END CORRECTIONS FOR SINGLE AND DOUBLE ORIFICE PARTITIONS IN A CIRCULAR TUBE

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CHAPTER I

INTRODUCTION

The acoustic impedance of a thin, square edged, circular orifice mounted in a circular tube, and that of two orifices of equal diameter mounted in a circular tube were studied. The acoustic impedance of an acoustic element such as an orifice or tube is defined as the complex ratio of the pressure differential across the element (measured in dynes per square centimeter in the cgs system of units) to the volume velocity (measured in cubic centimeters per second) through the element. The real part of this complex ratio is the acoustic resistance of the element and the imaginary part is the acoustic reactance of the element. The reactance is positive for an orifice, and is analogous to inductance in an electrical circuit. This positive reactance is termed the inertance of the element. The resistance of an orifice is analogous to the resistance in an electrical circuit.

The impedance of an orifice is dependent on the geometry of objects near the orifice and the baffling condition. The most familar condition is that of the orifice in an infinite baffle. This case was treated by Lord Rayleigh.[1] The impedance of an orifice or tube is expressed in terms of an end correction. This is the quantity which must be added to the actual length of the tube or orifice

[1] Lord Rayleigh, The Theory of Sound, (Dover Publications, New York, 1945), Vol. 2.

(thickness of the orifice plate) to obtain the correct value of the acoustic impedance.

This study is an experimental determination of the end correction necessary when the orifice is mounted in a circular tube, and also when two orifices are mounted in a circular tube and spaced symmetrically from the center line of the tube. End corrections are determined as a function of the spacing from the center line of the tube. The effect on the end corrections of the other parameters such as the viscosity of the fluid, the ratio of orifice to tube diameter, the frequency of fluid motion in the orifice, and the volume velocity through the orifice are considered. Three orifice diameters were studied. For each diameter several orifice plates were constructed with the orifices spaced at various distances from the center line of the enclosing tube.

Several theoretical investigations into the effect of acoustic interaction between two orifices have been made. Wolff and Malter[2] discussed only the effective inertance between two vibrating rigid, circular disks, whereas Klapman[3] evaluated both the inertance and resistance components of the interaction impedance of two circular pistons vibrating in an infinite baffle. In both cases the mutual impedance of two orifices was evaluated by a direct numerical integration of the pressure at the surface of the circular disk or plane piston of fluid. For the study at hand where the wave length of the

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[2] I. Wolff and L. Malter, "Sound Radiation from a System of Vibrating Circular Diaphragms," Physical Review, 33, No. 6, p. 1061, (1929).

[3] S. L. Klapman, "Interaction Impedance of a System of Circular Pistons," Journal of the Acoustical Society of America, 11, p. 289, Jan. (1940).

sound is long compared to the orifice diameter, Klapman's theory indicates there should be no resistance components of the mutual impedance. Pritchard[4] presents a similar method of determining both inertance and resistance components of the impedance. The solution again is not in closed form. However, it is consistent with that of Klapman.

Theoretical results which are compared to the experimental determinations are derived from considering an incompressible, viscous fluid moving periodically in a tube, and, also, the acoustic radiation from an orifice wherein the medium is compressible and assumed to move with piston like motion in the orifice. Expressions have been presented by Thurston[5] for the impedance per unit length of a tube of infinite length. These expressions when multiplied by the effective length of the orifice or tube (the actual length plus the appropriate end correction) give the values of the resistance and inertance of a tube or orifice.

In the past, there has been remarkable agreement between theory and experimental results from applying this method. The theory was first applied to the case of periodic fluid flow through circular tubes by Thurston.[5] However, this was primarily a study to test the validity of the theory for periodic flow of fluid through very long tubes wherein the end corrections coming from the theory of acoustic radiation are not dominate factors. These developments

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[4] R. L. Pritchard, "Directivity of Acoustic Linear Point Arrays," Technical Memorandum No. 21, Office of Naval Research Contract N50RI-76 Project Order X, Acoustics Research Lab., Harvard University, Cambridge, Mass., p. 134, Jan. 1951.

[5] G. B. Thurston, "Periodic Fluid Flow Through Circular Tubes," Journal of the Acoustical Society of America, 24, No. 6, p. 653, (1952).

will be presented later in more detail. In a later work[6] Thurston studied the periodic flow of fluid through orifices and found that the Rayleigh end correction is applicable to both the inertance and the resistance of the orifice. In the case of an orifice, the end correction is the predominant factor in the effective length of the orifice. Still later, the theory being slightly altered to account for the geometry of the acoustic element studied, the same general approach, by experimentation[7], was proven to be correct, where the periodic flow of fluid through rectangular tubes was studied. The final paper[8] published by Thurston, where the approach again proved successful, described the experimental determination of the end correction needed for an orifice of varying diameter mounted in a circular tube of fixed diameter. This study is yet another experiment where the same general method is applied. The resulting end corrections determined experimentally are compared with those determined theoretically by Ingard[9].

The impedance of orifices has been found to be a nonlinear function of volume velocity beyond a certain critical value of volume velocity.[6] This nonlinearity appears as an odd harmonic distortion of the pressure differential across the orifice. Also, the fundamental sine component of the pressure changes slope at a critical volume

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[6] G. B. Thurston and C. E. Martin, Jr., "Periodic Fluid Flow Through Circular Orifices," Journal of the Acoustical Society of America, 25, No. 1, p. 26, (1953).

[7] J. K. Wood and G. B. Thurston, "Acoustic Impedance of Rectangular Tubes," Journal of the Acoustical Society of America, 25, No. 5, p. 858, (1953).

[8] G. B. Thurston and J. K. Wood, "End corrections for a Concentric Circular Orifice in a Circular Tube," Journal of the Acoustical Society of America, 25, No. 5, p. 861, (1953).

[9] U. Ingard, "On the Theory and Design of Acoustic Resonators," Journal of the Acoustical Society of America, 25, No. 6, p. 1037, (1953).

velocity. This is to say that, given a sinusoidal volume velocity through the orifice, the pressure is not sinusoidal beyond the particular critical volume velocity. A plot of the peak pressure components for a single orifice mounted in a tube as a function of the peak volume velocity through it is shown in Figure 1. In the same figure, a similar plot appears, wherein the pressure components for an orifice pair mounted in a tube are presented as a function of the volume velocity per orifice. It can be seen that at a volume velocity of $U_M = 0.9$ cm^3/sec the fundamental pressure component begins to change slope from first power to second power, and that the third harmonic component of the pressure is of considerable magnitude. At this point the pressure component predominantly is due to a quantity in the pressure-volume velocity relation which varies as the square of the velocity of the volume. If, instead of the total fundamental pressure component being plotted against volume velocity as presented in the figure, the pressure component which is attributable to the resistance (the sine component of the pressure -- assuming the volume velocity to be sine varying) and that due to the inertance (the cosine component of the pressure) were plotted against the volume velocity it could be seen that the resistance component of pressure would change slope from first to second power at a much lower value of volume velocity than that of the total fundamental. The total fundamental component of the pressure as presented has not changed slope before this value of $U_M = 0.9$ cm^3/sec, where the nonlinear pressure component becomes dominant.

It can be seen in the figure that the pressure components of the single orifice are higher than that of the double orifice case.

Two factors are involved which explain a difference in the two cases. The first effect present to account for a difference is that of the compliance or stiffness of the hydrodynamical test system used in making these measurements. This effect evidently was not the predominant factor which influenced the magnitude of the pressure. This effect of system compliance will be discussed in more detail in a later chapter. However, it gives rise to a higher indicated pressure than the actual value. The second effect, which evidently was the dominant cause of the pressure components for the double orifice being higher than those of a single orifice, is in the impedance of the tube. To obtain the same volume velocity per orifice in the double orifice case it is necessary to have twice the total volume velocity as that of a single orifice. Hence the pressure component due to the impedance of the tube will be doubled for the double orifice case and the magnitude of the total component will be larger. It can be seen in the Figure 1 that both fundamental curves approach the same value at $U_M = 6.0$ cm^3/sec. This is the point where the orifice impedance so dominates the total impedance of orifice and tube combined that the effect of the tube is negligible. All of the data in this study were taken in the linear region of the pressure-volume velocity relations where the resistance term is linear and the harmonic distortion is vanishingly small.

CHAPTER II

THEORETICAL CONSIDERATIONS

An expression for the acoustic impedance of a circular tube of infinite length has been presented by Crandall[1] and was later modified by Thurston.[2] Expressions for the inertance and resistance of the tube per unit length are given in the latter's development. It is possible with these expressions to obtain the actual impedance of a tube or orifice of finite length by multiplying the expressions for the impedance per unit length by the effective length of the tube or orifice. To obtain the effective length of the tube or orifice it is necessary to add to the actual length the appropriate end correction. These end corrections affect a correction to the actual length by accounting for the geometry in the vicinity of the end of the orifice or tube, or in other words the baffling condition.

The expressions described above combine a theory for an incompressible fluid moving periodically in an infinite tube with the theory for acoustic radiation. In the theory for acoustic radiation a non-viscous, compressible fluid is taken as the medium. The fluid is assumed to move in a piston-like manner in a circular opening in a plane wall. The incompressible fluid for the long tube theory is

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[1] I. B. Crandall, Theory of Vibrating Systems and Sound, (D. Van Nostrand Co., Inc., 1926), p. 229.

[2] G. B. Thurston, "Periodic Fluid Flow Through Circular Tubes," Journal of the Acoustical Society of America, 24, No. 6, p. 635, (1952).

considered to be a viscous fluid and to move with a non-piston like velocity profile. This study is an extension of the previous applications of this approach to account for the end correction needed because of interaction of orifice and bounding tube walls and also that due to the interaction of one orifice with another and with the bounding tube walls. A development of the theory for the end corrections which is used here will be presented in the following sections.

(a) Hydrodynamic Theory for an Infinite Tube.

Consider the impedance of a tube of infinite length. Two cases must be distinguished according to whether the tube is effectively narrow or wide. The classifications depend not only on the relation of the constants of the fluid and the geometry, but also the frequency with which the liquid moves through the tube. The discriminating factor which determines these cases is

$$(Kr_o) = r_o\left[(-i\omega\rho)/\mu\right]^{1/2} \tag{2.1}$$

where K is the wave number, ρ the density of the fluid, μ the coefficient of viscosity, and r_o the radius of the tube. If (Kr_o) is less than unity the tube is effectively a "narrow" tube wherein the impedance which the tube presents to the driving force is predominantly resistive. In the other case where (Kr_o) is large compared to unity the tube is effectively "wide". Here the impedance of the tube is essentially inertial.

Thurston[2] in his modification of the results presented by Crandall, introduces a factor defined as follows.

$$Y_a = r_o\left[(\omega\rho)/\mu\right]^{1/2}. \tag{2.2}$$

The factor can be thought of as a measure of the relative importance of the boundary layer for a particular tube radius. The thickness of the boundary layer is $\Delta = \left[\frac{2\pi\mu}{\rho\omega}\right]^{\frac{1}{2}}$. Then $Y_a = \frac{r_o(2\pi)^{1/2}}{\Delta}$. It is obvious also that Y_a is the discriminating factor that determines whether the tube is effectively "wide" or "narrow."

The development of relations for an incompressible fluid moving periodically in a tube of infinite length is similar to the development of an equation for the motion of a circular membrane. The axial driving force is $\vec{\Psi}dx$, where $\vec{\Psi}$ is the negative driving force parallel to the axis of the tube. The total driving force on an annular ring of the fluid of volume $2\pi r dr dx$ is $\vec{\Psi}dx \cdot 2\pi r dr$. This is opposed by, one, an inertial reaction $i\omega\rho 2\pi r dr dx$ and, two, a resistive force due to friction $-2\pi r dx \cdot \mu \frac{\partial \dot{\xi}}{\partial x}$. Using the negative pressure gradient, the velocity decreases with r. The net force is then

$$\frac{\partial}{\partial r}\left(-2\pi r dx \mu \frac{\partial \dot{\xi}}{\partial r}\right) dr \tag{2.3}$$

Hence the equation of motion is

$$\left[i\omega\rho - \frac{\mu}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)\right]\dot{\xi} = \vec{\Psi} \tag{2.4}$$

If one recalls that $Y_a = r_o\left[\frac{\rho\omega}{\mu}\right]^{1/2}$ and takes into account the boundary condition that $\dot{\xi} = 0$ at $r = r_o$, the solution is

$$\dot{\xi}(r) = \frac{\Psi r_o}{i\mu Y_a}\left\{1 - \frac{J_o\left[(-1)^{1/2}\frac{r}{r_o}Y_a\right]}{J_o\left[(-1)^{1/2}Y_a\right]}\right\}. \tag{2.5}$$

The impedance per unit length $Z_{\alpha 1} = \Psi/(\pi r_o^2 \dot{\xi}')$, is

$$Z_{\alpha 1} = \frac{i\mu Y_a}{4r_o^4}\left\{1 - \frac{2J_1\left[(-1)^{1/2}Y_a\right]}{(-1)^{1/2}Y_a J_o\left[(-1)^{1/2}Y_a\right]}\right\} \tag{2.6}$$

where $\dot{\xi}'$ is the average velocity over the tube cross-section.

The following approximations may be made. For $Y_a < 1$ the impedance per unit length becomes

$$Z_{\alpha 01} = R_{\alpha 01} + i\omega L_{\alpha 01} \tag{2.7}$$

where $$R_{\alpha 01} = (8\mu)/(\pi r_o^4) \tag{2.8}$$

and $$L_{\alpha 01} = (4/3)(\rho)/(\pi r_o^2). \tag{2.9}$$

For the condition that $Y_a > 10$ these second approximations hold:

$$R_{\alpha m1} = (2)^{1/2}(\mu Y_a)/(\pi r_o^4) \tag{2.10}$$

and

$$L_{\alpha m1} = (\rho)/(\pi r_o^2). \tag{2.11}$$

It is convenient to express the results as follows:

$$\frac{R_\alpha}{R_{\alpha 0}} = \frac{Y_a}{16} \operatorname{Re}\left\{\frac{(i)^{3/2} Y_a J_1[iY_a]}{J_0[iY_a]} - \frac{i}{2} Y_a^2\right\}^{-1} \tag{2.12}$$

and

$$\frac{L_\alpha}{L_{\alpha 0}} = \frac{3}{8} Y_a^2 \operatorname{Im}\left\{\frac{(i)^{3/2} Y_a J_1[iY_a]}{J_0[iY_a]} - \frac{i}{2} Y_a^2\right\}^{-1} \tag{2.13}$$

These functions are plotted in Figures 2 and 3.

To obtain the impedance of a tube, determine $R_{\alpha 0}$ and $L_{\alpha 0}$ from equations (28) and (29) by multiplying these equations by the effective length of the tube. Then, having calculated the Y_a factor, determine $L_\alpha/L_{\alpha 0}$ and $R_\alpha/R_{\alpha 0}$ from the plotted functions. From these calculations come L_α and R_α.

(b) <u>End Corrections for a Single Orifice in an Infinite Baffle</u>

As has been stated above it is necessary to determine the appropriate end corrections to account for the geometry at the ends of the tube or orifice or the baffling conditions. By way of introduction to this matter of end corrections consider the infinite baffle end correction developed by Lord Rayliegh.[3] A development of this is also presented by Kinsler and Frey.[4] This end correction is for the condition that a circular opening be in an infinite baffle. The fluid in the

[3] Lord Rayleigh, <u>The Theory of Sound</u> (Dover Publications, New York, 1945), Vol. 2.

[4] L. E. Kinsler and A. R. Frey, <u>Fundamentals of Acoustics</u>, (John Wiley and Sons, Inc., New York, 1950), p. 187.

circular orifice is assumed to move periodically with plane, piston-like motion. The integrated effect of the acoustic radiation pressure caused by one differential area on another differential area of the piston is determined. The effect of this radiation pressure on the piston due to itself is the same as adding mass to the piston. Using this motion it is possible to determine what length should be added to the length of the tube to account for the effective increase in mass. This added length is the infinite-baffle end correction for one side of the baffle. In cases where there is symmetry of the geometry with respect to the plane of the infinite baffle the correction for one side must be applied to the second side. This is to say, it is necessary to double the end correction, determined as explained above, for one side and use this together with the actual length of the tube in order to determine the effective length of the tube and thence the impedance of the tube.

Although this end correction is derived by considering the effect of radiation pressure on an increase of the mass of the fluid moving in the orifice, that is, an increase of the inertance of the orifice, it has been found by experiment[5] to apply equally well to the resistance of the orifices.

(c) End Corrections for One and Two Orifices in an Infinite Tube

Ingard[6] discussed the interaction between two circular orifices contained in a circular tube. His results are expressed in the convenient form of an end correction. For this reason this experiment

[5] G. B. Thurston and C. E. Martin, Jr., "Periodic Fluid Flow Through Circular Orifices," Journal of the Acoustical Society of America, 25, No. 1 p. 26, (1953).

[6] U. Ingard, "On the Theory and Design of Acoustic Resonators," Journal of the Acoustical Society of America, 25, No. 6, p. 1037, (1953).

has been conducted so as to compare experimental results with those calculated by Ingard.

Ingard introduces the problem by means of a semi-quantitative discussion and gives a qualitative insight into it. Consider Figure 4. Here the two orifices are mounted in a tube. To illustrate the situation, consider these orifices to be mounted in an infinite baffle. If they were spaced far from each other so that the interaction would be negligible, each would have an infinite-baffle total end correction of $\delta_0 = .96(A_0)^{1/2}$ where A_0 is the area of the piston. Each would then have an approximate specific acoustic mass reactance of

$$X = (\rho\omega\delta_0)/(A_0) = (.96\rho\omega)/(A_0)^{1/2}. \qquad (2.14)$$

Now since there are two orifices, as it were, in parallel the combined mass reactance is $\frac{X}{2} = \frac{.48\rho\omega}{(A_0)^{1/2}}$. Suppose that the two orifices are brought together such that they coalesce--forming one orifice of twice the area of a single orifice. The mass reactance of this combination would then be $X' = \frac{.96\rho\omega}{(2A_0)^{1/2}} = (2)^{1/2}\frac{.48\rho\omega}{(A_0)^{1/2}}$. In other words, the mass reactance should increase by a factor of $(2)^{1/2}$ when the two orifices are brought together. A similar qualitative discussion can be given for the case where two orifices are mounted in a tube.

Ingard's approach to the problem is to obtain a solution of the wave equation for the velocity potential. It is assumed there is piston-like motion of the fluid in the orifice. The wave equation is given as

$$\nabla^2\Phi = \frac{1}{c^2}\frac{\partial^2\Phi}{\partial t^2} . \qquad (2.15)$$

The solution is obtained in cylindrical corrdinates and is made up of a combination of solutions of the type:

$$\Phi = \begin{matrix}\cos\\ \sin\end{matrix}(m\phi)\left[A_{mn}J_m(b_{mn}r)+B_{mn}N_m(b_{mn}r)\right]\exp\left[iK_{mn}z-2\pi i\nu t\right]. \quad (2.16)$$

Rather than solve directly for the velocity potential for the case of two orifices in a tube a solution is obtained for one eccentric orifice in a tube. This calculation can be put in the form of an end correction and is plotted in Figure 5. This is then the length which must be added to the actual length of the orifice (thickness of the orifice plate) to obtain the effective length of the orifice. The result is:

$$\delta_{11} = (A_0)^{1/2}\frac{4}{(\pi)^{1/2}}\frac{R}{r_0}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}F_{mn} \quad (2.17)$$

where

$$F_{mn} = \frac{\left[J_1(b_{mn}r_0)\,J_0(b_{mn}a)\right]^2}{\left[J_1(b_{mn}R)\right]^2\left[b_{mn}R\right]^3\left[1-(m)^2/(b_{mn}R)^2\right]} \quad (2.18)$$

and $b_{mn}R$ is the n th root of the equation $J'_{mn}(b_{mn}R)$. Having this, the velocity potential for one eccentrically placed orifice, it is possible to obtain the solution for two orifices by applying the same result to the second orifice. This then gives rise to a second end correction which is applied to a single orifice of a pair of orifices mounted in a tube. The result is as follows and is plotted in Figure 5.

$$\delta_{12} = (A_0)^{1/2}\frac{4}{(\pi)^{1/2}}\frac{R}{r_0}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}(-1)^n F_{mn} \quad (2.19)$$

(d) <u>Combination of The Theories</u>

It is now possible to determine the inertance and resistance of a single orifice in a tube or one of a pair of orifices in a tube. For the single orifice case, combine equation (2.9) and (2.13) with (2.17)

thus:

$$L_\alpha = (L_\alpha / L_{\alpha o}) L_{\alpha o1} (t + 2\delta_{11}) \tag{2.20}$$

and

$$R_\alpha = (R_\alpha / R_{\alpha o}) R_{\alpha o1} (t + 2\delta_{11}) \tag{2.21}$$

To determine the inertance and resistance of a single orifice which is one of a pair of orifices mounted in a tube use equations (2.9) and (2.13) with (2.19) thus:

$$L_\alpha = (L_\alpha / L_{\alpha o}) L_{\alpha o1} (t + 2\delta_{11} + 2\delta_{12}) \tag{2.22}$$

and

$$R_\alpha = (R_\alpha / R_{\alpha o}) R_{\alpha o1} (t + 2\delta_{11} + 2\delta_{12}) \tag{2.23}$$

where t is the length of the orifice or the orifice plate thickness.

CHAPTER III

EXPERIMENTAL EQUIPMENT

(a) The Hydrodynamical Test System.

The hydrodynamical test system is a device used to measure the impedance of acoustic elements. The model used for this experimental work is a modification of the original equipment designed by G. B. Thurston[1]. It is primarially designed for low frequency measurements (less than 700 cps.). The system will measure accurately the pressure and volume velocity of acoustic elements whose dimensions are small compared with the wavelength of the compressional wave set up in the fluid medium. A cross-sectional drawing of the system is shown in Figure 6. The wall of the main test chamber is one inch thick. Its inside diameter is two inches and the depth is three inches. It is made of brass. The Sylphon bellows attached to a geophone type driver provides the motion of the fluid through the orifice. Except for the very small amount of dilatation in the lower chamber, all of the fluid that is displaced by the driving bellows moves through the orifice. Thus the flow through the orifice may be measured at the driver. A second geophone mounted on the same driving shaft is used to measure the velocity of the driving geophone. If the displacement of the driver is $A \cos \omega t$ then the volume velocity set up by the driver is $SA \sin \omega t$ where A is the amplitude and S the effective area of the

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[1] G. B. Thurston, "Apparatus for Absolute Measurement of Analogous Impedance of Acoustic Elements," Jounal of the Acoustical Society of America 24, No. 6, p. 649, (1952).

driver, and ω the angular frequency of the motion.

(b) Calibration of the System.

The system is calibrated for the pressure sensitivity in two ways. The first is what might be referred to as the d.c. method. Here the lower chamber of the system is sealed by a thick plate mounted in the sample holder. The side valve is opened and the water level is adjusted to a selected height in the stand pipe (refer to Figure 5 for the location of the stand pipe). The d.c. potential of the output of the discriminator is read. The water level is then increased anywhere in the range from .5 cm to 10 cm, depending upon how sensitive the pressure pickup is, and the d.c. potential is read again. Hence, knowing the change in potential difference and the corresponding pressure change, the sensitivity of the pressure pickup can be determined. The second method of calibration which was used for most of this work involves the use of a standard tube 6.922 cm long and .707 cm diameter. Knowing the impedance of the tube and setting up a known volume velocity through it, a measurement of the resulting pressure signal gives the sensitivity.

The f.m. pressure system is shown in Figure 7. In adjusting sensitivity the plate separation of the sensing capacitor is changed; the internal variable capacitor of the discriminator then is changed to bring the oscillator back to the 10.7 mc central frequency. When the discriminator is properly adjusted, it has a discriminator characteristic which is linear to within 1% over a range of the characteristic from +10 to -10 volts, and over a range from +25 to -25 volts, the variation of the sensitivity is 6%.

The system is calibrated for volume-velocity sensitivity by using

the standard tube. Knowing the pressure developed for a given potential difference from the geophone flow meter, the volume velocity sensitivity can be determined. It is measured in cm^3/sec/volt.

Also incorporated in the test system (Figure 6) is a device to measure the displacement of the driver. The magnitude of the displacement of the driver is proportional to the charge in the capacitance of the capacitor as shown. This capacitor forms part of the tank circuit of a 4.5 mc oscillator the output of which goes through a frequency modulation discriminator. The electronic circuit is similar to that for the pressure sensitive discriminator of Figure 7. Because of the use made of this attachment, calibration is not necessary.

(c) Auxillary Equipment.

In addition to the equipment described above a cathode summing circuit is used in obtaining some of the data. The schematic diagram for this circuit is shown in Figure 7. This device allows the subtraction of either the total sine-fundamental pressure component, cosine-fundamental, or the total fundamental pressure component from the total pressure signal. Taking the volume velocity as the reference and assigning this to vary as $U_M \sin \omega t$, the driver displacement is $X_M \cos \omega t$. The use of voltages proportional to these quantities together with a phase shift of π radians in the adder circuit allows substration of that particular component of the pressure. This procedure of adding out one of the components of the pressure is accomplished by adjusting the total remaining signal for a minimum.

A General Radio Type 736-A Wave Analyzer was used to measure the pressure signal. A Hewlett Packard Model 400 AB vacuum tube voltmeter was used to measure the volume velocity signal. The wave form was

monitored on a Du Mont Cathode-Ray Oscillograph, Type 304-A. The geophone driver is powered by a Hewlett Packard Model 205 AG Audio Signal Generator.

(d) Analog of the Hydrodynamical Test System.

It is desirable to have an analog of the hydrodynamical test system which would be easier to analyze than the system itself. The most convenient analog available seems to be an electrical analog. The linear acoustic properties of the system and the nonlinear properties of the orifice or other acoustic elements which may be placed in the system are inertance, compliance, and resistance. These properties correspond respectively to inductance, capacitance, and resistance in an electrical circuit.

The acoustic inertance in the system appears by virtue of the mass of the fluid which moves through the orifice, mass of the brass in the capacitive hydrophone pressure detector, mass of fluid in the pressure detector, mass of moving driver assembly, and in general any mass associated with or connected to the system which moves or is capable of motion when the proper force is applied. Figure 6 will clarify where these various components are located.

The acoustic compliance characterizes any portion of the system which reacts to a displacement with a force which is proportional to the displacement. Compliance is found in the driving bellows as well as in the pressure detector bellows. The water is compliant as well as is the brass of the walls of the chamber and the orifice plate itself.

Acoustic resistance is found in the action of the bellows and the motion of the water, either against the brass of the system or in motion relative to itself. The resistance we consider is of such a nature that

a force proportional to the velocity is produced when there is motion at a particular place. This resistance has the property of being capable of dissipating mechanical energy in the same way that electrical resistance dissipates electrical energy.

Figure 8 represents the electrical analog of the hydrodynamic test system. In the analog the electric current will represent the volume velocity of the fluid and electrical potential difference will represent the pressure. Consider first the driver. This is thought of as a source of pressure, P_D . This pressure first acts through an inductor, L_D , which would represent the inertance of the driving bellows and attached moving geophone coil. The compliance of the driver, also reacts to restrain the driving pressure of the source and hence is represented as the capacitance C_D in series with the inductance, L_D . The resistance of the driver, R_D , also resists the motion of the bellows and is also in series with the capacitance and inductance.

The lower chamber of the hydrodynamical test system has resistance and compliance associated with it. Since this is below the orifice and tube combination, these acoustic properties are thought of in light of the electrical analog of the system, act as shunting impedances. These are referred to as R_S the shunting resistance and C_S the shunting compliance. The position of these elements relative to the orifice and the driver is shown in Figure 8.

(e) <u>Measurement of the Internal Impedances of the System.</u>

After arriving at an equivalent circuit which appears to describe the behavior of the system adequately it was found desirable to measure the magnitude of these acoustic properties of the system. Consider first the internal impedance of the driver. If the system is sealed off

with a stiff plate and the system is driven with a low frequency volume velocity, then the pressure measured across such a high impedance as the stiff plate termination is essentially the pressure at the driver. This pressure, P_D, is proportional to the current that flows in the driving coils. Hence P_D may be written as

$$P_D = K_D i_D . \tag{3.1}$$

It was then desired to determine K_D, the electro-mechanical coupling constant. This was done by measuring the pressure and the corresponding current which flowed in the driver coil. This measurement was made both by using a sinusoidally varying current and by applying a direct durrent to the driver.

To determine the resistance of the driver, R_D, the system was left open and the frequency of the driver was adjusted to resonance. The pressure at resonance and also the volume velocity were measured. Since at resonance the reactance is zero, the total pressure measured (from knowing the current in the coil) was that due only to the resistance of the driver. This is given by:

$$R_D = P_D / U_M . \tag{3.2}$$

The values of the inertance of the driver L_D and its compliance C_D were determined next. With the system terminated with a stiff plate and tuned for resonance, it follows that

$$L_D \omega_{01} = 1/(\omega_{01} C_D) \tag{3.3}$$

where ω_{01} is one particular resonant frequency. To obtain two simultaneous equations involving L_D and C_D the system was terminated with a long standard tube of known inertance L_T and resistance R_T. This combination was again tuned for resonance. This arrangement gives a second equation for the resonant condition:

$$(L_T + L_D)\omega_{02} = 1/(\omega_{02} C_D) . \tag{3.4}$$

Solve equations (3.3) and (3.4) for the L_D and C_D. This gives

$$L_D = (L_T \omega_{02}^2)/(\omega_{01}^2 - \omega_{02}^2) \tag{3.5}$$

and

$$C_D = 1/(L_D \omega_{01}^2) \tag{3.6}$$

Perhaps the most important of the parameters of the system is shunting compliance and shunting resistance. These properties were measured by using the cathode adder circuit which will be described in the following chapter. Briefly this method eliminates either the sine or cosine component of the pressure. By measuring the remaining component and knowing the volume velocity and frequency the values of shunting compliance C_S and shunting resistance R_S were determined.

The acoustic parameters of the system were measured by the methods outlined. The results of these measurements are as follows: $R_D = 5.75 \times 10^3$ cgs, $L_D = 1.30 \times 10^2$ cgs, and $C_D = 2.0 \times 10^{-7}$ cgs. For the stiff pressure sensitivity bellows $C_S = 4 \times 10^{-8}$ cgs and $R_S = 2.5 \times 10^4$ cgs. Most of the experimental work here was done with the softer bellows for which $C_S = 10 \times 10^{-8}$ cgs units. All of these measurements were made using water at a frequency of 22 cps.

CHAPTER IV

EXPERIMENTAL METHODS

(a) Resonant Circuit Method.

The resonant circuit method of taking data was used for most of the measurements made in this study. The method can be explained best in light of the electrical analog of the hydrodynamical test system. Recall that associated with the orifice and tube is a mass reactance and a resistance. If an air volume is placed above the orifice and tube combination as shown in Figure 6, a compliance is placed in series with the inertance of the orifice and tube. The resulting LCR circuit can then be tuned for resonance by adjusting the volume of air. This is done by withdrawing a known amount of water from the cone. The pressure component at resonance is totally resistive, being the sum of the resistance of the orifice of the tube, and of the air volume. An advantage of this method of taking data is that the effect of shunting impedance on the measured value of the pressure is almost negligible in most cases.

The cone used for the purpose described above has an interior angle of 53°. It is two inches in diameter at its base, two inches in height, and is mounted at the end of a four inch glass tube. Theoretical determinations for the compliance C_E and resistance R_E of an air volume have been developed by Daniels.[1]

[1] F. Daniels, "Acoustical Impedances of Enclosures," Journal of the Acoustical Society of America, 19, No. 4, p. 569 (1947).

Figures 9 and 10 are plots of the compliance and resistance of the cone used. These curves were computed by using the following constants of air: thermal conductivity $k = 5.706 \times 10^{-5}$ cal/cm sec °C; pressure (atmospheric) $p_o = 0.9812 \times 10^6$ dynes/cm^2; density $\rho = 0.001155$ gm/cm^3; ratio of the specific heats $\gamma = 1.403$; the specific heat at constant pressure $C_p = 0.240$ cal/gm °C.

The measured values of the inertance and resistance of a single orifice plate will be referred to as L_α and R_α. In the case where measurements are being made on a double-orifice plate sample it must be noted and distinguished that here the total measured resistance or inertance of the orifice plate is one-half of the value of one of the orifices taken singly. This can be more easily seen in light of the electrical analog of the system wherein the orifices of a double orifice combination are thought to be in parallel. For the double-orifice cases the inertance and resistance shall be referred to as $L_{\alpha 2}$ and $R_{\alpha 2}$, respectively. These are the total measured values of the double-orifice plate.

At resonance the reactance is zero. This gives the relation $\omega_o = (LC_E)^{-1/2}$. If C_E and ω_o are known, then L may be computed. If the inertance of the tube, L_T, is subtracted from this total computed inertance the remainder is the inertance of the single-orifice plate, L_α, or $L_{\alpha 2}$ for the double-orifice plate. Referring to the equations (2.20) and (2.22) of Chapter II on theoretical considerations, the inertance of the single orifice plate is

$$L_\alpha = (L_\alpha / L_{\alpha o}) \cdot L_{\alpha o1} (t + 2\delta_{11}) \tag{4.1}$$

and for the double-orifice plate, accounting for the fact that there are two orifices in parallel:

$$L'_{\alpha 2} = (1/2)\left[(L_\alpha / L_{\alpha 0})\, L_{\alpha 01}(t + 2\delta_{11} + 2\delta_{12})\right]. \tag{4.2}$$

If the frequency of the volume velocity, the dimensions of the orifice and the constants of the fluid are known, the Y_a factor can be computed from equation (2.2). This will give $L_\alpha / L_{\alpha 0}$ Also, having the orifice dimensions, $L_{\alpha 01}$ can be computed from equation (2.9). With these parameters the experimental value of the end correction can be determined. This would come from the following equations:

$$\delta_{11} = (1/2)\left[\frac{L_\alpha}{(L_\alpha / L_{\alpha 0})\, L_{\alpha 01}} - t\right] \tag{4.3}$$

and

$$\delta_{11} + \delta_{12} = (1/2)\left[\frac{L_{\alpha 2}}{(1/2)(L_\alpha / L_{\alpha 0})\, L_{\alpha 01}} - t\right] \tag{4.4}$$

for the single and double orifice cases respectively.

Having calibrated the system for pressure sensitivity, as discussed previously, it is possible to obtain the measured value of total resistance of the orifice-tube combination. Knowing the pressure and the volume velocity, this resistance is computed from $R = P_s / U_M$. From this total resistance, the resistance of the tube R_T is subtracted and the remainder is the resistance R_α of the single-orifice plate, or is $R_{\alpha 2}$ of the double-orifice plate. Knowing the frequency, dimensions of the orifice, constants of the

fluid, and the measured value of the resistance of the orifice plate, it is possible to compute the resistance end correction. To distinguish between the end correction computed from the inertance measurements and that computed from the resistance measurements, the resistance end correction shall be denoted by Δ_{11} for the single orifice and Δ_{12} for the end correction due to interaction between two orifices. These will correspond to the end corrections for inertance δ_{11} and δ_{12}. Referring to equations (2.21) and (2.23) for the single- and double-orifice cases the end corrections are given by:

$$\Delta_{11} = (1/2)\left[\frac{R_\alpha}{(R_\alpha/R_{\alpha 0})\,R_{\alpha 01}} - t\right] \tag{4.5}$$

and

$$\Delta_{11} + \Delta_{12} = (1/2)\left[\frac{R_{\alpha 2}}{(1/2)(R_\alpha/R_{\alpha 0})\,R_{\alpha 01}} - t\right] \tag{4.6}$$

respectively. Again as in the case of the inertance measurement the 1/2 factor is used for the double orifice case.

(b) <u>The Subtraction Method.</u>

In a second method of taking the data the cathode adder circuit described earlier is used. This method involves taking the signal from the volume velocity monitor or the displacement monitoring discriminator, shifting the phase π radians, and adding a portion of this to the total pressure signal. The remaining component can then be determined. If one knows the volume velocity, either the resistance or inertance can be determined. Once these values are

obtained the end corrections may be computed from equations (4.2) and (4.4), for the double-orifice case and from (4.1) and (4.3) for the single-orifice case.

This method of taking the data is very convenient to use. It has the disadvantage that the shunting impedance is an important factor in the final measured pressure, if the anti-resonant frequency is less than approximately five times the frequency of the volume velocity.

(c) Corrections Necessary to Account for System Interaction.

It was found necessary to correct for the shunting impedance of the system for some of the measurements taken. This was done for the study of the effect of frequency. It was necessary to make this correction because, though small at 22 cps where most of this work was done, the effect of shunting impedance increases with frequency. In these measurements water was used as the fluid and hence the resistive component of the pressure was very small. The shunting resistance was also neglected. Instead of determining the inertance of the orifice, as would be the case if the shunting impedance were very large compared to the inertance of the orifice, the impedance of the parallel combination of orifice inertance and shunting compliance was determined. Knowing the shunting impedance from previous measurement, it was possible to compute the inertance of the orifice. This is done with equation (4.7).

$$1/(iX_L) = 1/Z - 1/(iX_S) \tag{4.7}$$

where X_L is the reactance of the orifice, Z the measured impedance

and X_s the reactance of the system. Knowing the angular frequency, the inertance is computed. The determination of the end correction follows in the same way as described previously.

CHAPTER V

PRESENTATION OF THE DATA

Pressure-volume velocity data were taken for the double and single orifice plates for the three values of the ratio of orifice radius to containing tube radius (r_o/R). These were reduced and presented in the form of end corrections as functions of the ratio of orifice separation to tube radius (a/R). The major parameters considered were the ratio of orifice separation to tube radius (a/R) and second the Y_a variable. Variations of the Y_a variable actually involve the variation of four other parameters. These are the orifice diameter, the frequency of excursion of the fluid in the orifice, the density of the fluid, and the viscosity of the fluid. The latter two variables were actually taken as one, the kinematic viscosity of the fluid, or the ratio of the dynamic viscosity of the fluid, to the density of the fluid.

(a) Geometry Study

Experimental values of the resistance and inertance end corrections for the double orifice ($\frac{\delta_{11}+\delta_{12}}{\sqrt{A_o}}$, $\frac{\Delta_{11}+\Delta_{12}}{\sqrt{A_o}}$) and also for the single orifice sets ($\frac{\delta_{11}}{\sqrt{A_o}}$, $\frac{\Delta_{11}}{\sqrt{A_o}}$) as functions of the ratio of orifice separation to tube radius (a/R) are presented in Figures 11 and 12 respectively for the three orifice sizes

($r_o/R = .15$, $r_o/R = .075$, $r_o/R = .0375$). Here the fluid was water at approximately 25° C temperature, and the frequency of the fluid motion was 22 cps.

Consider first the end corrections for the single eccentric orifice as shown in Figure 11. It is to be noted that common to all the curves is the general shape. As the center of the orifice changes in position away from the center of the tube, the end correction increases (increasing a/R). This is the same, of course, as saying that the inertance and resistance has increased. The change in end correction with separation from the center of the tube is more pronounced for the case of the larger orifice diameter. As the orifice moves out the end correction increases until at the wall of the tube, the end correction is a maximum. For the intermediate size orifice the effect due to the closure of orifice with the wall is not apparent till the separation has increased beyond the point where this effect is noticeable for the larger orifice. Similarly the effect of interaction of wall and orifice for the smallest orifice is evident only at greater distances from the center of the tube. This is reasonable since the larger orifice dominates a larger percentage of the total area of the orifice plate contained in the tube and hence would experience the proximity of the wall at a lesser distance from the center than would a smaller orifice.

It should be noted that the end corrections for resistance and inertance are about the same magnitude. This indicates that the same end correction can be applied to compute the resistance as well as the inertance of an orifice. There is doubt that the curves for resistance and inertance end corrections should have different shapes for a particular orifice size. Any difference in shape of the curves presented here is probably due to experimental error.

The Y_a range covered in these data was 4, 8, and 16 for the small, intermediate, and large orifices respectively. It is probable that the end correction effectively does not change form with Y_a factor. These factors substantiate the validity of the theoretical approach to the problem. It is hence apparently correct to state that the orifice may be represented as a tube of finite length; the length being the sum of the end correction and thickness of orifice plate.

Consider now the data taken on the double orifice pair sets. The same range was covered as for the single orifices. The orifices were spaced symmetrically from the center of the tube. The resulting experimental end corrections are presented in Figure 12 for the resistance and inertance end correction. It can be seen from the graphs that the end corrections for the resistance and inertance are of essentially the same magnitude and shape.

The graphs indicate the fact that interaction takes place when the two orifices are in close proximity to one another and this effect of interaction falls off as the orifices are separated. The end corrections pass through a minimum, and then increase in magnitude as the effect of interaction with the walls of the containing tube becomes more pronounced.

As with the case of the single orifice the amount of interaction observed, whether with the walls of the tube or with the other orifice, decreases with orifice diameter. For the case of the smaller diameter, or smaller ratio of r_o/R, the end correction curves both resistance and inertance are essentially flat between the range of values from $a/R = .2$ to $a/R = .6$. This value of end correction approaches the value of end correction for an orifice in an infinite baffle as what might be expected. This value is within 20% of the value which is the same error apparent in all this resonate circuit data.

It was noticed that there were places in the curves where more data points were needed. New samples were fabricated and measurements were made on these with the thought of filling in the needed points. These data were somewhat rough. As a result, between the points in the portions of the curves where some of these supplementary data points appear, dashed lines which serve to indicate a trend and also that there is uncertainty in the actual shape. Points on either side

of the added data points were repeated and these were used to indicate the amount of adjustment needed in the magnitude of the new points to accomplish a satisfactory fit for new points.

Data on the composite graphs of Figures 11 and 12 were not all taken on the same day, or if they were on the same day, not at the same time. Some adjustment was necessary because it is impossible to keep all of the parameters of the experiment exactly the same when the time needed for taking the data is relatively long. For this reason there was not consistency in the order of which the individual plots appeared. To adjust for this and strive to present the graphs in the most correct order, a set of data was taken to establish this correct order of the curves associated with the three values of r_o/R. To do this one sample was taken from each set of orifice samples giving the three values of r_o/R. Measurements of pressure and volume velocity were made keeping the volume velocity the same for each sample. The only points in this set of data which had the correct order were those for the inertance end correction where $a/R=0$ for the single orifice set. This correct order follows from the theoretical and experimental values of the end correction for concentrically mounted orifices in a tube determined respectively by Ingard[1] and Thurston[2].

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[1] U. Ingard, "On the Theory and Design of Acoustic Resonators", Journal of the Acoustical Society of America, 25, No. 6, p. 1037, (1935)

[2] G. B. Thurston and J. K. Wood, "End Corrections for a Concentric Circular Orifice in a Circular Tube", Journal of the Acoustical Society of America, 25, No. 5, p. 861, (1953).

(b) Frequency Study

Measurements were made wherein the Y_a factor was varied by varying the frequency of excitation of the fluid in the orifice. To make these measurements an eccentric orifice and the corresponding double orifice samples were chosen. These orifices had ratio $r_o/R = .15$ and $a/R = .159$. The frequency was varied from 20 to 100 cps. Only the inertance end corrections were determined. These are presented in Figure 13.

The data presented have been corrected by assuming the system impedance to be a shunting compliance. It is felt that the reason the end corrections do not have a constant value as a function of frequency can be attributed to the fact that the effect of system shunting resistance becomes more pronounced with increasing frequency. This effect of shunting resistance was not taken into account when correcting these data. In this study varying the frequency over the range of 80 cps, caused the Y_a factor to vary about 9 units. The result of this study substantiated again the correctness of the theoretical approach.

(c) Viscosity Studies

Using a glycerol solution of known concentration measurements of the inertance end corrections for the double and single

orifices of $r_o/R = .15$ as functions of a/R were made. The curves are shown in Figure 14.

The resulting curves have the same general shape as those of water. This experiment presents further proof of the soundness of the theoretical approach to the problem, and indicates the effect of viscosity on the acoustic inertance of an orifice is entirely described by the Y_a factor.

(d) Discussion of the Errors

Because the resistance component was very small compared to the inertance component of the pressure in the linear pressure region, it was difficult to obtain data by the adder circuit method and thus the resonate circuit method was employed for the majority of the data taken in this study. It will be recalled that the advantage of the resonant circuit method was that the pressure signal to noise ratio that was present in the cathode circuit adder circuit method of taking the data was not present with this resonate circuit method.

In Figure 15 is presented the inertance end corrections for $r_o/R = .15$ (for double and single orifices) as a function of These data were taken using the cathode adder circuit. It will be noticed that the value of the point at $a/R = .4$ (single orifice)

is approximately 20% less than the corresponding point in Figure 11, where the data were taken by the resonate circuit method. A similar discrepancy can be noticed between the data taken to study the effects of viscosity and frequency as presented in Figures 14 and 13 and the data of Figure 11.

This discrepancy must be explained, however, no complete explanation has been reached. An attempt was made to correct this resonate circuit data by a consideration of an equivalent circuit which would take into account the resistance and compliance of the system. The resonant condition instead of being considered to be simply the resonance of the compliance of the air volume and the inertance of the orifice was taken to be the resonance of the inertance of the orifice and the air volume compliance and also the compliance of the system. This condition was expressed by letting the phase angle of the total effective impedance of the equivalent circuit equal zero. A sample calculation using this method brought about a correction to the impedance of the orifice which affected a decrease in the total magnitude by about 5% of the uncorrected value. This was not adequate to account for the approximate 20% variation between the resonate circuit method and the cathode adder method of taking the data.

It is felt that this variation should be thought of as an error in the resonate circuit method of taking the data. The reason for this assumption is the fact that the values for the end correction for concentrically mounted orifices in a tube have been determined theoretically by Ingard[1], and these values were experimentally validated by Thurston[2] as mentioned before. In this experimental work where the a/R variable equals zero is the case where the orifices are concentrically located with respect to the center of the containing tube. The end corrections at this point should be equal to those determined by Ingard and Thurston. When the cathode adder circuit method was employed in taking the data these correct values were obtained for these concentric points to within about 5% of the correct value. This provided more evidence of the assumption that the data of the resonate circuit method were in error. Certainly there are other factors present which effect the magnitude of the end correction other than those due to the properties of the hydrodynamic test system. It can be seen that the error of approximately 20% is a constant error and causes the value of the end correction to come out too large consistantly. Such an error might be due to a constant error introduced while taking the data. This might mean that the pressure measuring equipment used to determine the pressure

might have been in error, however, such a possibility is unlikely as the same equipment was used to take the similar readings with the other method of taking the data, and here such an error was not apparent. Also if these were error in the meters or the associated electronic equipment which might introduce an error, then the large discrepancies would appear only in the resistance end corrections and not in the inertance end correction. This is not the case, however as the large error is present in both the inertance and resistance end corrections.

The graphs of the values of the compliance and resistance of the air volumes appear in Figures 9 and 10. These are derived from a theoretical expression and used in determining the resistance and inertance of the orifice as described before. It is possible that these curves are erroneous but these were scrutinized and no large errors were found. All of these factors which could introduce constant errors in both the resistance and inertance of the orifice could possibly be the cause for the major portion of the large experimental error. It is possible to suppose that all of these factors taken collectively in the proper fashion could explain the large error.

CHAPTER VI

CONCLUSIONS AND RESULTS

A comparison of the experimental results and the theoretical predictions is now possible. Ingard has plotted the end correction for inertance for the case of the ratio of the orifice radius to containing tube radius, $r_0/R = .15$ as a function of a/R for double and single orifices. Presented in Figures 11 and 12 are the experimental end corrections for inertance and resistance end corrections and in Figure 5 are the theoretical curves. It will be noted in comparing these curves that there are marked differences in the shapes. Both the theoretical curves for the single orifice and double orifice indicate little interaction between orifice and wall. This corresponds to the larger values on . This is the most significant difference to be noted. The effect of interaction between the orifice and wall of the containing tube was found experimentally to be quite pronounced. Indeed, the data for the double orifice pairs indicates that this effect was more predominate than the orifice interaction and gave rise to a larger value of end correction than the interaction between the two orifices (small a/R values). The discrepancy in the data magnitude and the theoretical values has been discussed at some length in the preceding chapter.

The shape of the experimental curves has been substantiated by many trials and also by different methods of taking the data.

It can be concluded from this study that the general theoretical approach is valid, and it is possible to combine the theory for an incompressible fluid flowing in an infinite tube with that theory for acoustic radiation from a plane piston to describe the motion of an incompressible fluid in a thin orifice. The results have shown that the manner of describing the impedance of an orifice in terms of the Y_a factor is valid, over a large range of that parameter. The fact that the same end correction may be used to compute the inertance and resistance of an orifice was found to be correct.

It is suggested for further study that the effect of interaction among more than two orifices be studied. This could then be expanded to the study of screens, and finally the study and characterization of porous media.

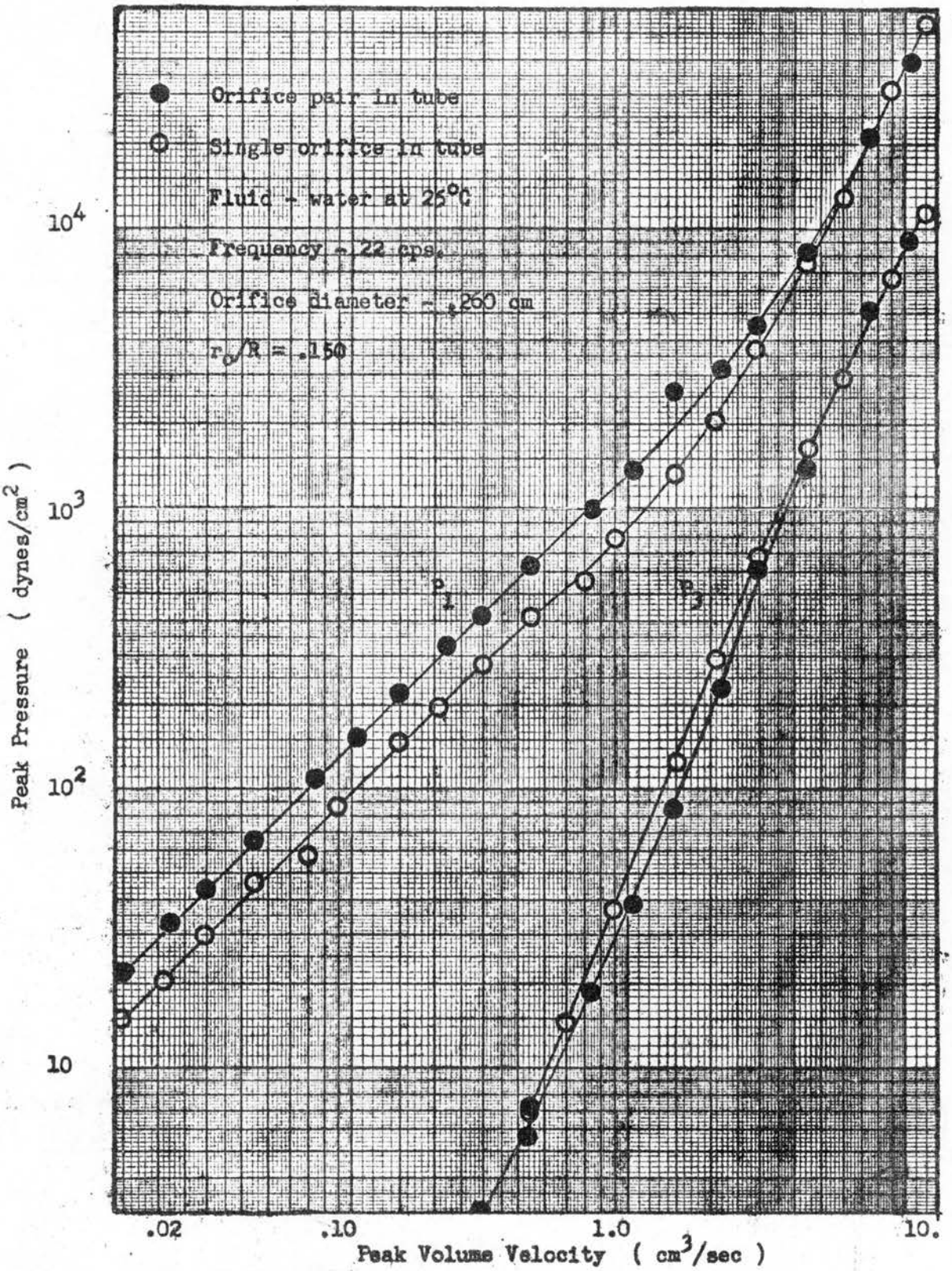


FIGURE 1. Fundamental and third harmonic pressure components as a function of peak volume velocity. For the double orifice these components are plotted against volume velocity per orifice.

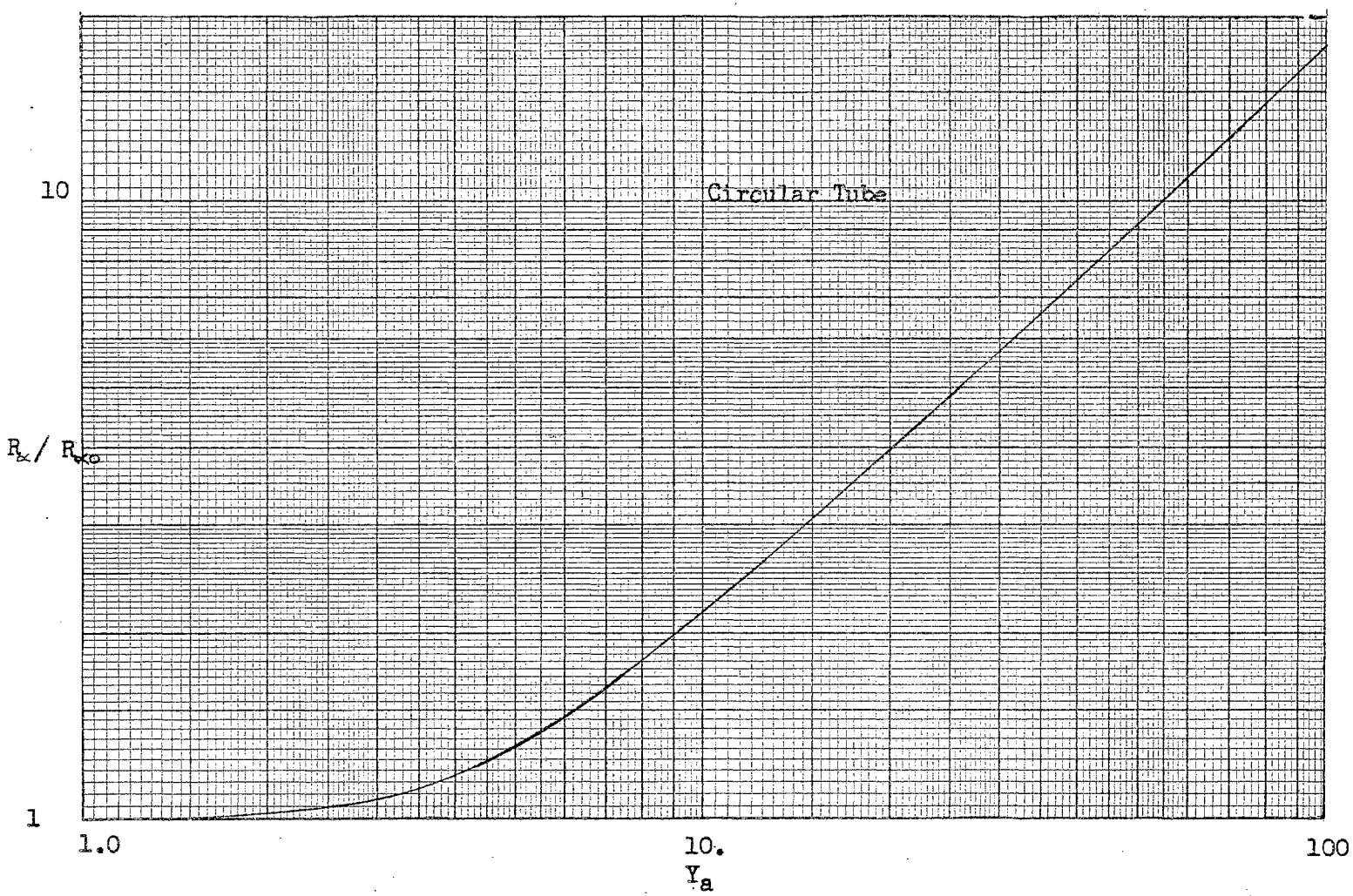


FIGURE 2

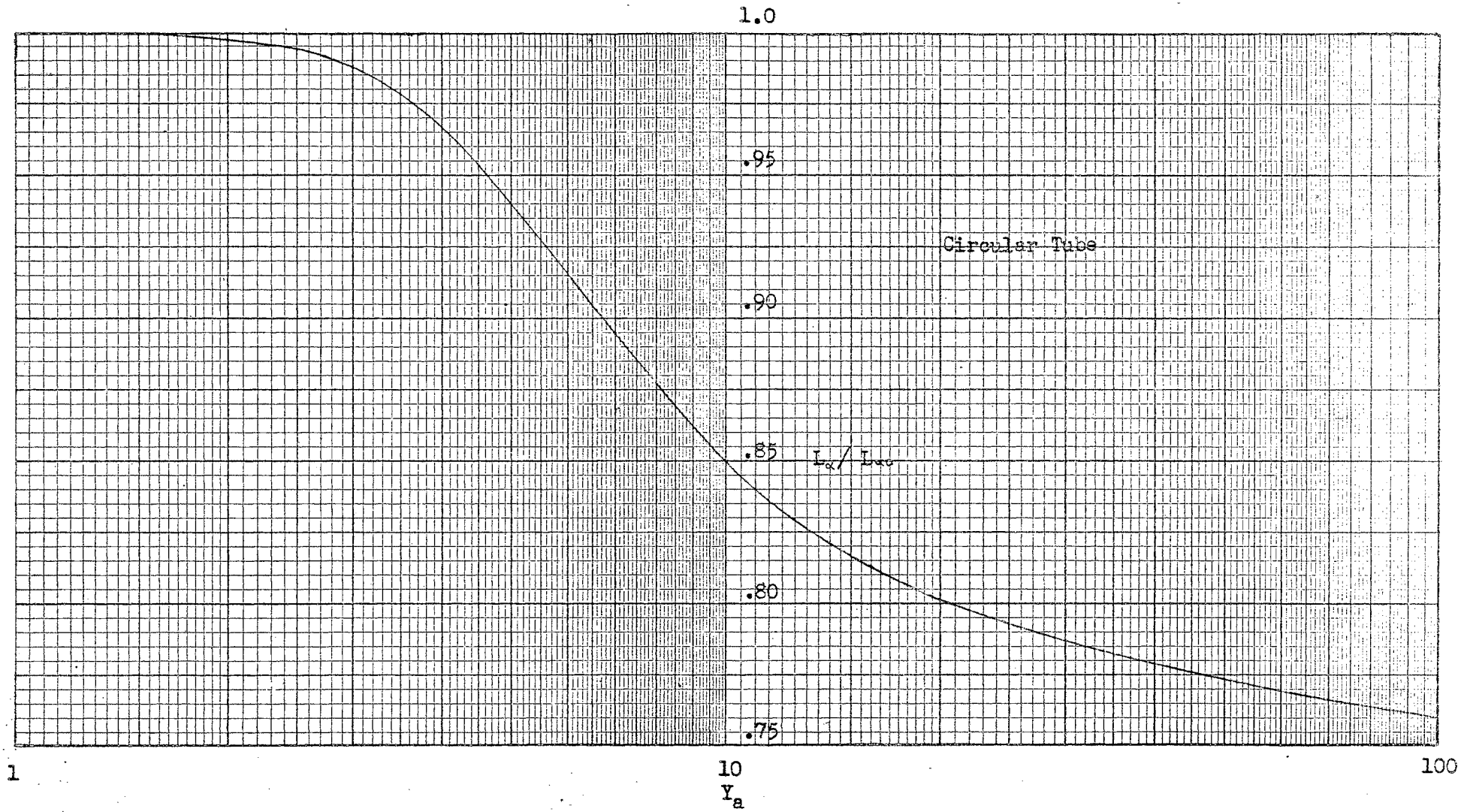


FIGURE 3

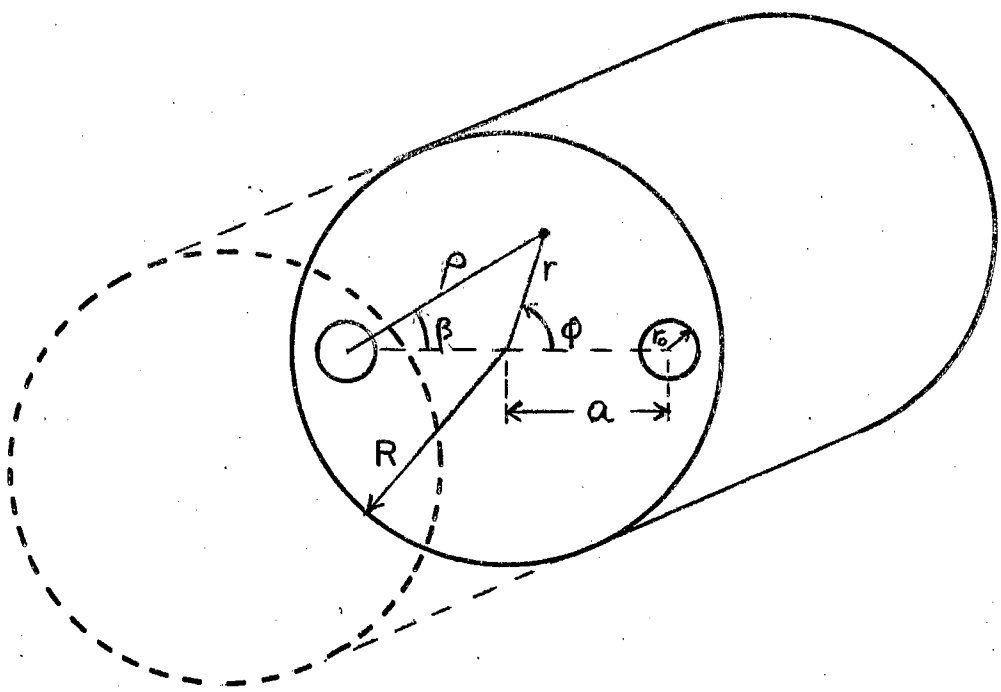


FIGURE 4. The geometry of the acoustic interaction study.

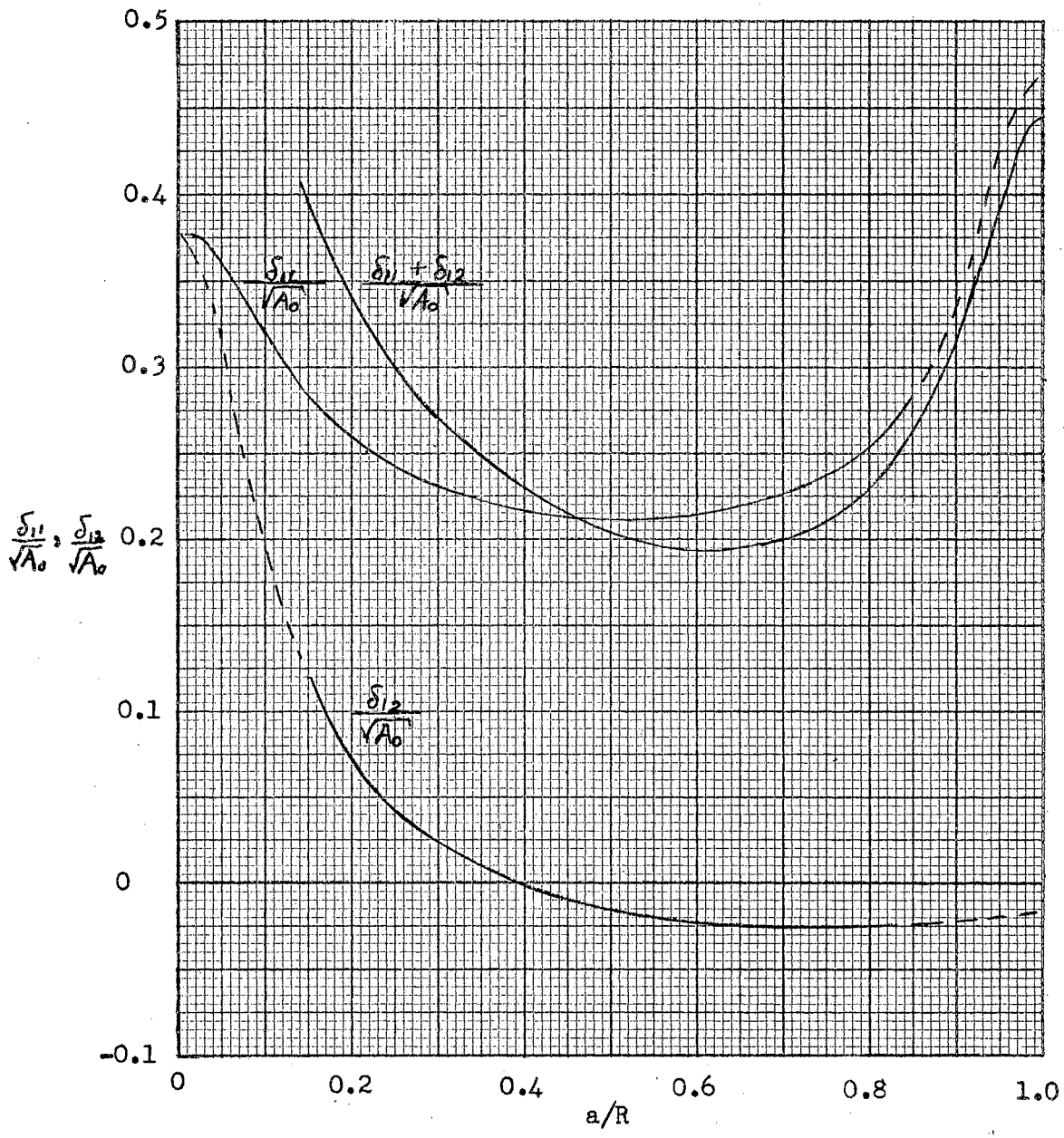


FIGURE 5. Theoretical end correction curves. $r_o/R = .15$

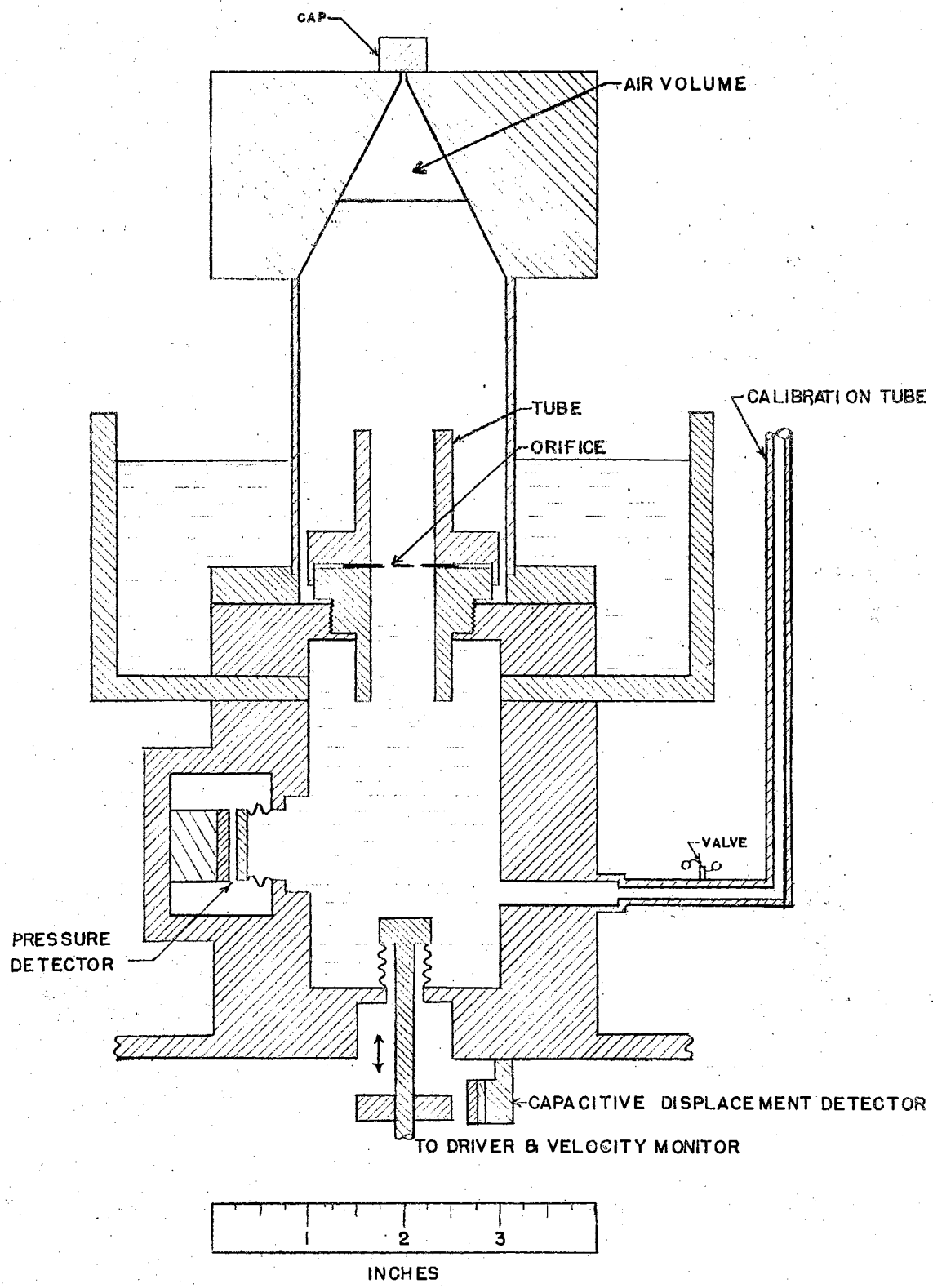
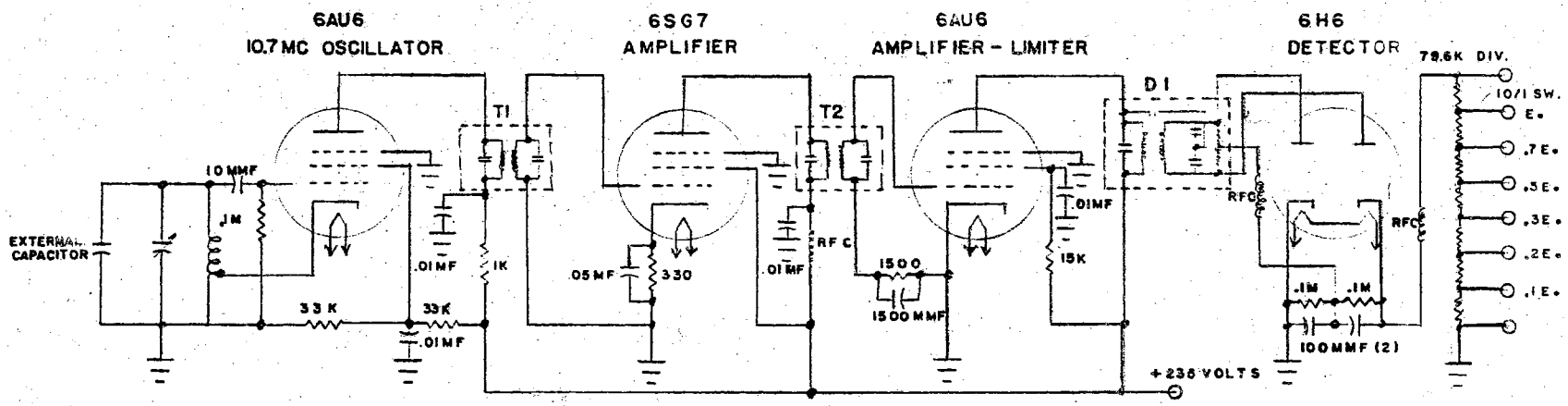
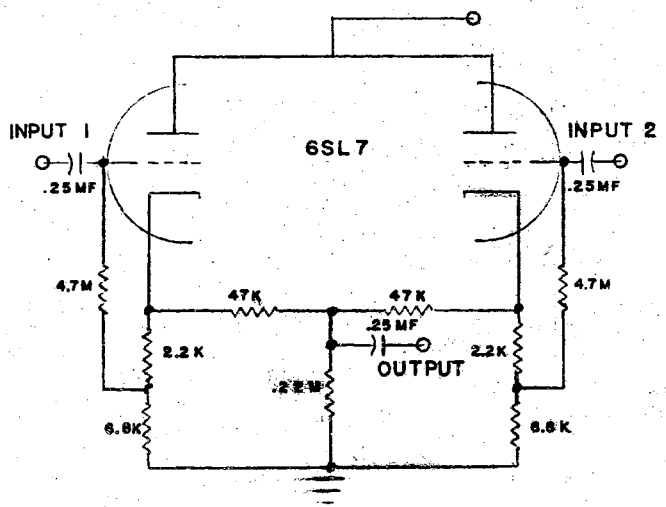


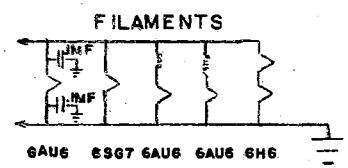
FIGURE 6. HYDRODYNAMICAL TEST SYSTEM

CAPACITIVE HYDROPHONE FM SYSTEM



CATHODE SUMMING CIRCUIT





LEGEND:

T1—MILLER TYPE 1451 INTERSTAGE TRANSFORMER

T2—SAME AS T1

D1—MILLER TYPE 1464 DISCRIMINATOR

FIGURE 7.

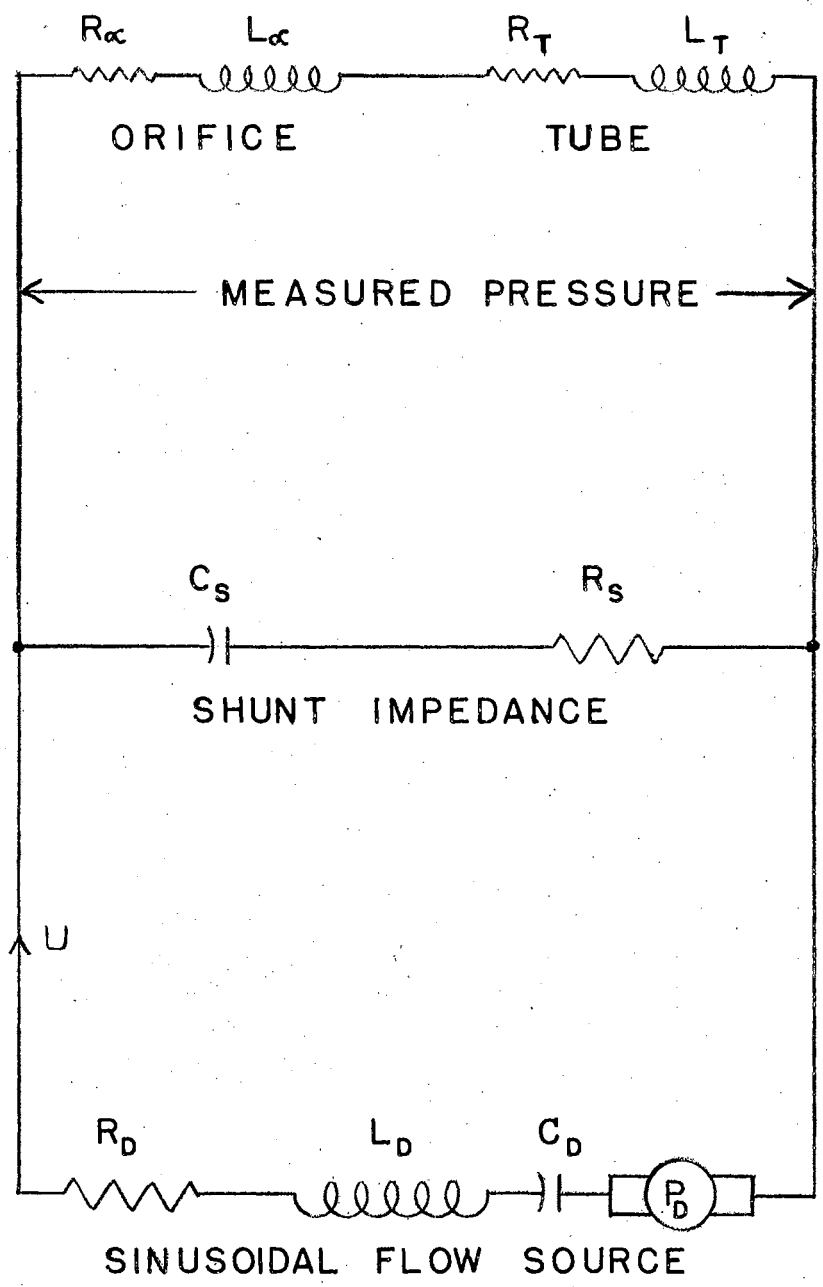


FIGURE 8. ELECTRICAL ANALOG OF THE HYDRODYNAMICAL TEST SYSTEM

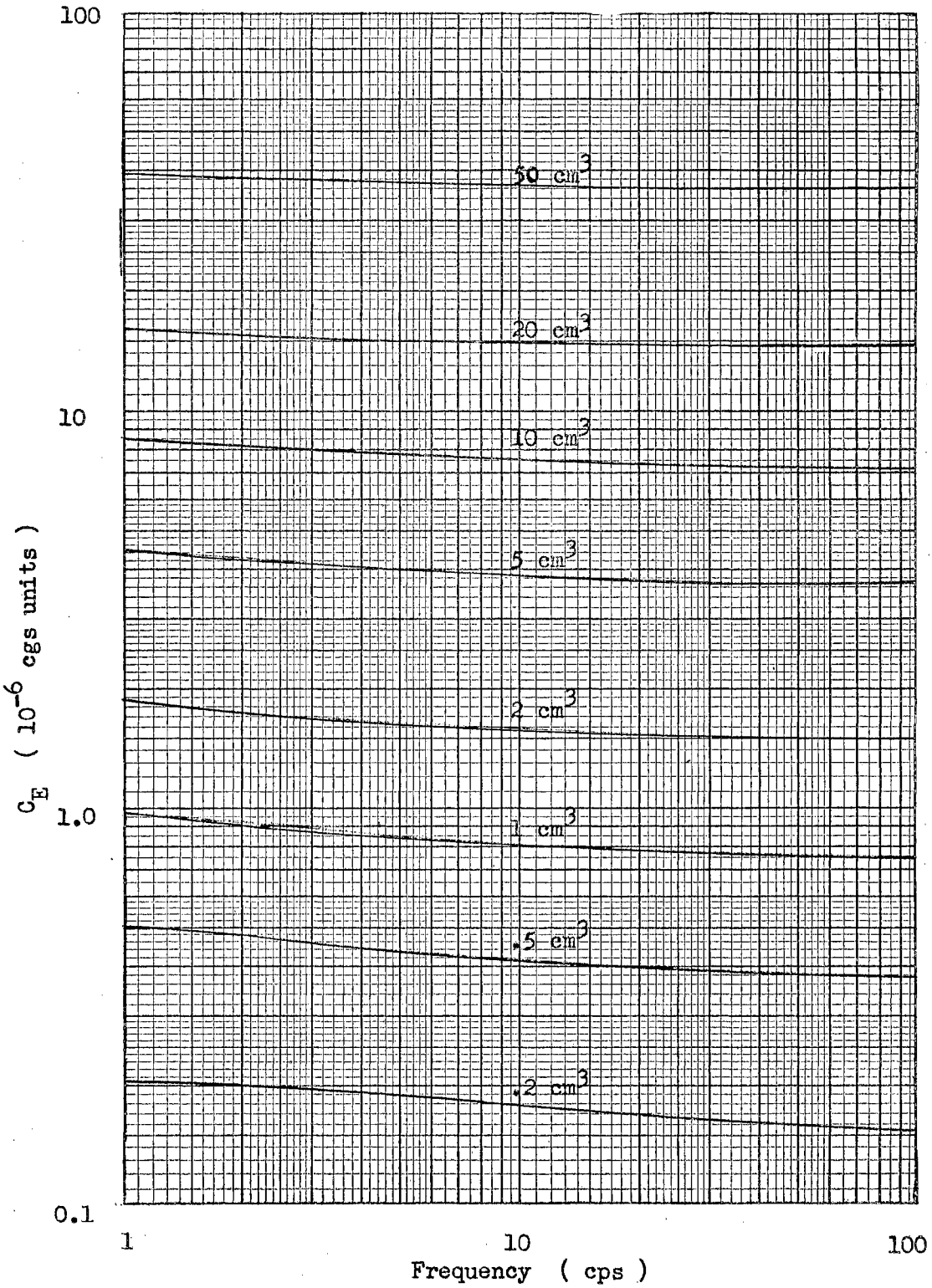


FIGURE 9. Compliance vs. Frequency for 53° cone.

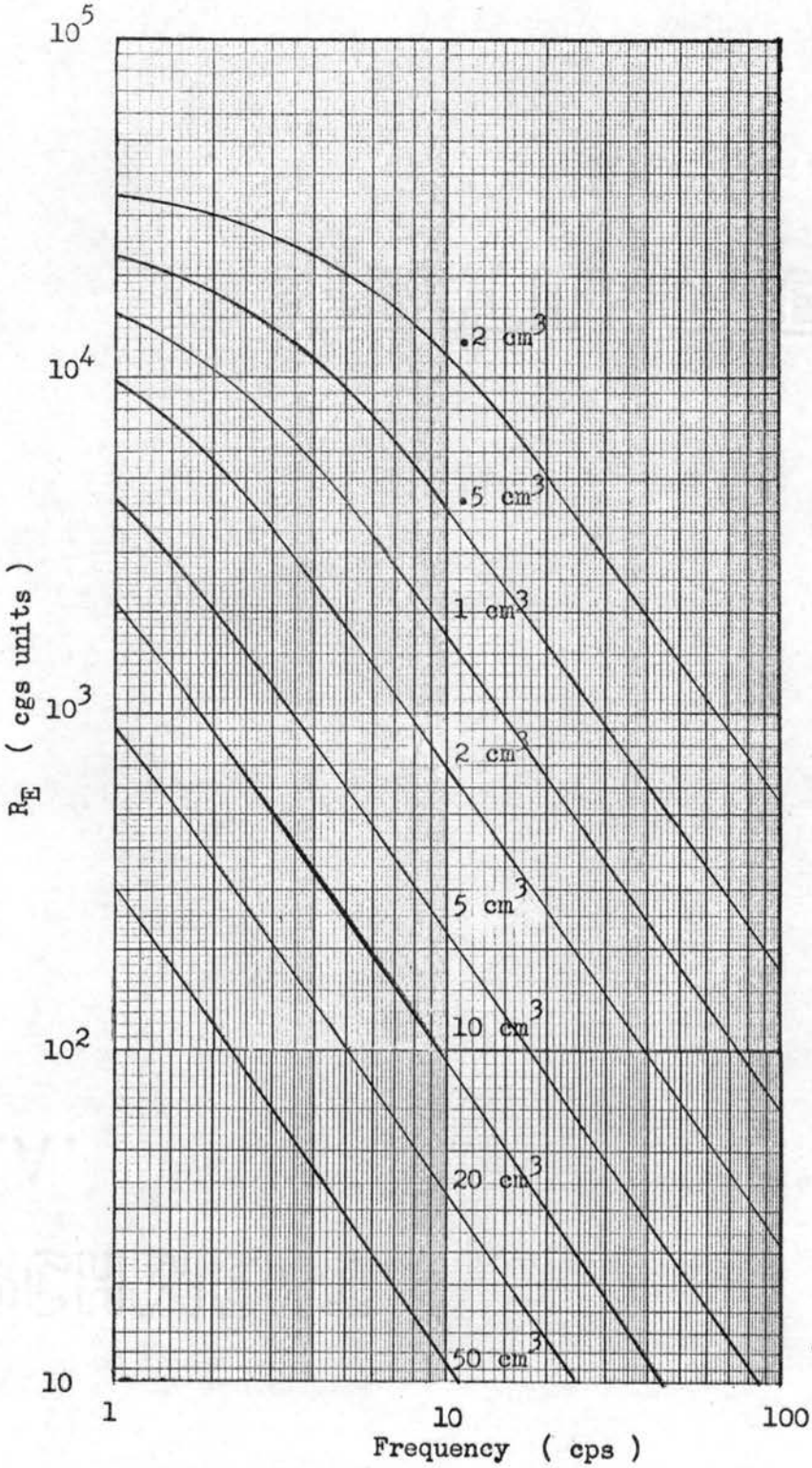


FIGURE 10. Resistance vs. Frequency for 53° cone

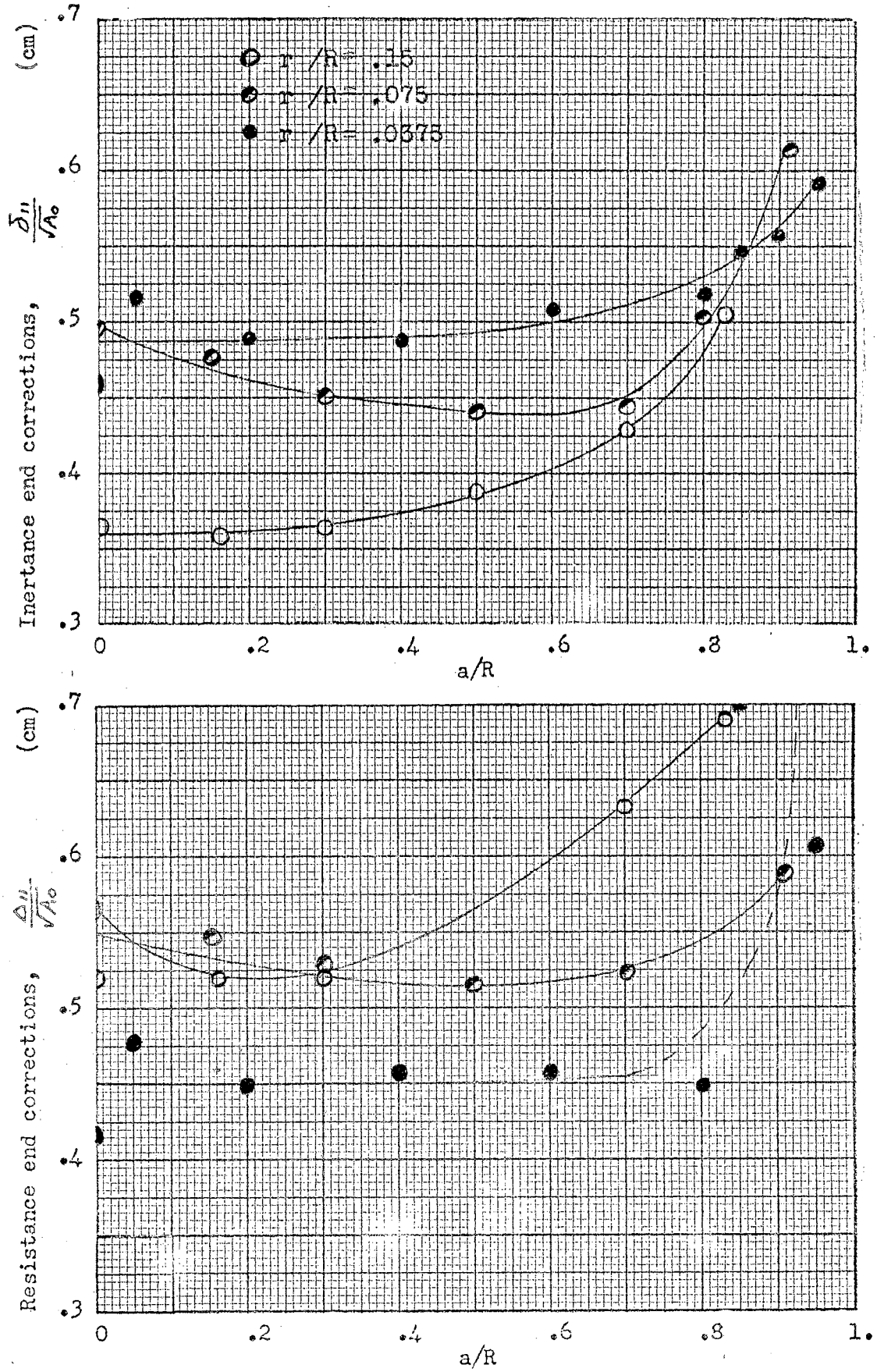


FIGURE 11. Single orifice end corrections of inertance and resistance as functions of a/R. Resonate circuit method, water at 25°C, f = 22 cps, U_M linear region.

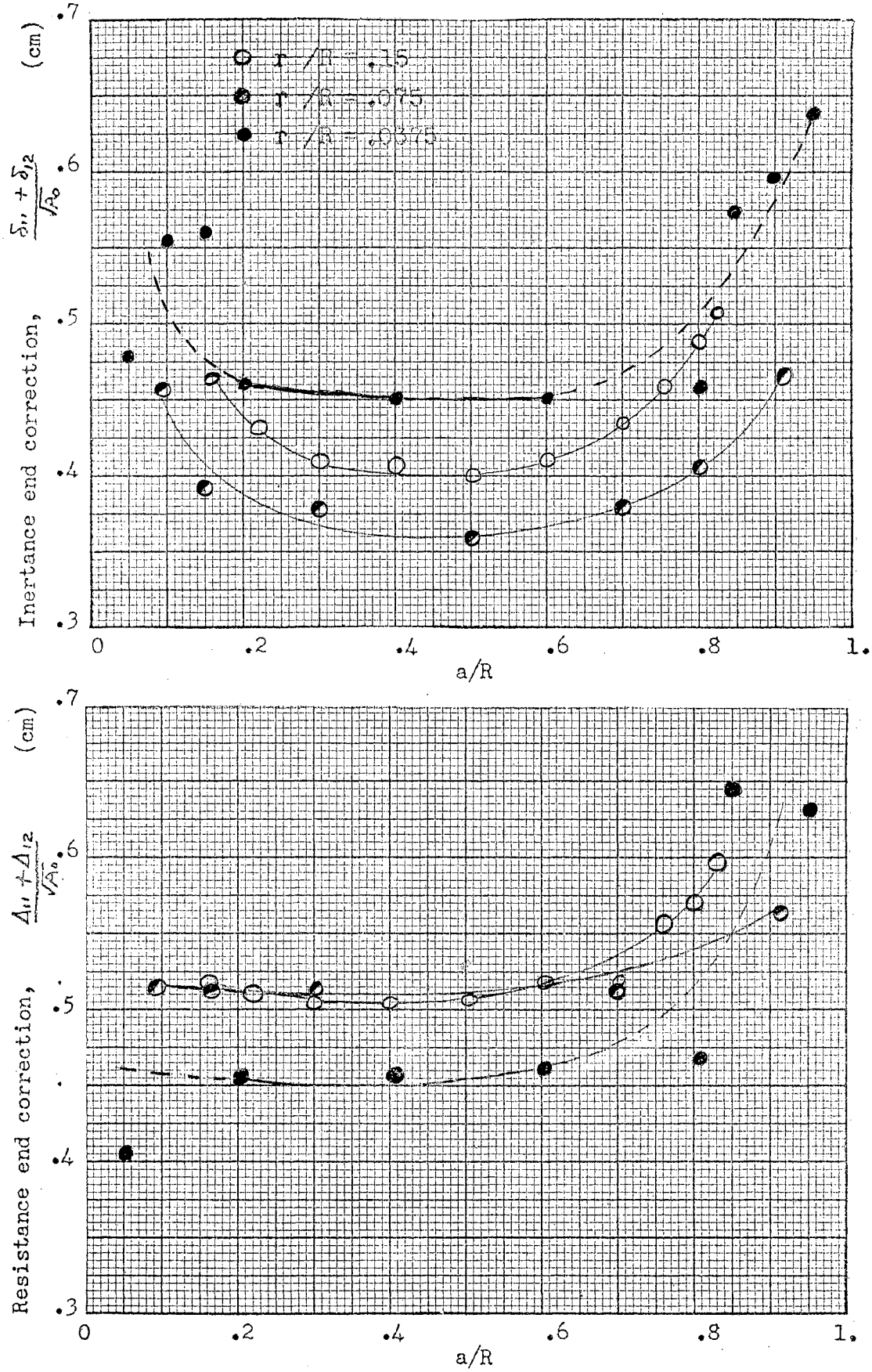


FIGURE 12. Double orifice end corrections of inertance and resistance as functions of a/R. Resonate circuit method, water at 25°C, f = 22 cps, U_M linear region.

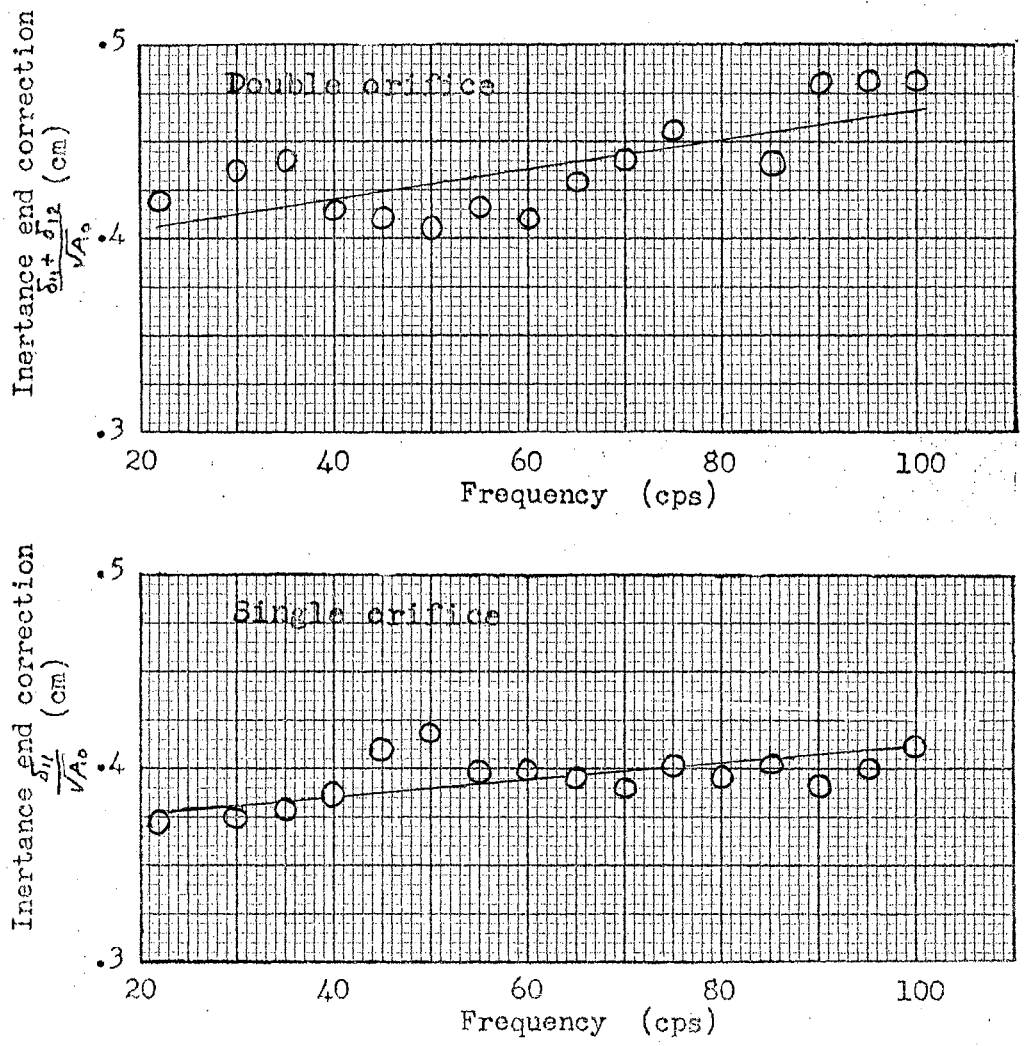


FIGURE 13. Inertance end corrections for single and double orifices as functions of frequency for water at 25°C, a/R = .159, r /R = .15, U_M = .891 cm^3/sec.

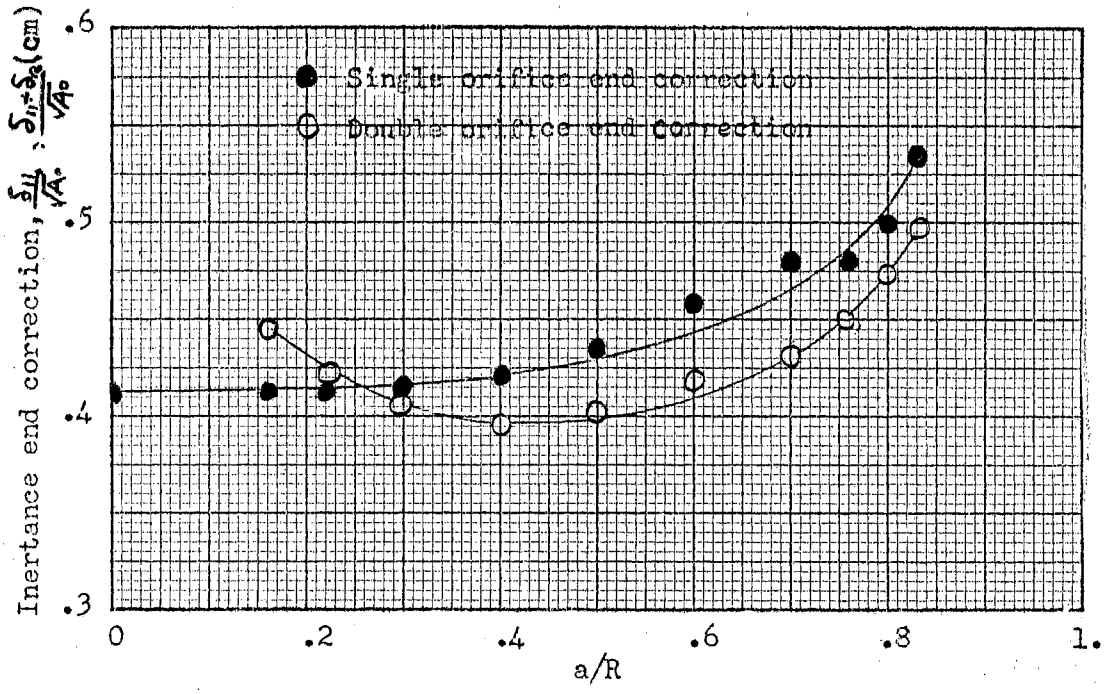


FIGURE 14. Single and double orifice end corrections of inertance as functions of a/R. Glycerol solution at 25°C, r /R = .15, U_M = .891 cm^3/sec, f = 22 cps.

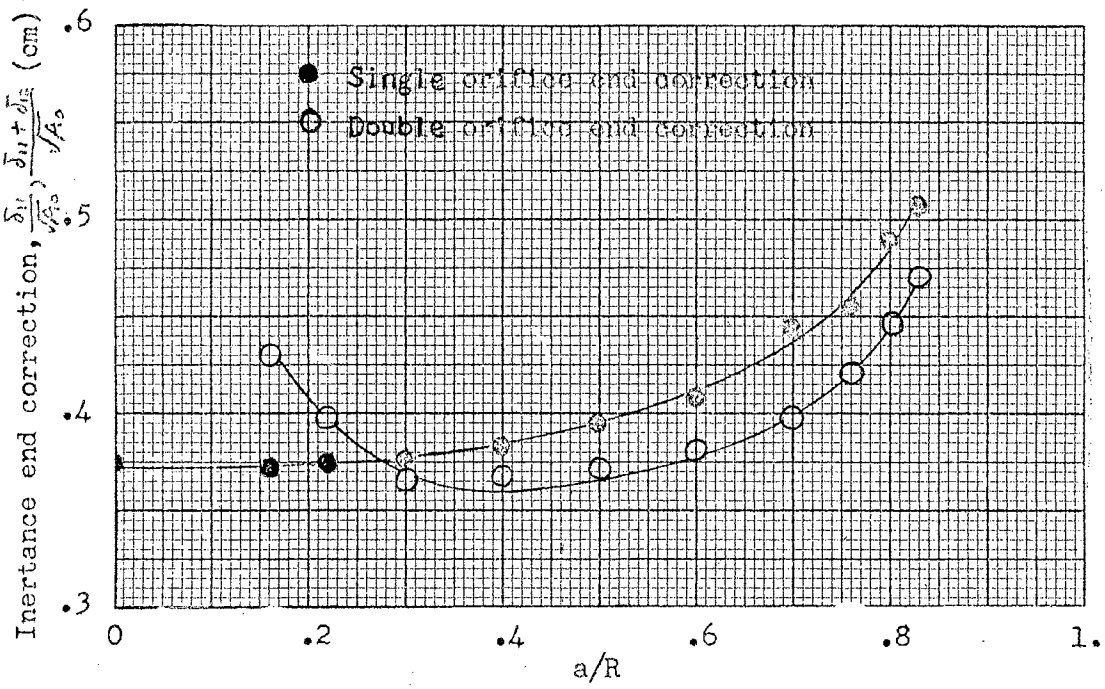


FIGURE 15. Single and double orifice end corrections of intance as functions of a/R. Water at 25°C, r /R = .15, U_M = .891 cm^3/sec, f = 22 cps.

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APPENDIX

THE INTERACTION OF TWO CIRCULAR APERTURES IN A CIRCULAR TUBE

The purpose of this appendix is to point out some of the details of Ingard's development[1] of the end correction for a single circular aperture in a circular tube and the extension of the theory for two apertures in a circular tube.[2]

The wave equation for a non-viscous ideal fluid is

$$\nabla^2 \Phi = \frac{1}{c^2} \frac{\partial^2 \Phi}{\partial t^2} \tag{1}$$

The solution[3] of the equation in cylindrical coordinates is made up of combinations of solutions of the type:

$$\Phi = \begin{matrix}\cos\\ \sin\end{matrix}(m\phi)\left[A_{mn} J_m(b_{mn} r) + B_{mn} N_m(b_{mn} r)\right] \exp\left[i K_{mn} Z - 2\pi i \nu t\right] \tag{2}$$

where J_m is a Bessel function of the first kind of order m, and N_m is a Neumann function of order m and is a second solution to Bessel's equation. The Neumann function approaches infinity as r approaches 0. For this reason these functions will not be used for the problem at hand, because they would give rise to infinite velocities and pressures

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[1] U. Ingard, "On the Theory and Design of Acoustic Resonators," Journal of the Acoustical Society of America, 25, No. 6, p. 1037, (1953).

[2] Many of the details of this appendix were supplied through a personal correspondence with Dr. Ingard. This assistance is gratefully acknowledged by the author.

[3] P. M. Morse, Vibration and Sound, (McGraw-Hill Book Co., Inc., New York, 1936).

at the origin of the coordinate system.

To reduce the solution of the wave equation presented in equation (2), first factor out the $e^{-2\pi i \nu t}$ term for this will be common throughout the development. There remains the solution for a wave propagation in the positive direction as

$$\phi_1 = \begin{matrix} COS \\ SIN \end{matrix} (m\phi) \left[A_{mn} J_m (b_{mn} r) \right] e^{i K_{mn} z} \tag{3}$$

where b_{mn} is the wave number in the r or radial direction of propagation and K_{mn} is the wave number in the z direction — the direction perpendicular to the plane. There is also the possibility of reflected waves and these would be given by:

$$\phi_2 = \begin{matrix} COS \\ SIN \end{matrix} (m\phi) \left[B_{mn} J_m (b_{mn} r) \right] e^{-i K_{mn} z} . \tag{4}$$

Combining these two solutions as given by equations (3) and (4) and summing on m and n yields:

$$\phi = \begin{matrix} COS \\ SIN \end{matrix} (m\phi) \left[\sum_{mn} (A_{mn} e^{i K_{mn} z} + B_{mn} e^{-i K_{mn} z}) J_m (b_{mn} r) \right] \tag{5}$$

Now it remains to choose either the $COS(m\phi)$ or $SIN(m\phi)$ to suit the problem. Refer to Figure 4 and note the way in which ϕ is measured. For the velocity to increase in the proper manner, it is necessary to choose the $COS(m\phi)$ term. Thus the solution reduces to

$$\phi = \sum_{mn} (A_{mn} e^{i K_{mn} z} + B_{mn} e^{-i K_{mn} z}) J_m (b_{mn} r) COS(m\phi) . \tag{6}$$

It is now desirable to express the solution in terms of one constant and a phase angle. Let

$$B_{mn} = -A_{mn} e^{-2\Psi_{mn}} \tag{7}$$

where Ψ_{mn} is the phase angle. Substitute in equation (6) and get:

$$\phi' = \sum_{mn} A_{mn} e^{-\Psi_{mn}} \left(e^{iK_{mn}z + \Psi_{mn}} - e^{-iK_{mn}z - \Psi_{mn}}\right) J_m(b_{mn} r) \cos(m\phi) . \tag{8}$$

Now recall the identity:

$$\sinh(x) = \tfrac{1}{2}\left(e^{x} - e^{-x}\right) . \tag{9}$$

By using this identity and incorporating the factor of 2 into the constant A_{mn}, the solution is reduced to

$$\phi' = \sum_{mn} A_{mn} e^{-\Psi_{mn}} \sinh(\Psi_{mn} + iK_{mn} z) J(b_{mn} r) \cos(m\phi) \tag{10}$$

which is the solution that Ingard starts with in his development.[1]

At $z=0$ the velocity distribution is $U = U(r,\phi)$. Expand this in Bessel functions thus:

$$U = \sum_{mn} a_{mn} J_m(b_{mn} r) e^{im\phi} . \tag{11}$$

In order to evaluate the constants A_{mn} it is first necessary to determine the constants a_{mn}. Multiply equation (11) by $J_m(b_{mn} r)$ and integrate over the limits $0 \le r \le R$ and $0 \le \phi \le 2\pi$. This gives:

$$\int_0^{2\pi}\int_0^R a_{mn} J_{mn}^2(b_{mn} r)\, r\, dr\, d\phi = \int_0^{2\pi}\int_0^R U J_m(b_{mn} r) e^{-im\phi} r\, dr\, d\phi \,. \tag{12}$$

$U=0$ everywhere but in the orifice and there the velocity is like that of a plane piston. Then change U to U_0 and evaluate the integral of equation (12) over the piston. This gives

$$a_{mn} = \frac{U_0}{2\pi\int_0^R J_m^2(b_{mn} r)\, r\, dr} \iint_{PISTON} J_m(b_{mn} r) e^{-im\phi} r\, dr\, d\phi \,. \tag{13}$$

In order to perform the integration of the above equation it is useful to use the addition theorem for Bessel functions. This involves expressing $J_m(b_{mn} r)e^{-im\phi}$ in terms of Bessel functions, referred to an axis through the center of the orifice, corresponding to the new polar coordinates ρ and β . These are shown in Figure 4 and the resulting tansformation gives

$$J_m(b_{mn} r)\, e^{-im\phi} = \sum_{p=-\infty}^{\infty} J_p(b_{mn} a)\, J_{m+p}(b_{mn}\rho)\, e^{-(p+m)\beta} \,. \tag{14}$$

Insert this into the integral over the piston in equation (13) and get

$$U_0 \int_0^{2\pi}\int_0^{r_0} \sum_{p=-\infty}^{\infty} J_p(b_{mn} a)\, J_{m+p}(b_n \rho) e^{-i(p+m)\beta} \rho\, d\rho\, d\beta \,. \tag{15}$$

Considering the component parts of the double integration, there is first

$$J_{mn} = \int_0^{r_0} J_{m+p}(b_{mn}\rho)\, \rho\, d\rho \tag{16}$$

which is easily evaluated in special cases.

Also there is the integral

$$\int_0^{2\pi} e^{-i(p+m)\beta}\, d\beta \tag{17}$$

which is equal to zero unless $p=-m$ and for $p=-m$ it becomes

$$\int_0^{2\pi} e^{-i(p+m)\beta}\, d\beta = \int_0^{2\pi} d\beta = 2\pi . \tag{18}$$

Hence equation (16) may be evaluated at $p=-m$. Otherwise equation (15) equals zero. Reference to equation (16) gives

$$J_{mn} = \frac{r_0}{b_{mn}} J_1(b_{mn} r_0) . \tag{19}$$

Consider now the other integral in the expression (13). This yields

$$\int_0^R J_m^2(b_{mn} r)\, r\, dr = \frac{1}{2} R^2 \left(1 - \frac{m^2}{b_{mn}^2 R^2}\right) J_m^2(b_{mn} R) \tag{20}$$

Substitute equations (16) and (19) into (13) and also (18) into (13) and obtain the relation

$$a_{mn} = 2U_0 \frac{r_0}{R} \frac{1}{b_{mn} R} \frac{J_{-m}(b_{mn} a)\, J_1(b_{mn} r_0)}{J_m^2(b_{mn} R)\left[1 - \frac{m^2}{(b_{mn} R)^2}\right]} \tag{21}$$

Now recall that

$$U = -\frac{\partial \phi'}{\partial z} \tag{22}$$

and that from equation (11)

$$U\Big|_{z=0} = \sum_{mn} {}_{mn} J_m(b_{mn} r)\, e^{im\phi}. \tag{23}$$

Then it is possible to evaluate the constants A_{mn}. To do this, differentiate equation (10) and set it equal to equation (11) at $z = 0$ and obtain

$$\sum_{mn} a_{mn} J_m(b_{mn} r)(\cos m\phi + i \sin m\phi) \tag{24}$$

$$= -\sum_{mn} A_{mn} e^{-\psi_{mn}} \cosh(\psi_{mn} + iK_{mn} z)(iK_{mn}) J_m(b_{mn} r) \cos(m\phi).$$

Since at $z=0$ the $iK_{mn} z = 0$ it follows that:

$$A_{mn} = \frac{a_{mn}}{-iK_{mn} \cosh(\psi_{mn}) e^{-\psi_{mn}}}. \tag{25}$$

From the total pressure P on the piston, obtained by integration of $P = i\omega\phi$ over the piston, the impedance $Z = \frac{P}{U_0}$ can be obtained. The expression for the pressure is

$$P = i\omega \sum_m \sum_n A_{mn} e^{-\psi_{mn}} \sinh(\psi_{mn} + iK_{mn} z) J_m(b_{mn} r) \cos m\phi. \tag{26}$$

This integral must be evaluated in the same way as that for the constant, by the use of the addition theorem of Bessel functions and a change of coordinates. The impedance of the piston is then

$$Z = \frac{P}{U_0} = \pi r_0^2 \rho c \sum_m \sum_n F_{mn}(r_0/R, a/R) \frac{1}{\delta_{mn}} \operatorname{TGH} \psi_m \tag{27}$$

where

$$F_{mn} = \frac{4\, J_1^2(b_{mn} r_0)\, J_m^2(b_{mn} a)}{(b_{mn} R)^2 \left[1-(m/b_{mn} R)^2\right] J_m^2(b_{mn} R)} \tag{28}$$

and

$$\delta_{mn} = \left[1 - b_{mn}/R\right]^{\frac{1}{2}} = i\,\frac{\omega_{mn}}{\omega}\left[1-(\omega/\omega_{mn})^2\right]^{\frac{1}{2}} . \tag{29}$$

The term δ_{mn} is the cut-off frequency for the (m, n) mode. This is the frequency where that particular mode of transmission no longer takes place. The term $m=0,\ n=0$ represents the contribution from the plane wave which is purely resistive and is not considered for the mass end correction.

For frequencies much lower than the lowest cut-off frequency for the higher order modes, that is, when $\omega << \omega_{mn}$, then $\delta_{mn} \simeq i\omega_{mn}/\omega \simeq i c b_{mn} \omega$. Furthermore, considering an infinitely long tube so that $TGH = 1$ the phase angle between the reflected and transmitted waves approaches infinity and the reflected wave vanishes. Then the expression for the impedance as given in equation (27) can be written as

$$Z = \pi r_0^2 (-i\omega) \rho\, \delta_{11} \tag{30}$$

where the mass end correction δ_{11} is

$$\delta_{11} = \frac{4}{\pi^{\frac{1}{2}}} (\pi r_0)^{\frac{1}{2}} \frac{1}{(r_0/R)} \sum_m{}' \sum_n{}' \frac{J_1^2(b_{mn} r_0)\, J_m^2(b_{mn} a)}{(b_{mn} R)^3 \left[1-(m/b_{mn} R)^2\right] J^2(b_{mn} R)} . \tag{31}$$

The prime on the summation sign means that the term $m=0, n=0$ is not

included. This expression reduces to the end correction for the concentric aperture, when $a=0$. In that case m is always zero.

Consider now an additional aperture of the same size and at the same distance a from the center of the tube. If the pressure field of the first piston is P_1 and of the second is P_2 then the total pressure is $P=P_1+P_2$. If the pressure caused by piston 1 at the surface of piston 2 is denoted by P_{12} the interaction impedance is

$$Z_{12} = \frac{1}{U_2} \int_{S_2} P_{12}\, dS_2 \tag{32}$$

where U_2 is the velocity of the second piston.

The total sound field has the same velocity potential as that of a single orifice. However, in determining the constants, as in equation (13), the integration must be extended over both of the orifices. The pressure of the first piston, using equations (10 and (21) and integrating over the limits indicated, gives:

$$P_1 = \rho c U_1 \sum \frac{2 r_0 \quad J_{-m}(b_{mn} a) J_1(b_{mn} r_0) \, SINH(\Psi_{mn} + i K_{mn} Z)}{R b_{mn} R \quad J_m^2(b_{mn} R)[1 - m^2/(b_{mn} R)^2] \delta_{mn} \, COSH \, \Psi_{mn}} J_m(b_{mn} r) \, COS \, m\phi. \tag{33}$$

The same expression holds for the pressure of the second piston except for the factor $(-1)^m$. Thus this pressure is

$$P_2 = \rho c U_2 \sum (-1)^m \frac{2 r_0 \quad J_{-m}(b_{mn} a) J_1(b_{mn} r_0) \, SINH(\Psi_{mn} + i K_{mn} Z)}{R b_{mn} R \, J_m^2(b_{mn} R)[1 - m^2/(b_{mn} R)^2] \delta_{mn} \, COSH \, \Psi_{mn}} J_m(b_{mn} r) \, COS \, m\phi. \tag{34}$$

The interaction impedance is then

$$Z_{12} = (-i\omega) \pi r_0^2 \rho \, \delta_{12} \tag{35}$$

where, for the condition $\omega << \omega_{mn}$

$$\delta_{12} = \frac{4}{(\pi)^{\frac{1}{2}}} (\pi r_0)^{\frac{1}{2}} \sum_m{}' \sum_n{}' (-1)^m \frac{J_1^2(b_{mn} r_0)\, J_m^2(b_{mn} a)}{(b_{mn} R)^3 [1 - (m/b_{mn} R)^2]\, J_m^2(b_{mn} R)} \tag{36}$$

The primes on the summation sign mean that the term $m=0, n=0$ is excluded. Both this function and the one for the end correction for the single orifice are plotted in Figure 5 for the case where $r_0/R = .15$.

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