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SMALL-GROUP LEARNING IN A UNIVERSITY PRECALCULUS COURSE

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APPROVED FOR THE DEPARTMENT OF
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BY

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CHAPTER ONE

INTRODUCTION

Educators strive to do all that they can to help their students learn. There has been much discussion, as well as research, on how best to do that. Most educators would agree that they want students to be actively involved in the learning process. Certainly this is true of mathematics educators. They want their students to think through and understand the procedures and the underlying concepts involved in a mathematical topic, rather than simply to memorize facts and procedures. They want their students to make connections between ideas and concepts and to develop a broader perspective on those ideas and concepts.

Some research has been carried out indicating that active approaches to helping students learn are more effective than approaches in which the student is a more passive recipient of information (Bonwell and Sutherland, 1996). Active learning is conceptualized in various ways. Bonwell and Eison (1991) indicated some characteristics likely to be common to most ideas of active learning. These include:

- Students are involved in more than listening;
- Less emphasis is placed on transmitting information and more on developing students' skills;
- Students are engaged in activities (e.g., reading, discussing, and writing); and

- Greater emphasis is placed on students' exploration of their own attitudes and values.

Active learning has been described as "...anything that 'involves students in doing and thinking about the things they are doing' " (Bonwell and Eison, 1991).

Instructors can use many different teaching strategies in the classroom to encourage students to take a more active part in their own learning. Cooperative learning is one such strategy. Increasing numbers of instructors are using cooperative learning strategies as means of involving students more with the learning process.

Students in college mathematics classrooms come from diverse social and educational backgrounds. They bring with them varied mathematical abilities and varied levels of confidence in their ability to learn mathematics. These students also have some characteristics in common. They are social beings. They have been talking for sixteen years or more, and using conversations to learn about the world in which they live. Teachers who use cooperative learning methods are striving to make use in a positive way of the differences as well as the similarities in the student population. Some believe that cooperative learning can lead to improved learning as well as more positive attitudes, under certain circumstances and for certain types of learning.

The more traditional mode of instruction in the college mathematics classroom has been the lecture. In the lecture, the instructor acts as the dispenser of knowledge and students act as the recipients of this knowledge. Lecturing has a number of strengths (Bonwell, 1996, p. 32). Some of these include:

- Lectures can present large amounts of information.
- Lectures allow the instructor maximum control of the learning process.
- Lectures present little risk for students.
- Lectures appeal to those who learn best by listening.

Lecturing also has some serious limitations. Some of the disadvantages of lecturing include the following:

- In lectures, students are often passive because there is no mechanism to ensure that they are intellectually engaged with the material.
- Students' attention wanes quickly after fifteen to twenty-five minutes.
- Information tends to be forgotten quickly when students are passive.
- Lectures emphasize learning by listening, which is a disadvantage for students who have other learning styles.

Students may be accustomed to learning from a lecturer, and may feel that there is less risk in such an environment. However, instructors seem clearly to have a responsibility to students to do all that those instructors can do to help the students learn. It seems likely logically and from discussion and research that students may learn more when they are actively involved with their learning.

Fogarty and Bellanca (1992) discuss a movement in classrooms towards new interaction models of learning that put the focus on the learner rather than the lecturer. They view the “traditional stand-up teaching model” as being at one end of a spectrum involving cooperative interactions. They view the “new school lecture” as being towards the other end of the spectrum, with more cooperative interaction between students. They mention the difficulty for teachers in making such a shift, and suggest:

> [T]he move toward the new school ‘lecture’, with its accent on student interactions, is made easier if seen as a gradual change. Student involvement is designed so that strategies increase student participation by degrees. In this way, teachers and students are able to adjust and adapt to the new model over time. Surprisingly and almost unfailingly, once the philosophical shift begins, once teachers begin implementing cooperative interactions, the evidence of student motivation becomes so overwhelmingly visible that teachers are encouraged to try more. ... Teachers using cooperative interactions in the classroom say the positive effects on student motivation, achievement, and self-concept are so immediately visible and so astonishingly dramatic that the incentives are there for novices to do more. (p. 84-86)

Some instructional situations seem to limit how much advantage individual class instructors can take of cooperative learning strategies. In some college mathematics courses, instructors are expected to cover a certain amount of material, so that the students are prepared for higher-level courses. Further, some college classes are taught using uniform sections, in which a course coordinator structures the course material so that students in all sections cover the same terms, concepts, and examples as well as taking identical exams throughout the semester. These two situations seem clearly to lead to constraints in what can be done with cooperative learning methods. However, it may be possible to use a form of cooperative learning in such classrooms despite those constraints.

This study will examine the potential benefits of implementing small-group work in a modest way both inside and outside of a university mathematics classroom in a situation in which the course both intends to prepare students for further courses and is uniformly structured by someone other than the instructor. As will be discussed in Chapter 2, research has shown that cooperative learning can lead to improved learning as well as more positive attitudes, under certain circumstances and for certain types of learning. This study intends to see if these benefits can be realized within the context of an instructional situation constrained as described above and with minimal intervention by the instructor. This intention leads to the research questions proposed for this study.

Research Questions

As will be seen, considerable research has been done on the use of cooperative learning in mathematics classrooms. Much of the research has been done using long-term groups, involving major changes in the structure of the classroom procedures, grading system, and assignments. However, there are difficulties with implementing major change in the way that mathematics is taught. Many of our students have been taught in traditional classrooms for years. Many instructors have taught in a traditional way for years. Change can be unsettling and disturbing, to both students and instructors. Many classroom situations are constrained by the amount of material that students must master and/or by external setting of goals, content, and assessment.

One benefit of the lecture style of teaching is that the instructor feels more in control of the pace of instruction. In situations where there is a set amount of material to be covered, the instructor may be more comfortable with lecturing, especially if the total amount of material to be covered is extensive and seems to require a rapid pace. Much of the research that has been done with cooperative learning in college mathematics classes has involved major changes in classroom teaching techniques as well as assessment methods. It is an open question as to whether modest changes implemented in uniform sections can make a significant difference in improving students' academic achievement and attitudes. This is the central question that will be investigated in this study.

Behind this study is an interest in the use of cooperative learning to help students learn mathematics by helping each other. This includes an interest in how cooperative learning can help students with different ability levels. It is also concerned with whether cooperative learning may lead to improved academic achievement on certain types of problems. However, the present study seeks to pursue those interests with a small intervention in a constrained instructional setting to see if the putative benefits of cooperative learning can be realized in that context.

This research will look at several research questions. These include the following central questions:

1. Will students who are involved in small-group work in class as well as outside the classroom (working on daily homework assignments) do as well on uniform multiple-choice exams as students who work on the same assignments individually? Will they do as well in their final course grade? That is, will students do as well in externally imposed assessment tasks in a constrained situation with minimal intervention to foster small-group work?
2. Does the use of small-group cooperative learning provide equal benefits for high, middle, and low achievers in comparison to their counterparts in control groups, particularly in constrained situations with minimal intervention?
3. Does the use of small-group cooperative learning lead to improvement for various types of mathematical problems, particularly in a constrained situation with minimal intervention?
4. Does the use of small-group cooperative learning lead to improved attitudes, attendance, or retention even in a constrained situation with minimal intervention to foster small group cooperative learning?

CHAPTER TWO

REVIEW OF THE LITERATURE

What is essential to understanding cooperative learning? Johnson and Johnson (1995) list four topics critical to understanding the importance of cooperative learning:

1. The theoretical foundation of cooperation
2. Research validating and refining the theory
3. Practical uses of cooperative efforts in education
4. Future prospects of cooperative learning

The first two of these topics will be discussed in this chapter since they seem relevant to the concerns addressed in this study. The third topic is interesting but beyond the immediate concerns of this study and so will not be addressed. The fourth topic will be discussed only in the context of Chapter 5 in which attention is turned to conclusions that can be drawn from the present study.

The Theoretical Foundation of Cooperation

As was stated earlier there has been considerable research done on cooperative learning in a variety of settings. Much of this research can be grounded in three general theoretical perspectives — the perspective of cognitive developmental theory, the perspective of behavioral learning theory, and the perspective of social interdependence theory (see Johnson and Johnson, 1995).

Cognitive Developmental Theory.

Piaget and Vygotsky initially developed cognitive developmental theory. Both were brilliant psychologists, both born in 1896. Although the field of cognitive development expanded considerable in the twentieth century, both of these founders continue to be the central figures behind this theoretical perspective. Their works are often presented as being at opposite ends of a spectrum but they actually have much in common. Piaget is generally seen as a cognitive psychologist who was interested in an individual child's thought processes and learning patterns. Vygotsky is seen as a social psychologist whose main focus was on knowledge as a social construct. However, in the area of the social aspect of learning and thinking the works of both men contain much in common (Smith, 1996).

Piaget, discussing an individual in the process of learning, states:

> The subject must be active, must transform things, and find the structure of his own actions on the objects. When I say 'active', I mean it in two senses. One is acting on material things. But the other means doing things in social collaboration, in a group effort. This leads to a critical frame of mind, where children must communicate with each other. This is an essential factor in intellectual development. (Ripple and Rockcastle, p. 4)

Clearly Piaget emphasizes a child's activity in learning but, equally clearly, he emphasizes "social collaboration."

In *The Moral Judgment of the Child* (1948), Piaget discusses cooperation and peer interaction further. He says:

> It is idle ... to try and transform the child's mind from outside, when his own taste for active research and his desire for cooperation suffice to ensure a normal intellectual development. The adult must therefore be a collaborator and not a master .... All moral and all logical norms are the result of cooperation. Let us therefore try to create in the school a place where individual experimentation and reflection carried out in common come to each other's aid and balance one another. (p. 412)

Again Piaget here stresses the centrality of cooperation to development and emphasizes that this centrality must be realized in a school setting. What Piaget postulates for the development of younger children may likely be a part of their continued development at later ages as well.

Duveen (1997) discusses Piaget's writings, saying:

> In *The Moral Judgement of the Child* (1932) Piaget makes a fundamental distinction between two forms of acquiring social knowledge. On the one hand there is knowledge which he describes as the product of social transmission, where it is the authority of a dominant or privileged figure which is the source of knowledge. As against this Piaget also argues that there is knowledge which is acquired through cognitive elaboration in a process of reconstruction. ... The latter ... can only occur in autonomous

> relations between equal partners, where each has the freedom to engage in argument and debate. (p. 74)

By Duveen's interpretation of Piaget's work, social transmission of knowledge by a dominant authority (for example, a lecturer) must work complementarily with a different process of development that involves partnership, argument, and debate. While this may not at first seem to be "cooperative" learning, this is precisely the social dimension of learning that is made real in small-group work and cooperative learning strategies.

Vygotsky's work is built around the notion that knowledge is developed through social activity. In *Mind in Society* (1978), Vygotsky discusses the role of social interaction in learning, as well as his idea of the "zone of proximal development." In particular, he states the following:

> Every function in the child's cultural development appears twice: first, on the social level, and later, on the individual level; first, between people (inter-psychological), and then inside the child (intra-psychological). This applies equally to voluntary attention, to logical memory, and to the formation of concepts. All the higher functions originate as actual relations between human individuals. (p. 57)

This view of individual development obviously puts a premium on social interaction as essential to growth.

Vygotsky goes on further to formalize the difference between what a child can do on his or her own in problem solving and what that same child can do in social interaction with a teacher or with more capable peers. He writes:

> When we determine a child's mental age by using tests, we are almost always dealing with the actual developmental level. In studies of children's mental development it is generally assumed that only those things that children can do on their own are indicative of mental abilities. ... Over a decade even the profoundest thinkers never questioned the assumption; they never entertained the notion that what children can do with the assistance of others might be in some sense even more indicative of their mental development than what they can do alone. ... When it was first shown that the capability of children with equal levels of mental development to learn under a teacher's guidance varied to a high degree, it became apparent that those children were not mentally the same age and that the subsequent course of their learning would obviously be different. This difference ... is what we call the zone of proximal development. It is the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers. (p. 85-86)

Vygotsky went so far as to make the social dimension of learning the criterion that distinguishes the intellectual growth of children from that of animals and thus is a central characteristic of human intellectual development. He writes:

> A primate can learn a great deal through training by using its mechanical and mental skills, but it cannot be made more intelligent, that is, it cannot be taught to solve a variety of more advanced problems independently. For this reason animals are incapable of learning in the human sense of the term; human learning presupposes a specific social nature and a process by which children grow into the intellectual life of those around them. (p. 88)

Baloche (1998) offers the comments of a high school chemistry student that illustrate this "zone of proximal development" and the potential of peer interaction. She presents the student as saying, "Sometimes, coming from the teacher it is a lot more technical. I know they try to bring it down to your level, but when you do it with your friends, you can just say 'Well, I don't understand' and they can rephrase it and they can help you." (p. 4)

Certainly both Piaget and Vygotsky emphasized the social dimension of human intellectual development and learning. While their theoretical developments may have differed, both found social interaction essential to learning according to their theories. As founders and shapers of the cognitive development perspective, those beliefs seem likely to carry into the work of later cognitive developmental researchers. In this sense, even when not talking about cooperative learning strategies in the modern or narrow sense, they discuss them in general and theoretical terms that offer strong support for the essential idea behind the more modern strategies.

Behavioral Learning Theory.

Moving to a very different perspective, behavioral learning theory with its emphasis on conditioned responses to stimuli would seem to be far from amenable to ideas of social and cooperative learning. However, while cooperative learning may be less commonly associated with behavioral learning theory, this theory does give useful insights and information about group processes and learning. The psychological theory of imitation and modeling are areas of behaviorism that can be used to guide research on cooperative learning.

Bandura (1986), discusses his views of modeling and social cognition theory. He writes:

> In the social cognitive view people are neither driven by inner forces nor automatically shaped and controlled by external stimuli. Rather, human functioning is explained in terms of a model of triadic reciprocality in which behavior, cognitive and other personal factors, and environmental events all operate as interacting determinants of each other. (p. 18)

In spite of the jargon, the essential point seems to be that that individuals are not shaped by their own independent or internal development but rather through this along with their interrelationships with others. Thus, even from a behavioral perspective, a social dimension of learning seems essential.

Bandura goes on to write:

> The external facilitators of modeling mentioned earlier accelerate observational learning of judgmental rules, as do the conceptual skills of observers. Children who have some facility at mastering concepts are quicker at inferring judgmental rules from modeled actions than those who lack conceptual skills (Ito, 1975; Sukemune, Haruki, and Kashiwagi, 1977). Of particular interest, however, is evidence that modeling improves conceptual functioning, even in children who are lacking such cognitive skills. With further aids to modeling, such children might well approximate the attainments of the more developmentally advanced. In short, the level of cognitive skill should be regarded as a reciprocally contributing influence, that is itself improvable by social learning, rather than simply a limiting condition in observational learning. (p. 102)

Again the emphasis is on learning through modeling, even for children who are not developmentally advanced, and that this type of social learning can overcome barriers to learning so that the level of individual cognitive skill is not an unchangeable barrier that prevents individual growth when the individual is in social settings. Again this theorizing is in the context of younger children, but principles of the role of social interaction in development and learning seem unlikely to terminate suddenly before students reach college age. These dynamics may be subtler but seem likely to have some continuity even into the learning of college students.

Kelly (1982) also discusses the observational learning process and modeled learning. He points out factors that improve learning from modeled behavior. He lists six factors that facilitate observational learning, writing:

Some of the factors that appear to facilitate observational learning include:

1. Age of the model, particularly in childhood and adolescence. Children are most likely to imitate the behavior of a model similar in age to the observer or slightly older. Social behaviors exhibited by younger models are less likely to be imitated.
2. Sex of the model, with models of the same sex as the observer exerting a stronger influence than opposite-sex models.
3. Likeability of the model, with models high in warmth and affectionate characteristics more salient in influence than cold, unaffectionate-appearing models.
4. Perceived similarity to the observer. If an observer perceives or is told that a model is similar to himself or herself, a greater degree of imitative learning will occur than if the model is seen as highly dissimilar.

5. Observed consequence to the model when the model engages in the social behavior. If the observer watches a model engage in a social behavior and also sees that the model achieves a positive outcome as a result of it, there is increased likelihood that the observer will imitate that behavior. This is termed 'vicarious reinforcement', since the observer sees the reinforcing consequence achieved by another person. On the other hand, observed (vicarious) punishment of a model's social behavior decreases the likelihood of imitative behavior.
6. The observer's own direct learning history for engaging in the same or similar social behavior as seen in the model. In many cases, the observer has had some direct personal experience handling situations similar to those in which the model is seen. The observer may have engaged in similar social responses as well. If the observer has a personal history of being rewarded for behaviors similar to those now exhibited by the model, it is more likely that the observer will actually exhibit the modeled social behavior than if the observer has been personally punished for behaviors now seen in the model. (p. 18-19)

This approach emphasizes the similarity of the "peer" model for behavioral learning, with increased similarity likely to facilitate learning by observing the peer. It also emphasizes the importance of the observer's personal experience and similarity to the one observed for the observer's learning. Certainly these conditions seem likely to continue into play with college age students in cooperative and small-group learning situations in which they can both observe and interact with close peers as they tackle mathematical tasks.

Johnson and Johnson (1995) mention the influences of the noted behaviorist B. F. Skinner and his writings on group contingencies. Skinner (1968) discusses contingencies, writing:

> Three variables compose the so-called contingencies of reinforcement under which learning takes place: (1) an occasion upon which behavior occurs, (2) the behavior itself, and (3) the consequences of the behavior. ... Special techniques have been designed to arrange what are called contingencies of reinforcement—the relations which prevail between behavior on the one hand and the consequences of that behavior on the other – with the result that a much more effective control of behavior has been achieved. ... So far as we are concerned here, teaching is simply the arrangement of contingencies of reinforcement. (p. 4-9)

At first glance, this seems to offer little to do with cooperative learning. However, in shaping the "consequences of the behavior", social interactions can play an important role in reinforcement that shapes behavioral development. This may well be the fundamental theory underlying what others from the behavioral perspective emphasize in discussions of modeling and observational learning.

We will not go into more detail on the works of Skinner. While he does not specifically address cooperative learning, he offers much to the ideas of shaping behavior in the individual as well as the group. He does make an interesting comment on one alternative to cooperative learning. He writes,

> Those who advocate competition as a useful social motive may wish to use the reinforcements which follow from excelling others, although there is the difficulty that in this case the reinforcement of one child is necessarily aversive to another. (1968, p. 20)

This at least implies that he regarded children's interactions with each other as relevant to reinforcing behavior and to learning and change. His comment does not deny the importance or even inevitability of a social dimension to behavioral learning but rather singles out the aversive effects inherent in competitive rather than cooperative learning.

Social Interdependence Theory.

Social interdependence theory is not as well known as cognitive developmental theory or behavioral theory. However, Smith and MacGregor (1992, p. 12) state that "...cooperative learning is based on the social interdependence theories of Kurt Lewin and Morton Deutsch (Deutsch, 1949; Lewin, 1935)." Johnson and Johnson (1995, p. 206), broadening this sentiment, state, "While the cognitive developmental and behavioral orientations have their followings, by far the most important work dealing with cooperation is social interdependence theory."

In his dissertation entitled *A Theory of Co-operation and Competition*, Deutsch (1947) described a theory of the effect of cooperation and competition upon small-group functioning. His basic premise was that the type of interdependence (positive, negative, or none) in a situation determines how people interact with each other. He derived some psychological implications from the concepts of cooperation and competition. He used the notions of cathexis (investment of mental or emotional energy in a person, object, or idea), inducibility (ability to be moved by persuasion or influence), and other concepts to develop a number of hypotheses concerning the effects of cooperation and competition on group process.

Deutsch's dissertation was quite theoretical in nature. However, the paper was written to provide a background for future experimental studies of the effects of cooperation and competition on small group functioning.

One of his students, D. W. Johnson, has done a great deal of research on cooperative learning and social interdependence. He has written volumes on the potential practical applications of the theory to education. In an interesting commentary on the 1995 article by Johnson and Johnson, Deutsch (1995) praises them for their work showing the positive effects of promotive interactions on student achievement, interpersonal relations, and psychological health. He also cautions:

> However, I note that the skills involved in teaching cooperative learning well are only acquired with considerable effort and time. ... [I]t takes much experience for people to acquire the knowledge, attitudes, and skill required to be effective cooperative members of the various groups to which they belong. The Johnsons rightly stress the many benefits to be derived from cooperation, but they do not emphasize sufficiently their realization of how much persistent, intelligent effort is required to develop and sustain effective cooperation. The Johnsons and I would surely agree that the effort is very worthwhile. (p. 257)

While this research of Deutsch and the Johnsons certainly argues for the relevance and importance of cooperative learning, there is a warning here of direct relevance to the research questions of this study. If developing effective cooperative groups takes time and practice, it may be that such groups are unlikely to effectively develop in constrained situations or with minimal interventions designed to foster such groups. This type of somewhat spontaneous cooperative group formation seems distant from that type envisioned by these researchers.

Research on Cooperative Learning

Research has been taking place on the relative effects of cooperation, competition, and individual efforts on learning since the 1920's (Johnson, Johns, Holubec and Roy, 1984). In particular, a great deal of research on cooperative learning has been conducted in the past 30 years, at all grade levels, with students of varied ethnicity and academic achievement level and in most school subjects. Slavin (1995) suggests that cooperative learning is one of the most extensively evaluated of all instructional innovations.

Interest in cooperative learning among college educators is also growing rapidly. This researcher conducted an ERIC search of “Cooperative Learning in Higher Education”. Table 2.1 shows the number of citations per year from 1985 to 1998.

Table 2.1. ERIC Citations for Phrase

“cooperative learning in higher education”

Year	‘85	‘86	‘87	‘88	‘89	‘90	‘91	‘92	‘93	‘94	‘95	‘96	‘97	‘98
Citations	36	37	45	66	109	138	153	177	207	232	236	236	229	243

While many of the articles listed are not experiments involving small groups and control groups, there is much thought and investigation that has taken place about the effectiveness of cooperative learning in colleges and universities in a variety of situations and on a variety of subjects. A cursory check of these citations seems to indicate that few of these articles involve studies with experimental groups and a control group. This researcher also conducted an ERIC search of "Cooperative Learning in Higher Mathematics Education" and found 156 citations. In a non-rigorous investigation of the quasi-experimental studies found among those citations, most showed either significant positive results of cooperative learning or else non-significant results with slightly higher achievement results for the cooperative groups.

Areas of Agreement Among Researchers

There is general agreement among reviewers of the cooperative learning literature about the potential positive effects on student achievement. Slavin (1992) discusses four reviews of cooperative learning research. He concludes from these reviews that:

> [C]ooperative learning methods can and usually do have a positive effect on student achievement. ... [T]here is almost as strong a consensus that the achievement effects are not seen for all forms of cooperative learning but depend on two essential features, at least at the elementary and secondary levels. One of these features is group goals, or positive interdependence: The cooperative groups must work together to earn recognition, grades, rewards, and other indicators of group success. Simply asking students to work together is not enough. The second essential feature is individual accountability: The groups' success must depend on the individual learning of all group members. (p. 97)

These conclusions do not bode well for the research questions investigated in the current study. First, there is unlikely to be recognition of group success in informal group work. Second, the groups are essentially being asked to work together, something that Slavin says is not enough to produce achievement effects. Finally, there are no real provisions for group accountability that will apply so that individual or group success is a contingency of course success.

Slavin (1995) conducted a review of cooperative learning in which small groups of elementary or secondary students worked together to learn. The review involved ninety studies, some of which used multiple comparisons. In order to be included in the review, a study had to meet a set of criteria. The criteria were as follows:

1. [S]tudies had to evaluate forms of cooperative learning in which small groups of elementary or secondary students worked together to learn.
2. Studies had to compare cooperative learning with control groups studying the same material.
3. Evidence had to be given that experimental and control groups were initially equivalent.
4. A study had to take at least four weeks (or twenty hours).
5. Achievement measures had to assess objectives taught in experimental as well as control classes. (p. 20)

A total of ninety-nine separate comparisons of cooperative learning and control methods were discussed in the review. Sixty-three (64 percent) of these comparisons significantly favored cooperative learning. Five (5 percent) of these comparisons significantly favored the control groups.

Davidson (1985) also conducted a selective review of the research that had been done on the use of small-group learning in mathematics. He states: "Considering all the studies comparing student achievement in small-group instruction and traditional methods in mathematics, the majority showed no significant difference. When significant differences were found, they almost always favored the small-group procedure." (p. 224)

There is also agreement on the potential positive effects on such affective outcomes as inter-group relations, acceptance of mainstreamed students, self-esteem, and attendance. Slavin (1995) states:

> Although not every study has found positive effects on every non-cognitive outcome, the overall effects of cooperative learning on student self-esteem, peer support for achievement, internal locus of control, time on-task, liking of class and of classmates, cooperativeness, and other variables are positive and robust. (p. 70)

These results seem somewhat mixed on the cognitive effectiveness of small group, cooperative work. The only certainty of effectiveness seemed to come from carefully controlled studies that met criteria unlikely to be met with the constrained conditions of the present study. There was further agreement on the non-cognitive positive effects of such cooperative work but, again, these may be difficult to realize with the minimal intervention in the present study.

Areas of Disagreement Among Researchers

There was also some controversy concerning cooperative learning research (Slavin, 1992). One concern is whether cooperative learning is as effective in colleges and universities as it is in lower grade levels. There has not been as much research done in Grades 10 and higher as there has for the lower grades, and the results are less consistent than those at the lower grades are. In a fairly extensive search of books on cooperative learning, this researcher found a number of studies (Sherman and Thomas, 1986; Fraser et al., 1977; Chang, 1977; Brechting and Hirsch, 1977) that indicated positive results of cooperative learning in senior high school and college settings.

Another area of debate is the effectiveness of cooperative learning for higher-order conceptual learning. Again, most research on cooperative learning has focused on basic skills. However, there are studies that showed significant positive results in creative writing as well as on higher-order understanding in social studies (Slavin, 1992).

A third issue is whether group goals and individual accountability are necessary at the college level in order for cooperative learning to work. Davidson (1985) cites several studies that resulted in significant differences in achievement (favoring the students working in groups) without the use of group rewards.

Summary

The result of this review of the relevant studies and theoretical perspectives is mixed. The survey of theoretical perspectives suggests that there are basic mechanisms of growth, development, and learning that should continue into work with college students. However, there is a considerable gulf between these theoretical perspectives and practical research on cooperative learning.

The review of studies of cooperative learning suggests that under appropriate conditions, such efforts can be effective both in improving achievement and attaining non-cognitive goals. However, the appropriate conditions appear to be quite demanding. They may well not be realized in the present constrained situation with minimal intervention to foster cooperative work and little accounting of group work.

Further, there are concerns expressed in the literature about the effectiveness of such cooperative work at the level of higher education, for more complex learning, and without an accountability component in the effort. All of these concerns would weigh against the hope of success in the present intervention. However, only an empirical investigation can finally verify whether the minimal interventions to be undertaken in the proposed constrained situation of uniform examinations and a "packed" syllabus will limit the effectiveness of cooperative learning.

CHAPTER THREE

METHOD

The experimental study undertaken for the present investigation involved a research protocol of limited intervention in a context that was heavily constrained and weighted against the success of cooperative learning. This situation essentially is an acid test of whether a "bare bones" approach to fostering cooperative learning can succeed in a situation in which it faces virtually all barriers to success that have been mentioned in research studies of cooperative learning. It is, however, the situation faced by many instructors, even those with interest or belief in cooperative learning, given the realities of many college mathematics classrooms and teaching situations.

Research Protocol

The participants in this study are students enrolled in the Elementary Functions course (Math 1523) at the University of Oklahoma in the fall semester of 1999. This course is typically taken by those who are preparing for the calculus sequence and contains the preparatory material that must be mastered to ensure success in calculus. It is also taken as a terminal course for a small proportion of students who are satisfying a general education requirement in "quantitative modes of thought." The course has many sections every semester and is coordinated by a full-time academic professional who also designs the syllabus and all tests. All tests are administered in large, uniform sections rather than in the self-contained "small" classes of about 35 in which lectures and discussions take place. All of the sections have been organized so that each section has the same syllabus, objectives, homework assignments, uniform examinations, grading scale, and course policies.

The syllabus and examinations, as well as course goals, are thus not under the control of an instructor for an individual section (such as the present researcher). The syllabus is designed to prepare for the calculus sequence those students who either have not had adequate preparatory work or who have failed to place directly into calculus on a university-wide mathematics placement test. Since it is the major preparation for calculus for those known to need more preparation, the syllabus is very crowded with a variety of topics having to do with advanced algebra, trigonometry, and elementary functions. This course clearly qualifies as a demanding instructional situation that is not under the direct control of the class instructor in many important aspects.

The majority of the students are first-semester freshmen, although some students who do even worse on the placement test must first take a prior course (Math 1503, Introduction to Elementary Functions) or even remedial work in beginning or intermediate algebra. Four sections of Math 1523 were used for the current study (Sections 005, 006, 011, and 012). The average mathematics ACT scores of the four classes, as well as the average mathematics ACT score of all participants in this study, is shown in the Table 3.1.

Table 3.1. Means Table

Effect: Pre-experimental Mathematics Ability
Dependent Variable: Math ACT Score

Section	005	006	011	012	Overall
Mathematics ACT score	22.2	25.1	24.1	24.3	24.15

An equivalence table (Moore, 1995) was used when the student's SAT score was available, rather than their ACT score. A copy of this table can be found in Appendix A.

The textbook used in Elementary Functions is *Contemporary Precalculus: A Graphing Approach*, 2nd edition (Hungerford, 1997). Students enrolled in the Elementary Functions course also use a *Study Guide* (1999). The *Study Guide* is prepared by the course coordinator and used for all sections of Math 1523. It contains a framework of the material to be covered by each instructor. Each lesson in the *Study Guide* includes a list of important terms, objectives, and example problems from the section or sections covered in that lesson.

The four sections of Math 1523 involved in the present study were taught by two different instructors (one the researcher), each of whom taught two different sections. This researcher prepared a commentary on the *Study Guide* for the two instructors to use. This commentary looks at each problem from each lesson of the

Study Guide and discusses how group work might be done with appropriate problems. This researcher also wrote a short document entitled "Tips for Teachers" to help the instructors develop a perspective to guide them during the semester. A copy of this document can be found in Appendix I.

Methodology

Two instructors, one the researcher, each taught two sections. Each instructor taught one experimental section and one control section. Students in the two experimental sections were assigned to formal groups. The other two sections (the control sections) did not do any sort of organized group work. These students were not being discouraged from working together on their homework or while studying for exams. All four sections had the same homework assignments and the same examinations, those common to all sections of the course.

An analysis of the mathematics ACT scores was be done using analysis of variance (ANOVA) and an F-test to test for differences in mathematics ability between the four classes at the beginning of the semester. It was decided that if there were significant differences in mathematics ACT scores among the sections analysis of covariance (ANCOVA) would be used to test for differences between the four classes on each of the three one-hour examinations and the final examination.

Treatment Measures

Students in the experimental sections worked as time permitted in their small groups during class on material in the *Study Guide*. (This was the same *Study Guide* that the control groups were using.) The *Study Guide* was made up of 42 lessons. Each of these lessons had a list of important terms, followed by a number of objectives, with several examples for each objective. Some of the examples were basic enough that the students with minimal explanations and suggestions could complete them. These examples were the ones that the two experimental classes working on during class. There was no documentation done regarding the amount of such work done in the experimental sections, nor of the amount and types of interaction done by the students while working on these examples. The working of examples by the experimental students in their groups during class was done as often as was deemed possible by the instructors, given time constraints and difficulty of material on certain days.

The assigned homework was to be turned in by each group in the experimental sections. Each instructor was responsible for determining a policy regarding turning in the homework as a group. This researcher planned to be somewhat flexible with this early on in the semester, but planned to deduct points for homework not turned in as a group. This researcher also planned to e-mail students who were not complying early in the semester to remind them of the policy, as well as reminding them in class. The quizzes and examinations were done individually by both the experimental and control groups. In the experimental sections, the instructors gave an individual score and a group score (average) on each quiz.

Students in the experimental sections were grouped in teams of two to four. There are small differences in opinions among researchers concerning the size of a group to use with cooperative learning, although Hagelgans, Reynolds, Schwingendorf, Vidakovic, Dubinsky, Shahin, and Wimbish (1995) state: "The authors recommend ... that each group contain three or four students. ... There is consensus in the literature on cooperative learning that the ideal size of a group is four students" (p. 23). The initial size of the groups in this study was thus set at three to four students. It is assumed that there would be students who dropped as well as students who added the course during the first two weeks. It was preferred that the group size not drop to two students, but this would be allowed if it seems to be the best arrangement under the given circumstances due to dropping students.

An information questionnaire was to be given the first week of class. (A copy of the questionnaire can be found in Appendix B.) Students were to be asked to name mathematics courses that they had taken in the past two years and the grade that they received in those courses. Students were also to be asked for their local address, as well as their major and hobbies or interests. The instructors were intended to use previous mathematics courses, students' interests, and local addresses as guidelines when forming the groups, as detailed below.

The instructors were to attempt when possible to group students in narrow-range mixed-ability groups. Some of the research done on cooperative learning has looked at the effects of grouping students by ability in a variety of ways. Webb (1985) discusses a group of five studies conducted on the effects of small-group work in a variety of grade levels and on a variety of topics. One of the key variables in all of the studies was ability composition grouping. In all of the studies, students were classified as high, medium, or low ability. These categories corresponded to the top 25 percent, the middle 50 percent, and the bottom 25 percent of each sample, using results from achievement tests given at the beginning of the studies. Students in each ability group were randomly assigned to either uniform-ability groups or to mixed-ability groups. There were two types of mixed-ability groups: students from two ability levels (high and medium levels or medium and low levels) or students from all three ability levels. The conclusions from the study were as follows:

> In summary, the results of group ability composition present a consistent picture. Mixed-ability groups with students from two ability levels seem to be beneficial for all students, whereas mixed-ability groups with students from three ability levels seem to be beneficial for the highest and the lowest students but not for those in the middle. In the former type of mixed-ability groups, all students seem to participate in the teacher-learner relationship, whereas in the latter type of mixed-ability group, the range of

> ability is great enough to allow students to make a distinction between students in considerable need of help (low-ability students) and students in moderate need of help (medium-ability students). Because the groups in these studies concentrated on the most needy, they ignored those who needed less help. A tentative recommendation for classroom practice is to compose groups with (1) the highest- and lowest-ability students in the class but not those with medium ability; (2) groups with a moderate range of ability (highs and mediums or mediums and low); and (3) groups with only medium-ability students. (p. 166-167)

The above article seems to suggest that heterogeneous grouping works best, although it is not a good idea to have too wide of a range of abilities in each group. Thus, the main priority when choosing group membership for this research project was to get students of varied abilities in the same group, without having the range of abilities too diverse. Due to the length of time that it took to obtain ACT and SAT scores, students' achievement in their most recent mathematics course was used when considering their ability level for forming groups. It is clear that a grade of a B, for example, for two students from different high schools in a pre-calculus course does not necessarily indicate equal abilities. However, the use of the students' most recent mathematics course seemed to be the best available indicator of ability under the circumstances.

An attempt will be made to place students who live near to each other in the same group, in an attempt to make it easier for students to get together outside of class. Hagelgans et al. (1995) writes:

> Especially on a commuter campus, taking into account the geographical data of the students will promote face-to-face student interaction in groups outside of the classroom .... In fact, students who have had two or three semesters of successful work in classes that have used cooperative learning groups have asserted that — from their perspective — the single most important criteria for forming a successful group is the ability to meet with their group members on a regular basis outside of class (p. 26).

Also, an attempt was made to put students with similar majors or interests in the same group. It was felt that this type of grouping might help promote group cohesion and camaraderie.

This was the extent of formal intervention in the experimental groups. These groups were carefully formed using the criteria discussed above. They worked together on in-class exercises from the *Study Guide* and were given a group grade on those exercises. They were reminded to work together as groups. A group as well as individual grade was given for quizzes. It was hoped that this would lead to other out-of-class group work but no formal mechanism was in place to ensure that this was true. This certainly seems to satisfy the condition of being a minimal intervention to foster group work in a demanding instructional situation.

Questionnaires

Much of the group work done by students in the experimental classes it is hoped occurred outside of class, both in doing the homework problems assigned each class period and in preparing for quizzes and exams. Questionnaires were given periodically during the semester to document the amount of time spent working in groups outside of the classroom, as well as with whom the students worked. (A copy of the questionnaire can be found in Appendix C.) This questionnaire was given to all four classes in order to have some record of group work being done outside of the classroom by the control classes as well as the experimental classes.

An attitudinal questionnaire was given at the start and end of the semester. (A copy of the questionnaire can be found in Appendix D.) This questionnaire looked at students' attitudes concerning mathematics and the classroom as well as their personal learning styles. This questionnaire used a nine-point Likert scale. There were twelve statements on the questionnaire. Each student rated his or her agreement or disagreement with the statements. Four of the statements concerned the student's perceptions of the teacher's role in the classroom. Four of the statements concerned the students' sense of how they learn by working with others and their preference for working with others or alone. Four of the statements concerned the student's perceptions of mathematics as a discipline. The results of the questionnaires were analyzed at the end of the semester to examine whether there were significant changes in any of the sections on any of the statements. Another attitudinal questionnaire was also given only to the two experimental sections at the end of the semester. (A copy of the questionnaire can be found in Appendix E.) This questionnaire asked for students' responses to questions about how difficult it was for them to get together during the semester, their feelings about doing group work during the semester, and how much group work they had done previously in their mathematics classes.

Summary of Analysis Plan

Attendance was monitored in all classes during the semester. An analysis of the attendance was done at the end of the semester to investigate if there were major differences in students' attendance in the different sections. The uniform examinations were three one-hour examinations and a comprehensive two-hour final examination. Kuder-Richardson Formula 21 was used to find an estimate for the internal-consistency reliability of the multiple-choice parts of each of the examinations. Approximately 75 percent of the points on the exams were made up of multiple-choice questions, with the other 25 percent of the points coming from the three open-ended questions on the final page of each examination. The course coordinator developed the examinations, using test questions submitted by instructors of the various Math 1523 sections. The students were told which sections of the text would be covered on each examination, as well as the structure of the examination and the number of questions on each examination.

An analysis of the exam scores was also done in order to investigate whether there were significant differences between the class averages as well as differences in the number of students who scored at least 90 percent or at most 60 percent. The back page of the examinations always consisted of three open-ended questions. An evaluation of inter-grader reliability was done on these open-ended questions to examine whether the two instructors graded the back page of each examination in a similar manner.

An analysis of scores on each question on all of the examinations was done to investigate whether students who did in-class small-group work tended to do better or worse on certain types of questions. There has been research done on the effects of cooperative learning on various types of mathematics problems. Brechting and Hirsch (1977) found that students in their small group treatment scored significantly higher on a test of manipulative skills than the students in their control group (taught by traditional methods).

Qualitative Methods

The data gathered for this study also had a qualitative component. The qualitative part of this study came from open-ended data gathering and analysis of this data. Several of the questionnaires used in this study had places for students to comment on statements or issues involving cooperative learning. These comments were analyzed to get some idea of the students' perspective on cooperative learning as well as on their experiences with working together during the semester. This researcher also interviewed two student volunteers each from his two sections taught. At the interview, the students were given several problems to solve. They were asked to talk out loud about their thinking and were questioned during and after their problem-solving tasks about how they used cooperative learning while studying mathematics during the semester. (A framework of the interview can be found in Appendix F.) An analysis of the comments and written work from these interviews was done to look for how students' thinking behavior was possibly influenced by working with others.

Chapter 4

RESULTS

Analysis of the data was carried out as described in Chapter 3, using the SAS package for statistical analyses (SAS Institute, 1996). Analyses examined background variables, test results, and qualitative data. Each will be discussed in turn.

Students, Examination Reliability, and Amount of Group Work

The first set of results reported concerns background variables and those necessary for assessing the instruments used and the extent that the intervention had an effect. This includes a comparison of demographic variables. It also includes assessment of the reliabilities of the multiple-choice portions of examinations and inter-grader reliability of the open-ended portions of the examinations as graded by the two instructors involved. Finally, data are presented on how much group work was actually done or reported done by the students.

Demographics

Students enrolled in the Math 1523 course are typically freshman or sophomores, since many of them need the course as a prerequisite for the calculus sequence or other courses. In all four of the sections used in this research, less than 10 percent of each class was composed of upperclassmen. No analysis was made of the age make-up of the four classes. An analysis of the gender make-up was done on each of the four sections. A chi-square test statistic was used. The results of the analysis are shown in Table 4.1.

Table 4.1. Gender Proportion Table

Effect: Gender
Dependent Variable: Class Roster (Oct. 7, 1999)

Gender	Section 005	Section 006	Section 011	Section 012	All Sections
Male	52.9 %	56.9 %	57.1 %	60.0 %	56.67 %
Female	47.1 %	43.1 %	42.9 %	40.0 %	43.33 %

There were no significant differences between the four sections with regards to gender make-up. (The chi-square p-value was 0.976.)

ACT Scores of the Four Sections

An ANOVA was done on the mathematics ACT scores of the four classes in an attempt to determine if the classes were similar in ability at the beginning of the semester. The results of this analysis are shown in Table 4.2.

Table 4.2. Mean Mathematics ACT Score by Section

Effect: Pre-experimental Mathematics Ability
Dependent Variable: Mathematics ACT Score

Section	Number of Students	Mean ACT Score	Standard Deviation	Standard Error
005	35	22.92	4.18	.3537
006	50	25.41	2.35	.8365
011	35	24.40	3.21	.6237
012	30	24.32	3.30	.5426

There were no significant differences between three of the classes. Section 005, however, was significantly lower on the mean mathematics ACT score than the other three classes ($p = 0.0236$). This section was one of the classes that had been chosen as one where students were to work in small groups during the semester. An analysis of covariance will be used in later analyses due to this difference.

Examination Reliability

The Kuder-Richardson 21 Formula was used to analyze the three one-hour examinations and the final examination for internal-consistency reliability. Examination 1 had a reliability of 0.8, Examination 2 a reliability of 0.59, and Examination 3 a reliability of 0.73. The Final Examination had a reliability of 0.73. All of these values indicated an at least marginally acceptable level of internal consistency reliability to allow for study of achievement effects due to the intervention.

The back page of each of the hour examinations consisted of three open-ended questions. (Examples of problems from the back pages are included in Appendix G.) Since two different graders were used, an analysis of inter-rater reliability was done using Kendell's Tau. Results of this analysis are shown in Table 4.3.

Table 4.3. Inter-grader Reliability on Open-ended Responses

Effect: Inter-rater Reliability (Kendall's Tau)
Dependent Variable: Scores on Open-ended Examination Problems

EXAMINATION	Problem 22	Problem 23	Problem 24
Examination 1	.88	.97	.94
Examination 2	.98	.72	.66
Examination 3	.98	.77	.87

The last question on Examination 2 had a value of Kendall's Tau that was below 0.7. The two instructors interpreted the answers to this question differently in a number of cases. The values of Kendall's Tau were nearly all above 0.7 (with all but three values above 0.86). The back page point total was only 24 percent of each examination. Therefore, it will be assumed that the inter-grader reliability was high enough to allow the examination scores of the four sections to be compared without adjusting for grading differences.

Group Work Done in Class

Part of the treatment for the two experimental classes was work done together in class on some of the examples from the *Study Guide*. The *Study Guide* consisted of forty-two lessons. Each lesson had a list of important terms, a set of objectives, and examples of problems that involved the objectives. At times the examples were basic enough that it was felt that the problem was within their "zone of proximal development" (see earlier discussion of Vygotsky). The experimental classes were given enough information to work through the examples in their groups and were instructed to do so. The instructors worked these same problems in the control sections. This type of in-class group work was not done every class period, and due to a combination of factors (quizzes, homework questions, inability of the instructor to arrive early due to a class the previous period), this type of work was not done as frequently as had been planned. This contributes further to defining the experimental intervention as minimal.

Group Work Reported By Students.

A questionnaire was given out during the semester to all four sections to document the amount of group work done outside of class by each section. This self-report data is the only evidence on out-of-class group work available in this study.

This questionnaire was given three times during the semester, approximately once each month. (A copy of this questionnaire can be found in Appendix C.) A Likert scale was used on the questionnaire. Students were asked to circle the appropriate number indicating the proportion of the time they had spent during the past week studying in their groups. For the end-of-semester questionnaire, they were asked to circle the appropriate number indicating the proportion of the time they had spent during the *semester* studying in their groups. They were given the following choices: 0 percent , 25 percent , 50 percent , 75 percent , and 90 percent or more.

A comparison was done between sections 005 and 006, which were taught by one instructor, and between sections 011 and 012, which were taught by this researcher. Sections 005 and 011 were the experimental sections; sections 006 and 012 were the control sections. (As a reminder, (E) will stand for "Experimental section" and (C) will stand for "Control section" in Tables 4.4 to 4.6.) The results of the questionnaire given after Examination 1 are shown in Table 4.4.

Table 4.4. Group Work Reported after Examination 1

Tally Table (Number of students reporting in a certain category.)
Effect: Proportion of time spent working in groups, September 20-24

Section	0 %	25 %	50 %	75 %	90% or more	Fischer's Exact *t*
005 (E)	23	10	0	0	0	0.286
006 (C)	25	21	0	0	1	
011 (E)	2	9	9	6	2	0.000135
012 (C)	11	5	0	1	0	

It should be noted that Section 006, one of the control sections, reported more group work done outside of class than did Section 005, the experimental section taught by the same instructor who taught both sections. This is a result of the minimal intervention with only self-report data to monitor implementation.

The results of the questionnaire given after examination 2 are shown in Table 4.5.

Table 4.5. Group Work Reported after Examination 2

Tally Table (Number of students reporting in a certain category.)
Effect: Proportion of time spent working in groups, October 18-22

Section	0 %	25 %	50 %	75 %	90% or more	Fischer's Exact *t*
005 (E)	12	9	2	0	1	0.593
006 (C)	23	17	2	0	0	
011 (E)	6	8	5	3	1	0.087
012 (C)	9	1	1	2	0	

The results of the questionnaire given on the last day of class are shown in the following table.

Table 4.6. Group Work Reported on Last Day of Class

Tally Table (Number of students reporting in a certain category.)
Effect: Proportion of time spent working in groups during this semester

Section	0 %	25 %	50 %	75 %	90% or more	Fischer's Exact t
005 (E)	14	11	1	1	0	0.273
006 (C)	21	10	0	0	0	
011 (E)	6	9	9	1	0	0.101
012 (C)	9	4	2	2	0	

Although none of the comparisons between sections 005 and 006 on any of the questionnaires is statistically significant, the amount of group work reported by section 005 (experimental group) was greater than that reported by section 006 (control group) on the second and third questionnaires. The amount of group work reported by section 011 (experimental group) was greater than that reported by section 012 (control group) on all of the questionnaires. It was only statistically significantly greater on the first questionnaire.

Achievement Results

The answers to the research questions of this study are embodied in whether the minimal intervention of group work in the demanding instructional situation produces enough difference to bring about changes in achievement or attitudes. The data thus far indicate that there were no statistically significant differences in the group work done by experimental and control sections outside of class. The only intentionally introduced differences were thus the group work introduced in class. The question is whether this was accompanied by differences in achievement results.

Analysis of Examination Results

Several statistical methods were used to examine the effects of the treatment and the amount of group work reported by the students on their academic achievement. First, an analysis of covariance (ANCOVA) was performed on each of the examinations as well as the point totals for the semester. An analysis of the achievement results used categories of NO for 0 percent, LOW for 25 percent, MED for 50 percent, and HI for 75 percent, with the categories listed at the end of the semester questionnaire used to identify each student's "group work" category. In the tables to follow (and in the SAS programs), GpWk stands for the "group work" category.

The analysis of the results from Examination One is shown Table 4.7. The analysis of the results from Examination Two is shown in Table 4.8. The analysis of the results from Examination Three is shown in Table 4.9. The analysis of the results from the Final Examination is shown in Table 4.10

Table 4.7. ANCOVA P-Values for Variables and Interactions on Examination One Scores

Variable	ACT	Treat-ment	ACT* Treat-ment	GpWk	ACT* GpWk	Treat-ment* GpWk	Three way
p-value	0.3689	0.2998	0.2877	0.8046	0.7959	0.7575	0.7254

Table 4.8. ANCOVA P-Values for Variables and Interactions on Examination Two Scores

Variable	ACT	Treat-ment	ACT* Treat-ment	GpWk	ACT* GpWk	Treat-ment* GpWk	Three way
p-value	0.7044	0.2727	0.2967	0.7873	0.7487	0.6901	0.7139

Table 4.9. ANCOVA P-Values for Variables and Interactions on Examination Three Scores

Variable	ACT	Treat-ment	ACT* Treat-ment	GpWk	ACT* GpWk	Treat-ment* GpWk	Three way
p-value	0.2698	0.1506	0.2496	0.0581	0.0475	0.7415	0.8973

Table 4.10. ANCOVA P-Values for Variables and Interactions on Final Examination Scores

Variable	ACT	Treat-ment	ACT* Treat-ment	GpWk	ACT* GpWk	Treat-ment* GpWk	Three way
p-value	0.8345	0.1693	0.2322	0.3003	0.2626	0.6712	0.7491

There were no significant results on the first two examinations. Analysis of Examination Three indicates that the amount of group work done by the students may have had a marginally significant impact on their examination score. Also, there was a significant p-value on the interaction between the students' mathematics ACT score and the amount of group work done. An analysis of the connection between students' ACT scores, amount of group work done during the semester, and their point total for the course will be done later.

Analysis with Frequently Absent Students' Data Deleted

There were some students in all of the sections used for this research who were frequently absent from class. Their absences could have skewed the results in some way, as those students who were in the experimental sections and missed class a lot were not benefiting fully from the in-class group work. A further analysis of Examination Three and the Final Examination was done, with students who missed class more than 50 percent of the time left out of the analysis. The results of this analysis are shown in Table 4.11.

Table 4.11. ANCOVA P-Values for Examination Three and Final Examination Scores with Frequently Absent Students' Data Deleted

Variable	ACT	Treat-ment	ACT* Treat-ment	GpWk	ACT* GpWk	Treat-ment* GpWk	Three way
Exam 3	0.2377	0.1434	0.2388	0.0412	0.0334	0.6600	0.8203
Final Exam	0.8929	0.1241	0.1793	0.1404	0.1245	0.5036	0.5919

Here there were significant results in the analysis of Examination Three, with the group work variable and the ACT score with group work interaction variables having p-values less than 0.05. There were no significant results in the analysis of the Final Examination results, though nearly all of the p-values were lower than those from the analysis of the Final Examination data from all students who took part in this study. A similar analysis will be done on the point totals for the semester, following a brief explanation of the points possible in the course.

Analysis of Point Totals for the Semester

As mentioned earlier, all sections of the Elementary Functions course (Math 1523) were taught using the same textbook, syllabus, examinations, point total, and grading scale. The point total for the course consisted of the three one-hour examinations that were worth 100 points each, the final examination which was worth 200 points, and a class work score. Students could receive up to 100 points for the class work. Instructors had some discretion about how to award points for class work.

This instructor picked up homework daily. There were ten quizzes given during the semester. Students in the experimental section were given an individual quiz score as well as a group quiz score, which was the average of the quiz scores from students in their group. At the end of the semester, the top two-thirds of all homework and quiz scores for each student were added, and their percent of the total possible became their class work score. Also, students were given 24 bonus "attendance" points at the start of the semester. Each absence cost them 4 points. After six absences, they lost no further points, having lost all of the 24 bonus points. So there were a total of 624 points possible for the course. Homework and quizzes made up less than one-sixth of this amount.

An analysis of the point totals for the four sections involved in this study was done at the end of the semester using ANCOVA. The results of this analysis are shown in Table 4.12. In this analysis, the sample size was 150.

Table 4.12. ANCOVA P-Values for Semester point totals (All Four Sections)

Variable	ACT	Treatment	ACT* Treatment	GpWk	ACT* GpWk	Treatment* GpWk	Three way
p-value	0.9883	0.1314	0.1824	0.1444	0.1224	0.7826	0.8379

An investigation of the treatment was carried out by looking at the least squares means of the experimental classes and the control classes. The least squares means was lower for the experimental classes than it was for the control classes, though not significantly lower ($p = 0.2553$).

An analysis of the point totals was also done with those students who were absent more than 50 percent of the time left out of the analysis. The results of this analysis are shown in Table 4.13. Again, the least squares means was lower for the experimental classes than it was for the control classes, though not significantly lower ($p = 0.2117$).

Table 4.13. ANCOVA P-Values for Semester Point Totals with Frequently Absent Students' Data Deleted

Variable	ACT	Treat-ment	ACT* Treat-ment	GpWk	ACT* GpWk	Treat-ment* GpWk	Three way
p-value	0.9413	0.0856	0.1266	0.0522	0.0451	0.6304	0.7281

In this analysis, the interaction between the ACT score and the amount of group work done by that student was significant. A further analysis of this interaction will be done shortly. The amount of group work was very close to being significant, and the treatment was marginally significant ($p < 0.1$). Both of these results seem to warrant more study in future research.

Analysis Using Ratio = (Semester Point Total)/(ACT Score)

Another analysis of the achievement results for the four sections was done by looking at the ratio of the semester point total to the ACT score for each student. This measure was thus essentially the number of points earned per mathematics

ACT score point. This index thus further equated results for students with differing mathematics ACT scores.

An ANCOVA was performed using this ratio as the dependent variable in the model. This analysis was done for all students in each of the sections as well as for only those students who attended class more than 50 percent of the time. The results of this analysis are shown in Table 4.14. In this analysis, the comparison is Control—Experimental. The experimental classes had a higher ratio than the control classes, for the comparison involving both the entire subject pool as well as the comparison involving the classes without those students who missed class frequently.

Table 4.14. ANCOVA Using Ratio = (Semester Point Total) / (ACT Score) (Comparison of Least Squares Means Using Control – Experimental)

Experimental Groups (by attendance)	Differences of Least Squares Means	Standard Error	p-value
All Students Included	-1.2304	1.2085	0.3114
Regular Attendance Only	-1.4145	1.0900	0.1976

To investigate the effects of the treatment further, an analysis was done using the same ratio as above, looking at the effects of each instructor. Using a model containing only the treatment, the ratio for the experimental classes was higher than that for the control classes for both instructors, though not significantly higher. (The p-value for this researcher was 0.4328; the p-value for the assisting instructor was 0.0798.) There were slightly different results when the frequently absent students were left out of the analysis, though nothing significant.

Comparison of Different Ability Levels

One of the questions of this research project was whether small-group work affects students with different abilities in varying amounts. An analysis for this question was done by placing students in three categories based on their ACT scores. The original plan for grouping students into low, medium, and high ability groups was to divide the total student population into three groups of fairly equal size. This was not possible, due to the large number of students with certain ACT scores. The decision was made to call students with an ACT score of 15 to 23 (inclusive) low-ability students. Students with ACT scores of 24, 25, or 26 were called medium-ability students, and students with an ACT score of 27 or higher were called high-ability students. The sizes of the three categories are shown in the following table.

Table 4.15. Population of Academic Ability Categories

Ability Level	Low	Medium	High
ACT Scores	15-23	24-26	27-34
Number of Students	40	63	29

An analysis was done using ANCOVA, with the category serving as the covariant. The results of the analysis are shown in Table 4.16. There was very little interaction between the Treatment and the Group Work variables in the previous model ($p = 0.9215$).

Table 4.16. ANCOVA Table (Achievement Ability as Covariant) for Total Semester Points (DDF = 128)

SOURCE	Type III F	p-value
Group Work	1.29	0.2834
Treatment	1.19	0.2781
ACT Category	0.46	0.4974
GroupWork *ACTCategory	1.76	0.1619
Treatment * ACTCategory	1.97	0.1644

A further analysis was done using ANCOVA and a model with only Group Work and ACT Category as the variables. The results of this analysis are shown in Table 4.17.

Table 4.17. ANCOVA Table (Achievement Ability as Covariant) for Total Semester Points (DDF = 128)

SOURCE	Type III F	p-value
Group Work	0.49	0.6867
ACT Category	7.29	0.0083
GroupWork*ACTCategory	1.20	0.3148

A similar analysis to the one described above was done, with the Group Work variable replaced in the model by the Treatment variable. The results of this analysis are shown in Table 4.18.

Table 4.68. ANCOVA Table (Achievement Ability as Covariant) for Total Semester Points (DDF = 128)

SOURCE	Type III F	p-value
Treatment	4.69	0.0322
ACT Category	5.62	0.0193
Treatment*Category	7.06	0.0089

An examination of the differences of least squares means at each category level reveals how the category level affects the achievement mean of the experimental and the control classes. (The difference is Control – Experimental.) Table 4.19 shows the results of this analysis.

Table 4.19. Differences of Least Squares Means between Experimental and Control Groups by Achievement Level for Semester Point Totals

Achievement Category	Differences of Least Squares Means	Standard Error	p-value
Low-ability	42.4842	28.2576	0.1352
Medium-ability	-23.3775	17.9610	0.1954
High-ability	-89.2393	32.8036	0.0074

Students in the high-ability category did significantly better with respect to total semester points in the experimental classes than in the control classes. In order to examine the interaction between the treatment and the ability level further an analysis was done for each instructor. The results of this analysis are shown in Table 4.20.

Table 4.20. ANCOVA Table (Achievement Ability as Covariant) by Instructor for Semester Point Totals (DDF = 128)

Instructor	SOURCE	Type III F	p-value
A	Treatment	3.95	0.0516
	Category	1.38	0.2451
	Treatment*Category	6.70	0.0121
B	Treatment	0.92	0.3400
	Category	3.42	0.0691
	Treatment*Category	0.97	0.3272

For the comparisons of the least squares means between the experimental and control groups by teacher, the low-ability students in both control classes had a higher value than those in the experimental classes. For the other two ability levels, both experimental classes had higher value than the two control classes. The only statistically significant comparison was with the comparisons of the high-ability students for this researcher ($p = 0.0063$).

Grade Proportions.

Although it is not always a good measure of what students have learned, students certainly are quite concerned about what grade they receive in a course. With the treatment being minimal, it was an open question as to whether some students in the experimental classes may have been helped by the treatment enough to result in their receiving an A rather than a B, a B rather than a C, and so on. An analysis of the proportion of the students in each group (experimental and control) who received a certain grade was done using a chi-square statistic. The results of this analysis are shown in Table 4.21. In this table, the ordering of the size of the proportions will be denoted using C for the control group and E for the experimental group. For the experimental sections, $\underline{n} = 70$, and for the control sections, $n = 80$. The proportion of a particular grade differed significantly only in three cases. The experimental class students received more A's on Examination 3 and the Final Examination (perhaps the cumulative effect of group work) while the control class students received more B's on Examination 2.

Table 4.21. P-values for Comparisons of Grade Proportions

Examination	A	B	C	D	F
Examination 1	.796 E>C	.353 C>E	.170 C>E	.835 C>E	.456 E>C
Examination 2	.237 E>C	.001 C>E	.532 E>C	.144 E>C	.593 E>C
Examination 3	.039 E>C	.095 C>E	.131 C>E	1.00	.758 C>E
Final Exam	.002 E>C	.670 C>E	.068 C>E	.564 C>E	.668 C>E
Course Grade	.239 E>C	.355 C>E	.056 C>E	.180 E>C	.513 C>E

Group Work – ACT Score Interaction.

Several statistical methods were used to investigate the interaction of group work with ACT score. The first method used was an analysis using the differences of the least squares means, looking at the differences between the various levels of group work reported. Because of the significant interaction (p-value = 0.0451) between group work and ACT score in the analysis with frequently absent students deleted, this researcher decided to hold the levels of ACT constant and explore the differences in the levels of group work at individual ACT levels. [This type of analysis is very similar to taking a partial derivative, where one holds x constant and looks at the change (or difference) in z.] As mentioned earlier, the analysis of the achievement results uses categories of NO for 0 percent, LOW for 25 percent, MED for 50 percent, and HI for 75 percent, with the categories listed on the end of the semester questionnaire used to identify each student's "group work" category. The results of this analysis are shown in Table 4.22.

Table 4.22. Ordering of Least Squares Means of Semester Point Totals by Group Work at each ACT Score with Frequently Absent Students' Data Deleted

ACT score	Least squares means rankings (highest to lowest)
15, 17-21	HI, NO, LOW, MED
23	HI, NO, MED, LOW
25	MED, NO, HI, LOW
27-30,33-34	MED, NO, LOW, HI

Not all of the differences between the highest and lowest values of the least squares means were significant. For students with an ACT score of 17, the value of the least squares means for the HI level of group work was significantly higher than the value of the least squares means for the MED level with a p-value of 0.0483. For students with ACT scores of 26 or higher, the values of the least squares means for the MED level of group work was significantly higher than the values of the least squares means for the LOW level, with p-values of less than 0.05 for each level of ACT score.

An analysis was done with the three ability categories described earlier. The treatment was left out of this model, and was replaced by the amount of group work reported. The rankings of the least squares means were examined at each of the three categories. Results were somewhat different than those in the previous analysis, and are shown in Table 4.23.

Table 4.23. Least Squares Means of Semester Point Totals by Ability

Achievement Ability Category	Least squares means ranking (highest to lowest)
Low	LOW HI NO MED
Medium	MED NO HI LOW
High	MED NO HI LOW

These analyses seem to indicate that for students with lower ACT scores, a high amount of group work is a better predictor of academic achievement in this course than are the other levels of group work. Also, for students with higher ACT scores, a medium amount of group work is a better predictor of academic achievement in this course than are the other levels of group work.

It is notable that those students who reported no group work done during the semester had least squares means values that were consistently higher than most of the other levels of group work. An examination of point totals and amount of group work for each level of ACT score shows that no students with an ACT score of 29 or higher reported any group work done during the semester. At each level of ACT score of 23 and above, there was always at least one student with no reported group work and a total of 549 or higher (out of 624 possible). At ACT scores of 20 and 21, the highest point totals for the semester were attained by students with no reported group work. These facts should not be interpreted as evidence that group work does not work. However, it seems clear that there are students who can do well at a variety of ability levels with little or no group work.

Examination Question Results, Experimental vs. Control Groups

Another of the questions this research project was investigating was whether students who studied mathematics in groups learned certain topics, procedures, or concepts better than students who didn't study in groups did. An analysis was done on each question from each of the examinations, using the chi-square statistic on the proportion of students in the experimental and control groups who got that question correct. There were no questions with a statistically significant difference in this analysis.

Qualitative Results

In addition to effects on achievement, the research questions and method for this study suggested that group work might affect student attitudes. It was also of interest to examine student feelings about and reactions to group work. These aspects were analyzed as part of the qualitative component of this study.

Attitudinal Results

A questionnaire was given to all four sections involved in this study during the first and last weeks of classes. (A copy of this questionnaire can be found in Appendix D.) This questionnaire looked at students' attitudes concerning mathematics and the classroom as well as their personal learning styles. There were twelve statements on the questionnaire. Students rated their agreement or disagreement with the questions. Four of the statements concerned the students' perceptions of the teacher's role in the classroom. Four of the statements concerned the students' sense of how they learn by working with others and their preference for working with others or alone. Four of the statements concerned the students' perceptions of mathematics as a discipline.

Unfortunately, students in two of the sections did not put their names on the questionnaire given at the beginning of the semester, so a statistical analysis of pre- versus post-experimental attitudes for those sections was not possible. Fortunately, the same instructor taught the two sections with complete attitudinal data. The results from these two sections were analyzed to examine whether there were significant changes on any of the statements. The results of this analysis are shown in Table 4.24.

Table 4.24. Attitudinal Questionnaire T-Test P-Values for Changes in Rating, POST – PRE, Comparing Experimental and Control Classes

Statement	1	2	3	4	5	6
p-value	.0286	.5610	.6492	.9582	.9854	.2131
Statement	7	8	9	10	11	12
p-value	.0209	.8465	.9412	.4429	.6005	.6945

Only two of the twelve comparisons resulted in a significant difference between the experimental and the control class. Statement One was, "*The teacher's job is to tell me how to do math.*" The mean difference (post – pre) in the control class was –1.0588, with a standard deviation of 2.07577. The mean difference in the experimental class was 0.1538, with a standard deviation of 1.4337. The decrease in the response indicates that students in the control class tended to disagree more (or agree less) with this statement at the end of the semester than they did at the beginning of the semester.

The other significant difference occurred with students' rating of Statement Seven, which said, "*The teacher's job is to help us think through a procedure or concept.*" The mean difference (post – pre) in the control class was –0.2941, with a standard deviation of 0.9852. The mean difference in the experimental class was 0.6923, with a standard deviation of 1.4905. Again, the responses of the control class tended to move towards less agreement with this statement at the end of the semester than at the beginning of the semester.

The mean change in response by the experimental class to Statement Seven was fairly large (0.6923). The only statements with a larger mean change in response by the experimental class were Statements Six (0.9231) and Eleven (-0.8846). Statement six was, "*Math can be a bewildering subject.*" There was an increase of agreement with this statement in both classes. Statement Eleven was, "*I spend most of lecture time writing what the teacher says (or writes).*" There was a decrease of agreement with this statement in both classes.

Experimental Sections Questionnaire

Another questionnaire was given to the experimental sections at the end of the semester, asking students a number of questions about their experiences with the group work done during the semester. (A copy of this questionnaire can be found in Appendix E.) The responses were fairly similar to all questions, with no significant differences between the two experimental classes.

The responses to several of the questions were somewhat noteworthy. Question One was "*How easy was it for you to get together with members of your group to study?*" The mean response for both of the experimental sections was around 6. A rating of a 5 translated as "somewhat difficult", while a rating of a 7 translated as "difficult". A rating of a 1 translated as "very easy", while a rating of a 3 translated as "easy". Only one student in one of the sections gave a rating of 1, 2, or 3. Question Two was "*What factors influenced your not working more with members of your group?*" There was a list of six possible factors following this question (including *Other*), and students were told to check all that apply. Out of fifty-four students who answered this question, thirty-nine checked the factor: *Difficult to make time to get together.* Only nine students checked the factor: *Don't like working in groups.*

Statement Seven concerned the small-group work that was done during class involving the *Study Guide*. It stated *Occasionally this semester you were to work on examples from the Study Guide, with some assistance from me. Please rate this method by circling the appropriate number.* The mean response on this statement was 7 or greater for both sections, with a response of 7 translating as "somewhat helpful" and a response of 9 translating as "very helpful". The lowest response to this statement was a 5 (which translates as neutral), and there were only four students who gave this response. The response to Statement Seven seems to indicate that the in-class part of the treatment was well-received by the majority of the students.

Statement six on this questionnaire was, "*Rate your experiences with small-group learning this semester.*" The mean response for both sections was around 6, which was between "neutral" (for a response of 5) and "positive" (for a response of 7). The majority of responses were a 5, but there were more responses toward the "very positive" end of the scale (a response of 9) than there were toward the "very negative" end (a response of 1). There were a total of 51 students who responded on this statement. None circled a 1 or a 2, and only six students circled a 3 or a 4. Twenty-one students circled a 6 or higher and there were six students who circled a 9 (very positive).

Opinions on Small-Group Work

On the student information questionnaire given at the beginning of the semester, students were asked to comment on the statement: "*Working together in small groups can help students learn.*" A sample of their comments can be found in Appendix H. The statement was worded positively, although an attempt was made (by using the word "can") to keep it somewhat neutral. Most of the students took the time to make a comment on this statement. Most of them agreed with the statement, but almost one-fourth of the students made comments to the effect that they either disagreed or both agreed and disagreed. There were a few students who strongly disagreed with this statement; their comments are worth noting, in that these students might have resisted working with other group members or might have dropped the course. In the two sections where identifying the students by name was possible, the two students in the experimental section who disagreed with this statement either dropped or received an F in the course. This is balanced by the student in the control section who disagreed and received a D in the course.

Student interviews

In order to seek a better understanding of students' reactions to and feelings toward group work, interviews were carried out with two students from the experimental sections and two students from the control sections. These four students had volunteered to be interviewed. These students were given problems similar to those they had been assigned as homework problems during the semester. They were asked to think out loud while working through these problems, and were questioned about their thought processes while working the problems. They were also asked questions about working in groups. The interviews were taped for later analysis.

All of the students experienced some difficulties in working through the problems on their own. When they became stuck while working through a problem, the interviewer would give hints and suggestions to help the student get started again. One of the students from the experimental class, when asked about how he and his roommate would study together, commented: "About half the time one of us would start working on it and the other one would review the book and then when they got stuck we'd help each other. That's how we studied for the final, pretty much." The other student from the experimental section also reported working with others in his group outside of the classroom.

Both of the students in the control sections reported working with others outside of the classroom, although one of these students reported that he only went once to the Housing Learning Center (a university-run tutoring center where free tutoring was available). This student did live out of town (approximately fifteen miles from the University) and he mentioned that he worked at a part-time job. The other student in the control group said that she studied with three other people on her dormitory floor who were taking the same course, and that they would sometimes get help from another person on their floor who was taking calculus.

One of the questions on the Student Information Questionnaire given at the beginning of the semester asked the students to indicate who they would prefer to have in their group, though it was stated that they might not end up in the same group with that person. Both of the students who were interviewed who were in the experimental sections had roommates who were in the same section of the course used in this study, and they had been allowed to be in the same group as their roommate. One of these students was asked how he felt about having a say in who was in his group, and he stated that he preferred having a say. The other student commented that he and his roommate tended to study together without the other two group members, though they would meet shortly before class to discuss the homework. He mentioned that the other two group members were getting tutoring through another university-run tutoring program. He commented that it was difficult for all four of them to find time to get together outside of class time.

There is some indication in the literature on cooperative learning that the instructor should not let the students choose the group membership, although most of the articles are written for and about cooperative learning in the primary and secondary schools. Matthews (1998) mentions comments made by the students during interviews she conducted. She states: "None of the three liked the instructor assigning groups, and felt strongly that students should be allowed to form groups of their own choosing." (p. 78)

Several of the students had done quite a bit of small-group work in their mathematics classes prior to coming to college. One student commented:

> We did a lot. Our teacher liked to use it. We met for an hour and a half every day, and we worked in groups during class. We had time to do calculator stuff in class instead of having to go home and try to figure it out.

The other student stated:

> The last time I did group work was in geometry in junior high school. ...

> We'd divide the work up among the group; say if we had 30 problems then each person would do 10. If [some of the problems] involved a totally new concept, then the person that did those problems would explain it to the rest of the group. ... Our homework was done in class, then we'd take home what we didn't get done, and grade it the next day.

When this student was asked if some students in his group (in junior high school) had not done their share of the work, replied "All the time. But then I wouldn't do the work so we'd get a zero on the homework, but I'd get it later, before the exam."

Chapter 5

DISCUSSION

The purpose of this study was to investigate whether a modest amount of small group work done (both inside and outside the classroom) could have a positive effect on students' achievement or attitudes in several university pre-calculus classes. It was decided not to investigate the effects of small group work by either gender, age, ethnicity, or major, due to the minimal nature of the intervention involved in this study as well as the inexperience of the two instructors with using cooperative learning in the classroom.

This researcher had not used cooperative learning as a teaching technique in the classroom prior to this research project, and had been involved with cooperative learning as a student only a very few times. The other instructor also had not used cooperative learning in the classroom before assisting with this research. This may be one reason for the small number of significant results. Another related factor that may have affected the results of the study was that both instructors had taught the course used in this study before, and as a result the presentations of material in the control classes may have been more polished than that done in the experimental classes. It is conjectured that instructors with little or no prior experience may well not find any significant differences between their small-group classes and their traditional classes. It is still an open question whether instructors with some experience in cooperative learning can expect to find positive results from using cooperative learning in their classroom using a minimal intervention.

Summary of Results

There were no consistent significant differences on achievement comparisons between the experimental group and the control group on any of the three one-hour examinations or on the final examination. This was also true for comparisons of total points accumulated during the semester. The results of these comparisons are shown in the following table.

Table 5.1. P-Values for Achievement Comparisons on Hour Examinations, Final Examination, and Total Points for the Semester.

Comparison	Treatment	Group Work
Examination 1	0.2998	0.8046
Examination 2	0.2727	0.7873
Examination 3	0.1506	0.0581
Final Examination	0.1693	0.3003
Total Points	0.1314	0.1444

On the analysis that was done with frequently absent students' data left out, the amount of group work showed a statistically significant effect on the results of Examination Three. There were no other significant differences, although the differences were more significant than with the previous comparisons. The results of these comparisons are shown in the following table.

Table 5.2. P-Values for Achievement Comparisons on Third Hour Examination, Final Examination, and Total Points for the Semester With Frequently Absent Students' Data Deleted.

Comparison	Treatment	Group Work
Examination Three	0.1434	0.0412
Final Examination	0.1241	0.1404
Total Points	0.0856	0.0522

While there is not a clear indication of the positive effects of either the treatment or the amount of group work reported by the students, the statistically significant p-value of the variable "Group Work" for the comparison between the experimental and control groups (from Table 5.2) on Examination Three may be meaningful. (The p-value for all students from this variable is marginally significant, with $p < 0.1$.) Also, the p-values from Table 5.1for the third one-hour examination, the final examination, and the total points for the semester are considerably lower than those for the first two one-hour examinations.

One possible explanation for these results may be that because of the fairly modest nature of the treatment, students tended to meet infrequently outside of class to study together (see Table 4.6). Because of this, their skills at working and studying as a group took most of the semester to improve to the point that these skills led to improved achievement results. It may also be that students did not take seriously the potential value of studying together until the third one-hour examination. The p-value for the variable "Group Work" for the final examination was not statistically significant. This does not disprove either of the above conjectures, since students may not have been able to study for the final examination with people that they were used to studying with, because of conflicting schedules due to other final examinations.

The p-values from Table 5.2 are somewhat lower than the comparable values from Table 5.1. This is a reasonable result, given that in general students who were frequently absent (more than 50 percent of the class periods) missed out on much of whatever positive benefits might have been found from the in-class group work, and may have missed out on working with others outside the classroom as well. Both the treatment variable and the group work variable are marginally significant ($p < 0.1$) on the comparisons using total semester points for students who attended class more than 50 percent of the time (Table 5.2). While this result is not strong enough for any definite conclusions about the positive effects of either the treatment or the amount of group work done by the students, it seems to indicate that further research with this treatment may be fruitful, especially in a course where there was not a set amount of material that had to be covered during each class period. One of the frustrations of both instructors in this study was the fast pace of the course and the relatively small amount of in-class group work that was done. Certainly these results make it questionable whether minimal group work interventions in demanding instructional situations will be more effective than what students spontaneously do on their own (as in the control sections of this study).

One of the questions being examined in this study was how group work affected students of differing abilities. Slavin (1995) notes:

> One particularly important question relates to whether cooperative learning is beneficial to students at all levels of prior achievement. ... The evidence from experimental studies that met the inclusion criteria for this review supports neither position. (p. 44)

Both analyses of Examination Three test scores (for all students as well as for those students who came to class more than half of the time) found statistically significant interaction between the students' ACT scores and the amount of group work reported being done outside of class. This result also occurred in the analysis involving semester point totals for those students who came to class more than half of the time. The students were grouped into low-ability, medium-ability, and high-ability categories, based on their ACT scores. There was some difficulty with an analysis using ANCOVA with the group work and ACT category in the model. In the ANCOVA with treatment and ACT category in the model, there were statistically significant differences in the treatment ($p = 0.0322$), ACT category ($p = 0.0193$), and interaction between the two ($p = 0.0089$).

An investigation of these results was done using differences of least squares means and semester point totals. It was interesting that the students in the low-ability category performed better in the control classes than the experimental classes, and students in the medium-ability and high-ability categories performed better in the experimental classes than the control classes. Davidson (1985) cites several studies with similar results, although these studies did not involve group rewards for individual learning. It may be that the use of a group quiz score and a group homework score may not have been a strong enough factor to serve as a true "group reward for individual learning".

It may also be that a number of the low-ability students were not working together both in and outside the classroom. This could have been due to the relatively small number of situations in the classroom where group work was encouraged and due to the fact that students could choose not to work together outside of the classroom (although they were frequently encouraged to do so). The p-value for the ACT score by group work interaction was statistically significant ($p = 0.0451$), when we use the data for only those students who attended class frequently and the point totals for the semester.

For those problems that were to be done in the classroom in the groups, the low-ability students may not have been able to recall the information necessary to start on the problem, and so did not benefit as much from this situation as the higher ability students. There was not as much time to devote to the in-class group work as had been hoped for when the semester began, and the explanations that the low-ability students received from their group may have been somewhat superficial. The instructors went over these problems after giving the students a few minutes to work together on them. However, it is arguable that some of the low-ability students in these situations in the experimental classes did not become as fully involved with the thought processes necessary to understand these problems as the low-ability students in the control classes. In the control classes, the instructor would remind the students what ideas, processes, or concepts were needed to begin working on these problems, and then demonstrate how this was done.

Students in the medium-ability and high-ability categories did better in the experimental classes than in the control classes, with the high-ability students in the experimental classes doing significantly better ($p = 0.0074$). High-ability students may have been more comfortable with being challenged to start on a problem that was somewhat new to them. Further, the reflection that they did in order to start on those problems may have helped them to better understand that kind of problem as well as the processes and concepts involved.

Qualitative Results

The results of the attitudinal questionnaire were flawed by the fact that students in two of the sections did not put their names on the questionnaire. For the other two sections, there were only two statements with significant differences between the experimental class and the control class. On the statement, "*The teacher's job is to tell me how to do math*" (Statement One), the responses of the control class tended to move toward the "disagree" end of the scale. The mean difference (post – pre) was –1.0588. The mean difference of the experimental class was 0.1538. The other statement (Statement Seven) with a significant difference between the classes was, "*The teacher's job is to help us think through a procedure or concept.*" On this statement, the responses of the control class again moved toward the "disagree" end of the scale. The mean difference (post – pre) was –0.2941. The responses of the experimental class tended to move towards a greater agreement with this statement. (The mean difference was 0.6923.)

It may be noteworthy that both of these statements concerned the students' perceptions of the teacher's role in the classroom. Statement One was written with the intention of implying that the student's role was more passive, since the teacher was "... tell[ing] me how to do math.". Statement Seven was similar in a way, but the wording here was intended to imply that the students were more in control of their learning, and the teacher was to help them think. The significant increase in the response of the experimental class could be interpreted as an increased belief for the class in the notion that they were in charge of their learning.

Within-Class Grouping

Students were assigned to groups at the end of the first week of class. There was a considerable amount of changing of group membership early on, although most of the groups retained most of their original membership throughout the semester. Most of the change in group membership was due to students dropping and adding the course after the first week, although attendance and other problems also played a part. This researcher originally assigned the thirty-eight students on the class roster in the experimental class to ten groups of three students each and two groups of four students each. Of these students, two students in one of the groups dropped the course, as well as two other students. Two students added this researcher's experimental class. The other instructor originally assigned his students in the experimental class to nine groups of four students each and one group of five students. Six of the students in this class dropped the course; two of these students were in the same group.

The ACT and SAT scores for the students were obtained late in the semester. These scores were used in the analyses described in Chapter Four. They were also used to examine the average of the point totals for the semester of each group, the ability makeup of these groups, and the amount of group work reported by students in each group. This information is shown in Table 5.3. As mentioned in Chapter Three, there were 600 points possible in the course. Also, students with an ACT score of between 15 and 23 were considered low-ability, those students with an ACT score of between 24 and 26 were considered medium-ability, and those with an ACT score of 27 or higher were considered high-ability.

The groups are listed with the group with the best average listed first, the group with the next-best average listed next, and so on. For the purpose of saving space in the table, low-ability students will be listed using an L, medium-ability students will be listed using an M, and high-ability students will be listed using an H. The "(U)" in the Group Work Reported column indicates that a student in that group did not fill out the questionnaire asking about amount of group work done that was given during the last week of class.

Table 5.3. Ability Levels of Group Members, Average Semester Point Total, and Amount of Group Work Reported, by Group in Experimental Class A

Group	Ability of Group Members	Average of Point Totals	Group Work Reported
1	M, M, M	564	Med., Med., Low
2	M, M, M, M	543	Med., Med., Med.,Low
3	M, M, H	542	No, No, Med.
4	M, H	510	Med., Med.
5	L, L, M, M	482	Low, Low, Low, (U)
6	L, M, M, H	477	Med., Low, Low, (U)
7	M, M, H	458	Low, Low, Low
8	L, L, M	426	None, None, (U)
9	L, M	388	Med., High
10	L, H	365	None, (U)
11	L, L, M	282	(U), (U), (U)

Some comments may help to provide a better perspective on the above information. Students in Groups 1 and 2 were usually talking with each other when the instructor would arrive for class. Further, this instructor had the students do "minute papers" several times during the semester. On these minute papers, students were asked what concept or ideas they had learned that day, what they were confused about, and to comment on how their group was getting along. Members from Groups 1 and 2 would make very positive comments about their group on these minute papers. Two of the three students in Group 3 reported doing no group work during the semester. (The one medium-ability student who reported doing group work mentioned studying with others that weren't in the same section.)

Group 4 had been four members, but two of the members were commuters; these two members were allowed to form their own group (Group 10), since it was difficult for them to get together with the other two members who lived on campus. The two members who lived on campus were roommates. (One of these students was a high-ability student, the other was a medium-ability student.) Two of the students in Group 5 were roommates; both of them were medium-ability students. The other two students in Group 5 were friends, and were involved in a tutoring program outside of class. From comments made on the minute papers and during an interview (with one of this group), it seems that all four of them never got together very often outside of the classroom, although the two pairs seemed to meet frequently.

Group 6 started with three students, and kept those three throughout the semester. The high-ability student in their group joined the group when one member from his original (three-member) group dropped the course and the other member was absent several times. The student who joined the group ended up with the lowest semester point total of the group, in spite of the fact that this person was from the high-ability group. Group 7 seemed to get along fairly well, judging from instructor observation and comments made on the minute papers. One of the group made a comment in an e-mail to the instructor that the group wasn't getting together as much as they had hoped to, and that they would try harder to find time to meet.

The medium-ability student in Group 8 commented in a minute paper that he wasn't comfortable having to teach the other two group members the material that they didn't pick up in class. He attended class infrequently, though more than half of the time. It seemed that a sense of camaraderie never developed for this group. Group 9 had originally started with four members but one student had dropped and another requested to be allowed to join another group. The low-ability student in this group ended up with a slightly higher semester point total, despite the fact that that person's ACT score was four points lower than the other group member's ACT score. Group 10 has been mentioned in an earlier paragraph; this was the two-person commuter group. Group 11 had one group member who missed class more than half of the class periods, and another student who missed frequently, though not as much. The third member of the group had the lowest ACT score of the group, but had the highest semester point total of the group, by a considerable margin.

An analysis of the makeup of each group in the assisting instructor's experimental class was also done; this analysis was similar to the one reported above. The group makeup was examined with regards to the ACT category, amount of group work reported by members of each group, and the average point total for the semester for each group was done with this class. The results of this analysis are shown in Table 5.4. (As in Table 5.3, low-ability students will be listed using an L, medium-ability students will be listed using an M, and high-ability students will be listed using an H. The "(U)" in the Group Work Reported column indicates that a student in that group did not fill out the questionnaire asking about amount of group work done that was given during the last week of class.

Table 5.4. Ability Levels of Group Members, Average Semester Point Total, and Amount of Group Work Reported, by Group for Experimental Class B

Group	Ability of Group Members	Average of Point Totals	Group Work Reported
1	L, L, M	537.33	No, No, (U)
2	H, H,Unknown,Unknown	523.38	No, No, (U), (U)
3	L, L, M, Unknown	475.75	No, Low, Low, Low
4	L, M, H, Unknown	438	No, No, No, High
5	L, M	421.25	Low, (U)
6	L, L, L, M, M	417.7	No, Low,Low,(U),(U)
7	L, L, Unknown	415	No, Low, Med.
8	M, M	398	No, (U)
9	L, Unknown	388.25	No, (U)
10	L, M, Unknown	351	No, Low, Low

Concluding Remarks

This study was an investigation of the possible effects of using a modest amount of small-group work inside and outside of a university pre-calculus classroom. It seems that a number of instructors are reluctant to change their methods of instruction. Small changes are easier to make, and easier to convince others to make. This investigation was designed with the use of a modest amount of group work specifically to address this issue.

Even with the minimal intervention used in a demanding mathematics instructional situation, there were some documentable effects of group work or so it seems. Certainly this study does not contribute to those studies that examine a more fully implemented and extensive use of formal small group work. On the other hand, the level of intervention sustained here is something that can be implemented even in a very constrained instructional setting by a single instructor even when it is not course policy.

This researcher certainly plans to use cooperative learning again in future classes. It is hoped that future teaching situations will not always have the built-in constraints that the sections used in this research project had. When time is at a premium, the lecture mode of teaching seems to be the easiest choice of teaching techniques; a shortage of time certainly limits what can be done in the classroom using cooperative learning. Also, assigning projects that require students to work together outside of the classroom would be a useful tool in enlarging the amount of work done outside of the classroom by the students in their groups. This tool was not an option for this research project, due to the uniformity of the sections. This researcher believes that there was more small-group work done outside of the classroom in his experimental section than in his control section, though the analysis of group work done outside of the classroom did not give a significant result.

While the results of this investigation are not profound, it may be that some of the results will encourage others to try using small-group work in their classrooms. This researcher believes that there were a fair number of students in his experimental section who had a positive experience from the cooperative learning done during the semester, and would recommend cooperative learning to any instructor willing to try this technique in their classroom. While there are risks in making changes and in trying new methods of teaching, the benefits can be well worth the risks.

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APPENDIX A

ACT and SAT Mathematics Score Equivalents

Table A-1
ACT and SAT Mathematics Score Equivalents

ACT: Mathematics	SAT: Math
1	200-250
2	260
3	270
4-5	280
6	290
7	300
8	310
9	320
10	330
11	340
12	350
13	360
14	370
15	380
16	390
17	400
18	410
19	420
20	430-440
21	450
22	460-470
23	480-490
24	500-510
25	520-530
26	540-550
27	560-580
28	590-600
29	610-620
30	630-640
31	650
32	660-670
33	680-690
34	700-710
35	720-730
36	740-800

Note: The correlation between ACT: Mathematics and SAT: Math based on Langston's (1987) sample of 12,526 students was .834 and was significant at $P < .01$.

APPENDIX B

Student Information Questionnaire

Student Information Questionnaire for MA1523

1. NAME: ____

2. MAJOR: ____

3. a) LOCAL ADDRESS: __________________________________

   ____

   b) Where you went to high school: ____________________________

   c) e-mail address: _____________________________________

4. AGE: ____________

5. GENDER (circle one): Male Female

6. a) What was the last math course you had? Where? When?
   Please give the grade you received in that class.

Course: _______________________ Where: _______________________

When: ____________________________ Grade: _________________

   b) Please list any math courses you've had in the past two years.

Course	Where?	When?	Grade?

7. Have you studied trigonometry in a previous math class?

Circle one: YES NO

If you circled yes, approximately how many weeks did you spend studying trigonometry? ________________________________

8. Have you worked in cooperative groups in any of your previous courses in high school or college? Circle one:

No Yes, high school Yes, college Yes, both

9. Have you used a graphing calculator before?

No Yes, a little Yes, some Yes, quite a bit

Comments on your answer: ______________________________

____

10. Are you involved in extracurricular activities (including work)?
______________ If so, what? ______________________________

____

Approximately how many hours per week do you work? ______________

11. Please list your interests or hobbies. ______________________________

____

12. (a) Please list names of anyone in this class you would like to work with. (We make no promises about group composition.)

____

____

(b) Please list names of anyone in this class you would NOT like to work with. (We make no promises about group composition.)

____

____

APPENDIX C

Group Work Questionnaire

Group Work Questionnaire

Please print your name on this. I will not look at these until after I turn in your grades; this information is only to help me more accurately interpret the effects of your small-group work on your exam scores. Try to be fairly accurate.

NAME (Print): ____

1. a) While doing your math homework this semester, about what proportion of the time did you work with others?
(Circle the most appropriate.)

None 25% 50% 75% 90% or more

b) How frequently did you meet?
(Circle the most appropriate.)

Never Once/week Twice/week 3 times/week More often

c) Did the amount of group work you did increase // stay the same // decrease as the semester progressed? (Circle the most appropriate.)

2. With whom did you meet?
(Circle ANY that are appropriate; put an asterisk (*) by the most commonly used.)

Class Member(s) Friend(s) (other than someone in class)

Housing Learning Center Private Tutor

Other ______________________________________

3. What did you do in your group study?
(Circle any that are appropriate.)

Work problems. Discuss material from the previous class.

Check answers. I don't work with others.

Other ____

APPENDIX D

Attitudinal Questionnaire

Questionnaire

Please rate these statements for math courses in general.

	strongly disagree		disagree		neutral		agree		strongly agree
The teacher's job is to tell me how to do math.	1	2	3	4	5	6	7	8	9
I don't really understand something till I explain it to others.	1	2	3	4	5	6	7	8	9
I enjoy studying mathematics.	1	2	3	4	5	6	7	8	9
I learn best when most of the class time is the teacher's lecture.	1	2	3	4	5	6	7	8	9
I'm more comfortable working on learning by myself.	1	2	3	4	5	6	7	8	9
Math can be a bewildering subject.	1	2	3	4	5	6	7	8	9
The teacher's job is to help us think through a procedure or concept.	1	2	3	4	5	6	7	8	9
I enjoy working with others inside and outside the classroom.	1	2	3	4	5	6	7	8	9
My math classes have been a positive experience.	1	2	3	4	5	6	7	8	9
I prefer to listen and write during class rather than to discuss.	1	2	3	4	5	6	7	8	9
I spend most of lecture time writing what the teacher says (or writes).	1	2	3	4	5	6	7	8	9
Math contains some interesting, useful, and powerful connections	1	2	3	4	5	6	7	8	9

APPENDIX E

Experimental Group Questionnaire

Questionnaire Sections 005 and 011

1) How easy was it for you to get together with members of your group to study?
(Circle the most appropriate number.)

Very Easy		Easy		Somewhat Difficult		Difficult		Very Difficult
1	2	3	4	5	6	7	8	9

2) What factors influenced your not working more with members of your group?
(Check all that apply.)

a) Don't like working in groups. ____________
b) Difficult to make time to get together. __________
c) I live out of town. ________
d) I missed class too much. _________
e) I didn't get along with one or more people in my group. _________
f) Other. (Please comment.)
____
____

3) How often did a person in your group not do their fair share?
(Circle the most appropriate number.)

Never		A little				A lot	Quite a lot	
1	2	3	4	5	6	7	8	9

4) Approximately how many hours per week did you spend preparing for this class?(Include time spent doing homework, reading your notes or the textbook, studying for exams, etc. .)
(Estimate as best you can.) ____________________

5) Approximately how many hours per week did you spend preparing for this class <u>with others</u>? (Include time spent doing homework, reading your notes or the textbook, studying for exams, etc..)
(Estimate as best you can.) ____________________
(More on the back of this page.)

6) Rate your experiences with small-group learning this semester.

(Circle the most appropriate answer.)

Very Negative		Negative		Neutral		Positive		Very Positive
1	2	3	4	5	6	7	8	9

Comment if you want to on your experiences (positive and negative) with small-group learning in this class.

____

____

7) Occasionally this semester you were to work on examples from the Study Guide, with some assistance from me. Please rate this method by circling the appropriate number.

Very Unhelpful		Somewhat Unhelpful		Neutral		Somewhat Helpful		Very Helpful
1	2	3	4	5	6	7	8	9

Comment if you want to on your thoughts & opinions about working together in class on examples from the Study Guide.

____

____

____

8) How much group work have you done in math classes before this one?
(Circle the most appropriate.)

None		A little		Some		A lot		Quite a bit
1	2	3	4	5	6	7	8	9

Please describe any positive or negative aspects of previous group work.

____

____

APPENDIX F

QUALITATIVE INTERVIEW QUESTIONS

1. (GIVE THE STUDENT A PROBLEM TO WORK.)
   Please solve this problem. Tell me OUT LOUD what you're doing at each step.

2. Do you recall solving a similar problem during this semester? Did you work with others in your group on such a problem?

3. How much group work have you done this semester?

4. Did you study others when doing your homework? How often?

5. Did you study with others when preparing for quizzes and examinations? How often?

6. How do you think studying with others affects your learning?

7. What (if anything) was frustrating about studying with others?

APPENDIX G

Sample Back Page Examination Problems

Sample Back Page Examination Problems

- An open-top box with a square base has a volume of 20 cu. ft. . What dimensions will minimize the amount of material required to produce the box? What is the total surface area? Round you answers to the nearest tenth.

- Prove the following identity: $(1 + \cos x) / \sin x + \sin x / (1 + \cos x) = 2 \csc x$.

- The second hand on a clock is 2 inches long. Answer the following questions; put your answers in units of radians, inches and seconds.

  A) What is the angular speed of the second hand? ____________________

  B) What is the linear speed of the second hand? ____________________

  C) How far does the tip of the second hand travel in 2 minutes? _______________

- Find the exact solution of the equation below. Approximations from your calculator will not earn full credit. For full credit, solve this problem algebraically.

$$2\text{\^{}}x = 3\text{\^{}}(x - 3)$$

APPENDIX H

Students' Comments About Working in Groups
(Comments Made at the Start of the Fall Semester 1999)

Group Work Comments (Start of Semester)

Section 012 (Control)

- "Ideas can be thrown out in groups that allow students to learn from the discussion."
- "I believe this statement because obviously 2 minds are better than one. When studying with a small group, one is able to assimilate more viewpoints which will lead to a more complete or better understanding. Also the theory of "the best way to learn a subject is to teach a subject" applies. By sharing your ideas, your confidence is also increased."
- "I agree; working in groups can help students learn & understand better because there is more than one opinion about a certain problem & there are more people trying to come up w/a solution to that problem. A single person would have more trouble & more time spent on a problem."
- "The reason this is true is because sometimes other classmates pick things up better than you and they can also help get you to understand a concept better by the way they explain it."
- "I strongly agree with this statement. I enjoy math but it is not one of my strongest subjects. It usually takes me longer than most other students to comprehend a new topic. I feel that if I don't understand the teacher's explanation, it's helpful to hear it from another student. It helps me to learn it when I have it shown several different ways. It also helps to know more than one way to solve problems."

- "I believe that if a student has trouble grasping a concept, a peer can be helpful. They can see where the other student is coming from and lead him or her in the right direction."
- "I definitely agree because it allows students to further 'cement' the newly learned concepts into their heads because they're learning to explain what they understand. (or question what they don't)"
- "I strongly agree because if one student does not understand something, another student may have just figured it out and can relate to the problems the other student may be having. The instructors sometimes forget the little problems that students sometimes have. Also, some students feel more comfortable asking a question to someone who they know because they could be shy." (This student dropped the (control) class.)
- "I think that working in groups does help you to understand more & better but, me, I think that sometimes (the majority) I learn better by myself first. Once I know exactly what I'm doing I understand it a little more once in a group."
- "Working together can bring diverse ideas to a group to help everyone better understand the problem better. However, groups can lead to distracting extraneous talking that veers from the subject at hand. I do not usually benefit from group experience."
- "Working in small groups can be sometimes a help to some students, but I prefer studying on my own because I can work at my own pace and understand a particular topic better."

- "No, I feel they make it more difficult to concentrate because of distractions & also force people to depend more on others."

Section 011 (Experimental)

- "I agree with this. A teacher explains math to the students, but when I leave the classroom I have only the info I could take in. When students work together they can still teach each other after class."
- "In a group, you can find other ways to solve a problem. Students learn well when they share how they remember a formula, etc. When a person works alone, they sometimes have 1 way to solve a problem."
- "If all of the students put forth an effort, then working in small groups can really help."
- "Yes sometimes but the best way to learn math for me is to do homework problems."
- "This statement is true in math as it is in any subject. The actual discussion of material allows students to confirm their ideas on the subject as well as providing a proving ground where they can support their ideas with knowledge provided by the course."
- "If someone in the group knows the material fairly well, then he/she can help the rest of the group, assuming the rest of the group wants to learn."
- "You can help each other start a problem or get through a tough part of it rather than just giving up when you're in a group instead of individual."
- "I agree because I can learn things better if I can explain them to others."

- “I totally agree with this statement. I get frustrated with math very easily and it helps me to have people to study with, to ask questions & to help guide me. Sometimes it helps me to learn a concept by explaining to others.”
- “I do believe that is true unless the group is pressed for time. In that case the strongest student or students are relied upon for the solutions and weaker students become passive.”
- “If someone in the group knows the material fairly well, then he/she can help the rest of the group, assuming the rest of the group wants to learn.”
- “It all depends on the person.”
- “Working in a group can be ok as long as someone in the group really knows what they are doing; if nobody understands what is going on then it really serves no purpose.”
- “I agree with this statement to some extent, but I strongly believe math is best learned by working problems & getting them right on your own.”
- “I disagree and agree. I’m a person who likes to work with people, but not too many. I disagree b/c I understand better working by myself. Too many people confuse me.”
- “I do not really agree with this statement. In my experience, one person does the entire group" work while others copy. This type of learning is not beneficial to me because frequently in math I work much slower than other students. I feel that I would be left behind.”
- “I think that working in small groups can be intimidating if it takes you awhile to understand what you are doing. I don’t like working in groups.” (This student dropped the (experimental) class.)

Section 006 (Control)

- "I agree because students are in the same situation and can relate to each other. They also communicate with each other better."
- "Sometimes yes & sometimes no. It really depends on the person & the material being learned."
- "This is true in most situations when working with people who are wanting to learn the material and strive to truly understand what they're doing not just getting the answer to that one problem."
- "Group work creates connections positively 99% of the time. Whoever said one head is better than two? No one."
- "If someone is working alone on a certain topic, and they become stuck on a problem or concept, then nobody can help them. They become frustrated and bored, and if they can't figure it out, they give up. If they were working in a group someone that understood the subject could help them figure it out."
- "This is mainly dependent on who one works with; I have had many occasions where no work was completed due to the fact that the people in the group did too much talking/socializing instead of focusing on their assignment."
- "Explaining how to work problems to other students helps reinforce what you just learned from the teacher, hearing it once & taking notes, then voicing what you heard impacts more what you just learned."
- "It really helps me to hear personal strategies and different things like that. It's also a lot less overwhelming when you can get private help."

- "If work actually gets done the potential for a better learning experience exists. It all comes down to what you and your group know and do."
- "It helps; people might be more likely to speak up if they don't understand."
- "Sometimes it's helpful but I generally prefer to work alone."
- "I think that it helps some people to discuss their ideas and hear ideas from other people. Most students need that interaction w/ other students to be motivated and to help them learn but that's not true for everyone."
- "I agree because other students may be able to explain a topic in a more understandable way to another student."
- "I do my best when the teacher explains how to do a certain concept and gives me examples. I then like to do my homework and then compare my answers with another student's to see if I am on the right track."
- "I believe small groups can be efficient if the members of the group are dedicated to helping each other learn & understand the material. However, if the members are not dedicated, distractions and conflicts will arise, & working alone would have been the wiser choice. I, personally, would prefer to work alone."
- "I agree because I can be stumped on a problem and having a little help is a big positive."
- "Working in groups can be helpful-just not if your are the only one doing the work. If it's going to be that way I would prefer to work alone."
- "It can help the lower end of the class learn and understand better, but the upper half who already understand can be slowed down by it. I worked in groups in several math classes, and my progress was often hindered by those who didn't understand

and always looked to me for explanation instead of the instructor."

- "I feel that small groups can help students to learn; however, I have always preferred to work alone. I have a systematic way of studying and when different personalities are involved, several time-infringing problems arise. I prefer to rely on the teacher when I have subject-specific questions rather than other students who may or may not grasp the subject matter."
- "Working in a group allows a student to interact with others for the purpose of exchanging experiences. Such as-if I do not understand a concept, another student may know how to get me to understand because not too long ago, they did not understand. The teacher, however, learned the concept years ago & does not remember <u>how</u> he/she learned it."
- "Groups are positive if the group actually communicates. If they sit like bumps on a log, it sucks. No one learns anything. But if the group gets along, then the group could learn a lot."
- "True. Most students need to verbally go through math not only to help others, but himself also"
- "Small groups are wonderful for gathering information and developing different concepts about the same problem; however, small groups should not be daily. Individuals need to learn from one another, but also themselves."
- "I hate group work. I mean I <u>really</u> <u>really</u> HATE it. It may be my least favorite aspect of school. I think it is very helpful for some people, but shouldn't be forced on everyone. Maybe they should offer 2 classes when they can, group oriented and individual based."

Section 005 (Experimental)

- "Sometimes when students don't understand a problem the teacher explains, they can usually understand from another student's point of view."
- "I strongly agree with the statement because from my experience I think it makes a student's mind & knowledge broad and also makes the class interesting."
- "True, working in small groups means you are discussing what you are leaning & have automatic 'study partners' to help you work through concepts you have problems with."
- "Very, very true statement; what this class is going to do will help students in the future."
- "I believe working in groups can improve understanding of concepts & explaining answers to other people is beneficial."
- "I agree because it may benefit to hear how your peers explain a concept compared to how your professor explains it."
- "I believe that working in a small group creates a positive environment-positive in that no one person will always have the correct answer. Ironic, maybe, but that allows the student to ask classmates, instead of asking aloud [so as] not to be embarrassed ... & getting snickers!"
- "I think this is true because students will ask questions more & will have someone there to explain. It is always easier to learn one on one than one on thirty-five. I think it will work out good."

- "Each individual student has an independent process of thinking about and solving problems. In the context of a math class, where there is definite solution to a problem, most likely some students will ride on the shoulders of others. Therefore will learn less than if they were to think for themselves. In English or philosophy it could work."
- "I believe that working in a group can help students learn-but I also believe that working alone creates better study habits. Group work sometimes creates laziness with certain group members-and those people struggle in work done alone. In the long run, I believe there should be a healthy mixture of group and alone work."
- "In some cases that statement is true, but in others it is not. Everyone works and learns in different ways. For some, working alone would be better because it forces the student to independently discover and completely understand the material. For others, it might be more helpful to have a peer help answer questions. Some people are too shy to ask questions & some learn better explaining what he/she knows about the material. Relying on one way to help students learn is not the best way. The best way to help students is by incorporating all the ways of learning into the class (Examples: visual, audio, group work, independent work)."
- "Working in groups in the past seems to liven things up allowing for a much more relaxed/fun environment. I have in the past worked in groups in a math class and it proved to be beneficial."
- "I believe group work to be quite beneficial when all members participate. Members of the group can benefit from each other's strengths and improve their weaknesses. Great idea for a math class, all of my Spanish classes use this method."

- "I agree with this statement. I am excited about the idea of working in groups in this class."
- "I worked in groups in my Algebra Two class in high school and it worked well for me. I did better and understood. It was easier having a peer explain it rather than the teacher sometimes."
- "I could see this as being an advantage because you can ask in a small group instead of out loud which can be embarrassing."
- "Sometimes, if all members of the group participate in the process and everyone has their own responsibility."
- "I agree with this statement. It just depends on how comfortable the student feels in the group and how much effort they put into it."
- "Yes I believe it would really help especially when a mathematics problem is difficult we can borrow ideas from each other."
- "I agree in that when working in groups you get the benefit of shared knowledge & understanding. However, some individuals succeed better solving and coming to conclusions better by themselves."
- "Depending on the student, group work can be varying levels of beneficial. It is good for everyone at times, although better for some students."

APPENDIX I

Tips for Teachers

Suggestions and Comments for Instructors

Teaching MA1523 Using Small-Group Work

Tips for Teachers

I plan to have a group size of 3-4 students. I plan to put my students to work on the (appropriate) examples in the Study Guide by themselves, after they get together in their groups. They will be instructed to look to each other for assistance before the group asks the instructor for help. If one or two members of the group is struggling with an example, they can observe what the other student (who may or may not have correct work) has done. They should not just copy down what the other person has done, since they don't learn how it was done, the other person doesn't learn by tutoring, and the other person's work may be wrong.

I plan to assign the homework to be done in groups, and pick up an assignment regularly but not on announced days (somewhat "randomly"). The homework will be a preparation for the exams and the quizzes, so students should understand how to do all of the problems. They may split the problems up, but they should explain to each other how to solve those problems. I would not encourage splitting up skill-type problems.

I will give individual quizzes, also on a "random" but regular basis. I also plan to give a "group grade" on each quiz— the average of the scores of the group members. The two types of quiz scores and the group homework score will all count towards the 100 points for the homework score, after I throw out approximately 1/3 of the lowest of these scores. This is a method I've worked out to try to balance individual accountability and positive interdependence (we want the group members to look out for each other). You're welcome to use whatever system you're comfortable for figuring the homework score.

It may be a good idea to give a "minute paper" on a regular basis. This is an (optionally) anonymous questionnaire given at the end of class, asking a) what was fairly clear about that day's lesson, b) what was still unclear about it, and c) any comments or questions. This gives the students the chance to communicate to you problems with their groups or their understanding without having to visit with you in person. (Hopefully, they would want to do that, but some may not be comfortable in doing so.)

What I hear, I forget.
What I see, I remember.
What I do, I understand. — Confucious

"At the beginning of a semester, the teacher explains that the method [group work] will place new responsibilities on the student and require them to learn new behavior." (Learning in Groups; Bouton & Garth, Ed.; page 34)

The following two paragraphs are comments of some teachers when asked what the most important advice you would give to teachers who are about to use cooperative learning for the first time.

The most important advice I could give a teacher who is planning to use cooperative learning is to be prepared! Study the handbook; thoroughly acquaint yourself with procedures, scoring, suggestions, and so on; thoroughly indoctrinate your students through practice sessions and demonstrations; get all the materials together far in advance of the actual implementation; be flexible; be prepared for frustration (yours and your students'); ... above all, enjoy the experience. Also, wait until you've gotten to know your students before you try to put them in groups. Again, be flexible; switch kids around until you've created good groups. (A comment from a middle school teacher about group work.)

I recommend that a teacher think big but move slowly! I would remind her that when a teacher tries a new skill she will actually feel less competent for a period of time. I read that a new teaching skill takes 20-30 practices before a teacher reaches a comfort zone in its use. (A comment from a 5th grade teacher about group work.)

Critical Elements of Cooperative Learning Methods:

1. Face-to-face interaction.
2. Positive interdependence: students work together to achieve a group goal.
3. Individual accountability.
4. Interpersonal and small-group skills: students must be taught effective means of working together and of discussing how well their groups are working to achieve their goals.

Basic Principles

1. Make sure you offer some kind of recognition or reward to successful teams.
2. Make each student responsible for his or her own performance.
3. Set up a scoring system that allows students of all performance levels to contribute meaningfully to the team scores or products.

Seven Rules for Students Working in Groups

1. I am critical of ideas, not people.
2. I remember that we are all in this together.
3. I encourage everyone to participate.
4. I listen to everyone's ideas, even if I do not agree with them.
5. I restate what someone said if it is not clear.
6. I try to understand both sides of the issue.
7. I first bring out all the ideas, then put them together.

(NOTE: The last two may not be so applicable to math groups.)

Three Important Goals to Accomplish with Active Learning

1. Team building: help students to become acquainted with each other and create a spirit of cooperation and interdependence.

2. On-the-spot assessment: learn about the attitudes, knowledge, and experience of the students.

3. Immediate learning involvement: create initial interest in the subject matter.