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Scope and Method of Study: This paper presents a discussion of the phenomenon that we refer to as interference patterns, alternating regions of light and darkness, that occur when two beams of light interact through a medium. Also, a mathematical explanation, consistent with a wave model of light, and methods of obtaining the coherent sources necessary for the observation of interference are introduced.

Certain opitcal phenomena can be interpreted only by thinking of light as having a corpuscular nature; other phenomena, interference being one, can be completely explained by use of the wave theory. A simple mechanical model is used to develop the mathematical relationships.

The interferometer is discussed briefly.

Findings and Conclusions: The wave theory does, indeed, offer the vehicle for a complete and satisfactory explanation of the interference in light.

The interferometer, which uses the principle of interference, may be used to make precise measurements of the wavelength of light.

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THE INTERFERENCE OF LIGHT

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INTRODUCTION

Two beams of light can be caused to interact in such a way that alternating regions of light and darkness are observed. These modifications of intensity are referred to as interference patterns. The effect generally occurs when several wave systems are being propagated at the same time through a medium. This paper presents a discussion of this phenomenon, a mathematical explanation consistent with a wave model of light, and methods of obtaining the coherent sources necessary for observation of interference phenomena. Finally, the application of interference to precision measurement by means of an interferometer will be briefly discussed.

There are certain optical phenomena which can be interpreted only by thinking of light as having a corpuscular nature. Other phenomena, interference being one, find a complete and satisfactory explanation by an ordinary wave theory and, for this reason, the wave model is employed in this paper. Because of its simplicity, a mechanical model will be used as an aid in developing mathematical relationships. All such relationships developed from this model are applicable to any wave phenomena including light.

In order to display the forms of waves effectively all curves drawn must necessarily be transverse to the direction in which the waves advance. However, this does not limit the discussion to waves of a transverse nature; or, for that matter, to waves in a material medium. Results are equally applicable to scalar quantities, the magnitudes of which at any point along a line may be represented graphically by displacements perpendicular to that line. With sound waves, for example, the pressure in the medium conveying the sound varies with time and position, and is a scalar quantity. With light waves the varying quantities are electric and magnetic intensities; these are vector quantities and are directed perpendicularly to the direction of propagation. In the discussion which follows the amplitude of the light wave will be treated as a constant quantity, though more often it decreases with increasing distance. The quantities defined retain the same significance when the amplitude is not constant.

HOW LIGHT TRAVELS

In order to understand better the optical phenomenon known as interference, and to make use of it for scientific measurements, we must understand how the wave principle applies and explains it. The energy transmitted from one point to another point can be explained by the wave theory as originally propounded by Huygens. We will consider the form of the wave to depend upon the vibratory motion transmitted to the particles of the medium, and although we do not think of a light wave as a disturbance in an elastic solid medium, the conclusions carry over into light and into all electromagnetic waves which are characterized by different ranges of wavelengths.

Let us consider the addition of several simple harmonic motions with specific attention to the combination which will be applicable to light waves and to the production of interference phenomena. When two or more waves cross the same region of a medium and are examined afterwards, the individual waves are found to come through unaltered. However, the instantaneous (optical) disturbance of the point where two or more (light) waves cross is the sum of the (optical) disturbance that would be produced by each wave separately. If the two displacements are given by $y_1 = f_1(t)$ and $y_2 = f_2(t)$ then $y = y_1 + y_2$. The resultant displacement y is the vector sum of the individual displacements. This result is known as the principle of superposition.

More specifically, and more applicable to the present discussion, if the wave forms are due to sources undergoing simple harmonic motion and having the same frequency, we may write:

$$Y_1 = B \sin(\omega t + \alpha_1): \quad Y_2 = C \sin(\omega t + \alpha_2)$$

Then $y = y_1 + y_2 = A \sin(\omega t - \theta)$. That is, the sum of the two simple harmonic motions having the same frequency is also a simple harmonic motion of that frequency. The amplitude A depends upon the two amplitudes, B and C, and the phase angle θ is given by $\theta = \alpha_2 - \alpha_1$.[1]

When a particle rotates with uniform angular velocity around a circular path its projection of a diameter is a linear oscillatory motion of a simple harmonic nature. A graph of the vertical displacement of the particle with respect to time is given in Figure 2. The vertical displacement of the particle can be expressed in terms of the angular position. Thus; $y = OQ = A \sin \theta$, and the resultant figure is a sine curve.

If t is the time reckoned from the instant when $\theta = 0$, the angular velocity of the radius A can be expressed as $\omega = \frac{\theta}{t}$, so $\theta = \omega t$; and $y = A \sin \theta = A \sin \omega t$. For the more general case when the particle is at some angle θ when $t = 0$ then the displacement from the center 0 along the y axis is given by $y = A \sin (\omega t + \theta)$. This is the general expression for the displacement of a single particle which experiences periodic motion of a simple harmonic nature. In a medium each particle in a line undergoes this same motion as the disturbance is transmitted to these particles. This simultaneous motion of all the particles gives rise to a wave form that is propagated through the medium. The waveform, not the particles, travel through the medium. The waveform travels a distance equal to one wavelength λ in a time τ. This characteristic

[1] See Appendix I for detailed derivation of this result.

time is called the period of the vibration and represents the time for a particle to complete one cycle. The velocity of the disturbance is then $v = \frac{\lambda}{\tau} = \lambda f$, where f is the frequency or the number of cycles per unit time. It follows from the definitions that $f = \frac{1}{\tau}$.

If the particle P is the first of a series of particles in an elastic media then the disturbance to this particle is, as previously noted, transmitted to succeeding particles. The displacement of a single oscillatory particle at any time t is given by $y = A \sin(\omega t + \theta)$. A disturbance started by particle O (see Figure 3) reaches another particle at some distance x from it in the time x/v and the displacement of this second particle is given by

$$y = A \sin (\omega t + \theta) = A \sin \left(\frac{2\pi}{t} + \theta\right) = A \sin \left[\frac{2\pi}{\tau} (t - \frac{x}{v}) + \theta\right].$$

We now have a more general expression giving the displacements of all particles situated along the path of the disturbance at any instant. The wave displacement y is given as a function of both position x and time t.

A more useful form when considering optical disturbances is

$$y = A \sin \left[\frac{2\pi}{\lambda} (vt - x) + \theta\right]. \tag{1-1}$$

The quantity

$$\left[\frac{2\pi}{\lambda} (vt - x) + \theta\right]$$

is known as the phase angle, or simply the phase of the motion. For sinusoidal waves of constant frequency, traveling at constant velocity, the phase of the motion depends on:

(1) The position of the wave system, that is, on x.

(2) The time t.

(3) The value of γ, the phase constant, which is the initial phase when t and x are zero.

The difference in phase between two similar disturbances is important in our consideration of interference phenomena. Consider two similar sources as shown in Figure 4 and their effect on the media at the point H. The sources are emitting the sinusoidal wave forms:

$$y_1 = B \sin\left[\frac{2\pi}{\lambda}(vt - x_1) + \theta_1\right];$$

$$y_2 = C \sin\left[\frac{2\pi}{\lambda}(vt - x_2) + \theta_2\right],$$

where $x_1 = S_1H$ and $x_2 = S_2H$. For reasons which will be discussed presently it is absolutely necessary to have the difference in phase between the two sources a constant value if the interference phenomena of light is to be observed. For convenience we will consider the case where $\theta_1 - \theta_2 = 0$.

Calling the phase difference δ we have

$$\delta = \left[\frac{2\pi}{\lambda}(vt - x_2) + \theta_2\right] - \left[\frac{2\pi}{\lambda}(vt - x_1) + \theta_1\right]$$

$$\delta = \left[\frac{2\pi}{\lambda}(x - x) + \theta_2 - \theta_1\right] \text{ giving}$$

$$\delta = \frac{2\pi}{\lambda})x_2 - x_1) \tag{1-2}$$

The term $(x_2 - x_1)$ is the difference in the optical paths from S_1 and S_2 to H. The optical path is defined as the product ηd, where η is the refractive index of the medium and d is the distance in that medium. The optical path represents the distance in a vacuum that the light would travel in the same time that it travels the distance d in the medium. When there are several segments of the light path d_1, d_2, . . . in substances having different indices η_1, η_2, . . . the optical path is given by

$$d = \eta_1 d_1 + \eta_2 d_2 + \ldots = \sum_L \eta_1 d_1 .$$

Equation (1-2) states that the phase difference at H is $\frac{2\pi}{\lambda}$ times the optical path difference. This is a very important result of which we shall make use.

If we choose a time $t = 0$ such that $\theta_1 = \theta_2 = 0$ we have for the disturbance at H;

$$y = B^2 + C^2 + 2BC \cos \left[\frac{2\pi}{\lambda} (x_2 - x_1)\right] \sin \left[\frac{2\pi}{\lambda} (vt - x_1) + \beta\right] .$$

The angle under the radical gives the difference in phase between y_1 and y_2 at H. If S_1 and S_2 are sources of light then y represents the optical disturbance at the point H and this is manifested by a certain intensity of the light there. The intensity -- indeed the intensities of all kinds of waves -- is proportional to the square of the amplitude,[2] and we may write $I = KA^2$. It will be convenient to consider the constant of proportionality between the square of the amplitude and the intensity to be unity. It will have this value for appropriately chosen units.

The amplitude of y is given by the term under the radical. It will be noted that it is independent of time but is dependent upon the phase difference δ. With the proper choice of units we have

$$I = A^2 = B^2 + C^2 + 2 \, BC \cos \delta . \tag{1-3}$$

Considering the case where $B = C = a$, we may write

$$I = 4 \, a^2 \cos \frac{2\delta}{2} \tag{1-4}$$

It is seen that the intensity at H can vary from a maximum of $4 \, a^2$ to a minimum of 0, depending upon the phase difference. Figure 5 is a plot of equation (1-4) for various values of δ and shows how the intensity varies as we pass through regions of maxima and minima.

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[2] See Appendix II for this derivation.

The intensity will be a maximum when $\cos\frac{\delta}{2}$ is ± 1 and a minimum when $\cos\frac{\delta}{2} = 0$. Let us see under what conditions we get these extremes of intensity.

I is a maximum for $\cos\frac{\delta}{2} = \pm 1$, that is, for $\frac{\delta}{2} = 0, \pi, 2\pi \ldots n\pi$ or $\delta = n\,2\pi$ [where $n = 1, 2, 3, \ldots k$]. This occurs when

$$\delta = 2\pi n = \frac{2\pi(x_2 - x_1)}{\lambda}, \text{ and } x_2 - x_1 = n\lambda. \tag{1-5}$$

I is a minimum when $\cos\frac{\delta}{2} = 0$; that is, for $\frac{\delta}{2} = (n + 1/2)\pi$ or $\delta = (2n + 1)\pi$, which occurs when

$$\delta = (2n + 1)\pi = \frac{2\pi(x_2 - x_1)}{\lambda},$$

or when

$$x_2 - x_1 = (n + 1/2)\lambda. \tag{1-6}$$

Or, more simply expressed, the crests superpose and combine for maximum intensity when the path difference is equal to an integral multiple of a wavelength, and, the waves are said to be in phase. The maximum positive and the maximum negative displacements fall together and give zero effect when the path difference is an odd number of half wavelengths, and the waves are said to be exactly out of phase.

In 1801, Thomas Young was the first person to observe interference phenomena using light. Young used a pin hole exposed to sunlight for his source S, Figure 6, and two more holes for S_1 and S_2. For simpler results, but with no essential difference, a bulb with a long straight filament is used for the source S and two parallel slits for S_1 and S_2. This setup provides cylindrical wave fronts which will give a pattern of parallel lines on the screen. The light from the two sources will arrive at some point J on the screen, out of phase due to the two paths of different lengths. At an adjacent point H the light arrives in phase. Thus, we have alternate dark and bright bands observed on the screen. The position of these bands can be obtained from considerations shown in Figure 7. An arc of length HS_1 is drawn about H; the distance between the two slits is very much smaller than the distance x so that the arc is verynnearly a straight line. The path difference, $d_1 = S_2H - S_1H$. The angle θ is very small so that $\sin\theta \cong \tan\theta$. For maximum intensity at H it is necessary that $d = n\lambda$, or that

$$n\lambda = d = 3\sin\theta \qquad \sin\theta \quad \tan\theta \quad \frac{y}{x}$$

$$n\lambda = e\,\frac{y}{x}$$

and the distance to a given bright band H becomes

$$y_H \quad n\lambda\,\frac{x}{e}\ .$$

Here, the symbol n represents the order of the interference bands or fringes. For a dark band J

$$(n + 1/2)\lambda = D\sin\theta = D\,\frac{y}{x}\ , \text{ and}$$

$$y_J = (n + 1/2)\lambda\,\frac{x}{D}\ .$$

From the symmetry of the figure it can be seen that the distance from S_1 and S_2 to point 0 on the screen is the same and that there is a bright fringe here of zero order.

From the above arguments we see that if the two beams arrive at a point exactly out of phase they interfere in such a way that the resultant intensity is zero. This is called destructive interference. When the beams arrive at a point to produce the maximum intensity the interference is referred to as constructive. The energy which disappears at the minima is still present at the maxima, where, as previously noted, the intensity is greater than would be produced by the two beams acting separately. The energy is not destroyed but is merely redistributed in the interference pattern. The average intensity is exactly that which would exist in the absence of interference. As shown in Figure 5 the intensity varies from 0 to $4a^2$. Each beam acting separately would contribute $2a^2$ as indicated by the broken line. To obtain the average value for n fringes we note that the average value for the $\cos^2$ is 1/2.[3] This gives $I = 2a^2$ and so there is no violation of the conservation of energy law.

We have shown mathematically that interference phenomena should occur and we have described Young's experiment in which interference patterns are observed. Why, then, do we not observe the phenomena whenever we have light from two sources striking a surface? Since interference is not readily observed in our daily lives we conclude that the realization of the phenomena depends upon the fulfillment of certain conditions; and this is indeed the case. To have a well defined pattern the intensity at a region corresponding to a dark band must remain zero, and at the region of bright band remain at a maximum. For this to be possible the following conditions must be satisfied:

1. The two sources must emit waves with a constant phase difference, that is, they must be coherent sources. This implies that, ultimately, the light must come from one source.

2. The light must be sensibly monochromatic, of a single wavelength, or else the optical paths for the two beams must be nearly equal to within a few wavelengths of light.

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[3]See Appendix III for derivations of this result.

3. The wave fronts of the interfering wave trains must be traveling in the same direction or make very small angles with each other.
The first two conditions are necessary for the production and the third is necessary for the observation of the phenomena.

The first condition is demanded by the requirement that there be sustained maxima and minima. To bring about this condition requires that the two interfering beams are always derived from the same source of light. Experimentally, it is impossible to obtain interference fringes from two separate sources. This failure is due to the fact that light from any one source is not an infinite train of waves. There are sudden changes of phase which occur very rapidly at a rate of perhaps 10^8 per second. In light sources the radiating atoms emit wave trains of finite length. Because of collisions or damping these are very short. Thus, although interference fringes might exist on a screen for a short interval, they will shift their position each time there is a phase shift, with the result that no fringes will be seen at all. On the other hand if the two sources producing interference are both derived from the same source they will always have a point to point correspondence of phase. When the phase of the light from a point on one source suddenly shifts, that of the light from the corresponding point in the other source must shift exactly and simultaneously. Thus the difference in phase between any pairs of points in the two sources always remains constant and the interference fringes remain stationary. This relation is called coherence and any interference experiment with light must use sources that possess this point to point phase relation.

Condition two is necessary to avoid the masking of all interference due to the presence of more than one wavelength. Cancellation occurs at different points for different wavelengths and though there may be a point of darkness for some wavelengths there is a partial or complete brightness for the neighboring wavelengths. So, for white light, there is constructive interference for all wavelengths at one point, that of zero path difference, and this fringe is achromatic. On either side there will be symmetrically located and colored fringes.

The third condition is necessitated by the limitation of the resolving power of the human eye. This may be readily seen by referring to Figure 8. The figure represents two plane wave fronts making a small angle with each other. The solid lines represent maximum positive displacement and the dotted lines represent maximum negative displacement. The regions of maximum constructive and destructive interference are represented by the lines CI and DI respectively. It follows that increasing the angle between the wave fronts decreases the spacing between the interference fringes. The fringes can thus be brought so close together as to be unobservable except under very high magnification.

METHODS OF DETECTING INTERFERENCE

It will be remembered that it is impossible to observe interference effects between light waves emitted by independent sources. Any light source consists of a large number of microscopic, uncorrelated sources; each of which is active for short periods of time, and quiescent the rest of the time. For simplicity, assume that during their active periods all the microscopic sources emit trains of sinusoidal waves of the same wavelength. The resultant optical disturbance produced by the source as a whole may be represented, as we have seen, by a sinusoidal function of time whose phase and amplitude change whenever one of the small sources goes on or off. Consequently, the optical disturbances produced by two such microscopic sources will have a phase difference of $\theta_1 - \theta_2$ that varies rapidly and irregularly with time. Such sources are incoherent. The position of the interference fringes will change as the phase difference changes. At a given instant a maximum intensity will occur where a minimum was present a short time before. Neither the eye nor the other optical devices can resolve such rapid fluctuations of intensity, so the observable result is a uniform illumination of the screen. Clearly, to observe the interference phenomena we must have sources that maintain a constant phase with respect to each other. It is possible to create such sources either by using a source and its optical image, or by using two different images of the same source. In this manner light beams originating from the same source meet at a given point after having followed different paths and they are suitable for interference.

Reference has already been made to the double slit method employed by Thomas Young. Light spreads out from the first slit, Figure 6, and illuminates the secondary slits S_1 and S_2. There will always be a constant phase difference, usually zero, between the waves as they emerge from the separate slits S_1 and S_2. Interference is, as we have seen, possible between the light waves coming from these slits.

A second method by which interference fringes may be obtained is through the use of two plane mirrors. The method was devised by Augustin Fresnel and is shown schematically in Figure 9.

In this apparatus two plane mirrors M and M' are inclines to each other at an angle of almost 180 degrees so that they lie almost in the same plane. A beam of light from a source S, which may be the focus of a lens or a slit parallel to the intersection of the surfaces of the mirrors, is allowed to fall on the mirrors. The beam is reflected from the two mirrors and after reflection the light appears to come from the two virtual images S' and S". In the region of space where the two beams overlap, interference phenomena occur and a series of equally spaced bright and dark fringes similar to those described in the previous chapter appears on a screen parallel to the intersection of the mirrors.

Fresnel also devised a method of obtaining two coherent sources by using two prisms placed together as shown in Figure 10. In this arrangement, what was done by reflection is now accomplished by refraction. Light from the source S, a slit or point source, falls on the prism ABCD. The edge B divides the incident light into two portions. The effect is to produce two virtual images of the source and these images act in every respect like the images formed by the double mirrors.

Interference is again observed in the region where the refracted beams overlap.

Using a single mirror Lloyd was able to obtain interference between light waves coming from the source and a virtual image of the source. This proves to be a more satisfactory geometry than Fresnel's mirrors or biprism. It is shown schematically in Figure 11. The mirror is placed so that rays from a slit source fall on it at nearly the grazing incidence, and the reflected rays appear to diverge from a virtual source S'. As before, the interference takes place where the two beams overlap. In this geometry the reflected rays cannot come below the plane of the surface of the mirror, so, in general, only one half of the complete system of bands is visible.

Two virtual sources may be obtained by subdividing a beam of light by refraction through two plates of the same nature and equal thickness, placed at an angle as shown in Figure 12. A source is placed on the bisector of the bisector of the angle. The light that falls on the plate N passes through it in the direction CD and emerges parallel to its original direction but displaced so that it appears to be coming from the point S'. Similarly, the light emerging from the plate M diverges from the virtual source S". The emerging cones are received by a lens and brought to the real foci s' and s". After diverging from s' and s" the beams will overlap and produce the desired fringes.

CLASSES OF INTERFERENCE

Previously we dealt with interference fringes obtained by superposing two wave trains, each of which came from the same source but by two different paths. In all the cases considered it was essential that a narrow source be used, otherwise, different parts of a broad source gave rise to maxima in different places, and uniform illumination would result. There are methods of obtaining interference fringes in which a broad source is necessary. Among these consideration will be given to thin films and parallel plane surfaces because of their application in the interferometer to be discussed later.

At any interface between two media light is partially reflected and partially transmitted. When such waves are suitably reunited interference phenomena are produced that are particularly easy to observe. We can interpret this phenomena as follows:

Monochromatic light from a source S, Figure 13, falls on a thin film and is reflected to a converging lens I which in turn forms an image on the screen. The lens may be an eye focused on the small region about P. The screen, or retina if the eye is used, receives two bundles of light rays from the same source but traveling by different paths, and thus having a phase difference. The image of P, is formed on the screen at P'. The intensity at P' may vary from a maximum to a minimum depending upon the phase difference.

Since the optical path lengths of the two rays between P and P' are equal, therefore, the two rays arrive at P' with the same phase difference that they possessed at P. In deriving the expression for this path difference we shall make reference to Figure 14. The film has an extremely small angle γ and the distance AC is small compared to the distance S from the film. Therefore we may draw AF perpendicular to SC and consider SA = SF. Then the optical path difference Δ is given by

$$\Delta = \eta(AB + BC) - \eta_o FC,$$

$$\text{let } \eta(AB + BC) = x_2, \text{ and } \eta_o FC = x_1.$$

$$\text{Draw } Ch \perp AB, \text{ then } X_1 = \eta_o FC = \eta AH,$$

$$\Delta = \eta(AH + HB) + \eta BC - \eta AH, \text{ and}$$

$$\Delta = \eta(HB + BC).$$

If we produce HB to J so that BJ = BC, then

$$\Delta = \eta(HB + BJ), = \eta HJ, = \eta CJ \cos \alpha .$$

For geometrical considerations CJ $\perp$ mM, making CK = KJ and $\alpha = \beta + \gamma$. Letting d represent the thickness of the film at C_1 we have

$$CJ = 2d, \text{ and therefore}$$

$$\Delta = 2\eta d \cos (\beta + \gamma).$$

But, since $\gamma \ll \beta$, we may write

$$\Delta = 2\eta d \cos \beta$$

We conclude that, for a given film, the path difference depends upon the film thickness at the point C and on the angle β.

As we have previously seen, the phase difference corresponding to a given optical path difference $x_2 - x_1$ is given by

$$\delta = \frac{2\pi}{\lambda_o} (x_2 - x_1); \text{ from which,} \qquad (1-2)$$

$$\delta = \frac{4\pi\eta d}{\lambda_o} = \cos \beta \qquad (3-2)$$

Equation (1-2) was derived to give the wavelength λ as that of the light in the medium which contains the source. In making use of this relationship here we have replaced λ by λ_o, which is the wavelength of the light in the medium of optical density η_o. Then, η_o contains the source as shown in Figure 14. The quantity

$$\frac{\lambda_o}{\eta} = \lambda,$$

where λ is the wavelength in the medium of optical density η, that is, of the film. Equation (3-2) then becomes

$$\delta = \frac{4\pi d}{\lambda} \cos \beta \ .$$

When reflection takes place at the interface of an optically denser medium a phase change of 180^o takes place. Thus, if η is either less than both the indices η_o and η_1 or greater than both of them the phase difference is given by

$$\delta = \frac{4\pi d}{\lambda} \cos \beta \pm \pi \ .$$

The interference will produce a maximum of intensity if the phase difference is an integral multiple of 2π and a minimum of intensity when it is an odd multiple of π. If we consider $\eta > \eta_o = \eta_1$ (as is the case for a film of air) then for an intensity maximum to occur

$$2\pi m = \frac{4\pi d}{\lambda} \cos \beta - \pi, \text{ and}$$

$$d = \frac{(2m + 1)}{\cos \beta} \cdot \frac{\lambda}{4}$$

$$m = 0, 1, 2, \cdot \cdot \cdot k \qquad (3\text{-}3)$$

For a minimum, the result is

$$(2m + 1)\pi = \frac{4\pi d}{\lambda} \cos \beta - \pi, \text{ and}$$

$$d = \frac{m}{\cos \beta} \cdot \frac{\lambda}{2}$$

$$m = 0, 1, 2, \cdots k \qquad (3\text{-}4)$$

Suppose parallel light is incident on a film wedge of small angle γ. Then β is constant and d is variable. Where the film has a thickness d satisfying equation (3-3) the light will interfere to produce a maximum intensity, and at positions where d satisfies equation (3-4) there is a minimum of intensity in the reflected light. The fringes will be parallel to the edge of the wedge. To see these fringes the angle γ must be very small and the source must be broad or extended in order to see an extended system of fringes. Light from a single source will reach the eye at some one point. An extended source supplies many points. Any one band of the alternate dark or bright fringes is the locus of points along constant film thickness, and hence are called fringes of constant thickness.

A somewhat different class of interference is the film or plate with plane parallel surfaces. Under these circumstances fringes can be obtained even with the surfaces very far apart, the separation may be several centimeters. This type of interference may be viewed with the naked eye or with a telescope focused at infinity. In this manner we can observe interference between parallel rays that arise from a single ray by reflection at the surfaces of the plate; the rays being brought together in the focal plane of the telescope or on the retina. Figure 15 represents two such plane parallel surfaces S_1 and S_2. For a ray incident on the plate there will be partial reflection and partial transmission at each surface, and this will give rise to a series of parallel reflected rays and a series of parallel transmitted rays. We shall only concern ourselves with the reflected rays and since the intensity of the rays that have undergone multiple reflection is greatly diminished their contribution is negligible and we shall consider the first two reflected rays only.

By a method similar to that given above, it can readily be shown that the optical path difference from the source to the point of convergence for these two rays is

$$\Delta = 2\eta d \cos \beta \ . \tag{3-5}$$

It will be noted that this result is identical with equation (3-1); however, (3-5) holds rigorously, while (3-1) is valid only for the case of small film thickness.

Since η and d are constants the path difference varies only with β. It follows that the resultant intensity at P varies with β, alternating from a maximum, when $2\eta \cos \beta$ equals a whole number of wavelengths, to a minimum, when it is equal to an odd number of half wavelengths. As in the previous case a broad source is used so that the plate is illuminated by rays coming from many directions. Since the phase difference between interfering rays is a function of the angle at which they are reflected from the plate, rays entering the lens at a given angle to its axis are focused at points on a circle lying in its focal plane. The broad source produces a system of fringes which are concentric circles with their center at the point where a perpendicular from the lens intersects the film. These fringes, due to a variation in the angle β, are known as fringes of constant inclination.

THE INTERFEROMETER

An instrument has been designed to make use of this interference phenomenon. Its primary function is to measure the wavelength of light in terms of some known standard of length, or to measure an unknown length in terms of the known wavelength of light. Such an instrument is called an interferometer. One of the best known and most widely used is the one designed by A. A. Michelson. It is quite simple in design and the basic arrangement is shown in Figure 16. The instrument is composed of a half silvered mirror A which divides the light from the source S into two beams of equal amplitude. One beam is reflected to the plane mirror M and the other beam is transmitted to the plane mirror M'. M is mounted on a carriage which can be moved along a precision machined track in the direction shown by the double arrow. The motion of M is controlled by a fine micrometer screw. Either or both mirrors are equipped with tilting screws so that they may be made accurately perpendicular to each other.

Light from the source S is rendered parallel by a lens L and strikes the half-silvered mirror A at a 45 degree angle. The divided beam proceeds to mirrors M and M, reflects back to A, and then to the eye at E. The two beams traveling toward E after reflection from M and M' are from the same source and hence are suitable for the production of interference.

The plate G is a compensating plate, which is exactly like A except that it is not silvered. G is inserted between A and M' in order to make the optical paths of both interfering beams identical. This is not necessary to produce fringes from a monochromatic source but is absolutely essential if white light is used. As shown in the diagram, the light reflected to M and then to E passes through A three times. The light transmitted to M' then reflected to E passes once through A and twice through G, making both paths equal. If A had been silvered on the front surface, G would have to be placed in the path from A to M. In manufacture, a single block is worked optically plane parallel, and then cut in half, forming the dividing plate A and the compensating plate G.

Using a telescope, one sights from E in the direction of M. M is then seen directly and a virtual image of M' is formed by reflection in A. This virtual image M' is parallel to M and its location with respect to M is determined by the respective distances of the two mirrors from the reflecting surface A. If M' is closer to A than M, the image of M', call it M", is in front of M, as shown in Figure 17. One of the interfering beams comes to E by reflection from M. The other appears to come by reflection from the virtual image M". If M and M' are accurately perpendicular, then the two planes of M and M" will be parallel. The interference fringes seen are the same as would be produced by an air film between M and M" with the difference that in a real film, multiple reflections may occur, while here there will be only the two reflections. These fringes are concentric circles similar to those produced by two reflecting parallel surfaces, as discussed in the previous chapter.

When M and M" are not parallel but their planes intersect we have the equivalent of a wedge-shaped film of variable thickness and the fringes, as we have seen, are lines of constant thickness of the wedge-shaped air film enclosed between M and M". In either case, if the mirror M is slowly displaced by means of the screw, the interference pattern gradually changes.

As an example, let us look at the circular fringes produced by the parallel mirror planes. Suppose that the instrument has been adjusted to give good fringes with monochromatic light. The mirror M is then moved slowly through distance d. The optical path of a light ray is increased by the length 2d, the center becomes alternately bright and dark, and at any given point a bright fringe changes from bright to dark to bright again, i.e. a shift of the fringe system occurs. Since each time the physical path difference changes by one half of a wavelength we go from a maximum to a minimum of intensity, another half wavelength change will bring the intensity back to a maximum again. It follows that the slow movement of M will cause a continuous shift of the fringes.

If we can measure the distance d by a micrometer screw we have a simple method of evaluating the wavelength of the light. If k fringes appear at the center (or k changes from bright to bright take place at a given point) while the mirror is moved a distance d, then $k\lambda = 2d$. Conversely, if the wavelength of the light emitted by the source is known, the same relationship provides a precise means of measuring small distances.

These techniques, first used by Michelson in 1880, furnished reliable methods of accurately studying light phenomena. To be consistent with the principle of relativity, a first requirement is that the velocity assigned to light shall be subject to the Einteinian transformation of space and time, when the frame of reference is altered. The relationship to Einstein's Theory of Relativity is best expressed in what is known as his second postulate: "The velocity of light is constant in all inertial systems."

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APPENDIX I

RESULTANT DISPLACEMENT DUE TO TWO PERIODIC WAVE MOTIONS AT THE SAME POINT

The principle of superposition states that the resultant displacement of any point at any instant is the vector sum of the amplitudes of the individual waves at the point in question at that time. In particular, the resultant motion of a particle at a given point H due to two simple periodic motions of the same frequency can be determined by the following argument. The individual displacements would be

$$y_1 = a_1 \sin \omega t;$$

$$y_2 = a_2 \sin (\omega t + \alpha) .$$

The resultant displacement is the vector sum of the individual displacements. This gives

$$\begin{aligned} y &= a_1 \sin \omega t + a_2 \sin (\omega t + \alpha) \\ &= a_1 \sin \omega t + a_2 \sin \omega t \cos \alpha + a_2 \sin \alpha \cos \omega t \\ &= (a_1 + a_2 \cos \alpha) \sin \omega t + a_2 \sin \alpha \cos \omega t \end{aligned}$$

We can reduce y to a more convenient form.

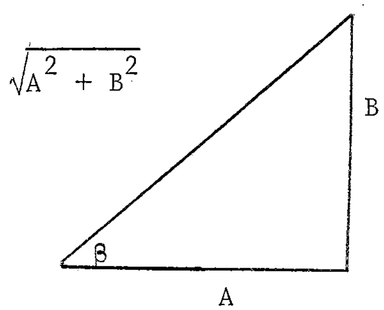
$$Y = A \sin \omega t + B \cos \alpha$$

multiplying by

$$\frac{\sqrt{A^2 + B^2}}{\sqrt{A^2 + B^2}}$$

$$Y = \sqrt{A^2 + B^2} \quad \frac{A}{\sqrt{A^2 + B^2}} \sin \omega t + \frac{B}{\sqrt{A^2 + B^2}} \cos \alpha$$

Referring to the following figure:



$$\cos \beta = \frac{A}{\sqrt{A^2 + B^2}} \quad (1) \qquad \sin \beta = \frac{B}{\sqrt{A^2 + B^2}} \quad (2)$$

$$\beta = \arctan \frac{B}{A} \quad (3)$$

$$Y = \sqrt{A^2 + B^2}\ [\cos \beta \sin \omega t + \sin \beta \cos \omega t]$$

$$= \sqrt{A^2 + B^2}\ \sin (\omega t + \beta)$$

By analogy, for y we can write:

$$A = a_1 + a_2 \cos \alpha \text{ and } B = a_2 \sin \alpha$$

$$y = \sqrt{(a_1 + a_2 \cos \alpha)^2 + (a_2 \sin \alpha)}\ \sin (\omega t + \beta)$$

$$Y = \sqrt{a_1^2 + a_2^2 + 2a_1 a_2 \cos \alpha}\ \sin (\omega t + \beta)$$

The effect at H is such that a particle has a displacement with a period the same as the two component waves, but with a phase determined by β and an amplitude determined by the term under the square root sign.

To obtain the displacement given on page 6

$$y_1 = B \sin \left[\frac{2\pi}{\lambda}(vt - x_1)\right] \text{ and } Y_2 = C \sin \left[\frac{2\pi}{\lambda}(vt - x_2)\right]$$

$$\frac{2\pi}{\lambda}(vt - x_1) = \omega t$$

$$\frac{2\pi}{\lambda}(vt - x_2) = \omega t + \delta$$

$$\delta = \frac{2\pi}{\lambda}(x_2 - x_1)$$

APPENDIX II

INTENSITY AMPLITUDE RELATIONSHIP IN A WAVE

The energy carried by a simple harmonic wave is related to the amplitude of the wave. The energy carried by an optical wave determines the intensity of the light at the point in consideration. We can determine this relationship from the following reasoning:

$$V = A\omega$$

$$V = A\omega \cos \theta$$

$$\cos \theta = \frac{\sqrt{A^2 - y^2}}{A}$$

$$V = \omega\sqrt{A^2 - y^2}$$

(See Figure 2). The instantaneous velocity of the particle moving with simple harmonic motion is:

$$V = \omega\sqrt{A^2 - y^2}$$

The maximum value is ωA when the particle is passing through the equilibrum position, $y = 0$, and it has only kinetic energy. This kinetic energy is proportional to v^2 and, therefore, to A^2.

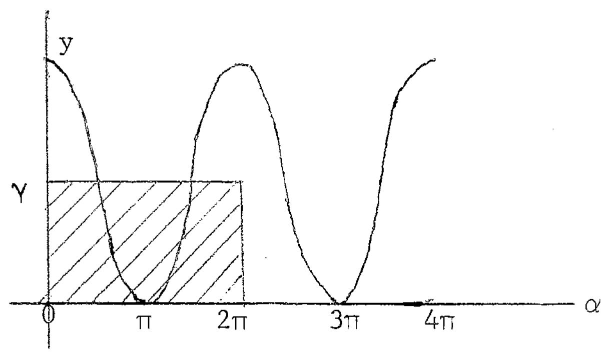
The intensity is defined as the time rate of flow of energy per unit area perpendicular to the direction of propagation and, therefore, proportional to A^2.

$$I = \frac{KE}{t \cdot area} = \frac{A^2}{t \cdot area}$$

APPENDIX III

THE AREA UNDER THE CURVE

Average value of $\cos^2 \alpha$. The function $y = \cos^2 \alpha$ is that shown in the figure, but extending to infinity in both directions.



The area under the curve can be obtained by integration between the limits for which the area is desired. Since the function is periodic it is necessary to work with only one period from 0 to 2π.

If $y = f(x)$, then the area under the curve is

$$A = \int_a^b f(x)dx = (b - a)Y .$$

We know that this area, in fact any area under a curve, is equal to the area of a rectangle of base b and a height Y, where Y is the arithmetic mean of the function between these limits. Thus, we find the area under the curve from 0 to 2 which will be equal to some rectangle of height Y as shown in the figure.

$$A = \int_0^{2\pi} \cos^2 \alpha d\alpha = (2\pi - 0)Y = (1/2\ \alpha + 1/4 \sin 2\alpha) \Big|_0^{2\pi}$$

$$A = \pi$$

therefore

$$2\pi Y = \pi \text{ or } Y = 1/2 \ .$$

Thus we have that the average value of $\cos^2$ function over an integral number of cycles is precisely 1/2.

APPENDIX IV

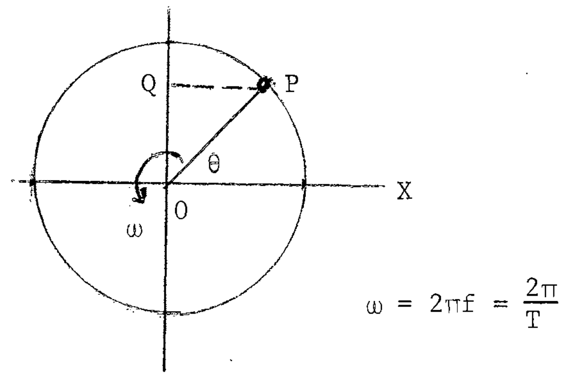


Figure 1. Simple Harmonic Motion.

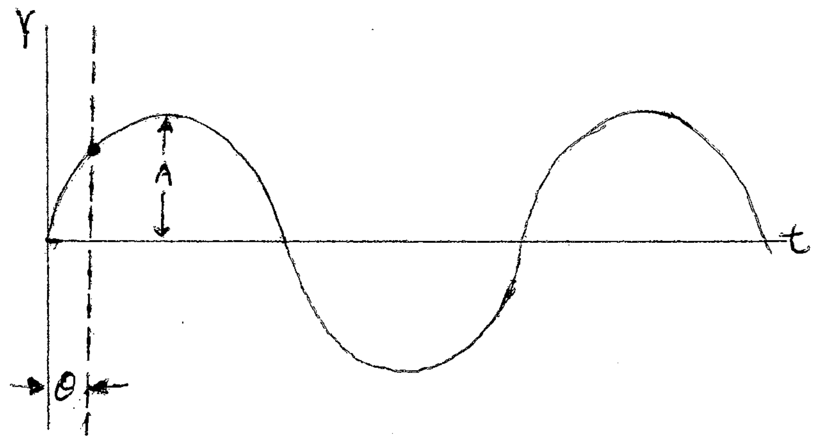


Figure 2. Displacement of Particle Moving with Simple Harmonic Motion.

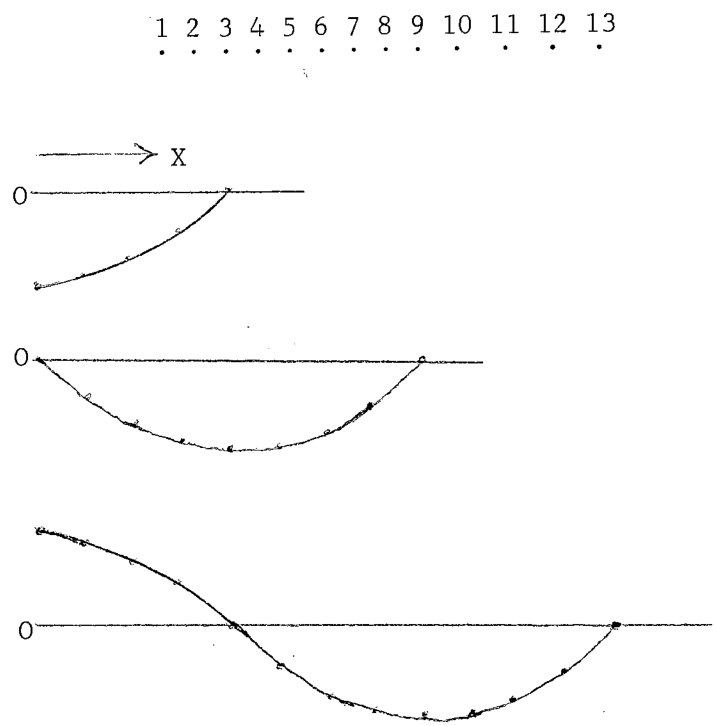


Figure 3. Displacement of Single Oscillatory Particles.

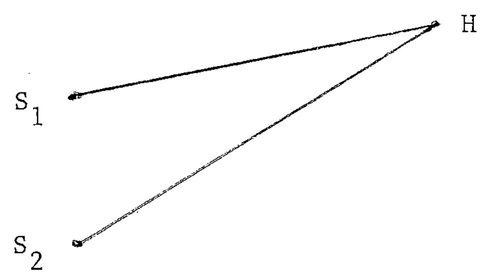


Figure 4. Interference from Two Similar Sources.

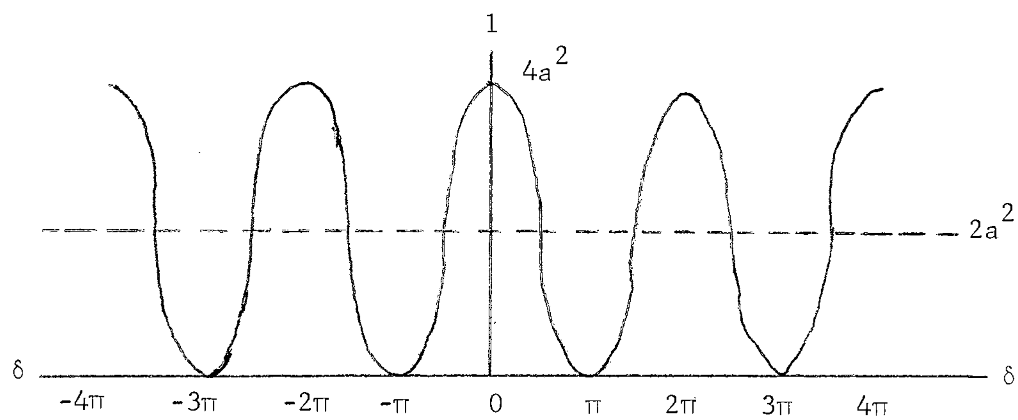


Figure 5. Intensity Distribution Produced by Interference of Two Waves.

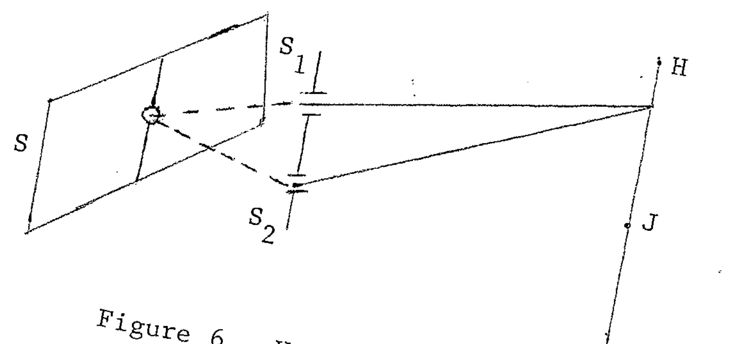


Figure 6. Young's Experiment.

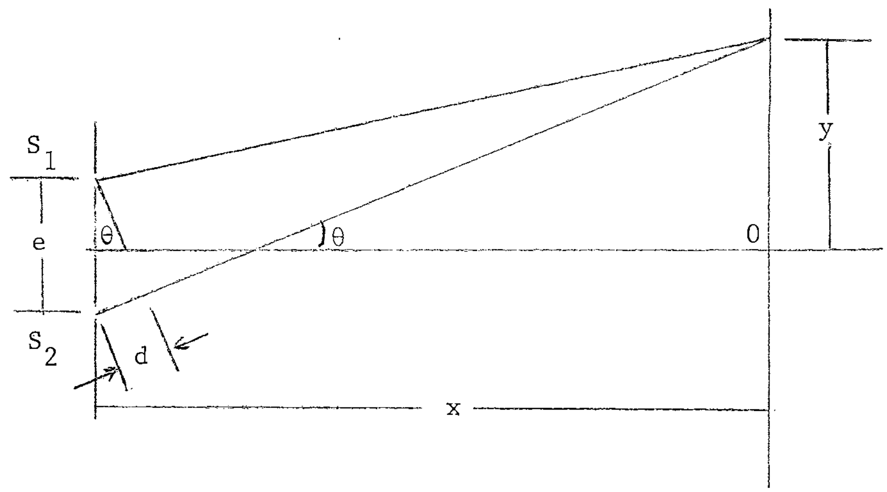


Figure 7. Destructive Interference.

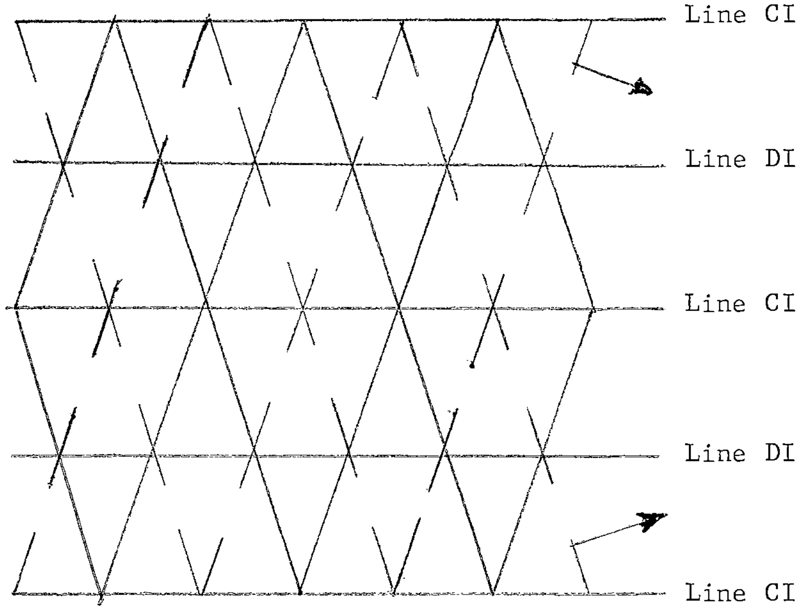


Figure 8. Two Interfering Wave Fronts Making
Small Angles with Each Other.

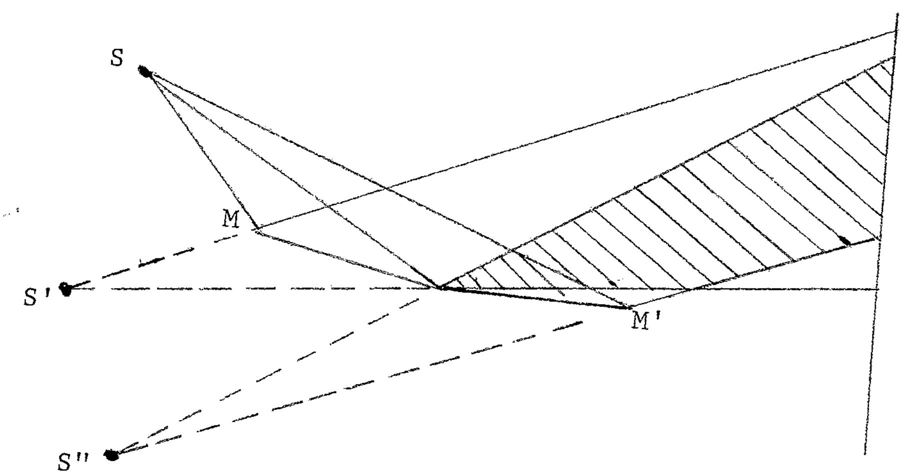


Figure 9. Fresnel Double Mirror.

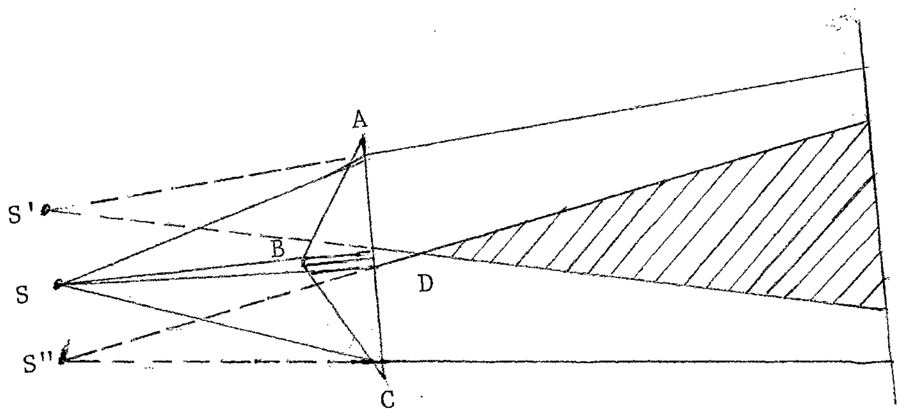


Figure 10. Fresnel Biprism.

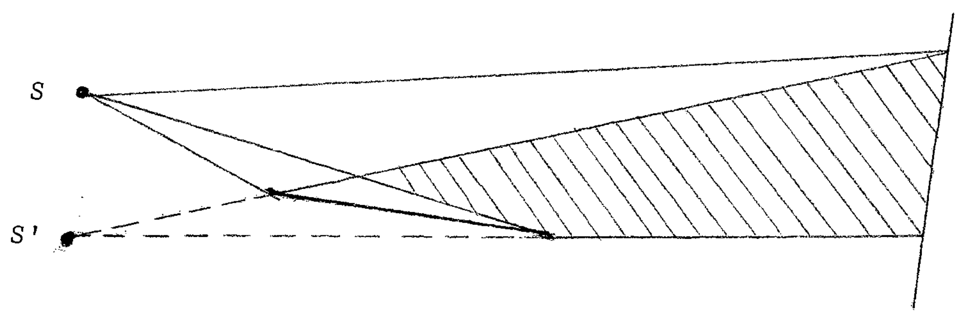


Figure 11. Lloyd's Mirror.

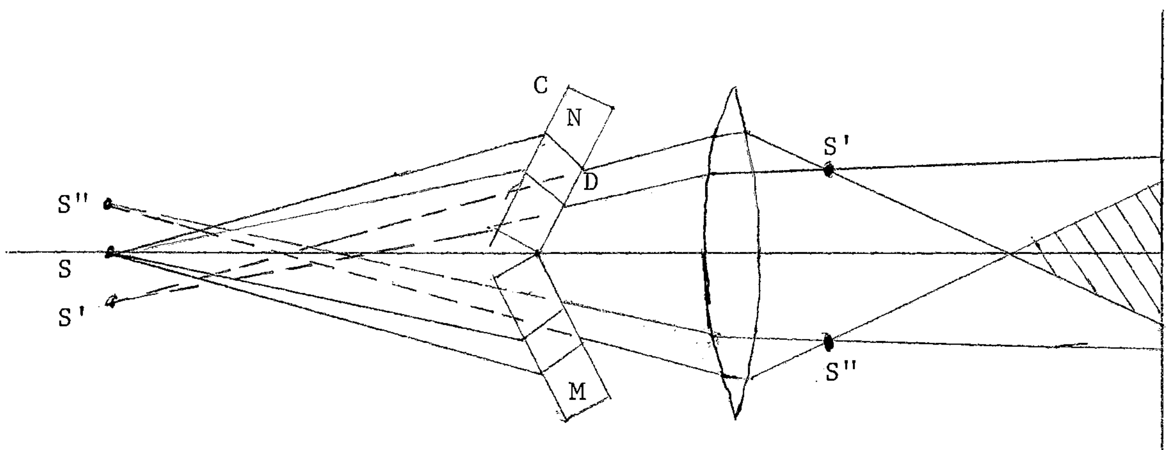


Figure 12. Bi-plates.

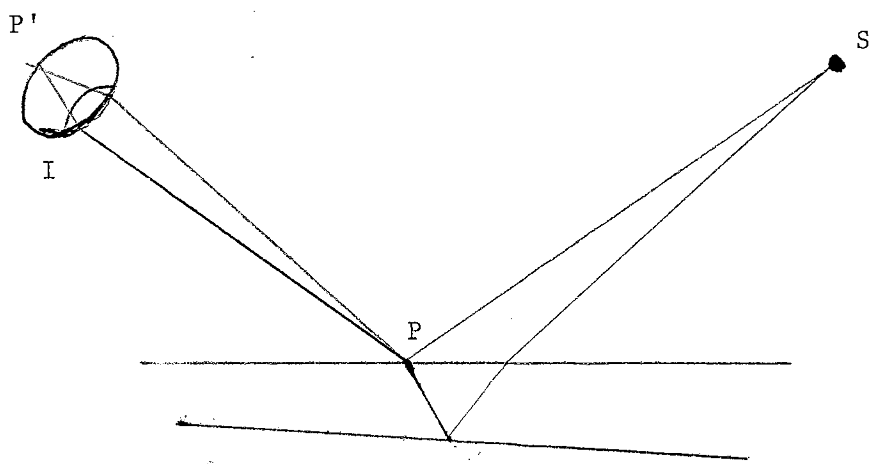


Figure 13. Reflection from a Thin Film.

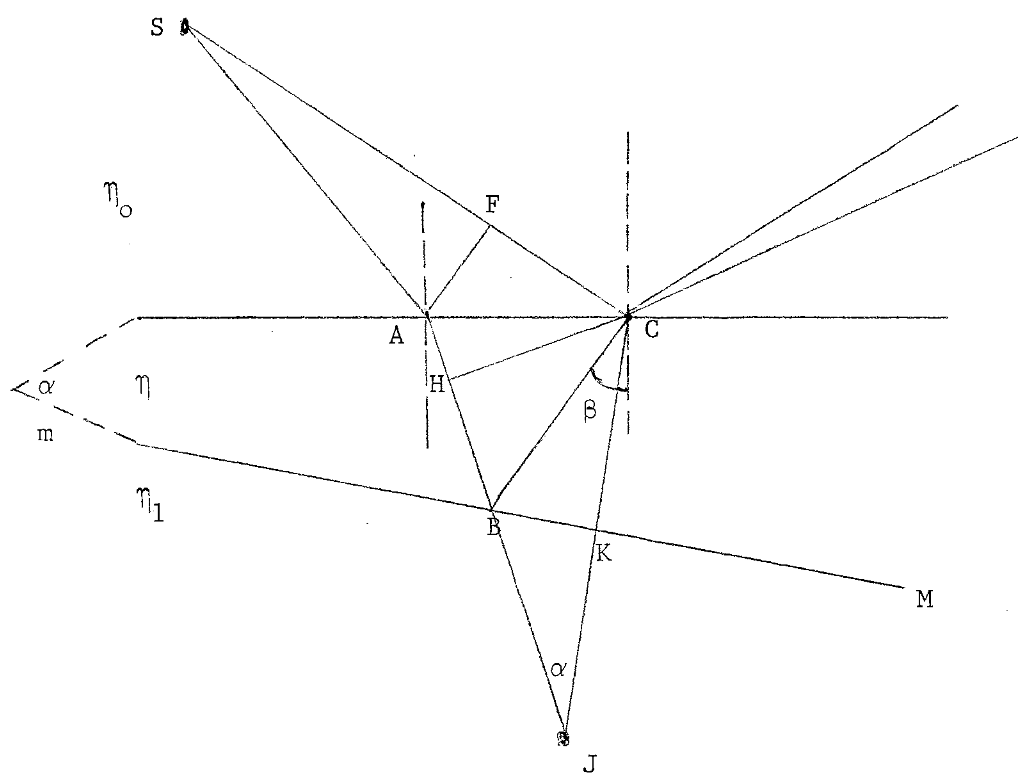


Figure 14. Interference from a Thin Film.

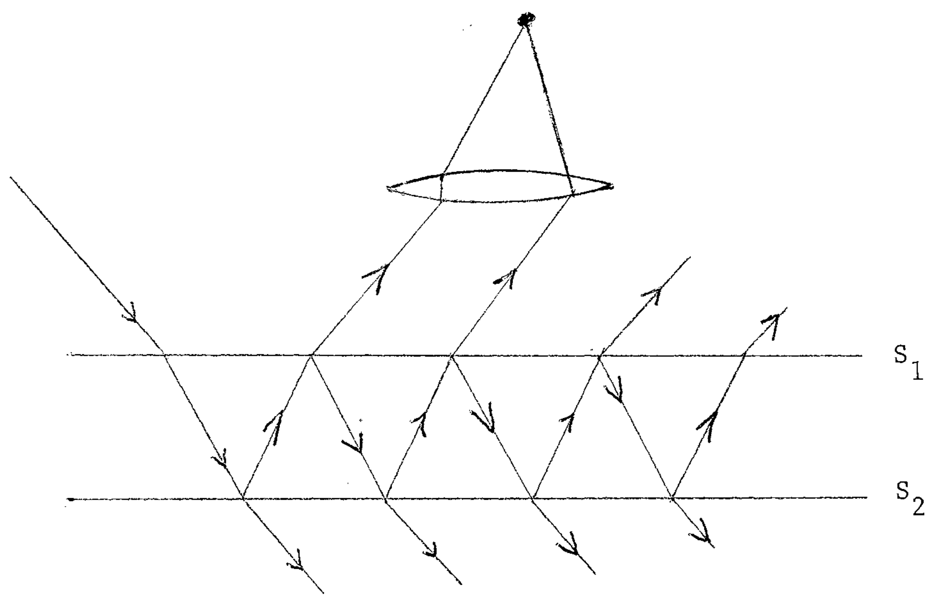


Figure 15. Multiple Reflections in a Plane Parallel Film.

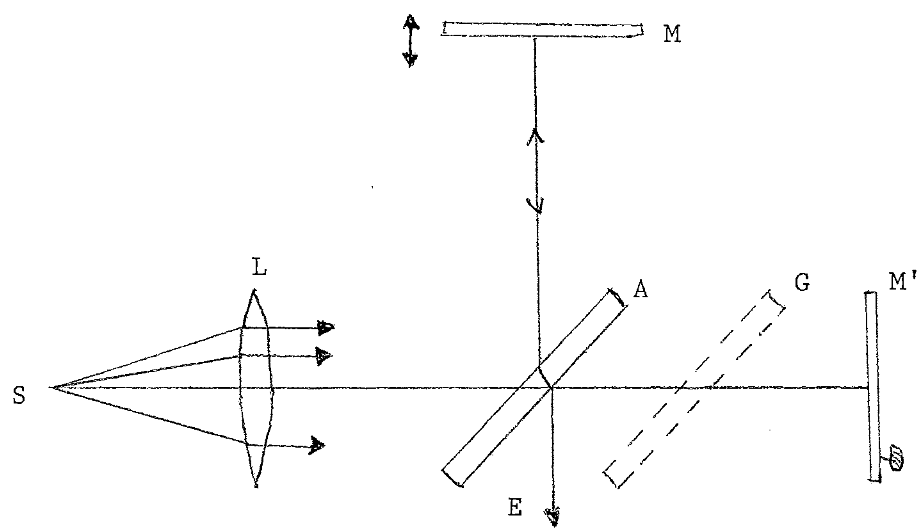


Figure 16. Optical System of the Michelson Interferometer.

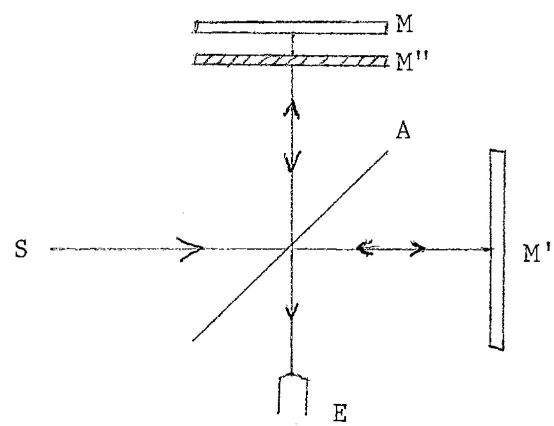


Figure 17. Telescope Used to Detect Interference Patterns.

VITA

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Master of Science

Report: THE INTERFERENCE OF LIGHT

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