EXISTENCE CRITERIA OF SINGLE AND MULTI-LOOP MECHANISMS WITH ONE GENERAL CONSTRAINT

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CHAPTER I

INTRODUCTION

Background and Purpose of Obtaining Existence Criteria

The concept of mobility was something of a mystery until it was mathematically formulated by Grübler (1, 2, 3)[1] in 1884, Delassus (4, 5, 6) in 1900, Malytcheff (7) in 1923, Bricard (8, 9) in 1927, and Kutzbach (10, 11, 12, 13) in 1929.

Given an arbitrary arrangement of rigid bodies connected by kinematic joints, Grübler's mobility criteria will determine the number of degrees of freedom or mobility of the system. Artobolevski and Dobrovolskii (14, 15) proposed more general mobility criteria which attempt to account for the existence of a number of overconstrained linkages. Sharikov (16) used the theory of screws to study the classification and existence of such linkages. Sharikov's method is geometrical in nature and it has its limitations. Voinea and Atanasiu (17) have examined the mobility of linkages by considering

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[1] Numbers in parentheses denote the references given in the Bibliography.

the relationship between the classical theory of screws and line geometry. This study, though incomplete, has influenced many of the later studies in this area.

Myard (18) and Goldberg (19) derived overconstrained linkages by combining Bennet linkages in such a manner that one or more members become redundant.

The existence of overconstrained linkages has also been studied by Soni (20, 21, 22, 23, 24) and by Soni and Harrisberger (25, 47). The basic tool used is the 3 x 3 screw matrix. The method consists in examining the residual coefficient matrix (RCM) of a linkage. The rank of RCM is directly related to the mobility of the linkage. The number of columns is related to the number of general constraints. The number of passive constraints or idle freedoms is represented by the difference between the number of rows and the number of columns. Using this procedure, Soni (21) has investigated the existence criteria of linkages with one general constraint by examining some of the six-link, six revolute mechanisms. The properties of the RCM also permit it to be used as a basis for the classification of mechanisms (21).

An alternate approach to the study of mechanism mobility is based on the use of vector algebra. A general method for obtaining the compatibility conditions of mechanisms by using this method has been proposed by Soni and Pelecudi (26, 46).

Moroshkin's (92) approach is based on the number of closed loops in a mechanism. In this method, transformation equations are used to describe the basic geometry of a mechanism. The number of independent transformation equations, which is also the rank of the system of equations, is determined by the configuration of the mechanism. The mobility of the mechanism is related to the number of degrees of freedom in all the joints and the rank of the system of the transformation equations.

Another method is based on the classical theory of screws. A detailed account of the theory has been given by Ball (112) in 1900. An excellent review of the theory has also been given by Henrici (114). Sharikov (16), Voinea and Atanasiu (17) have employed this theory to examine the mobility of the mechanisms. In this method, a mechanism is regarded as a group or a collection of screws in space. The screws define a screw system whose order is determined by the configuration of the mechanism and the pitch values of the screws. The mobility of the mechanism is related to the total number of screws in the mechanism and the order of the screw system formed by them.

Myard (18), Goldberg (19), Voinea and Antansiu (17), and Dimentberg and Yoslovich (29) are among those who have proposed various linkages with two general constraints. Using the five-bar linkage (5H) proposed by Voinea and Atanasiu (17) as a basis, Hunt

(30, 31, 32) and Waldron (33, 34, 35, 36, 37) have recently proposed a class of linkages derivable from this linkage for instantaneous mobility. Waldron has also proposed some single and multi-loop linkages by combining the known Delassus overconstrained three and four-link mechanisms.

The various methods described above for examining the mobility of mechanisms have contributed considerably to a better understanding of the nature of space mechanisms. However, all these methods suffer from one serious shortcoming, that they are all essentially dealing only with instantaneous or transitory mobility and not with finite mobility. This feature makes these methods unsuitable for examining the existence criteria of mechanisms in which there are conditions imposed not only on the twist angles, but also on the other constant kinematic parameters. This drawback is overcome by the passive coupling method developed by Dimentberg and first introduced by him in 1948 (38, 39, 40). In this method, the existence criteria of an overconstrained mechanism are obtained from the displacement relationships of an appropriate zero family mechanism (20, 21, 47) by imposing suitable passive coupling conditions on the latter, by making some of the joints passive. The method not only assures finite mobility, but is also capable of yielding the necessary conditions for the existence of the derived mechanism.

For finite mobility, one would therefore prefer to adopt the passive coupling technique proposed by Dimentberg (38, 39, 40). Dimentberg's passive coupling approach was extended by Pamidi (41) to develop the existence criteria of 5R spatial mechanism with two passive constraints. Further extension of the work led Soni, Pamidi and Dukkipati (42, 43) and Soni (27) to develop the necessary and sufficient existence criteria of four and five-link mechanisms with one and two passive couplings. Design procedures of mechanisms with a passive coupling are also recently proposed by Soni and Harrisberger (44, 45, 46).

The successful application of Dimentberg's technique to study passive coupling conditions of single loop four and five-link mechanisms with various types of pairing conditions (consisting of R, P, H, C and S pairs)[2] by Pamidi (41), Soni, Dukkipati and Pamidi (42, 43), and Soni (27) makes it possible to further extend its application to study passive coupling conditions of six-link, single and multi-loop spatial mechanisms. A systematic investigation of these mechanisms has been greatly hindered so far by the non-availability of closed-form displacement relationships of spatial six-link mechanisms. However, the results recently obtained by Soni and Dukkipati (120) make it possible to obtain the existence criteria of these mechanisms by using Dimentberg's passive coupling technique.

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[2] Throughout this study, R, P, H, C, and S are used to denote the revolute, prism, helical, cylinder and spherical pairs respectively.

The concept of general constraints suggests that there are certain specific geometrical conditions which must be imposed on a multi-loop kinematic chain if it is to have one degree of freedom. According to the mobility criteria of Artobolvski and Dobrovolskii (14, 15) and Voinea and Atanasiu (17) that one general constraint is defined by a specific orientation of the axes of the pairs along with some specific geometrical relationship between the constant kinematic parameters of the chain.

The mobility criteria permits us to enumerate all possible single and multi-loop mechanisms with or without passive couplings. For example, when there are no general constraints, Soni and Harrisberger (21, 23, 24) showed that there are one type and 28 different kinds of single-loop, six-link mechanisms with one general constraint. A systematic enumeration by Soni and Robertson (28) showed the possible existence of nearly 350 constrained kinematic chains possessing one general constraint. In a similar way, when there are no general constraints ($m = 0$), Huang and Soni (48) showed that there are seven different types and 494 different kinds of six-link, two-loop single degree of freedom space chains which do not have general constraints. In a similar way, Huang and Soni showed that there could exist a maximum of 4 different types and 287 different kinds of six-link, two-loop single degree of freedom mechanisms with one general constraint, and two different types and 119 kinds of six-link, two-loop single degree of freedom mechanisms with two general constraints, and one type and 36 different kinds of six-link, two-loop single degree of freedom mechanisms requiring three general constraints for mobility.

A systematic enumeration of the six-link, two loop space kinematic chains with Zero general constraint shows (48) the possible existence of nearly 365,025 constrained kinematic chains. A similar survey by Soni and Huang enumerated the possible existence of 146,313 constrained kinematic chains possessing one general constraint, 31,509 constrained kinematic chains possessing two general constraints and 2,430 constrained kinematic chains possessing three general constraints. Thus there is a possibility for the existence of 180,252 constrained kinematic chains possessing either 1, 2 or 3 general constraints. The necessary and sufficient existence criteria for these mechanisms are not yet known.

The objective of the present study is to investigate the mobility and the existence of single and multi-loop mechanisms with one general constraint. Linkages with two passive couplings are representative of the class of two-loop linkages. It is proposed to extend Dimentberg's theory of passive coupling and the 3 x 3 matrices with dual-number elements to develop a generalized approach to derive the existence criteria of multi-loop overconstrained mechanisms. Using this method it is proposed to investigate the existence of six-link, one and two-loop linkages with one general constraint and having lower kinematic pairs. The proposed method, besides being useful in the study of the mobility and existence of linkages, will also facilitate the closed form displacement relationships for the newly discovered mechanisms which can be utilized for their type determination, kinematic analysis and synthesis.

Specifically, the objectives of the present study are:

1. To obtain the existence criteria of six-link, single-loop, 3H+3P space mechanisms. Besides explaining the existence of known five and six-link mechanisms, the derived criteria should also reveal the existence of other mechanisms.

2. To obtain the existence criteria of six-link, two-loop, R-R-C-C-C-R-C, R-R-C-C-C-P-C, R-C-C-R-C-C-R, and R-C-C-R-C-C-P space mechanisms. The derived criteria should facilitate the investigation of the existence of such mechanisms.

In the next chapter, the Dimentberg's passive coupling method employed for the above purpose is discussed in detail. In the remaining chapters, the results of the objectives mentioned above are presented.

Definitions and Explanation of Terms

Some of the definitions of existence criteria used in this study are described below:

1. Mechanism: A closed kinematic chain in which one of the links fixed is called a mechanism.

2. Mobility: The mobility of a mechanism is the number of independent quantities required to specify its motion completely.

3. Constrained Motion: A mechanism with mobility one is said to have a constrained motion.

4. Constrained Mechanism: A mechanism with one degree of freedom (denoted by "F = 1" mechanism) is referred to as constrained mechanism.

5. Unconstrained Mechanism: A mechanism with multi-degree of freedom is referred to as an unconstrained mechanism.

6. Structure: A mechanism with zero degree of freedom is referred to as a structure.

7. Kinematic Pair: A kinematic pair can be defined as a (frictionless) joint which connects, and at the same time, constrains the relative motion between two rigid bodies. Geometrically, one may imagine a pair as two mating profiles, known as pairing elements or male and female elements.

8. Degree of freedom of a kinematic pair: The degree of freedom of a kinematic pair is the number of independent variables necessary to specify the relative position of two links connected by the pair.

9. Lower and higher kinematic pairs: If a male element of a kinematic pair makes, with its female element, either area or surface contact, the kinematic pair is called a lower kinematic pair. Examples of lower kinematic pairs include a revolute pair, a prism pair, a helical pair, a cylinder pair, a spherical pair, etc.

   If, however, male and female elements of a kinematic pair make either a line contact or a point contact, then this kinematic pair is called a higher kinematic pair. Examples of higher kinematic pairs are a cam-pair, a sphere-plane pair, etc. For a complete description and classification see reference (21).

   Lower kinematic pairs are efficient for transmitting higher forces. Higher kinematic pairs are used primarily for building motion transmitting devices rather than force transmitting devices.

10. Linkage configuration: The configuration of the mechanism, or linkage configuration, at a given instant during motion, is completely specified by the spatial polygon defined by the axes of the mechanism.

11. Constant kinematic parameters of a mechanism: The constant kinematic parameters of a mechanism are the link lengths, the twist angles, the constant offset distances (kink-links) and the

constant displacement angles. These parameters are constant for a given mechanism and remain unchanged during its motion.

12. Variable kinematic parameters of a mechanism: The variable kinematic parameters of a mechanism are the variable offset distances (translations) along its pair axes and the variable displacement angles. These parameters are not constant for a given mechanism, but vary during its motion.

13. Finite mobility: A mechanism is said to have finite mobility when it is capable of executing motion over a finite range. Thus, for example, a spherical four-link, four-revolute mechanism has a finite mobility of one.

14. Transitory or instantaneous mobility: A mechanism is said to have transitory or instantaneous mobility when it is capable of executing motion over only an infinitesimal range. Thus, for example, a spherical four-link, four helical mechanism (equal pitch values) has a transitory or instantaneous mobility of one (32). It may also be noted that instantaneous mobility at all instants may often lead to finite mobility (30, 35).

15. True mobility: A mechanism is said to have true mobility when it has finite mobility with all the freedoms in all of its joints active. Thus, for example, a plane four-link, four revolute mechanism has, except at its locking positions, a true mobility of one, but a five-link H-P-P-P-P space mechanism does not

have true mobility since its helical pair remains permanently locked. In the present study, a mechanism is said to "exist" when it has a true mobility of one.

16. Zero family mechanisms: Consider a two-loop, six-link space mechanism. Let p_k denote the number of kinematic pairs of class k in which the degree of freedom is k and $\Sigma p_k = 7$. Then Σp_k denotes the total number of degrees of freedom permitted at all the joints. When $\Sigma p_k = 13$, any random combination of constant kinematic parameters will, in general yield a two-loop mechanism with mobility one.

Similarly, let f_i denote the number of degrees of freedom permitted at the ith joint of a single-loop space mechanism. Then the total number of degrees of freedom permitted at all the joints is denoted by Σf_i. When $\Sigma f_i = 7$, any random combination of constant kinematic parameters will, in general, yield a single-loop mechanism with mobility one.

Such mechanisms in which there are no conditions imposed on the constant kinematic parameters are called zero family mechanisms. The 1R+6C mechanism, the 4R+3S mechanism, and the 1R+3P+3E mechanism are some examples of zero family mechanisms.

17. Overconstrained mechanism: Consider a two-loop, six-link space mechanism. When $\Sigma k\, p_k < 13$, a random combination of

constant kinematic parameters will, in general, yield a configuration which is a structure. Two-loop mechanisms with $\Sigma k\, p_k < 13$ can exist with mobility one only when their constant kinematic parameters satisfy certain definite mathematical relationships.

In a similar way when $\Sigma f_i < 7$, a random combination of constant kinematic parameters will, in general, give a single-loop configuration which is a structure.

Hence, such mechanisms in which conditions are imposed on the constant kinematic parameters are called overconstrained mechanisms.

18. Number of passive couplings: The number of passive couplings C_p in an overconstrained mechanism with two loops is given by the simple relationship

$$C_p = 13 - \Sigma k p_k$$

where $\Sigma k p_k$ denotes the total number of degrees of freedom permitted at all the joints of the six-link two-loop overconstrained space mechanism.

The number of passive couplings C_p in an overconstrained mechanism with one loop is given by the simple relationship

$$C_p = 7 - \Sigma k p_k$$

where $\Sigma k p_k$ denotes the total number of degrees of freedom permitted at all the joints of the six-link single-loop overconstrained space mechanism.

19. Existence criteria of an overconstrained mechanism: For the present study, the existence criteria of an overconstrained mechanism denotes a set(s) of conditions that are necessary for its existence. These conditions are equations relating to the constant kinematic parameters of the mechanism. An overconstrained mechanism of the prescribed type satisfies all of the conditions forming the existence criteria simultaneously.

20. Closure conditions: Closure conditions are algebraic equations between the parameters of a linkage which give the conditions required by the closure of a loop in a linkage.

21. Passive freedoms: Passive freedoms are the destroyed freedoms of the pairs as a result of certain geometric constraints (passive constraints). In practice the passive freedoms and also the redundant freedoms, may be kep in the mechanism rather than eliminating them by replacing the pairs possessing the passive freedoms with pairs of lower class. This is preferred to have ease in design, operation, and lubrication.

CHAPTER II

DIMENTBERG'S PASSIVE COUPLING METHOD ILLUSTRATED FOR A SPATIAL FIVE-LINK H-H-P-P-H MECHANISM

Nature of Dimentberg's Method

Dimentberg in 1948 introduced the method of passive coupling and illustrated the method of obtaining the existence criteria of a number of overconstrained four-link mechanisms (29, 38, 39, 40). Waldron (33, 34, 35, 36, 37), Ogino and Watanabe (51) however apparently unaware of the work of Dimentberg have recently used dual-number algebra to study the mobility of a spatial four-link chain with four cylinder pairs and have come-up with certain overconstrained four-link mechanisms.

The use of Dimentberg's method for obtaining the existence criteria of an overconstrained mechanism involves the following three steps:

1. Select a Parent Mechanism. It is, in general, possible to derive an overconstrained mechanism from more than one parent mechanism.

Thus, for example, the four-link RSRR mechanism can be derived from either the RSCR mechanism or the RSRC mechanism.

2. Develop the closed-form displacement relationships between independent and dependent displacement variables of the parent mechanism.

   If the parent mechanism has no helical pairs, the displacement relationships are algebraic in nature. If the parent mechanism has helical pairs, the displacement relationships are complicated in nature.

3. Impose the required passive coupling conditions on the parent mechanism so as to obtain the desired overconstrained mechanism. Thus, for example, passive coupling condition is imposed on the cylinder pair of the parent four-link RSCR mechanism in order to obtain the RSRR overconstrained mechanism. When the displacement relationships involved are algebraic in nature, this step very often involves examination of the conditions for common roots between two algebraic polynomials or between successive sets of two polynomials. The results obtained lead to conditions on the constant kinematic parameters of the parent mechanism and provide the necessary conditions for the existence of the desired overconstrained mechanism.

Example

In this section, the Dimentberg method of passive coupling technique is demonstrated to obtain the existence criteria of an H-H-P-P-H five-link mechanism. This is done by considering a five-link H-H-C-C-H mechanism as the parent mechanism.

An H-H-C-C-H five-link space mechanism with general proportions is shown in Figure 1, with helical pairs at joints A, B, E and cylinder pairs at joints C and D. The instantaneous configuration of the H-H-C-C-H mechanism as shown in Figure 1 is completely defined by two sets of five dual angles (38), each as follows:

1. Between adjacent pairing axes:

$$\hat{\alpha}_i = \alpha_i + \epsilon\, a_i \qquad (i = 1, 2, \ldots, 5) \qquad (2\text{-}1)$$

where α_i (i = 1 to 5) are the twist angles and a_i (i = 1 to 5) are the kinematic link lengths. Note that, by definition, $\epsilon^2 = 0$.

2. Between adjacent common perpendiculars:

$$\hat{\theta}_i = \theta_i + \epsilon\, s_i \qquad (i = 1, 2, \ldots, 5) \qquad (2\text{-}2)$$

with $$s_i = p_i\, \theta_i \qquad (i = 1, 2, 5) \qquad (2\text{-}3)$$

where θ_i (i = 1 to 5) are the angular displacements at the kinematic pairs, s_i (i = 1 to 5) are the translational displacements along the kinematic axes, and p_i (i = 1, 2, 5) are the finite pitch values of the helical pairs.

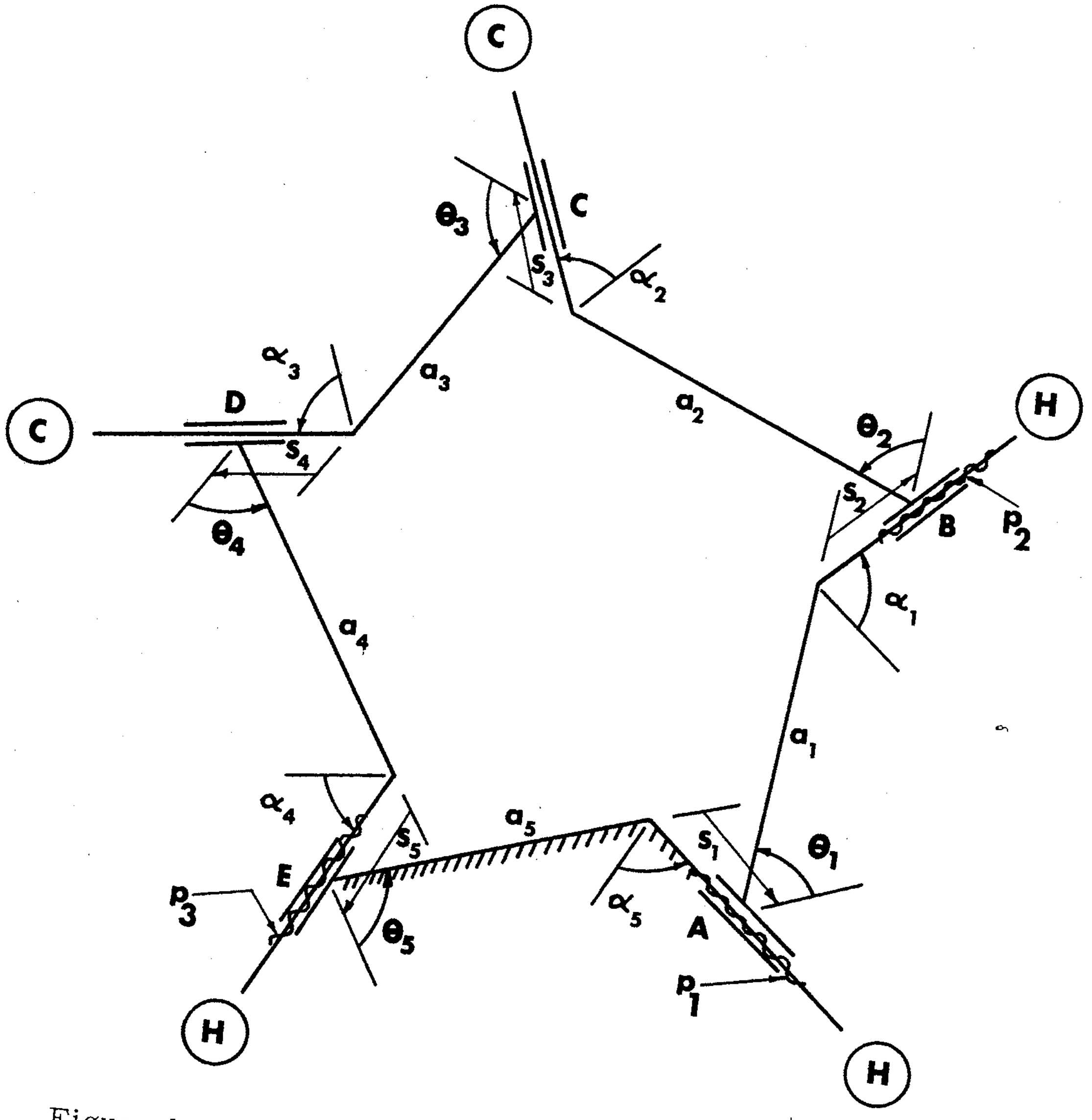

Figure 1. Five-link H-H-C-C-H Space Mechanism

In equation (2-2), the five angles, θ_i (i = 1 to 5) and the two sliding components along the cylindric axes (s_3 and s_4) constitute the seven independent linkage variables; among them θ_1 is the input angle and $\hat{\theta}_5$ is the output angle. The five dual angles, $\hat{\alpha}_i$ (i = 1 to 5) in equation (2-1) and the three finite pitch values of the helical pairs (p_1, p_2, p_5) constitute the thirteen real parameters necessary to specify an H-H-C-C-H mechanism of general proportions.

Consider the H-H-C-C-H five-link space mechanism shown schematically in Figure 2. This mechanism reduces to an H-H-P-P-H mechanism, as shown in Figure 3, if the rotational displacement angles θ_3 and θ_4 at the two cylinder pairs remain constant at all positions of the mechanism.

The dual-matrix loop closure equation for the H-H-C-C-H mechanism shown in Figure 2 is given by (120)

$$[\hat{\theta}_4]_3\ [\hat{\alpha}_3]_1\ [\hat{\theta}_3]_3\ [\hat{\alpha}_2]_1\ [\hat{\theta}_2]_3\ [\hat{\alpha}_1]_1\ [\hat{\theta}_5]_3\ [\hat{\alpha}_4]_1 = [I] \qquad (2\text{-}4)$$

where

$$[\hat{\theta}_i]_3 = \begin{bmatrix} C\hat{\theta}_i & S\hat{\theta}_i & 0 \\ -S\hat{\theta}_i & C\hat{\theta}_i & 0 \\ 0 & 0 & 1 \end{bmatrix}^{1}$$

[1] In this equation and in all the subsequent equations and tables throughout this study, C and S denote the cosine and sine of the respective angles.

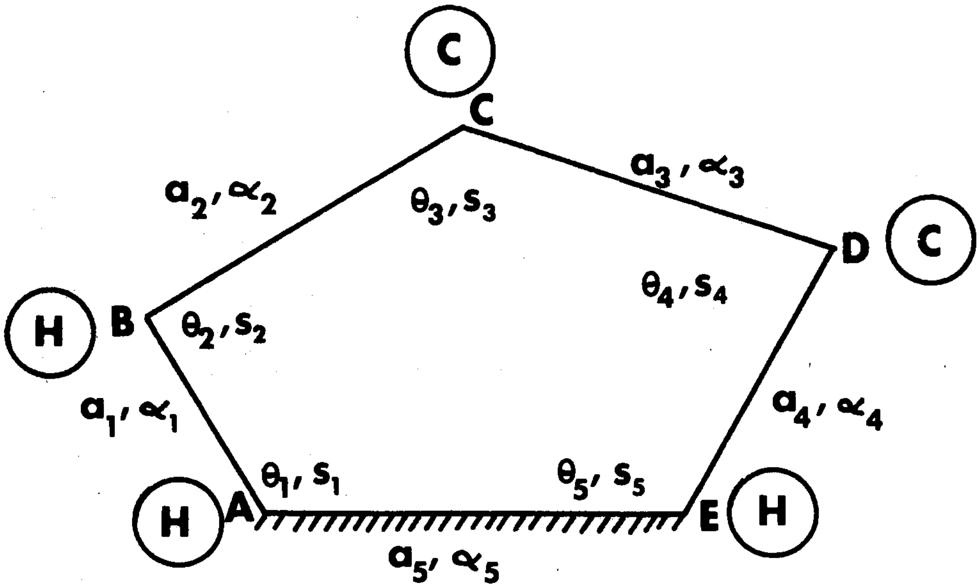

Figure 2. H-H-C-C-H Space Mechanism

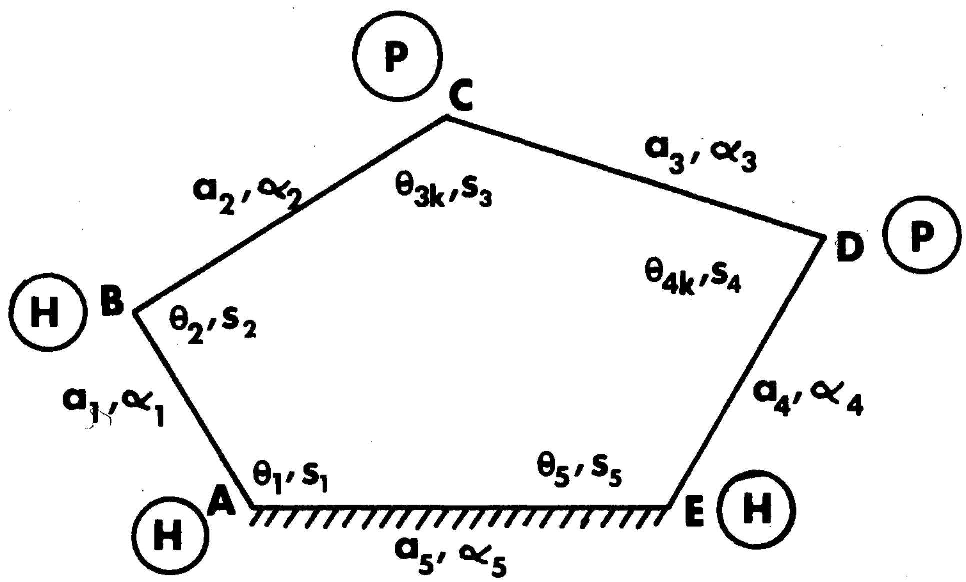

Figure 3. H-H-P-P-H Space Mechanism Obtained From the Mechanism in Figure 2 by Making $\theta_3 = \theta_{3k}$ = a Constant and $\theta_4 = \theta_{4k}$ = a Constant

$$[\hat{\alpha}_i]_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & C\hat{\alpha}_i & S\hat{\alpha}_i \\ 0 & -S\hat{\alpha}_i & C\hat{\alpha}_i \end{bmatrix}$$

and

$$[I] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

By arranging the loop-closure condition of the mechanism in three different ways, the following relationships can be obtained.

$$F(\hat{\theta}_4, \hat{\theta}_3, \hat{\theta}_1) = (S\hat{\alpha}_2 S\hat{\alpha}_4 S\hat{\theta}_3)S\hat{\theta}_4 - S\hat{\alpha}_4(C\hat{\alpha}_2 S\hat{\alpha}_3 + S\hat{\alpha}_2 C\hat{\alpha}_3 C\hat{\theta}_3)C\hat{\theta}_4 + C\hat{\alpha}_4(C\hat{\alpha}_2 C\hat{\alpha}_3 - S\hat{\alpha}_2 S\hat{\alpha}_3 C\hat{\theta}_3) - (C\hat{\alpha}_1 C\hat{\alpha}_5 - S\hat{\alpha}_1 S\hat{\alpha}_5 C\hat{\theta}_1) = 0 \tag{2-5}$$

$$f(\hat{\theta}_5, \hat{\theta}_4, \hat{\theta}_3) = [(S\hat{\alpha}_4 C\hat{\alpha}_5 + C\hat{\alpha}_4 S\hat{\alpha}_5 C\hat{\theta}_5)S\hat{\theta}_4 + S\hat{\alpha}_5 S\hat{\theta}_5 C\hat{\theta}_4](S\hat{\alpha}_2 S\hat{\theta}_3) + [S\hat{\alpha}_5 S\hat{\theta}_5 S\hat{\theta}_4 - (S\hat{\alpha}_4 C\hat{\alpha}_5 + C\hat{\alpha}_4 S\hat{\alpha}_5 C\hat{\theta}_5)C\hat{\theta}_4](C\hat{\alpha}_2 S\hat{\alpha}_3 + S\hat{\alpha}_2 C\hat{\alpha}_3 C\hat{\theta}_3) + (C\hat{\alpha}_4 C\hat{\alpha}_5 - S\hat{\alpha}_4 S\hat{\alpha}_5 C\hat{\theta}_5)(C\hat{\alpha}_2 C\hat{\alpha}_3 - S\hat{\alpha}_2 S\hat{\alpha}_3 C\hat{\theta}_3) - C\hat{\alpha}_1 = 0 \tag{2-6}$$

$$f(\hat{\theta}_4, \hat{\theta}_3, \hat{\theta}_2) = [(S\hat{\alpha}_3 C\hat{\alpha}_4 + C\hat{\alpha}_3 S\hat{\alpha}_4 C\hat{\theta}_4)S\hat{\theta}_3$$
$$+ S\hat{\alpha}_4 S\hat{\theta}_4 C\hat{\theta}_3] (S\hat{\alpha}_1 S\hat{\theta}_2) + [S\hat{\alpha}_4 S\hat{\theta}_4 S\hat{\theta}_3$$
$$- (S\hat{\alpha}_3 C\hat{\alpha}_4 + C\hat{\alpha}_3 S\hat{\alpha}_4 C\hat{\theta}_4) C\hat{\theta}_3] (C\hat{\alpha}_1 S\hat{\alpha}_2$$
$$+ S\hat{\alpha}_1 C\hat{\alpha}_2 C\hat{\theta}_2) + (C\hat{\alpha}_3 C\hat{\alpha}_4 - S\hat{\alpha}_3 S\hat{\alpha}_4 C\hat{\theta}_4)(C\hat{\alpha}_1 C\hat{\alpha}_2$$
$$- S\hat{\alpha}_1 S\hat{\alpha}_2 C\hat{\theta}_2) - C\hat{\alpha}_5 = 0 \qquad (2\text{-}7)$$

Note that each of the above equations relates the dual displacement angles θ_3 and θ_4 at the two cylinder pairs to a third dual displacement angle.

Let the rotational displacement angles θ_3 and θ_4 at the two cylinder pairs be now held constant at all positions of the mechanism. Denoting these constant values by θ_{3k} and θ_{4k} respectively, the primary parts of Eqs. (2-5), (2-6) and (2-7) give

$$A_c C\theta_1 + A_n = 0 \qquad (2\text{-}8)$$

$$B_s S\theta_5 + B_c C\theta_5 + B_n = 0 \qquad (2\text{-}9)$$

$$C_s S\theta_2 + C_c C\theta_2 + C_n = 0 \qquad (2\text{-}10)$$

The constants used in the above equations are functions of the constant kinematic parameters a_i, α_i and the constant displacement angles θ_{3k} and θ_{4k} of the mechanism are defined in Table I.

Note that each of the equations (2-8), (2-9) and (2-10) contains only one variable and must hold true at varying values of that variable. Their coefficients must, therefore, vanish. This gives

TABLE I

CONSTANTS FOR USE IN EQUATIONS (2-8) THROUGH (2-11)

$$A_c = S\alpha_1\, S\alpha_5$$

$$A_n = S\alpha_2[S\alpha_4\,(S\theta_{3k}\,S\theta_{4k} - C\alpha_3\,C\theta_{3k}\,C\theta_{4k})$$

$$- S\alpha_3\,C\alpha_4\,C\theta_{3k}] + C\alpha_2\,(C\alpha_3\,C\alpha_4 - C\alpha_3\,S\alpha_4\,C\theta_{4k})$$

$$- C\alpha_1\,C\alpha_5$$

$$B_s = S\alpha_5\,[S\alpha_2\,(S\theta_{3k}\,C\theta_{4k} + C\alpha_3\,C\theta_{3k}\,S\theta_{4k})$$

$$+ S\alpha_3\,C\alpha_2\,S\theta_{4k}]$$

$$B_c = S\alpha_5\,\{C\alpha_4\,[S\alpha_2\,(S\theta_{3k}\,S\theta_{4k} - C\alpha_3\,C\theta_{3k}\,C\theta_{4k})$$

$$- C\alpha_2\,S\alpha_3\,C\theta_{4k}] - S\alpha_4\,(C\alpha_2\,C\alpha_3 - S\alpha_2\,S\alpha_3\,C\theta_{4k})\}$$

$$B_n = C\alpha_5\,\{S\alpha_4\,[S\alpha_2\,(S\theta_{3k}\,S\theta_{4k} - C\alpha_3\,C\theta_{3k}\,C\theta_{4k})$$

$$- C\alpha_2\,S\alpha_3\,C\theta_{4k}] + C\alpha_4\,(C\alpha_2\,C\alpha_3 - S\alpha_2\,S\alpha_3\,C\theta_{3k})\} - C\alpha_1$$

$$C_s = S\alpha_1\,[S\alpha_4\,(C\theta_{3k}\,S\theta_{4k} + C\alpha_3\,S\theta_{3k}\,C\theta_{4k}) + S\alpha_3\,C\alpha_4\,S\theta_{3k}]$$

$$C_c = S\alpha_1\,\{C\alpha_2\,[S\alpha_4\,(S\theta_{3k}\,S\theta_{4k} - C\alpha_3\,C\theta_{3k}\,C\theta_{4k})$$

$$- S\alpha_3\,C\alpha_4\,C\theta_{3k}] - S\alpha_2\,(C\alpha_3\,C\alpha_4 - S\alpha_3\,S\alpha_4\,C\theta_{4k})\}$$

$$C_n = C\alpha_1\,\{S\alpha_2\,[S\alpha_4\,(S\theta_{3k}\,S\theta_{4k} - C\alpha_3\,C\theta_{3k}\,C\theta_{4k})$$

$$- S\alpha_3\,C\alpha_4\,C\theta_{3k}] + C\alpha_2\,(C\alpha_3\,C\alpha_4 - S\alpha_3\,S\alpha_4\,C\theta_{4k})\} - C\alpha_5$$

$$A_c = A_n = 0$$

$$B_s = B_c = B_n = 0 \qquad (2\text{-}11)$$

$$C_s = C_c = C_n = 0$$

The above equations provide the necessary conditions for the existence of an H-H-P-P-H mechanism. However, it is possible to further simplify the conditions given by Eqs. (2-11). For example, examination of Eqs. (2-11) yields the following relationships:

$$\alpha_1 = \alpha_5 = 0 \qquad (2\text{-}12)$$

and

$$S\alpha_3 \, (S\alpha_2 \, C\alpha_4 \, C\theta_{3k} + C\alpha_2 \, S\alpha_4 \, C\theta_{4k}) - C\alpha_3 \, (C\alpha_2 \, C\alpha_4 - S\alpha_2 \, S\alpha_4 \, C\theta_{3k} \, C\theta_{4k}) - S\alpha_2 \, S\alpha_4 \, S\theta_{3k} \, S\theta_{4k} + 1 = 0 \qquad (2\text{-}13)$$

Equation (2-12) shows that the axes of the three helical pairs are parallel to one another. Equation (2-13) is a definite closure condition relating the twist angles α_2, α_3 and α_4 of the mechanism with the constant displacement angles θ_{3k} and θ_{4k} at the two prismatic pairs (Figure 3). The H-H-P-P-H linkage is shown in Figure 4.

Note that the results have been obtained by considering only the primary parts of the dual displacement relationships of the parent H-H-C-C-H mechanism. Hence, the results will remain unaffected even if one or more of the helical pairs are replaced by

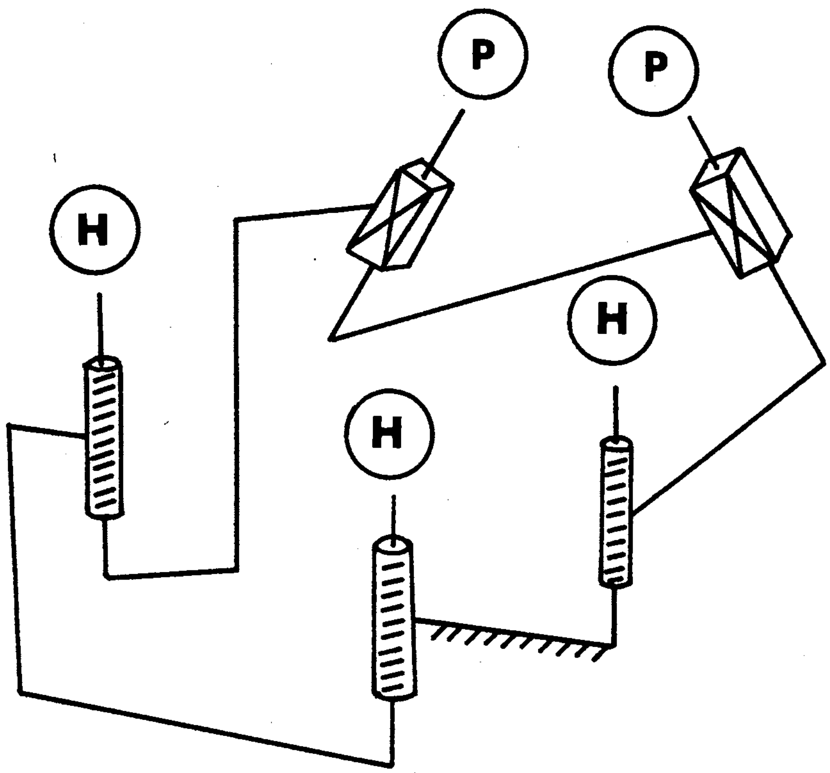

Figure 4. H-H-P-P-H Space Mechanism (30, 35, 119)

revolute pairs. Note further that the results obtained are independent of the link lengths involved. Hence, if one of the link lengths is taken to be zero, the results will apply with equal validity to four-link mechanisms derivable from the above five-link mechanism (29). The results obtained in the present example for the H-H-P-P-H mechanism also confirm the results obtained by Hunt (30), Waldron (35), Pamidi (41), and Pamidi, Soni and Dukkipati (119). The results of Hunt and Waldron were obtained by considering the 5H and 6H mechanisms of Voinea and Atanasiu (17) which are themselves overconstrained mechanisms. The results of Pamidi, Soni and Dukkipati were obtained by considering the more general zero family mechanisms, thus guaranteeing full-cycle mobility. Also, in addition to the parallelism of the axes, the existence derived in the present study gives definite closure conditions to be satisfied by the constant kinematic parameters of the respective mechanism.

Scope of Dimentberg's Method

Dimentberg has employed his method in those cases in which the translational freedom of a cylinder pair is made passive (29, 38, 39, 40). The method has been shown equally applicable to the cases in which the rotational freedom of a cylinder pair is made passive by Soni (27), Pamidi (41), and Dukkipati (122). Pamidi obtained the existence criteria of R-P-C-P and R-C-P-P mechanisms by imposing passive coupling conditions on the rotational freedom of the output cylinder pair of an R-C-C-C mechanism. Soni (27) obtained the existence criteria of an R-P-R-C-R five-link overconstrained mechanism from the parent R-C-R-C-R mechanism. Dukkipati (122) obtained the existence criteria of an R-S-P-R four-link overconstrained mechanism by imposing passive coupling on the rotational freedom at the cylinder pair of the parent R-S-C-R mechanism.

Extension of Dimentberg's method to five-link mechanisms led Pamidi, Soni and Dukkipati (119) to obtain the existence criteria of the five-link, five revolute mechanism, R-R-R-P-R mechanism, and 3H+2P, 2H+3P mechanisms.

Dimentberg's method also holds true for the case in which the entire freedom of a kinematic pair is made passive by Pamidi (41) and Dukkipati (122). The joint thus becomes locked and no motion is possible at that joint. The results obtained are in agreement with those obtained by Dimentberg and show that it is possible to obtain an overconstrained mechanism from more than one parent mechanism.

The extensions to Dimentberg's method as demonstrated by Soni, Pamidi and Dukkipati illustrate the immense scope of the method and show that the method can be employed to handle a variety of passive coupling conditions. The objective of the present study is to extend Dimentberg's method to single and multi-loop six-link mechanisms.

Passive Coupling Conditions Considered in Single-Loop Mechanisms in the Present Study

The passive coupling conditions considered in single-loop mechanisms in the present study are confined to those cases in which a passive coupling is imposed on a cylinder pair in order to obtain a prism pair. This involves examination of only the primary part of the various dual displacement relationships of the parent mechanism.

The cases proposed are summarized in Table II and fall into the following single category.

1. Passive coupling in a cylinder pair to obtain a prism pair. Thus passive coupling is imposed on the cylinder pair of the parent 3H+2P+1C space six-link mechanisms in order to reduce it to a prism pair of the overconstrained 3H+3P space mechanisms (see cases 1, 2, and 3 in Table II).

Passive Coupling Conditions Considered in Two-Loop Mechanisms in the Present Study

The passive coupling conditions considered in two-loop mechanisms in the present study are confined to those cases in which the required displacement relationships are algebraic in

TABLE II

PASSIVE COUPLING CONDITIONS CONSIDERED IN SINGLE-LOOP MECHANISMS IN THE PRESENT STUDY

(H: Helical pair, P: Prismatic pair, C: Cylinder pair)

Case	Kinematic pair selected for inducing passive coupling condition	Kinematic pair obtained because of passive coupling condition	Parent mechanism examined for inducing passive coupling condition	Overconstrained mechanism obtained because of passive coupling condition	Considered in
1	C	P	H-C-P-P-H-H*	H-P-P-P-H-H	
2	C	P	H-C-P-H-P-H	H-P-P-H-P-H	Chapter III
3	C	P	H-C-H-P-H-P	H-P-H-P-H-P	

*Here and throughout, this abbreviation refers to the sequence of kinematic pairs joining the links of a spatial mechanism, starting with the fixed link. See Figure 5.

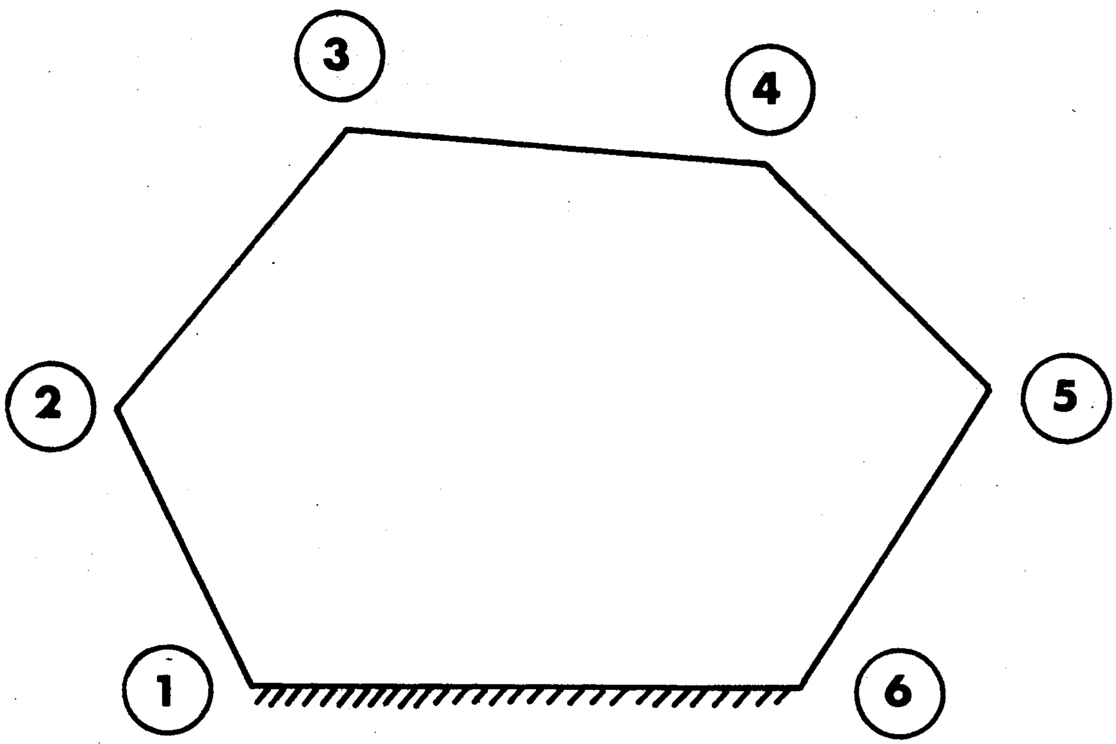

Figure 5. Schematic Representation of Six-link, Single-loop Space Mechanism ($\Sigma f_i = 7$)

nature. The cases considered are summarized in Table III and fall into the following three categories:

1. Passive coupling in two cylinder pairs (one in each loop) to obtain the revolute pairs (see cases 1, 3, and 4 in Table III).

2. Passive coupling in two cylinder pairs (one in each loop) to obtain one revolute pair and one prism pair (see cases 2 and 5 in Table III).

3. Passive coupling in two cylinder pairs (one in each loop) to obtain two prism pairs (see case 6 in Table III).

TABLE III

PASSIVE COUPLING CONDITIONS CONSIDERED IN TWO-LOOP MECHANISMS IN THE PRESENT STUDY

(R: Revolute pair, P: Prismatic pair, C: Cylinder pair)

Case	Kinematic pairs (one from each loop) selected for inducing passive coupling conditions	Kinematic pairs obtained because of passive coupling conditions	Parent mechanism examined for inducing passive coupling conditions	Overconstrained mechanism obtained because of passive coupling conditions	Considered in
1	C-C	R-R	R-C-C-C-C-C-C[1]	R-R-C-C-C-R-C[2]	Chapter IV
2	C-C	R-P	R-C-C-C-C-C-C	R-R-C-C-C-P-C	
3	C-C	R-R	R-C-C-C-C-C-C	R-R-C-C-C-R-C[3]	Appendix A
4	C-C	R-R	R-C-C-C-C-C-C	R-C-C-R-C-C-R	Appendix B
5	C-C	R-P	R-C-C-C-C-C-C	R-C-C-R-C-C-P	
6	C-C	P-P	R-C-C-C-C-C-C	R-P-C-P-C-P-C	
				R-P-P-C-C-P-C	

[1]Here and throughout, this abbreviation refers to the sequence of kinematic pairs joining the links of a six-link, two-loop spatial mechanism of Stephenson type, starting with the fixed link. See Figure 6.

[2]One kink-link assumed zero. (Special form of Case 3.)

[3]Non-zero kink-links. (General proportions.)

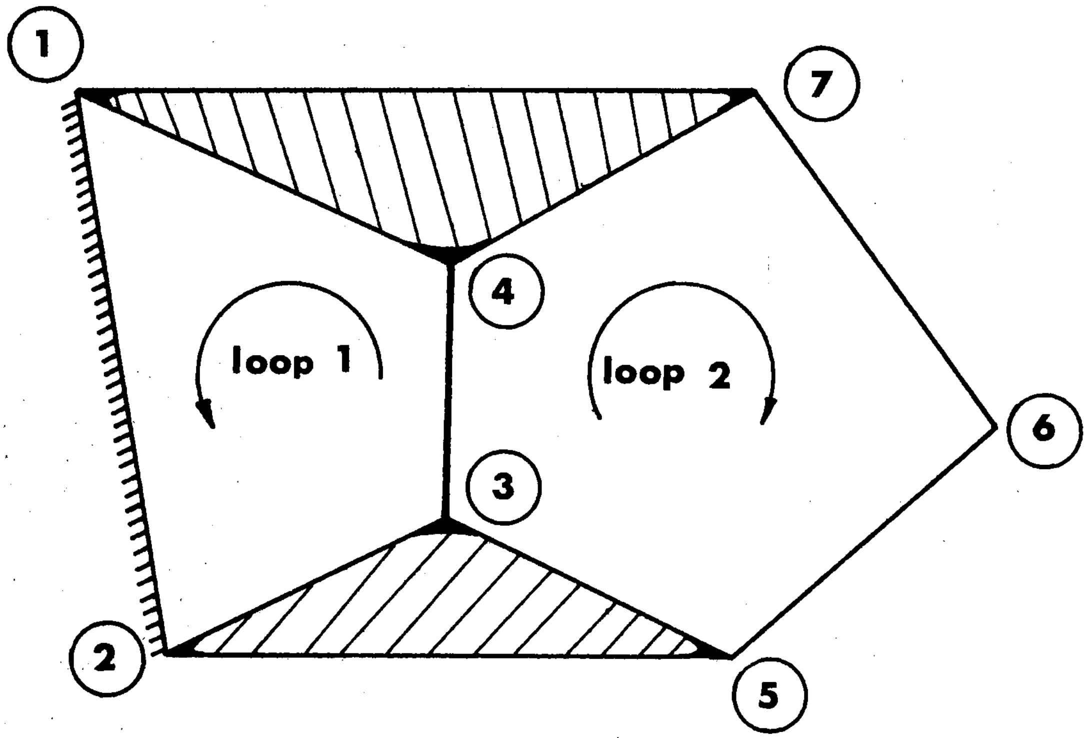

Figure 6. Schematic Representation of Six-link, Two-loop Space Mechanism of Stephenson Type ($\Sigma f_i = 13$)

CHAPTER III

EXISTENCE CRITERIA OF SINGLE-LOOP MECHANISMS

Displacement Relationships for Obtaining the Existence Criteria

The use of Dimentberg's method for obtaining the existence criteria of overconstrained mechanisms requires the displacement relationships of the appropriate parent mechanisms. The required relationships can always be obtained by suitably arranging the loop-closure condition of the parent mechanism.

Consider a general single-loop, six-link space mechanism consisting of helical, revolute, prismatic and cylinder pairs combined in such a way that the sum of the degrees of freedom in all the joints is equal to seven (Figure 7). Such a mechanism would necessarily have to have one cylinder pair. If the type of the remaining five pairs and the location of all the six pairs in the mechanism are properly chosen, this mechanism will serve as a parent mechanism for any overconstrained mechanism with one pressure coupling.

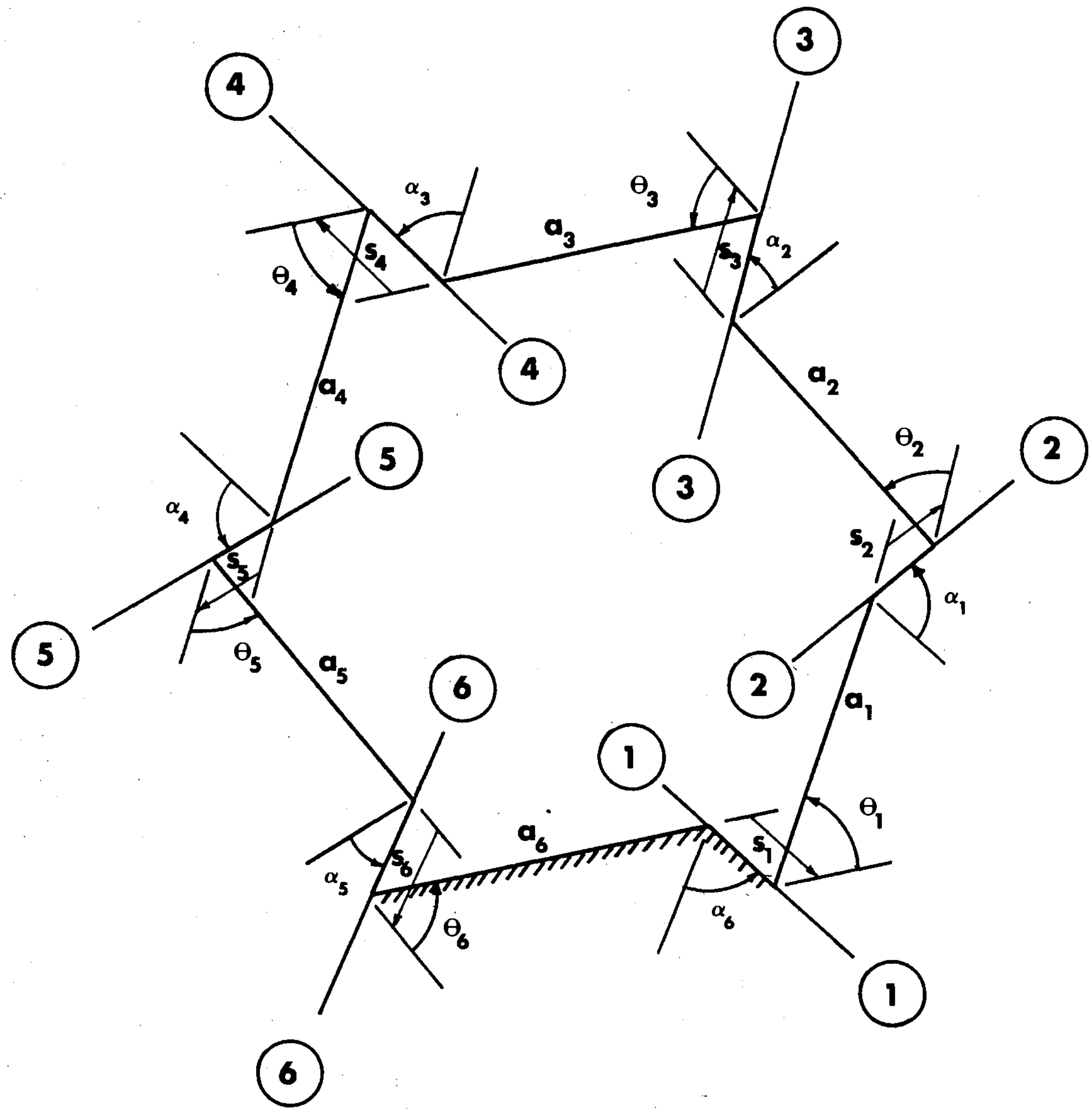

Figure 7. General Six-link, Single-loop Space Mechanism With Helical, Revolute, Prismatic and Cylinder Pairs ($\Sigma f_i = 7$)

The instantaneous configuration of the mechanism in Figure 7 is completely defined by two sets of six dual angles, each as follows:

1. Between adjacent pairing axes:

$$\hat{\alpha}_i = \alpha_i + \epsilon a_i \qquad (3\text{-}1)$$

where α_i (i = 1 to 6) are the twist angles and a_i (i = 1 to 6) are the link lengths. These twelve quantities are constant for any given mechanism. Note also, that by definition,

$$\epsilon^2 = 0 .$$

2. Between adjacent common perpendiculars:

$$\hat{\theta}_i = \theta_i + \epsilon s_i \qquad (3\text{-}2)$$

where θ_i (i = 1 to 6) are the angular displacements at the kinematic pairs and s_i (i = 1 to 6) are the translations along the kinematic axes. These quantities may be variable or remain constant depending upon the type of kinematic pairs used in the mechanism. For instance, in a prismatic pair, the angular displacement remains constant, while in a revolute pair, the translation along the axis is constant. In a helical pair, the translation along the axis and the angular displacement both vary in such a way that their ratio is always constant and equal to the pitch. In a cylinder pair, the translation along the axis and the angular displacement both vary and are independent of each other.

The dual-matrix loop-closure equation of the spatial six-link mechanism in Figure 7 is given by (120):

$$[\hat{\alpha}_1]_1\ [\hat{\theta}_1]_3\ [\hat{\alpha}_2]_1\ [\hat{\theta}_2]_3\ [\hat{\alpha}_3]_1\ [\hat{\theta}_3]_3\ [\hat{\alpha}_4]_1\ [\hat{\theta}_4]_3\ [\hat{\alpha}_5]_1\ [\hat{\theta}_5]_3$$
$$[\hat{\alpha}_6]_1\ [\hat{\theta}_6]_3 = [I] \tag{3-3}$$

where

$$[\hat{\alpha}_i]_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & C\hat{\alpha}_i & S\hat{\alpha}_i \\ 0 & -S\hat{\alpha}_i & C\hat{\alpha}_i \end{bmatrix} \tag{3-4}$$

$$[I] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

and

$$[\hat{\theta}_i]_3 = \begin{bmatrix} C\hat{\theta}_i & S\hat{\theta}_i & 0 \\ -S\hat{\theta}_i & C\hat{\theta}_i & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{3-5}$$

Three arrangements of Eq. (3-3) are useful in the study of existence criteria.

1. The relationship involving two adjacent dual displacement angles and the two dual displacement angles opposite to both of them.

In this arrangement of Eq. (3-3), six matrices are used on either side of the equality sign. Thus, for instance,

$$[\theta_5]_3\ [\alpha_4]_1\ [\theta_4]_3\ [\alpha_3]_1\ [\theta_3]_3\ [\alpha_2]_1$$
$$= [\theta_2]_3^{-1}\ [\alpha_1]_1^{-1}\ [\theta_1]_3^{-1}\ [\alpha_6]_1^{-1}\ [\theta_6]_3^{-1}\ [\alpha_5]_1^{-1} \tag{3-6}$$

Simplifying the above equation by using relations (3-4) and (3-5) and equating the "33" elements of the resultant matrix equation, we get

$$\begin{aligned} F_1(\hat{\theta}_1, \hat{\theta}_3, \hat{\theta}_4, \hat{\theta}_6) &= [S\hat{\theta}_4\, S\hat{\theta}_3\, S\hat{\alpha}_4\, S\hat{\alpha}_2 \\ &- C\hat{\theta}_4(C\hat{\theta}_3\, S\hat{\alpha}_4\, C\hat{\alpha}_3\, S\hat{\alpha}_2 + S\hat{\alpha}_4\, S\hat{\alpha}_3\, C\hat{\alpha}_2)] \\ &+ (-C\hat{\theta}_3\, C\hat{\alpha}_4\, S\hat{\alpha}_3\, S\hat{\alpha}_2 + C\hat{\alpha}_4\, C\hat{\alpha}_3\, C\hat{\alpha}_2) \\ &- [S\hat{\theta}_1\, S\hat{\theta}_6\, S\hat{\alpha}_1\, S\hat{\alpha}_5 - C\hat{\theta}_1(C\hat{\theta}_6\, S\hat{\alpha}_1\, C\hat{\alpha}_6\, S\hat{\alpha}_5 \\ &+ S\hat{\alpha}_1\, S\hat{\alpha}_6\, C\hat{\alpha}_5)] - (-C\hat{\theta}_6\, C\hat{\alpha}_1\, S\hat{\alpha}_6\, S\hat{\alpha}_5 \\ &+ C\hat{\alpha}_1\, C\hat{\alpha}_6\, C\hat{\alpha}_5) = 0 \end{aligned} \qquad (3\text{-}7)$$

Note that Eq. (3-7) involves the adjacent displacement angles $\hat{\theta}_1$ and $\hat{\theta}_6$ and the displacement angles $\hat{\theta}_3$ and $\hat{\theta}_4$ opposite to both of them.

Cyclic permutation permits Eq. (3-7) to be written in six different ways. It is, therefore, possible to get six equations of the form (3-7) involving different combinations of two adjacent angles and the two angles opposite to both of them.

2. Relationship involving three adjacent dual displacement angles and the dual displacement angle opposite to all three of them.

In this arrangement of Eq. (3-3), seven matrices are used on one side of the equality sign and five matrices on the other. Thus, we have, for instance,

$$[\hat{\theta}_4]_3\,[\hat{\alpha}_3]_1\,[\hat{\theta}_3]_3\,[\hat{\alpha}_2]_1\,[\hat{\theta}_2]_3$$

$$= [\hat{\alpha}_1]_1^{-1}\,[\hat{\theta}_1]_3^{-1}\,[\hat{\alpha}_6]_1^{-1}\,[\hat{\theta}_6]_3^{-1}\,[\hat{\alpha}_5]_1^{-1}\,[\hat{\theta}_5]_3^{-1}\,[\hat{\alpha}_4]_1^{-1} \qquad (3\text{-}8)$$

Simplifying Eq. (3-8) by using relations (3-4) and (3-5) and equating "33" elements of the resultant matrix equation, we get

$$F_2\,(\hat{\theta}_1,\,\hat{\theta}_3,\,\hat{\theta}_5,\,\hat{\theta}_6) = C\hat{\theta}_5\,[S\hat{\theta}_1\,S\hat{\theta}_6\,(S\hat{\alpha}_1\,C\hat{\alpha}_5\,S\hat{\alpha}_4)$$

$$+ C\hat{\theta}_1\,C\hat{\theta}_6\,(-S\hat{\alpha}_1\,C\hat{\alpha}_6\,C\hat{\alpha}_5\,S\hat{\alpha}_4) + C\hat{\theta}_1(S\hat{\alpha}_1\,S\hat{\alpha}_6\,S\hat{\alpha}_5\,S\hat{\alpha}_4)$$

$$+ C\hat{\theta}_6(-C\hat{\alpha}_1\,S\hat{\alpha}_6\,C\hat{\alpha}_5\,S\hat{\alpha}_4) + (-C\hat{\alpha}_1\,C\hat{\alpha}_6\,S\hat{\alpha}_5\,S\hat{\alpha}_4)]$$

$$+ S\hat{\theta}_5[S\hat{\theta}_1\,C\hat{\theta}_6\,S\hat{\alpha}_1\,S\hat{\alpha}_4 + C\hat{\theta}_1\,S\hat{\theta}_6\,S\hat{\alpha}_1\,C\hat{\alpha}_6\,S\hat{\alpha}_4$$

$$+ S\hat{\theta}_6\,C\hat{\alpha}_1\,S\hat{\alpha}_6\,S\hat{\alpha}_4] + [S\hat{\theta}_1\,S\hat{\theta}_6\,S\hat{\alpha}_1\,S\hat{\alpha}_5\,C\hat{\alpha}_4$$

$$+ C\hat{\theta}_1\,C\hat{\theta}_6(-S\hat{\alpha}_1\,C\hat{\alpha}_6\,S\hat{\alpha}_5\,C\hat{\alpha}_4) + C\hat{\theta}_1(-S\hat{\alpha}_1\,S\hat{\alpha}_6\,C\hat{\alpha}_5\,C\hat{\alpha}_4)$$

$$+ C\hat{\theta}_6(-C\hat{\alpha}_1\,S\hat{\alpha}_6\,S\hat{\alpha}_5\,C\hat{\alpha}_4) + C\hat{\alpha}_1\,C\hat{\alpha}_6\,C\hat{\alpha}_5\,C\hat{\alpha}_4]$$

$$- C\hat{\alpha}_3\,C\hat{\alpha}_2 + S\hat{\alpha}_3\,S\hat{\alpha}_2\,C\hat{\theta}_3 = 0 \qquad (3\text{-}9)$$

Note that Eq. (3-9) involves the three adjacent displacement angles $\hat{\theta}_1$, $\hat{\theta}_6$, and $\hat{\theta}_5$ and the displacement angle $\hat{\theta}_3$ opposite to all of them. Cyclic permutation allows Eq. (3-9) to be written in six different ways. It is, therefore, possible to obtain six equations of the form (3-9) involving different combinations of three adjacent angles and a fourth displacement angle opposite to them.

3. Relationship involving four adjacent dual displacement angles.

In this arrangement of Eq. (3-3), nine matrices are used on one side of the equality sign and three matrices on the other. The important point to note is that the matrix on the side containing three matrices involves only the constant kinematic parameters of the mechanism. Thus, we have, for instance,

$$[\hat{\alpha}_6]_1\ [\hat{\theta}_1]_3\ [\hat{\alpha}_1]_1\ [\hat{\theta}_2]_3\ [\hat{\alpha}_2]_1\ [\hat{\theta}_3]_3\ [\hat{\alpha}_3]_1\ [\hat{\theta}_4]_3\ [\hat{\alpha}_4]_1$$

$$= [\hat{\theta}_5]_3^{-1}\ [\hat{\alpha}_5]_1^{-1}\ [\hat{\theta}_6]_3^{-1} \qquad (3\text{-}10)$$

Note that the central matrix $[\hat{\alpha}_5]^{-1}$ on the right hand side involves only the constant kinematic parameters of the mechanism.

Simplifying the above equation by using relations (3-4) and (3-5) and equating the "33" elements of the resultant matrix, we get

$$F_3(\hat{\theta}_1,\ \hat{\theta}_2,\ \hat{\theta}_3,\ \hat{\theta}_4) = -S\hat{\theta}_1\ S\hat{\alpha}_4\ S\hat{\alpha}_6(-C\hat{\theta}_2\ S\hat{\theta}_4\ C\hat{\theta}_3$$

$$- C\hat{\theta}_4\ C\hat{\alpha}_3\ S\hat{\theta}_3 + S\hat{\theta}_2\ C\hat{\alpha}_2\ S\hat{\theta}_4\ S\hat{\theta}_3 - S\hat{\theta}_2\ C\hat{\alpha}_2\ C\hat{\theta}_4\ C\hat{\alpha}_3\ C\hat{\theta}_3$$

$$+ S\hat{\theta}_2\ C\hat{\theta}_4\ S\hat{\alpha}_3) + S\hat{\theta}_1\ S\hat{\alpha}_6\ C\hat{\alpha}_4(C\hat{\theta}_2\ S\hat{\alpha}_3\ S\hat{\theta}_3$$

$$+ S\hat{\theta}_2\ C\hat{\alpha}_2\ S\hat{\alpha}_3\ C\hat{\theta}_3 + S\hat{\theta}_2\ S\hat{\alpha}_2\ S\hat{\alpha}_3)$$

$$- C\hat{\theta}_1\ S\hat{\alpha}_6\ S\hat{\alpha}_4\ C\hat{\alpha}_1(-S\hat{\theta}_2\ S\hat{\theta}_4\ C\hat{\theta}_3 - S\hat{\theta}_2\ C\hat{\theta}_4\ C\hat{\alpha}_3\ S\hat{\theta}_3$$

$$- C\hat{\theta}_2\ C\hat{\alpha}_2\ S\hat{\theta}_4\ S\hat{\theta}_3 + C\hat{\theta}_2\ C\hat{\alpha}_2\ C\hat{\theta}_4\ C\hat{\alpha}_4\ C\hat{\theta}_3$$

$$- C\hat{\theta}_2\ S\hat{\alpha}_2\ C\hat{\theta}_4\ S\hat{\alpha}_3) + C\hat{\theta}_1\ S\hat{\alpha}_6\ S\hat{\alpha}_4\ S\hat{\alpha}_1(-S\hat{\alpha}_2\ S\hat{\theta}_4\ S\hat{\theta}_3$$

$$+ S\hat{\alpha}_2\ C\hat{\theta}_4\ C\hat{\alpha}_3\ C\hat{\theta}_3 + C\hat{\alpha}_2\ C\hat{\theta}_4\ S\hat{\alpha}_3)$$

$$+ C\hat{\theta}_1\ S\hat{\alpha}_6\ C\hat{\alpha}_4\ C\hat{\alpha}_1\ (S\hat{\theta}_2\ S\hat{\alpha}_3\ S\hat{\theta}_3 - C\hat{\theta}_2\ C\hat{\alpha}_2\ S\hat{\alpha}_3\ C\hat{\theta}_3$$

$$- C\hat{\theta}_2\ S\hat{\alpha}_2\ C\hat{\alpha}_3) - C\hat{\theta}_1\ S\hat{\alpha}_6\ C\hat{\alpha}_4\ C\hat{\alpha}_1\ (-S\hat{\alpha}_2\ S\hat{\alpha}_3\ C\hat{\theta}_3$$

$$+ C\hat{\alpha}_2\ C\hat{\alpha}_3) - C\hat{\alpha}_6\ S\hat{\alpha}_4\ S\hat{\alpha}_1\ (-S\hat{\theta}_2\ S\hat{\theta}_4\ C\hat{\theta}_3 - S\hat{\theta}_2\ C\hat{\theta}_4\ C\hat{\alpha}_3\ S\hat{\theta}_3$$

$$- C\hat{\theta}_2\, C\hat{\alpha}_2\, S\hat{\theta}_4\, S\hat{\theta}_3 + C\hat{\theta}_2\, C\hat{\alpha}_2\, C\hat{\theta}_4\, C\hat{\alpha}_3\, C\hat{\theta}_3$$

$$- C\hat{\theta}_2\, S\hat{\alpha}_2\, C\hat{\theta}_4\, S\hat{\alpha}_3) - C\hat{\alpha}_6\, S\hat{\alpha}_4\, C\hat{\alpha}_1\, (-S\hat{\alpha}_2\, S\hat{\theta}_4\, S\hat{\theta}_3$$

$$+ S\hat{\alpha}_2\, C\hat{\theta}_4\, C\hat{\alpha}_3\, C\hat{\theta}_3 + C\hat{\alpha}_2\, C\hat{\theta}_4\, S\hat{\alpha}_3)$$

$$+ C\hat{\alpha}_6\, C\hat{\alpha}_4\, S\hat{\alpha}_1\, (S\hat{\theta}_2\, S\hat{\alpha}_3\, S\hat{\theta}_3 - C\hat{\theta}_2\, C\hat{\alpha}_2\, S\hat{\alpha}_3\, C\hat{\theta}_3$$

$$- C\hat{\theta}_2\, S\hat{\alpha}_2\, C\hat{\alpha}_3 + C\hat{\alpha}_6\, C\hat{\alpha}_4\, C\hat{\alpha}_1\, (-S\hat{\alpha}_2\, S\hat{\alpha}_3\, C\hat{\theta}_3$$

$$+ C\hat{\alpha}_2\, C\hat{\alpha}_3) - C\hat{\alpha}_5 = 0 \qquad (3\text{-}11)$$

Note that Eq. (3-11) involves the four adjacent dual displacement angles $\hat{\theta}_1$, $\hat{\theta}_2$, $\hat{\theta}_3$, and $\hat{\theta}_4$.

Cyclic permutation allows Eq. (3-11) to be written in six different ways. It is, therefore, possible to obtain six equations of the form (3-11) involving different combinations of four adjacent dual displacement angles.

Observe that Eqs. (3-7), (3-9) and (3-11) are all dual equations. Each of them, therefore, represents two scalar equations. Since six equations of the form (3-7), six equations of the form (3-9), and six equations of the form (3-11) are possible, a total of thirty-six scalar equations are available. These thirty-six equations make it possible to obtain the existence criteria of all mechanisms with one passive coupling (and also many mechanisms with one or more passive couplings with number of links equal to or less than six).

Existence Criteria of the Six-Link 3H+3P Mechanisms

In the following sections, the Dimentberg's passive coupling technique has been employed to obtain the existence criteria of the six-link 3H+3P mechanisms. These criteria are obtained by considering only the primary parts of the displacement relationships of the appropriate parent mechanisms. They, therefore, lead to conditions on only the twist angles and constant displacement angles of the mechanisms considered and are independent of their link lengths and constant offset distances.

In a 3H+3P mechanism, the three three revolute pairs may be either adjacent to each other or be separated by one or two prismatic pairs. All possible types of 3H+3P mechanisms are, therefore, represented by the following mechanisms:

i) H-P-P-P-H-H Mechanism

ii) H-P-P-H-P-H Mechanism

iii) H-P-H-P-H-P Mechanism

Existence Criteria of the Six-Link H-P-P-P-H-H Mechanism

The existence criteria of an H-P-P-P-H-H mechanism can be obtained from the displacement relationships of an H-C-P-P-H-H mechanism.

Consider the H-C-P-P-H-H mechanism with general proportions shown schematically in Figure 8, with helical pairs at joints A, E, and F, cylinder pairs at joint B, and prismatic pairs at joints C and D. The instantaneous configuration of the H-C-P-P-H-H mechanism as shown in Figure 8 is completely defined by the two sets of six dual angles, each as follows:

1. Between adjacent pairing axes:

$$\hat{\alpha}_i = \alpha_i + \varepsilon a_i \qquad (i = 1, 2, \ldots, 6) \qquad (3\text{-}12)$$

where α_i (i = 1 to 6) are the twist angles and a_i (i = 1 to 6) are the kinematic link lengths.

2. Between adjacent common perpendiculars:

$$\hat{\theta}_i = \theta_i + \varepsilon s_i \qquad (i = 1, 2, \ldots, 6) \qquad (3\text{-}13)$$

with $$s_i = p_i \theta_i \qquad (i = 1, 5, 6)$$

where θ_i (i = 1 to 6) are the angular displacements at the kinematic pairs, s_i (i = 1 to 6) are the translational displacements along the kinematic axes, and p_i (i = 1, 5, 6) are the finite pitch values of the helical pairs.

In equation (3-13), the four angles, θ_i (i = 1, 2, 5, 6) and the three sliding components along the axes of the cylinder and prism pairs, s_i (i = 2, 3, 4) constitute the seven linkage variables; the six dual angles, $\hat{\alpha}_i$ (i = 1 to 6) in equation (3-12), the two constant displacement angles of the prism pairs at joints C and D, θ_i (i = 3, 4)

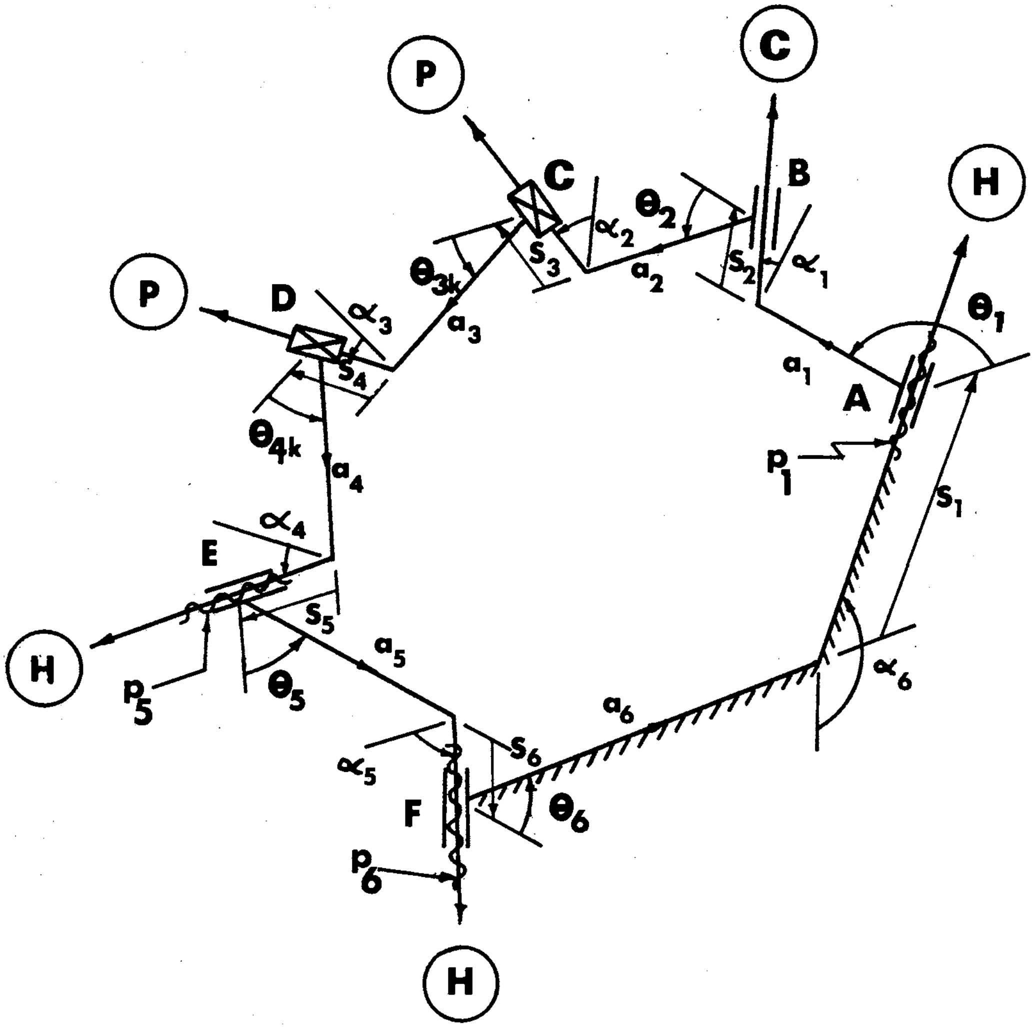

Figure 8. Six-link H-C-P-P-H-H Space Mechanism

in equation (3-13), and the three finite pitch values of the helical pairs, p_i (i = 1, 5, 6) constitute the seventeen real parameters necessary to specify an H-C-P-P-H-H mechanism of general proportions. This mechanism reduces to an H-P-P-P-H-H mechanism if the displacement angle θ_2 at the cylinder pair remains constant at all positions of the mechanism (Figure 10).

By considering the loop-closure condition of the mechanism in Figure 9 in three different ways, the following relationships can be obtained (120):

$$\begin{aligned}
F_2 (\hat{\theta}_2, \hat{\theta}_3, \hat{\theta}_4, \hat{\theta}_6) = {} & C\hat{\theta}_2 [S\hat{\theta}_4 S\hat{\theta}_3 (S\hat{\alpha}_4 C\hat{\alpha}_2 S\hat{\alpha}_1) \\
& + C\hat{\theta}_4 C\hat{\theta}_3 (-S\hat{\alpha}_4 C\hat{\alpha}_3 C\hat{\alpha}_2 S\hat{\alpha}_1) + C\hat{\theta}_4 (S\hat{\alpha}_4 S\hat{\alpha}_3 S\hat{\alpha}_2 S\hat{\alpha}_1) \\
& + C\hat{\theta}_3 (-C\hat{\alpha}_4 S\hat{\alpha}_3 C\hat{\alpha}_2 S\hat{\alpha}_1) + (-C\hat{\alpha}_4 C\hat{\alpha}_3 S\hat{\alpha}_2 S\hat{\alpha}_1)] \\
& + S\hat{\theta}_2 [S\hat{\theta}_4 C\hat{\theta}_3 (S\hat{\alpha}_4 S\hat{\alpha}_1) + C\hat{\theta}_4 S\hat{\theta}_3 (S\hat{\alpha}_4 C\hat{\alpha}_3 S\hat{\alpha}_1) \\
& + S\hat{\theta}_3 (C\hat{\alpha}_4 S\hat{\alpha}_3 S\hat{\alpha}_1)] + [S\hat{\theta}_4 S\hat{\theta}_3 (S\hat{\alpha}_4 S\hat{\alpha}_2 C\hat{\alpha}_1) \\
& + C\hat{\theta}_4 C\hat{\theta}_3 (-S\hat{\alpha}_4 C\hat{\alpha}_3 S\hat{\alpha}_2 C\hat{\alpha}_1) + C\hat{\theta}_4 (-S\hat{\alpha}_4 S\hat{\alpha}_3 C\hat{\alpha}_2 C\hat{\alpha}_1) \\
& + C\hat{\theta}_3 (-C\hat{\alpha}_4 S\hat{\alpha}_3 S\hat{\alpha}_2 C\hat{\alpha}_1) + (C\hat{\alpha}_4 C\hat{\alpha}_3 C\hat{\alpha}_2 C\hat{\alpha}_1)] \\
& - C\hat{\alpha}_6 C\hat{\alpha}_5 + S\hat{\alpha}_6 S\hat{\alpha}_5 C\hat{\theta}_6 = 0 \qquad (3\text{-}14)
\end{aligned}$$

$$\begin{aligned}
F_3 (\hat{\theta}_1, \hat{\theta}_2, \hat{\theta}_3, \hat{\theta}_4) = {} & -S\hat{\theta}_1 S\hat{\alpha}_4 S\hat{\alpha}_6 (-C\hat{\theta}_2 S\hat{\theta}_4 C\hat{\theta}_3 \\
& - C\hat{\theta}_4 C\hat{\alpha}_3 S\hat{\theta}_3 + S\hat{\theta}_2 C\hat{\alpha}_2 S\hat{\theta}_3 S\hat{\theta}_4 - S\hat{\theta}_2 C\hat{\alpha}_2 C\hat{\theta}_4 C\hat{\alpha}_3 C\hat{\theta}_3 \\
& + S\hat{\theta}_2 C\hat{\theta}_4 S\hat{\alpha}_3) + S\hat{\theta}_1 S\hat{\alpha}_6 C\hat{\alpha}_4 (C\hat{\theta}_2 S\hat{\alpha}_3 S\hat{\theta}_3
\end{aligned}$$

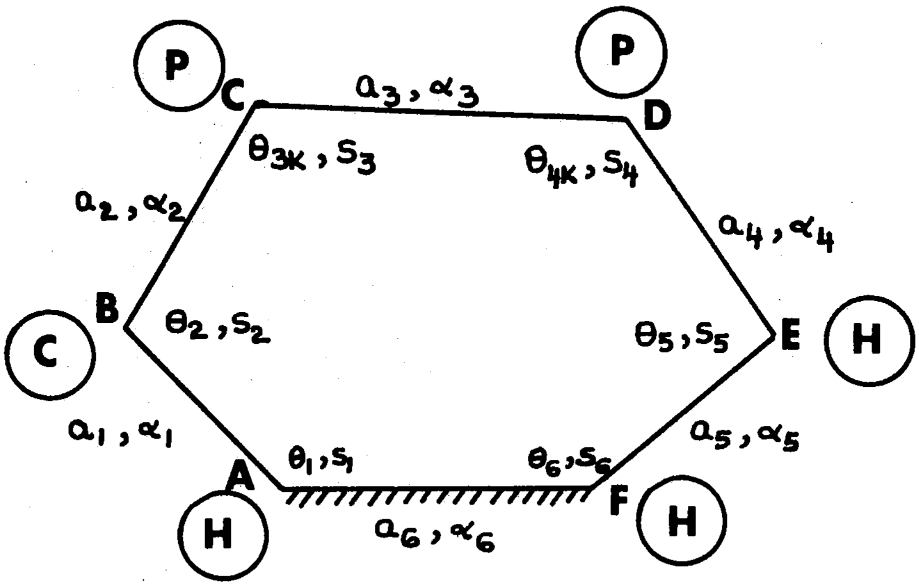

Figure 9. H-C-P-P-H-H Space Mechanism

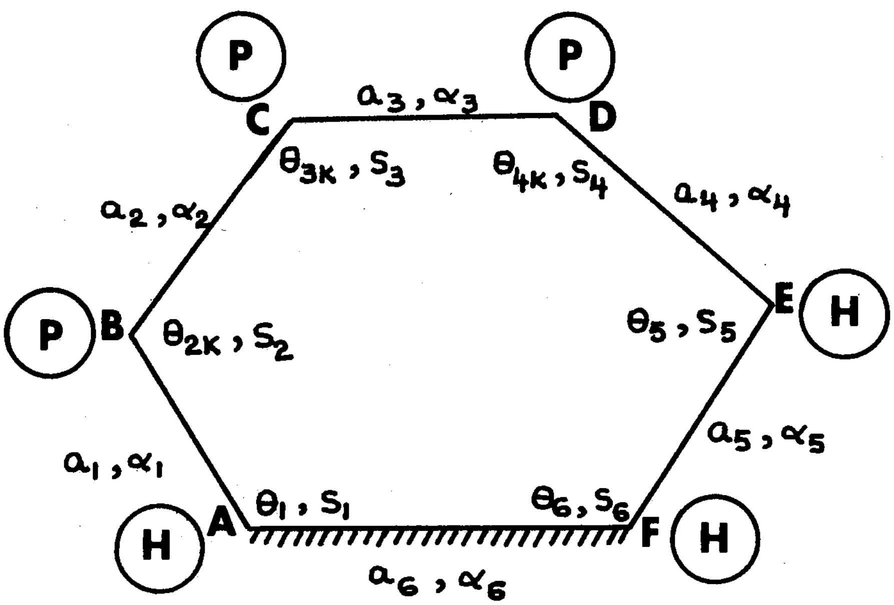

Figure 10. H-P-P-P-H-H Space Mechanism Obtained From the Mechanism in Figure 9 by Making $\theta_2 = \theta_{2k}$ = a Constant

$$+ S\hat{\theta}_2\, C\hat{\alpha}_2\, S\hat{\alpha}_3\, C\hat{\theta}_3 + S\hat{\theta}_2\, S\hat{\alpha}_2\, S\hat{\alpha}_3) -$$

$$- C\hat{\theta}_1\, S\hat{\alpha}_6\, S\hat{\alpha}_4\, C\hat{\alpha}_1 (-S\hat{\theta}_2\, S\hat{\theta}_4\, C\hat{\theta}_3 - S\hat{\theta}_2\, C\hat{\theta}_4\, C\hat{\alpha}_3\, S\hat{\theta}_3$$

$$- C\hat{\theta}_2\, C\hat{\alpha}_2\, S\hat{\theta}_3 + C\hat{\theta}_2\, C\hat{\alpha}_2\, C\hat{\theta}_4\, C\hat{\alpha}_3\, C\hat{\theta}_3 - C\hat{\theta}_2\, S\hat{\alpha}_2\, C\hat{\theta}_4\, S\hat{\alpha}_3)$$

$$+ C\hat{\theta}_1\, S\hat{\alpha}_6\, S\hat{\alpha}_4\, S\hat{\alpha}_1 (-S\hat{\alpha}_2\, S\hat{\theta}_4\, S\hat{\theta}_3 + S\hat{\alpha}_2\, C\hat{\theta}_4\, C\hat{\alpha}_3\, C\hat{\theta}_3$$

$$+ C\hat{\alpha}_2\, C\hat{\theta}_4\, S\hat{\alpha}_3) + C\hat{\theta}_1\, S\hat{\alpha}_6\, C\hat{\alpha}_4\, C\hat{\alpha}_1 (S\hat{\theta}_2\, S\hat{\alpha}_3\, S\hat{\theta}_3$$

$$- C\hat{\theta}_2\, C\hat{\alpha}_2\, S\hat{\alpha}_3\, C\hat{\theta}_3 - C\hat{\theta}_2\, S\hat{\alpha}_2\, C\hat{\alpha}_3 -$$

$$- C\hat{\theta}_1\, S\hat{\alpha}_6\, C\hat{\alpha}_4\, S\hat{\alpha}_1 (-S\hat{\alpha}_2\, S\hat{\alpha}_3\, C\hat{\theta}_3 + C\hat{\alpha}_2\, C\hat{\alpha}_3)$$

$$- C\hat{\alpha}_6\, S\hat{\alpha}_4\, S\hat{\alpha}_1 (-S\hat{\theta}_2\, S\hat{\theta}_4\, C\hat{\theta}_3 - S\hat{\theta}_2\, C\hat{\theta}_4\, C\hat{\alpha}_3\, S\hat{\theta}_3$$

$$- C\hat{\theta}_2\, C\hat{\alpha}_2\, S\hat{\theta}_4\, S\hat{\theta}_3 + C\hat{\theta}_2\, C\hat{\alpha}_2\, C\hat{\theta}_4\, C\hat{\alpha}_3\, C\hat{\theta}_3$$

$$- C\hat{\theta}_2\, S\hat{\alpha}_2\, C\hat{\theta}_4\, S\hat{\alpha}_3) - C\hat{\alpha}_6\, S\hat{\alpha}_4\, C\hat{\alpha}_1 (-S\hat{\alpha}_2\, S\hat{\theta}_4\, S\hat{\theta}_3$$

$$+ S\hat{\alpha}_2\, C\hat{\theta}_4\, C\hat{\alpha}_3\, C\hat{\theta}_3 + C\hat{\alpha}_2\, C\hat{\theta}_4\, S\hat{\alpha}_3)$$

$$+ C\hat{\alpha}_6\, C\hat{\alpha}_4\, S\hat{\alpha}_1 (S\hat{\theta}_2\, S\hat{\alpha}_3\, S\hat{\theta}_3 - C\hat{\theta}_2\, C\hat{\alpha}_2\, S\hat{\alpha}_3\, C\hat{\theta}_3$$

$$- C\hat{\theta}_2\, S\hat{\alpha}_2\, C\hat{\alpha}_3) + C\hat{\alpha}_6\, C\hat{\alpha}_4\, C\hat{\alpha}_1 (-S\hat{\alpha}_2\, S\hat{\alpha}_3\, C\hat{\theta}_3$$

$$+ C\hat{\alpha}_2\, C\hat{\alpha}_3) - C\hat{\alpha}_5 = 0 \tag{3-15}$$

$$F_3 (\hat{\theta}_2, \hat{\theta}_3, \hat{\theta}_4, \hat{\theta}_5) = - S\hat{\theta}_2\, S\hat{\alpha}_5\, S\hat{\alpha}_1 (-C\hat{\theta}_3\, S\hat{\theta}_5\, C\hat{\theta}_4$$

$$- C\hat{\theta}_5\, C\hat{\alpha}_4\, S\hat{\theta}_4 + S\hat{\theta}_3\, C\hat{\alpha}_3\, S\hat{\theta}_5\, S\hat{\theta}_4 - S\hat{\theta}_3\, C\hat{\alpha}_3\, C\hat{\theta}_5\, C\hat{\alpha}_4\, C\hat{\theta}_4$$

$$+ S\hat{\theta}_3\, C\hat{\theta}_5\, S\hat{\alpha}_4) + S\hat{\theta}_2\, S\hat{\alpha}_1\, C\hat{\alpha}_5 (C\hat{\theta}_3\, S\hat{\alpha}_4\, S\hat{\theta}_4$$

$$+ S\hat{\theta}_3\, C\hat{\alpha}_3\, S\hat{\alpha}_4\, C\hat{\theta}_4 + S\hat{\theta}_3\, S\hat{\alpha}_3\, S\hat{\alpha}_4)$$

$$- C\hat{\theta}_2\, S\hat{\alpha}_1\, C\hat{\alpha}_2 (-S\hat{\theta}_3\, S\hat{\theta}_5\, C\hat{\theta}_4 - S\hat{\theta}_3\, C\hat{\theta}_5\, C\hat{\alpha}_4\, S\hat{\theta}_4$$

$$- C\hat{\theta}_3 \, C\hat{\alpha}_3 \, S\hat{\theta}_5 \, S\hat{\theta}_4 + C\hat{\theta}_3 \, C\hat{\alpha}_3 \, C\hat{\theta}_5 \, C\hat{\alpha}_4 \, C\hat{\theta}_4$$

$$- C\hat{\theta}_3 \, S\hat{\alpha}_3 \, C\hat{\theta}_5 \, S\hat{\alpha}_4) + C\hat{\theta}_2 \, S\hat{\alpha}_1 \, S\hat{\alpha}_5 \, S\hat{\alpha}_2 \, (-S\hat{\alpha}_3 \, S\hat{\theta}_5 \, S\hat{\theta}_4$$

$$+ S\hat{\alpha}_3 \, C\hat{\theta}_5 \, C\hat{\alpha}_4 \, C\hat{\theta}_4 + C\hat{\alpha}_3 \, C\hat{\theta}_5 \, S\hat{\alpha}_4)$$

$$+ C\hat{\theta}_2 \, S\hat{\alpha}_1 \, C\hat{\alpha}_5 \, C\hat{\alpha}_2 \, (S\hat{\theta}_3 \, S\hat{\alpha}_4 \, S\hat{\theta}_4 - C\hat{\theta}_3 \, C\hat{\alpha}_3 \, S\hat{\alpha}_4 \, C\hat{\theta}_4$$

$$- C\hat{\theta}_3 \, S\hat{\alpha}_3 \, C\hat{\alpha}_4) - C\hat{\theta}_2 \, S\hat{\alpha}_1 \, C\hat{\alpha}_5 \, S\hat{\alpha}_2 \, (-S\hat{\alpha}_3 \, S\hat{\alpha}_4 \, C\hat{\theta}_4$$

$$+ C\hat{\alpha}_3 \, C\hat{\alpha}_4) - C\hat{\alpha}_1 \, S\hat{\alpha}_5 \, S\hat{\alpha}_2 \, (-S\hat{\theta}_3 \, S\hat{\theta}_5 \, C\hat{\theta}_4$$

$$- S\hat{\theta}_3 \, C\hat{\theta}_5 \, C\hat{\alpha}_4 \, S\hat{\theta}_4 - C\hat{\theta}_3 \, C\hat{\alpha}_3 \, S\hat{\theta}_5 \, S\hat{\theta}_4$$

$$+ C\hat{\theta}_3 \, C\hat{\alpha}_3 \, C\hat{\theta}_5 \, C\hat{\alpha}_4 \, C\hat{\theta}_4 - C\hat{\theta}_3 \, S\hat{\alpha}_3 \, C\hat{\theta}_5 \, S\hat{\alpha}_4)$$

$$- C\hat{\alpha}_1 \, S\hat{\alpha}_5 \, C\hat{\alpha}_2 \, (-S\hat{\alpha}_3 \, S\hat{\theta}_5 \, S\hat{\theta}_4 + S\hat{\alpha}_3 \, C\hat{\theta}_5 \, C\hat{\alpha}_4 \, C\hat{\theta}_4$$

$$+ C\hat{\alpha}_3 \, C\hat{\theta}_5 \, S\hat{\alpha}_4) + C\hat{\alpha}_1 \, C\hat{\alpha}_5 \, S\hat{\alpha}_2 \, (S\hat{\theta}_3 \, S\hat{\alpha}_4 \, S\hat{\theta}_4$$

$$- C\hat{\theta}_3 \, C\hat{\alpha}_3 \, S\hat{\alpha}_4 \, C\hat{\theta}_4 - C\hat{\theta}_3 \, S\hat{\alpha}_3 \, C\hat{\alpha}_4)$$

$$+ C\hat{\alpha}_1 \, C\hat{\alpha}_5 \, C\hat{\alpha}_2 \, (-S\hat{\alpha}_3 \, S\hat{\alpha}_4 \, C\hat{\theta}_4 + C\hat{\alpha}_3 \, C\hat{\alpha}_4) - C\hat{\alpha}_6 = 0 \qquad (3\text{-}16)$$

Observe that Eq. (3-14) is similar in form to Eq. (3-9) and Eqs. (3-15) and (3-16) are similar in form to Eq. (3-11). Note also that each of the above equations relates the dual displacement angles $\hat{\theta}_2$, $\hat{\theta}_3$, and $\hat{\theta}_4$ to a fourth dual displacement angle. The displacement angles θ_3 and θ_4 at the prismatic pairs are constant.

Let the displacement angle θ_2 at the cylinder pair be now held constant at all positions of the mechanism. Denoting the constant value of θ_2 by θ_{2k}, the primary parts of Eqs. (3-14), (3-15), and (3-16) give

$$A_s\, S\theta_6 + A_c\, C\theta_6 + A_n = 0 \qquad (3\text{-}17)$$

$$B_s\, S\theta_1 + B_c\, C\theta_1 + B_n = 0 \qquad (3\text{-}18)$$

$$C_s\, S\theta_5 + C_c\, C\theta_5 + C_n = 0 \qquad (3\text{-}19)$$

The constants in the above equations involve the constant kinematic parameters and are defined in Table IV.

Observe that each of the equations (3-17) through (3-19) contains only one variable and must hold true at varying values of that variable. This is possible only if their coefficients vanish. This gives

$$\begin{aligned} &A_s = A_c = A_n = 0 \\ &B_s = B_c = B_n = 0 \\ &C_s = C_c = C_n = 0 \end{aligned} \qquad (3\text{-}20)$$

Examination of Eqs. (3-20) gives the following relationships:

$$\alpha_5 = \alpha_6 = 0 \qquad (3\text{-}21)$$

$$\begin{aligned} C\theta_{2k}\, &[S\theta_{4k}\, S\theta_{3k}\, (S\alpha_4\, C\alpha_2\, S\alpha_1) + C\theta_{4k}\, C\theta_{3k}(-S\alpha_4\, C\alpha_3\, C\alpha_2\, S\alpha_1) \\ &+ C\theta_{4k}\, (S\alpha_4\, S\alpha_3\, S\alpha_2\, S\alpha_1) + C\theta_{3k}\, (-C\alpha_4\, S\alpha_3\, C\alpha_2\, S\alpha_1) \\ &+ (-C\alpha_4\, C\alpha_3\, S\alpha_2\, S\alpha_1)] + S\theta_{2k}\, [S\theta_{4k}\, C\theta_{3k}(S\alpha_4\, S\alpha_1) \\ &+ C\theta_{4k}\, S\theta_{3k}\, (S\alpha_4\, C\alpha_3\, S\alpha_1) + S\theta_{3k}\, (C\alpha_4\, S\alpha_3\, S\alpha_1)] \\ &+ S\theta_{4k}\, S\theta_{3k}\, (S\alpha_4\, S\alpha_2\, C\alpha_1) + C\theta_{4k}\, C\theta_{3k}(-S\alpha_4\, C\alpha_3\, S\alpha_2\, C\alpha_1) \end{aligned}$$

TABLE IV

CONSTANTS FOR USE IN EQUATIONS (3-17) THROUGH (3-20)

$$A_c = S\alpha_6\ S\alpha_5$$

$$A_n = C\theta_{2k}\ [S\theta_{4k}\ S\theta_{3k}\ (S\alpha_4\ C\alpha_2\ S\alpha_1) + C\theta_{4k}\ C\theta_{3k}\ (-S\alpha_4\ C\alpha_3\ C\alpha_2\ S\alpha_1)$$
$$+ C\theta_{4k}\ (S\alpha_4\ S\alpha_3\ S\alpha_2\ S\alpha_1) + C\theta_{3k}\ (-C\alpha_4\ S\alpha_3\ C\alpha_2\ S\alpha_1)$$
$$+ (-C\alpha_4\ C\alpha_3\ S\alpha_2\ S\alpha_1)] + S\theta_{2k}\ [S\theta_{4k}\ C\theta_{3k}\ (S\alpha_4\ S\alpha_1)$$
$$+ C\theta_{4k}\ S\theta_{3k}\ (S\alpha_4\ C\alpha_3\ S\alpha_1) + S\theta_{3k}\ (C\alpha_4\ S\alpha_3\ S\alpha_1)]$$
$$+ [S\theta_{4k}\ S\theta_{3k}\ (S\alpha_4\ S\alpha_2\ C\alpha_1) + C\theta_{4k}\ C\theta_{3k}\ (-S\alpha_4\ C\alpha_3\ S\alpha_2\ C\alpha_1)$$
$$+ C\theta_{4k}\ (-S\alpha_4\ S\alpha_3\ C\alpha_2\ C\alpha_1) + C\theta_{3k}\ (-C\alpha_4\ S\alpha_3\ S\alpha_2\ C\alpha_1)$$
$$+ (C\alpha_4\ C\alpha_3\ C\alpha_2\ C\alpha_1)] - C\alpha_6\ C\alpha_5$$

$$B_s = - S\alpha_4\ S\alpha_6\ (-C\theta_{2k}\ S\theta_{4k}\ C\theta_{3k} - C\theta_{4k}\ C\alpha_3\ S\theta_{3k}$$
$$+ S\theta_{2k}\ C\alpha_2\ S\theta_{3k}\ S\theta_{4k} - S\theta_{2k}\ C\alpha_2\ C\theta_{4k}\ C\alpha_3\ C\theta_{3k}$$
$$+ S\theta_{2k}\ C\theta_{4k}\ S\alpha_3) + S\alpha_6\ C\alpha_4\ (C\theta_{2k}\ S\alpha_3\ S\theta_{3k} + S\theta_{2k}\ C\alpha_2\ S\alpha_3\ C\theta_{3k}$$
$$+ S\theta_{2k}\ S\alpha_2\ S\alpha_3)$$

$$B_c = -S\alpha_6\ S\alpha_4\ C\alpha_1\ (-S\theta_{2k}\ S\theta_{4k}\ C\theta_{3k} - S\theta_{2k}\ C\theta_{4k}\ C\alpha_3\ S\theta_{3k}$$
$$- C\theta_{2k}\ C\alpha_2\ S\theta_{4k}\ S\theta_{3k} + C\theta_{2k}\ C\alpha_2\ C\theta_{4k}\ C\alpha_3\ C\theta_{3k}$$
$$- C\theta_{2k}\ S\alpha_2\ C\theta_{4k}\ S\alpha_3) + S\alpha_6\ S\alpha_4\ S\alpha_1\ (-S\alpha_2\ S\theta_{4k}\ S\theta_{3k}$$
$$+ S\alpha_2\ C\theta_{4k}\ C\alpha_3\ C\theta_{3k} + C\alpha_2\ C\theta_{4k}\ S\alpha_3)$$

TABLE IV (Continued)

$$+ S\alpha_6\ C\alpha_4\ C\alpha_1\ (S\theta_{2k}\ S\alpha_3\ S\theta_{3k} - C\theta_{2k}\ C\alpha_2\ S\alpha_3\ C\theta_{3k}$$

$$- C\theta_2\ S\alpha_2\ C\alpha_3) - S\alpha_6\ C\alpha_4\ S\alpha_1\ (-S\alpha_2\ S\alpha_3\ C\theta_{3k} + C\alpha_2\ C\alpha_3)$$

$$B_n = -C\alpha_6\ S\alpha_4\ S\alpha_1\ (-S\theta_{2k}\ S\theta_{4k}\ C\theta_{3k} - S\theta_{2k}\ C\theta_{4k}\ C\alpha_3\ S\theta_{3k}$$

$$- C\theta_{2k}\ C\alpha_2\ S\theta_{4k}\ S\theta_{3k} + C\theta_{2k}\ C\alpha_2\ C\theta_{4k}\ C\alpha_3\ C\theta_{3k}$$

$$- C\theta_{2k}\ S\alpha_2\ C\theta_{4k}\ S\alpha_3) - C\alpha_6\ S\alpha_4\ C\alpha_1\ (-S\alpha_2\ S\theta_{4k}\ S\theta_{3k}$$

$$+ S\alpha_2\ C\theta_{4k}\ C\alpha_3\ C\theta_{3k} + C\alpha_2\ C\theta_{4k}\ S\alpha_3)$$

$$+ C\alpha_6\ C\alpha_4\ S\alpha_1\ (S\theta_{2k}\ S\alpha_3\ S\theta_{3k} - C\theta_{2k}\ C\alpha_2\ S\alpha_3\ C\theta_{3k}$$

$$- C\theta_{2k}\ S\alpha_2\ C\alpha_3) + C\alpha_6\ C\alpha_4\ C\alpha_1\ (-S\alpha_2\ S\alpha_3\ C\theta_{3k} + C\alpha_2\ C\alpha_3)$$

$$- C\alpha_5$$

$$C_s = - S\theta_{2k}\ S\alpha_5\ S\alpha_1\ (-C\theta_{3k}\ C\theta_{4k} + S\theta_{3k}\ C\alpha_3\ S\theta_{4k})$$

$$- C\theta_{2k}\ S\alpha_1\ S\alpha_5\ C\alpha_2\ (-S\theta_{3k}\ C\theta_{4k} - C\theta_{3k}\ C\alpha_3\ S\theta_{4k})$$

$$+ C\theta_{2k}\ S\alpha_1\ S\alpha_2\ S\alpha_5\ (-S\alpha_3\ S\theta_{4k}) + C\alpha_1\ S\alpha_5\ S\alpha_2\ (S\theta_{3k}\ C\theta_{4k}$$

$$+ C\theta_{3k}\ C\alpha_3\ S\theta_{4k}) + C\alpha_1\ S\alpha_5\ C\alpha_2\ S\alpha_3\ S\theta_{4k}$$

$$C_c = - S\theta_{2k}\ S\alpha_5\ S\alpha_1\ (-C\alpha_4\ S\theta_{4k} - S\theta_{3k}\ C\alpha_3\ C\alpha_4\ C\theta_{4k} + S\theta_{3k}\ S\alpha_4)$$

$$- C\theta_{2k}\ S\alpha_1\ S\alpha_5\ C\alpha_2\ (-S\theta_{3k}\ C\alpha_4\ S\theta_{4k} + C\theta_{3k}\ C\alpha_4\ C\theta_{4k}$$

$$- C\theta_{3k}\ S\alpha_3\ S\alpha_4) + C\theta_{2k}\ S\alpha_1\ S\alpha_5\ S\alpha_2\ (S\alpha_3\ C\alpha_4\ C\theta_{4k} + C\alpha_3\ S\alpha_4)$$

$$- C\alpha_1\ S\alpha_5\ S\alpha_2\ (-S\theta_{3k}\ C\alpha_4\ S\theta_{4k} + C\theta_{3k}\ C\alpha_3\ C\alpha_4\ C\theta_{4k}$$

TABLE IV (Continued)

$$- C\theta_{3k}\, S\alpha_3\, S\alpha_4) - C\alpha_1\, S\alpha_5\, C\alpha_2\, (S\alpha_3\, C\alpha_4\, C\theta_{4k}$$

$$+ C\alpha_3\, S\alpha_4$$

$$C_n = S\theta_{2k}\, S\alpha_1\, C\alpha_5\, (C\theta_{3k}\, S\alpha_4\, S\theta_{4k} + S\theta_{3k}\, C\alpha_3\, S\alpha_4\, C\theta_{4k}$$

$$+ S\theta_{3k}\, S\alpha_3\, C\alpha_4) + C\theta_2\, S\alpha_1\, C\alpha_5\, C\alpha_2\, (S\theta_{3k}\, S\alpha_4\, S\theta_{4k}$$

$$- C\theta_{3k}\, C\alpha_3\, S\alpha_4\, C\theta_{4k} - C\theta_{3k}\, S\alpha_3\, C\alpha_4)$$

$$- C\theta_{2k}\, S\alpha_1\, C\alpha_5\, S\alpha_2\, (-S\alpha_3\, S\alpha_4\, C\theta_{4k} + C\alpha_3\, C\alpha_4)$$

$$+ C\alpha_1\, C\alpha_5\, S\alpha_2\, (S\theta_{3k}\, S\alpha_4\, S\theta_{4k} - C\theta_{3k}\, C\alpha_3\, S\alpha_4\, C\theta_{4k}$$

$$- C\theta_{3k}\, S\alpha_3\, C\alpha_4) + C\alpha_1\, C\alpha_5\, C\alpha_2\, (-S\alpha_3\, S\alpha_4\, C\theta_{4k} + C\alpha_3\, C\alpha_4) - C\alpha_6$$

$$+ C\theta_{4k}(-S\alpha_4 \, S\alpha_3 \, C\alpha_2 \, C\alpha_1) + C\theta_{3k}(-C\alpha_4 \, S\alpha_3 \, S\alpha_2 \, C\alpha_1)$$

$$+ (C\alpha_4 \, C\alpha_3 \, C\alpha_2 \, C\alpha_1)] - 1 = 0 \qquad (3\text{-}22)$$

The above relationships provide the necessary conditions for the existence of an H-P-P-P-H-H mechanism. Eq. (3-21) shows that the axes of the three helical pairs are parallel to each other. Eq. (3-22) is a closure condition relating the twist angles α_1, α_2, α_3, and α_4 of the mechanism with the constant displacement angles θ_{2k}, θ_{3k}, and θ_{4k} at the three prismatic pairs (Figure 10).

Existence Criteria of the Six-Link H-P-P-H-P-H Mechanism

The existence criteria of an H-P-P-H-P-H mechanism can be obtained from the displacement relationships of an H-C-P-H-P-H mechanism.

Consider the H-C-P-H-P-H space mechanism shown schematically in Figure 11, with helical pairs at joints A, D, and F, cylinder pairs at joint B, and prism pairs at joints C and E. The instantaneous configuration of the H-C-P-H-P-H mechanism as shown in Figure 11, is completely defined by the two sets of six dual-angles, each as follows:

1. Between adjacent pairing axes:

$$\hat{\alpha}_i = \alpha_i + \varepsilon \, a_i \qquad (i = 1 \text{ to } 6) \qquad (3\text{-}23)$$

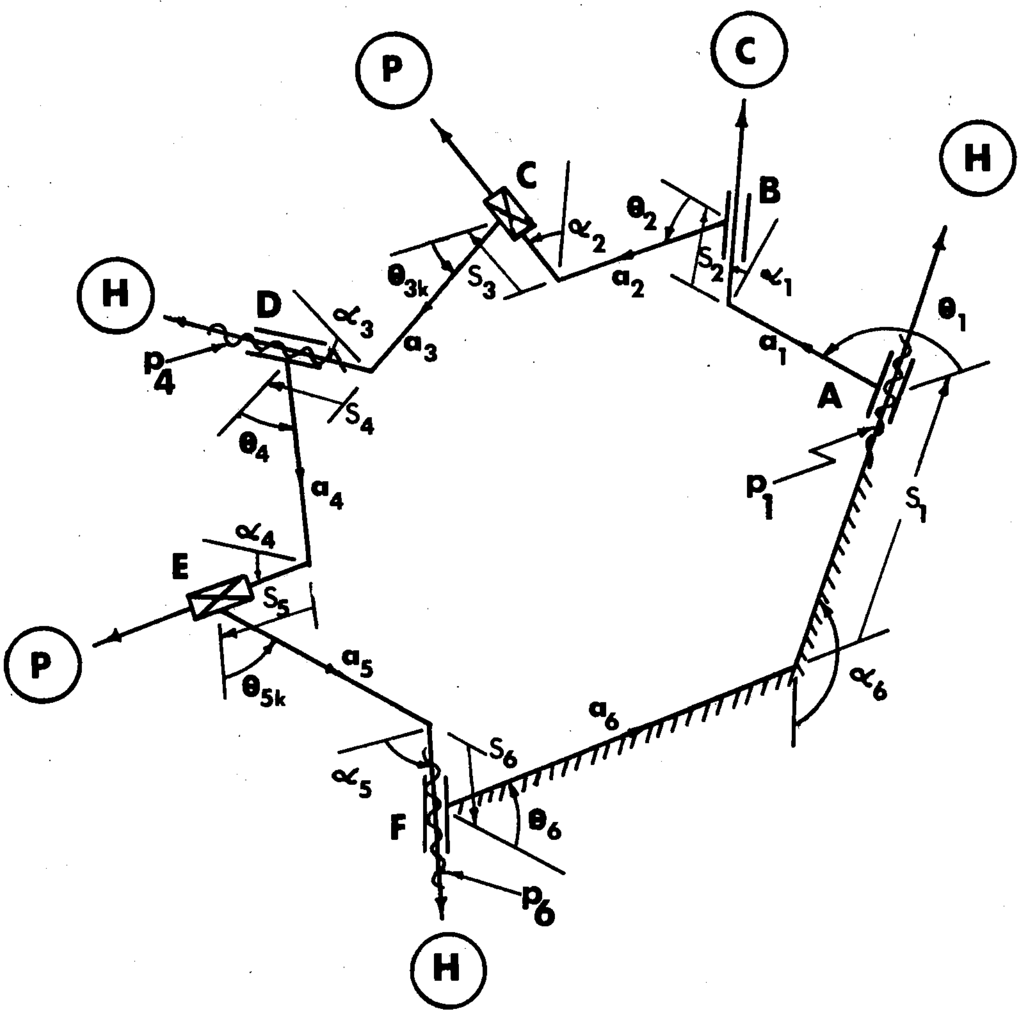

Figure 11. Six-link H-C-P-H-P-H Space Mechanism

where α_i (i = 1 to 6) are the twist angles and a_i (i = 1 to 6) are the kinematic link lengths.

2. Between adjacent common perpendiculars:

$$\hat{\theta}_i = \theta_i + \varepsilon s_i \qquad (i = 1 \text{ to } 6) \qquad (3\text{-}24)$$

with $$s_i = p_i \theta_i \qquad (i = 1, 4, 6)$$

where θ_i (i = 1 to 6) are the angular displacements at the kinematic pairs, s_i (i = 1 to 6) are the translational displacements along the kinematic axes, and p_i (i = 1, 4, 6) are the finite pitch values of the helical pairs.

In Eq. (3-24), the four angles, θ_i (i = 1, 2, 4, 6) and the three sliding components along the axes of the cylinder and prism pairs (s_2, s_3, and s_5) constitute the seven linkage variables of the H-C-P-H-P-H mechanism. The six dual angles $\hat{\alpha}_i$ (i = 1 to 6) in Eq. (3-23) and the two constant displacement angles θ_{3k} and θ_{5k} of the prismatic pairs at joints C and E and the three finite pitch values of the helical pairs (p_1, p_4, p_6) constitute the seventeen real parameters necessary to specify an H-C-P-H-P-H mechanism of general proportions.

Consider the H-C-P-H-P-H space mechanism shown schematically in Figure 12. This mechanism reduces to an H-P-P-H-P-H mechanism if the displacement angle θ_2 at the cylinder pair remains constant at all positions of the mechanism (Figure 13).

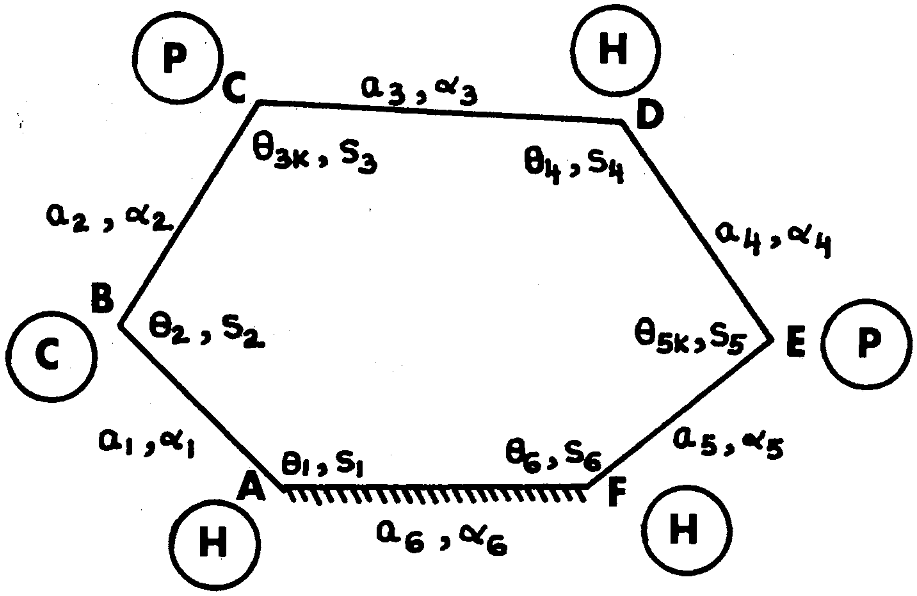

Figure 12. H-C-P-H-P-H Space Mechanism

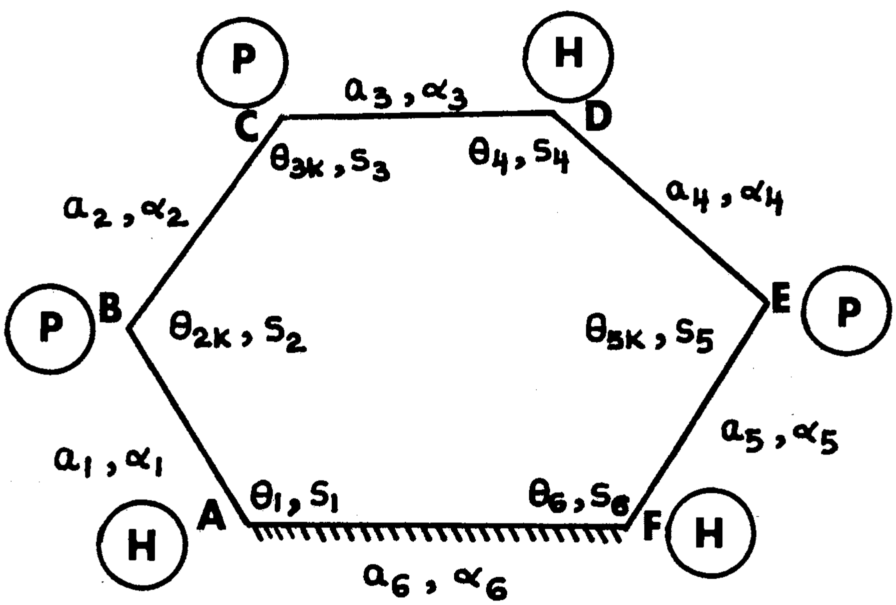

Figure 13. H-P-P-H-P-H Space Mechanism Obtained From the Mechanism in Figure 12 by Making $\theta_2 = \theta_{2k}$ = a Constant

By considering the loop-closure condition of the mechanism in Figure 12 in three different ways, the following relationships can be obtained (120):

$$F_1(\hat{\theta}_6, \hat{\theta}_5, \hat{\theta}_3, \hat{\theta}_2) = [S\hat{\theta}_3 S\hat{\theta}_2 S\hat{\alpha}_3 S\hat{\alpha}_1 - C\hat{\theta}_3(C\hat{\theta}_2 S\hat{\alpha}_3 C\hat{\alpha}_2 S\hat{\alpha}_1$$
$$+ S\hat{\alpha}_3 S\hat{\alpha}_2 C\hat{\alpha}_1)] + (-C\hat{\theta}_2 C\hat{\alpha}_3 S\hat{\alpha}_2 S\hat{\alpha}_1 + C\hat{\alpha}_3 C\hat{\alpha}_2 C\hat{\alpha}_1)$$
$$- [S\hat{\theta}_6 S\hat{\theta}_5 S\hat{\alpha}_6 S\hat{\alpha}_4 - C\hat{\theta}_6 (C\hat{\theta}_5 S\hat{\alpha}_6 C\hat{\alpha}_5 S\hat{\alpha}_4$$
$$+ S\hat{\alpha}_6 S\hat{\alpha}_5 C\hat{\alpha}_4)] - (C\hat{\theta}_5 C\hat{\alpha}_6 S\hat{\alpha}_5 S\hat{\alpha}_4$$
$$+ C\hat{\alpha}_6 C\hat{\alpha}_5 C\hat{\alpha}_4) = 0 \qquad (3\text{-}25)$$

$$F_2(\hat{\theta}_3, \hat{\theta}_2, \hat{\theta}_1, \hat{\theta}_5) = C\hat{\theta}_1 [S\hat{\theta}_3 S\hat{\theta}_2 (S\hat{\alpha}_3 C\hat{\alpha}_1 S\hat{\alpha}_6)$$
$$+ C\hat{\theta}_3 C\hat{\theta}_2 (-S\hat{\alpha}_3 C\hat{\alpha}_2 C\hat{\alpha}_1 S\hat{\alpha}_6) + C\hat{\theta}_3 (S\hat{\alpha}_3 S\hat{\alpha}_2 S\hat{\alpha}_1 S\hat{\alpha}_6)$$
$$+ C\hat{\theta}_2 (-C\hat{\alpha}_3 S\hat{\alpha}_2 C\hat{\alpha}_1 S\hat{\alpha}_6) + (-C\hat{\alpha}_3 C\hat{\alpha}_2 S\hat{\alpha}_1 S\hat{\alpha}_6)]$$
$$+ S\hat{\theta}_1 [S\hat{\theta}_3 C\hat{\theta}_2 (S\hat{\alpha}_3 S\hat{\alpha}_6) + C\hat{\theta}_3 S\hat{\theta}_2 (S\hat{\alpha}_3 C\hat{\alpha}_2 S\hat{\alpha}_6)$$
$$+ S\hat{\theta}_2 (C\hat{\alpha}_3 S\hat{\alpha}_2 S\hat{\alpha}_6)] + [S\hat{\theta}_3 S\hat{\theta}_2 (S\hat{\alpha}_3 S\hat{\alpha}_1 C\hat{\alpha}_6)$$
$$+ C\hat{\theta}_3 C\hat{\theta}_2 (-S\hat{\alpha}_1 S\hat{\alpha}_3 C\hat{\alpha}_2 C\hat{\alpha}_6) + C\hat{\theta}_3(-S\hat{\alpha}_3 S\hat{\alpha}_2 C\hat{\alpha}_1 C\hat{\alpha}_6)$$
$$+ C\hat{\theta}_2 (-C\hat{\alpha}_3 S\hat{\alpha}_2 S\hat{\alpha}_1 C\hat{\alpha}_6) + (C\hat{\alpha}_3 C\hat{\alpha}_2 C\hat{\alpha}_1 C\hat{\alpha}_6)]$$
$$- C\hat{\alpha}_5 C\hat{\alpha}_4 + S\hat{\alpha}_5 S\hat{\alpha}_4 C\hat{\theta}_5 = 0 \qquad (3\text{-}26)$$

$$F_3(\hat{\theta}_2, \hat{\theta}_3, \hat{\theta}_4, \hat{\theta}_5) = - S\hat{\theta}_2 S\hat{\alpha}_5 S\hat{\alpha}_1 (-C\hat{\theta}_3 C\hat{\theta}_5 C\hat{\theta}_4$$
$$- C\hat{\theta}_5 C\hat{\alpha}_4 S\hat{\theta}_4 + S\hat{\theta}_3 C\hat{\alpha}_3 S\hat{\theta}_5 S\hat{\theta}_4 - S\hat{\theta}_3 C\hat{\alpha}_3 C\hat{\theta}_5 C\hat{\alpha}_4 C\hat{\theta}_4$$
$$+ S\hat{\theta}_3 C\hat{\theta}_5 S\hat{\alpha}_4) + S\hat{\theta}_2 S\hat{\alpha}_1 C\hat{\alpha}_5 (C\hat{\theta}_3 S\hat{\alpha}_4 S\hat{\theta}_4$$

$$+ S\hat{\theta}_3\, C\hat{\alpha}_3\, S\hat{\alpha}_4\, C\hat{\theta}_4 + S\hat{\theta}_3\, S\hat{\alpha}_3\, S\hat{\alpha}_4)$$

$$- C\hat{\theta}_2\, S\hat{\alpha}_1\, S\hat{\alpha}_5\, C\hat{\alpha}_2\, (-S\hat{\theta}_3\, S\hat{\theta}_5\, C\hat{\theta}_4 - S\hat{\theta}_3\, C\hat{\theta}_5\, C\hat{\alpha}_4\, S\hat{\theta}_4$$

$$- C\hat{\theta}_3\, C\hat{\alpha}_3\, S\hat{\theta}_5\, S\hat{\theta}_4 + C\hat{\theta}_3\, C\hat{\alpha}_3\, C\hat{\theta}_5\, C\hat{\alpha}_4\, C\hat{\theta}_4$$

$$- C\hat{\theta}_3\, S\hat{\alpha}_3\, C\hat{\theta}_5\, S\hat{\alpha}_4) + C\hat{\theta}_2\, S\hat{\alpha}_1\, S\hat{\alpha}_5\, S\hat{\alpha}_2\, (-S\hat{\alpha}_3\, S\hat{\theta}_5\, S\hat{\theta}_4$$

$$+ S\hat{\alpha}_3\, C\hat{\theta}_5\, C\hat{\alpha}_4\, C\hat{\theta}_4 + C\hat{\alpha}_3\, C\hat{\theta}_5\, S\hat{\alpha}_4)$$

$$+ C\hat{\theta}_2\, S\hat{\alpha}_1\, C\hat{\alpha}_5\, C\hat{\alpha}_2\, (S\hat{\theta}_3\, S\hat{\alpha}_4\, S\hat{\theta}_4 - C\hat{\theta}_3\, C\hat{\alpha}_3\, S\hat{\alpha}_4\, C\hat{\theta}_4$$

$$- C\hat{\theta}_3\, S\hat{\alpha}_3\, C\hat{\alpha}_4) - C\hat{\theta}_2\, S\hat{\alpha}_1\, C\hat{\alpha}_5\, S\hat{\alpha}_2\, (-S\hat{\alpha}_3\, S\hat{\alpha}_4\, C\hat{\theta}_4$$

$$+ C\hat{\alpha}_3\, C\hat{\alpha}_4) - C\hat{\alpha}_1\, S\hat{\alpha}_5\, S\hat{\alpha}_2\, (-S\hat{\theta}_3\, S\hat{\theta}_5\, C\hat{\theta}_4 - S\hat{\theta}_3\, C\hat{\theta}_5\, C\hat{\alpha}_4\, S\hat{\theta}_4$$

$$- C\hat{\theta}_3\, C\hat{\alpha}_3\, S\hat{\theta}_5\, S\hat{\theta}_4 + C\hat{\theta}_3\, C\hat{\alpha}_3\, C\hat{\theta}_5\, C\hat{\alpha}_4\, C\hat{\theta}_4$$

$$- C\hat{\theta}_3\, S\hat{\alpha}_3\, C\hat{\theta}_5\, S\hat{\alpha}_4) - C\hat{\alpha}_1\, S\hat{\alpha}_5\, C\hat{\alpha}_2\, (-S\hat{\alpha}_3\, S\hat{\theta}_5\, S\hat{\theta}_4$$

$$+ S\hat{\alpha}_3\, C\hat{\theta}_5\, C\hat{\alpha}_4\, C\hat{\theta}_4 + C\hat{\alpha}_3\, C\hat{\theta}_5\, S\hat{\alpha}_4)$$

$$+ C\hat{\alpha}_1\, C\hat{\alpha}_5\, S\hat{\alpha}_2\, (S\hat{\theta}_3\, S\hat{\alpha}_4\, S\hat{\theta}_4 - C\hat{\theta}_3\, C\hat{\alpha}_3\, S\hat{\alpha}_4\, C\hat{\theta}_4$$

$$- C\hat{\theta}_3\, S\hat{\alpha}_3\, C\hat{\alpha}_4) + C\hat{\alpha}_1\, C\hat{\alpha}_5\, C\hat{\alpha}_2\, (-S\hat{\alpha}_3\, S\hat{\alpha}_4\, C\hat{\theta}_4$$

$$+ C\hat{\alpha}_3\, C\hat{\alpha}_4) - C\hat{\alpha}_6 = 0 \qquad (3\text{-}27)$$

Note that Eq. (3-25) is similar in form to Eq. (3-7), Eq. (3-26) is similar in form to Eq. (3-9) and Eq. (3-27) is similar in form to Eq. (3-11). Note also that each of the above equations relates the dual displacement angle $\hat{\theta}_2$, $\hat{\theta}_3$, and $\hat{\theta}_5$ to a fourth dual displacement angle. The displacement angles θ_3 and θ_5 of the prism pairs are constant. Let the displacement angle θ_2 at the cylinder pair be now made constant at all positions of the mechanism. Denoting the constant value of θ_2 by θ_{2k}, the primary parts of Eqs. (3-25), (3-26), and (3-27) give

$$D_s S\theta_6 + D_c C\theta_6 + D_n = 0 \tag{3-28}$$

$$E_s S\theta_1 + E_c C\theta_1 + E_n = 0 \tag{3-29}$$

$$F_s S\theta_4 + F_c C\theta_4 + F_n = 0 \tag{3-30}$$

The constants used in the above equations involve the constant kinematic parameters of the mechanism and are defined in Table V.

Observe that each of the equations (3-28) through (3-30) contains only one variable and must hold true at varying values of that variable. This is possible only if their coefficients vanish. This gives

$$\begin{aligned} D_s &= D_c = D_n = 0 \\ E_s &= E_c = E_n = 0 \\ F_s &= F_c = F_n = 0 \end{aligned} \tag{3-31}$$

Equation (3-31) represents the necessary conditions for the existence of H-P-P-H-P-H mechanism. It is, however, possible to further simplify the conditions given by Eq. (3-31). For example, examination of Eq. (3-31) together with the constants of Table V show that the following case is possible:

$$\alpha_6 = 0 \tag{3-32}$$

$$\alpha_4 \pm \alpha_5 = n\pi \qquad (n = 0, 1, 2, \ldots) \tag{3-33}$$

TABLE V

CONSTANTS FOR USE IN EQUATIONS (3-28) THROUGH (3-31)

$$D_s = - S\theta_{5k} S\alpha_6 S\alpha_4$$

$$D_c = (C\theta_{5k} S\alpha_6 C\alpha_5 S\alpha_4 + S\alpha_6 S\alpha_5 C\alpha_4$$

$$D_n = [S\theta_{3k} S\theta_{2k} S\alpha_3 S\alpha_1 - C\theta_{3k} (C\theta_{2k} S\alpha_3 C\alpha_2 S\alpha_1 + S\alpha_3 S\alpha_2 C\alpha_1)]$$
$$+ (-C\theta_{2k} C\alpha_3 S\alpha_2 S\alpha_1 + C\alpha_3 C\alpha_2 C\alpha_1) - (C\theta_{5k} C\alpha_6 S\alpha_5 S\alpha_4$$
$$+ C\alpha_6 C\alpha_5 C\alpha_4)$$

$$E_s = [S\theta_{3k} C\theta_{2k} (S\alpha_3 S\alpha_6) + C\theta_{3k} S\theta_{2k} (S\alpha_3 C\alpha_2 S\alpha_6)$$
$$+ S\theta_{2k} (C\alpha_3 S\alpha_2 S\alpha_6)$$

$$E_c = [S\theta_{3k} S\theta_{2k} (S\alpha_3 C\alpha_1 S\alpha_6) + C\theta_{3k} C\theta_{2k} (-S\alpha_3 C\alpha_2 C\alpha_1 S\alpha_6)$$
$$+ C\theta_{3k} (S\alpha_3 S\alpha_2 S\alpha_1 S\alpha_6) + C\theta_{2k} (-C\alpha_3 S\alpha_2 C\alpha_1 S\alpha_6)$$
$$+ (-C\alpha_3 C\alpha_2 S\alpha_1 S\alpha_6)]$$

$$E_n = [S\theta_{3k} S\theta_{2k} (S\alpha_3 S\alpha_1 C\alpha_6) + C\theta_{3k} C\theta_{2k} (-S\alpha_1 S\alpha_3 C\alpha_2 C\alpha_6)$$
$$+ C\theta_{3k} (-S\alpha_3 S\alpha_2 C\alpha_1 C\alpha_6) + C\theta_{2k} (-C\alpha_3 S\alpha_2 S\alpha_1 C\alpha_6)$$
$$+ (C\alpha_3 C\alpha_2 C\alpha_1 C\alpha_6)] - C\alpha_5 C\alpha_4$$

$$F_s = - S\theta_{2k} S\alpha_5 S\alpha_1 (-C\theta_{3k} C\theta_{4k} + S\theta_{3k} C\alpha_3 S\theta_{4k})$$
$$- C\theta_{2k} S\alpha_1 S\alpha_5 C\alpha_2 (-S\theta_{3k} C\theta_{4k} - C\theta_{3k} C\alpha_3 S\theta_{4k})$$
$$+ C\theta_{2k} S\alpha_1 S\alpha_5 S\alpha_2 (-S\alpha_3 S\theta_{4k}) - C\alpha_1 S\alpha_5 S\alpha_2 (-S\theta_{3k} C\theta_{4k}$$

TABLE V (Continued)

$$- C\theta_{3k}\, C\alpha_3\, S\theta_{4k}) - C\alpha_1\, S\alpha_5\, C\alpha_2\, (-S\alpha_3\, S\theta_{4k})$$

$$F_c = - S\theta_{2k}\, S\alpha_5\, S\alpha_1\, (-C\alpha_4\, S\theta_{4k} - S\theta_{3k}\, C\alpha_3\, C\alpha_4\, C\theta_{4k} + S\theta_{3k}\, S\alpha_4)$$

$$- C\theta_{2k}\, S\alpha_1\, S\alpha_5\, C\alpha_2\, (-S\theta_{3k}\, C\alpha_4\, S\theta_{4k} + C\theta_{3k}\, C\alpha_3\, C\alpha_4\, C\theta_{4k}$$

$$- C\theta_{3k}\, S\alpha_3\, S\alpha_4) + C\theta_2\, S\alpha_1\, S\alpha_5\, S\alpha_2\, (S\alpha_3\, C\alpha_4\, C\theta_{4k}$$

$$+ C\alpha_3\, S\alpha_4) - C\alpha_1\, S\alpha_5\, S\alpha_2\, (-S\theta_{3k}\, C\alpha_4\, S\theta_{4k} + C\theta_{3k}\, C\alpha_3\, C\alpha_4\, C\theta_{4k}$$

$$- C\theta_{3k}\, S\alpha_3\, S\alpha_4) - C\alpha_1\, S\alpha_5\, C\alpha_2\, (S\alpha_3\, C\alpha_4\, C\theta_{4k} + C\alpha_3\, S\alpha_4)$$

$$F_n = S\theta_{2k}\, S\alpha_1\, C\alpha_5\, (C\theta_{3k}\, S\alpha_4\, S\theta_{4k} + S\theta_{3k}\, C\alpha_3\, S\alpha_4\, C\theta_{4k}$$

$$+ S\theta_{3k}\, S\alpha_3\, C\alpha_4) + C\theta_{2k}\, S\alpha_1\, C\alpha_5\, C\alpha_2\, (S\theta_{3k}\, S\alpha_4\, S\theta_{4k}$$

$$- C\theta_{3k}\, C\alpha_3\, S\alpha_4\, C\theta_{4k} - C\theta_{3k}\, S\alpha_3\, C\alpha_4) -$$

$$- C\theta_{2k}\, S\alpha_1\, C\alpha_5\, S\alpha_2\, (-S\alpha_3\, S\alpha_4\, C\theta_{4k} + C\alpha_3\, C\alpha_4)$$

$$+ C\alpha_1\, C\alpha_5\, S\alpha_2\, (S\theta_{3k}\, S\alpha_4\, S\theta_{4k} - C\theta_{3k}\, C\alpha_3\, S\alpha_4\, C\theta_{4k}$$

$$- C\theta_{3k}\, S\alpha_3\, C\alpha_4) + C\alpha_1\, C\alpha_5\, C\alpha_2\, (-S\alpha_3\, S\alpha_4\, C\theta_{4k}$$

$$+ C\alpha_3\, C\alpha_4) - C\alpha_6$$

and

$$[S\theta_{3k}\, S\theta_{2k}\, S\alpha_3\, S\alpha_1 - C\theta_{3k}\,(C\theta_{2k}\, S\alpha_3\, C\alpha_2\, S\alpha_1$$
$$+ S\alpha_3\, S\alpha_2\, C\alpha_1)] + (-C\theta_{2k}\, C\alpha_3\, S\alpha_2\, S\alpha_1$$
$$+ C\alpha_3\, C\alpha_2\, C\alpha_1) = 0 \qquad (3\text{-}34)$$

The above relationships provide the necessary conditions for the existence of an H-P-P-H-P-H space mechanism. Equations (3-32) and (3-33) show that the axes of the three helical pairs are parallel to each other. Equation (3-34) is a closure condition relating the twist angles α_1, α_2, and α_3 of the mechanism with the constant displacement angles θ_{2k}, θ_{3k}, and θ_{5k} at the three prismatic pairs (Figure 13).

Existence Criteria of the Six-Link H-P-H-P-H-P Mechanism

The existence criteria of an H-P-H-P-H-P mechanism can be obtained from the displacement relationships of an H-C-H-P-H-P mechanism shown in Figure 14, with helical pairs at joints A, C, and E, cylinder pair at joint B, and prism pairs at joints D and F. The instantaneous configuration of the H-C-H-P-H-P mechanism as shown in Figure 14 is completely defined by the two sets of six dual angles, each as follows:

1. Between adjacent pairing axes:

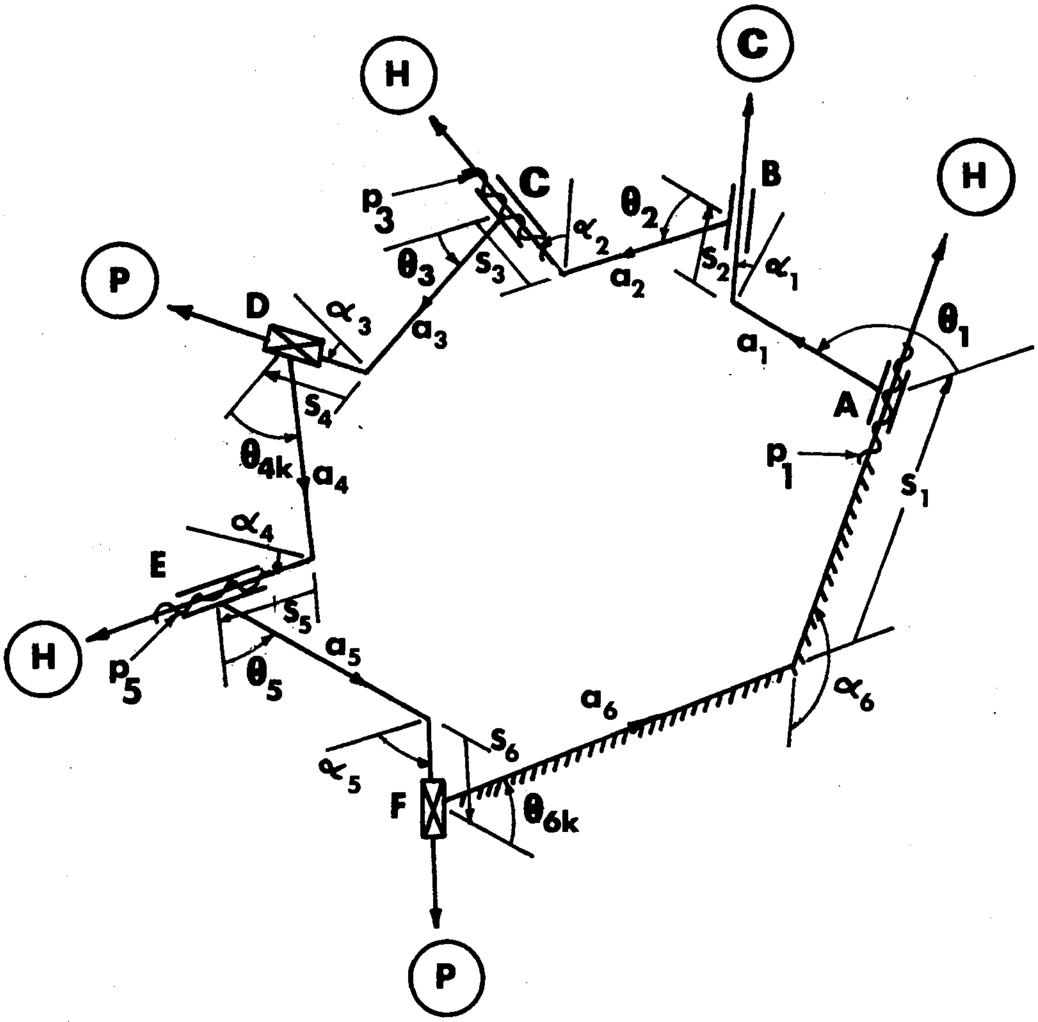

Figure 14. Six-link H-C-H-P-H-P Space Mechanism

$$\hat{\alpha}_1 = \alpha_i + \epsilon a_i \qquad (i = 1 \text{ to } 6) \qquad (3\text{-}35)$$

where α_i (i = 1 to 6) are the twist angles and a_i (i = 1 to 6) are the kinematic link-lengths.

2. Between adjacent common perpendiculars:

$$\hat{\theta}_i = \theta_i + \epsilon s_i \qquad (i = 1 \text{ to } 6) \qquad (3\text{-}36)$$

with $$s_i = p_i \theta_i \qquad (i = 1, 3, 5)$$

where θ_i (i = 1 to 6) are the angular displacements at the kinematic pairs, s_i (i = 1 to 6) are the translational displacements along the kinematic axes, and p_i (i = 1, 3, 5) are the finite pitch values of the helical pairs.

In Eq. (3-36), the four angles, θ_i (i = 1, 2, 3, 5), and the three sliding components along the axes of the cylinder and prism pairs (s_2, s_4, s_6) constitute the seven linkage variables of the H-C-H-P-H-P mechanism. The six dual angles $\hat{\alpha}$ (i = 1 to 6) in Eq. (3-35) and the two constant displacement angles θ_{4k} and θ_{6k} of the prismatic pairs at joints D and F and the three finite pitch values of the helical pairs (p_1, p_3, p_5) constitute the seventeen real parameters necessary to specify an H-C-H-P-H-P space mechanism of general proportions.

Consider the H-C-H-P-H-P space mechanism shown schematically in Figure 15. This mechanism reduces to an H-P-H-P-H-P mechanism if the displacement angle θ_2 at the cylinder pair remains constant at all positions of the mechanism (Figure 16).

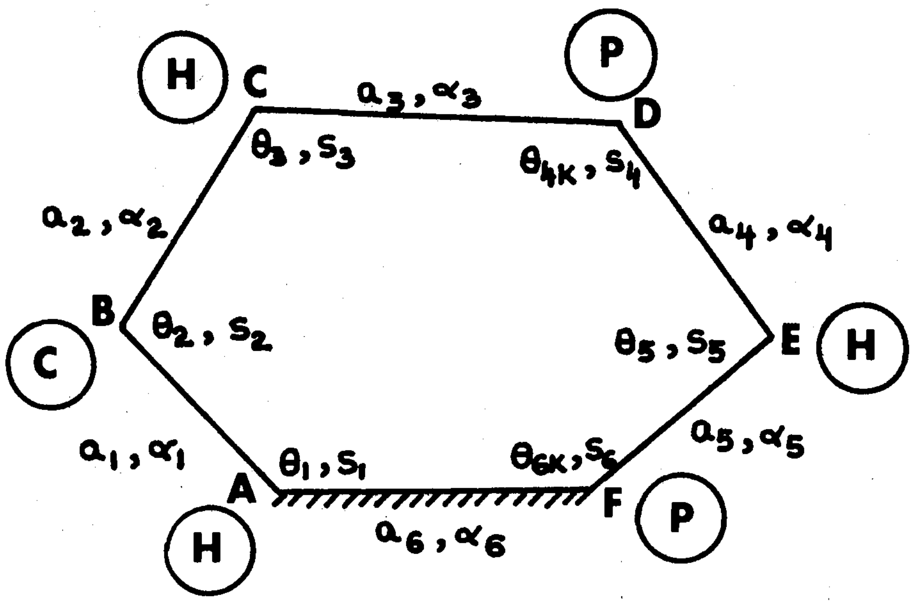

Figure 15. H-C-H-P-H-P Space Mechanism

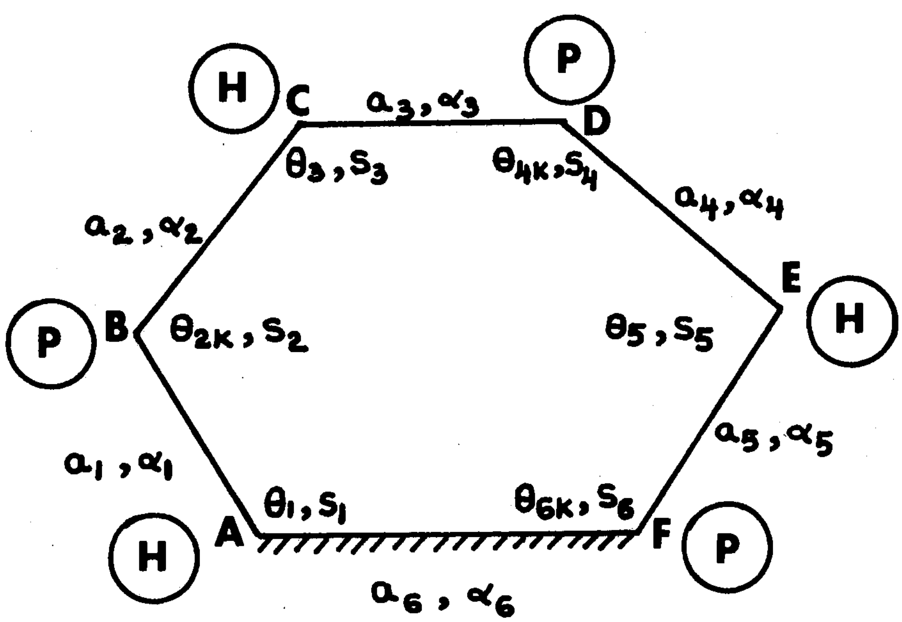

Figure 16. H-P-H-P-H-P Space Mechanism Obtained From the Mechanism in Figure 15 by Making $\theta_2 = \theta_{2k}$ = a Constant

By considering the loop-closure condition of the mechanism in Figure 15 in three different ways, the following dual displacement relationships can be obtained (120):

$$F_2(\hat{\theta}_4, \hat{\theta}_3, \hat{\theta}_2, \hat{\theta}_6) = C\hat{\theta}_2 [S\hat{\theta}_4 S\hat{\theta}_3 (S\hat{\alpha}_4 C\hat{\alpha}_2 S\hat{\alpha}_1)$$
$$+ C\hat{\theta}_4 C\hat{\theta}_3 (-S\hat{\alpha}_4 C\hat{\alpha}_3 C\hat{\alpha}_2 S\hat{\alpha}_1) + C\hat{\theta}_4 (S\hat{\alpha}_4 S\hat{\alpha}_3 S\hat{\alpha}_2 S\hat{\alpha}_1)$$
$$+ C\hat{\theta}_3 (-C\hat{\alpha}_4 S\hat{\alpha}_3 C\hat{\alpha}_2 S\hat{\alpha}_1) + (-C\hat{\alpha}_4 C\hat{\alpha}_3 S\hat{\alpha}_2 S\hat{\alpha}_1)]$$
$$+ S\hat{\theta}_2 [S\hat{\theta}_4 C\hat{\theta}_3 (S\hat{\alpha}_4 S\hat{\alpha}_1) + C\hat{\theta}_4 S\hat{\theta}_3 (S\hat{\alpha}_4 C\hat{\alpha}_3 S\hat{\alpha}_1)$$
$$+ S\hat{\theta}_3 (C\hat{\alpha}_4 S\hat{\alpha}_3 S\hat{\alpha}_1)] + [S\hat{\theta}_4 S\hat{\theta}_3 (S\hat{\alpha}_4 S\hat{\alpha}_2 C\hat{\alpha}_1)$$
$$+ C\hat{\theta}_4 C\hat{\theta}_3 (-S\hat{\alpha}_4 C\hat{\alpha}_3 S\hat{\alpha}_2 C\hat{\alpha}_1) + C\hat{\theta}_4 (-S\hat{\alpha}_4 S\hat{\alpha}_3 C\hat{\alpha}_2 C\hat{\alpha}_1)$$
$$+ C\hat{\theta}_3 (-C\hat{\alpha}_4 S\hat{\alpha}_3 S\hat{\alpha}_2 C\hat{\alpha}_1) + (C\hat{\alpha}_4 C\hat{\alpha}_3 C\hat{\alpha}_2 C\hat{\alpha}_1)]$$
$$- C\hat{\alpha}_6 C\hat{\alpha}_5 + S\hat{\alpha}_6 S\hat{\alpha}_5 C\hat{\theta}_6 = 0 \qquad (3\text{-}37)$$

$$F_2(\hat{\theta}_6, \hat{\theta}_5, \hat{\theta}_4, \hat{\theta}_2) = C\hat{\theta}_4 [S\hat{\theta}_6 S\hat{\theta}_5 (S\hat{\alpha}_6 C\hat{\alpha}_4 S\hat{\alpha}_3)$$
$$+ C\hat{\theta}_6 C\hat{\theta}_5 (-S\hat{\alpha}_6 C\hat{\alpha}_5 C\hat{\alpha}_4 S\hat{\alpha}_3) + C\hat{\theta}_6 (S\hat{\alpha}_6 S\hat{\alpha}_5 S\hat{\alpha}_4 S\hat{\alpha}_3)$$
$$+ C\hat{\theta}_5 (-C\hat{\alpha}_6 S\hat{\alpha}_5 C\hat{\alpha}_4 S\hat{\alpha}_3) + (-C\hat{\alpha}_6 C\hat{\alpha}_5 S\hat{\alpha}_4 S\hat{\alpha}_3)]$$
$$+ S\hat{\theta}_4 [S\hat{\theta}_6 C\hat{\theta}_5 (S\hat{\alpha}_6 S\hat{\alpha}_3) + C\hat{\theta}_6 S\hat{\theta}_5 (S\hat{\alpha}_6 C\hat{\alpha}_5 S\hat{\alpha}_3)$$
$$+ S\hat{\theta}_5 (C\hat{\alpha}_6 S\hat{\alpha}_5 S\hat{\alpha}_3)] + [S\hat{\theta}_6 S\hat{\theta}_5 (S\hat{\alpha}_6 S\hat{\alpha}_4 C\hat{\alpha}_3)$$
$$+ C\hat{\theta}_6 C\hat{\theta}_5 (-S\hat{\alpha}_6 C\hat{\alpha}_5 S\hat{\alpha}_4 C\hat{\alpha}_3) + C\hat{\theta}_6 (-S\hat{\alpha}_6 S\hat{\alpha}_5 C\hat{\alpha}_4 C\hat{\alpha}_3)$$
$$+ C\hat{\theta}_5 (-C\hat{\alpha}_6 S\hat{\alpha}_5 S\hat{\alpha}_4 C\hat{\alpha}_3) + (C\hat{\alpha}_6 C\hat{\alpha}_5 C\hat{\alpha}_4 C\hat{\alpha}_3)]$$
$$- C\hat{\alpha}_2 C\hat{\alpha}_1 + S\hat{\alpha}_2 S\hat{\alpha}_1 C\hat{\theta}_2 = 0 \qquad (3\text{-}38)$$

$$F_2 (\hat{\theta}_2, \hat{\theta}_1, \hat{\theta}_6, \hat{\theta}_1) = C\hat{\theta}_6 [S\hat{\theta}_2 S\hat{\theta}_1 (S\hat{\alpha}_2 C\hat{\alpha}_6 S\hat{\alpha}_5)$$

$$+ C\hat{\theta}_2 C\hat{\theta}_1 (-S\hat{\alpha}_2 C\hat{\alpha}_1 C\hat{\alpha}_6 S\hat{\alpha}_5) + C\hat{\theta}_2 (S\hat{\alpha}_2 S\hat{\alpha}_1 S\hat{\alpha}_6 S\hat{\alpha}_5)$$

$$+ C\hat{\theta}_1 (-C\hat{\alpha}_2 S\hat{\alpha}_1 C\hat{\alpha}_6 S\hat{\alpha}_5) + (-C\hat{\alpha}_2 C\hat{\alpha}_1 S\hat{\alpha}_6 S\hat{\alpha}_5)]$$

$$+ S\hat{\theta}_6 [S\hat{\theta}_2 C\hat{\theta}_1 (S\hat{\alpha}_2 S\hat{\alpha}_5) + C\hat{\theta}_2 S\hat{\theta}_1 (S\hat{\alpha}_2 C\hat{\alpha}_1 S\hat{\alpha}_5)$$

$$+ S\hat{\theta}_1 (C\hat{\alpha}_2 S\hat{\alpha}_1 S\hat{\alpha}_5)] + [S\hat{\theta}_2 S\hat{\theta}_1 (S\hat{\alpha}_2 S\hat{\alpha}_6 C\hat{\alpha}_5)$$

$$+ C\hat{\theta}_2 C\hat{\theta}_1 (-S\hat{\alpha}_2 C\hat{\alpha}_1 S\hat{\alpha}_6 C\hat{\alpha}_5) + C\hat{\theta}_2 (-S\hat{\alpha}_2 S\hat{\alpha}_1 C\hat{\alpha}_6 C\hat{\alpha}_5)$$

$$+ C\hat{\theta}_1 (-C\hat{\alpha}_2 S\hat{\alpha}_1 S\hat{\alpha}_6 C\hat{\alpha}_5) + (C\hat{\alpha}_2 C\hat{\alpha}_1 C\hat{\alpha}_6 C\hat{\alpha}_5)]$$

$$- C\hat{\alpha}_4 C\hat{\alpha}_3 + S\hat{\alpha}_4 S\hat{\alpha}_3 C\hat{\theta}_4 = 0 \qquad (3\text{-}39)$$

Observe that Eqs. (3-37) through (3-39) are similar in form to Eq. (3-9). Note also that each of the above equations relates the dual displacement angles $\hat{\theta}_2$, $\hat{\theta}_4$, and $\hat{\theta}_6$ to a fourth dual displacement angle. The displacement angles θ_4 and θ_6 at the prismatic pairs are constant for all positions of the mechanism.

Let the displacement angle θ_2 at the cylinder pair be now held constant at all positions of the mechanism. Denoting the constant value of θ_2 by θ_{2k}, the primary parts of Eqs. (3-37), (3-38), and (3-39) give respectively:

$$H_s S\theta_3 + H_c C\theta_3 + H_n = 0 \qquad (3\text{-}40)$$

$$I_s S\theta_5 + I_c C\theta_5 + I_n = 0 \qquad (3\text{-}41)$$

and

$$J_s S\theta_1 + J_c C\theta_1 + J_n = 0 \qquad (3\text{-}42)$$

The constants in the above equations involve the constant kinematic parameters of the mechanism and are defined in Table VI.

Observe that each of the equations (3-40) through (3-42) contains only one variable and must hold true at varying values of that variable. This is possible only if their coefficients vanish. This gives:

$$H_s = H_c = H_n = 0$$

$$I_s = I_c = I_n = 0 \qquad (3\text{-}43)$$

and

$$J_s = J_c = J_n = 0$$

Equations (3-43) represents the necessary conditions for the existence of H-P-H-P-H-P mechanism. It is, however, possible to further simplify the conditions given by Eq. (3-43). For example, examination of Eq. (3-43) together with the constants of Table VI show that the following case is possible:

$$\alpha_1 \pm \alpha_2 = p\pi$$

$$\alpha_3 \pm \alpha_4 = p\pi \qquad (p = 0,\ 1,\ 2,\ .\ .\ .) \qquad (3\text{-}44)$$

$$\alpha_5 \pm \alpha_6 = p\pi$$

Equations (3-44) give the necessary conditions for the existence of an H-P-H-P-H-P mechanism. All these conditions show that the axes of the helical pairs are parallel to one another and the axes of the prism pairs are randomly oriented.

TABLE VI

CONSTANTS FOR USE IN EQUATIONS (3-40) THROUGH (3-43)

$$H_s = C\theta_{2k}\,[S\theta_{4k}\,(S\alpha_4\,C\alpha_2\,S\alpha_1] + S\theta_{2k}\,[C\theta_{4k}\,(S\alpha_4\,C\alpha_3\,S\alpha_1)$$

$$+ (C\alpha_4\,S\alpha_3\,S\alpha_1)] + S\theta_{4k}\,S\alpha_4\,S\alpha_2\,C\alpha_1$$

$$H_c = C\theta_{2k}\,[C\theta_{4k}\,(-S\alpha_4\,C\alpha_3\,C\alpha_2\,S\alpha_1) + (-C\alpha_4\,S\alpha_3\,C\alpha_2\,S\alpha_1)]$$

$$+ S\theta_{2k}\,[S\theta_{4k}\,S\alpha_4\,S\alpha_1] + [C\theta_{4k}\,(-S\alpha_4\,C\alpha_3\,S\alpha_2\,C\alpha_1)$$

$$+ (-C\alpha_4\,S\alpha_3\,S\alpha_2\,C\alpha_1)]$$

$$H_n = C\theta_{2k}\,[C\theta_{4k}\,(S\alpha_4\,S\alpha_3\,S\alpha_2\,S\alpha_1) + (-C\alpha_4\,C\alpha_3\,S\alpha_2\,S\alpha_1)]$$

$$+ C\theta_{4k}\,(-S\alpha_4\,S\alpha_3\,C\alpha_2\,C\alpha_1) + (C\alpha_4\,C\alpha_3\,C\alpha_2\,C\alpha_1) - C\alpha_6\,C\alpha_5$$

$$+ S\alpha_6\,S\alpha_5\,C\theta_{6k}$$

$$I_s = C\theta_{4k}\,[S\theta_{6k}\,(S\alpha_6\,C\alpha_4\,S\alpha_3)] + S\theta_{4k}\,[C\theta_{6k}\,(S\alpha_6\,C\alpha_5\,S\alpha_3)$$

$$+ C\alpha_6\,S\alpha_5\,S\alpha_3] + S\theta_{6k}\,S\alpha_6\,S\alpha_4\,C\alpha_3$$

$$I_c = C\theta_{4k}\,[C\theta_{6k}\,(-S\alpha_6\,C\alpha_5\,C\alpha_4\,S\alpha_3) + (-C\alpha_6\,S\alpha_5\,C\alpha_4\,S\alpha_3)]$$

$$+ S\theta_{4k}\,[S\theta_{6k}\,S\alpha_6\,S\alpha_3] + [C\theta_{6k}\,(-S\alpha_6\,C\alpha_5\,S\alpha_4\,C\alpha_3)$$

$$+ (-C\alpha_6\,S\alpha_5\,S\alpha_4\,C\alpha_3)]$$

$$I_n = C\theta_{4k}\,[C\theta_{6k}\,S\alpha_6\,S\alpha_5\,S\alpha_4\,S\alpha_3 - C\alpha_6\,C\alpha_5\,S\alpha_4\,S\alpha_3]$$

$$+ [C\theta_{6k}\,(-S\alpha_6\,S\alpha_5\,C\alpha_4\,C\alpha_3) + (C\alpha_6\,C\alpha_5\,C\alpha_4\,C\alpha_3)] - C\alpha_2\,C\alpha_1$$

$$+ S\alpha_2\,S\alpha_1\,C\theta_{2k}$$

TABLE VI (Continued)

$$J_s = C\theta_{6k} [S\theta_{2k} S\alpha_2 C\alpha_6 S\alpha_5] + S\theta_{6k} [C\theta_{2k} S\alpha_2 C\alpha_1 S\alpha_5 + C\alpha_2 S\alpha_1 S\alpha_5]$$

$$+ S\theta_{2k} S\alpha_2 S\alpha_6 C\alpha_5$$

$$J_c = C\theta_{6k} [C\theta_{2k} (-S\alpha_2 C\alpha_1 C\alpha_6 S\alpha_5) + (-C\alpha_2 S\alpha_1 C\alpha_6 S\alpha_5)]$$

$$+ S\theta_{6k} [S\theta_{2k} S\alpha_2 S\alpha_5] + [C\theta_{2k} (-S\alpha_2 C\alpha_1 S\alpha_6 C\alpha_5)$$

$$+ (-C\alpha_2 S\alpha_1 S\alpha_6 C\alpha_5)]$$

$$J_n = C\theta_{6k} [C\theta_{2k} (S\alpha_2 S\alpha_1 S\alpha_6 S\alpha_5) + (-C\alpha_2 C\alpha_1 S\alpha_6 S\alpha_5)]$$

$$+ [C\theta_{2k} (-S\alpha_2 S\alpha_1 C\alpha_6 C\alpha_5) + (C\alpha_2 C\alpha_1 C\alpha_6 C\alpha_5)] - C\alpha_4 C\alpha_3$$

$$+ S\alpha_4 S\alpha_3 C\theta_{4k}$$

Summary and Extension of the Results to Other Mechanisms

The existence criteria derived in the above sections clearly show that the six-link, single loop 3H+3P mechanisms can exist only when the axes of the helical pairs are parallel to one another. Substitution of the existence criteria of 3H+3P mechanisms derived in the above sections into the displacement relationships of the respective parent mechanisms show that these mechanisms have two degrees of freedom. Note that the results have been obtained by considering only the primary parts of the displacement relationships of the respective parent mechanisms. Hence, the results will remain unaffected even if one or more of the helical pairs are replaced by revolute pairs. Such a replacement yields 18 different types of overconstrained mechanisms with helical, revolute, and prism pairs. The results are, therefore, equally valid for the six-link 3R+3P, 2R+1H+3P, and 2H+1R+3P mechanisms. Using the developed existence criteria, it becomes possible to write the existence conditions of the 18 mechanisms with one passive coupling. These 18 mechanisms and their existence conditions are described in Table VII.

Note further that, the results obtained are independent of the link lengths involved. Hence, if one of the link lengths is taken to be zero, the results will apply with equal validity to five-link

TABLE VII

EXISTENCE CONDITIONS OF OVERCONSTRAINED SIX-LINK SPATIAL MECHANISMS WITH HELICAL, REVOLUTE, AND PRISM PAIRS (ONE PASSIVE COUPLING)

Case	Parent Mechanism	Overconstrained Mechanism[1]	Existence Criteria
1	H-C-P-P-H-H	H-P-P-P-H-H[2]	Axes of helical and revolute parallel to one another and should satisfy Eq. (3-22)
2	R-C-P-P-R-R	R-P-P-P-R-R	
3	H-C-P-P-R-R	H-P-P-P-R-R	
4	R-C-P-P-R-H	R-P-P-P-R-H	
5	R-C-P-P-H-H	R-P-P-P-H-H	
6	H-C-P-P-H-R	H-P-P-P-H-R	
7	H-C-P-H-P-H	H-P-P-H-P-H[3]	Axes of helical and revolute pairs parallel to one another and should satisfy Eq. (3-34)
8	R-C-P-R-P-R	R-P-P-R-P-R	
9	R-C-P-H-P-R	R-P-P-H-P-R	
10	H-C-P-R-P-R	H-P-P-R-P-R	
11	R-C-P-R-P-H	R-P-P-R-P-H	
12	H-C-P-R-P-H	H-P-P-R-P-H	
13	R-C-P-H-P-H	R-P-P-H-P-H	
14	H-C-P-H-P-R	H-P-P-H-P-R	
15	H-C-H-P-H-P	H-P-H-P-H-P[4]	Axes of helical and revolute pairs parallel to one another
16	R-C-R-P-R-P	R-P-R-P-R-P	
17	H-C-R-P-R-P	H-P-R-P-R-P	
18	R-C-H-P-H-P		

[1] Mobility two (F = 2).

[2] See Figure 17.

[3] See Figure 19.

[4] See Figure 21.

mechanisms derivable from the above six-link mechanisms. Similarly, the criteria for four-link mechanisms derivable from the above six-link mechanisms can be obtained by taking two link lengths zero. Examples of five-link mechanisms deduced from the derived existence criteria of the above six-link mechanisms are shown in Figures 18, 20, and 22. The results of five-link mechanisms obtained in this manner also confirm the results obtained by Pamidi, Soni, and Dukkipati (119), Hunt (30), and Waldron (35). The results of Hunt and Waldron were obtained by considering the 5H and 6H mechanisms of Voinea and Atanasiu (17), which are themselves overconstrained mechanisms. The results of Soni, Pamidi, and Dukkipati, and also in the present study, on the other hand, have been obtained by considering the more general zero family mechanisms. Further, in addition to the parallelism of the axes, the present results also give definite closure conditions that must be satisfied by the several constant kinematic parameters of the respective mechanisms.

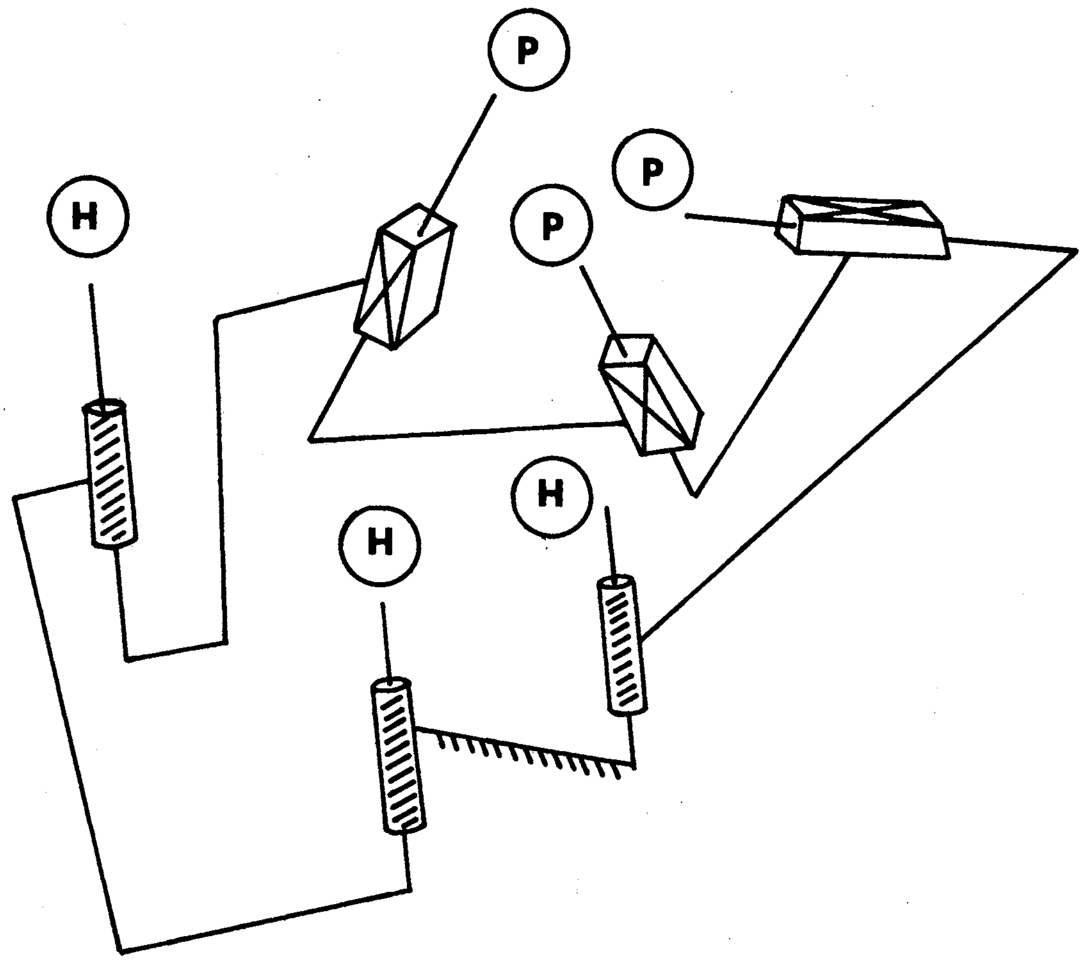

Figure 17. Six-link H-P-P-P-H-H Overconstrained Space Mechanism (F = 2). Case 1 in Table VII

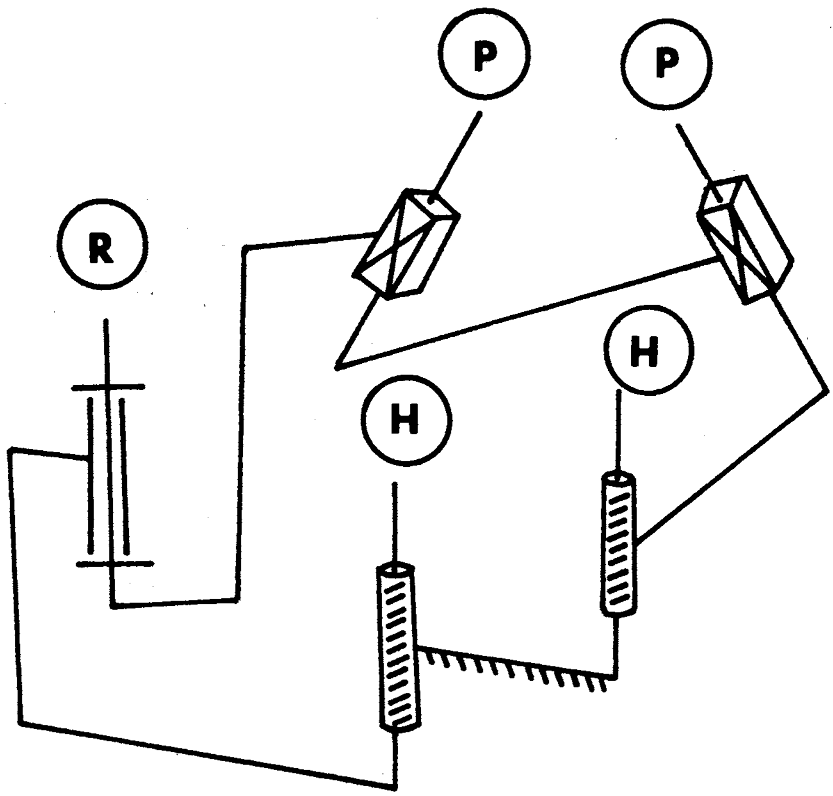

Figure 18. Five-link H-P-P-R-H Overconstrained Space Mechanism Obtained From the H-P-P-P-H-H Mechanism in Figure 17 by Making $\hat{\alpha}_2 = 0$ and $p_5 = 0$. (30, 35, 119)

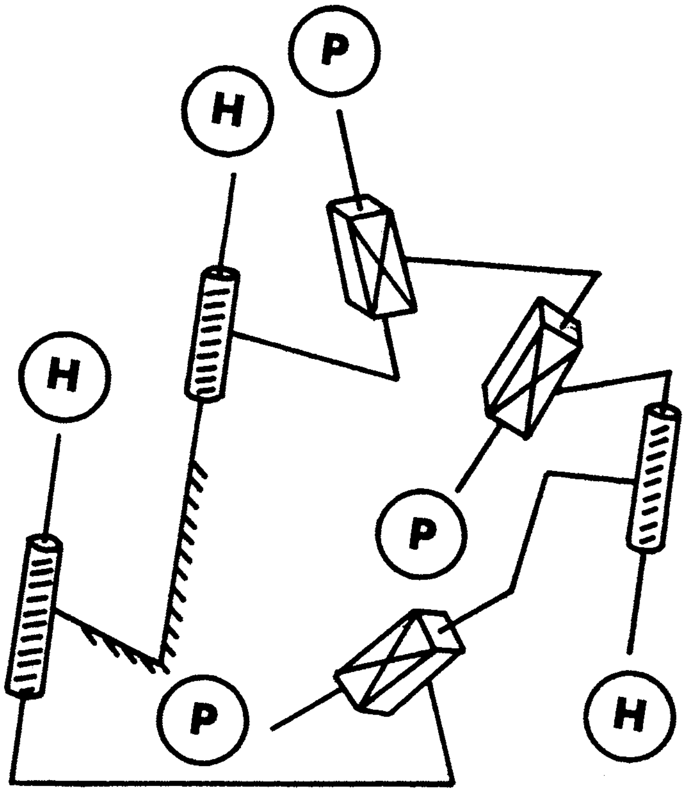

Figure 19. Six-link H-P-P-H-P-H Over-constrained Space Mechanism (F = 2). Case 7 in Table VII

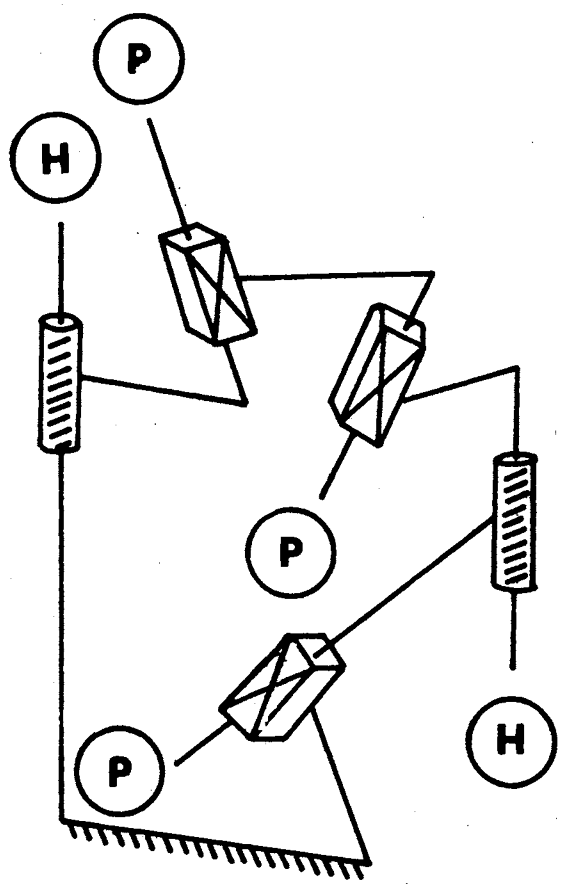

Figure 20. Five-link H-P-P-H-P Overconstrained Space Mechanism (F = 1) Obtained From Figure 19 by Making $\hat{\alpha}_5 = 0$ (30, 35, 119)

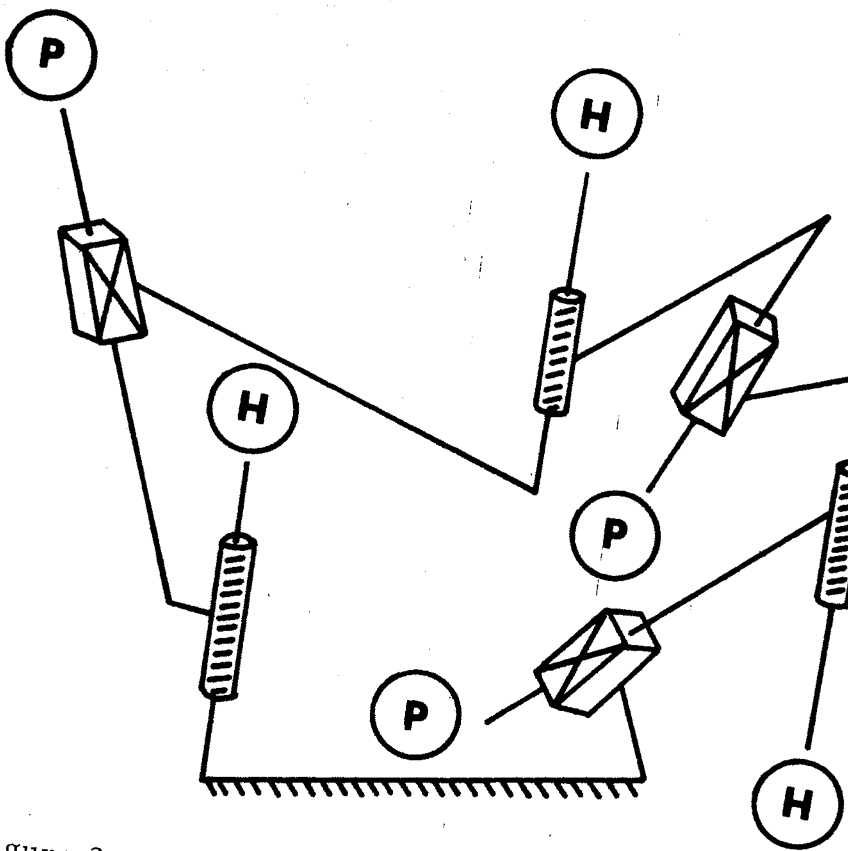

Figure 21. Six-link H-P-H-P-H-P Overconstrained Space Mechanism (F = 2). Case 15 in Table VII

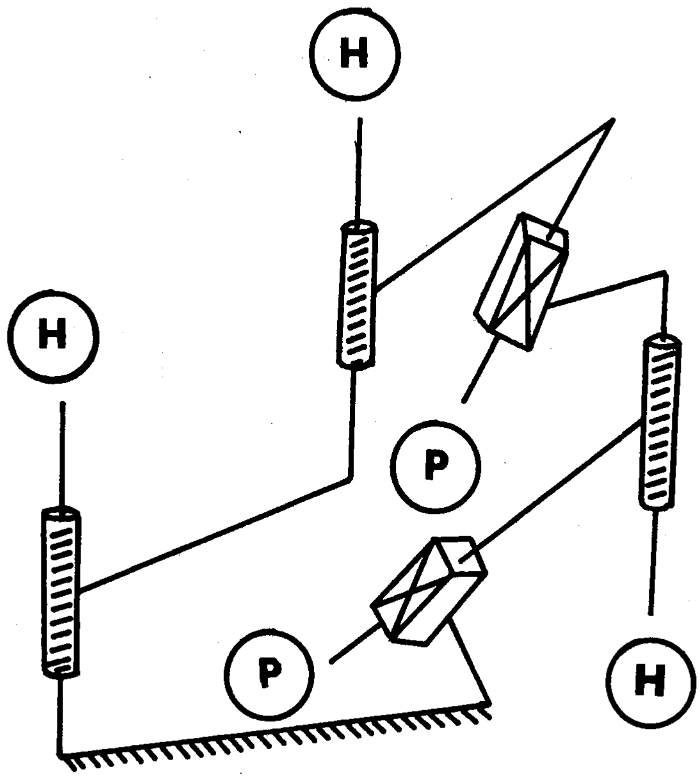

Figure 22. Five-link H-H-P-H-P Over-constrained Space Mechanism (F = 1) Obtained From the H-P-H-P-H-P Mechanism in Figure 21 by Making $\hat{\alpha}_1 = 0$. (30, 35, 119)

CHAPTER IV

EXISTENCE CRITERIA OF TWO-LOOP MECHANISMS

In this chapter, the Dimentberg passive coupling technique has been employed to obtain the existence criteria of the six-link, two-loop R-R-C-C-C-R-C (one kink-link zero) and R-R-C-C-C-P-C mechanisms. These criteria are obtained from the displacement relationships of the parent six-link, two-loop R-C-C-C-C-C mechanism (120). The procedure for obtaining the existence criteria of R-R-C-C-C-R-C, R-C-C-R-C-C-R, and R-C-C-R-C-C-P mechanisms from the parent R-C-C-C-C-C mechanism with general proportions is considered in Appendixes A and B. Appendix C deals with the conditions for the existence of two prism pairs in a two-loop mechanism.

Displacement Relationships for Obtaining the Existence Criteria

The use of Dimentberg's method for obtaining the existence criteria of overconstrained two-loop mechanisms requires the displacement of the appropriate parent mechanism. The required relationships can always be obtained by suitably arranging the loop-closure conditions of the parent mechanism.

Consider a general six-link, two-loop spatial mechanism of Stephenson type in Figure 23, with revolute pair at joint A and cylinder pairs at joints B, C, D, E, F, and G. Note that the sum of the degrees of freedom in all joints of the mechanism is thirteen. The mechanism has four binary links (AB, CD, EF, and FG) and two ternary links (AGD and BCE).

Definitions of a Spatial Ternary Link

The geometrical configuration bounded by three non-parallel and non-intersecting lines in space and a set of three uniquely drawn common perpendiculars--one between each two lines--is defined as a spatial ternary link. The three lines are defined as the axes of the ternary link; the three dual angles specifying the relative positions of the axes are called the sides of the ternary link. The three dual angles specifying the relative positions of the common perpendiculars are defined as the angles of the spatial ternary link.

Figure 24 shows a spatial ternary link A′A-B′B-C′C whose three axes A′A, B′B, and C′C are respectively specified by unit line vectors $\hat{s}_1$, $\hat{s}_2$, and $\hat{s}_3$. The three unit line vectors $\hat{\beta}$, $\hat{\gamma}$, and $\hat{\delta}$ are respectively coaxial with the common perpendiculars AB′,

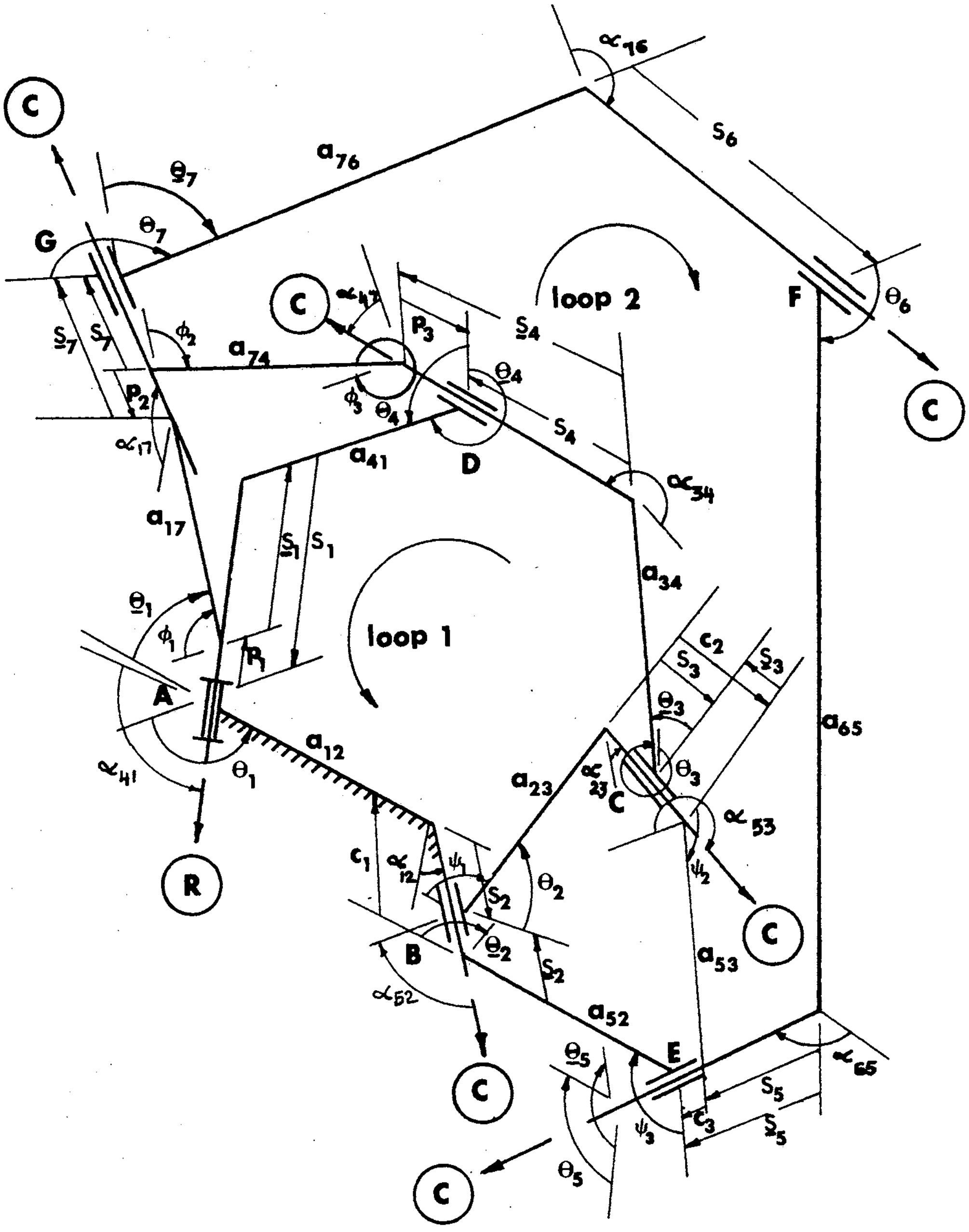

Figure 23. General Six-link, Two-loop R-C-C-C-C-C-C Space Mechanism of Stephenson Type

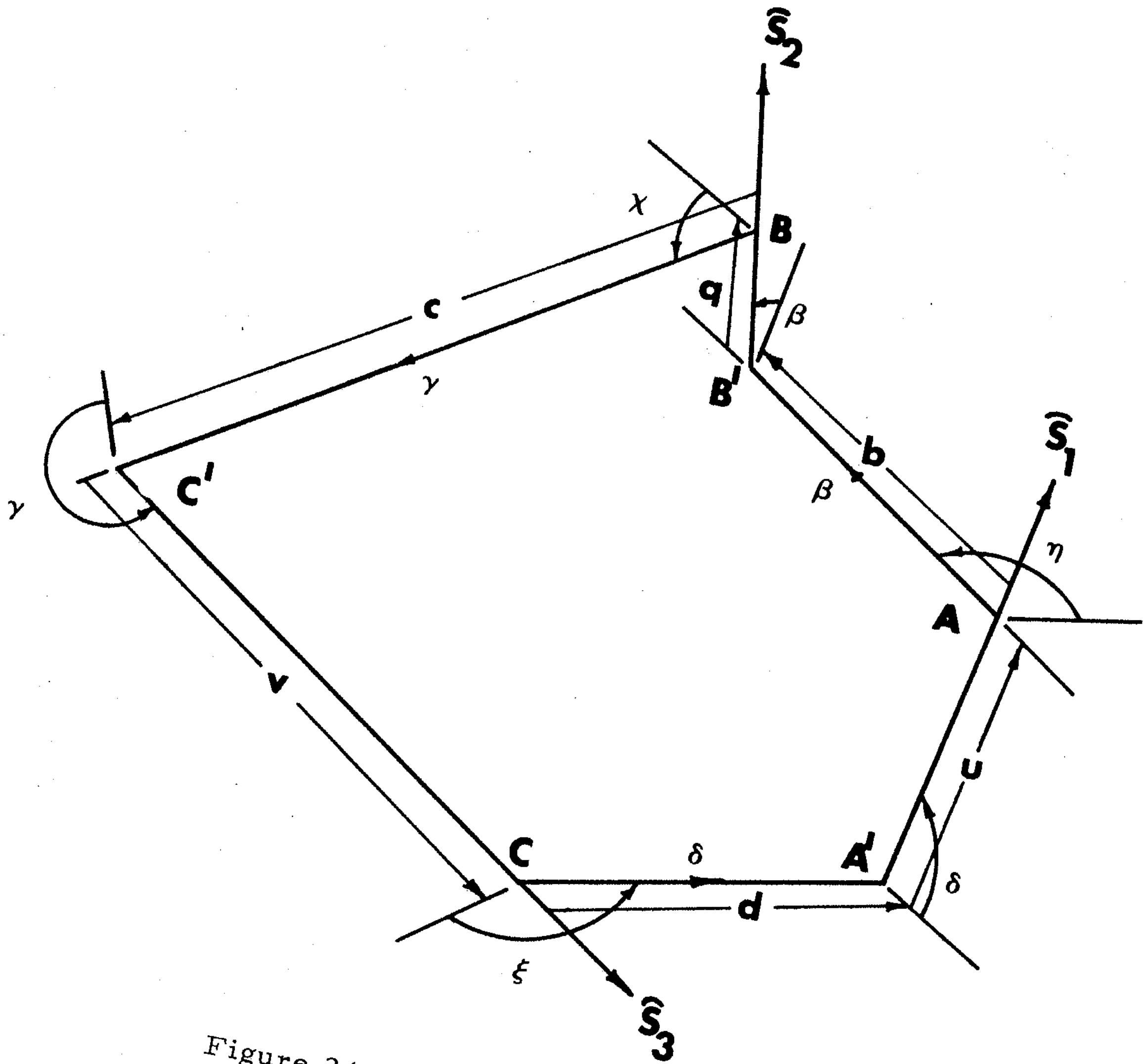

Figure 24. A Spatial Ternary Link

BC′, and C′A. The directions of the six unit line vectors forming the spatial ternary link may be chosen arbitrarily provided the sense of the dual angles is consistent with the directions of the unit line vectors.

In Figure 24, the directions are chosen in accordance with the following convention:

1. Designate AA′, BB′, and CC′ as axes 1, 2, and 3 respectively.
2. $\hat{\beta}$, $\hat{\gamma}$, and $\hat{\delta}$ are directed from axes 1 to 2, 2 to 3, and 3 to 1 respectively.
3. The directions of $\hat{s}_1$, $\hat{s}_2$, and $\hat{s}_3$ are chosen in such a way that the six unit line vectors of the spatial ternary link are so directed as to form a closed loop in space.

Thus, one may write the three sides of the spatial ternary link as

$$\hat{\beta} = \beta + \epsilon b$$
$$\hat{\gamma} = \gamma + \epsilon c \qquad (4\text{-}1)$$
$$\hat{\delta} = \delta + \epsilon d$$

where β, γ, and δ are the twist angles and b, c, and d are the kinematic link lengths.

The three angles of the spatial ternary link are

$$\hat{\eta} = \eta + \epsilon u$$
$$\hat{\chi} = \chi + \epsilon q \qquad (4\text{-}2)$$
$$\hat{\xi} = \xi + \epsilon v$$

where η, χ, and ξ are the constant rotational displacement angles and u, q, and v are the constant offset distances.

Using 3 x 3 matrices with dual number elements, the loop closure condition of the ternary link in Figure 24 is given by

$$[\hat{\xi}]_3 \, [\hat{\gamma}]_1 \, [\hat{\chi}]_3 \, [\hat{\beta}]_1 \, [\hat{\eta}]_3 \, [\hat{\delta}]_1 = [\hat{I}] \tag{4-3}$$

where

$$[\hat{\xi}]_3 = \begin{bmatrix} C\hat{\xi} & S\hat{\xi} & 0 \\ -S\hat{\xi} & C\hat{\xi} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[\hat{\gamma}]_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & C\hat{\gamma} & S\hat{\gamma} \\ 0 & -S\hat{\gamma} & C\hat{\gamma} \end{bmatrix}$$

$$[\hat{\chi}]_3 = \begin{bmatrix} C\hat{\chi} & S\hat{\chi} & 0 \\ -S\hat{\chi} & C\hat{\chi} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[\hat{\beta}]_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & C\hat{\beta} & S\hat{\beta} \\ 0 & -S\hat{\beta} & C\hat{\beta} \end{bmatrix}$$

$$[\hat{\eta}]_3 = \begin{bmatrix} C\hat{\eta} & S\hat{\eta} & 0 \\ -S\hat{\eta} & C\hat{\eta} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[\hat{\delta}]_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & C\hat{\delta} & S\hat{\delta} \\ 0 & -S\hat{\delta} & C\hat{\delta} \end{bmatrix}$$

and

$$[I] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(4-4)

In the case where the three axes A′A, B′B, and C′C in Figure 24 intersect at one point, say 0 (i.e., A′, B′, and C′ coincide at 0), the spatial ternary link is reduced to a spherical ternary link as shown in Figure 25; it is a configuration bounded by three arcs $\widehat{AB}$, $\widehat{BC}$, and $\widehat{CA}$ on the surface of a sphere of unit radius, with 0 as its center. Since the axes are intersecting, all the dual parts in Eqs. (4-1) and (4-2) become zero. Thus, the three sides of the spherical ternary link ABC are represented by β, γ, and δ and the three angles are η, χ, and ξ.

If the three axes in Figure 24 are parallel, then the spatial ternary link A′A-B′B-C′C becomes a planar ternary link, the plane P on which it lies is perpendicular to the three axes as shown in Figure 26. Since the axes are parallel, β, γ, and δ in Eq. (4-1) are equal to zero. Thus the sides of the plane ternary link A′B′C′ are represented by the pure dual numbers ϵb, ϵc, and ϵd. With the

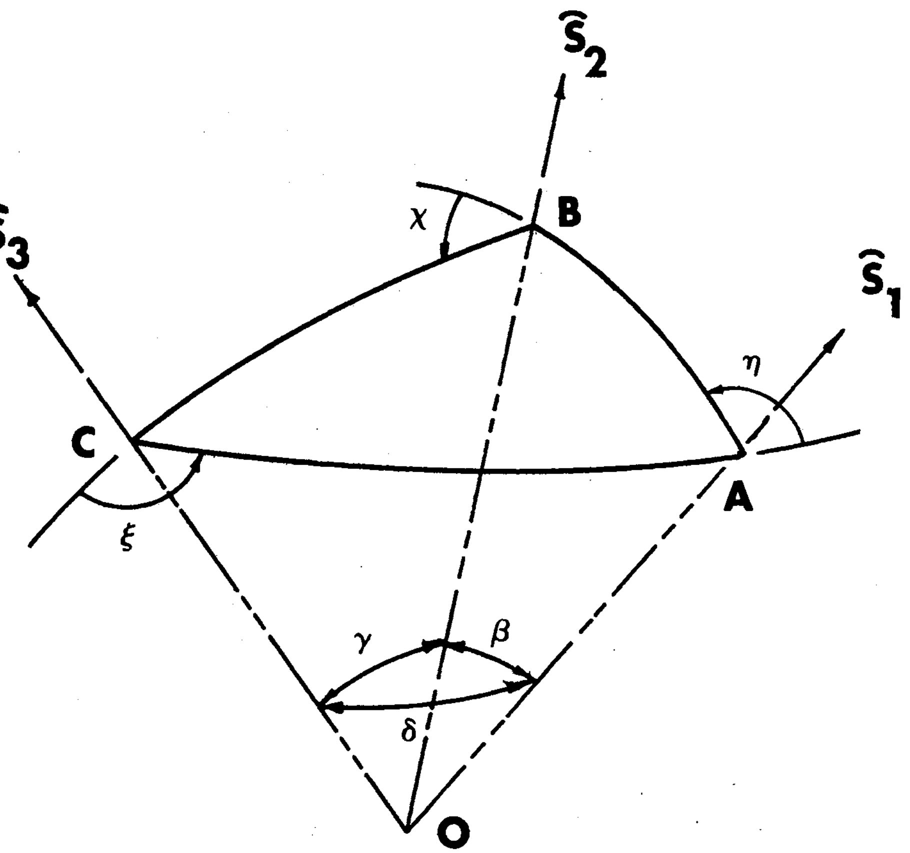

Figure 25. A Spherical Ternary Link

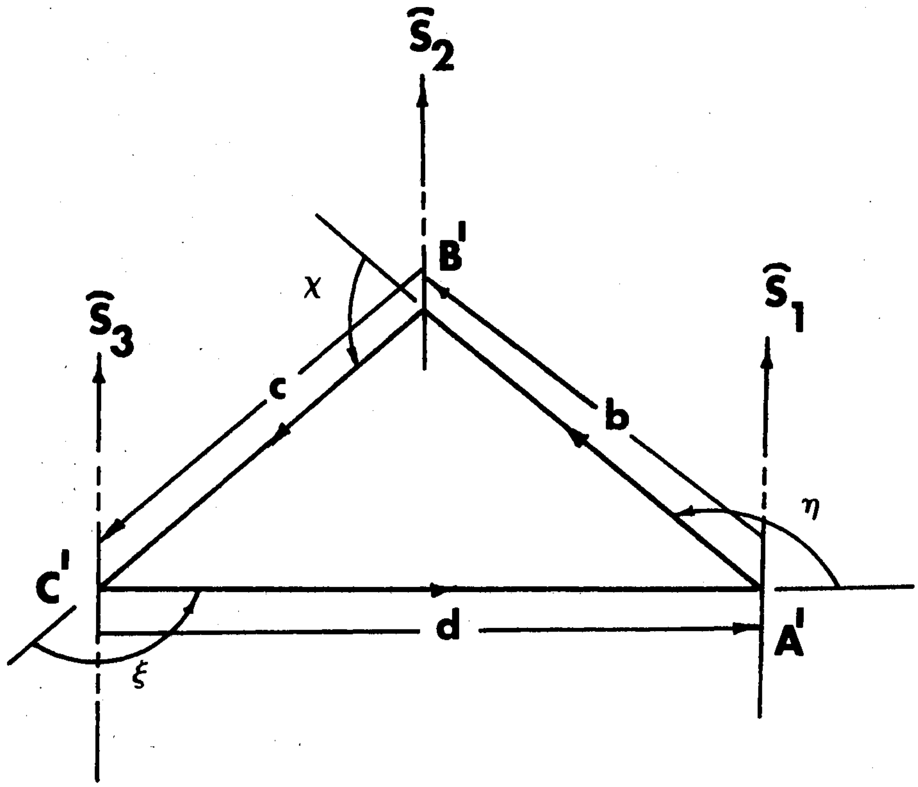

Figure 26. A Plane Ternary Link

three common perpendiculars lying in the same plane, s_1, s_2, and s_3 in Eq. (4-2) vanish and the angles are represented by the real numbers η, χ, and ξ.

Summarizing a spatial ternary link is completely specified by the relative positions of its three axes which in general, are non-parallel and non-intersecting. If the axes are intersecting, one obtains a spherical ternary link; if parallel, one obtains a plane ternary link.

The relative positions of the three axes of a spatial ternary link s_1, s_2, and s_3 may be expressed in terms of its three sides $\hat{\beta}$, $\hat{\gamma}$, and $\hat{\delta}$ and three angles $\hat{\eta}$, $\hat{\chi}$, and $\hat{\xi}$. However, these six dual numbers are not independent of one another--given any three of the six dual-numbers, the remaining ones can be determined by the closure condition of the ternary link. Thus, a spatial ternary link can be completely specified by any three out of its six elements--three sides and three angles.

The constant displacement angles η, χ, and ξ, and the constant offset distances u, q, and v of a spatial ternary link in Figure 24 for a given set of twist angles (β, γ, δ) and link lengths (b, c, d) can be derived in the following manner.

Equation (4-3) can be expressed as

$$[\hat{m}] = [\hat{n}]^{-1} \tag{4-5}$$

where

$$[\hat{m}] = [\hat{\delta}]_1$$

$$[\hat{n}] = [\hat{\xi}]_3\,[\hat{\gamma}]_1\,[\hat{\chi}]_3\,[\hat{\beta}]_1\,[\hat{\eta}]_3 \tag{4-6}$$

since $[\hat{\eta}]$ is an orthogonal matrix, $[\hat{\eta}]^{-1}$ is identical to its transposed matrix. When the matrix products are carried out, the dual-matrix loop equation for the spatial ternary link becomes:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & C\hat{\delta} & S\hat{\delta} \\ 0 & -S\hat{\delta} & C\hat{\delta} \end{bmatrix} = \begin{bmatrix} \hat{k}_4\,C\hat{\xi} - (\hat{k}_2\,C\hat{\gamma} - S\hat{\beta}\,S\hat{\gamma}\,S\hat{\eta})\,S\hat{\xi} & -\hat{k}_4\,S\hat{\xi} - (\hat{k}_2\,C\hat{\gamma} - S\hat{\beta}\,S\hat{\gamma}\,S\hat{\eta})\,C\hat{\xi} & \hat{L}_1\,S\hat{\eta} + S\hat{\gamma}\,C\hat{\eta}\,S\hat{\chi} \\ k_1\,C\hat{\xi} - (\hat{k}_3\,C\hat{\gamma} + S\hat{\beta}\,S\hat{\gamma}\,C\hat{\eta})\,S\hat{\xi} & -\hat{k}_1\,S\hat{\xi} - (\hat{k}_3\,C\hat{\gamma} + S\hat{\beta}\,S\hat{\gamma}\,C\hat{\eta})\,C\hat{\xi} & -\hat{L}_1\,C\hat{\eta} + S\hat{\gamma}\,S\hat{\eta}\,S\hat{\chi} \\ \hat{L}_2\,S\hat{\xi} + S\hat{\beta}\,S\hat{\chi}\,C\hat{\xi} & \hat{L}_2\,C\hat{\xi} - S\hat{\beta}\,S\hat{\chi}\,S\hat{\xi} & C\hat{\beta}\,C\hat{\chi} - S\hat{\beta}\,S\hat{\gamma}\,C\hat{\chi} \end{bmatrix} \tag{4-7}$$

where

$$\hat{k}_1 = S\hat{\eta}\,C\hat{\chi} + C\hat{\beta}\,C\hat{\eta}\,S\hat{\chi}$$

$$\hat{k}_2 = C\hat{\eta}\,S\hat{\chi} + C\hat{\beta}\,S\hat{\eta}\,C\hat{\chi}$$

$$\hat{k}_3 = S\hat{\eta}\,S\hat{\chi} - C\hat{\beta}\,C\hat{\eta}\,C\hat{\chi}$$

$$\hat{k}_4 = C\hat{\eta}\,C\hat{\chi} - C\hat{\beta}\,S\hat{\eta}\,S\hat{\chi}$$

$$\hat{L}_1 = S\hat{\beta}\,C\hat{\gamma} + C\hat{\beta}\,S\hat{\gamma}\,C\hat{\chi}$$

$$\hat{L}_2 = C\hat{\beta}\,S\hat{\gamma} + S\hat{\beta}\,C\hat{\gamma}\,C\hat{\chi}$$

Equating the elements "33" of both members of Eq. (4-7), we have

$$C\hat{\delta} = C\hat{\beta}\ C\hat{\gamma} - S\hat{\beta}\ S\hat{\gamma}\ C\hat{\chi} \tag{4-8}$$

where all the dual angles are already defined in Eqs. (4-1) and (4-2).

The primary part of Eq. (4-8) can be written as

$$C\chi = \frac{C\beta\ C\gamma - C\delta}{S\beta\ S\gamma} \tag{4-9}$$

The value of Cos χ corresponding to a set of twist angles (β, γ, δ) can be computed from Eq. (4-9). However, there are two ways to assemble such a ternary link since the angle χ is double-valued. The dual-part of Eq. (4-8) gives the constant offset distance q for a given set of β, γ, δ and b, c, d.

$$q = \frac{-d\ S\delta + b\ S\beta\ C\gamma + c\ S\gamma\ C\beta + C\chi\ (b\ C\beta\ S\gamma + c\ C\gamma\ S\beta)}{S\chi\ S\beta\ S\gamma} \tag{4-10}$$

To solve for the remaining ternary link parameters, we equate the corresponding dual elements "13", "23", "31", and "32" of both members of Eq. (4-7). Separate the resultant equation into two parts from which we may solve for:

$$S\eta = \frac{-\ S\gamma\ S\chi\ S\delta}{L_1^2 + S^2\gamma\ S^2\chi} \tag{4-11}$$

$$C\eta = \frac{-\ L_1\ S\delta}{L_1^2 + S^2\gamma\ S^2\chi} \tag{4-12}$$

$$S\xi = \frac{-\ S\beta\ S\chi\ S\delta}{L_2^2 + S^2\beta\ S^2\chi} \tag{4-13}$$

$$C\xi = \frac{-L_2\ S\delta}{L_2^2\ S^2\beta\ S^2\chi} \tag{4-14}$$

$$u = \frac{-b\ L_4\ S\eta + c\ (L_3\ S\eta - C\gamma\ S\chi\ C\eta) - q\ S\gamma\ (C\eta\ C\chi - S\eta\ S\chi\ C\beta)}{L_1\ C\eta - S\gamma\ S\chi\ S\eta} \tag{4-15}$$

$$v = \frac{b(L_3\ S\xi - C\beta\ S\chi\ C\xi) - c\ L_4\ S\xi - q\ S\beta\ (C\chi\ C\xi - S\chi\ S\xi\ C\gamma)}{L_2\ C\xi - S\beta\ S\chi\ S\xi} \tag{4-16}$$

where

$$\begin{aligned} L_1 &= S\beta\ C\gamma + C\beta\ S\gamma\ C\chi \\ L_2 &= C\beta\ S\gamma + S\beta\ C\gamma\ C\chi \\ L_3 &= S\beta\ S\gamma - C\beta\ C\gamma\ C\chi \\ L_4 &= C\beta\ C\gamma - S\beta\ S\gamma\ C\chi \end{aligned} \tag{4-17}$$

Thus the four parameters η, ξ, u, and v are uniquely determined from Eqs. (4-11) through (4-17).

The instantaneous configuration of the six-link, two-loop R-C-C-C-C-C-C mechanism, schematically shown in Figure 27, is completely defined by two sets of dual angles, each as follows:

1. Between adjacent pairing axes:

$$\hat{\alpha}_{ij} = \alpha_{ij} + \epsilon\ a_{ij} \tag{4-18}$$

where $\hat{\alpha}_{ij}$ is the dual angle between axes i and j, α_{ij} are the twist angles and a_{ij} are the link lengths as shown in Figure 27.

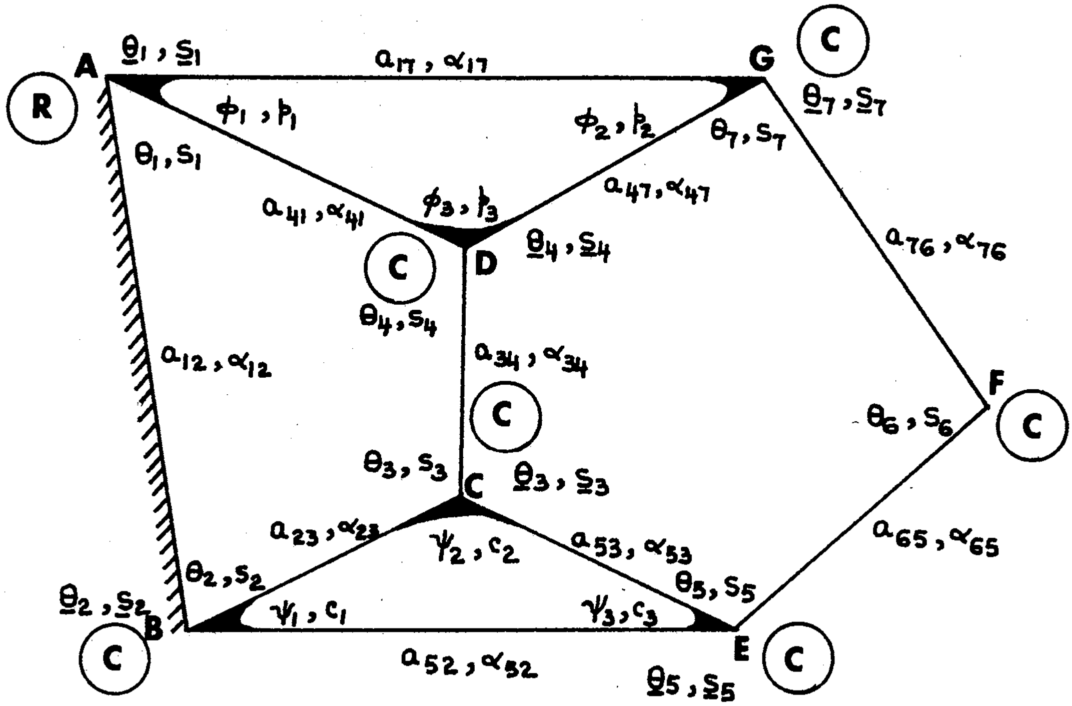

Figure 27. Six-link, Two-loop R-C-C-C-C-C-C Space Mechanism

2. Between adjacent common perpendiculars:

$$\hat{\theta}_i = \theta_i + \varepsilon s_i \tag{4-19}$$

where θ_i (i = 1 to 7) are the angular displacements of links, s_i (i = 2 to 7) are the linear displacements at the cylinder joints, and s_1 is the constant offset distance (kink-link) measured along the axis of the revolute pair.

There are 13 variables in Eq. (4-19), θ_1 is the input angle at the revolute pair A and θ_i, s_i (i = 2 to 7) are the other linkage variables. The 20 quantities in Eq. (4-18), α_{ij} and a_{ij} (ij = 12, 23, 34, 41, 17, 76, 65, 52, 53, 47) and the constant offset distance s_1 in Eq. (4-19), constitute the 21 constant real linkage parameters necessary to specify completely a six-link, two-loop space mechanism of Stephenson type with general proportions. The loop-closure condition of the mechanism can be written in three ways, one for each loop. It is to be noted that the mechanism has only two independent loops. Since $\underline{\theta}_i$, $\underline{s}_i$ (i = 1 to 7) are not independent of θ_i and s_i (i = 1 to 7) respectively, the relationship between $\hat{\underline{\theta}}_i$ and $\hat{\theta}_i$ can be obtained. Thus

$$\hat{\underline{\theta}}_i = \underline{\theta}_i + \varepsilon \underline{s}_i \tag{4-20}$$

$$\hat{\underline{\theta}}_1 = - \hat{\theta}_1 + \hat{\Phi}_1 + \pi$$

$$\hat{\underline{\theta}}_2 = - \hat{\theta}_2 + \hat{\psi}_1 + \pi$$

$$\hat{\underline{\theta}}_3 = - \pi + \hat{\theta}_3 - \hat{\psi}_2$$

$$\underline{\hat{\theta}}_4 = -\pi + \hat{\theta}_4 - \hat{\Phi}_3 \qquad (4\text{-}21)$$

$$\underline{\hat{\theta}}_5 = \hat{\theta}_5 + \hat{\psi}_3 - \pi$$

$$\underline{\hat{\theta}}_6 = \hat{\theta}_6$$

$$\underline{\hat{\theta}}_7 = \hat{\theta}_7 - \hat{\Phi}_2 + \pi$$

where

$$\hat{\Phi}_i = \Phi_i + \epsilon\, p_i \qquad (i = 1, 2, 3)$$

$$\hat{\psi}_i = \psi_i + \epsilon\, c_i \qquad (i = 1, 2, 3) \qquad (4\text{-}22)$$

Note that $\hat{\Phi}_i$ (i = 1, 2, 3) are the angles and $\hat{\alpha}_{17}$, $\hat{\alpha}_{74}$, $\hat{\alpha}_{41}$ are the sides of the ternary link AGD and $\hat{\psi}_i$ (i = 1, 2, 3) are the angles and $\hat{\alpha}_{23}$, $\hat{\alpha}_{35}$, $\hat{\alpha}_{52}$ are the sides of the ternary link BCE in Figure 23. The parameters of the six-link, two-loop R-C-C-C-C-C-C space mechanism of Stephenson type are described in Table VIII.

Using (3 x 3) matrices with dual number elements, closed form displacement relationships of the mechanism are derived by Soni, Dukkipati, and Huang (120).

Loop 1 (ABCDA)

The loop-closure condition of the mechanism in Figure 27 for the loop 1 (ABCDA) is given by (120):

$$[\hat{\theta}_1]_3\, [\hat{\alpha}_{12}]_1\, [\hat{\theta}_2]_3\, [\hat{\alpha}_{23}]_1\, [\hat{\theta}_3]_3\, [\hat{\alpha}_{34}]_1\, [\hat{\theta}_4]_3\, [\hat{\alpha}_{41}]_1 = [I] \qquad (4\text{-}23)$$

where

TABLE VIII

PARAMETERS OF SIX-LINK, TWO-LOOP R-C-C-C-C-C-C SPACE MECHANISM OF STEPHENSON TYPE

Constant Kinematic Parameters	Variable Kinematic Parameters
Independent Parameters:	Rotational Displacement Angles:
Kinematic Links:	θ_i (i = 1 to 7)
a_{ij} (ij = 12, 23, 34, 41, 17, 76, 65, 52, 53, 47)	Translational Displacements:
Twist Angles:	s_i (i = 2 to 7)
α_{ij} (ij = 12, 23, 34, 41, 17, 76, 65, 52, 53, 47)	Total: 13
Kink-Link: s_1	
Total: 21	

Dependent Parameters:

Constant Displacement Parameters:

Φ_i, ψ_i (i = 1 to 3)

Kink-Links:

p_i, c_i (i = 1 to 3)

Total: 12

$$[\hat{\theta}_i]_3 = \begin{bmatrix} C\hat{\theta}_i & S\hat{\theta}_i & 0 \\ -S\hat{\theta}_i & C\hat{\theta}_i & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$[\hat{\alpha}_{ij}]_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & C\hat{\alpha}_{ij} & S\hat{\alpha}_{ij} \\ 0 & -S\hat{\alpha}_{ij} & C\hat{\alpha}_{ij} \end{bmatrix}$$

and

$$[I] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad (4\text{-}24)$$

Two arrangements of Eq. (4-23) are useful in the study of existence criteria.

1. Relationship involving the adjacent dual displacement angles.

In this arrangement of Eq. (4-23), five matrices are used on one side of the equality sign and three matrices on the other. Thus, we have, for instance,

$$[\hat{\alpha}_{12}]_1 \, [\hat{\theta}_1]_3 \, [\hat{\alpha}_{41}]_1 \, [\hat{\theta}_4]_3 \, [\hat{\alpha}_{34}]_1$$

$$= [\hat{\theta}_2]_3^{-1} \, [\hat{\alpha}_{23}]_1^{-1} \, [\hat{\theta}_3]_3^{-1} \qquad (4\text{-}25)$$

Simplifying the above equation by using relations (4-24) and equating the "33" elements of the resultant matrix equation, we get

$$F_1(\hat{\theta}_1, \hat{\theta}_4) = (S\hat{\alpha}_{12}\, S\hat{\alpha}_{34}\, S\hat{\theta}_1)\, S\hat{\theta}_4 - [S\hat{\alpha}_{34}\, (S\hat{\alpha}_{41}\, C\hat{\alpha}_{12} + C\hat{\alpha}_{41}\, S\hat{\alpha}_{12}\, C\hat{\theta}_1)]\, C\hat{\theta}_4 - C\hat{\alpha}_{23} + C\hat{\alpha}_{34}\, (C\hat{\alpha}_{41}\, C\hat{\alpha}_{12} - S\hat{\alpha}_{41}\, S\hat{\alpha}_{12}\, C\hat{\theta}_1) = 0 \qquad (4\text{-}26)$$

Note that Eq. (4-26) involves the two adjacent dual displacement angles $\hat{\theta}_1$ and $\hat{\theta}_4$.

Cyclic permutation permits Eq. (4-26) to be written in four different ways. It is, therefore, possible to get four equations of the form (4-26) involving different combinations of two adjacent angles.

2. Relationship involving two displacement angles opposite to one another.

In this arrangement of Eq. (4-23), three matrices are used on one side of the equality sign and five matrices on the other. The important point to note is that the central matrix on the side containing three matrices involves only the variable kinematic parameters of the mechanism. Thus, we have, for instance,

$$[\hat{\alpha}_{12}]_1\, [\hat{\theta}_1]_3\, [\hat{\alpha}_{41}]_1 = [\hat{\theta}_2]_3^{-1}\, [\hat{\alpha}_{23}]_1^{-1}\, [\hat{\theta}_3]_3^{-1}\, [\hat{\alpha}_{34}]_1^{-1}\, [\hat{\theta}_4]_3^{-1} \qquad (4\text{-}27)$$

Note that the central matrix $[\hat{\theta}_1]_3$ on the left hand side only involves the variable kinematic parameters of the mechanism.

Simplifying Eq. (4-27) by using relations (4-24) and equating the "33" elements of the resultant matrix equation, we get

$$f_1(\hat{\theta}_1, \hat{\theta}_3) = C\hat{\alpha}_{21} C\hat{\alpha}_{14} + S\hat{\alpha}_{21} S\hat{\alpha}_{14} C\hat{\theta}_1 - C\hat{\alpha}_{43} C\hat{\alpha}_{32} - S\hat{\alpha}_{43} S\hat{\alpha}_{32} C\hat{\theta}_3 = 0 \quad (4\text{-}28)$$

Cyclic permutation allows Eq. (4-28) to be written in two different ways. It is, therefore, possible to obtain two equations of the form (4-28) involving different combinations of two opposite displacement angles.

Loop 2 (DGFECD)

The dual-matrix loop closure equation for loop 2 (DGFECD) is given by

$$[\hat{\theta}_4]_3 [\hat{\alpha}_{47}]_1 [\hat{\theta}_7]_3 [\hat{\alpha}_{76}]_1 [\hat{\theta}_6]_3 [\hat{\alpha}_{65}]_1 [\hat{\theta}_5]_3 [\hat{\alpha}_{53}]_1 [\hat{\theta}_3]_3 [\hat{\alpha}_{34}]_1 = [I] \quad (4\text{-}29)$$

Two arrangements of Eq. (4-29) are useful in the study of existence criteria.

1. Relationship involving two adjacent dual displacement angles and the dual displacement angle opposite to both of them.

In this arrangement of Eq. (4-29), five matrices are used on either side of the equality sign. Thus, we have, for instance,

$$[\hat{\alpha}_{47}]_1 [\hat{\theta}_7]_3 [\hat{\alpha}_{76}]_1 [\hat{\theta}_6]_3 [\hat{\alpha}_{65}]_1 = [\hat{\underline{\theta}}_4]_3^{-1} [\hat{\alpha}_{34}]_1^{-1} [\hat{\underline{\theta}}_3]_3^{-1} [\hat{\alpha}_{53}]_1^{-1} [\hat{\theta}_5]_3^{-1} \quad (4\text{-}30)$$

Simplifying the above equation by using relations (4-20), (4-21), (4-24) and equating the "33" elements of the resultant matrix equation, we get

$$F_2 (\hat{\theta}_3, \hat{\theta}_6, \hat{\theta}_7) = S\hat{\alpha}_{47} S\hat{\alpha}_{65} S\hat{\theta}_7) - S\hat{\alpha}_{65} (C\hat{\alpha}_{47} S\hat{\alpha}_{76} + S\hat{\alpha}_{47} C\hat{\alpha}_{76} C\hat{\theta}_7) C\hat{\theta}_6 + C\hat{\alpha}_{65} (C\hat{\alpha}_{47} C\hat{\alpha}_{76} - S\hat{\alpha}_{47} S\hat{\alpha}_{76} C\hat{\theta}_7) - (C\hat{\alpha}_{53} C\hat{\alpha}_{34} - S\hat{\alpha}_{53} S\hat{\alpha}_{34} C\hat{\underline{\theta}}_3) = 0 \tag{4-31}$$

Note that Eq. (4-31) involves the adjacent displacement angles $\hat{\theta}_6$ and $\hat{\theta}_7$ and the displacement angle $\hat{\underline{\theta}}_3$ opposite to both of them.

Cyclic permutation permits Eq. (4-30) to be written in five different ways. It is, therefore, possible to get five equations of the form (4-31) involving different combinations of two adjacent angles and the angle opposite to both of them.

2. Relationship involving three adjacent dual displacement angles.

In this arrangement of Eq. (4-29), seven matrices are used on one side of the equality sign and three matrices on the other. The important point to note is that the central matrix on the side containing three matrices involves only the constant kinematic parameters of the mechanism. Thus, we have, for instance,

$$[\hat{\alpha}_{76}]_1 [\hat{\theta}_7]_3 [\hat{\alpha}_{47}]_1 [\hat{\underline{\theta}}_4]_3 [\hat{\alpha}_{34}]_1 [\hat{\underline{\theta}}_3]_3 [\hat{\alpha}_{53}]_1 = [\hat{\theta}_6]_3^{-1} [\hat{\alpha}_{65}]_1^{-1} [\hat{\theta}_5]_3^{-1} \tag{4-31}$$

Note that the central matrix $[\alpha_{65}]_1^{-1}$ on the right hand side involves only the constant kinematic parameters of the mechanism.

Simplifying Eq. (4-31) by using relations (4-24) and equating the "33" elements of the resultant matrix equation, we get

$$\begin{aligned} f_2(\underline{\hat{\theta}}_3, \underline{\hat{\theta}}_4, \hat{\theta}_7) = {} & [(S\hat{\alpha}_{47}\, C\hat{\alpha}_{76} + C\hat{\alpha}_{47}\, S\hat{\alpha}_{76}\, C\hat{\theta}_7)\, S\underline{\hat{\theta}}_4 \\ & + S\hat{\alpha}_{76}\, S\hat{\theta}_7\, C\underline{\hat{\theta}}_4]\, (S\hat{\alpha}_{53}\, S\underline{\hat{\theta}}_3) + [S\hat{\alpha}_{76}\, S\hat{\theta}_7\, S\underline{\hat{\theta}}_4 \\ & - (S\hat{\alpha}_{47}\, C\hat{\alpha}_{76} + C\hat{\alpha}_{47}\, S\hat{\alpha}_{76}\, C\hat{\theta}_7)\, C\underline{\hat{\theta}}_4]\, (C\hat{\alpha}_{53}\, S\hat{\alpha}_{34} \\ & + S\hat{\alpha}_{53}\, C\hat{\alpha}_{34}\, C\underline{\hat{\theta}}_3) + (C\hat{\alpha}_{47}\, C\hat{\alpha}_{76} \\ & - S\hat{\alpha}_{47}\, S\hat{\alpha}_{76}\, C\hat{\theta}_7)\, (C\hat{\alpha}_{53}\, C\hat{\alpha}_{34} - S\hat{\alpha}_{53}\, S\hat{\alpha}_{34}\, C\underline{\hat{\theta}}_3) \\ & - C\hat{\alpha}_{65} = 0 \end{aligned} \tag{4-32}$$

Note that Eq. (4-32) involves the three adjacent displacement angles $\underline{\hat{\theta}}_3$, $\underline{\hat{\theta}}_4$, and $\underline{\hat{\theta}}_7$.

Cyclic permutation allows Eq. (4-31) to be written in five different ways. It is, therefore, possible to obtain five equations of the form (4-32) involving different combinations of three adjacent angles.

Loop 3 or Outer Loop (ABEFGA)

The loop-closure condition of the mechanism in Figure 27 for loop 3 is given by

$$\begin{aligned} & [\underline{\hat{\theta}}_1]_3\, [\hat{\alpha}_{17}]_1\, [\underline{\hat{\theta}}_7]_3\, [\hat{\alpha}_{76}]_1\, [\hat{\theta}_6]_3\, [\hat{\alpha}_{65}]_1\, [\underline{\hat{\theta}}_5]_3\, [\hat{\alpha}_{52}]_1 \\ & \qquad [\underline{\hat{\theta}}_2]_3\, [\hat{\alpha}_{21}]_1 = [I] \end{aligned} \tag{4-33}$$

Two arrangements of Eq. (4-33) are useful in the study of existence criteria. These arrangements are similar to the loop 2 considered above.

The first is the arrangement of five matrices on either side of the equality sign. Thus, we have, for instance,

$$[\hat{\alpha}_{17}]_1 \, [\hat{\underline{\theta}}_7]_3 \, [\hat{\alpha}_{76}]_1 \, [\hat{\theta}_6]_3 \, [\hat{\alpha}_{65}]_1$$

$$= [\hat{\underline{\theta}}_1]_3^{-1} \, [\hat{\alpha}_{21}]_1^{-1} \, [\hat{\underline{\theta}}_2]_3^{-1} \, [\hat{\alpha}_{52}]_1^{-1} \, [\hat{\underline{\theta}}_5]_3 \qquad (4\text{-}34)$$

Simplifying the above equation by using relations (4-24) and equating the "33" elements of the resultant matrix equation, we get

$$\begin{aligned} F_3 \, (\hat{\underline{\theta}}_2, \, \hat{\theta}_6, \, \hat{\underline{\theta}}_7) = {} & (S\hat{\alpha}_{17} \, S\hat{\alpha}_{65} \, S\hat{\underline{\theta}}_7) \, S\hat{\theta}_6 - S\hat{\alpha}_{65} \, (C\hat{\alpha}_{17} \, S\hat{\alpha}_{76} \\ & + S\hat{\alpha}_{17} \, C\hat{\alpha}_{76} \, C\hat{\underline{\theta}}_7) \, C\hat{\theta}_6 + C\hat{\alpha}_{65} \, (C\hat{\alpha}_{17} \, C\hat{\alpha}_{76} \\ & - S\hat{\alpha}_{17} \, S\hat{\alpha}_{76} \, C\hat{\underline{\theta}}_7) - (C\hat{\alpha}_{52} \, C\hat{\alpha}_{21} - S\hat{\alpha}_{52} \, S\hat{\alpha}_{21} \, C\hat{\underline{\theta}}_2) = 0 \end{aligned} \qquad (4\text{-}35)$$

Note that Eq. (4-35) involves the adjacent displacement angles $\hat{\underline{\theta}}_6$ and $\hat{\underline{\theta}}_7$ and the displacement angle $\hat{\underline{\theta}}_2$ opposite to both of them.

Cyclic permutation allows Eq. (4-34) to be written in five different ways. It is, therefore, possible to get five equations of the form (4-35) involving different combinations of two adjacent angles and the angle opposite to both of them.

The second is the arrangement of seven matrices on one side of the equality sign and the three matrices on the other. Thus, we have, for instance,

$$[\hat{\alpha}_{76}]_1 [\hat{\underline{\theta}}_7]_3 [\hat{\alpha}_{17}]_1 [\hat{\underline{\theta}}_1]_3$$

$$= [\hat{\theta}_6]_3^{-1} [\hat{\alpha}_{65}]_1^{-1} [\hat{\theta}_5]^{-1} [\hat{\alpha}_{21}]_1 [\hat{\underline{\theta}}_2]_3 [\hat{\alpha}_{52}]_1 \qquad (4\text{-}36)$$

Simplifying Eq. (4-36) by using relationships (4-24) and equating the "33" elements of the resultant matrix equation, we get

$$\begin{aligned} f_3 (\hat{\underline{\theta}}_1, \hat{\underline{\theta}}_2, \hat{\underline{\theta}}_7) = {} & [(S\hat{\alpha}_{17} \, C\hat{\alpha}_{76} + C\hat{\alpha}_{17} \, S\hat{\alpha}_{76} \, C\hat{\underline{\theta}}_6) \, S\hat{\underline{\theta}}_1 \\ & + S\hat{\alpha}_{76} \, S\hat{\underline{\theta}}_7 \, C\hat{\underline{\theta}}_1] \, [S\hat{\alpha}_{52} \, S\hat{\underline{\theta}}_2] + [S\hat{\alpha}_{76} \, S\hat{\underline{\theta}}_7 \, S\hat{\underline{\theta}}_1 \\ & - (S\hat{\alpha}_{17} \, C\hat{\alpha}_{76} + C\hat{\alpha}_{17} \, S\hat{\alpha}_{76} \, C\hat{\underline{\theta}}_7) \, C\hat{\underline{\theta}}_1] \, (C\hat{\alpha}_{52} \, S\hat{\alpha}_{21} \\ & + S\hat{\alpha}_{52} \, C\hat{\alpha}_{21} \, C\hat{\underline{\theta}}_2) + (C\hat{\alpha}_{17} \, C\hat{\alpha}_{76} \\ & - S\hat{\alpha}_{17} \, S\hat{\alpha}_{76} \, C\hat{\underline{\theta}}_7) \, (C\hat{\alpha}_{52} \, C\hat{\alpha}_{21} - S\hat{\alpha}_{52} \, S\hat{\alpha}_{21} \, C\hat{\underline{\theta}}_2) \\ & - C\hat{\alpha}_{65} = 0 \end{aligned} \qquad (4\text{-}37)$$

Note that Eq. (4-37) involves the adjacent displacement angles $\hat{\underline{\theta}}_1$ and $\hat{\underline{\theta}}_2$ and the displacement angle $\hat{\underline{\theta}}_7$ opposite to both of them.

Observe that equations (4-26), (4-28), (4-31), (4-32), (4-35), and (4-37) are all dual equations. Each of them, therefore, represents two scalar equations. Since four equations of the form (4-26), two of the form (4-28), and five each of the form (4-31), (4-32), (4-35), and (4-37) are possible; a total of fifty-two scalar equations are available. These fifty-two scalar equations make it possible to obtain the existence criteria of all mechanisms with one general constraint or two passive couplings.

Existence Criteria of the Six-Link R-R-C-C-C-R-C Mechanism

In this section, the Dimentberg passive coupling method has been used to obtain the existence criteria of an R-R-C-C-C-R-C mechanism with one kink-link zero from the displacement relationships of the parent R-C-C-C-C-C-C mechanism. The procedure for obtaining the existence criteria of the R-R-C-C-C-R-C mechanism with non-zero kink-links is given in Appendix A.

Derivation of the Existence Criteria

Consider the six-link, two-loop R-C-C-C-C-C-C space mechanism shown schematically in Figure 27. Note that the offset distance at the revolute pair at A is constant. If the translational displacement s_2 at the cylinder pair at B remains constant and the translational displacement s_6 at the cylinder pair at F reduces to zero at all positions of this mechanism, then it reduces to an R-R-C-C-C-R-C mechanism as shown in Figure 28.

By considering the loop-closure condition of the mechanism in Figure 27 in two different ways, one from loop 1 (ABCDA) and the other from outer loop (ABEFGA), the following displacement relationships can be obtained:

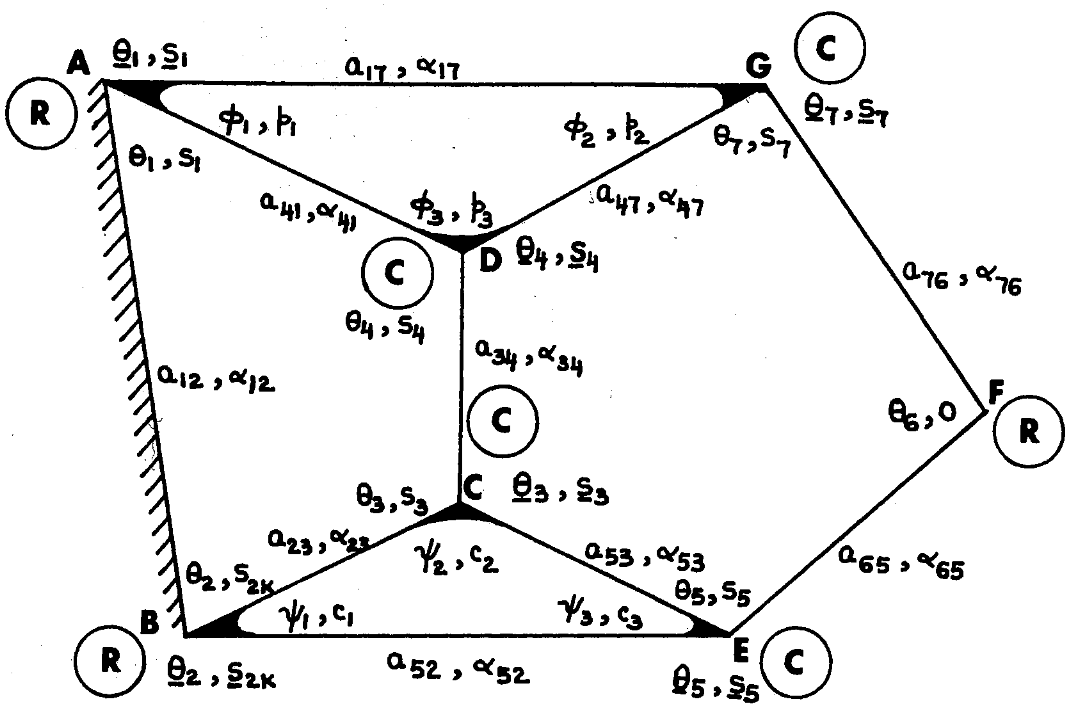

Figure 28. R-R-C-C-C-R-C Space Mechanism Obtained From the Mechanism in Figure 27 by Making $s_2 = s_{2k}$ = a Constant and $s_6 = 0$

$$F_1(\hat{\theta}_1, \hat{\theta}_2) = (S\hat{\alpha}_{23} S\hat{\alpha}_{41} S\hat{\theta}_2) S\hat{\theta}_1 - [S\hat{\alpha}_{41} (S\hat{\alpha}_{12} C\hat{\alpha}_{23} + C\hat{\alpha}_{12} S\hat{\alpha}_{23} C\hat{\theta}_2)] C\hat{\theta}_1 - C\hat{\alpha}_{34} + C\hat{\alpha}_{41} (C\hat{\alpha}_{12} C\hat{\alpha}_{23} - S\hat{\alpha}_{12} S\hat{\alpha}_{23} C\hat{\theta}_2) = 0 \quad (4\text{-}38)$$

$$F_3(\underline{\hat{\theta}}_1, \underline{\hat{\theta}}_2, \hat{\theta}_6) = (S\hat{\alpha}_{17} S\hat{\alpha}_{52} S\underline{\hat{\theta}}_2) S\underline{\hat{\theta}}_1 - S\hat{\alpha}_{17} (C\hat{\alpha}_{52} S\hat{\alpha}_{21} + S\hat{\alpha}_{52} C\hat{\alpha}_{21} C\underline{\hat{\theta}}_2) C\underline{\hat{\theta}}_1 + C\hat{\alpha}_{17} (C\hat{\alpha}_{52} C\hat{\alpha}_{21} - S\hat{\alpha}_{52} S\hat{\alpha}_{21} C\underline{\hat{\theta}}_2) - (C\hat{\alpha}_{76} C\hat{\alpha}_{65} - S\hat{\alpha}_{76} S\hat{\alpha}_{65} C\hat{\theta}_6) = 0 \quad (4\text{-}39)$$

Note that Eq. (4-38) is similar in form to Eq. (4-26) and Eq. (4-39) is similar to Eq. (4-35). Now, let the translation s_2 become constant equal to s_{2k} and the translation s_6 be zero at all positions of the mechanism. Using equations (4-20), (4-21) and (4-22) the dual part of Eq. (4-38) becomes

$$B_2(t_1)\, t_2^2 + B_1(t_1)\, t_2 + B_0(t_1) = 0 \quad (4\text{-}40)$$

where

$$t_1 = \tan(\theta_1/2)$$

$$t_2 = \tan(\theta_2/2)$$

and

$$B_2(t_1) = B_{22} t_1^2 + B_{21} t_1 + B_{20}$$

$$B_1(t_1) = B_{12} t_1^2 + B_{11} t_1 + B_{10} \quad (4\text{-}41)$$

$$B_0(t_1) = B_{02} t_1^2 + B_{01} t_1 + B_{00}$$

The constants in Eqs. (4-41) involve only the constant kinematic parameters of the mechanism and are defined in Table IX.

Eliminating the angle θ_6 from the primary and dual parts of Eq. (4-39) using Eqs. (4-20) through (4-22), we get

$$A_2(t_1)\, t_2^2 + A_1(t_1)\, t_2 + A_0(t_1) = 0 \qquad (4\text{-}42)$$

where

$$\begin{aligned} A_2(t_1) &= A_{22}\, t_1^2 + A_{21}\, t_1 + A_{20} \\ A_1(t_1) &= A_{12}\, t_1^2 + A_{11}\, t_1 + A_{10} \\ A_0(t_1) &= A_{02}\, t_1^2 + A_{01}\, t_1 + A_{00} \end{aligned} \qquad (4\text{-}43)$$

The constants in Eqs. (4-43) are defined in Table XI. If an R-R-C-C-C-R-C mechanism of the type under consideration is to exist, the quadratic equations (4-40) and (4-42) must have at least one common root. This gives the condition (102):

$$\begin{vmatrix} B_2(t_1) & B_1(t_1) & B_0(t_1) & 0 \\ 0 & B_2(t_1) & B_1(t_1) & B_0(t_1) \\ A_2(t_1) & A_1(t_1) & A_0(t_1) & 0 \\ 0 & A_2(t_1) & A_1(t_1) & A_0(t_1) \end{vmatrix} = 0 \qquad (4\text{-}44)$$

Equation (4-44) is a function of only the variable t_1. Expanding and simplifying it, we get

$$C_8\, t_1^8 + C_7\, t_1^7 + \ldots + C_1\, t_1 + C_0 = 0$$

or in short,

TABLE IX

CONSTANTS FOR USE IN EQUATION (4-41)

$$D_{002} = a_{41}\, C\alpha_{41}\, S\alpha_{23} + a_{23}\, C\alpha_{23}\, S\alpha_{41}$$

$$D_{001} = s_1\, S\alpha_{23}\, S\alpha_{41} + s_{2k}\, C\alpha_{12}\, S\alpha_{23}$$

$$D_{000} = s_{2k}\, S\alpha_{12}\, C\alpha_{41}\, S\alpha_{23}$$

$$E_{002} = s_{2k}\, S\alpha_{23}\, S\alpha_{41} + s_1\, C\alpha_{12}\, S\alpha_{23}$$

$$E_{001} = -a_{23}\, C\alpha_{23}\, C\alpha_{12} + a_{12}\, S\alpha_{12}\, S\alpha_{23}$$

$$E_{000} = -a_{12}\, C\alpha_{12}\, C\alpha_{41}\, S\alpha_{23} - a_{23}\, C\alpha_{41}\, C\alpha_{23}\, S\alpha_{12} + a_{41}\, S\alpha_{41}\, S\alpha_{12}\, S\alpha_{23}$$

$$F_{002} = s_1\, S\alpha_{41}\, S\alpha_{23}$$

$$F_{001} = -a_{41}\, C\alpha_{41}\, C\alpha_{23} + a_{23}\, S\alpha_{23}\, S\alpha_{41}$$

$$F_{000} = a_{34}\, S\alpha_{34} - C\alpha_{12}\,(a_{41}\, S\alpha_{41}\, C\alpha_{23} + a_{23}\, S\alpha_{23}\, C\alpha_{41}) - a_{12}\, S\alpha_{12}\, C\alpha_{41}\, C\alpha_{23}$$

$$B_{22} = E_{001} - E_{000} - F_{001} + F_{000}$$

$$B_{21} = -2\,(E_{002} - F_{002})$$

$$B_{20} = -E_{001} - E_{000} + F_{001} + F_{000}$$

$$B_{12} = -2\,(D_{001} - D_{000})$$

$$B_{11} = 4\, D_{002}$$

TABLE IX (Continued)

$$B_{10} = 2\,(D_{001} + D_{000})$$

$$B_{02} = -E_{001} + E_{000} - F_{001} + F_{000}$$

$$B_{01} = 2\,(E_{002} + F_{002})$$

$$B_{00} = E_{001} + E_{000} + F_{001} + F_{000}$$

TABLE X

CONSTANTS FOR USE IN TABLE XI

$$U_1 = \frac{a_{76}\, C\alpha_{65}}{S\alpha_{76}} + \frac{a_{65}\, C\alpha_{76}}{S\alpha_{65}}$$

$$U_2 = a_{76} \frac{C\alpha_{76}}{S\alpha_{76}} + a_{65} \frac{C\alpha_{65}}{S\alpha_{65}}$$

$$F_0 = U_1 - U_2\, C(\alpha_{52} - \alpha_{21} - \alpha_{17}) - (a_{52} - a_{21} - a_{17})\, S(\alpha_{52} - \alpha_{21} - \alpha_{17})$$

$$F_1 = -2\, S\alpha_{17}\, [s_1\, S(\alpha_{52} - \alpha_{21}) + s_{2k}\, S\alpha_{52}]$$

$$F_2 = U_1 - U_2\, C(\alpha_{52} - \alpha_{21} + \alpha_{17}) - (a_{52} - a_{21} + a_{17})\, S\,(\alpha_{52} - \alpha_{21} + \alpha_{17})$$

$$G_0 = 2\, S\alpha_{52}\, [s_1\, S\alpha_{17} + s_{2k}\, S(\alpha_{21} + \alpha_{17})$$

$$G_1 = 4\, S\alpha_{17}\, S\alpha_{52}\, (a_{17}\, Ct\, \alpha_{17} - a_{76}\, Ct\, \alpha_{76} - a_{65}\, Ct\, \alpha_{65} + a_{52}\, Ct\, \alpha_{52})$$

$$G_2 = -2\, S\alpha_{52}\, [s_1\, S\alpha_{17} - s_{2k}\, S(\alpha_{21} - \alpha_{17})]$$

$$H_0 = U_1 - U_2\, C(\alpha_{52} + \alpha_{21} + \alpha_{17}) - (a_{52} + a_{21} + a_{17})\, S(\alpha_{52} + \alpha_{21} + \alpha_{17})$$

$$H_1 = 2\, S\alpha_{17}\, [s_1\, S(\alpha_{52} + \alpha_{17}) + s_{2k}\, S\alpha_{52}]$$

$$H_2 = U_1 - U_2\, C(\alpha_{52} + \alpha_{21} - \alpha_{17}) - (a_{52} + a_{21} - a_{17}) S(\alpha_{52} + \alpha_{21} - \alpha_{17})$$

TABLE XI

CONSTANTS FOR USE IN EQUATION (4-43) AND TABLE XII

$$x_1 = \tan (\Phi_1/2)$$

$$y_2 = F_0 - F_1 x_1 + F_2 x_1^2$$

$$y_1 = 2 F_0 x_1 - F_1 x_1^2 + F_1 - 2 F_2 x_1$$

$$y_0 = F_0 x_1^2 + F_1 x_1 + F_2$$

$$w_2 = -G_0 + G_1 x_1 - G_2 x_1^2$$

$$w_1 = -2 G_0 x_1 + G_1 x_1^2 - G_1 + 2 G_2 x_1$$

$$w_0 = -G_0 x_1^2 - G_1 x_1 - G_2$$

$$z_2 = H_0 - H_1 x_1 + H_2 x_1^2$$

$$z_1 = 2 H_0 x_1 - H_1 x_1^2 + H_1 - 2 H_2 x_1$$

$$z_0 = H_0 x_1 + H_1 x_1 + H_2$$

$$x_2 = \tan (\psi_1/2)$$

$$A_{22} = x_2^2 y_2 + x_2 w_2 + z_2$$

$$A_{21} = x_2^2 y_1 + x_2 w_1 + z_1$$

$$A_{20} = x_2^2 y_0 + x_2 w_0 + z_0$$

$$A_{12} = 2x_2 (z_2 - y_2) + w_2 (x_2^2 - 1)$$

$$A_{11} = 2x_2 (z_1 - y_1) + w_1 (x_2^2 - 1)$$

TABLE XI (Continued)

$$A_{10} = 2x_2 (z_0 - y_0) + w_0 (x_2^2 - 1)$$

$$A_{02} = y_2 - x_2 w_2 + x_2^2 z_2$$

$$A_{01} = y_1 - x_2 w_1 + x_2^2 z_1$$

$$A_{00} = y_0 - x_2 w_0 + x_2^2 z_0$$

$$\sum_{i=0}^{8} c_i t_1^i = 0, \qquad i = 0, 1, 2, \ldots, 8 \qquad (4\text{-}45)$$

The constants in the above equation are defined in Table XII. Equation (4-45) must hold good at all values of the variable t_1. Its coefficient must, therefore, vanish. Thus, we have

$$c_i = 0, \qquad i = 0, 1, 2, \ldots, 8 \qquad (4\text{-}46)$$

Condition (4-46) represents nine equations among the 20 constant kinematic parameters of the R-R-C-C-C-R-C mechanism in Figure 28 (namely, the 8 link-lengths a_{76}, a_{65}, a_{52}, a_{17}, a_{34}, a_{41}, a_{23}, and a_{12}, the 8 twist angles α_{76}, α_{65}, α_{52}, α_{17}, α_{41}, α_{34}, α_{23}, and α_{12}, the 2 constant offset distances s_1, s_{2k} of the revolute pairs A and B, and the 2 constant displacement angles Φ_1 and ψ_1 at the two ternary links at joints A and B). These nine equations provide the necessary conditions for the existence of a six-link two-loop R-R-C-C-C-R-C mechanism with constant offset distances at the revolute pairs at A and B and zero offset distance at the revolute pair at F.

On Obtaining R-R-C-C-C-R-C Mechanism From the Derived Criteria

The existence criteria derived in the previous section can be used to obtain the constant kinematic parameters of the R-R-C-C-C-R-C mechanism.

TABLE XII

COEFFICIENTS FOR USE IN EQUATION (4-45)

$$c_8 = A_{12} A_{22} B_{02} B_{12} + (2A_{02} A_{22} - A_{12}^2) B_{02} B_{22} - A_{02} A_{22} B_{12}^2$$

$$+ A_{02} A_{12} B_{12} B_{22} - A_{22}^2 B_{02}^2 - A_{02}^2 - B_{22}^2$$

$$c_7 = A_{12} A_{22} (B_{01} B_{12} + B_{02} B_{11}) + (A_{11} A_{22} + A_{12} A_{21}) B_{02} B_{12}$$

$$+ (2A_{02} A_{22} - A_{12}^2) (B_{01} B_{22} + B_{02} B_{21}) + 2 (A_{01} A_{22}$$

$$+ A_{02} A_{21} - A_{11} A_{12}) B_{02} B_{22} - 2 A_{02} A_{22} B_{11} B_{12}$$

$$- (A_{01} A_{22} + A_{02} A_{21}) B_{12}^2 + A_{02} A_{12} (B_{11} B_{22} + B_{12} B_{21})$$

$$+ (A_{01} A_{12} + A_{02} A_{11}) B_{12} B_{22} - 2 [A_{22} B_{02} (A_{21} B_{02}$$

$$+ A_{22} B_{01}) + A_{02} B_{22} (A_{01} B_{22} + A_{02} B_{21})]$$

$$c_6 = A_{12} A_{22} (B_{00} B_{12} + B_{02} B_{10} + B_{01} B_{11}) + (A_{10} A_{22} + A_{12} A_{20}$$

$$+ A_{11} A_{21}) B_{02} B_{11} + (A_{11} A_{22} + A_{12} A_{21}) (B_{01} B_{12} + B_{02} B_{11})$$

$$+ (2A_{02} A_{22} - A_{12}^2) (B_{00} B_{22} + B_{02} B_{20} + B_{01} B_{21})$$

$$+ [2 (A_{00} A_{22} + A_{02} A_{20} + A_{01} A_{21} - A_{10} A_{12}) - A_{11}^2] B_{02} B_{22}$$

$$+ 2(A_{01} A_{22} + A_{02} A_{21} - A_{11} A_{12}) (B_{01} B_{22} + B_{02} B_{21})$$

$$- A_{02} A_{22} (2B_{10} B_{12} + B_{11}^2) - (A_{00} A_{22} + A_{02} A_{20}$$

$$+ A_{01} A_{21}) B_{12}^2 - 2(A_{01} A_{22} + A_{02} A_{21}) B_{11} B_{12}$$

$$+ A_{02} A_{12} (B_{10} B_{22} + B_{12} B_{20} + B_{11} B_{21}) + (A_{00} A_{12}$$

TABLE XII (Continued)

$$+ A_{02} A_{10} + A_{01} A_{11}) B_{12} B_{22} + (A_{01} A_{12} + A_{02} A_{11})(B_{11} B_{22}$$

$$+ B_{12} B_{21}) - A^2_{22} (2B_{00} B_{02} + B^2_{01}) - (2A_{20} A_{22} + A^2_{21}) B^2_{02}$$

$$- 4A_{21} A_{22} B_{01} B_{02} - A^2_{02} (2B_{20} B_{22} + B^2_{21}) - (2A_{00} A_{02}$$

$$+ A^2_{01}) B^2_{22} - 4A_{01} A_{02} B_{21} B_{22}$$

$$c_5 = A_{12} A_{22} (B_{00} B_{11} + B_{01} B_{10}) + (A_{10} A_{21} + A_{11} A_{20}) B_{02} B_{12}$$

$$+ (A_{11} A_{22} + A_{12} A_{21}) (B_{00} B_{12} + B_{02} B_{10} + B_{01} B_{11})$$

$$+ (A_{10} A_{22} + A_{12} A_{20} + A_{11} A_{21}) (B_{01} B_{12} + B_{02} B_{11})$$

$$+ (2A_{02} A_{22} - A^2_{12}) (B_{00} B_{21} + B_{01} B_{20}) + 2(A_{00} A_{21} + A_{01} A_{20}$$

$$- A_{10} A_{11}) B_{02} B_{22} + 2(A_{01} A_{22} + A_{02} A_{21} - A_{11} A_{12})(B_{00} B_{22}$$

$$+ B_{02} B_{20} + B_{01} B_{21}) + [2 (A_{00} A_{22} + A_{02} A_{20} + A_{01} A_{21}$$

$$- A_{10} A_{12}) - A^2_{11}] (B_{01} B_{22} + B_{02} B_{21}) - 2A_{02} A_{22} B_{10} B_{11}$$

$$- (A_{00} A_{21} + A_{01} A_{20}) B^2_{12} - (A_{01} A_{22} + A_{02} A_{21}) (2B_{10} B_{12}$$

$$+ B^2_{11}) - 2(A_{00} A_{22} + A_{02} A_{20} + A_{01} A_{21}) B_{11} B_{12}$$

$$+ A_{02} A_{12} (B_{10} B_{21} + B_{11} B_{20}) + (A_{00} A_{11} + A_{01} A_{10}) B_{12} B_{22}$$

$$+ (A_{01} A_{12} + A_{02} A_{11}) (B_{10} B_{22} + B_{12} B_{20} + B_{11} B_{21})$$

$$+ (A_{00} A_{12} + A_{02} A_{10} + A_{01} A_{11}) (B_{11} B_{22} + B_{12} B_{21})$$

$$- 2 [A^2_{22} B_{00} B_{01} + A_{20} A_{21} B^2_{02} + A_{21} A_{22} (2 B_{00} B_{02} + B^2_{01})$$

TABLE XII (Continued)

$$+ (2A_{20} A_{22} + A_{21}^2) B_{01} B_{02}] - 2[A_{02}^2 B_{20} B_{21} + A_{00} A_{01} B_{22}^2$$

$$+ A_{01} A_{02} (2B_{20} B_{22} + B_{21}^2) + (2A_{00} A_{02} + A_{01}^2) B_{21} B_{22}]$$

$$c_4 = A_{12} A_{22} B_{00} B_{10} + A_{10} A_{20} B_{02} B_{12} + (A_{11} A_{22}$$

$$+ A_{12} A_{21}) (B_{00} B_{11} + B_{01} B_{10}) + (A_{10} A_{21}$$

$$+ A_{11} A_{20}) (B_{01} B_{12} + B_{02} B_{11}) + (A_{10} A_{22} + A_{12} A_{20}$$

$$+ A_{11} A_{21}) (B_{00} B_{12} + B_{02} B_{10} + B_{01} B_{11}) + (2A_{02} A_{22}$$

$$- A_{12}^2) B_{00} B_{20} + (2A_{00} A_{20} - A_{10}^2) B_{02} B_{22} + 2(A_{01} A_{22}$$

$$+ A_{02} A_{21} - A_{11} A_{12}) (B_{00} B_{21} + B_{01} B_{20}) + 2(A_{00} A_{21}$$

$$+ A_{01} A_{20} - A_{10} A_{11}) (B_{01} B_{22} + B_{02} B_{21}) + [2(A_{00} A_{22}$$

$$+ A_{02} A_{20} + A_{01} A_{21} - A_{10} A_{12}) - A_{11}^2] (B_{00} B_{22} + B_{02} B_{20}$$

$$+ B_{01} B_{21}) - A_{02} A_{22} B^2_{10} - A_{00} A_{20} B_{12}^2 - 2(A_{01} A_{22}$$

$$+ A_{02} A_{21}) B_{10} B_{11} - 2(A_{00} A_{21} + A_{01} A_{20}) B_{11} B_{12}$$

$$- (A_{00} A_{22} + A_{02} A_{20} + A_{01} A_{21}) (2B_{10} B_{12} + B_{11}^2)$$

$$+ A_{02} A_{12} B_{10} B_{20} + A_{00} A_{10} B_{12} B_{22} + (A_{01} A_{12}$$

$$+ A_{02} A_{11}) (B_{10} B_{21} + B_{11} B_{20}) + (A_{00} A_{11} + A_{01} A_{10})(B_{11} B_{22}$$

$$+ B_{12} B_{21}) + (A_{00} A_{12} + A_{02} A_{10} + A_{01} A_{11}) (B_{10} B_{22}$$

$$+ B_{12} B_{20} + B_{11} B_{21}) - A_{22}^2 B_{00}^2 - A_{20}^2 B_{02}^2$$

TABLE XII (Continued)

$$- 4(A_{21} A_{22} B_{00} B_{01} + A_{20} A_{21} B_{01} B_{02}) - (2A_{20} A_{22}$$

$$+ A_{21}^2) (2B_{00} B_{02} + B_{01}^2) - A_{02}^2 B_{20}^2 - A_{00}^2 B_{22}^2$$

$$- 4(A_{01} A_{02} B_{20} B_{21} + A_{00} A_{01} B_{21} B_{22}) - (2A_{00} A_{20}$$

$$+ A_{01}^2) (2B_{20} B_{22} + B_{21}^2)$$

$$c_3 = A_{10} A_{20} (B_{01} B_{12} + B_{02} B_{11}) + (A_{11} A_{22} + A_{12} A_{21}) B_{00} B_{10}$$

$$+ (A_{10} A_{22} + A_{12} A_{20} + A_{11} A_{21}) (B_{00} B_{11} + B_{01} B_{10})$$

$$+ (A_{10} A_{21} + A_{11} A_{20}) (B_{00} B_{12} + B_{02} B_{10} + B_{01} B_{11})$$

$$+ (2A_{00} A_{20} - A_{10}^2) (B_{01} B_{22} + B_{02} B_{21}) + 2(A_{01} A_{22}$$

$$+ A_{02} A_{21} + A_{11} A_{12}) B_{00} B_{20} + [2(A_{00} A_{22} + A_{02} A_{20}$$

$$+ A_{01} A_{21} - A_{10} A_{12}) - A_{11}^2] (B_{00} B_{21} + B_{01} B_{20})$$

$$+ 2(A_{00} A_{21} + A_{01} A_{20} - A_{10} A_{11}) (B_{00} B_{22} + B_{02} B_{20}$$

$$+ B_{01} B_{21}) - 2A_{00} A_{20} B_{11} B_{12} - (A_{01} A_{22} + A_{02} A_{21}) B_{10}^2$$

$$- 2(A_{00} A_{22} + A_{02} A_{20} + A_{01} A_{21}) B_{10} B_{11} - (A_{00} A_{21}$$

$$+ A_{01} A_{20}) (2B_{10} B_{12} + B_{11}^2) + A_{00} A_{10} (B_{11} B_{22} + B_{12} B_{21})$$

$$+ (A_{01} A_{12} + A_{02} A_{11}) B_{10} B_{20} + (A_{00} A_{12} + A_{02} A_{10}$$

$$+ A_{01} A_{11}) (B_{10} B_{21} + B_{11} B_{20}) + (A_{00} A_{11} + A_{01} A_{10})(B_{10} B_{22}$$

$$+ B_{12} B_{20} + B_{11} B_{21}) - 2[A_{20}^2 B_{01} B_{02} + A_{21} A_{22} B_{00}^2$$

TABLE XII (Continued)

$$+ (2A_{20} A_{22} + A_{21}^2) B_{00} B_{01} + A_{20} A_{21} (2B_{00} B_{02} + B_{01}^2)]$$

$$- 2[A_{00}^2 B_{21} B_{22} + A_{01} A_{02} B_{20}^2 + (2A_{00} A_{02} + A_{01}^2) B_{20} B_{21}$$

$$+ A_{00} A_{01} (2B_{20} B_{22} + B_{21}^2)]$$

$$c_2 = A_{10} A_{20} (B_{00} B_{12} + B_{02} B_{10} + B_{01} B_{11}) + (A_{10} A_{22} + A_{12} A_{20}$$

$$+ A_{11} A_{21}) B_{00} B_{10} + (A_{10} A_{21} + A_{11} A_{20}) (B_{00} B_{11}$$

$$+ B_{01} B_{10}) + (2A_{00} A_{20} - A_{10}^2) (B_{00} B_{22} + B_{02} B_{20} + B_{01} B_{21})$$

$$+ [2(A_{00} A_{22} + A_{02} A_{20} + A_{01} A_{21} - A_{10} A_{12}) - A_{11}^2] B_{00} B_{20}$$

$$+ (A_{00} A_{21} + A_{01} A_{20} - A_{10} A_{11}) (B_{00} B_{21} + B_{01} B_{20})$$

$$- A_{00} A_{20} (2B_{10} B_{12} + B_{11}^2) - (A_{00} A_{22} + A_{02} A_{20}$$

$$+ A_{01} A_{21}) B_{10}^2 - 2(A_{00} A_{21} + A_{01} A_{20}) B_{10} B_{11}$$

$$+ A_{00} A_{10} (B_{10} B_{22} + B_{12} B_{20} + B_{11} B_{21}) + (A_{00} A_{12} + A_{02} A_{10}$$

$$+ A_{01} A_{11}) B_{10} B_{20} + (A_{00} A_{11} + A_{01} B_{10}) (B_{10} B_{21} + B_{11} B_{20})$$

$$- A^2{}_{20} (2B_{00} B_{02} + B_{01}^2) - (2A_{20} A_{22} + A_{21}^2) B_{00}^2$$

$$- 4A_{20} A_{21} B_{00} B_{01} - A_{00}^2 (2B_{20} B_{22} + B_{21}^2) - (2A_{00} A_{02}$$

$$+ A_{01}^2) B_{20}^2 - 4A_{00} A_{01} B_{20} B_{21}$$

$$c_1 = A_{10} A_{20} (B_{00} B_{11} + B_{01} B_{10}) + (A_{10} A_{21} + A_{11} A_{20}) B_{00} B_{10}$$

$$+ (2A_{00} A_{20} - A_{10}^2) (B_{00} B_{21} + B_{01} B_{20}) + 2(A_{00} A_{21}$$

TABLE XII (Continued)

$$+ A_{01} A_{20} - A_{10} A_{11}) B_{00} B_{20} - 2A_{00} A_{20} B_{10} B_{11}$$

$$- (A_{00} A_{21} + A_{01} A_{20}) B_{10}^2 + A_{00} A_{10} (B_{10} B_{21} + B_{11} B_{20})$$

$$+ (A_{00} A_{11} + A_{01} A_{10}) B_{10} B_{20} - 2[A_{20} B_{00} (A_{21} B_{00}$$

$$+ A_{20} B_{01}) + A_{00} B_{20} (A_{01} B_{20} + A_{00} B_{21})]$$

$$c_0 = A_{10} A_{20} B_{00} B_{10} + (2A_{00} A_{20} - A_{10}^2) B_{00} B_{20} - A_{00} A_{20} B_{10}^2$$

$$+ A_{00} A_{10} B_{10} B_{20} - A_{20}^2 B_{00}^2 - A_{00}^2 B_{20}^2$$

If the constant kinematic parameters are regarded as unknowns, it is possible to solve this system of equations (4-46) for the unknowns. The algebraic equations (4-46) describing the existence criteria of the mechanism are sufficiently complex to prevent from presenting any simplified geometric descriptions. In fact, the complexity extends far enough to prevent from presenting simplified explicit results in order to facilitate direct computations of the mechanism parameters. Hence it is not practical to solve the equations analytically. Instead, a numerical search technique (123) is preferred to solve for the constant kinematic parameters.

The numerical method used in the present study for solving the system of 9 consistent nonlinear algebraic equations representing the existence conditions of the R-R-C-C-C-R-C mechanism is that developed by Chandler (123). The listing of the computer program is given in Appendix D. Let

$$f_i(x_1, x_2, x_3, \ldots, x_n) = 0 \qquad i = 1, 2, \ldots, 9 \qquad (4\text{-}47)$$

represent a system of nonlinear equations in n unknowns where x_1, x_2, ..., x_n are the 20 unknowns (link lengths a_{76}, a_{65}, a_{52}, a_{17}, a_{41}, a_{23}, a_{34}, and a_{12}, and twist angles α_{76}, α_{65}, α_{52}, α_{17}, α_{41}, α_{34}, α_{23}, and α_{12}, constant offset distances s_1 and s_{2k}, and the two constant displacement angles Φ_1 and ψ_1 at the two ternary links).

An objective function:

$$Y = \sum_{i=1}^{9} f_i^2 (x_1, x_2, \ldots, x_{20})$$

is defined and is minimized such that $Y \approx 0$.

It is important to note that the equations given by (4-47) represents only necessary conditions for the existence of R-R-C-C-C-R-C mechanism. The conditions are not sufficient because satisfaction of the criteria does not itself guarantee an R-R-C-C-C-R-C space mechanism. This is because Eqs. (4-46) also have solutions that correspond to spherical and planar mechanisms. Such solutions are called here trivial solutions. See, for instance Table XV in Appendix D.

The triviality and non-triviality of the solutions of Eqs. (4-47) can be checked by substituting the values of the constant kinematic parameters in the original displacement relationships of the parent R-C-C-C-C-C-C mechanism (120). A non-trivial solution will give constant offset distance at the cylinder pair B, and zero offset distance at the cylinder pair F at all positions of the parent mechanism without, at the same time, affecting its true mobility. A trivial solution will not meet these requirements.

Using the proposed numerical technique, the following solution is obtained: (See Table XVI and Figure 35 in Appendix D.)

Twist-Angles:

$\alpha_{12} = 70.000^\circ$

$\alpha_{23} = 0.0^\circ$

$\alpha_{34} = 70.000^\circ$

$\alpha_{41} = 0.0^\circ$

$\alpha_{65} = 0.120^\circ$

$\alpha_{76} = 70.100^\circ$

$\alpha_{52} = 180.000^\circ$

$\alpha_{17} = 180.008^\circ$

Constant Displacement Angles:

$\Phi_1 = 30.00^\circ$

$\psi_1 = 80.00^\circ$

Kink-Links:

$s_1 = 0.4''$

$s_{2k} = 0.4''$

Link-Lengths:

$a_{12} = 2.00''$

$a_{23} = 1.72''$

$a_{34} = 2.5''$

$a_{41} = 3.0''$

$a_{65} = 10.0''$

$a_{76} = 10.0''$

$a_{52} = 0.5''$

$a_{17} = 0.5''$

Substitution of these parameters in the displacement relationships of R-R-C-C-C-R-C mechanism (120) shows zero translation s_6 and constant translation s_{2k} at the cylinder pairs F and B respectively.

From the extensive search carried out using this numerical technique, it shows that the system of Eqs. (4-47) appear to have narrow range of solutions for the R-R-C-C-C-R-C mechanism.

Existence Criteria of the Six-Link R-R-C-C-C-P-C Mechanism

The six-link, two-loop R-R-C-C-C-P-C mechanism can be derived, like the R-R-C-C-C-R-C mechanism, from the parent R-C-C-C-C-C-C mechanism.

In this section, the Dimentberg method has been used to obtain the existence criteria of the R-R-C-C-C-P-C mechanism with constant offset distances at its revolute pairs and constant displacement angle at the prismatic pair from the displacement relationships of an R-C-C-C-C-C-C mechanism.

Consider the R-C-C-C-C-C-C space mechanism shown schematically in Figure 27. If the translational displacement s_2 at the cylinder pair at B and the rotational displacement s_6 at the cylinder pair at F remain constant at all positions of this mechanism, then it reduces to an R-R-C-C-C-P-C mechanism as shown in Figure 29.

By considering the loop-closure condition of the mechanism in Figure 27 in two different ways, one from loop 1 (ABCDA), the other from outer loop (ABEFGA), the following relationships can be obtained.

$$F_1(\hat{\theta}_1, \hat{\theta}_2) = (S\hat{\alpha}_{23}\, S\hat{\alpha}_{41}\, S\hat{\theta}_2)\, S\hat{\theta}_1 - [S\hat{\alpha}_{41}\,(S\hat{\alpha}_{12}\, C\hat{\alpha}_{23} + C\hat{\alpha}_{12}\, S\hat{\alpha}_{23}\, C\hat{\theta}_2)]\, C\hat{\theta}_1 - C\hat{\alpha}_{34} + C\hat{\alpha}_{41}\,(C\hat{\alpha}_{12}\, C\hat{\alpha}_{23} - S\hat{\alpha}_{12}\, S\hat{\alpha}_{23}\, C\hat{\theta}_2) = 0 \qquad (4\text{-}48)$$

$$F_3 = (\hat{\underline{\theta}}_1, \hat{\underline{\theta}}_2, \hat{\theta}_6) = (S\hat{\alpha}_{17}\, S\hat{\alpha}_{52}\, S\hat{\underline{\theta}}_2)\, S\hat{\underline{\theta}}_1 - S\hat{\alpha}_{17}\,(C\hat{\alpha}_{52}\, S\hat{\alpha}_{21} + S\hat{\alpha}_{52}\, C\hat{\alpha}_{21}\, C\hat{\underline{\theta}}_2)\, C\hat{\underline{\theta}}_1 + C\hat{\alpha}_{17}\,(C\hat{\alpha}_{52}\, C\hat{\alpha}_{21} - S\hat{\alpha}_{52}\, S\hat{\alpha}_{21}\, C\hat{\underline{\theta}}_2) - (C\hat{\alpha}_{76}\, C\hat{\alpha}_{65} - S\hat{\alpha}_{76}\, S\hat{\alpha}_{65}\, C\hat{\theta}_6) = 0 \qquad (4\text{-}49)$$

Note that Eq. (4-48) is the same as Eq. (4-38) and Eq. (4-49) is the same as Eq. (4-39).

Now, let the translational displacement s_2 become constant and the rotational displacement θ_6 be also constant at all positions of the mechanism.

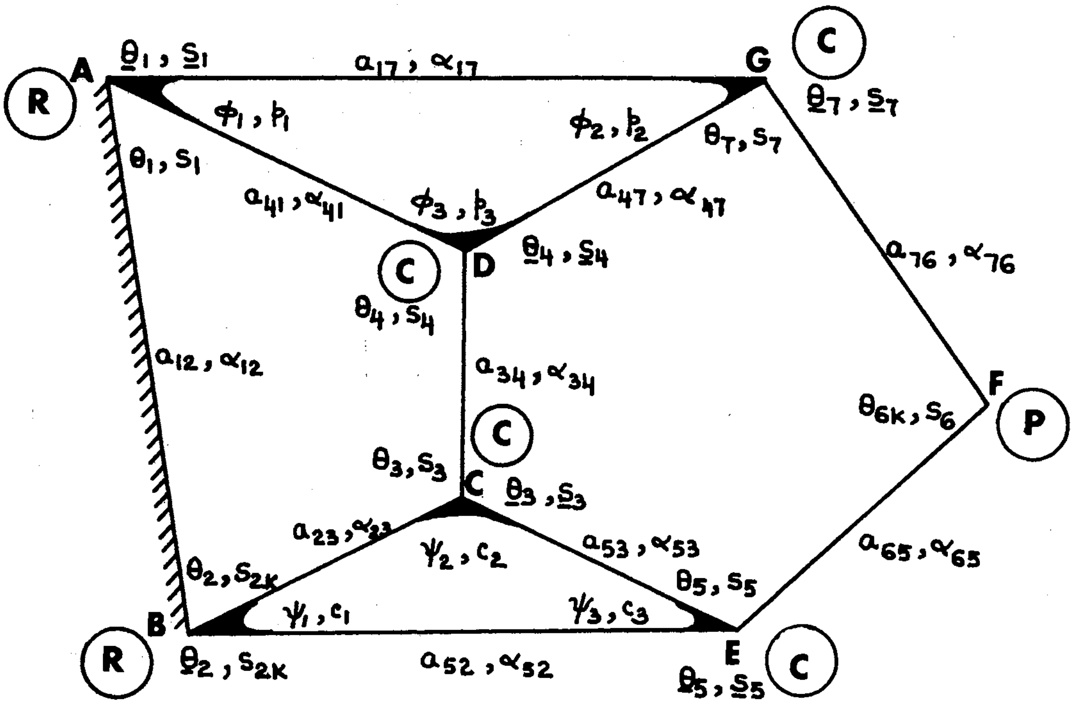

Figure 29. R-R-C-C-C-P-C Space Mechanism Obtained From the Mechanism in Figure 27 by Making $s_2 = s_{2k}$ = a Constant and $\theta_6 = \theta_{6k}$ = a Constant

The dual part of Eq. (4-48) after simplification using Eqs. (4-20) through (4-22) becomes

$$B_2\,(t_1)\,t_2^2 + B_1\,(t_1)\,t_2 + B_0\,(t_1) = 0 \qquad (4\text{-}50)$$

where $\quad t_1 = \tan\,(\theta_1/2) \qquad t_2 = \tan\,(\theta_2/2)$

and

$$B_2\,(t_1) = B_{22}\,t_1^2 + B_{21}\,t_1 + B_{20}$$

$$B_1\,(t_1) = B_{12}\,t_1^2 + B_{11}\,t_1 + B_{10} \qquad (4\text{-}51)$$

$$B_0\,(t_1) = B_{02}\,t_1^2 + B_{01}\,t_1 + B_{00}$$

Note that Eq. (4-50) is the same as Eq. (4-40) and the constants in Eqs. (4-51) involve only the constant kinematic parameters of the mechanism and hence are defined in Table IX.

Denoting the constant value of the angle θ_6 by θ_{6k}, the primary part of Eq. (4-49) becomes

$$M_2\,(t_1)\,t_2^2 + M_1\,(t_1)\,t_2 + M_0\,(t_1) = 0 \qquad (4\text{-}52)$$

where

$$M_2\,(t_1) = M_{22}\,t_1^2 + M_{21}\,t_1 + M_{20}$$

$$M_1\,(t_1) = M_{12}\,t_1^2 + M_{11}\,t_1 + M_{10} \qquad (4\text{-}53)$$

$$M_0\,(t_1) = M_{02}\,t_1^2 + M_{01}\,t_1 + M_{00}$$

The constants in Eqs. (4-53) also involve only the constant kinematic parameters of the mechanism and are defined in Table XIII.

The quadratic equations (4-50) and (4-53) represent two different forms of displacement relationships for the same mechanism.

TABLE XIII

CONSTANTS FOR USE IN EQUATIONS (4-53)

$$A_{002} = S\alpha_{17}\ S\alpha_{52}\ C\psi_1\ C\Phi_1 - S\psi_1\ S\alpha_{17}\ S\alpha_{52}\ C\alpha_{21}\ S\psi_1$$

$$A_{001} = -S\alpha_{17}\ S\alpha_{52}\ C\psi_1\ S\Phi_1 + C\Phi_1\ S\alpha_{17}\ S\alpha_{52}\ C\alpha_{21}\ S\psi_1$$

$$A_{000} = C\alpha_{17}\ S\alpha_{52}\ S\alpha_{21}\ S\psi_1$$

$$B_{002} = -S\alpha_{17}\ S\alpha_{52}\ S\psi_1\ C\Phi_1 - S\Phi_1\ C\psi_1\ S\alpha_{17}\ S\alpha_{52}\ C\alpha_{21}$$

$$B_{001} = S\Phi_1\ S\alpha_{17}\ S\alpha_{52}\ S\psi_1 + C\psi_1\ C\Phi_1\ S\alpha_{17}\ S\alpha_{52}\ C\alpha_{21}$$

$$B_{000} = C\alpha_{17}\ S\alpha_{52}\ S\alpha_{21}\ C\psi_1$$

$$C_{002} = S\Phi_1\ S\alpha_{17}\ C\alpha_{52}\ S\alpha_{21}$$

$$C_{001} = C\Phi_1\ S\alpha_{17}\ C\alpha_{52}\ S\alpha_{21}$$

$$C_{000} = C\alpha_{17}\ C\alpha_{52}\ C\alpha_{21} - C\alpha_{76}\ C\alpha_{65} + S\alpha_{76}\ S\alpha_{65}\ C\theta_{6k}$$

$$M_{22} = B_{001} - B_{000} - C_{001} + C_{000}$$

$$M_{21} = -2\ B_{002} + 2\ C_{002}$$

$$M_{20} = -B_{001} - B_{000} + C_{001} + C_{000}$$

$$M_{12} = -2\ A_{001} + 2\ A_{000}$$

$$M_{11} = 4\ A_{002}$$

$$M_{10} = 2\ A_{001} + 2\ A_{000}$$

TABLE XIII (Continued)

$$M_{02} = -B_{001} + B_{000} - C_{001} + C_{000}$$

$$M_{01} = 2\,B_{002} + 2\,C_{002}$$

$$M_{00} = B_{001} + B_{000} + C_{001} + C_{000}$$

They should, therefore, have at least one root in common between them.

The condition using Sylvester dialytic eliminant then becomes

$$\begin{vmatrix} B_2(t_1) & B_1(t_1) & B_0(t_1) & 0 \\ 0 & B_2(t_1) & B_1(t_1) & B_0(t_1) \\ M_2(t_1) & M_1(t_1) & M_0(t_1) & 0 \\ 0 & M_2(t_1) & M_1(t_1) & M_0(t_1) \end{vmatrix} = 0 \qquad (4\text{-}54)$$

It should be noted that Eq. (4-54) is a function of only the variable t_1.

Expanding and simplifying the above equation, we get

$$R_8 t_1^8 + R_7 t_1^7 + \ldots + R_1 t_1 + R_0 = 0$$

or in short

$$\sum_{i=0}^{8} R_i t_1^i = 0 \qquad (4\text{-}55)$$

Equation (4-55) is exactly similar in form to Eq. (4-45). Its coefficients R_i (i = 0 to 8) can be obtained from the coefficients of Eq. (4-45) replacing the constants A_{ij} by M_{ij}.

Equation (4-55) must hold true at all values of the variable θ_1. Its coefficients must, therefore, vanish (102). Thus, we have

$$R_i = 0, \qquad i = 0, 1, 2, \ldots, 8 \qquad (4\text{-}56)$$

Condition (4-56) represents nine equations among the 17 constant kinematic parameters of the R-R-C-C-C-P-C mechanism in Figure 29 (namely, the four link lengths a_{12}, a_{23}, a_{34}, and a_{41}, the eight twist angles α_{12}, α_{23}, α_{41}, α_{52}, α_{76}, and α_{65}, the three constant displacement angles θ_{6k}, Φ_1, and ψ_1, and the two constant offset distances s_1 and s_{2k}). The nine equations provide the necessary conditions for the existence of a six-link, two-loop R-R-C-C-C-P-C mechanism with constant offset distances at the revolute pairs at A and B, and constant displacement angle at the prismatic pair at F.

On Obtaining R-R-C-C-C-P-C Mechanism From the Derived Criteria

The existence criteria obtained above can be utilized to obtain the constant kinematic parameters of an R-R-C-C-C-P-C mechanism with constant offset distance at revolute pair B and constant displacement angle at the prismatic pair at F.

Considering the constant kinematic parameters as unknowns, the 9 equations given by condition (4-56) can be represented as

$$F_i\,(x_1, x_2, \ldots, x_n) = 0 \qquad\qquad i = 1 \text{ to } 9$$

The above equation represents a system of nine consistent nonlinear equations in the 17 unknown constant kinematic parameters of the mechanism. However, the high nonlinearity of the equations once again emphasizes the complex nature of the investigation and shows that the presenting of simplified explicit expressions for direct computation of the mechanism parameters is a problem by itself.

Like Eqs. (4-47), the above equation also has trivial solutions. As in the case of the R-R-C-C-C-R-C mechanism, the triviality or non-triviality of a solution can be checked by substituting the values of the constant kinematic parameters in the original displacement relationship of the parent R-C-C-C-C-C-C mechanism (120). A non-trivial solution will give constant rotational displacement (θ_{6k}) at the cylinder pair F and constant translational displacement (s_{2k}) at the cylinder pair B, at all positions of the mechanism, without at the same time, affecting its true mobility.

In an effort to obtain an overconstrained mechanism (non-trivial solution) over one thousand sets of mechanism parameters (initial guess values for the computer program) were tried, but none yielded an R-R-C-C-C-P-C space mechanism. Perhaps the parameters of the overconstrained R-R-C-C-C-P-C mechanism lie in a very narrow band of range, and can be discovered only by an extensive search.

CHAPTER V

SUMMARY AND CONCLUSIONS

The present work is devoted to exploring the application of Dimentberg's passive coupling technique and studying existence criteria of single and multi-loop mechanisms. In this study, the existence criteria of overconstrained mechanisms with one general constraint and consisting of helical, revolute, cylinder and prismatic pairs have been obtained by using Dimentberg's passive coupling method. This represents the first attempt in using this method to single and two-loop, six-link mechanisms after its usefulness in the case of four-link mechanisms was first demonstrated by Dimentberg, five-link mechanisms by Soni and Pamidi.

The mechanisms considered in this study are the six-link, single-loop 3H+3P mechanisms, two-loop R-R-C-C-C-R-C, R-R-C-C-C-P-C mechanisms, two-loop R-R-C-C-C-R-C, R-R-C-C-C-P-C, R-C-C-R-C-C-R and R-C-C-R-C-C-P mechanisms. The results obtained in the case of single-loop 3H+3P mechanisms confirm the findings of other investigators. The existence criteria of the two-loop mechanisms obtained in the study are new.

The principal results of the investigation are as follows:

1. The existence criteria of the six-link 3H+3P mechanisms obtained in the study show that these mechanisms (and others obtained by extending the results) exist if and only if the axes of the helical (and/or revolute) pairs are parallel to one another. When the axes of the helical (and/or revolute) pairs are parallel it was found that these mechanisms will have two degrees of freedom. When one of the link lengths is taken to be zero, the results will apply with equal validity to five-link mechanisms derivable from the above six-link mechanisms. This confirms the results that were obtained by Hunt and Waldron by considering the H-H-H-H-H and H-H-H-H-H-H mechanisms of Voinea and Atanasiu; Soni, Pamidi, and Dukkipati by considering the H-C-H-C-H and H-C-C-H-H mechanisms. The results in the present study have, however, been obtained by considering the more general zero family mechanisms and give, besides the parallelism of the axes, the definite closure conditions to be satisfied by the constant kinematic parameters of the mechanism concerned.

2. The existence criteria of the six-link, two-loop R-R-C-C-C-R-C mechanism with one zero offset distance were obtained as a set of 9 nonlinear algebraic equations in the 20 constant kinematic parameters of the mechanism. The number of

independent equations, however, is suspected to be less than 9 because of the method of elimination used. The derived criteria make it possible to investigate the existence of R-R-C-C-C-R-C mechanism. The algebraic expressions describing the existence criteria of the mechanism are sufficiently complex to prevent from presenting any simplified geometric descriptions. In fact, the complexity extends far enough to prevent from presenting simplified explicit results in order to facilitate direct computations of the linkage parameters. A numerical technique based on direct search technique was proposed to solve for the parameters of the R-R-C-C-C-R-C mechanism. The proposed numerical technique is illustrated by presenting an illustrative example of an R-R-C-C-C-R-C overconstrained mechanism.

3. The existence criteria of the six-link, two-loop R-R-C-C-C-P-C mechanism are obtained as a set of nine nonlinear equations in the 17 constant kinematic parameters of the mechanism. These equations make it possible to investigate the existence of R-R-C-C-C-P-C mechanisms. However, the high nonlinearity of the equations once again emphasizes the complex nature of the investigation and shows that presenting simplified explicit expressions for direct computation of the linkage parameters is a problem by itself. Hence numerical approach

appears to be the only route. The proposed numerical technique is tried using the derived existence criteria to obtain a compatible set of constant kinematic parameters of the R-R-C-C-C-P-C mechanism, but none yielded a non-trivial solution.

The present study provides a general mathematical approach to obtain the existence criteria of six-link, single and two-loop space mechanisms for a variety of passive couplings and/or general constraints. All the required displacement relationships (see, for instance, Chapters III and IV) for obtaining the existence criteria of six-link mechanisms for a variety of passive coupling conditions are developed. The displacement relationships are derived in dual form. They are valid for six-link, single and two-loop parent mechanisms consisting of helical, revolute, prism and cylinder pairs.

By using the derived displacement relationships and Dimentberg's passive coupling method the existence criteria conditions for the following cases are also studied. (Appendixes A, B and C)

1. The existence criteria of the six-link, two-loop R-R-C-C-C-R-C mechanism with general proportions are shown to be a set of seventeen conditions among the twenty-one constant kinematic parameters of the mechanism.

2. The existence criteria of the six-link, two-loop R-C-C-R-C-C-R mechanism of general proportions are shown to be a set of

385 conditions among the 22 constant kinematic parameters of the mechanism.

3. The existence criteria of the six-link, two-loop R-C-C-R-C-C-P mechanism of general proportions are shown as a set of 65 conditions among the 22 constant kinematic parameters of the mechanism.

4. It was shown that, in an R-C-C-C-C-C-C six-link, two-loop space mechanism, when one cylinder pair in loop 1 is reduced to a prismatic pair, another cylinder pair in that loop will also reduce to a prismatic pair. This result agrees with that by Dimentberg (29) in the case of four-link, single-loop R-C-C-C mechanism. It was also shown that the existence criteria of the six-link, two-loop R-P-C-P-C-P-C and R-P-P-C-C-P-C mechanisms (Appendix C) requires the axes of the revolute and cylinder pairs in both loops parallel to each other and the axes of the prism pairs are randomly oriented.

Except in very simple cases, the solution of the derived existence criteria conditions can be regarded as a problem by itself. Thus, for instance, the existence criteria of the R-C-C-R-C-C-R mechanism (Appendix B) with general proportions are expected to lead to 385 conditions among the 22 constant kinematic parameters of the mechanism. It can be seen that errors are apt to be introduced if such high order and large number of equations are not carefully handled. Again, the examination of the resultant conditions in order to obtain a compatible set of constant kinematic parameters presents a task of formidable proportions.

The concept of general constraints in mobility criteria for single or multi-loop mechanisms suggests there are certain geometrical conditions which must be imposed on a kinematic chain if it is to have one degree of freedom. The exact nature of this general constraint is not completely known (121). The mobility criteria predicts only the possible existence of mechanisms under the classification of general constraints. The nature and significance of general constraints can be realized only when all the kinematic chains under the specific general constraint domain are virtually explored for mobility. This is possible when general mathematical models for each type and kind of mechanism (48) are developed in terms of all of its constant kinematic parameters. By studying the degenerate cases and by exploring relationships between all the basic parameters, we can identify the general constraint criteria for mobility. The present work is another attempt in achieving this objective. It is then possible to construct physical models of most of these mechanisms and identification of the geometric conditions which create the general constraints. The possible components of general motion under the concept of general constraints can then be identified. Thus, for instance, for the case of one general constraint the components of general motion can be either 3 rotations and 2 translations or 2 rotations and 3 translations.

A previous study on the existence criteria of single-loop over-constrained four and five-link mechanisms (29, 38, 39, 40, 27, 41, 122, 119) and also the present study on six-link, single and two-loop mechanisms reveals certain important points. These points are presented below:

1. When the displacement relationships involved are algebraic in nature the Dimentberg method ultimately leads to one or more polynomial equations. The complexity and the order of these polynomials can be reduced by considering the entire spectrum of loop equations available by arranging the loop closure condition in various ways rather than by considering just a few of the available equations.

2. The primary part of a dual equation contains only the primary parts of its component terms. The dual part of a dual equation, however, involves both the primary and the dual parts of its component terms. The dual part of any dual equation is, therefore, always more complicated than its primary part. When passive coupling is imposed on a cylinder pair to reduce it to a prism pair (Chapters II and III), restrictions are put on only the rotation at the C pair and thus one has to deal with the primary parts of the concerned displacement relationships.

But when passive coupling is imposed on a cylinder pair to reduce it to a revolute pair, restrictions are placed on only the translation (see, for instance, Chapter IV) at the cylinder pair and thus one has to deal with the dual parts of the concerned displacement relationships. Thus the analytical work involved in reducing a cylinder pair to a prismatic pair is always much less complicated than in reducing that cylinder pair to revolute pair.

3. When the displacement relationships are algebraic in nature, the Dimentberg method often involves examination of the common roots between two polynomials or successive sets of two polynomials. In such cases, it is necessary to consider only one common root between the equations involved. It is however possible to consider more than one common root between these equations. The resultant conditions, however, represent only special cases of the more general case obtained by considering only one common root. When two equations have more than one common root, it implies that they have at least one common root.

4. If the parent mechanism contains helical pair, the derived existence criteria remain less complicated in nature if only the rotations at the helical pairs are involved. Thus in the present study, the existence criteria of the two-loop

mechanisms are less complicated in nature because the parent mechanism considered do not have any helical pairs.

5. When the existence criteria involve twist angles and constant displacement angles they can generally expected to be simple. In such cases, it is possible to examine the relationship between the equations analytically. This is illustrated in the examples of Chapters II and III.

When the existence criteria involve link lengths, kink-lengths in addition to twist angles and constant displacement angles, it may then become difficult to examine the relationships between the constant kinematic parameters of the derived mechanism analytically. In such cases the suitable numerical method is to be used to solve for the parameters of the newly discovered overconstrained mechanism from the derived criteria.

6. The derived criteria represents only necessary conditions for existence of a mechanism considered. The conditions are not sufficient because the criteria does not by itself guarantee an overconstrained mechanism of the desired type. The criteria is expected to provide trivial solutions that give mechanisms without a true mobility of one. Trivial solutions can be one of two types:

(1) A solution becomes trivial if the constant kinematic parameters yield an overconstrained mechanism with mobility greater than one. (See, for instance, Chapter III)

(2) A solution becomes trivial if the constant kinematic parameters yield an overconstrained mechanism of a higher family, that is, an overconstrained mechanism having more than the required number of passive couplings. (See, for instance, Appendix C)

The triviality and non-triviality of a solution can be examined by substituting the values of the constant kinematic parameters in the original displacement relationships of the parent mechanism. If the mobility is two or more, the variable kinematic parameters in the parent mechanism become indeterminate unless 2 or more variables are specified.

A locked joint is indicated by the fact that a pair variable corresponding to that joint becomes constant. The case represents a non-trivial solution only when either of the above conditions is present and gives an overconstrained mechanism of the desired type with a true mobility of one.

Since trivial solutions always exist, the existence criteria obtained by the present method represents a set of consistent equations. But all the equations in the system (representing the conditions among the constant kinematic parameters) may not in general be independent. This is especially true when the number of unknowns in the equations is more or less than the number of equations. In such cases it may not be possible to examine the relationship between the parameters analytically.

Although the existence criteria obtained using Dimentberg's method is often complicated, the method has certain definite points in its favor. For example, it

a. provides necessary and sufficient conditions for the existence of overconstrained mechanisms;

b. assures finite mobility to the newly discovered overconstrained mechanisms;

c. shows clearly that, in general, the mobility of overconstrained mechanisms is a function of the twist angles, link lengths, constant displacement angles and the constant offset distances;

d. permits the computation of the mechanism proportions from the existence criteria;

e. permits the introduction of different forms of passive coupling conditions in kinematic pairs; and

f. enables one to obtain the closed form displacement relationships for the newly discovered mechanisms which can be utilized for their type determination, kinematic analysis and synthesis.

The present study shows that the mobility of space mechanisms is a field of continued interest and challenge. In the coming years, the following important areas of research appear to offer great promise:

1. The development of a unified method for determining the existence of multi-loop mechanisms. This unified method utilizes passive coupling technique to allow derivation of results algebraically and screw systems theory to allow determination of results geometrically so as to express the criteria as both necessary and sufficient conditions among the constant kinematic parameters of the overconstrained mechanism in explicit form.
2. Use of this unified method to formulate the necessary and sufficient existence conditions of multi-link, multi-loop mechanisms with one, two and three general constraints.
3. Examination of the types of motion displayed by these overconstrained mechanisms.
4. Practical applicabilities of newly discovered overconstrained mechanisms.
5. Investigation of mathematical functions for which these mechanisms are best suited for function generation, three-dimensional path generation and rigid body guidance.

Because of the nature of the problems, the proposed investigation is expected to deal with an unusually high level of algebra and geometry.

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APPENDIX A

EXISTENCE CRITERIA OF THE SIX-LINK R-R-C-C-C-R-C MECHANISM WITH NON-ZERO KINK-LINKS

This appendix deals with the calculations necessary to derive the existence criteria of the six-link, two-loop R-R-C-C-C-R-C mechanism with general proportions mentioned in Chapter IV.

Referring to Figures 27 and 30, the same equations (4-38) and (4-39) are written down. Now let the translations s_1, s_2 and s_6 be constant at all positions of the mechanism. Since s_6 does not appear in equation (4-38), equation (4-40) remains the same.

Separating equation (4-39) into primary and dual parts, with the aid of equations (4-20) through (4-22) and then eliminating the angle θ_6 from these primary and dual parts, we get an equation of the form

$$A_4 (t_1) t_2^4 + A_3 (t_1) t_2^3 + A_2 (t_1) t_2^2 + A_1 (t_1) t_2 + A_0 (t_1) = 0 \qquad \text{(A-1)}$$

where $t_1 = \tan (\theta_1/2)$

$t_2 = \tan (\theta_2/2)$

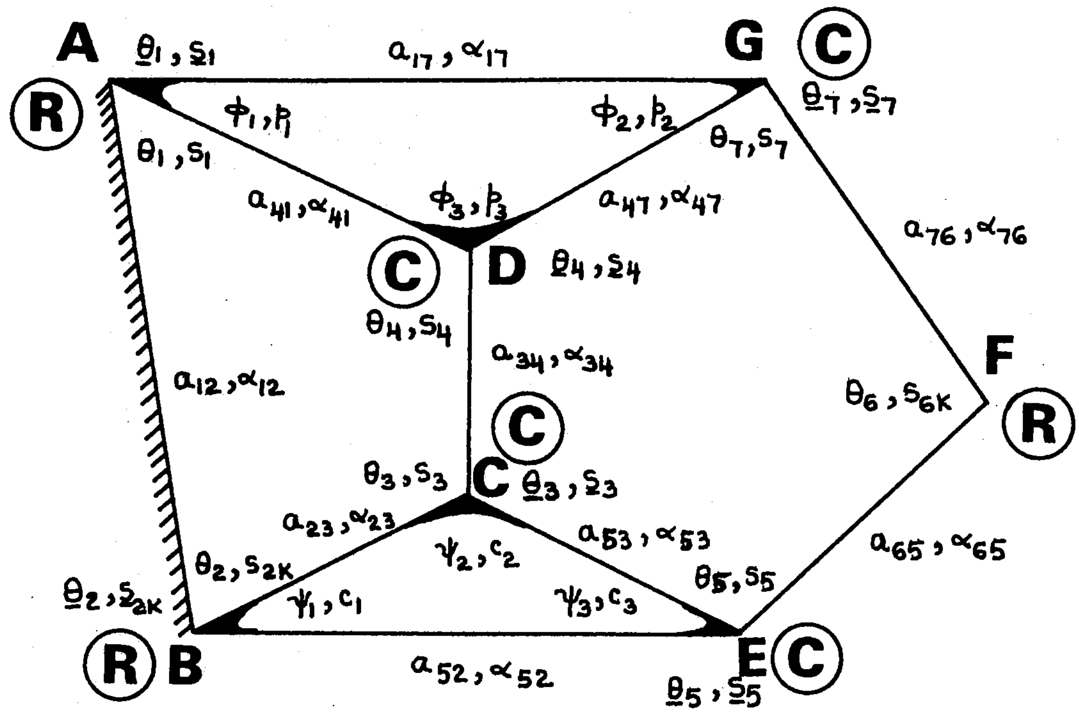

Figure 30. R-R-C-C-C-R-C Space Mechanism Obtained From the Mechanism in Figure 27 by Making $s_2 = s_{2k}$ = a Constant and $s_6 = s_{6k}$ = a Constant

and

$$A_i(t_1) = A_{i4}\, t_1^4 + A_{i3}\, t_1^3 + A_{i2}\, t_1^3 + A_{i2}\, t_1^2 + A_{i1}\, t_1 + A_{i0}$$

$$i = 0,\ 1,\ 2,\ 3,\ 4 \qquad \text{(A-2)}$$

The constants in equation (A-2) involve only the constant kinematic parameters of the mechanism in Figure 30. The equations (4-40) and (A-1) represent two different forms of displacement relationships for the same mechanism. They should, therefore, have at least one root in common between them. This gives the condition (102):

$$\begin{vmatrix} A_4(t_1) & A_3(t_1) & A_2(t_1) & A_1(t_1) & A_0(t_1) & 0 \\ 0 & A_4(t_1) & A_3(t_1) & A_2(t_1) & A_1(t_1) & A_0(t_1) \\ B_2(t_1) & B_1(t_1) & B_0(t_1) & 0 & 0 & 0 \\ 0 & B_2(t_1) & B_1(t_1) & B_0(t_1) & 0 & 0 \\ 0 & 0 & B_2(t_1) & B_1(t_1) & B_0(t_1) & 0 \\ 0 & 0 & 0 & B_2(t_1) & B_1(t_1) & B_0(t_1) \end{vmatrix} = 0 \qquad \text{(A-3)}$$

Equation (A-3) is a function of only the variable t_1. Expanding and simplifying it, we get

$$E_{16}\, t_1^{16} + E_{15}\, t_1^{15} + \ldots + E_1\, t_1 + E_0 = 0$$

or in short,

$$\sum_{i=0}^{16} E_i t_1^i = 0 \qquad \text{(A-4)}$$

Equation (A-4) consists of only the variable t_1 (or θ_1) describing the position of the mechanism in Figure 30 and must be satisfied at all positions of that mechanism. This equation must hold good at all values of the variable t_1. Thus, equating the coefficients to zero, we have,

$$E_i = 0 \qquad i = 0, 1, 2, \ldots, 16 \qquad \text{(A-5)}$$

Condition (A-5) represents seventeen equations among the twenty-one constant kinematic parameters of the mechanism in Figure 30 (namely, the eight link lengths a_{76}, a_{65}, a_{52}, a_{17}, a_{34}, a_{41}, a_{23} and a_{12}; the eight twist angles α_{76}, α_{65}, α_{52}, α_{17}, α_{34}, α_{41}, α_{23} and α_{12}; the three constant offset distances s_1, s_{2k} and s_{6k} of the revolute pairs at A, B, and F; and the two constant displacement angles Φ_1 and ψ_1 at the two ternary links at joints A and B). These seventeen equations provide the necessary conditions for the existence of an R-R-C-C-C-R-C mechanism with general proportions.

APPENDIX B

EXISTENCE CRITERIA OF THE SIX-LINK R-C-C-R-C-C-R AND R-C-C-R-C-C-P MECHANISMS

This appendix deals with the procedure for obtaining the existence criteria of six-link, two-loop R-C-C-R-C-C-R, R-C-C-R-C-C-P mechanisms with general proportions from the displacement relationships of the parent R-C-C-C-C-C-C mechanism mentioned in Chapter IV.

Existence Criteria of the Six-Link R-C-C-R-C-C-R Mechanism

Consider the R-C-C-C-C-C-C mechanism shown schematically in Figure 27. This mechanism reduces to an R-C-C-R-C-C-R mechanism if the translational displacements s_4 and s_7 of the cylinder pairs at D and G are forced to be constant at all positions of the mechanism (Figure 31).

By considering the loop-closure condition of the mechanism in Figure 27 for loop 1 (ABCDA) and outer loop (ABEFGA), the following dual relationships can be obtained:

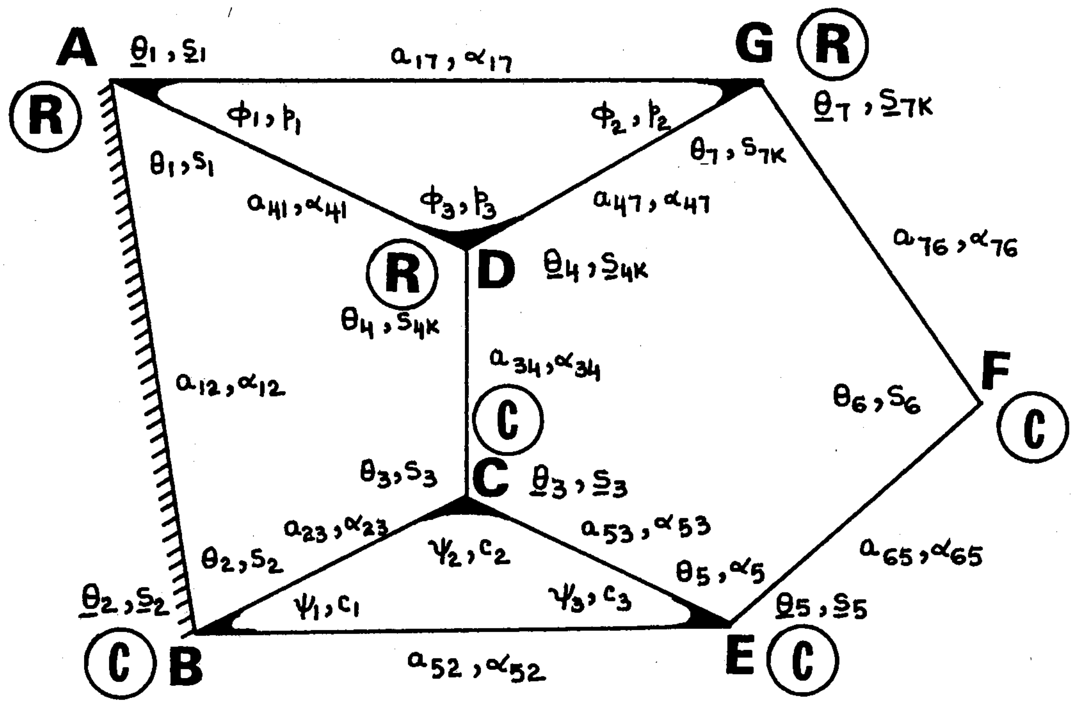

Figure 31. R-C-C-R-C-C-R Space Mechanism Obtained From the Mechanism in Figure 27 by Making $s_4 = s_{4k} =$ a Constant and $s_7 = s_{7k} =$ a Constant

$$F_1(\hat{\theta}_1, \hat{\theta}_2) = (S\hat{\alpha}_{23}\, S\hat{\alpha}_{41}\, S\hat{\theta}_2)\, S\hat{\theta}_1 - [S\hat{\alpha}_{41}\,(S\hat{\alpha}_{12}\, C\hat{\alpha}_{23} + C\hat{\alpha}_{12}\, S\hat{\alpha}_{23}\, C\hat{\theta}_2)]\, C\hat{\theta}_1 - C\hat{\alpha}_{34} + C\hat{\alpha}_{41}\,(C\hat{\alpha}_{12}\, C\hat{\alpha}_{23} - S\hat{\alpha}_{12}\, S\hat{\alpha}_{23}\, C\hat{\theta}_2) = 0 \qquad \text{(B-1)}$$

$$f_1(\hat{\theta}_2, \hat{\theta}_4) = C\hat{\alpha}_{14}\, C\hat{\alpha}_{43} + S\hat{\alpha}_{14}\, S\hat{\alpha}_{43}\, C\hat{\theta}_4 - C\hat{\alpha}_{32}\, C\hat{\alpha}_{21} - S\hat{\alpha}_{32}\, S\hat{\alpha}_{21}\, C\hat{\theta}_2 = 0 \qquad \text{(B-2)}$$

$$f_3(\hat{\underline{\theta}}_1, \hat{\underline{\theta}}_2, \hat{\underline{\theta}}_7) = [(S\hat{\alpha}_{17}\, C\hat{\alpha}_{76} + C\hat{\alpha}_{17}\, S\hat{\alpha}_{76}\, C\hat{\underline{\theta}}_7)\, S\hat{\underline{\theta}}_1 + S\hat{\alpha}_{76}\, S\hat{\underline{\theta}}_7\, C\hat{\underline{\theta}}_1]\,(S\hat{\alpha}_{52}\, S\hat{\underline{\theta}}_2) + [S\hat{\alpha}_{76}\, S\hat{\underline{\theta}}_7\, S\hat{\underline{\theta}}_1 - (S\hat{\alpha}_{17}\, C\hat{\alpha}_{76} + C\hat{\alpha}_{17}\, S\hat{\alpha}_{76}\, C\hat{\underline{\theta}}_7)\, C\hat{\underline{\theta}}_1]\,(C\hat{\alpha}_{52}\, S\hat{\alpha}_{21} + S\hat{\alpha}_{52}\, C\hat{\alpha}_{21}\, C\hat{\underline{\theta}}_2) + (C\hat{\alpha}_{17}\, C\hat{\alpha}_{76} - S\hat{\alpha}_{17}\, S\hat{\alpha}_{76}\, C\hat{\underline{\theta}}_7)\,(C\hat{\alpha}_{52}\, C\hat{\alpha}_{21} - S\hat{\alpha}_{52}\, S\hat{\alpha}_{21}\, C\hat{\underline{\theta}}_2) - C\hat{\alpha}_{65} = 0 \qquad \text{(B-3)}$$

Let the translational displacements s_4 and s_7 be now made constant for varying values of θ_1. Denoting the constant values of s_4 and s_7 by s_{4k} and s_{7k} respectively, and eliminating the angle θ_7 from the primary and dual parts of Equation (B-3), with the aid of equations (4-20) through (4-22), a polynomial of the form

$$\sum_{m,n=0}^{8} p_{mnj}\, t_1^m\, t_2^n\, s_2^j = 0 \qquad \text{(B-4)}$$

for $j = 0, 1, 2, 3, 4$

can be obtained, in which

$t_1 = \tan(\theta_1/2)$

$$t_2 = \tan(\theta_2/_2)$$

and

$$p_{mnj} = p_{mnj}(a_{\ell k}, \alpha_{\ell k}, s_1, s_{7k}, \Phi_1, \Phi_2, \psi_1)$$

for $\ell k = 17, 76, 65, 52, 21$ (B-5)

Similarly, by eliminating the angle θ_2 from the primary and dual parts of equation (B-1), a polynomial of the form

$$\sum_{m=0}^{8} q_{mj} t_1^m s_2^j = 0 \quad \text{(B-6)}$$

for $j = 0, 1, 2, 3, 4$

can be obtained, in which

$$q_{mj} = q_{mj}(a_{\ell k}, \alpha_{\ell k}, s_1) \quad \text{(B-7)}$$

for $\ell k = 23, 41, 12, 34$

Also eliminating the angle θ_4 from the primary and dual parts of equation (B-2), a polynomial of the form

$$\sum_{m=0}^{4} R_{mj} t_2^m s_2^j = 0 \quad \text{(B-8)}$$

for $j = 0, 1, 2$

can be obtained, in which

$$R_{mj} = R_{mj}(A_{\ell k}, \alpha_{\ell k}, s_{4k}) \quad \text{(B-9)}$$

for $\ell k = 41, 34, 23, 12$

Eliminating t_2, between equations (B-4) and (B-8) by Sylvester dialytic method (102),

$$\begin{vmatrix} U_0 & U_1 & U_2 & U_3 & U_4 & U_5 & U_6 & U_7 & U_8 & 0 & 0 & 0 \\ 0 & U_0 & U_1 & U_2 & U_3 & U_4 & U_5 & U_6 & U_7 & U_8 & 0 & 0 \\ 0 & 0 & U_0 & U_1 & U_2 & U_3 & U_4 & U_5 & U_6 & U_7 & U_8 & 0 \\ 0 & 0 & 0 & U_0 & U_1 & U_2 & U_3 & U_4 & U_5 & U_6 & U_7 & U_8 \\ V_0 & V_1 & V_2 & V_3 & V_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & V_0 & V_1 & V_2 & V_3 & V_4 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & V_0 & V_1 & V_2 & V_3 & V_4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & V_0 & V_1 & V_2 & V_3 & V_4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & V_0 & V_1 & V_2 & V_3 & V_4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & V_0 & V_1 & V_2 & V_3 & V_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & V_0 & V_1 & V_2 & V_3 & V_4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & V_0 & V_1 & V_2 & V_3 & V_4 \end{vmatrix} = 0 \qquad \text{(B-10)}$$

in which

$$U_n = \sum_{m=0}^{8} p_{mnj} \, t_1^m \, s_2^j \qquad \text{(B-11)}$$

$$V_n = \sum_{m=0}^{2} R_{mj} \, s_2{}^m \qquad \text{(B-12)}$$

Expanding and simplifying equation (B-10), a polynomial of the form,

$$\sum_{m,n=0}^{32} B_{mn} t_1^m s_2^n = 0 \qquad \text{(B-13)}$$

can be obtained, in which

$$B_{mn} = B_{mn} (a_{\ell k}, \alpha_{\ell k}, s_1, \Phi_1, \Phi_2, \psi_1, s_{4k}, s_{7k}) \qquad \text{(B-14)}$$

for ℓk = 12, 23, 34, 41, 17, 76, 52, 65

Eliminate s_2, between equations (B-6) and (B-13) by Sylvester dialytic method. The result will be a determinant of 36th order and hence the diagonal term of the determinant is of the order of 32(8) + 4(32) (= 384) in the half tangent of the input angle θ_1, or symbolically,

$$\sum_{m=0}^{384} W_m t_1^m = 0 \qquad \text{(B-15)}$$

in which

$$W_m = W_m (a_{\ell k}, \alpha_{\ell k}, \Phi_1, \Phi_2, \psi_1, s_1, s_{4k}, s_{7k}) \qquad \text{(B-16)}$$

and ℓk = 12, 23, 34, 41, 17, 76, 65, 52

Equation (B-15) is a function of only the variable θ_1. This equation must hold true at all values of the variable angle θ_1. Hence equating the coefficients of equation (B-15) to zero, gives

$$W_m = 0 \qquad m = 0, 1, 2, \ldots, 384 \qquad \text{(B-17)}$$

Condition (B-17) represents 385 equations among the 22 constant kinematic parameters of the mechanism in Figure 31 (namely the eight link lengths a_{12}, a_{23}, a_{34}, a_{41}, a_{17}, a_{76}, a_{65}, and a_{52}; the eight twist angles α_{12}, α_{23}, α_{34}, α_{41}, α_{17}, α_{76}, α_{65}, and α_{52}; and the three kink-links s_1, s_{4k} and s_{7k} and the three constant displacement angles Φ_1, ψ_1, and Φ_2). These 385 equations provide the necessary conditions for the existence of an R-C-C-R-C-C-R mechanism with general proportions.

Existence Criteria of the Six-Link R-C-C-R-C-C-P Mechanism

The existence criteria of an R-C-C-R-C-C-P space mechanism can be obtained from the displacement relationships of the R-C-C-C-C-C-C space mechanism. The R-C-C-C-C-C-C mechanism in Figure 27 reduces to an R-C-C-R-C-C-P mechanism, if the rotational displacement θ_7 and the translational displacement s_4 of the cylinder pairs at G and D respectively are forced to be constant at all positions of the mechanism (Figure 32).

The existence criteria of this mechanism can be obtained in the same manner as that of the R-C-C-R-C-C-R mechanism. It can be shown that the number of conditions for this mechanism are lower than that of the R-C-C-R-C-C-R mechanism, because the variable angle θ_7, which has to be eliminated, is kept constant in the present case.

From the primary part of equation (B-3), a polynomial of the form,

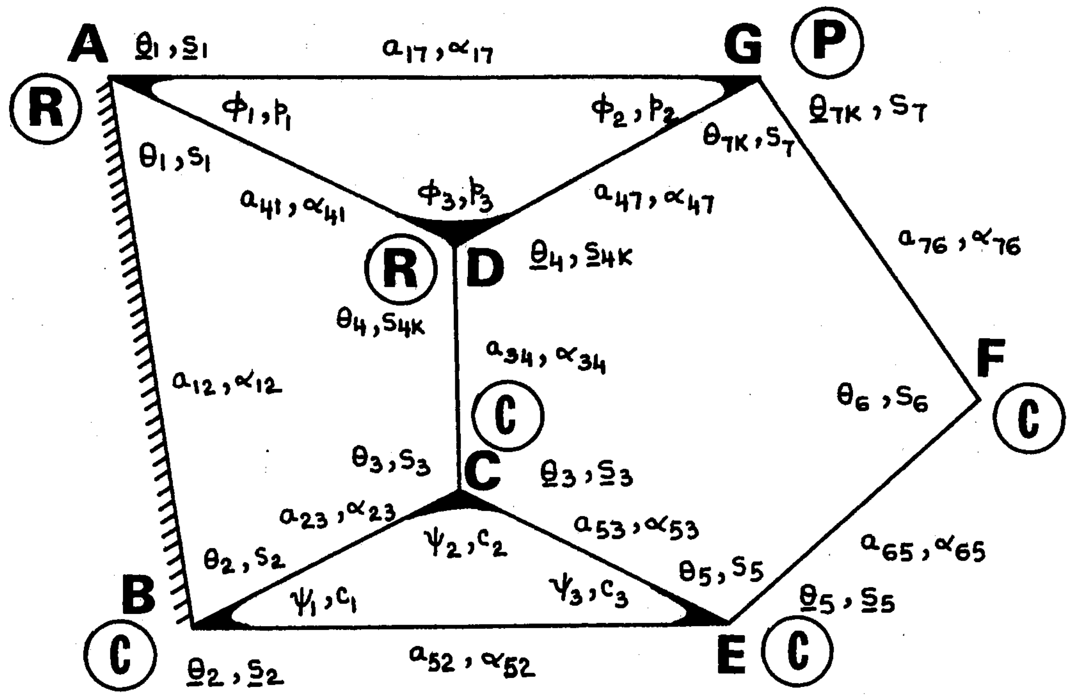

Figure 32. R-C-C-R-C-C-P Space Mechanism Obtained From the Mechanism in Figure 27 by Making $s_4 = s_{4k} =$ a Constant and $\theta_7 = \theta_{7k} =$ a Constant

$$\sum_{m,n=0}^{2} M_{mn} t_1^m t_2^m = 0 \tag{B-18}$$

can be obtained, in which

$$M_{mn} = M_{mn} (\alpha_{ij}, \Phi_1, \psi_1, \Phi_2, s_1, \theta_{7k})$$

$$ij = 17, 76, 52, 21, 65$$

and t_1 and t_2 are the same as in equations (B-5). Equations (B-6) and (B-8) remain unchanged for this mechanism since these equations do not involve θ_7 or s_7.

Eliminate θ_2 between equations (B-18) and (B-8) by Sylvester's dialytic method,

$$\begin{vmatrix} U_0 & U_1 & U_2 & 0 & 0 & 0 \\ 0 & U_0 & U_1 & U_2 & 0 & 0 \\ 0 & 0 & U_0 & U_1 & U_2 & 0 \\ 0 & 0 & 0 & U_0 & U_1 & U_2 \\ V_0 & V_1 & V_2 & V_3 & V_4 & 0 \\ 0 & V_0 & V_1 & V_2 & V_3 & V_4 \end{vmatrix} = 0 \tag{B-19}$$

in which

$$U_n = \sum_{m=0}^{2} M_{mn} t_1^m$$

and V_n is the same as in equation (B-12).

Expanding and simplifying equation (B-19), another polynomial of the form

$$\sum_{m=0}^{8} N_m \; t_1^m \; s_2^j \qquad j = 0 \text{ to } 4 \tag{B-20}$$

can be obtained, in which

$$N_m = N_m \,(a_{\ell k}, \alpha_{\ell k}, s_{4k}, s_1, \theta_{7k}, \Phi_1, \psi_1, \Phi_2) \tag{B-21}$$

for $\ell k = 17, 76, 52, 21, 65, 41, 34, 23.$

The polynomial equation in one variable θ_1 can be obtained by eliminating s_2 between equations (B-20) and (B-6) by the Sylvester dialytic method. The result will be a determinant of 8th order in which each diagonal element is a polynomial of 8th order in t_1. Hence the diagonal term of the determinant is of the order of 8 x 8 (= 64) in the half-tangent of the input angle θ_1, namely

$$\sum_{j=0}^{64} P_j \, t_1^j = 0 \tag{B-22}$$

where

$$P_j = P_j \,(a_{\ell k}, \alpha_{\ell k}, s_1, s_{4k}, \theta_{7k}, \Phi_1, \Phi_2, \psi_1) \tag{B-23}$$

for $\ell k = 17, 76, 52, 21, 65, 41, 34, 23.$

The above equation (B-22) must be valid for varying values of the variable t_1. Its coefficients must, therefore, vanish. This gives

$$P_j = 0 \qquad j = 0, 1, 2, \ldots, 64 \tag{B-24}$$

Condition (B-24) represents 65 equations among the 22 constant kinematic parameters of the mechanism in Figure 32, namely (the eight link lengths a_{17}, a_{76}, a_{65}, a_{52}, a_{21}, a_{41}, a_{34}, and a_{23}; the the eight twist angles α_{17}, α_{76}, α_{65}, α_{52}, α_{21}, α_{41}, α_{34} and α_{23}; the four constant displacement angles Φ_1, Φ_2, ψ_1 and ψ_2; and the two constant offset distances (kink-links) s_1 and s_{4k}).

These 65 equations provide the necessary conditions for the existence of an R-C-C-R-C-C-P mechanism with general proportions.

APPENDIX C

EXISTENCE CRITERIA OF THE SIX-LINK R-P-C-P-C-P-C, R-P-P-C-C-P-C MECHANISMS

In this appendix, Dimentberg's passive coupling technique has been employed to obtain the existence criteria of the six-link, two-loop R-P-C-P-C-P-C and R-P-P-C-C-P-C space mechanisms. These criteria are obtained by considering only the primary parts of the displacement relationships of the six-link, two-loop R-C-C-C-C-C-C space mechanism. They, therefore, lead to conditions on only the twist angles and constant displacement angles of the mechanism considered and are independent of their link lengths and constant offset distances.

Derivation of the Existence Criteria

The existence criteria of the R-P-C-P-C-P-C, and R-P-P-C-C-P-C mechanisms can be obtained from the displacement relationships of an R-C-C-C-C-C-C mechanism.

Consider the R-C-C-C-C-C-C space mechanism shown schematically in Figure 27. By suppressing the rotational freedom of the cylinder pairs at the joints B and F, it is possible to examine the conditions for the existence of two prismatic pairs in this mechanism at all positions of the mechanism.

By considering the loop-closure condition of the mechanism in Figure 27 for loop 1 (ABCDA) and outer loop (ABEFGA) in three different ways, the following dual displacement relationships can be obtained.

$$f_1 (\hat{\theta}_4, \hat{\theta}_2) = C\hat{\alpha}_{41} C\hat{\alpha}_{34} + S\hat{\alpha}_{41} S\hat{\alpha}_{34} C\hat{\theta}_4 - C\hat{\alpha}_{23} C\hat{\alpha}_{12} - S\hat{\alpha}_{23} S\hat{\alpha}_{12} C\hat{\theta}_2 = 0 \qquad \text{(C-1)}$$

$$F_1 (\hat{\theta}_2, \hat{\theta}_3) = (S\hat{\alpha}_{34} S\hat{\alpha}_{12} S\hat{\theta}_3) S\hat{\theta}_2 - [S\hat{\alpha}_{12} C\hat{\alpha}_{34} + C\hat{\alpha}_{23} S\hat{\alpha}_{34} C\hat{\theta}_3)] C\hat{\theta}_2 - C\hat{\alpha}_{41} + C\hat{\alpha}_{12} (C\hat{\alpha}_{23} C\hat{\alpha}_{34} - S\hat{\alpha}_{23} S\hat{\alpha}_{34} C\hat{\theta}_3) = 0 \qquad \text{(C-2)}$$

$$F_1 (\hat{\theta}_1, \hat{\theta}_2) = (S\hat{\alpha}_{23} S\hat{\alpha}_{41} S\hat{\theta}_2) S\hat{\theta}_1 - [S\hat{\alpha}_{41} (S\hat{\alpha}_{12} C\hat{\alpha}_{23} + C\hat{\alpha}_{12} S\hat{\alpha}_{23} C\hat{\theta}_2)] C\hat{\theta}_1 - C\hat{\alpha}_{34} + C\hat{\alpha}_{41} (C\hat{\alpha}_{12} C\hat{\alpha}_{23} - S\hat{\alpha}_{12} S\hat{\alpha}_{23} C\hat{\theta}_2) = 0 \qquad \text{(C-3)}$$

$$F_3 (\underline{\hat{\theta}}_2, \hat{\theta}_6, \hat{\theta}_1) = (S\hat{\alpha}_{17} S\hat{\alpha}_{52} S\underline{\hat{\theta}}_2) S\underline{\hat{\theta}}_1 - S\hat{\alpha}_{17} (C\hat{\alpha}_{52} S\hat{\alpha}_{21} + S\hat{\alpha}_{52} C\hat{\alpha}_{21} C\underline{\hat{\theta}}_2) C\underline{\hat{\theta}}_1 + C\hat{\alpha}_{17} (C\hat{\alpha}_{52} C\hat{\alpha}_{21} - S\hat{\alpha}_{52} S\hat{\alpha}_{21} C\underline{\hat{\theta}}_2) - (C\hat{\alpha}_{76} C\hat{\alpha}_{65} - S\hat{\alpha}_{76} S\hat{\alpha}_{65} C\hat{\theta}_6) = 0 \qquad \text{(C-4)}$$

$$f_3(\underline{\hat{\theta}}_2, \underline{\hat{\theta}}_6, \underline{\hat{\theta}}_5) = [(S\hat{\alpha}_{52}\, C\hat{\alpha}_{21} + C\hat{\alpha}_{52}\, S\hat{\alpha}_{21}\, C\underline{\hat{\theta}}_2)\, S\underline{\hat{\theta}}_5$$
$$+ S\hat{\alpha}_{21}\, S\underline{\hat{\theta}}_2\, C\underline{\hat{\theta}}_5]\,(S\hat{\alpha}_{76}\, S\hat{\theta}_6) + [S\hat{\alpha}_{21}\, S\underline{\hat{\theta}}_2\, S\underline{\hat{\theta}}_5$$
$$- (S\hat{\alpha}_{52}\, C\hat{\alpha}_{21} + C\hat{\alpha}_{52}\, S\hat{\alpha}_{21}\, C\underline{\hat{\theta}}_2)\, C\underline{\hat{\theta}}_5]\,(C\hat{\alpha}_{76}\, S\hat{\alpha}_{65}$$
$$+ S\hat{\alpha}_{76}\, C\hat{\alpha}_{65}\, C\hat{\theta}_6) + (C\hat{\alpha}_{52}\, C\hat{\alpha}_{21} - S\hat{\alpha}_{52}\, S\hat{\alpha}_{21}\, C\underline{\hat{\theta}}_2)$$
$$(C\hat{\alpha}_{76}\, C\hat{\alpha}_{65} - S\hat{\alpha}_{76}\, S\hat{\alpha}_{65}\, C\hat{\theta}_6) - C\hat{\alpha}_{17} = 0 \qquad \text{(C-5)}$$

$$F_3(\underline{\hat{\theta}}_2, \hat{\theta}_6, \underline{\hat{\theta}}_7) = (S\hat{\alpha}_{17}\, S\hat{\alpha}_{65}\, S\underline{\hat{\theta}}_7)\, S\hat{\theta}_6 - S\hat{\alpha}_{65}\,(C\hat{\alpha}_{17}\, S\hat{\alpha}_{76}$$
$$+ S\hat{\alpha}_{17}\, C\hat{\alpha}_{76}\, C\underline{\hat{\theta}}_7)\, C\hat{\theta}_6 + C\hat{\alpha}_{65}\,(C\hat{\alpha}_{17}\, C\hat{\alpha}_{76}$$
$$- S\hat{\alpha}_{17}\, S\hat{\alpha}_{76}\, C\underline{\hat{\theta}}_7) - (C\hat{\alpha}_{52}\, C\hat{\alpha}_{21} - S\hat{\alpha}_{52}\, S\hat{\alpha}_{21}\, C\underline{\hat{\theta}}_2) = 0$$
$$\text{(C-6)}$$

Observe that equations (C-2) and (C-3) are similar in form to equation (4-26), equations (C-4) and (C-6) are similar in form to equation (4-35), and equations (C-5) and (C-1) are similar in form to equations (4-37) and (4-28) respectively.

Note that each of the equations (C-1) through (C-3) relates the dual displacement angle $\hat{\theta}_2$ to a second dual displacement angle, and equations (C-4) through (C-6) relates the dual displacement angles $\hat{\theta}_2$ and $\hat{\theta}_6$ to a third dual displacement angle.

Let the displacement angles θ_2 and θ_6 at the cylinder pairs at B and F be now made constant at all positions of the mechanism. Denoting the constant values of θ_2 and θ_6 by θ_{2k} and θ_{6k} respectively, the primary parts of equations (C-1) through (C-6) give respectively,

$$A_c\, C\theta_4 + A_n = 0 \tag{C-7}$$

$$B_s\, S\theta_3 + B_c\, C\theta_3 + B_n = 0 \tag{C-8}$$

$$C_s\, S\theta_1 + C_c\, C\theta_1 + C_n = 0 \tag{C-9}$$

$$D_s\, S\underline{\theta}_1 + D_c\, C\underline{\theta}_1 + D_n = 0 \tag{C-10}$$

$$E_s\, S\underline{\theta}_5 + E_c\, C\underline{\theta}_5 + E_n = 0 \tag{C-11}$$

and

$$F_s\, S\underline{\theta}_7 + F_c\, C\underline{\theta}_7 + F_n = 0 \tag{C-12}$$

The constants in equations (C-7) through (C-12) involve only the constant kinematic parameters of the mechanism and are defined in Table XIV.

Note that each of the equations (C-7) through (C-12) contains only one variable and must be valid at varying values of that variable. This is possible only if their coefficients vanish. This gives

$$\begin{aligned}
&A_c = A_n = 0\\
&B_s = B_c = B_n = 0\\
&C_s = C_c = C_n = 0\\
&D_s = D_c = D_n = 0\\
&E_s = E_c = E_n = 0\\
\text{and}\quad &F_s = F_c = F_n = 0
\end{aligned} \tag{C-13}$$

Examination of equations (C-13) shows that the following cases are possible.

TABLE XIV

CONSTANTS FOR USE IN EQUATIONS (C-7) THROUGH (C-13)

$A_c = S\alpha_{41}\,S\alpha_{34}$

$A_n = C\alpha_{41}\,C\alpha_{34} - C\alpha_{23}\,C\alpha_{12} - S\alpha_{23}\,S\alpha_{12}\,C\theta_{2k}$

$B_s = S\alpha_{34}\,S\alpha_{12}\,S\theta_{2k}$

$B_c = -C\alpha_{12}\,S\alpha_{23}\,S\alpha_{34} - C\theta_{2k}\,S\alpha_{12}\,C\alpha_{23}\,S\alpha_{34}$

$B_n = -C\theta_{2k}\,[S\alpha_{12}\,(S\alpha_{23}\,C\alpha_{34})] - C\alpha_{41} + C\alpha_{12}\,C\alpha_{23}\,C\alpha_{34}$

$C_s = S\alpha_{23}\,S\alpha_{41}\,S\theta_{2k}$

$C_c = -\,[S\alpha_{41}\,(S\alpha_{12}\,C\alpha_{23} + C\alpha_{12}\,S\alpha_{23}\,C\theta_{2k})]$

$C_n = -C\alpha_{34} + C\alpha_{41}\,(C\alpha_{12}\,C\alpha_{23} - S\alpha_{12}\,S\alpha_{23}\,C\theta_{2k})$

$D_s = S\alpha_{17}\,S\alpha_{52}\,S\underline{\theta}_{2k}$

$D_c = -S\alpha_{17}\,(C\alpha_{52}\,S\alpha_{21} + S\alpha_{52}\,C\alpha_{21}\,C\theta_{2k})$

$D_n = C\alpha_{17}\,(C\alpha_{52}\,C\alpha_{21} - S\alpha_{52}\,S\alpha_{21}\,C\underline{\theta}_{2k}) - (C\alpha_{76}\,C\alpha_{65} - S\alpha_{76}\,S\alpha_{65}\,C\theta_{6k})$

$E_s = S\alpha_{76}\,S\underline{\theta}_{6k}\,(S\alpha_{52}\,C\alpha_{21} + C\alpha_{52}\,S\alpha_{21}\,C\underline{\theta}_{2k}) + S\alpha_{21}\,S\underline{\theta}_{2k}\,(C\alpha_{76}\,S\alpha_{65}$

$\qquad + S\alpha_{76}\,C\alpha_{65}\,C\theta_{6k})$

$E_c = S\alpha_{76}\,S\theta_{6k}\,(S\alpha_{21}\,S\underline{\theta}_{2k}) + (C\alpha_{76}\,S\alpha_{65} + S\alpha_{76}\,C\alpha_{65}\,C\theta_{6k})$

$\qquad - (S\alpha_{52}\,C\alpha_{21} + C\alpha_{52}\,S\alpha_{21}\,C\underline{\theta}_{2k})$

$E_n = (C\alpha_{52}\,C\alpha_{21} - S\alpha_{52}\,S\alpha_{21}\,C\underline{\theta}_{2k})(C\alpha_{76}\,C\alpha_{65} - S\alpha_{76}\,S\alpha_{65}\,C\theta_{6k}) - C\alpha_{17}$

TABLE XIV (Continued)

$$F_s = S\alpha_{17}\, S\alpha_{65}\, S\theta_{6k}$$

$$F_c = -S\alpha_{65}\, C\theta_{6k}\, S\alpha_{17}\, C\alpha_{76} - C\alpha_{65}\, S\alpha_{17}\, S\alpha_{76}$$

$$F_n = -S\alpha_{65}\, C\alpha_{17}\, S\alpha_{76}\, C\theta_{6k} + C\alpha_{6k}\, C\alpha_{17}\, C\alpha_{76} - C\alpha_{52}\, C\alpha_{21} + S\alpha_{52}\, S\alpha_{21}\, C\underline{\theta}_{2k}$$

1. $C\theta_{6k} < |1|$, $C\theta_{2k} < |1|$ (That is, $\theta_{6k} \neq m\pi$, $\theta_{2k} \neq m\pi$, $m = 0, 1, 2, \ldots$).

The only real solution possible in this case is given by

$$\begin{aligned} &\alpha_{12} = \alpha_{23} = \alpha_{34} = \alpha_{41} = 0 \\ &\alpha_{76} = \alpha_{65} = \alpha_{52} = \alpha_{17} = 0 \end{aligned} \tag{C-14}$$

Equation (C-14) shows that the kinematic axes are all parallel to each other. An R-P-C-C-C-P-C mechanism satisfying this condition, however, represents only a trivial solution since it yields a planar configuration in which the revolute and cylinder pairs remain locked.

2. $C\theta_{6k} = |1|$, $C\theta_{2k} = |1|$ (That is, $\theta_{6k} = m\pi$, $\theta_{2k} = m\pi$, $m = 0, 1, 2, \ldots$).

This gives

$$\alpha_{12} + \alpha_{23} = n\pi$$

$$\alpha_{41} \pm \alpha_{34} = n\pi$$

$$\alpha_{17} = 0$$

$$\alpha_{76} \pm \alpha_{65} = n\pi$$

and $\alpha_{52} \pm \alpha_{21} = n\pi$

for $n = 0, 1, 2, \ldots$ (C-15)

3. $C\theta_{6k} < |1|$, $C\theta_{2k} = |1|$ (That is, $\theta_{6k} \neq m\pi$, $\theta_{2k} = m\pi$, $m = 0, 1, 2, \ldots$).

This gives

$$\alpha_{12} - \alpha_{23} = n\pi$$

$$\alpha_{41} \pm \alpha_{34} = n\pi$$

$$\alpha_{76} = \alpha_{65} = \alpha_{17} = 0 \qquad \text{(C-16)}$$

and $\alpha_{52} \pm \alpha_{21} = n\pi$

4. $C\theta_{6k} = |1|$, $C\theta_{2k} < |1|$ (That is, $\theta_{6k} = m\pi$, $\theta_{2k} \neq m\pi$, $m = 0, 1, 2, \ldots$).

This gives

$$\alpha_{41} = 0 \text{ or } \pi$$

$$S\alpha_{12}\, S\alpha_{23}\, C\theta_{2k} - C\alpha_{12}\, C\alpha_{23} \pm C\alpha_{34} = 0$$

$$\alpha_{17} = 0$$

$$\alpha_{76} \pm \alpha_{65} = m\pi$$

$$\text{for } m = 0, 1, 2, \ldots \qquad \text{(C-17)}$$

Substitution of the relations given by equations (C-15) and (C-16) in the displacement equations of the parent R-C-C-C-C-C-C mechanism (120) show that cases 2 and 3 give a prismatic pair at joint D in addition to prismatic pairs at joints B and F. These solutions, therefore, given an R-P-C-P-C-P-C mechanism (Figure 33). They also show that the axes of the revolute and cylinder pairs are parallel to each other.

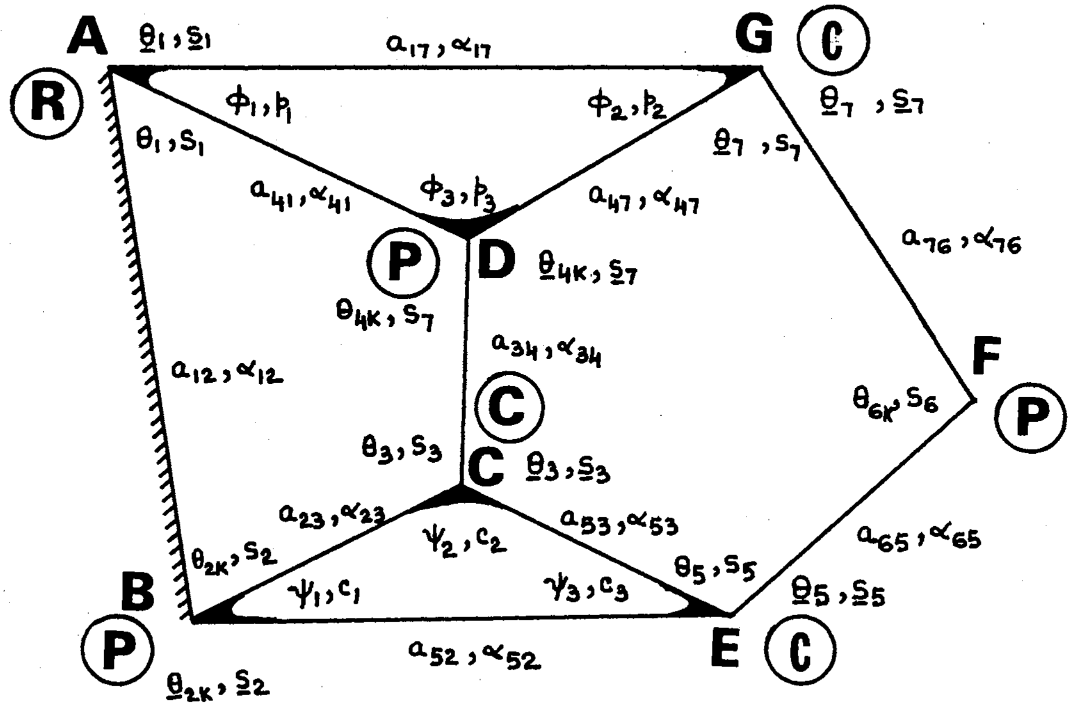

Figure 33. R-P-C-P-C-P-C Space Mechanism Obtained From the Mechanism in Figure 27 by Making $\theta_2 = \theta_{2k} =$ a Constant and $\theta_6 = \theta_{6k} =$ a Constant

Similarly, case 4 gives a prismatic pair at joint C in addition to the prismatic pairs at joints B and F. It, therefore, gives an R-P-P-C-C-P-C mechanism (Figure 34). It also shows that the axes of the revolute and cylinder pairs are parallel to each other.

The above results thus lead to the conclusion, that in an R-C-C-C-C-C-C mechanism, when one cylinder pair in loop 1 (path ABCDA in Figure 27) is reduced to a prismatic pair, another cylinder pair in that loop is also reduced to a prismatic pair. This result agrees with that by Dimentberg and Yoslovich (29) in the case of single loop, four-link mechanisms. Further, the axes of the revolute and cylinder pairs in both the loops are then parallel to each other.

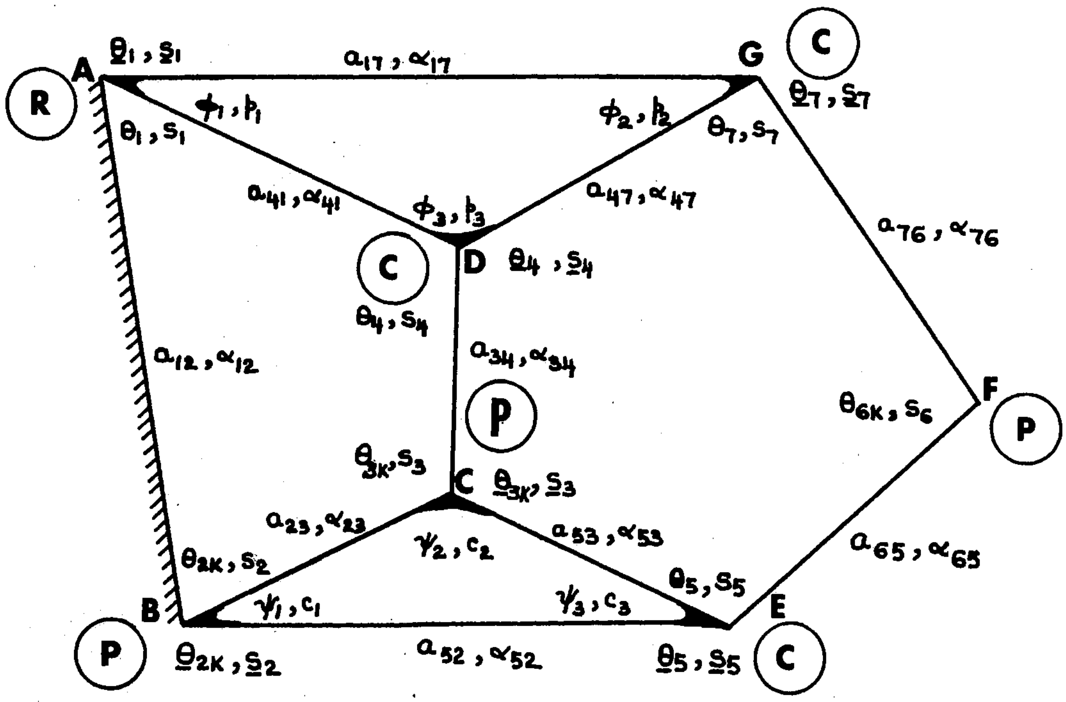

Figure 34. R-P-P-C-C-P-C Space Mechanism Obtained From the Mechanism in Figure 27 by Making $\theta_2 = \theta_{2k}$ = a Constant and $\theta_6 = \theta_{6k}$ = a Constant

APPENDIX D

COMPUTER PROGRAM

The following computer program is used for solving the system of nine consistent nonlinear algebraic equations representing the existence conditions of the R-R-C-C-C-R-C and R-R-C-C-C-P-C mechanisms. The program is that developed by Chandler (123) based on function minimization technique. Its usage is given as part of the listing.

```
//DVR79   JOB (12271,441-60-8386,20),'DUKKIPATI     ',CLASS=A
/*ROUTE PRINT HOLD
// EXEC FORTGCLG,REGION.GO=100K,TIME.GO=30
//FORT.SYSIN DD *
C      ******************************************************************
C      *                                                                *
C      *                                                                *
C      *                                                                *
C      *      TO SYNTHESIZE THE SIX-LINK,TWO-LOOP R-R-C-C-C-R-C SPACE   *
C      *      MECHANISM FROM THE EXISTENCE CRITERIA                     *
C      *                                                                *
C      *  DESCRIPTION OF PARAMETERS                                     *
C      *      N - NUMBER OF INDEPENDENT VARIABLES                       *
C      *      NP - CONVERGENCE MONITOR                                  *
C      *           =0 WILL NOT PRINT                                    *
C      *           =1 WILL PRINT EVERY ITERATION                        *
C      *      NN - TOTAL NUMBER OF ITERATIONS OR FUNCTION EVALUATIONS   *
C      *      DELTA - CURRENT STEP SIZE                                 *
C      *      F - MINIMUM STEP SIZE                                     *
C      *      ROW - REDUCTION FACTOR FOR STEP SIZE, < 1                 *
C      *      X - CURRENT BASE POINT                                    *
C      *      XL - LOWER BOUND OF SEARCH DOMAIN                         *
C      *      XR - UPPER BOUND OF SEARCH DOMAIN                         *
C      *                                                                *
C      *  USAGE                                                         *
C      *      REQUIRES THE FOLLOWING DATA CARDS                         *
C      *         CARD 1 - N,NP,NN,DELTA,F,ROW WITH 3I5,3D20.0           *
C      *         CARDS 2,3,4 - INITIAL VALUES FOR X(N) WITH 7D10.0      *
C      *         CARDS 5,6,7 - LOWER BOUND VALUES FOR XL(N) WITH 7D10.0 *
C      *         CARDS 8,9,10 -UPPER BOUND VALUES FOR XR(N) WITH 7D10.0 *
C      *                                                                *
C      *  SUBROUTINES REQUIRED                                          *
C      *      SUBROUTINE  PATRN                                         *
C      *      SUBROUTINE  FUNK                                          *
C      *      SUBROUTINE  STEPIT                                        *
C      *      SUBROUTINE  MERIT                                         *
C      *                                                                *
C      *  GENERAL REMARKS                                               *
C      *      VECTORS X(N),XL(N),XR(N) CONSISTS OF THE N PARAMETERS IN  *
C      *      THE FOLLOWING ORDER                                       *
C      *      TWIST ANGLES - ALPHA 12,ALPHA 23,ALPHA 34,ALPHA 41,       *
C      *      ALPHA 65,ALPHA 76,ALPHA 52,ALPHA 17                       *
C      *      KINK-LINKS - P1,C1                                        *
C      *      LINK-LENGTHS - A12,A23,A34,A41,A65,A76,A52,A17            *
C      *      ALL TWIST ANGLES ARE MEASURED IN DEGREES AND KINK-LINKS   *
C      *      AND LINK-LENGTHS ARE MEASURED IN INCH UNITS               *
C      *      IF REQUIRED SOME OF THE VARIABLES CAN BE FIXED BY         *
C      *      SETTING THE CORRESPONDING MASK(N) EQUAL TO 1 IN THE       *
C      *      SUBROUTINE PATRN                                          *
C      *      WITH SLIGHT MODIFICATIONS IN THE MAIN PROGRAM AND IN      *
C      *      THE SUBROUTINE MERIT THIS PROGRAM CAN ALSO BE USED FOR    *
C      *      SYNTHESIZING THE SIX-LINK,TWO-LOOP R-R-C-C-C-P-C          *
C      *      MECHANISM                                                 *
C      *                                                                *
C      *  REFERENCES                                                    *
C      *      CHANDLER,J.P., " STEPIT,PROGRAM NO.66,QUANTUM CHEMISTRY   *
C      *      PROGRAM EXCHANGE",DEPARTMENT OF CHEMISTRY,INDIANA         *
C      *      UNIVERSITY,BLOOMINGTON,INDIANA,47401                      *
C      *                                                                *
C      *      HOOKE,R.,AND JEEVES,T.A., " DIRECT SEARCH SOLUTION OF     *
C      *      NUMERICAL AND STATISTICAL PROBLEMS ", J.ASSOC. FOR        *
C      *      COMPUTING MACHINERY,NO.2,VOL.8,APR.1962,PP.212-230        *
C      *                                                                *
C      *                                                                *
C      *                                                                *
C      ******************************************************************
C
C
      IMPLICIT REAL * 8 (A-H,O-Z)
      DIMENSION  NAME1(2,10),NAME2(8)
      DIMENSION X(20),XL(20),XR(20),QP(9)
      DIMENSION XX(20),XXX(20)
      DATA NAME 1/'ALPH','A 12','ALPH','A 23','ALPH','A 34','ALPH',
     J'A 41','ALPH','A 65','ALPH','A 76','ALPH','A 52','ALPH','A 17','PH
     JI ' ,'1   ','SI 1','    '/
      DATA NAME2/'A 12','A 23','A 34','A 41','A 65','A 76','A 52','A 17'
     J /
      COMMON/QQ/Q1,Q2,Q3,Q4,Q5,Q6,Q7,Q8,Q9
      PI =3.14159265358979300
      RAD=PI/180.0
      DEG=180./PI
C
C                         READ INPUT DATA
C
      READ(5,1300) N,NP,NN,DELTA,F,ROW
      READ (5,55) (X (I),I=1,20)
      READ (5,55) (XL(I),I=1,20)
      READ (5,55) (XR(I),I=1,20)
 55   FORMAT(7D10.0)
 1300 FORMAT(3I5,3D20.0)
 1310 FORMAT(57X,'N     = ',I10,/,
     J57X,'NP    = ',I10,/,
     J57X,'NN    = ',I10,/,
     J57X,'DELTA = ',D10.3,/,
     J57X,'F     = ',D10.3,/,
     J57X,'ROW   = ',D10.3,///)
C
C                         PRINT THE INPUT DATA
C
      WRITE (6,1000)
 1000 FORMAT(1H1,30X,'" EXISTENCE CRITERIA OF SIX-LINK,TWO-LOOP R-R-C-C-
     JC-R-C SPACE MECHANISM "',/////,51X,' INITIAL VALUES OF THE VARIABL
     JES ',/,1H ,51X,32('-'),///)
      WRITE(6,1310) N,NP,NN,DELTA,F,ROW
      WRITE(6,1010)
 1010 FORMAT(39X,'|',14('-'),'|',3(13('-'),'|')  )
      WRITE(6,1200)
      WRITE(6,1020)
 1020 FORMAT(39X,'|',1X,'TWIST ANGLES |',6X,'X',6X,'|',4X,'XMIN',5X,'|',
     J4X,'XMAX',5X,'|'  )
      WRITE (6,1200)
 1200 FORMAT(39X,'|',14X,'|',3(13X,'|'))
 1210 FORMAT(38X,'|',12X,'|',3(13X,'|'))
      WRITE(6,1010)
      DO 1035 J=1,10
      WRITE(6,1030) (NAME1(K,J),K=1,2),X(J),XL(J),XR(J)
 1030 FORMAT(39X,'|',3X,2A4,3X,'|',3(1X,D11.4,1X,'|')  )
      WRITE (6,1200)
      WRITE(6,1010)
 1035 CONTINUE
      WRITE(6,1040)
 1040 FORMAT(////)
      WRITE(6,1050)
 1050 FORMAT(38X,'|',12('-'),'|',3(13('-'),'|')  )
      WRITE (6,1210)
      WRITE(6,1060)
 1060 FORMAT(38X,'| KINK LINKS |',6X,'X',6X,'|',4X,'XMIN',5X,'|',4X,'XMA
     JX',5X,'|'  )
      WRITE(6,1210)
```

```
      WRITE(6,1050)
      WRITE(6,1070) X(11),XL(11),XR(11)
 1070 FORMAT(38X,'|',5X,'S1',5X,'|',3(1X,D11.4,1X,'|'))
      WRITE (6,1210)
      WRITE(6,1050)
      WRITE(6,1080) X(12),XL(12),XR(12)
 1080 FORMAT(38X,'|',5X,'S2',5X,'|',3(1X,D11.4,1X,'|'))
      WRITE (6,1210)
      WRITE(6,1050)
      WRITE(6,1040)
      WRITE(6,1010)
      WRITE(6,1090)
 1090 FORMAT(39X,'|',' LINK-LENGTHS |',
     J                                   6X,'X',6X,'|',4X,'XMIN',5X,'|',4X,
     J'XMAX',5X,'|'   )
      WRITE (6,1200)
      WRITE(6,1010)
      DO 1110 II=1,8
      L=12+II
      WRITE(6,1100)NAME2(II),X(L),XL(L),XR(L)
 1100 FORMAT(39X,'|',5X,A4,5X,'|',3(1X,D11.4,1X,'|'))
      WRITE(6,1200)
      WRITE(6,1010)
 1110 CONTINUE
      DO 56 I=1,10
      X(I)=X(I)*RAD
      XL(I)=XL(I)*RAD
   56 XR(I)=XR(I)*RAD
C
C                        CALL PATRN TO MINIMIZE THE FUNCTION Y
C
                 CALL PATRN(      N,NP,DELTA,F,XL,XR,Y,X,ROW,NN)
      DO 22 I=1,10
      XL(I)=XL(I)*DEG
      XR(I)=XR(I)*DEG
   22 X(I)=X(I)*DEG
C
C                        Q1,Q2,Q3,........Q9 ARE THE NINE EXISTENCE CONDITIONS
C
      QP(1)=Q1
      QP(2)=Q2
      QP(3)=Q3
      QP(4)=Q4
      QP(5)=Q5
      QP(6)=Q6
      QP(7)=Q7
      QP(8)=Q8
      QP(9)=Q9
C
C                        PRINT THE FINAL VALUES OF THE VARIABLES
C
      WRITE(6,2000)
 2000 FORMAT(1H1,52X,'FINAL VALUES OF THE VARIABLES',/,1H ,52X,29('-'),/
     J//)
      WRITE(6,1010)
      WRITE(6,1020)
      WRITE(6,1200)
      WRITE(6,1010)
      DO 1075   J=1,10
      WRITE(6,1030) (NAME1(K,J),K=1,2),X(J),XL(J),XR(J)
      WRITE(6,1200)
      WRITE(6,1010)
 1075 CONTINUE
      WRITE(6,1040)
      WRITE(6,1050)
      WRITE(6,1210)
      WRITE(6,1060)
      WRITE(6,1050)
      WRITE(6,1070) X(11),XL(11),XR(11)
      WRITE(6,1210)
      WRITE(6,1050)
      WRITE(6,1080) X(12),XL(12),XR(12)
      WRITE(6,1210)
      WRITE(6,1050)
      WRITE(6,1040)
      WRITE(6,1010)
      WRITE(6,1090)
      WRITE(6,1200)
      WRITE(6,1010)
      DO 1330 II=1,8
      L=12+II
      WRITE(6,1100)NAME2(II),X(L),XL(L),XR(L)
      WRITE(6,1200)
      WRITE(6,1010)
 1330 CONTINUE
      WRITE(6,1040)
C
C                        PRINT THE FINAL VALUES OF THE EXISTENCE CONDITIONS
C
      WRITE(6,2020) (L,QP(L),L=1,9)
 2020 FORMAT(48X,'FINAL VALUES OF THE EXISTENCE CONDITIONS',///
     J                                                            ,(55X,'EQUA
     JTION ',I2,' = ',D11.4,/))
      STOP
      END
C
C
C
C
      SUBROUTINE PATRN (      N,NP,DELTA,F,XL,XR,Y,XX,ROW,NN)
C
C  INTERFACE ROUTINE TO MAKE STEPIT LOOK LIKE PATRN.
C  J. P. CHANDLER, COMPUTER SCIENCE DEPT., OKLAHOMA STATE UNIVERSITY.
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION XL(20),XR(20),XX(20)
      COMMON /CSTEP/ X(20),XMAX(20),XMIN(20),DELTX(20),DELMN(20),
     X   ERR(21,20),CHISQ,NV,NTRAC,MATRX,MASK(20)
      COMMON /FRODO/ NFMAX,NFLAT,JVARY,NXTRA
      EXTERNAL FUNK
C
C                          MOVE VARIABLES INTO STEPIT COMMON.
      NV=N
      NTRAC=NP
      NFMAX=NN
      DO 1 J=1,NV
      MASK(J)=0
      DELTX(J)=DELTA
      DELMN(J)=F
      XMIN(J)=XL(J)
      XMAX(J)=XR(J)
    1 X(J)=XX(J)
C                          CALL STEPIT TO MINIMIZE CHISQ.
      CALL STEPIT (FUNK)
C                          RETURN Y AND XX(J).
      Y=-CHISQ
      DO 2 J=1,NV
    2 XX(J)=X(J)
      RETURN
      END
C
C
```

```
C
C
C
      SUBROUTINE FUNK
C
C  INTERFACE ROUTINE TO MAKE MERIT LOOK LIKE FUNK.
C
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON /CSTEP/ X(20),XMAX(20),XMIN(20),DELTX(20),DELMN(20),
     X    ERR(21,20),CHISQ,NV,NTRAC,MATRX,MASK(20)
C
      CALL MERIT (      X,Y)
      CHISQ=-Y
      RETURN
      END
C
C
C
C
C
      SUBROUTINE MERIT (X,Y)
C
C                       ROUTINE TO CALCULATE THE MERIT FUNCTION Y DEFINED
C                       AS THE SUM OF THE SQUARES OF THE NINE EXISTENCE
C                       CONDITIONS Q1,Q2,Q3,......Q9 FOR THE SIX-LINK,
C                       TWO-LOOP R-R-C-C-C-R-C SPACE MECHANISM
C
      IMPLICIT REAL *8 (A-H,J-Z)
      REAL *8 DABS,DSIN,DCOS,DTAN
      DIMENSION   X(20),XX(20),XXX(20),XL(20),XR(20)
      COMMON/QQ/Q1,Q2,Q3,Q4,Q5,Q6,Q7,Q8,Q9
      PI=3.141592653589793
C
      AL12=X(1)
      AL23=X(2)
      AL34=X(3)
      AL41=X(4)
      AL65=X(5)
      AL76=X(6)
      AL52=X(7)
      AL21=-AL12
      AL17=X(8)
      P1=X(9)
      C1=X(10)
      S1=X(11)
      S2=X(12)
      A12=X(13)
      A23=X(14)
      A34=X(15)
      A41=X(16)
      A65=X(17)
      A76=X(18)
      A52=X(19)
      A21=-A12
      A17=X(20)
C
C                       CHECK FOR ZERO DENOMINATOR
C
      DO  43  I=1,10
      IF(X(I).EQ..5*PI) GO TO 48
      X(I)=X(I)
      GO TO 52
 48   X(I)=X(I)+.05
 52   CONTINUE
      IF(X(I).EQ.PI) GO TO 47
      X(I)=X(I)
      GO TO 51
 47   X(I)=X(I)+.05
 51   CONTINUE
      IF(X(I).EQ.1.5*PI) GO TO 49
      X(I)=X(I)
      GO TO 53
 49   X(I)=X(I)+.05
 53   CONTINUE
 43   CONTINUE
C
      SAL12=DSIN(AL12)
      SAL23=DSIN(AL23)
      SAL34=DSIN(AL34)
      SAL41=DSIN(AL41)
      SAL65=DSIN(AL65)
      SAL76=DSIN(AL76)
      SAL52=DSIN(AL52)
      SAL21=DSIN(AL21)
      SAL17=DSIN(AL17)
      CAL12=DCOS(AL12)
      CAL23=DCOS(AL23)
      CAL34=DCOS(AL34)
      CAL41=DCOS(AL41)
      CAL65=DCOS(AL65)
      CAL76=DCOS(AL76)
      CAL52=DCOS(AL52)
      CAL21=DCOS(AL21)
      CAL17=DCOS(AL17)
      SP1=DSIN(P1/2.)
      SC1=DSIN(C1/2.)
      CP1=DCOS(P1/2.)
      CC1=DCOS(C1/2.)
C
C                       CONSTANTS FOR USE IN EQ.(4-41) SUMMARISED IN TABLE
C                       IX OF THE THESIS
C
      D002=A41*CAL41*SAL23+A23*CAL23*SAL41
      D001=S1*SAL23*SAL41+S2*CAL12*SAL23
      D000=S2*SAL12*CAL41*SAL23
      E002=S2*SAL23*SAL41+S1*CAL12*SAL23
      E001=-A23*CAL23*CAL12+A12*SAL12*SAL23
      E000=-A12*CAL12*CAL41*SAL23-A23*CAL41*CAL23*SAL12+A41*SAL41*SAL12
     J    *SAL23
      F002=S1*SAL41*SAL23
      F001=-A41*CAL41*CAL23+A23*SAL23*SAL41
      F000=A34*SAL34-CAL12*(A41*SAL41*CAL23+A23*SAL23*CAL41)-A12*SAL12
     J      *CAL41*CAL23
      B22=E001-E000-F001+F000
      B21=-2.*(E002-F002)
      B20=-E001-E000+F001+F000
      B12=-2.*(D001-D000)
      B11=4.*D002
      B10=2.*(D001+D000)
      B02=-E001+E000-F001+F000
      B01=2.*(E002+F002)
      B00=E001+E000+F001+F000
C
C                       CONSTANTS FOR USE IN TABLE XI  .   THESE ARE
C                       SUMMARISED IN TABLE X IN THE THESIS
C
      U1=A76*CAL65/SAL76+A65*CAL76/SAL65
      U2=A76*CAL76/SAL76+A65*CAL65/SAL65
      Z0=AL52-AL21-AL17
      F0=U1-U2*DCOS(Z0)-(A52-A21-A17)*DSIN(Z0)
      Z1=AL52-AL21
      F1=-2.*SAL17*(S1*DSIN(Z1)+S2*SAL52)
```

```
      Z2=AL52-AL21+AL17
      F2=U1-U2*DCOS(Z2)-(A52-A21+A17)*DSIN(Z2)
      Z3=AL21+AL17
      G0=2.*SAL52*(S1*SAL17+S2*DSIN(Z3))
      CT17=DCOS(AL17)/DSIN(AL17)
      CT76=DCOS(AL76)/DSIN(AL76)
      CT65=DCOS(AL65)/DSIN(AL65)
      CT52=DCOS(AL52)/DSIN(AL52)
      G1=4.*SAL17*SAL52*(A17*CT17-A76*CT76-A65*CT65+A52*CT52)
      Z4=AL21-AL17
      G2=-2.*SAL52*(S1*SAL17-S2*DSIN(Z4))
      Z5=AL52+AL21+AL17
      Z6=AL52+AL17
      Z7=AL52+AL21-AL17
      H0=U1-U2*DCOS(Z5)-(A52+A21+A17)*DSIN(Z5)
      H1=2.*SAL17*(S1*DSIN(Z6)+S2*SAL52)
      H2=U1-U2*DCOS(Z7)-(A52+A21-A17)*DSIN(Z7)
C
C                    CONSTANTS FOR USE IN EQ. (4-43) AND TABLE XII.
C                    THESE ARE SUMMARISED IN TABLE XI OF THE  THESIS
C
      X1=SP1/CP1
      X2=SC1/CC1
      Y2=F0-F1*X1+F2*X1*X1
      Y1=2.*F0*X1-F1*X1*X1+F1-2.*F2*X1
      Y0=F0*X1*X1+F1*X1+F2
      W2=-G0+G1*X1-G2*X1*X1
      W1=-2.*G0*X1+G1*X1*X1-G1+2.*G2*X1
      W0=-G0*X1*X1-G1*X1-G2
      Z8=H0-H1*X1+H2*X1*X1
      Z9=2.*H0*X1-H1*X1*X1+H1-2.*H2*X1
      Z10=H0*X1*X1+H1*X1+H2
      A22=X2*X2*Y2+X2*W2+Z8
      A21=X2*X2*Y1+X2*W1+Z9
      A20=X2*X2*Y0+X2*W0+Z10
      A12=2.*X2*(Z8-Y2)+W2*(X2*X2-1.)
      A11=2.*X2*(Z9-Y1)+W1*(X2*X2-1.)
      A10=2.*X2*(Z10-Y0)+W0*(X2*X2-1.)
      A02=Y2-X2*W2+X2*X2*Z8
      A01=Y1-X2*W1        +X2*X2*Z9
      A00=Y0-X2*W0+X2*X2*Z10
C
C                    CONSTANTS FOR DEFINING THE NINE EXISTENCE CONDITIONS
C                    Q1,Q2,.....Q9 OF TABLE XII
C
      X81=A12*A22*B02*B12
      X82=-A12*A12*B02*B22
      X83=-A02*A22*B12*B12
      X84=A02*A12*B12*B22
      X85=A22*A22*B02*B02
      X85=-X85
      X86=2.0*(A02*A22*B02*B22)
      X87=-A02*A02*B22*B22
      X71=A11*A22*B02*B12+A12*A21*B02*B12+A12*A22*B01*B12+A12*A22*B02*B1
     11
      X72=-(2.0*A11*A12*B02*B22+A12*A12*(B01*B22+B02*B21))
      X73=-((A01*A22+A02*A21)*B12*B12+2.0*A02*A22*B11*B12)
      X74=A01*A12*B12*B22+A02*A11*B12*B22+A02*A12*B11*B22+A02*A12*B12*B2
     11
      X75=-(2.0*A21*A22*B02*B02+2.0*A22*A22*B01*B02)
      X76=2.0*(A01*A22*B02*B22+A02*A21*B02*B22+A02*A22*B01*B22+A02*A22*B
     102*B21)
      X77=-(2.0*A01*A02*B22*B22+2.0*A02*A02*B21*B22)
      X611=A10*A22*B02*B12+A12*A20*B02*B12+A12*A22*B00*B12+A12*A22*B02*B
     110
      X612=A11*(A21*B02*B12+A22*B01*B12+A22*B02*B11)+A12*(A22*B01*B11+A2
       X(I)=X(I)
       WRITE(6,1210)
     11*B02*B11+A21*B01*B12)
      X61=X611+X612
      X621=2.0*A10*A12*B02*B22+A12*A12*(B00*B22+B02*B20)
      X622=A11*A11*B02*B22+A12*A12*B01*B21+2.0*A11*A12*(BC1*B22+B02*B21)
      X62=-(X621+X622)
      X631=(A00*A22+A02*A20)*B12*B12+2.0*A02*A22*B10*B12
      X632=A01*A21*B12*B12+A02*A22*B11*B11+2.0*B11*B12*(A01*A22+A02*A21)
      X63=-(X631+X632)
      X641=A00*A12*B12*B22+A02*A10*B12*B22+A02*A12*B10*B22+A02*A12*B12*B
     120
      X642=A01*(A11*B12*B22+A12*B11*B22+A12*B12*B21)+A02*(A12*B11*B21+A1
     11*B12*B21+A11*B11*B22)
      X64=X641+X642
      X65=-(2.0*A20*A22*B02*B02+2.0*A22*A22*B00*B02+A22*A22*B01*B01+A21*
     1A21*B02*B02+4.0*A21*A22*B01*B02)
      X661=A00*A22*B02*B22+A02*A20*B02*B22+A02*A22*B00*B22+A02*A22*B02*B
     120
      X662=A01*(A21*B02*B22+A22*B01*B22+A22*B02*B21)+A02*(A22*B01*B21+A2
     11*B02*B21+A21*B01*B22)
      X66=2.0*(X661+X662)
      X67=-(2.0*A00*A02*B22*B22+2.0*A02*A02*B20*B22+AC1*A01*B22*B22+A02*
     1A02*B21*B21+4.0*A01*A02*B21*B22)
      X511=A12*A21*B01*B11+A11*A22*B01*B11+A11*A21*B02*B11+A11*A21*B01*B
     112
      X512=A10*(A21*B02*B12+A22*B01*B12+A22*B02*B11)+A20*(A11*B02*B12+A1
     12*B01*B12+A12*B02*B11)
      X513=B00*(A11*A22*B12+A12*A21*B12+A12*A22*B11)+B10*(A11*A22*B02+A1
     12*A21*B02+A12*A22*B01)
      X51=X511+X512+X513
      X521=2.0*A11*A12*B01*B21+A11*A11*(B01*B22+B02*B21)+2.0*A10*(A11*B0
     12*B22+A12*B01*B22+A12*B02*B21)
      X522=B00*(2.0*A11*A12*B22+A12*A12*B21)+B20*(2.0*A11*A12*B02+A12*A1
     12*B01)
      X52=-(X521+X522)
      X531=(A02*A21+A01*A22)*B11*B11+2.0*A01*A21*B11*B12
      X532=A00*(A21*B12*B12+2.0*A22*B11*B12)+A20*(A01*B12*B12+2.0*A02*B1
     11*B12)
      X533=2.0*B10*(A01*A22*B12+A02*A21*B12+A02*A22*B11)
      X53=-(X531+X532+X533)
      X541=A02*A11*B11*B21+A01*A12*B11*B21+A01*A11*B12*B21+A01*A11*B11*B
     122
      X542=A00*(A11*B12*B22+A12*B11*B22+A12*B12*B21)+A10*(A01*B12*B22+A0
     12*B11*B22+A02*B12*B21)
      X543=B10*(A01*A12*B22+A02*A11*B22+A02*A12*B21)+B2C*(A01*A12*B12+A0
     12*A11*B12+A02*A12*B11)
      X54=X541+X542+X543
      X55=A21*A22*B01*B01+A21*A21*B01*B02+A20*(A21*B02*B02+2.0*A22*B01*B
     102)+B00*(2.0*A21*A22*B02+A22*A22*B01)
      X55=-2.0*X55
      X561=A02*A21*B01*B21+A01*A22*B01*B21+A01*A21*B02*B21+A01*A21*B01*B
     122
      X562=A00*(A21*B02*B22+A22*B01*B22+A22*B02*B21)+A20*(A01*B02*B22+A0
     12*B01*B22+A02*B02*B21)
      X563=B00*(A01*A22*B22+A02*A21*B22+A02*A22*B21)+B2C*(A01*A22*B02+A0
     12*A21*B02+A02*A22*B01)
      X56=2.0*(X561+X562+X563)
      X57=A01*A02*B21*B21+A01*A01*B21*B22+A00*(A01*B22*B22+2.0*A02*B21*B
     122)+B20*(A02*A02*B21+2.0*A01*A02*B22)
      X57=-2.0*X57
      X411=A10*A20*B02*B12+A10*A22*B00*B12+A10*A22*B02*B10
      X412=A12*A20*B00*B12+A12*A20*B02*B10+A12*A22*B00*B10
      X413=A11*A21*(B00*B12+B02*B10)+A11*B01*(A20*B12+A22*B10)+A11*B11*(
     1A20*B02+A22*B00)
      X414=A21*B01*(A10*B12+A12*B10)+A21*B11*(A10*B02+A12*B00)+B01*B11*(
     1A10*A22+A12*A20)+A11*A21*B01*B11
      X41=X411+X412+X413+X414
C
```

```
 X421=A10*A10*B02*B22+2.0*A10*A12*(B00*B22+B02*B20)+A12*A12*B00*B20
 X422=A11*A11*(B00*B22+B02*B20)+2.0*A11*B01*(A10*B22+A12*B20)+2.0*A
111*B21*(A10*B02+A12*B00)+2.0*A10*A12*B01*B21+A11*A11*B01*B21
 X42=-(X421+X422)
 X431=A00*A20*B12*B12+A02*A22*B10*B10+2.0*B10*B12*(A00*A22+A02*A20)
 X432=2.0*A01*A21*B10*B12+B11*B11*(A00*A22+A02*A20)+2.0*A01*B11*(A2
10*B12+A22*B10)+2.0*A21*B11*(A00*B12+A02*B10)+A01*A21*B11*B11
 X43=-(X431+X432)
 X441=A00*A10*B12*B22+A00*A12*B10*B22+A00*A12*B12*B20
 X442=A02*A10*B10*B22+A02*A10*B12*B20+A02*A12*B10*B20
 X443=A01*A11*(B10*B22+B12*B20)+A01*B11*(A10*B22+A12*B20)+A01*B21*(
1A10*B12+A12*B10)
 X444=A11*B11*(A00*B22+A02*B20)+A11*B21*(A00*B12+A02*B10)+B11*B21*(
1A00*A12+A02*A10)+A01*A11*B11*B21
 X44=X441+X442+X443+X444
 X451=A20*A20*B02*B02+A22*A22*B00*B00+4.0*A20*A22*B00*B02
 X452=2.0*(A21*A21*B00*B02+A20*A22*B01*B01+2.0*A21*B01*(A20*B02+A22
1*B00))+A21*A21*B01*B01
 X45=-(X451+X452)
 X461=A00*A20*B02*B22+A00*A22*B00*B22+A00*A22*B02*B20
 X462=A02*A20*B00*B22+A02*A20*B02*B20+A02*A22*B0C*B20
 X463=A01*A21*(B00*B22+B02*B20)+A01*B01*(A20*B22+A22*B20)+A01*B21*(
1A20*B02+A22*B00)
 X464=A21*B01*(A00*B22+A02*B20)+A21*B21*(A00*B02+A02*B00)+B01*B21*(
1A00*A22+A02*A20)+A01*A21*B01*B21
 X46=2.0*(X461+X462+X463+X464)
 X471=A00*A00*B22*B22+A02*A02*B20*B20+4.0*A00*A02*B20*B22
 X472=2.0*(A01*A01*B20*B22+A00*A02*B21*B21+2.0*A01*B21*(A00*B22+A02
1*B20))+A01*A01*B21*B21
 X47=-(X471+X472)
 X311=A10*A21*B01*B11+A11*A20*B01*B11+A11*A21*B00*B11+A11*A21*B01*B
110
 X312=A12*(A21*B00*B10+A20*B01*B10+A20*B00*B11)+A22*(A11*B00*B10+A1
10*B01*B10+A10*B00*B11)
 X313=B02*(A11*A20*B10+A10*A21*B10+A10*A20*B11)+B12*(A11*A20*B00+A1
10*A21*B00+A10*A20*B01)
 X31=X311+X312+X313
 X321=2.0*A10*A11*B01*B21+A11*A11*(B01*B20+B00*B21)
 X322=2.0*A12*(A11*B00*B20+A10*B01*B20+A10*B00*B21)
 X323=B02*(2.0*A10*A11*B20+A10*A10*B21)+B22*(2.0*A10*A11*B00+A10*A1
10*B01)
 X32=-(X321+X322+X323)
 X331=(A00*A21+A01*A20)*B11*B11+2.0*A01*A21*B10*B11
 X332=A02*(A21*B10*B10+2.0*A20*B10*B11)+A22*(A01*B10*B10+2.0*A00*B1
10*B11)
 X333=2.0*B12*(A01*A20*B10+A00*A21*B10+A00*A20*B11)
 X33=-(X331+X332+X333)
 X341=A00*A11*B11*B21+A01*A10*B11*B21+A01*A11*B10*B21+A01*A11*B11*B
120
 X342=A02*(A11*B10*B20+A10*B11*B20+A10*B10*B21)+A12*(A01*B10*B20+A0
10*B11*B20+A00*B10*B21)
 X343=B12*(A01*A10*B20+A00*A11*B20+A00*A10*B21)+B22*(A01*A10*B10+A0
10*A11*B10+A00*A10*B11)
 X34=X341+X342+X343
 X35=-2.0*(A20*A21*B01*B01+A21*A21*B00*B01+A22*(A21*B00*B00+2.0*A20
1*B00*B01)+B02*(A20*A20*B01+2.0*A20*A21*B00))
 X361=A00*A21*B01*B21+A01*A20*B01*B21+A01*A21*B00*B21+A01*A21*B01*B
120
 X362=A02*(A21*B00*B20+A20*B01*B20+A20*B00*B21)+A22*(A01*B00*B20+A0
10*B01*B20+A00*B00*B21)
 X363=B02*(A01*A20*B20+A00*A21*B20+A00*A20*B21)+B22*(A01*A20*B00+A0
10*A21*B00+A00*A20*B01)
 X36=2.0*(X361+X362+X363)
 X37=-2.0*(A00*A01*B21*B21+A01*A01*B20*B21+A02*(A01*B20*B20+2.0*A00
1*B20*B21)+B22*(2.0*A00*A01*B20+A00*A00*B21))
 X211=A12*A20*B00*B10+A10*A22*B00*B10+A10*A20*B02*B10+A10*A20*B00*B
   X(I)=X(I)
  WRITE(6,1210)
112
 X212=A11*(A21*B00*B10+A20*B01*B10+A20*B00*B11)+A10*(A20*B01*B11+A2
11*B00*B11+A21*B01*B10)
 X21=X211+X212
 X221=2.0*A10*A12*B00*B20+A10*A10*(B00*B22+B02*B20)
 X222=A11*A11*B00*B20+A10*A10*B01*B21+2.0*A10*A11*(B00*B21+B01*B20)
 X22=-(X221+X222)
 X231=(A00*A22+A02*A20)*B10*B10+2.0*A00*A20*B10*B12
 X232=A01*A21*B10*B10+A00*A20*B11*B11+2.0*B10*B11*(A00*A21+A01*A20)
 X23=-(X231+X232)
 X241=A02*A10*B10*B20+A00*A12*B10*B20+A00*A10*B12*B20+A00*A10*B10*B
122
 X242=A01*(A11*B10*B20+A10*B11*B20+A10*B10*B21)+A00*(A10*B11*B21+A1
11*B10*B21+A11*B11*B20)
 X24=X241+X242
 X25=-(2.0*A20*A22*B00*B00+2.0*A20*A20*B00*B02+A21*A21*B00*B00+A20*
1A20*B01*B01+4.0*A20*A21*B00*B01)
 X261=A02*A20*B00*B20+A00*A22*B00*B20+A00*A20*B02*B20+A00*A20*B00*B
122
 X262=A01*(A21*B00*B20+A20*B01*B20+A20*B00*B21)+A00*(A20*B01*B21+A2
11*B00*B21+A21*B01*B20)
 X26=2.0*(X261+X262)
 X27=-(2.0*A00*A02*B20*B20+2.0*A00*A00*B20*B22+A00*A00*B21*B21+A01*
1A01*B20*B20+4.0*A00*A01*B20*B21)
 X11=A11*A20*B00*B10+A10*A21*B00*B10+A10*A20*B01*B10+A10*A20*B00*B1
11
 X12=-(2.0*A10*A11*B00*B20+A10*A10*(B00*B21+B01*B20))
 X13=-((A00*A21+A01*A20)*B10*B10+2.0*A00*A20*B10*B11)
 X14=A01*A10*B10*B20+A00*A11*B10*B20+A00*A10*B11*B20+A00*A10*B10*B2
11
 X15=-2.0*(A20*A21*B00*B00+A20*A20*B00*B01)
 X16=A01*A20*B00*B20+A00*A21*B00*B20+A00*A20*B01*B20+A00*A20*B00*B2
11
 X16=2.0*X16
 X17=-2.0*(A00*A01*B20*B20+A00*A00*B20*B21)
 X01=A10*A20*B00*B10
 X02=-A10*A10*B00*B20
 X03=-A00*A20*B10*B10
 X04=A00*A10*B10*B20
 X05=-A20*A20*B00*B00
 X06=2.0*A00*A20*B00*B20
 X07=-A00*A00*B20*B20
C
C               Q1,Q2,....Q9 ARE THE NINE EXISTENCE CONDITIONS
C               FOR THE SIX-LINK,TWO-LOOP R-R-C-C-C-R-C
C               SPACE MECHANISM
C
 Q1=X01+X02+X03+X04+X05+X06+X07
 Q2=X11+X12+X13+X14+X15+X16+X17
 Q3=X21+X22+X23+X24+X25+X26+X27
 Q4=X31+X32+X33+X34+X35+X36+X37
 Q5=X41+X42+X43+X44+X45+X46+X47
 Q6=X51+X52+X53+X54+X55+X56+X57
 Q7=X61+X62+X63+X64+X65+X66+X67
 Q8=X71+X72+X73+X74+X75+X76+X77
 Q9=X81+X82+X83+X84+X85+X86+X87
 Y=-(Q9*Q9+Q8*Q8+Q7*Q7+Q6*Q6+Q5*Q5+Q4*Q4  +Q3*Q3+Q2*Q2+Q1*Q1)
  RETURN
  END
C
C
C
C
C
```

```
      SUBROUTINE STEPIT(FUNK)
C
C  STEPT 7.0        A.N.S.I. STANDARD FORTRAN        JANUARY 1973
C  COPYRIGHT 1965 -- J. P. CHANDLER, PHYSICS DEPT., INDIANA UNIVERSITY.
C      (PRESENT ADDRESS ....  COMPUTER SCIENCE DEPT.,
C             OKLAHOMA STATE UNIVERSITY, STILLWATER, OKLAHOMA 74074)
C
C  STEPT (FORMERLY CALLED STEPIT) FINDS LOCAL MINIMA OF A SMOOTH
C  FUNCTION OF SEVERAL PARAMETERS.
C
C  -STEPIT IS A PHLEGMATIC METHOD OF SOLVING A PROBLEM.-
C           --  J. H. BURRILL, JR., -360 STEPIT - A USER-S MANUAL-
C
C  THIS SOURCE DECK AND A WRITE-UP ARE AVAILABLE AS PROGRAM NO. 66 FROM
C
C      QUANTUM CHEMISTRY PROGRAM EXCHANGE
C      DEPT. OF CHEMISTRY, INDIANA UNIVERSITY
C      BLOOMINGTON, INDIANA 47401
C
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C
C  INPUT VARIABLES.....  FUNK,NV,NTRAC,MATRX,MASK,X,XMAX,XMIN,
C                                DELTX,DELMN,NFMAX,NFLAT
C  OUTPUT VARIABLES....  CHISQ,X,ERR
C
C     FUNK        --  THE NAME OF THE SUBROUTINE THAT COMPUTES CHISQ
C                          GIVEN X(1),X(2),....,X(NV) (AN EXTERNAL
C                          STATEMENT IS REQUIRED IN THE CALLING PROGRAM
C                          FOR EACH SUCH SUBROUTINE)
C     NV          --  THE NUMBER OF PARAMETERS, X
C     NTRAC       --  =0 FOR NORMAL OUTPUT, =+1 FOR TRACE OUTPUT,
C                          =-1 FOR NO OUTPUT
C     MATRX       --  =0 FOR NO ERROR CALCULATION, =100+M FOR ERROR
C                          CALCULATION USING STEPS 10**M TIMES LARGER
C                          THAN THE LAST STEPS USED IN THE MINIMIZATION
C     CHISQ       --  THE VALUE OF THE FUNCTION TO BE MINIMIZED
C     MASK(J)     --  NONZERO IF X(J) IS TO BE HELD FIXED
C     X(J)        --  THE J-TH PARAMETER
C     XMAX(J)     --  THE UPPER LIMIT ON X(J)
C     XMIN(J)     --  THE LOWER LIMIT ON X(J)
C     DELTX(J)    --  THE INITIAL STEP SIZE FOR X(J)
C     DELMN(J)    --  THE LOWER LIMIT (CONVERGENCE TOLERANCE) ON THE
C                          STEP SIZE FOR X(J)
C     ERR(J,K)    --  RETURNS THE ERROR MATRIX IF -MATRX- IS NONZERO
C                            (ALSO USED FOR SCRATCH STORAGE)
C     NFMAX       --  THE MAXIMUM NUMBER OF FUNCTION COMPUTATIONS
C     NFLAT       --  NONZERO IF THE SEARCH IS TO TERMINATE WHEN ALL
C                          TRIAL STEPS GIVE IDENTICAL FUNCTION VALUES
C     JVARY       --  STEPT SETS JVARY NONZERO IF X(JVARY) IS THE ONLY
C                          X(J) THAT HAS CHANGED SINCE THE LAST CALL TO
C                          FUNK (THIS CAN BE USED TO SPEED UP FUNK)
C     NXTRA       --  USED BY SUBROUTINE SIMPLEX BUT NOT BY STEPIT
C
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C
      EXTERNAL FUNK
C
C  THE FOLLOWING STATEMENTS CONVERT STEPT TO DOUBLE PRECISION.
C  STEPT CONTAINS NO MIXED MODE STATEMENTS, NO MATTER WHETHER THE
C  VARIABLES BEGINNING WITH A-H AND O-Z ARE TYPE REAL OR ARE TYPE
C  DOUBLE PRECISION.
C
C     DOUBLE PRECISION X,XMAX,XMIN,DELTX,DELMN,ERR,CHISQ,
C    X   VEC,TRIAL,XSAVE,CHI,DX,OLVEC,SALVO,XOSC,
C    X   CHIOS,Q,RELAC,HUGE,RATIO,COLIN,CMPMX,ACK,FACUP,DELDF
C     DOUBLE PRECISION RZERO,RHALF,RUNIT,RTWO,DELX,XPLUS,COMPR,
C    X   A,SUB,P,CHSAV,CHOLD,SAVE,ADX,CHIME,DENOM,DEL,DXZ,DXU,
C    X   DCZ,DCU,ANUM,CINDR,AVEC,SUMO,SUMV,COSIN,COXCM,
C    X   CHBAK,STEPS,FAC,QSQRT,DSQRT
C
C  THE DIMENSIONS OF ALL VECTORS AND MATRICES (AS OPPOSED TO ARRAYS)
C  ARE NV, EXCEPT FOR ....
C       ERR(NV,MOSQ),XOSC(NV,MOSQ),CHIOS(MOSQ).
C  IF ERRORS ARE TO BE CALCULATED BY SUBROUTINE STERR, HOWEVER, THEN
C  ERR MUST BE DIMENSIONED AT LEAST      ERR(NV,MAX(NV,MOSQ)) .
C  TO REDUCE STORAGE TO A MINIMUM, SET MOSQ=0, REDIMENSION ERR(1,1),
C  XOSC(1,1), SALVO(1), AND CHIOS(1), DELETE THE OSCILLATION SEARCH
C  (SEE COMMENT CARDS BELOW), AND SUPPLY A DUMMY SUBROUTINE STERR.
C  OR, USE SUBROUTINE STP, WHICH HAS THESE DELETIONS PLUS DELETION OF
C  THE COLINEARITY CHECK.
C
      DIMENSION VEC(20),TRIAL(20),XSAVE(20),CHI(20),DX(20)
      DIMENSION OLVEC(20),SALVO(20),XOSC(20,5),CHIOS(5),JFLAT(20)
C
C  IF UNLABELLED COMMON AND SINGLE PRECISION ARE USED, SUBROUTINE STEPT
C  IS THEN WRITTEN ENTIRELY IN A.N.S.I. STANDARD BASIC FORTRAN.
C
      COMMON /CSTEP/ X(20),XMAX(20),XMIN(20),DELTX(20),DELMN(20),
     *   ERR(21,20),CHISQ,NV,NTRAC,MATRX,MASK(20)
C
      COMMON /FRODO/ NFMAX,NFLAT,JVARY,NXTRA
C
C  SET THE LIBRARY FUNCTION FOR SINGLE PRECISION (SQRT) OR FOR
C  DOUBLE PRECISION (DSQRT).  NO OTHER FUNCTIONS ARE USED, EITHER
C  EXTERNAL OR INTRINSIC.
C  THE ONLY SUBROUTINES CALLED ARE FUNK, STERR, AND DATSW.
C  STERR COMPUTES THE ERROR MATRIX ERR, IF MATRX IS NONZERO.
C  THE STATEMENT    CALL DATSW(NSSW,JUMP)    RETURNS JUMP=1 IF
C  SENSE SWITCH NUMBER -NSSW- IS ON, AND JUMP=2 IF IT IS OFF.
C  IF NO SENSE SWITCH IS TO BE USED, SUPPLY A DUMMY ROUTINE FOR DATSW.
C
C     QSQRT(Q)=DSQRT(Q)
      QSQRT(Q)=SQRT(Q)
C
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C
C  SET FIXED QUANTITIES ....
C
C                            KW ...  LOGICAL UNIT NUMBER OF THE PRINTER
      KW=6
C                            KTYPE ...  CONSOLE TYPEWRITER UNIT
      KTYPE=64
C                            NSSW ...  TERMINATION SENSE SWITCH NUMBER
      NSSW=6
C                            HUGE ...  A VERY LARGE REAL NUMBER
C                                (DEFAULT VALUE FOR XMAX AND -XMIN)
      HUGE=1.E37
C                            NVMAX ...  MAXIMUM VALUE OF NV
      NVMAX=20
C                            MOSQ ...  MAXIMUM DEPTH OF OSCILLATION
C                                 SEARCH
      MOSQ=5
C                            STCUT ...  RATIO OF STEP SIZE DECREASE
      STCUT=10.
C                            COLIN ...  COLINEARITY TOLERANCE
      COLIN=0.99
C                            CMPMX ...  UPPER BOUND ON COMPR
      CMPMX=.999
C                            NCOMP ...  MAXIMUM NUMBER OF CYCLES
```

```
C                                   WITHOUT ATTEMPTING A GIANT STEP
C                                   (NCOMP.LE.1 DISABLES THE COLINEARITY
C                                   CHECK)
      NCOMP=4
C                              ACK ...  RATIO OF STEP SIZE INCREASE
      ACK=2.0
C                              FACUP ...  IF MORE THAN FACUP STEPS ARE
C                                   TAKEN, THE STEP SIZE IS INCREASED
      FACUP=4.
C                              MXSTP ...  LOG2(MAXIMUM NUMBER OF STEPS)
      MXSTP=5
C                              DELDF ...  DEFAULT VALUE FOR DELTX(J)
      DELDF=.01
C
      RZERO=0.
      RHALF=.5
      RUNIT=1.
      RTWO=2.
      RTEN=10.
C
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C
C  NO REAL CONSTANTS ARE USED BEYOND THIS POINT.
C  CHECK SOME INPUT QUANTITIES, AND SET THEM TO STANDARD VALUES IF
C  DESIRED.  FIRST, MAKE SURE THAT THE TERMINATION SENSE SWITCH IS OFF.
C
      JUMP=2
      CALL DATSW (NSSW,JUMP)
      IF(JUMP-1)10,10,40
C                              ONLY USAGE OF THE CONSOLE TYPEWRITER....
   10 WRITE(KTYPE,20)NSSW
   20 FORMAT(/23H TURN OFF SENSE SWITCH I2//1H )
   30 CALL DATSW (NSSW,JUMP)
      IF(JUMP-1)30,30,40
C                              KWIT ...  TERMINATION SWITCH
   40 KWIT=0
      IF(NV)440,440,50
   50 IF(NV-NVMAX)60,60,440
C                              COMPUTE RELAC, THE RELATIVE ERROR OF THE
C                                   MACHINE AND PRECISION BEING USED.
C                                   RELAC IS USED IN SETTING DELMN(J) TO
C                                   A DEFAULT VALUE.
   60 RELAC=RUNIT
   70 RELAC=RELAC/RTEN
      XPLUS=RUNIT+RELAC
      IF(XPLUS-RUNIT)80,80,70
   80 IF(NCOMP)90,90,100
   90 NCOMP=1
  100 JVARY=0
C                              NACTV ...  NUMBER OF ACTIVE X(J)
      NACTV=0
      DO 260 I=1,NV
      SALVO(I)=RZERO
      IF(MASK(I))260,110,260
C                              CHECK THAT DELTX(I) IS NCT NEGLIGIBLE.
  110 IF(DELTX(I))120,140,120
  120 XPLUS=X(I)+DELTX(I)
      IF(XPLUS-X(I))130,140,130
  130 XPLUS=X(I)-DELTX(I)
      IF(XPLUS-X(I))170,140,170
  140 IF(X(I))150,160,150
  150 DELTX(I)=DELDF*X(I)
      GO TO 170
  160 DELTX(I)=DELDF
  170 IF(DELMN(I))190,180,200
  180 DELMN(I)=DELTX(I)*RELAC
      IF(DELMN(I))190,200,200
  190 DELMN(I)=-DELMN(I)
  200 IF(XMAX(I)-XMIN(I))210,210,220
  210 XMAX(I)=HUGE
      XMIN(I)=-HUGE
  220 NACTV=NACTV+1
C                              X(I)=AMAX1(XMIN(I),AMIN1(XMAX(I),X(I)))
      IF(X(I)-XMAX(I))240,240,230
  230 X(I)=XMAX(I)
  240 IF(X(I)-XMIN(I))250,260,260
  250 X(I)=XMIN(I)
  260 CONTINUE
C
C  COMPUTE COMPR.  THE PROBABILITY THAT THE COSINE OF THE ANGLE
C  BETWEEN TWO RANDOM DIRECTIONS EXCEEDS COMPR IS APPROXIMATELY
C  (1-COLIN)/2 .
C
      COMPR=RZERO
      IF(NACTV-1)440,310,270
  270 A=NACTV
      SUB=RTWO/(A-RUNIT)
      P=RTWO*(RUNIT/QSORT(A)/(RUNIT-RHALF**SUB)-RUNIT)
      COMPR=(RUNIT-(RUNIT-COLIN)**SUB)*(RUNIT+P*(RUNIT-CCLIN))
C
C                              COMPR=AMIN1(CMPMX,ABS(COMPR))
      IF(COMPR)280,290,290
  280 COMPR=-COMPR
  290 IF(COMPR-CMPMX)310,310,300
  300 COMPR=CMPMX
C
  310 IF(NTRAC)400,320,320
  320 WRITE(KW,330)
  330 FORMAT(56H1ENTER SUBROUTINE STEPT.  COPYRIGHT 1965, J. P. CHANDLER
     *      //19H INITIAL VALUES.... /1H )
      WRITE(KW,340)(MASK(J),J=1,NV)
  340 FORMAT(/10H MASK    =  9(I6,6X)/(4X9I12))
      WRITE(KW,350)(X(J),J=1,NV)
  350 FORMAT(/10H X       =  9E12.4/(10X 9E12.4))
      WRITE(KW,360)(XMAX(J),J=1,NV)
  360 FORMAT(/10H XMAX    =  9E12.4/(10X 9E12.4))
      WRITE(KW,370)(XMIN(J),J=1,NV)
  370 FORMAT(/10H XMIN    =  9E12.4/(10X 9E12.4))
      WRITE(KW,380)(DELTX(J),J=1,NV)
  380 FORMAT(/10H DELTX   =  9E12.4/(10X 9E12.4))
      WRITE(KW,390)(DELMN(J),J=1,NV)
  390 FORMAT(/10H DELMN   =  9E12.4/(10X 9E12.4))
  400 CALL FUNK
      CHSAV=CHISQ
      CALL FUNK
C                              NF ...  NUMBER OF FUNCTICN CALLS
      NF=2
      IF(CHISQ-CHSAV)410,430,410
  410 WRITE(KW,420)CHSAV,CHISQ,NF
  420 FORMAT(///3(/59H WARNING....  CHISQ IS NOT A REPRODUCIBLE FUNCTION
     * OF X(J).  )/5X 8HCHSAV = E22.14,5X 8HCHISQ = E22.14,5X5HNF = I5)
C
C                              JOCK ...  SWITCH USED IN SETTING JVARY
  430 JOCK=1
      IF(NTRAC)470,450,450
  440 KWIT=1
  450 WRITE(KW,460)NV,NACTV,MATRX,NCOMP,NFMAX,NFLAT,
     *     RELAC,STCUT,ACK,COLIN,COMPR,CHISQ
  460 FORMAT(//1X I3,11H VARIABLES,I3,8H ACTIVE.10X7HMATRX =I4,10X
     *     7HNCOMP =I2,10X7HNFMAX =I8,10X7HNFLAT =I2//
     *   8H RELAC =E10.3,8X7HSTCUT =E10.3,8X5HACK =E10.3,8X
     *   7HCOLIN =E10.3,8X7HCOMPR =E10.3///8H CHISQ =E16.9///
```

```
     *   23H BEGIN MINIMIZATION....    )
  470 IF(KWIT)480,480,2140
  480 IF(NTRAC)510,510,490
  490 WRITE(KW,500)
  500 FORMAT(//60(2H *)//10X29HTRACE MAP OF THE MINIMIZATION //1H )
C
  510 DO 520 I=1,NV
C                             VEC(J) ...  CURRENT VECTOR OF NUMBER OF
C                                  STEPS IN X(J)
      VEC(I)=RZERO
C                             DX(J) ...  CURRENT STEP SIZE FOR X(J)
  520 DX(I)=DELTX(I)
C                             CHOLD ...  BEST PREVIOUS VALUE OF CHISQ
      CHOLD=CHISQ
C                             NOSC ...  CURRENT DEPTH OF THE OSCILLATION
C                                  INFORMATION
      NOSC=0
C
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C
C  VARY THE PARAMETERS ONE AT A TIME.
C  THIS IS THE STARTING POINT USED EACH TIME THE STEP SIZE IS REDUCED
C  OR A SUCCESSFUL GIANT STEP IS COMPLETED.
C
C                             NCIRC ...  NUMBER OF CONSECUTIVE X(J)
C                                  WITHOUT SIZEABLE CHANGES
  530 NCIRC=0
C                             NZIP ...  NUMBER OF CONSECUTIVE CYCLES
C                                  WITHOUT A GIANT STEP
      NZIP=0
C
C  MAIN DO LOOP FOR CYCLING THROUGH THE VARIABLES.
C  FIRST TRIAL STEP WITH EACH VARIABLE IS SEPARATE.
C
C                             NACK ...  NUMBER OF ACTIVE X(J) CYCLED
C                                  THROUGH
  540 NACK=0
      DO 1770 I=1,NV
C                             JFLAT(J) ...  NONZERO IF CHANGING X(J) DID
C                                  NOT CHANGE CHISQ
      JFLAT(I)=0
C                             OLVEC(J) ...  OLD VECTOR OF NUMBER OF
C                                  STEPS IN X(J)
      OLVEC(I)=VEC(I)
      VEC(I)=RZERO
C                             TRIAL(J) ...  CHANGE IN X(J)
      TRIAL(I)=RZERO
      IF(MASK(I))550,560,550
  550 VEC(I)=-RZERO
      JFLAT(I)=1
      GO TO 1750
  560 NACK=NACK+1
C                             ADX=ABS(DX(I))
      ADX=DX(I)
      IF(ADX)570,580,580
  570 ADX=-ADX
  580 SAVE=X(I)
C                             CHECK THAT DX(I) IS NOT NEGLIGIBLE.
      XPLUS=SAVE+DX(I)
      IF(XPLUS-SAVE)590,600,590
  590 XPLUS=SAVE-0X(I)
      IF(XPLUS-SAVE)610,600,610
  600 JFLAT(I)=2
      GO TO 810
C                             STEP X(I).
  610 X(I)=SAVE+DX(I)
      JVARY=0
      IF(JOCK)630,630,620
  620 JOCK=0
      JVARY=I
C                             NFLAG ...  COUNTER USED IN SETTING JFLAG(J)
  630 NFLAG=1
      IF(X(I)-XMIN(I))650,640,640
  640 IF(X(I)-XMAX(I))660,660,650
  650 NFLAG=NFLAG+3
      GO TO 680
  660 CALL FUNK
      NF=NF+1
      JVARY=I
C                             SAVE OLD VALUE OF CHISQ FOR INTERPOLATION.
      CHIME=CHISO
      IF(CHISO-CHOLD)850,670,680
  670 NFLAG=NFLAG+1
C                             STEP X(I) THE OTHER WAY.
  680 XPLUS=X(I)
      X(I)=SAVE-DX(I)
      IF(X(I)-XMIN(I))820,690,690
  690 IF(X(I)-XMAX(I))700,700,820
  700 CALL FUNK
      NF=NF+1
      JVARY=I
      IF(CHISO-CHOLD)840,710,720
  710 NFLAG=NFLAG+1
  720 IF(NFLAG-3)730,800,820
C                             PERFORM PARABOLIC INTERPOLATION.
C                                  CHECK FOR ZERO DENOMINATOR, ETC.
C
  730 IF(CHISO-CHIME)740,820,740
  740 DENOM=(CHISQ-CHOLD)-(CHOLD-CHIME)
      IF(DENOM)750,820,750
  750 TRIAL(I)=DX(I)*(CHISQ-CHIME)/(RTWO*DENOM)
      VEC(I)=TRIAL(I)/ADX
      X(I)=SAVE+TRIAL(I)
      IF(X(I)-SAVE)770,760,770
  760 CHISQ=CHOLD
      GO TO 790
  770 CALL FUNK
      NF=NF+1
      IF(CHISQ-CHOLD)780,790,790
  780 CHOLD=CHISQ
      JOCK=1
      GO TO 830
  790 TRIAL(I)=RZERO
      VEC(I)=RZERO
      GO TO 820
  800 JFLAT(I)=1
  810 VEC(I)=-RZERO
  820 X(I)=SAVE
  830 NCIRC=NCIRC+1
      IF(NCIRC-NACTV)960,1840,1840
C                             FLIP DX(I) FOR MORE EFFICIENT OPERATION.
  840 DX(I)=-DX(I)
C
C  A LOWER VALUE OF CHISO HAS BEEN FOUND.  STEP, DOUBLE THE STEP SIZE,
C  AND REPEAT AS LONG AS CHISQ DECREASES, UP TO MXSTP TIMES.
C
  850 NCIRC=0
      DEL=DX(I)
      NSTP=0
  860 CHIME=CHOLD
      CHOLD=CHISQ
      VEC(I)=VEC(I)+DEL/ADX
```

```
      TRIAL(I)=TRIAL(I)+DEL
      NSTP=NSTP+1
      IF(NSTP-MXSTP)870,940,940
  870 DEL=ACK*DEL
      XPLUS=SAVE
      SAVE=X(I)
      X(I)=SAVE+DEL
      IF(X(I)-XMIN(I))950,880,880
  880 IF(X(I)-XMAX(I))890,890,950
  890 CALL FUNK
      NF=NF+1
      IF(CHISQ-CHOLD)860,900,900
C
C                            PERFORM PARABOLIC INTERPOLATION.
  900 DXZ=SAVE-XPLUS
      DXU=X(I)-SAVE
      DCZ=CHOLD-CHIME
      DCU=CHISQ-CHOLD
      DENOM=DCZ*DXU-DCU*DXZ
      IF(DENOM)910,950,910
  910 DEL=(DCZ*DXU**2+DCU*DXZ**2)/(RTWO*DENOM)
      X(I)=SAVE+DEL
      IF(X(I)-SAVE)920,960,920
  920 CALL FUNK
      NF=NF+1
      IF(CHISQ-CHOLD)930,950,950
  930 CHOLD=CHISQ
      TRIAL(I)=TRIAL(I)+DEL
      VEC(I)=VEC(I)+DEL/ADX
  940 JOCK=1
      GO TO 960
  950 X(I)=SAVE
C                            DO NOT INCREASE THE STEP SIZE PREMATURELY.
  960 IF(NZIP)970,970,990
  970 IF(NCOMP-I)980,980,1740
  980 IF(NACK-1)1740,1740,990
C                            AVEC=ABS(VEC(I))
  990 AVEC=VEC(I)
      IF(AVEC)1000,1010,1010
 1000 AVEC=-AVEC
 1010 IF(AVEC-FACUP)1080,1020,1020
C
C                            INCREASE THE STEP SIZE.
 1020 DX(I)=ACK*ADX
      VEC(I)=VEC(I)/ACK
      OLVEC(I)=OLVEC(I)/ACK
      IF(NOSC)1050,1050,1030
 1030 DO 1040 J=1,NOSC
 1040 ERR(I,J)=ERR(I,J)/ACK
 1050 IF(NTRAC)1080,1080,1060
 1060 WRITE(KW,1070)I,DX(I)
 1070 FORMAT(10H STEP SIZEI3,14H INCREASED TO E13.5)
C
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C
C  STEP ALONG A RESULTANT DIRECTION, IF POSSIBLE.
C  FIRST CHECK THE COLINEARITY OF VEC AND OLVEC.  SINCE THESE ARE
C  NUMBERS OF STEPS, THE TEST IS SCALE-INVARIANT.
C
C                            CHECK THE COLINEARITY OF VEC AND OLVEC.
 1080 SUMO=RZERO
      SUMV=RZERO
      DO 1090 J=1,NV
      SUMO=SUMO+OLVEC(J)**2
 1090 SUMV=SUMV+VEC(J)**2
      IF(SUMO*SUMV)1740,1740,1100
 1100 SUMO=QSQRT(SUMO)
      SUMV=QSQRT(SUMV)
      COSIN=RZERO
      DO 1110 J=1,NV
 1110 COSIN=COSIN+(OLVEC(J)/SUMO)*(VEC(J)/SUMV)
      IF(NZIP-NCOMP)1120,1150,1150
 1120 IF(NZIP-1)1740,1130,1140
 1130 IF(NACK-NACTV)1740,1140,1140
 1140 IF(COSIN-COMPR)1740,1150,1150
 1150 IF(VEC(I))1160,1740,1160
 1160 NONZR=0
      DO 1180 J=1,NV
      IF(VEC(J))1170,1180,1170
 1170 NONZR=NONZR+1
 1180 CONTINUE
      IF(NONZR-2)1740,1190,1190
C
C  SIMON SAYS, TAKE AS MANY GIANT STEPS AS POSSIBLE...
C
 1190 IF(MOSQ)1370,1370,1200
 1200 CONTINUE
C
C X X X X X X X X X X X X X X X X X X X X X X X X X X X X X X X X X X X
C
C  TO DELETE THE OSCILLATION SEARCH SECTION, SET MOSQ=0, REMOVE ALL
C  STATEMENTS BETWEEN THIS POINT AND THE NEXT COMMENT CARD OF X-S, AND
C  REMOVE THE STATEMENT SURROUNDED BY X-S FURTHER ON.
C
C                            KL ...  POINTER FOR OSCILLATION CHECK
      KL=1
C                            STORE OSCILLATION INFORMATION.
      NOSC=NOSC+1
      IF(NOSC-MOSQ)1240,1240,1210
 1210 NOSC=MOSQ
      IF(NOSC-1)1370,1240,1220
C                            THE STACK OF OSCILLATION INFORMATION IS
C                                 FULL.  PUSH IT DOWN, THROWING AWAY
C                                 THE OLDEST ITEM.
 1220 DO 1230 K=2,NOSC
      CHIOS(K-1)=CHIOS(K)
      DO 1230 J=1,NV
      XOSC(J,K-1)=XOSC(J,K)
 1230 ERR(J,K-1)=ERR(J,K)
C                            ADD THE NEW ITEM.
 1240 DO 1250 J=1,NV
      XOSC(J,NOSC)=X(J)
 1250 ERR(J,NOSC)=VEC(J)/SUMV
      CHIOS(NOSC)=CHOLD
      IF(NOSC-2)1370,1260,1260
C
C  SEARCH FOR A PREVIOUS SUCCESSFUL GIANT STEP IN A DIRECTION MORE
C  NEARLY PARALLEL TO THE DIRECTION OF THE PROPOSED STEP THAN WAS THE
C  IMMEDIATELY PREVIOUS ONE.  THIS MAY MEAN THAT THE DIRECTIONS OF THE
C  GIANT STEPS OSCILLATE PERIODICALLY (ZIG-ZAG).  TRY GIGANTIC
C  (OSCILLATION) STEPS OF DECREASING PERIOD, THEN ORDINARY GIANT STEPS.
C
 1260 COXCM=RZERO
      DO 1270 J=1,NV
 1270 COXCM=COXCM+ERR(J,NOSC)*ERR(J,NOSC-1)
      NAH=NOSC-1
 1280 DO 1310 K=KL,NAH
      NRETR=NAH-K
      COSIN=RZERO
      DO 1290 J=1,NV
 1290 COSIN=COSIN+ERR(J,NOSC)*ERR(J,K)
      IF(K-NAH)1300,1320,1320
```

```
 1300 IF(COSIN-COXCM)1310,1320,1320
 1310 CONTINUE
      GO TO 1370
 1320 KL=K+1
      IF(NTRAC)1350,1350,1330
 1330 NT=NOSC-K
      WRITE(KW,1340)NT,COXCM,COSIN
 1340 FORMAT(/1X8H********5X26HGIGANTIC STEP WITH PERIOD I2,
     *   35H BEING ATTEMPTED.   COXCM, COSIN = 2E13.4)
 1350 DO 1360 J=1,NV
      SALVO(J)=TRIAL(J)
 1360 TRIAL(J)=X(J)-XOSC(J,K)
      CHBAK=CHIOS(K)
      GO TO 1380
C
C X X X X X X X X X X X X X X X X X X X X X X X X X X X X X X X X X X
C
C  PERFORM GIANT STEPS OR GIGANTIC (OSCILLATION) STEPS.
C
 1370 CHBAK=CHI(I)
C                             NRETR ...  NUMBER OF OSCILLATION PERIODS
C                                  YET TO BE TESTED (=-1 IF A GIANT STEP
C                                  IS BEING TRIED)
      NRETR=-1
C                             NGIAN...  NUMBER OF GIANT OR GIGANTIC
C                                  STEPS COMPLETED
 1380 NGIAN=0
 1390 DO 1440 J=1,NV
      XSAVE(J)=X(J)
      IF(MASK(J))1440,1400,1440
 1400 X(J)=X(J)+TRIAL(J)
C                             X(J)=AMAX1(AMIN1(X(J),XMAX(J)),XMIN(J))
      IF(X(J)-XMAX(J))1420,1420,1410
 1410 X(J)=XMAX(J)
 1420 IF(X(J)-XMIN(J))1430,1440,1440
 1430 X(J)=XMIN(J)
 1440 TRIAL(J)=ACK*TRIAL(J)
      JOCK=0
      JVARY=0
      CALL FUNK
      NF=NF+1
      IF(CHISQ-CHOLD)1450,1520,1520
 1450 CHBAK=CHOLD
      CHOLD=CHISQ
      NGIAN=NGIAN+1
      IF(NTRAC)1390,1390,1460
 1460 IF(NGIAN-1)1470,1470,1500
 1470 WRITE(KW,1480)CHBAK,NF,(VEC(J),J=1,I)
 1480 FORMAT(//8H CHISQ =E16.8,8X4HNF =I7//5X16HNO. OF STEPS =  9E11.3/
     *    (21X9E11.3))
      WRITE(KW,1490)(XSAVE(J),J=1,NV)
 1490 FORMAT(9H X(I)..../(1X9E13.5))
 1500 WRITE(KW,1510)CHISQ,NF,(X(J),J=1,NV)
 1510 FORMAT(/8H CHISQ =E16.8,8X4HNF =I7/9H X(I)..../(1X9E13.5))
      GO TO 1390
C                             DO NOT INTERPOLATE AFTER AN UNSUCCESSFUL
C                                  GIANT STEP.
 1520 IF(NGIAN)1600,1600,1530
C                             PERFORM PARABOLIC INTERPOLATION.
C
 1530 DENOM=ACK*CHBAK-(ACK+RUNIT)*CHOLD+CHISQ
      IF(DENOM)1540,1600,1540
 1540 CINDR=(ACK**2*CHBAK-(ACK**2-RUNIT)*CHOLD-CHISQ)/(RTWO*ACK*DENOM)
      DO 1590 J=1,NV
      IF(MASK(J))1590,1550,1590
 1550 X(J)=XSAVE(J)+CINDR*TRIAL(J)
C                             X(J)=AMAX1(AMIN1(X(J),XMAX(J)),XMIN(J))
      IF(X(J)-XMAX(J))1570,1570,1560
 1560 X(J)=XMAX(J)
 1570 IF(X(J)-XMIN(J))1580,1590,1590
 1580 X(J)=XMIN(J)
 1590 CONTINUE
      JOCK=0
      JVARY=0
      CALL FUNK
      NF=NF+1
      IF(CHISQ-CHOLD)1670,1600,1600
 1600 DO 1610 J=1,NV
      TRIAL(J)=SALVO(J)
 1610 X(J)=XSAVE(J)
      IF(NTRAC)1640,1640,1620
 1620 WRITE(KW,1630)CHOLD,NGIAN
 1630 FORMAT(/8H CHISQ =E16.8,7H  AFTERI3,13H GIANT STEPS.  )
      WRITE(KW,1490)(X(J),J=1,NV)
 1640 IF(NGIAN)1650,1650,1700
 1650 CONTINUE
C
C X X X X X X X X X X X X X X X X X X X X X X X X X X X X X X X X X X
C
C  IF THE OSCILLATION SEARCH IS DELETED, DELETE THE FOLLOWING STATEMENT.
C
      IF(NRETR)1720,1660,1280
C
C X X X X X X X X X X X X X X X X X X X X X X X X X X X X X X X X X X
C
C                             IF ALL GIGANTIC STEPS WERE UNSUCCESSFUL,
C                                  TRY A GIANT STEP.
 1660 IF(NRETR)1720,1370,1720
C
 1670 CHOLD=CHISQ
      JOCK=1
      IF(NTRAC)1700,1700,1680
 1680 STEPS=NGIAN
      STEPS=STEPS+CINDR
      WRITE(KW,1690)CHOLD,STEPS
 1690 FORMAT(/8H CHISQ =E16.8,7H  AFTERE11.3,13H GIANT STEPS.   )
      WRITE(KW,1490)(X(J),J=1,NV)
 1700 IF(NRETR)530,1710,1710
 1710 CONTINUE
C                             A SUCCESSFUL GIGANTIC STEP HAS OCCURRED.
      GO TO 530
C                             AN UNSUCCESSFUL GIANT STEP HAS OCCURRED.
C                                  DELETE ITS OSCILLATION INFORMATION.
C                                  NOSC=MAX0(NOSC-1,0)
 1720 NOSC=NOSC-1
      IF(NOSC)1730,1740,1740
 1730 NOSC=0
 1740 CHI(I)=CHOLD
 1750 CONTINUE
C                             RETURN IF THE SENSE SWITCH IS ON.
      CALL DATSW (NSSW,JUMP)
      IF(JUMP-1)2110,2110,1760
C
 1760 IF(NF-NFMAX)1770,1770,2090
 1770 CONTINUE
C                             END OF THE MAIN DO LOOP.
C
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C
C  ANOTHER CYCLE THROUGH THE VARIABLES HAS BEEN COMPLETED.
C  PRINT ANOTHER LINE OF TRACES.
C
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```
      IF(NTRAC)1790,1790,1780
 1780 WRITE(KW,1480)CHOLD,NF,(VEC(J),J=1,NV)
 1790 IF(NZIP)1830,1800,1830
 1800 IF(NTRAC)1830,1830,1810
 1810 WRITE(KW,1490)(X(J),J=1,NV)
      WRITE(KW,1820)
 1820 FORMAT(1H )
 1830 NZIP=NZIP+1
      GO TO 540
C
C  A NEW BASE POINT HAS BEEN FOUND.  PRINT THE REMAINING TRACES.
C
 1840 IF(NTRAC)1860,1860,1850
 1850 WRITE(KW,1480)CHOLD,NF,(VEC(J),J=1,I)
      WRITE(KW,1490)(X(J),J=1,NV)
C
C  DECREASE THE SIZE OF THE STEPS FOR ALL VARIABLES.
C
 1860 CONTINUE
C                              RETURN IF THE SENSE SWITCH IS ON.
      CALL DATSW (NSSW,JUMP)
      IF(JUMP-1)2110,2110,1870
C
 1870 IF(NF-NFMAX)1880,1880,2090
C                              CHECK WHETHER ALL ABS(DX(J)) .LE. DELMN(J).
 1880 NGATE=1
      DO 1930 J=1,NV
      IF(MASK(J))1930,1890,1930
C                              ADX=ABS(DX(J))
 1890 ADX=DX(J)
      IF(ADX)1900,1910,1910
 1900 ADX=-ADX
 1910 IF(ADX-DELMN(J))1930,1930,1920
 1920 NGATE=0
 1930 DX(J)=DX(J)/STCUT
      IF(NGATE)1970,1970,1940
 1940 IF(NTRAC)2150,1950,1950
 1950 WRITE(KW,1960)
 1960 FORMAT(///65H TERMINATED WHEN THE STEP SIZES BECAME AS SMALL AS TH
     *E DELMN(J).    )
      GO TO 2150
C                              CHECK THE JFLAT(J).
 1970 IF(NFLAT)2060,2060,1980
 1980 JFLMN=5
      DO 2010 J=1,NV
      IF(MASK(J))2010,1990,2010
 1990 IF(JFLAT(J)-JFLMN)2000,2010,2010
 2000 JFLMN=JFLAT(J)
 2010 CONTINUE
      IF(JFLMN-1)2060,2020,2020
 2020 IF(NTRAC)2150,2030,2030
 2030 WRITE(KW,2040)
 2040 FORMAT(///49H TERMINATED WHEN THE FUNCTION VALUES AT ALL TRIAL
     *   23H POINTS WERE IDENTICAL.  )
      WRITE(KW,2050)(DX(J),J=1,NV)
 2050 FORMAT(///23H CURRENT STEP SIZES.... //(1X9E13.5))
      GO TO 2150
 2060 IF(NTRAC)530,530,2070
C                              PRINT THE DX(J) AND SEARCH SOME MORE.
C
 2070 WRITE(KW,2080)(DX(J),J=1,NV)
 2080 FORMAT(//60(1X1H*)//26H STEP SIZES REDUCED TO....//(1X9E13.5))
      GO TO 530
C
 2090 WRITE(KW,2100)NFMAX
 2100 FORMAT(///46H ABNORMAL TERMINATION....   MORE THAN NFMAX = I7,
     *   31H CALLS TO THE CHISQ SUBROUTINE.)
      GO TO 2130
C
 2110 WRITE(KW,2120)
 2120 FORMAT(///42H SUBROUTINE STEPIT TERMINATED BY OPERATOR.)
C
 2130 WRITE(KW,2050)(DX(J),J=1,NV)
C
C                              SET SWITCH FOR TERMINATION.
 2140 KWIT=1
C                              CALL FUNK WITH THE BEST SET OF X(J).
 2150 JVARY=0
      CALL FUNK
      IF(CHISQ-CHSAV)2170,2170,2160
 2160 WRITE(KW,420)CHSAV,CHISQ,NF
 2170 IF(NTRAC)2210,2180,2180
 2180 WRITE(KW,2190)NF,(X(J),J=1,NV)
 2190 FORMAT(//1XI5,23H FUNCTION COMPUTATIONS
     *   ///10X24HFINAL VALUES OF X(J)....//(1X5E22.14))
      WRITE(KW,2200)CHISQ
 2200 FORMAT(//24H FINAL VALUE OF CHISQ = E22.14//)
 2210 IF(KWIT)2260,2220,2260
C                              MATD=IABS(MATRX-100)
 2220 MATD=MATRX-100
      IF(MATD)2230,2240,2240
 2230 MATD=-MATD
 2240 IF(MATD-50)2250,2250,2260
C
C                              SKIP ERROR CALCULATION IF ANY MASK(J).NE.0.
 2250 IF(NACTV-NV)2260,2270,2260
 2260 RETURN
C                              SET THE STEP SIZES FOR SUBROUTINE STERR.
 2270 FAC=RTEN**(MATRX-100)
      DO 2280 I=1,NV
 2280 DX(I)=FAC*DX(I)
C                              CALL STERR TO COMPUTE AN APPROXIMATE
C                                   ERROR MATRIX.
C
      CALL STERR (FUNK,KW,NSSW,DX,NF,XSAVE,TRIAL)
      GO TO 2140
C  END STEPT.
      END
      BLOCK DATA
C
C  BLOCK DATA SUBPROGRAM FOR STEPIT, SIMPLEX, AND STP.
C  ELIMINATE IF COMMON IS UNLABELLED, AND SET THE VARIABLES BEFORE
C  CALLING STEPT.
C
      COMMON /FRODO/ NFMAX,NFLAT,JVARY,NXTRA
      DATA NFMAX/1000000/, NFLAT/1/, NXTRA/0/
      END
      SUBROUTINE DATSW (NSSW,JUMP)
C
C  DUMMY VERSION OF SUBROUTINE DATSW (ALL SWITCHES PERMANENTLY OFF).
C
      JUMP=2
      RETURN
      END
      SUBROUTINE STERR (FUNK,KW,NSSW,DX,NF,XSAVE,TRIAL)
C
C  STERR 1.0        A.N.S.I. STANDARD FORTRAN         JANUARY 1973
C  J. P. CHANDLER, COMPUTER SCIENCE DEPT., OKLAHOMA STATE UNIVERSITY
C
C  STERR COMPUTES AN APPROXIMATE ERROR MATRIX FOR A NONLINEAR
C  FITTING PROBLEM.
C  THE VALUES COMPUTED ARE OFTEN POOR APPROXIMATIONS.  THEY SHOULD BE
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```
C  CHECKED USING SUBROUTINE FIDO.
C
C  INPUT QUANTITIES.....  FUNK,KW,NSSW,DX,NF,X
C  OUTPUT QUANTITIES....  NF,ERR
C  SCRATCH STORAGE......  XSAVE,TRIAL
C
C  DX(J) ARE THE STEP SIZES FOR APPROXIMATING THE DERIVATIVES OF CHISQ
C  WITH RESPECT TO X(J) BY FINITE DIFFERENCES.  SEE STEPT FOR
C  DEFINITIONS OF ALL OTHER QUANTITIES.
C  XMAX, XMIN, AND MASK ARE IGNORED IN STERR.
C
C      DOUBLE PRECISION X,XMAX,XMIN,DELTX,DELMN,ERR,CHISQ,DX,TRIAL,XSAVE,
C     X   SECND,CHOLD,RZERO,RUNIT,TENLN,SNDET,DETLN,ABER,DENOM
C      DOUBLE PRECISION P,AP,Q,QSQRT,DSQRT,QLOG,DLCG,DXDEF,RTWO
C
      DIMENSION DX(20),XSAVE(20),TRIAL(20)
      DIMENSION SECND(2,2)
C
      COMMON /CSTEP/ X(20),XMAX(20),XMIN(20),DELTX(20),DELMN(20),
     *   ERR(21,20),CHISQ,NV,NTRAC,MATRX,MASK(20)
      COMMON /FRODO/ NFMAX,NFLAT,JVARY,NXTRA
C
C     QSQRT(Q)=DSQRT(Q)
      QSQRT(Q)=SQRT(Q)
C     QLOG(Q)=DLOG(Q)
      QLOG(Q)=ALOG(Q)
C
C                              DXDEF ...  DEFAULT VALUE FOR DX
      DXDEF=.001
      RZERO=0.
      RUNIT=1.
      RTWO=2.
      TENLN=2.303
C
      DO 5030 J=1,NV
      IF(DX(J))5020,5000,5030
 5000 DX(J)=DXDEF*X(J)
      IF(DX(J))5020,5010,5030
 5010 DX(J)=DXDEF
      GO TO 5030
 5020 DX(J)=-DX(J)
 5030 XSAVE(J)=X(J)
      CALL FUNK
      NF=NF+1
      CHOLD=CHISQ
      IF(NTRAC)5070,5040,5040
 5040 WRITE(KW,5050)
 5050 FORMAT(41HISIZES OF INCREMENTS TO BE USED BELOW....)
      WRITE(KW,5060)(DX(J),J=1,NV)
 5060 FORMAT(/(1X9E13.5))
C
C  COMPUTE THE (SYMMETRIC) MATRIX OF SECOND PARTIAL DERIVATIVES OF
C  CHISQ WITH RESPECT TO THE X(J).
C
C                              COMPUTE THE DIAGONAL PARTIALS FIRST.
 5070 DO 5090 I=1,NV
      JVARY=0
      DO 5080 J=1,2
      X(I)=XSAVE(I)+DX(I)
      CALL FUNK
      NF=NF+1
      JVARY=I
      SECND(1,J)=CHISQ
 5080 DX(I)=-DX(I)
      X(I)=XSAVE(I)
 5090 ERR(I,I)=((SECND(1,1)-CHOLD)-(CHOLD-SECNO(1,2)))/DX(I)**2
C
C                              COMPUTE THE OFF-DIAGONAL PARTIALS.  USE A
C                                   REDUNDANT FOUR-POINT RULE FOR GREATER
C                                   RELIABILITY.
      IF(NV-2)5140,5100,5100
 5100 DO 5130 I=2,NV
      IM=I-1
      DO 5130 J=1,IM
      DO 5120 K=1,2
      X(I)=XSAVE(I)+DX(I)
      JVARY=0
      DO 5110 L=1,2
      X(J)=XSAVE(J)+DX(J)
      CALL FUNK
      NF=NF+1
      JVARY=J
      SECND(K,L)=CHISQ
      X(J)=XSAVE(J)
 5110 DX(J)=-DX(J)
      X(I)=XSAVE(I)
C                              RETURN IF THE SENSE SWITCH IS ON.
      JUMP=2
      CALL DATSW (NSSW,JUMP)
      IF(JUMP-1)5580,5580,5120
C
 5120 DX(I)=-DX(I)
 5130 ERR(I,J)=((SECND(1,1)-SECND(1,2))-(SECND(2,1)-SECNO(2,2)))/
     *   (RTWO*DX(I)*RTWO*DX(J))
C
C                              END OF THE DERIVATIVE COMPUTATION
 5140 IF(NTRAC)5180,5150,5150
 5150 WRITE(KW,5160)
 5160 FORMAT(/////45H MATRIX OF THE SECOND PARTIAL DERIVATIVES.... /1H )
      DO 5170 I=1,NV
 5170 WRITE(KW,5060)(ERR(I,J),J=1,I)
C
 5180 DO 5190 I=1,NV
      DO 5190 J=1,I
      IF(ERR(I,J))5190,5200,5190
 5190 CONTINUE
      GO TO 5220
 5200 WRITE(KW,5210)
 5210 FORMAT(////46H THE ABOVE MATRIX CONTAINS ONE OR MORE ZEROES. /
     *   51H PERHAPS A LARGER VALUE OF -MATRX- SHOULD BE TRIED,
     *   31H TO SEE IF THEY ARE LEGITIMATE.    )
C
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C
C  INVERT THE MATRIX OF SECOND PARTIAL DERIVATIVES USING THE GAUSS-
C  JORDAN METHOD (F. L. BAUER AND C. REINSCH, P. 45 IN -LINEAR ALGEBRA-
C  BY J. H. WILKINSON AND C. REINSCH (SPRINGER-VERLAG, 1971)).
C  ONLY THE LOWER TRIANGLE OF ERR IS USED OR ALTERED.
C
 5220 DETLN=RZERO
      SNDET=RUNIT
C                              NOTPD ...  =1 IF THE MATRIX IS NOT
C                                   POSITIVE DEFINITE
      NOTPD=0
      DO 5350 KK=1,NV
      K=NV+1-KK
      P=ERR(1,1)
      AP=P
      IF(P)5250,5230,5260
 5230 WRITE(KW,5240)
 5240 FORMAT(////27H ERROR MATRIX IS SINGULAR.
     *      37H PERHAPS -MATRX- SHOULD BE INCREASED.   ////1H )
```

```
      GO TO 5580
 5250 NOTPD=1
      SNDET=-SNDET
      AP=-AP
 5260 DETLN=DETLN+QLOG(AP)/TENLN
      IF(NV-2)5320,5270,5270
 5270 DO 5310 J=2,NV
      Q=ERR(J,1)
      IF(J-K)5290,5290,5280
 5280 XSAVE(J)=Q/P
      GO TO 5300
 5290 XSAVE(J)=-Q/P
 5300 DO 5310 L=2,J
 5310 ERR(J-1,L-1)=ERR(J,L)+Q*XSAVE(L)
 5320 ERR(NV,NV)=RUNIT/P
      IF(NV-2)5350,5330,5330
 5330 DO 5340 J=2,NV
 5340 ERR(NV,J-1)=XSAVE(J)
 5350 CONTINUE
C
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C
C  PRINT THE ERRORS AND CORRELATIONS, AND RETURN.
C
      IF(NOTPD)5380,5380,5360
 5360 WRITE(KW,5370)
 5370 FORMAT(////44H THE ERROR MATRIX IS NOT POSITIVE DEFINITE.
     *       37H PERHAPS -MATRX- SHOULD BE DECREASED.   )
 5380 IF(NTRAC)5410,5390,5390
 5390 WRITE(KW,5400)DETLN,SNDET
 5400 FORMAT(////51H ALOG10(MAGNITUDE OF DETERMINANT OF ABOVE MATRIX) =
     *    E13.5,10X22HSIGN OF DETERMINANT = F4.1)
C
C                                THE ERROR MATRIX IS TWICE THE INVERSE OF
C                                     THE MATRIX OF SECOND DERIVATIVES.
 5410 DO 5480 I=1,NV
      DO 5420 J=1,I
      ERR(I,J)=ERR(I,J)*RTWO
C                                RETURN THE FULL MATRIX.
 5420 ERR(J,I)=ERR(I,J)
C                                XSAVE(I)=SIGN(SQRT(ABS(ERR(I,I))),ERR(I,I))
      ABER=ERR(I,I)
      IF(ABER)5430,5480,5440
 5430 ABER=-ABER
 5440 ABER=QSQRT(ABER)
      IF(ERR(I,I))5450,5460,5480
 5450 ABER=-ABER
 5460 WRITE(KW,5470)ERR(I,I)
 5470 FORMAT( ///50H NEGATIVE OR ZERO MEAN SQUARE ERROR ENCOUNTERED...
     *    3XE16.8/37H PERHAPS -MATRX- SHOULD BE DECREASED. ///1H )
 5480 XSAVE(I)=ABER
      IF(NTRAC)5580,5490,5490
 5490 WRITE(KW,5500)
 5500 FORMAT(/////20H STANDARD ERRORS.... )
      WRITE(KW,5060)(XSAVE(J),J=1,NV)
C
      IF(NV-1)5580,5580,5510
 5510 WRITE(KW,5520)
 5520 FORMAT(/////45H LOWER TRIANGLE OF THE CORRELATION MATRIX.... /1H )
      DO 5570 I=2,NV
      IM=I-1
      DO 5560 J=1,IM
      DENOM=XSAVE(I)*XSAVE(J)
      IF(DENOM)5540,5530,5550
 5530 TRIAL(J)=RZERO
      GO TO 5560
 5540 DENOM=-DENOM
 5550 TRIAL(J)=ERR(I,J)/DENOM
 5560 CONTINUE
 5570 WRITE(KW,5060)(TRIAL(J),J=1,IM)
C
 5580 RETURN
C  END STERR.
      END
//GO.SYSIN DD *
//
```

TABLE XV

PARAMETERS OF SPHERICAL SIX-LINK R-R-R-R-R-R-R MECHANISM

" EXISTENCE CRITERIA OF SIX-LINK,TWO-LOOP R-R-C-C-C-R-C SPACE MECHANISM "

INITIAL VALUES OF THE VARIABLES

N	=	20
NP	=	0
NN	=	99000
DELTA	=	0.500D-01
F	=	0.100D-04
ROW	=	0.500D 00

TWIST ANGLES	X	XMIN	XMAX
ALPHA 12	0.8500D 02	0.0	0.3600D 03
ALPHA 23	0.1200D 03	0.0	0.3600D 03
ALPHA 34	0.1900D 03	0.0	0.3600D 03
ALPHA 41	0.2200D 03	0.0	0.3600D 03
ALPHA 65	0.5500D 02	0.0	0.3600D 03
ALPHA 76	0.1750D 03	0.0	0.3600D 03
ALPHA 52	0.7200D 02	0.0	0.3600D 03
ALPHA 17	0.3120D 03	0.0	0.3600D 03
PHI 1	0.7000D 02	0.0	0.3600D 03
SI 1	0.1200D 03	0.0	0.3600D 03

TABLE XV (Continued)

KINK LINKS	X	XMIN	XMAX
S1	0.0	0.0	0.5000D 01
S2	0.0	0.0	0.5000D 01

LINK-LENGTHS	X	XMIN	XMAX
A 12	0.0	0.0	0.5000D 01
A 23	0.0	0.0	0.5000D 01
A 34	0.0	0.0	0.5000D 01
A 41	0.0	0.0	0.5000D 01
A 65	0.0	0.0	0.5000D 01
A 76	0.0	0.0	0.5000D 01
A 52	0.0	0.0	0.5000D 01
A 17	0.0	0.0	0.5000D 01

TABLE XV (Continued)

```
ENTER SUBROUTINE STEPIT.  COPYRIGHT 1965 J. P. CHANDLER. PHYSICS DEPT., INDIANA UNIVERSITY.

INITIAL VALUES....

MASK   =   0            0            0            0            0            0            0            0            0
           0            0            0            0            0            0            0            0            0
           0            0

X      =   0.1484D 01   0.2094D 01   0.3316D 01   0.3840D 01   0.9599D 00   0.3054D 01   0.1257D 01   0.5445D 01   0.1222D 01
           0.2094D 01   0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0
           0.0          0.0

XMAX   =   0.6283D 01   0.6283D 01   0.6283D 01   0.6283D 01   0.6283D 01   0.6283D 01   0.6283D 01   0.6283D 01   0.6283D 01
           0.6283D 01   0.5000D 01   0.5000D 01   0.5000D 01   0.5000D 01   0.5000D 01   0.5000D 01   0.5000D 01   0.5000D 01
           0.5000D 01   0.5000D 01

XMIN   =   0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0
           0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0
           0.0          0.0

DELTX  =   0.5000D-01   0.5000D-01   0.5000D-01   0.5000D-01   0.5000D-01   0.5000D-01   0.5000D-01   0.5000D-01   0.5000D-01
           0.5000D-01   0.5000D-01   0.5000D-01   0.5000D-01   0.5000D-01   0.5000D-01   0.5000D-01   0.5000D-01   0.5000D-01
           0.5000D-01   0.5000D-01

DELMN  =   0.1000D-04   0.1000D-04   0.1000D-04   0.1000D-04   0.1000D-04   0.1000D-04   0.1000D-04   0.1000D-04   0.1000D-04
           0.1000D-04   0.1000D-04   0.1000D-04   0.1000D-04   0.1000D-04   0.1000D-04   0.1000D-04   0.1000D-04   0.1000D-04
           0.1000D-04   0.1000D-04

 20 VARIABLES, 20 ACTIVE.          MATRX =   0         NCOMP = 5         NFMAX =   99000 .        NFLAT = 1

RATIO =  0.100D 02          ACK =  0.200D 01          COLIN =  0.990D 00          COMPR =  0.401D 00

CHISQ =  0.0

BEGIN MINIMIZATION....


TERMINATED WHEN THE STEP SIZES BECAME AS SMALL AS THE DELMN(J).

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        FINAL VALUES OF X(I)....

  0.148352986419520 01  0.209439510239320 01  0.331612557878920 01  0.383972435438750 01  0.959931088596880 00
  0.305432619099010 01  0.125663706143590 01  0.544542726622230 01  0.122173047639600 01  0.209439510239320 01
  0.0                   0.0                   0.0                   0.0                   0.0
  0.0                   0.0                   0.0                   0.0                   0.0

FINAL VALUE OF CHISQ =   0.0
```

TABLE XV (Continued)

FINAL VALUES OF THE VARIABLES

TWIST ANGLES	X	XMIN	XMAX
ALPHA 12	0.8500D 02	0.0	0.3600D 03
ALPHA 23	0.1200D 03	0.0	0.3600D 03
ALPHA 34	0.1900D 03	0.0	0.3600D 03
ALPHA 41	0.2200D 03	0.0	0.3600D 03
ALPHA 65	0.5500D 02	0.0	0.3600D 03
ALPHA 76	0.1750D 03	0.0	0.3600D 03
ALPHA 52	0.7200D 02	0.0	0.3600D 03
ALPHA 17	0.3120D 03	0.0	0.3600D 03
PHI 1	0.7000D 02	0.0	0.3600D 03
SI 1	0.1200D 03	0.0	0.3600D 03

KINK LINKS	X	XMIN	XMAX
S1	0.0	0.0	0.5000D 01
S2	0.0	0.0	0.5000D 01

TABLE XV (Continued)

LINK-LENGTHS	X	XMIN	XMAX
A 12	0.0	0.0	0.5000D 01
A 23	0.0	0.0	0.5000D 01
A 34	0.0	0.0	0.5000D 01
A 41	0.0	0.0	0.5000D 01
A 65	0.0	0.0	0.5000D 01
A 76	0.0	0.0	0.5000D 01
A 52	0.0	0.0	0.5000D 01
A 17	0.0	0.0	0.5000D 01

FINAL VALUES OF THE EXISTENCE CONDITIONS

EQUATION 1 = 0.0

EQUATION 2 = 0.0

EQUATION 3 = 0.0

EQUATION 4 = 0.0

EQUATION 5 = 0.0

EQUATION 6 = 0.0

EQUATION 7 = 0.0

EQUATION 8 = 0.0

EQUATION 9 = 0.0

TABLE XVI

PARAMETERS OF SPACE SIX-LINK R-R-C-C-C-R-C MECHANISM

" EXISTENCE CRITERIA OF SIX-LINK,TWO-LOOP R-R-C-C-C-R-C SPACE MECHANISM "

INITIAL VALUES OF THE VARIABLES

```
N     =        20
NP    =         0
NN    =     99000
DELTA = 0.500D-01
F     = 0.100D-16
ROW   = 0.500D 00
```

TWIST ANGLES	X	XMIN	XMAX
ALPHA 12	0.7000D 02	0.7000D 02	0.7000D 02
ALPHA 23	0.0	0.0	0.0
ALPHA 34	0.7000D 02	0.7000D 02	0.7000D 02
ALPHA 41	0.0	0.0	0.0
ALPHA 65	0.8000D 02	0.0	0.3600D 03
ALPHA 76	0.1200D 03	0.0	0.3600D 03
ALPHA 52	0.2000D 03	0.0	0.3600D 03
ALPHA 17	0.1110D 03	0.0	0.3600D 03
PHI 1	0.3500D 02	0.3000D 02	0.3600D 03
SI 1	0.8500D 02	0.8000D 02	0.3600D 03

TABLE XVI (Continued)

KINK LINKS	X	XMIN	XMAX
S1	0.1200D 01	0.4000D 00	0.1000D 02
S2	0.7000D 00	0.4000D 00	0.1000D 02

LINK-LENGTHS	X	XMIN	XMAX
A 12	0.2000D 01	0.2000D 01	0.1000D 02
A 23	0.1720D 01	0.1720D 01	0.1000D 02
A 34	0.2500D 01	0.2500D 01	0.1000D 02
A 41	0.3000D 01	0.3000D 01	0.1000D 02
A 65	0.4000D 01	0.5000D 00	0.1000D 02
A 76	0.3500D 01	0.5000D 00	0.1000D 02
A 52	0.6000D 01	0.5000D 00	0.1000D 02
A 17	0.4300D 01	0.5000D 00	0.1000D 02

TABLE XVI (Continued)

```
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INITIAL VALUES....

MASK   =     1            1            1            1            0            0            0            0            0
             0            0            0            1            1            1            1            0            0
             0            0

X      =  0.1222D 01   0.0          0.1222D 01   0.0          0.1396D 01   0.2094D 01   0.3491D 01   0.1937D 01   0.6109D 00
          0.1484D 01   0.1200D 01   0.7000D 00   0.2000D 01   0.1720D 01   0.2500D 01   0.3000D 01   0.4000D 01   0.3500D 01
          0.6000D 01   0.4300D 01

XMAX   =  0.1222D 01   0.0          0.1222D 01   0.0          0.6283D 01   0.6283D 01   0.6283D 01   0.6283D 01   0.6283D 01
          0.6283D 01   0.1000D 02   0.1000D 02   0.1000D 02   0.1000D 02   0.1000D 02   0.1000D 02   0.1000D 02   0.1000D 02
          0.1000D 02   0.1000D 02

XMIN   =  0.1222D 01   0.0          0.1222D 01   0.0          0.0          0.0          0.0          0.0          0.5236D 00
          0.1396D 01   0.4000D 00   0.4000D 00   0.2000D 01   0.1720D 01   0.2500D 01   0.3000D 01   0.5000D 00   0.5000D 00
          0.5000D 00   0.5000D 00

DELTX  =  0.5000D-01   0.5000D-01   0.5000D-01   0.5000D-01   0.5000D-01   0.5000D-01   0.5000D-01   0.5000D-01   0.5000D-01
          0.5000D-01   0.5000D-01   0.5000D-01   0.5000D-01   0.5000D-01   0.5000D-01   0.5000D-01   0.5000D-01   0.5000D-01
          0.5000D-01   0.5000D-01

DELMN     0.1000D-16   0.1000D-16   0.1000D-16   0.1000D-16   0.1000D-16   0.1000D-16   0.1000D-16   0.1000D-16   0.1000D-16
          0.1000D-16   0.1000D-16   0.1000D-16   0.1000D-16   0.1000D-16   0.1000D-16   0.1000D-16   0.1000D-16   0.1000D-16
          0.1000D-16   0.1000D-16

 20 VARIABLES, 12 ACTIVE.        MATRX =    0          NCOMP = 5          NFMAX =    99000          NFLAT = 1

RATIO =  0.1000 02          ACK =  0.2000 01          COLIN =  0.9900 00          COMPR =  0.4010 00

CHISQ =  0.14409228D 10

BEGIN MINIMIZATION....

TERMINATED WHEN THE STEP SIZES BECAME AS SMALL AS THE DELMN(J).

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FINAL VALUE OF CHISQ =   0.13943922372902D-10
```

TABLE XVI (Continued)

FINAL VALUES OF THE VARIABLES

TWIST ANGLES	X	XMIN	XMAX
ALPHA 12	0.7000D 02	0.7000D 02	0.7000D 02
ALPHA 23	0.0	0.0	0.0
ALPHA 34	0.7000D 02	0.7000D 02	0.7000D 02
ALPHA 41	0.0	0.0	0.0
ALPHA 65	0.1208D 00	0.0	0.3600D 03
ALPHA 76	0.7010D 02	0.0	0.3600D 03
ALPHA 52	0.1800D 03	0.0	0.3600D 03
ALPHA 17	0.1800D 03	0.0	0.3600D 03
PHI 1	0.3000D 02	0.3000D 02	0.3600D 03
SI 1	0.8000D 02	0.8000D 02	0.3600D 03

KINK LINKS	X	XMIN	XMAX
S1	0.4000D 00	0.4000D 00	0.1000D 02
S2	0.4000D 00	0.4000D 00	0.1000D 02

TABLE XVI (Continued)

LINK-LENGTHS	X	XMIN	XMAX
A 12	0.2000D 01	0.2000D 01	0.1000D 02
A 23	0.1720D 01	0.1720D 01	0.1000D 02
A 34	0.2500D 01	0.2500D 01	0.1000D 02
A 41	0.3000D 01	0.3000D 01	0.1000D 02
A 65	0.1000D 02	0.5000D 00	0.1000D 02
A 76	0.1000D 02	0.5000D 00	0.1000D 02
A 52	0.5000D 00	0.5000D 00	0.1000D 02
A 17	0.5000D 00	0.5000D 00	0.1000D 02

FINAL VALUES OF THE EXISTENCE CONDITIONS

EQUATION 1 = -0.2377D-06

EQUATION 2 = -0.8172D-06

EQUATION 3 = -0.3607D-06

EQUATION 4 = 0.1627D-05

EQUATION 5 = 0.6722D-06

EQUATION 6 = -0.2111D-05

EQUATION 7 = -0.9166D-06

EQUATION 8 = 0.2023D-05

EQUATION 9 = -0.7668D-06

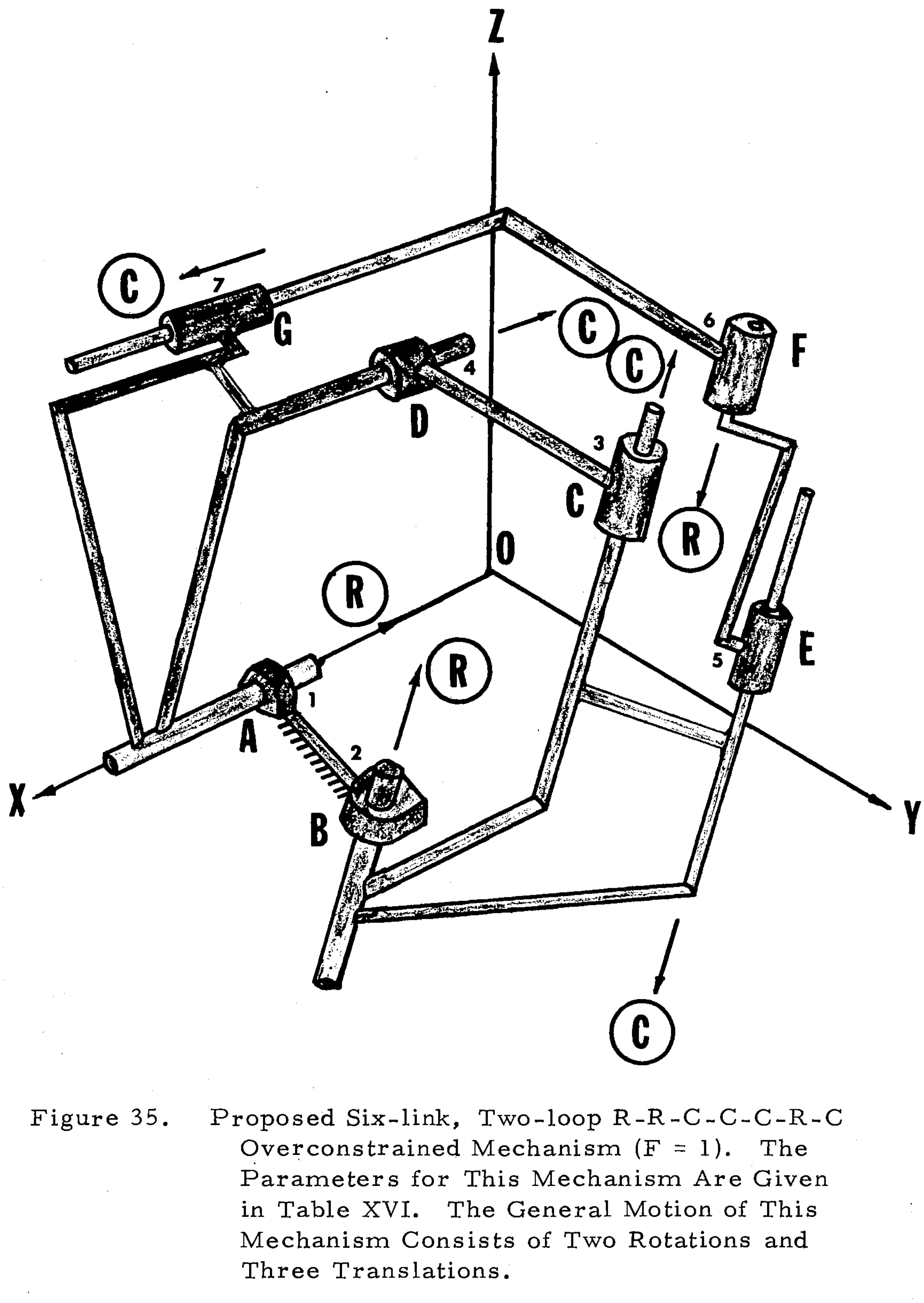

Figure 35. Proposed Six-link, Two-loop R-R-C-C-C-R-C Overconstrained Mechanism (F = 1). The Parameters for This Mechanism Are Given in Table XVI. The General Motion of This Mechanism Consists of Two Rotations and Three Translations.

VITA

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