EFFECT OF TEMPERATURE CHANGE IN CONTINUOUS RIGID FRAME BRIDGES

By

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IN CONTINUOUS RIGID FRAME

BRIDGES

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PREFACE

This study is the last part of the structural research program initiated at the Oklahoma Agricultural and Mechanical College in February, 1952. This program has been supported by Robberson Steel Company of Oklahoma City and directed by Professor Jan J. Tuma.

The author wishes to acknowledge and express his indebtedness to Professor Jan J. Tuma, his thesis advisor, for his invaluable suggestions and guidance throughout his undergraduate and graduate work, which made this work possible; to Professor Roger L. Flanders, Head of the Civil Engineering Department, for his aid in securing for him a Graduate Fellowship, which made his graduate work possible, and to the Professors in the Civil Engineering Department for their interest and friendship extended to him during his studies.

The author expresses his gratitude to his beloved wife, Catherine L. Simpson, for her constant encouragement and patience, which without this work would not have been possible.

The writer wishes to thank Mrs. Carolyn Bedingfield for her excellent work in typing and arranging this manuscript with its many equations.

N. G. S.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF ILLUSTRATIONS

NOMENCLATURE

T_0	Initial temperature
T_1	Final temperature at top
T_2	Final temperature at bottom
T_T	Change in temperature at top
T_B	Change in temperature at bottom
$\mathcal{E}_T$	Linear temperature strain at top
$\mathcal{E}_B$	Linear temperature strain at bottom
α	Coefficient of thermal expansion
$d\varphi_{TB}$	Angular temperature strain
R_{AY}, R_{BY}, R_{CY}	Vertical reactions at the respective joint
R_{AX}, R_{BX}, R_{CX}	Horizontal reaction at the respective end
H	Function of the horizontal reaction
M_{AB}, M_{BA}, M_{BC}	Bending moment at the respective end
Δ_{AX}, Δ_{BX}, Δ_{CX}	Horizontal displacement at the respective joint
Δ_{AY}, Δ_{BY}, Δ_{CY}	Vertical displacement at the respective joint
N_x	Normal force at x
V_x	Shearing force at x
M_x	Bending moment at x
U_{EXT}	External work due to loads and reactions
U_1	External work due to loads
U_2	External work due to reactions
U_{INT}	Internal work due to loads, reactions, temperature changes, and moisture changes.

NOMENCLATURE (CONTINUED)

U_3 Internal work due to loads and reactions

U_4 Internal work due to temperature and moisture changes

L Length

A_x Variable area of the cross-section of the beam

I_x Variable moment of inertia of the cross-section of the beam

E Modulus of elasticity of the material

G Modulus of rigidity of the material

$\mathcal{H}_x$ Variable shear coefficient of the cross-section of the beam

a, b, c, d, e, f, g, h, j Distribution factors

s Function of the distribution factors

X, Y Denominators of convergency

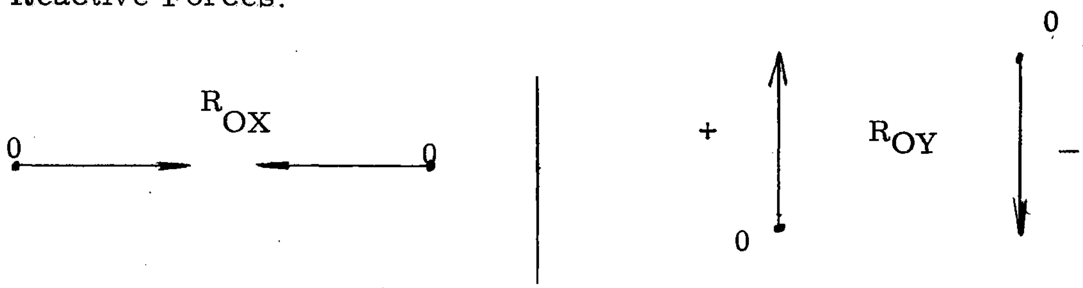
K_{AB}, K_{BA}, K_{BC} Stiffness factors of the respective member

K'_{AB}, K'_{BA}, K'_{BC} Modified stiffness factors of the respective member

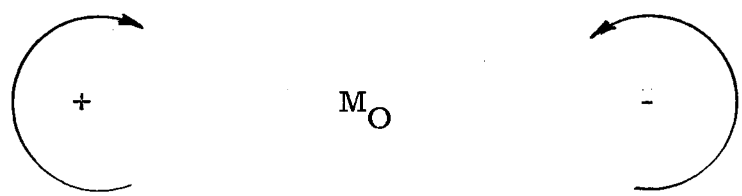
C_1, C_2, C_3, C_4, N, Equivalents

SIGN CONVENTION

Reactive Forces:

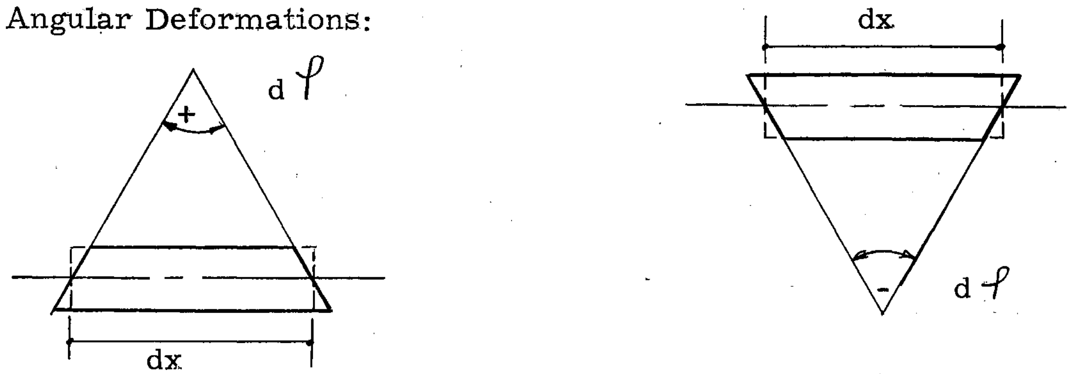


Reactive Moments:



Linear Deformations:

Angular Deformations:



PART I

INTRODUCTION

The stresses and deformations developed in rigid frame bridges are of two types:

1. Primary stresses and deformations (due to loads).
2. Secondary stresses and deformations (due to change in temperature, change in moisture content and displacement of supports).

In many cases, the secondary effects reach large magnitudes and are no longer secondary. This is particularly true in the case of a rigid frame bridge of which the topside is exposed to the sun radiation and the underside remains in the shade. The temperature moments become third power functions of the depth of the main girder and a first power function of the temperature differential.

The purpose of this thesis is the mathematical investigation of these moments in three and four span rigid frame bridges by means of infinite, geometric series. The results are summarized in two tables (VI and VII) and the suggested procedure is illustrated by two typical examples.

PART II

DERIVATION OF DEFORMATION EQUATIONS

A fixed end unsymmetrical beam of variable cross-section is considered (Fig. 1).

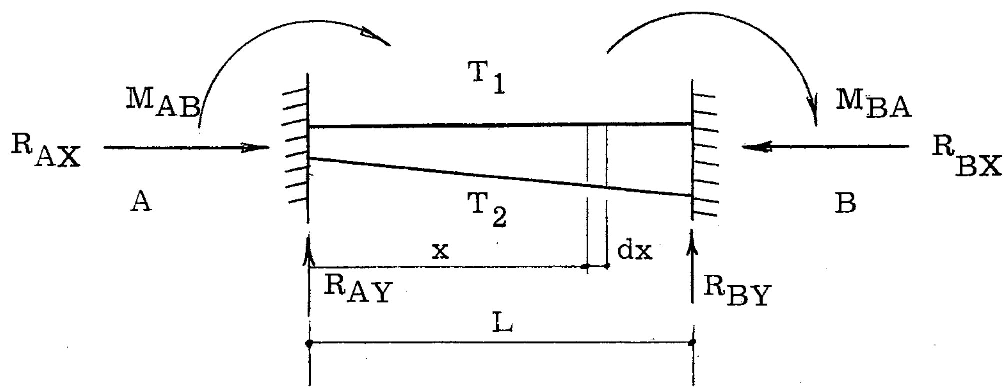


Fig. 1

Fixed End Beam

The change in temperature above the beam

$$T_T = T_1 - T_0 \tag{1}$$

and the change in temperature below the beam

$$T_B = T_2 - T_0 \, . \tag{2}$$

Where

T_0 = Initial temperature

T_1 = Final temperature top

T_2 = Final temperature bottom.

The linear temperature strain

$$\varepsilon_T = \alpha T_T \text{ and } \varepsilon_B = \alpha T_B \qquad (3) \text{ and } (4)$$

The angular temperature strain (if $T_T < T_B$) (Fig. 2).

$$d\varphi_{TB} = \alpha \frac{T_B - T_T}{h_x} dx \qquad (5)$$

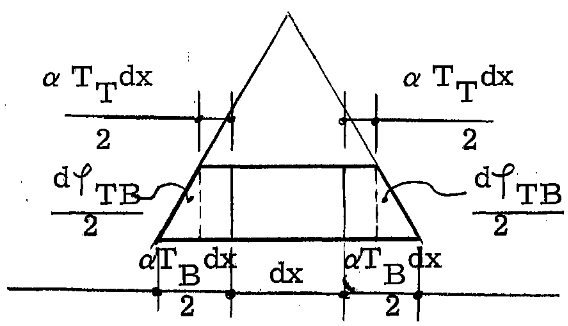


Fig. 2

Temperature Deformation of Element dx

The reactive forces in terms of end moments M_{AB}, M_{BA}, and the axial thrust H are:

$$R_{AY} = -\frac{M_{AB} + M_{BA}}{L} \qquad (6)$$

$$R_{BY} = \frac{M_{AB} + M_{BA}}{L} \qquad (7)$$

$$R_{AX} = -R_{BX} = H. \qquad (8)$$

The normal force at x

$$N^{(A)}_{x = 0 \to L} = -H. \tag{9}$$

The shearing force at x

$$V^{(A)}_{x = 0 \to L} = -\frac{M_{AB} + M_{BA}}{L} . \tag{10}$$

The bending moment at x in terms of x = x and $x' = L-x$,

$$M^{(A)}_{x = 0 \to L} = M_{AB}\,\frac{x'}{L} - M_{BA}\,\frac{x}{L} . \tag{11}$$

From the principle of minimum energy

$$\frac{\partial U_{INT}}{\partial M_{AB}} = \frac{\partial U_{EXT}}{\partial M_{AB}} \tag{12}$$

$$\frac{\partial U_{INT}}{\partial M_{BA}} = \frac{\partial U_{EXT}}{\partial M_{BA}} \tag{13}$$

$$\frac{\partial U_{INT}}{\partial H} = \frac{\partial U_{EXT}}{\partial H} . \tag{14}$$

Where

$$U_{EXT} = \begin{cases} U_1 = \text{External work due to loads} \\ U_2 = \text{External work due to reactions} \end{cases}$$

$$U_{INT} = \begin{cases} U_3 = \text{Internal work due to loads and reactions} \\ U_4 = \text{Internal work due to temperature and moisture change.} \end{cases}$$

If only the change in temperature is considered

$$U_1 = 0 \qquad\qquad U_2 = 0 \tag{15}$$

$$U_3 = \int_A^B \frac{N_x^2\,dx}{2A_xE} + \int_A^B \frac{\mathcal{H}_x V_x^2\,dx}{2A_xG} + \int_A^B \frac{M_x^2\,dx}{2EI_x} \tag{16}$$

$$U_4 = \int_A^B \left(\frac{\mathcal{E}_B + \mathcal{E}_T}{2}\right) N_x dx + \int_A^B M_x d\,\varphi_{TB} \tag{17}$$

The symbols in Equations (16) and (17) are

A_x = Variable area of the cross-section of the beam

I_x = Variable moment of inertia of the cross-section of the beam

E = Modulus of elasticity of the material

G = Modulus of rigidity of the material

$\mathcal{H}_x$ = Variable shear coefficient of the cross-section of the beam.

The minimum energy Equations (12, 13, 14) in terms of Equations (15, 16, 17) are:

$$0 = \int_A^B \frac{\mathcal{H}_x M_{AB}\,dx}{A_xGL^2} + \int_A^B \frac{\mathcal{H}_x M_{BA}\,dx}{A_xGL^2} + \int_A^B \frac{M_{AB}x'^2 dx}{EI_xL^2} - \int_A^B \frac{M_{BA}x'x\,dx}{EI_xL^2}$$
$$+ \int_A^B \frac{x'}{L}\,d\varphi_{TB} \tag{18}$$

$$0 = \int_A^B \frac{\mathcal{H}_x M_{AB}\,dx}{A_xGL^2} + \int_A^B \frac{\mathcal{H}_x M_{BA}\,dx}{A_xGL^2} - \int_A^B \frac{M_{AB}x'x\,dx}{EI_xL^2}$$
$$+ \int_A^B \frac{M_{BA}x^2 dx}{EI_xL^2} - \int_A^B \frac{x}{L}\,d\varphi_{TB} \tag{19}$$

$$0 = \int_A^B \frac{H\,dx}{A_xE} - \int_A^B \left(\frac{\mathcal{E}_B + \mathcal{E}_T}{2}\right) dx\ . \tag{20}$$

With new equivalents:

$$C_1 = \int_A^B \frac{\mathcal{H}_x \, dx}{A_x G L^2} + \int_A^B \frac{x^2 \, dx}{E I_x L^2}$$

$$C_2 = \int_A^B \frac{\mathcal{H}_x \, dx}{A_x G L^2} - \int_A^B \frac{x x' \, dx}{E I_x L^2}$$

$$C_3 = \int_A^B \frac{\mathcal{H}_x \, dx}{A_x G L^2} + \int_A^B \frac{x'^2 \, dx}{E I_x L^2}$$

$$C_4 = \int_A^B \frac{dx}{A_x E}$$

$$C_5 = \int_A^B \left(\frac{\mathcal{E}_B + \mathcal{E}_T}{2} \right) dx \qquad (21)$$

$$C_6 = \int_A^B \frac{x}{L} \, d\varphi_{TB}$$

$$C_7 = \int_A^B \frac{x'}{L} \, d\varphi_{TB}$$

the deformation Equations (18, 19, 20) become:

$$0 = C_3 M_{AB} + C_2 M_{BA} + C_7 \qquad (22)$$

$$0 = C_2 M_{AB} + C_1 M_{BA} - C_6 \qquad (23)$$

$$0 = C_4 H - C_5 \qquad (24)$$

Let

$$C_1 C_3 - C_2 C_2 = N \qquad (25)$$

After solving Equations (22) and (23) simultaneously:

$$M_{AB} = -\frac{C_1C_7 + C_2C_6}{N} \tag{26}$$

$$M_{BA} = \frac{C_3C_6 + C_2C_7}{N} \tag{27}$$

and from Equation (24)

$$H = \frac{C_5}{C_4} \quad . \tag{28}$$

If the cross-section of the beam is constant

$$A_x = A \qquad I_x = I \qquad \mathcal{H}_x = \mathcal{H} \qquad h_x = h$$

and the equivalents (21) become:

$$\begin{aligned}
C_1 &= \frac{\mathcal{H}}{LAG} + \frac{L}{3EI} \\
C_2 &= \frac{\mathcal{H}}{LAG} - \frac{L}{6EI} \\
C_3 &= \frac{\mathcal{H}}{LAG} + \frac{L}{3EI} \\
C_4 &= \frac{L}{AE} \\
C_5 &= \frac{\alpha L\,(T_B + T_T)}{2} \\
C_6 &= \frac{\alpha L\,(T_B - T_T)}{2h} \\
C_7 &= \frac{\alpha L\,(T_B - T_T)}{2h} \\
N &= \frac{1}{EI}\left(\frac{\mathcal{H}}{AG} + \frac{L^2}{12EI}\right)
\end{aligned} \tag{29}$$

If the cross-section of the beam is constant and the shearing deformation is neglected

$$
\begin{aligned}
C_1 &= C_3 = \frac{L}{3EI} \\
C_2 &= -\frac{L}{6EI} \qquad (30) \\
N &= \frac{L^2}{12\, E^2 I^2}
\end{aligned}
$$

and the final form of the redundant Equations (26, 27, 28) in terms of the simplified Equations (30) become:

$$M_{AB} = -\frac{\alpha\,(T_B - T_T)\, EI}{h} \qquad (31)$$

$$M_{BA} = \frac{\alpha\,(T_B - T_T)\, EI}{h} \qquad (32)$$

$$H = \frac{\alpha\,(T_B + T_T)\, AE}{2} \qquad (33)$$

PART III

DERIVATION OF NEW DISTRIBUTION FACTORS FOR A THREE SPAN BRIDGE FRAME

A typical three span cyclosymmetrical bridge frame with members of constant cross-section is considered (Fig. 3). The deck is freely supported at the abutments, restrained against translation at D and intergal with piers hinged at the bottom.

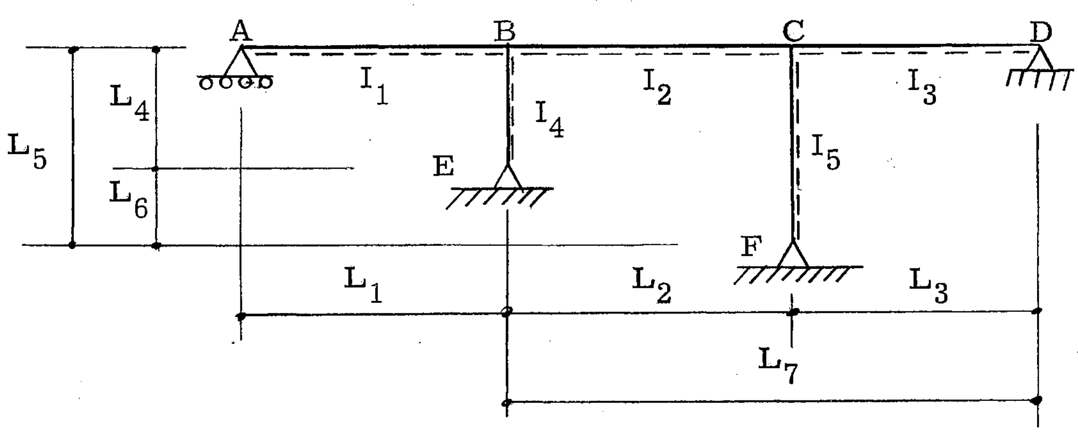


Fig. 3

Three Span Bridge Frame

A. Equal Temperature Deformations

The uniform temperature change $T_T = T_B = T$ is assumed for each member.

The fixed end moments due to the uniform temperature change $T_T = T_B = T$, (Fig. 4):

$$EM_{BA}^{(0)} = \frac{3EI_1}{L_1^2} \alpha T L_4$$

$$EM_{BE}^{(0)} = \frac{3EI_4}{L_4^2} \alpha T L_7$$

$$EM_{CD}^{(0)} = -\frac{3EI_3}{L_3^2} \alpha T L_5 \qquad (34)$$

$$EM_{CF}^{(0)} = \frac{3EI_5}{L_5^2} \alpha T L_3$$

$$FEM_{BC}^{(0)} = FEM_{CB}^{(0)} = \frac{6EI_2}{L_2^2} \alpha T L_6 \quad .$$

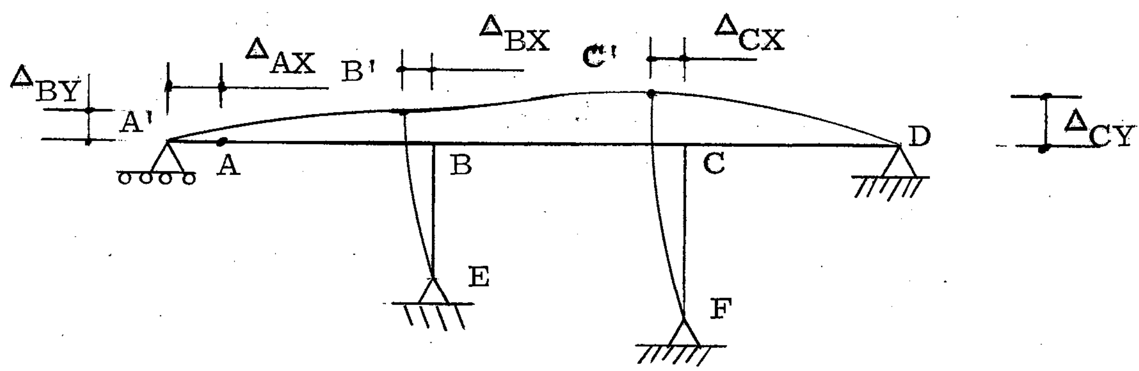


Fig. 4

Elastic Curve Due to Change in Temperature

and Rotational Restrain at B and C.

The distribution factors are designated as:

$$D_{BA} = a \qquad D_{CB} = 2c$$
$$D_{BC} = 2b \qquad D_{CD} = d \qquad (35)$$
$$D_{BE} = e \qquad D_{CF} = f$$

After recording the fixed end moments, all joints are successively unlocked and allowed to rotate gradually into their equilibrium position.

The unbalance at joint B

$$M_B^{(0)} = \alpha T \left(\frac{3EI_1}{L_1^2} L_4 + \frac{6EI_2}{L_2^2} L_6 + \frac{3EI_4}{L_4^2} L_7 \right) . \qquad (36)$$

The unbalance at joint C

$$M_C^{(0)} = \alpha T \left(\frac{6EI_2}{L_2^2} L_6 + \frac{3EI_3}{L_3^2} L_5 + \frac{3EI_5}{L_5^2} L_3 \right) . \qquad (37)$$

The balancing procedure in the algebraic form of a series is expanded for each moment in Table I and Table II.

The final moments due to the uniform temperature change at joint B are:

$$M_{BA} = EM_{BA}^{(0)} - aM_B^{(0)} - abcM_B^{(0)} - \dots$$
$$+ acM_C^{(0)} + abc^2M_C^{(0)} + \dots$$
$$= EM_{BA}^{(0)} - \frac{a}{X}M_B^{(0)} + \frac{ac}{X}M_C^{(0)} . \qquad (38)$$

$$M_{BE} = EM_{BE}^{(0)} - eM_B^{(0)} - ebcM_C^{(0)} - \dots$$
$$+ ecM_C^{(0)} + ebc^2M_C^{(0)} + \dots$$
$$= EM_{BE}^{(0)} - \frac{e}{X}M_B^{(0)} + \frac{ec}{X}M_C^{(0)}$$

and

$$M_{BC} = -M_{BA} - M_{BE}. \tag{38}$$

The final moments due to the uniform temperature change at joint C are:

$$\begin{aligned} M_{CD} &= EM_{CD}^{(0)} + dbM_B^{(0)} + db^2cM_B^{(0)} + \ldots \\ &\qquad - dM_C^{(0)} - dbcM_C^{(0)} - \ldots \\ &= EM_{CD}^{(0)} + \frac{db}{X} M_B^{(0)} - \frac{d}{X} M_C^{(0)} \quad . \end{aligned}$$

$$\begin{aligned} M_{CF} &= EM_{CF}^{(0)} + fbM_B^{(0)} + fb^2cM_B^{(0)} + \ldots \\ &\qquad - fM_C^{(0)} - fbcM_C^{(0)} - \ldots \\ &= EM_{CF}^{(0)} + \frac{fb}{X} M_B^{(0)} - \frac{f}{X} M_C^{(0)} \end{aligned} \tag{39}$$

and

$$M_{CB} = -M_{CD} - M_{CF}$$

where

$$X = 1 - bc.$$

The horizontal reaction at D

$$H = \frac{M_{BE}}{L_4} + \frac{M_{CF}}{L_5} \quad . \tag{40}$$

TABLE I

ALGEBRAIC DISTRIBUTION OF UNIT MOMENT AT B

Joints	B			C		
Members	BA	BE	BC	CB	CF	CD
Distribution Factors	a	e	2b	2c	f	d
First Distribution	-a	-e	-2b			
Carry Over				-b		
Second Distribution				+2bc	+bf	+bd
Carry Over			+bc			
Third Distribution	-abc	-ebc	$-2b^2c$			
Carry Over				$-b^2c$		
Fourth Distribution				$+2b^2c^2$	$+b^2cf$	$+b^2cd$
Carry Over			$+b^2c^2$			
Fifth Distribution	$-ab^2c^2$	$-eb^2c^2$	$-2b^3c^2$			
Carry Over				$-b^3c^2$		
Sixth Distribution				$+2b^3c^3$	$+b^3c^2f$	$+b^3c^2d$
	:	:	:	:	:	:
Infinite Distribution	0	0	0	0	0	0

TABLE II

ALGEBRAIC DISTRIBUTION OF UNIT MOMENT AT C

Joints	B			C		
Members	BA	BE	BC	CB	CF	CD
Distribution Factors	a	e	2b	2c	f	d
First Distribution				$-2c$	$-f$	$-d$
Carry Over			$-c$			
Second Distribution	$+ac$	$+ec$	$+2bc$			
Carry Over				$+bc$		
Third Distribution				$-2bc^2$	$-fbc$	$-dbc$
Carry Over			$-bc^2$			
Fourth Distribution	$+abc^2$	$+ebc^2$	$+2b^2c^2$			
Carry Over				$+b^2c^2$		
Fifth Distribution				$-2b^2c^3$	$-fb^2c^2$	$-db^2c^2$
Carry Over			$-b^2c^3$			
Sixth Distribution	$+ab^2c^3$	$+eb^2c^3$	$+2b^3c^3$			
	:	:	:	:	:	:
Infinite Distribution	0	0	0	0	0	0

B. Unequal Temperature Deformations

If the nonuniform temperature change Equations (1) and (2) is considered, the fixed end moments due to this change (assuming $T_B > T_T$) become:

$$
\begin{aligned}
FM_{BA} &= \frac{3EI_1}{L_1^2}\,\alpha T_B L_4 + \frac{3}{2}\,\frac{EI_1}{h_1}\,\alpha(T_B - T_T) \\
FEM_{BC} &= \frac{6EI_2}{L_2^2}\,\alpha T_B L_6 - \frac{EI_2}{h_2}\,\alpha\,(T_B - T_T) \\
FEM_{CB} &= \frac{6EI_2}{L_2^2}\,\alpha T_B L_6 + \frac{EI_2}{h_2}\,\alpha\,(T_B - T_T) \\
FM_{CD} &= -\frac{3EI_3}{L_3^2}\,\alpha T_B L_5 - \frac{3}{2}\,\frac{EI_3}{h_3}\,\alpha\,(T_B - T_T) \\
FM_{BE} &= \frac{3EI_4}{L_4^2}\,\alpha\left(\frac{T_B + T_T}{2}\right)L_7 \\
FM_{CF} &= \frac{3EI_5}{L_5^2}\,\alpha\left(\frac{T_B + T_T}{2}\right)L_3 \quad .
\end{aligned}
\tag{41}
$$

The temperature change below the girder is T_B and the columns undergo uniform change due to T_B only. Hereafter, the procedure of analysis is the same as in the previous case.

PART IV

DERIVATION FOR A FOUR SPAN BRIDGE FRAME

A typical four span cyclosymmetrical bridge frame with members of constant cross-section is considered (Fig. 5). The deck is freely supported at the abutments, restrained against translation at E and intergal with piers hinged at bottom.

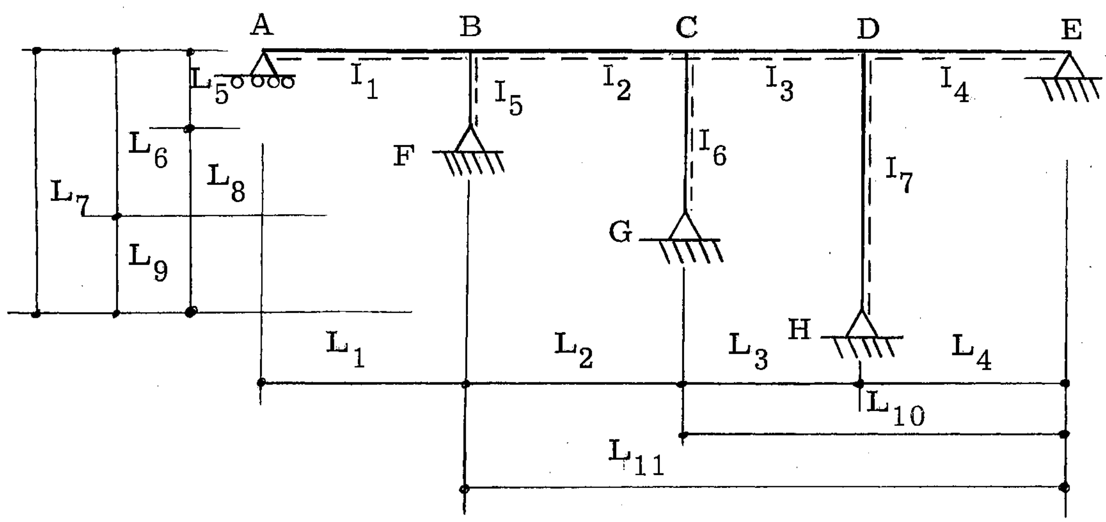


Fig. 5

Four Span Bridge Frame

A. Equal Temperature Deformations

The uniform temperature change $T_T = T_B = T$ is assumed for each member.

The fixed end moments due to this temperature change (Fig. 5) are:

$$EM_{BA}^{(0)} = \frac{3EI_1}{L_1^2}\ \alpha TL_5$$

$$EM_{CG}^{(0)} = \frac{3EI_6}{L_6^2}\ \alpha TL_{10}$$

$$EM_{BF}^{(0)} = \frac{3EI_5}{L_5^2}\ \alpha TL_{11}$$

$$EM_{DH}^{(0)} = \frac{3EI_7}{L_7^2}\ \alpha TL_4 \tag{42}$$

$$FEM_{BC}^{(0)} = FEM_{CB}^{(0)} = \frac{6EI_2}{L_2^2}\ \alpha TL_8$$

$$FEM_{CD}^{(0)} = FEM_{DC}^{(0)} = \frac{6EI_3}{L_3^2}\ \alpha TL_9$$

$$EM_{DE}^{(0)} = -\frac{3EI_4}{L_4^2}\ \alpha TL_7$$

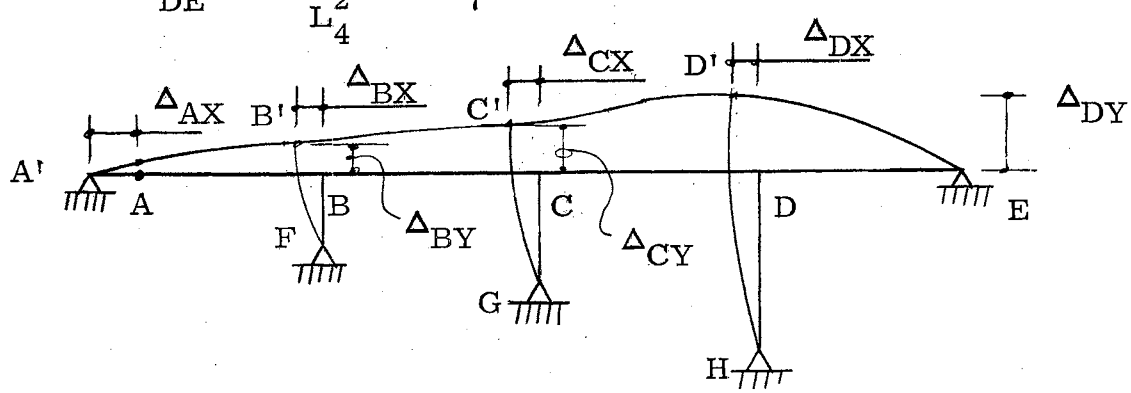


Fig. 6

Elastic Curve Due to Change in Temperature
and Rotational Restrain at B, C, and D.

The distribution factors are designated as:

$$
\begin{array}{lll}
D_{BA} = a & D_{CB} = 2c & D_{DC} = 2e \\
D_{BC} = 2b & D_{CD} = 2d & D_{DE} = f \\
D_{BF} = g & D_{CG} = h & D_{DH} = j
\end{array} \qquad (43)
$$

After recording the fixed end moments, all joints are successively unlocked and allowed to rotate gradually into their equilibrium position.

The unbalance at joint B

$$M_B^{(0)} = \alpha T \left(\frac{3EI_1}{L_1^2} L_5 + \frac{3EI_5}{L_5^2} L_{11} + \frac{6EI_2}{L_2^2} L_8 \right) . \qquad (44)$$

The unbalance at joint C

$$M_C^{(0)} = \alpha T \left(\frac{3EI_6}{L_6^2} L_{10} + \frac{6EI_2}{L_2^2} L_8 + \frac{6EI_3}{L_3^2} L_9 \right) . \qquad (45)$$

The unbalance at joint D

$$M_D^{(0)} = \alpha T \left(\frac{3EI_7}{L_7^2} L_4 + \frac{6EI_3}{L_3^2} L_9 - \frac{3EI_4}{L_4^2} L_7 \right) . \qquad (46)$$

The balancing procedure in the algebraic form of series is expanded for each moment (Table III, IV, V).

The final moments due to the uniform temperature change at joint B in terms of $Y = 1 - bc - de$ are:

$$M_{BA} = EM_{BA}^{(0)} - aM_B^{(0)} - \frac{abc}{Y} M_B^{(0)} + \frac{ac}{Y} M_C^{(0)} - \frac{ace}{Y} M_D^{(0)} \qquad (47)$$

$$M_{BF} = EM_{BF}^{(0)} - gM_B^{(0)} - \frac{gbc}{Y} M_B^{(0)} + \frac{gc}{Y} M_C^{(0)} - \frac{gce}{Y} M_D^{(0)} \qquad (48)$$

and

$$M_{BC} = - M_{BA} - M_{BF} . \qquad (49)$$

TABLE III

ALGEBRAIC DISTRIBUTION OF UNIT MOMENT AT B

Joints	B			C			D		
Members	BA	BF	BC	CB	CG	CD	DC	DH	DE
Distribution Factors	a	g	2b	2c	h	2d	2e	j	f
First Distribution	-a	-g	-2b						
Carry Over				-b					
Second Distribution				+2bc	+hb	+2bd			
Carry Over			+bc				+bd		
Third Distribution	-abc	-gbc	$-2b^2c$				-2ebd	-jbd	-fbd
Carry Over				$-b^2c$		-ebd			
Fourth Distribution				+2bcs*	+hbs*	+2bds*			
Carry Over			+bcs				+bds		
Fifth Distribution	-abcs	-gbcs	$-2b^2cs$				-2ebds	-jbds	-fbds
Carry Over				$-b^2cs$		-bds			
Sixth Distribution				$+2cbs^2$	$+hbs^2$	$+2bds^2$			
	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
Infinite Distribution	0	0	0	0	0	0	0	0	0

* s = bc + de

TABLE IV

ALGEBRAIC DISTRIBUTION OF UNIT MOMENT AT C

Joints	B			C			D		
Members	BA	BF	BC	CB	CG	CD	DC	DH	DE
Distribution Factors	a	g	2b	2c	h	2d	2e	j	f
First Distribution				-2c	-h	-2d			
Carry Over			-c				-d		
Second Distribution	+ac	+gc	+2bc				+2ed	+jd	+fd
Carry Over				+bc		+ed			
Third Distribution				-2cs*	-hs*	-2ds*			
Carry Over			-cs				-ds		
Fourth Distribution	+acs	+gcs	+2bcs				+2eds	+jds	+fds
Carry Over				+bcs		+eds			
Fifth Distribution				$-2cs^2$	$-hs^2$	$-2ds^2$			
Carry Over			$-cs^2$				$-ds^2$		
Sixth Distribution	$+acs^2$	$+gcs^2$	$+2bcs^2$				$+2eds^2$	$+jds^2$	$+fds^2$
	:	:	:	:	:	:	:	:	:
Infinite Distribution	0	0	0	0	0	0	0	0	0

* s = bc + de

TABLE V

ALGEBRAIC DISTRIBUTION OF UNIT MOMENT AT D

Joints	B			C			D		
Members	BA	BF	BC	CB	CG	CD	DC	DH	DE
Distribution	a	g	2b	2c	h	2d	2e	j	f
First Distribution							-2e	-j	-f
Carry Over						-e			
Second Distribution				+2ce	+he	+2de			
Carry Over			+ce				+de		
Third Distribution	-ace	-gce	-2bce				$-2de^2$	-jde	-fde
Carry Over				-bce		$-de^2$			
Fourth Distribution				+2ces*	+hes*	+2des*			
Carry Over			+ces				+des		
Fifth Distribution	-aces	-gces	-2bces				$-2de^2s$	-jdes	-fdes
Carry Over				-bces		-des			
Sixth Distribution				$+2ces^2$	$+hes^2$	$+2des^2$			
	:	:	:	:	:	:	:	:	:
Infinite Distribution	0	0	0	0	0	0	0	0	0

* s = bc + de

The final moments due to the uniform temperature change at joint C are:

$$M_{CB} = FEM_{CB}^{(0)} - bM_{B}^{(0)} - \frac{b^2c}{Y}M_{B}^{(0)} + \frac{2bc}{Y}M_{B}^{(0)} - \frac{2c}{Y}M_{C}^{(0)} + \frac{bc}{Y}M_{C}^{(0)} + \frac{2ce}{Y}M_{D}^{(0)} - \frac{bce}{Y}M_{D}^{(0)} \qquad (50)$$

$$M_{CG} = EM_{CG}^{(0)} + \frac{hb}{Y}M_{B}^{(0)} - \frac{h}{Y}M_{C}^{(0)} + \frac{he}{Y}M_{D}^{(0)} \qquad (51)$$

and

$$M_{CD} = -M_{CB} - M_{CG} \qquad (52)$$

The final moments due to the uniform temperature change at joint D are:

$$M_{DE} = EM_{DE}^{(0)} - \frac{fdb}{Y}M_{B}^{(0)} + \frac{fd}{Y}M_{C}^{(0)} - fM_{D}^{(0)} - \frac{fde}{Y}M_{D}^{(0)} \qquad (53)$$

$$M_{DH} = EM_{DH}^{(0)} - \frac{jdb}{Y}M_{B}^{(0)} + \frac{jd}{Y}M_{C}^{(0)} - jM_{D}^{(0)} - \frac{jde}{Y}M_{D}^{(0)} \qquad (54)$$

and

$$M_{DC} = -M_{DE} - M_{DH}.$$

The horizontal reaction at E:

$$H = \frac{M_{BF}}{L_5} + \frac{M_{CG}}{L_6} + \frac{M_{DH}}{L_7} \qquad (55)$$

B. Unequal Temperature Deformations

If the nonuniform temperature change Equations (1) and (2) is considered, the fixed end moments due to this change (assuming $T_B > T_T$) become:

$$EM_{BA} = \frac{3EI_1}{L_1}\alpha T_B L_5 + \frac{3EI_1}{2h_1}\alpha(T_B - T_T) \qquad (56)$$

$$FEM_{BC} = \frac{6EI_2}{L_2^2}\ \alpha T_B L_8 - \frac{EI_2}{h_2}\ \alpha\,(T_B - T_T)$$

$$EM_{BF} = \frac{3EI_5}{L_5^2}\ \alpha\left(\frac{T_B + T_T}{2}\right) L_{11}$$

$$FEM_{CB} = \frac{6EI_2}{L_2^2}\ \alpha T_B L_8 + \frac{EI_2}{h_2}\ \alpha\,(T_B - T_T)$$

$$FEM_{CD} = \frac{6EI_3}{L_3^2}\alpha T_B L_9 - \frac{EI_3}{h_3}\ \alpha\,(T_B - T_T) \qquad (56)$$

$$EM_{CG} = \frac{3EI_6}{L_6^2}\ \alpha\left(\frac{T_B + T_T}{2}\right) L_{10}$$

$$FEM_{DC} = \frac{6EI_3}{L_3^2}\ \alpha T_B L_9 + \frac{EI_3}{h_3}\ \alpha\,(T_B - T_T)$$

$$EM_{DE} = -\frac{3EI_4}{L_4^2}\alpha T_B L_7 - \frac{3EI_4}{2h_4}\ \alpha\,(T_B - T_T)$$

$$EM_{DH} = \frac{3EI_7}{L_7^2}\ \alpha\left(\frac{T_B + T_T}{2}\right) L_4$$

Hereafter the procedure of analysis is the same as in the uniform case.

PART V

TABLES

A. General Notes

Two general tables of end moments due to temperature change are presented in this part of the thesis.

Table VI - End Moments in Three Span Unsymmetrical Bridge Frame

Table VII - End Moments in Four Span Unsymmetrical Bridge Frame.

Each table is composed of the following major parts:

1. Description of Frame - Definition and structural identities.
2. Illustration of Frame - Figure containing symbols for all structural elements.
3. Algebraic Equivalents - Stiffness factors and distribution factors.
4. Final Moments - Algebraic moment coefficients known as new distribution factors.

B. Procedure

The procedure of analysis may be summarized in the following steps:

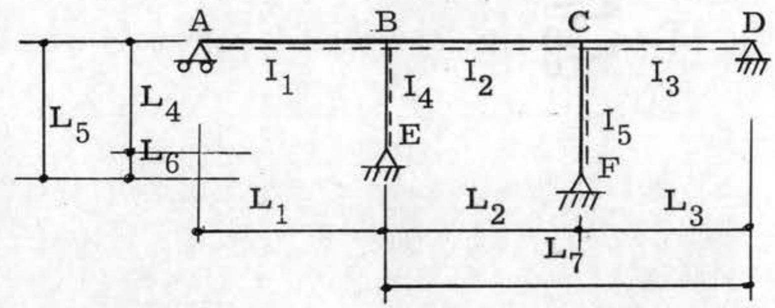
1. Select the table for the case to be investigated and adjust the symbols to those shown in table.
2. Compute all stiffness factors, distribution factors, and equivalents.

3. Compute the fixed and propped end moments due to the uniform and nonuniform change in temperature.
4. Substitute the equivalents, the fixed and propped end moments in the moment part of the respective table and compute the final end moments.
5. Check the final answers by means of moment equilibrium at the joint.

TABLE VI - END MOMENTS IN THREE SPAN UNSYMMETRICAL BRIDGE FRAME

Three span frame freely supported at abutments, restrained against translation at D, with piers hinged at bottom.

Constant Moment of Inertia.



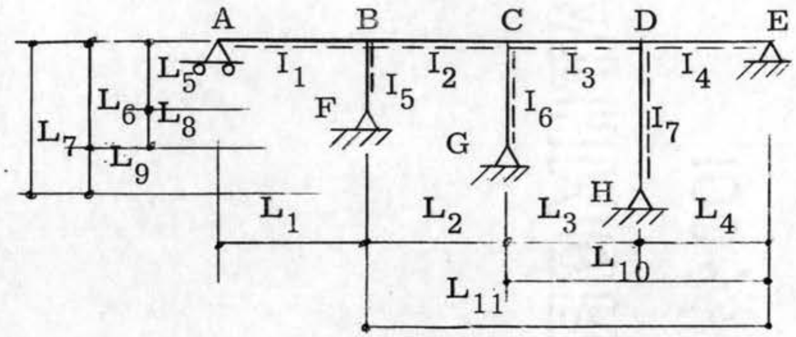
STIFFNESS FACTORS	DISTRIBUTION FACTORS
$K'_{BA} = \frac{3I_1}{4L_1}$ $\quad K'_{BE} = \frac{3I_4}{4L_4}$ $\quad K_{BC} = \frac{I_2}{L_2}$	$a = \frac{K'_{BA}}{\Sigma K_B}$ $\quad e = \frac{K'_{BE}}{\Sigma K_B}$ $\quad b = \frac{K_{BC}}{2\Sigma K_B}$
$\sum K_B = K'_{BA} + K'_{BE} + K_{BC}$	
$K_{CB} = \frac{I_2}{L_2}$ $\quad K'_{CF} = \frac{3I_5}{4L_5}$ $\quad K'_{CD} = \frac{3I_3}{4L_3}$	$c = \frac{K_{CB}}{2\Sigma K_C}$ $\quad f = \frac{K'_{CF}}{\Sigma K_C}$ $\quad d = \frac{K'_{CD}}{\Sigma K_C}$
$\sum K_C = K_{CB} + K'_{CF} + K'_{CD}$	$X = 1 - bc$

TABLE VI - (CONTINUED)			
FINAL MOMENTS			
Moment	End Moment	$EM_{BA} + EM_{BE} + FEM_{BC}$	$FEM_{CB} + EM_{CF} + EM_{CD}$
M_{BA}	EM_{BA}	$-\frac{a}{X}$	$+\frac{ac}{X}$
M_{BE}	EM_{BE}	$-\frac{e}{X}$	$+\frac{ec}{X}$
M_{BC}	FEM_{BC}	$-\left(\frac{2b - cb}{X}\right)$	$-\left(\frac{c - 2bc}{X}\right)$
M_{CB}	FEM_{CB}	$-\left(\frac{b - 2cb}{X}\right)$	$-\left(\frac{2c - bc}{X}\right)$
M_{CF}	EM_{CF}	$+\frac{fb}{X}$	$-\frac{f}{X}$
M_{CD}	EM_{CD}	$+\frac{db}{X}$	$-\frac{d}{X}$

TABLE VII - END MOMENTS IN FOUR SPAN UNSYMMETRICAL BRIDGE FRAME

Four span frame freely supported at abutments, restrained against translation at E, with piers hinged at bottom.

Constant Moment of Inertia.



STIFFNESS FACTORS

$$K'_{BA} = \frac{3I_1}{4L_1} \qquad K'_{BF} = \frac{3I_5}{4L_5} \qquad K_{BC} = \frac{I_2}{L_2}$$

$$\Sigma K_B = K'_{BA} + K_{BC} + K'_{BF}$$

$$K_{CB} = \frac{I_2}{L_2} \qquad K'_{CG} = \frac{3I_6}{4L_6} \qquad K_{CD} = \frac{I_3}{L_3}$$

$$\Sigma K_C = K_{CB} + K'_{CG} + K_{CD}$$

$$K_{DC} = \frac{I_3}{L_3} \qquad K'_{DH} = \frac{3I_7}{4L_7} \qquad K'_{DE} = \frac{3I_4}{4L_4}$$

$$\Sigma K_D = K_{DC} + K'_{DH} + K'_{DE}$$

DISTRIBUTION FACTORS

$$a = \frac{K'_{BA}}{\Sigma K_B} \qquad g = \frac{K'_{BF}}{\Sigma K_B} \qquad b = \frac{K_{BC}}{2\Sigma K_B}$$

$$c = \frac{K_{CB}}{2\Sigma K_C} \qquad h = \frac{K'_{CG}}{\Sigma K_C} \qquad d = \frac{K_{CD}}{2\Sigma K_C}$$

$$e = \frac{K_{DC}}{2\Sigma K_D} \qquad j = \frac{K'_{DH}}{\Sigma K_D} \qquad f = \frac{K'_{DH}}{\Sigma K_D}$$

$$X = 1 - bc$$

$$Y = 1 - bc - de$$

TABLE VII-(CONTINUED)

FINAL MOMENTS

Moment	End Moment	$EM_{BA}+EM_{BF}+FEM_{BC}$	$FEM_{CB}+EM_{CG}+FEM_{CD}$	$FEM_{DC}+EM_{DH}+EM_{DE}$
M_{BA}	EM_{BA}	$-\left(a + \frac{abc}{Y}\right)$	$+\frac{ac}{Y}$	$-\frac{ace}{Y}$
M_{BF}	EM_{BF}	$-\left(g + \frac{gbc}{Y}\right)$	$+\frac{gc}{Y}$	$-\frac{gce}{Y}$
M_{BC}	FEM_{BC}	$-\left(\frac{2b}{X} - \frac{bc}{Y}\right)$	$-\left(\frac{c - 2bc}{Y}\right)$	$+\left(\frac{ce - 2bce}{Y}\right)$
M_{CB}	FEM_{CB}	$-\left(\frac{b}{X} - \frac{2bc}{Y}\right)$	$-\left(\frac{2c - bc}{Y}\right)$	$+\left(\frac{2ce - bce}{Y}\right)$
M_{CG}	EM_{CG}	$+\frac{hb}{Y}$	$-\frac{h}{Y}$	$+\frac{he}{Y}$
M_{CD}	FEM_{CD}	$+\left(\frac{2db - dbe}{Y}\right)$	$-\left(\frac{2d - de}{Y}\right)$	$-\left(\frac{e}{X} - \frac{2de}{Y}\right)$

TABLE VII - (CONTINUED)

FINAL MOMENTS

Moment	End Moment	$EM_{BA}+EM_{BF}+FEM_{BC}$	$FEM_{CB}+EM_{CG}+FEM_{CD}$	$FEM_{DC}+EM_{DH}+EM_{DE}$
M_{DC}	FEM_{DC}	$+\left(\frac{db - 2dbe}{Y}\right)$	$-\left(\frac{d - 2de}{Y}\right)$	$-\left(\frac{2e}{X} - \frac{de}{Y}\right)$
M_{DH}	EM_{DH}	$-\frac{jdb}{Y}$	$+\frac{jd}{Y}$	$-\left(j + \frac{jde}{Y}\right)$
M_{DE}	EM_{DE}	$-\frac{fdb}{Y}$	$+\frac{fd}{Y}$	$-\left(f + \frac{fde}{Y}\right)$

PART VI

EXAMPLES

Two typical examples are introduced to demonstrate the application of moment coefficients recorded in Table VI. The reinforced concrete bridge considered in both cases is composed of prismatic members. The modulus of concrete

$$E = 3 \times 10^3 \quad \text{kip per in.}^2$$

and the coefficient of thermal expansion of concrete

$$\alpha \doteq 6.5 \times 10^{-6} \quad \text{per degree of Fahrenheit.}$$

The bridge deck integral with piers is supported by an expansion roller at A and by a hinge at D. The piers are hinged at bottoms. All values are given in inches, kips, or kip-inches.

<u>Example 1:</u> The effect of uniform change in temperature from

$$T_0 = 70^\circ \quad \text{to} \quad T_1 = T_2 = 120^\circ$$

in the bridge frame shown in Fig. 7 is investigated.

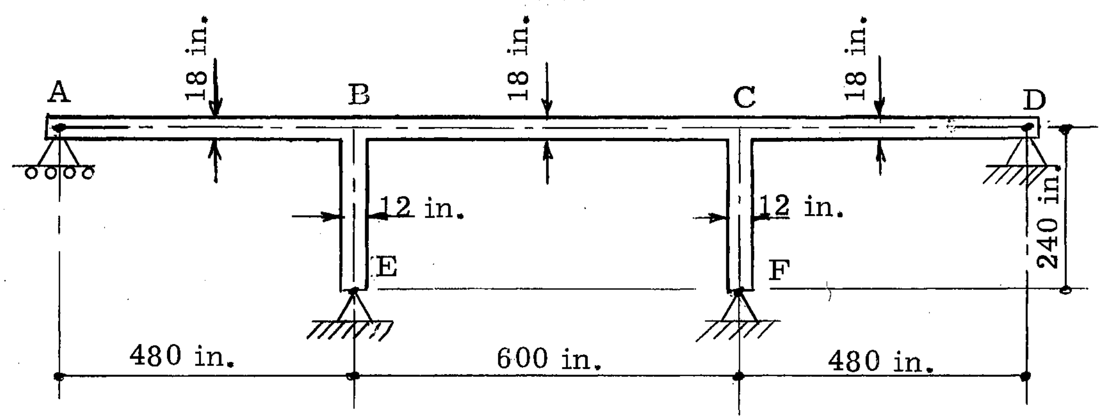


Fig. 5

Three Span Symmetrical Reinforced Concrete Bridge Frame

The procedure of analysis is outlined in Part V of this thesis. The moment coefficients are computed by means of Table VI.

1. <u>Stiffness Factors:</u>

$K'_{BA} = 109.4$ $\qquad K'_{BE} = 64.8$ $\qquad K_{BC} = 116.6$

$$\Sigma K_B = \Sigma K_C = 290.8$$

$K_{CB} = 116.6$ $\qquad K'_{CF} = 64.8$ $\qquad K'_{CD} = 109.4$

2. <u>Distribution Factors:</u>

$a = 0.378$ $\qquad e = 0.222$ $\qquad b = 0.200$

$c = 0.200$ $\qquad f = 0.222$ $\qquad d = 0.378$

$$X = 0.96$$

3. Moments (Equation 34)

$EM_{BA} = +17.75$ k-in. $EM_{CD} = -17.75$ k-in.

$EM_{BE} = +94.80$ k-in. $EM_{CF} = +42.10$ k-in.

$FEM_{BC} = 0$ $FEM_{CB} = 0$

$EM_{BA} + EM_{BE} + FEM_{BC} = +112.55$ k-in.

$FEM_{CB} + EM_{CF} + EM_{CD} = +24.30$ k-in.

4. Final Moments (Table VI)

$$M_{BA} = +17.75 - \frac{0.378}{0.96}(+112.55) + \frac{0.0756}{0.96}(+24.30) = -24.65 \text{ k-in.}$$

$$M_{BE} = +94.80 - \frac{0.222}{0.96}(+112.55) + \frac{0.0444}{0.96}(+24.30) = +69.90 \text{ k-in.}$$

$$M_{BC} = 0 - \frac{0.36}{0.96}(+112.55) - \frac{0.12}{0.96}(+24.30) = -45.25 \text{ k-in.}$$

$$M_{CB} = 0 - \frac{0.12}{0.96}(+112.55) - \frac{0.36}{0.96}(+24.30) = -23.20 \text{ k-in.}$$

$$M_{CF} = +42.10 + \frac{0.0444}{0.96}(+112.55) - \frac{0.222}{0.96}(+24.35) = +41.67 \text{ k-in.}$$

$$M_{CD} = -17.75 + \frac{0.0756}{0.96}(+112.55) - \frac{0.378}{0.96}(+24.35) = -18.47 \text{ k-in.}$$

Example 2: The effect of nonuniform change in temperature from

$$T_0 = 70^{\circ} \text{ to } \quad T_1 = 120^{\circ}, \quad T_2 = 70^{\circ}$$

in the bridge frame shown in Fig. 7 is investigated. The numerical constants computed in the Example 1 may be used, but new fixed and propped end moments must be calculated.

1. Moments (Equation 41)

$EM_{BA} = -474.00$ k-in. $FEM_{CB} = -316.00$ k-in.

$EM_{BE} = +47.40$ k-in. $EM_{CF} = +21.00$ k-in.

$FEM_{BC} = +316.00$ k-in. $EM_{CD} = +474.00$ k-in.

$EM_{BA} + EM_{BE} + FEM_{BC} = -110.60$ k-in.

$FEM_{CB} + EM_{CF} + EM_{CD} = +179.00$ k-in.

2. Final Moments (Table VI)

$$M_{BA} = -474.00 - \frac{0.378}{0.96}(-110.60) + \frac{0.0756}{0.96}(+179.00) = 416.30 \text{ k-in.}$$

$$M_{BE} = +47.40 - \frac{0.222}{0.96}(-110.60) + \frac{0.0444}{0.96}(+179.00) = +81.30 \text{ k-in.}$$

$$M_{BC} = +316.00 - \frac{0.36}{0.96}(-110.60) - \frac{0.12}{0.96}(+179.00) = +335.00 \text{ k-in.}$$

$$M_{CB} = -316.00 - \frac{0.12}{0.96}(-110.60) - \frac{0.36}{0.96}(+179.00) = -369.60 \text{ k-in.}$$

$$M_{CF} = +21.00 + \frac{0.0444}{0.96}(-110.60) - \frac{0.222}{0.96}(+179.00) = -25.40 \text{ k-in.}$$

$$M_{CD} = +474.00 + \frac{0.0756}{0.96}(-110.60) - \frac{0.378}{0.96}(+179.00) = +395.00 \text{ k-in.}$$

PART VII

SUMMARY AND CONCLUSIONS

The moments due to the effect of temperature in rigid frame bridges were investigated by the algebraic moment distribution. It was shown, that each moment is being formed by a series which is:

a.) Infinite

b.) Convergent

c.) Geometric

The sum of each series is a finite number and is being called the new distribution factor. The new distribution factor is a specific function of each member and is independent of loads or volume change.

The investigation is limited to three and four span bridge frames. The final results are general and precise.

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