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AN EXAMINATION OF THE GENESIS, MAINTENANCE, AND  
MANIFESTATION OF THE PEDAGOGICAL BELIEFS OF A NOVEL  
INSTRUCTOR – A CASE STUDY

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DEPARTMENT OF INSTRUCTIONAL LEADERSHIP AND ACADEMIC  
CURRICULUM

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For my children.  
Thank you for reminding me of the power of dreams.

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## Abstract

The great preponderance of research in undergraduate mathematics, especially as it pertains to students transitioning into post-secondary education, tends to problematize the student or the difference in the educational environment. What appears to be lacking, however, is research into the ways that the instructor may be exacerbating this transition. Fundamental to the concerns of the quality and appropriateness of contemporary post-secondary mathematics instruction is the ability to envision alternatives to traditional approaches. This case study identified one individual, whose practice fell well beyond the norms at the institution in which they found employment and explored how they developed their teacher-identity, how they maintained that teacher-identity as they entered into their department and the profession in general, and how they have engaged in praxis. Data collected included interviews, course observations, and course document analysis. Analysis of these data revealed that a number of personal and/or professional crises prompted the evolution of the participant's teacher- and researcher-identity and these evolutions resulted in the need to reimagine the enactment of his affordances regarding teaching, resulting in an envisioning of his praxis as an interrelated system of praxis as an application of the creative process and praxis as an application of mathematical problem solving.

Keywords: identity development, mathematics education, post-secondary mathematics, praxis, teacher-identity

## Chapter 1: Introduction

There is a growing push for students to pursue advanced mathematics in high school, and for good reason: students taking and achieving in advanced high school mathematics correlates with not just a higher probability of attending college, but also of attending a 4-year institution (Aughinbaugh, 2012). There are also many studies on the effect of advanced mathematics in high school and lifetime earnings (see: Altonji, 1995; Joensen and Nielsen, 2009; Levine and Zimmerman, 1995; Rose and Betts, 2004) with most showing a positive correlation.

In recent years, however, a dire trend has become apparent in US colleges and universities: traditional pedagogies are not providing student learning outcomes in mathematics that the students deserve (and modern society demands). About two-thirds of incoming community college students are being referred to courses in developmental mathematics with nearly three-fourths of those students never earning college-level mathematics course credit (Bahr, 2013). Across all post-secondary institutions, only about 59% of students entering schools in 2009 graduated in six years (National Center for Education Statistics, 2017).

The questions of whom education serves and for what purpose are at the very core of the great pendulum swings in education over the past century. Not the least affected in these shifts has been mathematics. In the early parts of the 20th century in the United States, mathematics was taught as a method of training the mind (Klein, 2003) but would soon shift, due significantly to war and industrialization, to being perceived as being useful as a support field for the great works of engineering and science (Tucker, 2013). Mathematics was no longer being taught primarily for its own sake and, due in part to the work of Edward Thorndike's stimuli-response approach to teaching and learning of the subject (Ellis & Berry, 2005); mathematics was a target for reform by the social efficiency progressives of the time. These reforms called for student tracking and mastery being measured by replication of results "learned" through rote methods. Meanwhile, in higher education, the

identity of the faculty member moved from a sharer of knowledge to a creator of knowledge (Tucker, 2013).

Further, notions that one can teach someone by means of lecture have been challenged by progressive thinkers for as long as the United States has been pushing toward democratizing education (e.g. Dewey, Piaget, Vygotsky). Despite this (and the myriad of reform efforts), the predominant method of teaching mathematics in universities has remained the lecture (Hora & Ferrare, 2013; Mesa, Celis, & Lande, 2014).

Hence, students may enter post-secondary institutions believing that learning occurs through the ability to produce a desired outcome (i.e., they may hold product-oriented educational goals as opposed to process-oriented educational goals), while many lecturers are providing learning experiences that tend primarily to value the understanding of the instructor, moderated significantly by the instructor's narrative prowess. Plainly, the conditioned beliefs of students and the practices of many lecturers are incongruent. These disparate goals may never lead students to the robust, flexible, mathematician-like thinking that is the goal of mathematics training (Dreyfus, 1991). This concern has been met with increasing calls for mathematics educators at all levels to re-evaluate their teaching practices (see: Conference Board of the Mathematical Sciences, 2016; National Council of Teachers of Mathematics, 1989, 2000, 2014; White House Office of Science and Technology Policy, 2016).

Among the calls that have stemmed from the increased attention to the teaching and learning of mathematics at the post-secondary level have been some very prominent calls for a shift to active-learning, which calls students to engage in higher-order cognitive tasks (Bonwell & Eison, 1991). That is, students *must* do more than merely listen and transcribe. Recently, these calls have come from the White House Office of Science and Technology Policy (2016) and the Conference Board of the Mathematical Sciences (2016), whose members include seventeen of the most prominent professional mathematics societies. This attention is not without good reason. Freeman et al. (2014), in a meta-analysis of 225 studies comparing active-learning to traditional lecture, plainly state that,

If the experiments analyzed here had been conducted as randomized controlled trials of medical interventions, they may have been stopped for benefit—meaning that enrolling patients in the control condition might be discontinued because the treatment being tested was clearly more beneficial. ... There were 29,300 students in the 67 lecturing treatments with data on failure rates. Given that the raw failure rate in this sample averaged 33.8% under traditional lecturing and 21.8% under active learning, the data suggest that 3,516 fewer students would have failed these STEM courses under active learning. (p. 4)

Among the many pedagogies that support active-learning are collaborative learning, project based learning, problem based learning, and inquiry based learning, which research has additionally indicated success in mitigating gender-biased outcomes in the learning of college mathematics (Laursen, Hassi, Kogan, & Weston, 2014).

Despite this, the predominant trend in post-secondary mathematics education is still lecture with some academicians constructing arguments in the defense of the practice (see: Bland, Saunders, and Frisch, 2007; Burgan, 2006; Webster, 2015). Many of these “defenses” rebuke a binary reality wherein lecture is or is not an acceptable method of teaching despite much of the research on active learning focusing on the dominant pedagogical actions (that is, they do not suggest that lecture should *never* occur, just that it should not be the dominant teaching practice). These kinds of slippery-slope arguments pang of the kind of reactionary fervor that often manifest when individuals are encouraged (or forced) to move from their comfortable beliefs and practices.

DeGuzmán, Hodgson, Robert, and Villani (1998) considered the issues that students face when transitioning to tertiary educational environments and found that the barriers to student transition could be classified as: epistemological and cognitive difficulties, sociological and cultural difficulties, and didactic difficulties. In this, DeGuzmán et al. use the word “didactic,” as an adjective signifying an connotatively neutral relationship to teaching. Most relevant of the identified didactic difficulties are the lack in pedagogical and teaching abilities, the lack of positive teaching role models, a disregard for the importance of methodology, and a lack of feedback procedures. That is, many of the instructors teaching first-year college-level mathematics courses at universities have little to no development in the art of

teaching, have role models who are gifted researchers (regardless of teaching ability), an unwillingness (or inability) to help students develop an understanding of how mathematicians do mathematics, and often rely on teaching monologues during which there is little to no meaningful feedback from the students on their learning.

### **Problem Statement**

Fundamental to the concerns of the quality and appropriateness of contemporary post-secondary mathematics instruction is the ability to envision alternatives to traditional approaches. If we are to achieve a reality wherein active-learning is commonplace in post-secondary mathematics classrooms, examples of excellent teaching must be identified, studied, and shared. Seeley referred to these examples as “pockets of wonderfulness” and justified their study by saying,

For teachers, learning about and trying out a student-centered classroom or giving an increased emphasis to understanding is part of healthy professional growth. Our challenge as educators is to capture wonderful ideas and multiply their payoff by working with colleagues to plan for and build on what each teacher does. When we do this, students’ learning becomes continuous and cumulative, resulting in achievement that grows by leaps and bounds from year to year (Seeley, 2004, N. pag.).

### **Purpose**

This study investigated the beliefs of such a “pocket of wonderfulness” as they are shared and demonstrated by a single, purposefully selected, instructor at a public Carnegie R-1 institution in the Southern United States who, despite implied incentives otherwise, has regularly delivered instruction in a manner that is consistent with his beliefs and research interests. Namely, these include Inquiry-Based Learning (IBL) (see: Cook, Murphy, and Fukawa-Connelly, 2016; Laursen, Hassi, Kogan, Hunter, and Weston, 2011) and creativity (see: Pólya, 1981; Sriraman, 2005; Wallas, 1926). Further, his pedagogical beliefs have created in his class a novel permutation on the implementation, promotion, and assessment of productive failure (see: Granberg, 2016; Kapur, 2014; Kapur and Bielaczyc, 2012). The

manifestation of these beliefs and interests in the day-to-day teaching of this instructor is beyond the instructional norms at the institution and the department at which the instructor at the time found himself a junior faculty member. As such, the pedagogical actions resulting from this steadfastness and fidelity to his beliefs provided some additional risk as he pursued tenure.

Given his attention to conceptual understanding, metacognition, equality, and students' identity as doers of mathematics, especially in a traditional university mathematics department, it is difficult to imagine that this instructor's classes are not precisely an example of what Seeley calls a "pocket of wonderfulness". As such, the goal of this proposed writing is to share the stories of this one instructor in hopes that it provides insight into the origin and maintenance of the beliefs that guide this instructor's approach to teaching and to provide an insight into those actions based in his beliefs.

Hence, this study sought to find answers to the following questions:

1. What experiences does this professor perceive to have been essential to the development of his identities as a teacher-learner?
2. How has the instructor integrated into to his university and in his department, especially with regard to his pedagogical actions within the context of his position and status at his university and in his department?
3. What kinds of pedagogical actions does this professor employ and how does he choose when and how to implement them into his teaching?

### **Approaches**

There are three main emphases in research on and with mathematics teachers: their knowledge, beliefs, and identity. These lines of research use somewhat different theoretical or conceptual frameworks. Most research on teachers' knowledge and beliefs is inspired by constructivism, although references to socio-cultural theory are increasing in number. In contrast, studies of teacher identity generally adopt a more social approach. (Skott, 2013, p. 547)

This study was conceptualized as one directed at teacher beliefs, but in order to address questions related to an individual's beliefs, it is prudent to first give an operationalization of the construct for use in the studies resulting from this questioning (see: Pajares, 1992; Skott, 2013). Any questioning of beliefs is fundamentally both epistemological and ontological in nature because beliefs serve as perceptual filters, meaning that the reality we perceive is filtered by the lens of our belief sets (Puchta, 2010). Hence, beliefs can be defined as the constructs and propositions that an individual, possibly subconsciously, holds true and uses to interpret and forecast occurrences in the world around them (Borg, 2001; Goodenough, 1963; Pajares, 1992; Richardson, 1996)(see also, Eisenhart, Shrum, Harding, and Cuthbert, 1988; Green, 1971). Because of this deeply intimate, interpretational relationship between the individual and their beliefs, it has historically been an area of rich research. At the time of this writing, Google Scholar returned some 2.35 million results to a query for 'teacher beliefs,' and 409,000 for 'math\* teacher beliefs.'

With this definition, it becomes apparent that to investigate an individual's beliefs means coming to experience life as that individual. Even then, the limits of language inherently limit the extent to which beliefs, if known by the believer, can be expressed.

Language is not a neutral medium that passes freely and easily into the private property of the speaker's intentions; it is populated—overpopulated—with the intentions of others. Expropriating it, forcing it to submit to one's own intentions and accents, is a difficult and complicated process. (Bakhtin, 1981, p.294)

Thus, I have come to imagine that belief is a schema that, as a whole, is just beyond one's ability to directly study. This research, then, has taken to considering the residual of beliefs upon other, more accessible, constructs of identity and praxis.

Two key conceptions of identity are the Eriksonian identity and the Meadian identity. "Erikson understood identity as an acquisition, something that one has and that becomes coherent and consistent. A Meadian identity is an action, it is something one does, and it is multiple, contradictory and socially constituted" (Darragh, 2016, p. 27). As such, it is significantly more appropriate to consider, for the sake of this research, the Meadian identity.

This choice is consistent with the perspectives of many key works that have been influential in the conception of this study (see: Boaler, 2002; Gee, 2000; Sfard and Prusak, 2005; Wenger, 1998). Claiming to adopt the Meadian conception of identity, however, does little to operationalize the conception for this study. Efforts to do so have been inconsistent in the literature, and, hence, this study adopts the approach of Grootenboer (2013) that identity is conceptualized as the development of the individual. Specific attention is often given to specific development in narrow, interrelated areas (i.e., teacher-identity, student-identity, outsider-identity, mathematician-identity, etc.). Additionally, these identities are “produced at the intersection of ... personal aspirations ... and the multiple external demands they encounter” in their experiences (Brown & McNamara, 2011, p.27). Clearly, if beliefs are the propositions that one holds to experience and make sense of the world, the interaction between beliefs and identity, then, occurs through experiences. Further, the re-storied experiences of an individual reveal insights into not only their identity but also their beliefs. This is congruent with the conception that identity can be represented through narratives (Bruner, 1997, 2004) or that story-telling itself is identity (Sfard & Prusak, 2005).

Knowledge, being a significant area of philosophical research itself, is a much more difficult concept to operationalize for this study. Given the tremendous historical effort put into defining knowledge, it is clear that any conception used in this study will be in many ways significantly limited. However, this research adopts the perspective that knowledge is the collection of propositions a person holds that a significant mass of a social group that a person self-identifies with also holds without need for further evidence or verification, and thus, the fact and the implications stemming from the fact can be taken for granted. As such, knowledge is a personal acquisition that is both socially-mediated and socially-validated (see: Berger and Luckman, 1966/1991). Because of the social aspect of knowledge, a person may hold contradictory ‘facts’ as true in disparate social situations. Similarly, there are also, then, propositions that serve as ‘facts’ in certain situations that do not in others.

Knowledge, with regard to teaching, has been conceptualized as three related constructs:

content knowledge, pedagogical knowledge, and pedagogical content knowledge (Shulman, 1986). The ways that these realms of knowledge manifest in the pedagogic practice of a teacher can be considered, then, through affordances. An affordance is created by the interplay of an individual's perception of the environment and possibilities of behavior (Hora, 2012, 2016). Such perceptions of the environment are reflections of that individual's beliefs and the imagining of possibilities of behavior are explicit manifestations of pedagogical knowledge.

### **Researcher Positionality**

Of course, as with all qualitative study, the researcher is the ultimate instrument. However, as a person, I am unable to divorce myself from my thoughts, my beliefs, my emotions, and my presuppositions. Thus, disciplined subjectivity must be utilized to ensure that the story being shared is that of the participant's experience and not of my own personal biases. LeCompte, Schensul, Weeks, and Singer (1999) defines "disciplined subjectivity" as

[T]he rigorous self-reflection about one's own impact on the field, as well as how one's preferences, prejudices, biases, hopes, and concerns affect the course and outcomes of research. (p.66)

Additionally, member checks will help to provide validity to the analysis of the study (Merriam, 2009). This recognized, it is prudent for me to be explicit about my own relationship with this participant and the origins of my interest in the questions explored here within.

I have been teaching professionally since Fall of 2008, when I was employed at a community college to teach developmental mathematics courses. That semester I taught two different courses. One, the lowest level of remediation offered, allowed students to work at their own pace and when they were ready, the students went to a centralized testing area to take an exam. Students often rushed through rote-work types of problems in order to get my approval to take a departmental exam. If they correctly answered a satisfactory number of questions, they would be permitted to move to the next "module" or would be forced to

repeat the same rote-work problems again in order to be permitted to retest.

The other class I taught that semester was the final class in the developmental sequence at the time and although the course content was standardized, none of the methods or assessments were. That class I taught almost exclusively through lecture at the beginning. Not knowing that other options could exist in college mathematics classrooms (due to time or space or perceived student constraints), I trudged forward with the fumes of the dry erase marker being the only feedback I had on my teaching. By mid-semester, I had to concede that I was not making the progress I wanted and that that was a personal failing and not one of my students. Emboldened, perhaps, by the freeform structure of the other course I implemented collaborative, competition-based pedagogical actions.

While I no longer believe that an artificially imposed competitive environment can result in optimal outcomes in the classroom, that experience stuck with me throughout my career. In many ways, it is this experience that has caused much personal consternation as the courses I have taught became more closely controlled by their coordinators.

For a number of years, I have been completely dismayed at the cage placed around me by this level of coordination. I had suppressed memories of my non-traditional practices in that first semester, which were by this point a painful reminder that teaching need not reflect the practice that I experienced as a student and that I did not and currently do not believe are adequate for students to deeply, richly, and flexibly learn mathematics.

Then I heard of an instructor that was described to me by a student as someone they hated, but when I asked if the student had learned much the response was an overwhelming “yes.” I was enamored with the idea that an instructor could be reviled while still being acknowledged as a superior instructor. Finally, in Spring 2016, I was able to observe this individual teach. It was euphoria to watch a third semester course in introductory calculus (focusing on topics such as series convergence and surface and contour integrals) that was not taught by way of lecture. That experience caused a shift in my perception of what could be done in lower-level mathematics courses and reawakened the interest I had in non-traditional

pedagogies in which I had many years earlier dabbled.

I soon learned that the instructor would, in the following Fall, be teaching a large enrollment section of a second semester course in introductory calculus (which focused on integrals and integration techniques). Further, he intended to deliver that section via the same inquiry-based pedagogies, which caused that student I first spoke with to despise the instructor but respect his teaching.

In the Fall semester of the following academic year, I was fortunate enough to observe daily the teaching of this instructor and his pedagogical practices were beyond my wildest conceptions of what was feasible in a large-enrollment course. The resultant research from that semester has been focused on providing insights into the alternate reality of that instructor's teaching. Reflecting on that experience, I found myself questioning the origins of the beliefs to which this instructor must hold. This study is the result of that line of questioning.

As to my relationship with this participant, I am acutely aware that I consider this individual a professional colleague, a mentor, and a friend. We have spent many hours working in close physical and conceptual proximity to each other. This rapport is simultaneously a benefit to this study and an area of concern as to the trustworthiness of it. Fortunately, since this study focuses primarily on the personal narratives that this instructor tells himself to justify his actions it is necessary that much of the story be shared in his own words and that it is the meaning that he draws from them that is of ultimate concern.

Hence, I have used reliable strategies for triangulation to serve to ensure trustworthiness in this case study. Merriam (2009) states that triangulation can include "the use of multiple methods of data collection, multiple sources of data, multiple investigators, or multiple theories to confirm emerging findings" (p. 216). To this end, this study utilizes several methods of data collection (interviews, observations, and document analyses) and multiple theories (dialogical self, social-cognitive theory, bi-directional acculturation model, communities of practice, creative process, multidimensional problem-solving framework).

As there is only a single participant, to assist this triangulation, questions have been revisited several times over the year and a half that the data have been collected. In doing so not only can subtle changes in the manifestation of these instructor beliefs be explored, but the consistency of the instructor's stated intentions can also be considered.

### **Organization**

The questions guiding this study serve to provide an outline for this report, which takes the form of three academic research articles. This is done in lieu of presenting a second chapter with a thorough, in-depth literature review, followed by a chapter with the complete methodology and description of the analytical tools employed by this research, and finally a chapter that consolidated the findings of all three avenues of inquiry before ending with a chapter to serve as the conclusion of the report. Hence, the presented "articles" that compose this dissertation are listed as chapters two, three, and four and the elements that would be found in what may be considered the more traditional style of dissertation are spread, as appropriate, across those chapters.

Thus, in chapter two, I consider the first question and explore the ways that this instructor's teacher-, student-, and mathematician-identities are negotiated and consider, specifically, the narratives that provide insight to the origins and development of a robust teacher-identity. Chapter three is centered on the ways in which this instructor has come to find place in his academic department and within his research interests. In doing so, I explore two facets of his budding professional identity: his researcher-identity and his teacher-identity. In chapter four, the final question is addressed with a concerted effort to map out the ways that praxis is manifested in the instructor's practice. Ultimately, chapter five discusses the findings of this study en masse as well as addressing the limitations of this study and directions for future work in the field.

## Chapter 2: Developing Teacher Identity

It has been well established that the development of a robust teacher identity is important for novice teachers (see Britzman, 2003; Freese, 2006; Jarvis-Selenger, Pratt, and Collins, 2010; Palmer, 1998; Pinnegar, 2005). This has contributed to a growing body of research, which is often confounded by the complexities of identity (Beauchamp & Thomas, 2009). As such, reviews of literature form a suitable basis to begin exploration as these complexities can be taken in aggregate and from which, perhaps more robust, implications can be drawn.

Kagan (1992) wrote a thorough review of research in an effort to conceptualize the body of literature dealing with professional development and growth for preservice and neophyte teachers. Inferences from her review are that novice teacher development should emphasize procedural (as opposed to theoretical) knowledge, self-reflection (especially as it pertains to their own personal experiences), extended clinical experiences (in lieu of additional theoretical coursework), and cognitive dissonance (wherein developing educators have to re-evaluate their beliefs and perceptions).

More recently, Noddings (2005) suggests a more holistic vision for teacher development:

[T]he most fundamental change required is to empower teachers as we want them to empower students. We do not need to cram their heads with specific information and rules. Instead we should help them learn how to inquire, to seek connections between their chosen subject and other subjects, to give up the notion of teaching their subject only for its own sake, and to inquire deeply into its place in human life broadly construed. (p. 178)

Anderson and Stillman (2013) examined specifically the literature pertaining to student teaching. Such clinical experiences, as were found crucial in Kagan's (1992) review, are widely promoted and even touted by some as the most important facet of preservice teacher education (Anderson & Stillman, 2013). Anderson and Stillman caution that direct linear causation is confounded by a plethora of factors that influence teacher thinking and that, fundamentally, the research on student teaching programs was considered to be too weak

(lacking both density and methodological rigor) to determine critical components of those experiences and how they help preservice teachers learn to teach.

Opfer and Pedder (2011) considered research related to teacher professional development and learning, finding that teachers bring their own experiences into account in not just their teaching, but also in how they approach their own learning. Further, “[c]hanges in [teacher] beliefs lead to changes in practice that bring changes in student learning that bring further changes in practice that result in additional changes in belief and so on. The relationship between these processes is also reciprocal with changes in one being contingent on changes in another” (p. 395). In other words, teachers, even student teachers, engage in praxis (see Freire, 1970/2005).

Beijaard, Meijer, and Verloop (2004) examined the literature centered on teachers’ professional identities. In their review, they conclude that research into narratives of teachers and their professional identities are compatible with the theoretical literature and form a sound basis upon which further work can be completed. They further define four major features of professional identity as an on-going process of development and (re-)interpretation, dependent on and implying both person and the context, composed of sub-identities, and dependent on agency.

Some, more contemporary, studies warrant mention as well. Arvaja (2016) examined how a single teacher’s professional identity is constructed over the course of a one-year course on pedagogical studies. In an examination of their participant’s personal narratives, as provided in a reflection diary, Arvaja (2016) found that teacher identity is not merely a siloed professional identity, existing in partition from more personal notions of self. Rather, personal histories have significant weight upon teacher identity. Battey and Franke (2008) conceived of the process of learning to teach and self-identifying as a teacher as occurring through a community of practice (see Wenger, 1998). Among the findings is that, “[t]eachers’ identities carry personal histories, emotion, values, and knowledge and they shape how teachers participate in professional development and their classrooms” (Battey & Franke, 2008, p.

145).

Flores and Day (2006), in a longitudinal study, found that teachers' identities changed from that of their beginnings through an interrelated process; the three main influences being past experiences, initial training, and context. Beginning teachers tended to have preconceptions of what constituted good or bad teaching through an 'apprenticeship of observation' as pupils themselves (Lortie, 1975). Additionally, teachers found their preparation for the classroom to be lacking; treating the university preparation as disjoint from the 'real-world' of the classroom. Lastly, the nature of students and the classroom served to push their focus from creativity to compliance (Flores & Day, 2006). The way that Flores and Day (2006) envisioned the relationship between these factors is shown in Figure 1.

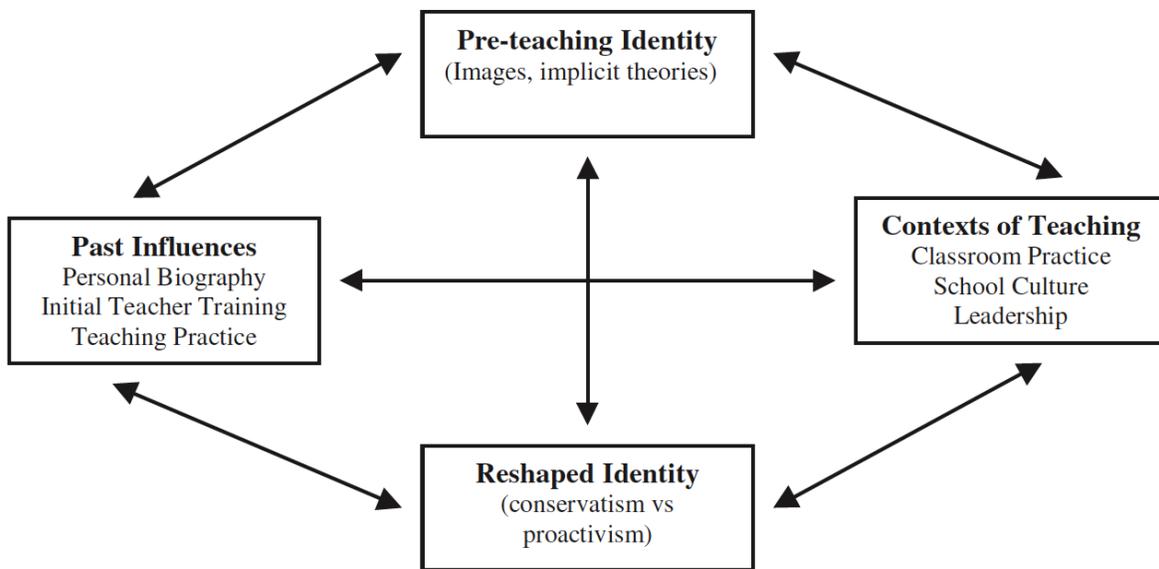


Figure 1: Key mediating influences on the formation of teacher identity.

*Note.* From "Contexts which shape and reshape new teachers identities: A multi-perspective study" by Flores, M. A., & Day, C. (2006). *Teaching and teacher education*, 22(2), 219-232.

Oleson and Hora (2013) examined the belief that teachers learn to teach primarily through this 'apprenticeship of observation' and determined that although instructors do learn to teach in part through modeling their practice, this is a very small part of the much more

complicated matter of how an instructor develops into the teacher that they become. Similar to what has been observed in PK–12 teachers, other factors include teaching experience, student feedback, and development as researchers.

The preponderance of teacher-identity research has focused on preservice PK–12 teachers. Research into how professors learn to teach exists and is a growing field (see Kagan, 1992; Kane, Sandretto, and Heath, 2002) but tends to focus on teacher beliefs and intentions. How influences and experiences interact in the formation of a teacher identity for those going into post-secondary education is currently lacking.

### **Dialogical Self**

Although identity itself has been the focus of many studies, there are significant difficulties in the understanding of it. These difficulties stem from a lack of a succinct and cohesive definition, exacerbated by the fact that it has been the focus of studies in myriad of disciplines including philosophy, psychology, and education, each having their own notions, specialized language, and conceptions (Beauchamp & Thomas, 2009). That said, there seems to exist three predominant perspectives of identity: psychological-developmental, socio-cultural, and poststructural (Grootenboer, Smith, & Lowrie, 2006). These perspectives are presented in Table 1.

Identity can be represented through narratives (Bruner, 1997, 2004) or can be conceived as a narrative itself (Sfard & Prusak, 2005). If narratives are, at least, expressions of identity, then they can be examined as the means through which identity is formed (Alsup, 2006; Beauchamp & Thomas, 2009). In order to do so, however necessitates adopting one of the aforementioned perspectives. The theoretical framework that this research adopts is that of the dialogical self (Akkerman & Meijer, 2011; Hermans, 2001).

The conception of the self as dialogical is an attempt to unify these disparate conceptions of the self and, thus, identity. That is, identity is both multiple and unitary, both social and individual, and both discontinuous and continuous. Identity has multiplicity in that there are a number of I-positions (a notion that has its roots in Bakhtin's 'multiple voices')

within a person’s self (Akkerman & Meijer, 2011). It is worth noting that the concept of an ‘I’ separate from ‘me’ in dialogical identity as adopted for this study has only nuanced differences to that of the Meadian ‘I’ or ‘me.’ That is, Mead treated the image of one’s self as separate from the awareness of it.

The ‘I’ is the response of the organism to the attitudes of the others; the ‘me’ is the organized set of attitudes of others which one himself assumes. The attitudes of the others constitute the organized “me,” and then one reacts toward that as an “I.” (Mead, 1934/1952, p. 175)

Table 1: Identity Discourses across Three Theoretical Perspectives

	<b>Psychological</b>	<b>Socio-Cultural</b>	<b>Poststructural</b>
<b>Locus of Identity</b>	<ul style="list-style-type: none"> <li>• individual</li> <li>• the “inner world”</li> <li>• self, self-concept, self-efficacy</li> <li>• intrapsychic cognitive structures</li> <li>• emotion</li> <li>• agency, autonomy</li> </ul>	<ul style="list-style-type: none"> <li>• relational self, the “outer world”,</li> <li>• connected to otherness</li> <li>• embodied</li> <li>• habitus, fields, cultural capital</li> <li>• belonging</li> </ul>	<ul style="list-style-type: none"> <li>• non-agentic,</li> <li>• a political posture,</li> <li>• no unified self,</li> <li>• all is relative</li> <li>• subjectivity rather than identity</li> <li>• positionings</li> <li>• becoming</li> </ul>
<b>Identity Formation</b>	<ul style="list-style-type: none"> <li>• internalised, behavioural repertoires</li> <li>• executive functions of the self (monitoring, choice-making)</li> <li>• searching for internal integrity, autonomy and competence</li> <li>• individual responsibility for who one is</li> </ul>	<ul style="list-style-type: none"> <li>• constructed and situated</li> <li>• communal consciousness and identification</li> <li>• sociocultural reproduction and framing</li> </ul>	<ul style="list-style-type: none"> <li>• Interior self is populated by others</li> <li>• a constant becoming</li> <li>• a function of difference</li> <li>• constituted by political and institutional processes</li> </ul>
<b>Theoretical Alignments</b>	Bandura, Erikson, Piaget	Wenger, Vygotsky, Bakhtin, Bourdieu, Bernstein	Foucault, Derrida

*Note.* From “Researching Identity in Mathematics Education: The Lay of the Land” by Grootenboer, P., Smith, T., and Lowrie, T. (2006). In P. Grootenboer, R. Zevenbergen, & M. Chinnappan (Eds.), *Identities cultures and learning spaces*. Proceedings of the 29th annual conference of the Mathematics Education Research Group of Australasia (Vol. 2, pp. 612–615). Canberra, Australia: MERGA.

Dialogical self, as a framework, is built to a great extent on the work of James and Bahktin (Hermans, 2001). That said, the choice to use the Meadian understanding of ‘I’ and ‘me’ over that of James is reasonable in that James considered the ‘I’ as the self-as-knower and ‘me’ as the self-as-known, which is not wholly different than the more accessible definition

of Mead. Where a dialogical approach to identity differs, though, is in the composition of 'I'. Instead of 'I' as a monolith of the self, each facet of identity is represented with a different 'I.' An individual who happens to be a teacher may have the sub-identity of that as a 'teacher,' but may also have other sub-identities as a 'spouse,' a 'graduate of a certain school,' as an 'environmentalist,' etc. Each of these sub-identities may, themselves, be well-defined dialogical identities and may even be in direct conflict with each other in ideology or intention (Akkerman & Meijer, 2011).

However, identity is also unitary in that all of these I-positions are in dialogue with each other I-position and that the negotiated space created by this dialogue is the singular identity.

The existence of multiple identities, or I-positions in terms of the Dialogical Self Theory, does not mean that the I moves to a particular position without taking into account any other positions. By taking turns, they are always in a dialogical relationship of inter-subjective exchange and temporary dominance. This dialogue is not necessarily harmonious, hence, the self is also a negotiated space. It is suggested that the coherence of self resides in the continuous attempt to synthesize the different parts. (Akkerman & Meijer, 2011, p. 312)

Identity is social in that the individuals with whom we interact, the communities we operate within (Wenger, 1998), the roles that we play (Goffman, 1959), and our expectations of the reactions we elicit within each of these contexts (Cooley, 1902/1922) mediate which I-position becomes dominant as well as how our identity develops. In this way, the community surrounding an individual shapes her identity (Akkerman & Meijer, 2011).

It is also individual in that a person can exert agency in social interactions; that one's self, being composed of myriad I-positions in a dialogic space with each other, "can introduce new voices into the communities in which they participate" (Akkerman & Meijer, 2011, p. 315). In doing so, the individual can alter the shared identity of the community.

Identity is discontinuous in that, as with Bahktin's concept of 'unfinalizeability,' the self and identity are always in flux. The self is composed of numerous I-positions, each I-position may manifest in different circumstances or differently within the same circumstance. As such,

it is not reasonable to consider one's actions at any given moment in relation to how they have behaved previously, as they may result from different arrangements of self (Akkerman & Meijer, 2011).

Finally, identity is continuous in that individuals strive to make sense of their self. This may be done through routines or socially constructed norms. Most notably, for this study, however, is that identity is also held together in continuity through narratives. Through narratives, accounts of memory are filtered through the dominant I-positions and voiced into a shared experience, with one's self or with a social other. Hence, the memories established in one arrangement of self are re-interpreted through a potentially different arrangement of self (Akkerman & Meijer, 2011). In narratives, 'I' speaks the story of 'me.'

For example, as a former military service member, I have an authoritarian I-position (I am a non-commissioned officer). As someone who has been the chief architect in a number of personal failings, I also have a timid I-position (I am a student who has failed out of graduate school). These are but two of the perhaps innumerable I-positions I hold. It is worth noting also, that in labeling these I-positions, I am limiting the scope of what they truly are, which is, in itself, disingenuous to some extent. However, these I-positions are in direct conflict with each other. How can I be both extrovert and introvert, both loud and quiet, both impulsive and calculating? When I enter my advisor's office, the circumstance prompts my self to cede dominance to my I-position of the failure. That does not mean that every time I meet with my advisor that I am meek. If that were the case, I would have had no autonomy in my course-of-study. As such, the interaction of these two facets of my identity shows that it is both multiple (there are at least two I-positions) while still being singular (these positions negotiate dominance within my self). It is both social (my interaction with my advisor prompts a particular I-position into dominance) while still being individual (I can exert pressure into the social relationship between my advisor and myself, thus changing that social relationship). It is both discontinuous (I am timid on one day, but assertive the next) yet continuous (I can share narratives about the development of those

I-positions and the flow from one to the next).

Although ideally any exploration of identity-building would be best situated in such a way as to explore these characteristics as both separate and inter-connected with each other, the necessities of language and story-telling, which reports of research are a form of, require a method of ordering. As such, it is reasonable to consider identity-building as a form of story telling (Sfard & Prusak, 2005), hence treating the facet of continuity/discontinuity as the background from which the facets of singularity/multiplicity and personal/social can be explored.

This study utilized an intrinsic case study design to explore the narratives that a single unique instructor, Ivan, tells to explain the origins of his practice. Ivan was purposefully chosen as the sole participant of this study due to his striking incongruence with the teaching practices normally found within his department. That is, Ivan teaches using Inquiry-Based Learning (IBL), whereas the norm within his department is exclusively lecture. As he was also junior faculty within his department there were perceived to be substantial normalizing pressures upon his teaching but he persisted in acting in accordance with his beliefs despite them. Hence, this study sought to address the following questions:

1. What experiences does this professor perceive to have been essential to the development of his pedagogical beliefs as a teacher-learner?
2. How did Ivan construct his identity as a teacher-learner?

## **Methodology**

Schwandt (2015) defined a case study as a “complicated arena [of qualitative inquiry] involving methodological choices directly related to goals or purposes of conducted case-based research, research traditions in different disciplines, and the ways in which investigators define a case” (p. 26). Additionally, “an intrinsic case study is the study of a case (e.g., person, specific group, occupation, department, organization) where the case itself is of primary interest in the exploration” (Grandy, 2009, p. 500). Further, the core phenomena of

interest were the experiences of Ivan. As such, this research was conceptualized as a narrative inquiry. In a narrative inquiry,

our principal interest in experience is the growth and transformation in the life story that we as researchers and our participants author. ... [I]n the construction of narratives of experience there is a reflexive relationship between living a life story, telling a life story, retelling a life story, and reliving a life story (Clandinin & Connelly, 2000, p. 71).

Since our beliefs are an intimate part of our personal identities, it is entirely reasonable to assert that a fundamental motivator of those beliefs are the stories that we tell ourselves about them and that those stories are the reality of beliefs' existence. Such stories, being the instructor's personal narratives, "had better be viewed not as a record of what happened (which is in any case a nonexistent record) but rather as a continuing interpretation and reinterpretation of our experience" (Bruner, 2004, pp. 691-692). As such, Clandinin and Connelly's (2000) framework of a three-dimensional narrative inquiry space (consisting of situation, continuity, and interaction) along with the four directions of inquiry (being inward, outward, backward, and forward) is an invaluable tool to situate a narrative researcher's work. "[T]o experience an experience—that is, to do research into an experience—is to experience it simultaneously in these four ways and to ask questions pointing each way" (Clandinin & Connelly, 2000, p. 50).

### *Participant and Setting*

Ivan was, at the time of this study, junior faculty in a fairly traditional mathematics department (with primary research interests in Algebra, Analysis, and Topology) at a Carnegie R-1 institution in the Southern United States, Current University (CU). Ivan represented fully half of the declared departmental research interest in undergraduate mathematics education (RUME) and it is in no small way that because of his interest in RUME that his teaching fell well outside of the norms at his institution. At the time of this study, Ivan had been at his institution for five years and was undergoing tenure review.

### *Data Collection and Analysis*

Eight semi-structured, audio-recorded interviews were conducted. Seven of these interviews occurred over the course of six months while the eighth was conducted two years prior. These interviews all occurred in Ivan’s faculty office, however, with the exception of two of the follow-up interviews, they continued, at Ivan’s invitation, past our scheduled interview time as we walked together to Ivan’s car. One of the two follow-up interviews that did not occur in Ivan’s faculty office was given entirely in Ivan’s car as he had invited me along to complete a personal errand of his. The goal of the initial interview was to establish a rough chronology of the pertinent, in his own opinion, events of Ivan’s journey, while the follow-up interviews systematically established vignettes of each of these events (see Table 2).

Table 2: Enumeration of Interviews

<b>Interview</b>	<b>Main Topics</b>	<b>Duration</b>	<b>Location</b>
Pre-Existing	Use of IBL	01:12:32	Ivan’s Office
Initial Interview	Establish Chronology	01:25:30	Ivan’s Office
Follow Up 1	Mentoring, Pre-professional Grantwork	00:56:14	Ivan’s Office, University Grounds
Follow Up 2	Motivations, Early Professional Work	00:42:27	Ivan’s Office, University Grounds
Follow Up 3	Aims and Goals of Education, Current Perspectives of Teaching	00:49:40	Ivan’s Office, University Grounds
Follow Up 4	Coming to Teach IBL, Modeling Behaviors	00:30:06	Ivan’s Office
Follow Up 5	Approach to Teaching, Reflection on Teaching	00:57:41	Ivan’s Office, University Grounds
Follow Up 6	Pre-Professional Grantwork, Influential Experiences as a Student, Concluding the Interviews	01:38:34	Ivan’s Personal Vehicle

Over the course of these interviews several questions and stories were revisited to establish stability in the narrative. Similarly, member checking has been used to further ensure

that the narrative co-created was, in fact, true to that which Ivan tells himself. These interviews were transcribed verbatim and then closed coded in accordance with Clandinin and Connelly's (2000) three-dimensions of narrative inquiry space. Those dimensions are "personal and social (interaction); past, present, and future (continuity); combined with the notion of place (situation)" (p. 50). Data were then open coded then subsequently axially coded. Using a constant comparative method, new data were compared with existing data for consistency and emergent themes were sought.

## **Findings**

Adopting narratives for this study necessitates either a topical or a chronological presentation of the results. Given the cumulative nature of Ivan's experiences, a brief chronology of relevant items is presented. In an effort to maintain trustworthiness and to not imply my own ownership of the narratives, they have been presented, where possible, with Ivan's own words and have been subjected to only minor editing to fit the format of this report and improve the flow, since topics were revisited many times in multiple interviews for this study.

### *Academic Origins*

Ivan grew up in a suburb near one of the great cities in the Midwest United States, which afforded him many opportunities growing up, including access to many Advanced Placement (AP) courses in secondary school and organized sports. Despite his father only having a high school educational attainment level and his mother an Associate's degree, Ivan determined fairly early that he was going to college,

Way back. I mean, even before the senior year. And I think that was just a combination of: I wanted to further my education and didn't know what I was going to do, coupled with the fact that a ton of my friends were going to college, no matter what. And so, I had to go to college, right? (Interview Data)

Ivan applied to several colleges in the area before setting on Midwestern Public University (MPU). Despite being offered acceptance to several schools in the region, his decisions were

strongly driven by the cost of attendance.

A lot of my friends went to college as well. Some of them went to [MPU]; there was an in-state tuition that was really attractive, right? I applied to a couple places; I got recruited to play soccer in a couple places. But [MPU] was the only place that offered a little bit more, which is in the form of a smallish scholarship, but also they gave me a ton of credits from my AP courses. I got ten or something credits that could help out with offsetting a little bit of cost. It was all a cost issue. Even the places I was recruited to play soccer they weren't giving me full-rides and so, it was one thing that I had to measure; how much money I'd pay versus what I would do. (Interview Data)

However, Ivan did not originally have a clear image of what he wanted to do when he got to university. As such, he tried several things before finally settling on mathematics.

I had an idea, I kind of wanted to do architecture, but I had no idea what I was getting into or what I was planning on and I look at students here at [CU] and they're so phenomenally framed! And I don't know what that is, if that's a product of the high school or whatnot, but they've got such a good head on their shoulders and some students really know what they want to do in life, and that's fantastic! I was not that student, and that may have contributed to some of the downfalls and pitfalls that I had. It definitely contributed to me shifting into math. I was in [MPU] and I started out as Architecture or Undeclared, I forgot which one, but I wanted to be an Architect. I was almost going to do Urban Planning and Architecture, then I changed to Sports Management and I took a class called Public Relations and I didn't do well in it. And I said, "Well, I don't know if I want to do this." And so I thought about what do I want to do, and I didn't know. It was really tough because in high school, I was very good at a lot of things and so you don't really sit down and concentrate on what really did I want to do.

Ivan's reasons for choosing an undergraduate major in mathematics were significantly affectual but, perhaps, superficial.

So, I had been taking classes. Like, I took [Calculus] 2, because I passed [Calculus] 1 via AP credit. And [Calculus] 2, I swear that I got an award for getting an A in [Calculus] 2, by the math department, and that might have been a little bit of it. But I remember thinking, "Wow, math!" You get sold on "you can do anything with math" and also I kept thinking, "math is uncool" and I'd done some things where it was trying to be unique or opposite. So I thought, "Well, math is uncool, maybe I could make it cooler," right? And so you can tell that some of these decisions are not necessarily reason-based or wholly reason-based. So, yeah, that's why I chose math. Because it was uncool. (Interview Data)

While at MPU, Ivan's grades were not the strongest, but were strong enough, due to his work ethic, for him to maintain employment as a resident assistant as well as to complete an honor's thesis.

I had these terrible grades, but I'd always go to office hours. I'd always try to learn and so I'd get a surface level learning or maybe I'd mimic my professor for some things or my professor would push me. [They] would half help me on a lot of things and I'd get B's and C's, sprinkled with A's, because I was a good computer. I was persistent though! I mean, you've got to give me that! I did my honors thesis in [number theory]. And so, I mean I really wanted to be a part of mathematics, but I had a lot of difficulty in proving. (Interview Data)

Given his known weakness in proving at the time, I inquired as to the reasons to pursuing graduate studies in math, to which Ivan provided an additional reason.

Because I wanted to be a Ph.D. I wanted to have a D-R in front of my name. I swear to you. I swear to you, these are my reasons. I wanted the fancy title. Once I got towards the end of undergrad, I thought to myself, "Well, I'd like to do a little bit more of this and I want the fancy title." And that's why I did the doctorate. (Interview Data)

### *A Stumbling Block*

Since he wanted to pursue graduate studies in mathematics, Ivan applied to three different schools and eventually settled on Pacific State Public University (PSPU).

[PSPU] was a pretty big change from everything else. I had grown up in the Midwest my whole life and then I went to a school in the Midwest. And so [PSPU] represented this totally different place. In fact, I applied to only three schools for [graduate] school: [New England State University], [PSPU], and I can't remember the third one. And [New England State University] because I was big into snowboarding and so I wanted to be close to a snowboarding capital. But, I only applied to three schools. And it was somewhat based on just getting a new change, right? I mean and why not? It's one of those things where I may never have an opportunity like that again and, I was twenty-two at the time and you know, what the heck? (Interview Data)

However, Ivan's cost-centric perspective encouraged him to make a number of counter productive decisions at PSPU. Ivan's lackluster undergraduate record meant that he didn't

receive financial support at PSPU. Thus, he paid for his education by working and taking out student loans. These decisions exacerbated his conceptual difficulties in mathematics.

I got accepted to [PSPU] without any [teaching assistantship] or any tuition waiver. I said that I would still do it, and so I took out a huge loan, but because I was taking out a huge loan, I wanted to maximize the amount of credits I could take in order to minimize the amount of years and, hence, the amount of money I would pay. So I came in and I had a suggested load and I said, "Well, I don't want to take the suggested load." And so, I think I ended up taking, at the beginning, fifteen credit hours of [graduate] coursework because I thought that it would be similar to [undergraduate school] and I took eighteen in [undergraduate school]. I dropped one, so I went down to twelve.

Secondly, because I didn't have a [teaching assistantship], I ended up getting a job as well, which I worked thirty hours for. You're seeing what's going on here. That is a recipe for disaster. But then couple that with the fact that I didn't know how to prove all that well, which is I don't know. I don't know if that's fault on my own, wholly. But I can take some responsibility for it. Anyways, these are all mixtures that are bad news. And couple that with trying to be assimilated into a new environment, right? It's just all bad. And I don't do well and it's because I tried mimicking my teachers and going to their office hours and everything like that. And I wasn't even in the ballpark of getting the reasoning down. The only reasoning I could do is if it were computational, but conceptual, not a clue, not a clue. (Interview Data)

As his grades slipped, Ivan was placed on academic probation and eventually was removed from PSPU, despite working hard to meet his goal of persisting in the program.

I get called in [to the graduate advisor's office and] we talk about probation and what that meant and everything else. The next semester, I take two courses: one course is on elementary number theory and the other course is an expansion of the real analysis course, Lebesgue Theory by the way. So I did Lebesgue Theory 1 and I'm doing Lebesgue Theory 2. And I needed to get an A and a B, in order to maintain 3.0. And they had A, A-, B+, so forth. So I needed an A and a B in order to retain and I was getting a B in Lebesgue Theory, even though I could look back at the stuff and I don't know about deserving that, but, in the Number Theory thing, I thought I was doing really well. I was making a lot of sense of some of these things. But I wasn't, right? Some parts I wasn't. And so, I ended up getting an A- or a B+, I forgot which, but it wasn't enough, and I went and I begged and I pleaded. And the professor said that maybe math isn't for me. (Interview Data)

The impact of this removal was clearly significant as evidenced by the softening of Ivan's

tone and his taking a moment to recompose himself as he told his story.

And so I failed out, right? It's one of my lowest points of my life. The hardest thing I've ever done in my life, other than having a kid. I wouldn't even put dissertation as hard as making that phone call to my parents that said that "I'm not only [a great many] miles away, but I've failed you all." Making that call to my grandpa too. All of them were supportive, which says a lot about them; says a lot about how much pressure I put on myself. (Interview Data)

It would be at PSPU that Ivan first experienced teaching as instructor-of-record, during his second semester.

[In high school], a friend of mine and I got to teach one day for an Algebra class, for another class, because we were both part of National Honor Society and part of that was to teach a day. And, I mean at the time it was mimicking our high school teacher and his nuances. It was supposed to be all in good fun and everything else. But [the second semester at PSPU] was when I got a one-class [tuition assistantship] at [PSPU] and I was in charge of thirty-something students. And, that class was in Pre-Calculus or Pre-Algebra. I don't even know. I think it was in College Algebra; something around there; the prerequisite for Pre-Calculus. And I taught one semester there and then I tried to figure out what my life is going to be, because I got kicked out of school. (Interview Data)

After failing out of PSPU, Ivan was left without direction and continued to work at his part-time job for the summer and, among his attempts to find employment, applied for a position teaching mathematics with the local public school district.

I was trying to find other jobs or trying to find a job, right? And I applied with the [Pacific Public Schools] to be a teacher and I got a call in November to come in and interview. (Interview Data)

It would be during this interview that Ivan had one of the first real exposures to the professional side of education and how little he truly knew of the art of teaching.

I had to take a test in this interview that asked me certain questions about my psychology. But I never knew what was going on, right? One of the questions was, "If a student comes into your class and cheats on the exam, what would you do?" And I said, "Well, I'd take them to the principal, immediately. And try to get them kicked out as quick as possible. Because they're violating this rule," right? And [the interviewer], at the end of this thing, he says, "You barely passed this exam." And I said, "What?" And he goes, "Listen, that question that I gave

you about the kid cheating on his exam, you don't know what that kid is going through! And you don't know if that kid isn't studying enough because they're certain things going on in their family life that is causing them to not have time to study. And so the lack of thought or awareness of that is a big issue. And you have got to think about that." And that was one big step back, a shock in my life. (Interview Data)

Ivan earned the position as an emergency hire and taught for a year at Pacific City High School (PCHS), but his teaching was still a far stretch from what it would become. Ivan described how he learned how to teach as an act of mimicking his recent experiences as a student.

I was an emergency hire, so this is something where they were just trying to fill in a spot. I taught three sections of Pre-Algebra and two sections of Algebra I. [As far as teaching, it was] not even learning on the job. I think my whole mindset was "I'm going to mimic what I've seen in my life." And, quite frankly, "what I've seen recently in my life." And that was more lecture. (Interview Data)

Aware of his youthful appearance, Ivan began trying to find ways to distinguish himself from his students.

I started dressing up in a tie when I started going to [PCHS] because I was twenty-three and students were seventeen and eighteen and I needed a way to physically differentiate myself from my students. I also grew out a beard at that time because I wanted to differentiate myself and because if I didn't do those things, I'd look sixteen! I just needed some way to distinguish who I was versus them. (Interview Data)

Further describing his time at PCHS, Ivan shared his beliefs at the time about the art of teaching, the development of teachers, and the nature of mathematics saying,

So there's a lot of surface-level things and that's kind of a theme with my beginnings of teaching, was that I was trying to do a lot of surface-level stuff. So for example, I'd introduce some kind of topic, but I'd introduce it the same exact way; the dry method that I thought of, but I'd throw in a joke. Or I'd talk about something and then I'd be like, "Oh, but here's some of the steps," right? It didn't have much from uh the students, although the students asked questions. It could have been considered an interactive thing and I would always have playful things like, "In this section I want you to do the prime numbers." So they had to do the problems that were prime and some of them did some problems that

weren't prime but they thought they were, so they did extra work. And I thought that, improving on teaching would be to improve on my jokes or improve on my handwriting, right? And while those are, I think, somewhat important to create an environment but they have little bearing with mathematics. It has perhaps some affectual bearing, but it has little to do with the mathematics involved. And so all the mathematics, I felt was kind of dry. I thought that if I honed those other parts, then the math teaching or my teaching was better.

I did do one thing, which I thought was interesting, even then. I held "office hours" but they were after school. And they were at a Starbucks that was located right around the school. And for one or two days a week, I would stay there and students would come to that Starbucks. And you know it was a densely populated city, so they would just walk over to Starbucks and they would work on their problems. (Interview Data)

Ivan shared his reasoning in leaving the PK-12 classroom and seeking a return to graduate studies, identifying both individual factors (such as goals and aspirations) as well as social ones.

I liked [teaching at PCHS]. I didn't like some of the bureaucracy, and I think I was naïve at that time. I was considering [teaching high school as a career path]. And I had to make a big decision. I can remember there was a headline on the local newspaper, and it was in the lounge and I looked at it. It said the average price of a home was some astronomical [amount of money] for a house in [Pacific State]! And I kept thinking to myself like, "First of all, do I want to stay here, live in [Pacific State]?" I loved it there, but I don't think I could have gotten any job, at least with my skill set and what I was motivated to do, that would have paid or would have been appropriate for me to live there. Teachers were still on this state salary and this regiment of "you only will get a salary increase every year of this amount and so forth." And that wouldn't have been affordable by any means. I loved the students. I loved the people around there and it was definitely something different, but I had thought about it. Yeah, for sure.

There were financial, but also socially motivated [factors]. If there's any way I can describe [Pacific State] it would be like Never-Never Land. Like the Peter Pan version, where it's like anything you dream or anything you want. [The weather was beautiful] and I felt like I was in Never-Never Land there. I was doing the same things week-after-week. We had our own routine. We'd go to the beach on Sundays and Monday through Friday, I had to work and I'd go out with people or have a good time, right? And I thought to myself, "Socially, am I okay with doing this for a long time?" And the answer was "No." It kept playing the same loop and I wanted to break that loop and so that motivated me to go back home and then look for other [graduate] schools. I can't think of anything else off the top of my head that dissuaded me [from persisting in secondary education].

I wasn't too pleased with teaching high school. I wanted to teach in college. And so, I thought that the only way I could do that is getting a Ph.D. in math so I reapplied. (Interview Data)

### *A Tipping Point*

Now with a third incentive for earning a Ph.D. in mathematics, Ivan was looking to again apply to a graduate program. He was encouraged by a former teammate from the MPU soccer team to apply to Western Land Grant University (WLGU), where he gained admission. Once he arrived at WLGU he was directed to take a course, Understanding and Constructing Proofs, with Glenn and Nan Tucker.

I applied to [WLGU], first of all, in June to start in August and, secondly, I applied to [WLGU] because a friend of mine from the [MPU] soccer team was at [WLGU] and said, "You should try applying here."

I get to my second [graduate] school and they suggested that because of my first [graduate] school and how well or poorly I did that I would take a course called Understanding and Constructing Proofs, a topics course, with [Nan and Glenn Tucker]. [Nan] was sick at the time, but [Glenn] was there. (Interview Data)

Understanding and Constructing Proofs was a topic of several of Ivan's stories about his time at WLGU. Ivan described the course in a way that clearly illustrated that he was very thankful for the experience, but was frustrated with it while he was a student, at least initially.

At first, it was, it was nerve-racking. And the course was different than anything else I have done. But it was so nice to have a class in which everything was slowed down and that you could focus on the proof. And I think that's one thing he tried to communicate all the time. We were focusing on the proof.

First of all there were one or two theorems that you had to do. And you had to do it for the next time, but you just tried them, and then we'd go in there and someone would show one of the theorems on the board. And then [Glenn] would come up and critique it. And then say, "Ok, rewrite it and do it again." Or if nobody had the theorem we would co-construct it. There was kind of a two-way thing going on. There's the "did you get the proof" and the other part was, "now can I just style it so that it molds what mathematicians do?" We never got feedback other than the presentations and then we'd go back home and redo them, and what the heck was going on? Like, where was the A? Where were the three points or whatever?

They would assign theorems without any pre-acknowledgement. And then we'd come in and we'd be like, "ugh," you know? Until that stuff clicked and then we were like, "oh, there's where it is!" And so it was tough for me. I needed external validation. (Interview Data)

Ivan described a shift in his mindset that allowed him to be successful in the Tuckers' course as well as empowered him as he continued through his graduate studies.

I needed to do well, but I was super open to any kind of new experience, and I think that might be a semi-qualifier for many students in IBL courses. Is to at least within the first few weeks be open that something's going to be okay, right? When I walked in and, and took that first class it was definitely different but I was in this mindset that I'm open to whatever experience is going to go on because I need to be successful.

So again it was still, "I need to satisfy what they want me to satisfy" and I guess in some sense that's always going to happen with an IBL course, but with some students that shift goes from extrinsic to intrinsic and it happened with me. In about a month and a half to two months of that class, something just clicked. That was huge! And I started getting successes and I was building my confidence. And on building confidence, you get more success. And so it changed a lot and I started presenting a lot more and I started being a part of it a lot more.

I was so empowered from that class that I passed all my [Qualifying Exams] first time, in Algebra, Complex Analysis, and Topology. I also passed my oral exam and my language exam and also my written oral, like the general exam that we all do at [CU]; all of those in mathematics and all of them easily. I was empowered to do other things. Like, I figured that fixing a car means a finite amount of steps and reasoning and therefore I could do it. And nowadays, I still do stuff like that. I fixed my refrigerator last year when it had a broken panel. I just went through online, researched it, looked it up, looked at some videos, looked at some things, and fixed it myself. Because I was empowered from that class, and that was the origin of me teaching this way. (Interview Data)

However, Ivan did not do much teaching at WLGU. He was admitted without assistance and was not guaranteed any teaching load with the department, which he directly contributes to his decision to apply to work on a National Science Foundation grant.

I didn't have many classes at [WLGU]. Every semester I was in the math department, I was getting reappointed on need, meaning I wasn't guaranteed a [teaching assistantship]. And this, part of me can understand it. I'm there, and they're worried about me even passing. And I was doing well, but they still were very hesitant, right? So, I wasn't guaranteed a [teaching assistantship] every

semester, and I was getting really mad about this. I was kind of getting frustrated at worrying about when the next paycheck or where the next paycheck is going to come from. And this email came through, addressed to all math grads and it said, “Would you like to be a part of this grant? You’ll be guaranteed a [teaching assistantship] for two years.” I said, “I know nothing about this; I want this.” And so I asked the chair if I could be a part of it, and he said, “sure” and then he forwarded my name and then we went from there. And so, I met [Bridget] first. [Bridget] asked me two questions. And then she started like telling me, “We need to get you a computer, we need to get you an iPod, we need to get you this, this, and this” And I was like, “Uh, am I hired?” Right, I was so worried about what was going on. (Interview Data)

Ivan described the grant in broad generalizations over the course of the interviews with the exception of when he discussed Susan, who also worked on the grant and shared an office with Ivan. Ivan claimed to have learned a tremendous amount from Susan, such as having a robust and communicable understanding of fundamental mathematics concepts.

I was starting with [Bridget] and the grant right then, right after passing my [qualifying exams]. I get to meet [Susan] a couple weeks into this. My first impressions of [Susan] were that she was very energetic. I can recall even what the office that we shared looked like, and just having those conversations. She was always called me out on my BS. [For example,] we were sitting in our office, and she asked me, “What’s a ratio?” And I said, “Weeeellllllllllll, uhhhhhhmmmmmm. Y’knoooooooow, I can’t tell you right now, but I’d know one if I see one,” right? “I know the colon and you compare things, or whatever.” I think she asked me this after I said that I was exasperated because the students didn’t know some property and I couldn’t believe that these seventh graders couldn’t know this property, right? It’s this permeating thought that students are not capable, right? And then afterward, she said, “How can you judge their capabilities on these concepts when you don’t necessarily know these concepts?” She didn’t say it like that; she was a little more colorful, but what it did for me is: it was one of the first steps of saying to myself, “Look, I’ve got to think about what the students are thinking mathematically. But I also have to be prepared.” And both of those things have to be in focus. (Interview Data)

It would be from Susan that Ivan believes that he developed a deep caring for his students; a kind of deep caring that she modeled in her interactions with him as well as her interactions with the students with whom their grant interacted.

She called me out on my BS, but she also supported me like you wouldn’t believe and she, I mean whether she did or didn’t, I don’t know, but I believe she

genuinely loved me and she loved [my partner]. And she wanted to do anything and everything for us. We still have a shelf, in our house from [Susan] because she was trying to get rid of everything when she was moving. And so it was more than just work. It was this kind of care about your livelihood, right? It was the first instance that I can think of that I could see of people genuinely caring about you holistically. And even with students. So I'm thinking about my own, where this comes from in my own teaching, I genuinely care about them. And it might have started from seeing how [Susan] did things. There's a lot of care and support that can be given even though I am a math teacher, right? And I think that I learned a lot of that from her.

[Susan] was one of these people where I saw how much a person would pour in and how much they believed in what they were doing, and that's the amount of care and belief I'd like to think I put in. She was wholly invested that these middle schoolers, that she had no clue who they were, were going to get some kind of education from her. (Interview Data)

Ivan further shared what he believed he gained from his experience working on that grant, especially as it pertains to his teaching, saying,

I was very fresh to math [education] when I got into [the grant]. Literally the first year I did [the grant] was the first year I got into mathematics education. Right before that, I just passed my Topology [qualifying exam]. I think reflecting on it has helped me tremendously. I one thing was that I got to see was I-don't-know-how-many math classrooms at the elementary to middle school level, which was an eye-opener to me and it still helps me with my understanding of horizon content knowledge. So I know slightly where they're coming from and I also know, and this is when I talk to teachers, especially pre-service teachers that have taken courses in undergraduate math [education], that I kind of have an inkling about what they are getting into. And so I can talk with them about certain issues and certain constructs.

I think that that experience in the classroom helped me out a ton. I think it helps me with empathy, more than anything. We worked in school districts that had, like, 92% free and reduced lunch and 98% Hispanic and-or Latino, Latina, Latinx. So, to me, coming from [where I come from,] I never had those kind of situations, and each one of those were sample points to help me figure out what I can do in my undergraduate math course and where my students might be coming from. (Interview Data)

After completing his qualifying exams, Ivan needed to select a dissertation advisor, however his interests were still spread among a number of topics in mathematics and his vision for what his future held was still unclear.

I was looking for a dissertation. And I was also thinking, “I’m doing these [qualifying exams] and I really like all of this stuff.” So there was one thing about the [education] stuff that was pushing me, but I also wanted to make myself marketable. So I actually pursued for the first year, a dual dissertation. I’d asked for permission and I pursued a dual dissertation. I said that I would do dissertation in Algebra, in non-commutative fields, skew fields, right? And RUME, because I admired the [Tuckers] and I wanted to learn more about proof and proving. I also loved Algebraic Number Theory. And so I approached [the Tuckers] thinking that I would think about this, and they were like, “Do a reading course with us.” And I did. And then I did a reading course with my Algebra advisor, and then, after a year, I dropped one the two, because it was getting really stressful. I chose to drop Algebra and chose the RUME side, because I could see myself being more successful with it. (Interview Data)

Ivan shared how he came to know what RUME was as a hands-on endeavor with the Tuckers. Ivan said,

I took up the [Tuckers], but I still didn’t know what RUME meant for a while. And what was really nice with the [Tuckers] was that, not only was I reading articles as topics classes, but they immediately immersed me in a research project with them. So I was collecting data for them every day or every Monday, Wednesday, Friday, and then Tuesdays we would meet. We would hook up the camera and we’d actually rewind or go over some part that we felt was very important. And we dissected it like you would a basketball play or something like that. It was incredible, because you saw the nuances of what they were thinking with what was going on. The way they dissected how they taught, both research-wise and teaching-wise, I was privy to, and the way they talked about how they were teaching was so fascinating. (Interview Data)

It was clear that Ivan had gained a tremendous amount from his advisors. He discussed what he gained from Glenn first and most explicitly, by virtue of knowing Glenn first as his teacher.

One thing that I love and I absorbed from [Glenn] is his patience. He claims that he’s slow at mathematics and therefore he has to take a long time to absorb what’s going on. So, when we presented something, he’s super deliberate about reading line for line and trying to make sense of it. And the fact that math doesn’t have to be fast, I saw in the flesh with [Glenn]. It’s one thing I kind of absorbed and so it brings up another point. I think that a lot of people get frustrated with content coverage with IBL, but I loved the fact that [Glenn] chose one or two problems max and took a lot of time and a lot of effort to do the explanation of those. (Interview Data)

Ivan met Nan some time later, but she knew of Ivan through the stories that Glenn had shared with her.

It was actually a math gathering, where I had met [Nan] first. She was ill that whole [first] semester and so it wasn't until later on, when I had met them at a math gathering. It was just this kind of informal, we have cookies and tea and something like that. It's like a tea that we have here [at CU]. And so I got to meet her and [Glenn] had talked to her about me because both myself and friend of mine took the class and we had both done extremely well, even on the final and so they had talked. So she knew me from the class. And so we got to meet there. (Interview Data)

When Ivan next shared what he had learned from his advisors, he talked about them as a complementary pair; one focusing on the details and the other considering the gestalt.

[Nan] and [Glenn Tucker], they're incredible. They are mentors to an incredible degree, and they worry about me. They were everything in terms of RUME. They introduced me to everything. In the teaching aspect, I would have never thought about IBL if it weren't for them and going through their class and experiencing that, and experiencing the successes, mathematics-wise. The way [Nan] approached what was going on with my writing, the scratching out and everything else, and the way [Glenn] approached my writing, which was very holistic, like, "This whole section needs to be here, because it doesn't flow as well." They were gigantic in my research aspect. (Interview Data)

However, Ivan and the Tuckers had different visions for his immediate future. Ivan spoke to this point as he discussed what he had learned from them.

They've shown me certain things. They claim that they didn't have time to show me other things. I kind of rushed with my dissertation and everything else, because of how things were lined up in my life. [My partner] and I wanted to get married and set up a family, et cetera, et cetera. We felt like we had a nice little plan to do that, but that would require me to get a job sooner.

I would assume [the Tucker's wanted me to stay] so that we could start publishing or maybe think about certain things in-depth and having more of a coherent plan. Because I had ideas but I didn't know where I was going to go with those ideas. And so, post graduation, I was told to publish what I had written in some journal, and then go from there. Post graduation, I had so many ideas and so many thoughts, but there were so difficult to try to spring into action. (Interview Data)

It is during this time that Ivan shifted his own teaching practice from instructor-centered

to student-centered. He described the first time he taught using IBL and where his actions came from saying,

I did it in Summer [semester]. It was the first time I was ever in the non-coordinated course and I had gone through the IBL conference [that year] with the [Tuckers], and I bought in. And also, this is while also working for the grant. Influenced by [the grant], because it was my first year and that summer was the end of my first year of [the grant]. Influenced by [Nan] and [Glenn]. Influenced by the IBL workshops and other conferences I went to via [the grant]. Literally my whole teaching is just a bunch of little snippets that I get from I-don't-know-where, that kind of congeal in my mind. (Interview Data)

Ivan was asked if there was anything that he recalled from the IBL conference, which he sometimes referred to as a workshop, that he found particularly influential and helped him decide it was time to implement IBL in his class. His response was,

I think [the IBL conference] was one of those things where it was even more buy in. It was solidifying some of these constructs and thoughts that I had. Coupled with some of the literature that the [Tuckers] and I read. I think that was a big part of the conference. I can't pinpoint to any moment but just like in my class or even in any of the teaching I've done, there's a lot of times where I can't pinpoint the moment, but holistically, it felt like it was comfortable and that a lot of people are sharing ideas. Now, they weren't always research-based, but they were definitely ideas. And, in turn, I think that kind of risk taking in sharing ideas probably reflects how much risk taking they're taking in their classroom. (Interview Data)

The class that Ivan first taught using IBL was described as one that centered on the submission of 'portfolios' of student work.

In the class, what I had them do was they had to do two problems for every section, and I mean it was a five week class and you were flying through sections. What I did was, they would have a group portfolio, I don't know how to describe it. It was a weeklong project, where they had to make up two problems from each section, they had to make up an easy problem and a hard problem. And I never defined either of those, right? And then, for each section, they had to recap as well. And I tried to promote it so that they could recap it however they wanted to. [For example, one student] did it as a rhyme and then she made up a problem that's associated to this. Her medium was through a blog, and so she would turn in a website and I would go into that website and I would grade it, right? And she had a recap right at the top. She was really surprised at how

well she did. I actually got to contact her a couple years ago, and I talked with her and a friend of hers. They said that it changed not only their perceptions of math but it changed how they proceeded with other classes, and I thought that was kind of cool. (Interview Data)

Ivan described how students managed the amount of work he assigned and his expectations given that this was his first experience teaching using IBL.

I honestly tried stuff but I also wanted to be aware of their time and my time. Meaning, if I assign too many problems then they may not do them or they'll do them if it were a machine. I think in this class, I assigned too little, but I was also aware that they were smashing things into five weeks and they have lives and at [WLGU], I think that some of the students do work outside of the classroom quite often. I felt that more so than at [CU]. But I mean, this is all relative. I know that a lot of students still work outside of classwork. Anyways, but that's why I did what I did there, but I think from the IBL workshop and from my own experiences with the [Tuckers]; I tried to figure out ways to incorporate those elements. (Interview Data)

After completing his Ph.D., Ivan was accepted into a postdoctoral position before landing at CU, where he continued to teach using IBL and challenge the pedagogical norms of the mathematics department.

## Discussion

Ivan's development of identity occurred as an accumulation of incremental changes accompanied by times of incredible development due to crisis. Crisis is "accepted as designating a necessary turning point, a crucial moment, when development must move one way or another, marshaling resources of growth, recovery, and further differentiation" (Erikson, 1968/1994, p. 16). Although there were several crises present in Ivan's narrative, the discussion of Ivan's developing teacher identity will be situated around the major crisis of his failing out of PSPU. It is with this crisis that Ivan provided the most narrative time and from which numerous other crises seem to have spawned. These crises form the instances upon which I examined Ivan's developing teacher identity.

### *Mirrored Student (Pre-Crisis)*

Prior to attending PSPU, Ivan had no formal experiences teaching, although we can glean some insight into his notions of teaching from his recounting of the teaching project he completed in high school. Ivan says, “at the time it was mimicking our high school teacher and his nuances,” which indicates that he had been passively engaged in an ‘apprenticeship of observation’ (Lortie, 1975). However, this ‘apprenticeship’ was not oriented toward pedagogical actions but rather the idiosyncrasies of his high school teachers. This is not at all surprising as it was not until Ivan was in the midst of his crisis that he develops the desire to teach as is evidenced by the fact that he doesn’t state intentions of teaching college until he was teaching at PCHS.

Additionally, many of Ivan’s early decisions are predicated on ease or convenience, but moderated by fiscal concerns. He attended MPU because the university was affordable, due in part to a scholarship, and provided him credit for his AP exams. It is during this time that Ivan seems to have a minor crisis as he tries to find a direction for his studies as well as adjust to the rigor of undergraduate coursework: “I thought about what I want to do, and I didn’t know. It was really tough because in high school, I was very good at a lot of things.”

Encouraged by praise from the mathematics department at MPU, Ivan develops a novice mathematician identity, which he held tightly to for a sense of his academic self. Ivan decided in undergraduate school that he would pursue a Ph.D. in mathematics, despite his struggles.

Hence, it is clear that among the I-positions that Ivan holds prior to his removal from his first graduate school are that of the student, the outsider (to the norms and university), the team-sports athlete, and the mathematician. Ivan’s student identity was in flux due to his difficulties adjusting to undergraduate work and he struggled to find success, “I had these terrible grades, but I’d always go to office hours; I was persistent though!”

With respect to his identity as a student as well as as a mathematician, there are some shared beliefs: that mathematics is routine and procedure, that learning mathematics occurs through modeling and replication, and that he could be successful (see Bandura, 1977).

Additionally, Ivan's budding mathematician identity maintained strong individual aspects, "I really wanted to be a part of math," but also strong social aspects: he sought help from his professors who would "half help [him] on a lot of things" and he entered into mathematics as a field of study through the social validation of an award. It is noteworthy, also, that although Ivan frames his entry into mathematical study with this social context, as his story progresses to the end of his undergraduate experience so does the context of his verbiage – it becomes increasingly focused on himself which indicates that his identity is likely turning inward at that time.

### *Failing (Crisis)*

Ivan arrived at PSPU with the I-positions he developed at MPU. As the coursework in mathematics became increasingly conceptual, his belief of the procedural basis of the subject and his focus on the monetary cost of his education, exacerbated by the lack of funding for his studies, were immensely detrimental to his ability to succeed. As such, he was removed from PSPU in fairly short order.

It would be while at PSPU though that Ivan first was forced to formally play the role of the teacher. Although he only received a teaching load of one class while at PSPU, Ivan doesn't describe himself as a teacher at all in that context. Rather, he mentions it only in passing as he introduces the crisis of being "kicked out of school." It seems that even though Ivan had played the teacher for his class, that his teacher identity was merely an extension of his student identity. This seems to be a reasonable response as his teaching status was inherently linked to his status as a student.

The violence in his chosen phrasing, "kicked out," followed by the softening of his voice and the extending of his pauses as he described having to call home to inform his family that he had failed out of graduate school are indication of how immensely traumatic the event was for Ivan: "it's one of my lowest points of my life. The hardest thing I've ever done in my life, other than having a kid." In this, Ivan is trying to re-establish a social assurance of self with his family since his status as a student was stripped of him.

Ivan was searching for a way to make a living with his skill set and training. Fortunately, his part-time job provided some security but it did not fit with his academic training. As such, he continued to apply for positions until he received placement as an emergency hire at PCHS.

Ivan described his interview for that position and the tone and tempo, “What?,” of his retelling provide evidence as to the scope of the perturbation in his belief set upon being criticized for his responses, “that was one big step back, a shock in my life.” This shock was incredibly important to his future development of a robust teacher identity in that it began to move the locus of his attention from himself to his students and it forced him to consider that other factors may be important in the student-teacher relationship beyond behavioral modeling and replication.

That said, it is clear that although Ivan now had an nascent teacher identity, since his position as a teacher was no longer contingent upon his position as a student it is reasonable to make this distinction now, it still mirrored his student identity in many ways. This is perhaps because Ivan was forming this identity via his past experience as a teacher, wherein his student I-position was a dominant actor. As such, Ivan describes his mindset upon entering the classroom as, “‘I’m going to mimic what I’ve seen in my life.’ And, quite frankly, ‘what I’ve seen recently in my life.’ And that was more lecture.” Although it is possible to connote that the phrasing of “And that was more lecture” serves to separate the actions that Ivan was taking in the classroom from his beliefs, it is important to remember that the teacher identity of Ivan was significantly different in his telling the story than whilst he was living it. As such, no further meaning has been drawn from this.

Ivan’s actions, as a reflection of his beliefs and (hence) his identity, reflected the behaviors he perceived as important in his schooling. “I held ‘office hours’ but they were after school. And they were at a Starbucks that was located right around the school,” which indicates the value he placed on students seeking help from their instructor as he had to do to successfully complete his undergraduate program. Additionally, he focused the improvement of

his craft upon superficial facets of teaching, such as his charisma and handwriting, “so all the mathematics, I felt was kind of dry. I thought that if I honed those other parts, then the math teaching or my teaching was better.” As such, it is seen that Ivan’s conception of mathematics had not been changed since his undergraduate days, which indicates that his identity as a mathematician was likely in stasis or possibly even regression.

Ivan decided not to reapply for his position or to seek certification to continue teaching secondary school. The factors that he contributed to that decision are both social and personal. “I liked it, didn’t like some of the bureaucracy,” which seems to imply that although he was personally fulfilled with his work there existed a (social) mismatch between his expectations of the profession and the interpersonal or procedural realities of it. Additionally, Ivan states that he (personal) was not going to be socially satisfied with the routinization of his life in Pacific City. Finally, he presented justification of his choice to leave in terms of finding affordable housing. This indicates a complex interconnectedness of his personal and social identities. It does seem to indicate that both personally and socially (as an educated professional) he believed that he would be presenting a self, through his lack of possession of a home, which was below his stature. It is unclear if the norm in Pacific State is home ownership and that teacher salaries lag far behind comparable salaries in the area, which implies that this was a, predominantly internalized, social want. Otherwise, if rental was the norm in Pacific State, then Ivan’s statement seems to indicate that this was a predominantly personal want.

Ivan recognized that he at that time held a nascent teacher identity, which he wanted to develop further, “I wanted to teach in college. And so, I thought that the only way I could do that is getting a Ph.D. in math so I reapplied.” Prior to this statement, Ivan had not shared a desire to teach at any level, which indicates that something had shifted within him. A new I-position, that of the teacher, was negotiating presence within the dialogical space of his self.

### *Redefinition (Post-Crisis)*

Ivan returned to graduate studies because of this desire to teach in college. Ivan's storying of his re-applying to graduate schools leaves unclear the reason of his late application. Had he not decided until June that he was not going to try to stay in Pacific State? Ivan seems to imply that he applied so late because that's when WLGU was recommended to him, "I applied to [WLGU], first of all, in June to start in August and, secondly, I applied to [WLGU] because a friend of mine from the [MPU] soccer team was at [WLGU] and said, 'You should try applying here.'" What is clear from this, through the use of the word "friend" and the fact that they still communicated even though they had not played soccer together for some time, is that Ivan still socially identified with his old teammate. Also, Ivan still did not have a clear vision of his goals or mathematical interests when he applied to WLGU. He applied there because of a friend, not because he wanted to work with a specific instructor or was interested in certain fields of mathematics. This seems to reinforce the observation that his mathematician identity was still immature and perhaps that he still believed that mathematics was performed, learned, and taught through routinization, modeling, and replication, due to there being no evidence of change in or impetus to change this belief.

Once accepted to WLGU, Ivan was told to enroll in Understanding and Constructing Proofs, which provided him a basis to re-define his mathematician and student identities, which were no longer suited to navigate this experience. Hence, this new experience can be viewed as a crisis itself. "At first, it was, it was nerve-wracking. And the course was different than anything else I have done."

Ivan describes personal reactions to the class. "When I walked in and, and took that first class it was definitely different but I was in this mindset that I'm open to whatever experience is going to go on because I need to be successful." This need to be successful stems from his desires: "I really wanted to be a part of mathematics," "I wanted to have a D-R in front of my name," and "I wanted to teach in college." Additionally, he was reeling

from the crisis of failing out of PSPU and needed success to stabilize his sense of self.

His reactions were also social especially early in the class. “I needed external validation;” “it was still, ‘I need to satisfy what they want me to satisfy.’” Additionally, Ivan mentions a friend from the class when he discusses how Glenn had introduced him to Nan. This friend was not mentioned previously nor later in Ivan’s narrative, which implies that the success that Ivan had in Understanding and Constructing Proofs was a social success as well as a personal one.

While in the Tuckers’ class, Ivan began to observe more nuanced aspects of Glenn’s teaching, “the fact that math doesn’t have to be fast, I saw in the flesh with [Glenn]. I loved the fact that [Glenn] chose one or two problems max and took a lot of time and a lot of effort to do the explanation of those.” Glenn served as a role model to Ivan who embodied contradictions to Ivan’s beliefs about mathematics. If mathematics was strictly procedural, then it could be fast *and* exact, however Glenn was not fast. Additionally, the effort perceived by Ivan that Glenn put into his explanations demonstrated that mathematics is not exclusively procedural, but also significantly conceptual.

It is apparent, also, that through this class a major shift occurs in the predominance relationships between Ivan’s I-positions.

In about a month and a half to two months of that class, something just clicked. That was huge! And I started getting successes and I was building my confidence. And on building confidence, you get more success. And so it changed a lot and I started presenting a lot more and I started being a part of it a lot more.

This shift extended beyond his student identity and into his generalized self and persisted, “I was empowered to do other things. Like, I figured that fixing a car means a finite amount of steps and reasoning and therefore I could do it. And nowadays, I still do stuff like that.” This is due, in part, to the previous ways that Ivan negotiated these relationships failing him both at PSPU and in Understanding and Constructing Proofs.

This shift also put his teacher identity in incongruence with his beliefs as a student and a mathematician. Ivan continued to teach in a similar manner to that which he did at

PSPU and PCHS even after the Understanding and Constructing Proofs course. However, because of the close relationship between his student identity and his teacher identity, Ivan was beginning to question his beliefs and actions as a teacher, “I was empowered from that class, and that was the origin of me teaching this way.”

In an effort to receive some financial stability, Ivan applied to work on a grant, which provided him the opportunity to meet Susan, who would act as another influence for Ivan.

[Susan] was one of these people where I saw how much a person would pour in and how much they believed in what they were doing, and that’s the amount of care and belief I’d like to think I put in.

When Ivan describes the care of Susan, he is referring to a genuine and holistic care for the students, their goals, their experiences, and their lives (see Noddings, 2005). Ivan clearly perceived Susan as a teaching role model, perhaps due to working proximity to Ivan but also due to her experience and expertise as a mathematics educator relative to his own. As such, Ivan began to model parts of his teaching identity from what he saw in her,

I genuinely care about [students]. And it might have started from seeing how [Susan] did things. There’s a lot of care and support that can be given even though I am a math teacher, right? And I think that I learned a lot of that from her.

Ivan’s restorying of his time with the grant and with Susan was set in an almost exclusively social context. He discussed at length the interaction between himself and Susan and the changes he believed he had made due to them. His pronoun shifts from singular to plural when talking about the grant, which implies that his time with the grant provided him a sense of community and belonging. In fact, in one of our follow-up interviews I had to ask Ivan explicitly what *he* did on the grant.

Ivan’s time with Susan was transformational in his teaching identity. Ivan shared multiple times the story of Susan confronting him about his irritation about students having difficulty with ratios. Perhaps this is the most poignant example on his mind of Susan “call[ing] [Ivan] out on [his] BS,” but it is also very informative as to the beliefs inherent within that teacher

identity at the time. “I can’t tell you right now, but I’d know one if I see one,” implies a superficial understanding at best that focuses on the instances of a definition as opposed to the meaning behind it. His shock that the seventh graders with whom the grant was working at the time didn’t know it indicates that his expectation was that the students have perfect mastery of previous content and have no limit to the content that they were ready and able to master (see Ball, Thames, and Phelps, 2008; Shulman, 1986). In Ivan’s own words: “It was one of the first steps of saying to myself, ‘Look, I’ve got to think about what the students are thinking mathematically. But I also have to be prepared.’”

Ivan’s teaching identity had been developing while working for the grant, however, he had not had much opportunity to exercise his practice. One summer, however, Ivan was finally ready to bring his teaching practice in coherence with his beliefs as a student, a mathematician, and as a teacher. He had been inundated with IBL research through his attendance at the IBL conference that year and knew that it could be successful through his own experience in the Tuckers’ class. He had also begun doing research with the Tuckers as well as readings with them. “It was the first time I was ever in the non-coordinated course and I had gone through the IBL conference [that year] with the [Tuckers], and I bought in. And also, this is while also working for the grant.”

Although his desire was strictly personal, Ivan’s efforts to align his teaching with his beliefs caused him to pull from experiences, which were numerous internalized social events. This class also represents the first clear indication that Ivan’s teacher identity was truly distinct from his student identity, although it was strongly mediated by it. Many of the tasks that Ivan had his students complete were not replications of the tasks that he did as a student (e.g.: focus on group work, open-form portfolios, etc.) and they were clearly influenced by Susan’s impressions upon him, “I wanted to be aware of their time; they have lives and at [WLGU], I think that some of the students do work outside of the classroom quite often.” Prior to this class, Ivan did not express this concern for his student’s external (to his class) lives.

## Conclusion

For Ivan, it seems that through the crisis of failing out of graduate school he was in the position to experience the challenge to the truth of his beliefs. This possibly made him more receptive to alternative perceptions of students as beings with agency in their own lives as well as circumstances that weigh upon the choices that such agency provides. This, coupled with his own development of self as he moved from his state of crisis opened him up to accept new models for instruction. Pushed to grow further by Susan, Ivan continued to develop a new teacher-identity, now extending it beyond the student-based teacher-identity that he once held. As such, Ivan was able to apply theoretical components into his envisioning of practice. It is only at this point that it becomes clear that Ivan has developed a well-defined, although immature, teacher-identity, rather than a mere mirroring of his student-identity.

It remains to question if Ivan's experiences are unique in this generalization as well as if other mathematics faculty have developed teacher-identities through similar crises. Similarly, in the absence of such traumatic crises, are the teacher-identities of academics significantly different from that of their own student-identities and, if so, what mechanisms have helped them forge these teacher-identities?

As this is the exploration of one instructor's experiences, it is clear additional research is needed before trends can be identified and theory can be developed. However, it is also clear from Ivan's narratives that the significant crisis afforded to him through failing out of PSPU and the smaller crisis through his brief experiences at PCHS provided the impetus for Ivan to create an independent and well-defined teacher-identity.

### Chapter 3: Acculturating into Professorship

Teachers of all types are faced with a number of pressures that impact their professional satisfaction. In a meta analysis of 34 studies, Borman and Dowling (2008) found that, among a number of other factors, “schools with a lack of collaboration, teacher networking, and administrative support” have higher attrition rates (p. 396). Although the studies in this meta-analysis focused on common education, they are worthy of note for two key reasons: these factors may also affect junior faculty in tertiary settings and because teacher characteristics explain more variance in student outcomes than any school resource variable in common education (Coleman et al., 1966).

Conventional wisdom and anecdotal evidence extends the importance of the teacher to postsecondary settings as well. However, the attempts to gauge teacher effectiveness at the postsecondary level have been disproportionately focused on student evaluations of instruction, which are wholly inappropriate measures as they can be easily manipulated (by adjusting the perceived difficulty of a course for example) and do not necessarily reflect a professor’s ability to promote deep or meaningful learning (Carrell & West, 2010). Research does tend to show that active learning (see Bonwell and Eison, 1991) has been beneficial in Science, Technology, Engineering, and Mathematics (STEM) fields (Freeman et al., 2014). This seems to indicate that at least the beliefs necessary for a postsecondary instructor to choose to implement active learning in their classroom may be a significant characteristic.

Of course, junior faculty finding themselves placed in postsecondary research institutions have additional concerns beyond just their teaching. They are expected to be members of their campus and professional communities as well as to be productive members of their researcher communities. Although junior faculty often have teaching loads smaller than the average PK–12 educator, the splintering of their focus causes significant strain. Sabagh, Hall, and Saroyan (2018) found, in a meta-analysis of 36 studies, that adverse job demands, such as workload, contributed to faculty burnout, which raises obvious concerns for the students

in classes taught by those faculty members. Additionally, non-tenured professors tended to perceive that their universities de-emphasize teaching, leading them to feel as though they have to focus their energy into research and service instead to be successful (Sabagh & Saroyan, 2014).

A department at a university may be conceived as a community where new faculty are having to establish their own identities, reconcile their workload, and adapt to the norms of that community or of the greater college or university communities. Similarly, new faculty members have to acculturate to their research communities, to which there may be very little localized membership. This is exacerbated when the individual has not had significant prior research experience while as a student or while in a postdoctoral position. The ways in which new faculty members acculturate into these communities makes for a question of interest that can be studied through the narratives of such an individual. However, the decision of how to envision this community is not a trivial matter.

### **Communities**

Most groups of any permanence, be they small bands bounded by face-to-face contact, modern nations divisible into smaller subregions, or even occupational associations or neighborhood gangs, may be treated as speech communities, provided they show linguistic peculiarities that warrant special study. (Gumperz, 2009, p.66)

These ‘linguistic peculiarities’ include values, perceptions, accepted practices, and beliefs, which are performed through verbal demonstrations of a number of grammatical rules. As such, it is entirely acceptable to consider an academic department as a speech community (Gumperz, 2009). However, if one is to focus on reflective accounts of the acculturation process, perhaps it is not the most suitable framework. A faculty member recalling their early experiences after having already adopted the linguistic norms of the community may likely reframe those early experiences using the, now internalized, grammatical rules. Additionally, speech communities require a level physical proximity that may not exist within the faculty member’s research interests.

Swales (1990) provides the conceptual framework of a discourse community, which he distinguishes from speech communities in three key ways. First, speech communities, being bound by the ability to vocalize amongst members, do not extend well to communities whose members communicate by writing or are spatially separated. Discourse communities produce texts which can transcend this limitation. Second, speech communities are a form of sociolinguistic grouping while discourse communities are socio-rhetorical groupings. That is, where speech communities forefront the language being used to serve the needs of the group, a discourse community “integrates the ways people use language with the ways they live in the world” (Robbins, 1996, p. 1). As such, “the communicative needs of the goals tend to predominate in the development and maintenance of discursal characteristics” (Swales, 1990, p. 24). Both academic departments and narrow research fields can also be envisioned as discourse communities with aims such as fostering student growth and producing texts, for example research articles and submission letters (Pogner, 2005).

Table 3: Comparison between different groups and communities in organizations

	<b>Aim</b>	<b>Participants\ Members</b>	<b>Basis of Cohesion</b>	<b>Duration</b>
<b>Discourse Community</b>	Producing and disseminating knowledge in discourses and texts	No formal membership. Persons working on similar problems and with shared interests  Fuzzy boundaries	Common (professional) interest. Participation in the joint discourse for problem-solving and knowledge production	As long as there is an interest in the problem. Even if members often change, the DC continues to exist.
<b>Community of Practice</b>	Production, extension and exchange of knowledge, enlargement of the capabilities of the members	Members select themselves on the basis of expertise and interest in the subject  Fuzzy boundaries	Passion, commitment and obligation; identification with the group and its expertise	As long as there is an interest in the continued existence of the group, in the subject and in learning

*Note.* Adapted from Pogner, K. H. (2005). Discourse Communities and Communities of Practice. In Paper presented at the 21st EGOS Colloquium.

Furthermore, communities of practice are presented as a nuanced variation upon discourse

communities. A key distinction is

whether texts are mainly there for purposes or whether text production is also the reason for the existence of the community. In the first case, the community would mainly be classified as a community of practice (with discourse as the means) and in the second case primarily as a discourse community (with discourse as the means and the end). (Pogner, 2005, p. 18)

In this distinction, Pogner is invoking Wenger's (1998) usage of 'reification' to mean, "the process of giving form to our experience by producing objects that congeal this experience into 'thingness'" (p. 58). As such, considering academic departments and fields of research as separate communities of practice is an appropriate choice to consider the experiences of individuals entering into academia.

### **Acculturation**

Social scientists have long been interested in the phenomenon of immigrants entering into established communities. "Acculturation comprehends those phenomena which result when groups of individuals having different cultures come into continuous first-hand contact, with subsequent changes in the original cultural patterns of either or both groups" (Redfield, Linton, & Herskovits, 1936, p. 149). In this study, acculturation takes on Gans' (1997) definition: "acculturation refers mainly to the newcomers' adoption of the culture (i.e., behavior patterns, values, rules, symbols etc.) of the host society (or rather an overly homogenized and reified conception of it)" (p. 877) (c.f. Park, 1928).

There have been two primary frameworks from which acculturation has been studied: unidirectional and bidirectional. Unidirectional acculturation is often used synonymously with assimilation, meaning that subordinate groups become absorbed into the dominant culture (Gans, 1997; Ngo, 2008; Sam & Berry, 2010). As such, there is only one monolithic culture of interest to the groups, the dominant culture. Bidirectional acculturation provides for the separate interests of the immigrant group: identity maintenance and acceptance into the dominant group (Berry, 1980; Ngo, 2008). In the intersection of each of these interests is a different mode of acculturation between the groups.

Additionally, the interaction of acculturation and identity can be examined at a number of different levels, individual, interactive, and societal (Liebkind, Mähönen, Varjonen, & Jasinskaja-Lahti, 2016). The individual level focuses on “psychological processes and personal characteristics” (Liebkind et al., 2016, p.31). That is, questions of a person’s ways of forming a negotiated identity within their acculturation and the expressions of that identity can be explored. The societal level turns attention toward the inter-group interactions of identity. Lastly, the interactional level is concerned with the ways in which inter-group relations interacts with personal characteristics, and vice-versa. A number of studies have been conducted at each of these levels and many have considered multiple levels of this interaction within the same study (Liebkind et al., 2016).

### **Community Culture**

Hence, the claim that new faculty members must acculturate into their community of practice (i.e., their academic departments) necessitates aligning the verbiage from the two fields of study. Wenger (1998) provides three dimensions of practice as properties of a community: mutual engagement, joint enterprise, and shared repertoire.

Mutual engagement is at the core of meaning of Wenger’s (1998) ‘community.’ A community is not just a grouping but, rather, a grouping that is engaged in negotiated meaning-making. This requires ways of making engagement possible, which may require knowledge of information not directly related to the enterprise of the community resulting from membership. Additionally, diversity is necessary for a community of practice where, “each participant ... finds a unique place and gains a unique identity, which is both further integrated and further defined in the course of engagement in practice” (Wenger, 1998, pp. 75–76). Thus, homogeneity is neither precursor to membership nor is it a consequence of membership within a community of practice. Key to the alignment of these frameworks is this recognition that homogeneity is not required for nor a product of a community of practice, as this works in opposition to the driving assimilatory force of the unidirectional model. This does not, however, indicate that assimilation in some facets of identity cannot occur. Lastly, communities

foster mutual relationships, with all of the social intricacies that develop with them.

Joint enterprises are the very things that direct a community and hold it together. First, as a collection of individuals, communities of practices must have enterprises that are negotiated between its members. These enterprises may be stated aims and goals, but may also be implicit. “[The community members’] understanding of their enterprise and its effects in their lives need not be uniform for it to be a collective product” (Wenger, 1998, p. 79). The enterprise is never dictated to the community but is generated from the community, although outside forces may influence it. Lastly, communities have ‘regimes of mutual accountability’ which provides members feedback as to how to react to situations that may arise. These may be reified as rules or policies or may be tacit such as behavioral norms. It is important to clarify that these ‘regimes’ dictate what is considered acceptable rather than how things are routinized. As such, they align well with Gans’ (1997) elements of culture from the adopted definition of acculturation, as they can be perceived as values and rules.

Lastly, Wenger (1998) states that communities of practice must have a shared repertoire. Due to the collective nature of the community, over time, resources are developed to assist in the negotiation of meaning. These resources need not be directly related to the joint enterprise and may include “routines, words, tools, ways of doing things, stories, gestures, symbols, genres, actions, or concepts that the community has produced or adopted in the course of its existence, and which have become part of its practice” (Wenger, 1998, p. 83). As such, it seems reasonable to align Wenger’s ‘shared repertoire’ as an element of culture within the adopted definition.

Hence, for the purposes of this study, given this alignment of the vocabulary of Wenger and Gans, a community of practice is envisioned as a collection of individuals, working together to negotiate meaning from a joint enterprise and having its own culture. Additionally, this study adopts Sam and Berry’s (2010) framework for understanding acculturation strategies, which aligns the attitudes of immigrant groups and the larger society with the desire to maintain heritage culture and identity and the desire to build relationships (see Figure 2).



them. Hence, this study sought to address the following questions:

1. How has he acculturated into his academic (i.e., institutional-departmental and research) communities?
2. How has his professional identity been altered or reinforced through his early experiences as a junior faculty member?

Schwandt (2015) defined a case study as a “complicated arena [of qualitative inquiry] involving methodological choices directly related to goals or purposes of conducted case-based research, research traditions in different disciplines, and the ways in which investigators define a case” (p. 26). Additionally, “an intrinsic case study is the study of a case (e.g., person, specific group, occupation, department, organization) where the case itself is of primary interest in the exploration” (Grandy, 2009, p. 500). Further, the core phenomena of interest were the experiences of Ivan. As such, this research was conceptualized as a narrative inquiry. In a narrative inquiry,

our principal interest in experience is the growth and transformation in the life story that we as researchers and our participants author. ... [I]n the construction of narratives of experience there is a reflexive relationship between living a life story, telling a life story, retelling a life story, and reliving a life story (Clandinin & Connelly, 2000, p. 71).

Since our beliefs are an intimate part of our personal identities, it is entirely reasonable to assert that a fundamental motivator of those beliefs are the stories that we tell ourselves about them and that those stories are the reality of beliefs’ existence. Such stories, being the instructor’s personal narratives, “had better be viewed not as a record of what happened (which is in any case a nonexistent record) but rather as a continuing interpretation and reinterpretation of our experience” (Bruner, 2004, pp. 691–692). As such, Clandinin and Connelly’s (2000) framework of a three-dimensional narrative inquiry space (consisting of situation, continuity, and interaction) along with the four directions of inquiry (being inward, outward, backward, and forward) is an invaluable tool to situate a narrative researcher’s work. “[T]o experience an experience—that is, to do research into an experience-is

to experience it simultaneously in these four ways and to ask questions pointing each way” (Clandinin & Connelly, 2000, p. 50).

### *Participant and Setting*

Ivan was, at the time of this study, junior faculty in a fairly traditional mathematics department (with primary research interests in Algebra, Analysis, and Topology) at a Carnegie R-1 institution in the Southern United States, Current University (CU). Ivan represented fully half of the declared departmental research interest in undergraduate mathematics education (RUME) and it is in no small way that because of his interest in RUME that his teaching fell well outside of the norms at his institution. At the time of this study, Ivan had been at his institution for five years and was undergoing tenure review.

### *Data Collection and Analysis*

Eight semi-structured, audio-recorded interviews were conducted. Seven of these interviews occurred over the course of six months while the eighth was conducted two years prior. These interviews all occurred in Ivan’s faculty office, however, with the exception of two of the follow-up interviews, they continued, at Ivan’s invitation, past our scheduled interview time as we walked together to Ivan’s car. One of the two follow-up interviews that did not occur in Ivan’s faculty office was given entirely in Ivan’s car as he had invited me along to complete a personal errand of his. The goal of the initial interview was to establish a rough chronology of the pertinent, in his own opinion, events of Ivan’s story, while the follow-up interviews systematically established vignettes of each of these events (see Table 4).

Over the course of these interviews several questions and stories were revisited to establish stability in the narrative. Similarly, member checking has been used to further ensure that the narrative co-created was, in fact, true to that which Ivan tells himself. These interviews were transcribed verbatim and then closed coded in accordance with Clandinin and Connelly’s (2000) three-dimensions of narrative inquiry space. Those dimensions are “personal and social (interaction); past, present, and future (continuity); combined with the

notion of place (situation)” (p. 50). Data were then open coded then subsequently axially coded. Using a constant comparative method, new data were compared with existing data for consistency and emergent themes were sought.

Table 4: Enumeration of Interviews

<b>Interview</b>	<b>Main Topics</b>	<b>Duration</b>	<b>Location</b>
Pre-Existing	Use of IBL	01:12:32	Ivan’s Office
Initial Interview	Establish Chronology	01:25:30	Ivan’s Office
Follow Up 1	Mentoring, Pre-professional Grantwork	00:56:14	Ivan’s Office, University Grounds
Follow Up 2	Motivations, Early Professional Work	00:42:27	Ivan’s Office, University Grounds
Follow Up 3	Aims and Goals of Education, Current Perspectives of Teaching	00:49:40	Ivan’s Office, University Grounds
Follow Up 4	Coming to Teach IBL, Modeling Behaviors	00:30:06	Ivan’s Office
Follow Up 5	Approach to Teaching, Reflection on Teaching	00:57:41	Ivan’s Office, University Grounds
Follow Up 6	Pre-Professional Grantwork, Influential Experiences as a Student, Concluding the Interviews	01:38:34	Ivan’s Personal Vehicle

In an effort to better understand Ivan’s development as a researcher, all of his publications from the first two years of his placement at CU were reviewed. These were stylistically compared with his dissertation and other publications from the journals they were released in.

### Findings

Adopting narratives for this study necessitates either a topical or a chronological presentation of the results. Given the cumulative nature of Ivan’s experiences as he acculturated into his academic communities, a brief chronology of relevant items is presented. Within these broad chronological categories many of the events occurred concurrently and the order

of the presentation of the events do not necessarily reflect their temporal ordering, except where such an ordering is implied by the narrative.

In an effort to maintain trustworthiness and to not imply my own ownership of the narratives, they have been presented, where possible, with Ivan's own words and have been subjected to only minor editing to fit the format of this report and improve the flow, since topics were revisited many times in multiple interviews for this study.

### *Becoming a Professional*

Ivan, having recently received his Ph.D. in mathematics, focusing on RUME, found himself in a postdoctoral position at Northern Midwest University (NMWU). This position was housed in the College of Education, but was primarily tasked with improving the teaching of graduate students in the mathematics department.

I taught a one-zero teaching load over there. So I had one class, but, instead of teaching, I was hired to be a [teaching assistant] professional developer. [And so,] I did professional development for [teaching assistants], and I went in and I did observations. I only looked at one metric: "What questions were being asked by the students?" Quality, amount, et cetera. But questions asked by the students was my metric. What that says to me, it says to me almost everything. It says environment, including social and socio-mathematical norms. It says what is valued, because a student is going to interplay for the first few weeks, based on what you value and what they see that you value this, they're going to do it, right? But when they start uttering conjectures out of nowhere, you've got them valuing mathematics.

Ivan's experiences in his postdoctoral position conditioned him to be cognizant of inter- and intra-departmental relationships, which he described as a product of the nature of his appointment.

[What I got out of my postdoctoral appointment was] noticing departmental things, and political things. I mean politics in terms of in-campus, because I had a triple appointment. Part of it was in teacher education, part of it was in math education and part of it was in mathematics. And there's a reason for all of these things. There were difficulties and people starting splitting off. And so, with that kind of triple threat comes a lot of complications, because some people have expectations for you while others have other expectations, right? So, that was part of what I gained from there. I also got to meet a lot of people in

[education]. My [postdoctoral appointment] was not about research, now that I look back at it. It was about connections. And it was about absorbing as much as I could from whoever I could. I knew that my time was temporary. (Interview Data)

Emboldened by his success teaching using IBL in graduate school, Ivan continued refining his practice while at NMWU as he was able.

I also taught a [Calculus] 1 course, but it was highly coordinated. And so we had to be on the same section and we all had the same tests and everything else.

I had success with [IBL] in a Geometry course at [NMWU]. This 3000-level Geometry course and I tried to model it the same way [I had previously]. Like, I modeled it after the [transformational proofs remediation course I had to take at my graduate school with my Ph.D. advisors Nan and Glenn Tucker] in terms of notes. I didn't have the same tests or anything[; that is, the course was not highly coordinated]. I had academic freedom.

It was a summer class, [as it had been with my previous experiences teaching with IBL]. In this kind of, Geometry, but it was called, Higher Geometry. And I wanted to teach it IBL-style. I wrote my own notes and this [class was] just basically, figuring out line ratios and you look at non-Euclidean geometry and we looked at a little Topology. You know, these kinds of things.

I did have a lot of the reins off and so I tried some stuff and there was a lot of success. (Interview Data)

Ivan described the way he approached homework in the Higher Geometry class as a locus of particular deviation from what is typical in mathematics classes. In doing so, he also described his naïveté in the adoption of technology in the post-secondary math classroom. Ivan had his students purchase Livescribe pens, which can create a video recording of the student's writing as they work with the audio track of their verbalized thinking.

I had students buy Livescribe pens as their textbook and I gave them paper and they had to turn in their pens. That was their homework. And then I had ten minute sessions where we'd pull up some of their work on the pen and we'd watch it, much like you study game film, which I thought that would be the next wave of teaching and that was really difficult. (Interview Data)

### *Rethinking Teaching*

After completing his postdoctoral assignment at NMWU, Ivan applied for several positions before landing at CU. Once at CU, he made it clear that he intended to continue to

teach in the manner he had grown accustomed to – using IBL.

When I got [to CU], before I started teaching, I went to our chair. I actually secured a meeting with him and had a thirty-minute conversation about what I would do. And I was pretty confident and I strolled in and I said, “Look, I’m doing this because of this paper, this paper, this paper. And I just want to let you know.” And that first semester was quite tumultuous. (Interview Data)

Ivan’s difficulties as an instructor during his first semester stemmed from his modeling his practice and expectations from the Higher Geometry course at NMWU. In the quote that follows, Ivan mentioned the submission of portfolios from the student. These portfolios were simply a group submission of the completed problems from Ivan’s published notes, which provided problems to the students, but not solutions. These submissions were encouraged to be in a creative format, such as a blog or a YouTube video, but this was not required.

[My first semester at CU], I was trying something that I had had success with in the Geometry course at [NMWU]. I must have supplied Livescribe pens because that’s a lot of money if they bought them. But I wanted Livescribe pens in the classroom and I wanted to collect all this data.

I did two Calculus sections and I taught it IBL, but I taught it using two-week increments and having just a loaded portfolio every two weeks. I had a lot of, I guess, faith placed upon the students and that’s okay, that’s good. It’s just, I think, two weeks is a long time, especially for students who are, mostly first-year students that were trying to figure out what everything was. I would have been the same student. And I would have deferred everything to Saturday night or whatever it is, right? This coupled with a grading scheme that wasn’t really well defined led to pretty disastrous evaluations and complaints to the department. It hit hardest when, four students formally complained about my teaching to the chair and I’ll be honest with you, every time I got an email from the chair, I felt like I was going to be fired in that email. Every time that email came, it was like, “we need to discuss some things.” (Interview Data)

Ivan’s troubles extended beyond his teaching. He maintained an ambitious research agenda and a willingness to accept meetings and service related appointments. These demands on his time as well as his growing family were additional stressors as he tried to find his place in his department at CU. Speaking directly to his issues as researcher, Ivan shared his story of the first paper he submitted to an academic journal.

I had submitted a paper, when I was in [NMWU] as a [postdoctoral scholar], from my dissertation. It was the biggest piece of my dissertation. Well, it was a piece that I concentrated a lot of time on, not the biggest, I guess. But, I wrote this paper and I submitted it to a very large, very good journal. I didn't know what was going on with the reviews, and so I emailed my advisors like seven months after I submitted it, and I said, "What should I do about the reviews? I haven't heard back from them at all." And they said to me, "Well, it's, it is pretty lengthy; it's been seven months. Maybe you should email them." I emailed them and got no response. This was at a part of my life where I did not know what the process was, so, I was refreshing ManuscriptCentral, like every week, to try to figure out what was going on. Now I know to just submit it. Okay, great, you have a hurrah for two seconds, and you go on to the next thing. Anyways, eleven months go by on this paper. Submitted to a top journal in our field and the reviews come back and reviewer 2 had the review, "This paper makes no contribution to mathematics education. Publishing it would rather be a serious set back to it," along with other stuff, let me put it that way. What did I do? I saw the review. I forwarded it straight to my advisors, which, by the way, they're the most fantastic human beings on the planet. Because what they did was they sent me an immediate email that said, "Don't worry, you're fine." I think even [Nan] called me because she was worried about my psyche and how I was. Then I had a talk with my [partner], and I said, "Look. This may not be for me. I may not be an academic." This really was big. I'd never published before and to get an email, a review like that was pretty disheartening. And I remember telling her, "I love making homemade pizzas. Maybe we could do that as a profession or I used to [work retail], so maybe I should do that." But I really felt like, it was my first semester here at [CU] and there's a lot of other things going on in my life and this was just the icing on the cake of that experience. (Interview Data)

Ivan pooled all of the other issues impacting his sense of self and belonging together, which reiterated that the primary difficulties he perceived were those of his teaching and his research. Ivan did, however, briefly share more about his research agenda and familial growth issues saying,

I remember that chalkboard that I had in [my first office] and I had, like, five projects written down. And I was like, "I'm going to do all these projects, this semester." You know, it's ambitious and I talked with the [Tuckers] about it and everything else. I get so excited but I also got overwhelmed a lot that first semester.

[My partner] was pregnant. We had just moved here from [NMWU] and we were just settling in our house. And our house, there was this heating problem, and I don't know if that happened then or later on, but that was all in my first year.

(Interview Data)

### *Finding Footing*

After his disastrous first semester, Ivan was in need of some successes. Fortunately, he found those successes by the end of his first year at CU and he was able to refocus his commitments. First, Ivan described the support he received from his department chair as being beneficial.

That first semester I got four formal complaints. But the chair backed me on every single one of them. That was a huge thing, because, he didn't have to. And I could have gone toward some other path. And even the chair suggested that I should maybe look at a little more traditional things, but he never said, "you have to," right? He said, "No matter what you do, I'll be supportive."

I've never reflected on his handling of my situation. I've always thought that I've been fortunate to have him, when I had that situation. I don't know what else would have happened and, honestly, if it would have been handled any other way, I would have gone into survival mode and I would have said, "Okay, I'm going to lecture right now." At some point you say to yourself, "I am paying for my kids and my family. I'm going to do whatever is needed." (Interview Data)

Additionally, Ivan described his discussion with his department chair prior to his teaching at CU as a factor that, in his estimation, helped his department chair be supportive.

I have got to give kudos to myself, and that's weird for me to say, but for me, not only believing in myself—that I could overcome this, but also for going up to him prior to this and having the thirty-minute meeting and establishing the ground rules. It's an aspect in which I don't think many people think about. When you get into a new scenario, you're establishing who you are, and that can carry almost anything in your future endeavors. (Interview Data)

There were a number of changes that Ivan implemented following that first semester at CU. However, in his second semester at CU, Ivan had an experience that served to solidify his belief that he could be an effective IBL educator.

I did, after that first semester, totally adjust my point schemes, how students did things, what students did things, and I had students turn in homework everyday. And I've continued that.

But that second semester, here's what happened. I was teaching Discrete Math and students knew the game, the routine. And so I had a person sub for me on a Friday. I had to go; I think I was going to [NMWU] for a retirement party, because this person was super influential in getting me a job at [NMWU], so I owed it to her. Okay, I come back and I didn't even ask the students, because I'm always in a mindset of "Oh, okay, what are we up to, what are we up to?" So I didn't even ask the students what happened on Friday. Thursday rolls around and we have our weekly meetings and the person that subbed for me didn't sub for me. They ended up going to a job interview and so I had nobody in the class on that Friday. So then, Friday I come in I said, "Alright, I went out of town on last Friday. What did you all do? Because it seemed like you didn't miss a beat." And, students can lie to me and maybe that happened, right? But they seemed so sincere and so honest and all of them. It was a forethought, right? They said, "Well, we all showed up here. We worked for fifty minutes on the problems and then we left." And I was just floored!

You talk about the success of IBL; when you teach people something and they do it on their own, that is just a gigantic step in a positive direction. When you can remove yourself from a classroom and the students are learning still. You've imparted from external pressures to some sort of internal engine. And that's a gigantic moment. These experiences, the first semester and the second semester, kind of solidified my IBL. So that first semester was an awakening, but I never swayed from my holistic thought that IBL was successful. (Interview Data)

As for the other issues that were negatively affecting him in that first semester at CU, Ivan described how he had come to find footing in those facets of his profession. Regarding his research Ivan shared,

[As for the research,] I talked about this to some colleagues, both in [my professional development cohort for new math Ph.D.s] and with [Nan] and [Glenn] and they were incredibly supportive. And I brought this up at [my professional development program] about feedback from feedback like this, and one of the people [there] was a journal editor and she said, "Why don't you submit something to our journal?" And I did, and I got some success from peer-reviews, which was kind of gigantic. These things are kind of big. (Interview Data)

Further, Ivan described his willingness to accept service-related appointments and his growing family as issues that were resolved once he prioritized his family-life and focused his scheduling.

I was scheduling meetings all over the place and [my partner] said, "you need to take one day and you need to just focus on research. You're at a research

institution. You're spending a lot of time on teaching; you're spending a lot of time on service. Focus that one day on just research." And so I did, and nowadays I have one day that I literally do not accept any other calls or any other meetings or any anything else.

And it has helped a ton because then I get three and then another three after lunch hours of dedicated work. Focused work. And so that's helped a lot, that I've figured out how to do a life-balance. Kids have helped a lot. What its helped with is my kids make time WAY more precious now. And so, when I do have an hour that's dedicated to research, I need to be inundated with it. I need to focus and put all my energies in that. And because of them, I come in at 8:30 and I go home at 4:30. And the rest of my night is dedicated to them until they go to bed. And then the rest of that night is dedicated to my [partner], until she goes to bed. And then, I don't know, I try to do something for me. (Interview Data)

## Discussion

Although Ivan's experiences form a gestalt sense of his development as an academic, it is prudent to separate them into two sets of experiences: that of the researcher and that of the instructor. In doing so, Ivan's perceptions of acculturation into his department as an instructor and acculturation into his research field can be explored.

### *Researcher Acculturation*

Ivan left graduate school with his Ph.D. and, with it, some exposure to research; however he was still very much a novice researcher and had no publications in his field of his own (outside of conference proceedings). As such, it is hard to imagine that he held any semblance of a robust researcher identity or any research culture beyond that related to his time as a graduate student. However, as his area of research interest was RUME, some evidence of the composite elements of this developing identity can be gleaned from his professional development while at NMWU. "I only looked at one metric: 'What questions were being asked by the students?' Quality, amount, et cetera. But questions asked by the students was my metric. What that says to me, it says to me almost everything." This utterance makes clear that at this time Ivan's conceptions of teaching effectiveness were centered upon the students, much like his more robust ideas about teaching.

This facet of Ivan's researcher-self manifested in his teaching as well: "I had students

buy Livescribe pens as their textbook and I gave them paper and they had to turn in their pens.” At the time of my interactions with Ivan, he still retained much of the unidentifiable LiveScribe data from his first semester at CU and during our time together we conjectured ways to use that data in research. This seems to imply that Ivan likely had not had a clear plan to use the data when it was collected – a possible testament to his inexperience at the time.

Given this lack of a researcher identity, it is reasonable to assume that Ivan had no desire to maintain his ‘heritage’ identity (see: Sam and Berry, 2010). As such, his attitudes toward acculturation into his research community were limited to assimilation or marginalization. That is, he would have to adopt the research culture of the larger society or be marginalized within it. A third possibility is also present, that Ivan refuses to adopt a research culture and is separated from the field. In terms of prospective employment concerns, both separation and marginalization would mean the end of Ivan’s position at CU. Also, given the lack of Ivan’s research identity, he could only perceive that the research field would project attitudes of homogenization or exclusion depending on if Ivan felt a belonging with the field or not.

Ivan’s first journal submission came while he was at NMWU, although he would not hear about it until he was at CU. The submission was near the core of his researcher-identity as he had invested a tremendous amount of effort into it, “I had submitted a paper from my dissertation. It was the biggest piece of my dissertation. Well, it was a piece that I concentrated a lot of time on, not the biggest, I guess.” He also submitted it to one of the top-tier journals in his field, which speaks either to his naïveté or his pride in his research. Both of these interpretations are supported by Ivan’s story given the length and detail in it.

When he finally received the review, it was scathing. The very claim that publishing his work would be “a setback to [mathematics education]” provided Ivan the perception that he would be excluded from the research community. Although his identity was reinforced by the concern of his advisors for his well-being, Ivan’s response was that of separation from the group. He did not have a robust researcher-identity, nor avenues to develop one apparent to

him. “I had a talk with [my partner], and I said, ‘Look. This may not be for me. I may not be an academic.’ This really was big. I’d never published before and to get an email, a review like that was pretty disheartening.”

In response to his rejection, Ivan reached out to members of the community of novice professionals entering into mathematics positions. In doing this, Ivan was invited to submit his work to a different journal by its editor. This outreach was perceived as an effort to establish a relationship between the research community and Ivan. It was with this submission that Ivan found his first success and shifted into a mode of assimilation into the group and he perceived that the group accepted him as a researcher and sought to build relationships with him. As he assimilated, he gained confidence, which may be in part to the his growing adherence to the norms (or ‘regimes of mutual accountability’) within the field in addition to direct antecedents in his sense of self.

Although this second experience with article submission is worthy of note, little can be inferred directly from reading the text of the article as it is significantly modeled after his dissertation and the presentation of himself as researcher in that role and at that time. A reading of his work beyond the publications originating from his dissertation or clearly modeled by it is also not immediately fruitful as Ivan soon found himself working with a group of young researchers interested in similar phenomena within their research field. As such, this research group can be conceived as a community of practice within a community of practice and the work resulting from it may be better analyzed as an examination of the group culture rather than of Ivan’s attempts to acculturate into the research field. That said, having had some success in the larger community, his research field, and gaining integration into a sub-community within it, it is clear that Ivan had acculturated into the RUME community by this time.

### *Teacher Acculturation*

In contrast to his researcher-identity, Ivan’s teacher identity was fairly robust upon leaving graduate school. Despite not having many teaching experiences as a graduate student, he

worked on a grant that developed small activities for middle-school teachers to implement into their classes. In this role he observed many classes being taught and worked to develop professional development opportunities for teachers using the grant's products. This, coupled with his experiences as a high school teacher for a year previous and his exposure to research as a graduate student, served as a catalyst for his young teacher-identity. Unlike the case of his very weak researcher-identity, he had a strong drive to maintain his teacher-identity and the culture of it. This was moderated significantly by a tangible need to remain gainfully employed, as he was the only salary-earner in his home.

While at NMWU, it is not appropriate to view Ivan's stories with regard to teaching through the lens of acculturation, as he would have experienced little pressure to acculturate while there; "I knew that my time was temporary." Additionally, with the success that he had with the Higher Geometry class while at NMWU, there would be no reason for him to question any facet of the identity he developed in graduate school.

Before he started at CU, Ivan secured a meeting with the chair of the mathematics department with the intent of discussing Ivan's teaching beliefs and practices. This was a clear expression of his desire to maintain his teaching culture and identity. It also provided Ivan a chance to gain an insight as to the attitudes of the department toward his teaching identity. However, his first semester was met with student discontent extreme enough to result in poor evaluations and multiple formal complaints. These complaints necessitated additional discussions with the department chair. Given Ivan's fear of losing his job, it is clear that his interest in maintaining his teaching culture was perceived to be at odds with the department's interest in building relationships with him; "I'll be honest with you, every time I got an email from the chair, I felt like I was going to be fired in that email. Every time that email came, it was like, 'we need to discuss some things.'" Because of the need to remain employed, Ivan's desire to build relationships with the department was absolute, thus the only thing that could shift was his desire to maintain his culture and identity as a teacher.

[I]f it would have been handled any other way, I would have gone into survival mode and I would have said, “Okay, I’m going to lecture right now.” At some point you say to yourself, “I am paying for my kids and my family. I’m going to do whatever is needed.”

The support of the department chair, being perceived as an arbiter of the ‘regime of mutual accountability’ within the department, was crucial to Ivan at this time,

That first semester I got four formal complaints. But the chair backed me on every single one of them. That was a huge thing, because, he didn’t have to. And I could have gone toward some other path. And even the chair suggested that I should maybe look at a little more traditional things, but he never said, “you have to,” right? He said, “No matter what you do, I’ll be supportive.”

In doing this, Ivan was interpreting the chair’s response as willingness to allow Ivan to maintain Ivan’s teaching culture in many ways, but was also stating that it exists beyond the tacit norms of the departmental community and that some negotiation upon those norms must occur. The chair was also perceived as amiable and willing to build relationships with Ivan. Hence, Ivan perceived that the department could become a multicultural teaching community and that he could integrate into it.

Ivan did change his own pedagogic practices in some ways in that he revisited his homework and assessment practices, which were reflections of his prior beliefs about the nature of work and learning. Ivan did not, however, simply assimilate to the existing teaching culture of the department; he never discussed his practices as the department’s. This change was validated by the response of his students the following semester in the anecdote of them holding class in the absence of an instructor.

### *Service Comments*

Although, the third facet of Ivan’s position at CU was the expectation that he perform service for the department, institution, and his field, it seems that to Ivan, this is a work-life balance issue more than a cultural negotiation. Ivan discussed his service only as it infringed on the amount of time he had to spend with his family and do research. As such, no further analysis has been conducted with an eye toward service despite being a professional

requirement that reinforces engagement in both his departmental-institutional and research communities.

## **Conclusions**

It is unclear if and to what extent Ivan's experiences are unique. Do individuals entering junior faculty positions with robust teaching identities experience acculturation in similar ways? Do novice researchers entering these positions experience acculturation in similar ways? How can we best mitigate difficulties with acculturation? Although Ivan acculturated significantly through recovering from rejections (by his desired peers in research and by his students in teaching), in the absence of these, how do junior faculty to acculturate to their communities? These questions are just a few of the ones that additional research will be needed to answer.

It is not appropriate to attempt to generalize from a case study, especially one with only a single participant. However, it is clear that some implications can be gleaned from Ivan's experiences. First, and perhaps foremost, is that although adults and professionals, new faculty members may be novices in one or more of the facets of their new positions. As such, they likely do not have robust identities, as was the case for Ivan's researcher-identity. The single, excoriating rejection was sufficient for Ivan to consider employment not just outside of research but outside of teaching as well. It is not my intent to suggest infantilizing new faculty members, but care should be exercised when offering criticisms as these may be interpreted as rejections from within the communities they are trying to acculturate into.

Second, in the absence of a robust identity, support communities and individuals from within communities that new faculty members are seeking membership with are beneficial. I cannot say what would have become of Ivan if it were not for his new math Ph.D. professional development group or without the support of Glenn and Nan Tucker. It is clear that without the development group, he would not have gotten any validation of membership into the community of RUME researchers as quickly as he did, which may have been just in time. If it weren't for the support of his doctoral advisors, he might not have even gotten that far.

In a career field where there may be perceived, implicit need to distinguish oneself (e.g., author ordering on publications) it is important to remember that all of academia (and life) is interdependent, from the work of the great thinkers of our time to the newest faculty member and everyone in between and beyond. The support given to Ivan manifested in both the researcher aspect and the teacher aspect of his stories as he acculturated into each of these respective communities. Nan, Glenn, and the members of the professional development group helped Ivan recover from his rejection and find successes. The department chair supported and worked with Ivan to re-envision his practices despite formal complaints and poor evaluations. The extension of community membership or support is tremendous to those trying to orient themselves to the reality of their lives.

With respect to the development of graduate students for professorship, it is clear that Ivan's researcher-identity was more ready to assimilate the dominant culture than was his teaching identity. As a community, it may be that we must determine if this assimilation is in our best interest. If it isn't, opportunities must be made available for novice researchers to engage in research and have their attempts validated from within the community. This is not to imply that rigor and trustworthiness are to be sacrificed but rather that these opportunities should occur early and often with the hopes that students will take advantage of them.

Similarly, Ivan's teacher identity was well established and directly contributed to the teaching practices which brought him to my attention for this study. It is clear in his story that without both support and the robust culture with which his teacher-identity was associated that he likely would have assimilated to the didactical norm of his academic department. As such, it seems that opportunities should be made available for graduate students to teach or to engage in mindful observation of teaching within the field that they will most likely find professional placement. Additionally, opportunities should be made available for these student-teachers to move beyond the regimented courses such as Ivan's first few experiences so that they may explore the enactment and resilience of teaching

environments congruent with their own beliefs.

## Chapter 4: Enacting Pedagogy and Praxis

There is a perception that professors learn to teach by modeling their practice after the ways that they were taught and that this may contribute to the didactical difficulties faced by students entering tertiary mathematics classes (DeGuzmán et al., 1998). This belief has been amazingly resilient due in part to the fact that most academics do not receive formal training prior to their first teaching experiences. Oleson and Hora (2013) examined this belief and determined that although instructors do learn to teach in part through modeling their practice, this is a very small part of the much more complicated matter of how an instructor develops into the teacher that they become. Similar to what has been observed in PK–12 teachers, other factors include teaching experience, student feedback, and development as researchers.

### Affordances

A convenient way to organize the existing literature on instructor beliefs is to consider the construct of affordances. An affordance is created by the interplay of an individual’s perception of the environment and possibilities of behavior (Greeno, 1998; Hora, 2012, 2016). A classic example of an affordance is to consider a table. If the table is perceptively sturdy enough, it may have the affordance of being able to be stood upon. This stand-able-ness is then a user’s affordance for the table. The table may simultaneously have other affordances such as being sit-able, lay-able, or stack-able. These affordances are defined by situated constraints, such as the table appearing sturdy in the first example—a flimsy table may not have the affordance of being stand-able. Additionally, Greeno (1998) considered “attunements to constraints and affordances” as providing “an alternative, more general form of explanation of activity” (p. 8). These attunements allow an individual to infer meaning from one situation to another situation based on the systematic relationships inherent in both (Barwise, 1989).

The use of affordances in this research adopts the framing provided by Hora (2012) wherein,

perception of the environment guides activity by suggesting to the viewer certain possibilities for behavior. ... In this way, perceptions of affordances can eliminate the prospect for particular behaviors by discouraging a particular use or limiting the range of possible actions. However, affordances are not solely informed by visual perception, but also by the energetic costs and benefits of particular actions (Proffitt, 2006) and cultural conventions regarding the acceptable or habitual use of certain objects (Norman, 1990). (pp. 211-212)

It is also important to note that in adopting Hora (2012)'s conception of affordances I am also adopting Hora's definition of constraints as restricting affordances, which differs from the use of the term by situative theorists, as described above, which promote affordances.

Hence, instructor beliefs on teaching and learning may be considered as being demonstrated through a set of affordances about teaching and learning. That is, an instructor may utilize their pedagogical content knowledge and apply the socio-cultural constraints of their environment in order to act in their practice. Such an affordance is a heuristic which enables instructors to execute decisions about novel instructional situations, often subconsciously (Hora, 2012). As such, affordances may be perceived based on the characteristics of a novel situation or may be inferred from previous attunements internalized as schema. However, this shift in what is meant by constraints then provides an avenue to explore the ways in which an affordance may be present in one circumstance, but not in another. To return to the table example, an otherwise stand-able table may not be perceived as such if it is in a room with the constraint of a low ceiling, uneven floor, or has a glass top.

Thus, each novel situation, no matter how similar to previous experiences, brings with it a number of situational constraints and affordances. How an instructor chooses to behave based on those affordances is what I have referred to as "enacting" affordances and how that behavior changes in similar situations due to attunement, from prior experience or some internal locus, or unforeseen constraints is what I call the "re-enacting" of affordances. Additionally, teaching is inherently a reflexive and relational art. As such, during periods

of forward-planning a teacher may presuppose a number of constraints about their classes and “envision” possible affordances. Continual reflection may then cause the need to “re-envision” possible affordances as the instructor’s schema shifts. With this in mind, two primary areas of interest are developed: the development and nature of instructor’s beliefs and the constraints that limit how those beliefs are enacted in practice.

### **Basis of Beliefs**

There is a perception that research on how beliefs impact teaching action must not rely solely on the instructor’s self reported behavior (Kane et al., 2002). Although this is well intended, the research shows that there is room for further study on the perceptions that post-secondary mathematics teachers have about their own teaching; about the interface between beliefs and actions as it exists solely within the instructor (Barnett & Guzmán-Valenzuela, 2017). As individuals situated in their exterior world, teachers, like all people, do not differentiate between what exists and what they perceive exists (McAlpine, Weston, Berhiau, & Fairbank-Roch, 2006). That said, it is an area in which researchers must proceed with caution as “studies that utilize a multiple-choice-type instrument to gather data about teachers’ beliefs may fall prey to a self-fulfilling prophecy” (Kane et al., 2002, p. 197) and that such educators might not have reflected deeply upon questions about the nature of teaching; efforts to assess their beliefs using direct questions in interviews may cause the teacher to merely “bring to mind whatever relevant information they could, and then construct some sort of reasoned, integrated response,” (Eley, 2006, p. 211) which, due to its improvisational origins, may be best considered a conception of teaching rather than a well-formed and resilient belief. As such, any attempt to try to find the elements or bounds of an instructor’s belief set may need to be achieved through self-reports of indirect approaches.

Many characterizations of the nature of instructor beliefs impose the dichotomy of being either teacher-centered or student-centered (Douglas, Powell, & Rouamba, 2016; Mesa et al., 2014; Saroyan, Dagenais, & Zhou, 2009). Teacher-centered (or content-centered) belief systems tend to view the acts of teaching and learning as being analogous to transmission and

reception. That is, the instructor is considered to be the holder of knowledge and is sharing it with the students who absorb it. This is contrasted by student-centered belief systems, which tend to view the student as individuals able to act on and develop their own understandings; that the students can develop expertise as opposed to being provided expertise (Saroyan et al., 2009). These differences in belief were apparent in teaching strategies that members of different content areas would exercise. For example, Mesa et al. (2014) noted that in ‘hard’ fields instructors tended to use more content-centered approaches than those in ‘soft’ disciplines. Mathematics was further identified as the most content-centered discipline of the 16 that were studied. This conclusion is congruent with that of Hora and Ferrare (2013) in which the mathematicians studied frequently relied on students to problem-solve, to memorize material through extensive practice, and by use of lecturing at a chalkboard. It is important to note that for this study Hora and Ferrare denotes any kind practice in which the students work a given problem as problem solving as opposed to any deeper meaning of the phrase.

Whatever a teacher’s beliefs are, one way they interact with action is through intention. Complicating discussion is Norton, Richardson, Hartley, Newstead, and Mayes’ (2005) study which shows that there is both a consistency and inconsistency between instructor beliefs and intentions. There is consistency in that instructors with teacher-centered beliefs tended to report that their intended purposes and outcomes of teaching were based in knowledge transmission, such as job training or imparting information. Similarly, student-centered instructors tended to report intentions relating to problem-solving, and motivation. The inconsistency is apparent in that all instructors in their study had intentions that tended to be more teacher-centered than their reported beliefs were (Norton et al., 2005).

Oleson and Hora (2013) determined that the major categories of experience that shape teaching practice are experience in the classroom, as an educator, and experience as a learner. It is not surprising that teaching practice is shaped by experiences of what has worked and in which situations it has worked in the past. It is also not surprising that some faculty

are not willing to learn and revise their teaching practice even when their experience shows its ineffectiveness, since these faculty members by-and-large hold teacher-centered beliefs. Similarly, based on their experience as learners, instructors will often project their style of learning onto their students. For example, an instructor who believed that as a learner they used rote memorization of basic facts and believed that it facilitated their entry to advanced mathematics will often give their students assignments and suggestions that emphasize rote memorization in lower-level courses (Oleson & Hora, 2013). Hora (2012) also identified one additional experience that had an impact on teacher beliefs, that of being a parent. Instructors with children in PK–12 schools tended to be more aware of pedagogical issues.

Of course, “beliefs in practice are often too multi-faceted to be reduced to a single ‘type’ and that they interact with prior experiences, goals, and perceived affordances to the organizational context to frame the teaching task and guide instructional decisions” (Hora, 2014, p. 63). Since both graduate students (Douglas et al., 2016) and experienced instructors (Hora, 2014) show that beliefs are not homogenous within groups nor are they necessarily reflected in practice, some researchers have examined the area through the construct of conceptions. Saroyan et al. (2009) examined the conceptions that instructors have with regard to teaching. To this end, they have defined conceptions of teaching and learning as being nearly synonymous with beliefs but are more accessible than beliefs. These conceptions included teaching as information transmission, preparing and managing instruction, promoting learning of course material, and promoting lifelong learning. They situated these concepts as hallmarks of a continuum between teacher-centered and student-centered beliefs and, further, each of the categories is built upon previous ones. That is, the conception of teaching as preparing and managing instruction is predicated on a pre-existing conception of teaching as knowledge transmission. This seems in some ways congruent to the perspective that all teaching is teacher-centered as it is the teacher who is the ultimate planner and conductor of the educational environment as it interfaces with students.

## Constraints on Action

Considering teacher beliefs as apparent in affordances, it is necessary to consider the constraints that limit and provide definition to the actions that an instructor views as possibilities. These constraints are structural or socio-cultural in nature (Hora, 2012). Structural constraints include considerations about the imposed course structure (often as evidenced as by a departmental syllabus), the infrastructure of the classroom, and the students in the classes. Socio-cultural constraints include professional pressures on advancement and normative pressures.

### *Structural Constraints.*

In any teaching, there are some topics that are perceptibly more difficult for students and some that are more accessible to students (Mesa et al., 2014). This, coupled with syllabi that often sequence the content in a course, have a tendency to push instructors to implement more teacher-centered activities despite teacher beliefs (Hora & Ferrare, 2013).

This has exacerbated introductory mathematics courses, which are often designed to service a maximal quantity of students with a limited set of institutional resources, such as instructor-hours and physical space. This results in large class sizes, which in turn require large rooms. Limited numbers of teachers in the room encourages more knowledge-transmission activities, which is also encouraged by the physical layout of the seating in rows oriented toward the front of the classroom. As such, small-group work is often impractical and other such options are limited (Hora, 2012). Further, pressures on pass-fail percentages actively encourage instructors to direct their instruction to middling levels of students, often ignoring the needs of both high and low achieving students (Hora, 2016).

Instructors also hold different conceptions of teaching at different levels of instruction (Norton et al., 2005). Moreover, this has impacts on the decisions that instructors make in their classes, with upper-division courses tending to include more student-centered activities than lower-division courses. This is reported by faculty members as occurring because upper

level students being perceived as more advanced and the courses have fewer mandated topics than their lower-level counterparts (Hora, 2012).

### *Socio-Cultural Constraints.*

Professional rank and opportunities for advancement are also factors that limit the possible actions of instructors. Graduate students often feel the need to exert authority in their classrooms because they are not professors and because they are similar in age to the majority of their students (Douglas et al., 2016). Faculty members often view their work as educators through the lens of their workload demands. With many departments putting more weight, either real or perceived, on research than on teaching for tenure considerations the tension caused by these pressures is palpable (Hora, 2016). These pressures are exacerbated by heavy teaching loads, which often fall to junior faculty who are in most need of flexible scheduling to accommodate the research demands on their professional aspirations (Hora, 2012). This emphasis on research tends to provide a normative pressure on non-tenured faculty members (Hora, 2012). These factors tend to homogenize the perception of successful, or even acceptable, teaching practices (Hora, 2012, 2016).

### **Praxis**

It is with this in mind that my attention shifts to how teachers come to envision their affordances and create new realities for their classrooms. As a teacher seeks to improve their practice, it is clear that while there may often be a Vygotskian ‘more knowledgeable other’ (Vygotsky, 1930/1978) there may not always be. As such, it is appropriate to consider the ways in which an instructor improves upon their art of teaching when not under the explicit guidance of such a ‘more knowledgeable other.’ Often the word ‘praxis’ is used to describe a reflexive process by which teachers re-envision and re-actualize their teaching.

The choice to use the word ‘praxis’ has roots with Aristotle, whose meaning is a point of some contention (see: Ackrill, 1978; Balaban, 1990; Belfiore, 1983; Squires, 2003). Often  $\pi\rho\alpha\chi\iota\varsigma$  (*praxis*) is translated as ‘action’ or ‘conduct.’ This is often described with relation to

ποιησις (*poiesis*), which is translated as ‘making.’ These translations are problematic because they fail to adequately highlight the difference in what Aristotle meant. Some clarity can be found in the τελος (*telos*) of an action, the ‘ends’ or ‘aims’. If the doing of an action is performed for the τελος of another state of being then it is ποιησις. However, if the doing of an action is performed for the τελος of itself, then it is πραξις (Balaban, 1990). The implication of this is that for πραξις, unlike ποιησις, the existence of an end state is not necessary for the τελος.

While there have been many approaches and conceptions to praxis (see: Cowley, 2008; Haug, 2000; Vogel, 2017), the particular interpretation of praxis used in this study has been influenced by Freire (1970/2005) who defines it as “reflection and action upon the world in order to transform it” (p. 51). In doing so, Freire is at risk of providing a τελος with a distinct end state (a transformed world) and, hence, Freirian *praxis* may be interpreted as ποιησις, which does little to clarify the concept. However, it is my interpretation that such a transformation is an action performed with the τελος of transformation itself and *praxis* can be conceived as πραξις of transformation by way of reflection and action (i.e., ποιησις). The τελος of transformation requires only the envisioning of alternate, presumably better, ways to act upon the world. That is, praxis depends upon the envisioning, re-envisioning, enactment, and re-enactment of affordances.

This study utilized an intrinsic case study design to explore the narratives that a single unique instructor, Ivan, tells to explain the origins of his practice. Ivan was purposefully chosen as the sole participant of this study due to his striking incongruence with the teaching practices normally found within his department. That is, Ivan teaches using Inquiry-Based Learning (IBL), whereas the norm within his department is exclusively lecture. As he was also junior-faculty within his department there was perceived to be substantial normalizing pressures upon his teaching but he persisted in acting in accordance with his beliefs despite them. Hence, this study sought to address the following questions:

1. What kinds of pedagogical actions does this professor employ?

2. How does he choose when and how to implement them into his teaching?

### Methodology

Schwandt (2015) defined a case study as a “complicated arena [of qualitative inquiry] involving methodological choices directly related to goals or purposes of conducted case-based research, research traditions in different disciplines, and the ways in which investigators define a case” (p. 26). Additionally, “an intrinsic case study is the study of a case (e.g., person, specific group, occupation, department, organization) where the case itself is of primary interest in the exploration” (Grandy, 2009, p. 500). Further, the core phenomena of interest were the experiences of Ivan. As such, this research was conceptualized as a narrative inquiry. In a narrative inquiry,

our principal interest in experience is the growth and transformation in the life story that we as researchers and our participants author. ... [I]n the construction of narratives of experience there is a reflexive relationship between living a life story, telling a life story, retelling a life story, and reliving a life story (Clandinin & Connelly, 2000, p. 71).

Since our beliefs are an intimate part of our personal identities, it is entirely reasonable to assert that a fundamental motivator of those beliefs are the stories that we tell ourselves about them and that those stories are the reality of beliefs’ existence. Such stories, being the instructor’s personal narratives, “had better be viewed not as a record of what happened (which is in any case a nonexistent record) but rather as a continuing interpretation and reinterpretation of our experience” (Bruner, 2004, pp. 691-692). As such, Clandinin and Connelly’s (2000) framework of a three-dimensional narrative inquiry space (consisting of situation, continuity, and interaction) along with the four directions of inquiry (being inward, outward, backward, and forward) is an invaluable tool to situate a narrative researcher’s work. “[T]o experience an experience—that is, to do research into an experience—is to experience it simultaneously in these four ways and to ask questions pointing each way” (Clandinin & Connelly, 2000, p. 50).

### *Participant and Setting*

Ivan was, at the time of this study, junior faculty in a fairly traditional mathematics department (with primary research interests in Algebra, Analysis, and Topology) at a Carnegie R-1 institution in the Southern United States, Current University (CU). Ivan represented fully half of the declared departmental research interest in undergraduate mathematics education (RUME) and it is in no small way that because of his interest in RUME that his teaching fell well outside of the norms at his institution. At the time of this study, Ivan had been at his institution for five years and was undergoing tenure review.

### *Data Collection and Analysis*

Eight semi-structured, audio-recorded interviews were conducted. Seven of these interviews occurred over the course of six months while the eighth was conducted two years prior. These interviews all occurred in Ivan's faculty office, however, with the exception of two of the follow-up interviews, they continued, at Ivan's invitation, past our scheduled interview time as we walked together to Ivan's car. One of the two follow-up interviews that did not occur in Ivan's faculty office was given entirely in Ivan's car as he had invited me along to complete a personal errand of his. The goal of the initial interview was to establish a rough chronology of the pertinent, in his own opinion, events of Ivan's journey, while the follow-up interviews systematically established vignettes of each of these events.

Over the course of these interviews several questions and stories were revisited to establish stability in the narrative. Similarly, member checking has been used to further ensure that the narrative co-created was, in fact, true to that which Ivan tells himself. These interviews were transcribed verbatim and then closed coded in accordance with Clandinin and Connelly's (2000) three-dimensions of narrative inquiry space. Those dimensions are "personal and social (interaction); past, present, and future (continuity); combined with the notion of place (situation)" (p. 50). Data were then open coded then subsequently axially coded. Using a constant comparative method, new data were compared with existing data

for consistency and emergent themes were sought.

Additionally, one course taught by Ivan was observed every day that it met en masse, including examination days. This course was a second semester course in introductory calculus, which primarily dealt with integration and integration techniques. This course had a maximum enrollment of 160 students (137 students completing). The instructional team included Ivan and three graduate teaching assistants who were purposively chosen due to their expressed interest in IBL. The observed course provided Ivan additional funding through an institutional grant, which was used, in part, to support research related activities and travel for the graduate assistants. These observations were conducted prior to this study, however the resulting observation notes and researcher memos further add to the body of data used here within. Clean copies of these items have been subjected to open and axial coding, as above. Lastly, copies of one syllabus per semester from courses taught by Ivan while at CU have been considered and coded using the same coding regime.

## **Findings**

Adopting narratives for this study necessitates either a topical or a chronological presentation of the results. Given the cumulative nature of Ivan's experiences as he learned to enact tasks congruent with his pedagogical beliefs, a brief chronology of relevant items is presented within three key (topical) aspects of his practice.

In an effort to maintain trustworthiness and to not imply my own ownership of the narratives, they have been presented, where possible, with Ivan's own words and have been subjected to only minor editing to fit the format of this report and improve the flow, since topics were revisited many times in multiple interviews for this study.

### *A Typical Day in Class*

Before delving too deeply into the narratives at the heart of this study, it is prudent to first consider the structure of an average day in this class that was observed. The class met three days per week (Monday, Wednesday, and Friday) at 8:30 AM in a large room designed

to facilitate instructor lecturing. Students were also enrolled in one of six discussion sections led by one of the three graduate teaching assistants for the course.

Student seating was arranged in tiers with long tables spanning the width of the classroom and two large chalkboards covered the front of the room. The students' seats were mounted, hinged in pairs sharing a common support with the tabletop and bolted to the floor, which made it difficult for students to turn toward a neighbor that shared the same support post. The rows of seats were so narrow as to make passing by a seated individual nearly impossible, with the exception of a row halfway up the seating area, which was designed to be wheelchair accessible.

At the front of the room was a stage, which held an instructor's station with public address equipment, a document camera, and projection controls. The station was situated just off-center of the stage, in front of the two large chalkboards, which were all but unusable when the projection screens were lowered. The room was also equipped with a ceiling mounted video camera, which was not used in this section, nor did it take up any space.

Ivan would arrive 20-25 minutes prior to the start of class and begin drafting a daily agenda to project at the start of the class. Ivan would be dressed in a button-down dress shirt, slacks, a belt, a long tie, and sneakers. As students began to arrive to the classroom, Ivan often greeted the students individually, by name, and carried on pleasant conversations with the students until the teaching assistants began to arrive (filtering in along with the students), at which point he redirected his conversation to his teaching team.

When the class began, Ivan would usually offer about five minutes of orienting direction to the class, such as reading through the agenda aloud and emphasizing the seating arrangement for the day, which changed regularly (such as having students sit with individuals they had not yet worked with or seating all the students from a particular discussion section in the first two rows). The students would then begin lengthy conversations with their peers, regarding some problems or results of interest from the provided notes and the assigned homework. These conversations often lasted around 20-25 minutes of the 50 minute class.

Occasionally the instructor would give individual students or the teaching assistants his undivided attention for most of this time, but the norm was for him to travel from group to group and listen in on their conversations, occasionally offering a question of his own as he went. He very rarely, if ever, offered direct responses to any questions he was asked, instead relying on his ability to ask reframing questions back to the group and encouraging them to try to answer it. His traveling from group to group, given the layout of the class, would regularly give him cause to physically mount the tables and walk across them to reach the interior of the classroom, something that only hardly ever gave him much pause.

After the students had been given ample time to discuss their findings, the call went out requesting a student to present a *productive failure* to their peers from the document camera on the stage at the front of the room. A student volunteer would place their work on the document camera and begin describing, to the best of their ability, their process as they worked through a given homework problem, being sure to highlight their mistakes as they went. Students were expected to also present how these mistakes eventually led them to find their own solution to the given problem. Students sitting in the crowd were permitted the opportunity to ask questions and offer suggestions to the presenter as they went. These presentations often lasted 5-10 minutes and would end with the students in the audience offering applause to the presenter. During these presentations the instructor would nearly always be found sitting in the crowd with the students observing their reactions to the presentation of productive failure as well as providing the presenter his attention.

If time permitted, the class would conjecture about any big questions that seemed to remain after their discussions with their peers, such as the existence of perfect circles or any observations that unified several key concepts, with students regularly being at the front of the class presenting their ideas.

#### *Ivan's Description of his Teaching*

Ivan described his teaching practice not as superior to other teaching actions but, instead, he focused on the facets of emphasis present in his teaching and how they valued the outcomes

that he desired.

There is no statement of me downgrading any other teaching style or philosophy, because I think that there's a large spectrum of teaching styles and I am one part of the spectrum, but that doesn't mean that another part of the spectrum can't be as effective or maybe even more effective. I think that belief and trust and justification of your teaching style goes a long way. Students will go with whatever exudes off of a teacher, whatever their style is, if they believe in that style. Addressing that, I believe in humans and that involves growth. Notice that my structure of my syllabus is homeworks, which are 20 percent, quizzes, which are 15 percent. Homeworks are mostly human, meaning they just have to attempt stuff and I'll want them to get that because I want pre-work before the class. Quizzes, I treat those as half human, half other, meaning I do have to do some summative assessments with them. Productive failures: totally human. And if there's anything, that may be an explicit difference between myself and a traditional lecturer, but even the structure of 10-20-30 with the 10% [of the final grade coming from the first test] and the 20% [coming from the second test and the final being worth 30%] being places where they can grow from. My traditional lectures were 20-20-30 or 30-30-30. Right? And after that first test students, especially those in these intro classes like calculus are still adjusting, physically, in a new environment and emotionally in a new environment. They're trying to meet new friends, maybe those friends aren't that good, right? They're trying to get settled into a new environment. And so to hammer a 30% test on them after week 6, it may do a disservice to those who take a little longer to understand material and get involved with the material.

With regard to his limited implementation of traditional lectures, Ivan was sensitive toward the desired learning outcomes he held for his students as well as what he considered to be reasonable expectations for how students manage and assimilate information, saying,

[One] thing about lecturing is that it's difficult sometimes to learn from a lecture, there's difficulty in remembering and learning all those things. You catch little glimpses. You catch little glimpses when you are doing stuff too. But, when I learned how to drive a car or ride a bike or swim, I had to do it. Even when I worked at [retail store], they could get me through all the trainings and show me videos of whatever it is, but when I got in there, that was when I made mistakes and had to learn from stuff. I gave the wrong change to somebody, what happens then, right? And you gain this experience. In a traditional lecture, and even a flipped classroom, the information seems like it's bestowed upon or coming down from somebody versus, hopefully in my class, it's being generative. The students are generating the information. It mainly boils down to human care.

More than anything, I know the way I teach is different, therefore I've got to adjust it differently and I know that I have to adjust depending on how my

students are. And those two things, being malleable and having a lot of formative assessment, I think goes extra to the lecture. Lecture, I control it, the whole time. Fifty minutes, I control it. There's going to be no perturbations, right? But when I'm teaching the way I'm teaching, I'm very good about wanting perturbations, because I think those are the learning experiences.

Ivan also described the manner and mindset he has when he enters into teaching relationships, as well as what he perceived as the aims and goals of his class in the greater experience of the collegiate studies of his students.

It seems weird, but it's faith-based, in some sense, that you really believe that this stuff is going to be okay and good, and you believe in your students. Both of those have to be possible. Either one of those crumbles and you might go back to lecturing. And you have to be okay with making a ton of mistakes and learning from them. Vulnerability and honesty seem to be two pillars that I kind of lean on. Like, I'm a mistake-maker but I'm also a learner and both of those go hand-in-hand. I think that some teachers think that they need to be pristine all of the time. I remember making a calculation error and my stomach would fill up with butterflies and I'd be the most embarrassed human being on the planet at that time. But I learned so much about letting go from that. I've screwed up so many times and I've misremembered things that pop up and I'll just send an email saying, "Look, I made a huge mistake." It's kind of embedded in the DNA now, because with the productive failures, right? It's MY productive failure and I can position it to students that way. So it's about care, it's about life, it's about belief first.

I think I need to create an environment where my students can fail now and I don't mean fail as in fail the class. I think that they'll do well in the class; actually a majority will do well in the class, but fail now in order to reflect on that later and utilize it. Think about a video game, where you, go through a level. You game over, but you game over at this spot where failed. You just restart the level, right? And you go on with the knowledge of that failing attempt. That's what I want college to be for them. A bunch of "let's try this out" or "let's think this out" or "let's do this," so that in the future there's not as big of stakes with that. Let's try this doesn't involve them getting fired. It should be an intellectual stimulation coupled with a safety net of sorts, where they can try these ideas and thoughts.

Ivan explicated the role of his experiences in shaping his perspectives on teaching and his role in his classroom, by sharing,

It's one of these things, where I'm where I'm at, which means that I had the capabilities of doing it, and yet I was not doing it all that well. I was getting

B's and C's and getting kicked out of [graduate] school. So if I really have the capability to do well, or to get a Ph.D., there must be some kind of shift that I need to do in order for others to have that capability as well, or the opportunity, right? I think my whole experience has a lot to do with how I frame teaching and how I measure success, because, to me, it came a couple years later, after graduating [undergraduate school], where things clicked. And so, in my classroom, I have safeguards and measures to try to let students take that time to allow them to play. I think it's massive. My whole life plays a massive part in how I reframe and approach things. (Interview Data)

### *Notes*

Ivan's notes were unlike notes that one would use to lecture from or notes that, if provided to a student, could be used exclusively to learn course content. They were conceived in the, perhaps scant, space between these two aims. As such, the notes were published and the students were required to read and work the problems specified in the notes. Ivan described the value of creating his notes saying,

I think when I create notes, I learn about the content so much more because I have to explain it to myself before I explain it to others. A great example of this was [Calculus] 4, which is the multivariable Calculus. Look, I passed it in undergrad. Stop. End of sentence, right? I didn't understand it all that much. I didn't understand Green's Theorem and Stoke's Theorem and what the heck was going on. And so I had to re-jump in and look at all that. Once I saw those connections, my gosh! It was like my head exploded in such beautiful thoughts and ideas. And teaching [Calculus] 4 and understanding, why we dot product with the vector field and what the heck is going on with these integrations along paths, that is beautiful stuff! Yet it didn't seem that way to me, so I tried my best in my notes to unpack all of these little things. (Interview Data)

The ways in which Ivan determined the content of the notes were an exercise in envisioning his goals and the abilities of the students in his class.

I put myself into the eyes of a student and I go, "What do I think my students can do?" and "what do I think they need a little bit of extra support with?" Scaffolding questions or something like that. Sometimes I'll put them in; sometimes I'll take them out. I don't want to scaffold them. I want to see the grey and I want to see how they play around in this sea of non-existence, right? I also think about, like, when creating the notes, what's out there. What are the questions that I can ask that are non-Google-able? One of my favorite questions of all time, and sometimes I put it in quizzes and sometimes I put it in the notes,

is, “Is there such a thing in life as a perfect circle?” What that question asks is so much about Calculus, that it’s not even funny. It’s so saturated in infinitesimals and errors and humans and math and everything else. I love it. I love it.

When I design my notes, I think about, it’s sort of Zone of Proximal Development by Vygotsky, how much can I push the students and to what brink can I push the students, and I try to estimate that. And for example, in [Calculus] 2, there’s the Fundamental Theorem of Calculus. I asked them to prove it, but then I say, “Look, you can look it up on page blah blah blah blah.” I think what happens is, if I can create the notes in such a tantalizing, for the lack of better term, manner, then it’s almost like a student wants to find the proof of that theorem. And will go online or will go anywhere to satiate the need for wanting to know why this thing is true. And that’s tough. It’s tough to generate that. At the same token, I feel like that’s where my lecturing comes in as well. What I believe is traditional lecture. It’s to satiate that thirst or hunger for the answer or for the knowledge, not the answer. For the knowledge that is going on. The notes are a lot of push and pull. (Interview Data)

Ivan also described the role of textbooks as subordinate to his notes in how he wanted his students to use the curriculum materials.

Depending on the class, if I have a textbook, I’ll integrate that with it, if I’m forced to have a textbook. If not, I’ll, give some optional textbooks for them to look at, usually open source. But I want them to concentrate on the notes, because I want them to be very meticulous about the reading and the writing. (Interview Data)

Ivan did not publish his complete notes all at once; this allowed him to adjust them based on how the class was going. These adjustments happened both between semesters as well as within each semester. Ivan described a particular instance of making adjustments between semesters;

That class that you saw, I actually did a new thing. So I had a set of notes that I already had, I taught it four times previously. So I had a set of notes, but this time, I wanted to try something different, and it was partly influenced by [Calculus] 4 and partly it’s just sort of a re-arrangement of things. I got frustrated at the time with techniques being siloed from what was going on with some of the other stuff we were learning. We were learning about transcendental functions and so I wanted to: if you’re taking the derivative of the inverse sine I want you to also take the integral of the inverse sine. But that requires me to teach integration by parts, right? And so I kind of wanted to mold all of these together. And so I did. (Interview Data)

However, when Ivan was teaching a class for the first time, he did not have previous notes to refine from. Thus, with regard to the way that Ivan approached the construction of notes for the first time, Ivan says,

When I'm teaching for the first time I'll probably write up to four weeks and then I'll go on a week or two week or three week basis. And I'll craft notes because I'm adjusting to students and then every subsequent time I teach, I only release about four weeks worth, then I'll adjust the notes. Copy and paste, make some new questions, always adjusting based on my previous experience. And so they're almost always formative. (Interview Data)

Thus, Ivan made adjustments to the notes in a similar manner regardless of if they were originally generated or merely being refined from previously created notes. That said, Ivan described the adjustments as occurring in a number of different ways.

Adjustments may not be totally with the notes and it may be with the notes, where I put in an extra few questions and I'll re-upload the notes. But usually my adjustments come with what piece of the notes I assign. Let's say I'm having students that are having, at that time, trouble with X content. Then what I do is I say, "Okay, look. Let's either assign a little bit less for next time or let's have a quiz next time and no homework." Someway to adjust for the fact that they're not doing so well in X content. And I know in the back of my mind, I can always catch up with everything, because if I set norms of understanding, piecing together yourself, and figuring things out, then I can just throw content at them and they'll understand it. It goes back to belief, so I adjust based upon what I assign usually.

That is, Ivan was not primarily concerned with the coverage of the course as much as he was concerned with promoting student learning of the assigned content (see: Kogan and Laursen, 2014; Yoshinobu and Jones, 2012).

Sometimes [the amount I assign is] slightly determined for me. For example, if I'm teaching Tuesday-Thursday, the Thursday assignment that's due on Tuesday, I do a lot more than I do with Tuesday to Thursday. But usually around four to five tasks that, this is the funny definition, the Academy of Inquiry Based Learning calls "rich," which I don't know what that means, but I'm going to go with, "That students have to think, quite a bit, about." And then coupled with if I think that they need it or if I think that they should have it, a couple tasks that are kind of quicker or more rote or something where they can have this basis. So I'm trying to find that perfect balance of difficulties, whatever difficult means, and still cover slash adhere to or adjust the content.

[As for adjustments that need to happen to the notes, as an example,] in Discrete [Math], I got this notion of actually, depending on the definition, ask for examples and I didn't on this one notion, and so this iteration, I put that example generation question down. The students last iteration had a real difficulty with thinking about what this definition meant. One thing that clues me in, I get emails from students saying, "What do you mean by definition N?" And when it becomes that much, one student might not be too significant but three or four students, when they get troubled, then I need to adjust. The difference though is that sometimes I'll leave definitions intentionally difficult, right? Just recently, there was a definition of a graph and it was the perfect time to tell the students, or remind them, that a definition tells you what it is and it also tells you what it isn't. One big thing that you should think about as a person reading a math definition is what is this thing really communicating, right? And how can I push its boundaries. And so, I think I adjust sometimes depending if I think that they can catch their own understanding of the definition, or I'm intentionally vague because I want to send a message of, "be careful of X, Y, or Z definition." It's a lot of cognitive disequilibrium in order to, for them to be aware of their own equilibrium. (Interview Data)

### *Productive Failures*

Ivan, having struggled with the role of participation in his classroom, which he attributed in part to the difficulties of his first semester at Current University, found his answer in his implementation of productive failures. That semester there were four formal complaints to the department chair with whom Ivan worked. Ivan was encouraged to rethink his approach to teaching as a result of these difficulties.

I tried [to have students submit] portfolios in my first year at [CU]. It was a DISASTER and so what I did was I said, "Look. It was a disaster." And my chair, who noted the disaster said, "Look. It was a disaster, you've got to adjust." And I said that already to myself, "Okay, I'll adjust." I went to gamifying, meaning it was for the most point-hungry of point-hungries. You get 300 points; you get a level up. And level up, you get this or this, right? I thought that was it. It came from the [experiences I gained working on a grant in graduate school]. (Interview Data)

In this gamification, being the implementation of game elements into learning environments (Deterding, Dixon, Khaled, & Nacke, 2011; Landers, 2015), Ivan introduced elements of game assessment, through the use of points to track progress and accomplishment, and game fiction, through the use of terms such as 'level-up' to mark milestones of student

progress (see: Bedwell, Pavlas, Heyne, Lazzara, and Salas, 2012). However, the behavior that this gamification encouraged (see: Landers, 2015) were not congruent with the learning goals that Ivan held for his students.

I noticed that there were some inequities with that, and I'm not saying I address any of those nowadays, but I definitely saw how students were getting points. One aspect was participation and I wanted to value participation in some way. So, students were getting points for every utterance in the classroom. Well, okay. Someone who saw that and goes, "I'm just going to talk a lot in the classroom." And I was giving them points for it. Also, there's a quality control, what does it mean to be an utterance, right? Once you dive into one thing, there are always five other things that you need to worry about. And so, it's just re-evaluating things, and looking at what I've got and what I'm going to do and what really I want to accomplish. And that tells me I need to change something. That's why productive failures are so nice, because it's a required participation that does more than just participation. So you have to participate, somehow. The question is: How does that participation help, right? (Interview Data)

Productive failures in the way that Ivan described them are, at least in his mind, a novel creation, which were constructed using his understanding of Ed Berger's valuing of failure and productive struggle (see: Kapur, 2014; Kapur and Bielaczyc, 2012). Here, Ivan referred to Berger as a way to give credit to an individual who served as a model for the enactment of productive failures as a formal pedagogic construct in the classroom. Ivan described the roots of his implementation of productive failures as,

[Productive failures] stemmed from a couple of things. One was that Ed Berger did this. I don't know if he did it the way I do it, but he gave five percent towards this thing called something like "epic failure." I mean it was so monstrous, such a failure that he gave five percent towards it, right? And I'd been to a number of IBL conferences and many people talk about productive failure but they talk about it as an implicit thing that goes into IBL. It was like, "Well we should value productive failures, et cetera." And I was like, "well, I get that, but how do I value it? Well, let's try it," right? (Interview Data)

Ivan believed that for a systematic implementation and valuing of productive failures, points must be allocated to the practice. Thus, he discussed the intentionality of those points as well as the push-and-pull of incentivizing behavior that he hoped his students would internalize.

I've been on a kick of, "If I want something to happen, with students then I'd better give some points to it," because they won't appreciate the subtle things unless they get a little bit of a helping hand. It's honestly, if there's anything that's like really frustrating in my mind, it is how much should I overtly do things versus how much should I let it cook for a while. What I mean by that is, should I call on students or encourage students to do something? Or should I not and see if free will allows them to produce some items and some materials, right? This is always been tough but I think the productive failure thing makes sure that I'm valuing their mistakes and, in turn, I hope they value their mistakes. So it's two things. It was the IBL and Ed Berger. (Interview Data)

The students in Ivan's class were provided the following description of productive failure in the syllabus. This description has been relatively unchanged across all syllabi examined in this study since its inception (five semesters). For the class, whose observations were used in this study, there were no additional instructions provided to students in the class, unless they occurred in the discussion sections.

I am giving 5% of your grade to a category called "productive failure." Every class period, one person from class will go up to the board or [document camera] and explain how s/he approached a problem or theorem incorrectly, and how s/he overcame that difficulty. (Artifact)

Ivan described his implementation of productive failures as addressing and promoting failure-tolerance (see: Clifford, 1984, 1988) as well as how he understood the risk of implementing such an abnormal facet to his teaching, especially as he was using it to evaluate student learning.

I didn't know what was going to happen with that first ever productive failure, but one thing that my [research] group and I came across was, we said to ourselves, "Does [a proof] have to be correct in order to be creative?" I kept saying, "It could be an incorrect proof and still be really, really creative." But, in math, there's a lot of binary things going on. It's either correct or incorrect, right? And so, a student may not have the same appreciation for an incorrect proof that I did and so part of the motivation was towards that. The other part of the motivation was that I felt like students weren't even getting to a productive part. They were just hitting the failure and stopping the cycle. And I felt like if students presented productive failures that others would see that they're not alone in their difficulties but that they can be productive after. It took a little bit of time but I felt the time was right. Meaning, I felt like at that moment, I

could try this and it's five percent of the grade. So if it screwed up, then that's okay, I still get 95 to play with. (Interview Data)

When asked about the course within which he first implemented productive failures, Ivan said,

It was an Abstract Algebra course the first time I did it. It may have been [because of] the Abstract Algebra course but I may have also been inundated with productive failure being good in the IBL conference that summer. And thinking about that coupled with thinking about my research about professors failing and how they couldn't care less. You know, they had this mentality that they could overcome that failure. It wasn't anything to them, let's put it that way. I think it was a ripe time for me to try it but I can't say with certainty it was because of the Abstract Algebra course because I did it with the Calculus course as well [in a subsequent semester]. And sometimes the sophistication of the productive failures are not as apparent [in the Calculus course], but that's alright with me. I think that no matter what, we're opening ourselves up to be vulnerable and overcome some of those vulnerabilities. (Interview Data)

The impact of this explicit valuing and sharing of productive failures on Ivan's students was significant to Ivan, who shared his perception of this impact saying,

Students were having difficulty because they didn't think that they were having a big enough failure. I didn't know that there was quality of failure, right? And I really love the fact that they themselves become arbiters of whether they're doing good or not, right? Because it shifts the onus of learning and of education from my goals and my beliefs to what you want your own goals to be. It's difficult for some, but for others, it's the most empowering thing. They harness this power through the lightning or whatever, right? It's incredibly empowering!

[But even] with the productive failures, there's a student who said, "Well, he values failures in class but doesn't value them on the test." Well, that's true. I'm a hypocrite sometimes, but I try not to be as much of hypocrite as I can be. (Interview Data)

### *Homework*

Ivan contributed a significant amount of his experienced difficulties teaching during his first semester at CU to the structure of his homework. Namely, he routinely described homework as being due every two weeks and that students would wait until immediately prior to the submission deadline to begin work on it. A review of his syllabi showed that

in that first semester, he required homework submission every week with group portfolios, “where [groups of students] create an answer key for the notes and turn it in via the web” (Artifact), due every four weeks. This submission schedule was drastically altered for all future courses taught by Ivan to have submissions due every class meeting, which he had upheld across all syllabi available for review.

My first semester I had students turn in homework every two weeks, much like I did [in the first IBL course I taught, Calculus] 1 and then the next semester I had students turn in homework everyday. And I’ve continued that. (Interview Data)

The homework assigned was a form of forward-homework, wherein the students were asked to complete homework on course content that had not yet been addressed formally in the class meetings (see: McGivney Jr., 1990). Ivan used forward-homework to incentivize students to review and engage the class notes, which, in turn, allowed them to engage in the peer-discussions the following class meeting.

The impact of homework on the students’ final grades, the partial components of that grade impact, and how the homework requirement has been provided to students have also been places of some change, although Ivan did not directly address these changes in the interviews (see Table 5).

<b>Semester</b>	<b>Course</b>	<b>Syllabus Statement</b>	<b>Grade Impact</b>
1–First Semester at CU	Calculus 2	“20% Creativity and 20% Sense-making” (Artifact)	40% of final grade
2–Gamification Semester 1	Calculus 2	“You will receive 5 points for completeness and up to 3 points on a problem in the set that I designate the grader to grade. You can get 1 point for every attempt on that problem and the remainder of the points for correctness. For example, if you attempt a problem 3 times (by crossing out the attempts so that the grader or I can see legitimate attempts) and do not get it correct I will give you the full 3 points.” (Artifact)	Not indicated in syllabus; likely varies.
3–Gamification Semester 2	Calculus 2	“You will receive 6 points for completeness and up to 2 points on a problem in the set that I designate the grader to grade. You can get 1 point for every attempt on that problem and the remainder of the points for correctness. For example, if you attempt a problem 2 times (by crossing out the attempts so that the grader or I can see legitimate attempts) and do not get it correct I will give you the full 2 points.” (Artifact)	Not indicated in syllabus; likely varies.

<p>4– Gamification Semester 3</p>	<p>Calculus 3</p>	<p>“You will receive 6 points for completeness and up to 2 points on a problem in the set that I designate the grader to grade. You can get 1 point for every attempt on that problem and the remainder of the points for correctness. For example, if you attempt a problem 2 times (by crossing out the attempts so that the grader or I can see legitimate attempts) and do not get it correct I will give you the full 2 points.” (Artifact)</p>	<p>Not indicated in syllabus; likely varies.</p>
<p>5–Productive Failures Semester 1</p>	<p>Abstract Algebra 1</p>	<p>“Homework is based on effort and correctness. If you did the homework ON YOUR OWN, no matter if it is correct or not you will receive 1 point. The other 1 point will be for correctness. You are more than welcome to redo the previous HW for the point on the subsequent HW. Also, I encourage you to do more than what is assigned. Look at the type of problems I assign/ask, and ask the same of yourselves.” (Artifact)</p>	<p>20% of final grade</p>
<p>6–Productive Failures Semester 2</p>	<p>Calculus 3</p>	<p>“Homework is based on both effort and correctness. If you did the homework ON YOUR OWN, no matter if it is correct or not you will receive 3 points. The other 2 points will be for correctness. You are more than welcome to redo the previous HW for the two points on the subsequent HW. Also, I encourage you to DO MORE than what is assigned. Look at the type of problems I assign/ask, and ask the same of yourselves.” (Artifact)</p>	<p>20% of final grade</p>

<p>7-Productive Failures Semester 3 (Observed)</p>	<p>Calculus 2</p>	<p>“My goals for homework are two-fold: 1. Reflection on the last course 2. Preparation for the next course</p> <p>Therefore, the requirements for homework are as follows:</p> <p>1. (2 pts.) Write a paragraph describing</p> <p style="padding-left: 40px;">a. What happened in the previous large class period (so if homework is due on a Wednesday, you need to write about what you understood/gained from Monday’s large class period)</p> <p style="padding-left: 40px;">b. Two questions: One has to be content-focused (Ex: ‘Where does the formula for integration by parts come from?’), the other has to be process-focused (Ex: ‘I’m doing well in computation but not in concepts. How can I problem-solve better?’). Use the questions to better yourself as well as let us know of mathematical concept/process problems you are having.</p> <p>2. (3 pts.) Attempt (<b>ON YOUR OWN!!!</b>) the problems assigned for the next class period</p> <p>Think about homework as trying out new math ideas, then class as refining/understanding, and the recap paragraph as reflecting on your learning.” (Artifact)</p>	<p>15% of final grade</p>
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8–Productive Failures Semester 4	Discrete Mathe- matical Structures	“You will receive 5 points for each single homework assignment posted on to Canvas and every week there will be comments posted by the grader and I on your homework about your proving style and improvements. <ul style="list-style-type: none"> <li>• (2 points) Create and ask two questions. One can be about the content, but think and question about your own proving process and your approach to mathematics as well.</li> <li>• (3 points) Attempt the proofs as best as you can. Try, make mistakes, involve yourself with the problems.” (Artifact)</li> </ul>	15% of final grade
9–Productive Failures Semester 5	Discrete Mathe- matical Structures	“You will receive 4 points for each single homework assignment due on Thursday and 8 points for homework due on Tuesday posted on to Canvas and every week there will be comments posted by the grader and/or I on your homework about your proving style and improvements. <ul style="list-style-type: none"> <li>• (2 points) Create and ask two questions. One can be about the content, but think and question about your own proving process and your approach to mathematics as well.</li> <li>• (2/6 points) Attempt the proofs as best as you can. Try, make mistakes, involve yourself with the problems.” (Artifact)</li> </ul>	15% of final grade

Table 5: Homework Grading Statements from Syllabi

Ivan described the shift in his policy from the first semester to the second semester at CU saying, “I think two weeks is a long time, especially for students who are mostly first-year students that were trying to figure out what everything was. I would have been the same student.”

With regard to the requirement that students create questions as part of their homework Ivan shared,

The day-to-day homeworks, they have to write two questions. Questions involve some type of curiosity. You're curious to figure out what the heck is going on either in the process or in the content. And so those questions may prompt other questions. Which then may prompt some answer, which may prompt other questions. So you see how, once you get that little bit of mathematical curiosity, which does involve some creativity as well, that I can literally go away and they can do everything. And in that, I feel like I've educated them, because I've been educated in that curiosity. (Interview Data)

Homework is a topic that Ivan spent quite a bit of time thinking about. As discussed in the subsection on the notes, he adjusted the amount of homework assigned based on the day-to-day occurrences in the classroom, while often adjusting the quality and types of questions between semesters.

I'm now toying with the idea of selecting one of the problems from the homework to be graded rigorously and to have a feedback loop, because I think now I have it as the wild-wild west, right? You attempt the problems and you get can get points from there, which I think is very good to help with self-reflection. I'm a big fan of students setting their own standards, because they usually set standards that are higher than mine. Remember, I said [in the first IBL course I taught, I had students come up with two problems, one easy and one hard, but I never defined either of those terms]? My God! The easy were usually what I would consider to be the medium and the hard were something that came out of the atmosphere.

I [also have] learned quite a bit from my colleagues. One thing I learned from my colleagues was: one colleague had a system where students started off at 50% and then just added percentage based upon the number of points and partial points they scored. And he had said, "Well, we have such a big gap from zero to 60." And that totally was imprinted in my mind, because now I use homework in a certain way because I know that there's a gap between zero and 60 that's sparse, right? (Interview Data)

## Discussion

There are a number of ways to consider Ivan's narratives with respect to his teaching. One of which is to explore the ways he envisions his affordances and enacts his beliefs in his teaching. In doing so I consider Ivan's praxis as creativity. A second way to explore Ivan's

narratives is to inquire as to the ways he chooses when and how to implement pedagogical actions congruent with his beliefs. It is clear from the review of the literature and with the limitations of using reflective accounts that all that can be said about the impromptu teaching actions Ivan implements in the classroom is that they are likely enactments of preconceived affordances (see Hora, 2012). However, considering the question as one of the actions that occur backstage (Goffman, 1959) to the teaching performance is more fruitful. In this, I consider Ivan's praxis as problem solving.

### *Creativity*

There is a notable lack of a uniform definition for creativity, with more than 100 definitions existing in the research literature (Mann, 2006). Additionally, many conceptions consider creativity either by its process or by the products of creativity (Amabile, 1983). Further complicating discussion about creativity is the long-standing debate over whether creativity is domain-general or domain-specific (Baer & Kaufman, 2005).

This research adopts the framework of Baer and Kaufman's (2005) Amusement Park Theoretical (APT) model of Creativity. In the APT model, which is a hierarchical model, there are both domain-general and domain-specific elements to creativity. This hierarchy is composed of Initial Requirements (innate or environmental conditions which are domain-general that provide for the possibility of creative performance), General Thematic Areas (broadly grouped areas containing related domains), Domains (the specific areas of creative performance), and Micro-Domains (specific tasks within a domain) (Baer, 2010).

As the adopted framework considers creativity as both domain-general and domain-specific, attention must be paid to the extant literature on mathematical-creativity. It is worth noting that research into pedagogical-creativity may likely be a more appropriate avenue of exploration, except that Ivan viewed himself first and foremost as a mathematician, thus the ways he engaged in creative performance, perhaps, was most strongly influenced by the domain of mathematics. Hadamard (1945) theorized that the creative performance of mathematicians followed the four-stage process described by Wallas (1926).

Wallas (1926) described the four stages of creative process that the thinker experiences: preparation, incubation, illumination, and verification. In doing so he laid forth the groundwork for a way to envision creativity as a problem solving process. Preparation is the stage wherein the thinker is gathering and assessing their mental resources in order to apply them to a given problem. As such, it is a cognitive and intentional process. However, there need not be just a single problem at a time, “[the thinker] will often (particularly if [they are] working on the very complex material of the social sciences) have several kindred problems in [their] mind, on all of which the voluntary work of preparation has been, or is being done...” (Wallas, 1926, p. 85). Incubation is the involuntary and subconscious stage, wherein a multitude of “mental events take place” (Wallas, 1926, p.86). That is, despite not actively working to find a solution to a problem, the mind is still working toward the resolution of the problem, primed with the resources pulled forth in preparation. Illumination occurs in the ‘fringe-consciousness,’ the periphery around a ‘focal’ consciousness, and is characterized by a ‘flash’ of insight which may be preceded by an intimation of the successful association. Finally is the stage of verification, wherein the conscious mind uses the rules and resources identified and gathered in the preparation phase to assess the association made in the illumination phase. Sriraman (2009) confirmed that the model of Hadamard (1945) was still applicable. Thus, for the purposes of this study, creativity is viewed primarily as a process with Ivan’s specific reflective utterances being able to be mapped to stages provided by Wallas (1926).

*Praxis as Creativity.* This envisioning of praxis as creativity holds in that preparation and verification are explicitly reflective, while incubation and illumination can be seen as actions that generate ways of enacting affordances. Ivan’s affordances, when enacted, become the basis for his actions in the classroom and, hence, this mapping of creativity to praxis fits the traditional ways of talking about a teacher’s praxis (see Figure 3).

Treating praxis as creativity can extend to the iterative nature of Ivan’s teaching, wherein he uses his reflections from previous classes to inform how he approaches future classes. The

unfortunate implication of this is that every modification would have to happen comprehensively, since the action of generation occurs in the sub- or fringe-conscious mind (hence Ivan has no direct agency over them). This seems to be contradictory to the ways in which Ivan makes incremental changes to his practice within each semester as well as the ways he makes adjustments between semesters, which necessitates a second conception of his praxis to accommodate these adjustments.

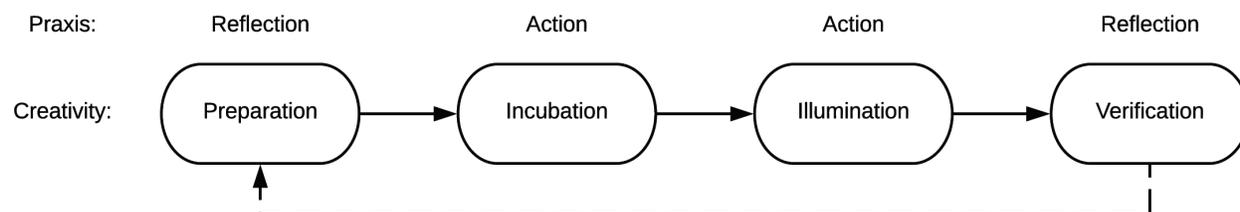


Figure 3: Creative generation of affordances as praxis.

### *Problem Solving*

Mathematical problem solving is an area that has existed for quite some time. Pólya (1957) considered questions of the nature of problem solving itself but more contemporary research tends to focus on the role of the solver in problem solving (see: Carlson and Bloom, 2005; Garofalo and Lester, 1985; Schoenfeld, 1992). For the sake of the discussion of Ivan's praxis being a form of mathematical problem solving, it is necessary to consider ways to conceptualize the process.

Pólya (1957) considered problem solving as a four-stage process but failed to explicitly consider the metacognitive facet of problem solving, leading Garofalo and Lester (1985) to create a four-stage model, built on the work of Pólya, but included specific metacognitive tasks. This 'cognitive-metacognitive framework' informed Carlson and Bloom's (2005) work, wherein they extended the 'cognitive-metacognitive framework' from a unidirectional process to a cyclic, iterative one. This 'multidimensional problem-solving framework' makes for an appropriate tool to examine Ivan's praxis due to the iterative nature of the framework as well as the focus on resources, heuristics, affect, and monitoring (i.e., multiple dimensions).

The first phase of the multidimensional problem-solving framework is that of orienting. This phase is similar to the preparation phase used by Wallas (1926) in creativity. In this phase the solver might determine the constraints of a problem, create organizational tools, engage in self-talk, etc. as they attempted to truly understand the problem. Then the solver would enter into the phase of planning. In planning, solvers conjecture possible solutions and strategies that could be used to solve the problem are imagined. This is followed by executing the plan and then checking the plan against the conceptions brought forth in orienting. The solver may return to planning several times in order to incrementally work through the problem at hand.

*Praxis as Problem Solving.* Praxis can be envisioned as problem solving in that orienting, planning, and checking are all phases of reflection, while executing is an action phase (see Figure 4). As this cycle can be conceptualized as one that permits Ivan to iteratively alter facets of his enactment of an affordance. However to first envision possible alternatives to the enactment of an affordance two things are necessary. First, that Ivan has reason to believe modifications are necessary. Second, there must be regular monitoring of the iterative changes proposed. As such, envisioning Ivan’s praxis as mathematical problem solving requires the problematization of an existing affordance.

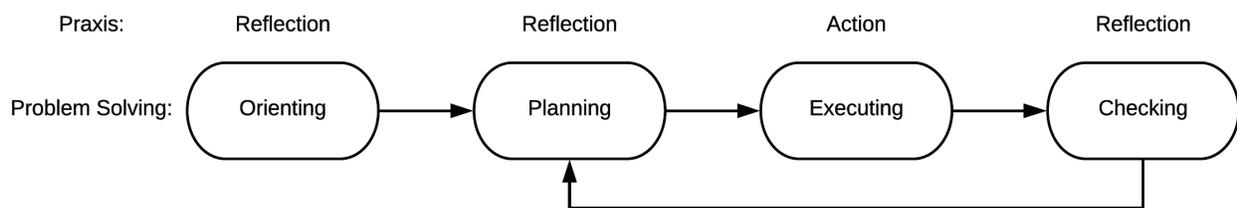


Figure 4: Refinement of enacted affordances as praxis.

*Ivan’s Praxis Cycle.*

It is clear that there are striking similarities between Carlson and Bloom’s (2005) ‘multi-dimensional problem-solving framework’ and the work of Wallas (1926) on creativity, despite being oriented about different tasks. As such, it is prudent to highlight the key distinctions

for the purposes of this analysis. First, and perhaps foremost, creativity is generative while problem-solving is investigative. That is, creativity is creates new affordances; problem solving refines them. Secondly, the nature of the processes are also different. Creativity emphasizes the location of the processes as being conscious, subconscious, or fringe-conscious. Problem-solving emphasizes the types of tasks being used (i.e., cognitive or metacognitive). Lastly, the iterative nature of problem solving allows envisioning the process of adjusting the implementation of tasks to be incremental and continuous. In creativity, with the generative action happening below consciousness, the individual has no means to assess validity of a conception of teaching until after generation is complete.

It is clear that the way Ivan engages in praxis is complicated even to himself. Ivan was unable to describe how he first chose to implement different tasks, “literally my whole teaching is a bunch of little snippets that I get from I-don’t-know-where, that kind of congeal in my mind,” which lends some credence to the interpretation that the generation of an affordance and initial reflection-action cycles of it’s implementation, Ivan’s praxis can be seen as creativity. Once revision is needed, however, it seems that Ivan shifts into a mode of problem-solving. As such, his praxis cycle can be conceptualized as a related process between these modes of reflection-action (see Figure 5).

In this process, when primed with a need to develop a new affordance, Ivan’s perceptions of the constraints incumbent with the circumstances of his teaching will be incorporated with his beliefs, attitudes, emotions, experiences, positions, and teaching conceptions into a creative planning phase. Ivan’s subconscious will continue to make associations and connections until such a time that a novel idea is generated, which is then checked against the resources pooled together in planning. This new affordance is enacted by Ivan and through formative assessment is regularly checked for its validity. If it is found to be valid, then the affordance continues to be maintained through a cycle of incubation, illumination (or not), and verification.

With the continual addition of experiences, Ivan’s subconscious continues to seek new

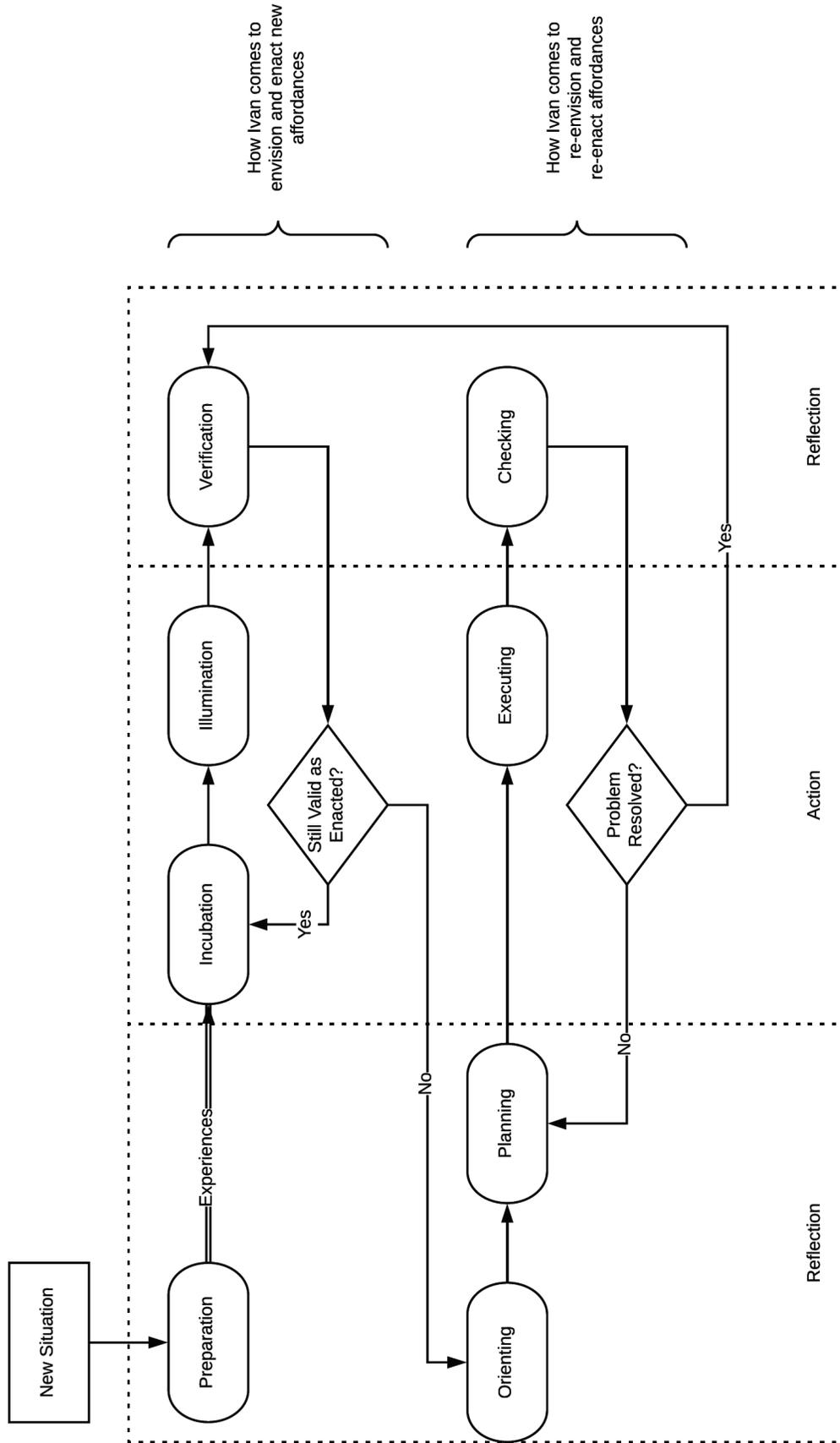


Figure 5: Ivan's praxis cycle

ways of enacting the affordance congruent with his beliefs. Illumination may not occur if the current enactment seems to be the most appropriate one based on the resources and may be skipped as Ivan receives more experiences with which to gauge his formative assessments.

Should he perceive of incongruence between his beliefs and the affordance, Ivan shifts into a cycle akin to mathematical problem solving wherein he first re-orientes himself to the, now problematized, affordance. Ivan then considers different ways of adjusting his enactment of the affordance before executing it. Finally, he checks to see that the problem, or the facet with which the adjustment was made, has been resolved. If not, the problem solving cycle continues as he plans subsequent rounds of adjustments. Once the problem is satisfactorily resolved, it returns to verification in the creativity cycle and then through incubation, illumination, and verification.

As this cycle is recursive, it stands to question how he stops enacting an affordance. Considering the ways that Ivan shifted as he changed his enactment of affordances related to attendance and participation, it is clear that there are at least two mechanisms for aborting an affordance. The first being crisis; something has gone so terribly wrong that all enactments of affordances warrant reflection. This is clear when Ivan completely abandons asking his students to complete portfolios after his first semester at CU. The second suggests a relationship between affordances in which Ivan constantly weighs them against his beliefs in an effort to choose which to enact. This is perhaps a breaking point in the cycle found in the incubation phase of the creative process, which is consistent with Ivan's narrative when he discusses the switch from participation points to productive failures, "It took a little bit of time, but I felt the time was right." Thus, the congruence between the perceived affordance of productive failures was greater than that of the enacted affordance of participation points based on utterances. This implies that the impetus for at least the creative cycle in Ivan's praxis need not be a real situation but could also be hypothetical.

*Notes.* Hence, when Ivan talks about how he goes about creating his notes, Ivan says, "I put myself into the eyes of a student and I go, 'What do I think my students can do?' and

‘what do I think they need a little bit of extra support with?’” In doing this he is entering into the preparation stage and among the mental resources he is gathering are his expectations of the students and their abilities, the content of the course, and his pedagogical content knowledge (see Shulman, 1986). Ivan describes his cognition during preparation specifically with respect to the observed course saying,

Take his example of teaching Calculus 4 the first time: “Look, I passed it in undergrad. Stop. End of sentence, right? I didn’t understand it all that much. I didn’t understand Green’s Theorem and Stoke’s Theorem and what the heck was going on.” Here Ivan is in preparation. He is taking into account what his experiences were, what the content was, and what his content knowledge was(n’t).

“And so I had to re-jump in and look at all that. Once I saw those connections...” Ivan’s use of the word “once” indicates that there was temporal distance between his exposure and the connections being made. It is perhaps the case that Ivan is considering the space since his undergraduate experience. It is just as likely that there was temporal space when he came across the theorems mentioned in his drafting of the notes. Regardless of the space, it seems that in this Ivan is talking about his incubation. Because incubation and illumination occur in the sub- or fringe-consciousness, it is not surprising that Ivan would have little to say about these stages. In fact, in all of the time I spoke with Ivan, he never mentioned explicitly needing to let the notes incubate.

In this space, it is suggested by Ivan’s next utterance, “my gosh! It was like my head exploded in such beautiful thoughts and ideas” that the resulting connection had an element of surprise and that it served to intimate additional insights (of other thoughts in incubation). Thus, Ivan is describing the illumination phase of his note generation. Of course, as he continued working on his notes and subsequently presented them to his students, verification must have occurred, because had it not (i.e., if the connections weren’t checked against the mental resources available to him) he would have had difficulty sequencing the content as well as meaningfully discussing the resultant insight with his students.

For Ivan's notes, the verification is continuous and occurs at two levels. First, is the verification of the envisioned modification with the experiences and expectations from the class. If it is, it becomes a crudely-developed affordance which will be refined through further revisions. These revisions are the outcome of the second kind of verification, wherein Ivan is verifying the appropriateness of the affordance within new classroom experiences.

Since he starts subsequent semesters with his notes from previous semesters (if such notes exist), but adjusts them in the manner described above, then his formative assessments offer a recurring verification against the new experiences. However, from time to time situations arise that force Ivan to address the validity of the affordance as implemented. This can be seen by Ivan's discussion of the Calculus 2 class that was observed as part of this study.

I wanted to try something different, and it was partly influenced by [Calculus] 4 and partly it's just sort of a re-arrangement of things. I got frustrated at the time with techniques being siloed from what was going on with some of the other stuff we were learning. We were learning about transcendental functions and so I wanted to: if you're taking the derivative of the inverse sine I want you to also take the integral of the inverse sine. But that requires me to teach integration by parts, right? And so I kind of wanted to mold all of these together. (Interview Data)

Here, Ivan identifies incongruence between his affordance, as enacted in the sequencing of topics in his notes, and his beliefs ("I got frustrated at the time with techniques being siloed") and experiences ("it was partly influenced by [Calculus] 4"). As such, the enactment of the affordance is problematized and Ivan moves to the orienting phase of the refining (i.e., problem solving) cycle. Some of the items he is consciously applying to the problem at hand: the content (integration techniques are traditionally presented separate from transcendental functions), his pedagogical beliefs (he didn't believe that this was necessary or beneficial), his affect (he was "frustrated"), and his content knowledge (the integral of the inverse sine requires integration by parts). He also continued the self-talk that that was characteristic of his entering into the preparation phase of the generative (i.e., creative) cycle.

This utterance also alludes to Ivan's planning phase, "if you're taking the derivative of

the inverse sine I want you to also take the integral of the inverse sine. But that requires me to teach integration by parts.” That is, Ivan is describing his mentally ‘playing’ with the sequence and conjecturing about how other topics might also need to be rearranged to accommodate this. Ivan discusses this as a reason for the ways his adjustments are made in general, “Adjustments may not be totally with the notes and it may be with the notes, where I put in an extra few questions and I’ll re-upload the notes. But usually my adjustments come with what piece of the notes I assign.” This illustrates that only major changes are shifted to the refining cycle, otherwise the pacing of the notes is adjusted rather than the notes themselves. It also shows that the changes in the refining cycle are clearly incremental; Ivan only discusses changing the quantity and quality of questions in the notes and re-uploading them. “Sometimes I’ll put them in; sometimes I’ll take them out. I don’t want to scaffold them. I want to see the grey and I want to see how they play around in this sea of non-existence.” As such, there must exist a monitoring feedback loop through either the generative cycle (resulting in a new affordance) or the refining cycle (resulting in additional modification).

*Productive Failure.* Being a novel development of the instructor, productive failure as a classroom characteristic has not been subject to significant changes. As such, this discussion of productive failures as evidence of a creative process as praxis will be centered upon the development of the affordance apparent in the characteristic rather than the ways in which praxis acts directly upon its envisioning.

Ivan maps his development of productive failures in his classroom back to his use of portfolios, which is a major point of insufficiency in his teaching during his first semester at CU, “I tried [to have students submit] portfolios in my first year at [CU]. It was a DISASTER...” From this, Ivan gamified his classroom, which was an affordance he had developed while working on a grant while in graduate school, “I went to gamifying, ... I thought that was [the solution]. It came from the [experiences I gained working on a grant in graduate school].”

Ivan began to notice that gamifying his classroom promoted student behaviors that did not align with his intentions,

One aspect was participation and I wanted to value participation in some way. So, students were getting points for every utterance in the classroom. Well, okay. Someone who saw that and goes, “I’m just going to talk a lot in the classroom.” And I was giving them points for it. (Interview Data)

As such, he began to consider alternate ways of valuing participation in the classroom. Hence, Ivan entered into a preparation stage wherein he reflected on his experiences and his pedagogical values. Ivan maintained a gamified classroom, which seems to indicate the sheer length of the temporal space within which incubation was occurring. That time is far too long for the problem of ‘how to better value student participation’ to have been constant on Ivan’s consciousness. Perhaps, also, it indicates a sort of dialogical relationship between the conscious and the subconscious in the early stages of creative thought because Ivan did not reach illumination until after he was exposed to Ed Berger’s work with epic failures. Ivan does not explicitly connect epic failures to his problematizing of participation, instead he relates it to the productive failures in IBL. “Ed Berger did this. I don’t know if he did it the way I do it ... And I’d been to a number of IBL conferences and many people talk about productive failure but they talk about it as an implicit thing that goes into IBL.” The final piece that Ivan identifies as catalyst for his envisioning of productive failures was his work on creativity and proving in mathematics.

Ivan doesn’t speak directly of the illumination of the affordance, but he does assert that the solution was intimated before it was revealed (see Wallas, 1926), “It took a little bit of time, but I felt the time was right. Meaning, I felt like at that moment, I could try this and it’s five percent of the grade. So if it screwed up, then that’s okay, I still get 95 to play with.” In this, Ivan is also directly speaking to both the structural constraints (e.g., pass-fail pressures) and socio-cultural constraints (e.g, normalizing didactical pressures).

Verification occurs as Ivan checks the affordance against the resources of experience and his values,

That's why productive failures are so nice, because it's a required participation that does more than just participation. ... I think that no matter what, we're opening ourselves up to be vulnerable and overcome some of those vulnerabilities. ... I really love the fact that they themselves become arbiters of whether they're doing good or not, right? Because it shifts the onus of learning and of education from like my goals and my beliefs to what you want your own goals to be. It's difficult for some, but for others, it's the most empowering thing. (Interview Data)

In this, it is evident that Ivan believes that the affordance of productive failures, as enacted, is not only fundamentally congruent with his values regarding participation but also his values regarding the very nature of education.

As with notes, there is evidence that this verification is an on-going process as well. Ivan is constantly reflecting on the value statements implicit in his enactment of productive failures which, perhaps, is the basis for subtle changes in the implementation of the affordance, which are undetectable by the tools of this study. “[T]here’s a student who said, ‘Well, he values failures in class but doesn’t value them on the test.’ Well, that’s true. I’m a hypocrite sometimes, but I try not to be as much of hypocrite as I can be.”

Ivan did not offer any words describing how productive failures have continued to change. This lends itself to the perception that Ivan had not yet had reason to refine the affordance and it may be simply a product of the generative cycle. This is supported by the stability of Ivan’s syllabus statement regarding the presentation productive failures.

*Homework.* Ivan did not envision his homework as a key component of his classroom per se, rather that it facilitated the social and sociomathematical norms (see: **yackel2000**; Yackel and Cobb, 1996) of the classroom (through his use of forward-homework and emphasizing attempts over correctness). However, the nature of the iterative changes that his homework expectations experienced indicate that it is an area which Ivan reconsiders often. The most notable change occurred between Ivan’s first and second semesters at CU.

In his first semester, he “had students turn in homework every two weeks,” which was partially a product of the portfolio-based grading that he had used previously. As such, his perceived affordances were limited to those of his teaching and IBL. Additionally, after having

tremendous difficulty that semester, Ivan had to re-conceptualize his practice, which pushed him to enter a phase of preparation. Among the resources that Ivan used in his creative process were his beliefs regarding what student work to value (i.e., process over product), what affordances he had perceived previously, and his experiences that first semester and prior. This is clear when Ivan discusses his problematizing of the situation: "I think two weeks is a long time [conception of expectational belief with implicit valuing of homework], especially for students who are mostly first-year students that were trying to figure out what everything was [conception of students populating his class]. I would have been the same student [reflection upon previous experience as a student]."

Ivan likely perceived the failings of his homework policy before the end of that first semester but he draws an analogy between the syllabus and rules to a game: "I feel like the syllabus is like the rules of the board game and the students are playing the game," which serves to provide rationale for why a change would not have occurred within that first semester. However, it is in this time that Ivan would have shifted into Incubation.

Illumination does not come by itself, but is catalyzed by his shift to gamification. Since gamification attempts to leverage elements of game design in the classroom, perhaps the most notable design element that can be observed in the homework is that of rewards. Games used frequent rewards as stimuli to trigger players' brains to increase production of dopamine (among other neurotransmitters) which produces pleasure within the player. The player's brain, or student's brain in a gamified classroom, begins to anticipate the increased dopamine from rewards, which is interpreted as an extrinsic motivation (the reward itself) becoming intrinsic (the want of the reward) (**nabar2018**). Hence, this possibly explains the quantity of points given being explicit and occurring frequently in Ivan's second semester at CU. Verification confirmed that the envisioned policy on homework is congruent with the resources gathered in preparation, otherwise its enactment would not have occurred, and on-going verification was used similar to that which was described for both the notes and the productive failures.

Interestingly, once Ivan's point scheme shifts along with his adoption of productive failures it becomes relatively stable with respect to point-values, but the work expectations continue to shift. This seems to imply a macroscopic refining process rather than a re-generation of the structure. An interesting example of this is the inclusion of the requirement that students "create and ask two questions."

Ivan identifies this requirement as one that was developed through the problematization of his homework policy wherein he believed that students should be provided the opportunity to create questions and have their curiosity fostered. This implied problematization and statements of belief are present when he says, "The day-to-day homeworks, they have to write two questions. Questions involve some type of curiosity. You're curious to figure out what the heck is going on either in the process or in the content. And so those questions may prompt other questions." Although Ivan does not provide explicit detail as to the implementation of this envisioning of homework, it stands to reason that it is the result of the refining cycle rather than the generative cycle in that the other facets of the enactment of homework, including the weight of the final grade provided to homework, were stable throughout this addition.

### **Conclusion**

Ivan's affordances are the result of a generative cycle which can be mapped onto the creative process (see: Wallas, 1926). When these affordances are enacted they become subject to a constant cycle of envision, re-envision, enactment, and re-enactment, which is praxis. This continued envisioning and enactment is a direct result of that creative process. However, when the enactment of affordances are found to be incongruent with his beliefs, Ivan shifts into a refining cycle which more closely resembles the multidimensional problem solving framework (see: Carlson and Bloom, 2005). This refining cycle can work incrementally on each facet of the problematized affordance until the incongruence can be resolved, at which point the rectified affordance is returned to the generative cycle for verification against extant beliefs.

It is unknown if the praxis cycles of other mathematics faculty follow similar patterns or if Ivan's praxis cycle is an artifact of his research interests or his previous exposure to issues in teaching. Additionally, although Figure 5 is presented in a deterministic manner, it is not my intent for it to be interpreted as such and even for the singleton case study of Ivan, more research would be needed to determine the robustness of the model. These constitute several directions for potential future research.

Although case studies should not be taken as grounds for generalization, they serve tremendous roles as existence proofs. As such, some conclusions can be gleaned from this study that warrant reiteration or additional attention. First, some mathematics faculty engage in deep, meaningful, and complex ways of praxis. In doing so, Ivan has had to generate his own affordances as well as to refine them. This calls into question the resilience and validity of the pervasive belief that professors teach as they were taught. Otherwise, it would seem that the praxis cycle of these instructors would be significantly simplified or nonexistent. Thus, it may be inappropriate to say that "professors teach as they were taught" instead of "professors *initially* teach as they were taught but some soon re-envision their teaching practice."

Additionally, the results of this study provide evidence against the monolithic perception that professors of mathematics do not reflect on their teaching. Although, as a case-study, this is not a rebuttal for every individual mathematics professor, it is clear that it not a rule for all such professors. As briefly mentioned in the directions for future research, this makes for an interesting question of 'for which types of professors does this perception hold?' Although, I would conjecture that all such professors engage in reflection on teaching, but some may be better equipped to refine their affordances based on their reflections.

Lastly, although only implicit in this study, by virtue of examining affordances this study is calling into question the norms of academic freedom in post-secondary mathematics departments. In examining Ivan's rich praxis it becomes immediately noticeable that it is by the freedom to implement actions in accordance with his beliefs that he is able to engage in

this praxis. With the preponderance of coordinated or moderated courses, especially in the lower levels of collegiate mathematics, it is worth questioning if the praxis of the instructors of these courses is being trivialized or even cut off. The subsequent, and more disturbing, corollary to this are questions of teaching quality and identity development/atrophy in the, possibly forced, absence of praxis. That is, what is a teacher deprived of praxis? As this is a study of post-secondary mathematics instruction, the most apparent facet of concern with regard to this questioning is in the development of praxis in graduate students. If praxis is seen as a valuable part of the teacher-identity, then it is incumbent upon us to push for more autonomy for experienced educators regardless of academic rank or status.

## Chapter 5: Conclusion

This study began as an inquiry into the beliefs that motivate Ivan's teaching practices. Ivan was asked to participate in this study because his practices appeared so inspired, yet foreign in the department within which he found himself. There is an exorbitant number of studies examining teacher beliefs; a search for 'teacher beliefs' in Google Scholar produced some 2.35 million results at the time of this report. However, the literature is rife with inconsistent definitions for the very topics of their study, as Kagan (1990) described, saying, "Terms such as teacher cognition, self-reflection, knowledge and belief can each be used to refer to different phenomena. Variation in the definition of a term can range from the superficial and idiosyncratic to the profound and theoretical" (p. 456). As such, this study has approached the concept of teacher beliefs not by trying to identify which propositions are or are not a belief Ivan holds, as it is apparent from the extant research that such a question is perhaps a more nuanced ontological question than I am currently equipped to sort through. Rather, the approach used in this study was to consider the peripheral elements of or relating to beliefs, knowing that the residue of Ivan's beliefs would manifest there within. These elements were the development of identity, the negotiating of personal identity within community, and praxis. These elements were explored by seeking answers the research questions posed by this study, namely:

1. What experiences does this professor perceive to have been essential to the development of his identities as a teacher-learner?
2. How has the instructor integrated into to his university and in his department, especially with regard to his pedagogical actions within the context of his position and status at his university and in his department?
3. What kinds of pedagogical actions does this professor employ and how does he choose when and how to implement them into his teaching?

Thus, I separated Ivan's story into three parts, each corresponding with one of the guiding questions of this study. As such, there may be myriad more insights to gain from Ivan's experiences simply by changing perspective and asking different questions.

Chapter two was temporally situated around Ivan's experiences prior to earning his doctorate, which served as a fairly arbitrary checkpoint, but seemed to coincide with the first evidence in Ivan's narratives that his teacher-identity was disparate from his student-identity and had become somewhat resilient. In doing so, I was able to explore Ivan's identity as a fluid, negotiated, construct using the framework of the dialogical self provided by Akkerman and Meijer (2011) and Hermans (2001). This exploration revealed a teacher-identity in Ivan that existed as a poorly defined replication of his student-identity despite his experiences as a high school teacher and as a graduate teaching assistant.

I found, also, that Ivan's teacher-, student-, and mathematician-identities maintained significant stability prior to the major crisis (see: Erikson, 1968/1994) of failing out of graduate school. It is in this crisis that Ivan was exposed to experiences that challenged his beliefs. Hence, he was able to accept new models for instruction through his 'apprenticeship of observation' (Lortie, 1975) in his Understanding and Constructing Proofs course as well as through exposure to an ethic of care (see: Noddings, 2005) while working on a grant while in graduate school. Although Ivan attributed these transformational experiences to 'serendipity' it was clear in his re-storying of his experiences that it was his recovery from crisis that provided the disentanglement from his existing beliefs needed for him to experience these events as he had. It is due to these experiences that Ivan first began to question the nature of teaching mathematics, which signifies the first true separation of his teaching and student identities, which would be reified when Ivan first teaches using his own, newly-formed beliefs about teaching as opposed to mirroring his student experiences.

After completing his doctorate, Ivan found placement in a postdoctoral position wherein his primary task was to create and implement a professional development program for graduate students. This position drew extensively on his previous grant experience but did little to

help Ivan develop a researcher identity. The interaction of his weak researcher-identity and his strong teacher-identity within the expectations of his subsequent placement at Current University (CU) formed the basis for the examination presented in chapter three.

Using the framework provided by Sam and Berry (2010) on acculturation strategies and envisioning research fields and academic departments as distinct communities of practice (see: Wenger, 1998) with cultures all their own (see: Gans, 1997), I considered the strength and resilience of Ivan's researcher- and teacher-identities as he began his professional career. This exploration revealed that Ivan's weak researcher-identity meant that he was needing to assimilate to the norms of his field for fear of being marginalized within it. This resulted in significant trauma to his researcher-self when he received a scathing response from his first submission to a professional academic research journal. This trauma led Ivan to question whether academia was truly a place he could belong. However, through outreach from within the larger community of individuals researching in undergraduate mathematics education (RUME), he was able to find the validation he needed to recover.

By contrast, Ivan's teacher-identity was fairly robust upon his arrival at CU. Ivan was teaching using Inquiry-Based Learning (IBL), which was congruent with his experiences, both as a student and as an instructor while in graduate school, and his beliefs, and had had some success with enacting pedagogic practices beyond what he himself had experienced as a student. Thus, when Ivan arrived at CU, he informed the department chair that he would continue teaching using IBL. However, his first semester had dismal outcomes in terms of student perceptions of their learning and of Ivan's teaching effectiveness. These culminated into multiple formal complaints to the department chair. The response in Ivan's teacher-identity to this difficulty can be seen through his reaction to the chair's discussions with him. Ivan wanted his teacher-identity to be integrated into the department's teaching culture. However, Ivan was the sole earner for his family, thus separation from the department was untenable. If Ivan perceived that maintenance of his teacher-identity was not desired by the department, then he would have to assimilate to the didactical norms of the department

or risk being marginalized, which would likely have resulted in him not receiving tenure. Fortunately, the department chair, being perceived by Ivan as the arbiter of the ‘regime of mutual accountability’ (see: Wenger, 1998), was supportive of Ivan. Thus, Ivan was able to integrate, albeit with some change to the enactment of his beliefs.

Ivan had acculturated into his communities of practice as a novice researcher and as a junior member of his academic department by the time I had met him. My questions leading to this study were centered upon the manifestation of his beliefs in his practice. The fourth chapter of this report connected Ivan’s practice, with respect to three hallmark features of his classroom that provide some insight into his envisioned and re-envisioned affordances, and the ways he negotiated enacting and re-enacting those affordances in his practice. That is, chapter four examined the ways that Ivan’s narratives, artifacts from his teaching, and my personal observations of one of his courses illuminated the ways Ivan engaged in praxis.

Ivan described the genesis of his affordances in a generative cycle that was mapped to Wallas’ (1926) work on creativity and was subject to constant re-evaluation for consistency with Ivan’s beliefs. When this evaluation necessitated modification to the enactment of the affordance, Ivan adopted a refining cycle, which was mapped onto the multidimensional problem solving framework (see: Carlson and Bloom, 2005), which allowed him to implement incremental changes until congruency with his beliefs was re-established.

Ivan’s rich ways of engaging in praxis were surprising given that the extant research has found that junior faculty members tend to prioritize their efforts on research and service over their teaching (Hora, 2012, 2016; Kagan, 1992; Kane et al., 2002; Sabagh & Saroyan, 2014). This would imply that the cognitive and metacognitive efforts of professional improvement would be focused on improving research and service, however, for Ivan, this was not necessarily the case.

### **Limitations**

Before addressing the implications of this study, it is prudent to first acknowledge issues that limit the generalizability of this study. First and foremost, this study was a case study

of a singleton instructor. As such, it is inappropriate to attempt to directly generalize the results of this study to any greater population. To this end, Merriam (2009) asserts that, for case studies, it “is the reader, not the researcher, who determines what can apply to his or her context” (p. 51). This is not to say that nothing can be gained from the research presented here within. Rather, case studies may be envisioned as ‘existence proofs’ or ‘counter-examples’ to existing results or conjectures.

It is also true that this research is centered about the narratives that Ivan has shared with me. As such, the results of this research have been drawn from my interpretations of Ivan’s moderated interpretations of his recollections of his perceptions. Hence, none of the events should be assumed to have occurred as they have been shared and subsequently reported. This is of concern as “the individual is likely to present himself in a light that is favorable to him” (Goffman, 1959, p. 7). Despite the efforts taken to return to Ivan’s narratives regularly, it is still likely that the whole of the narrative that Ivan tells himself to justify the existence and development of his beliefs was not, nor would ever be, shared with me.

### **Implications**

There are a number of things that can be gleaned from this study, however many of them can simply be taken as evidence of existence. For example, Ivan demonstrates that contrary to the perception that junior faculty tend to de-emphasize their teaching (see: Hora, 2012, 2016; Sabagh and Saroyan, 2014) should not be taken as a given; Ivan demonstrated robust ways of engaging in praxis as he approached integrating new pedagogical actions into his teaching. However, there are two particular implications that Ivan’s stories highlight. The first of which pertains to admissions into graduate programs, which operate as gatekeepers to advanced study and the second pertains to the very goals of a graduate education beyond the requisite coursework.

## *Gatekeepers*

It's one of these things, where I'm where I'm at, which means that I had the capabilities of doing it, and yet I was not doing it all that well. I was getting B's and C's and getting kicked out of [graduate] school. If I really had the capability to do well, to get a Ph.D., there must be some kind of shift that I need to do in order for others to have that capability as well. Or the opportunity. (Interview Data)

Entrance into a graduate program is a significant milestone in the lives of our prospective colleagues. With graduate programs evaluated in part by student degree completion outcomes and limited resources to support students in their studies it is clear that there should be some regulation of who is granted acceptance. However, it is also the case that Ivan clearly was not a strong candidate for graduate study at Pacific State Public University (PSPU) and even less so for Western Land Grant University (WLGU). It is by virtue of the admissions committee taking a chance on his application that Ivan was able to become the instructor presented in this study. How many other "pockets of wonderfulness" (see: Seeley, 2004) are never permitted the opportunity? In this, it is easy to be reminded of the famous quote by Adams (1907), presented with some context,

Not that his ignorance troubled him! He knew enough to be ignorant. His course had led him through oceans of ignorance; he had tumbled from one ocean into another till he had learned to swim; but even to him education was a serious thing. A parent gives life, but as parent, gives no more. A murderer takes life, but his deed stops there. *A teacher affects eternity; he can never tell where his influence stops.* (p. 261, emphasis added)

Just as Adams bemoans the impact on his students caused the fact the he was the weakest link in Harvard's history department, the impact of not affording an opportunity to a prospective student may reverberate eternally.

It is beyond the work of this study to offer suggestions for how to discern which candidates are most deserving of a second, or subsequent, opportunity. However, Ivan's case provides some conflicting implications as to the treatment of such students. At both PSPU and at WLGU Ivan was denied funding, at least initially. As such, his education was paid for by

student loans and outside employment. At PSPU, this was a direct contributor to Ivan's desire to rush through his coursework and exacerbated his difficulties with the coursework. However, at WLGU, this lack of support fed into Ivan's willingness to accept new pedagogies and ultimately led to him applying to be part of the grant from which he had learned a tremendous amount regarding the art of teaching. Additionally, it is not clear to what extent Ivan's failed experience at PSPU drove the change in Ivan's perspective regarding funding at WLGU. Although they are lingering questions, they hopefully give the community cause for additional deliberation before making admission decisions.

An additional question of access and equity is apparent in Ivan's story. Ivan chose his undergraduate school based almost exclusively on cost-related factors. It was in-state, offered an athletics scholarship, and credit for his Advanced Placement Exams. Ivan was also the first in his family to attend university, meaning he had to negotiate the social dynamics of that space for himself, without the aid of any familial reference. Thus, it is worth questioning if Ivan's background exacerbated his difficulties in undergraduate school, thus weakening his graduate school application, which resulted in him not receiving financial support. Plainly, if we envision education as a primary pathway to improve on one's socioeconomic caste in America (i.e., if education opens the door to the 'American dream') then is it the case that Ivan's caste provided him an inequitable share of difficulty in achieving his doctorate? It is not much of a stretch to conjecture the case given the readiness with which Ivan described finances as a primary motivator for many of his choices while a student. If there is even a modicum of truth to this conjecture, then there are significant equity questions that must be reconciled by admissions committees, unless we, as a community, are satisfied by only offering opportunity to the wealthiest (and often whitest) of potential candidates.

### *Goals of Graduate Education*

Doctoral programs hold a fairly unique position in higher education. It is these programs that not only develop students into novice researchers with utility in some narrow slice of the corpus of human knowledge but also that develop these students into colleagues and

potential replacements for the faculty that are so critical to that development. As such, insights can be gleaned into what the goals of graduate education are (possibly implicitly) or perhaps what they might be to help students in this development.

It is clear from Ivan's narratives that his initial goals for graduate school included attainment of a title and continued exposure to mathematics. Even after completing his Qualifying Exams, he did not have a clear vision of what the work of an academician was.

Once I got towards the end of undergrad I thought to myself, "Well, I'd like to do a little bit more of this and I want the fancy title." So that's why I did the doctorate. Even past my [Qualifying Exams] I didn't know what I was gonna do with my doctorate. So think about that. I had no clue! ... I was very good at mimicking, at redoing what the book had already done. I thought I was good at mathematics because of that. That is no where near what a mathematician actually does and even when I got to my dissertation and my professor was like, "You're gonna make new math." And I go, "No... no. I'm not gonna make new math." (Interview Data)

This calls into question the ways in which we advise, recruit, and retain students in these programs. Although there are a number of prospective employment outlets for holders of advanced degrees in mathematics, Ivan wanted to teach college. As such, perhaps an honest discussion between Ivan and any permanent member of the academic community within which he was studying should have occurred describing the true nature of the position toward which Ivan was working. It is hard to imagine that such a discussion would have negative consequences. If the realities of the work of a research mathematician were beyond what a student is willing to endure, surely such conversations would begin a dialog on how the student can meet their personal goals. Otherwise, such a conversation may have helped Ivan acculturate once he arrived at CU.

Additionally, Ivan claims to have had trouble getting started on his professional work,

Post graduation, I was told to publish what I had written in some journal, and then go from there. But, I had so many ideas and so many thoughts [regarding the kinds of research I wanted to do], but there were so difficult to try to spring into action. (Interview Data)

To this end, the Tuckers wanted Ivan to stay at WLGU for another year. As such, this also

is an area where either advising or opportunities to engage in research may have been lacking in Ivan's graduate school years.

These claims are predicated on the conception that graduate school (and perhaps all schooling), especially doctoral programs, should be keenly focused on the needs, wants, and life goals of the student. Faculty members are remiss to allow a student wishing to join academic faculty at research institutions without first having conversation with the student and sharing the nature of the field they are wishing to enter. It is certainly true that circumstances may differ from field to field or from institution to institution, but fundamental aspects of the job (e.g., research mathematicians "make new math") should not at all be a surprise. It is quite possibly the case that such conversations are best had very early or even prior to acceptance into such academic programs so as to mitigate negative reactions by the student as well as to more adequately manage the limited resources allocated to student support. It is perhaps advantageous to not think of doctoral students, and significantly more so of those attaining candidacy, as 'students' so much as 'prospective colleagues' and to interact with them, as much as is professionally appropriate, correspondingly. Just as it would be a disservice to the individual to not at least inform them of the nature of the job they are wishing to enter into, it is a disservice to those already in such positions who will interact most closely with them.

Further, as Ivan wanted to teach college-level mathematics and teaching was a key component for the majority of the positions that might have interested him, being able to engage in that art was critical for his development. However, it wasn't until Ivan was able to teach without the strong direction and uniform assessment criteria of a coordinated course that he was able to develop a distinct teacher-identity. If developing a teacher-identity is seen as essential in the development of the teacher (Britzman, 2003; Freese, 2006; Jarvis-Selenger et al., 2010; Palmer, 1998; Pinnegar, 2005), then it is perhaps the case that prospective colleagues should be allowed to have true autonomy in some of their classes. However, in Ivan's circumstance, he had to first be able to envision alternatives for the standard pedagogy he

had been exposed to. This gives cause to conjecture about the need for some exposure to alternative pedagogies for all prospective colleagues.

Noddings (2005) plainly asserted that

the most fundamental change required is to empower teachers as we want them to empower students. We do not need to cram their heads with specific information and rules. Instead we should help them learn how to inquire, to seek connections between their chosen subject and other subjects, to give up the notion of teaching their subject only for its own sake, and to inquire deeply into its place in human life broadly construed. (p. 178)

This study implies no less.

### **Ivan's Comments**

In the final preparation of this report, I contacted Ivan and asked him to read the second drafts of chapters two, three, and four of this dissertation. In this request, I indicated that I felt that although this work is my own and I fully stand behind my study and the analysis presented here within, the narrative had become a joint reconstruction between Ivan and myself. As such, I believed it necessary to allow Ivan a space to offer his reactions and rebuttals to my discussion and analysis. In the email response that I received to this request, Ivan said, "I'm really excited about discussing this; I went through quite a few stages of feelings throughout the reading." Thus, we set a meeting time for Ivan to share his response, which resulted in three key points of discussion: protecting his identity as much as possible, the analysis of the findings of this study, and the clarifications of factual events. Ivan discussed his perspective in reading the drafts he was provided saying,

Chapter two, I reread the most because it was the most personal out of the three, and I wanted to make sure that I was okay with everything, because it's really tough to be analyzed in terms of your difficulties.

From this part of our conversation, we discussed ways that wording could be adjusted that would be true to this research while serving to better protect his identity. As such, additional specifics such as names of the retailers for which Ivan had worked have been removed as well as generalizing the fields of study that he has previously engaged in.

Additionally, we had a discussion of my conception of the relationship between Wallas' (1926) creative process and Carlson and Bloom's (2005) multidimensional problem-solving framework. Specifically we discussed the role of creativity in problem solving as I have used them in chapter four to discuss Ivan's praxis cycle. The products of this conversation are still amorphous to me and will continue to be a point of reflection. That said, they do not directly affect the discussion of chapter four. Ivan also described his perception of the analyses presented in the chapters saying, "I'm coming from the most unique out of anyone to read this work, but I think the way you analyzed these things are incredibly novel; they're creative. And I really appreciated reading them."

Lastly, there were two factual errors, both occurring in chapter two, that Ivan clarified in our conversation. The first is with regard to the implied nature of Ivan's scholarship to his undergraduate program. One may infer from the text of chapter two that Ivan received an athletic scholarship. This was not so. Ivan received an academic scholarship and chose to play club soccer as an undergraduate student. The second clarification is that Ivan did, in fact, apply to multiple graduate schools upon deciding to leaving the secondary mathematics classroom. This fact was not shared during the original interviews as it had very little bearing upon Ivan's experiences. Although neither of these revelations affect the analysis and discussion presented in chapter two, I have made an effort to clarify these issues in the prose.

### **Future Directions**

Additional research into narratives of post-secondary mathematics instructors is needed if we are to hasten the rate at which we are improving student outcomes (Seeley, 2004). This research would be well served in approaching this in one of two separate pathways: adding additional small case studies as has been presented in this study or seeking to find commonalities in the narratives identified in such case studies across other 'pockets of wonderfulness' in post-secondary mathematics education.

More research is needed to simply share the things that 'wonderful' instructors do and

how they believe that they came to be the instructors that they perceive themselves to be. In doing so, we share their ideas and conceptions with our current and prospective colleagues in undergraduate mathematics education as well as turning an uncomfortable focus onto our own shortcomings as instructors of mathematics (DeGuzmán et al., 1998). The problem statement of this research began with what I believe is a fairly obvious statement, “Fundamental to the concerns of the quality and appropriateness of contemporary post-secondary mathematics instruction is the ability to envision alternatives to traditional approaches.” Additional research in this direction only serves to provide additional counter-examples to traditional practices.

Finally, this research calls to question the very uniqueness of Ivan. Is it that by nature and his formative development that he truly is a novel instructor? Or is he a novel instructor, like many others, who share some common experiences? Questions regarding who is privileged to enter graduate programs and what the goals of those programs should be as well as who is permitted to teach collegiate mathematics are all areas that would gain illumination with research seeking to generalize from narratives such the those presented in this study.

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