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CHAPTER I

INTRODUCTION

Most of the production functions previously used in econometric studies of production technology, e.g., Cobb-Douglas or Constant Elasticity of Substitution (C.E.S.), impose simplifying restrictions on the functional form. To simplify these models highly restrictive economic assumptions such as homotheticity, constancy of the elasticity of substitution, identical structure of the ex ante and ex post production technologies, etc., are used.

The purpose of this dissertation is to construct an econometric model which can test or empirically investigate the restrictive economic assumptions on the basis of empirical results through the techniques of statistical inference. The model will be based on the generalized Leontief production model recently developed and extensively used by Uzawa, Diewert, McFadden and Fuss.[1] More specifically, it will be constructed according to Fuss's

[1]H. Uzawa, "Production Functions with Constant Elasticities of Substitution," Review of Economic Studies, 29 (1962), 291-299; H. Uzawa, "Duality Principles in the

indirect functional form of the generalized Leontief production model. Restricted two-stage least squares method is used to estimate the unknown parameters since the model needs to restrict certain coefficients across equations.

The model is applied to the steam-electric generating industry. For a practical application of the model, a two-factor (capital and fuel), single time period case is considered in this study. The data used for empirical investigation consist of cross-section and time series observations on sixty-five steam-electric generating plants put into operation between 1958 and 1968.

In Chapter II, the development of the generalized Leontief production model and its main characteristics are discussed. The data-collecting procedure is briefly described in Chapter III, with the actual data used listed in Appendixes A and B. In Chapter IV a method is developed by which the assumption of homotheticity can be investigated and the appropriate functional form of non-homothetic production function is determined. In Chapter V, by simple

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Theory of Cost and Production," International Economic Review, 5 (1964), 216-219; W. E. Diewert, "An Application of the Shephard Duality Theorem: A Generalized Leontief Production Function," Journal of Political Economy, 79(3), May/June, 1971, 481-507; D. McFadden, "Cost, Revenue, and Profit Functions: A Cursory Review," Working Paper 86, IBER (Berkeley, 1966); Melvyn A. Fuss, "The Structure of Technology over Time: A Model for Testing the "Putty-Clay" Hypothesis," Discussion Paper No. 141, Harvard Institute of Economic Research, Harvard University, November, 1970.

manipulation of the model, isoquants are generated and identified. In Chapter VI the cross-price elasticities of factor demand and the Allen-Uzawa elasticities of factor substitution are estimated on the basis of the ex post generalized Leontief production function constructed in Chapter IV. In addition, the variability of the elasticity of substitution and the determinants of its variations are investigated. In Chapter VII, the structure of production technology is empirically tested based on the putty-semiputty model developed by Fuss and various cases for a two-factor model are explored. Finally, Chapter VIII states the conclusions.

CHAPTER II

DEVELOPMENT OF THE GENERALIZED LEONTIEF PRODUCTION MODEL

1. Introduction

In Cobb-Douglas or C.E.S. production functions, the production parameters are estimated directly from an arbitrarily chosen production function. In direct estimation of the production parameters in Cobb-Douglas or C.E.S. production functions, factor demand is treated as exogenous and factor prices are not explicitly included.

The generalized Leontief production function starts with an arbitrarily chosen cost function which satisfies the regularity conditions of the Shephard duality theorem.[1] Estimation of the parameters of the cost function provides an alternative way of indirectly estimating the production parameters. The simple relation between the total cost function of the producer and the corresponding derived factor demand functions is explored in the generalized Leontief production model. The main advantage of the generalized Leontief production model lies in its

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[1] Diewert, op. cit., pp. 484-495.

realistic view of microeconomic behavior of producers, where factor prices facing them are exogenous, and factor demand is endogenous. This indirect method of estimating the production parameters is particularly convenient when the cost function is relatively easy to formulate and estimate, and the underlying structure of technology cannot easily be summarized by a direct production function.

The following characteristics are considered the main contribution of the generalized Leontief production model in this dissertation:

(i) The generalized Leontief production function has a variable elasticity of factor substitution while C.E.S. production function has any arbitrary constant elasticity of factor substitution. (Cobb-Douglas and Leontief are considered the special cases of C.E.S. where the elasticities of factor substitution are 1 and 0 respectively.)

(ii) The variability of the elasticity of factor substitution in the generalized Leontief production function allows analysis of the determinants of variations in the elasticity of substitution. It permits empirically testing of 1) the Hicksian hypothesis[2] that the input ratio and the level of output are the dominant factors, e.g., $\sigma = f(K/F, y)$, and 2) the assumption of neutral vs.

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[2] J. R. Hicks, The Theory of Wages, 2nd edition (New York: St. Martin's Press, 1963), p. 132, 187-188.

non-neural technical change.

(iii) The generalized Leontief production function can deal with both non-constant returns to scale and non-homothetic type of production technology. The simulation of the model with various output scale functions allows the analysis of the homothetic assumption[3] and enhances the effort in determining the appropriate functional form of the output scale function used.

(iv) The generalized Leontief production function permits generating and identifying isoquants with simple manipulation.

(v) The generalized Leontief production function allows linkage of the ex post and the ex ante technologies with behavioral simplification so that the "putty-clay" hypothesis[4] can be tested directly.

The Shephard Duality Theorem will be briefly reviewed in section 2. In section 3, Diewert's functional form of the generalized Leontief production model will be discussed. Diewert is the main contributor in developing the generalized Leontief production model. In section 4, Fuss's functional form of the generalized Leontief production model which is a slight but very important variation of Diewert's model will be studied.

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[3]The term "homotheticity" or "non-homotheticity" is discussed in detail in section 2, Chapter IV.

[4]The "putty-clay" hypothesis is described in detail in section 1, Chapter VII.

Throughout this dissertation, Fuss's functional form of the generalized Leontief production model is used as the basic model for analysis of production technology.

2. The Duality of Cost and Production: The Shephard Duality Theorem

The generalized Leontief production model is based on the duality theorem of cost and production discussed by Shephard. The Shephard duality theorem[5] states that under certain assumptions there exists a one-to-one correspondence between the production possibility set and the cost function. Therefore, it is asserted by Diewert *et al.*[6] that estimation of the parameters of a cost function provides an alternative way of indirectly estimating the production parameters and thereby uncovering the structure of production technology.

3. Diewert's Functional Form of the Generalized Leontief Production Model

The producer's minimum total cost function (C) in

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[5]The theorem is based on a mathematical result concerning convex sets. For a mathematical treatment of the theorem, refer to R. W. Shephard, *Cost and Production Functions* (Princeton: Princeton University Press, 1953); The Shephard duality theorem is proved in detail by Diewert, *op. cit.*, pp. 483-497; a summary of Diewert's proof appears in Fuss, *op. cit.*, Appendix A.

[6]Diewert, *op. cit.*, p. 497; McFadden, *op. cit.*; Fuss, "The Time Structure of Technology: An Empirical Analysis of the Putty-Clay Hypothesis," unpublished doctoral dissertation, University of California, Berkeley, 1970, Chapter II.

producing the output (y), given factor price vector p = $(p_1,\ldots,p_n)$ is

$$C(y;p) = h(y) \sum_{i=1}^{n} \sum_{j=1}^{n} b_{ij}(p_i p_j)^{1/2} \qquad p_i \geq 0,\ y \geq 0, \qquad (2.1)$$[7]

where (i) h is a continuous, monotonically increasing function of y which tends to plus infinity as y tends to plus infinity with h(0) = 0; and (ii) B = (b_{ij}) is a symmetric n by n matrix with nonnegative elements (that is, $b_{ij} = b_{ji} \geq 0$).

h(y) = the output scale (or returns to scale) function,

b_{ij} = the (ex post) technological parameters.

The derived demand functions for each individual factor services (X_i) are obtained simply by partially differentiating the total cost function with respect to the appropriate factor price (p_i) under the assumptions of competitive factor markets and cost-minimizing behavior on the part of the production managers:[8]

$$X_i(y;p) = \frac{\partial C(y;p)}{\partial P_i} = h(y) \sum_{i=1}^{n} \sum_{j=1}^{n} b_{ij} \left(\frac{p_j}{p_i}\right)^{1/2} \qquad (2.2)$$

where $b_{ij} = b_{ji} \geq 0$ for $i \neq j$; $p_i > 0$.

The above n equations are called the cost-minimizing factor demand equations given factor prices and the level of output. Given knowledge of the output scale function h(y) and data on output, factor prices and factor demands,

[7]Diewert, op. cit., p. 497.

[8]Ibid., pp. 497-505.

the (ex post) technological parameters, b_{ij}, can be empirically estimated from equation (2.2).

Diewert's generalized Leontief production model represented by (2.1) and (2.2) has the following characteristics:

(i) The underlying production function exhibits non-constant returns to scale except when h(y) = y for all i,j.

(ii) The underlying production function is homothetic.

(iii) The main characteristic of Diewert's functional form is that it permits arbitrary degrees of substitutability between pairs of factors in a n factor production process. Both the cross-price elasticity of factor demand and the Allen-Uzawa partial elasticity of factor substitution are not necessarily constant, but can be variable.

a) The cross-price elasticity of factor demand is given by

$$E_{ij}(y;p) = \frac{1}{2} b_{ij} \left(\frac{p_j}{p_i}\right)^{\frac{1}{2}} \frac{h(y)}{X_i(y;p)} \qquad \text{for } i \neq j \qquad (2.3)$$

b) The Allen Uzawa partial elasticity of factor substitution is given by

$$\sigma_{ij}(y;p) = \frac{1}{2} b_{ij} \frac{h(y)}{X_i X_j (p_i p_j)^{\frac{1}{2}}} C(y;p) \qquad (2.4)$$

The restrictive economic assumption of homotheticity in the Diewert's form is the focal point of discussion in Chapter IV.

Chapter VI is assigned to analyze in depth the variability of the elasticities in the generalized Leontief production model.

4. Fuss's Functional Form of the Generalized Leontief Production Model

Melvyn Fuss's form of the generalized Leontief production model relaxes the homotheticity restriction employed by Diewert. For empirical use, the following cost and derived demand functions are suggested by Fuss.

Minimum total cost function, given the factor price vector $p = (p_1, \ldots, p_n)$ and output level y, is:

$$C(y;p) = \sum_{i=1}^{n} b_{ii} p_i h_i(y) + \sum_{\substack{i=1 \\ i \neq j}}^{n} \sum_{j=1}^{n} b_{ij} (p_i p_j)^{1/2} h(y) \qquad p_i \geq 0,\ y \geq 0, \qquad (2.5)$$

where $h(y)$ = the general output scale (or returns to scale) function

$h_i(y)$ = the i^{th} factor-related output scale function,

b_{ii}, b_{ij} = the (ex post) technological parameters.

His cost minimizing derived factor demand equations are:

$$X_i(y;p) = \frac{\partial C(y;p)}{\partial P_i} = b_{ii} h_i(y) + \sum_{\substack{j=1 \\ j \neq i}}^{n} b_{ij} \left(\frac{p_j}{p_i}\right)^{1/2} h(y) \qquad (2.6)$$

where $b_{ij} = b_{ji} \geq 0$ for $i \neq j$ by assumption.

If the output scale function $h_i(y)$, $h(y)$ is known and there is data on output, factor prices and factor demands, the (ex post) technological parameters can be empirically estimated, from equation (2.6).

Fuss's generalized Leontief production model represented by (2.5) and (2.6) has the following characteristics:

(i) The underlying production function exhibits non-constant returns to scale except when $h_i(y) = h(y) = y$ for all i,j.

(ii) The underlying production function is non-homothetic except when $h_i(y) = h_j(y) = h(y)$ for all i,j.

(iii) Both the cross-price elasticity of factor demand and the Allen-Uzawa (partial) elasticity of factor substitution are not necessarily constant, but can be variable and have exactly the same forms of (2.3) and (2.4), respectively.

In summary, Diewert's form is a special case of Fuss's more generalized form with respect to the homotheticity consideration. The advantage in using Fuss's form is that it permits evaluation of Diewert's homothetic assumption. Essentially, Fuss's form has all the advantages without the weaknesses inherent in Diewert's model and will be used in this dissertation.

CHAPTER III

THE MODEL APPLIED TO THE STEAM-ELECTRIC GENERATING INDUSTRY

1. The Advantages of Studying the Steam-Electric Generating Industry

In 1968, the production of electricity in the steam-electric generating industry, which is defined as those electric plants using fossil fuels (coal, oil and natural gas), accounted for more than 80 per cent of the total generation of electricity in the United States.[1] The remaining portion of electricity generation is derived from hydroelectric and nuclear power plants. However, the empirical analysis of this study is limited to steam-electricity generation because of the great qualitative difference between steam and hydroelectric-nuclear production of electricity.

The steam-electric generating industry is suitable for the application of the present model for several reasons.

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[1]Hans H. Landsberg and Sam H. Schurr, Energy in the United States: Sources, Uses, and Policy Issues, A Resources for the Future Study (New York: Random House, 1968), pp. 144-160.

Firstly, there have already been numerous studies of this industry. This fact allows comparison of the empirical results of the models presented in this dissertation with the earlier empirical results. This provides an idea of the consistency of the current model.

Second, data on the variables needed in constructing the generalized Leontief production model are readily available from the Federal Power Commission Reports.[2]

Third, the industry produces a single homogeneous and non-storable output--electricity,--which is convenient to treat in econometric studies. For instance, since the generation of electricity cannot be stored, there is no problem of estimating inventory in the model.

2. Previous Studies

Because of the easy availability of statistical data on the electricity generation industry, numerous empirical studies have been done of this industry. Some of the examples are the works of Barzel, Dhrymes-Kurz, Galatin, Komiya, Nerlove, McFadden and Fuss.[3]

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[2]Federal Power Commission, Statistics of Electric Utilities in the United States, 1958-1968, Classes A and B Privately Owned Companies (Washington, D.C.: U.S. Government Printing Office, 1959-69); Federal Power Commission, Steam-Electric Plant Construction Cost and Production Expenses, Annual Supplements, 1958-68 (Washington, D.C.: U.S. Government Printing Office, 1949-62).

[3]Y. Barzel, "The Production Function and Technical Change in the Steam-Power Industry," The Journal of Political Economy, Vol. 72 (April, 1964), pp. 133-150; P. J.

Excepting the works of McFadden and Fuss, however, most of the previous econometric studies of this industry applied either the Cobb-Douglas or the C.E.S. production functions and therefore assumed constant elasticity of substitution and homothetic production functions. Moreover, except for Galatin, they assumed that the structure of technology is putty-putty; i.e., that ex ante and ex post production possibilities are identical.

Broadly, three primary factors are used to produce electricity by steam-generation: capital, labor and fuel. Capital and fuel are the most important components of the total cost function. On the average, the capital cost amounts to 47 per cent, and the fuel cost comprises 49 per cent, of the total cost.[4] Production and maintenance

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Dhrymes and M. Kurz, "Technology and Scale in Electricity Generation," Econometrica, Vol. 32 (July, 1964), pp. 287-315; M. Galatin, Economies of Scale and Technological Change in Thermal Power Generation (Amsterdam: North-Holland, 1968); R. Komiya, "Technological Progress and the Production Function in the United States Steam Power Industry," Review of Economics and Statistics, Vol. 44 (May, 1962), pp. 156-166; M. Nerlove, "Returns to Scale in Electricity Supply," in Measurement in Economics--Studies in Mathematical Economics and Econometrics in Memory of Yehuda Grunfeld (Palo Alto: Stanford University Press, 1963); D. L. McFadden, "Notes on the Estimation of the Elasticity of Substitution," Working Paper No. 57, Institute of Business and Economic Research, University of California, Berkeley, December, 1964; M. Fuss, "Factor Substitution in Electricity Generation: A Test of the Putty-Clay Hypothesis," Discussion Paper No. 185, Harvard Institute of Economic Research, Harvard University, April, 1971.

[4]In McFadden's data (1958-61), capital and fuel costs amount to 45 per cent and 50 per cent of total cost, respectively. The remaining 5 per cent is labor cost.

labor is a substantially smaller component of total cost than capital and fuel. Since labor cost comprises only 4 per cent of the total cost, it is excluded from the production model in this study and fuel cost is included.

3. Description of Data

The data used for the empirical investigation consists of cross-section and time series observations on steam-electric generating plants in the United States. Specifically, the sample data consist of the relevant cost and production variables for 65 new steam-electric power plants put into operation between 1958 and 1968. Plant observations begin only after the plant has been in operation one full year and when no additional capacity is installed. For instance, if a new generating unit is added during an observation year, the plant is dropped from the data. The Federal Power Commission Reports are a unique source of statistics on a plant-by-plant basis. The data were collected from the above source and some of the original data were transformed for our purpose by the present writer.[6] The original and transformed data

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[6] Two types of electricity data collected previously were available to me in the beginning of the research. The first one was collected by A. Belinfante and used by M. Fuss for his dissertation. The observation years of this sample are between 1947 and 1959. The second one was listed in the Appendix A of McFadden's work. The observation years for McFadden's sample are between 1958 and 1961. However, the above two types of sample data are not current at the vintage point of view of 1970's.

used for actual estimation of the model are listed in Appendixes A and B.

4. Variables

y = plant output (net generation) in million kilowatt-hours in the first year of operation.

K = the cost-minimizing capital for a given output at specified factor prices.

K is observed as a net value of total cost of plant after depreciation in thousands of current dollars. Total cost of plant includes land and land rights, structures and improvements and equipment. Depreciation was calculated at 2.5 % per annum. Therefore, the cost-minimizing capital variable is a simple dollar aggregate of equipment and structures, etc. It is assumed that production is planned under the belief that capital will be maintained at a constant quality level throughout its lifetime.

F = the cost-minimizing fuel for a given output at specified factor prices.

F is observed as the amount of fuel in billions of B.t.u.'s. Coal, oil and gas are the principal fuels used for electricity generation. Converted to British thermal units they are not distinguished in this study.

r (or p_1) = price of capital services in the first year of operation.

r is observed as a proportionate rate of return (cash flow) to gross capital at original cost in service, for company's electricity operations. It was calculated by dividing ($V-wL$) by K where V, w, L, are Value added, Wage rate, and Labor force, respectively, during the first year of operation.

f (or p_2) = price of fuel in dollars per million B.t.u.'s.

Since many plants are set up to use more than one type of fuel, price of fuel is taken on a per million B.t.u. basis.

y_c = plant capacity (or capacity output) in million kilowatt-hours in the first year of operation.

ℓ = load (plant) factor = (output/capacity).

The load factor is defined as the actual yearly output divided by the capacity output which could have been produced while the turbin-generator was hot and connected to load for the full year.

A.C. = average capacity per boiler-generator unit.

A.C. can be found by dividing y_c by total number of operating (boiler-generator) units (u) in the plant.

v = vintage of plant (year of initial operation), 0 for 1957.

t-v = age of plant in years (= year of observation - year of initial operation); 1 for all sample in this study.

CHAPTER IV

AN EMPIRICAL INVESTIGATION OF HOMOTHETICITY AND RETURNS TO SCALE

1. Introduction

Various functional forms can be used to represent the production function. Once a specific functional form is arbitrarily chosen, the parameters of the function are estimated to investigate the relationship between output and factor inputs. For example, if a Cobb-Douglas functional form is chosen, the usual empirical procedure is to estimate the parameters from data on output and factor inputs, after first prejudging the issue of homotheticity (among others). Empirical verification that the production process under analysis is homothetic is not even considered within the context of the estimation procedure. Yet the procedure is only valid if and only if the production function is homothetic.

Thus the appropriate estimation procedure is to determine the relevance of the homotheticity assumption before using a specific production function, such as the Cobb-Douglas, to obtain the parameter estimates. The use of the ex post generalized Leontief production model, discussed in Chapter II allows analysis of the homothetic assumption. At the same time, the simulation of the model with various output scale functions enhances the effort in determining the proper form of the output scale function. An analysis of the empirical relevance of homotheticity and a simulation of the output scale function is conducted on the steam-electric generating industry discussed in Chapter III. In order to apply the generalized Leontief production model to the electricity generation industry, two primary assumptions are made: first, it is assumed that production managers in this industry seek to minimize cost, and, secondly, it is assumed that the firms face exogenously determined factor prices (which they have no power to influence). Thus, with the assumptions of cost-minimizing behavior and competitive factor markets, a model for the steam-electric generating industry can be built.

2. Definition of Homotheticity

The term "homotheticity" was originally used in geometry. It refers to the property that any two level surfaces (e.g., "isoquants") are related by a similarity transformation which preserves both angles and ratios of distances.[1]

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[1] David Gans, Transformations and Geometrics (New York: Appleton-Century-Crofts, 1969), p. 71.

Homothetic production functions and their characteristics were first introduced in 1953 by Shephard.[2] A production function is defined as homothetic if its marginal rates of technical substitution along the isoquants are dependent only on input proportions, and not on the level of output.

Therefore, a homothetic production function implies that along a ray from the origin crossing the isoquant map all the slopes of isoquants are equal. It is shown in Figure IV-A that the slopes of the tangent line (or the marginal rates of technical substitution between capital and fuel given factor prices) at the intersection between the rays (OT or OT') and the isoquant do not vary depending upon the output scale. It may be verified by looking at Figure IV-A that slopes at points P, Q, and R (or at P', Q', R') are the same.

An implicit functional form for the minimum total cost (C) function under the separability assumption[3] in a generalized Leontief production model is:

$$C = C(y, p_1, \ldots, p_n) = h(y) \cdot g(p_1, \ldots, p_n) \quad (4.1)$$

The above cost function is a natural simplification of the dependence of cost upon the level of output and factor prices of production. Shephard points out that homotheticity

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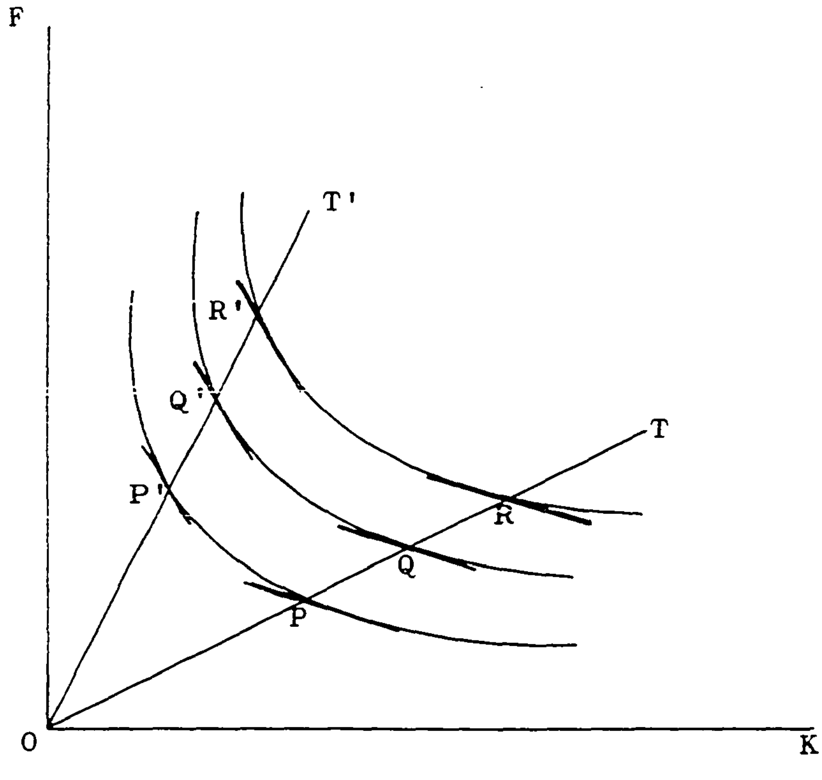
[2]Shephard, op. cit., pp. 42-43.

[3]For the description of the separability assumption, see ibid., p. 43.

FIGURE IV-A

HOMOTHETIC ISOQUANTS



of the production surfaces implies the separability of the minimum cost function.[4] In fact, the equality between the last two parts of equation (4.1) requires the function to possess the separability property. Then, the cost-minimizing input demand functions can be obtained using Shephard's Lemma[5];

$$X_i(y;p) = \frac{\partial C}{\partial p_i} = g_i(p_1, \ldots, p_i, \ldots, p_n) \cdot h(y) \tag{4.2}$$

$$X_k(y;p) = \frac{\partial C}{\partial p_k} = g_k(p_1, \ldots, p_k, \ldots, p_n) \cdot h(y) \tag{4.3}$$

where $p = (p_1, \ldots, p_i, \ldots, p_k, \ldots, p_n)$ is a factor price vector.

Then, the relative input intensities between two factors, e.g., factors i and k, can be obtained by dividing (4.2) by (4.3):

$$\frac{X_i}{X_k} = \frac{g_i(p_1,\ldots,p_i,\ldots,p_n) \cdot h(y)}{g_k(p_1,\ldots,p_k,\ldots,p_n) \cdot h(y)} = \frac{g_i(p_1,\ldots,p_i,\ldots,p_n)}{g_k(p_1,\ldots,p_k,\ldots,p_n)} \tag{4.4}$$

The output scale function, h(y), has no role in the final form of the relative input intensity equation (4.4). Since the relative input intensities are independent of the level

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[4] *Ibid.*

[5] By Shephard's Lemma, $\frac{\partial C(y;p)}{\partial p_i} = X_i(y;p)$. For a detailed explanation of Shephard's Lemma, see *ibid.*, p. 11. Also see Diewert, *op. cit.*, pp. 495-497.

of output, the production model represented in equation (4.1) is homothetic.

Two derived factor demand functions for factors j and k can be obtained from equation (2.2) which is Diewert's model. If these are set as ratios, the relative input intensities of factors j and k are derived as:

$$\frac{X_j}{X_k} = \frac{\sum b_{ji}\left(\frac{p_i}{p_j}\right)^{1/2}}{\sum b_{ki}\left(\frac{p_i}{p_k}\right)^{1/2}} \tag{4.5}$$

Therefore, the relative input intensities in a Diewert-type production model represented by equations (2.1) and (2.2) are independent of the level of output making the production function homothetic. Like the Cobb-Douglas, Diewert's production function presupposes that the production process under consideration is homothetic from the beginning. There is no way to refute the homothetic assumption when either of these production functions are chosen. One can only say that a "good fit" may indicate a correct assumption while a "bad fit" may indicate an incorrect assumption. However, since so many other explanations are possible, a "bad fit" alone cannot be considered as decisive evidence for rejecting the assumption of homotheticity. Another test is required and, as previously noted, Fuss has provided it.

As observed in Chapter II, Fuss's production function does not require homotheticity restriction. Functional forms of total cost and derived factor demand proposed by Fuss were represented in equation (2.5) and (2.6) in Chapter II. Using these equations, the relative input intensities of factors i and k are measured by

$$\frac{X_i}{X_k} = \frac{b_{ii}h_i(y) + \sum b_{ij}\left(\frac{p_j}{p_i}\right)^{\frac{1}{2}} \cdot h(y)}{b_{kk}h_k(y) + \sum b_{kj}\left(\frac{p_j}{p_k}\right)^{\frac{1}{2}} \cdot h(y)} \tag{4.6}$$

Therefore, Fuss's underlying production function becomes homothetic only under a special condition. The relative input intensity between i and k, $\frac{X_i}{X_k}$, becomes independent of the level of output (y) if and only if $h_i(y) = h_k(y) = h(y)$. This condition is called the homotheticity condition in a generalized Leontief production function. When Fuss's form satisfies the homotheticity condition, it becomes identical to Diewert's form.

3. Method of Estimation: The Two-Stage Least Squares Method

The two-stage least squares method is used to estimate the unknown parameters in the total cost function which uncover the underlying production function.[6] A good estimation

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[6]The computations were performed at the Merrick Computing Center, University of Oklahoma, by utilizing the "GLSQ" program in C. Liew and D. Kahng, "Computerized Econometric Analysis," Monograph No. 25, Bureau for Business and Economic Research, University of Oklahoma, 1971. The "GLSQ" program allows simple data transformations and estimates the parameter values by the two-stage least squares method.

of the total cost function is required for proper identification of the production function. The two-stage method is needed to take account of the restrictions across equations.

In stage 1, the parameters for the (unrestricted) derived demand equations (in this study, for capital and fuel demand equations) are estimated by the ordinary least squares estimation method and the standard error of the regression for each derived demand equation is calculated. The standard error of the regression is an estimator for the square root of the variance of the error terms in the equation. Using the estimated standard error of the regression, the variables in each equation are deflated.

In stage 2, all equations are properly stacked and the parameters of the stacked model are estimated by the ordinary least squares method. The stacked model may be estimated with restrictions across equations as well as without. If it is estimated without any restrictions across equations, the estimated parameter values are the same as those which are obtained from stage 1. Therefore, the advantage of using the two-stage least squares method lies in the ability to restrict the coefficients across equations.

This two-stage least squares method is based on the assumption of a zero off-diagonal terms in the variance-covariance matrix. For instance, in a two-equation model, it is assumed that $\sigma_{12} = \sigma_{21} = 0$ where $\mathrm{Cov}(e_1, e_2) = \sigma_{12} \cdot I$ and $\mathrm{Cov}(e_2, e_1) = \sigma_{21} \cdot I$, where I is an identity matrix.

In other words, to simplify the structure of the variance-covariance matrix, it is assumed that the residual terms in the capital demand equation is independent of the residual terms in the fuel demand equation.

In a matrix form,

$$V(e) = \begin{bmatrix} \sigma_{11} & 0 \\ 0 & \sigma_{22} \end{bmatrix} \cdot I \text{ where } e = \begin{bmatrix} e_1 \\ e_2 \end{bmatrix}$$

Whether or not there is a significant difference between the sums of squared residuals obtained from the stacked model with restrictions across equations and without can be statistically tested by F statistic.

Consider a two-equation model such as:

$y = Xb + e$

where

$$y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}, \qquad X = \begin{bmatrix} X & 0 \\ 0 & X \end{bmatrix}$$

$$\text{and } b = \left[X'(S \cdot I)^{-1}X\right]^{-1} X'(S \cdot I)^{-1}y.$$

b is the vector of coefficients to be estimated. e is the vector of the residuals having the properties $E(e) = 0$ and $Cov(e) = \sigma^2 \cdot I$.

The sum of squared residuals in the stacked model with no restrictions across equations, $e'e(NR)$, is

$$e'e(NR) = e'(S\cdot I)^{-1}e$$
$$= \frac{e_1'e_1}{S_{11}} + \frac{e_2'e_2}{S_{22}}$$

The sum of squared residuals in the stacked model with restrictions across equations is e'e(R).

The F statistic[7] can be found by

$$F = \frac{\dfrac{e'e(R) - e'e(NR)}{q}}{\dfrac{e'e(NR)}{n - k}}$$

where k = number of parameters in the unrestricted stacked model,

n = number of observations in stacked model,

q = number of parameters in the restricted stacked model.

F is distributed as Snedecor's F with q and n-k degrees of freedom. Significance levels of 5% or 1% are chosen as a decision rule in testing hypothesis.

If the null hypothesis that e'e(NR) = e'e(R) cannot be rejected at a chosen significance level, the two-stage least squares estimating method is applied when certain coefficients are restricted across equations. This method is the standard procedure in estimating the parameters for derived demand functions in the generalized Leontief production model throughout this dissertation.

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[7] C. R. Rao, Linear Statistical Inference and Its Applications (New York: John Wiley and Sons, Inc., 1965), pp. 237-240.

4. Empirical Relevance of Homotheticity and Returns to Scale

The ex post production function of the model (2.6) is homothetic if the homotheticity condition is satisfied. Otherwise the production function is non-homothetic.

Various functional forms of output scale can be tried with the help of a priori knowledge and previous works done on the specific industry under investigation. In this study, several functional forms are adopted for the capital-related output scale function ($h_1(y)$), the fuel-related output scale function ($h_2(y)$), and the general output scale function ($h(y)$); some with a constant returns to scale, some with an increasing returns to scale and some with a decreasing returns to scale.

All functional forms of the model (2.6) are simulated on the computer. Some of the functional forms are estimated under the homothetic assumption and some with the non-homothetic assumption. The former is called the First Group and the latter the Second Group. If some or all of the regression results in the Second Group statistically outperform those in the First Group, this indicates that the assumption of homotheticity does not hold. Simulation with various non-homothetic functional forms should then be tried and the one with the "best fit" can be selected for the final production model.

For an empirical application, six different functional forms of output scale are chosen under the homothetic assumption with a special consideration of variable returns to scale. Five of them have an exponential variation of actual output with an exponential interval of 1/2: y^0, $y^{1/2}$, y, $y^{3/2}$ and y^2. In addition, the capacity output ($y_c = y \cdot \ell^{-1}$) is used as the sixth variable in the output scale function. The capacity output is obtained by dividing the actual output by the utilization rate. Since we use six different functional forms of $h_1(y)$, $h_2(y)$ and $h(y)$, the total number of the regression equations under the non-homotheticity assumption is 210 (= $6^3 - 6$). The first stage regression results show that the R-squareds are high when the restrictions $h_1(y) = y_c$ and $h_2(y) = y$ are applied. The results of the stage one are given in Appendix C. Thus, based on the empirical results of the first stage, we restricted $h_1(y) = y_c$ and $h_2(y) = y$ and varied only the functional form of the general output scale for a non-homothetic simulation.

According to the estimating method described in section 3, the model is stacked in the second stage. The stacked model with restrictions across equations can be written in a matrix form as below:

<u>Stacked Model with restrictions ($b_{12} = b_{21}$) across equations</u>:

$$\begin{pmatrix} K \\ F \end{pmatrix} = \begin{bmatrix} h_1(y) & 0 & p\cdot h(y) \\ 0 & h_2(y) & \frac{1}{p}\cdot h(y) \end{bmatrix} \begin{pmatrix} b_{11} \\ b_{22} \\ b_{12} \end{pmatrix} + \begin{pmatrix} e_1 \\ e_2 \end{pmatrix} \qquad (4.7)$$

where $p = \left(\frac{p_2}{p_1}\right)^{1/2}$ and e_1, e_2 = residuals.

The regression results of the second stage with restrictions across equation are given in Tables IV-1 and IV-2.

The above set of ex post factor demand equations contain the estimated technological parameters, b_{ij}. For an empirical research, the maintained hypothesis is that the estimated production function is statistically closer to the true production function when the former is fitted better than other estimated production functions.[8] In other words, if the production function is homothetic, those regression equations with homothetic assumption should show better empirical results than those equations with non-homothetic assumption. In order to choose the appropriate

[8] Many empirical studies use the technique to identify the true relation. For example, Jorgenson and Stephenson selected the appropriate lag specification on the basis of the criterion of the minimum estimated value of the standard error of the regression. See D. W. Jorgenson and J. A. Stephenson, "Investment Behavior in U.S. Manufacturing, 1947-1960," <u>Econometrica</u>, 35(2), April, 1967, pp. 169-220.

TABLE IV-1[1]

Estimated Regression Results Associated with Equation (2.6) under the Assumption of Homotheticity[2]

Model	$h_1(y)=h_2(y)=h(y)$	S.E.R.	R^2	b_{11}	b_{22}	b_{12}
I	y^0	0.996659	.0005	30062. (8291.)	12485. (3306.)	-856. (4868.)
II	$y^{1/2}$	1.009853	.7414	841. (156.)	363. (63.)	56. (95.)
III	y	1.002596	.9921	21.5 (1.1)	9.41 (0.31)	0.10 (0.47)
IV	$y^{3/2}$	1.006577	.7748	0.318 (0.069)	0.147 (0.025)	0.061 (0.039)
V	y^2	1.035064	.3883	0.0051 (0.0020)	0.0024 (0.0007)	0.0012 (0.0011)
VI	y_c	0.996042	.8787	12.20 (1.79)	5.26 (0.69)	1.66 (1.07)

Note: See Table IV-2 for abbreviations and remarks.

TABLE IV-2[1]

Estimated Regression Results Associated with Equation (2.6) under the Assumption of Non-Homotheticity

Model	$h_1(y)$	$h_2(y)$	$h(y)$	S.E.R.	R[2]	b_{11}	b_{22}	b_{12}	b_{11}[3]	b_{22}[3]	b_{12}[3]
VII	y_c	y	y^0	1.004067	.9931	14.49 (0.43)	9.2 (0.08)	735.3 (197.0)			
VIII	y_c	y	$y^{1/2}$	1.009459	.9925	14.16 (0.49)	9.0 (0.16)	31.41 (10.46)			
IX	y_c	y	y	1.000200	.9919	14.75 (0.66)	9.38 (0.30)	0.142 (0.46)			
X	y_c	y	$y^{3/2}$	0.998214	.9926	15.63 (0.56)	9.90 (0.21)	-0.138 (0.006)	.0189 (.077)	.0329 (.653)	0
XI	y_c	y	y^2	0.998045	.9926	15.37 (0.49)	9.74 (0.14)	-0.0002 (0.0001)	.00036 (.011)	.00065 (.092)	0

Note: y = actual output

y_c = capacity output $(= \frac{y}{\ell})$ where ℓ is the load factor

S.E.R. = Standard error of the regression

The standard errors of the coefficients are given in parentheses.

[1]All computations were made by using the "GLSQ" program at the Merrick Computing Center, University of Oklahoma.

[2]Number of observations is 130.

[3]The estimated coefficients when b_{ij} is constrained to zero. The R-squareds are negative in these cases: -.8555 for Model X and -.8616 for Model XI.

specification the following criteria are used in this study: (a) R-squared[9] and (b) Consistency of sign and magnitude of the estimated regression coefficients, b_{ij}, for the output-weighted price variable. In most of the production models the sign of the factor price ratio coefficient is expected to be positive[10] since substitution of factors will dominate the production process. In this study, capital and fuel are expected to be substitutes from the knowledge of previous

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[9]In this study, the R-squared is employed as the criterion to choose the appropriate specification of the model (2.6). The estimated standard error of the two-stage estimation is not a correct indicator of the goodness-of-fit since

$$\hat{V}\begin{pmatrix} \dfrac{\varepsilon_1}{\hat{\sigma}_{11}} \\ \\ \dfrac{\varepsilon_2}{\hat{\sigma}_{22}} \end{pmatrix} = \begin{pmatrix} \left(\dfrac{\sigma^*_{11}}{\hat{\sigma}_{11}}\right)^2 \cdot I & 0 \\ 0 & \left(\dfrac{\sigma^*_{22}}{\hat{\sigma}_{22}}\right)^2 \cdot I \end{pmatrix} = k \cdot I$$

where $k \rightarrow$ unity as $\hat{\sigma}_{11} \rightarrow \sigma^*_{11}$ and $\hat{\sigma}_{22} \rightarrow \sigma^*_{22}$. $\hat{\sigma}_{11}$ and $\hat{\sigma}_{22}$ are the standard errors of the regression in the stage one equations and $\sigma^*_{11} I = \hat{v}(\varepsilon_1)$, $\sigma^*_{22} I = \hat{V}(\varepsilon_2)$ in the stage 2 equations.

[10]If $b_{ij} < 0$ for some i,j, factors i and j are complements in production which are rare cases in the production process. If $b_{ij} = 0$ the factors of production are independents. For a detailed analysis on the nature of b_{ij}, see Fuss, op. cit., Harvard Discussion Paper No. 141, Appendix B. Also see W. E. Diewert, "An Application of the Shephard Duality Theorem: A Generalized Leontief Production Function," Report 6921, Center for Mathematical Studies in Business and Economics, University of Chicago, June, 1969.

empirical studies[11] even though the elasticity of substitution between them are very low.[12] Moreover, the concavity restriction of the production possibility sets implies that the sign of b_{ij} must be nonnegative for $i \neq j$. If $b_{ij} < 0$ for $i \neq j$, it violates the concavity assumption of cost function. $\frac{\partial C}{\partial p_i} = X_i$ is the cost-maximizing derived demand function rather than a cost-minimizing one. Therefore, those models which possess negative technological parameters, b_{ij}, are rejected on a priori grounds.[13]

The Durbin-Watson statistic is calculated for each equation in stage 1. However, no problem of autocorrelation appears to be serious in all equations under simulation.

The empirical results show that models III, VII, VIII and IX provide the best empirical fits. These four models satisfy the non-negativity assumption of the technological parameters, and have R-squared values which are greater than 0.99 in all cases. Model III is an example of a homothetic and constant returns to scale production function.

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[11]Barzel, op. cit.; Dhrymes-Kurz, op. cit.; and Nerlove, op. cit.

[12]The estimation of the elasticities of substitution is dealt with in Chapter VI.

[13]If b_{kk} is negative, the factor k is nonessential for the production of output y. Therefore, non-negativity assumption of b_{kk} may be required for the factors to be essential for the production of non-zero output. This problem does not arise in our empirical results. See Diewert, op. cit., Report 6921.

Models VII, VIII and IX are the cases of non-homothetic and non-decreasing returns to scale production functions. Models I, X and XI violate the non-negativity assumption of the technological parameter. These three models were re-estimated with the constraint that $b_{12} = 0$. However, when the non-negativity constraints[14] of b_{12} are enforced, the R-squareds are low. This implies that the production function in the steam-electric generating industry does not follow the Leontief pattern (i.e., no substitution between capital and fuel). Models II, IV, V and VI - all homothetic cases - have R-squareds between 0.38 and 0.88.

In conclusion, the empirical results of this section support the contention that the production function in the steam-electric generating industry is non-homothetic[15] with non-decreasing returns to scale,[16] except in the case of model III.

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[14]In this special case (model 4.7), the inequality constrained (i.e., $b_{12} \geq 0$) least squares estimation leads to the ordinary least-squares estimation with the constraint (i.e., $b_{12} = 0$). See C. K. Liew, "The Stability Condition of the Inequality Constrained Least-Squares Estimation," Working Paper No. 20, University of Oklahoma, 1971.

[15]The mean value of estimated R-squareds of the First Group (e.g., homothetic models) is 0.629 while that of the Second Group (e.g., non-homothetic models) is 0.993.

McFadden's and Fuss's empirical results support the non-homothetic production function in the steam-electric generating industry. See McFadden, op. cit., pp. 38-39; Fuss, unpublished doctoral dissertation, p. 163.

[16]The empirical results of the following works support the increasing returns to scale in the steam-electric generating industry: R. Komiya, op. cit., pp. 156-66; McFadden, op. cit.; Fuss, op. cit.; Nerlove, op. cit.; Galatin, op. cit.

5. Summary and Remarks

1) Most of the production functions previously used in empirical works assume the production processes are homothetic in nature without any attempt to empirically substantiate this assumption. In order to obtain a realistic measure of the true production function, the assumption of homotheticity has to be tested prior to accepting any specific model. Therefore, the purpose of this chapter is to develop a method by which the homotheticity assumption can be investigated. Fuss's form of the generalized Leontief production function is used to determine if it satisfies this need.

2) The advantage of using a Fuss-type generalized Leontief production model is explored because it allows easy investigation of homothetic assumption. Also, if the homothetic assumption is rejected, it permits exploration of a non-homothetic form.

3) If a priori information about the type of technology and factor relationship in the industry (i.e., substitutes, complements, etc.) under study is available, the effort to determine the appropriate functional form of the output scale in a Fuss-type generalized Leontief production model can be greatly enhanced.

4) Based on the empirical evidence of the generalized Leontief production model, the production function of the steam-electric generating industry is shown to be non-homothetic and to have non-decreasing returns to scale.

CHAPTER V

AN EMPIRICAL MEASUREMENT OF THE ISOQUANTS FOR GENERALIZED LEONTIEF PRODUCTION FUNCTIONS

Given the following constraint for equations (5.1) and (5.2) the ex post factor demand functions for the two factor case are

$$X_1 = b_{11} \cdot h_1(y) + b_{12}p \cdot h(y) \tag{5.1}$$

$$X_2 = b_{22} \cdot h_2(y) + b_{12}\frac{1}{p} \cdot h(y) \tag{5.2}$$

where $p = \left(\frac{p_2}{p_1}\right)^{\frac{1}{2}}$. The constraint is $b_{12} = b_{21}$.

Substituting (5.1) into (5.2) through p, the isoquant forming equation is obtained:

$$X_2 = b_{22} \cdot h_2(y) + \frac{\left[b_{12} \cdot h(y)\right]^2}{X_1 - b_{11} \cdot h_1(y)} \tag{5.3}$$

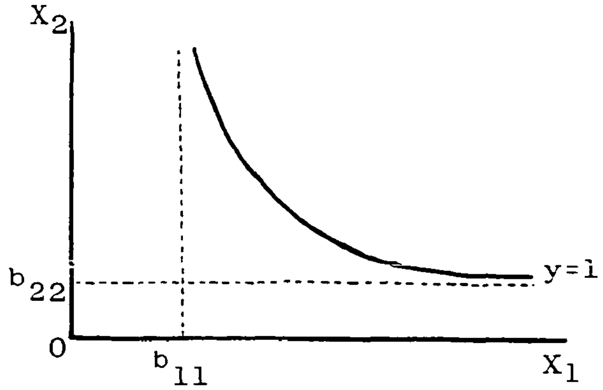
where $X_1 > b_{11} \cdot h_1(y)$ and $X_2 > b_{22} \cdot h_2(y)$ by assumption.

Since the functional forms of the output scales, $h_1(y)$, $h_2(y)$, and $h(y)$, are chosen in the model and the ex post parameters, b_{ij}, are estimated therefrom, the equation (5.3) generates an isoquant representing an ex post production possibility frontier. The shape of the isoquant will depend on the values taken by the ex post parameters, b_{ij}, and the functional forms of output scale.

In the case of a two-factor production function a diagram such as that sketched in Figure V-A can be drawn: The values of b_{11}, b_{22} determine the vertical and horizontal asymtotes, respectively, and the value of b_{12} affects the steepness of the slope.

FIGURE V-A

THE SHAPE OF THE ISOQUANT



Isoquants corresponding to Models I through IX are given at the end of this chapter.[1] Models X and XI are omitted due to their violation of the concavity restrictions required for production possibility sets.[2] As for isoquants

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[1]Despite their poor performances empirically, isoquants for the homothetic cases (e.g., Models II, IV, V and VI) are treated in this chapter in order to illustrate the differences between the homothetic and non-homothetic isoquants.

[2]As observed in Chapter IV, Model I also violates the concavity restriction. However, the isoquants for Model I are drawn here in order to illustrate the case of one thick isoquant.

for non-homothetic models, $\bar{\ell}$(the mean value of the load factor for 65 observations) is used in drawing all isoquants.

In all figures, each unit of capital services represents one thousand U.S. dollars, while each unit of fuel represents one billion B.t.u.'s (British thermal units). One unit of actual output (y) in the isoquant represents a net generation of one million kilowatt-hours. Because of space limitations, the figures represent only a portion of the isoquants (see Model II).

In Figure V-1 which represents Model I, a combination of 30,102 thousand dollars of capital and 30,803 billion B.t.u.'s of fuel produces a net generation of one million kilowatt-hours (at a point p). Model I has only one thick isoquant, as the diagram shows.

In Figure V-2 of Model II a combination of 851 thousand dollars worth of capital services and 676 billion B.t.u.'s of fuel produces a net generation of one million kilowatt-hours (at a point Q). However, as shown in the small diagram in the upper-righthand corner of Figure V-2, smaller and smaller units of capital and fuel are required as actual output increases from one unit to the next unit. Due to the increasing output scale function (i.e., $h_1(y) = h_2(y) = h(y) = \sqrt{y}$), reduced quantities of capital and fuel are required to produce an <u>additional</u> one million kilowatt-hours of electricity. Contradictory results to those of Model II are expected to come out in the cases of

Models IV and V since both models have decreasing output scale functions.

In Model III, the estimated b_{ij} value is so small that it appears to be similar to the isoquants of the Leontief production function. However, the isoquant is not exactly rectangular. Both Models III and VI have a constant returns to scale output function (In the case of Model VI, it is assumed for the generation of isoquant that $y_c = (y/\bar{\ell})$ where $\bar{\ell}$ is the mean value of ℓ; $\bar{\ell} = .67$ (constant) is used for the measurement of isoquants). For the isoquants corresponding to the models with constant output scale function, the increase or decrease of output does not affect the input requirements proportionately. For instance, suppose the optimum point of producing a net generation of one million kilowatt-hours is the combination of 20,200 dollars of capital services and fuel in the amount of 10.9 billion B.t.u.'s, this being point R in Figure V-6 representing Model VI. Then, the production of the net generation of 1 billion (instead of 1 million) kilowatt-hours would require a combination of capital of 20.2 million dollars and 10,900 billion B.t.u.'s of fuel exactly one thousand times as many inputs as required for producing 1 unit of output.

Model IX is a non-homothetic production model. However, it can generate homothetic isoquants rather than non-homothetic ones due to the fact that $\bar{\ell} = .67$ (constant) is used. Since ℓ itself is variable equation (5.3) cannot readily be usable because there will be three unknowns. The use of $\bar{\ell}$ permits generation of isoquants using only equation (5.3). The diagram of Model IX is similar to that of Model III. Both models turn out to be the cases of constant returns to scale.[3] In fact, the empirical results of Chapter IV show that Models III and IX are not substantially different from each other in a statistical sense.

The isoquants corresponding to non-homothetic production models, e.g., Models VII and VIII, cannot be drawn easily and the isoquant map is not shown to be systematic in terms of intervals as were the homothetic cases. In each non-homothetic case, the marginal rates of technical substitution depend not only on input proportions but on the scale of production. Whether the model has an increasing output scale function or not can be easily observed by drawing two or three isoquants. For instance, in the Figure V-8 of Model VIII, the net generation of 1 million kilowatt-hours may require a combination of the capital services of 26,000 dollars and fuel in the amount of 206 billion B.t.u.'s (at a point S) while the net generation of 2 million kilowatt-hours may require a combination of the capital services of 47,000 dollars and the fuel amount of 393 billion B.t.u.'s

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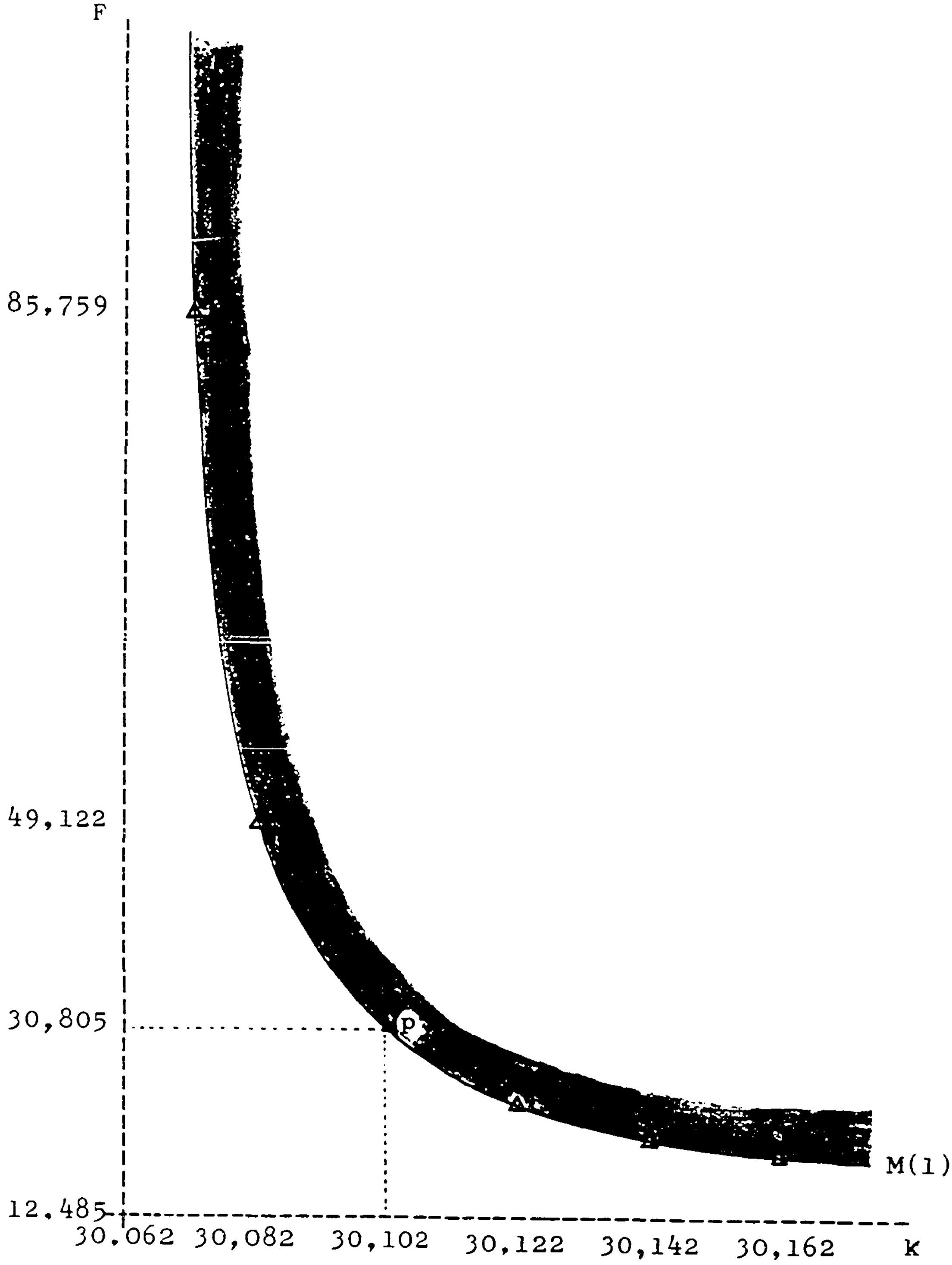
[3]It should be noted that Model IX becomes constant returns to scale case only because $\bar{\ell}$ (constant) is substituted for variable ℓ in order to make use of equation (5.3). Otherwise the returns to scale of Model IX is not determined offhand.

(at a point T). That is, the production of the second unit of output requires fewer capital services and smaller amounts of fuel than those needed for the first unit of output. It is obvious that the non-homothetic production model VIII has an increasing output scale function as a whole. The same can be said of Model VII and is easily observed in Figure V-7.

Contrary to the homothetic cases, the slopes of all the isoquants along the rays from the origin crossing the isoquant map (e.g., OP_1, OP_2 in Figure V-7 and OP_1, OP_2, OP_3 in Figure V-8) are not equal. Notice that the slopes of the tangent line (or the marginal rates of technical substitution between capital and fuel given input prices) at points U and V in Figure V-7 and those at points S and T in Figure V-8 are not equal. This fact can also be proved easily by the algebraic manipulation of equation (5.3) for each individual case.

FIGURE V-1

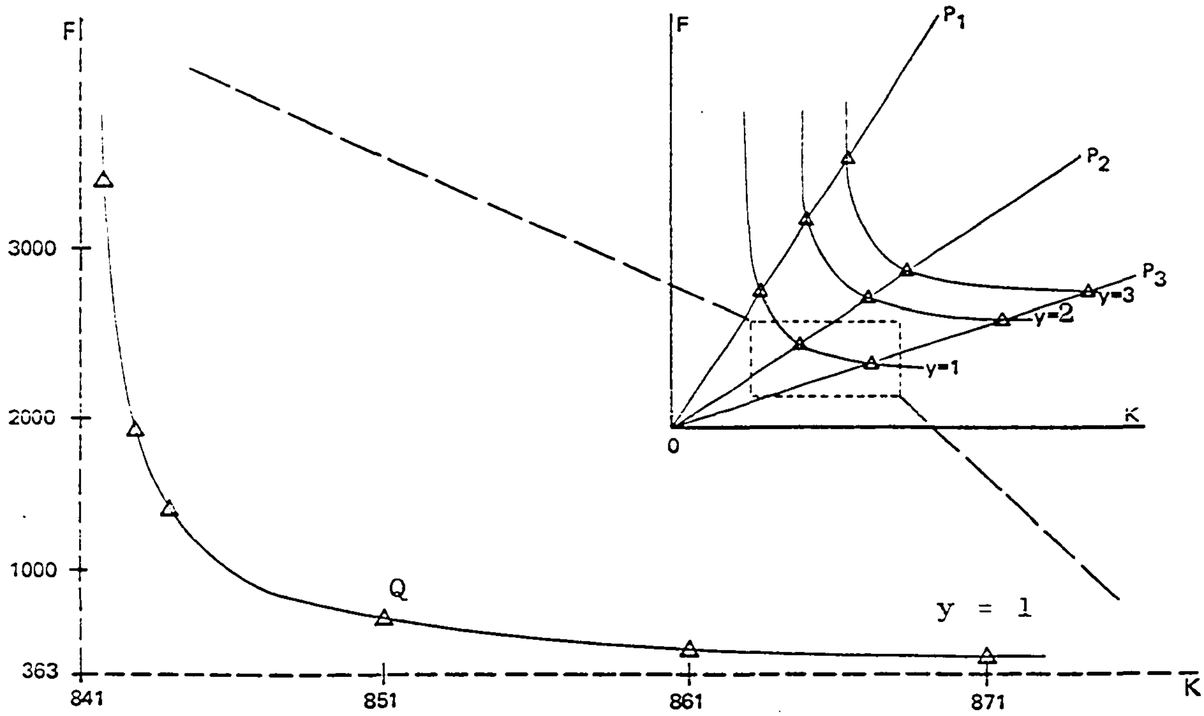
ISOQUANT CORRESPONDING TO MODEL I



$h_1(y) = h_2(y) = h(y) = y^0 = 1$

FIGURE V-2

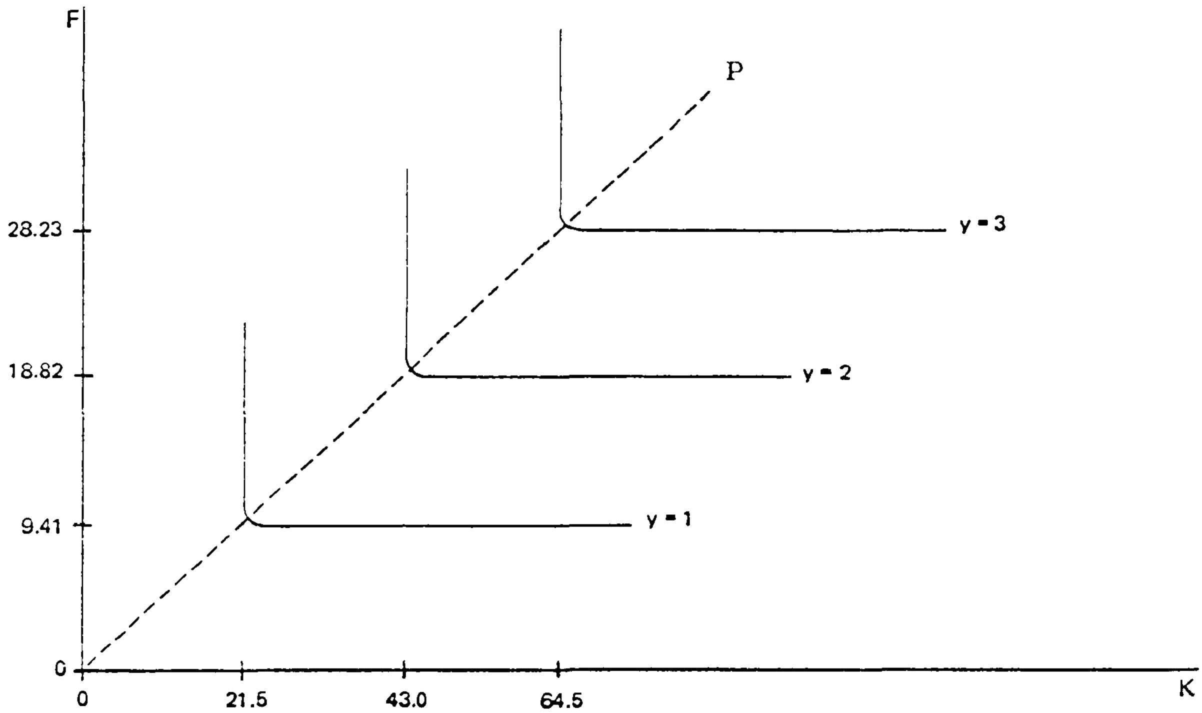
ISOQUANTS CORRESPONDING TO MODEL II



$h_1(y) = h_2(y) = h(y) = y^{1/2}$

FIGURE V-3

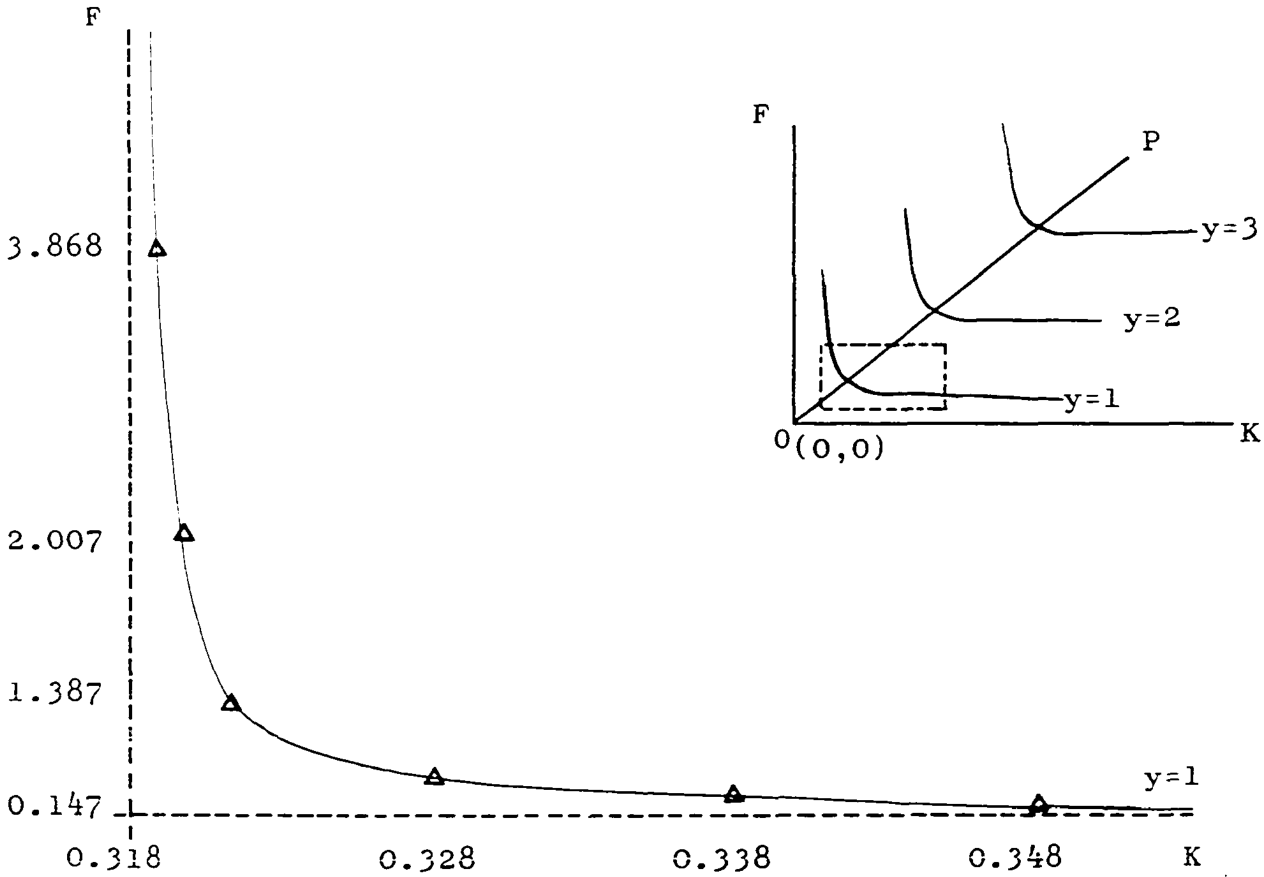
ISOQUANTS CORRESPONDING TO MODEL III



$h_1(y) = h_2(y) = h(y) = y$

FIGURE V-4

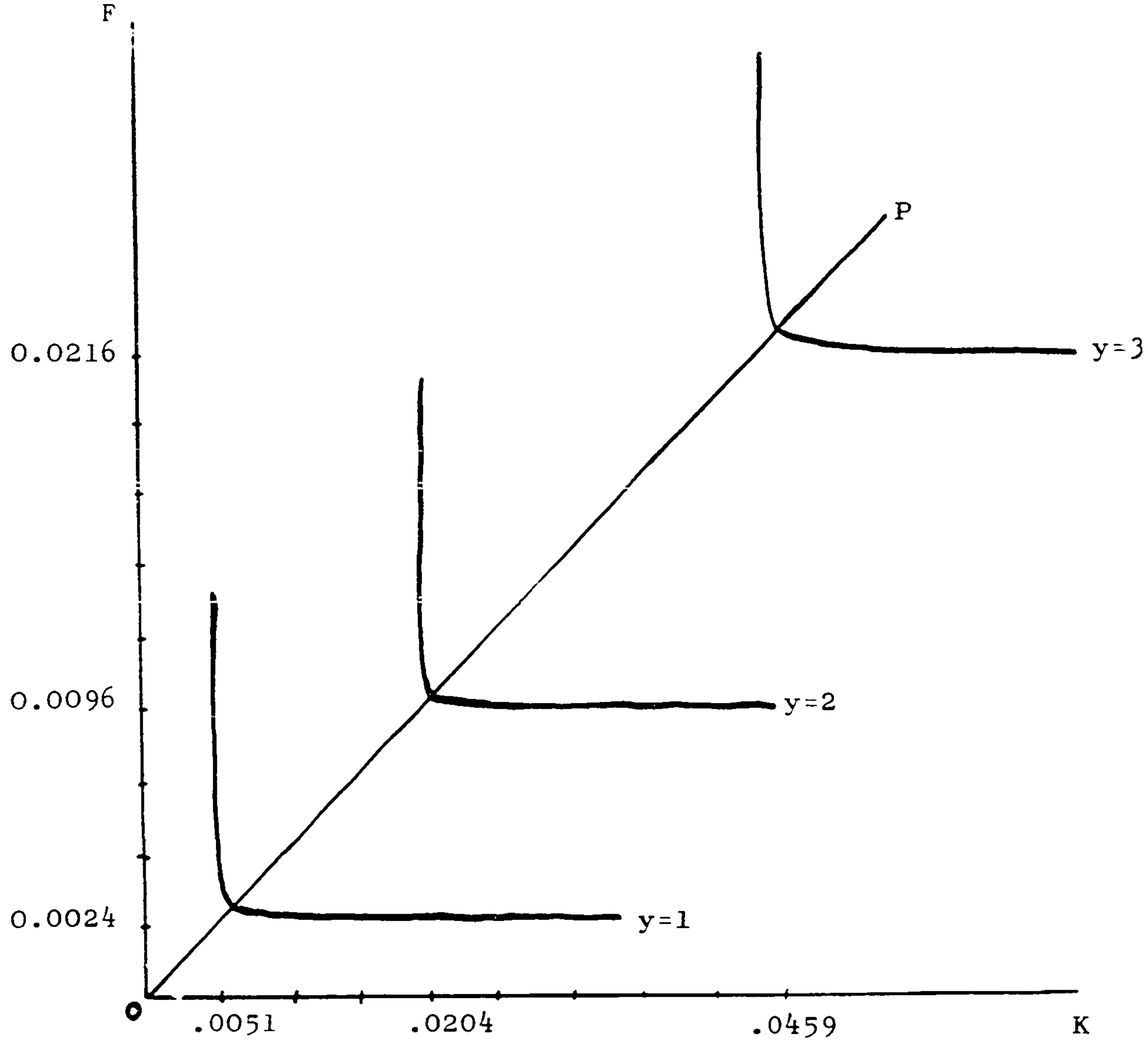
ISOQUANTS CORRESPONDING TO MODEL IV



$h_1(y) = h_2(y) = h(y) = y^{3/2}$

FIGURE V-5

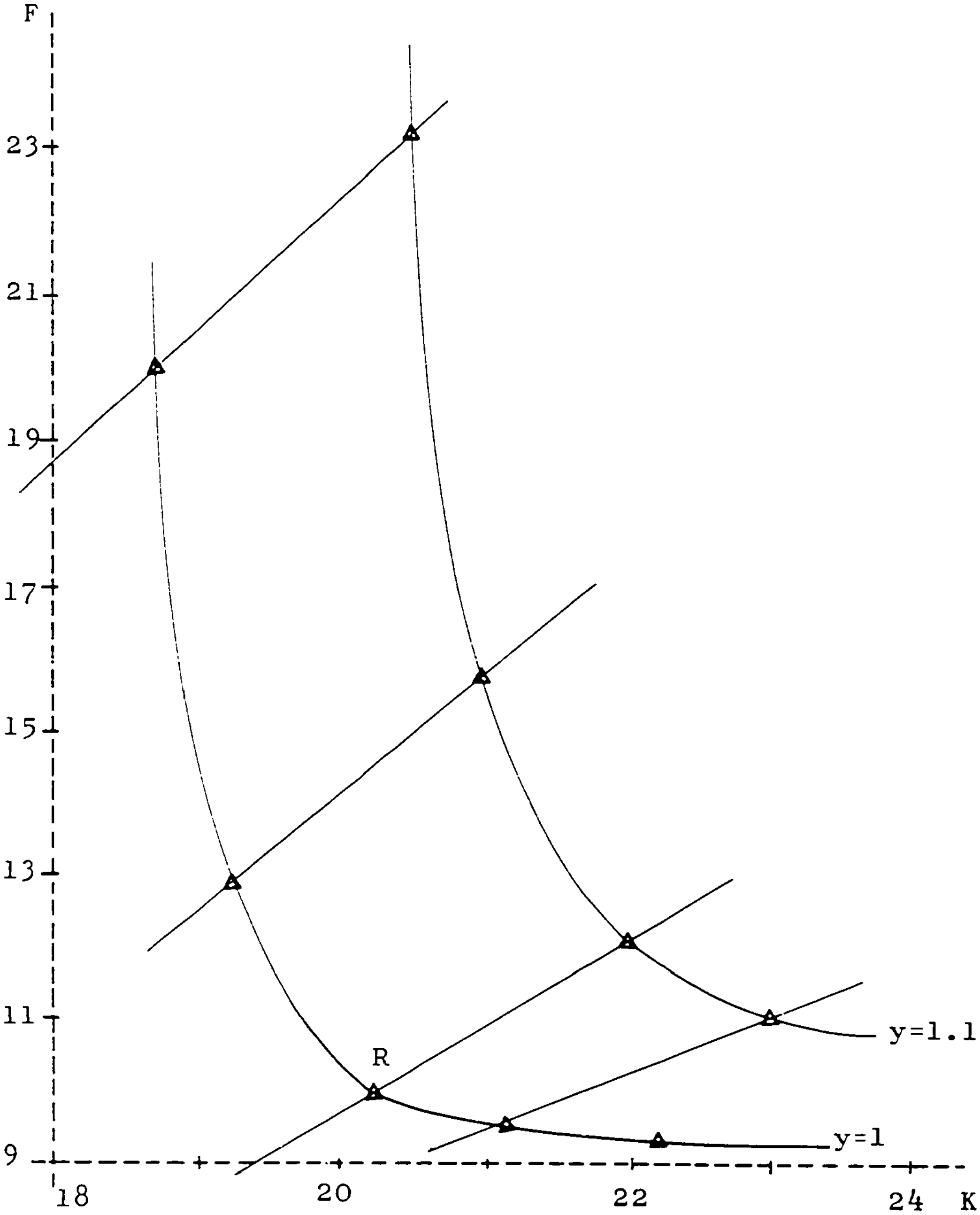
ISOQUANTS CORRESPONDING TO MODEL V



$h_1(y) = h_2(y) = h(y) = y^2$

FIGURE V-6

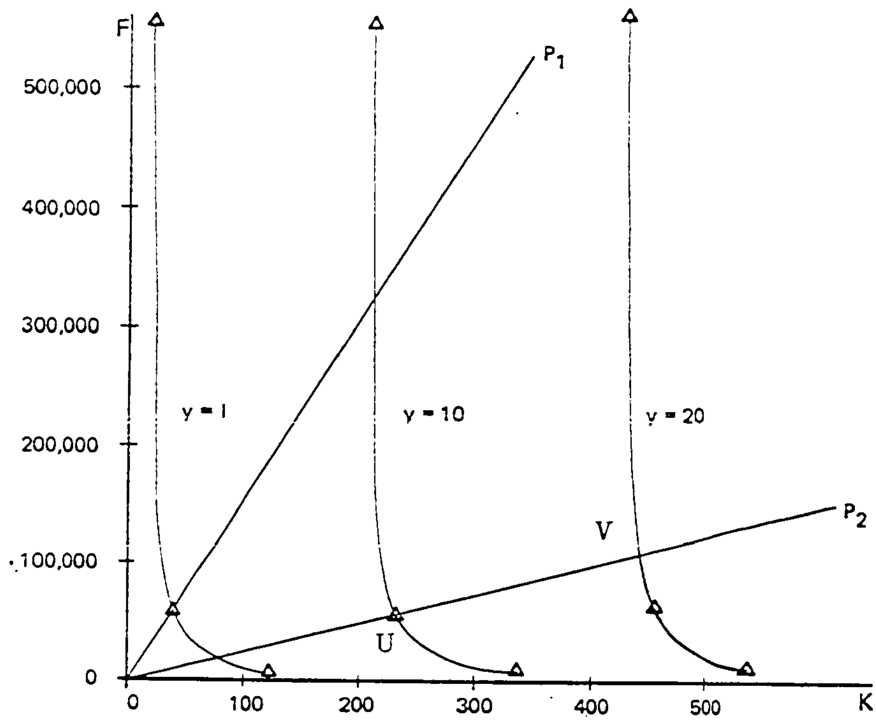
ISOQUANTS CORRESPONDING TO MODEL VI



$h_1(y) = h_2(y) = h(y) = y_c = y \cdot \bar{\ell}^{-1}$

FIGURE V-7

ISOQUANTS CORRESPONDING TO MODEL VII



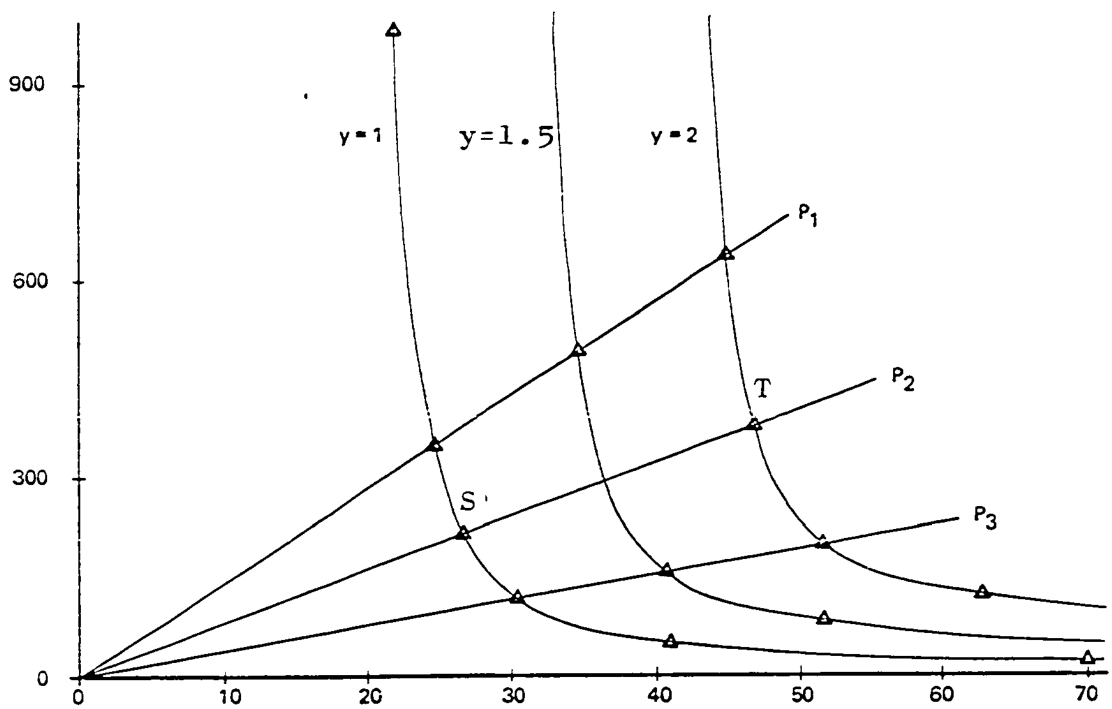
$$h_1(y) = y_c$$

$$h_2(y) = y$$

$$h(y) = y^0 = 1$$

FIGURE V-8

ISOQUANTS CORRESPONDING TO MODEL VIII



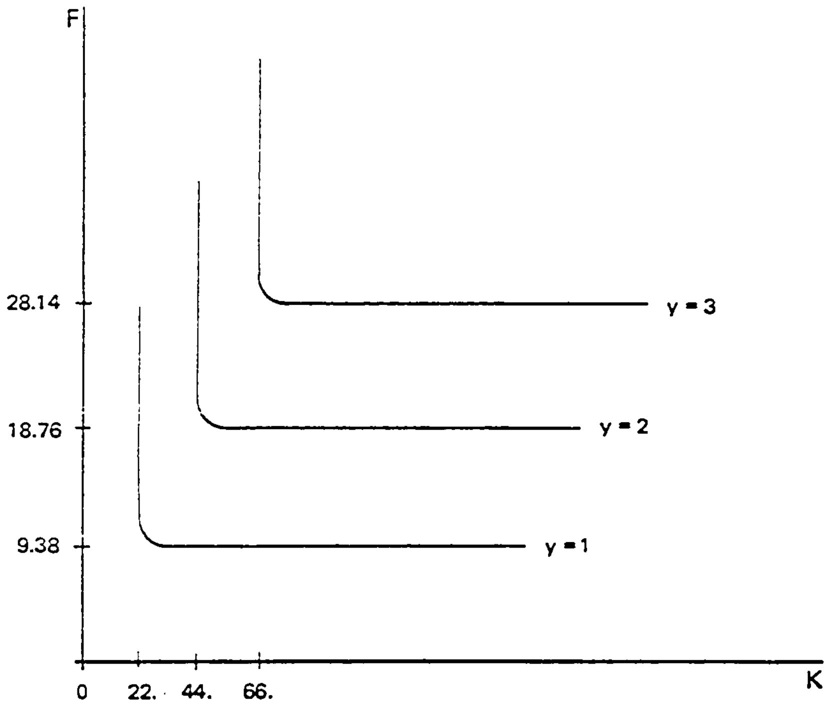
$$h_1(y) = y_c$$

$$h_2(y) = y$$

$$h(y) = y^{1/2}$$

FIGURE V-9

ISOQUANTS CORRESPONDING TO MODEL V-9



$$h_1(y) = y_c = y/\bar{\ell}$$

$$h_2(y) = h(y) = y$$

CHAPTER VI

THE VARIABILITY OF ELASTICITIES

1. Estimation of the Cross-Price Elasticities of Factor Demand and Elasticities of Factor Substitution

In this section, two types of elasticities are estimated with respect to the ex post generalized Leontief production function: (1) the cross-price elasticity of factor demand and (2) the Allen-Uzawa elasticity of factor substitution. The ex post production models I through XI which were estimated and analyzed in Chapter IV are used for an estimation of elasticities.

The (ex post) cross-price elasticity of factor demand (E_{ij}) is estimated from

$$E_{ij}(y;p) = \frac{\partial \log X_i(y;p)}{\partial \log p_j}$$

$$= \frac{1}{2} b_{ij} \left(\frac{p_j}{p_i}\right)^{1/2} \frac{h(y)}{X_i(y;p)} \qquad \text{for } i \neq j \qquad (6.1)$$

The cross-price elasticity of factor demand may be defined as an index of responsiveness of the cost-minimizing i^{th} input bundle to a change in the price of the j^{th} factor, all other prices held constant.

The (ex post) Allen-Uzawa (pair-wise partial) elasticity of factor substitution (σ_{ij}) is estimated from

$$\sigma_{ij}(y;p) = \frac{\left[C(y;p)\right]\left[\frac{\partial^2 C(y;p)}{\partial p_i \partial p_j}\right]}{\left[\frac{\partial C(y;p)}{\partial p_i}\right]\left[\frac{\partial C(y;p)}{\partial p_j}\right]}$$

$$= \frac{1}{2} b_{ij} \frac{h(y)}{X_i X_j (p_i p_j)^{1/2}} \cdot C(y;p) \qquad (6.2)$$

The Allen-Uzawa partial elasticity of factor substitution is defined for a total cost function.[1]

Since b_{ij} is estimated in Chapter IV and data on variables of factor prices, factor demands and actual output are assumed to be available in the generalized Leontief production model, the cross-price elasticity of factor demand and the Allen-Uzawa elasticity of substitution from equations (6.1) and (6.2) are easily estimated. The data on the total cost (C) were generated from $C(y;p) = \sum_{i=1}^{n} X_i p_i$ where n = the number of factors used in the model.

The different value of elasticities among the eleven models estimated is mainly dependent upon the magnitude of the ex post technological parameters, b_{ij}, which describe substitution possibilities and the functional form of the output scale used in each model. The estimated elasticities

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[1]R. G. D. Allen, Mathematical Analysis for Economists (London: Macmillan Co., 1964), p. 504; Uzawa, "Duality...," op. cit.; McFadden, op. cit., p. 6.

are variable across sample observations (and/or possibly over time). The variability of the elasticities of substitution and its determinants are subject-matter in the following section. Two types of elasticities given in Table VI-1 and Table VI-2 are the arithmetic mean value of respective elasticities of 65 observations and their corresponding standard deviations are also provided.

Note that the negative ex post technological parameters, b_{ij}, in models I, X and XI in Tables VI-1 and VI-2 cause their respective measured elasticities to adopt negative values. However, as noted earlier, this would violate the assumption of convexity of the production possibility sets. In order to be consistent with the assumptions of production function theory, no significance will be attached to the results of elasticity measurements of models I, X, and XI.

As for the rest of the models under investigation, all cross-price elasticities of factor demand are non-negative which indicates that capital and fuel are not complementary factors of production. All of the own price elasticities of factor demand are non-positive as required by concavity of cost function. In terms of the mean value, the cross-price elasticity of factor demand ranges between .0033 and .0715 in Table VI-1 while the Allen-Uzawa elasticity of substitution between capital and fuel ranges from .0066 to .1378 in Table VI-2. It can be observed that the

TABLE VI-1

Estimated Ex Post Cross-Price Elasticities of of Factor Demand (E_{kf})*

Model	b_{ij}	Elasticity Parameters: High Value	Low Value	Mean Value**	Standard Deviation
I	-856.8	-.0090	-.3825	-.039840	.005622
II	56.3	.1421	.0321	.062442	.003334
III	.1006	.0071	.0020	.003357	.000146
IV	.0605	.1543	.0072	.071489	.005493
V	.0012	.1872	.0011	.056177	.006464
VI	1.66	.0177	.0038	.009537	.000323
VII	735.3	.2933	.0065	.034191	.004824
VIII	31.41	.1128	.0128	.034837	.001860
IX	.1420	.0086	.0004	.004739	.000206
X	-.1380	-.0024	-.4322	-.162796	.012509
XI	-.0002	-.00002	-.0374	-.009363	.001077

*Subscripts k, f designate factors capital, fuel, respectively.

**The change in sign of each estimated cross-price elasticity parameter will give the estimated (ex post) own price elasticity of factor demand.

TABLE VI-2

Estimated Ex Post Allen-Uzawa Elasticities of Substitution Between Capital and Fuel (σ_{kf})*

Model	Elasticity Parameters			Standard Deviation
	High Value	Low Value	Mean Value	
I	-.0124	-.6550	-.0871	-.0124
II	.2779	.0552	.1272	.0053
III	.0096	.0032	.0066	.0002
IV	.2876	.0125	.1378	.0087
V	.3658	.0016	.1065	.0108
VI	.0838	.0126	.0201	.0011
VII	.5621	.0107	.0747	.0107
VIII	.1551	.0308	.0710	.0030
IX	.0135	.0045	.0093	.0002
X	-.0284	-.6549	-.3139	.0198
XI	-.0003	-.0610	-.0178	.0018

*Subscripts k, f designate factors capital, fuel, respectively.

Allen-Uzawa elasticity of factor substitution is approximately two times bigger than the cross-price elasticity in each case. The variations in output scale function do not seem to greatly affect the low values of estimated elasticities: In the case of cross-price elasticities, all of the estimated values are less than .08 while in the case of elasticities of substitution all are less than .14 in terms of the mean value no matter which output scale function is used in each model. In all cases, the standard deviations are fairly small. In addition, both E_{kf} and σ_{kf} remain positive in all sample observations. The largest individual value for the cross-price elasticity of factor demand is no greater than 0.30 and that for the Allen-Uzawa elasticity of substitution is less than 0.57.

One striking result obtained from the empirical measurements of this section is the variability of two estimated elasticities in each model. It is obvious from the estimated low values of σ_{kf} that the Cobb-Douglas production function is not an accurate measure of the ex post technology in the steam-electric generating industry. The variability of the Allen-Uzawa elasticity of substitution is given a special attention in the following section.

2. Testing the Variability of the Allen-Uzawa Elasticity of Substitution Between Capital and Fuel

This section investigates the determinants for the elasticity of substitution between capital and fuel. It tests the J. R. Hicks's hypothesis that the input ratio (in the case of the present model, the capital-fuel ratio) and the level of output will be the dominant factors causing the variations in the elasticity of substitution:

> If capital is increasing more rapidly than the supply of labor /If the capital-labor ratio increases7, a tendency towards a diminished elasticity of substitution will generally set in as capital grows /and vice versa7.[2]

> /There is a7 tendency for capital to shift from the less capitalistic to the more capitalistic trades, /i.e., to7 those which use a relatively large proportion of capital to labor making a unit of product /implying that the capital-labour ratio and the level of output have a positive relationship7.[3]

In principle, the elasticity of substitution can assume any value between zero and infinity. One of the primary weaknesses in using the Cobb-Douglas or the C.E.S. production functions is that their elasticities of substitution are by necessity constant. Specifically, in the case of a Cobb-Douglas production function, all Allen-Uzawa pair-wise partial elasticities of substitution are unity. For the C.E.S. production function the values for the

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[2] J. R. Hicks, op. cit., p. 132. Parentheses are mine.

[3] Ibid., pp. 187-188. Parentheses are mine.

elasticities of substitution are determined by the data; but, they are all constant and equal.[4] The variable elasticity of substitution (V.E.S.) production function aims at overcoming this disadvantage. The improvement from using a generalized Leontief production function is in its allowance for the variability of the elasticity of substitution.

From equation (6.2) it can readily be seen that the Allen-Uzawa elasticity of substitution may vary across the sample. Recently, McFadden and Revankar independently attempted to establish empirically the relationship between the elasticity of substitution and factors mostly affecting them.[5]

For instance, McFadden has considered the case where the elasticity of substitution of the C.E.S. varies linearly with time. Revankar has studied the variability of the elasticity of substitution and named the production function which allows it as the V.E.S. In Revankar's case, two more explanatory variables along with time are added to explain

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[4]See M. Fuss, Harvard Discussion Paper No. 141, p. 10.

[5]McFadden, op. cit.; N. S. Revankar, "A Class of Variable Elasticity of Substitution Production Functions," Econometrica, Vol. 39(1), January 1971, pp. 61-71. In Revankar's work, the elasticity of substitution parameter σ_{ij} of the V.E.S. production function varies (linearly with the capital-labor ratio) but only around the intercept term of unity. Moreover, the behavior of σ_{ij} is one-sidedness: It is either less or greater than unity over the sample as Revankar himself admitted.

the variation of the elasticity; namely, capital-labor ratio (K/L) and the level of output (y).

However, both McFadden's and Revankar's studies on the variability of the elasticity of substitution do not allow arbitrary returns to scale. In fact, they are dealing with only the constant returns to scale case. In contrast to this approach, the use of a generalized Leontief production function permits studying of the relationship between the elasticity of substitution and explanatory variables with <u>any arbitrary</u> returns to scale.

The models III, VII, VIII and IX are chosen for the empirical analysis of the determinants for the elasticity of substitution. Only the above four models are chosen from the eleven models presented in Chapter IV since these models yielded the best empirical results. Model III is the case of constant returns to scale with homothetic behavior. Models VII, VIII and IX are the cases of non-decreasing returns to scale with non-homothetic behavior.

Three step-wise regression models are constructed in order, first, to test whether or not the Allen-Uzawa elasticity of substitution between capital and fuel varies over time, and, second, to identify the dominant factors explaining this variation. A key characteristic of the three step-wise regressions outlined below is that they present the variability of the elasticity of substitution as the maintained hypothesis. The two limiting cases, Cobb-Douglas's unitary elasticity of substitution and C.E.S.'s constant elasticity of substitution are contained as testable hypotheses within the maintained hypothesis.[6]

In the beginning, each of the three regression models was estimated twice: One with the assumption of a linear relationship, and the other with the assumption of a log-linear relationship. The final three models are built upon the results of the latter, because the equations with a log-linear assumption statistically outperformed those with a simple linear assumption. The choice of the following models was based on the minimum estimated standard error of the regression:

The Three Models To Test the Variability of the Elasticity of Substitution

Model A $\quad \gamma = r e^{\lambda t} \exp u$

Model B $\quad \gamma = r e^{\lambda t} (K/F)^{\theta} \exp u$

Model C $\quad \gamma = r e^{\lambda t} (K/F)^{\theta}\, y^{\delta} \exp u$

where u is a stochastic residual term and E (exp u) $= \phi(1)$ where ϕ is the moment generating function of u. The

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[6] The Allen-Uzawa elasticity of substitution (σ_{ij}) in a two-factor case reduces to the direct elasticity of substitution (σ_{ij}^*): For instance,

$$\sigma_{ij} = \frac{C \cdot C_{rf}}{C_r \cdot C_f} = - \left.\frac{d \ln (C_r/C_f)}{d \ln (f/r)}\right|_{y \text{ fixed}}$$

$$= - \left.\frac{d \ln (K/F)}{d \ln (f/r)}\right|_{y \text{ fixed}} = \sigma_{ij}^*$$

random variable u enters in exponential form only because it is convenient to do so.

Taking the logarithm of both sides, it is found that:

Model A $\ln \hat{\sigma} = \ln \gamma + \lambda t + u$

Model B $\ln \hat{\sigma} = \ln \gamma + \lambda t + \theta \ln(K/F) + u$

Model C $\ln \hat{\sigma} = \ln \gamma + \lambda t + \theta \ln(K/F) + \delta \ln y + u$

where $E(u) = 0$.

Each function now exhibits linearity in its parameters except for γ. Therefore, the parameters can be estimated by the ordinary least squares estimation method (OLSQ).[7]

In model A, the following hypotheses can be set up:

	Hypothesis	Type of Production Function
(1)	If $\ln \gamma = \lambda = 0$,	Cobb-Douglas
(2)	If $\lambda = 0$,	Constant Elasticity of Substitution (with $\sigma = \gamma$)
(3)	If $\ln \gamma, \lambda \neq 0$,	Variable Elasticity of Substitution

Hypothesis (1) and (2) are null hypotheses independently against the alternative hypothesis (3). The rejection of the null hypothesis (1) at a significance level will automatically lead us to accept the alternative hypothesis which is the maintained hypothesis. Since it is needed to test a set of coefficients simultaneously, e.g., $\ln \gamma = \lambda = 0$, we have to use a F-statistic.

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[7]Liew and Kahng, op. cit., Chapter 2.

The estimation of the regression model A and the results of F test for related hypotheses are given in Tables VI-3 and VI-4, respectively.

The values of the t-statistic for the estimated coefficients are appropriate for testing the respective coefficients equal to 0. The values of the t-statistic are given in the parentheses below their respective coefficients in Tables VI-3, VI-5 and VI-7.

If $\lambda = 0$, it indicates that the elasticities of substitution between capital and fuel do not vary over time. In model A, the t-values in the four models are insignificant at 1 per cent level so that $\lambda = 0$ is accepted (t value with 63 degrees of freedom = 2.57). This would indicate that the elasticity of substitution between capital and fuel in the steam-electric generating industry does not vary significantly over time. No significant time trend is observed in the elasticity of substitution.

The sign of λ is inconsistent across the four models in Model A. The a priori expectation of the sign of λ is positive. The economic reasoning here is that as time passes, technology advances and results in the enhancement of opportunities of factor substitution. However, no judgment can be made without a more complete specification of the model.

Hypothesis (1) is rejected in all four models under investigation. In other words, the empirical result shows

TABLE VI-3

Regression Results of Model A

Model	Estimated ln γ	Coefficients λ	S.E.R.	R^2	D.W.	d.f.
III	-5.134 (-93.03)	.0186 (1.93)	.2276	.0556	2.104	63
VII	-2.591 (14.28)	-.0713 (-2.25)	.7485	.0744	2.078	63
VIII	-2.568 (-33.53)	-.0264 (-1.97)	.3159	.0581	1.929	63
IX	-4.790 (-86.79)	.0186 (1.93)	.2276	.0556	2.104	63

The value inside parentheses indicates the t-statistic for the respective coefficient.

TABLE VI-4

Results of F Test for Related Hypotheses in Model A

Hypothesis	Model				Degree of Freedom	Statistical Inference at 1% level*
	III	VII	VIII	IX		
(1)	31903.	1009.3	4743.4	27691.	(2,63)	Reject
(2)	3.71	5.06	3.89	3.71	(1,63)	Accept**
(3)	Alternative Hypothesis					Accept

*F values at a significance level of 1% from the F table are:
F(2,63) = 5.0
F(1,63) = 7.1

**Null hypothesis (2) can be rejected at a significance level of .10.

that the Cobb-Douglas production function would be a poor representation of the production technology in the steam-electric generating industry. Hypothesis (2) is accepted at 1 per cent level in all four models. At 5 per cent level, it can be rejected in models III, VIII and IX. In model VII, the hypothesis (2) can be rejected at 10 per cent level. As a whole, the characteristic of a constant elasticity of substitution between capital and fuel in model A can be either accepted or rejected depending upon the significance level chosen. However, the low R-squared values in four models indicate that model A is a very poor fit. Apparently time alone is not enough to explain the causes of variation in the Allen-Uzawa elasticity of substitution between capital and fuel. An improvement is needed in the specification of model A.

Past empirical works on the subject of the elasticity of substitution suggest that the input ratio, for instance, (K/F), may be a dominant factor in causing variations in the elasticity of substitution. This idea of including the capital-fuel ratio as an explanatory variable is followed in model B.

In model B, the following hypotheses can be set up:

Hypothesis	Type of Production Function
(1) If $\ln \gamma = \lambda = \theta = 0$,	Cobb-Douglas
(2) If $\lambda = \theta = 0$,	Constant Elasticity of Substitution
(3) If at least one of $\lambda, \theta \neq 0$,	Variable Elasticity of Substitution

Hypotheses (1) and (2) are null hypotheses independently against the alternative hypothesis (3). The two sets of null hypotheses can be simultaneously tested by the F-statistic. The estimation of the regression model B and the results of F test for related hypotheses are given in Tables VI-5 and VI-6, respectively.

The rejection of both null hypotheses (1) and (2) at the 1% level in all four models under investigation indicates that the alternative hypothesis of a variable elasticity of substitution should be accepted.

In model B, the relative importance of time variable in explaining variations of the elasticity of substitution is reduced in all four cases. As was the case in model A, the sign of λ in model B is inconsistent across the four models. Since R-squared values of models VII and VIII are still very low, .2604 and .1066 respectively, no significance will be attached in the interpretation of the signs of the estimated coefficients. The specification error itself may be responsible for the wrong sign.

In general, the R-squared values of four models in model B are considerably increased from those in model A by adding the capital-fuel ratio as a determining factor for the variability of the elasticity of substitution, especially, the R-squared values of models III and IX. In both cases, from .0556 in model A to .7571 in model B. At the same time, the standard errors of the regression of models III

TABLE VI-5

Regression Results of Model B

Model	Estimated Coefficients			S.E.R.	R^2	D.W.	d.f.
	$\ln \gamma$	λ	θ				
III	-4.597 (-93.65)	.0050 (0.99)	-.4977 (-13.38)	.1164	.7571	2.028	62
VII	-3.522 (-12.42)	-.0478 (-1.64)	.8614 (4.01)	.6724	.2649	1.803	62
VIII	-2.7639 (-21.13)	-.0214 (-1.60)	.1819 (1.83)	.3101	.1066	1.788	62
IX	-4.252 (-86.63)	.0050 (.99)	-.4977 (-13.38)	.1164	.7571	2.028	62

The value inside parentheses indicates the t-statistic for the respective coefficient.

TABLE VI-6

Results of F Test for Related Hypotheses in Model B

Hypothesis	Models				Degree of Freedom	Statistical Inference at 1% level*
	III	VII	VIII	IX		
(1)	122,260.	1266.8	4925.0	106150.	(3,62)	Reject
(2)	193.3	22.3	7.40	193.3	(2,62)	Reject
(3)	Alternative Hypothesis					Accept

*F values at a significance level of 1% from the F table are:
F(3,62) = 4.1
F(2,62) = 5.0

and IX, .1164, in model B is reduced to almost half those of the model A. Thus, a significant improvement is made in the specification of model B, especially when it is applied to models III and IX. The values of the Durbin-Watson statistic are between 1.8 and 2.0 which indicate the lack of serial correlation among the least squares residuals.

Because of their relatively higher R-squared values, models III and IX are analyzed for the sign of estimated coefficients. The positive sign of λ is observed in both models. This result fits a priori expectation on the basis of the economic reasoning explained earlier. The sign of coefficient for the capital-fuel ratio comes out as negative. This indicates that variations in the capital-fuel ratio causes inverse variations in the Allen-Uzawa elasticity of substitution between capital and fuel. In Revankar's study, the inverse relationship between capital-labor ratio and the elasticity of substitution was explained as follows:

> It is probably characteristic of a developed economy that a high capital-labor ratio at a given point of time represents a capital stock with a relatively larger proportion of new investment than does a low capital-labor ratio. This new investment naturally takes the form of new specialized machines and tools - and conceivably structures as well. These specialized machines, at the higher end of the scale (of the capital stock) call for specific skills and so allow very little substitutability. At the lower end, however, with older capital in use, the ease of substitution between capital and (understandably somewhat lower quality of) labor is likely to be higher.[8]

[8]N. S. Revankar, "Capital-Labor Substitution, Technological Change and Economic Growth: The U.S. Experience,

The same economic reasoning can be applied to the empirical results of models III and IX. A high capital-fuel ratio represents a relatively higher share of new investment in the capital stock than does a low capital-fuel ratio. The new investment may take the form of highly specialized machines (e.g., generators) which call for specific fuels and preassigned amounts of fuel and so allow for very little substitution. At the lower capital-fuel ratio, however, the older machines in operation may have a limited possibility of substitution between capital and fuel.

The empirical result of model B shows that a one per cent increase in the capital-fuel ratio results in almost a 50 per cent decrease in the elasticity of substitution between capital and fuel in models III and IX.

A significant role of the capital-fuel ratio as an explanatory variable is justified by a substantial increase of R-squared values in all four models. The t-values of θ are also considerably higher except those of model VIII.

The last attempt to explain the relationship between the variations in the elasticity of substitution and the factors causing them includes actual output (y) (of course, in logarithmic form) as an explanatory variable in addition to the variables used in model B.

1929-1953," Technical Report No. 11, Project for the Explanation and Optimization of Economic Growth, Institute of International Studies, University of California, Berkeley, 1968, pp. 25, 27.

Based on the model C, the following hypotheses are set up:

	Related Hypothesis	Type of Production Function
(1)	If $\ln \gamma = \theta = \delta = \lambda = 0$,	Cobb-Douglas with neutral technical change
(2)	If $\lambda = \theta = \delta = 0$,	Constant elasticity of substitution with neutral technical change
(3)	If $\lambda = 0$ and at least one of θ, and $\delta \neq 0$,	Variable elasticity of substitution with neutral technical change
(4)	If $\lambda \neq 0$ and at least one of $\ln \gamma$, θ, and $\delta \neq 0$.	Variable elasticity of substitution with nonneutral technical change

Hypotheses (1), (2) and (3) are testable hypotheses independently under the model C. Hypothesis (4) is the alternative hypothesis against the null hypothesis (1). Hypothesis (3) is a special case of hypothesis (4). Since a set of coefficients is tested simultaneously, a F test is performed against each null hypothesis as before.

The estimation of the regression model C and the results of F test for related hypotheses are given in the following Tables VI-7 and VI-8, respectively.

As a whole, the model C gives the minimum standard error of the regression (S.E.R.) in all four cases. The mean value of S.E.R.s of the three regression models is calculated below for comparison:

TABLE VI-7

Regression Results of Model C

Model	Estimated Regression Coefficients				S.E.R.	R^2	D.W.	d.f.
	$\ln \gamma$	λ	θ	δ				
III	-5.370 (-41.77)	-.0002 (-.058)	-.3633 (-10.05)	-.0988 (6.30)	.0913	.8529	2.008	61
VII	3.526 (27.43)	-.0002 (-.058)	-.3634 (-10.06)	-.9011 (-57.46)	.0931	.9867	2.008	61
VIII	.3735 (2.91)	-.0002 (-.058)	-.3633 (10.06)	-.4011 (-25.58)	.0913	.9238	2.008	61
IX	-5.025 (-39.09)	-.0002 (-.058)	-.3633 (-10.05)	-.0988 (6.30)	.0913	.8529	2.008	61

The value inside parentheses indicates the t-statistic for the respective coefficient.

TABLE VI-8

Results of F Test for Related Hypotheses in Model C

Hypothesis	Models				Degrees of Freedom	Statistical Inference at 1% level*
	III	VII	VIII	IX		
(1)	198680.	72010.	57473.	172500.	(4,61)	Reject
(2)	353.78	4513.7	739.64	353.78	(3,61)	Reject
(3)	.0034	.0034	.0034	.0034	(1,61)	Accept
(4)	Alternative Hypothesis					Accept

*F values at a significance level of 1% from the F table are:

$F(4,61) = 3.7$
$F(3,61) = 4.1$
$F(1,61) = 7.1$

Model	Mean Value of S.E.R.s of Three Models
A	.3799
B	.3038
C	.0913

The null hypotheses (1) and (2) are rejected by a F test in all four cases and, as a result, alternative hypothesis (4) is accepted. Therefore, the regression results of model C strongly support the maintained hypothesis of the variability of the elasticity of substitution. In model C, hypothesis (3) is considered as a special case of hypothesis (4). Therefore, a F test is performed against the null hypothesis (3) to see whether the time variable is an important factor causing variations in the elasticity of substitution between capital and fuel. The F-statistic in all four cases is .0034 which is definitely small enough to accept the null hypothesis that $\lambda = 0$. The t value of the estimated time variable coefficient, λ, is -0.058, in all four models which is definitely small enough to accept $\lambda = 0$. The t values for the other three coefficients are large enough to reject the hypothesis that the respective individual coefficient is equal to 0 at a significance level of .01. Since time turns out to be <u>not</u> important as an explanatory variable in model C, the sign of λ will not be considered seriously. The empirical results of Model C support the Hicksian hypothesis that the elasticity of substitution between capital and fuel is inversely related to both the capital-fuel ratio and the level of output. Notice that the sign of coefficients in all four models is positive. A one per cent increase in the capital-fuel ratio results in a 36.3 per cent decrease of the elasticity for all four models. The elasticity of substitution between capital and fuel is inversely related to the level of output. A one per cent increase in the level of output results in a 90 per cent decrease in the elasticity for model VII, a 40 per cent decrease for model VIII and a 9.8 per cent for models III and IX.

The empirical result of Model C also indicates that the level of output and the capital-fuel ratio have a positive relationship. In the generation of electricity the high level of output coincides with the high capital-fuel ratio. Notice that the sign of θ and δ are the same (Both are positive in model C).

For the least-squares regression results of model C, the values of R-squared are higher than .85 and the standard errors of the regression are considerably reduced in all four cases. The Durbin-Watson statistic is 2.008 in all four cases. These values indicate a fairly good least-squares fit and the lack of serial correlation among the least-squares residuals.

The empirical results in this section support the hypothesis that the production function in the steam-electric generating industry has a variable elasticity of substitution production function with neutral technical change. It also supports the contention that the Cobb-Douglas and the C.E.S. production functions do not accurately represent the ex post technical structure of electricity generation. For those reasons, the development and application of a generalized Leontief production function is justified.

3. Summary and Remarks

Several comments may be made here on the behavior of the Allen-Uzawa elasticity of factor substitution in a generalized Leontief production model.

1) In the generalized Leontief production function investigated, the elasticity of substitution is variable; neither unitary nor constant as the cases of the Cobb-Douglas or the C.E.S. production functions. It can easily be seen from the equation (6.2) that the elasticity of substitution varies across the sample (even at a given point of time).

2) Empirical works proved that the elasticity of substitution not only varies across the sample (at a given point of time) but may vary over time (at different time periods). In the steam-electric generating industry, however, the relative importance of time variable in explaining variations of the elasticity of substitution between capital and fuel has been greatly diminished (almost to none in

model C) by introducing other dominant explanatory variables.

3) The capital-fuel ratio and the level of output have been proved to be dominant factors causing variations in the elasticity of substitution between capital and fuel. The empirical study of the steam-electric generating industry shows that the Hicksian anticipation of the role of input ratio as well as the level of output in determining variations in the elasticity of substitution is correct.

4) The three regression models constructed in pages 61-2 have permitted testing empirically of that the Cobb-Douglas and the C.E.S. production functions are only special cases of the maintained hypothesis of the variable elasticity of substitution.

5) The use of a generalized Leontief production function with variable output scale functions does not limit the search for the dominant factors for the variable elasticity only to the case of a constant returns to scale. The replacement of a constant output scale function by a non-constant one will solve the problem of returns to scale. The current two-factor model may be extended to a multi-factor case: Multi-factor problem will require only more calculations.

6) The data on the steam-electric generating industry used in current study is both cross-section and time-series. The role of time in causing variations of the Allen-Uzawa

elasticity of substitution is complicated to a certain degree by the inclusion of dissimilar size of the cross-section in each period. More meaningful results would come out when purely time-series data are used. The future task of research is expected to prove this fact.

CHAPTER VII

A MODEL FOR TESTING THE "PUTTY-CLAY" HYPOTHESIS[1]

1. Introduction

The notion of "putty-clay" was introduced in relation to economic growth theory by several growth economists such as Johansen, Massell and Phelps in recent years.[2] Their main interest concerning "putty-clay" centers around the substitutability between capital (represented by, e.g., machines) and labor (represented by, e.g., man-hours) over the economic life of machines, especially in "vintage" growth models.

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[1] Chapter VII draws heavily on the description and construction of the "putty-semiputty" model outlined in Fuss's dissertation. Especially, section 2 of this chapter is a summary version of his development. Our model differs from Fuss's in that the former is a simplified two-factor model and does not require any a priori simplifying assumption of factor relationships. Moreover, our two-factor model attempts to provide a deeper insight on the various natures of "putty-semiputty" model.

[2] Leif Johansen, "Substitution Versus Fixed Proportion Coefficients in the Theory of Economic Growth: A Synthesis," Econometrica, 27 (April, 1959), 157-76.

Benton F. Massell, "Investment, Innovation and Growth," Econometrica, 30 (April, 1962), 239-52.

E. S. Phelps, "Substitution, Fixed Proportions, Growth and Distribution," International Economic Review, 4(3), September, 1963, 265-288.

Two types of structure of technology are classified: (1) Substitution between capital and labor before new investment (the installation of new machines) is made on the one hand and (2) Substitution between capital and labor after the machines are installed on the other hand. The former is called the ex ante structure of technology and the latter the ex post structure of technology.

Time concepts such as long-run and short-run considerations are related to the structure of technology, that is, substitutability between capital and labor. The ex ante technology is chosen according to some long-run criteria while the ex post technology is based on the short-run objectives subject to the constraints imposed by the ex ante choice of technique.

With the above-mentioned theoretical assumptions in mind, three discernible cases can be easily explained in production theory: (1) Putty-putty is the case where there is substitution before (ex ante) and after installation of new machines (ex post), (2) Putty-clay is the case where there is substitution before but fixed labor requirements after installation of new machines, and (3) Clay-clay is the special and rare case in which the capital-labor ratio is fixed both before and after installation of new machines.

The above discussions may be summarized as in the Table VII-1:

TABLE VII-1

The Time Structure of Technology

Substitution Between Factors in Ex Ante (long-run)	Ex Post (short-run)	Type of Structure of Technology
Yes	Yes	Putty-Putty
Yes	No	Putty-Clay
No	No	Clay-Clay

2. An Econometric Study of "Putty-Clay": M. Fuss's Solution:

Most previous works on the "putty-clay" thesis were highly theoretical and dealt in favor of developing vintage growth models and capital theory. In fact, few econometric studies of production were able to combine this thesis into practical models. The main obstacle probably lies in the dynamic concept of "putty-clay," involving the time dimension. Most of the production functions developed so far have been unable to link explicitly the ex post and ex ante technologies inside their production models.

Recently, Melvyn Fuss developed an extensive application of generalized Leontief production function to solve this problem.[3] The notion of the "two-level" nature of

[3]Empirically, Fuss has built a four-factor (e.g., equipment, structures, fuel and labor) ex ante-ex post model and devoted much of his energy in reducing the large number of parameters to be estimated in order to make the model manageable. To do so he needed a priori information

technology is introduced inside the model by the process of parameterization and through the role of expectation so that the model can directly test the "putty-clay" hypothesis. This subject is the main body of discussion in this chapter.

First of all, the ex post and the ex ante technologies are separated conceptually and secondly, the two technologies are explicitly linked through behavioral simplification.

The properties of Fuss's ex post generalized Leontief production model have been discussed in Chapter II. Therefore, only those characteristics which are directly related to linking it to the ex ante structure of technology will be discussed here.

The primary assumption of linkage is that the ex post parameters, b_{ij}, are conditional on the ex ante choice of technique. Assuming that at time $v(<t)$ an ex ante design is chosen, and the resultant ex post parameters are labeled b_{ij}^{v} which represent a technique of vintage v, the actual cost function, at time t, for a production unit chosen at time v, is

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about the nature of industry under specification. First of all, the putty-putty model is rejected by assumption, since the capital input is assumed fixed ex post. See his dissertation, Chapter IV. Also see M. Fuss, "Factor Substitution in Electricity Generation: A Test of the Putty-Clay Hypothesis," Discussion Paper No. 185, Harvard Institute of Economic Research, Harvard University, April, 1971.

$$C_t^v(y;p) = \sum_{i=1}^{n} b_{ii}^v p_{it} h_i(y_t) + \sum_{i \neq j}\sum b_{ij}^v (p_{it} \cdot p_{jt})^{1/2} \cdot h(y_t) \tag{7.1}$$

Another assumption is that the ex post parameters, b_{ij}^v, result from the technique which minimizes the present value of expected future costs over planning horizon. The expected present value if

$$V = \sum_{t=v+1}^{v+L} \rho_{t-v} \cdot {}_E C_t^v \tag{7.2}$$

where ${}_E C_t^v$ = the ex post cost expected at time t,

ρ_{t-v} = the discount rate expected at time t,

and L = the length of the planning period.

It is also assumed that the b_{ij}^v are conditional on a set of ex ante parameters, $a_{ij,kl}$. through the parameterization.

$$b_{ij}^v = \sum_{k=1}^{n} \sum_{l=1}^{n} \left(\frac{s_{kl}}{s_{ij}}\right)^{1/2} a_{ij,kl} \qquad i=j=1, \ldots, n \tag{7.3}$$

The parameters $a_{ij,kl}$ represent the common ex ante technology faced by producers in an industry at time v. The b_{ij}^v are dependent ex ante on the decision variables s_{ij}, s_{kl}.

The expected present value function (7.2) is minimized with respect to the decision variables s_{ij}, s_{kl} and the symmetry restrictions $a_{ij,kl} = a_{kl,ij}$ are applied:

$$s_{ii} = \sum_t \rho_{t-v}\, p^v_{it} \cdot h_i({}_E y^v_t) = q^v_{ii}$$

and

$$s_{ij} = \sum_t \rho_{t-v} (p^v_{it}\, p^v_{jt})^{1/2} \cdot h({}_E y^v_t) = q^v_{ij} \qquad i \neq j \qquad (7.4)^4$$

where p^v_{it}, p^v_{jt} = the expected future price of input i, of input j,

${}_E y^v_t$ = the expected level of output,

and q^v_{ij} = the output-weighted price variables.

The symmetry restrictions e.g., $a_{ij,kl} = a_{kl,ij}$ are analogous to the ex post restriction $b_{ij} = b_{ji}$.

Then, the optimal (ex ante cost-minimizing) ex post parameters take the form

$$b^v_{ij} = \sum_{k,l} \left(\frac{q^v_{kl}}{q^v_{ij}}\right)^{1/2} a_{ij,kl} \qquad i,j,k,l = 1,\ldots,n \qquad (7.5)$$

Thus, the choice of the ex post parameters, b^v_{ij}, depends on the expected time path of future factor prices and output requirements that are contained in the output-weighted price variables, q^v_{ij}. The specification of the expectations is an important part of the model. By equation (7.5), the

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[4]The proof of equation (7.4) is made in Fuss, Harvard Discussion Paper No. 141, pp. 19-20.

"two-level" nature of technology, ex ante and ex post, is explicitly linked in the model.

A set of a priori restrictions is imposed on the ex ante parameters under the assumption of cost-minimizing behavior just like it is done on the ex post parameters (the symmetry restrictions $b^v_{ij} = b^v_{ji}$):

$$a_{ij,kl} = a_{ji,kl} = a_{ij,lk} = a_{lk,ij} = a_{lk,ji} = a_{kl,ji} = a_{kl,ij} \tag{7.6}$$

If the only restrictions on the ex ante parameters are those contained in (7.6), the technology is called "putty-semiputty"[5] and the putty-semiputty is set up as the maintained hypothesis.

The three limiting cases (putty-putty, putty-clay and clay-clay) are contained as testable hypotheses within the maintained hypothesis as shown in Table VII-2.

TABLE VII-2

Related Hypotheses in Putty-Semiputty Model

Structure of Technology	Related Hypothesis
putty-putty	$a_{ij,kl} = 0$ unless $ij = kl$
putty-clay	$a_{ij,kl} = 0$ unless $i = j$ and $k = l$
clay-clay	$a_{ij,kl} = 0$ unless $i=j=k=l$
putty-semiputty	Alternative

[5]The putty-semiputty model allows for different sets of ex ante and ex post substitution possibilities, but does not specify a priori that either set is the null set. See Fuss, Harvard Discussion Paper No. 141, p. 3.

3. The Model

Suppose production units (steam-electric generating plants) use two inputs, capital (X_1) and fuel (X_2), and produce one homogeneous output y (electricity generation) in each time period with certainty, and have a planning horizon of one time period (one year). Then, using the notation of Fuss's "Putty-semiputty" model discussed in section 2, nine conclusions are derived:

(i) The assumption of perfect foresight for the expected output one year ahead results in $_E y_t^v = y_t$. In this case, the actual output is assumed to be the expected output so that expectations need not be estimated. As usual, it is a simplifying, but restrictive assumption.

(ii) The output-weighted price variables are

$$q_{ij}^v = (p_{it}^v \; p_{jt}^v)^{1/2} \cdot h(y_t)$$

$$q_{ii}^v = (p_{it}^v) \cdot h_i(y_t)$$

(iii) i,j,k,l = 1,2 and L = 1; t-v = 1 for all observations.

(iv) As for the ex ante parameters $a_{ij,kl}$ there can be $2^4 = 16$ different parameters originally since i,j,k,l = 2. However, using symmetry restrictions of equation (7.6), a total of 16 ex ante parameters can be reduced into six in a two factor model as below:

$a_{11,11}$

$a_{11,12} = a_{21,11} = a_{12,11} = a_{11,21}$

$a_{11,22} = a_{22,11}$

$a_{22,12} = a_{22,21} = a_{21,22} = a_{12,22}$

$a_{22,22}$

$a_{12,12} = a_{12,21} = a_{21,12} = a_{21,21}$

(v) There are 14 different cases in the putty-semiputty model assuming that $a_{11,11}$ and $a_{22,22}$ are always not equal to 0. If both or either of these coefficients are equal to 0, more cases have to be added to the table on the following page. Remarks on Table VII-3 are only an intuitive observation. Empirically they should be related to the estimated results of the ex post and ex ante elasticity of substitution in the future research.

(vi) The "two-level" nature of technology can be constructed:

(A) The ex post factor demand functions are

$$X_1 = b_{11}^v \cdot h_1(y) + b_{12}^v\left(\frac{p_2}{p_1}\right)^{1/2} \cdot h(y) + \mathcal{E}_1$$

$$X_2 = b_{22}^v \cdot h_2(y) + b_{21}^v\left(\frac{p_1}{p_2}\right)^{1/2} \cdot h(y) + \mathcal{E}_2 \qquad (7.7a)$$

where index 1 is for capital,

and index 2 is for fuel.

TABLE VII-3

The Ex Ante Parameters in a Two-Factor Model

Ex Ante Parameters	Cases													
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
$a_{11,11}$	$\neq 0$	$\neq 0$	$\neq 0$	$\neq 0$	$\neq 0$	$\neq 0$	$\neq 0$	$\neq 0$	$\neq 0$	$\neq 0$	$\neq 0$	$\neq 0$	$\neq 0$	$\neq 0$
$a_{11,12}$	0	0	0	$\neq 0$	$\neq 0$	$\neq 0$	0	$\neq 0$	$\neq 0$	0	$\neq 0$	0	$\neq 0$	0
$a_{11,22}$	0	$\neq 0$	0	$\neq 0$	0	0	0	0	$\neq 0$	$\neq 0$	$\neq 0$	$\neq 0$	$\neq 0$	$\neq 0$
$a_{22,12}$	0	0	0	$\neq 0$	$\neq 0$	0	$\neq 0$	$\neq 0$	0	$\neq 0$	$\neq 0$	0	0	$\neq 0$
$a_{22,22}$	$\neq 0$	$\neq 0$	$\neq 0$	$\neq 0$	$\neq 0$	$\neq 0$	$\neq 0$	$\neq 0$	$\neq 0$	$\neq 0$	$\neq 0$	$\neq 0$	$\neq 0$	$\neq 0$
$a_{12,12}$	$\neq 0$	0	0	$\neq 0$	$\neq 0$	$\neq 0$	$\neq 0$	0	0	0	0	$\neq 0$	$\neq 0$	$\neq 0$
Hypothesis	P-P	P-C	C-C	Putty-Semiputty										

Remarks: Cases (4) through (14) are "putty-semiputty.

(5),(6),(7),(13),(14)--closer to putty-putty.

(8),(9)--closer to putty-clay.

(10),(11)--closer to either putty-clay or clay-clay depending upon the t-values of $a_{11,12}$, $a_{11,22}$ and $a_{22,12}$.

(12)--closer to either putty-clay or putty-putty depending upon the t-values of $a_{11,22}$ and $a_{12,12}$; if $t(a_{11,22}) > t(a_{12,12})$, closer to putty-clay; if $t(a_{11,22}) < t(a_{12,12})$, closer to putty-putty.

All variables are flow variables which have the time dimension t. However, t is omitted here for the simplicity of notation.

With the constraints across equations ($b^v_{12} = b^v_{21}$), the model (7.7a) can be stacked as below:

$$\begin{pmatrix} X_1 \\ \\ X_2 \end{pmatrix} = \begin{pmatrix} h_1(y) & 0 & \left(\frac{p_2}{p_1}\right)^{1/2} \cdot h(y) \\ \\ 0 & h_2(y) & \left(\frac{p_1}{p_2}\right)^{1/2} \cdot h(y) \end{pmatrix} \begin{pmatrix} b^v_{11} \\ b^v_{22} \\ b^v_{12} \end{pmatrix} + \begin{pmatrix} \varepsilon_1 \\ \\ \varepsilon_2 \end{pmatrix} \qquad (7.7b)$$

(B) The ex ante choice of technique functions are derived from the equation (7.5):

$$b^v_{11} = a_{11,11} + 2a_{11,12}\left(\frac{q_{12}}{q_{11}}\right)^{1/2} + a_{11,22}\left(\frac{q_{22}}{q_{11}}\right)^{1/2} + e_1$$

$$b^v_{22} = a_{22,22} + 2a_{22,12}\left(\frac{q_{12}}{q_{22}}\right)^{1/2} + a_{11,22}\left(\frac{q_{11}}{q_{22}}\right)^{1/2} + e_2$$

$$b^v_{12} = 2a_{12,12} + a_{22,12}\left(\frac{q_{22}}{q_{12}}\right)^{1/2} + a_{11,12}\left(\frac{q_{11}}{q_{12}}\right)^{1/2} + e_3 \qquad (7.8a)$$

In a matrix form, (7.8a) can be written as:

$$\begin{pmatrix} b^v_{11} \\ \\ b^v_{22} \\ \\ b^v_{12} \end{pmatrix} = \begin{bmatrix} 1 & 2\left(\frac{q_{12}}{q_{11}}\right)^{\frac{1}{2}} & \left(\frac{q_{22}}{q_{11}}\right)^{\frac{1}{2}} & 0 & 0 & 0 \\ 0 & 0 & \left(\frac{q_{11}}{q_{22}}\right)^{\frac{1}{2}} & 2\left(\frac{q_{12}}{q_{22}}\right)^{\frac{1}{2}} & 1 & 0 \\ 0 & \left(\frac{q_{11}}{q_{12}}\right)^{\frac{1}{2}} & 0 & \left(\frac{q_{22}}{q_{12}}\right)^{\frac{1}{2}} & 0 & 2 \end{bmatrix} \begin{pmatrix} a_{11,11} \\ a_{11,12} \\ a_{11,22} \\ a_{22,12} \\ a_{22,22} \\ a_{12,12} \end{pmatrix} + \begin{pmatrix} e_1 \\ \\ e_2 \\ \\ e_3 \end{pmatrix} \tag{7.8b}$$

(C) The actual ex post input demand functions are obtained by substituting (7.8b) into (7.7b):

$$X_1 = a_{11,11}h_1(y) + a_{11,12}\left\{2\left(\frac{q_{12}}{q_{11}}\right)^{\frac{1}{2}}\cdot h_1(y) + \left(\frac{q_{11}}{q_{12}}\right)^{\frac{1}{2}}\left(\frac{p_2}{p_1}\right)^{\frac{1}{2}}\cdot h(y)\right\}$$

$$+ a_{11,22}\left(\frac{q_{22}}{q_{11}}\right)^{\frac{1}{2}}\cdot h_1(y) + a_{22,12}\left(\frac{q_{22}}{q_{12}}\right)^{\frac{1}{2}}\left(\frac{p_2}{p_1}\right)^{\frac{1}{2}} h(y)$$

$$+ 2a_{12,12}\left(\frac{p_2}{p_1}\right)^{\frac{1}{2}} h(y) + E_1$$

$$X_2 = a_{22,22}h_2(y) + a_{11,12}\left(\frac{q_{11}}{q_{12}}\right)^{\frac{1}{2}}\left(\frac{p_1}{p_2}\right)^{\frac{1}{2}} h(y) + a_{11,22}\left(\frac{q_{11}}{q_{22}}\right)^{\frac{1}{2}} h_2(y) +$$

$$a_{22,12}\left\{2\left(\frac{q_{12}}{q_{22}}\right)^{\frac{1}{2}}\cdot h_2(y) + \left(\frac{q_{22}}{q_{12}}\right)^{\frac{1}{2}}\left(\frac{p_1}{p_2}\right)^{\frac{1}{2}} h(y)\right\} + 2a_{12,12}\left(\frac{p_1}{p_2}\right)^{\frac{1}{2}} h(y)$$

$$+ E_2 \tag{7.9a}$$

In a matrix form, the above actual ex post input demand functions are:

$$\begin{pmatrix} X_1 \\ \\ X_2 \end{pmatrix} =$$

$$\left[\begin{array}{cccccc} h_1(y) & \left\{2\left(\frac{q_{12}}{q_{11}}\right)^{1/2} h_1(y) + \left(\frac{q_{11}}{q_{12}}\right)^{1/2}\left(\frac{p_2}{p_1}\right)^{1/2}\cdot h(y)\right\} & \left(\frac{q_{22}}{q_{11}}\right)^{1/2}\cdot h_1(y) & \left(\frac{q_{22}}{q_{12}}\right)^{1/2}\left(\frac{p_2}{p_1}\right)^{1/2} h(y) & 0 & 2\left(\frac{p_2}{p_1}\right)^{1/2} h(y) \\ 0 & \left(\frac{q_{11}}{q_{12}}\right)^{1/2}\left(\frac{p_1}{p_2}\right)^{1/2} h(y) & \left(\frac{q_{11}}{q_{22}}\right)^{1/2}\cdot h_2(y) & \left\{2\left(\frac{q_{12}}{q_{22}}\right)^{1/2} h_2(y) + \left(\frac{q_{22}}{q_{12}}\right)^{1/2}\left(\frac{p_1}{p_2}\right)^{1/2} h(y)\right\} & h_2(y) & 2\left(\frac{p_1}{p_2}\right)^{1/2} h(y) \end{array}\right] \begin{pmatrix} a_{11,11} \\ a_{11,12} \\ a_{11,22} \\ a_{22,12} \\ a_{22,22} \\ a_{12,12} \end{pmatrix}$$

$$+ \begin{pmatrix} E_1 \\ E_2 \end{pmatrix} \qquad (7.9b)$$

where $q_{11} = p_1^v \cdot h_1({}_E y) = p_1^v \cdot h_1(y) = p_1^v \cdot y_c$

$q_{22} = p_2^v \cdot h_2({}_E y) = p_2^v \cdot h_2(y) = p_2^v \cdot y$

$q_{12} = (p_1^v \cdot p_2^v)^{1/2} \cdot h({}_E y) = (p_1^v \cdot p_2^v)^{1/2} \cdot h(y)$

(vii) An estimation of the ex ante parameters in (7.9b) requires the estimation of expected factor prices, p_1^v, p_2^v. The expected factor prices are estimated by the method developed by Phillip Cagan.[6] The formula used in estimating expected capital and fuel prices is

$$p_t^v = \beta \sum_i (1-\beta+\alpha)^i \, p_{t-i} \qquad (7.10)$$

where p_{t-i} = a measured factor price at period t-i,

p_t^v = the expected level of p_t,

β = an adjustment parameter (also called the coefficient of expectation),

α = a parameter for the trend effect.

In empirical estimation, it is assumed that the parameter for the trend effect is equal to zero,[7] that the adjustment parameter is prior known and the β values of .25 and .50 are used in actual estimation with lagged period of 6.[8]

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[6]Phillip Cagan, "The Monetary Dynamics of Hyperinflation," in Milton Friedman, ed., Studies in the Quantity Theory of Money (Chicago: The University of Chicago Press, 1956), Chapter II, 25-117.

Price differential among different geographical regions should be given a special consideration in estimating expected factor prices. For instance, geographical location may be used as a dummy variable. However, it is not pursued in this dissertation.

[7]It is assumed that factor prices in electricity generation have no historical time trends.

[8]Five different values of β are originally estimated: .20, .25, .50, .75, .99. β values of .25 and .50 are chosen due to the behavior of their expected values fluctuating closely around the actual values. The other values result in extreme deviations from the actual values.

Since the current data are time series as well as cross-section, they are converted into purely time-series nature in order to estimate the expected factor prices based on (7.10). Conversion was done by the averaging method. The average expected prices are reconverted into time-series and cross-section by the distributive method. Since there is a different number of observations for each period in the sample used, the average factor price is calculated for each period. Based on these data of estimated average factor price ($\overline{p}_i$), the average expected price ($\overline{p}_i^v$) is estimated by using equation (7.10). Tables VII-4 and VII-5 give average actual and average expected factor prices for capital and fuel when the coefficient of expectation is .25 and .50. The estimated average expected factor price is distributed for each individual plant by:

$$p_{ik}^v = \left(\frac{\overline{p}_{it}}{\overline{p}_{it}^v} \right) \cdot p_{ik} \qquad (7.11)$$

where i index input, k index plant,

and k is one of the plants at period t.

The expected prices for capital and fuel of 65 observations thus estimated are listed in the Appendix B.

(viii) The production of electricity in the last two decades may be characterized by the technological progress (embodied in changes in factors) and increased efficiency in the use of factors of production. In this model, the

TABLE VII-4

The Average Actual and Average Expected Capital Prices

Year	Actual $\bar{P}_1$	Expected $\bar{P}_1^{V**}$	Expected $\bar{P}_1^{V***}$	n
53	0.1046	0.0843	0.1016	
54	0.1022	0.0843	0.1011	
55	0.1009	0.0840	0.1002	
56	0.1039	0.0844	0.1013	
57	0.1010	0.0840	0.1003	
58	0.1041	0.0845	0.1014	8
59	0.1019	0.0842	0.1008	12
60	0.0998*	0.0836	0.0996	3*
61	0.1157	0.0871	0.1068	8
62	0.1063	0.0873	0.1058	6
63	0.1050	0.0872	0.1046	7
64	0.1009	0.0860	0.1019	5
65	0.1147	0.0886	0.1075	5
66	0.1188	0.0917	0.1124	5
67	0.1047	0.0897	0.1074	3
68	0.1243	0.0936	0.1150	2

n = number of plants in each period.

*Four plants were observed in the original data, but one plant 60003 (J. H. Ward) was dropped in estimating the average price due to its uncomparably low value (.0186).

** $\beta = .25$; $\alpha = 0$; $i = 6$

*** $\beta = .50$; $\alpha = 0$; $i = 6$

TABLE VII-5

The Average Actual and Average Expected Fuel Prices

Year	Actual	Expected		n
	$\overline{p}_2$	$\overline{p}_2^{v**}$	$\overline{p}_2^{v***}$	
53	0.3249	0.2253	0.2811	
54	0.2305	0.2158	0.2539	
55	0.2495	0.2107	0.2493	
56	0.2602	0.2115	0.2527	
57	0.3308	0.2306	0.2899	
58	0.2268	0.2191	0.2565	8
59	0.2663	0.2165	0.2589	12
60	0.3179	0.2316	0.2866	4
61	0.2839	0.2336	0.2833	8
62	0.3024	0.2392	0.2908	6
63	0.2630	0.2304	0.2743	7
64	0.3003	0.2378	0.2855	5
65	0.2800	0.2365	0.2807	5
66	0.2700	0.2307	0.2729	5
67	0.2713	0.2282	0.2699	3
68	0.1758	0.2017	0.2205	2

** $\beta = .25$; $\alpha = 0$; i = 6

*** $\beta = .50$; $\alpha = 0$; i = 6

ex ante parameters are dealt as functions of the technological progress (or regress) and efficiency (or inefficiency) in the use of factors. Time variable (v) is used as a measure of technological progress[9] and average capacity of turbine generator unit (A.C.) is used as a measure of efficiency. For doing this, a simplifying assumption is made that only the diagonal elements $a_{11,11}$ and $a_{22,22}$ of the ex ante parameters differ from plant to plant. Therefore, it is assumed that

$$a_{11,11} = \alpha_1 + \beta_1 v + \delta_1(\text{A.C.}),$$
$$a_{22,22} = \alpha_2 + \beta_2 v + \delta_2(\text{A.C.}) \qquad (7.12)$$

(ix) By substituting (7.12) into (7.9b) and introducing $h_1(y) = y_c$ and $h_2(y) = y$, the final form of stacked model with constraints across equations testing the "putty-clay" hypothesis in the steam-electric generating industry is shown on the following page.

4. Empirical Results

Based on the empirical results in Chapter IV that the ex post production function in the steam-electric generating industry can be better represented by the non-homothetic functional form, only non-homothetic functional variations are used in testing the "putty-clay" hypothesis.

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[9] $\frac{\partial X_i}{\partial v}$ is called the factor augmenting condition in Fuss, dissertation, p. 176.

$$
\begin{pmatrix} X_1 \\ \\ X_2 \end{pmatrix} =
\begin{bmatrix}
y_c & vy_c & [A.C]y_c & 0 & 0 & 0 & \left\{2\left(\frac{q_{12}}{q_{11}}\right)^{1/2} y_c + \left(\frac{q_{11}}{q_{12}}\right)^{1/2}\left(\frac{p_2}{p_1}\right)^{1/2} h(y)\right\} & \left(\frac{q_{22}}{q_{12}}\right)^{1/2} y_c & \left(\frac{q_{22}}{q_{12}}\right)^{1/2}\left(\frac{p_2}{p_1}\right)^{1/2} h(y) & 2\left(\frac{p_2}{p_1}\right)^{1/2} h(y) \\
0 & 0 & 0 & y & vy & [A.C]y & \left(\frac{q_{11}}{q_{12}}\right)^{1/2}\left(\frac{p_1}{p_2}\right)^{1/2} h(y) & \left(\frac{q_{11}}{q_{22}}\right)^{1/2} y & \left\{2\left(\frac{q_{12}}{q_{22}}\right)^{1/2} y + \left(\frac{q_{22}}{q_{12}}\right)^{1/2}\left(\frac{p_1}{p_2}\right)^{1/2} h(y)\right\} & 2\left(\frac{p_1}{p_2}\right)^{1/2} h(y)
\end{bmatrix}
\begin{pmatrix} \alpha_1 \\ \beta_1 \\ \delta_1 \\ \alpha_2 \\ \beta_2 \\ \delta_2 \\ a_{11,12} \\ a_{11,22} \\ a_{22,12} \\ a_{12,12} \end{pmatrix}
+ \begin{pmatrix} e_1 \\ \\ e_2 \end{pmatrix} \qquad (7.13)
$$

Four models are experimented with variations in general output scale function. The regression results are given in Table VII-5. The general output scale functions used in four models are: 1, $y^{1/2}$, y, $y^{3/2}$ for models I through IV, respectively. The values in the parentheses below the estimated coefficients are t-statistics.

In order to choose the appropriate specifications, the criteria which were used for the ex post production functions are used: (a) R-squared and (b) Consistency of sign of the estimated regression coefficients $a_{ij,kl}$. The empirical results show that all four models have very high R-squared values ranging from .9936 in model I to .9940 in model III. As for the sign of the price variables, all four models have one or two negative signs. However, only models I and II, both of which are increasing returns to scale cases, give the negative price variable coefficient insignificant at 1% significance level. Model II is selected for our final model according to its higher R-squared value than that of model I and its relatively higher individual t-statistics than those of model I.

The second-stage regression results of model II are represented by the following system of factor demand equations as our final putty-semiputty model (p. 101):

TABLE VII-6

Estimated Regression Results for Testing the "Putty-Clay" Hypothesis

Estimated Coefficients	Models I	II	III	IV
α_1	15.887 (8.48)	15.134 (9.46)	17.190 (11.86)	18.376 (12.20)
β_1	-.37344 (-1.92)	-.29892 (-1.53)	-.31488 (-1.58)	-.35479 (-1.80)
δ_1	.000047 (0.08)	-.00030 (-.58)	-.00050 (-.93)	-.00040 (-.75)
α_2	9.3877 (17.51)	8.4109 (20.29)	9.4710 (23.76)	9.8240 (22.11)
β_2	.04253 (1.73)	.03926 (1.57)	.05138 (2.13)	.03862 (1.58)
δ_2	-.000067 (-.79)	-.00015 (-1.54)	-.00024 (-2.87)	-.00008 (-1.09)
$a_{11,12}$	10.674 (0.84)	-.21764 (-0.44)	.000002 (.01)	.000001 (1.57)
$a_{11,22}$	-.09788 (-.18)	2.2140 (2.18)	2.8784 (3.83)	1.0984 (2.54)
$a_{22,12}$	-3.8853 (-.49)	-.02049 (-.52)	.00004 (.63)	.0000003 (2.04)
$a_{12,12}$	336.86 (1.39)	1.5770 (0.17)	-1.4559 (-2.68)	-.02537 (-2.42)
R^2	.9936	.9937	.9945	.9940
S.E.R.	.993157	.997392	1.0043	1.0721
N	130	130	130	130

Remark: S.E.R. = Standard Error of the Regression
N = Number of observations in the stacked model
The t-statistics are given in the parentheses.
*All computations were made by using "GLSQ" program in the Merrick Computing Center, University of Oklahoma.

$$K = (\underset{(1.599)}{15.134} - \underset{(.196)}{.29892}\ v - \underset{(.0005)}{.0003}\ [\text{A.C.}])\ y_c$$

$$- \underset{(.495)}{.21764}\left\{2\left(\frac{q_{12}}{q_{11}}\right)^{1/2} y_c + \left(\frac{q_{11}}{q_{12}}\right)^{1/2}\left(\frac{p_2}{p_1}\right)^{1/2} y^{1/2}\right\} + \underset{(1.015)}{2.214}\left(\frac{q_{22}}{q_{11}}\right)^{1/2} y_c$$

$$- \underset{(.0391)}{.02049}\left(\frac{q_{22}}{q_{12}}\right)^{1/2}\left(\frac{p_2}{p_1}\right)^{1/2} y^{1/2} + \underset{(19.028)}{3.154}\left(\frac{p_2}{p_1}\right)^{1/2} y^{1/2}$$

$$F = (\underset{(.414)}{8.4109} + \underset{(.0249)}{.03926}\ v - \underset{(.000095)}{.00015}\ [\text{A.C.}])\ y$$

$$- \underset{(.495)}{.21764}\left(\frac{q_{11}}{q_{12}}\right)^{1/2}\left(\frac{p_1}{p_2}\right)^{1/2} y^{1/2} + \underset{(1.015)}{2.214}\left(\frac{q_{11}}{q_{22}}\right)^{1/2} y$$

$$- \underset{(.0391)}{.02049}\left\{2\left(\frac{q_{12}}{q_{22}}\right)^{1/2} y + \left(\frac{q_{22}}{q_{12}}\right)^{1/2}\left(\frac{p_1}{p_2}\right)^{1/2} y^{1/2}\right\} + \underset{(19.028)}{3.154}\left(\frac{p_1}{p_2}\right)^{1/2} y^{1/2} \qquad (7.14)$$

N = 130

R^2 = .9937

S.E.R. = .997392

The standard errors of the coefficients are given in parentheses.

The above set of estimated factor demand equations contain the estimation of the $a_{ij,kl}$ which are of primary interest.

Empirical results of putty-semiputty model contained in (7.14) support the hypothesis that:

(i) The <u>actual</u> production function in the steam-electric generating industry is non-homothetic with increasing returns to scale.

(ii) Since the partial derivative $\left(\frac{\partial K}{\partial v}\right)$ is negative, it indicates that as time passes by, less and less capital is used. On the other hand, $\left(\frac{\partial F}{\partial v}\right)$ is positive which indicates that as time goes by, more and more fuel is needed. (The null hypothesis that $\beta_2 = 0$ can be rejected only at 10% level.) The rate of technical progress embodied in changes in capital services is .30 while the rate of technical progress embodied in changes in fuel services is -.04. (There was capital-saving technological progress while no technological progress occurred in the use of fuel.)

(iii) A one per cent increase in the average capacity for a turbine-generator unit results in a .0003% decrease in demand for capital services while a one per cent increase in the average capacity for a turbine-generator unit results in a .00015% decrease in demand for fuel services.

(iv) Since all the partial derivatives $\frac{\partial X_i}{\partial (A.C.)}$ are negative, there was an increased efficiency in the use of capital and fuel for a given average capacity through the continual introduction of larger scale units in producing electricity during the sample period.

(v) The production technology in the steam-electric generating industry is putty-clay. Only $a_{11,22}$ is significantly different from 0 at 5% level. (T value for 5% level is 1.96.) Based on Table VII-3, the null hypothesis of case (3) is accepted. That is, capital and fuel are substitutable ex ante, but are not substitutable ex post.[10] The ex post production function is Leontief.

Suppose that the coefficients which have an <u>insignificant</u> negative coefficients are dropped and that $a_{12,12}$ is not exactly equal to 0. Then, the model is closer to case (12) which is one of the putty-semiputty cases. Still, the model is similar to putty-clay rather than putty-putty since $t(a_{11,22}) > t(a_{12,12})$.

By accepting the null hypotheses that $a_{11,12} = 0$ and $a_{22,12} = 0$, concavity properties are satisfied since the rest of $a_{ij,kl} \geq 0$. As a result, in the putty-semiputty model described in (7.14), the factors capital and fuel are substitutes in production.

---

[10] The results of previous empirical studies on the substitutability between capital and fuel in the electricity generation industry are:

	ex ante	ex post
Barzel, Dhrymes-Kurz, Nerlove	putty	putty
Komiya	clay	clay
Galatin, McFadden, Fuss	putty	clay
The present model	putty	clay

5. Summary and Remarks

1) Based on the empirical evidence of the "putty-semiputty" model developed in this chapter, the actual production function of the steam-electric generating industry is shown to be non-homothetic, have increasing returns to scale and the structure of production technology is putty-clay. These results are in agreement with previous studies.

2) Fuss's "putty-semiputty" model can be applied to more than the two-factor case. In a two-factor case, estimates of six ex ante parameters are required. The disadvantage of using the putty-semiputty model in a multifactor case is the need to estimate a large number of ex ante parameters. Therefore, a priori knowledge or some simplifying assumptions on the specific industry under investigation are usually required in order to reduce the number of estimated parameters to a manageable size. The advantage of the current two-factor "putty-semiputty" model represented by equation (7.9b) and Table VII-3 over the multifactor case is that it does not require any a priori assumptions on the structure of technology. Therefore, it is a "generalized" model and through it the structure of technology can be empirically tested.

3) Expectations play an important role in the construction of a putty-semiputty model. However, there is

still room for improvement in the methods of estimating expected values and including them into an econometric model.

CHAPTER VIII

SUMMARIES AND CONCLUSIONS

The empirical investigation of this dissertation suggests that the Fuss-type generalized Leontief production function has an important econometric property which allows us to investigate empirically the restrictive economic assumptions of the conventional production functions (e.g., Cobb-Douglas, Leontief, C.E.S.) on the basis of the statistical inference of the empirical results.

Specifically, the use of the Fuss-type generalized Leontief production model makes it possible to investigate the assumption of homotheticity. Moreover, it helps to determine the appropriate functional form of a production function with a non-homothetic nature. Through a simple manipulation of the model, isoquants can be generated and identified. The model is also utilized to investigate the determinants for the variability of the elasticity of substitution and test J. R. Hicks's hypothesis that the input ratio and the level of output will be the dominant factors causing variations in the elasticity of substitution. Furthermore, Fuss's "putty-semiputty" model, an extensive application of the generalized Leontief production model, can directly test the structure of technology (or the "putty-clay" hypothesis).

The empirical results of the model support the hypothesis that the production function of the steam-electric generating industry is non-homothetic, has non-decreasing returns to scale and has a non-constant elasticity of substitution with neutral technical change. They also support the contention that the Cobb-Douglas and the C.E.S. production functions do not accurately represent the ex post technical structure of electricity generation. Finally, the empirical results support the contention that the type of production technology is putty-clay.

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APPENDIXES

APPENDIX A

STEAM-ELECTRIC GENERATING PLANTS, ONE YEAR AFTER INITIAL OPERATION

No.	Plant	Company	Observation year	y_c	y
1	Cameo	Public Service Co. of Colorado	58	192.7	108.6
2	Cherokee	Public Service Co. of Colorado	58	876	714.5
3	E.M. Brown	Kentucky Utilities Co.	58	876	600.1
4	Michoud	New Orleans Public Service, Inc.	58	1009	689.5
5	Gulf Coast	Mississippi Power Co.	58	657	453.1
6	G.G. Allen	Duke Power Co.	58	2891	2192.
7	Cunningham	Southwestern Public Service Co.	58	613.2	217.1
8	Yorktown	Virginia Electric & Power Co.	58	1494	1336
9	Huntington	Southern California Edison Co.	59	3812	2812
10	P.L. Bartow	Florida Power Corp.	59	1060	619.6
11	Ft. Myer	Florida Power & Light Co.	59	1095	755.5
12	Clay Boswell	Minnesota Power Co.	59	560.6	412
13	Montrose	Kansas City Power & Light Co.	59	1369	1089
14	Lewis & Clark	Montana-Dakota Utilities Co.	59	438	156.3
15	Portland (Sandy Shore)	Metropolitan Edison Co.	59	1314	960.3
16	Silas McMeekin	South Carolina Electric & Gas Co.	59	2190	1691
17	Bates	Central Power & Light Co.	59	578.2	330.7
18	North Houston	Houston Lighting & Power Co.	59	578.2	41.7
19	W.A. Parrish	Houston Lighting & Power Co.	59	2737	2149
20	Clinch River	Appalachian Power Co.	59	3907	3438

No.	Plant	Company	Observation Year	y_c	y
21	Mandalay Beach	Southern California Edison Co.	60	3812	3061
22	Dan E. Karn	Consumers Power Co.	60	2321	1592
23	J. H. Warden	Upper Peninsula Power Co.	60	136.7	81
24	Nelson Dewey	Wisconsin Power & Light Co.	60	876	588.1
25	Ocotillo	Arizona Public Service Co.	61	1991	1559
26	South Bay	San Diego Gas & Electric Co.	61	1196	931.9
27	Norwalk Harbor	The Connecticut Light & Power Co.	61	1314	1083
28	E. D. Edwards	Central Illinois Light Co.	61	1095	811
29	Breed	Indiana & Michigan Electric Co.	61	3942	2953
30	Mt. Tom	Holyoke Water Power Co.	61	1095	1036
31	Newman	El Paso Electric Co.	61	714.8	456.2
32	Nichols	Southwestern Public Service Co.	61	876.	391.7
33	Helena	Arkansas Power & Light Co.	62	2847	1049
34	Cool-Water	The California Electric Power Co.	62	572	457
35	Little Gypsy	Louisiana Power & Light Co.	62	2164	1503
36	Crane	Baltimore Gas & Electric Co.	62	1668	1255
37	Northeastern	Public Service Co. of Oklahoma	62	1489	1033
38	Brunner Island	Pennsylvania Power & Light Co.	62	3183	1728
39	Cholla	Arizona Public Service Co.	63	1007	860.8
40	Bailly	Northern Indiana Public Service Co.	63	1699	1271
41	Sabine	Gulf States Utilities Co.	63	4193	1835
42	J. H. Campbell	Consumers Power Co.	63	2321	1596
43	B. L. England	Atlantic City Electric Co.	63	1191	805.3
44	Canadys	South Carolina Electric & Gas Co.	63	1191	941.4

No.	Plant	Company	Observation Year	y_c	y
45	Oak Creek	West Texas Utilities Co.	63	714.8	485
46	Cimarron River	Western Light & Telephone Co., Inc.	64	515.1	317.2
47	Big Sandy	Kentucky Power Co.	64	2321	2137
48	Tracey	Sierra-Pacific Power Co.	64	464.3	101.7
49	Ravenswood	Consolidated Edison Co. of N.Y., Inc.	64	7008	4589
50	Naughton	Utah Power & Light Co.	64	1430	807.7
51	Neal	Iowa Public Service Co.	65	1289	776.7
52	Wilkes	Southwestern Electric Power Co.	65	1572	1266
53	Sunrise	Nevada Power Co.	65	714.8	422.3
54	Hudson	Public Service Electric & Gas Co.	65	3984	2006
55	Ashville	Carolina Power & Light Co.	65	1810	1173
56	Cape Kennedy	Florida Power & Light Co.	66	3522	2055
57	Lansing Smith	Gulf Power Co.	66	1310	1007
58	Harllee Branch	Georgia Power Co.	66	2621	1558
59	Coffeen	Central Illinois Public Service Co.	66	2891	2258
60	Reid Gardner	Nevada Power Co.	66	995.1	778.5
61	Crystal River	Florida Power Corp.	67	3859	2089
62	Baxter Wilson	Mississippi Power Light Co.	67	4771	2473
63	Roxboro	Carolina Power & Light Co.	67	3599	2740
64	Petersburg	Indianapolis Power & Light Co.	68	2292	1686
65	Maddox	New Mexico Electric Service Co.	68	995.1	471.8

No.	Plant	ℓ	A.C.	K	F
1	Cameo	.5635	192.7	5,791	1,536
2	Cherokee	.8156	876	18,080	7,672
3	E.M. Brown	.6850	876	14,220	6,107
4	Michoud	.6832	1009	12,310	7,440
5	Gulf Coast	.6896	657	11,790	4,369
6	G.G. Allen	.7583	1445	36,900	20,160
7	Cunningham	.3540	613.2	9,253	2,543
8	Yorktown	.8944	1494	25,210	12,830
9	Huntington Beach	.7375	1906	53,920	26,540
10	P.L. Bartow	.5846	1060	22,660	6,348
11	Ft. Myer	.6900	1095	16,010	7,513
12	Clay Boswell	.7349	560.6	12,710	4,184
13	Montrose	.7951	1369	24,500	11,160
14	Lewis & Clark	.3568	438	10,990	2,202
15	Portland (Sandy Shore)	.7303	1314	30,270	8,989
16	Silas McMeekin	.7721	1095	32,440	15,430
17	Bates	.5720	578.2	8,308	3,925
18	North Houston	.0721	578.2	9,795	573.2
19	W.A. Parrish	.7851	1369	26,950	22,090
20	Clinch River	.8799	1953	59,680	31,060
21	Mandalay Beach	.8028	1906	50,230	28,230
22	Dan E. Karn	.6859	2321	47,900	14,650
23	J.H. Warden	.5927	136.7	5,280	1,338
24	Nelson Dewey	.6713	876	16,730	5,737
25	Ocotillo	.7831	995.6	25,220	15,530
26	South Bay	.7794	1196	21,690	9,103
27	Norwalk Harbor	.8240	1314	30,180	10,320
28	E.D. Edwards	.7406	1095	28,600	7,930
29	Breed	.7491	3942	72,790	26,050
30	Mt. Tom	.9462	1095	23,590	10,040
31	Newman	.6382	714.8	9,974	4,349
32	Nichols	.4471	876	14,120	4,055
33	Helena	.3684	2847	43,220	10,380
34	Cool-Water	.7989	572	12,230	3,392
35	Little Gypsy	.6947	2164	27,630	14,750
36	Crane	.7523	1668	38,000	11,690
37	Northeastern	.6934	1489	22,390	8,235
38	Brunner Island	.5430	3183	45,460	16,280
39	Cholla	.8545	1007	20,460	8,309
40	Bailly	.7477	1699	34,520	12,180
41	Sabine	.4375	2097	76,470	18,500
42	J.H. Campbell	.6874	2321	48,470	14,300
43	B.L. England	.6760	1191	23,710	7,905
44	Canadys	.7902	1191	19,450	8,531
45	Oak Creek	.6785	714.8	9,132	4,854
46	Cimarron River	.6158	515.1	7,232	4,134
47	Big Sandy	.9204	2321	38,090	19,210

No.	Plant	ℓ	A.C.	K	F
48	Tracey	.2190	464.3	7,700	1,346
49	Ravenswood	.6548	3504	113,700	44,230
50	Naughton	.5650	1430	26,570	8,380
51	Neal	.6027	1289	20,120	7,641
52	Wilkes	.8049	1572	14,380	12,590
53	Sunrise	.5908	714.8	11,470	4,209
54	Hudson	.5035	3984	72,360	18,170
55	Ashville	.6480	1810	23,900	10,970
56	Cape Kennedy	.5835	3522	29,550	19,470
57	Lansing Smith	.7683	1310	21,210	9,892
58	Harllee Branch	.5944	2621	33,100	15,130
59	Coffeen	.7812	2891	43,720	22,490
60	Reid Gardner	.7823	995.1	17,070	7,745
61	Crystal River	.5412	3859	43,940	20,420
62	Baxter Wilson	.5184	4771	44,990	24,420
63	Roxboro	.7612	3599	40,000	25,280
64	Petersburg	.7353	2292	34,310	15,790
65	Maddox	.4741	995.1	10,930	4,901

APPENDIX B

ACTUAL AND EXPECTED FACTOR PRICES*

No.	Plant	p_1	$\check{p}_1$	p_2	$\check{p}_2$
1	Cameo	.0947	.0923	.2292	.2592
2	Cherokee	.0928	.0905	.2096	.2370
3	E.M. Brown	.1088	.1060	.2327	.2632
4	Michoud	.1236	.1204	.1215	.1374
5	Gulf Coast	.1107	.1078	.2502	.2830
6	G.G. Allen	.1084	.1056	.3066	.3467
7	Cunningham	.0943	.0919	.1923	.2175
8	Yorktown	.0999	.0973	.2720	.3076
9	Huntington Beach	.0855	.0846	.3378	.3284
10	P.L. Bartow	.0969	.0959	.3316	.3223
11	Ft. Myer	.1206	.1193	.3530	.3431
12	Clay Boswell	.0910	.0901	.4163	.4047
13	Montrose	.0981	.0971	.2077	.2019
14	Lewis & Clark	.0932	.0923	.2121	.2062
15	Portland (Sandy Shore)	.1137	.1125	.3582	.3482
16	Silas McMeekin	.0964	.0954	.3003	.2919
17	Bates	.1064	.1053	.1730	.1682
18	North Houston	.1059	.1048	.1640	.1594
19	W.A. Parrish	.1061	.1050	.1640	.1594
20	Clinch River	.1091	.1080	.1772	.1722
21	Mandalay Beach	.0958	.0955	.3309	.2983
22	Dan E. Karn	.0920	.0917	.3329	.3001
23	J.H. Warden	.0187	.0186	.3310	.2984
24	Nelson Dewey	.1118	.1115	.2768	.2495
25	Ocotillo	.0879	.0812	.3341	.3334
26	South Bay	.1055	.0974	.3357	.3350
27	Norwalk Harbor	.0989	.0913	.3393	.3386
28	E.D. Edwards	.2064	.1906	.2314	.2309
29	Breed	.1021	.0943	.1868	.1864
30	Mt. Tom	.1023	.0945	.3360	.3353
31	Newman	.1164	.1075	.3265	.3258
32	Nichols	.1061	.0980	.1810	.1806
33	Helena	.0994	.0989	.2657	.2555
34	Cool-Water	.0944	.0939	.4584	.4408
35	Little Gypsy	.1057	.1052	.2300	.2212

*The expected factor prices listed are estimated from Phillip Cagan's formula (7.10) with $\alpha = 0$, $\beta = 0.5$ and the lagged period of 6.

No.	Plant	p_1	p_1^v	p_2	p_2^v
36	Crane	.1105	.1099	.3299	.3173
37	Northeastern	.1089	.1083	.2561	.2463
38	Brunner Island	.1190	.1184	.2745	.2640
39	Cholla	.0938	.0934	.2371	.2473
40	Bailly	.1151	.1146	.2714	.2831
41	Sabine	.0969	.0965	.2050	.2138
42	J.H. Campbell	.1089	.1085	.3200	.3338
43	B.L. England	.1046	.1042	.3132	.3267
44	Canadys	.0886	.0883	.2955	.3082
45	Oak Creek	.1270	.1265	.1986	.2071
46	Cimarron River	.1178	.1190	.3529	.3355
47	Big Sandy	.1196	.1208	.1591	.1513
48	Tracey	.0959	.0969	.4496	.4275
49	Ravenswood	.0855	.0863	.3480	.3309
50	Naughton	.0856	.0865	.1920	.1826
51	Neal	.1218	.1142	.2793	.2800
52	Wilkes	.1222	.1145	.1900	.1905
53	Sunrise	.1106	.1037	.3645	.3654
54	Hudson	.1154	.1082	.3097	.3105
55	Ashville	.1034	.0969	.2558	.2564
56	Cape Kennedy	.1309	.1238	.3230	.3264
57	Lansing Smith	.1046	.0990	.2507	.2534
58	Harllee Branch	.1119	.1058	.3061	.3093
59	Coffeen	.1243	.1176	.1720	.1738
60	Reid Gardner	.1225	.1159	.2980	.3012
61	Crystal River	.1101	.1135	.2932	.2916
62	Baxter Wilson	.1069	.1102	.2463	.2423
63	Roxboro	.0957	.0986	.2772	.2757
64	Petersburg	.1199	.1109	.1740	.2182
65	Maddox	.1287	.1191	.1775	.2226

APPENDIX C

1) THE FIRST-STAGE ESTIMATED REGRESSION RESULTS FOR CAPITAL DEMAND EQUATION UNDER THE ASSUMPTION OF HOMOTHETICITY

Model	b_{11}	b_{12}	R-squared	S.E.R.	D.W.
I	32,259.7 (10,036.0)	-2,209.3 (5,983.1)	.0022	20,073.7	1.92
II	494.7 (240.8)	273.0 (148.6)	.6687	11,565.8	1.84
III	15.47 (4.67)	3.83 (2.84)	.7564	9,918.0	1.56
IV	.4865 (.0029)	-.0407 (.0731)	.5371	13,672.1	1.07
V	.0121 (.0029)	-.0029 (.0017)	.2316	17,615.8	0.83
VI	12.39 (2.76)	1.54 (1.67)	.8415	8,000.5	1.31

Number of Observations = 65
S.E.R. = Standard Error of the Regression
D.W. = Durbin-Watson statistic.
The standard errors of the coefficients are given in parentheses.

2) THE FIRST-STAGE ESTIMATED REGRESSION RESULTS FOR FUEL DEMAND EQUATION UNDER THE ASSUMPTION OF HOMOTHETICITY

Model	b_{22}	b_{21}	R-squared	S.E.R.	D.W.
I	10,745.8 (5,546.2)	1,847.0 (8,459.7)	.0008	8,550.0	1.89
II	454.9 (79.0)	-86.2 (120.5)	.7902	3,917.8	1.61
III	9.48 (.31)	-.0062 (.4804)	.9933	699.6	1.99
IV	.1218 (.0294)	.1012 (.0464)	.8335	3,490.2	1.08
V	.0007 (.0009)	.0041 (.0014)	.5329	5,845.9	0.91
VI	5.22 (1.08)	1.74 (1.40)	.8985	2,724.6	1.77

Number of Observations = 65

The standard errors of the coefficients are given in parentheses.