STATIC AND DYNAMIC ANALYSIS OF
CYLINDRICAL STEEL STACKS

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Nomenclature for Guy Wire Analysis

A	cross-sectional area
a	constant of integration
b	constant of integration
	constant
	non-elastic portion of displacement
C	guy wire chord length
C_1	chord length for support displaced to point 1
ΔC	change in chord length
c	constant
d	constant
d( )	differential of quantity within parentheses
E	modulus of elasticity
e	constant
F_v	vertical force
f	constant
f( )	undefined function of quantity within parentheses
g	constant
H	horizontal component of guy wire force
H_1	horizontal component of guy wire force with support displaced to point 1
K	tangent to horizontal force-displacement curve at H
L	guy wire span

L_H	$\frac{dL}{dH}$
L_{HH}	$\frac{d^2L}{dH^2}$
ΔL	change in span length
R	guy wire rise
r	maximum sag of guy wire
S	guy wire inplace arc length
S_H	$\frac{dS}{dH}$
S_o	unstressed guy wire length
S_{ot}	thermally expanded unstressed length of guy wire
S_t	thermally expanded inplace length of guy wire
s	distance along guy wire arc
ΔS	change in arc length
T	guy wire tensile force
T_I	average initial tensile force in the guy wire
T_{avg}	average tensile force in the guy wire
Δt	change in temperature
W	total weight of guy wire
W_1	total weight of guy wire and ice coating
w	linear weight of guy wire and ice coating, if any
w_g	linear weight of guy wire
w_i	linear weight of ice coating on guy wire
x	abscissa for guy wire
x_r	abscissa of point at which guy wire sag is maximum
Δx	change in span length

y	ordinate for guy wire
y_r	ordinate of point at which guy wire sag is maximum
Δy	change in wire
γ	slope of guy wire chord
ϕ	tangent to guy wire curve

Nomenclature for Stack Analysis

A	arbitrary point on elastic curve
B	arbitrary point on elastic curve
C_d	drag coefficient
C_I	lift coefficient
C_I^2	mean square value of lift coefficient
C_p	pressure coefficient
C_s	seismic coefficient
c	probability related distribution factor
D	diameter of stack
	superscript denoting dynamic
D_N	diameter of top section of stack
D_n	diameter of stack over nth interval
d	subscript denoting drag
d( )	derivative of quantity within parentheses
E	modulus of elasticity
e_θ	soil flexibility
F_e	generalized force
$\mathcal{F}_e$	periodic generalized force
f	frequency

f_n	natural frequency
f_v	frequency of vortex shedding
$f(\ )$	undefined function of quantity within parentheses
$f^j(\ )$	jth derivative of the undefined function
G_f	gust factor
g	acceleration of gravity
$g(\ )$	undefined function of quantity within parentheses
H_R	horizontal reactions at guy support points
h_d	datum height for wind velocity
h_n	overall height of stack
$h_{(\ )}$	height at point designated by subscript
I	second moment of the area
I_n	second moment of the area over the nth interval
J_h	earthquake bending moment coefficient at height
K_e	kinetic energy proportionality constant
K_f	generalized force constant
k	load function constant
k_n	load function constant for nth interval
$\mathcal{L}$	dynamic amplification factor
l	subscript denoting lift
M	bending moment
M_n^d	earthquake dynamic bending moment
M_e	generalized mass
M_n^k	earthquake static bending moment
M_n^s	1g static moment of distributed stack weight above point n taken about that point

m	elemental mass
m_n	normalized bending moment at n
ΔM	change in moment
N	subscript denoting top of stack or Nth interval
n	subscript denoting nth point or nth interval
P(S)	spectral density of Strouhal number
p	frontal force of wind
	subscript denoting point p
Q_e	generalized force
q	time dependent distributed lateral load
q_h	pressure intensity at height h
$\bar{q}_1$	mean square value of time dependent lateral load
Re	Reynolds number
S	Strouhal number for stationary cylinder
S_o	Strouhal number for oscillatory cylinder
T	kinetic energy
t	temperature
U	unaltered wind velocity
U_d	wind velocity at datum
U_e	effective wind velocity at datum
U_h	effective wind velocity at height h
u_n	distributed weight of stack
$V_{( )}$	shearing force at height designated by subscript
v_n	normalized shearing force at n
Δv	change in shearing force
W	weight of stack

W_e	work energy
w_d	distributed wind drag force
w_l	distributed wind lift force
$w(h,t)$	distributed wind force as a function of time and height
y	static amplitude of translational displacement
y_D	dynamic amplitude of translational displacement
$y_{( )}$	static displacement at point designated by subscript
y_n	static displacement at top of stack
	static displacement of guy support points
Δy	change in deflection
Z	earthquake zone factor
α	arbitrary exponent
	coefficient of thermal expression
β	damping coefficient
γ	topology related exponent
δ_i	displacement at point i
δ_{ij}	displacement of i caused by a unit load at j
Δ	prefix denoting change in prefixed quantity
Δ_R	net displacement of guy support point
θ	slope or rotation
$\Delta\theta$	change in slope
ν	kinematic viscosity of the air
π	3.14159265
ρ	air density
σ	standard deviation
Σ	summation
ω	circular frequency

ω_n circular natural frequency

ω_v circular frequency of vortex shedding

Ω potential energy

CHAPTER I

INTRODUCTION

Why the Stack?

The cylindrical steel stack as compared to bridges, buildings and other such structures is unique in that it is conceptually simple. Consequently, the structure is oftentimes considered as one requiring little serious thought. Yet, regardless of its conceptual simplicity, the cylindrical steel stack does evidence certain behavioral characteristics which are not only interesting to study but challenging to analyze.

The author's serious consideration of the cylindrical steel stack as an engineering structure resulted from the author's employment by a company engaged in the commercial production thereof. An investigation made as a consequence of that employment revealed a remarkable sparseness of published literature pertaining to the methodology of stack analysis. From this finding ensued the author's motivation to produce the following discourse.

Philosophical Approach

> It is the mark of an intelligent mind to distinguish that degree of accuracy which the nature of the subject permits and not to seek exactness where only an approximation of the truth is possible. (Aristotle)

The analysis of each and every structure is approximate in that a number of assumptions are required to render the structure analyzable. It follows, of course, that the restriction of the number of assumptions to a minimum, with those being limited to the "best" possible, will produce an idealized structure which closely approximates the subject structure. In the analysis of proposed structures, however, the exercise is not to analyze with great mathematical precision, but rather to afford the structure sufficient strength to sustain with some degree of certainty the requirements of its function and environment. Thus, relative to the analysis of the cylindrical steel stack, the objective of the following presentation is to develop equations and to set forth analytical procedures which are comprehensible, which are neither inordinately difficult to implement or excessively time-consuming to evaluate and lastly which, if not theoretically exact, are reasonably close approximations.

Statement of the Problems

The first and primary problem addressed herein is the establishment of expressions which mathematically define the internal shearing forces, the internal bending moments and the deflected shape of the stack in terms of the environmental forces to which the stack is subjected. The qualifications for the expression to be developed are that it allow for discontinuities in the curvilinearly varying distributed loading, for discontinuities in the cross-sectional properties of the stack structure, for abrupt changes in shearing forces and moments, and that it maintain continuity in the rotational and translational displacements of the bending axis of the stack.

The second problem to be considered herein is the development of a set of mathematical expressions which define the configuration and the responsive behavior of a freely hanging guy wire and their application to the problem of the guy supported stacks. This set of expressions should be written in terms of the guy wire unstressed length and internal tensile force. The specific items of responsive behavior to be mathematically described are the changes in configuration and internal force occurring with support displacement, with ambient temperature fluctuation or with linear weight increase due to icing. The problem here, however, is not so much the establishment of exact relationships but rather the development of reasonably accurate approximations.

Summarily, the problems considered herein consist of developing and synthesizing the procedure for analyzing free standing stacks for static wind loads, for static earthquake loads and for dynamic response to the wind and also for analyzing guy supported stacks for static wind loads and for static earthquake loads. The problem also encompasses the estimation of the natural frequency of vibration for each type of stack.

Problem Origins

The need for a theoretical mathematical definition of lateral stack deflections stems from two sources. The first is a practice which is accepted generally by industry and which was contrived in an effort to insure the survivability of self supported stacks. This practice consists essentially of establishing controls relative to stack bending flexibility and is effected through restriction of the transitional displacements exhibited by a stack as it responds to the imposed, theoretically idealized, environmental forces. Thus, the practice as presented in essence establishes the translational displacements of the free standing stack as a gauge of stack flexibility and thereby as a measure of the survivability of the structure.

Relative to the guy wire, the need for a comprehensive mathematical definition of responsive behavior stems from the consideration of guyed stacks as statically indeterminate structures. Subsequently, the need for the approximation of the mathematical definition is established by the complexity of the exact expressions. Furthermore, a review of literature reveals that several of the mathematical expressions required by and consequently developed for the analytical approach employed herein were not available previously.

Proposed Method of Solutions

There are many analytical methods available by which the lateral displacements of bending structures may be determined. Some of these methods require either the lumping of loads or the representation of the structure as a number of discrete elements. Other methods involve repetitive and often cumbersome mathematical integrations. Regardless, none of the available methods express explicitly the displacements as continuous functions. Furthermore, the pertinent literature reviewed indicates no previous attempts to solve the problem as outlined. Therefore, considering the practical importance of the calculated lateral displacements and the relevance of the timely determination thereof, it is deemed highly desirable, if not imperative, to establish an easily evaluated expression for the determination of the lateral displacements of bending structures.

The first problem, that of establishing an easily evaluated expression which defines translational stack displacements and which conforms to the qualifications set forth above, is solved in Chapter II by expanding and building on a rather obscure method formulated by Hetenyi in 1917 (10). The method, as presented by Hetenyi, was developed for single span bending members of constant cross-section loaded uniformly along their entire length. The author recognized that Hetenyi's approach could be generalized and subsequently did so by implementing the tasks outlined as follows:

1. Replace the MacLaurin series expansion used by Hetenyi with the more general Taylor series expansion.

2. Replace the fourth order and higher differential expressions of the polynomial with the equivalent integral expression.

3. Introduce the integral variable function as some parameter raised to an undefined power.

4. Simplify the integral expressions through interpretation of their physical significance relative to bending members.

5. Evaluate the integral expressions in general terms and arrange those terms in a logical fashion for ease of comprehension.

The successful completion of the five tasks thus specified establishes an expression for the evaluation of lateral displacements. The applicability of this expression, however, is limited to those portions of bending members which have constant cross-sections and which experience no discontinuity in the curvilinearly varying distributed loading.

The generalization process is culminated by establishing a procedure whereby the displacements may be determined for a bending member having a stepped cross-section and discontinuous loads. By this procedure the discontinuities establish limits within which the developed expression are applicable. Continuity in displacements is maintained across the incremental limits by the terms within the developed expressions. Thus, the solution to the primary problem is established.

The theoretical expressions and analytical procedure thus developed in Chapter II are subsequently adapted, in Chapter III, to the analysis of the free standing stack. Specifically considered within Chapter III are the natural frequency of stacks, the static and dynamic response of the stack to theoretically idealized wind forces of empirically predicted magnitude, and the equivalent static response of stack to the earthquake. Specifically excluded, however, for consideration herein, is the dynamic response of the stack to the earthquake. This exclusion reflects the current status of the state-of-the-art of earthquake engineering which has not as yet progressed to the point at which an earthquake spectrum for a specific area can be predicted.

The problem of establishing easily evaluated mathematical expressions idealizing guy wire behavior is solved in Chapter IV by approximating the desired theoretical relationships developed in Appendix C for the catenary. This approach departs from that customarily employed whereby the catenary was first approximated and subsequently the desired expressions developed from the approximation.

In Chapter V, the expressions defining guy wire behavior and those defining lateral stack displacement are linked in a procedure for the analysis of guyed stacks. The procedure thus formed is applied to the determination of the natural frequency of the guyed stack and to the response of the stack to the equivalent static forces of the wind and the earthquake.

Finally, the application of the expressions and procedures developed in Chapters II through V is illustrated in Chapter VI by a systematic analysis of a free standing stack and of a guy supported stack.

Applicable Literature

The literature reviewed during the course of study fell into several topical categories: the theory of bending members, free standing stacks, guy wire or cable theory and guy supported stacks. The first finding of the review was that there was a great abundance of literature dealing with specific areas of concern. In the case of bending member theory, none of the number of analytical methods presented in literature met the qualifications previously stated nor followed the approach used in Chapter II.

With regard to the free standing stack, the literature reviewed revealed that applicable literature was almost exclusively in the form of magazine articles and symposium papers. Furthermore, the area of concern of this literature was found to be restricted to the establishment of environmental loads, oftentimes as functions of stack displacements which were left undefined.

The field of literature dealing with cable theory is rather broad in that it covers not only guy wires but also transmission lines and mooring lines for ships. The first finding of this part of the review was that the behavioral characteristics considered in guy wire analysis constitute special cases in the other categories and are not dealt with generally. The second finding was that invariably the guy wire configuration was approximated by a parabola with a horizontal chord, the behavioral characteristics defined in terms that approximation and subsequently modified to account for a non-horizontal chord (9).

As regards the guy supported stack, the literature reviewed dealt largely with guy wire theory and left the interaction of stack and guy wire undefined. The approach presented in Chapter V whereby the tangent modulus is used to approximate a nonlinear load-displacement relationship is not a new nor uncommon one. Yet, this approach was found nowhere in the reviewed literature dealing with guy supported stacks.

It should be noted that the literature cited in the Bibliography is only that portion of the literature reviewed which supports, contributes to or helps clarify the expressions and analytical procedures subsequently developed.

Structural Description and Idealization

The stack as an engineering structure serves as a vertical flue and may be made of either steel or masonry. Presently, all tall stacks being made using steel are of welded construction and are fabricated by joining in series cylindrical sections of one or more diameters. When the fabrication involves sections of differing diameters the dissimilar sections are joined in the order of a unidirectional change in diameter. The joining of these sections is effected through use of transition pieces formed as conical frustums.

Stacks made of welded steel are prefabricated in segmental lengths at an industrial facility. The segments are then transported to a designated site and subsequently assembled either prior to or during erection. Stacks thus produced are limited in physical size by

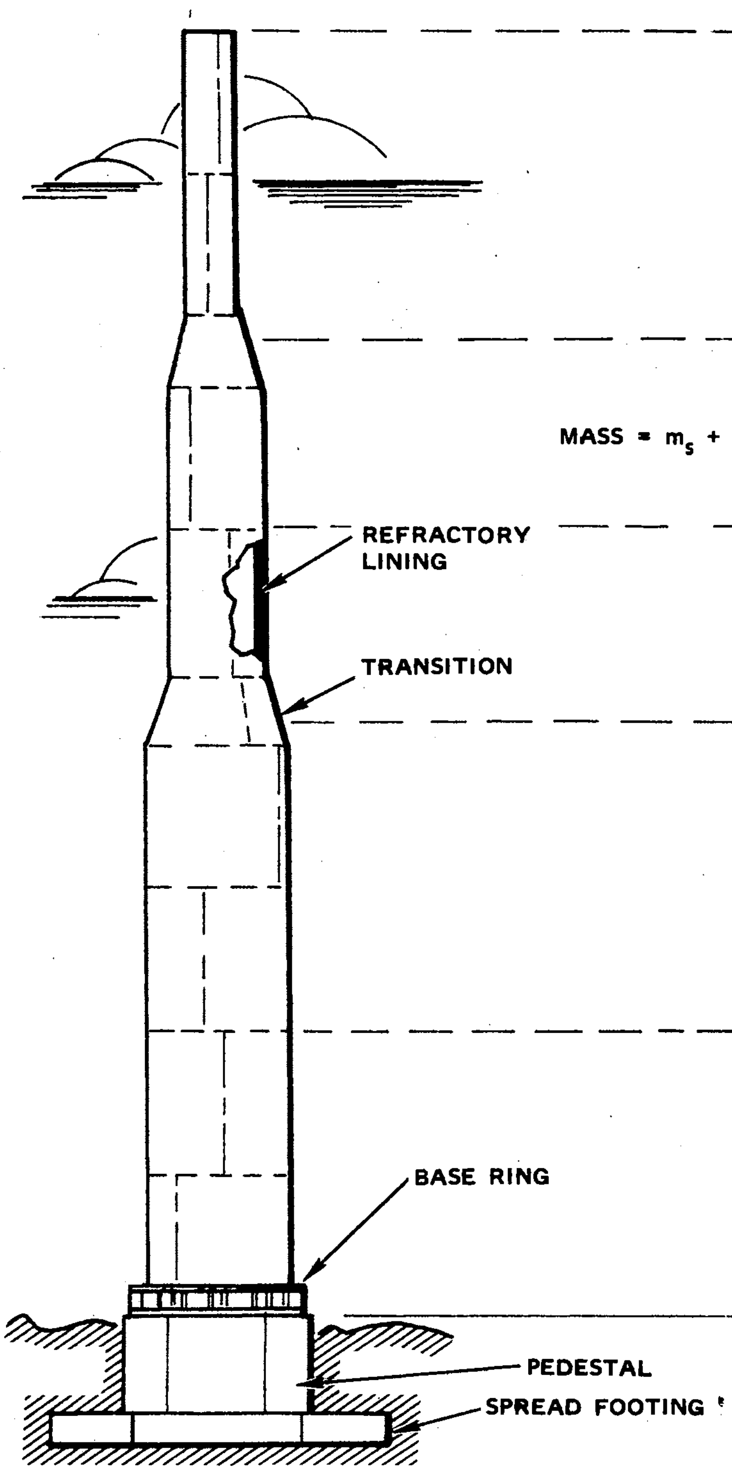

Figure 1. Typical Stack Configuration

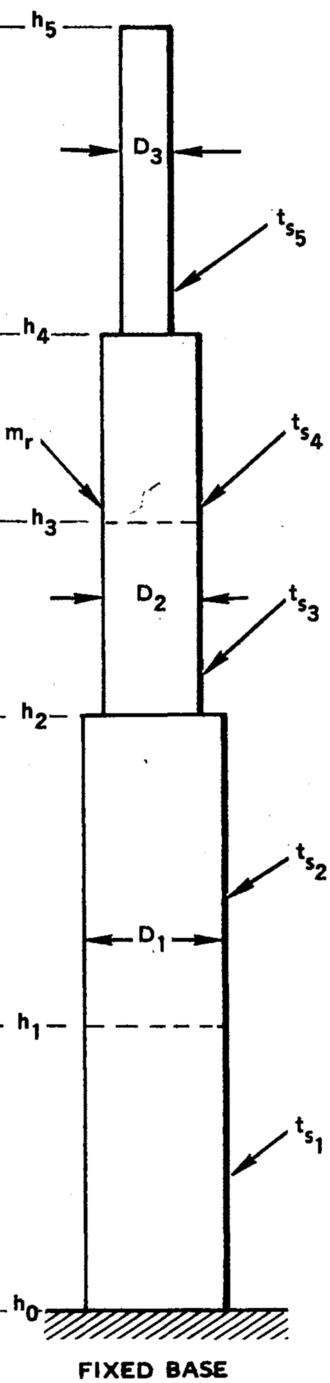

Figure 2. Idealized Stack

transportation constraints on the girth and erection restrictions on the height.

Stacks are often classified according to the manner in which they are provided lateral support. Thus, stacks that possess sufficient strength to stand alone are identified as self-supported or free standing stacks. Conversely, stacks which depend on some external means of lateral support are identified by their supportive structure (i.e., guy supported, derrick supported, etc.). Economics ordinarily dictates the support system chosen and for this reason the large majority of stacks are either self-supported or guyed.

Generally, stacks are supported vertically by reinforced concrete foundations. These foundations, usually octagonal in planform, are constructed as pedestals which rest upon and are made integrally with a spread footing. The connection of the stack to the foundation is effected through the use of bolts which are imbedded in the concrete and which mate with a ringlike structure placed at the base of the stack for that purpose.

Stacks as flues are used to vent gases into the atmosphere. These gases as products of combustion are sufficiently hot to cause physical damage to unprotected steel. For this reason, stacks are lined with refractory materials which significantly affect the mass of the structure but have little or no influence on the strength.[1]

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[1]Refractory materials, having a rate of thermal expansion different than that of steel, crack as the stack expands and contracts during use and consequently contribute little to stack strength and then only to the compressive strength. However, the presence of the refractory does help the stack to resist ovaling.

For analytical purposes, the stack as a structure will be idealized as shown in Figure 2. Thus, it is assumed that:

1. The stack is rigidly fixed against rotation and translation at the base; i.e., at the top of the foundation.

2. Changes in cross section occur abruptly.

3. Refractory material has significant mass but no strength.

4. All other assumptions ordinarily made in elastic analyses of bending members are applicable.

The Wind and Its Effects

By definition, the wind is simply the flow or movement of an air mass. As a moving mass, the wind possesses kinetic energy and this energy has as its ultimate source the sun. Simply stated, solar radiation, through a complex thermodynamic process, produces large systems of high and low pressure thereby establishing gradients. Consequently, the natural inclination toward an equalization of pressure induces the air movement to which we refer as the wind.

The wind by nature is a random phenomenon and as such exhibits continual fluctuations in velocity and direction. Additionally, the wind displays a variation in velocity with elevation, this characteristic being primarily attributable to surface friction. Furthermore, relative to specific locales, the effects of the wind are influenced by exposure of the site and by roughness of the terrain. Thus, since the representation of the wind as it actually occurs is precluded, it must be established through idealization.

Reported studies (5) have ascertained that the wind can be considered as being comprised of two components, one having a steady average velocity and the other a flucuating velocity. Furthermore, the studies established that it is the steady component which varies with elevation while the fluctuating component is more or less uniform. Finally, the studies showed that the steady component remains essentially constant for periods of significant duration.

The properties of the wind, as delineated above, are important relative to its idealized representation. First of all, the variation of the steady component with elevation permits this component to be expressed mathematically as a function of height thereby warranting the identification of a wind by its steady velocity at some datum elevation. Secondly, the constancy of the steady component allows the statistical timewise correlation of extreme winds for determination of recurrence intervals. Thus it becomes possible to formulate probability statements regarding the adverse wind conditions a structure might experience during its lifespan. Lastly, the altitudinal uniformity of the fluctuating component permits amplification of the steady component through application of a gust factor calculated as a function both of the dynamic properties of the structure and of the statistical distribution of the fluctuations and the effects thereof.

The wind, when traveling along the earth's surface, encounters obstacles which impede flow. As it alters speed and direction to go around these obstacles, a portion of the wind's kinetic energy is converted to potential energy in the form of pressure. The intensity of the pressure thus formed at any given point on the obstacle is a function of the velocity and density of the moving air, the size and shape of the obstacle and its orientation relative to the wind direction.

The vectorial accumulation of all the incremental forces produced by the individual pressures acting on the applicable areas determines the total force which the wind imposes on the obstacle. This vectorial accumulation is ordinarily effected empirically and presented in the form of dimensionless parameters characteristic of the individual shape.

In the analysis of wind sensitive structures, such as stacks, it is necessary to consider both the along-wind response and the cross-wind response of the structure to the wind. Consideration of the characteristics of the wind and of the observed behavior of structures has led to the treatment of the along-wind response as an equivalent-static condition, this treatment being implemented through application of the previously mentioned gust factor. The cross-wind response, however, must be treated dynamically since for some structures this condition is characterized by a distinctly oscillatory motion.

The oscillatory lateral response of wind sensitive structures as elicited by fluctuation of the lift forces is attributed to a fluid mechanism known as vortex shedding (5). This phenomenon is influenced not only by wind velocity and turbulence, downstream and upstream, but also by the motion of the structure itself. The property of vortex shedding which is of primary importance is the frequency of shedding. This characteristic, measured analytically by a parameter called the Strouhal number, has been related to wind conditions through correlation with Reynold number.

For analytical purposes, periodic vortex shedding is represented as a sine function. In the case of random shedding, the representation is effected through application of a time averaging process known as

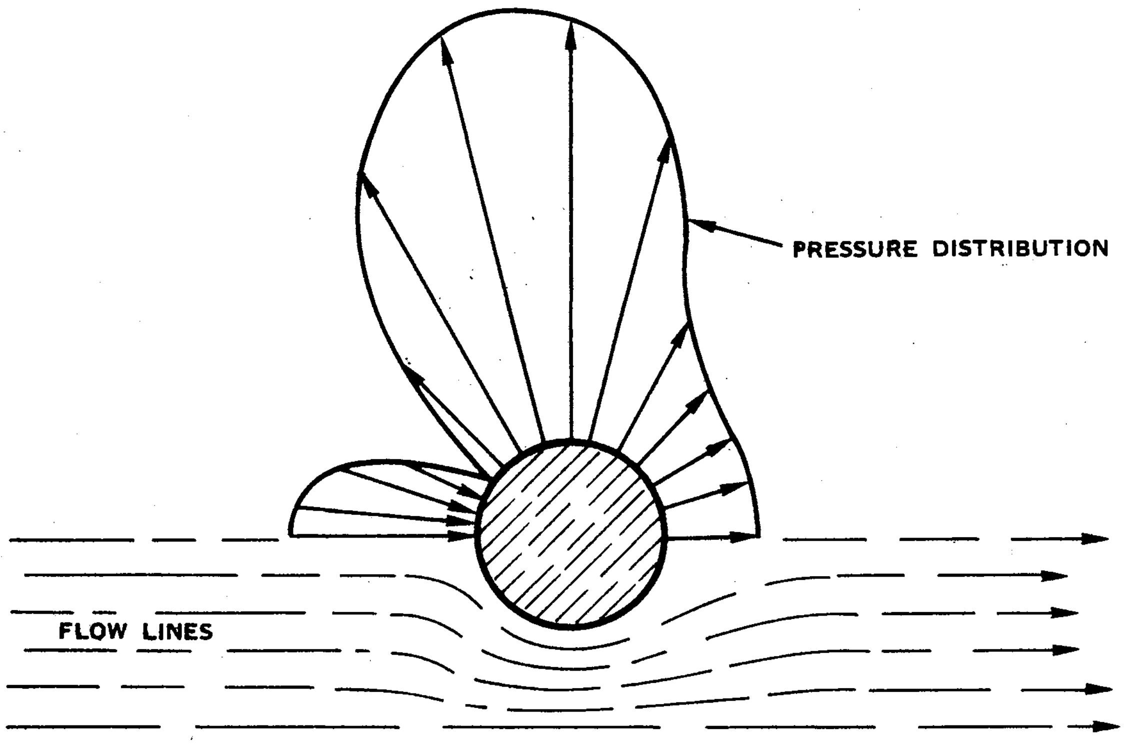


Figure 3. Laminar Flow Past a Cylinder

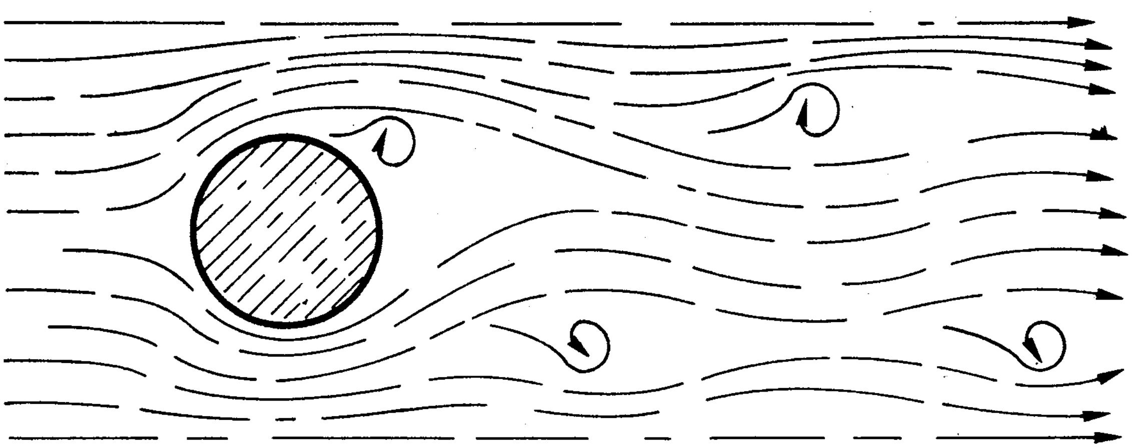


Figure 4. Vortex Street

the spectral density approach. The formulation of the mathematics applicable to vortex shedding is presented in Appendix A. The definition and evaluation of wind related parameters are presented in Appendix B.

Cylindrical stacks sometimes exhibit other modes of oscillatory response. These modes, along-wind and ovalling, have been observed but rarely and then the former lacks sufficient severity to warrant serious attention. The latter, however, may present a problem when stack diameters are large with respect to the thickness of shell. These cases may be handled as recommended in (5).

The Phenomenon of the Earthquake

Modern-day studies of the earth as a physical body are indicating that the crust of the earth is composed of a number of individual masses whose junctures occur at locations commonly known as faults. A continual geological activity within the earth encourages relative movement of the various masses (or earthplates), thus generating strains in and between the masses. These strains build to the limit of the crust's tolerance and at this point the crust ruptures. In a broad sense the earth is an elastic body and thus, when rupture occurs, the sudden consequent release of the built up strains causes the earth to rebound thereby setting up shock waves which propogate in all directions (20).

Earthquake generated shock waves are depicted generally as consisting of body waves, longitudinal and transverse, and of surface waves. Surface waves occur subsequent to and as a consequence of body waves in that the surface waves are set up as the body waves encounter and are reflected by the earth's surface. In essence, as these waves travel through the earth and along its surface, they cause the earth and objects located thereon to vibrate. Thus, the earthquake is simply a ground-vibration phenomenon.

The ground motion resulting from the earthquake is both random and complex. The motion is random in that there is no set pattern to the magnitudes of ground accelerations and decelerations or the time intervals over which they occur. The motion is complex in that the longitudinal transverse horizontal and vertical components of the motion are not in phase and are functions of the random accelerations. Furthermore, the intensity of the motion is dependent upon proximity and magnitude of the earthquake as well as the elastic properties and density of the earth in the general area. Thus, for the present level of the state of the art, the definitive prediction of the potential ground motion attributable to the earthquake is precluded (20). However, there are analytical approaches which are discussed subsequently in the text.

As stated above, structures experiencing earthquake generated ground motion are induced to vibrate. This type of vibration is labeled free vibration since there are no externally applied forces acting on the body of the structure causing the vibrations. Thus it is concluded that the internal forces in a structure subjected to an earthquake are some combination of gravity forces and motion related inertial forces.

CHAPTER II

FORCE-DISPLACEMENT RELATIONSHIPS

Theoretical Development

Mathematical Approach

The differential equations coupling internal forces to displacements are the fundamental relationships upon which are built the varied analytic approaches applicable to beamlike strutures. In general, when the deflection of a bending member at point h is defined as y, then the slope is

$$\theta = \frac{dy}{dh},$$

the bending moment is

$$M = EI\,\frac{d^2y}{dh^2}\,,$$

the shearing force is

$$V = \frac{d}{dh}\left(EI\,\frac{d^2y}{dh^2}\right),$$

and the intensity of the lateral loading causing the deflection is

$$w = \frac{d^2}{dh^2}\left(EI\,\frac{d^2y}{dh^2}\right). \tag{2.1}$$

Any of these differential equations can be legitimately used to mathematically describe physical systems. However, the nature of the environmental loadings to which stacks are subjected points to the last differential equation as the one suited best to the mathematical description of the physical system under consideration herein.

There are many methods available to effect the solution of this equation; however, one approach ideally suited to beamlike structures is one proposed by Hetenyi (10). Hetenyi's approach is based on the observation that "any deflection curve expressed as a polynomial can be written in the form of a MacLaurin series." Subsequent to and as a consequence of this observation, Hetenyi formulated through example the premise stated as "Relative to bending members, the MacLaurin series expansion of the solution to the differential equation is a polynomial which defines the deflection curve."

$$y = f(h) = f(0) + hf'(0) + \frac{h^2}{2!} f''(0) + \frac{h^3}{3!} f'''(0) + \ldots$$

where y is the lateral displacement at the point h along the bending axis from the point of expansion h = 0, f(0) is some function of h evaluated at h = 0 and $f'(0)$, $f''(0)$, . . . , $f^{\infty}(0)$ are derivatives of the function f(h) evaluated at h = 0.

For the purpose of the following development, the use of the MacLaurin series expansion is too restrictive. However, since the MacLaurin series is a specialization of the Taylor series at h = 0, the supposition is made that the latter can be substituted without violating the premise. Thus, the polynomial for the deflection curve can be written in the form

$$y = f(h) = f(h_p) + f'(h_p)(h-h_p) + \frac{f''(h_p)}{2!}(h-h_p)^2 + \ldots$$

where y is the lateral displacement at some point $(h-h_p)$ along the

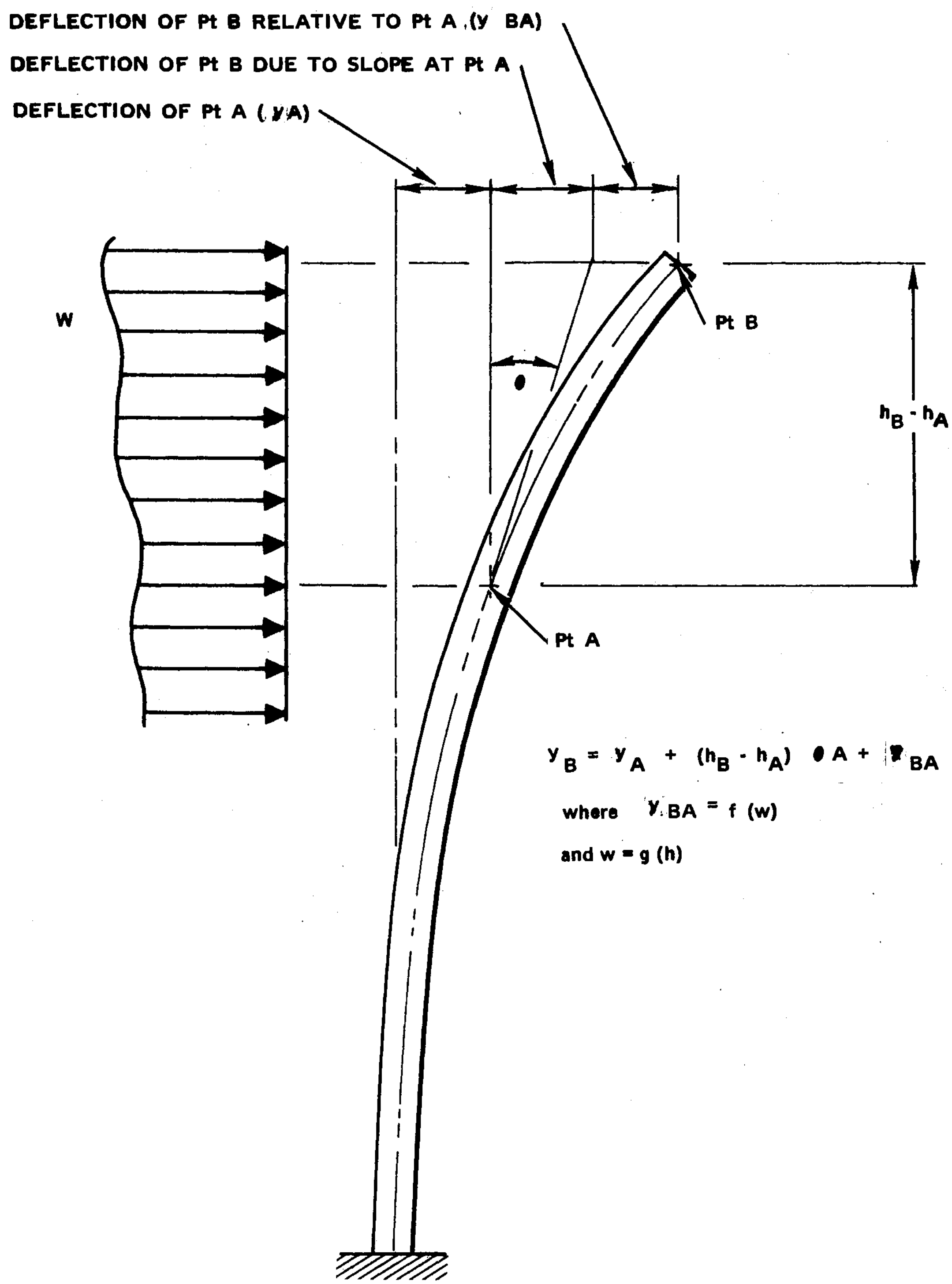


Figure 5. Items Contributing to Total Deflection

bending axis from the point of expansion $h = h_p$, $f(h_p)$ is some function of h evaluated at $h = h_p$ and $f'(h_p)$, $f''(h_p)$, . . . , $f^{\infty}(h_p)$ are derivatives of the function $f(h_p)$ evaluated at $h = h_p$.

One of the necessary conditions for the use of either the MacLaurin or Taylor series is that the function and its derivatives be continuous within the interval in which it is applied. Recognizing that stack loadings are continuous within specific intervals and that they are linear functions of a finite order, it is deduced that

$$\left[\frac{f^{IV}(h_p)}{4!}(h-h_p)^4 + \frac{f^{V}(h_p)}{5!}(h-h_p)^5 + \ldots\right] = \iiiint g(h)\,dh.$$

This deduction allows the solution to the differential equation to be expressed in the form

$$y = f(h_p) + f'(h_p)(h-h_p) + \frac{f''(h_p)}{2!}(h-h_p)^2 + \frac{f'''(h_p)}{3!}(h-h_p)^3 + \iiiint g(h)\,dh. \qquad (2.2)$$

Assigning specific quantities of undefined magnitude to the deflection, slope, bending moment, shearing force and distributed load at the point $h = h_p$ and restricting consideration to bending members of constant cross section within the interval containing the point, allows definition of the function and its derivatives at that point.

$$f(h_p) = y_p$$

$$f'(h_p) = \left(\frac{dy}{dh}\right)_p = \theta_p$$

$$f''(h_p) = \left(\frac{d^2y}{dh^2}\right)_p = \frac{M_p}{EI}$$

$$f'''(h_p) = \left(\frac{d^3y}{dh^3}\right)_p = \frac{V_p}{EI}$$

$$g(h) = \frac{w}{EI} .$$

Substitution of these relationships into Equation (2.2) and the subsequent simplification yields the solution to Equation (2.1) in the general form

$$y = y_p + \theta_p(h-h_p) + \frac{1}{EI}\left[\frac{M_p}{2}(h-h_p)^2 + \frac{V_p}{6}(h-h_p)^3 + \iiiint wdh\right] \tag{2.3}$$

Successive differentiation of Equation (2.3) defines the remainder of the desired expressions

$$\theta = \theta_p + \frac{1}{EI}\left[M_p(h-h_p) + \frac{V_p}{2}(h-h_p)^2 + \iiint wdh\right] \tag{2.4}$$

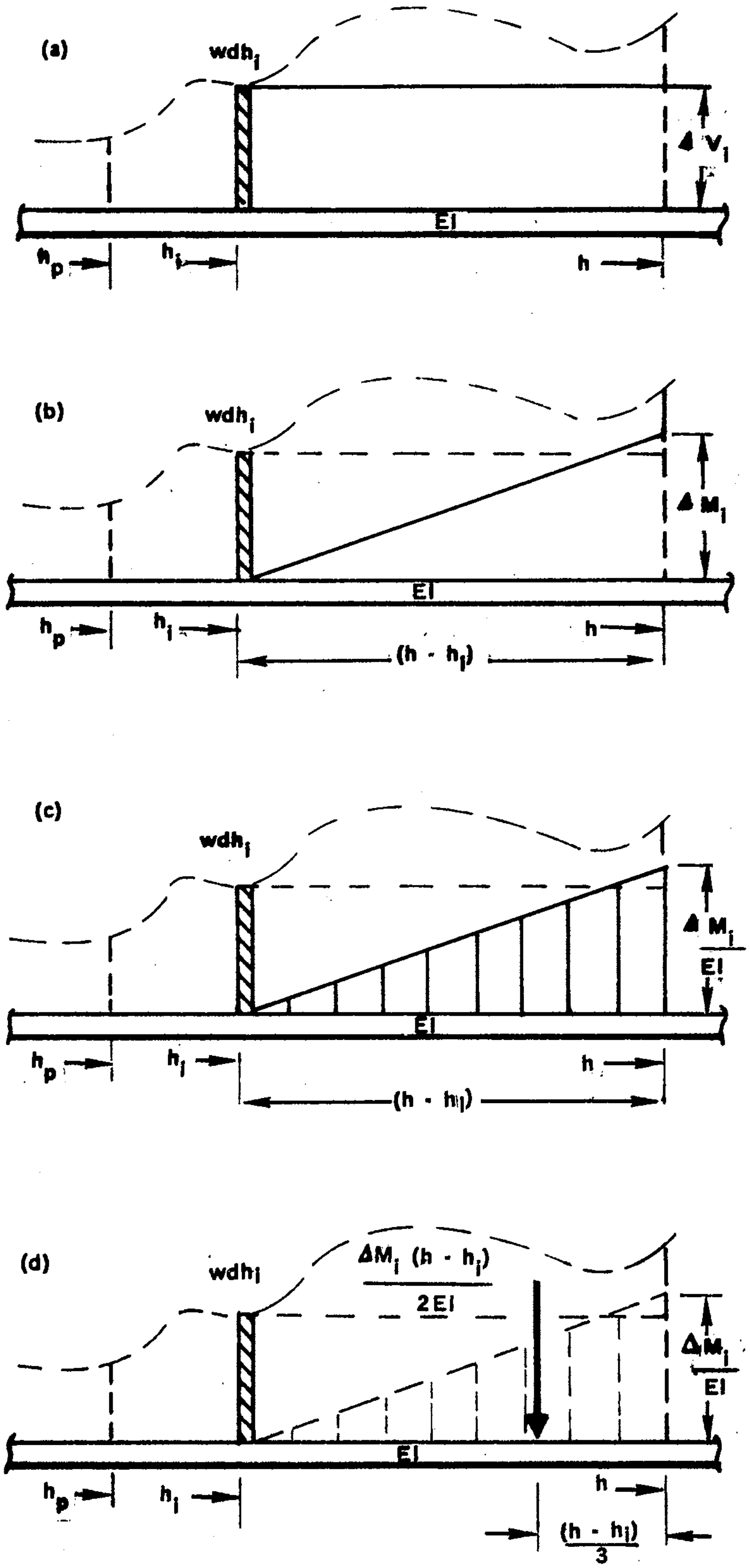
$$M = M_p + V_p(h-h_p) + \iint wdh \tag{2.5}$$

$$V = V_p + \int wdh. \tag{2.6}$$

The multiple integral terms appearing in these equations can be somewhat difficult to evaluate correctly, the difficulty being dependent upon the function w. However, this difficulty can be essentially eliminated if the physical significance of the integrals is now considered.

In Equation (2.6), the integral represents the change in shear over the interval effected by the distributed load in that interval. However, it is equally valid to consider the integral to represent the change in shear effected by the sum of all of the individual loads wdh within the interval. Coupling of this individual force concept with the physical significance of the subsequent multiple integrals leads to their mathematical simplification.

In Equation (2.5), the physical significance of the integral quantity is that it represents the change in the moment at point h effected



$$\Delta V_i = wdh_i$$

$$\Delta V_w = \sum_{h_p}^{h} (wdh_i)$$

$$\Delta M_i = (wdh_i)(h - h_i)$$

$$\Delta M_w = \sum_{h_p}^{h} (wdh_i)(h - h_i)$$

$$\Delta \theta_i = \frac{M_i}{2EI}(h - h_i)$$

$$\Delta \theta_w = \sum_{h_p}^{h} \frac{(wdh_i)(h - h_i)^2}{2EI}$$

$$\Delta y_i = \frac{\Delta M_i}{2EI} \frac{(h - h_i)^2}{3}$$

$$\Delta y_w = \sum_{h_p}^{h} \frac{(wdh_i)(h - h_i)^3}{6EI}$$

Figure 6. Individual Force Concept

by the distributed load in the interval. Now consider the quantity wdh_i to be an individual force acting at point h_i. The change in the moment at point h effected by this force is defined as $(wdh)(h-h_i)$. If all such forces wdh within the interval are considered, then the cumulative change in moment at point h is found by integrating h_i over the interval. Thus,

$$\Delta M_w = \int_{h_p}^{h} (h-h_i)(wdh_i).$$

Since $f(h_i) = f(h)$ and $f'(h_i) = f'(h)$, then this latter integral expression can be substituted for the one appearing in Equation (2.5), thus simplifying the potential evaluation problem.

Using the approach as above, consider now the triple integral quantity appearing in Equation (2.4). This integral represents the change in slope over the interval effected by the distributed loads. Now recall the first moment-area theorem which states "The change in slope of the tangents of the elastic curve between two points, A and B, is equal to the area under the M/EI curve between those two points" (34). Employing this relationship, the change of slope over the interval for the individual force wdh_i is found as $(wdh_i)(h-h_i)^2/2EI$. As before, the effect of all the individual forces within the interval is calculated as

$$\Delta\theta_w = \int_{h_p}^{h} \frac{(h-h_i)^2}{2EI} (wdh_i)$$

Finally, attention is given to the deflection equation. The integral quantity therein represents the change in deflection over the interval effected by the distributed load. However, the second moment-area theorem states "The departure of point B on the elastic curve from the tangent to the curve at point A is equal to the static moment about an axis through B of the area under the M/EI curve between points A and B" (34). Coupling this relationship with the single force idea establishes the change in the relative deflection due to that force as $(wdh_i)(h-h_i)^3/3EI$, and the change in the deflection attributable to all such forces within the interval as

$$\Delta y_n = \int_{h_p}^{h} \frac{(h-h_i)^3}{6EI} (wdh_i) \; .$$

Substitution of these three integral quantities into the applicable equations establishes the general force displacement equations as

$$y = y_p + \theta_p(h-h_p) + \frac{1}{EI}\left[\frac{M_p}{2} (h-h_p^2) + \frac{V_p}{6} (h-h_p)^2 + \int_{h_p}^{h} \frac{(h-h_p)^3}{6} (wdh_i)\right] \qquad (2.7)$$

$$\theta = \theta_p + \frac{1}{EI}\left[M_p(h-h_p) + \frac{V_p}{2} (h-h_p)^2 + \int_{h_p}^{h} \frac{(h-h_i)}{2} (wdh_i)\right] \qquad (2.8)$$

$$M = M_p + V_p(h-h_p) + \int_{h_p}^{h} (h-h_i)(wdh_i) \qquad (2.9)$$

$$V = V_p + \int_{h_p}^{h} wdh_i . \qquad (2.10)$$

Evaluation of the Load Function

At the beginning of this chapter, it was stated that the change in shearing force (or more commonly the distributed load) is some function g(h). Consideration of the environmental loadings to which stacks are subjected makes it desirable to define the function in the form of kh^{α}.

Selection of a particular function allows the integral quantities to be evaluated. Thus, the change in shear attributable to the distributed load is found as

$$\Delta V_w = \int_{h_p}^{h} (kh_i^{\alpha}) dh_i = k(h^{1+\alpha} - h_p^{1+\alpha})/(1+\alpha). \tag{2.11}$$

The change in moment caused by the distributed load is

$$\Delta M_w = \int_{h_p}^{h} (h-h_i)\ (kh_i^{\alpha}) dh_i = k\left(\frac{hh^{1+\alpha}}{1+\alpha} - \frac{h^{2+\alpha}}{2+\alpha}\right)\Bigg|_{h_p}^{h}$$

$$\Delta M_w = -(\Delta M - \Delta V_w h)$$

where

$$\Delta M = k(h^{2+\alpha} - h_p^{2+\alpha})/(2+\alpha). \tag{2.12}$$

The change in slope induced by the distributed load is

$$\Delta \theta_w = \int_{h_p}^{h} \frac{(h-h_i)^2\ (kh_i^{\alpha})}{2EI} dh_i = \frac{k}{2EI}\left(\frac{h^2 h_i^{1+\alpha}}{1+\alpha} - \frac{2hh_i^{2+\alpha}}{2+\alpha} + \frac{h_i^{3+\alpha}}{3+\alpha}\right)\Bigg|_{h_p}^{h}$$

$$\Delta \theta_w = \Delta \theta - \frac{h}{EI}\left(\Delta M - \frac{\Delta V_w h}{2}\right)$$

where

$$\Delta \theta = k(h^{3+\alpha} - h_p^{3+\alpha})/(2EI)(3+\alpha) \tag{2.13}$$

Finally, the change in deflection produced by the distributed load is

$$\Delta y_w = \int_{h_p}^{h} \frac{(h-h_i)^3\ (kh_i^{\alpha}) dh_i}{6EI}$$

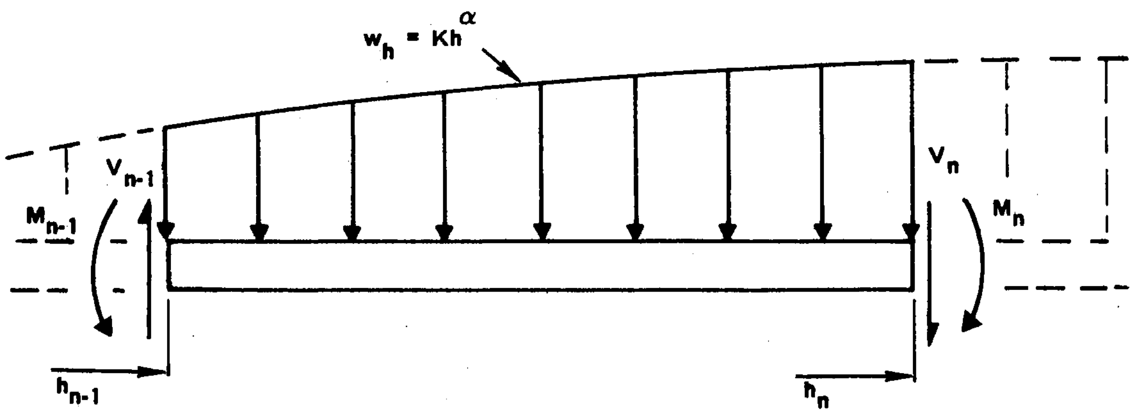


Figure 7. Equilibrium Forces on Elemental Length

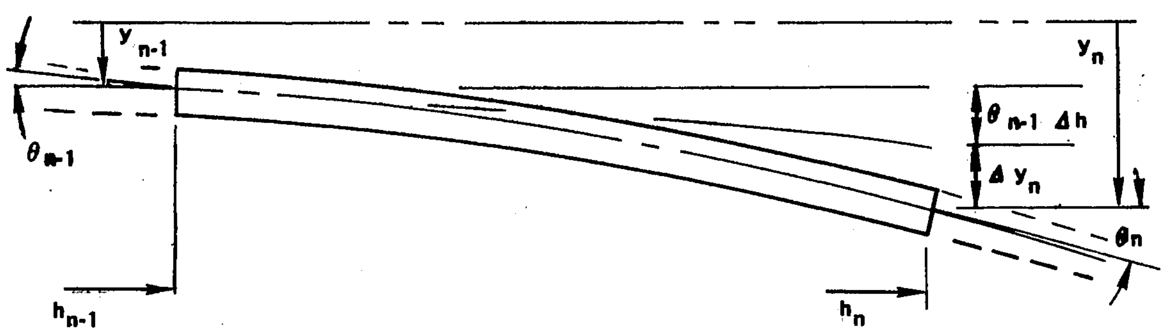


Figure 8. Elemental Displacements

$$\Delta y_w = \frac{k}{6EI}\left(\frac{h^3 h^{1+\alpha}}{1+\alpha} - \frac{3h^2 h^{2+\alpha}}{2+\alpha} + \frac{3hh^{3+\alpha}}{3+\alpha} - \frac{h^{4+\alpha}}{4+\alpha}\right)\Bigg|_{h_p}^{h}$$

$$\Delta y_w = -\Delta y + \Delta\theta h - \frac{h^2}{2EI}\left(\Delta M - \frac{\Delta V h}{3}\right)$$

where

$$\Delta y = k(h^{4+\alpha} - h_p^{4+\alpha})/(6EI)(4+\alpha) \qquad (2.14)$$

<u>General Expressions</u>

Substitution of the expressions developed above, for the integral quantities appearing in Equations (2.7) through (2.10) and the subsequent rearrangement of terms establishes the load-displacement equations in the following format.

$$V = (V_p + \Delta V) \qquad (2.15)$$

$$M = (M_p - \Delta M) + (V_p \Delta h + \Delta V h) \qquad (2.16)$$

$$\theta = (\theta_p + \Delta\theta) + \frac{(M_p \Delta h - \Delta M h)}{EI} + \frac{(V_p \Delta h^2 + \Delta V h^2)}{2EI} \qquad (2.17)$$

$$y = (y_p - \Delta y) + (\theta_p \Delta h + \Delta\theta h) + \frac{(M_p \Delta h^2 - \Delta M h^2)}{2EI} + \frac{(V_p \Delta h^3 + \Delta V h^3)}{6EI} \qquad (2.18)$$

where $\Delta h = (h-h_p)$ and for $w = kh^\alpha$, the quantities Δy, $\Delta\theta$, ΔM and ΔV are defined by Equations (2.11) through (2.14).

Application of Developed Expressions

Shear and Moment Expressions

Equations (2.15) and (2.16) are the general expressions developed for the shearing force and the bending moment at some arbitrary point on the bending axis. For the application of these equations to a specific physical system, it is necessary to find a point at which the magnitude of the defined quantities are known. Thus, in the case of a cantilevered beam, it is observed that at the free end of the beam the shearing force and the bending moment are zero.

Referring to the cited equations, let the point of reference p be identified as n and let the point of evaluation be n-1. Thus, progressing along the beam from the free end, the shearing force and bending moment are found at each point n-1 as

$$V_{n-1} = V_n + \Delta V_n \tag{2.19}$$

$$M_{n-1} = M_n - \Delta M_n + V_n \Delta h_n + \Delta V_n h_{n-1} \tag{2.20}$$

where

$$\Delta V_n = (h_{n-1}^{1+\alpha} - h_n^{1+\alpha})(k/1+\alpha)$$

$$\Delta M_n = (h_{n-1}^{2+\alpha} - h_n^{2+\alpha})(k/2+\alpha)$$

and

$$\Delta h_n = (h_{n-1} - h_n).$$

Applying these equations in each increment from n=N to n=0 establishes the shearing force and bending moment at each point of discontinuity in the cross-sectional properties of the beam as well as any other point of interest.

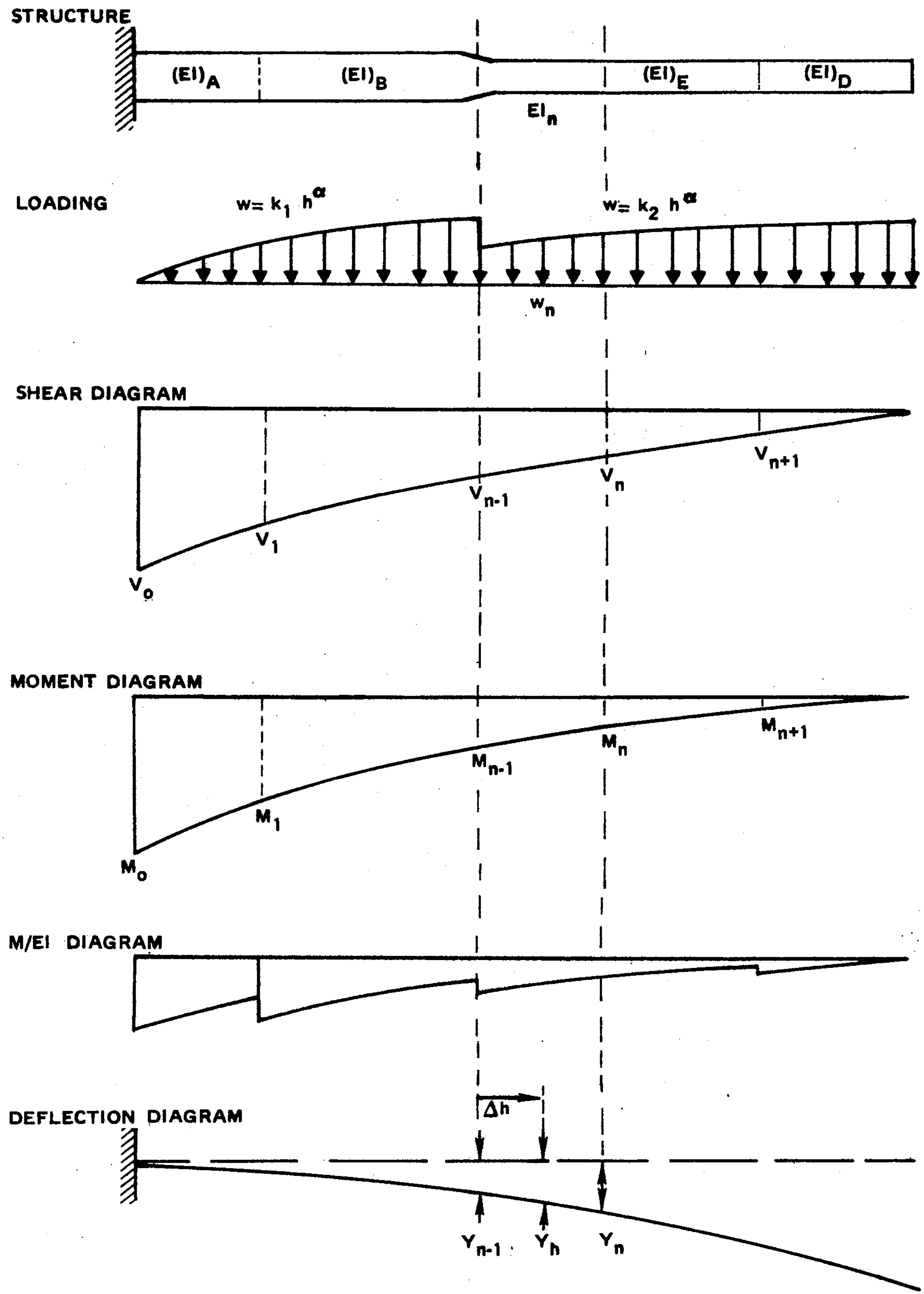


Figure 9. Typical Force-Displacement Curves

Although it was not specifically mentioned during the development, it should be noted that the sign convention employed was such that the defined quantities had positive values for an ever increasing function of h. It is observed that in this particular application, where the initial point occurs at the free end of the structure and the function of h decreases progressively away from the free end, the quantity ΔV_n and consequently V_n have only negative values. This need not and will not create a problem as long as the generated sign is carried through all subsequent calculations.

Displacement Expressions

Equations (2.17) and (2.18) are the general expressions for the rotation and translation at some arbitrary point on the bending axis. As before, it is necessary to establish a point at which the magnitudes of these quantities are known. Thus, in the case of the cantilevered beam, it is assumed that the base of the beam neither rotates nor translates.

For evaluation of the displacements, in the reference equations, define the point of reference p as n-1 and the point of evaluation as n. Thus, proceeding along the beam from the base, the rotation and translation at each point n are established as

$$\theta_n = (\theta_{n-1} + \Delta\theta_n) + \frac{(M_{n-1}\Delta h_n - \Delta M_n h_n)}{EI} + \frac{(V_{n-1}\Delta h_n^2 + \Delta V_n h_n^2)}{2EI} \tag{2.21}$$

$$y_n = (y_{n-1} - \Delta y_n) + (\theta_{n-1}\Delta h_n + \Delta\theta_n h_n) + \frac{M_{n-1}\Delta h_n^2 - \Delta M_n h_n^2}{2EI} + \frac{V_{n-1}\Delta h^3 + \Delta V_n h_n^3}{6EI} \tag{2.22}$$

where now

$$\left.\begin{aligned} \Delta V_n &= (h_n^{1+\alpha} - h_{n-1}^{1+\alpha})(k/1+\alpha) \\ \Delta M_n &= (h_n^{2+\alpha} - h_{n-1}^{2+\alpha})(k/2+\alpha) \\ \Delta \theta_n &= (h_n^{3+\alpha} - h_{n-1}^{3+\alpha})(k/3+\alpha)/(2EI) \\ \Delta y_n &= (h_n^{4+\alpha} - h_{n-1}^{4+\alpha})(k/4+\alpha)/(6EI) \\ \text{and} \qquad \Delta h_n &= (h_n - h_{n-1}). \end{aligned}\right\} \qquad (2.23)$$

Modified Form of Expressions

Consideration of the above equations indicates that some care must be taken in the correlation of Equations (2.19) and (2.20) with Equations (2.21) and (2.22). This warning results from the observation that the quantities ΔV_n, ΔM_n and Δh_n appearing in the former equations have the same magnitude but opposite sign of those same quantities which appear in the latter equations. Furthermore, the direction of V_{n-1} in the latter equations is opposite to that assumed in the development and thus a negative value should be used for the final ones along the beam.

It is possible to eliminate the inconsistency in the signs of the delta-quantities by simply multiplying one of the sets by a negative one. At the same time, it is possible to drop the requirement of carrying a negative sign with the shearing force by letting $V = -V$ wherever V occurs in the equations. Thus,

$$V_{n-1} = V_n + \Delta V_n \qquad (2.24)$$

$$M_{n-1} = M_n + \Delta M_n + V_n \Delta h_n - \Delta V_n h_{n-1} \qquad (2.25)$$

$$\theta_n = (\theta_{n-1} + \Delta\theta_n) + \frac{(M_{n-1}\Delta h_n - \Delta M_n h_n)}{EI} - \frac{(V_{n-1}\Delta h_n^2 - \Delta V_n h_n^2)}{2EI} \quad (2.26)$$

and

$$y_n = (y_{n-1} - \Delta y_n) + (\theta_{n-1}\Delta h_n + \Delta\theta_n h_n)$$

$$+ \frac{(M_{n-1}\Delta h_n^2 - \Delta M_n h_n^2)}{2EI} - \frac{(V_{n-1}\Delta h^3 - \Delta V_n h_n^3)}{6EI} \quad (2.27)$$

where the delta values are defined by Equation (2.23).

Equations (2.24) through (2.27) represent an efficient easily understood method of calculating forces and displacements for a cantilever beam which has discontinuous mass and section properties. However, the particular worth of the overall approach will be established only through consideration of the subsequent applications.

Special Cases

Limits of Load Function Exponent

Generally speaking, the exponent α of the load function will fall within the range of zero to one (5). Using zero for the value of α reduces the load function to a uniformly distributed load. Similarly, using the value of one for α produces the load function for a uniformly distributed loading. Reference (1) shows that both of these values are specified for loading conditions. For this reason, the load-displacement equations will be evaluated specifically for these α-values.

Uniformly Distributed Loading

To establish the load-displacement equations for a uniformly distributed loading, it is necessary to evaluate Equation (2.23) for $\alpha = 0$.

Thus,

$$\Delta V_n = (h_n - h_{n-1})(k_n)$$

$$\Delta M_n = (h_n^2 - h_{n-1}^2)(k_n/2)$$

$$\Delta \theta_n = (h_n^3 - h_{n-1}^3)(k_n/6EI)$$

$$\Delta y_n = (h_n^4 - h_{n-1}^4)(k_n/24EI).$$

Substitutions of these expressions into Equations (2.24) through (2.27) and the simplification of the resulting equations leads to the general load-displacement equations for a cantilever beam with a uniformly distributed load. Thus, from the free end to the fixed end, the force expressions when $w = k$ are

$$V_{n-1} = V_n + k_n \Delta h_n \tag{2.28}$$

$$M_{n-1} = M_n + V \Delta h_n + k_n \Delta h_n^2/2. \tag{2.29}$$

Similarly, from the fixed end to the free end the displacement expressions are found as

$$\theta_n = \theta_{n-1} + \left(\frac{\Delta h_n}{EI_n}\right)\left(M_{n-1} - \frac{V_{n-1}\Delta h_n}{2} + \frac{k\Delta h_n^2}{6}\right) \tag{2.30}$$

$$y_n = y_{n-1} + \theta_{n-1}\Delta h_n + \left(\frac{\Delta h_n^2}{2EI_n}\right)\left(M_{n-1} - \frac{V_{n-1}\Delta h_n}{3} + \frac{k_n \Delta h_n^2}{12}\right) \tag{2.31}$$

<u>Linearly Varying Loading</u>

The load-displacement equations for a linearly varying loading are determined by selecting $\alpha = 1$. For this value of α, Equation (2.23) assumes the following forms:

$$\Delta V_n = (h_n^2 - h_{n-1}^2)(k_n/2)$$

$$\Delta M_n = (h_n^3 - h_{n-1}^3)(k_n/3)$$

$$\Delta \theta_n = (h_n^4 - h_{n-1}^4)(k_n/8EI)$$

$$\Delta y_n = (h_n^5 - h_{n-1}^5)(k_n/30EI).$$

Substitutions of these expressions into Equations (2.24) through (2.27) and the subsequent simplification leads to the load-displacement equations for a cantilever beam with a linearly varying load. Thus, from the free end to the fixed end, the force expressions for $w_n = kh$ are

$$V_{n-1} = V_n + \frac{k\Delta h_n}{2}(h_n + h_{n-1}) \qquad (2.32$$

$$M_{n-1} = M_n + V_n \Delta h_n + \frac{k\Delta h_n^2}{6}(2h_n + h_{n-1}). \qquad (2.33)$$

And from the fixed end to the free end, the displacement expressions are

$$\theta_n = \theta_{n-1} + \left(\frac{\Delta h_n}{EI}\right)\left[M_{n-1} - \frac{V_{n-1}\Delta h_n}{2} + \left(\frac{k\Delta h^2}{24}\right)(h_n + 3h_{n-1})\right] \qquad (2.34)$$

$$y_n = y_{n-1} + \theta_{n-1}\Delta h_n + \left(\frac{\Delta h_n^2}{2EI}\right)\left[M_{n-1} - \frac{V_{n-1}\Delta h_n}{3} + \left(\frac{k\Delta h^2}{60}\right)\right.$$
$$\left.(h_n + 4h_{n-1})\right]. \qquad (2.35)$$

CHAPTER III

THE FREE STANDING STACK

Introduction

For analytical purposes, the free standing stack is classified as a vertical cantilever rather than as a column. This classification results from consideration of the nature of its primary loads and of its characteristic structural response to environmental excitation. Thus, the bending properties of the free standing stack are of primary importance.

As an exposed structure, the stack is expected to survive in general the forces of nature, but particularly and within specific bounds, the wind and the earthquake. When subjected to these natural phenomena, the stack displays several condition-dependent modes of response. For analytical purposes, these modes have been idealized and classified as static and forced dynamic.

There is one other phenomenon which elicits structural response of sufficient severity to warrant consideration. This phenomenon is the blast resulting from an explosion. Blast loading in general is a combination of overpressure and drag force and it elicits that mode of response which has been idealized and classified as the dynamic response to an impulsive force. This third phenomenon will not be treated in this study.

Static Analysis

The Static Wind

The drag component of wind force, though having dynamic characteristics, is converted to an equivalent static force through application of the gust factor. Making this conversion allows development of an equivalent static load as follows:

In Appendix B, the effective wind velocity is defined as

$$U_e = U_d \sqrt{G_f} \ ,$$

the wind profile as

$$U_h = U_e \ (h/h_d),$$

and the drag force as

$$w_d = C_d \ (pU_h^2/2)D.$$

The successive substitution of the former expressions into the latter yields an expression for the equivalent static loading in the form

$$w_h = K_h h^\alpha$$

where

$$K_h = (C_d \ D_n \ G_f \ pU_d^2/2h_d^\alpha) \qquad (3.1)$$

$$\alpha = 2\gamma$$

and γ is a topological related exponent defined in Reference (1).

Since this function is of the same form as that selected in Chapter II to represent the load function, then Equations (2.23) through (2.27) can be used to calculate the internal forces and subsequently the displacement for a free standing stack subjected to an effectively static wind loading.

The Static Earthquake

In the design of structures for earthquakes, the loadings specified by code do not represent expected conditions. Rather, they are supposed to result in the provision of sufficient strength to assure, with some probability, the survival of the structure. Thus it justifies the representation of the dynamic character of the earthquake as an equivalent static condition.

The procedure presented herein for representation of an equivalent static earthquake is based on the assumption that only the first mode of vibration is significant. This procedure, developed by the Structural Engineers Association of California, is widely accepted and is cited in building codes (1) and in texts (2, 11). It is not intended to derive or explain the procedure herein but rather to display the correlation of the equations developed in Chapter II with its use.

By the referenced procedure, the total dynamic lateral force is

$$V_o = Z\, K_s\, C_s\, W \tag{3.2}$$

where the seismic coefficient

$$C_s = 0.05\, \sqrt[3]{f_n} \tag{3.3}$$

the natural frequency f_n is defined in the next section of this chapter, the earthquake factor Z and the force factor K_s are constants established by the code and the total weight of the structure

$$W = \sum_{n=1}^{n} u_n\, \Delta h_i . \tag{3.4}$$

(Note that this expression can be calculated by evaluating Equation (2.28) at n = 0 when $k_n = u_n$.)

The dynamic lateral force V_o is to be distributed over the height of the stack as

$$q = (0.85\ V_o/M_o^s)\ u_n\ h = k_n^o h \qquad (3.5)$$

with 0.15 V_o acting as a fictitious load at the top of the stack and where M_o^s, the static gravity moment of stack weight taken about the base, is calculated by evaluating Equation (2.29) at n = 0 when $k_n = q_n$.

Noting that the equivalent static distributed lateral load q is of the form kh allows the calculation of the equivalent static shearing force V_n^d and moment M_n^d using Equations (2.32) and (2.33). Noting that $V_n = 0.15\ V_o$, then

$$V_{n-1}^d = V_n^d + (k_n \Delta h_n/2)(h_n + h_{n-1}) \qquad (3.6)$$

$$M_{n-1}^d = J_{n-1} M_{n-1}^k = J_{n-1}[M_n^k + V_n^d \Delta h_n + (K_n \Delta h_n^2/6)(2h_n + h_{n-1})] \qquad (3.7)$$

where

$$J_{n-1} = (0.6f^{2/3})[1 - (h_{n-1}/h_n)^3] + (h_{n-1}/h_n)^3.$$

The Stack and Dynamics

In the introduction to this chapter, one mode of dynamic stack response was designated. Further attention will be devoted to that classification to establish the relationship between the exciting function and the dynamic response of the stack, to idealize that relationship for analytical suitability and to define the limitations of the area of consideration. However, as a prerequisite for the exercise thus outlined, it is necessary to establish an expression for calculation of the natural frequency.

Natural Frequency

The fundamental natural frequency of the free standing stack will be calculated using the Rayleigh Method (see Appendix A). The implementation of this method is effected through the assumption of a deflected shape reasonably representative of the responsive configuration ensuing from dynamic excitation. The departure of this assumed configuration from the actual is reflected in an added amount of strain energy which consequently affects the calculation of a frequency greater in magnitude than the true value. Since the calculated frequency will exceed or at least equal the true frequency, the Rayleigh Method establishes an upper bound.

For the calculation of the fundamental natural frequency of a beam, it is customary to assume the shape defined by the static deflection of the structure under its own weight. In the case of the free standing stack it is observed that the mass of the structure is uniformly distributed within specific intervals and this consequently allows the static deflection curve to be defined by Equation (2.31). Thus, allowing the point n to become any arbitrary point within the interval (n - 1, n), the deflection for that point is

$$y = y_{n-1} + \theta_{n-1}\Delta h + (\Delta h^2/2EI)(M_{n-1}+V_{n-1}\Delta h/3 + k_n\Delta h^2/12) \tag{3.9}$$

where $k_n = u_n$, the linear weight within the nth interval, and the other terms are as defined by Equations (2.28), (2.29) and (2.30).

From Appendix A, the equation for the natural frequency is obtained as

$$f = (\tfrac{1}{2}\pi)\ \sqrt{\Omega_{max}/K_e} \tag{3.10}$$

where

$$\Omega_{max} = W_e = \frac{1}{2} \sum_{1}^{n} u_n \int_{o}^{\Delta hn} y \; d(\Delta h)$$

and

$$K_e = (\frac{1}{2g}) \sum_{1}^{n} u_n \int_{o}^{\Delta hn} y^2 \; d(\Delta h)$$

Substitution of the above expression for y into the latter two equations and the subsequent integration of both yields

$$\Omega_{max} = \sum_{1}^{n} (\frac{u_n \Delta h_n}{4})\Big[2y_{n-1} + \theta_{n-1}\Delta hn + \frac{M_{n-1}\Delta h_n^2}{3(EI)_n} - \frac{V_{n-1}\Delta h_n^3}{12(EI)_n}$$
$$+ \frac{U_n \Delta h_n^4}{60(EI)n}\Big] \tag{3.11}$$

$$K_e = \sum_{1}^{n} (\frac{U_n \Delta h_n}{2g})\Big[y_{n-1}^2 + (y_{n-1}\theta_{n-1}\Delta h_n) + (\theta_{n-1}^2 + \frac{U_{n-1}M_{n-1}}{3(EI)_n})$$
$$(\frac{\Delta h_n^2}{3}) - (\frac{y_{n-1}V_{n-1} - 3\theta_{n-1}M_{n-1}}{12(EI)_n})(\Delta h_n^3)$$
$$+ (\frac{y_{n-1}M_n - 4\theta_{n-1}V_{n-1}}{(EI)_n} + \frac{3M_{n-1}^2}{(EI)_n^2})(\frac{\Delta h_n^4}{60}) + (\frac{\theta_{n-1}u_n}{6} - \frac{V_{n-1}M_{n-1}}{3(EI)_n})$$
$$(\frac{\Delta h_n^5}{(12EI)_n}) + (\frac{u_n M_{n-1}}{2} + \frac{V_{n-1}^2}{3})(\frac{\Delta h_n^6}{84(EI)_n^2}) - (V_{n-1} - \frac{u_n \Delta h_n}{9})$$
$$(\frac{u_n \Delta h_n^7}{576(EI)_n^2})\Big] \tag{3.12}$$

The evaluation of these expressions and their substitution into Equation (3.10) now establishes the natural frequency of the mode shape assumed.

Dynamic Response

Vortex Shedding. In Chapter I, the fluid mechanism known as vortex shedding was presented as the phenomenon effecting fluctuations in the lift component of the wind force and consequently oscillatory structural response. Though not stated specifically, it was inferred and is hereby affirmed that vortex shedding even when random can elicit periodic response. The severity of the response, however, is dependent upon how nearly the frequency of the shedding coincides with natural frequency of the stack.[1] It is customary, therefore, to assume for analytical purposes that the frequency of vortex shedding is equal to the natural frequency of the stack (26). Consequently, it is possible to calculate first, the wind velocity corresponding to the assumed frequency of vortex shedding by making use of the Strouhal number and second, the internal dynamic forces.

In the reviewed literature, several theorems were set forth that are generally accepted yet lack sufficient evidence to establish them as facts. However, the use of these theorems is a necessity if the problem is to be solved in a timely manner. Thus, it is theorized that

1. Stacks vibrate at exactly their fundamental natural frequency (7, 28).

---

[1]See discussion in Appendix A.

2. The tendency is for the frequency of vortex shedding to be constant over a substantial part of the height at a value determined by the wind speed and diameter at the top of the stack (32).

3. Periodic shedding can be represented as a sinusoidal forcing function (13) even though the evidence dictates that the periodicity is otherwise (5, 25).

Periodic Excitation. In accordance with the last theorization, the exciting force for lateral oscillatory motion is established as a sinusoidal function when the lift component of the wind force is periodic (i.e., critical and transcritical ranges of Reynolds number). Thus,

$$w(h,t) = w_1 \sin \omega_v t$$

where ω_v is the frequency of vortex shedding. The general equations of motion may then be written in matrix notation as

$$[m]\{\ddot{y}\} + [2\ ]\{\dot{y}\} + [k]\{y\} = \{w(h,t)\} \tag{3.13}$$

Equating the frequency of vortex shedding to the fundamental natural frequency of the stack allows the solution of the equation (as in Appendix A) for the dynamic displacements. Thus

$$y_o = \left(\frac{F_e}{2\beta w^2\ M_e}\right)\left(\frac{y}{y_n}\right) \tag{3.14}$$

where (y/y_n) are lateral static displacements normalized on the tip displacement,

$$M_e = \left(\frac{1}{2g}\right) \sum_{1}^{n} u_n \int_{o}^{\Delta h} (y/y_n)^2\ d(\Delta h) = K_e/y_n^2 \tag{3.15}$$

and K_e is evaluated by Equation (3.11).

$$F_e = \sum_{1}^{n} \int_{h_{n-1}}^{h_n} w_1 (y/y_n) dh \; . \tag{3.16}$$

The expression for the linear lift force is developed from Equation (1.5) as

$$w_1 = \frac{1}{2} C_1 \; \rho \; D_n \; U_n^2 \; (h/h_n)^{\alpha} \; . \tag{3.17}$$

where $\alpha = 2\gamma$.

The expression defining y is given by an alternate form of Equation (3.9). Substituting $(h - h_{n-1})$ for the term Δh, the equation becomes

$$y_n = y_{n-1} + \theta_{n-1} (h-h_{n-1}) + \left(\frac{h-h_{n-1}}{2EI_n}\right)^2 \left[M_{n-1} - \frac{V_{n-1}(h-h_{n-1})}{3} + \frac{u_n (h-h_{n-1})^2}{12}\right]. \tag{3.18}$$

The quantity y_n is simply y_h evaluated at $h = h_n$ and thus a constant.

Substitution of Equations (3.17) and (3.18) into Equation (3.16) and the subsequent integration leads to an expression for the generalized force as

$$F_e = C_1 K_f \; U_n^2 / y_n \tag{3.19}$$

$$K_f = \frac{\rho}{2h_n^{\alpha}} \sum_{n=1}^{n} D_n \Big[\left(\frac{h_n^{1+\alpha} - h_{n-1}^{1+\alpha}}{1+\alpha}\right) \left(y_{n-1} - \theta_{n-1} \, h_{n-1} + \frac{M_{n-1} h_{n-1}^2}{2EI_n} + \frac{V_{n-1} h_{n-1}^3}{6EI_n} + \frac{u_n h_{n-1}^4}{24EI_n}\right) + \left(\frac{h_n^{2+\alpha} - h_{n-1}^{2+\alpha}}{2+\alpha}\right) \left(\theta_{n-1} - \frac{M_{n-1} h_{n-1}}{EI_n} - \frac{V_{n-1} h_{n-1}^2}{2EI_n} - \frac{u_n h_{n-1}^3}{6EI_n}\right) + \left(\frac{h_n^{3+\alpha} - h_{n-1}^{3+\alpha}}{3+\alpha}\right) \left(\frac{M_{n-1}}{2EI_n} + \frac{V_{n-1} h_{n-1}}{2EI_n}$$

$$+ \frac{u_n h_{n-1}^2}{4EI_n}) - (\frac{h_n^{4+\alpha} - h_{n-1}^{4+\alpha}}{4+\alpha})(\frac{V_{n-1} + u_n h_{n-1}}{6EI_n}) + (\frac{h_n^{5+\alpha} - h_{n-1}^{5+\alpha}}{5+\alpha})$$

$$(\frac{u_n}{24EI_n})] \cdot \tag{3.20}$$

Substitution of Equations (3.15) and (3.19) into Equation (3.14) leads to an expression for the dynamic displacements as

$$y_D = (\frac{K_f}{K_e})(\frac{C_1 y}{2\beta})(\frac{U_n}{w})^2 .$$

However, recalling that w is the frequency of vortex shedding equal to the natural frequency of the stack, then the term (U_n/w) may be written in terms of the Strouhal number S. Thus,

$$(U_n/w) = (2\pi D_n/S_D)$$

and then

$$y_D = (\frac{K_f}{K_e})(\frac{C_1 y}{2\beta})(\frac{2\pi D_n}{S_D})^2 \tag{3.21}$$

where

$$S_D = S/(1 + 1.54\ y_{Dn}/D_n) \tag{3.22}$$

and the quantity (y_{Dn}/D_n) is subject to the limitation specified in Appendix B.

The simultaneous solution of Equations (3.21) and (3.22) will yield an expression for y_{Dn}, consequently allowing calculation of the wind velocity at which the shedding frequency and the natural frequency are equal.

Random Excitation. In Chapter I it was stated that when the wind velocity was such that Reynolds number falls in the supercritical range $(3.0 \times 10^5 < Re < 3.5 \times 10^6)$, vortex shedding is random. For this situation, the frequency of vortex shedding is selected as the random variable and its mathematical representation is effected through the use of an approach from the theory of random functions known as spectral density (8). By the method outlined in Appendix A, the mean-square value of the dynamic load function is found as

$$\overline{q^2} = \left[\frac{Q_e}{2\beta w^2 M_e}\right]^2 \left[\frac{w\beta D_n \overline{C_l^2} P(S)}{2U_n}\right]$$

where

$$Q_e = \sum_{n=1}^{n} \int_{h_{n-1}}^{h_n} P_h \left(\frac{y}{y_n}\right) dh = K_f U_n^2 / y_n$$

and $\overline{C_l^2}$, the mean-square lift coefficient, and P(S), the normalized spectral density of the Strouhal number are defined in Appendix B.

The root-mean square value of the dynamic load function q is equivalent statistically to the standard deviation or σ-value. Thus, the 3σ-value of q represents a dynamic load which by probability will be exceeded only 0.27 percent of the time. Selection of the 3σ-value for the load function leads to the calculation of the corresponding maximum dynamic displacement as

$$y_D = (y/y_n)(3\sqrt{\overline{q^2}})$$

or

$$y_D = \mathcal{L}(y/y_n)$$

where the dynamic factor

$$\mathcal{L} = \left[\frac{3\; K_f D_n^2 y_n}{K_f}\right]\left[\frac{\overline{C_l^2} P(S)}{S^3}\right]^{0.5} .$$

<u>Internal Forces</u>. The forces V_n^D and M_n^D which occur internally at the time the stack is experiencing the dynamic displacements y_D are found as products of the forces corresponding to the normalized displacements and the dynamic factor $\mathcal{L}$. Establishing the normalized forces as

$$v_n = V_n(y_n/y_n)$$

and

$$m_n = M_n(y_n/y_n)$$

where V_n, M_n and y_n are defined by Equations (2.28), (2.29) and (2.31), and y_n is y_n at n = N when k_n is the weight per foot of the stack, then the dynamic forces are

$$V_n^D = \mathcal{L} v_n \quad (3.23)$$

$$M_n^D = \mathcal{L} m_n . \quad (3.24)$$

CHAPTER IV

THE GUY WIRE

Introduction

General Description

Structurally, the guy wire is a flexible tension member, ordinarily of steel, employed to provide lateral support for stacks and other tower-like structures. It may be constructed of a strand, a rope or a group of strands or ropes which act as a unit.

The construction of the guy wire is important in that it establishes physical characteristics which are pertinent to both its use and its analysis. By definition, a strand is an assembly of wires formed helically around a central wire in one or more symmetrical layers. Similarly, rope is an assembly of strands laid helically around a core typically composed of steel wires in the form of an independent wire rope or strand. The constructional difference thus defined influences the physical characteristics to the extent that strand possesses greater strength, a higher modulus of elasticity, less flexibility, and greater corrosion resistance than rope of the same diameter, material and protective coating (23).

As a structural member, the guy wire is unique in that its load-carrying capability is limited to resisting only those forces which introduce tension. The uniqueness of this property necessitates the

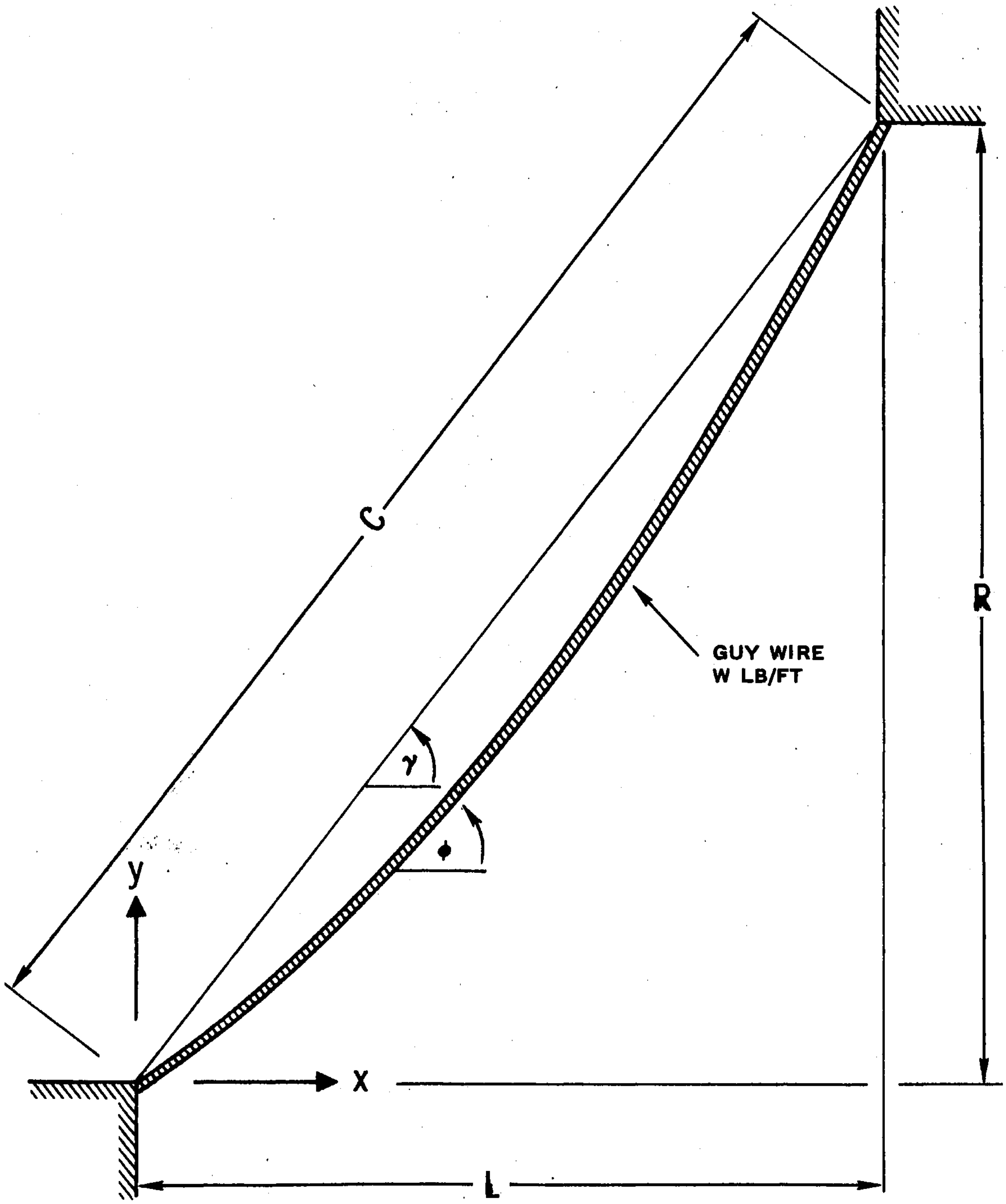


Figure 10. Freely Hanging Guy Wire

development of theoretical expressions to model mathematically the structural behavior of the guy wire. Relative to the support of stacks, the pertinent aspects of structural behavior are the general configuration, the internal tensile force as a function of configuration, the guy wire as a spring, and the change in internal load effected by either support movement or a change in the linear weight of the guy wire.

The Configuration

A guy wire loaded with its own weight and hanging freely between its supports assumes the shape of a curve called the "catenary" (1). The specific form of the equation of this curve as applied to guy wires is developed in Appendix C and is given as:

$$y = \frac{H}{w} [\cosh(\frac{wx}{H} + a) - \cosh a]$$

where

$$a = \sinh^{-1} [\frac{wR/2H}{\sinh (wL/2H)}] - \frac{wL}{2H} .$$

In the past this expression, which appears potentially cumbersome, was avoided by developing the equation of the curve for a guy wire with a horizontal chord (i.e., R = 0) and with the origin at the low point of the curve. Subsequently, the developed expression was adapted to the guy wire with an inclined chord by approximating the developed expression by that of a parabola, rotating the coordinate system and redefining some of the parameters. This approach and its subsequent development is known as the "General Cable Theory" (34).

The application of the parabola-for-catenary approach to the problem of inclined guy wires and the approximations necessitated by the subsequent development leads to potentially significant errors in the mathematical representation of the physical system. Hence, it is desirable to seek an alternate approach to the solution of the problem. In general, the analytical philosophy employed for the development of the "General Cable Theory" was to first approximate the catenary and then to expand the theory as based on that approximation. Conversely, the philosophy of the following approach is to expand the theory as based on the catenary and subsequently to approximate the developed expressions.

Assumptions

The assumptions made to facilitate the development are as follows:

1. The ability of the guy wire to resist or transmit axial loads and transverse loads is effected only through a change in the tensile force in the guy wire.

2. The guy wire is completely flexible and thus not capable of transmitting or resisting bending moments.

3. The only externally applied load acting on the guy wire between the supports is that induced by the force of gravity acting on the mass of the guy wire and any coating (such as ice) it might have.

4. The average initial tension in the guy wire is known.

These assumptions and the equations of the catenary as developed in Appendix C are applicable to any freely hanging cable or chain. However, since the following presentation is to be restircted to the "tight" guy wire, it is further assumed that

5. The tensile force in the guy wire is sufficiently large with respect to the weight of the guy wire as to restrict the values of the quantity (wL/H) to magnitudes much less than one.

6. The movement of one support relative to the other is sufficiently small to preclude a significant change in functions at the angle γ which defines the slope of the chord.

Approximated Expressions

Internal Force Versus Length

The stressed length of the freely hanging guy wire is defined by the relationship

$$S = [R^2 + \{\frac{2H}{w} \sinh (\frac{wL}{2H})\}^2]^{0.5} .$$

This relationship may be rewritten as

$$\sqrt{S^2 - R^2} = \frac{2H}{w} \sinh \frac{wL}{2H} .$$

Replacing the hyperbolic sine function with its power series representation yields

$$\sqrt{S^2 - R^2} = L\ [1 + \frac{1}{3!} (\frac{wL}{2H})^2 + \frac{1}{5!} (\frac{wL}{2H})^4 + \ldots + \frac{1}{n!} (\frac{wL}{2H})^{n-1}].$$

Since the quantity (wL/H) has been restricted to values much less than one, then the terms of the fourth order and higher are sufficiently small as to preclude their significance, thus

$$\sqrt{S^2 - R^2} = L\ [1 + \frac{1}{6} (\frac{wL}{2H})^2]$$

and

$$S^2 - R^2 = L^2\ [1 + \frac{1}{3} (\frac{wL}{2H})^2 + \cancelto{0}{\frac{1}{36} (\frac{wL}{2H})^4}] .$$

Employing the relationships $C^2 = L^2 + R^2$, $\sec\gamma = C/L$ and the approximation which states that as $k \rightarrow 0$, the quantity $(\sqrt{1 + k}) \rightarrow (1 + k/2)$, it is found that

$$S \cong C \left[1 + \frac{(wL/2H)^2}{6 \sec^2\gamma}\right].$$

Finally, letting $W = wL \sec\gamma$ and employing the relationship $T_r = H \sec\gamma$ allows the inplace length of the guy wire to be redefined as

$$S = C \left[1 + \frac{1}{24} \left(\frac{W}{T_r \sec\gamma}\right)^2\right]. \qquad (4.1)$$

Elastic Stretch

As fabricated, wire rope is an assembly of helically wound wires. During the fabrication process, the individual wires are partially yielded such that if released they would assume some configuration between their initial and as wound helical configurations. In the finished product, this captured "spring back" is responsible for a nonlinearly elastic characteristic.

For some applications of wire rope, the nonlinearly elastic characteristic is not important. For guy wires, however, it is undesirable because as wire rope is stressed, the magnitude of the captured "spring back" is reduced through further local yielding at the wires and thus some unpredictable amount of permanent set is encountered. Permanent set thus induced in guys can allow the occurrence of detrimental stresses in the supported structure. Consideration of this factor leads to the conclusion that it is highly desirable to prestress wire rope which is to be used for guy wires. Wire rope thus treated approaches being linearly elastic.

Assuming then that prestressed wire rope is to be used, the elastic stretch of the guy wire is defined essentially by the expression

$$\Delta S = T_{avg}\ S/AE$$

where

$$T_{avg} = \frac{H}{2S}\left[L + \frac{H}{w}\sinh\left(\frac{wL}{H}\right)\left\{1 + 2\left(\frac{wR/2H}{\sinh wL/2H}\right)^2\right\}\right].$$

Since this development is restricted to the consideration of "tight" guy wires and the consequent assumption that the quantity (wL/H) is much less than one, it is observed that as $(wL/H) \rightarrow 0$, $\sinh (wL/H) \rightarrow (wL/H)$ and thus,

$$T_{avg} \rightarrow \frac{H}{2S}\left[L + L\left\{1 + 2\left(\frac{R}{L}\right)^2\right\}\right] = \frac{HL}{S}\sec^2\gamma.$$

But, $H\sec\gamma = T_r$

and $L\sec\gamma = C$.

Thus, as

$$(wL/H) \rightarrow 0, \quad \Delta S \rightarrow \left(\frac{T_r C}{AE}\right). \tag{4.2}$$

This approximation will be used henceforth in the development.

The Unstressed Length

The unstressed length of the guy wire is found by subtracting the elastic stretch (Equation (4.2)) from the stressed length (Equation (4.1)). Thus,

$$S_o = C\left[1 + \frac{1}{24}\left(\frac{W}{T_r\sec\gamma}\right)^2 - \frac{T_r}{AE}\right]. \tag{4.3}$$

Further, when the initial tension is known or selected for a particular application, the magnitude of the unstressed length can be calculated.

Relative Movement of Supports

When one guy wire support moves a known amount relative to the other, the length and inclination of the chord are affected. Defining the support movement in terms of its components, Δx and Δy, allows the affected parameters to be expressed as

$$C_1 = [(R + \Delta Y)^2 + (L + \Delta x)^2]^{0.5},$$

$$\sec\gamma_1 = C_1/(L + \Delta x)$$

and

$$W_1 = wL_1 \sec\gamma_1 .$$

Since S_o, the unstressed length of the guy wire, is essentially independent of any subsequent variation in applied tensile force, then substitution of the above values into Equation (4.3) leads to a cubic equation in T_r, the guy wire tensile force and the only unknown. Thus,

$$T_r^3 + AE\left(\frac{S_o}{C_1} - 1\right) T_r^2 - \frac{1}{24} \frac{AEW_1^2}{\sec^2\gamma_1} = 0. \qquad (4.4)$$

Ambient Temperature Effects

Fluctuation in ambient temperatures induces changes in the internal forces of guyed structures. These force changes are brought about by changes in the rise R and guy wire length S with temperature while no change occurs in the span L. Defining the length of the guy wire at a temperature as

$$S_t = S(1 \pm \alpha\Delta t)$$

and the unstressed length at the temperature as

$$S_{o_t} = S_t - \Delta S.$$

Then $$S_o(1 \pm \alpha\Delta t) = S(1 + \alpha\Delta t) - \Delta S$$

or $$S_o = C_1\left[1 + \frac{1}{24}\left(\frac{W}{T_r \sec\gamma}\right)^2\right] - \frac{(T_r C_1/AE)}{(1 \pm \alpha\Delta t)} .$$

Rearranging yields a cubic equation in T_r as before

$$T_r^3 + AE(1 \pm \alpha\Delta t)\left(\frac{S_o}{C_1} - 1\right) T_r^2 - \frac{AE(1 \pm \alpha\Delta t)W_1^2}{24\sec^2\gamma_1} = 0 \qquad (4.5)$$

where

$$C_1 = [(R + \Delta y)^2 + L^2]^{0.5}$$

and

$$\Delta y = \pm(R\alpha\Delta t).$$

In northern regions, conditions favorable to icing will occur. In this case, not only do guy wires experience a thermal contraction but also an increase in linear weight. This effect is taken into account by letting $w = w_g + w_i$ where w_g and w_i are the linear weights of the guy wire and ice respectively, and w_i is assumed to be uniformly distributed. Then $W_i = (w_g + w_i)L\sec\gamma_1$.

General Displacement-Weight Expression

Since Equations (4.4) and (4.5) are both developed from Equation (4.3), they are quite similar. It follows that one equation may be written to represent both of the foregoing. Thus,

$$T_h^3 + fT_h^2 + g = 0.$$

The solution to this equation is found in Reference (16) as

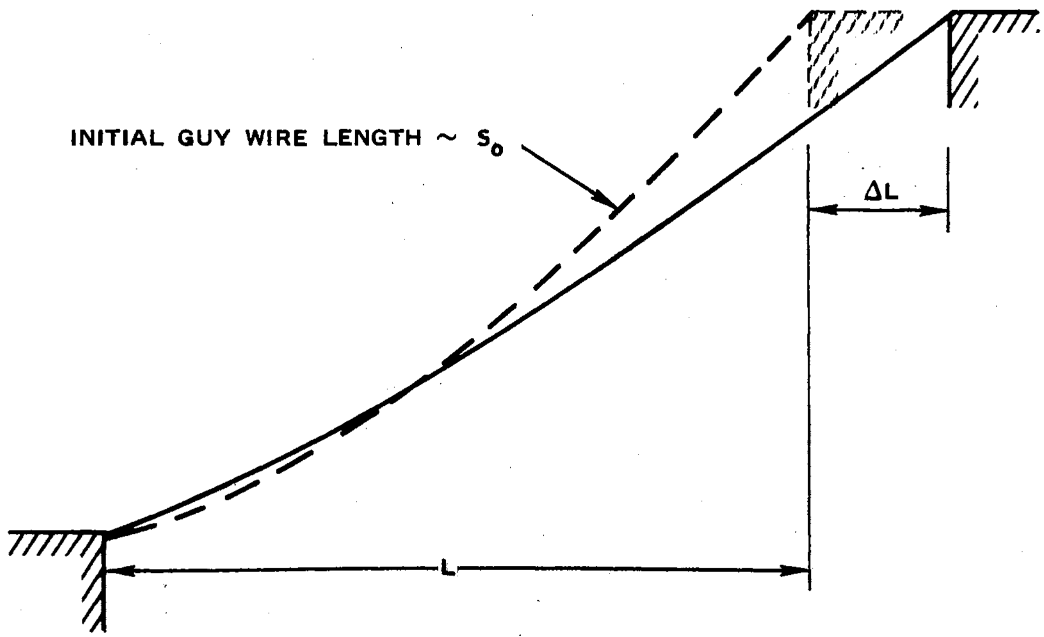


Figure 11. Horizontal Displacement of Support

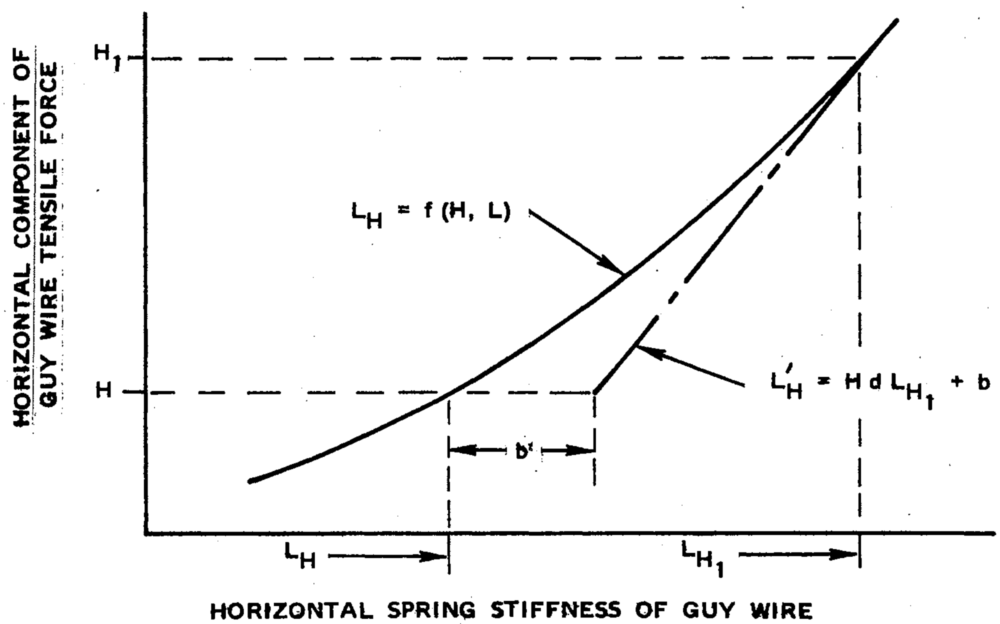


Figure 12. Horizontal Spring Stiffness of Guy Wire

$$T_h = b + c - (f/3)$$

where

$$b = [-0.5d + e]^{0.333}$$

$$c = [-0.5d - e]^{0.333}$$

$$d = (2f^3/27) + g$$

$$e = [(gf^3/27) + (g^2/4)]^{0.5}$$

$$f = (AE)(1 \pm \alpha\Delta t)[(S_o/C_1) - 1]$$

$$g = (AE)(1 \pm \alpha\Delta t)[W_1^2/24\sec^2\gamma_1]$$

$$C_1 = [R^2(1 \pm \alpha\Delta t \pm \Delta y/R)^2 + (L + \Delta x)^2]^{0.5}$$

$$\sec\gamma_1 = C_1/(L + \Delta x)$$

$$W_1 = (w_g + w_i)C_1.$$

<u>The Guy Wire as a Spring</u>

The guy wire behaves as a nonlinear spring in that with relative movement of supports both the curvature and the stressed length at the guy wire are affected. However, since the primary function of the guy wire is to provide lateral support, the movement considered will be restricted to the horizontal in the plane of the guy wire. Thus from Appendix C comes the expression

$$L_H = S_H \left(\frac{wS/H}{\sinh(wL/H)}\right) - \frac{2}{w} \tanh \frac{wL}{2H} + \frac{L}{H} . \qquad (C.18)$$

This expression defines the change in horizontal displacement as a curvilinear function of the total horizontal force. For analytical purposes, this form is not convenient and will be approximated by the tangent to the curve at the point (H_1, L_1). Thus,

$$L_H \cong L_{HH}(H - H_1) + L_{H_1}$$

where

$$L_{HH} = (SS_{HH} + S_H^2)\left(\frac{w/H_1}{\sinh(wL/H_1)}\right) - \left(\frac{2}{wH_1} \tanh \frac{wL}{2H_1}\right)$$

$$- \frac{2}{H_1}\left(L_{H_1} - \frac{L}{H_1}\right) - \frac{W}{H_1} \coth \frac{wL}{H_1}\left(L_H - \frac{L}{H_1}\right)^2.$$

For tight guy wires, the expressions for L_{H_1} and L_{HH} simplify to

$$L_{H_1} = \left(\frac{S_o \sec^2\gamma/AE}{1+\frac{1}{6}(wL/H_1)^2}\right) \tag{4.6}$$

and

$$L_{HH} = \left(\frac{S_o \sec\gamma/AE}{1+\frac{1}{6}(wL/H_1)^2}\right) - \left(\frac{L/H_1^2}{1-\frac{1}{12}(wL/H_1)^2}\right). \tag{4.7}$$

The horizontal displacement may now be expressed in the form

$$\Delta L = KH + b \tag{4.8}$$

where the elastic portion of the displacement is represented by KH where $K = L_{HH}$ at H_1. Similarly, the sag reduction portion of the displacement is represented by b where $b = (L_{H_1} - L_{HH}\,H)$ where L_{H_1} and L_{HH} are evaluated at H_1.

CHAPTER V

GUY SUPPORTED STACKS

Introduction

In the same manner that the free standing stack was considered as a vertical cantilever, the guy supported stack will be considered as a beam fixed at one end and resting on a number of flexible supports elsewhere. The idealization of the guy supported stack infers, as before, that it is the bending properties which are of primary importance.

Though the environmental forces to which the guyed stack is subjected are identical to those which the free standing stack experiences, the modes of response are different because of the presence of the guy wires. Thus, for this analysis, only the static response of the structure will be considered.

Assumptions for Structural Idealization

The guy supported stack is, of course, statically indeterminate, the degree of redundancy being a function of the number of levels of guy attachment and the number of guy wires attached at each level. The problem will be simplified, however, through the adoption of several assumptions which are as follows:

1. The guy attachment rings are infinitely rigid radially, and circumferentially.

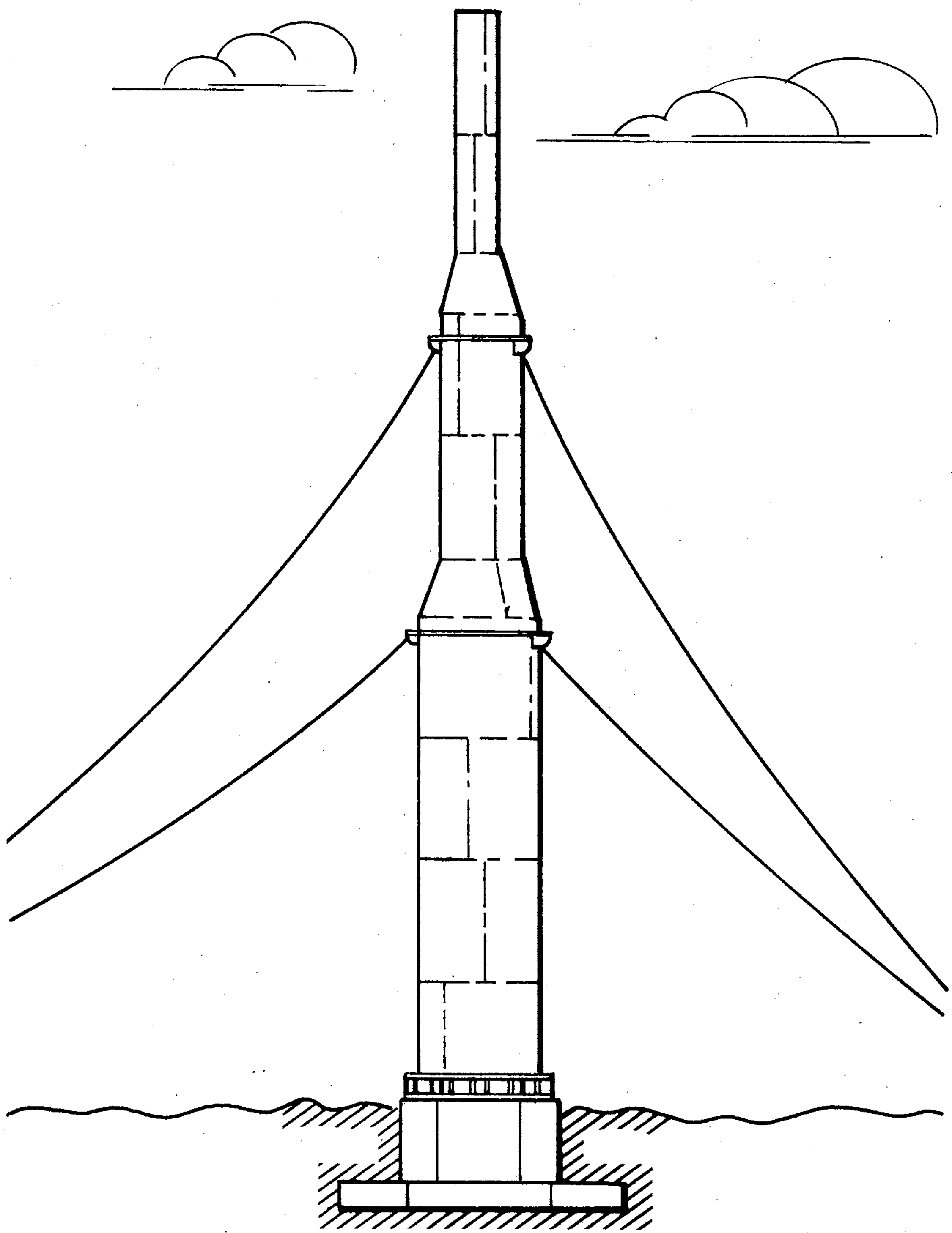


Figure 13. Guy Supported Stack

2. The vertical and rotational displacements of the attachment rings are sufficiently small as to be considered negligible.

3. The bending moments in the stack caused by the eccentric application of unequal guy wire forces are small with respect to the bending moments produced by the applied loads and thus need not be considered further.

4. The entire horizontal force acting at each level of guy attachment is carried by the tensioning guys (i.e., effectively no change in the load is considered for slackening guy).

The net effect of the assumptions above on the solution of the problem is that: a) it allows the stack to be treated as a two dimensional structure, and b) it restricts the unknown variables at each level of guy attachment to one horizontal displacement.

There is one other assumption which is ordinarily made relative to guy supported stacks. The assumption deals with the rotational fixity of the base. Customarily in structural analysis for reasons of expediency and for lack of better information, the bases of guyed stacks are assumed either fixed or pinned. It is recognized that in reality the fixity falls between these extremes and thus the question arises as to the better idealization. It is suggested, based on experience, that the assumption of a fixed base is closer to the true condition. It is recommended, however, that if there is any question on the part of the analyst, the assumption of a fixed base be made for analysis of the stack and the assumption of a pinned base be made for determination of maximum guy forces. Nevertheless, for the following development, the base of the stack will be regarded as having partial rotational fixity.

With the above assumptions, the problem has been established as having the same degree of redundancy as the number of levels of guywire attachment, the redundants selected being the net horizontal reactive forces of the guy supports. The idealization of a typical physical system is shown in Figure 14.

Orientation of the Guying System

If, as suggested above, the guy supported stack is idealized as having only one degree of freedom at each guy attachment level, then it becomes necessary to ascertain the orientation of the guy wire system relative to direction of the applied loads. The considerations responsible for this requirement are twofold. First, the use of only one degree of freedom at each guy support level necessitates the idealization of the guy wires as a composite system and the flexibility of such a system is a function of its orientation relative to the freedom direction. Second, since this is an exercise in design/analysis, the maximum guy wire internal forces are of primary interest and these too are a function of the system orientation.

For analytical purposes, the selected orientation of the guy wire system relative to the direction of the applied loads will be established through rationalization. For the three wire system pictured in Figure 15, it is argued that a loading system, whose bearing is defined by an angular departure of thirty degrees from the direction of one of the guy wires, will produce the maximum guy wire internal forces for that particular loading system. The rationale for this arrangement is that:

1. The portion of the load absorbed by the slackening guy is so small as to be considered negligible.

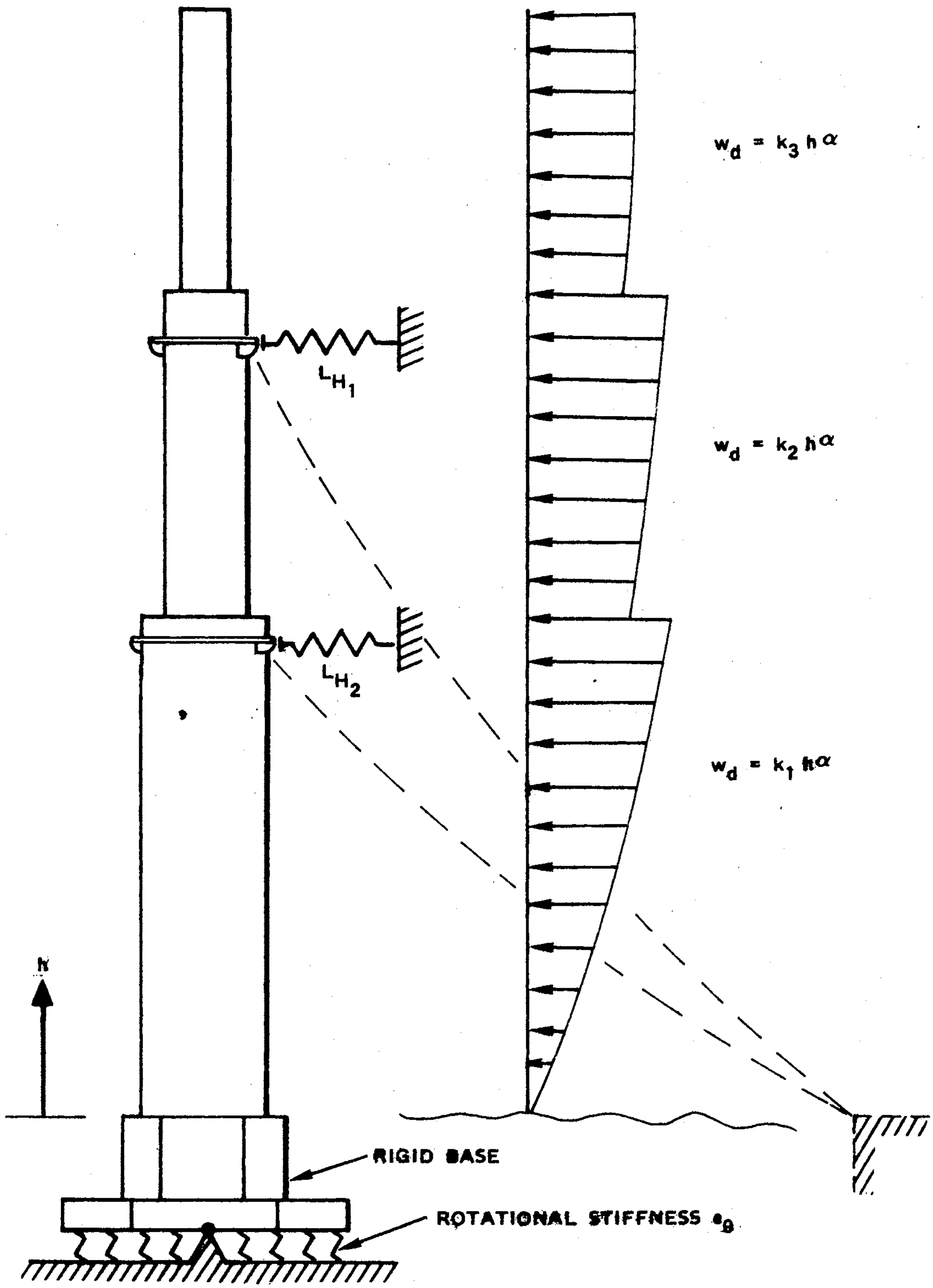


Figure 14. Idealized Guy Supported Stack

2. The guy directed normal to the load bearing has no load carrying capabilty in the direction of the loading system.

3. Since guy wires carry only tensioning loads and since the loading system is bearing thirty degrees relative to the direction of the only tensioning guy, then the loading must represent one component of the reactive force which falls in the vertical plane of the tensioning guy. Thus, the maximum horizontal component of guy wire force for a particular loading system is established as the reactive force for the guy composite system multiplied by the trigonometric secant function value for the thirty degree angular departure.

Analytical Approach

The method of analysis selected herein to effect the solution of the problem outlined above is the well-known Force Method (22). Implementation of this method entails the calculation of the displacements produced by unit forces applied at the redundant coordinates and the subsequent arrangement of those displacements into a rectangular array called the flexibility matrix. Employing the principle of superposition leads to the set of equations presented as

$$\begin{bmatrix} \delta_{11} & \delta_{12} & \ldots & \delta_{1s} \\ \delta_{21} & \delta_{22} & \ldots & \delta_{2s} \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \delta_{s1} & \delta_{s2} & \ldots & \delta_{ss} \end{bmatrix} \begin{Bmatrix} H_{R1} \\ H_{R2} \\ \cdot \\ \cdot \\ \cdot \\ H_{R} \end{Bmatrix} = \begin{Bmatrix} \Delta_1 \\ \Delta_2 \\ \cdot \\ \cdot \\ \cdot \\ \Delta_s \end{Bmatrix}$$

where the subscript s denotes the number of redundants and the Δ_j's are the net displacements in the direction of the redundants.

For this particular application of the Force Method, the Δ_j's are evaluated by employing the principle of superposition. Thus for each point j,

$$\Delta_j = \Delta_j^F - H_{Rj}\ \delta_j^R$$

where Δ_j^F is the free displacement of the loaded stack at the location of the redundants, H_{Rj} is the redundant force as above and δ_j^R is the displacement produced by a unit force on the supportive structure. The substitution of these expressions for the Δ_j's in the above equations and the subsequent rearrangement yields the following set of equations

$$\begin{bmatrix} (\delta_{11}+\delta_1^R) & \delta_{12} & \cdots & \delta_{1S} \\ \delta_{21} & (\delta_{22}+\delta_2^R) & \cdots & \delta_{2S} \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \delta_{S1} & \delta_{S2} & \cdots & (\delta_{SS}+\delta_S^R) \end{bmatrix} \begin{Bmatrix} H_{R1} \\ H_{R2} \\ \cdot \\ \cdot \\ \cdot \\ H_{RS} \end{Bmatrix} = \begin{Bmatrix} \Delta_1^F \\ \Delta_2^F \\ \cdot \\ \cdot \\ \cdot \\ \Delta_S^F \end{Bmatrix}$$

The simultaneous solution of these equations produces the values of the redundant reactive forces H_R thereby making the structure statically determinate and allowing computation of the reactive force at the base of the stack.

Application of the Approach

Relative to the analysis of the guy supported stack, it is observed that the displacement parameters δ_{ij}, δ_j^R and Δ_j^F represent the unit displacements of the stack, the unit displacements of the composite guy system and the free displacement of the loaded stack, respectively. These quantities will now be defined through theoretical formulation.

Unit Displacements for the Stack

Recalling that the unit displacements are those displacements produced by unit lateral forces acting at the guy supports allows the adaptation of Equations (2.28) through (2.31) to define those displacements. For this application it is recognized that the distributed load function k_n is zero and that the only applied lateral load is the unit force applied at node j. Thus for $h_j < h_n \leq h_N$,

$$V_n = M_n = 0$$

and for $0 \leq h_n \leq h_j$,

$$V_{n-1} = V_n = 1$$

and

$$M_{n-1} = M_n + V_n \Delta h_N$$

or

$$M_{n-1} = (h_j - h_{n-1}).$$

It follows that the moment at the base of the stack is $Mo = h_j$ and at the foundation-soil interface, $M_b = (h_j + h_b)$ where h_b is the height of the footing.

Considering the concrete base as rigid and the rotational flexibility of the soil to be e_θ, the displacements at the base of the stack are

$$\theta_o = \theta_b = M_b e_\theta = (h_j + h_b) e_\theta$$

and

$$Y_o = \theta_b h_b = h_b (h_j + h_b) e_\theta.$$

Using these as initial values, the displacements δ_{ij} can be determined using Equations (2.30) and (2.31).

Free Displacements of the Loaded Stack

The free displacements Δ_j^F of the loaded stack are calculated by following the procedure used in Chapter II for the free standing stack. In this particular application, however, the calculation is somewhat complicated relative to the determination of the natural frequency, the value of which is required for both the gust factor G_f and the seismic coefficient C_s.

The upperbound for the natural frequency of the guyed stack is evaluated using the Rayleigh method as presented in Appendix A. The first step in the implementation of this method is definition of the statically deflected mode shape in the restrained position. This step may be effected by following the analytical procedure currently being discussed where the loading is the static distributed weight of the stack acting normal to the longitudinal axis.

It should be noted for the application that the displacements at the base of the stack are not zero as before but rather are

$$\theta_o = \theta_b = M_b e_\theta = (Mo + VoH_b)e_\theta$$

$$Y_o = \theta_b h_b = (Mo + Voh_b)e_\theta h_b.$$

Unit Displacements of the Supportive System

The unit displacements for the composite guy system are defined in terms of the along-wind component of the extension of the tensioning guy. Reference to Figure 16 shows that for small displacements, the along-wind component of guy extension is the product of the guy extension and the secant value of the thirty degree angle of departure. Thus

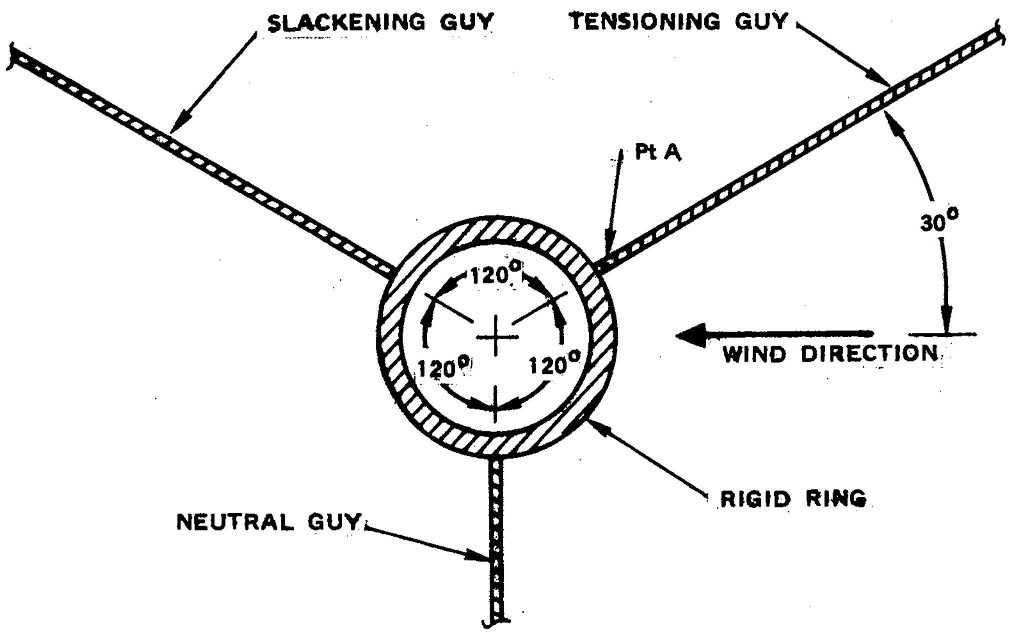


Figure 15. Wind Direction Versus Guy Wire Orientation

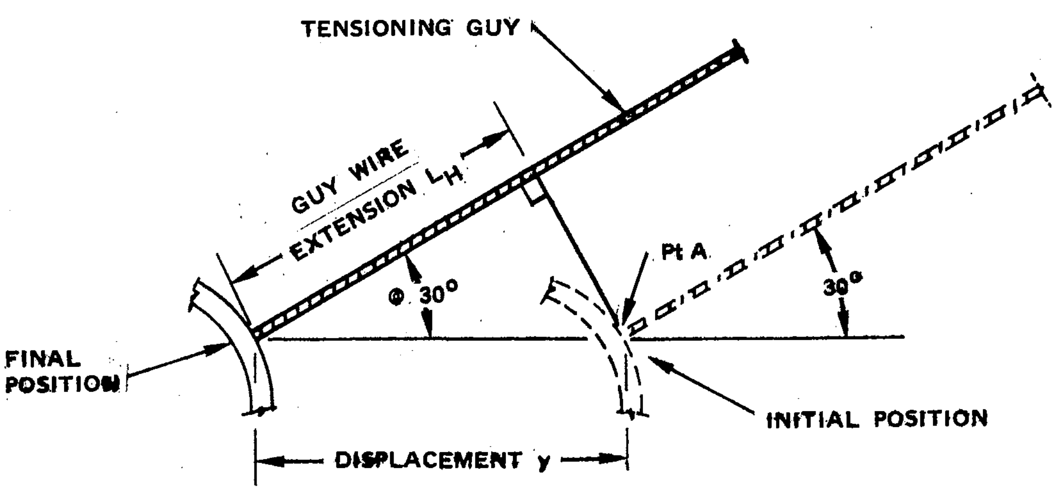


Figure 16. Horizontal Extension of Tensioning Guy

$$\delta_j^R = L_H \sec 30^\circ$$

where L_H, the horizontal component of guy wire extension is defined by Equation (4.10).

Mathematical Solution

Substitution of the displacement values determined by procedures established in the three previous paragraphs into Equations (5.1) establishes a set of simultaneous equations for the redundant reactive forces. These equations may be solved by any of the several conventional methods.

Internal Forces and Displacements

The internal forces and displacements for the guyed stack are evaluated for the along-force response of the structure by employing Equations (2.19) through (2.23) by the algebraic addition of the redundant reactive force to the shear forces at the appropriate points. Also, values of the tensile forces at point of maximum sag in the tensioning guy are found as

$$T_{ri} = H_{Ri} \sec 30^\circ \sec\gamma_i + T_{Ii}.$$

However, this does not complete this portion of the analysis since there is a lateral stack response which has not as yet been considered.

The presence of a lateral response of the guyed stack is determined by deductive reasoning. Thus, since the H_{Ri}'s are the horizontal along-force components of the forces in the tensioning guys, then there must be lateral components equal to $H_{Ri} \tan 30^\circ$. These lateral components must be reacted in part by the stack and in part by the neutral guy.

The analysis of the lateral response of the guyed stack is easily effected using the above analytical procedure. It is observed that

1. The unit displacements δ_{ij} for the stack are those calculated for the along-force portion of the analysis,

2. The free displacements δ_i^r for the loaded stack are evaluated as $\Delta_i^F = (H_{Ri} \tan 30°)\delta_{ii}$, and

3. The unit displacements δ_i^r for the supportive system are found as $\delta_i^R = L_{Hi}$.

Substitution of these latter displacement values into Equations (5.1) and the subsequent solution thereof establishes the lateral reactive forces. The internal forces and displacements are evaluated for the lateral response of the stack by using Equations (2.28) through (2.31). Subsequently, the total internal force at any particular point along the axis of the stack may be determined by the vectorial addition of the along-force and lateral components.

CHAPTER VI

SYSTEMATIC APPLICATION OF DEVELOPED EXPRESSIONS

Introduction

Customarily, the heights and the terminal diameters of stacks are established by the requirements of pollution codes relative to the composition of the waste stream vented by the stack into the atmosphere. It remains then for the structural analyst to determine the shell diameters and thicknesses which will provide strength adequate to withstand the environment specified by code. There are many "rules of thumb" used to establish the preliminary design and these will not be discussed. Rather, the procedural developments that follow will start with the assumption of a preliminarily designed structure. Concurrently, the implementation of these procedures is demonstrated through example problems.

The Free Standing Stack

Problem Definition

The stack pictured in Figure 17 is to be erected in a geographical location which is specified as a 125 mph Exposure C wind zone and an earthquake zone three (1). The process which the stack serves requires the stack height to be 150 feet and the terminal diameter to be four feet. The upper sections of the stack are to be lined with one and a half inches of refractory having a density of 90 pcf. The lower section is to be lined with three inches of the same refractory. The shell is to be made of ASTM A-283 Grade C steel (F_y = 30,000 psi, E = 29 x 10^6 psi, ρ_s = 490 pcf) having the shell thicknesses and diameter changes as shown in Figure 17.

Determination of the Fundamental Frequency

The first task to be accomplished in the analysis of the proposed stack is the determination of the fundamental frequency. This parameter is not calculated directly but rather is bounded by the following procedure. Intermediate numerical answers are displayed in Tables I and II.

Given Data

The data required to implement the analysis of the proposed stack are found in Figure 17 and in the paragraph entitled Problem Definition.

Step 1. Calculations

The following expressions are calculated by starting at the top of the stack and progressing to the base.

$$u_n = (n\rho_r/4)[D_n^2 - (D_n - 2t_{sn})^2] + (n\rho_r/4)[(D_n - 2t_{sn})^2 - (D_n - 2t_{sn} - 2t_{rn})^2]$$

$$V_{n-1} = V_n + U_n(h_n - h_{n-1})$$

$$M_{n-1} = M_n - \Delta h_n(V_n + \Delta V_n/2)$$

$$I_n = (\pi/64)[D_n^4 - (D_n - 2t_{sn})^4]$$

TABLE I

STEP 1. CALCULATED VALUES FOR DETERMINATION OF THE UPPER BOUND OF THE NATURAL FREQUENCY OF A FREE STANDING STACK

Point n (-)	Elevation of Point h (ft.)	Outside Diameter D (ft.)	Thickness: Shell t_s (in.)	Thickness: Refr. tr (in.)	Weight un (lb/ft)	Moment of Inertia I (in.^{4})	Shearing Force V (kips)	Bending Moment M (ft. kips)
5	150.0						0.0	0.0
		4.00	0.250	1.5	263	10689		
4	107.5						11.2	237.5
		6.75	0.250	1.5	448	51693		
3	55.0						34.7	1441.7
		6.75	0.375	1.5	555	77181		
2	32.5						47.2	2362.9
		9.00	0.375	3.0	1046	183584		
1	10.0						70.7	3689.4
		9.00	0.500	3.0	1187	243930		
0	0.0						82.5	4455.9

TABLE II

STEP 2. CALCULATED VALUES FOR DETERMINATION OF THE UPPER BOUND OF THE NATURAL FREQUENCY OF A FREE STANDING STACK

Point	Elevation on Point	Displacements		Energy Expressions	
		Rotational	Translational	Potential	Kinetic
n (-)	h (ft.)	θ (Radians)	y (ft.)	Ω (lb-ft)	K_e (lb/sec^2)
0	0.0	0.0	0.0	0.0	0.00
1	10.0	0.00083	0.00422	8.51	0.00
2	32.5	0.00264	0.04480	248.78	0.22
3	55.0	0.00536	0.13734	536.87	1.58
4	107.5	0.00908	0.54292	3814.13	43.61
5	150.0	0.01064	0.97851	4223.62	102.00
			Σ =	8831.97	147.41

Step 2. Calculations

The following expressions are evaluated by starting at the base of the stack and progressing to the top.

$$\theta_n = \theta_{n-1} + \frac{\Delta h_n}{EI_n}\left[M_{n-1} + \frac{\Delta h_n}{2}\left(-V_{n-1} + \frac{u_n \Delta h_n}{3}\right)\right]$$

$$y_n = y_{n-1} + \theta_{n-1}\Delta h_n + \left(\frac{\Delta h_n^2}{2EI_n}\right)\left[M_{n-1} + \frac{\Delta h_n}{3}\left(-V_{n-1} + \frac{u_n \Delta h_n}{4}\right)\right]$$

$$\sum_1^n \Omega_n = \Omega_{n-1} + \frac{u_n \Delta h_n}{4}\left[2y_{n-1} + \Delta h_n\left(\theta_{n-1} + \frac{\Delta h_n}{3EI_n}\right.\right.$$

$$\left.\left.\left\{M_{n-1} - \frac{V_{n-1}\Delta h_n}{4} + \frac{u_n \Delta h_n^2}{20}\right\}\right)\right]$$

$$\sum_1^n K_n = K_{n-1} + \frac{u_n \Delta h_n}{2g}\left[y_{n-1}^2 + \left(y_{n-1}\ \theta_{n-1}\Delta h_n\right)\right.$$

$$+ \left(\theta_{n-1}^2 + \frac{y_{n-1}M_{n-1}}{EI_n}\right)\frac{\Delta h_n^2}{3}$$

$$+ \left(\frac{-y_{n-1}V_{n-1} + 3\theta_{n-1}M_{n-1}}{12EI_n}\right)\Delta h_n^2$$

$$+ \left(y_{n-1}u_n - 4\theta_{n-1}V_{n-1} + \frac{3M_{n-1}^2}{EI_n}\right)\frac{\Delta h_n^4}{60EI_n}$$

$$+ \left(\theta_{n-1}u_n - \frac{2V_{n-1}M_{n-1}}{EI_n}\right)\frac{\Delta h_n^5}{72EI_n} + \left(\frac{u_n M_{n-1}}{2} + \frac{V_{n-1}^2}{3}\right)\frac{\Delta h_n^6}{84EI_n}$$

$$\left. + \left(-V_{n-1} + \frac{u_n \Delta h_n}{9}\right)\left(\frac{u_n \Delta h_n^7}{576(EI_n)^2}\right)\right].$$

The upper bound of the natural frequency is calculated as

$$f = \frac{1}{2\pi}\sqrt{\frac{\Sigma\Omega_n}{\Sigma K_n}} = \frac{1}{2\pi}\sqrt{\frac{8831.97}{147.41}} = 1.232 \text{ cps.}$$

Static Wind Analysis

The equivalent static forces experienced by the stack and the consequent displacements exhibited thereby as the stack is subjected to the theoretical force of a 125 mph wind are calculated following the procedure delineated below.

Gust Factor

Using the upper bound value of the natural frequency from above and the procedure set forth in Reference (1), the gust factor for the stack under consideration is found to be 1.431.

Wind Loading Constant

The wind loading constant K_n is defined by Equation (3.1) and evaluated using parameters from Reference (1). Thus,

$$K_h = C_d D_n G_f \rho U_d^2 / 2h_d^\alpha = 14.084\ D_n$$

when

$$C_d = 0.65$$

$$G_s = 1.431$$

$$\rho = 0.00512 \text{ lb/ft}^3$$

$$U_d = 125 \text{ mph}$$

$$h_d = 30 \text{ ft}$$

$$\alpha = 2\gamma = 2/7.$$

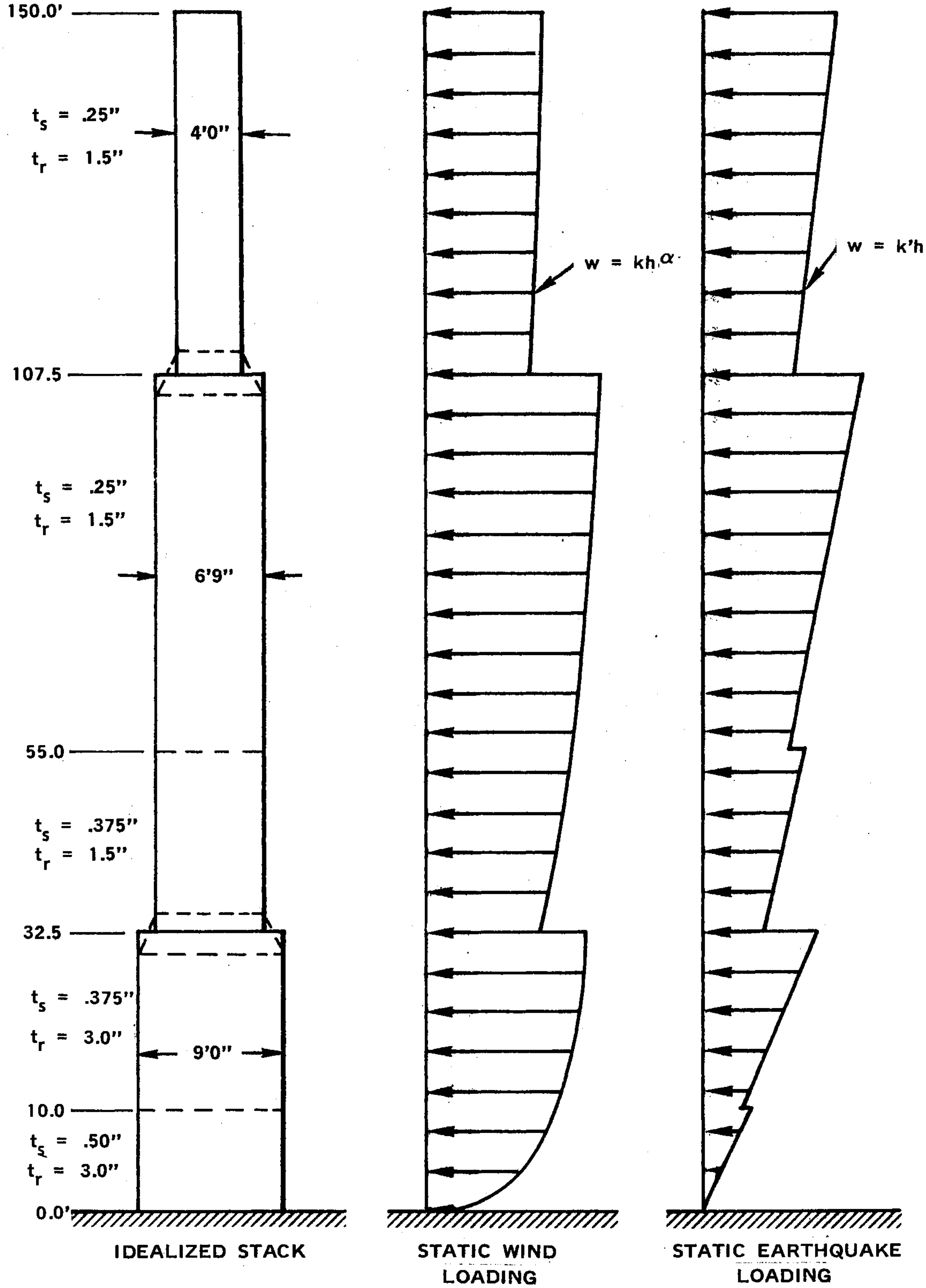


Figure 17. Idealized Free Standing Stack With Loadings

Static Wind Force

The equivalent static wind forces are calculated by applying Equations (2.23), (2.24) and (2.25). Beginning at the top of the stack where V_n and M_n are zero, and proceeding down the stack,

$$V_{n-1} = V_n + \Delta V_n$$

and

$$M_{n-1} = M_n + \Delta M_n + V_n \Delta h_n - \Delta V_n h_{n-1}$$

where

$$\Delta V_n = (h_n^{1+\alpha} - h_{n-1}^{1+\alpha})(K_n/1+\alpha)$$

$$\Delta M_n = (h_n^{2+\alpha} - h_{n-1}^{2+\alpha})(K_n/2+\alpha)$$

$$\Delta h_n = h_n - h_{n-1}$$

and as above $\alpha = 2\gamma = 2/7$.

Numerical values for these expressions related to the subject stack are given in Table III.

Static Wind Displacements

The displacements experienced by the stack when it is subjected to the equivalent static wind forces are determined by substituting the values calculated above into Equations (2.23), (2.26) and (2.27). Starting at the base of the stack where θ_0 and y_0 are zero and proceeding up the stack

$$\theta_n = \theta_{n-1} + \Delta\theta_n + \frac{(M_{n-1}\Delta h_n - \Delta M_n h_n)}{EI} - \frac{(V_{n-1}\Delta h_n^2 - \Delta V_n h_n^2)}{2EI}$$

and

TABLE III

STATIC WIND FORCE

Point n (-)	Elevation of Point h (ft)	Outside Diameter D (ft)	Loading Constant K_n (lb/ft^2)	Shearing Force ΔV (kips)	Shearing Force V (kips)	Bending Moment ΔM (ft-kips)	Bending Moment M (ft-kips)
5	150.0				0.00		0.0
		4.00	56.36	9.58		1508.4	
4	107.5				9.58		207.0
		6.75	95.06	17.47		1433.9	
3	55.0				27.05		1183.2
		6.75	95.06	6.28		276.7	
2	32.5				33.33		1864.4
		9.00	126.76	7.38		147.7	
1	10.0				40.71		2694.6
		9.00	126.76	1.91		10.7	
0	0.0				42.62		3106.4

$$y_1 = (y_{n-1} - \Delta y_n) + (\theta_{n-1}\Delta h_n + \Delta\theta_n h_n) + \frac{(M_{n-1}\Delta h_n^2 - \Delta M_n h_n^2)}{2EI} - \frac{(V_{n-1}\Delta h_n^3 - \Delta V_n h_n^3)}{6EI}$$

where

$$\Delta\theta_n = (h_n^{3+\alpha} - h_{n-1}^{3+\alpha})(K_h/3+\alpha)$$

$$\Delta y_n = (h_n^{4+\alpha} - h_{n-1}^{4+\alpha})(K_h/4+\alpha)$$

and the remaining terms are as defined above. Numerical values are shown in Table IV.

Static Earthquake Analysis

The equivalent static forces experienced by the stack when subjected to the theoretical forces of a zone three earthquake are calculated using the following procedure.

Total Static Lateral Load

The equivalent static inertial load acting on the stack is calculated as

$$V_o = ZK_sC_sW = 8.855^k$$

where from Reference (1), $Z = 1.0$, $K_s = 2$, the seismic coefficient $C_s = 0.05 \sqrt[3]{f} = 0.0536$, and the static weight W of the stack is obtained from the natural frequency calculations as 82.6^k.

Distribution of Lateral Load

The total static lateral load is distributed over the height of the stack in accordance with the expression

TABLE IV

STATIC WIND DISPLACEMENTS

Point n (-)	Elevation of Point n (ft)	Moment of Inertia I (in^4)	Slope* Δθ (Radians)	Slope* θ (Radians)	Deflection* Δy (in.)	Deflection* y (in.)
0	0.0			0.0		0.0
		243930	0.0		0.0	
1	10.0			0.00059		0.036
		183584	0.00005		0.005	
2	32.5			0.00197		0.393
		77181	0.00040		0.073	
3	55.0			0.00416		1.143
		51693	0.00595		2.044	
4	107.5			0.00728		5.104
		10689	0.03815		19.674	
5	150.0			0.00865		9.340

*Modulus of Elasticity $E = 29 \times 10^6$ psi

$$q_h = (0.85\ V_o/M_o^s)\ u_n h = K_n^D h$$

where $K_h^D = (1.689 \times 10^{-3})\ u_n$ and M_o^s is the static gravity moment of the linearly distributed stack weight u_n acting normal to the height and taken about the base of the stack. A fictitious concentrated load equal to 0.15 V_o acts at the top of the stack.

Static Earthquake Forces

The equivalent static forces for the earthquake are calculated in accordance with Reference (1) using Equations (2.32) and (2.33). Thus

$$V_{n-1}^D = V_n^D + K_n^D \Delta h_n (h_n + h_{n-1})$$

$$M_{n-1}^k = M_n^k + V_n^D \Delta h_n + (K_h \Delta h_n^2)(2h_n + h_{n-1})/6$$

$$M_{n-1} = J_{n-1} M_{n-1}^k$$

where $$J_{n-1} = (0.6\ f^{2/3})[1 - (h_{n-1}/h_n)^3] + (h_{n-1}/h_n)^3.$$

Numerical values for these quantities are given in Table V.

Dynamic-Wind Analysis

The wind mechanism known as vortex shedding elicits a periodic response from stacks. The severity of the response is measured by the magnitude of the concurring internal forces. For the stack structure previously proposed in this chapter, the maximum values for the internal forces are calculated using the approach developed in Chapter III.

Generalized Force Constant

The dynamic factor $\mathcal{L}$, defined by Equation (3.22), is a function of the generalized force constant K_f which has not as yet been evaluated.

TABLE V

STATIC EARTHQUAKE FORCES

Point n (-)	Elevation of Point h (ft)	Linear Weight u (lb/ft)	Loading Constant K^o (-)	Shearing Force V (kips)	Static Moment M^k (ft-k)	Moment Coefficient J (-)	Dynamic Moment M^o (ft = kips)
5	150.0			1.33	0.0	1.000	0.0
		263	0.444				
4	107.5			3.76	110.9	0.804	89.2
		448	0.757				
3	55.0			6.99	402.1	0.705	283.4
		555	0.937				
2	32.5			7.91	570.6	0.693	395.2
		1046	1.767				
1	10.0			8.75	759.7	0.690	523.9
		1187	2.005				
0	0.0			8.85	847.9	0.689	584.2

Referring to Equation (3.20), the generalized force constant is found as

$$K_f = \frac{\rho}{2h_n^{\alpha}} \sum_{n=1}^{n} D_n\Big[\Big(\frac{h_n^{1+\alpha} - h_{n-1}^{1+\alpha}}{1+\alpha}\Big)\Big(y_{n-1} - \theta_{n-1}h_{n-1} + \frac{M_{n-1}h_{n-1}^2}{2EI_n} + \frac{V_{n-1}h_{n-1}^3}{6EI_n} + \frac{u_n h_{n-1}^4}{24EI_n}\Big) + \Big(\frac{h_n^{2+\alpha} - h_{n-1}^{2+\alpha}}{2+\alpha}\Big)\Big(\theta_{n-1} - \frac{M_{n-1}h_{n-1}}{EI_n} - \frac{V_{n-1}h_{n-1}^2}{2EI_n} - \frac{u_n h_{n-1}^3}{6EI_n}\Big) + \Big(\frac{h_n^{3+\alpha} - h_{n-1}^{3+\alpha}}{3+\alpha}\Big)\Big(\frac{M_{n-1}}{2EI_n} + \frac{V_{n-1}h_{n-1}}{2EI_n} + \frac{u_n h_{n-1}^2}{4EI_n}\Big) - \Big(\frac{h_n^{4+\alpha} - h_{n-1}^{4+\alpha}}{4+\alpha}\Big)\Big(\frac{V_{n-1} + u_n h_{n-1}}{6EI_n}\Big) + \Big(\frac{h_n^{5+\alpha} - h_{n-1}^{5+\alpha}}{5+\alpha}\Big)\Big(\frac{u_n}{24EI_n}\Big)\Big]$$

or

$$K_f = \left(\rho/2h_n^{\alpha}\right) \sum_{h=1}^{n} K_f$$

where $\alpha = 2\gamma = 2/7$ and $\rho = 0.00512$ lb/ft^3.

Using the values in Tables I and II for the static gravity forces and displacements allows the values of K_f to be calculated as shown in Table VI. Subsequently, the generalized force constant is determined as

$$K_f = (0.00512/2 \text{ X } 150^{2/7})(976.6) = 3.982 \times 10^{-3} \text{ lbs.}$$

Initial Values for Wind Parameters

Rationalizing that the maximum response of the stack occurs when the frequency of vortex shedding dwells in the region of the fundamental frequency of the stack, these two frequencies are equated at the upper bound of the fundamental frequency, subsequently through an iterative

TABLE VI

VALUES FOR K_f

Point of Increment n (-)	Elevation of Point h (ft)	$\frac{K_c}{3 + \alpha}$ (ft)	$\frac{K_f}{3 + \alpha}$ (ft)
0	0.0	0.0	0.0
1	10.0	0.2	0.2
2	32.5	10.7	10.9
3	55.0	38.9	49.8
4	107.5	410.2	460.0
5	150.0	516.6	976.6

process initial values for the corresponding Strouhal number, wind velocity and Reynolds number are determined with the use of Equations (1.31), (1.32) and Figure 19. Thus

$$S = 0.22$$

$$U = f\ D_N/S = 22.4 \text{ fps}$$

and $$Re = U\ DN/\nu = 5.72 \times 10^5.$$

The lift coefficient C_l corresponding to Reynolds number is obtained from Figure 19 as 0.523. Also the spectral density P(S) corresponding to the above value of the Strouhal number is obtained using Equation (1.31) and is calculated as 1.136.

Dynamic Load Factor

The dynamic load factor $\mathcal{L}$, defined by Equation (3.22), is

$$\mathcal{L} = \left[\frac{3\pi K_f D_n^2 y_n}{K_e}\right] \left[\frac{\pi \overline{C_L^2} P(S)}{\beta S^3}\right]^{0.5}$$

where β is assumed to be 0.005 and all other terms have been previously defined. Thus

$$\mathcal{L} = \left[\frac{3\pi \times 3.982 \times 10^{-3} \times \overline{4^2} \times 0.9785}{147.41}\right] \left[\frac{\pi \times 0.\overline{523}^2 \times 1.136}{0.005 \times .\overline{22}^2}\right]^{0.5}$$

$$= 0.540$$

and $$y_b = \mathcal{L}(y_n/y_n) = 0.540 \text{ ft.}$$

Final Values of Wind Parameters

The procedure set forth in the two previous paragraphs is now repeated until the values of the Strouhal number, wind velocity and

Reynolds number converge. Using Equation (B.3) to evaluate the Strouhal number for the oscillating stack, the final values of the wind parameters are established as

$$S = 0.235$$

$$S_D = S/(1 + 1.54\ y_o/D) \tag{B.3}$$
$$S_D = 0.235/(1 + 1.54 \times 0.61/4) = 0.190$$

$$U = f_v D_N/S_o \tag{B.2}$$
$$U = 1.232 \times 4/0.190 = 25.9 \text{ fps}$$

$$Re = UD_N/2 \tag{B.1}$$
$$Re = 25.9 \times 4/1.567 \times 10^{-4} = 6.6 \times 10^5$$

$$P(S) = 4.8\ [\frac{1 + 682\ S_D^2}{(1 + 227\ S_D^2)^2}] \tag{B.4}$$
$$P(S) = 1.45$$

and $\mathcal{L} = 0.61$ ft.

Dynamic Forces and Displacements

The maximum dynamic displacements attributed to vortex shedding are determined using Equation (3.22). Subsequently, the internal forces corresponding to those displacements are calculated using Equations (3.24) and (3.25). Thus

$$V_n^D = \mathcal{L} v_n = \mathcal{L} V_n^S (y_n/y_n)$$

$$M_n^D = \mathcal{L} m_n = \mathcal{L} M_n^S (y_n/y_n).$$

Numerical values are given in Table VII.

TABLE VII

DYNAMIC FORCES AND DISPLACEMENTS

Point n (-)	Elevation h (ft.)	Dynamic Displacements Y_n^d (ft.)	Shearing Force V_n^D	Bending Moment M_n^D (ft.-kips)
5	150.0	0.61000	0.0	0.0
4	107.5	0.33845	7.0	148.1
3	55.0	0.08562	21.6	898.8
2	32.5	0.02793	29.4	1473.0
1	10.0	0.00263	44.1	2300.0
0	0.0	0.0	51.4	2777.8

The Guy Supported Stack

Problem Definition

The stack pictured in Figure 18 is to be erected to a geographical location which is specified as a 125 mph Exposure C wind zone and an earthquake zone three. The process which the stack serves requires the stack height to be 150 feet and the terminal diameter to be two feet. The upper sections of the stack are to be lined with one and a half inches of refractory having a density of 90 pcf. The lower section is to be lined with two inches of the same refractory. The shell is to be made of ASTM A-283 Grade C steel (F_y = 30,000 psi, E = 29 x 10^6 psi, ρ_s = 490 pcf) having the shell thicknesses and diameter changes as shown in Figure 18.

The stack is to be supported with guy wires using a dual three wire system as pictured in Figure 18. The corresponding guy wires from each of the two attachment elevations terminate at a common deadmen. The deadmen are located such that the attachment points for the guy wires occur at a distance of 120 feet from the center of the stack and at regular increments of 120° circumferentially.

The guy wires are to be fabricated using prestressed 5/8 inch diameter galvanized structural strand. The pertinent physical properties of this particular strand, obtained from almost any wire rope manufacturer's catalog, are listed as follows:

Breaking Strength =	45,600 lbs.
Cross-Sectional Metallic Area =	0.234 in^2.
Approximate Linear Weight =	0.82 lbs/ft.
Modulus of Elasticity =	24 x 10^6 psi.

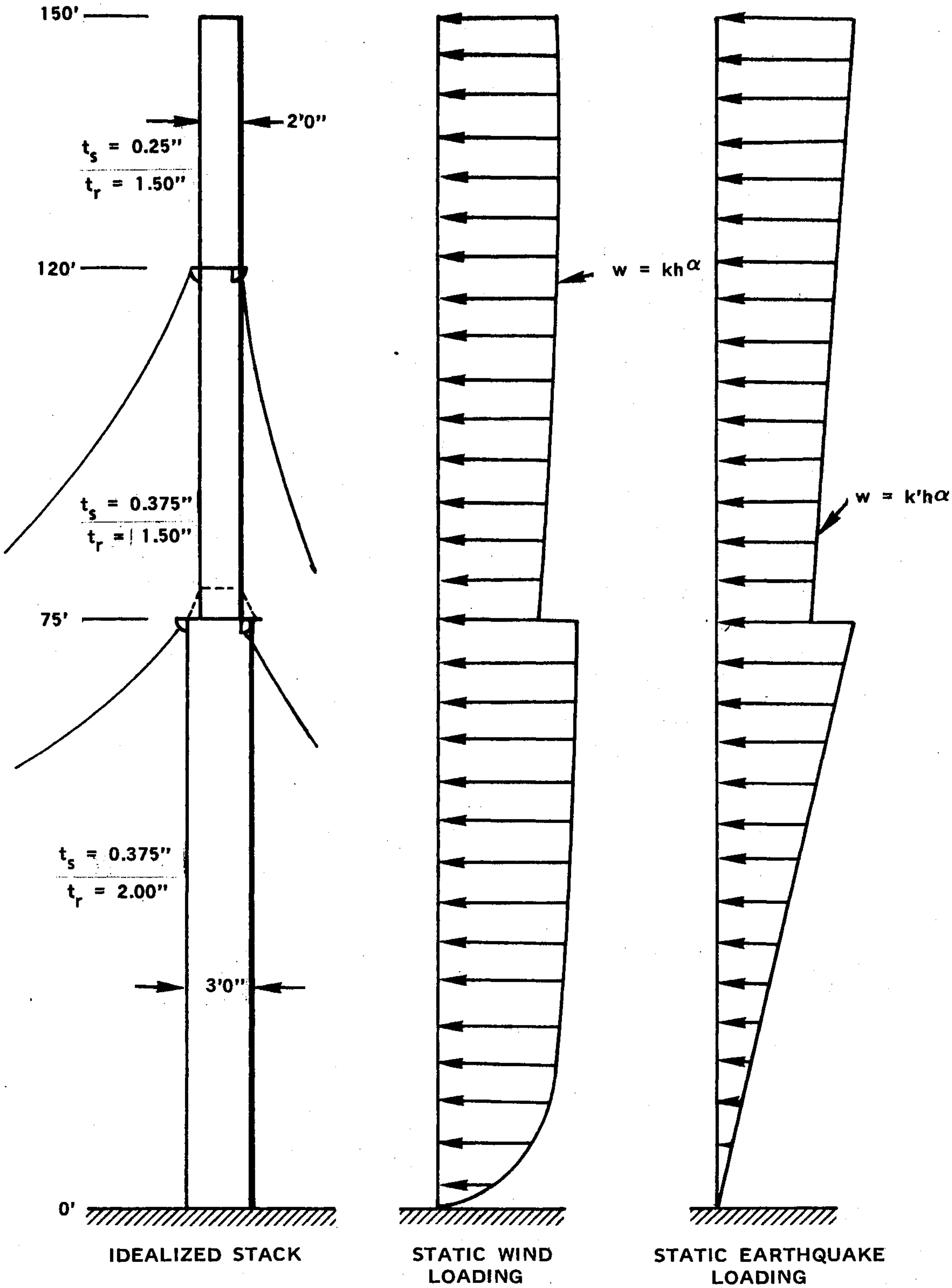


Figure 18. Idealized Guy Supported Stack With Loadings

Determination of the Fundamental Frequency

As a prerequisite for evaluation of the gust factor, the first task to be accomplished in the analysis of the proposed stack is the determination of the fundamental frequency. This quantity is not calculated directly but rather is equated to the upper bound which is evaluated using the Rayleigh method presented in Chapter III. For this calculation, the guy support points are treated as simple supports (i.e., fixed against translation). The motivation for employment of this restriction is simplification of the calculations. However, justification for the imposition of this restriction is found through a cursory study of the effect upon the gust factor, that effect being revealed to be negligible.

The fundamental frequency of the supported stack is established using the following procedure.

Step 1. Definition of Mode Shape. In accordance with the Rayleigh method, the fundamental mode shape is assumed to coincide with the statically deflected shape of the stack loaded laterally with its own weight. This shape is defined by analyzing the supported stack as a statically indeterminate structure. Thus, selection of the reactive forces of the guy supports as redundants, their subsequent release and the evaluation of the following expressions yields the values found in Tables VIII and IX.

$$I_n = (\pi/64)[D_n^4 - (D_n - 2t_{sn})^4]$$

$$u_n = \frac{\pi\rho_s}{4}[D_n^2 - (D_n - 2t_{sn})^2] + \frac{\pi\rho_r}{4}[(D_n - 2t_{sn})^2 - (D_n - 2t_{sn} - 2t_{rn})^2]$$

TABLE VIII

PHYSICAL CONSTANTS

Point n (-)	Elevation h (ft)	Increment h (ft)	Weight W (Lbs/ft)	Inertia I (in.^{4})
3	150			
		30	128.27	1315
2	120			
		45	159.51	1942
1	75			
		75	273.37	6659
0	0			

TABLE IX

FORCES AND DISPLACEMENTS FOR RELEASED STRUCTURE LOADED LATERALLY WITH OWN WEIGHT

	Forces		Displacements	
Point n (-)	Shearing Vn (kips)	Moment Mn (ft-kips)	Slope θ (radians)	Deflection Δn (ft)
3	0.0	0.0	0.08438	8.031
2	3.85	57.7	0.08220	6.116
1	11.03	392.4	0.05940	2.785
0	31.53	1988.0	0.0	0.0

TABLE X

FORCES AND DISPLACEMENTS FOR RELEASED STRUCTURE LOADED WITH UNIT FORCES

Point N (-)	Forces		Displacements	
	Shearing Vn (kips)	Moment Mn (ft-kips)	Slope θn (radians)	Deflection δn (ft.)
3	0.0	0.0	0.00720	0.70053
2	1.0	0.0	0.00720	0.48453
1	1.0	45	0.00461	0.19924
0	1.0	120	0.0	0.0
3	0.0	0.0	0.00210	0.26216
2	0.0	0.0	0.00210	0.19924
1	1.0	0.0	0.00210	0.10486
0	1.0	75	0.0	0.0

TABLE XI

FORCES AND DISPLACEMENTS FOR SUPPORTED STACK LOADED LATERALLY WITH OWN WEIGHT

Point n (-)	Forces		Displacements	
	Shearing Vn (kips)	Moment Mn (ft-kips)	Slope n (radians)	Deflection Yn (ft)
3	0.0	0.0	0.00363	-0.507
2^+	3.85	57.7	0.00145	0.000
2^-	-3.93			
1^+	3.25	92.3	-0.00120	0.000
1^-	-8.53			
0	11.97	170.9	0.0	0.000

TABLE XII

ENERGY VALUES FOR GUY SUPPORTED STACK

Point n (-)	Potential Ω (Lb.-ft.)	Kinetic Ke (Lb/sec^2)
0	0.0	0.0
1	167.3	0.115
2	2.6	0.357
3	79.6	0.166
	249.5	0.638

$$V_{n-1} = V_n + u_n(h_n - h_{n-1})$$

$$M_{n-1} = M_n + \Delta h_n(V_n + \Delta V_n/2)$$

$$\theta_n = \theta_{n-1} + \frac{\Delta h_n}{EI_n}\left[M_{n-1} + \frac{\Delta h_n}{2}\left(-V_{n-1} + \frac{u_n \Delta h_n}{3}\right)\right]$$

$$\Delta_n = \Delta_{n-1} + \theta_{n-1}\Delta h_n + \left(\frac{\Delta h_n^2}{2EI_n}\right)\left[M_{n-1} + \frac{\Delta h_n}{3}\left(-V_{n-1} + \frac{u_n \Delta h_n}{2}\right)\right].$$

Unit forces are placed on the released stack (one at a time) at each redundant location and the consequent displacements calculated using the above expressions in which all u_n's are zero. Mathematical values are given in Table X.

The redundant forces are now calculated using the approach presented in Chapter V. Thus

$$\begin{bmatrix} \delta_{11} & \delta_{12} \\ \delta_{21} & \delta_{22} \end{bmatrix} \begin{Bmatrix} H_{R1} \\ H_{R2} \end{Bmatrix} = \begin{Bmatrix} \Delta_1 \\ \Delta_2 \end{Bmatrix}$$

$$\begin{bmatrix} 0.48453 & 0.19924 \\ 0.19924 & 0.10486 \end{bmatrix} \begin{Bmatrix} H_{R1} \\ H_{R2} \end{Bmatrix} = \begin{Bmatrix} 6.116 \\ 2.785 \end{Bmatrix}$$

and

$$\begin{Bmatrix} H_{R1} \\ H_{R2} \end{Bmatrix} = \begin{Bmatrix} 7.78 \\ 11.78 \end{Bmatrix}.$$

The fundamental mode shape is now defined using the expression for Y above with the values of the parameters at interval terminals being given in Table XI.

Step 2. Fundamental Natural Frequency. The fundamental natural frequency is now determined using data from Table XI in conjunction with the expressions for $\sum_1^n \Omega_n$ and $\sum_1^n K_n$ given on page 75. The intermediate values for Ω_n and K_n are given in Table XII. Thus

$$f = \frac{1}{2\pi}\sqrt{\frac{\Sigma\Omega_n}{\Sigma K_n}} = \frac{1}{2\pi}\sqrt{\frac{249.5}{0.638}} = 3.147 \text{ cps.}$$

Static Wind Analysis

The equivalent static forces experienced by the stack and the consequent displacements exhibited thereby as the stack is subjected to the theoretical force of a 125 mph wind are calculated following the procedure delineated below. The stack is analyzed as a statically indeterminate structure using the method presented in Chapter V.

Wind Gust Factor. Using the upper bound value of the natural frequency from above and the procedure set forth in Reference (1), the gust factor for the stack under consideration is found to be 1.188.

Wind Loading Constant. The wind loading constant K_n is defined by Equation (3.1) and evaluated using parameters from Reference (1). Thus

$$K_n = C_d D_n G_f \, U_d^2 / 2h_d^{\alpha} = 11.688 \, D_n$$

when

$C_d = 0.65$ $\quad$ $U_d = 125$ mph

$G_f = 1.188$ $\quad$ $h_d = 30$ ft

$\rho = 0.00512$ lb/ft^3 $\quad$ $\alpha = 2\gamma = 2/7.$

Static Wind Force on the Released Stack. The equivalent static wind forces are calculated by applying Equations (2.23), (2.24) and

(2.25). Beginning at the top of the stack where V_n and M_n are zero, and proceeding down the stack,

$$V_{n-1} = V_n + \Delta V_n$$

and

$$M_{n-1} = M_n + \Delta M_n + V_n \Delta h_n - \Delta V_n h_{n-1}$$

where

$$\Delta V_n = (h_n^{1+\alpha} - h_{n-1}^{1+\alpha})(K_n/1+\alpha)$$

$$\Delta M_n = (h_n^{2+\alpha} - h_{n-1}^{2+\alpha})(K_n/2+\alpha)$$

$$\Delta h_n = h_n - h_{n-1}$$

and as above $\alpha = 2\gamma = 2/7$.

Numerical values for these expressions as related to the subject stack are given in Table XIII.

Static Wind Displacements for the Released Structure. The displacements experienced by the released stack when it is subjected to the equivalent static wind forces are determined by substituting the values calculated above into Equations (2.23), (2.26) and (2.27). Starting at the base of the stack where θ_0 and y_0 are zero and proceeding up the stack,

$$\theta_n = \theta_{n-1} + \Delta\theta_n + \frac{(M_{n-1}\Delta h_n - \Delta M_n h_n)}{EI} - \frac{(V_{n-1}\Delta h_n^2 - \Delta V_n h_n^2)}{2EI}$$

and

$$y_n = (y_{n-1} - \Delta y_n) + (\theta_{n-1}\Delta h_n + \Delta\theta_n h_n) + \frac{(M_{n-1}\Delta h_n^2 - \Delta M_n h_n^2)}{2EI_n} - \frac{(V_{n-1}\Delta h_n^3 - \Delta V_n h_n^3)}{6EI_n}$$

TABLE XIII

STATIC WIND FORCES ON RELEASED STACK

Point	Shearing Force		Bending Moment	
n (-)	ΔV (kips)	V (kips)	ΔM (ft-kips)	M (ft-kips)
3		0.0		0.0
2	3.56	3.56	480.0	53.9
1	4.86	8.42	476.0	325.8
0	8.19	16.61	345.7	1302.7

TABLE XIV

STATIC WIND DISPLACEMENTS OF RELEASED STACK

Point	Slope		Deflection	
N (-)	Δθ (radians)	θ (radians)	V (ft.)	Y (ft.)
0		0.0		0.0
1	0.06270	0.03970	2.848	1.809
2	0.04558	0.05454	1.540	4.018
3	0.00627	0.05557	0.120	5.677

where

$$\Delta\theta_n = (h_n^{3+\alpha} - h_{n-1}^{3+\alpha})(K_n/3+\alpha)$$

$$\Delta y_n = (h_n^{4+\alpha} - h_{n-1}^{4+\alpha})(K_n/4+\alpha)$$

and the remaining terms are as defined above. Numerical values are shown in Table XIV.

Unit Displacements of Released Stack. The displacements of the support point coordinates due to a unit force acting independently at each such coordinate are specified in Table X. The method of calculation for the tabulated values is presented in that section of this chapter dealing with the natural frequency.

The Guy Wire Support System. In addition to those previously specified, the parameters required for the mathematical representation of the guy wire are: a) the unstressed length S_o of the individual guy wires, b) the angles of inclination γ of the guy wire chords, and c) the guy wire spring constants.

Specifying the magnitude of the "as erected" (initial) tensile stress allows the use of Equation (4.3) to define the unstressed lengths S_o of the guy wires. Assigning a value of 5 ksi to this quantity, the referenced equation becomes

$$S_o = C[1 + \frac{1}{24}\left(\frac{W}{(5A)\sec\gamma}\right)^2 - \frac{(5A)}{AE}]$$

where

$$C = [R^2 + L^2]^{0.5}$$

$$\sec\gamma = C/L$$

$$W = wC$$

TABLE XV

GUY WIRE PARAMETERS

Guy No.	Rise R (ft.)	Span L (ft.)	Chord C (ft.)	Secγ (-)	Length So (ft.)
1	120.0	119.0	169.00	1.42016	168.967
2	75.0	118.5	140.14	1.18346	140.213

TABLE XVI

GUY WIRE SPRING CONSTANTS

Estimate No.	Guy No.	Estimate H + Initial (kips)	Spring Constants: L_H secγ30° (ft.)	Spring Constants: L_{HH} secγ30° (ft/kips)
1	1	8.54	0.07007	0.04745
	2	6.14	0.04038	0.03041
2	1	7.02	0.07007	0.04655
	2	6.09	0.04038	0.03041
3	1	6.72	0.07007	0.04629
	2	6.51	0.04038	0.03089

R = (Attach Point Elev.) - (Deadmen Elev.)

L = (Deadmen Radius) - (Radius of Stack Shell)

and A, E and w are as previously defined. Numerical values are displayed in Table XV.

Estimating the magnitudes of the tensile forces in the wind-loaded guy wires allows the use of Equations (4.5) and (4.7) to establish values for the guy wire spring constants. If the initial estimate is a poor approximation as revealed by the subsequent analysis, the process is iterated until convergence occurs. The calculated values of tensile forces from one iteration are used for the estimated values of the following one.

The magnitudes of the tensile forces in the loaded guy wires are estimated using the equations below. For the upper guy wire,

$$H_u = \frac{T}{\sec\gamma} = \frac{\text{Moment @ Base of Stack}}{(1/\sec 30°)\Sigma(R)} + \frac{T_I}{\sec\gamma}$$

where the initial tension T_I = (5ksi), A = 1.17 kips. Thus H_u = (1302.7 sec 30°)/(120 + 75) + (1.17/1.4202) = 8.54 kips. For the lower chord,

$$H_1 = \frac{2}{3}(H_u - T_I \cos \gamma_1) + T_I \cos \gamma_2$$

$$H_1 = 6.14 \text{ kips.}$$

The values of the spring constants are determined using Equations (4.6) and (4.7) and the magnitudes recorded in Table XVI.

$$L_H = \frac{S_o \sec^2\gamma/AE}{1 + \frac{1}{6}(wL/H_1)^2}$$

$$L_{HH} = \frac{S_o \sec\gamma/AE}{1 + \frac{1}{6}(wL/H_1)^2} - \frac{12L}{12\ H^2 - (wL)^2}$$

The use of these values allows the force-displacement relationships to be approximated as

$$L_H = [L_{HH} + (L_H - L_{HH}H_1)/H]\ H$$

and the unit displacement of the composite guy system given in Chapter V as

$$\delta_i^R = L_H \sec 30°$$

is expressed for the upper and lower sets as

$$\delta_1^R = (0.04745 - 0.335/H)\ H$$

and

$$\delta_2^R = (0.03041 - 0.145/H)\ H$$

Internal Loads Analysis. The redundant forces are calculated in accordance with the procedure set forth in Chapter V. Thus

$$\begin{bmatrix} (\delta_{11} + \delta_1^R) & \delta_{12} \\ \delta_{21} & (\delta_{22} + \delta_2^R) \end{bmatrix} \begin{Bmatrix} H_{R1} \\ H_{R2} \end{Bmatrix} = \begin{Bmatrix} \Delta_{R1} \\ \Delta_{R2} \end{Bmatrix}$$

Values for unit displacements δ_{ij} and the free displacements Δ_{Ri} are obtained from Tables X and XIV, respectively. Coupling these values with those given above for δ_i^R yield the following:

$$\begin{bmatrix} 0.53198 & 0.19924 \\ 0.19924 & 0.13527 \end{bmatrix} \begin{Bmatrix} H_{R1} \\ H_{R2} \end{Bmatrix} = \begin{Bmatrix} 4.314 \\ 1.925 \end{Bmatrix}$$

and

$$\begin{Bmatrix} H_{R1} \\ H_{R2} \end{Bmatrix} = \begin{Bmatrix} 6.20^k \\ 5.10^k \end{Bmatrix} .$$

Using calculated values of H (plus the initial amounts) as estimates for the recalculation of the spring constants and completing several iterations of the analytical procedure above leads to the values of H given as

$$\begin{Bmatrix} H_{R1} \\ H_{R2} \end{Bmatrix} = \begin{Bmatrix} 5.83^k \\ 5.76^k \end{Bmatrix} .$$

The forces and displacements for the guy supported stack are calculated using these last values of H as the guy reactive forces. Numerical values are given in Table XVII.

The guy wire tensile forces are now calculated as

$$T_{1avg} = H_{R1} \sec 30^\circ \sec {}_1 + T_I = 10.73 \text{ kips}$$

$$T_{2avg} = H_{R2} \sec 30^\circ \sec {}_2 + T_I = 9.04 \text{ kips.}$$

Static Earthquake Analysis

Procedurally, the earthquake analysis is identical to the wind analysis (differing only in the loading) and thus is not presented.

TABLE XVII

WIND FORCES AND DISPLACEMENTS FOR GUYED STACKS

Point	Forces		Displacements	
n (-)	Shear V (kips)	Moment M (ft-kips)	Slope θ_n (radians)	Defl. y_n (ft)
3	0.0	0.0	0.00152	0.083
2^+	3.56	53.9	0.00049	0.046
2^-	-2.27			0.046
1^+	2.69	63.45	0.00074	
1^-	-3.17			0.043
0	5.02	171.1	0	0

CHAPTER VII

SUMMARY AND CONCLUSIONS

Summary

The primary problem addressed in the preceding presentation was the development of an analytical procedure to be used in the determination of lateral displacements of bending members. This procedure, however, was required to conform to certain qualifications. As stated in Chapter I these qualifications are that the procedure must allow for the handling of:

1. Dicontinuities in its curvilinearly varying distributed load;
2. Discontinuities in cross-sectional properties of the stack;
3. Abrupt changes in the shearing force and the bending moment; and
4. That it maintain continuity in the rotational and translational displacements of the stack across the aforementioned discontinuities.

In addition to meeting the qualifications just stated, the developed procedure was required to be in keeping with the philosophical approach of this presentation. Thus, the developed procedure should be comprehensible and neither inordinately difficult to implement or excessively time consuming to evaluate.

A procedure complying with the qualifications thus stated was developed in Chapter II. Subsequently, this procedure was adapted to the analysis of the cylindrical steel stack in Chapters III and V for static wind and earthquake analysis as well as dynamic wind effects.

The secondary problem given consideration herein was the establishment of a set of mathematical expressions which define approximately the configuration and responsive behavior of the freely hanging guy wire. This problem was solved by developing the mathematical expression defining the catenary (the curve assumed by the freely hanging guy wire) and subsequently manipulating that expression to derive the expressions required to define the responsive behavior. The expressions thus developed in Appendix C were approximated in Chapter IV through series expansion of hyperbolic functions and the deletion of the higher order terms herein, the deletion being justified by restriction of applicability to tight guy wires (i.e., relatively flat catenary shape).

The expressions developed in Chapter IV were coupled with the stack displacement expressions developed in Chapter II to effect the internal loads analysis of the guy supported cylindrical steel stack.

The implementation of the procedure was demonstrated in Chapter VI by analyzing a free standing stack for static wind and earthquake loads and dynamic wind effects and by analyzing a guy supported stack for static wind and earthquake forces.

Concluding Remarks

The procedure developed herein for the determination of stack displacements is a powerful analytical tool. Its application is not restricted to the cylindrical steel stack but can be adapted to the analysis of any beam-type structure which is enveloped by the procedural limitations established by the developmental qualifications. Furthermore, the procedure is quite comprehensible when coupled with the concept of continuity of displacements present in elastically deformed physical systems.

The treatment of the guy wire problem does not compare favorably in elegance to that of the displacement solution. However, the expressions developed provide one means for the analyzation of guy wires and also help fill the gap between the highly complex theoretical approaches and the simplified potentially erroneous approximation approaches.

Recommendations

There are several areas of consideration recommended for further investigation and treatment. These areas are listed as follows:

1. Lower Bound for the Natural Frequency: In that Rayleigh method is applied herein to the determination of the upper bound of the natural frequency, it is desirable that a comparable method be developed for the lower bound thereby banding the fundamental natural frequency.

2. Structural and Aerodynamic Damping: The damping of oscillatory motion should be investigated so as to establish typical values to be used in the analysis of stacks.

3. Dynamic Earthquake Analysis: Much effort can be and is being addressed to earthquake analysis. It would be very beneficial if a theoretical normalized spectrum could be established and coupled with an empirical intensity factor which would account for proximity to the origin of motion. The utilization of the spectrum might be further qualified by a probability factor which accounts for structural life and for the endangerment of human life resulting from structural failure.

There are without doubt other areas which warrant investigation but those specifically mentioned are the ones in which the author is most interested.

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APPENDIX A

SOME FUNDAMENTALS OF DYNAMICS

Definition of a Vibrating System

The word vibration as applied to physical systems infers a cyclic motion of discernible magnitude. The maximum departure of any element of the system from its at-rest position is referred to as the amplitude of displacement for that element. The number of times that the element repeats the cyclic motion in a given time span is defined as the frequency. Furthermore, when all elements of the system are experiencing unforced cyclic motion of the same frequency, the motion is defined as a principal mode of vibration and its frequency as a "natural frequency."

One characteristic of natural frequencies is that they are frequencies at which vibratory motion is maintained with the least external energy. Thus, when energy is supplied in phase with and at the same frequency as the natural frequency, any energy in excess of the least required will effect an increase in the amplitude of the motion. Theoretically, for an undamped or frictionless system, the amplitude will increase indefinitely. In the case of material systems, the amplitude will increase to some physical limit beyond which some portion of the system will cease to function elastically. Therefore, the natural frequencies are significant primarily when considered in conjunction with some external source of energy.

Physical systems are considered as being comprised of a number of elements or individual mass particles. The relative position of these masses at any given instant is defined as the configuration of the system. The number of independent coordinates required to completely describe that configuration is referred to as the number of degrees of freedom. For each degree of freedom characteristic of the physical system, there is a corresponding natural frequency.

Man-made structures such as buildings are constructed using elastic materials. Since the materials are elastic, then the many elements of the structure are capable of relative motion. Thus, fabricated structures are classified dynamically as multi-degree of freedom systems and as such have an infinite number of freedoms with a corresponding number of natural frequencies.

It is not practical to consider an infinite number of degrees of freedoms and for this reason it is customary to consider only a limited number, or alternately to restrict the number of freedoms by employing a lumped mass idealization of the structure. It should be noted, however, that as the number of restrictions increase, there is a corresponding decrease in the conformance of the idealized system to the physical system.

Calculation of Natural Frequencies

The Method

There are many analytical methods for determining natural frequencies. Some, however, are particularly suited for application to relatively simple elastic systems having continuous distributed mass.

One such method is the Rayleigh method (31). This method is based on the proposition stated as "In a natural mode of vibration of a conservative system, the frequency of vibration is stationary."

The key to the application of Rayleigh's proposition lies in its restriction to conservative systems vibrating at natural modes. First, the restriction to conservative systems infers that the total energy in the vibrating system has a constant magnitude and therefore the maximum kinetic energy equals the maximum potential energy. Secondly, the consideration of only natural modes implies simple harmonic motion thereby establishing the kinetic energy as proportional to the square of the frequency. The conclusion is that since the kinetic proportionality constant and the potential energy both depend upon the mode configuration, then the natural mode is related to a particular frequency.

The Rayleigh method is implemented by assuming some characteristic shape or mode which is a reasonable representation of the structural configuration under dynamic conditions and calculating the kinetic constant and the potential energy for the system in that configuration. The natural frequency for that mode is then found as the square root of the ratio of the potential energy to the proportionality constant.

From the above description it can be deduced that the Rayleigh method is used primarily for the estimation of a few of the lower modes. This deduction is based on the observation that the number of modes is the same as the number of freedoms, and that a characteristic shape must be assumed for each mode considered. Furthermore, the assumption of a configuration which is reasonably representative becomes increasingly difficult for the higher modes.

The accuracy of the Rayleigh method is dependent upon the correspondence of the assumed configuration to the actual configuration of the system. However, if the assumed configuration is not exact, the resulting calculated frequency will be higher than the actual frequency. This occurrence is attributable to the fact that structures naturally assume the configuration of least potential energy and therefore any other must be a configuration of higher energy. Consequently, the calculated frequency must be greater. Thus, the Rayleigh method establishes an upper bound of the frequency for each mode considered.

Application of the Method

As mentioned above, the first step in the implementation of the Rayleigh method is the assumption of a deflected shape which is representative of the dynamic configuration. Customarily, the deflected shape assumed is that defined by the static deflection curve of a system under its own weight.

The maximum potential energy stored in a conservative system can be defined by equating it either to the strain energy stored in the displaced system or to the work energy required to attain that displaced configuration. In the latter case, the work energy is established as the work done by the static loads in attaining their respective displacements. Thus, considering a deflected beam as consisting of a series of mass points, then the work done by each individual mass in a linearly elastic system is

$$dWe = u_n \, y_n / 2$$

and for all such mass points

$$W_e = \sum u_n \; y_n/2.$$

Furthermore, when u_n is constant with specific intervals and there exist several such intervals, then

$$W_e = \frac{1}{2} \sum_{1}^{n} u_n \int_{o}^{\Delta h} y_n d(\Delta h) \qquad (A.1)$$

The kinetic energy for a vibrating system is a function of the mass and the square of velocity. In the case of simple harmonic motion, the velocity can be expressed in terms of the circular frequency W and the amplitude of displacement. Thus, considering the segmented beam once again, the maximum kinetic energy for each individual mass is

$$dT = u_n(w \; y_n)^2/2g.$$

Restricting consideration to principal modes at which all mass points exhibit the same simple harmonic motion, the kinetic energy for the system is found as

$$T_{max} = \sum u_n(w \; y_n)^2/2g.$$

Furthermore, when as before u_n is constant within specific intervals and there exist several such intervals, then

$$T_{max} = w^2 K_e \qquad (A.2)$$

where

$$K_e = (1/2g) \sum_{1}^{n} u_n \int_{o}^{\Delta h} y^2 d(\Delta h) \qquad (A.3)$$

and (u/g) is the mass per unit length.

Finally, for conservative systems it is recognized that the maximum kinetic energy equals the maximum potential energy and thus the natural frequency is found as

$$f = \frac{w}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{\Omega_{max}}{K_e}}$$

for $\Omega_{max} = W_e$

where W_e and K_e are as above.

Analysis of Oscillatory Motion

Equations of Motion

Written in matrix notation, the general equations of motion for the elemental masses m of a multi-degree of freedom system are expressed as

$$[m]\{\ddot{y}\} + [2\beta]\{\dot{y}\} + [k]\{y\} = \{q\} \qquad (A.5)$$

where B, K and q are the elemental damping, elemental stiffness and elemental forcing function, respectively. The solution to these partial differential simultaneous equations is obtained herein by a mathematical technique known as model analysis (31). The principle which serves as a basis for this approach is that when a system is vibrating at a natural frequency, it can be treated as a one degree of freedom system. Letting the dynamic displacements y be defined as a product of a time function and a normalized displacement function

$$y(h,t) = q(t) \cdot y(h)/y_n$$

allows Equation (A.5) to be written in the following form for each normal mode. Thus,

$$\ddot{q} + 2\beta w\,\dot{q} + w^2 q = (F_e/M_e) \qquad (A.6)$$

where

$$F_e = \sum_1^n \frac{1}{y_n} \int_{h_{n-1}}^{h_n} w_1(h,t)y(h)dh$$

$$M_e = K_e/y_n^2$$

and K_e is as defined in Equation (A.3).

Solution for a Periodic Exciting Force

When the load function w(h,t) is periodic and expressed mathematically as

$$w(h,t) = w_1 \sin \omega t$$

then the expression for the generalized force can be written as

$$\mathcal{F}_e = F_e \sin \omega t.$$

The solution of Equation (A.6) for the periodic load function can then be found as

$$q = F_e \cos \omega t/2\beta w^2 M_e$$

and q is obviously a maximum when cos wt = 1.

The dynamic displacements are now calculated as

$$y_o = (q)_{max} (y/y_n).$$

Solution for a Random Exciting Force

When the load function w(h,t) is random, the solution to Equation (A.6) is found using an approach taken from the theory of random functions. By this approach the frequency of vortex shedding for a cylinder is established as the random variable of the lift force and expressed in terms of the spectral density (8) of the Strouhal numbers.[1] Subsequently, assuming the steady state solution to be sinusoidally periodic, using the considerations common to the analysis of lightly damped systems (i.e., $w = w_n$) and using the modal analysis technique as above, the mean-square value of the random load function for an oscillating cylinder is found as

$$\overline{q^2} = \left[\frac{Q_e}{2\beta w^2 M_e}\right]^2 \left[\frac{wD_n\beta \overline{C_1^2}\, P(S)}{2U}\right] \tag{A.10}$$

where $\overline{C_1^2}$ and P(S) are the mean square lift coefficient and spectral density, respectively, and

$$Q_e = \sum_{h=1}^{n} \int_{h_{n-1}}^{h_n} P_h \left(\frac{y}{y_n}\right) dh.$$

If the random load function q has a normal distribution, then the root-mean-square value is equivalent to the statistical standard deviation σ-value. Consequently, the maximum probable load can be defined as

$$(q)_{max} = c\sqrt{\overline{q^2}}$$

when c is a factor related to the probability of occurrence. Thus, when c has a value of three, the calculated maximum load represents a σ-value which by probability will be exceeded only 0.27 percent of the time.

Finally, the dynamic displacements of a cylinder subjected to a random exciting force are calculated as

$$(y_D)_{max} = (y/y_n)(c\sqrt{\overline{q^2}}).$$

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[1] See Appendix B.

APPENDIX B

WIND RELATED PARAMETERS

Introduction

The values and functions presented below as definitions of wind parameters were gleaned from observations, recommendations and conclusions made relative to wind data empirically established. It should be noted that these evaluations are not exact but typical and their respective magnitudes should be qualified as dictated by engineering judgment.

Reynolds Number

By definition, Reynolds number is the dimensionless ratio of wind inertia forces to wind viscous forces. For cylinders, Reynolds number is defined as

$$Re = U\,D/\nu$$

where U is the unaltered steady component of wind velocity, D is the cross-wind frontal width (diameter) of the cylinder and ν is the kinematic viscosity of the air.

The correlation of Reynolds number with the phenomenon of vortex shedding has established the following characteristic divisions or ranges of Reynolds number (32). Thus, for Reynolds numbers less than 3×10^5, vortex shedding is fairly periodic and this range is known as the critical range. Similarly, for Reynolds numbers between 3×10^5 and 3.5×10^6, vortex shedding is random and this range is known as the supercritical range. Finally, for Reynolds numbers above 3.5×10^6, vortex shedding gradually changes from random to periodic and this range is called the transcritical range.

Strouhal Number

The Strouhal number is a dimensionless measure of the frequency of vortex shedding for a stationary structure. Mathematically, the number is defined as

$$S = f_v D/U$$

where f_v is the frequency of vortex shedding and the quantities D and U are as above. (It should be noted that two vortices, one each on alternate sides, are shedding each cycle. See Figure 4.)

Since the Strouhal number is a function of vortex shedding, its variation can be defined relative to the specified ranges of Reynolds numbers. Thus

1. For the critical range ($Re \leq 3 \times 10^5$), the Strouhal number has been found to have a relatively stable value generally reported at 0.2 (32).

2. For the supercritical range ($3 \times 10^5 < Re < 3.5 \times 10^6$), the variation of the Strouhal number is not firmly established. However, based on the referenced data, a tentative idealization is presented in Figure 19. (It can be argued that this tentative idealization is conservative.)

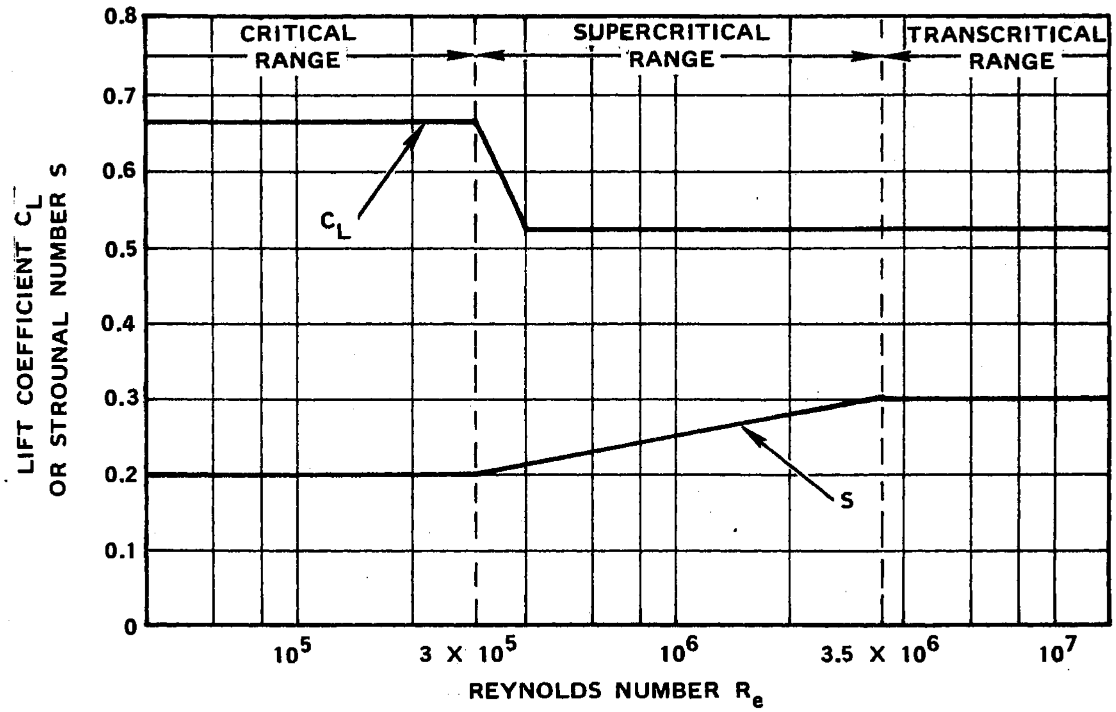

Figure 19. Lift Coefficient and Strouhal Number

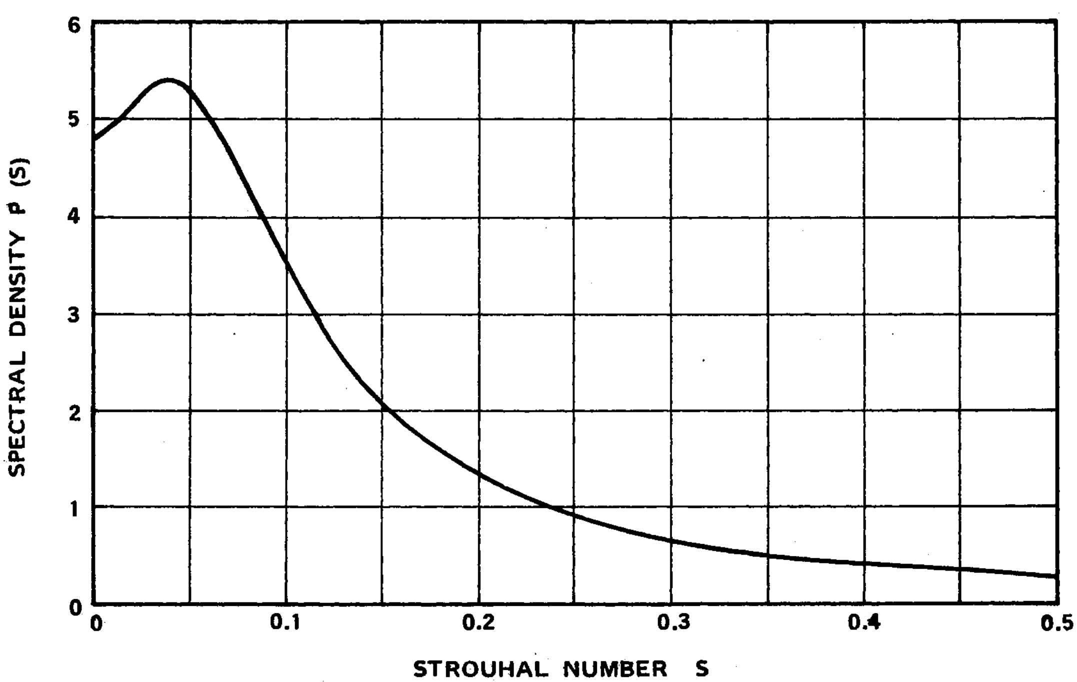

Figure 20. Normalized Spectral Density

3. For the transcritical range ($Re \geq 3.5 \times 10^6$), existing information indicates that the Strouhal number again becomes stable and for this range the value is approximately 0.3. It should be noted that these statements relating the Strouhal number to Reynolds number though accepted are generalities and should be constantly considered as such.

The Strouhal number as discussed is a measure of vortex shedding for a stationary structure. The argument is set forth by Vellozzi and Cohen (32) that the Strouhal number for an oscillatory structure should be

$$S_o = S/(1 + 1.54\ y/D)$$

where the limiting value of y/D is unity, this value being generally supported by experiment.

Lift Coefficient

The lift coefficient C_l is a correlation factor which relates wind velocity and structural cross-sectional configuration to the cross-wind component of wind force imposed on the structure. The variation of the lift force with wind velocity is established using Reynolds number.

In general, for the critical range of Reynolds number ($Re \leq 3 \times 10^5$), the lift coefficient is considered for analytical purposes to have a stationary value of 0.66 (7, 13) though in reality it varies from 0.15 to 1.30 (26). Similarly, for the supercritical range of Reynolds numbers ($3 \times 10^5 < Re < 3.5 \times 10^6$), empirical data (9) indicates that the lift coefficient decreases from the value of 0.66 at the lower bound to a value of 0.523 at a Reynolds number of 4×10^5. Thereafter for the remainder of the supercritical range and throughout the transcritical range ($Re \geq 3.5 \times 10^6$), the lift coefficient is considered to maintain a stable value of 0.523.

Spectral Density for Lift Force

Spectral density (8) is a mathematical process used to define a random function of time in terms of its frequency. In the case of the random fluctuation of lift forces, the frequency parameter selected by Fung (9) was the Strouhal number. The variation of the spectral density shown in Figure 20 was established as an idealization of empirically obtained data and was subsequently normalized such that

$$\int_0^\infty P(S)dS = 1$$

where

$$P(S) = 4.8\left[\frac{1 + 682\ S_o^2}{(1 + 227\ S_o^2)^2}\right]$$

Drag Coefficient

The drag coefficient C_d is a correlation factor which relates wind velocity and structural cross-sectional configuration to the along-wind component of the wind force imposed on the structure. In general, for cylinders having moderately smooth surfaces and height-to-diameter ratios in the neighborhood of fifteen, an accepted value of C_d is 0.65 and this holds for all values of Reynolds number. However, various wind codes (1) are more definitive and should be consulted.

Gust Factor

The gust factor G_f is a correlation factor which transforms the dynamic effects of gusting into an equivalent static loading. As such, it must account for the dynamic characteristics of the structure as well as fluctuations in wind velocity. Consideration of the velocity of the wind as consisting of two components, one having a steady average value and the other a fluctuating value, and the desire to relate the gust factor to the steady average component leads to the expression

$$G_f = (U_{avg} + f\ U_{fluc})/U_{avg}$$

or

$$G_f = 1 + 3\sigma$$

where the term σ represents the ratio of the standard deviation of the wind loading to the mean wind loading and accounts for the effects of wind gusts and for the dynamic characteristics of the structure.

Effective Wind Velocity and Wind Profile

The mathematical representation of the idealized wind has been formulated by following the proceeding rational process (1).

1. The steady state components and recurrence intervals for the extreme winds peculiar to given sites were established through the application of statistical principles to measured observations of the local winds.

2. Consideration of the function, life and wind sensitivity of a proposed structure and the consequences of possible failure lead to the selection of a recurrence interval and consequently a basic wind velocity U_d at datum elevation h_d.

3. The gust factor G_f is established through the statistical analysis of the fluctuating component and its correlation to the characteristic along-wind dynamic response of the structure under consideration.

4. The effective wind velocity at datum is established as

$$U_e = U_d \sqrt{G_f}$$

5. Finally, the wind profile is idealized as

$$U_h = U_e(h/h_d)^{\gamma}$$

where γ is a topology related factor which varies from 1/7 to 1/3 (1).

Wind Pressure and Distributed Wind Force

The intensity of wind pressure is a function of wind velocity and as such is defined by the expression

$$p_h = C_p \, \rho \, U_h^2/2$$

where C_p, the pressure coefficient, is a function of the size, shape and orientation of the obstacle and p is the density of the air.

As stated in Chapter I, the vectorial accumulation of all the incremental forces produced by the individual pressures acting on the applicable areas determines the total force which the wind imposes on the obstacle. Resolving this total force into components parallel to and normal to the direction of the wind flow and referring to these components as drag and lift respectively, the wind force components per unit length are defined as

$$w_d = C_d(\rho U_h^2/2)D$$

$$w_l = C_l(\rho U_h^2/2)D$$

where C_d and C_l are the nondimensional accumulation factors and D is a crosswind dimension representative of the particular planform shape of the obstacle.

APPENDIX C

THE MATHEMATICAL REPRESENTATION OF THE GUY WIRE

The General Configuration

The ensuing presentation formulates the equation of the catenary as applicable to the configuration of the guy wire pictured in Figure 21. Referring to the free body diagram of the elemental length of guy wire shown in Figure 22 and applying the laws of statics yields the expression

$$\Sigma F_V = 0, \quad V - wds = V - dV$$

and then

$$wds = dV.$$

Recalling the approximation

$$ds \cong dx \sqrt{1 + y_x^2} \tag{C.1}$$

and substituting above gives

$$wdx \sqrt{1 + y_x^2} = dV.$$

On rearranging

$$w \sqrt{1 + y_x^2} = V_x. \tag{C.2}$$

Now from the geometry of the free body it is observed that

$$V = H \tan \phi = Hy_x.$$

Differentiating with respect to x gives

$$V_x = Hy_{xx}. \tag{C.3}$$

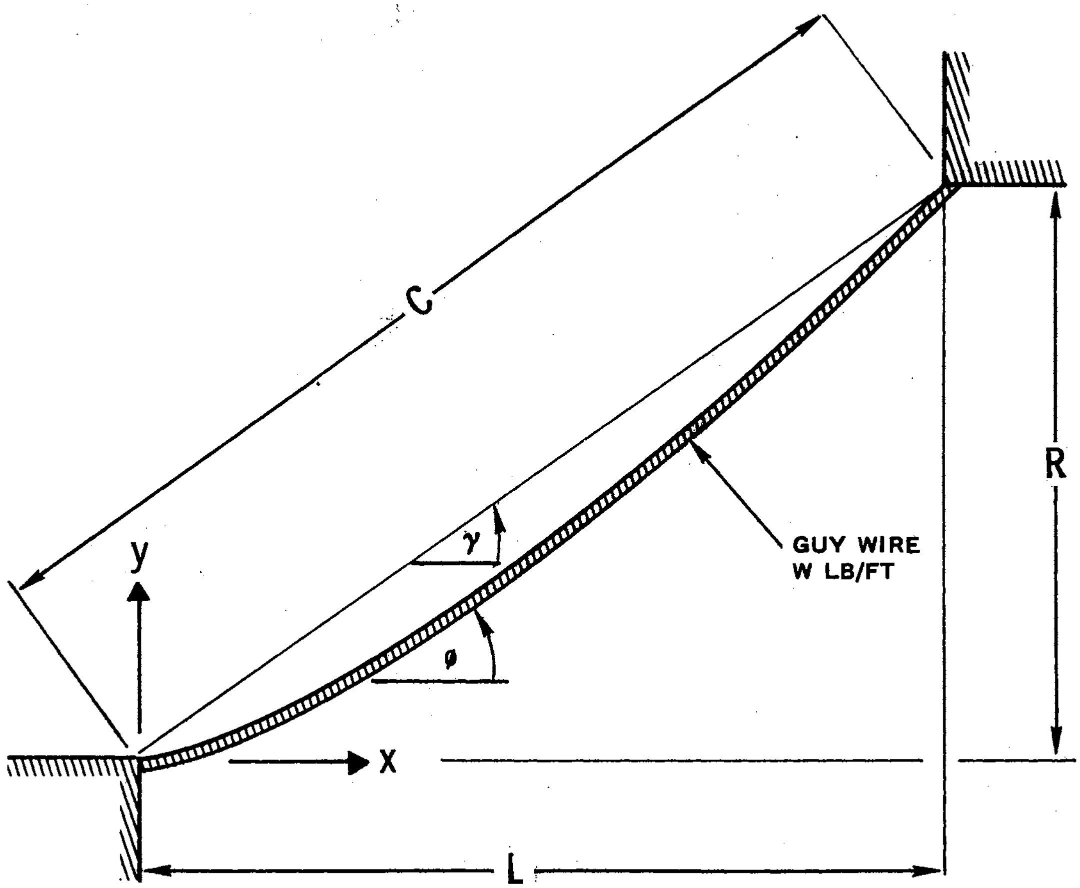

Figure 21. Freely Hanging Guy Wire

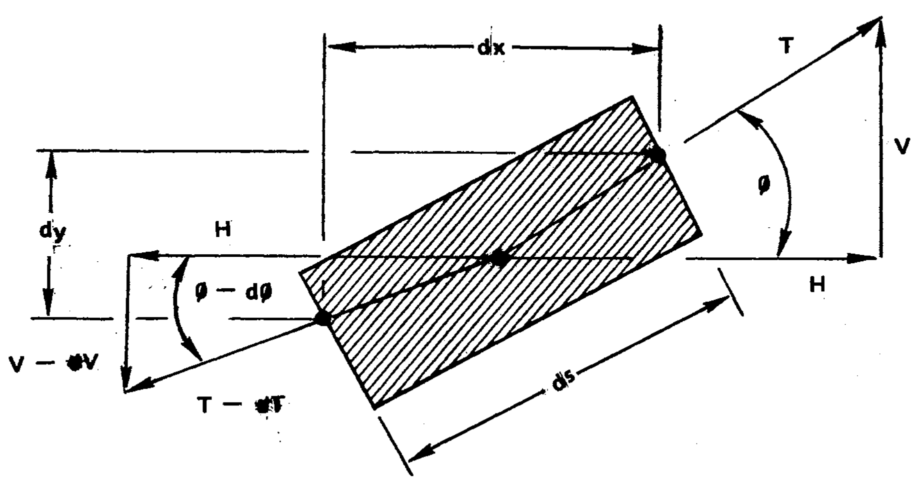

Figure 22. Elemental Length of Guy Wire

Equating then the expressions (C.2) and (C.3) and rearranging leads to the differential equation

$$\frac{y_{xx}}{\sqrt{1 + y_x^2}} = \frac{w}{H} .$$

Integrating now produces the expression

$$\ell n \left(y_x + \sqrt{1 + y_x^2}\right) = \frac{wx}{H} + a$$

Recognizing that

$$\ell n \left(y_x + \sqrt{1 + y_x^2}\right) = \sin h^{-1} y_x$$

allows the above equation to be rewritten as

$$y_x = \sin h \ \left(\frac{wx}{H} + a\right). \tag{C.4}$$

Integrating this expression leads to the equation of the catenary. Thus,

$$y = \frac{H}{w} \cos h \ \left(\frac{wx}{H} + a\right) + b. \tag{C.5}$$

The constants of integration must be evaluated from the boundary conditions as defined by the physical location of the supports. Therefore, from the condition that $y = 0$ at $x = 0$, it is found that

$$b = \frac{-H}{w} \cosh a$$

and then

$$y = \frac{H}{w} \left[\cosh \left(\frac{wx}{H} + a\right) - \cosh a\right]$$

or

$$y = \frac{2H}{w} \sinh \left(\frac{wx}{2H}\right) \sinh \left(\frac{wx}{2H} + a\right). \tag{C.6}$$

From the condition that $y = R$ at $x = L$, it is found that

$$a = \sinh^{-1} \left[\frac{wR/2H}{\sinh (wL/2H)}\right] - \frac{wL}{2H} . \tag{C.7}$$

Using this definition of the constant a in conjunction with the form of the Equation (C.6) establishes a specific form of the catenary equation which mathematically describes the physical configuration of the guy wire.

The Inplace Length

Idealizing the freely hanging guy wire as a line allows the use of the line integral to define its length S. Thus,

$$S = \int_0^c ds = \int_0^c \frac{ds}{dx}\, dx.$$

But from Equation (C.1) $\frac{ds}{dx} = \sqrt{1 + y_x^2}$ where y_x is defined by Equation (C.4). Thus,

$$\frac{ds}{dx} = \sqrt{1 + \sinh^2 \left(\frac{wx}{H} + a\right)} = \cosh \left(\frac{wx}{H} + a\right) \tag{C.8}$$

and

$$s = \int_0^L \cosh \left(\frac{wx}{H} + a\right) dx$$

Integrating yields the expression

$$S = \frac{H}{w} \left[\sinh \left(\frac{wL}{H} + a\right) - \sinh a\right].$$

Substituting the value of a as defined by Equation (C.7) and rearranging terms establishes the length of the tensioned guy wire as

$$S = \left[R^2 + \left\{\frac{2H}{w} \sinh \left(\frac{wL}{2H}\right)\right\}^2\right]^{0.5} \tag{C.9}$$

The Slope of the Catenary

By definition, the angle of the slope at any point on the curve is

$$\phi = \tan^{-1} y_x .$$

Substituting Equation (C.4) for y_x allows the slope of the catenary to be defined as

$$\phi = \tan^{-1} [\sinh (\frac{wx}{H} + a)] \qquad (C.10)$$

Guy Wire Sag

The sag of a freely hanging guy wire is defined as the maximum departure of the idealized guy wire from and in the direction normal to the chord connecting the supports. By inspection it is deduced that the maximum departure occurs at the point where $\tan\phi = \tan\gamma = R/L$. Defining x as the horizontal distance from the origin to the point at which the maximum distance occurs allows Equation (C.10) to be rewritten as

$$\frac{R}{L} = \sinh \left(\frac{wx_r}{H} + a\right).$$

Solving for x_r defines the abscissa for the point as

$$x_r = \frac{H}{w} [\sinh^{-1} \frac{R}{L} - a] \qquad (C.11)$$

Substituting this value at x_r into Equation (C.6) and simplifying yields the ordinate of the point

$$y_r = \frac{H}{w} [\sec\gamma - \cosh a] \qquad (C.12)$$

The maximum sag h of the cable is now defined as

$$r = \cos\gamma \, (x_r \tan\gamma - y_r)$$

or

$$r = x_r \sin\gamma - y_r \cos\gamma. \qquad (C.13)$$

Tensile Force in Guy Wire

Referring to Figure 22 it is observed that the tension T can be defined as T = H(ds/dx). Substitution of Equation (C.8) for the quantity (ds/dx) gives the desired form at the expression for T as

$$T = H \cosh \left(\frac{wx}{H} + a\right). \tag{C.14}$$

Average Tensile Force

The average tensile force in the guy wire is found by integrating the tensile forces over the length and then dividing by the length. Thus,

$$T_{avg} = \frac{1}{S} \int T ds = \frac{H}{S} \int_0^S \frac{ds}{dx} ds. \tag{C.15}$$

The employment of Equation (C.1) and a change of variables allows this expression to be rewritten as

$$T_{avg} = \frac{H}{S} \int_0^L \cosh^2 \left(\frac{wx}{H} + a\right) dx.$$

Integration and evaluation at limits produce

$$T_{avg} = \frac{H}{2S} \left[L + \frac{H}{2w} \left\{\sinh \left(\frac{2wL}{H} + 2a\right) - \sinh 2a\right\}\right].$$

Substitution of the value of a from Equation (C.7) and the subsequent simplification defines the average tensile force as

$$T_{avg} = \frac{H}{2S} \left[L + \frac{H}{w} \sinh \left(\frac{wL}{H}\right) \left\{1 + 2 \left(\frac{wR/2H}{\sinh wL/2H}\right)^2 \right\}\right] \tag{C.16}$$

Elastic Stretch of Guy Wire

Defining the elongation of an elemental length of guy wire as dS = (T/AE)ds establishes the elongation of the entire guy wire as

$$\Delta S = \int_0^S \frac{T}{AE}\, ds$$

But from Equation (C.15),

$$\int_0^S T ds = T_{avg}\, S$$

and thus

$$\Delta S = (T_{avg}\, S/AE) \tag{C.17}$$

Horizontal Force-Displacement Relationship

The horizontal force-displacement relationship is obtained through the consideration of Equation (C.9). Squaring this expression

$$S^2 = R^2 + \left(\frac{2H}{w} \sinh \frac{wL}{2H}\right)^2 \tag{C.18}$$

Imparting a small horizontal displacement ΔL to the span L leads to the deduction that the parameters consequently affected are the length S and the horizontal force component H. Differentiating the above with respect to H and simplifying yields

$$S \frac{dS}{dH} = \left(\frac{2H}{w} \sinh \frac{wL}{2H}\right) \left[\frac{2}{w} \sinh \frac{wL}{2H} + \cosh \frac{wL}{2H} \left(\frac{dL}{dH} - \frac{L}{H}\right)\right].$$

Solving now for (dL/dH) leads to

$$\frac{dL}{dH} = \frac{dS}{dH} \left[\frac{wS/H}{\sinh (wL/H)}\right] - \left(\frac{2}{w} \tanh \frac{wL}{2H} - \frac{L}{H}\right). \tag{C.19}$$

Similarly,

$$\frac{d^2L}{dH^2} = \left(\frac{dS}{dH}\right)^2 \left[\frac{w/H}{\sinh wL/H}\right] - \left(\frac{2}{wH} \tanh \frac{wL}{2H}\right)$$

$$- \frac{2}{H} \left(\frac{\partial L}{\partial H} - \frac{L}{H}\right) - w \coth \frac{wL}{H} \left(\frac{\partial L}{\partial H} - \frac{L}{H}\right)^2. \tag{C.20}$$

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