AN EXPERIMENTAL STUDY TO DETERMINE THE EFFECTIVENESS OF ONE TYPE OF COURSE IN COLLEGE MATHEMATICS FOR THE NON-MAJOR

By

SISTER MARY FIRMINA LAJEWSKI, C.S.S.F.

Bachelor of Science
Fordham University
Bronx, New York
1956

Master of Science
Oklahoma State University
Stillwater, Oklahoma
1959

Submitted to the Faculty of the Graduate School of
the Oklahoma State University
in partial fulfillment of the requirements
for the degree of
DOCTOR OF EDUCATION
May, 1966

AN EXPERIMENTAL STUDY TO DETERMINE THE EFFECTIVENESS

OF ONE TYPE OF COURSE IN COLLEGE MATHEMATICS

FOR THE NON-MAJOR

Thesis Approved:

Wm P Ewens
Thesis Adviser

B R Belden

Jeanne Agnew

Alan H. Hamilton

Dean of the Graduate School

ACKNOWLEDGMENTS

The writer wishes to express her gratitude to the Chairman of her Advisory Committee, Dr. W. Price Ewens, who assumed the major responsibility for directing this study, for his consistent thoroughness and valuable guidance, and to the members of the Committee, Dr. Jeanne L. Agnew, Dr. Bernard R. Belden, and Dr. O.H. Hamilton, for their interest and aid.

The writer also wishes to acknowledge her indebtedness to the many people who rendered their valuable assistance in the collection of data needed for the completion of this study: the heads of mathematics departments and professors who completed the questionnaires; the jury of 38 experts who helped to identify the most desired objectives of a college mathematics course for the non-major; the college mathematics teachers who sent copies of tests and examinations; and, in particular, all those who directed the testing that was done in the 10 colleges which constituted the random sample for this study.

The writer is humbly grateful to Reverend Mother Mary Virginette, the superior of the Felician Sisters of the Lodi Province, for the generous leaves-of-absence and the material aid given to make this work possible.

The writer also extends her heartfelt appreciation to Sister Mary Angelia and the Felician Sisters at Blackwell General Hospital for their moral support, encouragement, and material assistance, and to Mrs. Kathie Fitch who did the typing and offset work.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER I

THE PROBLEM

Origin and Statement of the Problem

Since the time of Euclid mathematics held an honored place in the curriculum and its value was generally recognized. It was assumed that mental training derived from mathematics carried over automatically to other situations, even non-mathematical ones. It was believed that the individual who acquired skill in reasoning in mathematics would be able to reason in any other subject as well; the training in memory obtained in mathematics would be helpful in any situation where memory was used. The Committee of Ten of 1893 did not question the disciplinary value of mathematics and the College Entrance Examination Board of 1900 continued to accept this point of view. There was no question about the basic importance of mathematics in the consideration of social, economic and technical problems.[1]

When experimental educational psychology commenced its course of development in the early decades of the present century, it took a critical attitude towards everything in educational theory and demanded that theories be tested by experiment. This led to many experiments to

[1]E.R. Breslich, "Importance of Mathematics in General Education," The Mathematics Teacher, Vol. XLIV (January, 1951), p. 4.

determine the amount of "transfer" of training in one subject to other situations.

The results were disturbing. As early as 1900 Thorndike and Woodsworth arrived at the conclusion that improvement in one mental function seldom brings about equal improvement in any other function no matter how similar.[2] Immediately other educators began to interpret these findings to mean that the idea of transfer may be disregarded as if it did not exist and that it could be written off as a value in the study of mathematics. Furthermore, influential leaders in a reform movement in mathematics said nothing of its disciplinary values. Felix Klein in Germany, John Perry in England and E.H. Moore in the United States suggested reforms in the selection, organization and presentation of instructional materials and in doing so they turned their attention primarily to the utilitarian aspects of the subject.[3] The idea of mathematics-for-its-own-sake was not mentioned and thus began the decline of mathematics from its generally recognized place as "the queen of the sciences" to the more humble position of a "tool study" or, as one author has aptly stated it, "the handmaiden of other sciences."[4]

About the same time, that is in the early part of the twentieth century, there began to occur a great change in our educational system. "This change has been nourished in the basic philosophy that education for

---

[2]Dom Thomas Verner Moore, Cognitive Psychology (New York, 1939), p. 473.

[3]Commission on Secondary School curriculum, Mathematics in General Education (New York, 1940), p. 5.

[4]Anthony Standen, Science Is A Sacred Cow (New York, 1950), p. 186.

all is not only democracy's obligation but its necessity."[5] Students began to be admitted to high schools and later on to colleges "en masse" regardless of their ability. Many could not cope with the traditional courses and so teachers and pupils alike began to question their usefulness. School administrators permitted students to drop these courses or to substitute for them newly designed, often "watered-down," courses in many different fields. As a result there developed the free elective system in many high schools and colleges. This situation further contributed to the decline of mathematics in the school curriculum. Since the "transfer" value of mathematics was supposedly disproved, students who were not interested in a professional use of mathematics saw no reason for taking a course in it especially since they had a secret dread of it anyway. In this decision they were often supported by their educators and gradually mathematics ceased to be considered an integral part of liberal education.[6]

During the depression days of the thirties it was especially difficult to justify mathematics for its own sake; to make the subject acceptable it was necessary to show that it was useful in daily-life activities. This gave rise to the general mathematics courses which placed primary emphasis on skills and application. Too often these courses degenerated into remedial arithmetic.[7]

Then came the Second World War and with it the insistent demands of the armed forces and industry for more mathematically trained people.

---

[5] F. Lynwood Wren, "The Merits and Content of a Freshman Mathematics Course," School Science and Mathematics, Vol. LII (November, 1952), p. 597.

[6] William Betz, "Five Decades of Mathematical Reform," The Mathematics Teacher, Vol. XLIII (December, 1950), p. 382.

[7] Jack D. Wilson, "What Mathematics for the Terminal Student?", The Mathematics Teacher, Vol. LIII (November, 1960), p. 520.

Mathematics began to enjoy a new prominence in education, but at the same time many heretofore unrecognized deficiencies in mathematics education were brought to light.[8] The general mathematics courses previously designed in an effort to reach all students, began to be looked upon with disfavor and in many schools they fell into complete disrepute. Those who were able pursued the traditional sequence of courses with more and more emphasis on skills. Since the layman had little use for technical mathematics, he objected to the naked and dry material that was usually presented. Thus mathematics, a field of learning which had contributed so richly to our culture, became a subject in which the average adult was relatively illiterate.[9]

In the forties with the addition of more and more new courses to the curricula of higher institutions of learning, the compartmentalization and proliferation in our educational system become more pronounced. A protest movement known as the general education movement which had started much earlier now began to gain momentum. Its object was to reunify the curriculum by providing a common core of educational experience that should be shared by all students, regardless of their special interest.[10]

Typical of the type of thinking that gave rise to programs of general education on the college level was the Basic College Program put into effect in 1944 at Michigan State College and described as follows:

> It is designed to build specialized training, where desirable, on a broader foundation. It is designed to give each student -- whether he be an eventual specialist or not--the opportunity

---

[8]Ina Lynch, "Forces That Have Influenced the School Mathematics Program," School Science and Mathematics, Vol. LXIV (April, 1964), p. 255.

[9]Ibid., p. 259.

[10]William P. Tolley et al, Cooperation in General Education (American Council on Education, Washington, D.C., 1947), p. 15.

> for knowledge, skill, understanding, appreciation, and thinking in diverse ways, so that he may develop as a well-rounded individual, capable of adjustment to changing situations; capable not only of rendering service on a job, but also of utilizing effectively those nonwork, nonsleep hours that constitute so important a part of the good life.[11]

Many other colleges began to develop similar programs which, in their opinion, would prepare the average citizen for his place in the world. In doing so they began to ask questions as to what should be the appropriate place, if any, for mathematics in such a program of general education and what content should be included in a course designed to fill that place.

Noteworthy among them was the College of the University of Chicago. In an attempt to answer the above questions, a mathematics course for general education was designed and offered in the autumn of 1943. It was a one-year course (two semesters, or three quarters) meeting five times per week, and presupposing a knowledge of elementary algebra and plane geometry.[12] The course was under constant scrutiny and revision. By 1947 one-fifth of the course was devoted to logic, two-fifths to algebra, and two-fifths to analytic geometry and trigonometry.[13] Thus in the late forties most of the content was still traditional.

The course described above seemed to be better suited to the student specializing in mathematics and its related fields rather than to the

---

[11] Paul L. Dressel et al., Comprehensive Examinations in a Program of General Education, (Michigan State College, 1948), p. 3.

[12] E.P. Northrop, "Mathematics in a Liberal Education," The American Mathematical Monthly, Vol. LII (March, 1945), p. 134.

[13] E.P. Northrop, "The Mathematics Program in the College of the University of Chicago," The American Mathematical Monthly, Vol. LV (January, 1948), pp. 3-6.

non-major. In the opinion of a number of mathematicians, it was too difficult for the purpose of general education.[14]

By 1949 many colleges had included what they called a cultural general mathematics course in their curriculum. This course aimed to prepare students to be intelligent citizens mathematically and to show the relation of mathematics to other great fields of learning. Its proponents claimed that it emphasized understanding of mathematical concepts rather than the acquisition of manipulative skills, knowledge of the history of mathematics, and an appreciation of the place of mathematics in the culture of the past and present. Although, in the opinions of over 900 teachers, there was some agreement as to the topics to be included in such a course there was a noticeable lack of uniformity in the interpretation of its goals. In many instances, it was a cultural general mathematics course in name only. In reality it was still basically the traditional introductory course.[15]

The status of mathematics-for-general-education at the beginning of the fifties could be summed up in the following statement:

> Mathematics for the Non-Scientific or General Student is becoming increasingly important in college education. A large number of schools give such a course or courses which are often required of a large percentage of students enrolled in a particular college or division. The content and organization of the mathematics for these students is best characterized by its variability.[16]

---

[14] Ibid., p. 7.

[15] Kenneth E. Brown, "The Content of a Course in General Mathematics--Teachers' Opinions," The Mathematics Teacher, Vol. XLIII (January, 1950), pp. 26-28.

[16] James H. Zant, "General Mathematics at the College Level," School Science and Mathematics, Vol. L (June, 1950), p. 477.

By the end of the decade mathematics gained a new over-all status. The reasons for this were numerous but a central factor was the success of the Russians with their "Sputnik." We were now in a new technological age--the space age--and mathematics teachers were forced to review and revise not only the mathematics taught, but also the methods of teaching that mathematics.[17] Widespread activity and interest in the development of new programs in mathematics from the kindergarten through high school was bound to have some influence on the type of mathematics generally required of the non-major.

At present the so called new approach to mathematics is no longer an experiment. It is reflected in most textbooks that have recently been printed for a college course in mathematics-for-general-education. In every one of them there appears to be a common theme: mathematics must be based on the understanding of ideas; skills should evolve out of these ideas.[18]

There is also an increase in the number of colleges that have added a mathematics requirement to the undergraduate program for the non-major who had traditionally shunned college mathematics. The course required is usually a basic concept course which includes topics such as elementary set theory, logic, and mathematical structure.[19]

There can be no doubt that the importance of mathematics in a

---

[17] Preston C. Hammer, "The Role and Nature of Mathematics," Pi Mu Epsilon, Vol. III (Spring, 1964), p. 501.

[18] Vincent H. Haag, Structure of Algebra (Reading, Mass., 1964), p. iii (preface).

[19] C.B. Lindquist, "Entering Levels and College Courses in Freshman Mathematics, " School Life, Vol. XLV (April, 1963), p. 14.

program of general education is now recognized more than it has ever been in the past; however, opinion still varies concerning the purposes and content of the mathematics that should be included in such a program. Several types of courses have been experimented with and continue to be offered. At least three types are generally recognized even though there is no sharp dividing line which would distinguish one from the others. They are usually referred to as traditional, remedial, or as mathematics-for-general-education which is neither traditional nor remedial in nature. If the number of textbooks for the latter course published in the last two years can be used as a criterion for making a judgment, then the mathematics-for-general-education type of course is the preferred one, or at least it is becoming so.

There is little evidence that any of these courses has been properly evaluated. The lack of agreement relative to purposes and content have probably discouraged any efforts directed toward measurement.

Since many colleges have included in their curriculum a mathematics-for-general education type of course on which there is at least some semblance of agreement, now would be the time to make an attempt at evaluation. In the words of Paul L. Dressel, "it has long been recognized that evaluation, both in a broad sense and in a more specialized application is necessary if the complex processes of higher education are to be administered most efficiently, effectively, and economically."[20]

This study has been designed to make such an evaluation of the mathematics-for-general-education type of course in its present form.

---

[20] Paul L. Dressel et al, Evaluation in Higher Education (Boston, 1961), p. ix (preface).

The writer does not presume that there is total agreement as to its objectives and content, but the writer does assume that there are enough areas of agreement, if they are identified, to warrant such an investigation.

This study will, therefore, seek to identify the more desirable objectives of a mathematics-for-general-education type of course and then seek to measure the effectiveness of the course in achieving these objectives.

Definition of Terms

1. General education: Those phases of nonspecialized and nonvocational education that should be the common possession of educated persons as individuals and as citizens in a free society.

2. Fused or correlated course: A mathematics course in which the content is made up of certain traditional topics such as algebra, trigonometry, analytic geometry, and calculus.

3. Remedial course: A course whose main objective is improvement in mathematical skills.

4. Mathematics-for-general-education type of course: A cultural type of course in which the content is generally made up of topics other than the traditional; it emphasizes the understanding of broad mathematical concepts rather than the acquisition of technical skills.

5. Non-major: A college student or graduate who has not majored in mathematics, engineering, or any of the physical sciences.

Major Assumptions

Four basic assumptions were made in the design of this study. They are as follows:

1. There are three types of college mathematics courses being offered to the non-major in a program of general education: (a) the remedial course; (b) the fused or correlated course; and (c) the mathematics-for-general-education type of course.

2. Students not majoring in mathematics or its related fields can enrich their general education background by a type (c) course. They are capable of developing a genuine appreciation for the language of mathematics and an abiding respect for the very important work of the specialist who needs the support of the average citizen, "for the strength of a culture depends in large measure on the extent to which its citizens understand and support the work of the experts."[21]

3. It is possible to compile a list of the most desired objectives of a college mathematics course in a program of general education. These objectives will be found in literature, research papers, college bulletins, and textbooks.

4. A reliable and valid device may be constructed for the purpose of measuring the attainment of these objectives by a mathematics-for-general-education type of course.

Need for the Study

During the last twenty years there has been a revival of interest in mathematics for general education. The work of clarifying the objectives and of the selection of content for a college mathematics course for non-majors has been a part of several research projects.

[21]Harold P. Fawcett, "Mathematics in General Education," Bulletin of the National Association of Secondary-School Principals, Vol. LIII (May, 1959), p. 33.

However, there is little evidence of any significant contribution having been made in the field of evaluation.

In an address presented at the annual meeting of the Mathematical Association of America in 1944 E.P. Northrop of the University of Chicago stated:

> I am impressed by the fact that only a very small number of writers are bold enough to question whether or not the objectives of mathematics in liberal education have yet been clearly stated; or, if stated, whether or not they have been measured by means of tests and examinations. Of this small group of critics, some believe that significant tests will eventually be found, but have not been found to date. A very few suspect that significant tests may never be found.[22]

In 1950 James H. Zant of the mathematics department of Oklahoma State University raised questions relative to the need for evaluation.

> "General education as a movement has attempted to correct the undue emphasis on specialization, on the acquisition of factual information and technical skills and the almost exclusive emphasis on the intellectual development of the student which has come to be associated with the more traditional programs. This movement--is designed to encourage the integration and retention of the knowledge and skills gained by the students and to provide opportunity for emphasis on long-term goals of instruction."
>
> If we think of mathematics in terms of the above statement, a method of evaluation becomes one of the primary problems involved.[23]

Zant also stated that to carry out a program of improvement it is necessary to clarify the goals of such a course; to improve existing evaluative devices and to develop new ones to provide information on student progress relative to common goals.[24]

---

[22] E.P. Northrop, "Mathematics in a Liberal Education," The American Mathematical Monthly, Vol. LII (March, 1945), p. 132.

[23] James H. Zant, "General Mathematics at the College Level," School Science and Mathematics, Vol. L (June, 1950), pp. 477-478.

[24] Ibid., p. 479.

Doctor Philip S. Jones of the mathematics department of the University of Michigan, in a panel discussion on experimental courses and texts for non-majors, made the following statement:

> I do believe that some course designed for the non-science, non-mathematics major is needed and will in time become the accepted thing. Its objectives and procedures need definition, study, experimentation, and clarification.[25]

More recently the need for methods of evaluating mathematics programs and the lack of measuring devices for this purpose was expressed by Donavan A. Johnson of the University of Minnesota:

> The evaluation of a school mathematics curriculum will need the judgment of many specialists from varied fields...It will also need a comprehensive testing program which includes tests which have not yet been devised.[26]

The statements of these experts indicate that although there is evidence of renewed and vigorous interest in college mathematics for general education, there is little evidence that what has been done in clarifying the objectives and content of a college mathematics-for-general-education type of course has been properly evaluated.

Purpose of the Study

The purpose of this study is:

1. To identify the desired objectives of a course in mathematics for the non-major in a program of general education.

2. To measure the effectiveness of the mathematics-for-general-education type of course in attaining these objectives.

---

[25] Philip S. Jones and Cleota Fry, "Development of a Junior College Mathematics Program for Non-Science, Non-Mathematics Majors," <u>School Science and Mathematics</u>, Vol. L (June, 1950), p. 441.

[26] Donavan A. Johnson, "Evaluating a School Mathematics Curriculum," <u>School and Society</u>, Vol. XC (December, 1962), p. 425.

Scope

This study is limited to the 530 institutions of higher learning designated in the Higher Education Directory of 1961-1962 by the letter (e), i.e., liberal arts, general, and teacher preparatory institutions. These are the institutions to which the preliminary questionnaire was sent and they represent almost every state in the union.

However, the random sample to which the measuring device was applied was selected from the 166 institutions of higher learning which, in response to the questionnaire, stated that they were currently offering a type (c) course and were willing to cooperate in the testing. Since the schools in the sample were not chosen from any particular section of the country, the study, in a limited sense, could be considered nation-wide.

Limitations

There are several limiting factors that are immediately apparent in this study. It is difficult to draw a sharp dividing line between the fused and the mathematic-for-general-education type of courses. There will always be some overlapping between the two.

Since a great variety of textbooks is used, the course content in almost every institution differs from that of other institutions. This makes it extremely difficult to produce a reliable measuring device.

The random selection of schools was made from a list of those schools that in response to the questionnaire stated that they offered a mathematics-for-general-education type of course and were willing to co-operate in the application of the measuring device to their students. Hence the study is not a truly representative one. Any inferences drawn from the data apply only to the population from which the random selection was made.

In order to attain the most desired objectives in a mathematics-for-general-education type of course, it would be necessary to extend the course to at least two semesters with a minimum of six credit-hours. Since the institutions willing to cooperate in the testing agreed to do so only if the test would require no more than one period of class time, the present study was limited to a one-semester course.

CHAPTER II

A REVIEW OF RELATED LITERATURE

The extensive published literature as well as the unpublished research of the past thirty years indicates that there has been a great amount of interest in the development of a college course in mathematics for general education. The research studies appear to be of two types. They are: (1) studies which considered the problems of determining objectives and of the selection of subject matter for a course in mathematics for general education on the college level; and (2) studies which were concerned with the evaluation of an already existing course.

The type (1) studies are rather numerous and so it was necessary to make a selection of only those which were particulary important and helpful in connection with the present investigation. Only three type (2) studies were found. All of them are reviewed here.

In 1941 the special committee of the American Association for the Advancement of Science on the improvement of science teaching in colleges and universities turned its attention to the problem of improving science instruction for the purpose of general education. In order to determine the point of view of science and mathematics teachers with respect to certain issues involved in this problem and also to discover the present practices in those courses designed primarily for purposes of general education, the committee sent out questionnaires to 500 colleges and universities 56 per cent of which were answered. The responses indicated a general agreement that the introductory course in mathematics is unsatisfactory for the non-specializing students and more appropriate for the students who later specialize in mathematics; and that this course should be significantly improved for the non-major.[1]

The following objectives of such a course were indicated as most desirable: (1) to develop the ability to do logical thinking, (2) to develop the ability to interpret mathematical data and to apply the mathematical principles related to one situation in other similar problems, and (3) to make students familiar with the facts, principles, and concepts of mathematics.[2]

In the questionnaire an attempt was made to determine the opinions of mathematics teachers concerning the importance of certain projects which may increase the effectiveness of mathematics instruction for the non-major. The general opinion was that "the most important problem which should receive consideration is the clarification of a point of view for teachers of mathematics with regard to the place of mathematics in general education at the college level."[3] The interest of teachers in this project seemed to indicate that at that time a general uncertainty existed concerning what mathematics instruction should do for the non-specializing student.

In 1942 Kenneth E. Brown tried to resolve this uncertainty. He

---

[1]C.A. Hutchinson, ed., "Mathematics Instruction for Purposes of General Education," The American Mathematical Monthly, Vol. XLVIII (March, 1941), p. 189.

[2]Ibid., pp. 193-194.

[3]Ibid., p. 198.

carried out an investigation[4] which was motivated by a desire to answer some of the following questions: What are the objectives that teachers have proposed for non-major mathematics students? What type of material are instructors offering in such courses? What success are these teachers having in attempting to meet life's mathematical needs of the large academic group of students who seek other fields of study after the freshman year? Briefly stated, the aims of his study were: (1) to trace the historical development of college general mathematics in the United States; (2) to show the present status of general mathematics in American colleges; and (3) to discover and point out certain trends in the development of college general mathematics.[5]

Brown obtained his data from a survey of pertinent literature in the field, a questionnaire answered by 458 colleges in the United States offering general mathematics, an analysis of more than fifty general mathematics classroom recitations, and the opinions of 1500 students enrolled in general mathematics classes.[6]

After analyzing the data he reached the following conclusions:

1. General mathematics courses should be classified as (a) preparatory, (b) cultural, or (c) combined (preparatory and cultural).

2. The preparatory type of general mathematics seemed to be in the most stable position.

3. The combination course was held in greatest disfavor by both teacher and student.

---

[4]Kenneth E. Brown, General Mathematics in American Colleges, (Teachers College, Columbia University, New York, 1943).

[5]Ibid., p. 2.

[6]Kenneth E. Brown, "The Content of a Course in General Mathematics -- Teachers' Opinions," The Mathematics Teacher, Vol. XLIII (January, 1950), p. 25.

4. The cultural general mathematics course, while not entirely satisfactory, was more nearly meeting the needs of the terminal student in mathematics than the traditional offerings.

5. There was a definite trend towards the adoption of a cultural general mathematics course for the terminal student. However, this trend might not continue because the national crisis might cause students to choose the traditional courses.[7]

Brown's interest in the development of general mathematics did not cease with the publication of his dissertation. In 1947 he sent a similar questionnaire to 480 colleges whose catalogues indicated that they offered a course in general mathematics for the non-major.

Of the 333 schools that returned their answers more than 200 reported that they once had or were now offering a course in cultural general mathematics. However, only sixty percent of them were now offering such a course. The reasons for dropping the course were the following:

1. Lack of proper teaching staff in the face of increased enrollments.

2. Lack of a desirable textbook.[8]

The conclusion reached by Brown at that time was that unless much progress is made in the development of proper textbooks "cultural general mathematics will take a mediocre place among the subjects of our college curriculum."[9]

In March of 1949 Brown decided to continue the pursuit of his investigations along the following lines:

1. What topics do the teachers consider important in a general mathematics course for the cultural student?

---

[7]Ibid.

[8]Kenneth E. Brown, "Is General Mathematics in the College on Its Way Out?" The Mathematics Teacher, Vol. XLI (April, 1948), p. 158.

[9]Ibid.

2. Now that teaching staffs are more readily available, is there a trend toward general mathematics?[10]

He sent a questionnaire to all the universities, colleges, teachers colleges and junior colleges for white students listed in the Educational Directory of the United States Office of Education. More than 900 institutions returned questionnaires that indicated topics considered important in a cultural course in general mathematics. Some of the comments on the questionnaires reflected a strong opposition to the cultural course. However, "three-fourths of the teachers expressed a favorable attitude toward the course if a proper textbook could be found."[11] Emphasis was placed on arithmetic, algebra, consumer problems, trigonometry, geometry and statistics as part of the content of general mathematics.

> Other topics suggested as very important in a cultural general mathematics course were: history of mathematics, number system, mathematical reasoning, meaning of the processes, nature of mathematics, logic, use of mathematics in civilization, approximate computation, business mathematics, appreciation of mathematics, field work, slide rule, empirical formulas and recreational problems.[12]

The prompt (within two weeks) return of 900 questionnaires containing many favorable comments indicated a great interest in the development of such a course. "These comments expressed the desire to make mathematics meaningful to the students rather than present a maze of symbols that were of interest only to the instructors."[13]

---

[10]Kenneth E. Brown, "The Content of a Course in General Mathematics - Teachers' Opinions," The Mathematics Teacher, Vol. XLIII (January, 1950), p. 26.

[11]Ibid., p. 29.

[12]Ibid., p. 30.

[13]Ibid.

In 1950 Adele Leonhardy conducted a study to determine what basic mathematical concepts and processes are found in some 35 textbooks which were currently used in three areas of general education, namely, physical sciences, biological sciences and the humanities.

She discovered that "three-fourths of the mathematics is quantitative, one-fifth is the mathematics of spacial relationships, and one-twentieth deals with logical structure."[14]

> The mathematics required for general education is relatively simple, for it is the arithmetic of elementary school and certain concepts and processes from each of the four years of high school mathematics.[15]

Leonhardy's study was based on the assumption that the mathematics content found in the textbooks used in three areas of general education was an indication of the content to be included in a mathematics for general education type of course. The conclusion derived from her study seems to indicate preference for a mathematical course that stresses functional competence.

Kathrine C. Mires attempted to determine the extent to which there is need at the college level for a course in mathematics for general education designed to improve functional competence in mathematics. She also investigated the nature of the content that should be included in such a course.

After administering the Davis Test of Functional Competence in

---

[13] Ibid.

[14] Adele Leonhardy, "The Mathematics Used in the Humanities, Social Science, and Natural Science Areas in a Program of General Education on the College Level," Science Education, Vol. XXXVI (October, 1952) p. 252.

[15] Ibid.

Mathematics[16] to 1811 entering college freshmen at six state colleges in Oklahoma in the fall of 1955, she concluded that "only a very few of those tested had a satisfactory understanding of and ability to use the essentials for functional competences in mathematics which were recommended as a part of the general education of all citizens by the Commission of Post-War Plans of the National Council of Teachers of Mathematics."[17]

The study indicated a definite need in the six state colleges of Oklahoma in the fall of 1955 for a college course in mathematics for general education which included work designed to improve functional competence. Relative to the content of such a course Mires suggested the following:

> A course in mathematics for general education on the college level might include work on any of the mathematical concepts covered by the Davis Test but emphasis should be placed on the ten topics on which greatest lack of competence was shown. These topics were: (1) drawing conclusions, (2) estimating answers, (3) measurement, (4) use of approximate numbers, (5) basic geometric concepts, (6) reading and interpreting tables, (7) use of formulas, (8) consumer problems involving per cents but also requiring some other knowledge, (9) basic algebraic simplification, and (10) ratio and proportion.[18]

In her study Mires recommends that each of the above-mentioned topics should not be taken up as a separate unit of the course but that these and other topics to be considered be organized about broad mathematical concepts as a means of unifying and making meaningful the

---

[16]Published by World Book, Co., (New York, 1951).

[17]Kathrine Carrie Mires, "The Need for and the Nature of One Type of Course in Mathematics for General Education at the College Level" (Unpublished Ed. D. dissertation, University of Oklahoma, 1956), p. 64.

[18]Ibid.

various specific topics. She concludes that it is important that the course be organized and presented in a manner that emphasizes the development of understanding of mathematical concepts rather than the improvement of mathematical skills and the memorization of facts.

In 1955 Zachary Taylor Gallion,[19] having assumed that teachers of college mathematics teach the material that is included in the textbooks, made a critical examination of seventeen textbooks in use at that time for a course in mathematics for general education. The seventy topics compiled from the texts were sent to members of an appraisal group for evaluation as to their worth for inclusion in a course in freshman general mathematics.

He discovered that there was intense dissatisfaction with the traditional courses of algebra and trigonometry for the general student, and that there was a definite trend away from them toward a course specially designed as a part of one's general education. He conceded that although there was a marked trend toward the organization of general mathematics courses since 1948, yet 54.3 per cent of the total pages of the textbooks studied were devoted to traditional topics.

In 1956 W. A. Lafferty[20] undertook the improvement and modernization of the course, Fundamentals of Mathematics, which was offered as a part of a rather strong general education program at Northwest Missouri State College. His study was concerned primarily with the development of

---

[19] Zachary Taylor Gallion, "A Determination and Appraisal of the Content of Freshman General Mathematics Courses in Selected Colleges and Universities. (Unpublished Ph.D. dissertation, Louisiana State University, 1955).

[20] William A. Lafferty, "The Selection of Subject Matter in Mathematics for General Education." (Unpublished Ed.D. dissertation, Teachers College, Columbia University, 1956).

a method for choosing subject matter for the above-mentioned course. With this end in view he set up and validated five criteria for this selection. Briefly stated, they are:

1. Subject matter should be neither too difficult nor too easy and not merely about mathematics but should require performance of mathematics on the part of the student.

2. A minimum prerequisite for the course would be one year of high school mathematics, either general mathematics or algebra.

3. It should reveal the spirit and nature of modern mathematics: what mathematics is and what it is that mathematicians do.

4. It should include the philosophic and aesthetic values of mathematics.

5. The general education offering in mathematics should be devised preferably by the staff who will teach it, with local conditions in mind.[21]

Of the six units of subject matter to which criteria were applied four were judged suitable for the course. They were: (1) The group concept, (2) The development of integers from the natural members, (3) Symbolic logic applied largely to particular statements, and (4) Analytic geometry.[22]

Means investigated the objectives of freshman and sophomore mathematics courses for seven liberal arts colleges in Texas. He found that objectives concerned with habits, appreciations, and attitudes were usually judged to be of greater relative importance than objectives concerned with strictly mathematical manipulation.

Having identified a list of recommended objectives, Means constructed a 23-item test which he administered to a sample of 35 students from the

---

[21]Ibid., pp. 38-52.

[22]Ibid., pp. 87-88.

seven colleges. He found that, in general, the highly desired objectives were not being fully realized.[23]

Milligan took the objectives listed in Mean's study, as well as a number of others, and classified them according to Bloom's Taxonomy of Educational Objectives. He then proceeded to determine 17 specific criteria for content selection. He applied these criteria to subject matter data obtained from a search of recent literature. Each topic was rated on one of four levels. The means of such ratings were then used to compile a list of suitable subject matter topics to be used in one specific freshman course, Introduction to Mathematical Analysis, at Mountaintown College.[24]

It is noteworthy that Milligan's process for the selection of subject matter could be applied to other mathematics courses as well as to courses in other fields.

In 1963 Dana J. Lefstad analyzed the mathematics courses identified as non-traditional and non-vocational in scope and offered by the junior colleges in the United States and Canada for general education. The review of literature pertinent to this study and data obtained from questionnaires revealed that there was no single list of objectives on which all authors were in agreement. However, the search did reveal that there were fourteen objectives on which there was substantial agreement. They may be stated as follows: a course in mathematics designed

---

[23] James Horatio Means, "Objectives of Mathematics Instruction in Seven Texas Colleges" (Unpublished Ed.D. dissertation, Oklahoma State University, 1958).

[24] Merle W. Milligan, "An Inquiry into the Selection of Subject Matter Content for College Freshman Mathematics" (Unpublished Ed.D. dissertation, Oklahoma State University, 1961).

for general education provides the sudent the opportunity to increase and/or develop his

1. Powers of critical and logical thinking.
2. Knowledge of and understanding of the terminology and basic ideas of mathematics.
3. Appreciation of the importance of mathematics in today's world.
4. Functional competence in mathematics.
5. Knowledge of the history of mathematics and understanding of the contributions mathematics has made to the development of civilization.
6. Ability to analyze and formulate problems; use scientific approach.
7. Ability to communicate by increasing skill and understanding in the use of mathematical symbols.
8. Understanding of the nature of proof, including ideas of truth and validity.
9. Knowledge of the role of mathematics in cultural activities.
10. Understanding of the function concept.
11. Understanding of the principles of probability and statistics.
12. Mathematical background as an aid to further study.
13. Appreciation of the utility and power of mathematics.
14. Power to do independent thinking.[25]

This study also found that there was little agreement on the topics included in the content of the courses offered, "Only four topics appeared in 70 per cent, or more, of the courses examined. These topics were: concept of numbers, fundamental operations, bases other

---

[25]Dana J. Lefstad, "An Analysis of the Junior College Program in Mathematics for General Education." (Unpublished Ed.D. dissertation, Washington State University, 1963) Dissertation Abstracts, p. 1097.

than ten, and the laws of algebra.[26]

The studies reviewed up to this point were relevant to the present study insofar as they were helpful in compiling a list of desirable objectives for a college course in mathematics for general education and in determining the content for such a course.

Only three studies were found which had as an aim the evaluation of an existing course. All three were carried out in a single institution of higher learning and were designed to measure the effectiveness of a course which was being offered in that institution at the time the study was undertaken. The first of these was done in the College of Agriculture at Cornell University in 1958, the second at the University of Buffalo in 1959, and the third one was completed in 1962 at Austin Peay State College.

The purpose of the first study was to evaluate the effectiveness of a one-semester experimental course in applied mathematics designed for college freshmen. The samples used were freshmen enrolled in the College of Agriculture at Cornell University during the years 1952 through 1955.

The course was evaluated by comparing an experimental and a control group in terms of subsequent grade-point averages, final marks in courses involving mathematics, gains in knowledge of mathematics as measured by the Cornell Mathematics Test, and tendency to remain enrolled at Cornell.

The statistical evidence did not indicate that the experimental course influenced subsequent achievement or ultimate attrition rate to

---

[26] Ibid.

any important degree.[27]

Prior to Second World War a course, The Significance of Mathematics, was developed at the University of Buffalo. Designed for freshmen as a terminal course, its aims were to survey the whole field of mathematics, to show the relation and application of mathematics to other areas of our culture, to examine the methods and tools used by mathematicians, and to develop appreciation rather than skills.[28]

In 1951 a new two-year Division of General and Technical Studies was established at the University of Buffalo. The new program included a course named Perspectives in Mathematics. The course was designed to study the place of mathematics in the modern world by discussing its methods, some of the important topics with which it is concerned, and its relation to science, philosophy, and human experience. The students enrolled in this course were encouraged to find out what mathematics is, what its tools are, what it has to do with other branches of knowledge, how the mathematician works with his tools, how he fashions new tools when he needs them, and how mathematics is used in modern science and industry.[29]

The development of a course of this type reflected a conviction on the part of the college faculty that the well-educated college student needs an appreciation of mathematics and that much more of value can be

---

[27]J. Stanley Ahmann and M.D. Glock, "An Evaluation of the Effectiveness of a Freshman Mathematics Course," Journal of Educational Psychology. Vol. L (1959), p. 41.

[28]Harriet F. Montague and Phyllis M. Henry, "The Case for a General Education Course in Mathematics," The Journal of General Education, Vol. XIII (July, 1961), p. 97.

[29]Ibid., p. 98.

learned from a cultural type of general education course than from a "skills" course.

This course was taught continuously from 1951 to 1959 when the Division of General Technical Studies was incorporated into the new University College. A new course, Significance of Mathematics, was then offered as a freshman course for students in the University College.

After the course was discontinued, a study to measure its effectiveness was undertaken by Harriet F. Montague and Phyllis M. Henry. The aim of the study was to review and evaluate the Perspectives in Mathematics course as far as the attainment of objectives was concerned in the hope that this study would be helpful to those who were shaping the Significance of Mathematics course, and to provide information to others who were considering the appropriateness of such a course in their own curricula.[30]

The study was based on data obtained from records of the students' grades and information obtained from students in some of the Perspectives in Mathematics classes concerning their attitudes toward the study of mathematics at the beginning of the course.

The total number of students from 1951-52 through 1958-59 who completed the course was included in the study. However, because of incomplete records and the unavailability of the necessary information the actual number of students used in the statistical analyses was always somewhat less than the possible total of 428.

Several variables which might have a relationship to success in the course, Perspectives in Mathematics, were investigated.

The final grade received in the course was related with: (1) the average grade received in high school mathematics subjects; (2) the

[30] Ibid., p. 99.

amount of mathematics taken in high school; (3) success, as measured by grades, in other college mathematics courses; (4) the grade point average of all work taken in the students' two-year programs leading to Associate Degrees; (5) and (6) the average of high school work, either reported in terms of New York State regents' average or, where this was not available, the school average; (7) the size of the high school graduating class; and (8) the placement in quintile of the student in high school graduating class. The Pearson product moment correlation formula was used.[31]

The results of the study demonstrated that with two exceptions some relationship existed between the variables tested and success in the Perspectives course. The exceptions were the correlation between the grade received in the course and the size of the high school graduation class, and the correlation between the grade in the Perspectives course and the number of years of mathematics in high school.

The correlation of .596 between the University of Buffalo grade point average and the Perspectives in Mathematics grade was the highest correlation obtained in the study. Interestingly, the second highest correlation was the .408 figure obtained from the seventy-five cases whose high school New York State regents' average was available. Thus, the correlation obtained between success in Perspectives in Mathematics and indicators of general scholastic achievement, both in high school and college, shows a closer relationship than the variables related to specific mathematical achievement.[32]

An attempt was made to study the correlation of the attitudes of the students at the beginning of the course with the grades received at the end of the course. It was found that the relationship that existed between these two variables was rather low. This result was somewhat unreliable due to the difficulties connected with the evaluation of attitudes.

The books that were used as texts for the Perspectives course

---

[31]Ibid.

[32]Ibid., p. 102.

at various times during the eight years that it was offered were:

Banks, J.Hoiston. Elements of Mathematics, Allyn and Bacon. 1956.

Kline, Morris. Mathematics in Western Culture. Oxford Press. 1953.

Kramer, Edna. The Main Stream of Mathematics. Oxford Press. 1951.

None of these books was found to be completely satisfactory, but when they were supplemented or edited any one of them could meet the desired aims of the course.

The general conclusions arrived at by the investigators in this study were the following:

> We believe that this general education, cultural course in mathematics has value and can be studied profitably and successfully by students regardless of their preparation or ability in mathematics or their attitudes toward mathematics. We believe that such a course gives the layman a sounder and broader idea of mathematics than skill courses in arithmetic, algebra, or similar subjects.[33]

The third study[34] of this type was undertaken in 1961 by the mathematics staff at Austin Peay State College in Clarksville, Tennessee under the direction of William G. Stokes, head of the mathematics department. The aim of this study was to investigate the effectiveness of the course, Fundamental Concepts of Mathematics, in achieving the following objectives:

1. Improvement of general mathematical ability.

2. Improvement of the ability to think critically in non-mathematical as well as in mathematical situations.

3. Introduction of new ideas in contemporary mathematics.[35]

---

[33] Ibid., p. 104.

[34] The Effectiveness of Mathematics 200 in Regard to Certain Desirable Objectives of Mathematics for General Education, (Mathematics Department of Austin Peay College, Tennessee, 1962), mimeographed report, pp. 1-10.

[35] Ibid., p. 2.

The course is one of the General Education Core requirements for the degree of Bachelor of Science and Bachelor of Science in Education. It carries three units of credit for each of two semesters. The study was based on the work of one semester. The textbook used in the course is Basic Concepts of Elementary Mathematics by William L. Schaaf.

The population used as a basis for the study consisted of experimental and control groups of students from the freshman class of 1961. The experimental group was composed of 45 students from three classes taught in the fall by three different members of the mathematics staff. The control group was made up of a sample of 49 freshmen students who in the fall of 1961 enrolled for the course Fundamental Concepts of Mathematics which was to be given in the winter quarter.

All incoming freshmen were given three tests the results of which were used in the study: (1) the Cooperative Mathematics Pre-Test for College Students, (2) the Watson-Glaser Thinking Appraisal and (3) the ACE Psychological Test. Tests (1) and (2) were re-administered to the experimental group at the end of the fall quarter. The difference between the initial and final scores made up the gain of the student on each of the two tests for the period in which he was enrolled in the Fundamental Concepts of Mathematics course.

All students who completed the course took a comprehensive final examination over the coursework at the end of the quarter. This test, designed to measure understanding of the course material, was constructed by the mathematics staff and consisted of 42 multiple choice items.

At the beginning of the winter quarter the Cooperative Mathematics Pre-Test and the Watson-Glaser Test were re-administered to the control group. The gain of the members of this group was available for the period of the fall quarter when these students were not enrolled in the Fundamental Concepts of Mathematics course. At the same time they took the comprehensive examination without first having been exposed to the course materials.

The following hypotheses were tested:

1. That the gain of the experimental group in the ability to think critically in non-mathematical situations as measured by the Watson-Glaser Test of Critical Thinking will be significantly higher than the gain of the control group.

2. That the gain of the experimental group in general mathematical understanding during the fall quarter as measured by the Cooperative Mathematics Pre-Test will be significantly higher than the gain of the control group.

3. That the understanding of the course material of Mathematics 200 as measured by a comprehensive examination over the material will be significantly higher for the experimental group.[36]

On the basis of test scores the following conclusions were drawn: (1) On the Watson-Glaser Critical Thinking Appraisal the difference in gains made by the two groups was not significant. Therefore, there was no evidence that the course contributed to the ability to think critically in non-mathematical situations; (2) the significantly higher gains made by the experimental group over the control group on the Cooperative Mathematics Pre-Test indicated that the course, Fundamental Concepts of Mathematics, contributed to the general mathematical ability of the students; and (3) the mean scores of 76.05 for the experimental group and 6.49 for the control group left no doubt that the course contributed to an understanding of new

---

[36] Ibid., pp. 4-5.

mathematical concepts.

In general it was concluded "that evidence of the study warrants continuation of the course in essentially its present form, that is as a course which stresses concepts of mathematics rather than techniques."[37] The lack of significant gains in critical thinking showed that greater effort should be made to teach applications of the principles of mathematical logic.

Summary

The problem of identifying objectives and determining the content for a college mathematics course in a program of general education has long been of interest but particularly so during the last 30 years.

Various types of mathematics courses considered appropriate for the non-major have been investigated. They range from the remedial, or review type with emphasis on the acquisition of computational skills to the cultural type with emphasis on appreciation.

The findings resulting from these investigations indicate that there seems to be a growing preference among college teachers of mathematics for a specially designed course for the non-major, a course which is neither traditional nor remedial in nature. However, there is still some disagreement as to what should be the objectives and content for such a course.

Only three studies aimed at measuring the effectiveness of an existing course were found. The general conclusion reached by them was that a college course in mathematics for general education has value

---

[37] Ibid., p. 7.

and can be studied profitably and successfully by students regardless of their ability in mathematics or their attitudes towards mathematics.

The material in this chapter has been presented as evidence to support the need for the present study. The literature and research reviewed here indicate that although much has been done toward this end, a completely satisfactory list of objectives has not yet been defined. This lack of agreement on objectives probably is one of the factors which has discouraged research in evaluation.

The changing character of our educational system and the new discoveries in the fields of science and mathematics make it imperative that the search for an appropriate course in college mathematics for the non-major continue.

It is assumed by the writer that if significant contributions are to be made toward this end, it is first necessary to evaluate existing courses and thus identify areas of content that are desirable and also those areas which need revision. Without evaluation continued research in the field could become meaningless.

CHAPTER III

PROCEDURE

The over-all purpose of this study was to measure the effectiveness of a course in college mathematics for the non-major. The actual measurement and evaluation was only the last step of the investigation which is described in this chapter.

The work of collecting the data pertinent to this study was done in four stages which consisted of: 1) the preliminary survey; 2) the identification of desired objectives; 3) the construction of a measuring instrument; and 4) the application of the measuring instrument to a random sample of colleges. The procedure is described in that order.

The Preliminary Survey

A preliminary survey was made in order to discover which schools offer a college mathematics-for-general-education, or a type (c), course for the non-major. In the writer's opinion the colleges most likely to offer such a course would be the liberal arts, general education, and teacher preparatory colleges. Therefore, in the spring of 1963 a questionnaire[1] was formulated and sent to the heads of the mathematics departments of the 530 institutions of higher learning designated in the Higher Education Directory of 1961-1962 by the letter (e), i.e. liberal

---

[1]See Appendix A.

arts and general, and teacher preparatory institutions.[2]

The number of those who completed the questionnaire was 410 or 77.4 per cent of the total. Of those who responded 217 schools indicated that they offer such a course. Approximately 30 others said that they were planning to introduce such a course in the near future. Typical comments made by the latter group were:

> In keeping with our philosophy of liberal arts we feel that the traditional integrated course leaves something to be desired. Beginning in the fall of 1965 we plan to offer a course similar to type (c).

> I would appreciate your letting me know how to obtain the summary of your results, as this might prove helpful in making plans to include such a course in our school.

The prompt responses and the many favorable comments made on the completed questionnaires indicated a genuine interest in the study and a willingness to cooperate if possible. Only in two instances an unfavorable attitude toward the course was expressed by the following remarks:

> No type (c) course contemplated. It is not considered very worthwhile.

> Any course made up of topics other than traditional must be a course about mathematics rather than a course in mathematics.

Of the 217 schools that claimed to offer a type (c) course, a total of 166 expressed a willingness to participate in the testing necessary to complete this study. The following were some typical reasons given by those who refused to cooperate:

> Since I do not know who will be instructing in the course next fall, I do not wish to commit the instructor without his consent.

---

[2] U.S. Department of Health, Education, and Welfare, Higher Education, Part 3, (Washington, D.C., 1962), p. 2.

> The course was an experiment this year. Until we feel more secure in its validity and until we find a suitable textbook I would hesitate to accept such testing.
>
> I fail to see how we can participate at no cost.
>
> I am of the opinion that the possible objectives of such a course are so different that it would be difficult to design a test to measure their effectiveness significantly.

One of the items on the questionnaire requested a listing of the textbooks currently used for a type (c) course. Ninety-seven different textbooks were reported in use by the 217 schools which offer such a course. A list of thirty-five of those reported in use by at least two schools is found in Appendix B.

After the responses to the questionnaire were collated, a letter of thanks and a summary of the responses[3] was sent to the head of the mathematics department of each of the 410 institutions that completed the questionnaire.

Formulation of a List of Objectives

The main emphasis of this study was centered on the evaluation of a mathematics-for-general-education type of course by developing a reliable measuring instrument and applying it to a sample selected from the group of 166 schools previously mentioned. To construct such an instrument, it was necessary to compile a list of objectives recommended for a college mathematics course in a program of general education. With this purpose in mind, the writer made a survey of periodicals, research papers, committee reports, doctoral theses and more than 30 textbooks.

---

[3]See Appendix B.

The search for objectives was at first directed toward the identification of the over-all objectives for general education. A number of sources contained lists of these. In a few instances the objectives were included in a definition of general education.

A rather comprehensive list was found in a report of the President's Commission on Higher Education. It is reproduced here in full.

1. To develop for the regulation of one's personal and civic life a code of behavior based on ethical principles consistent with democratic ideals.

2. To participate actively as an informed and responsible citizen in solving the social, economic, and political problems of one's community, State and Nation.

3. To recognize the interdependence of the different peoples of the world and one's personal responsibility for fostering international understanding and peace.

4. To understand the common phenomena in one's physical environment, to apply habits of scientific thought to both personal and civic problems, and to appreciate the implications of scientific discoveries for human welfare.

5. To understand the ideas of others and to express one's own effectively.

6. To attain a satisfactory emotional and social adjustment.

7. To maintain and improve his own health and to cooperate actively and intelligently in solving community health problems.

8. To understand and enjoy literature, art, music, and other cultural activities as expressions of personal and social experience, and to participate to some extent in some form of creative activity.

9. To acquire the knowledge and attitudes basic to a satisfying family life.

10. To choose a socially useful and personally satisfying vocation that will permit one to use to the full his particular interests and abilities.

11. To acquire and use the skills and habits involved in critical

and constructive thinking.[4]

It was observed that in very list of objectives and in almost every definition emphasis was placed on critical thinking. An example of this is the following statement which is a part of a definition of liberal education formulated by some members of the faculty of the University of Chicago.

> A liberal education is one which liberates the student's mind. It does so by providing him with intellectual disciplines of various kinds. He must be taught not only to read and write but to analyze and interpret what he has read, to look for premises and conclusions of arguments, to recognize when he has found them, and to discover the presuppositions which lead to the particular choice of premises used. He must become acquainted not only with a part of humanity's store of knowledge, but with the various methods by which premises lead to conclusions and with methods by which conclusions are validated.[5]

Critical thinking was also emphasized by the Harvard Committee in its report on general education. The committee stated that in a program of general education the abilities to be sought above all others are:

1. To think effectively.

2. To communicate thought.

3. To make relevant judgments.

4. To discriminate among values.[6]

An investigation of general education by the American Council on Education Studies resulted in a list of over-all goals for general

---

[4]President's Commission on Higher Education, "Establishing the Goals," Higher Education for American Democracy, Vol. 1 (Washington, 1947), p. 50.

[5]The Idea and Practice of General Education (Chicago, 1950), p. 200.

[6]Harvard Committee, General Education in a Free Society (Cambridge, Mass., 1945), p. 64.

education with additional sets of specific objectives for every field of content to be included in a program of general education. In particular, the Council emphasized the role of mathematics in helping the layman to understand his environment, to solve his daily problems, and to employ useful nonverbal methods of thought and communication.[7]

After going through whatever pertinent literature could be found and after eliminating obvious duplications, the investigator selected only those over-all objectives for general education which were significantly related to mathematics.

The next step in the study was to identify specific objectives for a course in mathematics-for-general-education. The writer found Means' doctoral dissertation[8] especially helpful because most of the objectives found in journals, college bulletins, and textbooks published prior to 1958 were listed in his study.

More recently formulated objectives were found in textbooks written by Banks[9], Wade and Taylor[10], Schaaf[11], Western and Haag[12], and Hafstrom[13]. None of these writers emphasized the

---

[7] A Design for General Education, American Council on Education Studies, Vol. VIII (Washington, D.C., 1944), p. 43.

[8] Means, pp. 13-22.

[9] J. Houston Banks, Elements of Mathematics (Boston, Mass., 1959).

[10] Thomas L. Wade and Howard E. Taylor, Fundamental Mathematics New York, 1961).

[11] William L. Schaaf, Basic Concepts of Elementary Mathematics (New York, 1960).

[12] Donald W. Western and Vincent H. Haag, An Introduction to Mathematics (New York, 1959).

[13] John E. Hafstrom, Basic Concepts in Modern Mathematics (Reading, Mass., 1961).

acquisition of skill in mathematical computations but seemed to attach much importance to reasoning, understanding of broad mathematical concepts, an appreciation of mathematics, and a respect for its methods. The following list is an example of the type of objectives found in the most recently written textbooks.

1. To become acquainted with certain modern mathematical concepts that will help us better to understand and use mathematics.

2. To use concepts such as set, mapping, relation, group and isomorphism to gain a stronger understanding of some familiar number systems and how these systems are related.

3. To develop an appreciation of the logic of mathematical arguments and an added respect for the powers of the human mind.

4. To make precise statements and to be able to defend them.[14]

All the objectives found were carefully analyzed, categorized as to knowledge, skills, and attitudes, and synthesized into a single list of 17 objectives. This list appears in Appendix C of this study.

Validation of Objectives by the Jury

A jury of 50 experts was chosen in order to validate the list of 17 objectives. The experts were selected at random from among the heads of the mathematics departments of the 217 schools who, in reply to the preliminary questionnaire, indicated that their schools offer a type (c) course. The names of the schools were arranged in the order of their listing in the Directory of Higher Education and then numbered from 1 to 217. A book of random digits[15] was then used to make the selection.

---

[14]Ibid., pp. 1-5.

[15]The Rand Corporation, A Million Random Digits (Glencoe, Illinois, 1955).

A letter and a copy of the objectives were sent for validation to each of the 50 experts. A four-point rating scale, described as follows, was placed at the top of the first page of the list of 17 objectives.

__4__ The objective is highly desirable.

__3__ The objective is of considerable value.

__2__ The objective is of slight value.

__1__ The objective is of no value.

Within three weeks 38 completed evaluation sheets were returned. This constituted 76 percent of the total number that was sent out. The data derived from these returns are found in Table I. The sum of the ratings assigned to each objective by the 38 judges is designated as the index value of that objective. The average rating is the index value divided by 38.

None of the objectives was unanimously rated "4" by the 38 judges. However, five of them had average ratings close enough to "4" to warrant an equivalent rating of "highly desirable." Index values of 114 or higher were possessed by 15 of the 17 objectives. This meant that they had an average rating of at least "3" that is "of considerable value." Index values lower than 114 were possessed by only two objectives. Their ratings of 2.8648 and 2.8421 were close enough to "3" to be rated as "of considerable value."

TABLE I

OBJECTIVES OF A COLLEGE MATHEMATICS-FOR-GENERAL-EDUCATION TYPE OF COURSE ARRANGED IN DESCENDING ORDER OF THEIR INDEX VALUES AS ASSIGNED BY THE 38 JUDGES

	Objective	Index Value	Average Rating	Equivalent Rating
1.	C-4	147	3.8684	4
2.	A-2	140	3.6842	4
3.	A-5	138	3.6316	4
4.	B-3	134	3.5263	4
5.	C-3	133	3.5000	4
6.	B-4	130	3.4210	3
7.	B-1	130	3.4210	3
8.	A-4	128	3.3684	3
9.	B-7	128	3.3684	3
10.	C-1	127	3.3431	3
11.	B-5	125	3.2897	3
12.	B-6	123	3.2368	3
13.	C-2	122	3.2105	3
14.	C-5	121	3.1842	3
15.	A-3	120	3.1579	3
16.	A-1	109	2.8648	3
17.	B-2	108	2.8421	3

The conclusion drawn from the data in Table I was that all the 17 objectives were judged to be of considerable value, or better, in a college mathematics-for-general-education type of course. It is noteworthy that the two objectives receiving the highest rating were concerned with methods of correct reasoning while the two lowest rated ones were the acquistion of knowledge of the history of mathematics and the development of skills to deal with nonverbal symbolism.

Of the 17 objectives validated by the judges only seven were selected as a basis for the evaluation of a type (c) course. The chosen objectives were those that were rated highest by the judges and which, in the opinion of the writer, could be adequately tested in a fifty-minute multiple-choice objective test. They are listed here in descending order of their index values.

A-2: Knowledge of the mathematical methods of reasoning.

A-5: Knowledge of the basic mathematical vocabulary and definitions necessary for the attainment of these objectives.

B-3: Ability to draw valid inferences from data; to distinguish between "valid" and "true."

B-4: Ability to make precise statements and to be able to defend them.

B-1: Ability to use modern mathematical concepts to gain a stronger understanding of some familiar number systems and how these systems are related.

A-4: Knowledge of the new approach to mathematics as well as of new ideas in contemporary mathematics.

B-7: Ability to interpret correctly some of the symbols and diagrams used in the literature of our technical culture.

It may be observed that if the seven most desired objectives were selected on the basis of their index values, then C-4 and C-3 should have been included in the list. In fact the highest rating was given to objective C-4: the attainment of a wholesome respect for correct reasoning and precise definitions. Fifth in rank was C-3: readiness to revise judgments and to change behavior in the light of reason. Both these objectives were concerned with attitudes. Their omission was justified, first of all, by the fact that the measurement of the attainment of the desired attitudes would be difficult due to the limitations of the test. Secondly, attitudes are usually evaluated in the following ways: by self-report procedures, such as responding to a questionnaire or rating scale, or by writing an essay; by observer-reports, in which another person observes and records behavior that gives evidence of the desired attitudes; and by the interview which involves both the student and the observer.[16] Such an evaluation could hardly be worked into an objective test. Thirdly, one must consider that inherent "in the study of attitudes toward mathematics is the idea that an attitude involves both cognitive and non-cognitive aspects - that is, both beliefs and feelings about the object of the attitudes."[17] The cognitive aspects of the two omitted objectives are certainly included in objectives A-3, B-3, and B-4. Thus their omission did away with some undesirable overlapping and repetition.

Construction of the Chart of Specifications for the Test

Consideration of the purposes of the test, the limitations as to the time alloted to the test and as to the length of the course to be evaluated, and the advantages of a multiple-choice type of test[18] led the writer to decide that the measuring instrument would be an objective test consisting of 50 multiple-choice items. Each item

---

[16] Mary Corcoran and E.G. Gibb, "Appraising Attitudes in the Learning of Mathematics," Twenty-Sixth Yearbook: (National Council of Teachers of Mathematics, Washington, 1961), p. 107.

[17] Ibid., p. 105.

[18] H.H. Remmers and N.L. Gage, Educational Measurement and Evaluation (New York, 1955), p. 95.

would have one correct response and four distractors. In most items one of the distractors would be "none of these."

Having decided on the form of the test and the objectives that would be used as a basis for the construction of it, the writer was faced with two questions: 1) What areas of content should be included in the test? and 2) What proportion of the test should be devoted to each objective and to each unit of content?

The answer to the first question was based, in part, on the conclusions reached by the investigators of the research studies reviewed in Chapter II. The four areas of subject matter validated by Lafferty's criteria for selection[19] and the four topics which appeared in 70 per cent of the courses examined by Lefstad[20] were considered. In addition, the writer made a study of the content of 10 textbooks published since 1958[21]. With the seven objectives as a criterion, the units of subject matter to be tested were selected from the previously mentioned sources. A topical outline of this content is found in Appendix H.

In order to answer the second question, a two-dimensional Chart of Specifications[22] was constructed. The units of content were listed on the horizontal axis and objectives on the vertical axis. Thus a grid was formed. It was then possible to fill in the rectangles of the grid with figures to indicate not only the apportionment of items in terms of content and in terms of objectives, but also how the two

---

[19] See Chapter II, p. 23.

[20] See Chapter II, p. 26.

[21] A listing of the books is found in Appendix D.

[22] See Table II, p. 47.

TABLE II

CHART OF SPECIFICATIONS FOR A TEST OF COLLEGE MATHEMATICS FOR GENERAL EDUCATION

OBJECTIVES \ CONTENT	Numeration; Decimal System; Nondecimal Systems; Change of Base	Subsystems of the Real Numbers; Counting Numbers, Whole Numbers, and Integers	The Real Numbers: Rational and Irrational	Complex Numbers	Types of Reasoning and Symbolic Logic	Sets and Variables	Relations, Functions, and Graphs	Abstract Systems: Groups and Fields	Totals for Objectives
KNOWLEDGE AND UNDERSTANDING OF									
1. Mathematical methods of reasoning	--	--	--	--	3	--	--	--	3
2. The new approach to mathematics as well as new ideas in contemporary mathematics	2	1	--	--	--	3	--	1	7
3. Basic mathematical vocabulary and definitions necessary for attainment of objectives.	--	3	2	1	1	1	2	1	11
SKILLS AND ABILITY TO									
4. Draw valid inferences from data; to distinguish between valid and true	--	1	1	--	2	1	--	--	5
5. Make precise statements and to be able to defend them	--	2	3	--	2	1	--	1	9
6. Use modern mathematical concepts to gain a stronger understanding of some familiar number systems	1	--	2	--	--	1	1	--	5
7. Interpret correctly some of the symbols and diagrams used in the literature of our technical culture	--	--	4	--	--	1	5	--	10
Totals for Content	3	7	12	1	8	8	8	3	50

were to be combined. The first figure to be entered was the one in the lower right hand corner, 50, which represents the total number of items in the test. Then the right hand column and the bottom row were filled in, giving the total number of items for each objective and the total for each area of content. These figures were based on the writer's judgment of the relative importance of the objectives and on the estimated percentage of pages allotted to the selected areas of content in the 10 textbooks that were reviewed for content. Then the figures for the inner rectangles on the chart were filled in. The number of items allotted to each objective in the vertical columns was determined by the relative importance of the objective in a particular area of content.

Preparation of an Item Pool for the Tryout Forms of the Test

In the preliminary questionnaire of this study, question four was a request for copies of tests and examinations used in the teaching of a mathematics-for-general-education type of course. The response surpassed the writer's expectations, because 117 copies of various tests and examinations submitted by 65 schools were received. These were carefully analyzed and the questions which fit into the chart of specifications were copied on index cards and classified as to objective and content. The aim was to obtain at least twice the number of items required by the chart for each objective as well as for each area of content. Missing test items were composed by the writer and items not of the multiple-choice type were revised. In this way an item pool of approximately 150 questions was formed.

From this item pool exactly 100 questions were selected for a tryout form of the test which was to be administered at Oklahoma State University to four groups of students who had completed a course in mathematics-for-general-education during the spring semester of 1964.

Since the test had to be given during one 50-minute period of class time, the 100 items were divided, according to the chart of specifications, into two sets of 50 items each. The title given to the test was: A Test of College Mathematics for General Education, Forms A and B.

These tryout forms of the test were then submitted to two members of the mathematics department at Oklahoma State University for review and correction. Their criticism and helpful suggestions resulted in the revision of about six items. The revisions consisted of a change of a word or phrase and, in two instances, some ambiguity was removed by changing one of the responses.

After a set of directions for the testees was formulated, the two tryout forms of the test were duplicated by the mimeograph process. For facility in scoring IBM answer sheets were provided. The score consisted of the number of items answered correctly. The directions, the two forms of the test, and a sample answer sheet are found in Appendices F and G.

The four groups of students at Oklahoma State University who took the test had been taught by three different teachers. In order to ascertain that all of the 100 items were tried out on each group, approximately one-half of each class received Form A and the other half, Form B. To insure uniformity in the administering of the test, a specific set of instructions was prepared for each teacher.

The tryout tests were administered in May of 1964, during the last week of the spring semester. A total of 98 students took the test: 53 took Form A and 45 took Form B.

Computation of Reliability Coefficients for the Two Tryout Forms of the Test

A measure of the internal consistency of each form of the test was determined by the Split-Half Method.[23] The Pearson r, or the reliability coefficient of one-half the test, was found to be .75 for Form A and .52 for Form B. The Spearman Brown prophecy formula[24] was then applied to find the coefficients of reliability of the entire test. These turned out to be .86 for Form A and .69 for Form B.

Item Analysis

To determine the merit of each of the 100 test items used in the tryout forms, test results were subjected to an item analysis. The computational procedures used were those recommended by Davis.[25] As a result of the item analysis four things were obtained concerning each item: (1) the difficulty of the item; (2) the discrimination index of the item; (3) the measure of the internal consistency of individual items, and (4) the effectiveness of the distractors.

After the test papers had been scored, the answer sheets to each

---

[23]N.M. Downie and R.W. Heath, Basic Statistical Methods (New York, 1959), pp. 83-85.

[24]Ibid., p. 193.

[25]Frederick B. Davis, Item-Analysis Data (Cambridge, Mass., 1949).

of the two forms of the test were arranged in rank order with respect to scores. The highest 27 percent and the lowest 27 percent for each test was then removed and used as the high-scoring and low-scoring groups for the item analysis.

The proportion of success in the highest and lowest 27 percent was computed for each item by means of the following formulas:

For the highest 27 percent of the sample:

$$P_H = \frac{R_H - \frac{W_H}{K - 1}}{N_H - NR_H}$$

For the lowest 27 percent of the sample:

$$P_L = \frac{R_L - \frac{W_L}{K - 1}}{N_L - NR_L}$$

where: N_H = the number of testees in the highest 27 percent of the sample,

R_H = the number of testees in the highest 27 percent of the sample that answered the item correctly,

W_H = the number of testees in the highest 27 percent of the sample that answer the item incorrectly,

NR_H = the number of testees in the highest 27 percent of the sample that do not reach the item in the time limit,

N_L = the number of testees in the lowest 27 percent of the sample,

R_L = the number of testees in the lowest 27 percent of the sample that answer the item correctly,

W_L = the number of testees in the lowest 27 percent of the sample that answer the item incorrectly,

NR_L = the number of testees in the lowest 27 percent of the sample that do not reach the item in the time limit,

K = the number of choices in the item.[26]

Having calculated P_H, the proportion of successes in the highest 27 percent, and P_L, the proportion of successes in the lowest 27 percent for each item, it was then possible to read the difficulty and discrimination indices from the Davis Item Analysis Chart.[27] The information thus obtained about each item is contained in Tables III and IV.

TABLE III

ITEM ANALYSIS DATA FOR THE 50 ITEMS OF A TEST OF COLLEGE MATHEMATICS FOR GENERAL EDUCATION - FORM A

Item Number	W_L	W_H	R_L	R_H	P_H	P_L	Discrim. Index	Difficulty Index	r
1.	14	9	0	5	.20	-.25	23	-10	.36
2.	9	9	5	5	.20	.20	0	32	.00
3.	8	9	6	5	.20	.28	-7	35	-.11
4.	7	3	7	11	.73	.37	23	52	.37
5.	11	8	3	6	.28	-.07	22	24	.35
6.	9	2	5	12	.82	.20	43	51	.61
7.	12	7	2	7	.37	-.07	28	28	.46
8.	10	11	4	3	.02	.11	-20	18	-.32
9	8	5	6	9	.55	.28	17	45	.28
10.	12	10	2	4	.11	-.07	12	7	.20

[26]Ibid., pp. 30-31.

[27]Frederick B. Davis, Item Analysis Chart (Cooperative Test Service of the American Council on Education, New York, 1944).

TABLE III (Continued)

Item Number	W_L	W_H	R_L	R_H	P_H	P_L	Discrim. Index	Difficulty Index	r
11.	13	6	1	8	.46	-.16	40	28	.58
12.	9	5	5	9	.55	.20	24	43	.38
13.	10	3	4	11	.73	.11	45	46	.74
14.	11	8	3	6	.32	.02	38	30	.63
15.	12	7	2	7	.37	-.07	35	28	.58
16.	12	5	2	9	.55	-.07	45	35	.74
17.	14	11	0	3	.02	-.25	-7	-25	-.12
18.	12	6	2	8	.46	-.07	40	32	.66
19.	8	6	6	8	.46	.28	11	43	.19
20.	12	8	2	6	.28	-.07	28	24	.44
21.	13	10	1	4	.11	-.16	13	-10	.21
22.	11	8	3	6	.28	.02	36	28	.54
23.	14	12	0	2	-.07	-.25	20	-28	.32
24.	12	10	2	4	.11	-.07	12	7	.20
25.	12	7	2	7	.37	-.07	35	28	.52
26.	13	10	1	4	.11	-.16	13	-10	.21
27.	11	6	3	8	.46	.02	48	35	.66
28.	7	0	7	14	.96	.37	63	60	.78
29.	9	7	5	7	.37	.20	12	38	.20
30.	9	5	5	9	.55	.20	24	44	.38
31.	12	7	2	7	.40	-.07	37	30	.55
32.	10	6	4	8	.50	.11	30	40	.46
33.	12	4	2	10	.64	-.07	51	38	.85

TABLE III (Continued)

Item Number	W_L	W_H	R_L	R_H	P_H	P_L	Discrim. Index	Difficulty Index	r
34.	12	9	2	5	.20	-.07	23	19	.36
35.	13	5	1	9	.60	-.16	49	34	.67
36.	5	2	9	12	.82	.55	20	60	.32
37.	7	1	7	13	.91	.37	41	58	.59
38.	9	8	5	6	.32	.20	9	36	.15
39.	11	4	3	10	.69	.02	61	42	.77
40.	9	5	5	9	.55	.21	23	44	.36
41.	7	1	7	13	.91	.40	39	59	.58
42.	13	4	1	10	.64	-.17	51	35	.69
43.	11	3	3	11	.73	.02	64	44	.79
44.	12	2	2	12	.82	-.08	63	43	.78
45.	11	4	3	10	.64	.02	57	41	.74
46.	14	7	0	7	.44	-.29	39	20	.57
47.	13	8	1	6	.33	-.19	31	19	.49
48.	10	8	4	6	.33	.12	18	34	.29
49.	12	5	2	9	.60	-.08	49	36	.81
50.	11	3	3	11	.79	.02	67	45	.81

TABLE IV

ITEM ANALYSIS DATA FOR THE 50 ITEMS OF A TEST OF COLLEGE MATHEMATICS FOR GENERAL EDUCATION - FORM B

Item Number	W_L	W_H	R_L	R_H	P_H	P_L	Discrim. Index	Difficulty Index	r
1.	9	5	3	7	.48	.06	37	37	.55
2.	10	5	2	7	.48	-.04	42	34	.60
3.	8	7	4	5	.32	.17	12	36	.20
4.	6	4	6	8	.59	.38	13	49	.21
5.	10	9	2	3	.06	-.04	5	7	.08
6.	8	2	4	10	.79	.17	43	49	.62
7.	9	7	3	5	.27	.07	22	30	.34
8.	5	4	7	8	.58	.48	6	52	.10
9.	11	8	1	4	.18	-.14	21	24	.34
10.	10	11	2	1	-.18	.04	-	-	-*
11.	8	6	4	6	.41	.17	18	38	.29
12.	11	9	1	3	.07	-.14	6	-13	.11
13.	11	4	1	8	.64	-.14	51	36	.69
14.	5	2	7	10	.79	.48	21	58	.34
15.	1	1	11	11	.98	.90	18	83	.30
16.	11	11	1	1	-.16	-.16	0	29	.00
17.	4	1	8	11	.98	.58	45	66	.74
18.	5	0	7	12	.96	.48	44	62	.73
19.	11	8	1	4	.20	-.14	20	10	.36
20.	6	4	6	8	.59	.38	13	49	.21

* The Davis Item Analysis Data do not provide discrimination and difficulty indexes when the P_H is negative and the P_L positive.

TABLE IV (Continued)

Item Number	W_L	W_H	R_L	R_H	P_H	P_L	Discrim. Index	Difficulty Index	r
21.	10	4	2	8	.58	-.04	48	37	.66
22.	5	5	7	7	.48	.48	0	49	.00
23.	8	4	4	8	.58	.17	29	44	.44
24.	7	8	5	4	.17	.27	-9	34	-.14
25.	8	5	4	7	.48	.17	22	41	.36
26.	12	11	0	1	-.14	-.25	10	-32	.17
27.	10	6	2	6	.38	-.04	36	29	.53
28.	10	5	2	7	.48	-.04	42	34	.60
29.	11	10	1	2	-.04	-.14	16	-22	.26
30.	9	6	3	6	.38	.06	31	34	.47
31.	12	9	0	3	.06	-.25	5	15	.08
32.	9	11	3	1	-.14	.06	-	-	-
33.	10	3	2	9	.69	-.04	54	42	.72
34.	12	4	0	8	.58	-.25	48	40	.66
35.	12	5	0	7	.48	-.25	42	36	.60
36.	11	5	1	7	.52	-.14	44	38	.62
37.	9	5	3	7	.52	.06	39	38	.57
38.	9	8	3	4	.18	.06	16	25	.26
39.	8	4	4	8	.64	.17	32	45	.49
40.	10	6	2	6	.41	-.04	37	34	.55
41.	11	9	1	3	.07	-.14	6	16	.11
42.	6	2	6	10	.86	.38	34	56	.51
43.	10	4	2	8	.64	-.04	51	41	.69
44.	9	9	3	3	.08	.06	4	19	.06

TABLE IV (Continued)

Item Number	W_L	W_H	R_L	R_H	P_H	P_L	Discrim. Index	Difficulty Index	r
45.	10	6	2	6	.45	-.04	39	35	.58
46.	8	4	4	8	.58	.17	29	44	.44
47.	8	8	4	4	.17	.18	0	30	.00
48.	9	7	3	5	.30	.06	25	31	.40
49.	6	7	6	5	.30	.38	-5	41	-.09
50.	7	0	5	12	.96	.27	56	56	.74

Selection of Items for the Final Form of the Test

The selection of items for the final form of A Test of College Mathematics for General Education was based on the item analysis data found in Tables III and IV, and on the requirements of the chart of specifications.

The item discrimination index is a measure of the difference in the proportion of individuals who answer the item correctly in the high group tested against the proportion in the low group. "If the difference is a significant one the item is accepted as being one which discriminates."[28] Davis claims that "items with discrimination indices above 20 will ordinarily be found to have sufficient discriminating power for use in most achievement and aptitude tests."[29]

---

[28]Downie and Heath, p. 203.

[29]Davis, p. 15.

The difficulty of an item may be defined as the proportion of the testees who answer an item correctly, or it may be defined as a proportion of those that actually know the answer to an item. The difficulty index on the Davis chart is an estimate of the proportion of the number of testees who know the answer to the item. This estimate is obtained from the data in only the highest and lowest 27 per cent of the testees and is based on the assumption that the regression of item score on the total test score is rectilinear.[30]

Theorist in the field differ somewhat as to what should be the distribution of the difficulty indices of the items. Dorothy A. Wood says that for most achievement tests the difficulty percentages of the items should be about 50 with a few items significantly easier and a few significantly more difficult to motivate the poorest students and to challenge the best.[31] Davis tends to agree with her.[32]

Nunnally suggests that a good rule is to pick a set of items whose average difficulty level is near the middle of the possible score range. In a multiple choice test with five alternatives for each item, one-fifth of the subjects would get the item correct by guessing alone. Consequently the minimum expected difficulty would be 20 per cent and the expected range of scores on a 50-item test would be from 10 to 50. In that case, the average difficulty would be about 60 per cent.[33]

---

[30] Ibid., pp. 3-6.

[31] Dorothy Adkins Wood, Test Construction, (Columbus, Ohio, 1960), p. 82

[32] Davis, p. 25

[33] Jum C. Nunnally, Jr., Tests and Measurements (New York, 1959), p. 146

In view of the above discussion, the writer decided to select those items that had discrimination indices of 20 or more, and difficulty indices that averaged about 60, with a range from about 35 to 80.

Since the two forms of the test in question were made up of a more or less homogeneous group of items it was appropriate to examine the internal consistency of the individual items.[34] The values of r found in Tables III and IV were read from Flanagan's Table of Values of the Product-Moment Coefficient of Correlation in a Normal Bivariate Population Corresponding to Given Proportions of Success.[35] These values of r are estimates of the correlation coefficient between each item and the test score, excluding the item in question. Thorndike claims that the values of r obtained from Flanagan's Table are the most satisfactory item validity indexes based on the upper and lower 27 per cent.[36]

The internal consistency data was used to eliminate unsatisfactory items rather than to identify the best items. Items with very low correlations with the test as a whole, and particularly those showing negative correlations were discarded. Preference was given to items with intermediate internal consistency values. Any item with an r value less than .30 was considered undesirable. However, in the selection of items Thorndike's advice was kept in mind:

> Exclusive preoccupation with item internal consistency may lead to an undue narrowing of the scope of the test...Internal consistency data must be used with discretion within the framework of

---

[34]Robert L. Thorndike, Personnel Selection: Test and Measurement Techniques (New York, 1949), p. 252.

[35]Ibid., p. 348.

[36]Ibid., p. 242.

> the original outline and specifications for a test. They cannot override the outline and they do not provide a substitute for a definite content outline for the test.[37]

In Test A, the number of items which, more or less, satisfied these criteria was 28, and in Test B there were 23. That gave a total of 51 items from which to construct the final 50-item test. However, out of the 51 items 15 had to be discarded because, in several instances, similar questions in the same area of content on both tests satisfied the desired criteria. Thus it became necessary to select 14 more questions from among those that did not satisfy one or more of the statistical criteria. The selection of these was based on the requirements of the chart of specifications. Whenever a choice of two possible items had to be made, the item-analysis data was consulted.

The 50 items selected for the final form of the test were now examined for the effectiveness of distractors. In the process of assembling the item-analysis data, the responses of the high group were counted for the different distractors for each test item and the same was done for the low group. Good distractors were considered to be those which were selected to a greater extent by the members of the low scoring group.[38] Since the distractors of the 50 selected items were found to be more or less of this type, no further revision of items was made.

The 50 selected items were then arranged according to subject matter and a copy of the final form of A Test of College Mathematics for General Education was typed. The test was then duplicated by the

---

[37]Ibid., p. 253.

[38]Downie and Heath, p. 205.

multilith process. A copy of the test and the directions for administering it are found in Appendix H.

Collection of the Data

The measuring device was now ready and it was possible to undertake the last step of the procedure, namely, the actual testing of the population used in this study. The population consisted of 166 colleges which, in response to the questionnaire, indicated that they offered a type (c) course to the non-major and were willing to participate in the testing.

Due to financial limitations it was impossible to administer the test in all the 166 colleges. Therefore, a random sample of 10 schools was selected. The schools were numbered from 1 to 166 according to the order of their listing in the Directory of Higher Education of 1961-1962. Than a book of random digits was used to make the selection. The list of 10 schools included in the random sample is found in Appendix I.

In August of 1964, a letter and a questionnaire[39] pertinent to the testing was sent to each school in the sample. The purpose of the questionnaire was to find out when the fall semester commenced, what were the possible dates for testing, and how many students would be taking the test.

After receiving the requested information, the investigator had the testing materials mailed to each of the 10 schools in time to be administered in September during the first week of the fall

---

[39]See Appendix J.

session before the students had been exposed to a type (c) course. The total number of students who took this pre-test was 595.

In January of 1965, the test materials were again sent for re-administration to the same schools. The number of students who took the test after completing the course was 483. The decrease in number was due, in a small measure, to absentees and dropouts. However, the greatest loss in the number of testees was due to the failure of the investigator to take into account the fact that three of the schools in the sample were conducted on a quarterly, rather than a semester basis. As a result, many of the students were not on the campus during the month of January.

The analysis of data described in Chapter IV was based only on the results obtained in the tests by the 483 students who took it twice, once at the beginning and then again at the end of the course.

Summary

The purpose of this study was to evaluate the effectiveness of a college course in mathematics-for-general-education by constructing a test to be used in measuring the attainment of certain desired objectives for such a course.

As a first step in the procedure a questionnaire was sent to 530 colleges to see which of them offered a type (c) course for the non-major. The number of schools that responded was 410, and of these 166 indicated that they offered such a course and were willing to cooperate in this study.

A survey of various forms of literature was made in order to compile a list of objectives recommended for a college course in mathematics-for-general-education. The results of the survey were synthesized into a list of 17 objectives, which list was sent to 50 judges for validation. On the basis of the rank they received on the rating sheets, seven of the objectives were selected as the most desirable ones for a type (c) course.

A measuring device in the form of a 50-item multiple choice objective test was then constructed according to the recommendations of theorists in the field. It was administered to a sample of 10 schools selected at random from the population of 166 schools that were willing to participate in the testing.

The test was administered twice, once at the beginning of the course and a second time at the end. The results obtained in the test by the 483 students who took both the pretest and the post-test constituted the data which was analyzed in Chapter IV.

CHAPTER IV

ANALYSIS OF DATA

In this chapter the empirical data of this study are analyzed in the following order: (1) reliability of the test for the group of 483 testees; (2) statistical data derived from test scores; (3) testing for significance of difference in means for pretest and post-test; (4) testing for significant differences of percentage of correct responses for test items related to the various general education objectives.

Reliability of the Test

The reliability coefficients for the tryout forms of the test were calculated during the test development period. They were found to be .856 for Form A and .692 for Form B. In order to see how the reliability of the final form of _A Test of College Mathematics for General Education_ compared with that of the tryout forms, the value of r was again calculated. By applying the Split-Half Method to the post-test scores the Pearson product-moment r was found to be .662 for one-half the test. To find the reliability of the entire test the Spearman-Brown prophecy formula was used.[1] The value thus obtained was .796 which was slightly more than the mean of the two values of r estimated for the tryout forms.

To determine how much confidence could be placed in the scores obtained

[1]Downie and Heath, p. 84.

in the test, the standard error of measurement was calculated by the formula

$$s_e = \sqrt{1 - r}\ .$$

The standard deviation, s, for the 483 post-test scores was 7.074. Substituting the values of s and r in the formula, it was found that

$$s_e = 7.074\ \sqrt{1 - .796}$$
$$= 3.197 \text{ or } 3.2$$

This small value of s_e is another measure of the reliability of the test. It indicates that an individual's obtained score on A Test of College Mathematics for General Education is not more than 3.2 units from the true score.

Statistical Data Derived from Test Scores

In this section the statistical data derived from test scores obtained from the two administrations of the test is presented. The score made by an individual on A Test of College Mathematics for General Education consisted of the number of items answered correctly. The possible score was fifty. A frequency distribution of pre- and post-test scores is given in Table V.

A study of Table V shows that the pretest scores ranged from zero to thirty-seven with a mean of 14.4, and the post-test scores ranged from three to forty-four with a mean of 22.0. The maximum frequency on the pretest was 194 and it occurred in the 10-14 interval. The next highest frequency, 123, was considerably smaller. On the post-test the maximum number of scores seemed to be evenly distributed within two intervals, namely 15-19 and 20-24. It may be observed that there was a frequency increase from pre- to post-test in all but the three lowest score intervals and the highest interval which contained no scores for either test.

TABLE V

FREQUENCY DISTRIBUTION OF PRETEST AND POST-TEST SCORES

Score Interval	Frequency Pretest	Post-Test
45 - 49	0	0
40 - 44	0	10
35 - 39	3	16
30 - 34	7	45
25 - 29	18	83
20 - 24	56	136
15 - 19	123	139
10 - 14	194	47
5 - 9	74	6
0 - 4	8	1
Range	37 - 0	44 - 3
Mean	14.4	22.0

Table V also shows that from pre- to post-test there was a mean gain of 7.6 but at the same time there was a four unit increase in range. This indicates that although the scores were higher on the post-test they were also more widely dispersed. This effect may be clearly seen in Figure 1 which contains frequency polygons of pre- and post-test scores.

An examination of the frequency polygons of Figure 1 shows that the scores on the post-test approximate a normal curve with a mean of 22 occurring just 1.5 units to the left of the midpoint (23.5) of the range of scores, whereas the scores on the pretest are skewed to the left with a mean of 14.4 occurring 4.1 units to the left of the midpoint (18.5) of the range and thus they depart somewhat from a normal distribution.

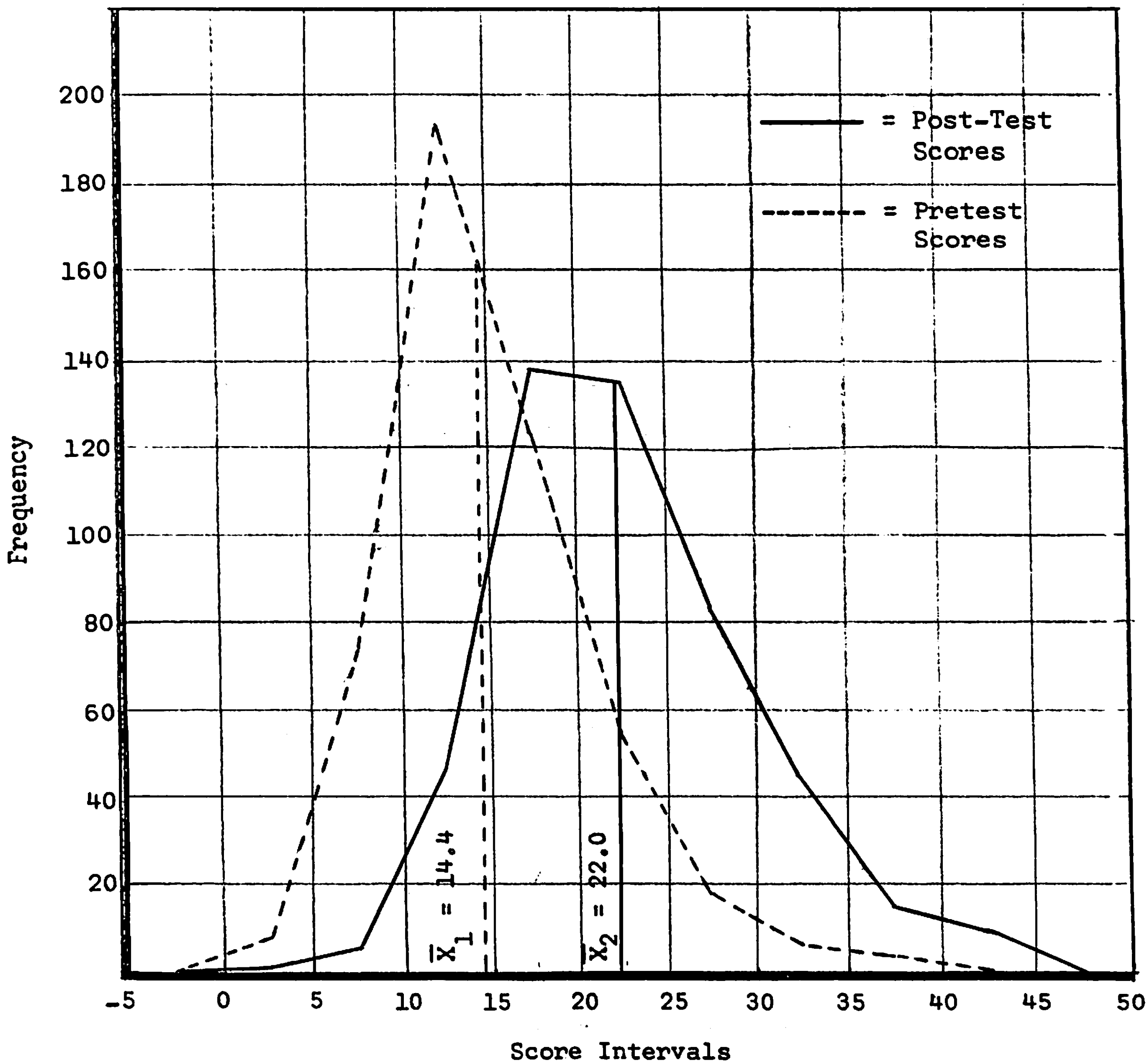


Figure 1. Frequency Polygons of Pretest and Post-Test Scores.

A study of the diagrams in Figure 1 shows that a definite change relative to the seven selected objectives seems to have occurred during the time between the two administrations of the test while the students were enrolled in a college mathematics-for-general-education type of course.

In order to get a better picture of the distribution of the scores in relation to each of the participating colleges, the means and standard deviations were calculated for each of the 10 schools in the sample as well as for the combined group. These statistics are given in Table VI.

TABLE VI

THE MEANS AND STANDARD DEVIATIONS OF THE PRETEST AND POST-TEST SCORES FOR THE 10 SCHOOLS IN THE SAMPLE

Code Number of School	N	Mean Pretest	Mean Post-Test	Standard Deviation Pretest	Standard Deviation Post-Test
1	9	13.33	21.67	6.87	9.68
2	11	11.54	13.91	4.01	3.21
3	25	19.52	29.68	6.37	9.40
4	13	17.92	26.69	8.19	7.65
5	36	11.75	21.05	3.84	7.05
6	71	15.82	23.50	5.97	7.16
7	75	12.81	19.07	5.13	6.03
8	83	13.20	21.40	4.55	6.15
9	50	15.16	24.04	5.98	6.18
10	110	14.94	21.43	5.45	6.12
Combined Group	483	14.44	22.01	5.68	7.07

The data in Table VI show that in all cases there is a difference between the statistics derived from the pre-scores and those derived from the post-scores. The means have apparently increased from pretest to post-test. Whether this increase is significant or not is the subject of the discussion in the next section.

It is interesting to note that there was an increase in the standard deviations for eight out of ten schools. This indicates a wider dispersion of scores on the post-test than on the pretest, which effect has been previously noted in the polygons of Figure 1. In only two instances did the standard deviation decrease.

Since each question in the measuring instrument developed in this study was to test a particular objective, it was especially interesting to find what percentage of the 483 students correctly answered each item on the post-test and then to examine the distribution of items according to these percentages. Table VII contains this information.

From a study of the table it is apparent that no item was answered by less than 15 percent of the subjects and none was answered by more than 89 percent. Item 45 was the only one answered by less then 20 percent and only items 16 and 33 were answered by more than 74 percent of the subjects.

The items seem to be clustered about the 40-44 percent interval. According to the Davis Item Analysis Chart, these items have an approximate difficulty index of 60 which is the mean difficulty index used by the investigator in the selection of test items. This shows that the results of the post-test seem to be in agreement with the data obtained previously from the item analysis of the tryout forms of the test.

TABLE VII

FREQUENCY DISTRIBUTION OF THE ITEMS BASED ON THE PERCENT OF 483 TESTEES CORRECTLY ANSWERING EACH ITEM*

Percent Intervals	Items in Each Interval Based on Percent of Correct Responses to Each Item	Number of Items in Each Interval
95 - 99		0
90 - 94		0
85 - 89	16, 33	2
80 - 84		0
75 - 79		0
70 - 74	2, 29	2
65 - 69	1, 8	2
60 - 64	15	1
55 - 59	25, 31, 32, 36	4
50 - 54	3, 4, 9, 11, 18, 34, 35	7
45 - 49	19, 23, 30, 41	4
40 - 44	6, 13, 20, 21, 24, 26, 27, 38	8
35 - 39	10, 39, 40, 42, 48, 50	6
30 - 34	5, 7, 22, 28, 47	5
25 - 29	12, 37, 49	3
20 - 24	14, 17, 43, 44, 46	5
15 - 19	45	1
10 - 14		0
5 - 9		0
0 - 4		0

*This table should be read in the following manner: Between 55 and 59 percent of the testees answered correctly Item 25, Item 31, Item 32, and Item 36.

Testing a Hypothesis About the Difference of Means

Since the data analyzed here represent pairs of measurements on the same individual, the statistical treatment designed for correlated samples was employed.[2] The number of pairs in the sample population, 483, was large enough to warrant the assumption of normal distribution.

The estimated standard error of the difference between initial and final means was calculated for each of the 10 schools in the sample as well as for the combined schools. To faciliate computation, the following formula for paired data was used:

$$s_{\bar{D}} = \frac{N\Sigma D^2 - (\Sigma D)^2}{N^2(N - 1)}$$

where $s_{\bar{D}}$ = standard error of the mean difference.

D = difference between every pair of scores.

$\bar{D}$ = mean difference between scores on the pretest and scores on the post-test.

The null hypothesis of no difference between means was tested for significance at the one percent level. The t-test was used because four of the schools had fewer than 30 students and had to be treated as small samples. The ratio, $t = D/s_{\bar{D}}$, was calculated for each school and for the combined schools. The data thus obtained is given in Table VIII.

When the calculated values of t were compared with the tabulated ones at the one percent level, it was found that in every instance the calculated values were in the critical region and, therefore, significant.

---

[2] Downie and Heath, pp. 127-134.

TABLE VIII

TESTING THE SIGNIFICANCE OF THE DIFFERENCE BETWEEN MEANS FOR THE 10 SCHOOLS INCLUDED IN THE SAMPLE

Code Number of School	N	Initial Mean	Final Mean	Mean Gain	t	$t_{.01}$	df
1	9	13.13	21.67	8.34	3.759**	3.355	8
2	11	11.54	13.91	2.37	3.991**	3.169	10
3	25	19.52	29.68	10.16	8.760**	2.797	24
4	13	17.92	26.69	8.77	9.250**	3.055	12
5	36	11.75	21.06	9.31	9.138**	2.727	35
6	71	15.82	23.51	7.69	15.078**	2.653	70
7	75	12.81	19.07	6.26	10.744**	2.650	74
8	83	13.20	21.40	8.20	13.863**	2.644	82
9	50	15.16	24.04	8.88	11.684**	2.680	49
10	110	14.94	21.43	6.49	15.107**	2.625	109

** Significant at the .01 level.

On the basis of the data given in Table VIII, it was concluded that the differences between the initial and final means were real and not just due to chance.

Testing a Hypothesis about the Difference of Two Percentages

In order to measure the effectiveness of the mathematics-for-general-education type of course in attaining the seven objectives identified as most desirable, the test questions were divided into seven groups each of which contained only those items which were constructed to test one of these objectives. The percentage of testees answering an item correctly was computed for both the pretest and post-test scores. Than a test of hypothesis was made on every pair of percentages in each of the seven groups.

The null hypothesis of no difference in the percentages of correct responses on the pretest and post-test was tested for significance at the .05 and .01 levels. The standard error of the difference between two proportions was calculated for every item by the formula for correlated data,

$$s_{D_p} = \sqrt{s^2_{p_1} + s^2_{p_2} - 2(r_{12})(s_{p_1})(s_{p_2})}$$

where s^2_p is the variance of a proportion and is equal to $p(1 - p)/N$,[3] and r_{12} is the Pearson product-moment coefficient of correlation between the pretest and post-test scores and which was calculated to be .70. The test statistic, $z = (p_2 - p_1)/s_{D_p}$, was then used to obtain the information presented in Table IX.

TABLE IX

COMPARISON OF THE PERCENTAGES OF STUDENTS ANSWERING CORRECTLY EACH ITEM IN A GROUP OF ITEMS ON THE PRETEST WITH THE PERCENTAGES OF STUDENTS ANSWERING CORRECTLY EACH ITEM IN A GROUP OF ITEMS ON THE POST-TEST

Group Number *	Item	X_1	X_2	p_1	p_2	p_2-p_1	z	$\bar{p}_1$	$\bar{p}_2$
1.	24	136	194	28.2	40.2	12.0	7.23**		
	27	161	210	33.3	43.5	10.2	5.96**		
	28	70	145	14.5	30.0	15.5	10.33**	25.3	37.9

* The numbers in this column agree with the numbers of the objectives in the chart of specifications on page 47.

[3] Downie and Heath, p. 137.

TABLE IX (Continued)

Group Number	Item	X_1	X_2	p_1	p_2	p_2-p_1	z	$\bar{p}_1$	$\bar{p}_2$
2.	5	71	156	14.7	32.3	17.6	11.58**		
	6	96	208	19.9	43.1	23.2	14.32**		
	10	94	191	19.5	39.5	20.0	12.42**		
	32	160	266	33.1	55.1	22.0	12.86**		
	34	145	244	30.0	50.5	20.5	11.79**		
	39	120	181	24.8	37.5	12.7	7.80**		
	49	66	140	13.7	29.0	15.3	10.34**	22.2	41.0
3.	1	151	335	31.3	69.4	38.1	23.37**		
	2	143	340	29.6	70.4	40.8	25.50**		
	3	93	250	19.2	51.8	32.6	19.88**		
	12	106	119	21.9	24.6	2.7	1.61		
	21	139	194	28.8	40.2	11.4	6.83**		
	23	126	234	26.1	48.4	22.3	13.35**		
	26	101	210	20.9	43.5	22.6	13.86**		
	36	183	271	37.8	56.1	18.3	10.58**		
	42	116	183	24.0	37.9	13.9	8.58**		
	44	93	109	19.2	22.6	3.4	2.31**		
	48	81	191	16.8	39.5	22.7	14.18**	25.1	45.8
4.	4	194	258	40.2	53.4	13.0	7.58**		
	16	367	422	76.0	87.4	11.4	8.14**		
	25	241	265	49.9	54.9	5.0	2.84**		
	30	210	219	43.5	45.3	1.8	1.02		
	33	279	426	78.5	88.2	9.7	7.23**	57.6	65.8

TABLE IX (Continued)

Group Number	Item	X_1	X_2	P_1	P_2	P_2-P_1	z	$\bar{P}_1$	$\bar{P}_2$
5.	8	216	326	44.7	67.5	22.8	13.41**		
	9	138	259	28.6	53.6	25.0	14.79**		
	11	147	248	30.4	51.3	20.9	12.29**		
	14	54	108	11.2	22.4	11.2	8.23**		
	20	126	209	26.1	43.3	17.2	10.36**		
	29	217	338	44.9	70.0	25.1	14.76**		
	31	196	274	40.6	56.7	16.1	9.20**		
	38	108	207	22.4	42.8	20.4	12.43**		
	50	162	168	33.5	34.8	1.3	0.77	31.4	49.2
6.	7	59	147	12.2	30.4	18.2	12.21**		
	13	76	199	15.7	41.2	25.5	15.93**		
	22	91	149	18.8	30.8	12.0	7.84**		
	35	169	241	34.9	49.9	15.0	8.72**		
	46	78	111	16.1	23.0	6.9	4.89**	19.6	35.1
7.	15	172	296	35.6	61.3	25.7	15.20**		
	17	105	102	21.7	21.1	-.6	-0.41		
	18	166	241	34.4	49.9	15.5	8.96**		
	19	184	218	38.1	45.1	7.0	4.04**		
	37	109	120	22.6	24.8	2.2	1.45		
	40	117	175	24.2	36.2	12.0	7.45**		
	41	156	215	32.3	44.5	12.2	7.18**		
	43	94	97	19.5	20.1	0.6	0.43		
	45	82	76	17.0	15.7	-1.3	-1.00		
	47	97	145	20.1	30.0	9.9	5.02**	26.5	34.9

The following is a key to the symbols used in Table IX:

X_1 = the number of subjects who answered correctly an item on the pretest.

X_2 = the number of subjects who answered correctly an item on the post-test.

p_1 = percentage of 483 students answering correctly an item on the pretest.

p_2 = percentage of 483 students answering correctly an item on the post-test.

$\bar{p}_1$ = percent of correct responses in a group of items on the pretest.

$\bar{p}_2$ = percent of correct responses in a group of items on the post-test.

* = significant at the .05 level.

** = significant at the .01 level.

In Table IX the objective of the first group of items was to gain knowledge of the mathematical methods of reasoning. The difference in percentage of correct responses for each of the three items in this group was significant at the .01 level. The greatest gain was made for Item 28 although the percentage of correct responses for that item was lowest on both applications of the test. The content of Item 28 was symbolic logic.

The second group of test questions had for its objective the acquisition of knowledge of the new approach to mathematics as well as of new ideas in contemporary mathematics. The gain in percentage of correct responses for each of the seven items in this group was significant at the .01 level. The greatest gain was made for Item 6 which required the changing of a number from base ten to base four. The smallest gain was made for Item 39 which required the finding of the solution set of a given inequality.

The objective tested in the third group of items was knowledge of basic mathematical vocabulary and definitions necessary for the attainment of the desired objectives. The gains in percentage were not significant at the .01 level for only two of the eleven items in the group. These t were Item 12 and Item 44. Item 12 required ability to distinguish between rational and irrational numbers. In addition to a knowledge of the definitions it required some skill in algebraic computation. The gain for Item 44 was significant at the .05 level. This item tested knowledge and understanding of the properties of an equivalence relation. In this group the greatest gain, 40.8, was made for Item 2 which tested knowledge of the distributive law. The smallest gain, 2.7, was made for Item 12.

The fourth group of items tested ability to draw valid inferences from data and to distinguish between "valid" and "true." For the five items in this group the gains in percentage were significant for items 4, 16, 25, and 33. The gain was not significant only for Item 30. This item required the drawing of a valid inference from a syllogism. The significant gains in this group were comparatively low. They ranged from 5 to 13.

The fifth group of items tested ability to make precise statements and to be able to defend them. There were significant gains for eight out of nine items, namely for items 8, 9, 11, 14, 20, 29, 31, and 38. The gain was not significant for Item 50. In this item the testees were asked to select a valid reason for not proving axioms in a mathematical system. The greatest gain, 25.1, was made for Item 29 which required a restatement of the contrapositive of an implication in the language of symbolic logic.

The objective of the sixth group of items was ability to use modern mathematical concepts to gain a stronger understanding of some familiar number systems. The gains in percentages for each of the five items in this group were significant at the .01 level. The greatest gain, 25.5, was made for Item 13 which required the testee to know that when fractions were added to the whole numbers to form the set of rational numbers a new property, namely closure under division, was added to the number system. The smallest gain, 6.9, was made for Item 46 which called for a description of the domain of a given function.

The seventh group of items tested ability to interpret correctly some of the symbols and diagrams used in the literature of our technical culture. The gains were not significant for four out of ten items in the group. Of the four, Item 17 dealt with logarithms, Item 37 contained several symbols but was aimed primarily at ability to interpret absolute values, Item 43 tested ability to distinguish a relation that is a function from one that is not, and Item 45 was based on the knowledge of mathematical symbols used to describe a function. Items 17 and 45 were the only two items out of fifty which exhibited a negative gain in percentage. A comparatively high gain, 25.7, was made for Item 15 which tested ability to work with exponents.

The over-all gains in percentages for each group ranged from 8.2 to 20.7. The greatest gains were made for groups two and three with increases in over-all percentages of 20.7 and 18.8 respectively. The smallest gains were made for groups four and seven with increases in over-all percentages of 8.2 and 8.4 respectively.

Summary and Interpretation of Findings

The statistical analysis presented in this chapter indicates that the mean gain shown on *A Test of College Mathematics for General Education* by the 483 students who completed a type (c) course was significant at the

.01 level. Therefore, there is evidence as indicated by the instrument used that, in general, the course contributed toward the achievement of the objectives for which the instrument was constructed.

More specifically, the gains in percentages of students answering an item correctly were significant for forty-two out of fifty items. The gains were significant for all items testing objectives 1, 2, and 3. This indicates that the course was effective in teaching methods of reasoning and that it contributed to a knowledge of, and an ability to use, modern mathematical concepts to gain a stronger understanding of some familiar number systems.

Four of the eight items for which the gains were not significant were in group seven, the objective of which was to acquire ability to interpret correctly symbols and diagrams used in the literature of our technical culture. This implies that the course was not very effective in this area.

Although the gains for two of the eleven items in group three were not significant at the .01 level, the overall gain in percentage of correct responses for the eleven items, 20.7, was higher than for any other group. On the basis of this evidence it was concluded that the course contributed to a knowledge of basic mathematical vocabulary and definitions necessary for the attainment of the desired objectives.

The gains were significant for five out of six items in group four. The overall gain, 8.2, was smaller than that of any other group; however, the overall percentages of correct responses were the highest on both the pretest and the post-test. It was concluded, therefore, that either the course contributed somewhat to the ability to draw valid inferences from data, or the test items were based on material with relation to which the students tested had previously acquired knowledge and ability.

In group five, the gains were significant for eight out of nine items. This evidence, plus the fact that the overall gain for the group, 17.8, was the third highest, led to the conclusion that the course contributed to the ability to make precise statements and to be able to defend them.

The greatest gains in over-all percentages for groups two and three seem to imply that during the time the 483 subjects were enrolled in a mathematics-for-general-education type of course greatest changes were effected relative to the knowledge and understanding of the new approach to mathematics and new ideas in contemporary mathematics, as well as to the knowledge and understanding of basic mathematical vocabulary and definitions necessary for the attainment for the seven selected objectives.

The smallest gains in over-all percentages for groups four and seven seem to indicate that the course was least effective in producing in the 483 subjects changes relative to the acquisition of skills and ability to draw valid inferences from data, and to interpret correctly some of the symbols and diagrams used in the literature of our technical culture.

CHAPTER V

SUMMARY AND CONCLUSIONS

Review of the Study

The primary aim of this study was to measure the effectiveness of a college mathematics-for-general-education type of course in achieving seven objectives indentified as most desirable.

In order to fulfill this purpose, several minor investigations were carried out. The first was a questionnaire sent to 530 higher institutions of learning in order to determine how many of them offered a mathematics-for-general-education type of course and were willing to cooperate in the study; the second was the formulation of a list of recommended objectives for a college mathematics course for the non-major and the validation of these objectives by 38 judges; the third was the development of a measuring instrument based on seven most desired objectives as determined by the ratings of the judges.

The measuring instrument titled *A Test of College Mathematics for General Education* was developed according to the usual procedure recommended by theorists in the field. A chart of specifications was drawn up according to which two tryout forms of the test were constructed. These tests were administered to 98 students at Oklahoma State University and then subjected to an item analysis according to the methods described by Davis in his Item Analysis Data.[1] On the basis of the resulting

[1] Davis, pp. 30-37.

information a fifty-item multiple-choice test was constructed. This test was administered twice, once at the beginning and then again after completing the course, to 483 students enrolled in 10 schools selected at random from among 166 colleges that offered a mathematics-for-general-education type of course and were willing to cooperate in the study. The 483 pre- and post-test scores thus obtained constituted the data that were subjected to a statistical analysis in this investigation.

The analysis of data included the determination of the reliability of the test for the 483 subjects, a comparison of frequency distributions of pre- and post-test scores, and statistical tests for significant differences in means and for significant differences in percentages of correct responses for test items related to the seven objectives.

The data gathered in this investigation have been analyzed and presented in expository and tabular form in the foregoing chapters. In the present chapter these findings are summarized and conclusions are drawn from the findings.

Findings and Tentative Conclusions

The following findings and tentative conclusions have resulted from this investigation:

1. Of the 410 colleges that completed the questionnaire fifty-three percent offered a mathematics-for-general-education type of course to the non-major. Approximately fifteen percent of those who did not now offer it planned to do so in the near future.

2. Ninety-seven different textbooks were reported in use. Seven of these were used by at least ten schools; thirty-five by at least two schools; and sixty-two were each used by just one school.

3. All of the 17 objectives submitted for validation to 38 judges

were rated as of considerable value or better. On the basis of this evidence it was concluded that there appears to be a concensus of opinion relative to the most desired objectives of a college mathematics course for the non-major.

4. It was found that the measuring instrument developed in this study had a reliability coefficient of .80 and a standard error of measurement equal to 3.2. In spite of this statistical evidence as to its reliability, the measuring instrument was subject to some immediately discernible imperfections. It embraced only seven of the seventeen highly desirable objectives. Also, the distribution of items relative to the seven objectives was not the best. In many instances a test item seemed to measure several objectives and the decision of placing it in one group rather than in another was a highly subjective one.

5. A comparison of the distribution of pre- and post-test scores obtained from the two applications of the measuring instrument to 483 students showed that a substantial change relative to the seven selected objectives had occurred during the time these students were enrolled in a mathematics-for-general-education type of course.

6. There were significant mean gains from pre- to post-test for each of the ten schools in the sample as well as for the combined group.

7. There were significant gains in percentage of correct responses for forty-two out of fifty items. The percentages, however, were rather low. On the post-test only 16 out of 50 items were answered correctly by more than 50 percent of the subjects.

8. The gains in percentages were significant for all items in groups one, two, and three. The gains were not significant for one item in groups four and five respectively, for two items in group six, and for four items in group seven. On the basis of this information it was

concluded that although the mathematics-for-general-education type of course appeared to make significant contributions toward the attainment of each of the seven selected objectives, it seemed to be less effective in certain areas than in others. The non-significant gains for four out of ten test items in group seven seemed to imply that either the course was not very effective in developing ability to interpret correctly symbols and diagrams used in the literature of our technical culture or the test items were faulty.

9. The over-all gains in percentages of correct responses for each group of items ranged from 8.2 to 20.7. The smallest gains were made for groups four and seven while the greatest gains were made for groups two and three. It is interesting to note that although the over-all gain for group four was the smallest, the percentages of correct responses for the group on both tests were the highest. It was concluded that either the course was not very effective in developing ability to draw valid inferences from data or the test questions in this group were based on material with relation to which the students tested had previously acquired knowledge and ability.

Recommendations

The investigator makes the following recommendations:

1. There is a lack of reliable measuring instruments for mathematics in a program of general education on the college level. More research is needed to improve existing evaluative devices and to develop new ones to provide information on student progress relative to common goals. The task, however, is too colossal for one investigator. Teamwork is recommended.

2. Due to the limitations of this study, the objectives related

to the development of certain highly desirable attitudes were not evaluated. Further study should be carried out in order to measure the effectiveness of the mathematics-for-general-education type of course as it relates to (1) the creation of interest in, and change in attitude toward mathematics; (2) a wholesome respect for correct reasoning and precise definitions; and (3) an appreciation for the aesthetic values of mathematics.

3. A possible reason for the relatively low percentages of correct responses on the post-test was that one semester did not provide enough time adequately to cover the content necessary for the achievement of the desired goals. It would be desirable to conduct an investigation of the effectiveness of a mathematics-for-general-education type of course which extends through at least two semesters and compare the results with those obtained in this study.

4. The statistical analysis of data in the present study indicated that the mathematics-for-general-education type of course does not seem to be very effective in developing ability to interpret correctly symbols and diagrams used in the literature of our technical culture. It would be interesting to investigate current literature to determine which symbols and diagrams are used enough to warrant their inclusion in the content of a college mathematics course for the non-major.

5. Since there exist other mathematics courses which are being offered to the non-major, it would seem desirable to collect and interpret evidence on the changes induced in students subjected to various programs in college mathematics for the non-major in relation to certain common goals and then to compare the results thus obtained.

SELECTED BIBLIOGRAPHY

Bartz, Albert E. Educational Measurement. Minneapolis, Minnesota: Burgess Company, 1963.

Betz, William. "Five Decades of Mathematical Reform." The Mathematics Teacher, XLIII (December, 1950), p. 382.

Breslich, E.R. "Importance of Mathematics in General Education." The Mathematics Teacher, XLIV (January, 1951), pp. 1-6.

Brown, Kenneth E. General Mathematics in American Colleges. New York: Bureau of Publications, Teachers College, Columbia University, 1943.

________. "Is General Mathematics in the College on Its Way Out?" The Mathematics Teacher, XLI (April, 1948), pp. 154-158.

________. "The Content of a Course in General Mathematics -- Teachers' Opinions." The Mathematics Teacher, XLIII (January, 1950), pp. 25-30.

Chicago, University of. The Idea and Practice of General Education; An account of the University of Chicago by present and former members of the faculty. Chicago: University of Chicago Press, 1950.

Commission on Secondary School Curriculum. Mathematics in General Education New York: Appleton-Century Company, 1940.

Committee on a Design for General Education. A Design for General Education. Washington, D.C.: American Council on Education Studies, 1944.

Corcoran, Mary and E.G. Gibb. "Appraising Attitudes in the Learning of Mathematics." Twenty-Sixth Yearbook. Washington, D.C.: National Council of Teachers of Mathematics, 1961. pp. 105-122.

Davis, Frederick B. Item-Analysis Data. Cambridge, Massachusetts: Harvard University Press, 1949.

Downie, N.M. and R.W. Heath. Basic Statistical Methods. New York: Harper, 1959.

Dressel, Paul L. et al. Comprehensive Examinations in a Program of General Education. East Lansing, Michigan: Michigan State College Press, 1948.

________. Evaluation in General Education. Dubuque, Iowa: W.C. Brown Company, 1954.

Dressel, Paul L. Evaluation in Higher Education. Boston: Houghton Mifflin, 1961.

Fawcett, Harold P. "Mathematics in General Education." Bulletin of the National Association of Secondary School Principals, LIII (May, 1959), p. 33.

Gallion, Zachary Taylor. "A Determination and Appraisal of the Content of Freshman General Mathematics Courses in Selected Colleges and Universities." (Unpublished Ph.D. Dissertation, Louisiana State University, 1955).

Haag, Vincent H. Structure of Algebra. Reading, Massachusetts: Addison-Wesly, 1964.

Hammer, Preston C. "The Role and Nature of Mathematics," Pi Mu Epsilon III (Spring, 1964), pp. 501-509.

Harvard Committee. General Education in a Free Society. Cambridge, Massachusetts: Harvard University Press, 1945.

Hutchinson, C.A. "Mathematics Instruction for Purposes of General Education." The American Mathematical Monthly, XLVIII (March, 1941), pp. 189-197.

Johnson, Donavan A. "Evaluating a School Mathematics Curriculum." School and Society, XC (December, 1962), pp. 424-426.

Lafferty, William A. "The Selection of Subject Matter in Mathematics for General Education." (Unpublished Ed.D. Dissertation, Teachers College, Columbia University, 1956).

Leonhardy, Adele. "The Mathematics Used in the Humanities, Social Science, and Natural Science Areas in a Program of General Education on the College Level." Science Education, XXXVI (October, 1952), pp. 252-253

Lindquist, C.B. "Entering Levels and College Courses in Freshman Mathematics," School Life, XLV (April, 1963), pp. 14-17.

Lynch, Ina. "Forces That Have Influenced the School Mathematics Program." School Science and Mathematics, LXIV (April, 1964), pp. 255-264.

Means, James Horatio. "Objectives of Mathematics Instruction in Seven Texas Colleges." (Unpublished Ed.D. Dissertation, Oklahoma State University, 1958).

Milligan, Merle W. "An Inquiry into the Selection of Subject Matter Content for College Freshman Mathematics." (Unpublished Ed.D. Dissertation, Oklahoma State University, 1961).

Mires, Kathrine Carrie. "The Need for and the Nature of One Type of Course in Mathematics for General Education at the College Level." (Unpublished Ed.D. Dissertation, University of Oklahoma, 1956).

Montague, Harriet F. and Phyllis M. Henry. "The Case for a General Education Course in Mathematics." The Journal of General Education XIII (July, 1961), pp. 97-112.

Moore, Dom Thomas Verner. Cognitive Psychology. New York: J.B. Lippincott Company, 1939.

Northrop, E.P. "MAthematics in a Liberal Education." The American Mathematical Monthly, LII (March, 1945), pp. 132-137.

________. "The Mathematics Program in the College of the University of Chicago." The American Mathematical Monthly, LV (January, 1948), pp. 1-7.

Nunnally, Jum C. Tests and Measurements. New York: McGraw-Hill, 1959.

Pixley, Loren W. "Mathematics in the Community Junior College." The Mathematics Teacher, LVII (May, 1964), pp. 313-315.

President's Commission on Higher Education. Higher Education for Democracy, I: Establishing the Goals. Washington, D.C., 1947.

Remmers, H.H. and N.L. Gage. Educational Measurement and Evaluation. New York: Harper Press, 1955.

Rosskopf, Myron F. "The Place of Mathematics in General Education." School Science and Mathematics, XLIX (October, 1949), pp. 565-570.

Russell, Thomas. The Search for a Common Learning: General Education 1800-1960. New York: McGraw-Hill, 1962.

Standen, Anthony. Science is a Sacred Cow. New York: Dutton Press, 1950.

Stokes, William G. "The Effectiveness of Mathematics 200 in Regard to Certain Desirable Objectives of Mathematics for General Education." (Unpublished Report of the Mathematics Department of Austin Peay College, Tennessee, 1962).

Sturm, Harold E. "Development of a Junior College Mathematics Program for Non-Science, Non-Mathematics Majors." School Science and Mathe-mathics, L (June, 1950), pp. 437-441.

Thorndike, Robert L. Personnel Selection: Test and Measurement Techniques. New York: J. Wiley Press, 1949.

Tolley, William P. et al. Cooperation in General Education. Washington, D.C.: American Council on Education, 1947.

Wilson, Jack D. "What Mathematics for the Terminal Student?" The Mathematics Teacher, LIII (November, 1960), pp. 518-523.

Wood, Dorothy Adkins. Test Construction. Columbus, Ohio: C.E. Merrill, 1960.

Wren, Lynwood F. "The Merits and Content of a Freshman Mathematics Course." School Science and Mathematics, LII (November, 1952), pp. 595-603.

Zant, James H. "General Mathematics at the College Level." School Science and Mathematics, L (June, 1950), pp. 477-479.

APPENDICES

APPENDIX A

THE PRELIMINARY QUESTIONNAIRE

Name of Institution of Higher Learning ______________________

Location ______________________

President, Dean, or Head of the Mathematics Department

____

Would you be willing to fill in the items above and to give the information asked for below? Please return the two sheets in the stamped envelope enclosed. These data are needed as a basis for a nation-wide study of the effectiveness of one type of mathematics course for the non-major* in the institutions of higher learning classified as liberal ar[ts] and general, and teacher preparatory. This study is sponsored by Oklahoma State University. Tabulated returns will be available as soon as all the above-mentioned institutions of higher learning have replied.

At least three types of mathematics courses are being offered to non-majors in a program of general education. They are: (a) the remedial course whose main objective is improvement in mathematical skills; (b) the fused or correlated course in which the content is made up of certain traditional topics such as algebra, trigonometry, analytic geometry and calculus; (c) the mathematics-for-general-education or cultural type of course in which the content is generally made up of topics other than the traditional.

1. Is a course in mathematics a prerequisite for graduation for all students enrolled in this institution of higher learning?

   No _____ Yes _____ Number of credit hours required _______

2. Do you offer non-majors a mathematics course classified as (c) above, that is, a mathematics-for-general-education type of course?

   No _____ Yes _____

   If your answer is YES, please complete the following:

* non-major: a college student or graduate who has not majored in mathematics, engineering, or any of the physical sciences.

Title of this course ______________________

Titles (s) of text (s) used and name (s) of author (s)

____

____

____

Number of credit hours given for this course ______

Total number of students enrolled in this course in the fall of 1962 ____

spring of 1963 ____

Additional Comments: ______________________

____

____

____

____

3. Would you be willing to cooperate in the successful completion of this study by permitting the students enrolled in a Type C course during the fall or spring of 1963-64 to be subjected to a test designed to measure the effectiveness of this course?

(Note: You will incur no cost. The results will be tabulated without particular reference to any school. Names of students will be kept in strict confidence.)

No _____ Yes _____

Comments: ______________________

____

____

4. If possible, please send a copy of one or more of the tests and examinations you have used to evaluate achievement in a mathematics-for-genera[l] education type of course.

Very truly yours,

Sister Mary Firmina, C.S.S.F.
Doctoral Student at Oklahoma Stat[e]
University

APPENDIX B

LETTER AND SUMMARY OF RESPONSES
TO THE QUESTIONNAIRE

1002 S. Walnut
Stillwater, Oklahoma
January 28, 1964

Dear Fellow Teacher:

I wish to thank you for your gracious cooperation in answering the questionnaire that was sent to you in the spring of 1963.

The questionnaire was sent to 530 institutions of higher learning and of these 410 or 77% responded. I am enclosing a summary of the results which might be of interest to you.

A total of 166 schools expressed a willingness to participate in the testing. This is a much greater number than I had anticipated. Because of limited finances the test will have to be administered to a sample population. Schools included in the sample will be notified by the first week of April, 1964.

If you are interested in a copy of the completed report, please let me know and I'll see if I can arrange to have one sent to you.

Sincerely yours,

Sister Mary Firmina, C.S.S.F.

A SUMMARY OF THE RESPONSES OBTAINED FROM THE QUESTIONNAIRE

Question 1: Is a course in mathematics a prerequisite for graduation for all students enrolled in this institution of higher learning?

No 277 Yes 133

Number of credit hours required:

12 semester hours....1 school
6 semester hours...41 schools
5 semester hours....4 "
4 semester hours....9 "
3 semester hours...54 "
2 semester hours....4 "

10 quarter hours....1 school
9 quarter hours....1 school
6 quarter hours....3 schools
5 quarter hours....6 "
4 quarter hours....3 "
3 quarter hours....4 "
2 quarter hours....1 school

Question 2: Do you offer non-majors a mathematics course classified as a mathematics-for-general-education type of course?

No 183 Yes 217

Number of credit hours:

8 semester hours....5 schools
7 semester hours....1 school
6 semester hours...75 schools
5 semester hours....6 "
4 semester hours...20 "
3 semester hours...75 "
2 semester hours....5 "
0 semester hours....1 school

10 quarter hours....1 school
9 quarter hours....3 schools
6 quarter hours....2 "
5 quarter hours....8 "
4 quarter hours....6 "
3 quarter hours....8 "
2 quarter hours....1 school

Title of the Course: Most course titles either corresponded with or were similar to the titles of the textbooks being used.

Textbooks: Ninety-seven different books were mentioned. The followin is a list of the 35 textbooks which were reported in use by at least two schools.

Title	Author	Publisher and Copyright	Number of Schools
Fundamentals of Mathematics	Moses Richardson	Macmillan 1958	24
Basic Concepts of Elementary Mathematics	William Schaaf	Wiley - 1960	17
Elements of Mathematics	J. H. Banks	Allyn and Bacon 1961	16
Fundamental Concepts of Elementary Mathematics	Brumfiel, Eicholz and Shanks	Addison-Wesley 1962	15
Fundamental Mathematics	Wade and Taylor	McGraw-Hill 1961	14
Finite Mathematical Structures, or Introduction to Finite Mathematics	Kemeny, Snell and Thompson "	Prentice-Hall 1959 Prentice-Hall 1957	13
Principles of Mathematics	Allendoerfer and Oakley	McGraw-Hill 1955 and 1963	12
A Modern Introduction to Mathematics	John Freund	Prentice-Hall 1956	9
An Introduction to the Elements of Mathematics	John Fujii	Wiley- 1961	8
Elementary Concepts of Mathematics	Burton Jones	Macmillan 1947	8
Fundamentals of Freshman Mathematics	Allendoerfer and Oakley	McGraw-Hill 1959	7
General College Mathematics	Ayres, Fry and Jonah	McGraw-Hill 1960	6
Foundation of Mathematics	Denbow and Goedicke	Harper - 1959	6
Introductory College Mathematics	Adele Leonhardy	Wiley - 1963 2nd Ed.	6
Mathematics for General Education	Trimble, Hamilton and Silvey	Prentice-Hall 1963	6
Mathematics: A Cultural Approach	Morris Kline	Addison-Wesley 1962	5

Title	Author	Publisher and Copyright	Number of Schools
A Modern Introduction to College Mathematics	Israel Rose	Wiley - 1959	5
Theory of Arithmetic	Peterson and Hashisaki	Wiley - 1963	4
Modern Mathematics	A. B. Evenson	Scott-Foresman 1962	3
Basic Concepts in Modern Mathematics	John Hafstrom	Addison-Wesley 1961	3
A Modern Introduction to Basic Mathematics	Mervin Keedy	Addison-Wesley 1963	3
Understanding Arithmetic	Swain	Rinehart - 1952	3
Introduction to Probability and Statistics	Alder and Roessler	Freeman - 1962	2
Modern Mathematics	Samuel Altwerger	Macmillan 1960	2
An Introduction to Foundations and Fundamental Concepts of Mathematics	Eves and Newsom	Rinehart 1957	2
Fundamental Mathematical Systems	Sister Vincent Ferrer, RSM	St. Xavier College Chicago	2
Introducing Mathematics	Floyd Helton	Wiley - 1958 2nd ed.	2
Elements of Modern Mathematics	K.O. May	Addison-Wesley 1959	2
Introductory Analysis	V. O. McBrien	Appleton	2
Understanding Basic Mathematics	Leslie Miller	Holt, Rinehart and Winston	2
Introduction to College Mathematics	Newsom and Eves	Prentice-Hall 1954	2
Basic College Mathematics	Sachs, Rasmusen and Purcell	Allyn and Bacon 1960	2

Title	Author	Publisher and Copyright	Number of Schools
Sets, Relations, Functions: An Introduction	Selby and Sweet	McGraw-Hill 1963	2
Mathematics, the Man Made Universe	S. K. Stein	Freeman - 1963	2
Basic Concepts of Mathematics	Webber and Brown	Addison-Wesley 1963	2

APPENDIX C

OBJECTIVES OF A COLLEGE COURSE IN MATHEMATICS

FOR GENERAL EDUCATION

Please indicate your opinion of the value of each objective for a mathematics-for-general-education type of course for the college student by placing 4, 3, 2,or 1 before the statement of the objective according to the following scale:

__4__ The objective is highly desirable.

__3__ The objective is of considerable value.

__2__ The objective is of slight value.

__1__ The objective is of no value.

A mathematics-for-general-education type of course should help a student to acquire

A. Knowledge

_____1. Of the natural origin and evolutionary growth of mathematics ideas from antiquity to the present.

_____2. Of the mathematical methods of reasoning.

_____3. Of the postulational development of mathematical systems.

_____4. Of the new approach to mathematics as well as of new ideas in contemporary mathematics.

_____5. Of the basic mathematical vocabulary and definitions necessary for the attainment of these objectives.

B. Skills and Abilities

_____1. To use modern mathematical concepts to gain a stronger understanding of some familiar number systems and how these systems are related.

_____2. To deal with nonverbal symbolism.

_____3. To draw valid inferences from data; to distinguish between "valid and "true."

_____4. To make precise statements and to be able to defend them.

_____5. To apply the logic and patterns of mathematical reasoning to the solution of problems.

_____6. To think critically in non-mathematical as well as in mathematical situations.

_____7. To interpret correctly some of the symbols and diagrams used in the literature of our technical culture.

C. Attitudes, such as:

_____1. The habit of approaching problems objectively.

_____2. Willingness to face facts and conclusions.

_____3. Readiness to revise judgments and to change behavior in the light of reason.

_____4. A wholesome respect for correct reasoning and precise definitions.

_____5. An appreciation for the aesthetic values of mathematics.

Fill in this space with any objectives which, in your opinion, would improve the given set. Use the same scale for evaluating them.

_____ ____

____

_____ ____

____

_____ ____

____

_____ ____

____

APPENDIX D

A LIST OF 10 TEXTBOOKS REVIEWED

FOR CONTENT

Author (s)	Title	Publisher and Copyright
Altwerger, Samuel I.	Modern Mathematics	Macmillan New York, 1960
Brumfiel, Charles F., R. R. Eicholz and M. E. Shanks	Fundamental Concepts of Elementary Mathematics	Wiley New York, 1962
Fruend, John E.	A Modern Introduction to Mathematics	Prentice-Hall Englewood Cliffs 1959
Hafstrom, John E.	Basic Concepts in Modern Mathematics	Addison-Wesley Reading, Mass., 1963
May, Kenneth O.	Elements of Modern Mathematics	Addison-Wesley Reading, Mass., 1959
Sachs, Rasmusen, Purcell	College Mathematics	Allyn and Bacon Boston, 1960
Schaaf, William L.	Elementary Mathematics	Wiley New York, 1962
Wade, Thomas L. and H.E. Taylor	Fundamental Mathematics	McGraw-Hill New York, 1961
Webber, G.C. and J. A. Brown	Basic Concepts of Mathematics	Addison-Wesley Reading, Mass., 1963
Western, Donald W. and V.H. Haag	An Introduction to Mathematics	Holt New York, 1959

APPENDIX E

AN OUTLINE OF CONTENT INCLUDED IN THE CHART OF SPECIFICATIONS FOR A TEST OF COLLEGE MATHEMATICS FOR GENERAL EDUCATION

I. Numeration

- Decimal System; place value in base 10
- Nondecimal systems; bases other than 10
- Binary and duodecimal numeration
- Change of base
- Numerals versus numbers

II. The Natural Numbers

- Operations for natural numbers
- Closure
- The commutative laws
- The associative laws
- The distributive laws
- Multiplicative identity

III. The Positive and Negative Integers and Zero

- Role of zero
- Inverse operations
- Division with zero
- Operations for integers
- Order of the integers
- Properties of the integers
- Factors and prime numbers
- Greatest common divisor
- Least common multiple
- Division algorithm

IV. Rational Numbers

- Rational numbers; an extension of the integers
- Addition of rational numbers
- Multiplication of rational numbers
- Reciprocal and division

V. Real Numbers

- Existence of irrational numbers
- The real number line
- Repeating and non-repeating, nonterminating decimals
- Operations with real numbers
- Properties of the operations
- Absolute value
- Inequality
- Powers and roots
- Logarithms
- Scientific notation
- Denumerability of the rational numbers
- Non-denumerability of the real numbers

VI. Complex Numbers

- The imaginary number
- Complex numbers: an extension of the real numbers
- Operations with complex numbers

VII. Types of Reasoning

- By analogy
- By induction
- By deduction
- Distinction between 'valid' and 'true'

VIII. Symbolic Logic

- Conjunctions
- Disjunctions
- Negations
- Conditional sentences
- Universally true sentences
- Truth tables for more complicated sentences
- Tautologies
- Logical inference
- Syllogism
- Contrapositives
- Applications of methods of proof
- Mathematical induction

IX. Sets and Variables

- The set concept
- Finite sets
- Infinite sets
- One-to-one correspondence
- Subsets
- Venn diagrams
- Set union and intersection
- Properties of the operations on sets
- Boolean algebra: some applications
- Variables
- Equations
- Inequalities
- Solution sets

X. Relations, Functions and Graphs

- Some common types of relations
- Relation defined
- Set notations for relations
- Equivalence relations
- Inverse of a relation
- Graphs of relations
- Relations and graphs in the set of real numbers
- Functions: a special class of relations
- Linear functions
- Polynomial functions

XI. Abstract Mathematical Systems

- Groups
- Fields
- Structure of a Mathematical System

APPENDIX F

A TEST OF COLLEGE MATHEMATICS FOR GENERAL EDUCATION

FORM A

Directions: Each question below is followed by five choices, only one of which is the correct answer. Find the correct answer and record it on the separate answer sheet. Make your answer marks heavy and black. If you change your mind about an answer, be sure to erase your first mark completely. Your score will be the total number of right answers.

Be sure to print your name on the answer sheet and indicate which form (A or B) of the test has been given to you.

1. What is the standard set which can be used to tell us 'how many' objects there are in any given set?

   (1) The set of whole numbers
   (2) The set of natural numbers
   (3) The set of all numerals
   (4) The set of rational numbers
   (5) The set of real numbers

2. The statement, "For each real number x, 3x + 5x = 8x"

   (1) is false because (3)(1) + (5)(2) ≠ (8)(3).
   (2) is false because (3)(2) + (5)(2) ≠ (8)(4).
   (3) is true because (3)(2) + (5)(2) = (8)(2).
   (4) is true because of the distributive law and 3 + 5 = 8.
   (5) is true because of the commutative law and 3 + 5 = 8.

3. Which of the following is an example of the associative law for additio

   (1) (2 + 3) + 5 = 2 + (3 +5)
   (2) (2 + 3)5 = 2 + (3)(5)
   (3) (2 + 3)5 = (2)(5) + (3)(5)
   (4) 2 + 3 = 3 + 2
   (5) none of these

4. How can we tell when a set is closed under addition?

   (1) It is closed when, in adding the first two elements, we get another element in the set.

(2) It is closed when the order in which addition is performed makes no difference.
(3) It is closed when the set possesses an identity element for addition
(4) It is closed when the sum of any two elements of the set is an eleme of the set.
(5) None of these answers is correct.

5. Which of the following statements is true about the symbol "1/0"?

(1) 1/0 = 1 because the numerator is 1 and the denominator is nothing.
(2) 1/0 = 0 because any number divided by 0 is 0.
(3) 1/0 is undefined.
(4) 1/0 is infinity.
(5) 1/0 is an arbitrary constant.

6. Let a and b represent two counting numbers. If the greatest common fact of a and b is 1, what is the least common multiple of a and b?

(1) Either a or b, whichever is greater.
(2) The product of a and b.
(3) Either a or b, whichever is a prime number.
(4) Either a or b, whichever is a composite number.
(5) None of these.

7. Which of the following sets is closed with respect to addition?

(1) The set of odd integers.
(2) The set of even integers.
(3) The set of integers from 0 to 10 inclusive.
(4) The set of integers from 1 to 10 inclusive.
(5) None of these.

8. When fractions were added to the whole numbers to form the set of ration numbers, what new property was added to the number system?

(1) The distributive property through substraction.
(2) The commutative property of division.
(3) Closure under division.
(4) The associative property of division.
(5) None of these.

9. Can every rational number be expressed in decimal notation?

(1) No. Some cannot.
(2) Yes. Every rational number can be expressed as a decimal numeral containing a limited number of digits.
(3) Yes. Every rational number can be expressed as a decimal numeral though some will contain an unlimited number of digits in random arrangement.
(4) Yes. Every rational number can be expressed as a decimal numeral which either repeats a single digit or a block of digits over and over again.
(5) None of these.

10. $-3/2 + 6/-7 =$

(1) 33/14 (2) -9/14 (3) -33/-14 (4) -9/-14 (5) None of these

11. The number $\sqrt{7}$ belongs to

(1) the set of natural numbers.
(2) the set of rational numbers, but not to the set of natural numbers
(3) the set of real numbers, but not to the set of rational numbers.
(4) the set of complex numbers, but not to the set of real numbers.
(5) none of the above sets.

12. 162,800 written in scientific notation is

(1) 1628×10^2 (2) $.1628 \times 10^6$ (3) 1.628×10^5
(4) 162.8×10^3 (5) 1.628×10^{-5}

13. $\log a + 2 \log b =$

(1) $2 \log ab$ (2) $\log 2ab$ (3) $\log ab^2$ (4) $\log (a^2b^2)$
(5) None of these

14. $\log_2 8 =$

(1) 16 (2) 4 (3) 6 (4) 3 (5) none of these

15. $4^{1/2} + 4^o + 4^{-1/2} + 4^2 =$

(1) 16 (2) 19 1/2 (3) 4 (4) 4 1/2 (5) none of these

16. The expression $(4^{1/3})^{3/2}$ is equal to

(1) 1/16 (2) 2 (3) -4 (4) 16 (5) none of these

17. The rational number 3.1414... is equivalent to

(1) 311/99 (2) π (3) 3.14/100 (4) 314/99 (5) none of these

18. The number 135_{10} would be written to base 4 as

(1) 2011_4 (2) 211_4 (3) 2013_4 (4) 214_4 (5) none of these

19. The binary number 1101 is equal to

(1) 8 in the decimal system.
(2) 13 in the decimal system.
(3) 550.5 in the decimal system.
(4) 2202 in the decimal system.
(5) none of these.

20. In what number base does 2(4 + 3) = 22?

(1) 4 (2) 5 (3) 6 (4) 7 (5) none of these

21. Are all words in a mathematical system defined?

(1) Yes, because mathematics is a science.
(2) Yes, because this is a distinguishing characteristic of mathematics
(3) No, because then we would have circular definitions.
(4) No, because some words are so simple they would be difficult to def but not impossible.
(5) It depends on the particular mathematical system.

22. In mathematics, axioms are never proved because

(1) the proofs are obvious.
(2) the proofs are difficult but the axioms are obvious.
(3) they are not true.
(4) it is impossible to prove them within the mathematical system.
(5) no other assumptions could be axioms.

23. Given the set A = {a, b, c} and a single operation * subject to the table on the right, which of the following statements is true about A and the given operation?

*	a	b	c
a	a	a	c
b	b	b	c
c	c	c	c

(1) It is a group.
(2) It is not a group because it does not contain a unique identity element.
(3) It is not a group because it does not have closure under *.
(4) It is not a group because it is not commutative under *.
(5) It is not a group because it is impossible to tell what the elements a, b, c represent.

24. If p and q are any statements and if p implies q, then

(1) p may be false and q true.
(2) either p and q are both true or else both are false.
(3) p may be true and q false.
(4) both p and q are always true.
(5) none of the above statements is true.

25. If all squares are rectangles and all rectangles are parallelograms, then

(1) if ABCD is not a rectangle, then ABCD is not a parallelogram.
(2) if ABCD is not a square, then ABCD is not a parallelogram.
(3) if ABCD is a parallelogram, then ABCD is a square.
(4) if ABCD is not a parallelogram, then ABCD is not a square.
(5) none of the above statements is true.

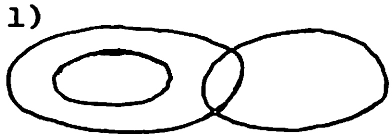
26. The contrapositive of the statement "If I am elected, you will have more playgrounds" is:

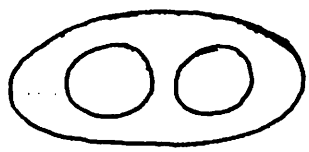
(1) If you will have more playgrounds then I will have been elected.
(2) If I am not elected, you will not have more playgrounds.
(3) If you do not have more playgrounds, I will have been defeated.
(4) If and only if I am elected will you have more playgrounds.
(5) None of these.

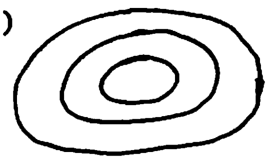
27. Given the following hypotheses: "Some students are not wealthy. All Pipers are wealthy." Which of the following conclusions is valid?

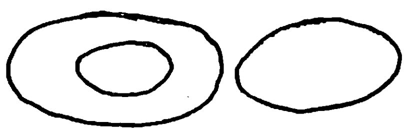
(1) Some students are not Pipers.
(2) Some Pipers (assuming there are Pipers) are not students.
(3) No Pipers are students.
(4) The conclusions (1), (2), (3) are all valid.
(5) The conclusions (1), (2), (3) are all invalid.

28. For the following argument select the diagram below which corresponds to the argument: "All voters are citizens, and no animals are citizens Therefore, no animals are voters."

(1)


(2)


(3)


(4)


(5)


29. Which of the following statements is equivalent to the statement "If a Mathematics 123 student studies, then he will pass?"

(1) If a Math 123 student passes, then he has studied.
(2) Only if a Math 123 student studies will he pass.
(3) To pass Math 123, it is necessary that a student study.
(4) Studying is a sufficient condition for a student to pass Math 123.
(5) None of these.

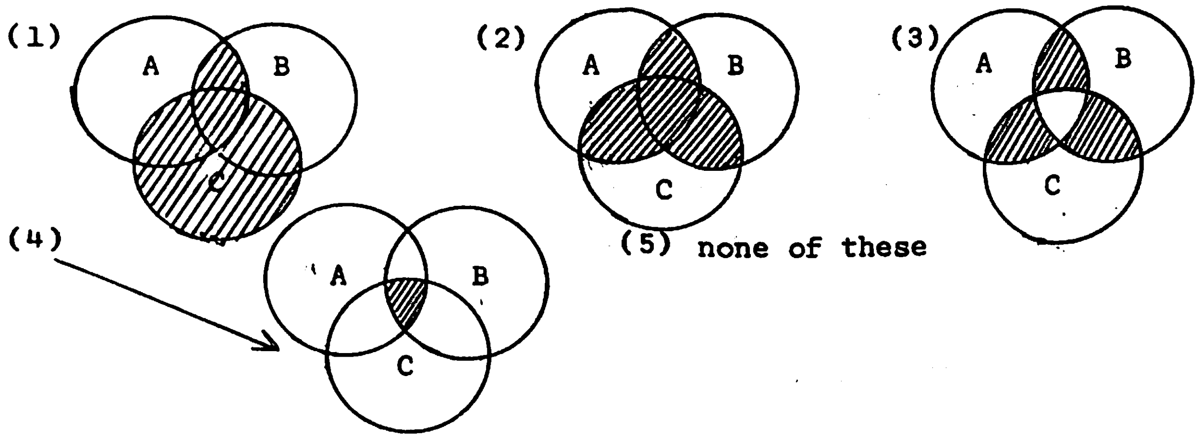
30. Let the symbol p mean "I am over 21 years old," and the symbol q mean "I can vote." Which of the following symbolic statements says "If I cannot vote, then I am not 21 years old"?

(1) $p \vee q$ (2) $\sim p \rightarrow \sim q$ (3) $\sim q \rightarrow \sim p$ (4) $\sim [p \wedge q]$ (5) none of thes

31. The statement "All brokers are honest" can be expressed algebraically by: (Note: $\bar{H}$ means "not honest")

(1) $B \cup H = \emptyset$ (2) $B \cap \bar{H} = \emptyset$ (3) $B \cap \bar{H} \neq \emptyset$ (4) $B \cap H \neq B$
(5) none of these

32. Which of the following Venn diagrams indicate $(A \cap B) \cup C$?



(5) none of these

33. Which of the following sets is finite?

(1) The set of rational numbers.
(2) The set of integers.
(3) The set of irrational numbers.
(4) The set of even integers.
(5) None of these.

34. If a set $A = \{1, 2, 3\}$ and $B = \{5, 6, 7\}$, then $A \cap B =$

(1) $\{3, 4, 5\}$ (2) $\{1, 2, 3, 5, 6, 7\}$ (3) $\emptyset$

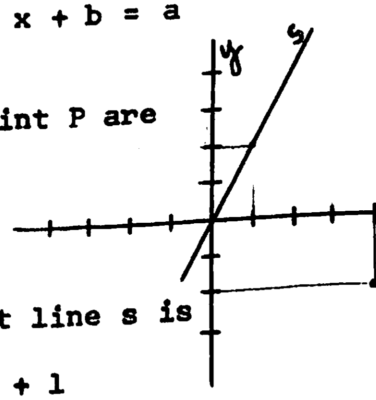
(4) $\{4\}$ (5) none of these

35. If $3/7 = (2x-1)/(x+5)$, then

(1) $x = 38/15$ (2) $x = 22/11$ (3) $x = 8/11$ (4) $x = -22/11$
(5) none of these

36. $a - b = x$ if and only if

(1) $b-a = x$ (2) $x - b = a$ (3) $x - a = b$ (4) $x + b = a$
(5) none of these



37. In the diagram on the right the coordinates of point P are

(1) (2,4) (2) (4,2) (3) (4,-2)

(4) (-2,-4) (5) (-2,4)

38. In the diagram above, the equation of the straight line s is

(1) $y = x$ (2) $y = 2x$ (3) $y = 1/2\ x$ (4) $y = x + 1$

(5) none of these

39. If a function is defined as a set of ordered pairs such that for each first component there is exactly one second component, which of the following sets is a function?

(1) The set containing (1,1), (1,-1), (4,2), (4,-2)
(2) The set containing (1,4), (2,7), (2,5)
(3) The set containing (2,2), (3,3), (4,2), (5,3)
(4) The set containing (2,3), (2,4), (1,5)

40. Using the definition of function given in question 39, which of the following equations has a solution set which is a function?

(1) $y = 3x^2 + 2x + 5$ (2) $x^2 + y^2 = 9$ (3) $x^2y^2 = 1$
(4) $x^2 + y^2 = 4$ (5) none of these

41. If $f(x) = 2x^2 - 3x + 4$ then $f(3) =$

(1) 31 (2) 13 (3) -6 (4) 24 (5) none of these

42. If $f(x) = 2x^2 - 3x + 4$ and $g(x) = 2x - 3$, then $g(f(3)) =$

(1) 3 (2) 5 (3) 59 (4) 24 (5) 23

43. Euler's formula for the regular polyhedra is $V - E + F = 2$. If a regular polyhedron has 4 faces and 8 edges, how many vertices does it have?

(1) 6 (2) 14 (3) 10 (4) 12 (5) none of these

44. Given the statement: "A pound of butter costs 10 cents more than a dozen eggs." If we let B = cost of a pound of butter in cents and E = cost of a dozen eggs in cents, which mathematical statement below is equivalent to the given statement?

(1) $E + 10 = B$ (2) $12E + 10 = B$ (3) $B + 10 = E$
(4) $B + 10 = 12E$ (5) none of these

45. The relation, less then, as applied to real numbers is

(1) symmetric, transitive, and reflextive.
(2) symmetric and transitive, but not reflexive.
(3) symmetric, but neither transitive nor reflexive.
(4) transitive, but neither symmetric nor reflexive.
(5) non-reflexive, non-symmetric, and non-transitive.

46. If x is a real number, the solution set of the inequality, $x^2 < 0$, consists of

(1) all real numbers less than zero.
(2) all positive real numbers.
(3) $\{0\}$.
(4) all non-negative real numbers.
(5) the empty set.

47. Which number is a solution of the quadratic equation: $x^2 + 4x + 5 = 0$?

(1) $2 + i$ (2) $1 + 2i$ (3) $-2 - i$ (4) $-1 + 2i$ (5) none of these

48. Find the set of real numbers which will make the following compound statement true. $\{x \mid 1 - 2x < 9\} \cap \{x \mid 3x = 1\}$

(1) The empty set
(2) All x less than - 4
(3) $\{1/3\}$
(4) All x greater than 4
(5) None of these

49. Given the set of objects {△, □} and the operation * defined by the table on the right, evaluate the statements (a), (b), and (c).
(a) The identity element is △.
(b) □ is its own inverse.
(c) The operation * is commutative.

*	△	□
△	△	□
□	□	△

(1) (a) is true, but (b) and (c) are false.
(2) (b) is true, but (a) and (c) are false.
(3) (c) is true, but (a) and (b) are false.
(4) (a) and (b) are true, but (c) is false.
(5) (a), (b), and (c) are all true.

50. The equation of the line passing through the points (1,0) and (2,1) is

(1) $x + y = 1$ (2) $2x - y = 2$ (3) $x - y = 1$

(4) $x = y$ (5) none of these

APPENDIX G

A TEST OF MATHEMATICS FOR GENERAL EDUCATION

FORM B

Directions: Each question below is followed by five choices, only one of which is the correct answer. Find the correct answer and record it on the separate answer sheet. Make your answer marks heavy and black. If you change your mind about an answer, be sure to erase the first mark completely. Your score will be the total number of right answers.

Be sure to print your name on the answer sheet and indicate which form (A or B) of the test has been given to you.

1. Which of the following is an example of the distributive law?

   (1) $(2 + 3) + 5 = 2 + (3 + 5)$
   (2) $(2)(5) + 3 = (2 + 3) + (5 + 3)$
   (3) $(2 + 3)5 = (2)(5) + (3)(5)$
   (4) $2 + 3 = 3 + 2$
   (5) none of these

2. Which pair of activities, if any, is commutative?

   (1) To comb your hair and put on your hat.
   (2) To pour hot water into a cup and to put a tea bag into the cup.
   (3) To study for an exam and to take the exam.
   (4) To change into a swimming suit and to dive into the pool.
   (5) None of these.

3. Two sets have the same cardinal number

   (1) if they are both infinite sets.
   (2) if they have the same ordinal number.
   (3) if their elements can be placed in one-to-one correspondence.
   (4) only if they are not infinite.
   (5) if they are both countable.

4. In a proof appears the following step: $(x \cdot a)b = x(a \cdot b)$. What is the justification for this step?

   (1) The commutative law for multiplication
   (2) The associative law for addition
   (3) The commutative law for addition
   (4) The associative law for multiplication
   (5) None of these

5. Which of the following laws is illustrated by the statement, "If $a \cdot 3 = 7 \cdot 3$ then $a = 7$"?

(1) Associative law for multiplication
(2) Commutative law for multiplication
(3) Distributive law
(4) Cancellation law for multiplication
(5) None of these

6. Why can we not divide six by zero?

(1) Because there is no number n such that $6 \cdot n = 0$.
(2) Because there is no number n such that $n \cdot 0 = 6$.
(3) Because any number multiplied by zero equals zero.
(4) We can divide six by zero; the result is zero.
(5) None of these answers is correct.

7. Which of the following sets is not closed under multiplication?

(1) 0, 2, 4, 6, 8, 10, 12, ...
(2) -2, -4, -6, -8, -10, -12, ...
(3) 1, 3, 5, 7, 9, 11, ...
(4) 3, 6, 9, 12, 15, 18, ...
(5) All of these sets are closed under multiplication.

8. If a, b, and c are numbers such that $a - b = c$ and if a is negative and b is positive, then

(1) c is negative.
(2) c is positive.
(3) c is zero.
(4) c is positive or zero.
(5) it is impossible to tell whether c is positive or negative.

9. Suppose the greatest common factor of two numbers is the same as the least common multiple. What must be true about the numbers?

(1) They must be the same number.
(2) They must be 1.
(3) They must be the same prime number.
(4) They must be composite numbers.
(5) Such numbers do not exist.

10. If the ordered pairs (a,b) and (c,d) represent rational numbers, then (a,b) + (c,d) =

(1) (ac,bd)
(2) (a + c, b + d)
(3) (ad + bc, bd)
(4) (ac - bd, ad + bc)
(5) none of these

11. The repeating decimal 0.575757... belongs to

(1) the set of natural numbers.
(2) the set of real numbers, but not to the set of rational numbers.
(3) the set of complex numbers but not to the set of real numbers.
(4) the set of rational numbers, but not to the set of natural numbers
(5) none of these.

12. $(1/x + 1/y)^{-1} =$

(1) $(x+y)/xy$ (2) $x + y$ (3) $xy/(x+y)$ (4) $(x^2+y^2)/xy$
(5) none of these.

13. Which of the following numbers is rational?

(1) $2\sqrt{5} - \sqrt{5}$ (2) $2\sqrt{2}$ (3) $(5+\sqrt{2})(5-\sqrt{2})$ (4) $2 + \pi$
(5) none of these.

14. In scientific notation, 0.000053 is written

(1) 5.3×10^5 (2) 5.3×10^{-5} (3) 53×10^{-6} (4) 530000
(5) none of these.

15. Logarithms may be used to

(1) shorten computation by changing a multiplication problem to a problem of addition.
(2) add a column of figures.
(3) find the length of a side of a right triangle since they give the ratio of sides in a right triangle.
(4) substract two numbers.
(5) add two numbers.

16. If $\log_{10}3 = 1.099$, then $\log_{10}9 =$

(1) 3.298 (2) 2.198 (3) 0.198 (4) 9.099 (5) none of these

17. The expression $(-8)^{2/3}$ is equal to

(1) 2 (2) -2 (3) 1/2 (4) -1/2 (5) none of these

18. The expression $b^x b^y$ is equal to

(1) b^{xy} (2) b^{x+y} (3) b^{x-y} (4) $(b^2)^{xy}$ (5) bxy

19. Which of the following is a non-denumerable set?

(1) The set of integers.
(2) The set of even numbers.
(3) The set of rational numbers.
(4) The set of real numbers.
(5) None of these.

20. In the diagram you have a set of x's. How would this number of x's be expressed in the base 8 system?

(1) 40 (2) 54 (3) 44

(4) 5.5 (5) none of these

X X X X X X X X X X X
X X X X X X X X X X X
X X X X X X X X X X X
X X X X X X X X X X X

21. In base two the number 10110 is equal to the following number written in base ten:

(1) 1011 (2) 222 (3) 20 (4) 22 (5) none of these

22. In base six arithmetic, 324 + 205 + 143 =

(1) 1120 (2) 672 (3) 3040 (4) 1110 (5) none of these

23. Express in simplest form the number $-\sqrt{-9}$.

(1) 3 (2) -9i (3) -3i (4) 3i (5) none of these

24. It is not necessary for a mathematical science to contain

(1) undefined words (2) defined words (3) numbers
(4) postulates (5) theorems

25. For the mathematical system consisting of four elements, a, b, c, d, and a single operation subject to the table on the right, the identity element is

(1) a (2) b (3) c (4) d

(5) 1

•	a	b	c	d
a	c	d	b	a
b	d	c	a	b
c	b	a	d	c
d	a	b	c	d

26. For the mathematical system of question 25 the inverse of b is

(1) a (2) b (3) c (4) d (5) -b

27. The statement, "For each real number x, 3x + 6 = 9x"

(1) is false because $(3)(2) + 6 \neq (9)(4)$.
(2) is false because $(3)(5) + 6 \neq (9)(5)$.
(3) is true because $(3)(1) + 6 = (9)(1)$.
(4) is true because of the distributive law and 3 + 6 = 9.
(5) is true because of the associative law and 3 + 6 = 9.

28. Given: "If it is a fish, then is is a vertebrate." Which of the following statements is equivalent to the given one?

(1) If it is a vertebrate then it is a fish.
(2) If it is not a vertebrate then it is not a fish.

(3) If it is not a fish it is not a vertebrate.
(4) None of the statements (1), (2), (3) is equivalent to the given one.
(5) All of the statements (1), (2), (3), are equivalent to the given one.

29. Any implication is equivalent to

(1) its converse.
(2) its opposite.
(3) its contrapositive.
(4) its inverse.
(5) none of these.

30. Which of the following statements (implications) is false?

(1) If triangles are squares, then monkeys are birds.
(2) If triangles are polygons, then 1 = 2.
(3) If wishes are horses, then the moon is made of cheese.
(4) Statements (1), (2), (3) are false.
(5) All three statements are true.

31. Which of the following compound sentences is true?

(1) July has 30 days or Christmas is December 25.
(2) The moon is larger than the earth and rabbits multiply rapidly.
(3) The first prime number is 4, or 8 is not a multiple of 2.
(4) The statements (1), (2), (3) are all true.
(5) The statements (1), (2), (3) are all false.

32. Given the hypotheses: No triangles are circles. All fortins are triangles. All ecktars are circles.
Which of the following is a valid conclusion?

(1) No fortins are ecktars.
(2) Some ecktars are not fortins.
(3) No fortins are circles.
(4) All are valid.
(5) All are invalid.

33. If all planets are heavenly bodies and the orbits of all planets are ellipses, then

(1) the orbits of all heavenly bodies are ellipses.
(2) if Ceres is a planet, then Ceres has an orbit which is an ellipse.
(3) if Ceres has an orbit which is an ellipse, then Ceres is a planet.
(4) all heavenly bodies are planets.
(5) none of the above statements is correct.

34. In 500 cases that we tried, treatment T cured disease D. From this data we assume that treatment T will always cure disease D. Which of the following statements is correct.

(1) The above agrument is deductive in nature.
(2) The above argument is inductive in nature.
(3) By trying 500 cases we proved that treatment T will always cure disease D.
(4) Statements (1) and (3) are true.
(5) Statements (2) and (3) are true.

35. If set A = $\{1, 2, 3\}$, set B = $\{3, 4, 5\}$, set C = $\{5, 6, 7\}$, then $(A \cap C) \cup B$ =

(1) $\{3, 4, 5\}$ (2) $\{5, 6, 7\}$ (3) $\{1, 2\ 3\}$
(4) $\emptyset$ (5) none of these

36. If S_1 is the set of even integers, and S_2 is the set of multiples of three, then the set of integers common to both S_1 and S_2

(1) is empty. (2) contains all even integers. (3) does not contain any even integers. (4) contains all the multiples of 3. (5) contains all the multiples of 6.

37. If $2x - 5k = kx - 1$, then

(1) $x = (5k-1)/(2-k)$ (2) $x = 1/5k - 2/k$ (3) $x = 3$ (4) $x = 4$
(5) none of these

38. Which of the following quadratic equations has $\{-3, 7\}$ as a solution set?

(1) $x^2 + 4x + 21 = 0$ (2) $x^2 + 4x - 21 = 0$ (3) $x^2 - 4x - 21 = 0$
(4) $x^2 - 4x + 21 = 0$ (5) $x^2 + 2x - 12 = 0$

39. If the universe is $\{0, 1/2, 1, 3, 4, 5, 9, 12\}$ what set of values of x will make the open (conditional) statement "$2x < 8$" true?

(1) The set of all real numbers less than 4.
(2) The set of all real numbers less than 8.
(3) $\{0, 1/2, 1, 3\}$.
(4) $\{0, 1/2, 1, 3, 4\}$.
(5) $\{0, 1/2, 1, 3, 4, 5\}$.

40. Which of the following ordered pairs (x,y) represents the point of intersection of the graphs of $5x + 3y = 7$ and $9x - y = 3$?

(1) (-4,9) (2) (1,6) (3) (-1,4) (4) (1/2, 3/2) (5) none of thes

41. Given the function described by the graph below, the set of numbers $\{1, 2, 3, 4, 5\}$ is called the

(1) variable. (2) open sentence. (3) domain.
(4) range. (5) none of these.

42. If $f(x) = 2x^2 - 3x + 4$, then $f(2) =$

(1) 26 (2) 14 (3) 0 (4) -2 (5) 6

43. If $f(x) = 2x^2 - 3x + 4$ and $g(x) = 2x - 3$, then $g(f(2)) =$

(1) 12 (2) 9 (3) 3 (4) 6 (5) 1

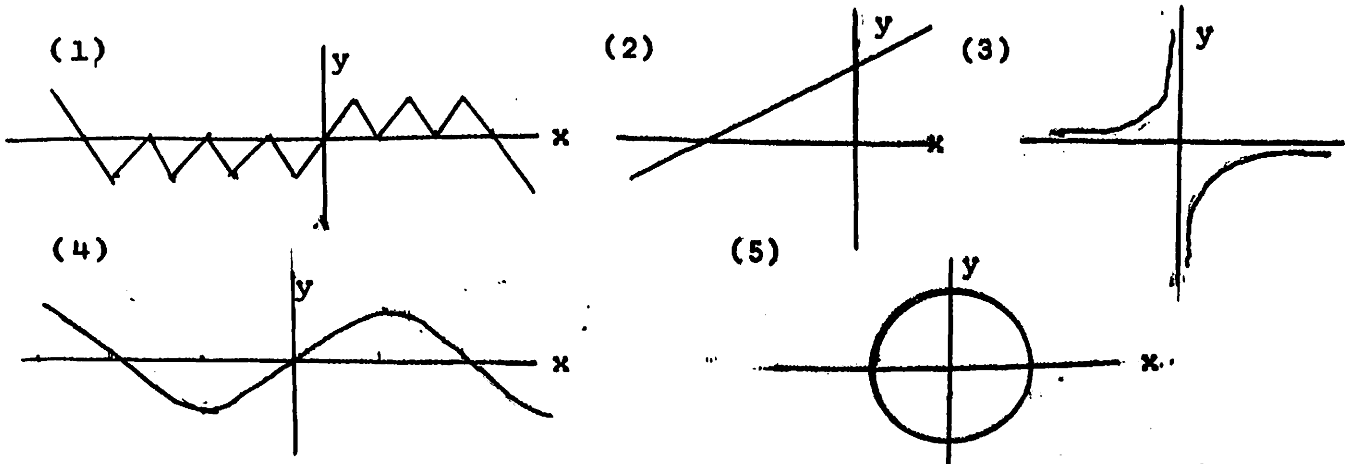
44. If x is a real number, what is the solution set of the open statement $|x| > 3$?

(1) $\{x|\ 0<x<3\}$ (2) $\{x|\ x>3\}$ (3) $\{x|\ -3<x<3\}$
(4) $\{x|\ x>3 \text{ and } x<-3\}$ (5) none of these

45. If f is a function and x a real number such that $f(x) = (2x+3)/(x-4)$, what is the largest possible domain for x which is a subset of real numbers ?

(1) All real numbers x.
(2) All real numbers x except $x = 0$.
(3) All rational numbers x.
(4) All real numbers except $x = 4$.
(5) All real numbers greater than zero.

46. The graphs below represent relations in which the universal set is the set of real numbers. Which of these relations is not a function?



47. Which of the following relations is neither reflexive, symmetric, nor transitive?

(1) is greater than (2) is the father of (3) is heavier than
(4) is not equal to (5) is a multiple of (positive whole numbers)

48. Given the statement, "Two dozen eggs and one pound of butter cost \$2.00." Let B = cost of pound of butter in cents and E = cost of a dozen eggs in cents. Which mathematical statement below is equivalent to the given statement?

(1) 2E + B = \$2.00 (2) 2E + B = 2 (3) 24E + B = \$2.00
(4) 2E + B = 200 (5) none of these

49. Perform the following operation and express the answer in the form of an ordered pair: (4 + 3i)/i(1-2i)

(1) 20i/5 (2) (14/5,6/5) (3) (8/5,3/5) (4) (11,2)
(5) none of these

50. The formula relating the Kelvin and Centigrade scales is K = C + 273.18. A mixture of alcohol and water that contains 39% of alcohol freezes at 244.5 K. What reading does this represent on the Centigrade scale?

(1) -67.7 C (2) -28.7 C (3) 10.3 C (4) 517.7 C
(5) none of these

APPENDIX H

A TEST OF COLLEGE MATHEMATICS FOR GENERAL EDUCATION

Directions: Each question below is followed by five choices, only one of which is the correct answer. Find the correct answer and record it on the separate answer sheet. Use either pen or pencil (preferably pencil for ease in erasing). Make your answer marks dark and so that they completely fill the space between the dotted lines. If you change your mind about an answer be sure to erase your first mark. (If you are using a pen place a cross X over the first mark). Your score will be the total number of right answers.

1. How can we tell when a set is closed under addition?

    (1) It is closed when, in adding the first two elements, we get another element in the set.
    (2) It is closed when the order in which addition is performed makes no difference.
    (3) It is closed when the set possesses an identity element for addition.
    (4) It is closed when the sum of any two elements of the set is an element of the set.
    (5) None of these answers is correct.

2. Which of the following is an example of the distributive law?

    (1) $(2 + 3) + 5 = 2 + (3 + 5)$
    (2) $(2)(5) + 3 = (2 + 3) + (5 + 3)$
    (3) $(2 + 3)5 = (2)(5) + (3)(5)$
    (4) $2 + 3 = 3 + 2$
    (5) none of these

3. Which pair of activities, if any, is commutative?

    (1) To comb your hair and put on your hat.
    (2) To pour hot water into a cup and to put a tea bag into the cup.
    (3) To study for an exam and to take the exam.
    (4) To change into a swimming suit and to dive into the pool.
    (5) None of these.

4. Let a and b represent two counting numbers. If the greatest common factor of a and b is 1, what is the least common multiple of a and b?

    (1) Either a or b, whichever is greater.

(2) The product of a and b.
(3) Either a or b, whichever is a prime number.
(4) Either a or b, whichever is a composite number.
(5) None of these.

5. In base two the number 10110 is equal to the following number written in base ten:

(1) 1011 (2) 222 (3) 20 (4) 22 (5) none of these

6. The number 135_{10} would be written to base 4 as

(1) 2011_4 (2) 211_4 (3) 2013_4 (4) 214_4 (5) none of these

7. In what number base does 2(4 + 3) = 22?

(1) 4 (2) 5 (3) 6 (4) 7 (5) none of these

8. In a proof appears the following step: $(x \cdot a)b = x(a \cdot b)$. What is the justification for this step?

(1) The commutative law for multiplication.
(2) The associative law for addition.
(3) The commutative law for addition.
(4) The associative law for multiplication.
(5) None of these.

9. Why can we not divide six by zero?

(1) Because there is no number n such that $6 \cdot n = 0$.
(2) Because there is no number n such that $n \cdot 0 = 6$.
(3) Because any number multiplied by zero equals zero.
(4) We can divide six by zero; the result is zero.
(5) None of these answers is correct.

10. Which of the following sets is not closed under multiplication?

(1) 0, 2, 4, 6, 8, 10, 12,...
(2) -2, -4, -6, -8, -10, -12,...
(3) 1, 3, 5, 7, 9, 11, ...
(4) 3, 6, 9, 12, 15, 18,...
(5) All of these sets are closed under multiplication.

11. Can every rational number be expressed in decimal notation?

(1) No. Some cannot.
(2) Yes. Every rational number can be expressed as a decimal numeral containing a limited number of digits.
(3) Yes. Every rational number can be expressed as a decimal numeral though some will contain an unlimited number of digits in random arrangement.
(4) Yes. Every rational number can be expressed as a decimal numeral which either repeats a single digit or a block of digits over and over again.
(5) None of these.

12. Which of the following numbers is rational?

(1) $2\sqrt{5} - \sqrt{5}$ (2) $2\sqrt{2}$ (3) $(5 + \sqrt{2})(5 - \sqrt{2})$ (4) $2 + \pi$
(5) None of these.

13. When fractions were added to the whole numbers to form the set of rational numbers, what new property was added to the number system?

(1) The distributive property through subtraction.
(2) The commutative property of division.
(3) Closure under division.
(4) The associative property of division.
(5) None of these.

14. If the ordered pairs (a,b) and (c,d) represent the rational numbers a/b and c/d, then (a,b) + (c,d) =

(1) (ac,bd) (2) (a+c, b+d) (3) (ad+bc, bd) (4) (ac - bd, ad + bc)
(5) None of these.

15. The expression $b^x b^y$ is equal to

(1) b^{xy} (2) b^{x+y} (3) b^{x-y} (4) $(b^2)^{xy}$ (5) bxy

16. a - b = x if and only if

(1) b-a = x (2) x - b = a (3) x - a = b (4) x + b = a
(5) none of these

17. log a + 2 log b =

(1) 2 log ab (2) log 2 ab (3) log ab^2 (4) log (a^2b^2)
(5) none of these

18. 162,800 written in scientific notation is

(1) 1628×10^2 (2) $.1628 \times 10^6$ (3) 1.628×10^5
(4) 162.8×10^3 (5) 1.628×10^{-5}

19. The expression $(4^{1/3})^{3/2}$ is equal to

(1) 1/16 (2) 2 (3) -4 (4) 16 (5) none of these

20. The statement, "For each real number x, 3x + 6 = 9x"

(1) is false because $(3)(2) + 6 \neq (9)(4)$.
(2) is false because $(3)(5) + 6 \neq (9)(5)$.
(3) is true because $(3)(1) + 6 = (9)(1)$.
(4) is true because of the distributive law and 3 + 6 = 9.
(5) is true because of the associative law and 3 + 6 = 9.

21. The number $\sqrt{7}$ belongs to

(1) the set of natural numbers.
(2) the set of rational numbers, but not to the set of natural numbers.
(3) the set of real numbers, but not to the set of rational numbers.
(4) the set of complex numbers, but not to the set of real numbers.
(5) none of the above sets.

22. Which of the following is a non-denumerable (uncountable) set?

(1) The set of integers.
(2) The set of even numbers.
(3) The set of rational numbers.
(4) The set of real numbers.
(5) None of these.

23. Express in simplest form the number $-\sqrt{-9}$.

(1) 3 (2) -9i (3) -3i (4) 3i (5) none of these

24. In 500 cases that we tried, treatment T cured disease D. From this data we assume that treatment T will always cure disease D. Which of the following statements is correct?

(1) The above argument is deductive in nature.
(2) The above argument is inductive in nature.
(3) By trying 500 cases we proved that treatment T will always cure disease D.
(4) Statements (1) and (3) are true.
(5) Statements (2) and (3) are true.

25. Given the following hypotheses: "Some students are not wealthy. All Pipers are wealthy." Which of the following conclusions is valid?

(1) Some students are not Pipers.
(2) Some Pipers (assuming there are Pipers) are not students.
(3) No Pipers are students.
(4) The conclusions (1), (2), (3) are all valid.
(5) The conclusions (1), (2), (3) are all invalid.

26. The contrapositive of the statement "If I am elected, you will have more playgrounds" is:

(1) If you will have more playgrounds then I will have been elected.
(2) If I am not elected, you will not have more playgrounds.
(3) If you do not have more playgrounds, I will have been defeated.
(4) If and only if I am elected will you have more playgrounds.
(5) None of these.

27. Any implication is equivalent to

(1) its converse.
(2) its opposite.

(3) its contrapositive.
(4) its inverse.
(5) none of these.

28. If p and q are any statements and if p implies q, then

(1) p may be false and q true.
(2) either p and q are both true or else both are false.
(3) p may be true and q false.
(4) both p and q are always true.
(5) none of the above statements is true.

29. Let the symbol p mean "I am over 21 years old," and the symbol q mean "I can vote." Which of the following symbolic statements says, "If I cannot vote, then I am not 21 years old."?

(1) $p \vee q$ (2) $\sim p \rightarrow \sim q$ (3) $\sim q \rightarrow \sim p$ (4) $\sim [p \wedge q]$
(5) none of these

30. If all planets are heavenly bodies and the orbits of all planets are ellipses, then

(1) the orbits of all heavenly bodies are ellipses.
(2) if Ceres is a planet, then Ceres has an orbit which is an ellipse.
(3) if Ceres has an orbit which is an ellipse, then Ceres is a planet.
(4) all heavenly bodies are planets.
(5) none of the above statements is correct.

31. Given: "If it is a fish, then it is a vertebrate." Which of the following statements is equivalent to the given one?

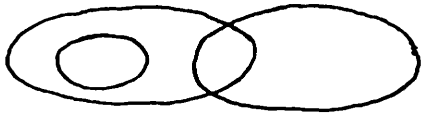
(1) If it is a vertebrate then it is a fish.
(2) If it is not a vertebrate then it is not a fish.
(3) If it is not a fish it is not a vertebrate.
(4) None of the statements (1), (2), (3) is equivalent to the given one.
(5) All of the statements (1), (2), (3) are equivalent to the given one.

32. If the universe is $\{0, 1/2, 1, 3, 4, 5, 9, 12\}$ what set of values of x will make the open (conditional) statement "$2x < 8$" true?

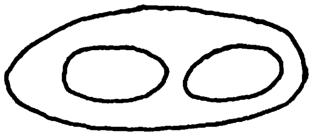
(1) The set of all real numbers less than 4.
(2) The set of all real numbers less than 8.
(3) $\{0, 1/2, 1, 3\}$.
(4) $\{0, 1/2, 1, 3, 4\}$.
(5) $\{0, 1/2, 1, 3, 4, 5\}$.

33. Select the diagram below which corresponds to the argument: "All voters are citizens, and no animals are citizens. Therefore, no animals are voters."

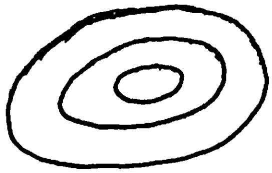
(1)



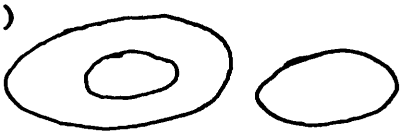
(2)



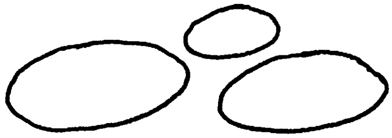
(3)



(4)

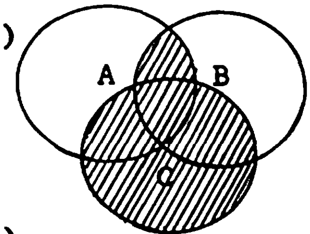


(5)

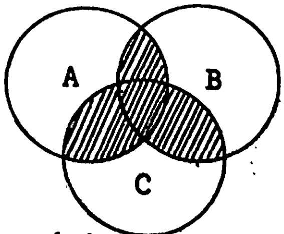


34. Which of the following Venn diagrams indicate $(A \cap B) \cup C$?

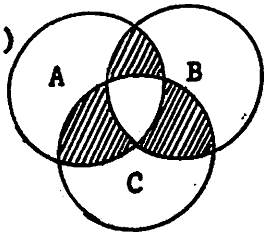
(1)



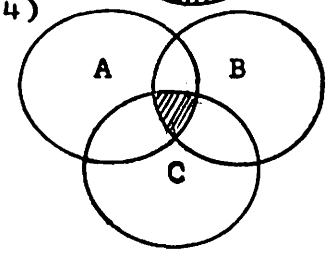
(2)



(3)



(4)



(5) none of these

35. If S_1 is the set of even integers, and S_2 is the set of multiples of three, then the set of integers common to both S_1 and S_2.

(1) is empty. (2) contains all even integers. (3) does not contain any even integers. (4) contains all the multiples of 3. (5) contains all the multiples of 6.

36. Which of the following sets is finite?

(1) The set of rational numbers.
(2) The set of integers.
(3) The set of irrational numbers.
(4) The set of even integers.
(5) None of these.

37. If x is a real number, what is the solution set of the open statement $|x| > 3$?

(1) $\{x \mid 0 < x < 3\}$ (2) $\{x \mid x > 3\}$ (3) $\{x \mid -3 < x < 3\}$
(4) $\{x \mid x > 3 \text{ and } x < -3\}$ (5) none of these

38. The statement: "All brokers are honest" can be expressed algebraically by: (Note: $\bar{H}$ means "not honest").

(1) $B \cap H = \emptyset$ (2) $B \cap \bar{H} = \emptyset$ (3) $B \cap \bar{\bar{H}} \neq \emptyset$ (4) $B \cap H \neq B$
(5) none of these

39. If x is a real number, the solution set of the inequality, $x^2 < 0$, consists of

(1) all real numbers less than zero.
(2) all positive real numbers.
(3) $\{0\}$.
(4) all non-negative real numbers.
(5) the empty set.

40. The equation of the line passing through the points (1,0) and (2,1) is

(1) $x + y = 1$ (2) $2x - y = 2$ (3) $x - y = 1$
(4) $x = y$ (5) none of these

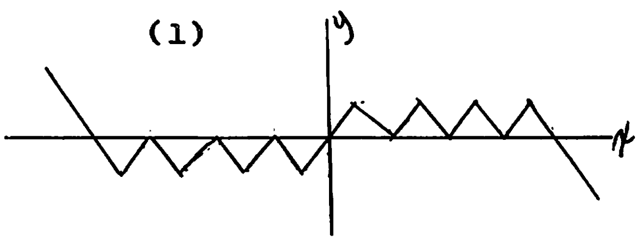
41. Euler's formula for the regular polyhedra is $V - E + F = 2$. If a regular polyhedron has 4 faces and 8 edges, how many vertices does it have?

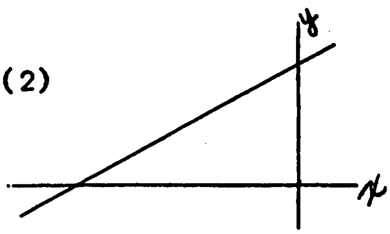
(1) 6 (2) 14 (3) 10 (4) 12 (5) none of these

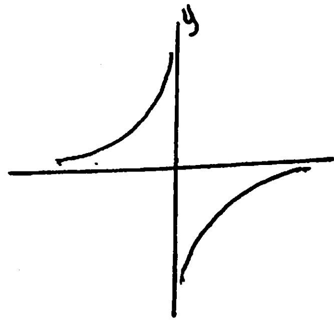
42. If a function is defined as a set of ordered pairs such that for each first component there is exactly one second component, which of the following sets is a function?

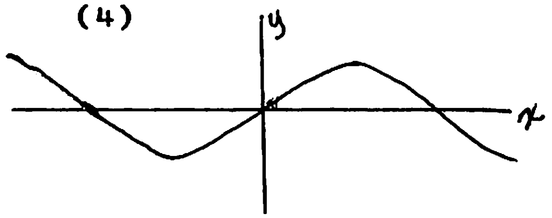
(1) the set containing (1,1), (1,-1), (4,2), (4,-2)
(2) the set containing (1,4), (2,7), (2,5)
(3) the set containing (2,2), (3,3), (4,2), (5,3)
(4) the set containing (2,3), (2,4), (1,5)

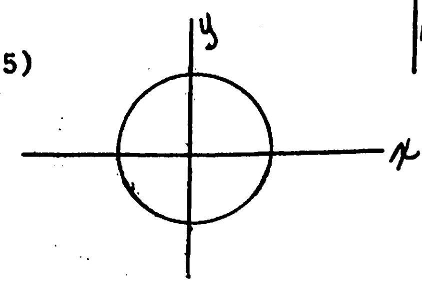
43. The graphs below represent relations in which the universal set is the set of real numbers. Which of these relations is not a function?

(1)


(2)


(3)


(4)


(5)


44. The relation, less than, as applied to real numbers is

(1) symmetric, transitive, and reflexive.
(2) symmetric and transitive, but not reflexive.
(3) symmetric, but neither transitive nor reflexive.
(4) transitive, but neither symmetric nor reflexive.
(5) non-reflexive, non-symmetric, and non-transitive.

45. If $f(x) = 2x^2 - 3x + 4$ and $g(x) = 2x - 3$, then $g(f(3)) =$

(1) 3 (2) 5 (3) 59 (4) 24 (5) 23

46. If f is a function and x a real number such that $f(x) = (2x-3)/(x-4)$, what is the largest possible domain for x which is a subset of real numbers.

(1) All real numbers x.
(2) All real numbers x except $x = 0$.
(3) All rational numbers x.
(4) All real numbers except $x = 4$.
(5) All real numbers greater than zero.

47. Which of the following ordered pairs (x,y) represents the point of intersection of the graphs of $5x + 3y = 7$ and $9x - y = 3$?

(1) (-4,9) (2) (1,6) (3) (-1,4) (4) (1/2, 3/2)
(5) none of these

48. For the mathematical system consisting of four elements a, b, c, d, and a single operation subject to the table on the right, the identity element is

∘	a	b	c	d
a	c	d	b	a
b	d	c	a	b
c	b	a	d	c
d	a	b	c	d

(1) a (2) b (3) c (4) d
(5) 1

49. Given the set of objects {△, □} and the operation * defined by the table on the right, evaluate the statements (a), (b) and (c).
(a) The identity element is △.
(b) □ is its own inverse.
(c) The operation * is commutative.

*	△	□
△	△	□
□	□	△

(1) (a) is true, but (b) and (c) are false.
(2) (b) is true, but (a) and (c) are false.
(3) (c) is true, but (a) and (b) are false.
(4) (a) and (b) are true, but (c) is false.
(5) (a), (b), and (c) are all true.

50. In mathematics, axioms are never proved because

(1) the proofs are obvious.
(2) the proofs are difficult but the axioms are obvious.
(3) they are not true.
(4) it is impossible to prove them within the mathematical system.
(5) no other assumptions could be axioms.

A TEST OF COLLEGE MATHEMATICS FOR GENERAL EDUCATION

Directions for Administering

A. SHIPMENT OF MATERIALS TO THE TEST USER

The following materials will be shippped to reach the test user before the designated testing date:

a. one test booklet for each student to be tested;

b. one answer sheet for each student scheduled to take the test plus some extra sheets;

c. scrap paper for students' use during the test;

d. a copy of Directions for Administering for each examiner;

e. items to be used for return of materials.

B. ADMINISTRATION OF TEST

1. 55 minutes will be adequate for the test; 10 minutes for instructions and distribution of materials and 45 minutes of actual testing time.

2. Distribute an answer sheet and a sheet of scrap paper to each examinee.

3. Instruct the examinees to print the information requested on the side of the answer sheet. Don't bother to write NAME OF TEST.

4. When all information has been entered on the side of the answer sheet distribute the test booklets.

5. When the test booklets have been distributed say:

Look at the directions on the first page while I read them aloud to you.

Note: The tests will be hand scored. That is why electrographic pencils have not been provided.

6. As soon as you are sure that everyone understands how to proceed, say:

> Do not write anything nor make any marks in the test booklets. Use the scrap paper provided. When you finish the test hand in your test booklet, answer sheet, and scrap paper. Leave quietly. If there are no questions you may begin.

7. Allow 45 minutes for the test.

C. <u>RETURN OF MATERIALS</u>

All test materials should be returned within <u>2 days</u> after the test has been administered.

1. <u>All</u> test booklets are to be returned by parcel post immediately <u>after</u> tests have been administered.

2. Check the answer sheets to determine whether all information has been completed by the individual taking the test. In the event that information has been omitted, it should be completed whenever possible.

3. Arrange the answer sheets in alphabetical order by the last name of the students. Place in envelope provided for this purpose and send it out by first class mail.

4. Please use the materials provided for the return of test materials.

APPENDIX I

ALPHABETICAL LIST OF SCHOOLS

INCLUDED IN THE SAMPLE

Carroll College	Waukesha, Wisconsin
Ellsworth Junior College	Iowa Falls, Iowa
Hanover College	Hanover, Indiana
Luther College	Decorah, Iowa
Macalester College	St. Paul, Minnesota
St. Joseph's College for Women	Brooklyn, New York
St. Martin's College	Olympia, Washington
Trevecca College	Nashville, Tennessee
Villa Madonna College	Covington, Kentucky
Wheaton College	Wheaton, Illinois

APPENDIX J

A LETTER AND QUESTIONNAIRE SENT TO THE SCHOOLS

IN THE RANDOM SAMPLE

Immaculate Conception Junior College
South Main Street
Lodi, New Jersey

August 24, 1964

Dear

In response to a questionnaire sent out in the spring of 1963 you agreed to have your institution of higher learning participate in a testing program designed as a part of my doctoral dissertation "A Study to Measure the Effectiveness of One Type of College Mathematics for the Non-Major." Originally a test was to have been administered to the participating schools at the end of the spring semester in 1964. However, in order to have a uniform basis for comparing the mathematical achievement of the students in certain areas before and after having been exposed to the course, the test will be given at the beginning of the semester in September of 1964 and again at the end of the semester in January of 1965.

The test will be administered to a sample of those schools which offer a mathematics-for-general-education type of course. Your school has been included in the sample.

The test covers a one-semester (3-5 credit hours) course. Since in many schools the same course is extended through two semesters, you will receive with the test materials an outline of the topics on which the test is based. The test materials will be sent to you at no cost. Testing should be done during the first week of school. Required testing time is 55 minutes which includes 10 minutes for instructions and distribution of materials and 45 minutes of actual testing time.

Please complete the enclosed questionnaire and send it out as soon as possible so that the tests could be mailed to you in time. Your prompt cooperation will be greatly appreciated.

I am doing my graduate work at Oklahoma State University but during the school year 1964-65 I will be teaching mathematics at Immaculate Conception Junior College in Lodi, New Jersey.

Sincerely yours,

Sister Mary Firmina

Please fill out this form and return it in the enclosed addressed and stamped envelope as soon as possible.

Name and location of school: ______________________________

____

____

What is the approximate number of students enrolled in a mathematics-for-general education type of course for the fall semester of 1964? ______

____

What is the title and catalog number of the course?

____

Semesters: ____

Credit hours: ____

What textbook will be used?

Title: ____

Authors: ____

Publisher: ____________________________Copyright: _______

On what day does the fall semester begin? ________________

There is no fixed date for the test. It should be given during the first week of school. On what day would you like to administer it? ______

____

To what name and address should the test materials be sent?

____

____

____

Additional Comments: ___________________________________

____

____

This form was completed by: ______________________________

APPENDIX K

A LETTER CONTAINING THE RESULTS OF THE PRETEST

AND INSTRUCTIONS FOR SCORING

THE POST-TEST

Immaculate Conception Junior College
South Main Street
Lodi, New Jersey

December 31, 1964

Dear

I wish to thank you for your excellent cooperation in administering A Test of College Mathematics for General Education to your students in September of 1964.

Included you will find a list of your students and their scores. The score consists of the number of correct answers. I waited till now to send you the scores because I thought that they would become meaningful only when you will have obtained the second set of scores in January and will use the first set for comparative purposes. I am also including a summary of the results obtained in the first test.

On January 2, I will put into the mail the booklets and materials necessary for the second testing which should take place as close to the end of the first semester as it is possible and at the same time most convenient for you. Dropouts are expected but I hope there won't be too many.

Some schools have expressed a desire to correct their own tests in order to have the results immediately available. Hence, I am including with the test materials an answer key and instructions pertinent to the administering and correcting of the test and the disposal of the materials.

Correction of the test is optional. Whether or not you correct the test, you will receive after the second testing another list of your students with both scores. As soon as my dissertation is completed you will receive a copy of it and you will thus get a more detailed analysis of the test results.

May God reward you for all your kindness. My prayerful wishes to you for a happy and successful New Year.

Sincerely yours,

Sister Mary Firmina

A Summary of the Results of the First Test

The sample consisted of 10 schools with a total of 595 students participating in the testing. The following table contains a summary of the results obtained in the first test which was given in September. The schools are numbered. Your school is starred.

Number of School	Median	Mean X	Variance s^2	Standard Deviation	Range
1	10	13.33	47.25	6.87	28 - 7
2	11	11.62	1.95	1.40	17 - 5
3	19	19.52	40.59	6.37	30 - 9
4	17	18.31	56.80	7.54	37 - 4
5	12	11.52	14.67	3.83	22 - 3
6	15	15.44	33.45	5.78	36 - 3
7	12	13.12	26.12	5.11	32 - 2
8	13	12.98	20.22	4.50	26 - 5
9	16	16.37	37.18	6.10	35 - 5
10	13	14.78	30.90	5.56	31 - 0
Combined Schools	14	14.64	33.15	5.76	37 - 0

Directions for Correcting

If you decide to correct your own tests you may do so in one of two ways.

First Method: Use the key and count the number of correct answers without placing any checks or marks on the answer sheet. Then write the number of correct answers on the line to the right of number 1, under "Scores."

Second Method: If you wish to obtain the scores for yourself and at the same time assist me, use a red pencil and whenever the answer is incorrect or missing draw a short red line through the double bars of the correct answer. As you do this count the number of incorrect answers and write it on the line to the right of number 5, under "Scores." Substract the number of incorrect answers from 50 and then write the number of correct answers on the line to the right of number 1, under "Scores." To make myself clear I am sending you a sample of a corrected paper.

VITA

Sister Mary Firmina Lajewski, C.S.S.F.

Candidate for the Degree of

Doctor of Education

Thesis: AN EXPERIMENTAL STUDY TO DETERMINE THE EFFECTIVENESS OF ONE TYPE OF COURSE IN COLLEGE MATHEMATICS FOR THE NON-MAJOR

Major Field: Higher Education

Biographical:

Personal Data: Born in Sayreville, New Jersey, April 16, 1916, daughter of Ladislaus and Alexandra Lajewski.

Education: Attended both public and Catholic elementary schools in Sayreville, New Jersey; graduated from Immaculate Conception High School, Lodi, New Jersey, in 1932; completed Immaculate Conception Junior College, Lodi, New Jersey, in 1940; attended Pope Pius X School of Music at Manhattanville College of the Sacred Heart, New York City, 1943-1947; received the Bachelor of Science degree from Fordham University, Bronx, New York, with a major in mathematics, in 1956; received the Master of Science degree from Oklahoma State University, with a major in natural science, in 1959; completed the requirements for the Doctor of Education degree at Oklahoma State University, May, 1966.

Professional Experience: Taught at various elementary schools in New Jersey, 1934-1953; taught mathematics, physics, and music at three New Jersey high schools: St. Joseph's, Camden, 1953-1956; St. Anthony, Jersey City, 1956-1957; Our Lady of the Lake Regional, Sparta, 1957-1958 and 1960-1963; attended National Science Foundation Academic Year Institute at Oklahoma State University, 1958-1959; received a three-summer grant from the National Science Foundation for graduate study at Oklahoma State University, 1960-1962; head of mathematics department at Immaculate Conception Junior College, Lodi, New Jersey, 1964-1965.

Professional Organizations: Albertus Magnus Guild, Association of Mathematics Teachers of New Jersey, Kappa Delta Pi, Mathematical Association of America, National Council of Teachers of Mathematics, New Jersey Catholic Round Table of Science, New Jersey Junior College Association, Pi Mu Epsilon.