AN ECONOMIC-BASED, ACTIVITY-LINKED PRODUCTIVITY MEASUREMENT MODEL

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PREFACE

This study extends the PLM model [Hansen, Mowen and Hammer, 1991] to separate technical and input-tradeoff (price) efficiency changes. In addition, it extends the PLM focus on unit-level productivity measurement to also include performance with respect to the productivity of setup and inventory management activities. Specifically, through an extended analysis that introduces the actual inputs used into the economic framework and compares them with the unobservable, optimal inputs, the PLM measure can be separated into two components: changes in technical efficiency and changes in input-tradeoff efficiency. With this supplemental information, the company can readily identify and prioritize specific opportunities for profit improvement through better adaptation to the technical efficiency of the operation as well as relative input prices. Furthermore, by incorporating concepts developed in activity-based costing (ABC) and in the economic order quantity (EOQ) model, this study has developed a conceptual framework for measuring batch level productivity. The resulting measures allow insights to be gained into aspects of batch level productivity performance which do not appear in the original PLM measure. These new insights center on the tradeoff between setup and inventory carrying costs as

well as the productivity with which setup resources are used. Furthermore, the measures provide a useful base to perform secondary analyses to provide guidance in profit improvement through better control of the technical process as well as better inventory management.

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CHAPTER I

THE RESEARCH PROBLEM

1.1 Introduction

Several factors determine long-term corporate success, but the most important one is probably the measurement of productivity. Productivity highlights improvements in the physical use of resources. For example, it improves when companies can produce the same outputs using fewer of all inputs than they would otherwise have used. Improvements of this type reduce unit costs and enhance companies'strength, viability, and profitability.

A number of procedures have been devised to measure a company's productivity. Some approaches use one or more partial indices, such as output per direct labor hour. Other approaches involve aggregate indices in which the output values of various sources, such as sales from several products, are combined into a single composite output indicator and simultaneously several input elements, such as materials, labor, energy, and capital, are combined into a single input indicator. The ratio of the composite output indicator to the composite input indicator is used as the productivity measure.

Although much has been written regarding productivity measurement, very few companies have designed a management information system that permits the measurement of productivity. A recent survey of 1,000 U.S. controllers by Steedle [1988] reveals that many companies do not measure productivity. In many of the respondents'companies, productivity is measured but in a limited way; most use a handful of unsophisticated measures at best. Interestingly, Steedle [1988] also found that some of the more sophisticated applications of productivity analysis reported in the literature are not common in practice.

Armitage and Atkinson [1990] summarize a great deal of the diverse literature on productivity and complement this review with a survey and seven site visits to Canadian companies. They conclude that aggregate productivity indices are rarely used while partial operating-based measures are widely used at the shop-floor level. Aggregate productivity indices were rejected by the practitioners as being misdirected, irrelevant, or too complex. The findings of this survey also indicate that operational productivity measures were not used by middle- and higher-level managers. At these levels, financial measures tended to be substituted for productivity measures.

Nanni, Dixon and Vollman [1990] gathered survey data from 150 managers at four plants of Northern Telecom, Inc. The results indicate that financial measures are perceived to be more important at the strategic level of the company than at lower levels.

Despite the failure of the aggregate productivity indices to gain acceptance and the tendency of middle- and upper-level managers to use financial numbers, companies find a need to develop productivity measures to supplement the information being reported by their internal accounting system. As Kaplan [1984] indicates, in the short run, profits can increase if output prices can be raised faster than input costs are rising; in the long run, however, only through productivity gains does the company have the chance for survival. Companies attempting to maintain profits through price recovery are unlikely to become or long remain world class competitors. Because profitability measures can obscure changes in overall productivity and, therefore, affect a company's ability to survive, productivity measurement and reporting is necessary.

1.2 Principal Issues

To benefit from productivity improvements, management need measurement methods for monitoring productivity performance and identifying improvement opportunities. These methods must yield accurate results and clearly indicate the impact of productivity changes on profits so that productivity improvement opportunities could be ranked according to their dollar impact on bottom-line profits.

In recent years, efforts have been made to develop productivity measures that relate productivity changes to changes in profitability of the company. Of the several profit-linked productivity measurement models, three have gained some recognition in the literature. These three models are the APC model developed by the American Productivity Center, the PPP model developed by Miller [1984], and the BDK model developed by Banker, Datar and Kaplan [1989].[1]

As aptly noted by Hansen, Mowen and Hammer [1991], one of the major problems of the existing profit-linked productivity measures is the absence of any theoretical underpinning justifying the productivity measurement models being used. By appealing to the economic theory of production, Hansen et al. [1991] show that the APC, PPP, and BDK models fail to accurately measure the productivity contribution to profitability.[2] They develop a profit-linked productivity measurement model (the PLM model) that is founded on the economic theory of production. The PLM model is basically a modification of the APC, PPP, and BDK

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[1]A detailed description of the APC model can be found in Belcher [1984]. The APC and PPP models have been applied in practice. The APC model has been used by at least 50 companies [Hansen et al., 1991]; the PPP model was developed and applied by Miller at the Ethyl Corporation and has been used by other manufacturing companies [Miller, 1984].

[2]Hansen et al. [1991] attribute this failure to the use of base-period prices. They note that using base-period prices not only improperly values changes in input efficiency attributable to input-tradeoff efficiency, but that the impropriety also extends to technical efficiency improvements because base-period prices do not reflect the opportunity cost of the improvements. They show that current input prices should be used for accurate profit-linked productivity measurement.

models. The modification increases the accuracy of profit-linked measurement and permits a connection to the operational and partial productivity measures. It also establishes an equivalency among the three models.

Unfortunately, changes in production approaches frequently involve tradeoffs among inputs. A decrease in the productivity of one input may be necessary to achieve an increase in the productivity of another input. This tradeoff is desirable only if the decline in the cost of one input is not more than offset by an increase in the cost of another input. Therefore, productivity improvement could also be achieved by improved input-tradeoff (price) efficiency.[3] Although valuing tradeoffs is embedded in the PLM measure, the value of the input-tradeoff efficiency improvement is not revealed by the PLM measure. Basically, having a productivity gain of a certain dollar amount provides only limited information regarding productivity changes. The likely changes in the technical process and in relative input prices mandate a profit-linked productivity measurement model that allows the measurement of performance

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[3]In the economic literature, economic efficiency is divided into two components: (1) technical efficiency and (2) input-tradeoff (price) efficiency. A company is technically efficient if for a given input combination it is not possible to get the same output using less of one inputs and no more of any other input; otherwise some inputs would be wasted. If a company succeeds in maximizing profits, i.e., it sets the value of the marginal product of each variable input equal to its price, then a company is price efficient.

with respect to both the technical efficiency of the operation and the adaptation to relative input prices.

An additional limitation of the PLM model relates to the implicit assumption that when unit volume doubles from the base period to the current period, so does the cost of all inputs consumed by a product (constant returns to scale). Since the only activities whose performance varies directly with the quantity of product units produced are unit level activities, the PLM model assumes that all activities are performed at that level. Cooper [1990] provides empirical evidence suggesting that the consumption of inputs by non-unit level activities is unrelated to the number of units produced or to the size of a production run. For example, doubling the size of a batch does not necessarily require doubling the number of setups or part orders. This failure to capture the economic nonproportionalities inherent in production and to accurately measure productivity contributions can lead to erroneous evaluations and decisions and, therefore, suboptimal results. For example, assume that a profit-linked measure indicates that the productivity contributions have been positive since a new productivity improvement program has been in place. Suppose, however, that the productivity contribution has actually remained unchanged over time but that the profit-linked measure is inaccurately measuring the contribution. Management may decide erroneously to maintain the program. Moreover, several of the other benefits of productivity measurement, e.g., better use of resources, and rewards and bonuses based on productivity may all suffer if the productivity measure is inaccurate and misleading.

1.3 Objectives of the Study

The effects of substitution among materials, labor, and other key variable (unit level) inputs on proposed productivity measures have not been investigated nor has the measurement of non-unit level productivity been the subject of any accounting research. In addition, because accounting scholars have not explored productivity measurement in any depth nor has productivity measurement been considered part of the information that will aid managers in their decision-making and control activities, much of the considerable knowledge accounting scholars have gained on the economic theory of production, on the operation and analysis of activity-based costing (ABC) systems, and on the economic order quantity (EOQ) model has not been applied to productivity measurement procedures.

Thus, the three principal objectives of this study are:

(1) To extend the unit-based PLM model by developing two new measures of productivity which allow assessment of the change in profits attributable to technical and input-tradeoff efficiency changes.

(2) To extend the PLM focus on unit level productivity measurement to also include productivity performance with respect to batch level inputs by developing a productivity measurement in which the productivity measure is not distorted by nonproportional consumption of inputs.

(3) To demonstrate the superiority of the model developed in this study to the PLM model.

1.4 Organization of the Study

The remainder of this study is organized as follows. Chapter II reviews the literature on productivity measurement. In Chapter III, the economic theory of production is used to develop a conceptual foundation for unit level productivity measurement. In Chapter IV, using the theoretical framework from Chapter III, two theoretically economic-based measures of productivity are derived which isolate the effect of technical and input-tradeoff efficiency changes on the change in profits. Chapter V derives two profit-linked measures of productivity that are consistent with the theoretical definitions of changes in productive efficiency and yet do not rely on explicit knowledge of the production function. Chapter VI develops a conceptual framework for batch level productivity measurement by incorporating concepts from ABC and from the EOQ model. Building on the theory from Chapter VI, Chapter VII describes the development of a batch level productivity measurement. Chapter VIII evaluates and compares the PLM model with the model developed in this study. The final chapter presents the summary and conclusions of this study.

CHAPTER II

REVIEW OF THE LITERATURE

2.1 Introduction

This chapter reviews the literature on productivity measurement. The theoretical underpinnings, the practical approaches to measurement, and the importance of such measurements are discussed. The chapter concludes with a discussion of four profit-linked productivity measurement models, the APC, PPP, BDK, and PLM models, followed by an evaluation of these four models to examine their strengths and weaknesses.

2.2 The Importance of Productivity Measurement

Productivity involves producing output efficiently and is usually defined as the ratio of outputs achieved to inputs consumed. Companies committed to productivity improvement understand that the goal of enhancing the output/input ratio is achieved by producing more output with the same inputs, achieving the same output with fewer inputs, trading off more costly inputs for less costly inputs, or a combination of the three. Productivity measurement is concerned with measuring changes in productivity so that efforts to improve productivity can be evaluated.[4] Measurement can also be prospective and serve as input for strategic decision making.

In recent years, the concept of productivity measurement and productivity improvement has received much attention in accounting literature and in the literature of related business fields. Careful analysis of the textbooks and the current literature in accounting journals concerning productivity leads to the conclusion that productivity is a critical success factor in today's complex business environments ([Wait, 1980], [Mammone, 1980a, 1980b], [Deming, 1982], [Kaplan, 1983], [Goldratt and Cox, 1984], [Hayes and Wheelwright, 1984], [Means, 1984], [Johnson and Kaplan, 1987], [Howell, Brown, Soucy, and Seed, 1987], [Banker, Datar, and Kaplan, 1989], and [Armitage and Atkinson, 1990]). The measurement, reporting, and control of productivity are critical for the long-term survival of companies. Eilon, Gold, and Soesan [1976] and Aggarwal

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[4]Traditionally, management accounting has concentrated on measuring intraperiod productivity by using standards and variance analysis. In fact, however, this approach may impede productivity improvement as standards may imply more knowledge of the production function than actually exists. Achieving standard then conveys the illusion of total productive efficiency, when, in reality, significant improvement is possible. Concentrating on quality improvement is a better approach. As a company reduces the number of defective units, quality is improved. As the number of defective units decreases less materials, labor, and overhead are used to produce the good output. By reducing the number of inputs used to produce the good output, productivity is improved.

[1980, 1984] identify four separate reasons for measuring productivity. Three of those reasons apply to the question addressed in this study. For strategic purposes, companies measure productivity in order to compare their performance with that of their competitors. For tactical purposes, measuring productivity allows companies to know the relative performance of their individual divisions. For planning purposes, companies measure productivity to compare relative benefits of various inputs.

2.3 Productivity Measurement Methods

A company's productivity can be measured by many different methods. Fortunately, most of the resulting methods can be classified into one of three broad categories: partial (component or individual) productivity measures, total factor (aggregate) productivity measures, and profit-linked productivity measures.

Partial Productivity Measures

Partial productivity measures are an analysis of aggregate output to a single input. Advocates of partial measures agree that there can be no single universal productivity measure that captures the true essence of productivity and that a series of separate and distinct indices of productivity trends is needed. If output or input is measured in dollars, then we have a financial productivity measure. If both output and input are measured in physical quantities, then we have an <u>operational</u> productivity measure.

Partial measures are seen by some as a good measure of a company's short-run effectiveness, efficiency, and competitiveness. They allow managers to concentrate on the use of a particular input. The best known partial productivity measure is output per direct labor hour. The ratio takes the form of output (measured by the physical quantity or the constant dollar value of units produced) divided by direct labor hours. Advocates of the measure tout its relative simplicity in comparison to the more complex aggregate productivity measures ([Greenberg, 1973] and [Rostas, 1955]).

According to Davis [1951, p.5], output per direct labor hour is appropriate as a company productivity measure only if one of two conditions exists: either (1) "labor time expended must be so large in relation to the other resources that the total is changed appreciably only by changes in the labor item," or (2) "changes in the other resource inputs must move in the same direction and at the same rate as labor." As Gold [1979, 1980, and 1981] and Wait [1980] indicate, the first condition is often violated and there is no evidence that the second condition should hold true for the majority of companies. Therefore, productivity measures based on labor inputs alone can serve only to obscure the contributions of and interactions among the other factors of production and therefore encourage the substitution of the other, typically more important (in terms of relative costs), factors of production for direct labor. The possible existence of input tradeoffs mandates a _total_ measure of productivity for evaluating the merits of productivity decisions. Only by looking at the total productivity effect of all inputs can managers draw any conclusions regarding productivity performance.

Total Factor Productivity Measures

Total factor productivity measures consider the contributions of all inputs to the company's productivity. Proponents of total factor measures believe that no single input factor can possibly account for the changes in productivity and that an analysis of an increase or decrease in overall productivity is possible only if output is related to all associated inputs. However, they differ in their opinions as to what variables to include and how to measure them in the output/input productivity equation.

Law [1972] suggests that only labor and capital inputs are relevant to a company's productivity. Materials input is ignored because it is a purchased good and, thus, represents the productive efforts of other companies. Productivity ratios based on this approach are often called "value-added" total factor measures. Kendrick [1977] argues that the "value-added" measures are inappropriate since managers are faced with the task of conserving all cost elements, including purchased materials. He, thus, supports a productivity measure based on materials, labor, and capital inputs.

Whether one favors the inclusion or the exclusion of materials input, one is still confronted with the unpleasant task of combining a diverse set of factors into a single number representing the company's total input. One proposition [Greenberg, 1973] is that all factors of production be restated in terms of equivalent labor hours (in much the same way as cost accountants calculate equivalent units of production). The problem with this approach is that purchased materials and capital are simply not denominated in terms of labor hours [Kendrick and Creamer, 1965]. Probably the best solution to the problem of combining input factors is that suggested by Davis [1951]: to measure all quantities (outputs and inputs) in terms of dollars.

Total factor measures are thought by their proponents to be useful in developing long-run, strategic implications for an economy, industry, or a company. Although approaches to measuring total factor productivity vary, the most common measure is the ratio of output (usually the dollar value of sales or value added by the company) divided by inputs (the dollar value of materials, labor, capital, and energy). Examples of the total factor branch of the productivity literature are (1) the classical approach from economics using indices based on the ratio of weighted current production to weighted base period production [Silver, 1982], (2) the Craig and Harris [1973] productivity model using the ratio of output to a weighted index of the various input factors (materials and purchased parts, labor, capital, and other goods and services), (3) other approaches to productivity involving comparing the aggregate value of total output to a weighted index of inputs ([Davis, 1954], [Kendrick and Creamer, 1965], [Mundel, 1976], [Kendrick, 1977], [Taylor and Davis, 1977], [Bain, 1982], [Brayton, 1983], and [Kendrick, 1984]), and (4) the value-added approach to defining the output component in the productivity equation ([Law, 1972] and [Coates, 1980]).

Although total factor measures are advantageous to partial measures as an accurate determination of the overall productivity, some scholars ([Kendrick and Creamer, 1965] and [Gold, 1979, 1980, and 1981]) argue that even total factor measures fall short of providing managers with information adequate to interpret productivity changes. They insist that a systematic examination of both partial and total factor measures is necessary to fully understand the meaning of productivity changes. Gold's [1979, 1980, 1981] productivity model, which partitions the return on investment ratio into five components: output value, average costs, capacity utilization, the productive yield of fixed costs, and the allocation of total investment capital between fixed investment and working capital, is an approach to the systematic analysis of productivity changes.

Profit-Linked Productivity Measures

Evaluating the effects of productivity changes on current profits is one way to value productivity changes. Profits change from period to another period; some of that profit change is caused by productivity changes. Evaluating the amount of profit change attributable to productivity change is defined as profit-linked productivity measurement.

Profit-linked productivity measurement relates total factor productivity change (i.e., change in materials, labor, capital and energy productivity) and the effects of these changes, taken together or individually, to the corresponding change in company profitability.[5] The motivation behind the development of the profit-linked productivity measurement is reflected in the following comment by Hansen, Mowen, and Hammer [1991, pp. 2-3]:

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[5]This approach arises naturally when a line-item income statement, with its separations of revenues and expenses, is the departure point for the analysis. Expanding the analysis one step further to isolate price and quantity effects, the change in revenue can be partitioned into the change in quantity sold multiplied by the change in the unit price, and the change in costs can be partitioned into a change in the quantity of inputs used multiplied by the change in the input prices. Thus, profit changes can be explained by a series of quantity variances and price variances, which, collectively, add up to the change in profits.

> As already noted, middle and upper-level managers are accustomed to dealing with profit-oriented performance reports; consequently, they are apt to pay little attention to measures of productivity unless they can see how much a productivity change affects bottom-line profits. This interest in the effect on profits is reinforced by the observation that bonuses, promotions, and salary increases are often tied to profit performance. Agency theory, of course, usually assumes that income is the jointly observable outcome on which contracting is based. Thus, it is in the manager's self-interest to be concerned with the level of profitability.

The advantages of profit-linked measures over the more traditional approaches are numerous. Since middle- and-upper level managers are used to reacting to financial data and dealing with balance sheet relationships, profit-linked measures are more readily understandable and more likely to be used. The ability to express productivity in the dollar and cents financial language permits the profit-linked measurement procedure to become a more viable performance monitoring and decision supporting tool and to become an important part of the overall management process of the company. Profit-linked measures allow companies to evaluate company profit plans to determine whether the productivity changes implied by their plans are overly ambitious, reasonable, or not sufficiently ambitious. They also allow companies to measure the extent to which the company's productivity performance is strengthening or weakening its overall competitive position relative to its competitors.

Several alternative models have been advocated in the literature for implementing this profit-linked approach, including the models developed by Kendrick [1961], Kendrick and Creamer [1965], Craig and Harris [1973], Mammone [1980a and b], Sumanth and Hassan [1980], Hammer, Hansen and Mowen [1981], Loggerenberg and Cucchiaro [1981-82], Ruch [1981], Chaudry [1982], Belcher [1984], Miller [1984, 1987], Sink [1984], Banker, Datar and Kaplan [1989] and Hansen, Mowen and Hammer [1991]. Three profit-linked measurement models have gained some prominence: The total factor model developed by the American Productivity Center (the APC model), the "Profitability = Productivity + Price Recovery" model developed by Miller [1984] (the PPP model), and the variance analysis model developed by Banker et al. [1989] (the BDK model). While each of these models has its unique features, they all develop around the fundamental logic embedded in the APC's original work; in particular, a company generates profits from two sources: productivity and/or from price recovery improvement.[6]

The APC Model. The APC model is based on quantifying the period-to-period change in the following relationship: Profitability = (Productivity)*(Price Recovery). The first

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[6]Such a decomposition is appealing because it ties the analysis of profitability performance to the strategy of the company. One corporate strategy framework suggests that companies follow either a low cost or a differentiation strategy [Porter, 1985]. A company attempting to become low cost producers should look to productivity improvements for its profit growth; a company implementing a differentiation strategy could also expect to see profit increases arising from price recovery increases as customers became willing to pay more for the special features and services delivered by the company's outputs.

step in productivity measurement under the APC model is the calculation of the quantity change ratios for output and inputs. An aggregate output quantity change ratio is calculated as weighted average of the individual output quantity change ratios, with the weights being the base period's share of output value for each product. The aggregate input quantity change ratio is calculated in an exactly same fashion, where the weights are the relative cost shares for each input. The productivity ratio is calculated by dividing the aggregate output quantity change ratio by the aggregate input quantity change ratio. The APC model defines the profitability ratio as the change in output value divided by the change in input value. The price recovery ratio equals the profitability ratio divided by the productivity ratio.

Although the APC model stresses ratios and indices in its development, it backs into dollar figures as an additional step of its calculation. For example, the productivity contribution is calculated in the APC model by subtracting each input <u>quantity</u> change ratio from the aggregate output quantity change ratio and multiplying the resulting number by the cost of base-period inputs. Mathematically, this contribution can be expressed as follows:

$$\text{Productivity} = C_t[S^D_{t+1}/S_t) - (C^D_{t+1}/C_t)]$$

where:

C_t = costs in the base period

S^D_{t+1} = sales in the current period deflated to the base period

S_t = sales in the base period

C^D_{t+1} = costs in the current period deflated to the base period

Once the productivity contribution is known, the price recovery contribution is calculated by subtracting the productivity contribution from the profitability contribution. The profitability contribution can be found by subtracting each input value change ratio from the aggregate output change ratio and multiplying the resulting number by the cost of base-period inputs. The formula for calculating the profitability contribution can be expressed as follows:

$$\text{Profitability} = C_t[(S_{t+1}/S_t) - (C_{t+1}/C_t)]$$

where:

C_t = costs in the base period

S_{t+1} = sales in the current period

S_t = sales in the base period

C_{t+1} = costs in the current period

The PPP Model. Unlike the APC model, the PPP model starts with and maintains dollar figures throughout its derivation of the final productivity outcomes. The PPP model is an additive relation and can be expressed as:

Profitability = Productivity + Price Recovery. In this model, the actual gross profit realized in the current period is compared with the gross profit that would have been realized had the company's profit margin remained unchanged. If the changes in the prices of inputs and outputs are removed, then the difference between the two gross profits is the productivity contribution. For the two-consecutive-period model, removing the price changes is the same as using base-period prices when calculating the current period's gross profit. To calculate the gross profit that would have been realized, the model first calculates the base period's gross profit ratio. This ratio is then multiplied by the current-period revenues, stated in base-period prices. The formula for calculating the PPP measure can be expressed as follows (for a single-product setting):[7]

$$\text{Productivity} = S^D_{t+1}[((S^D_{t+1} - C^D_{t+1})/S^D_{t+1}) - ((S_t - C_t)/S_t)]$$

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[7]Miller and Rao [1989] show that although the mathematical rationale behind the APC and PPP models is slightly different, the calculations implied by either model are the same. This is the case if only one single product is involved and needs deflating. The same calculations also apply when the models are used to compare two consecutive periods, regardless of the number of products involved. When multiple products and more than two time periods are involved, the APC and PPP models can produce significantly different numerical results due to different deflating methods. Specifically, APC uses a period-to-period inflation method to restate current figures to a base period year whereas PPP uses a cumulative deflation method.

where:

S^D_{t+1} = sales in the current period deflated to the base period

C^D_{t+1} = costs in the current period deflated to the base period

C_t = costs in the base period

S_t = sales in the base period

The price recovery component is found by subtracting the productivity component from the profitability contribution. The formula for calculating the profitability contribution can be expressed as follows:

$$\text{Profitability} = S_{t+1}[((S_{t+1} - C_{t+1})/S_{t+1}) - ((S_t - C_t)/S_t)]$$

where:

S_{t+1} = sales in the current period

C_{t+1} = costs in the current period

S_t = sales in the base period

C_t = costs in the base period

<u>The BDK Model.</u>[8] The BDK model combines the main features of the traditional standard cost system and the APC model. In contrast to the APC and PPP models, the BDK model

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[8]Banker et al. [1989] argue that previous models, such as the APC model, rely on using actual output and inputs and a constant product mix between periods; such a procedure can result in productivity improvements when none have actually occurred. False productivity improvements can be signaled simply by changes in the mix and volume of output and not by any change in the productivity of the production process. The BDK model is designed to overcome these problems.

requires the calculation and use of accounting variances (sales activity, productivity, and price recovery). The model assumes a separable, linear production function (a special case of a linearly homogeneous production function). The BDK model requires that technological standards for material, labor, and overhead be developed for the base and current periods. Usage variances are then calculated for each period, using the base-period prices to value the quantity variances. The change in individual usage variances, adjusted for any change in input-output standards, is defined as the productivity contribution to the change in profits. The formula for calculating this contribution can be expressed as follows (productivity > 0 implies a favorable contribution):

$$\text{Productivity} = [(\underline{AQ}_t - \underline{SQ}_t)\underline{P}'_t - (\underline{AQ}_{t+1} - \underline{SQ}_{t+1})\underline{P}'_t] + [\underline{SQ}^*_{t+1} - \underline{SQ}_{t+1}]\underline{P}'_t$$

where:

$\underline{AQ}_i$ = vector of actual input quantities for period i = t, t+1

$\underline{SQ}_i$ = vector of standard input quantities for period i, i = t, t+1

$\underline{SQ}^*_{t+1}$ = vector of standard input quantities for period t+1, using base-period standards

The price recovery component can be found by calculating price variances separately for outputs and inputs using base-period actual prices and then summing. To calculate the sales activity variance component, the BDK model first calculates the contribution margins for individual products using the actual prices and the standard usage quantities from the base period. The difference in the base and current-period's actual output quantities is then evaluated at the budgeted contribution margin for each output product and added together to produce the sales activity variance.

The PLM Model. Unlike the APC, PPP, and BDK models, the PLM model uses current prices to value productivity changes. The PLM model is based on the assumption that if productivity has changed, then the inputs that would have been used in the current period, holding productivity constant, will differ from those actually used. The difference in the inputs that would have been used in the absence of any productivity change and those that were used in the current period is a physical measure of the change in productivity between the two periods. The productivity contribution is calculated by multiplying each component of the change vector by its current input price and summing. The formula for calculating the PLM measure can be expressed as follows:

$$\text{Productivity} = (\underline{m}_t - \underline{m}_{t+1})(\underline{P}'_{t+1}) \cdot q_{t+1}$$

where:

$\underline{m}_i$ = productivity vector for period i, i = t, t+1

$\underline{P}'_{t+1}$ = transpose of the current price vector

q_{t+1} = current-period output

The price recovery component can be found by subtracting the productivity contribution from the total change in profits. This component is identical in concepts to that of Miller [1984] but is defined differently. The price recovery component consists of an input factor and a revenue factor. The input factor is the change in profitability due to input price and quantity changes assuming that productivity did not change. The revenue factor is the change in revenues from one period to the next. The combination of these two factors is referred to as the price recovery component.

2.4 Evaluation of the APC, PPP, BDK, and PLM Models

Hansen et al. [1991] analyze four profit-linked productivity measurement models, the APC, PPP, BDK, and PLM models, using four desirable criteria for a profit-linked productivity model: (1) accurately measures both the direction and magnitude of a productivity change, (2) links with operational measures, (3) links with partial measures of productivity, and (4) uses existing accounting information for its calculation. They show that the PLM model satisfies all four criteria; the other three models fail to satisfy all four criteria.

First, in their original form, the APC, PPP, and BDK models make no efforts to link with operational and partial measures. The PLM model, on the other hand, requires the calculation and use of operational and partial measures, establishing a direct linkage of operational measures with financial measurement. Second, all but the PLM model fail to accurately measure the productivity contribution. Third, the APC, PPP, and PLM calculations are based on existing accounting information; additional information would need to be generated to calculate the BDK measure since a standard costing system typically only calculates unit standards for materials and labor.

While PLM's superiority is based on greater measurement accuracy, its ability to link with operational and partial measures of productivity, and the use of existing accounting data for its calculation, PLM does have disadvantages. PLM does not measure the total loss being experienced by a company attributable to input-tradeoff inefficiency nor does it use a standard or optimal mix of outputs to assess this change. If a company experiences only changes in technical efficiency, partial productivity measures convey significant information. In this special case, an increase in a partial measure always indicates a productivity improvement and a decrease always indicates a decline in productivity. In general, however, productivity changes are not always caused by changes in technical efficiency. A decline in the productivity of one input may be necessary to achieve an increase in another input; this tradeoff is desirable if the overall cost of inputs declines. Therefore, productivity improvements are also caused by favorable tradeoffs among inputs; however, this is not revealed by the PLM measure nor does the manager know the dollar impact of the tradeoffs.

An additional limitation of the PLM measure arises when the consumption of inputs by unit level activities is not strictly proportional to the consumption of inputs by non-unit level activities. In this unfortunately not uncommon situation, the PLM measure becomes systematically distorted and difficult to interpret.

2.5 Summary and Conclusion

With the increased interest in using operational measures at the shop-floor level and the emergence of activity-based costing, it is necessary to have measures that link the operational measures to company-wide profitability while identifying and valuing tradeoffs among inputs and capturing the economic nonproportionalities inherent in production. The PLM model has been shown to have certain advantages over the traditional profit-linked models. Unfortunately, changes in production approaches often involve tradeoffs among inputs. The PLM model may signal an improvement in overall productivity but the source of the improvement is not revealed. Assuming constant returns to scale will further distort the PLM measure as the consumption of non-unit level inputs is unrelated to the number of units produced or to the size of a production run. The remainder of this thesis will develop a productivity measurement model that overcomes these problems.

CHAPTER III

AN ECONOMIC APPROACH TO THE VALUATION OF UNIT LEVEL PRODUCTIVITY

3.1 Introduction

Using the economic theory of production, this chapter demonstrates the types of productive inefficiency (technical and input-tradeoff), and shows how, in principle, changes in input efficiency can be partitioned into technical and input-tradeoff efficiency.

3.2 General Exposition of Economic Theory of Production

In classical economics, the company's objective is to maximize profits. Under perfect competition, the company is a price taker in both input and output markets. As a result, it has control only over production and is interested in achieving economic efficiency, i.e., producing the optimal output for the least cost. For a given technology, a company's production function is a technical constraint which describes the maximum output obtainable from every possible input combination or process. Once the level of output is given, the production function can be represented by an isoquant specifying all input combinations which yield the given level of output.[9] The optimal input combination is the one which maximizes a company's profits.[10]

If a company is not using input combinations in the best way possible, then productivity improvement is achievable through more efficient utilization of these inputs. The two sources of productive inefficiency for a given technology are <u>technical</u> inefficiency and <u>input-tradeoff</u> (price) inefficiency. To illustrate, we will assume that the objective of the company is to minimize the cost of production, x_1 and x_2 subject to a linearly homogeneous production function:

$$q = kx_1^{\alpha}x_2^{\beta} \tag{1}$$

where:

q = output quantity

x_1 = quantity of x_1 required

x_2 = quantity of x_2 required

k, α, β = suitable exponents and coefficients defined by the technical process

This functional form is, in general, assumed to be

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[9]For our purposes, the isoquant is purely an expository device. It is used to help identify a theoretical productivity standard against which actual results can be compared and to demonstrate how, in principle, changes in input efficiency can be valued.

[10]For expository purposes, we will assume that the production function and the optimal input combination are known only after the actual input combinations are chosen, thereby allowing for the possibility of productive inefficiency.

valid although the specific values assigned to the parameters are dependent on the quality of the inputs involved. Under these conditions, production costs will be minimized for any given level of output and a specified input quality level by solving a Lagrangian equation of the form:

Minimize $C = p_1x_1 + p_2x_2$

subject to $q = kx_1^{\alpha}x_2^{\beta}$ (2)

and, therefore, in Lagrangian form:

$C(x_1, x_2, l) = p_1x_1 + p_2x_2 - l(kx_1^{\alpha}x_2^{\beta} - q)$ (3)

where:

p_1, p_2 = prices of x_1 and x_2, respectively

3.3 Productivity Variance Analysis

Farrell [1957] develops concepts on the productivity measurement that have some direct application to this situation. Using his approach, the solution of this Lagrangian problem is the point O in Figure 1 when the following values are arbitrarily chosen for the unknown parameters:

$\alpha = \beta = \frac{1}{2}$; $k = 2$; $p_1 = \$1$; $p_2 = \$4$ and $q = 500$ units

Given these assumed parameter values, the algebraic solution is given in Table I.

In Figure I, units of input, x_1, are measured along the horizontal axis, and units of input, x_2, are measured along the vertical axis. The isoquant, IJ, corresponds to the actual level of output produced (500 units). Equation (8)

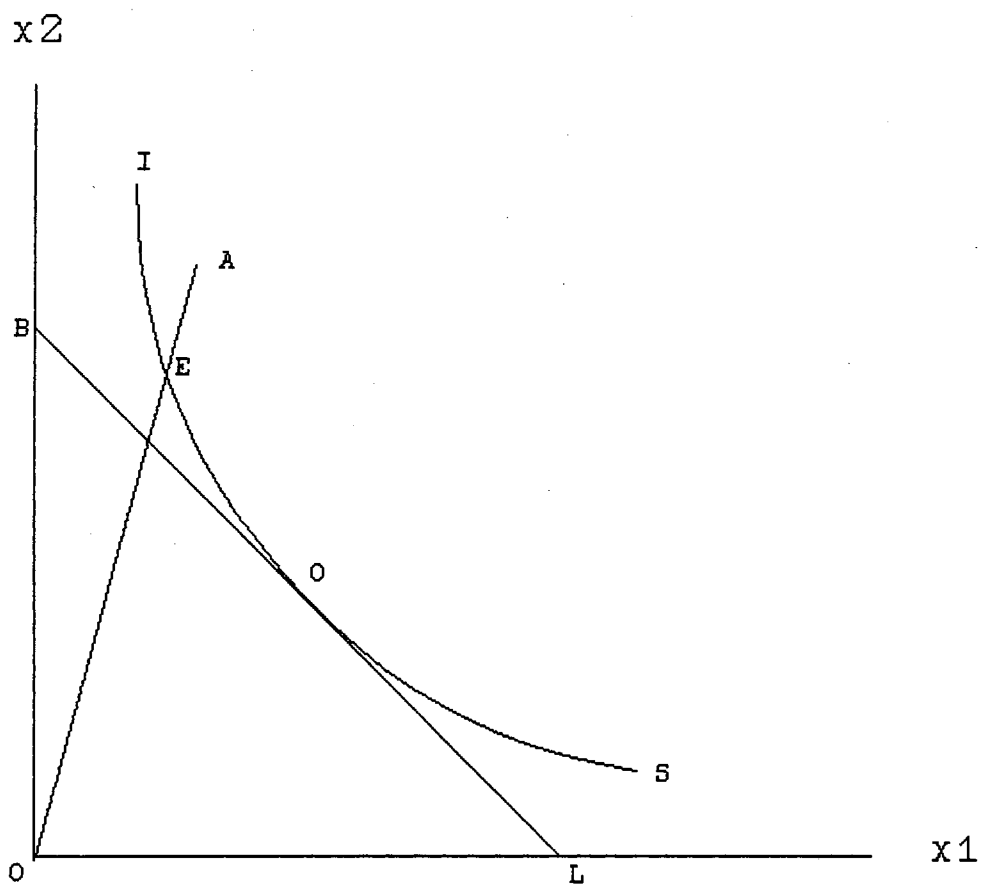

Figure 1. Technical and Input-Tradeoff Efficiency Illustrated

TABLE I

ILLUSTRATION OF OPTIMAL INPUT CALCULATIONS

The solution of the cost minimization problem:

Minimize $x_1 + 4x_2$

subject to $2x_1^{\frac{1}{2}}.x_2^{\frac{1}{2}} = 500$

requires the use of Lagrangian multipliers. Thus, the Lagrangian form:

$C(x_1, x_2, l) = x_1 + 4x_2 - l(2x_1^{\frac{1}{2}}.x_2^{\frac{1}{2}} - 500)$

Since the production function is convex to the origin, the global minimum is derived by setting all the first order partial derivatives to zero:

$$dc/dx_1 = 1 - lx_2^{\frac{1}{2}}.x_1^{-\frac{1}{2}} = 0 \quad (4)$$

$$dc/dx_2 = 4 - lx_1^{\frac{1}{2}}.x_2^{-\frac{1}{2}} = 0 \quad (5)$$

$$dc/dl = 2x_1^{\frac{1}{2}}.x_2^{\frac{1}{2}} - 500 = 0 \quad (6)$$

From (4)

$$l = x_1^{\frac{1}{2}}.x_2^{-\frac{1}{2}} \quad (7)$$

Substitute for l in (5)

$$4 - (x_1^{\frac{1}{2}}.x_2^{-\frac{1}{2}})(x_1^{\frac{1}{2}}.x_2^{-\frac{1}{2}}) = 0$$

or:

$$x_1 = 4x_2 \quad (8)$$

By substituting for x_1 in (6)

$$2(4x_2)^{\frac{1}{2}}.x_2^{\frac{1}{2}} - 500 = 0 \quad (9)$$

or

$$x_2 = 125, \; x_1 = 500$$

---

suggests that the point of optimality on this isoquant is given by the point of tangency between the output isoquant and budget line (BL). It is also easy to show that for a given level of output, the optimal input combination is the point at which the rate of technical substitution equals the ratio of the two input prices: $f_1/f_2 = p_1/p_2$, where f_i is the partial derivative with respect to input i, and p is the price of input i [Henderson and Quandt, 1980].

Under actual manufacturing conditions, deviations from this optimal point can arise in two ways: (1) through abnormal waste, the actual usage of inputs in quantities greater than that required by the production function (technical inefficiency) and (2) the incorrect choice of inputs given the relative prices of the inputs (input-tradeoff or price inefficiency). Technical inefficiency implies that the same output could have been produced with less inputs. It can occur for any number of reasons; for example, deficient managerial ability, poor training programs, redeployment of labor, etc. Input-tradeoff inefficiency means that the least-costly input combination on the isoquant could have been chosen. It can arise because of satisficing behavior, over-or under-valuation of the opportunity costs of the company, etc.

Assume further that the actual quantities of x_1 and x_2 used to produce the 500 units of output were 400 and 256, respectively (represented by the point A). The difference in costs between the point representing the actual quantities of the inputs used (point A) and the optimal point (point O) can be explained by:

(1) Technical Efficiency

= Costs at A less costs at E

(2) Input-Tradeoff Efficiency

= Costs at E less costs at O

Next assume that point E in Figure 1 represents the point on the output isoquant at which inputs in the same mix as that actually used would have been used had no technical inefficiency occurred. Graphically, it is the point at which a ray from the origin to the actual input combination (point A) intersects the isoquant for the actual output quantity. In this example, the slope of the ray is (x_2/x_1). As a result, values of x_2 located on the ray may be defined in terms of x_1 as:

$$x_2 = (256/400)x_1 = 0.64x_1 \tag{10}$$

The technically efficient input combination of inputs at the actual mix can be found by substituting into $q = 2x_1^{\frac{1}{2}}.x_2^{\frac{1}{2}}$ the actual quantity of output for q, and the definition of x_2 found in Equation (10) and solving for x_1. In this example, $x_1^e = 312.5$, $x_2^e = 200$. The calculation of variances for the hypothetical example is provided in Table II.

The variance analysis in Table II indicates that the primary cause of the deviation from the minimum cost is the technical inefficiency. The primary action needed to eliminate the productive inefficiency is a closer control of

TABLE II

ILLUSTRATION OF ECONOMIC APPROACH TO VARIANCE CALCULATION

a. Actual Costs at actual quantity used

$= \underline{P} \cdot \underline{q}^a$

where:

$\underline{P}$ = vector of input prices.

$\underline{q}^a$ = vector of actual input quantities

$= [1 \quad 4] \begin{bmatrix} 400 \\ 256 \end{bmatrix} = \$\ 1424$

b. Costs of most efficient use of inputs given actual mix

$= \underline{P} \cdot \underline{q}^e$

where:

$\underline{q}^e$ = vector of technically efficient input quantities

$= [1 \quad 4] \begin{bmatrix} 312.5 \\ 200.0 \end{bmatrix} = \$1,112.50$

c. Costs at the optimal mix given actual prices of inputs (assuming actual input prices = standard input prices(SP))

$= \underline{SP} \cdot \underline{q}^o$

where:

$\underline{q}^o$ = vector of optimal input quantities

$= [1 \quad 4] \begin{bmatrix} 500 \\ 125 \end{bmatrix} = \$1,000$

d. Technical Efficiency Variance

$= [(a)-(b)]$

$= \$\ 311.50$

TABLE II (Continued)

e.	Input-Tradeoff Efficiency Variance
	= [(b)-(c)]
	= $112.50
f.	Total Input Efficiency Variance
	= [(a)-(c)]
	= $424.00

the technical process. Minimizing scrap, waste, and rework are all ways in which managers can avoid the recurrence of the unfavorable technical efficiency variance. The input-tradeoff efficiency variance is unfavorable $112.50. If this variance is considered to be insignificant, no corrective action is needed. If it is considered to be significant, managers must also be concerned with the relative amounts used of each input. The least costly input combination on the isoquant should be chosen.

Unfortunately, productivity variance analysis, as a one-period retrospective analysis of productivity performance, can only explain differences between actual results and the productivity standard in a single period. It does not explain period-to-period changes in productivity; as a consequence, the same variances calculated in two different time periods may not be comparable since the standards in a period may be unrealistic or obsolete for the following periods. In addition, standards may imply more knowledge of the production function. Achieving standards then conveys the illusion of total productivity, when, in fact, significant improvement is possible. Pressures to achieve standard may also discourage productivity improvement. For example, a production manager might pass on a defective component so that the material usage standard can be met. However, this is not a productive act. Subsequent inspection or product failure (after the sale and delivery of the product) may mandate the use of additional company resources.

On the other hand, productivity measurement emphasizes period-to-period changes in productivity and, therefore, continual improvement over time.[11] The objective is to improve productive efficiency each period. Although productivity measurement has the advantage of generating comparable variances in successive time periods, its main problem is that it totally ignores the prespecified standards. Viewing productivity solely from the perspective of period-to-period change may conceal the source of productive efficiency or inefficiency. The company should first analyze intraperiod productivity by comparing actual

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[11]Typically, productivity measurement has been thought of as an interperiod phenomenon. This however, is not a necessary condition for productivity measurement.

results with the productivity standard. Only then can meaningful interperiod comparisons be made.

3.4 Summary and Conclusion

We introduced actual inputs used into the economic framework and compared them with the optimal inputs. This comparison permits us to demonstrate the types of productive inefficiency: technical inefficiency and input-trade-off (price) inefficiency, and to show how, in principle, changes in input inefficiency can be measured and valued. Relying on explicit knowledge of the production function, the next chapter relates technical and input-tradeoff efficiency variances developed in this chapter to the PLM model to show their complementary nature and derives two new profit-based measures of productivity that are consistent with the theoretical definitions of changes in productive efficiency.

CHAPTER IV

A MULTIPERIOD UNIT LEVEL PRODUCTIVITY MEASUREMENT SYSTEM

4.1 Introduction

Using the economic framework involving perfectly competitive input markets and knowledge of the production function, this chapter extends the PLM model to separate technical and input-tradeoff efficiency changes.

4.2 Profit-Linked Productivity Measures and Measures of Efficiency

The profit-linked productivity measurement is a different concept from the measures of efficiency (technical and input-tradeoff). However, there are some conceptual commonalities. In particular, both profit-linked productivity measures and efficiency measures are concerned with the efficiency of input usage. Profit-linked productivity measures tend to be interperiod measures and stress continual improvement over time; the idea is to motivate and evaluate attempts to improve productive efficiency each period. Efficiency measures, on the other hand, tend to be intraperiod measures and call for accomplishment of a standard; the calculation of trends in productivity is not stressed.

If the production function were known and, thus, the optimal mix of inputs, it would be possible to calculate the intraperiod level of productive inefficiency and to separate this total into its technical and input-tradeoff inefficiency components. In a subsequent section, we will show how the intraperiod measures of efficiency, comparing actual with budgeted, can be combined with an interperiod PLM model, comparing actual costs between successive periods, to provide a more systematic and comprehensive explanation of changes in profitability each period and over time. Before this can be done, the PLM model need to be defined.

PLM Model Defined

The PLM model defines a total physical productivity measure, $\underline{m} = (x_1/q, x_2/q)$, where q is the output (the elements of the productivity vector are the inverse of the average productivity of each input. The PLM model is based on the assumption that if $\underline{m}_t \neq \underline{m}_{t+1}$, then productivity has changed from period t to t+1. If productivity has changed, then the inputs that would have been used for the current period ($x^* = \underline{m}_t \cdot q_{t+1}$) in the absence of any productivity change will differ from those actually used. The difference in the cost of the inputs that would have been used in the absence of any productivity change and the cost of the inputs actually used is the amount by which profits changed because of productivity changes:

$$PLM = (\underline{m}_t - \underline{m}_{t+1})(\underline{P}'_{t+1})\cdot q_{t+1} \qquad (11)$$

where:

$\underline{P}' = (p_1, p_2)$, the input price vector (the prime on $\underline{P}$ signifies its transpose)

4.3 The Simple Model

We start by considering a single product with constant intrinsic qualities. For simplicity in exposition, we assume initially (1) two inputs, (2) two-consecutive-period setting, (3) equality of actual input prices and standard input prices, (4) equality of production and sales, (5) equality of base-period and current-period input prices, (6) equality of base-period and current period output quantity, (7) perfectly competitive input markets[12], (8) constant returns to scale, and (9) the absence of any technological progress between periods. Now refer to Figure 2.

In Figure 2, units of input, x_1, are measured along the horizontal axis, and units of x_2, are measured along the vertical axis. The isoquant, IS, corresponds to the actual

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[12]For <u>imperfect</u> input markets, the price of the input depends on the input quantity purchased. The assumption of perfectly competitive markets facilitates the valuation of the changes in input efficiency. It allows current actual input prices to be used to value input efficiency changes and has no effect on how the input usage is calculated. This assumption also appears to be operationally sound. For example, the larger the number of buyers of inputs, the less likely that any company's actions will influence the input's price - at least not enough to be concerned about the impact on valuing input efficiency.

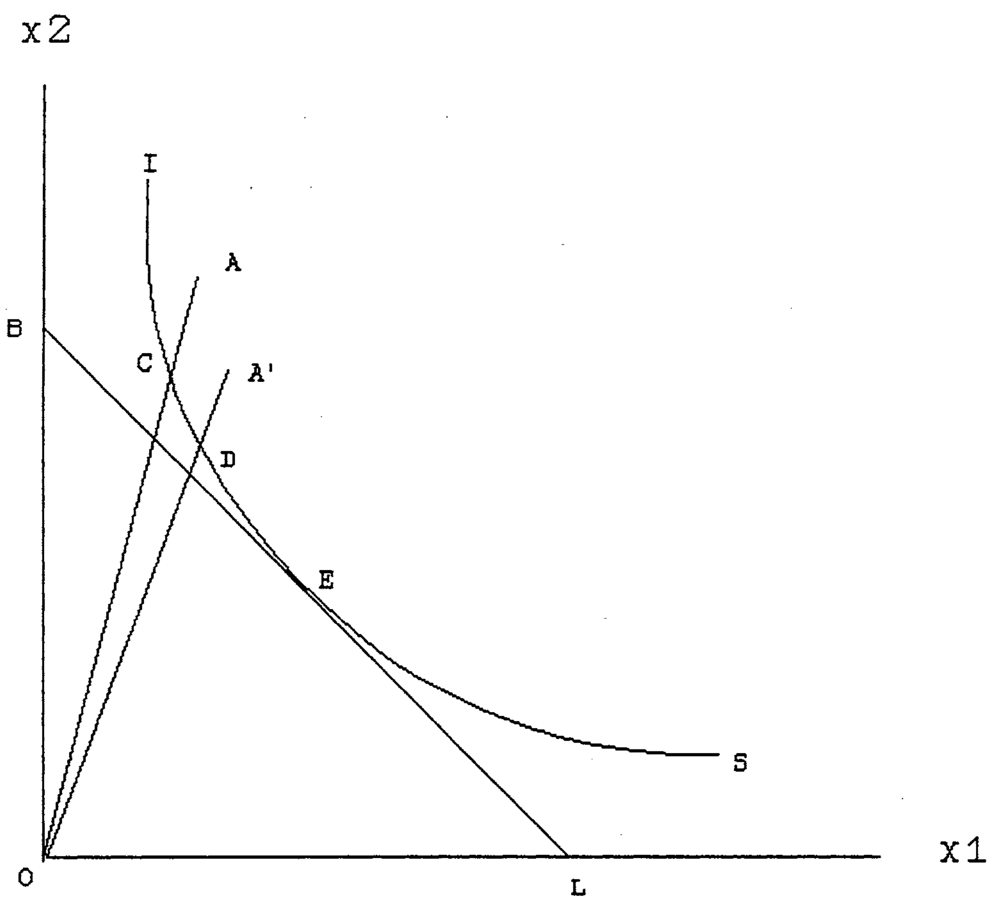

Figure 2. Technical, Input-tradeoff Inefficiencies and Valuation of Input Efficiency Changes (The Simple Model)

level output produced (same for period t and period t+1), the point A corresponds to the actual level of inputs used in period t, and the point A' corresponds to the actual level of inputs used in period t+1. It is clear that $\underline{m}_t = \underline{m}_{t+1}$, implying that productivity has changed from period t to period t+1. The change in productivity, in physical terms is equal to $\underline{A} - \underline{A}'$. Moving down the rays on which $\underline{A}$ and $\underline{A}'$ are located, we encounter $\underline{C} = (x_{1t}/q_t, x_{2t}/q_t)$, and $\underline{D} = (x^e_{1t+1}/q_{t+1}, x^e_{2t+1}/q_{t+1})$, a technically efficient combination of inputs for period t and period t+1, respectively. Note that both $\underline{C}$ and $\underline{D}$ require less of both inputs to produce the same amount of output. Therefore, productive efficiency can be improved by moving to $\underline{C}$ in period t and to $\underline{D}$ in period t+1. The difference in the physical quantities saved in period t and period t+1 is a physical measure of the change in technical efficiency between the two periods and is expressed by $(\underline{A} - \underline{C}) - (\underline{A}' - \underline{D})$.

Next let BL be the budget line for period t and period t+1. As the tangency point of the budget line reveals, point $\underline{E}$ is the optimal input combination for both periods. Staying either at point $\underline{C}$ in period t or point $\underline{D}$ in period t+1 would have been more costly. Economic efficiency can be improved by changing the input combination to $\underline{E}$. The difference in this savings is a physical measure of the change in input-tradeoff efficiency between the two periods and is measured by ($\underline{C}$ - $\underline{E}$) - ($\underline{D}$ - $\underline{E}$). With the choice of the current price vector, the productivity contribution to the change in profits (PLM) can be expressed as follows:

$$\text{PLM} = \{[(\underline{m}_t - \underline{m}_t^e) - (\underline{m}_{t+1} - \underline{m}_{t+1}^e)] + [(\underline{m}_t^e - \underline{m}_t^o) - (\underline{m}_{t+1}^e - \underline{m}_{t+1}^o)]\}(\underline{P}'_{t+1})\cdot q_{t+1} \quad (12)$$

where:

$\underline{m}_i^e$ = vector of technical efficiency for period i, i = t, t+1

$\underline{m}_i^o$ = vector of price (input-tradeoff) efficiency for period i, i = t, t+1

$\underline{P}'$ = (p_1, p_2), vector of current-period input prices (the prime on P signifies its transpose)

A Numerical Example.

Assume that a company produces a product with the following Cobb-Douglas production function:

$$f(x_1, x_2) = x_1^{\frac{1}{2}} \cdot x_2^{\frac{1}{2}} \quad (13)$$

The associated expansion path can be expressed as indicated below:

$$x_1 = (p_2/p_1) \cdot x_2 \quad (14)$$

Using the data from Table III, we know that for the base and current period, the expansion path is $x_1 = 9x_2$. Thus, the optimal combination of inputs for the base and current period's output ($q_t = q_{t+1} = 30$) is $x_1 = 90$ and $x_2 = 10$. Comparing this optimal combination with the actual inputs used in period t ($x_1 = 80$, and $x_2 = 20$) and in period

TABLE III

DATA FOR NUMERICAL EXAMPLE

	Base Period	Current Period
Output quantity	30.00	30.00
Output price	$15.00	$15.00
Price of Input 1	$ 1.00	$ 1.00
Price of Input 2	$ 9.00	$ 9.00
Quantity of Input 1	80.00	70.00
Quantity of Input 2	20.00	35.00

t+1 (x_1 = 70 and x_2 = 35), it is clear that productive inefficiencies exist in both periods. In addition, since the output of the current period is also 30 units, the input usage would have been x_1 = 80 and x_2 = 20, holding productivity constant. The inputs that would have been used in period t+1, holding productivity constant, however, are different than those actually used ($\underline{m}_t \neq \underline{m}_{t+1}$), implying a change in productivity from one period to the next.

In order to evaluate the sources of the inefficiencies in period t and period t+1 and the change in profits form one period to the next attributable to each of these sources, $\underline{m}_i^e$ (i = t, t+1) must be determined. Since the technically efficient quantity of inputs at the actual mix is the point at which the isoquant for the actual output quantity and a ray from the origin to the actual input combination intersects, values of x_{2i} located on the ray may be defined in terms of x_{1i} as:

$$x_{2t} = (1/4)x_{1t} \tag{15a}$$

$$x_{2t+1} = (1/2)x_{1t+1} \tag{15b}$$

The technically efficient combination of inputs at the actual mix can be calculated by substituting into Equation (13) the actual quantity of output for q (q = 30) and the definition of x_{2i} found in Equation (15a and 15b) and solving for x_{1i}. In this example, $x^e_{1t} = 60$, $x^e_{2t} = 15$, $x_{1t+1} = 30\sqrt{2}$, and $x_{2t+1} = 15\sqrt{2}$. Knowing these values, the calculation of the technical efficiency variance, the input-tradeoff efficiency variance, the change in technical efficiency, the change in input-tradeoff efficiency, and the total change in productive efficiency between the two periods is illustrated in Table IV. A graph demonstrating the problem is shown in Figure 3.

The analysis of productivity change just presented implicitly assumed a single-product setting and that the input prices and the output quantity remain unchanged from one period to another period. The analysis also implicitly assumed the absence of any technological progress between periods and that the output quantity has the same intrinsic qualities from one period to another period. Evaluation of productivity-induced profit changes becomes more complicated when these factors are allowed to vary. To derive technical

TABLE IV

CALCULATION OF PRODUCTIVITY VARIANCES, TECHNICAL AND INPUT-TRADEOFF EFFICIENCY CHANGES, AND PLM

Period t

Technical Efficiency Variance

$$= ([80/30 \quad 20/30] - [60/30 \quad 15/30])\begin{bmatrix} 1 \\ 9 \end{bmatrix}.30$$

= \$65

Input-tradeoff Efficiency Variance

$$= ([60/30 \quad 15/30] - [90/30 \quad 10/30])\begin{bmatrix} 1 \\ 9 \end{bmatrix}.30$$

= \$15

Period t+1

Technical Efficiency Variance

$$= ([70/30 \quad 35/30] - [30\sqrt{2}/30 \quad 15\sqrt{2}/30])\begin{bmatrix} 1 \\ 9 \end{bmatrix}.30$$

= \$151.65

Input-tradeoff Efficiency Variance

$$= ([30\sqrt{2}/30 \quad 15\sqrt{2}/30] - [90/30 \quad 10/30])\begin{bmatrix} 1 \\ 9 \end{bmatrix}.30$$

= \$53.35

Technical Efficiency Change

= \$65 - \$151.65

= \$(86.65)

Input-tradeoff Efficiency Change

= \$15 - \$53.35

= \$(38.35)

TABLE IV (Continued)

PLM

= Technical Efficiency Change + Input-tradeoff Efficiency Change

= $(86.65) + $(38.35)

= (125)

and input-tradeoff efficiency measures, we will continue to assume (1) a single product with constant intrinsic qualities, (2) equality of actual input prices and standard input prices, (3) equality of production and sales, and (4) the absence of any technological progress between periods, while allowing input prices and output quantity to change over time. As before, to evaluate the productivity contribution, a two-consecutive-period model will be used. In this two-period model, the objective is to explain the change in profits from period t to period t+1 attributable to technical and input-tradeoff efficiency changes. In addition to the two-consecutive-period assumption, we also continue the assumptions underlying the economic model of the previous section. Now consider Figure 4.

The actual output isoquants, IS1 and IS2, are shown in Figure 4. The budget line for period t is BL1 and the

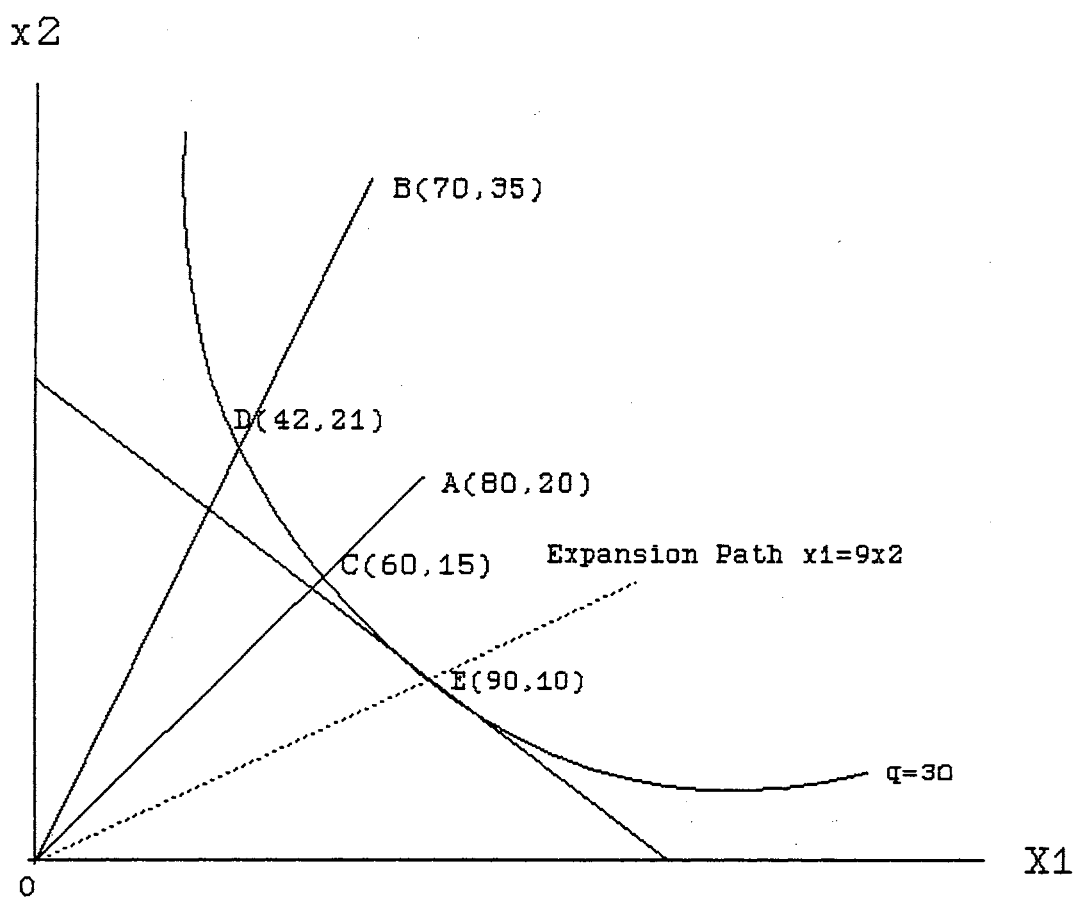

Figure 3. Numerical Example Illustrated

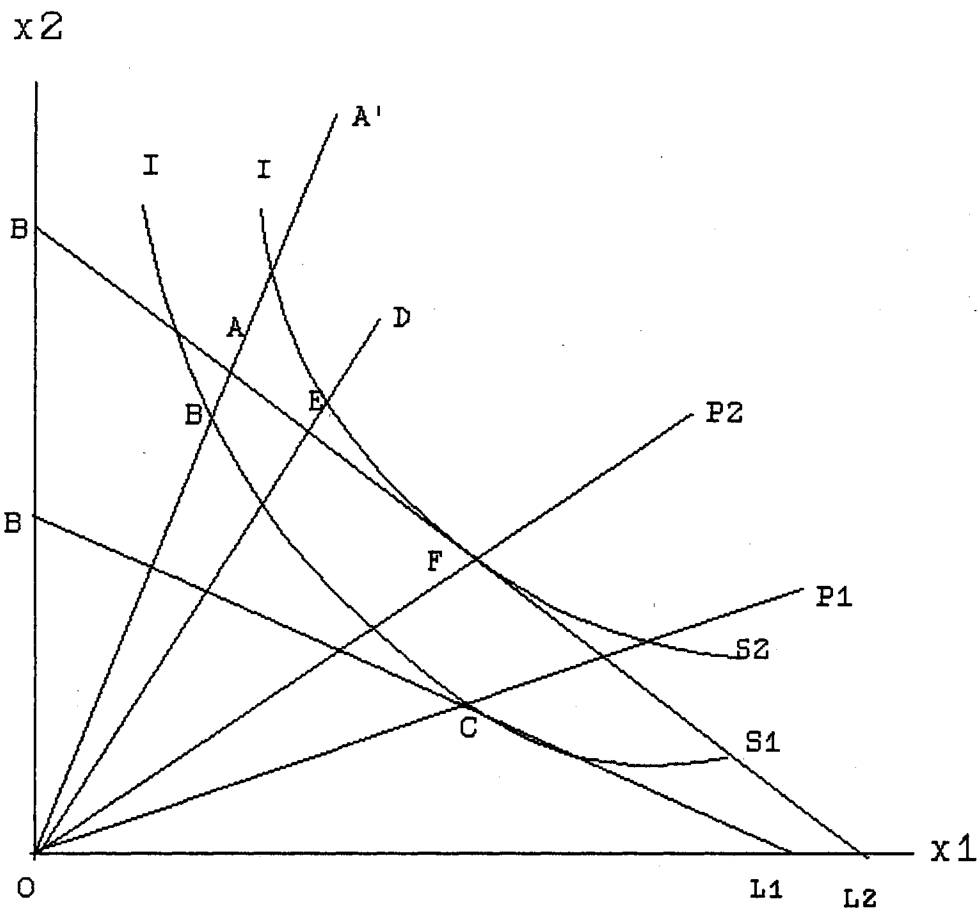

Figure 4. Technical, Input-tradeoff Inefficiencies and Valuation of Input Efficiency Changes (The Simple Model: An Extension)

budget line for period t+1 is BL2. The slopes of the budget lines are different, implying a change in input prices. Let $\underline{A}$ be the input mix actually used in period t, $\underline{A}'$ the input mix that would have been used in period t+1 in the absence of any productivity change, and $\underline{D}$ the input mix actually used in period t+1. Clearly, $\underline{m}_t \neq \underline{m}_{t+1}$; the total change in input efficiency, in physical terms, is equal to $\underline{A}' - \underline{D}$. Moving down the rays on which $\underline{A}$ and $\underline{D}$ are located, we encounter $\underline{B}$, a technically efficient mix of inputs for period t and $\underline{E}$, a technically efficient mix of inputs for period t+1. Productive efficiency can be improved by moving to $\underline{B}$ in period t and to $\underline{E}$ in period t+1. The difference in quantities saved in both periods is one measure of technical efficiency changes and is expressed by $[(\underline{A} - \underline{B}) \cdot q_{t+1}/q_t - (\underline{D} - \underline{E})]$. Similarly, moving down the ray OP_1, the optimal expansion path for period t, and the ray OP_2, the optimal expansion path for period t+1, we encounter $\underline{C}$, an optimal mix of inputs for period t and $\underline{F}$, an optimal mix of inputs for period t+1. Efficiency can be improved by moving to $\underline{C}$ in period t and to $\underline{F}$ in period t+1 (they are less costly than $\underline{B}$ and $\underline{E}$). The difference in the physical tradeoffs associated with the input-tradeoff inefficiency is one measure of the change in input-tradeoff efficiency between the two periods and is measured by $[(\underline{B} - \underline{C}) \cdot q_{t+1}/q_t - (\underline{E} - \underline{F})]$.

The technical and input-tradeoff efficiency measures are now analogous to those illustrated in the simple model.

The major innovation in this setting occurs in calculating a usage standards variance to control for any changes in the input consumption standards between period t and period t+1 attributable to changes in the input prices. If there is input-tradeoff inefficiency in the base and current period, then the usage variance measures a portion of the change in profits from one period to the next attributable to input-tradeoff efficiency changes obtained by moving from one optimal point to another. If, however, there is no input-tradeoff inefficiency in the base and current period, then the usage variance measures the change in profits from one period to the next attributable to input-tradeoff inefficiency changes. The calculation proceeds by evaluating, at the actual output level of period t+1, the difference in the standard consumption of inputs between period t and period t+1 $[(\underline{C}\cdot q_{t+1}/q_t - \underline{F})]$. With input prices and output quantity changing from one period to the next and with the choice of the current price vector, the PLM measure can now be decomposed as follows:

$$\begin{aligned} PLM = \{[(\underline{m}_t - \underline{m}_t^e) - (\underline{m}_{t+1} - \underline{m}_{t+1}^e)] + [(\underline{m}_t^e - \underline{m}_t^o) \\ - (\underline{m}_{t+1}^e - \underline{m}_{t+1}^o) + (\underline{m}_t^o - \underline{m}_{t+1}^o)]\}(\underline{P}_{t+1}')\cdot q_{t+1} \qquad (16) \end{aligned}$$

where:

$\underline{m}_i^e$ = vector of technical efficiency for period i, i = t, t+1

$\underline{m}_i^o$ = vector of price (input-tradeoff) efficiency for period i, i = t, t+1

$\underline{P}' = (p_1, p_2)$, vector of current-period input prices (the prime on P signifies its transpose)

A Numerical Example

As before, we will illustrate the analysis via a numerical example. The basic setup for the example in Table V is similar to the numerical example used earlier but we have changed the quantity produced of output, the quantities used of Input 1 and 2, as well as x_1's unit price. Also assume as in the first numerical example, that the production function is $f(x_1, x_2) = (x_1^{\frac{1}{2}})(x_2^{\frac{1}{2}})$. Using data from Table V, the expansion path for the base period is $x_1 = 9x_2$. Therefore, the optimal input combination for the base period's output ($q = 30$) is $x_1 = 90$, $x_2 = 10$. Comparing this optimal combination with the actual inputs used ($x_1 = 80$, $x_2 = 20$), it is clear that productive inefficiency exists in period t. Moving to the current period, we note that the input price of X_1 has changed. This, in turn, changes the expansion path to $x_1 = x_2$ and the optimal input combination to $x_1 = x_2 = 50$. This combination is also the period's actual input usage, indicating the achievement of perfect productive efficiency. A graph demonstrating the problem is shown in Figure 5. Given the data in Table V, the technical efficiency variance, the input-tradeoff efficiency variance, the usage standards variance, the change in technical efficiency, the change in input-tradeoff efficiency, and PLM can be calculated. This calculation is

TABLE V

DATA FOR NUMERICAL EXAMPLE

	Base Period	Current Period
Output quantity	30.00	50.00
Output price	$15.00	$15.00
Price of Input 1	$ 1.00	$ 9.00
Price of Input 2	$ 9.00	$ 9.00
Quantity of Input 1	80.00	50.00
Quantity of Input 2	20.00	50.00

shown in Table VI.

4.4 Derivation of the Measures: An Extension

In this section, we will expand our derivation of the efficiency measures to those settings where technological progress is allowed to take place between periods. Technological progress means that more output can be produced with a given set of input combination. In terms of output isoquants, technological progress means that they move toward the origin. In what follows, technological progress will be confined to process-improving innovations,

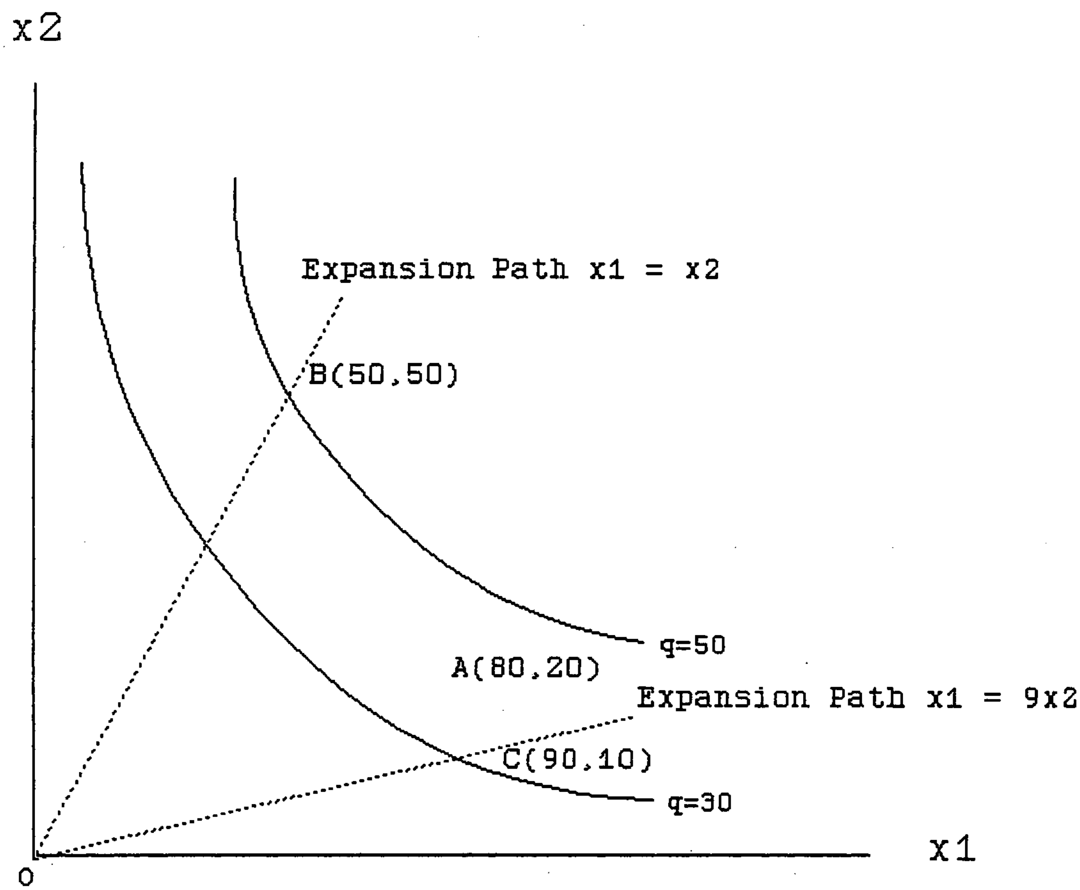

Figure 5. Numerical Example Illustrated

TABLE VI

CALCULATION OF PRODUCTIVITY VARIANCES, TECHNICAL AND INPUT-TRADEOFF EFFICIENCY CHANGES, AND PLM

Period t

Technical Efficiency Variance

$$= ([80/30 \quad 20/30] - [60/30 \quad 15/30])\begin{bmatrix} 9 \\ 9 \end{bmatrix}.30$$

= $225

Input-tradeoff Efficiency Variance

$$= ([60/30 \quad 15/30] - [90/30 \quad 10/30])\begin{bmatrix} 9 \\ 9 \end{bmatrix}.30$$

= $(225)

Period t+1

Technical Efficiency Variance

= Input-tradeoff Efficiency Variance

= 0

Usage Standards Variance

$$= ([90/30 \quad 10/30] - [50/50 \quad 50/50])\begin{bmatrix} 9 \\ 9 \end{bmatrix}.50$$

= $600

Technical Efficiency Change

= $225(50/30) - $0

= $375

Input-tradeoff Efficiency Change

= ($225)(50/30) - 0 + $600

= $225

TABLE VI (Continued)

PLM

= Technical Efficiency Change + Input-tradeoff Efficiency Change

= \$375 + \$225

= \$600

such as implementing just-in-time production systems.[13] We will assume that the product's intrinsic characteristics remain the same over time. In order to focus only on the productivity effect from changes in the manufacturing process between periods, we will assume that the input prices and the output quantity produced in each period are the same. As before, we assume (1) equality of actual input prices and standard input prices, and (2) equality of production and sales. Also assume initially the absence of any productive inefficiencies (technical and input-tradeoff) in both periods. Now refer to Figure 6.

The output isoquant has moved rather uniformly toward

[13]There are two types of technological progress: process innovation and product innovations. Process innovations are those innovations not apparent in the physical properties of the product. Product innovations, on the other hand, require some adjustment on the part of the consumer.

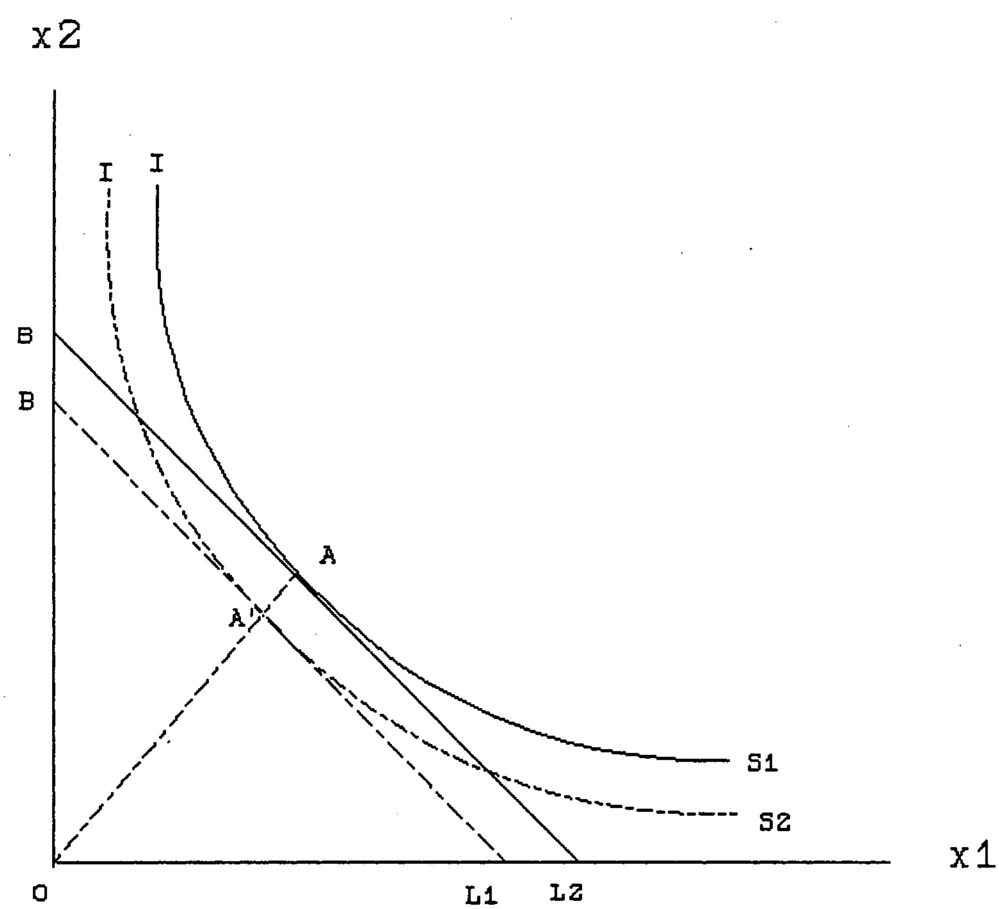

Figure 6. Input Efficiency in the Presence of Technological Progress and Valuation of Input Changes

the origin, implying that more can be produced in period t+1 with the same amounts of inputs than in period t.[14] The actual output isoquant for period t is IS1, and the actual output isoquant for period t+1 is IS2. The budget line for period t is BL1 and the budget line for period t+1 is BL2. The slopes of the budget lines are the same for both periods, implying no change in input prices from period t to period t+1. Next assume that the actual (optimal) input combination is point A for period t and point A' for period t+1 (we assumed the absence of any productive inefficiency in both periods). Notice also that A' requires less of both inputs to produce the same output. The move from A to A', thus, the physical quantities saved is a physical measure of the change in profits from one period to the next attributable to changes in the manufacturing process and is expressed by A - A'.

Recall that in the simple model, PLM was a function of technical and input-tradeoff efficiency changes. If the input prices change from one period to the next, the change in profits attributable to changes in input-tradeoff efficiency is the sum of the change in input-tradeoff efficiency and the change in the standards for input quantities per unit of output (due to input price changes).

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[14]This need not always be the case. Factor saving, for example, can only occur for one factor of production, say labor. This implies that the isoquant would have shifted downwards parallel to the labor axis and technological progress would be biased in the labor-saving direction.

The main innovation in this current setting is the calculation of a usage-standards variance to reflect changes in the input consumption standards from one period to the next attributable to technological changes. The tighter standards in period t+1 reflect technical efficiency improvements made during period t that need to be incorporated when calculating productivity. The calculation proceeds by assessing, at the current actual output level, the difference in the standard consumption of inputs between the two periods.

For those cases where productive inefficiency exists in each period, the analytical development (Equation (12)) would need to be altered since $[(\underline{m}_t - \underline{m}_t^e) - (\underline{m}_{t+1} - \underline{m}_{t+1}^e)]\cdot(\underline{P}'_{t+1})\cdot q_{t+1}$ would no longer accurately measure the technical efficiency contribution to the change in profits. Furthermore, PLM is also altered:

$$\begin{aligned}\text{PLM} = \{[(\underline{m}_t - \underline{m}_t^e) - (\underline{m}_{t+1} - \underline{m}_{t+1}^e)] + [(\underline{m}_t^o - \underline{m}_{t+1}^o) \\ + (\underline{m}_t^e - \underline{m}_t^o) + (\underline{m}_{t+1}^e - \underline{m}_{t+1}^o)]\}(\underline{P}'_{t+1})\cdot q_{t+1}\end{aligned} \quad (17)$$

where:

$\underline{m}_i^e$ = vector of technical efficiency for period i, i = t, t+1

$\underline{m}_i^o$ = vector of price (input-tradeoff) efficiency for period i, i = t, t+1

$\underline{P}'$ = (p_1, p_2), vector of current-period input prices (the prime on P signifies its transpose)

The derivation of Equation (17) assumed equality of base-period and current-period input prices and equality of base-period and current period output quantity. These two assumptions were made only for simplicity in exposition and all the results hold when input prices and output quantity are allowed to vary. Now consider Figure 7.

The actual output isoquant for period t, IS1, and the actual output isoquant for period t+1, IS2, are shown in Figure 7. The budget line for period t is BL1 and the budget line for period t+1 is BL2. The slopes of the budget lines are different, indicating a change in input prices from one period to the next. Assume further that the point A in Figure 7 corresponds to the input mix actually used in period t and the point A' corresponds to the input mix actually used in period t+1. For expository purposes, let C be the input mix that would have been used in the absence of any productivity change, IS3 and BL3, the output isoquant and the budget line corresponding to the input mix that would have been used without a productivity change.

Comparing the inputs that would have been used without a productivity change (point C) with the corresponding optimal input mix (point C'), it is clear that productive inefficiency exists in period t. Moving to the current period, we note that the input prices and the output quantity have changed; this in turn, changes the optimal input mix from C' to E. This combination, however, differs from the period's actual usage, implying the inability to achieve perfect productive efficiency. A physical measure of the change in input inefficiency between the two periods

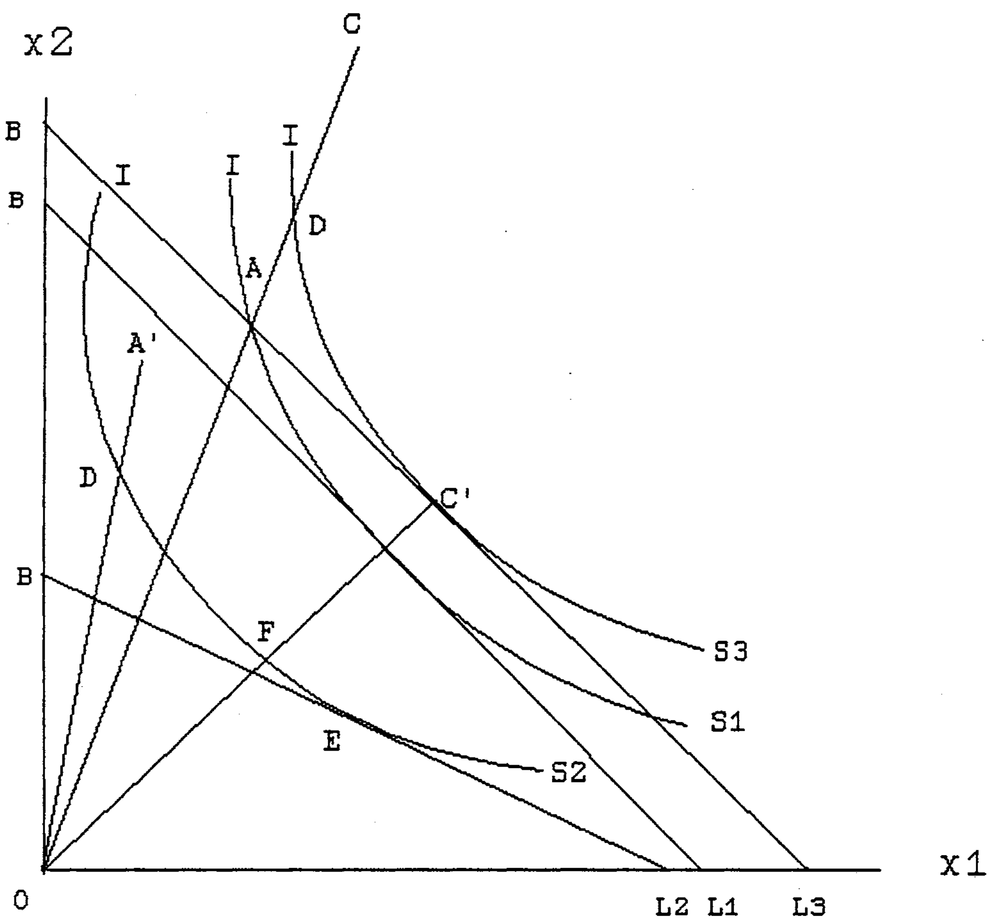

Figure 7. Technical and Input-Tradeoff Efficiency in the Presence of Technological Progress and Valuation of Input Efficiency Changes

is expressed by $\underline{C} - \underline{A}'$, which is the sum of technical and input-tradeoff efficiency changes, adjusted for any changes introduced in the standards for input quantities per unit of output due to changes in the input prices and changes in the manufacturing process between the two periods: $\{[(\underline{C} - \underline{D}) - (\underline{A}' - \underline{D}') + (\underline{C}' - \underline{F})] + [(\underline{D} - \underline{C}') - (\underline{D}' - \underline{E}) + (\underline{F} - \underline{E})]\}$. The calculation is now identical to Equation (17), but even here there is a critical difference. In this setting, the usage-standards variance ($\underline{C}' - \underline{E}$) not only measures a portion of the change in technical efficiency ($\underline{C}' - \underline{F}$) but also a portion of the change in input-tradeoff efficiency ($\underline{F} - \underline{E}$) obtained by moving from the base-period optimal point, adjusted for any change in output quantity (labeled as point $\underline{C}'$), to the current-period optimal point (point $\underline{E}$) The overall effect is to yield the correct net productivity contribution. With the input prices, the output quantity, and the technology changing, and with the choice of the current price vector, the PLM measure is altered as follows:

$$\begin{aligned} \text{PLM} = \{&[(\underline{m}_t - \underline{m}_t^e) - (\underline{m}_{t+1} - \underline{m}_{t+1}^e)]\cdot q_{t+1} \\ &+ [(\underline{m}_t^o \cdot q_{t+1} - x^e] \\ &+ [(\underline{m}_t^e - \underline{m}_t^o) - (\underline{m}_{t+1}^e - \underline{m}^o{}_{t+1})]\cdot q_{t+1} \\ &+ [x^e - (\underline{m}_{t+1}\cdot q_{t+1}]\}\cdot \underline{P}'{}_{t+1} \end{aligned} \tag{18}$$

where:

$\underline{m}_i^e$ = vector of technical efficiency for period i, i = t, t+1

$\underline{m}_i^o$ = vector of price (input-tradeoff) efficiency for period i, i = t, t+1

$\underline{P}'$ = (p_1, p_2), vector of current-period input prices (the prime on P signifies its transpose)

x^e = a technically efficient combination of inputs in the absence of any inefficiency

<u>A Numerical Example</u>

A simple example can be used to clarify and demonstrate these concepts. Assume that the production function of the base period is $f(x_1, x_2) = (x_1^{\frac{1}{2}})(x_2^{\frac{1}{2}})$ and that of the current period is $f(x_1, x_2) = 1.5(x_1^{\frac{1}{2}})(x_2^{\frac{1}{2}})$. Now consider the data in Table VII.

TABLE VII

DATA FOR NUMERICAL EXAMPLE

	Base Period	Current Period
Output quantity	30.00	60.00
Output price	$15.00	$15.00
Price of Input 1	$ 1.00	$ 9.00
Price of Input 2	$ 9.00	$ 9.00
Quantity of Input 1	80.00	50.00
Quantity of Input 2	20.00	50.00

Using the data from Table VII, the expansion path for the base period is $x_1 = 9x_2$. Thus, the optimal mix for the base period's output ($q = 30$) is $x_1 = 90$, $x_2 = 10$. Comparing this optimal mix with the actual input mix used ($x_1 = 80$, $x_2 = 20$), it is clear that there is productive inefficiency in period t. Moving to the current period, we note that the input prices and the output quantity have changed. This, in turn, changes the expansion path to $x_1 = x_2$ and the optimal mix to $x_1 = x_2 = 40$. This combination, however, differs from the period's actual input usage, ($x_1 = x_2 = 50$), indicating the inability to achieve perfect productive efficiency. A graph demonstrating the problem is shown in Figure 8. The calculation of the technical efficiency variance, the input-tradeoff efficiency variance, the usage standards variance due to technological changes, the usage standards variance due to input price changes, the change in technical efficiency, the change in input-tradeoff efficiency, and the PLM measure is illustrated in Table VIII.

4.5 Extensions to Multiple Inputs, Multiple Products

The two-input assumption was made initially only for simplicity in exposition. Extension to multiple inputs is possible by generalizing the Lagrangian-multiplier method to n input variables. This can be easily carried out by writing the input variables in subscript notation. The objective function will then be in the form:

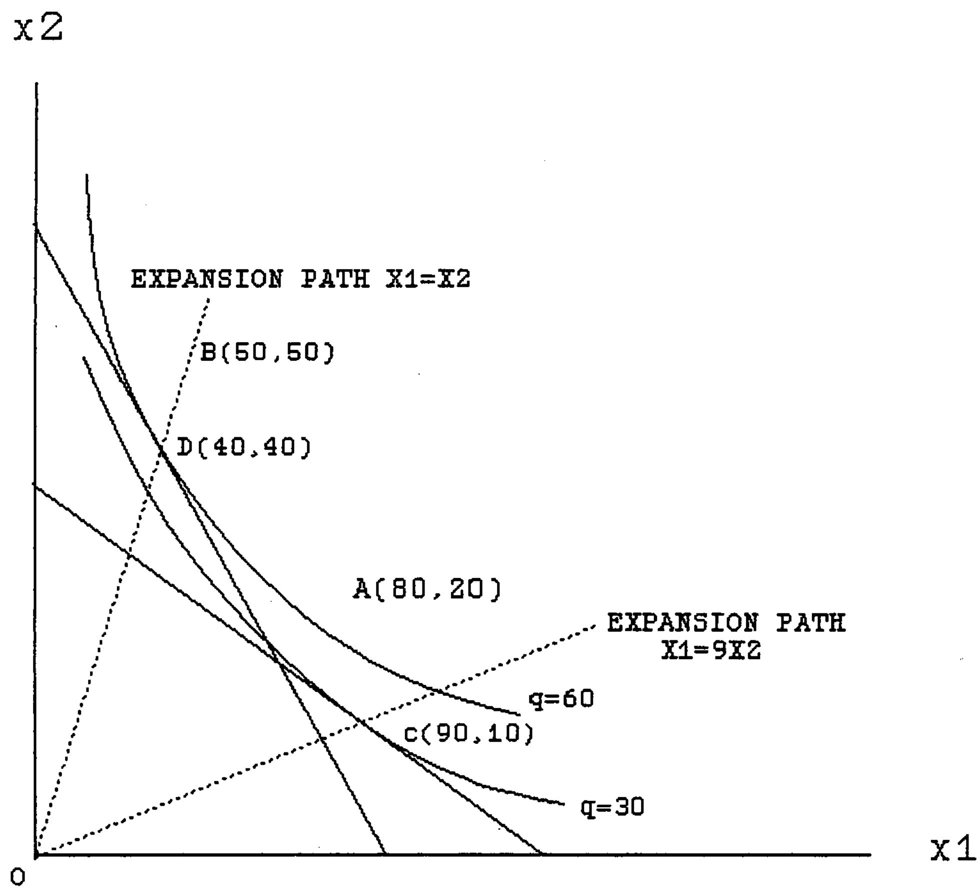

Figure 8. Numerical Example Illustrated

TABLE VIII

CALCULATION OF PRODUCTIVITY VARIANCES, USAGE STANDARDS VARIANCES, TECHNICAL AND INPUT-TRADEOFF EFFICIENCY CHANGES, AND PLM

Period t

Technical Efficiency Variance

$$= ([80/30 \quad 20/30] - [60/30 \quad 15/30])\begin{bmatrix} 9 \\ 9 \end{bmatrix}.30$$

= \$225

Input-tradeoff Efficiency Variance

$$= ([60/30 \quad 15/30] - [90/30 \quad 10/30])\begin{bmatrix} 9 \\ 9 \end{bmatrix}.30$$

= \$(225)

Period t+1

Technical Efficiency Variance

$$= ([50/60 \quad 50/60] - [40/60 \quad 40/60])\begin{bmatrix} 9 \\ 9 \end{bmatrix}.60$$

= \$180

Input-tradeoff Efficiency Variance

$$= ([40/60 \quad 40/60] - [40/60 \quad 40/60])\begin{bmatrix} 9 \\ 9 \end{bmatrix}.60$$

= \$0

Usage Standards Variance due to Technological Changes

$$= ([90/30 \quad 10/30] - [120/60 \quad (40/3)/60])\begin{bmatrix} 9 \\ 9 \end{bmatrix}.60$$

= \$600

Usage Standards Variance due to Input Price Changes

$$= ([120/60 \quad (40/3)/60] - [40/60 \quad 40/60])\begin{bmatrix} 9 \\ 9 \end{bmatrix}.60$$

= \$480

TABLE VIII (Continued)

Technical Efficiency Change
= \$225*(60/30) - \$180 + \$600
= \$870
Input-tradeoff Efficiency Change
= (\$225)(60/30) - 0 + \$480
= \$30
PLM
= Technical Efficiency Change + Input-tradeoff Efficiency Change
= \$870 + \$30
= \$900

$$C = f(x_{1j}, x_{2j}, \ldots, x_{nj})$$

subject to the technical constraint

$$g(x_{1j}, x_{2j}, \ldots, x_{nj}) = q_j$$

It follows that the Lagrangian function will be

$$C(x_{1j}, x_{2j}, \ldots, x_{nj}, l_j) = f(x_{1j}, x_{2j}, \ldots, x_{nj}) - l_j[g(x_{1j}, x_{2j}, \ldots, x_{nj}) - q_j)]$$

for which the first-order condition will consist of the following (n+1) simultaneous equations:

$$C_l = - g(x_{1j}, x_{2j}, \ldots, x_{nj}) + q_j = 0$$

$$C_{1j} = f_{1j} - lg_{1j} = 0$$

$$C_{2j} = f_{2j} - lg_{2j} = 0$$

$$\cdots\cdots\cdots\cdots\cdots\cdots$$

$$C_{nj} = f_{nj} - lg_{nj} = 0$$

where:

$$j = t, t+1$$

The solution to this Lagrangian problem is the period's optimal mix of inputs. The technically efficient combination of inputs at the actual mix can be found by defining values of $x_{ij}(i = 2, 3, \ldots, n)$ in terms of x_{1j} and substituting into $C = f(x_{1j}, x_{2j}, \ldots, x_{nj})$ the actual quantity of output for q_j and the definition of x_{ij} and solving for x_{1j}. Once the optimal input combination and the technically efficient combination of the base and current period are known, the component of the profitability change that is attributable to the change in technical efficiency and to the change in input-tradeoff efficiency can be calculated in the same way as the two-input setting. Therefore, whatever the calculations are that are needed for the two-input setting, they apply equally to the multiple-input setting.

Although the measures were developed in a single product setting, they are equally applicable to a multiple product setting. In such a setting, the efficiency measures are first calculated for each individual product (assuming constant intrinsic qualities) and then added together to arrive at a company-wide measure (PLM = ΣPLM_i = $\Sigma\delta TE_i$+ $\delta\Sigma IE_i$, where δTE_i = technical efficiency change, δIE_i = input-tradeoff efficiency change , i = 1, 2, ..., n). If, however, the intrinsic qualities of a product, e.g., design quality, alter significantly from the base period to the current period, the productivity measures (technical, input-tradeoff, and PLM) for this product may not be meaningful. For example, a change in design quality may require a completely different mix of inputs. Essentially, if intrinsic qualities are significantly changed, productivity for two different products is being evaluated. The solution is to redefine the base period for the new product and wait until another period has passed before measuring productive efficiency [Hansen et al., 1991].

4.6 Summary and Conclusion

Founded on the economic theory of production, this chapter has developed an extension of the PLM measure to separate technical and input-tradeoff efficiency changes. The separation requires assessment of the optimal input quantities each period; this requires knowledge of the company's production function. Since it is often difficult and very costly to assess the underlying productive relationships, a question could be raised concerning the model's practical usefulness. Fortunately, although the measures were derived using economic constructs that are unobservable and difficult to estimate, the measures themselves can be calculated from known observations - observations readily available from a company's existing accounting system. The next chapter will demonstrate a derivation of the observable measures (technical and input-tradeoff) that are consistent with the theoretical definitions of changes in productive efficiency and yet do not rely on knowledge of the production function.

CHAPTER V

DERIVATION OF THE OBSERVABLE PRODUCTIVITY MEASURES

5.1 Introduction

In general, the ability to calculate technical and input-tradeoff efficiency measures depends on explicit knowledge of a company's production function. Knowledge of the production function allows the determination of a technically efficient input combination as well as optimal output and optimal input combinations, which in turn determine the intraperiod standards against which actual performance can be compared. In practice, however, because of the presence of uncertainty, managers would not likely know the precise form of the production function and, thus, the most efficient way of using inputs (imperfect knowledge of the production function)[15]; therefore, the optimal output and input combinations would not likely be precisely known. The presence of uncertainty does not prevent managers from attempting to determine technical and input-tradeoff

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[15]Imperfect knowledge of the production function means that the manager does not know the maximum output possible for a specified set of inputs nor does he know exactly how to use the inputs to achieve this output if it were known.

efficiency changes. This chapter demonstrates a derivation of technical and input-tradeoff efficiency measures that are consistent with the theoretical definitions of changes in productive efficiency and yet do not rely on explicit knowledge of the production function. These measures will require only observable data as inputs and, thus, will be referred to as <u>observable</u> measures.

5.2 Derivation of the Observable Measures

We assume that a company operates in a perfectly competitive output market (i.e., the company can sell all its products). Next let $f(x_1, x_2)$ be a production function with two variable factors of production, x_1 and x_2 (for simplicity only two inputs are assumed). Assume that these two variable inputs are purchased in competitive markets at constant unit prices. Further assume that the production function is homogeneous of degree one (i.e., constant returns to scale). Now refer to Figure 9.

In Figure 9, units of input, x_1, are measured along the horizontal axis and units of inputs, x_2, are measured along the vertical axis. The isoquant, IS1, corresponds to the base-period actual level of output produced and the point <u>D</u> (20,80) corresponds to the actual combination of inputs used to achieve this level of output. The isoquant, IS2, corresponds to the current actual level of output produced and the point <u>D</u>'(40,40) corresponds to the actual combination of inputs used to achieve this level of output.

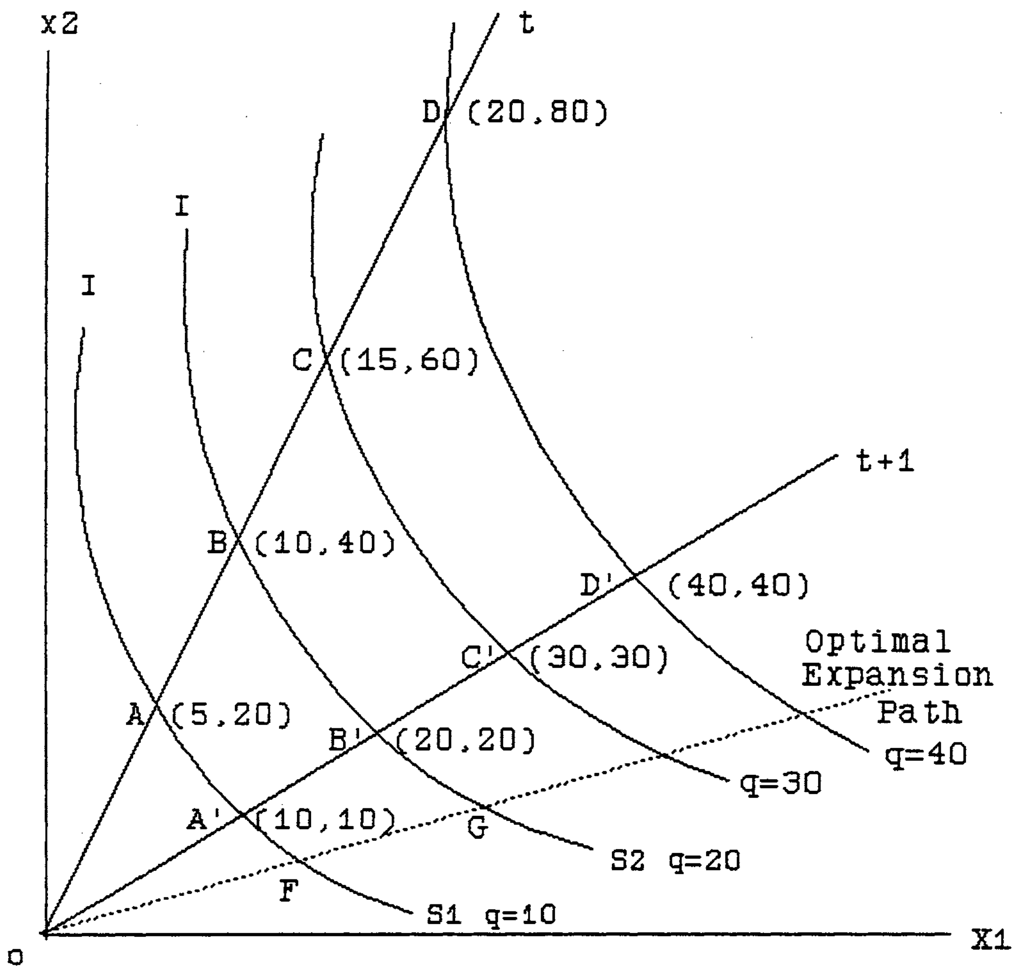

Figure 9. Technical and Input-Tradeoff Efficiency Changes Illustrated (in the Absence of a Knowledge of the Production Function)

The point A represents a technically efficient combination of inputs for the base-period's output. The point B' represents a technically efficient combination of inputs for the current period's output. The ray, OF, represents the optimal expansion path. For simplicity of presentation, assume initially that the input prices remain the same for both periods (i.e., $P_{it} = P_{it+1}$). Clearly, $m_D \neq m_A$ and $m_{D'} \neq m_{B'}$. An excess usage of inputs or waste has occurred and, thus, technical inefficiency. The difference in the inputs that would have been used, had no technically inefficiency occurred (a technically efficient combination of inputs), and those that were actually used is a physical measure of technical inefficiency and is measured by the ratio OD/OA (for period t) and OD'/OB' (for period t+1). This measure is made up of an observable (known) waste factor (e.g., breakage, shrinkage, theft, spoilage, or defect), and an unobservable waste factor (opportunity waste).

As will be shown later, with observable waste savings in the base period and with the assumption of perfectly competitive output market, the output will be increased and sold. Any increase in output from the base period to the current period must be due to changes in technical efficiency. Knowing the change in output quantities between periods and the quantities of observable waste of the base and current period allows the determination of a least cost technically efficient input combination of the base and current period. A technical efficiency variance is now possible by comparing the period's actual usage with the corresponding technically efficient input combination. The change in technical efficiency variances, adjusted for any change in output quantities, is defined as the technical efficiency contribution to the change profits. Once the technical efficiency component is known, the input-tradeoff efficiency component can be found by subtracting the technical efficiency component from the PLM measure.

To see this, assume that the base-period observable waste is $\underline{D} - \underline{C}$ ($W_O = (5,20)$) and the base-period total waste is $\underline{D} - \underline{A}$ ($W_T = W_O + W_U$, where W_U = unobservable waste). Assume further that q (=40 units) is the theoretical capacity and is not expandable in the short run. Also, assume that the base-period output is q_A (=10), the current-period output is q_B, (=20) and the base-period optimal input combination is $\underline{F}$. We assume initially that $W_O = (5,20)$ for period t and $W_O = (0, 0)$ for period t+1. For expository purposes, we will assume that $x^e_{1(i)}$, $x^e_{2(i)}$ is known.[16]

Using the above information, the calculation of the measure of technical inefficiency (TE), the total waste factor, and the observable waste factor for the base period is illustrated in Table IX.

The numerical example is only a theoretical construct and, in no way, should be interpreted as assuming that a

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[16]This assumption allows the determination of W_T and W_O, which are not known without (x^e_1, x^e_2).

TABLE IX

CALCULATION OF TECHNICAL EFFICIENCY, TOTAL WASTE, AND OBSERVABLE WASTE

<u>Period</u> <u>t</u>

$$TE_D = (\sqrt{(20)^2 + (80)^2})/(\sqrt{((5)^2 + (20)^2})$$
$$= \sqrt{6,800}/\sqrt{425} = 4$$
$$= 40/10 = q_D/q_A$$
$$TE_A = \sqrt{425}/\sqrt{425} = 1$$
$$W_T = 4 - 1 = 3$$
$$TE_C = (\sqrt{(15)^2 + (60)^2})/\sqrt{425}$$
$$= \sqrt{3,825}/\sqrt{425} = 3$$
$$W_O = TE_D - TE_C$$
$$= 4 - 3 = 1$$

---

manager knows (x_1^e, x_2^e), which is not observable. The assessment of TE_D, TE_A, W_T, TE_C and W_O can still be carried out even if (x_1^e, x_2^e) is not known. We know from the economic theory of production that $\underline{A} = (x_1^e, x_2^e)$ is the point on the isoquant at which inputs in the same mix as that actually used ($\underline{D} = (x_1^a, x_2^a) = (20, 80)$) should have been used had no technical inefficiency occurred, implying that $x_2^e = 4x_1^e$. Knowing $x_2^e = 4x_1^e$, TE_D, TE_C, and W_O are calculated as follows:

$$TE_D = \sqrt{6,800}/\sqrt{(x_1^2 + x_2^2)}$$
$$= (20\sqrt{17})/(x_1\sqrt{17})$$
$$= 20/x_1$$
$$TE_C = \sqrt{3,825}/\sqrt{(x_1^2 + x_2^2)}$$
$$= (15\sqrt{17})/(x_1\sqrt{17})$$
$$= 15/x_1$$
$$TE_D - TE_C = W_O$$
$$= 20/x_1 - 15/x_1$$
$$= 5/x_1$$

Now, with W_O savings in period t and with the assumption of perfectly competitive output markets, the output will be increased and sold and increased efficiency will result in increased output. Any increase in output because of the assumption of perfectly competitive output markets must be due to changes in technical inefficiency (δTE). Thus,

$$(q_{t+1}/q_t) - 1 = \delta TE$$
$$= TE_D - TE_C$$

(where $(q_{t+1}/q_t) - 1$ = waste savings)

$$(20/10) - 1 = 1$$
$$= TE_D - TE_C$$
$$= 5/x_1$$
$$x_{1t}^e = 5 \text{ and } x_{2t}^e = 4x_{1t}^e = 20.$$

When output quantity changes from the base period to the current period, the base period standard is adjusted to the new output quantity level. To calculate (δTE) and (δIE) (input-tradeoff efficiency changes), we will assume that $\underline{P}$ = (P_1, P_2) = (\$1, \$1), $\underline{F} = \underline{A}'$, $\underline{G} = \underline{B}'$, etc. (i.e., the optimal expansion path is OD' (OD' = OF), and hence, the absence of any input-tradeoff inefficiency in period t+1). Using this information and assuming that (x^e_{1t+1}, x^e_{2t+1}) = (20, 20) is known (which we can calculate), the calculation of PLM, δTE, and δIE is given in Table X.

TABLE X

CALCULATION OF PLM, TECHNICAL EFFICIENCY, INPUT-TRADEOFF EFFICIENCY, AND TECHNICAL AND INPUT-TRADEOFF EFFICIENCY CHANGES

$$\begin{aligned}
\text{PLM} &= (\underline{m}_t - \underline{m}_{t+1}) \cdot q_{t+1} \cdot (\$1, \$1) \\
&= [(20/10, 80/10) - (40/20, 40/20)] \cdot (20) \cdot (\$1, \$1) \\
&= (0, 6)(20)(\$1, \$1) \\
&= \$120
\end{aligned}$$

$$\begin{aligned}
\text{TE}_t &= [(20, 80).2 - (10, 40)] \cdot (\$1, \$1) \\
&= \$150
\end{aligned}$$

$$\begin{aligned}
\text{TE}_{t+1} &= [(40, 40) - (20, 20)] \cdot (\$1, \$1) \\
&= \$40
\end{aligned}$$

$$\begin{aligned}
\delta\text{TE} &= \text{TE}_t - \text{TE}_{t+1} \\
&= \$150 - \$40 = \$110
\end{aligned}$$

$$\begin{aligned}
\text{IE}_t &= [(10, 40) - (20, 20)] \cdot (\$1, \$1) \\
&= \$10
\end{aligned}$$

TABLE X (Continued)

$IE_{t+1} = 0$ (by assumption)

$$\delta IE = IE_t - IE_{t+1}$$
$$= \$10 - \$0$$
$$= \$10$$

$$\delta TE + \delta IE = \$110 + \$10$$
$$= \$120$$
$$= PLM$$

<u>Determination of Technically Efficient</u>

<u>Input Combination for Period t+1</u>

In period t (after observing q_{t+1}), we know (x_1^e, x_2^e) of period t.

$$TE_C = (\sqrt{(15)^2 + (60)^2})/(\sqrt{(5)^2 + (20)^2})$$
$$= \sqrt{(225 + 3{,}600)}/\sqrt{425} = 3$$

$$W_T = TE_D - 1$$
$$= 4 - 1 = 3$$

$$W_O = TE_D - TE_C$$
$$= 4 - 3 = 1$$

$$W_U = W_T - W_O$$
$$= 3 - 1 = 2$$

We now know the total waste, the sum of the observable and unobservable waste. This has tremendous implications for the "zero defects" concept. Focusing on eliminating the observable waste leaves the unobservable waste unattended. Knowing (x^e_{1t}, x^e_{2t}) also provides the information needed for calculating B', a point needed for calculating δTE and δIE (We could also get B by a process similar to that used to get B , but this requires three periods of data instead of two).

From (x^e_{1t}, x^e_{2t}), we know

$TE_D = 4$, $TE_C = 3$, and $TE_B = 2$

We know that if $W_{O(t+1)} = 0$), then q associated with the outcome will have $x_1 = 10$ and $x_2 = 40$ associated (assuming no change in input-tradeoff efficiency). Of course, observing q = 20 tells us that $\delta TE = 1$ ((20/10) - 1), implying that $x_1/5 = 1$, $x_1 = 5$, and that $(x_1 + 5, x_2 + 20) = (10, 40) = (x^e_{1t}, x^e_{2t}) + W_O$ where $W_O = (5, 20)$. For the new output level of 20, we can calculate TE_D and TE_C.

$$TE_D = \sqrt{6,800}/\sqrt{1,700}$$
$$= 2$$

which is the technical inefficiency attributable to the unobservable waste (for our initial assumption). Thus, even if the input-tradeoff efficiency changes in period t+1, the technical inefficiency in period t+1 still must be 2. Hence, for period t+1, with D' = (40, 40), the actual input combination, we have

$$TE_{D'} = \sqrt{((40)^2 + (40)^2)}/\sqrt{(x_1^2 + x_2^2)}$$
$$= \sqrt{3,200}/\sqrt{(2x_1^2)}$$
$$= 40/x_1$$
$$= 2,$$

$x^e_{1t+1} = 20$ and $x^e_{2t+1} = 20$.

5.3 Extension to Multiple Inputs

Extending the analysis of technical and input-tradeoff efficiency changes to multiple inputs is possible by assuming that the input proportions at the point of actual usage and those that would have been used, had no technical inefficiency occurred, are equal (a proportional excess usage (waste)). To see this, let f (x_1, x_2, x_3) be a production function with three variable inputs. As before, we assume that the production function exhibits constant returns to scale and that both the output market and the input market are perfectly competitive. Now consider Figure 10.

The optimal expansion path, OF, is shown in Figure 10. Assume that the input prices remain unchanged for both periods and that $(P_1 = P_2 = P_3 = \$1)$. Next assume that $\underline{D}$ represents the base-period actual input combination, $\underline{D} - \underline{C}$ represents the base-period observable waste (W_O = 5, 5, 40), and $\underline{C} - \underline{A}$ represents the base-period unobservable waste. Also assume that the theoretical capacity is 40 units and is not expandable in the short run. Next let q_A be the base-

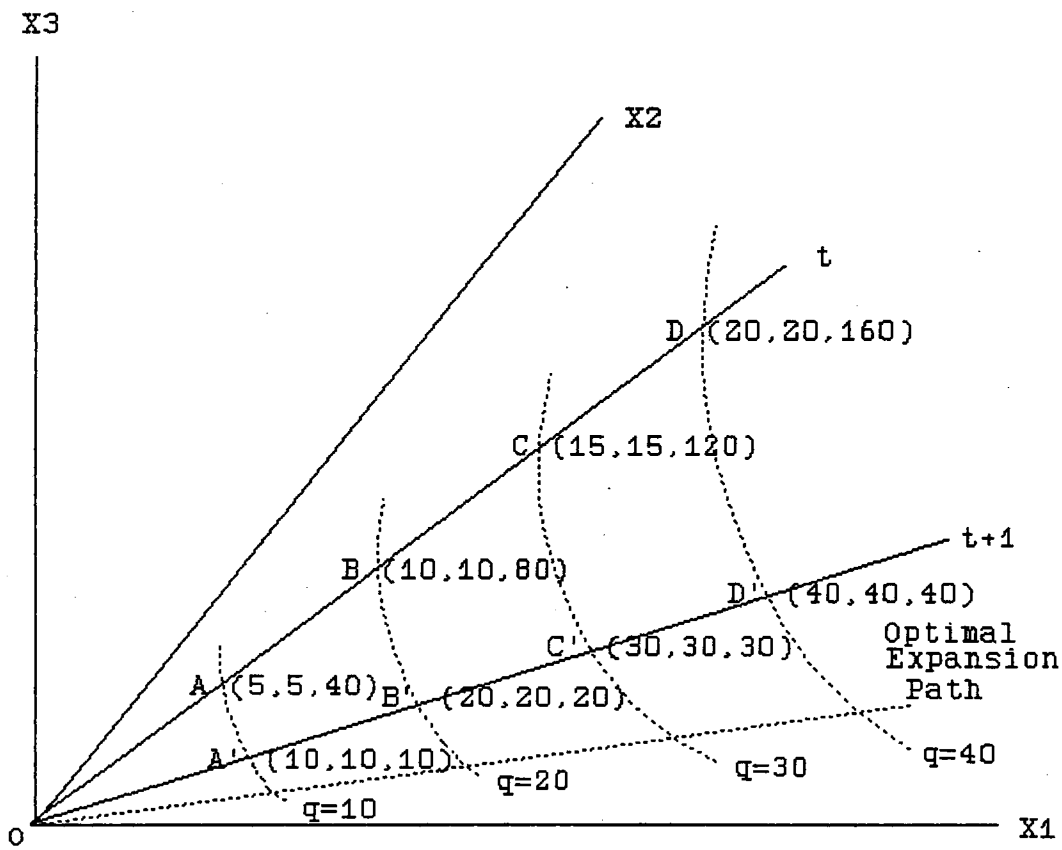

Figure 10. Technical and Input-Tradeoff Efficiency Changes Illustrated (in the Absence of a Knowledge of the Production Function): Multiple Inputs

period actual output, q_B, the current actual output, $\underline{F}$, the base-period optimal input combination, and $\underline{D}'$, the current actual input combination. Also assume that $W_O = (0, 0, 0)$ for period t+1. The calculation of PLM, δTE, and δIE is shown in Table XI.

TABLE XI

CALCULATION OF PLM, TECHNICAL EFFICIENCY, INPUT-TRADEOFF EFFICIENCY, AND TECHNICAL AND INPUT-TRADEOFF EFFICIENCY CHANGES

$$PLM = (\underline{m}_t - \underline{m}_{t+1})\cdot q_{t+1}\cdot(\$1, \$1)$$

$$= [(20/10, 20/10, 160/10)$$

$$- (40/20, 40/20, 40/20)].(20).(\$1, \$1, \$1)$$

$$= (0, 0, 14)(20)(\$1, \$1, \$1)$$

$$= \$280$$

$$TE_D = \sqrt{((20)^2 + (20)^2 + (160)^2)}/\sqrt{(x_1^2 + x_2^2 + x_3^2)}$$

$$= \sqrt{26,400}/(x_1\sqrt{66}) = 20/x_1$$

$$TE_C = \sqrt{(15)^2 + (15)^2 + (120)^2)}/\sqrt{(x_1^2 + x_2^2 + x_3^2)}$$

$$= \sqrt{14,850}/(x_1\sqrt{66}) = 15/x_1$$

$$TE_D - TE_C = W_O$$

$$= 20/x_1 - 15/x_1 = 5/x_1 = \delta TE_t$$

Since $q_{t+1}/q_t - 1 = \delta TE_t = TE_D - TE_C$

$$20/10 - 1 = 5/x_1$$

$$x^e_{1(t)} = 5,\ x^e_{2(t)} = 5,\ x^e_{3(t)} = 40,$$

TABLE XI (Continued)

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and $TE_B = (x^e_{1(t)}, x^e_{2(t)}, x^e_{3(t)}) + W_O$

$= (5, 5, 40) + (5, 5, 40)$

$= (10, 10, 80)$

For the new output level of 20,

$TE_D = \sqrt{(20)^2 + (20)^2 + (160)^2} / \sqrt{(10^2 + 10^2 + 80^2)}$

$= \sqrt{26{,}400} / \sqrt{6{,}600} = 2$

For period t+1, with D' = (40, 40, 40),

$TE_{D'} = \sqrt{(40)^2 + (40)^2 + (40)^2} / \sqrt{(x_1^2 + x_2^2 + x_3^2)}$

$= \sqrt{4{,}800} / (x_1\sqrt{3}) = 40/x_1 = 2,$

$x^e_{1(t+1)} = x^e_{2(t+1)} = x^e_{3(t+1)} = 20$

$TE_t = [(20, 20, 160).2 - (10, 10, 80)].(\$1, \$1)$

$= \$300$

$TE_{t+1} = [(40, 40, 40) - (20, 20, 20)].(\$1, \$1, \$1)$

$= \$60$

$\delta TE = TE_t - TE_{t+1}$

$= \$300 - \$60 = \$240$

$IE_t = [(10, 10, 80) - (20, 20, 20)].(\$1, \$1, \$1)$

$= \$40$

$IE_{t+1} = 0$ (by assumption)

$\delta IE = IE_t - IE_{t+1}$

$= \$40 - \$0 = \$40$

$\delta TE + \delta IE = \$240 + \$40$

$= \$280 = PLM$

Clearly, extending the analysis to multiple inputs does not alter the fundamental derivation of the measures. The model is still a two-period model with the measure of technical inefficiency calculated as:

$$TE = \frac{\sqrt{(x_{1j}^2 + x_{2j}^2 + \ldots + x_{ij}^2)}}{\sqrt{(x_{1j}^e)^2 + (x_{2j}^e)^2 + \ldots + (x_{ij}^e)^2}}$$

where:

$i = 1, 2, \ldots, n$

$j = t, t+1$

The point is that technical and input-tradeoff efficiency changes can be measured and valued regardless of the number of inputs involved.

5.4 Summary and Conclusion

This chapter has demonstrated a derivation of two observable measures: technical and input-tradeoff efficiency. No knowledge of the production function is required to calculate the measures. The measures can be calculated using data already present in a company's existing accounting system. Specifically, the following data are required to calculate the measures: (1) actual input quantities for the base and current period; (2) base and current period output; (3) actual (observable) waste for the base period, and (4) current input prices. Given these four inputs, the change in profitability from the base period to the current period attributable to technical efficiency can be calculated. Once the technical efficiency component is known, the input-tradeoff efficiency component can be calculated by subtracting the technical efficiency component from the PLM measure.

CHAPTER VI

AN ACTIVITY-BASED, ECONOMIC ORDER QUANTITY APPROACH TO THE VALUATION OF BATCH LEVEL PRODUCTIVITY

6.1 Introduction

This chapter has two objectives. First, it discusses the relationship of the activities performed in the production process to the consumption of inputs and to the production of products in a context of activity-based costing (ABC). Second, it develops a conceptual foundation for batch level productivity measurement by incorporating concepts from ABC and from the economic order quantity (EOQ) model.

6.2 Activity-Based Costing

ABC is a more recent development in product costing that focuses on the activities performed to produce products in the production process. Since ABC focuses on activities rather than products, it helps overcome the distorted product costs arising from the use of traditional cost systems.[17] ABC assumes that it is activities (such as setting up machines, supporting direct labor, and administering parts) which cause cost, not products, and it is products which create the demands for activities (e.g., direct labor hours, number of setups, and number of parts in the product). Thus, in the first stage of ABC systems, expenses of support resources are traced to the activities performed by these resources. The second stage involves tracing activity costs to products based on individual products'demand for each activity.[18]

6.3 Activity Identification and Classification

Activity identification is an integral part of the ABC process. Cooper and Kaplan [1991] classify manufacturing activities and their associated costs into four, mutually exclusive and exhaustive categories: (1) unit level activities, (2) batch level activities, (3) product level activities, and (4) facility level activities (see Figure

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[17]Underlying traditional product costing is the assumption that all activities are performed at the unit level and vary directly with the direct labor hours, direct labor dollars, or machine hours.

[18]Traditional product costing also consists of two stages. But in the first stage costs are assigned not to activities but to an organizational unit such as the plant or departments. In both traditional and activity-based costing, the second stage involves assigning costs to the product. The principal computational difference between the two methods is the number of cost drivers used. ABC uses a much larger number of cost drivers than the one or two unit-based cost drivers typical in a traditional system. Some of these bases are used to trace costs whose consumption varies proportionately with the number of units produced; others are used to trace costs whose consumption does not vary with unit volume.

11). Unit level activities are performed once for each unit produced. For example, machine hours and power are used each time a unit is produced. Direct materials and direct labor activities are also examples of unit level activities although they are not overhead costs. Batch level activities are performed once for each batch produced. These activities are common to each unit in the batch; for example, setups, material movements, and first part inspections. As more batches are produced, more batch level resources are consumed but the demands for the batch level resources are independent of the number of units produced after completing the batch level activities. Thus, batch level activities would typically be represented by a step function. Product level activities are related to individual models of a product. These activities are performed as needed to ensure the timely production of products. Product level activities are independent of how many units or batches of the product are produced and would be represented by a function of the form:

$$m_{ij}(q_j) = m_{ij} \text{ if } q_j > 0 \text{ and } m_{ij}(q_j) = 0 \text{ if } q_j = 0$$

where:

m_{ij} = the activity measure for the i^{th} cost pool and the j^{th} product

q = the vector of the company's outputs [Noreen, 1991]

Examples of product level activities are engineering product specifications, process engineering, and engineering change

Activities		Expenses
Facility Level Activities	<————	Plant Management
	<————	Building and Grounds
	<————	Heating and Lighting
Product Level Activities	<————	Process Engineering
	<————	Product Specifications
	<————	Engineering Change Notices
	<————	Product Enhancement
Batch Level Activities	<————	Setups
	<————	Material Movements
	<————	Purchase Orders
	<————	Inspection
Unit Level Activities	<————	Materials
	<————	Direct Labor
	<————	Machine Costs
	<————	Energy

(Source: Cooper and Kaplan [1991])

Figure 11. The Hierarchy of Factory Operating Expenses

notices. Facility level activities are performed to sustain a production facility. These activities are common to many different products and bear no clear relationship to the volume and mix of individual products. Examples of facility level activities are plant management, building and grounds maintenance, security, property taxes, heating and lighting, and plant depreciation.

The ABC hierarchy reveals that some costs vary in proportion to the production volume (unit level costs) and some costs do not (batch and product level costs). While batch and product level costs do not vary with the number of units produced nor can they be controlled at the unit level, they do vary with other measures of activity but not instantaneously. For example, the costs of the setup and the production control departments will vary directly with the number of setups and the number of different types of products being produced [Cooper and Kaplan, 1987]. Batch and product level costs can be decreased by process improvements, such as reducing setup times and implementing just-in-time production systems, or by reducing complexity or product diversity in the plant. Such process improvements and complexity reductions will reduce the demand for personnel in support departments, thereby allowing productivity gains with respect to batch and product level resources. Thus, a principal focus of productivity programs would be to arrive at the same amount of output with fewer batch and product level resources.

Facility level costs, on the other hand, are incurred just to have the ability to produce the products and it is not possible to identify individual products that consume these costs. Facility level costs will not increase in the short run if production volume increases nor can they be decreased without shutting down the facility completely. As a result, facility level costs are fixed costs and are not driven by any of the cost drivers found in any of the first three categories (unit, batch, and product). Banker, Datar, and Kaplan [1989] indicate that fluctuations in such costs are the result of relative price changes for these resources and not of using more or fewer of them. Consequently, productivity gains cannot be obtained through facility level resources. So one possible approach is to ignore them for productivity calculations.

Clearly, the hierarchical classification of activities gives the manager the ability to look at the relationship between activities and the resources they consume. The ability to signal the widely different demands that individual products made on resources used to perform non-unit level activities is a major advantage of ABC over traditional costing systems. The ABC hierarchy suggests that although constant returns to scale, an assumption explicitly made by the PLM model, has a logical basis at the unit level measurement, it does not have such a logical basis at the non-unit level measurement. Such an assumption is likely to impair the ability of the PLM model to measure the productivity contribution accurately whenever the quantity of inputs that a product consumes does not vary in direct proportion to the number of product units produced.

The failure of the PLM model to meet the ABC requirements suggests a refined model which avoids the assumption of constant returns to scale. The economic order quantity (EOQ) model allows us to do this very thing. If the annual demand were known and the optimal tradeoff between inventory carrying costs and setup costs could be identified, it would be possible to calculate the total intraperiod level of setup inefficiency and to break this total down into its technical and setup-inventory tradeoff inefficiency components. Before this can be done, the EOQ model need to be defined.

6.4 The EOQ Model

In selecting a lot size for production, managers are concerned with setup and inventory carrying costs. Setup costs are the costs of preparing machines and facilities for each production run. Examples include wages of idled production workers, the cost of idled production facilities, and the costs of test runs (materials, labor, and overhead). Carrying costs are the cost of carrying or lacking inventory. Examples of carrying costs are obsolescence, handling costs, and storage space. The total setup and carrying cost can be described by the following equation:

Total Costs = Setup Cost + Carrying Cost

$$TC = C_s.D/Q + C_c.Q/2 \qquad (19)$$

where:

TC = the total setup and carrying cost

C_s = the cost of setting up a production run

D = the known annual demand

Q = the lot size for production

C_c = the cost of carrying one unit of stock for one year

Maximizing profits requires that inventory-related costs be minimized. Minimization of carrying costs, however, favors producing in small lot sizes and, therefore, encourages small or no inventories. Minimization of setup costs, on the other hand, favors long, infrequent production runs and, therefore, encourages larger inventories. The objective of the EOQ model is to determine the lot size that equates these two sets of conflicting costs so that the total cost of carrying inventory and setting up a production run is minimized. Among the assumptions of the EOQ model are that (1) the unit production cost is constant and does not vary with changes in the lot size changes, (2) the demand rate is known with certainty and is a constant rate per unit of time, (3) the cost per setup is constant, (4) carrying cost is constant over the same time period as that of the demand and is measured in terms of dollars per unit, (5) stockout cost is so prohibitively high that inventory is replenished before stockouts can occur, (6) production quantity is constant per setup, (7) replenishments of inventory arrive before the inventory level reaches zero or the safety stock level is reached, and (8) lead time for setting up a production run is known with certainty and is constant. Since the optimal lot size is the quantity that minimizes Equation (19), a formula for calculating this quantity is expressed as follows:

$$EOQ = \sqrt{(2DC_s)/C_c} \tag{20}$$

Traditional Production Environment

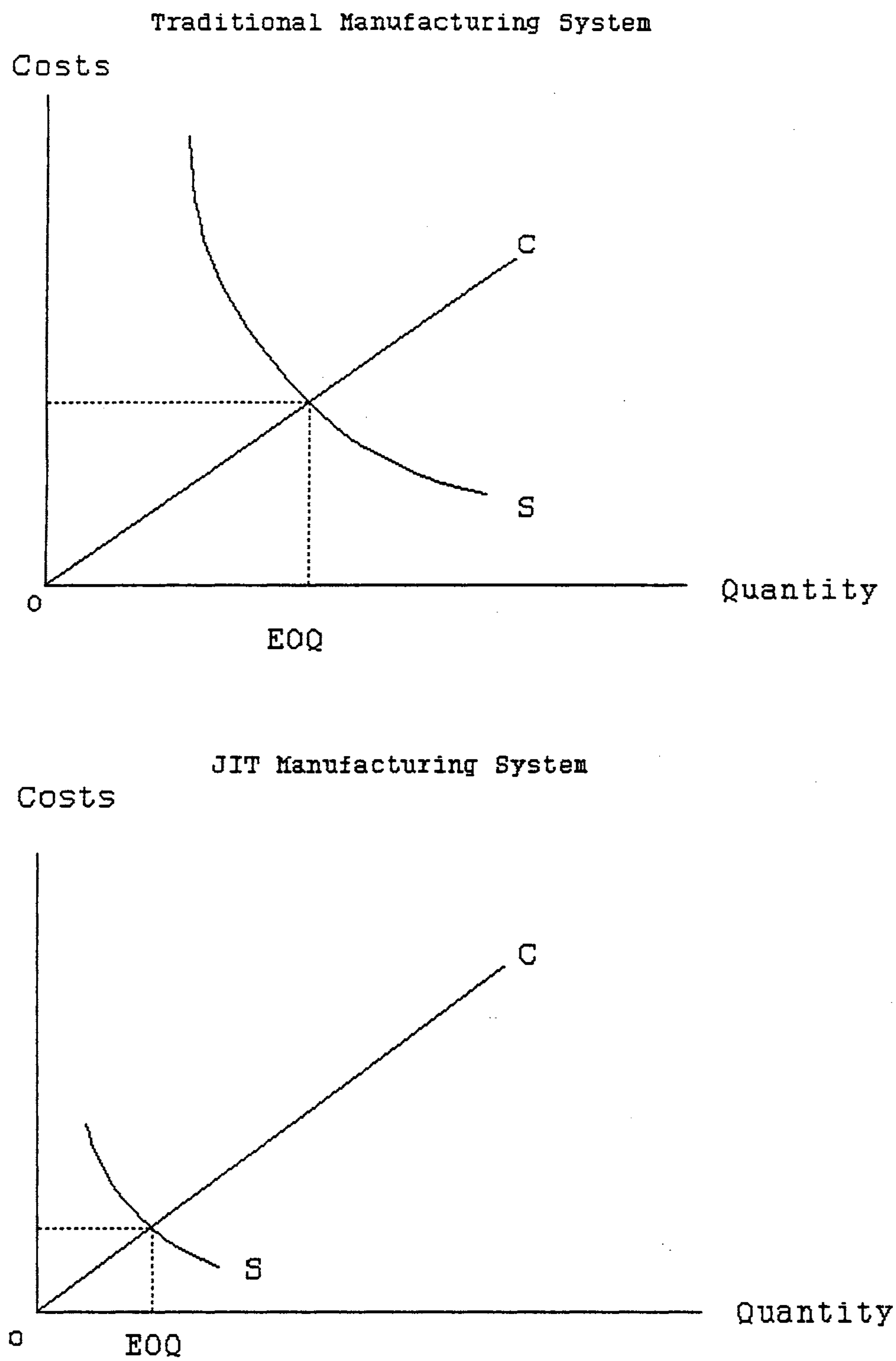
The significance of the EOQ model can be better appreciated by first understanding the nature of the traditional production environment. This environment is described by the mass production of a few standardized products with a high setup cost content. The high setup cost encourages a large batch size and long production runs. Furthermore, diversity is considered to be costly and is avoided. Producing variations of the product could be quite costly as additional, special features would typically require even more costly and frequent setups. Therefore, the traditional approach accepts setup costs as a given and then finds lot sizes that best balance the two categories of costs, setup and carrying costs.

Just-in-Time (JIT) Production Environment

JIT manufacturing and purchasing represents the continual pursuit of zero defects and zero inventories, therefore, productivity, through a commitment to a high level of quality and continuous improvement and the elimination of all activities that do not add value to a product. Under the JIT approach, inventories are viewed as a form of waste, a cause of delays, and a signal of production inefficiencies. Not only do inventories tie up resources such as cash, space, and labor, but also they obscure productive inefficiencies and increase the complexity of a company's information system. As a result, JIT takes a totally different approach to minimizing total carrying and setup costs. In contrast to the traditional approach, JIT does not accept the existence of setup costs. Rather, it attempts to drive the time it takes to set up a production run to zero. If transaction costs for acquiring inventory can be driven to an insignificant level, the only remaining cost to minimize is carrying cost. This is accomplished by reducing inventories to very low levels. In terms of the EOQ argument, JIT attempts to reduce and eventually eliminate setup times so that the optimal lot size (EOQ) approximates one (see Figure 12). With a lot size of one, the work can flow smoothly to the next stage without the need to move it into inventory and to schedule the next machine to accept this item.

6.5 Productivity Variance Analysis

In the discussion that follows, technical and setup-



(C = Carrying Costs, S = Setup Costs)

Figure 12. EOQ Under Traditional versus JIT Manufacturing

inventory tradeoff inefficiency and the valuation of the inefficiency will be illustrated within an EOQ framework involving perfect information regarding the annual demand for the product, time per setup, cost of setting up a production run, and unit carrying cost. Assume initially a traditional production setting. This assumption will be relaxed later to reflect the JIT philosophies.

Given perfect information regarding the annual demand, time per setup, cost per production run, and unit carrying cost, the ability to calculate the optimal lot size, therefore, the optimal number of setups, exists - so why would a company operate inefficiently in such an environment? The answer is that it would not. Imperfect information regarding either the annual demand or the time per setup or the cost of setting up a production run, or the unit carrying cost or a combination of the four must exist for productive inefficiency to be possible. We assume that the source of productive inefficiency is imperfect information regarding the annual demand, time per setup, cost of setting up a production run, and unit carrying cost. For analytical purposes, however, we can introduce the actual lot size, therefore, the actual number of setups, into the theoretical framework and compare them with the optimal lot size and the optimal number of setups. This comparison enables us to define and demonstrate the types of inefficiency (technical and setup-inventory tradeoff) and to show how, in principle, changes in productivity with respect to setup and inventory management activities can be valued. For expository purposes, we will assume that the optimal tradeoff between setup costs and carrying costs is known only after the actual lot size is selected for production, thus allowing the possibility of productive inefficiency.

To illustrate, assume that a company produces two products: Product A and Product B with several plants throughout the nation. Each plant produces all subassemblies necessary to assemble a particular model. The manager of the company's largest plant is convinced that the current lot sizes are too large and wants to identify the level of existing inefficiency. To assist him in his decisions, the controller has supplied the information provided in Table XII.

Next assume that the actual lot size (Q_A) is 64,000 for Product A and (Q_B) 36,000 for Product B. Dividing D_A (320,000) by Q_A (64,000) and D_B (180,000) by Q_B (36,000) produces the actual number of setups (per year) for Product A and Product B, which is 5 (320,000/64,000) and 5 (180,000/36,000), respectively. Multiplying the actual number of setups per year (10) by the cost of setup yields the total setup cost of \$180,000 (10*\$18,000).

The total carrying cost for the year is given by $C_c \cdot Q/2$; this expression is the same as multiplying the average inventory on hand (Q/2) by the carrying cost per unit (C_c). For a production run of 64,000 units of Product A and 36,000 units of Product B with carrying cost of \$6 per

TABLE XII

DATA FOR NUMERICAL EXAMPLE

	Product A	Product B
Annual Demand	320,000	180,000
Unit Carrying Cost	$6.00	$6.00
Standard Unit Carrying Cost	6.00	6.00
Actual Setup Time	1,200	1,200
Standard Setup Time	1,000	1,000
Setup Wage Rate	$15.00	$15.00

unit, the average inventory is 50,000 (100,000/2) and the total carrying cost for the year is $300,000 ($6*50,000). Applying Equation (19), the total cost is $480,000 ($180,000 + $300,000).

Next assume that 1,000 hours per setup at a cost of $15.00 per hour should have been used to produce each product. Using the above information, the optimal lot size for each product and, thus, the efficiency measures are calculated in Table XIII. As the calculation reveals, the actual lot size of 64,000 units of Product A and the actual lot size of 36,000 units of Product B are not the best choice since Product A and Product B could have been

TABLE XIII

ILLUSTRATION OF AN EOQ APPROACH TO VARIANCE CALCULATION

$EOQ = \sqrt{}(2DC_s/C_c$

$EOQ_A = \sqrt{}(2*320,000*15,000)/6$

$= 40,000$

$EOQ_B = \sqrt{}(2*180,000*15,000/6$

$= 30,000$

a.	Actual setup cost	$180,000
b.	Actual inventory carrying cost = Carrying cost that would have been incurred in the absence of any technical inefficiency	300,000
c.	Setup cost that would have been incurred in the absence of any technical inefficiency (10*1000*$15)	150,000
d.	Carrying cost that would have been incurred had the optimal lot size been chosen for production (14*1,000*$15)	210,000
e.	Setup cost that would have been incurred had the optimal lot size been chosen for production (35,000*$6)	$210,000

TABLE XIII (Continued)

f. Technical Efficiency Variance

= [(a) - (c)]

= $180,000 - $150,000

= $30,000

g. Setup-Inventory Tradeoff

Efficiency Variance

= [(b) + (c)] - [(d) + (e)]

= ($150,000 + $300,000)

- ($210,000 + $210,000)

= $30,000

h. Setup Usage Variance

= [(c) - (e)]

= $150,000 - $210,000

= $(60,000)

i. Inventory Efficiency Variance

= [(b) - (d)]

= $300,000 - 210,000

= $90,000

j. Total Efficiency Variance

= [(f) + (g)]

= $30,000 + $30,000

= $60,000

produced in batches of 40,000 and 30,000, respectively. In other words, the annual demand of 320,000 units of Product A can be satisfied using 8 batches (320,000/40,000) while the annual demand of 180,000 units of Product B can be satisfied using 6 batches (180,000/30,000). Substituting (EOQ_A (40,000) + EOQ_B (30,000)) as the value of Q in Equation (19) and summing yields a total cost of $420,000 [(14*1000*$15) + (35,000*$6). Comparing the optimal lot size of 40,000 units of Product A and 30,000 units of Product B with the actual lot size produced 64,000 units of Product A and 36,000 units of Product B, it is clear that productive inefficiency arises (a lot size of 40,000 of Product A and 30,000 of Product B are less costly than a lot size of 64,000 of Product A and 36,000 of Product B ($420,000 versus $480,000)). A graph illustrating the problem is shown in Figure 13.

In Figure 13, lot size per production run is measured along the horizontal axis, and setup and carrying costs are measured along the vertical axis. The setup cost curve is SC and the inventory carrying cost line is CC. Next assume that the actual setup cost is AD and that the setup cost that would have been incurred in the absence of any technical inefficiency is C'D. Productive efficiency can be improved by reducing setup time, therefore, setup cost to C'D. The setup cost saved is one measure of technical efficiency and is expressed by AD - C'D. Also notice that as lot size decreases, inventory carrying cost decreases and

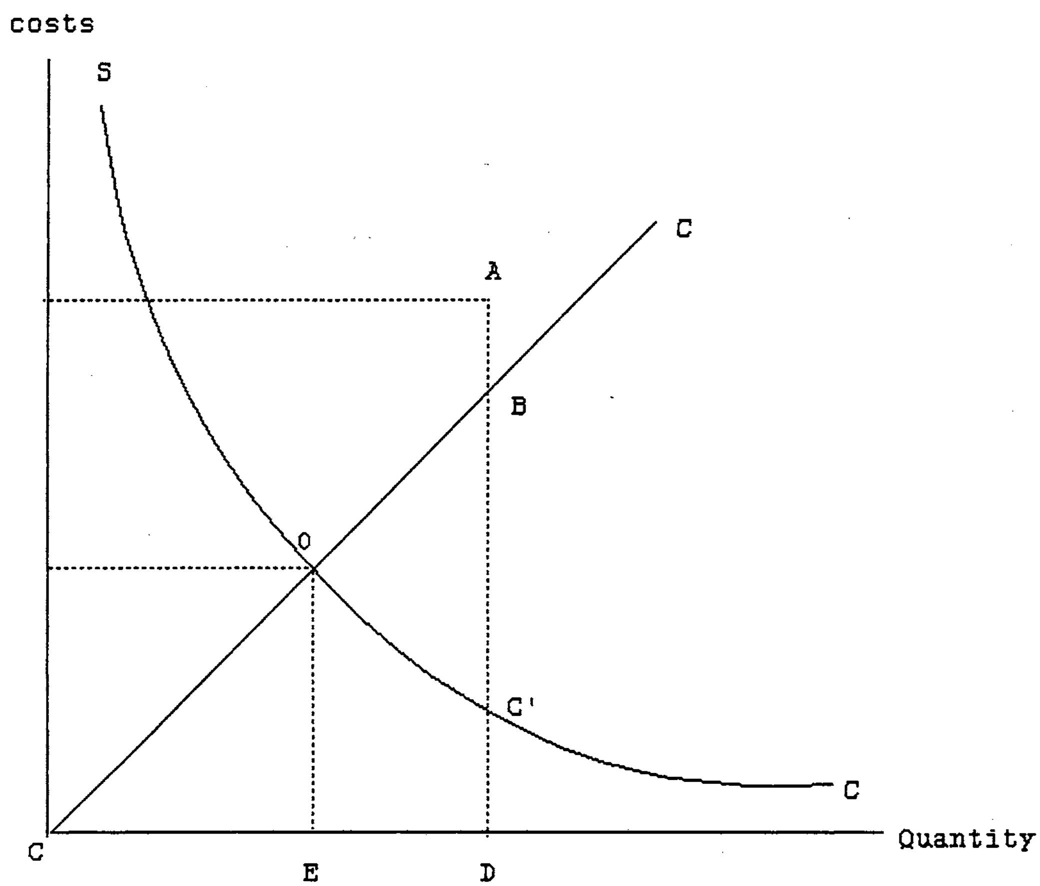


Figure 13. Technical and Setup-Inventory
Inefficiency Illustrated

setup cost increases. Eventually a point E is reached at which the carrying cost equals the setup cost and any additional decrease in lot size costs more than the corresponding reduction in inventory costs. Since CE is less costly than CD, economic efficiency can be improved by reducing the lot size from CD to CE. The tradeoffs associated with the price inefficiency are expressed by (BD + C'D) - 2OE. A measure of the total input inefficiency is (AD + BD) - 2OE, which is the sum of technical and setup-inventory tradeoff inefficiency measures: (AD - C'D) + [(BD +C'D) - 2OE]. Therefore, productivity improvement with respect to setup and inventory management activities can be achieved by using less of setup time (technical efficiency) as well as by trading off setup cost for carrying cost (setup-inventory efficiency).

If desired, the setup-inventory tradeoff efficiency measure can be partitioned into two components: setup usage and inventory efficiency. The setup usage component can be calculated by subtracting the setup cost that would been incurred had the optimal lot size been chosen for production from the setup cost that would have been incurred in the absence of any technical inefficiency. Similarly, the inventory efficiency component can be calculated by subtracting the carrying cost that would have been incurred had the optimal lot size been chosen for production from the carrying cost that would have been incurred had no technical inefficiency occurred.

In this example, an unfavorable technical efficiency variance of $30,000 arises because more setup time was used than expected. Redesigning the manufacturing process so that the same output can be produced with less setup time is one way in which efficiency can be improved. The setup-inventory tradeoff efficiency variance is unfavorable $30,000, indicating the inability to achieve perfect setup-inventory tradeoff efficiency. To help managers understand why this variance occurred, the setup-inventory tradeoff efficiency variance can be separated into two variances: the setup usage variance and the inventory efficiency variance. The setup usage variance is favorable ($60,000) because less was spent on setup than was budgeted. The inventory efficiency variance is unfavorable $90,000 because more was spent on inventory than was budgeted. Clearly, by breaking the total efficiency variance down into its component parts, managers can better analyze and control the total variance. They are able to identify the inception of inefficiencies and take appropriate corrective action.

6.6 Extension of Variance Analysis to Multiple Batch Level Activities

It may also be noted that if the tradeoff between setup and inventory carrying costs can be identified, the EOQ approach can be easily extended to multiple batch level activities. For example, setup costs and ordering costs are similar in nature - both represent costs that must be incurred to acquire inventory. They differ only in the nature of the prerequisite activity (configuring equipment and facilities versus filling out and placing an order). Thus, whatever is said for setup activities also applies equally to ordering activities. With respect to ordering activities, the technical efficiency variance provides a direct measure of the ability of a company to reduce the cost of purchased parts (transaction costs for acquiring inventory), e.g., by developing long- term contracts with suppliers so that the supplier base is reduced. The order-inventory tradeoff efficiency variance measures the ability of a company to solve the problem of resolving the conflict between ordering costs and carrying costs. This is achieved by selecting an inventory level that minimizes the sum of these costs.

6.7 Extension to a JIT Production Environment

Current frequently changing market demands require development of new products and variations of existing products. These changes in product mix typically result in shorter life cycles for products and smaller lot size as product diversity increases. As a result, setup costs become more important. With the JIT approach, lot size is not <ins>optimized</ins>; it is <ins>minimized</ins> by attempting to drive setup time to zero. The most obvious effect of reduced setup time is reduced cost per setup. We will assume that cost per setup is the product of time per setup and the setup wage rate and that any reduction in setup time translates into a directly proportional reduction in cost per setup. The relationship in Equation (20) implies that a reduction in setup time would result in a lower EOQ.

While the effect of reduced setup time on cost per setup is straightforward, the effect on total annual cost (setup cost + carrying cost) is less obvious. Although reducing setup time reduces the lot size, more setups per year are required to achieve the same volume of production. The question is: do more setups at less cost per setup increase or decrease total annual cost ? Substituting the EOQ formula (Equation(20)) into Equation (19) results in an expression that clearly shows the answer:

$$\begin{aligned} TC_s &= \sqrt{DC_cC_s/2} + \sqrt{DC_cC_s/2} \\ &= \sqrt{2(DC_cC_s)} \end{aligned} \qquad (21)$$

We see from Equation (21) that reducing setup time not only produces a reduction in total setup cost but also a reduction in inventory carrying cost. Table XIV shows how reducing setup time reduces the optimal lot size and total costs of setup and carrying inventory. The calculation is based on an EOQ type of formulation whereby the costs of setup are balanced against the costs of carrying inventory. At a cost of $1,000 in turning off a machine, cleaning it, putting on new dies and setting it up again with proper quality on the next run, the company would seek to minimize the number of times it would do that - in fact, 100 times per year (annual demand divided by the optimal lot size of

TABLE XIV

REDUCING SETUP COSTS: EOQ ILLUSTRATED

(Source: Cavinato, 1991)

Setup Cost	Annual Demand	Hold-ing Cost%	Opti-mal Lot	No. of Runs	Total Holding Costs	Total Setup Costs	Total Costs
$1,000	20,000	0.25	200	100	$100,000	$100,000	$200,000
800	20,000	0.25	179	112	89,443	89,443	178,885
600	20,000	0.25	155	129	77,460	77.460	154,919
400	20,000	0.25	126	158	63,246	63,246	126,491
200	20,000	0.25	89	224	44,721	44,721	89,443
100	20,000	0.25	63	316	31,623	31,623	63,246
50	20,000	0.25	45	447	22,361	22,361	44,721
25	20,000	0.25	32	632	15,811	15,811	31,623
10	20,000	0.25	20	1000	10,000	10,000	20,000
5	20,000	0.25	14	1414	7,071	7,071	14,142
1	20,000	0.25	6	3162	3,162	3,162	6,325
0.50	20,000	0.25	4	4472	2,236	2,236	4,472
0.25	20,000	0.25	3	6325	1,581	1,581	3,162
0.10	20,000	0.25	2	10000	1,000	1,000	2,000
0.05	20,000	0.25	1	14142	707	707	1,414

200). This would cost the company a total setup cost of $100,000 per year and a total setup and inventory cost of $200,000. The $100,000 setup cost represents machine time, labor, and overhead that could be otherwise used to produce goods rather than be involved in lost productive switchover efforts. The sensitivity of reduced setup costs is further shown in column A of Table XIV. A drop to a cost of $800 means the company would undergo 112 setups in a year, but the total setup costs would drop to $89,443 and total costs would drop by nearly $22,000. The new optimal lot is now 179 units. A reduction to $100 means the optimal lot size is 63 and total setups per year are now 317; the cost of switchovers are now only $31,623 per year. The setup costs at the bottom of Table XIV are in the cents ranges. At $0.05 the optimal lot is 1 unit, with a total setup and inventory cost of only $1,414 per year.

The use of JIT does not mean that productivity variance analysis is less useful. Indeed, it becomes more useful since it provides more accurate insights regarding the sources of productive inefficiency; these insights produce better decisions. Variance analysis, within a JIT framework, however, must be modified. The modification is straightforward. In this setting, the total variance is the difference between what was actually paid and what should have been paid to <u>minimize</u> lot sizes (i.e., to achieve lot sizes of one). This total variance can be broken down into two components: technical efficiency and setup-inventory tradeoff efficiency. Technical efficiency variance is the difference between what was actually paid and what should have been paid had no technical inefficiency occurred. Setup-inventory tradeoff efficiency variances is the difference between what should have been paid in the absence of any technical inefficiency and what should have been paid to minimize lot sizes.

6.8 Summary and Conclusion

By appealing to the ABC framework and to the EOQ model, this chapter has developed a conceptual model for measuring productivity with respect to setup and inventory management activities. The model allows insights to be gained into aspects of batch level productivity performance which do not appear in the original PLM model. These new insights center on inventory management activities as well as the efficiency with which setup resources are used. Technical efficiency variance indicates how well the company is in accomplishing setup. Setup-inventory tradeoff efficiency variance, on the other hand, indicates how efficient the company is in balancing conflicting setup and carrying costs. With this supplemental information, the company could readily identify and prioritize specific opportunities for profit improvement through better control of the technical process as well as better management of inventory. The next chapter will demonstrate how to make intraperiod variance analysis more dynamic to track changes in productivity between periods.

CHAPTER VII

MULTIPERIOD BATCH LEVEL PRODUCTIVITY MEASUREMENT SYSTEM

7.1 Introduction

Using the theoretical framework developed in Chapter V, this chapter develops an extension of the PLM model to measure changes in (batch level) productivity between periods in terms of changes in technical efficiency and changes in setup-inventory tradeoff efficiency.

7.2 Profit-Linked Productivity Measures and Measures of Efficiency

As already discussed, although profit-linked productivity measures and measures of efficiency are concerned with the efficient use of inputs, the goal of profit-linked productivity measures and that of measures of efficiency are different. Specifically, profit-linked measures are interperiod measures of efficiency and call for continual improvement over time; efficiency measures, on the other hand, are intraperiod measures of efficiency and tend to emphasize achievement of a standard. It is straightforward, by combining the best features from both approaches, to assess productivity performance each period and over time. In particular, if the demand for the product, time per setup, setup wage rate, and unit carrying cost were known, it would be possible to break the total intraperiod level of inefficiency down into two components: technical and setup-inventory tradeoff inefficiency. By knowing these sources of inefficiency (technical and setup-inventory tradeoff), the change in profitability from one period to the next attributable to technical and setup-inventory tradeoff efficiency changes can be determined.

PLM Redefined

Recall that the PLM model defines the productivity contribution as the difference between the cost of inputs that would have been used for the current period in the absence of a productivity change and the cost of the actual inputs. The inputs that would have been used assuming no productivity change can be determined by multiplying the current-period output by the inverse of the input's base-period productivity ratio. The determination of the inputs that would have been used in the absence of any productivity change assumes that the productive inefficiency present is also multiplied by the same constant (constant returns to scale). For example, if a company is using two units too many of an input and output doubles, then the inefficiency doubles to four units. Whenever the consumption of inputs by unit level activities is not directly proportional to the consumption of inputs by batch level activities, the PLM measure will be systematically distorted; in addition, its analytical development would need to be altered since ($\underline{m}.q_{t+1}$) would no longer yield the inputs (the number of setups) that would have been used had productivity remained unchanged. The alteration is, however, straightforward. We assume that in choosing a lot size for production, a company always chooses the one that minimizes the sum of setup costs and carrying costs (the optimal lot size) and that the selection of the optimal lot size depends on the company's expected annual demand and estimated setup time per production run. This optimal lot size will be referred to as the ex ante optimum. Assume further that the ex ante optimum cannot be adjusted as new information regarding the actual demand and setup time becomes available so as to allow for the possibility of productive inefficiency (i.e., the lot size that was optimal ex ante may not be optimal ex post. Next let $\hat{q}_t$ be the expected annual demand for period t, and k, a factor by which output is increased from the base period to the current period. But Q_t (the actual lot size) = $\sqrt{2\hat{q}_t\hat{C}_{st}/\hat{C}_{ct}}$ and so the formula for calculating the number of setups that would have been used (S^*), therefore, the average inventory on hand that would have been (I^*) with no change in productivity is as follows:

$$Q^*_{t+1} = \sqrt{2k\hat{q}_t\hat{C}_{st}/\hat{C}_{ct}}$$

$$S^*_{t+1} = k\hat{q}_t/Q^*_{t+1}$$

$$= k\hat{q}_t/(\sqrt{k}.\sqrt{2\hat{q}_t\hat{C}_{st}/\hat{C}_{ct}})$$

$$= \sqrt{k}S_t \tag{22a}$$

$$I^*_{t+1} = k\hat{q}_t/S^*_{t+1}$$

$$= k\hat{q}_t/2(\sqrt{k}.S_t)$$

$$= \sqrt{k}.Q_t/2 \tag{22b}$$

where:

S_t = actual number of setups for period t

Equation (22) suggests that if a company always selects the optimal lot size for production and if output is increased from the base period to the current period by a factor of k, then the number of setups, therefore, the average inventory on hand, will increase by a factor of $\sqrt{k}$.

7.3 Modified PLM: The Simple Model

Consider a company producing two products: A and B. The two products are produced in the same plants. Assume initially (1) a single batch level activity (setup), (2) two-consecutive-period, traditional production setting, (3) equality of actual and standard setup wage rate, (4) equality of base-period and current-period standard time per setup, (5) equality of actual and standard unit carrying cost, (6) equality of base-period and current-period setup wage rate, (7) equality of base-period and current-period unit carrying cost, (8) equality of base-period and current-period output quantity, and (9) equality of base-period and current-period standard time per setup, and (10) perfectly competitive input markets. Now refer to Figure 14.

The carrying cost line for period t and period t+1 is

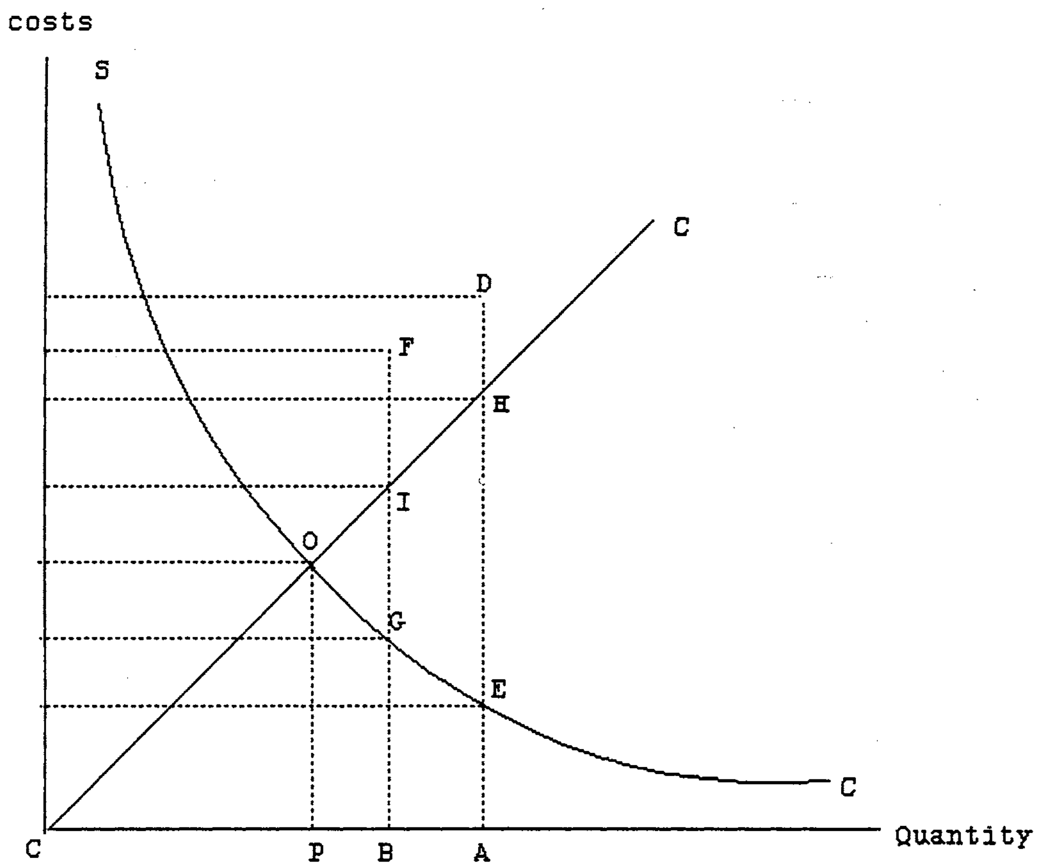


Figure 14. Input Efficiency and Valuation of Technical, Setup-Inventory Tradeoff, and Technological Efficiency Changes

CC and the setup cost curve for period t and period t+1 is SC. Next assume that the actual lot size is AC for period t and BC for period t+1. Clearly, costs at point A ≠ costs at point B; a productivity change has occurred. The value of the productivity change is equal to (AD + AH) - (BF + BI). Moving along the line AD, we encounter AE, the setup cost that would have been incurred had no technical inefficiency occurred in period t. Productive efficiency can be improved by reducing setup time, therefore, setup cost to AE. Moving to the current period, we note that the setup time has reduced; this in turn, drives setup cost to BF. Moving along the line BF, we encounter BG, the setup cost that would have been incurred in the absence of any technical inefficiency. Economic efficiency can be achieved by reducing setup time, therefore, setup cost to BG. The difference in the setup time saved in both periods, therefore, the setup cost, is one measure of the profitability change that is attributable to the change in technical efficiency and is measured by [(AD + AH) - (AE + AH)] - [(BF + BI) - (BG + BI)] = (AD - AE) - (BF - BG) (AH = actual carrying cost = carrying cost that would have been incurred in the absence of any technical inefficiency). Next let OP be the setup cost (carrying cost) that would have been incurred had the optimal lot size been selected for production. Economic efficiency can be improved in both periods by selecting a lot size of CP (it's less costly than AC and BC). The difference in the tradeoffs associated with the price inefficiency is one measure of the profitability change that is attributable to the change in setup-inventory tradeoff efficiency and is measured by [(AE + AH) - 2OP] - [(BG + BI) - 2OP]. A measure of the total change in input efficiency is (AD + AH) - (BF + BI), which is the sum of technical and setup-inventory tradeoff efficiency measures: [(AD - AE) - (BF - BG)] + {[(AE + AH) - 2OP] - [(BG + BI) - 2OP]}.

Modified PLM (MPLM)

$$= \{[(\underline{m}_t - \underline{m}_t^e) - (\underline{m}_{t+1} - \underline{m}_{t+1}^e)] + [(\underline{m}_t^e - \underline{m}_t^o) - (\underline{m}_{t+1}^e - \underline{m}_{t+1}^o)](\underline{P}'_{(t)t+1}) \cdot q_{t+1} \qquad (23)$$

where:

$\underline{m}$ = $(x_1/q, x_2/q)$, x_1 = number of setups, x_2 = average inventory on hand, and q = output

$\underline{P}'$ = transpose of the price vector (setup wage rate, unit carrying cost)

A Numerical Example

Assume that a company produces two products: A and B. In order to produce the products, special equipment must be set up. The standard cost per setup, the standard unit carrying cost, the actual cost per setup, the quantity produced of each product, and the actual lot size (calculated based on the ex ante optimum) are given in Table XV.

Using data from Table XV, the calculation of the actual number of setups used in each period is as follows:

TABLE XV

DATA FOR NUMERICAL EXAMPLE

	A	B
Standard Cost per Setup ($4*50 hours)	$200	$200
Standard (Actual) Unit Carrying Cost	$2	$2
Actual Cost per Setup (period t = $4*62.5 hours)	$250	$250
Actual Cost per Setup (period t+1 = $4*50 hours)	$200	$200
Output Quantity (Actual Demand)	20,000	45,000
Actual lot size for period t	2,500	4,500
Actual lot size for period t+1	2,000	3,000

	Period t	Period t+1
Product A	8 (20,000/2,500)	10 (20,000/2,000)
Product B	10 (45,000/4,500)	15 (45,000/3,000)

Multiplying the actual number of setups used to produce each product by the cost per setup and summing produces the total setup cost of $4,500 ($250*18) for period t and $5,000

(\$200*25) for period t+1. The total carrying cost for period t is given by [(2,500 + 4,500)/2]*\$2 = \$7,000 and that for period t+1 is given by [(2,000 + 3000)/2]*\$2 = \$5,000; thus, the total cost for period t is \$11,500 (\$4,500 + \$7,000) and that for period t+1 is \$10,000 (\$5,000 + \$5,000). A lot size of 2,500 units of Product A and 4,500 units of Product B with a total cost of \$11,500, however, is not the best choice for period t. Economic efficiency could be realized by producing some other quantity that produced a lower total cost. Using data from Table XV, the optimal lot size (ex post optimum) for each product is given by:

$$EOQ_A = \sqrt{(2*20{,}000*200)/2} = 2{,}000$$

$$EOQ_B = \sqrt{(2*45{,}000*200)/2} = 3{,}000$$

The optimal number of setups would be 10 (20,000/2,000) for Product A and 15 (45,000/3,000) for Product B; thus, the total setup cost is \$5,000 (25*\$200). The average inventory on hand is 2,500 (5,000/2) with a total carrying cost of \$5,000 (2,500*\$2). Thus, the total cost is \$10,000 (\$5,000 + \$5,000). Comparing the cost of the ex post optimum with the cost actually incurred (\$11,500), it is clear that productive inefficiency exists in period t. Moving to the current period, we note that the ex ante optimum is also the period's ex post optimum and that the cost of the ex ante optimum is exactly equal to the cost of the ex post optimum (\$10,000), indicating the achievement of perfect productive efficiency. In addition, since the quantities produced of Product A and Product B of the current period are also

20,000 units and 45,000 units, respectively, the number of setups would have been 18 and the average inventory on hand would have been 3,500, assuming no change in productivity. A graph illustrating the problem is shown in Figure 15. The calculation of the technical efficiency variance, the input-tradeoff efficiency variance, the change in technical efficiency, the change in setup-inventory tradeoff efficiency, and the total change in productive efficiency is illustrated in Table XVI.

TABLE XVI

CALCULATION OF PRODUCTIVITY VARIANCES, TECHNICAL AND SETUP-INVENTORY TRADEOFF EFFICIENCY CHANGES AND MODIFIED PLM

<u>Period t</u>

Technical Efficiency Variance

$$= [18*62.5/7{,}000 \quad 3{,}500/7{,}000] - [18*50/7{,}000 \quad 3{,}500/7{,}000])\begin{bmatrix} 4 \\ 2 \end{bmatrix}.7{,}000$$

= \$900

Setup-Inventory Tradeoff Efficiency Variance

$$= [18*50/7{,}000 \quad 3{,}500/7000] - [25*50/7{,}000 \quad 2{,}500/7{,}000])\begin{bmatrix} 4 \\ 2 \end{bmatrix}.7{,}000$$

= \$600

TABLE XVI (Continued)

Technical Efficiency Change

= 900 - 0

= $900

Setup-Inventory Tradeoff Efficiency Change

= $600 - 0

= $600

MPLM = Technical Efficiency Change + Setup-Inventory Tradeoff Efficiency Change

= $900 + $600

= $1,500

7.4 MPLM: An Extension

The simple model just presented assumed a two-product-traditional production setting, with setup wage rate and unit carrying cost, standard time per setup and output quantity remaining unchanged from one period to the next. We will expand our derivation of efficiency measures to those settings where setup wage rate and unit carrying cost are allowed to change. The objective will be to decide whether to use (1) base-period prices (setup wage rate, unit carrying cost), or (2) current actual prices to value setup-inventory tradeoff and technical efficiency changes. As

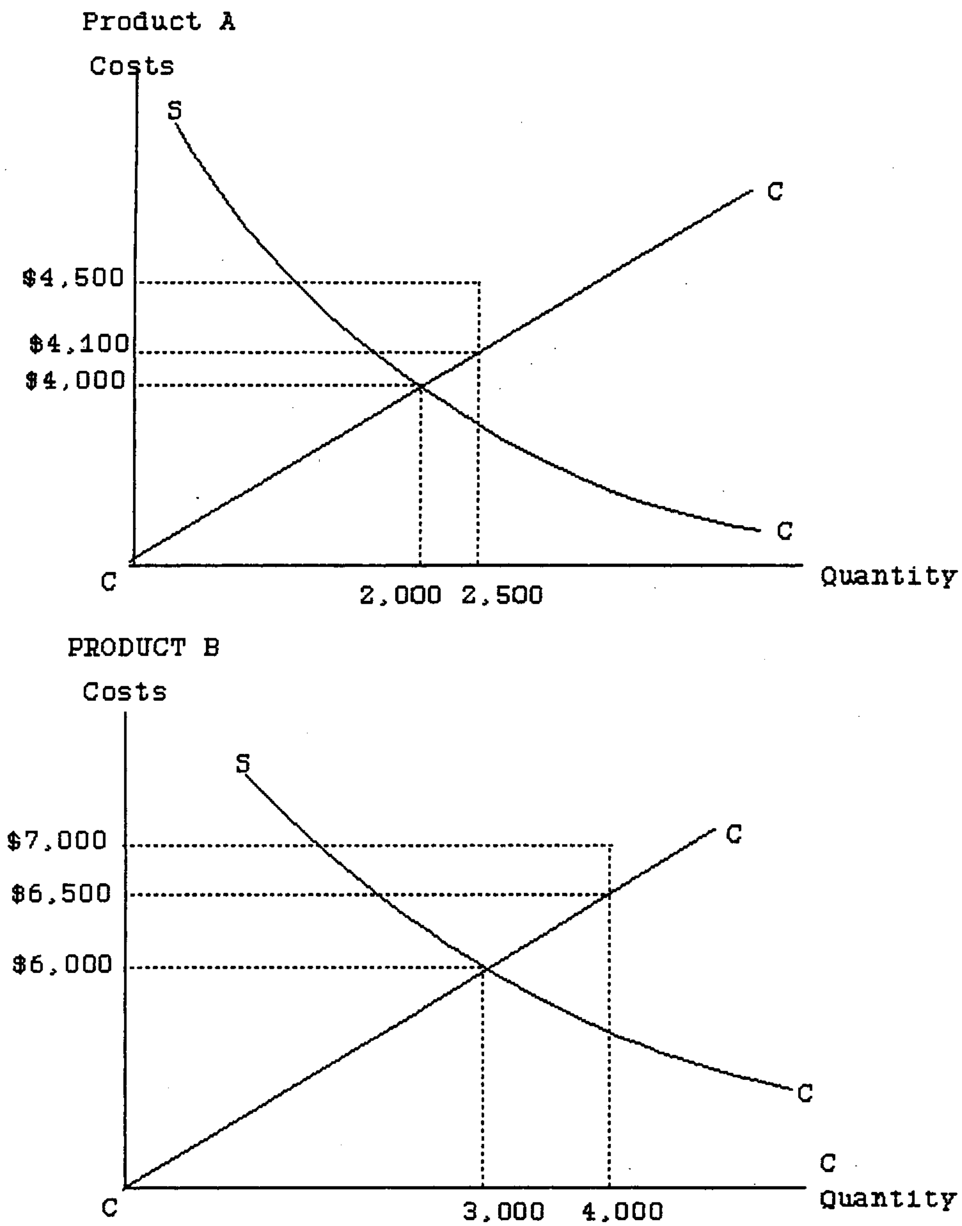


Figure 15. Numerical Example Illustrated

before, we will continue to assume (1) a two-product-traditional-production setting, (2) a single batch level activity (setup), (3) two-consecutive-period setting, (4) equality of actual and standard setup wage rate, (5) equality of actual and standard unit carrying cost, (6) equality of base-period and current-period output quantity, and (7) equality of base-period and current-period time per setup, while allowing setup wage rate and unit carrying cost to change over time. Also assume initially the absence of any productive inefficiency in both periods. Now consider Figure 16.

The carrying cost line for period t, CC1, and the carrying cost line for period t+1, CC2 are shown in Figure 16. The setup cost curve for period t is SC1 and period t+1 is SC2. The slopes of the carrying cost lines are different and the setup cost shifts leftward, implying a change in input prices (unit carrying cost and cost per setup) from one period to the next. Next assume that the lot size actually produced is AC for period t and BC for period t+1. As the intersection points of the carrying cost lines and the setup cost curves reveal, AC is the optimal lot size (ex post optimum = ex ante optimum) for period t and BC is the optimal lot size for period t+1. Producing AC units in period t+1 would have been more costly. Savings can be realized by changing the lot size to BC. This savings is the component of the profitability change attributable to the change in setup-inventory tradeoff efficiency.

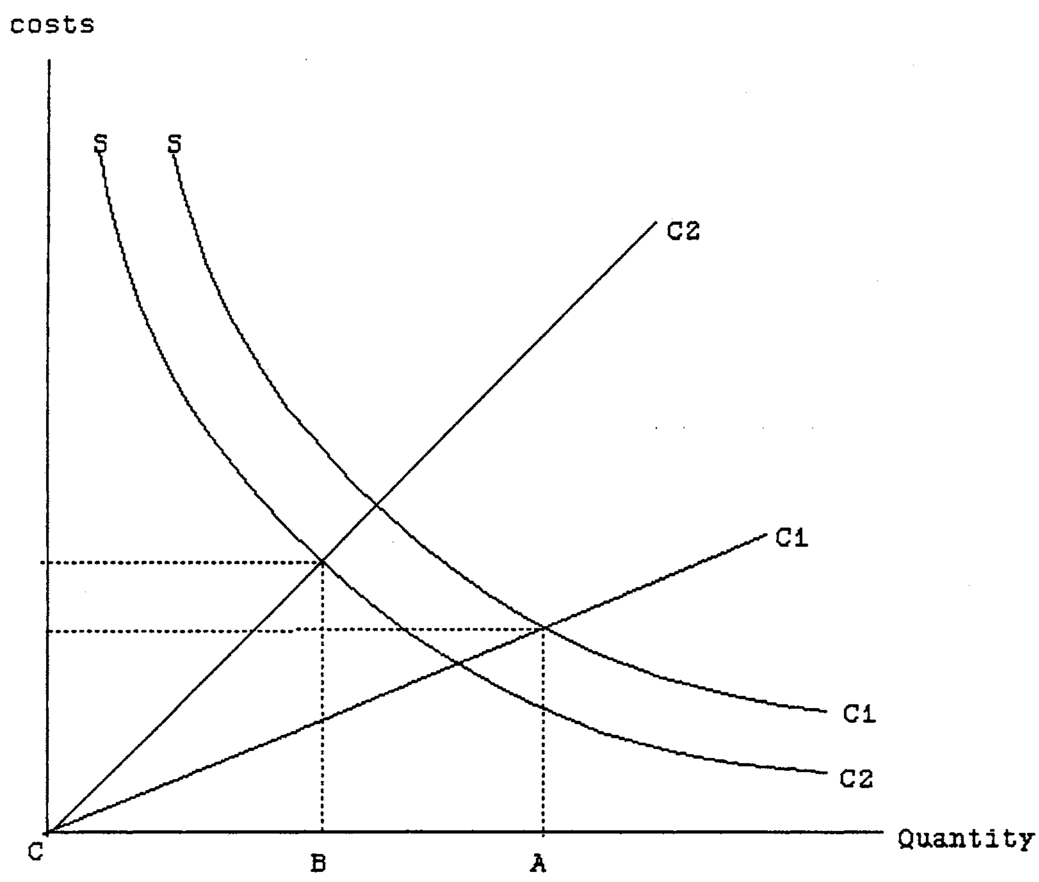


Figure 16. Setup-Inventory Tradeoff Efficiency and Valuation of Input Changes

The only remaining issue is the valuation of this savings. Two choices are available: (1) input prices from period t, and (2) input prices from period t+1. A strong theoretical argument can be made for the use of current prices. Indeed, the use of base-period prices will create an efficiency measure that provides erroneous signals regarding setup-inventory efficiency. If the setup-inventory tradeoff efficiency had been measured using base-period prices, the signal would have been a decline in profits due to the change in the input efficiency. This is because for the price vector, $\underline{P}_t$, the lot size AC is superior to the lot size BC. Yet we know that the shift from AC to BC is justified on the basis that AC is the least costly lot size, measured, however, with respect to the new input prices. Therefore, the need to value changes in input efficiency attributable to setup-inventory tradeoff efficiency requires the use of current prices. In addition, if changes in technical efficiency occur, the recommendation to use current input prices is strengthened. The opportunity cost of technical efficiency changes is measured by the current price vector ($\underline{P}_{t+1}$), not by $\underline{P}_t$.

Extending this result to the case where output quantity changes across periods is straightforward. We will assume that the standard time per setup is revised in period t+1 to reflect expected changes in the production process. To see this, we will continue to assume a two-product, traditional production setting, while allowing output quantity and time per setup to change over time. As before, to assess the productivity contribution, a two-consecutive-period model will be used. In this two-period model, the objective is to explain the change in profits from period t to period t+1 attributable to technical and setup-inventory tradeoff efficiency changes. In addition to the two-consecutive-period assumption, we also continue to assume (1) a single batch level activity (setup), (2) equality of actual and standard setup wage rate, (3) equality of actual and standard unit carrying cost, and the assumptions underlying the EOQ model. Now refer to Figure 17.

In Figure 17, lot size per production run is measured along the horizontal axis, and setup and inventory costs are measured along the vertical axis. Assume that the carrying cost line is CC1 for period t and CC2 for period t+1 and that the setup cost curve is SC1 for period t and SC2 for period t+1. Assume that CP1 in Figure 17 corresponds to the base-period ex post optimum, CB corresponds to the base-period ex ante optimum (base-period actual lot size), CE corresponds to the ex post optimum that would have been had input prices and standard time per setup remained the same, CH corresponds to the lot size that would have been produced, holding base-period expected time per setup, base-period setup wage rate and base-period expected unit carrying cost constant, CP2 corresponds to the current-period ex post optimum, and CK corresponds to the current-period ex ante optimum (current actual lot size). Comparing

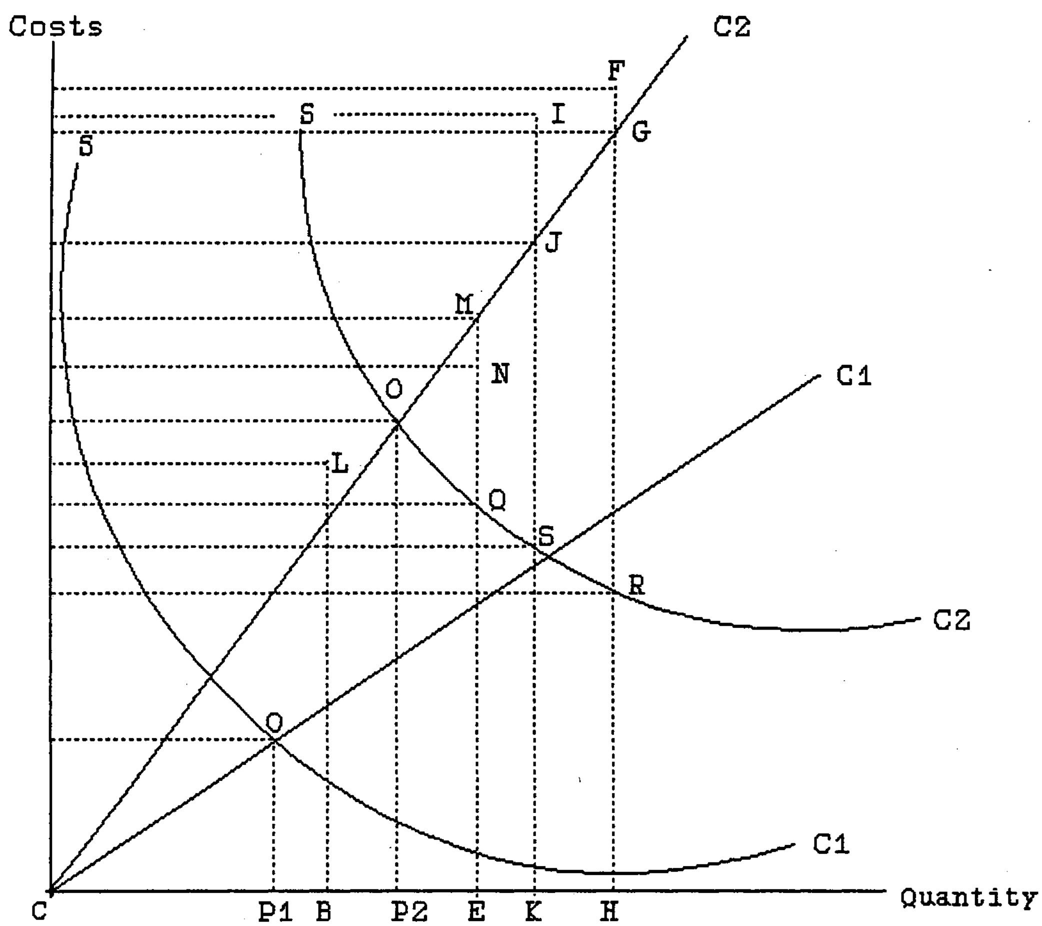


Figure 17. Technical and Setup-Inventory Efficiency and Valuation of Input Changes

the cost of the lot size that would have been produced in period t+1 in the absence of any productivity change (FH + GH) with the cost of the current actual lot size (IK + JK), it is clear that a productivity change has occurred. The change in input efficiency, in dollar terms, is equal to (FH + GH) - (IK + JK) and can be dichotomized as follows:

(1) Usage Standards Variance (due to Technical Efficiency)
= (EN + EM) - (EQ + EM)

(2) Usage Standards Variance (due to Setup-Inventory Tradeoff Efficiency)
= (EQ + EM) - (2OP2)

(3) Technical Efficiency Change
= [(FH + GH) - (HR + GH)] - [(IK + JK) - (KS + JK)] + Usage Standards Variance (due to Technical Efficiency)
= [(FH + GH) - (HR + GH)] - [(IK + JK) - (KS + JK)] + (EN + EM) - (EQ + EM)

(4) Setup-Inventory Tradeoff Efficiency Change
= [(HR + GH) - (EN + EM)] - [(KS + JK) - 2OP2)] + Usage Standards Variance (due to Setup-Inventory Tradeoff Efficiency)
= [(HR + GH) - (EN + EM)] - [(KS + JK) - 2OP2)] + (EQ + EM) - (2OP2)

Note that technical and setup-inventory tradeoff efficiency changes are calculated exactly as illustrated in the simple model. The main innovation occurs in calculating a usage standards variance to incorporate changes in setup wage rate, changes in unit carrying cost, as well as changes in time per setup between period t and period t+1. The calculation proceeds by evaluating, at the actual output level of period t+1, the difference in the cost of the base-period ex post optimum and that of the current-period ex post optimum. The usage standards variance can be broken down into two components: One component attributable to the technical efficiency and the other component attributable to the setup-inventory tradeoff efficiency. If there is technical and price (setup-inventory tradeoff) inefficiency in the base and current period, then the technical and price efficiency components measure the change in technical efficiency and the change in setup-inventory tradeoff efficiency from one period to the next as well as a portion of the change in usage standards attributable to technical and price efficiency changes. The overall effect is to yield the correct net productivity contribution. With the choice of the current price vector, MPLM can be expressed as follows:

$$\begin{aligned} \text{MPLM} = \{&[\sqrt{k_1}(\underline{x}_t - \underline{x}_t^e) - (\underline{x}_{t+1} - \underline{x}_{t+1}^e)] \\ &+ [\sqrt{k_2}(\underline{x}_t^o - S_t^o \cdot T_{t+1} - I_t^o)] \\ &+ [\sqrt{k_1}\underline{x}_t^e - \sqrt{k_2}\underline{x}_t^o) - (\underline{x}_{t+1}^e - \underline{x}_{t+1}^o)] \\ &+ [\sqrt{k_2}(S_t^o \cdot T_{t+1} - I_t^o) - \underline{x}_{t+1}^o]\}(\underline{P}'_{t+1}) \end{aligned} \qquad (24)$$

where:

$\underline{x} = (x_1, x_2)$, x_1 = number of setup hours (number of setups*time per setup), x_2 = average inventory on hand

S_t^o = base-period optimal number of setups

T_{t+1} = current standard time per setup

I_t^o = average inventory on hand that would have been had the optimal lot size been chosen for production in period t

$\underline{P}'$ = transpose of the price vector

$k_1 = q_{t+1}/\hat{q}_t$

$k_2 = q_{t+1}/q_t$

A Numerical Example

As before, we will illustrate the analysis via a numerical example. Assume that a company has two products: A and B. In order to produce the products, special equipment must be setup. The standard cost per setup, the standard unit carrying cost, the actual cost per setup, the quantity produced of each product and the lot size produced are given in Table XVII.

Based on the information provided in Table XVII, the calculation of the actual number of setups, the optimal lot size (ex post optimum), and the optimal number of setups is as follows:

TABLE XVII

DATA FOR NUMERICAL EXAMPLE

	A	B
Standard Cost per Setup (period t = $4*50 hours)	$200	$200
Standard Cost per Setup (period t+1= $5*15 hours)	$75	$75
Standard (Actual) Unit Carrying Cost (period t)	$2	$2
Standard (Actual) Unit Carrying Cost (period t+1)	$3	$3
Actual Cost per Setup (period t = $4*80 hours)	$320	$320
Actual Cost per Setup (period t+1 = $5*20 hours)	$100	$100
Output Quantity (Actual Demand, period t)	20,000	45,000
Output Quantity (Actual Demand, period t+1)	45,000	45,000
Actual lot size (period t)	2,500	3,000
Actual lot size (period t+1)	1,800	1,500

<u>Actual Number of Setups</u>

	Period t	Period t+1
Product A	8 (20,000/2,500)	25 (45,000/1,800)
Product B	15 (45,000/3,000)	30 (45,000/1,500)

<u>Optimal Lot Size</u>

	Period t	Period t+1
Product A	2,000	1,500
	(√(2*20,000*200)/2)	(√(2*45,000*75)/3)
Product B	3,000	1,500
	(√(2*45,000*200)/2)	(√(2*45,000*75)/3)

<u>Optimal Number of Setups</u>

	Period t	Period t+1
Product A	10 (20,000/2,000)	30 (45,000/1,500)
Product B	15 (45,000/3,000)	30 (45,000/1,500)

Assume further that the ex ante optimum (actual lot size) for period t is calculated based on $\hat{q}_A = 20,000$ and $\hat{q}_b = 45,000$. Since $q_{A(t+1)} = q_{B(t+1)} = 45,000$, so $k_{1A} = 2.25$ and $k_{1B} = 1$. The number of setups would have been 12 (45,000/√2.25*2,500) for Product A and 15 (45,000/3000) for Product B and the average inventory on hand would have been 1,875 (√2.25*2,500/2) for Product A and 1,500 (3,000/2) for product B, assuming no change in productivity. Since the optimal lot size (ex post optimum) is calculated based on the annual demand for each product and since $q_{A(t+1)} = q_{B(t+1)} = 45,000$, so $k_{2A} = 2.25$ and $k_{2B} = 1$. Knowing k_{2A} and k_{2B}, the optimal number of setups would have been 15 (45,000/√2.25*2,000) for Product A and 15 (45,000/3000) for

Product B and the average inventory on hand would have been 1,500 ($\sqrt{2.25*2,000}/2$) for Product A and 1,500 (3,000/2) for product B, holding productivity constant. A graph illustrating the problem is shown in Figure 18. The calculation of the technical efficiency variance, the setup-inventory tradeoff variance, the usage standards variances, the change in technical efficiency, the change in setup-inventory tradeoff efficiency, and MPLM is shown in Table XVIII.

7.5 Extension to a JIT Production Setting

Extension to a JIT production setting is possible and all the results hold when the model is extended to incorporate the JIT philosophies with the ex post optimum defined as the lot size of one. The point is that technical and setup-inventory tradeoff efficiency measurement can be measured with respect to any ex post optimum. The technical efficiency variance is calculated as the difference between the actual costs (setup and carrying) and the costs that would have been incurred for the actual lot size had the time per setup been driven to an insignificant level. The change in technical efficiency variances, adjusted for any change in usage standards, is defined as the technical efficiency contribution to the change in profits. The setup-inventory tradeoff variance, on the other hand, is calculated as the difference between the costs that would have been incurred to produce the actual lot size and the

PRODUCT A

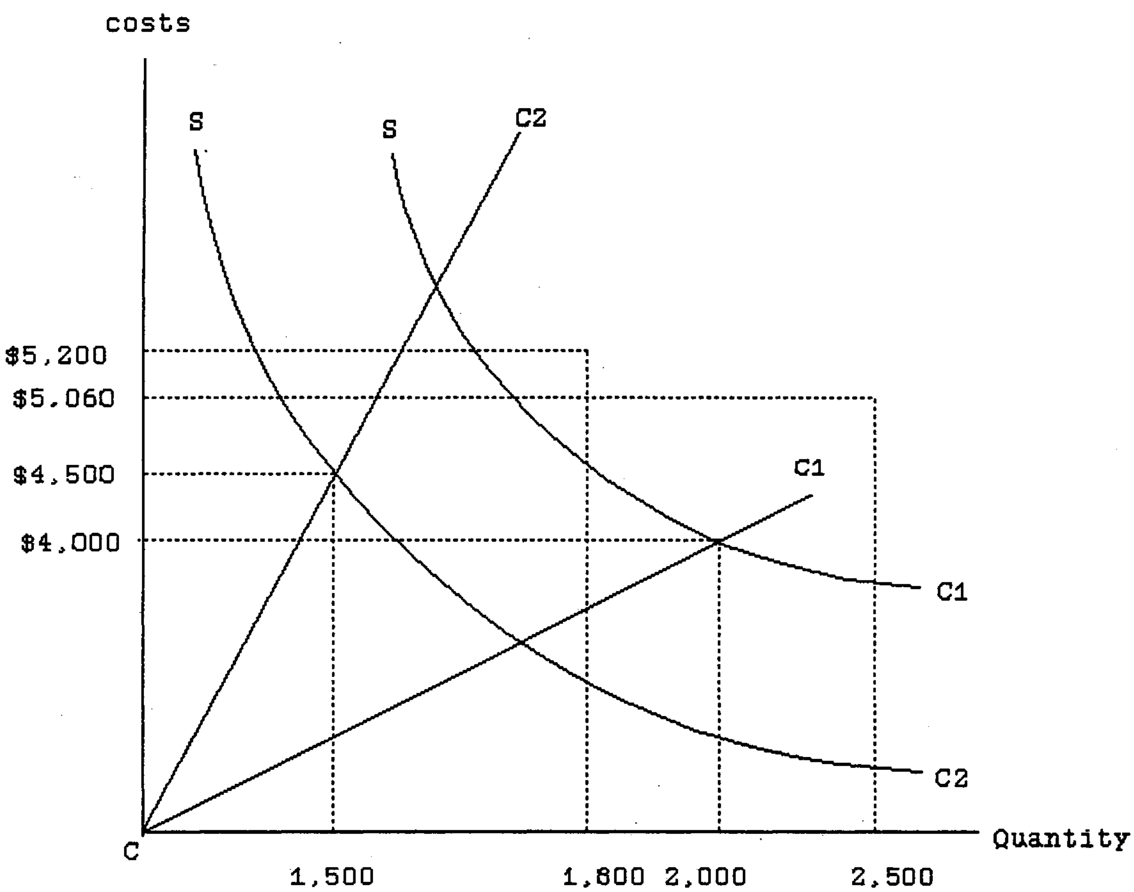


Figure 18. Numerical Example Illustrated

PRODUCT B

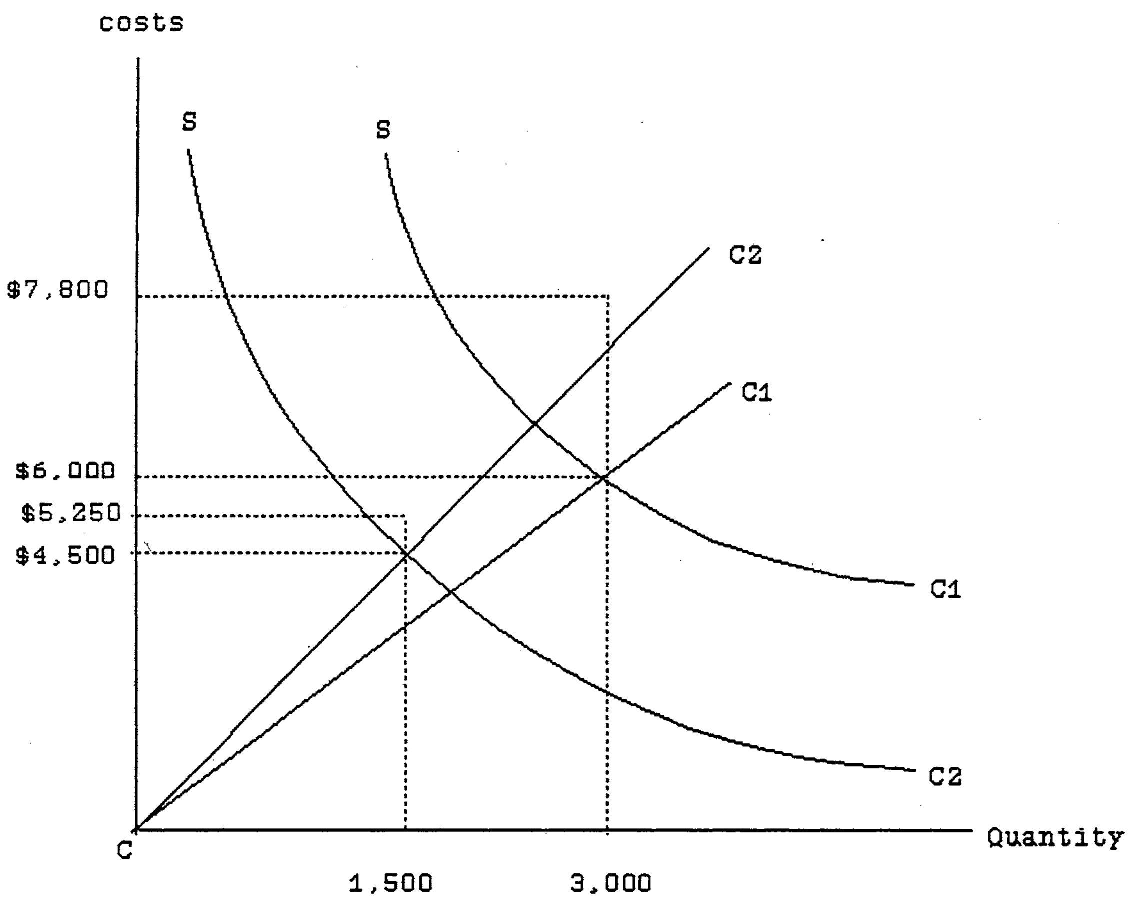


Figure 18. Numerical Example Illustrated
(Continued)

TABLE XVIII

CALCULATION OF PRODUCTIVITY VARIANCES, TECHNICAL AND SETUP-INVENTORY TRADEOFF EFFICIENCY CHANGES AND MODIFIED PLM

Period t

Technical Efficiency Variance

= {[(20,000/2,500)*80*$4 + (2,500/2)*$2]

- [(20,000/2,500)*50*$4 + (2,500/2)*$2]

+ [(45,000/3,000)*80*$4 + (3,000/2)*$2]

- [(45,000/3,000)*50*$4 + (3,000/2)*$2]

= $960 + $1,800

= $2,760

Setup-Inventory Tradeoff Efficiency Variance

= {[(20,000/2,500)*50*$4 + (2,500/2)*$2]

- [20,000/2,000)*50*$4 + (2,000/2)*$2] + 0

= $4,100 - $4,000

= $100

Period t+1

Technical Efficiency Variance

= {[(45,000/1,800)*20*$5 + (1,800/2)*$3]

- [(45,000/1,800)*15*$5 + (1,800/2)*$3]

+ [(45,000/1,500)*20*$5 + (1,500/2)*$3]

- [(45,000/1,500)*15*$5 + (1,500/2)*$3]

= $625 + $750

= $1,375

TABLE XVIII (Continued)

Setup-Inventory Tradeoff Efficiency Variance

= {[(45,000/1,800)*15*$5 + (1,800/2)*$3]

- [45,000/1,500)*15*$5 + (1,500/2)*$3] + 0

= $4,575 - $4,500

= $75

Usage Standards Variance (due to Technical Efficiency)

= {[(45,000/3,000)*50*$5 + (3,000/2)*$3]

- [(45,000/3000)*15*$5 + (3,000/2)*$3]}

+ {[(45,000/3,000)*50*$5 + (3,000/2)*$3]

- [(45,000/3000)*15*$5 + (3,000/2)*$3]}

= $2,625 + $2,625

= $5,250

Usage Standards Variance (due to Setup-Inventory Tradeoff Efficiency)

= {[(45,000/3,000)*15*$5 + (3,000/2)*$3]

- [(45,000/1,500)*15*5 + (1,500/2)*$3]}

+ {[(45,000/3,000)*15*$5 + (3,000/2)*$3]

- [(45,000/1,500)*15*5 + (1,500/2)*$3]}

= $1,125 + $1,125

= $2,250

TABLE XVIII (Continued)

Technical Efficiency Change

= {[(45,000/3,750)*80*$5 + (3,750/2)*$3]

- [(45,000/3,750)*50*$5 + (3,750/2)*$3]}

- {[(45,000/1,800)*20*$5 + (1,800/2)*$3]

- [(45,000/1,800)*15*$5 + (1,800/2)*$3]}

+ {[(45,000/3,000)*80*$5 + (3,000/2)*$3]

- [(45,000/3,000)*50*$5 + (3,000/2)*$3]}

+ $5,250

= $1,175 + $1,500 + $5,250

= $7,925

Setup-Inventory Tradeoff Efficiency Change

= {[(45,000/3,750)*50*$5 + (3,750/2)*$3]

- [(45,000/3,000)*50*$5 + (3,000/2)*$3]}

- {[(45,000/1,800)*15*$5 + (1,800/2)*$3]

- [(45,000/1,500)*15*$5 + (1,500/2)*$3]}

+ $2,250

= $300 + $2,250

= $2,550

MPLM = Technical Efficiency Change + Setup-Inventory Tradeoff Efficiency Change

= $7,925 + $2,550

= $10,475

costs that would have been incurred had the optimal lot size (ex post optimum) been chosen for production. The change in setup-inventory tradeoff variances, adjusted for any change in usage standards, is defined as the setup-inventory tradeoff efficiency contribution to the change in profits.

7.6 Summary and Conclusion

Using the EOQ model, this chapter has developed an extension of the PLM model to also include performance with respect to the productivity of setup and inventory management activities. The EOQ approach provides a useful base to perform secondary analyses to provide guidance in profit improvement through better control of the technical process as well as better inventory management. The derivation of the measures (technical and setup-inventory tradeoff efficiency) requires an assessment of the tradeoff between setup costs and inventory costs. Three inputs are required to calculate the measures: (1) base and current period output, (2) base and current period input prices (setup wage rate, unit carrying cost), and (3) actual lot size for the base and current period. These inputs should be available in a company's existing information system. Given these three inputs, the change in profitability from one period to the next attributable to technical and setup-inventory tradeoff efficiency can be calculated.

CHAPTER VIII

EVALUATION OF THE PLM AND MODIFIED PLM MODELS

8.1 Introduction

In this chapter, the performance of the PLM model will be evaluated vis-a-vis that of the modified PLM (MPLM) model using the following criteria identified by Hansen, Mowen, and Hammer [1991]: measurement accuracy, connection to partial and operational measures, and data requirements.

8.2 Assessment of Measurement Accuracy

A Numerical Example

To evaluate the measurement accuracy of the PLM and MPLM models, a simple numerical example will be used. Table XIX provides a summary statistics for a company producing two products with inputs of a single raw material (x_1), a single grade of labor (x_2), various and types of overhead. For simplicity in our numerical example, we assume that all overhead is driven by one type of transaction, such as number of setups (S), that $\hat{q}_t = Q_t$, and that the ex ante optimum (the actual lot size) and the ex post optimum for each period are the same (i.e., the absence of any technical

TABLE XIX

DATA FOR NUMERICAL EXAMPLE

	Base Period		Current Period	
	A	B	A	B
Output quantity	20.00	30.00	45.00	30.00
Price of x_1	2.00	2.00	8.00	8.00
Price of x_2	8.00	8.00	8.00	8.00
Quantity of x_1	40.00	35.00	67.50	25.00
Quantity of x_2	20.00	15.00	67.50	25.00
Cost of Setup	5.00	5.00	5.00	5.00
Unit Carrying Cost	2.00	2.00	2.00	2.00
Number of Setups (S)	2.00	3.00	3.00	3.00

and setup-inventory tradeoff inefficiency).

Using data from Table XIX, the PLM and the MPLM measures are given in Table XX. The PLM measure signals an improvement in productivity performance, valued at \$7.50. Yet we know from the properties of the problem that there has been no improvement in productivity from the base period to the current period; the PLM measure gives the wrong signal. The productivity gain of \$7.50 arises because the percentage increase in Product A's sales volume does not

TABLE XX

CALCULATION OF THE PLM and MPLM MEASURES

$$PLM = (\underline{m}_t - \underline{m}_{t+1})(\underline{P}_{t+1}) \cdot (q_{t+1})$$

<u>Product</u> <u>A</u>

$$PLM_A = [(40/20 \quad 20/20 \quad 2/20)$$

$$- (67.5/45 \quad 67.5/45 \quad 3/45)](\$8 \quad \$8 \quad \$5)(45)$$

$$= [(90 \quad 45 \quad 4.5) - (67.5 \quad 67.5 \quad 3)](\$8 \quad \$8 \quad \$5)$$

$$= (22.5 \quad -22.5 \quad 1.5)(\$8 \quad \$8 \quad \$5)$$

$$= \$7.5$$

<u>Product</u> <u>B</u>

$$PLM_B = [(35/30 \quad 15/30 \quad 3/30)$$

$$- (25/30 \quad 25/30 \quad 3/30)](\$8 \quad \$8 \quad \$5)(30)$$

$$= [(10 \quad -10 \quad 0)(\$8 \quad \$8 \quad \$5)$$

$$= \$0$$

$$PLM = \Sigma PLM_i$$

$$= \$7.5 + \$0$$

$$= \$7.5$$

Unit Level Measurement

$$MPLM = PLM$$

$$= (\underline{m}_t - \underline{m}_{t+1})(\underline{P}_{t+1})(q_{t+1})$$

<u>Product</u> <u>A</u>

$$MPLM_A = [(40/20 \quad 20/20)$$

$$- (67.5/45 \quad 67.5/45)](\$8 \quad \$8)(45)$$

$$= [(90 \quad 45) - (67.5 \quad 67.5)](\$8 \quad \$8)$$

$$= (22.5 \quad -22.5)(\$8 \quad \$8)$$

TABLE XX (continued)

$= \$0$

Product B

$MPLM_B = [(35/30 \quad 15/30)$

$\quad - (25/30 \quad 25/30)](\$8 \quad \$8)(30)$

$= [(10 \quad -10)(\$8 \quad \$8)$

$= \$0$

Batch Level Measurement

$S^*_{t+1} = \sqrt{k}S_t$

Product A

$S^*_{t+1(A)} = \sqrt{45/20}*2$

$= 1.5*2$

$= 3 = S_{t+1(A)}$

Product B

$S^*_{t+1(B)} = \sqrt{30/30}*3$

$= 3 = S_{t+1(B)}$

$MPLM_A = MPLM_B = 0$

---

require the same percentage increase in setup costs and has nothing to do with any improvement in productive efficiency. Thus, productivity calculations under the PLM model are strongly influenced by nonproportionality between the product's consumption of unit and non-unit level inputs.

The MPLM measure does not rely on constant returns to scale nor does it assume a proportional consumption of inputs. Rather, MPLM acknowledges that not all inputs are consumed in direct proportion to the quantity of product units produced. Furthermore, it relies on the EOQ model when calculating productivity. Thus, without any productivity change (the cost per setup in each period remains the same), the total number of setups would have been $S^*_{t+1} = 6$, which is also the period's actual number of setups, implying no change in productivity. The MPLM measure correctly signals a zero productivity contribution and values it correctly.

The MPLM model, however, has appeal even beyond the ability to accurately measure the direction and value of productivity changes. In fact, by appealing to the economic theory of production, to the ABC framework, and to the EOQ model, it can be shown that the MPLM measure can be partitioned further into technical and input-tradeoff (setup-inventory tradeoff) efficiency changes. Thus, MPLM's strength is based on its ability to direct management's attention to the real source of problems. Directing management's attention to the real cause of problems is important because the appropriate corrective action that must be taken differs depending on the problem suggested by the proposed measures. A decline in technical efficiency, for example, might suggest the need for additional training or better motivation for the employee as well as the need to redesign the manufacturing process so that more output can be produced with fewer inputs. A decline in input-tradeoff (setup-inventory tradeoff) efficiency, on the other hand, might suggest the need for managers to be more concerned with the relative amounts used of each input; the least-costly input combination should be chosen. This indepth analysis thus has the advantage of isolating productivity changes attributable to the better ability to control input usage (materials, labor, and setup resources) from those attributable to the better ability to trade off more costly inputs for less costly inputs and also establishing a standard by which a company's resource utilization can be evaluated.

8.3 Overall Assessment

The accuracy of the PLM model becomes an issue when the consumption of inputs by unit level activities and the consumption of inputs by batch level activities are not strictly proportional. By modifying the PLM model to reflect nonproportional consumption of inputs, the problems of accuracy can be overcome. At the unit level measurement, the MPLM model extends the PLM model to isolate technical and input-tradeoff efficiency changes. The technical efficiency measure assesses and motivates attempts to reduce rework by producing fewer defective units and, therefore, can be used to monitor and evaluate past decisions regarding proposed quality improvement programs. Reducing the number of defective units improves quality; moreover, this act simultaneously reduces the amount of resources used to produce and sell a company's output, and productive efficiency is also improved. The input-tradeoff efficiency measure, on the other hand, assesses the overall change in the tradeoffs of the individual inputs. Because input-tradeoff (price) inefficiency may occur even if there is no technical inefficiency, managers must also be concerned with the relative amounts used of each input. The least-costly input combination should be chosen.

As with the PLM model, the MPLM model represents a theoretically justifiable measure of productivity-induced profit changes. It is derived using the economic theory of production, and therefore, is not an arbitrary assessment. In addition, the productivity measures, $\underline{m}_{t(t+1)}$, $\underline{m}^{e}_{t(t+1)}$, $\underline{m}^{o}_{t(t+1)}$ are vectors of partial productivity measures whose components can be interpreted as operational productivity assessments of individual inputs. The differences in these components $(\underline{m}_t - \underline{m}^{e}_{t}) - (\underline{m}_{t+1} - \underline{m}^{e}_{t+1})$ and $(\underline{m}^{e}_{t} - \underline{m}^{o}_{t}) - (\underline{m}^{e}_{t+1} - \underline{m}^{o}_{t+1})$, adjusted for any change in input-output standards and multiplied by q_{t+1}, provide the changes in each input quantity attributable to technical and input-tradeoff efficiency changes. Finally, multiplying each of these differences by the transpose of the current price vector yields the technical and input-tradeoff efficiency contribution. Thus, as with the original PLM model, the MPLM model is the sum of partial productivity measures.

This means that MPLM is also directly connected with the operational productivity measures, allowing the manager to assess and monitor individual input contributions and to identify the sources of inefficiency.

The MPLM model also extends the PLM focus on unit level activities to also include performance with respect to the productivity of setup and inventory management activities. The setup-inventory tradeoff efficiency measure allows managers to focus on minimizing the total cost of setting up a production run and carrying inventory. The technical efficiency measure allows managers to assess the productivity consequences of process improvement. It is conceivable that a company could be producing a good with little or no defects but still have an inefficient process. By improving training, eliminating conflicts in employee assignments, and placing tools and dies in convenient locations, the time to set up a machine can be reduced and efficiency can be improved; this act is independent of quality. Valuing the economic consequences of this act is a key element in calculating the technical efficiency change and is embedded in the technical efficiency measure.

Although the advantages just described for the MPLM model are impressive, an important issue is whether the data required for its calculation are observable and readily available or whether the company would have to incur significant incremental costs to implement the model. Fortunately, although the measures (technical and input-tradeoff efficiency) were derived using economic constructs that are unobservable and difficult to estimate (e.g., the production function), the measures themselves can be calculated using information from known observations - observations readily obtainable from a company's existing accounting system (see Chapter VI for a derivation of the observable measures).

8.4 Summary and Conclusion

Using four desirable criteria for a profit-linked productivity measures [Hansen et al., 1991], this chapter has shown that the MPLM model satisfies all three criteria. The PLM model, however, fails to accurately signal a productivity change whenever the amount of unit level inputs that a product consumes does not vary in direct proportion to the amount of other inputs that it consumes. The reliance on the assumption of constant returns to scale is a major reason for the failure of the PLM model. In their original form, both the PLM and MPLM models make effort to link with operational and partial measures. Linking with operational partial measures provides an integrated and consistent productivity measurement system while permitting managers to evaluate and isolate productivity problems associated with particular inputs. Finally, like the PLM model, the MPLM model makes use of data already available in the existing accounting information system.

CHAPTER IX

SUMMARY AND CONCLUSIONS

9.1 Introduction

In recent years, efforts have been made to develop productivity measures that link productivity measures to profitability. Of the several profit-linked productivity measurement models, three have gained some prominence: The APC model developed by the American Productivity Center, the PPP model developed by Miller [1984], and the BDK model developed by Banker, Datar and Kaplan [1989]. Hansen et al. [1991] show that all three models fail to accurately measure the direction and magnitude of productivity changes. They attribute this failure to the use of base-period prices. Hansen et al. show that current input prices should be used for accurate profit-linked productivity measurement. They develop a profit-linked productivity measurement model (the PLM model) that is founded on the economic theory of production. The PLM model increases the accuracy of profit-linked measurement and allows a connection to the operational and partial productivity measures. It also establishes an equivalency among the three models.

In general, however, productivity improvement can be achieved by using less of each input (technical efficiency) as well as by trading off one input for the other (input-tradeoff efficiency); these sources of inefficiency, however, are not revealed by the PLM measure. Essentially, having a productivity gain of a certain dollar amount falls short of providing managers with information sufficient to interpret productivity changes. Directing management's attention to the real cause of problems is important because the appropriate corrective action that must be taken differs depending on the problem suggested by the two new measures (technical and input-tradeoff efficiency measures).

An additional limitation of the PLM model relates to the implicit assumption that if output doubles from the base period to the current period, then the manager would double the inputs, assuming there is no change in productivity. Unfortunately, empirical evidence from the accounting literature seems to indicate that the consumption of inputs by non-unit level activities is unrelated to the number of units produced or to the size of a production run [Cooper, 1990]. For example, doubling the size of a batch does not necessarily require doubling the number of setups. This failure to capture the economic nonproportionalities inherent in production and to accurately measure productivity contributions, can lead to bad evaluations and decisions, therefore, suboptimal results. In addition, many of the other benefits of measuring productivity , e.g., better use of resources, improved motivation and accountability, assessment of trends, comparison to competitors, and rewards and bonuses (based on productivity) may all suffer if the productivity measures are inaccurate.

9.2 Summary of Results

This thesis set out to fulfill three objectives:

(1) To extend the unit-based PLM model by developing two new measures of productivity which allow assessment of the change in profits attributable to technical and input-tradeoff efficiency changes.

(2) To extend the PLM focus on unit level productivity measurement to also include productivity performance with respect to batch level inputs by developing a productivity measurement in which the productivity measure is not distorted by nonproportional consumption of inputs.

(3) To demonstrate the superiority of the model developed in this study to the PLM model.

At the unit level measurement, our synthesis of PLM and variance analysis has yielded some useful insights. The intraperiod productivity variance analysis can be combined with an interperiod PLM measurement to provide a more systematic and comprehensive explanation of changes in productivity each period and over time. Overall changes in productivity can be partitioned into two components: changes in technical efficiency and changes in input-tradeoff (price) efficiency. A decline in technical efficiency might suggest the need for additional training or better motivation for the employee. A decline in input-tradeoff efficiency might suggest the need for the cost center supervisors to be more concerned with the relative amounts used of each input. This requires that the purchasing department be instructed to inform the cost center supervisors on changes in relative prices which affect their departments as soon as these changes occur. It would then be up to the supervisors to ensure that the impact of the relative price changes on their input mix is reflected in their production programs. The impact of the relative price changes on the respective quantities required of different types of materials, for example, should also be conveyed to the purchasing department so that the replenishment of inventories would be undertaken with knowledge of the changing requirements.

At the non-unit level measurement, our integration of the EOQ model into the PLM framework increases the accuracy of the PLM measure and extends the PLM focus on unit level inputs to also include productivity performance with respect to setup and inventory management activities. Measuring productivity performance with respect to setup resources should be especially valuable as setup resources consume a larger proportion of total production costs. The separation of overall productivity into its component parts, technical and setup-inventory tradeoff efficiency changes, permits better measurement and control over the consumption of the organization's setup resources as well as better management of inventory. A decline in technical efficiency may indicate an inefficient technical process. Improving training, eliminating conflicts in labor assignments, and placing tools and dies in convenient locations may reduce setup times, therefore decreasing the amount of labor input and improving productivity. A decline in setup-inventory tradeoff efficiency may suggest a company's inability to solve the problem of resolving the conflict between setup and inventory carrying costs; the lot size that minimizes the sum of these costs should be chosen.

9.3 Implications and Suggestions

This study has shown that, by introducing the actual inputs for each period into the theoretical framework and comparing them with the unobservable, optimal inputs each period, the PLM measure can be broken down into two components: One component attributable to technical efficiency changes and the other component attributable to input-tradeoff efficiency changes. Implications that may result from this suggested decomposition would be that it would be incorrect to attribute productivity changes entirely to changes in technical efficiency nor can managers accurately draw any conclusions regarding productivity performance only by looking at the total productivity effect. The possible existence of tradeoffs among inputs mandates a productivity measurement model that allows the assessment of the disaggregate financial consequences of technical and input-tradeoff efficiency changes. A decline in technical efficiency might indicate deficient managerial ability, poor training programs, redeployment of labor, etc. A decline in input-tradeoff efficiency, on the other hand, might indicate satisficing behavior or over-or under-valuation of the opportunity costs of the company. Whether the isolation of the PLM measure actually provides useful information for management is an empirical question which can only be answered by further research.

As with any other profit-linked productivity measurement model, our proposed model is not without its own significant problems. First, the calculation of the measures requires identification of the optimal input quantities each period and, therefore, knowledge of the production function. Since it is often difficult and very costly to assess the underlying production function, a question could be raised regarding the information required to permit an implementation of the model. Fortunately, even though the measures were derived using economic constructs that are unobservable and difficult to estimate, the measures themselves can be calculated from known observations - observations readily available from a company's existing accounting systems. The derivation of the observable measures, however, assumed the absence of any technological progress between periods. To properly assess technical and input-tradeoff efficiency improvements in the presence of technological progress between periods requires identification of the optimal input quantities each period. In theory, this requires knowledge of the production function. Therefore, one potential area in which the model can be refined is in the calculation of the observable measures that allow for technological progress and yet do not rely on explicit knowledge of the production function.

A second limitation of the model relates to its dependence on a restrictive assumption regarding the introduction of inefficiency or waste in production. The calculation of the technical efficiency (or waste) variance and, therefore, the change in technical efficiency, implies a proportional excess usage (waste) in both inputs by assuming that the input proportions at the point of actual usage and those represented by the production function are equal. This means that every time some breakage, shrinkage, theft, spoilage, or defect occurs in one input it will occur in the other in a fixed proportion. This is not likely to be in accord with the facts and suggests a refined model which avoids this assumption.

Furthermore, this study has shown that, by appealing to the ABC framework and to the EOQ model, the PLM measure fails to accurately measure the productivity contributions. The dependence on constant returns to scale is a major reason for the failure of the PLM model. Implications thus exist for the increased need to consider nonlinearities in the underlying production function when calculating productivity. This study also has shown that productivity performance with respect to setup and inventory management activities can be decomposed into two components: changes in technical efficiency and changes in setup-inventory tradeoff efficiency. The analysis, however, falls short of exhausting all the possible paths for measuring and analyzing non-unit level productivity performance. Future studies could develop an extension of the model to include performance with respect to the productivity of non-unit level activities other than setup activities.

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