

STRUCTURAL AND SPATIAL ANALYSES OF
THE FEEDER CATTLE SECTOR

By

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CHAPTER I

INTRODUCTION

In order to make intelligent decisions at the various structural levels, a basis for determining the consequences of alternative courses of action must be established. In particular the decision making process may be viewed as a succession of simple steps designed to identify the state of the world and to find the appropriate response. Economic models, which utilize the body of logic known as economic theory, provide the decision maker with a means of classifying the important variables relevant to a particular real world situation and the qualitative estimation of the future course of certain variables under assumed conditions. Given economic theory and the resulting models as a foundation stone in this process of gaining knowledge for choice purposes, statistical data and the modern methods of statistical inference combine to yield quantitative parameter estimates from which the consequences of alternative actions may be enumerated and the appropriate course of action selected.

A. Objectives of This Study

Within the general measurement objectives, the direction in which simplification proceeds depends primarily upon the particular goals to be pursued. In this study, the dual purposes are to (1) obtain estimates of the parameters associated with the supply and demand for feeder cattle and (2) determine optimum interregional flows for feeder cattle and regional price differentials that are consistent with the optimum shipment program. As a means of realizing the first objective, a structural model of the feeder

cattle sector will be established. The second goal will be reached through the development of a spatial price equilibrium model for feeder cattle. Thus, the feeder cattle economy will be viewed from two vantage points.

1. The structural objective

a. General considerations

Ignoring spatial aspects for the moment, the feeder cattle economy can be viewed as an interlocking system of relationships. Simplification begins by partitioning the structure in order to identify these relationships. Once the system is partitioned in this fashion, variables within each block must be defined and classified. Economic theory is the foundation for the partitioning and classification phases, along with knowledge of peculiarities of the particular sector not incorporated in the general theory.

Whereas partitioning and classification were formerly the end of economic research, they now constitute a means for measuring. In order to attain measurement, however, several further steps are required. These include (1) considering only those variables that are measurable, (2) obtaining data to reflect all variables not omitted by the first step, (3) postulating an algebraic form for the relationships and the distributional properties of the variables and (4) employing estimation techniques that do not violate the assumptions upon which the prior stages rest.

In the last decade, many advances have been made in providing satisfactory estimation techniques. These methods, which recognize the interdependency of economic structures, are largely due to Haavelmo,¹ Anderson

¹Haavelmo, T., "The Statistical Implications of a System of Simultaneous Equations," Econometrica, Vol. XI, 1943.

and Rubin,² Koopmans,³ and other members of the Cowles Commission for Research in Economics. The latest contribution in this area is due to Theil⁴ and Basmann.⁵ However, the methodology for measurement in economics is much less than satisfactory. Among the more pressing problems are: (1) a lack of suitable data and (2) an absence of decision criteria for postulating the functional form of economic relationships.

b. The particular problem

In this study a simultaneous equation model will be constructed to represent the feeder cattle sector of the economy. Secondary data will be chosen to represent each variable included in the model and appropriate techniques will be used to estimate the parameters associated with the supply and demand relationships for feeder cattle. The Limited Information Single Equation⁶ and Maximum Likelihood⁷ methods will be used where appropriate and the least-squares and Theil-Basmann⁸ methods will be employed for comparative purposes.

²Anderson, T. W. and Rubin, H., "Two Papers on the Estimation of the Parameters of a Single Equation in a Complete System of Stochastic Equations," Cowles Commission New Series, No. 36, Chicago, 1951.

³Koopmans, T., Rubin, H. and Leipnik, R. B., "Measuring the Equations of Dynamic Economics," Statistical Inference in Dynamic Economic Models, Cowles Commission Monograph No. 10, John Wiley and Sons, New York, 1956, Chapter II.

⁴Theil, H., "Estimation and Simultaneous Correlation in Complete Equations Systems," Central Plan Bureau, The Hague, June 1953, (mimeographed).

⁵Basmann, R. L., "A Generalized Classical Method of Linear Estimation of Coefficients in a Structural Equation," Econometrica, Vol. XXV, January 1957.

⁶Koopmans, T. C. and Hood, W. C., "The Estimation of Simultaneous Linear Economic Relationships," Studies in Econometric Methods, Cowles Commission Monograph No. 14, John Wiley and Sons, New York, 1953, Chapter VI.

⁷Ibid.

⁸For a computational approach for this method, see Judge, G. G. and Wallace, T. D., "Discussion of the Theil-Basmann Method of Estimating Equations in a Simultaneous System," Submitted for publication, Oklahoma State University, Department of Agricultural Economics, Stillwater, 1958.

The resultant estimates should be useful to the firm by providing answers to such questions as: (1) What will be the effect of an increase in the availability of feeder cattle on the price of the factor (feeder cattle)? (2) What will be the effect of an increase in the price of slaughter beef on the demands for feeder cattle? (3) What effect will a shortage of feed grains have on the demand for feeder cattle?

The results should also prove valuable to the policy planners, since the following types of questions can be handled. (1) How will the cattle feeding firms respond to a governmental pricing policy for slaughter cattle? (2) What will be the effect on feeder cattle supplies and demands if the government limits the production of feed grains?

Simultaneous equation models are especially useful to policy planners since the effect of proposed policy can be traced throughout as much of the economy as is reflected by the model. The single equation model is quite limited in this respect.

Previous studies of the feed-livestock sector have been made by Fox,⁹ Nordin, Judge, and Wahby,¹⁰ and Hildreth and Jarrett.¹¹ Fox was primarily interested in estimates of the elasticities of demand for a large number of farm products, therefore, he employed relatively simple models in each instance. Nordin, Judge and Wahby developed models for the beef, pork, and poultry sector and then employed various econometric procedures to estimate the relevant parameters. Hildreth and Jarrett estimated struc-

⁹Fox, K.A., "The Analysis of Demand for Farm Products," Technical Bulletin No. 1081, USDA, 1953.

¹⁰Nordin, J.A., Judge, G. G., and Wahby, O., "Application of Econometric Procedures to the Demand for Agricultural Products," Iowa Agricultural Experiment Station, Research Bulletin No. 410, Iowa State College, Ames, 1954.

¹¹Hildreth, C. and Jarrett, F. F., A Statistical Study of Livestock Production and Marketing, Cowles Commission Monograph No. 15, John Wiley and Sons, New York, 1955.

tural relations for all livestock products. Thus, they worked with large aggregated models and made no analyses relating to individual products. Judge¹² also made a study of the cattle feeding sector as it affected the Iowa cattle feeders.

This study differs from the previous ones in that (1) a considerable amount of post-war data will be used, (2) aggregation was held to a minimum, (3) several variables were considered that were ignored in the other studies, and (4) all the states that play a major role in cattle feeding were included.

1. The spatial objectives

a. General considerations

Formerly, space was ignored as a variable in the equilibrium theories of economics, in that all demands and supplies of a product were assumed to be concentrated at a point. However, recent contributions by Samuelson,¹³ Koopmans,¹⁴ and others allow space to be treated explicitly in the equilibrium framework. The problem of geographically separated markets, spatial pricing and optimum shipments is a generalization of the transportation problem of linear programming.¹⁵

Given the intensities of demands and supplies of a product at different localities and the cost of transporting the good among all points, the transportation problem can be stated as that of determining the set of

¹²Judge, G. G., "Determinants of the Extent and Type of Cattle Feeding in Iowa," Unpublished Masters Thesis, Iowa State College, Ames, 1949.

¹³Samuelson, P. A., "Spatial Price Equilibrium and Linear Programming," The American Economic Review, Vol. XLII, June, 1952, pp. 283-303.

¹⁴Koopmans, T. C. (ed.), Activity Analysis of Production and Allocation, Cowles Commission Monograph No. 13, John Wiley and Sons, New York, 1951, chapter III.

¹⁵Dantzig, G. B., "Application of Simplex Method to a Transportation Problem," Activity Analysis of Production and Allocation, John Wiley and Sons, New York, 1951, pp. 359-374.

product flows that will minimize total transport cost. As a corollary to the transportation problem, the spatial pricing problem is that of evaluating regional price differences that are consistent with the optimum set of flows.

b. The particular objective

To make the Samuelson-Hitchcock model operational for the feeder cattle economy, a regional demarcation of the United States will be postulated, data to reflect regional supplies and demands will be gathered and transportation cost for shipping feeder cattle among regions will be obtained. Then using the simplex technique, optimum interregional flows of feeder cattle will be established for various time periods. With the aid of the linear programming dual,¹⁶ regional price differences for feeder cattle will also be determined.

Once the normative models are established, it will be possible to assess the effect of changes in the basic factor upon the equilibrium scheme, such as: (1) a redistribution of available supplies, (2) a change in the transport cost between any i^{th} and j^{th} region that are in the optimum flow solution, and (3) a shift in the level and area of demand.

Due to the relative newness of spatial analysis, there have been very few empirical applications of this technique. Fox and Tauber¹⁷ developed some aggregative models of the feed livestock economy, and Groom¹⁸ applied the transportation problem to the feeder livestock and corn sec-

¹⁶Dorfman, R., Samuelson, P. A. and Solow, R. M., Linear Programming and Economic Analysis, McGraw Hill Book Co., New York, 1958, chapter V.

¹⁷Fox, K. A. and Tauber, R. C., "Spatial Equilibrium Models of the Livestock Feed Economy," The American Economic Review, Vol. XLV, pp. 584-608.

¹⁸Grooms, C. G. "An Application of the Transportation Model in Deriving Least Cost Interregional Flows of Feeder Livestock and Corn, Unpublished Masters Thesis, Iowa State College, Ames, 1958.

tors of the economy. Also, Judge and Wallace¹⁹ developed spatial models of the beef and pork sectors of the economy.

This investigation differs from Grooms' study in that the regional demarcation is more specific, the supplies and demands are defined differently, two time periods are considered, and the transport rates were generated from non-linear relationships.

B. The Setting

The construction of an economic model requires a priori knowledge of the sector under study and a knowledge of the economic theory relevant to that sector of the economy. A description of the feeder cattle industry will be presented in this section.

The geographical distribution of resources necessary for the production and finishing of feeder cattle is such that the sector is divided into two distinct areas. The abundant grass and range area makes the western states especially well suited for the production of cattle for feeding. Alternatively, surplus feeds, especially corn and hay, and the geographical distribution of the population are influential factors in locating the cattle feeding industry in the corn belt. Since it takes approximately four pounds of corn to produce one pound of meat, it was to the advantage of the cattle feeding firms to have the cattle shipped to the feed grain production area, rather than shipping the feeds to the area of production and then shipping the finished product back to the east for consumption.

Available data indicate that eight corn belt states (Ohio, Illinois, Indiana, Michigan, Wisconsin, Iowa, Nebraska, and Minnesota) accounted

¹⁹Judge, G. G. and Wallace, T. D., "A Spatial Price Equilibrium Analysis of the Beef Marketing System," Oklahoma Agricultural Experiment Station, Submitted for publication, Oklahoma State University, Stillwater, 1958.

for approximately 58 percent of the cattle on feed January 1, 1956, in the United States.²⁰ It was also determined that five other states (North Dakota, South Dakota, Missouri, Kansas, and Colorado) accounted for 18 percent of the number of cattle on feed January 1. Therefore these 13 states accounted for 76 percent of the cattle on feed January 1, 1956. Eleven western states (Texas, Oklahoma, Colorado, Kansas, Missouri, Nebraska, North Dakota, South Dakota, Wyoming, Montana, and New Mexico) supplied 89 percent of the cattle shipped into the corn belt states.²¹ These eleven states also produced 46 percent of the total production of beef calves in 1956 (stocks from which feeder cattle are obtained).²²

In recent years there has been an increasing trend toward a redistribution of the population from the east and midwest to the western part of the United States. Because of this shift the west coast is now feeding a great number of cattle. For example, California imported 1,175,000 head of feeder cattle in 1956 and ranked as the second largest importer of feeder cattle in the United States.²³ This increasing demand for feeder cattle in the west means that the corn belt cattle feeding firms must compete with the western states for the supply of feeder cattle.

Only a small number of the cattle feeding firms in the corn belt area produce the cattle they feed. Thus, markets and marketing agencies are used in transferring the majority of the feeder cattle from the producer

²⁰Livestock Market News Statistics and Related Data, AMS, Statistical Bulletin No. 209, USDA, Washington, D. C., 1956, p. 8.

²¹North Central Livestock Marketing Research Committee, "Marketing Feeder Cattle and Sheep," Nebraska Experiment Station, Bulletin No. 410, Lincoln, 1952, p. 13.

²²Livestock and Poultry Inventory, January 1, AMS, USDA, February, 1957, Washington, D. C.

²³California Department of Agricultural, "California Annual Livestock Report," Summary for 1956, Sacramento, 1956.

to the demanders. The predominant marketing channels for both selling and purchasing feeder cattle are: (1) terminal public market, (2) auctions, (3) local dealers, and (4) direct shipments. The most important of these marketing channels are the terminal market and direct shipments. The relative numbers of animals moving through each of these channels has shifted considerable over time. In 1951, 60 percent of the imports of feeder cattle into the corn belt states were shipped from terminal markets.²⁴ However, in 1956 only 48 percent of the imports were shipped through the terminal market.

Accurate data are not available on total numbers of cattle fed annually and for comparative purposes, the best available alternative is number of cattle on feed January 1. As indicated by this series, marked variations in the number of cattle on feed January 1, (figure 1) have occurred over time periods for which data are available. For example, there were 3,633,000 head of cattle on feed January 1, 1940, and ten years later there were 4,463,000 head on feed. Although this represents a large increase, there were 6,099,000 head on feed January 1, 1957.²⁵ Over time there has been a consistent trend upward in the number of cattle on feed, although there have been wide fluctuations in this number. For example, on January 1, 1952, there were 4,961,000 head of cattle on feed and one year later there were 5,754,000 head of cattle on feed, an increase of 793,000 head.²⁶ Similar examples can be obtained that show decreases in the number on feed.

Along with annual variations in numbers on feed seasonal fluctua-

²⁴Livestock Market News Statistics and Related Data, *ibid.*, p. 18.

²⁵*Ibid.*

²⁶*Ibid.*

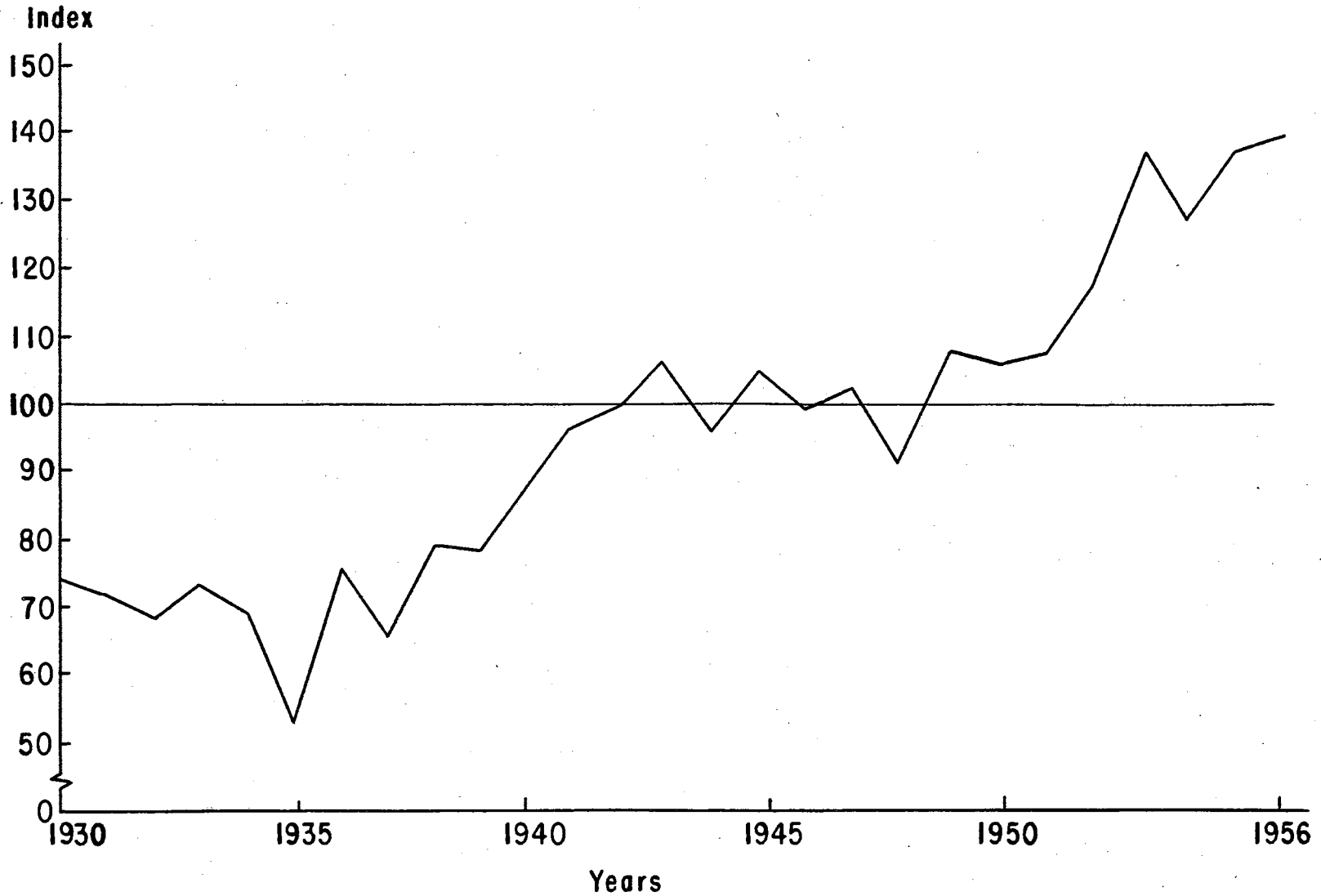


Figure 1. Cattle and Calves on Feed January 1, in the United States, Index 1947-49 = 100

Source: "Livestock Market News Statistics," AMS, USDA, CS-24-1946, page 7, CS-38-1948, page 6, Statistical Bulletins No. 118-1951, page 6 and No. 209-1956, page 8.

tions also occur. These fluctuations differ in that the seasonal fluctuations follow a definite pattern. Sales and purchases tend to concentrate more in August, September, October, and November with a seasonal peak in October (figure 2). This seasonal pattern can best be explained in that fall is the most logical time for the producing firms to sell the feeder cattle because of the availability of grass. In the feeding area cattle feeders prefer to buy in the fall. At that time they have some knowledge of the availability of corn and hay and the favorability of the alternative of feeding hogs. Since hogs and feeder cattle utilize the same factor (corn) in the finishing process the price of hogs should have a definite effect on the number of cattle fed annually.

The descriptive aspects of the feeder cattle sector as discussed in this section will be used as a basis for the construction of the sector and spatial models.

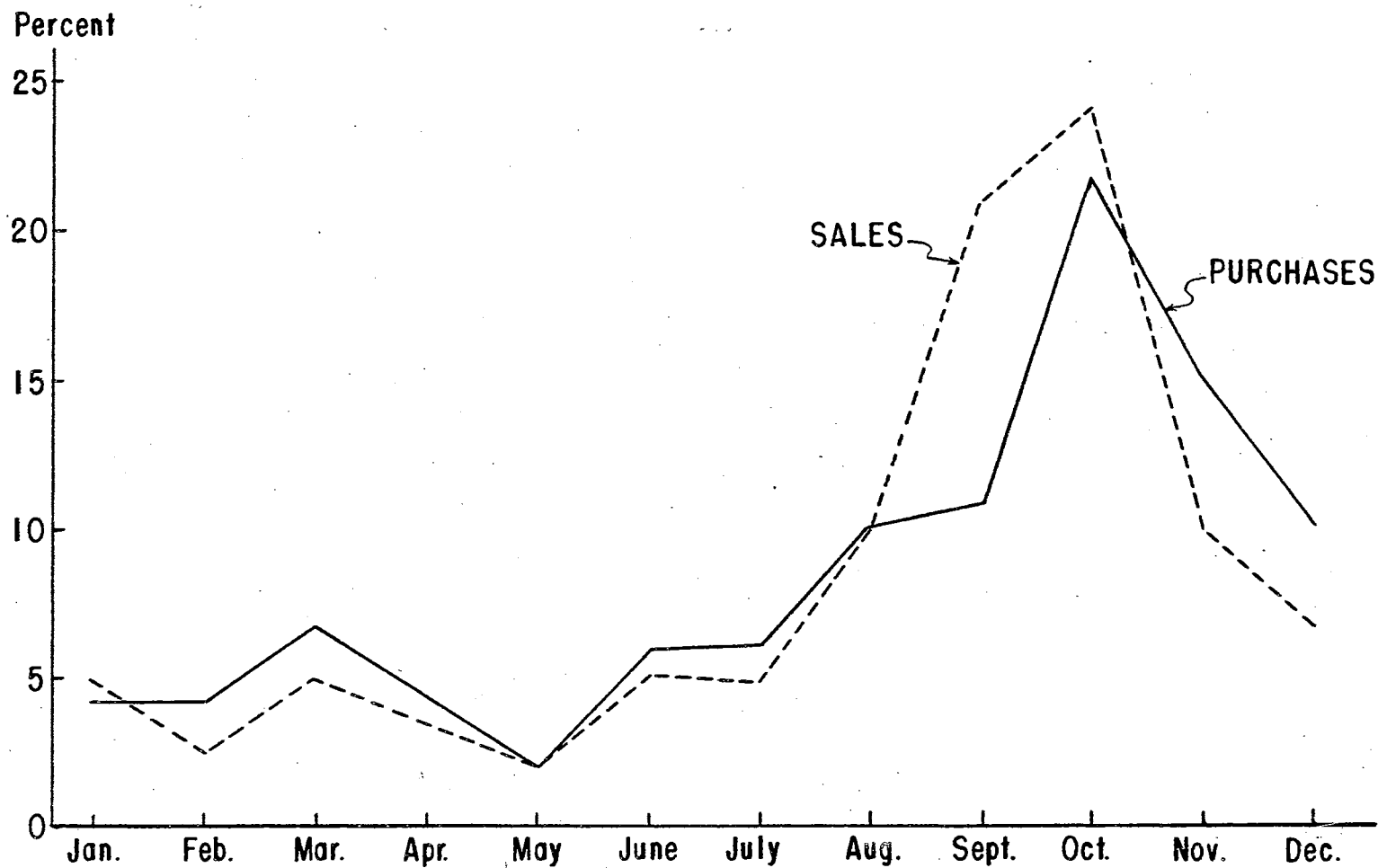


Figure 2. Proportion of Feeder Cattle and Calves Purchased and Sold, by Months

Source: "Marketing Feeder Cattle and Sheep," The Agricultural Experiment Station of Nebraska, Bulletin No. 410, Lincoln, 1952, page 23.

CHAPTER II

CONCEPTUAL FRAMEWORK

In order to reflect a complete conceptual framework for the cattle feeding sector, a theory of the firm, household, and market must be specified. However, since this study is basically oriented toward the behavior relationships of cattle feeding and producing firms, only the theory of the firm will be presented in detail. Presentations relating to the theory of the household and market are contained in the writings of Samuelson,²⁷ Mosak,²⁸ Klein,²⁹ and many other contemporary economists. Certain relevant parts of these theories will be reviewed when the economic models are presented.

A. Assumptions and Definitions

In order to estimate the relevant variables associated with the supply and demand of feeder cattle, it is necessary to construct models that reflect behavior patterns of decision making units in the feeder cattle sector. Information underlying the construction of economic models depends mainly upon two sources. These are: (1) a priori knowledge of the sector being investigated and (2) a knowledge of the economic theory relevant to that sector of the economy. Since in this study both the buyers and sellers are firms, it is necessary to derive the theory of behavior of the

²⁷Samuelson, P. A., Foundations of Economic Analysis, Harvard University Press, 1948, chapters IV and V.

²⁸Mosak, J. L., General Equilibrium Theory in International Trade, Cowles Commission Monograph No. 7, Principia Press, Bloomington, Ind., 1944.

²⁹Klein, L. R., Economic Fluctuations in the United States, Cowles Commission Monograph No. 14, John Wiley and Sons, New York, 1950, pp. 41-47.

feeder cattle sector within the framework of the theory of the firm.

In order to develop the structural behavior relationships of the feeder cattle sector, certain definitions and restrictive assumptions should be made explicit. The firm will be defined as a decision making unit in which factors are transformed into products. The assumptions underlying the analysis are:

(1) The firm wishes to maximize profit.

(2) The economy is one of perfect competition, which implies perfect factor mobility, infinite divisibility of resources, and instantaneous adjustment to change, etc.

Although the specified assumptions may not reflect the real world counterparts, certain degree of abstraction is a requirement in order to have an operational and manageable model.

B. Theory of the Firm

Given the definition and objectives of the firm and assuming that n factors are used to produce m products, the problem confronting the firm can be stated mathematically as maximizing,

$$\sum_{i=1}^k P_i x_i = R \quad \text{where } n + m = k,^{30} \quad (2.1)$$

subject to

$$f = f(x_1, \dots, x_k) = 0, \quad (2.2)$$

where m of the x_i 's represent products and n of the x_i ' denote factors. The price of i^{th} input or output is symbolized by P_i . To maximize equation (2.1) subject to (2.2) a constant of proportionality (Lagrange Multiplier) λ will be employed.³¹

³⁰Factor will be denoted by negative products. This notation was developed by Hicks, J. R., Value and Capital, Clarendon Press, Oxford, 1948, p. 319.

³¹Allen, R. G. D., Mathematical Economics, Macmillan and Co., London, chapter XVIII, 1956.

The necessary conditions for profit maximization is that

$$\frac{\partial (R - \lambda f)}{\partial x_i} = 0. \quad (2.3)$$

Secondly, for sufficient assurance that a maximum exists, the matrix of second order partials derivatives of equation (2.3) must be negative definite. Algebraically this can be stated as:

$$\frac{\partial^2 (R - \lambda f)}{\partial x_i^2} < 0. \quad (2.4)$$

Solving equation (2.3) yields:

$$P_i = \lambda f_i \quad i = 1, \dots, k \quad (2.5)$$

where f_i is the first order derivative of f with respect to the i^{th} x .

Thus eliminating λ , the following relations are obtained:

$$\frac{P_i}{f_i} = \frac{P_j}{f_j} \quad \begin{array}{l} i \neq j \\ i, j = (1, \dots, k) \end{array} \quad (2.6)$$

or

$$\frac{f_1}{f_k} = \frac{P_1}{P_k}, \quad \frac{f_2}{f_k} = \frac{P_2}{P_k}, \quad \dots, \quad \frac{f_{k-1}}{f_k} = \frac{P_{k-1}}{P_k}$$

These ratios indicate that the necessary conditions for maximum profits of a firm is that the marginal rate of substitution between any two factors, products, or a factor and a product is equal to their price ratio.

C. Supply Function

The equilibrium condition for the firm is given by equations (2.2) and (2.5) subject to equation (2.4). The variables x_i are determined as functions of all the prices (P_i), the demand function for factors and the supply function for products. The effect of any specified change in any price can be investigated by differentiating equations (2.2) and (2.4) with respect to that price. The solution of these equations is given by:

$$\frac{\partial x_i}{\partial P_t} = \frac{F_{it}}{F} \quad i, t = (1, 2, \dots, k)^{32} \quad (2.8)$$

which is the effect of one price change on the demand and supply of any x .

Mosak³³ has shown that from (2.8) the following can be obtained:

$$\bar{x}_{it} = -\frac{F_{it}}{F} \quad (2.9)$$

The bar is used to indicate the firm supply of products or demand for factors. Thus,

$$\frac{\partial x_i}{\partial P_t} = -\bar{x}_{it} \quad (2.10)$$

Then in particular,

$$\frac{\partial x_i}{\partial P_i} = -\bar{x}_{ii} > 0. \quad i = 1, 2, \dots, k \quad (2.11)$$

The direct effect of a price change on the demand or supply of the commodity is given by (2.10). For a product $\bar{x}_r > 0$ the condition is given by (2.11) and a price increase raises the supply which implies that the firm's supply curve for any product is positively sloped in regards to its price. For a factor $\bar{x}_i < 0$ the condition is

$$\frac{\partial(-\bar{x}_i)}{\partial P_i} = \bar{x}_{ii} < 0. \quad (2.12)$$

Thus, a price increase reduces the demand. Alternatively it can be interpreted that the firm's demand curve for any factor is negatively sloped with respect to its price.

Finally, the effect of a change in the i^{th} price on the p^{th} commodity is given by (2.7). The sign of \bar{x}_{it} (2.10) provides a definition of substitution and complementarity in firm production. If $\bar{x}_i > 0$, the x_i

³² F is the determinant of the matrix of the first and second order partial derivatives of equation (2.2). F_{it} is the cofactor of the f_{it} within F , where $f_{it} = \frac{\partial^2 f}{\partial x_i \partial x_t}$ for $i, t = 1, 2, \dots, k$.

³³ Mosak, J. L., *ibid.*, chapter VII.

and x_t commodities are substitutes. If x_i and x_t are products an increase in the price of one leads to a decrease in the output of the other. If they are factors, an increase in the price of one will result in an increase in the firm demand for the other. If $\bar{x}_{it} < 0$, the x_i and x_t are complements. Hicks³⁴ has discussed the effects of changing price relationships when two commodities are complements.

From the previous analysis it follows that the firm's demand of any factor or supply of any product are functions of the prices of all the commodities that the firm uses and sells. Since the equilibrium condition (2.3) is homogeneous of degree zero, a change in all prices will not affect the firm's demand for factors or supply of products. The supply function of the firm may be written in the form:

$$\bar{x}_i = \bar{x}_i(z_1, z_2, \dots, z_{k-1}). \quad (2.13)$$

Where $z_i = \frac{P_i}{P_k}$, which is the ratio of the price of the x_i to the numeraire x_k .³⁵ In order to obtain a market supply function the individual firm supply must be summed over all firms in the economy. The resulting function does not differ from equation (2.13) except that the quantities involved refer to the factors and products of all firms.

The theory of the firm that has been reviewed suggests the importance of including in the firm behavior relationships the price of the product or factor, the prices of other factors and the prices of alternative competing enterprises. This broad general framework then provides a basis for classifying the variables that logically should enter the

³⁴ Hicks, *ibid.*, p. 93.

³⁵ The numeraire serves as a standard in terms of which the price of all other commodities are expressed.

firm relations for the economic model to be constructed.

D. Spatial Equilibrium Market Analysis

In order that a model reflecting the optimum geographical product flows between regions might be obtained, the following restrictive and expository assumptions are made. Perfect competition assumptions dictate the requirements for regional pattern of prices and flows of the commodity. Therefore, each firm is assumed to have the objective of maximizing profits and thus will make shipment decisions which yield the greatest per unit net return. The supply source and market for each geographical area is assumed to be represented by a fixed point. All regions are connected by transport costs that are independent of the direction and volume of trade and flows of the product are unhampered by outside interference. It is also necessary to make the assumption that no negative shipments can occur.

As a basis for the theoretical analysis, the "transportation problem," originated by F. L. Hitchcock was formulated.³⁶

Samuelson has stated the minimum transportation cost problem as:

" . . . given at each of two or more localities a domestic demand and supply for a given product in terms of its market price at that locality. We are also given a constant transport cost for carrying one unit of the product between any two of the specified localities. What then will be the final competitive equilibrium of prices in all the markets, of amounts supplied and demanded at each place, and of exports and imports?"³⁷

Given the transport costs, the surplus and deficit regions, and the quantities of the commodity involved in each case, the problem of determining minimum cost flows may be treated as a linear programming problem.

³⁶ "The Distribution of a Product from Several Sources to Numerous Localities," Journal of Mathematics and Physics, Vol. XX, 1941, pp. 224-230.

³⁷ Samuelson, American Economic Review, *ibid.*, p. 283.

Given the a_i (excess demands) and b_j (excess supplies), $i = 1, \dots, n$; $j = 1, \dots, m$; $n + m = N$, the problem is one of satisfying all demands while exhausting the total supplies in such a way as to minimize the transport costs. The problem may be stated algebraically as finding a set of X_{ij} such that:

$$\sum_{j=1}^m \sum_{i=1}^n X_{ij} C_{ij} = \text{minimum}, \quad (2.14)$$

subject to:

$$\sum_{j=1}^m X_{ij} = a_i; \quad i = 1, \dots, n, \quad (2.15)$$

$$\sum_{i=1}^n X_{ij} = b_j; \quad j = 1, \dots, m, \quad (2.16)$$

$$\sum_{i=1}^n a_i = \sum_{j=1}^m b_j \quad (2.17)$$

and

$$X_{ij} \geq 0 \text{ for all } i, j. \quad (2.18)$$

where X_{ij} represents the amount of product shipped from the i^{th} surplus region to the j^{th} deficit region; a_i represents the amount of product available for export from the i^{th} surplus region; b_j is the amount demanded by the j^{th} deficit region and C_{ij} is the per unit cost of shipping from the i^{th} exporting region to the j^{th} importing region. There are many solutions to (2.15) and (2.16) subject to (2.17) and (2.18), and given any feasible solution of $n + m - 1$ or $N - 1$ shipments, an iterative procedure known as the simplex method provides a means of converging to the optimum program (the one that satisfies [2.14]).³⁸

³⁸ See Datzig, *ibid.*, for an example of the use of the simplex technique in obtaining a solution for the transportation problem.

Therefore, given the a_i 's and b_j 's (regional excess supplies and demands), the linear programming transportation model may be used to determine the optimum geographical flow of products. It can be shown that the resulting minimum cost set of flows is the one that would be determined under the conditions of perfect competition. This conclusion follows since the equilibrium prices are tied together by a specific set of transport costs. The solution obtained will be unique except for the case when two or more sources find two markets equally profitable. In this case more than one optimum shipment plan exist.

Up to this point, the programming problem, as such, has been solved without recourse to price differentials, the prime economic allocators of the regional distribution of the product. However, as with any linear programming problem, the solution implicitly places values on the various inputs and outputs involved. Therefore, with the aid of the duality theorem of linear programming, a unique set of price differentials may be derived which corresponds to the equilibrium set of flows. Thus, given a minimum cost transportation solution, the dual problem is concerned with deriving the vector of regional price differentials consistent with this solution.

To construct the dual of the transportation problem, let V_j be associated with the destinations and U_i be associated with the origins or supply points.³⁹ The problem may then be set forth in the following equations as that of maximizing:⁴⁰

³⁹ This section was originally developed by Judge and Wallace, "A Spatial Price Equilibrium Analysis of the Beef Marketing System," *ibid.*

⁴⁰ This development of the dual follows that given by Dorfman, Samuelson, and Solow, *ibid.*

$$S = \sum_{j=1}^m b_j V_j - \sum_{i=1}^n a_i U_i \quad (2.19)$$

subject to the restriction:

$$\begin{aligned} V_j - U_i &= C_{ij}, \quad i = 1, 2, \dots, n; \quad j = 1, 2, \dots, m; \\ U_i, V_j &> 0, \end{aligned} \quad (2.20)$$

Since equation (2.14) is equal to S , the total cost of transportation derived in the minimum formulation, the maximum problem may be thought of as finding the values of the U_i and V_j that will maximize the total gain in value of amount shipped subject to non-positive profits on each shipment. Within this framework, it is then possible to interpret the U_i as the value of the product at supply origin i and V_j as the value of the product delivered at destination j . Then equation (2.21) may be written:

$$V_j - U_i \leq C_{ij}, \quad (2.22)$$

stating that for any supply-destination pair, the value at the destination must be no greater than the value of the input at the supply point plus the transportation cost. For routes in the basis, destination value equals supply point value plus transportation costs. For those routes not in the basis, destination value is equal to or less than the supply point value plus transport costs. For any given problem, once the supply-destination pairs are known, then for that set of pairs, equation (2.22) may be written as:

$$V_j - U_i = C_{ij}.$$

This defines a set of linear equations involving $n + m$ unknown values of the U_i and V_j . Since there are only $n + m - 1$ observed unit transport cost in a basic solution, a unique solution to the set of equations requires assigning an arbitrary value to either one U_i or V_j . By choosing the value at the i^{th} supply point as equal to zero, a set of price dif-

ferentials is generated subject to this choice or base.⁴¹

In addition to providing objective estimates of the regional price differentials, the U_i and V_j also contain two types of useful economic information: (1) the values of U_i measure the comparative advantages of the surplus regions and (2) the values of the V_j give the delivered price differentials that correspond to the most economical allocation of the supply from the viewpoint of minimum aggregate transportation costs.

⁴¹ It should be mentioned that only the regional price differences are determined by this formulation. The regional equilibrium prices are plus or minus the price differential relative to the price in the base region.

CHAPTER III

THE ECONOMIC AND STATISTICAL MODELS

In this section a set of structural equations (sector model) and a spatial model describing the feeder cattle marketing segment of the economy will be discussed. To aid in the presentation of the sections to follow and to facilitate reading, various terms and concepts will be briefly defined.

A. Definitions and Concepts

1. Economic models

Construction of an economic model is an attempt to describe in a simplified way the underlying relations that reflect the observable economic phenomena in some segment of the economy. In this study an economic model will be defined as that specification which results from the consideration of economic theory and a priori knowledge relevant to the sector under study.

2. Types of equations

An economic model may contain four types of structural equations: (1) behavior equations, (2) identities, (3) technical equations, and (4) institutional relationships. The behavior equations represent the response of individuals or groups to an economic stimuli. The technical equations are used to express physical relationships or non-behavior relations. Institutional equations are hypothesized to represent institutional factors set down by laws or rules. The identity equations represent relationships that are true by definition.

3. Types of variables

For each of the four types of equations described above there exist four possible types of variables. They are: (1) endogenous; variables that are generated by the system in which they appear, (2) exogenous; variables that are assumed to be generated outside the system but affect the system, (3) predetermined or lagged; variables generated independently of the current structural relations, and (4) shock or disturbance; not directly observable random variables.

4. Statistical model

A priori knowledge and economic theory are employed in the formulation of the structural equations and classification of variables. However, in order to estimate the parameters of the structural equations other assumptions such as the algebraic form of the relations and the distributional properties of the variables must be specified. Making these and other assumptions the transition is made from the theoretical model to one in which the variables are quantifiable and may be represented by real world counterparts. The resulting specification must, of course, be consistent with the a priori assumptions of the study.

Within this general framework the set of structural equations may be represented by the following statistical model.

$$BY'_t + AZ'_t = U'_t \quad (3.1)$$

where B is a G x G coefficient matrix of the Y's; Y'_t is the transpose of a 1 x G vector of endogenous variables; A is a G x H coefficient matrix of the Z's; Z'_t is the transpose of a 1 x H vector of exogenous and/or predetermined variables; and U'_t is the transpose of a 1 x G vector of shock variables.

A single equation appearing in the general model may be expressed as:

$$(\beta, 0) Y'_t + (\alpha, 0) Z'_t = U_{it} \quad (3.2)$$

where $(\beta, 0)$ is a $1 \times G$ vector, the β partition being $1 \times g$ and 0 is the nullity partition of order $1 \times G - g$; and $(\alpha, 0)$ is a $1 \times H$ vector, the α partition being $1 \times h$ and 0 is the nullity partition of order $1 \times H - h$.

5. Identification

Given the statistical model, the identification problem is one of ascertaining (1) if each equation represents a definite economic relationship, and (2) if the estimation of its structural parameters is possible. Koopmans⁴² has derived the condition for identification of equations of a model.

Using the derivation of Koopmans the necessary condition for a single equation to be just identified is

$$H - h = g - 1.$$

That is the number of exogenous variables appearing in the system but not in the equation to be estimated must be equal to one less than the number of endogenous variables appearing in the structural equation to be estimated.

If the equation is underidentified there exist an infinity of possible solutions for the parameters to be estimated. The underidentified case exists when

$$H - h < g - 1.$$

When

$$H - h > g - 1$$

then the equation is said to be overidentified. Thus, the number of exogenous variables outside the structural equation to be estimated but within the system is greater than the number of endogenous variables in the structural equation less one.

B. General Sector Model

⁴² Koopmans and Hood, *ibid.*, chapter II.

In the presentation to follow a brief discussion of the logic underlying each equation and a description of each specified variable will be presented.

1. Demand for feeder cattle

From the theory of the firm, the demand for a factor is postulated as being a function of the factor price, price of the finished product, price of alternative competing enterprises, and price of availability of other factors of production that are used in the production of the final product. Because of the lack of available data concerning the numbers of cattle fed during the year, the number of cattle demanded was represented by the number of cattle on feed January 1. The price of slaughter cattle was included as an indicator of the demand for the finished product. Since hog feeding is the most important alternative enterprise competing with cattle feeding, the price of slaughter hogs was included to reflect this variable. Corn and hay are important factors in the cattle feeding process and thus, the availabilities of each should condition the behavior of the cattle feeding firm. Time was included to reflect possible technological developments and trends in cattle feeding over the sample period. Using this conceptual framework the firm demand for feeder cattle is postulated as:

$$\beta_{11}Y_{1t} + \beta_{12}Y_{2t} + \beta_{13}Y_{3t} + \alpha_{11}Z_{1t} + \alpha_{12}Z_{2t} + \alpha_{13}Z_{3t} = U_{1t} \quad (3.6)$$

where Y_{1t} is the number of cattle on feed January 1, in 13 corn belt states;⁴³ Y_{2t} is the price of feeder cattle at Kansas City; Y_{3t} denotes the price of slaughter steers at Chicago; Z_{1t} is a time variable; Z_{2t} is the production of corn plus stock on farm in 13 corn belt states; and

⁴³ See page 8 for an explanation and a list of these thirteen states.

Z_{3t} is the production of hay in these 13 corn belt states.

The Y_{it} 's represent endogenous variables, the Z_{it} 's denote exogenous or predetermined variables and U_{it} represents a not directly observable random variable resulting from incomplete specification.

2. Supply of feeder cattle

From the consideration of firm theory, the supply of the product (feeder cattle) will be postulated as depending on the price of the product, price of competing enterprises, the price of factors used in the production process and the firms' willingness to hold the product during time t . Since feeder cattle may go either to the feed lot or to slaughter, the price of slaughter steers was included to reflect this alternative competing demand. Time is postulated to reflect any technological changes that have taken place over the sample period. As an indicator of the number of cattle available for supply, the number of calves less than one year old and the number of steers greater than one year old on farms January 1, in 11 western states were included.⁴⁴ As an indicator of the firms' willingness to hold the cattle during the year, t , the lagged price of feeders and the range conditions were included in the equation. Thus, the supply of feeder cattle can be stated algebraically as:

$$\beta_{21}Y_{1t} + \beta_{22}Y_{2t} + \beta_{23}Y_{3t} + \alpha_{21}Z_{1t} + \alpha_{24}Z_{4t} + \alpha_{25}Z_{5t} + \alpha_{26}Z_{6t} = U_{2t} \quad (3.7)$$

where Z_{4t} is the total of the number of calves less than one year old and the number of steers greater than one year old on farms in 11 western range states; Z_{5t} is the condition of the range of the western states; and Z_{6t} is $Y_{2(t-1)}$. All other variables have been previously defined.

3. Demand at the farm for beef

The farm demand for beef is postulated as being a function of the farm

⁴⁴ See page 8 for an explanation and a list of these eleven states.

price of the commodity, retail price of the commodity, price of substitutes, and farm supply of the product, all acting interdependently to determine the farm demand for beef. The price of beef steers was postulated as the variable to indicate the farm price of beef. The price of slaughter hogs was included to indicate demand for an alternative enterprise. Time was included and can be interpreted similar to the previous uses in the previous equation. This relationship can be stated as:

$$\beta_{33}Y_{3t} + \beta_{34}Y_{4t} + \beta_{35}Y_{5t} + \beta_{36}Y_{6t} + \alpha_{31}Z_{1t} = U_{3t} \quad (3.8)$$

where Y_{5t} is the farm supply of beef; Y_{6t} is the retail price of beef, and the other variables are as defined previously.

4. Supply of beef at the farm (physical relationship)

The supply of beef at the farm is assumed to be a function of the beginning inventory of cattle, availability of feed grains and time. The inventory variable which reflects the resource stock of cattle is viewed as the number of beef cattle of farms January 1. The total United States production of corn is included to reflect the availability of one of the major factors of production. Time is again assumed to reflect any changes in technology that have occurred during the sample period. Thus, the postulated relationship can be stated as:

$$\beta_{45}Y_{5t} + \alpha_{41}Z_{1t} + \alpha_{47}Z_{7t} + \alpha_{48}Z_{8t} = U_{4t} \quad (3.9)$$

where Z_{7t} is the production plus stocks of corn (t-1); and Z_{8t} represents the number of beef cattle on farms January 1. All other variables have been previously defined.

5. Demand for beef at retail

From the theory of consumer choice, the retail demand for beef is postulated to be a function of the price of beef at retail, price of substitutes commodities, consumer incomes, and number of consumers. For purpose

of simplification only, the retail price of pork was used to indicate the demand for substitute commodities. The income variable was denoted by the total disposable income. The number of consumers is represented by estimates of the United States population. Time is included to reflect the possible shifts in consumer preference throughout the sample period. Thus, the retail demand for beef is postulated as:

$$\beta_{56}Y_{6t} + \beta_{57}Y_{7t} + \beta_{58}Y_{8t} + \alpha_{59}Z_{9t} + \alpha_{5,10}Z_{10t} = U_{5t} \quad (3.10)$$

where Y_{7t} is the retail supply of beef; Y_{8t} is the retail price of pork; Z_{9t} represents the total disposable income for the United States; and Z_{10t} is the total United States population.

6. Supply of beef at retail

The supply of beef at the retail has a direct relationship with the supply at the farm. Given the farm production, then what are the factors that enter into the farmer's decision as whether to sell or hold his stock of beef during a given time period, t ? Since price enters into the decision to either sell or hold, the retail price of beef was used as an indicator of the availabilities of beef to the retail demanders. Feeder cattle prices and production of corn were included to indicate the favorability of intensive cattle feeding, thus influencing the current production. Time was included to represent any trends that may have occurred over the sample period. Thus, the following equation represents the supply of beef at the retail market.

$$\beta_{62}Y_{2t} + \beta_{65}Y_{6t} + \beta_{66}Y_{6t} + \alpha_{61}Z_{1t} + \alpha_{67}Z_{7t} = U_{6t} \quad (3.11)$$

All variables have been previously defined.

7. Demand for pork at the farm

The logic presented for the farm demand of beef is also relevant for the farm demand for pork. Therefore, it is assumed that the farm price of pork is interdependently related to the retail price of pork, farm production

of pork, time and the price of substitute commodities (slaughter cattle) and is postulated as:

$$\beta_{73}Y_{3t} + \beta_{74}Y_{4t} + \beta_{78}Y_{8t} + \beta_{79}Y_{9t} + \alpha_{7,11}Z_{11t} = U_{7t} \quad (3.12)$$

where Y_{9t} is the farm production of pork. All other variables have been previously defined.

8. Supply of pork at the farm

The farm production of pork was postulated to be a function of the production and stocks of corn, time, and the potential available resource pool of hogs. The number of gilts and sows was used to indicate the potential availability for feeding. The logic used to arrive at this postulated relationship was similar to that used in equation (3.7). The supply equation for the farm supply of pork is postulated as:

$$\beta_{89}Y_{9t} + \alpha_{81}Z_{1t} + \alpha_{87}Z_{7t} + \alpha_{8,11}Z_{11,t} = U_{8t} \quad (3.13)$$

where $Z_{11,t}$ is the number of gilts and sows on farm January 1. All other variables have been previously defined.

9. Demand for pork at retail

The same analysis used in specifying the demand relations for the retail demand for beef was used to specify the retail demand for pork. The equation representing the demand for pork at the retail level is as follows:

$$\beta_{96}Y_{6t} + \beta_{98}Y_{8t} + \beta_{9,10}Y_{10,t} + \alpha_{91}Z_{1t} + \alpha_{99}Z_{9t} + \alpha_{9,10}Z_{10,t} = U_{9t} \quad (3.14)$$

where Y_{10t} is the supply of pork at the retail level (consumption). All other variables have been previously defined.

10. Supply of pork at retail

The retail supply of pork is postulated as being determined by the price of the commodity, availability of substitutes, and time. The equation representing the supply of pork at retail is:

$$\beta_{10,8}Y_{8t} + \beta_{10,9}Y_{9t} + \beta_{10,10}Y_{10t} + \alpha_{10,1}Z_{1t} = U_{10,t} \quad (3.15)$$

All variables have been previously defined.

The model presented comprises a complete system of equations. The system involves 10 equations, 10 random residuals denoted by U_{1t} and 10 endogenous or simultaneously observed variables denoted by Y_{it} .

C. Alternative Sector Models

As an alternative specification, supply and demand relationships will also be estimated for eight of the thirteen states considered in the general model.⁴⁵ Since these states accounted for 58 percent of the number of cattle on feed January 1, 1956, they therefore, play an important role in the demand for feeder cattle. In order to consider only eight states, Y_{1t} becomes the number of cattle on feed January 1, in eight corn belt states. It will also be necessary to change Z_{2t} (corn) and Z_{3t} (hay) to an eight-state basis.

A limited sample of data was available for eight states concerning the number of feeder cattle imported annually. Since a large percentage of the feeder cattle are imported during the last six months of the year, the imports from July to December were also used as an alternative specification of the demand of feeder cattle (Y_{1t}).

If the cattle feeding year is defined as being from July to July, then estimates of demand can be obtained with the imports for this period as Y_{1t} . To estimate the demand for feeder cattle using July to July imports, all price variables were put on a July-to-July basis.

In order to consider other admissible specifications, alternative time periods were used for some of the variables described in the general model section.

D. Data Relevant to Sector Models

⁴⁵ See page 8 for a list of these eight states.

1. Discussion of data

Time series data were used to reflect the basic variables contained in the models. Although secondary data are not the ideal form of data, in the absence of a controlled experiment, they were the only alternative. To obtain data by a controlled experiment, it would be necessary to conduct a large scale experiment imposing alternative prices and levels of incomes on consumers and producers, while observing their reactions. Since a controlled experiment involving the feeder cattle sector is not feasible, it must be assumed that an experiment of a similar type has been carried out automatically by the market mechanism of the sector under study.⁴⁶ Thus, accepting this assumption, the data were obtained from various publications of the United States Department of Agriculture and other governmental agencies.

2. Time periods of data and algebraic form of the equations

To make use of all the data available two sample periods were chosen. The sample period chosen to reflect data for the relations involving number of cattle on feed was from 1930 to 1957. The other sample period chosen for the relations involving imports of feeder cattle was from 1940 to 1956. For both sample periods, all available observations were used. This resulted in 28 observations for models involving the number of cattle on feed January 1, and 17 observations for the models in which imports were postulated to reflect demands. A description of the data used for the relevant variables is presented in Appendix A.

All the data were converted to a 1947-49 = 100 index with the exception of Z_{1t} , the time variable. In order to account for price level changes, all prices and the total disposable income variables were deflated

⁴⁶ Judge, G.G., "Econometric Analysis of the Demand and Supply Relationships for Eggs," Storrs Agr. Expt. Stat., Storrs, Connecticut, 1954, p.20.

by the index of prices received by farmers. This deflator was used because the prices which this study are concerned are primarily farm prices.

The algebraic form of the structural equations described in Chapter III must be specified in order that estimates of the parameters may be obtained. The two most likely functional forms existing are: (1) expressing the equation in natural units and (2) transforming the data to logarithms. There is little a priori reason for choosing one functional form over the other. However, the logarithmic form does have the advantage of flexibility and the resulting estimates can be interpreted as elasticities.⁴⁷ Thus, the indices of all the data, except the time variable, were transformed to a logarithmic form.

E. Spatial Model

To develop a spatial equilibrium model for the feeder cattle sector it is necessary to define the conceptual framework as it applies to the real world. How to divide an economic territory into geographical contiguous units is one of the unsolved problems of spatial analysis. Lacking an objective set of criteria in arriving at the final demarcation, the investigator seeks a regional model that will be both manageable and reasonable realistic or meaningful. A priori knowledge and use of the general criteria stated above were used in partitioning the feeder cattle sector into 25 geographically contiguous regions. States represent the smallest units for which data could be obtained (figure 3). Since data were not available for the Southern and Eastern States, they were not considered a part of the feeder cattle marketing sector of the United States. Each regional market or source of supply is represented by a point that

⁴⁷ Wallace, T. D., "An Econometric Study of the Beef Industry," Unpublished Masters Thesis, Oklahoma State University, 1957, p. 67.



<u>Region</u>	<u>State</u>	<u>Supply and Demand Point</u>
1	Arizona	Flagstaff
2	Colorado	Denver
3	California	Fresno
4	Indiana	Indianapolis
5	Illinois	Springfield
6	Idaho	Boise
7	Iowa	Des Moines
8	Kansas	Wichita
9	Minnesota	St. Paul
10	Montana	Billings
11	Missouri	Kansas City
12	Michigan	Lansing
13	Nevada	Austin
14	Nebraska	Grand Island
15	North Dakota	Bismark
16	New Mexico	Albuquerque
17	Oklahoma	Oklahoma City
18	Oregon	Portland
19	Ohio	Columbus
20	South Dakota	Aberdeen
21	Texas	Ft. Worth
22	Utah	Salt Lake City
23	Washington	Seattle
24	Wisconsin	Portage
25	Wyoming	Casper

Figure 3. The Regional Demarcation, Demand and Supply Points, Spatial Feeder Cattle Model

is identified with a certain city within each region.

Regional supplies and demands are taken as predetermined for a given time period. Also it is assumed that exports of feeder cattle outside the United States are negligible. In order to arrive at an optimum flow pattern, the excess supplies and excess demands must be defined. A region is designated as a supplier if the number of cattle available for supply exceeds the number of cattle fed for time period, t . Likewise, a region will be denoted a demander of feeder cattle if the available cattle for feeding are fewer than the number the region demands for some given time, t .

Since the transport rate for shipping feeder cattle between all possible combinations of regions are basic to the spatial solution, it is necessary to obtain estimates of the cost of shipping between the points representing each pair of regions. A large percentage of the feeder cattle are shipped by rail,⁴⁸ and the equation used to generate the rates between the market and supply points was postulated as:

$$C_{ij} = B_1 M_{ij} + B_2 M_{ij}^{1/2} + \epsilon \quad (3.16)$$

where C_{ij} is the cost of shipping feeder cattle in dollars per one hundred pounds from region i to region j ; M_{ij} is the rail mileage between region i and region j ; B_1 and B_2 are parameters to be estimated; and ϵ is an unobservable random error.

This functional form was postulated in the belief that rail rates are an increasing function of mileage but should increase at a decreasing rate. For obvious reasons, the function was postulated as having a zero intercept.

F. Data Relevant to Spatial Model

⁴⁸ See North Central Livestock Marketing Research Committee, *ibid.*, pp. 42-47, for an explanation of the relative importance of rail shipments.

1. Discussion of data

Given the regional demarcation, the transportation problem becomes one of determining the excess supplies and demands of each region. Since the most complete records of cattle numbers and cattle on feed are published by the Agricultural Marketing Service, data published by this agency were used to determine each region's supply and demand. Therefore, all estimates of excess demands and supplies were obtained directly from secondary data without recourse to regional demand and supply relationships. For a complete description of the data pertaining to the spatial model, see Appendix A.

The two time periods investigated were 1951 and 1956. These two time periods were chosen because they represent a contrast of the low and high years for imports of feeder cattle by the cattle feeding states.

2. Discussion of transport costs

The transport rates between all possible pairs of regions are also basic to the solution of a spatial model. The basing points were chosen because of their relative positions within the region or because of their importance in the feeder cattle marketing sector of the economy. This selection would have made individual transport rates between all possible pairs of regions very difficult to obtain. Therefore, the decision was made to generate the transport costs by estimation. Also, generating a transport rate function allowed a rate to be estimated between any possible pairs of basing points.

The functional relationship postulated in equation (3.16), the sample data, rail mileages, and the least squares procedure using moments about zero were employed to estimate the unknown parameters. The results are:

$$C_{ij} = .000571M_{ij} + .020752M_{ij}^{1/2} \quad (3.17)$$

$$R^2 = .982$$

for feeder cattle being shipped East;

and

$$C_{ij} = .000428M_{ij} + .025275M_{ij}^{1/2} \quad (3.18)$$

$$R^2 = .996$$

for feeder cattle being shipped West.⁴⁹

The estimated transport rates between all regions are presented in Table I.

⁴⁹ Rail rates for shipping feeder cattle were obtained from the General Offices of the Santa Fe Railroad Company in Oklahoma City; the corresponding rail mileages were taken from "Rand McNally Commercial Atlas and Marketing Guide," 86th edition, Rand McNally and Company, Chicago, 1955.

TABLE I

ESTIMATED TRANSPORT RATES FOR FEEDER CATTLE BETWEEN
 SPECIFIC POINTS, BY REGIONS, UNITED STATES
 (cents per one hundred pounds)

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	
1	-	.97	.92	1.91	1.80	1.57	1.65	1.31	1.98	1.67	1.46	2.06	.98	1.33	1.79	.59	1.25	1.64	2.17	1.76	1.26	1.21	1.79	1.86	1.28	
2		-	1.62	1.36	1.03	1.14	.93	.83	1.12	.91	.89	1.41	1.22	.63	1.17	.73	.99	1.55	1.55	1.39	1.05	.82	1.70	1.26	.56	
3			-	-	-	1.35	-	1.67	-	1.65	1.88	-	.78	1.90	2.05	1.22	1.63	1.16	-	1.90	1.72	1.18	1.31	-	1.43	
4				-	-	2.01	-	.97	-	1.63	.77	-	2.29	1.05	1.24	1.60	1.03	2.41	-	1.71	1.21	1.81	2.33	-	1.56	
5					-	1.92	-	.76	-	1.49	.53	-	1.72	.90	1.24	1.39	.89	2.24	-	1.11	1.09	1.69	2.16	-	1.34	
6						-	1.64	1.56	1.56	.76	1.67	2.10	1.19	1.35	1.17	1.56	1.70	.74	2.18	1.27	1.75	.65	.91	1.83	.82	
7							-	.68	-	1.26	.43	-	1.88	.52	.94	1.32	.82	2.00	-	.90	.95	1.38	2.02	-	1.11	
8								-	.94	1.44	.42	1.08	1.77	.79	1.34	.97	.37	2.04	1.17	1.22	.62	1.30	2.18	.98	1.14	
9									-	1.12	.73	-	2.17	.74	.68	1.54	1.07	1.92	-	.53	1.24	1.68	1.88	-	1.28	
10										-	1.27	1.63	-	1.08	-	-	-	-	1.74	.84	-	.92	-	1.35	-	
11											-	.91	1.47	.47	1.14	1.12	.58	2.04	.91	1.02	.79	1.41	2.03	.77	1.16	
12												-	2.38	1.06	1.21	1.68	1.18	2.50	-	1.08	1.36	1.90	2.40	-	1.58	
13													-	1.62	-	-	-	-	2.46	1.62	-	.85	-	2.31	-	
14														-	1.13	1.13	.88	1.91	1.17	1.02	1.05	1.09	1.86	.86	.81	
15															-	-	-	-	1.36	.45	-	1.11	-	.92	-	
16																		-	-	1.78	1.78	-	1.21	-	1.56	
17																		-	-	1.22	1.33	-	1.44	-	1.10	
18																			-	2.49	1.68	-	1.12	-	2.11	
19																				-	1.21	1.34	1.98	2.49	-	1.74
20																					-	1.47	1.09	1.63	.78	1.15
21																						-	1.50	-	1.28	-
22																							-	1.29	1.82	.81
23																								-	2.07	-
24																									-	1.37
25																										-

CHAPTER IV

EMPIRICAL RESULTS OF SECTOR MODELS

The parameter estimates of the equations relating to the sector model will be presented in this section. Equations not directly related to the ones postulated in the general model are noted in Appendix C. Economic and statistical tests for each relation will be given and economic implications of the parameter estimates will also be discussed.

A. Equations Relating to the Firm Demand for Feeder Cattle for Thirteen States

1. Limited information estimate

Since the feeder cattle demand relationship in the general model was overidentified, the limited information method of parameter estimation was employed. Estimation of this relationship resulted in the following parameter estimates:

$$\dot{Y}_1 = \frac{.0470Y_2}{(.192)^2} - \frac{.4695Y_3}{(.281)^3} + \frac{.0655Y_4}{(.177)^4} + \frac{.0365Z_1}{(.008)^1} + \frac{.4123Z_2}{(.071)^2} + \frac{.2316Z_3}{(.032)^3} + .1193 \quad (4.1)$$
$$s^2 = .0264$$

Since data are in logarithms the coefficients may be directly interpreted as the elasticities (except for the coefficient of the time variable, Z_1).

The standard errors of the estimated parameters appear directly below the coefficients in each instance. The estimated residual variance (s^2) appears directly below the equation. The dot over the Y_1 variable indicates an estimated relationship. This format will be followed for all estimated relations in this study unless otherwise stated.

These results indicate a positive relationship between the price of the factor (feeder cattle) and the quantity demanded. These estimates also indicate that there is a negative relationship between the demand for the factor and the price of the final product (slaughter cattle). They also indicate a positive relation between the price of the alternative competing factor (hogs) and the demand for feeder cattle. Thus, all three estimates of the coefficients of Y_2 , Y_3 and Y_4 are inconsistent with theoretical expectations with regard to sign. However, it should be noted that none of the coefficients of these variables are statistically significant at the 95 percent probability level. The estimates of the coefficients relating to the production of corn and hay are consistent with theory. The coefficient of the Z_1 variable indicates that there has been an upward trend in the demand for feeder cattle over the sample period.

The following are examples of the ceteris paribus conditional economic statements that may be made from the above estimates. A one percent increase in the production of corn would increase the demand for feeder cattle approximately 0.41 percent. Similarly, a one percent increase in the production of hay would result in approximately a 0.23 percent increase in the demand for feeder cattle.

To check the specification of the model, the likelihood ratio test for testing the overidentified restrictions is given by the following statistics:⁵⁰

$$T \log_e (1+v) \sim \chi^2_{(H-h-g+1)} \text{df} \quad (4.2)$$

where T refers to the sample size; v is the inverse of the largest characteristic root of the matrix associated with the final solution; and $(H-h-g+1)$ is the number of overidentifying restrictions (H, h and g were

⁵⁰ Anderson and Rubin, *ibid.*

defined in Chapter III). In this instance

$$1.2275 < \chi^2_{.01}(6)df. \quad (4.3)$$

Therefore the hypothesis that the coefficients of the exogenous variables assumed zero in the equation actually are zero, cannot be rejected at the .01 significance level for this equation.

2. Just identified estimates

By eliminating all exogenous variables from the system not appearing in the equation except Z_4 , Z_9 and Z_{10} , the necessary condition was met for the equation to be just-identified. Therefore, the results appearing in equation (4.4) were obtained from the reduced forms.⁵¹

$$\dot{Y}_1 = -.0923Y_2 - .4732Y_3 + .2352Y_4 + .0390Z_1 + .4860Z_2 + .2159Z_3 \quad (4.4)$$

The signs of the parameters obtained from the reduced form estimates are consistent with the limited information estimates except for the sign of the coefficient of variable Y_2 (price of feeder cattle). The magnitude of the parameter estimates are similar to those obtained in equation (4.1) and similar ceteris paribus statements can be made. Other just identified estimates are presented in Appendix C, section I, A.

3. Single equation estimates

With Y_1 chosen as the dependent variable, the least squares results of equation (4.1) gave the following parameter estimates.

$$\dot{Y}_1 = \begin{matrix} -.0895Y_2 & -.0292Y_3 & -.0526Y_4 & + .0477Z_1 & + .3677Z_2 & + .1994Z_3 & + 1.0630 \end{matrix} \quad (4.5)$$

$$\begin{matrix} (.132)^2 & (.173)^3 & (.090)^4 & (.017)^1 & (.077)^2 & (.081)^3 & \end{matrix}$$

$$R^2 = .939$$

The multiple correlation coefficient (R^2) is presented for each equation estimated by least squares technique. In comparing the results of least squares estimates with the limited information estimates the signs

⁵¹ Judge, G. G., "Econometric Analysis of the Demand and Supply Relationships for Eggs," *ibid.*, pp. 33-42.

of the coefficients pertaining to Y_2 and Y_4 have changed. All the signs of the coefficients obtained by least squares are consistent with theory except the coefficient of Y_3 . However, it should be noted that statistically this parameter estimate is not significantly different from zero.⁵² Also the parameters of Y_2 and Y_4 are not significantly different from zero. It should be noted that the magnitude of the coefficients of Z_1 , Z_2 and Z_3 are reasonable consistent for all three equations (4.1), (4.4) and (4.5).

The following ceteris paribus conditional statements may be formulated from (4.5) A one percent increase in the price of feeder cattle would result in approximately a 0.09 percent reduction in the demand for feeder cattle. A one percent increase in the price of slaughter hogs would result in a decrease in the demand of feeder cattle of approximately 0.05 percent. A one percent increase in the production of hay would result in approximately a 0.20 percent increase in the demand for feeder cattle.

The coefficients of Y_3 (price of slaughter cattle) and Y_4 (price of slaughter hogs) were not significant in either the estimates of equation (4.1) or (4.5), therefore, it was believed that they were adding very little to the explanation of the variation in Y_1 . From this hypothesis the following relationship was estimated with Y_3 and Y_4 omitted.

$$Y_1 = \frac{-.1168}{(.085)^2} Y_2 + \frac{.0490}{(.016)^1} Y_3 + \frac{.3783}{(.072)^2} Z_2 + \frac{.1896}{(.076)^3} Z_3 + .9770 \quad (4.6)$$

$$R^2 = .938$$

The results of (4.6) indicate that almost as much of the variation in Y_1 was explained by omitting Y_2 and Y_3 as there was by including them in the equation. The "t" value of the coefficient of Y_2 was the largest ob-

⁵² The statements concerning significant levels will in all instances refer to the 5 percent confidence level.

tained up to this point, but it was only significant at the 20 percent level. The signs agree with theoretical expectation and the magnitude of the coefficients are consistent with the previous estimates. These results lend support to the proposition that the price of slaughter cattle and price of the alternative and competing enterprise (hog feeding) have very little impact in the decision making process of cattle feeding firms.

In an effort to obtain a variable that would reflect the importance of the price of the finished product the difference between price of steers and price of feeder cattle was substituted for Y_3 in equation (4.5). The results are:

$$\dot{Y}_1 = \frac{-.0836Y_2}{(.103)^2} + \frac{.0191Y_{14}}{(.043)^{14}} - \frac{.0545Y_4}{(.090)^4} + \frac{.0502Z_1}{(.017)^1} + \frac{.3676Z_2}{(.076)^2} + \frac{.1910Z_3}{(.080)^3} + .9952 \quad (4.7)$$

$$R^2 = .939$$

The positive sign of the Y_{14} (price steers minus price of feeder cattle) indicates that as the differences increases the cattle feeding firms are willing to feed more cattle.

The signs of all the coefficients agree with their theoretical counterparts. However, the signs of the coefficients of the Y variables are not significantly different from zero.

Alternative least squares estimates of the demand for feeder cattle may be found in Appendix C, section I, B. These alternatives include different observation periods for variables such as Y_3 , Y_4 and Y_2 . Also parameters of other variables that were believed to influence the fluctuations in the number of cattle fed were estimated and the results are presented in this Appendix.

B. Estimation Representing Demands by Feeding Firms in Eight States

1. Equations with demands represented by number on feed January 1

a. Single equation estimates

Corresponding with equation (4.5) the following equation was estimated for eight states:

$$\dot{Y}_1 = \frac{-.1660Y_2}{(.115)^2} + \frac{.0682Y_3}{(.151)^3} - \frac{.0694Y_4}{(.079)^4} + \frac{.0571Z_1}{(.014)^1} + \frac{.3611Z_2}{(.072)^2} + \frac{.2031Z_3}{(.067)^3} + 1.0492 \quad (4.8)$$

$$R^2 = .954$$

The following are examples of ceteris paribus conditional statements that may be made for equation (4.8). Approximately a 0.16 percent decrease in the demand of feeder cattle will occur if there is a one percent increase in the price of feeder cattle. A one percent increase in the production of hay in these eight states will result in approximately a 0.20 percent increase in the demand for feeder cattle in these states.

The magnitude of the estimated coefficients of this equation are consistent with the estimates of (4.5) the similar relationship for the thirteen states.

Since the estimates of the coefficients of Y_3 and Y_4 were not significantly different from zero, the following equation was estimated with

Y_3 and Y_4 omitted.

$$Y_1 = \frac{-.1396Y_2}{(.075)^2} + \frac{.0560Z_1}{(.013)^1} + \frac{.3705Z_2}{(.070)^2} + \frac{.1992Z_3}{(.065)^3} + 1.0013 \quad (4.9)$$

$$R^2 = .952$$

The estimates are consistent with the results of (4.8) with regard to signs and magnitude. The results also further indicate the major importance of Y_2 (price of feeders), Z_1 (time), Z_2 (corn production) and Z_3 (hay production) in determining the demand for feeder cattle. In these estimates the coefficient of Y_2 approaches significance at the .05 percent level of confidence.

Other least squares estimates concerning the effect of a change in the period of observation for various variables and the effect of including other variables in specifying the relation, may be found in Appendix

C, section II, B.

b. Just identified estimates

To obtain an estimate of the interdependent nature of the demand for feeder cattle in eight states the following just identified equation was estimated.

$$\dot{Y}_1 = -.0727Y_2 - .2337Y_3 - .0181Y_4 + .0511Z_1 + .3900Z_2 + .2191Z_3 \quad (4.10)$$

Where Z_4 , Z_9 and Z_{10} were the exogenous variables considered in the remainder of the system. With the exception of the sign of the coefficient of Y_3 the estimates are consistent with (4.4) a similar equation for thirteen states.

For these estimates, ceteris paribus statements consistent with the ones made for equation (4.8) may be inferred. Other just identified relations using other sets of exogenous variables outside the structural equation are presented in Appendix C, section II, A.

2. Imports from July through December as reflectors of demand

Using imports as the Y_1 variable the following equation was obtained.

$$\dot{Y}_1 = \frac{-.1156Y_2}{(.349)^2} + \frac{.3471Y_3}{(.478)^3} - \frac{.2926Y_4}{(.175)^4} + \frac{.1170Z_1}{(.038)^1} + \frac{.2954Z_2}{(.225)^2} + \frac{.1724Z_3}{(.142)^3} + 1.0529 \quad (4.11)$$

$$R^2 = .805$$

All signs of the estimated coefficients agree with theory. Although the coefficients of Z_2 and Z_3 are not significantly different from zero, the magnitudes of the coefficients are consistent with previous estimates. The "t" value of the coefficient of Y_4 has increased thus giving an indication of the importance of the price of hogs to the cattle feeding firms of the corn belt. The magnitude of the coefficient of Y_3 has increased, but the error variance has increased accordingly such that the coefficient is not significantly different from zero. The time variable is significantly different from zero. The sign of the coefficient of Y_2 agrees with

its theoretical counterpart, although the coefficient is not significantly different from zero.

Depicting an alternative time period (October through July average price instead of the yearly average price) for Y_4 resulted in the following parameter estimates.

$$Y_1 = \frac{-.2241Y_2}{(.330)^2} + \frac{.5069Y_3}{(.456)^3} - \frac{.3156Y_4}{(.152)^4} + \frac{.1194Z_1}{(.036)^1} + \frac{.3748Z_2}{(.213)^2} + \frac{.1709Z_3}{(.133)^3} + .9652 \quad (4.12)$$

$$R^2 = .907$$

The change in the time period for the observation of Y_4 had very little affect on the signs, magnitude or significance of the coefficients.

Alternative estimates using imports as the indicator of demand may be found in Appendix C, section III.

3. July to July imports as a reflector of demand for feeder cattle

Specification of all the Y variables on a July to July basis resulted in the following parameter estimates:

$$Y_1 = \frac{-.9605Y''_2}{(.483)^2} + \frac{1.3805Y''_3}{(.620)^3} - \frac{.1199Y''_4}{(.188)^4} + \frac{.0693Z_1}{(.036)^1} + \frac{.2520Z_2}{(.200)^2} + \frac{.3463Z_3}{(.130)^3} + .2917 \quad (4.13)$$

$$R^2 = .923$$

The signs of the parameters agree with theoretical expectations but differ in magnitude relative to the other estimated equations. The parameter estimates of the variables, Y''_2 (price of feeder cattle), Y''_3 (price of slaughter steers), Z_1 (time) and Z_3 (hay production) are significant at or above the 10 percent probability level. Although the production of corn is not significant it should be noted that the magnitude of the coefficient is consistent with the previous estimates. An estimate of the affect of number of beef cattle on farms on the imports is presented in Appendix C, section IV.

C. Summary of Results of Estimated Demand Relations

Estimates for the equations representing the demand for feeder cattle

were obtained by four different methods and under four different variable specifications. However, the elasticity coefficients of corn and hay production associated with the demand for feeder cattle and the coefficient of the time variable were consistent over different equations and alternative techniques of estimation. Table II gives the range of these coefficients.

TABLE II

RANGE IN COEFFICIENTS OF THE VARIABLES Z_1 , Z_2 AND Z_3
FOR ALL DEMAND EQUATIONS

Time Periods	No. States	Method of Estimation	Number of Equations	Z_1	Z_2	Z_3
January 1	13	least squares	10	.016-.069	.309-.378	.185-.252
January 1	8	least squares	5	.055-.076	.363-.372	.199-.226
Imports July-December	8	least squares	4	.080-.119	.295-.393	.161-.171
Imports July-July	8	least squares	3	.069-.141	.252-.432	.216-.346
January 1	13	just identified	8	.016-.052	.202-.488	.163-.320
January 1	8	just identified	4	.054-.056	.367-.397	.203-.219

Since it was not possible to find a price variable that was significant over all demand equations it is believed that corn and hay production are the most important factors to consider in explaining the demand for feeder cattle. There has been an increasing demand for feeder cattle over the sample period considered other things being equal. Omitting Y_3 (price of slaughter steers) and Y_4 (price of slaughter hogs) from the analysis reduced the percentage explanation of Y_1 only slightly.

The choice of an equation to be used should depend upon the problem and the conditions one wishes to reflect.

D. Equations Relating to the Supply of Feeder Cattle

The Theil-Basmann method was used to estimate the parameters of the supply equation postulated in the general model, and the following results were obtained.

$$Y_1 = \frac{.2097Y_1'}{(.411)^2} - \frac{.3535Y_{12}}{(.446)^{12}} + \frac{.0884Z_1'}{(.066)^1} + \frac{.6704Z_4}{(.210)^4} + \frac{.7623Z_5}{(.403)^5} + \frac{.1763Z_6}{(.143)^6} - .9850 \quad (4.14)$$

$$s^2 = .0025$$

The signs of the parameters of the price of feeder cattle, slaughter steers, time and animals available for supply, agree with theoretical expectations. It was expected that an increase in the conditions of the range would result in producers holding their feeder cattle over a longer period. However, the positive sign of the coefficient of Z_5 (range conditions) indicates otherwise. Alternatively, the range condition variable could be highly correlated with the production of hay and corn, thus causing an increase in the demand for feeder cattle which is reflected in the supply equation. If this was the case then this increase in demand would result in an increase in the supply of feeder cattle.

A priori it was believed that the lagged price of feeder cattle would indicate the producer's willingness to hold his cattle over some time, t . The positive coefficient of Z_6 (lagged price of feeder cattle) indicates that as the price increased last year, the supply of feeder cattle would be increased the following year. Only the coefficient of Z_4 (number of calves less than one year old and number of steers greater than one year old on farms January 1) was significantly different from zero at the 5 percent significance level.

As a basis for comparison, equation (4.4) was estimated by least squares with the following results obtained:

$$\begin{aligned}
 Y_1 = & \frac{.1266Y_1}{(.362)^2} - \frac{.2715Y_{12}}{(.391)^{12}} + \frac{.0969Z_1}{(.063)^1} + \frac{.6531Z_4}{(.211)^4} + \frac{.8054Z_5}{(.398)^5} + \frac{.1810Z_6}{(.137)^6} - 1.0637 \\
 & R^2 = .822 \\
 & s^2 = .0026
 \end{aligned}
 \tag{4.15}$$

The signs, magnitudes, and significance of the coefficients estimated by least squares are consistent with those obtained by the Theil-Basmann method.

Alternative equations containing variables such as price of feeder sheep and production of hay in the range states were estimated and the results are presented in Appendix C, section V.

E. Summary of Results Obtained from Supply Relations

Since the primary emphasis of this study was concerned with obtaining parameter estimates of firm demand relations, only a limited number of specifications were concerned with the supply analysis. From the estimated results it would appear that the most important factors in determining the supply of feeder cattle are: (1) price of feeder cattle, (2) cattle available for supplying, (3) conditions of the range and (4) the changes over time. In most instances the signs of the estimated coefficients were reasonably consistent over the different methods of estimation used.

CHAPTER V

EMPIRICAL RESULTS OF SPATIAL MODELS

Employing the general spatial model presented in Chapter III and given the basic data necessary for the spatial solution, the empirical problem is one of determining the optimum geographical flows and regional price differentials for feeder cattle under alternative assumptions and different time periods. The time periods for which the spatial analysis is accomplished are the years 1951 and 1956. These years were used because they represent a relatively low and high period, respectively, for feeder cattle imports into the corn belt. For each year, four alternative models were estimated. The estimated models differed in that alternative methods were used to generate the regional availabilities of feeder cattle. Since regional demand relationships were not available, regional demands were estimated from basic data on cattle imports. Two model for each time period are presented and discussed in the text and the other four solutions are presented in Appendix D.

A. Spatial Analysis for 1956

1. Model I (1956)

The demand and supply for each region was determined as follows: for regions (3,4,5,7,9,12,14,19,20 and 24) data representing the annual imports of feeder cattle were available, therefore these regions were classified as deficit by the amount of the imports for that year. The classification and quantities involved for the other regions were obtained as follows: regions were designated as surplus regions if the number of calves and steers on farms January 1, time period t , denoted by (X_{1i}) not including

the number of steers and calves on feed January 1, time t , less the number of replacement heifers for time t (X_{2i}) and number of cattle fed during the year t (X_{3i}) is a positive quantity. If the quantity is negative, the regions were denoted as deficit. Algebraically this relation can be stated as follows, if:

$$X_{1i} - X_{2i} - X_{3i} > 0 \text{ then the } i^{\text{th}} \text{ region is surplus,} \quad (5.1)$$

and if

$$X_{1i} - X_{2i} - X_{3i} < 0 \text{ then the } i^{\text{th}} \text{ region is deficit.} \quad (5.2)$$

This relationship formed the basis for determining the supply and demand of each region. Since data were available on imports for South Dakota and Nebraska and other data were available that indicated that these states were also exporters of feeder cattle, they were included in this model as both a supplier and demander of feeder cattle in an effort to more nearly approach reality. Exports for South Dakota and Nebraska were determined from the following relationship.

$$X_{1i} - X_{2i} - X_{3i} + X_{4i} = \text{amount available for exports,} \quad (5.3)$$

$i = 14, 20$

where X_{4i} is the number of feeder cattle imported. Since these states were importing feeder cattle, it was necessary to account for them when the available exports were computed. The number imported adds to the availability and therefore it was necessary to account for this increase in the number of feeder cattle available for shipment.

Using the data as defined, the linear programming transportation model was used to derive the optimum geographical shipment pattern for feeder cattle. The optimum solution given the transport cost and excess demands and supplies is presented in Table III.

The cells of Table III in which underlined figures appear represent the activities obtain in the optimum solution, and the corresponding

TABLE III

SURPLUS AND DEFICITS, OPTIMUM FLOWS AND ESTIMATED REGIONAL PRICE DIFFERENTIALS FOR FEEDER CATTLE IN THE UNITED STATES, MODEL I (1956)
(shipments in thousands)

Destinations	Origins																Totals	V _i
	11	15	20	14	8	17	21	10	6	25	2	16	22	13	23	18		
19	.13	.03	<u>6</u>	.26	.11	.03	<u>206</u>	.40	1.14	.62	.61	1.11	1.11	1.99	1.49	1.64	212	.78
4	.15	.07	.06	.30	.07	<u>230</u>	.03	.45	1.13	.60	.58	1.09	1.10	1.98	1.49	1.72	230	.62
5	.05	.21	.20	.29	<u>1,037</u>	<u>267</u>	.05	.45	1.18	.52	.39	1.02	1.12	1.55	1.46	1.69	1,304	.48
12	.26	.01	<u>120</u>	.28	.15	.12	.15	.42	1.19	.59	.60	1.14	1.16	2.04	1.53	1.78	120	.65
24	.42	.02	<u>45</u>	.38	.35	.34	.37	.44	1.22	.68	.75	1.32	1.38	2.27	1.50	1.69	45	.35
9	.63	.03	<u>458</u>	.51	.56	.56	.58	.46	1.20	.84	.86	1.55	1.49	2.38	1.56	1.75	458	.10
7	.04	<u>82</u>	.08	<u>992</u>	.01	.02	<u>796</u>	.31	.99	.38	.38	1.04	.90	1.80	1.41	1.54	1,870	.39
20	1.12	<u>140</u>	4.67	.99	1.04	1.02	1.01	.38	1.11	.91	1.33	1.99	1.10	2.03	1.51	1.71	140	-.10
14	<u>433</u>	.11	.12	4.40	.04	<u>107</u>	.02	.05	.62	<u>89</u>	<u>14</u>	.77	.53	1.46	1.17	1.37	643	.47
1	.76	.54	.63	.50	.33	.14	<u>48</u>	.41	.61	.24	.11	<u>144</u>	.42	.59	.87	.87	192	.70
3	.79	.41	.38	.68	.30	.13	.07	<u>434</u>	<u>60</u>	<u>172</u>	.37	.24	<u>18</u>	<u>75</u>	<u>178</u>	<u>239</u>	1,176	1.09
Surplus	.56	.01	.13	.43	.28	.15	<u>658</u>	<u>84</u>	.30	.22	.40	.67	.47	.87	.34	.49	742	-.56
Totals	433	222	629	992	1,037	604	1,708	518	60	261	14	144	18	75	178	239	7,132	
U _i	0	-.55	-.43	-.13	-.28	-.41	-.56	-.56	-.26	-.34	-.16	.11	-.09	.31	-.22	-.07		

Total Shipments: 6,390,000 head or 44,730,000 pounds (from information obtained in the "Livestock Market News Statistics" the average weight for all feeder shipped was estimated to be 700 pounds).

Total Cost: \$38,647.476.00 (since the rates were in cents per one hundred pounds it was necessary to multiply the cost by 7 to convert the rates to a per head basis).

number represents the number (in thousands) of feeder cattle involved in the optimum flow solution. A total of 7,132,000 head of feeder cattle were available for exporting. Since there were only 6,390,000 head demanded a surplus of 742,000 head was obtained. The total cost of transporting the cattle from the supply regions to the demand regions was \$38,647,476.00. Some examples of these interregional movements are: Region 17 (Oklahoma) would supply 267,000 head of feeder cattle to Region 5 (Illinois). Because of their price disadvantage of being so far from the demand area, Region 21 (Texas) would have a surplus of 658,000 head and Region 10 (Montana) would absorb the remainder of the surplus of 84,000 head. Undoubtedly some of these would be sold as grass fat steers.

The numbers in light type indicates the indirect costs for activities not included in the final basic solution and refer to the opportunity cost of not including a particular activity (excess demand and excess supply combination) in the optimum solution. From the theory of the simplex method, an optimum cannot exist if the direct minus the indirect cost is less than zero. Thus, any change in the flow pattern as described in Table III would increase the total transport costs. For example, Region 11 (Missouri) could ship to Region 5 (Illinois) at only a 0.05 cents loss because of its advantage to other markets or alternatively by increasing the total transport costs by 0.05 cents per one hundred pounds of feeder cattle shipped. Alternatively, it could be inferred that the transport cost between Region 11 and Region 5 would have to decrease at least 0.05 cents per one hundred pounds, before any product would be shipped that direction.

Given the optimum flow solution the duality theorem may be employed to obtain a unique set of U_i and V_j once a base region has been selected. The resulting U_i and V_j obtained from the optimum solution contained two

types of useful economic information. First, the U_i measured the comparative locational advantage of the supply points relative to Region 11 (Missouri), the base region. For example, feeder cattle were priced at 0.13 cents per one hundred pounds more at Region 11 than they were at Region 14 (Nebraska). Alternatively, feeder cattle were priced at 0.31 cents per one hundred pounds less at Region 11 than at Region 13 (Nevada). Perhaps this can be explained in that Region 13 is very close to a large deficit area, Region 3 (California). Secondly, the values of the V_j denote the delivered price differentials in relation to the base region for the deficit regions. For example, the price of feeder cattle were priced at 1.09 cents per one hundred pounds higher in Region 3 than they were in the base region.

Since there is a movement of feeder cattle into Region 14 (Nebraska), a region which is a surplus producer, two sets of price differentials relative to the base region were obtained. As an exporter the price is 0.13 cents per one hundred pounds lower relative to the base region. As an importer the price is 0.47 cents per one hundred pounds higher than the price in the base region. This is one example of the inefficiency resulting from cross hauling. The cause of the different price differentials may be due to importers and exporters marketing at different times, marketing different weight animals, or the non homogeneity of the product (one region demands cattle from a particular region). Alternative price differentials were similarly obtained for Region 20 (South Dakota).

2. Model II (1956)

As in Model I, regions for which imports were available were classified as deficit regions. However on the supply side in this model an effort was made to take into account the relative low ratio of calves to cows on farms January 1, 1956, in the range states. In Model I, the

number of calves and steers on farms January 1, were used as the supply pool from which feeder cattle could be purchased. A closer investigation of the data revealed that on January 1, some of the range states could account for only a small percentage of the number of calves actually produced in that state. For example, in Region 16 (New Mexico) there were ten cows on farms for every four calves on farms January 1, 1956. Since Model I did not consider the number of calves that were shipped before January 1, an alternative model was specified. In this model a region was denoted a surplus region if:

$$X'_{1i} - X_{2i} - X_{3i} - X_{5i} > 0, \quad (5.4)$$

and a deficit region if:

$$X'_{1i} - X_{2i} - X_{3i} - X_{5i} < 0. \quad (5.5)$$

Where X'_{1i} is 83 percent of the beef cows on farms January 1, time t , in the i^{th} region, plus the steers on farms January 1, t , minus the number of steers and calves on feed January 1, t ; ⁵³ and X_{5i} is the number of steers not shipped to the feed lots from the i^{th} region in time t .

Nebraska (Region 14) was included only as an importer and specified deficit by the amount of their imports for 1956. Region 20 (South Dakota) was specified as both an exporter and an importer. The availabilities of Region 20 were computed from the following relationship:

$$X'_{1,20} - X_{2,20} - X_{3,20} - X_{5,20} + X_{4,20} = \text{availabilities} \quad (5.6)$$

Given the excess demands and supplies the optimum flow solution for Model II (1956) is presented in Table IV. For this flow solution 6,535,000 head of feeder cattle were available for shipment and since the demands

⁵³ 83 percent was used because the total number of beef calves on farms January 1, 1956, was 83 percent of the number of beef cows on farms in the 25 regions.

TABLE IV

SURPLUS AND DEFICITS, OPTIMUM FLOWS AND ESTIMATED REGIONAL PRICE DIFFERENTIALS FOR
FEEDER CATTLE IN THE UNITED STATES, MODEL II (1956)
(shipments in thousands)

Destinations	Origins													Totals	V _i
	11	15	20	8	17	21	10	25	16	22	13	23	18		
19	.15	.03	.01	.16	.08	<u>212</u>	.37	.64	.84	1.08	1.96	1.46	1.61	212	.76
4	.14	.04	.04	.09	.02	<u>230</u>	.39	.59	.79	1.04	1.92	1.43	1.66	230	.63
5	.02	.16	.16	<u>498</u>	<u>766</u>	<u>40</u>	.37	.49	.70	1.04	1.47	1.38	1.61	1,304	.51
12	.27	<u>1</u>	<u>118</u>	.19	.16	.14	.38	.60	.86	1.12	2.00	1.49	1.74	119	.64
24	.43	.01	<u>45</u>	.39	.38	.36	.40	.69	1.04	1.34	2.23	1.46	1.65	45	.34
9	.64	.02	<u>458</u>	.60	.60	.57	.42	.85	1.27	1.45	2.34	1.52	1.71	458	.09
7	.06	<u>33</u>	.09	.06	.07	<u>1,837</u>	.28	.40	.77	.87	1.77	1.38	1.51	1,870	.37
20	1.14	<u>140</u>	4.68	1.09	1.07	1.01	.35	.93	1.72	1.07	2.00	1.48	1.68	140	-.12
14	<u>21</u>	.09	.11	.07	.03	<u>326</u>	<u>47</u>	<u>249</u>	.48	.48	1.41	1.12	1.32	643	.47
6	1.52	.45	.68	1.16	1.17	1.02	<u>66</u>	.33	1.23	.36	1.30	.49	.47	66	.15
2	.67	.38	.73	.36	.39	.25	.08	<u>23</u>	.33	.46	1.26	1.21	1.21	23	.22
1	1.05	.81	.91	.65	.46	.27	.65	.53	<u>147</u>	.66	.83	1.11	1.11	147	.41
3	.84	.44	.42	.38	.21	.10	<u>440</u>	.05	<u>234</u>	<u>35</u>	<u>141</u>	<u>76</u>	<u>250</u>	1,176	1.04
Surplus	.61	.04	.17	.36	.23	.03	<u>102</u>	.27	.43	.47	.87	.34	.49	102	-.61
Totals	21	174	621	498	766	2,645	655	272	381	35	141	76	250	6,535	
U _i	0	-.57	-.44	-.25	-.38	-.58	-.61	-.34	-.18	-.14	.26	-.27	-.12		

Total Shipments: 6,433,000 head

Total Cost: \$43,325,735.00

were only 6,433,000 head of feeder cattle there was a surplus of 102,000 head. Region 10 (Montana) accounted for all the surplus under the optimum flow solution of Model II (1956). The total transport cost of satisfying all demands was \$43,325,745.00.

Region 20 (South Dakota) was included as both an exporter and an importer and the price differentials are: the importers pay a price of 0.12 cents less per one hundred pounds for feeder cattle relative to the base region (Missouri), and the exporters received a price of 0.44 cents per one hundred pounds less relative to the base region.

Comparing the solution of this model with the optimum flow pattern for Model I (1956) reveals that Region 2 (Colorado) and Region 6 (Idaho) were deficit regions, whereas in Model I they were surplus regions. The estimated price differentials are reasonably consistent with regard to sign and magnitude with those estimated for Model I. Some of the deviation between the models can be explained in that Model I indicated the total possibilities for shipments from the surplus regions, and does not take into account the number of steers that were not shipped to the feed lots during the year t , and the number of calves shipped before January 1.

3. Model III (1956)

Model III (1956) is an extension of Model I (1956). The two models differ in that the number of steers not shipped to the cattle feeding firms during the year t , are taken into account in Model III. This specification results in Model III being a deficit model (demands are greater than supplies). The optimum flow solution is presented in Appendix D, Table VII. In the optimum solution 4,670,000 head of feeder cattle were shipped for a total transport cost of \$26,585,986.00. In the optimum solution, Ohio (Region 19), Indiana (Region 4), Michigan (Region 12), Arizona (Region 1) and California (Region 3) could not completely satisfy

their demands. Undoubtedly some of the deficit can be attributed to the lack of data pertaining to the shipments of feeder cattle from the southern states and shipments within the corn belt states. Colorado (Region 2) and Utah (Region 22) are deficit regions whereas in Model I (1956) they were surplus regions. The estimated regional price differentials are consistent with these estimated for Model I (1956).

4. Model IV (1956)

Alternatively, Model IV (1956) compares with Model II (1956), except that Nebraska (Region 14) is both an exporter and importer. The supplies and demands were estimated by the relationship presented in equations (5.4) and (5.5). Nebraska (Region 14) availabilities were computed from equation (5.6) with Region 20 replaced by Region 14.

The optimum solution for Model IV (1956) is presented in Appendix D, Table VIII. In the optimum solution, 6,433,000 head of feeder cattle were transported from the surplus regions to the deficit regions at a total transport cost of \$42,221,592.00. The surplus of 532,000 head is absorbed by Regions 21 (Texas) and 10 (Montana). The estimated regional price differentials are consistent with regard to sign and magnitude with the estimates obtained for Model II (1956).

B. Spatial Analysis for 1951

1. Model I (1951)

Having investigated alternative equilibrium solutions for one period of time, it should prove useful to consider an alternative formulation which involves changes in the geographical distribution and level of production and demand. Therefore, a spatial analysis was carried out for the year 1951. Since data were not available for numbers of cattle fed annually previous to 1955, the number of cattle on feed January 1, $t + 1$, was used as an indicator of the number of cattle fed in t . Defining this

variable as X_{3i}^i the surplus and deficit regions can be determined from the relationship described in equations (5.1) and (5.2) with X_{3i}^i substituted for X_{3i} . Algebraically this relationship can be stated as:

$$X_{1i} - X_{2i} - X_{3i}^i > 0 \text{ then the } i^{\text{th}} \text{ region is a surplus region, (5.7)}$$

and if

$$X_{1i} - X_{2i} - X_{3i}^i < 0 \text{ then the } i^{\text{th}} \text{ region is a deficit region. (5.8)}$$

Regions for which import data were available were specified deficit by an amount equal to the number of cattle imported. The relations specified by equations (5.7) and (5.8) result in a large surplus of feeder cattle. Part of this large surplus may be the result of under estimating the number of cattle fed annually. The demands for states for which import data were not available were computed by the method used in Model I (1956).

Given the excess supplies and demands the equilibrium solution for Model I (1951) is presented in Table V. Model I (1951) and Model I (1956) differ in that Region 1 (Arizona) was a surplus region in Model I (1956) and is a deficit region in Model I (1951).

Estimates of the regional price differentials associated with the optimum for Model I (1951) are reasonably consistent with the estimates of the price differentials associated with the optimum for Model I (1956). Within the optimum flow pattern a peculiar situation occurred in that Region 16 (New Mexico) does not have a price advantage or disadvantage relative to the base region. This equal advantage can be in part explained in that surrounding regions are deficit.

There were 12 percent fewer feeder cattle shipped in Model I (1951) relative to Model I (1956) and a \$12,315,232.00 smaller total transport cost. Region 21 (Texas) absorbed 88 percent of the surplus that existed in Model I (1951).

TABLE V

SURPLUS AND DEFICITS, OPTIMUM FLOWS AND ESTIMATED REGIONAL PRICE DIFFERENTIALS FOR
FEEDER CATTLE IN THE UNITED STATES, MODEL I (1951)
(shipments in thousands)

Destinations	Origins																Totals	V _i	
	11	15	20	14	8	17	21	10	6	25	2	16	1	22	13	23			18
19	.13	.54	<u>79</u>	.23	.07	.03	.13	.53	1.27	.75	.61	1.00	1.69	1.24	2.12	1.62	1.77	79	.78
4	.15	.58	.06	.27	.03	<u>148</u>	.16	.58	1.26	.73	.58	.98	1.59	1.23	2.11	1.62	1.85	148	.62
5	.09	.76	.24	.30	<u>762</u>	.04	.22	.62	1.35	.69	.43	.95	1.66	1.29	1.72	1.63	1.86	762	.44
12	.26	.52	<u>61</u>	.25	.11	.12	.28	.55	1.32	.72	.60	1.03	1.71	1.29	2.17	1.66	1.91	61	.65
24	.42	.53	<u>31</u>	.35	.31	.34	.50	.57	1.35	.81	.75	1.21	1.81	1.51	2.40	1.63	1.82	31	.35
9	.63	.54	<u>232</u>	.48	.52	.56	.71	.59	1.33	.97	.86	1.44	2.18	1.62	2.51	1.69	1.88	232	.70
7	.07	.54	.11	<u>1,300</u>	<u>52</u>	.05	.16	.47	1.15	.54	.41	.96	1.59	1.06	1.96	1.57	1.70	1,352	.36
20	.61	<u>145</u>	4.16	.45	.49	.51	.63	<u>23</u>	.73	.53	.82	1.37	1.65	.72	1.65	1.13	1.33	168	.41
14	<u>510</u>	.62	.12	4.37	<u>38</u>	<u>131</u>	.15	.18	.75	.13	<u>11</u>	.66	1.16	.66	1.59	1.30	1.50	690	.47
3	.66	.79	.25	.52	.13	<u>74</u>	.07	<u>132</u>	<u>102</u>	<u>221</u>	.24	<u>199</u>	<u>43</u>	<u>27</u>	<u>68</u>	<u>86</u>	<u>123</u>	1,075	1.22
Surplus	.43	.39	<u>27</u>	.27	.11	.02	<u>1,521</u>	<u>151</u>	<u>30</u>	.22	.27	.43	.93	.47	.87	.34	.49	1,698	-.43
Totals	510	145	429	1,300	852	353	1,521	306	102	221	11	199	43	27	68	86	123	6,296	
U _i	0	-.04	-.43	-.16	-.32	-.41	-.43	-.43	-.13	-.21	-.16	0	.30	.04	.44	-.09	.06		

Total Shipments: 4,598,000 head

Total Cost: \$25,495,153.00

2. Model II (1951)

Since Model I (1951) did not take into account the number of calves that were shipped before January 1, an alternative model similar to Model II (1956) was postulated. The regions were classified as deficit or surplus from the results of the following relationships:

$$X''_{1i} - X_{2i} - X'_{3i} - X_{5i} > 0 \text{ then the } i^{\text{th}} \text{ region is surplus,} \quad (5.9)$$

and if

$$X''_{1i} - X_{2i} - X'_{3i} - X_{5i} < 0 \text{ then the } i^{\text{th}} \text{ region is deficit.} \quad (5.10)$$

Where X''_{1i} is 81 percent of the number of beef cows on farms January 1, 1951, plus the number of steers on farms January 1, 1951, not including the number of calves and steers on feed January 1, 1951.⁵⁴ All other variables have been previously defined. Regions for which imports were available were classified as deficit. Regions 20 (South Dakota) and 14 (Nebraska) were included as both an exporter and an importer. Their availabilities were computed in a similar fashion to Model II (1956).

Given the surplus and deficit regions and the transport cost, the equilibrium solution for Model II (1951) is presented in Table VI. The total cost of satisfying all the demands was \$29,684,494.00. The cattle feeding firms demands were 4,618,000 head of feeder cattle and numbers available for supplying were 5,239,000 head, thus leaving a surplus of 621,000 which was held by Region 21 (Texas) and Region 10 (Montana).

The optimum solution for Model II (1951) differs from Model I (1951) in that Region 2 (Colorado) has changed from a surplus to a deficit region. Most of the estimates of the regional price differentials are consistent with regard to sign of the estimates for Model I (1951). In the equilibrium

⁵⁴ 81 percent was used because the total number of calves on farms January 1, 1951, was 81 percent of the number of beef cows on farms in the 25 regions.

TABLE VI

SURPLUS AND DEFICITS, OPTIMUM FLOWS AND ESTIMATED REGIONAL PRICE DIFFERENTIALS FOR
FEEDER CATTLE IN THE UNITED STATES, MODEL II (1951)
(shipments in thousands)

Destinations	Origins																Totals	V _i
	11	15	20	14	8	17	21	10	6	25	16	1	22	13	23	18		
19	.18	.44	<u>78</u>	.29	.19	.11	.03	.43	1.17	.67	.90	1.59	1.14	2.02	1.52	1.67	78	.73
4	.14	.42	<u>32</u>	.27	.09	.02	<u>116</u>	.42	1.10	.59	.82	1.43	1.07	1.95	1.46	1.69	148	.63
5	.02	.54	.12	.24	<u>67</u>	<u>359</u>	<u>336</u>	.40	1.13	.49	.73	1.44	1.07	1.50	1.41	1.64	762	.51
12	.31	.42	<u>61</u>	.31	.23	.20	.18	.45	1.22	.64	.93	1.61	1.19	2.07	1.56	1.81	61	.60
24	.47	.43	<u>31</u>	.41	.43	.42	.40	.47	1.25	.73	1.11	1.71	1.41	2.30	1.53	1.72	31	.30
9	.68	.44	<u>232</u>	.54	.64	.64	.61	.49	1.23	.84	1.34	2.08	1.52	2.41	1.59	1.78	232	.05
7	.06	.38	.05	<u>596</u>	.06	.07	<u>756</u>	.31	.99	.40	.80	1.43	.90	1.80	1.41	1.54	1,352	.37
20	.76	<u>101</u>	4.26	.61	.71	.69	.63	.67	.73	.55	1.37	1.65	.72	1.65	1.13	1.33	168	.26
14	<u>153</u>	.47	.07	4.38	.07	.03	<u>294</u>	.03	.60	<u>243</u>	.51	1.01	.51	1.44	1.15	1.35	690	.47
2	.67	.76	.69	.26	.36	.39	.25	.11	.64	<u>21</u>	.36	.90	.49	1.29	1.24	1.24	21	.22
3	.81	.79	.35	.68	.35	.18	.07	<u>225</u>	<u>6</u>	.02	<u>353</u>	<u>134</u>	<u>54</u>	<u>136</u>	<u>28</u>	<u>139</u>	1,075	1.07
Surplus	.58	.39	.10	.43	.33	.20	<u>498</u>	<u>123</u>	.30	.24	.43	.73	.47	.87	.34	.49	621	-.58
Totals	153	101	434	596	67	359	2,000	415	6	264	353	134	54	136	28	139	5,239	
U _i	0	-.19	-.48	-.15	-.25	-.38	-.58	-.58	-.28	-.34	-.15	.15	-.11	.29	-.24	-.09		

Total Shipments: 4,618,000 head

Total Cost: \$29,684,494.00

solution of Model I (1951) Region 16 (New Mexico) has an equal price advantage relative to the base region. The results of Model II (1951) indicate that Region has a 0.15 cents per one hundred pounds price disadvantage relative to the base region. For Model II (1951) there were 20,000 more feeder cattle transported relative to Model I (1951), but the increase in transport cost was 4,189,431.00. The disproportionate increase in the transport costs can partially be explained in the relocation of calves, by using 81 percent of the beef cows on farm as the number of calves available for supply. This specification resulted in the western states having more feeder cattle available for shipping and the midwestern states, such as Kansas and Nebraska, have fewer available for export. Thus, the states near the deficit area have fewer to supply, and therefore the transport cost was disproportionately larger since the cattle were estimated as being shipped over longer distances.

Comparing Model II (1951) with a similar model for 1956 (Model II) reveals that Region 6 (Idaho) and Region 1 (Arizona) have changed from surplus to deficit. In most instances the estimated price differentials are comparable with regard to sign and magnitude with the estimated differentials for 1956. There were 28 percent more feeder cattle shipped under Model II (1956), and the costs were larger by \$13,641,240.00.

3. Model III (1951)

The surplus and deficit regions for Model III (1951) were classified as such by subtracting from the availabilities of Model I (1951) the number of steers not shipped to the feed lots for time t , (X_{5i}). Regions for which imports were available were classified as deficit regions. Given the deficit and surplus regions the equilibrium solution for Model III (1951) is presented in Appendix D, Table IX. The above specification results in a deficit model. The regions in which demands were not satisfied are:

Ohio (Region 19), Indiana (Region 4), Michigan (Region 12) and California (Region 3). There were 3,889,000 head of feeder cattle demanded, but there were only 2,888,000 head available for shipping. The total transport cost was \$17,653,776.00. The deficit supply situation may be due to the lack of information concerning shipments within corn belt states, shipments from the southern states to the corn belt states, or lack of information pertaining to the shipments of cattle prior to January 1.

This model is similar to Model III (1956) in that the deficit and surplus regions were classified by a similar method. The two models differ in that Region 6 (Idaho) is a deficit rather than a surplus region. The estimated price differentials are consistent with those estimated for Model III (1956).

4. Model IV (1951)

In an effort to account for underestimating the number of cattle fed annually, an alternative model was estimated. Since Model II (1951) is believed to be a reasonable approximation of reality, Model IV (1951) was estimated with supplies equal to demands. For regions for which imports were not available, the number of cattle fed during the year was estimated from the following relationship:

$$X'_{3i} + \frac{X'_{3i}}{p} \left(\frac{\sum_{i=1}^p X'_{3i}}{\sum_{i=1}^p X'_{3i}} \right) \left(\text{surplus for Model II (1951)} \right) \quad (5.11)$$

where p is the number of regions for which imports of feeder cattle were not available. Using this relationship to represent the number of cattle fed annually, the deficit and surplus regions were determined from equations (5.9) and (5.10). This model differs from Model II (1956) in that Regions 8 (Kansas) and 6 (Idaho) changed from surplus to deficit.

The equilibrium solution is presented in Appendix D, Table X. The

signs and magnitude of the estimated price differentials are consistent with the estimates of Model II (1951). There was an increase of \$1,883,049.00 in the transport cost of Model IV (1951) relative to Model II (1951).

C. Discussion of the Results of the Spatial Models

In the models estimated that were surplus (supplies greater than demands), Regions 10 (Montana) and 21 (Texas) absorbed the surplus in all instances. When deficit models occurred Regions 19 (Ohio), 4 (Indiana), 12 (Michigan), 1 (Arizona) and 3 (California) were deficit regions. The estimated regional price differentials of one region relative to the base region were reasonably consistent under alternative specifications. For example, the estimated price differential of region 17 (Oklahoma) relative to the base region ranged from 0.41 to 0.36 cents per one hundred pounds less for feeder cattle. There was also consistency with regard to the direction of shipments. For example, Region 17 (Oklahoma) shipped part of its surplus to Region 5 (Illinois) in seven of the eight models estimated. Likewise Region 21 (Texas) shipped part of its surplus to Region 7 (Iowa) in seven of the eight models estimated. Since data were not available on regional exports, there was no basis for assessing the accuracy of the results or the inefficiency of the actual transportation flow pattern.

CHAPTER VI

IMPLICATIONS OF THE RESULTS

Knowledge of the structure of a particular segment of the economy is useful if it aids in making the "best" decisions. Within this normative framework, knowledge of structural parameters and spatial flows, such as those estimated in this study provide one basis for depicting in advance the probable impact of various economic variables, given certain goals. The implications of the results of the behavior relations and spatial models, as they are related to decision making at the various structural levels will be presented in this section.

A. Behavior Relations

Knowledge of the interdependent nature of the sector under study along with the connecting parameter estimates is a necessary prerequisite for intelligent decision making. The accuracy and completeness of this information is of course conditioned by the decision or decisions being considered.

Thus by formulating, identifying and estimating structural equations the effect of certain decisions may be estimated and the uncertainty as to the consequences of these actions reduced.

1. Implications for the firm

Having knowledge of the future path of such factors as price of feeder cattle, price of slaughter steers, price of hogs, production of corn and hay, the cattle feeding firm could adjust production plans to meet the task of allocating resources in a manner more nearly compatible with profit and efficiency objectives. Knowledge of the demand relation should also be

useful to the producer of feeder cattle in his planning for future production. Likewise a knowledge of the supply relationship for feeder cattle should prove useful to the cattle feeding firm in the planning firm's operations. Whether the estimated structural parameter are sufficiently accurate for forecasting remains to be proven.

2. Implication for decision making by policy planners

Perhaps the area of economic policy offers the greatest possibilities for econometric analysis. In the making of decisions the policy maker should have a knowledge of the interdependent nature of the sector under study in order to properly evaluate proposed policies before they are put into operation. This is not meant to imply that all possible policy questions concerning the feeder cattle sector can be answered as a result of this study. However, these estimated relations should provide an objective means of analyzing such questions as what effect would a change in the feeder cattle pricing policy have on (1) demand for feeder cattle, thus influencing the production of beef, and (2) the demand for feeder hogs which would influence the supply of pork. Since the production of corn and hay are thought to be important variables influencing the demand for feeder cattle, a change in the policy regarding production of these factors could be viewed as it would affect the production of beef. Implications of other alternative policies could be analyzed, but in all cases the results are conditioned by the assumptions underlying each model.

B. Spatial Model

The solution of spatial models and their dual may be used to obtain insights into many of the theoretical problems involving the efficiency and competitive structure of individual sectors of the economy. In addition knowledge may be obtained relative to problems of industrial structure and

comparative statics when the consequences of change or action are desired.⁵⁵

1. Implications of results to the firm

In regard to the feeder cattle firm these analyses should suggest how changes in transport costs, geographical distribution and level of production and demands might affect regional feeder cattle prices and movements.

The set of regional price differentials corresponding to the optimum flow pattern may be used in determining the optimum geographical location of the feeder cattle producing and feeding firms.

2. Implications for decision making by policy planners

The perfect market concept used in formulating the spatial equilibrium model provides a standard of comparison whereby the pricing and the distribution of a product or a factor can be judged as efficient or inefficient relative to some base. By using the spatial model the policy maker can postulate a set of conditions and ascertain the effects when certain variables are allowed to deviate from the initial conditions. In particular, the model provides information basic to determining the consequences under changes in (1) transport rates and (2) geographical distribution of product supply on the levels and direction of geographical flows and prices. Other related questions can be answered such as: how would the optimum solution change if transport cost between each pair of regions increased or decreased by a given amount? The normative model could be very useful in policy making in that it depicts what could happen under given ends and assumptions.

⁵⁵ Judge and Wallace, "Econometric Analysis of the Beef Sector of the Economy," *ibid.*

CHAPTER VII

SUMMARY

The primary objectives of this study were: (1) to evaluate those factors which appear to have the most influence on the fluctuations on demand and supply of feeder cattle and (2) to estimate a spatial equilibrium model for the feeder cattle sector of the economy.

A sector model was postulated to reflect the behavior of the feeder cattle firms. Secondary data were selected to reflect the variables of the models and alternative techniques of estimation were employed to estimate the relevant parameters. Statistical and nonstatistical tests were performed to assess the validity of the estimated parameters of the behavior relations.

Alternatively, spatial models were postulated to depict the optimum geographical flows and regional price differentials of the feeder cattle sector. Since data were not available, regional demands were computed from basic data. Then, given the deficit and surplus regions the general method of linear programming was employed to derive the optimum flow solution (minimum transport cost).

Finally, the implications of the results for decision making and action by the government and firms were reviewed. The reliability of the results and conclusions depend upon the assumptions of the models, the data, and the method of estimation employed.

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APPENDIX A

DESCRIPTION OF DATA

A. Description of data relating to the sector model

Y_{1t} = Log of the (1947-49) index of the number of cattle and calves on fed January, 1 for thirteen or eight states; or the number of stockers and feeders received in the eight corn belt states. In each instance one of these three alternatives will be used depending on which one is chosen as the reflector of demand.

Y_{2t} = Log of the (1947-49) index of the July through December average price of stockers and feeders at Kansas City.⁵⁶

Y'_{2t} = Same as Y_{2t} except is the August through December average price instead of the July through December average.

Y''_{2t} = Same as Y_{2t} except the variable reflects the July to July average price of feeder cattle.

Y_{3t} = Log of the (1947-49) index of the yearly average price of choice grade steers sold out first hand for slaughter at Chicago.

Y'_{3t} = Same as Y_{3t} except the variable reflects a February through July average price.

Y''_{3t} = Same as Y_{3t} except the variable reflects a July to July average price.

Y_{4t} = Log of the (1947-49) index of the yearly average price of medium weight barrows and gilts at Chicago.

⁵⁶ All prices and income variables were deflated by the index of the prices received by farmers for all products (1910-14 = 100) as published by the Agricultural Marketing Service (unless stated otherwise).

Y_{4t}^0 = Same as Y_{4t} except the variable reflects the October through July average price.

Y_{4t}'' = Same as Y_{4t} except the variable reflects the July to July average price.

Y_{11t} = Log of the (1947-49) index of the August through December average price of No. 3 yellow corn at Chicago.

Y_{12t} = Log of the (1947-49) index of the yearly average price of all weights and grades of beef steers sold out first hand for slaughter at Chicago.

Y_{13t} = Log of the (1947-49) index of the August 15th price of alfalfa hay for eight corn belt states.

$$Y_{14t} = Y_{3t} - Y_{2t}.$$

Y_{15} = Log of the (1947-49) index of the yearly average price of all grades and weights of feeder sheep at Omaha.

Z_{1t} = Time, Linear, Decimal.

Z_{1t}' = Time, Linear, in Logs.

Z_{2t} = Log of the (1947-49) index of the October 1 estimate of production plus stocks of corn for thirteen or eight states depending on the specification of the Y_{1t} variable.

Z_{3t} = Log of the (1947-49) index of the September 1 estimate of production of alfalfa hay for thirteen states or eight states depending on the specification of the Y_{1t} variable.

Z_{4t} = Log of the (1947-49) index of the number of calves less than one year old on farms January 1 and the number of steers greater than one year old on farms January 1, for eleven western states.

Z_{4t}^0 = Same as Z_{4t} except it relates to fourteen western states.

Z_{5t} = Log of the (1947-49) index of the July through September average condition of the ranges in the Western States.

$$Z_{6t} = Y_{2(t-1)}$$

Z_{7t} = Log of the (1947-49) index of the October 1 estimate of production plus stocks of corn on farms in the United States.

Z_{8t} = Log of the (1947-49) index of the number of beef animals on farms January 1, in the United States.

Z'_{8t} = Log of the (1947-49) index of the number of beef cows on farms January 1, in thirteen or eight states depending on the specification of the variable Y_{1t} .

Z''_{8t} = Log of the (1947-49) index of the number of beef animals on farms January 1, in thirteen or eight states depending on the specification of the variable Y_{1t} .

Z_{9t} = Log of the (1947-49) index of the yearly total personal disposable income for the United States.

Z_{10t} = Log of the (1947-49) index of the yearly population of the United States (including members of the armed forces).

Z_{11t} = Log of the (1947-49) index of the total number of sows and gilts greater than six months old on farm January 1, in the United States.

Z_{12t} = Log of the (1947-49) index of the number of cattle on feed in the range states January 1, not including those states used as reflectors of demand.

Z_{14t} = Log of the (1947-49) index of the September 1 estimate of production of all hay, in fourteen range states.

$$Z_{15t} = Z'_{4t} - Z_{12t}$$

Z_{16t} = The (1947-49) index of the estimated past profits of the cattle feeders in the corn belt states. This profit is the difference between cost of inputs and revenue received from outputs. It does not include payment to labor, etc.

B. Description of data relating to the spatial models

X_{1i} = Number of calves and steers on farms January 1, time period t , minus the number of calves and steers on feed January 1, time t , (when data were not available the number of calves and steers was postulated to be 80 percent of the number of cattle of feed January 1, time t).

X'_{1i} = 83 percent of the number of cows on farms January 1, time t , plus the number of steers on feed January 1, time t , minus the number of calves and steers on feed January 1, time t , (if data were not available the number of calves and steers on feed January 1 was computed as described in X_{1i}).

X''_{1i} = Same as X'_{1i} except 81 percent was used instead of 83 percent of the cows as reflectors of the number of calves born in each state.

X_{2i} = The number of heifers on farms January 1, time $(t + 1)$ minus the number of heifers on feed January 1, time $(t + 1)$, was used to reflect the number of heifers kept for replacement for time t .

X_{3i} = The number of cattle fed annually for time t .

X^0_{3i} = The number of cattle on feed January 1 time $(t + 1)$. This variable was used as an indicator of X_{3i} when data were not available.

X_{4i} = The imports of feeder cattle during the year t .

X_{5i} = The number of steers on farms January 1, time $(t + 1)$ minus the number of steers on feed January 1, time $(t + 1)$, (when data were not available, 60 percent of the total number of cattle on feed January 1, time $(t + 1)$ was used as an indicator of the number of steers on feed January 1, time $[t + 1]$).

APPENDIX B

SOURCE OF DATA DESCRIBED IN APPENDIX A

A. Source of Data for Sector Models

Y_{1t} : "Livestock Market News Statistics and Related Data," Livestock Division, AMS, USDA, Statistical Bulletins, CS-24, 1946, pages 7 and 25, CS-38, 1948, pages 6 and 21, No. 118, 1951, pages 6 and 13 and No. 209, 1956, pages 8 and 18.

Y_{2t} : "Livestock and Meat Situation," Livestock Division, AMS, USDA, various issues published February, 1946, page 32, March, 1957, page 28 and July, 1958, page 11.

Y'_{2t} : Same source as Y_{2t} .

Y''_{2t} : Same source as Y_{2t} .

Y_{3t} : Same source as Y_{2t} .

Y'_{3t} : Same source as Y_{2t} .

Y''_{3t} : Same source as Y_{2t} .

Y_{4t} : "Price of Hogs and Hog Products," AMS, USDA, Statistical Bulletin No. 205, March 1957, page 7.

Y'_{4t} : Same source as Y_{4t} .

Y''_{4t} : Same source as Y_{4t} .

Y_{11t} : "Grain and Feed Statistics Through 1956," Agricultural Economics Division, AMS, USDA, Statistical Bulletin No. 159, page 38; and a supplement to Statistical Bulletin No. 159, 1958, page 18.

Y_{12t} : Same source as Y_{1t} , Statistical Bulletin No. 208, 1956, page 38.

Y_{13t} : "Crops and Markets," AMS, USDA, issues published from 1930 to

1957 were used to obtain this series.

Y_{14t} ; See Y_{3t} and Y_{2t} .

Y_{15t} ; Same source as Y_{2t} , March, 1949, March, 1956, page 31 and March, 1958, page 28.

Z_{1t} ; No source of publication.

Z_{1t}^1 ; No. source of publication.

Z_{2t} ; Same source as Y_{13t} .

Z_{3t} ; Same source as Y_{13t} .

Z_{4t} ; "Livestock and Poultry Inventory, January 1," Crop Reporting Board, AMS, USDA, Statistical Bulletins, No. 88-1950 and No. 117-1956; also "Livestock and Poultry on Farms and Ranches, January 1," published by the same source, February, 1957, and February, 1958.

Z_{4t}^1 ; Same source as Z_{4t} .

Z_{5t} ; Same source as Y_{11t} , page 89.

Z_{6t} ; See Y_{2t} .

Z_{7t} ; Same source as Y_{11t} , pages 7 and 25 and pages 5 and 11 in the supplement.

Z_{8t} ; Same source as Z_{4t} .

Z_{8t}^1 ; Same source as Z_{4t} .

Z_{8t}^2 ; Same source as Z_{4t} .

Z_{9t} ; "Personal Income by States," Office of Business Economics, United State Department of Commerce, a supplement to the "Survey of Current Business, page 146; also "The Survey of Current Business," published by the same source, June 1958, page 1.

Z_{10t} ; Same source as Z_{9t} , page 144 in the "Personal Income by States", and page 11 in the "Survey of Current Business".

Z_{11t} ; Same source as Z_{4t} .

Z_{12t} ; Same source as Y_{1t} .

Z_{14t} ; Same source as Y_{13t} .

Z_{15t} ; See Z_{4t}' and Z_{12} .

Z_{16t} ; "The Livestock and Meat Situation," August, 1957, Table 4, page 11, was used as a guide for computing this variable.

B. Source of Data for Spatial Models

X_{1i} ; Same source as Y_{1t} , Statistical Bulletins No. 209, 1956, page 8, and "Number of Cattle on Feed by Classes," Crop Reporting Board, AMS, USDA, AMS-147, 1956 and 1958 supplement to this bulletin.

X_{1i}' ; Same source as Z_{4t}' , Statistical Bulletin No. 117 and supplement published February, 1957; also same source as X_{1i} .

X_{1i}'' ; Same source as X_{1t} , Statistical Bulletin No. 127, 1952, page 6, and same source as Z_{4t}' , Statistical Bulletin No. 117.

X_{2i} ; Same source as X_{1i}' .

X_{3i} ; Same source as X_{1i} .

X_{3i}' ; Same source as X_{1i} .

X_{4i} ; Same source as Y_{1t} , Statistical Bulletin No. 209, 1956, page 18.

X_{5i} ; Same source as X_{1i}' .

APPENDIX C

EMPIRICAL RESULTS OF EQUATIONS RELATING TO THE SECTOR MODEL

I. Equations Postulated to Reflect the Firm Demand for Feeder Cattle for Thirteen States

A. Just Identified Estimates

The necessary condition that a just identified relationship exist is that the number of exogenous variables appearing in the system, not in the equation to be estimated, be one less than the number of endogenous variables appearing in the structural equation to be estimated. The equations will be presented and the exogenous variables considered that appeared in the system but not in the equation will be listed.

$$1. \dot{Y}_1 = -.1144Y_2 - .0054Y_3 - .0502Y_4 + .0486Z_1 + .3731Z_2 + .1974Z_3 \quad (C.1)$$

Y_{3t} and Y_{4t} were considered as exogenous variables and Z_{4t} was the exogenous variable considered that did not appear in the equation, but did appear in the system.

$$2. \dot{Y}_1 = -.3143Y_2 + .2651Y_4 + .0528Z_1 + .4844Z_2 + .1632Z_3 \quad (C.2)$$

The exogenous variables outside the structural equation are Z_{4t} and Z_{9t} .

$$3. \dot{Y}_1 = .2682Y_2 - 1.2415Y_3 + .1864Y_4 + .0165Z_1 + .4886Z_2 + .3015Z_3 \quad (C.3)$$

Z_{4t} , Z_{9t} and Z_{11t} were the variables considered outside the structural equation.

$$4. \dot{Y}_1 = .4879Y_2 - .6377Y_3 - .5782Y_4 + .0238Z_1 + .2029Z_2 + .3199Z_3 \quad (C.4)$$

Z_{4t} , Z_{8t} and Z_{10t} were the variables considered outside the structural equation.

$$5. \dot{Y}_1 = -.7813Y_2 + .9955Y_3 + .3282Y_4 + .0819Z_1 + .4809Z_2 + .0523Z_3 \quad (C.5)$$

The exogenous variables not considered are Z_{4t} , Z_{8t} and Z_{9t} .

$$8. \dot{Y}_2 = \frac{-.2379Y_1}{(.352)^1} + \frac{.9302Y_3}{(.196)^3} + \frac{.0871Y_4}{(.147)^4} + \frac{.0448Z_1}{(.031)^1} - \frac{.3017Z_2}{(.147)^2} - \frac{.0309Z_3}{(.048)^3} - .1769 \quad (C.15)$$

$$R^2 = .778$$

Equation (C.15) was equation (4.5) with Y_2 chosen as the dependent variable.

C. Theil-Basman Method

$$1. \dot{Y}_1 = \frac{.0399Y_1}{(.411)^2} + \frac{.4883Y_{12}}{(.446)^{12}} - \frac{.1998Y_4}{(.065)^4} - \frac{.0972Y_5}{(.210)^5} + \frac{.1513Z_1}{(.403)^1} - \frac{.1020Z_{16}}{(.143)^{16}} + 1.5950 \quad (C.16)$$

$$s^2 = .0036$$

This equation is similar to equation (C.8).

II. Equations Postulated to Reflect the Demand of Feeder Cattle in Eight States

A. Just Identified Estimates

$$1. \dot{Y}_1 = -.2051Y_2 + .0355Y_4 + .0562Z_1 + .3980Z_2 + .2031Z_3 \quad (C.17)$$

Z_{4t} and Z_{9t} were the variables considered outside the structural equation.

$$2. \dot{Y}_1 = -.1202Y_2 - .0490Y_3 - .0654Y_4 + .0547Z_1 + .3674Z_2 + .2100Z_3 \quad (C.18)$$

Y_{3t} and Y_{4t} were considered as exogenous variables and Z_{4t} was the variables considered outside the structural equation.

$$3. \dot{Y}_1 = -.1825Y_2 + .0561Z_1 + .3866Z_2 + .2045Z_3 \quad (C.19)$$

Z_{4t} was the variable considered outside the structural equation.

B. Single Equation Estimates

$$1. \dot{Y}_1 = \frac{-.1687Y_2}{(.099)^2} + \frac{.0714Y_3}{(.148)^3} - \frac{.0879Y_4}{(.017)^4} + \frac{.0556Z_1}{(.032)^1} + \frac{.3693Z_2}{(.070)^2} + \frac{.2260Z_3}{(.070)^3} + 1.0159 \quad (C.20)$$

$$R^2 = .956$$

$$2. \dot{Y}_1 = \frac{-.2509Y_2}{(.153)^2} + \frac{.1289Y_3}{(.148)^3} - \frac{.0978Y_4}{(.067)^4} - \frac{.0761Z_1}{(.032)^1} + \frac{.3627Z_2}{(.070)^2} + \frac{.2250Z_3}{(.070)^3} - \frac{.0928Z_8}{(.131)^8} + 1.2329 \quad (C.21)$$

$$R^2 = .962$$

$$3. \dot{Y}_1 = \frac{-.1259Y_2}{(.075)^2} - \frac{.0756Y_3}{(.064)^4} + \frac{.0548Z_1}{(.013)^1} + \frac{.3720Z_2}{(.069)^2} + \frac{.2044Z_3}{(.064)^3} + 1.0816 \quad (C.22)$$

$$R^2 = .953$$

III. Demand Equations Reflected by Imports from July through December into Eight Corn Belt States (single equation estimates)

$$1. \dot{Y}_1 = \frac{.2356Y_2}{(.472)^2} - \frac{.0813Y_3}{(.351)^3} - \frac{.2895Y_4}{(.170)^4} + \frac{.0915Z_1}{(.204)^1} + \frac{.3930Z_2}{(.313)^2} + \frac{.1716Z_3}{(.158)^3}$$

$$+ \frac{.1531Z_8}{(.734)^8} + .6854 \quad (C.23)$$

$$R^2 = .897$$

$$2. \dot{Y}_1 = \frac{-.1040Y_2}{(.509)^2} + \frac{.3470Y_3}{(.504)^3} - \frac{.2927Y_4}{(.184)^4} + \frac{.1104Z_1}{(.204)^1} + \frac{.3020Z_2}{(.311)^2} + \frac{.1708Z_3}{(.157)^3}$$

$$+ \frac{.0238Z_8}{(.727)^8} + .9832 \quad (C.24)$$

$$R^2 = .896$$

$$3. \dot{Y}_1 = \frac{-.1558Y_2}{(.479)^2} + \frac{.5083Y_3}{(.161)^3} - \frac{.3190Y_4}{(.480)^4} + \frac{.0804Z_1}{(.190)^1} + \frac{.3544Z_2}{(.295)^2} + \frac{.1606Z_3}{(.149)^3}$$

$$+ \frac{.1425Z_8}{(.689)^8} + .5517 \quad (C.25)$$

$$R^2 = .908$$

IV. Demand Equations Reflected by Imports from July to July into Eight Corn Belt States (single equation estimates)

$$1. \dot{Y}_1 = \frac{-.0312Y_2}{(.512)^2} + \frac{.3316Y_3}{(.506)^3} - \frac{.2048Y_4}{(.185)^4} + \frac{.0357Z_1}{(.205)^1} + \frac{.3750Z_2}{(.313)^2} + \frac{.2163Z_3}{(.158)^3}$$

$$+ \frac{.2753Z_8}{(.731)^8} + .0935 \quad (C.26)$$

$$R^2 = .901$$

$$2. \dot{Y}_1 = \frac{-.5542Y_2}{(.596)^2} + \frac{1.3314Y_3}{(.612)^3} - \frac{.0838Y_4}{(.188)^4} - \frac{.1405Z_1}{(.188)^1} + \frac{.4325Z_2}{(.253)^2} + \frac{.3339Z_3}{(.128)^3}$$

$$+ \frac{.7221Z_8}{(.637)^8} + 1.9185 \quad (C.27)$$

$$R^2 = .932$$

V. Equations Postulated as Reflectors of the Supply of Feeder Cattle (single equation estimates)

$$1. \dot{Y}_1 = \frac{.2236Y_2}{(.127)^2} - \frac{.0276Y_3}{(.231)^3} - \frac{.0625Z_1}{(.267)^1} + \frac{.9223Z_4}{(.052)^4} + \frac{.6870Z_5}{(.219)^5} + \frac{.4370Z_{12}}{(.224)^{12}}$$

$$- 2.2547 \quad (C.28)$$

$$R^2 = .932$$

$$2. \dot{Y}_1 = \frac{.1033Y}{(.140)^2} - \frac{.0352Y}{(.142)^3} - \frac{.0352Z}{(.243)^1} + \frac{.3435Z^1}{(.142)^4} - \frac{.0167Z}{(.189)^5} + \frac{.7900Z}{(.108)^{12}} - .1695 \quad (C.29)$$

$$3. \dot{Y}_1 = \frac{.1409Y}{(.309)^2} + \frac{.0505Y}{(.215)^3} - \frac{.2868Y}{(.255)^{15}} + \frac{.0726Z}{(.030)^1} + \frac{.0089Z^1}{(.291)^4} + \frac{.4211Z}{(.166)^{14}} + 1.2003 \quad (C.30)$$

$$R^2 = .970$$

$$4. \dot{Y}_1 = \frac{.1600Y}{(.289)^2} + \frac{.0452Y}{(.213)^3} - \frac{.2930Y}{(.250)^{15}} + \frac{.0704Z}{(.028)^1} + \frac{.0338Z}{(.263)^4} + \frac{.4142Z}{(.158)^{14}} + .1563 \quad (C.31)$$

$$R^2 = .910$$

$$5. \dot{Y}_1 = \frac{.1960Y}{(.338)^2} + \frac{.1947Y}{(.250)^3} - \frac{.3477Y}{(.269)^{15}} + \frac{.0785Z}{(.032)^1} + \frac{.3653Z^1}{(.257)^4} + \frac{.6566Z}{(.325)^5} - .2070 \quad (C.32)$$

$$R^2 = .901$$

APPENDIX D

EMPIRICAL RESULTS OF ALTERNATIVE SPATIAL ANALYSES

TABLE VII

SURPLUS AND DEFICITS, OPTIMUM FLOWS AND ESTIMATED REGIONAL PRICE DIFFERENTIALS FOR
FEEDER CATTLE IN THE UNITED STATES, MODEL III (1956)
(shipments in thousands)

Destinations	Origins														Deficit	Totals	V _i
	11	15	20	14	8	17	21	10	25	16	13	23	18				
19															212	212	.56
4															230	230	.56
5	52				741	452	59									1,304	.53
12															120	120	.56
24		13							32							45	.48
9		156	302													458	.24
7				38				1,372	380	79						1,869	.39
6									18							18	-.11
2										105						105	-.16
1										17	108				160	285	.56
22									24							24	.05
3												35	108	156	877	1,176	.56
Totals	52	169	302	38	741	452	1,431	454	201	108	35	108	156	1,599	5,846		
U _i	0	-.44	-.29	-.13	-.23	-.36	-.56	-.87	-.72	-.03	-.22	-.75	-.60	.56			

Total Shipments: 5,846,000 head

Total Cost: \$26,585,986.00

TABLE VIII

SURPLUS AND DEFICITS, OPTIMUM FLOWS AND ESTIMATED REGIONAL PRICE DIFFERENTIALS FOR
FEEDER CATTLE IN THE UNITED STATES, MODEL IV (1956)
(shipments in thousands)

Destinations	Origins														Totals	V _i
	11	15	20	14	8	17	21	10	25	16	22	13	23	18		
19							212								212	.76
4							230								230	.63
5					498	766	40								1,304	.51
12		1	118												119	.64
24			45												45	.34
9			458												458	.09
7		33		430			1,407								1,870	.37
20		140													140	-.12
14	21						373		249						643	.47
6								66							66	.18
2									23						23	.22
1										147					147	.44
3								440		234	35	141	76	250	1,176	1.07
Surplus							383	149							532	-.58
Totals	21	174	621	430	498	766	2,645	655	272	381	35	141	76	250	6,965	
U _i	0	-.57	-.44	-.15	-.25	-.38	-.58	-.58	-.34	-.15	-.11	.29	-.24	-.09		

Total Shipments: 6,433,000 head

Total Cost: \$42,221,592.00

TABLE IX

SURPLUS AND DEFICITS, OPTIMUM FLOWS AND ESTIMATED REGIONAL PRICE DIFFERENTIALS FOR
FEEDER CATTLE IN THE UNITED STATES, MODEL III (1951)
(shipments in thousands)

Destinations	Origins														Deficit	Totals	V _i
	11	15	20	14	8	17	21	10	6	25	16	13	23	18			
19						87	11								78	78	.67
4					415	98									50	148	.67
5	249															762	.53
12															61	67	.67
24		9						22								31	.50
9		100	132													232	.26
7				102			985	20		57						1,352	.41
2										122						1,220	-.14
1											23					23	.04
22										5						5	-.03
3									5		135	35	27	61	812	1,075	.67
Totals	249	109	132	102	415	185	996	230	10	179	158	35	27	61	1,001	3,889	
U _i	0	-.42	-.27	-.11	-.23	-.36	-.54	-.85	-.68	-.70	-.55	-.11	-.64	-.49	.67		

Total Shipments: 3,889,000 head

Total Cost: \$17,563,776.00

TABLE X

SURPLUS AND DEFICITS, OPTIMUM FLOWS AND ESTIMATED REGIONAL PRICE DIFFERENTIALS FOR
FEEDER CATTLE IN THE UNITED STATES, MODEL IV (1951)
(shipments in thousands)

Destinations	Origins														Totals	V ₁
	11	15	20	14	17	21	10	25	16	1	22	13	23	18		
19			56			22									78	.76
4						148									148	.63
5					334	428									762	.51
12			61												61	.63
24			31												31	.33
9			232												232	.08
7				449		903									1,352	.37
20		82					86								168	.23
14	94					446	1	150							691	.47
8					9										9	-.01
6							18								18	.15
2								107							107	.22
3							301		345	106	40	130	21	132	1,075	1.04
Totals	94	82	380	449	343	1,947	406	257	345	106	40	130	21	132	4,732	
U ₁	0	-.22	-.45	-.15	-.38	-.58	-.61	-.34	-.18	.12	-.14	.26	-.27	-.12		

Total Shipments: 4,732,000 head

Total Cost: \$31,567,543.00

VITA

Willard Robert Sparks

Candidate for the Degree of

Master of Science

Thesis: STRUCTURAL AND SPATIAL ANALYSES OF THE FEEDER CATTLE SECTOR

Major Field: Agricultural Economics

Biographical:

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