A DYNAMIC NON-LINEAR MODEL OF AN URBAN SITUATION WITH PROBABILISTIC TYPE CAUSAL MODELS

By

WILLIAM EDWARD HOGAN JR.

Bachelor of Science
Oklahoma State University
Stillwater, Oklahoma
1965

Master of Science in Electrical Engineering
Southern Methodist University
Dallas, Texas
1969

Submitted to the Faculty of the Graduate College
of the Oklahoma State University
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Thesis Approved:

Thesis Adviser

Dean of the Graduate College

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GLOSSARY OF SYMBOLS AND DEFINITIONS

AVR	- Actual vacancy rate of housing in the residential sector
BCE	- Blue collar exogenous economic units comprised of households who work for industry exogenous to the minority community
BCI	- Blue collar endogenous economic units comprised of households who work for industry endogenous to the minority community
CPI	- Community projection index which is a relative measure of attractiveness of the urban area
DE	- Declining employment parameters based on normal attrition rate for the minority work force
E	- Age group in each economic unit comprised of individuals 15-19 years old
Endogenous	- As used here, events or variables originating from or due to causes internal to the minority race
EU	- Economic unit which is defined as a household. In this urban model, there are five types of economic units which are BCI, WCI, BCE, WCE and UEMP.
Exogenous	- As used here, events or variables originating from or due to causes external to the minority race.
ε	- Is an element of
F	- Age group in each economic unit comprised of individuals 20-44 years old
FA	- Failure rate of industry exogenous to the minority community
FR_ℓ	- Fraction of total minority households which own or rent their home where $\ell = 1$ for homeowners and $\ell = 2$ for renters
HH	- Connotes a household and is used in reference with parameters indicating how many of a particular age group exist in a given household
HV	- Vacancy rate in housing which assumes the value of the actual vacancy rate AVR during model simulation

LOF - Limited opportunity factor which controls flow between economic units endogenous and exogenous to the minority community and economic units exogenous to the minority community. This variable reflects disparities encountered by the minority in the labor market.

MI_n - Migration into the urban area for the n^{th} economic unit

MO_n - Migration out of the urban area for the n^{th} economic unit

MPIWI - Minority owned industry which is greater than five years old

NH - New housing source for single family homes in the residential sector

NM_n - Net migration for the n^{th} economic unit and equals the difference between migration in and out of the urban area

NR - New housing source for rental units in the residential sector

OF - Opportunity factor which controls movement between economic units endogenous to the minority community

OR - Control for new industry exogenous to the minority community

PIWI - Minority industry

PEWIE - Majority industry

RU - Rental units in the residential sector

S_n - Age group comprised of individuals 45 and above. The age group is divided in this model into 45-64 and 65 and above where n = 1, 2 to designate these groups.

S - Source for industry exogenous to the minority community

SEV - Socio-economic variable

SF<15 - Single family homes which are priced below $15,000

SF>15 - Single family homes which are priced above $15,000

SH - Slum housing

SMSA - Standard metropolitan statistical area as defined by the Bureau of Census

TH - Total housing in the residential sector

TMHH - Total number of minority households

TMP - Total minority population

UE_n - Unemployment rate for the n^{th} economic unit

UEMP - All unemployed economic units

UP - Parameter which represents hiring practices of white collar economic units

WCE - White collar exogenous economic unit employed by industry exogenous to the minority community

WCEUE - White collar exogenous economic units that are unemployed

WCI - White collar endogenous economic units employed by industry endogenous to the minority community

WCIUE - White collar endogenous economic units which are unemployed

Y - Age group in each economic unit comprised of individuals 0-14 years old

CHAPTER I

INTRODUCTION

One of the most complex systems in existence today is the urban complex of today's cities. This system is composed of many sectors which contribute to the interactions that occur in an urban area. The various sectors could be classified as Demographic, Transportation, Residential, Industrial and Educational components. The sectors (subsystems) named are a fraction of the many sectors which could be used to describe an urban system; but, with these sectors, an overview of the total urban system can be provided.

Meaningful contributions have been made to analysis of the urban problem, but few contributions to date have been made in mathematical equations which lend themselves to mathematical analysis. Equations which describe social phenomena must be derived and techniques such as those studied in connection with linear and non-linear systems in engineering need to be applied for better insight into the urban problem. Because of the urban crisis which exists today, we are forced by the exigencies of time to try and understand the interactions within an urban area. The author being an electrical engineer with studies inclined toward the system science area, the purpose of this research has been to develop a model along with insights and such analysis techniques peculiar to the physical sciences which can be applied to the economic and social interaction of urban areas.

Many models directed towards urban research have appeared recently. The four general areas covered in these research efforts have been Transportation, Land Use, Demography, and Economic activity. In Steinitz (76), page 20, a table is shown listing twenty planning models. Of utmost importance in this table are the function, theory and method of each model.

The functions listed are projection, allocation, and derivation. Projection means to estimate the future of the system. Allocation occurs when the modeling process divides the system into subsystems (sectors). Derivation occurs when the modeling process transforms the system model by deriving another system from it.

The purpose of a model is the performance of one or more of these functions, possibly in varying combinations, in order to simulate the subject whether it be Land Use, Transportation, Demography, etc.

Statement of the Problem

Within each large metro-urban complex, there exists today what is often called a ghetto (68). This is normally defined as being a conclave of impoverished people. Harold Rose (68) defines a ghetto to be an area to which individuals are confined due to economic, social, or any other socially imposed restrictions.

Many authors have stated that the problems that exist in these urban complexes are due, in part, to the existence of the ghetto; but, no one has attempted an intuitive mathematical analysis and parameter estimation of these areas (68,17,26,82).

The impoverished areas are inhabited in large measure by minorities and most of these minorities are made up of people of the Negro race (68).

In this research, reference to the minority may be interpreted as reference to the Negro race. Because of the previous statements, it seems proper that a model should be constructed of the inner city utilizing intuitive concepts that underlie the social and economic phenomena, which are perhaps peculiar to this area, and affect the minority.

As an indicator of the differences between standard urban models and one specifically derived to apply to minority groups, the model conceived by Forrester (27) and colleagues makes assumptions about several variables which should be reexamined when considering minority groups in an urban area. As an example, the statement is made that "increasing managers in proportion to managerial jobs increases the likelihood of establishing new enterprise". Because of the education and business experience lag of the minority, and alienation from the majority race, more minority group managers do not, necessarily, constitute a force to create new enterprise within the minority community (17,68).

Consider for a moment the availability of minority managers. The mere fact that managers exist does not affect demand for a service, or any particular enterprise (17). The provision for service, which is a very high risk enterprise, may occur if funds can be appropriated. In this risk situation, investment has to occur in spite of the fact that the investment may never be recovered (17). This is confirmed when studies are undertaken concerning the spending in blighted areas (52).

There are other generalizations by Forrester which must be questioned. One of these is the statement which is made concerning the mechanization of agriculture.

> One occasionally hears that the mechanization of southern agriculture, which has forced the immigration of the Negro to the cities, is responsible for the plight of the American city. While such an external pressure can intensify urban

> difficulties, the earlier chapters of this book make clear that the basic causes of urban slums are internal. Another common source of blame for urban decay is low outward mobility, a result of rejection of Negroes by the more affluent urban and suburban sections.

In these statements, Forrester has stated that the problems of the urban area are internal to the slum. If this was true, one must ask the question, why did he not model the problem?

In this research, the actual problem areas are modeled with particular reference to the minority groups in the inner city or slum area, whichever is appropriate. The actual troubled urban areas are modeled because it is thought that its problems can be solved only if the areas are understood. One should not infer that the author thinks the problem lies within the inner city. For whatever proportion of the problem which does reside in the inner city, urban research will be enhanced by studying this problem. The author does believe that invisible boundaries, along with other exogenous inputs, are partially responsible for the socio-economic problems which exist in America's urban areas.

Research Objectives

The research, covered in this dissertation, is done to provide an economic activity model which deals with the minority group in the inner city. Concepts borrowed from engineering systems and probability theory are applied to the model to present a definitive analysis of the system. Probabilistic models are developed for parameters which are components of the state variables of the system.

The conception of the model is based on existing data and intuitive understanding of the causal relationships which direct economic growth forces. The propensity for growth within the urban system is conditioned on the education, residential, and demographic indexes indicated, as time is projected forward.

The model must reflect the economic activity. The economic activity consists of industry which is exogenous or endogenous to the minority community. The minority community consists of all minority population, regardless of what spatial area they occupy.

The theory of this model is based on macro-analytic growth forces, with the dynamics of decision activity based on group behavior rather than the individual. An example of this is the consideration by the model to generate a new business. This does not occur because of individual actions, but it does occur when the entire market place (community) exhibits a demand to support such an activity.

The mathematical equations which comprise the model consist of linear and non-linear equations of state. At this point, a definition of the concept of state presented by Gupta is appropriate (35). Any variable of a system defining some performance of the system and subject to change can be thought of as a state variable of the system. Knowing all these variables for a particular system completely defines the state of the system. Once the state is defined for an element in the system, the state of that element can be determined at any other time by knowing the initial value and any input applied (35). In electrical networks, the state variables may sometimes be chosen by inspection; while in urban systems, the variables are chosen because they represent that part of the system which is to be observed.

Consider a generalized linear differential equation:

$$a_n \frac{d^n y}{dt^n} + a_{n-1} \frac{d^{n-1} y}{dt^{n-1}} + \dots + a_1 \frac{dy}{dt} + a_0 y = u(t) \quad . \qquad (1.1)$$

A set of state variables may be defined as follows: let

$$x_1 = y \tag{1.2}$$

$$x_2 = \frac{dy}{dt} = \frac{dx_1}{dt} = \dot{x}_1 \tag{1.3}$$

$$\vdots$$

$$x_n = \frac{d^{n-1}y}{dt^{n-1}} = \dot{x}_{n-1} \quad . \tag{1.4}$$

The definitions of (1.2) - (1.4) yield a set of first order differential equations

$$\dot{x}_1 = x_2 \tag{1.5}$$

$$\dot{x}_2 = x_3 \tag{1.6}$$

$$\vdots$$

$$\dot{x}_{n-1} = x_n \tag{1.7}$$

$$\dot{x}_n = -\frac{a_{n-1}}{a_n} x_n - \dots - \frac{a_1}{a_n} x_2 - \frac{a_0}{a_n} x_1 + \frac{1}{a_n} u(t) \quad . \tag{1.8}$$

Using matrix theory notation, states $x_1, \dots, x_n$ define a state vector

$$\underline{x} = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} \quad . \tag{1.9}$$

Equations (1.5) through (1.8) can be rewritten as

$$\begin{bmatrix} \dot{x}_1 \\ \vdots \\ \\ \dot{x}_n \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & --- & 0 \\ 0 & 0 & 1 & --- & 0 \\ --- & --- & --- & --- & --- \\ 0 & 0 & --- & --- & 1 \\ -\frac{a_0}{a_n} & -\frac{a_1}{a_n} & --- & --- & -\frac{a_{n-1}}{a_n} \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ \\ x_n \end{bmatrix} + \begin{bmatrix} 0 \\ \vdots \\ \\ \frac{1}{a_n} \end{bmatrix} u \quad (1.10)$$

or

$$\dot{\underline{x}} = [A]\underline{x} + [B]\underline{u} \quad . \quad (1.11)$$

Using the definition of a derivative, a difference equation may be written for each differential equation in the matrix equation (1.10). The definition of a derivative is (32,69),

$$\dot{x} = \lim_{\Delta t \to 0} \frac{x(t+\Delta t) - x(t)}{\Delta t} \quad (1.12)$$

which may be approximated as

$$\dot{x} = \frac{\Delta x}{\Delta t} = \frac{x(k+1) - x(k)}{\Delta t} \quad (1.13)$$

as shown in Goldberg (32) where

$$\Delta t = (k+1) - (k) \quad (1.14)$$

as a time increment, and

$$\Delta\, x(k) = x(k+1) - x(k) \quad . \quad (1.15)$$

Using Equation (1.13), each equation in matrix Equation (1.10) is rewritten in difference form and expressed as a matrix.

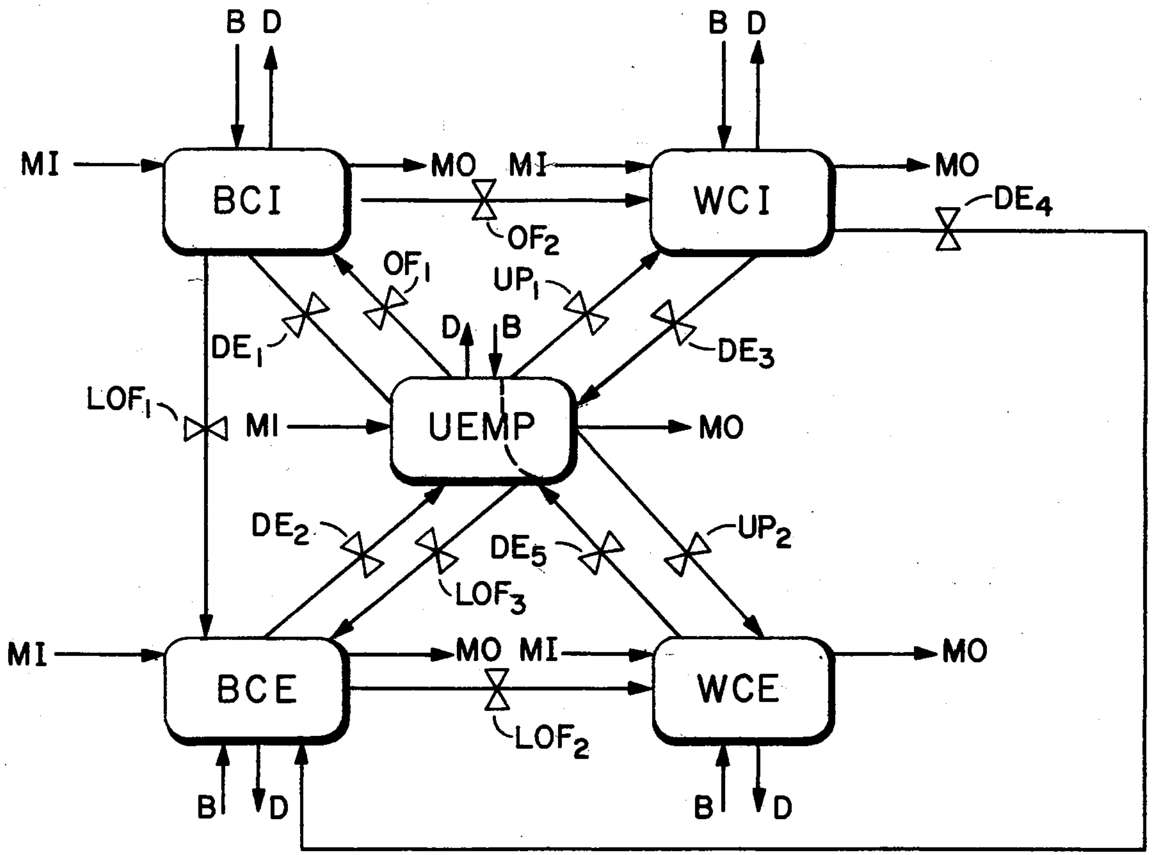
$$\frac{\Delta\, \underline{x}(k)}{\Delta t} = [A]\underline{x}(k) + [B]\underline{u}(k) \quad (1.16)$$

which is the general form for all state vectors. The equations just discussed were linear and a general solution may be written. The majority of the equations in this model are non-linear for which no general solution is known.

The probabilistic estimation of parameters appears to be a new concept as well as the subject of the model. For the first time socio-economic variables are quantified on the basis of the axiomatic definition of probability with a high degree of success. These probabilistic models are then structured as series-parallel models with intuitive logic dictating the type of structure. Other socio-economic scientists have tried to use the axiomatic probability basis for decision making with limited success (9). Savage is one of the latest to attempt this, but, he used it for personal decision making or alternative policy (70). This limits the application considerably. In this dissertation parameters which affect society are considered, not the logical thought process.

A brief outline of the model before its detail is presented. The urban model consists of four sectors which are Demographic, Educational, Residential, and Industrial.

The demographic sector, shown in Figure 1, consists of five types of economic categories or units. These economic units (EU) are Blue Collar Endogenous (BCI), White Collar Endogenous (WCI), Blue Collar Exogenous (BCE), White Collar Exogenous (WCE), and the Unemployed (UEMP). The terms exogenous and endogenous refer to the industrial firms where the economic categories are employed. Firms which are owned by the minority race are endogenous to the minority community and employ BCI and WCI economic units. Firms which are owned by the majority race are exogenous to the minority community and employ BCE and WCE economic units.



MI = MIGRATION IN
MO = MIGRATION OUT
D = DEATHS
B = BIRTHS

Figure 1. Demographic Sector

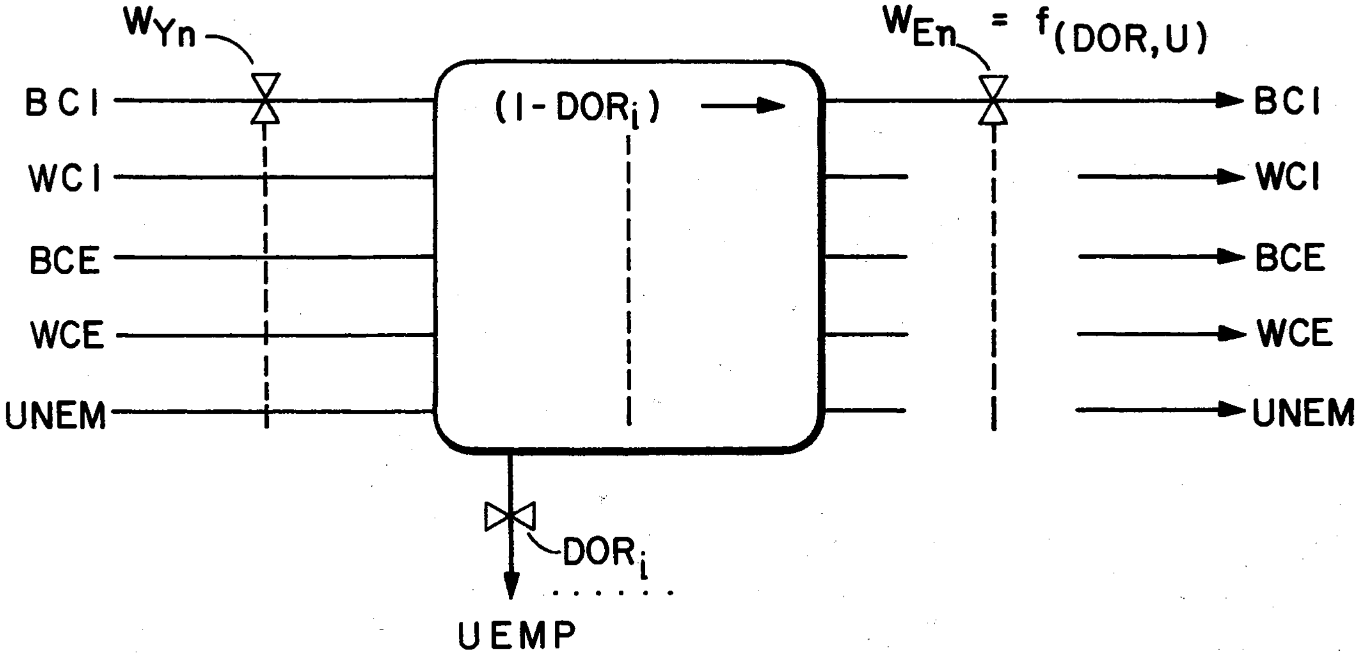
The unemployed category consists of BCI, WCI, BCE, and WCE. The white collar economic units are separated from the blue collar economic units within the UEMP category as indicated by the dashed line in Figure 2. The separation of the economic units was done because of the vast differences which exist between white collar and blue collar employment categories in the form of qualifications, promotional opportunities, and unemployment rates.

The values which control the flow, in Figure 2, between economic units are controlled by four types of variables. The variables are LOF_n (limited opportunity factor), OF_n (opportunity factor), DE_n (declining employment factor), and UP_n (upward opportunity factor.

The LOF_n controls the flow between economic units endogenous to the minority community to economic units exogenous to the community, and reflects disparities such as discrimination in the job market. The OF_n controls flow between economic units endogenous to the minority community. Because of the differences minorities face when seeking employment within the community or when seeking employment outside of the community, separate factors LOF_n and OF_n were used in the model. DE_n represents declining employment due to normal attrition, layoffs, etc. UP_n represents promotion opportunity when dealing with white collar employment.

The educational sector is shown in Figure 2 and represents the secondary educational process, i.e., grades 9 through 12. Each employment category is handled separately because of possible differences in drop-out-rates (DOR_n). The parameters W_{Yn} and W_{En} are transport factors which move individuals into and out of the educational process based on age.

The residential sector, shown in Figure 3, consists of four types of



DOR_i : SIGNIFIES DROPOUT RATE FOR THE ith GROUP

U : DEATH RATES

Figure 2. Educational Sector

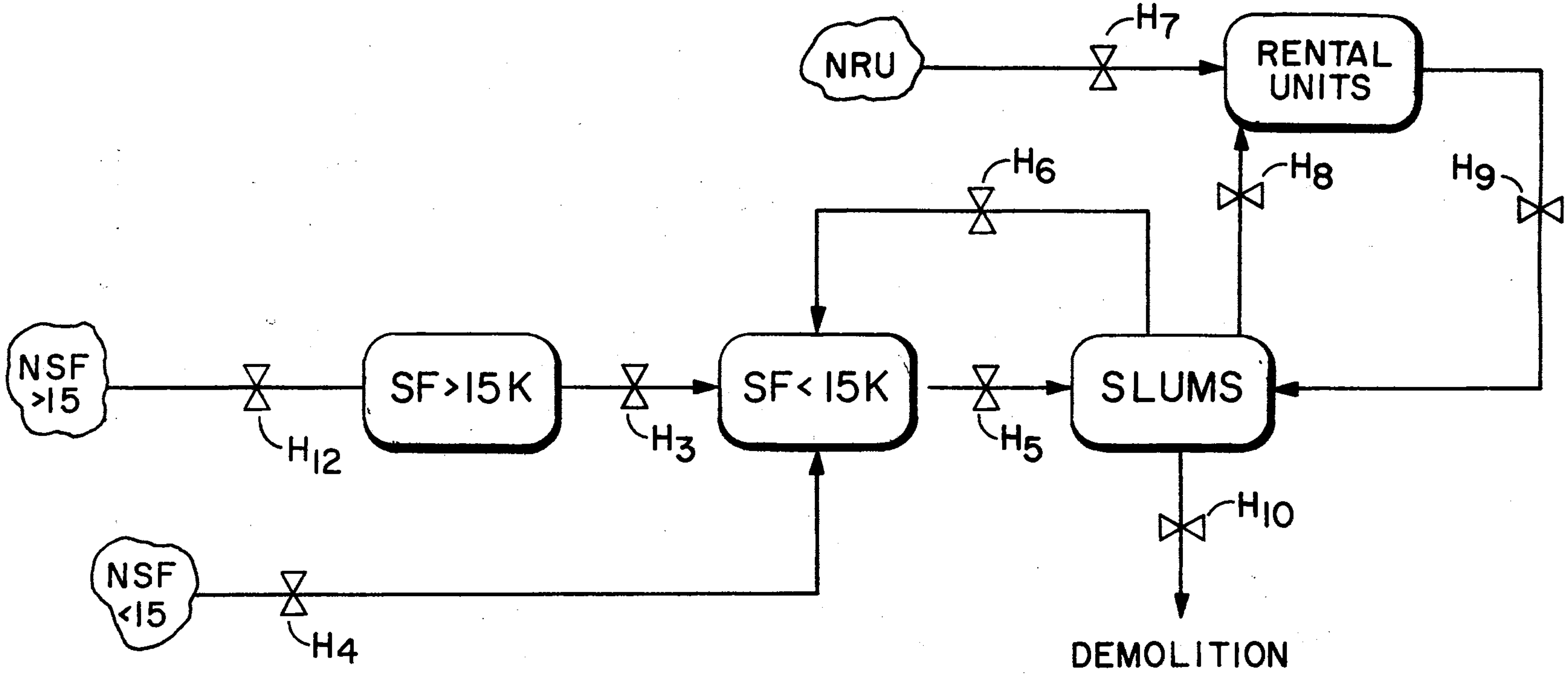


Figure 3. Residential Sector

housing. They are single family units less than \$15,000 (SF<15), single family units greater than \$15,000 (SF>15), rental units (RU), and slum housing (SH). The sources of new housing shown as NSF≥15, NSF<15, and NRU are controlled by variables H_{12}, H_4, and H_7, respectively and these sources and their control variables are functions of housing vacancies and population within the urban area.

The parameters H_3, H_5, H_9 provide the filtering process which occurs in housing due to age, maintenance, etc. H_6, H_8, and H_{10} provide a gradual form of urban renewal which rehabilitate slums in the urban area.

In the industrial sector, shown in Figure 4, there are two types of industry. They are profits-exogenous (PEWIE) and profits-endogenous (PIWI). The PEWIE firm or industry is controlled by the majority and employs BCE and WCE economic units, while the PIWI firm is controlled by the minority and employs BCI and WCI economic units. In this dissertation the number of exogenous firms is held constant because the interest lies in studying the minority industrial base. New PIWI firms (XNF) are controlled by a function of existing PIWI, UB_1, and all minority EU. UB_2 is the failure rate of new minority firms. UB_3 is a transport factor which changes a PIWI to a mature firm (MPIWI) after five years. UB_4 represents the failure rate of MPIWI firms, and UB_5 represents reinvestment by mature industry to form new endogenous firms.

Concerning the PEWIE firm, OR_1 controls the source XNEF (new exogenous firms) and FA_1 is their failure rate.

This completes the description of the various sectors of the urban model which is shown in its entirety in Figure 5.

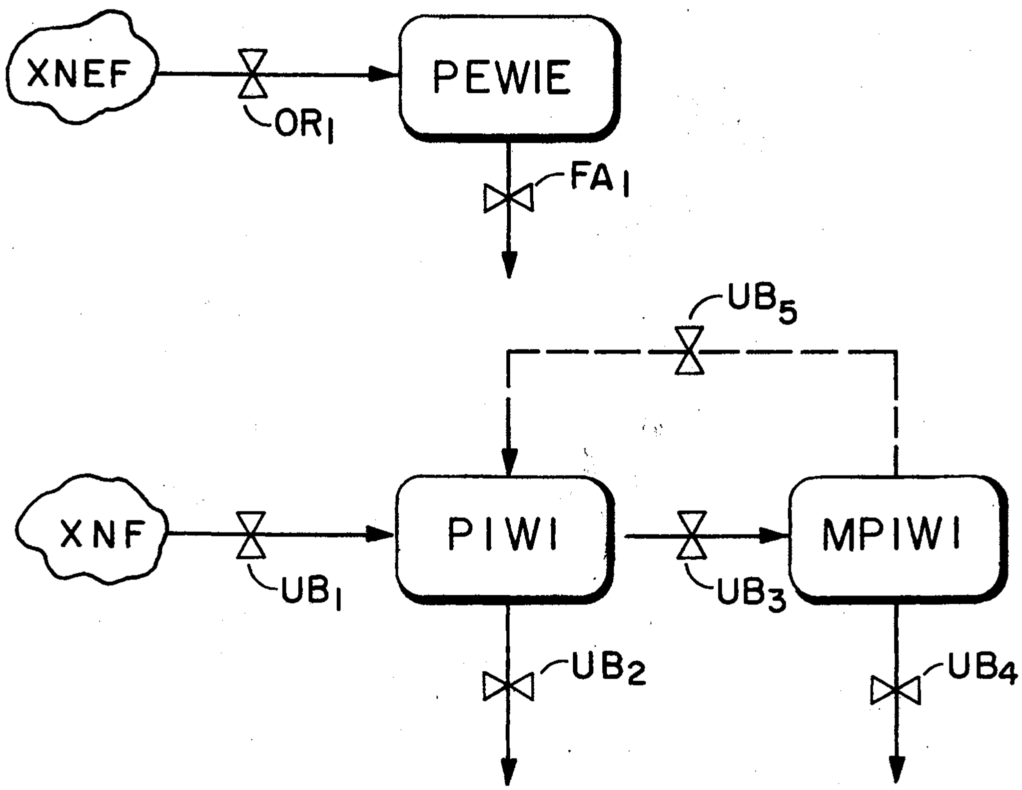


Figure 4. Industrial Sector

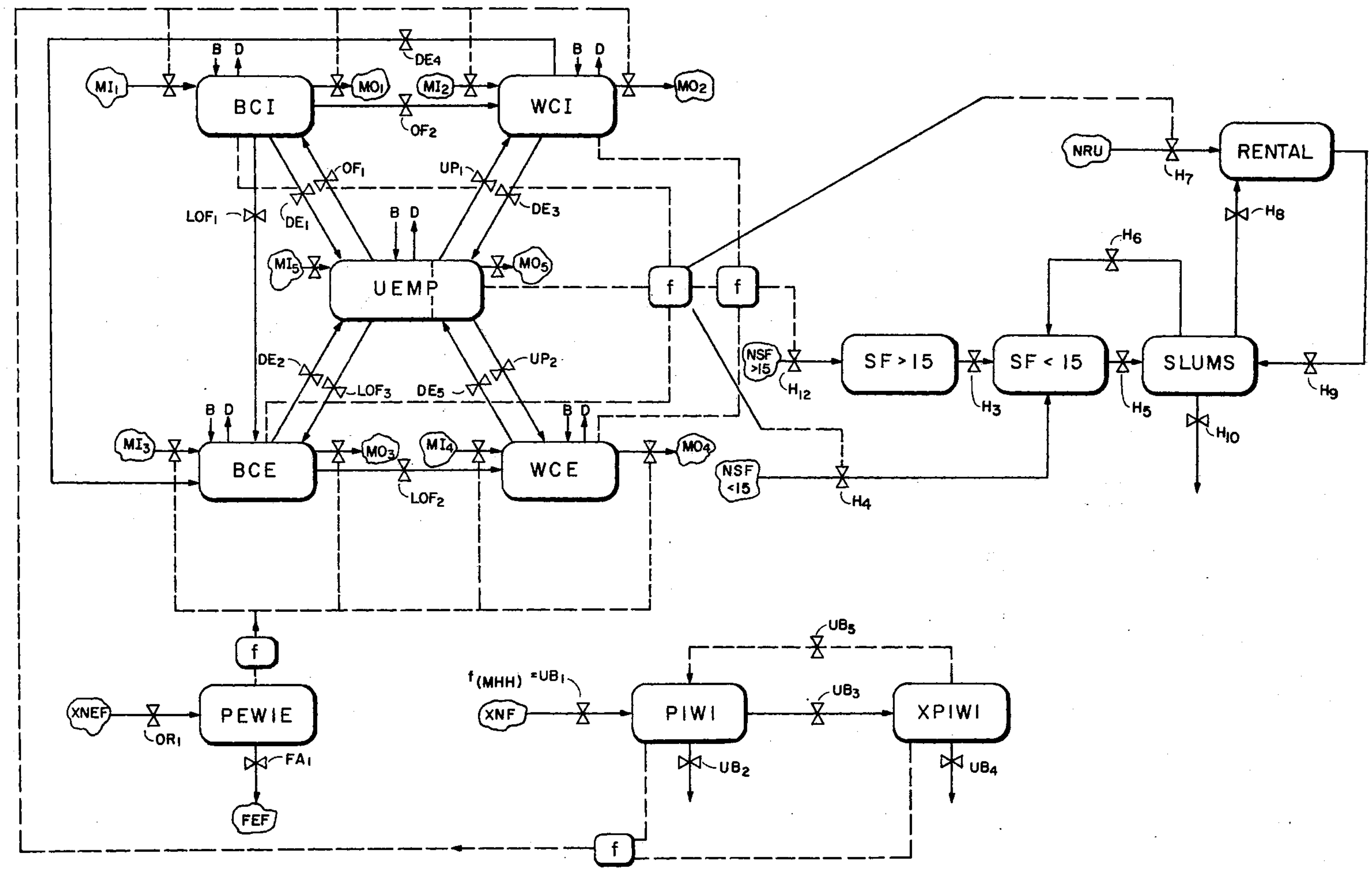


Figure 5. Urban Model

Chapter Organization

Chapter II contains a review of applicable literature. Chapters III and IV contain the development of the urban model by sectors. This also consists of deriving the state equations which mathematically describe the model. Chapter V reviews axiomatic probability and introduces the unique concept of probabilistic causal models. Sensitivities are derived and used in selection of the structure of the causal models. Chapter VI discusses operation of the model with respect to experiments in migration patterns, education, industrial expansion, etc.

Chapter VII discusses conclusions and future research which should be done in the area of urban systems.

CHAPTER II

A REVIEW OF APPLICABLE LITERATURE

Applicable Urban Research

There is a great amount of urban research occurring today. A major fraction of this work has been directed toward study of the minority groups in this country. The largest portion of this minority group research, in the urban setting, has been directed toward the Negro race.

Portions of the research have dealt directly with politics, economy, residential aspects and other important facets of any community (68,17, 82). Harold Rose (68) demonstrates knowledge of all phases of urban institutions. This includes discussions concerning the demographic transition in an urban area, along with an overview of the residential, industrial, and educational basis for the ghetto. He labeled his work a spatial behavioral perspective. This dissertation is a spatial behavioral systems analysis. Theordore Cross (17) discusses the ineptitude of the minority industrial base to compete with the economic mainstream of America. This is explained as being caused by education, inexperience, and lack of a financial base for initial investment. The normal financial base is lacking for minority industry because of the high risk involved. This forces many enterprises to close, after some initial phases of operation, and others are never started.

In order for some of the urban problems to be solved, the minority industrial base must be strengthened (82). In comparing the area modeled in this research with that of an underdeveloped country, it is necessary that the underdeveloped area develop a demand for its services (1). This demand must be met with more than just labor. This necessitates that the urban area create demand endogenous and exogenous to the community. A recent study has shown that demand endogenous to the minority community is very weak (52).

One of the basic themes, which is constantly underlined in all minority research, is the unequal position of the minority within society (21,87,83,84,89,39). Bowen (12) shows the disadvantages in the labor force. Taeuber (84) investigates the effects of this disadvantage in residential markets in general. Numerous government documents display this disparity and programs implemented to correct the same (23,48,53). This dissertation deals on a mathematical basis with the limiting of minority opportunities. The Coleman report (15) in education is also along these same lines. In several reports, Coleman, as well as others, state that one of the largest differentials in our educational system occurs because of a lack of expectation of success in the environment exogenous to the minority community (15,54). This has never before been represented mathematically in a causal relationship, but is attempted in this dissertation.

Discussion of the socio-economic factors which pervade urban research could continue indefinitely, but would accomplish nothing more than what has already been said. It is obvious that a urban minority model needs to be constructed to enable urban researchers to so order their thought processes, that constructive programs may be implemented.

Urban Modeling

Urban modeling has been attempted by many individuals. The areas modeled have been residential, industrial, demographic, educational, etc. (2,4,11,14,15,22,27,28,40,67,47,71,75,91). The models represent a fair cross-section of all models and the strategies used.

The model by Arrow (2) concerning discrimination in the job market is the only model seen by the author that attempts to quantify discrimination. Kenneth Arrow (2) modeled racial discrimination in the labor market. He describes a model by which an employer can purchase labor at a fixed price, but for which he must choose some point on an indifference curve between wages and the proportion of whites in the firm. Disparity in job opportunities often result when making this decision.

Residential Models

Residential development has been studied by a number of individuals (14,57,37,76). The approach, to the method of study, has likewise been varied. Chapin (14) has simulated residential development from the standpoint of having fertile cells of land. These cells represent the metropolitan area divided into a grid system. By then allocating different site amenities to the cells, attractiveness patterns are determined for various households. The residential decision process was conditioned on: (1) intensity of residential development; (2) accessibility; (3) household budget; (4) household activity patterns; and (5) household taste patterns. The residential development process is allowed to take place in a probabilistic manner. This development process entails Monte Carlo techniques and is not used as a basis for causal relationships, but is used to distribute possible households. Chapin's model again demonstrates modeling of the final behavior expected, and not the underlying causes of the behavior.

Robinson, Wolfe and Barringer (67) produced a simulation model having to do with urban renewal. "The operation of the model is based upon a matching, within the computer, of existing stocks of space in the city with potential users of the space, on the basis of the relative need or desire for particular types of space by categories of users." This is similar to the previously discussed model, but differs in that it has manipulative mathematical equations in which the variables may be adjusted for expected results. The question that must be asked is, do the equations actually explain causal relationships, or are they linear regression type equations with no meaning to the constants and the manner in which they are adjusted?

Stochastic models of residential development have also been studied (37). They leave a lot to be desired, however, in that few scientists have a firm grasp of the complexity introduced when using stochastic processes.

Demographic Models

Schweitzer and Dienes (71) made a significant contribution to the theory of demographic modeling. To some extent their model is similar to the demograhpic sector model in this research effort. They explained population growth as being a kinetic process. After dividing the population into age groups, differential equations were written around each age group. The propensity to move from one age group to another is automatic because of the aging process and the birth and death rates are accounted for. They however fail to account for non-uniform distributions in their populations which move between age groups as well as effects of migration, education, etc. Non-uniform distributions, and the previous effects mentioned are accounted for in this dissertation.

Lowry (47) confronts the idea of computer models dealing with migration and metropolitan growth. Two models were constructed. The first model deals with migrational flows between New York and Chicago. The first model explains some of the behavioral factors that govern migration movements. The second model deals with migration into particular metropolitan areas. Both of the models use linear regression techniques to fit statistical data.

Beshers (7), in 1968, edited a book containing various approaches to the modeling of large scale social systems. The major thrust of this literature is demographic projections. Computer methods were devised for detailed population estimates and projections by age, sex, etc. This text discusses many of the current treatments of the data which are necessary for a demographic projection along with the applicable computer language.

DeCani (18), in 1961, constructed stochastic models of population and growth. He considered three models which were increasingly complex. The first model is concerned with pure migration from one region to another. The results were a contribution in that the limiting distributions of the populations for each region were binomial as a function of time. The limiting distributions are the probability distributions of the population after an infinite time has elapsed. The second model also dealt with migration, but it also considered births and deaths within the system. The population distribution was again studied and the limiting distributions did not exist, even though the influence of the initial population distribution decreased with time. The third model is a predator-prey-type model and because of the assumptions the findings are as expected; the predators eventually take over the region.

Educational Models

Stone (80), in 1966, outlined several models for the educational process. The scope of one of the models includes all forms of education, training and retraining in which he regards the educational system as a system of connected processes, as in input-output analysis. The demographic characteristics of the student population were found to control the initial activity levels; but, as time progressed the decisions between students and advisers became the main determinant. In a later model, Stone (79) makes an effort to connect demographic, educational and manpower statistics. Here, the basic building block is a population accounting matrix. His major objective was the combining of the educational system with exogenous parameters such as migration.

Industrial Models

Several researchers have attempted to model the structure of various components of the economic activity which occurs in urban areas (5,27, 41).

Berry (5) approaches this phase of modeling under the guise of the retail market. He discusses models drawn from two sources. One of the models was prepared for the city of Chicago, in conjunction with its Community Renewal Program. The other model was prepared for the six county area surrounding Chicago. The first model helped to describe the deteriorating conditions which existed, while the second was used to predict retail patterns into the future. Both models used regression analysis.

Forrester (27) used an industrial component in his urban model which was composed of three phases of industry. They were new, mature and declining industrial firms. The methods of state-space analysis were used.

One of the most complete documented works on industrial modeling and locational analysis was done by Isard, et al. (41). Throughout the text are references to model construction, input-output analysis, etc. In Chapter 11, specific reference is made to gravity models as a means of defining the interaction which occurs between people, households, industrial complexes and others. In these gravity models, the region is conceived as a mass. Interregional reaction is then equated with the interaction which occurs between masses in physical science. Gravity models in all phases of urban research have been important in that this is a possible link between the physical laws which govern our universe with the abstract concepts which are so often present in the social sciences.

Urban-Regional Models

There are a number of contributors or researchers in the area of urban-regional models (11,16,22,27,28,40,63). Echenique (22) uses the principle found in gravity models, and other concepts, to look at urban stocks and the activity which occurs between these stocks. The urban structure is defined as the outcome of various processes which allocate physical objects and activities to sites within an area. The static model differentiates three activity groups: employment, residential and service activities; and two physical components: total quantity of floor space, and a transportation network. The simple dynamic model adjusts the location of residence and services in accordance with changes in employment, transportation, and availability of space. The complex dynamic model introduces different classes within each activity group which locate, and are constrained in location by the stocks which are differentiated by condition and structural type.

The models, which have caused so much controversy in the past year, have been those done by J. W. Forrester (27,28). As stated in the introduction Forrester has made some questionable assumptions concerning the causal factors which control the behavior in urban areas. He makes the assertion that he has modeled the inner city; but, the assumptions are not necessarily valid for the minority group which is modeled here. The model which he conceived has three basic sectors. They are Demographic, Housing and Business with three classes in each sector. The equations for simulation are differential equations which can be written by examining the flow diagrams of the model. The sources for the three sectors are independent sources, or infinite sources. Gray (33), in a critique of Forrester's work states that the business and housing sector were rejected because of the assumed constant life expectancy of each class of business. The argument against this was, the fixed "filtration" model constrains the population of housing and business and thus causes equilibrium in the system.

Gray's major criticisms of the model are the following: (1) the model does not model the suburbs; (2) he is precise about how many people are in a family but does not specify any objective function; (3) the model is a highly damped continuous time-invariant system. The fixed land constrains it to continuity, and the filtration model of business and housing causes a unique equilibrium. This is not the case in this research; because, the parameters are justified, the structure of the model is non-linear, and the filtration in the housing sector is a variable conditioned on conditions of the urban system. The objective of the model here is to describe the inner city.

Another very detailed model of urban structures is given by Crecine (16). This model is very similar to Lowry's (46), "Model of a Metropolis". Regression analysis is used in both models to fit existing data. Exogenous and endogenous variables are defined and correspond to the usual variables used by other urban researchers.

The variables represent residential, industrial and commercial sectors. Their economic activity deals primarily with retail trade. The underlying theory of the "Model of a Metropolis" is the gravity model concept. Crecine's (16) model uses regression analysis. A discussion of twenty planning models is presented by Kilbridge (43). In the discussion, models are aggregated by their subject, function, theory, and method. The subjects are Land, Transportation, Population and Economic activity. These subjects cover the gamut of most planning models. The interesting insight provided is the function of each model. The function can either be projection, allocation, derivation or a combination of two or all three. Only five out of the twenty models perform all of the functions above and because of this, they are the most complex. In the final portion of Kilbridge (43), references are given for forty-eight other urban models.

Sociological Dependent Variable Causal Models

Causal models, as used in the social sicences, are models involving one way causation and can be handled by recursive sets of equations. These models provide a heuristic device for broadening the scope of simple regression approaches that commonly focus on a single dependent variable. In the social sciences, the causal models are used as a systematic way out of the impasee reached when trying to bridge the gap between research theories, on the one hand, and research techniques on the other.

Blalock (9), in 1971, edited some excellent papers concerning causal models in the social sciences. References are made to three or more variable causal models. Methods of analyzing these models with path analysis are given. Path analysis is a way of depicting regression equations by a simple diagram. Path analysis is similar to the kinetic model or general engineering systems block diagram, which facilitates the writing of differential equations with ease. One of the papers presented in Blalock is "Deductions From Axiomatic Theory". The term axiomatic, as it is used here, refers to a set of propositions which summarize the knowledge in a given field and can be used for finding further knowledge deductively. Using first order partial regression equations, the flow of causal models is determined. Zetterberg (95) is responsible for most of this deductive logic.

Up to the present date, sociological scientists have not based their variables on axiomatic probability. In Chapter Five of this dissertation variable (causal) models are derived; but, for the first time, these models are based on the axiomatic concept of probability. As is stated in Buckley (13), "There were those who refused to give up hope of successful axiomatization of 'personal probability', so as to extend the strictly objectivistic view adhered to by the majority". This resulted in an interesting formulation by Savage (70). This formulation deals with personal probability (in decision-making). It does not deal with trying to establish a probabilistic basis for the variable models.

The research here structures, constructs an axiomatic space, and determines the sensitivity of what are hoped to be the new causal models in urban research.

CHAPTER III

DEMOGRAPHIC AND EDUCATIONAL SECTORS OF URBAN MODEL

Introduction

As described before, the model is divided into four major sectors. These sectors are Demographic, Residential, Education, and Industrial (or Economic). The first three are used as growth indices and are constructed before the economic sector because their growth provides the propensity for economic growth in the socio-economic system being modeled. The system described by this model is, as stated before, confined to a minority community. In this model, the Negro community is discussed whenever an example is employed.

Demographic Sector

In the demograhpic sector, the population is divided into five categories. They are blue-collar endogenous (BCI), blue-collar exogenous (BCE), white-collar endogenous (WCI), white-collar exogenous (WCE) and the unemployed sub-sector (UEMP).

Blue- and white-collar endogenous are those people, who work for industry which is endogenous to the minority being modeled. Blue- and white-collar exogenous individuals are thereby defined as the category of individuals who are employed by firms or institutions not owned within the minority. The unemployed sector is very important. This sector is composed of those individuals who are chronically unemployed as well as temporarily unemployed individuals.

Within each one of these categories, there are several phenomena which have to be dealt with in order to describe the movement between categories. These are the following:

(1) births and deaths;

(2) new households formed or dissolved as a result of:

(a) migration into the area;

(b) migration out of the area;

(c) promotion which allows economic mobility from one category to another; and

(d) decay of existing industry.

Birth-Death Sub-Model

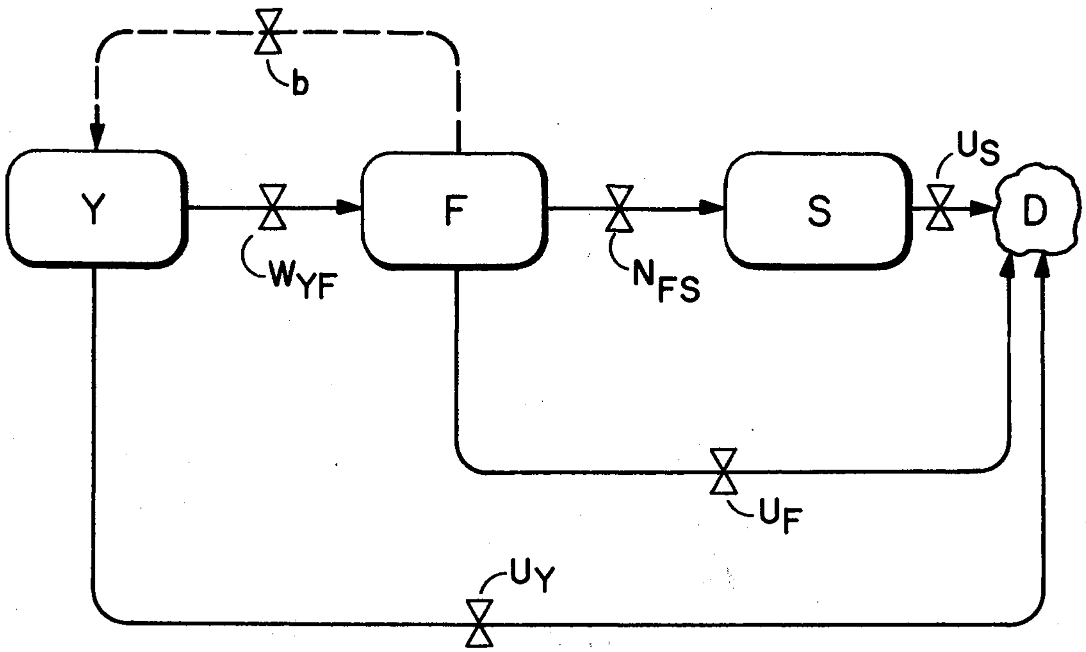
To properly handle the births and deaths, which will have to be allocated for each demographic category, a sub-model is used in the following manner. Births or deaths are computed for each economic category and allocated to the various categories at the end of each year. This process is repeated annually in order to generate the age distribution for each unit of time within the model.

The basic birth-death sub-model is shown in Figure 6. By examining this figure, the equations for each level in the model are written in continuous form.

$$\dot{Y} = -Y(w_{YF} + u_Y) + bF \qquad (3.1)$$

$$\dot{F} = -F(w_{FS} + u_F) + w_{YF}Y \qquad (3.2)$$

$$\dot{S} = -Su_S + w_{FS}F \qquad (3.3)$$



Y : THE NUMBER OF PREPRODUCTIVE PEOPLE

F : THE NUMBER OF REPRODUCTIVE PEOPLE

S : THE NUMBER OF POST PRODUCTIVE PEOPLE

D : THE NUMBER OF DEATHS

b : THE BIRTH RATE REFERENCED TO FERTILE GROUP (VARIES WITH THE ECONOMIC CATEGORY BEING CONSIDERED)

W_{ij}: MOBILITY FACTOR DEPENDENT UPON AGE WHICH ALLOWS THE TRANSFER OF INDIVIDUALS FROM GROUP i TO GROUP j WITHIN A GIVEN CATEGORY

U_i : MORTALITY RATE OF THE GROUP i, $i = Y, F, S$

Figure 6. Birth Death Sub-Model

$$N = Y + F + S \quad , \tag{3.4}$$

where N is the total population at any time.

In this research, difference equations will be used, primarily because the existing data bases are in discrete rather than continuous form. Therefore the equations must be converted into the form of difference equations.

In order to accomplish this transformation, the continuous derivative must be approximated.

First the definition of the derivative is recalled as being:

$$\dot{x} = \frac{dx}{dt} = \lim_{\Delta t \to 0} \frac{x(t + \Delta t) - x(t)}{\Delta t} \quad . \tag{3.5}$$

By using this definition, the differential equations for the Birth-Death sub-model may be approximated,

$$\frac{Y_{k+1} - Y_k}{\Delta t} = -Y_k(w_{YF} + u_Y) + bF_k \tag{3.6}$$

where

$$Y_{k+1} = Y_k[1.0 - (w_{YF} + u_Y)\Delta t] + bF_k \quad . \tag{3.7}$$

k is the time index and in this model Δt is one year. Therefore, with the various rates taken on a per-year basis, it is not necessary to carry it in further computation. Performing the same approximation for the remainder of the equations, we have the following:

$$F_{k+1} = F_k(1.0 - w_{FS} - u_F) + w_{YF}Y_k \tag{3.8}$$

$$S_{k+1} = S_k(1.0 - u_S) + w_{FS}F_k \tag{3.9}$$

$$N_k = Y_k + F_k + S_k \quad . \tag{3.10}$$

All of the variables correspond to their continuous counterparts in the Birth-Death model with the exception of w_{ij} which provided the mechanism for movement between age groups. To define w_{ij}, a possible age distribution for an age group is shown in Figure 7.

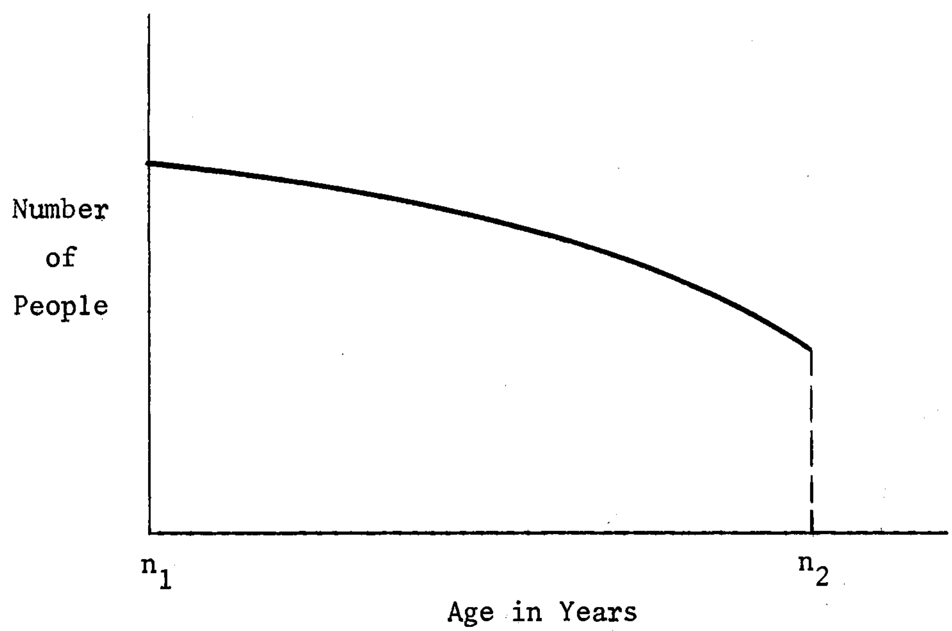


Figure 7. Age Distribution Dynamics

The number of people who pass n_2 is approximately

$$\frac{Z}{n_2 - n_1}\left(1 - \left(\frac{n_2 - n_1}{2}\right) u_Z\right) \tag{3.11}$$

where

Z = total under the distribution curve;

$\frac{Z}{n_2 - n_1}$ = average number of people in each year from n_1 to n_2; and

U_Z = death rate.

Therefore,

$$w_{ij} = \frac{1}{n_2 - n_1} \left[1 - (n_2 - n_1) \frac{u_i}{2}\right] \quad . \qquad (3.12)$$

BCI Model

Using the previous model as an overall basis, the demographic model for each economic group is formed. Each economic group is described in terms of a model within itself. Overall demographic projections are based on the basic birth-death predictions.

The first economic group considered is made up of the blue-collar endogenous workers, previously denoted as BCI. These individuals (as defined before) work for businesses within the minority economic sphere.

There are several factors which affect the BCI, and other economic categories, which must be reflected in the model. They are:

(1) births and deaths;

(2) formation of new households;

(3) migration in;

(4) migration out;

(5) mobility between economic categories; and

(6) new employment from ranks of the unemployed, and the reverse process (6,24).

The births and deaths are handled as shown previously using documented birth and mortality rates related to the age and economic group under consideration (44,60,61,62,73).

New households will be formed whenever either or both of the following conditions is met:

(1) members of the Y group remain in the area (area here connotes geographic region as well as minority status); and

(2) heads of households migrate into the area.

Throughout this study, households are the basic economic units and are numerated by head of household for simplification in the modeling process.

Migration into a given spatial area is a result of several conditions, but the most influential are listed below (6,47,78):

(1) supply of housing;

(2) unemployment rate;

(3) number of exogenous firms which support the specific economic groups; and

(4) relative attractiveness of the urban area.

The supply of housing will be based on the housing vacancy function which is developed later in the residential sector. When the demographic sector is simulated in isolation, the housing vacancy rate in the census data is used (20).

The percent unemployment is used as found in the data base (21,68, 89). Projections of the exogenous unemployment rate are inputs to the model and are entered as needed. As stated previously, the number of exogenous firms, which support specific economic categories, is a constant in this research. This is done to enable the study of the endogenous economic base.

Net migration is expressed as the difference between migration in and out. The initial net migration is assumed, while migration in and out are functions of conditions within the urban area. Migration in bears a direct relation to housing supply, number of supporting industrial firms, and a function of the community projection index CPI (CPI is a probabilistic measure of attractiveness of the urban area, and is derived in Chapter V.). In this model, the number of households that migrate into the area is a function of: housing vacancies, existing households (HH) in the area, industrial base, and the CPI. Migration out is a function of HH in the area and the unemployment rate.

Let EU_n = number of heads of household in an economic unit in the urban area where n = 1 for BCI, 2 for WCI, 3 for BCE, 4 for WCE and 5 for UEMP.

Now expressing the net migration rate (NM_n) for each EU, we have

$$NM_n = \frac{MI_n - MO_n}{EU_n} \qquad n = 1, \ldots, 5 \tag{3.13}$$

where

MO_n = migration out; and

MI_n = migration in.

Let HV = housing vacancy rate, TH = total housing supply, and assume a constant exogenous industrial base (PEWIE) such that $\dot{PEWIE} = 0$. PEWIE is discussed in the industrial sector (Chapter IV). Express

$$MI_n = HV \left[TH \cdot \frac{EU_n}{\sum_{n=1}^{5} EU_n} \right] \cdot f(CPI) \cdot \alpha m_n \tag{3.14}$$

where αm_n is a calibration factor which adjusts the relative attractiveness of the urban area with respect to other areas to zero for initial migration patterns.

Now

$$TH = (1 + HV) \sum_{n=1}^{5} [EU_n \cdot \frac{1 \text{ house}}{EU_n}] \tag{3.15}$$

which gives

$$MI_n = HV(1 + HV)f(CPI) \cdot EU_n \cdot \alpha m_n \quad . \tag{3.16}$$

Let

$$UE_n = \text{unemployment rate for } EU_n \quad .$$

Then

$$MO_n = \frac{UE}{\beta} \cdot EU_n$$

where β is a calibration factor which denotes the migration out given that an EU has an umemployment rate of one. For this model, $\beta = 8$ and is an assumption made by the author. NM has to be calibrated for zero net migration with existing conditions in the urban area to reflect a neutral attractiveness with respect to other areas. For this case,

$$NM_n = \frac{MI_n - MO_n}{EU_n} \quad . \tag{3.17}$$

For $NM_n = 0$, $MI_n = MO_n$ which produces,

$$\frac{UE_n \cdot EU_n}{\beta} = HV[1 + HV]f(CPI) \cdot EU_n \cdot \alpha m_n \tag{3.18}$$

and

$$\alpha m_n = \left[\frac{UE_n}{\beta \cdot HV[1 + HV]f(CPI)}\right] \quad . \tag{3.19}$$

This formulation provides the urban model with a dependent finite source and sink for migration patterns. The source for each EU_n, $n = 1, \ldots, 5$, is $\alpha m_n \cdot EU_n$ and the control for the value which controls the flow is, $HV[1 + HV]f(CPI)$ and the control for the migration out is UE_n/β. This is illustrated in Figure 8.

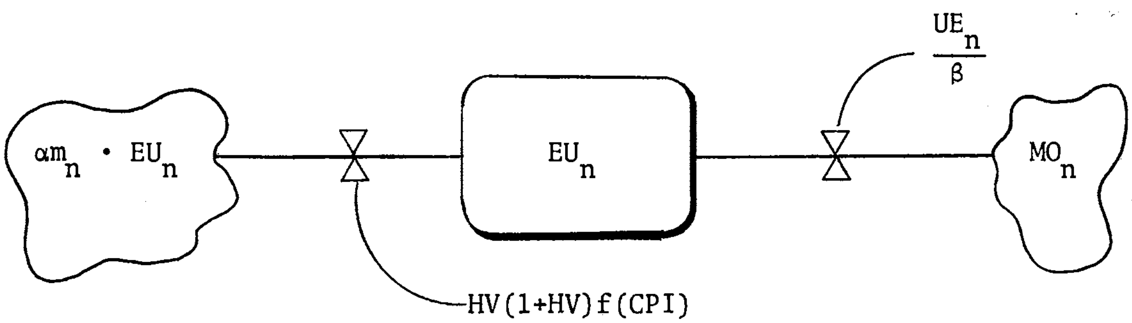


Figure 8. Basic Economic Unit Model

Economic mobility between EU's is input as a variable which may be changed to concur with existing opportunities and policies. This variable has two functions as well as two names.

When the mobility occurs endogenous to the community, the variable is called the opportunity factor. When mobility occurs from categories which are endogenous to the community to those which are exogenous or between exogenous categories, the variable will be titled the limited opportunities factor (LOF). These variables require careful formulation as they also vary with the economic units, between which they control flow.

Utilizing the previous relationships, two block diagrams are constructed. Figure 9 illustrates the general internal structure of each economic unit category, with particular reference to the BCI category in this case. Figure 10 shows the overall structure of the BCI category.

From the block diagrams, the equations are derived. The subscripts attached to the various age groups indicate the economic category. For example, Y_n = Y for the n^{th} category. The equations are, (for n = 1)

$$\dot{Y}_1 = -Y_1(\alpha_{12} + w_{Y1}) + b_1 \cdot E_1 + b_2 \cdot E_1 + \alpha_{11} \tag{3.20}$$

$$\dot{E}_1 = -E_1(\alpha_{14} + w_{E1}) + w_{Y1} \cdot Y_1 + \alpha_{13} \tag{3.21}$$

$$\dot{F}_1 = -F_1(\alpha_{16} + w_{F1}) + w_{E1} \cdot E_1 + \alpha_{15} \tag{3.22}$$

$$\dot{S}_{11} = -S_{11}(w_{S11} + u_{S11}) + w_{F1} \cdot F_1 + \alpha_{17} \tag{3.23}$$

$$\dot{S}_{12} = -S_{12} \cdot u_{S12} + w_{S11} \cdot S_{11} \tag{3.24}$$

where

$$BCI(0) = [Y_1(0) + E_1(0) + F_1(0) + S_1(0)] \tag{3.25}$$

$$\dot{BCI} = -BCI[LOF_1 + OF_2 + DE_1] + OF_1 \cdot UEMP \quad . \tag{3.26}$$

BCI(0) is the initial condition for the differential equation (3.26).

The α_{nj}, j = 1, ..., 7 are functions of the migration functions discussed earlier.

The symbol α_{n8} = (1/number of individuals per household$_n$). The remaining parameters are (n = 1),

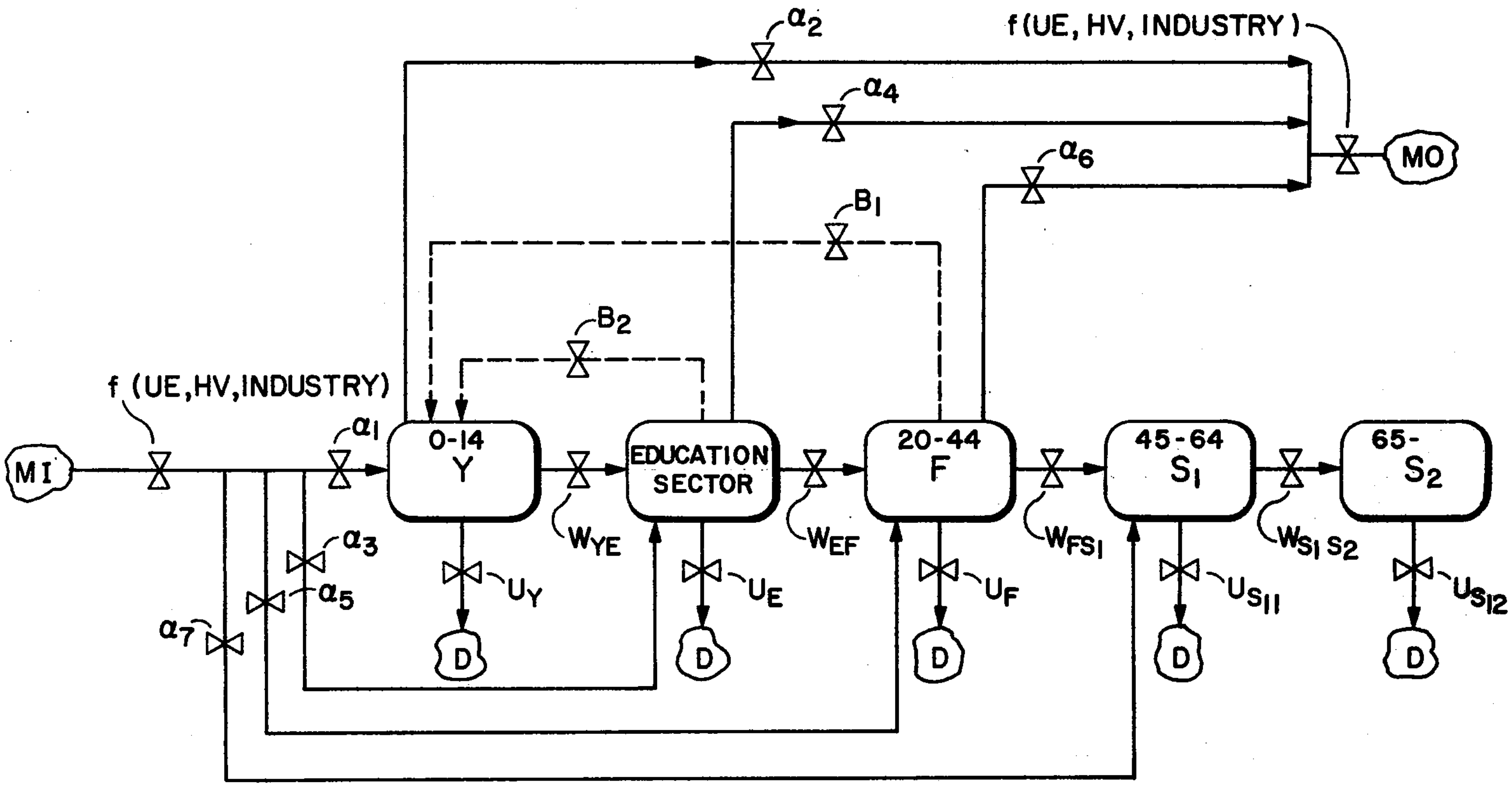
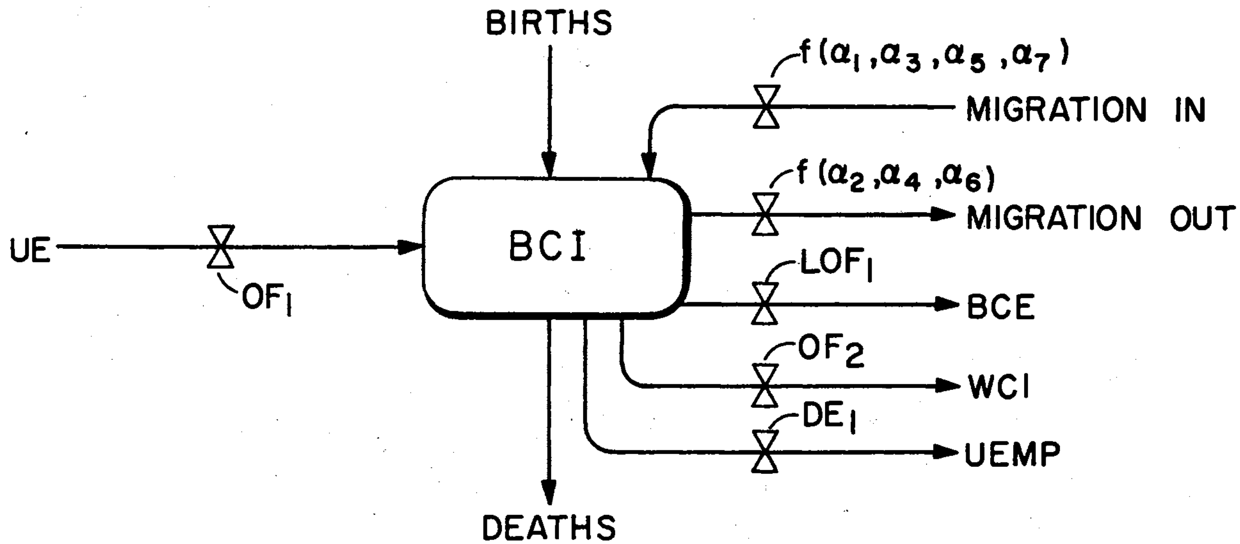


Figure 9. Basic Demographic Sector Sub-Model



DE_1 : DECLINING OPPORTUNITY FACTOR
OF_1 : OPPORTUNITY FACTOR
OF_2 : OPPORTUNITY FACTOR
LOF_1 : LIMITING OPPORTUNITY FACTOR
UE : UNEMPLOYED
BCE : BLUE COLLAR ENDOGENOUS
WCI : WHITE COLLAR ENDOGENOUS

Figure 10. BCI Compartment Model

$$\alpha_{n1} = HV[1 + HV]EU_n \cdot f(CPI) \cdot \alpha[\frac{Y + E}{HH}] \cdot \beta \quad (3.27)$$

$$\alpha_{n2} = \frac{[EU_n \cdot \frac{UE_n}{8} [\frac{Y + E}{HH}] \cdot \beta]}{Y_n} \quad (3.28)$$

$$\alpha_{n3} = HV(1 + HV)EU_n \cdot f(CPI)[\frac{Y + E}{HH}] \cdot \gamma \quad (3.29)$$

$$\alpha_{n4} = \frac{[EU_n \cdot \frac{UE_n}{8} \cdot \frac{Y + E}{HH} \cdot \gamma]}{E_n} \quad (3.30)$$

$$\alpha_{n5} = HV(1 + HV)EU_n \cdot f(CPI) \cdot \alpha[\frac{E}{HH}] \quad (3.31)$$

$$\alpha_{n6} = \frac{[EU_n \cdot \frac{UE_n}{8} \cdot [\frac{F}{HH}]]}{F_n} \quad . \quad (3.32)$$

$\alpha_{n7} = 0$ unless studying migration patterns of older households and is used as an experimental constant when needed. The constant is $\alpha_{n7} = 0$ in this research. β and γ are the fraction of the Y population which is undergoing migration (expressed in ages).

$$\beta = \frac{Y \text{ age span}}{(Y + E) \text{ age span}} \quad (3.33)$$

$$\gamma = \frac{E}{Y + E} \quad (3.34)$$

The remaining constants and variables are,

$$w_{Y1} = (\frac{1}{14})(1 - (\frac{u_{Y1}}{2})14) \quad (3.35)$$

$$w_{E1} = (\frac{1}{4})(1 - (\frac{u_{E1}}{2})4)(1 - DOR_1) \quad (3.36)$$

$$w_{F1} = (\frac{1}{25})(1 - (\frac{u_{F1}}{2})25) \tag{3.37}$$

$$w_{S11} = (\frac{1}{20})(1 - (\frac{u_{S1}}{2})20) \tag{3.38}$$

$$u_{Y1} = .003 \tag{3.39}$$

$$u_{E1} = .0023 \tag{3.40}$$

$$u_{F1} = .0079 \tag{3.41}$$

$$u_{S11} = .0279 \tag{3.42}$$

$$u_{S12} = .005 \tag{3.43}$$

$$b_1 = .0231 \tag{3.44}$$

$$b_2 = .0158 \tag{3.45}$$

The birth rates are from the Oklahoma Department of Health and Kiser (44,60,61,62). The death rates are from other vital statistics (73).

The parameters LOF and DOR are explained in Chapter V. The OF factor is conditioned on the growth of endogenous business. Forming the OF variable, we have the following.

$$OF = f(PIWI, EU \text{ per } PIWI) \tag{3.46}$$

where PIWI (discussed in Chapter IV) are firms endogenous to the area.

$$OF(k) = \frac{PIWI(k) - PIWI(k - 1)}{EU(k)} \cdot \frac{EU_n}{PIWI} \tag{3.47}$$

where $EU_n/PIWI$ is a constant derived from the data base (50). If the growth in PIWI is zero or negative, the OF factor assumes a constant value conditioned on normal declining employment (DE). DE is a constant conditioned on an assumed attrition rate of workers due to not wanting to work, sickness, welfare pays more than their job, etc.

Before proceeding to the remaining categories, conversion of the BCI differential equations to difference form and a matrix notation will be developed. Rewriting the differential equations, we have:

$$Y_1(k+1) = Y_1(k)(1-\alpha_{12}-w_{Y1}) + b_1 \cdot F_1(k) + b_2 \cdot E_1(k) + \alpha_{11} \quad (3.48)$$

$$E_1(k+1) = E_1(k)(1-\alpha_{14}-w_{E1}) + w_{Y1} \cdot Y_1(k) + \alpha_{13} \quad (3.49)$$

$$F_1(k+1) = F_1(k)(1-\alpha_{16}-w_{F1}) + w_{E1} \cdot E_1(k) + \alpha_{15} \quad (3.50)$$

$$S_{11}(k+1) = S_{11}(k)(1-w_{S11}) + w_{F1} \cdot F_1(k) + \alpha_{17} \quad (3.51)$$

$$S_{12}(k+1) = -S_{12}(k)u_{S12} + w_{S11} \cdot S_{11}(k) \quad (3.52)$$

along with,

$$BCI(k) = (Y_1(k) + E_1(k) + F_1(k) + S_1(k))\alpha_{18} \quad (3.53)$$

where BCI(0) is the initial condition for the difference equation.

$$BCI(k+1) = BCI(k)[1 - [LOF_1 + OF_2 + DE_1]] + OF_1(k)UEMP(k) \; . \quad (3.54)$$

At each increment in time, BCI(k) is updated and helps to form the EU_1 for the next increment in time. That is,

$$BCI(k+1) = \Delta EU_1(k) + BCI(k) \quad . \quad (3.55)$$

For the next increment in time, the $\Delta EU_1(k)$ is allocated to each age group in accordance with its former age distribution.

Representing the age groups in an economic unit in matrix form, we

$$\Delta\begin{bmatrix} Y_n(k) \\ E_n(k) \\ F_n(k) \\ S_{n1}(k) \\ S_{n2}(k) \end{bmatrix} = \begin{bmatrix} -(\alpha_{n2}+w_{Yn}) & b_2 & b_1 & 0 & 0 \\ w_{Yn} & -(\alpha_{n4}+w_{En}) & 0 & 0 & 0 \\ 0 & w_{En} & -(\alpha_{n6}+w_{Fn}) & 0 & 0 \\ 0 & 0 & w_{Fn} & -w_{S11} & 0 \\ 0 & 0 & 0 & w_{S11} & -u_{S12} \end{bmatrix}$$

$$\cdot \begin{bmatrix} Y_n(k) \\ E_n(k) \\ F_n(k) \\ S_{n1}(k) \\ S_{n2}(k) \end{bmatrix} + \begin{bmatrix} \alpha_{n1} \\ \alpha_{n3} \\ \alpha_{n5} \\ \alpha_{n7} \\ 0 \end{bmatrix} \quad . \qquad (3.56)$$

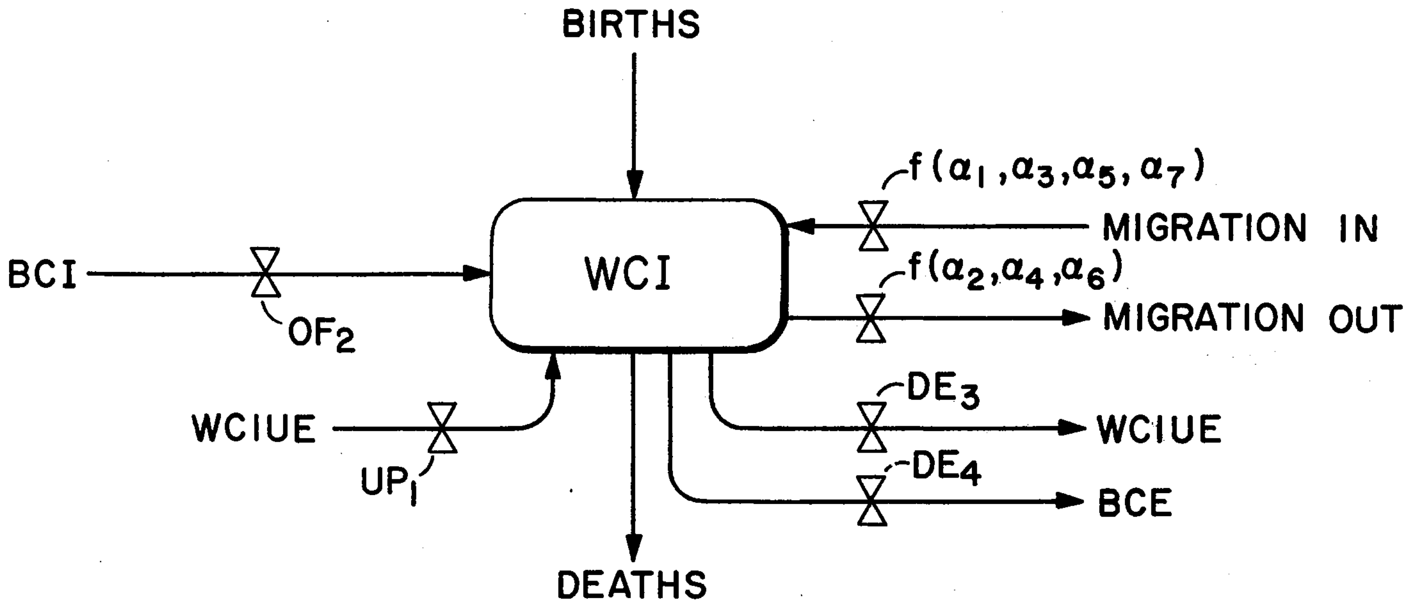
The matrix equation (3.56) is represented in matrix notation as,

$$\Delta \underline{EU}_n(k) = [A(\alpha_n, w_n, b_{n1}, b_{n2}, u_{Sn})]\underline{EU}_n(k) + [B(\alpha_n)] \quad . \qquad (3.57)$$

WCI Model

The remaining categories of the demographic sector are derived using the previous results derived for the blue collar endogenous sector.

The block diagram presented in Figure 9 will be conceptually the same with the exception of the necessary constants which have to change to reflect the changing economic situation. The overall block diagram will change however and is shown in Figure 11. This represents the white collar endogenous (WCI) employee who normally works inside of the geographical area in which he lives and, for the most part, is manager or owns a type of firm which is the employer of the economic category BCI. This firm is the minority firm and will be labelled PIWI. More will be



OF_n : OPPORTUNITY FACTOR

UP_1 : UPWARD ECONOMIC OPPORTUNITY FACTOR

DE_n : DECLINING ECONOMIC OPPORTUNITY FACTOR

Figure 11. WCI Compartment Model

said about characterization of firms in Chapter IV.

The equations for the WCI category are presented, noting that the form of the equations is similar to the BCI category and abbreviated in matrix form.

Writing the equations for Figure 11, we have the following for n = 2,

$$\underline{\dot{EU}}_2 = [A(\alpha_2, w_2, b_{21}, b_{22}, u_{S21})]\underline{EU}_2 + [B(\alpha_2)] \tag{3.58}$$

$$WCI(0) = [Y_2(0) + E_2(0) + F_2(0) + S_{21}(0)]\alpha_{28} \tag{3.59}$$

$$\dot{WCI} = -WCI(DE_3 + DE_4) + OF_2 \cdot BCI + UP_1 \cdot WCIUE \tag{3.60}$$

where

DE_n = declining employment;

UP_n = upward mobility in employment; and

WCIUE = unemployed normally employed in the WCI category.

The variable coefficients w_n and α_n are as stated before with possible differences occuring in death rates, birth rates and density per household.

Presenting these results in difference equation form gives,

$$\Delta\underline{EU}_2(k) = [A(\alpha_2, w_2, b_{31}, b_{32}, u_{S21})]\underline{EU}_2(k) + [B(\alpha_2)] \tag{3.61}$$

$$WCI(k) = [Y_2(k) + E_2(k) + F_2(k) + S_{21}(k)]\alpha_{28} \tag{3.62}$$

$$WCI(k+1) = WCI(k)]1-(DE_3+DE_4)]+OF_2(k)\cdot BCI(k)+UP_1\cdot WCIUE(k) \; . \tag{3.63}$$

BCE Model

The next economic category to be considered is made up of Blue Collar Exogenous (BCE) individuals. These employees work outside of the community in which they live and are employed by a different type of firm than the firms discussed previously, which employ the BCI. This firm (PEWIE) is different with respect to the profit and wage distribution process. By examining Figure 12, the equations for the economic category are,

$$\underline{\dot{EU}}_3 = [A(\alpha_3, w_3, b_{31}, b_{32}, u_{S31})]\underline{EU}_3 + [B(\alpha_3)] \quad (3.64)$$

$$BCE(0) = QY_3(0) + E_3(0) + F_3(0) + S_{31}(0)]\alpha_{38} \quad (3.65)$$

$$\dot{BCE} = -BCE(LOF_2 + DE_2) + LOF_3 \cdot UEMP + LOF_1 \cdot BCI \quad . \quad (3.66)$$

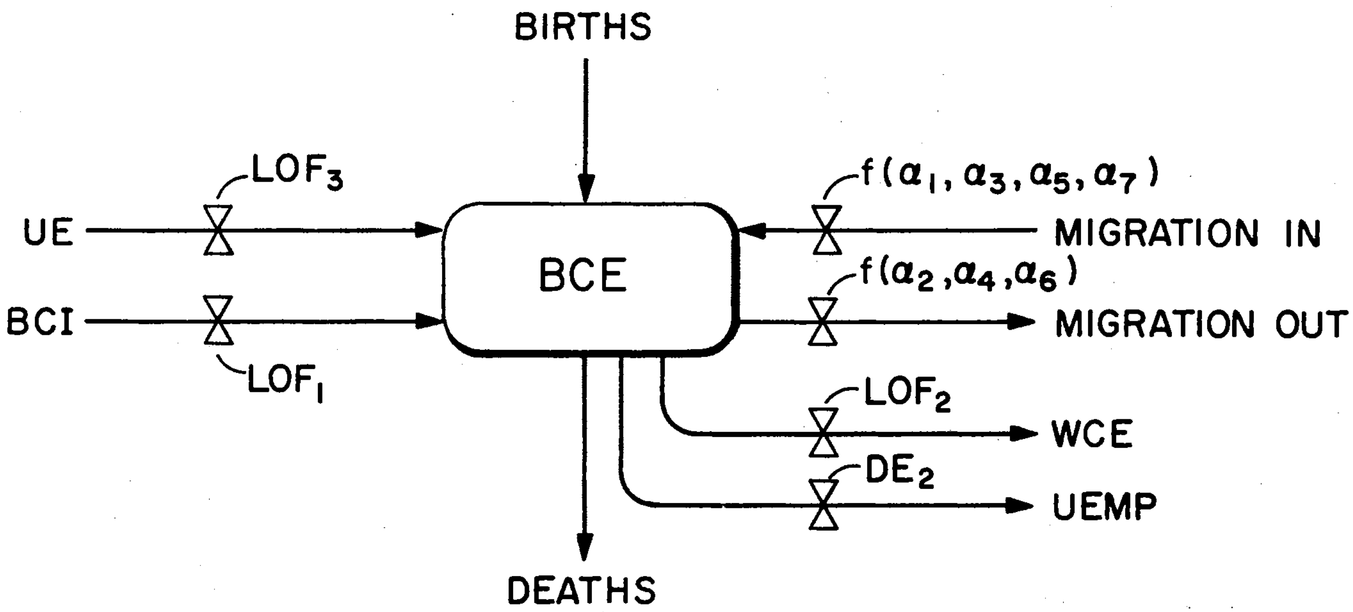
Again, the variable coefficients w_n and α_n (n = 1, 2, ...) have possibly changed due to differences in birth-death rates and density per household. Converting the continuous differential equations into difference form, we have the following:

$$\Delta\underline{EU}_3(k) = [A(\alpha_3, w_3, b_{31}, b_{32}, u_{S31})]\underline{EU}_3(k) + [B(\alpha_3)] \quad (3.67)$$

with

$$BCE(k) = [Y_2(k) + E_3(k) + F_3(k) + S_{31}(k)]\alpha_{38} \quad (3.68)$$

$$BCE(k+1) = BCE(k)[1-(LOF_2(k)+DE_2)]+LOF_3(k)\cdot UEMP(k)+LOF_1(k)\cdot BCI(k) . \quad (3.69)$$



LOF_n : LIMITED OPPORTUNITY FACTOR

DE : DECLINING OPPORTUNITY FACTOR

Figure 12. BCE Compartment Model

WCE Model

The remaining economic category of employed that has be be modeled is comprised of the white collar exogenous (WCE). These individuals represent professional employment in industries (PEWIE) not managed by the minorities. Their place of residence is normally within the minority neighborhood because of possible restrictions to their locating elsewhere in regards to housing. This could change however, because of the fair housing practices being implemented and with these changes the spatial patterns for black communities, as well as the economic structure, would change. The integration index (see Chapter V) would be increased thereby increasing the exogenous opportunity factors which should cause an increase in the hiring of the minority.

The firms for which the WCE and BCE individuals work are industries whose profit/wage distribution far exceeds that of the minority owned firms, in both size and effectivity concerning the formulation of public policy.

The equations which control this economic category are similar in structure with the exception of the values of constants and variables.

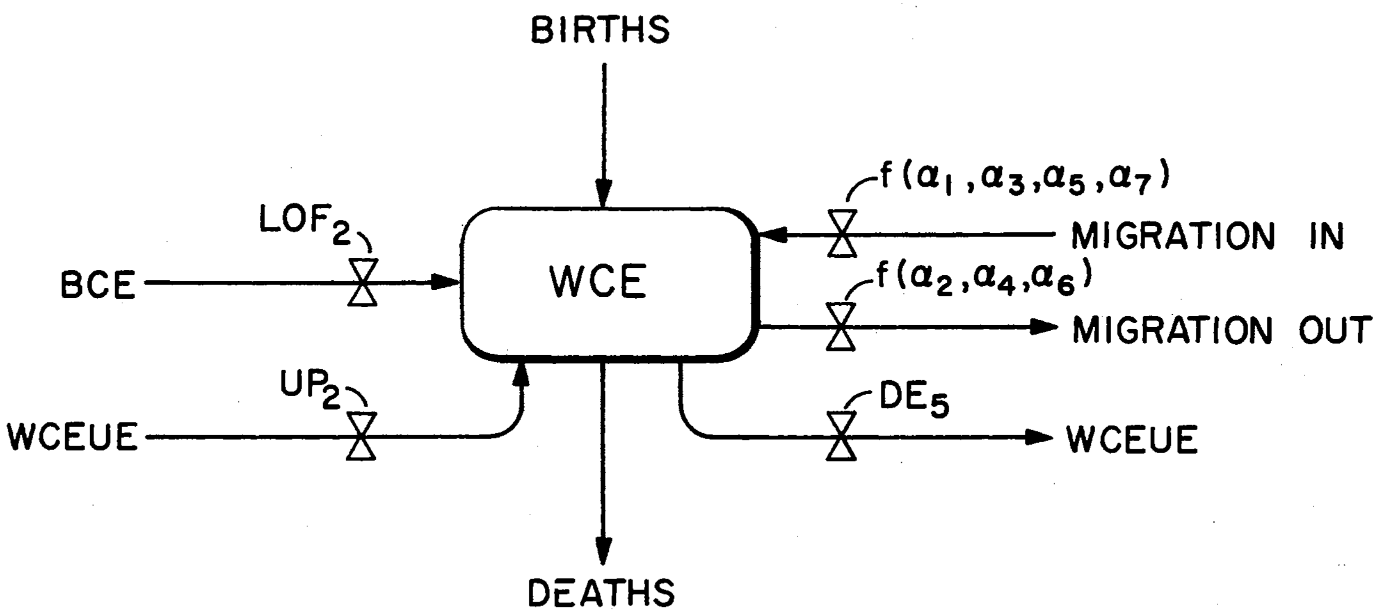
By examining Figure 13, we arrive at the equations which control the WCE category. They are:

$$\underline{\dot{EU}}_4 = [A(\alpha_4, w_4, b_{41}, b_{42}, u_{S41})]\underline{EU}_4 + [B(\alpha_4)] \qquad (3.70)$$

with

$$WCE(0) = (Y_4(0) + E_4(0) + F_4(0) + S_{41}(0)]\alpha_{48} \qquad (3.71)$$

$$\dot{WCE} = -WCE \cdot DE_5 + LOF_2 \cdot BCE \qquad (3.72)$$



DE_5 : DECLINING OPPORTUNITY

UP_2 : UPWARD ECONOMIC OPPORTUNITY FACTOR

LOF_2 : LIMITED OPPORTUNITY FACTOR

Figure 13. WCE Compartment Model

where the α and w constants are as explained before. The LOF_2 is derived in Chapter V but perhaps a word of explanation should accompany the choice of the constant (exogenously adjusted) DE_5. This is similar to other DE_n (n = 1, 2, ...) variables and is derived from existing employment statistics for this economic category (21).

Converting these to difference equation form,

$$\Delta \underline{EU}_4(k) = [A(\alpha_4, w_4, b_{41}, b_{42}, u_{S41})]\underline{EU}_4(k) + [B(\alpha_4) \qquad (3.73)$$

with

$$WCE(k) = [Y_4(k) + E_4(k) + F_4(k) + S_{41}(k)]\alpha_{48} \qquad (3.74)$$

$$\Delta WCE(k) = -WCE(k) \cdot DE_5 + LOF_2(k) \cdot BCE(k) \quad . \qquad (3.75)$$

UEMP Model

The last economic category to be modeled in the demographic sector is the unemployed. This category is considered to be one of the most significant in that it contains most of the people in a poverty status. This might be a temporary situation or the chronic poverty in which so many people are entrenched. This category does not contain the retired even though they may well be in poverty status. The block diagram is shown in Figure 14. The blue collar endogenous and exogenous have access to a common unemployed sector while the white collar endogenous and exogenous have a common unemployed sector. These two sectors of unemployment do not interact. This was done to facilitate better handling of the unemployment of professional and managerial people.

Migration of unemployed individuals into an area is dependent upon different factors than those used for the other economic categories. The

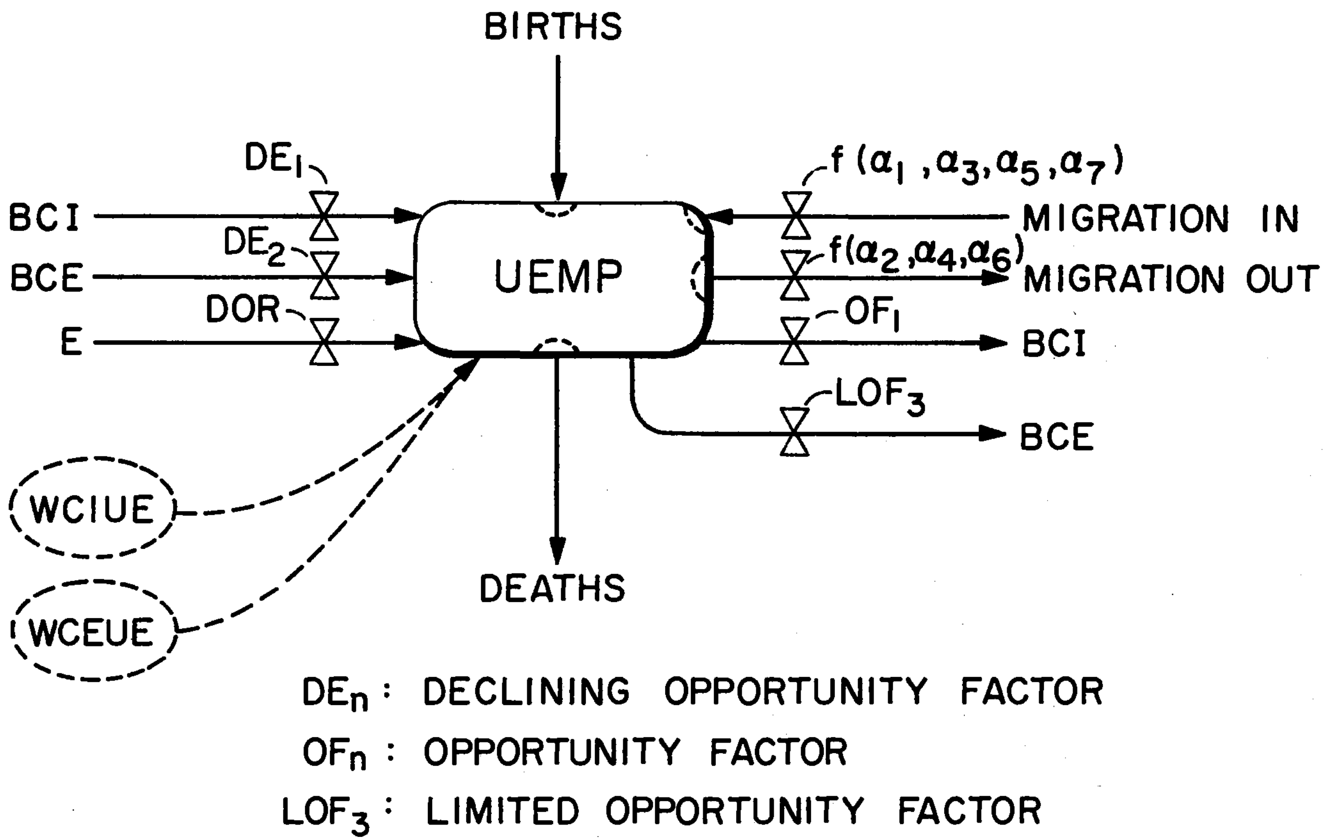


Figure 14. UEMP Compartment Model

factors to be used in this model are:

(1) low-income housing vacancy rate; and

(2) low-income maintenance (employment).

These causal factors compare with the other factors which cause immigration in that they are supportive housing and employment. As an example consider the implications of no income. Without this support, it would be ridiculous to assume a sustained flow of unemployed people into an area. Occasionally one hears of relatives supporting new households because of unemployment, but this cannot exist for a prolonged period of time. The economic burden of taking care of the average family is too much even for the relatively well-off.

The unemployment already present in a community tends to give rise to more unemployment. This is often overstated in the "principle" that poverty breeds poverty (58). To reverse the logic, a well-to-do community completely employed will not attract the unemployed because the latter cannot properly compete for the limited resources keeping in mind that there is a great disparity in the income.

The available low-cost housing plays a crucial role in that there should be some minimal-cost housing which individuals on welfare can acquire. In recent years, this has assumed the form of federal low-cost housing projects. (Whether or not these low-cost housing projects are good for any specific urban area can not be stated as fact at this point in time (58).)

The unemployed individuals in the WCI and WCE unemployment subsector are numerated along with all other unemployment remain with their respective economic groups in the sense that they are in their distinct economic category pool, because of the vast differences in qualifications of those in these pools.

Now by examining Figure 14, the mathematical relationships for the unemployed are derived.

$$\underline{\dot{EU}}_5 = [A(\alpha_5, w_5, b_{51}, b_{52}, u_{S51})]\underline{EU}_5 + B(\alpha_5) \tag{3.76}$$

with

$$UEMP(0) = [Y_5(0)+E_5(0)+F_5(0)+S_{51}(0)]\alpha_{58}+WCIUE(0)+WCEUE(0) \tag{3.77}$$

$$\dot{UEMP} = -UEMP[LOF_3+OF_1]+DE_1 \cdot BCI+DE_2 \cdot BCE+W\dot{CI}UE+WC\dot{E}UE+\frac{1}{2}\sum_{n=1}^{5}\left[\frac{DOR_n}{1-DOR_n}\right]w_{En} \cdot E_n \tag{3.78}$$

where DOR is derived in Chapter V and

$$\sum_{n=1}^{5} \frac{DOR_n}{1-DOR_n} \cdot w_{En} \cdot E_n$$

is the number of individuals not completing the educational process each year. The remaining equations are:

$$WC\dot{I}UE = DE_3 \cdot WCI - UP_1 \cdot WCI \tag{3.79}$$

$$WC\dot{E}UE = DE_5 \cdot WCE - UP_2 \cdot WCE \quad . \tag{3.80}$$

The variable coefficients w_n and α_n (n = 1, 2, ...) are as stated before with differences occurring in death rates and density in household.

Converting the previous equations to difference equation form,

$$\Delta\underline{EU}_5(k) = A(\alpha_5, w_5, b_{51}, b_{52}, u_{S51})\underline{EU}_5(k) + B(\alpha_5) \tag{3.81}$$

with

$$UEMP(k) = [Y_5(k)+E_5(k)+F_5(k)+S_{51}(k)]\alpha_{58}+WCIUE(k)+WCEUE(k) \quad (3.82)$$

$$UEMP(k+1) = UEMP(k)[1-LOF_3+OF_1)]+DE_1\cdot BCI+DE_2\cdot BCE+\Delta WCEUE(k)$$

$$+ \Delta WCIUE(k)+ \frac{1}{2}\sum_{n=1}^{5} \frac{DOR_n(k)}{1-DOR_n(k)} \cdot w_{En}\cdot E_n(k) \quad (3.83)$$

where

$$WCEUE(k+1) = WCEUE(k) + WCE(k)[DE_3 - UP_1] \quad (3.84)$$

$$WCIUE(k+1) = WCIUE(k) + WCI(k)[DE_5 - UP_2] \quad . \quad (3.85)$$

Educational Sector Model

The completion of the demographic sector paves the way for other sectors to be modeled. The next sector consists of the educational process.

The reason for developing the educational sector is to enable the model to show the effects of an efficient or deficient educational process. Many philosophers have thought for some time that with more education of the masses, there would be a drastic reduction in many of the current urban problems (36,39,66,86). Certainly, this is not the complete answer, but by incorporating education into the model, some of its effects can be studied.

The educational sector is embedded within the demographic sector. This is done because the enumeration of school age people must be fairly accurate in order to properly represent the movement into and out of the educational process.

From the demographic sector, we obtain the numbers of children in the educational system. The total number of individuals in the educational system is the sum of each educational variable in each economic category. The educational sector is shown as a system in Figure 15. This diagram is a composite of all educational systems within each economic category.

It is assumed here that if an individual completes the educational process, he will either remain in the community to form a household and join the labor force, or he will migrate from the area. Failure in the educational process routes an individual into the ranks of the unemployed. Forming the differential equations, we have,

$$\dot{E}_1 = -E_1[\alpha_{14} + W_{E1}] + W_{Y1} \cdot Y_1 + \alpha_{13} \qquad (3.86)$$

$$\dot{E}_2 = -E_2[\alpha_{24} + w_{E2}] + w_{Y2} \cdot Y_2 + \alpha_{23} \qquad (3.87)$$

$$\vdots$$

$$\dot{E}_n = -E_n[\alpha_{2n} + w_{En}] + w_{Yn} \cdot Y_n + \alpha_{n3} \qquad (3.88)$$

and for this model n = 1, 2, ..., 5, where w_{E_n} is,

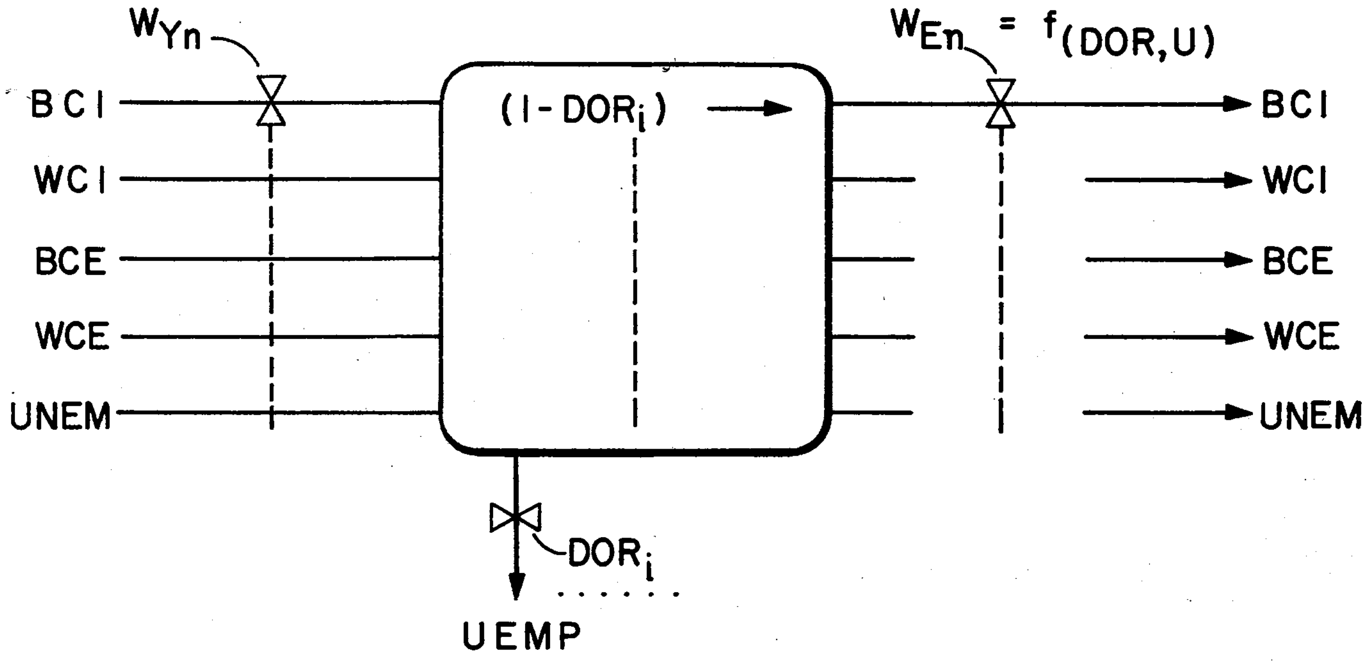
$$w_{E_n} = \frac{1}{4}[1 - u_{E_n}(\tfrac{4}{2})][1 - DOR_n] \qquad (3.89)$$

where

U_{E_n} = death rate;

DOR_n = cumulative drop out rate for four years.

The DOR function is modeled in Chapter V. Again an approximation for the death rate effect has been used to provide the transport mechanisms from the sector. The difference equations in general form are



DOR_i : SIGNIFIES DROPOUT RATE FOR THE ith GROUP

U : DEATH RATES

Figure 15. Educational Sector

$$\Delta \underline{E}_n(k) = [1-[\alpha_{n4}+w_{En}]]\underline{E}_n(k) + \alpha_{n3} \qquad n = 1, \ldots, 5 \qquad (3.90)$$

and the total number of individuals (E) in the educational process is,

$$E(k) = \sum_{n=1}^{5} E_n(k) \quad . \qquad (3.91)$$

This completes the Demographic and Educational sectors leaving the Residential and Industrial sectors to be considered in Chapter IV.

CHAPTER IV

RESIDENTIDAL AND INDUSTRIAL SECTORS OF URBAN MODEL

Residential Sector

Introduction

To present a proper perspective of the distribution of wealth within a community, an appraisal must be made of the housing within the area. To accomplish this requires a demonstration of knowledge of the economic (supply and demand) dynamics which are incorporated in the housing sector.

On the demand side for housing, there exist several important factors (55). They include: (1) number of households; (2) vacancies; (3) mortgage loan interest; and (4) construction costs (55). Certainly the number of households are an indicator of demand. If the population increases, then it is natural that the supply of housing must increase, approximately in a direct proportion.

At this time a point should be raised concerning how the housing projections will take place. Many economists and demographers use the economic base analysis concept in which the projections for housing are conditioned on the employment available (64,91). In this research, housing estimates will be conditioned on demographic projections which rely heavily on available employment. This, however, is only one portion of the complex housing demand question.

The existing vacancies constitute another influence on demand. If there were no vacancies, an immediate shortage in housing would result, creating a demand for construction as well as raising the price of what could be construed to be a scarce resource. If there were a large number of vacancies, then demand for construction would be virtually zero.

The items listed as being influences on demand are only a portion of the total, which include taxes, availability of mortgage financing, etc. In this model, the factors used are those which are believed to have the most relevance with respect to minority housing. As an example, consider what happens if there is an increase in taxes on real estate. The price of housing either directly in the tax or indirectly in rent increases, would automatically increase. Therefore, to show the effects of taxation, it suffices to show the effects of increasing housing prices and for the homeowner, each is an increase in the average expense to live. The interest and availability of mortgage loans also have a decided impact on the kind of housing minority classes can afford. One way to restrict the movement of minorities into an exclusive neighborhood is to restrict the flow of mortgage funds which apply to a particular residential area or type of housing.

The cost of construction which affects the price of housing directly must also be considered. As construction costs increase, the price of housing increases which hinders a family's ability to purchase, if increases are not offset by increases in income. The ability of families to pay for housing is directly linked with the Industrial Sector. This is apparent from the fact that as more exogenous jobs are created, more income flows into the area and the purchasing power of the minority community is increased.

This completes the demand function leaving the supply function to be dealt with. The supply of housing is modeled by a source dependent upon: (1) existing housing in the area; (2) increase in population; and (3) exogenously desired vacancy rate (55). Expressing the actual vacancy rate (AVR) as a function of existing and incremental households and existing housing gives the following:

$$AVR = \frac{[(TH + NH) - \sum_{n=1}^{5} (EU_n + \Delta EU_n)]}{(TH + NH)} \qquad (4.1)$$

where

TH = total housing units;

EU_n = economic unit (n = 1, 2, ..., 5);

NH = new housing structures; and

$\Delta EU_n = E_n(k+1) - E_n(k)$ where k is the time index (in years) in the model.

The model is supplied with an exogenous desired vacancy rate (DVR) which the housing sector attempts to satisfy. DVR is discussed in Appendix A. When the urban model is operating, the housing sector compares the DVR and AVR and if DVR<AVR new construction is zero. If the DVR>AVR, new construction occurs.

Expressing NH as a function of DVR and AVR,

$$NH = |DVR - AVR| TH \cdot f\left(\frac{EU_n}{\sum EU_n}\right) \cdot FR_\ell \qquad \ell = 1, 2 \qquad (4.2)$$

where new construction satisfies a fraction of the |DVR - AVR| leaving DVR>AVR. This is done to satisfy the assumption that the construction industry has imperfect information about the housing market, i.e., the number of houses to construct, and does not satisfy AVR = DVR. The $f(EU_n/\sum EU_n)$ is chosen to favor building for the economic unit which most supports a particular type of housing. Therefore the dependent source NH in Figure 16 consists of $TH \cdot f(EU_n/\sum EU_n)$ where the valve control is $|DVR - AVR|$.

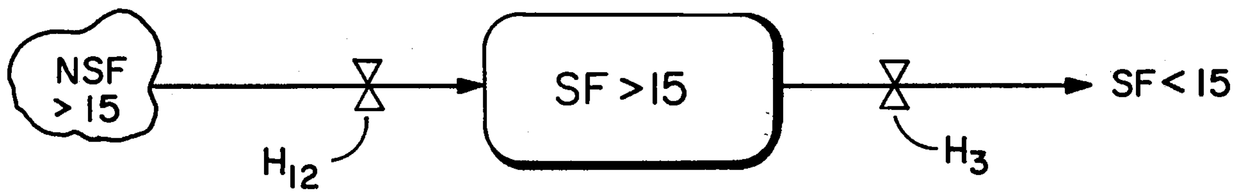
The constant FR_ℓ is the fraction of the total economic units which own or rent their home where ℓ equals one for homeowners and two for renters.

Using the previously described supply and demand functions, the residential sector is formed around these principles. The supply and demand functions vary through time partly because of lags in construction and expected vacancies.

In the residential sector, four types of housing are categorized. They are: (1) single family units > $15,000 [SF>15]; (2) single family units < $15,000 [SF<15]; (3) rental units [RU]; and (4) slums [SH].

The first three are self-explanatory but the last item requires considerable discussion. In this model, slums comprise that area where there is a high concentration of low-income households, unemployment, and excessive age of structures. The slum neighborhood is not measured as a specific area, but is comprised of a certain economic strata (68,89).

In this model, the Trickle-Down Theory will be used (34,64,88). This theory means that premium housing eventually decays and after a period of decades becomes lower priced housing. In this manner housing is provided for the economically disadvantaged or lower income groups, and in some cases, this is the only manner in which their housing is derived.



H_{12} : |DVR - AVR|

H_3 : TRANSPORT MECHANISM = f (PHYSICAL LIFE EXPECTANCY, EDUCATIONAL LEVEL

Figure 16. SF>15 Compartment Model

SF>15 Model

The block diagram for SF>15 is shown in Figure 16. The coefficient H_{12} is arranged such that $H_{12} = |DVR - AVR|$ if DVR>AVR and zero elsewhere.

The source (NH) which was derived previously is a function of existing housing, and minority population. The control value for this source is a function of AVR and DVR. The state variable equation from Figure 16 is,

$$[\dot{SF>15}] = -H_3[SF>15] + H_{12} \cdot NH_1 \tag{4.3}$$

$$NH_1 = \left[\frac{\sum_{n=1}^{4} EU_n}{\sum_{n=1}^{5} EU_n} \right] TH \cdot FR_1 \tag{4.4}$$

where $FR_1 = .573$ (20) for the Oklahoma City SMSA and equals fraction of households who own their houses. For the derivation of H_3, see Appendix A.

H_3 = f (physical life expectancy of structure, educational ratio of occupying household)

$$= \frac{1}{40\left(\frac{E}{E_0}\right)^{x_1}} \quad .$$

Conversion of the differential equation into difference form produces,

$$[SF>15](k+1) = [SF>15](k)(1 - H_3) + (H_{12})NH_1(k) \quad . \tag{4.5}$$

SF<15 Model

The next category of housing is similar to the higher cost housing, the major difference being that the supply is partially determined by the decline in stock of the SF>15 category which was just discussed. A block diagram of this new category which consists of single family housing less than $15,000 (SF<15) is shown in Figure 17.

All inputs to this category are similar with the exception of replacement housing. Replacement units are dependent upon urban renewal and sociological trends of a given urban area. As an example, in an urban area which is less than 50 years old, the tendency to renovate housing is generally less than a city which is 200 years old, simply because the land area available for construction has dwindled tremendously in the latter. The parameter H_4 equals $|DVR - AVR|$ for DVR>AVR, and zero otherwise.

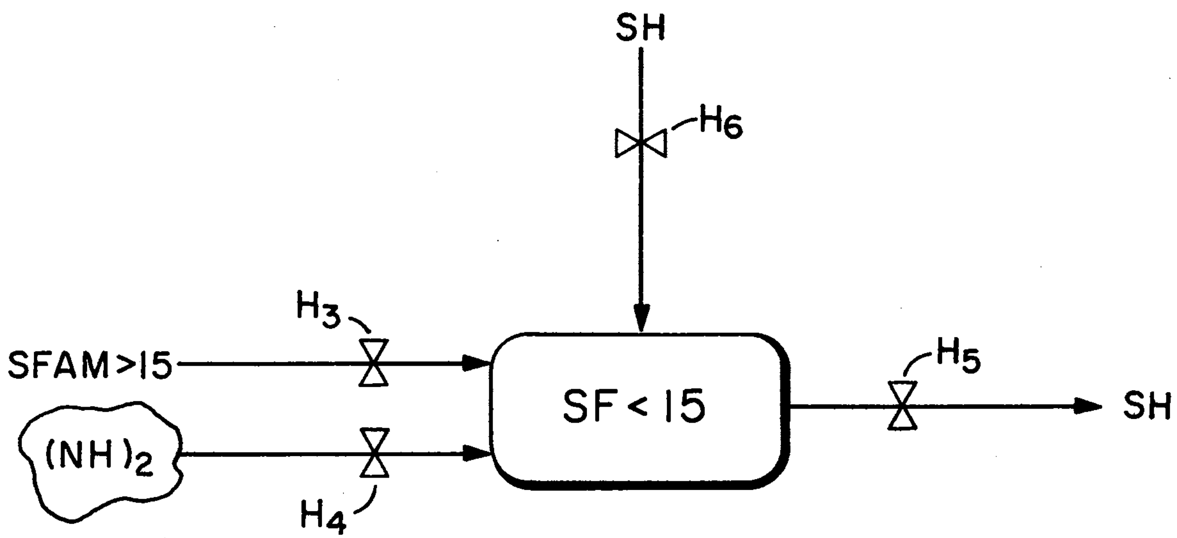
H_5 (see Appendix A for derivation) is a function of physical life expectancy of the structures coupled with the educational level of the occupants. H_5 has a shorter age factor (30 years) before the filtering effect transports the structures to slum housing.

The finite dependent source NH_2 is dependent upon existing economic units and housing is the following expression:

$$NH_2 = \frac{\sum\limits_{n=1,5} EU_n}{\sum\limits_{n=1}^{5} EU_n} \; TH \cdot FR_1 \quad . \tag{4.6}$$

From Appendix A, $H_6 = 0$ until social programs dictate a change.

$$H_5 = \frac{1}{[30(\frac{E}{E_0})^{x_2}]} \tag{4.7}$$



H_3: DEFINED PREVIOUSLY

H_4: $|DVR-AVR|$

H_5: TRANSPORT MECHANISM = f (PHYSICAL LIFE EXPECTANCY, EDUCATIONAL LEVEL)

Figure 17. SF<15 Compartment Model

from Appendix A. Writing the equations for Figure 17,

$$[\dot{SF<15}] = -[SF<15] \cdot H_5 + H_3 \cdot [SF>15] + H_4 \cdot NH_2 + H_6 \cdot SH \ . \quad (4.8)$$

In difference form, the results are

$$[SF<15](k+1) = [SF<15](k)(1-H_5)+H_3 \cdot [SF>15](k)+H_4 \cdot NH_2(k)+H_6 \cdot SH(k) \ . \quad (4.9)$$

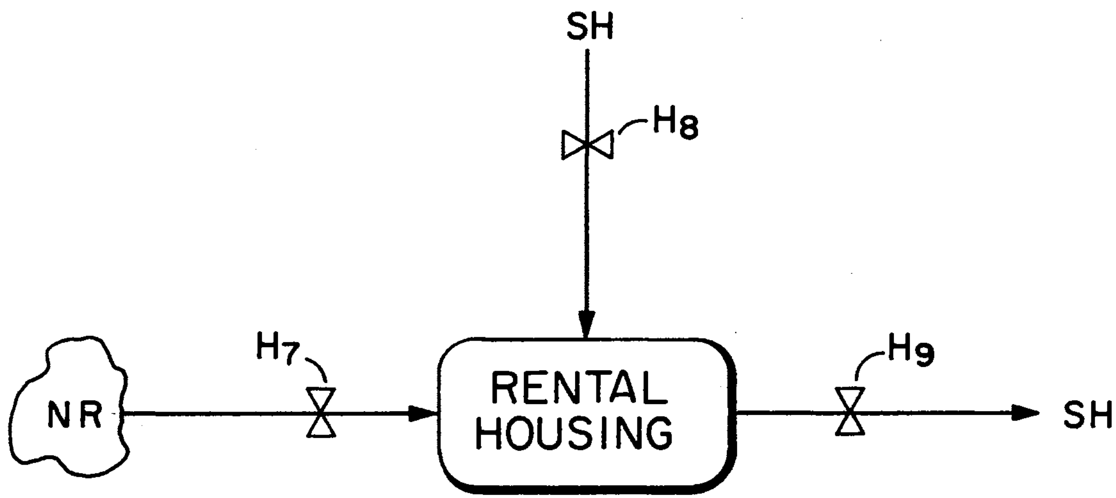
Rental Model

This category of housing, for the most part, contains apartment or project dwellings, normally multifamily units. The importance of their inclusion into the model is evident when you take into consideration that the census (20,49) for the year 1970, shows 8,148 owner occupied units and 6,632 renter occupied units in the Oklahoma City SMSA, which means that 42.7% of the minority rent their housing.

In some respects, the wealth or lack of it of an ethnic group may be determined by the number of rental units it occupies. This is a measure of capital retention in a spatial area in that wealth normally flows entirely from the area in the case of rentals, and is partially retained in owner-occupied housing.

The variables which affect this category of housing are similar to owner-occupied units, but differ in that the independent investors examine the demand equation in greater detail in order to assure a reasonable rate of return on his investment. In spite of this caution, rental vacancy rates are frequently ten times that of owner occupied categories.

A block diagram is shown in Figure 18. New construction is dependent upon demand which is a function of: (1) rental vacancy rates; (2) increase in construction costs; and (3) increase in minority households in lower economic categories which are not qualified economically for



H_7 : |DVR-AVR|

H_8 : URBAN RENEWAL CONTROL

H_9 : TRANSPORT MECHANISM = f (PHYSICAL LIFE EXPECTANCY, EDUCATIONAL LEVEL)

Figure 18. Rental Compartment Model

home ownership.

One of the phenomena which must be dealt with when modeling the housing sector is the choice that the individual makes to rent or buy when he moves into the urban sector. In this model percentages, based on initial conditions, determine the division between owner-occupied housing and rentals, with the change in this division dependent upon increasing the higher economic strata in the community. In this model, this is an exogenous parameter, and will be changed in future research. As the fraction of higher paid individuals is increased, more income will flow into the area, more housing capital will be retained, and as a consequence there will be more capital growth in the minority community.

In Figure 18, (NR) new rental units is shown as a finite dependent source. This source is dependent upon existing rental housing and the fraction of households which support this category. The source NR is expressed as,

$$NR = \left[\frac{\sum_{n=1}^{4} EU_n}{\sum_{n=1}^{5} EU_n} \right] TH \cdot FR_2 \tag{4.10}$$

where FR_2 = .427 for the Oklahoma City SMSA and equals the fraction of households in rental units (20,49).

The equations describing the rental units are derived by examining Figure 18. They are

$$\dot{RU} = -RU \cdot H_9 + H_7 \cdot NR + H_8 \cdot SH \tag{4.11}$$

where

H_7 = $|DVR - AVR|$ if DVR>AVR and zero elsewhere;

H_8 = urban renewal; this is derived in Appendix A and equals (1/60); and

H_9 = transport mechanism derived in Appendix A and equals $1/80[(E/E_0)^{x_2}]$.

Presented in difference form,

$$RU(k+1) = -H_9 \cdot RU(k) + H_7 \cdot NR(k) + H_8 \cdot SH(k) \quad . \qquad (4.12)$$

Slum Model

Housing eventually declines from its initial standard to that of slum housing. This category of housing, by its presence is clearly the most important indicator of the lack of economic and social well being. Slums are formed by deterioration of physical structures and economic depravation working in conjunction with social restriction. The block diagram of this housing category is shown in Figure 19.

The equation describing this model is,

$$\dot{SH} = - SH(H_6 + H_8 + H_{10}) + H_5 \cdot [SF<15] + H_9 \cdot RU \qquad (4.13)$$

where

H_6, H_8, H_{10} = urban renewal parameters (derived in Appendix A);

H_6 = 0 in the absence of urban renewal;

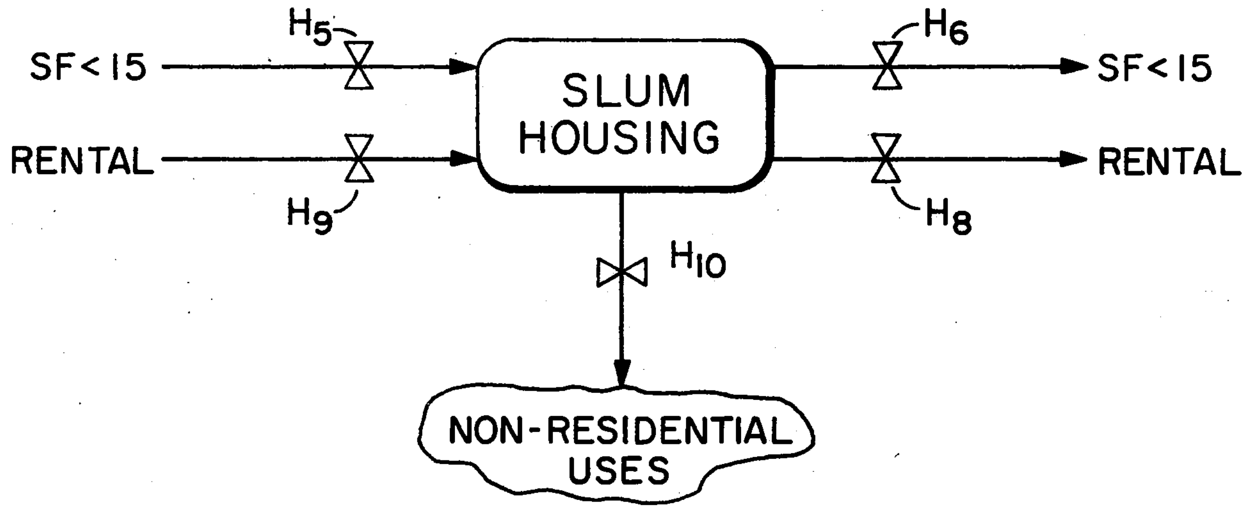
H_8 = (1/60) as derived in Appendix A;

H_{10} = (1/60) as derived in Appendix A; and

H_5 and H_9 have been explained in connection with other housing.

Converting this to difference form we have,

$$SH(k+1) = SH(k)[1-(H_6+H_8+H_{10})] + H_5 \cdot [SF<15](k) + H_9 \cdot RU(k) \quad . (4.14)$$



H_6 : URBAN RENEWAL CONVERTED TO SF<15

H_8 : URBAN RENEWAL CONVERTED TO RENTAL UNITS

H_{10}: URBAN RENEWAL CONVERTED TO NON-RESIDENTIAL USES

Figure 19. Slum Compartment Model

Industrial Sector

PEWIE and PIWI Compartment Model

Of utmost importance is the business or industrial sector in any economy. It is the means by which all economic units derive their sustenance. The availability of jobs is one of the dominant influences which causes migration into or out of any urban complex.

Within the industrial sector, there will be two types of firms or industries. The industries are characterized here by the financial flow of their profits. They are either profits exogenous (PE) or profits endogenous (PI). The PE firms pays wages both exogenous and endogenous, but is characterized by exogenous ownership. The profits exogenous firm provides less income retention within the minority economic sphere. Some geographers and economists have expressed a belief that the way to raise the standard of living within an underdeveloped country is by the retention of real income, through the creation of demand for the products and services produced by that country (1,26,68). This analogy can be applied to the minority group and the assumption was made that its economic development is similar to that of an underdeveloped country.

The firm with profits endogenous is a minority owned firm. The creative and sustaining economic and social forces for this type of firm are intricate, as characterized by its high failure rate, lack of capital, small numbers and limited access to the economic marketplace (17, 38,65,68,82). The author believes that the future of economic growth within the minority community will be largely dependent on the success or failure of these firms.

Within the industrial sector there are several phenomena which have to be modeled. They are: (1) new minority firms due to mature firms reinvesting; (2) new minority firms due to local entrepreneureship and government intervention; and (3) high failure rates.

For this model the two kinds of firms treated as state variables are PEWIE (profits exogenous, wages endogenous and exogenous) and PIWI (profits endogenous and wages endogenous). The employees which they support were established in Chapter III. In this research effort, the number of PEWIE firms is assumed constant while the number of PIWI firms is varied. The PEWIE firms are constant in this research effort, but if one wishes to study the effects of growth of exogenous firms, the ratio PEWIE(k+1)/PEWIE(k) is inserted in the migration in equation in the demographic sector for economic categories BCE and WCE. This would alter the calibration constant αm_n in the net migration equation. In the Oklahoma City Area, there are estimated 146 PIWI firms (64). This amounts to approximately 3% of the total industry there.

In accord with Figure 20, the equation which describes the PEWIE firm is developed. It is,

$$\dot{PEWIE} = -FA_1 \cdot PEWIE + OR_1 \cdot S \tag{4.15}$$

where

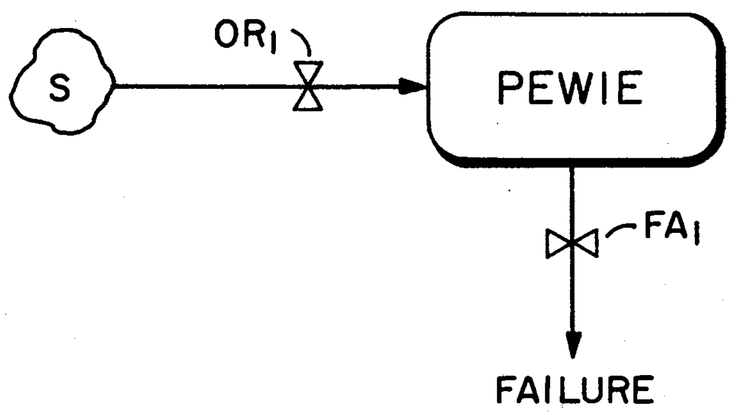
$OR_1 \cdot S$ = rate of increase due to new industrial firms; and

FA_1 = average failure rate.

In difference form,

$$PEWIE(k+1) = PEWIE(k)[1 - FA_1] + OR_1 \cdot S(k) \tag{4.16}$$

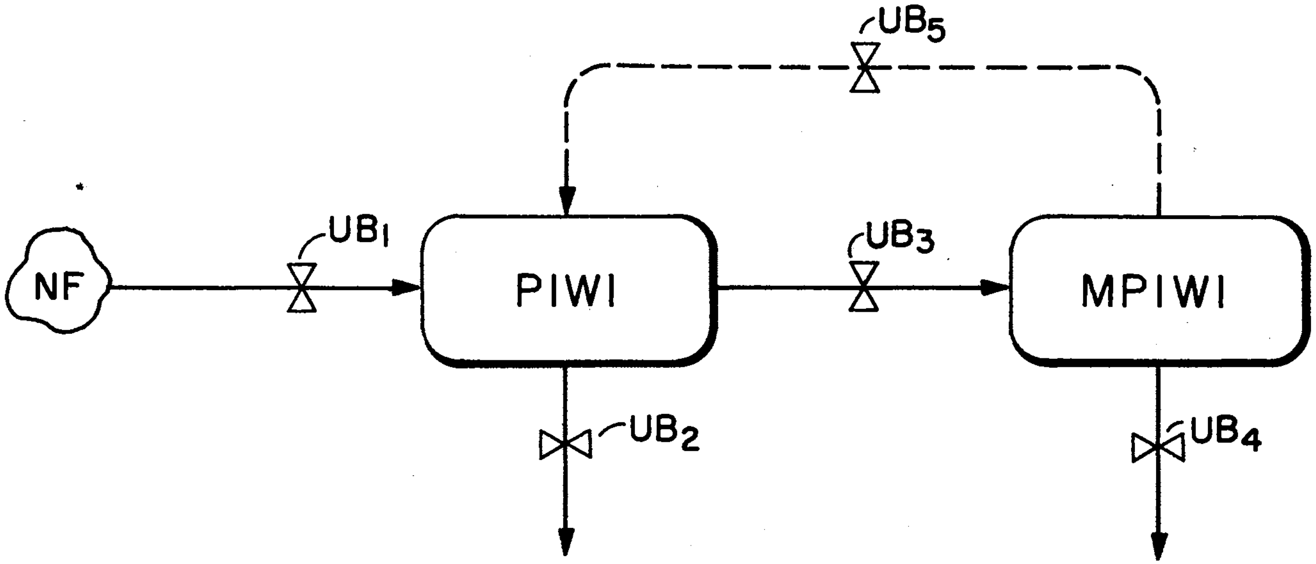
The block diagram for PIWI type firms is shown in Figure 21. There are two states shown on the diagram. The first is new minority business



$OR_I \cdot S$: NEW FIRMS/YEAR

FA_I : FAILURE RATE

Figure 20. PEWIE Compartment Model



UB_1 : NORMALLY 1

UB_2 : FAILURE RATE FOR STARTING MINORITY BUSINESSES

UB_3 : AGE FACTOR FOR BECOMING MATURE BUSINESS

UB_4 : FAILURE RATE FOR MATURE MINORITY BUSINESS

UB_5 : BUSINESS REINVESTMENT PARAMETER

NF : f(ΔEU, EXISTING FIRMS)

Figure 21. PIWI Compartment Model

and the second is mature minority business, defined as that business which has existed for five or more years.

By examining the block diagram, the equations are:

$$\dot{PIWI} = -PIWI(UB_2 + UB_3) + UB_1 \cdot NF + UB_5 \cdot MPIWI \qquad (4.17)$$

$$\dot{MPIWI} = -MPIWI \cdot UB_4 + UB_3 \cdot PIWI \qquad (4.18)$$

where

$$UB_1 = \Delta \sum_{n=1}^{4} EU_n$$

$UB_1 \cdot NF$ = new firms generated as a function of existing firms and minority population,

where

$$NF = \frac{PIWI(0) + MPIWI(0)}{\sum_{n=1}^{4} EU_n(0)}$$

where

$PIWI(0) + MPIWI(0) = 146$ as stated previously;

$UB_2 = .35$ (estimated failure of endogenous firms;

$UB_3 = 1/5\ [1-(UB_2/2)5]$;

$UB_4 = .01$ (assumed failure rate of mature endogenous firms; and

$UB_5 = .087$ (assumed and is experimental value of new firms due to reinvestment of mature firms.

The parameters UB_2, UB_4, and UB_5 are estimated values because there was no available data base which dealt with these parameters.

This completes the discussion of the urban model and paves the way for developing causal model in Chapter V.

CHAPTER V

PROBABILISTIC CAUSAL MODELS

Introduction

Causal models are used to model social conditions and present a systematic way out of the impasse reached when trying to bridge social research theory and technique. These models will be based on novel concepts of probability and structures are presented for these models based on sensitivities of the modeled variable. Throughout this chapter, the concept of probability is used as a mathematical basis. It is therefore necessary that the theory be reviewed.

Probability Theory

The axiomatic definition of probability will be used (56). To understand the axiomatic definition several concepts will be needed. For a given experiment we denote by η the certain event. This is the event that occurs in every trial. Given two events α and β, then $\alpha + \beta$ denotes the event that occurs if α or β or both occur. α and β are mutually exclusive if the outcome of one from a given trial excludes the occurrence of the other. In axiomatic probability theory the probability of an event α is a number $P(\alpha)$ and $P(\alpha)$ obeys the follwing postulates:

$$P(\alpha) \geq 0 \tag{5.1}$$

$$P(\eta) = 1 \tag{5.2}$$

If α and β are mutually exclusive, then

$$P(\alpha + \beta) = P(\alpha) + P(\beta) \quad . \tag{5.3}$$

The theory of probability deals with the results of an experiment and with events defined in terms of these results. In some applications, complex experiments are formed from two or more experiments. To analyze these combined experiments, the Cartesian product space is used.

This space is developed as follows: given two sets S_1 and S_2 consisting of elements s_1 and s_2,

$$s_1 \in S_1$$

and

$$s_2 \in S_2 \quad ,$$

an ordered pair (s_1,s_2) can be formed as a new single element. If all such pairs were formed from the sets S_1 and S_2 then a new set S_3 would be formed. This set $S_3 = S_1 \times S_2$ is the Cartesian product of S_1 and S_2 and defines a Cartesian product space.

Now given two experiments F_1 and F_2 with outcomes s_1 and s_2 from sets S_1 and S_2, we denote the probabilities of events from these two experiments by $P_1(s_1)$ and $P_2(s_2)$, respectively. The occurrence of both experiments provide a new experiment, F_3, whose outcomes are all pairs of objects (s_1,s_2).

By forming the space $S_3 = S_1 \times S_2$, the combined experiment is written in the form of $F_1 \times F_2 = F_3$. In the experiment F_3, events are all subsets S_3 of the form $\alpha \times \beta$ where α and β are events defined on the experiments F_1 and F_2. To complete the specification of the experiment

F_3, the probabilities of its various events are defined as:

$$P(\alpha \times \beta) = P(\alpha)P(\beta|\alpha) \tag{5.4}$$

where

$P(\beta|\alpha)$ is the probability of the conditional event β, given α

$$P(\alpha \times S_2) = P_1(\alpha) \tag{5.5}$$

$$P(S_1 \times \beta) = P_1(\beta) \quad . \tag{5.6}$$

If the experiments F_1 and F_2 are independent then,

$$P(\alpha \times \beta) = P(\alpha)P(\beta) \quad . \tag{5.7}$$

It should be noted that given multiple independent events $\alpha_1, \alpha_2, \ldots, \alpha_n$ then

$$P_{1,\ldots,n}(\alpha_1,\alpha_2,\ldots,\alpha_n) = \prod_{i=1}^{n} P_i(\alpha_i) \quad . \tag{5.8}$$

This introduces the concept of independence and the concept of axiomatic probability, and forms the basis for most of the ensuing discussion. For more discussion of the concepts of probability, the reader is directed, for example, to Papoulis (56).

Model Structure

The modeling of critical socio-economic variables (SEV) is undertaken with concepts founded in probability and engineering. There are three structures for models introduced and they are similar to switching networks in their logic.

Series Model

The series model is shown in Figure 22. P_s stands for probability of success and P_f for probability of failure. From the axioms,

$$P_s + P_f = 1 \tag{5.9}$$

$$P_s \text{ and } P_f \geq 0 \tag{5.10}$$

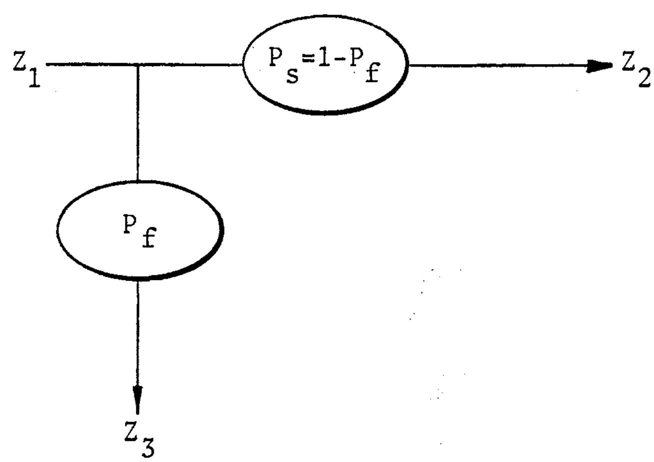


Figure 22. Series Causal Model

The flow through of the model is perfect if $P_s = 1$ and zero for $P_s = 0$. A transfer function can be derived for the model in the following manner,

$$Z_2 = (P_s)Z_1 = (1 - P_f)Z_1 \tag{5.11}$$

which produces

$$\frac{Z_2}{Z_1} = P_s = 1 - P_f \tag{5.12}$$

and

$$\frac{Z_3}{Z_1} = P_f \quad . \tag{5.13}$$

Each component of the SEV causal model is modeled to define a channel of possible flow. Intuitively, if a parameter of the SEV <u>must</u> be satisfied for flow through the system to occur, then it should be in series. If a parameter of a SEV is an alternative means of success, it should be modeled in parallel.

A multivariable series model is shown in Figure 23.

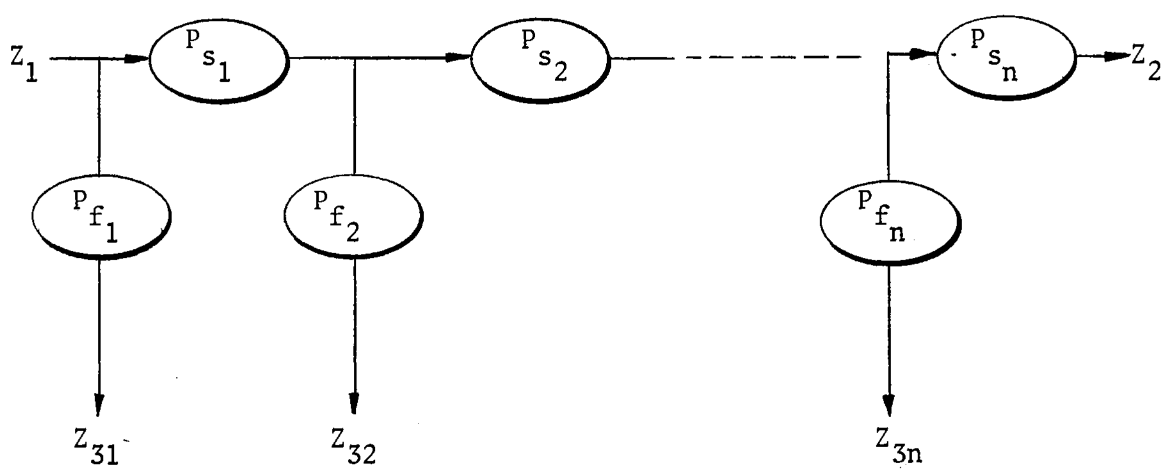


Figure 23. Multi-Parameter Series Causal Model

The transfer function for this probabilistic model is,

$$P(s) = \frac{Z_2}{Z_1} = \prod_{i=1}^{n} P_{s_i} \quad . \tag{5.14}$$

Now P_{s_i} is a function of only one x_i (where x_i is the urban area parameter being modeled probabilistically), and represents a variable which varies between 0 and 1 in a given urban system. Therefore,

$$P_{s_i} = f(x_i) \qquad i = 1, \ldots, n \quad . \tag{5.15}$$

Taking the $\partial/\partial x_j$, and since $\partial P_{s_i}/\partial x_j = 0$ for $i \neq j$,

$$\frac{\partial P(s)}{\partial x_j} = \frac{\partial P(s)}{\partial P_{s_j}} \cdot \frac{\partial P_{s_j}}{\partial x_j} \tag{5.16}$$

$$= \frac{\prod_{i=1}^{n} P_{s_i}}{P_{s_j}} \cdot \frac{\partial P_{s_j}}{\partial x_j} \quad . \tag{5.17}$$

Equation (5.17) may be expressed as,

$$\frac{\partial P(s)}{\partial x_j} = \frac{P(s)}{P_{s_j}} \cdot \frac{\partial P_{s_j}}{\partial x_j} \quad . \tag{5.18}$$

Using the properties of differentials when all but one variable is held constant (42), the equation above may be written as

$$\frac{\partial P(s)}{P(s)} = \frac{\partial P_{s_j}}{P_{s_j}} \quad . \tag{5.19}$$

The fractional change in Equation (5.19) may be used to define sensitivity. The definition of sensitivity $S_{P(s)}^{P_{s_j}}$ for all models is (81),

$$S_{P(s)}^{P_{s_j}} \triangleq \frac{\partial P(s)}{P(s)} \cdot \frac{P_{s_j}}{\partial P_{s_j}} \qquad (5.20)$$

and is a measure of sensitivity of P(s) with respect to P_{s_i}. Using Equation (5.20), the sensitivity for the series model is,

$$S_{P(s)}^{P_{s_j}} = \frac{\partial P(s)}{P(s)} \cdot \frac{P_{s_j}}{\partial P_{s_j}} = 1 \quad . \qquad (5.21)$$

For a further discussion of the concepts of sensitivity, the reader is referred to Su (81). This equation says that a percentage change in P_{s_j} yields an equal percentage change in P(s). This is moderately sensitive as we will see with the analysis of other model structures.

Parallel Model

A parallel model is shown in Figure 24.

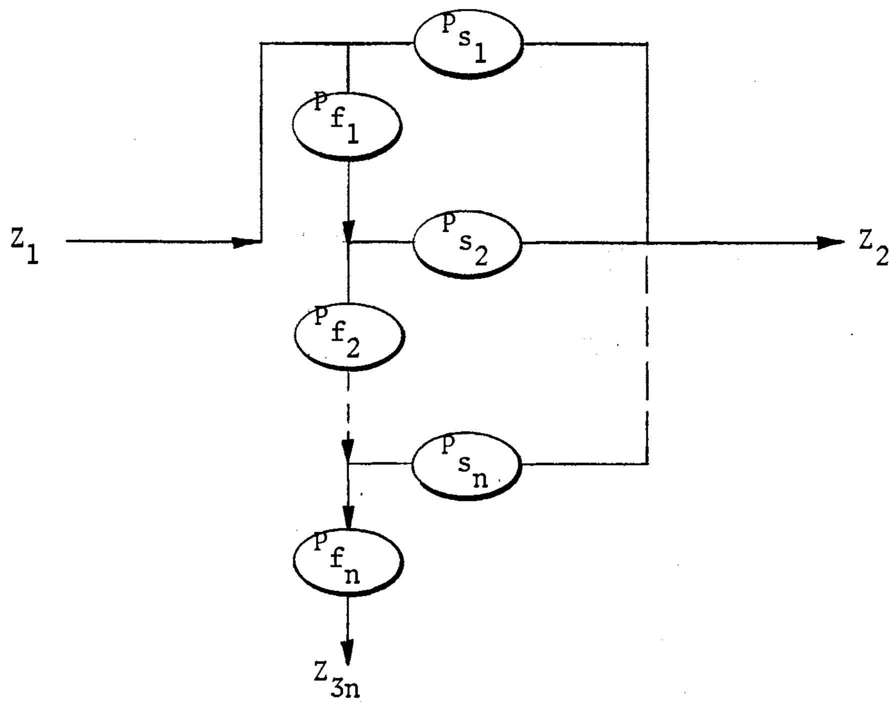


Figure 24. Parallel Causal Model

The transfer function for the parallel probabilistic model is,

$$P(s) = 1 - \prod_{i=1}^{n} (1 - P_{s_i}) \quad . \tag{5.22}$$

In order to examine sensitivities of the model structure, partial differentiation is performed and produces the following: let

$$P_{s_i} = f(x_i) \qquad i = 1, 2, \ldots, n \tag{5.23}$$

i.e., there is a one to one correspondence between P_{s_i} and x_i implied by $f(x_i)$. Then taking the $\partial/\partial x_j$, and since $\partial P_{s_i}/\partial x_j = 0$, $i \neq j$,

$$\frac{\partial P(s)}{\partial x_j} = \frac{\partial P(s)}{\partial P_{s_j}} \cdot \frac{\partial P_{s_j}}{\partial x_j} \tag{5.24}$$

$$\frac{\partial P(s)}{\partial x_j} = \frac{\prod_{i=1}^{n} (1 - P_{s_i})}{1 - P_{s_j}} \cdot \frac{\partial P_{s_j}}{\partial x_j} \quad . \tag{5.25}$$

Combining Equations (5.22) and (5.25),

$$\frac{\partial P(s)}{\partial x_j} = \frac{1 - P(s)}{1 - P_{s_j}} \cdot \frac{\partial P_{s_j}}{\partial x_j} \quad . \tag{5.26}$$

Using the properties of differentials,

$$\frac{\partial P(s)}{1 - P(s)} = \frac{\partial P_{s_j}}{1 - P_{s_j}} \tag{5.27}$$

which can be written as,

$$\frac{\partial P(s)}{P(s)} \left[\frac{P(s)}{1 - P(s)} \right] = \frac{\partial P_{s_j}}{P_{s_j}} \left[\frac{P_{s_j}}{1 - P_{s_j}} \right] \tag{5.28}$$

and by using Equation (5.20),

$$S_{P(s)}^{P_{s_j}} = \frac{\partial P(s)}{P(s)} \cdot \frac{P_{s_j}}{\partial P_{s_j}} \tag{5.29}$$

$$S_{P(s)}^{P_{s_j}} = \left[\frac{P_{s_j}}{1 - P_{s_j}} \right] \left[\frac{1 - P(s)}{P(s)} \right] \tag{5.30}$$

$$= \frac{\left[\frac{1}{P(s)} - 1 \right]}{\left[\frac{1}{P_{s_j}} - 1 \right]} \tag{5.31}$$

where

$$S_{P(s)}^{P_{s_j}} \leq 1$$

for

$$P(s) \geq P_{s_j} \qquad ,$$

which is always the case.

Series-Parallel Structure

A series-parallel probabilistic model is shown in Figure 25.

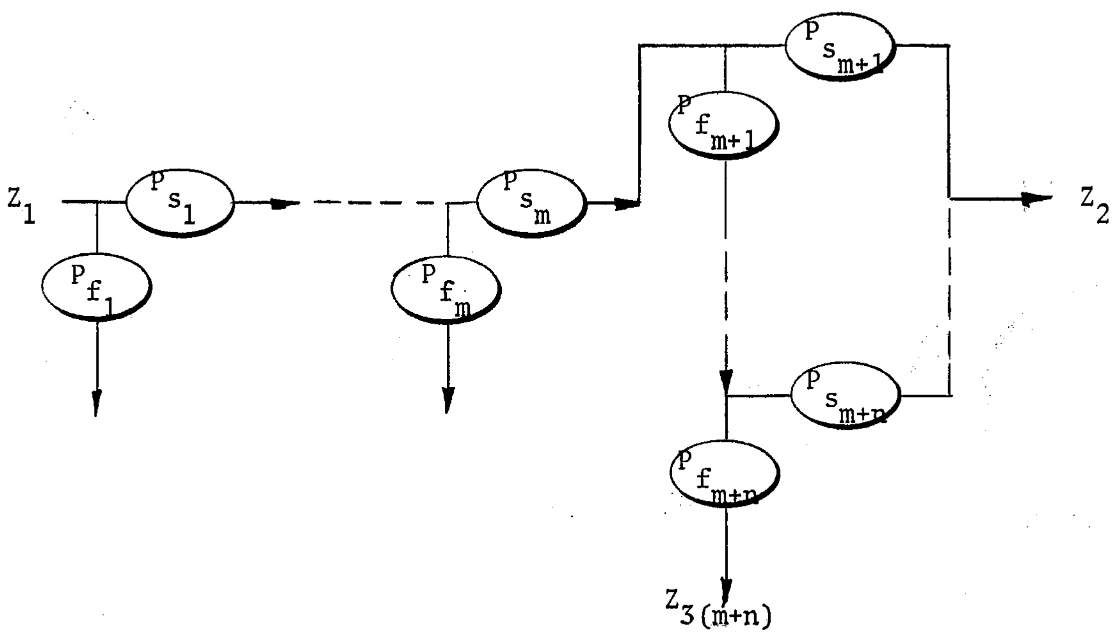


Figure 25. Series-Parallel Causal Model

Writing the transfer function for this structure gives,

$$P(s) = \prod_{i=1}^{m} P_{s_i} \left[1 - \prod_{j=m+1}^{m+n} (1 - P_{s_j}) \right] \quad . \tag{5.32}$$

There are two sensitivities to be examined. The first ($S_{p(s)}^{P_{s_k}}$) equals the sensitivity encountered when the variable is a series member of the model. The second ($S_{p(s)}^{P_{s_k}}$) equals the sensitivity encountered when the variable is a member of the parallel structure of the model.

Case I Series Member.

$$\frac{\partial P(s)}{\partial x_k} = \frac{\partial P(s)}{\partial P_{s_k}} \cdot \frac{\partial P_{s_k}}{\partial x_k} \quad \text{where} \quad \frac{\partial P_{s_i}}{\partial x_k} = 0 \qquad i \neq k \tag{5.33}$$

$$= \frac{\prod_{i=1}^{m} P_{s_i} \left[1 - \prod_{j=m+1}^{m+n} (1 - P_{s_j}) \right]}{P_{s_k}} \, \frac{\partial P_{s_k}}{\partial x_k} \quad . \tag{5.34}$$

Now

$$\frac{\partial P(s)}{\partial x_k} = \frac{P(s)}{P_{s_k}} \cdot \frac{\partial P_{s_k}}{\partial x_k} \quad . \tag{5.35}$$

Again using the concept of differentials, the sensitivity, from Equation (5.20) is,

$$S_{P(s)}^{P_{s_k}} = \frac{\partial P(s)}{P(s)} \cdot \frac{P_{s_k}}{\partial P_{s_k}} = 1 \tag{5.36}$$

which says that percentage changes in P(s) are equal to percentage changes in P_{s_k}.

Case II Parallel Member. Performing partial differentiation,

$$\frac{\partial P(s)}{\partial x_k} = \frac{\partial P(s)}{\partial P_{s_k}} \cdot \frac{\partial P_{s_k}}{\partial x_k} \quad \text{where} \quad \frac{\partial P_{s_j}}{\partial x_k} = 0 \qquad j \neq k \quad . \tag{5.37}$$

This produces

$$\frac{\partial P(s)}{\partial x_k} = \left[\frac{\prod_{i=1}^{m} P_{s_i} \prod_{j=m+1}^{m+n} (1 - P_{s_j})}{1 - P_{s_k}} \right] \frac{\partial P_{s_k}}{\partial x_k} \quad . \tag{5.38}$$

Rewriting this equation,

$$\frac{\partial P(s)}{\partial x_k} = \left[\frac{\prod_{i=1}^{m} P_{s_i} - P(s)}{1 - P_{s_k}} \right] \frac{\partial P_{s_k}}{\partial x_k} \quad , \tag{5.39}$$

where

$$\frac{\partial P(s)}{\prod_{i=1}^{m} P_{s_i} - P(s)} = \frac{\partial P_{s_k}}{P_{s_k}} \left[\frac{P_{s_k}}{1 - P_{s_k}} \right] \quad . \tag{5.40}$$

This equals

$$\frac{\partial P(s)}{P(s) \left[\frac{\prod_{i=1}^{m} P_{s_i}}{P(s)} - 1 \right]} = \frac{\partial P_{s_k}}{P_{s_k}} \left[\frac{P_{s_k}}{1 - P_{s_k}} \right] \quad . \tag{5.41}$$

Now the sensitivity may be expressed as,

$$S_{P(s)}^{P_{s_k}} = \frac{\partial P(s)}{P(s)} \cdot \frac{P_{s_k}}{\partial P_{s_k}} = \left[\frac{P_{s_k}}{1 - P_{s_k}} \right] \left[\frac{\prod_{i=1}^{m} P_{s_i}}{P(s)} - 1 \right] \qquad (5.42)$$

$$= \frac{\left[\frac{\prod_{i=1}^{m} P_{s_i}}{P(s)} - 1 \right]}{\left[\frac{1}{P_{s_k}} - 1 \right]} \quad , \qquad (5.43)$$

where

$$S_{P(s)}^{P_{s_k}} \leq 1 \qquad (5.44)$$

because

$$P(s) \geq P_{s_k} \left[\prod_{i=1}^{m} P_{s_i} \right] \quad .$$

Comparison of Probabilistic Model Sensitivities

There is a wide variation in sensitivity in the structures previously discussed. A comparison is made at this point and provides a guide for the intuitive SEV modeling which follows.

The sensitivity for the series model, from Equation (5.21),

$$S_{P(s)}^{P_{s_j}} = 1 \quad . \qquad (5.45)$$

The parallel model from Equation (5.31),

$$S_{P(s)}^{P_{s_j}} = \frac{\left[\frac{1}{P(s)} - 1\right]}{\left[\frac{1}{P_{s_j}} - 1\right]} \leq 1 \quad . \tag{5.46}$$

For the series-parallel model, series arm

$$S_{P(s)}^{P_{s_k}} = 1 \tag{5.47}$$

and for the series-parallel model, parallel arm

$$S_{P(s)}^{P_{s_k}} = \frac{\left[\frac{\prod_{i=1}^{m} P_{s_i}}{P(s)} - 1\right]}{\left[\frac{1}{P_{s_j}} - 1\right]} \leq 1 \quad . \tag{5.48}$$

Between the parallel and series model, the series model is the more sensitive. A variable modeled in the series, arm of a series-parallel model has more impact than a variable in the parallel structure.

Using the evaluations of sensitivity along with intuitive logic, and the concepts of axiomatic probability, the SEV variables DOR and LOF are now modeled.

LOF Model

In the development of the urban model, a variable was introduced as the LOF (limited opportunities factor). This variable is used to show the effect of obstacles faced by minorities, whether they be social or economic. The LOF variable modifies realtive economic mobility for the control of economic unit transfers from endogenous to exogenous jobs and between exogenous jobs. The LOF variable is used to represent several phenomena which are intrinsic to the problems of the minority. They include: (1) average education level; and (2) discrimination.

The education attained by an individual is self explanatory, but discrimination needs to be explicitly defined.

Discrimination is assumed to be a function of the following factors because they show a high correlation with the degree and amount of discrimination (10,25,59,87): (1) the percent educated (beyond high school) in the majority population (%ED) (10); (2) the integration index (II) which is defined as one minus the segregation index as derived by Taeuber (84); (3) the percent ultraconservative vote (%CV) in the total population (11); and (4) the ratio of minority to total urban population (MR).

The first item, percent educated in the majority population is addressed to the numeration of how many individuals have high school education and beyond. The assumption here is that it is less likely for prejudice to be demonstrated with more education.

Certainly education is not the only indicator of discrimination. For this reason, the integration index has been added. Barriers which seem unsurmountable even with advanced education often topple when people live in a common environment. Therefore, a measure of integration is introduced into the model.

The ultra-conservative vote is included because experience suggests people who vote strictly ultra-conservative do not favor integration (11). At most, this is an indicator as to the degree of racial harmony which can or does exist. If every person who is a member of the majority race were to believe in racial segregation, then opportunities for minority race members would tend to be very small.

The ratio of minority to total population obviously gives information relative to the relative number of members of a given minority that must exist in an area before a positive program is implemented for equal employment opportunities.

The LOF is a function of all four conditions and is now modeled using the concept of probability of gaining employment as a result of the aforementioned factors. The LOF is formed as a series-parallel causal model as shown in Figure 26.

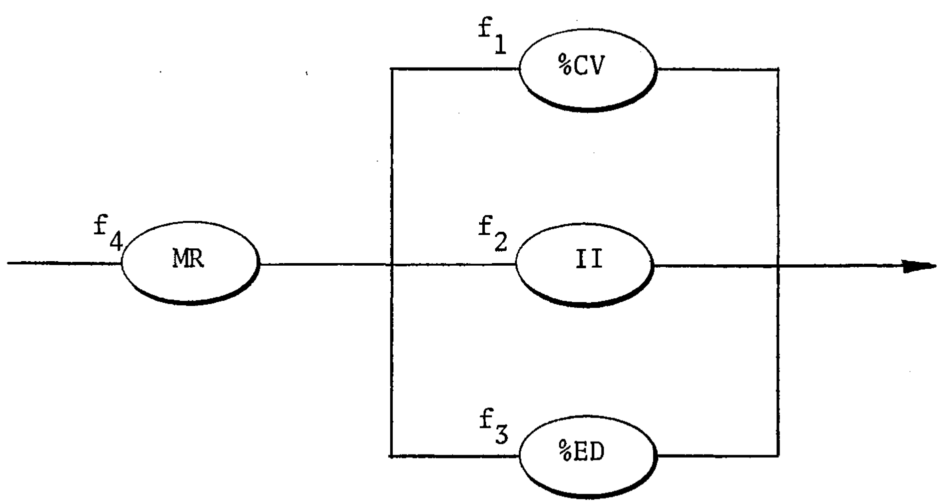


Figure 26. LOF Causal Model

This structure is intuitively apparent upon examination of the variables within the causal model. Certainly the minority ratio should be a series component, because if the model is applied to an urban area in which there is no minority present then the minority ratio is one.

The other variables, %CV, II, and %ED are in parallel because they are alternative means for the minority of achieving success in the exogenous environment. The parallel segment of the causal model is intuitive in that if either or all of the variables in the parallel segment demonstrate probabilities of success of one then the LOF becomes one.

Each variable is assigned a probability of success (P_s) as follows:

$$\begin{aligned} P_{s_1} &= f_1 \text{ (\%CV)} \\ P_{s_2} &= f_2 \text{ (II)} \\ P_{s_3} &= f_3 \text{ (\%ED)} \\ P_{s_4} &= f_4 \text{ (MR)} \end{aligned} \qquad (5.49)$$

In the discussion of the LOF and before calibration, one should note the clarity and plausibility the LOF causal model offers urban research. The concept of probabilistic modeling provides a link between qualitative theory and quantitative urban conditions. Probabilistic modeling as developed in this dissertation is a unique contribution to urban modeling, and can be used when modeling any socio-economic or biological system.

The curves for each P_s are modeled to fit three points. The maximum P_s equals one and the minimum P_s is zero in accordance with the axiomatic definition of probability. The third point on each P_s curve is established to concur with existing social conditions. It would be equally justifiable to model each curve with any curve-fitting technique as long as the three points previously mentioned, along with the definition of axiomatic probability, are satisfied. The comments in this paragraph are applicable to all causal models introduced in this dissertation.

The LOF is equal to one for optimum social conditions and is formulated as a series-parallel model. Proceeding with the formulation, we have,

$$LOF = [1 - \prod_{n=1}^{3} (1 - P_{s_n})]P_{s_4} \qquad (5.50)$$

where P_{s_n} and P_{s_4} have value 1 for optimum social conditions.

The LOF is calibrated using the ratio of labor force participation rates of minority to majority for existing societal conditions. Denote the minority labor force participation rate as BLFPR and the majority rate as WLFPR. From page 55 of Bowen (12), the WLFPR = 99.1 and the BLFPR = 97.54. This is after correction for possible underenumeration of the minority. This is a national rate, used because the appropriate factors for Oklahoma City SMSA are unknown.

Calibrating the LOF, we have

$$LOF = [1 - \prod_{n=1}^{3} (1 - P_{s_n})]P_{s_4} = [\frac{BLFPR}{WLFPR}] \qquad (5.51)$$

$$= \frac{97.54}{99.1} = .984 \qquad . \qquad (5.52)$$

Equal weighting is placed on each probability-like factor because there is no data base to suggest otherwise. Let

$$\alpha^3 = 1 - \prod_{n=1}^{3} (1 - P_{s_n}) \qquad (5.53)$$

and

$$\beta = P_{s_4} \qquad , \qquad (5.54)$$

then $\alpha^3\beta = .984$ for existing socio-economic conditions. Then assuming $\alpha = \beta$,

$$\alpha^4 = .984 \qquad (5.55)$$

$$\alpha = .996 \qquad (5.56)$$

which gives

$$P_{s_4} = .996 \quad . \qquad (5.57)$$

Now

$$\alpha^3 = 1 - \prod_{n=1}^{3} (1 - P_{s_n}) \qquad (5.58)$$

so

$$\prod_{n=1}^{3} (1 - P_{s_n}) = 1 - \alpha^3$$

$$= 1 - .988 = .012 \quad . \qquad (5.59)$$

Therefore with equal weighting on each factor

$$P_{s_n} = 1 - (.012)^{1/3}$$

$$= .771 \qquad (5.60)$$

for existing socio-economic systems.

The success factors, P_{s_n} and P_{s_4}, are modeled to equal one for optimum conditions which are defined as being 0%CV, II = 1, %ED = 100, and MR = 1.

Modeling P_{s_1} as a function of %CV with a declining exponential,

$$P_{s_1} = e^{-C_1 \left(\frac{\%CV}{100}\right)} \qquad (5.61)$$

[1] State and County Officers and Election Returns for 1968, State Election Board, Oklahoma City, Oklahoma (1970).

where C_1 is the calibration constant. Now for existing conditions,[1] %CV/100 = .18, producing

$$e^{-C_1(.18)} = .771$$

$$C_1 = \frac{-\ell n\ .771}{.18} = 1.4448 \qquad . \qquad (5.62)$$

Hence the model is

$$P_{s_1} = e^{-1.4448\ (\frac{\%CV}{100})} \qquad (5.63)$$

and is shown in Figure 27.

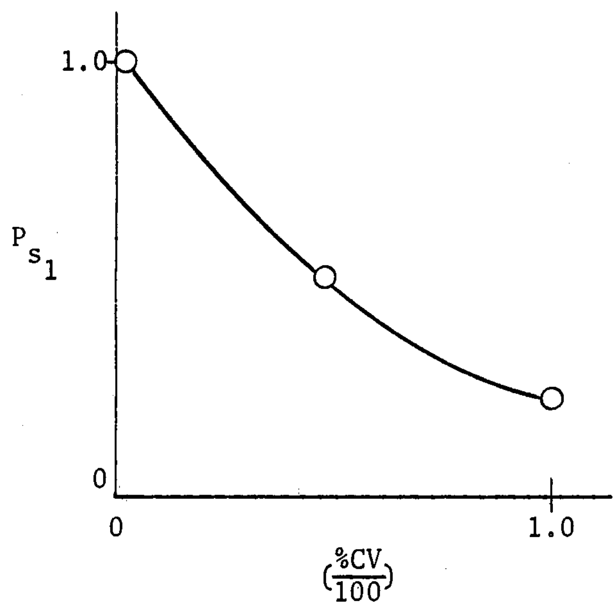


Figure 27. Probability Measure vs %CV

Modeling P_{s_2} as a power of the II, $P_{s_2} = (II)^{C_2}$ where C_2 is a calibration point for existing socio-economic conditions. Let

$$(II)^{C_2} = .771 \tag{5.64}$$

where from Taueber (84),

$$II = .125 \quad .$$

Solving for

$$C_2 = \frac{\ln .771}{\ln .125} \tag{5.65}$$

$$= .125 \quad .$$

Therefore $P_{s_2} = (II)^{.125}$ and is shown in Figure 28.

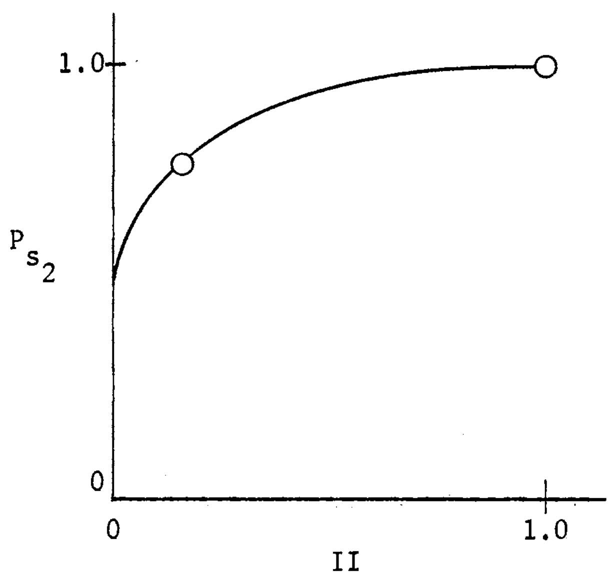


Figure 28. Probability Measure vs II

P_{s_3} is the success factor determined by the %ED where 0 < %ED ≤ 1. Forming the P_{s_3} model as a power of %ED/100, we have

$$P_{s_3} = \left(\frac{\%ED}{100}\right)^{C_3} \quad . \tag{5.66}$$

For today's conditions (%ED/100) = .619 (21). Therefore,

$$.771 = .619^{C_3} \tag{5.67}$$

$$C_3 = \frac{\ell n\ .771}{\ell n\ .619} \tag{5.68}$$

$$= .542$$

producing

$$P_{s_3} = \left(\frac{\%ED}{100}\right)^{.542} \tag{5.69}$$

and is shown in Figure 29.

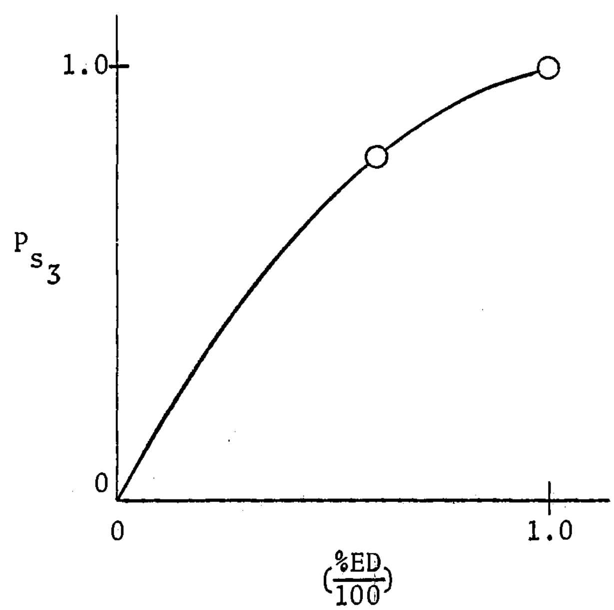


Figure 29. Probability Measure vs %ED

The minority ratio is the last probability type parameter to be modeled. This factor is modeled as the series section of the LOF model because it is dependent upon the existence of the minority population. Without a minority population, the LOF should not exist. When the LOF model is applied to the majority population, $P_{s_4} = 1$.

Modeling P_{s_4} as a power of the MR, we have the following.

$$P_{s_4} (MR)^{C_4} = .996 \quad . \qquad (5.70)$$

The minority ratio to the total population in the Oklahoma City SMSA tod today is .091 (21).

Calibrating, we have

$$C_4 = \frac{\ell n\ .996}{\ell n\ .091} = .001672 \quad . \qquad (5.71)$$

Therefore

$$P_{s_4} = (MR)^{.001672} \quad ; \qquad (5.72)$$

its variation is shown in Figure 30.

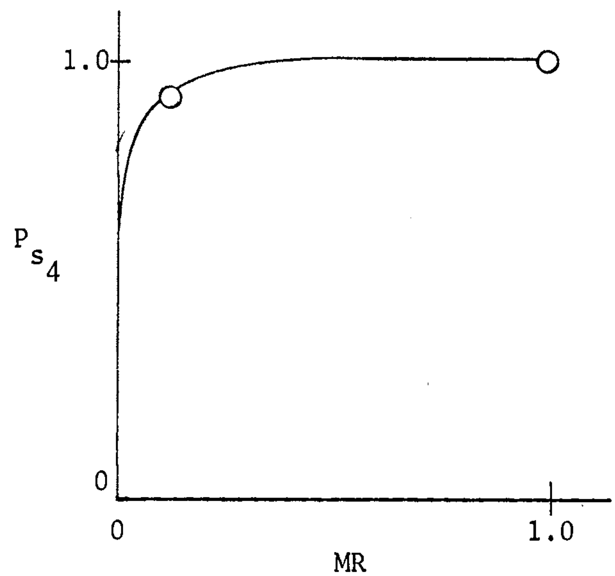


Figure 30. Probability Measure vs MR

Having modeled the various factors, the LOF is formed as a series-parallel model.

$$LOF = P_{s_4}[1 - \prod_{n=1}^{3} (1 - P_{s_n})]$$

$$= MR^{1.6(10^{-3})}[1 - [1 - e^{-26(\frac{\%CV}{100})}][1 - II^{.125}][1 - (\frac{\%ED}{100})^{.542}]] \quad (5.73)$$

To properly control the movement between economic units, the LOF multiplies a function of the unemployment rate, f(UE). According to Monney (51) LFPR should increase 3% for a 1% decrease in the unemployment rate. Approximating this type of relation, the $f(UE) = e^{-20UE}$ where for UE = 1, f(UE) $\cong$ 0 and for UE = zero, f(UE) = 1.

Therefore the economic movement is controlled by LOF $\cdot$ e^{-20UE}. When this model is applied to the majority community LOF = 1 and the job movement would be a function of unemployment only.

DOR Model

In the development of the educational sector, a variable was formed which was called the DOR which denotes the dropout rate. The DOR is assumed to be a function of the following: (1) density in the home (DH); (2) average parents education (PAE); (3) pupil/teacher ratio (P/T); and (4) community projection index (CPI). These factors are listed in the Coleman report (15). The first three factors are self-explanatory, but the CPI needs some explanation.

In several references, the idea that achievement of the minority is a direct function of success of their peers within the community is expressed (15,54). This idea is modeled by the CPI and is a function of values of the economic categories, WCI and WCE, which reside in the minority community.

The DOR model is modeled as a series-parallel model as shown in Figure 31.

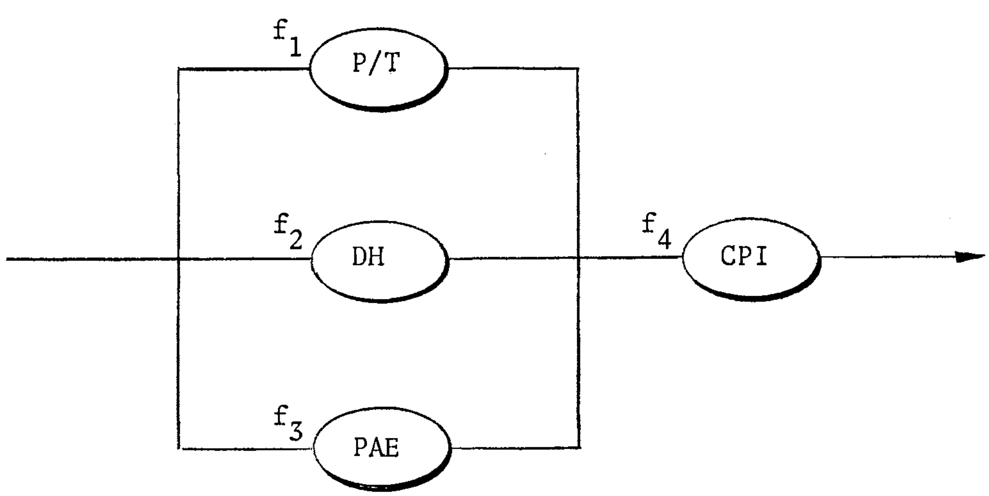


Figure 31. DOR Causal Model

The CPI is modeled as a series component of the model in order that the DOR model may be applied to the majority community. Intuitively, if the DOR model was applied to the majority community the CPI would equal one and the DOR would become a function of the variables in the parallel segment of the model.

The parallel segment of the DOR model is modeled as a function of P/T, DH, and PAE because these are alternative paths for success within the educational system. It is plausible to place the former variables in the parallel portion of the model because if either or all variables (P/T, DH, PAE) demonstrate a probability of one, then the DOR should become zero because optimum social conditions have been attained.

Each variable is assigned a probability of success (P_s) as follows:

$$
\begin{aligned}
P_{s_1} &= f_1\ (P/T) \\
P_{s_2} &= f_2\ (DH) \\
P_{s_3} &= f_3\ (PAE) \\
P_{s_4} &= f_4\ (CPI)
\end{aligned}
\qquad (5.74)
$$

Before calibrating the DOR probabilistic model, it should be noted that P_{se_n} is used to designate a probability of success referenced to the exogenous community. P_{si_n} denotes a probability of success referenced to the endogenous community.

The DOR model is a series-parallel model. The series portion is a function of the CPI while the parallel portion of the model consists of the previously listed variables.

The DOR model is expressed as,

$$DOR = 1 - P_{s_4} \cdot [1 - \prod_{n=1}^{3} (1 - P_{s_n})] \quad . \tag{5.75}$$

The strategy for calibrating the DOR consists of: (1) calibrate the DOR model for the majority community by assuming P_{se_4} = CPI = 1 for the majority community; (2) determine the P_{se_n} for n = 1, 2, 3 using the DOR exogenous (DOR_e) to the minority community; (3) by substituting the values for the variables from the minority community, P_{si_n} (n = 1, 2, 3) and the minority drop-out rate (DOR_i) the minority CPI is obtained. Using the DOR_e for the exogenous community,

$$1 - \prod_{n=1}^{3} (1 - P_{se_n}) = 1 - DOR_e \tag{5.76}$$

$$\prod_{n=1}^{3} (1 - P_{se_n}) = DOR_e \quad . \tag{5.77}$$

Let

$$x^3 = \prod_{n=1}^{3} (1 - P_{se_n}) = DOR_e \tag{5.78}$$

which places equal emphasis on each variable. DOR_e = .304 for the Oklahoma City SMSA (21) producing,

$$x = DOR_e^{1/3}$$

$$= .672 \quad . \tag{5.79}$$

Now

$$1 - P_{se_n} = .672 \tag{5.80}$$

for each P_{se_n}. Hence P_{se_n} = .328 is the calibration point for existing socio-economic conditions.

Modeling P_{se_1} as a function of P/T, we have that the pupil teacher ratio, for the Oklahoma City SMSA, is approximately 22.7 for both the minority and the majority community (74). Following intuition, P_{se_1} is modeled with a decaying exponential. We have,

$$P_{se_1} = e^{-C_1 \frac{P}{T}} \tag{5.81}$$

where $P/T \geq 1$ where for calibration $e^{-22.7\, C_1} = .328$. This produces,

$$C_1 = \frac{-\ell n\ .328}{22.7}$$

$$= .0491 \tag{5.82}$$

Therefore $P_{se_1} = e^{-.0491\ P/T}$; $P_{se_1} = P_{si_1}$ of the minority community. P_{se_1} vs P/T is illustrated in Figure 32.

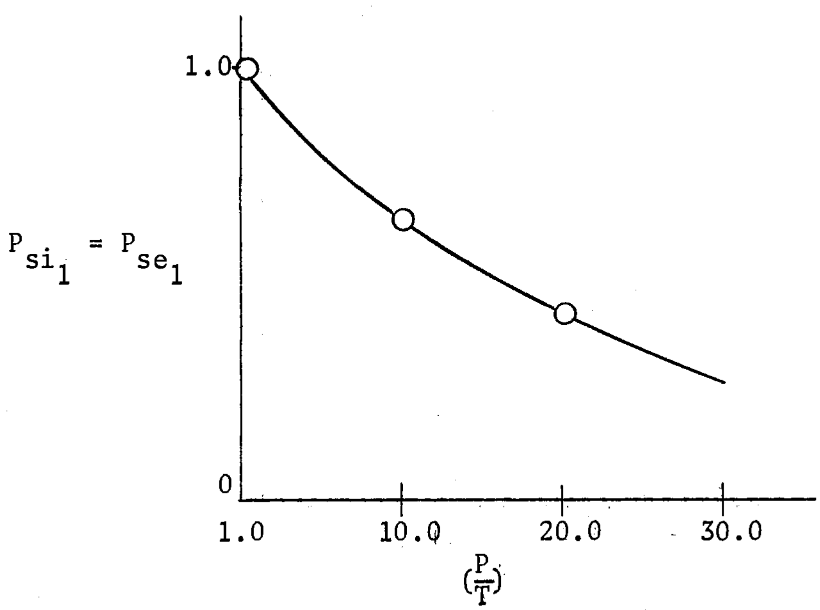


Figure 32. Probability Measure vs $\frac{P}{T}$

Modeling P_{se_2} as a function of DH, the approximate DH for the majority group in the Oklahoma City is approximately 3.0, and 3.3 for the minority group (20). Given $DH_e = 3.0$ and using a decaying exponential for the model, we have,

$$P_{se_2} = e^{-C_2[DH-2]} \quad , \qquad (5.83)$$

which is valid only for DH $\geq$ 2. Calibrating for the exogenous community,

$$C_2 = \frac{-\ln .328}{[DH-2]} = \frac{-\ln .328}{1} = 1.115 \quad . \qquad (5.84)$$

Therefore

$$P_{se_2} = e^{-1.115[DH-2]} \quad . \qquad (5.85)$$

Substituting the DH value for the minority produces

$$P_{si_2} = e^{-1.115(1.3)} = .2347 \quad . \qquad (5.86)$$

A plot of this model is shown in Figure 33.

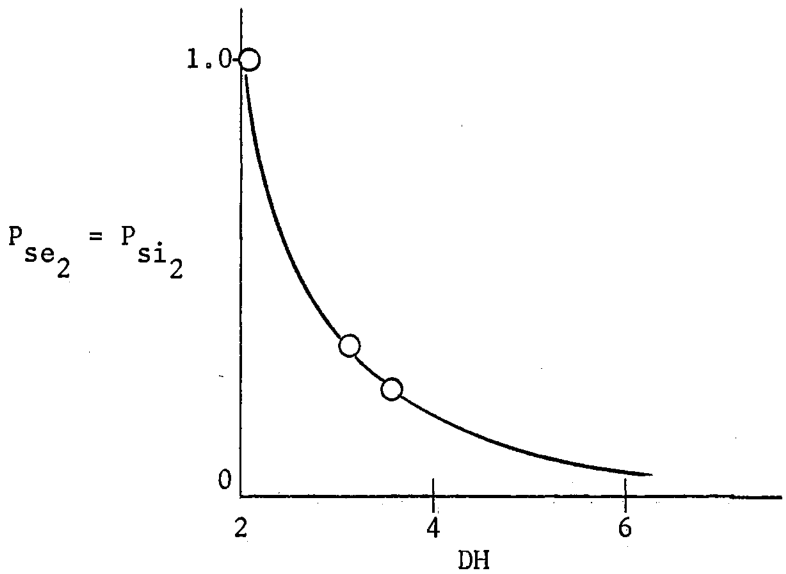


Figure 33. Probability Measure vs DH

Modeling P_{se_3} as a function of the parents average education (PAE), Lagrange interpolation is used to find the expression.

In the Oklahoma City SMSA the approximate PAE for the majority is 12.33 years and for the minority is 11.4 years (21). The model is defined over a range of $0 \leq \text{PAE} \leq 20$. With Lagrange interpolation,

$$P_{se_3} = \sum_{k=1}^{3} \prod_{\substack{i=1 \\ i \neq k}}^{3} \frac{(x-x_i)}{(x_k-x_i)} \cdot y_k$$

$$= \frac{(x-x_1)(x-x_3)}{(x_1-x_2)(x_1-x_3)} \cdot y_1 + \frac{(x-x_1)(x-x_3)}{(x_2-x_1)(x_2-x_3)} \cdot y_2$$

$$+ \frac{(x-x_1)(x-x_2)}{(x_3-x_1)(x_3-x_2)} \cdot y_3 \qquad (5.87)$$

where

x = PAE; and

$y = P_{se_3}$.

$$P_{se_3} = PAE[3.0507(10^{-3})PAE-.0110] \quad . \qquad (5.88)$$

For the minority community,

$$P_{si_3} = .271 \quad . \qquad (5.89)$$

A plot of the model is shown in Figure 34.

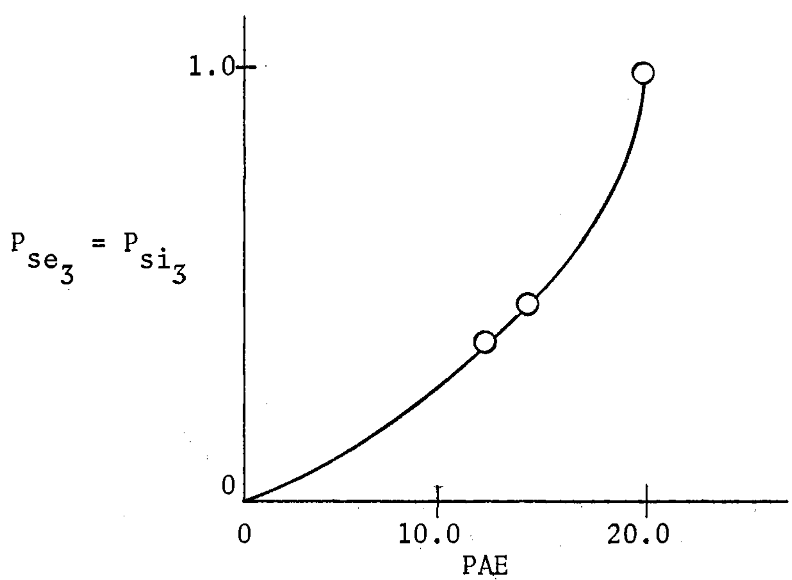


Figure 34. Probability Measure vs PAE

The last parameter to be modeled (P_{s_4}) was initially considered by the author to be very important. This parameter depends on the community projection index (CPI). Define

$$CPI = \frac{WCI + WCE}{TMHH} \qquad (5.90)$$

where TMHH equals the total minority households. The calibration of the CPI is done by equating its effect and the previous models to the drop-out rate endogenous to the community (DOR_i). As stated previously the DOR_i for the minority community is .382. Using the model previously developed with minority values,

$$P_{s_4} = (CPI)^{C_4} \tag{5.91}$$

where

$$P_{s_4}[1 - \prod_{n=1}^{3} (1 - P_{s_4})] = 1 - DOR_i \tag{5.92}$$

$$P_{s_4}[1 - (1 - P_{si})(1 - P_{si_2})(1 - P_{si_3})] = 1 - DOR_i \tag{5.93}$$

$$P_{s_4}[1 - (1 - .328)(1 - .234)(1 - .271)] = 1 - .382 \tag{5.94}$$

$$P_{s_4}[1 - .374] = .618 \tag{5.95}$$

$$P_{s_4} = \frac{.618}{.626} = .987 \quad . \tag{5.96}$$

The CPI for existing socio-economic conditions is, .13 (21).

Therefore the CPI for the minority community is,

$$P_{si_4} = (CPI)^{C_4} = .987 \tag{5.97}$$

$$C_4 = \frac{\ln .987}{\ln .13}$$

$$= 6.413(10^{-3}) \tag{5.98}$$

where the values for this function are,

CPI	P_{si_4}
.13	.987
.5	.995
1.0	1.0

.

P_{si_4} is plotted in Figure 35.

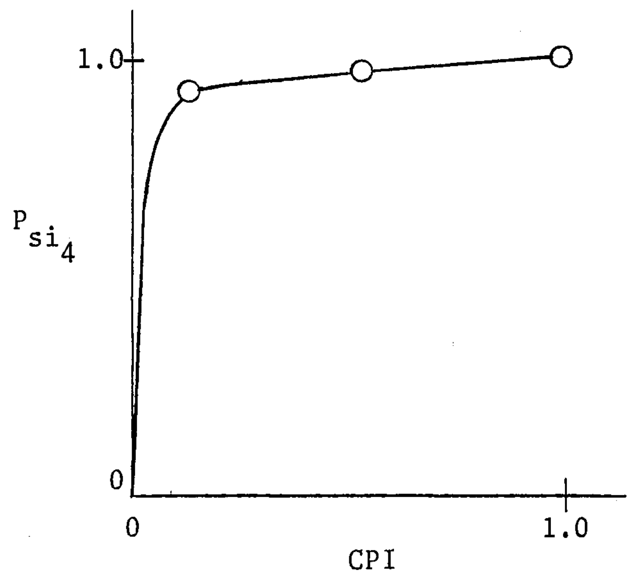


Figure 35. Probability Measure vs CPI

For this urban area, the DOR model accurately describes the two communities without the CPI, as noted by the insensitivity shown in Figure 35.

Now the DOR model may be expressed as,

$$1 - DOR = P_{si_4}[1 - \prod_{n=1}^{3} (1 - P_{si_n})]$$

$$= CPI^{6.413(10^{-3})}[1 - [1 - e^{-.0491\ P/T}]$$

$$[1 - e^{-1.1147[DH-2]}][1 - PAE[3.05(10^{-3})PAE-.0110]]] \quad . \quad (5.99)$$

This completes the formulation of the probabilistic causal models used in this urban model and paves the way for the simulation of the model described in the next chapter.

CHAPTER VI

MODEL SIMULATION

Introduction

The urban model conceived in this thesis performed very well under simulation. Simulations were performed in four ways. They were independent sector simulations which consisted of: (1) isolated demographic sector; (2) isolated residential sector; (3) isolated industrial sector; and (4) total simulation of the entire urban model.

For all of the isolated sectors, simulations were performed at the equilibrium point for the entire model. This point produces almost zero absolute growth with respect to the exogenous categores of the model. The exogenous categories of the model are BCE and WCE. All simulations with exception of the industrial sector, when performed by sectors, were initiated with neutral attractiveness to the area with respect to other urban areas. Neutral attractiveness was defined in Chapter III as being zero net migration.

Demographic Sector Simulation

This sector was run on the computer with constants for all exogenous inputs to the model. The first simulation is for zero net migration and the equilibrium conditions for the overall urban model. The equilibrium conditions result when the following conditions are true:

$$LOF_1 = LOF_3 = LOF * e^{(-32.35)(UEMP_{1,3})} \tag{6.1}$$

$$LOF_2 = LOF * e^{(-129.4)(UEMP_2)} \tag{6.2}$$

along with the constants for bith and death rates and declining employment.

In the first simulation as well as others, the initial conditions for each economic category were derived from the census data (21). The population was divided into blue-collar and white-collar economic categories. The census data percentage of exogenous to endogenous employment was then applied to obtain the four economic categories. The age distribution was accomplished in the same manner by applying percentages for a particular age group of the entire population to each economic category.

The average number of individuals per household was (4.47) with allowance for larger families (6.47) in the unemployed sector because of observed differences in fertility trends (44) and known underenumeration in census data.

Looking at Figure 36, the results are seen for the first demographic simulation. These results are also tabulated in Appendix B, Table I.

The BCI category declines and then builds due to no migration in and static economic conditions exogenous to the community. The WCI category continues to grow in the industrial base endogenous to the community.

The BCE and WCE categories are essentially at an equilibrium point, because of no net migration in due to neutral attractiveness of the area.

The UEMP category builds due to population growth, drop-outs from the educational process, and the stagnant economy. Until the channels are opened for flow into the mainstream of economic affairs, the UEMP

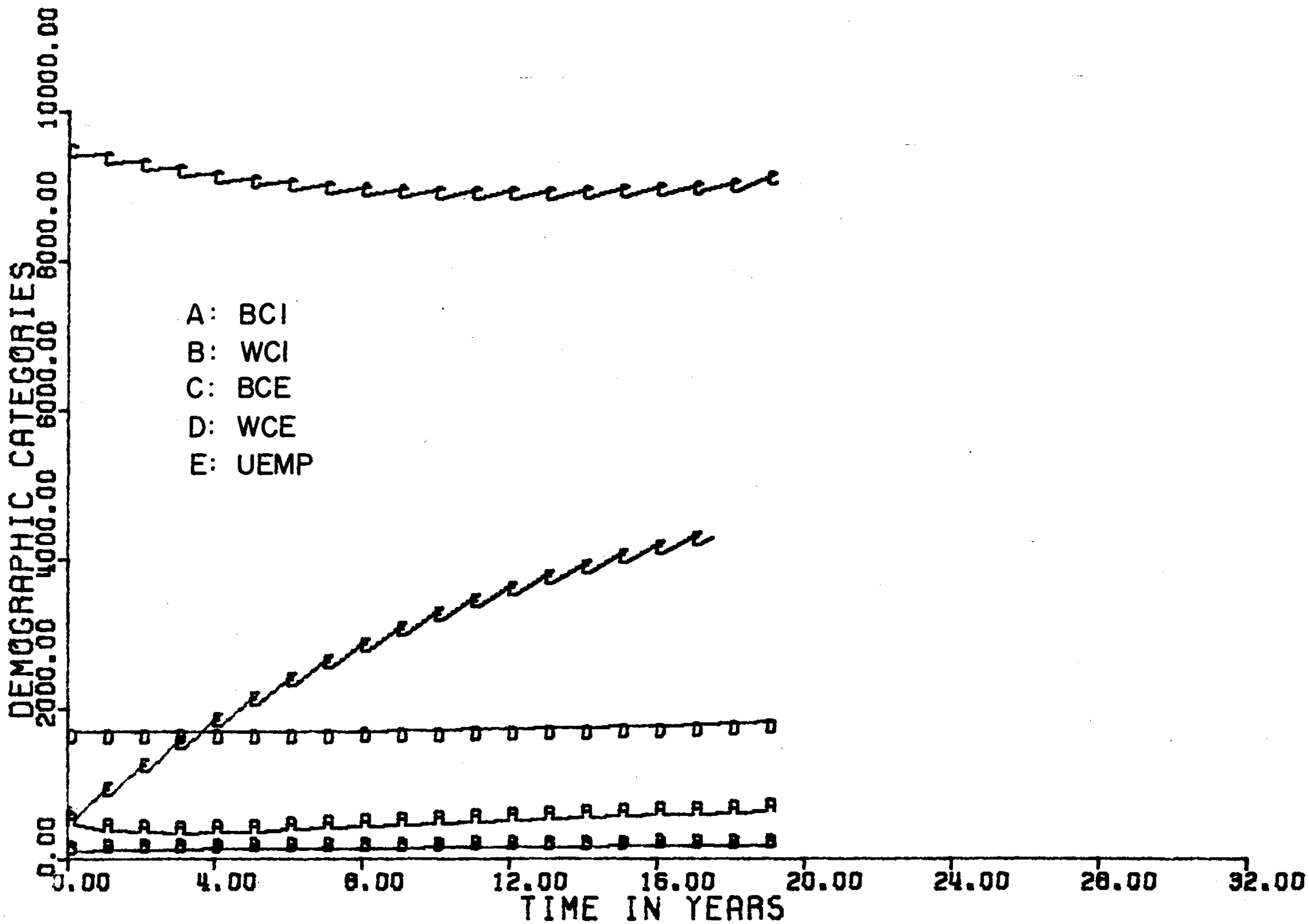


Figure 36. Demographic Simulation I, Demographic Sector

category will grow at a faster rate than the employed categories.

The second demographic simulation, shown in Figure 37 and listed in Table II, was performed by creating a net in-migration into the model by assuming the initial housing vacancies to be numerous. Starting at conditions of essentially equilibrium, as previously described, the value which controls the rate of flow into the area, αm_n, was increased by .048 and the resultant in-migration is obvious from Figure 37. The delta increase in housing vacancy rate required to obtain this in-migration was .2 of 1%. This vacancy rate was maintained for the twenty year period, however, which accounts for constant rise in in-migration. The CPI is also changing and as a result, there is not a constant proportionality between housing vacancies and net migration in.

Note that in the second simulation, the unemployed category was again rapidly increasing.

These simulations are helpful in understanding the basic relationships in the demographic sector, but fail to give the total picture because the residential and industrial sectors are not present to compensate for actual dynamics present in the urban system.

Residential Sector Simulation

Two situations are simulated concerning the residential sectors. They are: (1) no new construction; and (2) slum housing is removed from the system by massive urban renewal.

In Figure 38 and Table III, the simulation is shown for a residential area in which there is no new construction. This is due to the fact that the DVR is set equal to the actual vacancy rate. This is not the case in an actual urban area or in the urban model when run with all

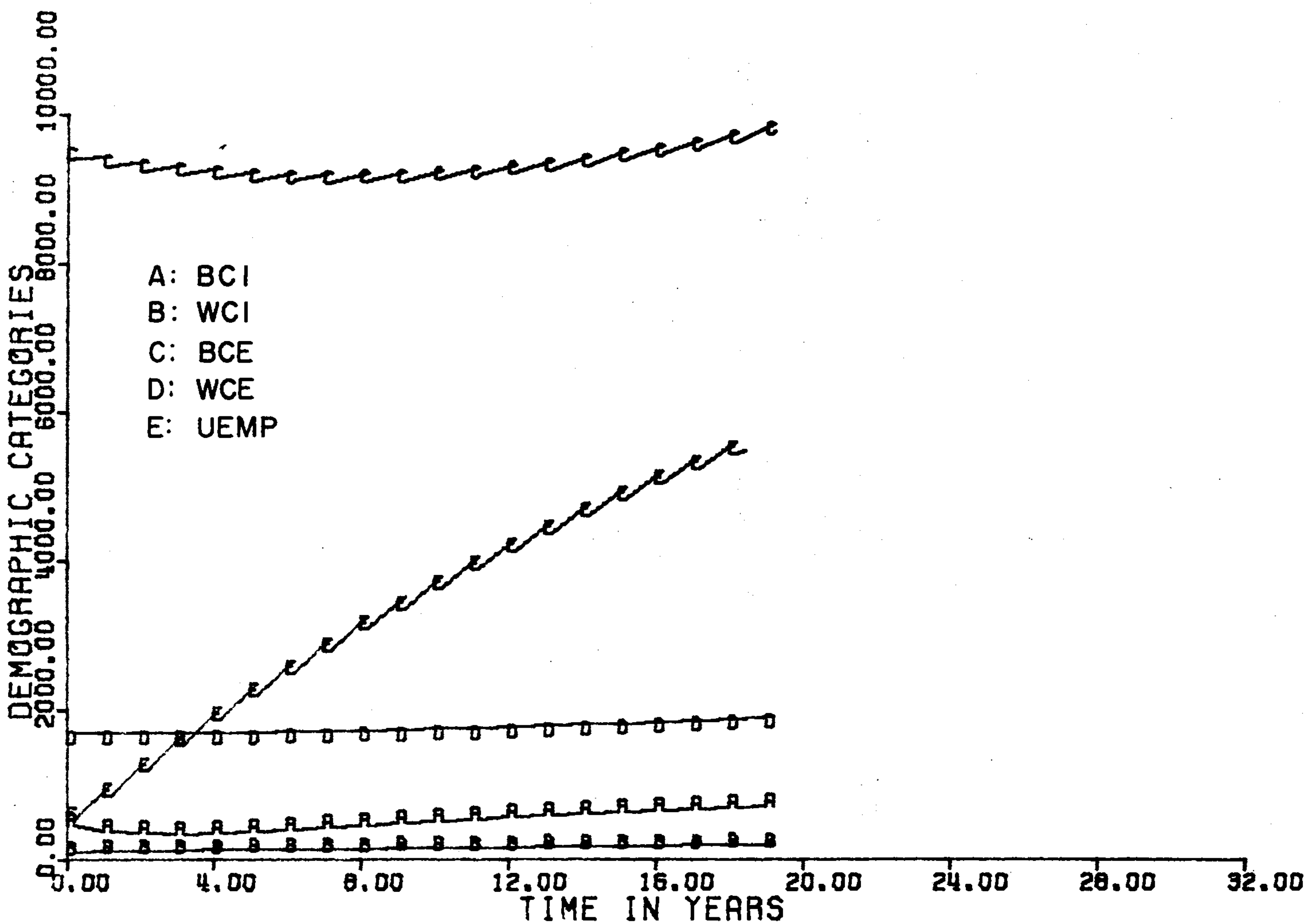


Figure 37. Demographic Simulation II, Demographic Sector

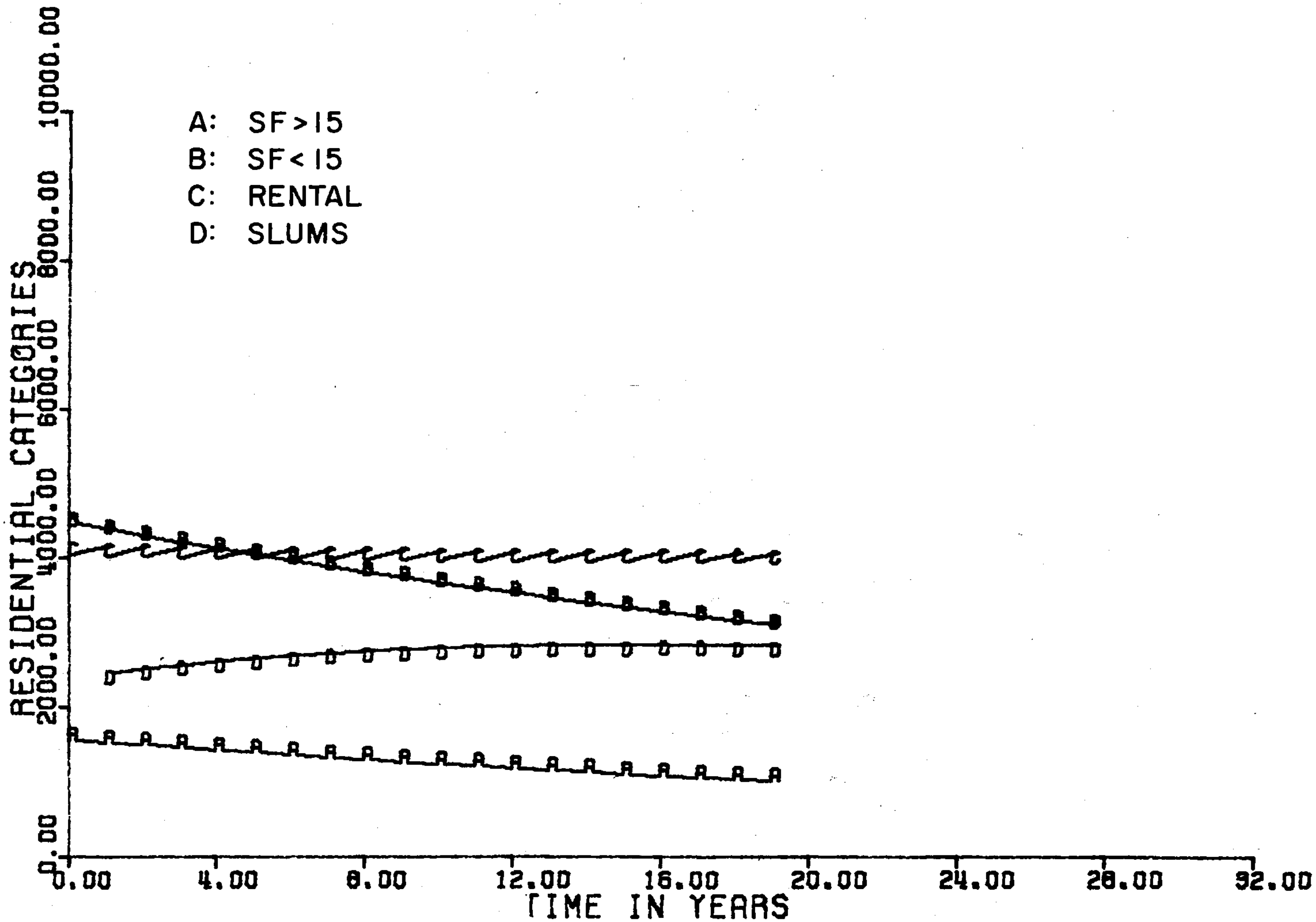


Figure 38. Residential Simulation I, Residential Sector

sectors. This was done to examine the flow of housing through the various categories and show the debilitating effects which result when there is no new construction and there is no effective urban renewal. The main result is shown as being an accumulation of slum housing and a drop in single family housing less than $15,000.

Given the same actual vacancy rate and desired vacancy rate along with massive urban renewal, the slum housing can be reduced. Examining the second simulation, shown in Figure 39 and Table IV, the final slum housing is considerably less. This is still not a realistic simulation however because the other sectors are not present to stimulate the construction of housing by showing a decrease in the actual vacancy rate. One effect that is noted however is that decreasing the slum housing, decreases the number of residential units which may be converted back into other residential categories. In so doing, the actual vacancy rate is decreased by more than the effect of just the number of units demolished because of a decrease in the units which are available for conversion from slums into other residential categories. The slum housing was reduced in the second simulation by destroying all but 1000 of the slum housing units in one year.

Industrial Sector Simulation

The industrial sector, consisting of two types of industry, was simulated for a net growth in the endogenous community, remembering that the number of exogenous firms was held constant throughout the simulation. For an annual growth of .6 percent in the minority households, taking into consideration the effects of failure of firms and new starts, the growth rate shown in the minority industrial base is approximately

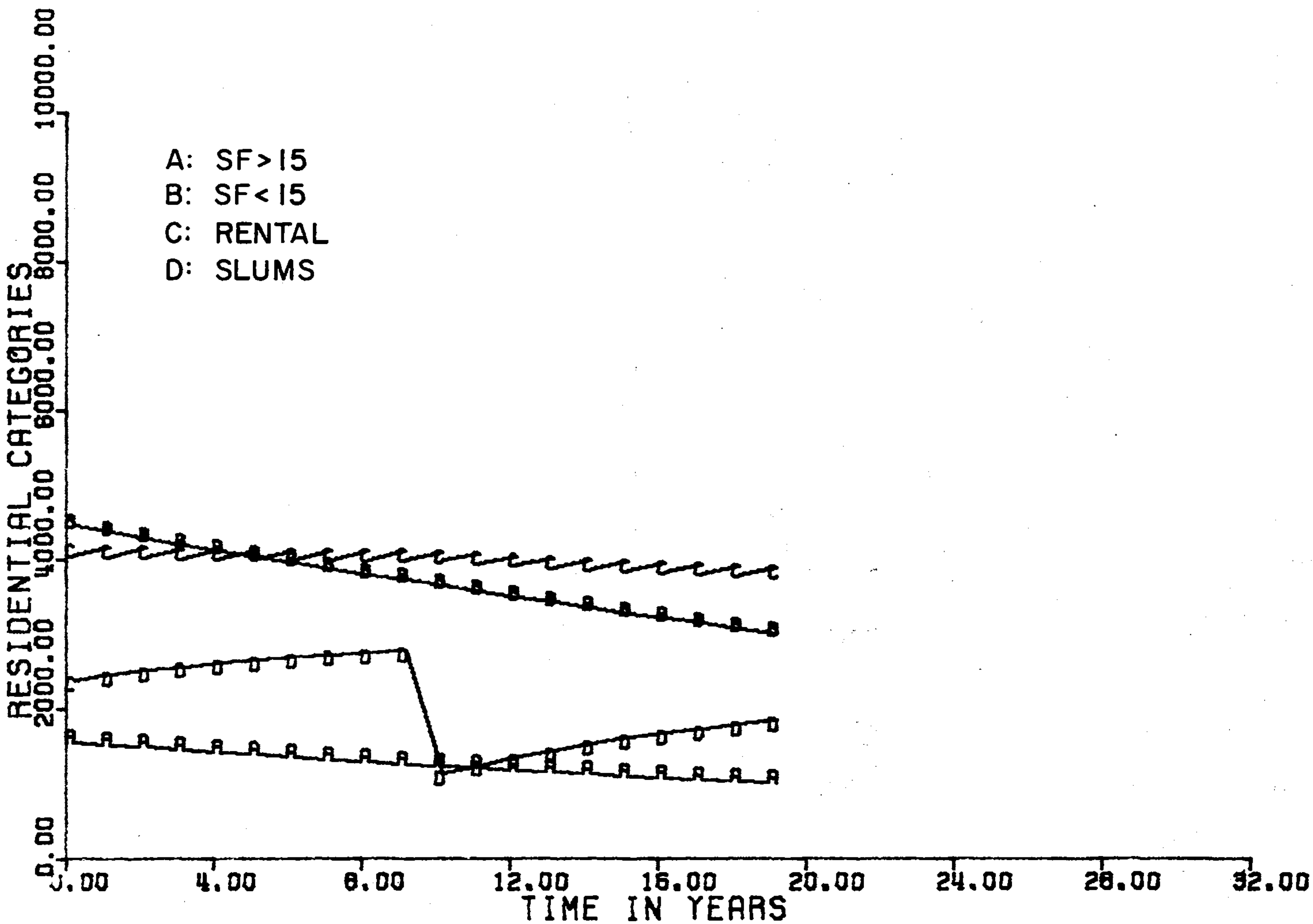


Figure 39. Residential Simulation II, Residential Sector

3%. This requires that the existing industrial base in the minority community provide 10% of the existing mature firms as new firms each year through government financing, general entrepreneurship, etc. This is believed by the author to be a realistic assumption from the standpoint of not requiring too much of the minority businessman. The startling fact is that if this were done, the industrial base of the minority community is almost doubled while the actual number of minority households has increased by 10 percent.

The results of this simulation are shown in Figure 40 and Table V. The PIWI are growing at a controlled geometric rate.

This completes the discussion of the sector simulations and paves the way for the simulation of the entire model.

Dynamic Non-Linear Urban Model Simulation

Combining the previously described sectors, the entire urban area is next simulated. The three simulations performed were: (1) setting the LOF at zero to simulate worst case social conditions; (2) assuming an equilibrium point which was wrong for the system to observe the effects of moving economic units through the urban system at the wrong rate and to invalidate the assumption; and (3) having determined the equilibrium point of the urban system under fixed exogenous assumptions, a simulation was performed to observe the interactions which take place within the minority community.

The first case simulates the situation which results when the worst social conditions are assumed to prevail in the urban area. These conditions require CV = 100%, II = 0, and the percent educated of the majority dropping to zero.

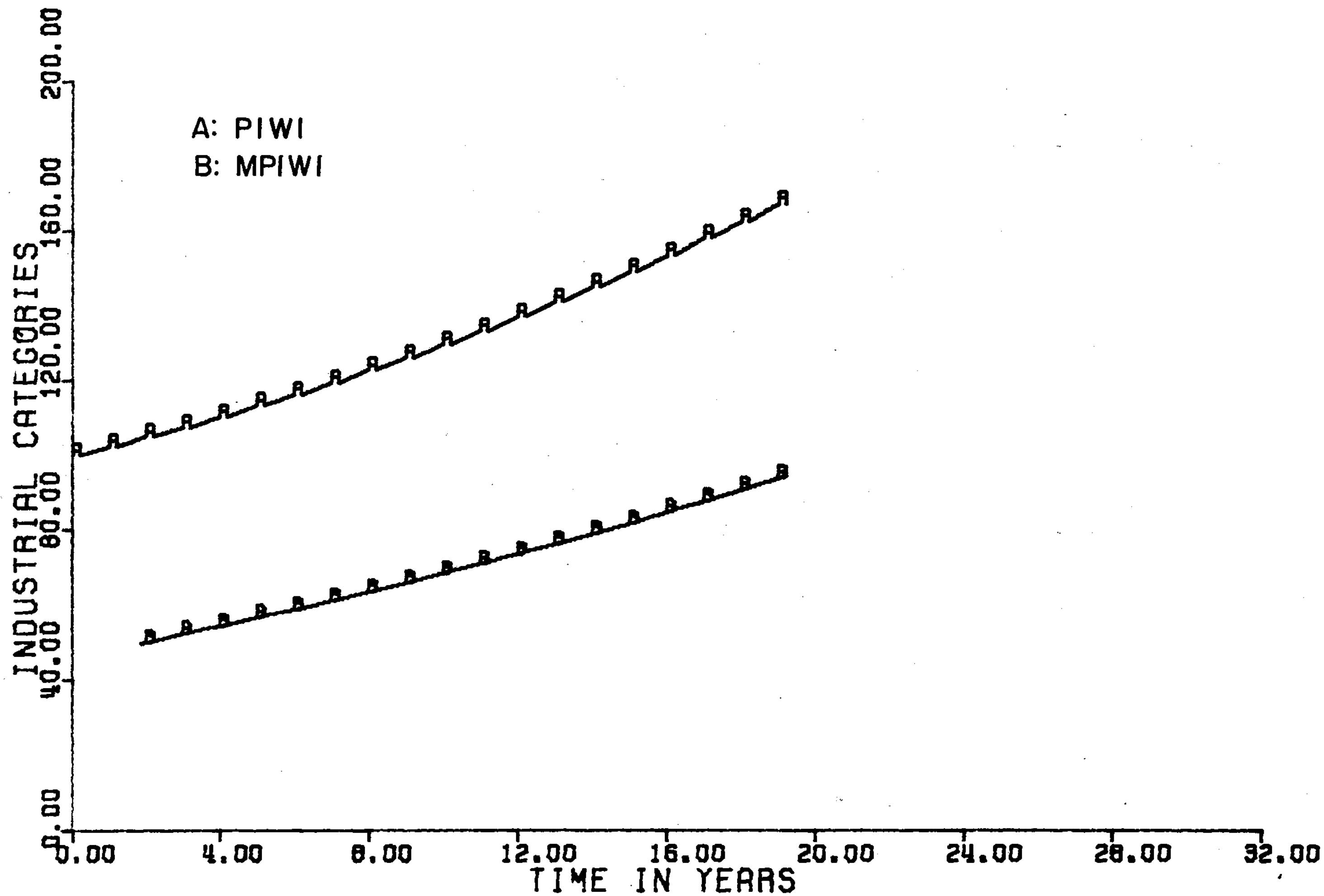


Figure 40. Industrial Simulation, Industrial Sector

The results of this simulation are shown in Figures 41-44 and Tables VI-IX. Because of the social conditions, the WCE and BCE decrease. The socio-economic conditions cause a net migration out of the urban area of the exogenous minority work force.

The amazing result is the reaction of the minority industrial base. As compared with other simulations (equilibrium), the industrial base of the minority community has shown more growth due to restrictions present in the exogenous community. This result is counterintuitive to the author's preliminary studies. A possible explanation is that the WCE and possibly BCE, who are perhaps better trained than their endogenous counterparts were forced to turn their abilities into endogenous business. The number of PIWI and XPIWI firms are a substantial number in urban simulation I where social conditions are the worse and approach the number in the third simulation performed with urban equilibrium conditions.

An expected result is that the unemployed sector grew at a rapid rate because there was no flow of unemployed to exogenous industry. The unfortunate or distressing condition which is implied by this simulation is the number of unemployed in the area grow faster than under acceptable social conditions.

A counterintuitive point is that there are fewer slums under LOF = 0 conditions than when the system equilibrium is simulated. The author attributes the fewer slums to an increase in the physical life expectancy of SF 15 and rental units due to increasing the educational level within the housing sector. One possible explanation is that of the WCE and BCE returning to the minority industrial base.

This simulation does follow intuition in that there are more

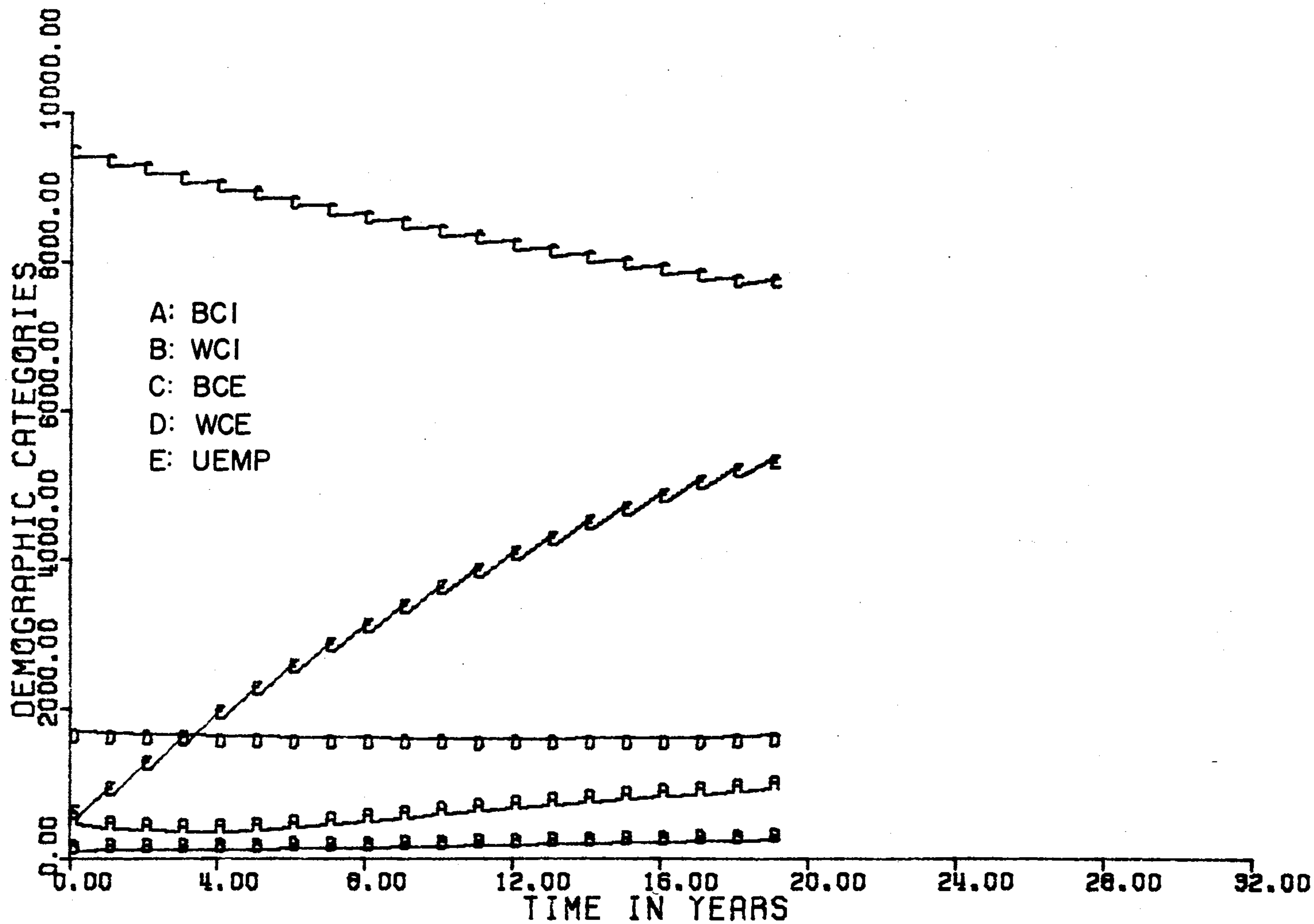


Figure 41. Urban Simulation I, Demographic Sector

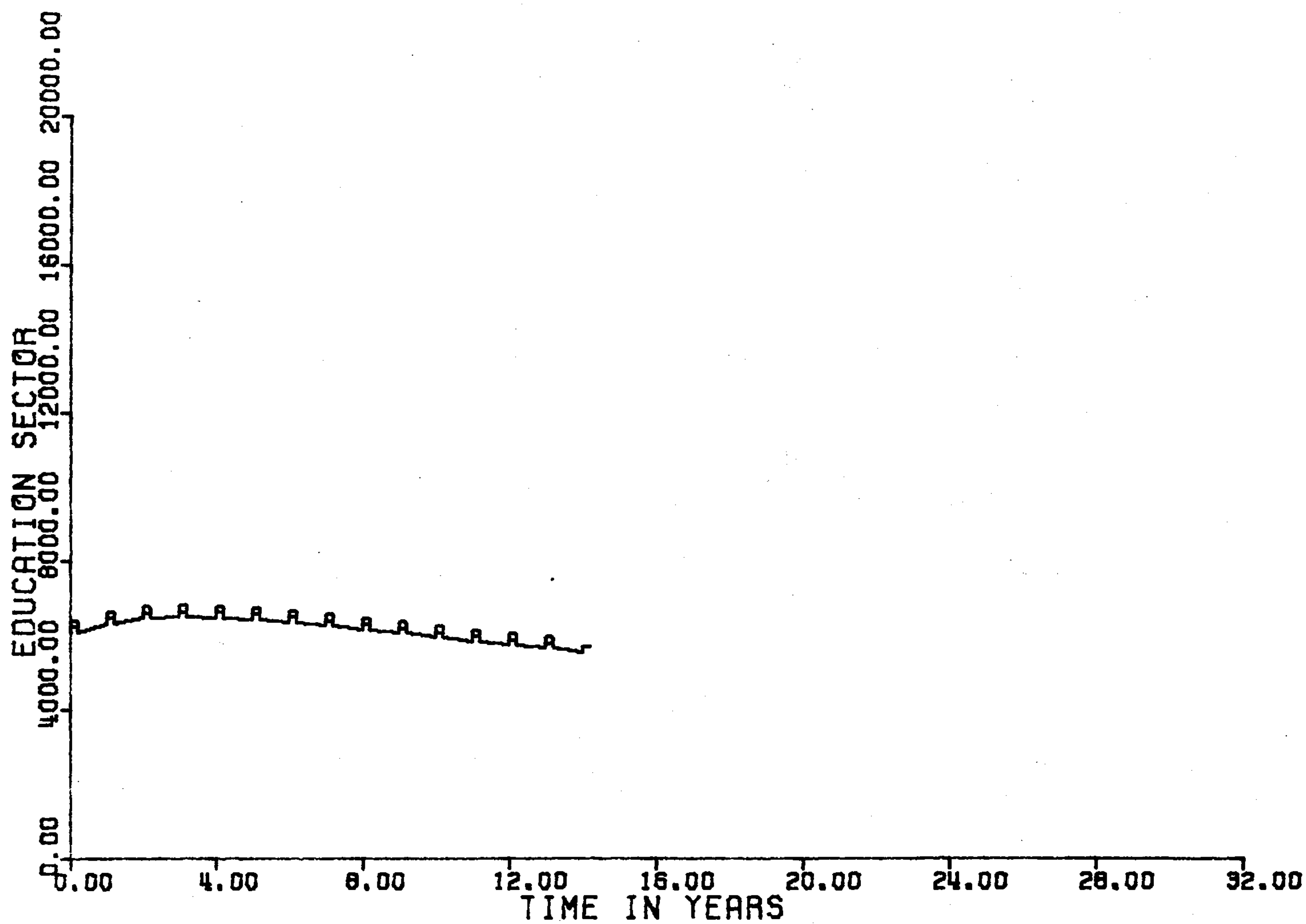


Figure 42. Urban Simulation I, Educational Sector

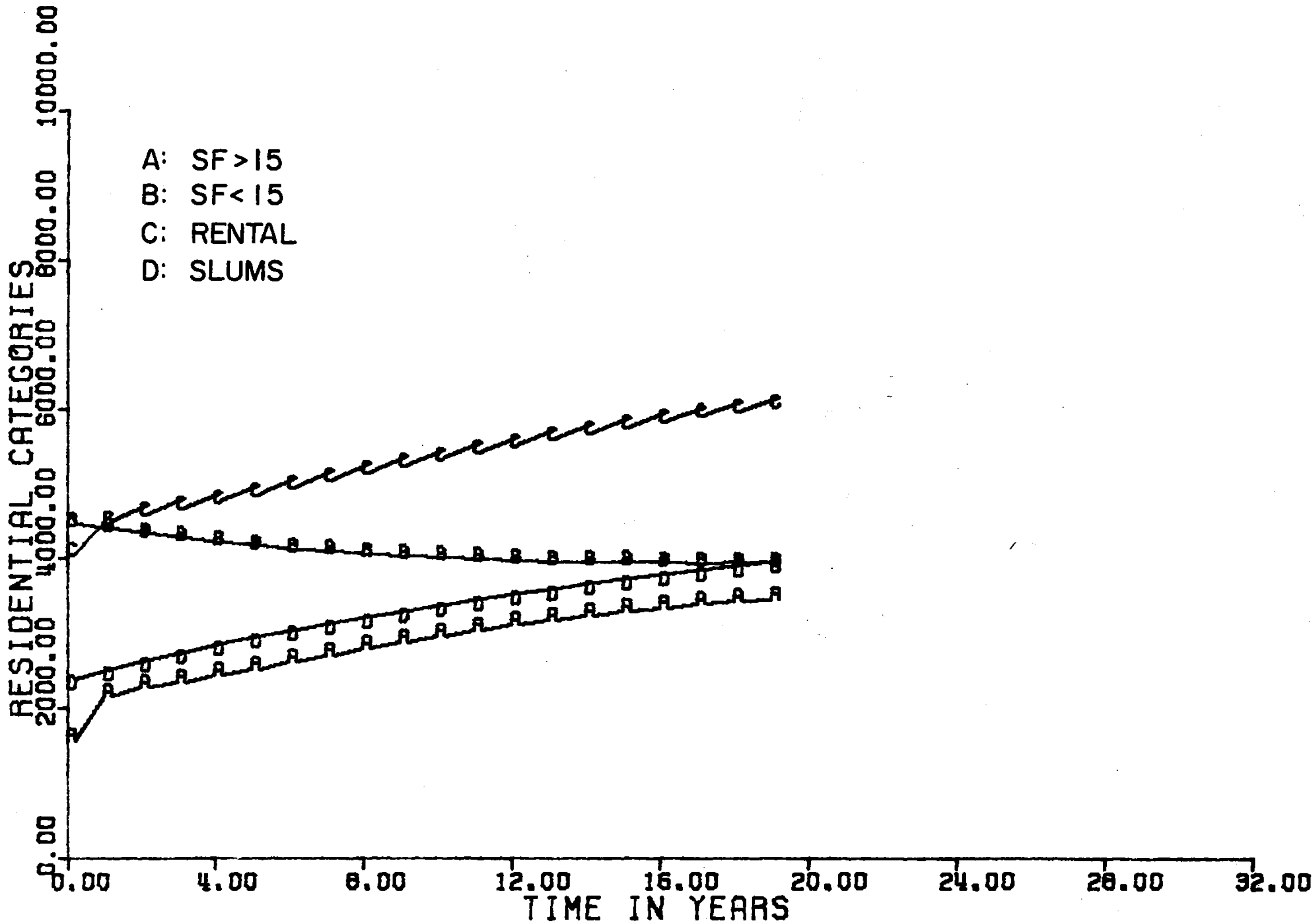


Figure 43. Urban Simulation I, Residential Sector

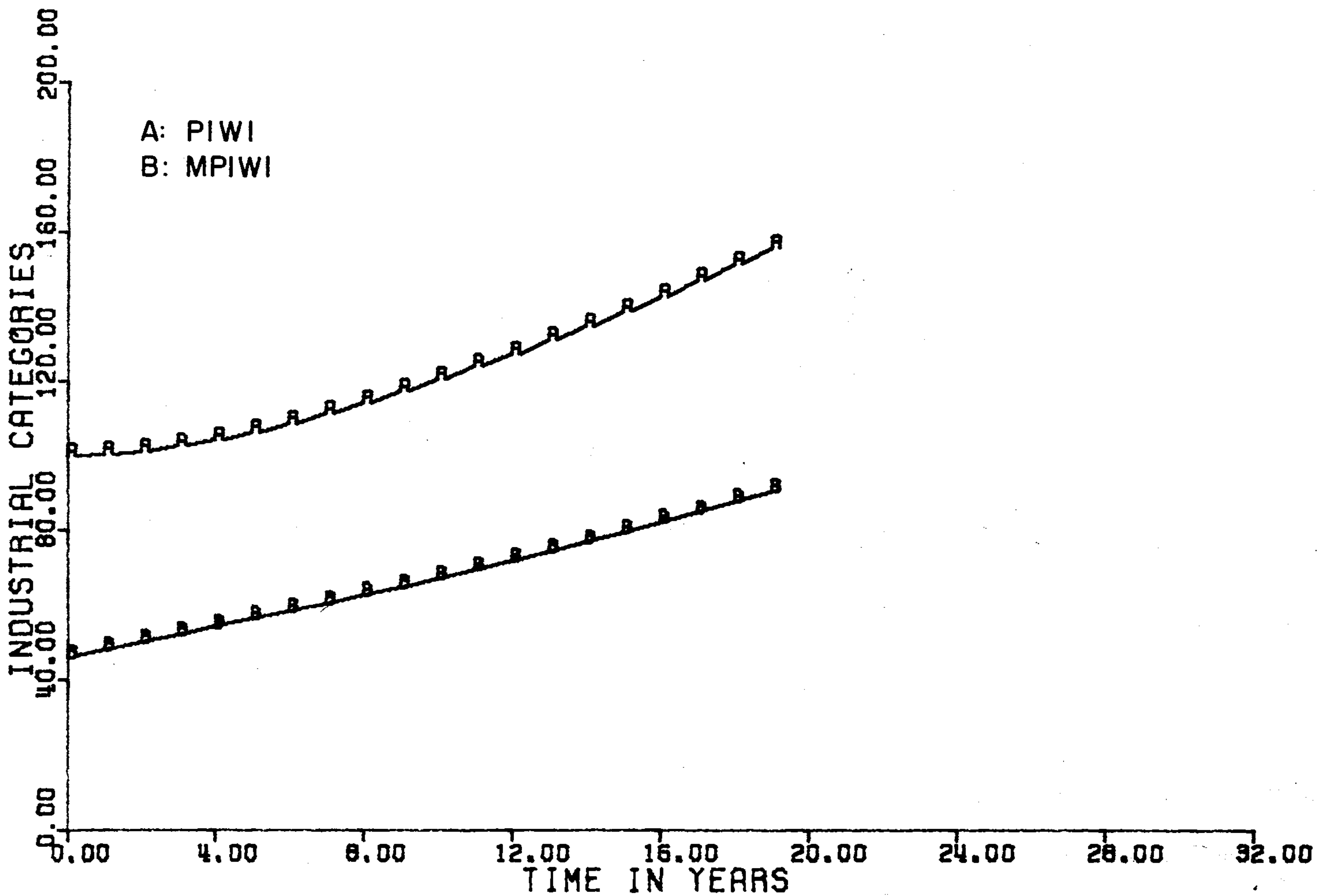


Figure 44. Urban Simulation I, Industrial Sector

unemployed and a decrease of the exogenous economic units. The decreasing number of new firms in the industrial sector was due to the decreasing minority population. The new firms were fewer in number, while the mature firms were essentially the same.

The educational sector in Figure 42 shows the increase in the beginning of the earlier parts of the simulation. It is fairly constant however because of net migration out offsetting births over deaths. The DOR, which is a function of CPI, was decreased however due to a net increase in white collar employment in the minority community.

The second simulation was performed testing the equilibrium point of the urban minority system with the exogenous urban area. An equilibrium point was estimated for the urban model by setting the inputs and outputs of the unemployed sector equal. This provided,

$$LOF_n = LOF \cdot e^{-21.11 \cdot UE} \qquad n = 1, 2, 3 \qquad (6.3)$$

where UE is unemployed rate. This does not result in equilibrium, however, when used in the model. Because $e^{-21.11 \cdot UE}$ allows people to move from economic unit to economic unit at too fast a rate, the LOF_n was perturbed to a new value:

$$LOF_n = LOF \cdot e^{-30\ UE} \qquad n = 1, 2, 3 \quad . \qquad (6.4)$$

This provides for 3.8% movement between economic units. This is still not the equilibrium point however as demonstrated by the model.

Using the latter definition of LOF_n, a simulation was performed. The results of this simulation are shown in Figures 45-48 and Tables X-XIII.

The WCE increased at a rate faster than an actual urban system would

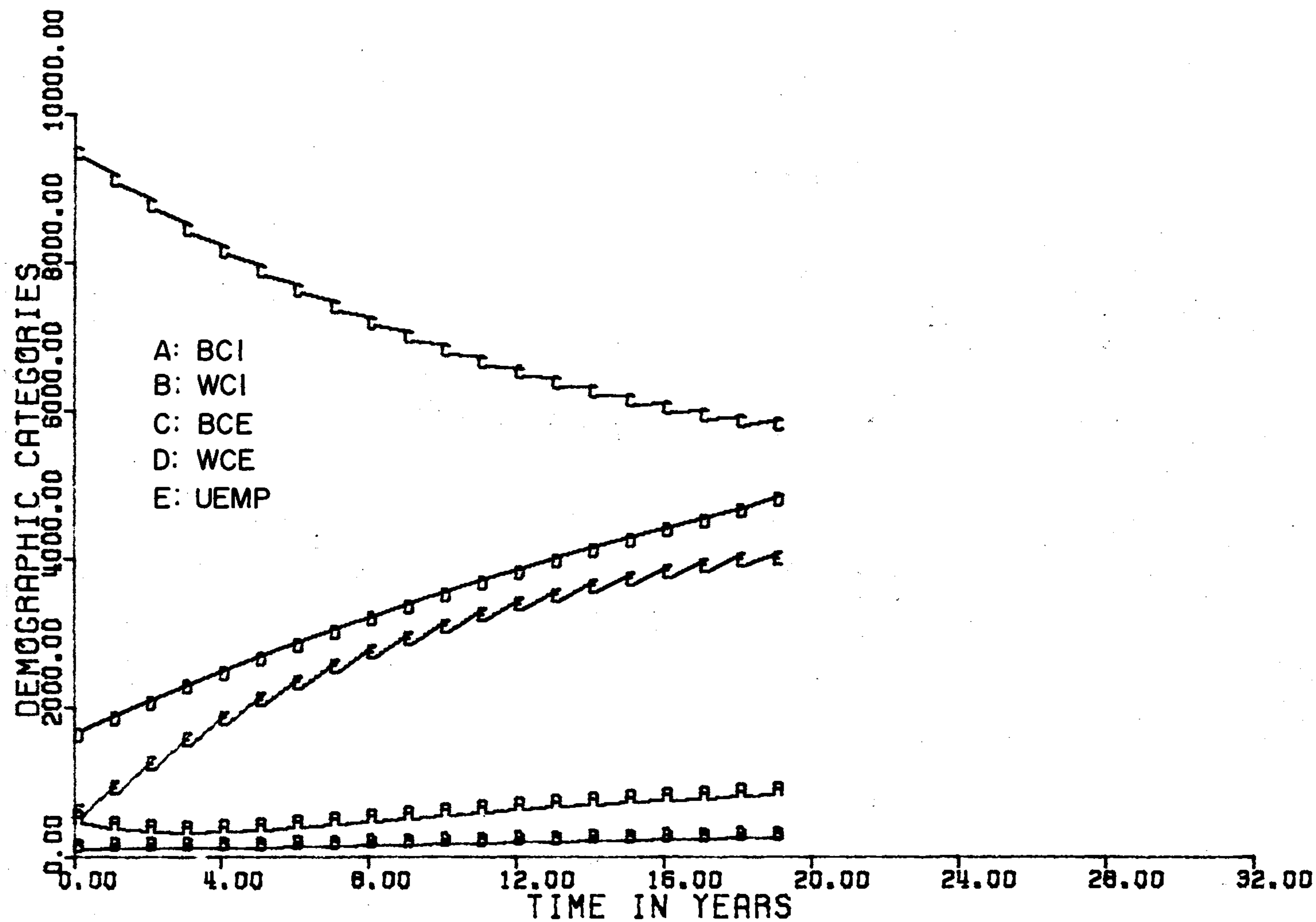


Figure 45. Urban Simulation II, Demographic Sector

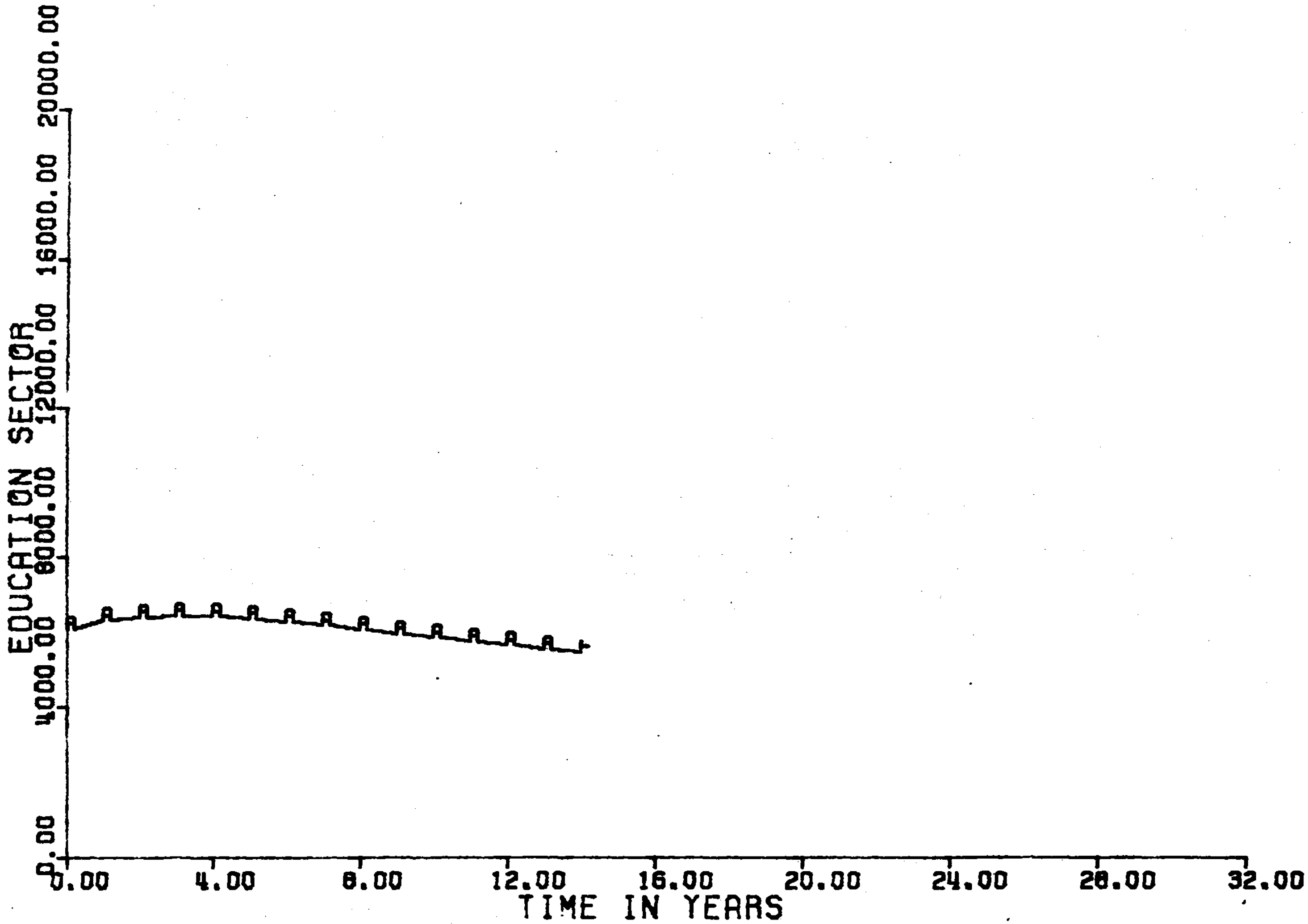


Figure 46. Urban Simulation II, Educational Sector

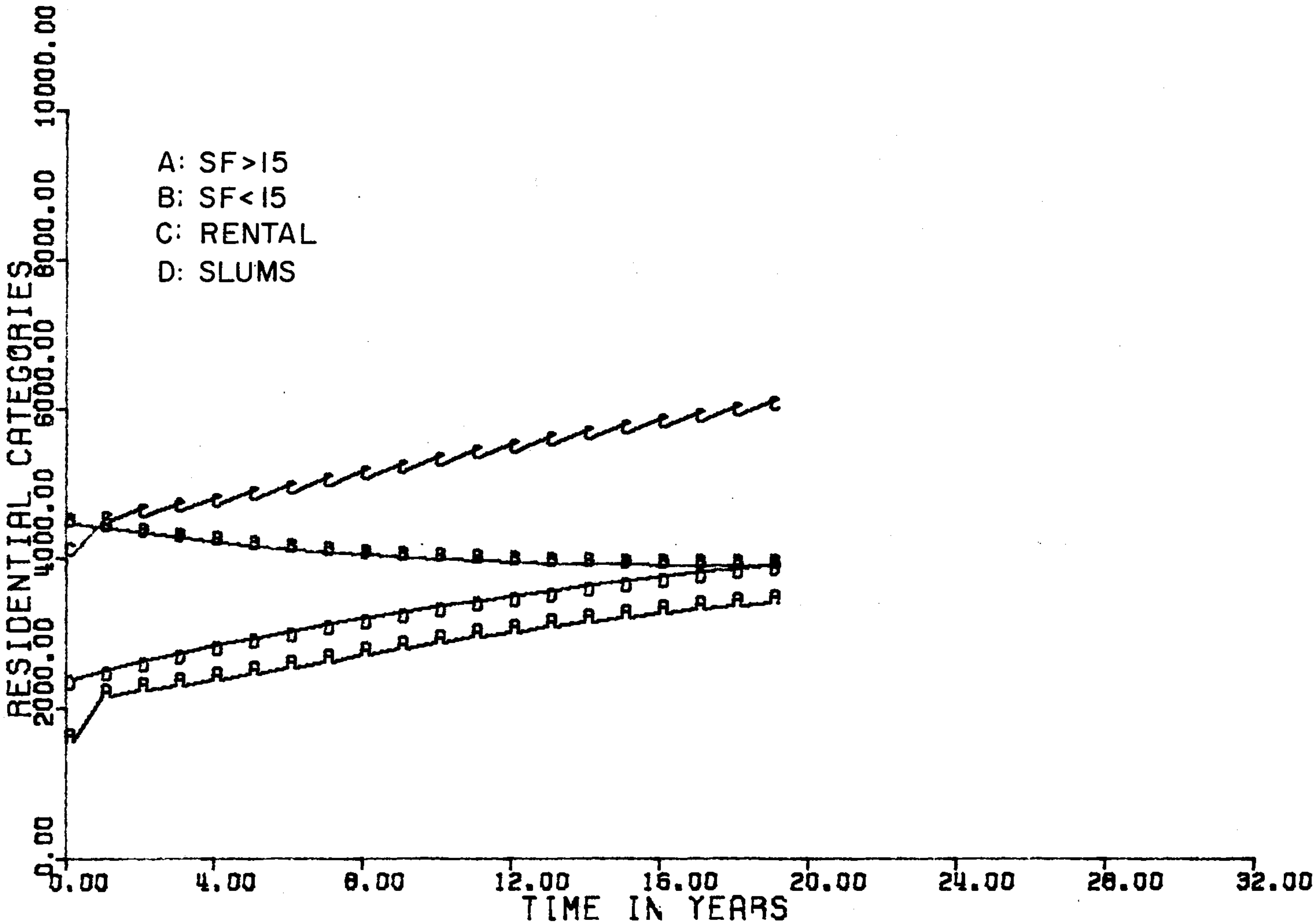


Figure 47. Urban Simulation II, Residential Sector

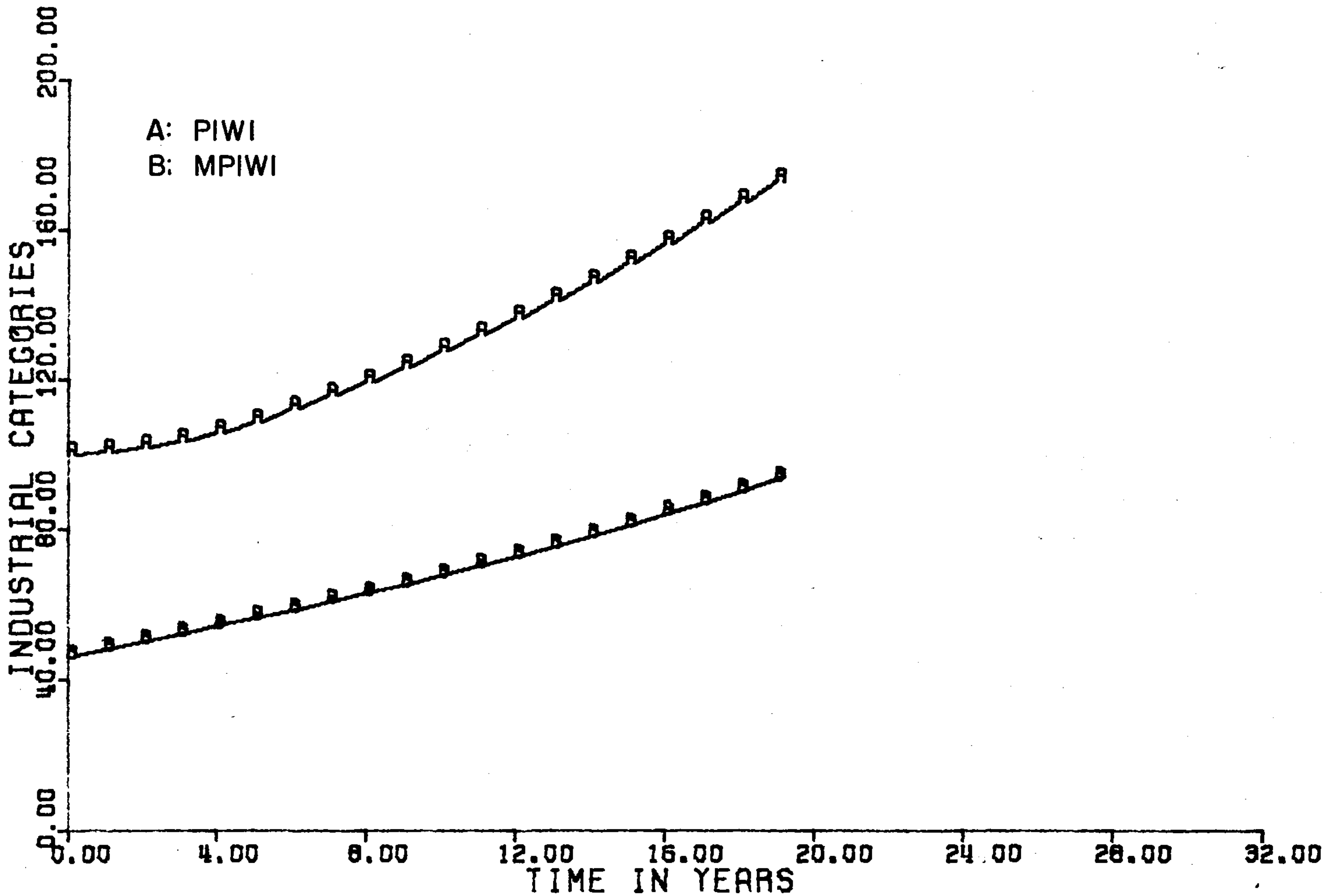


Figure 48. Urban Simulation II, Industrial Sector

allow. The number of BCE and WCE differ by a very small percentage. The model is reacting as it should in that it does not support the incorrect assumption that all of the LOF's are equal. Under this incorrect assumption, and the resulting increase in WCE, the model did produce fewer slums. This is because the CPI was higher.

This shows that the model gives plausible results for plausible assumptions and helps to validate the model.

The last case describes the model's reaction under equilibrium conditions. Equilibrium conditions for the model have been previously defined but they are with respect to fixed exogenous factors. They would best be represented by,

$$\frac{d}{dT}\ (\text{exogenous EU}) \cong 0 \qquad . \qquad (6.5)$$

The equilibrium conditions for this model are

$$LOF_1 = LOF_3 = LOF \cdot e^{-32.35(UE_{1,3})} \qquad (6.6)$$

and

$$LOF_2 = LOF \cdot e^{-129.4(UE_2)} \qquad . \qquad (6.7)$$

The multiplier effect of four shown in exponentiation in LOF_2 represent the approximate ratio of BCE/WCE encountered in the exogenous economic sub-sectors.

The results of this simulation are shown in Figures 49-52 and Tables XIV-XVII. As is shown in Figure 49 and Table XIV, the exogenous economic units, WCE and BCE are fairly constant. Growth is shown endogenous to the minority community however and follows intuitive reasoning.

This condition represents a fairly stagnant economy with practically

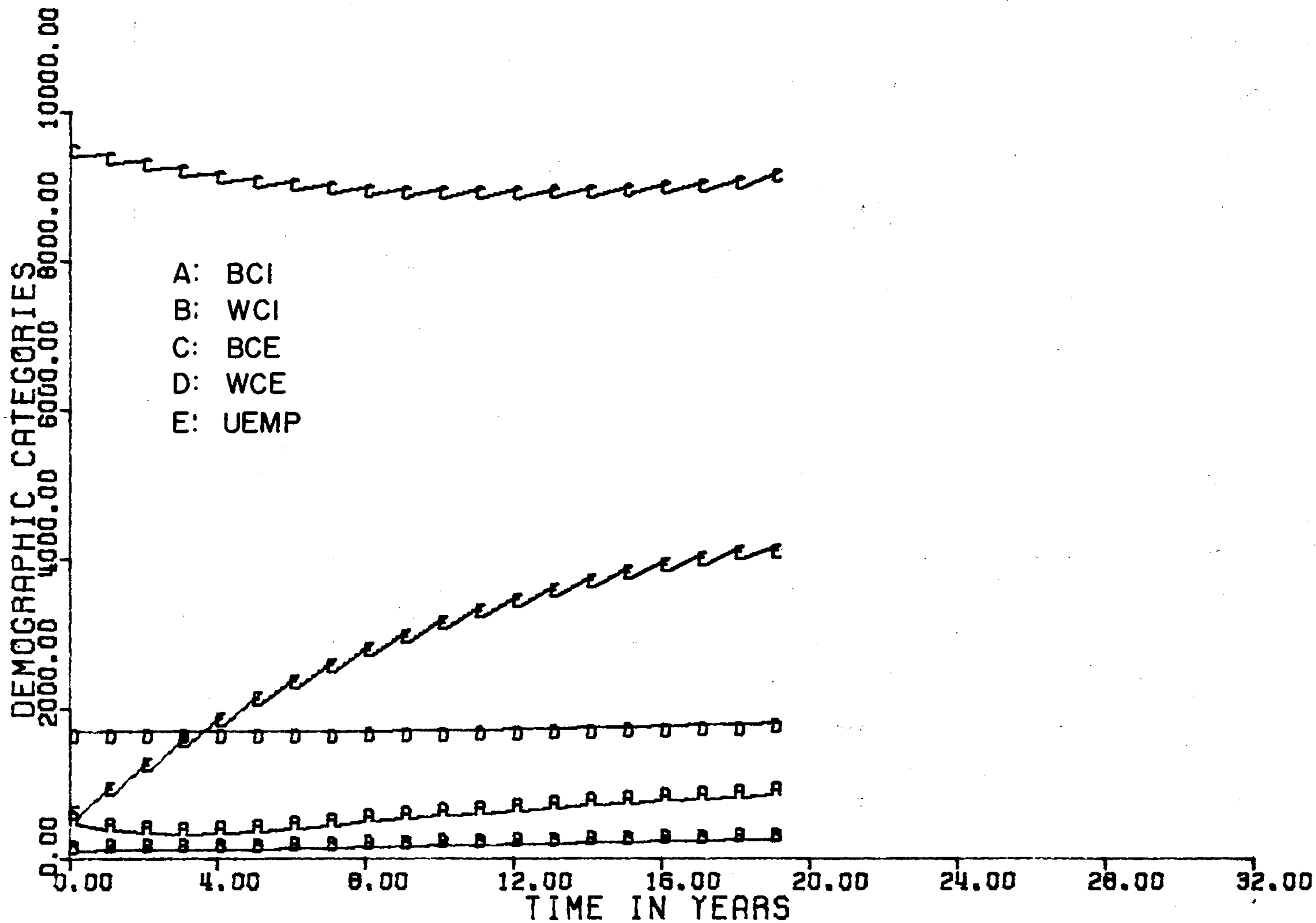


Figure 49. Urban Simulation III, Demographic Sector

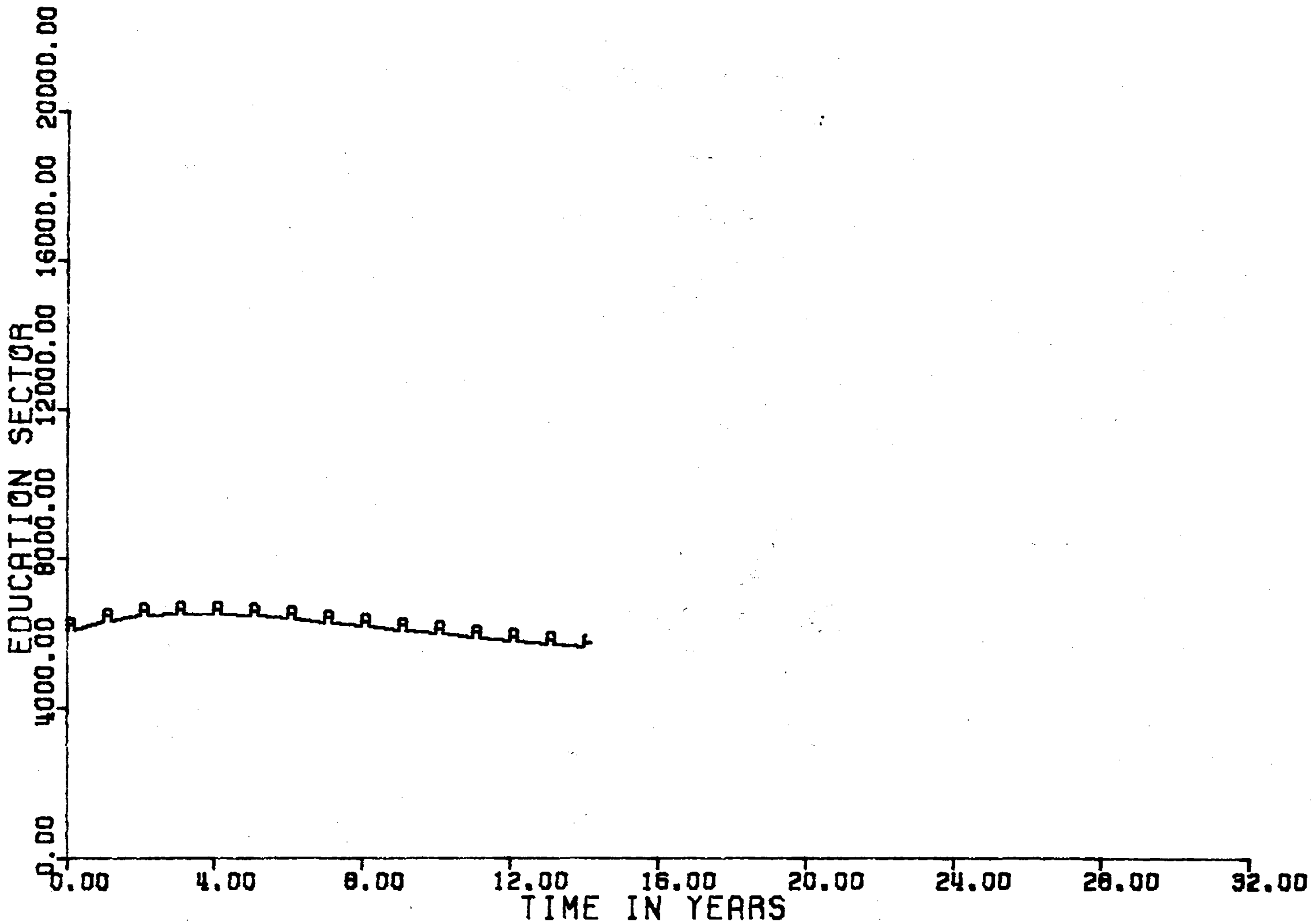


Figure 50. Urban Simulation III, Educational Sector

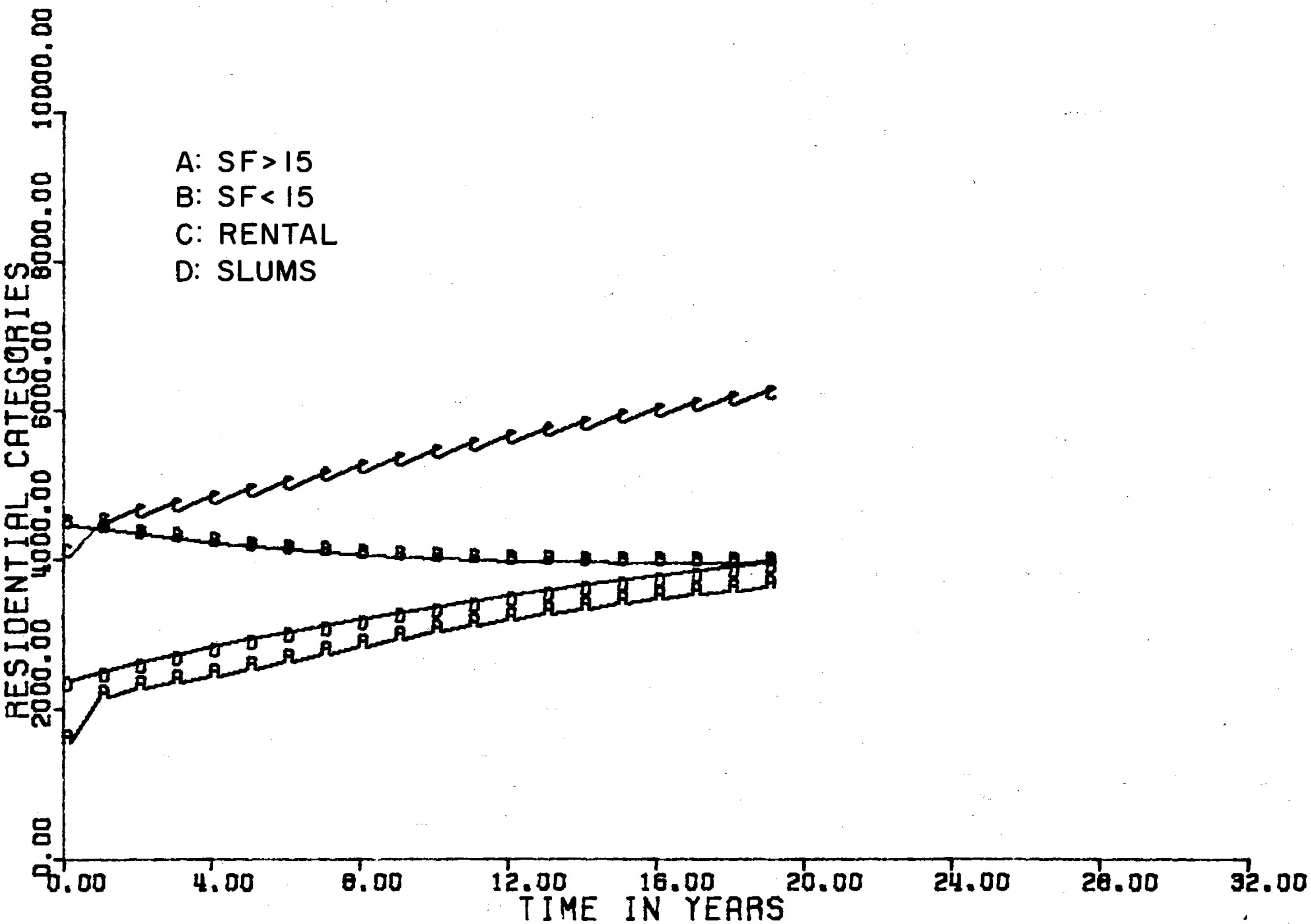


Figure 51. Urban Simulation III, Residential Sector

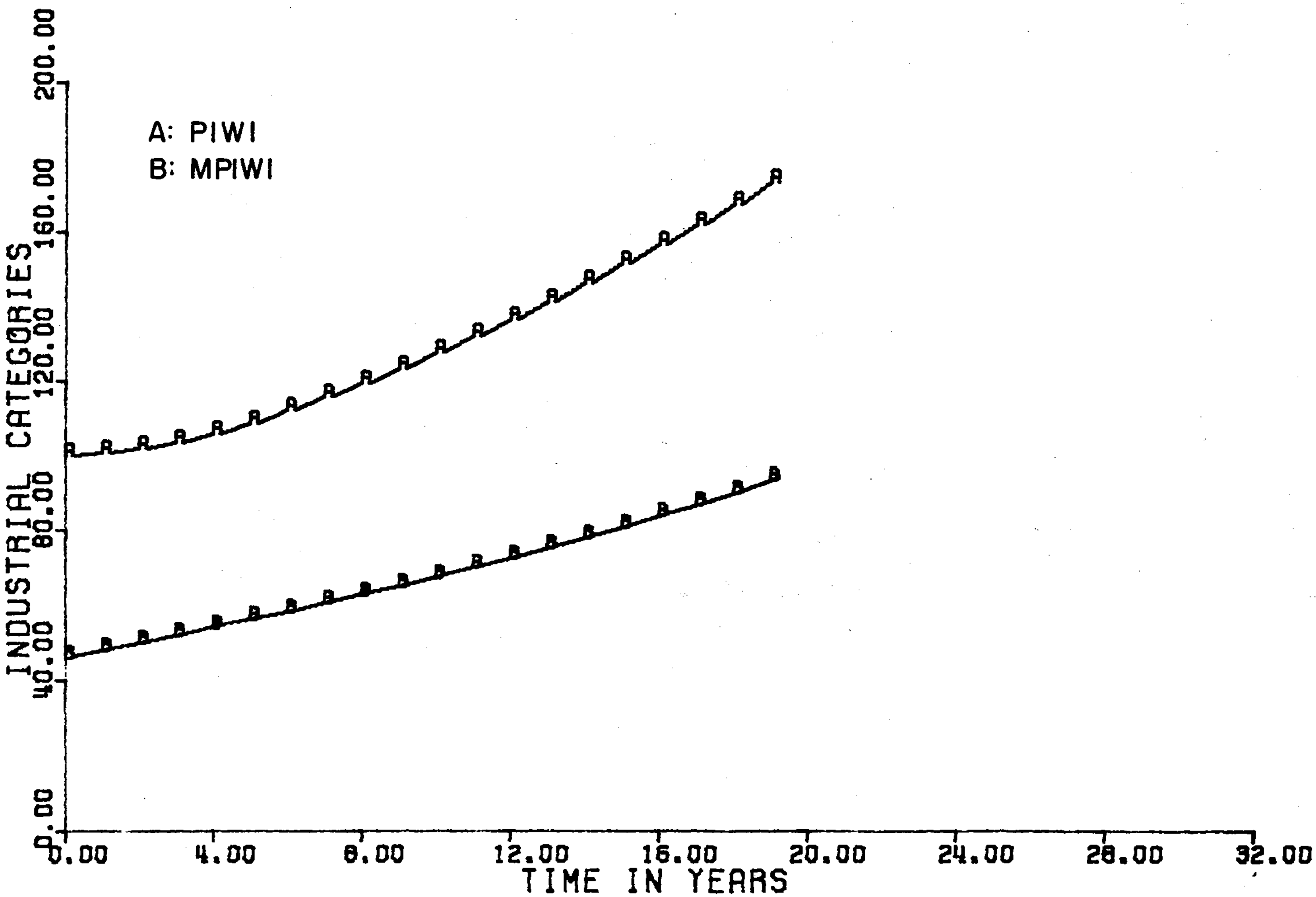


Figure 52. Urban Simulation III, Industrial Sector

no net flows between the minority economic system and majority economic system.

The SF<15 and slums show more growth than in other simulations, whereas the rental units show less. This is due to a greater growth of WCI coupled with a minute growth in the WCE category.

The urban renewal under this simulation (as in others) assumes .75% of slums being demolished each year and 2.25% being returned to SF 15 and rental units.

Other situations, not represented here, have been simulated and were fairly successful. In particular, some of these simulations suggest that urban renewal concentrated toward SF<15 units can do harm to an urban area if the rate of their construction is not carefully controlled. They create vacancies in the model, thereby stalling construction of other units and this can produce more slums.

This concludes the discussion of simulations performed. Because of these simulations, the author believes that urban systems can be modeled intuitively and give plausible solutions.

CHAPTER VII

SUMMARY AND CONCLUSIONS

Summary

This thesis has been devoted to the modeling of the minority urban community using the techniques of systems theory and to developing a systematic approach to the modeling of socio-economic variables. The modeling of the minority urban area has been needed for some period of time. There are a number of discussions concerning the minorities in the literature review, but there was no mathematical system's analysis or model which dealt with the minorities directly. To understand the interactions of an area, the minority community should be understood.

A dynamic nonlinear model of the minority was formed with four sectors. The four sectors, demographic, educational, residential, and industrial, were modeled using economic principles uncommon to the exogenous community and socio-economic variables which distinguish the minority from the majority.

In developing the various sectors or units, models were used and described by difference equations. Dependent sources were derived where needed as a function of the existing minority population and other endogenous urban conditions.

In developing the demographic sector, a non-proliferating model was used and was segmented according to age distribution. The impetus for moving through the demographic sector was aging. The educational sector was embedded within the demographic sector in the appropriate age segment or compartment.

The residential sector was developed using a variable trickle-down theory. This variable was conditioned on the average education which occupied a given residential category. With the life expectancy of physical structures subject to variation, the probability of reaching an assumed equilibrium point was reduced.

The industrial sector was developed in compartment fashion with minority enterprise being based on the growth of the minority population and the existing industrial base.

A unique concept which uses a probabilistic basis was introduced as a method of modeling socio-economic variables. These probabilistic models were based on the axiomatic concept of probability. Structures were devised for these variables based on: (1) whether they were alternate or direct means of achieving success and (2) the sensitivities which the variables needed to accurately describe a portion of the overall urban system.

Two of the variables modeled using this concept were the drop-out rate (DOR) and the limited-opportunity factor (LOF). The DOR was formed to characterize the educational sector. It was noted that after the DOR variable was modeled, it produced intuitively satisfying results for both communities.

The LOF was also a probabilistic type model which was designed to alter the normal economic stream due to disparities which exist in the economic job stream. Opportunity factors based on growth of the industrial base <u>within</u> the minority community were developed.

Simulations were run with each sector of the urban area individually, and then combined to produce the complete minority urban system. These simulations produced some counter-intuitive but plausible results and other results which were more than expected.

Conclusions

The primary objective of this research was to develop methods to enable urban modeling and model a minority urban system. A part of this objective was to perform simulations with this model. The results of this research can be evaluated based on the following observations.

(1) The demographic sector, compartmentally modeled, portrayed the demographic movement expected.

(2) Net migration patterns were plausibly represented by finite dependent sources controlled by existing conditions within the area being modeled.

(3) Endogenous and exogenous opportunity factors were found necessary when modeling minorities.

(4) The educational sector of the model is necessary, in that some of the phenomena which occur in urban areas are explained here by the presence, or absence, of adequate or increasing numbers of educated minority.

(5) A variable filtering chain when modeling the residential sector is necessary because the actual conditions in an urban residential area do vary with time and by implementing this, more reality was introduced into the simulations.

(6) The industrial base in the minority community is necessary and its growth can apparently be substantially large with exogenous investment.

(7) The socio-economic variables based on concepts of axiomatic probability produced plausible quantitative models.

(8) The overall urban model based on systems methods augmented by intuition provides the treatment needed for research in under-defined areas. The new causal models developed here provide the foundation for a methodical and logical approach to urban modeling.

(9) The simulations produced several counterintuitive results. They are:

(a) The disparities in minority opportunities that exist in the exogenous economic system can cause the minority to strengthen its own economic base but causes higher unemployment.

(b) The disparity mentioned previously could possibly cause the physical life expectancy of structures to increase because the educational level increases inside the community apparently as a result of talented minority having to accept work in the endogenous economic system.

(c) Urban renewal, in other simulations, has to be controlled if directed towards low-income housing because of the SF<15 category's place in the housing filtering chain. If not controlled, slums can be inadvertently created.

(d) Under slum demolition, more vacancies were eliminated than just the units demolished, because of a multiplier effect. This happens because the normal pool used for conversion of housing is also reduced.

Recommendations for Further Research

There are a number of extensions or desirable investigations of this research which should be considered. They are the following.

(1) Using the model to simulate other urban areas (Oklahoma City

data were used here).

(2) A disaggregation of the state variables which effect each economic unit, residential category and industrial units.

(3) A restructuring of birth and death rates by economic category based on probabilistic models of the type developed in this research.

(4) A linking of this urban model with another model which structures the majority community to allow variable exogenous factors.

(5) An in depth analysis of the effects of controlability and observability principles (found in systems engineering) as to their effect in controlling divergencies encountered in simulations. The author did extensive work in this area but reached no conclusions concerning global conditions.

Before this work is done, the data base for the urban area needs to be refined. The author believes that enough data exists to characterize an urban area, but perhaps other methods of extraction of relationships from this data need to be developed.

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APPENDIX A

RESIDENTIAL FILTERING MODEL AND DISCUSSION OF THE DESIRED VACANCY RATE

This appendix is devoted to the derivation of values of the variable life expectancies of physical structures and a discussion of the desired vacancy rate.

Residential Filtering Model

> It is a well recognized phenomena that housing tends to move downward in the quality and value scales as it ages. Thus housing that is introduced at or near the top descends gradually through successively lower value strata. It is often contended that the needs for additional housing on the part of the lower income groups can be met by the production of an adequate supply of new housing for the upper income groups. Thus, used homes would be released to be passed down to successively lower levels until the effect reached the bottom of the market. This process is popularly referred to as filtering down and is described most simply as the changing of occupancy as the housing that is occupied by one income group becomes available to the next lower income group.

Thus Ratcliff describes the concept that has been modified for use in this study (64).

In this model, the filtering process is represented as a function of the life expectancy of a physical structure which is variable. This variable tendency is dependent on a function of the educational level of those occupying each residential category.

To establish the physical lifetime of individual structures, a plot shown in Figure 53 is the distribtuion of housing in the Oklahoma City SMSA by age (20). From Grebler (34), we estimate the total life expectancy of housing to be 100 years. From Ratcliff (64), rental units are understood to filter faster and the estimate of their life expectancy is 100 years.

Now by examining Figure 53, the first 40 years are allocated to premium housing because of the density function which results from this distribution function. The remaining 60 years are divided equally

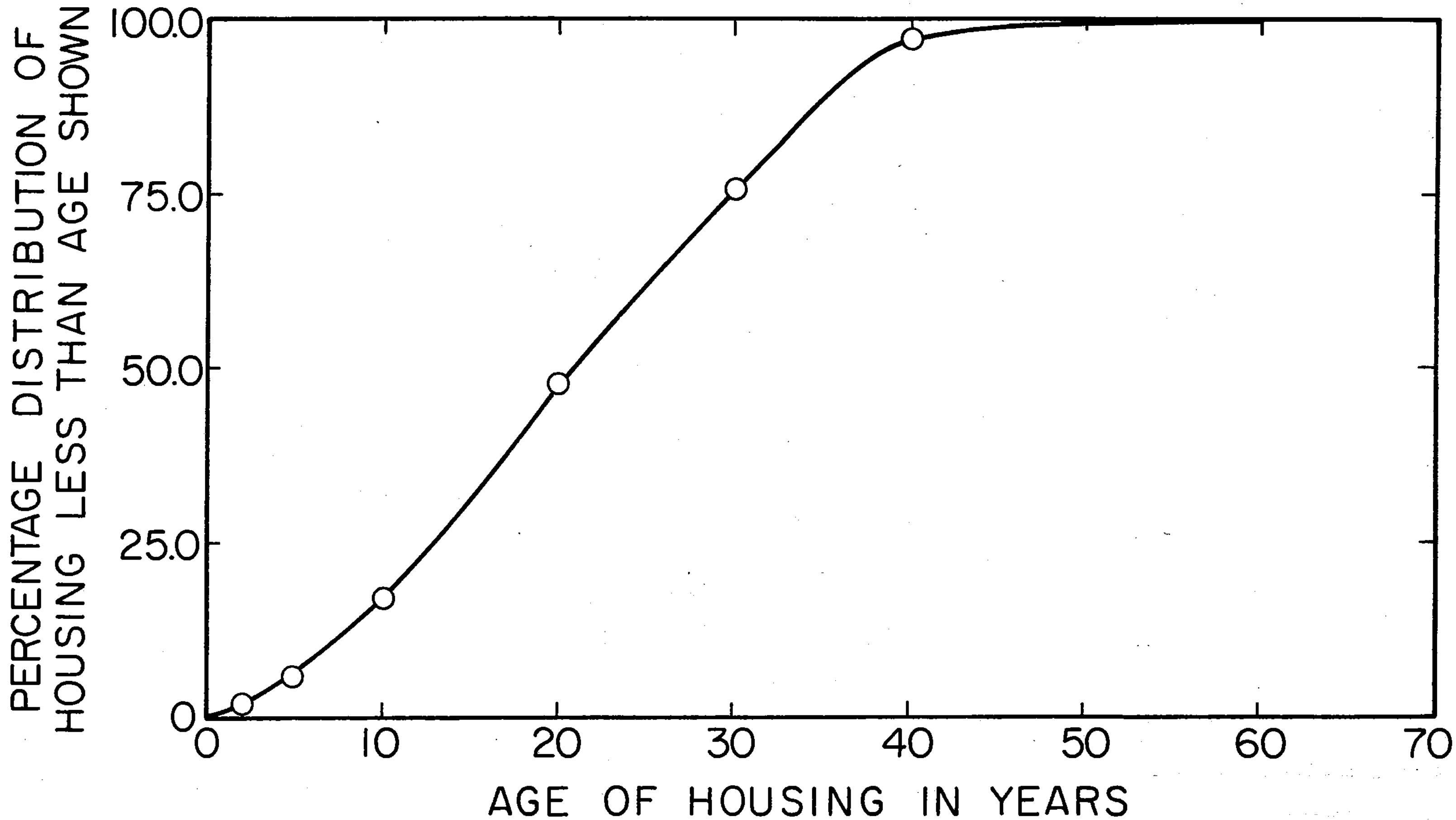

Figure 53. Distribution of Housing vs Age

between SF<15 and slum housing.

Expressing this filtering process in a mathematical equation,

$$\frac{1}{H_3} + \frac{1}{H_5} + \frac{1}{H_6 + H_8 + H_{10}} = 100 \text{ years} \tag{A.1}$$

and

$$\frac{1}{H_9} = 80 \text{ years} \tag{A.2}$$

where the coefficients are assumed from Figure 53.

This filtering is constant as now expressed above and to express it as a variable function of education, Equations (A.1) and (A.2) become,

$$\frac{1}{H_3(\frac{E}{E_0})^{x_1}} + \frac{1}{H_5(\frac{E}{E_0})^{x_2}} + \frac{1}{H_6 + H_8 + H_{10}} = 100 \tag{A.3}$$

$$\frac{1}{H_9(\frac{E}{E_0})^{1/2}} = 80^{1} \tag{A.4}$$

where

$$(\frac{E}{E_0})^{x_n} = 1 \quad \text{for} \quad n = 1, 2 \quad . \tag{A.5}$$

Deriving each coefficient, at time zero,

$$\frac{1}{H_3(\frac{E}{E_0})^{x_1}} = 40 \tag{A.6}$$

[1]The author has assumed that rental units filter slower in the minority community because of the prevalence of slums. This indicates that the structures are allowed to stand longer.

$$H_3 = \frac{1}{40(\frac{E}{E_0})^{x_1}} \tag{A.7}$$

and for the other coefficients,

$$H_5 = \frac{1}{30(\frac{E}{E_0})^{x_2}} \tag{A.8}$$

$$H_6 + H_8 + H_{10} = \frac{1}{30} \tag{A.9}$$

$$H_9 = \frac{1}{80(\frac{E}{E_0})^{x_2}} \quad . \tag{A.10}$$

The coefficients $H_6 + H_8 + H_{10}$ which sum to 1/30 are divided empirically to produce the conversion of slum housing into demolition, rental units, or low income family housing.

The derivation of the coefficients $(E/E_0)^{x_n}$, n = 1, 2 is accomplished by using the ratio of the population 25 year old and over who have completed the educational process (ED) to the total minority population (TMP). The coefficients E and E_0 are,

> E = ratio of successfully educated people to the total minority population at each increment, in time.

$$E(k) = \frac{ED(k)}{TMP(k)} \tag{A.11}$$

where

$$ED(k+1) = ED(k) + \sum_{n=1}^{4} w_{E_n} \cdot E_n \quad . \tag{A.12}$$

E_0 = ratio when the urban model is initiated, namely

$$E_0 = .19 \quad .$$

The ratio $(E/E_0)^{x_n}$ prolongs the lifetime of a structure if it increases. This is based on the assumption that with more education in a household, the structure will last longer due to more attention to maintenance, and probable income to provide maintenance.

Income as a function of education is demonstrated in the census data (21). The powers x_n, n = 1, 2 are introduced as an assumption that the average life of a physical structure is prolonged ten years if (E/E_0) doubles. To increase the lifetime any more than that would not agree with current economist's estimates (34,64).

Solving for x_1 for this derivation, the average lifetime of a SF>15 house is 40 years. Therefore,

$$40\left(\frac{E}{E_0}\right)^{x_n} = 50 \tag{A.13}$$

when

$$\frac{E}{E_0} = 2 \quad . \tag{A.14}$$

Hence

$$2^{x_1} = 1.25 \tag{A.15}$$

$$x_1 = .321 \quad . \tag{A.16}$$

Doing this same for rental units where the average lifetime is 80 years,

$$80\left(\frac{E}{E_0}\right)^{x_2} = 90 \qquad (A.17)$$

when

$$\frac{E}{E_0} = 2 \quad , \qquad (A.18)$$

producing

$$2^{x_2} = 1.125 \qquad (A.19)$$

$$x_2 = .169 \quad . \qquad (A.20)$$

This completes the residential filtering model.

Desired Housing Vacancy Rate (DVR)

The DVR for the urban model is an exogenous input. From the census data, the housing vacancy rate for the total Oklahoma City SMSA is given as 2.25% (20). This is not true however for minority areas in which the observed vacancy rate in slum neighborhoods is four to five times higher. Therefore, in the model, the desired vacancy rate was used as

$$.1125 = \frac{(2.25\%)5}{100} \quad .$$

Throughout the model, exogenous conditions are held constant. This was done to examine the interaction of the minority community.

APPENDIX B

COMPUTER LISTINGS OF SIMULATIONS

TABLE I

DEMOGRAPHIC CATEGORIES, DEMOGRAPHIC SIMULATION I

DEMOGRAPHIC CATEGORIES				
BCI	WCI	BCE	WCE	UEMP
467.5613	81.2080	9395.9727	1564.4294	540.9580
389.3804	93.4099	9304.5430	1558.1104	851.6318
351.9199	100.0724	9220.1758	1552.8723	1172.5215
342.9565	104.1263	9144.4766	1548.9685	1483.9504
351.5798	107.2525	9078.4648	1546.4973	1778.3887
369.7449	110.2818	9022.4883	1545.4670	2054.5229
392.4734	113.5426	8976.4727	1545.8374	2313.0315
416.9036	117.1050	8940.0977	1547.5496	2554.8079
441.4438	120.9228	8912.8789	1550.5242	2780.5771
465.2317	124.9094	8894.2500	1554.6716	2990.9338
487.8191	128.9728	8883.5586	1559.9009	3186.4292
508.9934	133.0317	8880.1367	1566.1143	3367.6211
528.6758	137.0204	8883.3086	1573.2136	3535.1021
546.8625	140.8884	8892.3984	1581.1038	3689.4944
563.5920	144.5990	8906.7500	1589.6934	3831.4421
578.9268	148.1272	8925.7578	1598.8909	3961.6145
592.9402	151.4570	8948.8281	1608.6128	4080.6753
605.7124	154.5795	8975.3984	1618.7786	4189.2734
617.3232	157.4915	9004.9570	1629.3149	4288.0703
633.1807	161.5518	9107.4180	1652.8689	4328.7891

TABLE II

DEMOGRAPHIC CATEGORIES, DEMOGRAPHIC SIMULATION II

DEMOGRAPHIC CATEGORIES				
BCI	WCI	BCE	WCE	UEMP
467.5613	81.2080	9395.9727	1564.4294	540.9580
389.3804	93.4099	9304.5430	1558.1104	851.6318
354.1519	100.7071	9252.0469	1555.3350	1198.6365
347.9417	105.3895	9208.5938	1554.0300	1545.2610
359.8765	109.1667	9175.4609	1554.3376	1882.6848
381.8511	112.8855	9153.2773	1556.3040	2208.7920
408.8235	116.8821	9142.1914	1559.9236	2523.6763
437.8760	121.2294	9142.1094	1565.1687	2827.7261
467.3647	125.8822	9152.7617	1571.9875	3121.2034
496.3828	130.7532	9173.7383	1580.3208	3404.2561
524.4431	135.7490	9204.5625	1590.0979	3677.0198
551.3005	140.7863	9244.7109	1601.2434	3939.6609
576.8486	145.7964	9293.6289	1613.6794	4192.4023
601.0596	150.7262	9350.7617	1627.3296	4435.5156
623.9519	155.5363	9415.5586	1642.1140	4669.3359
645.5701	160.1985	9487.4805	1657.9583	4894.2344
665.9729	164.6941	9566.0078	1674.7910	5110.6094
685.2271	169.0113	9650.6523	1692.5403	5318.8828
703.4021	173.1442	9740.9375	1711.1409	5519.4688
721.9978	177.4262	9878.5859	1741.2813	5537.2969

TABLE III

RESIDENTIAL CATEGORIES, RESIDENTIAL SIMULATION I

HOUSING CATEGORIES

SFAM<15K	SFAM>15K	SLUMS	RENTAL
4439.0000	1562.0000	2252.0000	4034.0000
4348.9688	1526.0916	2319.0247	4023.3542
4261.5977	1491.0808	2379.0056	4013.9587
4176.7656	1456.9453	2432.4578	4005.6802
4094.3613	1423.6631	2479.8608	3998.3960
4014.2668	1391.2131	2521.6609	3991.9929
3936.3794	1359.5742	2558.2727	3986.3667
3860.6023	1328.7263	2590.0823	3981.4209
3786.8450	1298.6497	2617.4485	3977.0671
3715.0222	1269.3250	2640.7051	3973.2239
3645.0544	1240.7334	2660.1633	3969.8164
3576.8665	1212.8567	2676.1113	3966.7756
3510.3870	1185.6768	2688.8181	3964.0388
3445.5500	1159.1763	2698.5337	3961.5479
3382.2927	1133.3384	2705.4900	3959.2498
3320.5559	1108.1465	2709.9031	3957.0964
3260.2842	1083.5842	2711.9741	3955.0437
3201.4246	1059.6362	2711.8896	3953.0510
3143.9275	1036.2869	2709.8232	3951.0818
3087.7461	1013.5212	2705.9363	3949.1028

TABLE IV

RESIDENTIAL CATEGORIES, RESIDENTIAL SIMULATION II

HOUSING CATEGORIES

SFAM<15K	SFAM>15K	SLUMS	RENTAL
4439.0000	1562.0000	2252.0000	4034.0000
4348.9688	1526.0916	2319.0247	4023.3542
4261.5977	1491.0808	2379.0056	4013.9587
4176.7656	1456.9453	2432.4578	4005.6802
4094.3613	1423.6631	2479.8608	3998.3960
4014.2668	1391.2131	2521.6609	3991.9929
3936.3794	1359.5742	2558.2727	3986.3667
3860.6023	1328.7263	2590.0823	3981.4209
3786.8450	1298.6497	2617.4485	3977.0671
3715.0222	1269.3250	2640.7051	3973.2239
3645.0544	1240.7334	1000.0000	3969.8164
3563.0317	1212.8567	1112.7908	3939.1062
3483.9858	1185.6768	1215.8843	3910.6597
3407.7546	1159.1763	1309.9734	3884.2871
3334.1858	1133.3384	1395.7034	3859.8123
3263.1377	1108.1465	1473.6743	3837.0723
3194.4780	1083.5842	1544.4446	3815.9160
3128.0828	1059.6362	1608.5332	3796.2039
3063.8357	1036.2869	1666.4238	3777.8062
3001.6289	1013.5212	1718.5659	3760.6033

TABLE V

INDUSTRIAL SIMULATION

INDUSTRIAL CATEGORIES

PEWIE	PIWI	XPIWI
1000.0000	100.0000	46.0000
999.9998	102.4715	48.0400
999.9995	105.0587	50.1214
999.9993	107.7623	52.2466
999.9990	110.5832	54.4182
999.9988	113.5225	56.6386
999.9985	116.5815	58.9102
999.9983	119.7617	61.2357
999.9980	123.0647	63.6173
999.9978	126.4922	66.0578
999.9976	130.0465	68.5595
999.9973	133.7295	71.1250
999.9971	137.5436	73.7570
999.9968	141.4914	76.4580
999.9966	145.5755	79.2307
999.9963	149.7987	82.0778
999.9961	154.1640	85.0020
999.9958	158.6746	88.0060
999.9956	163.3337	91.0928
999.9954	168.1450	94.2652

TABLE VI

URBAN SIMULATION I, DEMOGRAPHIC SECTOR

DEMOGRAPHIC CATEGORIES

BCI	WCI	BCE	WCE	UEMP
467.5613	81.2080	9395.9680	1564.4280	540.9580
406.8967	93.4099	9286.0780	1547.4870	858.4180
375.9485	99.0189	9176.6830	1531.9240	1197.6930
368.4675	103.1747	9067.8550	1518.0330	1537.7750
378.6987	106.9874	8960.3120	1505.9430	1868.5100
400.8364	111.1284	8854.5780	1495.6850	2185.9810
430.4370	115.9317	8751.0190	1487.2200	2489.2880
466.7788	121.9703	8649.8510	1480.4860	2775.6940
507.2769	129.1983	8551.2420	1475.3830	3046.1560
549.6155	137.3645	8455.2570	1471.8070	3302.4000
592.2646	146.1884	8361.8980	1469.6540	3545.7970
634.4016	155.4361	8271.0930	1468.8220	3777.1650
675.6658	164.9394	8182.7460	1469.2160	3996.9480
715.9434	174.5857	8096.7140	1470.7340	4205.3940
755.2285	184.3014	8012.8630	1473.2860	4402.7420
793.5552	194.0389	7931.0460	1476.7790	4589.2220
830.9656	203.7671	7851.1280	1481.1230	4765.1210
867.5034	213.4663	7772.9800	1486.2350	4930.7460
903.2085	223.1241	7696.4880	1492.0350	5086.4100
947.6440	235.1964	7680.3120	1510.0070	5196.7260

TABLE VII

URBAN SIMULATION I, RESIDENTIAL SECTOR

HOUSING CATEGORIES

SFAM<15K	SFAM>15K	SLUMS	RENTAL
4439.0000	1562.0000	2252.0000	4034.0000
4374.3200	2150.1380	2375.3240	4469.5000
4309.6600	2271.7280	2495.2690	4579.0540
4250.5030	2352.3360	2608.2710	4662.0620
4197.2730	2438.4130	2714.6940	4751.1870
4149.9250	2530.1810	2815.2340	4846.3240
4108.2570	2623.5290	2910.5150	4944.3040
4071.9610	2715.4190	3001.0670	5042.7610
4040.7070	2804.0030	3087.3370	5140.1870
4014.1280	2888.1530	3169.7080	5235.6320
3991.8350	2967.2210	3248.5190	5328.5190
3973.4390	3040.8550	3324.0600	5418.5030
3958.5580	3108.8370	3396.5880	5505.3470
3946.8270	3171.0540	3466.3240	5588.9140
3937.9020	3227.4560	3533.4610	5669.0970
3931.4590	3278.0640	3598.1650	5745.8590
3927.1990	3322.9480	3660.5780	5819.1990
3924.8430	3362.2260	3720.8270	5889.1440
3924.1340	3396.0520	3779.0170	5955.7570
3924.8330	3424.5970	3835.2440	6019.1130

TABLE VIII

URBAN SIMULATION I, INDUSTRIAL SECTOR

INDUSTRIAL CATEGORIES		
PEWIE	PIWI	XPIWI
1000.0000	100.0000	46.0000
1000.0000	101.5020	48.0400
1000.0000	103.1439	50.0971
1000.0000	104.9238	52.1747
1000.0000	106.8399	54.2760
1000.0000	108.8931	56.4043
1000.0000	111.2280	58.5625
1000.0000	113.8393	60.7576
1000.0000	116.6855	62.9960
1000.0000	119.7267	65.2831
1000.0000	122.9359	67.6235
1000.0000	126.2988	70.0206
1000.0000	129.8094	72.4779
1000.0000	133.4659	74.9983
1000.0000	137.2692	77.5849
1000.0000	141.2210	80.2408
1000.0000	145.3233	82.9689
1000.0000	149.5787	85.7723
1000.0000	153.9898	88.6540
1000.0000	158.5595	91.6172

TABLE IX

URBAN SIMULATION II, DEMOGRAPHIC SECTOR

DEMOGRAPHIC CATEGORIES

BCI	WCI	BCE	WCE	UEMP
467.5613	81.2080	9395.9680	1564.4280	540.9580
389.3801	93.4099	9036.6250	1778.8470	851.6318
349.4353	98.4372	8698.9140	1987.6660	1173.6660
338.1826	101.8446	8385.3120	2190.5760	1486.4170
347.0442	105.0211	8095.6670	2387.4300	1780.9870
373.2378	109.7419	7829.1990	2578.2850	2049.6790
410.0486	116.3270	7584.8590	2763.2880	2294.1420
451.6421	124.5396	7361.3980	2942.6660	2519.1720
494.2705	133.9191	7157.5350	3116.6950	2728.8540
536.0933	144.0254	6971.9250	3285.6660	2925.5140
576.4702	154.5381	6803.2690	3449.8760	3110.2000
615.3181	165.2522	6650.2730	3609.6050	3283.3940
652.7310	176.0423	6511.7180	3765.1130	3445.4460
688.8196	186.8296	6386.4370	3916.6390	3596.7510
723.6731	197.5623	6273.3510	4064.3930	3737.7630
757.3638	208.2065	6171.4490	4208.5700	3868.9880
789.9551	218.7404	6079.7920	4349.3510	3990.9430
821.5085	229.1514	5997.5190	4486.8820	4104.1480
852.0845	239.4338	5923.8350	4621.3200	4209.1130
887.4243	251.3597	5893.1790	4790.7420	4212.6090

TABLE X

URBAN SIMULATION II, RESIDENTIAL SECTOR

HOUSING CATEGORIES			
SFAM<15K	SFAM>15K	SLUMS	RENTAL
4439.0000	1562.0000	2252.0000	4034.0000
4374.3200	2150.1380	2375.3240	4469.5000
4308.7380	2249.2590	2495.0690	4563.0660
4248.3160	2311.0220	2607.5090	4632.4760
4193.5890	2384.9060	2713.0880	4712.6790
4144.6320	2470.2750	2812.5900	4803.0030
4101.4490	2562.3740	2906.7040	4899.8120
4063.8880	2657.4670	2996.0190	5000.2180
4031.7020	2753.5640	3081.0440	5102.6600
4004.5880	2849.6070	3162.2310	5206.2770
3982.2250	2944.8410	3239.9790	5310.4640
3964.2890	3038.6510	3314.6400	5414.7140
3950.4630	3130.5190	3386.5200	5518.6210
3940.4450	3220.0390	3455.8900	5621.8390
3933.9450	3306.8990	3522.9860	5724.1130
3930.6890	3390.8730	3588.0160	5825.2300
3930.4190	3471.8050	3651.1600	5925.0420
3932.8930	3549.5980	3712.5790	6023.4490
3937.8810	3624.1850	3772.4150	6120.3590
3945.1710	3695.5530	3830.7920	6215.7300

TABLE XI

URBAN SIMULATION II, INDUSTRIAL SECTOR

INDUSTRIAL CATEGORIES		
PEWIE	PIWI	XPIWI
1000.0000	100.0000	46.0000
1000.0000	101.5020	48.0400
1000.0000	103.1439	50.0971
1000.0000	104.9508	52.1747
1000.0000	107.2714	54.2767
1000.0000	110.1090	56.4157
1000.0000	113.3731	58.6043
1000.0000	116.9623	60.8526
1000.0000	120.8045	63.1681
1000.0000	124.8605	65.5565
1000.0000	129.1132	68.0224
1000.0000	133.5560	70.5700
1000.0000	138.1861	73.2032
1000.0000	143.0020	75.9258
1000.0000	148.0025	78.7415
1000.0000	153.1872	81.6542
1000.0000	158.5559	84.6673
1000.0000	164.1090	87.7845
1000.0000	169.8479	91.0094
1000.0000	175.7740	94.3454

TABLE XII

URBAN SIMULATION III, DEMOGRAPHIC SECTOR

DEMOGRAPHIC CATEGORIES

BCI	WCI	BCE	WCE	UEMP
467.5613	81.2080	9395.9680	1564.4280	540.9580
389.3801	93.4099	9304.5390	1558.1090	851.6318
349.4353	98.4372	9220.1670	1552.8710	1174.9170
338.1079	101.8065	9144.3000	1548.9800	1488.5820
346.7805	104.9029	9078.0420	1546.5300	1783.0490
372.6873	109.5144	9021.8200	1545.5190	2050.0600
409.0718	115.9626	8975.6750	1545.8650	2290.9090
450.0427	124.0054	8939.3900	1547.4670	2510.2080
491.8096	133.1751	8912.5660	1550.2270	2711.9530
532.5015	143.0236	8894.6950	1554.0510	2898.4380
571.4648	153.2242	8885.1710	1558.8490	3070.7360
608.6113	163.5678	8883.3550	1564.5290	3229.3920
644.0327	173.9256	8888.5820	1570.9990	3374.8460
677.8435	184.2170	8900.1790	1578.1690	3507.5950
710.1370	194.3896	8917.5190	1585.9490	3628.2230
740.9927	204.4092	8939.9800	1594.2500	3737.3620
770.4817	214.2546	8966.9720	1602.9950	3835.6680
798.6736	223.9138	8997.9330	1612.1030	3923.7930
825.6372	233.3825	9032.3550	1621.5040	4002.3780
859.3701	245.0998	9139.9960	1643.7000	4018.7200

TABLE XIII

URBAN SIMULATION III, RESIDENTIAL SECTOR

HOUSING CATEGORIES

SFAM<15K	SFAM>15K	SLUMS	RENTAL
4439.0000	1562.0000	2252.0000	4034.0000
4374.3200	2150.1380	2375.3240	4469.5000
4309.4720	2274.3840	2495.2770	4581.1710
4250.2220	2360.6540	2608.3170	4668.5420
4197.0030	2454.0500	2714.8240	4763.3350
4149.8120	2553.7540	2815.4970	4864.6910
4108.5350	2655.3730	2910.9450	4969.1670
4072.9120	2755.5300	3001.6870	5074.1210
4042.5750	2852.5830	3088.1730	5178.2100
4017.1150	2945.7820	3170.8010	5280.7920
3996.1150	3034.6690	3249.9210	5381.4490
3979.1700	3118.9000	3325.8390	5479.8780
3965.8920	3198.2120	3398.8230	5575.8240
3955.9190	3272.4370	3469.1050	5669.1050
3948.9140	3341.4760	3536.8880	5759.6010
3944.5630	3405.3190	3602.3460	5847.2420
3942.5760	3464.0020	3665.6340	5932.0030
3942.6840	3517.6090	3726.8850	6013.8900
3944.6400	3566.2480	3786.2170	6092.9290
3948.2180	3610.0580	3843.7320	6169.1670

TABLE XIV

URBAN SIMULATION III, INDUSTRIAL SECTOR

INDUSTRIAL CATEGORIES

PEWIE	PIWI	XPIWI
1000.0000	100.0000	46.0000
1000.0000	101.5020	48.0400
1000.0000	103.1439	50.0971
1000.0000	104.9467	52.1747
1000.0000	107.2619	54.2766
1000.0000	110.0931	56.4154
1000.0000	113.3490	58.6035
1000.0000	116.9273	60.8512
1000.0000	120.7541	63.1658
1000.0000	124.7893	65.5530
1000.0000	129.0146	68.0172
1000.0000	133.4225	70.5624
1000.0000	138.0094	73.1923
1000.0000	142.7730	75.9106
1000.0000	147.7116	78.7208
1000.0000	152.8239	81.6264
1000.0000	158.1092	84.6307
1000.0000	163.5675	87.7371
1000.0000	169.1994	90.9489
1000.0000	175.0063	94.2694

VITA

William Edward Hogan Jr.

Candidate for the Degree of

Doctor of Philosophy

Thesis: DYNAMIC NON-LINEAR MODEL OF AN URBAN SITUATION WITH PROBABILISTIC TYPE CAUSAL MODELS

Major Field: Electrical Engineering

Biographical:

Personal Data: Born in Montgomery, Alabama, September 30, 1942, the son of Mr. and Mrs. William E. Hogan.

Education: Graduated from Lincoln High School, Dallas, Texas; attended National Science Foundation Summer Workshop in Mathematics at Prairie View A & M College, Prairie View, Texas; received the Bachelor of Science degree from Oklahoma State University, Stillwater, Oklahoma in August, 1965, with a major in Electrical Engineering; received a Master of Science degree from Southern Methodist University, Dallas, Texas in December, 1969, with a major in Electrical Engineering; completed requirements for the Doctor of Philosophy degree at Oklahoma State University, with a major in Electrical Engineering, in July, 1973.

Professional Experience: Assistant Engineer, U. S. Naval Electronics Lab., McAlester, Oklahoma, Summer, 1963; Assistant Engineer, Collins Radio Co., Dallas, Texas, Summer, 1964; Design Engineer, Collins Radio Co., Dallas, Texas, 1965-1968; Project Engineer, Collins Radio Co., Dallas, Texas, 1968-1970; Graduate Research Assistant, Oklahoma State University, Stillwater, Oklahoma, 1970-1973.