LIGHTNING PROTECTION REQUIREMENT
OF UNIT-CONNECTED GENERATORS

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CHAPTER I

INTRODUCTION

The goal that dictates the power system design is continuity of service and safety of operation. The major task of a system engineer is therefore adequate protection of the system against abnormal conditions. There are two system disturbances of common occurrence, namely, overcurrent and overvoltage. The overcurrent condition on a system is caused by faults of various nature, which, if allowed to persist, may induce equipment damage and impair service continuity. Fortunately, by proper application of protective relays and fast switching, it is possible to avoid equipment failure and localize the fault area to a minimum. The overvoltage disturbance on a system is produced by one of the following:

1. Unbalanced faults in the system.
2. Switching surges.
3. Lightning surges.

The overvoltage caused by unbalanced faults is of power frequency; in a properly designed system, overvoltage of this sort is usually within the tolerance of the low frequency test value. The overvoltage due to switching or lightning is of surge nature: the voltage wave has a steep front and a long tail. The crest magnitude of the switching surge is limited, and it is only in systems with more than one step reduction in insulation level that it causes concern. The crest magnitude of the lightning surge may, however, reach thousands of kilovolts and the source of the surge is beyond human control. For this reason, the art of system overvoltage protection has been centered against the lightning surges.

The philosophy that governs the overvoltage protection of the power system is to coordinate apparatus insulation with the levels afforded by the protective devices. The lightning arrester, which is the most effective device known to date, limits the surge to its discharge voltage. By assigning to the electrical apparatus a basic insulation level higher than its arrester discharge voltage, the equipment will be protected in service. The process is called insulation coordination and has been adopted successfully throughout the industry in the past two decades. The standard of the basic insulation levels is shown in Table 1 and is defined by the Joint AIEE-EEI-NEMA Committee as "... reference levels expressed in impulse crest voltage with a standard wave not longer than 1.5 by 40 microseconds wave. Apparatus insulation as demonstrated by suitable tests

TABLE I

BASIC IMPULSE INSULATION LEVELS

Reference Class KV.	Standard Basic Impulse Level KV		Reduced Insulation Levels in use - KV
1.2	30*	45**	...
2.5	45*	60**	...
5.0	60*	75**	...
8.7	75*	95**	...
15	95*	110**	...
23	150		...
34.5	200		...
46	250		...
69	350		...
92	450		...
115	550		450
138	650		550
161	750		650
196	900		...
230	1050		900
287	1300		...
345	1550		...

* For distribution class equipment.
** For power class equipment.

"Electrical Transmission and distribution Reference Book."
Westinghouse Electric Corporation, East Pittsburgh, Pa.

shall be equal to or greater than the basic insulation levels....."

While the standard basic insulation levels have been applied successfully to the insulators and the static oil-immersed apparatus, the windings of the rotating machines are exceptions. The insulation for the rotating machines such as generators, motors, etc.., is held to a minimum due to space limitation. Also, since the impulse strength of the dry insulation is not much higher than the peak of the 60 cycle voltage breakdown, it becomes impossible for the rotating machines to meet the standard basic insulation levels. These vulnerabilities make the protection of the rotating machine against surge overvoltage a special problem.

In a rotating machine, two phases of the insulation stress are of concern: namely, the winding-to-ground stress and the turn-to-turn stress. The impulse stress on the insulation between the winding and machine frame is determined by the magnitude of the incoming surge voltage to ground. The stress on the turn insulation, on the other hand, depends more on steepness of the surge voltage that penetrates the winding. The protection of the rotating machine winding consists thus of limiting the surge voltage magnitude to a safe value and sloping or flattening the wave front of the penetrating surge. A special station type lightning arrester, designed to have a low protective ratio, connected between the machine terminal to ground is required to limit the surge voltage crest value; whereas, flattening of the wave front is usually accomplished by connecting a shunt capacitor in parallel with the arrester.

In metropolitan systems where the electric power is distributed at the generator voltage, the lightning surge has generally direct access to the generator terminals. For such systems, it is recommended practice to install the capacitor-arrester combination to reduce the hazard of the lightning surges. (1) With the development of the high-voltage long distance transmission, the unit-connected scheme became prevalent. Along with the trend, the problem of the surge protection of the unit-connected generator arose.

Owing to the uncertainty of available data and lack of a straightforward method of surge transfer analysis, the power engineer was conservative in the selection of the surge protective equipment for the unit-connected generator. Those in favor of the protective equipment held the opinion that the cost of the added insurance of the protective equipment is small when compared with the investment of the generator. In other words, insurance policy rather than the strict engineering requirement governed the selection. However, the necessity of the protective equipment has always been questioned, and there is a growing trend in disfavor of such equipment on the ground that due to the high reliability necessary in the operation of large unit connected turbine generator, addition of any equipment not absolutely essential can not be justified from the standpoint of continuity of service and safety.

The problem of the surge transfer study in a transformer is not new. As early as 1932, Palueff [2] had investigated the problem with transformer design as a main objective. Later, Bellaschi [3] conducted extensive oscillograph tests of the surge transfer on actual transformers with various terminal conditions. Recently, Howard, Armstrong and Johnson [4] made field tests on unit-connected generators and verified the results with their transient analyser. Hayward, Dillard and Hilman[7] have also tested three generators and compared the results with their Anacom study. In England, similar tests were made by Dr. Robinson[6]. Unfortunately none of the authors attempted to make a generalized study to embrace the majority of cases that are likely to be encountered in practice.

The purpose of this paper is to make a study of the performance of the unit-connected turbine-generators upon impact of the lightning surges by mathematical analysis, with an application study to cases usually encountered in present-day power systems. The problem has been treated in three steps:

1. Analysis of the lightning-surge transfer through power transformers.

2. Performance study of the turbine-generator under the impact of the transferred surge.

3. Comparison of surge-voltage along the generator winding with the dielectric strength of the winding insulation.

CHAPTER II

ANALYSIS OF SURGE TRANSFER THROUGH POWER TRANSFORMER

Part I

Statement of Problem Conditions

In the past decade, the average size of the power system has been greatly increased as a result of integration and expansion. This rapid growth of the system created a need for large size turbine-generator units; today, turbine-generators of 300,000 KVA unit capacity are common and the manufacturers are encouraged to produce units of even greater size. The bulk power supplied by these large generators is seldom distributed at generator voltage; instead, it is stepped up to higher voltage for transmission. In earlier installations, generators of a station were bussed together before connecting to the step-up transformer banks. The idea was based on flexibility of operation and continuity of service. With the present size of the generators, the cost of the generator-bus becomes prohibitive. Also, since modern turbine-generators are designed to an availability factor comparable to that of a static transformer, the advantages of the generator-bus are somewhat lost. These factors make the generator-bus undesirable for high reliability. The present trend of station connection is the unit scheme with each generator feeding its own step-up transformer. The generator and the transformer are permanently connected together without a circuit breaker and form an integral unit independent of the other units of the station. Figure 1 shows a typical unit-connected generator scheme. It presents the advantages of economy, simplicity and reliability and is gaining increasing favour among the station designers.

This paper uses a unit-connected generator scheme as the basis of study. The transformer is a wye-delta bank, with the delta side connected to the generator. The wye side is connected to a transmission line with its neutral solidly grounded. Station-type lightning arresters are mounted on the casing of the transformer adjacent to the high voltage bushings. The transformer and the transmission line are adequately shielded with ground wires in such a manner that the probability of a direct lightning stroke is remote. The voltage rating of the generator and the delta side of the transformer is assumed to be 13.8 Kv., the wye side voltage rating varies from 69 Kv. to 230 Kv.. Generators studied range in size from 100,000 KVA to 200,000 KVA. Single-turn winding is assumed since this type of construction is now used exclusively throughout the industry for machines of the size considered.

Part II

Characteristics of the Protective Devices

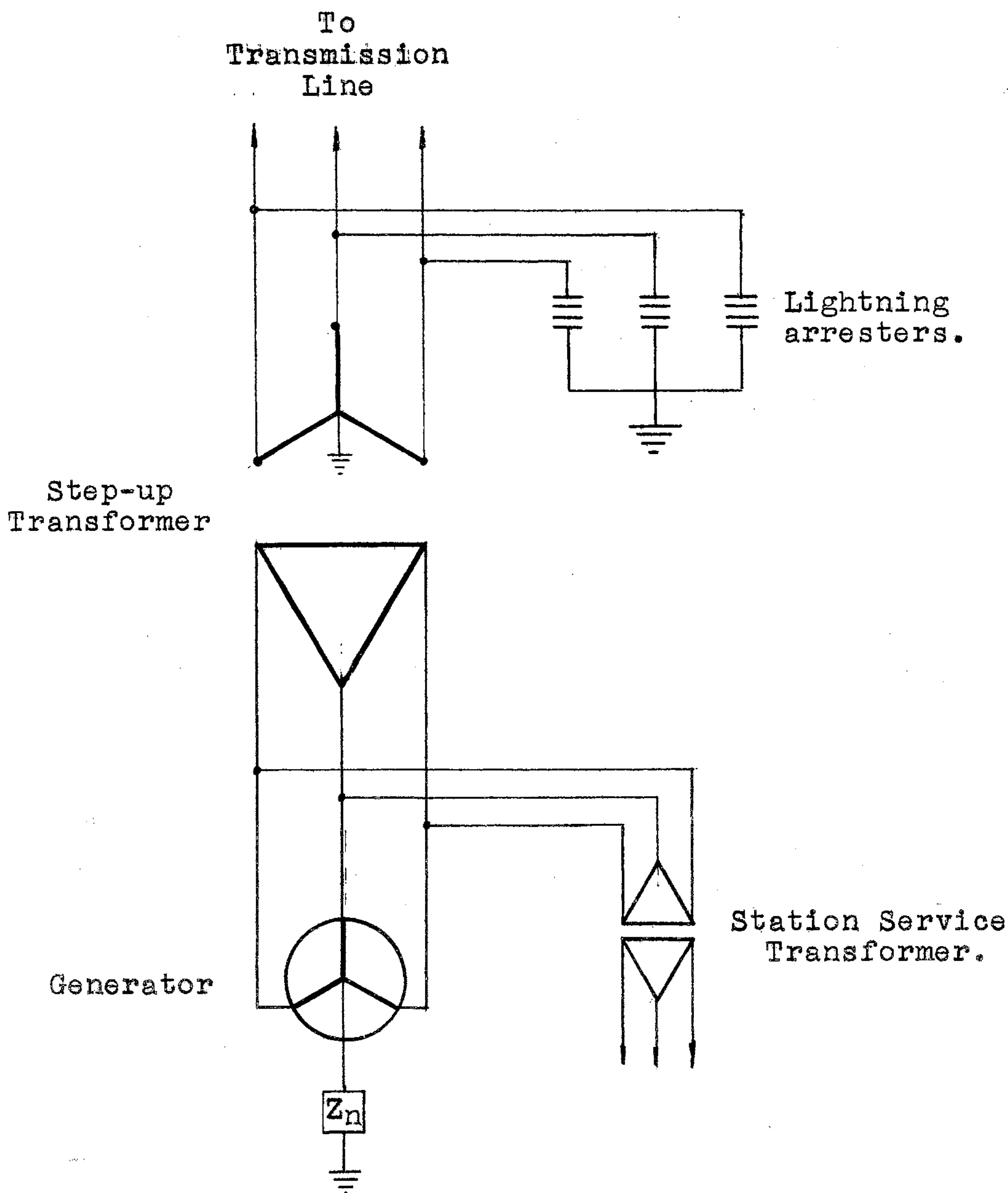
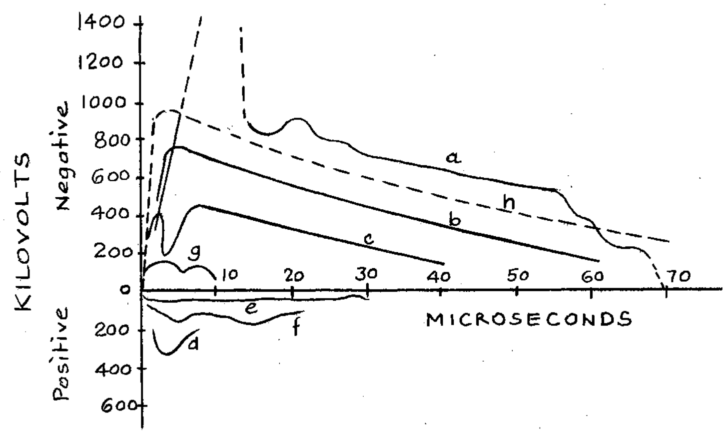


Figure 1

Typical Unit-connected Generator Scheme.

Before the surge transfer study can be made, it is necessary to determine the wave shape and the crest magnitude of the surge that can appear at the high voltage terminals of the transformer. Surge overvoltages in a power system are caused by two sources: namely, switching and lightning. The discharge voltage across the present station-type valve arresters is so low at switching surge currents that it is considered of second importance when compared with that of the lightning surges. Surge overvoltages caused by switching are therefore not considered in this study.

The wave shape of an actual lightning surge varies widely. Figure 2 shows typical lightning surges recorded with cathode-ray oscillographs. They were obtained on steel tower lines, most of which had ground wires and the surge originated at appreciable distances from the recording point. These curves, though different in form and magnitude, had the general shape of a steep front and a flattened tail; the fronts varied from 2 to 7 microseconds and the time to half of the crest voltage varied from 4 to 33 microseconds. The EEI-NEMA Joing Committee had adopted as standard a 1.5 by 40 microseconds wave for test of electrical equipment, one such wave (h) is also plotted in Figure 2 for comparison. It can be seen that the standard wave closely simulates the actual lightning surges and it would be logical to use this standard 1.5 by 40 microseconds wave as basis of comput-



Surge No.	Rate of Rise, KV. per micro-seconds	Crest KV.	Time: To Crest micro-seconds.	Time: To ½ Maximum Voltage micro-seconds	Where Taken
a	143	Unknown; 1000 recorded	7 sec. to 1000 Kv.	....	N.J.
b	183	750	5	33	Tenn.
c	270	550	6	26	N.J.
d	137	– 325	3	4	N.J.
e	30	180	5	15	W.Va.
f	...	45	...	30	N.J.
g	167	160	2.5	8	W.Va.
h	500	1000	2	43	*

* Westinghouse Trafford Laboratory Test.

Figure 2

Cathode-ray oscillographs
of
Typical Lightning Surges.

"Electrical Transmission and Distribution Reference Book"
Westinghouse Electric Corporation, Pittsburgh, Pa.

ation. This paper adopted, however, a 0 by 40 micro-seconds wave for sake of simplicity. The infinite front of the wave represents a more severe surge than any of the above recorded shapes; therefore calculations based on this wave yield more conservative results.

When a surge voltage of sufficient magnitude is applied across the terminals of an arrester, it is discharged through the arrester and the voltage at the terminals of the arrester is limited to the IR drop or discharge voltage of the arrester. Since the arresters are mounted adjacent to the high voltage bushings of the transformer, the surge voltage that appears at the transformer terminals is also limited to the discharge voltage of the arresters. The discharge voltage is a function of the arrester rating which in turn is determined by the system voltage and the effectiveness of the system grounding.

Lightning arresters are designed to discharge surge currents. They have no ability to discharge power current of normal frequency for even a few cycles. During an arrester operation, after the surge current is discharged, there is usually a follow-current of power frequency. This follow-current must be interrupted by the arrester series-gap to prevent destruction of the arrester elements. The series-gap can interrupt the follow current if the system voltage applied across the arrester during the time of operation does not exceed its rating. Proper choice of an arrester consists, therefore, of selecting a rating higher than the maximum overvoltage that can occur in the system; yet it should be lowest possible to limit the surge voltage to the greatest degree.

On an ungrounded system, full neutral shift during a single-phase short circuit is expected, the voltage at the arrester terminals may therefore attain the line-to-line value; moreover, a five percent allowance is customarily made for operation at a system voltage in excess of normal. An arrester installed in an ungrounded system should therefore be rated at 105 percent of the rated line-to-line voltage for satisfactory operation. The system considered in this paper is solidly grounded, thus the maximum line-to-ground voltage during fault condition is expected not to exceed eighty percent of the line voltage. For such a system it is general practice to use an arrester rated at about eighty-four percent of normal line-to-line voltage.

Table 2 gives the recommended arrester rating for various system voltages. The one hundred percent ratings apply to the ungrounded or non-effectively grounded systems; the eighty percent ratings apply to effectively grounded systems.

A standard of discharge-voltage characteristics has been published by the National Electrical Manu-

TABLE 2

RECOMMENDED STANDARD ARRESTER RATING

BASED ON

SYSTEM VOLTAGE AND NEUTRAL GROUNDING CLASS

System Voltage, Kv. Nominal	Max.	80% Arrester KV.	100% Arrester KV.
12	13	12	15
14.4	15	12	15
23	25	20	25
27.6	30	25	30
34.5	37	30	37
46	50	40	50
69	73	60	73
92	97	80	97
115	121	97	121
138	145	121	145
161	169	145	169
230	242	195	242
330	347	276	

" Report on Lightning Arrester Applications for Stations and Substations", AIEE Committee Report, Transaction American Institute of Electrical Engineers August 1957, p. 614.

facturers Association under the date of September 1955. Since then, arrester characteristics have been improved. Table 3 shows the maximum discharge characteristics for station-type valve arresters published by one of the largest manufacturers. The values given in Table 3 are somewhat lower than the NEMA Standard, since they are characteristics of currently available arresters. These maximum discharge voltages have been used in this paper as basis of calculation.

The discharge voltages in Table 3 are given in three arrester discharge currents, i.e. 5000 amperes, 10,000 amperes and 20,000 amperes. The magnitude of discharge current to be used in determining the discharge voltage depends on the severity of the lightning surges that the arresters are required to discharge. An extensive survey and analysis of data conducted by the AIEE Committee in a recent report (8) on lightning arrester applications revealed that reasonably severe upper limits for current crest and rates of rise of currents at stations would be 5000 amperes crest and 5000 amperes per microsecond. These two limits, if they should occur simultaneously would be more severe than over ninty-nine percent of the currents recorded to date at stations or substations. The 5000 amperes per microsecond wave has a faster rate of rise than the standard 10 by 20 microsecond wave. After comparison it is considered that a 10,000

TABLE 3

MAXIMUM PERFORMANCE CHARACTERISTICS

OF STATION TYPE LIGHTNING ARRESTERS

Arrester rating	Impulse sparkover in front of wave	Maximum Discharge Voltage, Kv crest, for discharge current 10 ×20 microsec. wave shape, ampere crest.		
Kv.	Kv. crest	5000 A.	10,000 A.	20,000 A.
3	12	8.5	9	10
6	24	17	19	20
9	35	24	26	28
12	45	32	35	38
15	55	40	44	47
20	72	55	60	65
25	90	65	71	76
30	105	80	87	94
37	125	96	105	113
40	130	104	114	123
50	155	130	142	153
60	190	160	174	189
73	230	195	212	230
90	285	240	262	283
97	310	258	280	304
109	360	282	306	333
121	390	320	350	378
133	430	350	380	410
145	460	375	408	440
169	540	450	490	530
182	585	470	510	552
195	610	500	545	588
242	770	640	695	755
258	830	685	745	805
264	860	700	765	825
276	900	730	795	860

A.M. Opsahl, "Insulation Coordination and New Arrester Characteristics," Transaction AIEE, August 1957, p. 481.

amperes, 10 by 20 microseconds current wave would give a discharge voltage in close approximation to that given by the 5000 amperes crest 5000 amperes per microsecond wave. Accordingly, 10,000 amperes, 10 by 20 microseconds current wave has been used in this paper to establish the arrester discharge voltages.

Figure 3 shows a plot of the maximum surge voltages that can appear across the high voltage terminals of the transformer and ground versus rated system voltages. Curve A applies to ungrounded or non-effectively grounded systems, and Curve B is for effectively grounded systems.

Part III

Surge Transfer Through Power Transformers

The winding and insulation structure of transformers and generators present very complex systems of inductances, capacitances and resistances. A surge which enters the winding along the copper conductor will find innumerable other paths along which it billows. The problem is somewhat similar to a cloud vapor entering a labyrinth and is not solvable by our present tools of mathematics.

The complex field problem has been simplified and is treated as a circuit problem based on transmission line theory. A transformer is then considered as a distributive network of capacitances, self-and mutual-inductances, resistances, and leakage conductances. Figure 4 shows a typical network representation for a single phase transformer. The voltage e_2 at the secondary of the transfor-

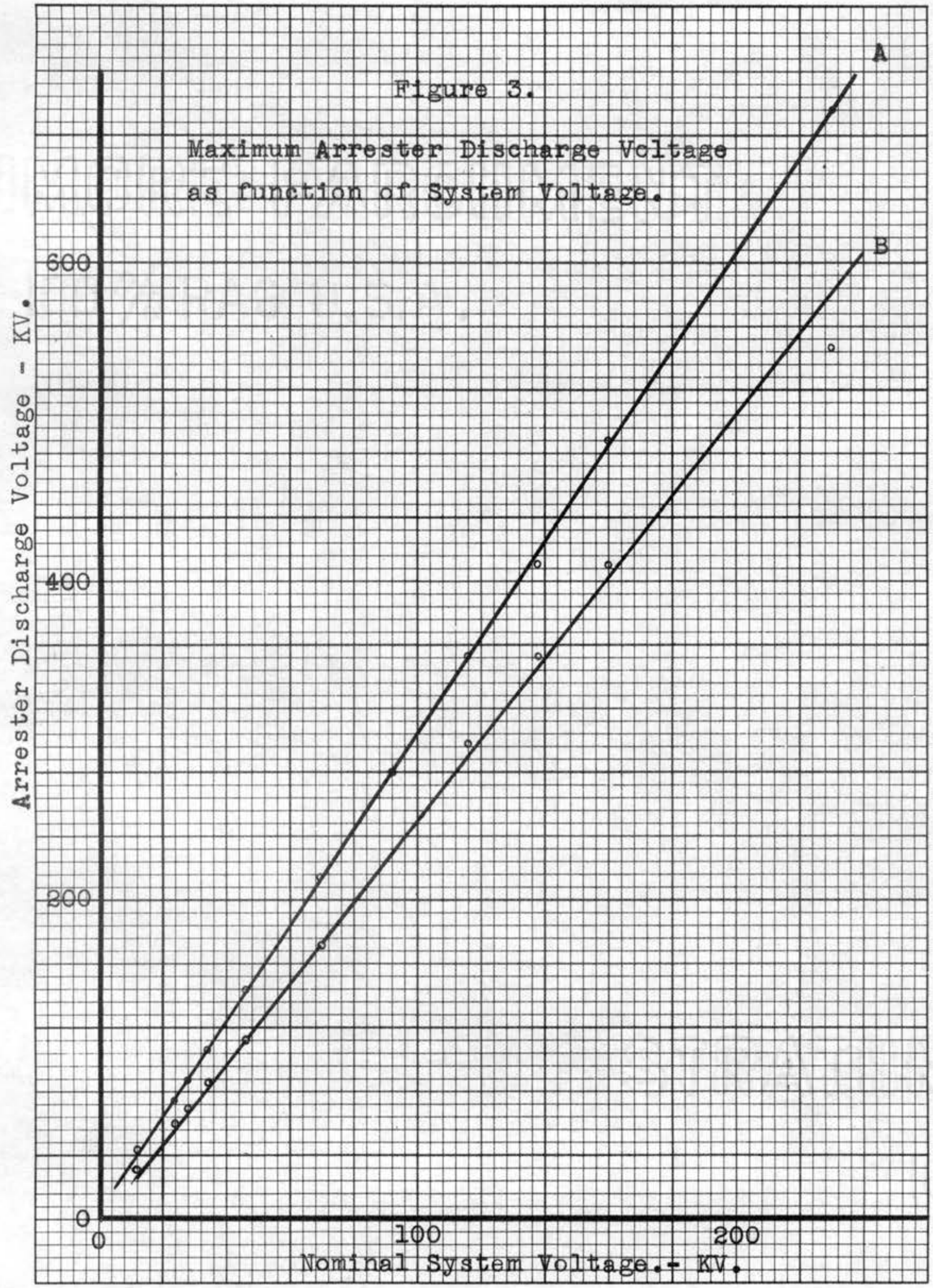


Figure 3.
Maximum Arrester Discharge Voltage
as function of System Voltage.
A
B
600
400
200
0
0
100
200
Arrester Discharge Voltage - KV.
Nominal System Voltage.- KV.

mer can be expressed as a function of the primary surge voltage e_1. Bewley [13] in early thirties had attempted the solution and found an 8th order partial differential equation of the form:

$$A_1 \frac{\partial^8 e_2}{\partial x^8} + A_2 \frac{\partial^8 e_2}{\partial x^6 \partial t^2} + A_3 \frac{\partial^8 e_2}{\partial x^4 \partial t^4} + A_4 \frac{\partial^6 e_2}{\partial x^4 \partial t^2} + A_5 \frac{\partial^6 e_2}{\partial x^2 \partial t^4} + A_6 \frac{\partial^4 e_2}{\partial t^4} = 0$$

This differential equation together with the boundary conditions imposed by the terminal apparatus yields an explicit solution of the transformer secondary voltage. In the unit-connected scheme, the transformer secondary is connected to the generator which itself is a complex distributive network. This imposes as one boundary condition an impedance function G(L,M,C,r,g,) and makes the solution of the equation so complicated that no attempt to solve it has been noted in published literature.

The constants which make the impedance function can only be calculated to a limited accuracy and the input lightning wave which would appreciably affect the solution is uncontrollable and varied in nature. Since the accuracy of the solution cannot be better than any of its input data, effort spent in the solution of such a complex equation is judged worthless; instead, further simplifications which do not greatly sacrifice the accuracy of the solution have been sought. Various steps toward simplification are summarized as follows:

1. Separate the surge transfer analysis into subtransient and transient periods.
2. Replace the distributive network of transformer by a lumped-constant equivalent circuit.
3. Replace the generator impedance function by a surge impedance of constant resistance value.

Experimental measurements of the surge voltage produced at the low voltage terminals by a lightning wave on the primary winding of a transformer indicate that it is possible to consider it the result of superposition of four components :

1. At the instant of the impact of the lightning wave, an electrostatic voltage is produced in the secondary winding. This voltage is in direct proportion to the electrostatic coupling between the two windings and is independent of the turn-ratio.
2. A free oscillatory component in the primary which induces a corresponding oscillation in the secondary winding.
3. A free oscillation in the secondary winding proper.
4. A purely electromagnetic component which is the result of induction between the two windings. This voltage depends solely on the

turn-ratio.

Since in a unit-connected scheme the transformer secondary is permanently connected to a generator of low impedance, the magnitudes of the second and third oscillatory components are, according to Palueff [2], low and can be neglected in most cases. The electrostatic component appears as a spike at the first instant of surge application. Its magnitude may reach high value depending on the capacitive coupling, but it usually subsides within one or two microseconds. The electromagnetic component has a larger time constant and generally appears after the electrostatic component has vanished.

In view of these facts, great simplification in analysis can be realized by treating the subtransient and transient periods separately. In the subtransient period, which can usually be set to expire at the end of one or two microseconds, the effect of the transformer capacitances predominates. In the transient period the effect of the inductances is more prominent.

This division of the transition periods permits segregation of the transformer network into two distinct circuits. In the subtransient period, the inductances act as an open circuit due to their inertia effect. The whole transformer can be regarded as a pure capacitive network. (Figure 5). At the end of the subtransient period, the capacitances become charged and behave like an open circuit; the effect of the inductances takes over and the transformer can then be considered as a pure inductive network. (Figure 6). This segregation of network permits separate analysis of electrostatic and electromagnetic components.

Since the terminal voltages only are of interest, it would be desirable to replace the complex impedance function of the generator by a single circuit element that behaves similarly to the generator under a surge impact. Experiments show that the surge response of a generator is practically similar to that of a transmission line and therefore has a definite value of surge impedance to traveling waves.

Various investigators have attempted to determine the value of the surge impedance of generators. The method of Abetti, Johnson and Schultz [5] is to connect a variable resistance across the open neutral end of one phase winding and to adjust this resistance so that there is no reflection of the traveling wave at that point. The value of the resistance represents then the surge impedance of the generator. Figure 7 shows the empirical curve derived from the results of their measurements; the surge impedance decreases with the KVA rating and increases with the KV rating of the generator.

Another method used to measure the surge impedance consists of finding the ratio of the surge voltage and the corresponding surge current at the same instant. Tests reveal that the initial response of a generator to

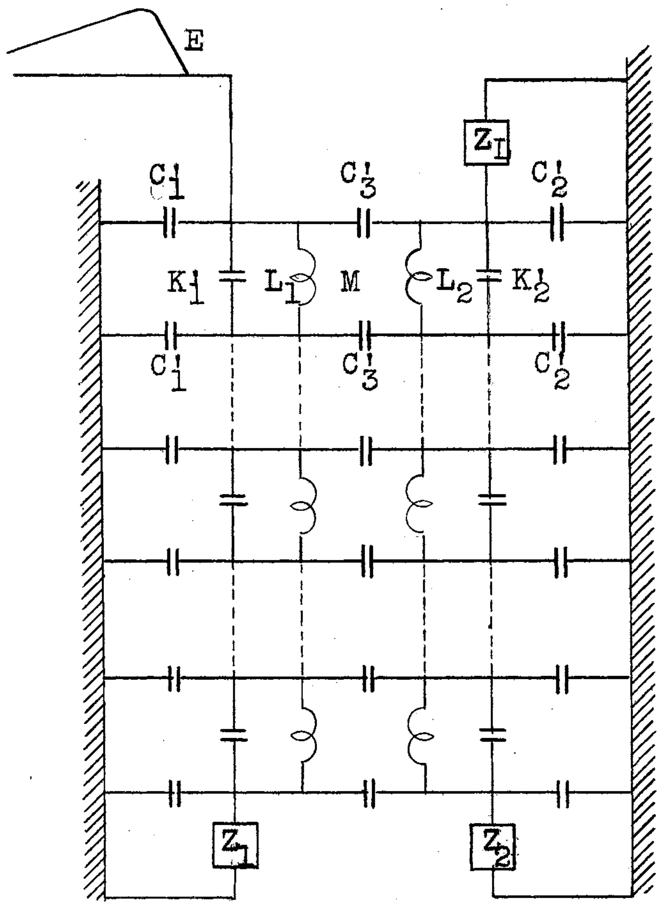


Figure 4.

Distributive Network Representation
of a
Single-phase Transformer.

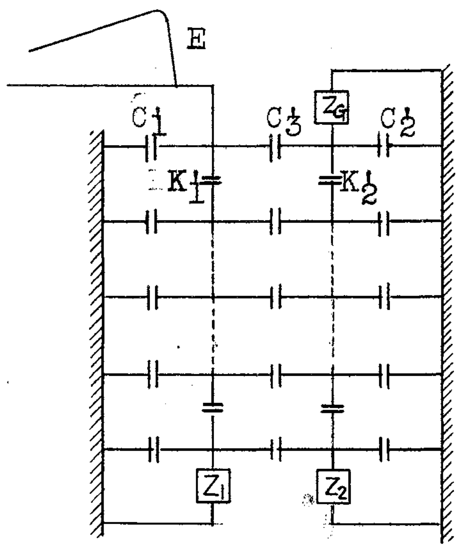


Figure 5

Capacitive Network Representation
of a
Single-phase Transformer.

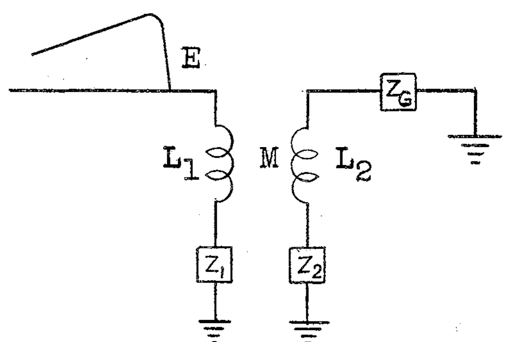


Figure 6

Inductive Network Representation
of a
Single-phase Transformer.

a surge voltage does not behave like a surge impedance of constant value; instead, it has the characteristics shown in Figure 8. At the first instant of application of a surge wave, the surge impedance is low; it then oscillates around an exponential axis to a final value which corresponds to that measured by resistance method. Investigators attribute this phenomenon to the effect of mutual induction between phase windings.

Synchronograph tests conducted by Dillard, Hayward and Hileman (7) have further shown that the surge impedance varies with the wave shape of the applied surge, with the method of surge application, and with the resistance value in series with the generator winding. (Figure 9). Fortunately, the curves also reveal that sharp variations in surge impedance are associated with steeper wave forms. With the flatter wave that can appear at the secondary terminals of the transformer, more uniform surge impedance value can be obtained. Besides, as it is seen later, limited variation of surge impedance has little effect on the magnitude of the transferred secondary voltage. The empirical curve of Figure 7 is thus adequate for the present purpose and has been used throughout the study.

A. Electrostatic Surge Transfer

At the instant a surge wave enters the high voltage terminal of a transformer, the coils which possess induc-

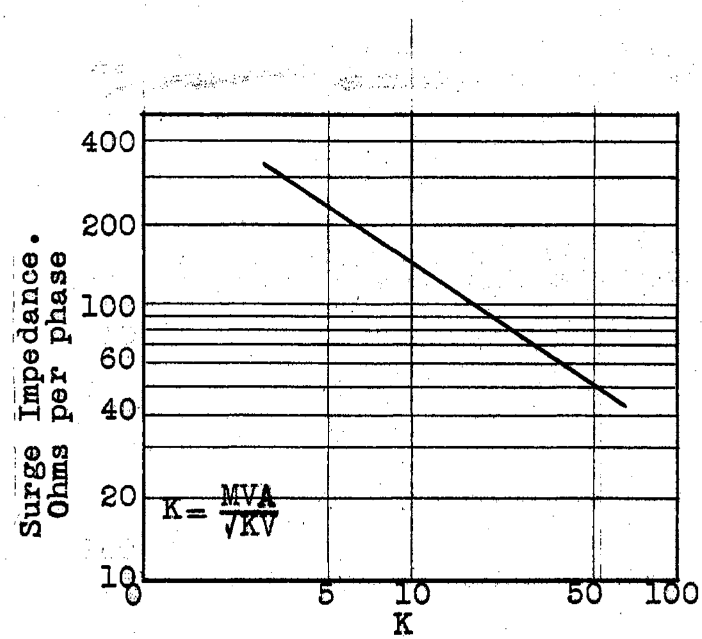


Figure 7.

Surge Impedance per phase for single turn Bar type winding Turbine Generator.

"Surge Phenomena in Large Unit-connected Steam Turbine Generators". P.A. Abetti, I.B. Johnson and A.J. Schultz. Transactions American Institute of Electrical Engineers. Volume 71, Part III, December 1952, pp. 1035-1047.

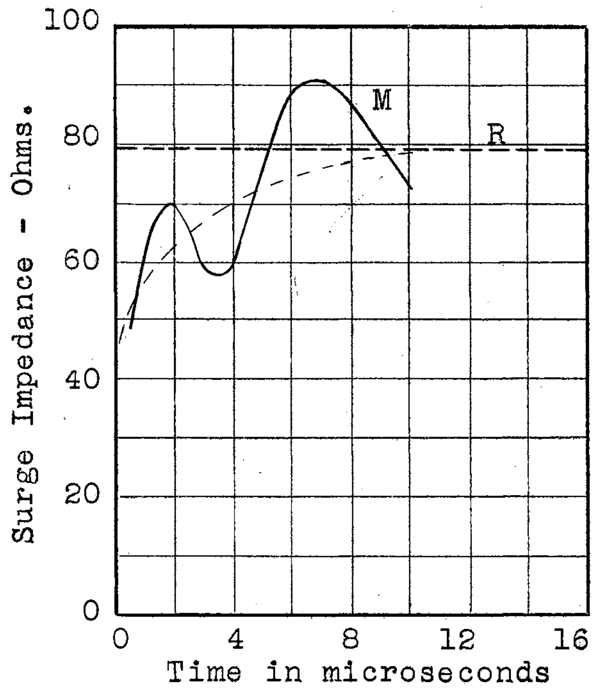


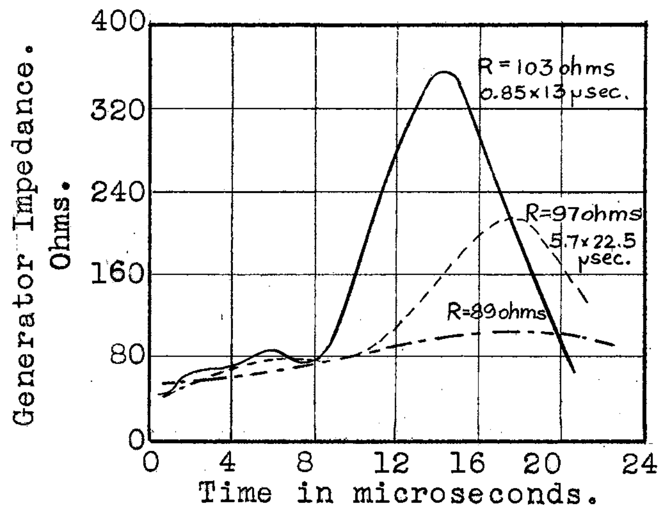
Figure 8.

Surge Impedance per Phase as function of time for a Typical Large Steam Turbine Generator.

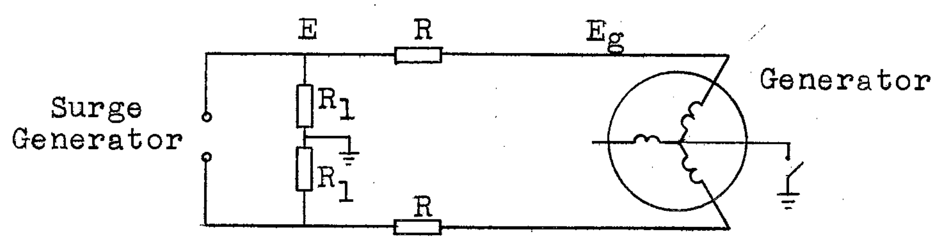
*M - Impedance as determined from measurements at input terminal.
*R - Value of resistance resulting in half open circuit voltage at neutral.

Machine Nameplate Data.
100,000 Kva-0.9p.f.- 13.8Kv. - 3600Rpm.

"Surge Phenomena in Large Unit-connected Steam Turbine Genrators" P. A. Abetti, I. B. Johnson and A. J. Schultz. Transactions American Institute of Electrical Engineers. Volume 71, Part III, December 1952, pp. 1035-1047.



Surge impedance as a function of time of a 125Mw. generator for a phase-to-phase applied surge showing variation of surge impedance with applied surge wave shape.



Circuit used to apply surges of various wave shapes to the generator:

Figure 9.

"Lightning Protection of Unit-connected Turbine Generators - Field and Laboratory Studies." A. P. Hayward, J. K. Dillard, A. R. Hileman. Trans. AIEE, February 1957, pp. 1370-1380.

tance oppose the flow of surge current and act as an open circuit. The dielectric coupling between the coils of the primary winding, between coils and ground, and between coils of the primary and secondary windings react, however, as short circuit and present low resistance paths to the surge voltage. The initial distribution of voltage and current along the winding is, therefore, determined largely by the capacitance coupling between various parts of the transformer. The distributive network of Figure 4 can thus be replaced by Figure 5 for electrostatic analysis.

In Figure 5, C_1' and C_2' represent the capacitance between ground and coil elements of the primary and secondary windings respectively, K_1' and K_2' represent the coil-to-coil capacitance of the primary and secondary windings and C_3' is the capacitance coupling between primary and secondary coils.

Capacitance C_1', C_2', and C_3' can be determined by actual measurements on a transformer. The primary winding, the secondary winding and the ground constitute three equipotential planes. The capacitances between these planes can be measured with a capacitance bridge. The delta capacitances C_{HG}, C_{HL}, and C_{LG} are then calculated as shown in Figure 10. These capacitances are related to the elementary capacitances as follows:

$$C_1' = C_{HG}\,dx \quad , \qquad C_2' = C_{LG}\,dx \quad , \qquad C_3' = C_{HL}\,dx \quad .$$

Capacitances K_1' and K_2' are not susceptible to measurements and should be calculated from the transformer design data.

Appendix IA gives a mathematical analysis of the electrostatic voltage distribution along the secondary winding of a single phase transformer for a rectangular wave applied at the terminal of the primary winding. One case of interest is a transformer with both primary and secondary neutrals grounded and its secondary line terminal connected to a generator of surge impedance Z. With the capacitance of the connecting bus or cable between the generator and the transformer neglected for the moment, the terminal voltage e_2 of the secondary winding is given as:

$$e_2 = \frac{mn[\beta \sinh\alpha \cosh\beta - \alpha \sinh\beta \cosh\alpha]}{n\beta \sinh\alpha \cosh\beta - m\alpha \sinh\beta \cosh\alpha} \cdot E\epsilon^{-\gamma t} \qquad (16)$$

$$\gamma = \frac{(n-m)\sinh\alpha \sinh\beta}{ZK_2(n\beta \sinh\alpha \cosh\beta - m\alpha \sinh\beta \cosh\alpha)} \qquad (17)$$

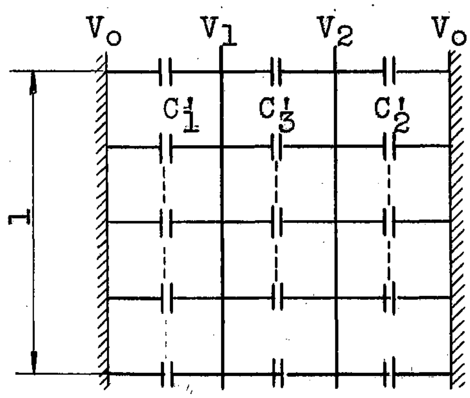
where the values of α, β, m and n are given in Appendix IA. In practice, α and β are generally large so that:

$$\cosh\alpha \cong \sinh\alpha \quad , \qquad \cosh\beta \cong \sinh\beta \ .$$

The secondary terminal voltage is then simplified to:

$$e_2 = \frac{mn(\beta-\alpha)}{n\beta - m\alpha} \cdot E\epsilon^{-\gamma t}$$

$$\gamma = \frac{n-m}{ZK_2(n\beta - m\alpha)}$$



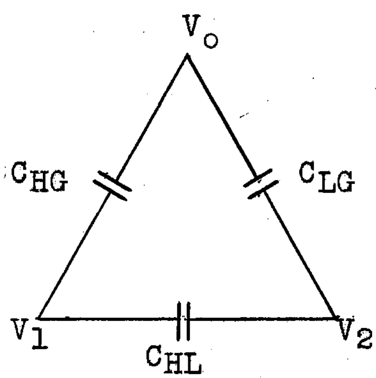


Figure 10.

Capacitance Measurement in a Single-phase Transformer.

Lack of published data on the transformer capacitances prevented a generalized study of the magnitude of electrostatically transferred voltage for transformers of various capacity and voltage classes. Results of study on two transformers are discussed: Both transformers are three-phase, grounded wye-delta connected and represent the type most commonly used in the unit-connected scheme. The voltage and capacity ratings are typical of recent installations. Capacitances C_1, C_2, C_3, K_1, and K_2 were calculated from the data found in the literature [5] and supplied by manufacturers.[15] A rectangular wave of infinite slope was applied between the high voltage terminal and ground of the transformer. The amplitude of the wave corresponded to the discharge voltage of the arrester of respective classes as given in Figure 3.

Consider first the single-phase transformer, that is, take one phase of the grounded-wye and the corresponding phase of the delta side disconnected from the other two phases. The electrostatic voltage e_2 at the secondary terminal was computed with Formula (16) and (17) in kilovolts and in per-unit. Data of the transformers and results of calculations are shown in the following tabulation:

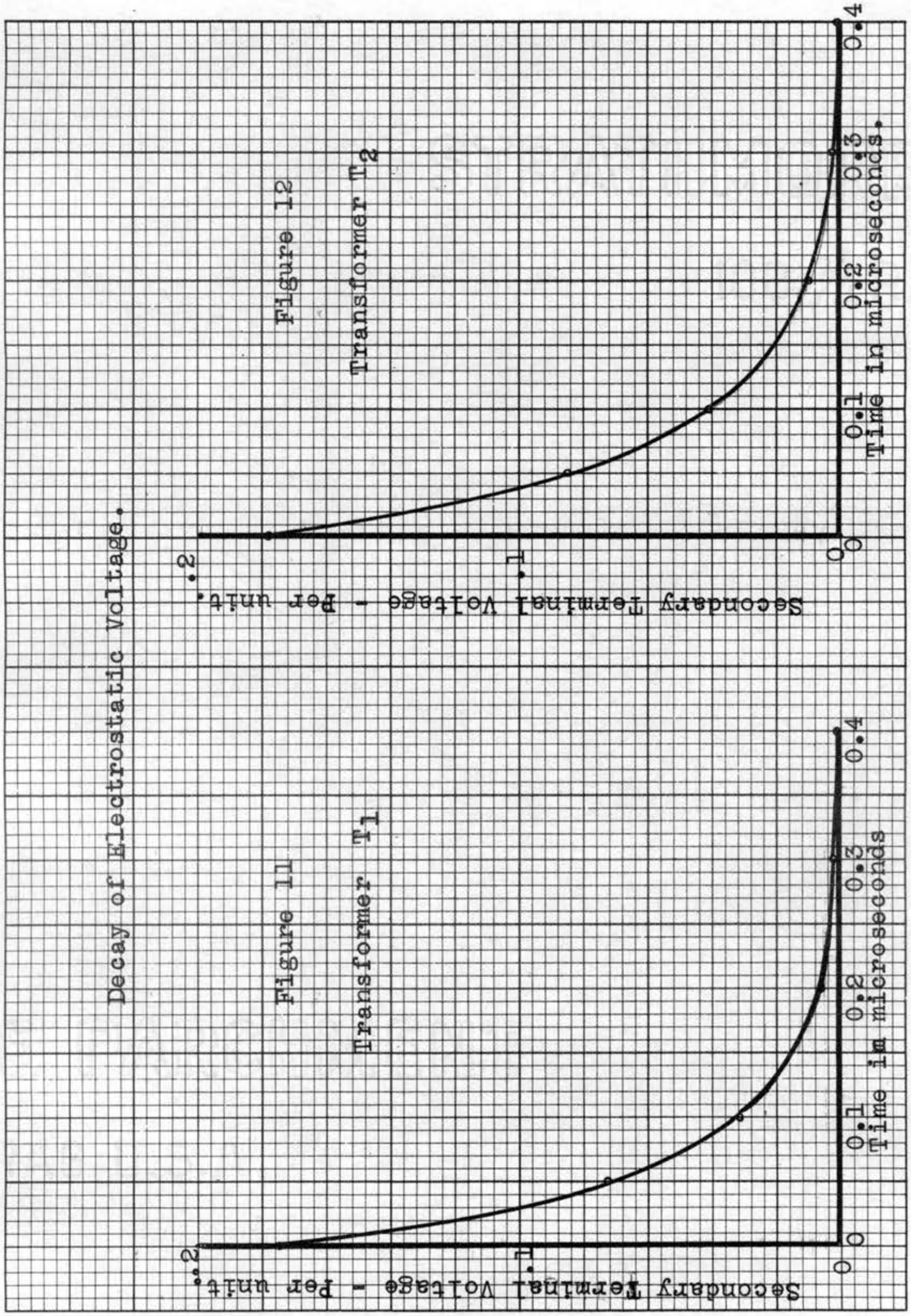
Transformer No.	1 [5]	2 [15]
Capacity MVA	145	100
Voltage: KV.		
primary	129	138
secondary	15	13.8

Phase	3	3
Connection. primary secondary	Grounded wye-delta	Grounded wye-delta
Capacitance per phase micro-microfarads		
C_1	2000	4500
C_2	6000	13200
C_3	3000	7000
K_1	20	*50
K_2	40	*100
Generator Surge Impedance Ohms	100	100
Applied Surge, E. KV.	330	350
Electrostatic Voltage e_2		
in Kv.	$57.7\epsilon^{-17.2t}$	$62.3\epsilon^{-14.65t}$
in per unit.	$0.175\epsilon^{-17.2t}$	$0.178\epsilon^{-14.65t}$

*estimated.

Plots of the secondary terminal voltage, in per-unit, versus time are shown in Figure 11 and 12. It is noted that in both cases the crest-magnitude of the electrostatically transferred voltage e_2 reaches instantly to approximately eighteen percent of the applied primary surge. It decays rapidly and drops in 0.2 microsecond to less than one percent of the applied surge crest value.

The foregoing analysis is valid for a single phase transformer without shunt capacitance in the secondary circuit. The external capacitance at the secondary ter-


Decay of Electrostatic Voltage.
Figure 11
Transformer T_1
Figure 12
Transformer T_2
Secondary Terminal Voltage - Per unit.
Secondary Terminal Voltage - Per unit.
Time in microseconds
Time in microseconds.
0
0.1
0.2
0.3
0.4
1
2

minal, the method of multiphase connection, the value of generator surge impedance, and the voltage class of the primary winding all affect the secondary electrostatic voltage to a greater or lesser extent. These are discussed in the following.

Effect of secondary capacitance:

In modern generating stations, the power transformers are usually located outdoors. The distance between the generator terminals and the transformer secondary terminals is kept as short as possible, varying between 50 to 250 feet in usual design. The connection can be made of cables in parallel or of isolated-phase-bus. Both methods of connection introduce appreciable capacitance in the secondary circuit which cannot be neglected in the analysis. Average capacitance of the isolated phase bus is 25 micro-microfarads per foot, while the capacitance of cable varies with its type, voltage class and size. The following tabulation indicates the characteristics of some most used cable type and sizes for generator-transformer connection. Several cables in parallel are generally required to carry the generator current.

Characteristics of 15Kv. Single conductor, paper-insulated cables.

Cable size MCM	Current Carrying Capacity(12) Amperes.	Capacitance C. mmfds per foot
750	797	200
1000	939	265

1500	1176	285
2000	1368	320

The capacitance introduced by the generator leads to the transformer ranges therefore from 1250 to over 200,000 micro-microfarads.

In order to visualize the effect of external capacitance, it is convenient to represent the distributive capacitive network of the transformer by an equivalent lumped-constants pi section. A method of conversion is given in Appendix IB. A pi section is formed with lumped capacitance C_H', C_{HL}' and C_L' so that it gives the same transient state and steady state solution as the distributive capacitive network at the secondary terminal. The complicated distributive network is thus simplified to the equivalent circuit of Figure A-2. The lumped capacitances C_{HL}' and C_L' are calculated from the distributive constants as follows:

$$C_{HL}' = \frac{K_2 mn[\beta \sinh\alpha \cosh\beta - \alpha \sinh\beta \cosh\alpha]}{(n-m)\sinh\alpha \cosh\beta} \qquad (22)$$

$$C_L' = \frac{K_2[n\beta(1-m)\sinh\alpha\cosh\beta - m\alpha(1-n)\sinh\beta\cosh\alpha]}{(n-m)\sinh\alpha\cosh\beta} \qquad (23)$$

For large angle of α and β :

$$C_{HL}' = \frac{K_2 mn(\beta - \alpha)}{n-m} \qquad (22a)$$

$$C_L' = \frac{K_2[n\beta(1-m) - m\alpha(1-n)]}{n-m} \qquad (23a)$$

The crest value of the electrostatic component at the secondary terminal is:

$$e_2 = \frac{C_{HL}'}{C_{HL}' + C_L'} E$$

If an external capacitance C_x is introduced at the secondary terminal, the voltage e_2 is reduced to:

$$e_2 = \frac{C'_{HL}}{C'_{HL} + C'_L + C_x} E$$

Figures 13 and 14 show the variation of the crest magnitude of the electrostatic voltage e_2 with terminal capacitance C_x. Figure 13 shows also, in black dots, results of actual measurements (5) on transformer T_1. Assuming that the generator leads are made of 50 feet of isolated-phase-bus, which constitutes a combination introducing least capacitance, the electrostatic component is reduced from 17.5 percent to 5.6 percent of the primary surge. If cables of same length were used for connection and assuming a generator of 100,000 KVA capacity with four 1,500 MCM cables per phase, the electrostatic component at the secondary terminal would be confined to 0.17 percent only of the primary surge value.

Effect of three-phase connection.

When a three-phase transformer with delta secondary is used as in most unit-connected schemes, additional capacitance is introduced by the delta side of the transformer. Appendix IC gives a method of computing the equivalent capacitive circuit of three-phase grounded wye-delta transformer. If each phase of the transformer has symmetrical arrangement, the crest magnitude of the electrostatically transferred surge is given as:

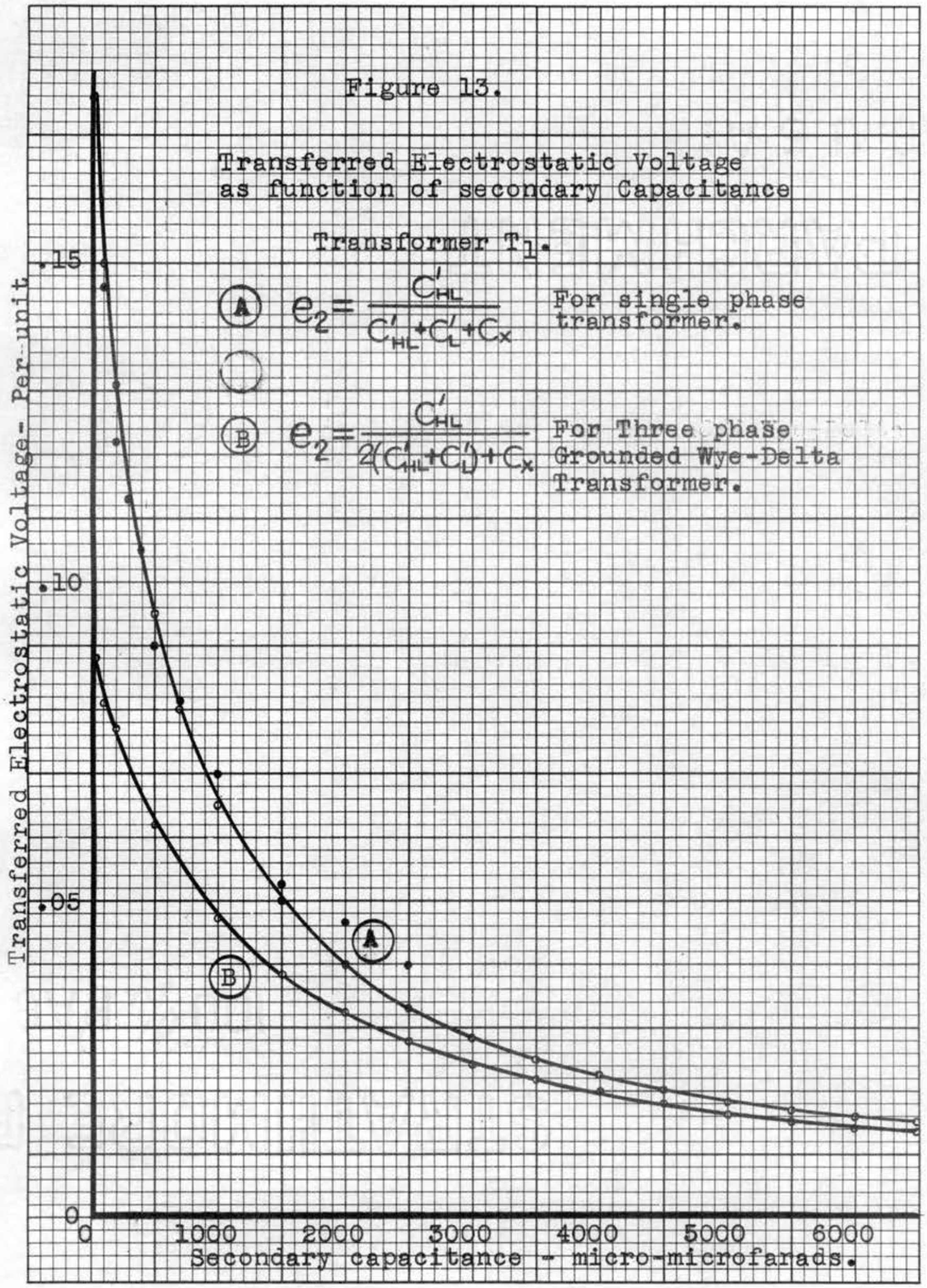


Figure 13.
Transferred Electrostatic Voltage
as function of secondary Capacitance
Transformer T_1.
(A) $e_2 = \frac{C'_{HL}}{C'_{HL}+C'_L+C_x}$ For single phase transformer.
(B) $e_2 = \frac{C'_{HL}}{2(C'_{HL}+C'_L)+C_x}$ For Three phase Grounded Wye-Delta Transformer.
Transferred Electrostatic Voltage- Per-unit
.15
.10
.05
0
0
1000
2000
3000
4000
5000
6000
Secondary capacitance - micro-microfarads.
A
B

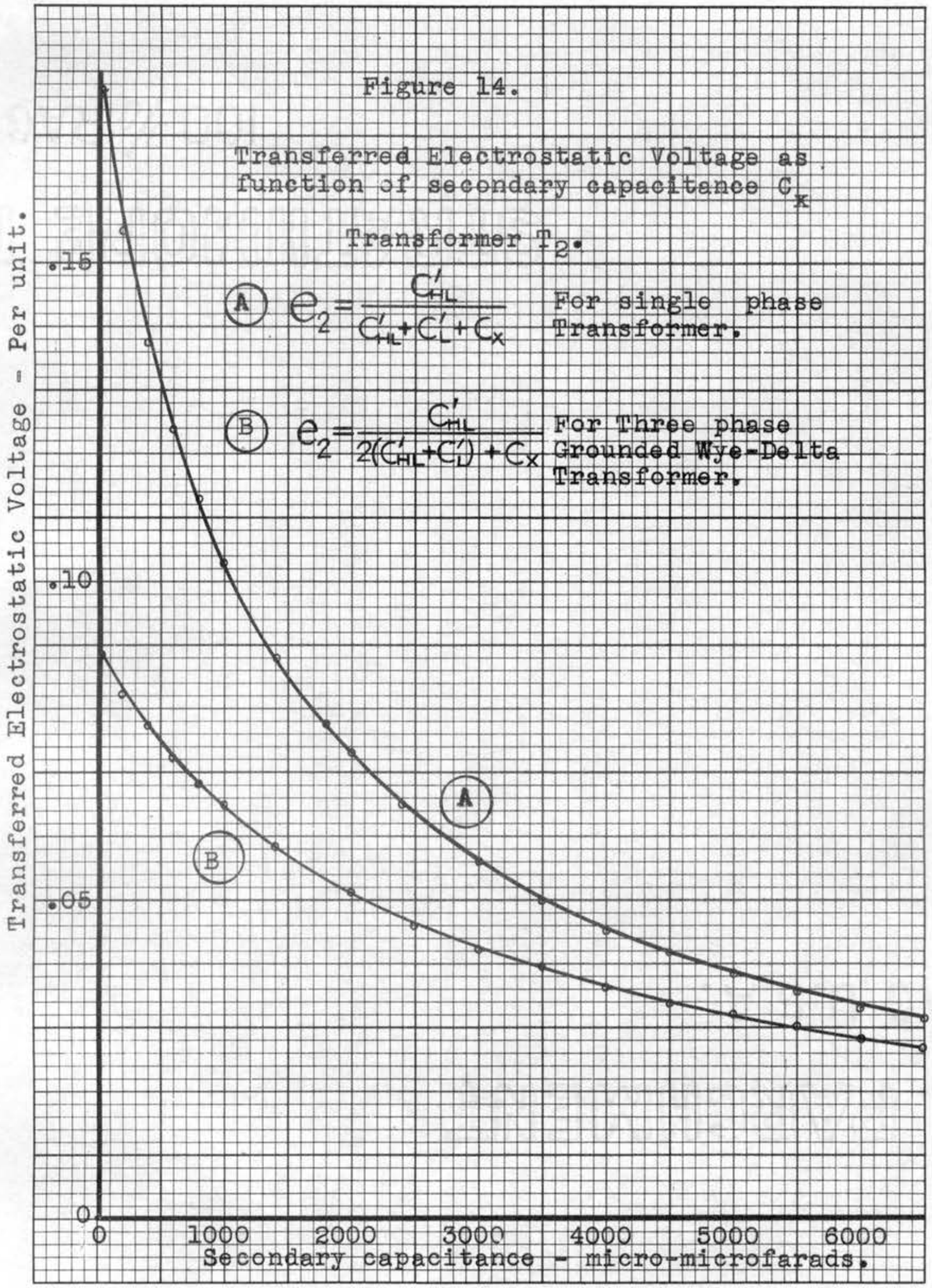


Figure 14.

Transferred Electrostatic Voltage as function of secondary capacitance C_x

Transformer T_2.

(A) $e_2 = \dfrac{C'_{HL}}{C'_{HL} + C'_L + C_x}$ For single phase Transformer.

(B) $e_2 = \dfrac{C'_{HL}}{2(C'_{HL} + C'_L) + C_x}$ For Three phase Grounded Wye-Delta Transformer.

$$e_2 = \frac{C'_{HL}}{2(C'_{HL} + C'_L) + C'_x} \cdot E$$

If there was no external capacitance C_x connected to the secondary terminal, the magnitude of transferred surge e_2 in a three-phase grounded wye-delta transformer would be half of that in a single phase transformer. Curve B of Figures 13 and 14 indicate the variation of the crest magnitude of the transferred surge with external capacitance C_x. The difference in magnitude between single-phase and three-phase transformers becomes less prominent for external capacitance C_x larger than 2000 micro-micro-farads.

Effect of generator surge impedance Z_g

In Equations (16) and (17) the constants m, n, α, and β are functions of transformer capacitances only; therefore the crest magnitude of the electrostatic component e_2 is not affected by the value of the generator surge impedance. The time constant γ of the exponential function is, however, directly proportional to the secondary surge impedance Z_g; low surge impedance produces faster transient.

The range of surge impedance contemplated in this study varies from 50 to 200 ohms. Owing to the variable characteristics of the surge impedance during the first few microseconds of the surge application, initial surge impedance of the average generator at 0.1 to 0.2 microsecond seldom exceeds 100 ohms. The 100-ohm impedance assumed in the previous calculation is therefore conservative; an average generator coupled to the transformer in question would give faster decrements.

Effect of primary voltage class

For the two transformers studied, the maximum electrostatically transferred surge voltage at the secondary terminal is 8.8 and 8.9 percent. With the maximum primary surge limited to 330 and 350 KV., this corresponds to 29 and 31 KV respectively.

What would be the magnitude of the transferred surge e_2 if a different turn-ratio transformer, say 230 to 15 Kv., were used in place of transformer 1? From Equation 16, the transferred surge e_2 is proportional to the incoming surge E whose value increases with the primary voltage in accordance with Figure 3. On the other hand, transformers of different turn-ratio have different physical proportions. This physical difference results in different capacitances C_1, C_2, and C_3... which in turn affect the constants m, n, α, and β of Equation 16.

The overall effect of a change of turn-ratio on the magnitude of the transferred surge e_2 cannot be determined without a prior knowledge of a definite relationship between the capacitance of a transformer and its turn-ratio. Owing to the variety of coil grouping design in transformer construction, such definite relationship cannot be established. Each transformer has therefore to be studied individually.

The results of the two transformers studied show that the electrostatically transferred component does not exceed 31 Kv. without external capacitance or 21 Kv. with 1250 micro-microfarads of bus capacitance. The component decays rapidly and vanishes within 0.4 microsecond. Its effect on the generator winding insulation is, therefore, negligible.

B. Electromagnetic Surge Transfer

Since transformer is designed to transfer electrical energy from one winding to another by electromagnetic induction, it is natural that the electromagnetically transferred surge is the most important of all components. The magnitude and wave form of the electromagnetic component depend on the transformer ratio, its impedance and the characteristics of the secondary connected apparatus.

This section is devoted to make a generalized study of the magnitude and wave form of the secondary voltage due to electromagnetic induction for all cases that are likely to be encountered in power utility practice. Severe surges and conservative machine constants are selected to yield relatively pessimistic results and thus provide adequate safety margin.

It has been shown previously that after the subtransient electrostatic component dies out, the transformer may be represented by a purely inductance network as shown in Figure 6. The circuit can be modified to one shown in Figure 15 for a unit-connected scheme. If its winding resistance and exciting component are neglected, the transformer can be represented by a single leakage inductance L. The overhead line to which the transformer is connected can be represented by a surge impedance Z_L' when the effect of the wave reflection is not considered. The connection between the generator and the transformer has negligible resistance and inductance, however, its capacitance is appreciable if cable connection is used; for cable of moderate length, it can be represented by a fictitious lumped capacitor connected across the generator windings . The generator itself is represented by its surge impedance Z_g. The circuit of Figure 15 can thus be simplified to the schematic diagram of Figure B-1-A. The diagram applies to the case when one phase of the transformer is struck by a lightning surge. This surge is discharged through the lightning arrester mounted adjacent to the bushings. The maximum surge voltage that appears at the transformer terminals is limited to the sparkover voltage at the first instant and then to the maximum discharge voltage of the arresters. This discharge voltage is given in Figure 3 for various voltage ratings of the connecting lines.

There are cases where two lines of a three-phase system are simultaneously struck by lightning. This is shown in the schematic diagram of Figure B-2-A. The surge are

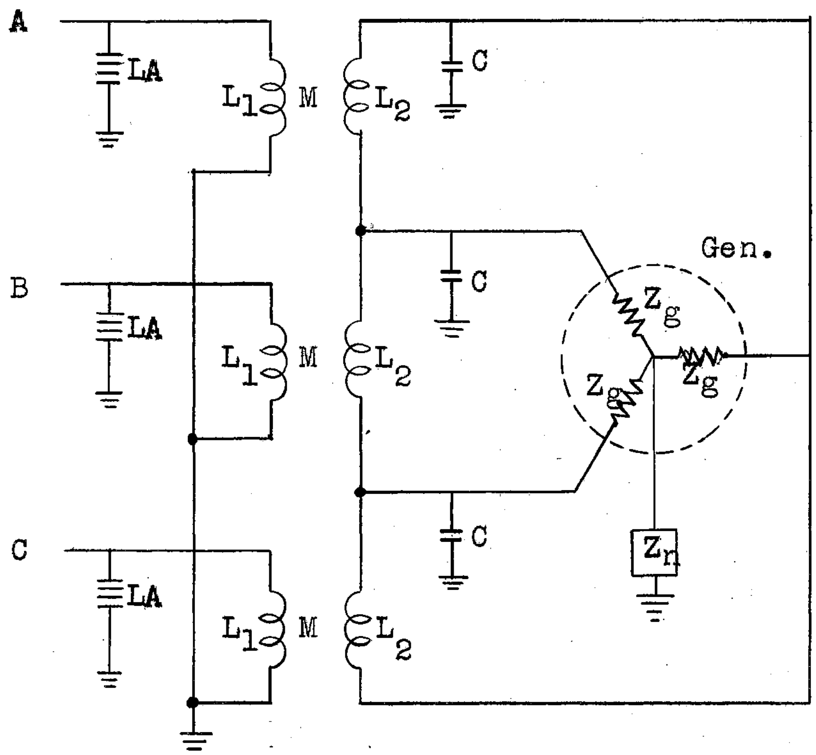


Figure 15.

Inductive Network of Unit-connected Generator with Grounded Wye-delta Transformer bank.

assumed simultaneous and of equal magnitudes.

When the surges enter by all three phases of a transformer at the same instant, a circulating surge current is induced within the delta winding of the secondary; however, no surge voltage will appear across the secondary terminals.

An analysis of the electromagnetic surge transfer through the transformer winding is included in Appendix II. It is noted in Figure B-1-A that, by symmetry, the currents I_2 and I_3 are equal in magnitude, consequently no current flows in line a; since line a is connected to the ground through capacitor C, phase a and the neutral of the generator are both at ground potential whether the generator neutral is actually grounded or isolated. This important characteristic is used in Part III for study of the surge voltage distribution along the generator winding.

With the surge striking one phase only, the schematic diagram of Figure B-1-A can be reduced to the equivalent circuit of Figure B-1-B as a result of the circuit equations (1) and (2). The secondary terminal voltage E_2 is expressed in Laplace transform form as:

$$E_2(s) = \frac{\sqrt{3}}{LCN} \cdot \frac{\left(s + \frac{Z_L}{L}\right) E_1(s)}{s^3 + s^2\left(\frac{Z_L}{L} + \frac{1}{CZ_g}\right) + s\left(\frac{Z_L}{LCZ_g} + \frac{3}{LC}\right) + \frac{2Z_L}{L^2C}} \quad (3)$$

Similar analysis is made for the case when the surges penetrate two phases. By the same reasoning, the generator neutral is at ground potential irrespective of its method of grounding. The equivalent circuit becomes that of Figure B-2-B. The secondary terminal voltage E_2 is:

$$E_2(s) = \frac{\sqrt{3}}{LCN} \cdot \frac{(s + \frac{Z_L}{L})E_1(s)}{s^3 + s^2(\frac{Z_L}{L} + \frac{1}{CZ_g}) + s(\frac{Z_L}{LCZ_g} + \frac{3}{LC}) + \frac{Z_L}{L^2C}} \qquad (6)$$

Expressions (3) and (6) are alike except that the last term of the denominator for the one-phase surge is double that of two-phase surge. This term is inversely proportional to the square of the undamped time constant of the secondary voltage. On the basis of a same primary surge, the voltage E_2 resulted from a two-phase surge has a time constant 1.41 times that of a single-phase surge, in other words, the voltage E_2 from one-phase surge is steeper. On the other hand, comparison of equivalent circuits of Figures B-1-B and B-2-B shows that the magnitude of the secondary voltage for a two-phase surge may be twice that of a one-phase surge if the line surge impedance Z_L is made infinite. The two voltages are equal if $Z_L = 0$. For other finite value of Z_L, the secondary voltage for a two-phase surge lies between one-hundred and two-hundred percent of that of a one-phase surge. Thus the two-phase surge yields higher secondary voltage for all practical cases.

In order that the computed results closely simulate the performance of the actual unit-connected systems found in practice, the following data and assumptions are adopted as basis of computation.

1. Magnitude and wave form of primary surge

The incoming surge magnitude is assumed to be limited to the maximum discharge voltage of the lightning arrester corresponding to the respective voltage class of the transmission line. This is shown by curve B of Figure 3 for various system voltages. The wave form is assumed to be 0 by 40 microseconds; it has an infinite front and decays to half of its crest magnitude in 40 microseconds. The surge is expressed mathematically as:

$$E_1(t) = E_1 \epsilon^{-.01734t} \quad KV.$$

where t is in microseconds and E_1 is the maximum discharge voltage of the arrester in Kv.

2. Two-phase surge was assumed in the calculation for higher secondary voltage.

3. Line surge impedance.

The surge impedance of an overhead power transmission line is around 400 ohms irrespective of its voltage rating and length. Accordingly, a line surge impedance $Z_L' = 400$ ohms is selected. This impedance, converted to the secondary side, is $Z_L = \frac{3Z_L'}{N^2}$ where N is the ratio of the primary line voltage to the secondary line voltage.

4. Generator capability

Three typical generator sizes, namely, 100,000; 150,000; and 200,000 Kva are selected for study. The generator voltage is assumed to be 13.8 KV. for all.

5. Generator surge impedance.

The generator surge impedance Z_g is approximated from the empirical curve shown in Figure 7.

6. Transmission line voltage.

Voltage classes included in the study are 69, 92, 115, 138, 161, 196 and 230 KV. The group represents most commonly used voltages for transmission of power. It should be noted that the economical voltage of transmission increases as the amount of power to be transmitted increases.

7. Capacitance of the generator-transformer connection.

Two cases are considered: C=0 and C=0.3 microfarad. The zero capacitance case simulates the transformer-generator connection using isolated-phase-bus which introduces negligible capacitance. When cable is used for connection, appreciable capacitance may result. A capacitance of 0.3 microfarad is considered as a limit for actual application: it is equivalent to five 1500 MCM cables in parallel per phase, each running 200 feet in length with a total current carrying capacity of approximately 5000 amperes. Higher capacity requires more cables in parallel and becomes economically less competitive with isolated phase-bus arrangement. It is thus believed justified to adopt 0.3 microfarads as the upper limit of C.

8. Transformer impedance.

Table 4 shows the standard range in impedance for two-winding transformers published by one leading manufacturer[12]. The customer's specification which requires deviation from the range usually means extra cost. Since the voltage regulation of the transmission system is usually accomplished by reactive power adjustment instead of relying on the low transformer impedance, there is no justification of a lower impedance than the above standard. Table 4 is thus an adequate representation of the range of transformer impedance found in practice.

Low transformer impedances contribute to steeper and higher transferred surge; therefore minimum impedance value listed in column 3 of the Table are used in the following study in order to produce maximum secondary voltage.

The above assumptions are summarized in Table 5A and 5B with all variables of Equation 6 calculated. For completeness, each generator capacity has to be studied against all possible primary voltages, i.e. 21 cases with C=0 and another 21 cases with C=0.3 microfarad. Since the economical transmission voltage increases as the amount of energy to be transmitted increases, it is less likely in practice that a low transmission voltage would be selected to transmit the power generated, say, by one or more units of 200,000 KVA capacity each. With the less probable cases eliminated, the study is confined to twelve more practical cases shown within the rectangles in Table 5A.

Case of generator-transformer connection with negligible capacitance.

TABLE 4

STANDARD RANGE IN TRANSFORMER IMPEDANCES FOR TWO-WINDING POWER TRANSFORMERS RATED AT 55°C RISE

High Voltage Winding Insulation Class KV.	Low-Voltage Winding Insulation Class KV.	Impedance Limit in percent: Class OA OW OA/FA OA/FA/FOA		Class FOA FOW	
		MIN.	MAX.	MIN.	MAX.
69	34.5	7.0	10.0	10.5	15.0
	46	8.0	11.0	12.0	16.5
92	34.5	7.5	10.5	11.25	15.75
	69	8.5	12.5	12.75	18.75
115	34.5	8.0	12.0	12.0	18.0
	69	9.0	14.0	13.5	21.0
138	34.5	8.5	13.0	12.75	19.5
	69	9.5	15.0	14.25	22.5
161	46	9.5	15.0	13.5	21.0
	92	10.5	16.0	15.75	24.0
196	46	10.0	15.0	15.0	22.5
	92	11.5	17.0	17.25	25.5
230	46	11.0	16.0	16.5	24.0

"Electrical Transmission and Distribution Reference Book." Westinghouse Electric Corporation, East Pittsburgh, Pa.

TABLE 5A

BASIC DATA USED FOR ELECTROMAGNETIC TRANSFER STUDY

Voltage Class Transmission System	Transformer Voltage Ratio	Minimum Transformer Impedance	*Transformer Inductance millihenry per phase on 13.8 KV. basis for Transf. Capacity		
KV.	N	Percent.	100 MVA	150 MVA	200 MVA
69	5	7	0.355	0.237	0.178
92	6.66	7.5	0.380	0.254	0.190
115	8.33	8.0	0.406	0.271	0.203
138	10	8.5	0.431	0.288	0.215
161	11.66	9.5	0.482	0.321	0.241
196	14.2	10	0.507	0.338	0.253
230	16.6	11	0.558	0.372	0.279
Generator Surge Impedance			80	50	48

* The transformer inductance per phase is calculated by the following relation.

$$L = 5050 \frac{\%\ \text{Impedance}}{\text{KVA}} \quad \text{Millihenry per phase}$$

TABLE 5B

BASIC DATA USED FOR ELECTROMAGNETIC TRANSFER STUDY

Voltage Class Transmission System	Maximum Primary Surge Voltage KV. Crest	Transformer Voltage Ratio N	Line Surge Impedance on 13.8Kv. Basis. Ohms
69	174	5	48
92	240	6.66	27
115	300	8.33	17.3
138	350	10	12
161	408	11.66	8.8
196	490	14.2	6.0
230	545	16.6	2.3

From Appendix II, the Laplace tranform of the secondary voltage $E_2(s)$ for a two-phase surge is:

$$E_2(s) = \frac{\sqrt{3}\,E_1(s)Z_g}{LN} \cdot \frac{\left(s + \frac{Z_L}{L}\right)}{s^2 + s\left(\frac{Z_L}{L} + \frac{3Z_g}{L}\right) + \frac{Z_L Z_g}{L^2}} \qquad (8)$$

Since there is no capacitance in the circuit, the solution is aperiodic. With $E_1(s)$ replaced by its appropriate Laplace transform and the denominator factored, equation (7) can be reduced to the form:

$$E_2(s) = \frac{\sqrt{3}\,E_1 Z_g}{LN} \cdot \frac{s + a_o}{(s+\gamma)(s+\alpha)(s+\beta)} \qquad (8a)$$

with its inverse Laplace transform:

$$E_2(s) = \frac{\sqrt{3}\,E_1 Z_g}{LN}\left[\frac{a_o-\gamma}{(\alpha-\gamma)(\beta-\gamma)}\epsilon^{-\gamma t} + \frac{a_o-\alpha}{(\gamma-\alpha)(\beta-\alpha)}\epsilon^{-\alpha t} + \frac{a_o-\beta}{(\gamma-\beta)(\alpha-\beta)}\epsilon^{-\beta t}\right] \qquad (9)$$

$$= A\epsilon^{-\gamma t} + B\epsilon^{-\alpha t} + C\epsilon^{-\beta t}$$

The values of A, B, C, α, β, and γ are calculated for each of the 12 cases. The secondary voltage $E_2(t)$ thus obtained is plotted in curve form as shown in Figures 16A, 16B, and 16C. It is noted that curve 1 for case 1 has the highest magnitude of all 12 cases; it rises to a crest of 32 KV. in 30 microseconds and decays slowly to zero.

The difference in crest magnitude and wave form of the secondary voltage is chiefly attributed to the relative effect of the line surge impedance. It is seen from Table 5 that, at 69Kv., the line surge impedance, referred to the secondary, is 48 ohms while at 230 KV., it is only 2.3 ohms. The higher the surge impedance Z_L is, the higher the crest magnitude of the secondary surge would be.

Calculations are also made by neglecting the surge impedance Z_L. Equation (7) is then reduced to:

$$E_2(s) = \frac{\sqrt{3}E_1(s)Z_g}{LN} \cdot \frac{1}{s + \frac{3Z_g}{L}} = \frac{\sqrt{3}E_1 Z_g}{LN} \cdot \frac{1}{(s+\gamma)(s+\frac{3Z_g}{L})} \qquad (10)$$

The inverse Laplace transform of (10) is:

$$E_2(t) = \frac{\sqrt{3}E_1 Z_g}{LN} \cdot \frac{1}{(\frac{3Z_g}{L} - \gamma)} \left(\epsilon^{-\gamma t} - \epsilon^{-\frac{3Z_g}{L}t} \right) \qquad (11)$$

Six cases are calculated and results plotted in dotted line in Figure 16A, 16B and 16C. It is noted that the difference becomes much less prominent. The crest magnitude lies within the narrow range of 16.8 KV. to 18.8 KV. and the time to crest is around 6 microseconds for all cases.

Case of Generator-transformer connection with capacitance C=0.3 microfarad.

The Laplace transform of the secondary voltage for a two-phase surge is:

$$E_2(s) = \frac{\sqrt{3}E_1(s)}{NLC} \cdot \frac{s + \frac{Z_L}{L}}{s^3 + s^2\left[\frac{Z_L}{L} + \frac{1}{CZ_g}\right] + s\left[\frac{Z_L}{LCZ_g} + \frac{3}{LC}\right] + \frac{Z_L}{L^2C}} \qquad (6)$$

With the range of the transformer inductances contemplated, a capacitance of 0.3 microfarad is more than sufficient to

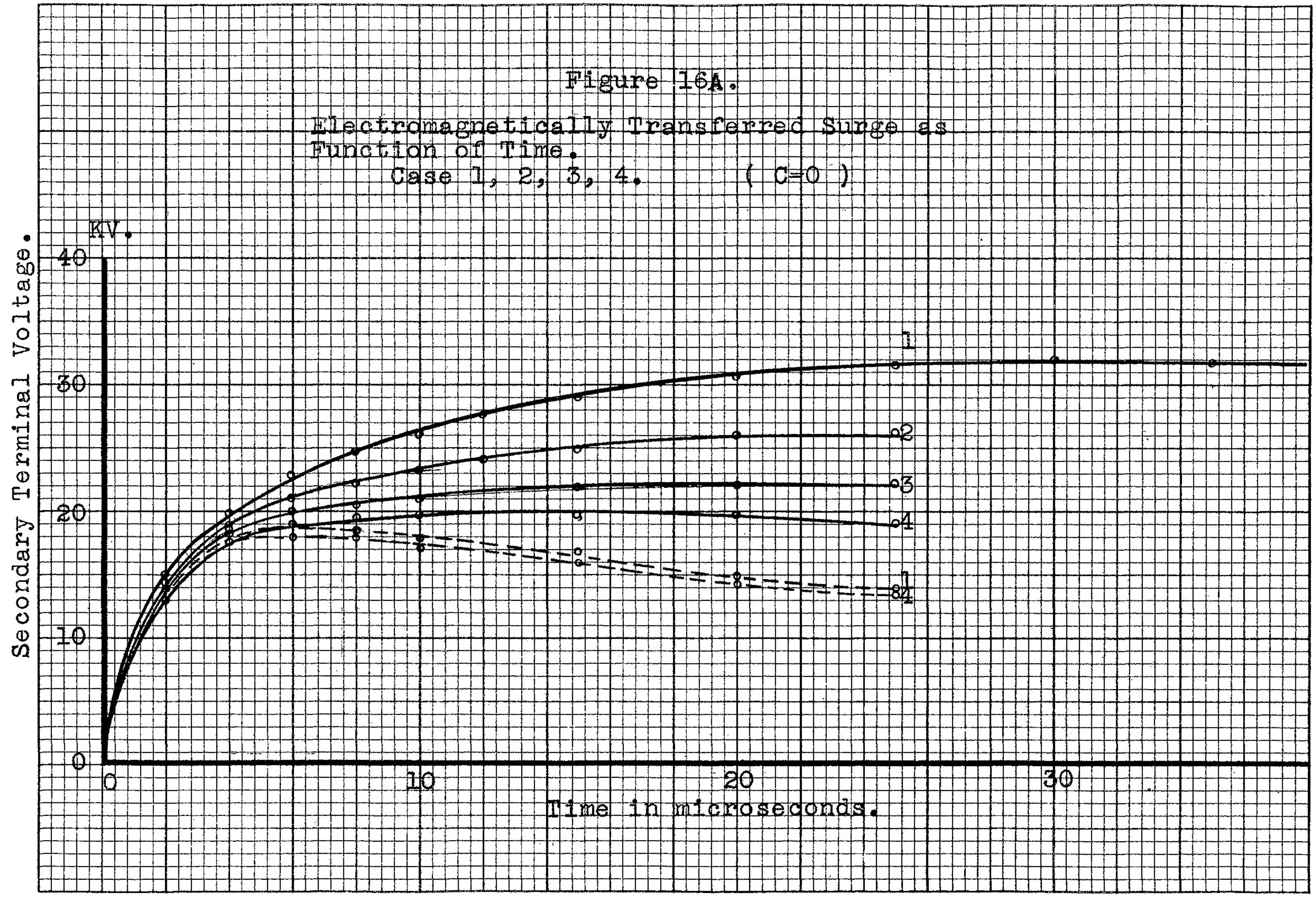


Figure 16A.

Electromagnetically Transferred Surge as Function of Time.
Case 1, 2, 3, 4. (C=0)

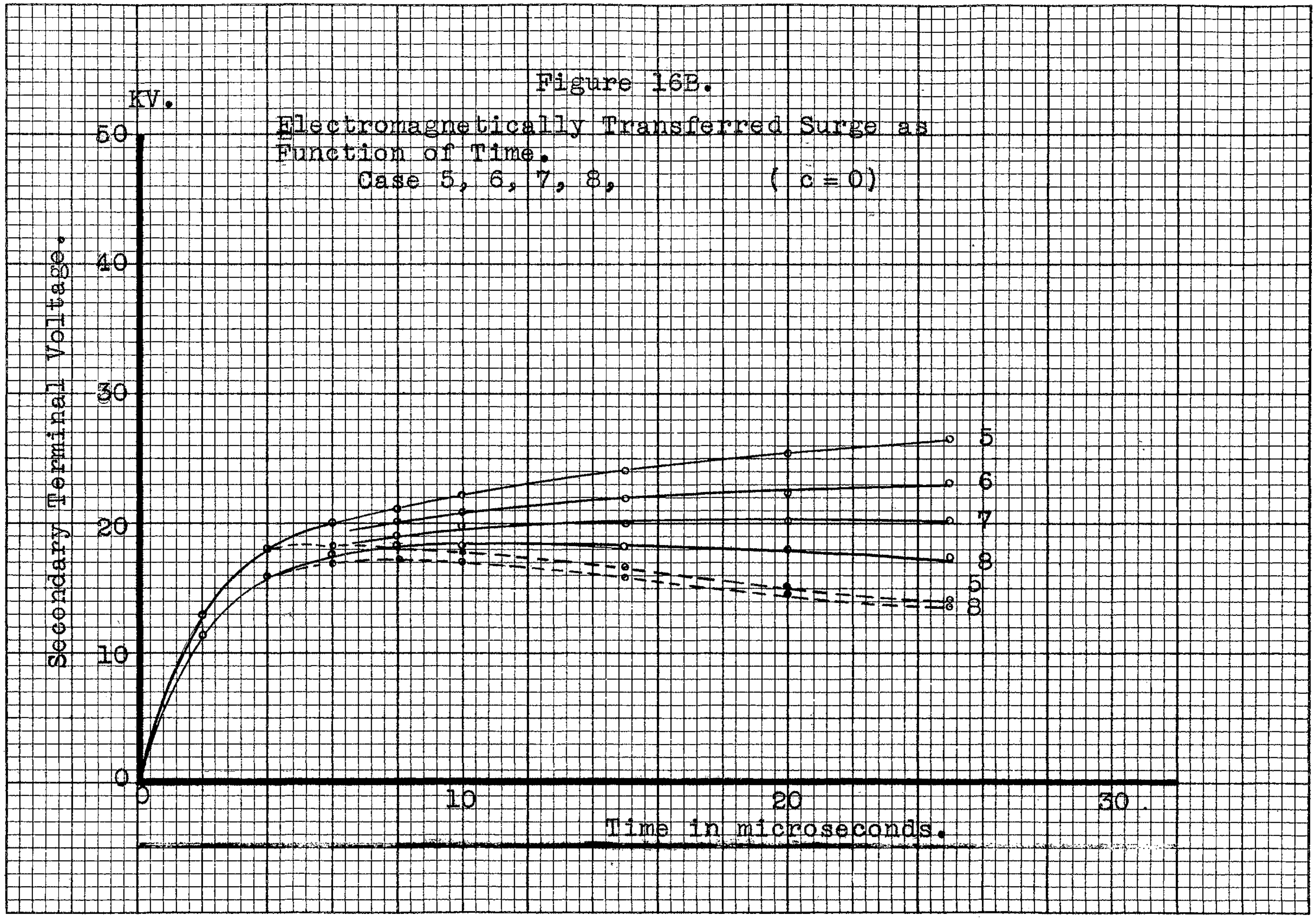


Figure 16B.

Electromagnetically Transferred Surge as Function of Time.
Case 5, 6, 7, 8, (c = 0)

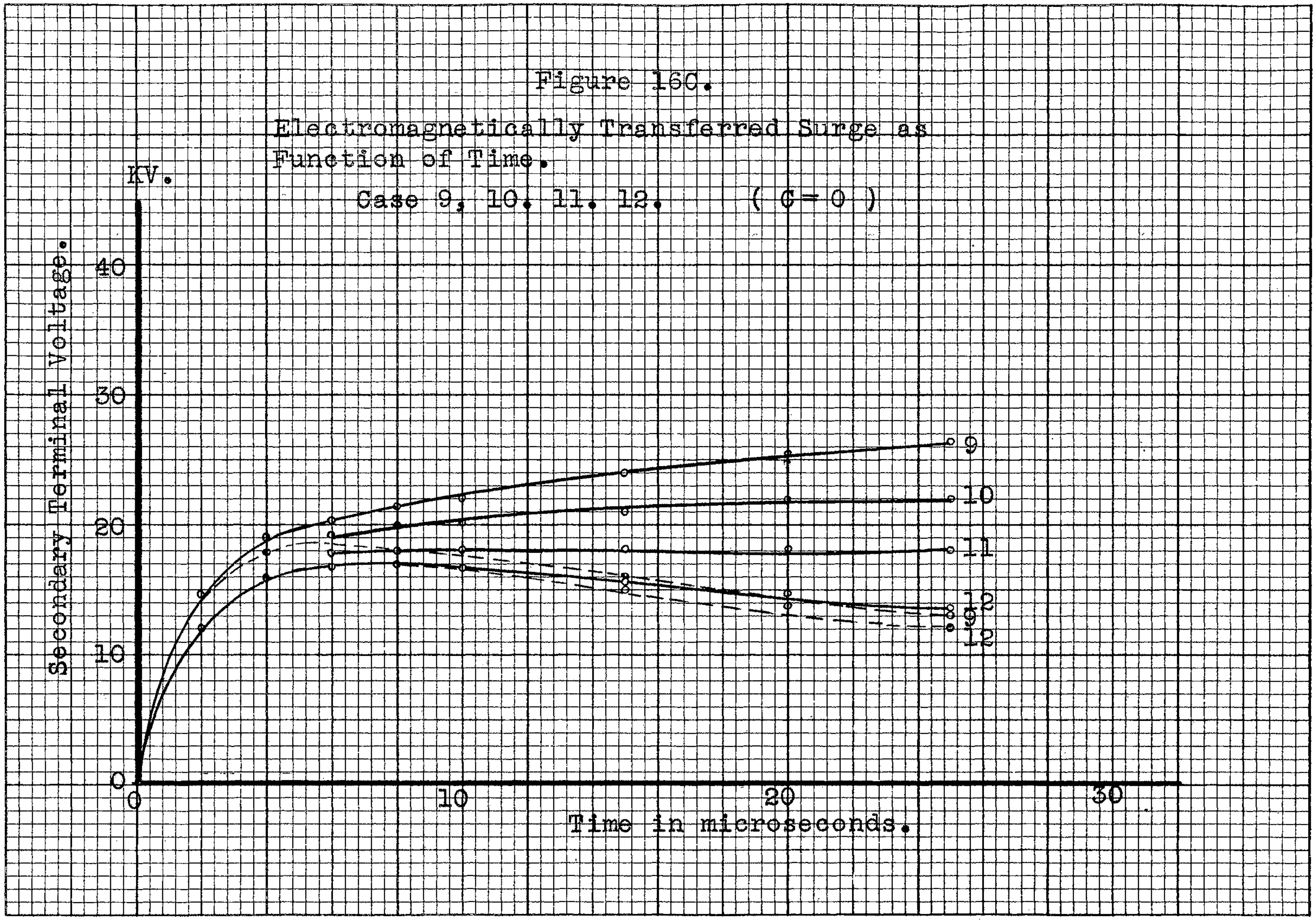


Figure 16C.

Electromagnetically Transferred Surge as Function of Time.

Case 9, 10, 11, 12. (C = 0)

make the secondary voltage oscillatory. The denominator of (6) can thus be factored into one real and two complex conjugate roots as follows:

$$E_2(s) = \frac{\sqrt{3}\,E_1}{LCN} \cdot \frac{s + a_0}{(s+\gamma)(s+\delta)\left[(s+\alpha)^2+\beta^2\right]} \qquad (6a)$$

The inverse Laplace transform of (6a) is:

$$E_2(t) = A_1\epsilon^{-\gamma t} + B_1\epsilon^{-\delta t} + C_1\epsilon^{-\alpha t}\sin(\beta t+\psi) \qquad (7)$$

Values of A_1, B_1, C_1, α, β, γ, δ, ψ are given in Appendix II. Numerical values of A_1, B_1, ...ψ are calculated for each of the 12 cases considered. The secondary voltage thus obtained is plotted in Figures 17A, 17B, and 17C.

Comparison of Figures 16A, 16B, 16C and 17A, 17B, 17C shows that the curves in group with capacitance C=0.3 mfd. have a less steep wave front and a higher crest magnitude. In both groups, curve 1 gives the highest crest voltage and sharpest rate of rise; the crest voltage for C=0 is 32 KV. against 40 KV. for C=0.3 mfd. The maximum rate of voltage rise for C=0 is approximately 11KV. per microsecond compared to about 2.5 KV. per microsecond for C=0.3 mfd.

The effect of the secondary capacitance is thus to flatten the wave front and increase the crest magnitude of the secondary surge voltage.

The above results are obtained from analysis where distributive constants are replaced by their surge imped-

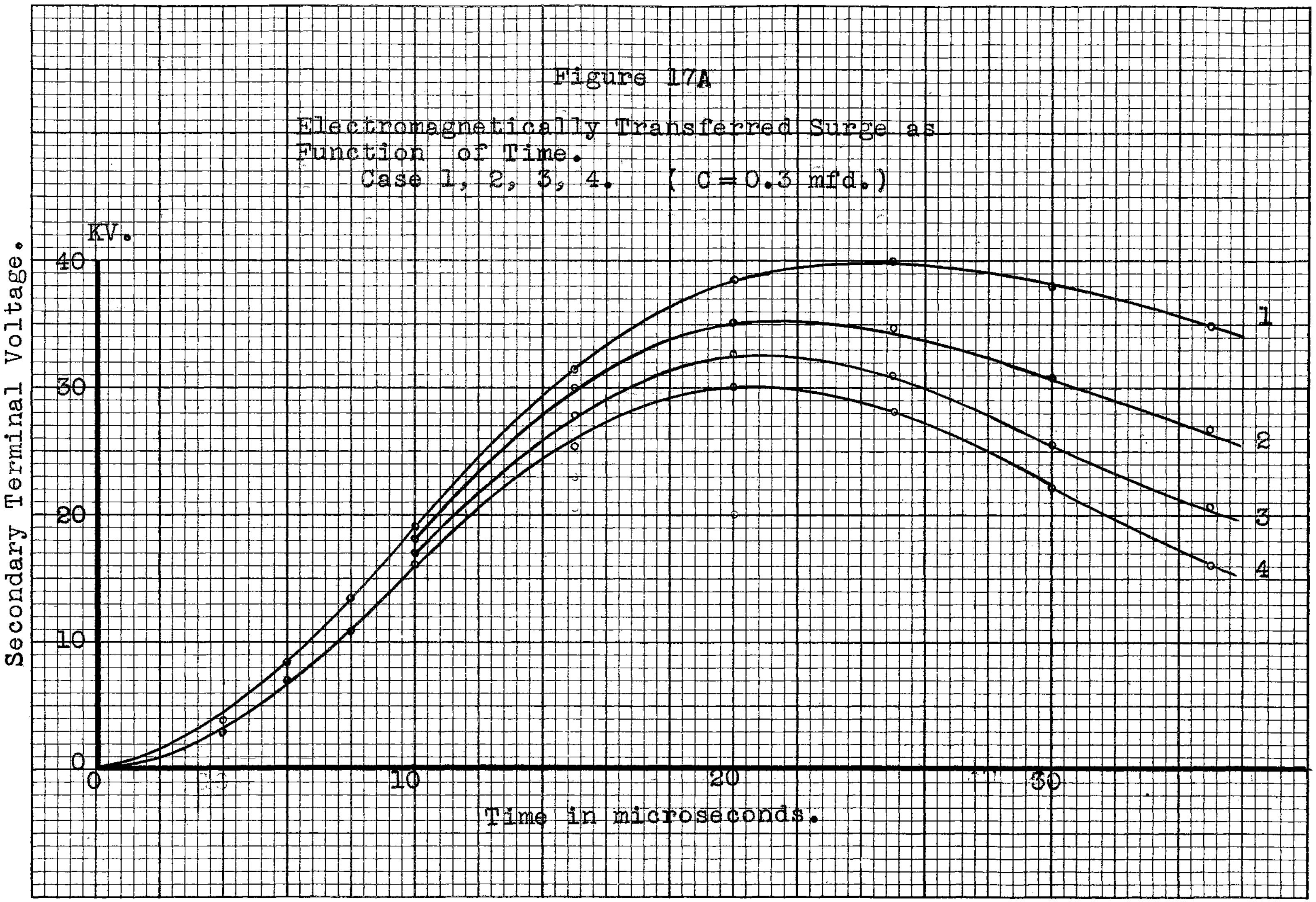


Figure 17A
Electromagnetically Transferred Surge as Function of Time.
Case 1, 2, 3, 4. (C = 0.3 mfd.)
Secondary Terminal Voltage.
KV.
40
30
20
10
0
0
10
20
30
Time in microseconds.
1
2
3
4

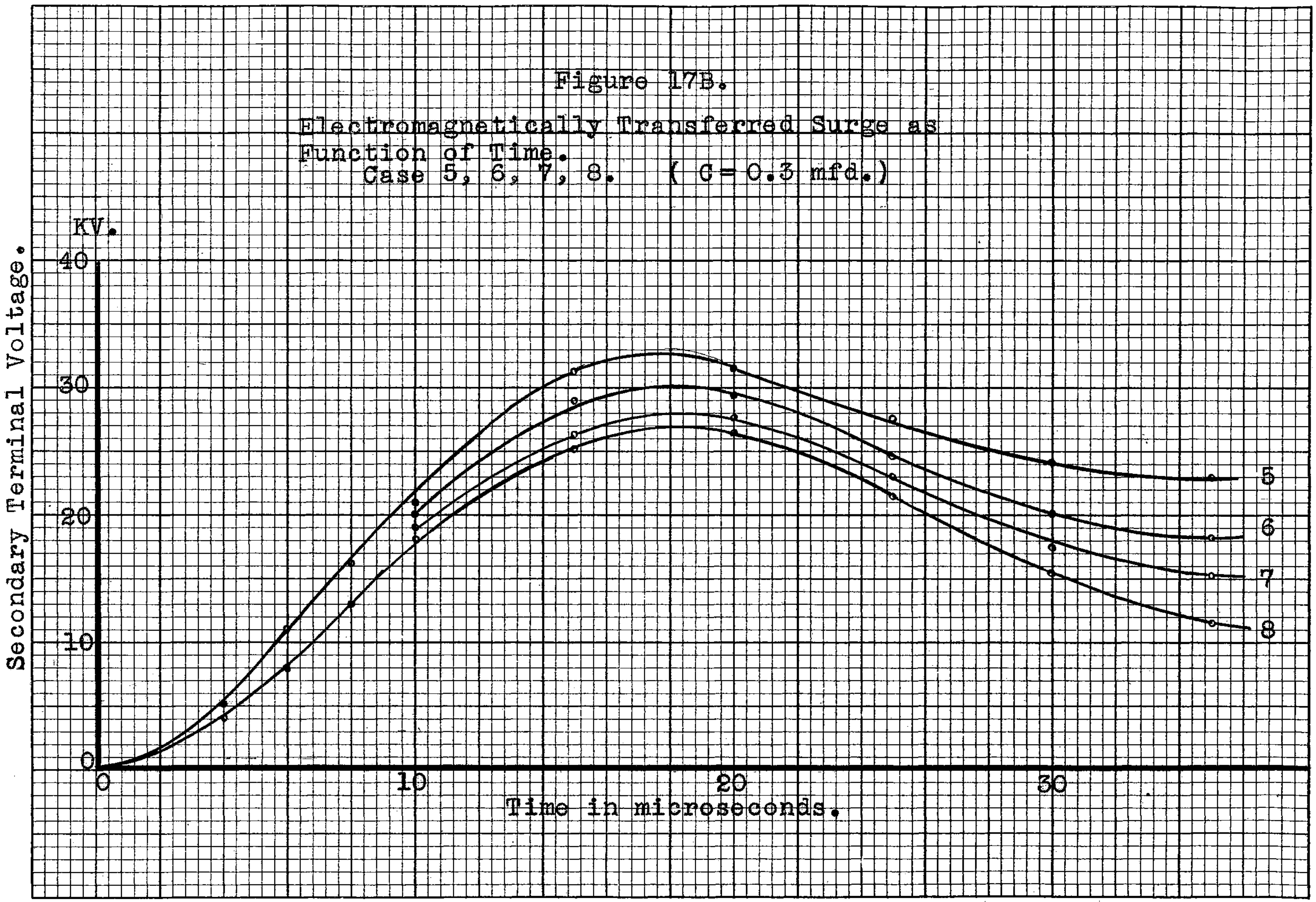


Figure 17B.

Electromagnetically Transferred Surge as Function of Time.
Case 5, 6, 7, 8. (C = 0.3 mfd.)

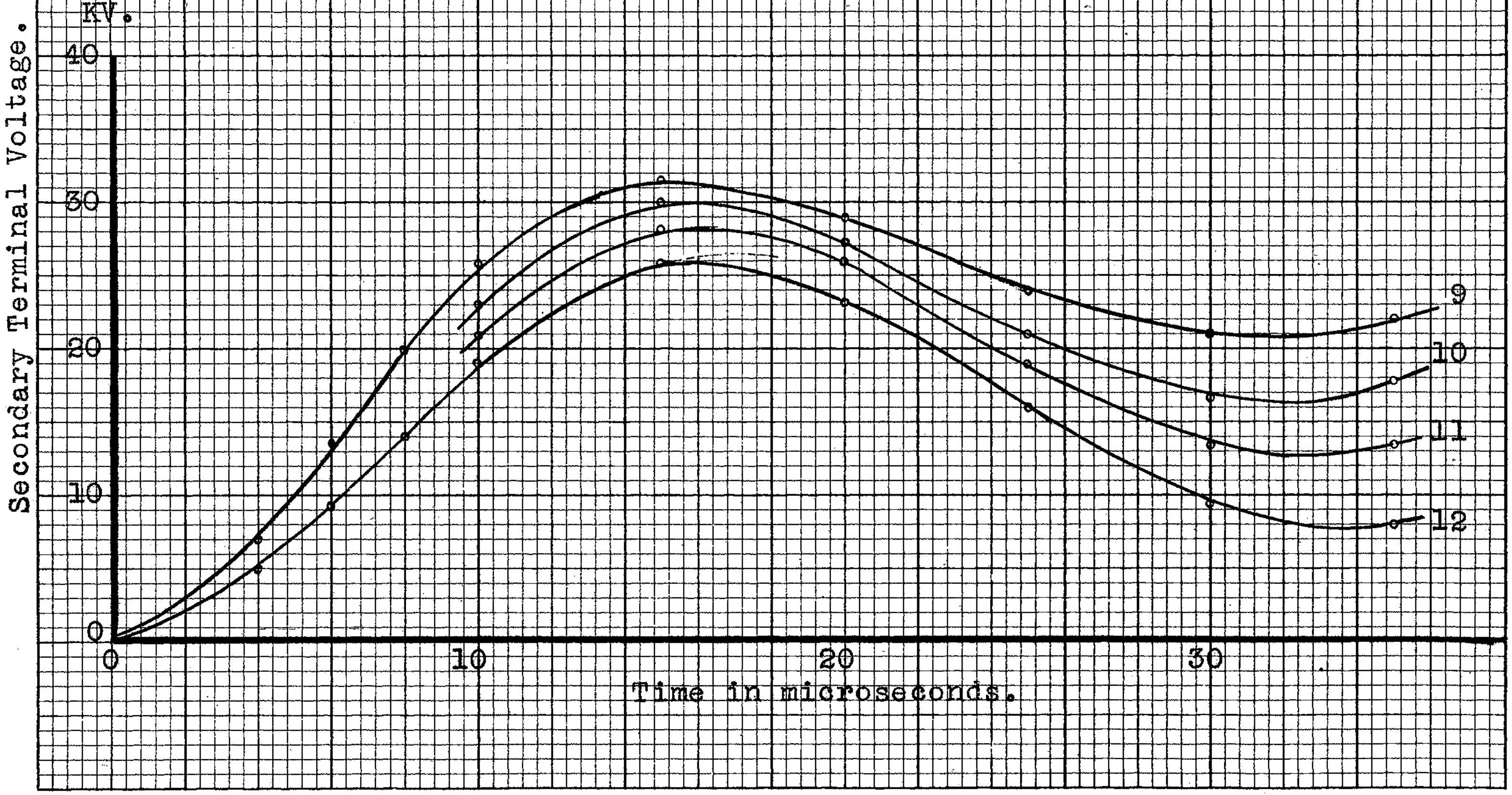


Figure 17C.

Electromagnetically Transferred Surge as Function of Time.
Case 9, 10, 11, 12. (C= 0.3 mfd.)

ances. In other words, the lines and the generator are considered to have an infinite electrical length so that no wave is reflected. In an actual generator of finite electrical length, when a surge voltage travels to the generator neutral, it sees a short circuit and is reflected with a wave of equal magnitude and opposite polarity. This reflected wave returns to the generator terminals after a time equal to twice the electrical length of the generator and is superposed on the incident wave. The result is a decrease of the terminal voltage e_2. Assume for example in the case 1 with C=0, the generator has an electrical length of 5 microseconds. The negative reflected wave would return to the generator terminals at the end of 10 microseconds and make the total voltage to decrease from the 11th microsecond on. The crest voltage would thus be limited to 26 KV. instead of 32 KV. as shown on curve 1. Unfortunately, the electrical length of a generator can be determined only by actual field measurement. It can not be accurately calculated without a knowledge of the total conductor length of the winding. However, whatever the electrical length of the machine, the crest voltages shown on curves (1) to (12) could not be exceeded. These curves thus represent the maximum voltages that can be attained under the foregoing assumptions.

Another simplification used in the analysis was the resistanceless or lossless character of the circuit. Although a lightning surge possesses tremendous power, it carries only relatively little energy due to its short duration. This energy is not replenishable and once it is dissipated, the surge becomes attenuated. The resistance and corona loss of the circuit dissipate an appreciable amount of the surge energy which is manifested by a decrease of the surge magnitude. This phenomenon largely accounts for the difference in magnitude between the above computed results and the experimental data obtained by the various investigators. Fortunately, all these factors contribute to the fact that the results given by the circuit analysis are conservative and the conclusions derived safer.

In summary, for the twenty-four cases analyzed, the secondary terminal voltage does not exceed 40 KV. in crest magnitude and a rate of rise of approximately 10 KV. per microsecond. A wave of 40 KV. crest with a linear front of 10 Kv. per microsecond could therefore be used as envelope of all the surge voltages studied. Such a wave will be used in the analysis of the voltage distribution along the generator winding.

CHAPTER III

SURGE BEHAVIOR OF LARGE GENERATORS.

Since the winding of generators is dry insulated and the impulse strength of the dry insulation is much lower than that of the oil immersed insulation; the generator winding, when compared to the oil immersed transformers, insulators, etc., becomes apparently the most vulnerable part of a system to lightning surge attack.

In a unit-connected system, the transformer acts as a reducer of surge magnitude and a flattener of wave front. Its effect is thus similar to that of a capacitor-arrester combination and compensates to a more or less degree the low impulse strength of the generator winding. In order to investigate whether this compensating effect is sufficient to make the generator winding to withstand the transferred surge voltage, answers to the following questions have been sought:

1. How high a voltage can each point of a generator winding reach upon impact of the transferred surge at its terminals?
2. How high a voltage is the winding able to withstand?

The first part of this chapter deals with an analysis of the voltage distribution along a generator winding for a given terminal surge voltage. The impulse characteristics of modern generator insulation are then discussed and compared to the transferred surge magnitude.

Part I

Transient Voltage Distribution Along a Generator Winding.

Although the generator stator winding is physically different from a transformer winding, they are electrically similar; each phase winding possesses a self- inductance, a mutual-inductance, a turn-to-turn capacitance, a turn-to-ground capacitance, a resistance and a leakance. In a single-turn-coil generator winding, the mutual-inductance between turns and the turn-to-turn capacitance are small and negligible. Figure C-1 of Appendix III shows the equivalent circuit of one phase of a stator winding, this is similar to that of Figure 4 for a single-phase transformer except that one winding only is considered.

Appendix III gives a mathematical analysis of the voltage distribution along the generator winding when the terminal is struck by an impulse voltage $E_1(t)$. The equation of the voltage at the point x along the winding at any time t, and for a rectangular surge E_1 is:

$$e(x,t) = E_1 x + E_1 \sum_{n=1}^{\infty} A_n \sin(n\pi x) \cos \omega_n t \qquad (21)$$

With a lossless generator, the step voltage E_1 travels without attenuation along the winding to the neutral where it sees a short circuit and is reflected with a step voltage E of opposite polarity. The voltage at point x is therefore either E or 0 depending on the time t. The change from E to 0 or 0 to E is abrupt. When the generator possesses resistance, the coefficient A_n includes a decaying exponential term and the Fourier series term of Equation (21) would eventually die out so that the final voltage distribution along the winding is linear. The voltage is maximum at the terminal and zero at the neutral.

If, instead of the rectangular wave, another surge of shape E(t) is applied at the generator terminal, the Duhamel's superposition theorem should be used to find the new voltage distribution function. Equation (22) of Appendix III gives one form of the Duhamel Theroem which is:

$$e'(x,t) = E_1(t)\phi(o) + \int_0^t E_1(\tau)\frac{\partial}{\partial t}\phi(t-\tau)d\tau \qquad (22)$$

where $E_1(t)$ is the equation of the impinging surge, $\phi(t)$ is the voltage distribution function if the impinging surge were a rectangular wave, in other words, it is e(x,t) of Equation (21), and $e'(xt)$ is the voltage distribution function for a surge $E_1(t)$ at the generator terminals.

It has been concluded in the preceding chapter that the study of twenty-four cases resulted in an envelope voltage of 40 KV. in crest magnitude and a linear front of 10 KV. per microsecond. The envelope is thus a 4 by infinity wave. This wave can be expressed mathematically by superposing two ramp functions of 10 KV. per microsecond slope and of opposite polarity with a time displacement of 4 microseconds.

The first ramp function is expressed as:

$$E(t) = \frac{E}{F}t = \frac{40t}{4} = 10t \quad \text{KV.}$$

where t is expressed in microseconds.

The second ramp function is:

$$E(t) = \frac{E}{F}(F-t).U(t-F) = 10(4-t).U(t-4) \quad \text{Kv.}$$

The generator terminal voltage is thus:

$$e_1 = E_1(t) + E_2(t) = 10t + 10(4-t).U(t-4).$$

it increases linearly from zero to 40 KV. in the first 4 microseconds and levels off at the end of 4 microseconds to a constant value of 40 KV.

By applying the superposition theorem twice, the voltage distribution along the generator winding is obtained as:

$$e'(x,t)_{t<4} = \frac{E_1}{F}xt + \frac{E_1}{F}\sum_{n=1}^{\infty}\frac{A_n}{\omega_n}\sin n\pi x.\sin\omega_n t \qquad (23)$$

$$e'(x,t)_{t>4} = xE_1 + E_1\sum_{n=1}^{\infty}\frac{A_n \sin\omega_n \frac{F}{2}}{\omega_n \frac{F}{2}}.\sin n\pi x.\sin\omega_n t \qquad (24)$$

for a lossless generator,

$$A_n = \frac{2 \cos n\pi}{n\pi} \quad ; \quad \omega_n = \frac{n\pi}{\sqrt{LC}}$$

Equation (23) and (24) are used to calculate the voltage distribution along one phase-winding of a 100 MVA generator. Assuming the effective reactance of the generator for the time interval considered to be ten percent and a surge impedance of 80 ohms, it is found that:

L = 0.507 millihenry per phase

C = 0.079 microfarad per phase

$\frac{1}{LC} = v$ = 0.158 numeric per microsecond

With the above linear-front voltage applied at the generator terminal, the voltagesat points distant $x = 0.9$, 0.8, 0.7,... are calculated to fifth harmonic for different time t after the surge application. The result is plotted in Figure 18.

The plot reveals that at any time t, the voltage at any point x does not exceed the terminal voltage of 40 KV. This voltage oscillates because of wave reflection at both terminals. Since the generator is assumed ideal and resistanceless, the oscillation does not attenuate. In a practical generator with resistance, the voltages at various points do finally settle to definite values E_x and result in a linear distribution.

For a lightning wave with exponentially decaying tail, the voltage distribution would be somewhat different in form. Moses and Alke (11) have conducted extensive measurements on both simulated and actual generator windings. Figures 19 and 20 show oscillographs taken on two generator windings with two different waves. The results indicate in common that for a generator whose neutral is at ground potential, the voltage at the intermediate points along the winding does not exceed the terminal voltage. The terminal voltage thus represents the maximum voltage that any part of a generator winding has to withstand.

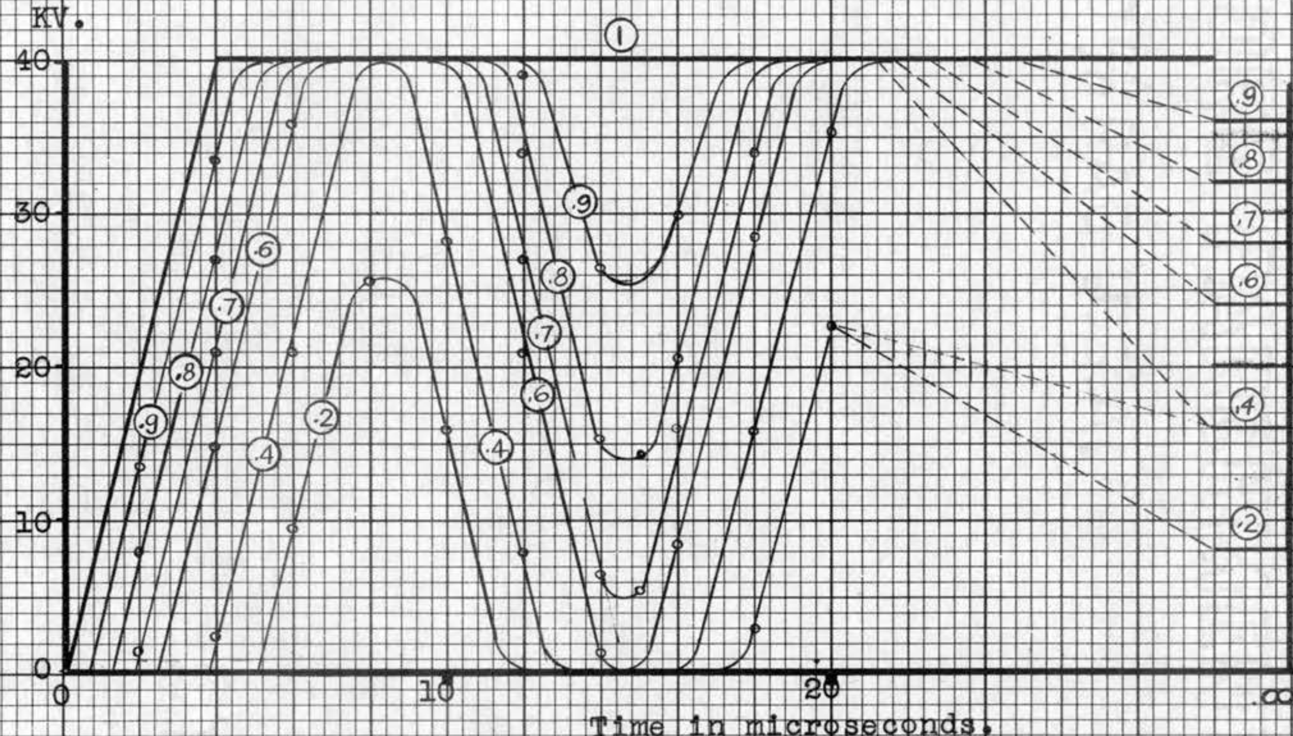
Part II

Voltage Endurance Characteristics of the Generator Winding Insulation

Considerable work has been done in the past several years to explore the voltage endurance characteristics of the high voltage generator winding insulation. Typical performance of such insulation is shown in Figure 21.[10] From the data, it can be seen that both the alternating- and direct-voltage endurance curves are essentially straight lines with negative slope when plotted on semilog coordinates: The breakdown voltage of the insulation decreases with increase in time of voltage application. The slope of the alternating-voltage endurance line is such that if the time of voltage application is decreased to one-tenth, the dielectric strength of the insulation increases by ten percent. The direct-voltage endurance curve is much flatter. This indicates that the time of direct-voltage application is less important than that of the alternating-volt-

Figure 18.

Surge Voltage Distribution Along a 100,000 KVA. Generator winding.



* Figure in circle denotes per unit distance from the neutral point.

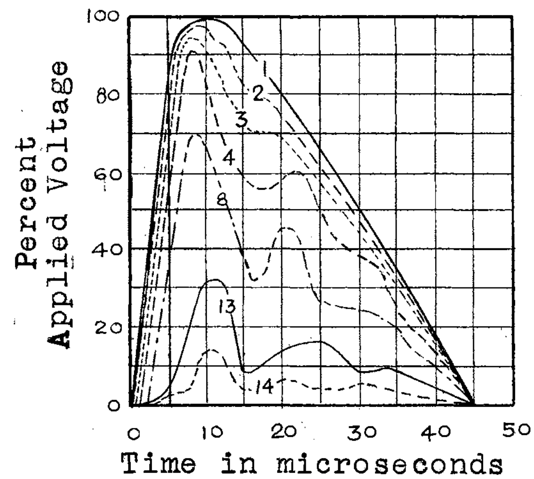


Figure 19

Surge voltage distribution along a 50 Mw. generator with 10 by 50 microseconds applied wave.

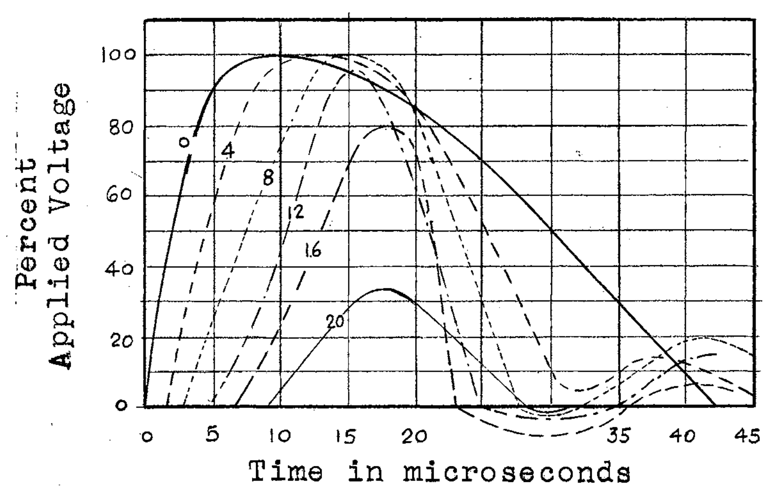


Figure 20.

Surge voltage distribution along a 60 Mw. generator with 8 by 30 microseconds applied wave.

"Studies of Impulse Strength and Impulse Testing Problems on High-Voltage Generators". G. L. Moses and R. J. Alke, Transaction American Institute of Electrical Engineers. April 1953, pp. 123-131.

age on the insulation breakdown voltage. For a given time of voltage application, the direct-voltage breakdown value is higher than the alternating-voltage value. The two curves converge as the time of voltage application becomes shorter so that for the range of impulse voltage created by lightning surges, the insulation breakdown value is practically same for the direct and alternating voltages.

Moses and Alke[11] gave the insulation characteristics for the 15 KV. class generators as shown in Figure 22. The average A-C breakdown level represents the alternating-voltage at which half of the tested coils broke down while the other half survived the test. The lower 3σ is a statistical limit and denotes the voltage at which the tested coil has a 99.73 percent probability to pass the test.

The basic insulation level of the rotating machines has been the subject of considerable discussion in the past several years. Though there is no official standard issued to date, both users and manufacturers of rotating michinery seem to agree that a 50 KV. level in the range from 1 to 100 microseconds for 15 KV. class rotating machines would be desirable. This is in contrast to the 110 KV. BIL Standard required of the switching equipment and oil immersed power apparatus. The 50 KV. level was mainly determined for coordination with the present day available

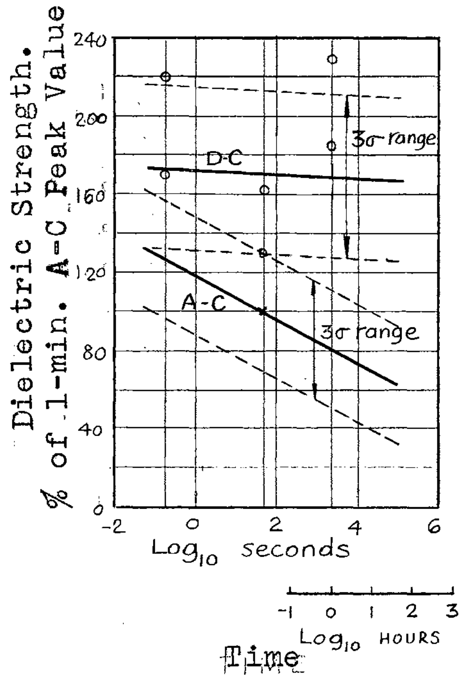


Figure 21.

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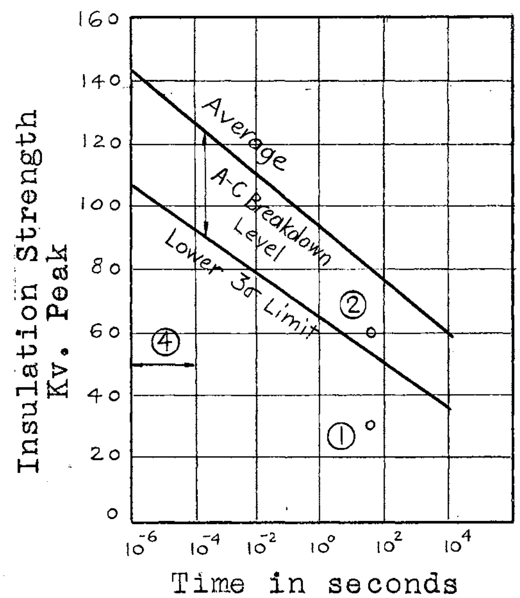
Alternating and Direct Voltage Endurance Characteristic of High Voltage Generator Winding Insulation.



** Figure 22.

Insulation Withstanding Characteristics of 15 Kv. class Machines.

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* "Alternating and Direct Voltage Endurance Studies on mica Insulation for Electrical Machinery" G.L. Moses, Transaction AIEE, 1951, Part I, pp. 763-769.

**"Studies of Impulse Strength and Impulse Testing Problems on High Voltage Generators". G. L. Moses and R. J. Alke, Transactions AIEE, April 1953, pp. 123-130.

special type station lightning arrester designed for the surge protection of rotating machines. These arresters have a breakdown-and a discharge-voltage of approximately 40 KV. on a 10 by 20 microseconds and 1500 Amp. current wave. The desired BIL is indicated as line 4 in Figure 22.

The ASA Standard C-50 on Rotating Electrical Machinery stipulates that a factory A-C proof test of 2E + 1000 volts rms for one minute is required of all rotating machines. This is indicated as point 1 in Figure 22. Point 2 is the one minute direct-voltage winding proof test value which is 3E + 1500 or 1.5 of the A-C proof test value. This test is not required by the ASA standard, but has been applied by manufacturers after the machine is assembled.

In view of the negative slope characteristic of the voltage endurance curve, a generator winding that passed the factory A-C or D-C proof test must have a surge withstanding strength of at least equal to the direct-voltage test value, which for the 15 KV. class machines is 60 KV. The direct voltage proof test acts as an impulse test to prove that the generator winding does have an impulse level of at least the desired 50 KV. crest.

From the foregoing picture, it can be concluded that if the maximum transferred surge of 40 KV. derived previously is applied at the terminals of the generator, there is ample reason to believe that the insulation will withstand alone without any auxiliary protective device. In other words, the winding insulation is well coordinated with its connecting transformer and one set of lightning arresters applied at the terminals of the transformer will protect the ensemble.

The transferred surge of 40 KV. has been maximized; all the conditions of calculation have been taken on the pessimistic side and the effect of wave reflection and energy loss in the circuit were neglected. This magnitude would be materially reduced in an actual system where there is loss and wave reflection. As the present day available station-type lightning arresters for rotating machines have a breakdown and discharge voltage of approximately 40 KV., such arresters, if installed, will have no chance to operate on a transferred surge. The function of the surge protective equipment at the generator terminals is thus at most to provide a back up protection for rare emergency.

Whether the surge protective equipment should be installed becomes therefore a decision based on insurance policy and other factors rather than an engineering requirement.

SUMMARY AND CONCLUSIONS

In a unit-connected generator scheme where the transformer is protected by lightning arresters installed adjacent to its high voltage terminals, the maximum surge voltage that can appear at the high voltage terminals are limited to the discharge voltage of the arresters given in Figure 3.

The surges can be transferred electrostatically and electromagnetically through the transformer winding to the generator terminals.

The electrostatic transfer is governed by the capacitive network of the transformer windings. Formulas for an equivalent circuit are derived for simple computations of the effect of the terminal conditions. This component is of subtransient nature. For the two cases studied, the crest magnitude of the electrostatic component does not exceed 21 KV. This voltage decays rapidly and vanishes within 0.4 microsecond. Its effect is negligible.

The electromagnetic transfer is analyzed for the unit-connected scheme and formulas are derived for the transferred surge voltage at the generator terminals. Twelve cases most commonly met in practice were analyzed with and without terminal capacitance. The result is an envelope voltage of 10 KV. per microsecond linear front and 40 KV. crest magnitude which is not exceeded in any of the investigated cases.

The voltage distribution along the generator winding is analyzed by classical transmission line theory. An actual distribution computed for a 100 MVA. generator with the above envelope voltage at the terminals showed that no points within the winding attain a voltage higher than the terminal voltage of 40 KV. crest.

The alternating- and direct-voltage endurance characteristics of the generator winding insulation indicate that if a generator withstood the ASA standard A-C proof test, it has a basic insulation level of at least 50 KV. crest.

Comparison of the winding insulation level with its maximum attainable surge voltage revealed that in a unit-connected scheme, the generator insulation is well coordinated with its connecting transformer. The lightning arresters at the transformer terminals protect the transformer and the generator as well.

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APPENDIX I

ELECTROSTATIC TRANSFER OF SURGE VOLTAGE THROUGH TRANSFORMERS

Part A

Single Phase Transformer

Figure A-1 represents the capacitance network of a single-phase transformer valid at the first instant of surge application. Consider a section dx anywhere inside the transformer, the charge across the elementary capacitances $\frac{K_1}{dx}$, $C_1 dx$ and $C_3 dx$ are:

$$q_{K_1} = \int i_{K_1}\, dt = K_1 \frac{\partial e_1}{\partial x} \; ; \qquad i_{K_1} = K_1 \frac{\partial^2 e_1}{\partial x \partial t} \qquad (1)$$

$$q_{C_1} = \int \frac{\partial i_1}{\partial x}\, dx\, dt = C_1 e_1 dx \; ; \qquad \frac{\partial i_1}{\partial x} = C_1 \frac{\partial e_1}{\partial t} \qquad (2)$$

$$q_{C_3} = \int \frac{\partial i_3}{\partial x} dx\, dt = C_3 dx (e_1 - e_2) \; ; \qquad \frac{\partial i_3}{\partial x} = C_3 \frac{\partial}{\partial t}(e_1 - e_2) \qquad (3)$$

At the node e_1:

$$\frac{\partial i_1}{\partial x} dx + \frac{\partial i_3}{\partial x} dx + i_{K_1} - \frac{\partial i_{K_1}}{\partial x} dx = i_{K_1}$$

$$\frac{\partial i_1}{\partial x} + \frac{\partial i_3}{\partial x} = \frac{\partial i_{K_1}}{\partial x} \qquad (4)$$

Replacing $\frac{\partial i_1}{\partial x}$, $\frac{\partial i_3}{\partial x}$, $\frac{\partial i_K}{\partial x}$, of equation (4) by their values from (1), (2) and (3) gives:

$$C_1\frac{\partial e_1}{\partial t} + C_3\frac{\partial e_1}{\partial t} - C_3\frac{\partial e_2}{\partial t} = K_1\frac{\partial^3 e_1}{\partial x^2 \partial t} \qquad (5)$$

Let $\frac{\partial}{\partial t}$ be replaced by operator p, regrouping and dividing through by p:

$$(C_1 + C_3)e_1 - K_1\frac{\partial^2 e_1}{\partial x^2} = C_3 e_2 \qquad (6)$$

By symmetry, similar relations can be obtained for the secondary side:

$$(C_2 + C_3)e_2 - K_2\frac{\partial^2 e_2}{\partial x^2} = C_3 e_1 \qquad (7)$$

Equations (6) and (7) give the set of simultaneous differential equations for e_1 and e_2. Eliminating e_1 between (6) and (7), the following results:

$$\frac{\partial^4 e_2}{\partial x^4} - \left(\frac{K_1(C_2+C_3)+K_2(C_1+C_3)}{K_1 K_2}\right)\frac{\partial^2 e_2}{\partial x^2} + \left(\frac{C_1C_2+C_1C_3+C_2C_3}{K_1 K_2}\right)e_2 = 0 \qquad (8)$$

This is the equation of the voltage distribution along the secondary winding at the instant the surge enters the primary coil. By merely replacing e_2 by e_1, the equation becomes the primary voltage distribution. The solutions for the primary and secondary distributions will, however, be different due to the different boundary conditions.

Since only x is shown as variable in equation (8), it can be treated as an ordinary fourth order linear differential equation with the following solutions:

$$e_1 = A_1\epsilon^{\alpha x} + B_1\epsilon^{-\alpha x} + C_1\epsilon^{\beta x} + D_1\epsilon^{-\beta x} \qquad (9)$$

$$e_2 = A_2\epsilon^{\alpha x} + B_2\epsilon^{-\alpha x} + C_2\epsilon^{\beta x} + D_2\epsilon^{-\beta x} \qquad (10)$$

where:

$$\alpha^2 = \frac{K_1(C_2+C_3) + K_2(C_1+C_3) + \sqrt{[K_1(C_2+C_3) - K_2(C_1+C_3)]^2 + 4K_1K_2C_3^2}}{2K_1K_2} \qquad (11)$$

$$\beta^2 = \frac{K_1(C_2+C_3) + K_2(C_1+C_3) - \sqrt{[K_1(C_2+C_3) - K_2(C_1+C_3)]^2 + 4K_1K_2C_3^2}}{2K_1K_2} \qquad (12)$$

A_1, A_2, B_1, ... and D_2 are constants of integration.
By replacing e_1 abd e_2 from (9) and (10) in (6) we have:

$$(C_1+C_3)[A_1\epsilon^{\alpha x}+B_1\epsilon^{-\alpha x}+C_1\epsilon^{\beta x}+D_1\epsilon^{-\beta x}] - K_1[A_1\alpha^2\epsilon^{\alpha x}+B_1\alpha^2\epsilon^{-\alpha x}+C_1\beta^2\epsilon^{\beta x}+D_1\beta^2\epsilon^{-\beta x}]$$
$$= C_3(A_2\epsilon^{\alpha x} + B_2\epsilon^{-\alpha x} + C_2\epsilon^{\beta x} + D_2\epsilon^{-\beta x}$$

equating like terms:

$$A_2 = \frac{C_1 + C_3 - K_1\alpha^2}{C_3}A_1 = mA_1 \quad ; \quad C_2 = \frac{C_1 + C_3 - K_1\beta^2}{C_3}C_1 = nC_1$$

$$B_2 = \frac{C_1 + C_3 - K_1\alpha^2}{C_3}B_1 = mB_1 \quad ; \quad D_2 = \frac{C_1 + C_3 - K_1\beta^2}{C_3}D_1 = nD_1$$

and equations (9) and (10) are reduced to:

$$e_1 = A\epsilon^{\alpha x} + B\epsilon^{-\alpha x} + C\epsilon^{\beta x} + D\epsilon^{-\beta x} \qquad (9a)$$

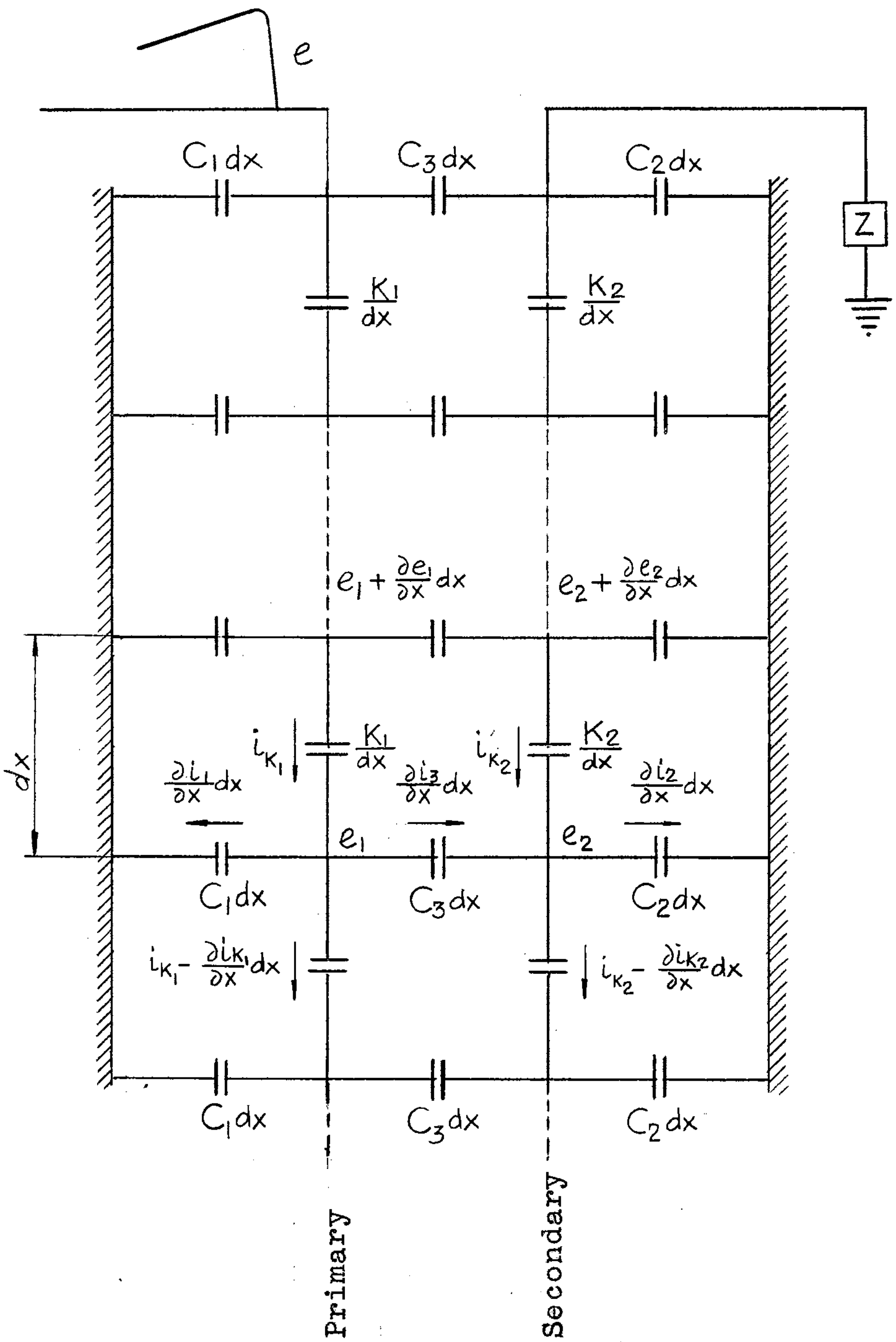


Figure A-1.

Capacitive Network of Single-phase Transformer.

$$e_2 = mA\epsilon^{\alpha x} + mB\epsilon^{-\alpha x} + nC\epsilon^{\beta x} + nD\epsilon^{-\beta x} \qquad (10a)$$

Two cases will be considered:

CASE 1: <u>PRIMARY AND SECONDARY NEUTRAL GROUNDED, SECONDARY LINE TERMINAL CONNECTED TO A GENERATOR OF SURGE IMPEDANCE Z_g</u>.

Thr boundary conditions are:

At the line terminals where x = 1:

$$e_1 = E \quad ; \quad e_2 = Z_g i_{K_2} = Z_g K_2 \frac{\partial^2 e_2}{\partial x \partial t} = Z_g p K_2 \frac{\partial e_2}{\partial x}$$

At the Neutral: $e_1 = e_2 = 0$

Applying the boundary conditions to equations (9a) and (10a), there results:

$$A = -B = \frac{n(\sinh\beta + b\cosh\beta)E}{2[n\sinh\alpha(\sinh\beta + b\cosh\beta) - m\sinh\beta(\sinh\alpha + a\cosh\alpha)]} \qquad (13)$$

$$C = -D = \frac{-m(\sinh\alpha + a\cosh\alpha)E}{2[n\sinh\alpha(\sinh\beta + b\cosh\beta) - m\sinh\beta(\sinh\alpha + a\cosh\alpha)]} \qquad (14)$$

where

$$a = Z_g p K_2 \alpha \quad ; \quad b = Z_g p K_2 \beta$$

The secondary voltage e_2 is:

$$e_2 = \frac{mn[(\sinh\beta + b\cosh\beta)\sinh\alpha x - (\sinh\alpha + a\cosh\alpha)\sinh\beta x].E}{n\sinh\alpha(\sinh\beta + b\cosh\beta) - m\sinh\beta(\sinh\alpha + a\cosh\alpha)} \qquad (15)$$

At the secondary terminal where x = 1,

$$e_{2T} = \frac{mnZ_g K_2(\beta \sinh\alpha \cosh\beta - \alpha \sinh\beta \cosh\alpha)\, p}{(n-m)\sinh\alpha \sinh\beta + Z_g K_2(n\beta \sinh\alpha \cosh\beta - m\alpha \sinh\beta \cosh\alpha)\, p} \cdot E$$

The coefficient of E represents the transfer function of the transformer and is of the form:

$$F(p) = \frac{Ap}{Bp + C}$$

the solution of which is:

$$e_2 = \frac{mn[\beta \sinh\alpha \cosh\beta - \alpha \sinh\beta \cosh\alpha]}{n\beta \sinh\alpha \cosh\beta - m\alpha \sinh\beta \cosh\alpha} \cdot E\epsilon^{-\gamma t} \qquad (16)$$

$$\gamma = \frac{(n-m)\sinh\alpha \sinh\beta}{Z_g K_2(n\beta \sinh\alpha \cosh\beta - m\alpha \sinh\beta \cosh\alpha)} \qquad (17)$$

CASE 2: <u>PRIMARY AND SECONDARY NEUTRAL GROUNDED, SECONDARY LINE TERMINAL CONNECTED TO A CAPACITOR C.</u>

The boundary conditions become:

At the line terminals where x = 1:

$$e_1 = E \quad ; \quad e_2 = Z(p)\, i_{K_2} = \frac{K_2}{Cp} \frac{\partial^2 e_2}{\partial x \partial t} = \frac{K_2}{C} \frac{\partial e_2}{\partial x}$$

at the neutral terminal: $e_1 = e_2 = 0$

Applying the boundary conditions to equations (9a) and (10a) gives values of constants A, B, C, and D identical to equations (13) and (14), but in this case:

$$a = \frac{K_2}{C}\alpha \quad ; \quad b = \frac{K_2}{C}\beta$$

The terminal voltage e_2 is:

$$e_2 = \frac{mn[(\sinh\beta + b\cosh\beta)\sinh\alpha x - (\sinh\alpha + a\cosh\alpha)\sinh\beta x]E}{n\sinh\alpha(\sinh\beta + b\cosh\beta) - m\sinh\beta(\sinh\alpha + a\cosh\alpha)}$$

at the secondary terminals where x = 1:

$$e_{2T} = \frac{mnK_2[\beta\sinh\alpha\cosh\beta - \alpha\sinh\beta\cosh\alpha]E}{C(n-m)\sinh\alpha\sinh\beta + K_2(n\beta\sinh\alpha\cosh\beta - m\alpha\sinh\beta\cosh\alpha)} \quad (18)$$

Part B

Equivalent Electrostatic Network of Single Phase Transformer

Since we are mostly interested in the terminal voltage of the transformer secondary rather than the internal distribution, the distributive capacitive network of the transformer may be replaced by a lumped pi section as shown in Figure A-2. The capacitances C'_H, C'_{HL} AND C'_L are the equivalent capacitances of the high voltage winding to ground, between the high and low voltage winding, and the low voltage winding to ground respectively.

Consider again the case of a transformer with the secondary terminal connected to a generator of surge impedance Z_g. Since the wave comes from the high voltage winding, the capacitance C'_H has no effect on the secondary voltage and is therefore left for simplicity. The simplified equivalent circuit is as shown in Figure A-3.

The network equations are:

$$E_1 = \frac{C'_{HL} + C'_L}{C'_{HL}C'_L}\int i_1\,dt - \frac{1}{C'_L}\int i_2\,dt$$

$$0 = -\frac{1}{C'_L}\int i_1\,dt + Z_g i_2 + \frac{1}{C'_L}\int i_2\,dt$$

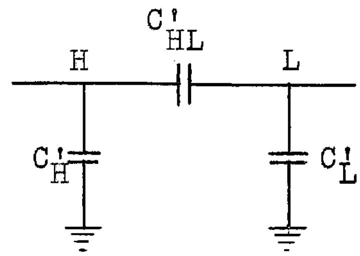


Figure A-2

Equivalent Capacitance Network
of
Grounded Single-phase Transformer.

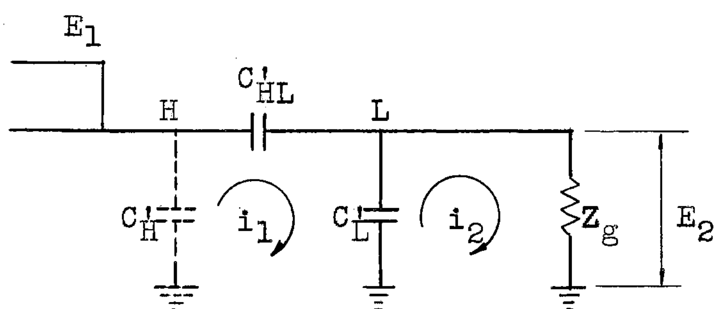


Figure A-3

Equivalent Capacitance Network
of
Grounded Single-phase Transformer
with Secondary connected to a surge impedance Z_g.

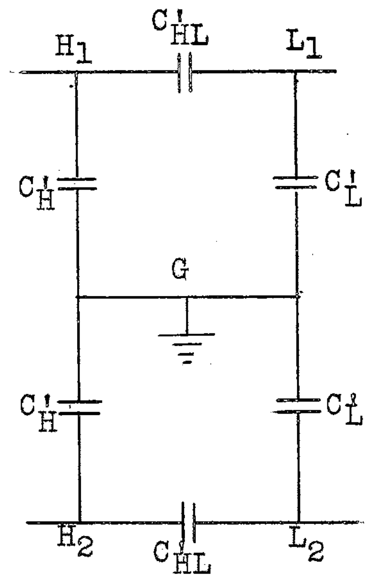


Figure A-4

Equivalent Capacitance Circuit
of
Ungrounded Single-phase Transformer.

The Laplace transforms of the above two equations are:

$$\frac{E_1}{S} = I_1 \frac{1}{S} \frac{C'_{HL} + C'_L}{C'_{HL} C'_L} - I_2 \frac{1}{S C'_L} \tag{19}$$

$$0 = - I_1 \frac{1}{S C'_L} + I_2 \left(Z_g + \frac{1}{S C'_L} \right) \tag{20}$$

Eliminating I_1, the secondary voltage $E_2(s)$ is obtained:

$$E_2(s) = Z_g I_2 = \frac{C'_{HL}}{C'_{HL} + C'_L} \frac{1}{S + \frac{1}{Z_g (C'_{HL} + C'_L)}} E_1$$

whose inverse Laplace transform is:

$$e_2 = \frac{C'_{HL}}{C'_{HL} + C'_L} E_1 \epsilon^{-\frac{1}{Z_g (C'_{HL} + C'_L)} t} \tag{21}$$

Comparing this equation with Equations (16) and (17), we obtain:

$$C'_{HL} = \frac{K_2 mn [\beta \sin\alpha \cosh\beta - \alpha \sinh\beta \cosh\alpha]}{(n-m) \sinh\alpha \cosh\beta} \tag{22}$$

$$C'_L = \frac{K_2 [n\beta(1-m) \sinh\alpha \cosh\beta - m\alpha(1-n) \sinh\beta \cosh\alpha]}{(n-m) \sinh\alpha \cosh\beta} \tag{23}$$

The above equivalent diagram for a single-phase transformer is based on the assumption that both neutrals of the high and low voltage windings are grounded. In case none of the neutrals are grounded, the equivalent diagram can be modified as shown in Figure A-4. This latter diagram is derived on the basis that the transformer should show same capacitance when it is looked from either terminals of the high voltage winding. The same holds true for either terminals of the low voltage winding. This representation of the transformer is not perfect, since the series capacitance of the high voltage winding, i.e. capacitance between terminals H_1 and H_2, is definitely lower than $\frac{C'_H}{2}$. However, the diagram represents correctly the capacitances between the terminals $H_1 - L_1$, $H_1 - G$, $L_1 - G$, $H_2 - L_2$, $H_2 - G$ and $L_2 - G$. This symmetrical representation of the transformer is used in the derivation of the equivalent circuit for three-phase transformer.

Part C

Equivalent Electrostatic Network of Wye-Delta Connected Three-phase Transformer.

The complete capacitive network of a three-phase grounded Wye-Delta transformer is shown in Figure A-5, where Z_g is the surge impedance of the generator connected to the transformer secondary and C_x is the capacitance introduced by the generator leads. If each phase is represented by its equivalent circuit derived from the preceding section, the network is reduced to Figure A-6. A surge entering phase A sees first the capacitance to ground of the high voltage winding C'_H, and the coupling capacitance C'_{HL}, then C'_{LA} of phase A, C'_{LB} of phase B, the coupling capacitance C'_{HLB} of phase B and load Z_g and C_x. All other capacitances of the transformer have little effect on the surge. This leads to the equivalent diagram of Figure A-7. The magnitude of the transferred surge is expressed as:

$$e_2 = \frac{C'_{HLA}}{C'_{HLA} + C'_{HLB} + C'_{LA} + C'_{LB} + C_x} E = \frac{C'_{HL}}{2(C'_{HL} + C'_L) + C_x} E \qquad (24)$$

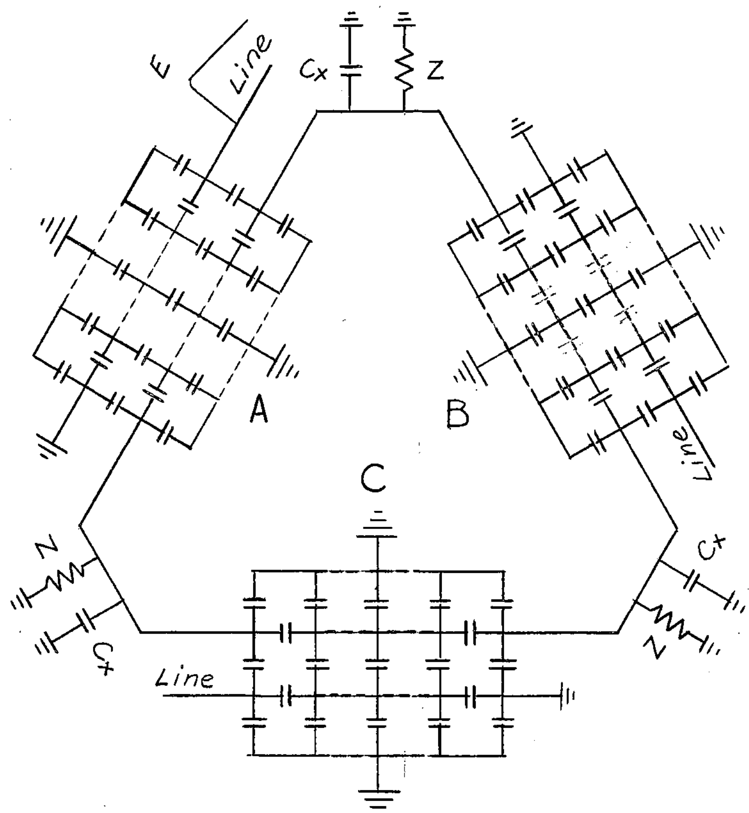


Figure A-5

Capacitive Network of a

Three-phase Grounded Wye-delta Transformer.

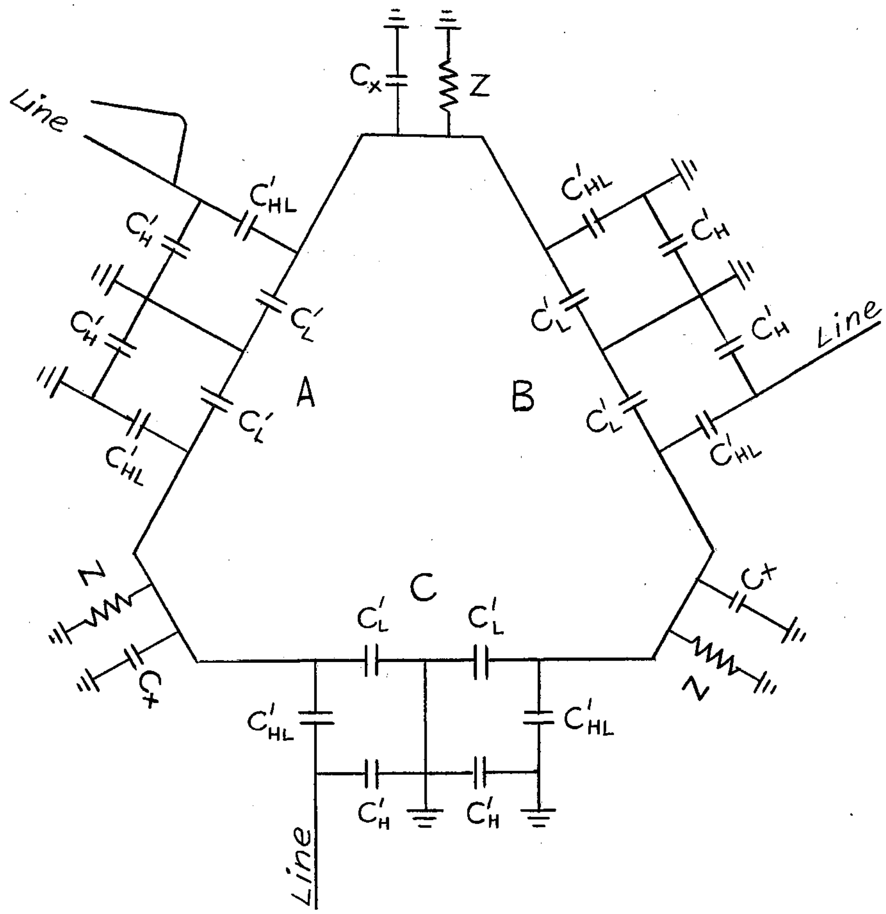


Figure A-6

Equivalent Capacitance Network
of
Three-phase Grounded Wye-delta Transformer.

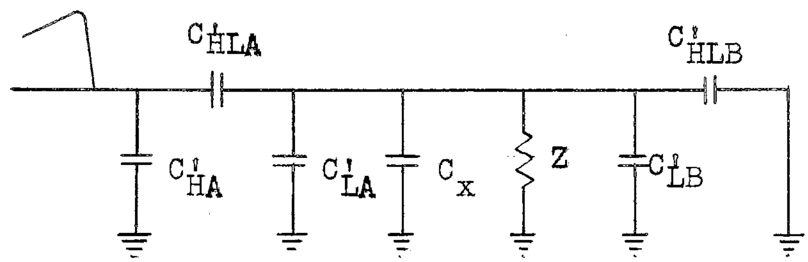


Figure A-7

Equivalent Capacitance Diagram
of a
Three-phase Grounded Wye-delta Transformer.

APPENDIX II

ELECTROMAGNETIC TRANSFER OF SURGE VOLTAGE THROUGH TRANSFORMERS

The following is a mathematical analysis of the electromagnetic transfer through transformer windings:

Two cases are considered: The case of the wave reaching only one phase of the transformer and the case when two phases of the high voltage winding are struck simultaneously by lightning surges of equal magnitudes.

Part A

Electromagnetic Transfer of Lightning Wave in Case of One-Phase Surge

Figure B-1-A is a schematic diagram of the unit-connected generator scheme with all the constants and voltages referred to the secondary basis. The transformer is of grounded wye-delta type, its impedance is represented by the leakage inductance L, the generator is represented by its surge impedance Z_g, the bus capacitance C_x is shown connected in parallel with the generator. The intact lines are represented by a line surge impedance Z'_L which is converted to the secondary basis as:

$$Z_L = Z'_L \frac{3}{N^2}$$

The magnitude and wave form of the incoming surge is $E_1(t)$

and is numerically equal to the maximum discharge voltage of the arresters installed adjecent to the high voltage bushings of the transformer. This voltage corresponds to $\frac{\sqrt{3}.E(t)}{N}$ when referred to the secondary basis. After the surge struck phase **A**, the voltage and current distributions in the transformer and the generator circuits are as shown in Figure B-1-**A**. The surge causes current I_1 to flow in the phase **A** winding of the wye side of the transformer. To counteract it, a corresponding current I_1 flows in phase **A** of the delta winding. The latter current, in flowing out of phase **A** winding, is split into two paths: a portion flows through the generator-capacitor circuit and the remainder flows through the phases B and C of the delta winding. The current flow in phases B and C of the delta winding causes corresponding current to flow in the phases B and C of the wye winding which returns by the intact lines of surge impedance Z_L.

From the foregoing and the Figure B-1-**A**, the circuit equations are:

$$LpI_1 + e_A = \frac{\sqrt{3}\,E_1(t)}{N}$$

$$e_A = I_2 Z(p) + I_3 Z(p) = Z(p)[I_2 + I_3]$$

$Z(p)$ being the combined impedance function of C_x and Z_g in parallel:

$$Z(p) = \frac{Z_g}{1 + pCZ_g}$$

Eliminating e_A :

$$LpI_1 + (I_2 + I_3) Z(p) = \frac{\sqrt{3}\,E_1(t)}{N}$$

For the high voltage side of the phases B and C:

$$e_C = (I_1 - I_3)(Z_L + Lp)$$
$$e_B = (I_1 - I_2)(Z_L + Lp)$$

For the low voltage side of the phases B and C:

$$e_C = -Z(p)(I_2 - I_3) + Z(p)I_3 = Z(p)(2I_3 - I_2)$$
$$e_B = Z(p)I_2 + Z(p)(I_2 - I_3) = Z(p)(2I_2 - I_3)$$

From the diagram, it is seen that the phases B and C of the delta winding act as a potentiometer for voltage e_A, by symmetry: $e_B = e_C$; $I_2 = I_3$ and

$$I_1 \frac{Lp}{2} + I_2 Z(p) = \frac{\sqrt{3}E_1(t)}{2N} \tag{1}$$

$$I_1(Z_L + Lp) - I_2[Z_L + Lp + Z(p)] = 0 \tag{2}$$

Inspection of equation (1) and (2) suggests the equivalent circuit of Figure B-1-B.

Taking the Laplace transforms of equations (1) and (2) and solving for I_2:

$$I_2(s) = \frac{\sqrt{3}E_1(s)}{N} \frac{Z_L + Ls}{L^2s^2 + 3LZ(s)s + Z_L[Ls + 2Z(s)]}$$

The voltage at the secondary terminal is:

$$E_2(s) = I_2(s)Z(s) = \frac{\sqrt{3}E_1(s)}{LCN} \frac{s + \frac{Z_L}{L}}{s^3 + s^2\left(\frac{1}{CZ_g} + \frac{Z_L}{L}\right) + s\left(\frac{Z_L}{LCZ_g} + \frac{3}{LC}\right) + \frac{2Z_L}{L^2C}} \tag{3}$$

If the wave form of the incoming surge $E_1(s)$ is known and its Laplace transform substituted in (3), the secondary terminal voltage in function of time can be determined by the inverse Laplace transform of equation (3).

Part B

Electromagnetic Transfer of Lightning Wave in Case of Two-Phase Surge

Figure B-2-A is a schematic diagram showing current distribution at the various points of the unit-connected generator when both phases A and C are struck simultaneously by lightning surges of equal magnitude and wave form. By symmetry of the circuit, it can be assumed that the current and the induced voltage in phase A and C are equal:

$$I_A = I_C = I_1 \qquad ; \qquad e_A = e_C$$

since the voltages are in phase, we have also:

$$e_B = e_A + e_C = 2e_A$$

The circuit equations are:

For phase A and C:

$$I_1 Lp + e_A = \frac{\sqrt{3}\,E_1(t)}{N}$$

$$I_2 Z(p) = e_A$$

$$\therefore \quad I_1 Lp + I_2 Z(p) = \frac{\sqrt{3}\,E_1(t)}{N} \qquad (4)$$

For phase B:

$$(I_1 - I_2)(Z_L + Lp) = e_B = 2e_A$$

$$2Z(p)I_2 = e_B$$

$$\therefore \quad I_1(Z_L + Lp) - I_2\left[Z_L + Lp + 2Z(p)\right] = 0$$

Or dividing through by 2:

$$I_1\left(\frac{Z_L}{2} + \frac{L}{2}p\right) - I_2\left[\frac{Z_L}{2} + \frac{L}{2}p + Z(p)\right] = 0 \qquad (5)$$

Inspection of equation (4) and (5), suggests the equivalent circuit diagram of Figure B-2-B

To find the expression for the secondary terminal voltage E_2, take the Laplace transform of both equation (4) and (5), and eliminating I_1:

$$I_2(s) = \frac{\sqrt{3}E_1(s)}{N} \frac{Ls + Z_L}{L^2s^2 + 3Z(s)Ls + Z_L[Z(s)+Ls]}$$

The secondary terminal voltage E_2 is:

$$E_2(s) = I_2(s)Z(s) = \frac{\sqrt{3}E_1(s)}{NLC} \frac{s + \frac{Z_L}{L}}{s^3 + s^2\left(\frac{1}{CZ_g} + \frac{Z_L}{L}\right) + s\left(\frac{Z_L}{LCZ_g} + \frac{3}{LC}\right) + \frac{Z_L}{L^2C}} \quad (6)$$

For an incoming surge of infinite front and exponential tail, its Laplace transform is:

$$E_1(s) = \frac{E_1}{s+\gamma}$$

The denominator of equation (6) may have three distinct roots or one real and two complex conjugate roots depending on the relative values of L and C. When the generator-transformer connection is of cable type, the capacitance is usually sufficient to make the secondary voltage oscillatory. The secondary voltage can then be written as:

$$E_2(s) = \frac{\sqrt{3}\,E_1}{NLC} \frac{s + a_o}{(s+\gamma)(s+\delta)[(s+\alpha)^2+\beta^2]} \quad (6a)$$

where $a_o = \frac{Z_L}{L}$

$\frac{1}{\gamma}$ = time constant of the impinging surge,

α, β, δ are factors of the denominator.

The inverse Laplace transform of (6a) is:

$$E_2(t) = A\epsilon^{-\gamma t} + B\epsilon^{-\delta t} + C\epsilon^{-\alpha t}\sin(\beta t + \psi) \qquad (7)$$

where $A = \frac{\sqrt{3}\,E_1}{NLC}\frac{a_o - \gamma}{(\delta-\gamma)[(\alpha-\gamma)^2+\beta^2]}$; $B = \frac{\sqrt{3}\,E_1}{NLC}\frac{a_o - \delta}{(\gamma-\delta)[(\alpha-\delta)^2+\beta^2]}$

$$C = \frac{\sqrt{3}\,E_1}{NLC}\frac{1}{\beta}\sqrt{\frac{(a_o-\alpha)^2+\beta^2}{[(\delta-\alpha)^2+\beta^2][(\gamma-\alpha)^2+\beta^2]}}\,; \quad \psi = \tan^{-1}\frac{\beta}{a_o-\alpha} - \tan^{-1}\frac{\beta}{\gamma-\alpha} - \tan^{-1}\frac{\beta}{\delta-\alpha}$$

If the capacitance C of the secondary is negligible, equation (6) is reduced to:

$$E_2(s) = \frac{\sqrt{3}\,E_1(s)\,Z_g}{LN}\;\frac{s + \frac{Z_L}{L}}{s^2 + s\left(\frac{Z_L}{L} + \frac{3Z_g}{L}\right) + \frac{Z_L Z_g}{L^2}} \qquad (8)$$

The solution is aperiodic as there is no capacitance in the circuit, and the denominator possesses two distinct roots so that equation (8) can be expressed as:

$$E_2(s) = \frac{\sqrt{3}\,E_1 Z_g}{LN}\;\frac{s + a_o}{(s+\gamma)(s+\alpha)(s+\beta)}$$

a_o and γ are as previously defined; α and β are the roots of the quadratic of the denominator.

The inverse Laplace transform of this is:

$$E_2(t) = \frac{\sqrt{3}\,E_1 Z_g}{LN}\left[\frac{a_o-\alpha}{(\gamma-\alpha)(\delta-\alpha)}\epsilon^{-\alpha t} + \frac{a_o-\gamma}{(\alpha-\gamma)(\delta-\gamma)}\epsilon^{-\gamma t} + \frac{a_o-\delta}{(\alpha-\delta)(\gamma-\delta)}\epsilon^{-\delta t}\right] \qquad (9)$$

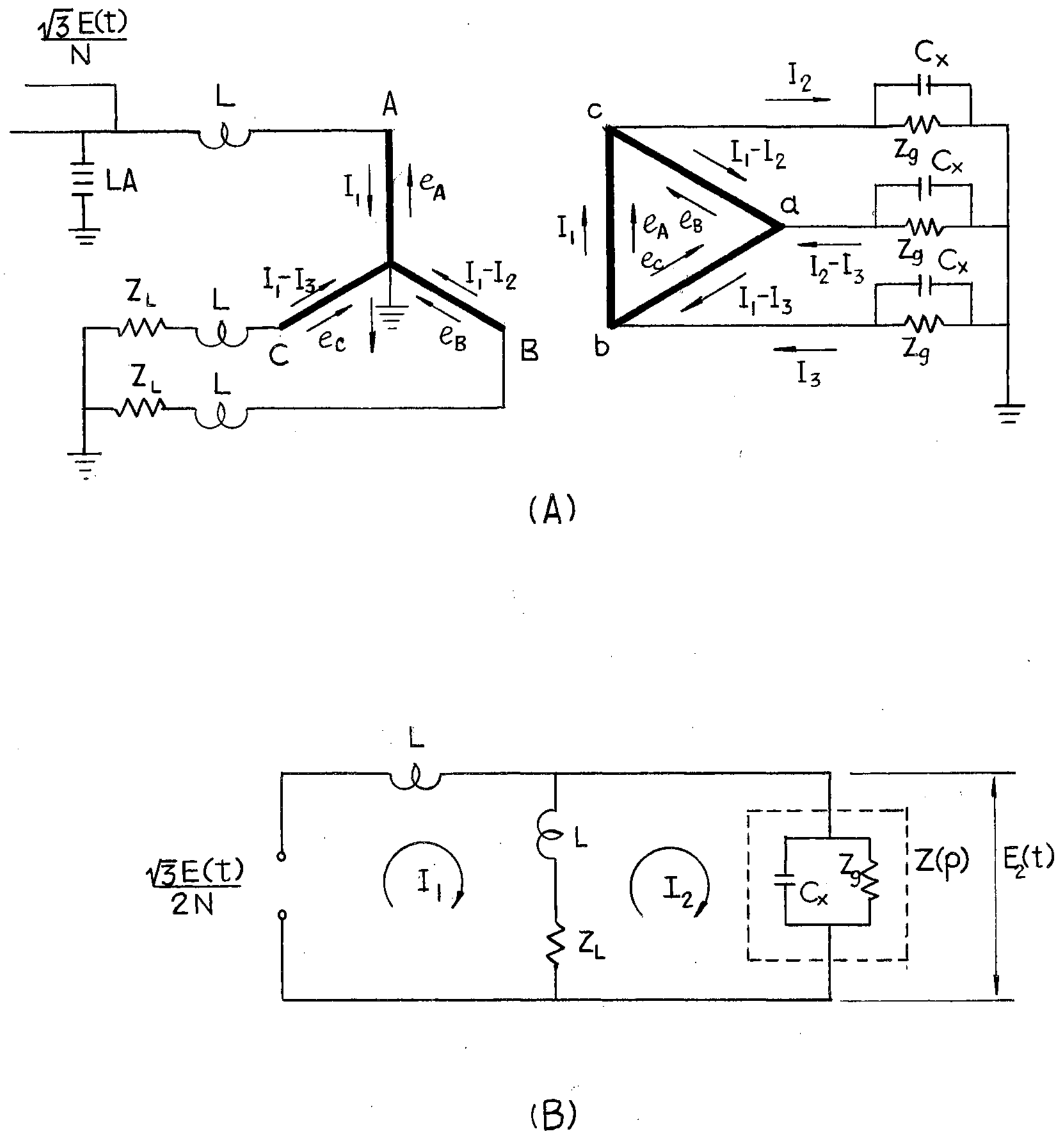
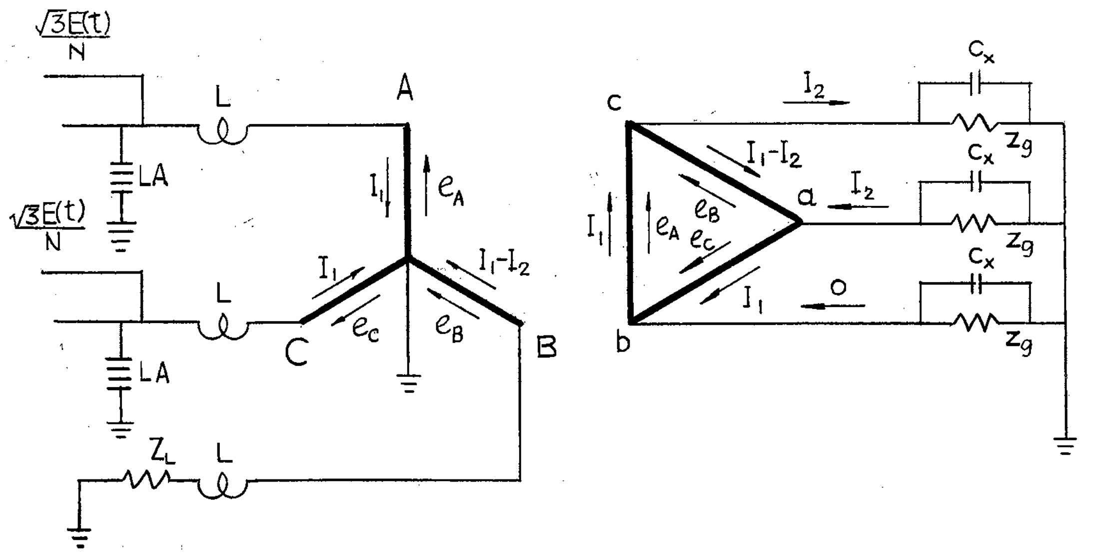
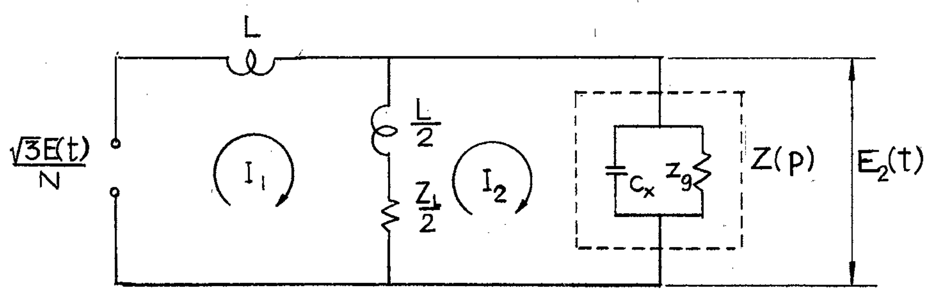


Figure B-1

Current Distribution Diagram and Equivalent Circuit of Unit-connected Transformer-Generator with surge voltage E(t) impinging one phase.



(A)



(B)

Figure B-2

Current Distribution Diagram and Equivalent Circuit of Unit-connected Transformer-Generator with surge voltage E(t) impinging two phases.

APPENDIX III

SURGE VOLTAGE DISTRIBUTION ALONG THE GENERATOR WINDING

Each phase of the generator can be represented by a ladder network as shown in Figure C-1, the voltage across the element MN = Δx is :

$$\Delta e = \frac{1}{\frac{K}{\Delta x}}\int i_1 \, dt$$

$$\left.\frac{\Delta e}{\Delta x}\right|_{\Delta x \to 0} = \frac{\partial e}{\partial x} = \frac{1}{K}\int i_1 \, dt$$

The currents i_1, i_2. . .in the various circuit elements are:

$$i_1 = K \frac{\partial^2 e}{\partial t^2} \qquad (1)$$

$$i_3 = \frac{g}{\Delta x} \cdot \Delta e = g \left.\frac{\Delta e}{\Delta x}\right|_{\Delta x \to 0} = g \frac{\partial e}{\partial x} \qquad (2)$$

$$\Delta i = G \Delta x . e + C \Delta x \frac{\partial e}{\partial t}$$

$$\left.\frac{\Delta i}{\Delta x}\right|_{\Delta x \to 0} = \frac{\partial i}{\partial x} = Ge + C \frac{\partial e}{\partial t} \qquad (3)$$

But: $$i = i_1 + i_2 + i_3 \quad ; \quad \frac{\partial i}{\partial x} = \frac{\partial}{\partial x}(i_1 + i_2 + i_3) \qquad (4)$$

Substituting $\frac{\partial i}{\partial x}$, i_1, and i_3 in (4)

$$\frac{\partial i_2}{\partial x} = Ge + C \frac{\partial e}{\partial t} - K \frac{\partial^3 e}{\partial x^2 \partial t} - g \frac{\partial^2 e}{\partial x^2} \qquad (5)$$

From Figure C-1:

$$\Delta e = r \Delta x i_2 + L \Delta x \frac{\partial i_2}{\partial t}$$

Or

$$\left.\frac{\Delta e}{\Delta x}\right|_{\Delta x \to 0} = \frac{\partial e}{\partial x} = r i_2 + L \frac{\partial i_2}{\partial t}$$

The derivative of which with respect to x gives:

$$\frac{\partial^2 e}{\partial x^2} = r\frac{\partial i_2}{\partial x} + L\frac{\partial^2 i_2}{\partial x \partial t} \qquad (6)$$

Combining (5) and (6):

$$LK\frac{\partial^4 e}{\partial x^2 \partial t^2} + (Lg+Kr)\frac{\partial^3 e}{\partial x^2 \partial t} + (1+gr)\frac{\partial^2 e}{\partial x^2} - LC\frac{\partial^2 e}{\partial t^2} - (LG+rC)\frac{\partial e}{\partial t} - rGe = 0 \qquad (7)$$

The generator is a high efficiency machine, r, g, and G can usually be neglected without great error:

$$LK\frac{\partial^4 e}{\partial x^2 \partial t^2} - \frac{\partial^2 e}{\partial x^2} - LC\frac{\partial^2 e}{\partial t^2} = 0 \qquad (8)$$

In modern turbine-generators, single-turn coil or bar-type construction is used, the capacitance between turns is limited to the coil-end part which is usually small and negligible. For such generators, equation (8) can be further simplified to:

$$\frac{\partial^2 e}{\partial x^2} = LC\frac{\partial^2 e}{\partial t^2} \qquad (9)$$

The solution of equation (9) has been attempted with in the following since generators of bar-type construction have been considered throughout the study. Assume first that the applied surge was a rectangular wave of amplitude E_1.

It has been noted in Chapter II on electromagnetic surge transfer that the neutral of the generator is always at ground potential irrespective of the method of grounding used. The

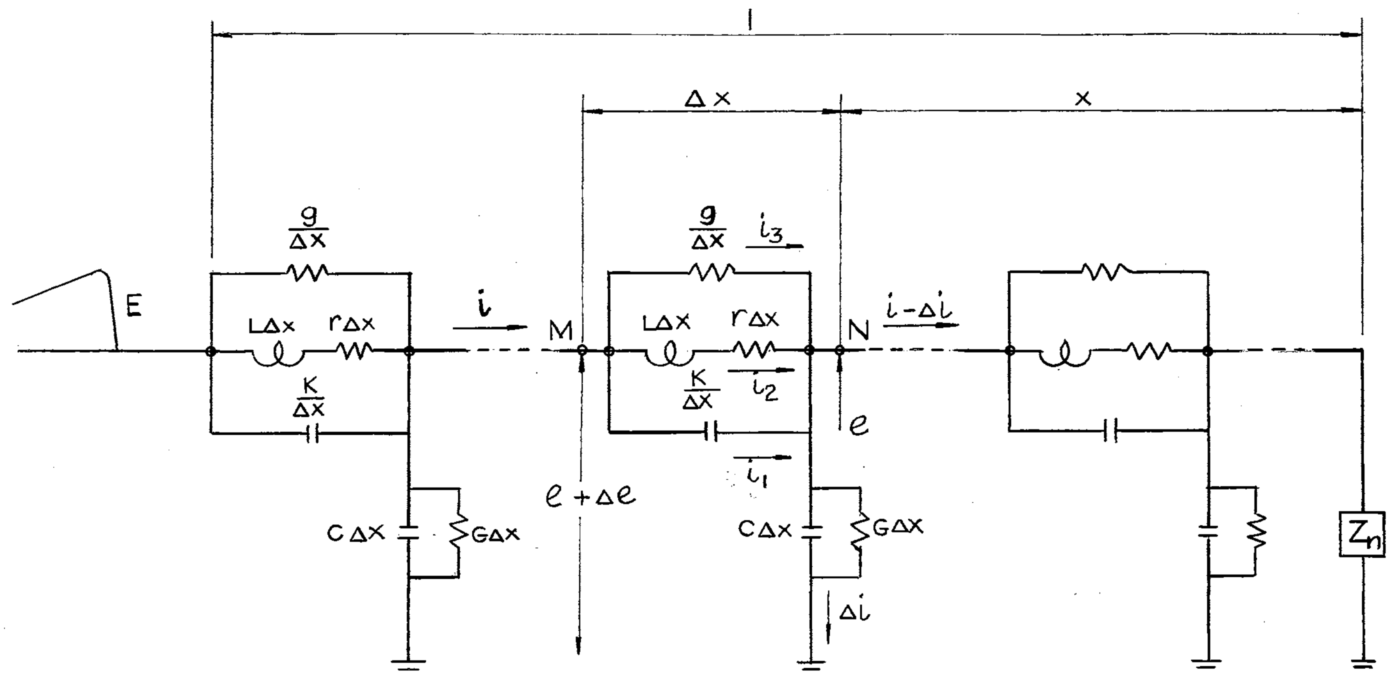


Figure C-1

Equivalent Circuit of Generator Phase Winding.

boundary conditions are therefore:

$$e(1, t) = E_1 \tag{10}$$

$$e(o, t) = o \tag{11}$$

The initial distribution of the surge voltage along the winding is:

$$e(x, o) = O \tag{12}$$

The final distribution of the surge voltage along the winding for a rectangular wave is:

$$e_s(x) = E_1 x$$

The solution of the differential equation (9) is composed of two parts:

$$e(x, t) = e_s(x) + e_t(x, t)$$

$e_s(x)$ is the final or steady state distribution of the surge voltage along the generator winding after the transient phenomena die out. $e_t(x,t)$ is the transient component whose boundary values are:

At the neutral point:

$$e_t(o, t) = e(o, t) - e_s(o) = o - o = o \tag{13}$$

At the generator terminal

$$e_t(1, t) = e(1, t) - e_s(1) = E_1 - E_1 = o \tag{14}$$

The initial value of e_t is:

$$e_t(x,o) = e(x,o) - e_s(x) = o - E_1 x = -E_1 x \qquad (15)$$

Due to the inertia effect of the generator winding inductance L, the initial rate of voltage variation is zero or:

$$\left.\frac{\partial e}{\partial t}\right]_{t=o} = 0 \qquad (16)$$

Assume a solution of $e_t(x,t)$ is of the form:

$$e_t(x,t) = X(x).T(t) \qquad (17)$$

replacing in equation (9) and dividing through by X.T

$$\frac{\ddot{X}}{X} = LC\frac{\ddot{T}}{T} = -k^2$$

This represents a set of two linear differential equations whose solutions are:

$$X(x) = C_1 \sin kx + C_2 \cos kx$$

and

$$T(t) = C_3 \sin\frac{k}{\sqrt{LC}}t + C_4 \cos\frac{k}{\sqrt{LC}}t$$

The transient voltage e_t is thus:

$$e_t(x,t) = X.T = (C_1 \sin kx + C_2 \cos kx)\left(C_3 \sin\frac{k}{\sqrt{LC}}t + C_4 \cos\frac{k}{\sqrt{LC}}t\right)$$

The constants C_1, C_2, C_3, and C_4, are determined by applying the boundary and initial conditions (13) through (16).

The boundary condition (13) gives:

$$e_t(o,t) = C_2\left(C_3 \sin\frac{k}{\sqrt{LC}}t + C_4 \cos\frac{k}{\sqrt{LC}}t\right) = o \qquad \therefore\ C_2 = o$$

The boundary condition (14) gives:

$$e_t(l,t) = C_1 \sin k\left(C_3 \sin\frac{k}{\sqrt{LC}}t + C_4 \cos\frac{k}{\sqrt{LC}}t\right)$$

in order that e_t has a non trivial solution, C_1 cannot be made zero, therefore

$$\sin k = 0 \quad ; \qquad k = n\pi \qquad n = 1, 2, 3 \ldots$$

and

$$e_t(x,t) = C_1 \sin n\pi x \left(C_3 \sin \frac{n\pi}{\sqrt{LC}} t + C_4 \cos \frac{n\pi}{\sqrt{LC}} t\right)$$

The initial condition (16) gives:

$$\left.\frac{\partial e_t}{\partial t}\right]_{t=0} = C_1 \sin n\pi x . \frac{C_3\sqrt{LC}}{n\pi} = 0$$

since $C_1 \neq 0$; $C_3 = 0$

and

$$e_t(x,t) = C_n \sin n\pi x \cos \frac{n\pi}{\sqrt{LC}} t \qquad (18)$$

The solution of $e_t(x,t)$ is many-valued depending on the value of n. It has been proved in Fourier analysis that if each value of the equation (18) for n = 1,2,3...is a solution of e_t, their sum is also a solution:

$$e_t(x,t) = \sum_{n=1}^{\infty} C_n \sin n\pi x \cos \frac{n\pi t}{\sqrt{LC}} \qquad (19)$$

The remaining initial condition (15) cannot satisfy (19) unless it is expressed itself in the Fourier series form. Expanding equation (15) into a half-range sine series:

$$e_t(x,0) = -E_1 x = \sum_{n=1}^{\infty} b_n \sin n\pi x \qquad (20)$$

where:

$$b_n = -2E_1 \int_0^1 x \sin n\pi x \, dx = 2E_1 \frac{\cos n\pi}{n\pi}$$

Equating (19) and (20) for time t = 0:

$$e_t(x,t) = \sum_{n=1}^{\infty} \frac{2E\cos n\pi}{n\pi} \sin n\pi x = \sum_{n=1}^{\infty} C_n \sin n\pi x$$

$$C_n = \frac{2E\cos n\pi}{n\pi}$$

The complete solution e(x,t) is:

$$e(x,t) = E_1 x + E_1 \sum_{n=1}^{\infty} A_n \sin n\pi x \cos \omega_n t \qquad (21)$$

$$A_n = \frac{2\cos n\pi}{n\pi} \quad ; \quad \omega_n = \frac{n\pi}{\sqrt{LC}}$$

Equation (21) is valid for a rectangular wave applied at the generator terminals. For another wave form E_1 (t), the Duhamel's superposition theorem should be used to find the new voltage function.

One form of the Duhamel theorem is:

$$e'(x,t) = E_1(t)\,\phi(0) + \int_0^t E_1(\tau)\frac{\partial}{\partial t}\phi(t-\tau)\,d\tau \qquad (22)$$

where $E_1(t)$ is the equation of the applied wave.

$\phi(t)$ is the voltage function resulted from a rectangular wave, i. e. e(x,t) of equation (21).

For the linear front and infinite tail wave contemplated in this study, the voltage response can be found by superposing two ramp functions of opposite sign and displaced by t=F.

The equation for the first ramp function is:

$$E_1(t) = \frac{E_1}{F}t$$

The equation for the second ramp function is:

$$E_2(t) = -\frac{E_1}{F}(t - F)U(t-F) = \frac{E_1}{F}(F - t)U(t-F).$$

By applying the Duhamel theorem (22), the following results are obtained:

$$e_{t<F}(x,t) = x\frac{E_1}{F}t + \frac{E_1}{F}\sum_{n=1}^{\infty}\frac{A_n}{\omega_n}\sin n\pi x \sin \omega_n t \qquad (23)$$

$$e_{t>F}(x,t) = xE_1 + E_1\sum_{n=1}^{\infty}A_n\left(\frac{\sin \omega_n \frac{F}{2}}{\omega_n \frac{F}{2}}\right)\sin n\pi x \cos \omega_n\left(t - \frac{F}{2}\right) \qquad (24)$$

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