GARCH OPTION PRICING, VALUING THE TARGET PRICE SUPPORT PROGRAM, AND A NEW EFFICIENCY CRITERION

By

TAEHOON KANG

Bachelor of Economics
Seoul National University
Seoul, Korea
1983

Master of Economics
Seoul National University
Seoul, Korea
1985

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Thesis Approved:

Wade Brorsen

Thesis Adviser

Brian Adam

Stephen R. Koontz

Jim Krehbiel

Thomas C. Collins

Dean of the Graduate College

PREFACE

This thesis consists of three separate essays. The first essay, titled "Conditional Heteroskedasticity, Asymmetry, and Option Pricing", introduces an asymmetric Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model. Commodity futures price changes are not distributed normally. Their distribution is leptokurtic and asymmetric. Volatility of price changes is not constant over time. Black's option pricing model which is based on normality and constant volatility is known to systematically misprice actual option premiums. The biases in Black's model may result from not considering stochastic volatility and non-normality. A GARCH option pricing model using Monte Carlo integration meets this objective. However, a limitation of the GARCH process is that it does not model skewness. The asymmetric GARCH process captures skewness in the mean equation. This essay also seeks to determine the most descriptive model of daily wheat futures price distributions among several models that consider non-normality and conditional heteroskedasticity. It also shows whether time-varying volatility and conditional non-normality can explain the biases in Black's option pricing model. One of the primary contributions of the essay is its new approach to modeling skewness. This essay also extends past research by considering non-normality as well as stochastic volatility and analyzing their effects on option pricing.

The second essay, titled "Valuing Target Price Support Programs with GARCH Average Option Pricing", seeks to determine the implicit premium of the government target price support program. The government deficiency payment program can be characterized as a subsidized put option except that acreage harvested is restricted. By participating in the program, farmers are protected from prices falling below the target price but can benefit from prices rising above the target. However, opportunity costs arise from the acreage restriction. In cases where revenue through acreage restriction exceeds the revenue from participating in the program, farmers would lose money from participating. Therefore, measuring the revenue from the government program is important for farmers to decide whether to participate in the program or not. Also, USDA could use the results to project participation, government cost, and in calculating advance deficiency payments. This essay improves on previous research in several ways. One is that it explicitly includes the non-normality and conditional volatility into the model using GARCH process. The other is that it considers average option pricing while past research did not. Past research considers the time value only for the period from the time of program sign-up until harvest, although the time value also occurs during the period of five (soon to be ten) months after harvest. A regression model based on the simulation results is provided for the GARCH average option pricing model to be easily used to project deficiency payments. The results can be used by extension to help farmers' decision making whether to participate in the program and by the USDA to project participation, government cost, and to calculate advance deficiency payments.

The third essay, titled "A New Efficiency Criterion: The Mean-Separated Target Deviations Risk Model", develops a new risk efficiency model which can order risky choices for decision makers whose monotonically increasing utility function lies within specified range. Conventional measures of risk do not distinguish between below-target and above-target outcomes, or else impose risk neutrality for above-target outcomes. The model is motivated by the intuition that decision makers respond in different ways to potential outcomes below a target return than to potential outcomes above a target return. The model measures return as expected value and risk as weighted sum of deviations below a target return and deviations above the target return, where the weight is determined by the decision maker's risk preferences. One contribution of this essay is the new risk measure. A second contribution is its provision for interval analysis. Like stochastic dominance with respect to a function, MSD can effectively reduce the efficient set by using appropriate ranges of absolute risk aversion coefficients. The MSD model is shown to be congruent with expected utility and also consistent with Stochastic Dominance rules. The MSD model also considers skewness in ranking alternatives. An empirical evaluation of a decision maker's choice of wheat marketing strategies shows that the criterion yields a smaller efficient set than alternative efficiency

criteria.

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ESSAY I

CONDITIONAL HETEROSKEDASTICITY, ASYMMETRY AND OPTION PRICING

CONDITIONAL HETEROSKEDASTICITY, ASYMMETRY AND OPTION PRICING

Abstract

Commodity futures price changes are not distributed normally. Their distribution is leptokurtic and asymmetric. Volatility of price changes is not constant over time. Black's option pricing model which is based on normality and constant volatility is known to systematically misprice actual option premiums. The biases in Black's model may result from not considering stochastic volatility and non-normality. A GARCH option pricing model using Monte Carlo integration meets this objective. However, a limitation of the GARCH process is that it does not model skewness. This paper introduces an asymmetric GARCH model that considers skewness. Results show that the asymmetric GARCH(2,1)-t process fits the data better than alternative models of Kansas City wheat futures prices. The GARCH Monte Carlo integration shows that Black's model underprices deep out-of-the-money put and call options relative to GARCH option pricing model when the true underlying process is a GARCH process. Differences between Black's model and the GARCH option model increase as time to maturity increases. When used to forecast actual option premiums, the mean squared error of the GARCH option pricing model for deep out-of-the-money put option is significantly smaller than that of Black's model. However, Black's model is sometimes better for at-the-money or in-the-money options.

Key Words : Asymmetry, GARCH, EGARCH, Options, Mispricing, Monte Carlo, Futures Prices, Kansas City Wheat.

CONDITIONAL HETEROSKEDASTICITY, ASYMMETRY AND OPTION PRICING

Introduction

The distribution of commodity futures prices is not normal but leptokurtic (Hall et al. 1989; Hudson et al. 1987). That is, there are more observations around the mean and more extreme values than with a normal distribution. Skewness and serial dependence of successive price changes are also well documented (Taylor 1985; Yang and Brorsen 1993). Ignoring the observed non-normality and stochastic volatility is likely to lead to biased estimates of option premia. Therefore, a correct option pricing model should model not only the stochastic volatility but also the conditional non-normality. It is well documented that the Black-Scholes option pricing model (OPM) based on constant volatility yields systematically biased estimates of deep in-the-money and deep out-of-the-money options (Black 1975; Johnson and Shanno 1987; Hull and White 1987). These inaccuracies may be due to the inappropriate assumptions about the futures price distribution. However, departures from normality have not been considered extensively in commodity option pricing because there is no clearly superior distributional specification (e.g. Eales and Hauser 1990).

Time-varying variance models can explain nonlinear dependence and leptokurtosis. The generalized autoregressive conditional heteroskedasticity (GARCH) process of Bollerslev (1986) was developed as an effective way of modeling the dynamics of volatility. Using a more general distributional assumption than normality, this model can be extended to capture the observed non-normality. Bollerslev (1987) suggested the GARCH(1,1)-t model, with one lag of the conditional variance and one of the squared residuals and with a conditional t distribution as a simple and very useful model. This GARCH(1,1)-t process fits most empirical data better than the GARCH(1,1)-normal or a mixed diffusion-jump process (Yang 1989). Myers and Hanson (1993) provided evidence that the GARCH OPM with a student distribution performs better than Black's OPM in predicting soybean option premia.

However, one limitation of the GARCH process is that it does not model the observed skewness. An exponential GARCH (EGARCH) model that captures skewness was suggested by Nelson (1991). The EGARCH model considers skewness by allowing the ARCH process to be asymmetric. Asymmetry in the dynamics of the mean return has not been considered in past research. This study introduces an asymmetric GARCH model that captures skewness in the mean equation and determines the most descriptive model of daily Kansas City wheat futures price distributions among several models that consider non-normality and conditional heteroskedasticity. The study also seeks to determine whether time-varying volatility and conditional non-normality can explain the biases in Black's option pricing model.

One of the primary contributions of the paper is its new approach to modeling skewness. The GARCH-t process fails to consider skewness and the

EGARCH process models skewness only in the variance equation. The asymmetric GARCH-t process enables the GARCH process to model skewness and the asymmetric EGARCH-t process models skewness both in the mean and the variance equations. The GARCH-t, the asymmetric GARCH-t, and the asymmetric EGARCH-t processes are estimated and the most likely model is selected.

This study improves upon Yang's methods in several ways. Yang assumed the length of lags in the mean equation was ten and that the t-distribution had ten degrees of freedom. He also assumed a GARCH(1,1) process. Such restrictive assumptions are avoided in this paper. Further, this study provides more accurate statistical tests of the i.i.d. assumption and goodness-of-fit. Also, Yang did not consider option pricing.

Previous option pricing models allowing stochastic volatility (e.g., Hull and White 1987) other than Myers and Hanson (1993) have not explicitly considered a conditional non-normal distribution. This paper considers non-normality as well as stochastic volatility and analyzes their effects on option pricing. This study extends Myers and Hanson in several ways. Significance tests are conducted on the performances of Black's OPM and GARCH OPM, and the simulated biases between the two OPM's are analyzed in terms of time to maturity effects and futures-exercise price ratio effects. Most important is that this paper uses models which can capture skewness while Myers and Hanson did not.

Procedures

The GARCH and EGARCH processes with and without asymmetry are estimated using maximum likelihood. The models are selected using likelihood ratio tests or the Schwarz model selection criterion. Then the effects on the option pricing of the model allowing time-varying volatility and conditional non-normality are analyzed for wheat options at the Kansas City Board of Trade. To examine the systematic mispricing error in Black's OPM, a Monte Carlo simulation and an out of sample comparison with actual option premiums are provided.

Econometric Models and Skewness

Skewness of the rate of return has received increasing attention in the past decade, as portfolio theory has been extended to include skewness along with mean return and variance to explain security preferences (Conine and Tamarkin 1981; Beedles and Simkowitz 1980; Junkus 1991; Kang et al. 1993). Skewness in daily futures returns is well documented. Twenty two of the thirty six futures price changes considered by Yang showed significant skewness. However, the observed skewness has not been well modeled with most models of asset price distributions.

Many competing statistical distributions have been proposed to model the departures from normality: a symmetric stable Paretian distribution (Mandelbrot 1963; Fama 1965), student t-distribution (Blattberg and Gonedes 1974), a mixture of normal distributions (Kon 1984), and a mixed diffusion-jump process (Akgiray and Booth 1988). However, since these models assume the independence of successive asset returns, they are inconsistent with empirical data that is known to be linearly or nonlinearly dependent. Further, these models are focused on capturing leptokurtosis. Jorion (1988) found that combining a jump process with a simple ARCH process provides a significantly better fit of the distribution of weekly exchange rates than either process alone. The mixed jump-diffusion process also models skewness. Combining the GARCH(1,1)-t process, however, with a jump process is not significantly better than the GARCH-t process alone (Brorsen and Yang 1992). Thus, a GARCH(1,1)-t process is used as the benchmark model, and other alternative models are compared with it.

While the GARCH model elegantly captures the volatility clustering in asset returns, it ignores the possible asymmetric response of variance to positive and negative residuals and restricts the parameters in the variance equation to be non-negative. Nelson (1991) suggested an Exponential GARCH (EGARCH) model that meets these objections. LeBaron (1989) reported that the EGARCH model explains skewness better in the distribution of weekly and monthly stock indices than the GARCH model. However, Nelson's EGARCH model considers skewness only in the variance equation.

In this paper, the GARCH under a student t distribution and the EGARCH under student t distributions will be considered. Each model will be estimated with and without asymmetry in the mean equation. Specifying the dynamics in the mean equation as asymmetric together with the GARCH or the EGARCH process might capture skewness more effectively. The GARCH process can model well-documented market anomalies such as the day-of-the-week effect (Chiang and Tapley 1983; Junkus 1986) and seasonality (Anderson 1985; Kenyon et al. 1987) both in means and variances and maturity effect (Milonas 1986) in the variance equation. In the GARCH process, the futures price changes, R_t, can be expressed as a stochastic process:

(1) $R_t = f(I_{t-1};\theta) + \varepsilon_t$

and $f(I_{t-1};\theta)$ denotes a function of I_{t-1} (the information set at time t-1) and the parameter vector θ. ε_t has a discrete time stochastic process, (ε_t), of the form;

(2) $$\varepsilon_t = \begin{cases} z_t h_t & \text{in the GARCH-normal model and} \\ ((v-2)/v)^{1/2}\omega_t h_t & \text{in the GARCH-t model.} \end{cases}$$

where z_t is i.i.d. normal with $E(z_t)=0$ and variance $Var(z_t)=1$ and ω_t is i.i.d. student with degrees of freedom v, $E(\omega_t)=0$ and $Var(\omega_t)=v/(v-2)$. Therefore, h^2_t is the time varying variance of ε_t. The GARCH(p,q) model expresses h^2_t as a linear function of past variance and past squared values of the process,

(3) $$h_t^2 = \alpha_0 + \alpha_i\sum_{i=1}^{q}\varepsilon_{t-i}^2 + \beta_j\sum_{j=1}^{p}h_{t-j}^2 .$$

Equation (1) is the mean equation and equation (3) is the variance equation. If h^2_t is to be the conditional variance of ε_t, it must be nonnegative. The GARCH model ensures nonnegativity by making h^2_t a linear combination of positive random variables. The EGARCH model ensures that h^2_t remains nonnegative by replacing h^2_t by $\ln(h^2_t)$ in equation (3)[1]. For example, the conditional variance of EGARCH(1,0)-t with one lag on the conditional variance is:

$$(4)\ \ln(h^2_t) = \alpha_0 + \beta h^2_{t-1} + \eta(\varepsilon_{t-1}/h_{t-1}) + \varphi(\mid \varepsilon_{t-1}/h_{t-1} \mid - (2/\pi)^{1/2}).$$

Over the range $0 < \varepsilon_{t-1}/h_{t-1} < \infty$, $\ln(h_t^2)$ is linear in $\varepsilon_{t-1}/h_{t-1}$ with slope $\eta + \varphi$, and over the range $-\infty < \varepsilon_{t-1}/h_{t-1} < 0$, $\ln(h_t^2)$ is linear in $\varepsilon_{t-1}/h_{t-1}$ with slope $\eta - \varphi$. If $\eta = 0$ $\ln(h_{t2})$ responds symmetrically to $\varepsilon_{t-1}/h_{t-1}$, but if $\eta \neq 0$ $\ln(h_{t2})$ responds asymmetrically. Thus the coefficient η is related to asymmetry in the variance equation.

In the GARCH process, the mean and the variance equations to be estimated are, respectively,

$$(5)\quad R_t = a_0 + \sum_{i=1}^{m} \lambda_i R_{t-i} + a_1 D_{MON} + a_2 D_{TUE} + a_3 D_{WED} + a_4 D_{THU} + a_5 SIN(2\pi K/252) + a_6 COS(2\pi K/252) + a_7 SIN(2\pi K/126) + a_8 COS(2\pi K/126) + \varepsilon_t,$$

$$(6)\quad h^2_t = \alpha_0 + \alpha_i \sum_{i=1}^{q} \varepsilon^2_{t-i} + \beta_j \sum_{j=1}^{p} h^2_{t-j} + b_1 D_{MON} + b_2 D_{TUE} + b_3 D_{WED} + b_4 D_{THU} + b^5 SIN(2\pi K/252) + b_6 COS(2\pi K/252) + b_7 SIN(2\pi K/126) + b_8 COS(2\pi K/126) + b_9 TTM,$$

where R_t is the logarithmic difference of daily returns at time t, the λ_i's are the

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[1] EGARCH(p,q) is $\ln(h_t^2) = (1+\sum_{i=1}^{q} \psi_i L^i)(1-\sum_{j=1}^{p} \Delta_j L^j)^{-1} g(z_{t-1})$, where L is a lag operator and $g(z_t)$ is a zero-mean i.i.d. random sequence that allows the conditional variance h_t^2 to asymmetrically respond to price falling and rising.

coefficients of lagged price changes, and m is the length of lags. The length of lags in the mean equation is identified with the Schwarz criterion[2]. D denotes dummy variables for each day of the week; $D_{MON} = 1$ if Monday and 0 otherwise, $D_{TUE} = 1$ if Tuesday and 0 otherwise, $D_{WED} = 1$ if Wednesday and 0 otherwise, and $D_{THU} = 1$ if Thursday and 0 otherwise. SIN and COS represent the sine and cosine functions, respectively, and π is approximated as 3.14. K in the sine and cosine functions is the number of trading days after January 1 of the particular year. Denominators in the sine and cosine functions are the specified cycle length in trading days, so 252 indicates a one year cycle and 126 a half year cycle. TTM is the time to maturity measured in the number of trading days prior to maturity.

The asymmetric pricing model has been extensively applied to the structure of farm-retail price transmission (e.g., Kinnucan and Forker 1987; Boyd and Brorsen 1988). The asymmetric model can be used as an appropriate way to capture the asymmetric responses of current variables to the change of past shock variables. Restrictions on short selling, asymmetries in information, preferences of investors, and market psychology might cause the differing responses of price to the past price rising or falling. The asymmetric GARCH model is a special case of the Threshold Autoregressive model of Tong and Lim

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[2] Schwarz's SC criterion is obtained by $SC(m) = \ln(SSE_m) + Q_m * \ln(T)/T$, where m is the length of lags, SSE_m is the squared sum of residuals, Q_m is the number of parameters and T is the number of observations. The value of m that minimizes SC is selected as the length of lags in the model.

(1980).

The model with asymmetry in the mean equation is obtained by segmenting the lagged price changes into one set for rising changes and another set for falling changes. The logarithmic changes in returns, R_t, are segmented as,

$$RP_t = \begin{cases} R_t\,,\ R_t \geq 0 \\ 0, \text{ otherwise,} \end{cases}$$

$$RN_t = \begin{cases} R_t\,,\ R_t < 0 \\ 0\,, \text{ otherwise,} \end{cases}$$

and the asymmetric model is specified as,

$$(7)\ R_t = a_0 + \sum_{i=1}^{m} \delta_i RP_{t-i} + \sum_{i=1}^{m} \omega_i RN_{t-i} + a_1 D_{MON} + a_2 D_{TUE} + a_3 D_{WED} + a_4 D_{THU} + a_5 SIN(2\pi K/252) + a_6 COS(2\pi K/252) + a_7 SIN(2\pi K/126) + a_8 COS(2\pi K/126) + \varepsilon_t.$$

where δ_i and ω_i represent the net effect of the i^{th} positive and negative changes of R_t, respectively.

The variance equation to be estimated in the GARCH model is (6) and that for the EGARCH model is obtained by adding the day-of-the-week effect dummies, seasonality, and time to maturity variables into equation (4).

The maximum likelihood estimates of alternative models are obtained using the statistical software package GAUSS. For the GARCH model, since the GARCH terms (α and β in the variance equation) are restricted to be nonnegative, inequality restrictions are imposed by taking the exponential of the parameters. Degrees of freedom is restricted to be greater than three for computational concerns. The variance of the first 15 observations of price changes was used as initial variance. The starting algorithm is Polak-Ribiere-type Conjugate Gradient method which performs well initially when the starting point is poor and a step size of one. After a few iterations, the algorithm is switched to the Davidon-Fletcher-Powell method and the Brent step-size method is used. The final estimates are obtained with the Newton method so that the Hessian is used to estimate the information matrix. All derivatives are calculated numerically.[3]

The asymmetry hypothesis is tested in two ways. One is that the total impact of past price increases is the same as that of past price decreases:

(8) $H_0: \quad \sum_{i=1}^{m} \delta_i = \sum_{i=1}^{m} \omega_i$

H_A : H_0 is not true.

The other is that the speed of adjustment to price increases and to price decreases are the same (Boyd and Brorsen).

(9) $H_0: \quad \delta_i = \omega_i$, for all i

H_A : H_0 is not true.

For the hypothesis tests of equations (8) and (9), Wald-F statistics are used. The Wald test is not invariant in nonlinear models, but it is still asymptotically valid (Dagenais and Dufour 1991). Estimated t-ratios of the parameters of the skewness term (η) in equation (4) provide tests of asymmetry in the variance

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[3]The estimation time took about 3 to 15 hours on a 486DX2/66MHZ computer. Estimation time depends heavily on the starting values.

equation of the EGARCH model.

Model Selection and Validation

Selecting between the GARCH-t and the asymmetric GARCH-t models is performed by the likelihood ratio test, since the models are nested. Model selection between the asymmetric GARCH-t and the asymmetric EGARCH-t models are conducted by the difference of the Schwarz criteria of the two models. The Schwarz criterion penalizes the model with more parameters and thus is useful for selecting among nonnested models with different numbers of parameters.

If the GARCH models are well-specified and fit the sample data, the standardized residual generated by the GARCH models should be i.i.d. normal or student. The Ljung-Box and McLeod-Li test statistics are provided to determine if the serial dependence is revealed in the rescaled residuals $(\varepsilon_t/\hat{h}_t)$ and squared rescaled residuals $((\varepsilon_t/\hat{h}_t)^2)$ of the selected model. The Brock-Dechert-Scheinkman (BDS) test (Brock et al. 1991) is used to test the null hypothesis that $\{R_t\}$ is i.i.d.. The BDS statistic is based on the correlation integral. The statistic for raw data is asymptotically distributed as a standard normal random variable under the null hypothesis. However, Brock et al. have shown that the distribution of the BDS statistic is not standard normal when the data are GARCH residuals. Therefore, the test in this study is more accurate than the test by Yang which assumed the BDS statistic with GARCH residuals had a standard normal distribution. The tables in Brock et al.(p.279) are used to obtain critical values of the test statistic.

The Kolmogorov-Smirnov goodness-of-fit test is used to determine if the residuals have a t-distribution. If the largest absolute deviation between the cumulative distributions of the rescaled residuals and the theoretical student distribution is bigger than the critical value, the null hypothesis of no significant difference between the two distributions is rejected. The rescaled residuals are multiplied by $(v/(v-2))^{1/2}$, where v is the estimated degrees of freedom of the model. This adjustment is needed because the variance of a t-distribution is $v/(v-2)$. Yang's test is biased, since he used the rescaled residuals without multiplying by $(v/(v-2))^{1/2}$.

Implications for Option Pricing

Using implied volatility, McBeth and Merville (1979) argued that the Black-Scholes formula overprices out-of-the-money options and underprices in-the-money options. However, Rubinstein (1985) argued that their results were not always true. Johnson and Shanno (1987) obtained numerical results for general cases in which the instantaneous variance obeys some stochastic processes. Under the situation where the volatility is a separate stochastic variable from the stock price, Hull and White (1987) have shown that the Black-Scholes formula overprices options that are at- or close-to-the-money and underprices options that are deep in- and deep out-of-the-money. This paper examines how well Black's OPM estimates option premia under non-normality as well as conditional heteroskedasticity.

Under the assumption of risk-neutrality, Black's (1976) commodity option price is a function of five underlying parameters: the current futures price (F_t), the exercise price of the option (X), the time to maturity of the option (T-t), the risk-free rate of interest (r), and the variance of futures prices (σ^2).

$$(10) \qquad B = \begin{cases} X^*N(-d_2) - F_t^*N(-d_1) & \text{for put,} \\ F_t^*N(d_1) - X^*N(d_2) & \text{for call,} \end{cases}$$

where $d_1 = [\ln(F_t/X) + (\sigma^2/2)(T-t)]/\sigma(T-t)^{1/2}$,

$d_2 = [\ln(F_t/X) - (\sigma^2/2)(T-t)]/\sigma(T-t)^{1/2}$, and

N() is normal cumulative density function.

GARCH OPM yields the expected option prices at maturity using a Monte Carlo integration. Two sets of T-t random numbers are generated: one from a t-distribution with v degrees of freedom and another from a standard normal distribution. Time is measured in number of trading days. The time-varying conditional variances are generated for T-t periods using estimates from the selected model. Then, with the conditional variances, the futures prices F_t are simulated for T-t periods to get the futures price at maturity. Denoting this price at maturity $\{F_T\}_i$, the simulated option prices are

$$(11) \qquad G = \begin{cases} e^{-r(T-t)}(1/n)\sum_{i=1}^{n} \max[X - \{F_T\}_i ,0] & \text{for call,} \\ e^{-r(T-t)}(1/n)\sum_{i=1}^{n} \max[\{F_T\}_i - X ,0] & \text{for put,} \end{cases}$$

where n = 10000 is the number of replications of this procedure.

One efficient way to improve the accuracy of this calculation is the control variate technique (Hammersley and Handscomb 1964; Boyle 1977). This technique replaces the problem under consideration by a similar but simpler problem which has an analytical solution. The solution of the simpler problem is used to increase the accuracy of the solution to the more complex problem. For this purpose, the GARCH option price, G in equation (11), is replaced by Black's option price, B in equation (10). The biases in B are reduced by an error correction term which is the difference between the GARCH option price G_1 and the control variate G_2 that mimics the behavior of GARCH option price and can be easily evaluated (see Boyle for details). The GARCH option price under control variate technique is, then,

(12) $G^* = B + (G_1 - G_2)$.

In the simulation, B is obtained analytically using the Black's OPM, and G_1 and G_2 are obtained from Monte Carlo methods as given in equation (11).

<u>Simulated Differences</u>. In the Monte Carlo simulation, the unconditional variance of the GARCH process ($\hat{\alpha}_0/(1 - \sum_{i=1}^{q} \hat{\alpha}_i - \sum_{j=1}^{p} \hat{\beta}_j)^{-1}$) is used as an initial volatility to generate conditional variances for G_1, and as a constant volatility for Black's analytical solution B and for Monte Carlo integration G_2 that mimics Black's analytical solution. The seasonality and the day of the week effects are not included in the Monte Carlo integration. Two sets of T-t random numbers are generated: one from a student distribution with degrees of freedom $\hat{v}$ and the other from a standard normal distribution. The student

random numbers are used for G_1 and the standard normal random numbers are used for G_2. These random numbers are generated using the same seed so that the random errors in G_1 and G_2 are positively correlated.

The issue in this portion of the paper is to determine the differences between Black's OPM and GARCH OPM. The difference between the Black's OPM and GARCH OPM may be caused by not considering the observed conditional heteroskedasticity and non-normality in Black's OPM. The extent of the difference can be measured by the absolute difference $(B - G^*)$ or by the percentage difference $\xi = (B - G^*)/G^*$. The differences for short-lived option premia differ from long-lived option premia. The differences are also different by how much the option is in the money or out of the money. Commodity futures options are mostly within 10% in or out of the money. For the case of a put option, the out-of-the-money option is examined at the levels of 1.10 and 1.05 in the futures-exercise price ratio (F_t/X), the at-the-money option at the level of 1.0, and the in-the-money option at the level of 0.95 and 0.90. For the case of a call option, the out-of-the-money option is examined at the levels of 0.90 and 0.95 in the futures-exercise price ratio, the at-the-money option at the level of 1.0, and the in-the-money option at the level of 1.05 and 1.10. In the Monte Carlo simulation, exercise price is set equal to \$1.00. The differences are measured from six months prior to through one half month to maturity.

The asymptotic t-statistics for the simulated differences are provided.

These could be used to determine whether the reported pricing biases are significantly different from zero. They are computed by the ratios of the simulated differences to the standard deviations of the differences.

Actual Differences. To examine the ability of the GARCH OPM and Black's OPM to predict actual premia, option prices are estimated for 1991 Kansas City wheat futures for two month periods prior to maturity for each March, May, July, September, and December contract. The ranges of futures prices during the simulation period, exercise prices for in-, at-, and out-of-the-money options are shown in Table 7. The risk-free interest rate is assumed constant during the simulation period at $r=5.5\%$[4]. In this out of sample simulation, 20 day historical volatilities are used for generating unconditional variances in the GARCH integration process G_1, for Black's analytical solution B, and for Black's integration process G_2.

Results are given for both put and call options. The mean errors and root mean squared errors (RMSE) of Black's OPM and GARCH OPM are computed. Further, the Ashley-Granger-Schmalensee test is used to determine if the mean squared error of Black's OPM is equal to that of the GARCH OPM. Statistical tests were based on White's (1980) heteroskedastic-consistent covariance matrix because of likely heteroskedasticity due to differences in maturity and

[4]During the simulation period, the range of the rate of return on Treasury bills was (0.052, 0.058).

also based on Newey-West's (1987) autocorrelation-consistent matrix because the Ashley-Granger-Schmalensee test assumes independence.

Sample Data

To estimate the alternative statistical models, the first differences of the natural logarithms of the daily futures closing prices of wheat at Kansas City Board of Trade are used. The data are for the period of Jan. 1982 to Sep. 1990. The data were created using Continuous Contractor from Technical Tools. Kansas City wheat futures contracts are traded based on five maturities: March, May, July, September, and December. The price series used is a continuous combination of the five contracts. The rollover date is the 21st day of the month prior to delivery. Log differences are taken before splicing so that no outlier is created at the rollover date.

Table 1 shows summary statistics for daily logarithmic changes in the closing prices of wheat futures contracts at Kansas City Board of Trade. The departures from normality are apparent from the high kurtosis and skewness. The daily put option premia over the simulation period are collected from the Kansas City Grain Market Review.

Empirical Results

Model Selection and Validation

Table 2 shows the estimated log-likelihoods and the test statistics associated with the null hypotheses of no asymmetry. In the asymmetric GARCH model, the total impact of past rising prices is not significantly different from that of past falling prices. However, the speed of adjustment for price rising is significantly different from that of price falling, implying significant asymmetries in mean returns. The absolute values of coefficients of the lagged falling prices are greater than those of lagged rising prices in Table 4. Significant asymmetry is present in the mean equation of the asymmetric EGARCH model (Table 2). The skewness term in the EGARCH model is not significantly different from zero.

Table 3 contains the test statistics of model selection. Likelihood ratio statistics and differences in Schwarz criteria are used to select between nested models and between nonnested models, respectively. The asymmetric EGARCH(1,0)-t process is not selected as better than the asymmetric GARCH(1,1)-t process[5]. The asymmetric GARCH(2,1)-t is selected over the asymmetric GARCH(1,1)-t which is selected over the GARCH(1,1)-t. The asymmetric GARCH(2,1)-t is also favored over the asymmetric GARCH(2,2)-t and over the asymmetric GARCH(3,1)-t. Thus, the asymmetric GARCH(2,1)-t process is selected as the best fit of the data and so its estimates are used to

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[5] The choice between the asymmetric GARCH(1,1)-t and the asymmetric EGARCH(1,0)-t is based on the difference in Schwarz criteria. The Schwarz criterion has a heavy penalty for additional parameters, especially when the sample size is big. However, in this case, the maximized log-likelihood value of the asymmetric EGARCH(1,0)-t is smaller than that of the GARCH(1,1)-t, and thus it can not be said that the asymmetric EGARCH(1,0)-t is better than the asymmetric GARCH(1,1)-t, even without referring to the Schwarz criterion.

obtain option prices.

Table 4 reports estimates and test statistics of the asymmetric GARCH(2,1)-t model. The estimated GARCH terms are all positive and significant[6]. The sum of GARCH terms (α, β_1, and β_2) is less than one implying stationarity[7]. Mean returns differ by day of the week, but variances do not. Significant seasonal patterns are revealed both in the mean and in volatility. The Ljung-Box and McLeod-Li tests (Table 4) do not detect any linear or second moment autocorrelations over time with the standardized data, which implies the GARCH-t process removed all the correlation in the first and second moments. The BDS statistics show that the null hypothesis of i.i.d. is rejected with the raw data (Table 4), implying that Kansas City wheat futures price changes are not i.i.d.. For the rescaled residuals, however, the BDS statistics, do not identify nonlinear dependence. The null hypothesis that the GARCH rescaled residuals follow a student distribution is not rejected at the 5% significance level. Yang did not adjust by the degrees of freedom and found

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[6] The parameters α, β_1, and β_2 are all restricted to be greater than or equal to zero. Thus the null hypothesis that the parameter is zero lies on the boundary of the parameter space. Wald-type test of null hypotheses which lie on the boundary of the parameter space do not have the usual asymptotic normal distribution (Moran 1971). Also the inequality constraint was imposed on the parameter α using an exponential transformation, so the t ratio of α is computed as $t = e^{\hat{\alpha}}/(e^{\hat{\alpha}} s^2_{e^{\hat{\alpha}}} e^{\hat{\alpha}})^{1/2}$, where $s^2_{e^{\hat{\alpha}}}$ is the standard error of $e^{\hat{\alpha}}$. But, unfortunately, Wald tests are not invariant to nonlinear transformations. Thus, the hypothesis tests should be interpreted with caution.

[7] $\alpha + \beta_1 + \beta_2 < 1$ is a sufficient condition, although not a necessary condition. Most past research is not clear on this point.

significance level. Yang did not adjust by the degrees of freedom and found that a GARCH-t model could be rejected. Thus the adjustment is shown to be important.

Differences between Black's Option Pricing and GARCH Option Pricing

Table 5 presents absolute and percentage differences between put option premiums with Black's OPM and GARCH OPM. The Black's OPM yields significantly lower premiums than the GARCH OPM for deep in- and deep out-of-the-money put options. The put option value depends on the left tail of the terminal distribution. Therefore, Black's OPM based on the normal distribution tends to yield lower option premiums than the GARCH OPM. Absolute differences increase as time to maturity increases in deep in- and deep out-of-the-money put options (Table 5, Panel A). Percentage differences for deep in-the-money option also increase as time to maturity increases, but those for deep-out-of-the money option decrease as time to maturity increases (Table 5, Panel B). As time to maturity decreases, the time-value of deep out-of-the-money option decreases very fast and eventually becomes zero. Therefore, deep out-of-the-money options close to maturity show extremely high percentage differences. Black's OPM yields at-the-money option premia significantly higher than does the GARCH OPM. Percentage differences for at-the-money put options decrease as time to maturity increases.

Table 6 presents absolute and percentage differences between call option premiums by Black's OPM and GARCH OPM. The Black's OPM yields significantly lower premiums than the GARCH OPM for deep out-of-the-money call options. Since the call option value depends on the right tail of the terminal distribution, Black's OPM based on the normal distribution tends to yield lower option premiums than the GARCH OPM. Absolute differences increase as time to maturity increases in deep out-of-the-money call options (Table 6, Panel A). However, percentage differences decreases as time to maturity increases (Table 6, Panel B), because as time to maturity decreases, the time-value of deep out-of-the-money options decreases very fast and eventually becomes zero. Black's OPM prices at-the money call options higher than GARCH OPM.

The simulation results confirm Hull and White's (1987) findings that the Black-Scholes model underprices in- and out-of-the-money options when stochastic volatility is present. Their argument that the Black-Scholes model overprices close-to-the-money options is also confirmed. The absolute differences are small, which agrees with Hull and White, and absolute differences between the two OPM's are larger for at-the-money options than for in- or out-of-the-money options.

The asymptotic t-statistics (Tables 5 and 6) provide some evidences that the reported option pricing errors are not due to sampling errors. In particular, differences between Black's option price and GARCH option price for deep out-of-the-money options are always significantly different from zero.

Performance of Black's and GARCH Option Pricing

Table 8 shows the result for an out of sample simulation. Performance of each model for at-, in-, and out-of-the-money options is shown. For out-of-the-money put options, the mean error and root mean squared error (RMSE) of the GARCH OPM are smaller than those of Black's OPM. For out-of-the-money call options, mean error of GARCH OPM is smaller than that of Black's OPM, but RMSE of GARCH OPM is larger than that of Black's OPM. For at-, and in-the-money put and call options, however, mean error and RMSE of Black's OPM are smaller than those of GARCH OPM. Black's OPM performs worse than GARCH OPM for deep-out-of-the money options, but performs better for at- and in-the-money options.

Ashley-Granger-Schmalensee (AGS) test (Table 9) supports the hypothesis of mean squared error of Black's OPM being significantly larger than that of GARCH OPM for out-of-the-money put options, but does not support it for out-of-the-money call options. AGS test also shows that mean squared errors for at- and in-the-money put options and at-the-money call options by GARCH OPM are greater than those by Black's OPM. AGS test for in-the-money call options is inconclusive. Therefore, GARCH OPM performs better than Black's OPM for out-of-the-money put options, but Black's OPM still performs better than GARCH OPM for other cases.

Conclusions

This paper introduces an asymmetric GARCH model that captures asymmetries in the mean equation, and determines the most likely distribution among alternative autoregressive conditional heteroskedasticity models. The asymmetric GARCH(2,1)-t with two lags of the conditional variance and one lag on the squared residuals, which considers asymmetry in the mean equation, was selected as the most likely among the alternative models. The alternative models considered were the GARCH-t and the asymmetric EGARCH-t models.

The Monte Carlo integration using the estimated asymmetric GARCH(2,1)-t parameters shows that Black's model values deep out-of-the-money put options and deep out-of-the-money call option less than the GARCH option pricing model does. However, Black's option pricing model values at-the-money put and call option premiums higher than the GARCH option pricing model does. Differences between Black's model and the GARCH option model increase as time to maturity increases, which confirms Hull and White's findings. The GARCH option pricing model predicts actual option premiums more accurately than Black's model for deep out-of-the-money option, but Black's model is still at least as good as the GARCH option pricing model in other cases.

Table 1. Summary Statistics of Daily Kansas City Wheat Futures Prices over January 1982 through August 1990.[a]

	Statistics
Sample Size(n)	2191
Mean (μ)	-0.0109
Standard Deviation (σ)	0.9774
Skewness[b]	0.6471*d
Kurtosis[c]	11.1585*

[a] Units are percentages. $R_t = [\ln(P_t) - \ln(P_{t-1})]*100$.

[b] Skewness is computed by $\sum_{t=1}^{n} (R_t-\mu)^3/(n-1)\sigma^3$.

[c] Excess kurtosis is computed by $\sum_{t=1}^{n} (R_t-\mu)^4/(n-1)\sigma^4 - 3$.

[d] Asterisks denote the null hypothesis of normality (i.e., zero skewness and zero kurtosis) are rejected at a 5% level based on the critical values by Snedecor and Cochran (1980).

Table 2. Estimated Log-likelihoods and Tests of Asymmetry with Alternative Models of Daily Futures Prices of Kansas City Wheat

Model	Maximized Log-Likelihood	Statistics for Asymmetries		
		Mean[a]		Variance[b]
		Total	Speed	
Asymmetric GARCH(1,1)-t	-2521.3	3.22	5.93*[c]	na[d]
Asymmetric GARCH(2,1)-t	-2515.4	3.33	5.86*	na
GARCH(2,1)-t	-2520.6	na	na	na
Asymmetric GARCH(2,2)-t	-2515.0	3.50	6.05*	na
Asymmetric GARCH(3,1)-t	-2514.7	0.87	3.75*	na
Asymmetric EGARCH(1,0)-t	-2523.5	8.08*	21.23*	1.39

[a] Statistics for asymmetries in the mean equations are distributed as $F(1,2191)$ for total impact and $F(3,2191)$ for the speed of adjustment under the null hypothesis that there is no asymmetries.

[b] Statistics for asymmetries in the variance equations are the t-statistics of the parameter representing skewness (η in equation (4)).

[c] Asterisks denote rejection of the null hypothesis of no asymmetry at the 5% significance level.

[d] Not applicable.

Table 3. Test Statistics of Model Selection with Alternative Models

Hypotheses		
Null	Alternative	Statistics
Asymmetric GARCH(1,1)-t	Asymmetric EGARCH(1,0)-t[b]	-3.29
Asymmetric GARCH(1,1)-t	Asymmetric GARCH(2,1)-t[a]	11.80[*b]
GARCH(2,1)-t	Asymmetric GARCH(2,1)-t[a]	10.40[*]
Asymmetric GARCH(2,1)-t	Asymmetric GARCH(2,2)-t[a]	0.42
Asymmetric GARCH(2,1)-t	Asymmetric GARCH(3,1)-t[a]	1.40

[a] Likelihood ratio test statistic is obtained by $2T*(L_1 - L_0)$, where T is the number of observations, L_1 is the loglikelihood values under alternative hypothesis, and L_0 under null hypothesis.

[b] The statistic reported is the difference in Schwarz criteria which is obtained by $2T*(L_1 - L_0)-(K_1 - K_0)*\ln(T)$, where K_1 and K_0 are the number of parameters under alternative and null hypothesis, respectively.

[c] Asterisk denotes rejection of the null hypothesis in favor of the alternative hypothesis at the 5% significance level.

Table 4. Statistics and Test Results of the Estimated Asymmetric GARCH(2,1)-t Process

	Estimated Coefficients	(t-ratio)
Mean		
Intercept	-0.125*[a]	(-3.05)
Lag 1 positive	0.027	(0.64)
Lag 1 negative	0.151*	(3.69)
Lag 2 positive	0.010	(0.26)
Lag 2 negative	-0.134*	(-3.51)
Lag 3 positive	0.050	(1.23)
Lag 3 negative	-0.071	(-1.81)
D_{MON}	0.041	(0.88)
D_{TUE}	0.077	(1.68)
D_{WED}	0.152*	(3.55)
D_{THU}	0.036	(0.82)
SIN252	-0.018	(-0.81)
COS252	0.061*	(2.79)
SIN126	0.025	(1.19)
COS126	-0.043*	(-2.09)
Variance		
Intercept	0.061	(1.74)
Alpha	0.160*	(2.47)[b]
Beta1	0.190*	(2.27)[b]
Beta2	0.592*	(7.25)[b]
D_{MON}	0.048	(0.81)
D_{TUE}	0.014	(0.22)
D_{WED}	-0.096	(-1.63)
D_{THU}	-0.039	(-0.70)
SIN252	0.006	(1.03)
COS252	-0.018*	(-2.47)
SIN126	0.007	(1.21)
COS126	-0.003	(-0.53)
Maturity	-0.001	(-0.31)
Degrees of Freedom		
v	7.31[c]	
Wald F statistics		
Day of Week in Mean	3.66*	
Seasonality in Mean	3.12*	
Day of Week in Variance	1.26	
Seasonality in Variance	2.92*	

(Table 4 Continued)

Ljung-Box and Mcleod-Li[d]	
$\varepsilon_t/h_t(12)$	13.66
$\varepsilon^2_t/h^2_t(12)$	20.41
BDS tests $(\varepsilon=\sigma)$[e]	
Raw Data	
Dimension = 3	13.04*
Dimension = 6	17.58*
Dimension = 9	24.53*
Rescaled Data	
Dimension = 3	0.47
Dimension = 6	-0.57
Dimension = 9	-0.07
Goodness-of-fit[f]	
D_{max}	0.013

a Asterisks denote the rejection of the null hypothesis at the 5% significance level. Values in parentheses are the t-statistics.

b Since the GARCH terms are restricted to be positive, the null hypothesis lies on the boundary of the parameter space. Under the assumptions of Moran (1971), the t-statistic is distributed as a mixture of a degenerate distribution and a half t-distribution. Hypothesis tests can still be conducted in the usual fashion with t-tests. The t ratio of α is computed as

$t = e^{\hat{\alpha}}/(e^{\hat{\alpha}} s^2_{e^{\hat{\alpha}}} e^{\hat{\alpha}})^{1/2}$, where $s^2_{e^{\hat{\alpha}}}$ is the standard error of $e^{\hat{\alpha}}$, because the inequality constraint was imposed on the parameter α using an exponential transformation.

c The degrees of freedom is restricted to be greater than three.

d Both null hypotheses that ε_t/h_t are not autocorrelated and that ε^2_t/h^2_t are not autocorrelated are tested with twelve degrees of freedom. Test statistics are distributed asymptotically as $\chi^2_{(12)}$ under the null hypothesis.

e The null hypothesis is that the standardized residuals are i.i.d. The hypothesis test is based on Table F.4 in Brock et al. (p.279).

f The critical value of this test is $D_c = 1.36/T^{1/2} = 0.0299$ where T is the sample size (Shannon, 1975).

Table 5. Differences between Black's Option Pricing and GARCH Option Pricing for Put Options When the True Process Is GARCH

	Time to maturity (months)					
	0.5	1	1.5	3	4.5	6
Panel A: Absolute differences[a]						
Deep In-the-money	-0.002	-0.008	-0.018	-0.030	-0.047	-0.059
($F_t/X=0.90$)	(-0.20)	(-0.66)	(-1.09)	(-1.30)	(-1.71)	(-1.91)
In-the-money	-0.001	-0.007	-0.005	-0.019	0.059	-0.010
($F_t/X=0.95$)	(-0.15)	(-0.59)	(-5.36)	(-0.88)	(2.39)	(-0.39)
At-the-money	0.051	0.052	0.078	0.070	0.069	0.040
($F_t/X=1.0$)	(7.89)	(5.26)	(6.37)	(3.78)	(3.16)	(1.54)
Out-of-the-money	-0.021	-0.015	-0.017	0.008	0.061	0.024
($F_t/X=1.05$)	(-5.19)	(-2.45)	(-1.81)	(0.55)	(3.46)	(1.14)
Deep Out-of-money	-0.009	-0.019	-0.036	-0.025	-0.028	-0.036
($F_t/X=1.10$)	(-3.67)	(-5.35)	(-5.92)	(-2.58)	(-2.00)	(-1.99)
Panel B: Percentage differences (%)[b]						
Deep In-the-money	-0.02	-0.08	-0.18	-0.30	-0.46	-0.57
($F_t/X=0.90$)	(-8.11)	(-9.85)	(-6.09)	(-4.08)	(-9.87)	(-0.16)
In-the-money	-0.03	-0.14	-0.10	-0.32	0.95	-0.17
($F_t/X=0.95$)	(-14.88)	(-9.20)	(-8.83)	(-3.09)	(3.48)	(-0.83)
At-the-money	4.89	3.47	4.28	2.67	2.17	0.01
($F_t/X=1.0$)	(0.40)	(0.58)	(0.41)	(0.91)	(1.01)	(0.96)
Out-of-the-money	-31.67	-6.73	-4.06	0.86	4.42	1.31
($F_t/X=1.05$)	(-11.35)	(-0.43)	(-2.69)	(1.50)	(0.91)	(1.27)
Deep Out-of-money	-97.67	-64.92	-44.54	-9.05	-5.04	-4.20
($F_t/X=1.10$)	(-5.58)	(-12.46)	(-12.95)	(-4.48)	(-0.63)	(-3.66)

a Black's option price minus GARCH option price when exercise price is set equal to $1.00. The absolute differences are measured in ¢/bushel.

b [(Black's price - GARCH price)/GARCH price]*100.

Asymptotic t-statistics are in parentheses.

Table 6. Differences between Black's Option Pricing and GARCH Option Pricing for Call Options When the True Process Is GARCH

	Time to maturity (months)					
	0.5	1	1.5	3	4.5	6
Panel A: Absolute differences (cents per bushel)[a]						
Deep Out-of-money	-0.003	-0.016	-0.037	-0.046	-0.036	-0.061
($F_t/X = .90$)	(-2.63)	(-4.96)	(-5.62)	(-4.39)	(-2.41)	(-2.75)
Out-of-the-money	-0.026	-0.028	-0.016	0.027	0.002	0.017
($F_t/X = .95$)	(-7.42)	(-4.24)	(-1.24)	(1.69)	(0.07)	(0.66)
At-the-money	0.050	0.063	0.074	0.093	0.061	-0.037
($F_t/X = 1.0$)	(7.32)	(5.77)	(5.05)	(4.65)	(2.03)	(-1.03)
In-the-money	0.0003	-0.016	0.013	-0.0004	-0.022	0.003
($F_t/X = 1.05$)	(0.03)	(-1.09)	(0.80)	(-0.01)	(-0.64)	(0.07)
Deep In-the-money	0.003	-0.006	-0.066	-0.011	-0.001	-0.043
($F_t/X = 1.10$)	(0.24)	(-0.37)	(-3.09)	(-0.36)	(-0.03)	(-1.01)
Panel B: Percentage differences (%)[b]						
Deep Out-of-money	-98.37	-79.15	-61.96	-21.92	-8.68	-9.02
($F_t/X = .90$)	(-3.32)	(-9.53)	(-14.97)	(-3.61)	(-2.38)	(-2.11)
Out-of-the-money	-43.25	-14.32	-4.41	3.24	0.12	1.03
($F_t/X = .95$)	(-1.28)	(-8.23)	(-4.03)	(5.28)	(0.81)	(1.35)
At-the-money	4.79	4.19	4.03	3.61	1.91	-0.99
($F_t/X = 1.0$)	(0.92)	(0.95)	(-0.97)	(0.95)	(0.07)	(-1.12)
In-the-money	0.01	-0.30	0.26	-0.01	-0.35	0.04
($F_t/X = 1.05$)	(13.00)	(-4.61)	(4.89)	(-2.35)	(-0.47)	(1.57)
Deep In-the-money	0.03	-0.06	-0.66	-0.11	-0.01	-0.41
($F_t/X = 1.10$)	(-1.00)	(-13.95)	(-14.33)	(-6.96)	(-4.04)	(-0.39)

a Black's option price minus GARCH option price when exercise price is set equal to $1.00. The absolute differences are measured in ¢/bushel.

b [(Black's price - GARCH price)/GARCH price]*100.

Asymptotic t-statistics are in parentheses.

Table 7. Ranges of Futures Prices During the Out of Sample Simulation Period and Strike Prices of In-, At-, and Out-of-the-Money Options for Each Contract[a]

Maturity	March	May	July	September	December
Panel A: Put Option					
Price Ranges	(2.55,2.65)	(2.70,2.90)	(2.85,2.95)	(2.70,3.00)	(3.10,3.80)
Out-of-Money	2.40	2.60	2.70	2.60	2.70
At-the-Money	2.60	2.90	2.90	2.90	3.40
In-the-Money	2.90	3.20	3.00	3.10	3.80
Panel B: Call Option					
Price Ranges	(2.55,2.65)	(2.70,2.90)	(2.85,2.95)	(2.70,3.00)	(3.10,3.80)
Out-of-Money	2.40	2.60	2.70	2.60	2.70
At-the-Money	2.60	2.90	2.90	2.90	3.40
In-the-Money	2.90	3.20	3.20	3.30	3.60

a Units are in $/bushel.

Table 8. Forecastimg Performance of Black and GARCH Option Pricing for 1991 Kansas City Wheat Options

	Black			GARCH		
Moneyness[a]	Out	At	In	Out	At	In
panel A: Put Option						
Mean Error[b]	-0.21	-0.68	0.08	-0.13	-0.70	0.16
Root Mean Squared Error[b]	0.73	1.77	2.39	0.71	1.85	2.44
panel B: Call Option						
Mean Error[b]	-0.38	-0.54	0.08	-0.36	-0.58	0.16
Root Mean Squared Error[b]	0.71	2.54	1.83	0.75	2.72	1.81

a The precise exercise prices for the in-, at-, and out-of-the-money options are given in Table 7.

b Mean errors and root mean squared errors are in cents per bushel.

Table 9. Ashley-Granger-Schmalensee Test of the Performance of Black and GARCH Option Pricing for 1991 Kansas City Wheat Options

	β_1[a]	β_2[a]	F statistics[b]	Conclusion	Model Favored
Panel A: Put Option					
Out-of-Money[c]	0.0837*	-0.00005	na[e]	reject H_0	GARCH
	(9.661)	(-0.006)			
At-the-Money[d]	0.0174	0.0225*	5.94*	reject H_0	Black
	(1.076)	(3.442)			
In-the-Money[d]	0.0850*	0.0086	14.49*	reject H_0	Black
	(5.076)	(1.869)			
Panel B: Call Option					
Out-of-Money[c]	0.0165*	-0.0418	na	not reject H_0	none
	(0.845)	(-1.864)			
At-the-Money[d]	0.0436*	0.0168*	3.33*	reject H_0	Black
	(2.020)	(2.487)			
In-the-Money[d]	0.0738*	-0.0084*	na	inconclusive	none
	(5.957)	(-2.999)			

a The Ashley-Granger-Schmalensee test is based on following regression results: $\Delta_t = \beta_1 + \beta_2(\Sigma_t - \bar{\Sigma}_t) + u_t$.

Here, $\Delta_t = \begin{cases} e^B_t - e^G_t & \text{for out-of-the-money option} \\ e^G_t - e^B_t & \text{for at- and in-the-money options,} \end{cases}$

where e^B_t is Black's option price minus actual option price and e^G_t is GARCH option price minus actual price. $\Sigma_t = e^B_t + e^G_t$, assuming $\bar{e}_t^B > 0$ and $\bar{e}_t^G > 0$. $\bar{\Sigma}_t$, $\bar{e}_t^B$, and $\bar{e}_t^G$ are means of Σ_t, e_t^B, and e_t^G , respectively.

b If both β_1 and β_2 are negative, then F test with one half of regular significance level can be used. However, if one of β_1 and β_2 is negative, then t test of the other coefficient should be used. If one of β_1 and β_2 is negative and significantly different from zero, then the test is inconclusive.

c H_0: Mean squared error of Black's OPM is equal to that of GARCH OPM,
H_1: Mean squared error of Black's OPM is greater than that of GARCH OPM.

d H_0: Mean squared error of GARCH OPM is equal to that of Black's OPM,
H_1: Mean squared error of GARCH OPM is greater than that of Black's OPM.

e In cases where both parameters are not positive, F test is not applicable.

* Asterisks denote significance at 5% level.

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ESSAY II

VALUING TARGET PRICE SUPPORT PROGRAMS WITH GARCH AVERAGE OPTION PRICING

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WITH GARCH AVERAGE OPTION PRICING

Abstract

The U.S. government deficiency payment program stabilizes farm income by transferring income from taxpayers to farmers. When revenue lost due to the acreage restriction exceeds the revenue gained from participating in the program, farmers would lose money by participating. Therefore, measuring the expected revenue from the government program is important for farmers to decide whether to participate in the program or not. This essay uses a GARCH average option pricing model to predict the implicit premium of the U.S. government deficiency payment program. The GARCH average option pricing model combines the GARCH process that considers both stochastic volatility and a nonnormal distribution with the average option pricing model that considers the average price of the underlying asset over a fixed period. A regression model based on the simulation results is provided for the GARCH average option model to be easily used to project deficiency payments. The results can be used by extension economist to help producers decide whether to participate in the program and by USDA to project participation, government

cost, and to calculate advance deficiency payments.

Key words : Farm Program, Deficiency Payments, GARCH, Exotic Option, Monte Carlo, Wheat

VALUING TARGET PRICE SUPPORT PROGRAMS WITH GARCH AVERAGE OPTION PRICING

Introduction

The U.S. government's deficiency payment program stabilizes net prices received by farmers and thus farmers can stabilize income by participating in the program. The program transfers income from taxpayers to farmers who participate in the program. The government deficiency payment program can be characterized as a subsidized put option except that acreage harvested is restricted (Gardner 1977; Irwin et al. 1988). Farmers that participate in the program are protected from prices falling below the target price but can benefit from prices rising above the target. However, opportunity costs arise from the acreage restriction. When revenue lost through the acreage restriction exceeds the revenue gained from participating in the program, farmers would lose money by participating. Therefore, measuring the expected revenue from the government program is important for farmers to decide whether to participate in the program or not. Also, USDA could use more accurate estimates of expected deficiency payments to project participation, government cost, and in calculating advance deficiency payments.

In extension, the expected deficiency payment is computed by the difference between the target price and the expected harvest price at the time of program sign-up (Anderson et al.). Such an approach considers only the intrinsic value[1] and ignores the time value of the contingent claim. The time value can be critical especially when prices are near the targets. Measuring the revenue from the program will likely become more important as the program is squeezed by the budget deficit. The objective of this paper is to help farmers decide whether to participate in the government's wheat program by determining the expected deficiency payment and to provide tools the U.S. government can use to project government cost, and calculate advance deficiency payments.

Based on the relationship between the target price program and a put option, past research used Black's option pricing model (Witt and Reid 1987; Turvey et al.1988) or a variant of Black-Scholes option pricing model (Marcus and Modest 1986) to determine the revenue from participating in the program. Black's option pricing model assumes constant volatility and lognormally distributed price changes. However, conditional nonnormality, conditional heteroskedasticity, and asymmetry in the distribution of wheat prices are also well documented (Yang and Brorsen 1992; Kang and Brorsen 1993). Monte Carlo studies showed that Black-Scholes and Black option pricing models underprice in- and out-of-the-money options relative to models considering nonnormality and heteroskedasticity (Johnson and Shanno 1987; Hull and

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[1]Actually the intrinsic value is the difference between the expected seasonal average and the target price. Since the expected seasonal average is not known at decision time, the difference between the expected harvest price and the target price is used as a proxy of the intrinsic value of the program.

White 1987; Kang and Brorsen 1993). Therefore, an option pricing model considering not only the non-normality but also stochastic volatility is needed.

This essay improves on previous research in several ways. One is that it explicitly considers the non-normality and stochastic volatility by using the GARCH option pricing model. In past research using Black or Black-Scholes option pricing formulae, the expected deficiency payment was computed based on the expected harvest price, which considers the time value only of the period between program sign-up and harvest. However, the time value of the period between harvest and the program maturity should also be considered. In this essay, the expected deficiency payment is computed based on the intrinsic value plus the time value of both the periods before and after harvest as well as based on stochastic volatility.

Another extension is that the deficiency payment program is modelled correctly as an average option. The amount of government transfer under the target price program is computed based on the difference between the target price and the average market price received by producers during the first five (ten) months of the marketing year. Therefore, an option pricing model considering the average price of the underlying asset over a fixed period provides a more accurate measure of the value of the target price program than one that assumes deficiency payments are based on harvest prices.

As a generalization of Black's option pricing, Bergman (1985) suggested an option pricing path that provides a closed form solution for the premium of an average value option. However, it oversimplifies the pricing model by assuming the exercise price equal to zero. Kemna and Vorst (1990) showed that an analytical solution to the more general problem is not possible and suggested a numerical solution to the average value option pricing model for European options.

Kemna and Vorst demonstrated how to use Monte Carlo integration to obtain premiums of average value options. However, Kemna and Vorst's average value option pricing model is still based on the assumptions of normality and constant volatility. This essay goes beyond Kemna and Vorst by combining average option pricing and GARCH option pricing which considers the nonlinear dynamics and stochastic volatility of the distribution of cash and futures price changes. Myers and Hanson (1993) showed that the prediction error of the GARCH (generalized autoregressive conditional heteroskedasticity) option model with a student t distribution is significantly smaller than that of Black's option pricing model. Duan (1991) has shown that the Black-Scholes option pricing model underestimates out-of-the-money call option premium when the underlying process is a GARCH process. Therefore, a marriage of the average option pricing with the GARCH process should provide a more accurate measure than average option pricing alone.

Theoretical Model and Method

Treating deficiency payments as a contingent claim enables computing the value of the program to producers and the cost to the government in a straightforward way. This essay determines the expected deficiency payments using an average value option pricing model which considers both conditional non-normality and stochastic volatility.

A GARCH process considering observed non-normality and seasonality is estimated using maximum likelihood. Then, the value of the target price program under alternative scenarios is obtained using Monte Carlo simulation.

The Theoretical Model

The farmer is assumed to be a taker of a stochastic price P for the unit output which is produced at a known cost of C. Total output Q may all be harvested if the farmer does not participate in the government deficiency payments program, or a portion of that Q_0 may be set aside if participating. The farmer is assumed to be a maximizer of expected profit $E(\pi)$. The profit depends upon the production cost, price, quantity harvested and the deficiency payments DP. The profit is,

(1) $\pi_1 = PQ - CQ$ if not participating in the program,

$\pi_2 = P(Q - Q_0) - C(Q - Q_0) + R(Q_0) + DP((1 - \eta_f)Q^* - Q_0^*)$ if participating,

where $R(Q_0)$ is net returns on set-aside acres, and DP = min[max(TP-AP, 0), max(TP-LR,0)], where TP is the target price, AP is the five (ten) month average price after harvest, LR is the loan rate, η_f is the portion of flexible acres, and $(Q^* - Q_0^*)$ is total program yield on harvested acres. The farmer is assumed to maximize expected profit:

(2) Max [$E(\pi_1)$, $E(\pi_2)$].

Let $R = \pi_2 - \pi_1 = DP((1 - \eta_f)Q^* - Q_0^*) + R(Q_0) - (P - C)Q_0$, then the farmer would participate in the program if $E(R) \geq 0$, but not participate if $E(R) < 0$. Therefore, it is critical to obtain the value of DP and P which are stochastic. E(P) can be obtained by adjusting futures prices or by other conventional means. The expected value of DP, E{ min[max(TP-AP,0), max(TP-LR)] } will be obtained in this paper using stochastic dynamic simulation and Monte Carlo integration.

The Statistical Model

The GARCH process with residuals following a student distribution has been selected as the most likely among alternative nonlinear dynamic models such as a diffusion jump process and deterministic chaos (Yang and Brorsen 1992). Kang and Brorsen (1993) suggested an asymmetric GARCH-t process that considers asymmetries in the mean equation and found that it is more likely than the GARCH-t of Bollerslev (1986) and than the exponential GARCH of Nelson (1991).

A GARCH model and an asymmetric GARCH model under a student distribution are estimated. The GARCH process can model well-documented seasonality in the variance (Kenyon et al. 1987). In the GARCH-t process, let the conditional distribution of price changes, y_i, be generalized t with mean $\mathbf{x}_i\boldsymbol{\theta}$, variance h_i^2, and degrees of freedom v:

(3) $y_i = \mathbf{x}_i\boldsymbol{\theta} + \varepsilon_i,$

where $\mathbf{x}$ is the vector of independent variables and $\boldsymbol{\theta}$ the parameter vector. Therefore, error term (ε_i) has a student t distribution with zero mean and variance h_i^2 with v degrees of freedom. ε_i can be specified as:

(4) $\varepsilon_i = ((v - 2)/v)^{1/2} z_i h_i,$

where z_i follows a t distribution which has mean zero and variance $v/(v - 2)$.

The asymmetric GARCH-t model is the same as the GARCH-t model except the lagged price changes in the mean equation are segmented into rising changes and falling changes. The logarithmic changes in returns, y_i, are segmented as,

$$yp_i = \begin{cases} y_i\ , & y_i > 0 \\ 0, & \text{otherwise,} \end{cases}$$

$$yn_i = \begin{cases} y_i\ , & y_i < 0 \\ 0\ , & \text{otherwise.} \end{cases}$$

The data for this study consist of two sets; futures prices and cash prices and cover the program years from 1981 through 1991. Futures prices are used as a proxy for expected cash prices over the period prior to harvest when there are no cash prices for new crop wheat. For the futures price data, evidences of cyclical seasonalities have been reported in the variance equation (Choi and Longstaff 1985) but not in the mean equation (Murphy 1987). A spline function is used to model seasonality in the mean equation. A spline function is a piecewise function in which the pieces are joined together in a continuous manner (Poirier 1976). The asymmetric GARCH-t model is specified as,

(5) $y_{j,i} = a_0 + \sum^{M}_{m=1} \delta_m yp_{j,i-m} + \sum^{M}_{m=1} \omega_m yn_{j,i-m} + W_{j,i} + \varepsilon_{j,i}$,

(6) $h^2_{j,i} = \alpha_0 + \alpha\varepsilon^2_{j,i-1} + \beta h^2_{j,i-1} + b_1 SIN(2\pi K/252)_j + b_2 COS(2\pi K/252)_j + b_3 SIN(2\pi K/126)_j + b_4 COS(2\pi K/126)_j$,

for j = 1, 2,...,J program years and for i = 1,2,...,I observations in each program year. δ_m and ω_m represent the net effect of the m^{th} positive or the m^{th} negative changes of y_i, respectively, and M is the length of lags. The length of lags in the mean equation is identified with the Schwarz criterion[2]. SIN and COS represent the sine and cosine functions, respectively, and π is approximated as 3.14. K in the sine and cosine functions is the number of trading days after January 1 of the particular year. Denominators in the sine and cosine functions are the specified cycle length in trading days, so 252 indicates a one year cycle and 126 a half year cycle. Equation (5) is the mean equation and equation (6) is the variance equation. $W_{j,i}$ captures the seasonality in the mean equation. A spline function with a cubic polynomial is used to impose no seasonality until harvest:

(7) $$W_{j,i} = \begin{cases} 0 & \text{if } \lambda \text{ prior to harvest,} \\ a_1 + a_2\lambda + a_3\lambda^2 + a_4\lambda^3 & \text{if } \lambda \text{ post harvest,} \end{cases}$$

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[2] Schwarz's SC criterion is obtained by $SC(m) = \ln(SSE_m) + Q_m*\ln(T)/T$, where m is the length of lags, SSE_m is the squared sum of residuals, and Q_m is the number of parameters and T is the number of observations. The value of m that minimizes SC is selected as the length of lags in the model.

where $\lambda = 1$ on June 1 and so $\{\lambda\} = \ldots -2, -1, 0, 1, 2, \ldots \lambda$. With restrictions of $0 = a_1 + a_2\lambda + a_3\lambda^2 + a_4\lambda^3$, when $\lambda =$ June 1 by imposing continuity, and $a_2 + 2a_3\lambda + 3a_4\lambda^2 = 0$, when $\lambda =$ June 1 by imposing differentiability, (7) can be rewritten as,

$$W_{j,i} = \begin{cases} 0 & \text{if } \lambda \leq 0, \\ (1 - 2\lambda + \lambda^2)a_3 + (2 - 3\lambda + \lambda^3)a_4 & \text{if } \lambda > 0, \end{cases}$$

The loglikelihood function of the conditionally heteroskedastic model with errors following a student distribution was derived by Bollerslev (1987). Since the data are cross-section time series, each set of data for one program year is used for computing the likelihood for each program year and then summed to obtain the likelihood for the whole data set. The log likelihood function used by Bollerslev is adapted to consider the cross-section time-series data:

$$(8) \qquad L(\theta,\dot{\theta}|x,\dot{x},y) = -\frac{1}{2} J*I \ln[\pi(\nu-2)] - J*I \ln[\frac{\Gamma(\nu/2)}{\Gamma((\nu+1)/2)}]$$

$$-\frac{1}{2}\sum_{j=1}^{J}\sum_{i=1}^{I}\ln[h_{j,i}] - \frac{\nu+1}{2}\sum_{j=1}^{J}\sum_{i=1}^{I}\ln[1+\frac{\varepsilon_{j,i}^2}{(\nu-2)h_{j,i}^2}],$$

where θ and $\dot{\theta}$ are vectors of parameters in the mean and the variance equations, respectively, **x** and $\dot{x}$ are vectors of independent variables in the mean and the variance equations, respectively, and y is the dependent variable in the mean equation. J is the number of program years, I is the number of observations in each program year and Γ() denotes the gamma function.

The maximum likelihood estimates are obtained using the statistical software package GAUSS (Aptech Systems Inc. 1992). Inequality restrictions are imposed on the GARCH terms (α and β in the variance equation) by taking the exponential of the parameters. The starting algorithm is Polak-Ribiere-type Conjugate Gradient method which performs well initially when the starting point is poor and a step size of one. After a few iterations, the algorithm is switched to the Davidon-Fletcher-Powell method and a BRENT step length method is used. The final estimates are obtained with the Newton method so that the Hessian is used to estimate the information matrix. All derivatives are calculated numerically. The model was considered as converged if the relative gradient (gradient ÷ parameter estimate) was less than 10^{-4}.

If the GARCH-t models are well specified, the standardized residual generated from the GARCH-t model should be i.i.d. student. The Ljung-Box and McLeod-Li test statistics are provided to check if the serial dependence is revealed in the rescaled residuals of the selected model. The Brock-Dechert-Sheinkman (BDS) test (Brock et al. 1991) is used to test the hypothesis that the daily futures and cash price changes are identically and independently distributed. The Kolmogorov-Smirnov goodness-of-fit test is used to determine if the residuals follow a student distribution. The rescaled residuals are adjusted by the standard deviation of the distribution, i.e., $(\varepsilon_{j,i}/h_{j,i})(v/(v-2))^{0.5}$, where v is the estimated degrees of freedom so that the rescaled residuals have variance of $v/(v-2)$.

Monte Carlo Integration

In a risk-neutral world, Black-Scholes option pricing model determines the option price that determines the value of a riskless hedge portfolio for a stock paying no dividend (Black and Scholes 1973). Therefore, the return to the hedge portfolio must be equal the riskless rate r. Merton (1973) extended Black-Scholes model allowing the stock to pay a dividend at a constant rate g so that the expected growth rate of the stock is (r-g). Black (1976) applied Merton's modification to commodity futures option pricing. Since a futures contract requires no initial investment, the return to the riskless hedge portfolio of a futures contract must be zero in a risk-neutral world. By setting the riskless rate equal to zero, the commodity option pricing model is obtained.

If we assume that the cash price has the same lognormal property as the stock and the futures price do, the expected deficiency payment can be approximated by using Black-Scholes and Black's option pricing models. In past research, the expected deficiency payment was computed based on the expected harvest price, which considers the time value only of the period between program sign-up and harvest. However, the time value of the period between harvest and the program maturity should also be considered. In this essay, the expected deficiency payment is computed based on the intrinsic value plus the time value of both the periods before and after harvest as well as based on stochastic volatility.

The cash price is expected to rise proportionally to the carrying cost and thus stored wheat is an asset just like a stock. Therefore, the expected deficiency payment after harvest can be approximated using the Black-Scholes-type model. The carrying cost is the sum of interest rate and the rate of storage cost. Since the commodity does not exist before harvest, carrying cost before harvest is zero. Assuming that the farmer has rationally expected harvest prices at decision time, no price increase should be expected during the period before harvest. Therefore, the expected deficiency payment before harvest can be obtained assuming no investment like in the Black model. However, unlike option trading in speculative markets, riskless arbitrage is not possible with the government deficiency payment program. Moreover, allowing stochastic volatility, it is not possible to solve the option pricing models in a closed form since the stochastic volatility adds risk which can not be diversified into a riskless hedge portfolio (Johnson and Shanno 1987; Hull and White 1987; Scott 1987; Myers and Hanson 1993). Therefore, the GARCH option pricing model using Monte Carlo integration is used. The expected deficiency payment is obtained by generating the estimated mean equation at maturity.

Since a farmer's decision making begins with planting wheat, one program year is defined as the period from planting time through the last date of the fifth (or tenth) month after harvest. One program year consists of fifteen (or twenty) months. For wheat, planted in September and harvested the next

June, 1992 program year has the period from September 1, 1991 through October 31, 1992 for five month average or through March 31, 1993 for ten month average. The program sign-up is usually between March 1 and April 15. One program year has six important points in time:

T_0 : planting time (September 1 in the previous calendar year),

T_1 : program sign-up start (March 1),

T_2 : program sign-up due (April 15),

T_3 : harvest time (June 1),

T_4 : five months after harvest (October 31),

T_5 : ten months after harvest (March 31 in the next calendar year).

The farmer can sign up in the program any time between T_1 and T_2. For simplicity, it is assumed that the farmer signs up at T_1 or at T_2 and that the Agricultural Stabilization and Conservation Service (ASCS) pays the deficiency payment at the end of the program year (T_4 or T_5).

A realization of the value, V_τ, is calculated as

(9) $V_\tau = \max\{ \min[TP-A_\tau, TP-LR], 0 \}$,

where τ is the maturity date, TP is the target price, and LR is the loan rate. To simplify the analysis in simulation, the loan rate is not considered. When prices are close to loan rate, the decision of whether to participate or not is easy. A_τ is the average market price computed as:

(10) $A_\tau = \frac{1}{(\tau - T)} \sum_{t=T_3}^{\tau} S_t$,

where S_t is the market price of the underlying commodity at time t. The target price program is a European option and thus American options are not considered.

In this analysis, the GARCH option model is used to obtain the market price S_t at time t for the period between τ and T_0. The GARCH model is estimated using data available at time T_0. Time is measured in number of trading days. The way to simulate S_t is different before harvest than it is after harvest. While S_t post harvest is obtained with the mean equation in (5), S_t prior to harvest is obtained using the mean equation with zero intercept which does not allow any seasonal patterns[3].

(11) $S_t = S_{t-1}\exp(g_{t-1})$,

where

$$g_{t-1} = \begin{cases} \sum_{m=1}^{M} \tilde{\delta}_m gp_{t-m} + \sum_{m=1}^{M} \tilde{\omega}_m gn_{t-m} + \tilde{h}_t z_t \left(\frac{\tilde{v} - 2}{\tilde{v}}\right)^{1/2} & \textit{before harvest} \\ \tilde{a}_0 + \sum_{m=1}^{M} \tilde{\delta}_m gp_{t-m} + \sum_{m=1}^{M} \tilde{\omega}_m gn_{t-m} + \tilde{W}_t + \tilde{h}_t z_t \left(\frac{\tilde{v} - 2}{\tilde{v}}\right)^{1/2} & \textit{after harvest} \end{cases}$$

$\tilde{\delta}_m$, $\tilde{\omega}_m$, and $\tilde{a}_0$ are estimated coefficients in equation (5). z_t is random number generated from student distribution with $\tilde{v}$ degrees of freedom, and $\tilde{W}_t$ is the estimated spline function in equation (7). g_t is segmented as:

[3]The futures prices as expected cash prices tend to fall before harvest during the observation period. Futures prices seem to reflect the possibility of a catastrophic events such as that which occurred in 1973. Futures prices before harvest fell possibly because no catastrophic event occurred during the observation period. Therefore, the market is assumed to be efficient (i.e., expectations are rational) in the sense of Fama (1970) which is contrary to what Wisner (1991) concluded based on similar findings.

$$gp_t = \begin{cases} g_t \,, & g_t > 0 \\ 0, & \text{otherwise,} \end{cases}$$

$$gn_t = \begin{cases} g_t \,, & g_t < 0 \\ 0 \,, & \text{otherwise.} \end{cases}$$

When the asymmetry in the mean equation is not significant, g_t does not have to be segmented.

S_t of the period between τ and T_3 is used to get the average price at maturity, A_τ, and one realization of the value from the program, V_τ. Denoting this value at maturity $\{V_\tau\}_i$, the simulated value is

$$(12) \quad V = \frac{1}{n}\sum_{i=1}^{n} \{V_\tau\}_i \,,$$

where n = 20000 is the number of replications of this procedure[4]. V is not discounted so that it can be directly compared at different decision making times.

One efficient way to improve the accuracy of this calculation is the control variate technique. The control variate technique adds a term with expected value of zero that is negatively correlated. The control variate in this case is the difference between the analytical solution of a simpler problem and a Monte Carlo integration (using the same random number seeds) of the problem which has an analytical solution. By doing this, the variance of calculation can be effectively reduced (Hammersley and Handscomb 1964; Boyle 1977).

Different control variates are used before and after harvest because

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[4]The calculation took 87.5 hours on a 486DX2/66MHZ computer.

different analytical solutions are used in each period. For the period of before harvest, an analytical solution is obtained by the standard Black option pricing model as follows:

(13) $B = TP*N(-d_2) - S_{T0}*N(-d_1)$,

where $d_1 = \{ \ln[S_{T0}/TP] + \sigma^2/2)*(T_3-T_0) \}/\{ \sigma(T_3-T_0)^{1/2}\}$, and

$d_2 = \{ \ln[S_{T0}/TP] - \sigma^2/2)*(T_3-T_0) \}/\{ \sigma(T_3-T_0)^{1/2}\}$,

where σ^2 is the volatility of the price changes and N() is the cumulative probability density function of a standard normal distribution. The Monte Carlo integration corresponding to this analytical solution B′ is generated using standard normal random numbers and constant volatility. Therefore, the control variate for the period before harvest is the difference between B in equation (13) and its corresponding Monte Carlo integration B′.

The deficiency payment is calculated based on the average of prices between harvest (June 1) and program maturity. Since an option based on an arithmetic average cannot result in an analytical expression for the value of an option (Kemna and Vorst 1990), the geometric average option pricing model is used. Following Kemna and Vorst, the geometric average put option premium is:

(14) $G = TP*N(-d_2) - e^q*S_{T3})N(-d_1)$,

where $q = ((r+c)*((\tau-T_3)/(\tau-T_0)) - \sigma^2/6)*(\tau - T_3)/2$,

$d_1 = \{ \ln[S_{T3}/TP] + ((r+c)*((\tau-T_3)/(\tau-T_0)) + \sigma^2/6)*(\tau - T_3) \}/\{ \sigma[(\tau - T_3)/3]^{1/2}\}$, and

$d_2 = \{ \ln[S_{T3}/TP] + ((r+c)*((\tau-T_3)/(\tau-T_3)) - \sigma^2/6)*(\tau - T_3) \}/\{ \sigma[(\tau - T_3)/3]^{1/2}\}$,

where r is the riskless interest rate, and c is storage cost as a percentage of target price. The interest rate is assumed to be constant at 5.5% and the storage cost 2.5¢/bushel per month so that c is constant at 7.6% per annum. The Monte Carlo integration for the post harvest period is obtained using standard normal random numbers and constant volatility and using a geometric average. The control variate is the difference between G in equation (14) and its corresponding Monte Carlo integration G′.

The accuracy in calculating V is increased by adding the two control variates. The GARCH average option price with the control variate technique is:

(15) $V^* = V + (B - B') + (G - G')$,

where B and G are analytical solutions from equations (13) and (14), respectively, and B′ and G′ are corresponding Monte Carlo integrations. Two sets of τ-T_0 random numbers are generated: one from a student distribution with $\tilde{v}$ degrees of freedom which is estimated from the model and another from a standard normal distribution. These are generated using the same seed and therefore the random errors in B′ and G′ are positively correlated with the random error in V.

Sensitivity analyses are conducted by varying the initial volatility, expected harvest prices at program sign-up, the time of program sign-up, and the time of program end (five month average or ten month average), etc. The expected deficiency payment will be calculated at the expected harvest prices of $4.50, $4.00, $3.50, and $3.00 per bushel under the target price of $4.00 per bushel. Twenty day volatility will be used as initial volatility with the levels of 2.0, 1.5, 1.0, 0.5, and 0.25[5]. Time to maturity effects will be detected by comparing the expected payments based on five month average and those on ten month average. Three decision points will be considered: September 1, March 1, and April 15. All the sensitivity analyses will be conducted in one run. The same seed is used for each expected initial prices in the same initial volatility to reduce variability of results.

Finally, a rule of thumb will be provided so that the GARCH average option premium model can be easily used to predict the expected deficiency payment. A regression model will be estimated based on the data from the result of the sensitivity analysis[6].

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[5]The historical daily volatility is measured as the standard deviation of the percentage changes of the five major market index is usually in the range of (0.5, 1.8). The range corresponds to the annual volatility of (0.08,0.28).

[6] A quadratic model on all the independent variables was considered. Since the quadratic relationship was significant only for IV and TTM, the quadratic terms and corresponding interaction term only for IV and TTM are considered. Deleting insignificant parameters is appropriate here since the goal is prediction.

(16) $DP = c_0 + c_1IV + c_2IV^2 + c_3V + c_4IV*TTM + c_5TTM + c_6START_{SEP} + c_7START_{MAR} + e,$

where

DP = Simulated deficiency payment,

IV = Difference between target price and the expected harvest price,

V = Initial 20 trading day volatility,

$$TTM = \begin{cases} 1 \text{ if DP is based on 10 month average price,} \\ 0\text{, otherwise,} \end{cases}$$

$$START_{SEP} = \begin{cases} 1 \text{ if decision making occurred on Sep. 1,} \\ 0\text{, otherwise,} \end{cases}$$

$$START_{MAR} = \begin{cases} 1 \text{ if decision making occurred on Mar. 1,} \\ 0\text{, otherwise.} \end{cases}$$

Intrinsic value is max(0, TP - SA), where SA is the expected seasonal average of the period between harvest and maturity. Since SA is not known at decision time, max(0, TP - EHP) is used as a proxy for intrinsic value, where EHP is expected harvest price. EHP is readily available and currently used as the basis of decisions.

Sample Data

Data for estimating the statistical model are divided into two sets : one is futures prices (July contract) and another is cash prices. Evidence suggests that wheat futures price is a good estimator of future cash price (Just and

Rausser 1981; Denbaly 1993). Since only changes are used, the basis is not a concern. The futures prices (July contracts) are used as a proxy of expected cash prices for the period between the time of planting (September 1 of the previous calendar year) and harvest time (June 1). The data are a form of cross-section time series, with the cross-sectional units being the program year.

First differences of the natural logarithms of the daily futures closing prices and of cash wheat prices are used[7]. An equally-weighted average between Kansas City and Chicago Board of Trade are used as futures price data, and five major market average cash prices as cash price data. The five major markets consist of Kansas City hard red winter wheat, Minneapolis dark northern spring wheat, Minneapolis hard amber durum wheat, Portland white wheat, and St. Louis soft red winter wheat. The futures price data are obtained from Technical Tools and the cash major market index from ASCS. The cash prices are collected for the five (or ten) month period from harvest time. The data cover eleven program years from 1981 to 1991.

Table 1 shows the summary statistics for the daily logarithmic changes in five major market average cash prices and average futures prices between Kansas City and Chicago Board of Trade. The departures from normality are obvious from the high kurtosis and skewness in both prices. The means of the

[7]Fama(1965) provides reasons for using log differences. First, the log difference is the return, with continuous compounding, from holding the asset. Second, while the variability of simple price changes increases as the price level increases, log differences neutralize price level effects.

two price series are not significantly different. The standard deviations of futures prices is significantly smaller than that of cash prices, which confirms Milonas (1986)[8]. Cash prices are negatively skewed, while futures prices are positively skewed, which is similar to Yang's (1989) findings for other commodities. The kurtosis of futures prices may be smaller because of price limits[9].

Results

Model Estimation

The null hypothesis that the asymmetric GARCH model fits the data better than the GARCH model was rejected based on the likelihood ratio test since the skewness in the mean equation may be neutralized because of combining two series of price changes which have opposite directions of skewness. Table 2 reports estimates and test results of the GARCH model. The estimated GARCH terms (α and β) are all positive and significant. The sum of α and β is less than one implying stationarity[10]. The spline terms are individually not significantly

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[8]Milonas found the volatility of commodity futures prices increases as the time to maturity decreases. Since the cash commodity can be delivered at any time, the maturity effect found by Milonas suggest that volatility of cash prices should be greater than that of futures prices.

[9]During the observation period, the Kansas City wheat futures prices hit the lower price change limit (-\$0.25) 4 times and it hit the the upper limit (\$0.25) 5 times. Chicago wheat futures prices hit the lower price change limit (-\$0.20) 8 times and it hit the upper limit (\$0.20) 6 times.

[10]$\alpha + \beta < 1$ is a sufficient condition but not a necessary condition for stationarity.

different from zero, but the null hypothesis that both spline terms (a_3 and a_4) are zero was rejected and therefore seasonality exists in the mean equation. Seasonal volatility in the variance equation is also significant.

The Ljung-Box test shows that the model removes linear dependence in the mean equation but the McLeod-Li tests for the squared standardized data reveals second moment dependence[11]. The BDS statistics show that the null hypothesis of i.i.d. is rejected in all dimensions for the raw data (Table 2), implying that the raw data are not random but dependent. For the rescaled residuals, the BDS statistics still identify nonlinear dependence, but the BDS statistics are considerably lower. The Kolmogorov-Smirnov goodness-of-fit test shows that the null hypothesis that the GARCH rescaled residuals follow a student distribution is not rejected at the 5% significance level.

<u>The Expected Deficiency Payment</u>

In this essay, the deficiency payment program is called in-the-money program if target price is greater than the expected harvest price, at-the-money if equal to the expected harvest price, or out-of-the-money if smaller than the expected harvest price. Table 3 reports the expected deficiency payment at various levels of expected initial prices, initial volatility, 5 month average or 10

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[11]First-order autocorrelation among squared residuals is statistically significant in spite of the GARCH(1,1) model having converged. Longer lags in the GARCH equation were not helpful in removing the remaining dependence. The McLeod-Li test may not be robust with respect to the nonnormality remaining in standardized residuals.

month average, and different decision time with a target price of $4.00 per bushel.

The payment increases as the program moves from out-of-the-money to in-the-money in all cases. Expected deficiency payment generally increases as the initial volatility increases because higher volatility implies higher time value. The later the decision making occurs, the lower the deficiency payment in most cases except in the out-of-the-money program.

Time to maturity effect is not the same between when program is at- or out-of-the-money and when in-the-money. When the program is in-the-money and the volatility is below 1.5, the deficiency payment of ten months is slightly smaller than that of five months. For the deep in-the-money case ($3.00 target price), the expected deficiency payment based on a ten month average is always smaller than that based on a five month average. However, when the program is at- or out-of-the-money, deficiency payment based on a ten month average is greater than that based on a five month average. Even when the program is in-the-money (for example, $3.50 target price), the expected deficiency payment based on a ten month average is bigger than that based on a five month average with initial volatilities higher than 1.5.

Table 4 contains the expected deficiency payment computed using Black's option pricing model as suggested by Witt and Craig (1987) and by Turvey et al. (1988). The moneyness effect, the effect of initial volatility, and the effect of decision time making are the same as the case of GARCH average option pricing model: the payment increases as the program moves from out-of-the money to in-the-money; the higher volatility, the bigger deficiency payment; and the later the decision making occurs, the lower the deficiency payment in all cases.

Since the approach using Black's option pricing model considers the time value only for the period from the time of decision making to harvest, the expected deficiency payment by Black's model is smaller than that with the GARCH average option pricing model which considers the time value not only for the period before harvest but also for the period after harvest[12]. However, since Black option pricing model is based on the expected harvest price (EHP) while the GARCH option pricing model is based on the expected seasonal average (SA), where EHP > SA, the expected deficiency payment by Black' model is greater than that with GARCH option pricing model. In our case, the expected deficiency payment computed by Black's option pricing model is generally smaller than that with the GARCH average option pricing model. Thus the time value effect dominates the expected price effect.

[12]Black's option pricing model assumes normality in price change distributions. However, cash price change distributions have thicker tails than a normal distribution. Like a put option, the expected deficiency payment depends on the left tail of the terminal distribution. Therefore, GARCH option pricing model with a t-distribution tends to yield a higher expected deficiency payment.

A Rule of Thumb

The estimated regression model is reported in Table 5. The dependent variable is the expected deficiency payment obtained from the simulation using GARCH average option pricing, and the independent variables are corresponding levels of intrinsic value of the program (IV), squared intrinsic value (IV^2), initial volatility (V), time to maturity (TTM), interaction term between IV and TTM (IVTTM), and the time of decision making ($START_{SEP}$ and $START_{MAR}$). The time to maturity and the decision time were estimated as dummy variables. Estimated coefficients are all significant except that of the dummy variable $START_{MAR}$ and the adjusted R-square is 0.9854.

This regression model is given so that the GARCH average option pricing can be easily used by extension economist to predict the expected deficiency payment at the point of decision making and by the government to project participation, government cost, and also to calculate advance payments. For example, with the target price of $4.00, expected harvest price of $3.80, and initial volatility of 0.8, the expected deficiency payment would be $0.44 per bushel if it is based on a ten month average and the decision was made at planting time.

Concluding Remarks

The government deficiency payment program contributes to farm income. Extension personnel have predicted the expected revenue from the government program using the expected harvest price. A Black-type option pricing model can also be used because the deficiency payment is a subsidized put option. However, Black's option premium is based on inappropriate assumptions such as a normal distribution and constant volatility. Also, since the payment is calculated based on the five (soon to be ten) month major market average prices, an average option pricing model provides a more accurate measure of the expected deficiency payment. This essay uses a GARCH (generalized autoregressive conditional heteroskedasticity) average option pricing model to meet these needs. The GARCH average option pricing model is a Monte Carlo integration combining the GARCH process and an average option pricing model.

Results show that the expected deficiency payment increases as the intrinsic value increases and/or the volatility increases. The expected deficiency payment based on a ten month average is smaller than that based on a five month average when the program is in-the-money and the initial volatility is low. Therefore, government cost can in most cases be reduced by switching the calculation basis of deficiency payment from a five month average to a ten month average, but the cost reduction is not large. A regression model based on the simulation results is provided for the GARCH average option model to be easily used to project the expected deficiency payment. The results can be used by extension to help producers decide whether to participate in the program or not and by USDA to project participation, government cost, and to calculate advance deficiency payments.

Table 1. Summary Statistics of Daily Cash and Futures Prices of Wheat: 1981 - 1991[a]

Statistics	Cash[b]	Futures[c]
No. observations	2317	2065
Mean	-0.001	-0.023
Std Dev	0.965	0.941
Skewness[d]	-0.48207*	0.28521*
Kurtosis[e]	27.5879*	7.05359*

a Units are percentages. y_i = [$\ln(y_i)$ - $\ln(y_{i-1})$]*100.
b Five major market average.
c Arithmetic average of futures prices of Chicago and Kansas City Board of Trade.

d Skewness is computed by $\frac{1}{n-1}\sum_{t=1}^{n}(y_t - \mu)^3/\sigma^3$.

e Kurtosis is computed by $\frac{1}{n-1}\sum_{t=1}^{n}(y_t - \mu)^4/\sigma^4 - 3$.

* Asterisks denote the null hypothesis of normality are rejected at 5% significance level. The critical values are based on Snedecor and Cochran (1980, p.492).

Table 2. Statistics from the GARCH(1,1)-t Process Estimated with Equal-weighted Kansas City and Chicago Board of Trade Futures Prices and Five Major Market Cash Price Index, 1981 - 1991.

	Estimated Coefficient	(t-ratio)
Estimated Loglikelihood	-4648.24	
Mean		
Intercept	-0.029*[a]	(-2.10)
Lag 1	0.089*	(5.58)
a_3	0.066	(1.59)
a_4	-0.024	(-1.10)
Variance		
Intercept	0.225*	(3.83)
Alpha	0.105*	(2.51)[b]
Beta	0.872*	(46.36)[b]
SIN252	0.012*	(3.17)
COS252	-0.002	(-0.67)
SIN126	-0.000	(-0.13)
COS126	-0.003	(-1.02)
Degrees of Freedom		
v[c]	5.03*	
Wald F statistics		
Spline in Mean	4.61*	
Seasonality in Variance	2.84*	
Ljung-Box and Mcleod-Li[d]		
$\varepsilon_t/h_t(12)$	14.79	
$\varepsilon^2_t/h^2_t(12)$	26.13*	
BDS tests ($\varepsilon=\sigma$)[e]		
Raw Data		
Dimension = 3	10.39*	
Dimension = 6	13.18*	
Dimension = 9	16.94*	
Rescaled Data		
Dimension = 3	1.61*	
Dimension = 6	1.18*	
Dimension = 9	1.94*	
Goodness-of-fit[f]		
D_{max}	0.018	

(Table 2 Continued)

a Asterisks denote the rejection of the null hypothesis at the 5% significance level.

b Since inequality constraints were imposed on the parameter α using exponential transformation,the t ratio of α is computed as

$$t = e^{\hat{\alpha}}/(e^{\hat{\alpha}} S^2_{e^{\hat{\alpha}}} e^{\hat{\alpha}})^{1/2}$$, where $S^2_{e^{\hat{\alpha}}}$ is the standard error of $e^{\hat{\alpha}}$.

c Degrees of freedom is restricted to be greater than three for computational concerns.

d The null hypotheses that ε_t/h_t and ε^2_t/h^2_t are not autocorrelated are tested with twelve degrees of freedom.

e The null hypothesis is that the standardized residuals are i.i.d. The hypothesis test is based on Table F.4 in Brock et al. (p.279).

f The critical value of this test is $D_c = 1.36/T^{1/2} = 0.02$ where T is the sample size.

Table 3. The Expected Deficiency Payment with GARCH Average Option Pricing Model at Various Levels of Expected Initial Prices, Initial Volatility, Time to Maturities, and Decision Time and a $4.00/bushel Target Price

Decision Time	TTM[a]	Initial Volatility[b]	Expected Initial Price[c] 450	400	350	300
		2.00	27.86	55.79	75.90	111.92
	10	1.50	23.62	49.37	68.67	108.39
	Months	1.00	19.32	41.73	61.62	105.52
		0.50	15.29	31.85	54.28	102.57
Sep 1		0.25	13.77	25.14	51.74	101.45
		2.00	18.37	49.95	74.38	113.13
	5	1.50	14.55	44.14	68.56	110.06
	Months	1.00	10.58	37.21	62.86	107.41
		0.50	6.65	28.16	56.96	104.80
		0.25	5.05	25.77	54.66	103.78
		2.00	23.32	45.90	63.98	106.33
	10	1.50	21.48	40.97	58.57	103.86
	Months	1.00	20.03	34.92	53.47	101.76
		0.50	19.34	27.27	48.29	99.24
Mar 1		0.25	19.40	21.89	46.77	98.26
		2.00	11.58	38.69	62.19	107.63
	5	1.50	10.00	34.29	58.36	105.61
	Months	1.00	8.71	28.81	54.90	103.70
		0.50	7.96	21.32	51.66	101.48
		0.25	7.92	15.79	50.58	100.59
		2.00	23.06	41.83	62.12	106.09
	10	1.50	22.17	37.68	57.32	103.71
	Months	1.00	21.71	32.58	52.68	101.64
		0.50	21.81	26.32	47.67	99.21
Apr 15		0.25	22.01	22.05	45.92	98.22
		2.00	10.70	34.17	60.24	107.46
	5	1.50	10.02	30.52	57.11	105.52
	Months	1.00	9.66	25.90	54.26	103.61
		0.50	9.66	19.58	51.53	101.44
		0.25	9.76	15.00	50.52	100.55

(Table 3 Continued)

a Time to maturity. Deficiency payment is currently calculated based on 5 month average, but soon to be 10 month average.
b Historical daily volatility is used as the measure of the volatility in the analytical solutions and corresponding Monte Carlo integrations.
c Farmers' expected harvest prices at the time of decision making. Unit is ¢/bushel.

Table 4. The Expected Deficiency Payment by Black's Option Pricing Model at Various Levels of Expected Initial Prices, Initial Volatility, and Decision Time and a $4.00/bushel Target Price

Decision Time	Initial Volatility[a]	Expected Initial Price[b] 450	400	350	300
	2.00	13.53	30.73	60.42	101.99
	1.50	10.02	26.62	57.51	101.00
Sep 1	1.00	6.20	21.75	54.42	100.30
	0.50	2.19	15.38	51.37	100.01
	0.25	0.49	10.88	50.24	100.00
	2.00	3.61	17.90	52.41	100.06
Mar 1	1.50	2.26	15.51	51.41	100.01
	1.00	1.02	12.66	50.57	100.00
	0.50	0.15	8.96	50.06	100.00
	0.25	0.01	6.33	50.01	100.00
	2.00	0.99	12.56	50.54	100.00
Apr 15	1.50	0.49	10.88	50.24	100.00
	1.00	0.15	8.88	50.06	100.00
	0.50	0.01	6.28	50.00	100.00
	0.25	0.00	4.44	50.00	100.00

a Historical daily volatility is used as the measure of the volatility in the analytical solutions and corresponding Monte Carlo integrations.

b Farmers' expected harvest prices at the time of decision making. Unit is ¢/bushel.

Table 5. Regression Results of the Expected Deficiency Payment on the Explanatory Variables

Variables	Estimated Coefficient	(t-ratio)
Intrinsic Value(IV)[a]	0.4782*	(39.23)
IV^2	0.0030*	(19.58)
Initial Volatility	7.7870*	(13.19)
Time to Maturity(TTM)	5.6127*	(6.78)
IV*TTM	-0.0907*	(-6.71)
$START_{SEP}$	5.7455*	(6.21)
$START_{MAR}$	0.5953	(0.64)
Intercept	17.4230*	(16.21)
Standard Error of Estimation	4.1409[b]	
Adj. R^2	0.9854	
Mean of Dependent Variable	50.25[b]	

a The difference between target price and expected harvest price is used as a proxy for intrinsic value.

b Unit is cents/bushel.

* Asterisks denote significance at 5% level.

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ESSAY III

A NEW EFFICIENCY CRITERION: THE MEAN-SEPARATED TARGET DEVIATIONS RISK MODEL

A NEW EFFICIENCY CRITERION: THE MEAN-SEPARATED TARGET DEVIATIONS RISK MODEL

Abstract

This paper develops a new risk efficiency model, Mean - Separated Target Deviations (MSD). MSD can be an interval analysis that orders risky choices for a decision maker whose monotonically increasing utility function lies within a specified range. Conventional measures of risk do not distinguish between below-target and above-target outcomes, or else impose risk neutrality for above-target outcomes. The model is motivated by the intuition that although decision makers in an investment environment are comfortable with expected value as a measure of return, they respond in different ways to potential outcomes below a target return than to potential outcomes above a target return. The measure of risk is a weighted sum of below-target deviations and above-target deviations. The weights are determined by decision maker's risk attitude. MSD is a special case of a von Neumann-Morgenstern expected utility function and of stochastic dominance. Unlike the mean-variance criterion, the MSD model considers skewness in ranking alternatives. An empirical evaluation of a decision maker's choice of wheat marketing strategies shows that the criterion yields a smaller efficient set than alternative efficiency criteria.

Key Words: Efficiency Criteria, Risk, Mean-Variance, Target, Stochastic Dominance

A NEW EFFICIENCY CRITERION: THE MEAN-SEPARATED TARGET DEVIATIONS RISK MODEL

Introduction

Risk and return models are commonly used to analyze decisions under uncertainty. The most common of these models is the mean-variance (E-V) model, in which return is measured as the mean and risk as the variance of the outcome distribution. In spite of its computational and graphical advantages (Hazell and Norton, p.80) and its attractive dichotomy between risk and return (Holthausen 1981), E-V analysis has several well-known theoretical shortcomings[1]. Tobin (1957) argues that E-V analysis is relevant when the utility function is quadratic, or when net returns are normally distributed[2]. Two main limitations of quadratic utility are: (a) the utility function is not monotonically nondecreasing, and (b) it displays increasing absolute risk aversion. Further, the assumption of normality often does not hold since actual returns are often skewed and leptokurtic.

Recognizing problems with the E-V criterion, alternative efficiency criteria have been introduced. These criteria include E-S (Expected Value-Semivariance) (Markowitz 1952; Mao 1970; Porter 1974), Target Risk-Return

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[1]See Fishburn (1977) for a list of relevant articles; also Meyer 1987.

[2]Meyer (1987) shows that a location-scale condition, of which normality is a special case, is the condition for ensuring that E-V analysis is consistent with expected utility.

(Fishburn 1977; Holthausen 1981), Mean-Gini criterion (Yitzhaki 1982; Buccola and Subaei 1984), First-degree Stochastic Dominance (Quirk and Spasonik 1962), Second-degree Stochastic Dominance (Hanoch and Levy 1969; Hadar and Russell 1969), and Stochastic Dominance with respect to a Function (SDWRF) (Meyer 1977; King and Robison 1983). Although these criteria overcome some of the disadvantages of the E-V criterion they have some limitations. For example, most require the assumption of everywhere risk aversion[3].

Moreover, Fishburn (1977) contends that the E-V model uses an unrealistic measure of risk. He noted, following Markowitz (1959), Mao (1970), and others, that decision makers (DM) "...frequently associate risk with failure to attain a target return," (p.117) suggesting that a measure of dispersion around a parameter which changes from distribution to distribution -- such as variance -- is not a suitable measure of risk. To address these shortcomings of the E-V model, Fishburn proposed a mean-risk model which generalized the mean-target semivariance model (Markowitz 1959, Mao 1970, Hogan and Warren 1972, Porter 1974). Fishburn's model measured return as the mean of the outcomes, but defined risk as weighted deviations of outcomes below a target, where the weight was related to the DM's risk preferences. Holthausen (1981) adapted Fishburn's model by using the same measure of risk but defining return as

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[3]SDWRF can allow risk preferring utility, and FSD does not require assumptions on DM's risk attitude.

weighted deviations above the target rather than as the mean.

This paper builds on Fishburn's and Holthausen's models by proposing a mean-risk model which generalizes Fishburn's model. It is shown that the model is consistent both with expected utility axioms and with stochastic dominance criteria. The model measures return as expected value and risk as deviations below a target return minus deviations above the target return, with both kinds of deviation weighted by the DM's risk preferences.

One contribution of this paper is the new risk measure. As indicated, the traditional measure of risk, variance, has been criticized as unrealistic (Markowitz 1952). Also, as Hanoch and Levy (1969, p.344) note, "The identification of riskiness with variance, or with any other single measure of dispersion, is clearly unsound. There are many obvious cases where more dispersion is desirable, if it is accompanied by an upward shift in the location of the distribution, or by an increasing positive asymmetry."[4] Alternative risk measures such as mean-target semivariance (Mao 1970, Porter 1974) or more generally weighted below-target deviations (Fishburn 1977, Holthausen 1981), also consider information only on outcomes below a target and ignore information on outcomes above the target. The proposed model, however, considers information on outcomes both below and above the target. Therefore, it is consistent with Hanoch and Levy's observation that outcomes

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[4]Tronstad and McNeill (1989) incorporated asymmetric price risk (only unfavorable deviations) into an econometric model of supply response.

above a target can reduce risk. Thus, the proposed measure is affected by the skewness of outcome distributions.

A second contribution of the proposed model is its provision for interval analysis. The interval analysis is to order risky choices for DM whose monotonically increasing utility function lies within specified ranges. Previous mean-risk models do not consider the interval analysis technique. Like stochastic dominance with respect to a function (SDWRF), MSD can effectively reduce the efficient set by using appropriate ranges of DM's risk attitude. Unlike SDWRF, however, MSD allows different ranges of risk attitude above and below a target return.

The MSD model goes beyond Fishburn's and Holthausen's model in several ways. Fishburn's model assumes risk neutrality above the target. This restriction is avoided in the MSD model as it is in Holthausen's model. Holthausen's model measures return as above-target deviations, while MSD measures return as expected return as in Fishburn's model. Expected return is the most common measure of return and decision makers are satisfied with it as a measure of return (Baumol 1963). More important is that the MSD model extends Fishburn's and Holthausen's models by allowing interval analysis.

Porter (1974), Fishburn (1977), Holthausen (1981), and Yizhaki (1982) have shown that their risk efficiency models are congruent with expected utility theory and consistent with stochastic dominance rules. This paper extends their results to show that the MSD model is also congruent with expected utility theory and consistent with stochastic dominance rules under some conditions.

The following sections introduce the model in more detail and show its relationship with expected utility theory and stochastic dominance criteria. Then, MSD is used to identify risk-efficient marketing strategies for wheat producers. The empirical study provides MSD efficient sets for given ranges of a DM's absolute risk aversion coefficients, and compares these results with those obtained from alternative efficiency criteria.

The Mean-Separated Target Deviations (MSD) Risk Model

There is usually a point on the abscissa at which something unusual happens to the individual's utility function (Fishburn and Kochenberger 1979). Therefore, using a target in specifying a utility function seems to be appropriate. Fishburn generalized the Expected value-Semivariance model by associating risk with below-target returns. In Fishburn's (1977) model, return is measured by expected return and risk is measured by the dispersion below a target,

$$(1)\quad \rho(\mathrm{F}) = \int_{-\infty}^{t} \phi(t - \pi)\,dF(\pi),$$

where $\phi(\mathrm{y})$, for $\mathrm{y} \geq 0$, is a nonnegative nondecreasing function in y with $\phi(0) = 0$, and $\mathrm{F}(\pi)$ is the probability of that return will not exceed π. Without loss of reality $\mathrm{F}(\pi)$ is assumed to be bounded as $\mathrm{F}(\pi_1) = 0$ and $\mathrm{F}(\pi_2) = 1$ for some real π_1 and π_2.

A special form of (1) is the α-t model, in which risk is measured by

$$(2) \quad r(F) = \int_{-\infty}^{t} (t - \pi)^{\alpha} dF(\pi) \ , \qquad \alpha > 0.$$

Fishburn showed that the α-t model is congruent with the expected utility model under the utility function:

$$(3) \quad U(\pi) = \begin{cases} \pi & \text{for all } \pi \geq t \\ \pi - k(t - \pi)^{\alpha} & \text{for all } \pi \leq t \text{ and } k > 0. \end{cases}$$

If $\alpha > 1$, the individual is risk averse below t, if $\alpha < 1$, risk averse below t, and if $\alpha = 1$, risk neutral below t.

Holthausen (1981) derived an α-β-t model with both risk and return measured as deviations from a target return so that the utility function for the above-target outcomes need not be linear. Risk in the α-β-t model is defined as in Fishburn's model, but return is defined as above-target deviations,

$$(4) \quad \Pi(F) = \int_{t}^{\infty} \theta(\pi - t) dF(\pi), \ ,$$

where $\theta(y)$, for $y \geq 0$, is nonnegative nondecreasing function in y with $\theta(0) = 0$. A specific form of (4) along with risk measure (2) gives the α-β-t model in which the return is measured as

$$(5) \quad R(F) = \int_{t}^{\infty} (\pi - t)^{\beta} dF(\pi). \ , \quad \beta \geq 0.$$

Using these measures of risk and return, Holthausen also showed that the α-β-t model is congruent with the expected utility model in which the utility function is:

$$(6)\quad U(\pi) = \begin{cases} (\pi - t)^{\beta} & \text{for all } \pi \geq t \\ -k(t - \pi)^{\alpha} & \text{for all } \pi \leq t \text{ and } k > 0. \end{cases}$$

where k is a constant for a given utility function and α and β reflect the risk profile of decision makers. If $\alpha < 1$ ($\alpha > 1$), then the individual is risk seeking (averse) below target and if $\beta < 1$ ($\beta > 1$), then the individual is risk averse (seeking) above target.

Whereas Fishburn's utility specification is linear in outcomes above the target, imposing risk neutrality on above-target returns, Holthausen's is nonlinear since it is weighted by a parameter which depends on the DM's risk preferences. Holthausen's specification allows the DM to have different risk preferences for outcomes above and below the target. Following Fishburn, the proposed MSD model measures return as expected value. Holthausen did not include expected value in his measure of return, suggesting that its use is somewhat redundant. However, Baumol (1963) has noted that DMs are generally satisfied with expected value as a measure of return.

Model Specification: The Mean-Separated Target Deviations Model

The Mean-Separated Target Deviations (MSD) model is motivated by the intuition that although decision makers in an investment environment are comfortable with expected value as a measure of return, they respond in different ways to potential outcomes below a target return than to potential outcomes above a target return. MSD uses an alternative to variance or semivariance as the measure of risk. The basic idea of MSD is that dispersion is separated into two parts: below-target deviations (BTD) and above-target deviations (ATD). BTD reduce the DM's expected utility, but ATD increase the DM's expected utility. Therefore, a higher level of dispersion of a distribution (e.g., variance) does not necessarily lower a DM's utility.

The general measure of risk is:

$$(7) \quad \Omega(F) = \int_{-\infty}^{t} \phi(t - \pi)dF(\pi) - \int_{t}^{\infty} \theta(\pi - t)dF(\pi) ,$$

where $\phi(\pi)$ and $\theta(\pi)$ are nonnegative nondecreasing function in π with $\phi(0) = \theta(0) = 0$, and F(π) is the cumulative probability distribution function over outcomes π.

In many ways, Ω is a more intuitive definition of risk than measures such as variance in E-V, semivariance in E-S, or below-target returns in the Target Risk Return model. Fishburn (1977) and Holthausen (1981) used only below target deviation (the first term in (8)) as the measure of risk, and Holthausen used above target deviation (the second term in (8)) as the measure of return. The measure of risk used here, Ω, is below-target deviations (BTD) less above-target deviations (ATD), with both weighted by probability and DM's risk attitude. Ω increases as BTD increases and decreases as ATD increases. This implies that the more negatively skewed the distribution, the higher the risk;

and the more positively skewed, the lower the risk. Since deviations are measured from the DM's target return, "skewness" from the DM's point of view may be more appropriately measured as skewness around the target[5]. Thus, this risk measure captures the skewness of the probability distribution and, as shown below, is very flexible, allowing one to incorporate various levels of risk attitude into the model.

A specific form of (7) is the MSD model which allows easy estimation. In the MSD model, risk is defined by

$$(8) \qquad SD_t(F) = \int_{-\infty}^{t} (t-\pi)^{\alpha} dF(\pi) - \int_{t}^{\infty} (\pi-t)^{\beta} dF(\pi), \quad \alpha,\beta>0.$$

<u>Congruence with Expected Utility</u>. Combining $\Omega(F)$ from equation (7) with expected value E_F gives a preference relationship in which the DM's preferences depend only on E_F and $\Omega(F)$. This section extends Fishburn's and Holthausen's results to show the relationship between the MSD model and the expected utility model. Let $U(E_F, \Omega(F))$ be a real-valued function such that, for all relevant distributions F and G,

F is preferred to G if and only if

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[5] Skewness around the mean is measured as $(1/n)\sum(\pi-\mu)/\sigma^3$, where μ is mean and σ is standard deviation, and skewness around the target is measured as $(1/n)\sum(\pi\text{-t})/s^3$, where t is the specified target return and s is standard deviation around the target instead of around the mean.

$U(E_F, \Omega(F)) > U(E_G, \Omega(G))$

where U is increasing in E and decreasing in Ω.

THEOREM 1: Suppose that, for all bounded distribution functions F and G, the MSD model with risk defined by (7) is congruent with expected utility in the sense that

(9) $U(E_F, \Omega(F)) > U(E_G, \Omega(G))$ *if and only if*

$$\int_{-\infty}^{\infty} U(\pi)dF(\pi) > \int_{-\infty}^{\infty} U(\pi)dG(\pi).$$

Then with $U(t) = 0$, $U(t-1) = t-1-\delta$ and $U(t+1) = t+1+\lambda$, there exist positive constants δ and λ such that

(10) $$U(\pi) = \begin{cases} \pi - \delta\phi(t - \pi) & \text{for all } \pi \leq t, \\ \pi + \lambda\theta(\pi - t) & \text{for all } \pi \geq t. \end{cases}$$

(Proof is given in the Appendix)

Fishburn's utility function in (3) is a special case where $\lambda = 0$.

The expected utility is:

(11) $$\int_{-\infty}^{\infty} U(\pi)dF(\pi) = E_F - \Omega(\pi)$$

When the MSD model is used, (10) gives

(12) $$U(\pi) = \begin{cases} \pi - \delta(t - \pi)^{\alpha} & \text{for all } \pi \leq t, \\ \pi + \lambda(\pi - t)^{\beta} & \text{for all } \pi \geq t. \end{cases}$$

δ is a unique solution to $U(t-1) = t-1-\delta$ and λ is a unique solution to $U(t+1) =$

$t+1+\lambda$. The utility function (12) can display various shapes depending on the values of α, β, δ, and λ. Some possible shapes of utility function according to (12) are given in Figure 1. The curve of $\alpha < 1$ is convex or risk preferring, the curve of $\alpha > 1$ is concave or risk averse, and the curve of $\alpha = 1$ is linear or risk neutral, all below target. The curve of $\beta < 1$ is concave or risk averse, the curve of $\beta > 1$ is convex or risk preferring, and the curve of $\beta = 1$ is linear or risk neutral, all above target. Even if $\alpha = 1$ and also $\beta = 1$, the individual is still risk averse if $\delta > \lambda$, and risk preferring if $\delta < \lambda$, because the utility function is kinked around the target.

Friedman and Savage (1948) suggested a three-segment utility function which is initially risk averse, then risk preferring, and then risk averse. Kahneman and Tversky (1979) suggested a function which is usually convex below target and concave above target. Fishburn and Kochenberger (1979) examined thirty empirically assessed utility functions with target points and found that the power functions as in (12) were substantially better than either the exponential or the linear functions. They also found that the majority of below-target functions were risk preferring and the majority of above-target functions were risk averse. Therefore, a utility function as (12) can be a sound candidate for a DM's utility functions.

The Arrow-Pratt absolute risk aversion (ARA) coefficient is defined as $r(\pi) = -U''(\pi)/U'(\pi)$, where U' and U" are the first and second derivatives of a von Neumann-Morgenstern utility function. The ARA coefficient can be used as an indicator of an individual's risk aversion. The larger the ARA coefficient, the more risk averse. If the ARA coefficient is negative, the individual is risk preferring, and if it is positive, the individual is risk averse. The absolute risk aversion function is invariant to positive linear transformations of the utility function. Therefore, upper and lower bounds on a DM's ARA function define an interval representation (King and Robison 1981). Like stochastic dominance with respect to a function, MSD can order risky choices based on interval representation of absolute risk aversion function.

From equation (12), the ARA coefficients below and above t are,

(13) $r_1 = \alpha(\alpha - 1)\delta(t - \pi)^{\alpha-2}/[1 + \alpha\delta(t - \pi)^{\alpha-1}], \quad \pi \leq t,$

$r_2 = -\beta(\beta - 1)\lambda(\pi - t)^{\beta-2}/[1 + \beta\lambda(\pi - t)^{\beta-1}], \quad \pi \geq t.$

If $\alpha > 1$ and $\beta > 1$, then the individual is risk averse below t ($r_1 > 0$) and risk preferring above t ($r_2 < 0$). If $0 < \alpha < 1$ and $0 < \beta < 1$, then the individual is risk preferring below target ($r_1 < 0$) and risk averse above target ($r_2 > 0$). α and β can be expressed as functions of r_1 and r_2, respectively. Since α and β are invariant to the level of wealth, the difference between π and t should be constant in solving (13) for α and β. For simplicity, letting the difference between π and t be unity and $\delta = \lambda = 1$[6],

(14) $\alpha = [1 + r_1 + ((1 + r_1)^2 + 4r_1)^{1/2}]/2, \quad \pi \leq t,$

$\beta = [1 - r_2 + ((1 - r_1)^2 - 4r_1)^{1/2}]/2, \quad \pi \geq t.$

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[6] α and β can also be obtained numerically, that is, the values of α and β that minimize $[r_1 - \alpha(\alpha-1)\delta/(1+\delta\alpha)]^2$ and $[r_2 + \beta(\beta-1)\lambda/(1+\lambda\beta)]^2$, respectively, are the parameters that reflect DMs' risk attitude. In this way, $\delta = \lambda = 1$ is not required.

The more risk averse below target, the bigger the value of α, and thus the more weighted the BTD is. The more risk preferring above target, the bigger the value of β, and thus the more weighted the ATD is. Therefore, the more negatively skewed the return distribution, the higher the risk in the MSD model.

A utility function of the type given in equation (12) can take a number of ranges depending on the values of r_1 and r_2. The power of the function can be determined from given values of r_1 and r_2. Given ranges of r_1 and r_2, the utility function in (12) can take a range, and thus interval analysis is possible. That is, MSD provides an efficient set for a utility function which lies within specified ranges. The MSD efficient set can be simply obtained using a computer spread sheet. The ranges of r_1 and r_2 corresponding to risk averse, risk neutral, and risk preferring DM can be obtained by eliciting the typical DMs' utility functions and computing the absolute risk aversion coefficients, or by using an interval approach (Meyer, 1977) to establish a range of absolute risk aversion. Alternatively, absolute risk aversion coefficients elicited in other studies might be used as proxies (Harris and Mapp, 1986). Depending on the context and the circumstances of the DM or his firm, t might be formulated as a target return such as one which yields the zero profit, return from an insured investment, or the return necessary to cover all variable costs of production, including the cost of borrowed capital, etc.

MSD efficient sets of each interval of ARA coefficient are obtained here by grid search method[7]. In the MSD model, the distribution F is better than another distribution G, if and only if the expected utility of F in (11) is greater than that of G. For each point of r_1 and r_2, one choice is selected as the best. For given ranges of r_1 and r_2, any choices selected as the best are included in the MSD efficient set.

Consistency of MSD with Stochastic Dominance Efficiency. Porter (1974) has demonstrated that the E-S efficient set is a subset of second degree stochastic dominance. Yitzhaki (1982) has shown that the M-G criterion is a necessary condition for first-and second-degree stochastic dominance. Fishburn (1977) and Holthausen (1981) have also shown that their models are consistent with the stochastic dominance rules. This section extends Fishburn's and Holthausen's results to the MSD model, and shows that MSD is consistent with stochastic dominance rules. The first, second, and third degree stochastic dominance rules are defined as:

F FSD G if and only if $F \neq G$ and $F(\pi) \leq G(\pi)$ for all π,

F SSD G if and only if $F \neq G$ and $F_1(\pi) \leq G_1(\pi)$ for all π, and

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[7]For example, for an individual who is risk averse below target and risk preferring above target, suppose the intervals of ARA coefficients are (0.56,2.80) below target and (-1.68,-0.56) above target. Using 0.01 as a grid size, the ARA coefficient below target can be 0.56,0.57,...,2.79,2.80, and the ARA coefficient above target can be -1.68,-1.67,..., -0.55,-0.56. The union of all sets obtained using all possible combinations of ARA coefficients below and above targets is the MSD efficient set for an individual who is risk averse below target and risk preferring above target. Evaluating only at points of ARA coefficients near boundaries of intervals may reduce computing time substantially.

F TSD G if and only if F $\neq$ G and $F_2(\pi) \leq G_2(\pi)$ for all π,

letting F FSD G, F SSD G, and F TSD G denote F dominate G by FSD, SSD, and TSD, respectively. Here, $F_1(\pi) = \int_{-\infty}^{\pi} F(x)dx$, so $F_1(\pi)$ is the area under $F(\pi)$ up to π, and $F_2 = 2\int_{-\infty}^{\pi} F_1(x)dx$, so $F_2(\pi)$ is twice the area under F_1 up to π.

The expected utility under distribution F can be defined as $E(u,F) = \int_{-\infty}^{\infty} u(\pi)dF(\pi)$. By Fishburn's (1977) Lemma 1, if F FSD G, then $E_F \geq E_G$ and E(U,F) $\geq$ E(U,G) for every utility function with U′ $\geq$ 0; if F SSD G, then $E_F \geq E_G$ and E(U,F) $\geq$ E(U,G) for every utility function with U′ $\geq$ 0 and U" $\leq$ 0; and if F TSD G, then $E_F \geq E_G$ and E(U,F) $\geq$ E(U,G) for every utility function with U′ $\geq$ 0, U" $\leq$ 0, and U‴ $\geq$ 0, where U′, U", and U‴ are the first, second, and third derivatives of utility function, respectively. Thus FSD corresponds to nondecreasing utility functions, SSD to nondecreasing and concave utility functions, and TSD to nondecreasing and concave utility function with U‴ $\geq$ 0. The following theorem shows the relationship between the MSD model and stochastic dominance rules.

THEOREM 2 : Except for risky alternatives with identical mean and separated target deviations, every MSD efficient set is a subset of the FSD set for $\alpha \geq 0$ and $\beta \geq 0$; every MSD efficient set is a subset of SSD set for $0 \leq \alpha \leq 1$ and $0 \leq \beta \leq 1$; and every MSD efficient set is a subset of TSD set for $\alpha \geq 2$ and

$\beta \geq 2$.

(Proof is given in the Appendix).

The remainder of this paper uses MSD to determine the efficient set of marketing strategies using basis as an indicator and compares the MSD efficient set with those determined using other criteria.

An Empirical Application

This section of the paper selects risk efficient marketing strategies for wheat producers. Several routine marketing strategies, including cash-only strategies and routine hedges, are compared with selective strategies using basis predictors. Since hedging shifts price risk in cash and futures markets to basis risk, unexpected changes in basis affect hedging outcomes. This empirical application considers whether producers can increase expected utility by using basis predictors, or indicators, to select marketing strategies.

Basis Indicator

Since futures prices and spot prices generally move in the same direction, price risk can be reduced by taking opposite positions in cash and futures markets. However, as Working (1953) noted, the movements of spot and futures prices do not show complete parallelism. A hedger can use this inequality between the movements of spot and futures prices to increase profits by using basis as an indicator for efficient marketing strategies. The unequal movement of spot and futures prices may provide indicators useful for reducing risk and improving profit. Net returns for several routine marketing strategies are computed each year for 1975 through 1990, as well as returns developed from selective hedging based on basis indicators. The returns from each of the strategies are compared using E-V, FSD, SSD, SDWRF, E-S, Mean-Gini, and MSD criteria.

The model used assumes that cash wheat and wheat futures contracts are traded in two time periods: at harvest time (period 0) and when the futures contracts are liquidated (period 1). For the two period model, net return per bushel is :

(15) $R = xC^0 + (1\text{-}x)C^1 + (F^1 - F^0)y - (1\text{-}x)CC,$

where R = net return (cents/bu)

C^n = cash price in period n, n=0,1.

F^n = futures contract price in period n, n=0,1.

x = 1 if sell wheat at harvest

x = 0 if store wheat at harvest

y = 1 if buy futures contract at harvest

y = -1 if sell futures contract at harvest

y = 0 if no action is taken in the futures market

CC = carrying cost; number of months multiplied by the sum of monthly storage and interest cost.

Three alternative marketing methods are considered in this analysis.

(a) The **CASH** method involves selling wheat at harvest and taking no additional action (x=1, y=0). The net return for this marketing method is:

(16) $R_i = C_i^0$, i = 74,75,,,91 ,

where R_i is net return for year i, and C_i^0 is the local cash price in period 0 (harvest time) in year i.

(b) The **SPECULATION** method involves selling wheat and buying futures contracts for speculation at period 0 (at harvest), and then liquidating the futures contract at period 1 (x=1,y=1). The net return is:

(17) $R_i = C_i^0 + (F_i^1 - F_i^0)$, i = 74,75,,,91,

where F_i^n = price of futures contract in period n in year i.

(c) The **SHORT HEDGING** method involves storing wheat and selling futures contracts in period 0, and then selling wheat and buying back the futures contracts in period 1 (x=0, y=-1). The net return is :

(18) $R_i = C_i^1 - (F_i^1 - F_i^0) - CC$, i = 74,75,,,91,

where C_i^1 is the local cash price in period 1 in year i. A fourth alternative, storing wheat at harvest for later sale (**STORAGE**), could be considered. However, this will yield approximately the same result as the SPECULATION strategy, except that the producer has to pay additional storage and interest charges.

A routine strategy is defined as one which follows a particular marketing method every year. The selective strategies considered here use expected changes in basis as an indicator to choose the best marketing method in each year. Instead of using the same marketing method every year, basis indicators are used to select one marketing method from among the three alternatives every year (See Table 1).

Current basis (CB) is defined as the difference between the harvest-time cash price and the harvest-time futures price for a given contract month. Expected basis (EB) is defined as the producer's expectation of the difference between the cash price and the futures price on the day the producer would liquidate any futures contracts and sell any cash commodity. In period 0, at harvest time, the producer forms an expectation of the period 1 basis.

Various forecasting models have been developed in previous studies, but several simple alternatives are presented here in an attempt to find marketing strategies that many producers could use. Two proxies, or forecasts, for expected basis (EB), as well as an indicator that combines the information obtained from both of these two proxies for expected basis, are presented. The first proxy assumes that an average of period 1 bases from previous years is a good predictor of this year's period 1 basis. Historical Expected Basis (HEB) is the average of period 1 bases for all years from 1974 up to year i. In each year, the average of daily futures prices for the month of liquidation is subtracted from the average of daily cash prices for that month. These monthly average period 1 bases are averaged together from 1974 to the year i to get an average period 1 basis. This average is used as a forecast of the year i's period 1 basis. A second proxy assumes that the historical average of period 0 (harvest-time) bases (Historical Average of Current Bases, HCB) is a good indicator of whether the current basis will increase or decrease by period 1. If CB is greater than its historical average (HCB), it is more likely to decrease than increase from period 0 to period 1. Conversely, if CB is smaller than its historical average (HCB), it is more likely to increase than decrease. Thus, HCB can be used to represent EB. As before, if CB is larger (smaller) than HCB, CB is larger (smaller) than EB.

Neither of the two proxies provides a perfect forecast of basis, so each proxy is adjusted by a measure of its variability. In Case I, each proxy is adjusted by its standard deviation. Thus, instead of $CB < EB$ and $CB \geq EB$, the adjusted indicators are $CB < EB - \sigma$ and $CB \geq EB + \sigma$ (σ denotes the standard deviation). In Case II, each proxy is adjusted by one-half standard deviation, that is, $CB < EB - \sigma/2$ and $CB \geq EB + \sigma/2$. The effect of these adjustments is to make a strategy other than CASH less likely to be used.

The strategies using the basis indicator are divided into three cases: Using Historical Expected Basis only (HEB), Using Historical Average of Current Bases Only (HCB), Using both HEB and HCB (HCEB). In mathematical form, the net returns from each of the adjusted strategies using the first two indicators are as follows: if $CB_i \geq EB_i + \sigma_{EBi}$, then the net return for year i is as shown in equation (16), where σ_{EBi} is the standard deviation of EB_i; if $CB_i < EB_i - \sigma_{EBi} - CC$, then the net return in year i is as shown in equation (18). Substituting HEB or HCB for EB gives the net return of the strategy using HEB, or the net return of the strategy using HCB, respectively.

The third indicator makes the conditions for taking a futures position even more strict by requiring that both proxies give the same sign before any marketing method other than CASH is taken. If CB is larger than or equal to not only HEB but also HCB, then choose the CASH method. That is, if $CB_i \geq \max \{ (HEB_i + \sigma_{HEBi}), (HCB_i + \sigma_{HCBi}) \}$, then the net return in year i is as shown in equation (16). On the other hand, if CB is smaller than both HEB and HCB, then the SHORT HEDGING method is the best. Therefore, if $CB_i < \min \{ (HEB_i - \sigma_{HEBi} - CC), (HCB_i - \sigma_{HCBi}) \}$, then the net return in year i is as shown in equation (18).

Procedures

Cash and futures prices for Hard Red Winter wheat are used to calculate daily basis between central Oklahoma and Kansas City for the period 1975-1990. Each year, June 20 basis (CB) is compared to the historical monthly basis for the month the underlying futures contract is liquidated (HEB), to the historical average daily basis on June 20 (HCB), and to both of them (HCEB). Daily bases are used for CB. Producers are assumed to make marketing decisions at harvest time (June 20) observing daily data. Monthly bases are used for expected basis (EB). In computing net returns, the closing cash and futures prices on June 20, as well as the first trading day in the month that the futures contract is liquidated, are used.

Carrying cost (CC) is defined as monthly storage cost plus monthly interest cost multiplied by the number of months wheat is stored. Assuming all wheat is stored in commercial storage, the monthly storage cost is assumed to be 2.5¢ per bushel. Defining interest cost as the interest savings from paying off a loan, the production loan rate of a commercial bank is used as the interest rate in computing CC.

The three most actively traded futures contracts (December, March, and May) are used in this analysis. As noted previously, cash wheat and futures contracts are assumed to be traded in two periods; at harvest time and when the futures contracts are liquidated. The futures contracts established at harvest are assumed to be liquidated on the first working day in either October, December, or March.

The net returns from using the three routine strategies and from choosing among the strategies using the basis indicators are computed for each year from 1975 through 1990. A total of 68 strategies are considered. Since some strategies have exactly the same return distributions, only 46 strategies with unique return distributions are analyzed. Each strategy is a set of net returns for 16 years. Table 1 shows summary statistics of the distributions of returns, whichever included in any efficient set considered. These observations are evaluated using E-V, E-S, FSD, SSD, SDWRF, Mean-Gini, and MSD.

The Generalized Stochastic Dominance Program (Cochran and Raskin) is used to perform the stochastic dominance analysis. Arrow-Pratt coefficients elicited at whole-farm income levels are adjusted to evaluate strategy choices described in terms of per bushel net returns. Risk aversion intervals (-1.68, -0.01), (-0.01,0.01), (0.01, 2.80), and (2.80. 5.60) are used to represent risk-preferring, risk-neutral, slightly risk-averse, and strongly risk-averse DM, respectively[8]. The levels of α and β in the MSD criterion are obtained by substituting the values of ARA coefficients into (14). In this analysis the cash prices at harvest are used as the target levels of net returns.

Results

The strategies included in each efficiency rule are indicated in Table 2. Individual strategies are described in a footnote to Table 2. The E-V efficient set consists of 13 strategies (Table 3). In FSD, 19 of 46 strategies are undominated. The SSD efficient set includes five strategies. The results of the SDWRF analysis show that one to four strategies are included for given ranges of ARA coefficients. Note that for the entire range (-1.68, 5.60), nine distinct strategies are included. Two strategies, $05HCB3_1$ and $12HCB1_2$, which did not appear in any of the smaller ranges are included in this entire range[9]. Using

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[8] ARA coefficients for wheat farmers elicited by King and Oamek (1983) are used in this analysis. The scales of the ARA coefficients are adjusted by the unit of outcome scale following Raskin and Cochran (1986).

[9]Given three ranges of ARA, $A=(r_1,r_2)$, $B=(r_3,r_4)$, and $C=(r_1,r_4)$, where $r_1 < r_2 < r_3 < r_4$. There exist some choice H which is undominated in range C. However, H can be dominated for smaller ranges of ARA, say A and B. Therefore, in this example, SDWRF ignores some choices in smaller ranges.

E-S, two strategies are included in the efficient set. Both of them are also in the SSD set, which is consistent with Porter's (1974) result. Two strategies are included in the M-G set, both are in the SSD set, which is consistent with results by Yitzhaki (1982), and by Buccola and Subaei (1984).

Twelve ranges of ARA coefficient are considered for evaluation under MSD (see Table 4). The first three cases are for a DM who is risk preferring below target, the next three ranges are for a DM who is risk neutral below target, the following three ranges are for a DM who is risk averse below target, and the last three ranges are for a DM who is strongly risk averse below target. Each group consists of three ranges of risk preferring, risk neutral, and risk averse above target. The interval of ARA coefficient below target is restricted to be greater than -0.17 and the interval of ARA coefficient above target to be smaller than 0.17 for α and β to be real numbers[10].

The MSD efficient set includes only one strategy $05HCB2_2$ for any given range of ARA coefficient (Table 4). The strategy selected under MSD is the least negatively skewed (Table 2). The strategies selected by the MSD criterion is also in the FSD set, which is consistent with THEOREM 2. However, MSD reduces the FSD efficient set more than 90%. The strategy selected by a DM who is risk averse both below and above target is also included in SSD efficient

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[10] If α and β are obtained numerically (See footnote 7), the ranges of r_1 and r_2 may not have to be restricted.

set in Table 3, which is consistent with THEOREM 2. The efficient set under MSD of an everywhere risk averse DM is smaller than the efficient set with SSD. The SSD efficient set includes five strategies, but the MSD efficient set for everywhere risk averse DM includes one.

The last row in Table 4 provides a comparison with the last row of the SDWRF results in Table 3. Both efficiency measures consider the same range of ARA, between -1.68 and 5.60. However, MSD uses a different range of ARA for below target (r_2 = (-1.68, 0.17)) and above target (r_1 = (-0.17, 5.60)), while SDWRF does not allow such a separation. For the entire range of r_2 = (-1.68, 0.17) and r_1 = (-0.17, 5.60), MSD contains one strategy. Since SDWRF includes nine distinct strategies for the entire range of (-1.68, 5.60) (Table 3), the efficient set of SDWRF is larger than that of MSD. Moreover, two strategies which are not included in any of the smaller intervals in SDWRF appear in the entire range (Table 3).

Conclusions

This paper develops a new risk efficiency model, Mean - Separated Target Deviations (MSD). MSD can be an interval analysis that orders risky choices for a decision maker whose monotonically increasing utility function lies within a specified range. Conventional measures of risk do not distinguish between below-target and above-target outcomes, or else impose risk neutrality for above-target outcomes. The model is motivated by the intuition that although decision makers in an investment environment are comfortable with expected value as a measure of return, they respond in different ways to potential outcomes below a target return than to potential outcomes above a target return. Dispersion as a measure of risk is separated into two parts: below-target deviations and above-target deviations. The risk measure in the model is below-target deviations minus above-target deviations, each term is weighted by probability and decision maker's risk attitude. Separating above-target returns from below-target returns allows the model to implicitly reflect the relationship between risk and skewness.

The MSD model is a generalization of Fishburn's (1977) model. Fishburn's model assumes risk neutrality above target. The MSD model avoids such a restriction. The MSD model is different from Holthausen's (1981) model in that it uses expected return as a measure of return as in Fishburn's while Holthausen uses above-target return as a measure of return. The MSD model goes beyond Fishburn's and Holthausen's model by allowing interval analysis.

The MSD model is shown to be congruent with von Neumann-Morgenstern expected utility theory. The model is also shown to be consistent with stochastic dominance. All the efficient strategies derived from MSD are also included in the first-degree stochastic dominance efficient set. The MSD efficient set for an individual who is everywhere risk averse is also a subset of second-degree stochastic dominance.

Efficient sets were determined for alternative marketing strategies to evaluate the usefulness of MSD. Results reveal that the MSD efficient set contains one strategy among forty six possible strategies for any given ranges of decision maker's absolute risk aversion coefficients and reduces the efficient set by more than 90% relative to FSD. For corresponding ranges of risk preferences, MSD yields a smaller efficient set than stochastic dominance rules.

Figure 1. Shapes of Utility Function in (12) for the Possible Values of α, β and $\delta > \lambda > 0$.

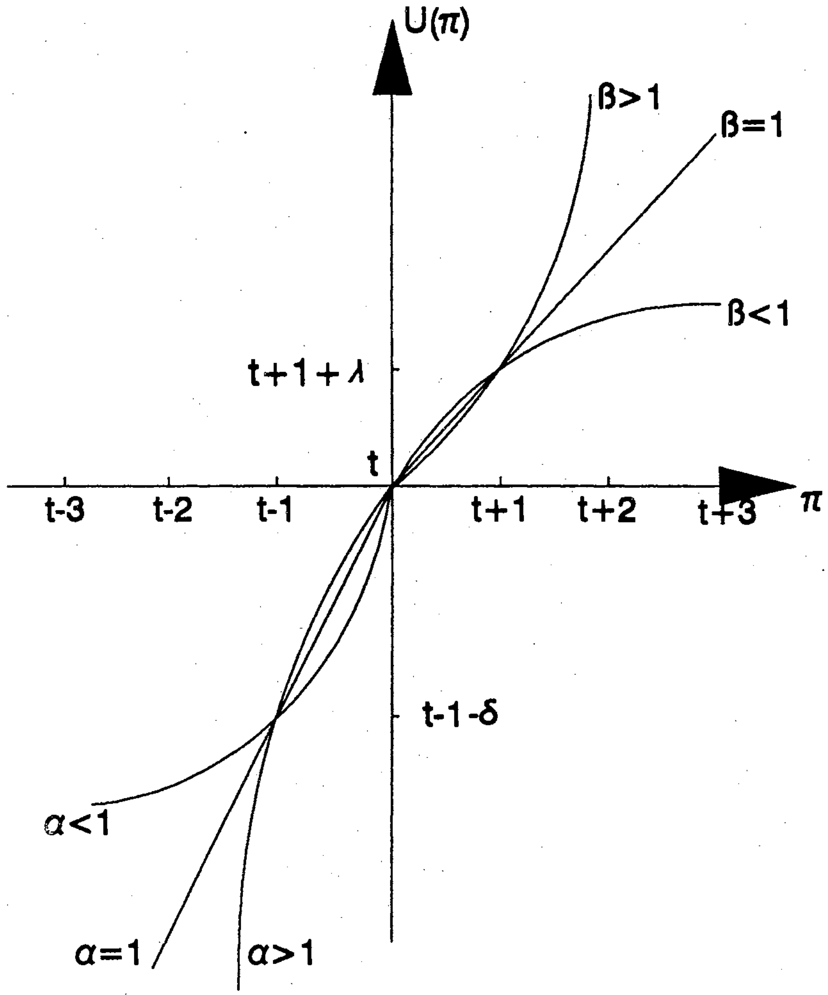

Table 1. Marketing Strategies, Basis Indicators, and Expected Basis Proxies Considered

Marketing Strategies	**Description**
Routine CASH	Sell cash commodity at harvest each year
Routine SPECULATION	Sell cash commodity and buy December, March, or May futures contract at harvest each year; sell contract in pre-expiration month
Routine SHORT HEDGING	Store cash commodity and sell December, March, or May futures contract at harvest; sell cash commodity and buy contract in pre-expiration month
Routine STORAGE	Store cash commodity at harvest; sell cash commodity in November, February, or April
Basis Indicator: CASH or SHORT HEDGING	Each year choose CASH or SHORT HEDGING strategy based on basis indicator
Basis Indicators: k = 1/2 or 1	**Chosen Strategy**
CB < (EB - carry - $k\sigma_{EB}$)	SHORT HEDGING
otherwise	CASH
Expected Basis Proxies	
EB = HCB	HCB = historical average of period 0 bases
EB = HEB	HEB = historical average of period 1 bases
EB = HCB and HEB	Basis Indicator for SHORT HEDGING must be satisfied with both EB = HCB and EB = HEB

Table 2. Summary Statistics of Alternative Strategies Included in Efficient Sets.

Strategies	Mean ($/bu)	Std. Dev. of Return	Max Return ($/bu)	Min Return ($/bu)	Skewness Around Mean[a]	Skewness Around Target[b]
CASH	3.1594	0.5458	4.00	2.15	-0.4935	-0.0267
STORAGE1	3.0519	0.4910	3.85	2.07	-0.1603	-0.4266
12LONG1	3.1163	0.5267	4.09	2.34	0.3022	-0.4816
12SHORT1	3.0950	0.5470	4.14	1.89	-0.4693	-0.0968
$12HCB1_1$	3.1282	0.5257	4.00	2.15	-0.4591	-0.0603
$12HEB1_1$	3.1362	0.5279	4.00	2.15	-0.4950	-0.0565
03LONG1	3.0827	0.6471	4.02	2.19	-0.0529	-0.3369
03SHORT3	3.0215	0.5853	3.88	1.85	-0.3202	-0.1629
03SHORT1	3.1285	0.7402	4.20	2.01	0.0187	-0.3883
$03HCB3_1$	3.1870	0.5687	4.00	2.15	-0.4701	-0.0229
05LONG2	3.0187	0.5795	3.97	1.94	-0.1742	-0.2820
05LONG3	2.9290	0.5275	3.78	2.00	0.0165	-0.6121
05LONG1	3.0705	0.6182	4.09	2.23	0.1143	-0.2820
05SHORT2	3.1353	0.6171	4.11	2.05	-0.1923	-0.0795
05SHORT3	3.0618	0.5911	4.05	1.96	-0.2159	-0.1412
05SHORT1	3.1407	0.7368	4.27	1.90	-0.0834	-0.4781
$05HCB2_1$	3.1952	0.5772	4.01	2.15	-0.4334	-0.0225
$05HCB3_1$	3.1994	0.5855	4.05	2.15	-0.3824	-0.0211
$05HCB1_1$	3.1987	0.6220	4.20	2.15	-0.1103	-0.1057
$12HCB2_2$	3.1946	0.5701	4.08	2.15	-0.4792	-0.0204
$12HCB1_2$	3.1530	0.5360	4.14	2.15	-0.4279	-0.0582
$12HEB1_2$	3.1260	0.5239	4.00	2.15	-0.4600	-0.0609
$03HEB2_2$	3.1845	0.5596	4.00	2.15	-0.5066	-0.0255
$03HEB1_2$	3.1702	0.5554	4.00	2.15	-0.4746	-0.0241
$03HCEB2_2$	3.1887	0.5623	4.00	2.15	-0.5125	-0.0225
$05HCB2_2$	3.2046	0.5879	4.11	2.15	-0.3844	-0.0195

* The name of the strategy consists of four parts: the first two digits denoting the underlying contract (i.e., 03 is March, 05 is May, and 12 is December contract); a name of a routine strategy (CASH, STORAGE, SPEC, or SHORT) or basis indicator used (HCB, HEB, or HCEB); a digit denoting the period when the contract is liquidated (1 = October 1 , 2 = December 1 of the current year, and 3 = March 1 of the following calendar year); at the end, the subscripts '1' denoting that the indicator is adjusted by one standard deviation (Case I) and '2' denoting that the indicator is adjusted by half standard deviation (Case II). For example, $12HCB1_1$ is the strategy of choosing one of the two alternative marketing methods according to the HCB indicator adjusted by one standard deviation (Case I) in each year and using the December contract which is liquidated on October 1 of the current year.

a Skewness around the mean is measured as $(1/n)^* \sum(\pi-\mu)^3/\sigma^3$, where μ is mean and σ is the standard deviation.

b Skewness around the target is measured as $(1/n)^* \sum(\pi-t)^3/s^3$, where t is the specified target return and s is the standard deviation around the target.

Table 3. Efficient Strategies with E-V, FSD, SSD, SDWRF, E-S, and M-G.

CRITERIA	RANGE OF ARA[a]	EFFICIENT SET[b]				
E-V	(0, ∞)	CASH	STORAGE1	12HCB1$_1$	12HEB1$_1$	
		05HCB2$_1$	05HCB3$_1$	12HCB1$_2$	12HCB2$_2$	
		12HEB1$_2$	03HEB12	03HEB2$_2$	03HCEB2$_2$	
		05HCB2$_2$				
FSD	(-∞,∞)	STORAGE1	12LONG1	12SHORT1	03SHORT1	05LONG3
		03SHORT3	03LONG1	05LONG1	05LONG2	05HCB1$_1$
		05SHORT1	05SHORT2	05SHORT3	03HCB3$_1$	05HCB2$_2$
		05HCB3$_1$	12HCB1$_2$	12HCB2$_2$	03HCEB2$_2$	
SSD	(0,∞)	12LONG1	12HCB1$_2$	12HCB2$_2$	03HCEB2$_2$	05HCB2$_2$
SDWRF	(-1.68, -0.01)	03SHORT1	05SHORT1	05HCB1$_1$	05HCB2$_2$	
	(-0.01, 0.01)	05HCB2$_2$				
	(0.01, 2,80)	12LONG1	12HCB2$_2$	03HCEB2$_2$	05HCB2$_2$	
	(2,80, 5.60)	12LONG1				
	(-1.68, 5.60)	03SHORT1,	05SHORT1,	12LONG1,	05HCB1$_1$,	05HCB2$_2$
		05HCB3$_1$*	12HCB1$_2$*	12HCB2$_2$	03HCEB2$_2$	
E-S	(0, ∞)	12HCB2$_2$	05HCB2$_2$			
M-G	(0, ∞)	12HCB2$_2$	05HCB2$_2$			

a ARA is the Arrow-Pratt absolute risk aversion coefficient
b See notes to table 2 for names of strategies.
* These strategies do not appear in any smaller range of SDWRF (See footnote 10 for details).

Table 4. Efficient Strategies with the MSD Criterion.

Absolute Risk Aversion Coefficient Intervals Below Target(r_1)	Above Target(r_2)	Efficient Set[a]	Stochastic Dominance
(-0.17,-0.01) risk preferring	(-1.68,-0.01) risk preferring	$05HCB2_2$	FSD
	(-0.01,0.01) risk neutral	$05HCB2_2$	FSD
	(0.01,0.96) risk averse	$05HCB2_2$	FSD
(-0.01,0.01) risk neutral	(-1.68,-0.01) risk preferring	$05HCB2_2$	FSD
	(-0.01,0.01) risk neutral	$05HCB2_2$	FSD
	(0.01,0.96) risk averse	$05HCB2_2$	FSD
(0.01,2.80) risk averse	(-1.68,-0.01) risk preferring	$05HCB2_2$	FSD
	(-0.01,0.01) risk neutral	$05HCB2_2$	FSD
	(0.01,0.17) risk averse	$05HCB2_2$	FSD(SSD)
(2.80.5.60) strongly risk averse	(-1.68,-0.01) risk preferring	$05HCB2_2$	FSD
	(-0.01,0.01) risk neutral	$05HCB2_2$	FSD
	(0.01,0.01) risk averse	$05HCB2_2$	FSD(SSD)
(-0.17,5.60)	(-1.68,0.17)	$05HCB2_2$	FSD

a See notes to Table 2 for names of strategies.

Appendix

Proof of Theorem 1: The basic idea of the proof follows that of Theorem 2 in Fishburn (1977). For notational simplicity, let $t=0$ with $U(0)=0$. For $\pi < -1$, let h exceed $-\pi$ and consider two gambles : F_1 is the fifty-fifty gamble for -1 or $g > 0$, and F_2 is the distribution which has probability $\phi(1)/2\phi(-\pi)$ for π and probability $[2\phi(-\pi) - \phi(1)]/2\phi(-\pi)$ for h, where $g = \phi(1)/\phi(-\pi) + h[2\phi(-\pi) - \phi(1)]/\phi(-\pi) + 1$ and $\theta(g) = \theta(h)[2\phi(-\pi) - \phi(1)]/\phi(-\pi)$. Then $E(F_1) = E(F_2)$ and $\Omega(F_1) = \Omega(F_2)$. Therefore, $(1/2)U(-1) + (1/2)U(g) = U(\pi)\phi(1)/2\phi(-\pi) + U(h)[2\phi(-\pi) - \phi(1)]/2\phi(-\pi)$. Solving for $U(\pi)$ yields $U(\pi) = \pi - \delta\phi(-\pi)$.

For $-1 < \pi < 0$, let h exceed $-\pi$ and consider two gambles : F_1 is the fifty-fifty gamble for π or h, and F_2 is the distribution which has probability $\phi(-\pi)/2\phi(1)$ for -1 and probability $[2\phi(1) - \phi(-\pi)]/2\phi(1)$ for g, where $g > \phi(-\pi)/[2\phi(1)-\phi(\pi)]$. Define $h = 2g - (1 + g)\phi(-\pi)/\phi(1) - \pi$ and $\theta(h) = \theta(g)[2\phi(1) - \phi(-\pi)]/\phi(1)$. Then $E(F_1) = E(F_2)$ and $\Omega(F_1) = \Omega(F_2)$. Therefore, $(1/2)U(\pi) + (1/2)U(h) = U(-1)\phi(-\pi)/2\phi(1) + U(g)[2\phi(1) - \phi(-\pi)]/2\phi(1)$. Solving for $U(\pi)$ yields $U(\pi) = \pi - \delta\phi(-\pi)$.

For $0 < \pi < 1$, consider two gambles : F_1 is fifty-fifty gamble for π or h, where $h < -\pi$, and F_2 gives 1 with probability $\theta(\pi)/\theta(1)$ and g with $[\theta(1) - \theta(\pi)]/\theta(1)$, where $g < -\theta(\pi)/[\theta(1)-\theta(\pi)]$. Define $h = 2\theta(\pi)/\theta(1) + 2g - g\theta(\pi)/\theta(1) - \pi$ and $\phi(-h) = 2\phi(-g) - 2\phi(-g)\theta(\pi)/\theta(1) - \theta(\pi)\lambda/\delta$. Then $E(F_1) = E(F_2)$ and $\Omega(F_1) = \Omega(F_2)$ so that $(1/2)U(\pi) + (1/2)U(h) = U(1)\theta(\pi)/\theta(1) + U(g)[\theta(1) - \theta(\pi)]/\theta(1)$. Solving for $U(\pi)$ yields $U(\pi) = \pi + \lambda\theta(\pi)$.

For $\pi > 1$, let $g < -1$ and consider two gambles : F_1 is fifty-fifty gamble for 1 or g, F_2 gives π with probability $\theta(1)/\theta(\pi)$ and h with $[\theta(\pi) - \theta(1)]/\theta(\pi)$, where $h < -\pi\theta(1)/[\theta(\pi)-\theta(1)]$. Define $g = 2\pi\theta(1)/\theta(\pi) + 2h - 2h\theta(1)/\theta(\pi) - 1$ and $\phi(-g) = 2\phi(-h) - 2\delta\phi(-h)\,\theta(1)/\theta(\pi) - \lambda\theta(1)$. Then $E(F_1) = E(F_2)$ and $\Omega(F_1) = \Omega(F_2)$ so that $(1/2)U(1) + (1/2)U(g) = U(\pi)\theta(1)/\theta(\pi) + U(h)[\theta(\pi) - \theta(1)]/\theta(\pi)$. Solving for $U(\pi)$ yields $U(\pi) = \pi + \lambda\theta(\pi)$. Therefore, proof is completed.■

Proof of Theorem 2: If F dominates G by FSD, then $F(\pi) \leq G(\pi)$ for all values of π, where $F(\pi)$ and $G(\pi)$ are the cumulative distribution functions of return on alternative risky actions F and G, respectively.

(19)

$$\Delta = SD_t(F) - SD_t(G) = \int_{-\infty}^{t} (t-\pi)^{\alpha}[dF(\pi)-dG(\pi)] - \int_{t}^{\infty} (\pi-t)^{\beta}[dF(\pi)-dG(\pi)].$$

Integrating (19) by parts,

(20)

$$\Delta = (t-\varepsilon_1)^{\alpha}[F(-\infty)-G(-\infty)] + \alpha\int_{-\infty}^{t} (t-\pi)^{\alpha-1}[F(\pi)-G(\pi)]d\pi + \beta\int_{t}^{\infty} (\pi-t)^{\beta-1}[F(\pi)-G(\pi)]d\pi,$$

since $F(\infty) = G(\infty) = 1$. ε_1 is the lower limit of integration. By FSD, $F(\pi) \leq G(\pi)$ for all π. Therefore, equation (20) is nonpositive for $\alpha \geq 0$ and $\beta \geq 0$. If, for any probability density functions, F dominates G by FSD, then the mean of F is at least as large as that of G for $\alpha \geq 0$, and $\beta \geq 0$. Under the assumption that F and G differ in either mean or separated semivariance, the above result is sufficient to guarantee that F dominates G by the MSD criterion.

Integrating (20) by parts,

(21)

$$\Delta = \alpha(t-\varepsilon_1)^{\alpha-1}[F_1(-\infty)-G_1(-\infty)] + \alpha(\alpha-1)\int_{-\infty}^{t}(t-\pi)^{\alpha-2}[F_1(\pi)-G_1(\pi)]d\pi$$

$$+ \beta(\varepsilon_2-t)^{\beta-1}[F_1(\infty)-G_1(\infty)] + \beta(\beta-1)\int_{t}^{\infty}(\pi-t)^{\beta-2}[F_1(\pi)-G_1(\pi)]d\pi,$$

where ε_2 is the lower and upper limits of integrations. $F_1(\pi) \leq G_1(\pi)$ for all π by SSD. Therefore, equation (21) is nonpositive and thus SSD implies MSD for $0 \leq \alpha \leq 1$ and $0 \leq \beta \leq 1$.

Integrating (21) by parts,

(22)

$$\Delta = \alpha(t-\varepsilon_1)^{\alpha-1}[F_1(-\infty)-G_1(-\infty)] + \alpha(\alpha-1)(t-\varepsilon_1)^{\alpha-2}[F_2(-\infty)-G_2(-\infty)]$$

$$+ \alpha(\alpha-1)(\alpha-2)\int_{-\infty}^{t}(t-\pi)^{\alpha-3}[F_2(\pi)-G_2(\pi)]d\pi + \beta(\varepsilon_2-t)^{\beta-1}[F_1(\infty)-G_1(\infty)]$$

$$+ \beta(\beta-1)(\varepsilon_2-t)^{\beta-2}[F_2(\infty)-G_2(\infty)] + \beta(\beta-1)(\beta-2)\int_{t}^{\infty}(\pi-t)^{\beta-3}[F_2(\pi)-G_2(\pi)]d\pi.$$

$F_2(\pi) \leq G_2(\pi)$ for all π by TSD. Therefore, equation (22) is nonpositive and thus TSD implies MSD for $\alpha \geq 2$ and $\beta \geq 2$.■

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VITA

Taehoon Kang

Candidate for the Degree of

Doctor of Philosophy

Thesis: GARCH OPTION PRICING, VALUING THE TARGET PRICE SUPPORT PROGRAM, AND A NEW RISK EFFICIENCY CRITERION

Major Field: Agricultural Economics

Biographical:

Personal Data: Born July 10, 1960, in Seoul, Korea, the son of Myongjae Lee and Shinbong Kang.

Education: Graduated from High School in the College of Education at Dongkook University, Seoul, Korea in 1979; Bachelor of Economics from the Department of Agricultural Economics at Seoul National University, Seoul, Korea in 1983; Master of Economics from the Department of Agricultural Economics at Seoul National University, Seoul, Korea in 1985; completed requirements for the Doctor of Philosophy degree at Oklahoma State University in July, 1993.

Professional Experience: Teaching Assistant, Department of Agricultural Economics, Seoul National University, March, 1983 - August, 1985; Assistant Researcher, Korea Rural Economics Institute, July, 1983 - December, 1985; Korean Army, December, 1985 - February, 1988; Customer Credit Analyst, Korea Housing Bank, Seoul, Korea, March, 1988 - May, 1989; Research Assistant, Department of Agricultural Economics, Oklahoma State University, August, 1989 - August, 1993.

Professional Memberships: American Agricultural Economic Association, Western Agricultural Economic Association, Southern Agricultural Economic Association, Asian Society of Agricultural Economists, Korean Agricultural Economic Association.

Awards: The 1993 Outstanding Graduate Student Paper Award, Southern Agricultural Economic Association.