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Scope of Study: The object of this study is the prediction of flow rates and pressures in a complex piping network. Use is made of the principles of linear graph theory to derive general system equations to relate flow rate and pressure in a piping network. This method is compared with the method of analysis now being used. It is shown that iteration is required to converge on the correct values. The theory is demonstrated by working both planar and nonplanar examples.

Findings and Conclusions: The method of analysis formulated appears superior to those now being used. Both planar and nonplanar networks can be analyzed by this method without modification.

ADVISER'S APPROVAL ______________________

NONPLANAR PIPING
NETWORK ANALYSIS

By

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NOMENCLATURE

f	Friction factor.
g	Gravitational constant.
h	Pressure loss due to friction of flowing fluid.
h_b	Column matrix denoting branch head losses.
h_{bs}	Column matrix denoting specified branch head losses.
h_c	Column matrix denoting chord head losses.
h_{cs}	Column matrix denoting specified chord head losses.
k	Component equation constant.
k_b	Diagonal matrix denoting branch component equation constants.
k_c	Diagonal matrix denoting chord component equation constants.
q	Flow rate.
q_b	Column matrix denoting branch flow rates.
q_{bs}	Column matrix denoting specified branch flow rates.
q_c	Column matrix denoting chord flow rates.
q_{cs}	Column matrix denoting specified chord flow rates.
x	Exponent of flow rate in component equation.
A, B, E, F, G, J, K, and M . . .	Matrices.
C	Arbitrary constant.
D	Pipe diameter.

H_b . . . . Column matrix denoting elevation head in the branches.

H_c . . . . Column matrix denoting elevation head in the chords.

I . . . . Unity Matrix.

L . . . . Length.

P . . . . Pressure.

T . . . . Superscript denoting transposed matrix.

cfs . . . . Cubic feet per second.

gpm . . . . Gallon per minute.

$|q_b^{x-1}|$ . . . . Diagonal matrix with the absolute value of each element raised to the x-1 power.

ϵ . . . . Pipe roughness.

ν . . . . Kinematic viscosity.

π . . . . Constant. (3.1415926)

INTRODUCTION

Prediction of mean flow and pressure in piping networks is necessary for the proper design and control of these networks. For many years, engineers have used the method of analysis presented by Hardy Cross [1]* to predict the behavior of piping networks. However, one of the drawbacks to this method is the difficulty of analyzing nonplanar networks. A planar network, as opposed to nonplanar, is one which can be mapped onto a plane with no elements crossing.

The topic of this report is the formulation of the system equations for a piping network using a matrix method presented by Koenig and Blackwell [2] founded on the notions of linear graph theory. High speed digital machinery was used to work examples comparing the matrix method to the method presented by Cross. No attempt was made to improve the hydraulic equations relating flow and head loss for individual piping components. Rather, a method of analyzing complex piping networks, using these basic relations, is presented.

The matrix method used in this report can be used to identify an independent set of system equations. Writing these equations is straightforward, and the procedure used will apply to any piping network. Thus, an automatic procedure can be established which can be used each time in analysis of either planar or nonplanar networks.

The Hardy Cross method is an iteration procedure based on identifying each pipe element of a network with a specific loop. As the

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*Numbers in brackets refer to listings in the Bibliography.

Hardy Cross method is usually used, it is necessary to identify each pipe element with only two loops. This restricts the Hardy Cross method to planar networks [3, 4] unless modifications are made.

Two illustrative examples were worked using the matrix method presented. Results for the planar example compared favorably with those obtained by the Hardy Cross method. The same formulation procedure and digital computer program as used for the planar example were used to work a nonplanar example. It is possible, but not practical, to work nonplanar networks using the Hardy Cross method. Consequently, the results of the nonplanar example were not compared with those of any other method. It was found that computer time used to work identical examples by the two methods was less for the program utilizing the linear graph technique.

The symbols used in this report may not in all cases conform to standard notation and may at times be confusing. The reader is referred to the Nomenclature for clarity in these instances.

CHAPTER I

SYSTEM EQUATIONS

The two basic equations of linear graph theory will be used to derive the system equations for a piping network. These equations are the circuit and the cut set equations which can be written in symbolic form as Equations (1.1) and (1.2). [see Appendix A]

$$\begin{bmatrix} K & A & I & O \\ J & B & O & I \end{bmatrix} \begin{bmatrix} h_{bs} \\ h_b \\ h_c \\ h_{cs} \end{bmatrix} = 0 \qquad (1.1)$$

$$\begin{bmatrix} I & O & -K^T & -J^T \\ O & I & -A^T & -B^T \end{bmatrix} \begin{bmatrix} q_{bs} \\ q_b \\ q_c \\ q_{cs} \end{bmatrix} = 0 \qquad (1.2)$$

Where A, B, K, and J are submatrices and the subscripts b, c, and s denote branch, chord, and specified, respectively. The across variable is represented by h (pressure difference), and the through variable is represented by q (flow rate).

1.1 Hydraulics

The specified variable for a piping network may be either pressure or flow rate known at any point in the network. Both variables may be specified with the linear graph technique. However, the usual specified variable for a water or oil piping network is flow rate. Thus, the only specified variable which is used in this report is the through variable, flow rate. When only the through variable is specified, Equations (1. 1) and (1. 2) reduce to Equations (1. 3) and (1. 4).

$$\begin{bmatrix} A & I & O \\ B & O & I \end{bmatrix} \begin{bmatrix} h_b \\ h_c \\ h_{cs} \end{bmatrix} = 0 \qquad (1.3)$$

$$\begin{bmatrix} I & -A^T & -B^T \end{bmatrix} \begin{bmatrix} q_b \\ q_c \\ q_{cs} \end{bmatrix} = 0 \qquad (1.4)$$

If the across variables are to be specified, Equations (1. 3) and (1. 4) would be modified correspondingly.

The flow in many actual networks is greatly influenced by the elevation differences within these networks. If the elevation difference between any two vertices is expressed as a head difference, one can directly add the head difference due to elevation to the friction head in the column matrix of Equation (1. 3) as expressed in Equation (1. 5).

$$\begin{bmatrix} A & I & O \\ B & O & I \end{bmatrix} \begin{bmatrix} h_b + H_b \\ h_c + H_c \\ h_{cs} \end{bmatrix} = 0 \qquad (1.5)$$

H_b and H_c are the heads due to elevation for the branches and chords respectively.

The equation which is often used describing the pressure loss due to friction of flowing water is the Hazen-Williams equation [5].

It can be written as Equation (1.6)

$$h = \frac{4.727\ L}{D^{4.87}} \left(\frac{q}{c}\right)^{1.852} \tag{1.6}$$

where h denotes head loss, q denotes flow rate, L denotes length, D denotes diameter, and C is an arbitrary constant.

Another equation which is often used to relate pressure loss and flow rate is the Darcy-Weisbach equation.

$$h = f\,\frac{8\ L\ q^2}{\pi^2 g^2 D^5} \tag{1.7}$$

The friction factor, f, in this equation is an arbitrary constant determined by the pipe parameters. Moody [6] presented a graph for finding the friction factor based on the open form Colebrook-White function. This graph is usually used to determine the friction factor; however, it is not ideally suited to use on a digital computer. Moody also presented an approximate formula for the friction factor in closed form which closely approximates the Colebrook-White function [7]. Substitution of this approximate f into the Darcy-Weisbach equation results in Equation (1.8).

$$h = 1.393 \times 10^{-4}\left[1 + \left(20{,}000\frac{\epsilon}{D} + 785{,}398\frac{D\nu}{q}\right)^{1/3}\right]\frac{Lq^2}{D^5} \tag{1.8}$$

In Equation (1.8), ϵ and ν represent the pipe roughness and the kinematic viscosity respectively.

Both Equation (1.6) and (1.8) can be expressed in symbolic form as Equation (1.9).

$$h = kq^x \tag{1.9}$$

Nearly all other hydraulic equations relating pressure loss and flow rate, such as those for valves and fittings, can also be expressed as Equation (1.9). In some instances, such as Equation (1.8), the constant k may be dependent upon the flow rate. This will cause no serious difficulty. An iteration process must be used to work the problem, and a new value for k can be found for each flow rate.

1.2 Equation Formulation

Equation (1.9) is to be used as the component equation for a general piping network. This equation can be expressed as Equation (1.10) with the head losses assuming the same sign as the flow.

$$\begin{bmatrix} h_b \\ h_c \end{bmatrix} = \begin{bmatrix} k_b \left|q_b^{x-1}\right| & 0 \\ 0 & k_c \left|q_c^{x-1}\right| \end{bmatrix} \begin{bmatrix} q_b \\ q_c \end{bmatrix} \qquad (1.10)$$

Equations (1.3) and (1.4) can be rewritten as

$$Ah_b + h_c = 0 \qquad (1.11)$$

and

$$q_b = A^T q_c + B^T q_{cs}. \qquad (1.12)$$

Substituting Equation (1.10) into (1.11) gives Equation (1.13).

$$A\, k_b \left|q_b^{x-1}\right| q_b + k_c \left|q_c^{x-1}\right| q_c = 0 \qquad (1.13)$$

Substituting Equation (1.12) into (1.13) and rearranging gives Equation (1.14).

$$\left[Ak_b \left|q_b^{x-1}\right| A^T + k_c \left|q_c^{x-1}\right|\right] q_c = - Ak_b \left|q_b^{x-1}\right| B^T q_{cs} \qquad (1.14)$$

Symbolically Equation (1.14) is

$$\left[G A^T + M\right] q_c = - G B^T q_{cs}. \qquad (1.15)$$

Thus,

$$q_c = - \left[GA^T + M\right]^{-1} \left[GB^T q_{cs}\right]. \quad (1.16)$$

Since the flow in the chord is included in the matrix on the right side of Equation (1.16), it is necessary to assume an initial value for the chord flow and approach the solution by iteration. The iteration procedure will involve the following steps:

1. Assume the flow in each chord.
2. Solve for the branch flow from Equation (1.12).
3. Solve for k if it is a function of the flow.
4. Solve Equation (1.16) for the new chord flow.
5. Substitute the new value for the assumed value of chord flow and repeat the procedure.

If the difference between the assumed chord flow and the calculated value is less than the required accuracy, the solution is complete. If the sign of the calculated chord flow is found to be exactly opposite the assumed value, the iteration process can be speeded up by taking the average of the two as the next assumed value.

If the Darcy-Weisbach equation is used, the component equation constants must be evaluated each iteration since they are dependent upon the flow rate. However, the constants need be calculated only once if the Hazen-Williams equation is used.

From Equation (1.3) we can write Equation (1.17).

$$h_{cs} = - Bh_b \quad (1.17)$$

This is the pressure necessary to sustain the flow which was specified. Knowing the pressure difference across the source necessary for flow and the pressure loss for each pipe element, it is a matter of addition and subtraction to obtain the pressure at any point.

CHAPTER II

ILLUSTRATIVE EXAMPLES

For illustrative purposes, two examples, a planar and nonplanar, were worked. An IBM FORTRAN computer program was written based on the theory presented and used the Hazen-Williams Equation as the component equation. This program, which is listed in Appendix B, was used to work both examples. Bartholet [8] presented another IBM FORTRAN program, using the Hardy Cross method, which was used to compare numerical results.

2.1 Planar Example

The planar example is one which is also worked using the Hardy Cross method by Steel [9]. The example is a three loop water distribution network shown in Figure 2-1. The system parameters are given in Table 2-1 and the linear graph for this network can be represented by Figure 2-2. The darker lines represent branches.

The only equations needed to describe this network are the pipe component equations and either the cut set or circuit equations. The component equation constants can be determined from the system parameters. Writing the circuit equations will develop the submatrices A and B which are utilized in the system equations. The circuit equations are given in Equation (2.1) for this particular tree. The numerical answers will not depend on the tree selection; however, the number of iterations required to reach

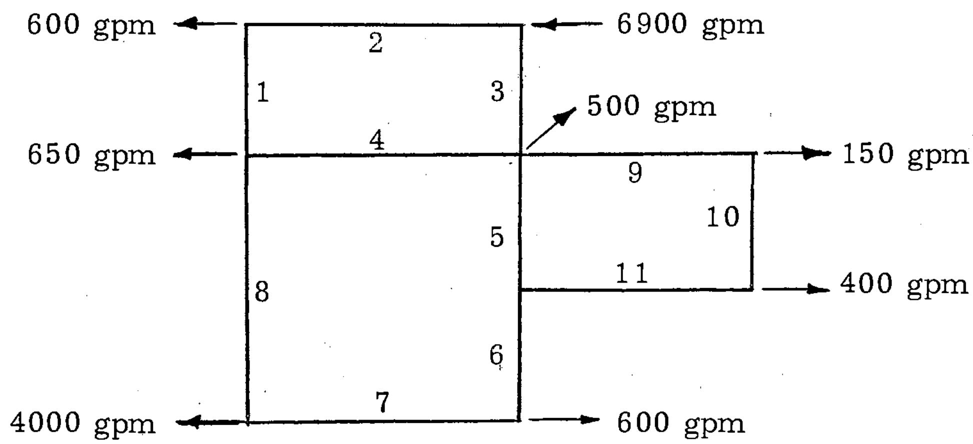

Figure 2-1. Physical System

Pipe No.	Length (ft.)	Diameter (in.)	Pipe No.	Length (ft.)	Diameter (in.)
1	3000	12	7	3000	12
2	3000	12	8	4000	12
3	4000	16	9	1500	8
4	3200	10	10	1000	8
5	1000	14	11	1500	8
6	2000	14			

Table 2-1. System Parameters

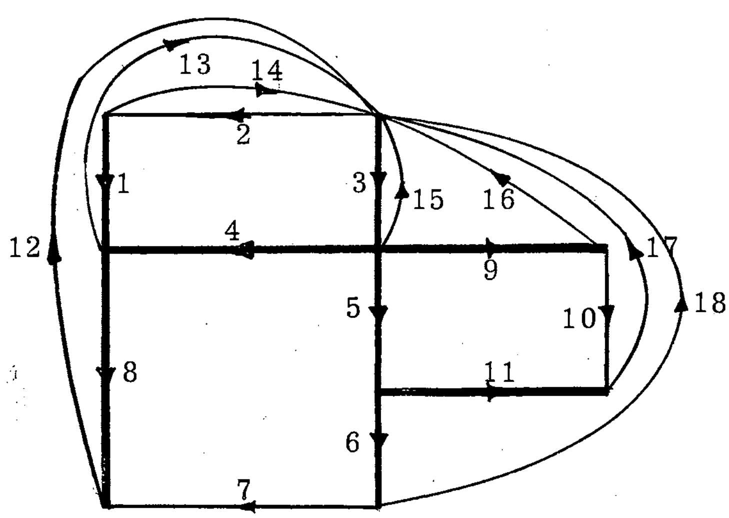

Figure 2-2. System Graph

$$\left[\begin{array}{cccccccc|cccccccccc}
1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 1 & 1 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 0 & -1 & -1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
-1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{array}\right]
\begin{bmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \\ h_5 \\ h_6 \\ h_7 \\ h_8 \\ h_9 \\ h_{10} \\ h_{11} \\ h_{12} \\ h_{13} \\ h_{14} \\ h_{15} \\ h_{16} \\ h_{17} \\ h_{18} \end{bmatrix} = 0 \qquad (2.1)$$

the correct value will. It is usually best to choose a tree with as many chords on the outer edge of the network as possible. As explained in Chapter 1, it is necessary to assume an initial flow in the unspecified chords. The number of iterations required to reach the correct value will depend upon the accuracy of this assumption.

The resulting flows are tabulated for comparison in Table 2-2.

Pipe No.	Flow Rate (gpm)		
	Steel	Hardy Cross	Linear Graph
1	1750	1726	1726
2	2350	2326	2327
3	4550	4573	4573
4	900	880	879
5	2700	2743	2745
6	2650	2642	2643
7	2050	2042	2042
8	1950	1957	1956

Pipe No.	Flow Rate (gpm)		
	Steel	Hardy Cross	Linear Graph
9	450	448	448
10	300	298	298
11	100	101	101

Table 2-2

The example was hand worked by Steel, and the accuracy of convergence required was 50 gpm. Both the Hardy Cross and linear graph results were obtained by use of a high speed digital computer. The accuracy required was 0.01 cfs, or approximately 4.49 gpm. This means that the results obtained from the three sources will not necessarily be the same. However, the results obtained by the linear graph technique are well within the accuracy specified, when compared with the results obtained by the Hardy Cross method or by Steel.

2.2 Nonplanar Example

To illustrate the usefulness of the linear graph technique for nonplanar networks, the network shown in Figure 2-3 was worked. The system parameters are given in Table 2-3 and the linear graph in Figure 2-4.

The required accuracy of iteration for this example was 0.01 cfs. The assumed flows for pipes 6, 7, 8, and 9 were 1000.0 gpm, 3000.0 gpm, 6000.0 gpm, and 3000.0 gpm respectively. Three iterations were required to converge to the values tabulated in Table 2-4.

Pipe No.	Flow Rate (gpm)	Head Loss (ft.)
1	2429.7	103.6
2	2963.5	221.8
3	2984.7	113.7
4	2545.3	334.7
5	2061.4	114.6
6	439.0	4.4
7	4052.4	334.0

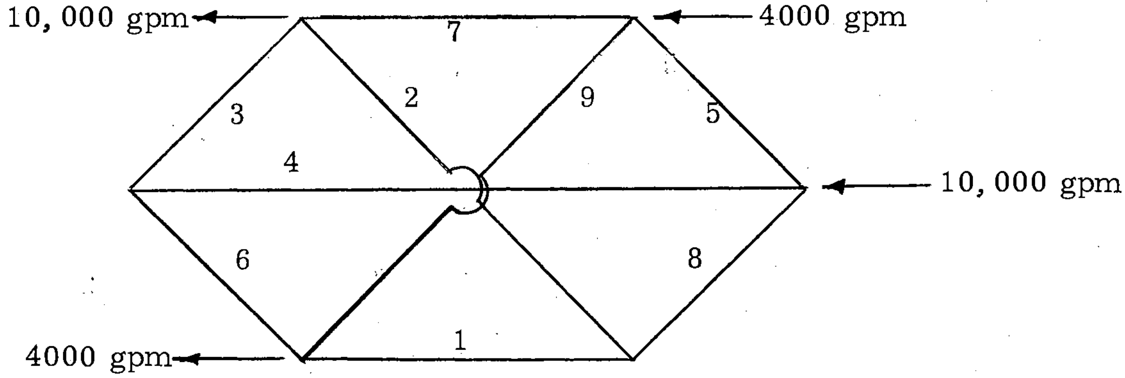

Figure 2-3. Physical System

Pipe No.	Length (ft.)	Diameter (in.)	Pipe No.	Length (ft.)	Diameter (in.)
1	2000	10	6	2000	10
2	1000	8	7	2500	10
3	1500	10	8	1000	10
4	2000	8	9	2000	8
5	3000	10			

Table 2-3. System Parameters

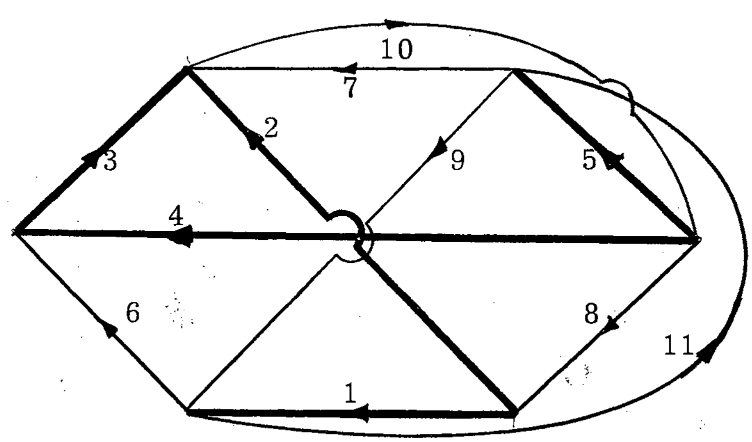

Figure 2-4. System Graph

Pipe No.	Flow Rate (gpm)	Head Loss (ft.)
8	5395.3	226.8
9	2008.7	215.9

Table 2-4

The two criteria which these results must satisfy are: the sum of the flows into or out of a vertex must be zero; and the sum of the head losses around any closed loop must be zero. These values do satisfy these conditions. This nonplanar network was not worked by the Hardy Cross method because it is impractical. Since the results do meet the necessary conditions and no other method was available to work this problem, the results were assumed correct.

CHAPTER III

SUMMARY AND CONCLUSIONS

3.1 Summary

Both planar and nonplanar networks can be analyzed using the linear graph technique. The procedure presented in this report can be used to analyze any piping network for which the basic component equations are known. The Hardy Cross iterative procedure, however, must be modified to work nonplanar networks.

It was found that fewer iterations were required using the linear graph technique if the tree selection was one which included the most possible chords on the outer edge of the network. For this type of tree, total computer time used was less for the program using the linear graph technique than the one using the Hardy Cross method.

Either the cut set or the circuit equations must be written for each network when using the linear graph technique. This can be very easily done after a small amount of practice; however, this means that more data must be assembled for working a problem using the linear graph technique.

For the Hardy Cross method, the flow in each element must be assumed and the flow into or out of each vertex must be zero before the iteration procedure is started. Only the flows in the unspecified chords must be assumed when using the linear graph technique. Due to the more complicated matrix manipulations involved, more space in the computer was needed to formulate and solve the equations for the linear graph method.

3.2 Conclusion

It is concluded that the theory presented in this report could be of significant benefit in repeated analysis of complex piping networks. Due to the necessary matrix inversion, use of digital machinery greatly facilitates the application of the analysis method presented.

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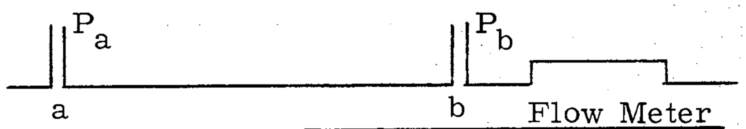
APPENDIX A

INTRODUCTION TO LINEAR GRAPH THEORY

Linear graph theory is a formulation technique for mathematically combining the individual component equations of a physical system. The nomenclature is outlined in the front of this report and is based on that used by Koenig of Blackwell [2]. A complete discussion of linear graph theory can also be found in many other texts dealing with networks and linear graphs [3, 4].

The linear graph technique is based on the graph of the physical system. Each component of a physical system can be represented by an oriented graph, the terminal graph. Two measurements are necessary to specify the state of this two terminal component and the relationship between these fundamental variables is the terminal equation. In the fluid system these variables might be flow rate and pressure. These two variables are known as the through variable and the across variable as described by Firestone [10]. The orientation of the terminal graph determines the reference for the variable measurement. For example, a



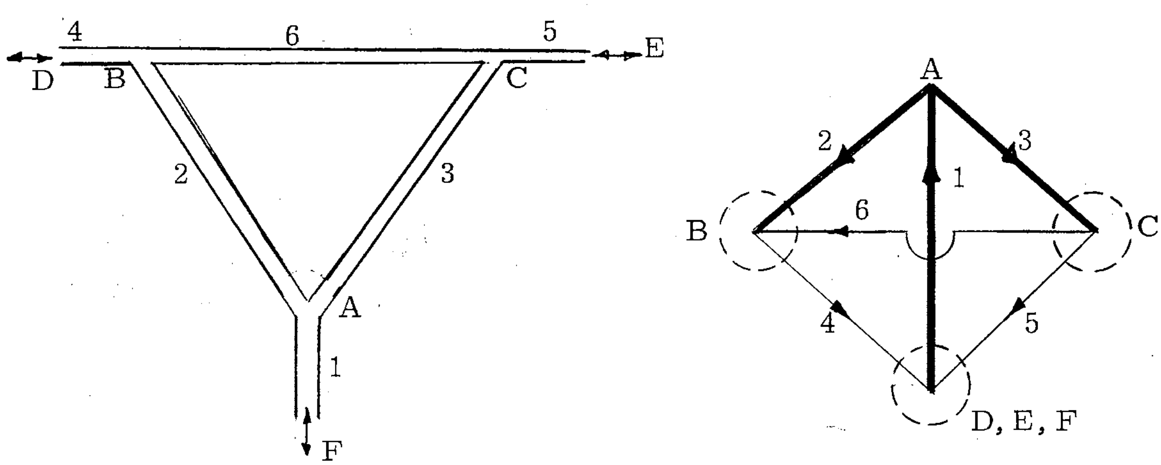
hydraulic element could be represented as $a \rightarrow b$. Positive pressure would be $P = P_a - P_b$ and positive flow would be from a to b.

The physical system consists of a collection of connected components, and the system graph is a collection of connected terminal graphs. A vertex is defined as the end of a terminal graph, this means each junction in the system graph is a vertex. System equation formulation depends upon the two postulates of linear graph theory. Postulate

I states that the sum of the through variables either entering or leaving a vertex is zero. Postulate II states that the sum of the elementary across variables is zero for any oriented closed circuit.

To insure independance of the equations which result from the use of these postulates, the tree of the system graph will be defined. The tree of a connected network is a connected portion of the network containing all of the vertices, but no closed circuits. The elements which form the compliment of the tree are chords. Each chord, together with only branches of the tree, forms a closed circuit whose orientation may be taken as that of the single chord. This idea of defining each chord as an element in only one loop is used to write an independent set of equations called the circuit equations.

To illustrate, consider the system shown in Figure A-1 where D, E, and F are at atmospheric pressure.



Physical System

System Graph

Figure A-1.

The branches are indicated by heavy lines. Postulate II is used to write the circuit equations for this graph using h as the across variable. Using matrix notation and arranging so that the branch variables are first, it is easy to show the independance of these equations:

The circuit equations are

$$\begin{bmatrix} 1 & 1 & 0 & \vdots & 1 & 0 & 0 \\ 1 & 0 & 1 & \vdots & 0 & 1 & 0 \\ 0 & -1 & 1 & \vdots & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \\ h_5 \\ h_6 \end{bmatrix} = 0. \qquad (A.1)$$

Equation (A. 1) can be symbolically represented by Equation (A. 2).

$$\begin{bmatrix} E & I \end{bmatrix} \begin{bmatrix} h_b \\ h_c \end{bmatrix} = 0 \qquad (A.2)$$

Where h_b and h_c represent the across variable for the branches and chords respectively. Since h_b cannot always be expressed in terms of h_c, specified across variables must be included as branches.

Postulate I can be used to determine the cut set equations. The sum of the through variables entering or leaving each cut set must be zero. Cut sets are defined and used with Postulate I to insure the use of an independent set of equations. The cut set can be defined as a portion of the system graph across which only one branch crosses. For convenience, positive flow entering or leaving the cut set will be taken as that of the cut branch. For the example, the cut sets are indicated by the dashed lines in Figure A-1. The cut set equations are given in Equation (A. 3).

$$\begin{bmatrix} 1 & 0 & 0 & \vdots & -1 & -1 & 0 \\ 0 & 1 & 0 & \vdots & -1 & 0 & 1 \\ 0 & 0 & 1 & \vdots & 0 & -1 & -1 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ q_5 \\ q_6 \end{bmatrix} = 0 \qquad (A.3)$$

The independent set of equations in Equation (A. 3) can be symbollically represented by Equation (A. 4).

$$\begin{bmatrix} I & F \end{bmatrix} \begin{bmatrix} q_b \\ q_c \end{bmatrix} = 0 \qquad \text{(A. 4)}$$

Since q_c cannot always be expressed in terms of q_b, all specified through variables must be included as chords.

It can be shown the $E = - F^T$ when both the cut set and circuit equations are written using the same tree. We can now rewrite Equations (A. 2) and (A. 4) as Equations (A. 5) and (A. 6) with the specified variables included.

$$\begin{bmatrix} K & A & I & O \\ J & B & O & I \end{bmatrix} \begin{bmatrix} h_{bs} \\ h_b \\ h_c \\ h_{cs} \end{bmatrix} = 0 \qquad \text{(A. 5)}$$

$$\begin{bmatrix} I & O & -K^T & -J^T \\ O & I & -A^T & -B^T \end{bmatrix} \begin{bmatrix} q_{bs} \\ q_b \\ q_c \\ q_{cs} \end{bmatrix} = 0 \qquad \text{(A. 6)}$$

The subscript s represents specified.

To arrive at Equations (A. 5) and (A. 6) in this form each time, the specified through variables are always placed in the chord system and included as the last elements of the column matrix. The specified across variables are placed in the branch system and included as the first elements of the column matrix.

APPENDIX B

IBM FORTRAN Program

Utilizing Linear Graph Theory

```
      DIMENSION L(10,20),QS(20),BC(20),CC(10)
      DIMENSION QB(20),QC(10),TQB(20),B(11,11)
    1 READ 700,NB,NC,C,FC,E
      DO 8 I=1,NC
      DO 8 J=1,NB
    8 READ 700 ,L(I,J)
      DO 10 I=1,NB
      READ 702,K,D,AL,QS(K)
      QS(K)=QS(K)/FC
   10 BC(K)=4.7256*AL/(((D/12.0)**4.87)*(C**1.852))
      DO 12 I=1,NC
      READ 702,K,D,AL,QC(K)
      PRINT 705,K,QC(K)
      QC(K)=QC(K)/FC
   12 CC(K)=4.7256*AL/(((D/12.0)**4.87)*(C**1.852))
      ITER=0
   15 DO 35 I=1,NB
      QB(I)=QS(I)
      DO 30 J=1,NC
      IF(L(J,I))20,30,25
   20 QB(I)=QB(I)-QC(J)
      GO TO 30
   25 QB(I)=QB(I)+QC(J)
```

```
30 CONTINUE
35 TQB(I)=BC(I)*QB(I)**0.852
   DO 60 I=1,NC
   DO 55 J=1,NC
   B(I,J)=0.0
   DO 50 K=1,NB
   IF(L(I,K))42,50,44
42 IF(L(J,K))48,50,46
44 IF(L(J,K))46,50,48
46 B(I,J)=B(I,J)-TQB(K)
   GO TO 50
48 B(I,J)=B(I,J)+TQB(K)
50 CONTINUE
55 B(J,I)=B(I,J)
60 B(I,I)=B(I,I)+CC(I)*QC(I)**0.852
   N=NC
   B(1,N+1)=1.0
   DO 70 I=1,N
70 B(I+1,N+1)=0.0
   DO 77 K=1,N
   DO 71 J=1,N
71 B(N+1,J)=B(1,J+1)/B(1,1)
   DO 76 I=2,N
   IF(N-K-I+2)72,72,73
72 CONST=B(I,1)
   GO TO 74
73 CONST=-B(I,1)
```

```
74  JCOL=I-1
    DO 75 J=JCOL,N
75  B(I-1,J)=B(I,J+1)+CONST*B(N+1,J)
    DO 76 J=1,JCOL
76  B(I,J)=B(J,I)
77  B(N,N)=B(N+1,N)
    DO 90 I=1,NC
    B(I,N+1)=0.0
    DO 90 J=1,NB
    IF(L(I,J))82,90,84
82  B(I,N+1)=B(I,N+1)+TQB(J)*QS(J)
    GO TO 90
84  B(I,N+1)=B(I,N+1)-TQB(J)*QS(J)
90  CONTINUE
    STOR=0.0
    DO 198 I=1,NC
    B(N+1,I)=0.0
    DO 94 J=1,NC
94  B(N+1,I)=B(N+1,I)+B(I,J)*B(J,N+1)
    H=0.5*(B(N+1,I)+QC(I))
    ANS=ABS(H-QC(I))
96  IF(STOR-ANS)97,98,98
97  STOR=ANS
98  QC(I)=H
198 CONTINUE
    IF(STOR-E)100,100,99
99  ITER=ITER+1
```

```
    PRINT 705,ITER,STOR
    IF(SENSE SWITCH 1)100,15
100 DO 105 I=1,NB
    H=BC(I)*QB(I)*QB(I)**0.852
    QB(I)=QB(I)*FC
105 TYPE 705,I,QB(I),H
    DO 110 I=1,NC
    H=CC(I)*QC(I)*QC(I)**0.852
    QC(I)=QC(I)*FC
110 TYPE 705,I,QC(I),H
    GO TO 1
700 FORMAT (I4,I4,E14.8,E14.8,E14.8)
702 FORMAT (I4,E14.8,E14.8,E14.8)
705 FORMAT (I4,2X F14.3,2X F14.3)
    END
```

VITA

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Master of Science

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