688.67	0	2,189.0	0	260.65	0	882.00	0	2,576.4
8	February	10.55	777.53	0	622.02	0	1,977.1	0	235.42	0	796.64	0	2,327.0
9	March	16.66	860.84	0	688.67	0	2,189.0	0	260.65	0	882.00	0	2,576.4
10	April	24.22	833.07	0	666.45	0	2,118.4	0	252.24	0	853.54	0	2,493.3
11	May	36.71	860.84	0	688.67	0	2,189.0	0	260.65	0	882.00	0	2,576.4
12	June	41.95	833.07	0	666.45	0	2,118.4	0	252.24	0	853.54	0	2,493.3

Source: U.S. Department of the Interior, Southwestern Power Administration, Tulsa, Oklahoma.

These values indicate that the analysis starts and finishes with the six reservoirs filled up to the level of storage corresponding to the top of the power pool for each of the six reservoirs.

The Trial Trajectories

In Oklahoma, the flow of the rivers depends primarily on the rainfall, which is less predictable than snow. It is convenient to assume that the reservoirs in the system are filled up to the top of the power pool level at the beginning of the summer. In this geographical area, this season coincides with the low stream flows, but at the same time, it is the period in which the electric-energy demand is the highest during the year.

With these considerations in mind, July is selected as the initial month. At the beginning of July, all of the reservoirs are filled up to the top of the power pools. At the end of a period of 12 months, the reservoirs should have the same storage level to start another year of operation.

In this study, the analysis uses two different trial trajectories. This is done in order to determine if optimal solution is unimodal.[13] If the values of the optimal policies using the two trial trajectories coincide a global maximum has been reached; otherwise, a local maximum is obtained.

The first trial trajectory keeps the storages in the reservoirs constant all along the year at the top of the power pool. These values are presented in Table XVI for the system of reservoirs.

In the second trial trajectory, we start at July with the reservoirs filled up to the top of the power pool, then from August through May the storage capacity increases up to 95 per cent of the total storage capacity of the reservoirs. Finally, in June it goes down again to a storage level equivalent to the top of the power pool, then the next year starts with the reservoirs filled up to these levels. The values for this trial trajectory are shown in Table XVII.

Use of the Constraints

At every month when we introduce the increments DX and create the subdomain S(k) around the trial trajectory value for the state variable, every neighbor value is checked against the state variables constraints Stomin and Stomax for each of the reservoirs in the system; if one of these neighbor values violates these constraints, it is deleted from any further analysis.

The optimal releases found in corridor C_h are checked for feasibility according to the Relmin and Relmax constraints values for each month. We proceed in the following way:

if:

$$u_i(k) < \text{Relmin}_i(k), \tag{25}$$

this inequality indicates that if the optimal releases from the i^{th} reservoir at month k is less than the minimum

TABLE XVI

FIRST TRIAL POLICY INDICATING THE MONTHLY VOLUME OF WATER STORED IN EACH RESERVOIR IN THOUSANDS OF ACRE-FEET

k	Month	Keystone	Fort Gibson	Webbers Falls	Tenkiller-Ferry	Eufaula	Robert S. Kerr
1	July	618.0	365.2	160.0	655.1	2,329.7	473.7
2	August	618.0	365.2	160.0	654.1	2,329.7	473.7
3	September	618.0	365.2	160.0	654.1	2,329.7	473.7
4	October	618.0	365.2	160.0	654.1	2,329.7	473.7
5	November	618.0	365.2	160.0	654.1	2,329.7	473.7
6	December	618.0	365.2	160.0	654.1	2,329.7	473.7
7	January	618.0	365.2	160.0	654.1	2,329.7	473.7
8	February	618.0	365.2	160.0	654.1	2,329.7	473.7
9	March	618.0	365.2	160.0	654.1	2,329.7	473.7
10	April	618.0	365.2	160.0	654.1	2,329.7	473.7
11	May	618.0	365.2	160.0	654.1	2,329.7	473.7
12	June	618.0	365.2	160.0	654.1	2,329.7	473.7
13	July	618.0	365.2	160.0	654.1	2,329.7	473.7

Source: U.S. Department of the Interior, Southwestern Power Administration, Tulsa, Oklahoma.

TABLE XVII

SECOND TRIAL POLICY INDICATING THE MONTHLY VOLUME OF WATER STORED IN EACH RESERVOIR IN THOUSANDS OF ACRE-FEET

k	Month	Keystone	Fort Gibson	Webbers Falls	Tenkiller-Ferry	Eufaula	Robert S. Kerr
1	July	618.0	365.2	160.0	654.1	2,329.7	473.7
2	August	2,078.0	1,625.3	160.0	1,294.4	5,751.4	473.7
3	September	2,078.0	1,625.3	160.0	1,294.4	5,751.4	473.7
4	October	2,078.0	1,625.3	160.0	1,294.4	5,751.4	473.7
5	November	2,078.0	1,625.3	160.0	1,294.4	5,751.4	473.7
6	December	2,078.0	1,625.3	160.0	1,294.4	5,751.4	473.7
7	January	2,078.0	1,625.3	160.0	1,294.4	5,751.4	473.7
8	February	2,078.0	1,625.3	160.0	1,294.4	5,751.4	473.7
9	March	2,078.0	1,625.3	160.0	1,294.4	5,751.4	473.7
10	April	2,078.0	1,625.3	160.0	1,294.4	5,751.4	473.7
11	May	2,078.0	1,625.3	160.0	1,294.4	5,751.4	473.7
12	June	2,078.0	1,625.3	160.0	1,294.4	5,751.4	473.7
13	July	618.0	365.2	160.0	654.1	2,329.7	473.7

releases mandatory from the i^{th} reservoir at this month, we let

$$u_i(k) = Relmin_i(k) \quad . \tag{26}$$

Now if we consider the other extreme release situation where

$$u_i(k) > Relmax_i(k) \tag{27}$$

that is, when the optimal release from reservoir i^{th} at month k is greater than the maximum allowable release at this stage, we let

$$u_i(k) = Relmax_i(k). \tag{28}$$

Interpolation Procedure

The values of energy rate, energy capacity, and surface as a function of the volume of water for the storage reservoirs; the tailwater elevation, and the energy capacity as a function of the total release for the run-of-river reservoir are stored in the computer as tables indicating chosen values of a given function relating pairs of these variables.

Under the impossibility of tabulating all the values of a function, or even a very large set of values, some interpolation procedure must be used permitting us to re-create a general value from a few chosen values.

In order to calculate the energy generated at the power plant of a storage reservoir by a given sequence of releases, it is necessary to know the energy rate and energy capacity at the storage levels associated with these releases. Also, the energy capacity and tailwater elevation related to a sequence of total releases are required in order to calculate the energy generated at the power plant of a run-of-river reservoir.

If the storage or total release values are not tabulated, an interpolation scheme is used to determine the energy related values associated with these particular values. The interpolation procedure adopted in this study is a cubic spline fitting which is a type of piecewise polynomial fitting.[14]

The name spline comes from a mechanical device used by draftsmen to fit a curve of minimum curvature through successive pairs of point of a set.

Spline fitting does not find an analytical function which passes through each of a given set of points. Instead, the interval is broken into a number of nonoverlapping subintervals and the points in each subinterval are fitted by a polynomial.[15, 16]

The spline fitting has the property of minimum curvature, in this sense the spline fits provide the "smoothest" interpolating functions for each subinterval.[17]

Conditions for Convergence

The convergence of the discrete differential dynamic programming algorithm depends upon the choice of the incremental value DX of the state variables and on the conditions of convergence, or the stopping criterion for the algorithm. Choosing a value of DX too small or a stopping condition too large may result in missing the global optimum, although the algorithm may converge to a local optimum as it was indicated in Chapter II.

The procedure followed in this study is the following: we assign a value of DX "large enough" to guarantee a good sweeping of the state space in the first iterations. Then this value is cut progressively by half, and the iteration process continues until DX reaches pre-determined small value of convergence.

The initial DX value considered in this study is for the critical period and the average inflows

$$DX = 32 \text{ KAF}$$

and for the high inflows

$$DX = 256 \text{ KAF}.$$

The value of DX is larger for the high inflows in order to make the convergence process faster; we are dealing with large volumes of water, so if the DX is small the convergence process takes a long time.

In order to determine the convergence of the algorithm, two stopping conditions are defined.

E_D = stopping condition for the increment value DX,

E_J = stopping condition for the total return J.

By using these two stopping conditions at the same time we could achieve the convergence of the algorithm.

Convergence tests are performed only for optimal solutions that converged for a given value of DX. A solution converges for a certain value of DX when the optimal solution, optimal sequence of releases, are the same for two successive iterations with a given DX value.

Then, every time we reach convergence for a certain DX, we will test to see if

$$DX < E_D. \tag{29}$$

If DX if smaller, we stop the calculations; otherwise, we continue the iterative process.

The stopping condition value related with the DX value to be used in this study is set up to the following value:

$$E_D = 1.0 \text{ KAF}.$$

In order to discuss the stopping condition associated with the performance criterion return, we define the following index:

$$\frac{J(\text{new}) - J(\text{old})}{J(\text{old})} . \tag{30}$$

This index indicates the relative change in the return for two successive solutions which converged for certain values of DX. J(new) indicates the return obtained from the latter convergence solution, and J(old) indicates the return for the preceding converged solution.

Then, if the value of this index is smaller than the value E_J, we stop the calculations; otherwise, we continue the iteration process in the following way:

$$\frac{J(\text{new}) - J(\text{old})}{J(\text{old})} < E_J. \tag{31}$$

The value for E_J used in this study is:

$$E_J = 0.0001.$$

An "or" relationship between the two stopping conditions is used. In this sense, if only one of the stopping conditions is met, the iterative process is halted. If neither of the stopping conditions is met, a new value for DX is defined, the previous value is cut in half and the iterative process continues.

The flow-chart of the computer program performing the discrete differential dynamic programming algorithm used in this study is shown in Figure 8.

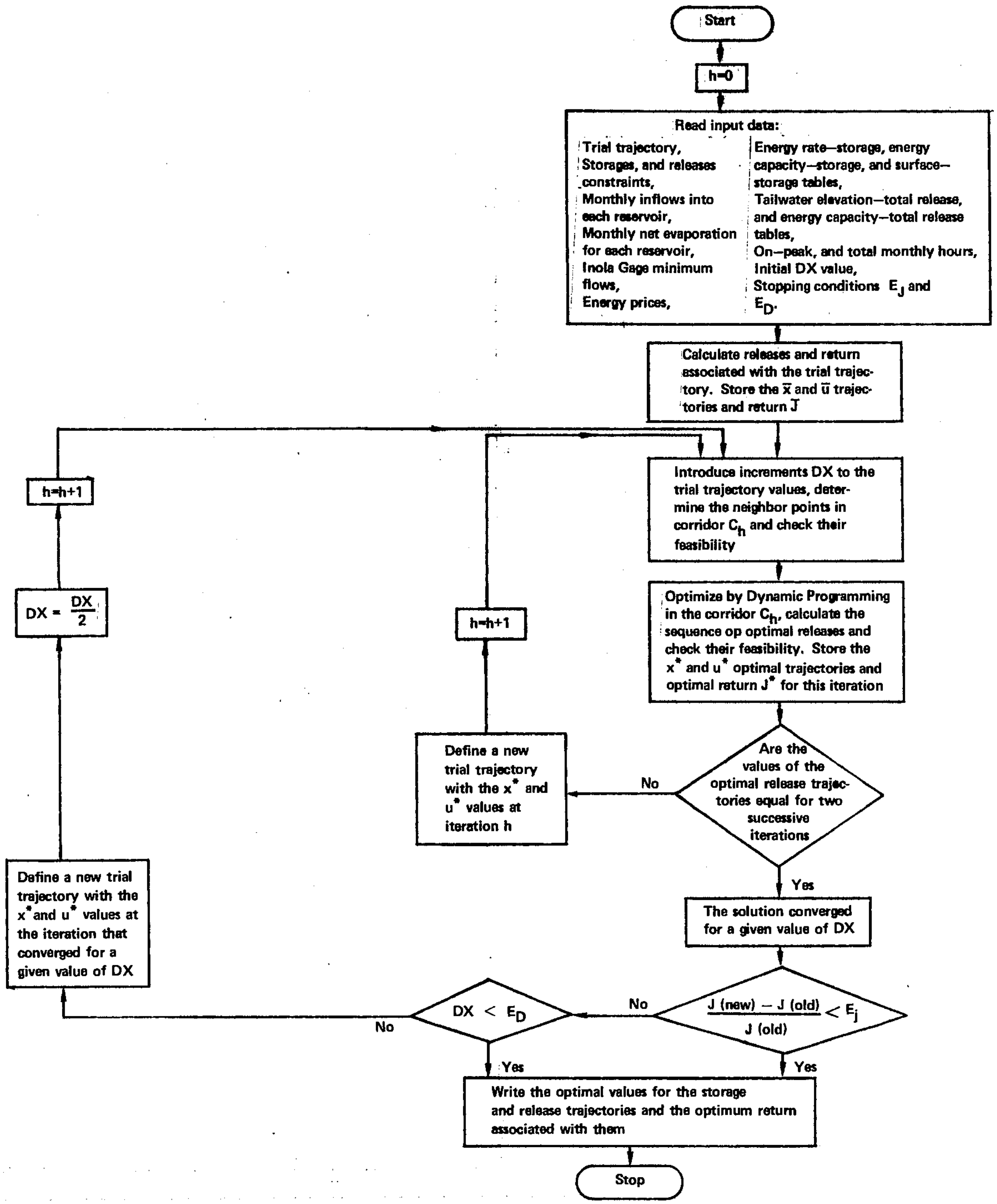

Figure 8. Flow Chart Indicating the Steps of the Discrete Differential Dynamic Programming Algorithm Employed in this Study

FOOTNOTES

[1]man(k-1) = natural flow at Inola Gage, Oklahoma plus the releases from Oologah when necessary.

[2]See pp. 34-35.

[3]W. A. Hall, A. J. Askew, and W. W-G. Yeh, "Use of the Critical Period in Reservoir Analysis," Water Resources Research, V (December, 1969), p. 1205.

[4]U. S. Army Corps of Engineers, Southwestern Division, Basic Data, I (Dallas, Texas, 1970), pp. 72, 77-81.

[5]Ibid.

[6]One mill is equal to one tenth of a cent or $0.001.

[7]R. K. Linsley and J. B. Franzini, Water Resources Engineering (New York, New York, 1964), p. 453.

[8]L. D. James and R. R. Lee, Economics of Water Resources Planning (New York, New York, 1971), pp. 329-337.

[9]T. C. Koopmans, "Water Storage Policy in a Simplified Hydroelectric System," In Proceedings of the First International Conference on Operations Research (Baltimore, Maryland, 1957), pp. 196-197.

[10]T. G. Roefs and L. D. Bodin, "Multireservoir Operation Studies," Water Resources Research, VI (1970), pp. 413-414.

[11]A. J. Fredrich, Digital Simulation of an Existing Water Resources System, Paper presented at the IEEE Joint National Conference on Major Systems (Los Angeles, California, 1971), p. 14.

[12]W. A. Hall and R. W. Shepard et al., Optimum Operations for Planning of a Complex Water Resources System, Water Resources Center, Contribution No. 102 (University of California, Los Angeles, California, 1967), p. 44.

[13]For a definition of a unimodal function, see p. 40.

[14]A cubic spline fitting computer program was developed by Mr. Lynn R. Ebbesen of the School of Mechanical and Aerospace Engineering at Oklahoma State University.

[15]D. A. Handscomb, "Spline Functions," Methods of Numerical Approximations, ed. D. C. Handscomb (London, England, 1966), pp. 163-167.

[16]J. A. Jacquez, A First Course in Computing and Numerical Methods (Reading, Massachusetts, 1970), pp. 264.266.

[17]For a proof of minimum curvature, see Ibid., p. 266.

CHAPTER IV

THE OPTIMAL OPERATIONAL POLICIES

In this chapter, the optimal operational policies are presented for the system of six reservoirs under the three hydrological conditions considered in the analysis. The optimal return for the system and the optimal storage and release policies for each particular reservoir are presented and discussed for the critical period, average, and high inflows. Also, the hydroelectric generation at each power plant is introduced.

Critical Period Inflows

The optimal returns for the system of six reservoirs associated with the critical period inflows, and the two trial trajectories are

First trial trajectory = $841,398

Second trial trajectory = $838,259

There is a difference of $3,139 between the optimal returns. In relative terms, this difference represents a 0.37% variation from the smallest optimal return value.

The convergence of the solution toward the optimal return is shown in Figure 9 for the two trial trajectories.

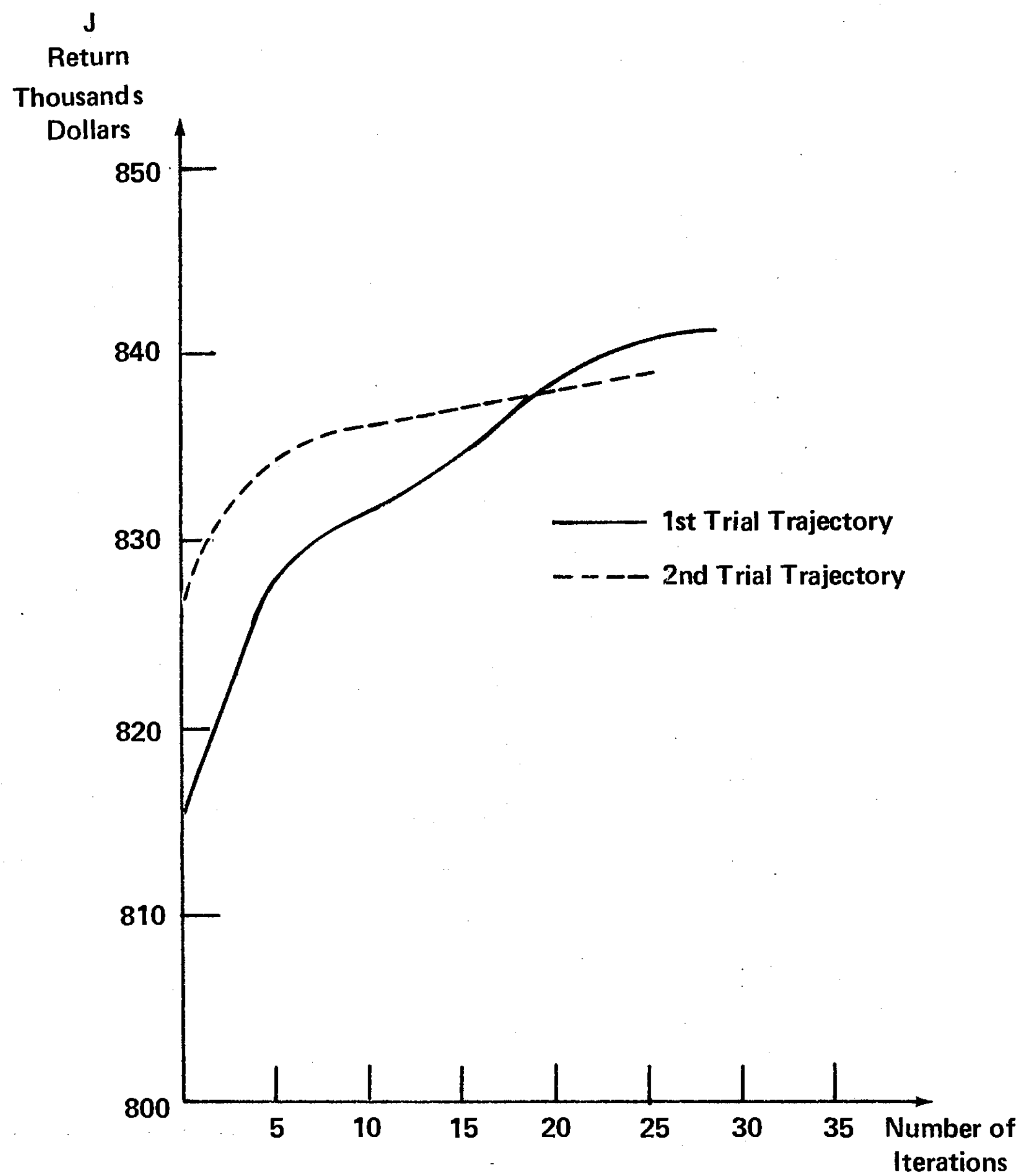

Figure 9. Total Return for the System of Six Reservoirs as a Function of the Number of Iterations for the Critical Period of Inflows

The number of iterations required for convergence were 36 for the first trial trajectory and 25 for the second trial trajectory.

The total processing time required to optimize the operation of the system for a period of 12 months is presented in Table XVIII.

TABLE XVIII

TOTAL PROCESSING TIME* REQUIRED FOR THE ALGORITHM TO OPTIMIZE THE OPERATION OF THE SYSTEM

Trial Trajectory	Number of Iterations	Total Processing Time, Minutes	Processing Time per Iteration, Minutes
1st	36	13.04	0.362
2nd	25	4.49	0.179

*IBM System 360 Model 65.

The first trial trajectory takes almost three times more than the second trial trajectory to converge to the optimal solution.

Following, the optimal storage and release policies for each reservoir are presented.

Keystone Reservoir

The optimal storage policy for Keystone reservoir is presented in Table XIX for the first trial trajectory, in Table XX for the second trial trajectory, and graphically in Figure 10.

As we can observe in Figure 10, the optimal storage policy of this reservoir did not meet the final boundary condition. This is due to the fact that regardless of evaporation losses, the yearly critical period inflows totaled 181 KAF and the yearly minimum releases add up to 325.19 KAF. The total minimum releases are bigger than the total yearly inflows.

Keystone starts with a storage of 618 KAF and at the end of 12 months operation finishes with a storage of 418.7 KAF. There has been a loss of 32.3% of the water in storage after a year of operation. In order to avoid this loss of water, the minimum releases from this reservoir must be eliminated, or set equal to the inflows minus evaporation, and in this way, the reservoir storage remains constant.

The optimal releases for the two trial trajectories are the same, and equal to the minimum releases required du during each month, as it is shown in Table XXI for the first trial trajectory and in Table XXII for the second trial trajectory.

Fort Gibson Reservoir

The optimal storage policies for Fort Gibson reservoir

TABLE XIX

OPTIMAL STORAGE POLICY FOR THE CRITICAL PERIOD INFLOWS AND THE TRIAL TRAJECTORY IN THOUSANDS OF ACRE-FEET

k	Month	Keystone	Fort Gibson	Webbers Falls	Tenkiller-Ferry	Eufaula	Robert S. Kerr
1	July	618.0	365.2	160.0	654.1	2,329.7	473.7
2	August	594.6	337.0	160.0	658.1	2,300.9	473.7
3	September	545.3	333.2	160.0	663.7	2,269.4	473.7
4	October	506.4	329.8	160.0	668.9	2,255.5	473.7
5	November	481.7	327.4	160.0	671.8	2,244.0	473.7
6	December	470.6	327.5	160.0	675.1	2,252.7	473.7
7	January	461.8	329.7	160.0	681.7	2,267.3	473.7
8	February	463.11	330.9	160.0	714.2	2,279.8	473.7
9	March	466.3	331.8	160.0	751.4	2,293.3	473.7
10	April	461.4	333.9	160.0	789.6	2,291.8	473.7
11	May	453.4	353.2	160.0	789.2	2,439.7	473.7
12	June	433.9	351.8	160.0	807.6	2,254.0	473.7
13	July	418.7	365.2	160.0	654.1	2,329.7	473.7

TABLE XX

OPTIMAL STORAGE POLICY FOR THE CRITICAL PERIOD INFLOWS AND THE SECOND TRIAL TRAJECTORY IN THOUSANDS OF ACRE-FEET

k	Month	Keystone	Fort Gibson	Webbers Falls	Tenkiller-Ferry	Eufaula	Robert S. Kerr
1	July	618.0	365.2	160.0	654.1	2,329.7	473.7
2	August	594.6	365.2	160.0	658.1	2,052.9	473.7
3	September	545.3	337.4	160.0	663.1	2,023.4	473.7
4	October	506.4	330.0	160.0	668.1	2,010.5	473.7
5	November	481.7	327.6	160.0	670.1	2,000.0	473.7
6	December	470.6	327.7	160.0	673.1	2,008.7	473.7
7	January	461.8	329.9	160.0	679.1	2,023.3	473.7
8	February	463.1	331.1	160.0	711.1	2,034.8	473.7
9	March	466.3	332.0	160.0	748.1	2,048.3	473.7
10	April	461.4	334.1	160.0	786.1	2,047.8	473.7
11	May	453.4	353.2	160.0	801.1	2,196.7	473.7
12	June	433.9	351.9	160.0	808.1	2,253.7	473.7
13	July	418.7	365.2	160.0	654.1	2,329.7	473.7

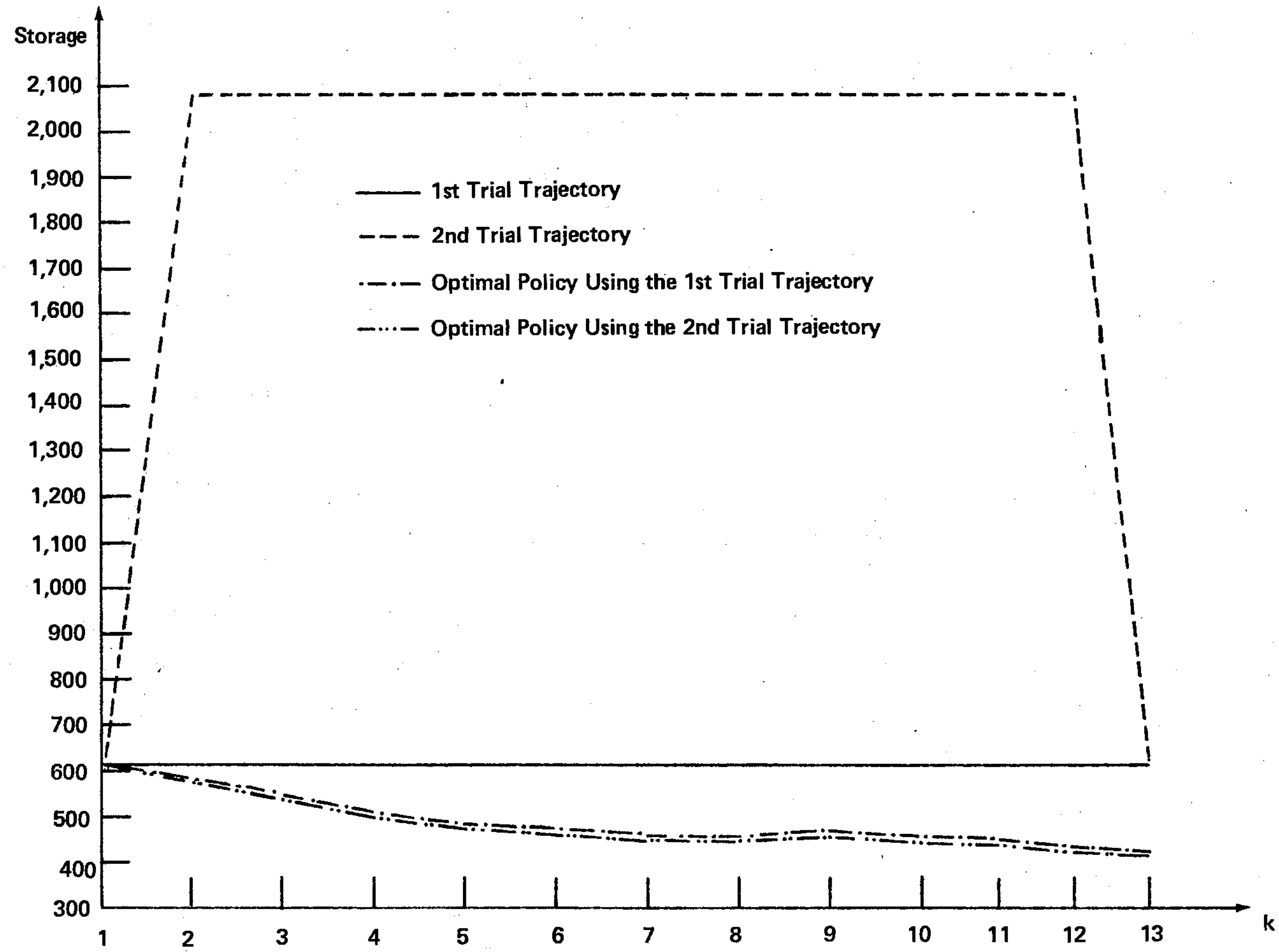

Figure 10. Keystone Reservoir: Optimal Storage Policies for the Critical Period Inflows

TABLE XXI

OPTIMAL RELEASE POLICY FOR THE CRITICAL PERIOD INFLOWS AND THE FIRST TRIAL TRAJECTORY IN THOUSANDS OF ACRE-FEET

k	Month	Keystone	Fort Gibson	Webbers Falls	Tenkiller-Ferry	Eufaula	Robert S. Kerr
1	July	45.01	0.83	58.40	0.05	248.00	295.58
2	August	50.05	24.00	91.26	0.54	0.74	91.41
3	September	35.52	4.27	49.60	0.26	0.21	50.66
4	October	25.03	0.19	35.20	0.81	0.01	38.26
5	November	16.13	0.11	26.41	0.33	0.11	34.39
6	December	11.68	0.05	22.35	0.57	0.20	32.60
7	January	11.68	0.04	38.25	0.58	0.96	47.89
8	February	10.55	0.06	23.04	0.11	0.12	40.94
9	March	16.66	0.16	47.71	0.25	0.14	55.75
10	April	24.22	0.41	64.80	0.58	0.23	103.30
11	May	36.71	0.11	55.45	11.41	0.13	95.42
12	June	41.95	0.74	75.03	167.23	0.11	251.12

TABLE XXII

OPTIMAL RELEASE POLICY FOR THE CRITICAL PERIOD INFLOWS AND THE SECOND TRIAL TRAJECTORY IN THOUSANDS OF ACRE-FEET

k	Month	Keystone	Fort Gibson	Webbers Falls	Tenkiller-Ferry	Eufaula	Robert S. Kerr
1	July	45.01	29.00	86.57	0.00	0.00	75.70
2	August	50.05	0.46	67.73	0.00	0.00	66.59
3	September	35.52	0.31	45.66	0.00	0.00	46.24
4	October	25.03	0.20	35.21	0.00	0.00	37.45
5	November	16.13	0.11	26.41	0.00	0.00	33.96
6	December	11.68	0.05	22.35	0.00	0.00	31.83
7	January	11.68	0.04	38.25	0.00	0.00	46.35
8	February	10.55	0.06	23.04	0.00	0.00	40.71
9	March	16.66	0.17	47.72	0.00	0.00	55.37
10	April	24.22	0.22	64.61	16.00	0.00	118.31
11	May	36.71	0.26	55.61	0.03	242.00	326.07
12	June	41.95	0.56	74.85	166.72	0.45	250.76

are shown in Table XIX for the first trial trajectory and in Table XX for the second trial trajectory, and graphically in Figure 11.

According to these results, the storage decreases from July to December when it reaches the smallest value; from then on, it increases until it reaches the final boundary condition for this reservoir.

The optimal release policies for the two trial trajectories are presented in Table XXI and XXII. The two policies are different specially for the first three months of operation; using the first trial trajectory, the greatest release occurs in August; for the 2nd trial trajectory, the biggest release takes place in July, the first month of operation.

Tenkiller-Ferry Reservoir

The optimal storage policies for Tenkiller-Ferry reservoir are presented in Tables XIX and XX, fir the first and second trial trajectories, respectively, and graphically in Figure 12.

The values of the optimal storages for the two trial trajectories are very similar, the biggest difference in values encountered corresponds to 3.5 KAF in the month of April. The optimal storages indicated that the volume of water is never smaller than the storage corresponding to a full power pool or 654.1 KAF, they always increase up to

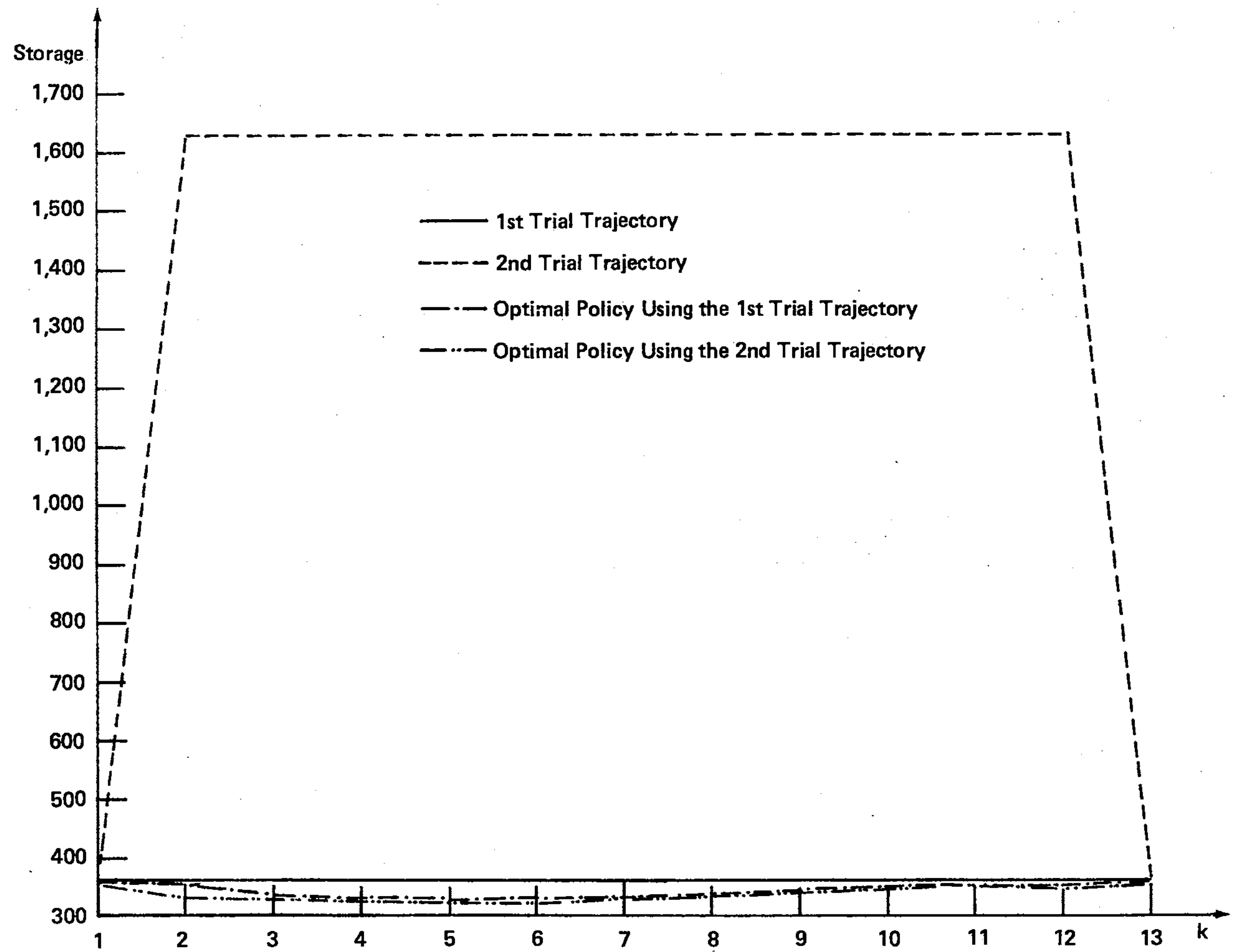

Figure 11. Fort Gibson Reservoir: Optimal Storage Policies for the Critical Period Inflows

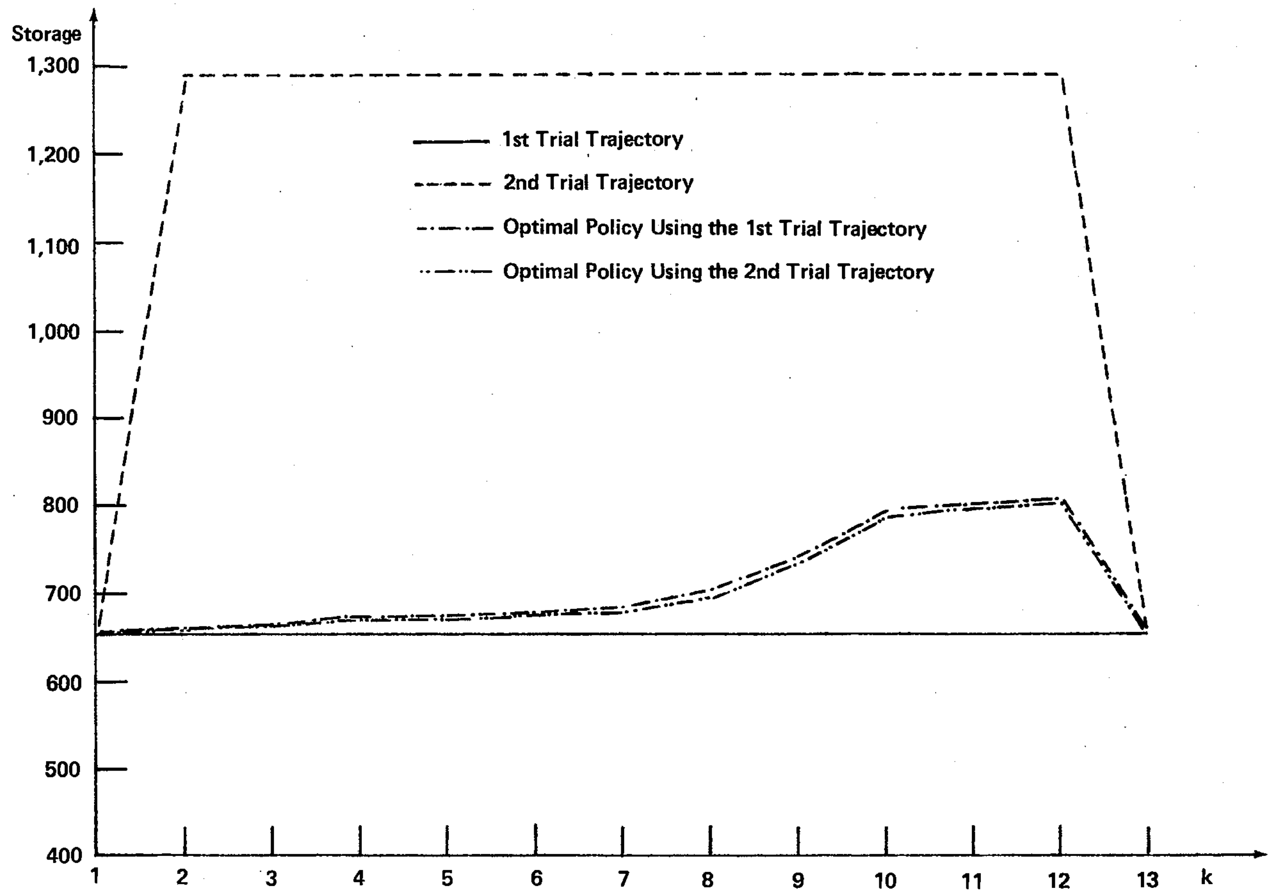

Figure 12. Tenkiller-Ferry Reservoir: Optimal Storage Policies for the Critical Period Inflows

June, and then decrease to meet the final boundary condition value.

The optimal release policy for the first trial trajectory is shown in Table XXI, for the second trial trajectory in Table XXII. The optimal release policies differ for the two trial trajectories; for the first trial trajectory, water is always released in small amounts along the year; according to the results for the second trial trajectory, there are releases only for three months in the period: April, May, and June and nothing during the rest of the year.

Eufaula Reservoir

The optimal storage policies for Eufaula reservoir are shown in Table XIX for the first trial trajectory and in Table XX for the second trial trajectory, and graphically in Figure 13. The values of the two optimal policies differ markedly for more than 200 KAF in each month.

These results indicate that the storage decreases from July to November, when it reaches the lowest value, and then increases until the final boundary is reached.

The optimal release policies also differ for the two trial trajectories. For the first trial trajectory, the biggest releases occurs in July; and in the rest of the year, only small amounts of water are released. The opposite occurs using the second trial trajectory, the biggest

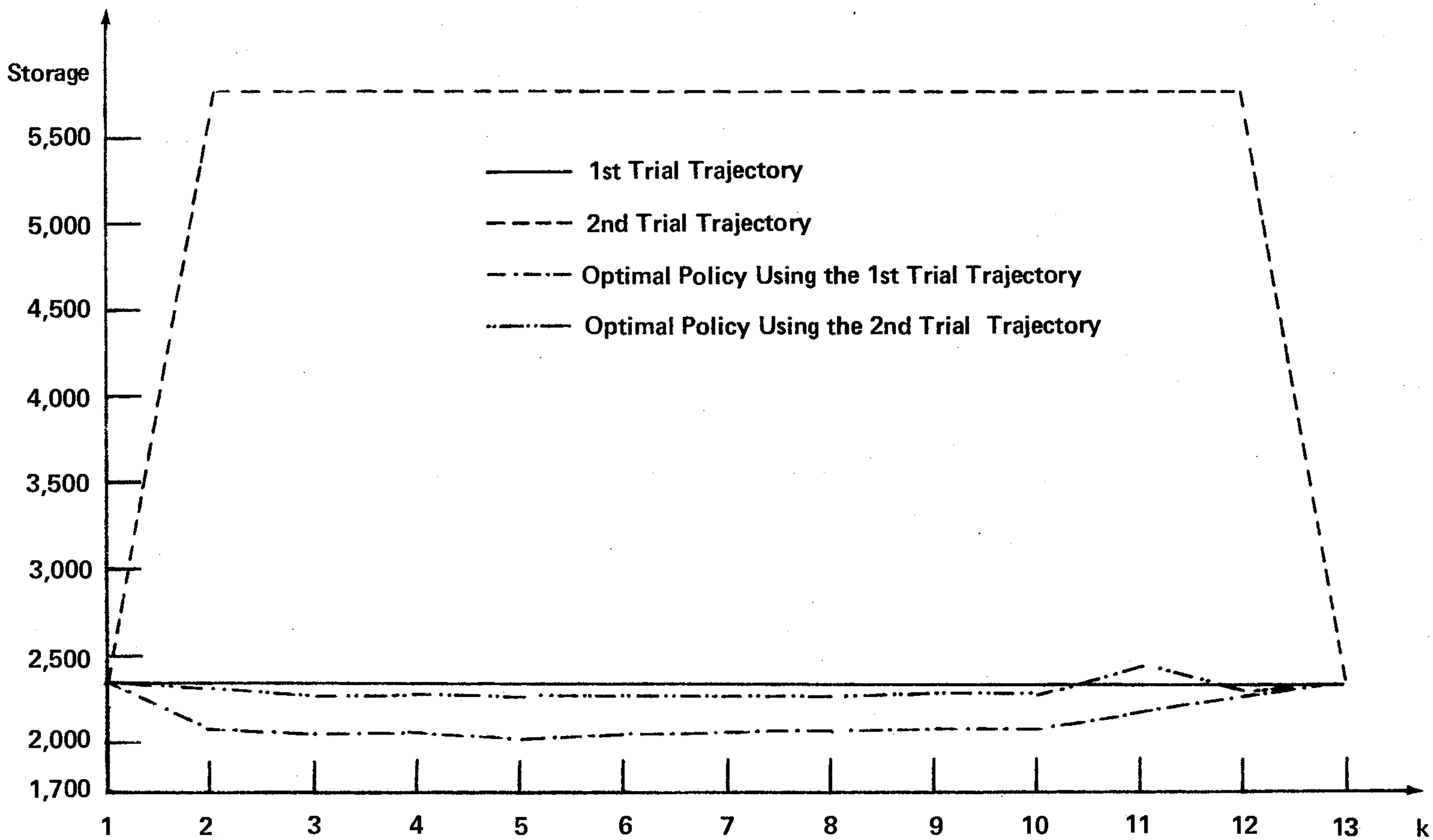

Figure 13. Eufaula Reservoir: Optimal Storage Policies for the Critical Period Inflows

release takes place in May at the end of the period of operation.

Webbers Falls and Robert S. Kerr Reservoirs

As it was stated in the empirical model presented in Chapter III, the storage in these two run-of-river reservoirs is going to be kept constant and equal to 160 KAF for Webbers Falls and 473.7 KAF for Robert S. Kerr.

The required released to keep the storage volumes at these two levels are shown in Table XXI for the first trial trajectory and in Table XXII for the second trial trajectory.

Energy Generation

The annual energy generation for the system of six reservoirs is shown in Table XXIII for the two trial trajectories. As it was expected, all the hydroelectric energy is generated on-peak, there is no off-peak energy production. More than 48% of the annual energy is generated at the Robert S. Kerr power plant for both trial trajectories.

The monthly energy generation at each power plant for the critical period is presented in Table XXXVI for Keystone, in Table XXXVII for Fort Gibson, in Table XXXVIII for Webbers Falls, in Table XXXIX for Tenkiller-Ferry, in Table XL for Eufaula, and in Table XLI for Robert S. Kerr (see Appendix A).

TABLE XXIII

ANNUAL ENERGY PRODUCTION IN KILOWATT-HOURS FOR THE SYSTEM OF SIX RESERVOIRS AND THE CRITICAL PERIOD INFLOWS

Power Plant	1st Trial Trajectory	2nd Trial Trajectory
	On-Peak	On-Peak
Keystone	8,277,580	8,277,580
Fort Gibson	77,364	77,450
Webbers Falls	15,303,616	15,316,298
Tenkiller-Ferry	13,867,790	13,884,619
Eufaula	10,717,241	10,729,127
Robert S. Kerr	45,337,033	44,946,791
System Total	93,580,624	93,231,865

Average Inflows

The optimal returns obtained from operating the system are for the two trial trajectories:

First trial trajectory = $6,492,249

Second trial trajectory = $6,491,650.

There is a difference of $599 between the two optimal trajectories; in relative terms, the difference represents a 0.0092% variations from the smallest return.

The rate of convergence toward the optimal return for each trial trajectory is presented in Figure 14. The first trajectory required 39 iterations to reach the optimal solution, and 46 iterations the second trial trajectory.

The total processing time required to optimize the operation of the six reservoirs for a period of 12 months is presented in Table XXIV for the two trial trajectories.

TABLE XXIV

TOTAL PROCESSING TIME* REQUIRED FOR THE ALGORITHM TO OPTIMIZE THE OPERATION OF THE SYSTEM

Trial Trajectory	Number of Iterations	Total Processing Time, Minutes	Processing Time per Iteration, Minutes
1st	39	59.09	1.515
2nd	46	48.68	1.058

*IBM System 360 Model 65

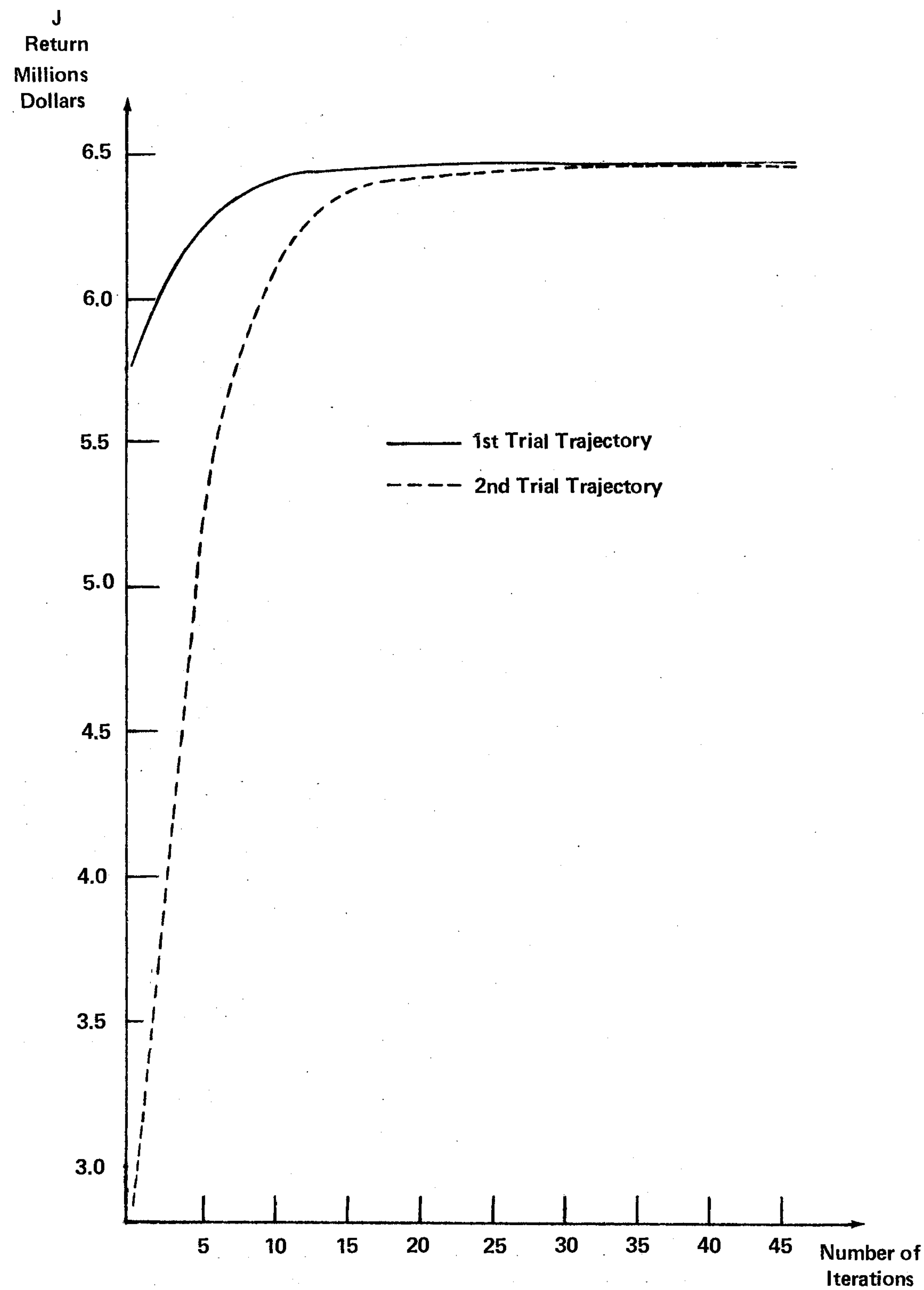

Figure 14. Total Return for the System of Six Reservoirs as a Function of the Number of Iterations and the Average Inflows

The optimal storage and release policies for each reservoir in the system are presented next.

Keystone Reservoir

The optimal storage policies for Keystone reservoir are shown in Table XXV for the first trial trajectory and in Table XXVI for the second trial trajectory, and graphically in Figure 15.

The values of the optimal storage policies are very similar for the two trial trajectories. The largest difference corresponds to November with a difference of 20.3 KAF between the two trial trajectories results.

The pattern of storage management indicated by the two optimal storage policies shows that the storage increases from July to September; decreases from September to October; increases again from October to December; decreases from December to February, the month at which the storage reaches its lowest value; increases from February to June; and finally, decreases to reach the final boundary condition value.

The optimal release policies are presented in Table XXVII for the first trial trajectory and in Table XXVIII for the second trial trajectory. The release values are similar for both trajectories, only three months show differences: August, October, and November; the largest difference encountered in these three months is 35.02 KAF and it occurs in October.

TABLE XXV

OPTIMAL STORAGE POLICY FOR THE AVERAGE INFLOWS AND THE FIRST TRIAL TRAJECTORY IN THOUSANDS OF ACRE-FEET

k	Month	Keystone	Fort Gibson	Webbers Falls	Tenkiller-Ferry	Eufaula	Robert S. Kerr
1	July	618.0	365.2	160.0	654.1	2,329.7	437.7
2	August	850.0	312.2	160.0	699.1	2,332.7	473.7
3	September	694.0	312.2	160.0	739.1	2,204.7	473.7
4	October	671.0	330.2	160.0	664.1	2,170.7	473.7
5	November	857.0	312.2	160.0	659.1	2,226.7	473.7
6	December	912.0	312.2	160.0	645.1	2,168.7	473.7
7	January	670.0	329.2	160.0	599.1	2,030.7	473.7
8	February	327.0	348.2	160.0	558.1	1,985.7	473.7
9	March	407.0	330.2	160.0	549.1	1,936.7	473.7
10	April	427.0	362.2	160.0	566.1	1,971.7	473.7
11	May	656.0	428.2	160.0	635.1	2,241.7	473.7
12	June	810.0	493.2	160.0	722.1	2,811.7	473.7
13	July	618.0	365.2	160.0	654.1	2,329.7	473.7

TABLE XXVI

OPTIMAL STORAGE POLICY FOR THE AVERAGE INFLOWS AND THE SECOND TRIAL TRAJECTORY IN THOUSANDS OF ACRE-FEET

k	Month	Keystone	Fort Gibson	Webbers Falls	Tenkiller-Ferry	Eufaula	Robert S. Kerr
1	July	618.0	365.2	160.0	654.1	2,329.7	473.7
2	August	850.1	393.3	160.0	699.7	2,333.1	473.7
3	September	679.9	402.8	160.0	586.2	2,205.5	473.7
4	October	656.2	420.5	160.0	614.1	2,172.1	473.7
5	November	877.3	312.1	160.0	660.7	2,228.5	473.7
6	December	911.5	332.3	160.0	646.9	2,172.5	473.7
7	January	669.2	349.4	160.0	601.8	2,034.3	473.7
8	February	328.4	368.8	160.0	558.1	1,989.5	473.7
9	March	408.6	350.6	160.0	549.4	1,940.1	473.7
10	April	427.4	382.9	160.0	566.6	1,975.4	473.7
11	May	656.0	449.1	160.0	636.4	2,246.1	473.7
12	June	810.0	514.2	160.0	722.4	2,815.4	473.7
13	July	618.0	365.2	160.0	654.1	2,329.7	473.7

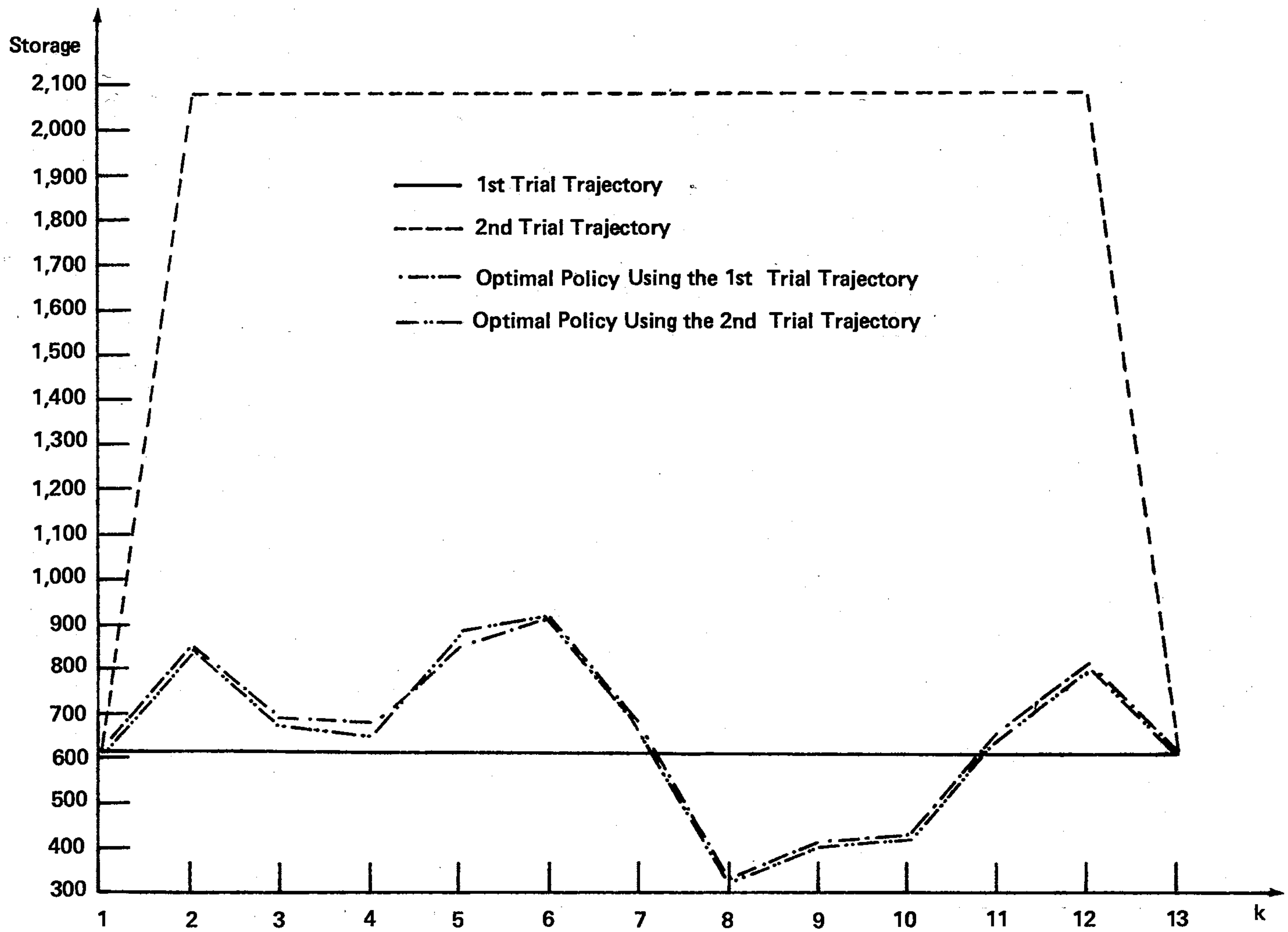

Figure 15. Keystone Reservoir: Optimal Storage Policies for the Average Inflows

TABLE XXVII

OPTIMAL RELEASE POLICY FOR THE AVERAGE INFLOWS AND THE FIRST TRIAL TRAJECTORY IN THOUSANDS OF ACRE-FEET

k	Month	Keystone	Fort Gibson	Webbers Falls	Tenkiller-Ferry	Eufaula	Robert S. Kerr
1	July	45.16	81.13	184.55	0.65	261.39	537.91
2	August	307.78	10.77	354.66	0.45	261.12	659.60
3	September	206.64	0.66	238.72	102.95	252.93	640.98
4	October	60.85	43.22	147.95	51.72	261.32	549.84
5	November	76.26	21.78	148.31	71.24	263.38	552.78
6	December	332.03	0.30	400.56	116.98	340.65	942.37
7	January	423.83	0.61	478.97	122.52	260.96	942.55
8	February	11.40	42.20	177.73	103.65	312.43	688.98
9	March	104.26	0.39	176.95	105.08	261.40	707.97
10	April	61.12	0.64	177.83	101.87	252.90	739.29
11	May	325.28	0.61	491.12	88.69	260.87	1,079.01
12	June	635.22	189.77	935.24	184.01	1,053.46	2,323.26

TABLE XXVIII

OPTIMAL RELEASE POLICY FOR THE AVERAGE INFLOWS AND THE SECOND TRIAL TRAJECTORY IN THOUSANDS OF ACRE-FEET

k	Month	Keystone	Fort Gibson	Webbers Falls	Tenkiller-Ferry	Eufaula	Robert S. Kerr
1	July	45.01	0.00	103.27	0.00	261.00	455.59
2	August	322.05	0.00	358.16	154.00	260.67	816.20
3	September	207.39	0.00	238.81	0.44	252.34	537.97
4	October	25.83	133.00	202.71	0.19	260.89	552.61
5	November	97.05	1.50	148.83	71.10	261.40	551.17
6	December	332.32	0.23	400.78	116.10	340.90	941.95
7	January	421.65	0.19	476.37	125.13	260.76	942.37
8	February	11.18	42.29	177.60	103.38	312.82	688.97
9	March	105.47	0.00	177.76	104.89	216.02	708.22
10	April	61.44	0.27	177.78	101.10	252.24	737.81
11	May	325.33	0.39	490.96	89.69	261.54	1,080.51
12	June	635.22	210.45	955.95	184.31	1,057.13	2,347.94

Fort Gibson Reservoir

The optimal storage policies for this reservoir are shown in Table XXV for the first trial trajectory and in Table XXVI for the second trial trajectory, graphically these optimal policies are presented in Figure 16. The optimal storage values are different for both trial trajectories, and only in the month of November are they almost the same.

There is no general pattern of behavior like the one found for Keystone reservoir. In general, it could be said that the first trial trajectory indicates that the storage decreases from July to September, increases in October, decreases again from October to December, and then increases until June. Finally, the storage decreases for both trajectories to meet the final boundary condition.

The optimal release policies for Fort Gibson reservoir are presented in Table XXVII for the first trial trajectory and in Table XXVIII for the second trial trajectory. The optimal release values for both trajectories differ markedly. The biggest difference is found in the first four months of the period. The first trial trajectory solution indicates that there are releases in the first four months; according to the second trial trajectory, there are no releases in the first three months, they are concentrated in the fourth month.

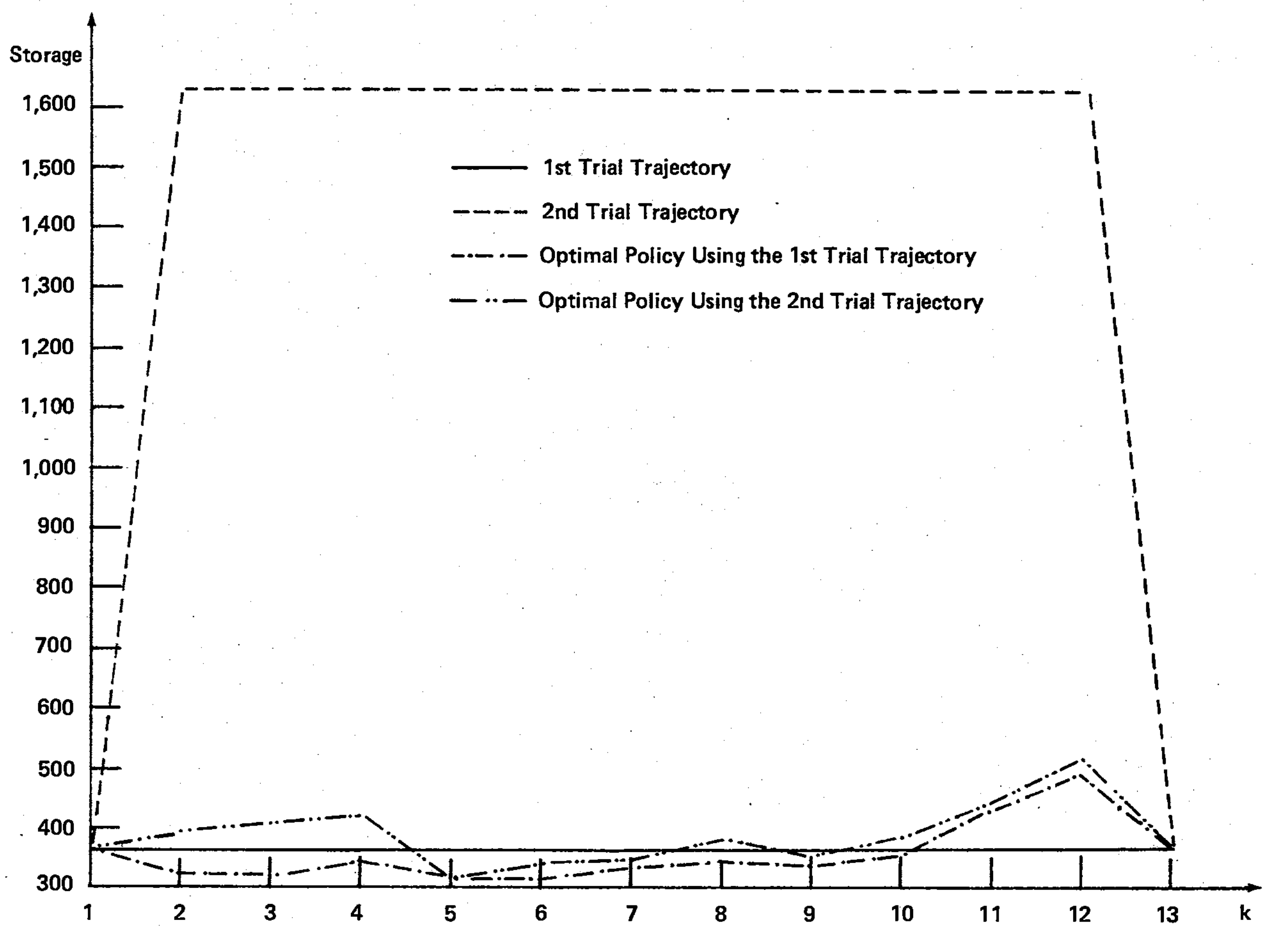

Figure 16. Fort Gibson Reservoir: Optimal Storage Policies for the Average Inflows

Tenkiller-Ferry Reservoir

The optimal storage policies for this reservoir are shown in Table XXV for the first trial trajectory and in Table XXVI for the second trial trajectory, and graphically in Figure 17. The optimal storage values are very similar for both trajectories, with the exception of September in which there is a difference of 152.9 KAF, and in October with a difference of 50 KAF.

The general pattern for the first trial trajectory solution indicates that the storage should increase from July to October, decrease from October to March, and increase again until June. The second trial trajectory indicates a different behavior for September and October; according to this trial trajectory, the volume increases from July to August, decreases from August to September, increases from September to November; and from November on, the optimal storage policy follows the same pattern found in the first trial trajectory solution.

The optimal release policies are presented in Table XXVII for the first trial and in Table XXVIII for the second trial trajectory. The optimal values of the optimal releases are different from July to October, but very similar the rest of the period. During the first four months, the optimal releases are concentrated in September for the first trial trajectory and in August for the second trial trajectory.

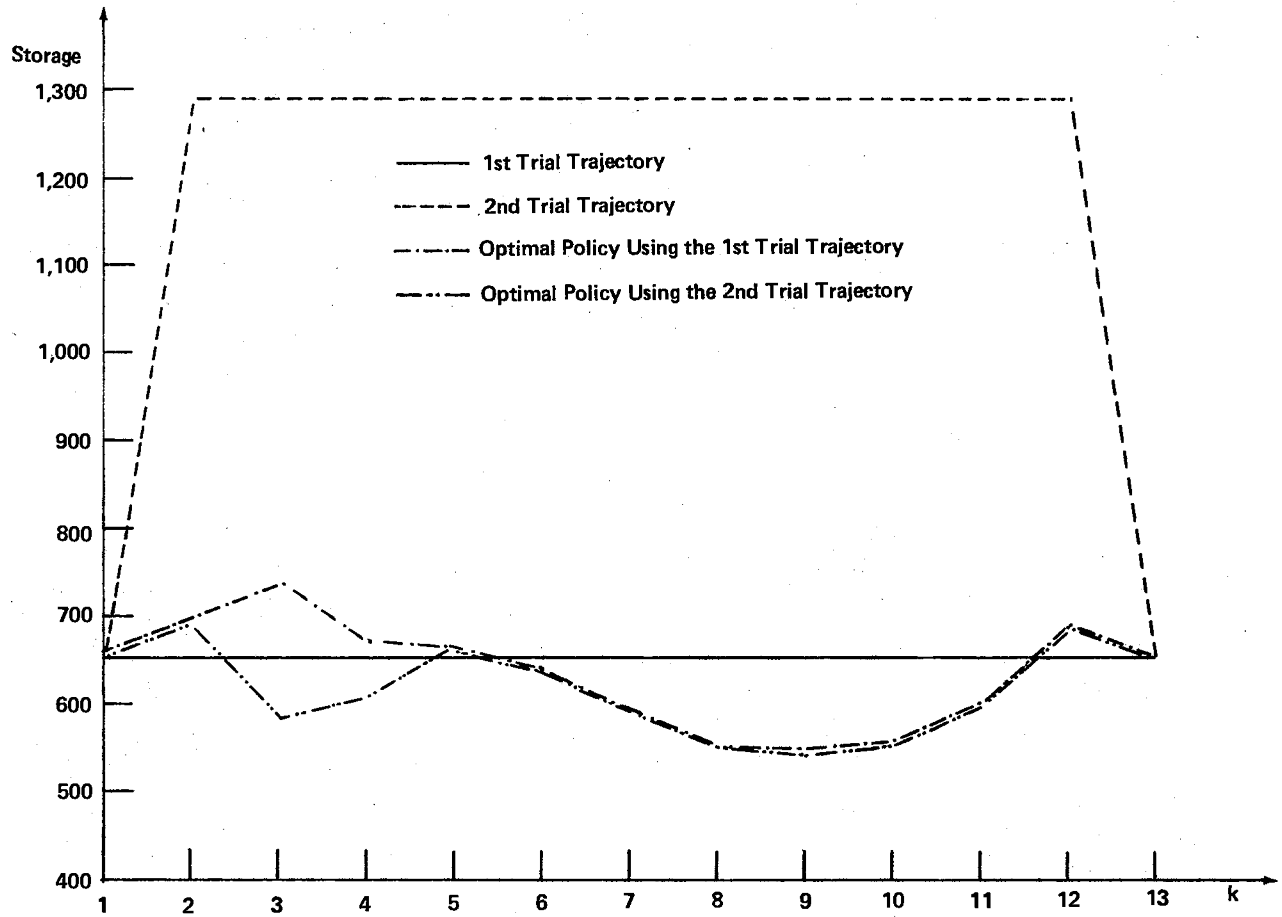

Figure 17. Tenkiller-Ferry Reservoir: Optimal Storage Policies for the Average Inflows

Eufaula Reservoir

The optimal storage policies for Eufaula reservoir are shown in Table XXV for the first trial trajectory and in Table XXVI for the second trial trajectory, graphically the optimal storages are shown in Figure 18.

The optimal storage values are very similar for both trial trajectories, the largest difference encountered between these values corresponds to only 4.4 KAF occurring during May.

The general pattern of storage operation for this reservoir is the following: storage increases from July to August, decreases from August to October, increases in November, decreases from November to March, increases again from March to June, and then decreases to meet the final boundary condition.

The optimal release policies for this reservoir are presented in Table XXVII for the first trial trajectory and in Table XXVIII for the second trial trajectory. The optimal release values for the two trial trajectories are very similar with a maximum difference of 3.67 KAF during June.

Webbers Falls and Robert S. Kerr Reservoirs

The required releases from these two reservoirs in order to keep their storages constant at 160 KAF in Webbers Falls and 473.7 KAF in Robert S. Kerr are shown in Table

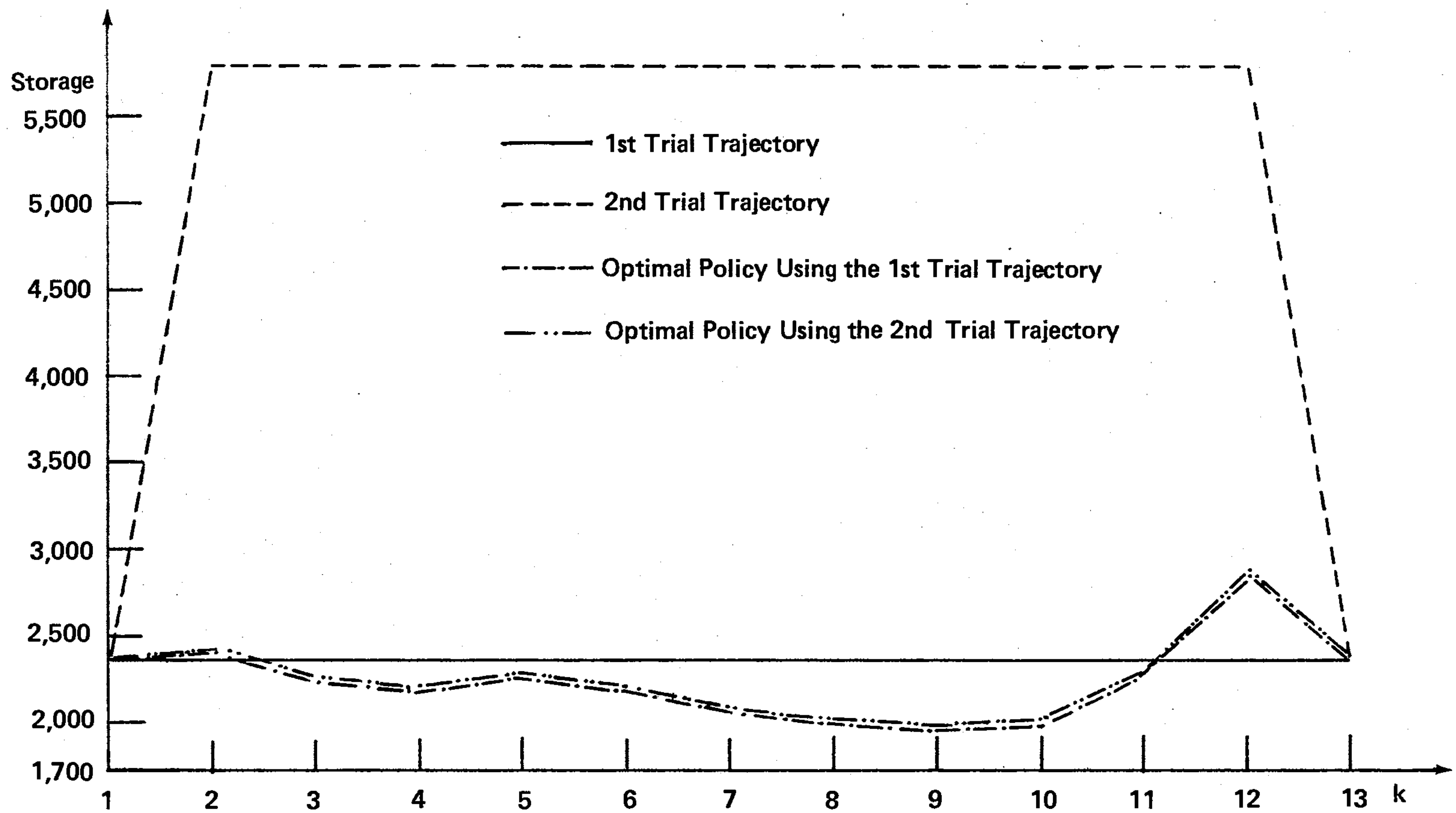

Figure 18. Eufaula Reservoir: Optimal Storage Policies for the Average Inflows

XXVII for the first trial trajectory and in Table XXVIII for the second trial trajectory.

Energy Generation

The annual hydroelectric energy generated for the system of six power plants is shown in Table XXIX. It can be observed that as the volume of infows increases off-peak energy starts being produced.

Using the first trial trajectory 769,195,538 kilowatt-hours are generated; from this amount 704,310,607 kilowatt-hours, or 91.56% are generated on-peak; and 64,884,931 kilowatt-hours, or 8.44% are generated off-peak. Employing the second trial trajectory 770,616,771 kilowatt-hours are generated; from this total energy 703,630,519 kilowatt-hours, or 91.31% are generated on-peak, and 66,986,252 kilowatt-hours, or 8.69% are generated off-peak.

Robert S. Kerr reservoir produces the greatest share of energy of the system, 49% of the on-peak energy, and more of the 49% of the off-peak energy. The same occurs for the critical period inflows.

The monthly energy generation on-peak and off-peak are shown in Table XLII for Keystone, in Table XLIII for Fort Gibson, in Table XLIV for Webbers Falls, in Table XLV for Tenkiller-Ferry, in Table XLVI for Eufaula, and in Table XLVII for Robert S. Kerr power plants (see Appendix A).

TABLE XXIX

ANNUAL ENERGY PRODUCTION IN KILOWATT-HOURS FOR THE SYSTEM OF SIX RESERVOIRS AND THE AVERAGE INFLOWS

Power Plant	1st Trial Trajectory		2nd Trial Trajectory	
	On-Peak	Off-Peak	On-Peak	Off-Peak
Keystone	98,785,012	135,261	98,948,245	144,899
Fort Gibson	4,233,437	—	6,551,936	—
Webbers Falls	69,674,490	29,557,840	67,714,958	30,900,107
Tenkiller-Ferry	64,095,562	39,668	63,995,880	82,883
Eufaula	122,094,586	2,774,995	122,195,420	2,782,635
Robert S. Kerr	345,427,520	32,377,168	344,224,080	33,075,728
System Total	704,310,607	64,884,931	703,630,519	66,986,252

High Inflows

The optimal returns associated with the equation of the system of six reservoirs under the high inflows and for the two trial trajectories are:

First trial trajectory = \$9,542,004

Second trial trajectory = \$9,551,575.

There is a difference of \$9,571 between the two optimal returns. In relative terms, this difference represents a 0.10% variation from the smallest optimal return value.

The rate of convergence of the two trial trajectories toward the optimal returns is shown in Figure 19. In order to reach convergence, the first trial trajectory required 53 iterations, and the second trial trajectory required 58 iterations.

The total processing time employed to optimize the operation of the system for the two trial trajectories and a period of twelve months are presented in Table XXX.

The optimal storage and release policies for each individual reservoir under the high inflows are presented next.

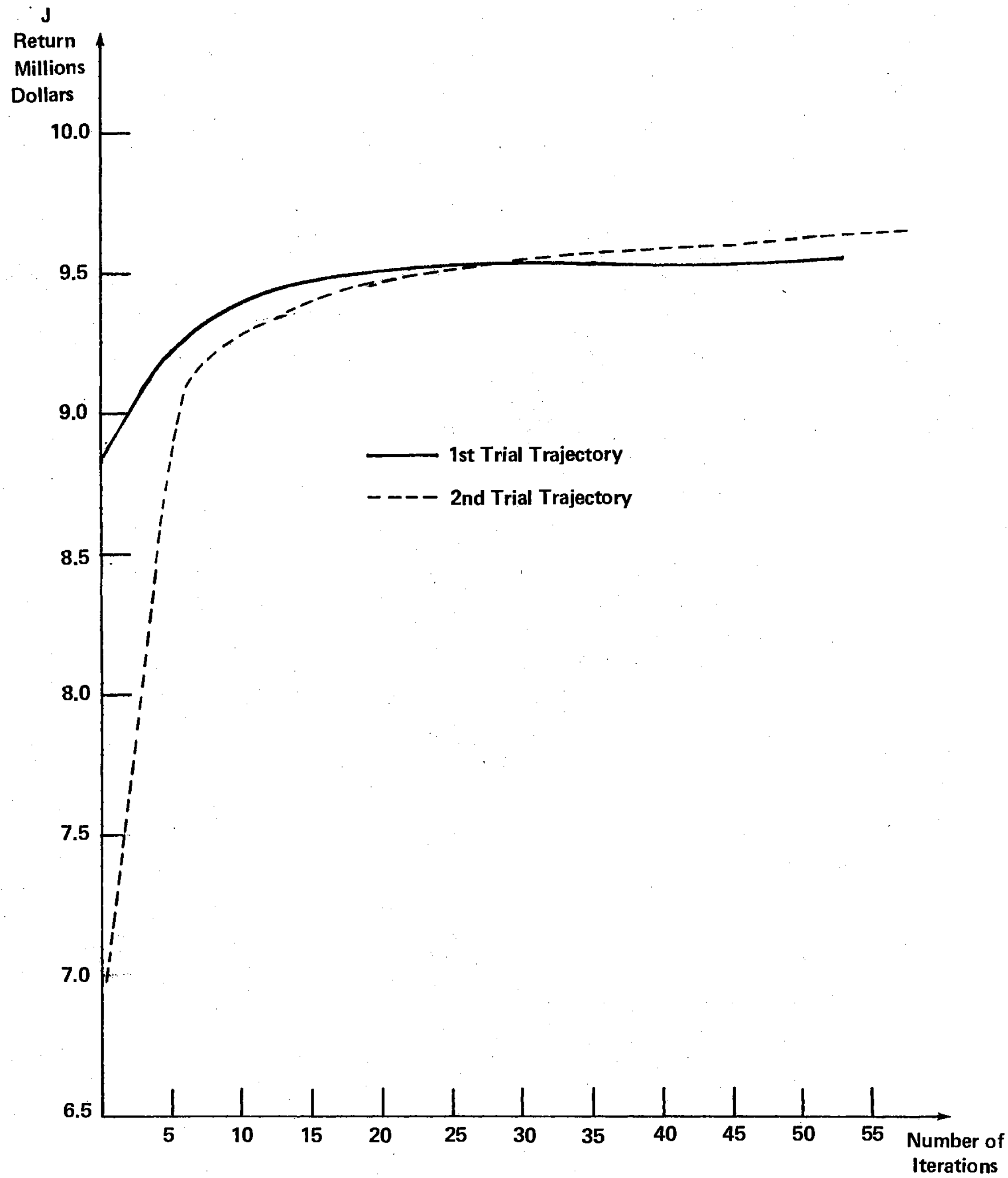

Figure 19. Total Return for the System of Six Reservoirs as a Function of the Number of Iterations and the High Inflows

TABLE XXX

TOTAL PROCESSING TIME* REQUIRED FOR THE ALGORITHM TO OPTIMIZE THE OPERATION OF THE SYSTEM

Trial Trajectory	Number of Iterations	Total Processing Time, Minutes	Processing Time per Iteration, Minutes
1st	53	92.40	1.743
2nd	58	72.48	1.249

*IBM System 360 Model 65

Keystone Reservoir

The optimal storage policies are presented in Table XXXI for the first trial trajectory and in Table XXXII for the second trial trajectory, and graphically in Figure 20.

The optimal storage values are similar from July to February, from then on the values vary between the two trial trajectories for as much as 251 KAF in March.

According to these optimal storages, the storage increases from July to August, decreases from August to October, increases from October to December, decreases from December to April to June, and then decreases to meet the final boundary condition. The difference between the results of the two trial trajectories is the rate at which storage decreases from January to April. The storage decreases more rapidly using the first trial trajectory than it does the the second trial trajectory.

TABLE XXXI

OPTIMAL STORAGE POLICY FOR THE HIGH INFLOWS AND THE FIRST TRIAL TRAJECTORY IN THOUSANDS OF ACRE-FEET

k	Month	Keystone	Fort Gibson	Webbers Falls	Tenkiller-Ferry	Eufaula	Robert S. Kerr
1	July	618.0	365.2	160.0	654.1	2,329.7	473.7
2	August	868.0	451.2	160.0	536.1	1,850.7	473.7
3	September	716.0	456.2	160.0	439.1	1,746.7	473.7
4	October	684.0	504.2	160.0	501.1	1,922.7	473.7
5	November	1,114.0	568.2	160.0	537.1	2,603.7	473.7
6	December	1,241.0	622.2	160.0	582.1	2,864.7	473.7
7	January	1,145.0	656.2	160.0	598.1	3,069.7	473.7
8	February	797.0	696.2	160.0	630.1	2,989.7	473.7
9	March	749.0	743.2	160.0	725.1	2,388.7	473.7
10	April	592.0	527.2	160.0	555.1	2,202.7	473.7
11	May	697.0	674.2	160.0	796.1	2,294.7	473.7
12	June	853.0	816.2	160.0	775.1	2,588.7	473.7
13	July	618.0	365.2	160.0	654.1	2,329.7	473.7

TABLE XXXII

OPTIMAL STORAGE POLICY FOR THE HIGH INFLOWS AND THE SECOND TRIAL TRAJECTORY IN THOUSANDS OF ACRE-FEET

k	Month	Keystone	Fort Gibson	Webbers Falls	Tenkiller-Ferry	Eufaula	Robert S. Kerr
1	July	618.0	365.2	160.0	654.1	2,329.7	473.7
2	August	866.0	451.4	160.0	534.1	1,855.0	473.7
3	September	713.5	484.3	160.0	436.6	1,724.3	473.7
4	October	681.0	532.8	160.0	498.8	1,899.4	473.7
5	November	1,110.0	597.6	160.0	538.6	2,580.4	473.7
6	December	1,236.0	652.2	160.0	583.9	2,841.4	473.7
7	January	1,140.0	686.8	160.0	600.4	3,046.3	473.7
8	February	1,007.0	417.0	160.0	633.4	3,035.4	473.7
9	March	1,000.0	464.5	160.0	729.4	2,391.4	473.7
10	April	554.0	527.0	160.0	806.4	1,969.4	473.7
11	May	659.0	674.4	160.0	795.4	2,313.4	473.7
12	June	815.0	816.1	160.0	774.4	2,554.4	473.7
13	July	618.0	365.2	160.0	654.1	2,329.7	473.7

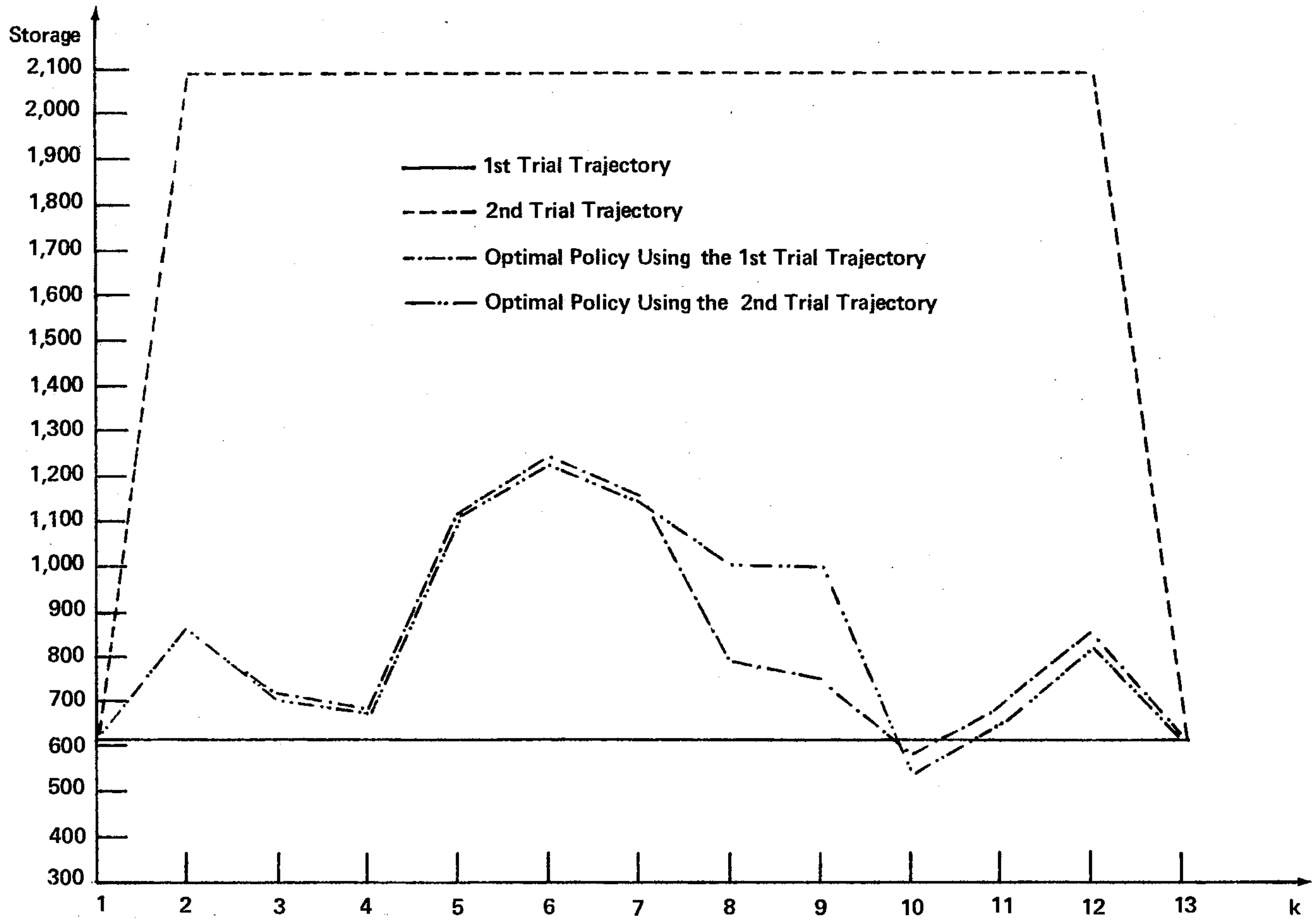

Figure 20. Keystone Reservoir: Optimal Storage Policies for the High Inflows

The optimal release policies are shown in Table XXXIII for the first trial trajectory and in Table XXXIV for the second trial trajectory. The optimal releases are similar from July to December, vary from January to March, are similar again in April and May, and vary again in June.

Fort Gibson Reservoir

The optimal storage policies for Fort Gibson reservoir are presented in Table XXXI for the first trial trajectory and in Table XXXII for the second trial trajectory, and graphically in Figure 21.

The optimal storage values are very similar for August and from April to June, varying for not more than 0.2 KAF. The rest of the values are not similar. The biggest differences are found in February and March.

The optimal results of the first trial trajectory indicate that the storage increases from July to March, decreases in April, increases from April to June, and decreases in July. According to the second trial trajectory results, the storage increases from July to January, decreases in February, increases from February to June, and decreases in July to meet the final boundary condition value.

The optimal release values are shown in Table XXXIII for the first trial trajectory and in Table XXXIV for the second trial trajectory. The value and distribution of the optimal releases are different. According to the results

TABLE XXXIII

OPTIMAL RELEASE POLICY FOR THE HIGH INFLOWS AND THE FIRST TRIAL TRAJECTORY IN THOUSANDS OF ACRE-FEET

k	Month	Keystone	Fort Gibson	Webbers Falls	Tenkiller-Ferry	Eufaula	Robert S. Kerr
1	July	392.06	0.23	509.55	225.05	1,083.28	2,042.51
2	August	516.32	27.92	630.46	217.53	487.81	1,473.37
3	September	401.10	0.73	469.63	0.45	300.23	942.40
4	October	261.00	0.96	356.63	82.33	260.77	978.27
5	November	183.12	0.73	308.92	90.63	252.73	845.62
6	December	276.14	0.68	483.44	131.01	261.34	1,060.27
7	January	504.68	0.24	621.25	127.52	640.82	1,568.79
8	February	275.13	0.53	776.48	90.68	1,228.37	2,326.51
9	March	397.84	277.68	854.20	417.38	843.39	2,576.63
10	April	552.71	0.73	853.70	117.19	1,052.84	2,493.23
11	May	860.96	0.64	1,251.34	385.92	1,293.79	3,405.98
12	June	1,156.24	598.78	1,983.33	381.33	1,455.19	4,201.13

TABLE XXXIV

OPTIMAL RELEASE POLICY FOR THE HIGH INFLOWS AND THE SECOND TRIAL TRAJECTORY IN THOUSANDS OF ACRE-FEET

k	Month	Keystone	Fort Gibson	Webbers Falls	Tenkiller-Ferry	Eufaula	Robert S. Kerr
1	July	394.00	0.00	511.27	227.00	1,079.00	2,041.89
2	August	516.85	0.00	603.07	218.07	514.45	1,473.15
3	September	401.65	0.00	469.48	0.27	301.23	943.06
4	October	261.98	0.00	356.66	78.52	260.84	974.57
5	November	184.12	0.00	309.19	90.33	252.74	845.62
6	December	276.14	0.00	482.77	130.52	261.44	1,059.21
7	January	289.69	310.00	716.01	126.51	571.76	1,593.50
8	February	233.58	0.67	735.08	89.68	1,271.35	2,327.10
9	March	685.56	0.71	864.96	170.38	1,079.38	2,576.37
10	April	553.01	0.28	853.56	368.64	802.01	2,493.71
11	May	861.16	0.96	1,251.86	385.92	1,346.73	3,459.44
12	June	1,118.49	598.67	1,945.62	380.64	1,421.12	4,128.52

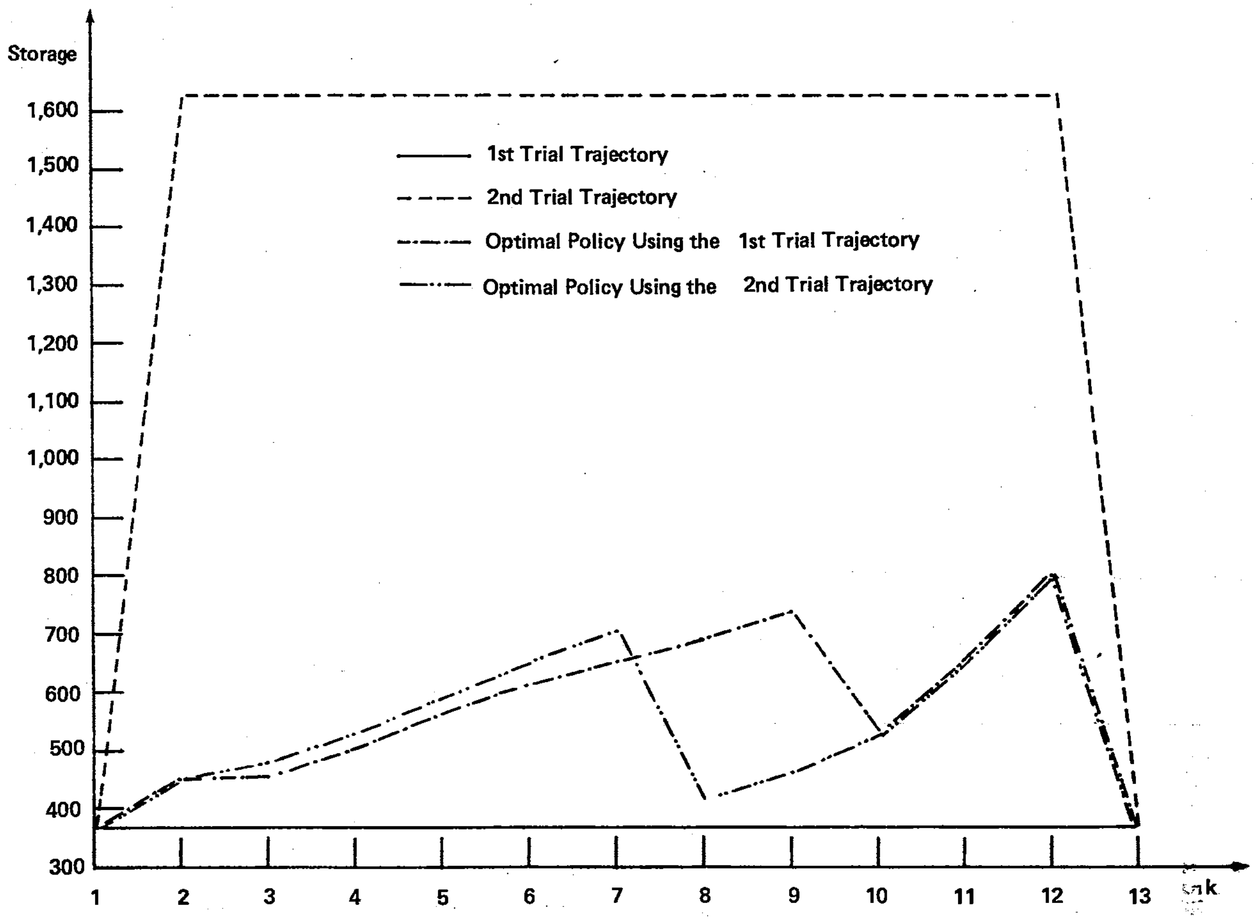

Figure 21. Fort Gibson Reservoir: Optimal Storage Policies for the High Inflows

for the second trial trajectory, there are no releases from July to September; this fact does not occur for the first trial trajectory. The optimal releases indicate the releases are concentrated in two months of the year, March and June, for the first trial trajectory, and January and June for the second trial trajectory. The value of the June release almost has the same value for the two trajectories.

<u>Tenkiller-Ferry Reservoir</u>

The optimal storage values are presented in Table XXXI for the first trial trajectory, in Table XXXII for the second trial trajectory, and graphically in Figure 22.

The optimal storage values for the two trial trajectories are very similar, with the storage in April being the only value which varies drastically. The greatest difference found for the other months is of 4.3 during March.

The results of the first trial trajectory indicate that the storage decreases from July to September, increases from September to March, decreases in April, increases in May, and decreases from May to July. The difference with the second trial trajectory is that the storage increases in the period from September to April, and then decreases.

The optimal release values are shown in Table XXXIII for the first trial trajectory and in Table XXXIV for the second trial trajectory. With the exception of the releases for March and April, the optimal releases values are very

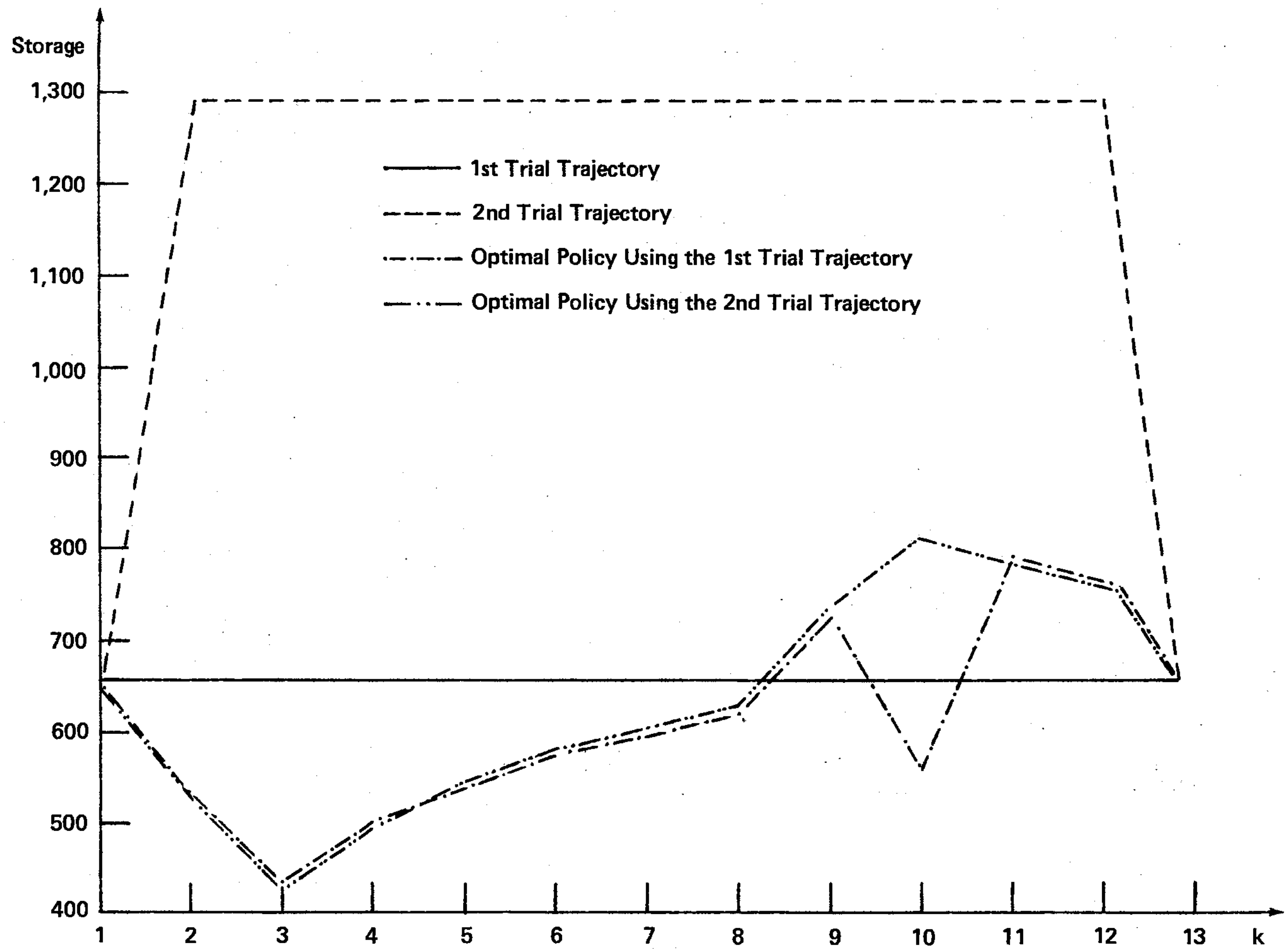

Figure 22. Tenkiller-Ferry Reservoir: Optimal Storage Policies for the High Inflows

similar for both trial trajectories. Besides, the distribution of the releases is very similar for the two trial trajectories, with the exception of March and April in which the order of the release is inverted for the two trajectories.

Eufaula Reservoir

The optimal storage values are shown in Table XXXI for the first trial trajectory and in Table XXXII for the second trial trajectory, and Figure 23 presents them graphically.

The optimal storage values are similar for both trial trajectories, the only month that shows a significant difference is April in which the optimal values have a variation of 233.4 KAF which corresponds to a 10% of the initial storage of this reservoir. The largest difference for the other month is only 45.7 KAF in February.

The general pattern of management for this reservoir should be the following: the storage decreases from July to September, increases from September to February, decreases from February to April, increases from April to June, and finally decreases to meet the final boundary condition value.

The optimal release values are presented in Table XXXIII for the first trial trajectory and in Table XXXIV for the second trial trajectory. The value and distribution of the optimal values are different in March and April in which the values are inverted for the two trial trajectories.

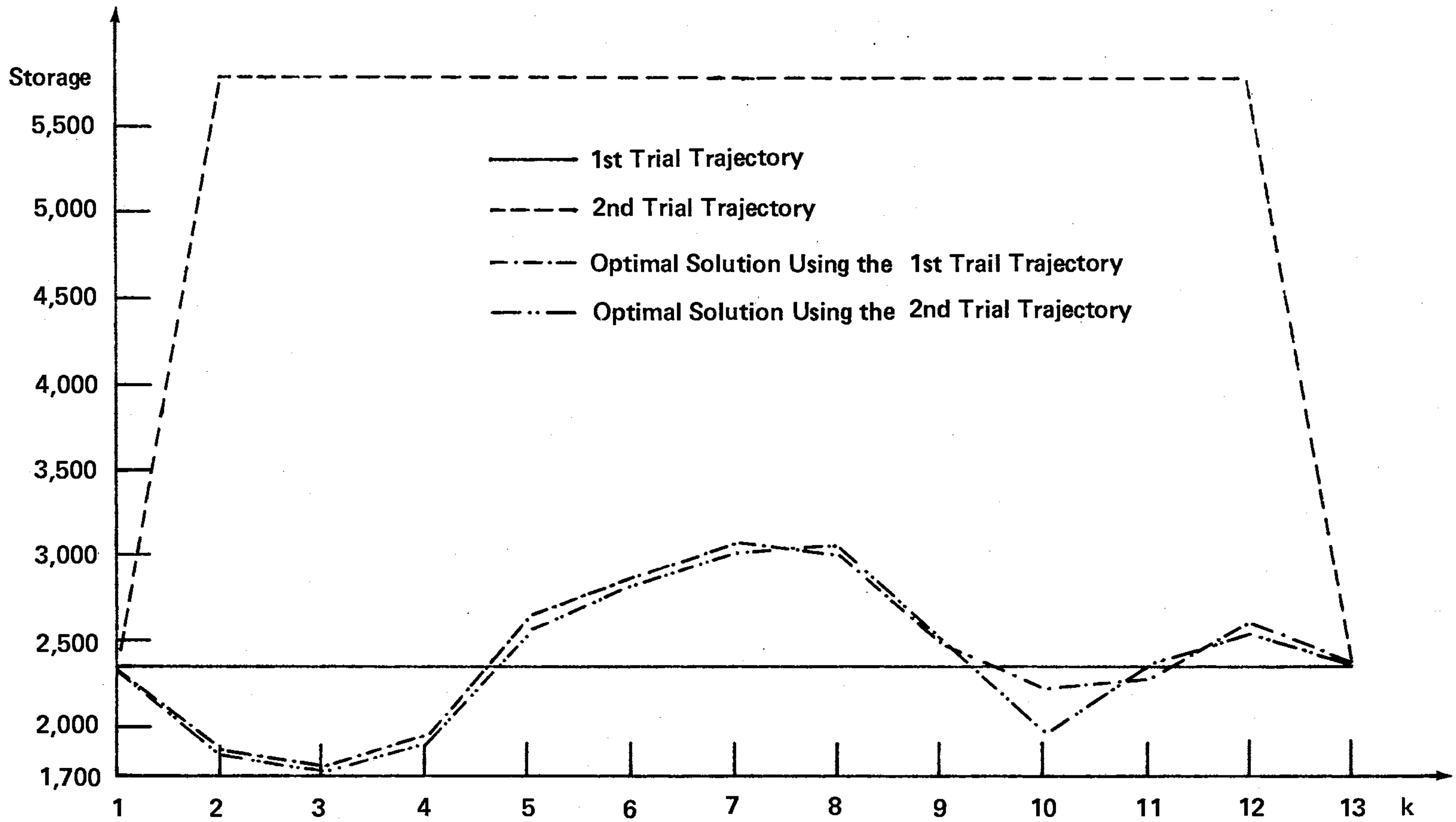

Figure 23. Eufaula Reservoir: Optimal Storage Policies for the High Inflows

Webbers Falls and Robert S. Kerr Reservoirs

The required releases to keep the storage in Webbers Falls constant at 160 KAF, and also constant in Robert S. Kerr at 473.7 KAF are shown in Table XXXIII for the first trial trajectory and in Table XXXIV for the second trial trajectory.

Energy Generation

The annual energy generated by the system of power plants under the high inflows is presented in Table XXXV. We observe from these results that when the amount of water flowing into the reservoir increases, the amount of energy generated also increases. The system is producing more than 1.4 billion kilowatt-hours under the high inflows compared with the 0.7 billion generated under the average inflows.

From the total of more than 114 billion kilowatt-hours produced using either of the two trial trajectories, on-peak energy represents more than 63%, and the off-peak represents more than 36%. When compared in relative terms with the average inflows generation, it is observed that the on-peak energy generation decreased from a 91% for the average inflows to 63% for the high inflows, and the off-peak generation increased from 8% for the average inflows to 36% for the high inflows.

The Robert S. Kerr power plant, as it occurred for the

TABLE XXXV

ANNUAL ENERGY PRODUCTION IN KILOWATT-HOURS FOR THE SYSTEM OF SIX RESERVOIRS AND THE HIGH INFLOWS

Power Plant	1st Trial Trajectory		2nd Trial Trajectory	
	On-Peak	Off-Peak	On-Peak	Off-Peak
Keystone	171,571,070	51,851,712	171,636,890	53,884,194
Fort Gibson	31,916,459	66,583	31,686,926	49,088
Webbers Falls	77,486,222	117,402,652	77,486,237	118,423,927
Tenkiller-Ferry	82,643,335	63,216,281	82,537,839	67,845,255
Eufaula	129,175,502	3,022,286	127,975,933	2,912,978
Robert S. Kerr	425,538,432	284,964,688	425,923,008	285,310,464
System Total	918,331,020	520,524,202	917,246,833	528,425,906

critical period and average inflows, is the one that generates more energy, representing more than 46% of the on-peak energy and more than 53% of the off-peak energy.

The monthly energy generation on-peak and off-peak is presented in Table XLVIII for Keystone, in Table XLIX for Fort Gibson, in Table L for Webbers Falls, in Table LI for Tenkiller-Ferry, in Table LII for Eufaula, and in Table LIII for Robert S. Kerr (see Appendix A).

CHAPTER V

SUMMARY AND CONCLUSIONS

This chapter is divided in three parts. The first presents the objectives and the procedure followed in this study, the second presents the main finding and conclusions, and finally the third indicates the limitations of the study and provided suggestions for further research in the area of optimization methods applied to water resources systems.

Objectives and Procedures

The main objectives of this study are the presentation and application of a new optimization technique called differential dynamic programming which drastically reduces the high speed memory requirements of the dynamic programming algorithm. The application of this optimization tool uses a complex water resources system composed by six multi-purpose reservoirs in Eastern Oklahoma. The reservoirs included in the analysis are Keystone, Fort Gibson, Webbers Falls, Tenkiller-Ferry, Eufaula, and Robert S. Kerr.

Considering the actual operation of the system and the nature of the water contracts, hydroelectric energy is considered in this research as the only "marketable" use of the water with an associated monetary return. Then, the performance criterion to be maximized was the return obtained by selling hydroelectric energy. A distinction is made between energy on-peak and off-peak, depending if the energy was generated during a high or a low demand period, respectively. This differentiation is important because in 1974 the price charged for the on-peak energy is 3.6 times higher than the off-peak price. An empirical model was developed as a multi-stage sequential decision process amenable to optimization by discrete differential dynamic programming. This model has a built-in procedure to transform the water released from the reservoirs into hydroelectric energy, and at the same time to differentiate between the on-peak and off-peak energy generation.

A deterministic approach is adopted in the analysis: the monthly inflows and the net evaporation rates are known quantities for the twelve months period under analysis.

In order to estimate the sensitivity of the optimal operational policy of the system under different hydrological conditions, the analysis is performed for three levels of monthly inflows corresponding to the critical period, average, and high inflows.

Findings and Conclusions

The findings and conclusions of this study are presented in two parts. The first presents the reduction in core memory achieved using this algorithm, the second part presents and discusses the optimal operational policies obtained applying the technique to the system of six reservoirs.

High Speed Memory Requirements

A drastic reduction in core memory was achieved by using differential dynamic programming. But before the results are presented, it is necessary to clarify some characteristics of the system under study.

In this dissertation, we are dealing with six reservoirs or six state variables; but actually, after the storages of Webbers Falls and Robert S. Kerr reservoirs were assigned constant values, the problem becomes a four state variables problem. In this sense, a decision has to be made for Keystone, Fort Gibson, Tenkiller-Ferry, and Eufaula reservoirs.

If the state policy space for these four reservoirs presented in Table XIV is quantized in a grid with a step size of 32 KAF, 60 grid points are necessary for Keystone, 45 grid points for Fort Gibson, 33 grid points for Tenkiller-Ferry, and 161 grid points for Eufaula reservoir. Then, according to formula (12) on page 33, the minimum number of memory locations required to solve this problem by programming are:

$$\begin{aligned} M &= 24 \cdot 60 \cdot 45 \cdot 33 \cdot 161 && [\text{bytes}] \\ &= 344{,}282{,}400 && [\text{bytes}] \end{aligned}$$

Dynamic programming requires 336,213 K[bytes][1] of high speed memory to solve this problem. For the same problem the differential dynamic programming required only 52 K-bytes , or 0.015% of the requirements of dynamic programming.

Another major obstacle found in applying dynamic programming is the computer time required for the computations and comparisons that must be made for each value of the control variable at each stage. If the control feasible space for these four variables is quantized in a grid with a step size of 16 KAF, then 46 grid points are required for Keystone, 39 grid points for Fort Gibson, 15 grid points for Tenkiller-Ferry, and 50 grid points for Eufaula reservoir. The total combinations of decisions provided by these points is given by the product of these values or

$$46 \cdot 39 \cdot 15 \cdot 50 = 1{,}345{,}500.$$

More than 1.3 million of combinations of decisions are required to optimize this problem using dynamic programming. If the optimization is limited to the neighborhood of the trial trajectory, this amount is greatly reduced. Besides, if the system is invertible like in this study, the number of combinations to take the system from $x(k-1)$ to a particular $x(k)$ is given by the formula T^n, or the number of incremental values raised to the number of state variables power. In this problem where three incremental values are considered (+DX, 0, -DX), and there are four state variables, the number of decisions at each stage is 3^4, or 81 decisions need to be considered instead of more than 1.3 million for the classical algorithm.

The Optimal Operational Policies

As the monthly inflows for the system increase, so does the value of the performance criterion. The optimal return associated with the critical period inflows is approximately 0.84 million, for the average inflows is 6.49 millions, and for the high inflows i 9.54 millions. The annual return increases dramatically by 672.6% between the critical period and the average inflows, and only by a 47% between the average and the high inflows.

More than 93 millions of kilowatt-hours are generated in a year under the critical period inflows, and all the hydroelectric generation is on-peak energy. The annual energy generation increases to 770 millions of kilowatt-hours for the average inflows from which more than 91% is produced on-peak and only approximately 8.5% is off-peak energy. For the high inflows, the annual energy generation is over 1.4 billion kilowatt-hours of which more than 63% is generated on-peak, and more than 36% is off-peak energy.

For every level of monthly inflows considered, the results indicate that Robert S. Kerr power plant generates over 48% of the annual total hydroelectric energy.

The optimal operational policies obtained for each of the six reservoirs in the system for the two trial trajectories employed in the analysis indicate that the optimal solutions are not unimodal for any of the three levels of inflows, the global optimal value of the performance criterion is unknown, as the optimal value of DX is also unknown.

Given the proximity of the optimal solutions, it is estimated that the algorithm reached alternative local optimum solutions. This characteristics of the algorithm was discussed in Chapter II, where an application of this technique was proposed for a fictitious water resource system.

These results may be considered only as approximate values of the optimum, but this is the trade-off that has to be paid to achieve such large reductions in core memory requirements. A few suggestions to improve this algorithm are given in the next section.

Limitations and Suggestions for Further Research

The limitations are mainly two: the first, associated with the stochastic nature of the inflows; and the second with the selection of the trial trajectory and the value of incremental values DX to form the subdomain S(k) at each stage of the analysis. Therefore, the suggestions for further research presented in the following section are related to procedures designed to overcome the obstacles presented by these two limitations.

Selection of the Trial Trajectory and the Incremental Values

In their work on discrete differential dynamic programming Gershwin and Jacobson state that in order to guarantee the convergence of the algorithm to a global maximum, it is necessary to choose a "sufficiently good" trial trajectory with respect to the objective of the research.[2] In this sense, "a priori knowledge" on the behavior of the performance criterion is a prerequisite for the application of the optimization technique.

When the return obtained by operating a reservoir is maximized using an iterative optimization technique, it is necessary to have some knowledge about the behavior of return under different operational policies and hydrological conditions for each individual reservoir in a system. Using this information, a "trial trajectory" which is "close" to the unknown optimal operational policy of the system can be used as the reference policy to start the optimization algorithm. This information is not available at the present for the six reservoirs considered in this study.

Optimization techniques used in management decision-making require basic economic data on the individual components of a system. As the complexity of the optimization method increases, so does the requirements for good basic data. This represents a very productive area of research in future water resources studies.

The other important requirement of the discrete differential dynamic programming algorithm required for convergence to a global optimum is the selection of the state variable incremental value DX in order to determine the subdomain S(k) at each stage in the analysis. If a global maximum is to be reached the incremental value should be chosen "properly," otherwise, the algorithm could converge to a local maximum, or minimum.

Jacobson proposes an interesting method to determine the value of the state variable increment.[3] Basically, the method consists in making the size of the increment DX a function of the stage variable. Then by choosing stage k_1 near k_f in the interval $[k_1, k_f]$ where $k_o \leq k_1 < k_f$, the state increment DX can be forced to be as small as wanted. This method is also effective for large values of the control variable.

Briefly, it can be said that there exists a stage k_1 close to the final stage k_f in the interval $k_o \leq k_1 < k_f$, such that the trial trajectory solution is followed from k_o to k_1, and then the variational performance criterion given by Equation (11) on page 30 is calculated applying the principle of optimality from k_1 to k_f. Solving this equation for DX provides a state increment value small enough as required by the algorithm. This procedure could be applied at the beginning of each iteration or when the performance criterion obtained from two successive iterations shows little or no improvement.

In order to check if the incremental value DX obtained by this method is small enough Jacobson[4] and Gershwin and Jacobson[5] propose the following test based on the value of the performance criterion

$$\Delta J_h = J_h^* - J_h$$

where J_h^* is the value of the performance criterion with the trial trajectory with the increments at iteration h, and J_h indicates the value of the performance criterion associated with the trial trajectory at iteration h.

If the value of ΔJ_h is close to zero, it can be inferred that the DX value is small enough as required by the algorithm.

This procedure to obtain the incremental value DX was not employed in this study. Provided the results of this research, in future applications of differential dynamic programming, it seems advisable to calculate the value of DX by the method proposed in this section.

The problems presented by the choice of a "good" trial trajectory, and a "proper" value for the state variable increment are not only limited to differential dynamic programming, they are also shared by other iterative optimization methods like the gradient and second variations method.[6, 7]

Stochastic Nature of the Inflows

The reservoir problem is an inventory problem in which the water flowing into the reservoir in a given period of time is a random variable. The stochastic nature of the monthly inflows was not considered in this study, as a means to overcome this problem, instead the flows were included at three different levels in order to estimate the sensibility of the system under different hydrological conditions.

Rainfall is the main source of water for the streamflows in Oklahoma. Given the fact that rainfall cannot be predicted accurately, it may be advisable to include the inflows as random variables in the analysis. In order to do this, it is necessary before hand to study the probability distributions describing the inflows during a given period of time. Furthermore, the dependence in time of the inflows must be studied carefully, because the dynamic programming optimization of systems that show complex dependence becomes very cumbersome.[8]

FOOTNOTES

[1]For the IBM system 360: 1[K bytes] = 1,024 [bytes].

[2]S. B. Gerschwin and D. H. Jacobson, A Discrete-Time Differential Application to Optimal Orbit Transfer, Division of Engineering and Applied Physics, Harvard University, Technical Report No. 566 (Cambridge, Massachusetts, 1968), p. 30.

[3]D. H. Jacobson, "New Second Order and First Order Algorithms for Determining Optimal Control," Journal of Optimization Theory and Applications, II (November, 1968), pp. 420-423.

[4]Ibid., p. 421.

[5]Gerschwin and Jacobson, pp. 19-20.

[6]D. J. Wilde, Optimum Seeking Methods (Englewood Cliffs, New Jersey, 1964), pp. 107-114, and Chapter 5.

[7]H. M. Wagner, Principles of Operations Research (Englewood Cliffs, New Jersey, 1969), Chapters 14 and 15.

[8]G. L. Nemhauser, Introduction to Dynamic Programming (New York, New York, 1966), pp. 129-155, 158.

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APPENDIX A

MONTHLY ENERGY GENERATION BY POWER PLANT IN THE SYSTEM

TABLE XXXVI

KEYSTONE POWER PLANT: MONTHLY ENERGY PRODUCTION IN KILOWATT-HOURS FOR THE CRITICAL PERIOD INFLOWS

k	Month	1st Trial Trajectory	2nd Trial Trajectory
		On-Peak	On-Peak
1	July	1,474,572	1,474,572
2	August	1,571,479	1,571,479
3	September	1,002,397	1,002,397
4	October	636,068	636,968
5	November	378,123	378,123
6	December	262,806	262,806
7	January	253,801	253,801
8	February	230,474	230,474
9	March	368,686	368,686
10	April	525,516	525,516
11	May	770,004	770,004
12	June	803,654	803,654
	Total	8,277,580	8,277,580

TABLE XXXVII

FORT GIBSON POWER PLANT: MONTHLY ENERGY PRODUCTION IN KILOWATT-HOURS FOR THE CRITICAL PERIOD INFLOWS

k	Month	1st Trial Trajectory	2nd Trial Trajectory
		On-Peak	On-Peak
1	July	2,050	69,157
2	August	56,809	1,251
3	September	11,812	861
4	October	507	525
5	November	207	279
6	December	125	129
7	January	111	114
8	February	171	177
9	March	446	462
10	April	1,128	607
11	May	208	504
12	June	3,727	3,348
	Total	77,364	77,450

TABLE XXXVIII

WEBBERS FALLS POWER PLANT: MONTHLY ENERGY PRODUCTION IN KILOWATT-HOURS FOR THE CRITICAL PERIOD INFLOWS

k	Month	1st Trial Trajectory	2nd Trial Trajectory
		On-Peak	On-Peak
1	July	1,521,401	2,228,740
2	August	2,350,727	1,763,786
3	September	1,292,036	1,189,238
4	October	917,062	917,246
5	November	688,050	688,151
6	December	582,460	582,509
7	January	996,530	996,569
8	February	600,283	600,345
9	March	1,242,918	1,243,075
10	April	1,687,696	1,682,896
11	May	1,444,484	1,448,536
12	June	1,979,969	1,975,307
	Total	15,303,616	15,316,298

TABLE XXXIX

TENKILLER-FERRY POWER PLANT: MONTHLY ENERGY PRODUCTION IN KILOWATT-HOURS FOR THE CRITICAL PERIOD INFLOWS

k	Month	1st Trial Trajectory	2nd Trial Trajectory
		On-Peak	On-Peak
1	July	3,281	—
2	August	34,248	—
3	September	16,707	—
4	October	52,517	—
5	November	21,686	—
6	December	37,270	—
7	January	38,152	—
8	February	7,809	—
9	March	17,644	—
10	April	43,137	1,120,836
11	May	862,845	77,225
12	June	12,732,494	12,686,558
	Total	13,867,790	13,884,619

TABLE XL

EUFAULA POWER PLANT: MONTHLY ENERGY PRODUCTION IN KILOWATT-HOURS FOR THE CRITICAL PERIOD INFLOWS

k	Month	1st Trial Trajectory	2nd Trial Trajectory
		On-Peak	On-Peak
1	July	10,563,416	—
2	August	28,163	—
3	September	7,776	—
4	October	204	—
5	November	4,034	—
6	December	7,388	—
7	January	36,491	—
8	February	4,620	—
9	March	5,402	—
10	April	8,639	—
11	May	5,121	10,669,062
12	June	45,987	60,065
	Total	10,717,241	10,729,127

TABLE XLI

ROBERT S. KERR POWER PLANT: MONTHLY ENERGY PRODUCTION IN KILOWATT-HOURS FOR THE CRITICAL PERIOD INFLOWS

k	Month	1st Trial Trajectory	2nd Trial Trajectory
		On-Peak	On-Peak
1	July	11,560,034	3,013,222
2	August	3,639,090	2,688,409
3	September	2,049,283	1,871,791
4	October	1,550,450	1,517,709
5	November	1,394,626	1,376,929
6	December	1,322,177	1,291,193
7	January	1,938,618	1,876,638
8	February	1,658,824	1,649,456
9	March	2,252,217	2,238,929
10	April	4,151,270	4,705,889
11	May	3,838,511	12,748,385
12	June	9,981,933	9,968,242
	Total	45,337,033	44,946,791

TABLE XLII

KEYSTONE POWER PLANT: MONTHLY ENERGY PRODUCTION IN KILOWATT-HOURS FOR THE AVERAGE INFLOWS

k	Month	1st Trial Trajectory		2nd Trial Trajectory	
		On-Peak	Off-Peak	On-Peak	Off-Peak
1	July	1,479,451	—	1,474,540	—
2	August	13,272,297	—	13,891,538	—
3	September	7,597,199	—	7,483,890	—
4	October	2,169,049	—	901,180	—
5	November	3,321,707	—	4,300,087	—
6	December	15,119,986	8,907	15,119,986	15,974
7	January	15,087,885	—	14,993,015	—
8	February	75,741	—	14,993,015	—
9	March	1,726,191	—	1,763,729	—
10	April	1,135,589	—	1,143,845	—
11	May	11,339,949	7,234	11,339,950	9,805
12	June	26,459,968	119,120	26,459,968	119,120
	Total	98,785,012	135,261	98,948,245	144,899

TABLE XLIII

FORT GIBSON POWER PLANT: MONTHLY ENERGY PRODUCTION IN KILOWATT-HOURS FOR THE AVERAGE INFLOWS

k	Month	1st Trial Trajectory	2nd Trial Trajectory
		On-Peak	On-Peak
1	July	197,913	—
2	August	3,921	—
3	September	237	—
4	October	119,460	1,893,358
5	November	7,935	752
6	December	111	899
7	January	1,655	680
8	February	97,255	130,409
9	March	1,068	—
10	April	1,453	1,869
11	May	9,025	6,563
12	June	3,793,404	4,517,406
	Total	4,233,437	6,551,936

TABLE XLIV

WEBBERS FALLS POWER PLANT: MONTHLY ENERGY PRODUCTION IN KILOWATT-HOURS FOR THE AVERAGE INFLOWS

k	Month	1st Trial Trajectory		2nd Trial Trajectory	
		On-Peak	Off-Peak	On-Peak	Off-Peak
1	July	4,775,955	—	2,689,143	—
2	August	9,185,424	—	9,276,231	—
3	September	6,207,574	—	6,209,955	275
4	October	3,850,754	—	3,863,975	1,383,412
5	November	3,860,184	—	3,863,982	9,650
6	December	6,209,932	4,188,145	6,209,932	4,193,762
7	January	6,209,921	6,211,111	6,209,926	6,144,244
8	February	4,622,974	1,564	4,621,103	—
9	March	4,604,035	—	4,622,974	28,194
10	April	4,622,974	3,939	4,622,974	2,887
11	May	4,622,941	8,110,951	4,622,941	8,106,956
12	June	10,901,822	11,042,130	10,901,822	11,030,722
	Total	69,674,490	29,557,840	67,714,958	30,900,107

TABLE XLV

TENKILLER-FERRY POWER PLANT: MONTHLY ENERGY PRODUCTION IN KILOWATT-HOURS FOR THE AVERAGE INFLOWS

k	Month	1st Trial Trajectory		2nd Trial Trajectory	
		On-Peak	Off-Peak	On-Peak	Off-Peak
1	July	41,181	—	—	—
2	August	30,727	—	10,378,845	—
3	September	7,280,100	—	24,624	—
4	October	3,315,052	—	11,023	—
5	November	4,533,100	—	4,534,973	—
6	December	7,292,998	296	7,256,881	4,705
7	January	7,048,818	—	7,235,273	—
8	February	5,459,998	18,304	5,459,997	—
9	March	5,441,672	—	5,435,665	—
10	April	5,459,996	21,068	5,445,604	—
11	May	5,440,324	—	5,459,998	53,787
12	June	12,751,596	—	12,752,997	23,883
	Total	64,095,562	39,668	63,995,880	82,883

TABLE XLVI

EUFAULA POWER PLANT: MONTHLY ENERGY PRODUCTION IN KILOWATT-HOURS FOR THE AVERAGE INFLOWS

k	Month	1st Trial Trajectory		2nd Trial Trajectory	
		On-Peak	Off-Peak	On-Peak	Off-Peak
1	July	11,135,892	—	11,119,385	—
2	August	11,161,749	—	11,163,152	—
3	September	10,334,118	—	10,337,272	—
4	October	9,049,485	1,493,790	9,051,770	1,497,211
5	November	9,136,007	1,281,204	9,138,737	1,285,424
6	December	10,535,204	—	10,550,670	—
7	January	9,946,549	—	9,962,667	—
8	February	8,797,021	—	8,812,956	—
9	March	9,506,560	—	9,522,990	—
10	April	9,361,865	—	9,378,780	—
11	May	10,822,317	—	10,839,106	—
12	June	12,307,819	—	12,317,935	—
	Total	122,094,586	2,774,994	122,195,420	2,782,635

TABLE XLVII

ROBERT S. KERR POWER PLANT: MONTHLY ENERGY PRODUCTION IN KILOWATT-HOURS FOR THE AVERAGE INFLOWS

k	Month	1st Trial Trajectory		2nd Trial Trajectory	
		On-Peak	Off-Peak	On-Peak	Off-Peak
1	July	20,482,816	—	17,522,688	—
2	August	24,862,736	—	30,317,136	—
3	September	24,239,776	—	20,556,928	—
4	October	20,984,624	—	21,048,928	—
5	November	21,090,896	—	21,033,104	—
6	December	34,653,872	—	34,639,648	—
7	January	34,660,032	—	34,654,032	—
8	February	25,932,480	304	25,932,368	—
9	March	25,932,464	665,536	25,932,464	709,184
10	April	25,932,480	1,758,352	25,932,464	1,706,816
11	May	25,932,464	13,303,232	25,932,464	13,353,760
12	June	60,722,880	16,649,744	60,721,856	17,305,968
	Total	345,427,520	32,377,168	344,224,080	33,075,728

TABLE XLVIII

KEYSTONE POWER PLANT: MONTHLY ENERGY PRODUCTION IN KILOWATT-HOURS FOR THE HIGH INFLOWS

k	Month	1st Trial Trajectory		2nd Trial Trajectory	
		On-Peak	Off-Peak	On-Peak	Off-Peak
1	July	12,811,295	—	12,875,185	—
2	August	22,679,968	20,464	22,679,968	6,832
3	September	15,119,986	35,884	15,120,000	12,353
4	October	9,449,974	20,668	9,449,974	18,104
5	November	9,449,991	1,298	9,449,991	33,153
6	December	15,119,986	37,735	15,119,986	4,770
7	January	15,119,986	11,346,862	15,119,986	33,767
8	February	11,339,989	12,365	11,339,990	3,197
9	March	11,339,989	4,279,227	11,339,990	21,815,008
10	April	11,339,959	5,958,937	11,341,903	4,627,586
11	May	11,339,979	20,432,656	11,339,949	18,824,032
12	June	26,459,968	9,705,616	26,459,968	8,505,392
	Total	171,571,070	51,851,712	171,636,890	53,884,194

TABLE XLIX

FORT GIBSON POWER PLANT: MONTHLY ENERGY PRODUCTION IN KILOWATT-HOURS FOR THE HIGH INFLOWS

k	Month	1st Trial Trajectory		2nd Trial Trajectory	
		On-Peak	Off-Peak	On-Peak	Off-Peak
1	July	568	—	—	—
2	August	437,473	—	—	—
3	September	12,008	—	—	—
4	October	19,776	—	—	—
5	November	18,208	—	—	—
6	December	18,960	—	—	—
7	January	7,152	—	9,679,704	—
8	February	16,864	—	9,393	—
9	March	9,399,997	12,039	12,281	—
10	April	16,156	—	6,152	—
11	May	19,313	—	29,412	—
12	June	21,949,984	54,544	21,949,984	49,088
	Total	31,916,459	66,583	31,686,926	49,088

TABLE L

WEBBERS FALLS PLANT: MONTHLY ENERGY PRODUCTION IN KILOWATT-HOURS FOR THE HIGH INFLOWS

k	Month	1st Trial Trajectory		2nd Trial Trajectory	
		On-Peak	Off-Peak	On-Peak	Off-Peak
1	July	12,419,842	763,046	12,419,842	807,307
2	August	9,314,865	6,970,073	9,314,865	6,295,940
3	September	6,209,926	5,970,177	6,209,926	5,966,403
4	October	3,863,961	5,398,257	3,863,961	5,399,330
5	November	3,863,968	4,163,530	3,863,965	4,170,797
6	December	6,209,921	6,326,225	6,209,921	6,309,014
7	January	6,209,910	9,866,936	6,209,904	12,263,024
8	February	4,622,924	15,413,716	4,622,924	14,360,960
9	March	4,622,924	17,382,160	4,622,924	17,654,048
10	April	4,622,924	17,365,584	4,622,924	17,365,760
11	May	4,622,949	17,825,520	4,622,949	17,825,152
12	June	10,902,108	9,957,428	10,902,128	10,006,192
	Total	77,486,222	117,402,652	77,486,237	118,423,927

TABLE LI

TENKILLER-FERRY POWER PLANT: MONTHLY ENERGY PRODUCTION IN KILOWATT-HOURS FOR THE HIGH INFLOWS

k	Month	1st Trial Trajectory		2nd Trial Trajectory	
		On-Peak	Off-Peak	On-Peak	Off-Peak
1	July	14,216,005	—	14,339,055	—
2	August	10,920,003	884	10,894,591	—
3	September	16,257	—	9,678	—
4	October	3,750,834	—	3,552,789	370
5	November	4,561,236	—	4,563,003	—
6	December	7,293,000	286	7,292,725	—
7	January	7,293,000	29,845	7,292,998	3,507
8	February	5,460,000	53,961	5,460,000	24,731
9	March	5,460,000	23,555,984	5,460,000	6,449,516
10	April	5,460,000	692,350	5,460,000	22,558,368
11	May	5,460,000	23,555,984	5,460,000	23,546,784
12	June	12,753,000	15,326,987	12,753,000	15,261,979
	Total	82,643,335	63,216,281	82,537,839	67,845,255

TABLE LII

EUFAULA POWER PLANT: MONTHLY ENERGY PRODUCTION IN KILOWATT-HOURS FOR THE HIGH INFLOWS

k	Month	1st Trial Trajectory		2nd Trial Trajectory	
		On-Peak	Off-Peak	On-Peak	Off-Peak
1	July	11,147,179	—	11,147,179	—
2	August	9,079,060	—	9,100,872	—
3	September	8,253,464	—	8,132,059	—
4	October	8,627,114	811,407	8,589,213	734,717
5	November	9,647,914	2,055,239	9,615,677	2,014,571
6	December	12,866,519	—	12,801,747	—
7	January	13,414,415	—	13,354,331	—
8	February	11,927,609	—	12,036,039	—
9	March	11,207,313	155,640	11,211,607	160,918
10	April	10,326,501	—	9,351,332	—
11	May	11,022,143	—	11,088,409	2,772
12	June	11,656,271	—	11,547,468	—
	Total	129,175,502	3,022,286	127,975,933	2,912,978

TABLE LIII

ROBERT S. KERR POWER PLANT: MONTHLY ENERGY PRODUCTION IN KILOWATT-HOURS FOR THE HIGH INFLOWS

k	Month	1st Trial Trajectory		2nd Trial Trajectory	
		On-Peak	Off-Peak	On-Peak	Off-Peak
1	July	69,255,392	—	69,237,504	—
2	August	51,961,744	—	51,986,720	—
3	September	34,654,064	—	34,660,976	15,888
4	October	21,631,472	14,233,616	21,631,472	14,108,992
5	November	21,631,472	9,730,144	21,631,472	9,730,016
6	December	34,660,960	3,951,456	34,660,960	3,916,112
7	January	34,661,040	20,332,080	34,661,072	21,072,368
8	February	25,933,728	51,446,576	25,933,712	51,459,968
9	March	25,926,336	58,233,840	25,926,336	58,233,552
10	April	25,927,808	56,009,296	25,927,776	56,008,896
11	May	25,316,832	53,654,400	25,193,296	53,428,384
12	June	53,977,584	17,373,280	54,471,712	17,336,288
	Total	425,538,432	284,964,688	425,923,008	285,310,464

APPENDIX B

THE COMPUTER PROGRAM

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C THIS PROGRAM CALCULATES THE OPTIMAL OPERATIONAL POLICY FOR A SYSTEM OF
C  SIX MULTI-PURPOSE RESERVOIRS
       COMMON/DATA/X1(30),ER1(30),EC1(30),SURF1(30),X2(21),ER2(21),
      1EC2(21),SURF2(21),X3(43),ER3(43),EC3(43),SURF3(43),X4(26),
      2ER4(26),EC4(26),SURF4(26),U5(11),EC5(11),TEL5(11),ER5,SURF5,
      3U6(16),EC6(16),TEL6(16),ER6,SURF6,NX1,NX2,NX3,NX4,NX5,NX6
       COMMON/SPLNE/WER1(60),WEC1(60),WSURF1(60),WER2(42),WEC2(42),
      1WSURF2(42),WER3(86),WEC3(86),WSURF3(86),WER4(52),WEC4(52),
      2WSURF4(52),WEC5(22),WTEL5(22),WEC6(32),WTEL6(32)
       COMMON/MISC/ISTAGE,NSTAGE,NOGOOD,NITER,NITMAX,XK1(6),DX,DXMIN,
      1XNOM(4,13),XK(6),H1(6,12),H2(12),PIK(81),X5,X6,IN(6,12),EV(6,12),
      2RELMAX(6,12),PIK1(81),RMIN1(12),STOMIN(4),STOMAX(4),INOLA(12),
      3UNOW(6),U(6,12),PI,PIBEST,LPI,PICUM,PU,PV
       COMMON/LATIC/CONTRL(81,4),LCHG(81),LNOM(81,10),LBAK(11)
       COMMON/POWER/EUS(12,6),EVS(12,6)
       DIMENSION XX(1),YY(1)
       REAL IN, INOLA
C  INITIALIZE TIMER
       CALL ELAPSE(ITIME)
C  READ CONTROL PARAMETERS
       READ(5,910)NSTAGE,NITMAX,DXMIN,DX,TIMET,DELJ
  910  FORMAT(2I5,4F10.0)
C  READ OR GENERATE INITIAL STORAGE TRAJECTORY
       READ(5,930)(XNOM(I,1),I=1,4),X5,X6,TEMP
       IF(TEMP.EQ.0.0)GO TO 8
       DO 7 I=1,NSTAGE
    7  READ(5,930)(XNOM(J,I+1),J=1,4)
       GO TO 9
    8  K=1
       DO 200 I=2,13
       DO 200 J=1,4
  200  XNOM(J,I)=XNOM(J,K)
C  READ DATA DESCRIBING EACH RESERVOIR
C  KEYSTONE=RESERVOIR 1
C  FORT GIBSON=RESERVOIR 2
C  TENKILLER-FERRY=RESERVOIR 3
C  EUFAULA =RESERVOIR 4
C  WERBBERS FALLS= RESERVOIR 5
C  ROBERT S. KERR=RESERVOIR 6
    9  READ(5,920)NX1
  920  FORMAT(I2,30H                              )
       DO 10 I=1,NX1
       READ(5,930)SURF1(I),EC1(I),ER1(I),X1(I)
   10  CONTINUE
  930  FORMAT(8F10.0)
       READ(5,920)NX2
       DO 20 I=1,NX2
       READ(5,930)SURF2(I),EC2(I),ER2(I),X2(I)
   20  CONTINUE
       READ(5,920)NX3
       DO 30 I=1,NX3
       READ(5,930)SURF3(I),EC3(I),ER3(I),X3(I)
   30  CONTINUE
       READ(5,920)NX4
       DO 40 I=1,NX4
       READ(5,930)SURF4(I),EC4(I),ER4(I),X4(I)
   40  CONTINUE
       READ(5,950)NX5
  950  FORMAT(I2,30H                              )
```

```
      DO 50 I=1,NX5
      READ(5,930)TEL5(I),U5(I),EC5(I)
   50 CONTINUE
      READ(5,950)NX6
      DO 60 I=1,NX6
      READ(5,930)TEL6(I),U6(I),EC6(I)
   60 CONTINUE
      READ(5,930)SURF5,SURF6
C  CREATE THE NEIGHBOR IDENTIFICATION MATRIX
      DO 70 I=1,81
      DO 70 J=1,4
   70 CONTRL(I,J)=0.0
      DO 80 I=1,79,3
      CONTRL(I,1)=-1.0
   80 CONTRL(I+2,1)=1.0
      DO 90 I=1,73,9
      CONTRL(I,2)=-1.0
      CONTRL(I+1,2)=-1.0
      CONTRL(I+2,2)=-1.0
      CONTRL(I+6,2)=1.0
      CONTRL(I+7,2)=1.0
   90 CONTRL(I+8,2)=1.0
      DO 100 I=1,9
  100 CONTRL(I,3)=-1.0
      DO 110 I=28,36
  110 CONTRL(I,3)=-1.0
      DO 120 I=19,27
  120 CONTRL(I,3)=1.0
      DO 130 I=1,40
  130 CONTRL(41+I,3)=-CONTRL(41-I,3)
      DO 140 I=1,27
      CONTRL(I,4)=-1.0
  140 CONTRL(I+54,4)=1.0
      DO 150 I=1,81
  150 LCHG(I)=1.0
      DO 160 I=1,79,3
  160 LCHG(I)=LCHG(I)+1
      DO 170 I=1,72,9
  170 LCHG(I)=LCHG(I)+1
      DO 180 I=1,55,27
  180 LCHG(I)=LCHG(I)+1
C READ INFLOW DATA
      DO 210 I=1,12
  210 READ(5,930)(IN(J,I),J=1,6)
C  READ EVAPORATION DATA
      DO 220 I=1,12
  220 READ(5,930)(EV(J,I),J=1,6)
C  READ RELEASE CONSTRAINTS
      DO 230 I=1,12
  230 READ(5,930)RMIN1(I),(RELMAX(J,I),J=1,6)
C  READ STORAGE CONSTRAINTS
      DO 240 I=1,4
  240 READ(5,930)STOMIN(I),STOMAX(I)
C  READ INOLA INFLOWS
      READ(5,930)(INOLA(I),I=1,7)
      READ(5,930)(INOLA(I),I=8,12)
C  READ ON AND OFF PEAK HOURS AND PRICES
      DO 250 I=1,12
  250 READ(5,930)(H1(J,I),J=1,6)
```

```
      READ(5,930)H2
      READ(5,930)PU,PV
C   CONVERT DATA TO CORRECT UNITS
      PU=PU*0.001
      PV=PV*0.001
      TEMP=1.0/12.0
      DO 260 I=1,12
      DO 260 J=1,6
  260 EV(J,I)=TEMP*EV(J,I)
      DO 270 I=1,NX1
  270 SURF1(I)=SURF1(I)*0.001
      DO 280 I=1,NX2
  280 SURF2(I)=SURF2(I)*0.001
      DO 290 I=1,NX3
  290 SURF3(I)=SURF3(I)*0.001
      DO 300 I=1,NX4
  300 SURF4(I)=SURF4(I)*0.001
      SURF5=SURF5*0.001
      SURF6=SURF6*0.001
C   INITIALIZE ALL THE SPLINE FITS FOR THE DATA
      CALL SPLINE(X1,SURF1,WSURF1,XX,YY,X1(1),1.0,NX1,1,0,IER)
      CALL SPLINE(X2,SURF2,WSURF2,XX,YY,X2(1),1.0,NX2,1,0,IER)
      CALL SPLINE(X3,SURF3,WSURF3,XX,YY,X3(1),1.0,NX3,1,0,IER)
      CALL SPLINE(X4,SURF4,WSURF4,XX,YY,X4(1),1.0,NX4,1,0,IER)
      CALL SPLINE(X1,ER1,WER1,XX,YY,X1(1),1.0,NX1,1,0,IER)
      CALL SPLINE(X2,ER2,WER2,XX,YY,X2(1),1.0,NX2,1,0,IER)
      CALL SPLINE(X3,ER3,WER3,XX,YY,X3(1),1.0,NX3,1,0,IER)
      CALL SPLINE(X4,ER4,WER4,XX,YY,X4(1),1.0,NX4,1,0,IER)
      CALL SPLINE(X1,EC1,WEC1,XX,YY,X1(1),1.0,NX1,1,0,IER)
      CALL SPLINE(X2,EC2,WEC2,XX,YY,X2(1),1.0,NX2,1,0,IER)
      CALL SPLINE(X3,EC3,WEC3,XX,YY,X3(1),1.0,NX3,1,0,IER)
      CALL SPLINE(X4,EC4,WEC4,XX,YY,X4(1),1.0,NX4,1,0,IER)
      CALL SPLINE(U5,EC5,WEC5,XX,YY,U5(1),1.0,NX5,1,0,IER)
      CALL SPLINE(U6,EC6,WEC6,XX,YY,U6(1),1.0,NX6,1,0,IER)
      CALL SPLINE(U5,TEL5,WTEL5,XX,YY,U5(1),1.0,NX5,1,0,IER)
      CALL SPLINE(U6,TEL6,WTEL6,XX,YY,U6(1),1.0,NX6,1,0,IER)
C   DETERMINE THE INITIAL RELEASES AND RETURN
      XK(5)=X5
      XK(6)=X6
      XK1(5)=X5
      XK1(6)=X6
      PICUM=0.0
      DO 310 ISTAGE=1,NSTAGE
      J=ISTAGE+1
C   DEFINE THE K AND K-1 STORAGE VECTOR
C     XK=X(K) AND XK1=X(K-1)
  304 DO 301 I=1,4
      XK(I)=XNOM(I,J)
  301 XK1(I)=XNOM(I,ISTAGE)
      CALL UPONE
C    IS THE CONTROL OK
      IF(NOGOOD.EQ.0)GO TO 309
C   BAD CONTROL - ALTER STORAGE TRAJECTORY AND TRY AGAIN
      IF (UNOW(1).GE.RMIN1(ISTAGE)) GO TO 302
      XNOM(1,J)=XNOM(1,J)-(RMIN1(ISTAGE)-UNOW(1))
  302 DO 303 I=2,4
      IF (UNOW(I).GE.0.0) GO TO 303
      XNOM(I,J)=XNOM(I,J)+UNOW(I)
  303 CONTINUE
```

```
      GO TO 304
C   GOOD CONTROL - INCREMENT RETURN
  309 PICUM=PICUM+PI
      DO 310 I=1,6
  310 U(I,ISTAGE)= UNOW(I)
C   WRITE OUT INITIAL STORAGE, RELEASES, AND RETURN
      NITER=0
      WRITE(6,960) NITER, PICUM
  960 FORMAT ('1ITERATION NUMBER',I4,/,'0THE RETURN IS',F12.2,/)
      WRITE(6,971)
  971 FORMAT ('0THE STATE TRAJECTORY IS',/)
      WRITE(6,980)
  980 FORMAT(1X,'STAGE',5X,'KEYSTONE',6X,'FORT GIBSON',2X,'TENKILLER-FER
     $RY',4X,'EUFAULA',5X,'WEBBERS FALLS',2X,'ROBERT S. KERR',/)
      NSTG1=NSTAGE+1
      DO 320 I=1,NSTG1
  320 WRITE(6,990)I,(XNOM(J,I),J=1,4),X5,X6
  990 FORMAT (1X,I3,6F15.4)
      WRITE (6,1000)
 1000 FORMAT('0THE CONTROL TRAJECTORY IS ',/)
      WRITE(6,980)
      DO 330 I=1,NSTAGE
  330 WRITE(6,990)I,(U(J,I),J=1,6)
C   CHECK INITIAL DX VALUE
      IF(DX.EQ.0.0) DX=0.1*AMIN1(XK(1),XK(2),XK(3),XK(4))
      NN=NSTAGE-2
C   INITIALIZE TIMING VARIABLES
      CALL ELAPSE(ITIME)
      ITIMET=TIMET*60000.0
      ITIMEC=ITIME
      ITIMEF=ITIME
      ITIMEI=ITIME
C   BEGIN OPTIMIZATION LOOP
      DO 520 NITER=1,NIT MAX
C   UPDATE TIMING VARIABLES
      CALL ELAPSE(ITIME)
      ITIMEC=ITIMEC+ITIME
      TIME=ITIMEC
      TIME=0.001*TIME
      MINC=TIME
      MINC=MINC/60
      SECC=MINC
      SECC=TIME-60.0*SECC
      ITIMEI=ITIMEI+ITIME
      TIME=ITIMEI
      TIME=0.001*TIME
      MIN=TIME
      MIN=MIN/60
      SEC=MIN
      SEC=TIME-60.0*SEC
      ITIMEI=0
C   OUTPUT TIME FOR LAST ITERATION
      WRITE (6,1100) MIN,SEC,MINC,SECC
 1100 FORMAT ('0ITERATION TIME =',I2,' MIN',F6.2,' SEC',/,'0ELAPSE TIME ='
     $  =',I3,' MIN',F6.2,' SEC')
      IF (NITER.EQ.1) GO TO 335
      LEFT=ITIMET-ITIMEC
      ITER=(ITIMEC-ITIMEF)/(NITER-1)
      ITER=ITER+ITER/4
```

```
C   CHECK REMAINING TIME
        IF (LEFT.GE.ITER) GO TO 335
C   END DUE TO TIME
        WRITE (6,1110)
 1110 FORMAT ('0THE PROGRAM IS TERMINATING DUE TO TIME.')
        GO TO 525
C CALCULATE ALL RETURNS FROM STAGE 1 TO 2
  335 ISTAGE=1
        DO 340 I=1,4
  340 XK1(I)=XNOM(I,1)
        CALL UPONE
        DO 360 I=1,81
        J=LCHG(I)
C   GENERATE STAGE 2 NEIGHBORS
        DO 350 K=1,J
  350 XK(K)=XNOM(K,2)+CONTRL(I,K)*DX
        PIK(I)=-1.0
C   CHECK NEIGHBORS FOR STORAGE CONSTRAINTS
        DO 355 J=1,4
        IF (XK(J).GT.STOMAX(J)) GO TO 360
        IF (XK(J).LT.STOMIN(J)) GO TO 360
  355 CONTINUE
C   GOOD POINT - FIND RETURN
        PIK(I)=0.0
        CALL UPTWO
C   IF RELEASE IS OK STORE RETURN
        IF (NOGOOD.EQ.0) PIK(I)=PI
  360 CONTINUE
C   END OF STAGE 1 TO 2 CALCULATIONS
C   BEGIN STAGE 2 TO NSTAGE-1 CALCULATIONS
        DO 405 KK=1,NN
        ISTAGE=KK+1
        L=KK+2
C   STORE RETURNS UP TO STAGE ISTAGE
C   ZERO THE RETURN AT STAGE K ARRAY
        DO 370 I=1,81
        PIK1(I)=PIK(I)
  370 PIK(I)=0.0
C   GENERATE ALL NEIGHBORS AT STAGE K
        DO 375 I=1,4
        DO 374 J=1,3,2
        PIBEST=XNOM(I,L)+DX*(J-2)
C   CHECK FOR STAGE K STORAGE CONSTRAINTS
        IF (PIBEST.LT.STOMIN(I)) GO TO 371
        IF (PIBEST.LE.STOMAX(I)) GO TO 374
C   IF A STORAGE IS VIOLATED ELIMINATE THIS NEIGHBOR AT STAGE K
  371 PIBEST=J-2
        DO 372 K=1,81
        IF (CONTRL(K,I).EQ.PIBEST) PIK(K)=-1.0
  372 CONTINUE
  374 CONTINUE
  375 CONTINUE
        DO 402 JJ=1,81
        J=LCHG(JJ)
C   GENERATE STAGE K-1 NEIGHBORS
        DO 380 I=1,J
  380 XK1(I)=XNOM(I,ISTAGE)+DX*CONTRL(JJ,I)
C   SKIP CALCULATIONS IF THIS NEIGHBOR IS NO GOOD
        IF (PIK1(JJ).LE.0.0) GO TO 402
```

```
C  CALL UPONE FOR CALCULATIONS DEPENDENT ON STAGE K-1
      CALL UPONE
      DO 400 LL=1,81
      J=LCHG(LL)
C  GENERATE STAGE K NEIGHBORS OF K-1 POINT
      DO 390 I=1,J
  390 XK(I)=XNOM(I,L)+CONTRL(LL,I)*DX
C  SKIP BAK STAGE K POINTS
      IF (PIK(LL).LT.0.0) GO TO 400
C  FIND RETURN AND RELEASES
      CALL UPTWO
C  IF THE RELEASES ARE OK COMPARE RETURNS
      IF (NOGOOD.NE.0) GO TO 400
      PI=PI+PIK1(JJ)
C  SAVE STAGE K NEIGHBOR GIVING LARGEST RETURN
      IF (PIK(LL).GE.PI) GO TO 400
      PIK(LL)=PI
C  SAVE LARGEST RETURN LOCATION
      LNOM(LL,KK)=JJ
  400 CONTINUE
  402 CONTINUE
  405 CONTINUE
C  PERFORM FINAL STAGE CALCULATIONS
      DO 410 I=1,4
  410 XK(I)=XNOM(I,NSTG1)
      PIBEST=0.0
      ISTAGE=NSTAGE
C  GENERATE FINAL STAGE LESS ONE NEIGHBORS
      DO 430 KK=1,81
      J=LCHG(KK)
      DO 420 K=1,J
  420 XK1(K)=XNOM(K,ISTAGE)+CONTRL(KK,K)*DX
C SKIP BAD NEIGHBORS
      IF (PIK(KK).LE.0.0) GO TO 430
C  FIND RETURN AND RELEASES
      CALL UPONE
      PI=PI+PIK(KK)
C  SKIP IF RELEASES ARE BAD
      IF(NOGOOD.LT.0)GO TO 430
C  SAVE BEST RETURN AND RETURN LOCATION
      IF(PI.LE.PIBEST)GO TO 430
      PIBEST=PI
      LPI=KK
  430 CONTINUE
C  CHECK FOR CONVERGENCE FOR CURRENT DX
      LBAK(NSTAGE-1)=LPI
      LPI=IABS(LPI-41)
      DO 440 I=1,NN
      J=NSTAGE-1-I
      LBAK(J)=LNOM(LBAK(J+1),J)
  440 LPI=LPI+IABS(LBAK(J)-41)
      IF(LPI.NE.0)GO TO 450
C  CONVERGENCE FOR DX - CHECK DXMIN AND CHANGE IN RETURN
      IF(DX.LE.DXMIN)GO TO 530
      IF((PIBEST-PICUM)/PICUM.LE.DELJ)GO TO 530
      PICUM=PIBEST
      TEMP=0.5*DX
      WRITE(6,1010)DX, TEMP
 1010 FORMAT('0THE SOLUTION HAS CONVERGED FOR DX= ',1PE14.6,/,'0THE NEW
```

```
     $DX VALUE =',E14.6)
      DX=TEMP
      GO TO 520
C  RECOVER THE  NEW NOMINAL STORAGES
  450 K=NSTAGE-1
      DO 470  ISTAGE=1,K
      J=LBAK(ISTAGE)
      DO 460 I=1,4
      XK1(I)=XNOM(I,ISTAGE)
      XK(I)=XNOM(I,ISTAGE+1)+CONTRL(J,I)*DX
  460 XNOM(I,ISTAGE+1)=XK(I)
      CALL UPONE
C  STORE NEW NOMINAL RELEASES
      DO 470 I=1,6
  470 U(I,ISTAGE)=UNOW(I)
      DO 480 I=1,4
      XK1(I)=XK(I)
  480 XK(I)=XNOM(I,NSTG1)
      ISTAGE=NSTAGE
      CALL UPONE
      DO 490 I=1,6
  490 U(I,NSTAGE)=UNOW(I)
C  WRITE THE RESULTS OF THIS ITERATION
      WRITE(6,960)NITER, PIBEST
      WRITE (6,971)
      WRITE(6,980)
      DO 500 I=1,NSTG1
  500 WRITE(6,990)I,(XNOM(J,I),J=1,4),X5,X6
      WRITE(6,1000)
      WRITE(6,980)
      DO 510 I=1,NSTAGE
  510 WRITE(6,990)I,(U(J,I),J=1,6)
  520 CONTINUE
C  WRITE APPROPRIATE TERMINATION MESSAGE
  525 WRITE (6,1020)
 1020 FORMAT('OTHE SOLUTION DID NOT CONVERGE.')
      GO TO 535
  530 WRITE(6,1030)
 1030 FORMAT('OTHIS IS THE FINAL TRAJECTORY.')
C  PUNCH CURRENT CONTROL PARAMETERS AND STORAGE FOR RERUN
  535 WRITE(7,1050)NSTAGE,NITMAX,DXMIN,DX,TIMET,DELJ
      WRITE(7,1040)(XNOM(J,1),J=1,4),X5,X6,X5
      DO 540 I=1,NSTAGE
  540 WRITE(7,1040)(XNOM(J,I+1),J=1,4),X5,X6
 1040 FORMAT(8F10.3)
 1050 FORMAT(2I5,2F10.3,1P2E10.4)
C  WRITE OUT POWER PRODUCED ON LAST ITERATION
      WRITE(6,1080)
 1080 FORMAT('1THE POWER PRODUCED IS',/)
      WRITE(6,980)
      DO 560 I=1,NSTAGE
      WRITE(6,990)I,(EUS(I,J),J=1,6)
  560 WRITE(6,1090)(EVS(I,J),J=1,6)
 1090 FORMAT(4X,6F15.4,/)
      STOP
      END
      SUBROUTINE UPONE
      COMMON/DATA/X1(30),ER1(30),EC1(30),SURF1(30),X2(21),ER2(21),
     1EC2(21),SURF2(21),X3(43),ER3(43),EC3(43),SURF3(43),X4(25),
```

```
      2ER4(26),EC4(26),SURF4(26),U5(11),EC5(11),TEL5(11),ER5,SURF5,
      3U6(16),EC6(16),TEL6(16),ER6,SURF6,NX1,NX2,NX3,NX4,NX5,NX6
       COMMON/SPLNE/WER1(60),WEC1(60),WSURF1(60),WER2(42),WEC2(42),
      1WSURF2(42),WER3(86),WEC3(86),WSURF3(86),WER4(52),WEC4(52),
      2WSURF4(52),WEC5(22),WTEL5(22),WEC6(32),WTEL6(32)
       COMMON/MISC/ISTAGE,NSTAGE,NOGOOD,NITER,NITMAX,XK1(6),DX,DXMIN,
      1XNOM(4,13),XK(6),H1(6,12),H2(12),PIK(81),X5,X6,IN(6,12),EV(6,12),
      2RELMAX(6,12),PIK1(81),RMIN1(12),STOMIN(4),STOMAX(4),INOLA(12),
      3UNOW(6),U(6,12),PI,PIBEST,LPI,PICUM,PU,PV
       COMMON/POWER/EUS(12,6),EVS(12,6)
       DIMENSION ECC(6),EVAP(6),ERR(6)
       DIMENSION XX(1)
       REAL IN,INOLA
C   DETERMINE ER, EC, AND SURF FOR STAGE K-1
       CALL SPLINE(X1,ER1,WER1,XX,ERR(1),XK1(1),1.0,NX1,1,1,IER)
       CALL SPLINE(X2,ER2,WER2,XX,ERR(2),XK1(2),1.0,NX2,1,1,IER)
       CALL SPLINE(X3,ER3,WER3,XX,ERR(3),XK1(3),1.0,NX3,1,1,IER)
       CALL SPLINE(X4,ER4,WER4,XX,ERR(4),XK1(4),1.0,NX4,1,1,IER)
       CALL SPLINE(X1,EC1,WEC1,XX,ECC(1),XK1(1),1.0,NX1,1,1,IER)
       CALL SPLINE(X2,EC2,WEC2,XX,ECC(2),XK1(2),1.0,NX2,1,1,IER)
       CALL SPLINE(X3,EC3,WEC3,XX,ECC(3),XK1(3),1.0,NX3,1,1,IER)
       CALL SPLINE(X4,EC4,WEC4,XX,ECC(4),XK1(4),1.0,NX4,1,1,IER)
       CALL SPLINE(X1,SURF1,WSURF1,XX,EVAP(1),XK1(1),1.0,NX1,1,1,IER)
       CALL SPLINE(X2,SURF2,WSURF2,XX,EVAP(2),XK1(2),1.0,NX2,1,1,IER)
       CALL SPLINE(X3,SURF3,WSURF3,XX,EVAP(3),XK1(3),1.0,NX3,1,1,IER)
       CALL SPLINE(X4,SURF4,WSURF4,XX,EVAP(4),XK1(4),1.0,NX4,1,1,IER)
C   CHANGE EVAP TO KAF
       DO 40 I=1,4
   40  EVAP(I)=EVAP(I)*EV(I,ISTAGE)
       EVAP(5)=SURF5*EV(5,ISTAGE)
       EVAP(6)=SURF6*EV(6,ISTAGE)
       ENTRY UPTWO
C   CALCULATE REQUIRED RELEASE
       DO 50 I=1,4
   50  UNOW(I)=XK1(I)-XK(I)+IN(I,ISTAGE)-EVAP(I)
       UNOW(5)=IN(5,ISTAGE)-EVAP(5)+UNOW(1)+UNOW(2)+INOLA(ISTAGE)
       UNOW(6)=IN(6,ISTAGE)-EVAP(6)+UNOW(3)+UNOW(4)+UNOW(5)
C    CHECK RELEASE CONSTRAINTS
       IF (UNOW(4).LT.-0.001) GO TO 20
       IF (UNOW(3).LT.-0.001) GO TO 20
       IF (UNOW(2).LT.-0.001) GO TO 20
       IF (UNOW(1).LT.0.999*RMIN1(ISTAGE)) GO TO 20
C   DETERMINE ER AND EC FOR RESERVOIRS 5 AND 6
       CALL SPLINE(U5,TEL5,WTEL5,XX,ERR(5),UNOW(5),1.0,NX5,1,1,IER)
       CALL SPLINE(U6,TEL6,WTEL6,XX,ERR(6),UNOW(6),1.0,NX6,1,1,IER)
       CALL SPLINE(U5,EC5,WEC5,XX,ECC(5),UNOW(5),1.0,NX5,1,1,IER)
       CALL SPLINE(U6,EC6,WEC6,XX,ECC(6),UNOW(6),1.0,NX6,1,1,IER)
C   DETERMINE THE RETURN
       PI=0.0
       HEL=489.5
       DO 80 I=1,6
C   DETERMINE RELEASE USED FOR POWER GENERATION
       UN=UNOW(I)
       IF(UN.GT.RELMAX(I,ISTAGE))UN=RELMAX(I,ISTAGE)
       IF(I.LE.4)GO TO 72
C   CALCULATE EN FOR RESERVOIRS 5 AND 6
       EN=883.5248*UN*(HEL-ERR(I))
       HEL=459.5
       GO TO 74
```

```
C  CALCULATE EN FOR RESERVOIRS 1 TO 4
   72 EN=ERR(I)*UN
C  CHECK RELATIONSHIPS FOR EN, EUH, EVH, EMAX, AND EPEAK
   74 EMAX=ECC(I)*H2(ISTAGE)
      EPEAK=ECC(I)*H1(I,ISTAGE)
      IF (EN.LE.EPEAK) GO TO 70
      IF (EN.GT.EMAX) EN=EMAX
      EUH=EPEAK
      EVH=EN-EPEAK
      IF(EVH.LT.0.0)EVH=0.0
      GO TO 75
   70 EUH=EN
      EVH=0.0
C  SAVE EUH AND EVH
   75 EVS(ISTAGE,I)=EVH
      EUS(ISTAGE,I)=EUH
C  ADD RETURN FOR RESERVOIR I-TH
   80 PI=PI+PU*EUH+PV*EVH
      NOGOOD=0
      RETURN
C  RELEASE CONSTRAINTS VIOLATED - RETURN NOGOOD NOT ZERO
   20 NOGOOD=-1
      RETURN
      END
C  CUBIC SPLINE INTERPOLATION RETURN
C REFERENCE : A FIRST COURSE IN COMPUTING AND NUMERICAL METHODS, JOHN A.
C      JACQUEZ, READING, MASSACHUSETTS, ADDISON-WESLEY PUBLISHING CO.,
       1970, PP. 263-266.
C  CALLING ARGUMENTS
C      X - ARRAY OF INDEPENDENT VARIABLE DATA LENGTH N
C      Y - ARRAY OF DEPENDENT VARIABLE DATA LENGTH N
C      W - WORKING ARRAY LENGTH 2N UNIQUELY ASSOCIATED WITH A GIVEN (X,Y)
C          SET OF DATA
C      XVAL - ARRAY OF NVAL RETURNED X VALUES AT WHICH THE INTERPOLATING
C             FUNCTION HAS BEEN EVALUATED
C      YVAL - ARRAY OF NVAL RETURNED Y VALUES DETERMINED BY EVALUATING
C             THE INTERPOLATING FUNCTION:  YVAL(I)=F(XVAL(I))
C      XVAL1 - THE VALUE TO BE USED FOR XVAL(1)
C      DX - INCREMENT IN XVAL: XVAL(I)=XVAL1+(I-1)*DX
C      N - NUMBER OF DATA POINTS IN X
C      NVAL - NUMBER OF INTERPOLATED VALUES TO BE DETERMINED
C      NVFLAG - CONTROL FLAG
C               0 - THIS IS THE FIRST CALL FOR A UNIQUE (X,Y,W) TRIPLE
C               1 - THIS IS A SUBSEQUENT CALL WITH A UNIQUE  (X,Y,W)
C                   TRIPLE
C               NOTE: IF X, Y, OR W IS MODIFIED BETWEEN CALLS NVFLAG=0
C                     MUST BE USED
C               IER - RETURN CODE
C                     -1   XVAL IS NOT IN THE RANGE OF X
C                      0   NO ERRORS
C                      1   ONLY IER INTERPOLATED VALUES WERE CALCULATED
C                          BECAUSE XVAL(IER+1) WOULD HAVE BEEN OUT OF THE
C                          THE RANGE OF X
C  NOTE:  WHENEVER NVFLAG=0 THE X AND Y DATA IS REORDERED IN INCREASING
C         X ORDER.
      SUBROUTINE SPLINE(X,Y,W,XVAL,YVAL,XVAL1,DX,N,NVAL,NVFLAG,IER)
C
      DIMENSION X(1),Y(1),W(1),XVAL(1),YVAL(1)
C
```

```
      IF (NVFLAG.NE.0) GO TO 60
C
      J=N-1
      DO 20 K=1,J
      I = K
      T1 =X(I)
      L=K+1
      DO 10 M=L,N
      IF(X(M).GE.T1) GO TO 10
      I=M
      T1 =X(I)
   10 CONTINUE
      IF(I.EQ.K) GO TO 20
      X(I) = X(K)
      X(K) = T1
      T1 = Y(I)
      Y(I) = Y(K)
      Y(K) = T1
   20 CONTINUE
      T1 = X(2) - X(1)
      J=J-1
      DO 30 I=1,J
      K=I+I
      L=I+2
      T = T1
      T1 = X(L) -X(K)
      T2 = 1.0/T1
      W(K) = T*T2
   30 W(N+K) = 6.0*T2*(T2*(Y(L)-Y(K))-(Y(K)-Y(I))/T)
      W(1) =0.0
      W(N+1)=0.0
      DO 40 I=1,J
      K=I+1
      T1=W(K)
      T=-1.0/(2.0+T1+T1+T1*W(I))
      W(K)=T
      K=K+N
   40 W(K)=T*(T1*W(K-1)-W(K))
      T=X(N)-X(N-1)
      W(N)=0.0
      J=J+1
      DO 50 I=1,J
      K=N-I
      W(K)=W(K)*W(K+1)+W(K+N)
      T1=X(K+1)-X(K)
      IF (T1.GT.T) T=T1
   50 W(K+N)=T1
      W(N+N)=T
   60 IER=-1
      IF (XVAL1.LT.X(1)) GO TO 130
      IF (XVAL1.GT.X(N)) GO TO 130
      T=(XVAL1-X(1))/W(N+N)
      I=T
      IF (I.LT.1) I=1
      K=N
   70 J=(I+K)/2
      IF (XVAL1-X(J)) 72,78,74
   72 K=J
      GO TO 76
```

```
 74 I=J
 76 IF (K-I.GT.1) GO TO 70
    L=I
    GO TO 80
 78 L=J
 80 J=1
    IER = 0
    X3=XVAL1
100 T=W(N+L)
    T4=Y(L)/T-T*W(L)/6.0
    I=L+1
    T3 = Y(I)/T-T*W(I)/6.0
    T1=1.0/(6.0*T)
    T2=W(I)*T1
    T1=W(L)*T1
    T = X(L)
    T5 = X(I)
110 X1 = T5 - X3
    X2 = X3-T
    XVAL(J) = X3
    YVAL(J) = T1*X1*X1*X1 +T2*X2*X2*X2+T3*X2+T4*X1
    IF(J.GE.NVAL) GO TO 130
    X3 = X3+ DX
    J=J+1
    IF(X3.LE.T5) GO TO 110
120 L=L+1
    IF (L.GE.N) GO TO 140
    IF(X3-X(L+1)) 100,120,120
140 IER = J-1
130 RETURN
    END
```

VITA

Arturo Le-Blanc Palma

Candidate for the Degree of

Doctor of Philosophy

Thesis: OPTIMAL OPERATIONAL POLICIES FOR A SYSTEM OF SIX MULTI-PURPOSE RESERVOIRS BY DISCRETE DIFFERENTIAL DYNAMIC PROGRAMMING

Major Field: Business Administration

Biographical:

Personal Data: Born in Santiago, Chile, December 12, 1941, the son of Waldo M. and Teresa L-B. Palma.

Education: Graduated from Manuel de Salas High School, Santiago, Chile, December, 1970; recieved the Ingeniero Agronomo degree from the Universidad Catolica de Chile, Santiago, Chile in December, 1968; received the Master of Science degree from Oklahoma State University in May, 1971; completed the requirements for the Doctor of Philosophy degree at Oklahoma State University in July, 1974.

Professional Experience: Research Assistant, Economics Research Center and Agricultural Economics Department, Universidad Catolica de Chile from October, 1966 to December, 1968; Economist in the Ministry of Agriculture, Santiago, Chile from December, 1968 to July, 1969; Teaching Assistant in the Administrative Sciences Department at Oklahoma State University from January, 1971 until May, 1972; Research Assistant in the Systems Sciences Center at Oklahoma State University from August, 1972 to May, 1974.

Organizations: Member of the Institute of Management Sciences and the Institute of Electrical and Electronics Engineers.