

**MULTI-AXIS PILOT MODELING FOR
A DAMAGED AIRCRAFT**

By

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TABLE OF CONTENTS

Chapter	Page
I. INTRODUCTION	1
1.1 Problem Statement	1
1.2 Organization	2
II. LITERATURE REVIEW	4
2.1 Cooper-Harper Rating Scale and Pilot Opinion Ratings.....	4
2.2 The Development of Pilot Models	5
2.3 Correlation of Pilot Model and POR	7
2.4 Comparison of Classical and Modern Approaches of the Pilot Model	10
III. OVERVIEW OF THE DEVELOPED METHODOLOGY	11
3.1 Details of the Utilized Aircraft	11
3.2 Definitions of Maneuvers	11
3.3 Control Surface Damage	15
3.4 Strategic Procedures of the Methodology	15
IV. THEORETICAL DEVELOPMENT	17
4.1 Modeling of Control Surface Damage	17
4.2 Dynamic Models of the Specific Aircraft with Damage	32
4.3 Derivation of Single-axis Transfer Functions	45
4.4 The Logic of Optimal Pilot Model	45
4.5 Multi-axis POR	51
V. DETAILS OF IMPLEMENTATION	54
5.1 The Assigned Constants and Range of Study	54
5.2 Sample Case	56
VI. ANALYSIS OF RESULTING DATA	61
VII. CONCLUSIONS	70
REFERENCES	72

Chapter	Page
APPENDICES	74
APPENDIX A - DERIVATION OF RATE OF TURN FOR COMBINATIONAL MANEUVER	75
APPENDIX B - FLIGHT CONDITIONS, AERODYNAMIC, INERTIAL, GEOMETRIC DATA	77
APPENDIX C - DIMENSIONAL STABILITY DERIVATIVES.....	79
APPENDIX D - INPUT MATRIX	82
APPENDIX E - PLANT MATRICES	84
APPENDIX F - RESULTING PORS	92
APPENDIX G - MATLAB PROGRAM FOR THE OPTIMAL PILOT MODEL	96

LIST OF TABLES

Table	Page
I. Mathematical Definitions of Maneuvers	16
II. Cooper-Harper Handling Qualities Rating Scale	20
III. Pilot Model Parameter Values	61

LIST OF FIGURES

Figure	Page
1. Gates Learjet Model 24B Dimensions and Geometry	12
2. Half-sided Sketch of a Control Surface	18
3. Aircraft Coordinate System	33
4. Definition of Stability Axes	37
5. Block Diagram of Optimal Control Model of Human Pilot	46
6. Control-theoretic Model of Optimal Human Behavior	47
7. Computation Flow of the Optimal Pilot Model	50
8. POR vs. Performance Index	52
9. POR of Straight and Level flight vs. Damage	62
10. POR of Symmetric Pull-up at $\Theta_0 = 15^\circ$ vs. Damage	63
11. POR of Steady Level Turn at $\Phi_0 = 15^\circ$ vs. Damage	64
12. POR of Combinational Maneuver at $\Theta_0 = 15^\circ$, $\Phi_0 = 15^\circ$ vs. Damage	65
13. POR of Steady Level Turn at $\Phi_0 = 30^\circ$ vs. Damage	66
14. POR of Combinational Maneuver at $\Theta_0 = 30^\circ$, $\Phi_0 = 30^\circ$ vs. Damage	67

NOMENCLATURE

α	angle of attack (perturbed)
$\dot{\alpha}$	rate of change in α (perturbed)
β	angle of sideslip (perturbed)
$\dot{\beta}$	rate of change in β (perturbed)
δ_a	angle of aileron (perturbed)
δ_e	angle of elevator (perturbed)
δ_r	angle of rudder (perturbed)
ε	angle of downwash
Φ_0	angle of bank (steady state)
ϕ	angle of bank (perturbed)
$\dot{\phi}$	rate of change in ϕ (perturbed)
η_h	dynamic pressure ratio at the horizontal tail
Λ	angle of sweep
Σ	Kalman filter factor
Θ_0	angle of pitch (steady state)
θ	angle of pitch (perturbed)
$\dot{\theta}$	rate of change in θ (perturbed)
ρ_{yi}	full attention observation noise to signal ratio
ρ_{uci}	motor noise ratio
σ_c^2	mean square output error due to the driving noise
τ	reaction time of pilot
τ_l	lead time constant of pilot

τ_E	angle of attack effectiveness of the elevator
τ_N	neuro-muscular constant of pilot
$\dot{\psi}_0$	rate of turn (steady State)
A	aspect ratio
A	plant matrix of dynamic model
a	half span
\underline{A}	aspect ratio of control surface damage with damage
\underline{a}	half span with damage
\underline{A}_h	aspect ratio of horizontal tail with damage
\underline{A}_w	aspect ratio of wing with damage
\underline{A}_v	aspect ratio of vertical tail with damage
B	input matrix of dynamic model
b	span of control surface
\underline{b}	span of control surface with damage
C	output matrix of dynamic model
\bar{C}	chord of control surface
$C_{D\alpha}$	variation of drag coefficient with α
$\underline{C}_{D\alpha}$	variation of drag coefficient with α after damage
$C_{L\alpha}$	variation of lift coefficient with α
$\underline{C}_{L\alpha}$	variation of lift coefficient with α after damage
$C_{L\alpha} \dot{\alpha}$	variation of lift coefficient with $\dot{\alpha}$
$\underline{C}_{L\alpha} \dot{\alpha}$	variation of lift coefficient with $\dot{\alpha}$ after damage
$\underline{C}_{L\alpha h}$	variation of lift coefficient of horizontal tail with α after damage
$\underline{C}_{L\alpha w}$	variation of lift coefficient of wing with α after damage
$C_{L\alpha WB}$	variation of lift coefficient of wing box with α

$\underline{C_{L\alpha}}$	variation of lift coefficient of vertical tail with α after damage
$C_{l\beta}$	variation of rolling moment coefficient with β
$\underline{C_{l\beta}}$	variation of rolling moment coefficient with β after damage
C_{lp}	variation of rolling moment coefficient with p
$\underline{C_{lp}}$	variation of rolling moment coefficient with p after damage
$C_{L\delta_e}$	variation of lift coefficient with δ_e
$\underline{C_{L\delta_e}}$	variation of lift coefficient with δ_e after damage
C_{Lq}	variation of lift coefficient with q
$\underline{C_{Lq}}$	variation of lift coefficient with q after damage
$C_{m\alpha}$	variation of pitching moment with α
$\underline{C_{m\alpha}}$	variation of pitching moment with α after damage
$C_{\dot{m}\alpha}$	variation of pitching moment with $\dot{\alpha}$
$\underline{C_{\dot{m}\alpha}}$	variation of pitching moment with $\dot{\alpha}$ after damage
C_{mq}	variation of pitching moment with q
$\underline{C_{mq}}$	variation of pitching moment with q after damage
$C_{n\beta}$	variation of yawing moment with β
$\underline{C_{n\beta}}$	variation of yawing moment with β after damage
$C_{n\delta_r}$	variation of yawing moment with δ_r
$\underline{C_{n\delta_r}}$	variation of yawing moment with δ_r after damage
C_{nr}	variation of yawing moment with r
$\underline{C_{nr}}$	variation of yawing moment with r after damage
$C_{y\beta}$	variation of side force with β
$\underline{C_{y\beta}}$	variation of side force with β after damage
$C_{y\delta_r}$	variation of side force with δ_r
$\underline{C_{y\delta_r}}$	variation of side force with δ_r after damage

C_{yr}	variation of side force with r
\underline{C}_{yr}	variation of side force with r after damage
f_i	attention fraction
F_x	aerodynamic force in X-axis
F_y	aerodynamic force in Y-axis
F_z	aerodynamic force in Z-axis
g	acceleration of gravity
I_x	moment of inertia about the X-axis
I_y	moment of inertia about the Y-axis
I_z	moment of inertia about the Z-axis
I_{xy}	XY product of inertia
I_{yz}	YZ product of inertia
I_{xz}	XZ product of inertia
i_h	incidence angle of horizontal tail
J	single-axis performance index before normalization
\bar{J}	normalized single-axis performance index
L	root length of control surface
l	tip length of control surface
\underline{l}	tip length of control surface after damage
\dot{l}	feedback gain of the Optimal Pilot Model
L_β	dimensional variation of rolling moment about the X-axis with β
L_{δ_a}	dimensional variation of rolling moment about the X-axis with δ_a
L_{δ_r}	dimensional variation of rolling moment about the X-axis with δ_r
L_p	dimensional variation of rolling moment about the X-axis with p
L_r	dimensional variation of rolling moment about the X-axis with r
M	Mach number

m	mass of the aircraft
m_x	aerodynamic moment in X-axis
m_y	aerodynamic moment in Y-axis
m_z	aerodynamic moment in Z-axis
M_α	dimensional variation of pitching moment about the Y-axis with α
$M_{\dot{\alpha}}$	dimensional variation of pitching moment about the Y-axis with $\dot{\alpha}$
M_{δ_e}	dimensional variation of pitching moment about the Y-axis with δ_e
M_q	dimensional variation of pitching moment about the Y-axis with q
M_u	dimensional variation of pitching moment about the Y-axis with u
n	load factor
N_β	dimensional variation of yawing moment about the Z-axis with β
N_{δ_a}	dimensional variation of yawing moment about the Z-axis with δ_a
N_{δ_r}	dimensional variation of yawing moment about the Z-axis with δ_r
N_p	dimensional variation of yawing moment about the Z-axis with p
N_r	dimensional variation of yawing moment about the Z-axis with r
P_0	rate of roll about the X-axis (steady state)
p	rate of roll (perturbed)
\dot{p}	rate of change in p (perturbed)
q	rate of pitch (perturbed)
Q_0	rate of pitch about the Y-axis (steady state)
q	rate of pitch (perturbed)
\dot{q}	rate of change in q (perturbed)
R_0	rate of yaw about the Z-axis (steady state)
R_t	radius of turn
r	rate of yaw (perturbed)
\dot{r}	rate of change in r (perturbed)

S	area of control surface
\underline{S}	area of control surface after damage
S_h	area of horizontal tail
\underline{S}_h	area of horizontal tail after damage
S_w	area of wing
\underline{S}_w	area of wing after damage
S_v	area of vertical tail
\underline{S}_v	area of vertical tail after damage
T_i	perception indifference threshold in i-axis
U_0	forward velocity along the X-axis (steady state)
u	forward velocity along the X-axis (perturbed)
\dot{u}	rate of change in u (perturbed)
v	side velocity (perturbed)
V_{yi}	noise intensity in i-axis
W	weight of the aircraft
W_0	downward velocity (steady state)
w	downward velocity (perturbed)
X_α	dimensional variation of X-force with α
X_{δ_e}	dimensional variation of X-force with δ_e
\overline{X}_{cg}	location of center of gravity in terms of \overline{C}
X_u	dimensional variation of X-force with u
Y_β	dimensional variation of Y-force with β
Y_{δ_a}	dimensional variation of Y-force with δ_a
Y_{δ_r}	dimensional variation of Y-force with δ_r
Y_p	dimensional variation of Y-force with p
Y_r	dimensional variation of Y-force with r

- Z_{α} dimensional variation of Z-force with α
- Z_{δ_e} dimensional variation of Z-force with δ_e
- Z_u dimensional variation of Z-force with u

CHAPTER I

INTRODUCTION

1.1 Problem Statement

The assessment of loss of control of a control surface damaged aircraft should receive further attention because of the safety and economic considerations. In many critical situations, the knowledge about the flying control status is desperately needed so that no human life is sacrificed for saving a seriously damaged aircraft, and the valuable aircraft is not abandoned while it can still survive. It is obviously not practical to obtain such knowledge by carrying out real test flights for each specific type of aircraft with various types and degrees of control surface damage. In this research, a computer simulation methodology is developed to obtain the data associated with the flying control status of various multi-axis maneuvers for a specific aircraft with various types and degrees of damage.

In this methodology, for a specific maneuver, a dynamic model of each axis is first developed for every specific control surface damage. Each resulting dynamic model is input into a computer program, the Optimal Pilot Model (OPM), to obtain the associated performance index, also known as cost. By the utilization of the cost function/ rating correlation model, the obtained performance index is then transformed into the corresponding value of pilot opinion rating (POR). For a specific control surface damage, the POR of each axis is thus integrated into the associated multi-axis POR of each maneuver by the use of the product rule.

Finally, by using the Cooper-Harper scale for the interpretation of the resulting multi-axis POR, the specific aircraft with a specific control surface damage is determined to be in whether a state of loss of control or a state of in-control during a specific maneuver.

Based on some valuable previous works, which are reviewed in Chapter II, this research develops a systematic methodology to achieve the following goals: (1) to develop the multi-axis dynamic models for a specific aircraft with various types and degrees of damage, (2) to relate the Cooper-Harper pilot ratings to the performance index for pilot closed loop control of damaged aircraft dynamics by use of the optimal control pilot model, (3) to monitor and predict the states of loss of control for various maneuvers performed by a specific aircraft with various types and degrees of damage, and (4) to establish a set of safety criteria for operating a specific aircraft; where the maneuver can be a straight and level flight, a steady level turn, a symmetrical pull-up, or a combination of a steady level turn and a symmetrical pull-up.

1.2 Organization

The pilot opinion rating is a proper tool for determining the state of control of a specific aircraft which is performing a specific maneuver. However, it is not likely to utilize flight tests to obtain the POR for a damaged aircraft. In fact, some existing experimental data can provide enough information for obtaining POR if a proper methodology is established. The methodology developed in this research is applied to a specific aircraft, Gates Learjet 24B, for the assessment of loss of control and a series of reasonable results are achieved. The foundation of this methodology is briefly illustrated in Chapter II which includes the flying quality evaluation scale, the different approaches of human pilot model, the correlations

between performance index and pilot opinion rating from different experimental data.

Chapter III introduces the configuration of the specific aircraft which induces several aerodynamic characteristics. This chapter also presents the mathematical definitions of the four typical maneuvers performed by a subsonic aircraft. The types and degrees of control surface damage are also defined numerically in this chapter. Also, Chapter III primarily provides a concise skeleton of the developed methodology.

Moreover, Chapter IV illustrated the theoretical development of the methodology on a step-by-step basis. In this chapter, the control surface damage is first mathematically modeled. The six rigid-body equations of motion are employed and developed into the general form of a dynamic model which consists of a pilot input matrix and an aircraft plant matrix. The single-axis transfer function, is thus derived from the obtained dynamic model. This single-axis transfer function is input into a software of the human pilot model, which is also briefly illustrated in this chapter, to obtain the corresponding single-axis performance index. The obtained single-axis performance is then transformed into the associated single-axis pilot opinion rating. The procedure of integrating single-axis PORs into the corresponding multi-axis POR is then presented in the end of this chapter.

Chapter V presents some details for applying the methodology to the specific aircraft. Several constants are given in this chapter. Moreover, the driving noise shaping filter employed in this methodology are also introduced. The analytical results are plotted and discussed in Chapter VI. Based on the resulting data, a set of safety criteria for operating the specific aircraft is then presented in this chapter. Chapter VII summarizes the principal conclusion of this thesis and contains suggestions for further research.

CHAPTER II

LITERATURE REVIEW

The response of a pilot to the encountered situation is practically based on the state of control rather than directly on the type and degree of damage that occurs to the aircraft. However, when a failure takes place, the type and amount of the damage dictates the controllability of the aircraft. Therefore, the relationship between failure and flying quality should be established. This chapter presents the previous works which provide the foundation of this research and make the contribution of this research possible.

2.1 Cooper-Harper Rating Scale and Pilot Opinion Ratings

Cooper and Harper [1] provide the standard of evaluating the controllability of aircraft based on their flying and engineering specialties. This scale represents a very successful attempt to relate pilot comments about the ease or difficulty with which aircraft can be controlled in certain flight situations to a numerical rating. The resulting rating, named POR, ranges from 1 (best) to 9 or 10 (worst), which is dominated by the pilot's mental and physical workload required to achieve the performance implied by a given mission phase.

Based on the Cooper-Harper pilot opinion rating scale, Dander [2] and Mitchell, Aponso, and Hoh [3] have reported experimental findings from single and multi-axis tracking tasks. Dynamically independent single, two, and/or three axis tracking experiments are conducted, and then subjective PORs are given for

each task. Although the best way to make the extension is not evident, several methodologies are proposed by Dander to predict multi-axis PORs based on single-axis results. Nevertheless, the Dander data is still the best data base upon which to test out theories.

2.2 The Development of Pilot Models

The analytical techniques for modeling the human pilot generally fall into two categories, the classical and modern approaches. For classical approaches, it relies heavily upon frequency domain representation such as a quasi-linear model of the human as a controller of single-input, single-output (SISO) systems developed by McRuer and Krendel [4]. The most common form of the classical human pilot model is

$$G_p(s) = K_p \frac{\tau_1 s + 1}{\tau_2 s + 1} \frac{e^{-Ts}}{\tau_n s + 1}$$

The last term models inherent human limitations of reaction time delays, and lags attributed to the neuro-muscular system. The remaining term represents the human's equalization characteristics and are adjustable in accordance with the type and difficulty of the control task.

The frequency domain pilot models are somewhat limited to single-input, single-output systems. Multiloop models have been implemented by Magdaleno, McRuer, and Stepelford [5] in which subsequential loop closure techniques are used with some limited success. The single loop block diagram of a roll tracking (hold wing level) task is also studied by Swaim [6]. It is seen that the pilot transfer function could be combined with aileron actuator transfer function and aileron to roll transfer function to yield a single block composite transfer function to be used

in further analysis and synthesis of any necessary stability augmentation system (SAS) for this roll tracking task.

For the modern approach, it is rooted in modern control and optimization theory based on the time domain. It is capable of treating multi-variable systems within a single conceptual framework using state space techniques. A flying quality evaluation technique called performance index is developed by Kleinman, Baron, and Levison. The basic assumption implied in this approach is that, subject to the pilot's inherent limitations, the well-trained, well-motivated pilot behaves in an optimal manner where "optimal" refers to a specific quadratic index of performance expressed as the following equation:

$$J = E \left\{ \lim_{T \rightarrow \infty} \int_0^T \frac{1}{T} [\underline{\mathbf{y}}^T(t) \mathbf{Q} \underline{\mathbf{y}}(t) + \dot{\underline{\mathbf{u}}}^T(t) \mathbf{R} \dot{\underline{\mathbf{u}}}(t)] dt \right\}$$

where $E\{\cdot\}$ = the expected value of $\{\cdot\}$,

$\underline{\mathbf{y}}(t)$ = vector of displayed and perceived variables

$\dot{\underline{\mathbf{u}}}(t)$ = vector of pilot control rate

\mathbf{Q} = weighting matrix of $\underline{\mathbf{y}}(t)$

\mathbf{R} = weighting matrix of $\dot{\underline{\mathbf{u}}}(t)$

The resulting performance index is essentially a weighted sum of the mean square deviation of a linear combination of the state variables from their desired values, and the mean square deviation of the commanded control rate from the desired pilot control rate. When the elements in \mathbf{Q} are chosen, the elements of \mathbf{R} have to

be changed in iterative fashion until the desired neuromuscular time constant is generated in the model.

2.3 Correlation of Pilot Model and POR

For the classical approach, McRuer, Ashkenas, and Guerre [8] correlate POR with parameters of the classical form of the pilot model. This is one of the earliest references of the relationship between the form of the pilot transfer function and POR for single-axis tracking. The concept of minimum pilot rating is introduced by Anderson [9]. The hypothesis is the pilot adopts a control strategy that maximizes his impression of the vehicle's handling qualities, or equivalent, minimizes his numerical opinion rating under a given particular vehicle and control task. This concept is then applied in a longitudinal helicopter hover task. By using Miller and Vinje's data, the general form of the pilot transfer function is obtained. Moreover, a workable expression for numerical POR is developed as the following equation:

$$\text{POR} = R_1 + R_2 + R_3 + 1.0$$

where R_1 was an explicit linear function of RMS performance, and R_2 and R_3 were linear functions of the lead time constants. This rating expression is then tested by using data from seven other simulations of a similar hover task with different vehicles. In this test, the pilot parameters are selected so that the POR is minimized. The resulting predicted ratings of this test are quite consistent with the actual ratings obtained from the seven pilot-participated simulations.

A lower frequency performance leads to a lower oscillation with smaller overshoot. However, a pilot is generally assumed to want to acquire the target

quickly and predictably with minimum overshoot and oscillation. The assumption is applied by Neal and Smith [10] to 51 basic short period configurations (pitch tracking tasks) to determine the optimal frequency of performance which can be suggested by the obtained PORs and resulting pilot's compensation. The result of the test suggests that a low frequency is favored for performance.

Another analytical technique for human pilot model is the modern approach. With the application of optimal control and estimation theory, the modern pilot model is first developed by Kleinman, Baron, and Levison [7]. The model consists of several components which are individually expressed by the associated mathematical forms. This pioneering work establishes such a model that modern control theorists refer to as a linear quadratic Gaussian control formulation which yields a numerical value of the performance index smaller than that for any other linear feedback structure.

Furthermore, a mathematical relationship between the performance index and POR is established by Dillow and Picha [11]. In their research, a longitudinal hover simulation is performed by the V/STOL aircraft and the following consistent relationship between actual pilot ratings and performance index J for the specific vehicle configurations of each task is obtained:

$$\text{POR} = \sqrt{J}$$

The performance index also can be related to POR in another mathematical form which is provided by Hess [12] under a rating hypothesis. This rating hypothesis states as that the numerical value of the performance index resulting from the modeling procedure can be related to the numerical pilot rating which the pilot assigns to the vehicle and task if (1) the performance index and model parameters in the optimal pilot modeling procedure yield a dynamically representative model

of the human pilot, (2) the variables selected for inclusion in the performance index are directly observable by the pilot, (3) the weighting coefficients in the performance index are chosen as the squares of the reciprocals of maximum "allowable" deviations of the respective variables, and these deviations are consonant with the task as perceived by the pilot. The rating hypothesis was tested by Hess using McDonnell's data from seven pitch attitude tracking tasks, Duffy's data from two longitudinal hover tasks, Arnold's data from five pitch attitude tracking tasks, and Miller and Vinje's data from five longitudinal hover tasks; and the correlation between POR and performance index is plotted.

The relationship between the performance index obtained by Arnold's and the POR obtained by Neal and Smith [10] is established by Schmidt [13] for the specific fourteen aircraft configurations. This relationship can be expressed as

$$\text{POR} = \log_{10}(J) + 4$$

for a single-axis tracking task performed by a conventional aircraft. And for a single-axis tracking task of high-order configuration dynamics, is expressed as

$$\text{POR} = \log_{10}(J)$$

where the slope of regression of the plot appears to be greater than the conventional one due to aeroelastic or other low damped mode. Moreover, by use of Dander's data, McRuer and Schmidt [14] correlate experimental PORs with single- and multi-axis performance index to give the following equation:

$$\text{POR} = 3.7 \log_{10} \left(\frac{J}{\sigma_c^2} \right) + 7.7$$

By the utilization of the Large Amplitude Multimode Aerospace Research Simulator (LAMARS) of the U.S. Air Force, Mitchell, Aponso, and Hoh [3] obtain the PORs of single and multi-axis tracking tasks performed by a low altitude, high-speed, fighter-type aircraft. Coincidentally, the same result as that of McRuer and Schmidt is obtained.

2.4 Comparison of Classical and Modern Approaches of the Pilot Model

The classical approach is applied to human pilot model in the early stage of development of controllability evaluation. Nevertheless, there are some pitfalls in the mentioned applications of classical pilot models to flying quality prediction: (1) the selection of the appropriate pilot model loop structure is often not straightforward, (2) the selection of particular parameter values in the model tends to be artful, and (3) incorporation of pilot model parameters, such as lead-time constants, in handling qualities metric can lead to difficulties because different pilot models are applied to different pilot tasks. In converse, the optimal approach tends to minimize some of the pitfalls associated with the handling qualities prediction schemes based upon classical techniques: (1) no a priori pilot loop structure has to be hypothesized, and (2) handling quality metric was an integral part of the optimal modeling procedure itself, i.e., the performance index.

CHAPTER III

OVERVIEW OF THE DEVELOPED METHODOLOGY

3.1 Details of the Utilized Aircraft

To implement the development of the methodology, the Gates Learjet 24B aircraft is utilized because this aircraft is representative of a medium sized conventional jet. The dimensions and geometry are illustrated in Figure 1. It can be seen that this specific aircraft can be characterized for (1) low wing location: the dihedral effect is decreased negatively by the fuselage interference so that a geometric dihedral angle is built to compensate the lift which produce the rolling moment;

(2) low wing sweep angle: the stability of the aircraft is enhanced because of the less negative rolling moment due to sideslip, C_{lr} ;

(3) high horizontal tail location: the downwash effect is reduced and the interference from the engine exhaust flow is decreased.

3.2 Definitions of Maneuvers

The specific maneuvers used are straight and level flight, steady level turn, symmetrical pull-up, and the combination of symmetrical pull-up and steady level turn. These maneuvers can be mathematically defined. For straight and level flight, rate of roll, rate of pitch, and rate of yaw are zero. For a steady level turn, rate of roll, angle of pitch are zero. For a symmetrical pull-up, rate of roll, rate of yaw, and angle of bank are zero. The steady state downward velocity, and side

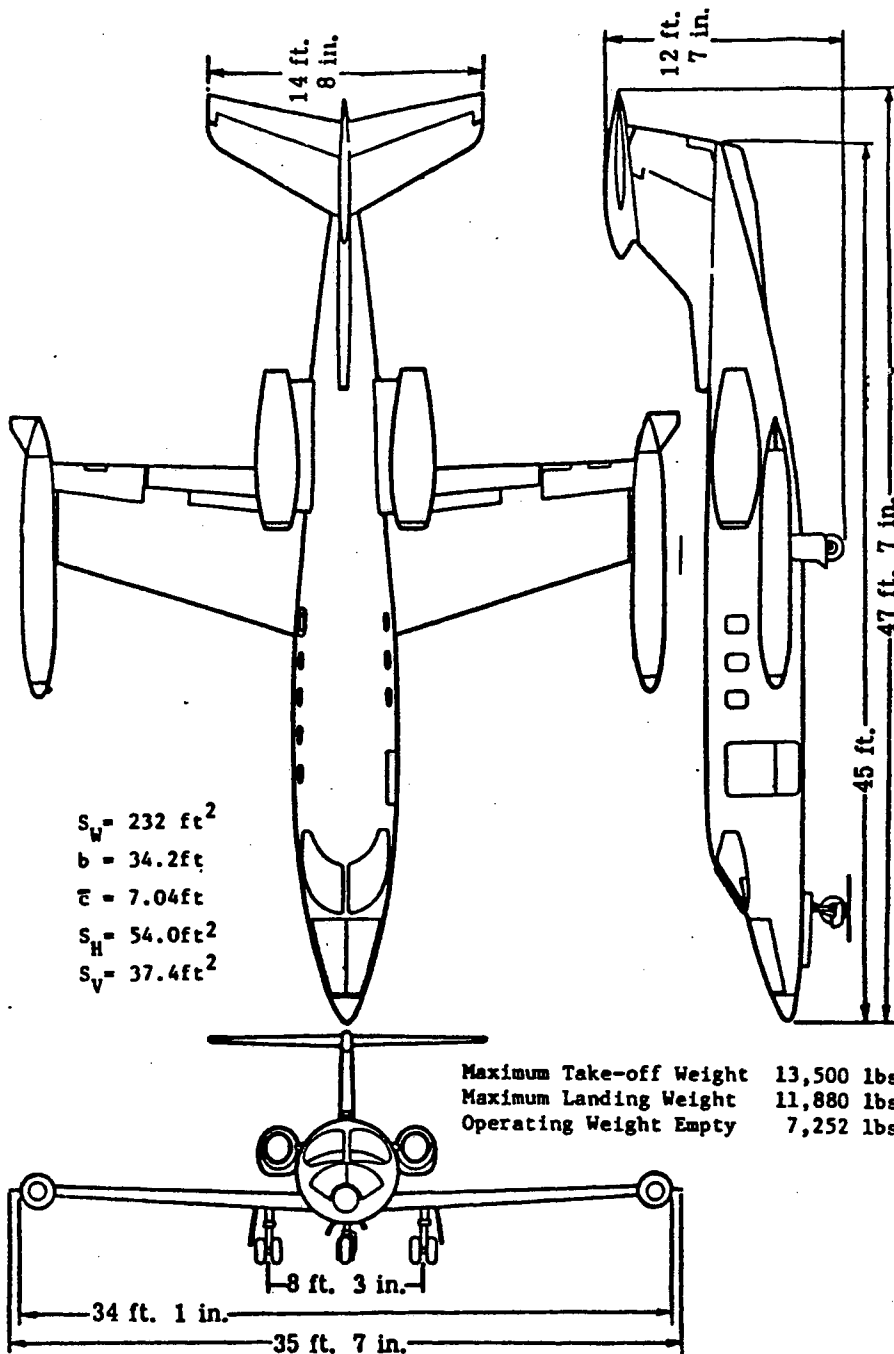


Figure 1. Gates Learjet Model 24B Dimensions and Geometry

velocity are zero for all the four maneuvers. These parameters of each defined maneuvers are summarized as Table 1. Moreover, it is important to know that P_0 , Q_0 , and R_0 are not zero in general.

TABLE I
MATHEMATICAL DEFINITION OF MANEUVERS

Parameters	Types of Maneuvers			
	1	2	3	4
P_0	0	0	0	√
Q_0	0	√	√	√
R_0	0	√	0	√
U_0	√	√	√	√
V_0	0	0	0	0
W_0	0	0	0	0
Φ_0	0	√	0	√
Θ_0	0	0	√	√

Maneuver 1 represents straight and level flight,

Maneuver 2 represents steady level turn,

Maneuver 3 represents straight and level flight,

Maneuver 4 represents combination of Maneuver

1 and 2; where √ sign indicates the parameter is not zero.

These three parameters are formulated and are given for each maneuver as follows:

(1) Straight and level flight:

$$P_0 = 0$$

$$Q_0 = 0$$

$$R_0 = 0$$

(2) Steady level turn:

$$P_0 = 0$$

$$Q_0 = \dot{\Psi}_0 \sin \Phi_0$$

$$R_0 = \dot{\Psi}_0 \cos \Phi_0$$

$$\text{,where } \dot{\Psi}_0 = \frac{g \tan \Phi_0}{U_0}$$

(3) Symmetrical pull-up:

$$P_0 = 0$$

$$Q_0 = \frac{g(n-1)}{U_0}$$

$$R_0 = 0$$

,where n is load factor and is assigned to be 2.

(4) Combination of steady level turn and symmetrical pull-up:

$$P_0 = -\dot{\Psi}_0 \sin \Theta_0$$

$$Q_0 = \dot{\Psi}_0 \cos \Theta_0 \sin \Phi_0$$

$$R_0 = \dot{\Psi}_0 \cos \Theta_0 \cos \Phi_0$$

$$\text{, where } \dot{\Psi}_0 = \frac{g \tan \Phi_0 \sqrt{\sec \Theta_0}}{U_0}$$

The derivation of the rate of turn, $\dot{\Psi}_0$, is presented in Appendix A.

3.3 Control Surface Damage

The specific types and degrees of damage are damage on wings, horizontal tail, or vertical tail, and each with increment of 6.25% of loss of the specific control surface. It is important to know that the wing area is based on the areas of the both wings; and the horizontal tail area is based on both horizontal tails. Moreover, the maximum loss of any control surface is limited to be 50% for practical reasons.

3.4 Strategic Procedures of the Methodology

The strategy for assessment of loss of control of this aircraft performing the specified maneuvers with various types and degrees of control surface damage can be portrayed using the following procedures:

- (1) Develop the multi-axis dynamic model of each maneuver for each type and degree of damage, which includes a plant matrix and an input matrix.
- (2) Transform the developed dynamic models into associated three single-axis transfer functions by using the single-input-single-output (SISO) method.
- (3) Input the resulted single-axis transfer functions and disturbance function to the Optimal Pilot Model (OPM), a C-language computer program developed by Kim [15], to obtain the single-axis performance index, J.

- (4) Transform the single-axis J into its corresponding single-axis pilot opinion rating (POR) by the utilization of the cost function /rating correlation model [14].
- (5) Use the Product Rule [3] to integrate the single-axis PORs of a specific type and degree of damage into the associated multi-axis POR.
- (6) Determine the state of control of the aircraft by using the Cooper-Harper rating scale [1] to interpret the resulting multi-axis POR. The aircraft is said to exhibit total loss of control if its POR is larger than 9; otherwise, the aircraft is controllable. The Cooper-Harper rating scale is given in Table 2.

TABLE II.

COOPER-HARPER HANDLING QUALITIES RATING SCALE

Aircraft Characteristics	Demands on the Pilot In Selected Task or Required Operation*	Pilot Rating
Excellent Highly Desirable	Pilot Compensation Not a Factor for Desired Performance	1
Good Negligible Deficiencies	Pilot Compensation Not a Factor for Desired Performance	2
Fair - Some Mildly Unpleasant Deficiencies	Minimal Pilot Compensation Required for Desired Performance	3
Minor But Annoying Deficiencies	Desired Performance Requires Moderate Pilot Compensation	4
Moderately Objectionable Deficiencies	Adequate Performance Requires Considerable Pilot Compensation	5
Very Objectionable But Tolerable Deficiencies	Adequate Performance Requires Extensive Pilot Compensation	6
Major Deficiencies	Adequate Performance Not Attainable With Maximum Tolerable Pilot Compensation. Controllability Not in Question	7
Major Deficiencies	Considerable Pilot Compensation Is Required for Control	8
Major Deficiencies	Intense Pilot Compensation Is Required to Retain Control	9
Major Deficiencies	Control Will Be Lost During Some Portion of Required Operation	10

CHAPTER IV

THEORETICAL DEVELOPMENT

To implement the described methodology, some theoretical-development tasks are required. The first is the modeling of control surface damage. Second of all is the establishment of dynamic models of the aircraft with various types and degrees of damage for different maneuvers which are needed in procedure (1) of the methodology mentioned in Section 3.4. The third part of this chapter is the transformation of the dynamic model into its three associated single-axis transfer functions; and this is procedure (2) of the methodology. The fourth part is the brief illustration of the logic of the Optimal Pilot Model software which is employed to obtain the single-axis performance index with the input of single-axis transfer functions. The fifth part presented in this chapter is to integrate the single-axis PORs into the associated multi-axis POR by using the product rule [3].

4.1 Modeling of Control Surface Damage

The mathematical models of control surface damages are necessary for the derivations of the expressions of stability derivatives with damage. A half-sided control surface, except vertical tail, can be generalized as Figure 2. By applying geometric relation,

the following equation are given:

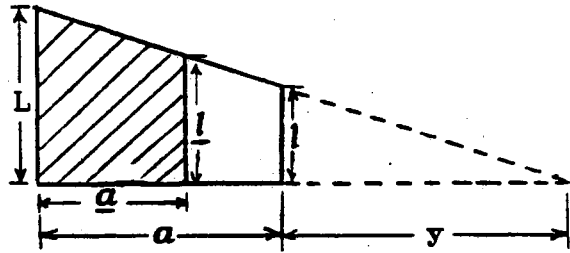


Figure 2. Half-sided Sketch of a Control Surface

$$\frac{y}{y+a} = \frac{l}{L}$$

then,

$$y = \frac{a \cdot l}{L - l} \quad (1)$$

also,

$$\frac{y+a-a}{y+a} = \frac{l}{L} \quad (2)$$

By substituting equation (1) into equation (2), the following equation is obtained:

$$l = L - \frac{a \cdot (L - l)}{a} \quad (3)$$

The remaining area after damage, \underline{S} , is given by:

$$\underline{S} = \frac{\underline{a} \cdot (L+l)}{2} + \frac{S}{2} \quad (4)$$

Substituting equation (2) and (3) into equation (4) to have

$$\underline{S} = \frac{-\underline{a}^2 \cdot (L-l)}{2 \cdot a} + L \cdot \underline{a} + \frac{S}{2} \quad (5)$$

Rearranging equation (5), it gives

$$\frac{L-l}{2 \cdot a} \cdot \underline{a}^2 + L \cdot \underline{a} + (0.5 \cdot S - \underline{S}) = 0 \quad (6)$$

By solving equation (6) to have

$$\underline{a} = \frac{L-l - \sqrt{L^2 + \frac{2 \cdot (L-l)(0.5 \cdot S - \underline{S})}{a}}}{\frac{L-l}{a}} \quad (7)$$

To obtain the span after damage, \underline{b} , the following geometric relation is used:

$$\underline{b} = 0.5 \cdot b + \underline{a}$$

where b is the span of the control surface.

Then

$$\underline{b} = \frac{L - \sqrt{L^2 + \frac{4 \cdot (L-l)(0.5 \cdot S - \underline{S})}{b}}}{\frac{2 \cdot (L-l)}{b}} + \frac{b}{2} \quad (8)$$

By use of the definition of aspect ratio,

$$A = \frac{b^2}{S}$$

it gives the aspect ratio after damage:

$$\underline{A} = \frac{b \left[1 + L - \sqrt{L^2 + \frac{4 \cdot (L-l)(0.5 \cdot S - \underline{S})}{L-l}} \right]^2}{2 \cdot \underline{S} \cdot (L-l)} \quad (9)$$

Applying the known geometric data of Gates Learjet 24B aircraft [16], the after-damage aspect ratio of each control surface is obtained:

$$\underline{A}_w = \frac{\left[17 + \frac{8.95 - \sqrt{80.1025 + 0.5141 \cdot (115 - \underline{S}_w)}}{0.2571} \right]^2}{\underline{S}_w} \quad (10)$$

$$\underline{A}_h = \frac{\left[7.30 + \frac{5.36 - \sqrt{28.7682 + 0.9173 \cdot (27 - \underline{S}_h)}}{0.4587} \right]^2}{\underline{S}_h} \quad (11)$$

$$\underline{A}_v = \frac{\left[\frac{8.95 - \sqrt{80.1025 + 1.2030 \cdot \underline{S}_v}}{0.6015} \right]^2}{\underline{S}_v} \quad (12)$$

The lift curve slope of a subsonic conventional aircraft is given by Roskam[16]:

$$C_{L\alpha} = \frac{2 \cdot \pi \cdot A}{2 + \sqrt{\frac{A^2}{K^2} \cdot (1 - M^2 - \tan^2 \Lambda) + 4}} \quad (13)$$

The after-damage lift curve slope is obtained by replacing A by \underline{A} in equation (14):

$$\underline{C}_{L\alpha} = \frac{2 \cdot \pi \cdot \underline{A}}{2 + \sqrt{\frac{\underline{A}^2}{K^2} \cdot (1 - M^2 - \tan^2 \Lambda) + 4}} \quad (14)$$

Therefore, for the Gates Learjet 24B aircraft,

$$\underline{C}_{L\alpha w} = \frac{2 \cdot \pi \cdot \underline{A}_w}{2 + \sqrt{\frac{\underline{A}_w^2}{K^2} \cdot (1 - 0.7^2) + 4}} \quad (15)$$

$$\underline{C_{L\alpha h}} = \frac{2 \cdot \pi \cdot \underline{A_h}}{2 + \sqrt{\frac{A_h^2}{K^2} \cdot (1 - 0.7^2 - 0.05) + 4}} \quad (16)$$

$$\underline{C_{L\alpha v}} = \frac{2 \cdot \pi \cdot \underline{A_v}}{2 + \sqrt{\frac{A_v^2}{K^2} \cdot (1 - 0.7^2 - 0.33) + 4}} \quad (17)$$

Then, by knowing $\underline{C_{L\alpha}}$ and \underline{s} , a series of equations in terms of the factors between the damaged and undamaged stability derivatives can be derived. Referred to Roskam [16], the undamaged longitudinal stability derivatives of horizontal tail are expressed as follows:

$$C_{L\delta\epsilon} = C_{L\alpha h} \cdot \frac{S_h \cdot \eta_h}{S_w} \cdot \left(1 - \frac{d\epsilon}{d\alpha}\right) \quad (18)$$

$$C_{m\delta\epsilon} = C_{L\alpha h} \cdot \frac{S_h \cdot \eta_h}{S_w} \cdot (\overline{X_{ac}} - \overline{X_{cg}}) \cdot \tau_E \quad (19)$$

$$C_{L\alpha} = C_{L\alpha WB} \cdot \frac{C_{L\alpha h} \cdot S_h \cdot \eta_h}{S_w} \cdot \left(1 - \frac{d\epsilon}{d\alpha}\right) \quad (20)$$

$$C_{m\alpha} = C_{L\alpha WB} \cdot (\overline{X_{cg}} - \overline{X_{ac}}) - \frac{C_{L\alpha h} \cdot \eta_h \cdot S_h \cdot (\overline{X_{ac}} - \overline{X_{cg}}) \cdot \left(1 - \frac{d\epsilon}{d\alpha}\right)}{S_w} \quad (21)$$

$$C_{L\alpha} = \frac{2 \cdot C_{Lah} \cdot \eta_h \cdot S_h \cdot (\overline{X_{ac}} - \overline{X_{cg}})}{S_w} \cdot \frac{d\varepsilon}{d\alpha} \quad (22)$$

$$C_{m\alpha} = \frac{-2 \cdot C_{Lah} \cdot \eta_h \cdot S_h \cdot (\overline{X_{ac}} - \overline{X_{cg}})^2}{S_w} \cdot \frac{d\varepsilon}{d\alpha} \quad (23)$$

$$C_{Lq} = \frac{2 \cdot C_{Lah} \cdot \eta_h \cdot S_h \cdot (\overline{X_{ac}} - \overline{X_{cg}})}{S_w} \quad (24)$$

$$C_{mq} = \frac{-2 \cdot C_{Lah} \cdot \eta_h \cdot S_h \cdot (\overline{X_{ac}} - \overline{X_{cg}})^2}{S_w} \quad (25)$$

$$C_{D\alpha} = \frac{2 \cdot C_{Lah} \cdot C_{Ll} \cdot \eta_h \cdot S_h}{\pi \cdot A \cdot e \cdot S_w} \cdot \left(1 - \frac{d\varepsilon}{d\alpha}\right) \quad (26)$$

Referring to equation (16), let

$$K_{1h} = \frac{C_{Lah}}{C_{Lah}}$$

$$K_{2h} = \frac{S_h}{S_h}$$

$$K_h = K_{1h} \cdot K_{2h}$$

the equation (18) becomes

$$\underline{C_{L\delta\epsilon}} = \underline{C_{L\alpha h}} \cdot \frac{S_h \cdot \eta_h}{S_w} \cdot \left(1 - \frac{d\epsilon}{d\alpha}\right) \quad (27)$$

Substituting K_{1h} , K_{2h} , K_h and equation (18) into equation (27), it gives

$$\underline{C_{L\delta\epsilon}} = K_h \cdot C_{L\delta\epsilon} \quad (28)$$

By using the same logic to have

$$\underline{C_{m\delta\epsilon}} = K_h \cdot C_{m\delta\epsilon} \quad (29)$$

$$\underline{C_{L\alpha}} = K_h \cdot C_{L\alpha} \quad (30)$$

$$\underline{C_{m\alpha}} = K_h \cdot C_{m\alpha} \quad (31)$$

$$\underline{C_{Lq}} = K_h \cdot C_{Lq} \quad (32)$$

$$\underline{C_{mq}} = K_h \cdot C_{mq} \quad (33)$$

$$\underline{C_{D\alpha}} = K_h \cdot C_{D\alpha} \quad (34)$$

Since

$$1 - \frac{d\epsilon}{d\alpha} = \tau_E = 0.5$$

Equation (18) can be substituted into equation (20) to give

$$C_{L\alpha} = C_{L\alpha WB} + C_{L\delta\epsilon} \quad (35)$$

Therefore, by use of the same logic for transforming equation (13)

into equation (14), for horizontal tail with damage,

$$\underline{C_{L\alpha}} = C_{L\alpha WB} + \underline{C_{L\delta e}} \quad (36)$$

Equation (36) subtracts equation (35) to give

$$\underline{C_{L\alpha}} = C_{L\alpha} - (C_{L\delta e} - \underline{C_{L\delta e}}) \quad (37)$$

Substituting equation (19) into equation (21) to have

$$C_{m\alpha} = C_{L\alpha WB} \cdot (\overline{X_{cg}} - \overline{X_{ac}}) + C_{m\delta e} \quad (38)$$

Again, for damaged horizontal tail,

$$\underline{C_{m\alpha}} = C_{L\alpha WB} \cdot (\overline{X_{cg}} - \overline{X_{ac}}) + \underline{C_{m\delta e}} \quad (39)$$

Equation (38) is subtracted by equation (39) and gives

$$\underline{C_{m\alpha}} = C_{m\alpha} - (C_{m\delta e} - \underline{C_{m\delta e}}) \quad (40)$$

If damage occurs to the vertical tail, the following stability derivatives are significantly effected by the lift curve slope, $\underline{C_{L\alpha v}}$, and surface area of vertical tail, $\underline{S_v}$:

$$C_{y\beta}, C_{l\beta}, C_{lp}, C_{n\beta}, C_{yr}, C_{nr}, C_{n\delta r}, C_{y\delta r}.$$

Referring to equation (16), let

$$K_{lv} = \frac{C_{L\alpha v}}{C_{L\alpha v}}$$

$$K_{2v} = \frac{S_v}{S_v}$$

$$K_v = K_{1v} \cdot K_{2v}$$

Then, by using K_{1v} , K_{2v} , K_v and undamaged lateral-directional stability derivatives, and following the same logic used for horizontal tail with damage, the stability derivatives with significance of damaged vertical tail [16] are

$$\underline{C_{y\beta}} = -\frac{\eta_v}{S_w} \cdot \left(1 + \frac{d\sigma}{d\beta}\right) \cdot \underline{C_{L\alpha}} \cdot \underline{S_v} \quad (41)$$

$$\underline{C_{l\beta}} = -\frac{\eta_v \cdot Z_v}{S_w \cdot b_w} \cdot \left(1 - \frac{d\sigma}{d\beta}\right) \cdot \underline{C_{L\alpha}} \cdot \underline{S_v} \quad (42)$$

$$\underline{C_{lp}} = -2 \cdot \left(\frac{Z_v}{b_w}\right)^2 \cdot \frac{\eta_v}{S_w} \cdot \left(1 + \frac{d\sigma}{d\beta}\right) \cdot \underline{C_{L\alpha}} \cdot \underline{S_v} \quad (43)$$

$$\underline{C_{m\beta}} = \frac{\eta_v}{S_w \cdot b_w} \cdot \left(1 - \frac{d\sigma}{d\beta}\right) \cdot (l_v \cos \alpha + Z_v \sin \alpha) \cdot \underline{C_{L\alpha}} \cdot \underline{S_v} \quad (44)$$

$$\underline{C_{yr}} = \frac{2 \cdot \eta_v}{S_w \cdot b_w} \cdot \left(1 + \frac{d\sigma}{d\beta}\right) \cdot (l_v \cos \alpha + Z_v \sin \alpha) \cdot \underline{C_{L\alpha}} \cdot \underline{S_v} \quad (45)$$

$$\underline{C_{yr}} = \frac{-2 \cdot K \cdot \eta_v}{S_w \cdot b_w^2} \cdot \left(1 + \frac{d\sigma}{d\beta}\right) \cdot (l_v \cos \alpha + Z_v \sin \alpha)^2 \cdot \underline{C_{L\alpha}} \cdot \underline{S_v} \quad (46)$$

$$\underline{C_{n\delta r}} = \frac{-\alpha_{\delta v} \cdot \eta_v \cdot X_v \cdot \underline{C_{L\alpha v}} \cdot \underline{S_v}}{S_w \cdot b_w} \quad (47)$$

$$\underline{C_{n\delta r}} = \frac{\eta_v \cdot X_v \cdot \underline{C_{L\alpha v}} \cdot \underline{S_v}}{S_w} \quad (48)$$

Therefore,

$$\underline{C_{y\beta}} = K_v \cdot C_{y\beta} \quad (49)$$

and

$$\underline{C_{l\beta}} = K_v \cdot C_{l\beta} \quad (50)$$

$$\underline{C_{lp}} = K_v \cdot C_{lp} \quad (51)$$

$$\underline{C_{n\beta}} = K_v \cdot C_{n\beta} \quad (52)$$

$$\underline{C_{yr}} = K_v \cdot C_{yr} \quad (53)$$

$$\underline{C_{nr}} = K_v \cdot C_{nr} \quad (54)$$

$$\underline{C_{n\delta r}} = K_v \cdot C_{n\delta r} \quad (55)$$

$$\underline{C_{y\delta r}} = K_v \cdot C_{y\delta r} \quad (56)$$

Considering the lift force decreases due to wing damage, the angle of attack, α , should be increased to compensate the loss of lift

force. Referring to the expression of lift coefficient [16]:

$$\underline{C_L} = \frac{W}{q \cdot \underline{S}} \quad (57)$$

Also,

$$\underline{C_L} = C_{LOWB} + \underline{C_{LaWB}} \cdot \alpha + \frac{C_{Lah} \cdot \eta_h \cdot S_h}{S_w} \cdot [\alpha - (\varepsilon_0 + \frac{d\varepsilon}{d\alpha} \cdot \alpha) + i_h + \tau_E \cdot \delta_e] \quad (58)$$

Substituting equation (15) and (57) into (58) to obtain

$$\alpha = \frac{\frac{W}{q \cdot \underline{S}} - C_{LOWB} + \frac{C_{Lah} \cdot \eta_h \cdot S_h}{S_w} \cdot [(\varepsilon_0 + \frac{d\varepsilon}{d\alpha} \cdot \alpha) + i_h + \tau_E \cdot \delta_e]}{\underline{C_{LaWB}} + \frac{C_{Lah} \cdot \eta_h \cdot S_h}{S_w}} \quad (59)$$

Referring to equation (15), let

$$K_{1w} = \frac{S_w}{S_w}$$

$$K_{w2} = \frac{b}{b}$$

$$K_w = K_{1w} \cdot K_{2w}$$

Then, by using K_{1w} , K_{2w} , K_w and stability derivatives derived by Finck and Hoak [17], and following the same logic used for

horizontal tail with damage, the stability derivatives with significance of damaged wing are

$$\underline{C_{L\delta\epsilon}} = \frac{C_{L\delta\epsilon}}{K_{1w}} \quad (60)$$

$$\underline{C_{yr}} = \frac{C_{yr}}{K_{1w}} \quad (61)$$

$$\underline{C_{lp}} = \frac{C_{lp}}{K_w \cdot K_{2w}} \quad (62)$$

$$\underline{C_{l\delta\alpha}} = \frac{C_{l\delta\alpha} \cdot S_f}{S_w} \quad (63)$$

$$\underline{C_{n\delta\alpha}} = 0.1124 \cdot \underline{C_{l\delta\alpha}} \quad (64)$$

$$\underline{C_{m\delta\epsilon}} = \frac{C_{m\delta\epsilon}}{K_{1w}} \quad (65)$$

$$\underline{C_{L\dot{\alpha}}} = \frac{C_{L\dot{\alpha}}}{K_{1w}} \quad (66)$$

$$\frac{C_{m\dot{\alpha}}}{K_{1w}} = \frac{C_{m\dot{\alpha}}}{K_{1w}} \quad (67)$$

$$\frac{C_{Lq}}{K_{1w}} = \frac{C_{Lq}}{K_{1w}} \quad (68)$$

$$\frac{C_{mq}}{K_{1w}} = \frac{C_{mq}}{K_{1w}} \quad (69)$$

$$\frac{C_{D\alpha}}{K_{1w}} = \frac{C_{D\alpha}}{K_{1w}} \quad (70)$$

$$\frac{C_{y\beta}}{K_{1w}} = \frac{C_{y\beta}}{K_{1w}} \quad (71)$$

$$\frac{C_{y\delta r}}{K_{1w}} = \frac{C_{y\delta r}}{K_{1w}} \quad (72)$$

$$\frac{C_{l\delta r}}{K_w} = \frac{C_{l\delta r}}{K_w} \quad (73)$$

$$\frac{C_{n\beta}}{K_w} = \frac{C_{n\beta}}{K_w} \quad (74)$$

$$\frac{C_{n\delta r}}{K_w} = \frac{C_{n\delta r}}{K_w} \quad (75)$$

$$\frac{C_{lr}}{K_w \cdot K_{w2}} = \frac{C_{lr}}{K_w \cdot K_{w2}} \quad (76)$$

$$\frac{C_{np}}{K_w \cdot K_{w2}} = \frac{C_{np}}{K_w \cdot K_{w2}} \quad (77)$$

$$\frac{C_{m\alpha}}{S_w} = \frac{C_{LaWB}}{S_w} \cdot (\overline{X_{cg}} - \overline{X_{acWB}}) - \frac{C_{Lah} \cdot \eta_h \cdot S_h \cdot (1 - \frac{d\varepsilon}{d\alpha}) (\overline{X_{ach}} - \overline{X_{cg}})}{S_w} \quad (78)$$

$$\text{where } \overline{X_{cg}} = \frac{\overline{X_{acWB}} + \frac{C_{Lah} \cdot \eta_h \cdot S_h \cdot \overline{X_{ach}} \cdot (1 - \frac{d\varepsilon}{d\alpha})}{\frac{C_{LaWB}}{S_w}}}{1 + \frac{C_{Lah} \cdot \eta_h \cdot S_h \cdot (1 - \frac{d\varepsilon}{d\alpha})}{\frac{C_{LaWB}}{S_w}}} - 0.1$$

As a summary, equation (28-34), equation (37), and equation (40) are the stability derivatives of horizontal tail with damage; equation (49-56) are those of vertical tail with damage; and equation (60-78) are those of wing with damage.

4.2 Dynamic Models of the Specific Aircraft with Damage

The six rigid-body equations of motion and the three Euler angles kinematic equations are given by Roskam [16]:

$$F_x = m \cdot (\dot{U} + Q \cdot W - R \cdot V) + m \cdot g \cdot \sin \Theta \quad (79)$$

$$F_y = m \cdot (\dot{V} + R \cdot U - P \cdot W) + m \cdot g \cdot \cos \Theta \cdot \sin \Phi \quad (80)$$

$$F_z = m \cdot (\dot{W} + P \cdot V - Q \cdot U) + m \cdot g \cdot \cos \Theta \cdot \cos \Phi \quad (81)$$

$$M_x = \dot{P} \cdot I_x + Q \cdot R \cdot (I_z - I_y) - (\dot{R} + P \cdot Q) \cdot I_{xz} - (\dot{Q} - P \cdot R) \cdot I_{xy} - (Q^2 - R^2) \cdot I_{yz} \quad (82)$$

$$M_y = \dot{Q} \cdot I_y + P \cdot R \cdot (I_x - I_z) - (\dot{P} + R \cdot Q) \cdot I_{xy} - (\dot{R} - P \cdot Q) \cdot I_{yz} - (R^2 - P^2) \cdot I_{xz} \quad (83)$$

$$M_z = \dot{R} \cdot I_z + P \cdot Q \cdot (I_y - I_x) - (\dot{Q} + P \cdot R) \cdot I_{yz} - (\dot{P} - Q \cdot R) \cdot I_{xz} - (P^2 - Q^2) \cdot I_{xy} \quad (84)$$

$$\dot{\Phi} = P + Q \cdot \tan \Theta \cdot \sin \Phi + R \cdot \tan \Theta \cdot \cos \Phi \quad (85)$$

$$\dot{\Theta} = Q \cdot \cos \Phi - R \cdot \sin \Phi \quad (86)$$

$$\dot{\Psi} = \frac{1}{\cos \Theta} \cdot (R \cdot \cos \Phi + Q \cdot \sin \Phi) \quad (87)$$

U, V, W are the linear velocities and P, Q, R are the angular velocities with respect to the body axes, and are illustrated in Figure 3. Moreover, steady state flight is defined as a flight condition for which all motion variables remain

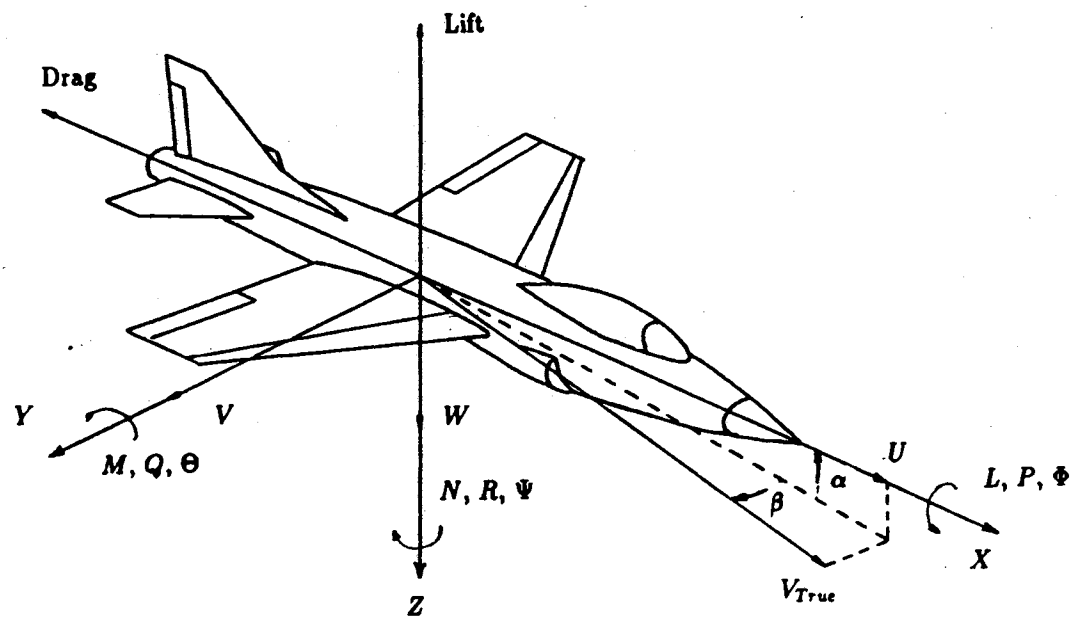


Figure 3. Aircraft Coordinate System

constant with time relative to a body fixed axes system. The steady state and small perturbation variables can be defined by the following equations:

$$U = U_0 + u \quad (88)$$

$$V = V_0 + v \quad (89)$$

$$W = W_0 + w \quad (90)$$

$$P = P_0 + p \quad (91)$$

$$Q = Q_0 + q \quad (92)$$

$$R = R_0 + r \quad (93)$$

$$\Theta = \Theta_0 + \theta \quad (94)$$

$$\Phi = \Phi_0 + \phi \quad (95)$$

$$\Psi = \Psi_0 + \psi \quad (96)$$

$$F_x = F_{x0} + f_x \quad (97)$$

$$F_y = F_{y0} + f_y \quad (98)$$

$$F_z = F_{z0} + f_z \quad (99)$$

$$M_x = M_{x0} + m_x \quad (100)$$

$$M_y = M_{y0} + m_y \quad (101)$$

$$M_z = M_{z0} + m_z \quad (102)$$

Substituting equation (88-102) into equation (79-87) and neglecting the products of the perturbation terms as small and making the sine, cosine, and tangent small

angle approximations, gives the steady-state equation (103-111) and the perturbation equation (112-120):

$$F_{x_0} = m \cdot (W_0 \cdot Q_0 - V_0 \cdot R_0 + g \cdot \sin \Theta_0) \quad (103)$$

$$F_{y_0} = m \cdot (U_0 \cdot R_0 - W_0 \cdot P_0 - g \cdot \cos \Theta_0 \cdot \sin \Phi_0) \quad (104)$$

$$F_{z_0} = m \cdot (V_0 \cdot P_0 - U_0 \cdot Q_0 - g \cdot \cos \Theta_0 \cdot \cos \Phi_0) \quad (105)$$

$$M_{x_0} = Q_0 \cdot R_0 \cdot (I_z - I_y) - P_0 \cdot Q_0 \cdot I_{xz} + P_0 \cdot R_0 \cdot I_{xy} - (Q_0^2 - R_0^2) \cdot I_{yz} \quad (106)$$

$$M_{y_0} = P_0 \cdot R_0 \cdot (I_x - I_z) - R_0 \cdot Q_0 \cdot I_{xy} + P_0 \cdot Q_0 \cdot I_{yz} - (R_0^2 - P_0^2) \cdot I_{xz} \quad (107)$$

$$M_{z_0} = P_0 \cdot Q_0 \cdot (I_y - I_x) - P_0 \cdot R_0 \cdot I_{yz} + Q_0 \cdot R_0 \cdot I_{xz} - (P_0^2 - Q_0^2) \cdot I_{xy} \quad (108)$$

$$\dot{\Phi}_0 = P_0 + Q_0 \cdot \tan \Theta_0 \cdot \sin \Phi_0 + R_0 \cdot \tan \Theta_0 \cdot \cos \Phi_0 \quad (109)$$

$$\dot{\Theta}_0 = Q_0 \cdot \cos \Phi_0 - R_0 \cdot \sin \Phi_0 \quad (110)$$

$$\dot{\Psi}_0 = \frac{1}{\cos \Theta_0} \cdot (R_0 \cdot \cos \Phi_0 + Q_0 \cdot \sin \Phi_0) \quad (111)$$

$$f_x = m \cdot (\dot{u} + W_0 \cdot q + Q_0 \cdot w - V_0 \cdot r - R_0 + g \cdot \theta \cdot \cos \Theta_0) \quad (112)$$

$$f_y = m \cdot (\dot{v} + U_0 \cdot r + R_0 \cdot u - W_0 \cdot p - P_0 \cdot w - g \cdot \phi \cdot \cos \Theta_0 \cdot \cos \Phi_0 + g \cdot \theta \cdot \sin \Theta_0 \cdot \sin \Phi_0) \quad (113)$$

$$f_z = m \cdot (\dot{w} + V_0 \cdot p + P_0 \cdot v - U_0 \cdot q - Q_0 \cdot u + g \cdot \phi \cdot \cos \Theta_0 \cdot \sin \Phi_0 + g \cdot \theta \cdot \sin \Theta_0 \cdot \cos \Phi_0) \quad (114)$$

$$m_x = \dot{p} \cdot I_x + (Q_0 \cdot r + R_0 \cdot q) \cdot (I_z - I_y) - (\dot{r} + Q_0 \cdot p + P_0 \cdot q) \cdot I_{xz} - (\dot{q} - P_0 \cdot r - R_0 \cdot p) \cdot I_{xy} - 2 \cdot (Q_0 \cdot q - R_0 \cdot r) \cdot I_{yz} \quad (115)$$

$$m_y = \dot{q} \cdot I_y + (P_0 \cdot r + R_0 \cdot p) \cdot (I_x - I_z) - (\dot{p} + Q_0 \cdot r + R_0 \cdot q) \cdot I_{xy} - (\dot{r} - P_0 \cdot q - Q_0 \cdot p) \cdot I_{yz} - 2 \cdot (R_0 \cdot r - P_0 \cdot p) \cdot I_{xz} \quad (116)$$

$$m_z = \dot{r} \cdot I_z + (P_0 \cdot q + Q_0 \cdot p) \cdot (I_y - I_x) - (\dot{q} + P_0 \cdot r + R_0 \cdot q) \cdot I_{yz} - (\dot{p} - Q_0 \cdot r - R_0 \cdot q) \cdot I_{xz} - 2 \cdot (P_0 \cdot p - Q_0 \cdot q) \cdot I_{xy} \quad (117)$$

$$\dot{\phi} = p + \phi \cdot (Q_0 \cdot \cos \Phi_0 - R_0 \cdot \sin \Phi_0) \cdot \tan \Theta_0 + q \cdot \sin \Phi_0 \cdot \tan \Theta_0 + r \cdot \cos \Phi_0 \cdot \tan \Theta_0 \quad (118)$$

$$\dot{\theta} = q \cdot \cos \Phi_0 - r \cdot \sin \Phi_0 - \phi \cdot (Q_0 \cdot \sin \Phi_0 + R_0 \cdot \cos \Phi_0) \quad (119)$$

$$\dot{\psi} = q \cdot \sin \Phi_0 \cdot \sec \Theta_0 + r \cdot \cos \Phi_0 \cdot \sec \Theta_0 + \phi \cdot (Q_0 \cdot \cos \Phi_0 - R_0 \cdot \sin \Phi_0) \cdot \sec \Theta_0 + \theta \cdot (Q_0 \cdot \sin \Phi_0 + R_0 \cdot \cos \Phi_0) \cdot \tan \Theta_0 \cdot \sec \Theta_0 \quad (120)$$

Equation (103-120) are linear with respect to the perturbation variables and allow an arbitrary steady-state maneuver by specifying U_0 , V_0 , W_0 , P_0 , Q_0 , and R_0 . Equation (112-119) can be further simplified by the following steps:

- (1) Referring to Figure 4, w_0 can be reduced to zero by replacing body axes with stability axes orientation to express the aerodynamic forces which consists of, drag, and side force.

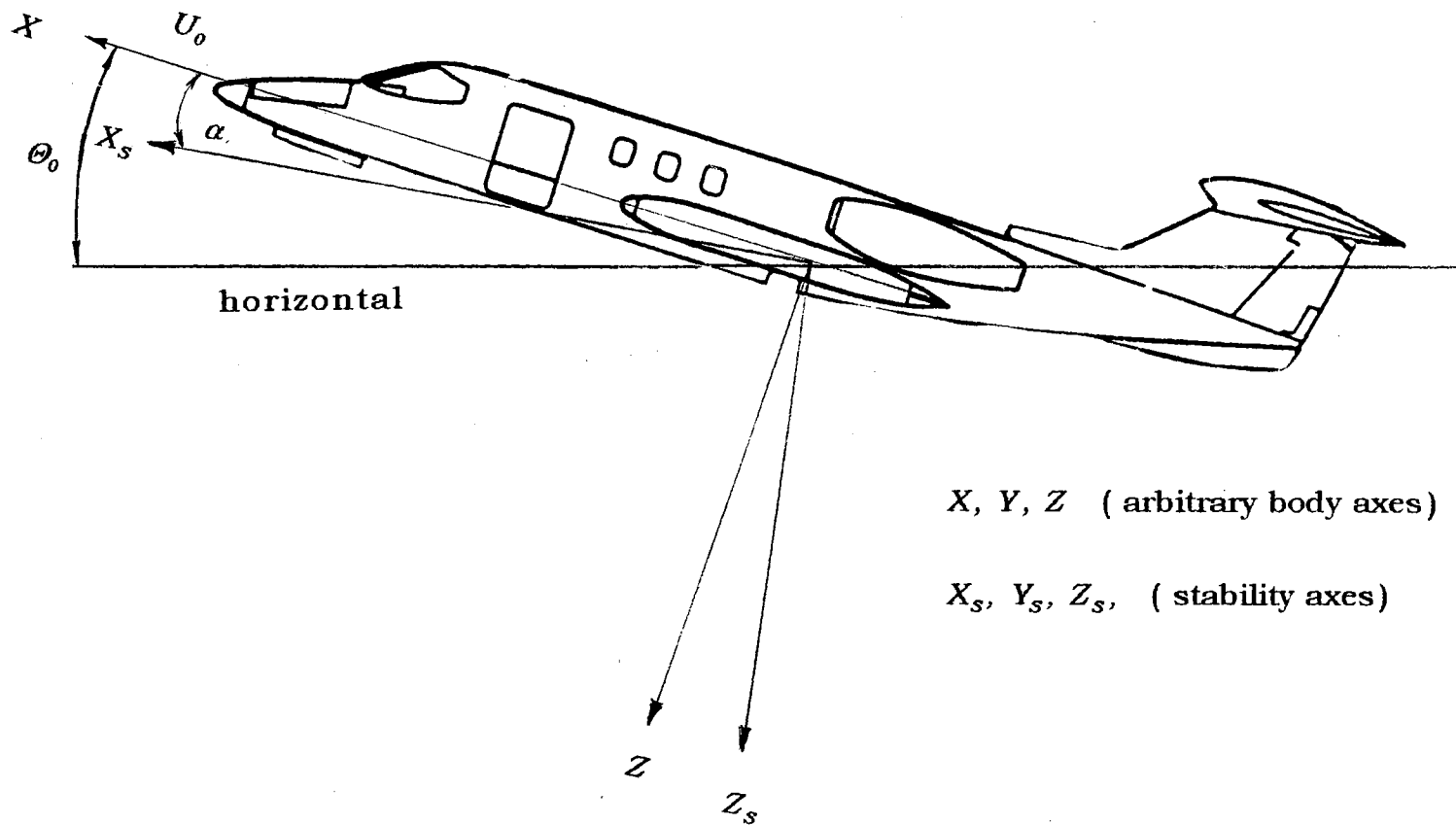


Figure 4. Definition of Stability Axes

(2) The angle of attack, α , and sideslip angle, β , are so small that they can be expressed in the following forms:

$$\alpha = \frac{w}{U_0}$$

$$\beta = \frac{v}{U_0}$$

then all the w/U_0 or v/U_0 related terms can be transformed into α or β related terms.

(3) After the implementation of step (1) and (2), equation (112-114), equation (115), equation (116), and equation (117) are divided by I_x , I_y , and I_z , respectively. However, the cross-products of mass moments of inertia in these equations remain, as they will not be zero for a damaged aircraft where the xz -plane will not be a plane of symmetry.

Consequently, the equations of motion are obtained as equation (121-126):

$$\frac{\dot{u}}{U_0} = -Q_0 \cdot \alpha + R_0 \cdot \beta - \frac{g \cdot \theta}{U_0} \cos \Theta_0 + \frac{f_x}{m \cdot U_0} \quad (121)$$

$$\begin{aligned} \dot{\alpha} = & -P_0 \cdot \beta + Q_0 \cdot \frac{u}{U_0} + q - \frac{g}{U_0} \cdot (\phi \cdot \cos \Theta_0 \sin \Phi_0 + \theta \cdot \sin \Theta_0 \cdot \cos \Phi_0) \\ & + \frac{f_z}{m \cdot U_0} \end{aligned} \quad (122)$$

$$\begin{aligned} \dot{\beta} = & P_0 \cdot \alpha + R_0 \cdot \frac{u}{U_0} + r - \frac{g}{U_0} \cdot (\phi \cdot \cos \Theta_0 \cos \Phi_0 + \theta \cdot \sin \Theta_0 \cdot \sin \Phi_0) \\ & + \frac{f_y}{m \cdot U_0} \end{aligned} \quad (123)$$

$$\begin{aligned} \dot{p} = & \frac{p}{I_x} \cdot (Q_0 \cdot I_{xz} - R_0 \cdot I_{xy}) + \frac{q}{I_x} \cdot [2 \cdot Q_0 \cdot I_{yz} + P_0 \cdot I_{xz} - R_0 \cdot (I_z - I_y)] \\ & - \frac{r}{I_x} \cdot [2 \cdot R_0 \cdot I_{yz} + P_0 \cdot I_{xy} + Q_0 \cdot (I_z - I_y)] + \frac{m_x}{I_x} \end{aligned} \quad (124)$$

$$\begin{aligned} \dot{q} = & \frac{q}{I_y} \cdot (R_0 \cdot I_{xy} - P_0 \cdot I_{yz}) + \frac{r}{I_y} \cdot [2 \cdot R_0 \cdot I_{xz} + Q_0 \cdot I_{xy} - P_0 \cdot (I_x - I_z)] \\ & - \frac{p}{I_{xy}} \cdot [2 \cdot P_0 \cdot I_{xz} + Q_0 \cdot I_{yz} + R_0 \cdot (I_x - I_z)] + \frac{m_y}{I_y} \end{aligned} \quad (125)$$

$$\begin{aligned} \dot{r} = & \frac{-r}{I_z} \cdot (Q_0 \cdot I_{xz} - P_0 \cdot I_{yz}) + \frac{p}{I_z} \cdot [2 \cdot P_0 \cdot I_{xy} - Q_0 \cdot (I_y - I_x)] \\ & - \frac{q}{I_z} \cdot [2 \cdot Q_0 \cdot I_{xy} + R_0 \cdot I_{xz} + P_0 \cdot (I_y - I_x)] + \frac{m_z}{I_z} \end{aligned} \quad (126)$$

The perturbation aerodynamic terms in equation (121-126) can be expressed as functions of stability derivatives. The stability derivatives are given by Swaim [18] as

$$f_x = \frac{\rho \cdot U_0^2 \cdot S}{2} \cdot \left[\frac{-u \cdot (C_{Du} + 2 \cdot C_{D0})}{U_0} + (-C_{D\alpha} + C_{L0}) \cdot \alpha - C_{D\delta e} \cdot \delta_e \right] \quad (127)$$

$$f_y = \frac{\rho \cdot U_0^2 \cdot S}{2} \cdot \left[\frac{b \cdot (C_{yp} \cdot p + C_{yr} \cdot r)}{2 \cdot U_0} + C_{y\beta} \cdot \beta + C_{y\delta a} \cdot \delta_a + C_{y\delta r} \cdot \delta_r \right] \quad (128)$$

$$f_z = \frac{\rho \cdot U_0^2 \cdot S}{2} \cdot \left[\frac{-c \cdot (C_{L\alpha} \cdot \dot{\alpha} + C_{Lq} \cdot q)}{2 \cdot U_0} - \frac{-u \cdot (C_{Lu} + 2 \cdot C_{L0})}{U_0} \right. \\ \left. + (-C_{L\alpha} + C_{D0}) \cdot \alpha - C_{L\delta e} \cdot \delta_e \right] \quad (129)$$

$$m_x = \frac{\rho \cdot U_0^2 \cdot S \cdot b}{2} \cdot \left[\frac{b \cdot (C_{lp} \cdot p + C_{lr} \cdot r)}{2 \cdot U_0} + C_{l\beta} \cdot \beta + C_{l\delta a} \cdot \delta_a + C_{l\delta r} \cdot \delta_r \right] \quad (130)$$

$$m_y = \frac{\rho \cdot U_0^2 \cdot S \cdot c}{2} \cdot \left[\frac{-c \cdot (C_{m\alpha} \cdot \dot{\alpha} + C_{mq} \cdot q)}{2 \cdot U_0} - \frac{u \cdot (C_{mu} + 2 \cdot C_{m0})}{U_0} \right. \\ \left. + C_{m\alpha} \cdot \alpha - C_{m\delta e} \cdot \delta_e \right] \quad (131)$$

$$m_z = \frac{\rho \cdot U_0^2 \cdot S \cdot b}{2} \cdot \left[\frac{b \cdot (C_{np} \cdot p + C_{nr} \cdot r)}{2 \cdot U_0} + C_{n\beta} \cdot \beta + C_{n\delta a} \cdot \delta_a + C_{n\delta r} \cdot \delta_r \right] \quad (132)$$

Equation (121-126) are substituted by equation (127-132) to give the aerodynamic forces and moments in terms of the dimensional stability derivatives as defined by Roskam[19] (given in Appendix C):

$$\frac{\dot{u}}{U_0} = \frac{-q_0 \cdot S \cdot (C_{Du} + 2 \cdot C_{D0})}{m \cdot U_0^2} \cdot u + \left[\frac{-q_0 \cdot S \cdot (C_{D\alpha} - C_{L0})}{m \cdot U_0} - Q_0 \right] \cdot \alpha$$

$$-\frac{g \cdot \cos \Theta_0}{U_0} \cdot \theta + R_0 \cdot \beta - \frac{q_0 \cdot S \cdot C_{D\delta e}}{m \cdot U_0} \cdot \delta_e \quad (133)$$

$$\begin{aligned} \dot{\alpha} = & \left[\frac{-q_0 \cdot S \cdot (C_{L\alpha} + 2 \cdot C_{L0})}{m \cdot U_0^2} + Q_0 \right] \cdot u + \left[\frac{-q_0 \cdot S \cdot (C_{L\alpha} - C_{D0})}{m \cdot U_0} - Q_0 \right] \cdot \alpha + q \\ & - \frac{g \cdot \sin \Theta_0 \cdot \cos \Phi_0}{U_0} \cdot \theta - P_0 \cdot \beta - \frac{g \cdot \cos \Theta_0 \cdot \sin \Phi_0}{U_0} \cdot \phi - \frac{q_0 \cdot S \cdot C_{L\delta e}}{m \cdot U_0} \cdot \delta_e \quad (134) \end{aligned}$$

$$\begin{aligned} \dot{q} = & \left[\frac{q_0 \cdot S \cdot \bar{c} \cdot (C_{m\alpha} + 2 \cdot C_{m0})}{I_y \cdot U_0} + \frac{q_0 \cdot S \cdot \bar{c}^2 \cdot C_{m\alpha} \cdot Q_0}{2 \cdot I_y \cdot U_0^2} - \frac{q_0^2 \cdot S^2 \cdot \bar{c}^2 \cdot C_{m\alpha} \cdot (C_{L\alpha} + 2 \cdot C_{L0})}{2 \cdot m \cdot I_y \cdot U_0^3} \right] \cdot u \\ & + \left[\frac{q_0 \cdot S \cdot \bar{c} \cdot C_{m\alpha}}{I_y} - \frac{q_0^2 \cdot S^2 \cdot \bar{c}^2 \cdot C_{m\alpha} \cdot (C_{L\alpha} + 2 \cdot C_{D0})}{2 \cdot m \cdot I_y \cdot U_0^2} \right] \cdot \alpha \\ & + \left[\frac{q_0 \cdot S \cdot \bar{c}^2 \cdot (C_{m\alpha} + C_{mq})}{2 \cdot I_y \cdot U_0} + \frac{R_0 \cdot I_{xy} - P_0 \cdot I_{yz}}{I_y} \right] \cdot q \\ & - \left[\frac{2 \cdot P_0 \cdot I_{xz} + Q_0 \cdot I_{yz} + R_0 \cdot (I_x - I_z)}{I_y} \right] \cdot p \\ & - \left[\frac{2 \cdot R_0 \cdot I_{xz} + Q_0 \cdot I_{xy} - P_0 \cdot (I_x - I_z)}{I_y} \right] \cdot r \\ & + \left[\frac{q_0 \cdot S \cdot \bar{c} \cdot C_{m\delta e}}{I_y} - \frac{q_0^2 \cdot S^2 \cdot \bar{c}^2 \cdot C_{m\alpha} \cdot C_{L\delta e}}{2 \cdot m \cdot I_y \cdot U_0^2} \right] \cdot \delta_e \quad (135) \end{aligned}$$

$$\dot{\theta} = q \cdot \cos \Phi_0 - 2 \cdot (Q_0 \cdot \sin \Phi_0 + R_0 \cdot \cos \Phi_0) \cdot \phi - r \cdot \sin \Phi_0 \quad (136)$$

$$\begin{aligned} \dot{\beta} = & \frac{-R_0 \cdot u}{U_0} + P_0 \cdot \alpha - \frac{g \cdot \theta}{U_0} \cdot \sin \Theta_0 \cdot \cos \Phi_0 + \frac{g \cdot \phi}{U_0} \cdot \cos \Theta_0 \cdot \cos \Phi_0 \\ & \frac{q_0 S \cdot C_{y\beta}}{m \cdot U_0} \cdot \beta + \frac{q_0 S \cdot b \cdot C_{y\beta}}{2 \cdot m \cdot U_0^2} \cdot p + \left(\frac{q_0 S \cdot b \cdot C_{yr}}{2 \cdot m \cdot U_0^2} - 1 \right) \cdot r \\ & + \frac{q_0 S \cdot C_{y\delta a}}{m \cdot U_0} \cdot \delta_a + \frac{q_0 S \cdot C_{y\delta r}}{m \cdot U_0} \cdot \delta_r \end{aligned} \quad (137)$$

$$\begin{aligned} \dot{\phi} = & p + q \cdot \sin \Phi_0 \cdot \tan \Theta_0 + [(Q_0 \cdot \cos \Phi_0 - R_0 \sin \Phi_0) \cdot \tan \Theta_0] \cdot \phi \\ & + r \cdot \cos \Phi_0 \cdot \tan \Theta_0 \end{aligned} \quad (138)$$

$$\begin{aligned} \dot{p} = & \left[\frac{2 \cdot Q_0 \cdot I_{yz} + P_0 \cdot I_{xz} - R_0 \cdot (I_z - I_y)}{I_x} \right] \cdot q + \frac{q_0 \cdot S \cdot b \cdot C_{lp}}{I_y} \cdot \beta \\ & + \left[\frac{q_0 \cdot S \cdot b^2 \cdot C_{lp} + 2 \cdot U_0 \cdot (Q_0 \cdot I_{xz} - R_0 \cdot I_{xy})}{2 \cdot I_x \cdot U_0} \right] \cdot p + \frac{q_0 \cdot S \cdot b \cdot C_{l\delta a}}{I_x} \cdot \delta_a \\ & + \frac{q_0 \cdot S \cdot b \cdot C_{l\delta r}}{I_x} \cdot \delta_r + \left[\frac{q_0 \cdot S \cdot b^2 \cdot C_{lr}}{2 \cdot I_x \cdot U_0} - \frac{2 \cdot R_0 \cdot I_{yz} + P_0 \cdot I_{xy} + Q_0 \cdot (I_z - I_y)}{I_x} \right] \cdot r \end{aligned} \quad (139)$$

$$\begin{aligned} \dot{r} = & - \left[\frac{2 \cdot Q_0 \cdot I_{xy} + R_0 \cdot I_{xz} - P_0 \cdot (I_y - I_x)}{I_z} \right] \cdot q + \frac{q_0 \cdot S \cdot b \cdot C_{nr\beta}}{I_z} \cdot \beta \\ & + \left[\frac{q_0 \cdot S \cdot b^2 \cdot C_{nr} - 2 \cdot U_0 \cdot (Q_0 \cdot I_{xz} - P_0 \cdot I_{yz})}{2 \cdot I_z \cdot U_0} \right] \cdot p + \frac{q_0 \cdot S \cdot b \cdot C_{nr\delta a}}{I_z} \cdot \delta_a \\ & + \frac{q_0 \cdot S \cdot b \cdot C_{nr\delta r}}{I_z} \cdot \delta_r + \left[\frac{q_0 \cdot S \cdot b^2 \cdot C_{nr}}{2 \cdot I_z \cdot U_0} - \frac{2 \cdot R_0 \cdot I_{xy} + Q_0 \cdot (I_y - I_x)}{I_z} \right] \cdot p \end{aligned} \quad (140)$$

Finally, the whole set of equations of motion of the undamaged aircraft is obtained and given as equation (136), equation (138) and equation (141-146):

$$\frac{\dot{u}}{U_0} = X_u \frac{u}{U_0} + \left(\frac{X_\alpha}{U_0} - Q_0 \right) \alpha - \left(\frac{g}{U_0} \cos \Theta_0 \right) \theta + R_0 \beta + \frac{X_{\delta_e}}{U_0} \delta_e \quad (141)$$

$$\begin{aligned} \dot{\alpha} = & (Z_u + Q_0) \frac{u}{U_0} + \frac{Z_\alpha}{U_0} \alpha + q - \left(\frac{g}{U_0} \sin \Theta_0 \cos \Phi_0 \right) \theta - P_0 \beta \\ & - \left(\frac{g}{U_0} \cos \Theta_0 \sin \Phi_0 \right) \phi + \frac{Z_{\delta_e}}{U_0} \delta_e \end{aligned} \quad (142)$$

$$\begin{aligned} \dot{q} = & [U_0 M_u + M_\alpha (Q_0 + Z y_u)] \frac{u}{U_0} + \left(M_\alpha + \frac{M_\alpha Z_\alpha}{U_0} \right) \alpha \\ & + \left[M_\alpha + M_q + \frac{1}{I_y} (R_0 I_{xy} - P_0 I_{yz}) \right] q - \left(\frac{M_\alpha g}{U_0} \cos \Theta_0 \cos \Phi_0 \right) \phi \\ & - \frac{1}{I_y} [2P_0 I_{xz} + Q_0 I_{yz} + R_0 (I_x - I_z)] p \\ & + \frac{1}{I_y} [2R_0 I_{xz} + Q_0 I_{xy} - P_0 (I_x - I_z)] r + \left(M_{\delta_e} + \frac{M_\alpha Z_{\delta_e}}{U_0} \right) \delta_e \end{aligned} \quad (143)$$

$$\dot{\theta} = q \cos \Phi_0 - (Q_0 \sin \Phi_0 + R_0 \cos \Phi_0) \phi - r \sin \Phi_0 \quad (136)$$

$$\begin{aligned} \dot{\beta} = & \frac{-R_0}{U_0} u + \alpha P_0 - \frac{g}{U_0} \theta \sin \Theta_0 \sin \Phi_0 + \frac{g}{U_0} \phi \cos \Theta_0 \cos \Phi_0 \\ & + \frac{Y_\beta \beta + Y_p P + (Y_r - U_0) r + Y_{\delta_a} \delta_a + Y_{\delta_r} \delta_r}{U_0} \end{aligned} \quad (144)$$

$$\dot{\phi} = p + q \sin\Phi_0 \tan\Theta_0 + [(Q_0 \cos\Phi_0 - R_0 \sin\Phi_0) \tan\Theta_0] \phi + r \cos\Phi_0 \tan\Theta_0 \quad (138)$$

$$\begin{aligned} \dot{p} = & \frac{1}{I_x} [2Q_0 I_{yz} + P_0 I_{xz} - R_0 (I_z - I_y)] q + L_{\beta} \beta \\ & + [L_p + \frac{1}{I_x} (Q_0 I_{xz} - R_0 I_{xy})] p + L_{\alpha} \delta_a + L_{\delta r} \delta_r \\ & \{L_r - \frac{1}{I_x} [2R_0 I_{yz} + P_0 I_{xy} + Q_0 (I_z - I_y)]\} r \end{aligned} \quad (145)$$

$$\begin{aligned} \dot{r} = & \frac{-1}{I_z} [2Q_0 I_{xy} + R_0 I_{xz} - P_0 (I_y - I_x)] q + N_{\beta} \beta \\ & + [N_r - \frac{1}{I_z} (Q_0 I_{xz} - P_0 I_{yz})] r + N_{\alpha} \delta_a + N_{\delta r} \delta_r \\ & + \{N_p + \frac{1}{I_z} [2P_0 I_{xy} - Q_0 (I_y - I_x)]\} p \end{aligned} \quad (146)$$

These equations could also be applied to the aircraft with control surface damage by updating the dimensional stability derivatives from Section 3.1, as well as the masses and moment inertia with damage. Moreover, the equations of motion are re-written in state space form and the dynamic model, the plant matrix, A , and input matrix, B , is therefore obtained (Appendix D).

4.3 Derivation of Single-axis Transfer Functions

The single-axis transfer functions of pitch, roll, and yaw are obtained by using the following equation (147) which are given by Kuo [20]:

$$G(s) = C(sI - A)^{-1}B \quad (147)$$

where $G(s)$ = transfer function of a specific axis

I = unit matrix

C = output matrix

4.4 The Logic of Optimal Pilot Model

The Optimal Pilot Model (OPM) is a C-language computer program developed by Kim [15]. This software is developed to obtain the single-axis performance index from an input single-axis dynamic model of an aircraft which can be expressed in state space form or transfer function form. In this research, to directly apply Dander's data, transfer-function formed dynamic models are used as input.

When disturbances occur to an aircraft, the resulting deviation is displayed on the instrument and visually perceived by the pilot with observation noise. After reaction time delay, the pilot evaluates the delayed situation and predicts the future situation and responds. The feedback with motor noise made by the pilot is delayed by his neuro-muscular system and input into the vehicle and produces an output status of the aircraft. Figure 5 illustrates these procedures. The dashed block is the so-called human operator model, or optimal pilot model, which includes the procedures from the visual perception to the neuro-muscular delayed feedback of the pilot. A more detail diagram of the optimal model is given as Figure 6.

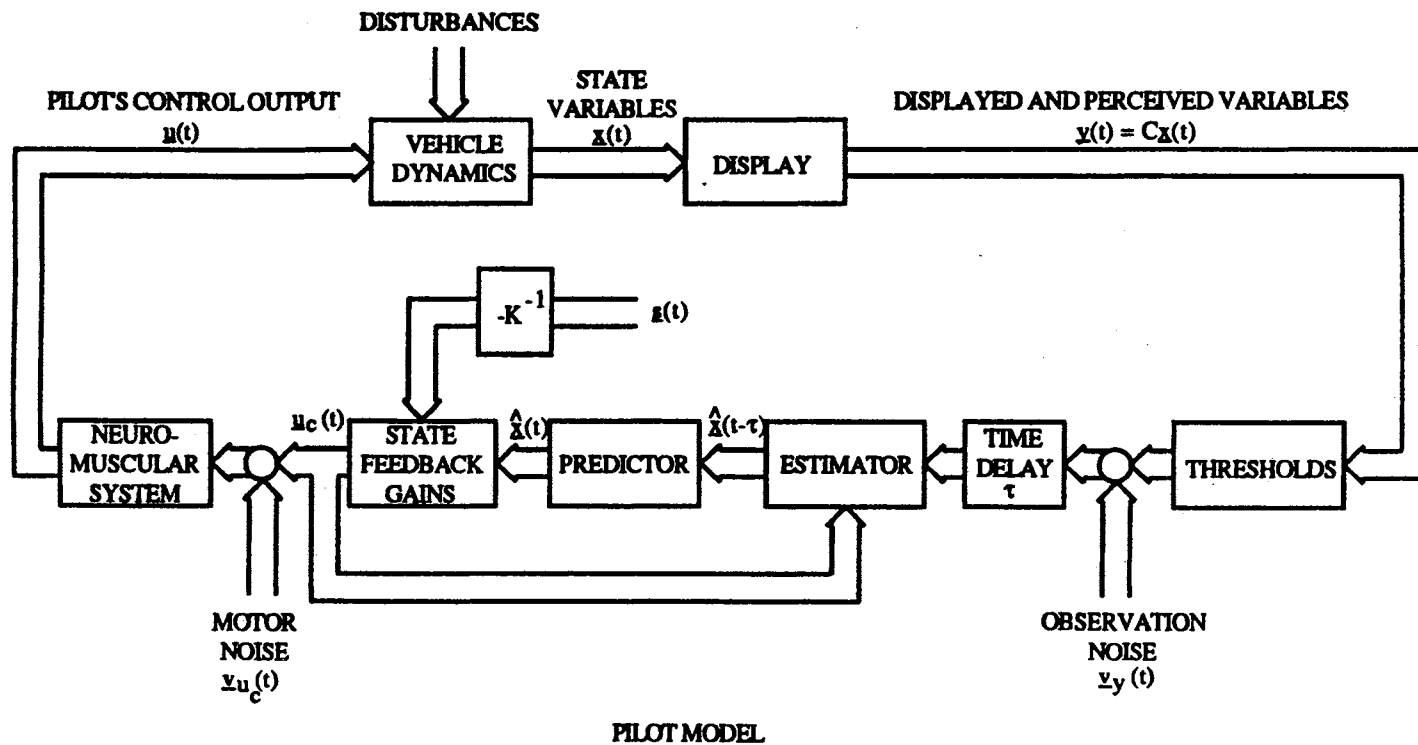


Figure 5. Block Diagram of Optimal Control Model of Human Pilot

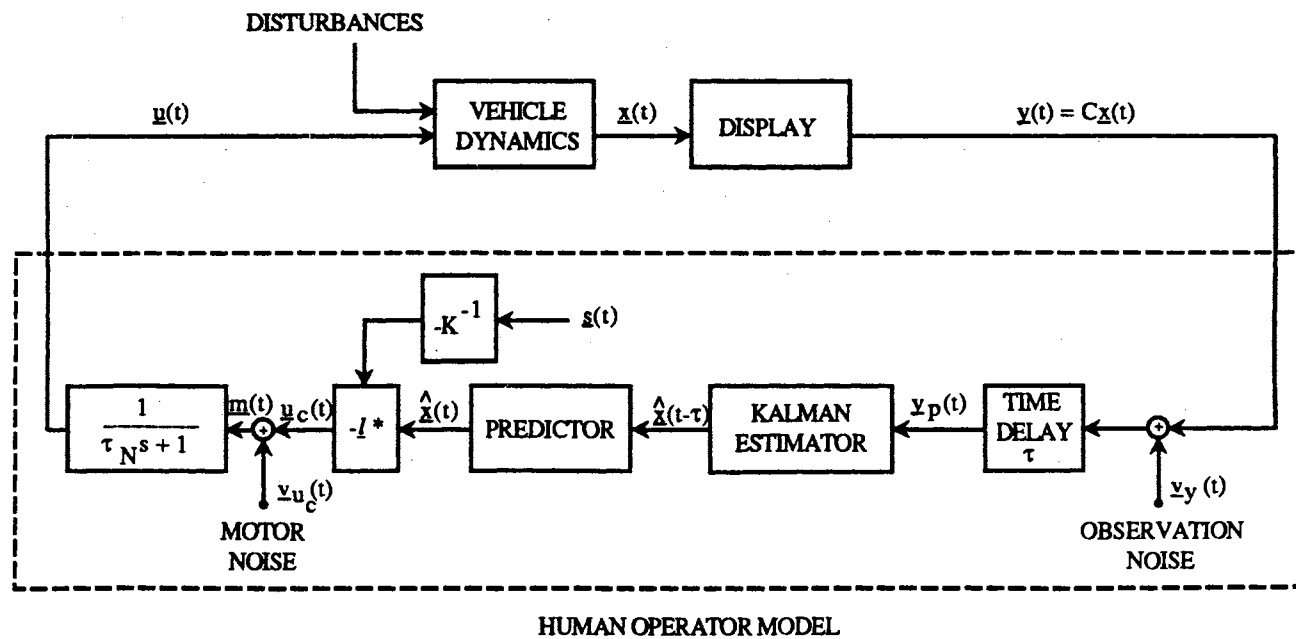


Figure 6. Control-theoretic Model of Optimal Human Behavior

Based on the illustrated logic, the Optimal Pilot Model (OPM) software is developed. Because the pilot does not have perfect observation of the instrumental output, which is expressed as vector $\mathbf{y}(t)$, he sees a delayed and noisy vector given by equation (148):

$$\underline{\mathbf{y}}_p(t) = \mathbf{C}\underline{\mathbf{X}}(t - \tau) + \underline{\mathbf{V}}_y(t - \tau) \quad (148)$$

The observation noise vector $\underline{\mathbf{v}}_y$ has a white noise diagonal auto-covariance matrix given by

$$E[\underline{\mathbf{V}}_{y_i}(t) \cdot \underline{\mathbf{V}}_{y_i}(\tau)] = \mathbf{V}_{y_i} \delta(t - \tau), \quad i = 1, 2, 3 \quad (149)$$

where

$$\mathbf{V}_{y_i} = \frac{\rho_{y_i} \cdot \pi \cdot E(y_i^2)}{f_i \cdot [\operatorname{erfc}(\frac{T_i}{\sqrt{2 \cdot E^2(y_i)}})]^2}$$

A Kalman filter is used to estimate the delayed state vector $\dot{\underline{\mathbf{x}}}(t - \tau)$. The Kalman filter (estimator) equations are

$$\dot{\underline{\mathbf{X}}}(t - \tau) = \mathbf{A} \cdot \hat{\underline{\mathbf{X}}}(t - \tau) + \mathbf{B} \cdot \hat{\underline{\mathbf{u}}}_c(t - \tau) + \Sigma \cdot \mathbf{C}^T \cdot \mathbf{V}_y^{-1} \cdot [\underline{\mathbf{y}}_p(t) - \mathbf{C} \hat{\underline{\mathbf{X}}}(t - \tau)] \quad (150)$$

where τ is the pilot's reaction time delay

$$\mathbf{A} \cdot \Sigma + \Sigma \cdot \mathbf{A}^T - \Sigma \cdot \mathbf{C}^T \cdot \mathbf{V}_y^{-1} \cdot \mathbf{C} \cdot \Sigma + \mathbf{E} \cdot \mathbf{W} \cdot \mathbf{E}^T = 0 \quad (151)$$

The estimated state $\dot{\underline{\mathbf{X}}}(t)$ can be obtained by a linear least squares predictor as the form

$$\dot{\hat{\mathbf{X}}}(t) = e^{A \cdot \tau} \cdot \dot{\hat{\mathbf{X}}}(t - \tau) \quad (152)$$

where $e^{A \cdot \tau}$ is a constant state transition matrix.

The feedback response without motor noise is given by

$$\underline{\mathbf{u}}_c = -I^* [\hat{\mathbf{X}} - K^{-1} \underline{\mathbf{s}}] = \mathbf{R}^{-1} \cdot \mathbf{B}^T [K \cdot \hat{\mathbf{X}} - \underline{\mathbf{s}}] \quad (153)$$

The following feedback response with motor noise is given by

$$\underline{\mathbf{m}}(t) = \underline{\mathbf{u}}_c(t) + \underline{\mathbf{V}}_{uc}(t) \quad (154)$$

The motor noise auto-covariance matrix is

$$E[\underline{\mathbf{V}}_{uci}(t) \cdot \underline{\mathbf{V}}_{uci}(\tau)] = V_{uci} \delta(t - \tau), \quad i = 1, 2, 3 \quad (155)$$

where $V_{ui} = \rho_{uci} \cdot \pi \cdot E(u_{ci}^2)$

The solution of the Kalman filter and predictor to obtain $\hat{\mathbf{X}}(t)$ and $\hat{\mathbf{y}}(t)$ requires an iterative procedure. This procedure, illustrated in Figure 7, is to select values for the intensities v_{yi} and v_{uci} solve the single-axis filter and predictor equations (150-152) to yield $\hat{\mathbf{x}}(t)$, compute the values $E(y_i^2)$ and $E(u_{ci}^2)$ from the computer simulation. Repeat this procedure until equation (149) and equation (155) are satisfied to a sufficient numerical accuracy for the v_{yi} and v_{uci} values.

Then, the single-axis performance index J is obtained by equation (156):

$$J = \mathbf{Q} \cdot E(y_i^2) + \mathbf{R} \cdot E(U_{ci}^2) \quad (156)$$

The single-axis performance index of the associated axis are then obtained and are ready for being transformed into the associated single-axis PORs. This transformation is discussed in the next section.

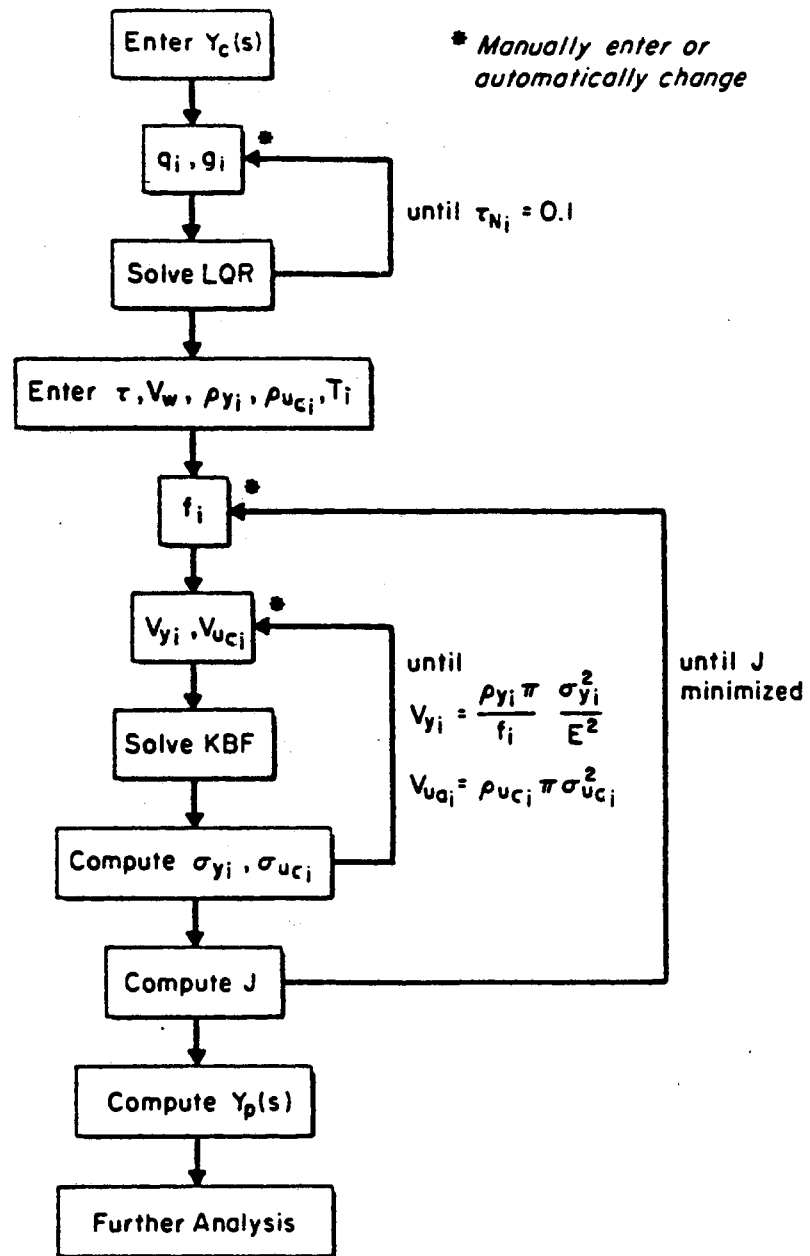


Figure 7. Computation Flow of the Optimal Pilot Model

4.5 Multi-axis POR

In this research, multi-axis POR is used to determine the flying quality of the specific aircraft, Gates Learjet 24B. As mentioned in section 3.3, the multi-axis POR is obtained by the integration of the associated three single-axis PORs which are resulted from the transformation of single-axis performance index. The three single-axis performance indices are first normalized by the following equation:

$$\bar{J} = \frac{J_i}{\sigma_{ci}^2} \quad (157)$$

and σ_{ci}^2 is the mean square output error due to the driving noise for axis i . The normalized performance index for the i axis, \bar{J}_i , is transformed into the corresponding single-axis POR by use of the cost function/rating correlation model [4], which is illustrated in Figure 8.

The linearity between single-axis normalized performance index and single-axis POR displayed in Figure 8 can be formulated as the following equation:

$$POR_i = 7 \bar{J} + 3 \bar{J} \cdot \log_0(\bar{J}_i) \quad (158)$$

The final procedure is to integrate the obtained single-axis PORs into the corresponding multi-axis POR by the utilization of the product rule [3]:

$$R_m = 10 + \frac{-1^{(m+1)}}{8.3^{(m-1)}} \prod_{i=1}^m (R_i - 10) \quad (159)$$

where R_m = the multi-axis POR
 R_i = the POR in the axis i
 m = the number of axes

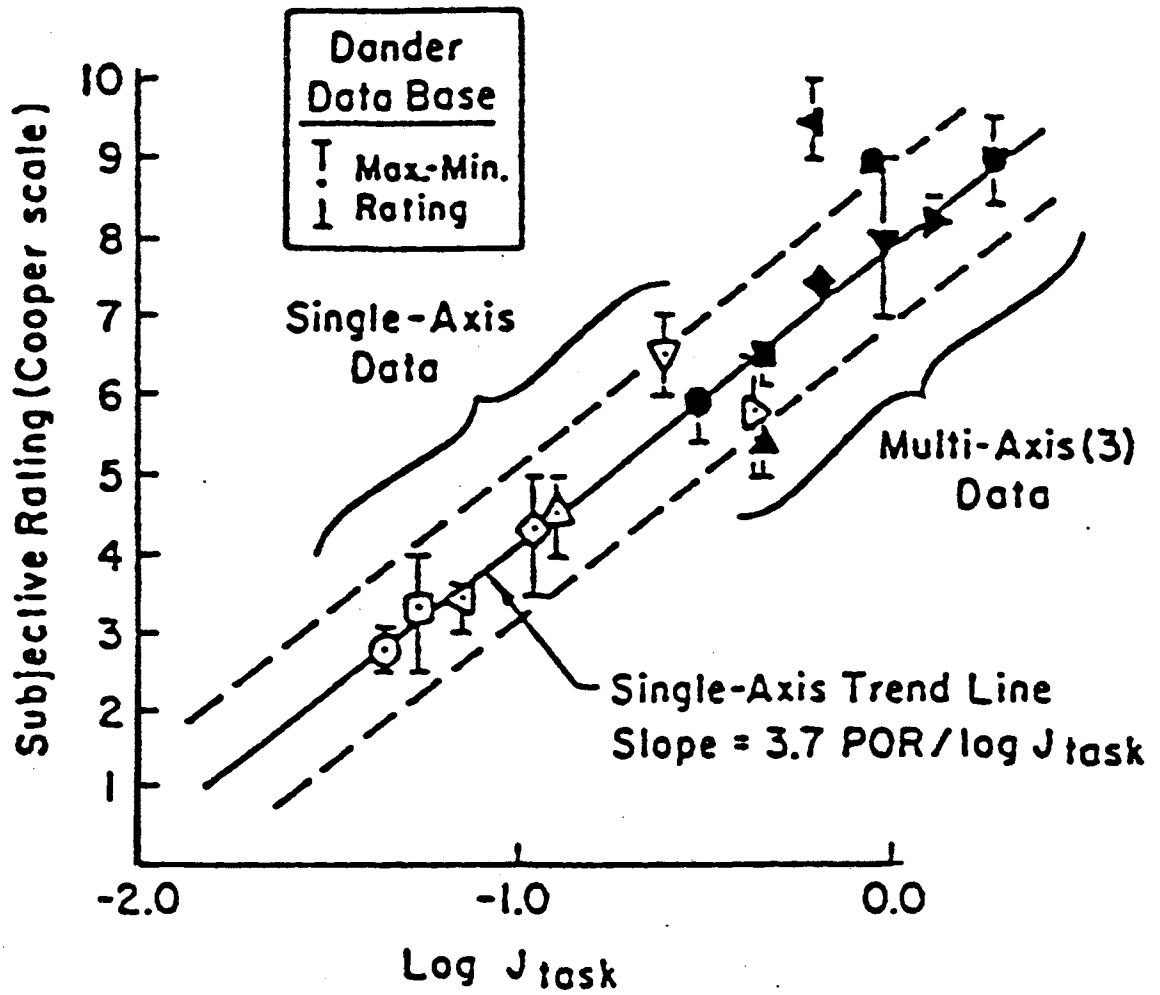


Figure 8. POR vs. Performance Index

The controllability of the aircraft which is performing a specific maneuver is then determined by using the Cooper-Harper rating scale [1], shown in Table II, to interpret the resulting multi-axis POR. The aircraft is said to exhibit total loss of control if the obtained POR is larger than 9; otherwise, the aircraft is controllable.

CHAPTER V

DETAILS OF IMPLEMENTATION

5.1 The Assigned Constants and Range of Study

The Gates Learjet 24B aircraft is utilized for this research. The flight conditions, geometry and inertia, steady state coefficients, and stability and control derivatives of this aircraft are given in Appendix B. The specific types and degrees of damage are damage on wings, horizontal tail, or vertical tail, and each with increment of 6.25% of loss of the specific control surface, and are mathematically defined. Moreover, the maximum loss of any control surface is limited to be 50%. In addition, because this specific aircraft is a commercial one rather than a fighter, the pitch angle Θ_0 and bank angle Φ_0 are not supposed to be larger than 30 degrees in practical situations. Therefore, the pitch and/or bank angles of the specified maneuvers are set to be 15 or 30 degrees. In addition, the flight speed is set to be the cruise speed which is 677 feet per second.

To perform the third procedure of the methodology mentioned in Section 3.4, several parameters are required to operate Kim's Optimal Pilot Model software [6]. They are neuro-muscular constant, τ_N , driving noise intensity, V_w , human reaction time delay, τ , noise ratios, ρ_{y1} , ρ_{y2} , and ρ_{y3} , indifference threshold, T_1 and T_2 , and fractional attention, f . The values of these parameters except f are given in Table III [21]. The value of f is selected based on the data from the optimal pilot model of McRuer and Schmidt [4] and is between 0 and 1.

Table III
PILOT MODEL PARAMETER VALUES

	V_w	τ	τ_N	ρ_{y1}	ρ_{y2}	ρ_{uc}	T_1	T_2
θ	1	0.2	0.1	0.01	0.01	0.01	0.015	0.025
ϕ	1	0.2	0.1	0.01	0.01	0.01	0.750	1.500
β	1	0.2	0.1	0.01	0.01	0.01	0.070	0.140

Also, in the third procedure, the input of driving noise shaping filters ($Y_{W,i}$ of the i -axis, i can be θ , ϕ or β) for the three axes are required by Kim's optimal pilot model software. These driving noise shaping filters are modeled as a second-order Markov process [4] based on Dander's data and are given as follows:

$$Y_{w,\theta}(s) = \frac{0.2219}{s^2 + 0.7s + 0.25} \quad (160)$$

$$Y_{w,\phi}(s) = \frac{13.3}{s^2 + 0.7s + 0.25} \quad (161)$$

$$Y_{w,\beta}(s) = \frac{0.53}{s^2 + 0.7s + 0.25} \quad (162)$$

The commanded attitude of the associated axis is thus the product of the driving noise shaping filter of the associated axis and the external disturbance, and is calculated by the software automatically. It is important to know that Gaussian white noise is used as the external disturbance. Moreover, in this research, the external disturbance is Gaussian distributed with a mean of zero and a variance which is a delta function of time.

5.2 Sample Case

A methodology for determining the controllability of a specific aircraft with specific control surface damage has been developed completely. In this section, a sample case is presented to provide an environment for understanding this methodology. In this sample case, a damage occurs to the specific aircraft and causes a loss of 43.75% of the area of the horizontal tail. Moreover, the damage occurs while the aircraft is performing a combinational maneuver defined in Section 3.2 with both Θ_o and Φ_o are constant at 15° .

Before determining the damaged case, the parameters C_{ip} and C_{nr} are first calibrated by setting the POR to be 4.5, a reasonable controllability, while the aircraft is performing straight and level flight, the simplest maneuver. The calibrated C_{ip} and C_{nr} are -3.17 and -13.34, respectively; and the undamaged $C_{L\alpha h}$ is 4.26. The procedure of implementation is then illustrated step by step:

(1) Obtain \underline{A}_h :

Substituting $\underline{S}_h = 30.38 \text{ (ft}^2\text{)}$ into equation (11) to give

$$\underline{A}_h = 2.08.$$

(2) Obtain $\underline{C}_{L\alpha h}$:

Substituting the obtained \underline{A}_h into equation (16) to give

$$\underline{C}_{L\alpha h} = 2.59.$$

(3) Obtain K_h :

$$K_h = \frac{C_{L\alpha h} \cdot S_h}{C_{L\alpha h} \cdot S_h} = 0.34$$

(4) Substitute the obtained K_h into equation (27-34) and equation (37) and equation (40) to give the stability derivatives with damage:

$$\begin{bmatrix} \frac{C_{L\delta e}}{C_{m\delta e}} \\ \frac{C_{L\dot{\alpha}}}{C_{m\dot{\alpha}}} \\ \frac{C_{Lq}}{C_{mq}} \\ \frac{C_{D\alpha}}{C_{L\alpha}} \\ \frac{C_{m\alpha}}{C_{m\alpha}} \end{bmatrix} = \begin{bmatrix} 0.16 \\ -0.43 \\ 0.76 \\ -2.30 \\ 1.61 \\ -5.32 \\ 0.10 \\ 5.54 \\ 0.18 \end{bmatrix}$$

(5) Substitute the stability derivatives with damage into Plant matrix, A , and input matrix, B :

$$A = \begin{bmatrix} -.019 & .009 & 0 & -.046 & .012 & 0 & 0 & 0 \\ -.135 & -.633 & 1 & -.012 & .003 & -.012 & 0 & 0 \\ .631 & 1.675 & -1.324 & .005 & -.001 & .018 & .012 & -.003 \\ 0 & 0 & .966 & 0 & 0 & -.013 & 0 & -.259 \\ -.012 & -.003 & 0 & -.003 & -.083 & .044 & 0 & -.999 \\ 0 & 0 & .069 & 0 & 0 & 0 & 1 & .259 \\ 0 & 0 & -.012 & 0 & -4.152 & 0 & -3 & .148 \\ 0 & 0 & -.001 & 0 & 2.843 & 0 & .005 & -7.5 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 & 0 \\ -0.024 & 0 & 0 \\ -6.204 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & .016 \\ 0 & 0 & 0 \\ 0 & 6.718 & .717 \\ 0 & -.448 & -1.656 \end{bmatrix}$$

(6) Obtaining the three single-axis transfer functions:

By use of equation (147) the transfer functions for three axes are given:

$$G_{\theta} = \frac{-5.168(s+0.026)(0.631)}{(s-0.074)(s-0.715)(s^2+1.928s+0.126)}$$

$$G_{\phi} = \frac{6.67(s+0.449)}{(s+3.027)(s^2+0.458s+0.057)}$$

$$G_{\psi} = \frac{1.677(s+0.031)}{(s+7.093)(s^2+0.458s+0.057)}$$

(7) Input the single-axis transfer functions and driving noise shaping filter, equations (160-162), into the Optimal Pilot Model and have

$$J_{\theta} = 0.074$$

$$J_{\phi} = 29.873$$

$$J_{\beta} = 0.048$$

(8) Use equation (157) to obtain the normalized single-axis performance index with $\sigma_{\alpha\delta}^2 = 0.14$, $\sigma_{\alpha\phi}^2 = 500$, $\sigma_{\alpha\beta}^2 = 0.81$ which are given by Thompson and McRuer [20]:

$$\bar{J}_{\theta} = 0.53$$

$$\bar{J}_{\phi} = 0.06$$

$$\bar{J}_{\beta} = 0.059$$

(9) Use equation (158) to obtain single-axis PORs

$$POR_{\theta} = 6.68$$

$$POR_{\phi} = 3.18$$

$$POR_{\beta} = 3.15$$

(10) Use the product rule, equation (159), to obtain the multi-axis POR as

$$POR = 10 + \frac{(-1)^4 (6.68 - 10)(3.18 - 10)(3.15 - 10)}{8.3^2} = 7.74$$

(11) Determine the state of control:

From the Cooper-Harper rating scale [1], it is clear that the aircraft is still in control after the damage.

The sample case thus provides an example of applying the methodology. By use of this methodology, a series of resulting data is obtained. The resulting data is then summarized and analyzed in the next chapter.

CHAPTER VI

ANALYSIS OF RESULTING DATA

The resulting data is plotted for straight and level flight, steady level turn, symmetrical pull-up, and the combination of steady level turn and symmetrical pull-up as shown in Figures 9, 10, 11, 12, 13, and 14. In addition, the numerical PORs are provided in Appendix E. From these figures, the computer simulation shows the same trend for all these maneuvers that the scale of loss of control increases while the control surface damage on wing, horizontal tail, or vertical tail increases.

The resulting data suggests that the Gates Learjet 24B aircraft is capable of taking serious control surface damage. To perform tasks with small pitch and bank angles which are not more than 15 degrees, the aircraft remains controllable even though the control surface damage of either type is about 43.75% loss. In addition, coincidentally, with the same scale of control surface loss, the specific aircraft is more sensitive to wing damage than to tail surface damage if the damage scales are all less than 37.50% for all four types of maneuvers. However, if the damage scales are more than 37.50%, the aircraft is more sensitive to horizontal tail loss. This dramatic increase in sensitivity to horizontal tail loss occurs because the variation of pitching moment coefficient with α , $C_{m\alpha}$, changes its sign from negative into positive while the damage scale is 37.50%; and this change abruptly reduces the controllability of the aircraft. However, among all types of control surface damage, the resulting data suggests that wing loss is still the most serious condition for this aircraft for all types of maneuvers studied.

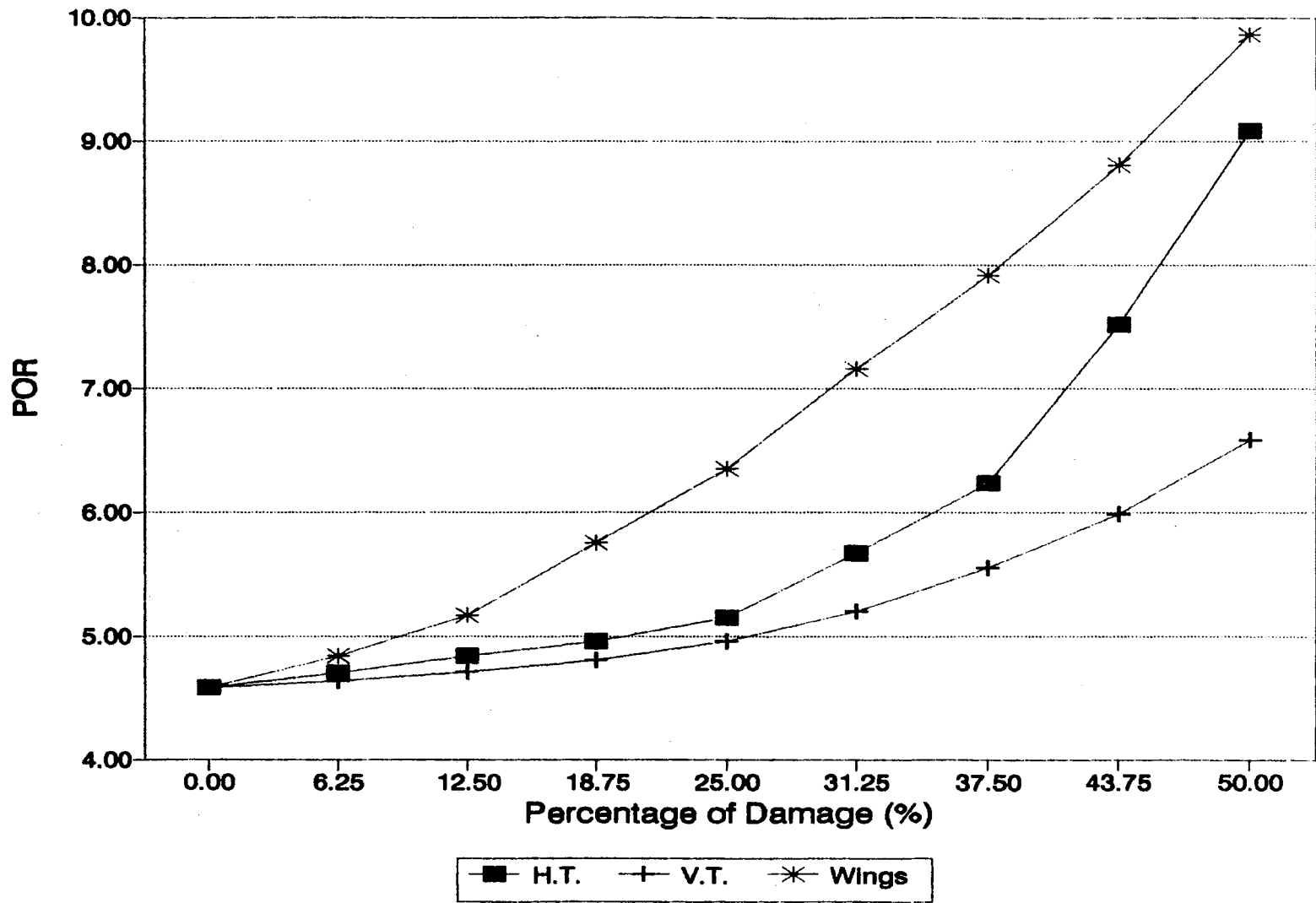


Figure 9. POR of Straight and Level Flight vs. Damage

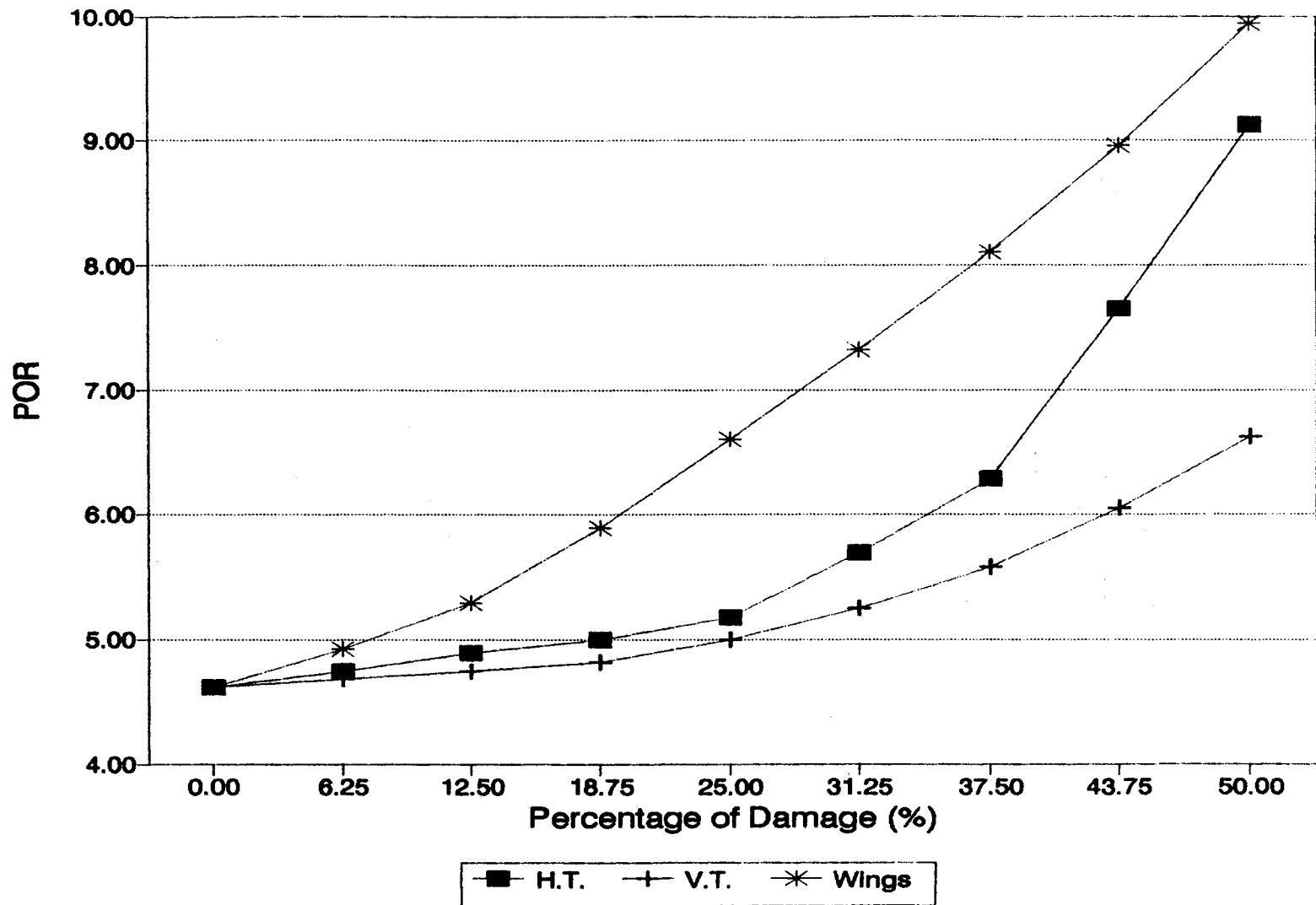


Figure 10. POR of Symmetric Pull-up at $\Theta_0 = 15^\circ$ vs. Damage

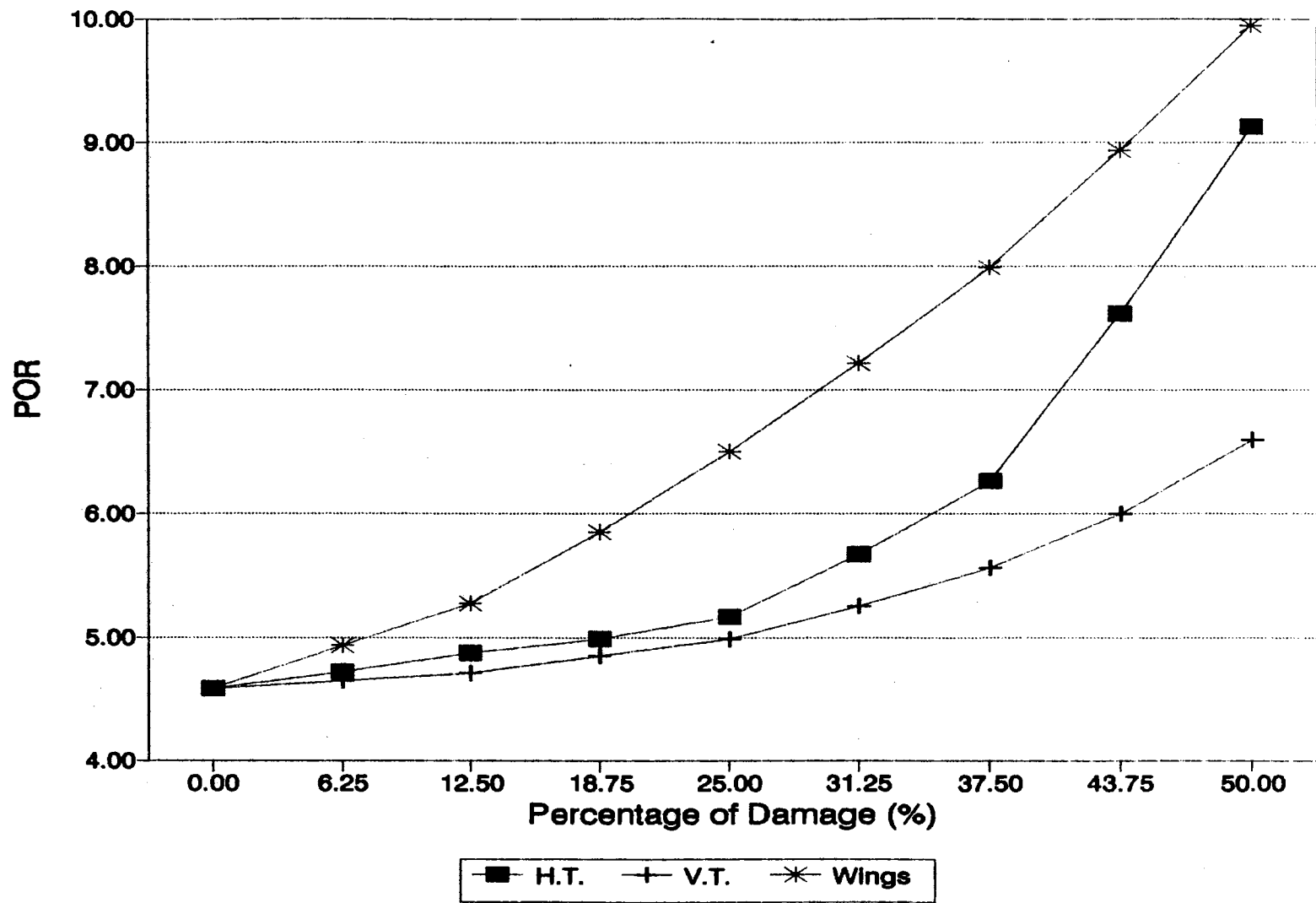


Figure 11. POR of Steady Level Turn at $\Phi_0 = 15^\circ$ vs. Damage

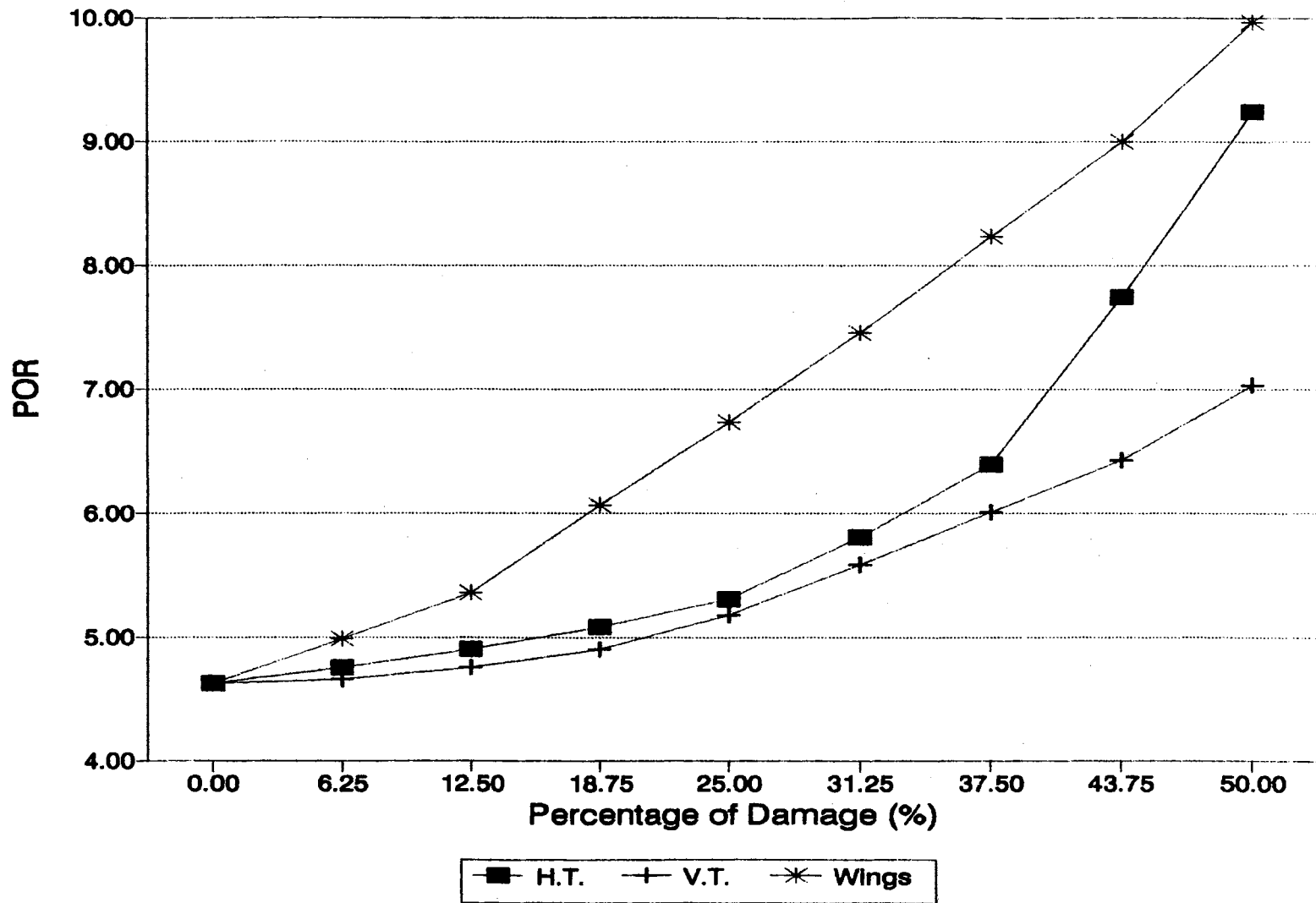


Figure 12. POR of Combinational Maneuver at $\Theta_0 = 15^\circ$, $\Phi_0 = 15^\circ$ vs. Damage

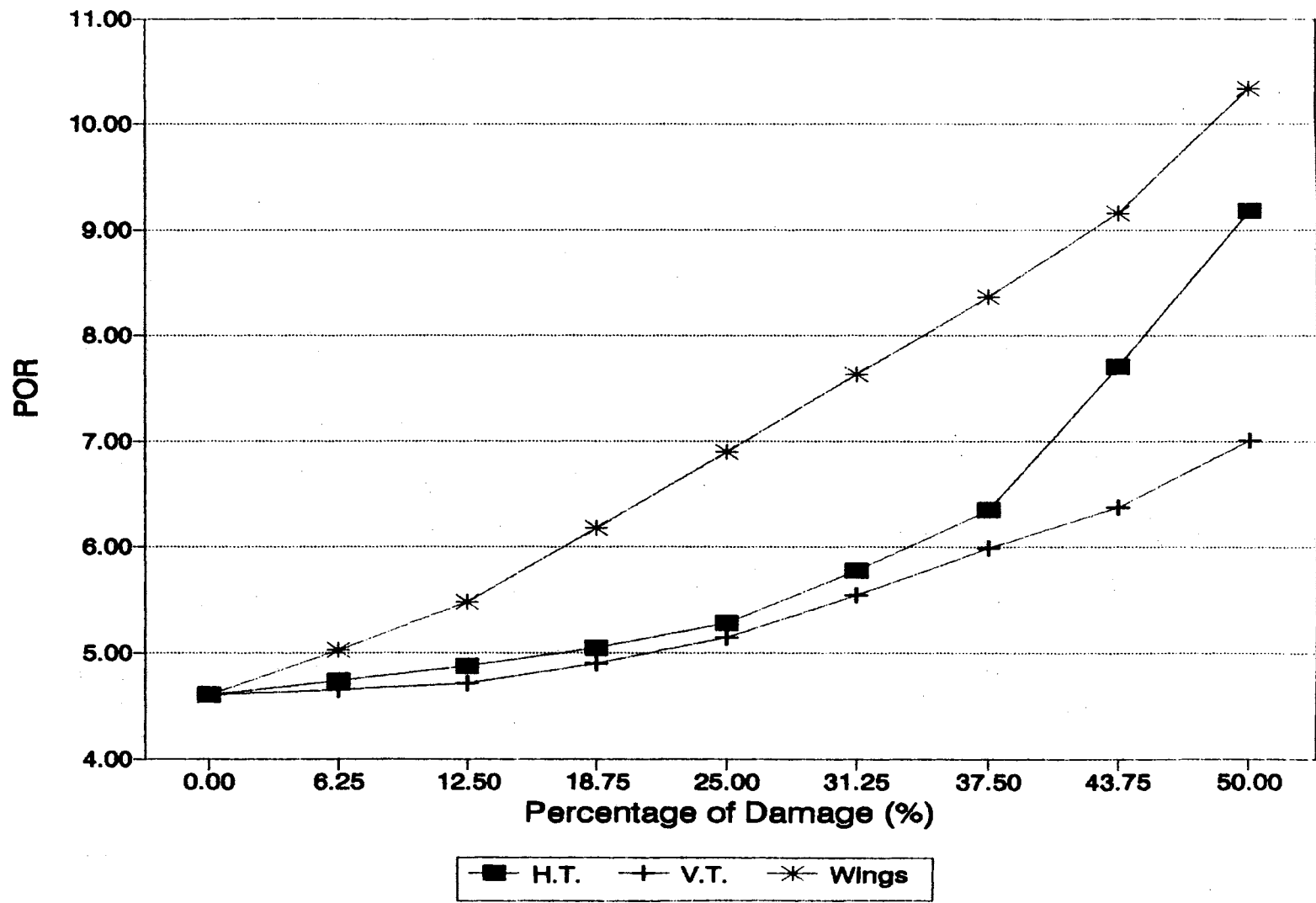


Figure 13. POR of Steady Level Turn at $\Phi_0 = 30^\circ$ vs. Damage

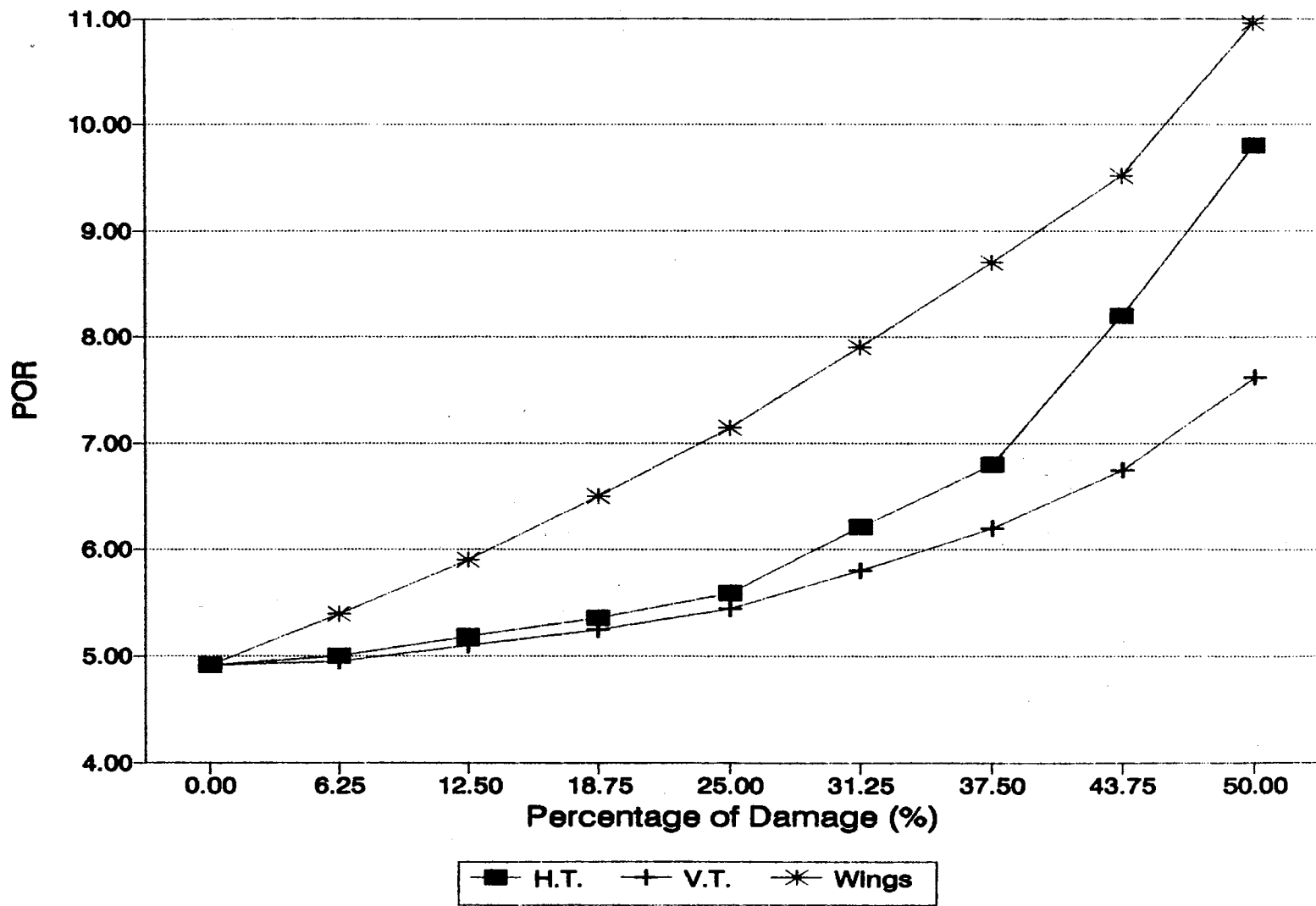


Figure 14. POR of Combinational Maneuver at $\Theta_0 = 30^\circ$, $\Phi_0 = 30^\circ$ vs. Damage

Moreover, based on the resulting data, some criteria can be established for operating this aircraft, Learjet 24B, as follows:

(1) When performing straight and level flight, symmetrical pull-up with Θ_0 no more than 30 degrees, steady level turn with Φ_0 no more than 30 degrees, or the defined combinational maneuver with neither Θ_0 nor Φ_0 is more than 30 degrees, the specific aircraft should be abandoned if

- (a) the wing loss is more than 43.75%, or
- (b) the horizontal tail loss is about 50%;

(2) When performing steady level turn with Φ_0 between 15 and 30 degrees, the specific aircraft should reduce its Φ_0 to 15 degrees or less if

- (a) the wing loss is about 42%, or
- (b) the horizontal tail loss is about 49%;

(3) When performing the defined combinational maneuver with both Θ_0 and Φ_0 are between 15 and 30 degrees, the specific aircraft should reduce both its Θ_0 and Φ_0 to 15 degrees or less, or simply reduce its maneuver to the steady level turn with the same Φ_0 if

- (a) the wing loss is about 40%, or
- (b) the horizontal tail loss is about 45%;

(4) When performing straight and level flight, symmetrical pull-up with Θ_0 no more than 30 degrees, steady level turn with Φ_0 no more than 30 degrees, or the defined combinational maneuver with neither Θ_0 nor Φ_0 is more than 30 degrees, the specific aircraft should not be abandoned even though the vertical tail loss is 50% or less.

These criteria can be easily applied when a failure identification system is built in the specific aircraft. Furthermore, to minimize the interference of human factors such as emotion, psychological stress, misjudgment, and so on, the criteria are expected to be built in a warning system which can respond to the identified

damage immediately. The methodology developed in this research thus provides for the specific aircraft a foundation for the establishment of a flying safety system.

CHAPTER VII

CONCLUSIONS

The assessment of loss of control of aircraft should be conducted so that the tragedies that human lives are sacrificed for saving a seriously damaged aircraft can be avoided. Furthermore, the damaged aircraft is not abandoned while it can still survive. This research thus develops a systematic methodology to evaluate the controllability for a specific aircraft, Learjet B24, with/without various types and degrees of control surface damage.

The general dynamic models of the aircraft with/without various types and degrees of damage have been developed as the bases of this research. These developed dynamic models, Kim's optimal pilot model, the cost function/rating correlation model, and the Cooper-Harper rating scale are then successfully integrated into this methodology. And consequently, based on computer simulation, the controllability of the specific aircraft with specific control surface damage is monitored and predicted while this aircraft is performing straight and level flight, symmetrical pull-up, steady level turn, or the combination of symmetrical pull-up and steady level turn. Moreover, a set of safety criteria for operating the specific aircraft is established. With these simple and clear criteria, the pilot can make a decision quickly and correctly to save the aircraft, or his life. This assumes the pilot has knowledge of the type and amount of damage from a failure identification system.

It is important to know that in a practical situation, the surface damage may occur in combinational loss of surface failure. The system breakdown can also

cause the loss of control of an aircraft. In fact, the pilot will encounter a more serious situation when both system and control surface failures occur. All these cases are complicated and ought to receive further attention. The methodology developed in this research provides a fair starting point for the future study on these cases.

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APPENDICES

APPENDIX A

DERIVATION OF RATE OF TURN FOR
COMBINATIONAL MANEUVER

Referring to dynamic equations given by Roskam[16]

$$L \cdot \cos \phi_0 \cdot \cos \Theta_0 = W = m \cdot g \quad (162)$$

$$L \cdot \sin \phi_0 = m \cdot \dot{\psi}_0^2 \cdot R_1 = m \cdot \dot{\psi}_0^2 \cdot \frac{U_0^2}{g \cdot \tan \phi_0} \quad (163)$$

Equation (163) is divided by Equation (162) to give

$$\tan \phi_0 \cdot \sec \Theta_0 = \frac{\dot{\psi}_0^2 \cdot U_0^2}{g^2 \cdot \tan \phi_0}$$

Then

$$\dot{\psi}_0 = \frac{g \cdot \tan \phi_0 \cdot \sqrt{\sec \Theta_0}}{U_0} \quad (164)$$

The kinematic equations for general steady state cases are:

$$P_0 = \dot{\phi}_0 - \dot{\psi}_0 \cdot \sin \Theta_0 \quad (165)$$

$$Q_0 = \dot{\Theta}_0 \cos \phi_0 + \dot{\psi}_0 \cos \Theta_0 \sin \phi_0 \quad (166)$$

$$R_0 = \dot{\psi}_0 \cdot \cos \phi_0 \cdot \cos \Theta_0 - \dot{\Theta}_0 \cdot \sin \phi_0 \quad (167)$$

For the defined combinational maneuver,

$$\dot{\Theta}_0 = \dot{\phi}_0 = 0$$

therefore, Equation (165) - (167) becomes

$$P_0 = -\dot{\psi}_0 \cdot \sin \Theta_0$$

$$Q_0 = \dot{\psi}_0 \cos \Theta_0 \sin \phi_0$$

$$R_0 = \dot{\psi}_0 \cos \phi_0 \cos \Theta_0$$

APPENDIX B

FLIGHT CONDITIONS, AERODYNAMIC, INERTIAL,
GEOMETERIC DATA

(A) Flight Conditions, Aerodynamic, Inertial, Geometric Data

Velocity = 677.0 ft/sec

Altitude = 40,000.0 ft

Air Density = 0.000588 slug/ft³

Weight = 13,000.0 lbs

 $I_{xx} = 28,000.0 \text{ slug/ft}^2$ $I_{yy} = 18,800.0 \text{ slug/ft}^2$ $I_{zz} = 47,000.0 \text{ slug/ft}^2$ $I_{xz} = 13,00.0 \text{ slug/ft}^2$ Wing Area = 230.0 ft²

Wing Span = 34.0 ft

Wing Mean Geometric Chord = 7.0 ft

 $C_{L0} = 0.41$ $C_{D0} = 0.03$ $C_{m\alpha} = -0.64$ $C_{\dot{m}\alpha} = -6.70$ $C_{mq} = -15.50$ $C_{Lu} = 0.40$ $C_{L\alpha} = 5.84$ $C_{\dot{L}\alpha} = 2.20$ $C_{Lq} = 4.70$ $C_{D\alpha} = 0.30$ $C_{L\delta_e} = 0.46$ $C_{m\delta_e} = -1.24$ $C_{l\beta} = -0.11$ $C_{lp} = -0.45$ $C_{lr} = 0.16$ $C_{l\delta_a} = 0.18$ $C_{l\delta_r} = 0.02$ $C_{n\beta} = 0.13$ $C_{np} = -0.01$ $C_{nr} = -0.20$ $C_{n\delta_a} = -0.02$ $C_{n\delta_r} = -0.07$ $C_{y\beta} = -0.73$ $C_{yp} = 0.00$ $C_{yr} = 0.40$ $C_{y\delta_a} = 0.00$ $C_{y\delta_r} = 0.14$

APPENDIX C

DIMENSIONAL STABILITY DERIVATIVES

(1) Longitudinal Dimensional Stability Derivatives

$$\begin{aligned}
 X_u &= \frac{-\bar{q}_1 S (C_{D_u} + 2C_{D_1})}{mU_1} \quad (\text{sec}^{-1}) \\
 X_{T_u} &= \frac{\bar{q}_1 S (C_{T_{x_u}} + 2C_{T_{x_1}})}{mU_1} \quad (\text{sec}^{-1}) \\
 X_\alpha &= \frac{-\bar{q}_1 S (C_{D_\alpha} - C_{L_1})}{m} \quad (\text{ft sec}^{-2}) \\
 X_{\delta_E} &= \frac{-\bar{q}_1 S C_{D_{\delta_E}}}{m} \quad (\text{ft sec}^{-2} \text{ or } \text{ft sec}^{-2} \text{deg}^{-1}) \\
 Z_u &= -\frac{\bar{q}_1 S (C_{L_u} + 2C_{L_1})}{mU_1} \quad (\text{sec}^{-1}) \\
 Z_\alpha &= -\frac{\bar{q}_1 S (C_{L_\alpha} + C_{D_1})}{m} \quad (\text{ft sec}^{-2}) \\
 Z_\alpha^* &= -\frac{\bar{q}_1 S C_{L_\alpha} \bar{c}}{2mU_1} \quad (\text{ft sec}^{-1}) \\
 Z_q &= -\frac{\bar{q}_1 S C_{L_q} \bar{c}}{2mU_1} \quad (\text{ft sec}^{-1}) \\
 Z_{\delta_E} &= -\frac{\bar{q}_1 S C_{L_{\delta_E}}}{m} \quad (\text{ft sec}^{-2} \text{ or } \text{ft sec}^{-2} \text{deg}^{-1}) \\
 M_u &= \frac{\bar{q}_1 S \bar{c} (C_{m_u} + 2C_{m_1})}{I_{yy} U_1} \quad (\text{ft}^{-1} \text{sec}^{-1}) \\
 M_{T_u} &= \frac{\bar{q}_1 S \bar{c} (C_{m_{T_u}} + 2C_{m_{T_1}})}{I_{yy} U_1} \quad (\text{ft}^{-1} \text{sec}^{-1}) \\
 M_\alpha &= \frac{\bar{q}_1 S \bar{c} C_{m_\alpha}}{I_{yy}} \quad (\text{sec}^{-2}) \\
 M_{T_\alpha} &= \frac{\bar{q}_1 S \bar{c} C_{m_{T_\alpha}}}{I_{yy}} \quad (\text{sec}^{-2}) \\
 M_\alpha^* &= \frac{\bar{q}_1 S \bar{c}^2 C_{m_\alpha^*}}{2I_{yy} U_1} \quad (\text{sec}^{-1}) \\
 M_q &= \frac{\bar{q}_1 S \bar{c}^2 C_{m_q}}{2I_{yy} U_1} \quad (\text{sec}^{-1}) \\
 M_{\delta_E} &= \frac{\bar{q}_1 S \bar{c} C_{m_{\delta_E}}}{I_{yy}} \quad (\text{sec}^{-2} \text{ or } \text{sec}^{-2} \text{deg}^{-1})
 \end{aligned}$$

(2) Lateral-Directional Dimensional Stability Derivatives

$Y_{\beta} = \frac{\bar{q}_1 S C_{y_{\beta}}}{m} \quad (\text{ft sec}^{-2})$	$L_{\delta_A} = \frac{\bar{q}_1 S b C_{\ell_{\delta_A}}}{I_{xx}} \quad (\text{sec}^{-2} \text{ or } \text{sec}^{-2} \text{ deg}^{-1})$
$Y_p = \frac{\bar{q}_1 S b C_{y_p}}{2mU_1} \quad (\text{ft sec}^{-1})$	$L_{\delta_R} = \frac{\bar{q}_1 S b C_{\ell_{\delta_R}}}{I_{xx}} \quad (\text{sec}^{-2} \text{ or } \text{sec}^{-2} \text{ deg}^{-1})$
$Y_r = \frac{\bar{q}_1 S b C_{y_r}}{2mU_1} \quad (\text{ft sec}^{-1})$	$N_{\beta} = \frac{\bar{q}_1 S b C_{n_{\beta}}}{I_{zz}} \quad (\text{sec}^{-2})$
$Y_{\delta_A} = \frac{\bar{q}_1 S C_{y_{\delta_A}}}{m} \quad (\text{ft sec}^{-2} \text{ or } \text{ft sec}^{-2} \text{ deg}^{-1})$	$N_{T_{\beta}} = \frac{\bar{q}_1 S b C_{n_{T_{\beta}}}}{I_{zz}} \quad (\text{sec}^{-2})$
$Y_{\delta_R} = \frac{\bar{q}_1 S C_{y_{\delta_R}}}{m} \quad (\text{ft sec}^{-2} \text{ or } \text{ft sec}^{-2} \text{ deg}^{-1})$	$N_p = \frac{\bar{q}_1 S b^2 C_{n_p}}{2I_{zz} U_1} \quad (\text{sec}^{-1})$
$L_{\beta} = \frac{\bar{q}_1 S b C_{\ell_{\beta}}}{I_{xx}} \quad (\text{sec}^{-2})$	$N_r = \frac{\bar{q}_1 S b^2 C_{n_r}}{2I_{zz} U_1} \quad (\text{sec}^{-1})$
$L_p = \frac{\bar{q}_1 S b^2 C_{\ell_p}}{2I_{xx} U_1} \quad (\text{sec}^{-1})$	$N_{\delta_A} = \frac{\bar{q}_1 S b C_{n_{\delta_A}}}{I_{zz}} \quad (\text{sec}^{-2} \text{ or } \text{sec}^{-2} \text{ deg}^{-1})$
$L_r = \frac{\bar{q}_1 S b^2 C_{\ell_r}}{2I_{xx} U_1} \quad (\text{sec}^{-1})$	$N_{\delta_R} = \frac{\bar{q}_1 S b C_{n_{\delta_R}}}{I_{zz}} \quad (\text{sec}^{-2} \text{ or } \text{sec}^{-2} \text{ deg}^{-1})$

APPENDIX D

INPUT MATRIX

All elements of the matrix are zeros except follows:

Input matrix B for all maneuvers:

$$b_{11} = X_{\delta e} / U_0$$

$$b_{21} = Z_{\delta e} / U_0$$

$$b_{31} = M_{\delta e} + \frac{M \cdot Z_{\delta e}}{a} / U_0$$

$$b_{53} = Y_{\delta r} / U_0$$

$$b_{72} = L_{\delta a}$$

$$b_{73} = L_{\delta r}$$

$$b_{82} = N_{\delta a}$$

$$b_{83} = N_{\delta r}$$

APPENDIX E
PLANT MATRICES

All elements of all matrices are zeros except as follows:

(1) Plant matrix **A** for straight and level flight:

$$a_{11} = X_u$$

$$a_{12} = X_\alpha / U_0$$

$$a_{14} = -g / U_0$$

$$a_{21} = Z_u$$

$$a_{22} = Z_\alpha / U_0$$

$$a_{23} = 1$$

$$a_{31} = U_0 M_u + M_\alpha Z_u$$

$$a_{32} = M_\alpha + \frac{M_\alpha Z_\alpha}{U_0}$$

$$a_{33} = M_\alpha + M_q$$

$$a_{36} = -M_\alpha g / U_0$$

$$a_{43} = 1$$

$$a_{55} = Y_\beta / U_0$$

$$a_{56} = g / U_0$$

$$a_{58} = \frac{Y_r}{U_0} - 1$$

$$a_{67} = 1$$

$$a_{75} = L_\beta$$

$$a_{77} = L_p$$

$$a_{78} = L_r$$

$$a_{85} = N_\beta$$

$$a_{87} = N_p$$

$$a_{88} = N_r$$

(2) Plant matrix A for steady level turn

$$a_{11} = X_u$$

$$a_{12} = \frac{X_\alpha}{U_0} - Q_0$$

$$a_{14} = -g/U_0$$

$$a_{15} = R_0$$

$$a_{21} = Z_u + Q_0$$

$$a_{22} = Z_\alpha/U_0$$

$$a_{23} = 1$$

$$a_{26} = \frac{-g}{U_0} \sin \Phi_0$$

$$a_{31} = U_0 M_u + M_\alpha (Q_0 + Z_u)$$

$$a_{32} = M_\alpha + \frac{M_\alpha Z_\alpha}{U_0}$$

$$a_{33} = M_\alpha + M_q + \frac{I_{xy} R_0}{I_y}$$

$$a_{36} = \frac{-M_\alpha g}{U_0} \cos \Phi_0$$

$$a_{37} = \frac{-[Q_0 I_{yz} + R_0 (I_x - I_z)]}{I_y}$$

$$a_{38} = \frac{2R_0 I_{xz} + Q_0 I_{xy}}{I_y}$$

$$a_{43} = \cos \Phi_0$$

$$a_{46} = -(Q_0 \sin \Phi_0 + R_0 \cos \Phi_0)$$

$$a_{48} = -\sin \Phi_0$$

$$a_{51} = -R_0$$

$$a_{55} = Y_\beta / U_0$$

$$a_{56} = \frac{g}{U_0} \cos \Phi_0$$

$$a_{57} = Y_p / U_0$$

$$a_{58} = \frac{Y_r}{U_0} - 1$$

$$a_{67} = 1$$

$$a_{73} = \frac{2Q_0 I_{yz} - R_0 (I_z - I_y)}{I_x}$$

$$a_{75} = L_\beta$$

$$a_{77} = L_p + \frac{Q_0 I_{yz} - R_0 I_{xy}}{I_x}$$

$$a_{78} = L_r - \frac{2R_0 I_{yz} + Q_0 (I_z - I_y)}{I_x}$$

$$a_{83} = \frac{-(2Q_0 I_{xy} + R_0 I_{xz})}{I_z}$$

$$a_{85} = N_\beta$$

$$a_{87} = N_p + \frac{-Q_0 (I_y - I_x)}{I_z}$$

$$a_{88} = N_r - \frac{Q_0 I_{xz}}{I_z}$$

(3) Plant matrix A for symmetrical pull-up:

$$a_{11} = X_u$$

$$a_{12} = \frac{X_\alpha}{U_0} - Q_0$$

$$a_{14} = \frac{-g}{U_0} \cos \Theta_0$$

$$a_{21} = Z_u + Q_0$$

$$a_{22} = Z_\alpha / U_0$$

$$a_{23} = 1$$

$$a_{24} = \frac{-g}{U_0} \sin \Theta_0$$

$$a_{31} = U_0 M_u + M_\alpha (Q_0 + Z_u)$$

$$a_{32} = M_\alpha + \frac{M_\alpha Z_\alpha}{U_0}$$

$$a_{33} = M_\alpha + M_q$$

$$a_{36} = \frac{-M_\alpha g}{U_0} \cos \Theta_0$$

$$a_{37} = \frac{-Q_0 I_{yz}}{I_y}$$

$$a_{38} = \frac{Q_0 I_{xy}}{I_y}$$

$$a_{43} = 1$$

$$a_{55} = Y_\beta / U_0$$

$$a_{56} = \frac{g}{U_0} \cos \Theta_0$$

$$a_{57} = Y_p / U_0$$

$$a_{58} = \frac{Y_r}{U_0} - 1$$

$$a_{66} = Q_0 \tan \Theta_0$$

$$a_{67} = 1$$

$$a_{68} = \tan \Theta_0$$

$$a_{73} = \frac{2Q_0 I_{yz}}{I_x}$$

$$a_{75} = L_{\beta}$$

$$a_{77} = L_p + Q_0 I_{xz} / I_x$$

$$a_{78} = L_r - Q_0 (I_z - I_y) / I_x$$

$$a_{83} = -2Q_0 I_{xy} / I_z$$

$$a_{85} = N_{\beta}$$

$$a_{87} = N_p - Q_0 (I_y - I_x) / I_z$$

$$a_{88} = N_r - Q_0 I_{xz} / I_z$$

(4) Plant matrix A for combinational maneuver:

$$a_{11} = X_u$$

$$a_{12} = \frac{X_{\alpha}}{U_0} - Q_0$$

$$a_{14} = \frac{-g}{U_0} \cos \Theta_0$$

$$a_{15} = R_0$$

$$a_{21} = Z_u + Q_0$$

$$a_{22} = Z_{\alpha} / U_0$$

$$a_{23} = 1$$

$$a_{24} = \frac{-g}{U_0} \sin \Theta_0 \cos \Phi_0$$

$$a_{25} = -P_0$$

$$a_{26} = \frac{-g}{U_0} \cos \Theta_0 \sin \Phi_0$$

$$a_{31} = U_0 M_u + M_{\alpha} (Q_0 + Z_u)$$

$$a_{32} = M_{\alpha} + \frac{M_{\alpha} Z_{\alpha}}{U_0}$$

$$\begin{aligned}
a_{33} &= M_{\alpha} + M_q + \frac{R_0 I_{xy} - P_0 I_{yz}}{I_y} \\
a_{36} &= \frac{-M_{\alpha} g}{U_0} \cos \Theta_0 \cos \Phi_0 \\
a_{37} &= \frac{-[2P_0 I_{xz} + Q_0 I_{yz} + R_0 (I_x - I_z)]}{I_y} \\
a_{38} &= \frac{2R_0 I_{xz} + Q_0 I_{xy} - P_0 (I_x - I_z)}{I_y} \\
a_{43} &= \cos \Phi_0 \\
a_{46} &= -(Q_0 \sin \Phi_0 + R_0 \cos \Phi_0) \\
a_{48} &= -\sin \Phi_0 \\
a_{51} &= -R_0 \\
a_{52} &= P_0 \\
a_{54} &= \frac{-g}{U_0} \sin \Theta_0 \sin \Phi_0 \\
a_{55} &= Y_{\beta} / U_0 \\
a_{56} &= \frac{g}{U_0} \cos \Theta_0 \cos \Phi_0 \\
a_{57} &= Y_p / U_0 \\
a_{58} &= \frac{Y_r}{U_0} - 1 \\
a_{63} &= \sin \Phi_0 \tan \Theta_0 \\
a_{66} &= (Q_0 \cos \Phi_0 - R_0 \sin \Phi_0) \tan \Theta_0 \\
a_{67} &= 1 \\
a_{68} &= \cos \Phi_0 \tan \Theta_0 \\
a_{73} &= \frac{2Q_0 I_{yz} + P_0 I_{xz} - R_0 (I_z - I_y)}{I_x} \\
a_{75} &= L_{\beta} \\
a_{77} &= L_p + \frac{Q_0 I_{xz} - R_0 I_{xy}}{I_x} \\
a_{78} &= L_r - \frac{2R_0 I_{xz} + P_0 I_{xy} + Q_0 (I_z - I_y)}{I_x}
\end{aligned}$$

$$a_{83} = \frac{-[2Q_0I_{xy} + R_0I_{xz} + P_0(I_y - I_z)]}{I_z}$$

$$a_{85} = N_\beta$$

$$a_{87} = N_p + \frac{2P_0I_{xy} - Q_0(I_y - I_x)}{I_z}$$

$$a_{88} = N_r - \frac{Q_0I_{xz} - P_0I_{yz}}{I_z}$$

APPENDIX F
RESULTING PORS

D% = percentage of control surface area that is lost

H.T. = horizontal tail

V.T. = vertical tail

W. = wings

TABLE IV
STRAIGHT AND LEVEL FLIGHT

D%	0.00	6.25	12.50	18.75	25.00	31.25	37.50	43.75	50.00
H.T.	4.58	4.70	4.83	4.95	5.14	5.66	6.23	7.52	9.08
V.T.	4.58	4.63	4.71	4.80	4.95	5.20	5.55	5.98	6.58
W.	4.58	4.84	5.17	5.75	6.35	7.15	7.91	8.80	9.86

TABLE V
SYMMETRICAL PULL-UP ($\theta_0 = 15^\circ$)

D%	0.00	6.25	12.50	18.75	25.00	31.25	37.50	43.75	50.00
H.T.	4.62	4.74	4.89	5.00	5.18	5.70	6.29	7.65	9.13
V.T.	4.62	4.68	4.75	4.82	5.00	5.25	5.58	6.05	6.62
W.	4.62	4.92	5.29	5.90	6.60	7.32	8.10	8.96	9.94

TABLE VI
STEADY LEVEL TURN ($\Phi_0 = 15^\circ$)

D%	0.00	6.25	12.50	18.75	25.00	31.25	37.50	43.75	50.00
H.T.	4.58	4.72	4.87	4.98	5.17	5.67	6.26	7.61	9.12
V.T.	4.58	4.64	4.71	4.85	4.98	5.25	5.56	6.00	6.59
W.	4.58	4.93	5.27	5.85	6.50	7.21	7.98	8.93	9.94

TABLE VII
COMBINATIONAL MANEUVER
($\Theta_0 = 15^\circ$, $\Phi_0 = 15^\circ$)

D%	0.00	6.25	12.50	18.75	25.00	31.25	37.50	43.75	50.00
H.T.	4.62	4.75	4.90	5.08	5.30	5.80	6.39	7.74	9.24
V.T.	4.62	4.65	4.75	4.90	5.18	5.58	6.01	6.42	7.03
W.	4.62	4.98	5.36	6.06	6.73	7.45	8.23	9.00	9.96

TABLE VIII
STEADY LEVEL TURN ($\Phi_0 = 30^\circ$)

D%	0.00	6.25	12.50	18.75	25.00	31.25	37.50	43.75	50.00
H.T.	4.60	4.74	4.87	5.05	5.28	5.78	6.35	7.70	9.18
V.T.	4.60	4.65	4.71	4.90	5.15	5.55	5.99	6.38	7.01
W.	4.60	5.02	5.48	6.18	6.90	7.63	8.36	9.16	10.0

TABLE IX
COMBINATIONAL MANEUVER
($\Theta_0 = 30^\circ$, $\Phi_0 = 30^\circ$)

D%	0.00	6.25	12.50	18.75	25.00	31.25	37.50	43.75	50.00
H.T.	4.91	5.00	5.19	5.36	5.60	6.21	6.80	8.20	9.80
V.T.	4.91	4.95	5.10	5.25	5.45	5.80	6.20	6.75	7.62
W.	4.91	5.40	5.90	6.50	7.15	7.90	8.70	9.52	10.0

APPENDIX G
MATLAB PROGRAM FOR THE OPTIMAL
PILOT MODEL

```
% fname: PILOT.M
% This program is for the analysis of pilot opinion ratings
%   using an optimal control model(OCM).
% This should be excuted externally by the user.

HEAD;
while 1
  MAINMENU;
  if MenuNo==1
    INPUTSYS;
  elseif MenuNo==2
    OPTIMAL;
  elseif MenuNo==3
    FDOMAIN;
  elseif MenuNo==4
    TDOMAIN;
  elseif MenuNo==5
    CUTOFF;
  elseif MenuNo==6
    RESULTS;
  elseif MenuNo==7
    OUTPUT;
  elseif MenuNo==8
    break;
  end
  clear MenuNo;
end
```

```

% fname: HEAD.M
% general description of this program.

clc
fprintf('\n -----
--')
fprintf('-----\n')
fprintf('      Jounguk Kim\n')
fprintf('      Oklahoma State University\n')
fprintf('      Pilot Opinion Ratings Using an Optimal Pilot
Model\n')
fprintf('-----
--')
fprintf('---\n\n')
fprintf('      - This program was developed for understanding and
\n')
fprintf('      interpretation of the OCM (Optimal Control Model)
\n')
fprintf('      results in the pilot opinion ratings by using Matlab
\n')
fprintf('      program.\n')
fprintf('      - A clear comparision between the OCM and the
describing \n')
fprintf('      transfer function of the human pilot is possible
\n')
fprintf('      in this program.\n')
fprintf('      - The estimation of the pilot opinion ratings \n')
fprintf('      (Cooper-Harper rating system) is also possible \n')
fprintf('      in both time domain and frequency domain.\n\n')
fprintf('> Hit any key to continue : ')
pause;

```



```
% fname: INPUTSYS.M
% Input system equation
% (transfer function or state space equation)

while 1
    clc;
    fprintf('\n\n\n\n')
    fprintf('                                S Y S T E M   E Q U A T I O
N\n\n\n\n\n\n')
    fprintf('                                1. Transfer function \n')
    fprintf('                                2. State space equation \n')
    fprintf('                                3. quit \n\n\n\n')
    No=input('> Select a number : ');
    if No==1
        TRANSF;
    elseif No==2
        SYSEQN;
    elseif No==3
        break;
    end
end
end
```

```

% fname: TRANSFER.M
% input data for the vehicle and filter transfer function
%
% 1)  $y(s) = G_s(s)u(s) + G_w(s)w(s)$ 
% 2)  $y(s) = -G_s(s)u(s) + G_w(s)w(s)$ 
%    $G_s(s)$ : vehicle transfer function
%    $G_w(s)$ : filter transfer function
%    $u(s)$  : optimal input
%    $w(s)$  : white noise
%    $y(s)$  : error to the command signal

yes=1;
while (yes)
    clear;
    clc;
    fprintf('-----\n')
    fprintf('  SYSTEM TRANSFER FUNCTION\n')
    fprintf('-----\n')
    fprintf('  1)  $y(s) = G_s(s)u(s) + G_w(s)w(s)$ \n')
    fprintf('  2)  $y(s) = -G_s(s)u(s) + G_w(s)w(s)$  \n')
    fprintf('       $G_s(s)$  : Vehicle transfer function\n')
    fprintf('       $G_w(s)$  : Filter transfer function\n\n');
    fprintf('> Select a number : ')
    case=input(' ');
    fprintf('> IMPORTANT:  $G_w(s)$  should be at least 2nd order ? (y/n)')
    check=input(' ', 's');
    if check=='n' | check=='N'
        fprintf('> Use state space form.\n\n')
        fprintf('> Hit any key to continue : ')
        pause;
        break;
    end
    fprintf('> Enter num of  $G_s(s)$  in [ descending powers of s ] :')
    nSystem=input(' ');
    fprintf('> Enter den of  $G_s(s)$  in [ descending powers of s ] :')
    dSystem=input(' ');
    fprintf('> Enter num of  $G_w(s)$  in [ descending powers of s ] :')
    nFilter=input(' ');
    fprintf('> Enter den of  $G_w(s)$  in [ descending powers of s ] :')
    dFilter=input(' ');
    fprintf('> Input data is correct ? (y/n) : ')
    s=input(' ', 's');
    fprintf('- processing...')
    if s=='Y'|s=='y'
        yes=0;
    else
        yes=1;
    end
end
end
while(check=='Y' | check=='y')
    % zeros and poles of Gsystem, Gfilter
    [zSystem,pSystem,gSystem]=tf2zp(nSystem,dSystem);
    [zFilter,pFilter,gFilter]=tf2zp(nFilter,dFilter);
    % transfer function to state space equation (system eqn)
    [As,BEs,Cs,Ds]=GETSS(nSystem,dSystem,nFilter,dFilter,case);
    [A,Bm,C,Dm]=minreal(As,BEs,Cs,Ds);
    B=Bm(:,1);
    E=Bm(:,2);

```

```

D=Dm(:,1);
% original state eqn matrix coefficients
Ao=As;
Bo=BES(:,1);
Eo=BES(:,2);
Co=Cs;
Do=Ds(:,1);
size1=max(size(A));
size2=max(size(Ao));
clc;
fprintf('-----\n')
fprintf('  STATE SPACE EQUATION \n')
fprintf('-----\n')
fprintf('  dx(t) = Ao x(t) + Bo u(t) + Eo w(t)\n')
fprintf('  y(t)  = Co x(t) + Do u(t)\n\n')
Ao
fprintf('\n> Hit any key to continue : ')
pause;
fprintf('\n\n')
Bo
fprintf('\n> Hit any key to continue : ')
pause;
fprintf('\n\n')
Eo
fprintf('\n> Hit any key to continue : ')
pause;
fprintf('\n\n')
Co
fprintf('\n> Hit any key to continue : ')
pause;
fprintf('\n\n')
Do
fprintf('\n> Hit any key to continue : ')
pause;
if size1 < size2
  clc;
  fprintf('-----\n')
  fprintf('  MINIMAL STATE SPACE EQUATION \n')
  fprintf('-----\n')
  fprintf('  dx(t) = A x(t) + B u(t) + E w(t)\n')
  fprintf('  y(t)  = C x(t) + D u(t)\n\n')
  A
  fprintf('\n> Hit any key to continue : ')
  pause;
  fprintf('\n\n')
  B
  fprintf('\n> Hit any key to continue : ')
  pause;
  fprintf('\n\n')
  E
  fprintf('\n> Hit any key to continue : ')
  pause;
  fprintf('\n\n')
  C
  fprintf('\n> Hit any key to continue : ')
  pause;
  fprintf('\n\n')
  D

```

```
fprintf('\n> Hit any key to continue : ')\npause;\nelse\nA=Ao; B=Bo; C=Co; D=Do; E=EO;\nend\nbreak;\nend\nclear size1 size2 s yes check Bm Dm As BEs Cs Ds;\nsave dumfile;
```

```

% fname: GETSS.M
% from transfer function to state space equation

function [Ac,BEc,Cc,Dc]=GETSS(nSystem,dSystem,nFilter,dFilter,case)
[A1,B1,C1,D1]=SS2(nFilter,dFilter);
[A2,B2,C2,D2]=SS2(nSystem,dSystem);
sizeF=size(A1);
n1=sizeF(1,1);
sizeS=size(A2);
n2=sizeS(1,1);
Ac=[A1,zeros(n1,n2);
    zeros(n2,n1),A2];
Bc=[zeros(n1,1);B2];
Ec=[B1;zeros(n2,1)];
BEc=[Bc,Ec];
if n1>1 & n2>1
    if case==1
        Cc=[eye(2),zeros(2,n1-2),eye(2),zeros(2,n2-2)];
    elseif case==2
        Cc=[eye(2),zeros(2,n1-2),-eye(2),zeros(2,n2-2)];
    end
    Dc=[zeros(2)];
elseif n1>1 & n2==1
    if case==1
        Cc=[eye(2),zeros(2,n1-2),[1,A2]'];
        Dc=[0 0;B2 0];
    elseif case==2
        Cc=[eye(2),zeros(2,n1-2),[-1,-A2]'];
        Dc=[0 0;-B2 0];
    end
end
end
% [A, Bm, C, Dm]=minreal(Ac,BEc,Cc,Dc);
%B=Bm(:,1);
%E=Bm(:,2);
%D=Dm(:,1);

```

```

% fname: SS2.m
% second canonical form (phase variable)

function [A,B,C,D]=ss2(num,den)
sizeN=size(num);
n1=sizeN(1,2);
sizeD=size(den);
n2=sizeD(1,2);
if n1>n2,..
    fprintf('Error : order of numerator is greater than denominator\n');..
    break;..
end
if n2==2
    A=-den(2)/den(1);
    B=num/den(1);
    C=1;
    D=0;
else
    A=zeros(n2-1);
    A(1:n2-2,2:n2-1)=eye(n2-2);
    A(n2-1,:)=-den(n2:-1:2);
    if n2>n1,..
        NUM=zeros(1,n2);..
        NUM(1,n2-n1+1:n2)=num;..
    else,..
        NUM=num;..
    end
    end
    b=zeros(n2,1);
    b(1)=NUM(1);
    for i=2:n2;..
        for j=1:i-1,..
            b(i)=-den(1+j)*b(i-j)+b(i);..
        end,..
        b(i)=b(i)+NUM(i);..
    end
    B=b(2:n2,1);
    C=zeros(1,n2-1);
    C(1,1)=1;
    D=b(1);
end
end

```

```

% filename: SYSEQN.M
% input system equation as state space form

yes=1;
while(yes)
    clear;
    clc;
    fprintf('-----\n')
    fprintf(' STATE SPACE EQUATION\n')
    fprintf('-----\n')
    fprintf(' dx(t) = Ao x(t) + Bo u(t) + Eo w(t) \n')
    fprintf(' y(t) = Co x(t) + Do u(t) \n\n')
    fprintf('> Enter Ao matrix : ')
    Ao=input(' ');
    fprintf('> Enter Bo matrix : ')
    Bo=input(' ');
    fprintf('> Enter Eo matrix : ')
    Eo=input(' ');
    fprintf('> Enter Co matrix : ')
    Co=input(' ');
    fprintf('> Enter Do matrix : ')
    Do=input(' ');
    fprintf('> Input data is correct ? (y/n) : ')
    s=input(' ', 's');
    if s=='Y'|s=='y'
        yes=0;
    else
        yes=1;
    end
end
end
BEO=[Bo,Eo];
DEO=[Do,zeros(2,1)];
[A,Bm,C,Dm]=minreal(Ao,BEO,Co,DEO);
B=Bm(:,1);
E=Bm(:,2);
D=Dm(:,1);
size1=max(size(A));
size2=max(size(Ao));
if size1 < size2
    clc;
    fprintf('-----\n')
    fprintf(' REDUCED STATE SPACE EQUATION \n')
    fprintf('-----\n')
    fprintf(' dx(t) = A x(t) + B u(t) + E w(t) \n')
    fprintf(' y(t) = C x(t) + D u(t)\n\n')
    A
    fprintf('\n> Hit any key to continue : ')
    pause;
    fprintf('\n\n')
    B
    fprintf('\n> Hit any key to continue : ')
    pause;
    fprintf('\n\n')
    E
    fprintf('\n> Hit any key to continue : ')
    pause;
    fprintf('\n\n')
    C

```

```

fprintf('\n> Hit any key to continue : ')
pause;
fprintf('\n\n')
D
fprintf('\n> Hit any key to continue : ')
pause;
end
[nSys,dSys]=ss2tf(A,B,C(1,:),D(1,:),1);
[nFil,dFil]=ss2tf(A,E,C(1,:),zeros(1),1);
[nSystem,dSystem]=minreal(nSys,dSys);
[nFilter,dFilter]=minreal(nFil,dFil);
[zSystem,pSystem,gSystem]=tf2zp(nSystem,dSystem);
[zFilter,pFilter,gFilter]=tf2zp(nFilter,dFilter);
if gSystem > 0
    case=1;
else
    case=2;
end
clc;
fprintf('-----\n')
fprintf('  TRANSFER FUNCTION  \n')
fprintf('-----\n')
fprintf(' y(s)= Gs(s)u(s) + Gw(s)w(s)\n')
fprintf(' Gs(s) : Vehicle transfer function\n')
fprintf(' Gw(s) : Filter transfer function\n\n')
gSystem
fprintf('\n> Hit any key to continue : ')
pause;
fprintf('\n\n')
zSystem
fprintf('\n> Hit any key to continue : ')
pause;
fprintf('\n\n')
pSystem
fprintf('\n> Hit any key to continue : ')
pause;
fprintf('\n\n')
gFilter
fprintf('\n> Hit any key to continue : ')
pause;
fprintf('\n\n')
zFilter
fprintf('\n> Hit any key to continue : ')
pause;
fprintf('\n\n')
pFilter
fprintf('\n> Hit any key to continue : ')
pause;
clear yes s BEO DEO Bm Dm size1 size2 nSys dSys nFil dFil;
save dumfile;

```



```
% fname: OPTIMAL.M
% This is a menu for the calculation of optimal control model.

while 1
    clc;
    fprintf('\n\n\n')
    fprintf('          C O M P U T A T I O N   F L O W\n\n\n')
    fprintf('          1. solve L Q R\n')
    fprintf('          2. solve K B F/linear predictor\n')
    fprintf('          3. quit \n\n\n')
    No=input('> Select a number : ');
    if No==1
        SOLVELQR;
    elseif No==2
        SOLVEKBF;
    elseif No==3
        break;
    end
end
end
```

```

% fname: SOLVELQR.M
% This is to solve the Linear Quadratic Regulator problem.
%
% augmented system matrix : A0, B0
% weighting matrix : Q0 for xhat, Q for yhat

while 1
    clc;
    fprintf('-----\n')
    fprintf('  LINEAR QUADRATIC REGULATOR \n')
    fprintf('-----\n')
    fprintf(' J(u) = E[ q1 y1(t)^2 + q2 y2(t)^2 + G udot(t)^2 ] : ')
    fprintf('cost function\n')
    fprintf(' y1(t) y2(t) : error and error rate for single axis\n')
    fprintf(' udot(t)      : optimal pilot control rate\n')
    fprintf(' q1, q2       : weights for error and error rate\n')
    fprintf(' G           : weight for control rate \n\n')
    fprintf('> Enter the weights Q for y(t) in [q1 q2] : ')
    q=input(' ');
    fprintf('> Enter the neuro-muscular time constant (Tn) : ')
    Tn=input(' ');
    fprintf('- Recommended interval to solve G : [ 0.000001 1 ]\n')
    fprintf('> Enter the initial interval for G in [ g1 g2 ] : ')
    intvG=input(' ');
    fprintf('- Processing...')
    Q=[q(1,1)  0      ;
        0      q(1,2)];
    G1=intvG(1);
    G2=intvG(2);
    sizeA=size(A);
    ns=sizeA(1,1);% no of states
    A0=zeros(ns+1);
    A0(1:ns,:)=A,B;
    B0=zeros(ns+1,1);
    B0(ns+1,1)=1;
    Q0=zeros(ns+1);
    Q0(1:ns,1:ns)=C'*Q*C;
    for i=1:ns
        for j=1:ns
            if abs(Q0(i,j))<0.000000000001
                Q0(i,j)=0;
            end
        end
    end
end
[L,K0]=LQR(A0,B0,Q0,G1);
F1=1/L(ns+1)-Tn;
[L,K0]=LQR(A0,B0,Q0,G2);
F2=1/L(ns+1)-Tn;
if F1*F2<0
    fprintf('\n- Initial interval for G is O.K.\n')
    fprintf('> processing')
    break;
else
    fprintf('- Initial interval of G is NOT appropriate for bisection')
    fprintf(' method.\n\n')
    fprintf('> Hit any key to continue : ')
    pause;
end
end

```

```
end
while abs((G1-G2)/G1)>0.001
  [L,K0]=LQR(A0,B0,Q0,G1);
  F1=1/L(ns+1)-Tn;
  G3=(G1+G2)/2;
  [L,K0]=LQR(A0,B0,Q0,G3);
  F3=1/L(ns+1)-Tn;
  if F1*F3<0
    G2=G3;
    fprintf('.')
  else
    G1=G3;
    fprintf('.')
  end
end
end
G=G1;
L1=L(1:ns);
L2=L(ns+1);
fprintf('\n> Hit any key to continue : ')
pause;
clear i j sizeA G1 G2 G3 intvG F1 F2 F3 ;
save dumfile;
```

```

% fname: SOLVEKBF.M
% This is to solve the Kalman Bucy Filter and linear predictor problem.
%
% augmented system matrix : A1, B1, C1 for Kalman Filter
% covariance matrix for noise : Vy, Vua, W

yes=1;
while (yes)
    clc;
    fprintf('-----\n')
    fprintf('  KALMAN BUCY FILTER /LINEAR PREDICTOR\n')
    fprintf('-----\n\n')
    fprintf('> Enter the pure perceptual time delay : ');
    delay=input(' ');
    fprintf('> Enter the covariance of w(t) : ');
    W=input(' ');
    fprintf('> Enter the noise ratios for [y1(t) y2(t) ua(t)] : ');
    ratios=input(' ');
    ratio1=ratios(1); ratio2=ratios(2); ratio3=ratios(3);
    fprintf('> Enter the observation thresholds for [y1(t) y2(t)] : ');
    treshd=input(' ');
    T1=treshd(1); T2=treshd(2);
    fprintf('> Enter the fractional attentions for [y1(t) y2(t)] : ');
    fractn=input(' ');
    f1=fractn(1); f2=fractn(2);
    fprintf('> Input data is correct ? (y/n) : ')
    s=input(' ', 's');
    if s=='Y' | s=='y'
        yes=0;
    else
        yes=1;
    end
end
% initial values for iteration
clc;
fprintf('\n-----\n')
fprintf('  KALMAN BUCY FILTER/LINEAR PREDICTOR\n')
fprintf('-----\n')
fprintf('- processing...')
check1=1;
while (check1)
    fprintf('\n> Do you want to try with default initial values')
    fprintf('\n for [var(y1) var(y2) var(ua)] ? (y/n)')
    s1=input(' ', 's');
    if s1=='Y' | s1=='y'
        vary1=0.1;
        vary2=0.1;
        varua=0.1;
    else
        fprintf('> Enter initial values for [var(y1) var(y2) var(ua)] : ')
        invar=input(' ');
        vary1=invar(1); vary2=invar(2); varua=invar(3);
    end
    fprintf('> processing')
    check2=1;
    check3=0;
    count1=0;
    count2=0;
end

```

```

while (check2)
%construct augmented matrix
    fprintf(' ');
    count1=count1+1;
    count2=count2+1;
    erfc1=1-ERF(T1/(sqrt(2)*sqrt(vary1)));
    erfc2=1-ERF(T2/(sqrt(2)*sqrt(vary2)));
    Vy1=pi*ratio1*vary1/(f1*erfc1^2);
    Vy2=pi*ratio2*vary2/(f2*erfc2^2);
    Vy=[Vy1 0 ;
        0 Vy2];
    Vua=pi*ratio3*varua;
    A1=A0;
    A1(ns+1,ns+1)=-L2;
    B1=zeros(ns+1,1);
    B1(ns+1,1)=L2;
    C1=[C,D];
    W1=zeros(ns+1);
    W1(1:ns,1:ns)=E*W*E';
    E1=eye(ns+1);
    W1(ns+1,ns+1)=L2*Vua*L2';%time domain performance
    [H1,sigma1]=LQE(A1,E1,C1,W1,Vy);
    Lopt=[L1/L2 0];
    [y1]=matitg(A1,W1,delay);
% [y1,co]=simpson('func',0,delay,A1,W1);
    E1=expm(A1*delay)*sigma1*expm(A1'*delay)+y1;
    A3=A1-B1*Lopt;
    W3=expm(A1*delay)*H1*Vy*H1'*expm(A1'*delay);
    Xhat1=lyap(A3,W3);
    X1=E1+Xhat1;
    Y=C1*X1*C1';
    Ua=Lopt*Xhat1*Lopt';
    if abs((vary1-Y(1,1))/max(vary1,Y(1,1))) > 0.01..
        | abs((vary2-Y(2,2))/max(vary2,Y(2,2))) > 0.01
        check2=1;
    elseif abs((varua-Ua)/max(varua,Ua)) > 0.01
        check2=1;
    else
        check1=0;
        check2=0;
        check3=1;
    end
    if count2==1
        temp1=vary1;
        temp2=vary2;
        temp3=varua;
    elseif count2==2;
        temp4=vary1;
        temp5=vary2;
        temp6=varua;
    end
    vary1=Y(1,1);
    vary2=Y(2,2);
    varua=Ua;
    if count2==3
        fprintf('\n iter no :          %g          %g
%g\n',...
            count1-2,count1-1,count1)

```

```

fprintf(' var(y1) :%15.7f%15.7f%15.7f\n',temp1, temp4, vary1)
fprintf(' var(y2) :%15.7f%15.7f%15.7f\n',temp2, temp5, vary2)
fprintf(' var(ua) :%15.7f%15.7f%15.7f\n',temp3, temp6, varua)
count2=count2-3;
fprintf('\n> Do you want more iterations ? (y/n)')
s2=input(' ','s');
if s2=='N' | s2=='n'
    check1=0;
    check2=0;
else
    fprintf('> Try with another set of initial values ? (y/n)')
    s3=input(' ','s');
    if s3=='Y' | s3=='y'
        fprintf('> Enter initial values : [vary1 vary2 varua]');
        invar=input(' ');
        vary1=invar(1); vary2=invar(2); varua=invar(3);
    end
    fprintf('> processing')
end
end
end
end
end
if check3==1
    fprintf('\n- K B F/linear predictor was solved.')
    fprintf('\n> Hit any key to continue : ')
    pause;
    xlsize=size(X1);
    n1=xlsize(1,1);
    U=X1(n1,n1);
    Uhatdot=L*Xhat1*L'+[zeros(1:ns),L2]*E1*[zeros(1:ns),L2]';
    J=trace(Y*Q)+trace(Uhatdot*G);
end
clear yes s s1 s2 s3 i j ratios treshd fractn invar count1 count2;
clear erfc1 erfc2 sizeA xlsize n1;
clear check1 check2 check3 vary1 vary2 varua temp1 temp2 temp3;
save dumfile;

```

```
% fname: MATITG.M
% matrix integral with time delay

function [y1]=matitg(A,W,delay)
Asize=size(A);
n=Asize(1,1);
[P,A1]=eig(A);
W1=inv(P)*W*(inv(P))';
for i=1:n
    for j=1:n
        temp=A1(i,i)+A1(j,j);
        if abs(temp) > 0.00001
            Jd(i,j)=W1(i,j)/(temp)*(exp(temp*delay)-1);
        else
            Jd(i,j)=W1(i,j)*delay;
        end
    end
end
end
y1=real(P*Jd*P');
```

```

% fname: FDOMAIN.M
% to find pilot transfer function

% 4th order Pade approximation of the delay
clc;
fprintf('-----\n')
fprintf('  FREQUENCY DOMAIN PERFORMANCE \n')
fprintf('-----\n')
fprintf('- Processing...')
[nPade,dPade]=pade4(delay);
[zPade,pPade,gPade]=tf2zp(nPade,dPade);
[Ap,Bp,Cp,Dp]=tf2ss(nPade,dPade);
A2=zeros(2*ns+7);
A2(1:ns+1,1:2*ns+6)=[A1-H1*C1,-B1*Dp*Lopt,B1*Cp];
A2(ns+2:2*ns+2,1:2*ns+2)=[-expm(A1*delay)*H1*C1,A1-B1*Lopt];
A2(2*ns+3:2*ns+6,ns+2:2*ns+6)=[-Bp*Lopt, Ap];
A2(2*ns+7,ns+2:2*ns+7)=[-L2*Dp*Lopt,L2*Cp,-L2];
B2=zeros(2*ns+7,2);
B2(1:2*ns+2,1:2)=[H1;expm(A1*delay)*H1];
C2=zeros(1,2*ns+7);
C2(1,2*ns+7)=1;
E2=zeros(2*ns+5,1);
E2(2*ns+7,1)=L2;
A3=A2;
B3=B2(:,1)+A2*B2(:,2);
C3=C2;
D3=C2*B2(:,2);
% pilot transfer function
[nPilot,dPilot]=ss2tf(A3,B3,C3,D3,1);
if case==1
    [zPilot,pPilot,gPilot]=tf2zp(-nPilot,dPilot);
elseif case==2
    [zPilot,pPilot,gPilot]=tf2zp(nPilot,dPilot);
end
[nPilotM,dPilotM]=minreal(nPilot,dPilot);
[zPilotM,pPilotM,gPilotM]=tf2zp(nPilotM,dPilotM);
n1=1;
d1=1;
if case==1
    n2=-nPilotM;
elseif case==2
    n2=nPilotM;
end
d2=dPilotM;
n3=nSystem;
d3=dSystem;
n4=-1;
d4=1;
nblocks=4;
BLKBUILD
q=[1 0 0;
    2 1 4;
    3 2 0;
    4 3 0];
iu=[1];
iy=[3];
[Ac1 Bc1 Cc1 Dc1]=CONNECT(a,b,c,d,q,iu,iy);
fprintf('- Processing...');

```



```
omega=logspace(-2,2,100);
if case==1
    [mag1,phase1]=bode(-nPilot,dPilot,omega);
elseif case==2
    [mag1,phase1]=bode(nPilot,dPilot,omega);
end
[mag2,phase2]=bode(nSystem,dSystem,omega);
[mag4,phase4]=bode(Ac1,Bc1,Cc1,Dc1,1,omega);
lmag1=20*log10(mag1);
lmag2=20*log10(mag2);
lmag3=lmag1+lmag2;
phase3=phase1+phase2;
lmag4=20*log10(mag4);
if phase1(1)==180
    phase1=phase1-360;
end
if phase2(1)==180
    phase2=phase2-360;
    phase3=phase3-360;
end
%for ascii data
d1=[omega',lmag1,lmag2,lmag3,lmag4,mag4];
d2=[omega',phase1,phase2,phase3,phase4];
clear nblocks n1 n2 n3 n4;
save dumfile;
save a.1 d1 /ascii /tabs
save a.2 d2 /ascii /tabs
fprintf('\n- Frequency domain performance was solved.\n')
fprintf('> Hit any key to continue : ');
pause
```

```

% fname: PADE.M
% 4th order Pade approximation for the pure time delay

function [nu,de]=pade4(delay)
A1=[1 -delay/2 (delay^2)/(2*3) -(delay^3)/(2*3*4);
    1 -delay/3 (delay^2)/(3*4) -(delay^3)/(3*4*5);
    1 -delay/4 (delay^2)/(4*5) -(delay^3)/(4*5*6);
    1 -delay/5 (delay^2)/(5*6) -(delay^3)/(5*6*7)];
b1=-(delay^4)*[1/(2*3*4*5) 1/(3*4*5*6) 1/(4*5*6*7) 1/(5*6*7*8)];
de=[inv(A1)*b1;1];
A2=[1 -delay (delay^2)/2 -delay^3/(3*2);
    0 1 -delay (delay^2)/2 ;
    0 0 1 -delay ;
    0 0 0 1 ];
b2=[(delay^4)/(4*3*2) -(delay^3)/(3*2) (delay^2)/(2) -delay]';
nu=[A2*de(1:4)+b2;1];
de=de';
nu=nu';

```

```

% fname: TDOMAIN.M
% This is for the time domain performance of OCM.

clc;
fprintf('-----\n')
fprintf('  STEP RESPONSE \n')
fprintf('-----\n')
fprintf('> Enter time interval for step response : ')
dt=input(' ');
fprintf('> Enter final time for step response : ')
tf=input(' ');
fprintf('- Processing...')
t5=(0:dt:tf)';
size5=max(size(t5));
u5=ones(size5,1);
iu=[1];
iy=[2];
[Ac2,Bc2,Cc2,Dc2]=connect(a,b,c,d,q,iu,iy);
[y5,x5]=lsim(Ac1,Bc1,Cc1,Dc1,u5,t5);
clear x5
[y6,x5]=lsim(Ac2,Bc2,Cc2,Dc2,u5,t5);
clear x5
plot(t5,y5),xlabel('time (sec)'),ylabel('y(t)'),grid
title('Step Input Response')
pause;
plot(t5,y6),xlabel('time (sec)'),ylabel('u(t) (pilot input)');
title('Step Input Response')
pause
[overshoot,i]=max(y5);
peaktime=t5(i);
[risemag1,j1]=min(abs(y5(1:i)-1));
risetime1=t5(j1);
[risemag2,j2]=min(abs(y5(1:j1)-0.8));
risetime2=t5(j2);
d3=[t5,y5,y6];
save a.3 d3 /ascii /tabs
clear iu iy tf dt x5 u5 size5 i j1 j2 risemag1 risemag2
fprintf('\n> Hit any key to continue : ');
pause

```

```

% fname: CUTOFF.M
% Bode ideal cutoff analysis
clc;
fprintf('-----\n');
fprintf(' BODE IDEAL CUTOFF ANALYSIS\n');
fprintf('-----\n');
fprintf('- Find the break frequency for disturbance and sensor
noise.\n');
fprintf('> Hit any key to continue : ');
pause;
axis('square')
semilogx(omega,lmag1),grid,title('Pilot transfer function');
xlabel('w (rad/sec)'),ylabel('magnitude (dB)');
pause;
fprintf('\n- Break frequency for disturbance \n')
fprintf('> Enter No (local max=1, local min=2, skip=3) : ')
no=input(' ');
if no==1 | no==2
    fprintf('> Enter subinterval for disturbance break freq : [w1 w2]');
    rangel=input(' ');
    [m1,Wd]=maxmin1(nPilot,dPilot,rangel,no);
end
[mSystem1,pSystem1]=bode(nSystem,dSystem,Wd);
mPilot1=20*log10(m1);
feedback=mPilot1+20*log10(mSystem1);
fprintf('> Hit any key to continue : ');
pause;
semilogx(omega,lmag1),grid,title('Pilot transfer function');
xlabel('w (rad/sec)'),ylabel('magnitude (dB)');
pause;
fprintf('\n- Break frequency for sensor noise \n')
fprintf('> Enter No (local max=1, local min=2, skip=3) : ')
no=input(' ');
if no==1 | no==2
    fprintf('> Enter subinterval for sensor noise break freq : [w1 w2]');
    range2=input(' ');
    fprintf('- Processing...');
    [m2,Wn]=maxmin1(nPilot,dPilot,range2,no);
end
[m21,p21,Wn]=bode(nSystem,dSystem,Wn);
[m31,p31,Wn]=bode(nPilot,dPilot,Wn);
mPilot2=20*log10(m2);
mPS=mPilot2+20*log10(m21);
pPS=p21+p31;
[m3,i]=max(mag4);
range3=[omega(i-1) omega(i+1)];
[peak,Wp]=maxmin2(Ac1,Bc1,Cc1,Dc1,range3,1);
logpeak=20*log10(peak);
range4=[0.001 Wp];
[m4,Wdr]=maxmin2(Ac1,Bc1,Cc1,Dc1,range4,2);
droop=1-m4;
ldroop=20*log10(m4);
[Gm,Pmargin,Wcg,Wcp]=margin(mag1.*mag2,phase3,omega);
Gmargin=20*log10(Gm);
% Bode Ideal Cutoff Analysis
x=Gmargin;
y=Pmargin/180;
Wl=Wcp/Wd;

```

```
W3=Wn/Wd;
slope=-12*(1-y);
highpole=1+abs(max(size(nSystem))-max(size(dSystem)));
%W2=W1*2^(-x/slope);
W2=2*(1-y)*W3/highpole;
Wc=W2*Wd;
maxFB=12*(1-y)*(1+log10(W2)/log10(2))-x;
percentFB=feedback/maxFB*100;
save dumfile;
clear range1 range2 range3 range4 m1 m2 m3 m4 i
fprintf('\n- Cutoff analysis was done.\n');
fprintf('> Hit any key to continue : ');
pause
```

```
% fname: MAXMIN1.M
% Local minimum and maximum

function [m,w]=maxmin1(num,den,range,case)
yes=1;
while(yes)
    omega=range(1):(range(2)-range(1))/20:range(2);
    [mag,phase]=bode(num,den,omega);
    if case==1
        [m,i]=max(mag);
    elseif case ==2
        [m,i]=min(mag);
    end
    w=omega(i);
    if (omega(i+1)-omega(i))/omega(i)<0.05
        yes=0;
    end
    range=[omega(i-1) omega(i+1)];
end
```

```
% fname: MAXMIN2.M
% Local maximum and minimum

function [m,w]=maxmin2(A,B,C,D,range,case)
yes=1;
while(yes)
    omega=range(1):(range(2)-range(1))/20:range(2);
    [mag,phase]=bode(A,B,C,D,1,omega);
    if case==1
        [m,i]=max(mag);
    elseif case ==2
        [m,i]=min(mag);
    end
    w=omega(i);
    if (omega(i+1)-omega(i))/omega(i)<0.05
        yes=0;
    end
    range=[omega(i-1) omega(i+1)];
end
```

```
% fname: RESULTS.M
% Results menu for time domain and frequency domain performance

while 1
    clc;
    fprintf('\n\n\n')
    fprintf('
                                     R E S U L T\n\n\n')
    fprintf('
1. Check input data\n')
    fprintf('
2. Time/frequency domain performance \n')
    fprintf('
3. Pilot transfer function\n')
    fprintf('
4. Bode plot \n')
    fprintf('
5. quit
                                     \n\n\n\n')
    No=input('> Select a number : ');
    if No==1
        CHECKIN;
    elseif No==2
        TFRRESULT;
    elseif No==3
        PTF;
    elseif No==4
        BODEPLOT;
    elseif No==5;
        break;
    end
end
end
```



```

% fname: CHECKIN.M
% check input data is correct or not to confirm results

clc;
fprintf('- SYSTEM TRANSFER FUNCTION : Gs(s)')
fprintf('\n Gain : %g\n',gSystem)
fprintf(' Zeros : \n')
s=size(zSystem);
imax=s(1,1);
for i=1:imax,...
    fprintf('          (%15.7f) + (%15.7f)j\n',real(zSystem(i)),...
        imag(zSystem(i)));..
end
s=size(pSystem);
imax=s(1,1);
fprintf(' Poles : \n')
for i=1:imax,...
    fprintf('          (%15.7f) + (%15.7f)j\n',real(pSystem(i)),...
        imag(pSystem(i)));..
end
fprintf('\n- FILTER TRANSFER FUNCTION : Gw(s)')
fprintf('\n')
fprintf(' Gain : %g\n',gFilter)
fprintf(' Zeros : \n')
s=size(zFilter);
imax=s(1,1);
for i=1:imax,...
    fprintf('          (%15.7f) + (%15.7f)j\n',real(zFilter(i)),...
        imag(zFilter(i)));..
end

```

```

fprintf(' Poles : \n')
s=size(pFilter);
imax=s(1,1);
for i=1:imax,..
    fprintf('          (%15.7f) + (%15.7f)j\n',real(pFilter(i)),...
        imag(pFilter(i)));..
end
fprintf('\n\n> Hit any key to continue : ');
pause
clc
fprintf('\n- INPUT PARAMETERS FOR OCM\n\n')
fprintf(' Covariance of disturbance (W)      : %g\n',W)
fprintf(' Objective function weights (Q)      : [ %g %g ]\n',...
        Q(1,1),Q(2,2))
fprintf(' Neuromuscular lag (Tn)              : %g\n',Tn)
fprintf(' Observation delay (Td)               : %g\n',delay)
fprintf(' Observation noise ratio (r1)         : %g\n', ratio1)
fprintf(' Neuromuscular noise ratio (r2)      : %g\n', ratio3)
fprintf(' Observation threshold (T1,T2)       : [ %g %g ]\n',T1,T2)
fprintf(' Fractional attention (f1,f2)       : [ %g %g ]\n\n\n',...
        f1,f2)
fprintf('> Hit any key to continue : ');
pause;

```

```

% fname: TRESULTS.M
% Results of time and freq domain performance on the screen

clc;
fprintf('\n- TIME DOMAIN PERFORMANCE\n\n')
fprintf(' Objective function weight (G)      : %g\n',G)
fprintf(' Covariance of u          (U)         : %g\n',U)
fprintf(' Covariance of ye          (y1,y2)      : %g %g\n',...
      Y(1,1),Y(2,2))
fprintf(' Covariance of ua          (Ua)        : %g\n',Ua)
fprintf(' Covariance of uhatdot     (Udot)      : %g\n',Uhatdot)
fprintf(' Optimal cost              (Jopt)     : %g\n',J)
fprintf(' Overshoot                 (os)       :
%g\n',overshoot)
fprintf(' Peak time                 (pt)       :
%g\n',peaktime)
fprintf(' Rise time at 100 percent  (rt1)    : %g\n',risetime1)
fprintf(' Rise time at 80 percent   (rt2)    : %g\n\n\n',risetime2)
fprintf('> Hit any key to continue : ');
pause;
clc;
fprintf('\n- FREQUENCY DOMAIN PERFORMANCE\n\n')
fprintf(' Gain margin              (Gm)       : %g\n',Gmargin)
fprintf(' Crossover frequency      (Wcg)     : %g\n',Wcg)
fprintf(' Phase margin            (Pm)       : %g\n',Pmargin)
fprintf(' Crossover frequency      (Wcp)     : %g\n',Wcp)
fprintf(' Feedback                 : %g\n',feedback)
fprintf(' Working band            (Wd)       : %g\n',Wd)
fprintf(' Pilot gain at Wd        : %g\n',mPilot1)
fprintf(' Noise cutoff frequency  (Wn)      : %g\n',Wn)
fprintf(' Pilot gain at Wn        : %g\n',mPilot2)
fprintf(' Pilot/System gain at Wn : %g\n',mPS)
fprintf(' Pilot/System phase at Wn : %g\n',pPS)
fprintf(' Resonant peak          : %g ( %g dB)\n',...
      peak, logpeak)
fprintf(' Peak frequency          (Wp)       : %g\n',Wp)
fprintf(' Droop                   : %g ( %g dB)\n\n',...
      droop, ldroop)
fprintf('> Hit any key to continue : ');
pause;
clc;
fprintf('\n- BODE IDEAL CUTOFF ANALYSIS\n\n')
fprintf(' Maximum available feedback : %g\n',maxFB)
fprintf(' Percent feedback at Wb     : %g\n',percentFB)
fprintf(' Gain slope at Wcp (dB/octave) : %g\n',slope)
fprintf(' Ideal phase crossover freq (Wc) : %g\n\n',Wc)
fprintf('> Hit any key to continue : ');
pause;

```

```
% fname: PTF.M
% Show pilot transfer function on the screen.

clc;
fprintf('- PILOT TRANSFER FUNCTION : Gpilot(s)\n')
fprintf(' Gain : %g\n',gPilot)
fprintf(' Zeros : ')
s=size(zPilot);
imax=s(1,1);
for i=1:imax,..
    fprintf('\n          (%15.7f) + (%15.7f)j',real(zPilot(i)),...
        imag(zPilot(i))),..
end
fprintf('\n> Hit any key to continue : ')
pause
clc;
fprintf('\n Poles : ')
s=size(pPilot);
imax=s(1,1);
for i=1:imax,..
    fprintf('\n          (%15.7f) + (%15.7f)j',real(pPilot(i)),...
        imag(pPilot(i))),..
end
fprintf('\n> Hit any key to continue : ')
pause
```

```
% fname: PTF.M
% Show pilot transfer function on the screen.

clc;
fprintf('- PILOT TRANSFER FUNCTION : Gpilot(s)\n')
fprintf(' Gain : %g\n',gPilot)
fprintf(' Zeros : ')
s=size(zPilot);
imax=s(1,1);
for i=1:imax,..
    fprintf('\n          (%15.7f) + (%15.7f)j',real(zPilot(i)),...
        imag(zPilot(i))),...
end
fprintf('\n> Hit any key to continue : ')
pause
clc;
fprintf('\n Poles : ')
s=size(pPilot);
imax=s(1,1);
for i=1:imax,..
    fprintf('\n          (%15.7f) + (%15.7f)j',real(pPilot(i)),...
        imag(pPilot(i))),...
end
fprintf('\n> Hit any key to continue : ')
pause
```

```
% fname: BODEPLOT.M
% open-loop and closed-loop Bode plot on the screen

axis('square')
semilogx(omega,lmag1,omega,lmag2,omega,lmag3),grid,xlabel('w
(rad/sec)');
ylabel('magnitude (dB)'),title('open-loop transfer function')
pause;
semilogx(omega,phases1,omega,phase2,omega,phase3),grid,xlabel('w
(rad/sec)');
ylabel('phase (degree)'),title('open-loop transfer function')
pause
semilogx(omega,mag4),grid,title('closed-loop transfer function')
xlabel('w (rad/sec)'),ylabel('magnitude')
pause
semilogx(omega,lmag4),grid,title('closed-loop transfer function')
xlabel('w (rad/sec)'),ylabel('magnitude (dB)')
pause
semilogx(omega,phase4),grid,title('closed-loop transfer function')
xlabel('w (rad/sec)'),ylabel('phase (degree)')
pause
```

```

% fname: OUTPUT.M
% write output data file

clc;
fprintf('-----\n')
fprintf(' WRITE OUTPUT DATA \n')
fprintf('-----\n')
fprintf('> Enter the configuration No : ')
confNo=input(' ','s');
fprintf('> Enter the output file name : ')
fn=input(' ','s');
fprintf('- processing...')
sizeNo=size(confNo);
n=max(sizeNo);
fprintf(fn,'\n-----')
for i=1:n
    fprintf(fn,'-')
end
fprintf(fn,'\nCONFIGURATION NO : ')
fprintf(fn,confNo)
fprintf(fn,'\n-----')
for i=1:n
    fprintf(fn,'-')
end
fprintf(fn,'\n')
if case==1
    fprintf(fn,'\n\n ye(s) = Gs(s) u(s) + Gw(s) w(s) \n\n')
elseif case==2
    fprintf(fn,'\n\n ye(s) = -Gs(s) u(s) + Gw(s) w(s) \n\n')
end

```

```

fprintf(fn,'    ye    : tracking error \n')
fprintf(fn,'    u     : pilot control input \n')
fprintf(fn,'    w     : plant processing driving noise \n')
fprintf(fn,'    Gs(s) : system transfer function \n')
fprintf(fn,'    Gw(s) : filter transfer function \n\n')
fprintf(fn,'System Transfer Function : Gs(s)')
fprintf(fn,'\n\nGain : %g\n\n',gSystem)
fprintf(fn,'Zeros : \n')
s=size(zSystem);
imax=s(1,1);
for i=1:imax,..
    fprintf(fn,' (%15.7f) + (%15.7f)j\n',real(zSystem(i)),...
        imag(zSystem(i)));..
end
s=size(pSystem);
imax=s(1,1);
fprintf(fn,'\nPoles : \n')
for i=1:imax,..
    fprintf(fn,' (%15.7f) + (%15.7f)j\n',real(pSystem(i)),...
        imag(pSystem(i)));..
end
fprintf(fn,'\n\nFilter Transfer Function :
Gw(s)')
fprintf(fn,'\n\n')
fprintf(fn,'Gain : %g\n\n',gFilter)
fprintf(fn,'Zeros : \n')
s=size(zFilter);
imax=s(1,1);

```



```

for i=1:imax,..
    fprintf(fn,' (%15.7f) + (%15.7f)j\n',real(zFilter(i)),...
        imag(zFilter(i)));..
end
fprintf(fn,'\nPoles : \n')
s=size(pFilter);
imax=s(1,1);
for i=1:imax,..
    fprintf(fn,' (%15.7f) + (%15.7f)j\n',real(pFilter(i)),...
        imag(pFilter(i)));..
end
fprintf(fn,'\n\nI n p u t   P a r a m e t e r s   f o r   O C M\n\n')
fprintf(fn,' Covariance of disturbance (W)      : %g\n',W)
fprintf(fn,' Objective function weights (Q)      : %g %g\n',...
    Q(1,1),Q(2,2))
fprintf(fn,' Neuromuscular lag (Tn)           : %g\n',Tn)
fprintf(fn,' Observation delay (Td)                : %g\n',delay)
fprintf(fn,' Observation noise ratio (r1)           : %g\n',ratio1)
fprintf(fn,' Neuromuscular noise ratio (r2)        : %g\n',ratio3)
fprintf(fn,' Observation threshold (T1,T2)         : %g %g\n',T1,T2)
fprintf(fn,' Fractional attention (f1,f2)          : %g %g\n\n',...
    f1,f2)
fprintf(fn,'T i m e   D o m a i n   P e r f o r m a n c e\n\n')
fprintf(fn,' Objevctive function weight (G)       : %g\n',G)
fprintf(fn,' Covariance of u (U)                   : %g\n',U)
fprintf(fn,' Covariance of ye (y1,y2)              : %g %g\n',...
    Y(1,1),Y(2,2))
fprintf(fn,' Covariance of ua (Ua)                : %g\n',Ua)
fprintf(fn,' Covariance of uhatdot (Udot)           : %g\n',Uhatdot)
fprintf(fn,' Optimal cost (Jopt)                    : %g\n',J)
fprintf(fn,' Overshoot (os)                         : %g\n',overshoot)
fprintf(fn,' Peak time (pt)                          : %g\n',peaktime)
fprintf(fn,' Rise time at 100 percent (rt1)          : %g\n',risetime1)
fprintf(fn,' Rise time at 80 percent (rt2)           : %g\n\n',risetime2)
fprintf(fn,'F r e q u e n c y   D o m a i n   P e r f o r m a n c
e\n\n')
fprintf(fn,' Gain margin (Gm)                     : %g\n',Gmargin)
fprintf(fn,' Crossover frequency (Wcg)              : %g\n',Wcg)
fprintf(fn,' Phase margin (Pm)                      : %g\n',Pmargin)
fprintf(fn,' Crossover frequency (Wcp)             : %g\n',Wcp)
fprintf(fn,' Feedback : %g\n',feedback)
fprintf(fn,' Working band (Wd)                       : %g\n',Wd)
fprintf(fn,' Pilot gain at Wd : %g\n',mPilot1)
fprintf(fn,' Noise cutoff frequency (Wn)           : %g\n',Wn)
fprintf(fn,' Pilot gain at Wn : %g\n',mPilot2)
fprintf(fn,' Pilot/System gain at Wn : %g\n',mPS)
fprintf(fn,' Pilot/System phase at Wn : %g\n',pPS)
fprintf(fn,' Resonant Peak : %g ( %g dB)\n',...
    peak, logpeak)
fprintf(fn,' Peak frequency (Wp)                    : %g\n',Wp)
fprintf(fn,' Droop : %g ( %g dB)\n\n',...
    droop, ldroop)
fprintf(fn,'Bode ideal cutoff analysis\n\n')
fprintf(fn,' Maximum available feedback : %g\n',maxFB)
fprintf(fn,' Percent feedback at Wb : %g\n',percentFB)
fprintf(fn,' Gain slope at Wcp (dB/octave) : %g\n',slope)
fprintf(fn,' Ideal phase crossover freq (Wc) : %g\n',Wc)
fprintf(fn,'Pade approximation for observation delay : Gpade(s)\n')

```

```

fprintf(fn,'\nGain : 1\n')
fprintf(fn,'\nZeros : ')
s=size(zPade);
imax=s(1,1);
for i=1:imax,..
    fprintf(fn,'\n (%15.5f) +
(%15.5f)j',real(zPade(i)),imag(zPade(i))),..
end
fprintf(fn,'\n\nPoles : ')
s=size(pPade);
imax=s(1,1);
for i=1:imax,..
    fprintf(fn,'\n (%15.7f) +
(%15.7f)j',real(pPade(i)),imag(pPade(i))),...
end
fprintf(fn,'\n\nPilot Transfer function : Gpilot(s)\n\n')
fprintf(fn,'Gain : %g\n\n',gPilot)
fprintf(fn,'Zeros : ')
s=size(zPilot);
imax=s(1,1);
for i=1:imax,..
    fprintf(fn,'\n (%15.7f) + (%15.7f)j',real(zPilot(i)),..
    imag(zPilot(i))),..
end
fprintf(fn,'\n\nPoles : ')
s=size(pPilot);
imax=s(1,1);
for i=1:imax,..
    fprintf(fn,'\n (%15.7f) + (%15.7f)j',real(pPilot(i)),..
    imag(pPilot(i))),..
end
fprintf('\n- Writing was completed in ')
fprintf(fn)
fprintf('\n> Hit any key to continue : ')
pause;
clear i imax s fn fn1 fn2 d1 d2;
clc;

```

2

VITA

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