MOMENTS OF DISTRIBUTIONS OF ROOTS OF QUADRATIC EQUATIONS WHOSE COEFFICIENTS FOLLOW THE BIVARIATE NORMAL

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PREFACE

The desire for more information concerning the roots of algebraic equations is never ending. For the special case of the quadratic equation, much information may be obtained without much difficulty. When the coefficients are real, the roots may be either real or complex conjugates.

The case under study here is the quadratic equation with real coefficients which follow the bivariate normal distribution. The purpose of this paper is to exhibit some of the moment values of the marginal probability density functions of the roots of this equation.

Indebtedness is acknowledged to my colleagues, Doyle McCown, Fred Turner, and Arthur E. Oldehoeft, for the aid derived from the use of their programs in the preparation of this paper.

I especially wish to thank Dr. John W. Hamblen, who served as my advisor and suggested this topic. Throughout the course of my graduate studies he was a constant inspiration and offered invaluable constructive criticism. His assistance and guidance are deeply appreciated and gratefully acknowledged.

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CHAPTER I

INTRODUCTION

Very little work has been done concerning the problem of finding the distributions of roots of algebraic equations whose coefficients are random variables.

Consider the polynomial equation

(1.1) $$z^n - a_1 z^{n-1} + a_2 z^{n-2} + \cdots + (-1)^n a_n = 0$$

where the a_p are complex random variables (i.e. the real and imaginary parts of the a_p have a given joint probability density function). The joint p.d.f. of the roots may be found since the a_p are functions of the z_p. Girshick [1], in his note in the Annals of Mathematical Statistics, exhibited this function, and, as an example, took the case where the u_p and the v_p are normally and independently distributed with zero means and the same variance σ^2. It appears that he was interested in finding ". . . the probability that one or more roots of the equation (1.1) will lie in a specified region of the complex plane."

In this paper we shall be concerned solely with the quadratic equation with real coefficients which follow the bivariate normal distribution. The quadratic written in the form

(1.2) $$\eta^2 - \xi_1 \eta + \xi_2 = 0$$

has roots η_1 and η_2 which are random variables and are either both real or are complex conjugates. Various equations necessary in the development of the joint p.d.f. of the roots of (1.2) will be given explicitly. Details leading to these equations have been omitted since they appear in an excerpt from Dr. Hamblen's Ph.D. thesis published in the Annals of Mathematical Statistics [2].

The primary purpose of this paper is to find the mean, variance, γ_1, and γ_2 for various marginal densities, $g_1(v_1|R)$ and $g_2(v_2|R)$, of the real roots of (1.2). In order to do this, values of $g_1(v_1|R)$ and $g_2(v_2|R)$ must first be calculated for several points. Then, using these values, we will be able to calculate values of the mean, variance, γ_1, and γ_2 for each of these functions by numerical methods.

Attempts will also be made to recognize any trends which might appear in γ_1 and γ_2 with respect to changes in values of the parameters of the joint p.d.f. of the coefficients of (1.2).

CHAPTER II

ESSENTIAL FORMULATION

In the quadratic equation of the form

$$\eta^2 - \xi_1 \eta + \xi_2 = 0 \tag{2.1}$$

where ξ_1 and ξ_2 are real random variables, we know that the roots η_1 and η_2 are random variables associated with ξ_1 and ξ_2 by the relationships

$$\begin{aligned} \eta_1 &= \frac{\xi_1}{2} + \sqrt{\frac{\xi_1^2}{4} - \xi_2} \\ \eta_2 &= \frac{\xi_1}{2} - \sqrt{\frac{\xi_1^2}{4} - \xi_2} \end{aligned} \tag{2.2}$$

and

$$\xi_1 = \eta_1 + \eta_2, \quad \xi_2 = \eta_1 \cdot \eta_2 \tag{2.3}$$

having $|J| = \left| \frac{\partial(\xi_1, \xi_2)}{\partial(\eta_1, \eta_2)} \right| = |\eta_1 - \eta_2| = (\eta_1 - \eta_2)$.

Here η_1 and η_2 are either both real or are complex conjugates.

We shall mean by the probability density function of a complex random variable the joint p.d.f. of the real and the imaginary parts of the complex variable. We see, therefore, that it is necessary to consider the real roots and the complex roots separately, since the joint p.d.f. of the real roots is of only two arguments while that of the complex roots will be of four. Hence it must be possible to partition the coefficient plane into two parts--one containing all points for which the roots are real, and the other containing all other points in the (ξ_1, ξ_2) plane.

It is easily seen that all points on the concave side of the parabola $\xi_2=\xi_1^2/4$ will yield complex roots, while all the remaining points in the plane give real roots.

Consider now the joint p.d.f., $f(x,y)$, of ξ_1 and ξ_2, where $f(x,y)$ is of the continuous type. By truncating along the parabola $\xi_2=\xi_1^2/4$, we obtain conditional p.d.f.'s relative to the hypotheses $\xi_2>\xi_1^2/4$ and $\xi_2\leq\xi_1^2/4$. If we let $P(R)=P(\xi_2\leq\xi_1^2/4)$ and $P(C)=P(\xi_2>\xi_1^2/4)$, then $P(R)$ and $P(C)$ are the probabilities of real and complex roots, respectively. They are given by

$$(2.4) \qquad P(R)=\iint_{y\leq x^2/4} f(x,y)\ dy\ dx,$$

and

$$(2.5) \qquad P(C)=\iint_{y>x^2/4} f(x,y)\ dy\ dx,$$

where $P(R)+P(C)=1$. Because of this relationship we consider only the conditional p.d.f. given by

$$(2.6) \qquad f(x,y|R)=f(x,y|y\leq x^2/4)=\frac{f(x,y)}{P(R)}$$

for all $y\leq x^2/4$ and $0<P(R)\leq 1$. If $P(R)=0$, then $f(x,y|R)\equiv 0$.

For $\xi_2\leq\xi_1^2/4$, the roots of (2.1) are real and have a joint p.d.f. which is uniquely determined by the joint p.d.f. of the coefficients ξ_1 and ξ_2. We will let $g(v_1,v_2|R)$ denote the joint p.d.f. of the real roots when $0<P(R)\leq 1$. The functions (2.2) and (2.3) satisfy the sufficient conditions for a change of variables in a continuous type density

function. Hence

$$(2.7) \qquad g(v_1, v_2 | R) = \frac{f(v_1+v_2, v_1 v_2)}{P(R)} \cdot |J|$$

for all $v_1 \geq v_2$ and $0 < P(R) \leq 1$, where $|J| = (v_1 - v_2)$. [3]

If now we let $g_1(v_1|R)$ and $g_2(v_2|R)$ be the marginal density functions of the real roots η_1 and η_2, we have

$$(2.8) \qquad g_1(v_1|R) = \int_{-\infty}^{v_1} g(v_1, v_2|R)dv_2, \text{ for all } v_1,$$

and

$$(2.9) \qquad g_2(v_2|R) = \int_{v_2}^{\infty} g(v_1, v_2|R)dv_1, \text{ for all } v_2.$$

We now consider the bivariate normal distribution which is given by

$$f(x,y) = \frac{1}{2\Pi\sigma_1\sigma_2\sqrt{1-\rho^2}} \exp\left\{-\frac{1}{2(1-\rho^2)}\left[\left(\frac{x-\mu_1}{\sigma_1}\right)^2 - 2\left(\frac{x-\mu_1}{\sigma_1}\right)\left(\frac{y-\mu_2}{\sigma_2}\right) + \left(\frac{y-\mu_2}{\sigma_2}\right)^2\right]\right\}$$

where $-\infty < x, y < \infty$.

Eliminating the details and writing down directly the equations defining $P(R)$, $g_1(v_1|R)$ and $g_2(v_2|R)$, we have

$$(2.10) \qquad P(R) = \int_{-\infty}^{\infty}\int_{-\infty}^{\theta(u)} (2\Pi)^{-1} \exp\left[-\tfrac{1}{2}(t^2+u^2)\right] \, dt \, du$$

where $\theta(u) = \left\{[(\sigma_1 u + \mu_1)^2 - 4\mu_2]/4\sigma_2 - \rho u\right\}/\sqrt{1-\rho^2}$

$$(2.11) \qquad g_1(v_1|R) = \int_{-\infty}^{v_1} \frac{(v_1 - v_2)}{2\Pi\sigma_1\sigma_2\sqrt{1-\rho^2}\cdot P(R)} \exp\left\{-\frac{1}{2(1-\rho^2)}\left[m_1^2(v_1)\cdot v_2^2 - 2m_1(v_1)\cdot m_2(v_1)\cdot v_2 + m_3(v_1)\right]\right\} dv_2$$

and

$$(2.12)\qquad g_2(v_2|R)=\int_{v_2}^{\infty}\frac{(v_1-v_2)}{2\Pi\sigma_1\sigma_2\sqrt{1-\rho^2}\cdot P(R)}\exp\left\{-\frac{1}{2(1-\rho^2)}\right.$$

$$\left.[m_1^2(v_2)\cdot v_1^2-2m_1(v_2)\cdot m_2(v_2)\cdot v_1+m_3(v_2)]\right\}dv_1$$

where $m_1^2(v_1)=(v_1/\sigma_2-\rho/\sigma_1)^2+(1-\rho^2)/\sigma_1^2$,

$$m_1(v_1)\cdot m_2(v_1)=\rho v_1^2/\sigma_1\sigma_2-(1/\sigma_1^2+\rho\mu_1/\sigma_1\sigma_2-\mu_2/\sigma_2^2)$$

$$+(\mu_1/\sigma_1^2-\rho\mu_2/\sigma_1\sigma_2),$$

and $m_3(v_1)=(v_1/\sigma_1-\mu_1/\sigma_1+\rho\mu_2/\sigma_2)^2+(1-\rho^2)\mu_2^2/\sigma_2^2$. [3]

Values of (2.10), which are given in Table I, were computed using the program written by Mr. McCown [4].

Table II contains values of the functions (2.11) and (2.12) for various combinations of the parameters μ_1, μ_2, σ_1, σ_2, and ρ. A program written by Mr. Turner [5] was used in computing these values.

Now to more completely describe the distributions of the roots of the quadratic equation for given values of the parameters μ_1, μ_2, σ_1, σ_2, and ρ, we calculate the mean, variance, γ_1, and γ_2 where, for a p.d.f. g(v), these are given by

$$(2.13)\qquad \mu_1=\int_a^b vg(v)dv \qquad\qquad \text{(Mean)}$$

$$(2.14)\qquad \mu_2=\int_a^b (v-\mu_1)^2 g(v)dv \qquad\qquad \text{(Variance)}$$

$$(2.15)\qquad \mu_3=\int_a^b (v-\mu_1)^3 g(v)dv \qquad\qquad \text{(Third Central Moment)}$$

$$(2.16)\qquad \mu_4=\int_a^b (v-\mu_1)^4 g(v)dv \qquad\qquad \text{(Fourth Central Moment)}$$

(2.17) $\gamma_1 = \frac{\mu_3}{\sigma^3}$ (Measure of Skewness)

(2.18) $\gamma_2 = \frac{\mu_4}{\sigma^4} - 3$ (Measure of Kurtosis)

Approximate values of the functions (2.13), (2.14), (2.17), and (2.18), for each function $g_1(v_1|R)$ and $g_2(v_2|R)$ listed in Table II, are given in Table III for reference under the headings of $\overline{v}$, σ_v^2, γ_1, and γ_2, respectively.

The program used in computing these values was written by Mr. Arthur E. Oldehoeft and is in the files of the Oklahoma State University Computing Center under the title of "Moments".

CHAPTER III

MOMENTS

In Table III are listed approximate values of μ, σ^2, γ_1, and γ_2 for each function given in Table II. Should more accuracy be desired, the program which computes g(v) may be rerun using a smaller increment for the abscissa, v. This then will give more values of the function g(v), which may be used in the program which computes values of the moments.

The values given for the moments are meant as aids in mentally picturing the functions and also aids in perhaps recognizing certain trends in γ_1 and γ_2 with respect to changes in the parameters of g(v).

On inspection of the functions $g_1(v_1|R)$ and $g_2(v_2|R)$ we find that for the set of parameters $\mu_1=\mu_2=0$ and $\sigma_1=\sigma_2=1$, a reflexive property exists. That is, g_1 for ρ is the mirror image of g_2 for $-\rho$. This symmetry is due to the fact that $g_1(v_1|R;\mu_1,\mu_2,\sigma_1,\sigma_2,\rho)=g_2(-v_2|R;\mu_1,\mu_2,\sigma_1,\sigma_2,-\rho)$ and $v_1=-v_2$ since $n(x,y;\mu_1,\mu_2,\sigma_1,\sigma_2,\rho)=n(-x,y;-\mu_1,\mu_2,\sigma_1,\sigma_2,-\rho)$.

In studying Table III, it is found that for the set of parameters $\mu_1=\mu_2=0$, and $\sigma_1=\sigma_2=1$, the symmetry, which is expected, is not easily recognized. This apparently is due to the accumulation of error in the calculations of g_1 and g_2 and to the fact that these functions were calculated for different sets of points. Also in these calculations no attempt was made to find the critical values of the functions g_1 and g_2. Thus, the fact that a relative maximum or minimum might appear at different points for g_1 and g_2 could easily account for the poor symmetry.

Symmetry is also evident between g_1 of section (e) and g_2 of section (f) of Table III. However, the same difficulty encountered before exists for g_2 of section (e) and g_1 of section (f). Even though these discrepancies exist, much information is available if reference is made to Table III.

In the cases considered, the moment values seem to indicate that the functions are well behaved when $|\rho| \leq .6$ and in most instances for all ρ.

After correcting the graphs of g_1 and g_2 given in Dr. Hamblen's dissertation on pages 178-189, it was noted that in the majority of the cases the curves appeared to be of Pearson's Type III, and in many instances close to the normal curve [3]. No special attempt was made, however, to classify the curves precisely. Should anyone desire more accurate classifications, the problem is not difficult, but should certainly be interesting.

Of course, much more work can be done towards exploring the properties of the distributions roots of algebraic equations with random coefficients. In particular, for the case of the quadratic, accurate and useful information can be easily obtained.

CHAPTER IV

COMPUTATIONS

Numerical methods of integration were employed for the calculations of the functions (2.10) through (2.18). The I.B.M. 650 Computer in the Computing Center of Oklahoma State University was used to carry out the computations which otherwise would have been virtually impossible. In all cases, Simpson's one-third rule for integration was employed and it proved very satisfactory for the problem at hand.

For P(R), evaluation was carried out between the limits of -4 and +4 with an increment of integration of one tenth, and the results are believed to be correct to six decimal places. Dr. John W. Hamblen [3], working with the Datatron 205 in the Statistical Laboratory at Purdue University, computed the same values using an increment of one eighth. In most instances, the two sets of answers agree to four decimals.

In the case of the functions (2.11) and (2.12), some discrepancies were noted when the results were compared with those computed by Dr. Hamblen [3]. Values which he computed for $g_1(v_1|R)$ and $g_2(v_2|R)$ yielded two modes in some cases, while all values listed in Table II for a particular g(v) define a one moded function. It is believed that these differences are due to the different methods used in calculating the functional values. Dr. Hamblen, in his calculations, used a fixed number of points, while the values in Table II were attained using a fixed increment for the variable. Since in the former method the increment would vary, the values in Table II are presumably more accurate.

For the functions (2.13), (2.14), (2.17), and (2.18), values of the argument, v, were entered as seven decimal numbers and values of the function, g(v), were entered as four decimal numbers. Hence the accuracy afforded is rather limited. Results for these functions were given as follows: μ_1 with nine decimals, σ^2 with seven decimals, μ_3 with six decimals, μ_4 with five decimals, and γ_1 and γ_2 with ten decimals. These figures were then rounded to the decimals given in Table III since they were believed no more accurate.

Several of these values were checked on a calculator and they agreed with the computed values to at least three decimals for μ and σ^2 and to at least two decimals for γ_1 and γ_2. However, this accuracy affords a good description of the functions $g_1(v_1|R)$ and $g_2(v_2|R)$ without the aid of graphs.

TABLE I

SOME VALUES OF P(R) WHEN f(x,y) IS THE BIVARIATE NORMAL

(a) $\mu_1=0,\ \mu_2=0,\ \sigma_1=1,\ \sigma_2=1$

ρ	P(R)
.9	.523745 806
.8	.545324 929
.6	.569821 131
.4	.581651 474
.2	.587320 772
0	.589026 320
-.2	.587320 772
-.4	.581651 474
-.6	.569821 131
-.8	.545324 929
-.9	.523745 806

(b) $\mu_1=3,\ \mu_2=10,\ \sigma_1=1,\ \sigma_2=2$

ρ	P(R)
.9	.000000 038
.8	.000000 145
.6	.000016 193
.4	.000287 593
.2	.001330 776
0	.003442 998
-.2	.006607 584
-.4	.010644 829
-.6	.015334 075
-.8	.020473 837
-.9	.023159 727

(c) $\mu_1=10,\ \mu_2=10,\ \sigma_1=1,\ \sigma_2=1$

ρ	P(R)
.9	.999927 077
.8	.999927 076
.6	.999925 869
.4	.999910 908
.2	.999856 954
0	.999739 182
-.2	.999537 933
-.4	.999238 792
-.6	.998831 233
-.8	.998308 239
-.9	.998001 111

TABLE I (Continued)

(d) $\mu_1=3$, $\mu_2=3$, $\sigma_1=1$, $\sigma_2=1$

ρ	P(R)
.9	.217763 433
.8	.259344 349
.6	.305742 576
.4	.330835 569
.2	.346779 452
0	.357913 221
-.2	.366174 368
-.4	.372566 449
-.6	.377666 756
-.8	.381832 584
-.9	.383642 231

(e) $\mu_1=10$, $\mu_2=3$, $\sigma_1=2$, $\sigma_2=1$

ρ	P(R)
.9	.999927 075
.8	.999926 236
.6	.999894 589
.4	.999776 183
.2	.999545 419
0	.999194 152
-.2	.998720 736
-.4	.998126 596
-.6	.997415 086
-.8	.996590 605
-.9	.996126 301

(f) $\mu_1=-10$, $\mu_2=3$, $\sigma_1=2$, $\sigma_2=1$

ρ	P(R)
.9	.996126 301
.8	.996590 605
.6	.997415 086
.4	.998126 596
.2	.998720 736
0	.999194 152
-.2	.999545 419
-.4	.999776 183
-.6	.999894 589
-.8	.999926 236
-.9	.999927 075

TABLE II

ORDINATES OF $g_1(v_1|R)$ AND $g_2(v_2|R)$ WHEN f(x,y) IS THE BIVARIATE NORMAL

1. $g_1(v_1|R)$

(a) $\mu_1=0$, $\mu_2=0$, $\sigma_1=1$, $\sigma_2=1$

v_1	g_1 ($\rho=+.9$)	g_1 ($\rho=+.8$)	g_1 ($\rho=+.6$)
-1.8	0.0000	0.0000	0.0000
-1.3	0.0000	0.0000	0.0000
-0.8	0.0000	0.0000	0.0004
-0.3	0.0051	0.0183	0.0448
0.2	0.4622	0.4738	0.4735
0.7	1.7276	1.3543	0.9865
1.2	0.0327	0.1815	0.3834
1.7	0.0035	0.0225	0.0906
2.2	0.0012	0.0046	0.0199
2.7	0.0003	0.0009	0.0038
3.2	0.0000	0.0001	0.0006
3.7	0.0000	0.0000	0.0001
4.2	0.0000	0.0000	0.0000
4.7	0.0000	0.0000	0.0000

v_1	g_1 ($\rho=+.4$)	g_1 ($\rho=+.2$)	g_1 ($\rho=0$)
-1.8	0.0000	0.0000	0.0000
-1.3	0.0000	0.0000	0.0001
-0.8	0.0017	0.0042	0.0075
-0.3	0.0679	0.0878	0.1043
0.2	0.4587	0.4367	0.4103
0.7	0.8029	0.6903	0.6133
1.2	0.4549	0.4767	0.4798
1.7	0.1585	0.2116	0.2508
2.2	0.0436	0.0704	0.0972
2.7	0.0097	0.0182	0.0289
3.2	0.0017	0.0037	0.0066
3.7	0.0002	0.0006	0.0012
4.2	0.0000	0.0001	0.0002
4.7	0.0000	0.0000	0.0000

TABLE II (Continued)

$\rho=-.2$

v_1	g_1
-1.8	0.0000
-1.3	0.0002
-0.8	0.0117
-0.3	0.1164
0.2	0.3809
0.7	0.5579
1.2	0.4765
1.7	0.2801
2.2	0.1225
2.7	0.0409
3.2	0.0105
3.7	0.0021
4.2	0.0003
4.7	0.0000

$\rho=-.4$

v_1	g_1
-1.8	0.0000
-1.3	0.0005
-0.8	0.0162
-0.3	0.1213
0.2	0.3489
0.7	0.5181
1.2	0.4723
1.7	0.3032
2.2	0.1462
2.7	0.0540
3.2	0.0154
3.7	0.0034
4.2	0.0006
4.7	0.0001

$\rho=-.6$

v_1	g_1
-1.8	0.0000
-1.3	0.0010
-0.8	0.0198
-0.3	0.1127
0.2	0.3144
0.7	0.4921
1.2	0.4712
1.7	0.3241
2.2	0.1693
2.7	0.0682
3.2	0.0213
3.7	0.0052
4.2	0.0010
4.7	0.0001

$\rho=-.8$

v_1	g_1
-1.8	0.0000
-1.3	0.0017
-0.8	0.0160
-0.3	0.0749
0.2	0.2769
0.7	0.4837
1.2	0.4804
1.7	0.3490
2.2	0.1949
2.7	0.0846
3.2	0.0286
3.7	0.0075
4.2	0.0015
4.7	0.0002

$\rho=-.9$

v_1	g_1
-1.8	0.0000
-1.3	0.0016
-0.8	0.0055
-0.3	0.0345
0.2	0.2569
0.7	0.4902
1.2	0.4940
1.7	0.3673
2.2	0.2113
2.7	0.0947
3.2	0.0333
3.7	0.0091
4.2	0.0019
4.7	0.0003

TABLE II (Continued)

(b) $\mu_1=3$, $\mu_2=10$, $\sigma_1=1$, $\sigma_2=2$

$\rho=+.4$

v_1	g_1
1.1	0.0020
1.5	0.0074
1.9	0.0275
2.3	0.0971
2.7	0.2796
3.1	0.5491
3.5	0.6699
3.9	0.5323
4.3	0.3074
4.7	0.1387
5.1	0.0507
5.5	0.0153
5.9	0.0038
6.3	0.0008
6.7	0.0001

$\rho=+.2$

v_1	g_1
1.1	0.0004
1.5	0.0030
1.9	0.0187
2.3	0.0915
2.7	0.2949
3.1	0.5553
3.5	0.6238
3.9	0.4785
4.3	0.2805
4.7	0.1326
5.1	0.0519
5.5	0.0170
5.9	0.0047
6.3	0.0011
6.7	0.0002

$\rho=0$

v_1	g_1
1.1	0.0001
1.5	0.0010
1.9	0.0110
2.3	0.0770
2.7	0.2883
3.1	0.5471
3.5	0.6035
3.9	0.4701
4.3	0.2880
4.7	0.1451
5.1	0.0613
5.5	0.0220
5.9	0.0067
6.3	0.0018
6.7	0.0004

$\rho=-.2$

v_1	g_1
1.1	0.0000
1.5	0.0003
1.9	0.0054
2.3	0.0599
2.7	0.2731
3.1	0.5322
3.5	0.5885
3.9	0.4716
4.3	0.3032
4.7	0.1625
5.1	0.0739
5.5	0.0288
5.9	0.0096
6.3	0.0028
6.7	0.0007

TABLE II (Continued)

$\rho=-.4$

v_1	g_1
1.1	0.0000
1.5	0.0000
1.9	0.0019
2.3	0.0425
2.7	0.2551
3.1	0.5148
3.5	0.5744
3.9	0.4746
4.3	0.3193
4.7	0.1812
5.1	0.0881
5.5	0.0369
5.9	0.0134
6.3	0.0042
6.7	0.0011

$\rho=-.6$

v_1	g_1
1.1	0.0000
1.5	0.0000
1.9	0.0003
2.3	0.0252
2.7	0.2375
3.1	0.4969
3.5	0.5606
3.9	0.4767
4.3	0.3342
4.7	0.1995
5.1	0.1028
5.5	0.0460
5.9	0.0179
6.3	0.0061
6.7	0.0018

$\rho=-.8$

v_1	g_1
1.1	0.0000
1.5	0.0000
1.9	0.0000
2.3	0.0087
2.7	0.2226
3.1	0.4789
3.5	0.5465
3.9	0.4770
4.3	0.3470
4.7	0.2166
5.1	0.1176
5.5	0.0558
5.9	0.0232
6.3	0.0084
6.7	0.0027

$\rho=-.9$

v_1	g_1
1.1	0.0000
1.5	0.0000
1.9	0.0000
2.3	0.0020
2.7	0.2178
3.1	0.4700
3.5	0.5395
3.9	0.4770
4.3	0.3526
4.7	0.2247
5.1	0.1249
5.5	0.0608
5.9	0.0260
6.3	0.0098
6.7	0.0032

TABLE II (Continued)

(c) $\mu_1=10$, $\mu_2=10$, $\sigma_1=1$, $\sigma_2=1$

v_1	g_1 ($\rho=+.9$)	g_1 ($\rho=+.8$)	g_1 ($\rho=+.6$)
3.2	0.0000	0.0000	0.0000
4.0	0.0000	0.0000	0.0001
4.8	0.0002	0.0003	0.0006
5.6	0.0030	0.0036	0.0049
6.4	0.0227	0.0249	0.0286
7.2	0.0976	0.1060	0.1105
8.0	0.2615	0.2643	0.2630
8.8	0.4192	0.3915	0.3726
9.6	0.3629	0.3230	0.3026
10.4	0.1589	0.1386	0.1337
11.2	0.0340	0.0305	0.0324
12.0	0.0036	0.0035	0.0045
12.8	0.0002	0.0002	0.0004
13.6	0.0000	0.0000	0.0000

v_1	g_1 ($\rho=+.4$)	g_1 ($\rho=+.2$)	g_1 ($\rho=0$)
3.2	0.0000	0.0000	0.0000
4.0	0.0001	0.0003	0.0004
4.8	0.0010	0.0015	0.0020
5.6	0.0064	0.0079	0.0095
6.4	0.0322	0.0356	0.0389
7.2	0.1141	0.1169	0.1196
8.0	0.2600	0.2564	0.2529
8.8	0.3624	0.3521	0.3448
9.6	0.2962	0.2906	0.2884
10.4	0.1366	0.1396	0.1441
11.2	0.0362	0.0393	0.0421
12.0	0.0055	0.0061	0.0067
12.8	0.0004	0.0005	0.0006
13.6	0.0000	0.0000	0.0000

TABLE II (Continued)

ρ=-.2

v_1	g_1
3.2	0.0000
4.0	0.0006
4.8	0.0026
5.6	0.0112
6.4	0.0418
7.2	0.1215
8.0	0.2498
8.8	0.3367
9.6	0.2832
10.4	0.1468
11.2	0.0439
12.0	0.0076
12.8	0.0008
13.6	0.0000

ρ=-.4

v_1	g_1
3.2	0.0000
4.0	0.0007
4.8	0.0033
5.6	0.0129
6.4	0.0448
7.2	0.1240
8.0	0.2467
8.8	0.3319
9.6	0.2799
10.4	0.1516
11.2	0.0455
12.0	0.0089
12.8	0.0009
13.6	0.0000

ρ=-.6

v_1	g_1
3.2	0.0000
4.0	0.0009
4.8	0.0040
5.6	0.0145
6.4	0.0475
7.2	0.1260
8.0	0.2453
8.8	0.3266
9.6	0.2719
10.4	0.1569
11.2	0.0469
12.0	0.0104
12.8	0.0010
13.6	0.0001

ρ=-.8

v_1	g_1
3.2	0.0000
4.0	0.0011
4.8	0.0047
5.6	0.0163
6.4	0.0488
7.2	0.1280
8.0	0.2545
8.8	0.3275
9.6	0.2443
10.4	0.1602
11.2	0.0514
12.0	0.0102
12.8	0.0011
13.6	0.0001

ρ=-.9

v_1	g_1
3.2	0.0000
4.0	0.0012
4.8	0.0048
5.6	0.0167
6.4	0.0461
7.2	0.1274
8.0	0.2725
8.8	0.3216
9.6	0.2171
10.4	0.1394
11.2	0.0570
12.0	0.0077
12.8	0.0011
13.6	0.0001

TABLE II (Continued)

(d) $\mu_1=3$, $\mu_2=3$, $\sigma_1=1$, $\sigma_2=1$

$\rho=+.9$

v_1	g_1
-0.2	0.0003
0.3	0.0056
0.8	0.0083
1.3	0.0144
1.8	0.1376
2.3	0.5547
2.8	0.5926
3.3	0.3923
3.8	0.1920
4.3	0.0721
4.8	0.0210
5.3	0.0047
5.8	0.0008
6.3	0.0001
6.8	0.0000

$\rho=+.8$

v_1	g_1
-0.2	0.0001
0.3	0.0027
0.8	0.0098
1.3	0.0359
1.8	0.2000
2.3	0.5282
2.8	0.5511
3.3	0.3753
3.8	0.1909
4.3	0.0747
4.8	0.0227
5.3	0.0054
5.8	0.0010
6.3	0.0001
6.8	0.0000

$\rho=+.6$

v_1	g_1
-0.2	0.0000
0.3	0.0010
0.8	0.0063
1.3	0.0407
1.8	0.2167
2.3	0.4928
2.8	0.5247
3.3	0.3808
3.8	0.2089
4.3	0.0889
4.8	0.0295
5.3	0.0076
5.8	0.0015
6.3	0.0002
6.8	0.0000

$\rho=+.4$

v_1	g_1
-0.2	0.0000
0.3	0.0004
0.8	0.0035
1.3	0.0334
1.8	0.2034
2.3	0.4638
2.8	0.5121
3.3	0.3939
3.8	0.2316
4.3	0.1061
4.8	0.0381
5.3	0.0107
5.8	0.0024
6.3	0.0004
6.8	0.0001

$\rho=+.2$

v_1	g_1
-0.2	0.0000
0.3	0.0001
0.8	0.0017
1.3	0.0250
1.8	0.1860
2.3	0.4384
2.8	0.5006
3.3	0.4048
3.8	0.2526
4.3	0.1238
4.8	0.0478
5.3	0.0145
5.8	0.0035
6.3	0.0007
6.8	0.0001

$\rho=0$

v_1	g_1
-0.2	0.0000
0.3	0.0000
0.8	0.0007
1.3	0.0174
1.8	0.1690
2.3	0.4163
2.8	0.4891
3.3	0.4125
3.8	0.2710
4.3	0.1409
4.8	0.0581
5.3	0.0189
5.8	0.0049
6.3	0.0010
6.8	0.0002

TABLE II (Continued)

ρ=-.2

v_1	g_1
-0.2	0.0000
0.3	0.0000
0.8	0.0002
1.3	0.0109
1.8	0.1532
2.3	0.3968
2.8	0.4774
3.3	0.4172
3.8	0.2865
4.3	0.1569
4.8	0.0685
5.3	0.0238
5.8	0.0066
6.3	0.0014
6.8	0.0002

ρ=-.4

v_1	g_1
-0.2	0.0000
0.3	0.0000
0.8	0.0000
1.3	0.0056
1.8	0.1386
2.3	0.3796
2.8	0.4661
3.3	0.4196
3.8	0.2996
4.3	0.1717
4.8	0.0791
5.3	0.0291
5.8	0.0085
6.3	0.0020
6.8	0.0004

ρ=-.6

v_1	g_1
-0.2	0.0000
0.3	0.0000
0.8	0.0000
1.3	0.0018
1.8	0.1252
2.3	0.3645
2.8	0.4550
3.3	0.4206
3.8	0.3104
4.3	0.1855
4.8	0.0895
5.3	0.0348
5.8	0.0108
6.3	0.0027
6.8	0.0005

ρ=-.8

v_1	g_1
-0.2	0.0002
0.3	0.0000
0.8	0.0000
1.3	0.0001
1.8	0.1131
2.3	0.3508
2.8	0.4442
3.3	0.4197
3.8	0.3193
4.3	0.1979
4.8	0.0997
5.3	0.0407
5.8	0.0135
6.3	0.0035
6.8	0.0007

ρ=-.9

v_1	g_1
-0.2	0.0000
0.3	0.0000
0.8	0.0000
1.3	0.0000
1.8	0.1084
2.3	0.3444
2.8	0.4390
3.3	0.4197
3.8	0.3251
4.3	0.2020
4.8	0.1022
5.3	0.0426
5.8	0.0150
6.3	0.0044
6.8	0.0009

TABLE II (Continued)

(e) $\mu_1=10$, $\mu_2=3$, $\sigma_1=2$, $\sigma_2=1$

v_1	g_1 ($\rho=+.9$)	g_1 ($\rho=+.8$)	g_1 ($\rho=+.6$)
1.0	0.0000	0.0000	0.0000
2.0	0.0001	0.0001	0.0001
3.0	0.0005	0.0006	0.0008
4.0	0.0028	0.0031	0.0036
5.0	0.0114	0.0120	0.0130
6.0	0.0339	0.0354	0.0366
7.0	0.0808	0.0810	0.0808
8.0	0.1554	0.1446	0.1398
9.0	0.2286	0.2015	0.1895
10.0	0.2476	0.2173	0.2015
11.0	0.1928	0.1804	0.1679
12.0	0.1072	0.1148	0.1095
13.0	0.0434	0.0559	0.0558
14.0	0.0133	0.0207	0.0222
15.0	0.0033	0.0059	0.0069
16.0	0.0007	0.0013	0.0017

v_1	g_1 ($\rho=+.4$)	g_1 ($\rho=+.2$)	g_1 ($\rho=0$)
1.0	0.0000	0.0000	0.0000
2.0	0.0002	0.0002	0.0002
3.0	0.0010	0.0013	0.0015
4.0	0.0042	0.0047	0.0053
5.0	0.0139	0.0149	0.0158
6.0	0.0377	0.0387	0.0397
7.0	0.0812	0.0815	0.0820
8.0	0.1385	0.1373	0.1365
9.0	0.1860	0.1834	0.1817
10.0	0.1967	0.1938	0.1922
11.0	0.1633	0.1609	0.1600
12.0	0.1061	0.1043	0.1036
13.0	0.0538	0.0524	0.0519
14.0	0.0212	0.0204	0.0203
15.0	0.0065	0.0061	0.0062
16.0	0.0015	0.0014	0.0015

TABLE II (Continued)

$\rho=-.2$

v_1	g_1
1.0	0.0000
2.0	0.0001
3.0	0.0017
4.0	0.0058
5.0	0.0166
6.0	0.0406
7.0	0.0822
8.0	0.1355
9.0	0.1795
10.0	0.1905
11.0	0.1587
12.0	0.1024
13.0	0.0518
14.0	0.0209
15.0	0.0068
16.0	0.0018

$\rho=-.4$

v_1	g_1
1.0	0.0000
2.0	0.0001
3.0	0.0019
4.0	0.0063
5.0	0.0175
6.0	0.0415
7.0	0.0826
8.0	0.1347
9.0	0.1781
10.0	0.1903
11.0	0.1574
12.0	0.1005
13.0	0.0521
14.0	0.0225
15.0	0.0078
16.0	0.0021

$\rho=-.6$

v_1	g_1
1.0	0.0000
2.0	0.0000
3.0	0.0021
4.0	0.0069
5.0	0.0182
6.0	0.0423
7.0	0.0829
8.0	0.1336
9.0	0.1759
10.0	0.1927
11.0	0.1539
12.0	0.0968
13.0	0.0546
14.0	0.0257
15.0	0.0089
16.0	0.0022

$\rho=-.8$

v_1	g_1
1.0	0.0000
2.0	0.0000
3.0	0.0023
4.0	0.0073
5.0	0.0190
6.0	0.0432
7.0	0.0816
8.0	0.1340
9.0	0.1693
10.0	0.2079
11.0	0.1390
12.0	0.0896
13.0	0.0626
14.0	0.0319
15.0	0.0083
16.0	0.0016

$\rho=-.9$

v_1	g_1
1.0	0.0000
2.0	0.0000
3.0	0.0023
4.0	0.0076
5.0	0.0191
6.0	0.0437
7.0	0.0740
8.0	0.1408
9.0	0.1523
10.0	0.2402
11.0	0.1204
12.0	0.0774
13.0	0.0769
14.0	0.0361
15.0	0.0045
16.0	0.0017

TABLE II (Continued)

(f) $\mu_1=-10$, $\mu_2=3$, $\sigma_1=2$, $\sigma_2=1$

$\rho=+.9$

v_1	g_1
-1.7	0.0051
-1.6	0.0069
-1.5	0.0096
-1.4	0.0135
-1.3	0.0193
-1.2	0.0282
-1.1	0.0426
-1.0	0.0666
-0.9	0.1076
-0.8	0.1812
-0.7	0.3138
-0.6	0.5598
-0.5	0.9955
-0.4	1.6854
-0.3	2.4255
-0.2	2.4310
-0.1	1.0277
0.0	0.0683
0.1	0.0000
0.2	0.0000
0.3	0.0000

$\rho=+.8$

v_1	g_1
-1.7	0.0043
-1.6	0.0060
-1.5	0.0084
-1.4	0.0118
-1.3	0.0170
-1.2	0.0252
-1.1	0.0384
-1.0	0.0608
-0.9	0.0998
-0.8	0.1705
-0.7	0.3017
-0.6	0.5494
-0.5	0.9996
-0.4	1.7172
-0.3	2.4836
-0.2	2.4273
-0.1	0.9951
0.0	0.0658
0.1	0.0001
0.2	0.0000
0.3	0.0000

$\rho=+.6$

v_1	g_1
-1.7	0.0030
-1.6	0.0043
-1.5	0.0060
-1.4	0.0087
-1.3	0.0128
-1.2	0.0192
-1.1	0.0300
-1.0	0.0489
-0.9	0.0831
-0.8	0.1478
-0.7	0.2739
-0.6	0.5256
-0.5	1.0048
-0.4	1.7985
-0.3	2.6179
-0.2	2.4450
-0.1	0.8983
0.0	0.0603
0.1	0.0003
0.2	0.0000
0.3	0.0000

$\rho=+.4$

v_1	g_1
-1.7	0.0019
-1.6	0.0028
-1.5	0.0041
-1.4	0.0059
-1.3	0.0089
-1.2	0.0137
-1.1	0.0220
-1.0	0.0370
-0.9	0.0655
-0.8	0.1224
-0.7	0.2410
-0.6	0.4900
-0.5	1.0001
-0.4	1.8835
-0.3	2.7951
-0.2	2.4567
-0.1	0.7968
0.0	0.0551
0.1	0.0005
0.2	0.0000
0.3	0.0000

$\rho=+.2$

v_1	g_1
-1.7	0.0011
-1.6	0.0016
-1.5	0.0025
-1.4	0.0037
-1.3	0.0056
-1.2	0.0089
-1.1	0.0147
-1.0	0.0257
-0.9	0.0477
-0.8	0.0945
-0.7	0.2004
-0.6	0.4444
-0.5	0.9876
-0.4	1.9895
-0.3	2.9995
-0.2	2.4262
-0.1	0.7016
0.0	0.0497
0.1	0.0010
0.2	0.0000
0.3	0.0000

$\rho=0$

v_1	g_1
-1.7	0.0005
-1.6	0.0008
-1.5	0.0013
-1.4	0.0019
-1.3	0.0031
-1.2	0.0050
-1.1	0.0086
-1.0	0.0156
-0.9	0.0307
-0.8	0.0657
-0.7	0.1529
-0.6	0.3801
-0.5	0.9519
-0.4	2.1071
-0.3	3.2358
-0.2	2.3994
-0.1	0.5934
0.0	0.0444
0.1	0.0015
0.2	0.0000
0.3	0.0000

TABLE II (Continued)

$\rho=-.2$

v_1	g_1
-1.7	0.0002
-1.6	0.0003
-1.5	0.0005
-1.4	0.0008
-1.3	0.0013
-1.2	0.0022
-1.1	0.0040
-1.0	0.0077
-0.9	0.0162
-0.8	0.0380
-0.7	0.1007
-0.6	0.2943
-0.5	0.8805
-0.4	2.2384
-0.3	3.5689
-0.2	2.3297
-0.1	0.4868
0.0	0.0390
0.1	0.0021
0.2	0.0001
0.3	0.0000

$\rho=-.4$

v_1	g_1
-1.7	0.0000
-1.6	0.0001
-1.5	0.0001
-1.4	0.0002
-1.3	0.0004
-1.2	0.0007
-1.1	0.0013
-1.0	0.0026
-0.9	0.0058
-0.8	0.0156
-0.7	0.0495
-0.6	0.1855
-0.5	0.7357
-0.4	2.3934
-0.3	3.9771
-0.2	2.1954
-0.1	0.3790
0.0	0.0337
0.1	0.0028
0.2	0.0003
0.3	0.0000

$\rho=-.6$

v_1	g_1
-1.7	0.0000
-1.6	0.0000
-1.5	0.0000
-1.4	0.0000
-1.3	0.0000
-1.2	0.0001
-1.1	0.0002
-1.0	0.0004
-0.9	0.0009
-0.8	0.0029
-0.7	0.0121
-0.6	0.0689
-0.5	0.4753
-0.4	2.5102
-0.3	4.6586
-0.2	1.9657
-0.1	0.2772
0.0	0.0284
0.1	0.0034
0.2	0.0006
0.3	0.0001

$\rho=-.8$

v_1	g_1
-1.7	0.0000
-1.6	0.0000
-1.5	0.0000
-1.4	0.0000
-1.3	0.0000
-1.2	0.0000
-1.1	0.0000
-1.0	0.0000
-0.9	0.0000
-0.8	0.0000
-0.7	0.0003
-0.6	0.0032
-0.5	0.0994
-0.4	2.2959
-0.3	5.8071
-0.2	1.5592
-0.1	0.1834
0.0	0.0231
0.1	0.0040
0.2	0.0009
0.3	0.0003

$\rho=-.9$

v_1	g_1
-1.7	0.0000
-1.6	0.0000
-1.5	0.0000
-1.4	0.0000
-1.3	0.0000
-1.2	0.0000
-1.1	0.0000
-1.0	0.0000
-0.9	0.0000
-0.8	0.0000
-0.7	0.0000
-0.6	0.0000
-0.5	0.0031
-0.4	1.5308
-0.3	6.8494
-0.2	1.2713
-0.1	0.1403
0.0	0.0204
0.1	0.0042
0.2	0.0012
0.3	0.0004

TABLE II (Continued)

2. $g_2(v_1|R)$

(a) $\mu_1=0$, $\mu_2=0$, $\sigma_1=1$, $\sigma_2=1$

$\rho=+.9$

v_2	g_2
-4.6	0.0005
-4.1	0.0027
-3.6	0.0120
-3.1	0.0418
-2.6	0.1135
-2.1	0.2407
-1.6	0.3981
-1.1	0.5069
-0.6	0.4647
-0.1	0.1908
0.4	0.0218
0.9	0.0044
1.4	0.0010
1.9	0.0000

$\rho=+.8$

v_2	g_2
-4.6	0.0004
-4.1	0.0022
-3.6	0.0100
-3.1	0.0362
-2.6	0.1019
-2.1	0.2235
-1.6	0.3802
-1.1	0.4949
-0.6	0.4594
-0.1	0.2236
0.4	0.0555
0.9	0.0113
1.4	0.0009
1.9	0.0000

$\rho=+.6$

v_2	g_2
-4.6	0.0002
-4.1	0.0014
-3.6	0.0070
-3.1	0.0274
-2.6	0.0835
-2.1	0.1969
-1.6	0.3573
-1.1	0.4898
-0.6	0.4708
-0.1	0.2681
0.4	0.0852
0.9	0.0123
1.4	0.0004
1.9	0.0000

$\rho=+.4$

v_2	g_2
-4.6	0.0001
-4.1	0.0008
-3.6	0.0047
-3.1	0.0202
-2.6	0.0673
-2.1	0.1729
-1.6	0.3393
-1.1	0.4965
-0.6	0.5002
-0.1	0.2999
0.4	0.0886
0.9	0.0093
1.4	0.0002
1.9	0.0000

$\rho=+.2$

v_2	g_2
-4.6	0.0001
-4.1	0.0005
-3.6	0.0030
-3.1	0.0141
-2.6	0.0520
-2.1	0.1477
-1.6	0.3194
-1.1	0.5087
-0.6	0.5436
-0.1	0.3235
0.4	0.0813
0.9	0.0062
1.4	0.0001
1.9	0.0000

$\rho=0$

v_2	g_2
-4.6	0.0000
-4.1	0.0002
-3.6	0.0017
-3.1	0.0091
-2.6	0.0376
-2.1	0.1201
-1.6	0.2930
-1.1	0.5233
-0.6	0.6024
-0.1	0.3399
0.4	0.0691
0.9	0.0036
1.4	0.0000
1.9	0.0000

TABLE II (Continued)

$\rho=-.2$

v_2	g_2
-4.6	0.0001
-4.1	0.0001
-3.6	0.0009
-3.1	0.0052
-2.6	0.0244
-2.1	0.0896
-1.6	0.2553
-1.1	0.5364
-0.6	0.6833
-0.1	0.3486
0.4	0.0546
0.9	0.0018
1.4	0.0000
1.9	0.0000

$\rho=-.4$

v_2	g_2
-4.6	0.0000
-4.1	0.0000
-3.6	0.0004
-3.1	0.0025
-2.6	0.0133
-2.1	0.0574
-1.6	0.2000
-1.1	0.5386
-0.6	0.8009
-0.1	0.3480
0.4	0.0388
0.9	0.0006
1.4	0.0000
1.9	0.0000

$\rho=-.6$

v_2	g_2
-4.6	0.0000
-4.1	0.0000
-3.6	0.0001
-3.1	0.0009
-2.6	0.0054
-2.1	0.0272
-1.6	0.1216
-1.1	0.4997
-0.6	0.9928
-0.1	0.3347
0.4	0.0221
0.9	0.0001
1.4	0.0000
1.9	0.0000

$\rho=-.8$

v_2	g_2
-4.6	0.0000
-4.1	0.0000
-3.6	0.0000
-3.1	0.0002
-2.6	0.0012
-2.1	0.0062
-1.6	0.0320
-1.1	0.3036
-0.6	1.3869
-0.1	0.2991
0.4	0.0063
0.9	0.0000
1.4	0.0000
1.9	0.0000

$\rho=-.9$

v_2	g_2
-4.6	0.0000
-4.1	0.0000
-3.6	0.0000
-3.1	0.0001
-2.6	0.0004
-2.1	0.0015
-1.6	0.0045
-1.1	0.0785
-0.6	1.8207
-0.1	0.2623
0.4	0.0009
0.9	0.0000
1.4	0.0000
1.9	0.0000

TABLE II (Continued)

(b) $\mu_1=3$, $\mu_2=10$, $\sigma_1=1$, $\sigma_2=2$

$\rho=+.4$

v_2	g_2
0.0	0.0027
0.3	0.0071
0.6	0.0210
0.9	0.0644
1.2	0.1813
1.5	0.4160
1.8	0.7116
2.1	0.8666
2.4	0.7307
2.7	0.4089
3.0	0.1390
3.3	0.0248
3.6	0.0019
3.9	0.0000

$\rho=+.2$

v_2	g_2
0.0	0.0011
0.3	0.0057
0.6	0.0271
0.9	0.1047
1.2	0.2979
1.5	0.5912
1.8	0.8160
2.1	0.7832
2.4	0.5123
2.7	0.2125
3.0	0.0489
3.3	0.0052
3.6	0.0002
3.9	0.0000

$\rho=0$

v_2	g_2
0.0	0.0006
0.3	0.0058
0.6	0.0370
0.9	0.1539
1.2	0.4074
1.5	0.7064
1.8	0.8393
2.1	0.6912
2.4	0.3791
2.7	0.1227
3.0	0.0193
3.3	0.0012
3.6	0.0000
3.9	0.0000

$\rho=-.2$

v_2	g_2
0.0	0.0004
0.3	0.0065
0.6	0.0504
0.9	0.2081
1.2	0.5020
1.5	0.7760
1.8	0.8268
2.1	0.6110
2.4	0.2889
2.7	0.0722
3.0	0.0072
3.3	0.0002
3.6	0.0000
3.9	0.0000

TABLE II (Continued)

$\rho=-.4$

v_2	g_2
0.0	0.0003
0.3	0.0076
0.6	0.0668
0.9	0.2643
1.2	0.5799
1.5	0.8157
1.8	0.7998
2.1	0.5440
2.4	0.2223
2.7	0.0398
3.0	0.0020
3.3	0.0000
3.6	0.0000
3.9	0.0000

$\rho=-.6$

v_2	g_2
0.0	0.0002
0.3	0.0089
0.6	0.0864
0.9	0.3208
1.2	0.6421
1.5	0.8355
1.8	0.7693
2.1	0.4895
2.4	0.1691
2.7	0.0178
3.0	0.0003
3.3	0.0000
3.6	0.0000
3.9	0.0000

$\rho=-.8$

v_2	g_2
0.0	0.0033
0.3	0.0098
0.6	0.1088
0.9	0.3748
1.2	0.6917
1.5	0.8416
1.8	0.7370
2.1	0.4466
2.4	0.1224
2.7	0.0037
3.0	0.0000
3.3	0.0000
3.6	0.0000
3.9	0.0000

$\rho=-.9$

v_2	g_2
0.0	0.0000
0.3	0.0097
0.6	0.1210
0.9	0.3999
1.2	0.7114
1.5	0.8411
1.8	0.7214
2.1	0.4289
2.4	0.0988
2.7	0.0004
3.0	0.0000
3.3	0.0000
3.6	0.0000
3.9	0.0000

TABLE II (Continued)

(c) μ_1=10, μ_2=10, σ_1=1, σ_2=1

ρ=+.9

v_2	g_2
0.8	0.0000
0.9	0.0014
1.0	0.6234
1.1	6.0147
1.2	3.0236
1.3	0.3421
1.4	0.0259
1.5	0.0022
1.6	0.0003
1.7	0.0000
1.8	0.0000
1.9	0.0000
2.0	0.0000

ρ=+.8

v_2	g_2
0.8	0.0003
0.9	0.0680
1.0	1.5298
1.1	4.4963
1.2	2.9385
1.3	0.7926
1.4	0.1437
1.5	0.0239
1.6	0.0043
1.7	0.0009
1.8	0.0002
1.9	0.0001
2.0	0.0000

ρ=+.6

v_2	g_2
0.8	0.0263
0.9	0.4435
1.0	2.0297
1.1	3.2709
1.2	2.4688
1.3	1.1585
1.4	0.4112
1.5	0.1294
1.6	0.0396
1.7	0.0127
1.8	0.0044
1.9	0.0016
2.0	0.0007

ρ=+.4

v_2	g_2
0.2	0.0000
0.5	0.0000
0.8	0.1161
1.1	2.6841
1.4	0.5795
1.7	0.0399
2.0	0.0035
2.3	0.0005
2.6	0.0001
2.9	0.0000
3.2	0.0000
3.5	0.0000
3.8	0.0000

ρ=+.2

v_2	g_2
0.2	0.0000
0.5	0.0000
0.8	0.2378
1.1	2.3368
1.4	0.6686
1.7	0.0749
2.0	0.0092
2.3	0.0016
2.6	0.0004
2.9	0.0001
3.2	0.0000
3.5	0.0000
3.8	0.0000

ρ=0

v_2	g_2
0.2	0.0000
0.5	0.0001
0.8	0.3606
1.1	2.1007
1.4	0.7160
1.7	0.1108
2.0	0.0176
2.3	0.0036
2.6	0.0010
2.9	0.0003
3.2	0.0001
3.5	0.0000
3.8	0.0000

TABLE II (Continued)

$\rho=-.2$

v_2	g_2
0.2	0.0000
0.5	0.0003
0.8	0.4711
1.1	1.9226
1.4	0.7400
1.7	0.1439
2.0	0.0277
2.3	0.0066
2.6	0.0020
2.9	0.0007
3.2	0.0002
3.5	0.0000
3.8	0.0000

$\rho=-.4$

v_2	g_2
0.2	0.0000
0.5	0.0008
0.8	0.5641
1.1	1.7794
1.4	0.7465
1.7	0.1728
2.0	0.0388
2.3	0.0103
2.6	0.0033
2.9	0.0012
3.2	0.0004
3.5	0.0000
3.8	0.0000

$\rho=-.6$

v_2	g_2
0.2	0.0000
0.5	0.0020
0.8	0.6335
1.1	1.6727
1.4	0.7489
1.7	0.1980
2.0	0.0501
2.3	0.0146
2.6	0.0050
2.9	0.0019
3.2	0.0007
3.5	0.0001
3.8	0.0000

$\rho=-.8$

v_2	g_2
0.2	0.0000
0.5	0.0041
0.8	0.6943
1.1	1.5737
1.4	0.7446
1.7	0.2189
2.0	0.0613
2.3	0.0193
2.6	0.0070
2.9	0.0028
3.2	0.0010
3.5	0.0001
3.8	0.0000

$\rho=-.9$

v_2	g_2
0.2	0.0000
0.5	0.0055
0.8	0.7212
1.1	1.5359
1.4	0.7441
1.7	0.2271
2.0	0.0668
2.3	0.0217
2.6	0.0081
2.9	0.0033
3.2	0.0013
3.5	0.0002
3.8	0.0000

TABLE II (Continued)

(d) $\mu_1=3$, $\mu_2=3$, $\sigma_1=1$, $\sigma_2=1$

v_2	g_2 ($\rho=+.9$)	g_2 ($\rho=+.8$)	g_2 ($\rho=+.6$)
-0.6	0.0051	0.0035	0.0018
-0.3	0.0072	0.0063	0.0051
0.0	0.0074	0.0111	0.0177
0.3	0.0076	0.0270	0.0870
0.6	0.0178	0.1263	0.4420
0.9	0.2063	0.7308	1.0974
1.2	1.4847	1.3756	1.0400
1.5	1.2442	0.8115	0.5002
1.8	0.3384	0.2180	0.1286
2.1	0.0221	0.0193	0.0126
2.4	0.0001	0.0002	0.0002
2.7	0.0000	0.0000	0.0000

v_2	g_2 ($\rho=+.4$)	g_2 ($\rho=+.2$)	g_2 ($\rho=0$)
-0.6	0.0009	0.0004	0.0001
-0.3	0.0040	0.0028	0.0018
0.0	0.0242	0.0307	0.0371
0.3	0.1672	0.2590	0.3552
0.6	0.6964	0.8654	0.9722
0.9	1.1276	1.0874	1.0341
1.2	0.8445	0.7212	0.6352
1.5	0.3712	0.2972	0.2473
1.8	0.0891	0.0650	0.0478
2.1	0.0080	0.0049	0.0027
2.4	0.0001	0.0001	0.0000
2.7	0.0000	0.0000	0.0000

TABLE II (Continued)

$\rho=-.2$

v_2	g_2
-0.6	0.0000
-0.3	0.0010
0.0	0.0435
0.3	0.4511
0.6	1.0391
0.9	0.9819
1.2	0.5719
1.5	0.2101
1.8	0.0341
2.1	0.0012
2.4	0.0000
2.7	0.0000

$\rho=-.4$

v_2	g_2
-0.6	0.0000
-0.3	0.0004
0.0	0.0498
0.3	0.5432
0.6	1.0794
0.9	0.9340
1.2	0.5232
1.5	0.1804
1.8	0.0226
2.1	0.0004
2.4	0.0000
2.7	0.0000

$\rho=-.6$

v_2	g_2
-0.6	0.0000
-0.3	0.0001
0.0	0.0557
0.3	0.6312
0.6	1.1018
0.9	0.8921
1.2	0.4850
1.5	0.1553
1.8	0.0125
2.1	0.0001
2.4	0.0000
2.7	0.0000

$\rho=-.8$

v_2	g_2
-0.6	0.0000
-0.3	0.0000
0.0	0.0619
0.3	0.7117
0.6	1.1133
0.9	0.8537
1.2	0.4546
1.5	0.1335
1.8	0.0038
2.1	0.0000
2.4	0.0000
2.7	0.0000

$\rho=-.9$

v_2	g_2
-0.6	0.0000
-0.3	0.0000
0.0	0.0653
0.3	0.7501
0.6	1.1147
0.9	0.8363
1.2	0.4415
1.5	0.1243
1.8	0.0007
2.1	0.0000
2.4	0.0000
2.7	0.0000

TABLE II (Continued)

(e) $\mu_1=10$, $\mu_2=3$, $\sigma_1=2$, $\sigma_2=1$

$\rho=+.9$ v_2	g_2	$\rho=+.8$ v_2	g_2	$\rho=+.6$ v_2	g_2
-0.4	0.0002	-0.4	0.0001	-0.4	0.0000
-0.3	0.0004	-0.3	0.0003	-0.3	0.0001
-0.2	0.0012	-0.2	0.0009	-0.2	0.0006
-0.1	0.0042	-0.1	0.0040	-0.1	0.0034
0.0	0.0204	0.0	0.0231	0.0	0.0284
0.1	0.1403	0.1	0.1834	0.1	0.2772
0.2	1.2713	0.2	1.5592	0.2	1.9649
0.3	6.8490	0.3	5.8039	0.3	4.6532
0.4	1.5256	0.4	2.2932	0.4	2.5094
0.5	0.0031	0.5	0.0994	0.5	0.4753
0.6	0.0000	0.6	0.0032	0.6	0.0689
0.7	0.0000	0.7	0.0003	0.7	0.0121
0.8	0.0000	0.8	0.0000	0.8	0.0029
0.9	0.0000	0.9	0.0000	0.9	0.0009
1.0	0.0000	1.0	0.0000	1.0	0.0004
1.1	0.0000	1.1	0.0000	1.1	0.0002
1.2	0.0000	1.2	0.0000	1.2	0.0001
1.3	0.0000	1.3	0.0000	1.3	0.0000
1.4	0.0000	1.4	0.0000	1.4	0.0000
1.5	0.0000	1.5	0.0000	1.5	0.0000
1.6	0.0000	1.6	0.0000	1.6	0.0000
1.7	0.0000	1.7	0.0000	1.7	0.0000
1.8	0.0000	1.8	0.0000	1.8	0.0000
1.9	0.0000	1.9	0.0000	1.9	0.0000
2.0	0.0000	2.0	0.0000	2.0	0.0000
2.1	0.0000	2.1	0.0000	2.1	0.0000
2.2	0.0000	2.2	0.0000	2.2	0.0000
2.3	0.0000	2.3	0.0000	2.3	0.0000

TABLE II (Continued)

	ρ=+.4		ρ=+.2		ρ=0
v_2	g_2	v_2	g_2	v_2	g_2
-0.4	0.0000	-0.4	0.0000	-0.4	0.0000
-0.3	0.0000	-0.3	0.0000	-0.3	0.0000
-0.2	0.0003	-0.2	0.0001	-0.2	0.0000
-0.1	0.0028	-0.1	0.0021	-0.1	0.0015
0.0	0.0337	0.0	0.0390	0.0	0.0443
0.1	0.3789	0.1	0.4860	0.1	0.5908
0.2	2.1919	0.2	2.3239	0.2	2.3935
0.3	3.9733	0.3	3.5670	0.3	3.2351
0.4	2.3931	0.4	2.2383	0.4	2.1071
0.5	0.7357	0.5	0.8805	0.5	0.9519
0.6	0.1855	0.6	0.2943	0.6	0.3801
0.7	0.0495	0.7	0.1007	0.7	0.1529
0.8	0.0156	0.8	0.0380	0.8	0.0657
0.9	0.0058	0.9	0.0162	0.9	0.0307
1.0	0.0026	1.0	0.0077	1.0	0.0156
1.1	0.0013	1.1	0.0040	1.1	0.0086
1.2	0.0007	1.2	0.0022	1.2	0.0050
1.3	0.0004	1.3	0.0013	1.3	0.0031
1.4	0.0002	1.4	0.0008	1.4	0.0019
1.5	0.0001	1.5	0.0005	1.5	0.0013
1.6	0.0001	1.6	0.0003	1.6	0.0008
1.7	0.0000	1.7	0.0002	1.7	0.0005
1.8	0.0000	1.8	0.0001	1.8	0.0003
1.9	0.0000	1.9	0.0001	1.9	0.0002
2.0	0.0000	2.0	0.0000	2.0	0.0001
2.1	0.0000	2.1	0.0000	2.1	0.0000
2.2	0.0000	2.2	0.0000	2.2	0.0000
2.3	0.0000	2.3	0.0000	2.3	0.0000

TABLE II (Continued)

$\rho=-.2$ v_2	g_2	$\rho=-.4$ v_2	g_2	$\rho=-.6$ v_2	g_2
-0.4	0.0000	-0.4	0.0000	-0.4	0.0000
-0.3	0.0000	-0.3	0.0000	-0.3	0.0000
-0.2	0.0000	-0.2	0.0000	-0.2	0.0000
-0.1	0.0010	-0.1	0.0005	-0.1	0.0002
0.0	0.0492	0.0	0.0533	0.0	0.0556
0.1	0.6965	0.1	0.7901	0.1	0.8929
0.2	2.4224	0.2	2.4553	0.2	2.4449
0.3	2.9994	0.3	2.7951	0.3	2.6179
0.4	1.9895	0.4	1.8835	0.4	1.7985
0.5	0.9876	0.5	1.0001	0.5	1.0048
0.6	0.4444	0.6	0.4900	0.6	0.5256
0.7	0.2004	0.7	0.2410	0.7	0.2739
0.8	0.0945	0.8	0.1224	0.8	0.1478
0.9	0.0477	0.9	0.0655	0.9	0.0831
1.0	0.0257	1.0	0.0370	1.0	0.0489
1.1	0.0147	1.1	0.0220	1.1	0.0300
1.2	0.0089	1.2	0.0137	1.2	0.0192
1.3	0.0056	1.3	0.0089	1.3	0.0128
1.4	0.0037	1.4	0.0059	1.4	0.0087
1.5	0.0025	1.5	0.0041	1.5	0.0060
1.6	0.0016	1.6	0.0028	1.6	0.0043
1.7	0.0011	1.7	0.0019	1.7	0.0030
1.8	0.0007	1.8	0.0013	1.8	0.0021
1.9	0.0004	1.9	0.0009	1.9	0.0014
2.0	0.0002	2.0	0.0005	2.0	0.0009
2.1	0.0001	2.1	0.0003	2.1	0.0005
2.2	0.0001	2.2	0.0001	2.2	0.0003
2.3	0.0000	2.3	0.0001	2.3	0.0001

TABLE II (Continued)

$\rho=-.8$		$\rho=-.9$	
v_2	g_2	v_2	g_2
-0.4	0.0000	-0.4	0.0000
-0.3	0.0000	-0.3	0.0000
-0.2	0.0000	-0.2	0.0000
-0.1	0.0000	-0.1	0.0000
0.0	0.0543	0.0	0.0504
0.1	0.9941	0.1	1.0277
0.2	2.4273	0.2	2.4310
0.3	2.4836	0.3	2.4255
0.4	1.7172	0.4	1.6854
0.5	0.9996	0.5	0.9955
0.6	0.5494	0.6	0.5598
0.7	0.3017	0.7	0.3138
0.8	0.1705	0.8	0.1812
0.9	0.0998	0.9	0.1076
1.0	0.0608	1.0	0.0666
1.1	0.0384	1.1	0.0426
1.2	0.0252	1.2	0.0282
1.3	0.0170	1.3	0.0193
1.4	0.0118	1.4	0.0135
1.5	0.0084	1.5	0.0096
1.6	0.0060	1.6	0.0069
1.7	0.0043	1.7	0.0051
1.8	0.0031	1.8	0.0037
1.9	0.0022	1.9	0.0026
2.0	0.0015	2.0	0.0018
2.1	0.0009	2.1	0.0012
2.2	0.0005	2.2	0.0006
2.3	0.0002	2.3	0.0002

TABLE II (Continued)

(f) $\mu_1=-10$, $\mu_2=3$, $\sigma_1=2$, $\sigma_2=1$

$\rho=+.9$

v_2	g_2
-18.0	0.0000
-17.0	0.0004
-16.0	0.0017
-15.0	0.0045
-14.0	0.0361
-13.0	0.0769
-12.0	0.0774
-11.0	0.1204
-10.0	0.2402
- 9.0	0.1523
- 8.0	0.1408
- 7.0	0.0740
- 6.0	0.0437
- 5.0	0.0191
- 4.0	0.0076
- 3.0	0.0023
- 2.0	0.0000

$\rho=+.8$

v_2	g_2
-18.0	0.0001
-17.0	0.0003
-16.0	0.0016
-15.0	0.0083
-14.0	0.0319
-13.0	0.0626
-12.0	0.0896
-11.0	0.1390
-10.0	0.2079
- 9.0	0.1693
- 8.0	0.1340
- 7.0	0.0816
- 6.0	0.0432
- 5.0	0.0190
- 4.0	0.0073
- 3.0	0.0023
- 2.0	0.0000

$\rho=+.6$

v_2	g_2
-18.0	0.0001
-17.0	0.0004
-16.0	0.0022
-15.0	0.0089
-14.0	0.0257
-13.0	0.0546
-12.0	0.0968
-11.0	0.1539
-10.0	0.1927
- 9.0	0.1759
- 8.0	0.1336
- 7.0	0.0829
- 6.0	0.0423
- 5.0	0.0182
- 4.0	0.0069
- 3.0	0.0021
- 2.0	0.0000

$\rho=+.4$

v_2	g_2
-18.0	0.0001
-17.0	0.0004
-16.0	0.0021
-15.0	0.0078
-14.0	0.0225
-13.0	0.0521
-12.0	0.1005
-11.0	0.1574
-10.0	0.1903
- 9.0	0.1781
- 8.0	0.1347
- 7.0	0.0826
- 6.0	0.0415
- 5.0	0.0175
- 4.0	0.0063
- 3.0	0.0019
- 2.0	0.0001

$\rho=+.2$

v_2	g_2
-18.0	0.0001
-17.0	0.0004
-16.0	0.0018
-15.0	0.0068
-14.0	0.0209
-13.0	0.0518
-12.0	0.1024
-11.0	0.1587
-10.0	0.1905
- 9.0	0.1795
- 8.0	0.1355
- 7.0	0.0822
- 6.0	0.0406
- 5.0	0.0166
- 4.0	0.0058
- 3.0	0.0017
- 2.0	0.0001

$\rho=0$

v_2	g_2
-18.0	0.0000
-17.0	0.0003
-16.0	0.0015
-15.0	0.0062
-14.0	0.0203
-13.0	0.0519
-12.0	0.1036
-11.0	0.1600
-10.0	0.1922
- 9.0	0.1817
- 8.0	0.1365
- 7.0	0.0820
- 6.0	0.0397
- 5.0	0.0158
- 4.0	0.0053
- 3.0	0.0015
- 2.0	0.0002

TABLE II (Continued)

$\rho=-.2$

v_2	g_2
-18.0	0.0000
-17.0	0.0003
-16.0	0.0014
-15.0	0.0061
-14.0	0.0204
-13.0	0.0524
-12.0	0.1043
-11.0	0.1609
-10.0	0.1938
- 9.0	0.1834
- 8.0	0.1373
- 7.0	0.0815
- 6.0	0.0387
- 5.0	0.0149
- 4.0	0.0047
- 3.0	0.0013
- 2.0	0.0002

$\rho=-.4$

v_2	g_2
-18.0	0.0000
-17.0	0.0003
-16.0	0.0015
-15.0	0.0065
-14.0	0.0212
-13.0	0.0538
-12.0	0.1061
-11.0	0.1633
-10.0	0.1967
- 9.0	0.1860
- 8.0	0.1385
- 7.0	0.0812
- 6.0	0.0377
- 5.0	0.0139
- 4.0	0.0042
- 3.0	0.0010
- 2.0	0.0002

$\rho=-.6$

v_2	g_2
-18.0	0.0000
-17.0	0.0003
-16.0	0.0017
-15.0	0.0069
-14.0	0.0222
-13.0	0.0558
-12.0	0.1095
-11.0	0.1679
-10.0	0.2015
- 9.0	0.1895
- 8.0	0.1398
- 7.0	0.0808
- 6.0	0.0366
- 5.0	0.0130
- 4.0	0.0036
- 3.0	0.0008
- 2.0	0.0001

$\rho=-.8$

v_2	g_2
-18.0	0.0000
-17.0	0.0002
-16.0	0.0013
-15.0	0.0059
-14.0	0.0207
-13.0	0.0559
-12.0	0.1148
-11.0	0.1804
-10.0	0.2173
- 9.0	0.2015
- 8.0	0.1446
- 7.0	0.0810
- 6.0	0.0354
- 5.0	0.0120
- 4.0	0.0031
- 3.0	0.0006
- 2.0	0.0001

$\rho=-.9$

v_2	g_2
-18.0	0.0000
-17.0	0.0001
-16.0	0.0007
-15.0	0.0033
-14.0	0.0133
-13.0	0.0434
-12.0	0.1072
-11.0	0.1928
-10.0	0.2476
- 9.0	0.2286
- 8.0	0.1554
- 7.0	0.0808
- 6.0	0.0339
- 5.0	0.0114
- 4.0	0.0028
- 3.0	0.0005
- 2.0	0.0000

TABLE III

SOME VALUES OF THE MEAN, VARIANCE, γ_1, AND γ_2 OF $g_1(v_1)$ AND $g_2(v_2)$ FOR VARIOUS VALUES OF THE PARAMETERS μ_1, μ_2, σ_1, σ_2, AND ρ

(a) $\mu_1=0$, $\mu_2=0$, $\sigma_1=1$, $\sigma_2=1$

	v_1				v_2			
ρ	$\bar{v}_1$	σ_v^2	γ_1	γ_2	$\bar{v}_2$	σ_v^2	γ_1	γ_2
+.9	0.8545	0.1024	-1.48	0.81	-1.2275	0.6115	-0.43	0.19
+.8	0.7631	0.0978	-0.53	3.42	-1.1521	0.6469	-0.32	0.14
+.6	0.7612	0.1890	0.48	2.10	-1.0599	0.6343	-0.32	0.07
+.4	0.8046	0.2828	0.55	1.13	-0.9976	0.5856	-0.36	0.06
+.2	0.8531	0.3693	0.51	0.69	-0.9451	0.5258	-0.41	0.12
0	0.9004	0.4484	0.46	0.41	-0.8939	0.4580	-0.45	0.19
-.2	0.9476	0.5204	0.40	0.22	-0.8398	0.3863	-0.50	0.36
-.4	0.9990	0.5853	0.36	0.12	-0.7770	0.3081	-0.54	0.53
-.6	1.0620	0.6339	0.32	0.09	-0.6939	0.2250	-0.63	0.80
-.8	1.1582	0.6375	0.35	0.17	-0.5463	0.1294	-1.04	1.70
-.9	1.2371	0.5957	0.48	0.24	-0.4441	0.0640	-1.19	4.09

(b) $\mu_1=3$, $\mu_2=10$, $\sigma_1=1$, $\sigma_2=2$

	v_1				v_2			
ρ	$\bar{v}_1$	σ_v^2	γ_1	γ_2	$\bar{v}_2$	σ_v^2	γ_1	γ_2
+.4	3.8036	0.5545	-0.82	-0.18	2.2136	0.2913	-1.02	0.19
+.2	3.6178	0.4664	-0.01	0.05	1.9312	0.2361	-0.38	-0.08
0	2.6128	0.4678	0.29	0.16	1.7926	0.2190	-0.18	-0.18
-.2	3.6483	0.4822	0.44	0.20	1.6999	0.2088	-0.11	-0.26
-.4	3.6958	0.4999	0.54	0.21	1.6284	0.2009	-0.09	-0.34
-.6	3.7467	0.5187	0.61	0.20	1.5710	0.1945	-0.09	-0.44
-.8	3.7965	0.5371	0.68	0.18	1.5233	0.1896	-0.12	-0.54
-.9	3.8245	0.5479	0.68	0.14	1.5029	0.1866	-0.11	-0.63

TABLE III (Continued)

(c) $\mu_1=10$, $\mu_2=10$, $\sigma_1=1$, $\sigma_2=1$

	v_1				v_2			
ρ	$\bar{v}_1$	σ_v^2	γ_1	γ_2	$\bar{v}_2$	σ_v^2	γ_1	γ_2
+.9	9.7678	1.8673	-1.37	-0.35	1.2280	0.0158	-1.08	-1.63
+.8	9.1439	1.1824	-0.76	0.15	1.1610	0.0088	-0.14	0.57
+.6	8.8841	1.1589	-0.15	0.15	1.1396	0.0169	0.67	1.29
+.4	8.8604	1.2269	-0.11	0.07	1.3909	0.1009	-0.95	-1.93
+.2	8.8242	1.2939	-0.06	0.25	1.3099	0.0609	-0.77	-1.05
0	8.8480	1.3617	-0.15	0.17	1.2691	0.0548	-0.28	0.38
-.2	8.8230	1.4273	-0.13	0.20	1.2456	0.0582	0.23	1.63
-.4	8.8700	1.5021	-0.26	0.23	1.2282	0.0646	0.65	2.54
-.6	8.8978	1.5808	-0.33	0.29	1.2209	0.0733	0.93	3.17
-.8	8.8992	1.6401	-0.37	0.27	1.2116	0.0820	1.16	3.51
-.9	8.5413	1.6100	0.38	0.24	1.2109	0.0870	1.23	3.68

(d) $\mu_1=3$, $\mu_2=3$, $\sigma_1=1$, $\sigma_2=1$

	v_1				v_2			
ρ	$\bar{v}_1$	σ_v^2	γ_1	γ_2	$\bar{v}_2$	σ_v^2	γ_1	γ_2
+.9	2.8671	0.4837	0.21	0.72	1.3194	0.0730	-0.80	5.54
+.8	2.8179	0.5282	0.31	0.42	1.1924	0.1056	-0.27	1.20
+.6	2.8475	0.5703	0.38	0.23	1.0645	0.1208	-0.09	0.43
+.4	2.9104	0.5993	0.43	0.17	0.9733	0.1257	0.05	0.23
+.2	2.9761	0.6246	0.47	0.12	0.9050	0.1287	0.15	0.05
0	3.0398	0.6485	0.49	0.06	0.8516	0.1302	0.21	-0.12
-.2	3.0985	0.6712	0.51	0.02	0.8088	0.1304	0.25	-0.26
-.4	3.1546	0.6942	0.52	-0.02	0.7732	0.1295	0.28	-0.37
-.6	3.2095	0.7184	0.52	-0.07	0.7433	0.1278	0.29	-0.47
-.8	3.2604	0.7459	0.50	-0.11	0.7169	0.1252	0.30	-0.55
-.9	3.2796	0.7578	0.50	-0.09	0.7056	0.1241	0.31	-0.59

TABLE III (Continued)

(e) $\mu_1=10,\ \mu_2=3,\ \sigma_1=2,\ \sigma_2=1$

	v_1				v_2			
ρ	$\bar{v}_1$	σ_v^2	γ_1	γ_2	$\bar{v}_2$	σ_v^2	γ_1	γ_2
+.9	10.8295	5.2697	-1.28	-0.34	0.3336	0.0043	-2.02	11.00
+.8	10.4776	4.6695	-0.96	-0.24	0.3232	0.0048	-1.31	4.00
+.6	10.0231	4.2473	-0.44	-0.14	0.3188	0.0080	0.02	2.33
+.4	9.8111	4.1614	-0.21	-0.09	0.3201	0.0120	0.70	3.29
+.2	9.6803	4.1567	-0.08	-0.05	0.3254	0.0165	1.10	4.70
0	9.6361	4.2108	-0.06	-0.04	0.3295	0.0214	1.39	5.71
-.2	9.5947	4.2872	0.00	-0.02	0.3348	0.0267	1.58	6.27
-.4	9.5952	4.4015	0.00	-0.01	0.3395	0.0322	1.75	6.96
-.6	9.6067	4.5479	0.01	-0.02	0.3441	0.0380	1.84	6.93
-.8	9.5348	4.7049	0.13	-0.08	0.3492	0.0439	1.91	7.06
-.9	9.3561	4.7842	0.38	-0.13	0.3519	0.0467	1.94	7.03

(f) $\mu_1=10,\ \mu_2=3,\ \sigma_1=2,\ \sigma_2=1$

	v_1				v_2			
ρ	$\bar{v}_1$	σ_v^2	γ_1	γ_2	$\bar{v}_2$	σ_v^2	γ_1	γ_2
+.9	-0.3519	0.0432	-1.69	4.67	- 9.3743	4.8281	-0.39	-0.08
+.8	-0.3491	0.0409	-1.68	4.79	- 9.5507	4.7476	-0.14	-0.03
+.6	-0.3448	0.0362	-1.63	4.87	- 9.6281	4.6073	-0.02	0.03
+.4	-0.3397	0.0314	-1.56	4.91	- 9.6161	4.4597	-0.01	0.06
+.2	-0.3340	0.0266	-1.46	4.79	- 9.6139	4.3414	-0.01	0.05
0	-0.3276	0.0218	-1.30	4.40	- 9.6509	4.2522	0.04	0.02
-.2	-0.3208	0.0173	-1.06	3.69	- 9.6945	4.1971	0.07	0.00
-.4	-0.3126	0.0131	-0.76	2.29	- 9.8259	4.2045	0.20	-0.03
-.6	-0.3036	0.0094	-0.37	1.00	-10.0389	4.2963	0.42	-0.10
-.8	-0.2849	0.0067	-0.44	0.00	-10.4891	4.7120	0.95	-0.23
-.9	-0.2535	0.0063	-1.11	0.00	-10.8353	5.2913	1.27	-0.34

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