

A THEORETICAL ANALYSIS OF KEYNESIAN LABOR MARKET  
ASSUMPTIONS AND THE MACROECONOMIC  
ADJUSTMENT PROCESS

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## PREFACE

The labor market played a very important role in Keynes original analysis of the macroeconomic system. It is ironic that his followers while ostensibly accepting Keynesian labor market assumptions in practice have largely ignored the labor market. As a consequence Keynes' labor market assumptions are not always properly incorporated into neo-Keynesian analysis.

The theoretical proposition and graphical techniques that are used in this study have evolved gradually over the last four or five years. I wish to thank my colleagues at Northern Michigan University for their comments and suggestions on preliminary drafts of various chapters. I wish to single out Mokhlis Y. Zaki, Howard R. Swaine, and Neil W. Carlson in particular for their helpful editorial comments. Thanks are also due to Miss Peggy Madden for her service in typing the dissertation.

However, my primary indebtedness is to Dr. Frank G. Steindl, my dissertation adviser; without his guidance and patient encouragement this study could not have been completed. Dr. Michael R. Edgmand and the other members of my committee made a number of helpful suggestions.

Thanks are also due to my wife, Ruth Ann Holmstrom; without her support it would have been impossible to complete this paper.

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## LIST OF SYMBOLS

Y	- Real income
X	- Output
QD	- Ex ante quantity demanded
N	- Employment
MP	- Marginal product of labor
W	- Real wage
W <sub>0</sub>	- Money wage
P	- Price level
M	- Real money supply
M <sub>0</sub>	- Nominal money supply
L	- Demand for real cash balances
r	- Rate of interest



CHAPTER I  
INTRODUCTION

A reconsideration and comparison of Keynesian and neo-Keynesian labor market assumptions is the prevailing theme of this dissertation. Over the years, a subtle divergence has evolved between Keynes' original system and what has become the orthodox neo-Keynesian model.

It is standard practice for modern neo-Keynesian theorists to treat the labor market as being of a second order of importance. For example, in the majority of modern textbooks, consideration of the labor market is postponed until after the commodity and money markets have been fully developed. The relative effectiveness of monetary and fiscal policy is usually discussed without reference to the labor market. Recently, however, neo-Keynesian theoretical interest in the labor market has shown signs of revival. For example, a well known neo-Keynesian monetary economist maintains that the primary difference between the neo-Keynesian and the modern-quantity-theory approaches to macroeconomics lies in differing assumptions with respect to the flexibility of the money wage and the handling of the labor market in general.<sup>1</sup>

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<sup>1</sup>Ronald L. Teigen, "A Critical Look at Monetarist Economics," Federal Reserve Bank of St. Louis Review, (January, 1972), pp. 15-17.

The objectives of this paper are the following. First, to demonstrate that the orthodox neo-Keynesian model does not accurately reflect Keynes' original labor market assumptions. It is shown that Keynes' labor market assumptions imply a positive rather than a negative relationship between the price level and quantity demanded. Second, it is demonstrated that the orthodox neo-Keynesian analysis of "excess demand" and "excess supply" is based on aggregate demand and supply relationships which are inconsistent. Third, a number of new techniques for graphically illustrating the Keynesian system are suggested. These have the effect of bringing to light labor market aspects of Keynes' model which have been suppressed by conventional neo-Keynesian methodology.

#### Classical Labor Market Postulates -- A Review

Keynes' labor market model is based in part on Classical assumptions. Classical theory rests on two fundamental postulates. The first, which Keynes accepted and his followers also supposedly accept, is "that the  $\overline{\text{real}}$  wage is equal to the marginal product of labor." The second is "that the utility of the  $\overline{\text{real}}$  wage when a given amount of labor is employed is equal to the marginal disutility of that amount of employment."<sup>2</sup>

For both Classical and Keynesian analysis these postulates have important dynamic connotations. In the Classical model these condi-

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<sup>2</sup>John M. Keynes, The General Theory of Employment Interest and Money (New York, 1936), p. 5.

tions are assumed to hold during the shortest interval of economic time. This means that an autonomous change in aggregate demand results in an equiproportional adjustment in both the money wage and the price level, the real wage therefore remaining constant. Thus, employment, output and real income are in a sense independent of aggregate demand. The Classical approach implicitly assumes that money wages and prices have high adjustment velocities relative to variations in employment, output, and income.

Keynes was able to establish a theoretical link between employment, output, income and aggregate demand only by rejecting the second Classical postulate.<sup>3</sup> Instead of assuming a high adjustment velocity for money wages, Keynes argued that money wages are relatively slow to adjust. On the other hand, Keynes explicitly accepted the assumption of equality between the real wage and the marginal product of labor not only at the equilibrium position but during the adjustment process as well.

In emphasising our point of departure from the classical system, we must not overlook an important point of agreement. For we shall maintain the first postulate as heretofore, subject only to the same qualifications as in the classical theory; and we must pause, for a moment, to consider what this involves.

It means that, with a given organisation, equipment and technique, real wages and the volume of output (and hence of employment) are uniquely correlated, so that, in general, an increase in employment can only occur to the accompaniment of a decline in the rate of real wages. Thus I am not disputing this vital fact which the classical economists have (rightly) asserted as indefeasible. In a given state of organisation, equipment and technique, the real wage earned by a unit of labour has a unique (inverse) correlation

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<sup>3</sup>Ibid., pp. 8, 11-12.

with the volume of employment. Thus if employment increases, then, in the short period, the reward per unit of labour in terms of wage-goods must, in general, decline and profits increase. This is simply the obverse of the familiar proposition that industry is normally working subject to decreasing returns in the short period during which equipment, etc. is assumed to be constant; so that the marginal product in the wage-good industries (which govern real wages) necessarily diminishes as employment is increased. So long, indeed, as this proposition holds, any means of increasing employment must lead at the same time to a diminution of the marginal product and hence of the rate of wages measured in terms of this product.<sup>4</sup>

The acceptance of the first postulate means that at any given level of employment the real wage equals the marginal product of labor. Both Keynes and his followers acknowledge this conclusion.

The acceptance of the first postulate plays an important part in the conclusions reached in this paper. It must be acknowledged that the assumption that the real wage equals the marginal product of labor continuously is by no means sacrosanct. Some authors have developed models which explicitly drop the first postulate.<sup>5,6</sup> The orthodox neo-Keynesian model, however, ostensibly accepts the first postulate.<sup>7</sup>

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<sup>4</sup>Ibid., pp. 17-18.

<sup>5</sup>Don Patinkin, Money, Interest and Prices, 2nd ed., (New York, 1965), pp. 313-334.

<sup>6</sup>Robert J. Barro and Herschel I. Grossman, "A General Disequilibrium Model of Income and Employment," American Economic Review, (March, 1971), pp. 83-93.

<sup>7</sup>Harry G. Johnson, Macroeconomics and Monetary Theory, (Chicago, 1972), pp. 9-10, 190-192.

## Hypothesis of the Dissertation

It is the hypothesis of this dissertation that the orthodox neo-Keynesian model does not fully reflect Keynes' labor market assumptions and in particular the role played by the labor market in the Keynesian theory of demand. In other words, it is hypothesized that if the complete Keynesian model is based on the assumption that the real wage equals the marginal product of labor, at all moments in time, then it is not legitimate to develop a theory of aggregate demand without reference to the labor market. Neo-Keynesian analysis of Keynes invariably notes an aggregate demand function in which there is a negative relationship between the price level and quantity demanded.<sup>8,9</sup> It is hypothesized that "aggregate demand" curves of this type require a violation of the labor market conditions implied by the first postulate. Furthermore, it is hypothesized that a positive rather than a negative relationship between the price level and quantity demanded is consistent with Keynesian labor market assumptions.

One way in which the labor market is connected to quantity demanded in the Keynesian system is via the real money supply and the rate of interest. Neo-Keynesian money market analysis usually ignores the labor market. Yet it can be demonstrated that the first postulate

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<sup>8</sup>Joseph P. McKenna, Aggregate Economic Analysis (Homewood, Ill., 1971).

<sup>9</sup>The expression "demand" curve is a misnomer with reference to curves of this type. This point is explained in detail in Chapter 4 on pages 54 through 60.

implies a direct connection between the level of employment and the equilibrium rate of interest.

Robert A. Mundell has developed a variation of the neo-Keynesian model that focuses on the interrelationship between the endogenous variables in the commodity and the labor markets.<sup>10</sup> Thus, Mundell's system is an improvement over orthodox neo-Keynesian analysis. It is hypothesized, however, that Mundell's system can be improved by a more effective treatment of the money market. Two new varieties of monetary equilibrium curves are developed in this paper which make it possible to avoid Mundell's assumption of a pegged rate of interest.<sup>11</sup> Mundell's analysis of disequilibria situations is based on the assumption that the commodity market adjusts more rapidly than the labor market. If, on the other hand, it is assumed that the commodity market adjustment lags behind the labor market, as Keynes assumed in the General Theory, Mundell's conclusions are reversed. In other words, Mundell's results are shown to be dependent on his particular assumptions about the relative adjustment velocities of the two markets.

#### Scope of the Dissertation

The scope of this dissertation is limited to the analysis of the issues mentioned above. The important problem of anticipated inflation is ignored in this paper as it is in the conventional neo-Keynesian

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<sup>10</sup>Robert A. Mundell, "An Exposition of Some Subtleties in the Keynesian System," Macroeconomics Selected Readings ed. by Walter L. Johnson and David R. Kamerschen (New York, 1970), pp. 32-38.

<sup>11</sup>Ibid., pp. 36-37.

model. For simplicity, the analysis is conducted in terms of a no-government model except that a monetary authority is assumed to control the money supply. The real balance effect is also assumed away.

The next chapter documents the fact that the Keynesian theory of the adjustment process is founded on the assumption of a close relationship between quantity demanded and employment and that the Keynesian theory of aggregate demand implies a positive relationship between the price level and quantity demanded.

Chapter 3 expands the analysis to include the money market and the rate of interest. A monetary equilibrium relation which takes cognizance of previously neglected labor market considerations is developed. In Chapter 4 the orthodox neo-Keynesian interpretation of Keynes' theory of aggregate demand is demonstrated to be defective. In addition to demonstrating that the neo-Keynesian aggregate demand function is inconsistent with Keynes' labor market assumptions, it is also shown that the neo-Keynesian theory of demand is based on assumptions which are contrary to those used in deriving the neo-Keynesian aggregate supply function.

In Chapter 5, Robert Mundell's article is reviewed.<sup>12</sup> Mundell develops a graphical model capable of demonstrating the interrelationship between the labor market and the commodity market equilibrium. However, Mundell's model is defective in one important respect. The Mundell model does not effectively include the money market. A method of avoiding this weakness is suggested. Also it is demonstrated that

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<sup>12</sup>Ibid.

alternative assumptions about the relative adjustment velocities of the labor and commodity markets have the effect of reversing Mundell's conclusions.

#### Summary

It is the primary objective of this paper to demonstrate that the conventional neo-Keynesian model does not make manifest the full implications of Keynes' labor market assumptions. The neo-Keynesian model implies a negative relationship between quantity demanded and the price level while Keynesian labor market assumptions indicate a positive relationship between these variables. It is also shown in this paper that careful attention to Keynesian labor market assumptions leads to a modification of orthodox neo-Keynesian money market analysis.



## CHAPTER II

### THE LABOR MARKET AND AGGREGATE DEMAND

The present chapter begins the analysis with the development of a simplified Keynesian model. This simplified model is contrived so as to emphasize aspects of Keynes' analysis which are not reflected in other interpretations of his work.

For this reason the orthodox neo-Keynesian procedure of developing the commodity and money markets prior to consideration of the labor market is altered. It is convenient instead to revert to Keynes' original procedure of starting with the labor market and a simplified theory of aggregate demand.

The term "neo-Keynesian" covers a wide variety of analytical approaches. The most popular representation of neo-Keynesian theory was developed by Joseph P. McKenna.<sup>1</sup> Other authors have since developed numerous variations based on McKenna's methodology. In fact, this approach is so popular that the paradigm of McKenna can be satisfactorily taken as the orthodox neo-Keynesian model.

The usual neo-Keynesian methodology involves dividing the three Keynesian markets into two subsets. The money and commodity market are combined to produce an "aggregate demand" function. The "aggregate

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<sup>1</sup>Joseph P. McKenna, Aggregate Economic Analysis, 4th ed., (Hinsdale, Ill., 1972).

supply" function is based exclusively on the labor market. The independently derived aggregate "demand" and "supply" functions are subsequently used to define "excess demand" and "excess supply" conditions and to determine the equilibrium price level and real income.

This approach is now so common that it is accepted almost without question as an accurate representation of Keynesian theory. Nevertheless, it is important to notice that the approach rests on the assumption that the money and commodity markets determine the level of aggregate demand without reference to the labor market. Yet in the General Theory it is explicitly assumed that real consumption is directly related to the level of employment.

Since we are here concerned in determining what sum will be spent on consumption when employment is at a given level, we should, strictly speaking, consider the function which relates the former quantity (C) to the latter (N). It is more convenient, however, to work in terms of a slightly different function, namely, the function which relates the consumption in terms of wage-units ( $C_w$ ) to the income in terms of wage-units ( $Y_w$ ) corresponding to a level of employment N...In general it is a good approximation to regard  $Y_w$  as uniquely determined by N.<sup>2</sup>

When Keynes wrote the General Theory, national income accounting was not fully developed. Keynes believed that it was not possible to satisfactorily measure aggregate real income and aggregate output. Accordingly his aggregate demand and supply functions were measured in "wage units."<sup>3</sup> The level of employment was used as the independent

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<sup>2</sup>John M. Keynes, The General Theory of Employment Interest and Money, (New York, 1936), p. 90.

<sup>3</sup>Ibid., p. 41.

variable in the aggregate demand as well as the aggregate supply function.<sup>4</sup>

The primary objective of this chapter is to develop a simplified Keynesian theory of employment. The model is Keynesian in the sense that aggregate demand is related to the level of employment. On the other hand, this model makes use of the modern convention of assuming that output and real income are operational variables. The preliminary model developed in this chapter is a simplified version of the Keynesian system. It is based on the assumption that quantity demanded is independent of the rate of interest. This allows consideration of the money market to be postponed until the next chapter.

The functional relationship between quantity demanded and the price level that is derived in this chapter implies a positive relationship between these variables. This result is interesting because neo-Keynesian analysis implies a negative relationship between quantity demanded and the price level.

Both the Keynesian and the neo-Keynesian models are based on the assumption that the money wage is constant. It can be shown, however, that the existence of involuntary unemployment does not require that the money wage be fixed in an absolute sense. It is sufficient that the money wage is slow to adjust relative to other endogenous variables in the system.

In the next section the Keynesian theory of employment is reviewed and is related to a simple neo-Keynesian model of income determination. In the third section the Keynesian theory of the real wage and the

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<sup>4</sup>Ibid., p. 29.

price level is incorporated into the model developed in the second section. It is shown that the Keynesian system implies a positive relationship between quantity demanded and the price level. The fourth section demonstrates that the Keynesian model requires sticky but not necessarily rigid money wages. The last section contains a summary of the chapter.

#### A Review of Selected Aspects of Keynes'

##### Theory of Employment

The objective of this section is to review Keynes' theory of employment in order to establish the fact that the level of employment is the independent variable in Keynes' aggregate demand and aggregate supply functions. In the next section the Keynesian approach is related to the neo-Keynesian income-expenditure approach.

Keynes points out that in any economy where production occurs over time, employment and output decisions are necessarily based on expectations as to the future state of demand.<sup>5</sup> These demand forecasts are based on sales results of the recent past. "It is sensible for producers to base their expectations on the assumption that the most recently realized results will continue."<sup>6</sup> It should also be noted that Keynes assumed a lag between changes in aggregate demand and subsequent adjustments in output and employment. "The actually realized results of the production and sale of output will only be relevant

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<sup>5</sup>Ibid., p. 47.

<sup>6</sup>Ibid., p. 51.

to employment insofar as they cause a modification of subsequent expectations."<sup>7</sup> In other words, current employment, output and real income in Keynes' system are independent of current quantity demanded. "When employment increases, aggregate real income is increased. The psychology of the community is such that when aggregate real income is increased, aggregate consumption is increased."<sup>8</sup> Accordingly, Keynes defined his aggregate demand function as relating: "various hypothetical quantities of employment to the proceeds [measured in wage units] which their outputs are expected to yield."<sup>9</sup>

To sum up, the Keynesian system is based on the assumption that "income (both money-income and real-income) depends on the volume of employment."<sup>10</sup> Therefore, employment is uniquely related to quantity demanded. Employment is the "independent" variable for Keynes' aggregate demand and supply functions.<sup>11</sup> Neo-Keynesian economists have converted Keynes' theory of employment into a theory of income determination. In the next section a simple neo-Keynesian model of income determination is related to a modified Keynesian theory of employment.

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<sup>7</sup>Ibid., p. 47.

<sup>8</sup>Ibid., p. 27.

<sup>9</sup>Ibid., p. 55.

<sup>10</sup>Ibid., p. 28.

<sup>11</sup>Ibid., p. 29.

The Neo-Keynesian Theory of Income and the  
Keynesian Theory of Employment

In this section a modified Keynesian theory of employment is related to an elementary neo-Keynesian theory of income determination. The objective of this section is to demonstrate the similarity between the Keynesian theory of employment and the simplest kind of income-expenditure model and to establish the framework for the analysis of the price level which follows in the next section. The analysis in this section is based on the following set of equations.

$$QD = \theta(Y) \quad \text{Quantity demanded is a function of income} \quad (2.1)$$

$$Y \equiv X \quad \text{The definitional equality between income and output} \quad (2.2)$$

$$QD = X \quad \text{The equilibrium condition} \quad (2.3)$$

$$X = T(N) \quad \text{The production function} \quad T'(N) > 0 \quad (2.4)$$

It is also assumed that  $0 < \frac{dQD}{dY} < 1$  and that  $\frac{d^2X}{dN^2} < 0$ . Verbally these conditions mean that the marginal propensity to consume is less than one but greater than zero and that the marginal product of labor curve has a negative slope.

A simple neo-Keynesian system is represented by equations (2.1), (2.2), and (2.3). This set contains three unknowns, QD, Y, and X and three equations. It is a market equilibrium system. Equation (2.1) is a neo-Keynesian aggregate demand function. It relates quantity demanded to real income. This curve is represented graphically by the  $QD_y$  curve on the right side of Figure (1). The  $45^\circ$  line in Figure (1) represents equation (2.2). This line indicates the output which corresponds

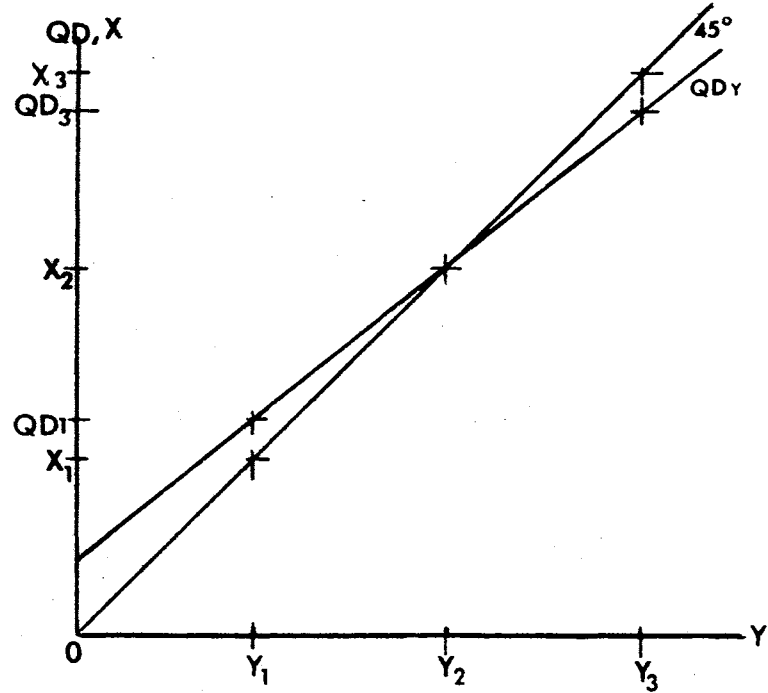
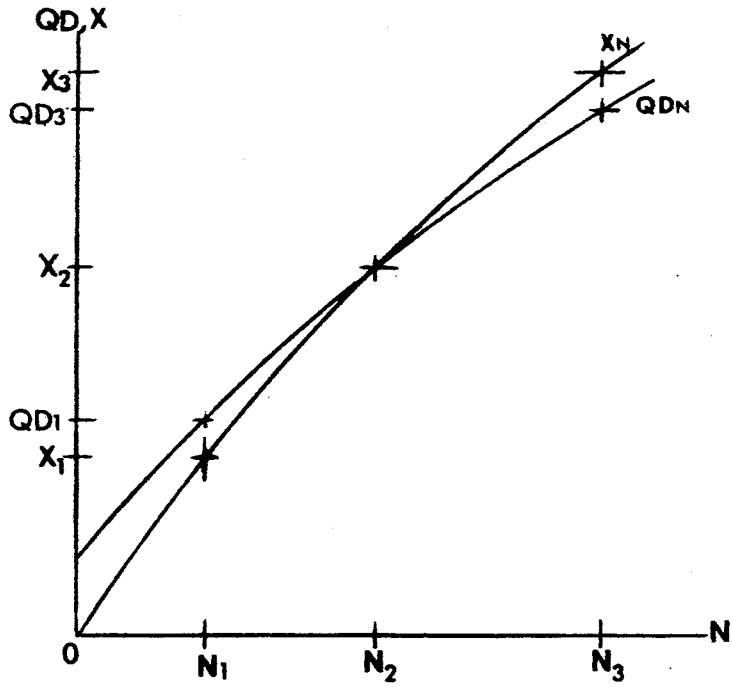


Figure 1. Keynesian Employment and Neo-Keynesian Income-Expenditure Models

to alternative levels of real income. The model's equilibrium conditions are expressed by equation (2.3). The diagram shows that equation (2.3) is satisfied only at income  $Y_2$ . At income  $Y_1$ , for example, quantity demanded  $QD_1$  is greater than output  $X_1$ . This means that at income  $Y_1$  businessmen experience unexpected decreases in their inventories and this leads them to increase employment and output. A rise in output means a rise in real income. The rise in income leads to an increase in quantity demanded. However, since by assumption  $\frac{dQD}{dY} < 1$ , the increase in output is less than the increase in income. Eventually at income  $Y_2$  quantity demanded equals output.

The "Keynesian" employment model is represented on the left side of Figure (1). This model is based on equations (2.1), (2.2), (2.3), and (2.4). The endogenous variables are  $QD$ ,  $Y$ ,  $X$ , and  $N$ . Mathematically this model differs from the simple neo-Keynesian system only by the addition of equation (2.4).

The  $X_n$  curve on the "Keynesian" left side of Figure (1) represents the production function. The  $QD_n$  curve in the same graph is a "Keynesian" aggregate demand curve. This demand function is based on equations (2.1), (2.2), and (2.4). These three equations include four endogenous variables,  $QD$ ,  $Y$ ,  $X$ , and  $N$  and three equations. This set may be reduced to one equation in two unknowns by substituting equation (2.4) into (2.2) and (2.2) into (2.1). Equation (2.5), which is obtained in this way represents the  $QD_n$  curve.

$$QD = \phi \sqrt{\bar{T}(N)} \quad (2.5)$$



The slope of the  $QD_n$  curve is equal to:

$$\frac{dQD}{dN} = \left( \frac{\partial QD}{\partial Y} \right) \left( \frac{\partial X}{\partial N} \right) \quad (2.6)$$

This result is obtained by taking the total differential of equations (2.1), (2.2), and (2.4). Equation (2.6) is then obtained by substitution.

The relationship between the Keynesian theory of employment and the neo-Keynesian theory of income can also be demonstrated graphically. The 45° line on the right side shows that each income is uniquely related to a particular output. For example, income  $Y_1$  implies output  $X_1$ . On the left side the  $X_n$  curve shows that an output of  $X_1$  occurs only when employment is  $N_1$ . If  $N_1$  is uniquely related to  $X_1$  and  $X_1$  implies an income of  $Y_1$ , it follows that  $N_1$  and  $Y_1$  are also uniquely related. At income  $Y_1$  quantity demanded is  $QD_1$  and therefore quantity demanded is also  $QD_1$  at employment  $N_1$ . In the same way employment  $N_3$  is related to income  $Y_3$  and therefore quantity demanded is  $QD_3$  at both  $Y_3$  and  $N_3$ . Other points on the  $QD_n$  are similarly related to points on the  $QD_y$  curve.

It should be noted that the slope of the  $QD_n$   $\left[ \left( \frac{\partial QD}{\partial Y} \right) \left( \frac{\partial X}{\partial N} \right) \right]$  is less than the slope of the production function  $\left( \frac{\partial X}{\partial N} \right)$  since it is assumed that  $\frac{\partial QD}{\partial Y} < 1$ .

The relationship between the Keynesian employment model on the left and the neo-Keynesian income model on the right is now quite clear. The  $QD_n$  curve on the left corresponds to the  $QD_y$  curve on the right. The production function corresponds to the 45° line in the neo-Keynesian model. In both models equilibrium occurs at that value

of the appropriate "independent" variable (either employment or income) where quantity demanded equals output.

In terms of the employment model on the left the adjustment process can be explained in the following way. At employment  $N_1$ , quantity demanded  $QD_1$  exceeds output  $X_1$ . Thus, business inventories decline at  $N_1$ . This leads employers to increase employment and output. As employment increases along the horizontal axis output rises along the  $X_n$  curve. The rise in output, however, means an increase in income and this means a rise in quantity demanded. At any level of employment less than  $N_2$ , quantity demanded exceeds output and consequently there is a tendency for employment to increase until the system reaches  $N_2$ .

It should be noticed that the equilibrium level of employment in this system is determined without direct reference to the price level. Keynes did not believe that the price level played a major role in his analysis. "We shall find that the Theory of Prices falls into its proper place as a matter which is subsidiary to our general theory."<sup>12</sup> It is not surprising, therefore, that it is not necessary to include the price level in a preliminary version of the "Keynesian" theory of employment. In the following section additional labor market equations are added to the system. These additional equations provide for the determination of the price level, the real wage and the marginal produce of labor.

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<sup>12</sup>Ibid., p. 32.

The Price Level, Quantity Demanded, Employment, and Real Income  
During the Adjustment Process and in Equilibrium

In the previous section the  $QD_n$  curve illustrated a relationship between employment and quantity demanded. More fundamentally, however, this relationship is a manifestation of the underlying relationship between income and quantity demanded. Because employment is uniquely related to output, and income equals output, quantity demanded may be expressed as a function of employment.

In this section quantity demanded is related to the price level. The resulting association is interesting because it indicates a positive relationship between quantity demanded and the price level, whereas the orthodox aggregate demand function which is discussed in Chapter 4 indicates a negative relationship between these variables. It is worth repeating that the positive relationship between the price level and quantity demanded developed in this paper is not indicative of a causal influence of price level on quantity demanded. Rather, this kind of phenomena merely reflects the underlying relationship between quantity demanded and income.

The analysis in this section is based on the following set of equations.

$$QD = \phi(N) \quad \text{The } QD_n \text{ curve} \quad (2.5)$$

$$X = T(N) \quad \text{The production function} \quad (2.4)$$

$$MP = \Omega(N) \quad \text{The marginal product of labor is a function of employment} \quad (2.7)$$

$$MP = \frac{W_0}{P} \quad \text{This equation expresses Keynes' assumption that the real wage}$$

equals the marginal product  
of labor (2.8)

QD = X      The condition necessary for  
equilibrium (2.3)

Both the Keynesian and the neo-Keynesian "aggregate supply" curves are based on equations (2.4), (2.7), and (2.8). This set contains four unknowns, X, N, MP, and P. It was previously established that in the Keynesian system employment is the "independent" variable in the aggregate supply function. Keynes noted that, "For every value of N there is a corresponding marginal productivity of labor.../and this/...determines the real wage."<sup>13</sup> Assuming a declining marginal product of labor "if employment increases, then, in the short period, the reward per unit of labor in terms of wage-goods must, in general, decline and profits increase."<sup>14</sup> Thus, Keynes argues that the real wage must decline as employment increases. If the money wage is assumed to be constant then the price level is positively related to the level of employment. Since output is also positively related to employment, output and the price level are likewise positively related.

This analysis is illustrated in Figure (2). Parts (2B) and (2C) represent the Keynesian theory of employment and the neo-Keynesian theory of income determination which were discussed in the previous section. In part (2E) the MP curve (equation (2.7)) represents the assumed negative relationship between the marginal product and employment. The RW curve in part (2D) refers to the relationship between the

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<sup>13</sup>Ibid., p. 29.

<sup>14</sup>Ibid., p. 17.

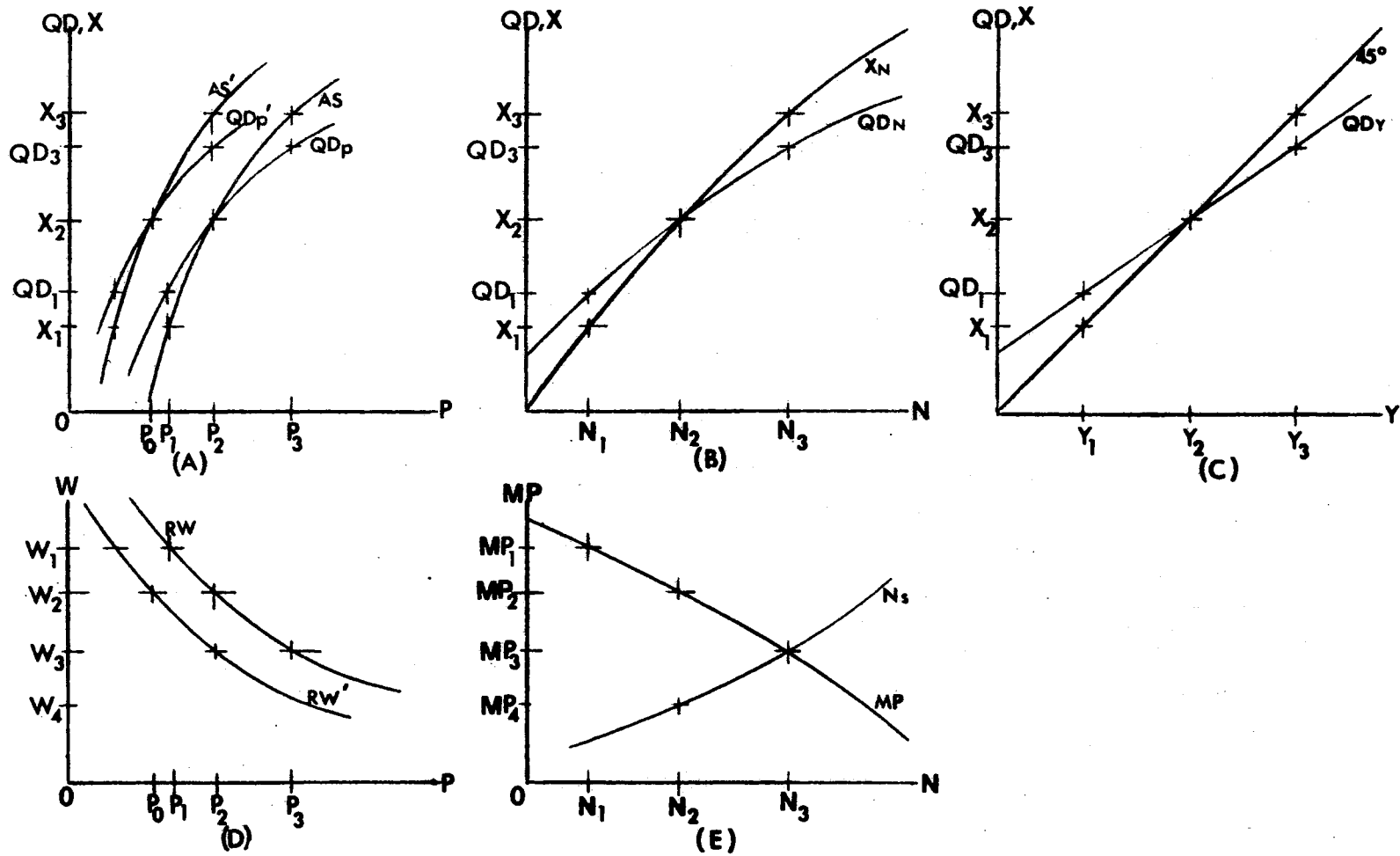


Figure 2. Quantity Demanded and the Price Level

real wage and the price level assuming a constant money wage. This relationship is expressed mathematically by the right side of equation (2.8).

Assume that a level of employment  $N_1$  is selected at random. The MP schedule in part (2E) shows that the marginal product is  $MP_1$  at  $N_1$ . Since, by assumption,  $MP = \frac{W_0}{P}$ , the real wage equals  $W_1$ . The RW curve indicates that a price level of  $P_1$  is required for a real wage of  $W_1$ . Thus employment  $N_1$  is associated with a price level of  $P_1$ . On the other hand, the  $X_n$  curve in part (2B) indicates that an output of  $X_1$  is associated with employment  $N_1$ . It follows that output  $X_1$  combined with price level  $P_1$  represents one point on the aggregate supply curve in part (2A). Other points can be obtained by selecting alternative levels of employment.

The slope of the aggregate supply curve in part (2A) can be obtained by taking the total differentials of equations (2.4), (2.7), and (2.8) and by proper substitution. This slope is represented by equation (2.9).

$$\frac{dX}{dP} = - \frac{W_0}{P^2} \cdot \frac{\frac{\partial X}{\partial N}}{\frac{\partial MP}{\partial N}} \quad (2.9)$$

Since  $\frac{\partial MP}{\partial N}$  is negative, this equation implies that the slope of the aggregate supply curve is positive.

The  $QD_p$  curve in part (2A) represents quantity demanded at alternative price levels. This curve is based on equations (2.5), (2.7), and (2.8). This set contains four unknowns,  $QD$ ,  $N$ ,  $MP$ , and  $P$ .

The graphical derivation of the  $QD_p$  can be explained in the following way. Select employment  $N_3$  at random. The MP curve at  $N_3$  indicates a marginal product of  $MP_3$  and the RW curve in part (2D) implies a real wage of  $W_3$  and a price level of  $P_3$ . The  $QD_n$  curve shows that quantity demanded at  $N_3$  is  $QD_3$ . The combination  $(P_3, QD_3)$  is one point on the  $QD_p$  curve. Other points on the  $QD_p$  can be obtained by repeating the same process for other levels of employment. The slope of the  $QD_p$  curve is obtainable from the total differentials of equations (2.5), (2.7), and (2.8). Equation (2.10) represents the slope of the  $QD_p$  curve.

$$\frac{dQP}{dP} = - \frac{W_0}{P^2} \cdot \frac{\frac{\partial QD}{\partial N}}{\frac{\partial MP}{\partial N}} \quad (2.10)$$

The slope of the aggregate supply curve is similar to the slope of the  $QD_p$  curve except that  $\frac{\partial QD}{\partial N}$  replaces  $\frac{\partial X}{\partial N}$  in the numerator of  $\frac{dQD}{dP}$  function. Since the  $QD_n$  curve is flatter than the  $X_n$  curve,  $\frac{\partial QD}{\partial N} < \frac{\partial X}{\partial N}$ . It follows, therefore, that the slope of the  $QD_p$  curve is flatter than the slope of the aggregate supply curve in part (2A) of Figure (2).

The complete model is represented by equations (2.3), (2.4), (2.5), (2.7), and (2.8). These five equations contain five unknowns,  $QD$ ,  $N$ ,  $X$ ,  $MP$ , and  $P$ .

It is worthwhile at this point to summarize the analysis. Keynes assumed a unique relationship between employment and output. At employment  $N_1$  in part (2B), output is  $X_1$  and therefore income is  $Y_1$  in part

(2C). At the same time, Keynes' labor market assumptions imply that an employment level of  $N_1$  means a price level of  $P_1$ . Thus, an employment level of  $N_1$  implies both an income of  $Y_1$  and a price level of  $P_1$ . It follows, therefore, that the "excess demand" condition that prevails at  $Y_1$  in part (2C) also exists at employment  $N_1$  in part (2B) and at price level  $P_1$  in part (2A).

The  $QD_y$  curve in part (2C) represents the fundamental Keynesian and neo-Keynesian assumption that quantity demanded is positively related to income. Output equals income by definition. Output is also uniquely related to employment. It follows, therefore, that employment is uniquely related to income. This means that quantity demanded is likewise related to employment. The  $QD_n$  curve shows this relationship. But if quantity demanded is positively related to employment and employment is positively related to the price level, it follows that quantity demanded is positively related to the price level.

#### Unemployment and the Money Wage

It is instructive to consider the relationship between unemployment and the level of money wages in the kind of model discussed in the previous section. Leijonhufvud<sup>15</sup> has pointed out that it is not necessary for "unemployment equilibrium" to assume that money wages are fixed in an absolute sense but only that they were relatively slow to adjust.

In part (2E) the supply of labor curve  $N_s$  cuts the MP curve at

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<sup>15</sup>Axel Leijonhufvud, On Keynesian Economics and the Economics of Keynes (New York, New York, 1968), p. 67.



employment  $N_3$ . Employment  $N_3$  is defined as the full employment level of income. Given the current position of the  $QD_y$  curve, involuntary unemployment or underemployment is equal to  $(N_3 - N_2)$  regardless of workers willingness to accept any given cut in their money wage.

For example, assume that the economy is in equilibrium at price level  $P_2$ , and output  $X_2$ . The real wage is  $W_2$  and the price level is  $P_2$ . Assume that workers accept a sharp cut in their money wage sufficient to lower the real wage to  $W_3$  at a price level of  $P_2$ . The  $RW'$  curve represents the real wage curve corresponding to the lower money wage.

Although the model assumes that output decisions are based exclusively on recent sales results, assume for the point of argument, that the temporary fall in real wage to  $W_3$  encourages employers to increase employment to  $N_3$  and output to  $X_3$ . However, at employment  $N_3$  quantity demanded is less than output. Therefore, output and the price level tend to decline along the  $AS'$  curve until equilibrium is reestablished at price level  $P_0$  and output  $X_2$ . The real wage is once again  $W_2$  and employment is again  $N_2$ . As long as the money wage is assumed to adjust more slowly than the price level the real wage always returns to the  $W_2$  level.<sup>16</sup>

On the other hand, if the model had been based on the assumption that the money wage adjusted rapidly relative to the price level, then the equilibrium real wage would be  $W_4$  on the  $N_s$  curve at  $N_2$ . At point  $(W_4, N_2)$  on the supply of labor function in part (2E) there is full employment in the sense that the marginal disutility of labor is equal

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<sup>16</sup>This example closely parallels a similar discussion of the impact of a cut in money wages by Keynes, p. 261.

to the real wage but this condition does not represent full employment in the usual sense of the term.

#### Summary

This chapter stresses the interrelationship between output, employment and quantity demanded in the Keynesian system. Keynes' theory of employment is shown to be closely related to an elementary neo-Keynesian model of income determination.

The analysis in this chapter demonstrates that a positive relationship between output and the price level derives from Keynes' labor market assumptions and this in turn implies a positive relationship between the price level and income. Since in this model income is the sole determinant of quantity demanded, it follows that quantity demanded is positively associated with the price level.

The model can also be used to demonstrate why Keynes argued that a slowly falling money wage cannot restore full employment. The primary weakness of the model used in this chapter is that it ignores the money market and it assumes away the causal impact of the rate of interest on quantity demanded. In the next chapter the model is expanded to take these factors into consideration.

## CHAPTER III

### QUANTITY DEMANDED IN THE COMPLETE KEYNESIAN SYSTEM

In the previous chapter a simple neo-Keynesian income-expenditure model was analyzed in conjunction with Keynes' labor market assumptions. The positive relationship between quantity demanded and the price level that resulted from this analysis is in conflict with a widely held view that quantity demanded and the price level are negatively related. It should be pointed out, however, that the model used in the previous chapter assumed quantity demanded to be independent of the rate of interest. It is also true that those models which indicate a negative relationship between the price level and quantity demanded rely on changes in the rate of interest to account for this phenomenon.

It is the primary objective of this chapter to modify the approach introduced in the last chapter so as to incorporate the money market and the rate of interest into the analysis. It is demonstrated below that while the inclusion of the interest rate does moderate the positive relationship between quantity demanded and the price level, it does not eliminate it. It is demonstrated that quantity demanded and the price level are positively related in any model which accepts Keynes' contention that quantity demanded is primarily (if not exclusively) a function of income.

In order to accomplish the primary goal of this chapter it is necessary to modify the usual neo-Keynesian analysis of the money market. The standard LM curve approach is based on the assumption of a constant price level. However, if real income equals output and output is positively related to the price level, it follows that income is also positively related to the price level. A positive relationship between income and the price level means that given a constant nominal money supply the real money supply is inversely related to income. In this chapter the conventional LM curve approach is modified to take this factor into consideration.

In the following section a more general variety of monetary equilibrium curve is developed to replace the LM curve. In place of the usual assumption of a constant price level this new relation is based on the assumption that the real wage is equal to the marginal product of labor.

The third section demonstrates that the inclusion of the rate of interest in the Keynesian system does not reverse the positive relationship between income and the price level which was uncovered in the last chapter. On the other hand, it can also be shown that if Keynes' assumption that income has a dominating influence on quantity demanded is dropped, and it is likewise assumed that the rate of interest has a relatively strong impact on quantity demanded, it is possible for quantity demanded to be negatively related to the price level.

The fourth section demonstrates that the assumption of a positively sloping IS curve causes no important modifications in the analysis.

The fifth section is devoted to a mathematical check of the conclusions reached in earlier sections and the last section contains a summary of the chapter.

### The LMN Curve

This section is concerned with the graphical derivation of a new variety of monetary equilibrium curve. The "LMN" curve differs from the orthodox neo-Keynesian "LM" curve in that the price level is not treated as autonomous. Instead the price level is assumed to equal the constant money wage over the marginal product of labor. The equations representing the model used in this chapter are listed below.

$$QD = \Psi(Y, r) \quad \text{Quantity demanded is a function of income and the rate of interest} \quad (3.1)$$

$$X \equiv Y \quad \text{The identity between real income and output} \quad (3.2)$$

$$QD = X \quad \text{The conditions necessary for equilibrium in the commodity market} \quad (3.3)$$

$$X = T(N) \quad \text{The production function} \quad (3.4)$$

$$MP = \Omega(N) \quad \text{The marginal product of labor is a function of employment} \quad (3.5)$$

$$MP = \frac{W_o}{P} \quad \text{The real wage equals the marginal product of labor} \quad (3.6)$$

$$L(Y, r) = \frac{M_o}{P} \quad \text{The money market equilibrium equation} \quad (3.7)$$

This system contains seven equations and seven unknowns. The unknowns are  $QD$ ,  $Y$ ,  $r$ ,  $X$ ,  $N$ ,  $MP$ , and  $P$ . Equation (3.1) differs from the demand function used in the previous chapter in that the rate of interest is included as a causal variable. It is assumed that  $\frac{dQD}{dr} < 0$ .

Equation (3.7), the monetary equilibrium equation has also been added to the system. It is taken for granted that  $\frac{dL}{dr} < 0$  and  $\frac{dL}{dY} > 0$ .

The LMN curve is based on equations (3.2), (3.4), (3.5), (3.6), and (3.7). This set contains five equations and six endogenous variables. The endogenous variables are  $X$ ,  $Y$ ,  $N$ ,  $MP$ ,  $P$ , and  $r$ . The analysis is illustrated in Figure (3). In part (3B) of Figure (3) the  $L_{Y1}$ ,  $L_{Y2}$ , and  $L_{Y3}$  curves represent the demand for money with respect to the rate of interest at alternative levels of income. Mathematically these curves represent the left side of equation (3.7). The right side of equation (3.7) is represented by the RMS curve in part (3E). This curve measures the real money supply corresponding to alternative price levels assuming a given nominal money supply. The RW curve in part (3F) which is similar to the RW curve used in Chapter 2 represents the relationship between the real wage and the price level assuming a constant money wage. The RW curve refers to the right side of equation (3.6). The marginal product curve is shown in part (3H). The production function is represented by the  $X_n$  curve in part (3J).

The derivation and analysis of the  $QD_p$  and  $QD_n$  curves is postponed until the next section.

Assume that after taking into consideration quantity demanded in the previous period, producers set output for the current period at  $X_1$ . This means income is  $Y_1$ . Part (3J) shows that an output of  $X_1$  requires a labor input of  $N_1$ . In part (3H) the marginal product of labor is  $MP_1$ . Given the constant money wage, the real wage curve in part (3F) indicates a price level of  $P_1$  will equate the real wage and

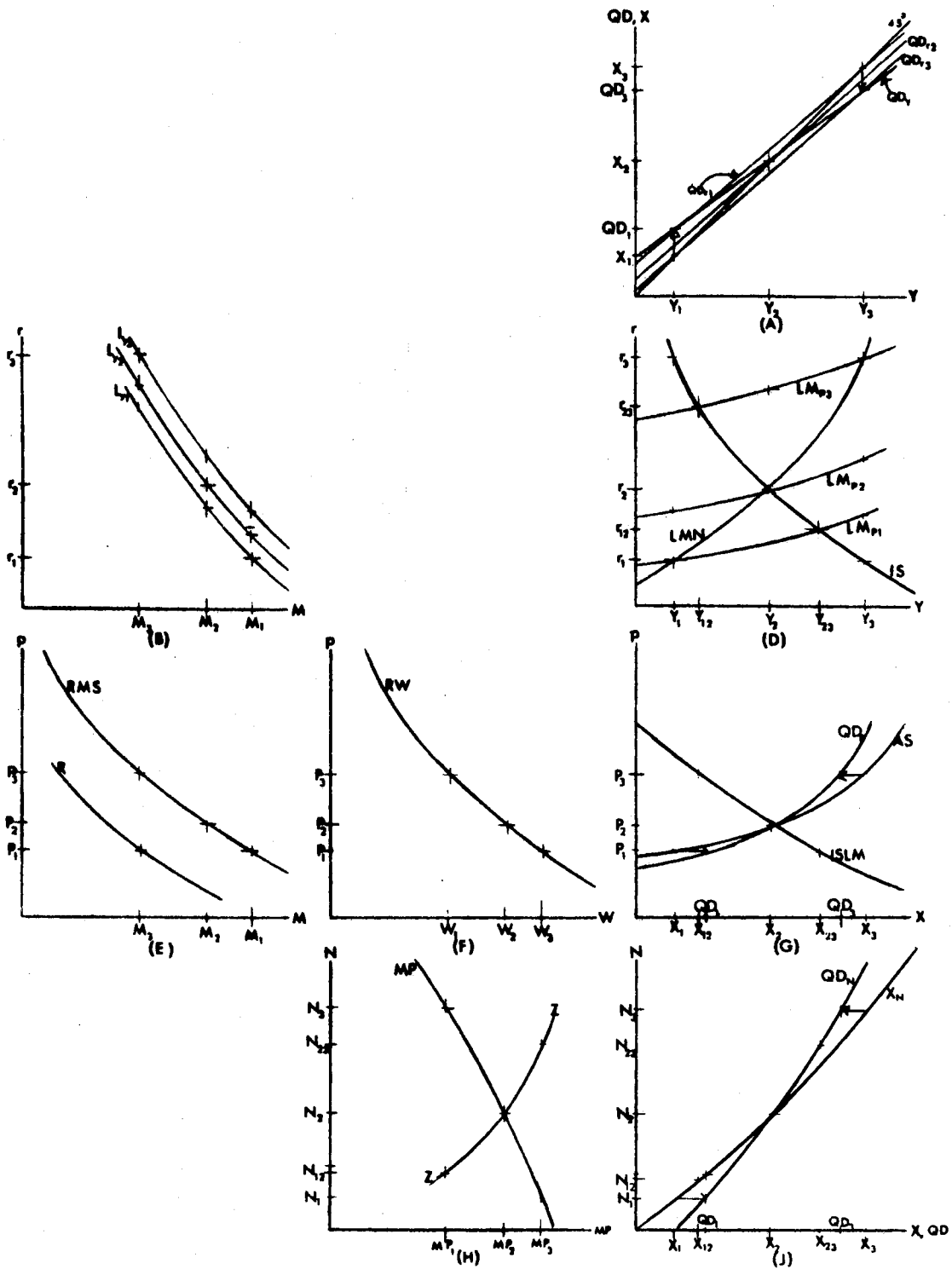


Figure 3. The Complete Keynesian System

the marginal product of labor. The RMS curve in part (3E) shows that a real money supply of  $M_1$  is consistent with a price level of  $P_1$ . Since the current output of  $X_1$  implies a real income of  $Y_1$ , the  $L_{y1}$  curve in part (3B) represents the demand for money. An interest rate of  $r_1$  is consistent with money market equilibrium in this situation. Combining interest rate  $r_1$  with income  $Y_1$  yields one point on the LMN curve. On the other hand, if production in the current period is set at  $X_2$ , then the price level must be  $P_2$  in order to satisfy equation (3.6). The increase in the price level to  $P_2$  reduces the real money supply to  $M_2$  while the increase in income to  $Y_2$  shifts the demand for money function to the  $L_{y2}$  position. The real money supply,  $M_2$  cuts the  $L_{y2}$  curve at interest rate  $r_2$ . The combination of interest rate  $r_2$  and income  $Y_2$  is another point on the LMN curve. Other points can be obtained in a similar way.

The LMN curve represents alternative combinations of the rate of interest and the level of income consistent with equilibrium in the money market while at the same time the real wage is equal to the marginal product of labor. The LMN curve differs from the conventional neo-Keynesian monetary equilibrium curve in the latter respect.

#### The $QD_p$ and $QD_n$ Curves

This section is concerned with the graphical derivation of the  $QD_p$  curve in part (3G) and the  $QD_n$  curve in part (3J). These curves are based on equations (3.1), (3.2), (3.4), (3.5), (3.6), and (3.7). This set includes six equations and seven unknowns. The  $QD_p$  curve represents the relationship between quantity demanded and the price



level and the  $QD_n$  curve illustrates the relationship between quantity demanded and employment. It can be observed that equation (3.3), the commodity market equilibrium condition, has been excluded from this system.

It should be recalled that it was determined in Chapter 2 that the Keynesian system is based on the assumption of a lag between changes in quantity demanded and subsequent adjustments in output and employment.

Given the LMN curve derived in the previous section, it is now quite easy to describe the derivation of the  $QD_p$  and  $QD_n$  curves. Assume that current output is set at  $X_1$ . The LMN curve indicates that a rate of interest of  $r_1$  clears the money market at income  $Y_1$  and price level  $P_1$ . The  $QD_{r_1}$  curve in part (3A) indicates that quantity demanded is  $QD_1$  at income  $Y_1$ . By transferring quantity demanded  $QD_1$  to price level  $P_1$ , and to employment  $N_1$ , points on the  $QD_p$  and the  $QD_n$  curves are obtained. These curves represent the relationship between changes in the designated variables and quantity demanded during the adjustment process. If current output is set at  $X_3$  and therefore income is  $Y_3$ , the interest rate in the current period would be  $r_3$ . Quantity demanded at income  $Y_3$  is  $QD_3$ . By transferring quantity demanded  $QD_3$  to the price level, and to the employment associated with output  $X_3$ , another set of points on the QD curves is obtained.

The adjustment process can be explained in the following way. Assume that the "R" curve in part (3E) represents the RMS curve. In this case the LMN curve (not shown) cuts the IS curve at interest rate  $r_3$  and income  $Y_1$ . The equilibrium level of employment is  $N_1$  and the

real wage is  $W_3$ . The price level is  $P_1$  and the real money supply is  $M_3$ .

For convenience assume that price level  $P_1 = 1.0$ . In this case the real and nominal money supply is  $M_3$ . Now assume that the nominal money stock is increased from  $M_3$  to  $M_1$ . Prior to any change in  $P$ , the increase in the money supply lowers the rate of interest down the  $L_{y1}$  curve to  $r_1$ . The fall in the rate of interest causes an upward shift of the  $QD_r$  curve in part (3A) to the  $QD_{r1}$  position. The excess of quantity demanded over production at income  $Y_1$  leads to an increase in output in the following period. As output increases employment increases along the  $X_n$  curve, the price level rises along the  $QD_p$  curve, and the rate of interest climbs along the LMN curve. However, as long as quantity demanded exceeds output the expansion continues. Equilibrium is reestablished at output  $X_2$ , interest rate  $r_2$ , employment  $N_2$  and price level  $P_2$ .

The positive relationship between quantity demanded and the price level occurs because under Keynesian labor market assumptions quantity demanded is positively related to income and income is positively related to the price level. It must be acknowledged, however, that these results are dependent on Keynes' assumption that income dominates quantity demanded. If, instead, it is assumed that the rate of interest dominates quantity demanded and income is of secondary importance, a positive relationship between the price level and quantity demanded is entirely possible even under Keynesian labor market requirements.

To demonstrate this possibility, Figure 4 is constructed on the assumption that the rate of interest is the dominating variable influencing quantity demanded. The  $QD_r$  curves are not steeply sloped and the shift in the  $QD_r$  curves is relatively large for a given change in the rate of interest. These alterations imply a relatively weak income response and a relatively strong interest rate impact on quantity demanded as compared to Figure 3.

At income  $Y_1$  for example, quantity demanded equals  $QD_1$  if the rate of interest is  $r_1$ . The AS curve indicates a price level of  $P_1$  is associated with output  $X_1$ . Quantity demanded in this situation is  $QD_1$ . Similarly at price level  $P_3$  quantity demanded is  $QD_3$ . As a result the  $QD_p$  curve has a negative slope.

The dotted  $QD_y$  curve in the upper graph represents quantity demanded at alternative income levels assuming that the interest rate is that indicated by the LMN curve. It should be noted that the  $QD_y$  curve has a negative slope implying a negative relationship between income and quantity demanded. The negative slope of the  $QD_y$  reflects the fact that a diagram is drawn so that the interest rate rather than income is the primary variable influencing quantity demanded. There is, of course, nothing wrong with such an assumption except that this kind of model should not be labeled "Keynesian" since it is based on non-Keynesian assumptions.

Finally, it should also be pointed out that although their appearance is similar, the  $QD_p$  curve in Figure 4 and the McKenna "aggregate demand" curve that is discussed in the next chapter are distinctly different. The two curves are based on different sets of assumptions.

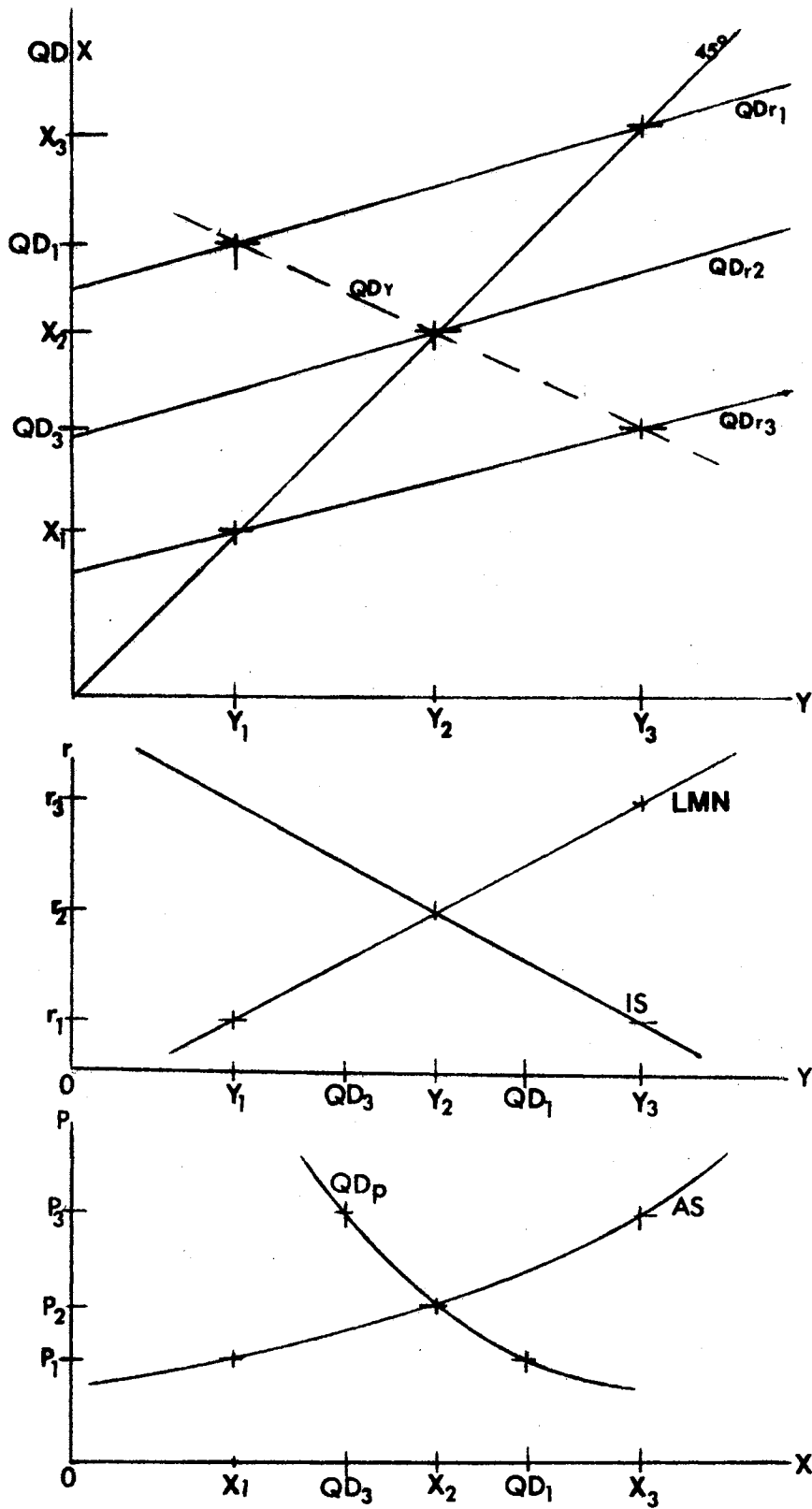


Figure 4. A Negative Sloping  $QD_p$  Function

Quantity Demanded in Conjunction with a  
Positive Sloping IS Curve

So far the analysis has proceeded on the assumption that the IS curve has a "normal" negative slope. However, in recent years, increasing support has been forthcoming for the contention that the IS frequently slopes upward to the right rather than downward to the left. It has been shown that a positively sloped IS curve requires the  $QD_r$  curves in the  $45^\circ$  line diagram have slopes in excess of unity.<sup>1</sup>

In Figure 5 the derivation of the  $QD_p$  curve with reference to a positively sloped IS curve is demonstrated. In the upper graph the  $QD_r$  curves corresponding to the indicated rate of interest have slopes in excess of one. The IS curve is derived in the normal way.

Assume that current output is  $X_2$  and current income is  $Y_2$ . At income  $Y_2$  the LMN curve indicates an interest rate of  $r_1$ . Since the  $QD_{r1}$  curve is above the  $45^\circ$  line at that income, interest rate  $r_1$  is obviously too low for equilibrium at income  $Y_2$ . The quantity  $QD_2$  in the upper graph measures quantity demanded at income  $Y_2$  and interest rate  $r_1$ . At income  $Y_4$  and interest rate  $r_5$  quantity demanded is  $QD_4$ . By transferring these amounts to the lower graphs the  $QD_p$  curve is obtained. It can be noted that the  $QD_y$  curve slopes upward and has a slope less than one even though the individual  $QD_r$  curves have slopes in excess of one! If the  $QD_y$  curve in the upper graph had a slope in

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<sup>1</sup>Frank G. Steindl, "Giffen Goods, IS Curves and Macroeconomic Stability," Metroeconomica, Vol. XXII, Maggio-Agosto, 1970, pp. 165-169.

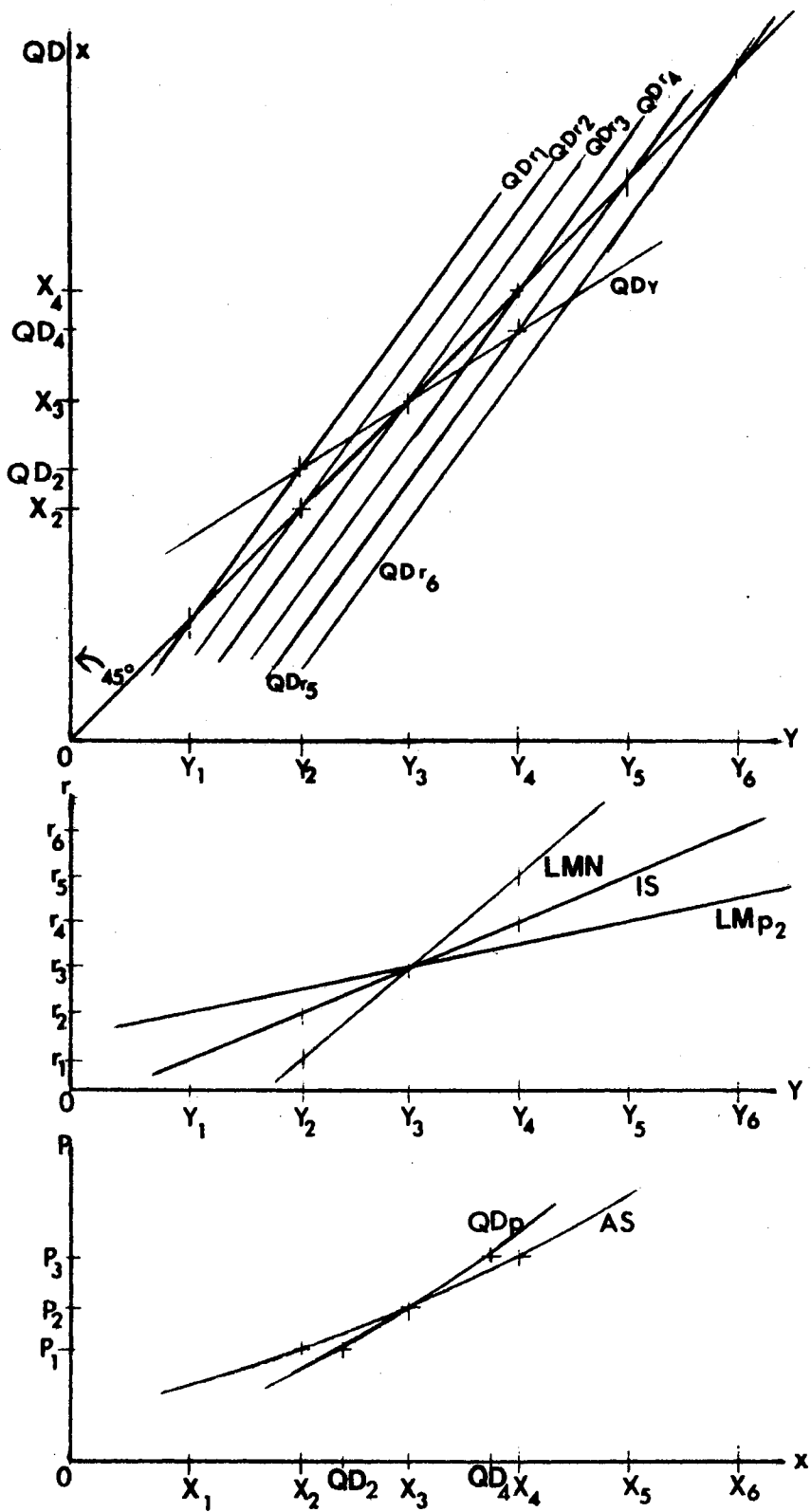


Figure 5. A Positive Sloping  $QD_p$  Curve With A Positive Sloping  $IS$  Curve

in excess of unity a stable equilibrium could not exist. The previous conclusions are not seriously affected by the assumption of a positively sloping IS curve.

### Mathematical Analysis

This section is concerned with the derivation and analysis of the slopes of the LMN curve, the  $QD_p$  curve, the  $QD_n$  curve and the  $QD_y$  curve. The equations used in this chapter are listed below.

$$QD = \Psi(Y, r) \quad (3.1)$$

$$X \equiv Y \quad (3.2)$$

$$QD = X \quad (3.3)$$

$$X = T(N) \quad (3.4)$$

$$MP = \Omega(N) \quad (3.5)$$

$$MP = \frac{W_0}{P} \quad (3.6)$$

$$L(Y, r) = \frac{M_0}{P} \quad (3.7)$$

### The Slope of the LMN

The LMN curve is based on equations (3.2), (3.4), (3.5), (3.6), and (3.7). The endogenous variables in this set are X, Y, N, MP, P, and r. Equations (3.8) through (3.12) represent the differentials of the LMN curve equations.

$$dX = dY \quad (3.8)$$

$$dX = \frac{\partial X}{\partial N} dN \quad (3.9)$$

$$dMP = \frac{\partial MP}{\partial N} dN \quad (3.10)$$

$$dMP = \frac{-W_0}{p^2} dP \quad (3.11)$$

$$\frac{\partial L}{\partial Y} dY + \frac{\partial L}{\partial r} dr = \frac{-M_0}{p^2} dP \quad (3.12)$$

Equation (3.13) is obtained by solving (3.9) for  $dN$  and substituting the result in equation (3.10).

$$dMP = \frac{\frac{\partial MP}{\partial N}}{\frac{\partial X}{\partial N}} dY \quad (3.13)$$

Equation (3.13) is substituted into equation (3.11).

$$dY \frac{\frac{\partial MP}{\partial N}}{\frac{\partial X}{\partial N}} = \frac{-W_0}{p^2} dP \quad (3.14)$$

When equation (3.14) is solved for  $dP$  and substituted into equation (3.12) equation (3.15) results.

$$\frac{\partial L}{\partial Y} dY + \frac{\partial L}{\partial r} dr = \frac{M_0}{W_0} \frac{\frac{\partial MP}{\partial N}}{\frac{\partial X}{\partial N}} dY \quad (3.15)$$

When equation (3.15) is solved for  $\frac{dr}{dY}$ , equation (3.16) is obtained. This equation represents the slope of the LMN curve.

$$\frac{dr}{dY} = - \frac{\frac{\partial L}{\partial Y}}{\frac{\partial L}{\partial r}} + \frac{1}{\frac{\partial L}{\partial r}} \frac{M_0}{W_0} \frac{\frac{\partial MP}{\partial N}}{\frac{\partial X}{\partial N}} \quad (3.16)$$

The term  $-\frac{\frac{\partial L}{\partial Y}}{\frac{\partial L}{\partial r}}$  represents the slope of the orthodox neo-Keynes-

ian LM curve. This term is always positive since it has a negative



coefficient and  $\frac{\partial L}{\partial r}$  is negative by definition. The denominator of the second term on the right side is negative because  $\frac{\partial L}{\partial r}$  is negative. The numerator is likewise negative since  $\frac{\partial MP}{\partial N}$  is negative. This makes the overall term positive.

Thus, the slope of the LMN is unambiguously positive. Since the last term on the right side is positive, it follows that the slope of the LMN is greater than the slope of the neo-Keynesian LM curve. This conclusion corresponds with previous graphical analysis.

#### The Slope of the $QD_y$ Curve

The  $QD_y$  curve, part (3A) of Figure (3), is based on equations (3.1), (3.2), (3.4), (3.5), (3.6), and (3.7). This set contains seven endogenous variables which are  $QD$ ,  $Y$ ,  $r$ ,  $X$ ,  $N$ ,  $MP$ , and  $P$ . Equation (3.17) is the differential of equation (3.1).

$$dQD = \frac{\partial QD}{\partial Y} dY + \frac{\partial QD}{\partial r} dr \quad (3.17)$$

Equation (3.18) is obtained from equation (3.16).

$$dr = \left[ \begin{array}{c} \frac{\partial L}{\partial Y} \\ - \frac{\partial L}{\partial L} \end{array} + \frac{Mo}{Wo} \frac{\frac{\partial MP}{\partial N}}{\frac{\partial L}{\partial r} \frac{\partial X}{\partial N}} \right] dY \quad (3.18)$$

Substituting equation (3.18) into equation (3.17) and solving for  $\frac{dQ}{dY}$  yields equation (3.19).

$$\frac{dQD}{dY} = \frac{\partial QD}{\partial Y} + \frac{\partial QD}{\partial r} \left[ \begin{array}{c} \frac{\partial L}{\partial Y} \\ - \frac{\partial L}{\partial L} \end{array} + \frac{\frac{\partial MP}{\partial N}}{\frac{\partial X}{\partial N} \frac{\partial L}{\partial r}} \frac{Mo}{Wo} \right] \quad (3.19)$$

Equation (3.19) represents the slope of the  $QD_y$  curve. The expression inside the brackets on the right side of equation (3.19) is the same as the right side of equation (3.16). Equation (3.16) equals the slope of the LMN curve. It was previously shown that this expression is always positive. Since  $\frac{\partial QD}{\partial r}$  is negative, it follows that the slope of the LMN times  $\frac{\partial QD}{\partial r}$  is also negative. If  $\frac{\partial QD}{\partial r} = 0$ , or if  $\frac{\partial L}{\partial r} = \infty$  the right hand term equals zero and the slope of the  $QD_y$  curve is reduced to  $\frac{\partial QD}{\partial Y}$ . If the slope of the LMN times  $\frac{\partial QD}{\partial Y}$  is greater in absolute terms than  $\frac{\partial QD}{\partial Y}$ , then  $\frac{dQD}{dY}$  is negative. Such a condition occurs when  $\left| \frac{\partial QD}{\partial r} \right|$  is relatively large and  $\left| \frac{\partial L}{\partial r} \right|$  is relatively small. In other words, when the rate of interest has a relatively strong effect on the  $QD_r$  curves in part (3A) of Figure (3), and when the  $L_y$  curves in part (3B) are relatively inelastic.

A negative sloping  $QD_y$  curve implies that as income increases the negative effect on quantity demanded that results from the concomitant increase in the rate of interest more than offsets the positive impact caused by the rise in income. A model based on this type of assumption is not consistent with the traditional Keynesian and neo-Keynesian emphasis on income as the primary determinant of quantity demanded.

#### The $QD_p$ Curve

The  $QD_p$  curve is based on the same set of equations and endogenous variables used to analyze the  $QD_y$  curve. Equation (3.20) which is ob-

tained from equation (3.19) represents the differential of quantity

$$dQD = \left\{ \frac{\partial QD}{\partial Y} + \left( -\frac{\frac{\partial L}{\partial Y}}{\frac{\partial L}{\partial r}} + \frac{Mo}{Wo} \frac{\frac{\partial MP}{\partial N}}{\frac{\partial X}{\partial N} \frac{\partial L}{\partial r}} \right) \right\} dY \quad (3.20)$$

demanded. By solving equation (3.14) for  $dY$ , equation (3.21) is obtained. If equation (3.21) is substituted in equation (3.20) and the

$$dY = -\frac{Wo}{p^2} \frac{\frac{\partial X}{\partial N}}{\frac{\partial MP}{\partial N}} dP \quad (3.21)$$

result solved for  $\frac{dQD}{dP}$ , the equation for the slope of the  $QD_p$  curve is obtained.

$$\frac{dQD}{dP} = \left\{ \frac{\partial QD}{\partial Y} + \frac{\partial QD}{\partial r} \left( -\frac{\frac{\partial L}{\partial Y}}{\frac{\partial L}{\partial r}} + \frac{Mo}{Wo} \frac{\frac{\partial MP}{\partial N}}{\frac{\partial X}{\partial N} \frac{\partial L}{\partial r}} \right) \right\} \left\{ \frac{Wo}{p^2} \frac{\frac{\partial X}{\partial N}}{\frac{\partial MP}{\partial N}} \right\} \quad (3.22)$$

The expression inside the brackets on the right side of equation (3.22) is the slope of the  $QD_y$  curve and the second term is equal to the slope of the aggregate supply curve. Since the slope of the aggregate supply curve is always positive, it follows that the slope of the  $QD_p$  is positive unless the slope of the  $QD_y$  is negative. Therefore, the  $QD_p$  curve would have a positive slope only under the non-Keynesian assumption that the rate of interest rather than real income dominates the determination of quantity demanded.

#### The $QD_n$ Curve

The  $QD_n$  curve is based on the same equations used to determine the  $QD_y$  and the  $QD_p$  curves. Consider once again equation (3.20).

$$dQD = \left\{ \frac{\partial QD}{\partial Y} + \frac{\partial QD}{\partial r} \left( \frac{\partial L}{\partial Y} + \frac{M_0}{W_0} \frac{\frac{\partial MP}{\partial N}}{\frac{\partial X}{\partial N} \frac{\partial L}{\partial r}} \right) \right\} dY \quad (3.20)$$

Now equation (3.8) is substituted into (3.9) and the result into equation (3.20). When this result is solved for  $\frac{dQD}{dY}$ , equation (3.23) is obtained.

$$\frac{dQD}{dN} = \left\{ \frac{\partial QD}{\partial Y} + \frac{\partial QD}{\partial r} \left( \frac{\partial L}{\partial Y} + \frac{M_0}{W_0} \frac{\frac{\partial MP}{\partial N}}{\frac{\partial X}{\partial N} \frac{\partial L}{\partial r}} \right) \right\} \frac{\partial X}{\partial N} \quad (3.23)$$

Equation (3.23) represents the slope of the  $QD_n$  curve in part (3J) of Figure (3). The value of  $\frac{\partial X}{\partial N}$  is positive. The term inside the brackets on the right side of equation (3.23) is the same as the slope of the  $QD_y$  curve. This expression is positive unless negative effect of rise in the rate of interest on quantity demanded more than offsets the positive effect of the increase in income.

#### Summary

A new variety of monetary equilibrium curve has been introduced in this chapter. This new curve differs from the conventional neo-Keynesian LM curve in that the neo-Keynesian procedure of assuming a constant price level is discarded. Instead, the price level is defined as equaling the money wage over the price level. This definition of the price level is implied by Keynes' assumption that the real wage equals the marginal product of labor.

It was demonstrated that the addition of the rate as a second causal variable did not alter the basic conclusion of the previous chapter that quantity demanded and the price level are positively related in the Keynesian system. It was shown that only by rejecting Keynes' assumption that income is the dominating causal variable influencing quantity demanded could the positive relationship between the price level and quantity demanded be reversed. These conclusions were demonstrated by graphical analysis as well as by a mathematical analysis of the slopes of the relevant functions.

## CHAPTER IV

### THE NEO-KEYNESIAN SYSTEM RECONSIDERED

Up to this point it has been established that Keynes' labor market assumptions imply a positive relationship between quantity demanded and employment. In addition, it has been demonstrated that the Keynesian system is consistent with a positive relationship between quantity demanded and the price level.

In this chapter these conclusions are contrasted with the orthodox neo-Keynesian interpretation of Keynesian theory. The orthodox neo-Keynesian model is based on the assumption that quantity demanded is independent of employment. Moreover, the neo-Keynesian model implies a negative relationship between quantity demanded and the price level.

It is the primary objective of this chapter to demonstrate that the neo-Keynesian "aggregate demand" function is not, in a fundamental sense, a "true" demand curve. It is demonstrated that the neo-Keynesian curve is a "market equilibrium" curve rather than a "demand curve." Patinkin's analysis of the distinction between a "demand curve" and a "market equilibrium" curve is used as the criterion for this determination.<sup>1</sup>

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<sup>1</sup>Don Patinkin, Money, Interest, and Prices, 2nd edition (New York, New York, 1965), pp. 11-12, 48-50, 387-392.

The neo-Keynesian theory of aggregate demand is based on Hicksian IS-LM curve analysis.<sup>2</sup> An important difference between Hicks' original formulation and the neo-Keynesian approach lies in the specification of the price level. Hicks followed Keynes in defining the price level as being equal to the money wage over the marginal product of labor.<sup>3</sup> The neo-Keynesian IS-LM curve model, on the other hand, is based on the assumption that the price level is a constant.

It is also demonstrated that the neo-Keynesian analysis of "excess demand" and "excess supply" is based on a defective model. It is shown that the neo-Keynesian "aggregate demand" and "aggregate supply" curves cannot legitimately be used to simultaneously represent quantity demanded and quantity supplied at alternative price levels. The problem is that these curves are based on contrary specifications for the relationship between real income (or output) and the price level. In other words, the neo-Keynesian aggregate "demand" and aggregate "supply" curve diagram implies that real income (or output) can equal two different amounts at the same moment in time.

While some neo-Keynesian IS-LM curve models are explicitly based on the assumption of a horizontal aggregate supply curve, others make no direct reference to the underlying aggregate supply relationships.<sup>4</sup>

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<sup>2</sup>John R. Hicks, "Mr. Keynes and the Classics," Macroeconomics: Selected Readings edited by Edward Shapiro, pp. 197-210.

<sup>3</sup>Ibid., p. 199.

<sup>4</sup>Joseph P. McKenna, Aggregate Economic Analysis, 4th edition (Homewood, Ill., 1971), p. 38.

Nevertheless, it is shown that all neo-Keynesian IS-LM curve models that assume a constant price level are either explicitly or implicitly based on the assumption of a perfectly elastic aggregate supply curve.

The following section reviews Patinkin's analysis of the distinction between "market-equilibrium" curves and "demand curves." In the third section the neo-Keynesian IS-LM curve model is amplified so as to take the horizontal aggregate supply curve assumption more explicitly into consideration. The fourth section illustrates that the derivation of the neo-Keynesian aggregate "demand" curve involves a series of "market-experiments." Thus, the neo-Keynesian curve is shown to be a "market-equilibrium" curve rather than a "demand" curve. The fifth section includes a mathematical analysis of the slope of the neo-Keynesian aggregate "demand" function. It is demonstrated that the neo-Keynesian aggregate demand model implies a negative relationship between output and the price level. The sixth section is concerned with a modified version of the neo-Keynesian system. It is shown that provided the Keynesian requirement that the real wage equal the marginal product of labor is relinquished and it is also assumed that the commodity and money markets adjust more rapidly than the price level, the neo-Keynesian aggregate "demand" curve can be used to represent quantity demanded at alternative price levels. This modified model, however, requires the rejection of Keynes' theory of aggregate supply. The seventh section contains a summary of the chapter.



## Demand Curves and Market Equilibrium Curves

It is the objective of this section to review Patinkin's analysis of the distinction between "demand" curves and "market equilibrium" curves. A "demand" curve is the result of a series of "individual-experiments" in which consumers are conceptually exposed to arbitrary variations in a causal variable.<sup>5</sup> The causal variable is endogenous from the point of view of the system as a whole but it is treated as independent in the derivation of the demand function. A demand function is represented mathematically by a structural equation.<sup>6</sup>

A "market-experiment," by way of contrast, involves the complete system. Each "market-experiment" determines one and only one equilibrium value for each of the endogenous variables in the system.<sup>7</sup> The solution for each of the endogenous variables can be expressed mathematically by an appropriate "reduced-form" equation in which only variables which are autonomous to the system as a whole appear on the right side of the equation.<sup>8</sup> It is meaningless to refer to the relationship between quantity demanded and the price level with respect to the complete system since by definition the system has meaning with reference to one price level and one level of quantity demanded.<sup>9</sup>

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<sup>5</sup>Patinkin, p. 12.

<sup>6</sup>Ibid., p. 392.

<sup>7</sup>Ibid., p. 389.

<sup>8</sup>Ibid., p. 392.

<sup>9</sup>Ibid., p. 391.

Thus, any change in the price level or quantity demanded from the point of view of the system as a whole requires a change in one or more of the variables which are autonomous with respect to the complete system.

### The Labor Market and the Neo-Keynesian

#### IS-LM Model

It is the object of this section to demonstrate that neo-Keynesian IS-LM curve analysis is based on the assumption of a horizontal aggregate supply curve. It is also shown that the equilibrium quantity demanded determined by the IS-LM curve model represents the result of a "market-experiment" rather than an "individual experiment."

The equations used in this section are listed below.

$$QD = \Psi(Y, r) \quad (4.1)$$

$$X \equiv Y \quad (4.2)$$

$$QD = X \quad (4.3)$$

$$X = h(N) \quad (4.4)$$

$$MP = h \quad (4.5)$$

$$MP = \frac{W_0}{P} \quad (4.6)$$

$$L(Y, r) = \frac{M_0}{P} \quad (4.7)$$

This set contains seven equations and seven endogenous variables. The endogenous variables include QD, Y, r, X, N, MP, and P.

It is not standard practice to include labor market equations in a neo-Keynesian IS-LM model. It is, nevertheless, appropriate for the

labor market to be included because the neo-Keynesian IS-LM model implies a perfectly elastic aggregate supply function. Most neo-Keynesian IS-LM models are limited to the equivalents of equations (4.1), (4.2), (4.3), and (4.7). This set contains four equations and five endogenous variables --  $Y$ ,  $r$ ,  $X$ ,  $QD$ , and  $P$ . The standard neo-Keynesian approach is to assume that the price level is a constant determined outside of the system.

It is often overlooked that this set of equations has subtle labor market implications. Notice that equation (4.2) defines real income as being equal to output. Given equation (4.2), any variation in real income implies a corresponding variation in output. A model which assumes a constant price level together with a variable output necessarily also assumes a perfectly elastic aggregate supply curve.

Neo-Keynesian model builders do not specify the labor market model that underlies their IS-LM curve analysis. Any labor market system that results in a constant price level together with a variable output would seem to be acceptable. Equations (4.4), (4.5), and (4.6) meet these specifications. Equation (4.4) indicates that output is a linear function of employment. Equation (4.5) shows that the slope of the production function "h" is equal to the marginal product of labor. Equation (4.6) is consistent with Keynes' requirement that the real wage equal the marginal product of labor. Equation (4.6) implies that the price level is equal to the money wage over the marginal product of labor. Since both the money wage and the marginal product of labor are assumed to be constant the price level is likewise constant regardless of the level of employment and output. Equation (4.8), the

"reduced form equation" for the price level is obtainable by substituting the slope of the production function (h) into equation (4.6) and solving for P.

$$P = \frac{W_o}{h} \quad (4.8)$$

If equation (4.8) is substituted into equation (4.7), equation (4.9) results.

$$L(Y,r) = \frac{Moh}{W_o} \quad (4.9)$$

If equation (4.9) is combined with equations (4.1), (4.2), and (4.3), the neo-Keynesian system is reduced to four equations and four unknowns. The unknowns are Y, r, X, and QD. This is a "market-equilibrium" set of equations. It is clear from equation (4.8) that any change in the price level requires a change in either h or  $W_o$ . The terms h and  $W_o$  are autonomous with respect to the system as a whole.

A set of neo-Keynesian IS-LM curves and their supporting functional relationships are illustrated in Figure (6). It has been established that the assumption of a constant price level together with a variable income implies a perfectly elastic aggregate supply curve. The linear production function  $X_{n2}$  in part (6J) represents equation (4.4). The constant slope of the  $X_{n2}$  curve means a constant marginal product of labor. The vertical  $MP_2$  curve in part (6H) corresponds to the slope of the  $X_{n2}$  curve. A real wage of  $W_2$  is consistent with a marginal product of  $MP_2$ . The RW curve in part (6F) shows that a price level of  $P_2$  is consistent with a real wage of  $W_2$  given the assumed money wage.

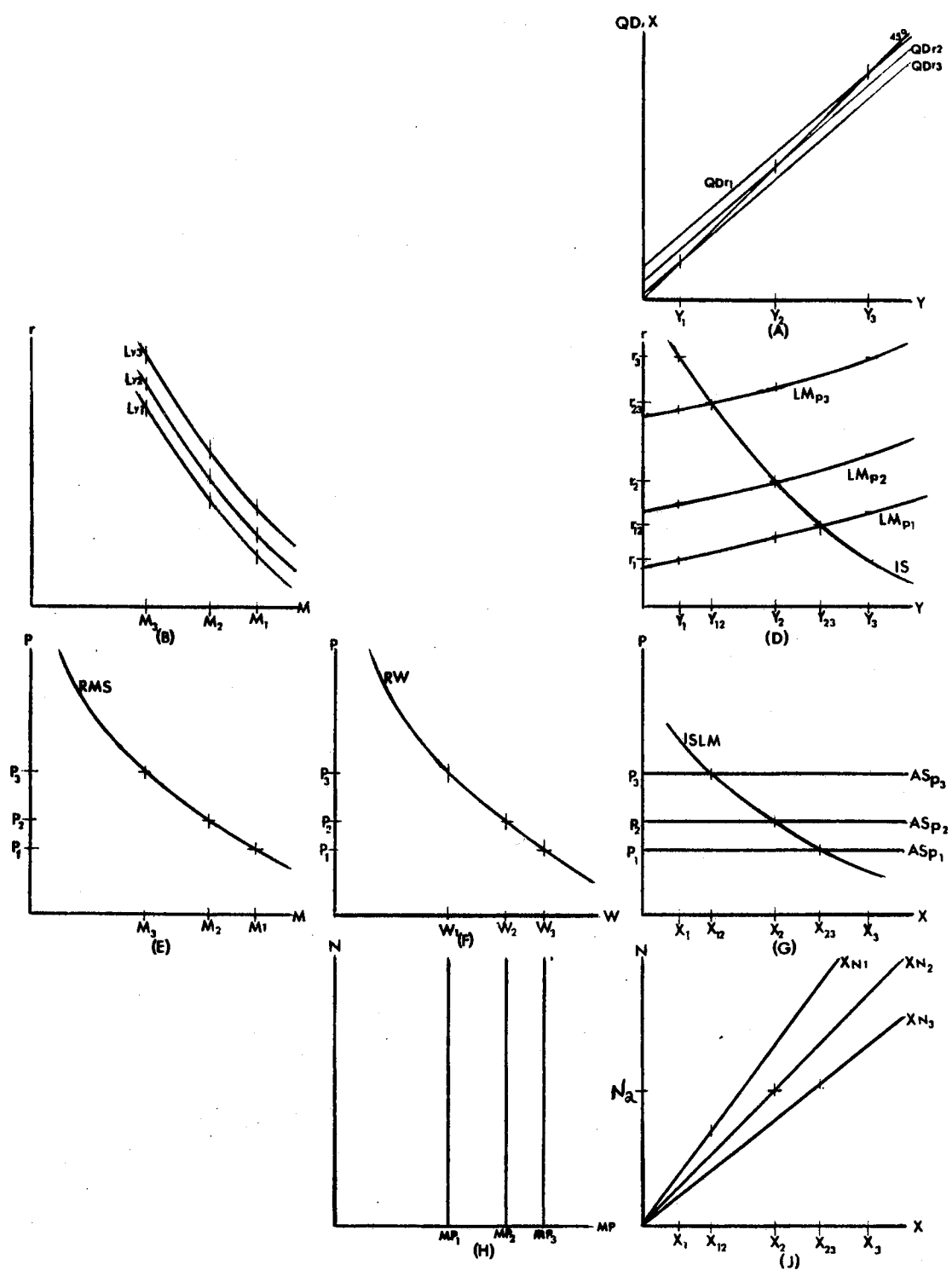


Figure 6. The Derivation of the Neo-Keynesian "Demand" Curve

The  $AS_2$  curve is the resulting aggregate supply function.

The horizontal aggregate supply curve  $AS_2$  in part (6G) means that the real money supply in part (6E) equals  $M_2$  at any real income. Under these conditions the  $LM_{p2}$  curve represents alternative combinations of the rate of interest and real income which are consistent with equilibrium in the money market while at the same time the real wage equals the marginal product of labor. The intersection of the LM curve with the IS curve at  $Y_2$  defines the equilibrium income. In part (6A) it should be noted that the  $QD_{r2}$  curve cuts the  $45^\circ$  line at  $X_2$ . The  $X_{n2}$  curve shows that an output of  $X_2$  implies an employment of  $N_2$ .

Thus, the neo-Keynesian IS-LM model not only defines equilibrium values for the endogenous variables in the commodity and money markets but it also defines equilibrium values for employment, the real wage, and the price level. In the next section the derivation of the neo-Keynesian "aggregate demand" function is shown to be the result of a series of "market-experiments" resulting from shifts in the horizontal aggregate supply curve.

#### The Neo-Keynesian Aggregate Demand Curve Reconsidered

The primary objective of this section is to demonstrate that the neo-Keynesian aggregate "demand" curve is a "market-equilibrium" curve rather than a demand curve. The neo-Keynesian aggregate "demand" curve is usually derived without explicit reference to the labor market. It was demonstrated in the previous section, however, that the IS-LM curve model is implicitly based on subtle but rather restrictive labor market assumptions.

It is shown in this section that the neo-Keynesian derivation of the aggregate "demand" function amounts to a series of "market-experiments" in which arbitrary changes in autonomous factors in the labor market are assumed to shift the IS-LM models perfectly elastic aggregate supply curve to a series of new positions. After each shift of the supply curve the system is allowed to return to equilibrium. The resulting relationship between alternative price levels and corresponding equilibrium levels of income is the neo-Keynesian aggregate "demand" curve. One obvious weakness of the neo-Keynesian IS-LM model is that it is based on the assumption of a perfectly elastic aggregate supply curve whereas the Keynesian system is based on the assumption of a positively sloping aggregate supply curve. It is not necessary to use the IS-LM curve-fixed price level approach to derive the neo-Keynesian aggregate demand curve. It is demonstrated in this section that the neo-Keynesian aggregate "demand" function can also be derived in terms of a model that uses conventional Keynesian labor market assumptions. The use of a conventional labor-market model in the derivation of the neo-Keynesian curve has the advantage of demonstrating that the neo-Keynesian aggregate "demand" function is the "locus of intersection points of demand curves and their corresponding supply curves."<sup>10</sup> This kind of curve "must be sharply distinguished conceptually from a true demand curve."<sup>11</sup>

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<sup>10</sup>Ibid., p. 48.

<sup>11</sup>Ibid., p. 50.

To sum up, it is shown up to this point that the so called neo-Keynesian aggregate "demand" curve is not a true demand curve. The neo-Keynesian curve in question results from a series of "market-experiments" in which the economy is moved from one equilibrium position to the next by shifts in the economy's aggregate supply curve.

The graphical derivation of the neo-Keynesian aggregate "demand" function can be demonstrated in terms of Figure (6). The analysis in the previous section was based on the assumption that the production function was located at the  $X_{n2}$  position and the vertical marginal product schedule was positioned at  $MP_2$ . These assumptions imply a horizontal aggregate supply curve at price level  $P_2$ . The  $LM_{p2}$  curve represents the appropriate money market equilibrium curve in this case. The  $LM_{p2}$  curve cuts the IS curve at interest rate  $r_2$  and income  $Y_2$ . The combination of price level  $P_2$  and income  $Y_2$  is one point on the neo-Keynesian aggregate demand curve.

Now assume a shift in the perfectly elastic aggregate supply curve to price level  $P_1$ . Neo-Keynesians ordinarily do recognize that a change in the price level requires a change in either the money wage or the marginal product of labor. It should be recalled that equation (4.8) defines the price level in terms of these two variables.

In part (6J) of Figure (6) assume that the  $X_n$  curve is shifted to the  $X_{n3}$  position and as a result the vertical marginal product of labor curve shifts to the  $MP_3$  position. These changes imply a shift of the aggregate supply curve to price level  $P_1$  and an increase in the real money supply to  $M_1$ . A constant real money supply of  $M_1$  is consistent with the  $LM_{p1}$  curve. The  $LM_{p1}$  curve cuts the IS curve at



$Y_{23}$ . The combination  $(P_1, Y_{23})$  represents a second position on the neo-Keynesian aggregate "demand" curve. It should be noticed that the second position on the curve was obtained by what Patinkin calls a market experiment in which a change in one of the autonomous variables in the labor market (in this case the slope of the production function) is assumed to shift the supply curve. After the economy had fully adjusted to the change, the new equilibrium income together with the new price level represents a second point on the curve. Other points may be obtained by making similar arbitrary variations in the supply curve.

The derivation of the neo-Keynesian "demand" curve just described follows the labor market requirements of the IS-LM curve model. Each point on the neo-Keynesian "demand" curve represents alternative combinations of output and the price level consistent with equilibrium in the money and commodity markets. Accordingly, the curve is henceforth referred to as the "ISLM" curve. It is interesting to notice, however, that it is not necessary to alter Keynes' assumption of a declining marginal product of labor and a positive sloping aggregate supply curve in order to obtain the neo-Keynesian curve in question.

The set of equations listed below is the same set that was used to represent the complete Keynesian system in Chapter 3.

$$QD = \Psi(Y, r) \quad (4.1)$$

$$X \equiv Y \quad (4.2)$$

$$QD = X \quad (4.3)$$

$$X = T(N) \quad (4.10)$$

$$MP = \Omega(N) \quad (4.11)$$

$$MP = \frac{W_o}{P} \quad (4.6)$$

$$L(Y,r) = \frac{M_o}{P} \quad (4.7)$$

This set contains seven equations and seven endogenous variables,  $Y$ ,  $r$ ,  $X$ ,  $QD$ ,  $N$ ,  $MP$ , and  $P$ . Like the IS-LM model, it represents a market equilibrium system. It is assumed in this case, however, that the marginal product of labor is negatively related to employment.

The derivation of the neo-Keynesian "demand" curve in terms of this kind of model is illustrated in Figure (7). Notice in part (7F) three alternative RW curves are illustrated. The position of the RW curve depends on the assumed value for  $(W_o)$ , the money wage. The money wage is an autonomous variable in the Keynesian system.

Assume that the current money wage locates the RW curve at the  $RW_2$  position. The derivation of the LMN curve was explained in Chapter 3. The  $LMN_2$  curve corresponds with real wage curve  $RW_2$ . The  $LMN_2$  curve cuts the IS curve at income  $Y_2$  and interest rate  $r_2$ . In part (7G) the  $QD_{p_2}$  curve represents the relationship between quantity demanded and the price level under the assumed conditions. The  $QD_{p_2}$  curve cuts the  $AS_2$  curve at  $Y_2$ . Notice that when the economy is at  $(P_2, Y_2)$  it is also on the IS curve at  $(r_2, Y_2)$ . Point  $(r_2, Y_2)$  is on the IS curve and positions on the IS curve imply that quantity demanded equals output. Price levels  $P_2$  and output  $X_2$  represent one point on the neo-Keynesian "demand" curve.

Now assume an increase in the money wage shifts the RW curve to the  $RW_3$  position. As a result the LMN curve shifts upward to the left to  $LMN_3$ . The equilibrium income falls to  $Y_{13}$  and output to  $X_{13}$ . The

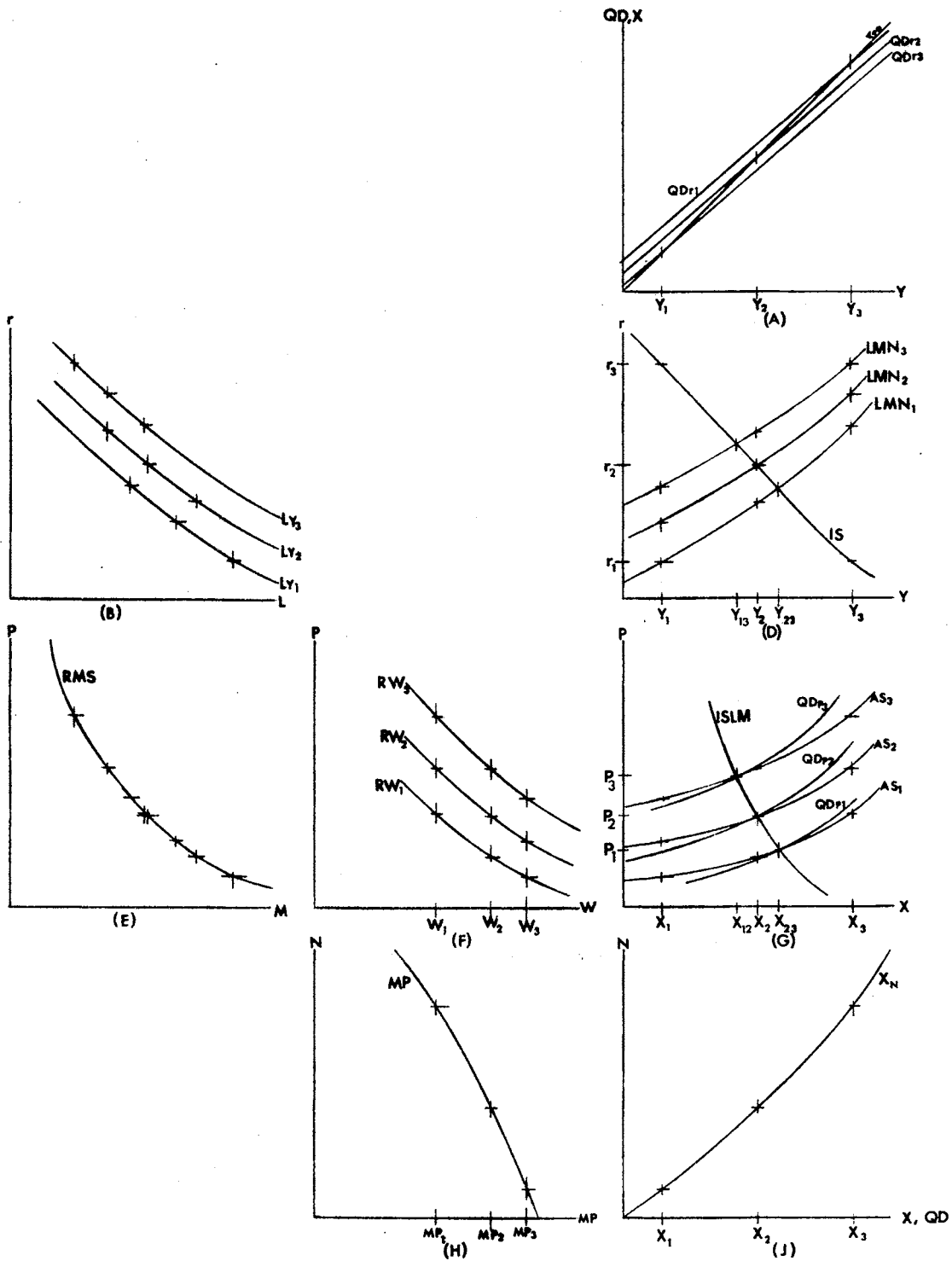


Figure 7. A Keynesian Derivation of the Neo-Keynesian Aggregate Demand Curve

AS and  $QD_p$  curves in part (7G) are shifted to  $AS_3$  and  $QD_{p3}$  respectively. In part (7G), quantity demanded equals output at output  $X_{13}$  and the new equilibrium price level is  $P_3$ . Other points on the neo-Keynesian "demand" curve can be obtained by a similar procedure.

It now is clear that the neo-Keynesian curve is not a demand curve. Instead it represents "the locus of intersection points of demand curves and their corresponding supply curves."<sup>12</sup> The neo-Keynesian curve represents alternative combinations of the price level and output (or real income) which are consistent with quantity demanded being equal to output in the commodity market while at the same time the quantity of real balances demanded equals the amount real balances in existence. It makes no sense to use such a curve to measure "excess demand" or "excess supply" at alternative price levels as neo-Keynesians often do since each point on the curve implies that output equals quantity demanded. The following section is concerned with a mathematical analysis of the slope of the ISLM curve.

#### The Slope of the Neo-Keynesian Aggregate

##### "Demand" Function

This section is concerned with a mathematical analysis of the slope of the neo-Keynesian aggregate "demand" function. It is demonstrated that the neo-Keynesian curve cannot legitimately be used to represent quantity demanded at alternative price levels in conjunction with a Keynesian aggregate supply curve.

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<sup>12</sup>Ibid., p. 48.

Equations (4.1) through (4.7) represent the conventional neo-Keynesian IS-LM model. This set contains seven equations and seven endogenous variables. From the point of view of the system as a whole neither the price level nor output (nor quantity demanded nor income) can change as long as the autonomous variables  $W_0$ ,  $M_0$ , and  $h$  do not change.<sup>13</sup> It is "meaningless to inquire as to the effect of a change in  $\bar{P}$  on  $\bar{X}$  or vice versa."<sup>14</sup> Price and output cannot change unless one of the independent variables changes.

In practice, however, neo-Keynesians have included only equations (4.1), (4.2), (4.3), and (4.7) in their analysis of aggregate demand. This set contains four equations but five unknowns. The endogenous variables are  $Y$ ,  $r$ ,  $QD$ ,  $X$ , and  $P$ . The difficulty with this approach is that output ( $X$ ) in the Keynesian system is uniquely related to employment via the production function. Each point on a production function defines a particular marginal product of labor. The price level in the Keynesian system is defined as being equal to the money wage over the marginal product of labor.<sup>15</sup> Thus the Keynesian model implies that output and the price level are related in a way that is inconsistent with the neo-Keynesian approach which assumes that the price level is independent of the level of employment.

Nevertheless, for point of argument, assume that (4.1), (4.2), (4.3), and (4.7) can legitimately be used as an independent set of

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<sup>13</sup>Ibid., p. 391.

<sup>14</sup>Ibid.

<sup>15</sup>Hicks, p. 199.

equations without reference to the labor market. The differentials of these equations are listed in the following set.

$$dQD = \frac{\partial QD}{\partial Y} dY + \frac{\partial QD}{\partial r} dr \quad (4.12)$$

$$dX = dY \quad (4.13)$$

$$dQD = dX \quad (4.14)$$

$$\frac{\partial L}{\partial Y} dY + \frac{\partial L}{\partial r} dr = \frac{-M_0}{p^2} dP \quad (4.15)$$

If equation (4.15) is solved for  $dr$  the result is equation (4.16).

$$dr = \frac{-M_0}{\frac{\partial L}{\partial r}} \left\{ \frac{ModP}{p^2} + \frac{\partial L}{\partial Y} dY \right\} \quad (4.16)$$

The expression  $dQD$  can be substituted for  $dY$  in equations (4.16) and (4.12). If (4.16) is then substituted into (4.12) and the latter solved for  $\frac{dQD}{dP}$  the slope of the neo-Keynesian aggregate "demand" curve is obtained.

$$\frac{dQD}{dP} = \frac{-M_0}{p^2} \frac{\frac{\partial QD}{\partial r}}{\frac{\partial L}{\partial r} \left( 1 - \frac{\partial QD}{\partial Y} \right) + \frac{\partial QD}{\partial r} \frac{\partial L}{\partial Y}} \quad (4.17)$$

The left hand term of this solution is negative. The numerator of the right hand term is negative. Both expressions in the denominator of the right hand term are likewise negative. Since both the numerator and the denominator are negative, the right hand term is positive. This makes the overall expression negative. This conclusion is consistent with the traditional neo-Keynesian assumption of a

negative relationship between quantity demanded and the price level. By substituting  $dX$  for  $dQD$  in equation (4.16) the same expression can now be used to express the relationship between output and the price level or by equation (4.13) the price and real income.

$$\frac{dY}{dP} = \frac{dX}{dP} = \frac{-M_0}{P^2} \frac{\frac{\partial QD}{\partial r}}{\frac{\partial L}{\partial r} \left(1 - \frac{\partial QD}{\partial Y}\right) + \frac{\partial QD}{\partial r} \frac{\partial L}{\partial Y}} \quad (4.18)$$

It may be recalled that Keynesian labor market assumptions imply a negative relationship between output and the price level. The specifications implied by equation (4.17) and Keynesian labor market assumptions are in conflict. Either one or the other must be rejected. Output (income) cannot be negatively and positively related to the price level at the same moment in time.

#### The Rejection of Keynesian Labor Market Assumptions

It is demonstrated in this section that if Keynesian labor market specifications are relinquished and at the same time it is assumed that the money and commodity markets adjust more rapidly than the price level, it is possible to obtain a model in which the neo-Keynesian aggregate "demand" curve can be considered a legitimate demand curve. However, it is necessary in this case to alter Keynes' labor market model.

The assumption of a sticky price level implies a rejection of Keynes' assumption that the real wage equals the marginal product of labor at all points in time. The equations included in the current analysis of aggregate demand include the following.

$$QD = \Psi (Y, r) \quad (4.1)$$

$$X \equiv Y \quad (4.2)$$

$$QD = X \quad (4.3)$$

$$X = T(N) \quad (4.4)$$

$$MP = \Omega (N) \quad (4.11)$$

$$L(Y, r) = \frac{M_0}{P} \quad (4.7)$$

$$W = \frac{W_0}{P} \quad (4.18)$$

It should be noted that the equation expressing equality between the real wage and the marginal product of labor is not included. Equation (4.18) defines the real wage as being equal to the money wage over the price level. In the present case it is assumed that the price level is relatively slow to adjust as compared with money and commodity market variables.

This set includes seven equations and eight endogenous variables. In principle it is possible by substitution to obtain a structural equation relating quantity demanded to the price level. The resulting relationship between quantity demanded and the price level would be identical to the neo-Keynesian aggregate "demand" function. Since the model used in this case does not include the requirement that the real wage equal the marginal product of labor it is interesting to observe the relationship between the real wage and employment implied by this system.

In Figure (8) assume that the price level is  $P_3$ . Given the assumption that the commodity and money markets adjust more rapidly than the price level, it is assumed that real output is  $X_{12}$  on the ISLM



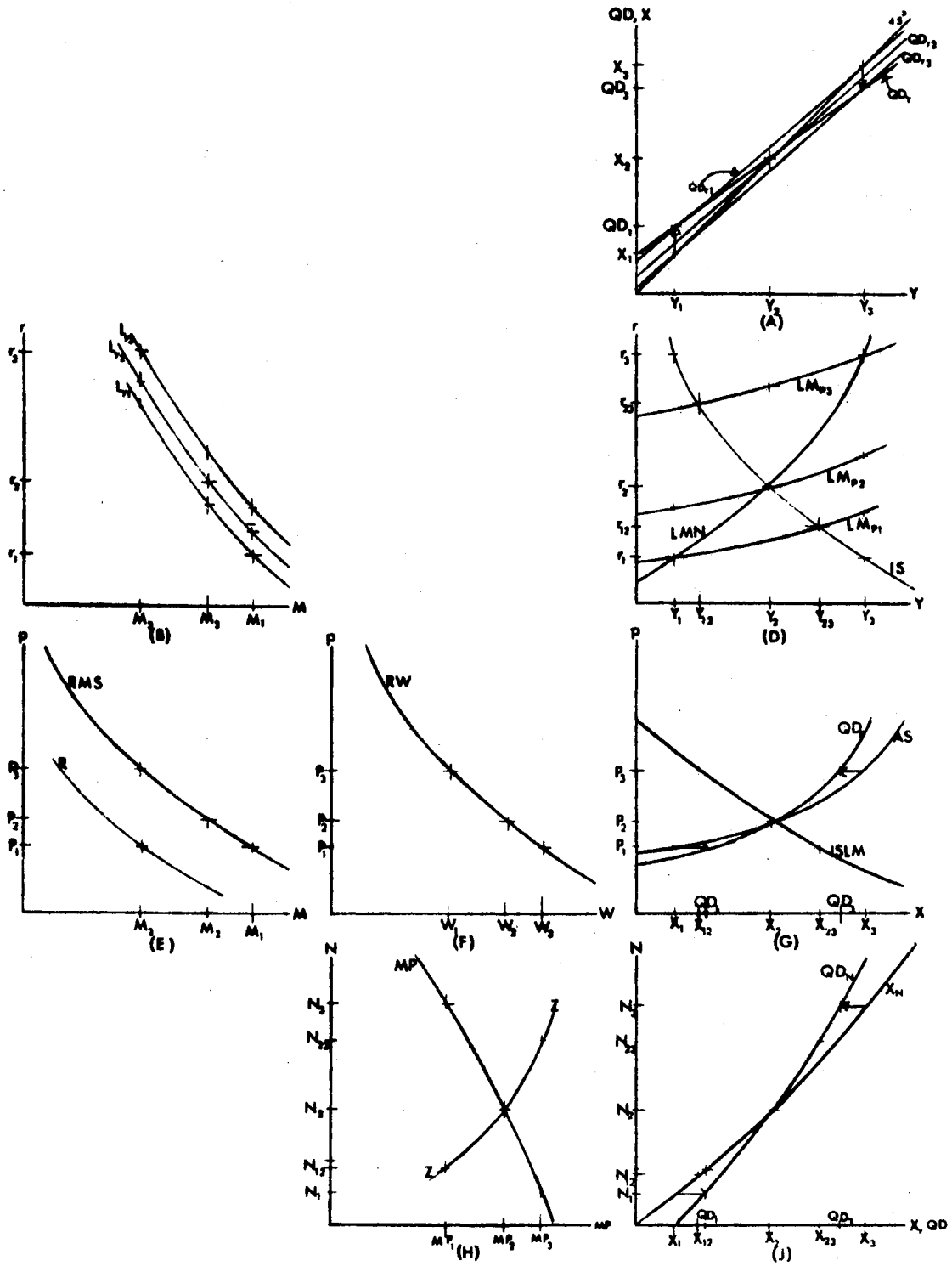


Figure 8. The Derivation of the ZZ Curve

curve rather than  $X_3$  on the AS curve. Note that in part (8F) a price level of  $P_3$  implies a real wage of  $W_1$ . Also note that output  $X_{12}$  requires a labor input of  $N_{12}$ . The combination of  $N_{12}$  and  $W_1$  represents one point on the ZZ curve in part (8H). Point  $(P_1, X_{23})$  on the ISLM curve is similarly related to point  $(N_{23}, W_3)$  on the ZZ curve. The ZZ curve represents the relationship between the real wage and employment that is implied by positions on the neo-Keynesian aggregate "demand" curve. The slope of the ZZ curve implies a positive relationship between employment and the real wage. The slope of the ZZ curve can be obtained by taking the differentials of the equations included in this analysis and solving for  $\frac{dW}{dN}$ . The relevant differentials are the following:

$$dQD = \frac{\partial QD}{\partial Y} dY + \frac{\partial QD}{\partial r} dr \quad (4.12)$$

$$dX = dY \quad (4.13)$$

$$dQD = dX \quad (4.14)$$

$$\frac{\partial L}{\partial Y} dY + \frac{\partial L}{\partial r} dr = \frac{-Mo}{p^2} dP \quad (4.15)$$

$$dX = \frac{\partial X}{\partial N} dN \quad (4.19)$$

$$dMP = \frac{\partial MP}{\partial N} dN \quad (4.20)$$

$$dW = \frac{-Wo}{p^2} dP \quad (4.21)$$

By appropriate substitution this set can be solved for  $\frac{dN}{dW}$ .

$$\frac{dN}{dW} = \frac{Mo}{Wo} \frac{1}{\frac{\partial X}{\partial N}} \left( \frac{\frac{\partial QD}{\partial r}}{\frac{\partial L}{\partial r} \left( 1 - \frac{\partial QD}{\partial Y} \right) + \frac{\partial QD}{\partial r} \frac{\partial L}{\partial Y}} \right) \quad (4.22)$$

Since  $\frac{\partial X}{\partial N}$  is positive the left hand term in the solution is positive. It was previously demonstrated in connection with equation (4.17) that the right hand term is positive. It follows, therefore, that the slope of the ZZ curve is positive.

Thus, the same set of assumptions which are necessary to make the ISLM curve a legitimate demand curve also imply a positive relationship between quantity demanded and the price level. Not only is it necessary to reject the assumption that real wage equals the marginal product of labor, but in addition the ISLM curve implies a positive relationship between the real wage and employment.

To sum up, it is now clear that if the ISLM curve is to be considered a demand curve it is necessary to reject Keynes' labor market model.

#### Summary

This chapter has demonstrated that the neo-Keynesian aggregate "demand" curve is not a legitimate demand curve. The neo-Keynesian derivation of the curve in question amounts to a series of market experiments. Autonomous changes in the labor market have the effect of shifting a perfectly elastic aggregate supply curve to a series of alternative price levels. The locus of alternative price levels with their corresponding equilibrium incomes is the neo-Keynesian "demand" curve. Points on the curve represent alternative equilibrium positions for the complete system.

It is also demonstrated, however, that it is not necessary to alter Keynes' labor market specifications in order to derive the neo-

Keynesian "demand" function. The same result can be obtained by shifting a positive sloping Keynesian type aggregate supply curve and observing resulting changes in equilibrium position of the economy.

Since at each point on the neo-Keynesian "demand" curve quantity demanded equals output it is illogical to use the same curve to measure "excess demand" or "excess supply." Excess demand and excess supply are by definition zero at all points on the curve.

The mathematical analysis of the set of equations used to derive the Keynesian "demand" function shows that this aggregate demand model implies not only a negative relationship between quantity demanded and the price level, but a negative relationship between output (and income) and the price level as well. Both these assumptions are inconsistent with the well known Keynesian labor market implication of a positive relationship between output and the price level. The neo-Keynesian approach implies that output and income can equal two different amounts at the same moment in time. Only by rejecting Keynes' labor market model can the neo-Keynesian curve be legitimately used as a demand curve.

## CHAPTER V

### PROFESSOR ROBERT MUNDELL'S MACRO MODEL

#### Introduction

In the previous chapter it was demonstrated that the neo-Keynesian graphical model is a defective tool for the analysis of "excess demand" and "excess supply." The difficulty results from the fact that the neo-Keynesian aggregate "demand" function has labor market implications which are inconsistent with the Keynesian theory of aggregate supply.

The present chapter is concerned with another graphical representation of the static Keynesian model and with the questionable use of the model for analysis of disequilibrium. Professor Robert A. Mundell has attempted to provide a "simple, compact and generalized exposition" that integrates supply conditions with the "Keynesian theory of income and interest rate determination."<sup>1</sup> Although not stated as a primary objective of his paper, Mundell, on the basis of his static model, conducts an analysis of disequilibria.

In one respect Mundell's methodology is an improvement over the orthodox neo-Keynesian model. His primary innovation is the develop-

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<sup>1</sup>Robert A. Mundell, "An Exposition of Some Subtleties in the Keynesian System," Macroeconomics; Selected Readings edited by Walter L. Johnson and David R. Kamerschun (New York, 1970), pp. 32-38.

ment of a new type of equilibrium curve which takes cognizance of the interrelationship between quantity demanded and the level of employment. In other respects, however, Mundell is less than completely successful in accomplishing his stated objectives.

The primary objective of this chapter is to relate Mundell's work to the analysis of aggregate demand just completed. In addition to demonstrating the relationship of Mundell's approach to the previous analysis Mundell's model is expanded and improved. Two new equilibrium curves are introduced which have the effect of integrating the Keynesian money market more effectively into Mundell's graphical apparatus.

Another objective of this chapter is to show that the validity of Mundell's conclusions about disequilibria are contingent upon his assumption about the relative adjustment velocities of the commodity market and the price level. In other words, as in the case of the neo-Keynesian aggregate "demand" function, Mundell's conclusions apply only if the price level is assumed to be relatively "sticky." It is demonstrated on the other hand that the Keynesian assumption of rapid adjustment of the price level relative to commodity market results in a reversal of Mundell's dynamic conclusions. Mundell's "overproduction" situations become Keynesian "underproduction" situations and visa versa.

Mundell's interest is somewhat wider than the standard Keynesian system. He also applies his graphical apparatus to situations where other than Keynesian labor market conditions prevail. Four alternative labor market situations are considered: case (a) prices and wages are both flexible, case (b) prices are flexible but wages are fixed, case (c) prices are constant but wages are flexible, and case (d) where both wages and prices are constant.

The procedure in each case is to incorporate the stated labor market assumptions into a static model which includes Keynesian commodity and money markets. This chapter is primarily concerned with Mundell's analysis of case (b) which represents conventional Keynesian labor market conditions. Case (a) represents the classical approach and case (d) is the equivalent to the McKenna's "fixed-price" and "fixed-wage" IS and LM curve model which was discussed in Chapter 4. The analysis of Mundell's case (c) model is not included in this paper since the labor market assumptions under which it is formulated are non-Keynesian.<sup>2</sup>

Mundell's treatment of the money market is non-Keynesian. Mundell fails to complement his graphical demonstration of the interrelationship between commodity market and labor market equilibrium with a parallel analysis of the interrelationship between money market and labor market equilibrium. As a result, in order to illustrate an under full employment equilibrium situation, in terms of his apparatus, Mundell resorts to the expedient of assuming that the rate of interest is "pegged" by the monetary authorities.<sup>3</sup> The "pegged" rate of interest is an interesting phenomena that occurs at times in the real world but such an assumption is not found in the General Theory.

To sum up, the objective of this chapter is to review Mundell's model and to suggest modifications which make it more representative of the Keynesian system. It is demonstrated that Mundell's dynamic

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<sup>2</sup>An analysis of the case (c) model is available from the author on request.

<sup>3</sup>Mundell, p. 36.

conclusions are reversed when Keynesian assumptions about the relative adjustment velocities are substituted for his own.

The next section reviews Mundell's analysis in terms of a pegged interest rate model. The third section suggests a way in which his approach can be modified so that the pegged interest rate assumption can be eliminated. The final section contains a summary and review.

### The Fixed Interest Rate Model

In this section Mundell's fixed interest rate version of the Keynesian system is reviewed. It is shown to differ significantly from the original Keynesian system. It is likewise demonstrated that even in terms of his own money market assumptions, Mundell's dynamic conclusions are dependent on assumptions about relative adjustment velocities that differ significantly from those found in the General Theory.

Mundell's mathematical model represents the complete Keynesian system. The set of equations that Mundell uses to represent the Keynesian system are highly condensed. It is convenient for present purposes to use the same set of equations that were used to represent the Keynesian system in Chapter 4. These equations are not in any way inconsistent with those used by Mundell.

$$QD = \Psi (Y, r) \quad (5.1)$$

$$X \equiv Y \quad (5.2)$$

$$QD = X \quad (5.3)$$

$$X = T (N) \quad (5.4)$$

$$MP = \Omega (N) \quad (5.5)$$



$$MP = \frac{W_0}{P} \quad (5.6)$$

$$L(Y, r) = \frac{M_0}{P} \quad (5.7)$$

$$W = \frac{W_0}{P} \quad (5.8)$$

This set of eight equations contains eight unknowns. The endogenous variables are  $Y$ ,  $r$ ,  $X$ ,  $QD$ ,  $N$ ,  $MP$ ,  $W$ , and  $P$ . In this case the definitional equation for the real wage has been added for convenience.

Although Mundell includes equation (5.7) in his mathematical model in his graphical analysis he assumes the interest rate is pegged by the monetary authorities. This procedure has the effect of eliminating equation (5.7) from the analysis and at the same time it reduces the number of unknowns to seven. The model now includes equations (5.1) through (5.6) plus equation (5.8). The essence of Mundell's graphical model is contained in his RR curve which is derived in Figure (9).

The "RR" curve indicates alternative combinations of the interest rate and the real wage which are consistent with equilibrium in the commodity market and with equality between the real wage and the marginal product of labor. In the Keynesian system the interest rate is endogenous. Mundell alters the usual Keynesian and neo-Keynesian procedure by treating the rate of interest rather than the nominal money supply as autonomous. The RR curve indicates the equilibrium real wage that is determined by the interest rate set by the monetary authorities. From Patinkin's point of view the RR curve represents the locus of interest rate - real wage combinations resulting from a market-experiment in which the system is conceptually subjected to

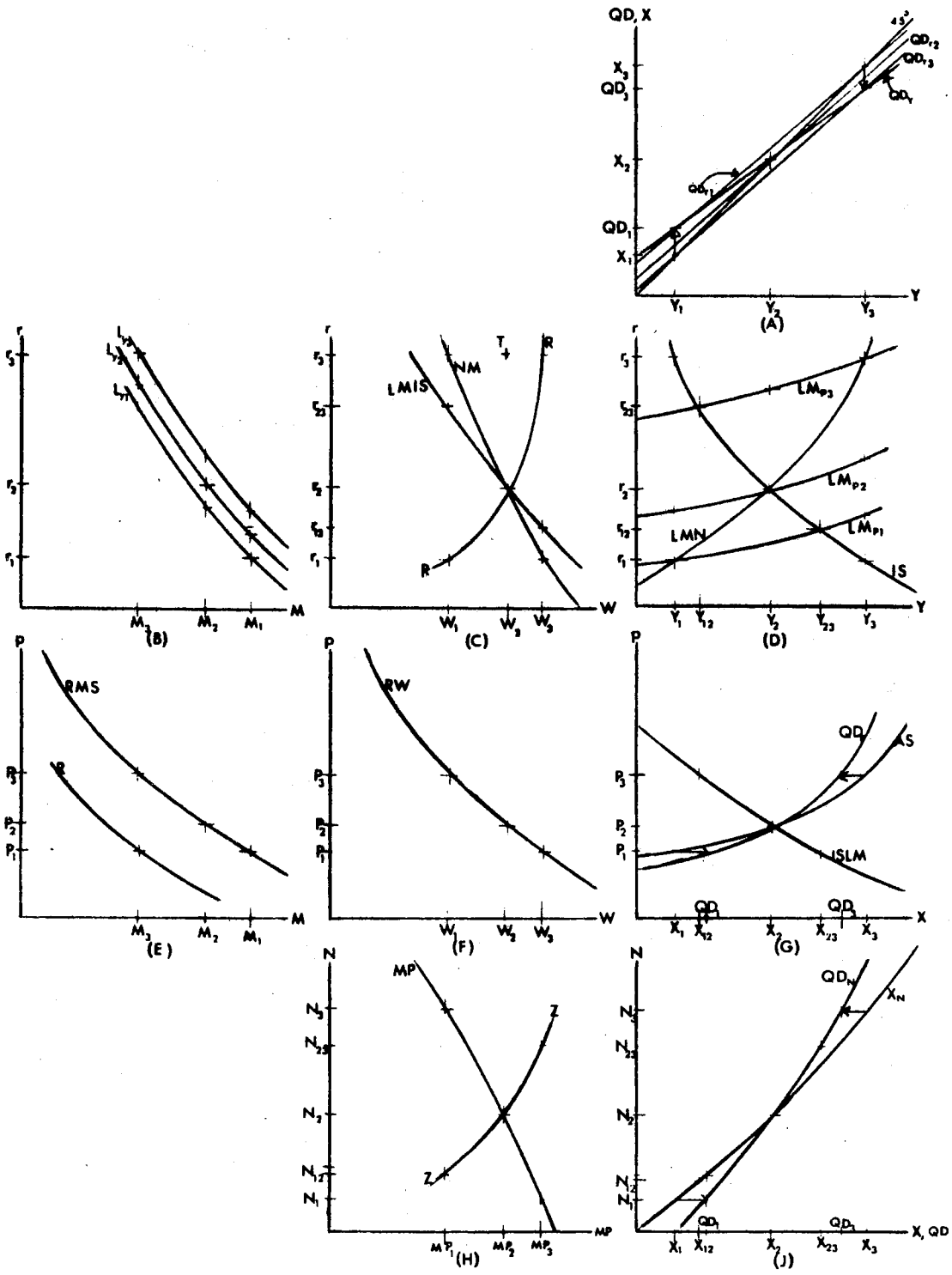


Figure 9. The Derivation of the RR, the MN, and the LMIS Curves

changes in the interest rate.

The RR curve can be obtained graphically in the following way. Select interest rate  $r_3$  at random. Given the interest rate the IS curve in part (9D) indicates that an income of  $Y_1$  is necessary to equate output and quantity demanded. An income of  $Y_1$  means an output of  $X_1$ . The production function in part (9J) shows that  $N_1$  units of labor are necessary to produce output  $X_1$ . The marginal product of labor at  $N_1$  is  $MP_3$ . This means that a real wage of  $W_3$  is required to satisfy equation (5.6). By combining  $r_3$  with  $W_3$ , one point on the "RR" curve is obtained. Other points can be obtained by repetition. The LMIS and NM curves should be ignored for now. They are discussed in the next section.

Mundell interprets the RR to mean the following. It represents "the relation between the interest rate and the real wage rate at which the employment levels implied by profit maximization (under competition) will yield outputs which can all be sold."<sup>4</sup> The RR curve is interesting because it demonstrates the interrelationship between the commodity market and labor equilibrium conditions. In this way the RR curve is superior to the McKenna approach. The model, however, differs significantly from the conventional Keynesian system.

Instead of tackling the problem of incorporating the money market into his graphical analysis, Mundell, in effect, assumes away the Keynesian money market. The interest rate is assumed to be determined outside the system. Given the rate of interest, the RR curve defines the equilibrium real wage. The equilibrium level of employment, output

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<sup>4</sup>Ibid., p. 35.

and income are likewise easy to obtain. It should be acknowledged that Mundell's assumption of an autonomous interest rate does simplify the Keynesian system. However, the absence of the money supply from the model is a serious weakness. In the following section, a modification of Mundell's approach is suggested that incorporates the money market. In this section, however, the analysis proceeds on the basis of the pegged interest rate assumption.

The reader is reminded that Mundell's model is a market equilibrium system. Logically, therefore, nothing can be said on the basis of this model about points which lie above or below the RR curve except that such points imply that either the commodity market or the labor market or possibly both markets are out of adjustment. Mundell, however, goes much further than this. He argues that points above the RR curve imply an excess of the marginal product of labor over the real wage. Accordingly, he labels all positions above the RR curve as representing "underproduction" conditions.

The basis for this contention can be demonstrated by considering point "T" which is above the RR curve. At point "T" the rate of interest is  $r_3$  and the real wage is  $W_2$ . Mundell reasons that if the interest rate is  $r_3$  then output will equal quantity demanded only if income is  $Y_1$ . An income of  $Y_1$  means an output of  $X_1$  and the production function shows that an output of  $X_1$  requires a labor input of  $N_1$ . At  $N_1$  the marginal product of labor is  $MP_3$ . Since  $MP_3$  is greater than the assumed real wage  $W_2$ , "underproduction" exists in Mundell's sense of the term.

Ostensibly this analysis seems straight forward and unobjectionable. It should be noticed, however, that this interpretation of point "T"

rests on the assumption that the commodity adjusts more rapidly than the labor market.

Mundell is entitled to make any assumptions about relative adjustment velocities he cares to make. However, his assumptions are not consistent with those found in the General Theory. Keynes assumed a lag between changes in quantity demanded and subsequent adjustments in employment, output, and income. On the other hand, Keynes assumed that the real wage equals the marginal product of labor at all moments in time. Thus, Mundell's assumptions are the reverse of those found in the General Theory. The dynamic implications of point "T" and other points above the RR curve are reversed when Keynesian relative adjustment velocities are substituted for those used by Mundell.

From a Keynesian point of view, point "T" represents a condition of "overproduction" rather than "underproduction." In Figure (9), under the Keynesian assumption that the real wage equals the marginal product of labor, a real wage of  $W_2$  would mean a marginal product of  $MP_2$ . This would mean that employment is  $N_2$  and output is  $X_2$ . At real income  $Y_2$ , which corresponds to output  $X_2$ , the  $QD_{r3}$  curve lies below the  $45^\circ$  line in part (9A), indicating an excess of output over quantity demanded. This means that inventories are rising and that employers will, therefore, decrease employment and output. This example indicates the importance of relative adjustment velocities in the analysis of disequilibria. This point can be made even more explicit by considering another possibility.

Assume, in order to make a point, that neither the commodity market nor the labor market are in adjustment. Assume that the economy is at

point "T" but that the current level of employment is  $N_{12}$ . At point "T" the real wage is  $W_2$  which is less than the marginal product of labor at  $N_{12}$ . Therefore, from Mundell's point of view, employers are induced to increase employment and output. On the other hand, an employment of  $N_{12}$  means that output is  $X_{12}$  and income is  $Y_{12}$ . At  $Y_{12}$  the  $QD_{r3}$  curve is below the  $45^\circ$  line. This means an excess of output over quantity demanded and therefore from a Keynesian point of view the conditions represented by point "T" would lead to a decrease in output and employment. It would be difficult to find a more striking example of the importance of relative adjustment velocities to the interpretation of the economic meaning of disequilibrium positions.

The slope of the RR curve can be obtained by taking the differentials of equations (5.1) through (5.6) plus (5.8) and then solving for  $\frac{dr}{dW}$ . This procedure results in equation (5.9).

$$\frac{dr}{dW} = \frac{\frac{\partial X}{\partial N} \left( 1 - \frac{\partial QD}{\partial Y} \right)}{\frac{\partial MP}{\partial N} \frac{\partial QD}{\partial r}} \quad (5.9)$$

It is clear from equation (5.9) that the RR has a positive slope provided that  $\left( 1 - \frac{\partial QD}{\partial Y} \right) > 0$ . Under these conditions the numerator is positive. Since both terms in the denominator are negative, the denominator is positive and, therefore, the overall expression is positive. If, however,  $\frac{\partial QD}{\partial Y} > 1$ , the slope of the RR curve is negative.

In the next section the RR curve is analyzed in conjunction with a system which includes the money market. In the complete Keynesian system the interest rate is an endogenous variable.

### The RR Curve in the Keynesian System

The objective in this section is to incorporate Mundell's RR curve into a model of the complete Keynesian system. To this end two new varieties of curves are introduced. The NM curve represents alternative combinations of the rate of interest and the real wage which satisfy the requirement that real wage equal the marginal product of labor and which are also consistent with equilibrium in the money market. The NM curve is based on equations (5.2), (5.4), (5.5), (5.6), (5.7), and (5.8). Only two commodity market equations are omitted. Assume a real wage of  $W_1$  is selected at random. In order to satisfy equation (5.6) the marginal product of labor must be  $MP_1$ . This means that employment is  $N_3$ , output is  $X_3$ , and income is  $Y_3$ . A real wage of  $W_1$  according to the RW curve in part (9F) also means that the price level is  $P_3$ . It can also be observed that according to equation (5.8), once the real wage is set the price level is also determined. In part (9E) the RMS curve shows that the real money supply is  $M_3$  at price level  $P_3$ . The  $L_{y3}$  curve in part (9B) shows that an interest rate of  $r_3$  is required for equilibrium in the money market. The combination  $r_3$  and  $W_1$  is one point on the NM curve. Other points can be obtained in a similar manner.

The NM curve is useful for both static and dynamic analysis. From a static point of view, the NM curve combined with the RR curve defines the equilibrium interest rate and real wage rate. In the present instance the NM curve cuts the RR curve at  $(r_2, W_2)$ . A decrease in the nominal money supply which shifted the RMS curve to the R position would have the effect of shifting the NM curve upward to the right.

Thus, leading to both a higher interest rate and a higher real wage. In this way Mundell's apparatus can be used as an alternative method of illustrating Keynes' static system.

In Keynes' dynamic model a lag exists between changes in quantity demanded and changes in output. Therefore, Mundell's equation (5.3), the commodity market equilibrium equation, does not hold during the adjustment process. Keynes also assumes, on the other hand, that the real wage equals the marginal product of labor and that the quantity of money demanded equals the existing quantity of money at all points in time. Since these conditions correspond with the specifications for the NM curve it follows that the NM curve represents the rate of interest that will prevail at alternative real wage levels during the adjustment process under Keynesian assumptions. For example, a real wage of  $W_3$  would imply an interest rate of  $r_1$  under the specified dynamic assumptions. A real wage of  $W_3$  means a marginal product of labor of  $MP_3$ , an employment of  $N_1$  and an output of  $X_1$ . This means income is  $Y_1$ . The  $QD_{r_1}$  curve is above the  $45^\circ$  line at  $Y_1$  and this implies that output will rise. As employment increases the marginal product of labor in part (9H) decreases. According to Keynesian assumptions this requires a corresponding decrease in the real wage and an increase in the interest rate along the NM curve. Equilibrium in part (9C) is established at  $(r_2, W_2)$ . This scenario is founded on the Keynesian assumption of equality between the real wage and the marginal product of labor. It should be observed that the NM and the LMN curves are complementary since they are based on the same set of equations.



It would be consistent with Mundell's approach, on the other hand, to assume that the money and commodity markets adjust more rapidly than the labor market. In that case, equation (5.6) does not hold in disequilibria. The model is then based on equations (5.1), (5.2), (5.3), (5.7), and (5.8). This set contains five equations but six endogenous variables which are  $Y$ ,  $r$ ,  $X$ ,  $QD$ ,  $P$ , and  $W$ .

The fact that the  $LM_{p_3}$  curve in part (9D) cuts the IS curve at  $(r_{23}, Y_{12})$  means that this combination of interest rate and income is consistent with equilibrium in the money and commodity markets providing that the price level is  $P_3$ . In part (9F) a price level of  $P_3$  means a real wage of  $W_1$  at the prevailing money wage. The point  $(r_{23}, W_1)$  is one point on the LMIS curve in part (9C). Other points can be obtained in a similar way.

Notice that the LMIS curve in conjunction with the RR or the NM curve is capable of defining the interest rate -- real wage combination which is consistent with equilibrium in the static model. Any two of the curves in part (9C) are sufficient for this purpose.

Under present assumptions the LMIS curve also has dynamic implications. If the commodity and money market adjust relatively rapidly, then the LMIS curve instead of the NM curve indicates the interest rate that prevails at alternative levels of real wages during the adjustment process. For example, if the real wage is  $W_3$ , then the price level is  $P_1$ . Assuming that the commodity and money markets are in equilibrium the interest rate must be  $r_{12}$ . The IS curve indicates that income is  $Y_{23}$ . It follows that output is  $X_{23}$  employment is  $N_{23}$ . Since the marginal product at  $N_{23}$  is less than the assumed real wage ( $W_3$ ) over-

production exists in Mundell's sense of the term. In the previous chapter it was demonstrated that the ISLM curve can represent quantity demanded at alternative price levels only if it is also assumed that the commodity and money markets adjust rapidly relative to the labor market. Under these assumptions the ISLM curve represents the relationship between output and the price level as well as the relationship between quantity demanded and the price level during the adjustment process. At the same time the LMIS curve indicates the interest rate that would prevail at alternative levels of the real wage. In this way the orthodox neo-Keynesian approach to the theory of aggregate demand is related to Mundell's assumptions about the adjustment process. However, in both cases the assumption of a lag in the real wage -- marginal product of labor adjustment is inconsistent with the General Theory.

The slope of the NM and the LMIS curves may be obtained from the differentials of the equations associated with each curve.

The NM curve is based on equations (5.2), (5.4), (5.5), (5.6), (5.7), and (5.8). The endogenous variables are X, Y, r, N, MP, P, and W. Equation (5.10) which equals the slope of the NM is obtained from the differentials of these equations.

$$\frac{dr}{dW} = - \frac{\frac{\partial L}{\partial Y} \frac{\partial X}{\partial N}}{\frac{\partial MP}{\partial N} \frac{\partial L}{\partial r}} + \frac{Mo}{\frac{\partial L}{\partial r} W_o} \quad (5.10)$$

Since  $\frac{\partial L}{\partial r}$  is negative the right hand term is negative. The numerator of the left hand term is positive. The denominator is positive since both  $\frac{\partial MP}{\partial N}$  and  $\frac{\partial L}{\partial r}$  are negative. Given the negative coefficient

this makes the left hand term negative. Since both terms are unambiguously negative the slope of the NM is negative.

The LMIS curve is based on equations (5.1), (5.2), (5.3), (5.7), and (5.8). The differentials of these equations can be used to obtain equation (5.11).

$$\frac{dW}{dr} = \frac{W_0}{M_0} \left( \frac{\partial L}{\partial r} + \frac{\frac{\partial L}{\partial Y} \frac{\partial QD}{\partial r}}{1 - \frac{\partial QD}{\partial Y}} \right) \quad (5.11)$$

The term  $\frac{\partial L}{\partial r}$  is negative by definition. The numerator of the right hand expression is negative because  $\frac{\partial QD}{\partial r}$  is negative by definition. The denominator is positive if  $\frac{\partial QD}{\partial Y} < 1$ . In this case the LMIS curve is negative sloping. If  $\frac{\partial QD}{\partial Y} > 1$ , then the right hand term inside the brackets is negative. If

$$\left| \frac{\frac{\partial L}{\partial Y} \frac{\partial QD}{\partial r}}{\left(1 - \frac{\partial QD}{\partial Y}\right)} \right| > \frac{\partial L}{\partial r}$$

then the slope of the LMIS is positive.

#### Summary

This chapter has analyzed a graphical apparatus suggested by Robert Mundell. It has been suggested that Mundell's model could be improved by a more effective treatment of the money market. Two new varieties of monetary equilibrium curves are suggested. These curves combine the equilibrium requirements of the money and labor markets on

the one hand and the money and commodity markets on the other hand.

It has also been suggested that Mundell's analysis of disequilibria in connection with his graphical model represents only one of at least two interpretations. Mundell's approach is non-Keynesian in the sense that he assumes that the money and commodity markets adjust more rapidly than the labor market. It is shown that when Keynesian assumptions about relative adjustment velocities are substituted for those of Keynes, Mundell's dynamic conclusions are reversed.

## CHAPTER VI

### SUMMARY

The primary objective of this paper was to demonstrate that two neo-Keynesian graphical representations of the Keynesian system are based on labor market assumptions which are inconsistent with the General Theory. In the case of the orthodox neo-Keynesian model, the theory of aggregate supply and the theory of aggregate demand were shown to be based on contradictory assumptions. As a result, those who use neo-Keynesian type aggregate supply and demand curves to measure "excess demand" or "excess supply" do not correctly specify quantity demanded. Mundell's model excludes the money market and his dynamic analysis is based on assumptions about relative adjustment velocities which conflict with those found in the General Theory.

A key consideration that is overlooked in orthodox neo-Keynesian models is the fact that each level of income in the commodity market implies a particular level of employment and output. Each level of employment is uniquely related to a particular marginal product of labor. Keynesian labor market assumptions imply that the price level is equal to the money wage over the marginal product of labor. It follows, therefore, that each level of income in the commodity market is linked to a particular price level.

In Chapter 2 it was demonstrated that when a simple neo-Keynesian income-expenditure dynamic model is combined with Keynesian labor market assumptions, the resulting relationship between quantity demanded and the price level is positive. It was pointed out that in this kind of model there is no direct causal relationship between quantity demanded and the price level. Instead the level of employment, output, and the price level are determined by recent sales results. Since output in the current period equals income and the level of income determines quantity demanded in the current period, both quantity demanded and the price level in the current period are ultimately determined by quantity demanded in the recent past. In other words, the price level and output are functions of the same independent variable and this fact accounts for the positive relationship between the price level and quantity demanded. It is the rise in income that occurs simultaneously with the rise in the price level that causes quantity demanded to increase as the price level increases.

In the third chapter the money market is incorporated into the system. In the preliminary model quantity demanded is positively related to a rise in the price level because Keynesian labor market assumptions imply that a rise in income occurs concomitantly with a rise in the price level. It is a widely known fact that a rise in the price level has the effect of decreasing the real money supply and thereby increasing the rate of interest at any level of income which is consistent with money market equilibrium. This rise in the interest rate has the effect of moderating the positive relationship between quantity demanded and the price level. It was demonstrated, however, that if

Keynes' assumption that income is the dominating factor in the determination of quantity demanded is retained, the price level and quantity demanded continue to be positively related.

In Chapter 4 it was shown that the neo-Keynesian aggregate "demand" function is a market equilibrium curve rather than a demand curve. The neo-Keynesian aggregate demand function was shown to be the result of a series of conceptual experiments in which the Keynesian system was moved from one equilibrium position to another as a result of autonomous changes in the labor market. The neo-Keynesian "demand" function was shown to represent the locus of intersection points of alternative aggregate supply curves and their corresponding  $QD_p$  curves.

Chapter 4 also demonstrated the logical inconsistency contained in those neo-Keynesian models which presume that quantity demanded at a particular price level can be determined without reference to output and employment. It is shown that aggregate supply and demand curves of the McKenna variety in effect assume that output and income can be two different amounts at the same time. It follows, therefore, that the orthodox neo-Keynesian aggregate demand-aggregate supply model is logically defective. If the neo-Keynesian aggregate "demand" curve truly represents quantity demanded at alternative price levels it is necessary to reject the Keynesian theory of aggregate supply and what amounts to the same thing one must reject the assumption that the real wage is equal to the marginal product of labor. If Keynesian labor market assumptions are retained, then it is necessary to reject the McKenna negative sloping aggregate demand curve.

It is shown in Chapter 5 that Mundell's graphical diagram is not an accurate model of the Keynesian system. Mundell's exposition of the Keynesian system is incomplete because it assumes away the money market. Instead, the rate of interest is treated as an exogenous variable. Chapter 4 demonstrated how Mundell's model can be altered so as to correct this deficiency.

In addition to the development of a static model of the Keynesian system, Mundell attempts to describe the adjustment process in terms of his static model. It was shown in Chapter 5 that Mundell's dynamic analysis is based on the assumption that there is a lag in the labor market adjustment between the real wage and the marginal product of labor. Keynes, on the other hand, accepted the Classical assumption that the real wage equals the marginal product of labor at all points in time.

Mundell's dynamic analysis was shown to depend on the assumption that output equals quantity demanded in the product market. Keynes argued that there would be a lag between changes in quantity demanded and subsequent changes in output and employment. It is shown in Chapter 5 that when Keynesian assumptions about the relative adjustment velocities of the labor and commodity markets are substituted for those made by Mundell, Mundell's dynamic conclusions are reversed. Disequilibrium situations which Mundell argues will lead to an expansion of output and employment ("underproduction"), from a Keynesian perspective represent "overproduction" conditions.



#### SELECTED BIBLIOGRAPHY

- Bailey, Martin J. National Income and The Price Level. New York: McGraw-Hill Book Company, 1962.
- Barrett, Nancy S. The Theory of Macroeconomic Policy. Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1972.
- Branson, William H. Macroeconomic Theory and Policy. Evanston: Harper and Row, 1972.
- Barro, Robert J., and Grossman, Herschel I. "A General Disequilibrium Model of Income and Employment." American Economic Review, Vol. 61, pp. 82-93.
- Carson, Deane. Money and Finance: Readings in Theory, Policy, and Institutions. New York, John Wiley & Sons, Inc., 1966.
- Conard, Joseph W. Introduction to Theory of Interest. Los Angeles: University of California Press, 1959.
- Dean, Edwin. The Controversy over the Quantity Theory of Money. Chicago: D.C. Heath and Company, 1965.
- Dernburg, Thomas F. and McDougall, Duncan M. Macroeconomics: The Measurement, Analysis, and Control of Aggregate Economic Activity. New York: McGraw-Hill Book Company, 1972. (4th edition).
- Friedman, Milton. "A Theoretical Framework for Monetary Analysis." Journal of Political Economy, Vol. 78 (March/April 1970), pp. 193-238.
- Hicks, John. Capital and Growth. New York: Oxford University Press, 1965.
- Hicks, John R. "Mr. Keynes and the 'Classics': A Suggested Interpretation." Macroeconomics, Selected Readings, Edward Shapiro, Chicago: Harcourt Brace & World, Inc., 1970, pp. 197-209.
- Holbrook, Robert S. "The Interest Rate, the Price Level, and Aggregate Output." Readings in Money, National Income, and Stabilization Policy. Ed. by W. L. Smith and R.L. Teigen, Homewood, Illinois: Richard D. Irwin, Inc., pp. 43-65.

- Keynes, John Maynard. The General Theory of Employment Interest and Money. New York: Harcourt, Brace and World, Inc., 1936.
- Klein, Lawrence R. The Keynesian Revolution, 2nd Edition. New York: The Macmillan Company, 1966.
- Leftwich, Richard H. The Price System and Resource Allocation, 4th Edition. Hinsdale, Illinois: The Dryden Press, Inc., 1970.
- Leijonhufvud, Axel. On Keynesian Economics and the Economics of Keynes. New York: Oxford University Press, 1968.
- Lindauer, John. Macroeconomics. New York: John Wiley & Sons, Inc., 1968.
- Lindauer, John. Macroeconomics, 2nd Edition. New York: John Wiley & Sons, Inc., 1971.
- Lindauer, John. Macroeconomic Readings. New York: The Free Press, 1968.
- McKenna, Joseph P. Aggregate Economic Analysis, 4th Edition. Hinsdale, Illinois: The Dryden Press, Inc., 1972.
- Modigliani, Franco. "Liquidity Preference and the Theory of Interest and Money." Readings in Monetary Theory, The American Economic Association Committee, Homewood, Illinois: Richard D. Irwin, Inc., 1960, pp. 186-239.
- Mundell, Robert A. "An Exposition of Some Subtleties in the Keynesian System." Macroeconomics, Selected Readings. Edward Shapiro, Chicago: Harcourt, Brace and World, Inc., pp. 210-221.
- Patinkin, Don. Money Interest and Prices, 2nd Edition. New York: Harper and Row, 1965.
- Pesek, Boris P. and Saving, Thomas R. The Foundations of Money and Banking. New York: The Macmillan Company, 1968.
- Shapiro, Edward. Macroeconomic Analysis, 2nd Edition. Chicago: Harcourt, Brace and World, Inc., 1970.
- Shapiro, Edward. Macroeconomics: Selected Readings. Chicago: Harcourt, Brace and World, Inc., 1970.
- Smith, W. L. and Teigen, R. L. Readings in Money, National Income and Stabilization Policy (revised edition). Homewood, Illinois: Richard D. Irwin, Inc., 1970.
- The Federal Reserve Bank of Boston -- Monetary Conference of June 8-10, 1969, Nantucket Island. Controlling Monetary Aggregates. Boston, Massachusetts: September, 1969.

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