PRECISION WITH WHICH SELECTED PHYSICAL PROPERTIES OF SIMILAR SOILS CAN BE ESTIMATED

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CHAPTER I

INTRODUCTION

Interest in the movement of water and its constituents through the soil has increased significantly in recent years. This increase has occurred principally because of a soil's natural capacity to receive and purify industrial and municipal wastes as well as potential contaminants from agricultural chemicals. Currently, application rates of wastes and chemicals and the subsequent movement of soil water and its constituents are based upon experience in a given region, and are generally not applicable to other areas or different watersheds without additional research. Consequently, the spatial variability of soils must be considered.

The availability of finite-difference solutions to soil-water and solute transport equations have made it possible to describe the movement and distribution of chemical materials for large watersheds. This was impossible in the past owing to the limited number of analytical solutions to naturally occurring field problems and the general unavailability of electronic computers to process the myriad of calculations required in finite-difference solutions. The validity of these finite-difference solutions is limited, however, by the precision of the soil physical parameters used in the solutions. Therefore, characterization of the input parameters and their spatial variability for a watershed must be known and quantitized before the precision of the finite-difference solutions can be estimated.

Measurements of various soil-water properties have been made in situ. These measurements have also been made on disturbed and undisturbed soil cores in the laboratory. Measurements involving disturbed materials only approximate the natural field soil because of changes in soil structure during sampling. In order to establish the limitations of the soil-water parameters obtained, the inherent spatial variation associated with in situ measurements as well as those on undisturbed cores must be quantitized.

Soil bulk density and texture are other physical measurements exhibiting spatial variability. Subsequently, the precision of these properties must also be characterized in order to establish the precision one may expect when using them in a numerical solution.

The objectives of this study were:

1. Evaluation of the spatial variation associated with the soil-water conductivity versus soil-water content relations used by hydrologists and others to calculate soil-water fluxes in watersheds and small agronomic fields.
2. Compare in situ measurements with other procedures which use simplifying assumptions to calculate soil-water conductivity, and determine the validity of these procedures for watershed studies.
3. Evaluate the precision of soil-water content measurements at 0.1 and 15 bars matric suction, bulk density, fraction of sand, silt, and clay and determine the optimum number of samples required to obtain a given precision.

CHAPTER II

LITERATURE REVIEW

The use of chemicals for industrial, municipal, and agricultural use has resulted in a growing concern about their influences on the environment. Of special concern is the movement of chemicals through soil resulting in the pollution of groundwater from agricultural pesticides and land spreading of wastes. The recent availability of electronic computers makes finite-difference solutions of mass-transit equations for the boundary conditions generally applicable to areas of land large enough to be of practical significance. For the above solutions to be applied validly, their precision needs to be known.

Literature, where the parameters relating to water retention and flow through the soil are characterized, are for small experimental plots and can not be generally applied to larger land areas. In addition, manuscripts quantitizing the variability of soil physical properties are not numerous owing to the large number of samples and work required for sample processing. Therefore, a literature review involving the variability of soil physical properties is more qualitative than quantitative. Wherever possible, quantitative results will be given. The quantitative measures of precision are the variance or standard error and, thus, are the quantities with which this review is most concerned.

The soil is inherently variable as a result of its development with time and parent materials, relief, weather conditions, and biological habitation. Obviously, the manner in which the above factors interact to produce the soil variations is complex and is not the subject of this review. Therefore, this study is limited to only the spatial variation of specific soil physical properties across and within similar soil areas, as delineated by soil classification.

The variance[1] of the soil-water content, θ, was shown to decrease with increasing soil depth by Andrew and Sterns (1963), Bethlahmy (1963), Carlson (1959), Sartz (1972) and van Bavel et al. (1968). Salter and Williams (1965), Sartz (1972), Towner (1968), and Webster (1966) report a large Var (θ). Standard errors, SE, as large as 50% of the mean have been reported for sampling sites of 0.81 to 14.18 ha for the same soil series by Hammond et al. (1958). Taylor, Evans, and Kemper (1961) found a SE of approximately 7% of the mean θ on small experimental plots. Wilcox (1959) has shown the Var (θ) to be different for different soil types. Towner (1968) found the Var (θ) to be large for swelling soils and Reinhart (1961) showed that the Var (θ) increases with stoniness.

Since the hydraulic conductivity is often inferred from the unsaturated soil-water content, which can be expressed as a function of matric suction, the Var (θ) at a given matric suction is important. Holtan et al. (1968) and Prince and Raney (1961) give sufficient data to document the fact that 1/3, 1, and 15-bar θ are highly variable within soil series. In comparing Var (1/3-bar θ) and Var (15-bar θ)

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[1] Hereafter, Var ( ) will denote the variance of the quantity in parenthesis. Symbol θ refers to volumetric soil water content unless otherwise noted.

in the A2 horizon for soils in the Blue Ridge Mountains of Georgia, Ike and Cutter (1968) estimated the reduction in variance following a grouping by soil series was 41 and 42%, respectively. Further grouping into soil type did not reduce the variance estimates. They grouped the B horizon by soil series and showed a reduction in Var (1/3-bar θ) and Var (15-bar θ) of 13 and 24% respectively. Further grouping of the B horizon by soil type gave no additional reduction in the variance estimates. Aljiburg and Evans (1961) studied two soil series in Oregon using 7.29 ha sites and showed the Var (0.1 - bar θ, weight %) to be 23.62 among the 68.60 within sites at 7.5 to 15 cm depths and 32.39 among and 23.80 within sites at 30 to 37.5 cm depths. Their estimates of the Var (15-bar θ, weight %) was 4.75 among and 1.80 within sites at 7.5 to 15.0 cm depths and 13.35 among and 5.52 within sites at 30 to 37.5 cm depths. Broadfoot and Burke (1958) grouped soils by texture and found the Var (θ) increased for finer textured soils.

The variability of soil bulk density, BD, is well documented within a soil series by Halton et al. (1968) in the New England Area, Prince and Raney (1961) in the Northeastern United States, and Rouke and Beck (1967) in Maine. No difference in the Var (BD) was found by Stutzbeck et al. (1972) among or within 0.08 ha sites in the Southeastern Adirondack Mountain Region in New York, by Aljibury and Evans (1961) among or within 7.29 ha sites in Oregon, or by Broadfoot and Burke (1958) within similar soils grouped by texture. Reinhart (1961) indicated that Var (BD) increased with stoniness.

Because soil texture is one of the soil physical properties which influences the movement of soil-water and its constituents, it is one of the most frequently measured soil physical properties. The variability of soil texture is well documented within a soil series by Holton et al. (1968) for the New England area, by Prince and Raney (1961) for the Northeastern United States, and by Rouke and Beck (1968) for Maine. Ike and Cutter (1968) for soils in the Blue Ridge Mountains of Georgia estimated Var (% Sand), Var (% Silt), and Var (% Clay) in the A2 horizon was reduced by grouping the soil series by 35, 39, and 21% respectively. Further grouping to soil type resulted in additional reductions. This is to be expected since the A2 texture is the criterion for dividing soil series into types. Similarly in an analysis of the B horizon, grouping into soil series reduced the Var (% Sand), Var (% Silt), and Var (% Clay) by 71, 42, and 37% respectively. Further grouping of the B horizon to soil type gave no appreciable reduction in variance over grouping as to soil series.

Rourke and Beck (1968) show that saturated hydraulic conductivities exhibit a wide range within a soil series. The Var (saturated hydraulic conductivity) was found to increase as the area sampled increased in a study of Derr et al. (1969). Mason et al. (1957) showed that the between sites variance was two to three times that observed within sites and that the SE was proportional to the mean.

Land use and cultural practices contribute to the variability of soil physical properties, also. Bethlahmy (1963) has found the Var (θ) to be different under different vegetative covers. Cultural practices which add organic material to the soil has been shown by Salter and Haworth (1961) and Salter and Williams (1963) to increase the variability of the soil-water potential relations for the Ap horizon. Broadfoot and Burke (1958) report the same SE for BD of soils grouped by land use. Another study by Soane (1970) showed that the

Var (BD) can be increased as a result of differential compaction.

No quantative estimates were reported for the variance of the unsaturated hydraulic conductivity versus soil-water content relations, K(θ). However, examples of the variability between sites within small experimental plots is given by Van Bavel, Strict, and Brust (1968). Davidson et al. (1969), Hillel, Krentos, and Stylianou (1972), Rose, Stern, and Drummond (1965), Rose and Stern (1967), and van Bavel, Strict, and Brust (1968) all give examples of the Var K(θ) with depth or the examples can be extracted from their work. The work of Davidson et al. (1969) and van Bavel, Brust, and Strick (1968) show that the SE of K(θ) decreases with increasing soil depth.

Laboratory methods for approximating K(θ) from soil-water characteristics were reviewed by Green and Corey (1971). They proposed a new method (a detailed description is presented later in this paper) which incorporated a matching factor to match experimental with calculated values at one point. Using published data they concluded that incorporating a matching factor was better than previously proposed methods. Black, Gardner, and Thurtell (1969) described a procedure (a detailed description is presented later in this paper) for approximating K(θ) in situ using some simplifying assumptions. Davidson et al. (1969) showed that the procedure of Black et al. (1969) was applicable to some nonhomogenous soils, but not for all soils.

Various conclusions have been made concerning the variance of soil physical properties. Becket and Webster (1971) concluded that the variance of a soil physical property can be obtained from an area as small as one m^2. Ike and Cutter (1968) concluded that sampling "model" pedons will produce accurate estimates of population means, but would not reflect the variance of the soil physical properties included in a soil series or type. Andrew and Sterns (1963) state that extensive sampling is necessary to characterize soil physical properties.

No estimates were found in the literature for Var (BD), Var (15-bar θ), Var (% Sand), Var (% Clay), Var (% Silt), or Var (0.1-bar θ) for areas approaching one m^2 nor were any quantitative estimates of the Var $K(\theta)$ found. Additionally no published results were found of comparisons of the procedure of Green and Corey (1971) with _in situ_ procedures and only one (Davidson et al., 1969) was found which compared the Black et al. (1969) procedure to _in situ_ measurements in nonhomogenous soils. These results are needed to indicate the sampling error of these quantities to test the accuracy of the procedures of Green and Corey (1971) and Black et al. (1969), and to add validity to estimates of $K(\theta)$ for given areas of soil. Results from this study will be used to give quantitative estimates of the factors outlined above.

CHAPTER III

MATERIALS AND METHODS

Selection of Experimental Locations

Eight sites on areas of Teller soil series were selected for this study. Detailed soil descriptions for each site are given in Appendix A. Three sites were located within a 300 m radius and the other five additional sites were selected within a 7.2 km radius of the three previous sites.

Experimental Procedures

The procedure used by Davidson et al. (1969) to evaluate soil hydraulic properties of various soils was used in this study. Plots were 3 m square, level, and were enclosed with a 15 cm high berm. Water was ponded on the soil surface until hydraulic equilibrium through the soil profile was established. Equilibrium was defined as the condition when a constant flux across the soil surface and a constant hydraulic head throughout the profile were attained for 4 hr. After this time, no additional water was applied and the soil surface was covered with a polyethylene plastic sheet to prevent evaporation from the surface during the drainage period. Straw and loose soil were placed on the plastic to minimize temperature changes during the drainage period. Hydraulic heads were recorded periodically using mercury-manometer tensiometers located in the center of each plot.

Tensiometers were located at 15 cm increments to a depth of 90 cm and at 30 cm increments from a depth of 90 to 150 cm. Three tensiometers were placed at each depth. Soil core samples were taken at each depth and were collected from a pit dug adjacent to each plot or in the experimental plot. Core samples were in the shape of a right cylinder measuring 7.6 cm by 7.6 cm. Three such samples were taken at each depth. In addition, composite samples were obtained from the soil surrounding the cores. The soil-water characteristics for desorption and bulk density were determined for each soil depth increment at every location using the undisturbed soil cores. Soil texture (by the hydrometer method) and 15-bar soil-water content values were determined for each soil depth increment at every location from the composite of disturbed soil taken from around each core (Black, 1964, p. 133-137, 375-377, 562-565).

Calculation ~~i~~of $K(\theta)$

Darcy's equation describes the drainage flux V_D(cm/hr) at depth D (cm) and is given by:

$$V_D = -K_D(\theta) \left.\frac{\partial H}{\partial Z}\right|_D \quad \text{(cm/hr)} \tag{1}$$

where $K_D(\theta)$ (cm/hr) is the hydraulic conductivity, θ (cm^3/cm^3) is the volumetric water content, H (cm) is the hydraulic head and Z (cm) is defined as the positive downward vertical distance from the soil surface. Assuming H consists of only the gravitational potential, Z (cm), the matric suction, h (cm), gives:

$$H = -h - Z \quad \text{(cm)} \tag{2}$$

Substituting equation (2) in (1) gives:

$$V_D = K_D(\theta)\left(\frac{\partial h}{\partial Z} + 1\right)\Bigg|_D \qquad \text{(cm/hr)}. \tag{3}$$

For a vegetative-free soil body with zero water flux across the surface, no net lateral soil-water flow, and the lower boundary at depth D, the equation of continuity for a semi-infinite profile can be written as:

$$-\frac{\partial V}{\partial Z} = \frac{\partial \theta}{\partial t} \tag{4}$$

Integrating equation (4) from $Z=0$ to $Z=D$ gives

$$\int_0^D -\frac{\partial V}{\partial Z}\, dZ = \int_0^D \frac{\partial \theta}{\partial t}\, dZ \tag{5}$$

Evaluating the left hand side of equation (5) results in

$$-V_D + V_0 = \int_0^D \frac{\partial \theta}{\partial t}\, dZ \tag{6}$$

where $V_o=0$ since the water flux across the soil surface is zero.

Integrating equation (6) with respect to time gives

$$\int_{t_\tau}^{t_{\tau+1}} - V_D \, dt = \int_{t_\tau}^{t_{\tau+1}} \left(\int_0^D \frac{\partial\theta}{\partial t} \, dZ \right) dt \quad \text{(cm)} \tag{7}$$

where t_τ, $t_{\tau+1}$ (hr) are consecutive observation times.

If $\int_0^D \frac{\partial\theta}{\partial t} \, dZ$ is continuous throughout the interval $0 < Z < D$, it is continuous throughout any part. Define the i^{th} depth increment as $Z_{i-1} < Z < Z_i$ where $Z_0 = 0$ and $Z_i (i \neq 0)$ are consecutive depths at which data were collected. It follows that

$$\int_0^D \frac{\partial\theta}{\partial t} \, dZ = \sum_{i=1}^{n} \int_{Z_{i-1}}^{Z_i} \frac{\partial\theta}{\partial t} \, dZ \tag{8}$$

where $Z_n = D$.

Substituting equation (8) in (7) gives

$$\int_{t_\tau}^{t_{\tau+1}} - V_D \, dt = \int_{t_\tau}^{t_{\tau+1}} \left(\sum_{i=1}^{n} \int_{Z_{i-1}}^{Z_i} \frac{\partial\theta}{\partial t} \, dZ \right) dt \quad \text{(cm)} \tag{9}$$

Substituting equation (3) in (9) and solving for $K_D(\theta)$ gives

$$K_D(\theta) = \left[- \sum_{i=1}^{n} \int_{t_\tau}^{t_{\tau+1}} \int_{Z_{i-1}}^{Z_i} \frac{\partial\theta}{\partial t} \, dZ \, dt \right] \left[\left(\frac{\partial h}{\partial Z} + 1 \right) \Bigg|_D (t_{\tau+1} - t_\tau) \right]^{-1} \tag{10}$$

Equation (10) was evaluated by finite difference approximation using the following simplifying assumptions.

a. The θ at Z_0 at time t_τ, $\left(\theta_{0,\tau}\right)$ is equal to θ at Z_1 at time t_τ, $\left(\theta_{1,\tau}\right)$.

b. The hydraulic gradient between Z_0 and Z_1 is equal to the hydraulic gradient between Z_1 and Z_2.

c.
$$\int_{t_\tau}^{t_{\tau+1}} \left(\int_{Z_j}^{D_j} \frac{\partial\theta}{\partial t} \, dZ \right) dt = \tfrac{1}{4}(Z_{j+1} - Z_j) \left[\theta_{j,\tau} + \theta_{(j+a),\tau} - \theta_{j,(\tau+1)} - \theta_{(j+1),\ (\tau+1)}\right] \qquad (11)$$

where

j refers to the maximum depth in question and i will be used to refer to intermediate depth increments

$D_j = \tfrac{1}{2}\,(Z_j + Z_{j+1})$,

$\theta_{j,\tau}$ = θ evaluated at depth Z_j and time t_τ.

d.
$$\int_{t_\tau}^{t_{\tau+1}} \left(\int_{Z_{i-1}}^{Z_i} \frac{\partial\theta}{\partial t} \, dZ \right) dt = \tfrac{1}{2}\,(Z_i - Z_{i-1}) \left[\theta_{i,\tau} + \theta_{(i-1),\tau} - \theta_{i,(\tau+1)} - \theta_{(i-1),\ (\tau+1)}\right] \qquad (12)$$

e. $$\left(\frac{\partial h}{\partial z}+1\right)_{D_j} = \tfrac{1}{2}\,(z_{j+1}-z_j)^{-1}\Big[h_{(j+1),\tau}+h_{(j+1),\,(\tau+1)} - h_{j,\tau} - h_{j,(\tau+1)} + z_{j+1} - z_j\Big] \tag{13}$$

where $h_{j,\tau}$ = h evaluated at z_j and time t_τ.

f. $$\theta = \tfrac{1}{4}\Big[\theta_{j,\tau} + \theta_{j,(\tau+1)} + \theta_{(j+1),\tau} + \theta_{(j+1),\,(\tau+1)}\Big] \tag{14}$$

where θ is evaluated at depth D_j.

With these assumptions equation (7) may be written

$$K_{D_j}(\theta) = \Big\{\tfrac{1}{2}f(j)\sum_{i=1}^{j}\Big[\theta_{i,\tau} + \theta_{(i-1),\tau} - \theta_{i,(\tau+1)} - \theta_{(i-1),\,(\tau+1)}\Big] \cdot \Big[z_i - z_{(i-1)}\Big] + \tfrac{1}{4}\Big[\theta_{(j+1),\tau} + \theta_{j,\tau} - \theta_{(j+1),\,(\tau+1)} - \theta_{j,(\tau+1)}\Big]\Big[z_{j+1} - z_j\Big]\Big\}\,(z_{j+1} - z_j)\Big\{\tfrac{1}{2}\Big[t_{(\tau+1)} - t_\tau\Big] \cdot \Big[h_{(j+1),\tau} + h_{(j+1),\,(\tau+1)} - h_{j,\tau} - h_{j,(\tau+1)} + z_{j+1} - z_j\Big]\Big\}^{-1} \quad \text{(cm/hr)} \tag{15}$$

where $f(j) = \begin{cases} 0 & j = 1 \\ 1 & j > 1 \end{cases}$

Black et al. (1969) used the assumptions that the change in soil-water content with time (τ to $\tau+1$) was the same at all depths and the component of hydraulic gradient due to matric suction was zero. Substituting the above assumptions into equation (10) reduces the equation to:

$$K_{z_n}(\theta) = -(\theta_{n,(\tau+1)} - \theta_{n,\tau})(z_n)(t_{\tau+1} - t_{\tau})^{-1} \quad (16)$$

Green and Corey (1971) used the following equation to calculate $K(\theta)$ from a soil-water characteristic plus a matching factor

$$K(\theta)_i = \frac{K_s}{K_{sc}} \cdot \frac{30\ \gamma^2}{\rho g \eta} \cdot \frac{\varepsilon^p}{n^2} \sum_{j=1}^{m} (2_j + 1 - 2i)h_j^{-2}$$

$$i = 1, 2, \ldots, m \text{ (cm/hr)} \quad (17)$$

where

$K(\theta)_i$ is the calculated conductivity for a specified water content or pressure (cm/hr),

θ is the water content (cm^3/cm^3),

i denotes the last water content class on the wet end, e.g., $i = 1$ identified the pore class corresponding to the maximum water content, and $i=m$ identified the pore class corresponding to the lowest water content for which conductivity was calculated

K_s/K_{sc} is the matching factor (measured maximum conductivity/ calculated maximum conductivity),

γ is the surface tension of water (g/hr^2)

ρ is the density of water (g/cm^3),

g is the gravitational constant (cm/hr^2),

η is the viscosity of water ($g/cm\ hr^{-1}$),

ε is the porosity (cm^3/cm^3),

p is a parameter that accounts for interaction of pore class.

n is the total number of pore classes, and

h_j is the pressure for a given class of water-filled pores (cm).

Hydraulic heads recorded with tensiometers in the experimental plot were used to calculate $h_{i,\tau}$ which was subsequently used with a soil-water characteristic to evaluate $\theta_{i,\tau}$ (Appendix B). In situ $K(\theta)$ were calculated with equation (15).

Equation (16) was used by Black et al. (1969) to calculate $K(\theta)$ and will be used here and the results compared to in situ measurements for a given depth increment.

Equation (17) was used herein to approximate $K(\theta)$ using

$$p = 2$$

$$m = 50$$

$$n = 50\ \theta_h/(\theta_h - \theta_L).$$

and linear interpolation for values between observed soil-water pressures and water contents. Results will be compared to in situ measurements for a given depth increment.

Precision of Soil Physical Properties

The statistical analysis for 0.1 and 15 bar water contents, textural components and bulk density are summarized by the AOV given in Table I. Subsequently, the SE for different arrangements are as follows:

1. Among field samples which are defined to be a single soil core and composite of soil surrounding the core

$$\pm\sqrt{MS_5} \tag{18}$$

2. Locations across depths, $\pm\sqrt{MS_5+(MS_2-MS_5)/24}$ (19)

TABLE I

AN AOV FOR SOIL-WATER CONTENT AT 100 CM MATRIC SUCTION, 15 BAR WATER CONTENT, TEXTURAL COMPONENTS, OR BULK DENSITY

Line No.	Line Entry	df	SS	MS	Expected Mean Square
1	Total Corrected	191	SS_1		
2	Location, L	7	SS_2	MS_2	$\sigma^2_{FS} + 24\sigma^2_L$ [1]
3	Depth, D	7	SS_3	MS_3	$\sigma^2_{FS} + 3\sigma^2_{D*L} + 24\ \Delta^2_D$ [2]
4	D*L	49	SS_4	MS_4	$\sigma^2_{FS} + 3\sigma^2 D*L$
5	Among Field Samples, FS (D,L)	128	SS_5	MS_5	σ^2_{FS}

[1] The subscript on the variance component, σ^2 indicates quantity with which it is associated.

[2] Δ^2_D is the variance among the depths used in this study.

3. Depths[1] across locations, $\pm\sqrt{MS_5 + (MS_3 - MS_4)/24}$ (20)

To estimate the SE of a new sampling arrangement, the exact sampling scheme must be specified. The only scheme evaluated here is from m new locations at a given depth with one field sample collected at each location. The SE of the mean of m samples collected in this way is

$$\sqrt{\left(MS_5 + \frac{MS_2 - MS_5}{24} + \frac{MS_4 - MS_5}{3}\right)/m.} \qquad (21)$$

Various ways can be used to calculate the optimum sampling allocations. Assigning costs to different levels of sampling with a total amount of funds available, sampling for a given probability of an error within a given percentage of the mean, or for a SE which is equal to a given percentage of the mean. The example illustrated here is for the number of samples for a SE which is less than or equal to 10% of the estimated mean. The equation from which this is computed is

$$0.1\ (\text{Mean of Soil Physical Property}) = \sqrt{\left(MS_5 + \frac{MS_2 - MS_5}{24} + \frac{MS_4 - MS_5}{3}\right)/m} \qquad (22)$$

which is subsequently solved for m.

The analysis of the spatial error associated with $K(\theta)$ was made

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[1] Since depths are fixed in this study, all references to depths applied only to the depths actually sampled.

by fitting the linear model[1]:

$$\log K(\theta)_{ij} = \mu + \lambda_i + \bar{\beta}_1\theta_j + \beta_{1_i} \quad \lambda_i\theta_j + \varepsilon_{ij} \tag{23}$$

where:

μ = the overall mean,

λ_i = the response due to location i and whose estimate is given by L_i,

β_{1_i} = the coefficient of a θ within a given location i and whose estimate is given by β_1.

$\bar{\beta}_1$ = the coefficient of θ for all locations within a given depth and whose estimate is given by $\bar{\beta}_1$, and

ε_{ij} = the random error associated with the j^{th} observation at the i^{th} location and whose estimate is given by the square root of MS_3 in Table II.

A straight line with slope $\bar{B}_1$ was made to pass through the average log K(θ) (=$\overline{LK}$) and $\bar{\theta}$, for all locations depths, or morphological horizons from which $\bar{B}_1$ was estimated and is shown graphically in Figure 5 as the dashed line. The AOV's are given in Tables II and III. SE's of the estimate of $\log K(\theta)|_{\theta_a}$ within depth, d, are given by

$$\pm\sqrt{MS_{3_d}\left[\frac{1}{N} + C_{ii} \quad (\theta_a - \bar{\theta})^2\right] + \frac{MS_{1_d} - MS_{3_d}}{\bar{k}_d}} \tag{24}$$

where:

N = total number of observations used to fit the line,

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[1] The same equations apply to horizons and is obtained by replacing d with h and L with D in equations (23), (24), and (25).

TABLE II

AN AOV FOR ESTIMATING LOG K(θ) ACROSS LOCATION WITHIN DEPTH d

Line No.	Line Entry	df	MS	Expected Mean Square
1	Total Corrected	$\sum_j n_{ij} -1$		
2	Locations, L (adusted for Water Content)	$\ell-1$	MS_{1_d}	$\sigma_R^2 + + \bar{k}_d \sigma_L^2$
3	Water Content (linear) WC	1		
4	L*WC (linear)	$\ell-1$		
5	Residual, R	$\sum_j (n_{ij}-2)$	MS_{3_d}	σ_R^2

Where ℓ = Number of Locations

n_{dj} = number of observations at location j and depth d

$\bar{k}_d = \left[1/(\ell-1)\right] \left[\sum_j n_{dj} - (\sum_j n_{dj}^2)/(\sum_j n_{dj})\right]$ (Snedecor and Cochran 1967, page. 290).

TABLE III

AN AOV FOR ESTIMATING LOG K(θ) ACROSS LOCATION WITHIN MORPHOLOGICAL HORIZON h

Line No.	Line Entry	df	MS	Expected Mean Square
1	Total	$\sum_j n_{hj}-1$		
2	Location, L (Adjusted for Water Content)	$\ell-1$	MS_{1_h}	$\sigma_R^2 + \bar{k}_h\sigma_L^2$
3	Water Content (linear), WC	1		
4	L*WC (linear)	$\ell-1$		
5	Residual, R	$\sum_j (n_{hj}-2)$	MS_{3_h}	σ_R^2

Where ℓ = number of locations

n_{hj} = number of observations at location j and horizon h

$$\bar{k}_h = \left[1/(\ell-1)\right] \left[\sum_j n_{hj} - (\sum_j n_{hj}^2)/(\sum_j n_{hj})\right]$$

c_{ii} = diagonal entry of the $(X'X)^{-1}$ corresponding to water content linear,

$\bar{\theta}$ = mean of all water contents where observations were used to fit the line,

$\log K|_{\theta_a}$ denotes the value of log K evaluated at θ_a.

Since these SE's are dependent upon the water content, graphical presentation is made as in Figure 5. Note the solid lines are for ± one SE. The $SE|_{\theta_a}$ can be obtained by passing a line parallel to the log K axis through θ_a and measuring the distance of this line from the dashed line to either one of the solid lines in Figure 5. To obtain the SE of the flux, V_D, take the antilogarithm of the intersection of the above with the solid line and use equation (1) or equation (3) to calculate V_D. Follow the same procedure with the intersection of the dashed line and the difference between the two calculated fluxes will be an estimate of the SE of the flux. The experimental points are graphed in Figure 5 as the number of the location where the data were collected. The SE of the estimate of $\log K(\theta)|_{\theta_a}$ for m new locations is given by

$$\pm\sqrt{MS_3\left[\frac{1}{N} + c_{ii}\ (\theta|_a - \bar{\theta})^2\right] + (1/m)\ \left[MS_3 + (1/\bar{k}_h)(MS_1 - MS_3)\right]} \qquad (25)$$

Setting the above equal to $0.1 \log K(\theta)|_{\theta_a}$ and solving for m gives the required number of new locations to sample for their $SE|_{\theta_a}$ to be within 10% of the mean $\log K(\theta)|_{\theta_a}$.

CHAPTER IV

RESULTS AND DISCUSSION

Tables IV through XI summarize the data for soil-water content versus soil-water pressure, textural components, and bulk densities from the eight locations of similar soils (Teller series). This soil is heterogeneous, vertically and horizontally, with respect to its physical properties. The spacial variation of textural components, BD and 0.1-and 15-bar soil-water content is given in Table XII. If comparisons of $\hat{\sigma}^2_{FS}$, $\hat{\sigma}^2_{DxL}$, $\hat{\Delta}^2$, $\hat{\sigma}^2_{L}$ in Table XII are made with each other, it must be remembered that these quantities are not independent and using these quantities to answer questions such as: "Is the variance of % sand with the depth larger than with location" is a hazardous procedure because the precision of the estimated variance is not known. As a result, inferences about population parameters may be invalid. There are procedures by which their precision can be approximated, but these procedures are beyond the scope of this study. $\hat{\sigma}^2_{FS}$ is an estimate of the inherent variation of the indicated soil physical property at a given location and depth with attendant variation in laboratory technique. It can be thought of as the "error of determination" of a soil physical property at a given location and depth. The SE_{FS} is an estimate of the "average error" as a result of the "error of determination" and therefore includes the uncertainty of both field and laboratory measurements.

TABLE IV

VALUES OF SOIL-WATER CONTENT VERSUS SOIL WATER PRESSURE, BULK DENSITY % SAND, % SILT, % CLAY, AND 15 BAR WATER CONTENT FOR LOCATION NUMBER 1

Depth (cm)	15	30	45	60	75	90	120	150
SOIL WATER PRESSURE (cm)				SOIL-WATER CONTENT (cc/cc)				
- 4	0.306	0.324	0.320	0.318	0.293	0.266	0.268	0.282
- 20	0.302	0.321	0.316	0.315	0.292	0.264	0.265	0.277
- 40	0.300	0.312	0.306	0.308	0.286	0.260	0.261	0.270
- 60	0.294	0.303	0.299	0.302	0.280	0.253	0.236	0.260
- 80	0.280	0.297	0.293	0.298	0.275	0.245	0.252	0.248
-100	0.262	0.292	0.289	0.294	0.271	0.239	0.244	0.238
-160	0.227	0.279	0.279	0.288	0.263	0.227	0.221	0.216
-190	0.217	0.274	0.275	0.286	0.260	0.224	0.216	0.208
				SOIL BULK DENSITY (gm/cc)				
	1.73	1.63	1.66	1.70	1.76	1.82	1.84	1.80
				% SAND				
	55.60	50.33	48.33	49.03	57.53	66.43	72.10	74.50
				% SILT				
	32.10	31.50	31.00	29.47	23.27	18.00	12.83	12.47
				% CLAY				
	12.30	18.17	20.67	21.60	19.27	15.60	15.13	13.03
				15 BAR WATER CONTENT (100) (g/g)				
	4.90	7.91	9.12	9.83	8.52	6.80	6.10	5.55

TABLE V

VALUES OF SOIL-WATER CONTENT VERSUS SOIL WATER PRESSURE, BULK DENSITY, % SAND, % SILT, % CLAY, AND 15 BAR WATER CONTENT FOR LOCATION NUMBER 2

Depth (cm)	15	30	45	60	75	90	120	150
SOIL WATER PRESSURE (cm)	SOIL-WATER CONTENT (cc/cc)							
- 4	0.317	0.339	0.344	0.318	0.278	0.270	0.262	0.309
- 20	0.312	0.337	0.342	0.314	0.274	0.265	0.249	0.304
- 40	0.303	0.330	0.337	0.311	0.267	0.261	0.244	0.302
- 60	0.288	0.323	0.331	0.305	0.255	0.250	0.237	0.299
- 80	0.276	0.320	0.329	0.301	0.248	0.242	0.231	0.296
-100	0.269	0.319	0.327	0.300	0.245	0.237	0.227	0.295
-130	0.249	0.312	0.322	0.293	0.232	0.222	0.212	0.287
-190	0.243	0.310	0.320	0.291	0.228	0.218	0.207	0.283
	SOIL BULK DENSITY (gm/cc)							
	1.67	1.64	1.67	1.74	1.79	1.82	1.86	1.78
	% SAND							
	52.90	43.57	45.03	55.43	65.47	69.10	72.67	71.00
	% SILT							
	29.53	31.93	27.50	20.33	17.37	17.23	16.20	13.03
	% CLAY							
	19.27	26.20	27.87	24.27	17.17	13.33	11.13	19.30
	15 BAR WATER CONTENT (100) (g/g)							
	6.19	10.17	10.94	9.74	7.07	5.58	5.09	8.59

TABLE VI

VALUES OF SOIL-WATER CONTENT VERSUS SOIL WATER PRESSURE, BULK DENSITY % SAND, % SILT, % CLAY, AND 15 BAR WATER CONTENT FOR LOCATION NUMBER 3

Depth (cm)	15	30	45	60	75	90	120	150
SOIL WATER PRESSURE (cm)				SOIL-WATER CONTENT (cc/cc)				
- 4	0.328	0.350	0.352	0.346	0.314	0.280	0.288	0.272
- 20	0.327	0.344	0.347	0.342	0.312	0.278	0.282	0.268
- 40	0.324	0.323	0.331	0.329	0.308	0.272	0.271	0.260
- 60	0.318	0.315	0.324	0.324	0.305	0.266	0.261	0.254
- 80	0.301	0.309	0.319	0.321	0.302	0.259	0.248	0.247
-100	0.271	0.300	0.313	0.317	0.298	0.249	0.231	0.237
-170	0.235	0.286	0.303	0.309	0.291	0.237	0.208	0.222
-190	0.229	0.285	0.301	0.309	0.290	0.235	0.204	0.220
				SOIL BULK DENSITY (gm/cc)				
	1.69	1.56	1.59	1.66	1.82	1.84	1.88	1.83
				% SAND				
	51.60	40.67	35.67	43.33	55.00	66.77	73.27	73.93
				% SILT				
	31.70	33.77	35.53	26.53	24.83	19.77	16.93	14.33
				% CLAY				
	16.80	24.60	25.87	30.20	20.17	13.50	9.83	11.77
				15 BAR WATER CONTENT (100) (g/g)				
	5.56	9.17	10.58	11.65	9.58	6.56	4.44	5.38

TABLE VII

VALUES OF SOIL-WATER CONTENT VERSUS SOIL WATER PRESSURE, BULK DENSITY, % SAND, % SILT, % CLAY, AND 15 BAR WATER CONTENT FOR LOCATION NUMBER 4

Depth (cm)	15	30	45	60	75	90	120	150
SOIL WATER PRESSURE (cm)				SOIL-WATER CONTENT (cc/cc)				
- 4	0.335	0.364	0.352	0.338	0.323	0.299	0.262	0.243
- 20	0.330	0.353	0.337	0.327	0.311	0.295	0.258	0.236
- 40	0.325	0.336	0.318	0.313	0.300	0.293	0.246	0.222
- 60	0.319	0.321	0.305	0.303	0.293	0.291	0.242	0.202
- 80	0.309	0.310	0.298	0.297	0.288	0.289	0.237	0.186
-100	0.295	0.300	0.290	0.293	0.285	0.288	0.234	0.174
-135	0.278	0.286	0.281	0.286	0.281	0.286	0.229	0.159
-180	0.262	0.272	0.270	0.279	0.277	0.285	0.224	0.149
				SOIL BULK DENSITY (gm/cc)				
	1.63	1.47	1.51	1.57	1.62	1.80	1.87	1.83
				% SAND				
	42.43	38.27	33.27	33.10	33.30	50.63	71.60	84.73
				% SILT				
	44.87	42.90	46.03	44.17	41.87	22.57	11.17	6.87
				% CLAY				
	12.77	18.87	20.70	22.73	24.93	26.80	17.27	8.43
				15 BAR WATER CONTENT (100) (g/g)				
	4.87	7.23	7.68	8.32	9.20	10.65	7.37	3.50

TABLE VIII

VALUES OF SOIL-WATER CONTENT VERSUS SOIL WATER PRESSURE, BULK DENSITY, % SAND, % SILT, % CLAY, AND 15 BAR WATER CONTENT FOR LOCATION NUMBER 5

Depth (cm)	15	30	45	60	75	90	120	150
SOIL WATER PRESSURE (cm)				SOIL-WATER CONTENT (cc/cc)				
	0.237	0.339	0.324	0.306	0.337	0.293	0.252	0.333
- 20	0.231	0.322	0.314	0.301	0.334	0.290	0.246	0.323
- 40	0.226	0.311	0.301	0.294	0.332	0.287	0.236	0.299
- 60	0.221	0.296	0.291	0.290	0.331	0.283	0.217	0.263
- 80	0.213	0.287	0.285	0.287	0.324	0.278	0.202	0.239
-100	0.203	0.280	0.280	0.285	0.319	0.276	0.191	0.225
-135	0.194	0.272	0.274	0.283	0.317	0.274	0.181	0.212
-170	0.187	0.266	0.270	0.281	0.317	0.273	0.176	0.205
				SOIL BULK DENSITY (gm/cc)				
	1.78	1.62	1.62	1.64	1.76	1.81	1.84	1.72
				% SAND				
	64.97	50.47	48.97	28.83	49.20	67.43	81.90	87.33
				% SILT				
	20.43	30.80	32.33	40.10	21.33	11.80	7.53	5.07
				% CLAY				
	11.27	19.07	18.70	31.07	29.47	20.77	10.57	7.60
				15 BAR WATER CONTENT (100) (g/g)				
	3.90	6.60	7.64	9.50	11.76	11.06	4.21	2.70

TABLE IX

VALUES OF SOIL-WATER CONTENT VERSUS SOIL WATER PRESSURE, BULK DENSITY, % SAND, % SILT, % CLAY, AND 15 BAR WATER CONTENT FOR LOCATION NUMBER 6

Depth (cm)	15	30	45	60	75	90	120	150
SOIL WATER PRESSURE (cm)	SOIL-WATER CONTENT (cc/cc)							
- 4	0.309	0.349	0.323	0.318	0.296	0.278	0.245	0.258
- 20	0.306	0.338	0.316	0.314	0.292	0.276	0.243	0.256
- 40	0.304	0.313	0.303	0.305	0.286	0.273	0.240	0.252
- 60	0.296	0.291	0.287	0.294	0.278	0.270	0.236	0.245
- 80	0.279	0.270	0.272	0.282	0.269	0.265	0.232	0.234
-100	0.260	0.255	0.261	0.273	0.263	0.262	0.229	0.228
-160	0.229	0.222	0.234	0.250	0.248	0.254	0.222	0.212
-190	0.219	0.211	0.224	0.241	0.242	0.252	0.220	0.207
	SOIL BULK DENSITY (gm/cc)							
	1.68	1.52	1.58	1.62	1.69	1.76	1.84	1.83
	% SAND							
	64.73	59.60	55.13	53.03	48.87	45.77	61.00	73.87
	% SILT							
	28.37	27.17	29.93	31.60	33.83	32.43	19.23	11.90
	% CLAY							
	10.23	13.27	14.97	15.37	17.30	21.83	19.77	14.23
	15 BAR WATER CONTENT (100) (g/g)							
	4.74	6.13	6.31	6.29	7.03	8.53	8.39	5.65

TABLE X

VALUES OF SOIL-WATER CONTENT VERSUS SOIL WATER PRESSURE, BULK DENSITY, % SAND, % SILT, % CLAY, AND 15 BAR WATER CONTENT FOR LOCATION NUMBER 7

Depth (cm)	15	30	45	60	75	90	120	150
SOIL WATER PRESSURE (cm)				SOIL-WATER CONTENT (cc/cc)				
- 4	0.340	0.298	0.313	0.314	0.301	0.290	0.274	0.267
- 20	0.333	0.291	0.301	0.303	0.296	0.290	0.272	0.261
- 40	0.292	0.263	0.274	0.287	0.291	0.290	0.270	0.249
- 60	0.248	0.232	0.253	0.275	0.285	0.289	0.266	0.230
- 80	0.228	0.207	0.237	0.266	0.280	0.287	0.263	0.214
-100	0.216	0.190	0.226	0.259	0.277	0.286	0.260	0.204
-160	0.192	0.150	0.204	0.242	0.268	0.282	0.254	0.188
-190	0.185	0.135	0.198	0.236	0.264	0.281	0.252	0.183
				SOIL BULK DENSITY (gm/cc)				
	1.59	1.66	1.64	1.64	1.71	1.78	1.81	1.77
				% SAND				
	78.93	79.93	68.83	65.13	55.00	50.33	67.53	77.53
				% SILT				
	14.67	16.53	19.57	19.73	22.67	25.83	20.43	14.13
				% CLAY				
	6.40	4.87	11.60	15.13	22.33	23.83	12.07	8.30
				15 BAR WATER CONTENT (100) (g/g)				
	3.16	2.05	4.33	5.76	7.82	9.14	7.60	4.85

TABLE XI

VALUES OF SOIL-WATER CONTENT VERSUS SOIL WATER PRESSURE, BULK DENSITY, % SAND, % SILT, % CLAY, AND 15 BAR WATER CONTENT FOR LOCATION NUMBER 8

Depth (cm)	15	30	45	60	75	90	120	150
SOIL WATER PRESSURE (cm)	SOIL-WATER CONTENT (cc/cc)							
- 4	0.307	0.331	0.35[illegible]	0.335	0.338	0.319	0.318	0.322
- 20	0.303	0.322	0.334	0.321	0.324	0.311	0.315	0.310
- 40	0.298	0.307	0.291	0.292	0.297	0.294	0.302	0.285
- 60	0.294	0.286	0.263	0.274	0.277	0.278	0.280	0.258
- 80	0.284	0.271	0.241	0.257	0.260	0.265	0.256	0.233
-100	0.264	0.257	0.222	0.242	0.245	0.253	0.237	0.217
-135	0.241	0.237	0.201	0.224	0.226	0.237	0.208	0.191
-170	0.229	0.221	0.186	0.211	0.211	0.226	0.195	0.179
	SOIL BULK DENSITY (gm/cc)							
	1.69	1.60	1.53	1.59	1.54	1.61	1.65	1.62
	% SAND							
	63.53	60.37	58.87	66.13	64.50	63.60	75.40	79.43
	% SILT							
	24.87	28.73	29.70	21.50	22.00	20.27	11.50	9.87
	% CLAY							
	10.33	10.90	11.43	12.37	13.50	16.13	13.10	10.70
	15 BAR WATER CONTENT(100) (g/g)							
	3.48	4.02	4.38	4.76	5.17	6.01	4.72	4.60

TABLE XII

ESTIMATED VARIANCE COMPONENTS OF SOME SOIL PHYSICAL PROPERTIES AND SE ACROSS LOCATION WITHIN DEPTH

Physical Property	$\hat{\sigma}^2_{FS}$	$\hat{\sigma}^2_{D*L}$	$\hat{\Delta}^2$	$\hat{\sigma}^2_{L}$	Standard errors from Equation (21)
Bulk Density (g/cm^3)	0.00157	0.00231	0.00718	0.00230	0.06
0.1 Bar θ (cm^3/cm^3)	0.0002	0.0007	0.0004	0.0002	0.0210
% Sand	19.1375	84.9221	102.1550	36.9990	7.49
% Silt	17.2862	27.0200	53.4105	18.9925	6.02
% Clay	19.4558	27.7843	14.0026	9.1730	5.39
15 bar θ (100*g/g)	0.4666	3.2306	1.6942	1.3070	1.33

A summary of the sampling procedure and evaluation of the required number of locations to be sampled to achieve a given precision is outlined in equation (22), and presented in Table XIII. In this particular case, the estimated total number of locations to be sampled, if the standard error is to be within 10% of the mean, is conditioned to a large degree by the means of the parameters used in the calculations. An estimated number of locations of one for bulk density is a reflection of the small variation in BD at a given depth in this soil. An example of the influence of the mean on the number of required sample locations is obtained by comparing the number of locations required to estimate % clay within 10% of the mean with the number required to estimate % sand within 10% of the mean. The difference in the number of locations required for the two quantities is almost totally due to the differences in the mean.

Saturated infiltration rates at $z = 0$ were found to be 0.494, 0.561, 0.466, 1.08, 0.276, 1.09, 1.29, and 0.947 cm/hr for locations 1 thru 8, respectively, with a mean of 0.775 cm/hr and SE of 0.369 cm/hr. Locations 1, 2, and 3 were within 0.15 km of each other and had a SE of approximately 7% of their mean compared to a SE of approximately 48% of the mean for all locations. This indicates that the stratification of large areas into smaller ones may reduce the variation.

The soil-water pressure distribution with depth after the cessation of infiltration for locations 1 and 8 is given in Figures 1 and 2, respectively. Location 1 is the least uniform with depth while location 8 is the most uniform. Figures 3 and 4 give the corresponding water content distributions with depth and represent the

TABLE XIII

THE NUMBER OF NEW LOCATIONS TO BE SAMPLED AND MEANS REQUIRED TO EVALUATE EQUATION (22)

	Means of Soil Physical Properties Across Location Within Depth					
Depth (cm)	BD (g/cm^3)	0.1 Bar θ (cm^3/cm)	% Sand	% Silt	% Clay	15 bar Water Content (100 g/g)
15	1.68	0.2551	59.33	28.32	12.42	4.60
30	1.54	0.2740	52.90	30.42	16.99	6.66
45	1.65	0.2761	49.26	31.45	18.98	7.62
60	1.65	0.2828	48.39	27.77	23.86	8.23
75	1.71	0.2754	53.60	25.90	20.52	8.27
90	1.78	0.2613	60.00	20.99	18.98	8.04
120	1.82	0.2306	71.93	14.48	13.61	5.95
150	1.77	0.2266	77.79	10.96	11.67	5.10
	Number of New Locations to be sampled for a SE within 10% of the above means[1/]					
15	1	2	4	8	37	24
30	1	2	5	7	20	11
45	1	1	8	6	16	9
60	1	1	6	8	10	7
75	1	2	5	9	13	7
90	1	2	4	14	16	8
120	1	2	3	30	30	14
150	1	2	2	53	41	19

[1/] Numbers for BD are reported as the smallest integer greater than the estimated number and all other numbers are rounded to the nearest integer.

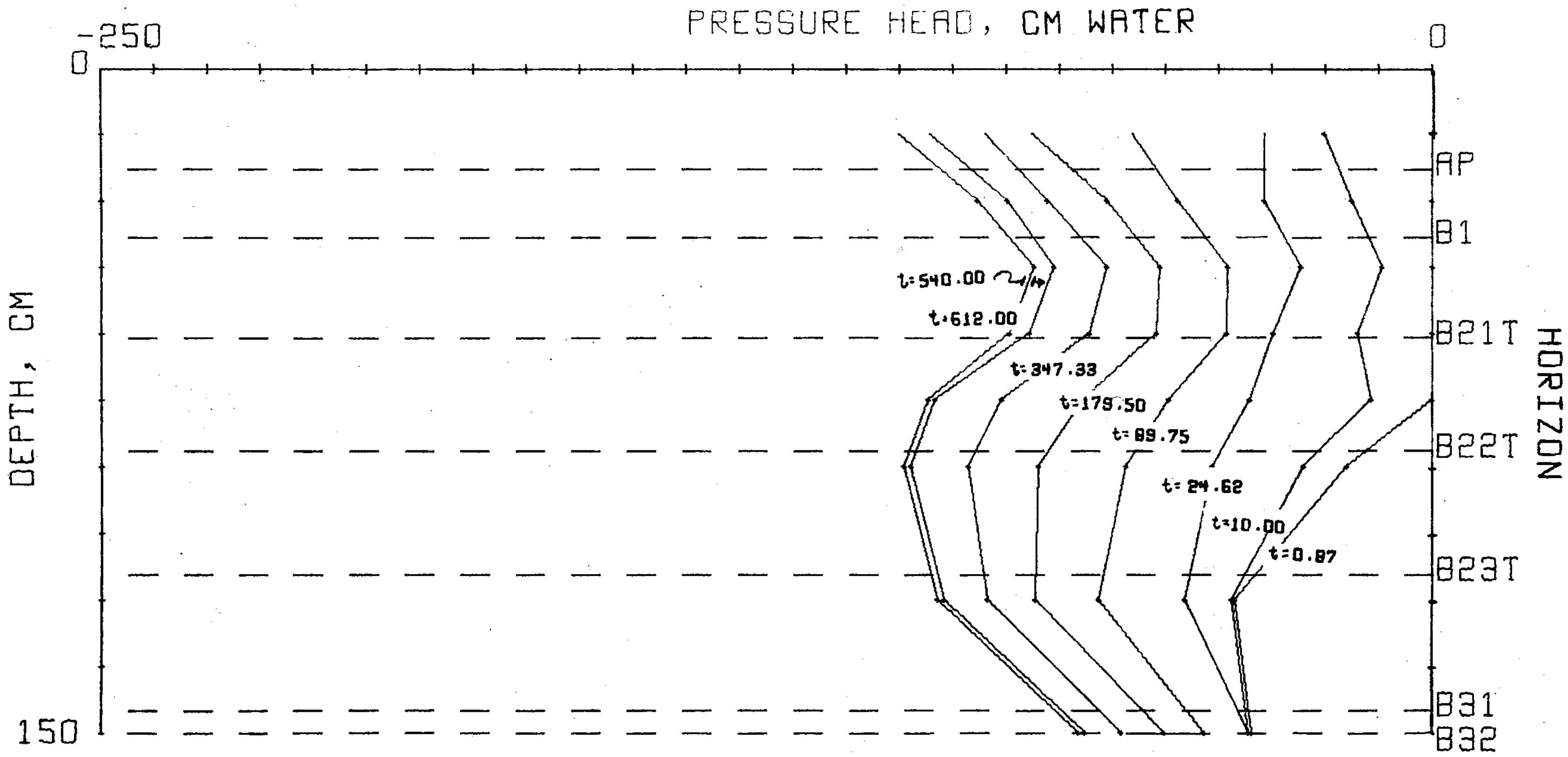

Figure 1. Pressure Head Distribution With Depth at Various Times, (t,hrs) After the Cessation of Infiltration at Location 1.

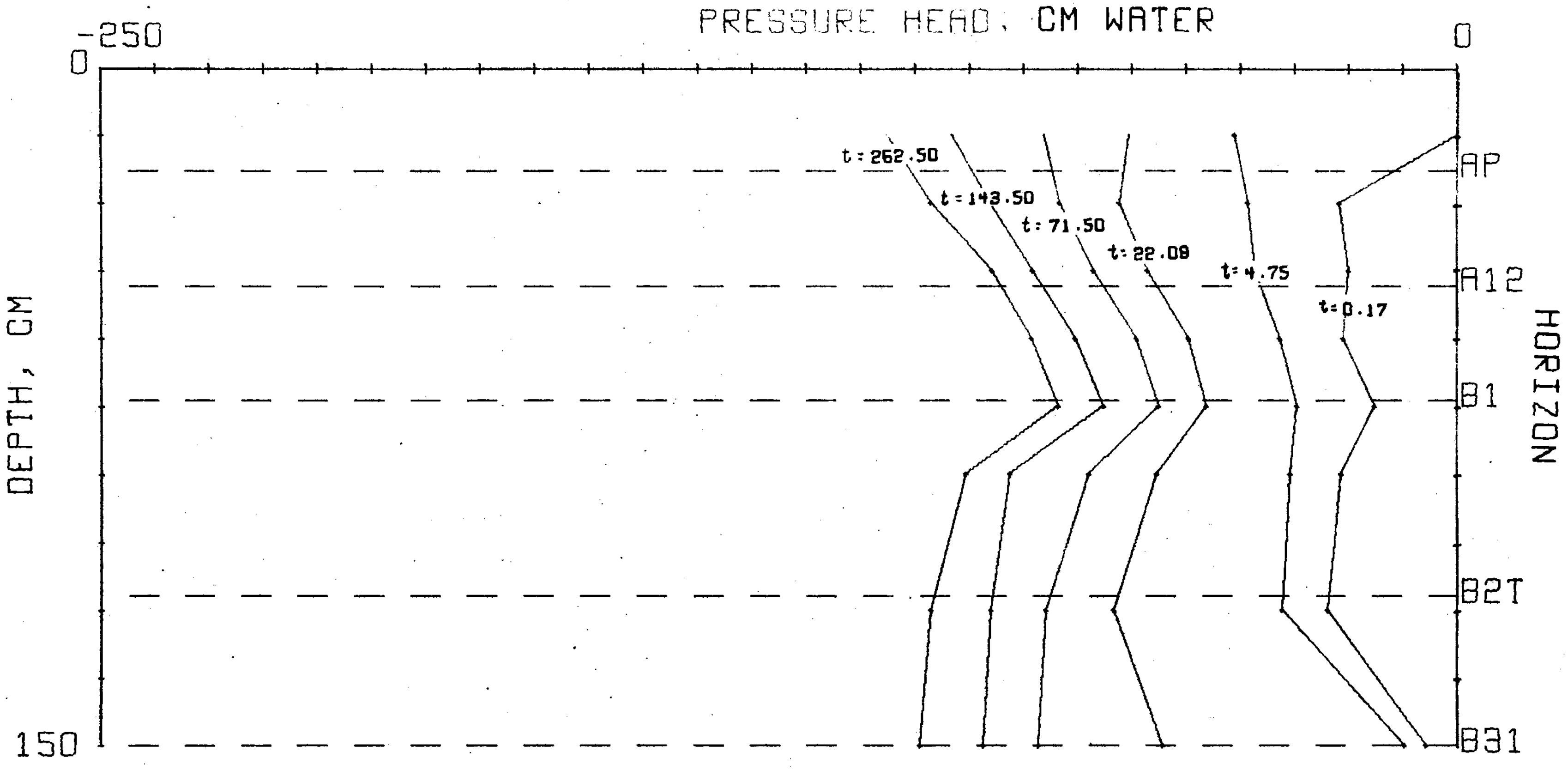

Figure 2. Pressure Head Distribution With Depth at Various Times (t,hrs) After the Cessation of Infiltration at Location 8.

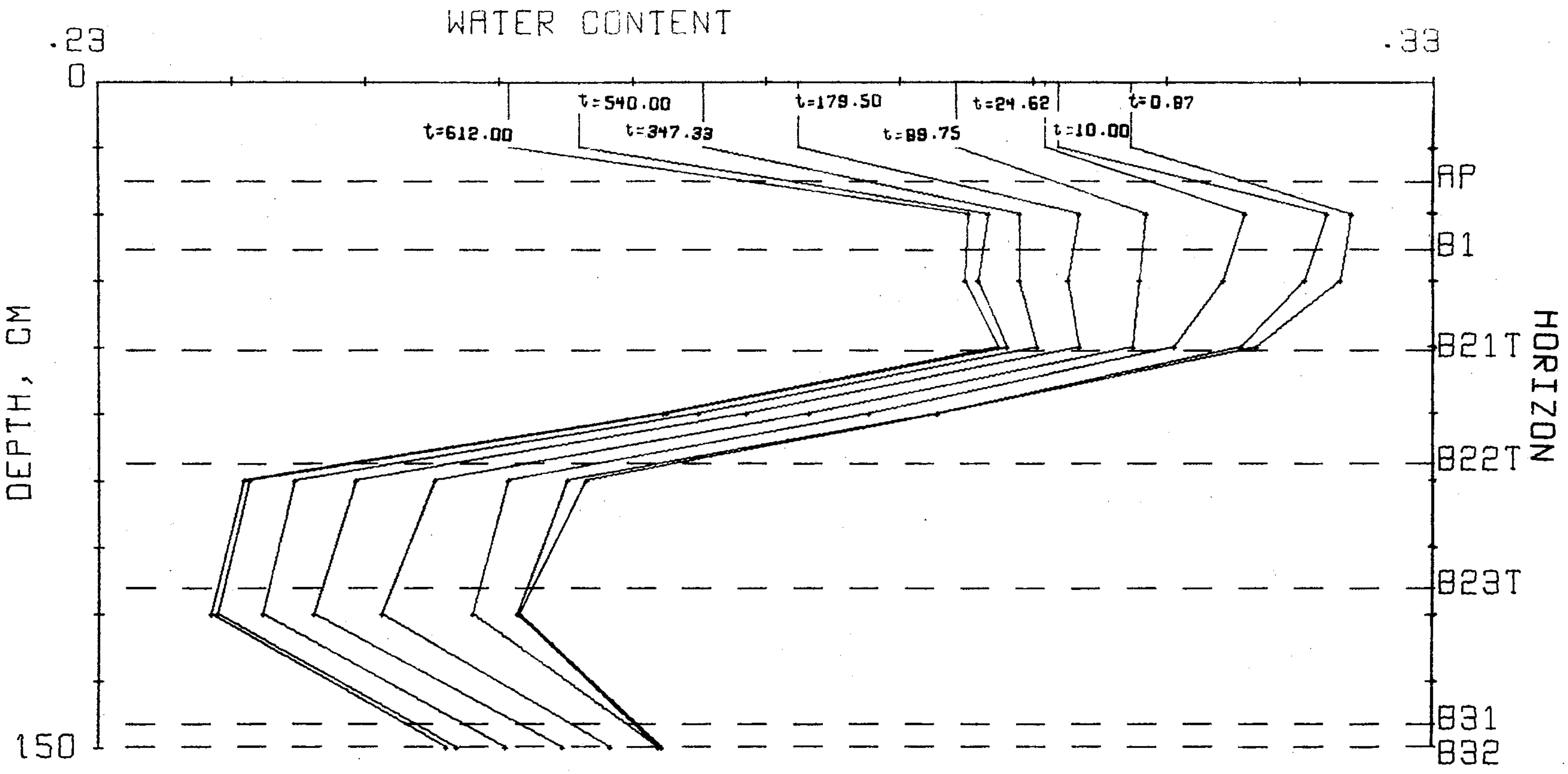

Figure 3. Soil-Water Content Distribution With Depth at Various Times (t,hrs) After the Cessation of Infiltration at Location 1.

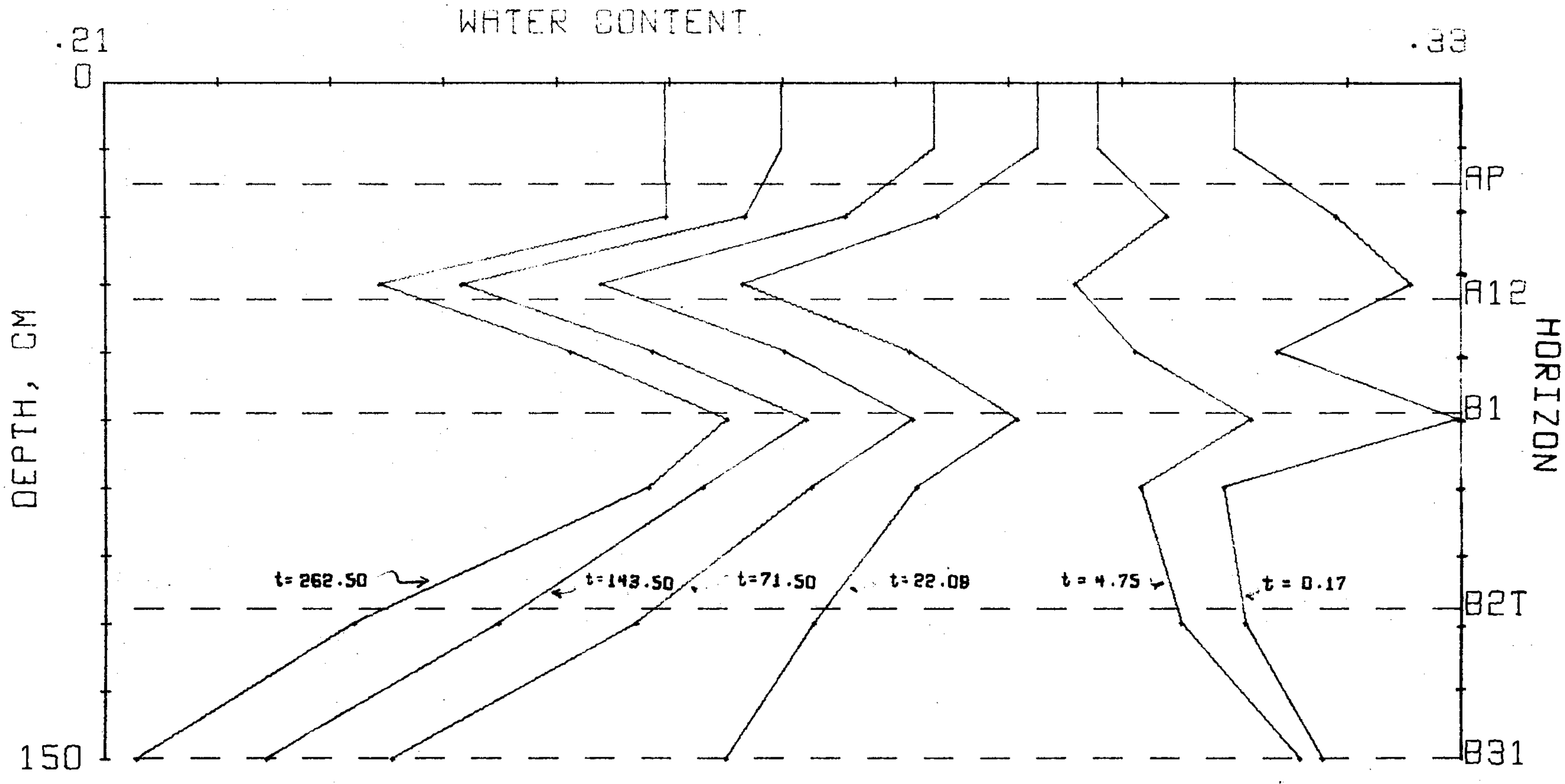

Figure 4. Soil-Water Content Distribution With Depth at Various Times (t,hrs) After the Cessation of Infiltration at Location 8.

extremes of the other water content with depth distributions for all locations. Neither location 1 nor 8 exhibited a constant pressure head or constant change in water content with time for all depths. Similar observations at the other six locations illustrate the fact that heterogenity exists with location and depth.

Subsequently, the basic assumptions for the soil-water conductivity solution of Black et al. (1969) were not met. However, it is possible to estimate the bias expected from using the assumption of a uniform water content change above a given depth. These data are summarized for locations 1 and 8 in Table XIV. This assumption was also used in the finite difference approximation for the 0 to 15 cm depth increment and therefore, accounts for a deviation of zero. The range in bias is from a low of approximately -41% to a high of approximately 88%. Figures 1 and 2 illustrate that $\frac{\partial h}{\partial z}$ is not zero and therefore, the assumption of unit gradient was not met either. It is possible for the two assumptions to be violated in such a manner that their total violations negate each other. An example where this might occur is for location 1 between depths 60 and 75 cm. The change in water content is too low while the $\frac{\partial h}{\partial z}$ is much larger than zero. The total effect of the assumptions might yield a soil water conductivity value approximately equal to the finite difference approximation.

The lateral variability of the soil-water conductivity soil-water content relation for all locations for a given depth is summarized in Figures 5 through 11. The dashed line is for the relation $\text{Log } K(\theta) = \overline{LK} + \overline{B}_1 (\theta - \bar{\theta})$ and the solid lines are for $\text{Log } K(\theta) \pm SE$ of $\text{Log } K(\theta)$ evaluated from equation (23). The numbers indicate the location at which the experimental data was collected. The fact that

TABLE XIV

A COMPARISON OF THE CUMULATIVE WATER FLOW PAST A GIVEN DEPTH AS PREDICTED BY A FINITE DIFFERENCE APPROXIMATION AND BY THE ASSUMPTION OF BLACK ET AL. (1969)

	Location 1 Time 540 hrs			Location 8 Time 217.17 hrs		
Depth (cm)	Finite Difference Approximation (cm/hr)	Black's Assumption (cm/hr)	%[1/] Deviation	Finite Difference Approximation (cm/hr)	Black's Assumption (cm/hr)	%[1/] Deviation
15	0.6998	0.6998	0	0.7549	0.7549	0
30	1.2660	0.8650	-31.7	1.5772	1.7791	12.8
45	1.6937	1.2688	-25.2	1.8846	2.7395	45.4
60	2.0512	1.2168	-40.7	2.7317	3.7560	37.5
75	2.3530	1.5571	-33.8	3.6618	4.8535	32.5
90	2.7012	2.3103	-14.5	4.7283	4.5791	-3.2
120	3.4333	2.7762	-19.1	6.7623	9.4616	39.9
150	4.0213	2.4100	-40.1	8.4401	15.7406	87.5

[1/] %deviation = 100 Black's Assumption - Finite difference approximation) divided by the finite difference approximation.

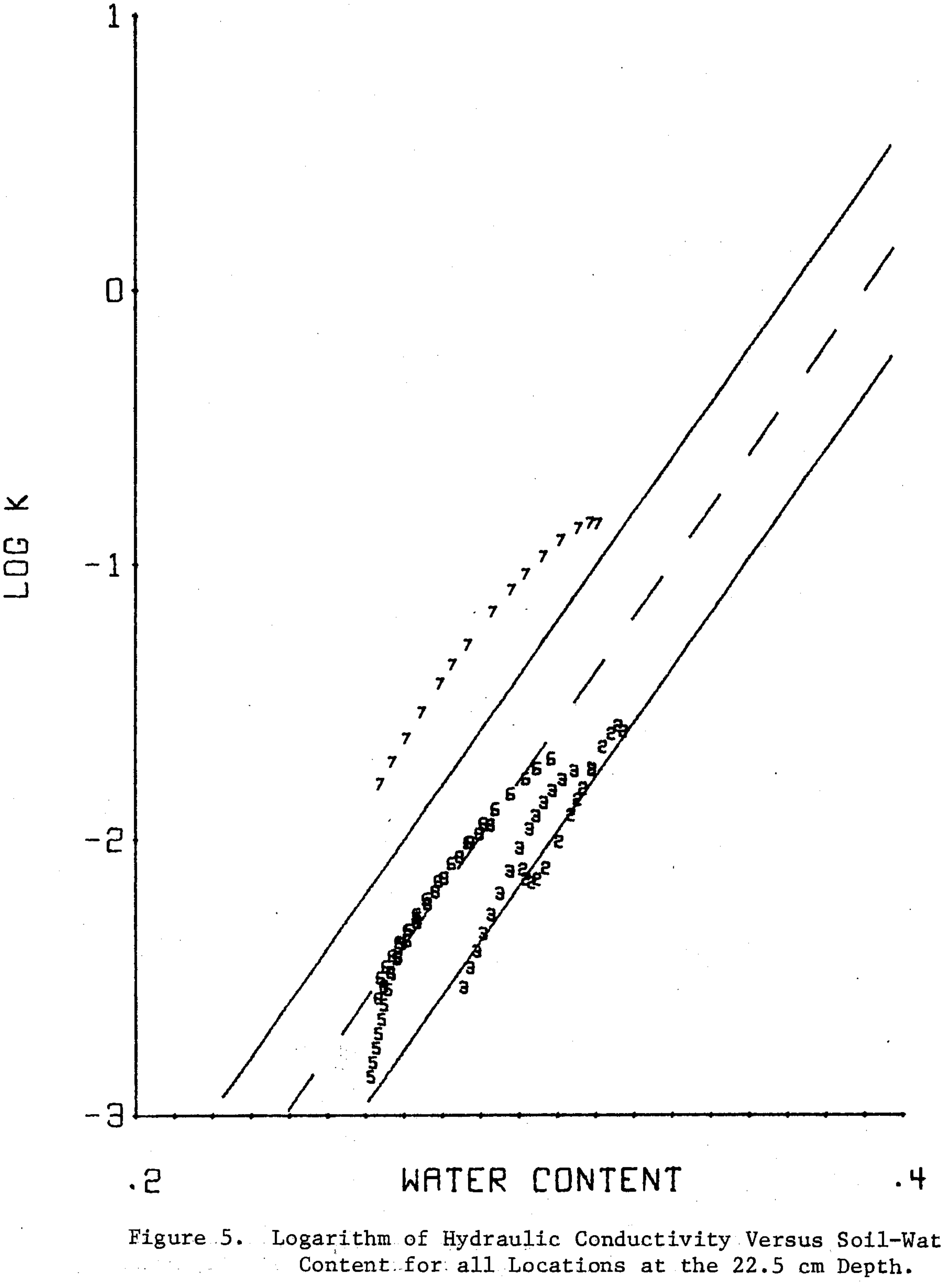

Figure 5. Logarithm of Hydraulic Conductivity Versus Soil-Water Content for all Locations at the 22.5 cm Depth.

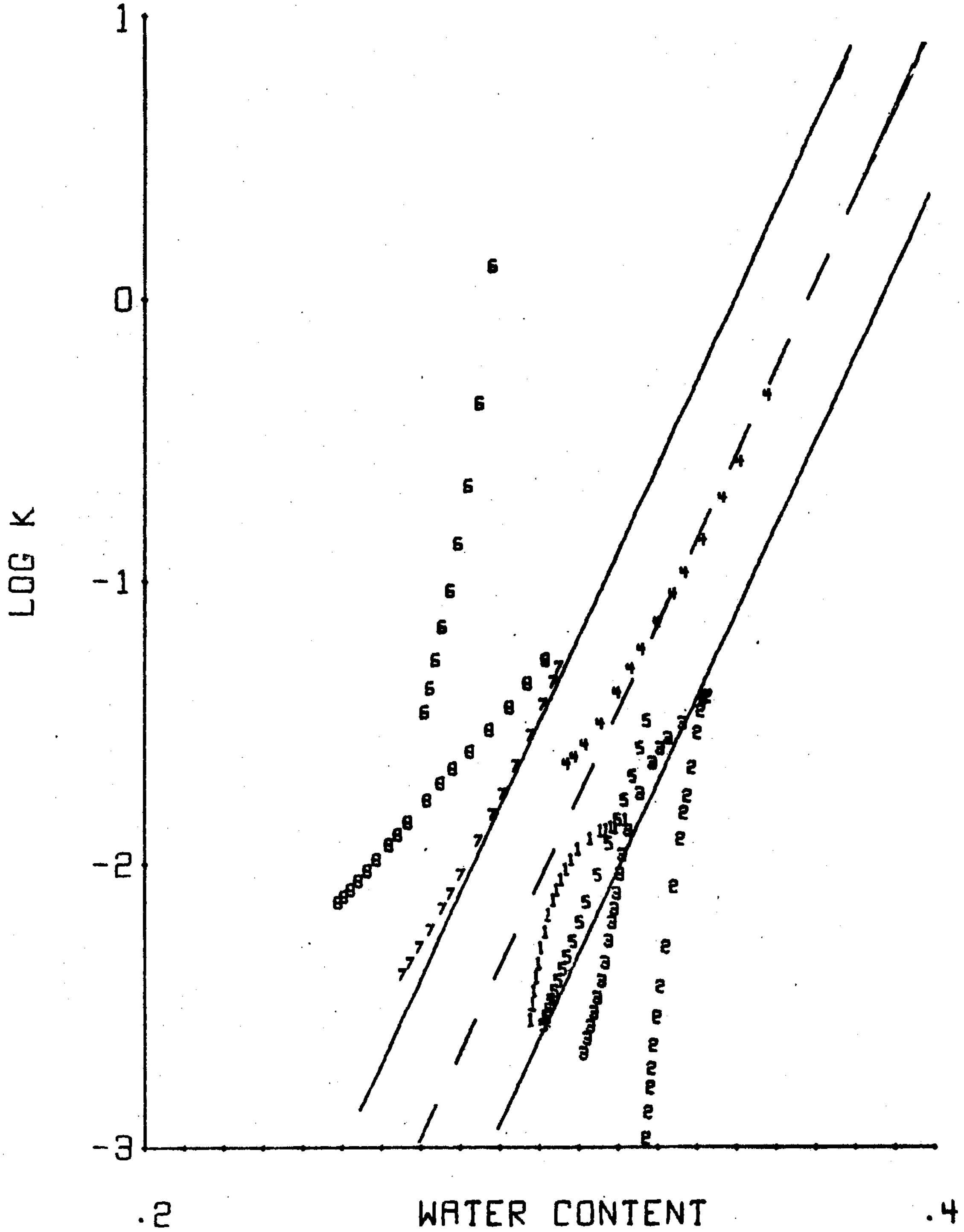

Figure 6. Logarithm of Hydraulic Conductivity Versus Soil-Water Content for all Locations at the 37.5 cm Depth.

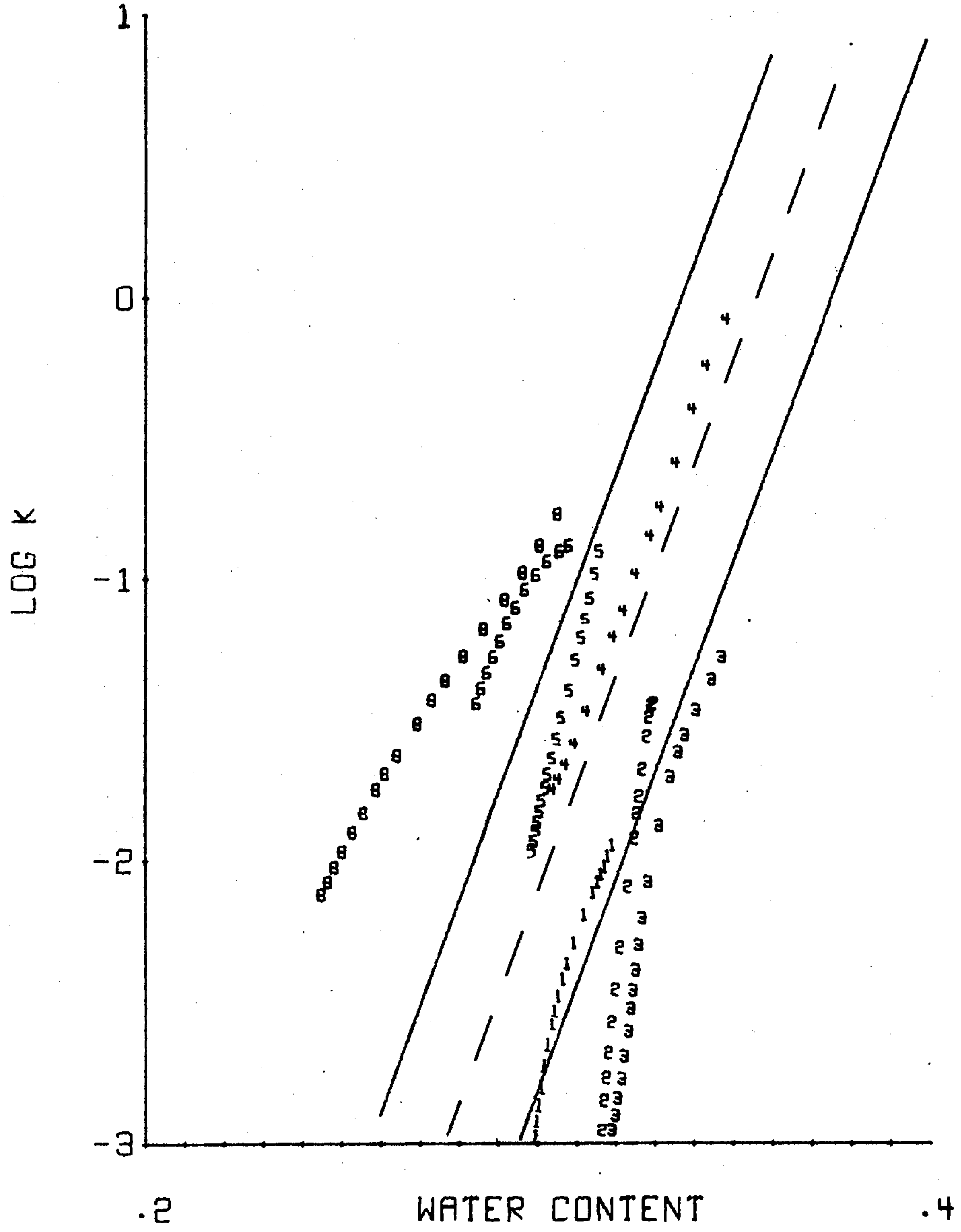

Figure 7. Logarithm of Hydrualic Conductivity Versus Soil-Water Content for all Locations at the 52.5 cm Depth.

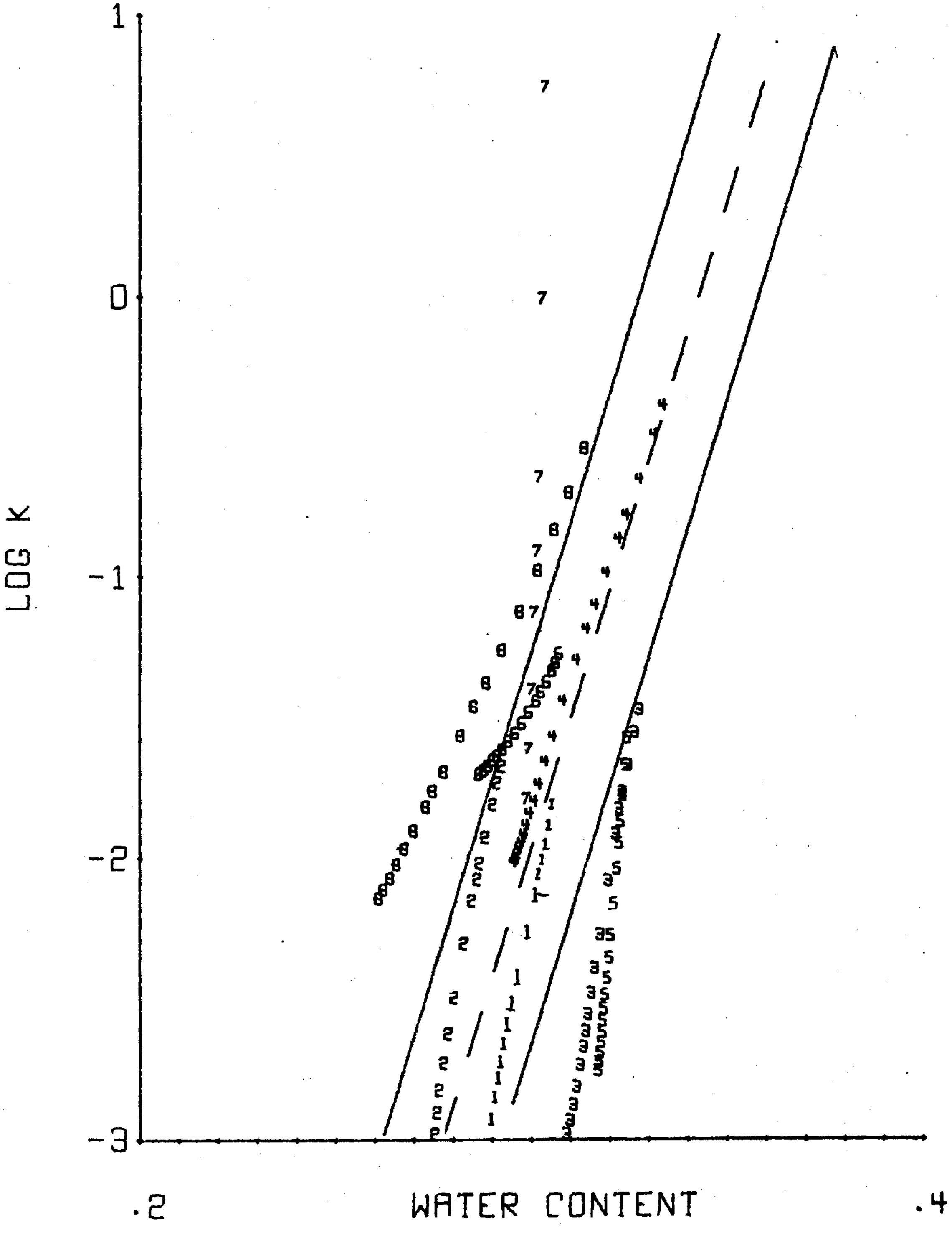

Figure 8. Logarithm of Hydraulic Conductivity Versus Soil-Water Content for all Locations at the 67.5 cm Depth.

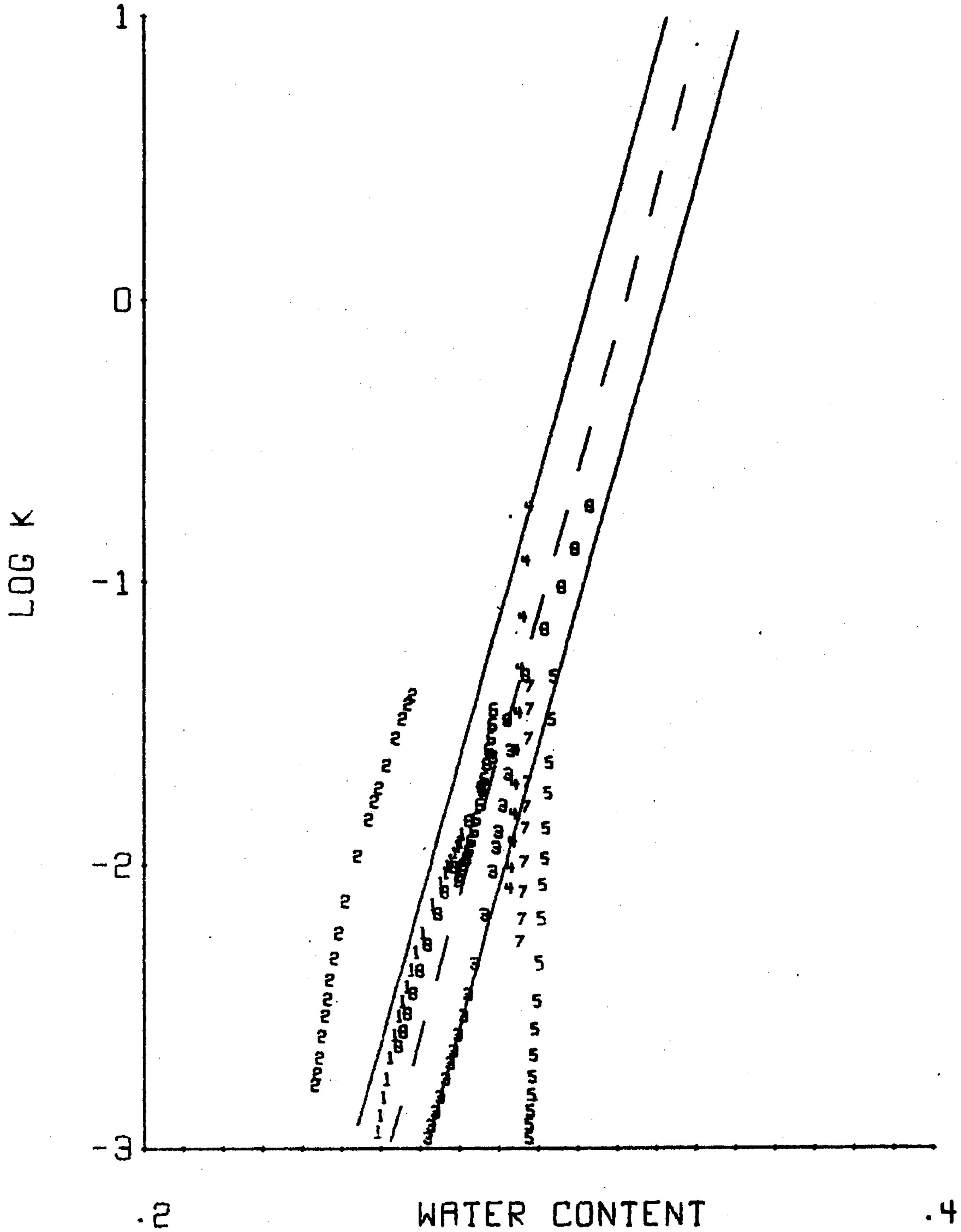

Figure 9. Logarithm of Hydraulic Conductivity Versus Soil-Water Content for all Locations at the 82.5 cm Depth.

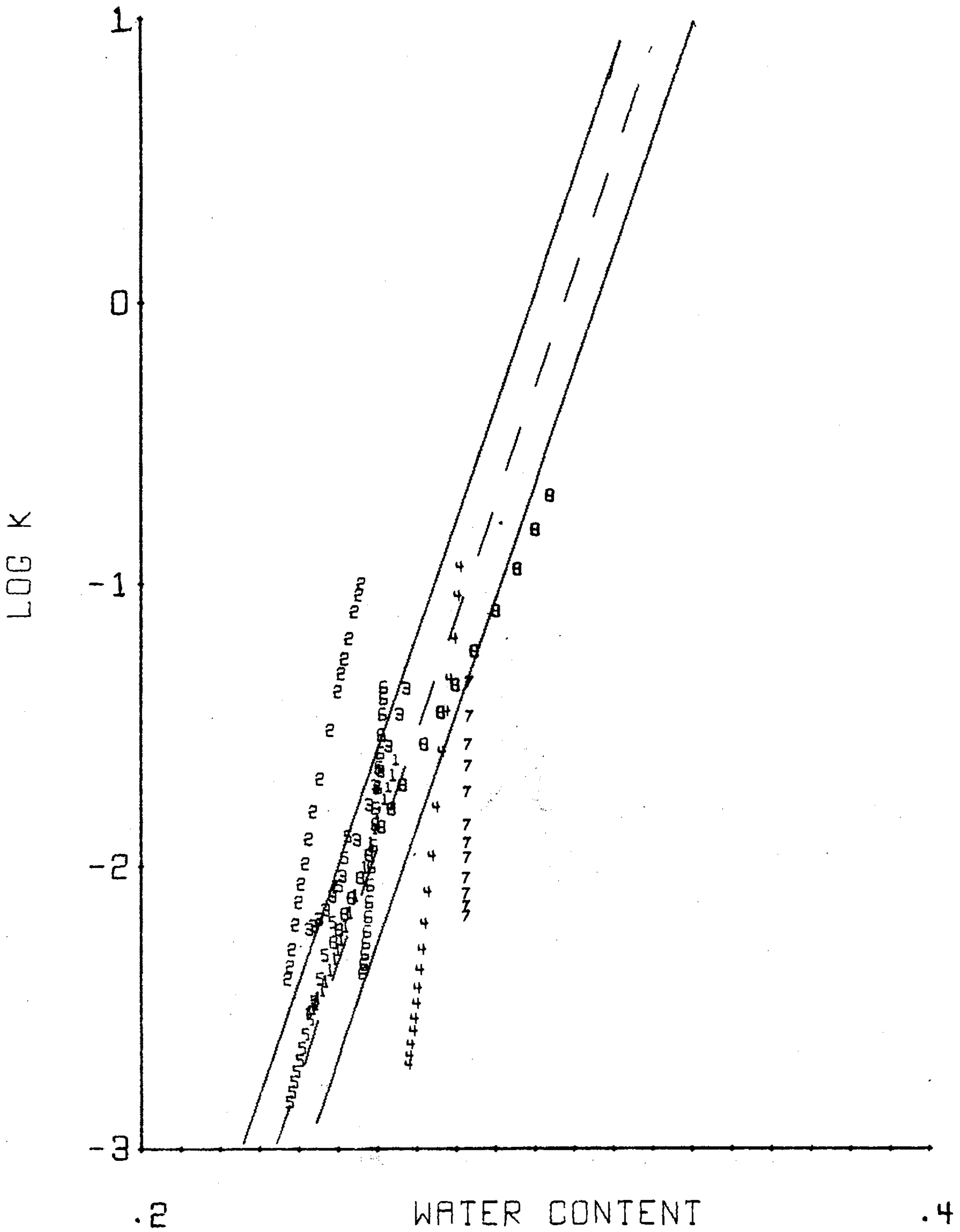

Figure 10. Logarithm of Hydraulic Conductivity Versus Soil-Water Content for all Locations at the 105 cm Depth.

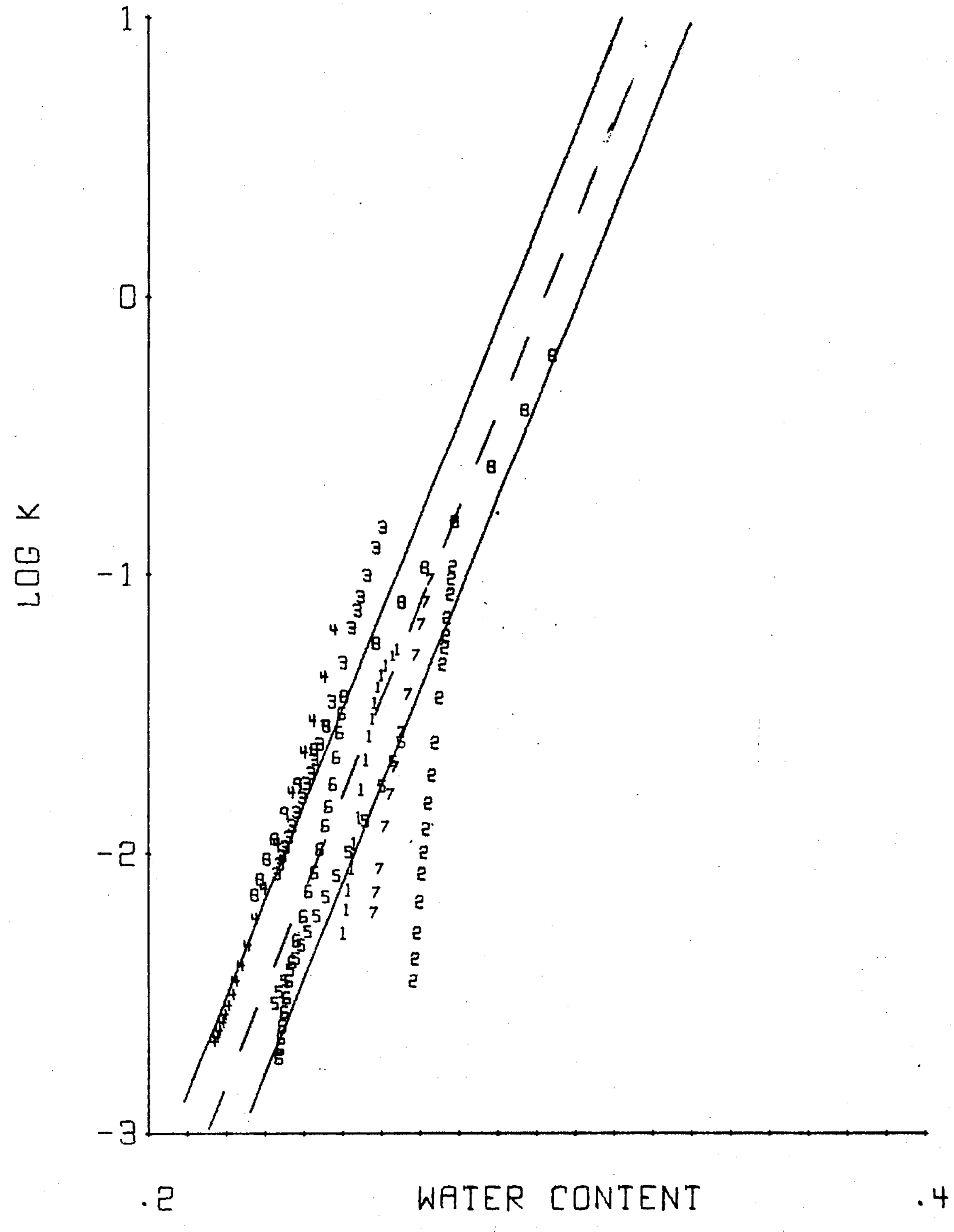

Figure 11. Logarithm of Hydraulic Conductivity Versus Soil-Water Content for all Locations at the 135 cm Depth.

the ± SE lines are straight indicate that the major component of the SE is due to location $\left(\frac{MS_{1_d} - MS_{3_d}}{\bar{k}_d}, \text{ equation } [23]\right)$ since the other component is responsible for curvature.

Additional calculations of the SE intervals with the curvature component equal to zero verified the fact that this component could be neglected without causing any appreciable error. (It was not neglected in any of the data presented).

Examining the width of the SE interval for Figures 5 through 11 indicates that the error decreases with depth since the width of the SE interval decreases with depth.

Examination of Figures 12 through 17 indicate the variability of the soil-water conductivity soil-water content relation for a radius of 0.15 km. The change in the widths of the SE interval compared to Figures 5 through 11 indicates that stratification of sampling to an area 0.15 km in radius will reduce the variation of the soil-water conductivity soil-water content relation except for the 82.5, 105 and 135 cm depths.

Figures 18 through 24 are for the lateral variability of an area 5.6 km in radius. Compared to the 0.15 km radius, the SE intervals are large except for the deeper depths. The constant width of the SE intervals again indicate that the location component is responsible for most of the error. A decreasing interval width with increasing depth is evident indicating that the error decreases as depth increases. In comparisons with corresponding depth for Figures 5 through 11, the SE interval are approximately of the same width indicating that stratification of sampling to areas as large as 5.6 km in radius does not reduce the variation.

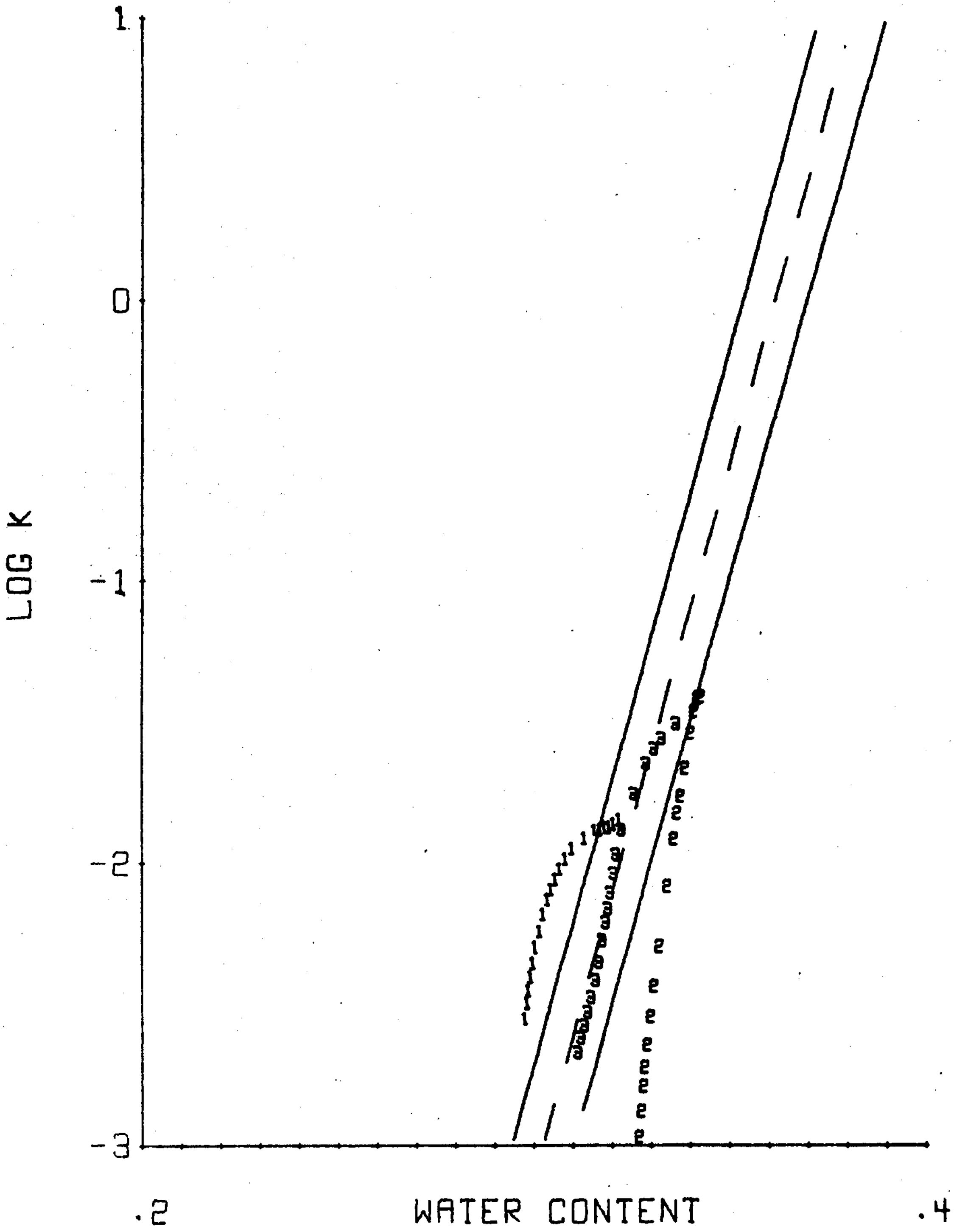

Figure 12. Logarithm of Hydraulic Conductivity Versus Soil-Water Content for an Area 0.15 km in Radius at the 37.5 cm Depth.

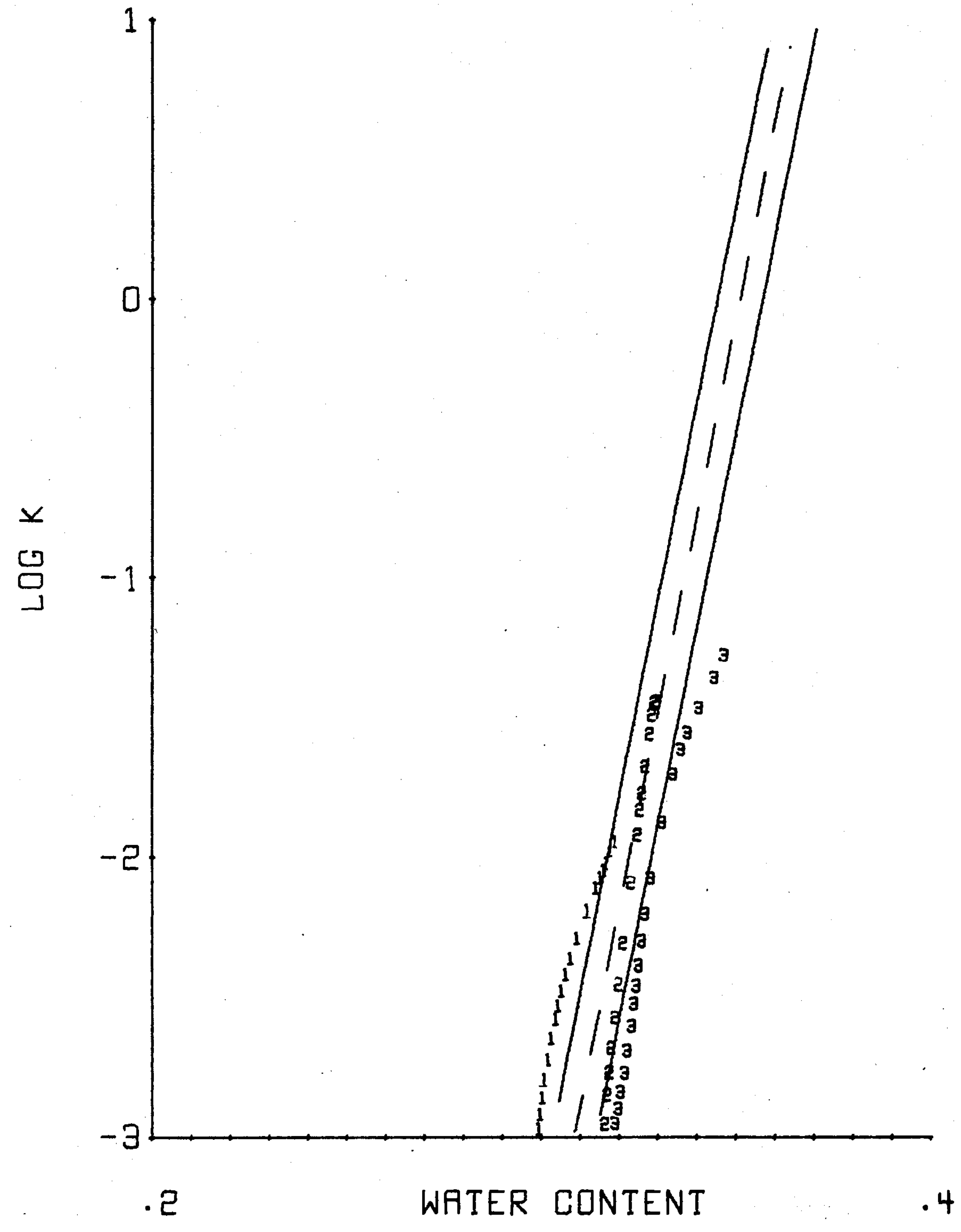

Figure 13. Logarithm of Hydraulic Conductivity Versus Soil-Water Content for an Area 0.15 km in Radius at the 52.5 cm Depth.

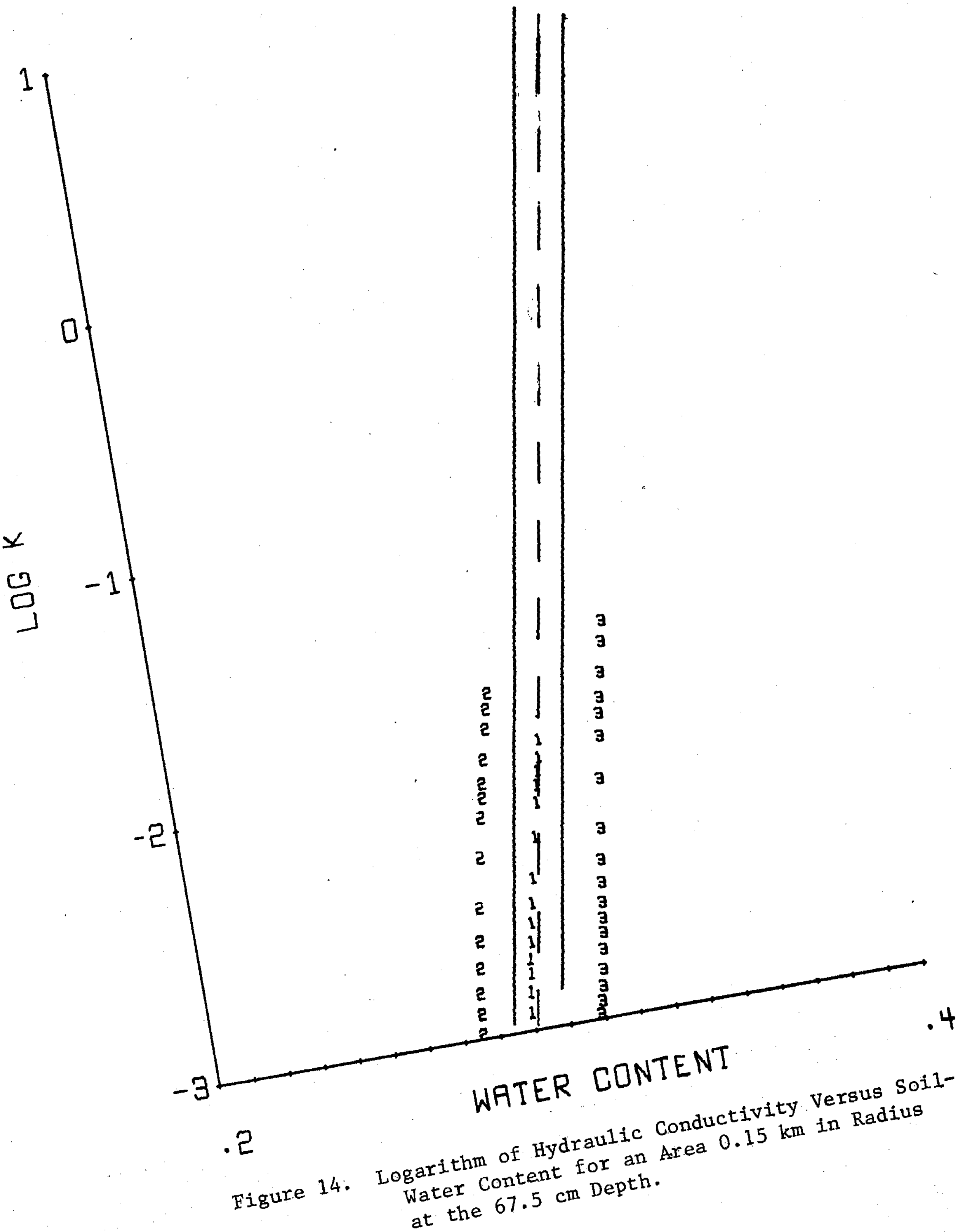

Figure 14. Logarithm of Hydraulic Conductivity Versus Soil-Water Content for an Area 0.15 km in Radius at the 67.5 cm Depth.

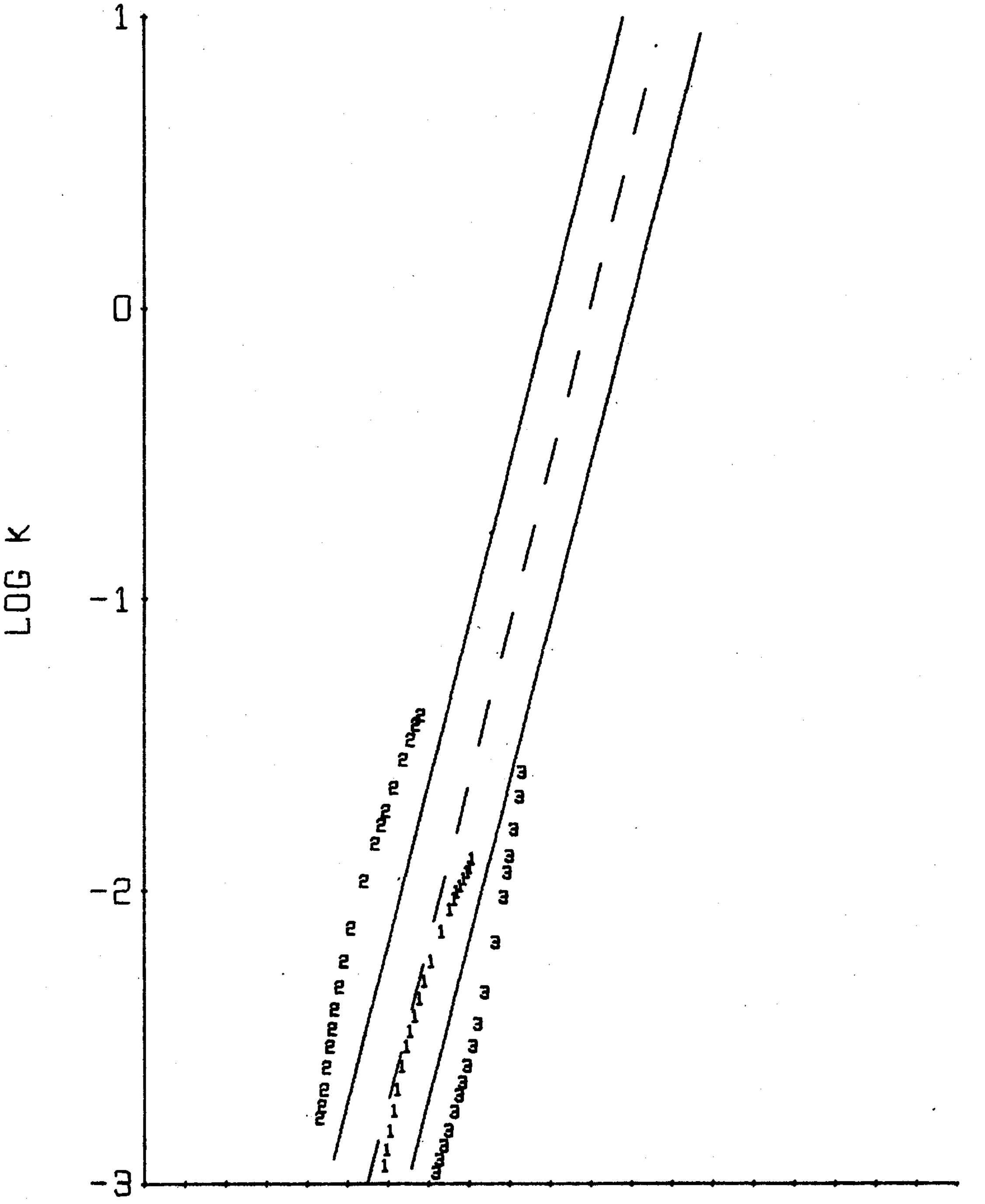

Figure 15. Logarithm of Hydraulic Conductivity Versus Soil-Water Content for an Area 0.15 km in Radius at the 82.5 cm Depth.

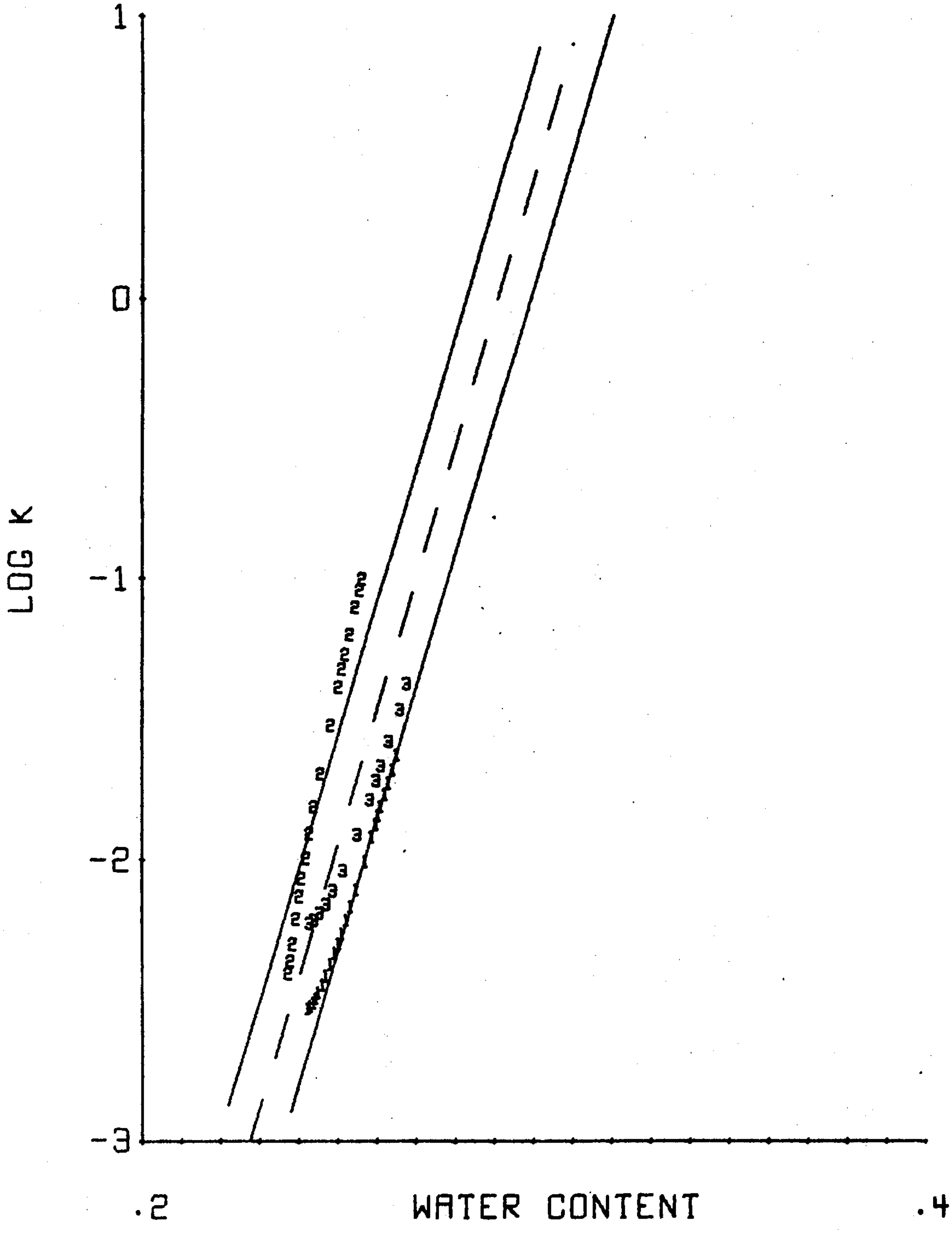

Figure 16. Logarithm of Hydraulic Conductivity Versus Soil-Water Content for an Area 0.15 km in Radius at the 105 cm Depth.

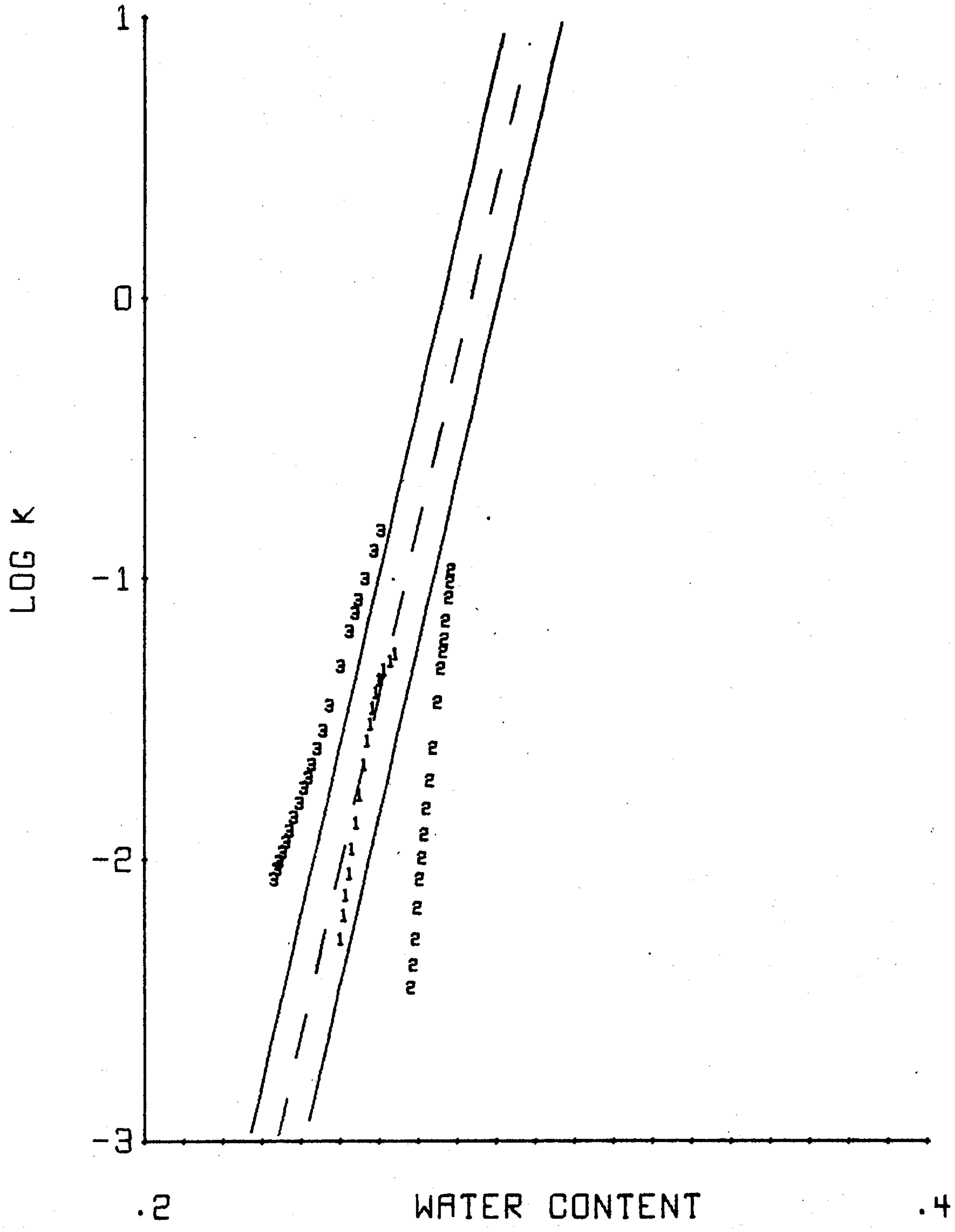

Figure 17. Logarithm of Hydraulic Conductivity Versus Soil-Water Content for an Area 0.15 km in Radius at the 135 cm Depth.

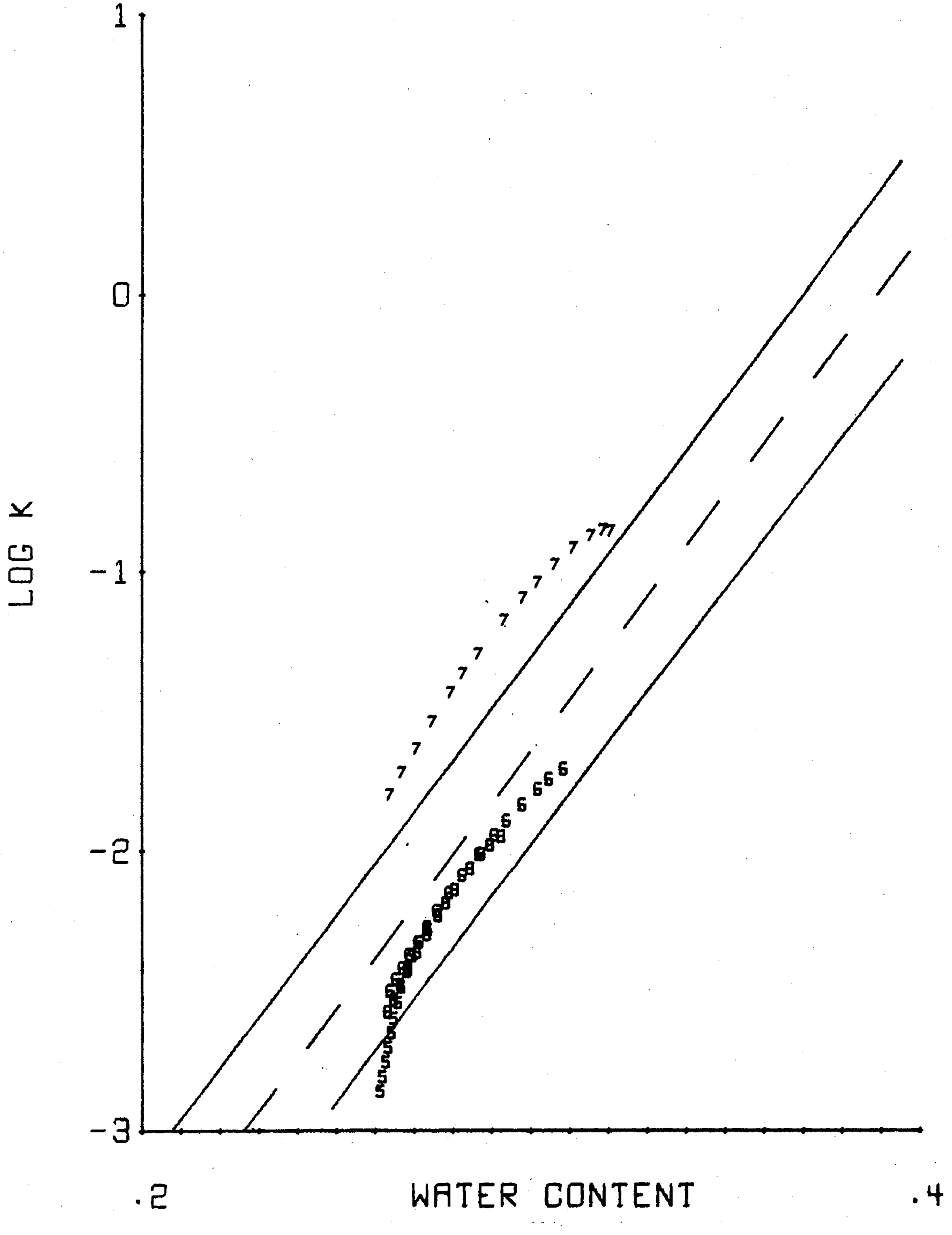

Figure 18. Logarithm of Hydraulic Conductivity Versus Soil-Water Content for an Area 5.6 km in Radius at the 22.5 cm Depth.

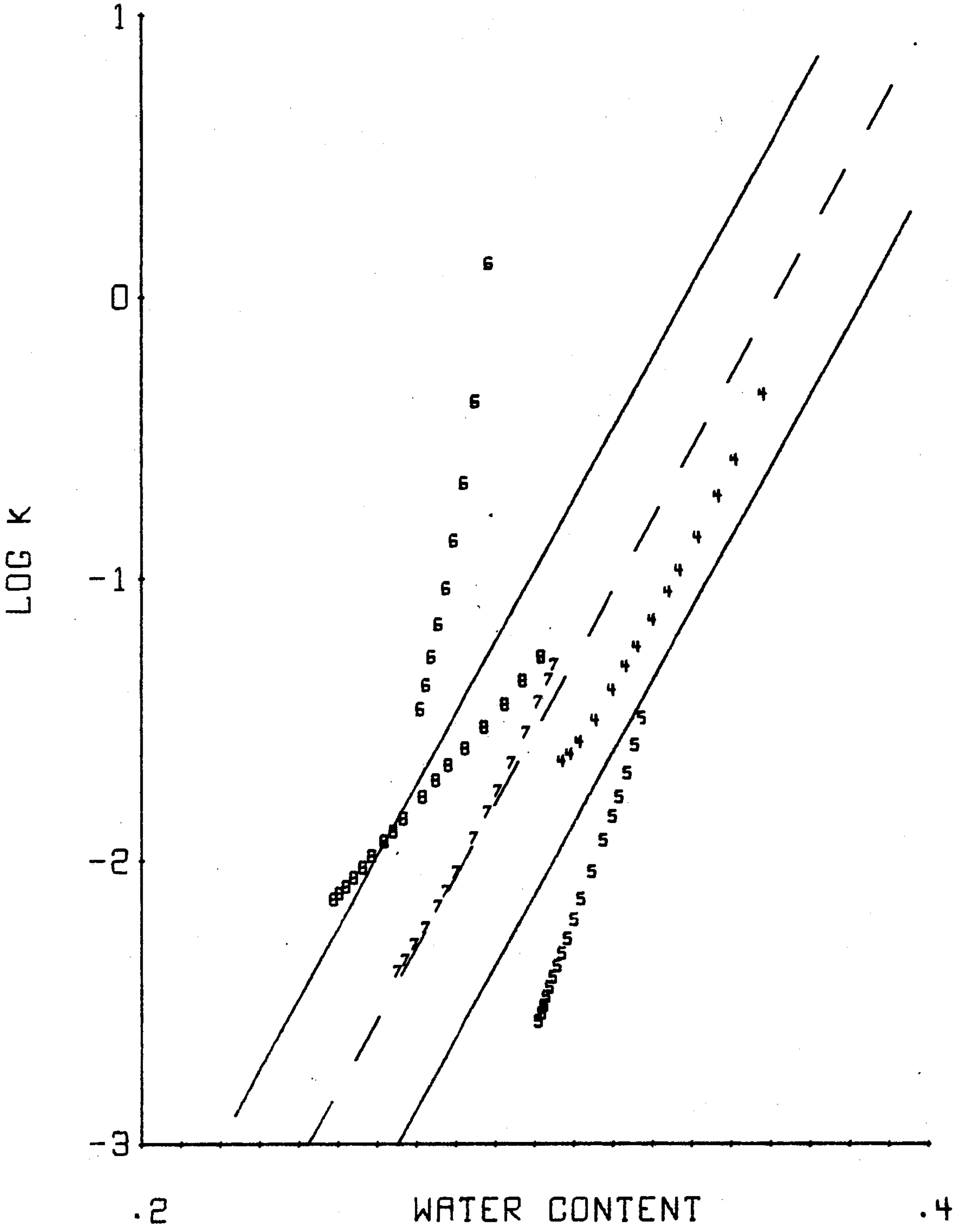

Figure 19. Logarithm of Hydraulic Conductivity Versus Soil-Water Content for an Area 5.6 km in Radius at the 37.5 cm Depth.

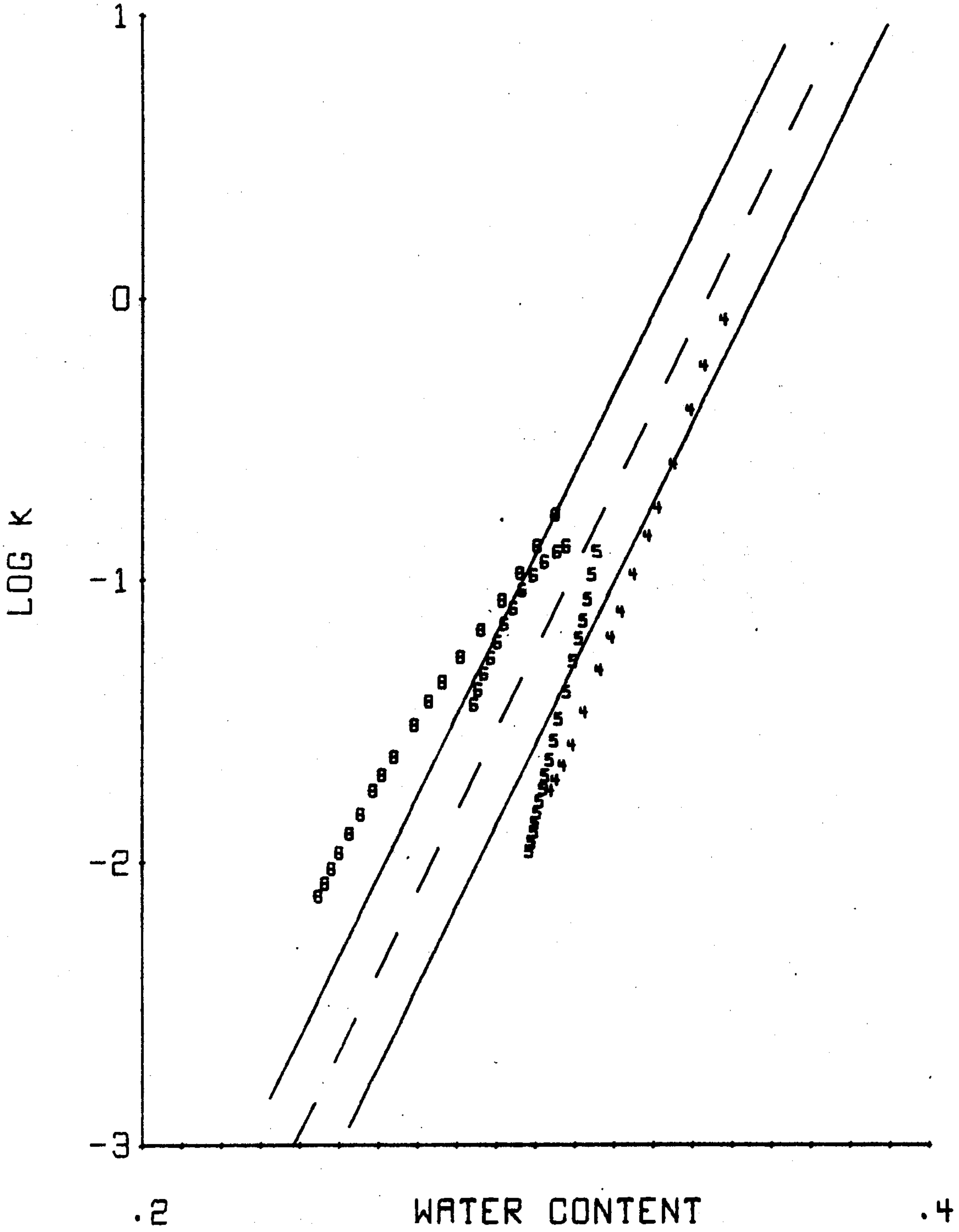

Figure 20. Logarithm of Hydraulic Conductivity Versus Soil-Water Content for an Area 5.6 km in Radius at the 52.5 cm Depth.

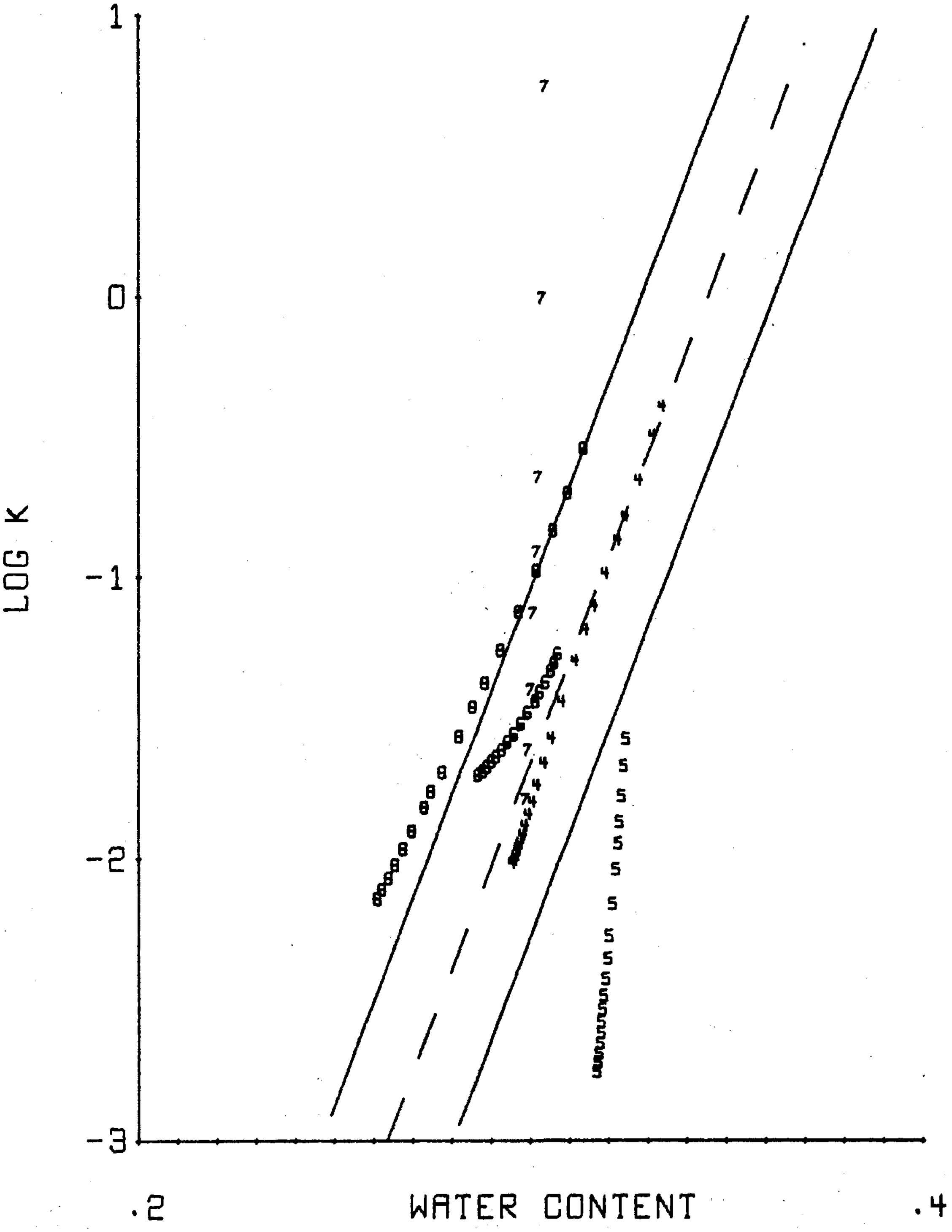

Figure 21. Logarithm of Hydraulic Conductivity Versus Soil-Water Content for an Area 5.6 km in Radius at the 67.5 cm Depth.

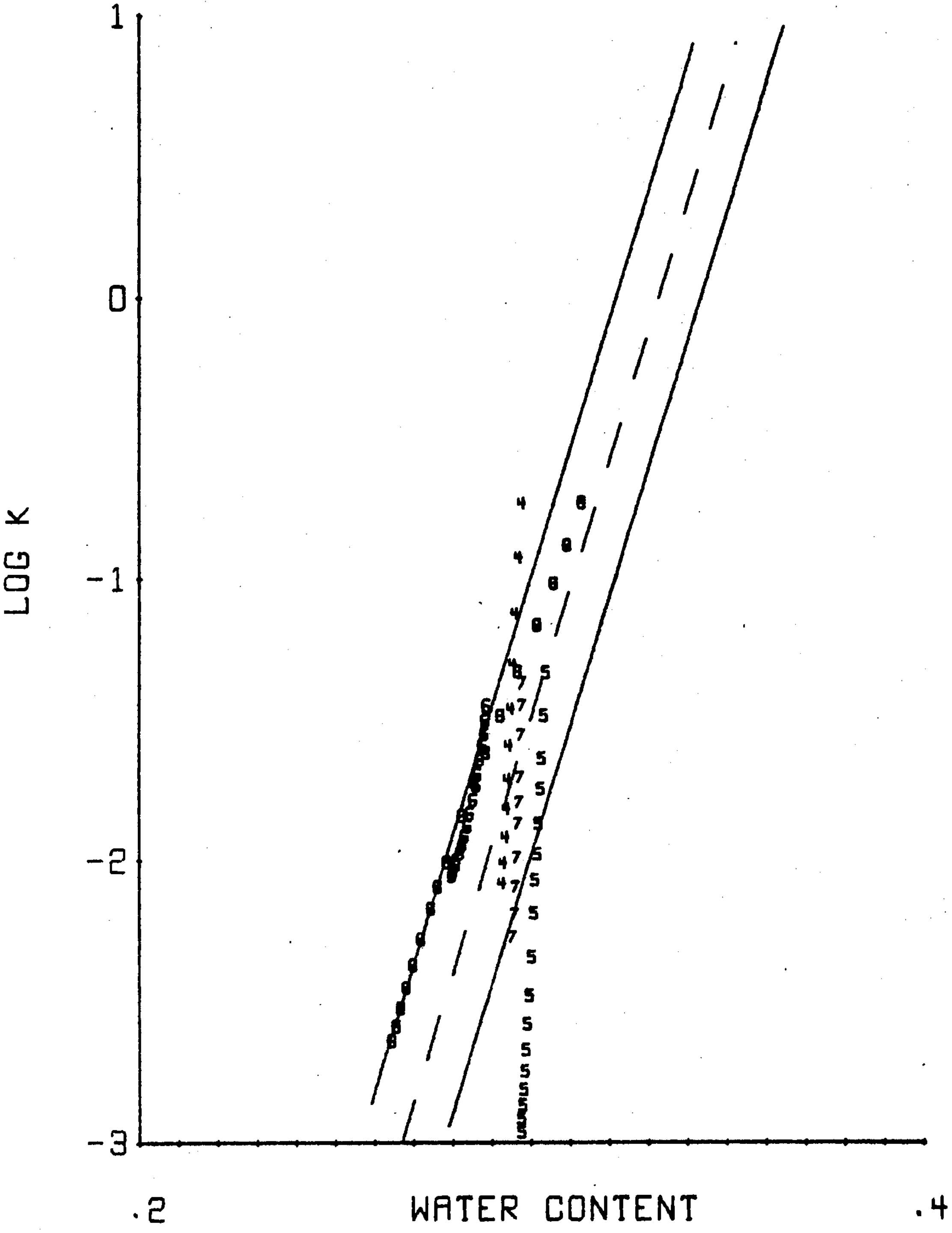

Figure 22. Logarithm of Hydraulic Conductivity versus Soil-Water Content for an Area 5.6 km in Radius at the 82.5 cm Depth.

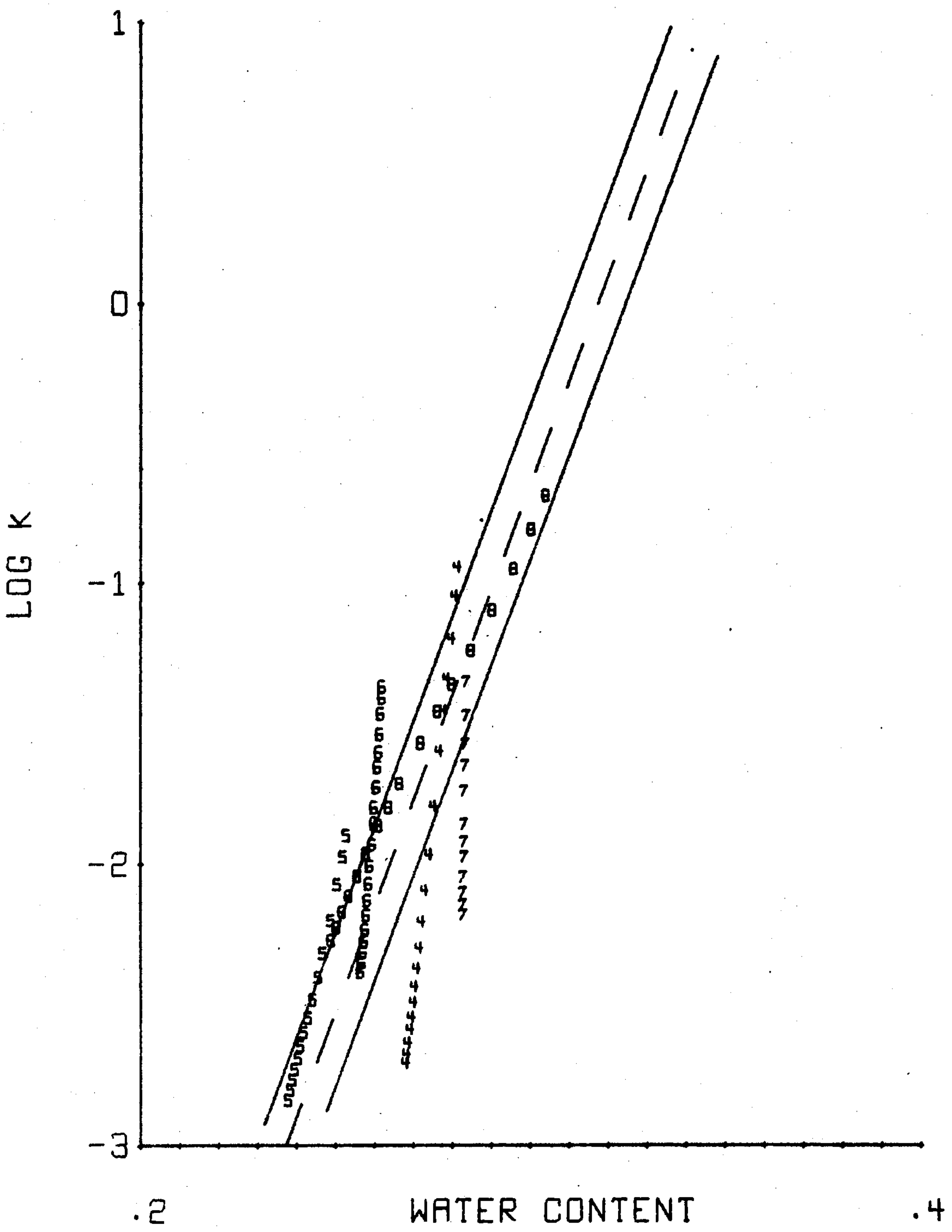

Figure 23. Logarithm of Hydraulic Conductivity Versus Soil-Water Content for an Area 5.6 km in Radius at the 105 cm Depth.

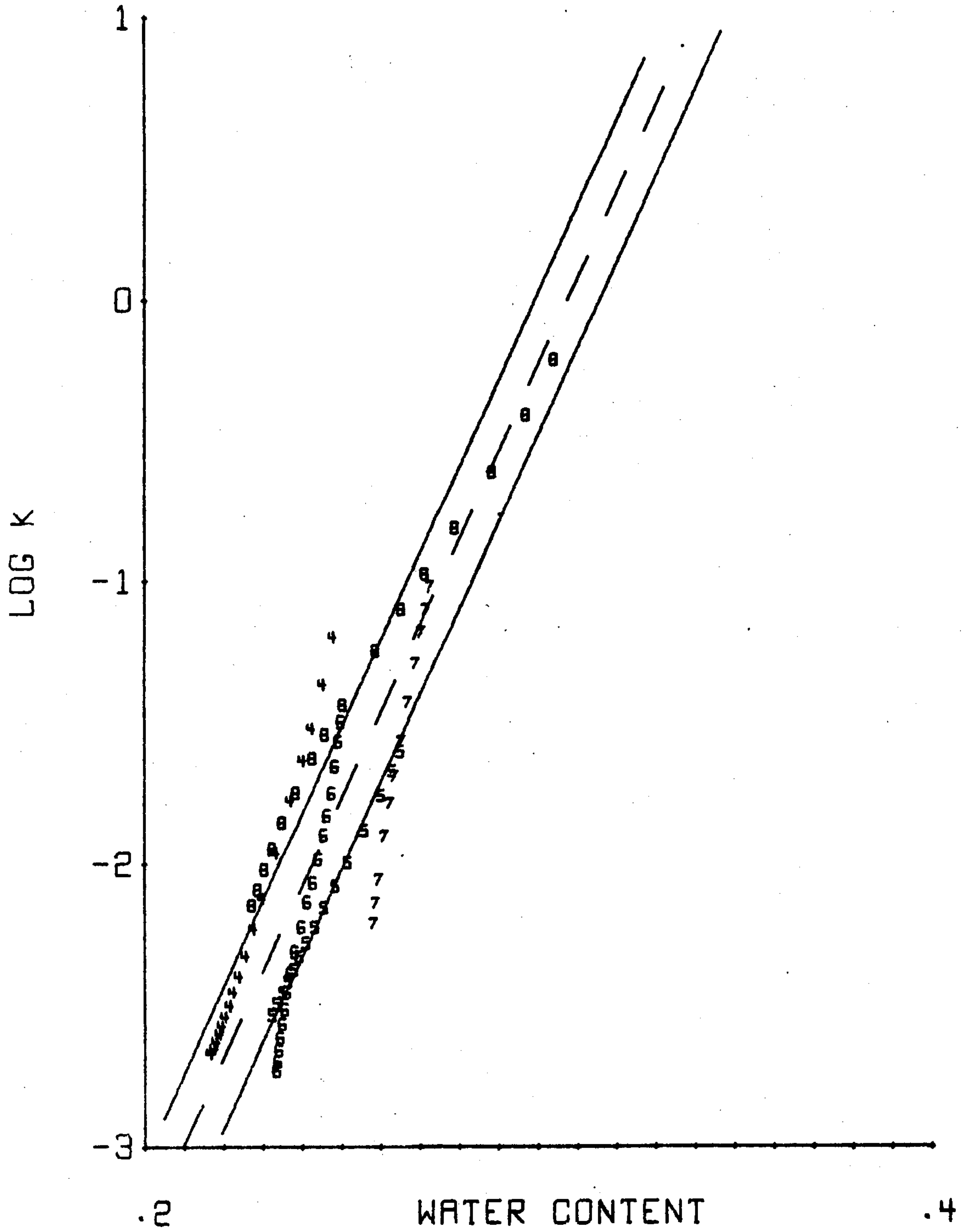

Figure 24. Logarithm of Hydraulic Conductivity Versus Soil-Water content for an Area 5.6 km in Radius at the 135 cm Depth.

Figures 25 and 26 are for the same morphological horizon. Locations 1, 2, and 3 are within 0.15 km of each other and location 5 is 7.2 km from location 1, 2 and 3. The prediction lines and SE intervals are essentially the same whether location 5 is included (Figure 25) or not (Figure 26). This result indicates that in the future, tensiometers should be near morphological horizon boundaries permitting soil-water conductivities to be calculated for morphological horizons rather than for a given depth increment. This procedure will give a more precise estimate of the soil-water conductivity versus soil-water content.

The vertical variability of the soil-water conductivity versus soil-water content is illustrated by the difference in $\log K(\theta) = \overline{LK} + \overline{B}_1 (\theta - \overline{\theta})$ (the dashed lines) in Figures 5 through 11, 12 through 17, and 18 through 24.

It should be noted that the ± SE lines in Figures 5 through 26 are for the data collected. Equation (25) should be used for the SE of new locations and therefore, would give different intervals than those shown. Evaluation of the number of new locations within a given depth by equation (25) such that the $SE|_{\theta}$ was equal to or less than 0.1 $\log K(\theta)|_{\theta}$ gave an estimate of one sample location.

Calculations involving Green and Corey's (1971) procedure gave soil-water conductivity soil-water content relations which were matched with field conductivities (Table XV) between the depths for which finite-difference conductivities were calculated. Since the water content for each finite-difference conductivity was an average of the water contents at the two depths where Green and Corey's (1971) procedure was used to evaluate the soil-water conductivity, the results for two consecutive depths will by definition bracket the finite

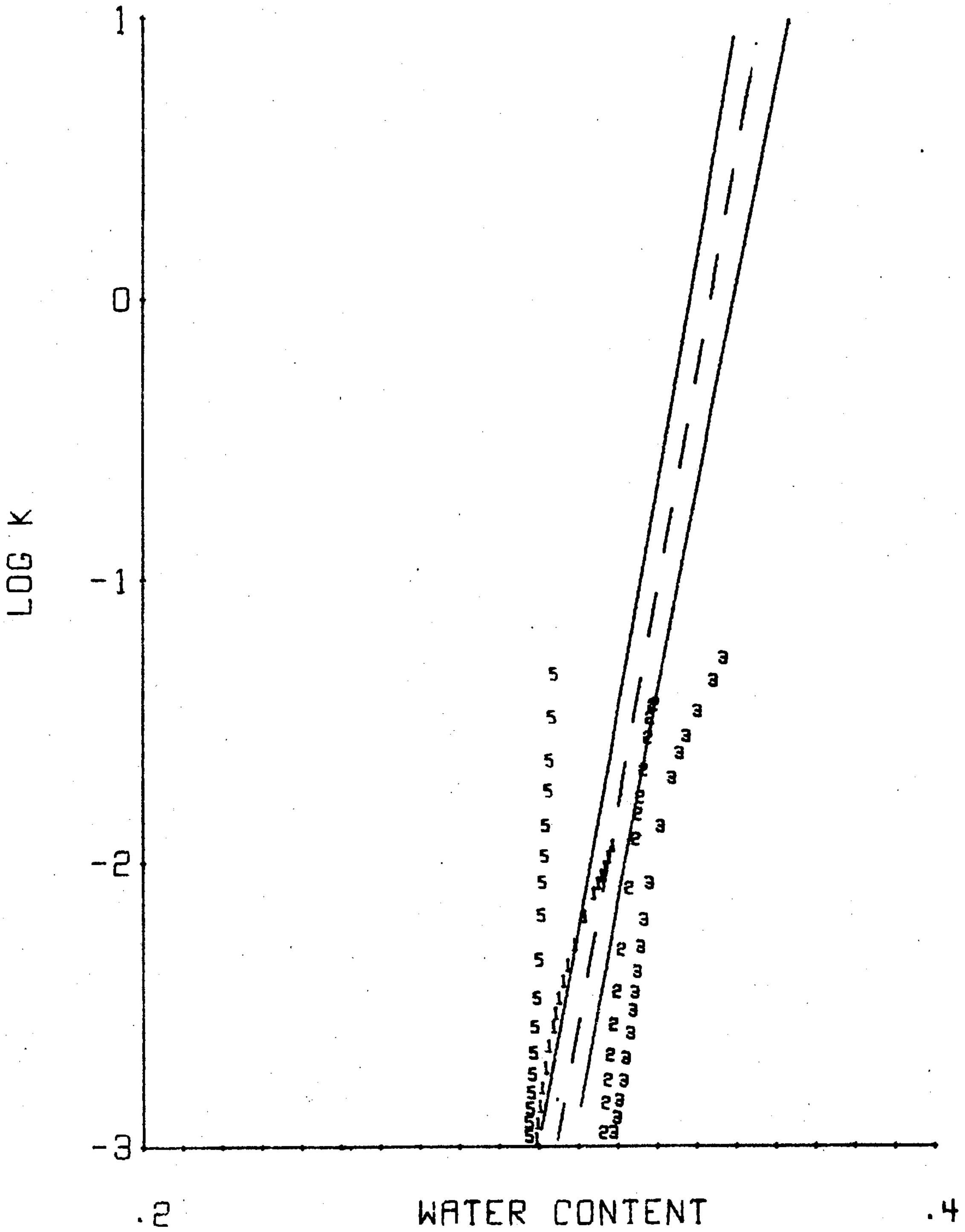

Figure 25. Logarithm of Hydraulic Conductivity Versus Soil-Water Content for the B21t Horizon for an Area 0.15 km in Radius.

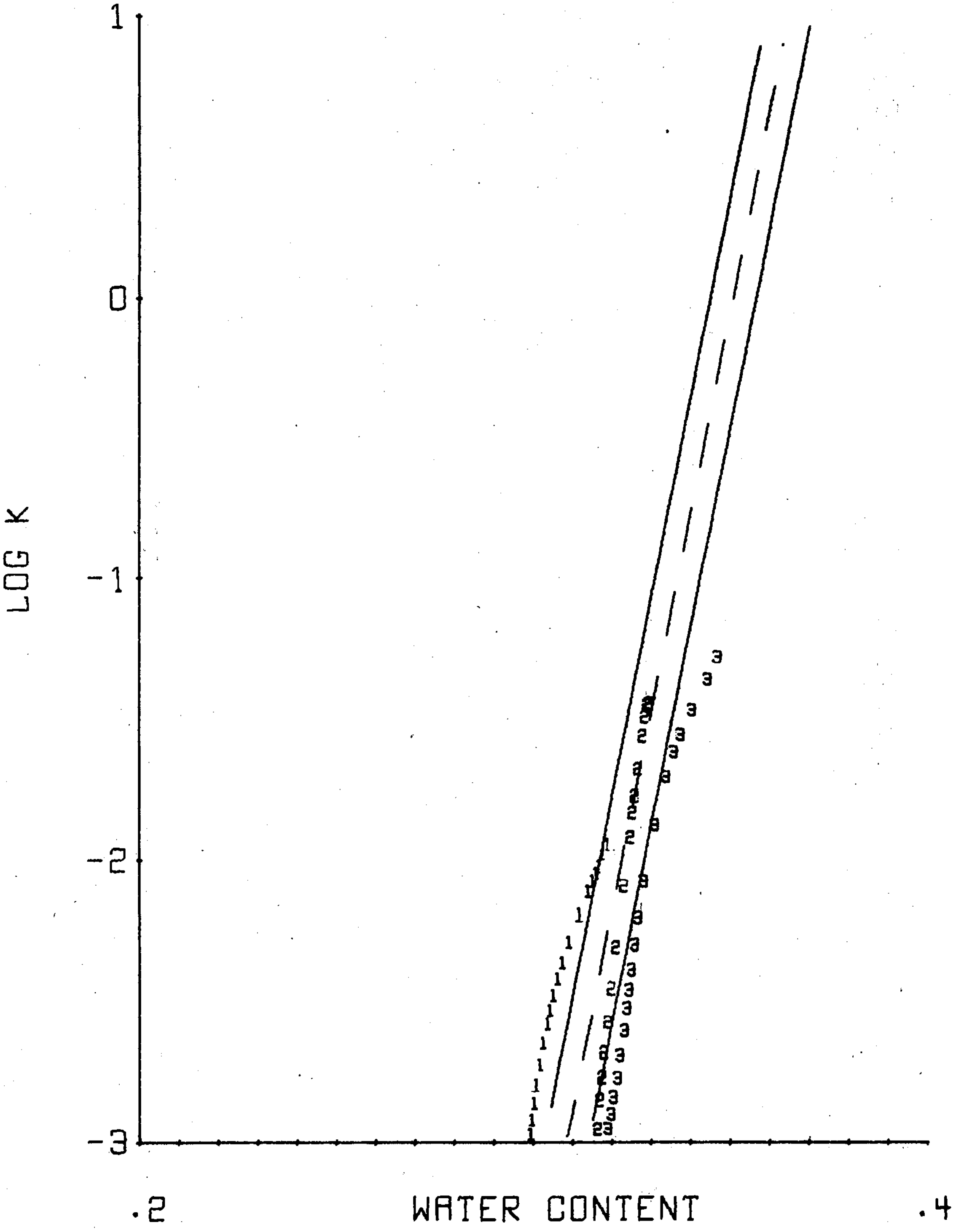

Figure 26. Logarithm of Hydraulic Conductivity Versus Soil-Water Content for the B21t Horizon for an Area 7.2 km in Radius.

TABLE XV

VALUES OF MAXIMUM HYDRAULIC CONDUCTIVITIES MATCHING FACTORS, AND SOIL WATER CONTENTS OF WHICH MAXIMUM HYDRAULIC CONDUCTIVITY OCCURRED

Location	Depth (cm)	K_s cm/hr)	K_s/K_{sc}	Water Content (cm^3/cm^3)
1	15	0.3222	0.6563	0.306
	30	0.2245	0.5231	0.324
	45	0.2117	0.4454	0.320
	60	0.2117	0.7469	0.318
	75	0.2117	1.2029	0.293
	90	0.1778	1.1859	0.265
	120	0.1976	1.7840	0.261
	150	0.2470	2.8589	0.271
2	15	0.3659	0.3326	0.317
	30	0.2550	1.0484	0.339
	45	0.2404	1.3870	0.344
	60	0.2404	1.1232	0.318
	75	0.2404	0.4176	0.278
	90	0.2657	0.8769	0.268
	120	0.2244	0.1577	0.262
	150	0.1870	0.9584	0.309
3	15	0.3039	0.4802	0.328
	30	0.2118	0.1491	0.350
	45	0.1997	0.2580	0.352
	60	0.1997	0.3405	0.346
	75	0.1997	1.8282	0.314
	90	0.1553	0.4848	0.280
	120	0.1864	0.7890	0.268
	150	0.2796	2.1130	0.264
4	15	0.7037	1.0073	0.335
	30	0.4905	0.1900	0.364
	45	0.4624	0.1374	0.352
	60	0.4624	0.2659	0.338
	75	0.4624	0.2900	0.323
	90	0.4855	4.3516	0.299
	120	0.4624	1.0427	0.262
	150	0.4316	0.2625	0.243

TABLE XV "CONTINUED"

Location	Depth (cm)	K_s (cm/hr)	K_s/K_{sc}	Water Content (cm^3/cm^3)
5	15	0.1800	0.5084	0.236
	30	0.1255	0.1569	0.328
	45	0.1183	0.1900	0.320
	60	0.1183	0.3007	0.306
	75	0.1183	1.7183	0.337
	90	0.1242	1.2490	0.293
	120	0.1274	0.3571	0.245
	150	0.1274	0.1037	0.317
6	15	0.7090	1.4977	0.309
	30	0.4941	0.1304	0.349
	45	0.4659	0.3537	0.323
	60	0.4659	0.9904	0.318
	75	0.4659	1.8362	0.296
	90	0.3913	7.7224	0.278
	120	0.4348	4.8600	0.245
	150	0.5435	2.3759	0.258
7	15	0.8394	0.1570	0.340
	30	0.5850	0.8573	0.298
	45	0.5516	0.1780	0.313
	60	0.5516	0.3100	0.314
	75	0.5516	3.1541	0.301
	90	0.4633	546.84991	0.290
	120	0.5148	14.5557	0.275
	150	0.6435	0.5972	0.267
8	15	0.618	1.0541	0.307
	30	0.430	0.3687	0.322
	45	0.406	0.1030	0.334
	60	0.406	0.2694	0.320
	75	0.406	0.1874	0.330
	90	0.426	0.4882	0.311
	120	0.437	0.3902	0.312
	150	0.437	0.1168	0.319

difference approximation for the included depth. If this is sufficient criteria for evaluating the accuracy of Green and Corey's (1971) technique, then it will be concluded directly that their method yields accurate results.

To predict a soil-water conductivity soil-water content relationship applicable to soil areas large enough for practical usage, Rogowski (1972) proposed a mathematical model based on:

1. saturated soil-water conductivity,
2. 15 bar soil-water content, and
3. soil-water content at air entry.

Clearly, these population parameters may or may not represent the unsaturated soil-water conductivities of the natural soil for which predicitions are to be made. Subsequently, invalid predictions will result since a "prediction" model should be based upon samples from the inference population. In comparison to the model of Rogowski (1972), the model used in this study was based upon a measured population. The estimates of the unsaturated soil-water conductivity are unbiased (i.e. the expected value of these estimates is the population mean), and the errors are quantized.

It should be noted that "prediction" models apply only to the population sampled. Therefore, each population of interest must be sampled to define a "valid" prediction equation.

CHAPTER V

SUMMARY AND CONCLUSIONS

Hydrologists and others who must predict soil-water status with time within watersheds are doing so at the present time based on their experiences in a given region. Therefore, the results of one region are not applicable to another without additional research. An analysis of the spatial variability of soil-water content versus soil-water conductivity was made during this study in order to estimate the precision of the calculated soil-water fluxes for a large and small land area. Also, soil-water conductivity measurements made _in situ_ were compared with laboratory procedures to evaluate their suitability. The spatial variability of textural components, bulk density, 0.1 and 15-bar soil-water content was also investigated so that the precision of these characteristics for small and large land areas could be estimated.

The following conclusions can be made based on the soil used in this study (Teller Soil Series):

1. The standard error of the logarithm of the soil-water conductivity may:
    a. be approximated closely by the component of error arising from location influences.
    b. be reduced for the soils used in this study by restricting the sampling area to 0.15 km in radius except for the soil depths of 82.5, 105, and 135 cm.

   c. be reduced by restricting sampling within morphological horizons.

2. Unsaturated soil-water conductivity calculations were made using the simplifying assumptions, (a) the change in water content with time was the same at all depths and (b) the gradient due to matric suction was zero. Results showed these calculations were biased owing to the nonhomogeniety of the soil unless the above assumptions negated each other.

3. The soil-water conductivity versus soil-water content relationships obtained using a soil-water characteristic plus a matching factor for two consecutive soil depths "bracketed" the average soil-water conductivity versus soil-water content relations obtained _in situ_ for these two depths.

4. Using the constraint that the standard error was equal to 10% of the mean for a soil physical property within a given depth, the number of estimated new locations to be sampled was conditioned by the mean of the soil physical property. In this study 1 or 2 locations were sufficient to estimate bulk density and 0.1 bar water content, 2 to 8 for % Sand, 6 to 53 for % Silt, 10 to 41 for % Clay, and 7 to 24 for 15 bar water content.

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APPENDIX A

SOIL DESCRIPTIONS OF EXPERIMENTAL LOCATIONS

Experimental Location Number 1[1]

Location: Payne County, Oklahoma; about 1 mile west and 1 mile north of Perkins; about 3/8 mile south and 80 ft. east of the north west corner of the NW ¼ Sec. 36 T. 18 N R. 2 E. Slope: 1-3%, (Colors are for moist soil unless otherwise stated).

Horizon	Depth (Inches)	Description
Ap	0-9	Dark brown (7.5YR 3/2) loam: weak fine granular structure; very friable; few fine roots; slightly acid; clear smooth boundary.
B1	9-15	Dark brown (7.5YR 3/3) light clay loam; weak medium prismatic structure parting to fine subangular blocky structure; friable; few fine roots; patchy clay films on faces of peds; neutral; gradual smooth boundary.
B21t	15-24	Dark brown (7.5YR 3/4) light clay loam; weak medium prismatic structure parting to medium subangular blocky structure; firm; few fine roots; nearly continuous clay films on faces of peds; few fine black concretions; neutral; gradual smooth boundary.
B22t	24-34	Reddish brown (5YR 4/4) sandy clay loam; moderate medium prismatic structure parting to weak subangular blocky structure; friable; few fine roots; nearly continuous clay films on faces of peds; few fine black concretions; neutral; gradual smooth boundary.
B23t	34-45	Reddish brown (5YR 4/4) light sandy clay loam; few fine distinct dark red mottles; moderate coarse prismatic structure; friable; nearly continuous clay films on faces of peds; neutral; gradual smooth boundary.

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[1] Soil descriptions through the courtesy of USDA, SCS. Soil Correlator, Jimmie W. Frie.

Horizon	Depth (Inches)	Description
B31	45-57	Coarsely mottled reddish brown (5YR 4/4), red (2.4YR 4/6) and brown (7.5YR 5/4) sticky sandy loam; weak coarse prismatic structure; very friable; patchy clay films on faces of peds; faces of some peds coated with brown sand grains; neutral; diffuse wavy boundary.
B32	57-70	Coarsely mottled reddish brown (5YR 4/4), red (2.5YR 4/6) and brown (7.5YR 5/4) sandy loam, with thin bands of sticky sandy loam; weak coarse prismatic structure; very friable; bands are firm; nearly continuous clay films on faces of peds; faces of some peds coated with brown sand grains; neutral.

Classification: Udic Argiustolls, fine-loamy, mixed, thermic

Series: *Teller

(*This is a taxadjunct to the series because of mottles in the lower B horizons.)

Experimental Location Number 2[1]

Location: Payne County, Oklahoma; about 1 mile west and 1 mile north of Perkins; about 3/8 mile south and 250 ft. east of the north west corner of the NW ¼ Sec. 36 T. 18 N R. 2 E. Slope: 1-3%, (Colors are for moist soil unless otherwise stated.)

Horizon	Depth (Inches)	Description
Ap	0-10	Very dark brown (7.5YR 2/2) loam; weak fine granular structure; very friable; many fine roots; slightly acid; clear smooth boundary.
B1	10-17	Dark brown (7.5YR 3/2) light clay loam; weak medium prismatic structure parting to moderate fine and medium subangular blocky structure; friable; few roots; few patchy clay films on faces of peds; few fine black concretions; slightly acid; gradual smooth boundary.
B21t	17-26	Dark reddish brown (5YR 3/4) clay loam; weak medium prismatic structure parting to medium subangular blocky structure; friable; few roots; nearly continuous clay films on faces of peds; few fine black concretions; slightly acid; gradual smooth boundary.
B22t	26-36	Dark reddish brown (5YR 3/4) sandy clay loam; many fine and medium distinct brown (10YR 4/3) and strong brown (7.5YR 4/6) mottles; weak medium prismatic structure parting to medium subangular blocky structure; friable; few roots; nearly continuous clay films on faces of peds; slightly acid; gradual smooth boundary.

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[1] Soil descriptions through the courtesy of USDA, SCS. Soil Correlator, Jimmie W. Frie.

Horizon	Depth (Inches)	Description
B31	36-51	Brown (10YR 4/3) sticky sandy loam; common fine through coarse distinct strong brown (7.5YR 4/6) mottles and few fine distinct reddish brown mottles; weak coarse prismatic structure; few roots; patchy clay films on faces of peds; few fine black concretions; slightly acid; clear wavy boundary.
B32	51-70	Yellowish red (5YR 4/6) sticky sandy loam; few fine through coarse distinct grayish brown (10YR 5/2) mottles and few medium faint strong brown (7.5YR 4/6) mottles; weak coarse prismatic structure; firm; nearly continuous clay films on faces of peds; slightly acid.

Classification: Udic Argiustolls, fine-loamy, mixed, thermic

Series: *Teller

(*This is a taxadjunct to the series because of mottles in the lower B horizons.)

Experimental Location Number 3[1/]

Location: Payne County, Oklahoma; about 1 mile west and 1 mile north of Perkins; about 3/4 mile south and 450 ft. east of the north west corner of the NW ¼ Sec. 36 T. 18 N R. 2 E. Slope 1-3%, (Colors are for moist soil unless otherwise stated.)

Horizon	Depth (Inches)	Description
Ap	0-9	Dark brown (7.5YR 3/2) loam; weak fine granular structure; very friable; many fine roots; slightly acid; clear smooth boundary.
B1	9-15	Dark brown (7.5YR 3/2) heavy loam; moderate fine subangular blocky structure; friable; common fine roots; neutral; gradual smooth boundary.
B21t	15-28	Dark brown (7.5YR 3/4) clay loam; weak medium prismatic structure parting to moderate medium subangular blocky structure; firm; few fine roots; few black concretions; nearly continuous clay films on faces of peds; neutral; gradual smooth boundary.
B22t	28-35	Dark brown (7.5YR 4/4) clay loam; common fine distinct yellowish red mottles; weak medium prismatic structure; friable; few fine roots; few fine and medium black concretions; nearly continuous clay films on faces of peds; neutral; gradual smooth boundary.
B31	35-57	Dark brown (7.5YR 4/4) sandy loam; common fine distinct yellowish red mottles; weak medium prismatic structure; very friable; few roots; few fine black concretions; patchy clay films on faces of peds; neutral; clear smooth boundary.

[1/] Soil descriptions through the courtesy of USDA, SCS. Soil Correlator, Jimmie W. Frie.

Horizon	Depth (Inches)	Description
B32	57-72	Dark reddish brown (5YR 3/4) sticky sandy loam; common fine distinct yellowish red mottles; weak coarse prismatic structure; firm; few fine black concretions; nearly continuous clay films on faces of peds; very slightly acid.

Classification: Udic Argiustolls, fine-loamy, mixed, thermic

Series: *Teller

(*This is a taxadjunct to the series because of mottles in the lower B horizons and brownish B2t horizons.)

Experimental Location Number 4[1/]

Location: Payne County, Oklahoma; about 10 miles west of Perkins; about 3/8 mile N and 80 ft. west of the southwest corner of the SW ¼ Sec. 4 T. 17 N R. 1 E. Slopes 0-1%, (Colors are for moist unless otherwise stated.)

Horizon	Depth (Inches)	Description
Ap	0-9	Dark brown loam, weak fine granular structure; friable; many roots, common pores; medium acid; plow boundary.
A12	9-21	Dark brown (7.5YR 3.5/2) loam; moderate coarse prismatic breaking to moderate medium granular structure; friable; common worm casts; many pores and roots; slightly acid; gradual smooth boundary.
B1	21-31	Brown (7.5YR 4/2) loam: moderate medium prismatic breaking to weak coarse subangular blocky structure; friable; thin clay films on ped surfaces in lower part; few Fe-Mn oxide concretions; many roots and pores; clay percentage gradationally increases with increasing depth; medium acid; gradual smooth boundary.
B21t	31-38	Brown (7.5YR 4/3) sandy clay loam; strong medium subangular blocky structure; firm; continuous clay films on ped surfaces; few pores; roots between peds and in pores; few Fe-Mn oxide concretions; medium acid; gradual smooth boundary.
B22t	38-45	Brown (7.5YR 4/4) sandy clay loam; moderate coarse subangular blocky structure firm grading to friable, continuous clay films on ped surfaces; common pores; roots primarilly between ped surfaces; common pores;

[1/] Soil description through the courtesy of USDA, SCS. Soil Specialist, Earl C. Nance.

Horizon	Depth (Inches)	Description
B22t (con't)	38-45	roots primarilly between ped surfaces and in pores; ped surfaces some color but slightly darker; clay percentage gradationally decreases with increasing depth; few Fe-Mn oxide concretions; few vertical columns of brown (7.5YR 5/4) fine sandy loam about 1" diameter with few clean sand grains; medium acid; gradual smooth boundary.
B3	45-55	Brown (7.5YR 4/4) fine sandy loam; moderate coarse prismatic structure; friable; thin clay films on ped surfaces; ped surface color is some but slightly darker; many pores, few roots; few strong brown (7.5YR 5/6) vertical columns about 1" diameter, of fine sandy loam with few clean sand grains; clay percentate gradationally decreases with increasing depth; slightly acid; clear smooth boundary.
C	55-75	Yellowish red (5YR 5/6) loamy fine sand; few coarse distinct strong brown (7.5YR 5/6) mottles in upper part; weak very coarse prismatic structure; very friable; few reddish yellow (5YR 6/6) vertical 1" diameter columns of loamy fine sand, with few clean sand grains; few clean sand grains in matrix; few roots in medium acid.

Pachic Angiustolls, fine-loamy mixed, thermix - would respond to management like Teller soils; suggest Taxadjunct to Teller series instead of the Milan Series (of Kansas). Milan has not been used in Oklahoma and Teller has been correlated in many counties.

Experimental Location Number 5[1]

Location: Payne County, Oklahoma; about 9 miles west and 1 mile north of Perkins; about 3/8 mile south and 80 ft. west of the northeast corner of the NE ¼ Sec. 33 T. 18 N R. 1 E. Slope: 0-1%, (Colors are for moist unless otherwise stated.)

Horizon	Depth (Inches)	Description
Ap	0-9	Dark brown (7.5YR 3/2) moist crushed loam; weak fine granular structure; very friable; pH 6.5; plow boundary.
A12	9-18	Dark brown (7.5YR 3/2) moist crushed loam; moderate medium and fine granular structure; friable; pH 6.5; gradual smooth boundary.
B1t	18-26	Brown (7.5YR 4/2) moist crushed loam; moderate coarse subangular blocky structure; firm; clay films on faces of peds; pH 6.9; clay increases gradually with increasing depth; colors are slightly darker in upper part; gradual smooth boundary.
B21t	26-36	Brown (7.5YR 4/4) moist crushed loam; moderate medium and coarse subangular blocky structure firm; continuous clay films on ped faces; pH 7.4, clean smooth boundary.
B22t s	36-51	Brown (7.5YR 4/4) moist crushed fine sandy loam; compound weak coarse prismatic structure breaking to weak coarse subangular blocky structure; friable; few coarse distinct yellowish brown (10YR 5/6) moist bodies and streaks that increase in amount with increasing depth; pH 7.0; gradual smooth boundary.
B3	51-68	Brown (7.5YR 5/4) moist sandy loam; weak coarse prismatic structure, very friable, sand grains coated; common coarse distinct yellowish brown (10YR 5/6) bodies and streaks; pH 7.0; diffuse smooth boundary.

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[1] Soil description through the courtesy of USDA, SCS. Soil Specialist Earl C. Nance.

Horizon	Depth (Inches)	Description
C	68-75	Brown (7.5YR 5/4) moist loamy sand; weak coarse prismatic structure; very friable; pH 7.0.

Tentative classification - Udic-Argiustolls fine-loamy, mixed, thermic

Series: Naron

Experimental Location Number 6[1]

Location: Payne County, Oklahoma; about 2 miles west of Perkins; about 150 feet north and 80 feet west of the southeast corner of the SE ¼ of Sec. 34 T. 18 N R. 2 E. Slope: 0-1%, (Colors are for moist unless otherwise stated.)

Horizon	Depth (Inches)	Description
Ap	0-9	Dark brown (7.5YR 3/2) average, the upper 3" is (7.5YR 3.5/2) and lower 6" is (7.5YR 3/2), loam; weak fine granular structure upper 3" and weak coarse prismatic and granular structure in lower 6"; friable; medium acid; abrupt plow boundary.
A12	9-22	Dark brown (7.5YR 3.5/2 loam; weak coarse prismatic structure; friable; common pores and earthworm casts; medium acid; gradual smooth boundary.
B1	22-34	Brown (7.5YR 4/3) loam; moderate coarse prismatic structure; friable; common pores and earthworm casts; few clean sand grains in matrix; few thin clay films on peds in lower part; clay gradationally increases with increasing depth; few fine Fe-Mn concretions; slightly acid; clear smooth boundary.
B2t	34-49	Brown (7.5YR 4/3) clay loam; moderate coarse prismatic breaking to moderate medium subangular blocky structure; firm; thin continuous clay films on ped surface; ped surface is (7.5YR 4/2), few fine and medium Fe-Mn oxide concretions; roots primarily in pores and on ped faces; few earthworm casts and pores; slightly acid; clear smooth boundary.

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[1] Soil descriptions through the courtesy of USDA, SCS. Soil Specialist, Earl C. Nance.

Horizon	Depth (Inches)	Description
B31	49-59	Brown (7.5YR 4/4) fine sandy laom, moderate coarse prismatic structure; friable, thin clay films on ped surfaces; few nearly vertical columns of light brown (7.5YR 6/4) about 3/4" diameter with some clean sand grains; few dark reddish brown (5YR 3/4) bodies that are slightly more clayey; common pores, few roots, few clean sand grains in matrix, few fine Fe-Mn oxide concretions; clay percentage gradationally decreases with increasing depth; medium acid; gradual smooth boundary.
B32	59-72	Reddish brown (5YR 4/4) fine sandy loam; weak coarse prismatic structure; hard dry, very friable moist; most sand grains coated, few clean sand grains in matrix; common nearly vertical columns of light reddish brown (5YR 6/4) fine sandy loam with few clean sand grains that are about 3/4" diameter, few medium bodies of dark reddish brown (5YR 3/4) that are slightly more clayey; many pores; few Fe-Mn oxide concretions; medium acid.

Pachic Angiustolls, fine-loamy mixed thermic - would respond to management like Teller soils; suggest that this be considered a Taxadjunct to the Teller series instead of the Milan Series - Milan (Kansas series) has not been used in Oklahoma.

Experimental Location Number 7[1]

Location: Payne County, Oklahoma; about 5½ miles west of Perkins; about 3/8 mile east and 80 feet north of the southwest corner of the SW ¼ T. 18 N R. 2 E. Slope: 1 + %, (Colors are for moist unless otherwise stated.)

Horizon	Depth (Inches)	Description
Ap	0-9	Brown (7.5YR 4/3) fine sandy loam; weak fine granular structure; friable; winnowed, lighter colored layers have been mixed by tillage and occur irregularly throughout matrix; a brown (7.5YR 5/4) winnowed layer is continuous at the 8 to 9 inch depths; slightly acid; plow boundary.
A12	9-16	Brown (7.5YR 4/3) fine sandy loam, weak fine granular structure; very friable; many pores, many roots; slightly acid; abrupt smooth boundary.
B1	16-21	Dark reddish brown (5YR 3/4) fine sandy loam; weak coarse prismatic structure; friable; many roots and pores; few earthworm casts; few bodies of reddish brown (5YR 4/4); clay percentage gradationally increases with increasing depth; slightly acid; gradual smooth boundary.
B21t	21-28	Reddish brown (5YR 4/4) fine sandy loam; weak coarse prismatic structure; friable; common pores and roots; few clean sand grains on ped surfaces; thin clay films on ped surfaces; few earthworm casts; few Fe-Mn oxide concretions; few fine vertical pores of reddish brown (5YR 5/4) fine sandy loam that has few clean sand grains; clay percentage gradationally increases with increasing depth; medium acids; gradual smooth boundary.

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[1] Soil descriptions through the courtesy of USDA, SCS. Soil Specialist Earl C. Nance.

Horizon	Depth (Inches)	Description
B22t	28-47	Reddish brown (5YR 4/4) sandy clay loam, moderate, medium prismatic breaking to moderate coarse subangular blocky structure; friable, clay films continuous on ped surfaces, few fine Fe-Mn oxide concretions; ped faces slightly darker but same color; few earthworm casts; gradual smooth boundary.
B31	47-55	Yellowish red (5YR 4/6) fine sandy loam, weak coarse prismatic breaking to weak coarse subangular blocky structure; friable; thin clay films on ped surfaces; common pores and few roots; few clean sand grains on ped surfaces; ped surfaces are same color but slightly darker; few Fe-Mn oxide concretions; few vertical columns of yellowish red (5YR 5/6) fine sandy loam that contains a few clean sand grains; clay percentage gradationally decreases with increasing depth; slightly acid; gradual smooth boundary.
B32	55-69	Yellowish red (5YR 4/6) fine sandy loam few medium distinct brown (7.5 YR 4/4) mottles; weak coarse prismatic structure; very friable; thin discontinuous clay films on ped surfaces; many pores; few clean sand grains in matrix; few columns of reddish yellow (5YR 6/6) fine sandy loam, about 1 inch diameter, with a few clean sand grains; few fine Fe-Mn oxide concretions; medium acid; abrupt smooth boundary.
C	69-75	Reddish brown (5YR 4/4) loamy fine sand; few medium faint eight reddish brown (5YR 6/4) mottles; weak very coarse prismatic structure; very friable; few fine Fe-Mn oxide concretions; few clean sand grains in matrix; medium acid.

This pedon classifies ultic Haplustalfs, fine-loamy mixed - and would Key to the Konawa Series except for the absence of an A2 horizon and the presence of a B1 horizon.

It is suggested that this soil has been Teller that has been mismanaged since the mollicepipedon is not present. It will manage like the Teller series. Therefore suggest Taxadjunct to Teller Series.

Experimental Location Number 8[1]

Location: Payne County, Oklahoma; about 4 miles west of Perkins; about 80 feet west and 80 feet north of the southeast corner of the SE¼ Sec. 32. T. 18 N R. 2 E. Slope 1 + %, (Colors are for moist unless otherwise stated.)

Horizon	Depth (Inches)	Description
Ap	0-9	Dark brown (7.5YR 3/2) moist crushed fine sandy loam weak fine granular structure; very friable; pH 6.5; plow boundary.
A12	9-19	Dark brown (7.5YR 3/2) moist crushed fine sandy loam; moderate medium and fine granular structure; pH 7.0; gradual smooth boundary.
B1	19-29	Brown (7.5YR 4/3) moist crushed fine sandy loam; compound weak coarse prismatic structure parting to coarse subangular blocky and granular structure, friable; pH 7.0; gradual smooth boundary.
B2t	29-46	Brown (7.5YR 5/5) moist crushed fine sandy loam; compound weak coarse prismatic structure parting to weak coarse subangular blocky structure; friable; thin clay films on faces of peds and coating sand grains, pH 7.8, gradual smooth boundary.
B31	46-65	Light yellowish brown (10YR 6/4) moist crushed fine sandy loam; weak coarse prismatic structure; friable; sand grains coated; many coarse, distinct, vertical to diagonal streaks and bodies of strong brown (7.5YR 5/8) streaks and bodies; pH of matrix 7.4, pH of streaks and bodies 7.2; few clean sand grains in channels in matrix; diffuse smooth boundary.

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[1] Soil descriptions through the courtesy of USDA, SCS. Soil Specialist, Earl C. Nance.

Horizon	Depth (Inches)	Description
B32	65-78	Strong brown (7.5YR 5/6) moist fine sandy loam; weak coarse prismatic structure; friable; sand grains coated; many medium and coarse distinct, vertical to diagonal bodies and streaks of light yellowish brown (10YR 6/4) moist; few bodies and streaks are dark yellowish brown (10YR 4/4) moist; pH 6.5; few clean sand grains in streaks and bodies; clay content is slightly lower in most streaks and bodies.

Tentative classification - Udic Haplustolls coarse loamy mixed thermic.

Similar to Canadian but not Canadian because of no flooding.

New Series

APPENDIX B

ESTIMATION OF WATER CONTENT FROM SOIL-WATER CHARACTERISTIC DATA

The soil-water contents for one soil core were measured at 8 increasing h (pressure head) increments. As a result, the observations of θ for progressive increments of h are serially correlated (Finney, 1964, p. 294). Additionally, the variance of θ, Var (θ), is not constant for all h. Therefore, the generally used least squares analysis procedure which assumes the errors are independent and identically distributed will be incorrect for use in this case.

To develop an analysis which will correctly describe the data, a special mathematical model is defined:

$$\theta_{ij} = \beta_o + \beta_1 h_{ij} + \beta_2 h_{ij}^2 + \ldots + \beta_{K-1} h_{ij}^{k-1} + e_{ij} \qquad (1)$$

$$1 \leq i \leq 8, \; 1 \leq j \leq 3$$

where i denotes the i^{th} h increment,

j denotes the j^{th} soil core from a given location and depth,

h_{ij} denotes the i^{th} increment of pressure head due to matric suction on the j soil core (cm),

θ_{ij} denotes the volumetric water content at the i^{th} increment of h on the j soil core (cm^3/cm^3),

β_n denotes unknown coefficients (cm^3/cm^{m+3}), and

e_{ij} denotes the random error associated with θ_{ij} (cm^3/cm^3).

In order that further analysis will be tractable, matrices[1] will be defined and used in the subsequent development. Units, where they occur, will be the same as those in equation (1) and are omitted.

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[1] Matrix multiplication, addition, and equations are defined as in Shields (1968).

Let $X^{j}_{8\cdot k}$ be an observation matrix defined as:

$$X^{j}_{8\cdot k} = \begin{bmatrix} 1 & h_{1j} & (h_{1j})^2 & \dots & (h_{1j})^{k-1} \\ 1 & h_{2j} & (h_{2j})^2 & \dots & (h_{2j})^{k-1} \\ 1 & h_{3j} & (h_{3j})^2 & \dots & \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ 1 & h_{8j} & (h_{8j})^2 & \dots & (h_{8j})^{k-1} \end{bmatrix} \qquad (2)$$

Let $X_{24\cdot k}$ be an observation matrix defined as:

$$X_{24\cdot k} = \begin{bmatrix} X^{1}_{8\cdot k} \\ X^{2}_{8\cdot k} \\ X^{3}_{8\cdot k} \end{bmatrix} \qquad (3)$$

Let $\theta_{24\cdot 1}$ be a matrix of observed water contents defined as:

$$\theta_{24\cdot 1} = \begin{bmatrix} \theta_{11} \\ \theta_{21} \\ \theta_{31} \\ \cdot \\ \cdot \\ \cdot \\ \theta_{81} \\ \theta_{12} \\ \theta_{22} \\ \theta_{32} \\ \cdot \\ \cdot \\ \cdot \\ \theta_{82} \\ \theta_{13} \\ \theta_{23} \\ \theta_{33} \\ \cdot \\ \cdot \\ \cdot \\ \theta_{83} \end{bmatrix} \qquad (4)$$

Let $\beta_{k\cdot 1}$ be a matrix of undefined parameters defined as:

$$\beta_{k\cdot 1} = \begin{bmatrix} \beta_o \\ \beta_1 \\ \beta_2 \\ \cdot \\ \cdot \\ \cdot \\ \beta_{k-1} \end{bmatrix} \qquad (5)$$

Let $\varepsilon_{24\cdot 1}$ be a matrix of random errors such that for the i^{th} row entry in $\theta_{24\cdot 1}$, θ_{ij}, the i^{th} row entry in $\varepsilon_{24\cdot 1}$, e_{ij}, is the corresponding random error of the observation.

In matrix notation the mathematical model becomes

$$\theta_{24\cdot 1} = X_{24\cdot k} \quad \beta_{k\cdot 1} + \varepsilon_{24\cdot 1} \tag{6}$$

Assume $\varepsilon_{24\cdot 1}$ is a random matrix of errors distributed as an multivariate normal with mean matrix $\phi_{24\cdot 1}$ (where ϕ[1] denotes the zero matrix (Shields, 1968 p. 113) and covariance matrix $V_{24\cdot 24}$. In matrix notation this becomes $\varepsilon_{24\cdot 1} \sim M\ V\ N\ (\phi_{24\cdot 1}, V_{24\cdot 24})$.

Since observations on the same soil core are independent of those taken on other cores, $\theta_{24\cdot 1}$ can be partitioned into three submatrices,

$$\theta_{j_{8\cdot 1}} = \begin{bmatrix} \theta_{1j} \\ \theta_{2j} \\ \theta_{3j} \\ \cdot \\ \cdot \\ \cdot \\ \theta_{8j} \end{bmatrix} \tag{7}$$

such that

$$\theta_{24\cdot 1} = \begin{bmatrix} \theta_{1_{8\cdot 1}} \\ \\ \theta_{2_{8\cdot 1}} \\ \\ \theta_{3_{8\cdot 1}} \end{bmatrix} \tag{8}$$

and θ_1, θ_2, and θ_3 are mutually independent and identically distributed.

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[1] In the sequel ϕ will denote a matrix with all elements equal to zero.

Partition $V_{24\cdot 24}$, the covariance matrix, such that

$$V_{24\cdot 24} = \left[\begin{array}{c:c:c} V_{11_{8\cdot 8}} & V_{12_{8\cdot 8}} & V_{13_{8\cdot 8}} \\ \hdashline V_{21_{8\cdot 8}} & V_{22_{8\cdot 8}} & V_{23_{8\cdot 8}} \\ \hdashline V_{31_{8\cdot 8}} & V_{32_{8\cdot 8}} & V_{33_{8\cdot 8}} \end{array}\right] \qquad (9)$$

then $V_{ij_{8\cdot 8}}$ $(i \neq j) = \phi$ and $V_{11} = V_{22} = V_{33}$.

To estimate V_{ii} consider a matrix

$$\theta_{8\cdot 3} = \begin{bmatrix} \theta_{1_{8\cdot 1}} & \theta_{2_{8\cdot 1}} & \theta_{3_{8\cdot 1}} \end{bmatrix} \qquad (10)$$

then an estimate of V_{ii} is given by

$$(\theta_{8\cdot 3} - \bar{\theta}_{8\cdot 3})\ (\theta_{8\cdot 3} - \bar{\theta}_{8\cdot 3})' \qquad (11)$$

where $(\theta_{8\cdot 3} - \bar{\theta}_{8\cdot 3})$ is a matrix of deviations from the mean,

$$\bar{\theta}_{8\cdot 3} = 1/3 \begin{bmatrix} \Sigma\theta_{1j} & \Sigma\theta_{1j} & \Sigma\theta_{1j} \\ \Sigma\theta_{2j} & \Sigma\theta_{2j} & \Sigma\theta_{2j} \\ \Sigma\theta_{3j} & \Sigma\theta_{3j} & \Sigma\theta_{3j} \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \Sigma\theta_{8j} & \Sigma\theta_{8j} & \Sigma\theta_{8j} \end{bmatrix} \qquad (12)$$

$\Sigma\theta_{1j}$ is the sum of the j^{th} row of the matrix in equation (10), and $(\theta_{8\cdot 3}-\bar{\theta}_{8\cdot 3})'$ is the transpose (Shields, 1968, p. 158) of $(\theta_{8\cdot 3} - \bar{\theta}_{8\cdot 3})$.

Since this estimate of V_{ii} has rank[1] ≤ 3, an analysis of the correlation coefficients was performed to obtain an estimate with rank $=8$. To exhibit the matrix of estimated correlation coefficients define D_{ii} to be a diagonal matrix (Shields, 1968, p. 135) such that $d_{ii} = v_{ii}$, where v_{ii} denotes a diagonal element of the matrix in equation (11). Then it follows that

$$D_{ii}^{-1} (\theta_{8\cdot 3} - \bar{\theta}_{8\cdot 3}) (\theta_{8\cdot 3} - \bar{\theta}_{8\cdot 3})' D_{ii}^{-1} = R_{ii} \tag{13}$$

where D_{ii}^{-1} denotes the inverse (Shields, 1968, p. 143) of D_{ii} and R_{ii} denotes the matrix of estimated correlation coefficients whose elements will be denoted in the sequel as r_{ij}. Since there were only three observations with which to estimate r_{ij}, the r_{ij} $(i\neq j)$, were assumed to be equal. Under this assumption to obtain the best estimate of $r_{ij}(i\neq j)$ a transformation to Fisher's z (Sendecor, Cochran, 1967, p. 135) was made;

$$z_{ij} = \tfrac{1}{2}\left[\log_e (1 + r_{ij}) - \log_e (1-r_{ij})\right] \quad (i\neq j). \tag{14}$$

As z is distributed almost normally, the average of the z_{ij}, $\bar{z}$, is the best estimate of the mean.

An inverse transformation of $\bar{z}$ gives $\bar{r}$. Redefining R_{ii} such that

$$r_{ij} = \begin{cases} \bar{r} & , i\neq j \\ 1 & , i=j \end{cases} \tag{15}$$

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[1] The rank of a matrix refers to the number of independent rows.

Premultiplying and post multiplying R_{ii} by D_{ii} gives

$$\hat{V}_{ii} = D_{ii} R_{ii} D_{ii} \tag{16}$$

where $\hat{V}_{ii}$ is the estimated covariance matrix V_{ii}.

The generalized least squares solution of equation (6) is given by:

$$B_{k \cdot 1} = \left[X'_{k \cdot 24} \quad \hat{V}^{-1}_{24 \cdot 24} \quad X_{24 \cdot k} \right]^{-1} X'_{k \cdot 24} \quad \theta_{24 \cdot 1} \tag{17}$$

(Graybill, 1961, p. 143) where $B_{k \cdot 1}$ is the estimate of $\beta_{k \cdot 1}$

The estimated θ is given by:

$$\theta|_h = X_{1 \cdot k}|_h \quad B_{k \cdot 1} \tag{18}$$

where $\theta|_h$ denotes θ evaluated at h.

Equation (18) was used to estimate the water contents from soil-water characteristics in this study.

VITA

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Candidate for the Degree of

Doctor of Philosophy

Thesis: PRECISION WITH WHICH SELECTED PHYSICAL PROPERTIES OF SIMILAR SOILS CAN BE ESTIMATED

Major Field: Soil Science

Biographical:

Personal Data: Born in Marianna, Arkansas, March 25, 1945, the son of Clymer and Eula Keisling.

Education: Graduated from T. A. Futrall High School, Marianna, Arkansas, in May, 1963; received the Bachelor of Science in Agriculture degree, with a major in Agronomy, from the University of Arkansas in January, 1967; completed the requirements for the Master of Science degree at Oklahoma State University in May, 1972, with a major in Agronomy; completed the requirements for the Doctor of Philosophy degree July, 1974.

Professional Experience: Employed by University of Kentucky, Lexington, Kentucky, September 1966 to December 1966; employed by Belleville Schools, Belleville, Arkansas, January 1967 to July 1968; employed by Plainview Schools, Plainview, Arkansas, September 1968 to May 1969; employed by University of Arkansas at Fayetteville, Arkansas, August 1969 to August 1970; NDEA Fellow at Oklahoma State University, Stillwater, Oklahoma, September 1970 to August 1973; Research Assistant at Oklahoma State University, Stillwater, Oklahoma, September 1973 to January 1974; Soil Physicist, Department of Agronomy, University of Georgia, Tifton, February 1974 to present.