A STUDY OF BOURDON TUBE DEFLECTION
USING A NUMERICAL ANALYSIS
SOLUTION

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A STUDY OF BOURDON TUBE DEFLECTION USING A NUMERICAL ANALYSIS SOLUTION

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PREFACE

For many years investigations have been made of the performance of the Bourdon Tube in the measurement of pressure. To date no satisfactory theory has been evolved to predict the action of the Bourdon Tube within reasonable design limits. The object of this investigation is to establish an empirical equation relating the change in radius, ΔR, of a Bourdon Tube to the applied pressure and other tube parameters.

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CHAPTER I

INTRODUCTION

The Bourdon Tube is one of the most widely used elements in the measurement of pressure. It was invented in 1849 by Schinz and marketed by E. Bourdon in 1850. Although many attempts have been made to analyze and predict its performance, to date no satisfactory theory has been determined that will predict the action within reasonable design limits.

Wuest [(1)*], Wolf [(2)], Clark, Gilroy, and Reissner [(3)] have all made important contributions in Bourdon Tube research. Wuest based his theory on the bulging deflection of the Bourdon Tube. His analysis is approximate and is particularly suited to tubes whose cross-sections have a greater width than height. The pointed-arc shape and the idealized section in Figure 1 are best fitted to his theory. The idealized section as used by Wuest was based on a tube with ends which "are infinitely rigid to all stress except unbending--to which they are perfectly flexible. This means that all bulging occurs in the horizontal walls__." [(1)]

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*Superior numbers in parentheses refer to numbers in the Bibliography.

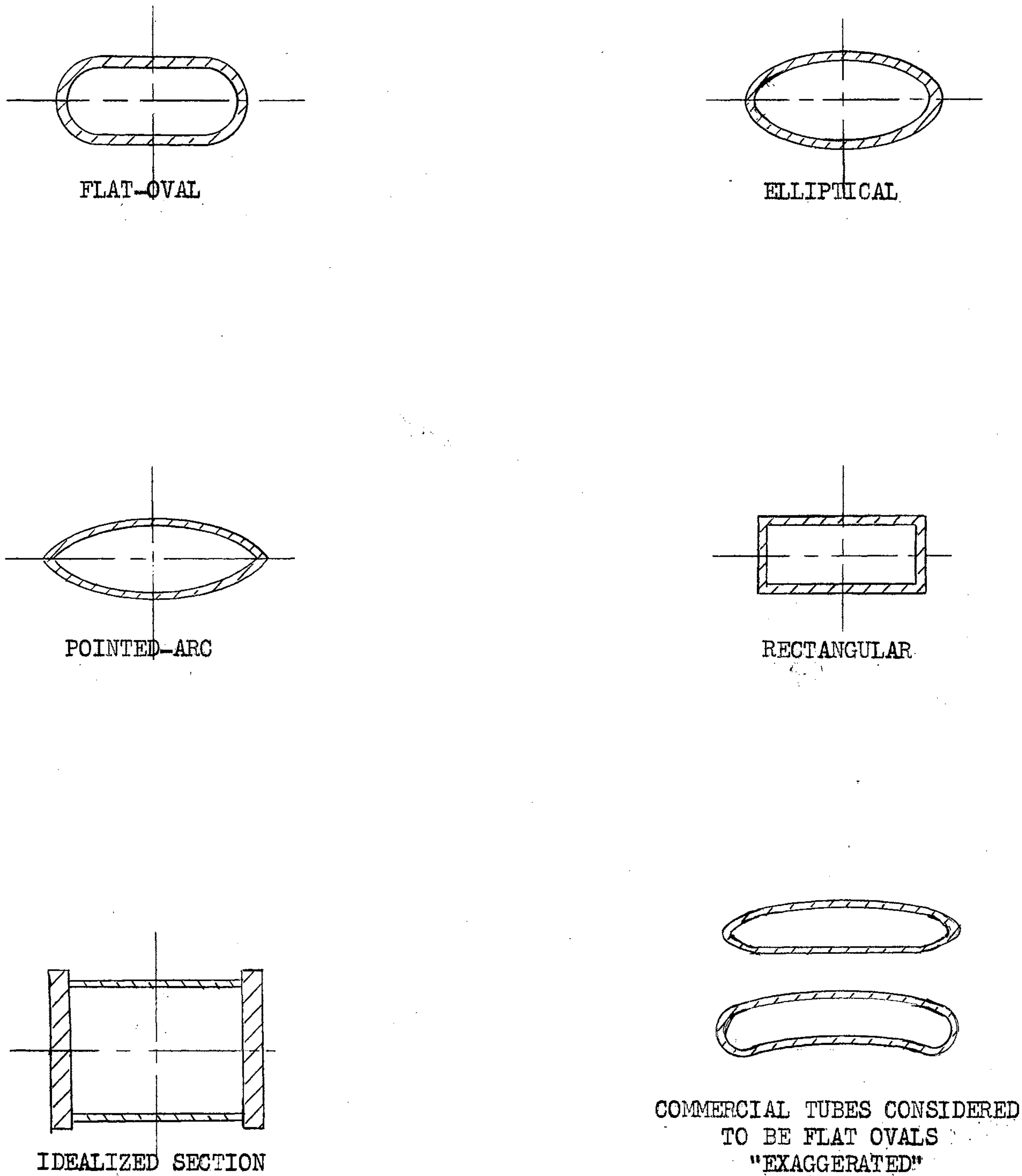

Fig. 1. Bourdon Tube Cross-Sections

Jennings [4] in discussing Wuest's work states:

Comparison shows that the pointed-arc has much greater sensitivity than the idealized section and also that it has much lower stiffness. These results are highly significant because they demonstrate the critical influence of section shape on tube performance. This is one reason why it is not easy to get consistent experimental data on these pressure detectors.

Wolf bases his theory of Bourdon Tube deflection on the assumption of an approximately correct bulging formation of the flat-oval section. Since Wolf has used a bulging function similar to that of pressure in a straight tube, his results are not expected to be accurate for highly curved tubes.

Using the Fourier series and asymptotic approximation methods, Clark, Gilroy, and Reissner have analyzed Bourdon Tubes with elliptical cross-sections. Jennings in comparing the different Bourdon Tube theories says:

The curves for elliptical tube stress and stiffness ratio check approximately with Wuest's and Wolf's curves for large values of the tube parameter. For highly curved Bourdon Tubes, this theory indicates the stress to be higher and the stiffness lower for elliptical tubes than for straight-sided tubes. However, study of both the sensitivity and stress curves shows that the elliptical section should give a greater deflection per unit stress than the straight-sided tube.

Both Wuest and Jennings in independent analyses found that by applying the idealized section theory to flat-oval tubes, it was possible to eliminate height to width ratio as an independent variable.

For the practical use of Bourdon Tubes the interest has been in the distance the tip travels, the angle through which it turns, and the effect of external forces acting on the tip. It was the purpose of this study to establish a method, resulting in equations, which will predict the tip travel of a Bourdon Tube in terms of the applied pressure and the measurable tube dimensions, of sufficient accuracy that they may be used in the design of new tubes.

CHAPTER II

STATEMENT OF THE PROBLEM

The objective of this investigation was to establish an empirical equation relating the change in radius, ΔR, of a Bourdon Tube to the applied pressure and other tube parameters. The value ΔR is necessary in order to predict the tip travel of a Bourdon Tube as will be shown in this report.

Due to the complex shape of a Bourdon Tube, it is difficult to form analytically, a differential equation that will describe the deflection of the tube, and it is much more difficult to integrate the equation once it has been formed. In this study, careful measurements of applied pressures and deflections of various sizes of tubes were made, and the changes in radii were computed. When a Bourdon Tube expands with applied pressure within the tube, the radius increases, and at the same time the angle subtended by the end points decreases. A relationship between these two values must be established in order that the 'tip travel' of the tube may be calculated. The 'tip travel' within very close limits may be considered as a function of the change in radius and the initial angle of the tube alone.

CHAPTER III

PROCEDURE

Twenty-five Bourdon Tubes selected at random and furnished by manufacturers of Bourdon Tubes were measured for this study. The measuring technique used in this study, (i.e., using a milling machine, microscope, and gage blocks to very accurately establish a circle of reference through three points that coincide with the centroids of the cross-section at these points), made it possible to measure the deflections of the tubes without the effects of the end fastenings and closures. A list of equipment used to obtain these measurements and a list of symbols and abbreviations are included in the appendix. This technique also provides a direct comparison of the values $\frac{\Delta R}{R_o}$ and $\frac{\Delta \psi}{\psi_o}$ (ratios of: change in radius to radius at zero pressure, and change in tube angle to tube angle at zero pressure), two values necessary for the completion of this study.

All measurements of coordinate points, $x_i y_i$, on the side of the tube that coincided with the central axis are correct within $\pm$ 0.0001 of an inch. All pressure measurements were made with a dead weight tester that was calibrated by the U. S. Bureau of Standards just prior to its use in this project. All pressures as reported are correct within 0.01 psi. No material samples were available to establish Young's Modulus (E) for the tube materials. The values used for E are values reported by H. L. Mason.[5]

The tubes used in this study were not true geometric shapes, i.e., the arcs of the tubes were not true circular arcs, and the cross-sections were not true flat-ovals. These deviations from true forms affected the accuracy of the derived expressions for ΔR. The material thicknesses used in the computations were the values reported by the tube manufacturers rather than true average values of thicknesses.

Each tube was examined to establish a uniform section removed from the transition shapes near mountings and end connections. After the uniform section was established, careful measurements were made of w and t (the major and minor axes of the tube cross-sections) and an average value of these quantities was noted. Three reference points were next established on the side of the tube, great care being taken that these reference points, approximately evenly spaced, coincided with the major diameters, w. These reference points are in the nature of 60° cones indented on the side of the tube. The base diameters of these conical points are approximately 0.001 inch. An assumption was made at this time that the cross-sections of the tubes are true flat-ovals. The reference points are located within an accuracy of $\pm$ 0.0005 inch of lying on the edge of the cylindrical surface that contains all the major axes and the central axis of the tube. These three reference points are used to determine the circle of the central axis of the tube.

For the second operation, the tube was rigidly mounted on the platen of a milling machine, in a vertical plane parallel to the platen of the mill. The tube was next exercised 25 times by applying and completely removing a pressure 10% greater than the rated pressure of the tube.

The third operation was one of measuring deflection versus pressure. Each tube was measured at 0%, 25%, 50%, 75%, and 100% of rated pressure, or pressures near to those just mentioned. Any deviations of pressure from those mentioned above were to facilitate the use of the large weights of the dead weight tester. The sequence of measurements, Figure 2, was as follows: at 0 psi, points x_2y_2, x_3y_3, and x_4y_4 were established; again at 25% psi, points x_2y_2, x_3y_3, and x_4y_4 were established and etc. The odd subscripts on the location points were necessary because originally five location points were to be used and three circles were to be measured on each tube, at each pressure, using points 1, 2, 3; 2, 3, 4; and 3, 4, 5. Circles established by points 1, 2, 3 and 3, 4, 5 were discarded because they would be highly influenced by the end conditions of each individual tube. In this study, the effects of end conditions were to be eliminated.

The results of the above measurements were used in Figure 2, Equations 2 and 3 to calculate the locations of all the centers of all the circles (a_x, b_y). These results were then used to calculate the radii of all the circles (Fig. 2, Eq. 1) and to establish the values of Δ R for each increment of pressure.

The results of Equations 1, 2, and 3 of Figure 2 were used in Figure 3 to establish Equations 4, 5, and 6, and values of $\frac{\Delta R}{R_o}$ and $\frac{\Delta \psi}{\psi_o}$ for purposes of comparison. The information from equations 5 and 6 was used to develop the equation for 'tip travel' in Figure 4, Equation 7.

The symbols and values for the major and minor axes of the tubes are different from values and symbols used by previous investigators. The symbols w and t were substituted for $b/2$ and $a/2$, major and minor semi-axes. The values w and t are overall measurements that can be measured with a micrometer. All equations developed and used in this paper are reported in algebraic form. All solutions involving these equations were programmed and solved using the IBM 650 computer located on the campus of the Oklahoma State University.

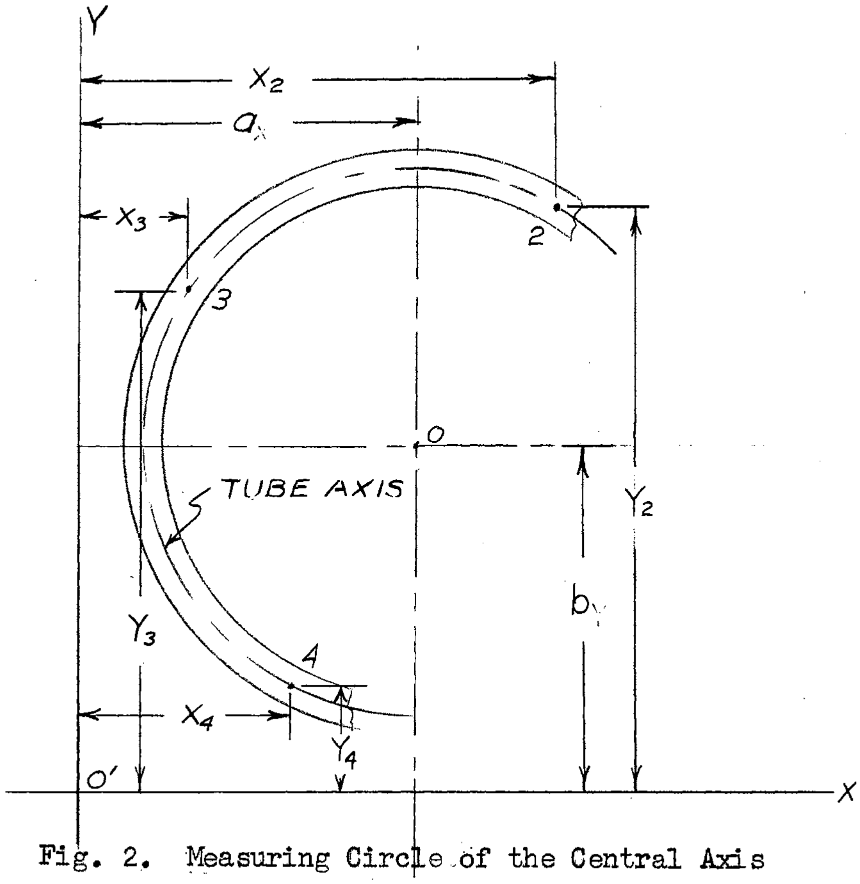

Fig. 2. Measuring Circle of the Central Axis

$$(X-a)^2 + (Y-b)^2 = r^2 \quad \text{General Equation (1)}$$

$$X^2 - 2aX + a^2 + Y^2 - 2bY + b^2 = r^2$$

Eq. for three points

$$X_2^2 - 2aX_2 + a^2 + Y_2^2 - 2bY_2 + b^2 = r^2$$

$$X_3^2 - 2aX_3 + a^2 + Y_3^2 - 2bY_3 + b^2 = r^2$$

$$X_4^2 - 2aX_4 + a^2 + Y_4^2 - 2bY_4 + b^2 = r^2$$

Rearrange terms

$$X_2^2 - 2aX_2 + Y_2^2 - 2bY_2 = r^2 - a^2 - b^2 = \text{constant for any one circle}$$

$$X_3^2 - 2aX_3 + Y_3^2 - 2bY_3 = C$$

$$X_4^2 - 2aX_4 + Y_4^2 - 2bY_4 = C$$

Solve above for a & b in terms of measured x's & y's.

$$a(X_4 - X_2) + b(Y_4 - Y_2) = \frac{Y_4^2 - Y_2^2}{2} + \frac{X_4^2 - X_2^2}{2} \qquad (2)$$

$$a(X_3 - X_2) + b(Y_3 - Y_2) = \frac{Y_3^2 - Y_2^2}{2} + \frac{X_3^2 - X_2^2}{2} \qquad (3)$$

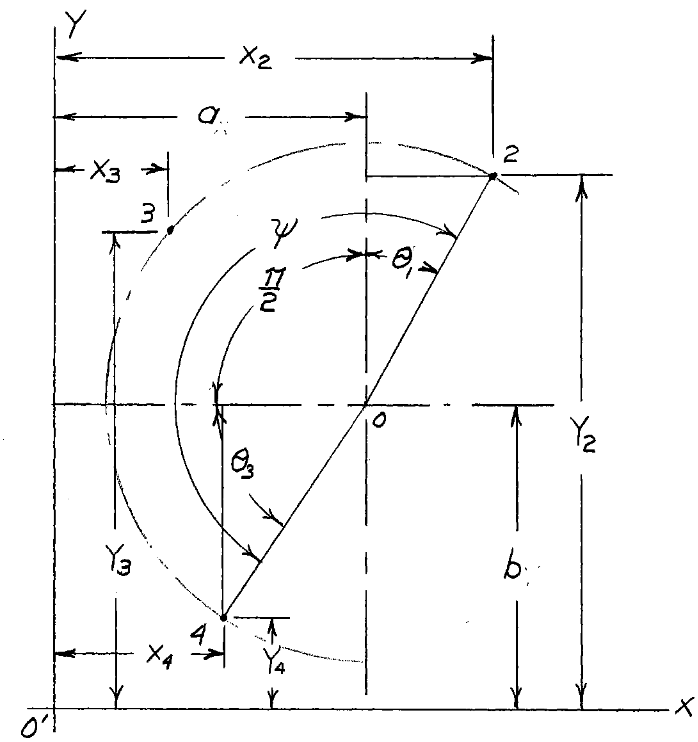

Fig. 3. Angle of Arc of Central Axis

$$\theta_1 = \tan^{-1} \frac{X_2 - a}{Y_2 - b}$$

$$\theta_3 = \tan^{-1} \frac{b - Y_4}{a - X_4}$$

$$\psi = \theta_1 + \frac{\pi}{2} + \theta_3$$

$$S_o = \psi_o R_o \quad ; \quad S_i = \psi_i R_i$$

From calculated results, Table II.

$$\varepsilon = S_i - S_o \approx 0 \tag{4}$$

$$\Delta\psi = \psi_o - \psi_i$$

$$\frac{\Delta\psi}{\psi_o} = \frac{\psi_o - \psi_i}{\psi_o} \tag{5}$$

$$\frac{\Delta R}{R_o} = \frac{R_i - R_o}{R_o} \tag{6}$$

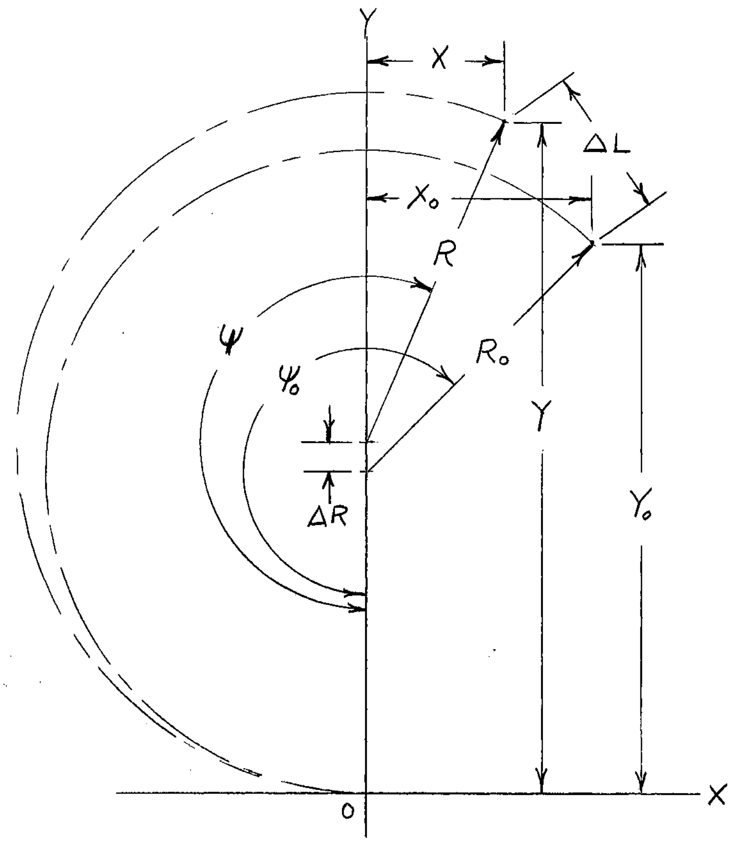

Fig. 4. Bourdon Tube Geometry

$$X = -R\sin\psi \quad ; \quad Y = R - R\cos\psi = R(1-\cos\psi); \; \pi < \psi < \tfrac{3}{2}\pi$$

$$dX = -R\cos\psi\, d\psi - \sin\psi\, dR$$

$$dY = R\sin\psi\, d\psi + (1-\cos\psi)\, dR$$

For small differences, differential equations may be written as difference equations.

$$\Delta X = -R\cos\psi\,\Delta\psi - \sin\psi\,\Delta R$$

$$\Delta Y = R\sin\psi\,\Delta\psi + (1-\cos\psi)\,\Delta R$$

From computed values - Table II.

$$\frac{\Delta R}{R_o} \approx \frac{\Delta\psi}{\psi_o} \; ; \; \therefore \Delta\psi \approx \frac{\psi_1}{R_o}\Delta R \qquad \text{— } \psi\text{'s in radians}$$

$$R = R_o + \Delta R, \qquad \Delta\psi = \psi_o - \psi \;\text{ or }\; \psi = \psi_o - \Delta\psi$$

$$\therefore \quad \psi = \psi_0 - \frac{\psi_0}{R_0}\Delta R$$

ΔR and measurable initial values were of interest — making the substitutions —

$$\Delta x = -(R_0 + \Delta R)\psi_0 \frac{\Delta R}{R_0} \cos\left(\psi_0 - \psi_0 \frac{\Delta R}{R_0}\right) - \Delta R \sin\left(\psi_0 - \psi_0 \frac{\Delta R}{R_0}\right)$$

$$\Delta Y = (R_0 + \Delta R)\psi_0 \frac{\Delta R}{R_0} \sin(\psi_0 - \psi_0 \frac{\Delta R}{R_0} + \Delta R\left[1 - \cos(\psi_0 - \psi_0 \frac{\Delta R}{R_0}\right]$$

From trigonometry

$$\sin(A - B) = \sin A \cos B - \cos A \sin B$$

$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$

making the substitutions

$$\Delta X = -(R_0 + \Delta R)\psi_0 \frac{\Delta R}{R_0}\left[\cos\psi_0 \cos\frac{\Delta R}{R_0} + \sin\psi_0 \sin\psi_0 \frac{\Delta R}{R_0}\right] -$$
$$-\Delta R\left[\sin\psi_0 \cos\psi_0 \frac{\Delta R}{R_0} - \cos\psi_0 \sin\psi_0 \frac{\Delta R}{R_0}\right]$$

$$\Delta Y = (R_0 + \Delta R)\psi_0 \frac{\Delta R}{R_0}\left[\sin\psi_0 \cos\psi_0 \frac{\Delta R}{R_0} - \cos\psi_0 \sin\psi_0 \frac{\Delta R}{R_0}\right]$$
$$+\Delta R - \Delta R\left[\cos\psi_0 \cos\psi_0 \frac{\Delta R}{R_0} + \sin\psi_0 \sin\psi_0 \frac{\Delta R}{R_0}\right]$$

$$\sin\psi_0 \frac{\Delta R}{R_0} \approx \psi_0 \frac{\Delta R}{R_0} \quad ; \quad \cos\psi_0 \frac{\Delta R}{R_0} \approx 1$$

making the substitutions

$$\Delta X = -(R_0 + \Delta R)\psi_0 \frac{\Delta R}{R_0}\left[\cos\psi_0 + \psi_0 \frac{\Delta R}{R_0}\sin\psi_0\right] - \Delta R\left[\sin\psi_0 - \psi_0 \frac{\Delta R}{R_0}\cos\psi_0\right]$$

$$\Delta Y = (R_0 + \Delta R)\psi_0 \frac{\Delta R}{R_0}\left[\sin\psi_0 - \psi_0 \frac{\Delta R}{R_0}\cos\psi_0\right] + \Delta R - \Delta R\left[\cos\psi_0 + \psi_0 \frac{\Delta R}{R_0}\sin\psi_0\right]$$

Expand both equations — collect terms — neglect all terms higher than terms of the first order.

$$\Delta X = -\Delta R\,(\psi_0 \cos\psi_0 + \sin\psi_0)$$

$$\Delta Y = \Delta R\,(1 + \psi_0 \sin\psi_0 - \cos\psi_0)$$

$$\Delta L = \left[\overline{\Delta X}^2 + \overline{\Delta Y}^2\right]^{\frac{1}{2}}$$

$$\Delta L = \Delta R\left[\psi_0^2 + 2 + 2(\psi_0 \sin\psi_0 - \cos\psi_0\right]^{\frac{1}{2}} \qquad (7)$$

CHAPTER IV

SELECTION OF THE MODEL EQUATION

A deflection equation that will predict the 'tip travel' of a Bourdon Tube is of the form $\Delta L = f(\psi_o \Delta R)$ in Figure 4. In order to devise a numerical analysis solution for Bourdon Tube deflection, deflection equations for other geometric shapes were examined for form.

A deflection equation for a simple beam is of the form $\delta = C \frac{P L^3}{E I}$, where deflection is a function of load, stiffness, and the geometry of the member. At the beginning of this study it was the opinion of the author of this paper that the deflection of a Bourdon Tube could be described in terms of load, stiffness of the material, and the geometry of the tube. Since the cross-sectional shape of the tube changes in a manner difficult to describe, an attempt to analyze the deflection in terms of a curved beam theory was abandoned.

For simple forms of deflection equations, the terms are all in the forms of products and powers, so a model equation of products and powers was devised and tried. This model equation recognized ΔR as a function of P, E, R_o, w, t, and h.

A model equation may be one of many forms. It may be linear, $y = a_o + a_1 x$; it may be parabolic, $y = a_o + a_1 x + a_2 x^2$; it may be cubic, $y = a_0 + a_1x + a_2x^2 + a_3x^3$; or it may be an exponential, logarithmic, or some other mathematical form. The form of a model equation may be selected in many different ways. Experience may dictate the form of a model equation, or plotting some values from experimental results may indicate a trend which will help select a model. No exact method is known for selecting the correct model of an equation on the first attempt. The only criterion for an acceptable model equation is the result obtained by testing it.

Since no parameter of one tube was equal to a corresponding parameter of another tube, that is, the major axis, minor axis, radius, and wall thickness varied from tube to tube, it was necessary to devise a curve fitting program that would yield an equation that would best fit the measured parameters of the tubes and the experimental results. ΔR was taken as the dependent variable and the values P, E, R_0, w, t, and h were used as independent variables.

The large number of independent variables in this study limits the possible variety of model equations that may be tried because of the lengthy computations involved in testing the various models. The first model tested was a simple product and power type: $\Delta R = P^{a_1} E^{a_2} R^{a_3} w^{a_4} t^{a_5} h^{a_6}$. This model was abandoned due to the lack of dimensional homogeneity.

The second model tested was of the same type as the first with one exception, a constant was added and the model took the form: $\Delta R = KPE^{-1} R^{a_1} w^{a_2} t^{a_3} h^{a_4}$. This model was discarded because the values a_1, a_2, a_3, etc. obtained in the solution appeared to be improbable values, i.e., values such as 11.6, 21.2, etc. The work at this stage was very difficult to compute, using slide rule accuracy. It will be noted here that an attempt was being made to keep the equations in a form similar to a simple beam equation. The Buckingham π theorem was tried and discarded at the beginning of the study. This was necessary because dimensionless groupings could not be formed.

The third model tested was of the form:
$\Delta R = e^{a_0} P^{a_1} E^{a_2} R_o^{a_3} w^{a_4} t^{a_5} h^{a_6}$. This form appeared promising so an attempt was made to refine the equation.

The manner in which the above equations were tested was simple, but involved numerous lengthy computations. All values except the a's were known from experimental work. Data were taken from as many tubes as there are unknowns in the model equation. As an example, there are six a's in the model equation above; therefore, six tubes were selected from the entire group of tubes. For each tube a ΔR and its corresponding pressure were selected, in this case the maximum pressure and ΔR were chosen. The natural logarithm was taken of both sides of the equation, and the equation took the following form:

$$\ln \Delta R = a_0 + a_1 \ln P + a_2 \ln E + a_3 \ln R_o + a_4 \ln w + a_5 \ln t + a_6 \ln h.$$

Six simultaneous equations were set up and carefully solved for the a's and the a's were found to have values that appeared to be reasonable, indicating that this might be an acceptable model equation. The values of the a's were substituted back into the model equation and the equation was tested solving for the same Δ R's that were used while solving for the a's. The model equation would predict these Δ R's within 15 to 20 percent of their measured values.

The next step in the solution was a curve fitting program to obtain the best possible values for the a's. The IBM 650 computer located on the campus was designed to calculate automatically the constants and coefficients for a wide range of polynomial regression equations. It will also indicate to the program user how good, in a statistical sense, is the equation which was selected to represent the data.

If it can be assumed that experimental errors are normally distributed, it can be shown that the "best fit is that one which minimizes the sum of squares of deviation of the observations from the function. For example

algebraically: $\Sigma(y_o - y_c)^2 = \text{minimum}$

where y_o = observed value

and y_c = calculated value.

If, for example, it is assumed that the functional relationship between y and x_1, x_2, x_3 can be expressed as a linear function, $y = a_o + a_1 x_1 + a_2 x_2 + a_3 x_3$, the best fit in the sense of least squares requires:

$$\Sigma(y_o - y_c)^2 = \Sigma\left[y_o - (a_o + a_1 x_1 + a_2 x_2 + a_3 x_3)\right]^2 = \text{minimum}$$

where the summation is over the set of observations. It should be pointed out that the x's need not be independent of each other. Thus, the x's can themselves be functionally related. For example one could have:

$x_2 = x_1^{\,2}, \; x_3 = x_1^{\frac{1}{2}}$, etc.

As mentioned above, the criterion of least squares requires a set of a's which minimizes

$$\Sigma \left[y - (a_o + a_1 x_1 + a_2 x_2 + a_3 x_3) \right]^2 .$$

From the calculus, this minimum may be found by equating to zero each of the partial derivatives with respect to the a's. This leads to a system of linear algebraic equations which may be solved for the a's.

Using the example with three independent variables, one obtains:

$$n\, a_o + a_1 \Sigma x_1 + a_2 \Sigma x_2 + a_3 \Sigma x_3 = \Sigma y \qquad (1)$$

$$a_o \Sigma x_1 + a_1 \Sigma x_1^2 + a_2 \Sigma x_1 x_2 + a_3 \Sigma x_1 x_3 = \Sigma y x_1 \qquad (2)$$

$$a_o \Sigma x_2 + a_1 \Sigma x_1 x_2 + a_2 \Sigma x_2^2 + a_3 \Sigma x_2 x_3 = \Sigma y x_2 \qquad (3)$$

$$a_o \Sigma x_3 + a_1 \Sigma x_1 x_3 + a_2 \Sigma x_2 x_3 + a_3 \Sigma x_3^2 = \Sigma y x_3 \qquad (4)$$

where n = number of observations

a_o, a_1, a_2, a_3 = coefficients to be determined

x_1, x_2, x_3 = independent variables

y = dependent variable

The solution of these so-called normal equations yields the desired coefficients.(6)

Two expressions relating ΔR to the tube parameters and pressures were developed for this paper. They are:

$$\Delta R = \frac{e^{-10.447}\, P^{0.913}\, w^{1.702}\, t^{0.6014}}{\left(\frac{E}{10^6}\right)^{1.652} R_o^{0.601}\, h^{2.538}} \qquad (A)$$

and

$$\triangle R = \frac{e^{-4.14}\ P\ w^{0.86}\ t^{1.82}}{\frac{E}{10^6}\ R_o^{1.46}\ h^{1.16}} \qquad (B)$$

A comparison of both calculated and measured $\triangle R$'s using Equations A and B above appears in Table 2 of the calculated results.

These equations were both established using the same model. Equation A was established by allowing the IBM 650 computer to predict all the exponents for a maximum accuracy curve-fit using the least squares or RAP program. Equation B was developed by arbitrarily fixing the exponents on P and E and then allowing the IBM 650 computer to predict the remaining exponents. Table 2 compares the results of the two equations within slide rule accuracy.

CHAPTER V

OBSERVED AND CALCULATED DATA

The observed data of this investigation are presented in Table 1. The calculated data appear in Table 2. In nearly all cases, $\triangle$ R was almost a linear function of P, as the $\triangle$ R values that were calculated from the measured data show.

The $\triangle$ R values that were calculated by the two expressions developed in this study do not appear linear with P.

The model equation that assumed $\triangle$ R linear with P yielded calculated values that differed from experimental values in a range from 4.5% to 4050%. This lack of correlation between experimental values and values computed from the empirical expression (P linear) indicated that the particular model equation was a poor model and that it yielded a poor fit to the true deflection curve.

The model equation that assumed $\triangle$ R not linear with P yielded calculated values of $\triangle$ R that differed from experimental values of $\triangle$ R in a range from 0% to 83%. Sixty per cent of the calculated values agreed with the experimental values within 25%. In cases where the per cent errors between experimental values and calculated values of $\triangle$ R were all larger than 25% for all the $\triangle$ R's on a particular tube, the tube was re-examined for cross-sectional shape. In every case where the errors between experimental and calculated values were large for all the ΔR's on a particular tube, the tube deviated from a flat-oval cross-sectional shape (Fig. 1).

Table 2 contains a column for values of $t/_{w}$. This column was used to determine whether a correlation existed between $t/_{w}$ and per cent error. No correlation was discovered.

Empirical Equation A appeared to be the preferred expression, more nearly fitting the true deflection curve.

TABLE I

OBSERVED DATA

Tube No. and rated P	P psi	E psi	W in.	t in.	h in.	x_2 in.	y_2 in.	x_3 in.	y_3 in.	x_4 in.	y_4 in.
	0	$28.5x10^6$	0.921	0.157	0.022	2.6840	4.5963	1.2091	4.0881	0.8435	2.5486
	20					2.6707	4.6111	1.1994	4.0900	0.8418	2.5481
I	40					2.6581	4.6265	1.1911	4.0916	0.8403	2.5474
100 psi	60					2.6452	4.6417	1.1824	4.0934	0.8396	2.5469
	80					2.6318	4.6569	1.1738	4.0949	0.8376	2.5463
	100					2.6189	4.6722	1.1656	4.0963	0.8371	2.5460
	0	$28.5x10^6$	0.714	0.184	0.025	3.4194	4.2063	1.7330	4.4825	0.7408	2.8525
II	50					3.4129	4.2312	1.7216	4.4886	0.7389	2.8519
200 psi	100					3.4051	4.2566	1.7106	4.4949	0.7376	2.8514
	150					3.3974	4.2815	1.6989	4.5008	0.7349	2.8500
	200					3.3886	4.3066	1.6876	4.5069	0.7333	2.8495
	0	$28.5x10^6$	0.716	0.170	0.030	3.4532	4.2399	1.3863	4.2556	0.8977	2.4165
	110					3.4427	4.2774	1.3724	4.2597	0.8967	2.4155
III	210					3.4300	4.3116	1.3595	4.2635	0.8965	2.4145
300 psi	310					3.4181	4.3456	1.3468	4.2670	0.8951	2.4135
	410					3.4052	4.3796	1.3332	4.2707	0.8944	2.4125
	0	$28.5x10^6$	0.700	0.214	0.030	3.4626	4.2694	1.3451	4.2417	0.8513	2.4965
IV	110					3.4531	4.2995	1.3324	4.2446	0.8506	2.4957
400 psi	210					3.4448	4.3268	1.3231	4.2473	0.8498	2.4948
	310					3.4357	4.3535	1.3134	4.2497	0.8496	2.4941
	410					3.4270	4.3808	1.3048	4.2526	0.8486	2.4931
	0	$28.5x10^6$	0.710	0.187	0.035	3.4016	4.2583	1.2547	4.1167	0.8505	2.5104
	110					3.3941	4.2802	1.2477	4.1182	0.8504	2.5097
V	210					3.3883	4.3000	1.2410	4.1197	0.8501	2.5089
500 psi	310					3.3811	4.3196	1.2349	4.1211	0.8497	2.5084
	410					3.3744	4.3399	1.2286	4.1227	0.8492	2.5079
	510					3.3672	4.3593	1.2221	4.1242	0.8485	2.5075
	0	$28.5x10^6$	1.111	0.155	0.0095	3.1797	4.4496	1.7240	4.5104	0.7546	3.3707
VI	10					3.1520	4.5103	1.6940	4.5278	0.7443	3.3695
15 psi	20					3.1260	4.5725	1.6629	4.5445	0.7343	3.3676
	30					3.0934	4.6345	1.6313	4.5614	0.7241	3.3661
	40					3.0583	4.6977	1.5986	4.5780	0.7139	3.3642
	0	$28.5x10^6$	0.907	0.172	0.016	2.8900	4.6024	1.4304	4.3850	0.7605	3.1967
VII	30					2.8651	4.6414	1.4077	4.3928	0.7531	3.1947
60 psi	60					2.8366	4.6808	1.3837	4.4002	0.7456	3.1925
	90					2.8067	4.7217	1.3599	4.4072	0.7380	3.1901
	120					2.7765	4.7606	1.3358	4.4143	0.7305	3.1879
	0	$28.5x10^6$	0.687	0.205	0.040	3.3908	4.2715	1.3287	4.2082	0.9606	2.1912
VIII	210					3.3811	4.2975	1.3204	4.2108	0.9609	2.1903
600 psi	410					3.3746	4.3217	1.3119	4.2129	0.9617	2.1896
	610					3.3643	4.3460	1.3035	4.2154	0.9619	2.1890
	810					3.3549	4.3701	1.2943	4.2179	0.9626	2.1885
	0	$28.5x10^6$	0.684	0.219	0.045	3.6221	4.1253	1.5513	4.3436	0.8866	2.4519
IX	210					3.6201	4.1443	1.5462	4.3463	0.8867	2.4515
800 psi	410					3.6165	4.1625	1.5410	4.3490	0.8877	2.4508
	610					3.6140	4.1810	1.5352	4.3518	0.8877	2.4502
	810					3.6095	4.1994	1.5289	4.3545	0.8883	2.4501
	0	$28.5x10^6$	0.681	0.211	0.049	3.7566	4.0711	1.7979	4.4256	1.0000	2.5150
X	250					3.7543	4.0881	1.7932	4.4287	1.0006	2.5146
1000 psi	500					3.7532	4.1054	1.7892	4.4318	1.0012	2.5142
	750					3.7502	4.1220	1.7835	4.4348	1.0023	2.5138
	1000					3.7479	4.1388	1.7786	4.4379	1.0025	2.5130

TABLE I (CONTINUED)

Tube No. and rated P	P psi	E psi	W in.	t in.	h in.	x_2 in.	y_2 in.	x_3 in.	y_3 in.	x_4 in.	y_4 in.
	0	$15.5x10^6$	0.835	0.248	0.018	5.6128	4.3026	2.2195	5.4415	1.2373	1.8456
	10					5.6140	4.3457	2.2051	5.4491	1.2378	1.8460
XI	20					5.6146	4.3898	2.1920	5.4566	1.2378	1.8460
50 psi	30					5.6148	4.4342	2.1766	5.4639	1.2378	1.8457
	40					5.6133	4.4811	2.1618	5.4715	1.2372	1.8454
	50					5.6126	4.5248	2.1444	5.4793	1.2368	1.8452
	0	$15.5x10^6$	0.854	0.296	0.028	5.7115	3.6926	2.5897	5.5489	0.8139	2.3631
XII	30					5.7200	3.7429	2.5716	5.5613	0.8129	2.3627
125 psi	60					5.7280	3.7932	2.5557	5.5739	0.8119	2.3621
	90					5.7350	3.8443	2.5370	5.5862	0.8105	2.3612
	130					5.7438	3.9119	2.5140	5.6023	0.8091	2.3604
	0	$15.5x10^6$	0.850	0.281	0.025	5.4975	4.0161	2.1174	5.5054	0.7270	2.2298
XIII	30					5.5034	4.0835	2.0945	5.5184	0.7258	2.2292
100 psi	60					5.5078	4.1505	2.0656	5.5310	0.7246	2.2284
	90					5.5119	4.2161	2.0481	5.5435	0.7232	2.2287
	120					5.5147	4.2861	2.0258	5.5561	0.7219	2.2268
	0	$15.5x10^6$	0.849	0.272	0.023	5.2216	4.1512	2.0178	5.5490	0.5253	2.2589
XIV	20					5.2248	4.2069	1.9976	5.5610	0.5238	2.2580
75 psi	40					5.2271	4.2628	1.9787	5.5731	0.5229	2.2574
	60					5.2289	4.3188	1.9581	5.5847	0.5219	2.2571
	80					5.2295	4.3736	1.9389	5.5962	0.5208	2.2564
	0	$15.5x10^6$	0.876	0.310	0.038	5.3014	3.9874	2.1130	5.5084	0.5642	2.2482
XV	60					5.3032	4.0363	2.0952	5.5193	0.5630	2.2474
225 psi	120					5.3068	4.0844	2.0786	5.5300	0.5621	2.2470
	180					5.3100	4.1344	2.0604	5.5406	0.5611	2.2464
	240					5.3123	4.1832	2.0438	5.5514	0.5602	2.2458
	0	$15.5x10^6$	0.863	0.296	0.030	5.6558	3.8272	2.1535	5.4773	0.7592	2.3152
XVI	40					5.6668	3.8898	2.1335	5.4886	0.7576	2.3148
140 psi	80					5.6743	3.9495	2.1135	5.4992	0.7541	2.3136
	120					5.6819	4.0095	2.0945	5.4999	0.7547	2.3126
	160					5.6877	4.0695	2.0749	5.5205	0.7537	2.3119
	0	$15.5x10^6$	0.877	0.299	0.035	5.1185	4.0229	1.8068	5.5349	0.3457	2.2809
XVII	50					5.1234	4.0714	1.7904	5.5447	0.3448	2.2805
200 psi	100					5.1278	4.1202	1.7743	5.5547	0.3440	2.2799
	150					5.1315	4.1685	1.7576	5.5642	0.3427	2.2793
	200					5.1345	4.2174	1.7412	5.5739	0.3419	2.2788
	0	$15.5x10^6$	0.867	0.304	0.032	5.3047	3.9942	2.0827	5.5365	0.5462	2.1915
XVIII	40					5.3090	4.0424	2.0642	5.5470	0.5455	2.1912
150 psi	80					5.3136	4.0911	2.0492	5.5579	0.5447	2.1908
	120					5.3173	4.1400	2.0308	5.5682	0.5438	2.1903
	160					5.3206	4.1890	2.0156	5.5788	0.5431	2.1899
	0	$15.5x10^6$	0.825	0.265	0.016	5.2913	4.0705	1.9758	5.5265	0.5482	2.2551
XIX	10					5.2953	4.1755	1.9441	5.5437	0.5461	2.2542
30 psi	20					5.3011	4.2346	1.9242	5.5556	0.5454	2.2536
	30					5.3024	4.2937	1.9031	5.5669	0.5442	2.2531
	40					5.3040	4.3534	1.8826	5.5784	0.5431	2.2527
	0	$15.5x10^6$	0.835	0.272	0.020	5.3107	3.9701	1.9608	5.4858	0.5156	2.2774
XX	20					5.3202	4.0526	1.9336	5.5022	0.5146	2.2768
60 psi	40					5.3274	4.1376	1.9069	5.5189	0.5135	2.2761
	60					5.3332	4.2230	1.8777	5.5354	0.5119	2.2753
	80					5.3355	4.3115	1.8488	5.5517	0.5107	2.2771

TABLE I (CONTINUED)

Tube No. and rated P	P psi	E psi	W in.	t in.	h in.	x_2 in.	y_2 in.	x_3 in.	y_3 in.	x_4 in.	y_4 in.
	0	28×10^6	0.634	0.290	0.038	4.1113	5.8671	0.8999	4.6357	1.8239	1.5601
XXI	200					4.0938	5.8901	0.8902	4.6356	1.8234	1.5600
600 psi	400					4.0754	5.9122	0.8802	4.6336	1.8229	1.5593
	600					4.0576	5.9337	0.8694	4.6313	1.8228	1.5588
	800					4.0389	5.9567	0.8587	4.6241	1.8224	1.5579
	0	28×10^6	0.637	0.296	0.055	4.4202	5.7533	1.0233	4.8058	1.8676	1.4925
XXII	250					4.4146	5.7618	1.0196	4.8051	1.8675	1.4922
2000 psi	500					4.4085	5.7704	1.0150	4.8043	1.8674	1.4919
	750					4.4023	5.7790	1.0110	4.8036	1.8674	1.4917
	1000					4.3962	5.7875	1.0067	4.8031	1.8672	1.4914
	0	28×10^6	0.647	0.279	0.048	4.3528	5.7895	0.9578	4.8019	1.9289	1.4337
XXIII	250					4.3414	5.8051	0.9496	4.8004	1.9288	1.4334
1000 psi	500					4.3302	5.8206	0.9403	4.7992	1.9286	1.4330
	750					4.3129	5.8354	0.9338	4.7979	1.9283	1.4325
	1000					4.3071	5.8512	0.9263	4.7964	1.9284	1.4323
	0	28×10^6	0.848	0.295	0.033	4.0934	5.8949	0.9183	4.6169	1.8511	1.5449
XXIV	50					4.0773	5.9137	0.9075	4.6144	1.8508	1.5440
200 psi	100					4.0617	5.9323	0.8992	4.6126	1.8506	1.5433
	150					4.0457	5.9507	0.8897	4.6108	1.8503	1.5429
	200					4.0299	5.9689	0.8810	4.6087	1.8501	1.5421
	0	28×10^6	0.845	0.277	0.024	3.8268	5.9213	0.8042	4.6006	1.8416	1.4336
XV	20					3.8095	5.9397	0.7942	4.5986	1.8411	1.4325
60 psi	40					3.7915	5.9586	0.7841	4.5965	1.8408	1.4321
	60					3.7750	5.9775	0.7738	4.5945	1.8411	1.4315
	80					3.7565	5.9960	0.7649	4.5920	1.8406	1.4311

TABLE II

DATA COMPUTED

Tube No.	P psi	a in.	b in.	R in.	ΔR Exp in.	$\Delta R / R_o$	ψ rad.	$\Delta\psi$ rad.	$\Delta\psi / \psi_o$	ε in.	ΔR_A Calc. in.	Error %	ΔR_B Calc. in.	Error %	t/w
I	0	2.413	2.989	1.630			2.012				0				0.17
	20	2.415	2.996	1.636	0.006	0.003	2.005	0.007	0.003	0.000	0.006	0	0.017	180	
	40	2.420	3.000	1.643	0.013	0.008	1.996	0.016	0.008	0.000	0.012	7.7	0.034	161	
	60	2.424	3.007	1.650	0.020	0.012	1.988	0.024	0.012	0.000	0.018	10.0	0.051	155	
	80	2.429	3.012	1.658	0.028	0.017	1.978	0.033	0.017	0.000	0.023	17.8	0.069	146	
	100	2.4334	3.018	1.665	0.035	0.021	1.969	0.042	0.021	0.000	0.028	20.0	0.086	146	
II	0	2.354	2.987	1.619			2.373								0.258
	50	2.359	2.992	1.626	0.007	0.004	2.362	0.011	0.004	0.000	0.007	0	0.035	400	
	100	2.364	2.998	1.663	0.014	0.008	2.352	0.021	0.008	0.000	0.014	0	0.070	400	
	150	2.369	3.003	1.641	0.022	0.013	2.341	0.031	0.013	0.001	0.021	4.5	0.101	359	
	200	2.373	3.008	1.648	0.029	0.018	2.331	0.042	0.018	0.000	0.027	6.9	0.140	383	
III	0	2.410	2.999	1.621			2.637								0.237
	110	2.418	3.006	1.632	0.011	0.007	2.619	0.018	0.007	0.001	0.009	18.2	0.054	391	
	210	2.424	3.014	1.641	0.020	0.013	2.604	0.033	0.013	0.000	0.016	20.0	0.103	415	
	310	2.431	3.021	1.652	0.031	0.019	2.588	0.050	0.019	0.000	0.023	25.8	0.150	384	
	410	2.437	3.029	1.662	0.041	0.025	2.573	0.065	0.025	0.000	0.031	24.4	0.200	388	
IV	0	2.420	2.995	1.646			2.564								0.306
	110	2.426	3.003	1.655	0.008	0.005	2.552	0.012	0.005	0.002	0.010	25.0	0.079	888	
	210	2.432	3.008	1.663	0.017	0.010	2.539	0.025	0.010	0.002	0.018	5.9	0.150	782	
	310	2.438	3.014	1.671	0.024	0.015	2.527	0.037	0.014	0.001	0.026	8.3	0.220	817	
	410	2.444	3.018	1.680	0.034	0.020	2.514	0.050	0.020	0.001	0.033	2.9	0.290	752	
V	0	2.408	2.972	1.625			2.517								0.264
	110	2.413	2.977	1.631	0.006	0.004	2.507	0.010	0.004	0.001	0.006	0	0.053	783	
	210	2.418	2.981	1.637	0.012	0.008	2.498	0.019	0.008	0.001	0.012	0	0.100	733	
	310	2.422	2.985	1.643	0.018	0.011	2.488	0.029	0.011	0.001	0.016	11.2	0.150	733	
	410	2.427	2.989	1.650	0.025	0.015	2.478	0.039	0.015	0.001	0.022	12.0	0.200	700	
	510	2.431	2.993	1.656	0.031	0.019	2.469	0.048	0.019	0.001	0.026	16.1	0.240	674	
VI	0	2.388	2.963	1.684			1.815								0.140
	10	2.404	2.977	1.705	0.022	0.013	1.792	0.023	0.013	0.000	0.039	77.0	0.021	4.55	
	20	2.425	2.989	1.732	0.048	0.029	1.767	0.048	0.027	0.004	0.074	54.0	0.043	10.4	
	30	2.442	3.004	1.756	0.072	0.043	1.743	0.072	0.040	0.003	0.107	48.6	0.064	11.1	
	40	2.461	3.020	1.781	0.097	0.057	1.718	0.097	0.053	0.003	0.139	43.3	0.085	12.4	
VII	0	2.372	3.071	1.616			1.819								0.190
	30	2.381	3.083	1.632	0.015	0.009	1.804	0.016	0.009	0.002	0.022	46.6	0.055	267	
	60	2.389	3.097	1.646	0.030	0.018	1.788	0.031	0.017	0.003	0.040	33.3	0.110	267	
	90	2.400	3.108	1.664	0.047	0.029	1.769	0.050	0.028	0.002	0.059	25.5	0.165	251	
	120	2.408	3.122	1.679	0.063	0.039	1.753	0.066	0.037	0.003	0.076	20.6	0.220	249	

TABLE II (CONTINUED)

Tube No.	P psi	a in.	b in.	R in.	ΔR Exp in.	$\frac{\Delta R}{R_o}$	ψ rad.	Δψ rad.	$\frac{\Delta\psi}{\psi_o}$	ε in.	ΔR_A Calc. in.	Error %	ΔR_B Calc. in.	Error %	t/w
XV	0	2.931	3.122	2.523			3.145								0.358
	60	2.938	3.130	2.533	0.010	0.004	3.132	0.013	0.004	0.002	0.012	20.0	0.075	650	
	120	2.945	3.138	2.544	0.021	0.008	3.118	0.027	0.008	0.001	0.023	9.5	0.150	615	
	180	2.953	3.146	2.556	0.032	0.013	3.104	0.041	0.013	0.002	0.033	3.1	0.225	510	
	240	2.961	3.154	2.567	0.044	0.017	3.091	0.054	0.017	0.002	0.043	2.3	0.300	582	
XVI	0	3.189	3.132	2.563			3.192								0.343
	40	3.200	3.141	2.578	0.015	0.006	3.173	0.018	0.006	0.001	0.015	0	0.059	293	
	80	3.208	3.149	2.593	0.029	0.011	3.156	0.036	0.011	0.001	0.027	6.9	0.119	310	
	120	3.222	3.151	2.606	0.042	0.017	3.133	0.058	0.018	0.002	0.040	4.8	0.178	324	
	160	3.228	3.169	2.619	0.056	0.022	3.124	0.067	0.021	0.001	0.053	5.4	0.238	325	
XVII	0	2.727	3.167	2.540			3.154								0.341
	50	2.735	3.174	2.552	0.011	0.004	3.140	0.014	0.004	0.001	0.013	18.2	0.064	482	
	100	2.743	3.182	2.563	0.022	0.009	3.126	0.028	0.009	0.000	0.024	9.1	0.129	486	
	150	2.751	3.189	2.574	0.034	0.013	3.113	0.041	0.013	0.000	0.035	2.9	0.193	467	
	200	2.759	3.197	2.586	0.045	0.018	3.099	0.055	0.017	0.000	0.045	0	0.259	475	
XVIII	0	2.911	3.131	2.545			3.173								0.350
	40	2.919	3.139	2.555	0.011	0.004	3.160	0.013	0.004	0.000	0.013	18.2	0.058	427	
	80	2.927	3.146	2.567	0.022	0.009	3.146	0.027	0.009	0.000	0.024	9.1	0.116	427	
	120	2.935	3.154	2.578	0.034	0.013	3.132	0.041	0.013	0.001	0.035	2.9	0.174	412	
	160	2.944	3.161	2.590	0.045	0.018	3.118	0.055	0.017	0.000	0.045	0	0.232	415	
XIX	0	2.918	3.168	2.539			3.146								0.321
	10	2.934	3.181	2.562	0.022	0.009	3.113	0.033	0.011	0.014	0.016	27.2	0.024	18.2	
	20	2.946	3.190	2.577	0.037	0.015	3.096	0.050	0.016	0.012	0.031	16.2	0.049	32.4	
	30	2.955	3.200	2.590	0.051	0.020	3.080	0.066	0.021	0.012	0.045	11.7	0.073	43	
	40	2.965	3.210	2.604	0.064	0.025	3.063	0.083	0.026	0.012	0.058	9.4	0.098	53	
XX	0	2.912	3.128	2.543			3.145								0.326
	20	2.926	3.141	2.562	0.019	0.008	3.122	0.023	0.007	0.001	0.019	0	0.025	31.6	
	40	2.941	3.154	2.581	0.039	0.015	3.097	0.047	0.015	0.001	0.037	5.1	0.050	28.2	
	60	2.955	3.168	2.601	0.059	0.023	3.074	0.071	0.023	0.001	0.053	10.2	0.075	27.1	
	80	2.970	3.183	2.621	0.078	0.031	3.049	0.096	0.030	0.005	0.069	11.5	0.100	28.2	
XXI	0	3.128	3.629	2.445			2.993								0.458
	200	3.129	3.636	2.452	0.007	0.003	2.984	0.009	0.003	0.000	0.008	14.3	0.100	1330	
	400	3.130	3.642	2.459	0.014	0.006	2.975	0.018	0.006	0.001	0.016	14.3	0.200	1330	
	600	3.131	3.649	2.466	0.021	0.008	2.968	0.025	0.008	0.001	0.022	4.8	0.300	1330	
	800	3.134	3.655	2.473	0.028	0.012	2.957	0.036	0.012	0.004	0.029	3.6	0.400	1330	

TABLE II (CONTINUED)

Tube No.	P psi	a in.	b in.	R in.	ΔR Exp in.	$\frac{\Delta R}{R_o}$	ψ rad.	$\Delta\psi$ rad.	$\frac{\Delta\psi}{\psi_o}$	ε in.	ΔR_A Calc. in.	Error %	ΔR_B Calc. in.	Error %	t/w
VIII	0	2.399	2.971	1.636			2.719								0.298
	210	2.405	2.976	1.643	0.007	0.005	2.705	0.014	0.005	0.002	0.009	28.6	0.099	1315	
	410	2.411	2.981	1.651	0.015	0.009	2.693	0.026	0.009	0.001	0.015	0	0.193	1187	
	610	2.416	2.986	1.658	0.022	0.014	2.681	0.038	0.014	0.003	0.022	0	0.286	1200	
	810	2.421	2.992	1.665	0.029	0.018	2.670	0.049	0.018	0.003	0.029	0	0.381	1214	
IX	0	2.453	2.964	1.648			2.676								0.320
	210	2.459	2.967	1.654	0.006	0.004	2.667	0.010	0.004	0.000	0.007	16.7	0.095	1485	
	410	2.463	2.970	1.659	0.011	0.007	2.659	0.018	0.007	0.000	0.012	9.1	0.187	1600	
	610	2.468	2.973	1.665	0.016	0.010	2.649	0.027	0.010	0.001	0.017	6.2	0.278	1636	
	810	2.472	2.977	1.669	0.022	0.013	2.641	0.034	0.013	0.000	0.022	0	0.369	1576	
X	0	2.549	2.990	1.621			2.709								0.31
	250	2.554	2.993	1.625	0.005	0.003	2.700	0.008	0.003	0.001	0.006	20.0	0.099	1880	
	500	2.559	2.995	1.631	0.010	0.006	2.691	0.017	0.006	0.001	0.011	10.0	0.199	1890	
	750	2.563	2.998	1.634	0.014	0.009	2.685	0.024	0.009	0.002	0.016	14.3	0.299	2038	
	1000	2.568	3.001	1.639	0.019	0.012	2.677	0.032	0.012	0.001	0.021	10.5	0.399	2000	
XI	0	3.355	3.199	2.513			3.256								0.295
	10	3.362	3.207	2.523	0.010	0.004	3.243	0.013	0.004	0.000	0.013	33.3	0.019	90	
	20	3.370	3.214	2.534	0.020	0.008	3.229	0.027	0.008	0.001	0.024	20.0	0.039	95	
	30	3.378	3.221	2.544	0.031	0.012	3.216	0.040	0.012	0.001	0.035	12.9	0.053	71	
	40	3.386	3.228	2.555	0.042	0.017	3.201	0.055	0.017	0.002	0.045	7.1	0.077	83	
	50	3.393	3.237	2.566	0.053	0.021	3.189	0.067	0.020	0.002	0.055	3.8	0.097	83	
XII	0	3.244	3.096	2.538			3.197								0.343
	30	3.252	3.104	2.549	0.011	0.004	3.184	0.014	0.004	0.000	0.014	27.2	0.048	336	
	60	3.260	3.112	2.560	0.022	0.009	3.170	0.028	0.009	0.001	0.027	22.8	0.096	336	
	90	3.267	3.121	2.571	0.033	0.013	3.156	0.041	0.013	0.001	0.037	12.1	0.145	340	
	130	3.278	3.131	2.586	0.048	0.019	3.138	0.060	0.019	0.001	0.051	6.3	0.210	338	
XIII	0	3.100	3.155	2.547			3.169								0.330
	30	3.111	3.166	2.562	0.015	0.006	3.150	0.019	0.006	0.000	0.017	13.3	0.049	227	
	60	3.120	3.179	2.577	0.030	0.012	3.133	0.036	0.011	0.003	0.034	13.3	0.099	230	
	90	3.132	3.188	2.593	0.046	0.018	3.113	0.056	0.018	0.000	0.047	2.2	0.148	222	
	120	3.143	3.198	2.609	0.062	0.024	3.093	0.076	0.024	0.001	0.061	1.6	0.198	219	
XIV	0	2.887	3.171	2.532			3.113								0.320
	20	2.896	3.180	2.545	0.013	0.005	3.097	0.016	0.005	0.001	0.014	7.7	0.035	169	
	40	2.905	3.189	2.558	0.026	0.010	3.081	0.032	0.010	0.000	0.027	3.8	0.069	166	
	60	2.915	3.198	2.571	0.039	0.016	3.065	0.047	0.015	0.001	0.038	2.6	0.104	167	
	80	2.924	3.207	2.584	0.052	0.021	3.050	0.063	0.020	0.001	0.050	3.8	0.139	167	

TABLE II (CONTINUED)

Tube No.	P psi	a in.	b in.	R in.	ΔR Exp in.	$\frac{\Delta R}{R_o}$	ψ rad.	$\Delta\psi$ rad.	$\frac{\Delta\psi}{\psi_o}$	ε in.	ΔR_A Calc. in.	Error %	ΔR_B Calc. in.	Error %	t/w
XXII	0	3.192	3.594	2.484			3.097								0.465
	250	3.193	3.596	2.487	0.002	0.001	3.093	0.003	0.001	0.000	0.001	50.0	0.083	4050	
	500	3.192	3.599	2.489	0.005	0.002	3.091	0.006	0.002	0.000	0.002	60.0	0.166	3220	
	750	3.193	3.601	2.491	0.007	0.003	3.088	0.009	0.003	0.000	0.003	57.0	0.249	3459	
	1000	3.193	3.603	2.494	0.010	0.004	3.085	0.012	0.004	0.000	0.004	60.0	0.332	3220	
XXIII	0	3.146	3.609	2.492			3.137								0.430
	250	3.146	3.613	2.497	0.004	0.002	3.132	0.005	0.002	0.000	0.005	25.0	0.088	2100	
	500	3.145	3.618	2.501	0.009	0.003	3.127	0.009	0.003	0.003	0.010	11.1	0.176	1856	
	750	3.144	3.621	2.504	0.011	0.005	3.120	0.016	0.005	0.006	0.015	36.2	0.264	2300	
	1000	3.146	3.626	2.509	0.017	0.007	3.116	0.021	0.007	0.001	0.019	11.8	0.350	1390	
XXIV	0	3.164	3.621	2.456			2.966								0.348
	50	3.163	3.627	2.462	0.006	0.002	2.960	0.006	0.002	0.002	0.011	83.3	0.038	533	
	100	3.165	3.633	2.468	0.012	0.005	2.952	0.014	0.005	0.001	0.021	75.0	0.077	542	
	150	3.165	3.639	2.474	0.018	0.007	2.946	0.020	0.007	0.002	0.031	72.2	0.115	538	
	200	3.166	3.644	2.480	0.024	0.010	2.938	0.028	0.009	0.001	0.040	66.6	0.154	541	
XXV	0	3.049	3.582	2.465			2.951								0.328
	20	3.050	3.588	2.471	0.006	0.003	2.943	0.008	0.003	0.000	0.005	16.7	0.020	233	
	40	3.050	3.595	2.478	0.013	0.005	2.936	0.015	0.005	0.000	0.009	30.8	0.039	200	
	60	3.051	3.601	2.484	0.019	0.008	2.929	0.022	0.007	0.003	0.013	31.6	0.059	210	
	80	3.052	3.607	2.491	0.026	0.010	2.920	0.031	0.010	0.001	0.017	34.6	0.078	200	

CHAPTER VI

CONCLUSIONS AND RECOMMENDATIONS

It was found from this study that the model equations

$$\Delta R = P^{a1} E^{a2} R_o^{a3} w^{a4} t^{a5} h^{a6} \qquad \text{(a)}$$

and

$$\Delta R = K P E^{-1} R_o^{a3} w^{a4} t^{a5} h^{a6} \qquad \text{(b)}$$

were unsatisfactory. Equation (b) did not produce the best possible agreement with the experimental values, and Equation (a) was not dimensionally homogeneous.

The model equation

$$\Delta R = e^{a_o} P^{a1} E^{a2} R_o^{a3} w^{a4} t^{a5} h^{a6}$$

formed by introducing unknown exponents for P. E, and the arbitrary constant produced calculated values of ΔR that agree with experimental values of ΔR with errors of less than 25%, for more than 60% of the values tested.

From the knowledge gained in this investigation, it is recommended that:

This investigation be made using carefully selected Bourdon Tubes, i.e., tubes with cross-sections of the same shapes within close limits.

Five tube parameters, i.e., E, R_o, w, t, and h should not vary simultaneously from tube to tube.

In the event future research is conducted to discover an improved model equation, special tubes should be manufactured for the study. These tubes should be manufactured in sets with only one parameter varying at a time. In this manner the effect each parameter has on the change of radius may be investigated.

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APPENDIX A

LIST OF ABBREVIATIONS AND SYMBOLS

The symbols used in this report have the following significance:

E	Modulus of Elasticity, psi
R_o	Radius of the central curved axis of the tube in the unpressurized condition, in.
R	Radius at any pressure, in.
ΔR	Change in radius of the tube, in.
ψ_o	Angle subtended by the end reference points of the tube in the unpressurized condition, radians
ψ	Angle subtended by the end reference points of the tube at any pressure, radians
$\Delta\psi$	Change in tube angle, radians
$x_i y_i$	Coordinates of location points measured from zero reference, in.
a,b,	Coordinates of center of tube circle measured from zero reference, in.
w	Width of the tube corresponding to the major diameter of the flat-oval shape, in.
t	Thickness of the tube corresponding to the minor diameter of the flat-oval shape, in.
h	Thickness of the tube material, in.
ΔL	Length of the chord of the arc of the 'tip travel', in.
RAP	Regression Analysis Program
P	Internal tube pressure, psi

APPENDIX B

LIST OF EQUIPMENT

1. Gage Blocks (1)

   Manufacturer: Fonda Gage Company Inc., Stamford, Connecticut.

   Type: 845 Unit Set

2. Milling Machine (1)

   Manufacturer: Elgin Tool Works, Chicago, Illinois

   Type: Bench, vertical

3. Dead Weight Gauge Tester (1)

   Manufacturer: Manning, Maxwell, and Moore Inc., Stratford, Conn.

   Type No. 1300

4. Vernier Height Gage (1)

   Manufacturer: Brown and Sharp Mfg. Co., Providence, R. I.

   Type: 12-inch

5. Micrometer (1)

   Manufacturer: L. S. Starrett Co., Athol, Mass.

   Type: One-inch equipped with sperical contacts

6. Micrometer (1)

   Manufacturer: L. S. Starrett Co., Athol, Mass.

   Type: Two-inch

7. Surface Plate (1)

   Manufacturer: R. A. D. Laboratory, Stillwater, Oklahoma

   Type: 12-in. X 14-in. precision

APPENDIX B (Continued)

8. Angle Plate (1)

Manufacturer: R. A. D. Laboratory, Stillwater, Oklahoma

Type: Ninety degree

9. Dial Indicator (2)

Manufacturer: Ames, Waltham, Mass.

Type: 0-0.025 in. Least reading = 0.001 in.

10. Dial Indicator (1)

Manufacturer: L. S. Starrett Co., Athol, Mass.

Type: Last word

11. Surface Gage (1)

Manufacturer: Lufkin Co., Lansing, Michigan

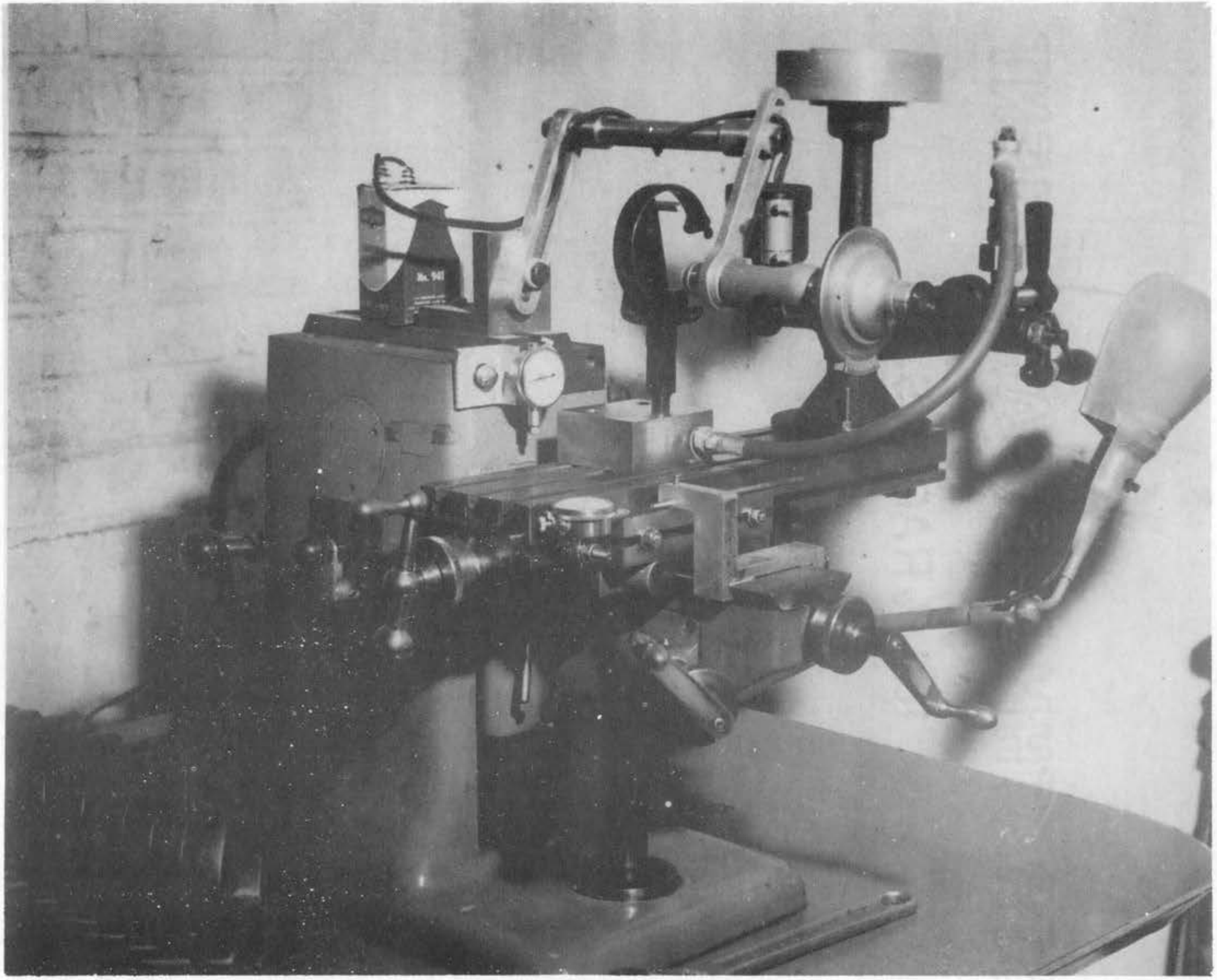
Type: Twelve-inch

12. Auxiliary Height Gage

Manufacturer: Ford Motor Company, Detroit, Michigan

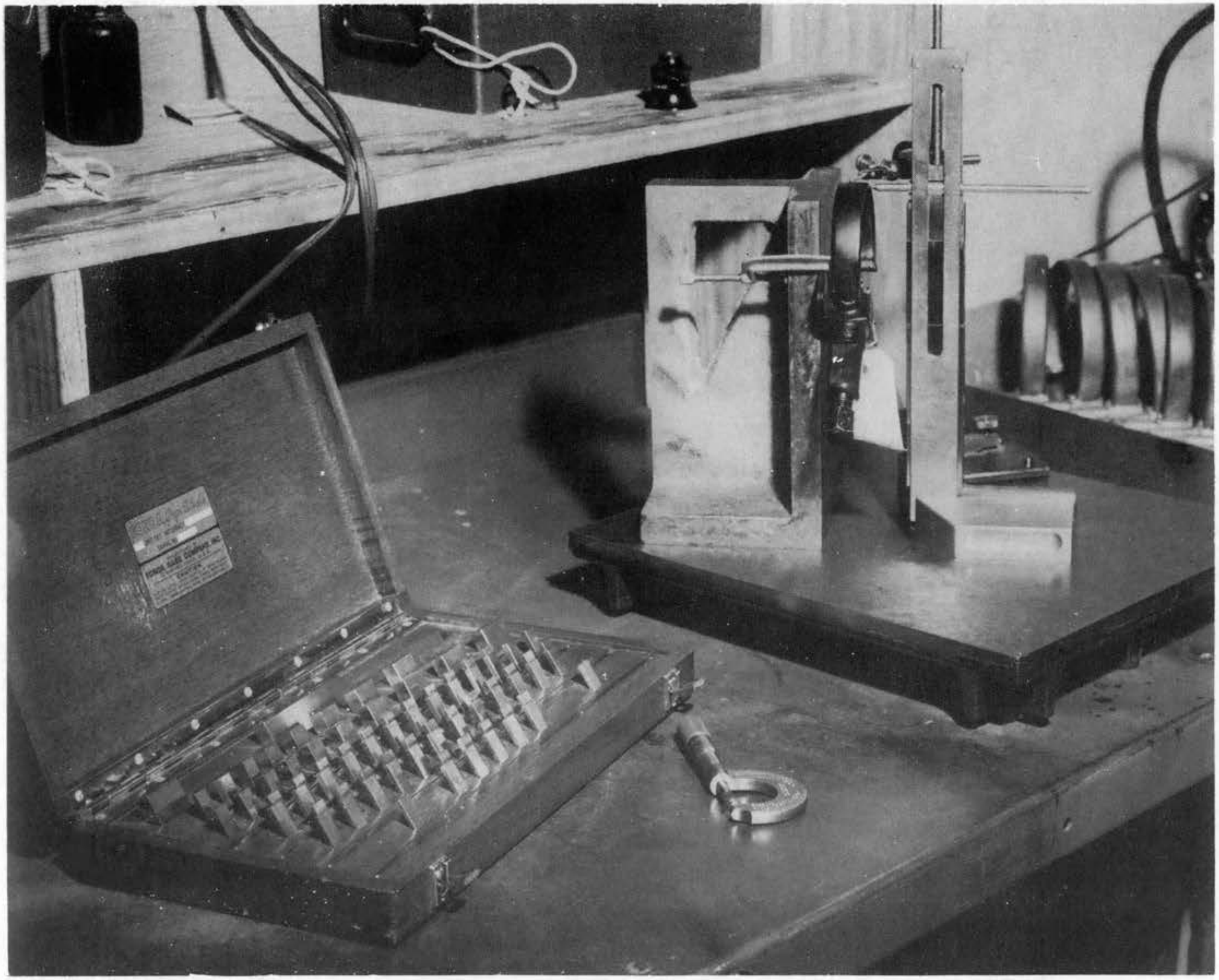
Type: To be used with Gage Blocks

PLATE I



The Measuring Setup

PLATE II



Measuring Devices for Setting Location Points

VITA

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