ANALYSIS OF TWO COLUMN SYMMETRICAL BENTS WITH EXTERNAL TIES BY THE CARRY-OVER MOMENT PROCEDURE

By

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EXTERNAL TIES BY THE CARRY-OVER
MOMENT PROCEDURE

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INTRODUCTION

A general method of analysis of multi-story, rectangular, two-legged, symmetrical frames with external ties is introduced in this thesis. The recommended procedure is a numerical approximation which may be carried out to a desired accuracy. All derivations are based on the principle of elastic deformation and the slope deflection equations for straight members are introduced as the basis of analysis. From these equations the three joint moment equation is derived and used as a mathematical model for the procedure suggested above.

The thesis is divided into six main parts. The first part is the derivation of the three joint moment equation. The physical interpretation of the elastic constants is explained in the second part. A typical shear equation is written in part three. A procedure of analysis is recommended in part four, and this procedure is applied to a numerical problem in part five. Part six is the summary or conclusion of the thesis.

The material presented in this thesis is the extension of the carry-over moment procedure presented by Tuma(1). The writer became interested in this investigation in Graduate Seminar C.E.-620 (2) where the possibility of this extension was suggested.

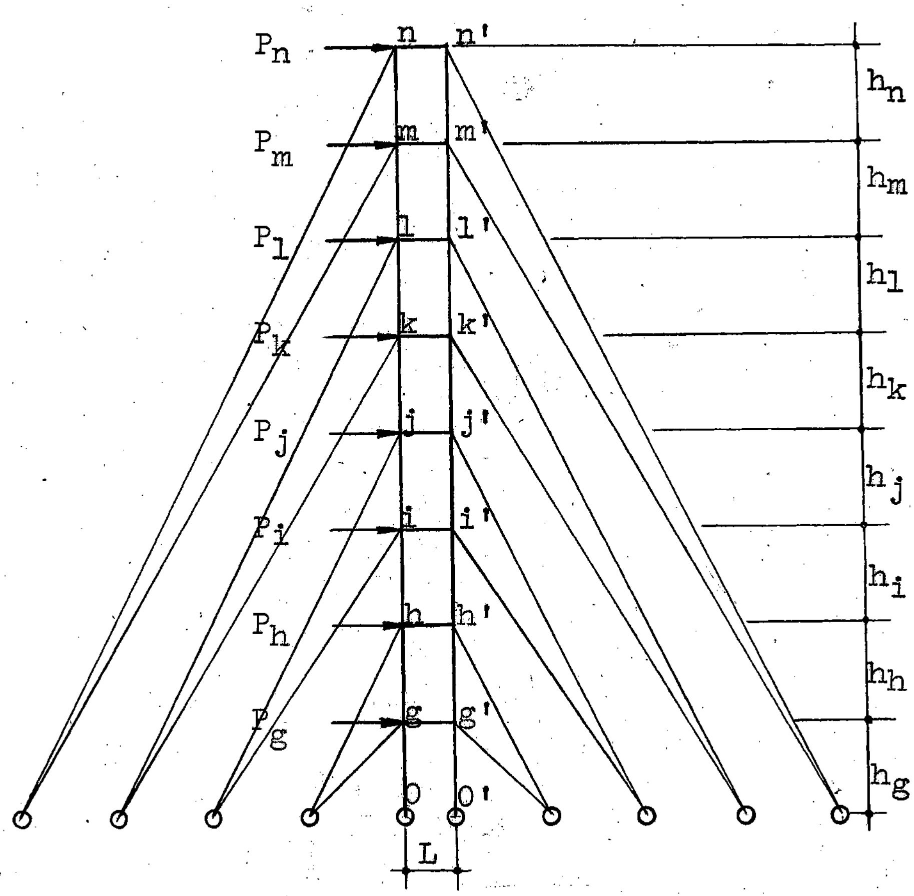
The contribution of the writer is the derivation of the three joint moment equation for this particular problem in connection with the spring constants of the ties. This work is the continuation of research work done by some other graduate students. Gregory (3) investigated rigid frames with joint translation prevented by carry-over slopes and carry-over moments. Sturm (4) considered the application of the carry-over moment procedure to the analysis of multi-story frames and Heller (5) reported the extension of the carry-over moment method to complex frames.

CHAPTER I

GENERAL THREE JOINT MOMENT EQUATION

1-1. Statement of the Problem

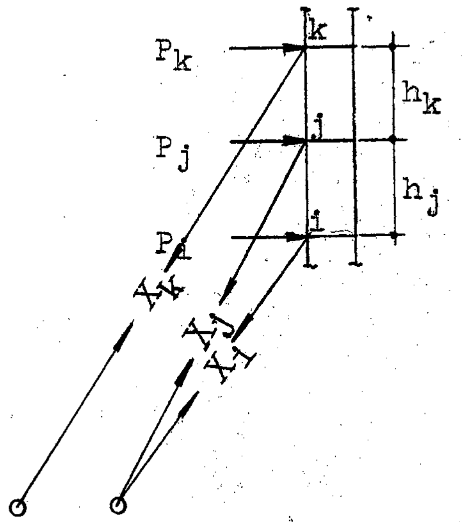
A multi-story frame with external ties and members of constant cross section is considered (Fig. 1-1).



Multi-Story Frame With External Ties

Fig. 1-1

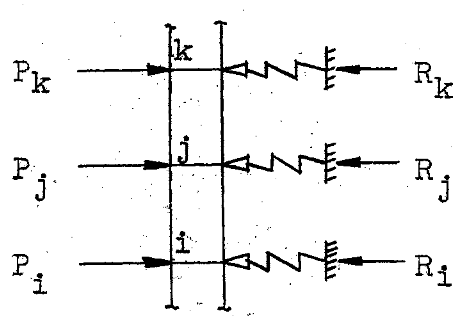
A typical section is removed from the frame (Fig. 1-2). Since the ties cannot take compression, only those in tension are shown.



Typical Section $\overline{ijk}$ of Multi-Story Frame

Fig. 1-2

The ties shown in Fig. (1-2) will be replaced by an equivalent spring (Fig. 1-3).



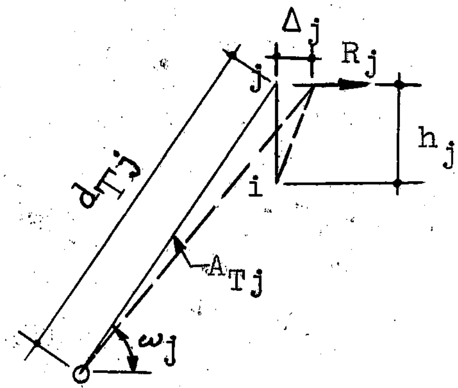
Typical Section $\overline{ijk}$ With Equivalent Springs

Fig. 1-3

The derivation of the three joint moment equation for the section shown in Fig. (1-3) will be made.

1-2. Axial Flexibility of the Tie

The tie at the joint j is selected as a typical tie (Fig. 1-4). The length of this tie is d_{Tj} and the area is A_{Tj}. The modulus of elasticity of the tie is denoted as E.



Tie at Joint j

Fig. 1-4

The normal force of the tie (Fig. 1-4) may be expressed as

$$N_j = \frac{R_j}{\cos\omega_j}$$

where R_j (Fig. 1-3) is the equivalent spring for the tie and ω_j is the angle between the tie and the horizontal (Fig. 1-4).

In order to determine Δ_j (Fig. 1-4), the energy of strain, U_j, is used.

$$U_j = \tfrac{1}{2}N_j \frac{N_j d_{Tj}}{A_{Tj}E} = \tfrac{1}{2}N_j^2 \lambda_j$$

where

$$\lambda_j = \frac{d_{Tj}}{A_{Tj}E}$$

By taking the first partial derivative of the strain energy with respect to the reaction, the value of Δ_j (Fig. 1-4) is obtained.

$$\frac{\partial U_j}{\partial R_j} = \Delta_j = N_j \frac{\partial N_j}{\partial R_j} \lambda_j = \frac{R_j}{\cos\omega_j} \cdot \frac{1}{\cos\omega_j} \cdot \lambda_j$$

The value of R_j is equal to the spring constant for the story j times the value of Δ_j.

$$R_j = C_j \Delta_j \tag{1-1}$$

where

C_j = spring constant for the story j.

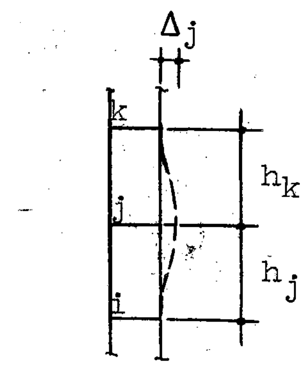
From Eq. (1-1):

$$C_j = \frac{(\cos\omega_j)^2}{\lambda_j} \tag{1-2}$$

The spring constant for any other joint of the structure (Fig. 1-1) may be written by its similiarity to that of C_j.

1-3. Slope Deflection Equations

The elastic curve of the section shown in Fig. (1-3) is assumed (Fig. 1-5).



Elastic Curve of Typical Section $\overline{ijk}$

Fig. 1-5

The slope deflection equations for the removed section (Fig. 1-3) assuming the elastic curve of Fig. (1-5) are as follows:

$$\left.\begin{aligned}
M_{kj} &= \underbrace{4EK_k\theta_k}_{K^*_{kj}} + \underbrace{2EK_k\theta_j}_{C^*_{jk}K^*_{jk}} + \underbrace{6EK_k\frac{\Delta_j}{h_k}}_{FM^*_{kj}} \\
M_{jk} &= \underbrace{4EK_k\theta_j}_{K^*_{jk}} + \underbrace{2EK_k\theta_k}_{C^*_{kj}K^*_{kj}} + \underbrace{6EK_k\frac{\Delta_j}{h_k}}_{FM^*_{jk}} \\
M_{jj} &= \underbrace{6EK_{jj}\theta_j}_{K^*_{jj}} \\
M_{ji} &= \underbrace{4EK_j\theta_j}_{K^*_{ji}} + \underbrace{2EK_j\theta_i}_{C^*_{ij}K^*_{ij}} - \underbrace{6EK_j\frac{\Delta_j}{h_j}}_{FM^*_{ji}} \\
M_{ij} &= \underbrace{4EK_j\theta_i}_{K^*_{ij}} + \underbrace{2EK_j\theta_j}_{C^*_{ji}K^*_{ji}} - \underbrace{6EK_j\frac{\Delta_j}{h_j}}_{FM^*_{ij}}
\end{aligned}\right\} \qquad (1\text{-}3)$$

1-4. Deformation Equations

Using the equivalents shown in Eq. (1-3) the deformation equations become:

$$\left.\begin{aligned}
M_{kj} &= K^{*}_{kj}\,\theta_k + C^{*}_{jk}K^{*}_{jk}\theta_j + FM^{*}_{kj} \\
M_{jk} &= K^{*}_{jk}\,\theta_j + C^{*}_{kj}K^{*}_{kj}\theta_k + FM^{*}_{jk} \\
M_{jj} &= K^{*}_{jj}\,\theta_j \\
M_{ji} &= K^{*}_{ji}\,\theta_j + C^{*}_{ij}K^{*}_{ij}\theta_i + FM^{*}_{ji} \\
M_{ij} &= K^{*}_{ij}\,\theta_i + C^{*}_{ji}K^{*}_{ji}\theta_j + FM^{*}_{ij}
\end{aligned}\right\} \qquad (1\text{-}4)$$

1-5. Three Joint Moment Equation

A new term "joint moment" is introduced. The joint moments at i, j, and k (Fig. 1-3) are:

$$\left.\begin{aligned}
JM_i &= \sum K^{*}_{i}\theta_i \\
JM_j &= \sum K^{*}_{j}\theta_j \\
JM_k &= \sum K^{*}_{k}\theta_k
\end{aligned}\right\} \qquad (1\text{-}5a)$$

From Eqs. (1-5a) the angular rotations of the respective joints in terms of the joint moments are:

$$\left.\begin{aligned} \theta_i &= \frac{JM_i}{\sum K_i^*} \\ \theta_j &= \frac{JM_j}{\sum K_j^*} \\ \theta_k &= \frac{JM_k}{\sum K_k^*} \end{aligned}\right\} \qquad (1\text{-}5)$$

In order for the joint j to be in equilibrium, the sum of all moments at j must be equal to zero.

$$M_{jk} + M_{jj} + M_{ji} = 0 \qquad (1\text{-}6a)$$

Substituting the value from Eq. (1-4) into Eq. (1-6a) gives:

$$\begin{aligned} C^*_{kj}K^*_{kj}\theta_k + K^*_{jk}\theta_j + K^*_{jj}\theta_j + K^*_{ji}\theta_j & \\ + C^*_{ij}K^*_{ij}\theta_i + FM^*_{jk} + FM^*_{ji} = 0 & \end{aligned} \qquad (1\text{-}6b)$$

which may be written as Eq. (1-6c) by substituting the values of θ from Eq. (1-5).

$$\begin{aligned} C^*_{kj}\frac{K^*_{kj}}{\sum K^*_k} JM_k + (K^*_{jk} + K^*_{jj} + K^*_{ji}) \frac{JM_j}{\sum K^*_j} & \\ + C^*_{ij} \frac{K^*_{ij}}{\sum K^*_i} JM_i + \sum FM_j = 0 & \end{aligned} \qquad (1\text{-}6c)$$

Let:

$$M^*_j = - \sum FM^*_j$$

$$-C^*_{kj}\frac{K^*_{kj}}{\sum K^*_k} = -C^*_{kj}D^*_{kj} = r^*_{kj}$$

$$-C^*_{ij}\frac{K^*_{ij}}{\sum K^*_i} = -C^*_{ij}D^*_{ij} = r^*_{ij}$$

Equation (1-6c) may then be simplified to:

$$- r^*_{ij}JM_i + JM_j - r^*_{kj}JM_k - M^*_j = 0 \qquad (1\text{-}6)$$

By solving Eq. (1-6) for JM_j the three joint moment equation (1-7) is obtained.

$$JM_j = r^*_{ij}JM_i + M^*_j + r^*_{kj}JM_k \qquad (1\text{-}7)$$

1-6. Final End Moments

Substituting the values of the angular rotations from Eq. (1-5) into the deformation equations (Eq. 1-4) and simplifying, the final end moments are written (Eq. 1-8).

$$\left.\begin{aligned}
M_{kj} &= D^*_{kj}JM_k - r^*_{jk}JM_j + FM^*_{kj} \\
M_{jk} &= D^*_{jk}JM_j - r^*_{kj}JM_k + FM^*_{jk} \\
M_{jj} &= D^*_{jj}JM_j \\
M_{ji} &= D^*_{ji}JM_j - r^*_{ij}JM_i + FM^*_{ji} \\
M_{ij} &= D^*_{ij}JM_i - r^*_{ji}JM_j + FM^*_{ij}
\end{aligned}\right\} \qquad (1\text{-}8)$$

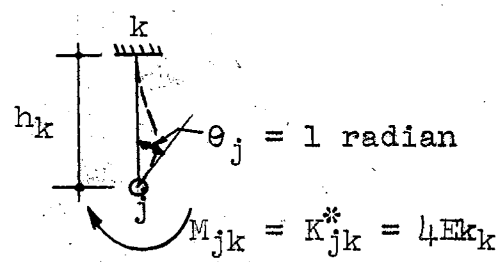
CHAPTER II

ELASTIC CONSTANTS

2-1. Modified Stiffness Factors

a) Columns:

The modified stiffness factor of the column is the moment required to produce a unit rotation of the near end of the member in question if the far end is fixed (Fig. 2-1).



Modified Stiffness Factor of Columns

Fig. 2-1

$$M_{jk} = K^*_{jk} = 4EK_k\theta_j + 2EK_k\theta_k$$

If θ_j is equal to one and θ_k is equal to zero, then

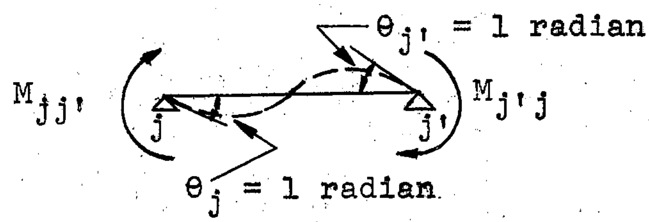
$$K^*_{jk} = 4EK_k$$

From the above it follows that:

$$\left.\begin{aligned} K^*_{kj} &= K^*_{jk} = 4EK_k \\ K^*_{ij} &= K^*_{ji} = 4EK_j \end{aligned}\right\} \qquad (2\text{-}1)$$

b) Girders:

The modified stiffness factor of the girder is the moment required at the near end of a simply supported member to produce a unit rotation of that end (Fig. 2-2).



Modified Stiffness Factor of the Girder

Fig. 2-2

$$M_{jj} = K^{*}_{jj} = 4EK_j\theta_j + 2EK_j\theta_j$$

Since θ_j is equal to 1 radian, then $\theta_{j'}$ is equal to one radian due to anti-symmetry and:

$$K^{*}_{jj} = 6EK_j \qquad (2\text{-}2)$$

2-2. Joint Stiffness

The joint stiffness is the sum of all modified stiffnesses of the near end of members meeting at a joint.

$$\sum K^{*}_{j} = K^{*}_{jk} + K^{*}_{jj} + K^{*}_{ji} \qquad (2\text{-}3)$$

2-3. Modified Distribution Factor

The modified distribution factor is the ratio of the modified stiffness factor of the near end of a member meeting at a joint to the joint stiffness.

$$\left.\begin{aligned} D^*_{kj} &= \frac{K^*_{kj}}{\sum K^*_k} \\ D^*_{ij} &= \frac{K^*_{ij}}{\sum K^*_i} \end{aligned}\right\} \qquad (2\text{-}4)$$

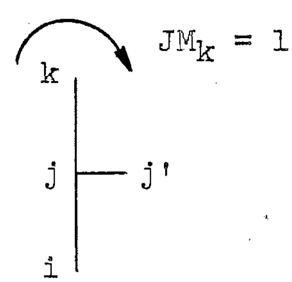
2-4. Modified End Moment Carry-Over Factor

The modified end moment carry-over factor is the moment developed at the near end of a member due to a unit moment applied at the far end of the member.

$$\left.\begin{aligned} C^*_{kj} &= C^*_{jk} = +\tfrac{1}{2} \\ C^*_{ij} &= C^*_{ji} = +\tfrac{1}{2} \end{aligned}\right\} \qquad (2\text{-}5)$$

2-5. Joint Carry-Over Factor

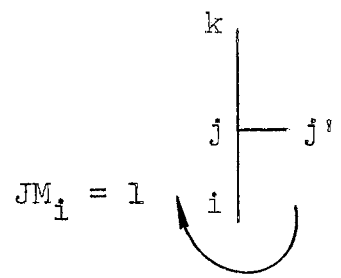
The joint carry-over factor, r^*_{kj}, is the joint moment at j due to a unit joint moment applied at k (Fig. 2-3).



Joint Carry-Over Factor r^*_{kj}

Fig. 2-3

The joint carry-over factor, r^*_{ij}, is the joint moment at j due to a unit joint moment applied at i (Fig. 2-4).



Joint Carry-Over Factor r^*_{ij}

Fig. 2-4

$$\left.\begin{aligned} r^*_{kj} &= -C^*_{kj} D^*_{kj} \\ r^*_{ij} &= -C^*_{ij} D^*_{ij} \end{aligned}\right\} \qquad (2\text{-}6)$$

2-6. Starting Moment

The starting moment is the joint moment equal to the negative sum of all fixed end moments.

$$M^*_j = -\sum FM^*_j = -6EK_k \frac{\Delta j}{h_k} + 6EK_j \frac{\Delta j}{h_j} \qquad (2\text{-}7)$$

2-7. Joint Moment

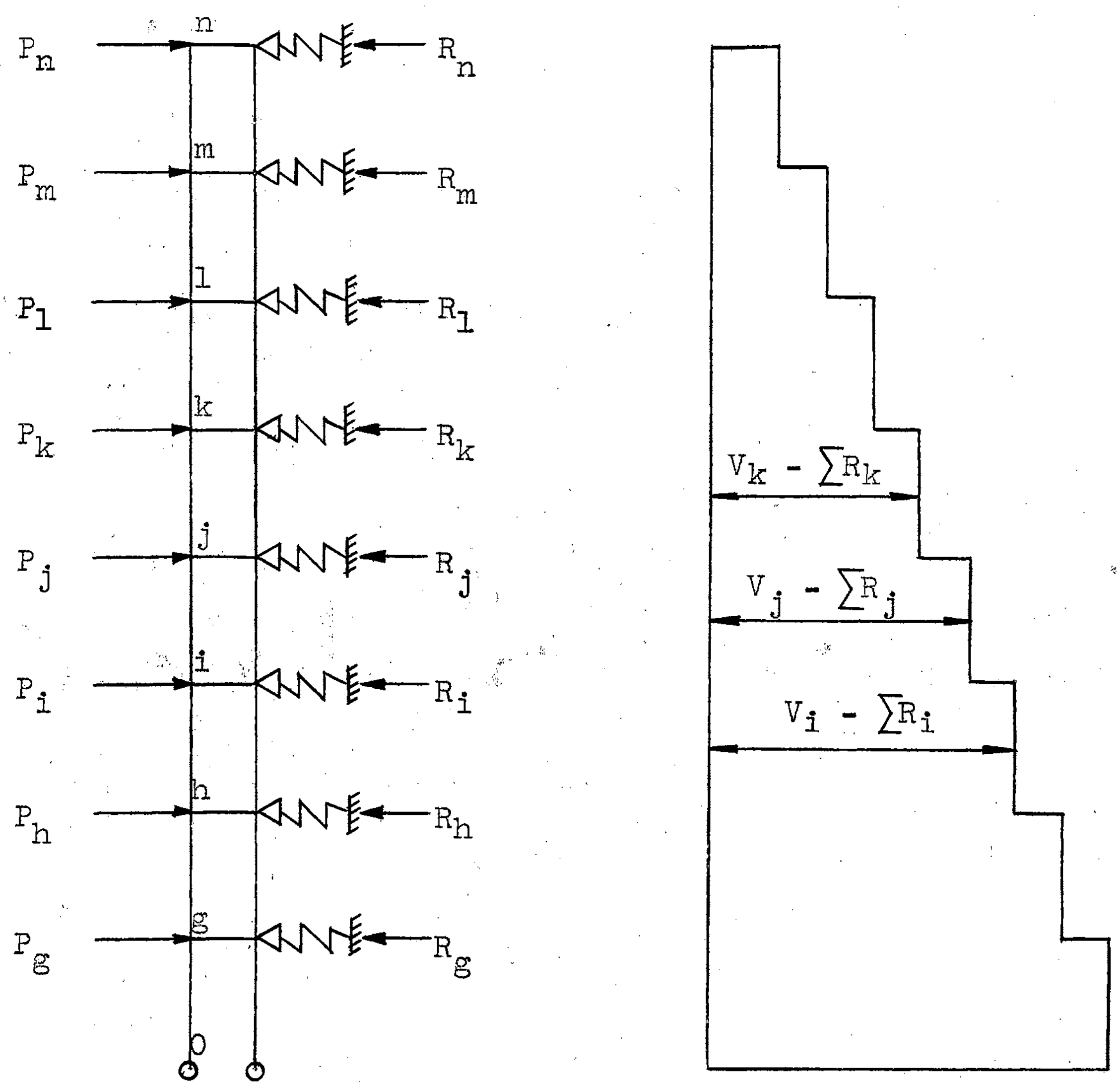
The joint moment is the product of the joint stiffness and the total angle of rotation, θ_j, of the joint.

$$JM_j = \sum K^*_j \theta_j \qquad (2\text{-}8)$$

CHAPTER III

SHEAR EQUATIONS

The shear diagram is shown in Fig. (3-1).



Shear Diagram

Fig. 3-1

The P values shown in Fig. (3-1) are concentrated loads acting at the joints. The R's are described in Fig. (1-3).

The designations used in the shear diagram of Fig. (3-1) are:

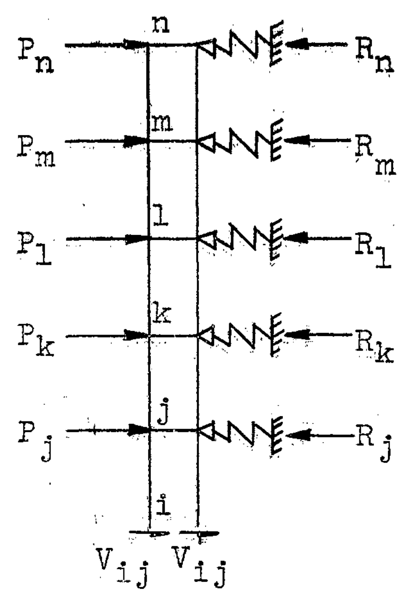
$$V_j = P_n + P_m + P_l + P_k + P_j \qquad (3\text{-}1a)$$

and

$$\sum R_j = R_n + R_m + R_l + R_k + R_j \qquad (3\text{-}1b)$$

The number of shear equations required for a frame of the type shown in Fig. (1-1) is equal to the number of independent translations. One shear equation can be written for each story of the frame.

A typical shear equation is written for the section shown in Fig. (3-2).



Section of Frame Above Story i

Fig. 3-2

In order for the section (Fig. 3-2) to be in equilibrium, the sum of all horizontal forces must be equal to zero.

$$\vec{P}_n + \vec{P}_m + \vec{P}_l + \vec{P}_k + \vec{P}_j + \vec{V}_{ij} + \vec{V}_{ij}$$
$$- \overleftarrow{R}_n - \overleftarrow{R}_m - \overleftarrow{R}_l - \overleftarrow{R}_k - \overleftarrow{R}_j = 0 \qquad (3\text{-}1c)$$

With notations (3-1a, b, and 1-1) and from the anti-symmetry, Eq. (3-1c) becomes:

$$V_j + 2V_{ij} - C_n\Delta_n - C_m\Delta_m - C_l\Delta_l$$
$$- C_k\Delta_k - C_j\Delta_j = 0 \qquad (3\text{-}1)$$

Since there are no loads acting on the members ij or i'j', it follows that:

$$V_{ij} = V'_{ij} = \frac{M_{ij} + M_{ji}}{h_j} \qquad (3\text{-}2)$$

Eq. (3-1) may now be rewritten as:

$$\frac{V_j h_j}{2} + M_{ij} + M_{ji} - \frac{C_n h_j}{2}\Delta_n - \frac{C_m h_j}{2}\Delta_m$$
$$- \frac{C_l h_j}{2}\Delta_l - \frac{C_k h_j}{2}\Delta_k - \frac{C_j h_j}{2}\Delta_j = 0 \qquad (3\text{-}3)$$

All other shear equations are similiar.

CHAPTER IV

PROCEDURE OF ANALYSIS

The procedure of analysis is:

1. Compute:

   a) The spring constants of the ties (Eq. 1-2)

   b) The modified stiffness factors (Eqs. 2-1, 2)

   c) The modified distribution factors (Eq. 2-4)

   d) The joint carry-over factors (Eq. 2-6)

   e) The starting moments (Eq. 2-7)

2. From the starting moments calculate the joint moment in as many tables as there are independent translations.

3. Perform a numerical check of the joint moments obtained (Eq. 1-7).

4. Write the end moments in terms of joint moments and fixed end moments (Eq. 1-8).

5. Write a shear equation for each independent translation (Eq. 3-3).

6. Solve the shear equations simultaneously for the Δ values.

7. Evaluate the results of step 3 in terms of the true values of the Δ's which gives the final end moments.

8. Make a numerical check of the results. The results must give equilibrium of joints as well as equilibrium of shears.

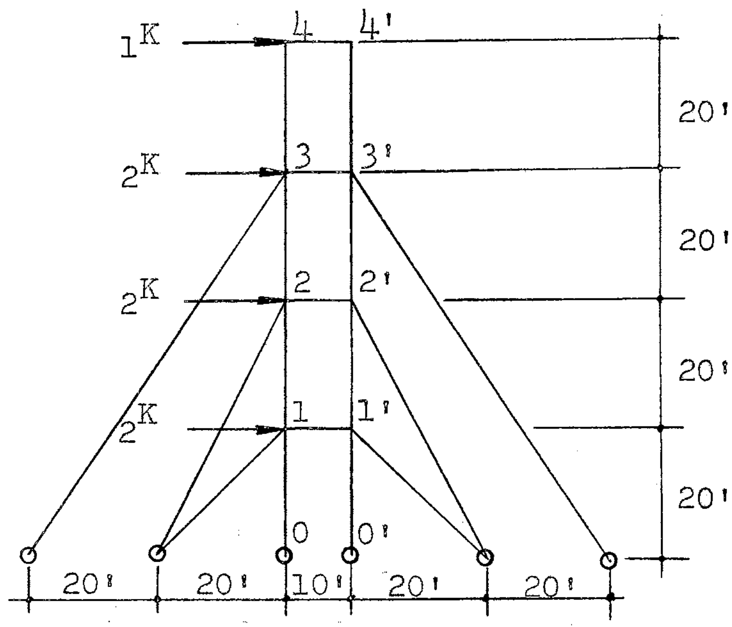
CHAPTER V

EXAMPLE

The application of the principles and procedures derived and discussed in the first four chapters will now be applied to one example problem. All values are given in feet, kips or kip feet.

5-1. Statement of the Problem

A four story frame with external ties and members of constant cross-section loaded as shown (Fig. 5-1) is analyzed.



Four Story Frame With External Ties

Fig. 5-1

A preliminary analysis was made for the frame (Fig. 5-1) and the following members and cable were selected.

a) Columns:

$\overline{01}$ - 10 WF 29

$\overline{12}$ - 10 WF 29

$\overline{23}$ - 10 WF 21

$\overline{34}$ - 10 WF 21

b) Girders:

$\overline{11'}$ - 10 WF 21

$\overline{22'}$ - 10 WF 21

$\overline{33'}$ - 10 WF 21

$\overline{44'}$ - 8 WF 17

c) Cable:

3/4" diameter cable

The modulus of elasticity of the columns and girders is 4,320,000 k/ft.^{2} while that of the ties is 3,460,000 k/ft.^{2}.

Although there would definitely be other loads on this structure, only the wind forces are considered.

5-2. Spring Constants of the Ties (Eq. 1-2)

$\omega_1 = 45^\circ$

$\omega_2 = 63.4^\circ$

$\omega_3 = 56.3^\circ$

$$d_{T1} = \sqrt{(20)^2 + (20)^2} = 28.3'$$

$$d_{T2} = \sqrt{(20)^2 + (40)^2} = 44.8'$$

$$d_{T3} = \sqrt{(40)^2 + (60)^2} = 72.2'$$

$$A_{T1} = A_{T2} = A_{T3} = \frac{(3.14)\ (.75)^2}{(4)\ (144)} = .00306$$

$$\lambda_1 = \frac{d_{T1}}{A_{T1}E} = \frac{28.3}{(.00306)\ (3.46)\ (10)^6} = .00267$$

$$\lambda_2 = \frac{d_{T2}}{A_{T2}E} = \frac{44.8}{(.00306)\ (3.46)\ (10)^6} = .00423$$

$$\lambda_3 = \frac{d_{T3}}{A_{T3}E} = \frac{72.2}{(.00306)\ (3.46)\ (10)^6} = .00682$$

$$C_1 = \frac{(\cos 45^\circ)^2}{.00267} = 187.5$$

$$C_2 = \frac{(\cos 63.4^\circ)^2}{.00423} = 47.5$$

$$C_3 = \frac{(\cos 56.3^\circ)^2}{.00682} = 45.2$$

5-3. <u>Modified Stiffness Factors (Eqs. 2-1, 2)</u>

$K'^{*}_{10} = 1.98E$

$K^{*}_{11'} = 5.32E$

$K^{*}_{12} = K^{*}_{21} = 2.63E$

$K^{*}_{22'} = 5.32E$

$K^{*}_{23} = K^{*}_{32} = 1.77E$

$K^{*}_{33'} = 5.32E$

$K^{*}_{34} = K^{*}_{43} = 1.77E$

$K^{*}_{44'} = 2.82E$

5-4. Joint Stiffness (Eq. 2-3)

$\sum K_1^* = 9.33E$ $\sum K_3^* = 8.86E$

$\sum K_2^* = 9.72E$ $\sum K_4^* = 4.59E$

5-5. Modified Distribution Factors (Eq. 2-4)

$D_{10}^* = .199$ $D_{32}^* = .200$

$D_{11'}^* = .536$ $D_{33'}^* = .600$

$D_{12}^* = .265$ $D_{34}^* = .200$

$D_{21}^* = .271$ $D_{43}^* = .386$

$D_{22'}^* = .547$ $D_{44'}^* = .614$

$D_{23}^* = .182$

5-6. End Moment Carry-Over Factor (Eq. 2-5)

$$C_{12}^* = C_{21}^* = C_{23}^* = C_{32}^* = C_{34}^* = C_{43}^* = +\tfrac{1}{2}$$

$$C_{01}^* = C_{10}^* = C_{11'}^* = C_{22'}^* = C_{33'}^* = C_{44'}^* = 0$$

5-7. Joint Carry-Over Factor (Eq. 2-6)

$$r_{01}^* = r_{10}^* = r_{11'}^* = r_{22'}^* = r_{33'}^* = r_{44'}^* = 0$$

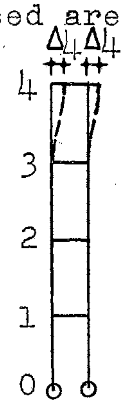
$r_{12}^* = -.133$ $r_{32}^* = -.100$

$r_{21}^* = -.136$ $r_{34}^* = -.100$

$r_{23}^* = -.091$ $r_{43}^* = -.193$

5-8. Starting Moments Due to Δ_4 (Eq. 2-7)

All the joints of the frame are held rigid except joint 4 which is allowed to translate (Fig. 5-2). The only starting moments produced are in member $\overline{34}$.



Elastic Curve of the Frame For Translation Δ_4

Fig. 5-2

$$M_4 = + \frac{(6)\ (30)\ (10)^3\ (1.77)}{(20)\ (12)} \Delta_4 = +1330\ \Delta_4$$

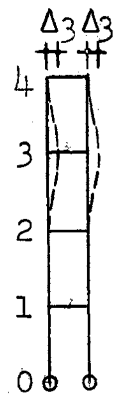
$$M_3 = + \frac{(6)\ (30)\ (10)^3\ (1.77)}{(20)\ (12)} \Delta_4 = +1330\ \Delta_4$$

$$M_2 = 0$$

$$M_1 = 0$$

5-9. Starting Moment Due to Δ_3 (Eq. 2-7)

All the joints of the frame are held rigid except joint 3 which is allowed to translate (Fig. 5-3). Therefore, the only starting moments produced are in members $\overline{23}$ and $\overline{34}$.



Elastic Curve of the Frame For Translation Δ_3

Fig. 5-3

$$M_4 = -\frac{(6)\ (30)\ (10)^3\ (1.77)}{(20)\ (12)}\Delta_3 = -1330\ \Delta_3$$

$$M_3 = -\frac{(6)\ (30)\ (10)^3\ (1.77)}{(20)\ (12)}\Delta_3$$

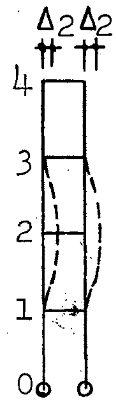
$$+\frac{(6)\ (30)\ (10)^3\ (1.77)}{(20)\ (12)}\Delta_3 = 0$$

$$M_2 = +\frac{(6)\ (30)\ (10)^3\ (1.77)}{(20)\ (12)}\Delta_3 = +1330\ \Delta_3$$

$$M_1 = 0$$

5-10. Starting Moment Due to Δ_2 (Eq. 2-7)

Joint 2 is allowed to translate while the rest of the joints are held rigid (Fig. 5-4). Since fixed end moments exist only in members $\overline{12}$ and $\overline{23}$, there will be a starting moment at joints 1, 2 and 3 due to translation Δ_2.



Elastic Curve of the Frame For Translation Δ_2

Fig. 5-4

$$M_4 = 0$$

$$M_3 = -\frac{(6)\ (30)\ (10)^3\ (1.77)}{(20)\ (12)}\Delta_2 = -1330\ \Delta_2$$

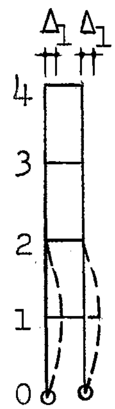
$$M_2 = -\frac{(6)\ (30)\ (10)^3\ (1.77)}{(20)\ (12)}\Delta_2$$

$$+\frac{(6)\ (30)\ (10)^3\ (2.63)}{(20)\ (12)}\Delta_2 = +640\ \Delta_2$$

$$M_1 = +\frac{(6)\ (30)\ (10)^3\ (2.63)}{(20)\ (12)}\Delta_2 = +1970\ \Delta_2$$

5-11. Starting Moment Due to Δ_1 (Eq. 2-7)

As in the three preceding articles, only one joint, joint 1, is allowed to translate (Fig. 5-5). Since member $\overline{01}$ was modified and only joint 1 is allowed to translate, there is a starting moment at joints 1 and 2.



Elastic Curve of Frame For Translation Δ_1

Fig. 5-5

$$M_4 = 0$$

$$M_3 = 0$$

$$M_2 = -\frac{(6)\ (30)\ (10)^3 (2.63)}{(20)\ (12)}\Delta_1 = -1970\ \Delta_1$$

$$M_1 = -\frac{(6)\ (30)\ (10)^3 (2.63)}{(20)\ (12)}\Delta_1$$

$$+\frac{(6)\ (30)\ (10)^3 (2.63)}{(2)\ (20)\ (12)}\Delta_1 = -985\ \Delta_1$$

5-12. Carry-Over Procedure Due to Δ_4 (Table 5-1)

Table 5-1 - Carry-Over Table For Δ_4				
Joint	4	3	2	1
Carry-Over Factors	-.193	-.100 -.100	-.091 -.136	-.133
Starting Moments	+1330	+1330		
1. C.O.M.		- 256		
2. C.O.M.	- 107		- 107	
3. C.O.M.		+ 10		+ 15
		+ 21		
4. C.O.M.	- 3		- 3	
			- 2	
5. C.O.M.				+ 1
Final Joint Moments	+1220	+1105	- 112	+ 16

5-13. Numerical Check (Eq. 1-7)

$$JM_4 = +1330 - .100\ (1105) = +1220\ \Delta_4$$

$$JM_3 = +1330 - .193\ (1220) - .091\ (-112) = +1105\ \Delta_4$$

$$JM_2 = 0 - .100\ (1105) - .133\ (15) = -112\ \Delta_4$$

$$JM_1 = 0 - .136\ (-112) = +16\ \Delta_4$$

5-14. Carry-Over Procedure Due to Δ_3 (Table 5-2)

Table 5-2 – Carry-Over Table For Δ_3				
Joint	4	3	2	1
Carry-Over Factors	→ -.193	← -.100 → -.100	← -.091 → -.136	← -.133
Starting Moments	-1330		+1330	
1. C.O.M.		- 121		- 181
		+ 256		
2. C.O.M.	- 14		- 14	
			+ 24	
3. C.O.M.		- 1		- 1
		+ 3		
Final Joint Moments	-1344	+ 137	+1340	- 182

5-15. Numerical Check (Eq. 1-7)

$$JM_4 = -1330 - .100\ (+137) = -1344\ \Delta_3$$

$$JM_3 = 0 - .193\ (-1344) - .091\ (+1340) = +137\ \Delta_3$$

$$JM_2 = +1330 - .100\ (+137) - .133\ (-182) = +1340\ \Delta_3$$

$$JM_1 = 0 - .136\ (+1340) = -182\ \Delta_3$$

5-16. Carry-Over Procedure Due to Δ_2 (Table 5-3)

Table 5-3 - Carry-Over Table For Δ_2

Joint	4	3	2	1
Carry-Over Factors	→ -.193	← -.100 → -.100	← -.091 → -.136	← -.133
Starting Moments		-1330	+ 640	+1970
1. C.O.M.	+ 133		+ 133	
			- 262	
2. C.O.M.		- 47		- 70
		- 26		
3. C.O.M.	+ 7		+ 7	
			+ 9	
4. C.O.M.		- 1		- 2
		- 1		
Final Joint Moments	+ 140	-1405	+ 527	+1898

5-17. Numerical Check (Eq. 1-7)

$$JM_4 = 0 - .100\ (-1405) = +140\ \Delta_2$$

$$JM_3 = -1330 - .193\ (+140) - .091\ (+527) = -1405\ \Delta_2$$

$$JM_2 = +640 - .100\ (-1405) - .133\ (+1898) = +527\ \Delta_2$$

$$JM_1 = +1970 - .136\ (+527) = +1898\ \Delta_2$$

5-18. Carry-Over Procedure Due to Δ_1 (Table 5-4)

Table 5-4 - Carry-Over Table For Δ_1

Joint	4	3	2	1
Carry-Over Factors	-.193	-.100 -.100	-.091 -.136	-.133
Starting Moments			-1970	- 985
1. C.O.M.		+ 180		+ 268
2. C.O.M.	- 18		- 18	
			+ 95	
3. C.O.M.		- 7		- 11
		+ 3		
4. C.O.M.			+ 1	
Final Joint Moments	- 18	+ 176	-1892	- 728

5-19. Numerical Check (Eq. 1-7)

$$JM_4 = 0 - .100\ (+176) = -18\ \Delta_1$$

$$JM_3 = 0 - .193\ (-18) - .091\ (-1892) = +176\ \Delta_1$$

$$JM_2 = -1970 - .100\ (+176) - .133\ (-728) = -1891\ \Delta_1$$

$$JM_1 = -985 - .136\ (-1892) = -728\ \Delta_1$$

5-20. Final End Moments in Terms of Δ's (Table 5-5)

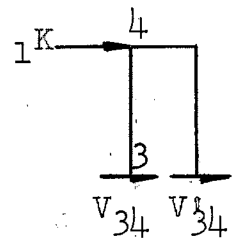
The final end moments in terms of Δ's, which are found by substituting the joint moments and fixed end moments into Eq. (1-8), are written below (Table 5-5).

Table 5-5 - End Moments in Terms of Δ's				
M	Δ_4	Δ_3	Δ_2	Δ_1
44'	+748	- 825	+ 86	- 11
43	-748	+ 825	- 86	+ 11
34	-874	+1098	- 254	+ 31
33'	+663	+ 82	- 843	+ 106
32	+211	-1180	+1097	- 137
23	+ 90	-1072	+1286	- 326
22'	- 62	+ 733	+ 289	-1035
21	- 28	+ 339	-1575	+1361
12	- 11	+ 134	-1395	+1520
11'	+ 8	- 98	+1017	- 390
10	+ 3	- 36	+ 378	-1130

5-21. Shear Equations (Eq. 3-3)

Since there are four independent translations, a shear equation is written for each story level.

a) The first shear equation is written for the section shown in Fig. (5-6).



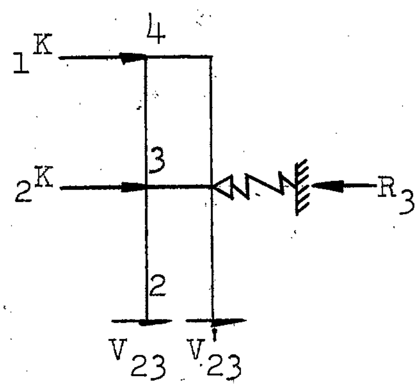
Section of Frame Above Story 3

Fig. 5-6

$$\sum F_x = 0 \quad \Big| \quad \frac{V_4 h_4}{2} + M_{34} + M_{43} = 0$$

$$-1622\ \Delta_4 + 1923\ \Delta_3 - 340\ \Delta_2 + 42\ \Delta_1 = -10$$

b) The second shear equation is written for Fig. (5-7).



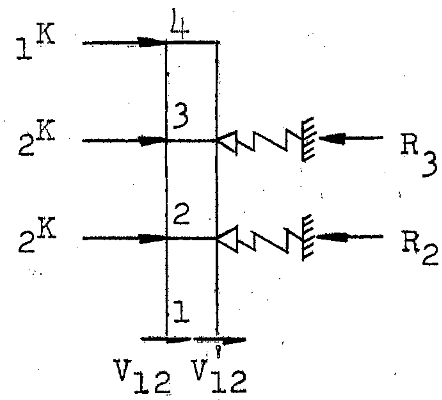
Section of Frame Above Story 2

Fig. 5-7

$$\sum F_x = 0 \quad \Big| \quad \frac{V_3 h_3}{2} + M_{23} + M_{32} - \frac{C_3 h_3}{2} \Delta_3 = 0$$

$$+301\ \Delta_4 - 2704\ \Delta_3 + 2383\ \Delta_2 - 463\ \Delta_1 = -30$$

c) The third shear equation is written for Fig. (5-8).



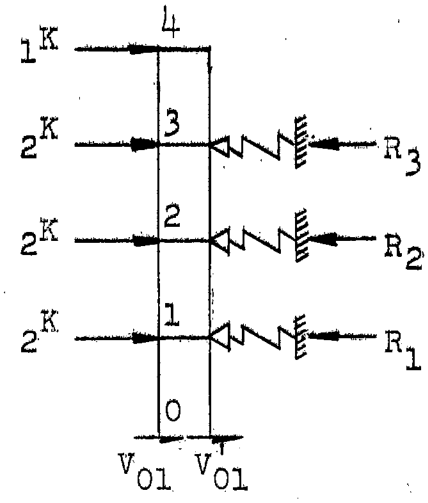
Section of Frame Above Story 1

Fig. 5-8

$$\sum F_x = 0 \quad \Big| \quad \frac{V_2 h_2}{2} + M_{12} + M_{21} - \frac{C_3 h_2}{2}\Delta_3 - \frac{C_2 h_2}{2}\Delta_2 = 0$$

$$-39\,\Delta_4 + 21\,\Delta_3 - 3445\,\Delta_2 + 2881\,\Delta_1 = -50$$

d) The fourth shear equation is written for the section shown in Fig. (5-9).



Section of Frame Above Hinged Supports

Fig. 5-9

$$\sum F_x = 0 \quad \Big| \quad \frac{V_1 h_1}{2} + M_{01} + M_{10} - \frac{C_3 h_1}{2}\Delta_3 - \frac{C_2 h_1}{2}\Delta_2 - \frac{C_1 h_1}{2}\Delta_1 = 0$$

$$+3\,\Delta_4 - 488\,\Delta_3 - 97\,\Delta_2 - 3005\,\Delta_1 = -70$$

5-22. Deformations

Solving the four shear equations simultaneously, the following Δ values are obtained.

$$\Delta_1 = +\ .0162 \qquad \Delta_3 = +\ .0389$$

$$\Delta_2 = +\ .0278 \qquad \Delta_4 = +\ .0470$$

5-23. Final End Moments (Table 5-6)

The final end moments are obtained by multiplying the values of Table (5-5) by their respective Δ values and summing the results algebraically for each point (Table 5-6).

Table 5-6 - Final End Moments

M	M	Due to Δ_4	Due to Δ_3	Due to Δ_2	Due to Δ_1	Final Moments
01	0'1'	0	0	0	0	0
10	1'0'	+ 1.4	- 1.4	+10.5	-18.4	- 7.9
11'	1'1	+ 3.8	- 3.8	+28.3	- 6.3	+22.0
12	1'2'	- 5.2	+ 5.2	-38.8	+24.7	-14.1
21	2'1'	- 1.3	+13.2	-43.8	+22.1	- 9.8
22'	2'2	- 2.9	+28.5	+ 8.0	-16.8	+16.8
23	2'3'	+ 4.2	-41.7	+35.8	- 5.3	- 7.0
32	3'2'	+ 9.8	-45.9	+30.5	- 2.2	- 7.8
33'	3'3	+31.2	+ 3.2	-23.4	+ 1.7	+12.7
34	3'4'	-41.0	+42.7	- 7.1	+ 0.5	- 4.9
43	4'3'	-35.2	+32.1	- 2.4	+ 0.2	- 5.3
44'	4'4	+35.2	-32.1	+ 2.4	- 0.2	+ 5.3

5-24. <u>Final</u> <u>Check</u>

A. Equilibrium of Shears:

1) Shear Equation I

-1622(.047) + 1923(.0389) - 340(.0278) + 42(.0162) + 10 = 0

$-.3 \doteq 0$

2) Shear Equation II

+301(.047) - 2704(.0389) + 2383(.0278) - 463(.0162) + 30 = 0

$-2.2 \doteq 0$

3) Shear Equation III

-39(.047) + 21(.0389) - 3445(.0278) + 2881(.0162) + 50 = 0

0 = 0

4) Shear Equation IV

+3(.047) - 488(.0389) - 97(.0278) - 3005(.0162) + 70.0 = 0

$-.1 \doteq 0$

B. Equilibrium of Joints:

1) $\sum M_1 = 0$	-7.9 + 22.0 - 14.1 = 0
2) $\sum M_2 = 0$	-9.8 + 16.8 - 7.0 = 0
3) $\sum M_3 = 0$	-7.8 + 12.7 - 4.9 = 0
4) $\sum M_4 = 0$	-5.3 + 5.3 = 0

CHAPTER VI

CONCLUSIONS

The general three joint moment equation for a two column symmetrical bent with external ties and members of constant cross section is derived. The analysis of frames of this type by the carry-over procedure is outlined and one example problem is included.

The carry-over moment procedure is a numerical approximation which may be carried out to a desired accuracy. The results obtained by this method are adequate for their application in engineering practice.

The procedure outlined in this thesis is applicable to frames with any number of stories. However, the labor increases with the number of Δ values and becomes prohibitive when the number of Δ values reach six.

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