FLAT PLATES BY SUCCESSIVE APPROXIMATIONS

By

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PREFACE

A numerical procedure of successive approximations for the analysis of flat elastic plates, simply supported, is presented in this thesis. The plate equations are expressed in finite differences; the solution is achieved by iteration. The accuracy of the solution is dependent upon the number of cycles of iteration.

The first part of the thesis is a sketch of the development of the general theory, which may be found in standard texts on flat plates.

The second part is a numerical analogy to the algebraic solution of thin plates prepared for the McDonnell Aircraft Corporation.[1] The numerical approach involves the use of the algebraic basic series only; the carry-over and circulatory series are solved numerically.

The writer wishes to acknowledge his indebtedness to his faculty and thesis advisor, Professor Jan J. Tuma, whose invaluable personal and professional counsel during the course of the writers graduate work supplied much incentive for further study.

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[1] Tuma, Jan J, et al, McDonnell Aircraft Corporation Research Project No. 1, Oklahoma State University Library

Special acknowledgement and appreciation is extended to the McDonnell Aircraft Corporation for their permission to use the ideas developed in their sponsored research project at Oklahoma State University as the nucleus of this thesis, as well as the final results of the project report as a numerical check on this approach.

August 1958
Stillwater, Oklahoma

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NOMENCLATURE

A	Improved Carry-over Factor Parallel to X-Axis
A'	Modified Carry-over Factor Parallel to X-Axis
B	Improved Carry-over Factor Parallel to Y-Axis
B'	Modified Carry-over Factor Parallel to Y-Axis
D	Improved Diagonal Carry-over Factor
D'	Modified Diagonal Carry-over Factor
D_f	Flexure Rigidity of a Flat Plate
E	Hooke's Modulus for the Plate Material
I	Moment of Inertia
M	Intermediate Moment Function
M_x	Moment per Unit Length in X-W Plane
M_{xy}	Twisting Moment Per Unit Length in Y-W Plane
M_y	Moment Per Unit Length in Y-W Plane
M_{yx}	Twisting Moment Per Unit Length in X-W Plane
P	Equivalent Concentrated Load
Q_x	Shear Per Unit Length on an X-W Plane
Q_y	Shear Per Unit Length on a Y-W Plane
R	Reaction at the Corner of a Simply Supported Plate
X	Denominator of Convergency.
a	Basic Carry-over factor parallel to X-Axis
b	Basic Carry-over Factor Parallel to Y-Axis
f_b	Convergency Factor
h	Thickness of the Plate
n	Ratio of Finite Difference Divisions

q ..... Intensity of Load Per Unit Area

w ..... Plate Deflection From Plane Surface.

λ ..... Load Function

μ ..... Poisson's Ratio

ψ ..... Function of Plate Material and Dimensions

CHAPTER

I

BASIC EQUATIONS

CHAPTER I BASIC EQUATIONS

1. INTRODUCTION

The object of this study is to present a procedure for the solution of the differential equations of second order for flat elastic plates. The equations are expressed in the form of finite differences equations; the solution is accomplished by iteration. Convergency of the iteration series is improved by a preliminary algebraic series solution of a basic set of 5 points.

The application of finite differences to flat plates was introduced by Dr. H. Marcus.(1)* The solution was achieved by Gauss's elimination or iteration. These methods are treated in standard texts in college algebra.

In order to reduce the error in the finite differences approximation to the elastic surface of flat plates to less than one percent, at least an eight strip division must be used. The resulting matrix yields 49 variables with their resulting 49 equations, the iteration solution of which becomes quite laborious.

The process of algebraic iteration developed by Tuma(2)(3)(4) affords a means to improve the rate of convergency of the iteration series. The improved rate of convergency considerably reduces the

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* Numbers in parenthesis refer to bibliography at end of thesis.

time and labor required to solve the matrix of equations. At the same time, the purely mechanical procedure of iteration is preserved.

Of the three series introduced by Tuma, the basic, circulatory and carry-over series, only the basic series is used herein as an algebraic series. The other series must exist, but are treated numerically rather than algebraically.

The writer's interest in this subject stems from the formal course in flat plates taught by Professor Tuma at Oklahoma State University. The application of the algebraic series to accelerate the numerical procedure is an outgrowth of the flat plates research project at Oklahoma State University.sponsored by the McDonnell Aircraft Corporation. The project was directed by Professor Tuma; the writer was a member of the research group.

The project report(5) contains the full development and application of the algebraic series to flat rectangular plates having simply supported edges.

2. THE PLATE EQUATIONS

In the general derivation of the plate equation, the following assumptions were made:

a.) The material is homogeneous and isotropic.

b.) Deflections are so small in comparison to the dimensions of the plate that they do not affect the geometry of the plate.

c.) The material follow Hooke's Law, and all deformations are within the range of Hooke's Law for the material.

d.) The edges are free to move in the plane of the plate, hence the edge reactions act normal to the plate.

The coordinate axes are shown in Fig. 1; the arrows indicate positive sense.

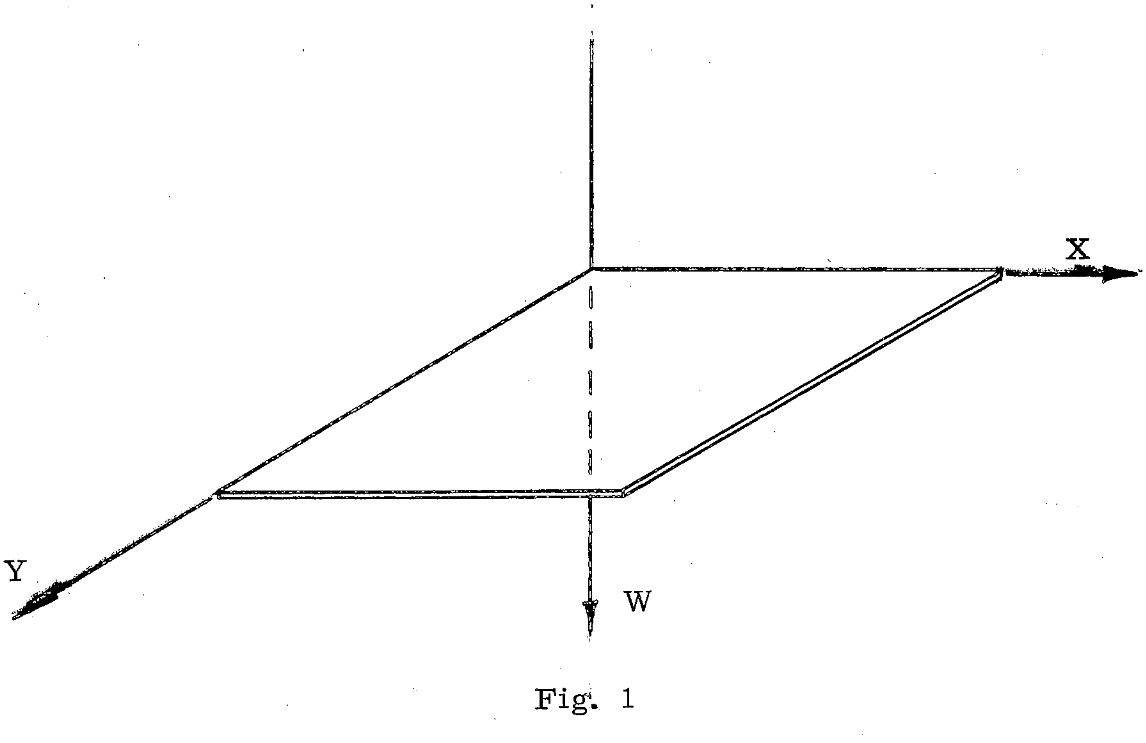


Fig. 1

Coordinate Axes For
The Plate Equation

The development of the general plate equations may be found in standard texts on flat plates.

The general plate equation is :

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{q}{D_f}, \qquad (I-1)$$

where w is the deflection of the plate, the positive sense taken downward,

q is the intensity of load per unit area.

D_f is the flexure rigidity of the plate :

$$D = \frac{Eh^3}{12(1-\mu^2)},$$

where E is Hooke's Modulus for the material,

h is the plate thickness,

and μ is Poisson's ratio for the material.

The shears and moments on a particle are shown in Fig. 2 . They are shown in positive sense.

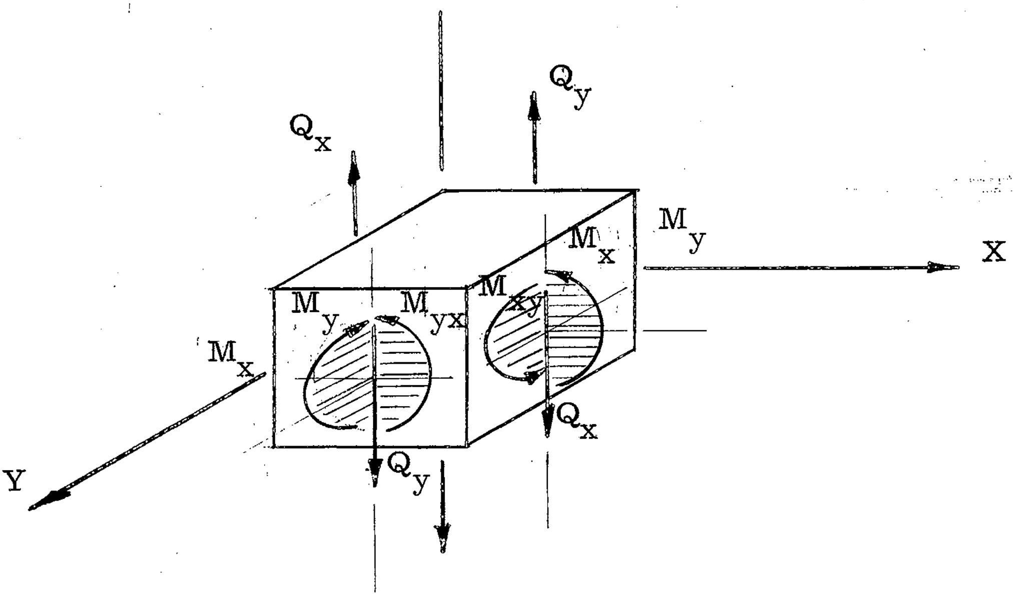


Fig. 2

Shears and Moments on a Particle.

Shears and moment are given by the following equations:

$$M_x = -D_f \left(\frac{\partial^2 w}{\partial x^2} + \mu \frac{\partial^2 w}{\partial y^2} \right) \qquad (I - 2)$$

$$M_y = -D_f \left(\frac{\partial^2 w}{\partial y^2} + \mu \frac{\partial^2 w}{\partial x^2} \right) \qquad (I - 3)$$

$$M_{xy} = -M_{yx} = D_f (1 - \mu) \frac{\partial^2 w}{\partial x \partial y} \qquad (I - 4)$$

$$Q_x = -D_f \frac{\partial}{\partial x} \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right) \qquad (I - 5)$$

$$Q_y = -D_f \frac{\partial}{\partial y} \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right) \qquad (I - 6)$$

The reaction at a corner of a simply supported plate is given by:

$$R = 2D_f (1 - \mu) \frac{\partial^2 w}{\partial x \partial y} \qquad (I - 7)$$

where R is the vertical reaction, positive sense taken upward.

Eq. (I - 1) can be expressed as two second-order equations. Summing equations (I - 2) and (I - 3),

$$M_x + M_y = -D_f (1 + \mu) \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right) ,$$

whence

$$\frac{M_x + M_y}{(1 + \mu)} = -D_f \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right) .$$

Calling

$$M = \frac{M_x + M_y}{D(1+\mu)} .$$

then

$$-\frac{M}{D_f} = \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) . \qquad (\text{I-8})$$

In operator form, the general plate equation is

$$q = D_f\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) .$$

Substituting,

$$q = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)(-M) ,$$

hence,

$$-q = \frac{\partial^2 M}{\partial x^2} + \frac{\partial^2 M}{\partial y^2} . \qquad (\text{I-9})$$

The solution of Eq. (I-8) and Eq. (I-9) will be presented in this study.

3. The Plate Equations In Finite Differences

The Laplacian equations (I-8) and (I-9) may be expressed by finite differences. The plate is divided into strips as shown in Fig. 3).

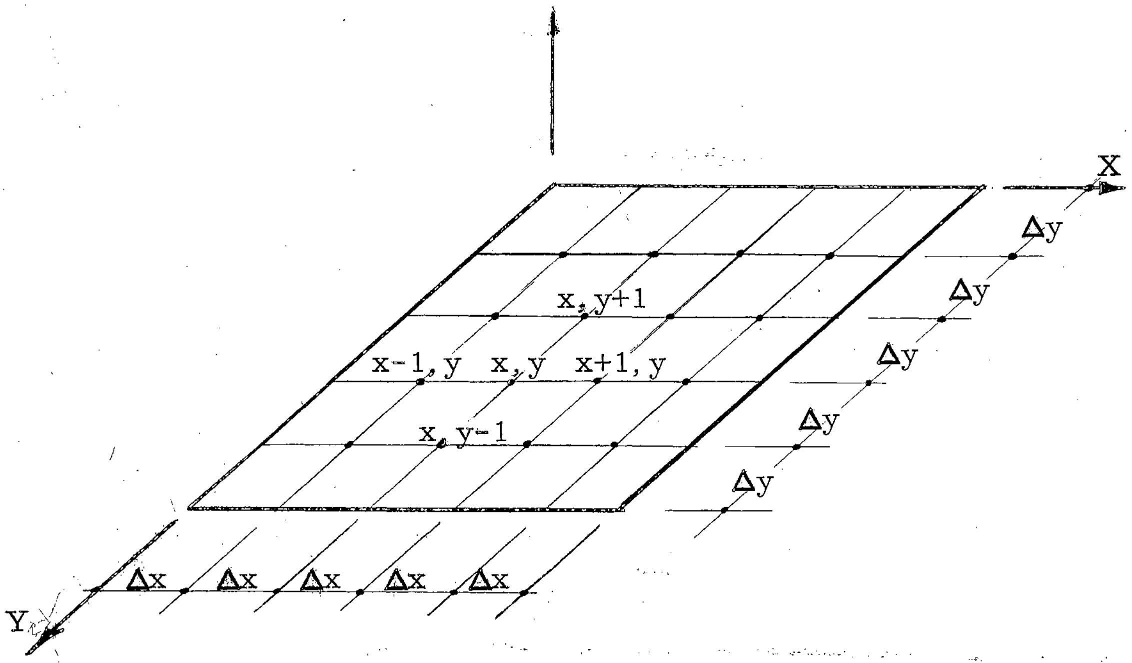


Fig. 3

General Point Numbers

In finite differences, for Eq. (I-9) at point (x, y) ,

$$\frac{\partial^2 M_{(x,y)}}{\partial x^2} = \frac{M_{(x-1,y)} - 2M_{(x,y)} + M_{(x+1,y)}}{\Delta x^2} ,$$

and

$$\frac{\partial^2 M_{(x,y)}}{\partial y^2} = \frac{M_{(x,y-1)} - 2\,M_{(x,y)} + M_{(x,y+1)}}{\Delta y^2} .$$

Summing, and denoting $\frac{\Delta x}{\Delta y} = n$,

$$-q\,\Delta x\,\Delta y = \left\{ \begin{array}{l} + \frac{1}{n}\,M_{(x-1,\,y)} + n\,M_{(x,\,y-1)} \\[2ex] - \frac{2+2n^2}{n}\,M_{(x,\,y)} \\[2ex] + \frac{1}{n}\,M_{(x+1,\,y)} + n\,M_{(x,\,y+1)} \end{array} \right\} .$$

In final form, Eq. (I-9) is :

$$-\frac{P_{(x,\,y)}}{N} = \left\{ \begin{array}{l} + a\,M_{(x-1,\,y)} + b\,M_{(x,\,y-1)} \\[2ex] - M_{(x,\,y)} \\[2ex] + a\,M_{(x+1,\,y)} + b\,M_{(x,\,y+1)} \end{array} \right\} , \qquad (\text{I-10})$$

where the symbols are :

$$P_{(x,\,y)} = q_{(x,\,y)}\,\overline{\Delta x}\;\overline{\Delta y} \qquad N = \frac{2+2\,n^2}{n}$$

$$a = \frac{\frac{1}{n}}{N} = \frac{1}{2+2n^2} \qquad b = \frac{n}{N} = \frac{n^2}{2+2n^2} .$$

Similarily, Eq, (I-8) in finite differences :

$$-\frac{M_{(x,\,y)}}{\psi} = \left\{ \begin{array}{l} + a\,w_{(x-1,\,y)} + b\,w_{(x,\,y-1)} \\[2ex] - w_{(x,\,y)} \\[2ex] + a\,w_{(x+1,\,y)} + b\,w_{(x,\,y+1)} \end{array} \right\} , \qquad (\text{I-11})$$

where $\psi = \dfrac{D_f N}{\Delta x \, \Delta y}$.

In finite differences, the cross-sectional elements of Eq's (I-2, 3, 4) are :

$$M_{x(x,y)} = -\psi \left\{ \begin{array}{l} + \; a \, w_{(x-1,y)} + \mu \, b \, w_{(x,y-1)} \\ \\ - \; 2 \, (a+\mu b) \; w_{(x,y)} \\ \\ + \; a \, w_{(x+1,y)} + \mu \, b \, w_{(x,y+1)} \end{array} \right\} , \qquad (\text{I-12})$$

$$M_{y(x,y)} = -\psi \left\{ \begin{array}{l} + \; \mu \, a \, w_{(x-1,y)} + b \, w_{(x,y-1)} \\ \\ - \; 2 \, (\mu \, a + b) \, w_{(x,y)} \\ \\ + \; \mu \, a \, w_{(x+1,y)} + b \, w_{(x,y-1)} \end{array} \right\} , \qquad (\text{I-13})$$

and

$$M_{xy(x,y)} = \psi \left(\frac{1-\mu}{4N} \right) \left[\begin{array}{l} + \; w_{(x-1,\, y-1)} - w_{(x+1,\, y-1)} \\ \\ + \; w_{(x+1,\, y+1)} - w_{(x-1,\, y+1)} \end{array} \right] . \qquad (\text{I-14})$$

Substituting Eq. (I-8) into Eq's (I-5, 6) , and expressing in finite differences,

$$Q_{x(x,y)} = \frac{\partial M_{(x,y)}}{\partial x} = \frac{1}{2 \, \Delta x} \left[M_{(x-1,y)} - M_{(x+1,y)} \right] , \qquad (\text{I-15})$$

and

$$Q_{y(x,y)} = \frac{\partial M_{(x,y)}}{\partial y} = \frac{1}{2 \, \Delta x} \left[M_{(x,y-1)} - M_{(x,y+1)} \right] . \qquad (\text{I-16})$$

The corner reaction given by Eq. (I-7) is :

$$R = 2\,M_{xy}$$

where M_{xy} is evaluated at the corner point.

Eq's (I - 10 and I - 11) are applied in the following numerical example.

CHAPTER

II

THE SOLUTION OF THE FINITE DIFFERENCES EQUATIONS.

CHAPTER II

THE SOLUTION OF THE FINITE DIFFERENCES EQUATIONS

1. NUMERICAL ITERATION

The diagonal system of finite differences equations stated in terms of Eq's (I-10) or (I-11) may be solved by methods of successive approximations, among which the most popular are :

a.) Gauss-Seidel Iteration
b.) Southwell Relaxation .

The application of the Gauss-Seidel numerical iteration to the analysis of a plate problem follows.

A flat rectangular plate of constant thickness, loaded by a single transverse force P applied at its center is considered. The plate is simply supported along all edges on a rigid foundation, and divided into 36 rectangular finite strips as shown (Fig. 4).

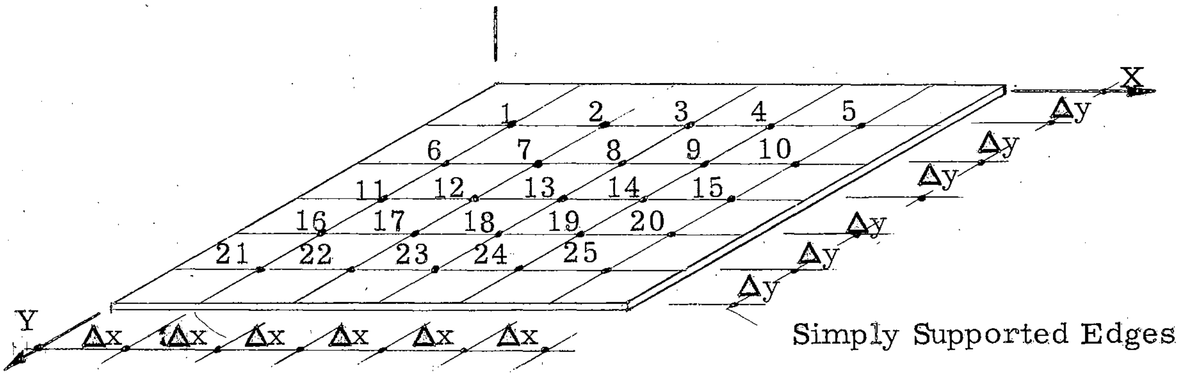


Fig. 4

Point Numbers for
Finite Differences Equations

The numerical constants of Eq's (I-10, 11) are :

$$\left|\begin{array}{lll} n & = & \frac{\Delta x}{\Delta y} = 1 \\ a & = & \frac{1}{4} \end{array}\right. \qquad \left|\begin{array}{lll} N & = & 4 \\ b & = & \frac{1}{4} \ . \end{array}\right.$$

In terms of these values, Eq's (I-10, 11) become:

$$-\lambda_{(x,y)} = \left\{\begin{array}{l} +\frac{1}{4} M_{(x-1,y)} + \frac{1}{4} M_{(x,y-1)} \\ - \quad M_{(x,y)} \\ +\frac{1}{4} M_{(x+1,y)} + \frac{1}{4} M_{(x,y+1)} \end{array}\right\} \qquad (II-1)$$

$$-\frac{M_{(x,y)}}{\psi} = \left\{\begin{array}{l} +\frac{1}{4} w_{(x-1,y)} + \frac{1}{4} w_{(x,y-1)} \\ - \quad w_{(x,y)} \\ +\frac{1}{4} w_{(x+1,y)} + \frac{1}{4} w_{(x,y-1)} \end{array}\right\}, \qquad (II-2)$$

where $\lambda_{13} = \frac{P}{N}$ and all other λ's $= 0$.

Eq. (II-1) is stated for each point of the plate (Fig. 4) . The system of these equations is shown in Table 1a , and the solution of this system by numerical iteration follows in Table 1b. The results of table 1b are in terms of the M-coefficient $k_{(x,y)}$ at the point divided by N :

$$M_{(x,y)} = \frac{k_{(x,y)}}{N} \ .$$

A typical equation in Table 1a is the equation for point 13. It reads:

$$\frac{1}{4} M_8 + \frac{1}{4} M_{12} - M_{13} + \frac{1}{4} M_{14} + \frac{1}{4} M_{18} = - 1\lambda ;$$

The constant is to the right of the equality sign.

It is noted here that as far as any single iteration is concerned, the only points involved are those immediately to the left, right top, and bottom of the point being iterated, as indicated in Fig. 5. This observation is put to use later, in the algebraic iteration.

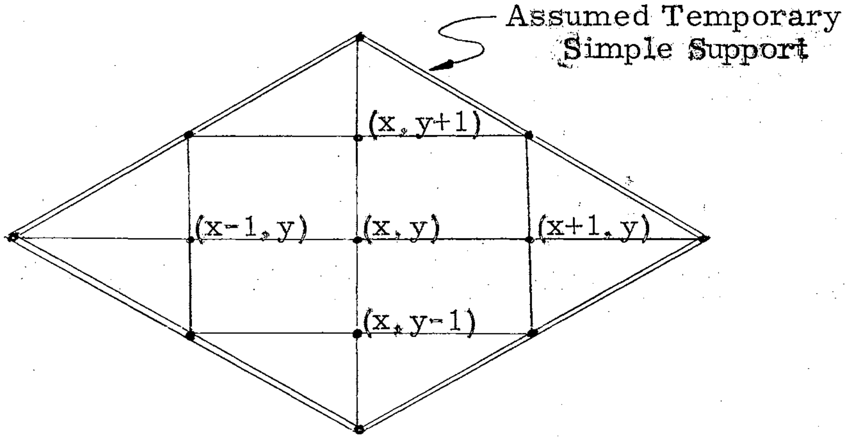


Fig. 5

Assumed Edge Conditions
for Single Iteration Step

	M_1	M_2	M_3	M_4	M_5	M_6	M_7	M_8	M_9	M_{10}	M_{11}	M_{12}	M_{13}	M_{14}	M_{15}	M_{16}	M_{17}	M_{18}	M_{19}	M_{20}	M_{21}	M_{22}	M_{23}	M_{24}	M_{25}	λ
1	-1	$\frac{1}{4}$				$\frac{1}{4}$																				
2	$\frac{1}{4}$	-1	$\frac{1}{4}$				$\frac{1}{4}$																			
3		$\frac{1}{4}$	-1	$\frac{1}{4}$				$\frac{1}{4}$																		
4			$\frac{1}{4}$	-1	$\frac{1}{4}$				$\frac{1}{4}$																	
5				$\frac{1}{4}$	-1					$\frac{1}{4}$																
6	$\frac{1}{4}$					-1	$\frac{1}{4}$				$\frac{1}{4}$															
7		$\frac{1}{4}$				$\frac{1}{4}$	-1	$\frac{1}{4}$				$\frac{1}{4}$														
8			$\frac{1}{4}$				$\frac{1}{4}$	-1	$\frac{1}{4}$				$\frac{1}{4}$													
9				$\frac{1}{4}$				$\frac{1}{4}$	-1	$\frac{1}{4}$				$\frac{1}{4}$												
10					$\frac{1}{4}$				$\frac{1}{4}$	-1					$\frac{1}{4}$											
11						$\frac{1}{4}$					-1	$\frac{1}{4}$				$\frac{1}{4}$										
12							$\frac{1}{4}$				$\frac{1}{4}$	-1	$\frac{1}{4}$				$\frac{1}{4}$									-λ
13								$\frac{1}{4}$				$\frac{1}{4}$	-1	$\frac{1}{4}$				$\frac{1}{4}$								
14									$\frac{1}{4}$				$\frac{1}{4}$	-1	$\frac{1}{4}$				$\frac{1}{4}$							
15										$\frac{1}{4}$				$\frac{1}{4}$	-1					$\frac{1}{4}$						
16											$\frac{1}{4}$					-1	$\frac{1}{4}$				$\frac{1}{4}$					
17												$\frac{1}{4}$				$\frac{1}{4}$	-1	$\frac{1}{4}$				$\frac{1}{4}$				
18													$\frac{1}{4}$				$\frac{1}{4}$	-1	$\frac{1}{4}$				$\frac{1}{4}$			
19														$\frac{1}{4}$				$\frac{1}{4}$	-1	$\frac{1}{4}$				$\frac{1}{4}$		
20															$\frac{1}{4}$				$\frac{1}{4}$	-1					$\frac{1}{4}$	
21																$\frac{1}{4}$					-1	$\frac{1}{4}$				
22																	$\frac{1}{4}$				$\frac{1}{4}$	-1	$\frac{1}{4}$			
23																		$\frac{1}{4}$				$\frac{1}{4}$	-1	$\frac{1}{4}$		
24																			$\frac{1}{4}$				$\frac{1}{4}$	-1	$\frac{1}{4}$	
25																				$\frac{1}{4}$				$\frac{1}{4}$	-1	

Table 1a

SYSTEM OF MOMENT EQUATIONS, 25 POINT PLATE

1													1													
2								.250				.250		.250				.250								
3			.063				.125		.125		.063		.250		.063		.125		.125				.063			
4		.047		.047		.047		.141		.047		.141		.141		.047		.141		.047		.047		.047		
5	.024		.059		.024		.094		.094		.059		.141		.059		.094		.094		.024		.059		.024	
6		.044		.044		.044		.097		.044		.097		.007		.044		.097		.044		.044		.044		
7	.022		.046		.022		.071		.071		.046		.097		.046		.071		.071		.022		.046		.022	
8		.035		.035		.035		.071		.035		.071		.071		.035		.071		.035		.035		.035		
9	.018		.035		.018		.053		.053		.035		.071		.035		.053		.053		.018		.035		.018	
10		.027		.027		.027		.054		.027		.054		.054		.027		.054		.027		.027		.027		
11	.014		.027		.014		.041		.041		.027		.054		.027		.041		.041		.014		.027		.014	
12		.020		.020		.020		.040		.020		.040		.040		.020		.040		.020		.020		.020		
13	.010		.020		.010		.030		.030		.020		.040		.020		.030		.030		.010		.020		.010	
$\sum$	.088	.173	.260	.173	.088	.173	.414	.653	.414	.173	.250	.653	1.653	.653	.250	.173	.414	.653	.414	.173	.088	.173	.260	.173	.088	

Table 1b

NUMERICAL MOMENT ITERATION, 25 POINT PLATE

Final moments are :

$M_1 = .088\,\lambda_{13}$
$M_2 = .173\,\lambda_{13}$
$M_3 = .250\,\lambda_{13}$
$M_6 = .173\,\lambda_{13}$
$M_7 = .414\,\lambda_{13}$
$M_8 = .653\,\lambda_{13}$
$M_{11} = .250\,\lambda_{13}$
$M_{12} = .653\,\lambda_{13}$
$M_{13} = 1.653\,\lambda_{13}$

These results may be checked by substitution into Eq. (II-1). For example :

<u>At point 13</u>

$$-(1.653\,\lambda_{13}) + \frac{4}{4}(.653\,\lambda_{13}) \doteq \lambda_{13}$$

$$-1.653 + .653 \doteq 1$$

<u>Error = 0%</u>

<u>At point 3</u>

$$-(.250\,\lambda_{13}) + \frac{2}{4}(.173\,\lambda_{13}) + \frac{1}{4}(.653) \doteq 0$$

$$-.250 + .865 + .163 \doteq 0$$

<u>Error = 0%</u>

<u>At point 1</u>

$$-(.088\,\lambda_{13}) + \frac{2}{4}(.173\,\lambda_{13}) \doteq 0$$

$$-.088 + .0865 \doteq 0$$

<u>Error = 5%</u>

This error was incurred due to a limited number of cycles of iteration.

The results of table 1b are then used as starting values for the second iteration operation. Eq. (II-2) is stated for each point of the plate.

The system of Eq. 's (II-2) is recorded in Table 2a and solved in Table 2b. The results of Table 2b are in terms of the deflection coefficient $j_{(x, y)}$ at the point multiplied by $\frac{1}{N \psi}$. In general form,

$$w_{(x, y)} = \frac{j_{(x, y)}}{N\psi} \quad .$$

	w_1	w_2	w_3	w_4	w_5	w_6	w_7	w_8	w_9	w_{10}	w_{11}	w_{12}	w_{13}	w_{14}	w_{15}	w_{16}	w_{17}	w_{18}	w_{19}	w_{20}	w_{21}	w_{22}	w_{23}	w_{24}	w_{25}	C
1	-1	$\frac{1}{4}$				$\frac{1}{4}$																				.088
2	$\frac{1}{4}$	-1	$\frac{1}{4}$				$\frac{1}{4}$																			.173
3		$\frac{1}{4}$	-1	$\frac{1}{4}$				$\frac{1}{4}$																		.250
4			$\frac{1}{4}$	-1	$\frac{1}{4}$				$\frac{1}{4}$																	.173
5				$\frac{1}{4}$	-1					$\frac{1}{4}$																.088
6	$\frac{1}{4}$					-1	$\frac{1}{4}$				$\frac{1}{4}$															.173
7		$\frac{1}{4}$				$\frac{1}{4}$	-1	$\frac{1}{4}$				$\frac{1}{4}$														.414
8			$\frac{1}{4}$				$\frac{1}{4}$	-1	$\frac{1}{4}$				$\frac{1}{4}$													.653
9				$\frac{1}{4}$				$\frac{1}{4}$	-1	$\frac{1}{4}$				$\frac{1}{4}$												.414
10					$\frac{1}{4}$				$\frac{1}{4}$	-1					$\frac{1}{4}$											.173
11						$\frac{1}{4}$					-1	$\frac{1}{4}$				$\frac{1}{4}$										.250
12							$\frac{1}{4}$				$\frac{1}{4}$	-1	$\frac{1}{4}$				$\frac{1}{4}$									.653
13								$\frac{1}{4}$				$\frac{1}{4}$	-1	$\frac{1}{4}$				$\frac{1}{4}$								1.65
14									$\frac{1}{4}$				$\frac{1}{4}$	-1	$\frac{1}{4}$				$\frac{1}{4}$							.653
15										$\frac{1}{4}$				$\frac{1}{4}$	-1					$\frac{1}{4}$						.250
16											$\frac{1}{4}$					-1	$\frac{1}{4}$				$\frac{1}{4}$					.173
17												$\frac{1}{4}$				$\frac{1}{4}$	-1	$\frac{1}{4}$				$\frac{1}{4}$				.414
18													$\frac{1}{4}$				$\frac{1}{4}$	-1	$\frac{1}{4}$				$\frac{1}{4}$			.653
19														$\frac{1}{4}$				$\frac{1}{4}$	-1	$\frac{1}{4}$				$\frac{1}{4}$		.414
20															$\frac{1}{4}$				$\frac{1}{4}$	-1					$\frac{1}{4}$	.173
21																$\frac{1}{4}$					-1	$\frac{1}{4}$				.088
22																	$\frac{1}{4}$				$\frac{1}{4}$	-1	$\frac{1}{4}$			.173
23																		$\frac{1}{4}$				$\frac{1}{4}$	-1	$\frac{1}{4}$		.250
24																			$\frac{1}{4}$				$\frac{1}{4}$	-1	$\frac{1}{4}$	.173
25																				$\frac{1}{4}$				$\frac{1}{4}$	-1	.088

Table 2a

SYSTEM OF DEFLECTION EQUATIONS, 25 POINT PLATE

1	.088	.173	.250	.173	.088	.173	.414	.653	.414	.173	.250	.653	1.653	.653	.250	.173	.414	.653	.414	.173	.088	.173	.250	.173	.088	
2	.086	.189	.250	.189	.086	.189	.412	.684	.412	.189	.350	.684	.656	.684	.250	.189	.412	.684	.412	.189	.086	.189	.250	.189	.086	
3	.094	.188	.265	.188	.094	.188	.436	.433	.436	.188	.265	.433	.684	.433	.265	.188	.436	.433	.436	.188	.094	.188	.265	.188	.094	
4	.094	.199	.202	.199	.094	.199	.316	.455	.316	.199	.202	.455	.433	.455	.202	.199	.316	.455	.316	.199	.094	.199	.202	.199	.094	
5	.100	.153	.214	.153	.100	.153	.328	.316	.328	.153	.214	.316	.455	.316	.214	.153	.328	.316	.328	.153	.100	.153	.214	.153	.100	
6	.076	.161	.150	.161	.076	.161	.224	.332	.224	.161	.150	.332	.316	.332	.150	.161	.224	.332	.224	.161	.076	.161	.150	.161	.076	
7	.080	.113	.163	.113	.080	.113	.241	.224	.241	.113	.163	.224	.332	.224	.163	.113	.241	.224	.241	.113	.080	.113	.163	.113	.080	
8	.056	.121	.112	.121	.056	.121	.168	.244	.168	.121	.112	.244	.224	.244	.112	.121	.168	.244	.168	.121	.056	.121	.112	.121	.056	
9	.060	.084	.121	.084	.060	.084	.182	.168	.182	.084	.121	.168	.244	.168	.121	.084	.182	.168	.182	.084	.060	.084	.121	.084	.060	
10	.042	.091	.084	.091	.042	.091	.126	.182	.126	.091	.084	.182	.166	.182	.084	.091	.126	.182	.126	.091	.042	.091	.084	.091	.042	
11	.046	.073	.091	.073	.046	.073	.137	.125	.137	.073	.091	.125	.182	.125	.091	.073	.137	.125	.137	.073	.046	.073	.091	.073	.046	
12	.036	.068	.067	.068	.036	.068	.098	.157	.098	.068	.067	.157	.125	.157	.067	.068	.098	.157	.098	.068	.036	.098	.067	.068	.036	
13	.034	.051	.073	.051	.034	.051	.112	.098	.112	.051	.073	.098	.157	.098	.073	.051	.112	.098	.112	.051	.034	.051	.073	.051	.034	
$\sum$	.892	1.663	2.042	1.663	.892	1.663	3.104	4.071	3.104	1.663	2.042	4.071	5.626	4.071	2.042	1.663	3.104	4.071	3.104	1.663	.892	1.663	2.042	1.663	.892	

Table 2b

NUMERICAL DEFLECTION ITERATION, 25 POINT PLATE

The results may again be checked by Eq. (II-2). For example :

At point 13

$$-(5.626) + \frac{4}{4}(4.071) \doteq -1.653$$

$$1.555 \doteq 1.653$$

Error = 5.9 %

At point 3

$$-(2.042) + \frac{2}{4}(1.663) + \frac{1}{4}(4.071) \doteq -.250$$

$$.192 \doteq .250$$

Error = 23.2 %

At point 1

$$-(.892) + \frac{2}{4}(1.663) \doteq .088$$

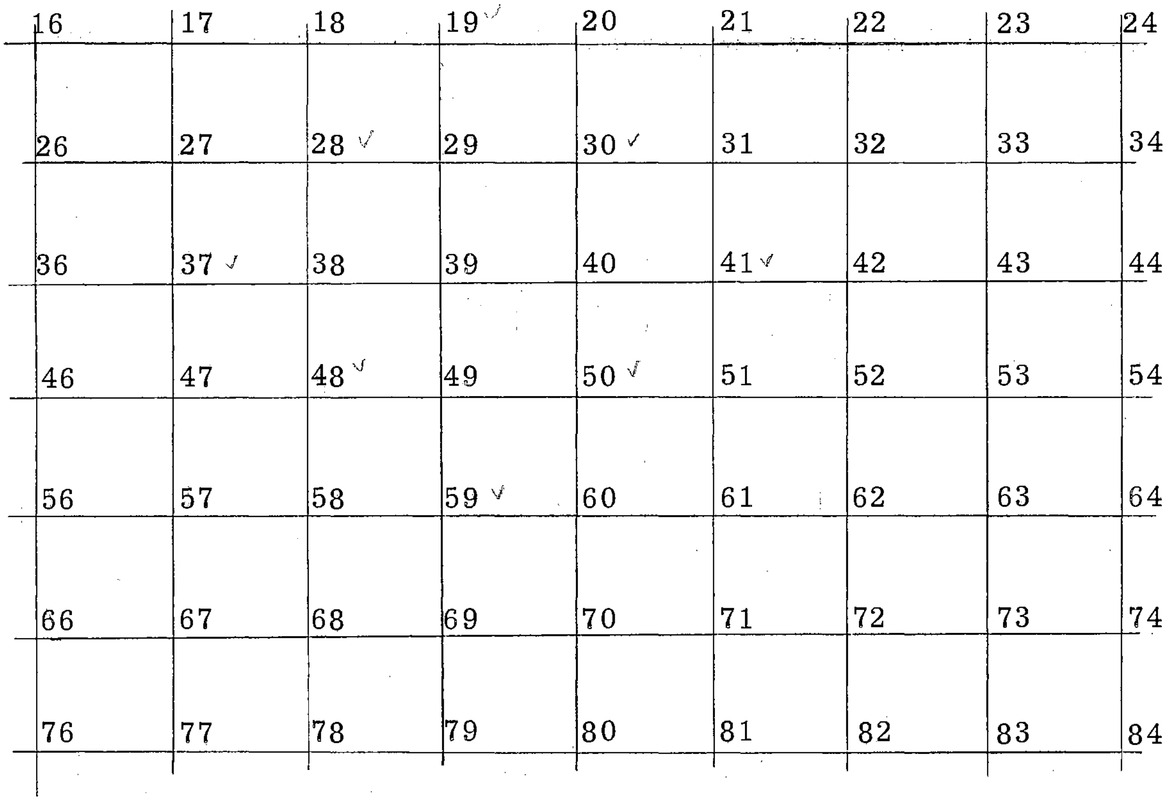
$$.050 \doteq .088$$

Error = 43.2 %

2. ALGEBRAIC ITERATION

The analysis of a flat plate by numerical iteration is a laborious and time-consuming procedure. The convergency of the procedure can be considerably improved by means of special equations based on study presented in Ref. (5). The derivation of these equations follows.

In order to simplify the symbols, an arbitrary point numbering system is used; the lattice and point numbers are shown in Fig. 6.



16	17	18	19 ✓	20	21	22	23	24
26	27	28 ✓	29	30 ✓	31	32	33	34
36	37 ✓	38	39	40	41 ✓	42	43	44
46	47	48 ✓	49	50 ✓	51	52	53	54
56	57	58	59 ✓	60	61	62	63	64
66	67	68	69	70	71	72	73	74
76	77	78	79	80	81	82	83	84

Fig. 6

Point Numbers for General Algebraic Equations

The matrix of equations Eq. (I-10) follows in Table 3. A load $\lambda_{50} = 1$ is taken at point 50. All other loads are zero.

			M_{40}	M_{41}	M_{42}		M_{48}	M_{49}	M_{50}	M_{51}	M_{52}		M_{58}	M_{59}	M_{60}		C
Equation for Point		•				•						•				•	
	40	•	-1	a		•			b			•				•	
	41	•	a	-1	a	•				b		•				•	
	42	•		a	-1	•					b	•				•	
		•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•
	48	•				•	-1	a				•	b			•	
	49	•				•	a	-1	a			•		b		•	
	50	•	b			•		a	-1	a		•			b	•	-1
	51	•		b		•			a	-1	a	•				•	
	52	•			b	•				a	-1	•				•	
		•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•
	58	•				•	b					•	-1	a		•	
	59	•				•		b				•	a	-1	a	•	
	60	•				•			b			•		a	-1	•	
		•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•

Table 3

GENERAL ALGEBRAIC SYSTEM OF EQUATIONS

A five point set (40, 41, 50, 51 and 60) is isolated by assuming

$$M_{30} = M_{41} = M_{52} = M_{61} = M_{70} = M_{59} = M_{48} = M_{59} = 0 .$$

Then the algebraic iteration is performed in n- cycles.

From the first cycle :

$$M_{40}^{(1)} = M_{60}^{(1)} = a$$

$$M_{49}^{(1)} = M_{51}^{(1)} = b$$

$$M_{50}^{(1)} = 1 .$$

From the second cycle :

$$M_{40}^{(2)} = M_{60}^{(2)} = 2(a^2 + b^2) a$$

$$M_{49}^{(2)} = M_{51}^{(2)} = 2(a^2 + b^2) b$$

$$M_{50}^{(2)} = 2(a^2 + b^2) .$$

From the n-th cycle :

$$M_{40}^{(n)} = M_{50}^{(n)} = 2(a^2 + b^2)^{n-1} a$$

$$M_{49}^{(n)} = M_{51}^{(n)} = 2(a^2 + b^2)^{n-1} b$$

$$M_{50}^{(n)} = 2(a^2 + b^2)^{n-1} .$$

If this procedure is repeated an infinite number of times, the values developed at each point (40, 60, 49, 51 and 50) form a series which is :

a. Infinite

b. Convergent

c. Geometric .

The sum of such a series is easily computed and for this particular case

$$M_{40}^{(1+2+\ldots+\infty)} = M_{60}^{(1+2+\ldots+\infty)} = \frac{a}{x_{22}} ,$$

$$M_{49}^{(1+2+\ldots+\infty)} = M_{51}^{(1+2+\cdots+\infty)} = \frac{b}{x_{22}} \qquad (II-3)$$

$$M_{50}^{(1+2+\cdots+\infty)} = \frac{1}{x_{22}} ,$$

where

$$x_{22} = 1 - 2a^2 - 2b^2 .$$

It may be observed from Fig. 7 that the same result is obtained by the relaxation of unit value at 50 and by carrying-over the relaxed value to the adjacent points by means of a and b respectively. This two step procedure forms the basis of the proposed procedure.

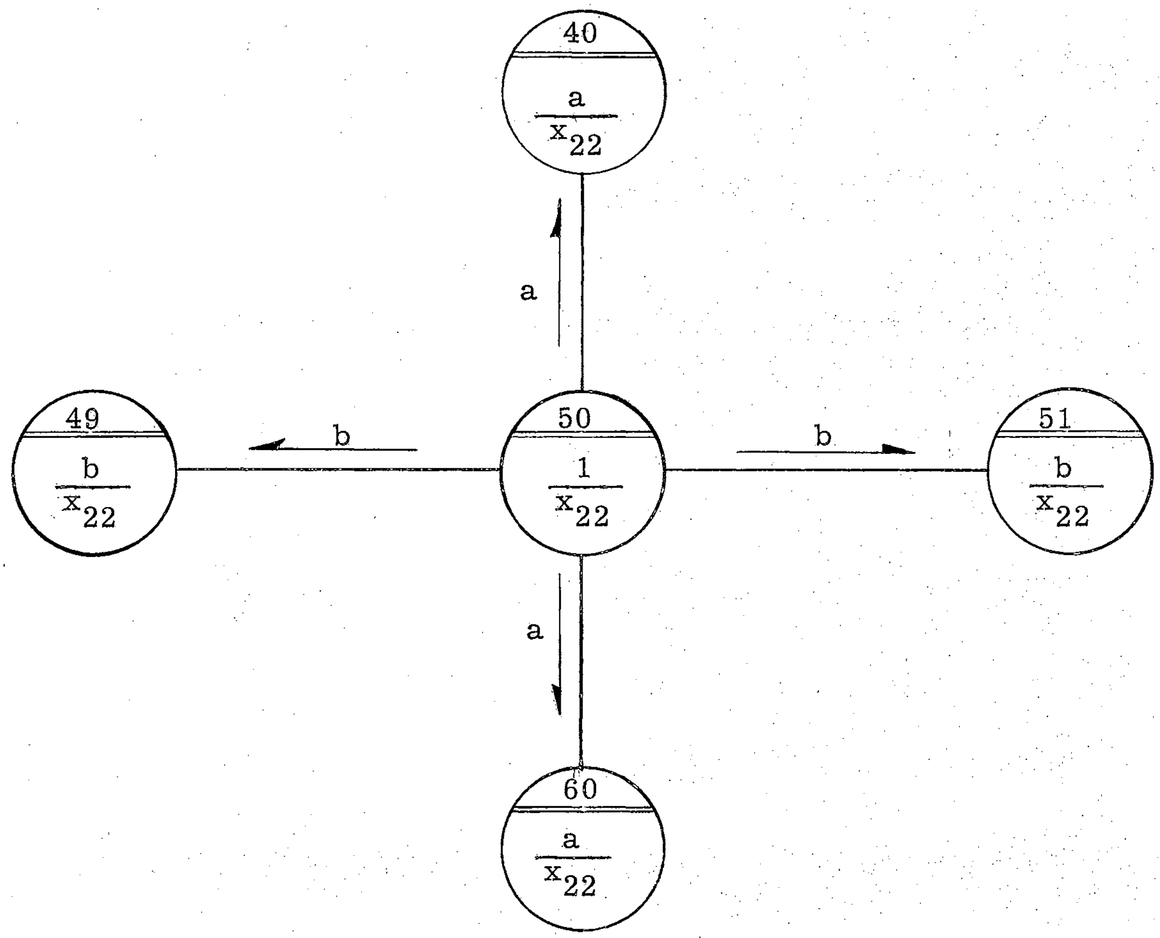


Fig. 7

Relaxation and Carry-over
Diagram

Fig. 7 represents not only a complete solution of the five point set (30, 49 50, 51, 60,) , but of any five point set isolated by a similar assumption.

If now a new five point set is assumed in the upper left corner (29 38, 39 40 and 49) , the involved starting value at $\overset{39}{29}$ (derived from the assumption of zero moments at 19, 30 41, 50, 59, 48, 37, and 28) is :

$$M^{(1)}_{\underset{39}{29}} = \left.\frac{2ab}{x_{22}}\right| \qquad \text{(II-4)}$$

and similarly at the other corner points the involved starting values are :

$$\left.\begin{aligned} M_{41}^{(1)} &= \frac{2ab}{x_{22}} \\ M_{59}^{(1)} &= \frac{2ab}{x_{22}} \\ M_{61}^{(1)} &= \frac{2ab}{x_{22}} \end{aligned}\right\} . \qquad (\text{II-4})$$

Finally, four more five point sets may be isolated adjacent to point 50. The left side five point set with the center at 48, the right side five point set with the center at 52, the upper side five point set at 30 and the lower side five point set at 70. The starting values at these points then are :

$$\left.\begin{aligned} M_{30}^{(1)} &= \frac{a^2}{x_{22}} \\ M_{70}^{(1)} &= \frac{a^2}{x_{22}} \end{aligned}\right\} \qquad (\text{II-5})$$

$$\left.\begin{aligned} M_{48}^{(1)} &= \frac{b^2}{x_{22}} \\ M_{52}^{(1)} &= \frac{b^2}{x_{22}} \end{aligned}\right\} . \qquad (\text{II-6})$$

Thus it becomes apparent that the carry-over set values from the center of the inital five point set (50) to the centers of adjacent sets are of three types :

a.) Vertical $= a^2 = A$

b.) Horizontal = b^2 = B

c.) Diagonal = 2ab = D .

The graphical interpretation of these carry-over set values is shown in Fig. 8 .

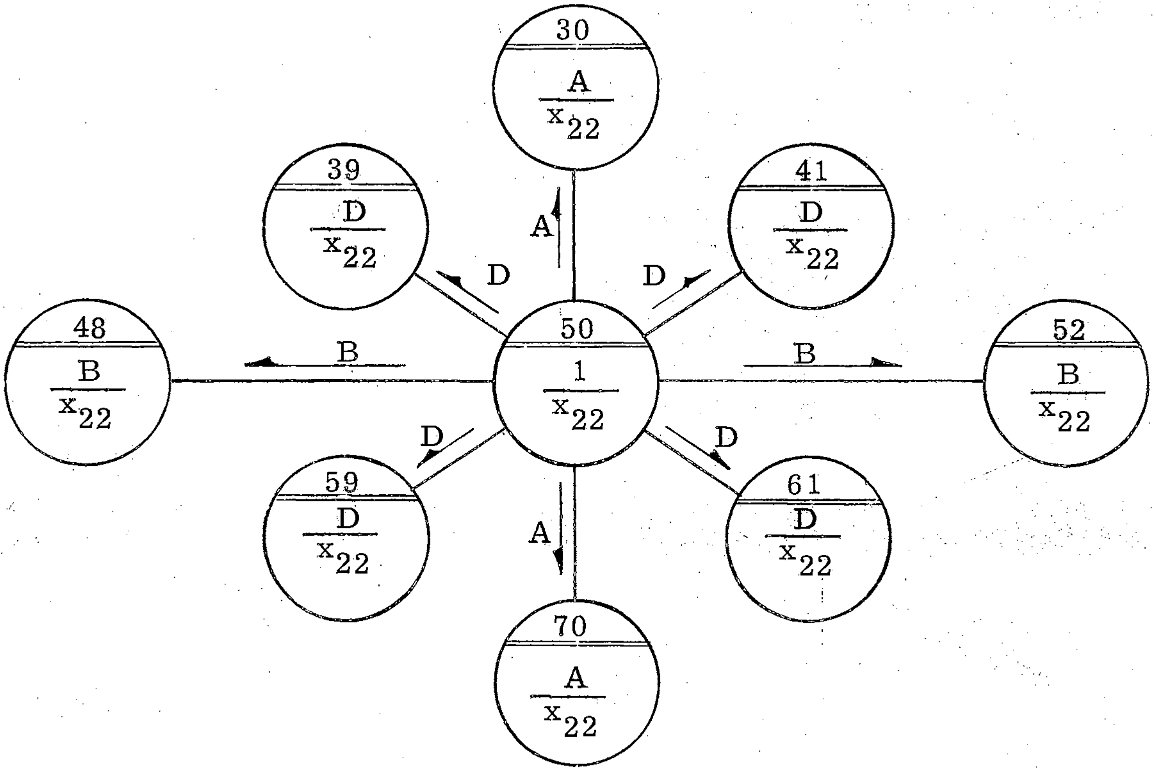


Fig. 8

Carry-Over Set
Diagram

With these new starting values the algebraic iteration may be repeated and the same results as in the case of set (50) must be obtained. The Eq's (II-4, 5 and 6) are completely general and apply for any five point set in the plate.

3. BOUNDARY CONDITIONS - DIRECT

Three special conditions occur in the application of Eq's (II-4, 5, and 6) which necessitate modifying the equations.

The conditions are :

a.) Left edge and right edge

b.) Upper edge and lower edge

c.) Corners.

If the center of the point set is placed adjacent to the left edge or right edge as shown in Fig. 9a and b, the edge point always produces a zero moment, and thus does not contribute any values to the iteration.

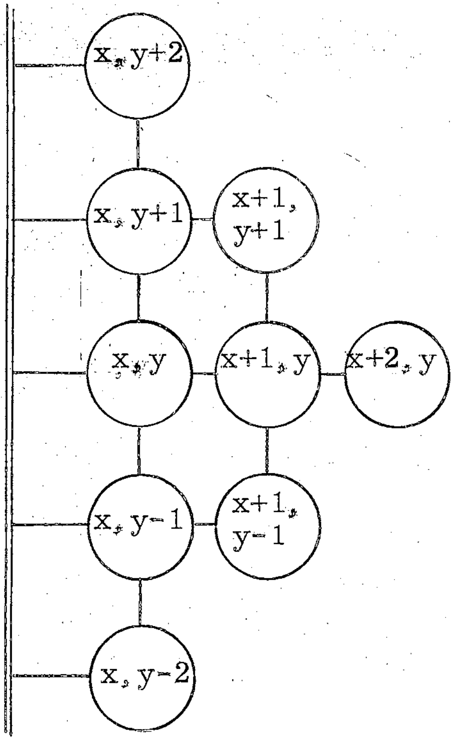


Fig. 9a

Point Set at Left Edge

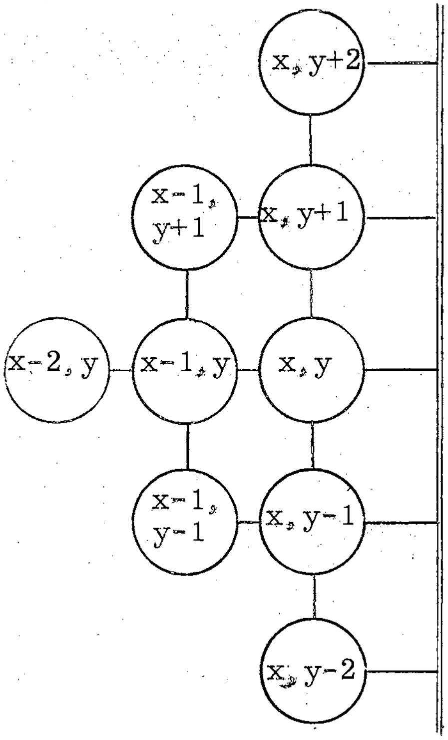


Fig. 9b

Point Set at Right Edge

For the left edge (Fig. 9a), Eq. (II-5) reduces to :

$$\left.\begin{aligned} M_{(x,y-1)} &= M_{(x,y+1)} = \frac{b}{x_{12}} \\ M_{(x-1,y)} &= 0 \ , \quad M_{(x+1,y)} = \frac{a}{x_{12}} \\ M_{(x,y)} &= \frac{1}{x_{12}} \end{aligned}\right\} \qquad \text{(II-7a)}$$

where

$$x_{12} = 1 - a^2 - 2b^2 \ .$$

The carry-over factors then become (from a point adjacent to a left edge) :

$$\left.\begin{aligned} A^{(L)}_{(\text{left})} &= 0 & \qquad A^{(R)}_{(\text{left})} &= a^2 \\ B^{(U)}_{(\text{left})} &= b^2 & \qquad B^{(D)}_{(\text{left})} &= b^2 \\ D^{(L)}_{(\text{left})} &= 0 & \qquad D^{(R)}_{(\text{left})} &= 2ab \end{aligned}\right\} \qquad \text{(II-8a)}$$

where, for example, $B^{(D)}_{(\text{left})}$ reads:

> Carry-over factor downward from a point adjacent to the left edge.

The set of values obtained by Eq's (II-7) will hereafter be referred to as the "relaxation set": the values obtained by Eq's (II-8) will be referred to as the "carry-over set" .

For the right edge (Fig. 9b), Eq (II-5) is cyclosymmetrical.

$$M_{(x,y-1)} = M_{(x,y+1)} = \frac{b}{x_{12}} \qquad \text{(II-7b)}$$

$$M_{(x+1, y)} = 0 , \quad M_{(x-1, y)} = \frac{a}{x_{12}}$$

$$M_{(x, y)} = \frac{1}{x_{12}} \qquad \text{(II-7b)}$$

The carry-over factors are also cyclosymmetrical.

$A^{(L)}_{(right)} = a^2$ $\qquad$ $A^{(R)}_{(right)} = 0$

$B^{(U)}_{(right)} = b^2$ $\qquad$ $B^{(D)}_{(right)} = b^2$

$D^{(L)}_{(right)} = 2ab$ $\qquad$ $D^{(R)}_{(right)} = 0$

The diagrams of Eq's (II-7a and b) and Eq's (8a and b) are shown in Fig's 10 a and b, and Fig's 11a and b, respectively.

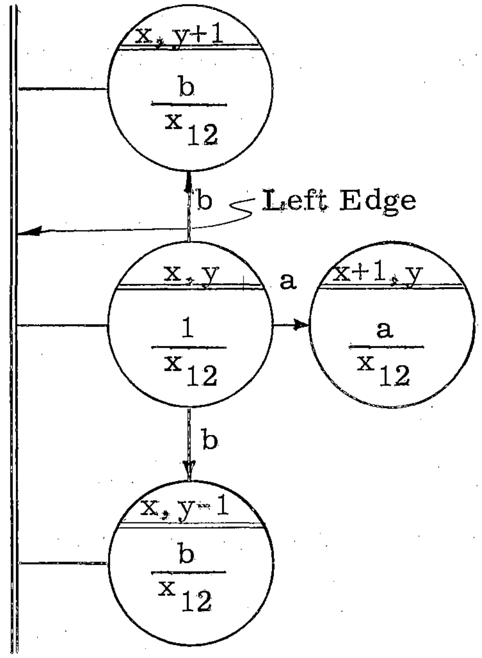


Fig. 10a

Relaxation Set Diagram — Left Edge

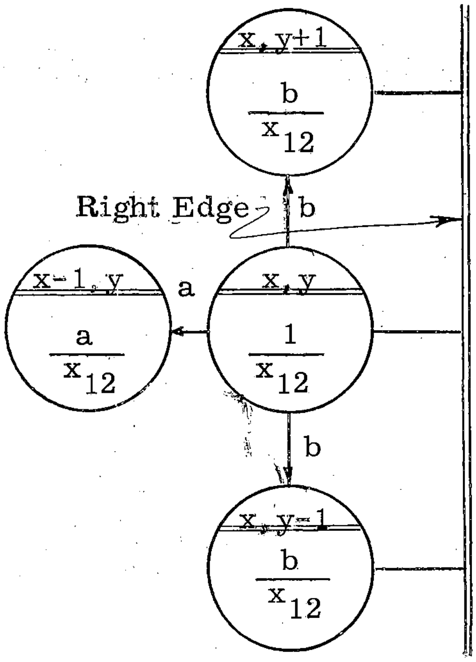
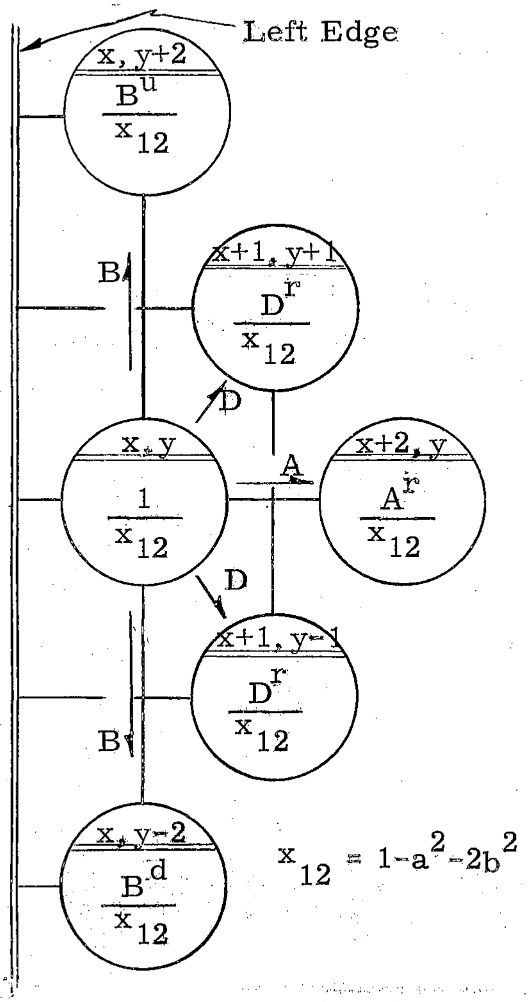


Fig. 10b

Relaxation Set Diagram — Right Edge



$$x_{12} = 1-a^2-2b^2$$

Fig. 11a

Diagram of Left
Carry-over Set

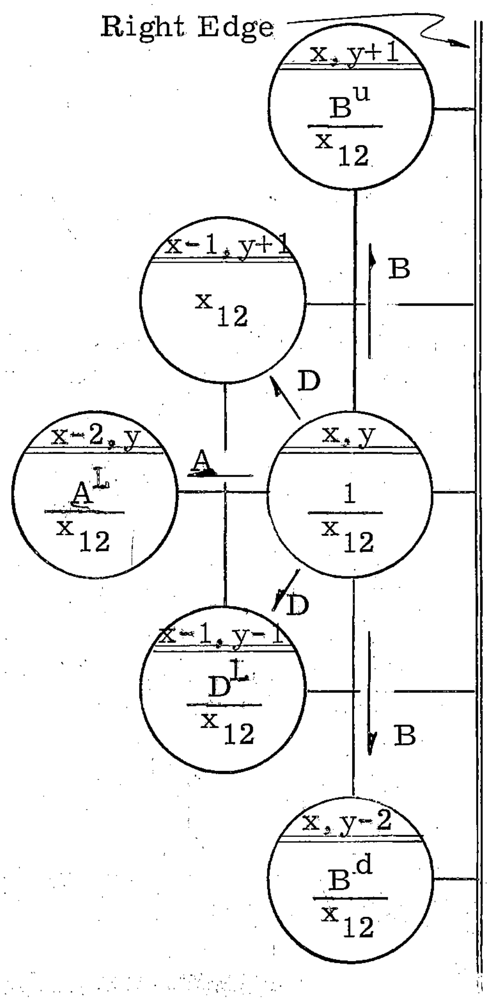
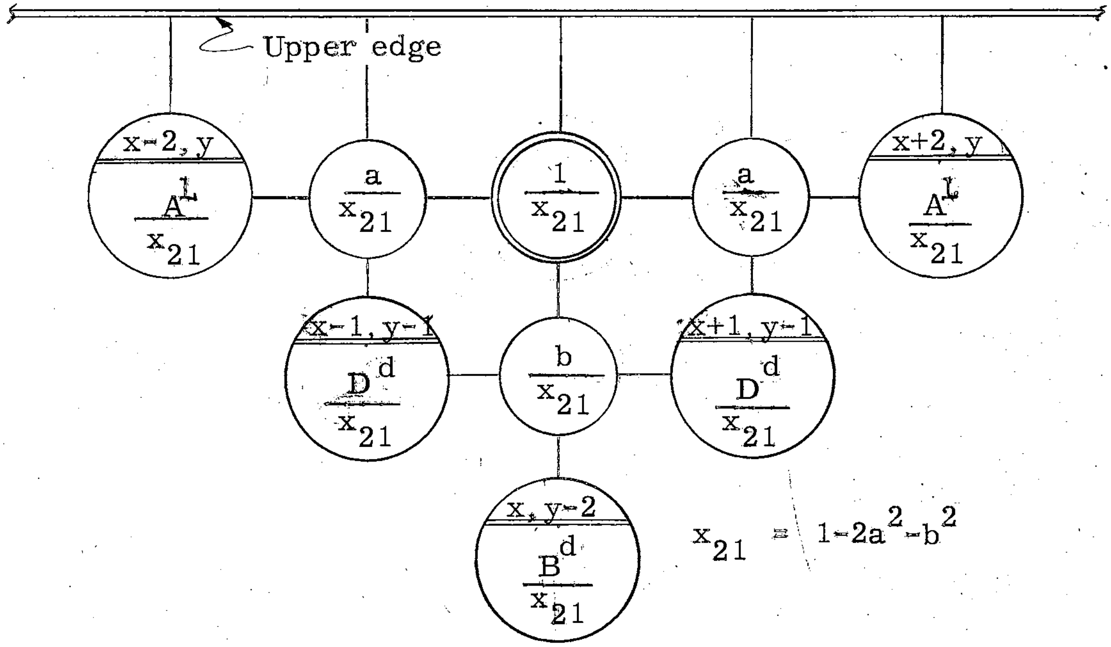


Fig. 11b

Diagram of Right
Carry-over Set

Similar relationships can be derived for the upper and lower edges, and for a corner. The final results for upper and lower edges are shown without derivation in Fig's 12a and b ; the final results for any corner is shown in Fig. 13. For Fig's 12a and b and for Fig. 13, and all similar succeeding charts, the values in the smaller rings are the relaxation set and those in the larger ring are the carry-over set. The double ring indicates the point where the starting value was iterated.



$$x_{21} = 1-2a^2-b^2$$

Fig 12a

Diagram of Upper Edge Relaxation
and Carry-over Sets

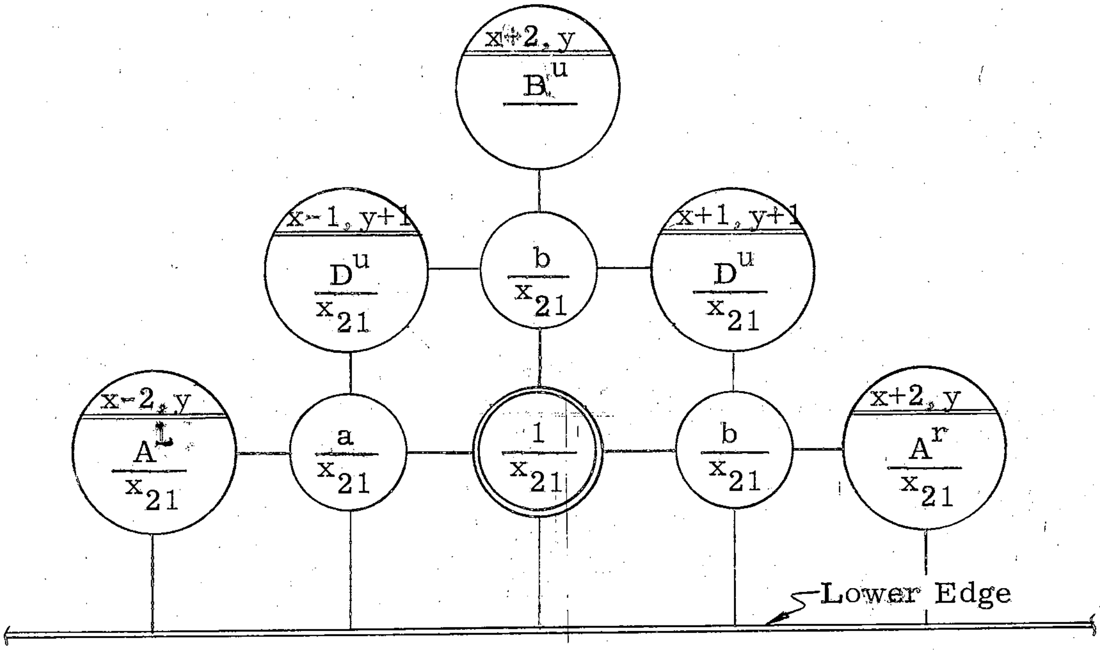


Fig. 12b

Diagram of Lower Edge Relaxation
and Carry-over Sets

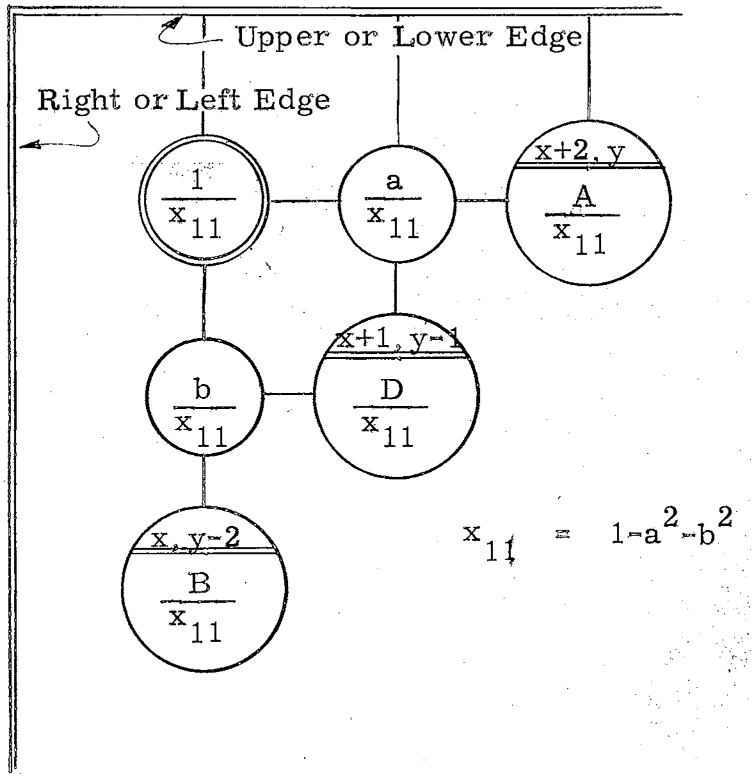


Fig. 13

Diagram of Corner Relaxation
and Carry-Over Sets

4. BOUNDARY CONDITIONS - INDIRECT

The foregoing derivations for boundary conditions yield direct results, but when $\Delta x \neq \Delta y$, they require the use of three additional convergency factors, three additional vertical carry-over factors, three additional horizontal carry-over factors, and three additional diagonal carry-over factors. Their use in a numerical application is quite cumbersome and creates a high probability of error.

An alternate, though indirect, method has been found to yield the same results with a much more mechanical operation. Only one extra factor (which is -1) is required, which greatly simplifies the application. The derivation follows.

At any edge, simply supported on a rigid foundation,

$$M_x = 0 , \quad \text{or} \quad M_y = 0$$

and

$$w = 0 .$$

From Eq. (I-2) ,

$$-\frac{M_x}{D} = \frac{\partial^2 w}{\partial x^2} + \mu \frac{\partial^2 w}{\partial y^2} .$$

Along a y-y boundary, the curvature ($\frac{\partial^2 w}{\partial y^2}$) is also equal to zero, hence :

$$\frac{\partial^2 w}{\partial x^2} = 0 .$$

In finite differences,

$$w_{(x-1,y)} - 2w_{(x,y)} + w_{(x+1,y)} = 0$$

Since $w_{(x,y)} = 0$ at a rigid support,

$$w_{(x-1,y)} = - w_{(x+1,y)} \quad . \qquad (\text{II-9a})$$

Similarly, along an x-x boundary,

$$w_{(x,y-1)} = - w_{(x,y+1)} \quad . \qquad (\text{II-9b})$$

Eq. (II-9a) is shown graphically in Fig. 14.

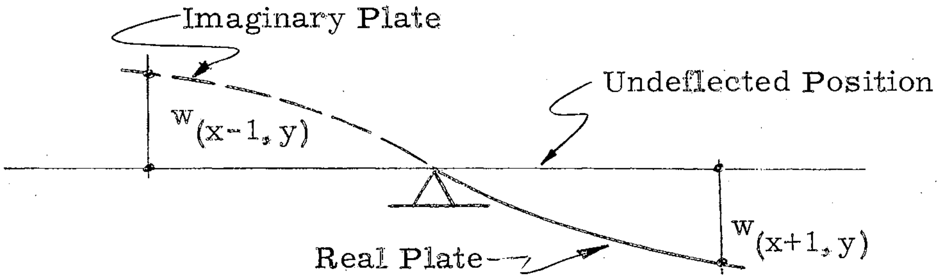


Fig. 14

Edge Conditions Along y-y Boundary.

Fig. 14 implies that an inflection point along a line of zero deflection is developed at a simple support. This can be accomplished physically by placing an imaginary plate at each of the four sides; each imaginary plate is identical to the real plate, but is anti symmetrically loaded. A sketch of the real and imaginary plates for a concentrated load is shown in Fig. 15.

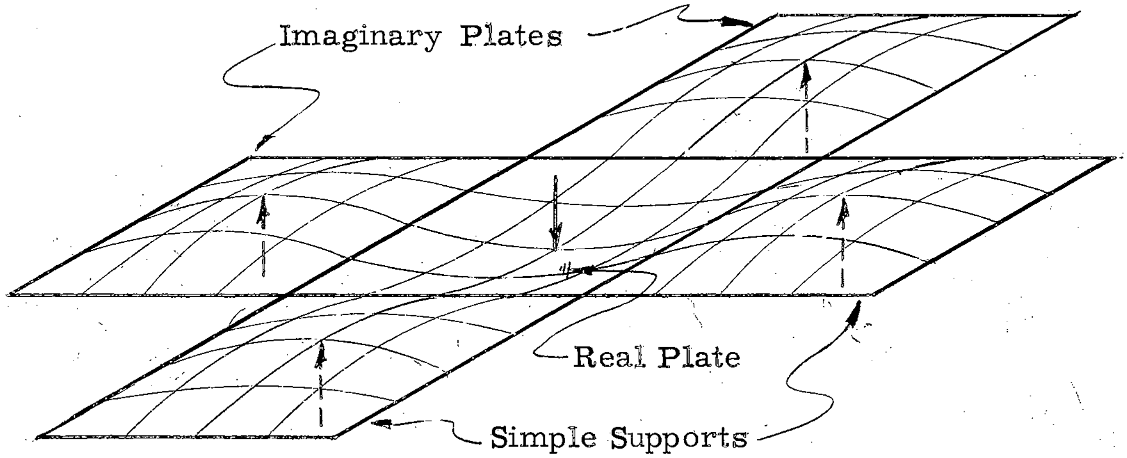


Fig. 15

Deflections of Real and Imaginary Plates

Consider a continuous plate over a simple support. A load $\lambda = 1$ is released at a point adjacent to the support. Sumultaneously a load $\lambda = -1$ is released from a point adjacent to the support but on the other side of the support, as indicated in Fig. 16.

From Fig. 16, the total value at the support is zero. At (x+1, y), there is a final value of $\frac{1}{x_{22}}$, plus a value $-\frac{A}{x_{22}}$ which must be iterated.

To achieve this result mechanically, when the point (x+1, y) is iterated, the λ is multiplied by $\frac{1}{x_{22}}$, which yields a fully iterated value for that load. the value $\frac{1}{x_{22}}$ is carried over to the imaginary point (x-1, y) by the regular carry-over factor A in the regular carry-over procedure. When desired, the point (x-1, y) can be iterated, but it carries only into (x+1, y), and by a carry-over factor -1 .

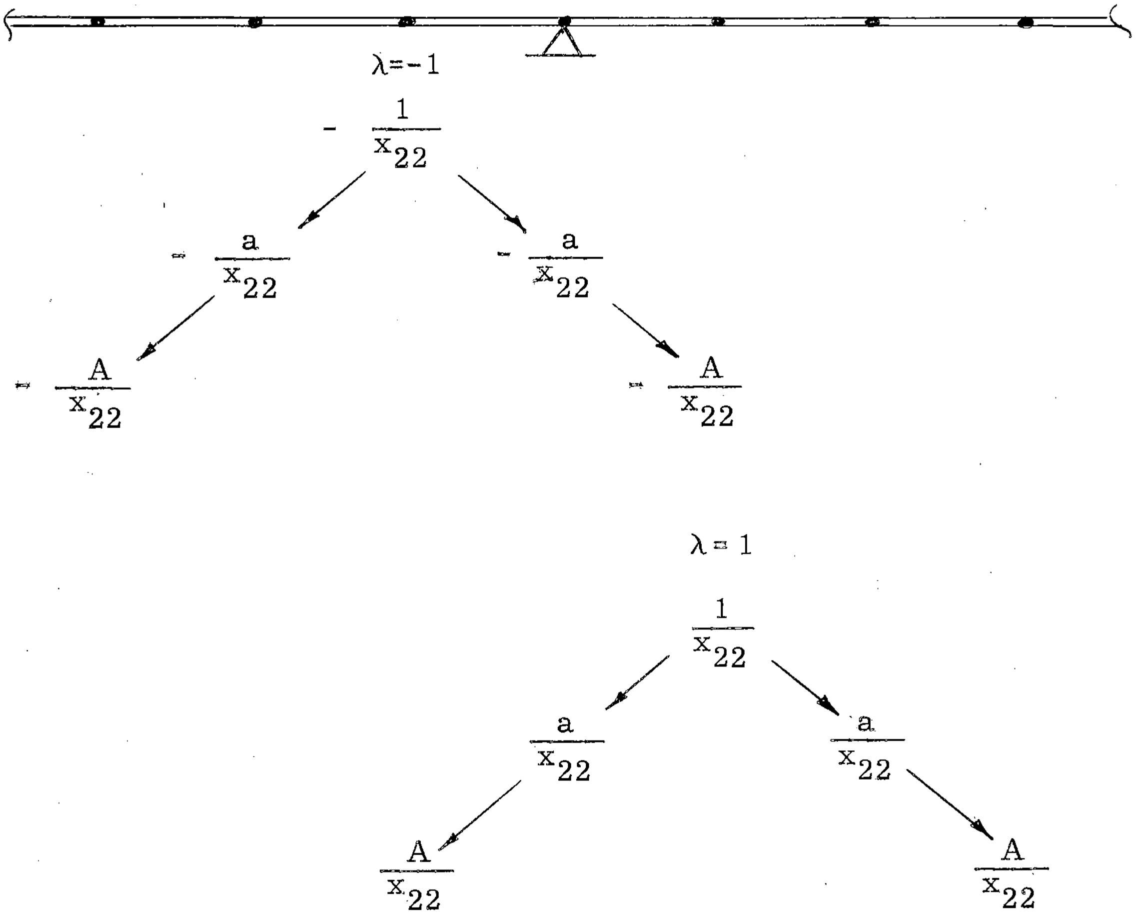


Fig. 16

Basic Series at an Inflection Point

Thus the final result is the same as indicated in Fig. 16 , i. e., the sum of values at (x+1, y) is $\frac{1}{x_{22}}$ which is fully iterated, plus $-\frac{A}{x_{22}}$, which must be iterated.

The proof will now be given that the resulting totally iterated values obtained by this procedure are the same as those given by Eq's (II-7a) .

Fig. 17 shows a starting value $-\frac{A}{x_{22}}$. It is then iterated through the basic series which yields a new starting value $+ \frac{A^2}{x_{22}^2}$.

This value is iterated through the basic series and the next starting value $- \frac{A^3}{x_{22}^2}$ is obtained.

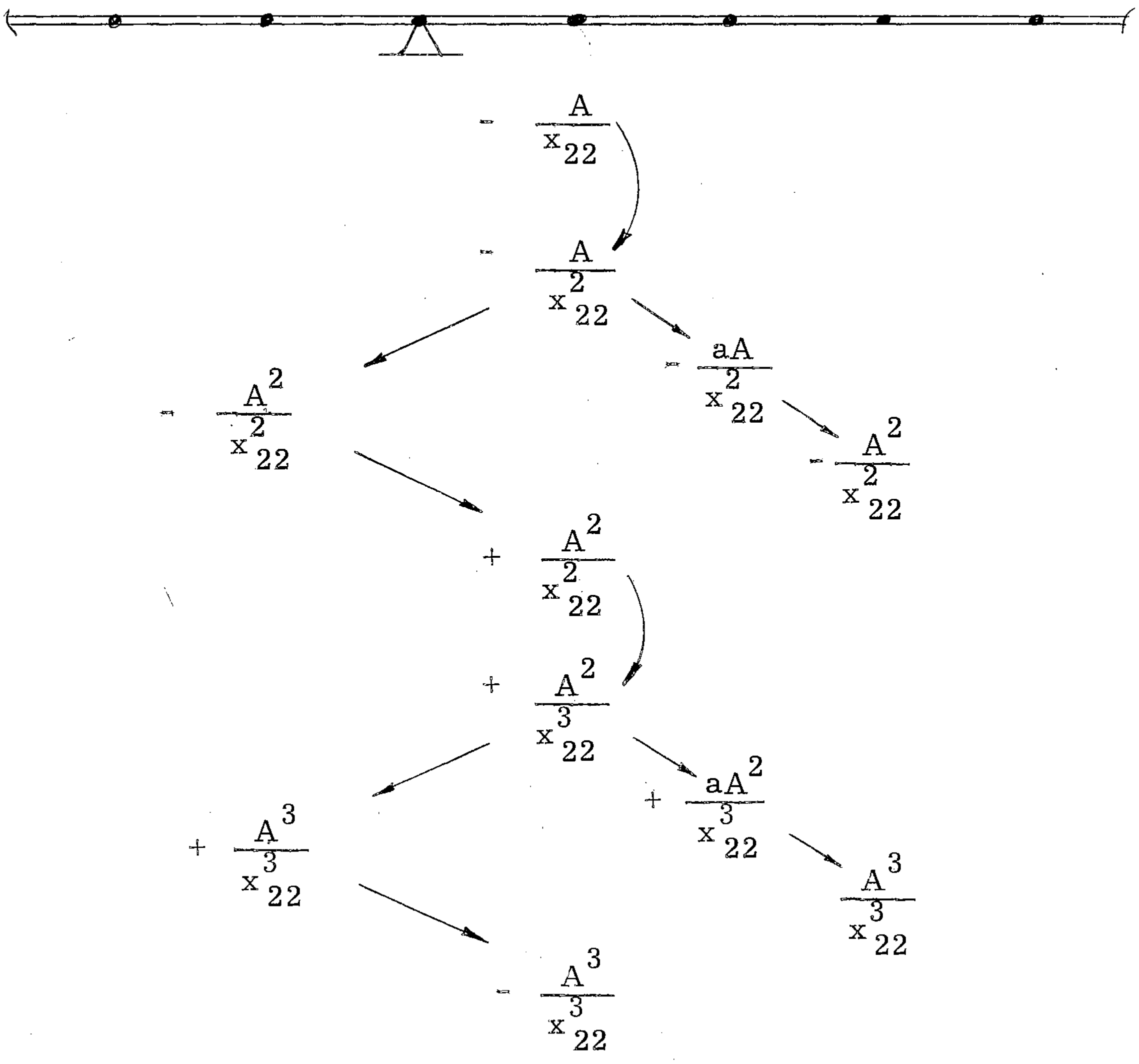


Fig. 17

Indirect Algebraic Edge Series

The iteration is repeated an infinite number of times. The resulting series at (x+1, y) is :

$$M_{(x+1,y)} = \frac{1}{x_{22}} - \frac{A}{x_{22}^{2}} + \frac{A^{2}}{x_{22}^{3}} - \frac{A^{3}}{x_{22}^{4}} + \cdots\cdots .$$

As before, the series is an infinite convergent geometric series, Its sum is :

$$M_{(x+1,y)} = \frac{1}{x_{22}} \left[\frac{1}{1-\frac{-A}{x_{22}}} \right] .$$

Simplifying, where $A = a^{2}$, and $x_{22} = 1-2a^{2}-2b^{2}$,

$$M_{(x+1,y)} = \frac{1}{x_{22}} \left[\frac{x_{22}}{x_{22}+a^{2}} \right] = \frac{1}{1-a^{2}-2b^{2}}$$

The result is then equal to $\frac{1}{x_{12}}$ as it was defined. Similarly, the relaxation set values and carry-over set values can be shown to be equal to those of Eq's (II-7a) .

The entire procedure can also be shown to be true by cyclosymmetry for an upper or lower edge, (simply substitute B for A and b for a in the derivation and the succeeding proof .)

At a corner, the series is developed in Fig. 18.

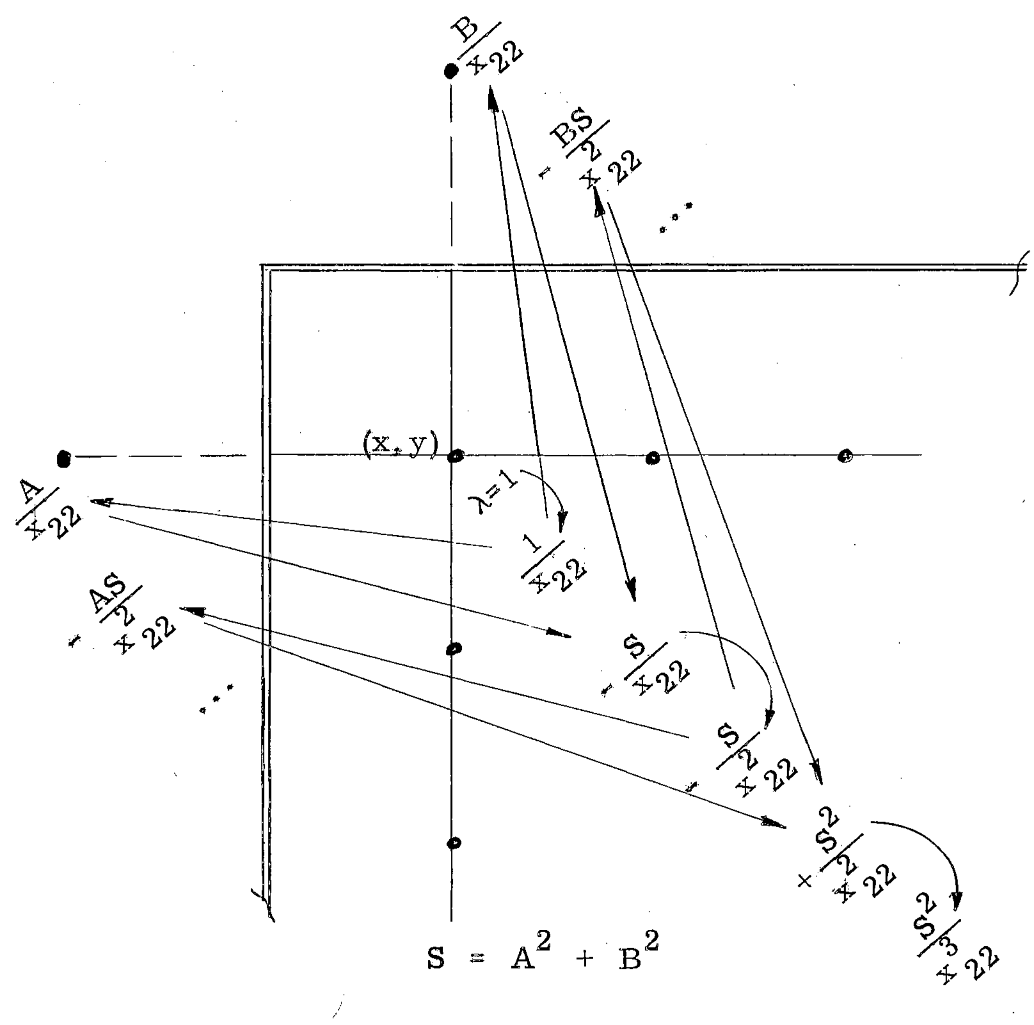


Fig. 18

Indirect Algebraic Corner Series

The series at the corner point is :

$$M_{(x,y)} = \frac{1}{x_{22}} - \frac{S}{x_{22}^2} + \frac{S^2}{x_{22}^3} - \ldots\ldots \quad .$$

The sum is, where $S = A + B = a^2 + b^2$;

$$M_{(x,y)} = \frac{1}{x_{22}} \left[\frac{1}{1 - \frac{-S}{x_{22}}} \right] = \frac{1}{x_{22} + S}$$

$$= \frac{1}{1 - a^2 - b^2} = \frac{1}{x_{11}} \quad .$$

which is the result from Fig. 13.

5. PROPOSED PROCEDURE

Before the tools previously developed are applied, it must be noted that the series yields two types of values :

a.) The fully iterated values of the relaxation set, which will hereafter be called balanced values, and

b.) The carry-over set values which must be iterated, which will be hereafter called unbalanced values .

Fig. 19 is a combination of Figs. 7 and 8. A value α is to be iterated at the center point. When multiplied by the balance factor $\frac{1}{x_{22}}$, it becomes $f_b \alpha$, where

$$f_b = \frac{1}{x_{22}} ,$$

and is called the balance factor. This balanced value is then carried into the relaxation set by a or b, and into the carry-over set by A, B, or D.

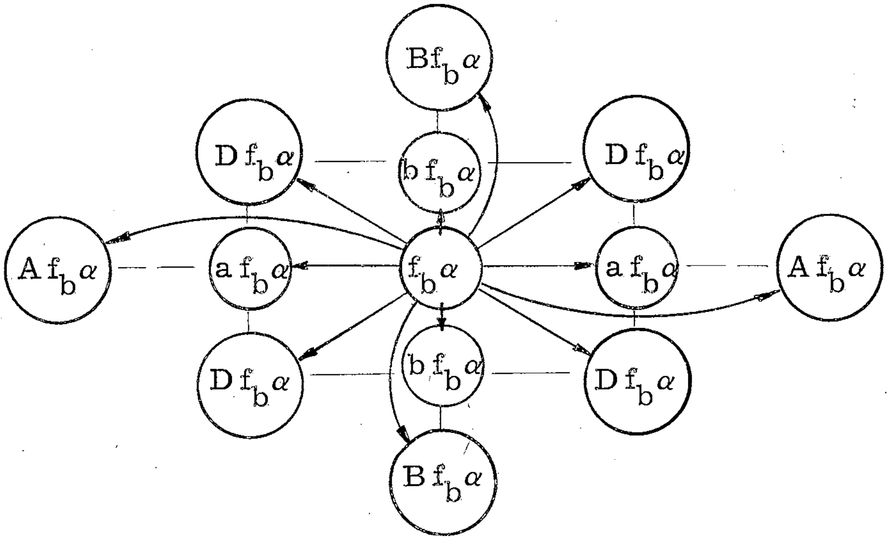


Fig. 19

Distribution of Unbalanced Value

In Fig. 19, those values in the smaller rings (the relaxation set) are fully iterated, and those in the larger rings (the carry-over set) become the new unbalances α_1, α_2, α_3, etc. Each of these α's must be balanced (multiplied by f_b) and then distributed. The new unbalances thus formed in the new carry-over sets must then be balanced and distributed. The process is repeated until the error is within limits.

The carry-over factors and balance factor for an interior point are shown in Fig. 20, for a perimeter point in Fig. 21 and for a corner point in Fig. 22. For these figures, $\alpha' = f_b \alpha$; n is equal to unity, or, $\Delta x = \Delta y$.

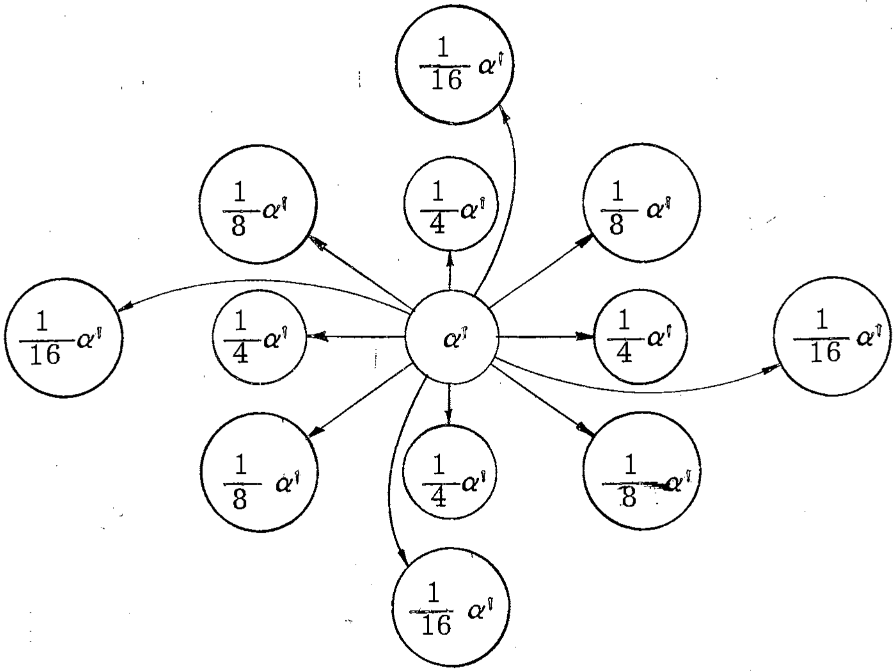


Fig. 20

Carry-Over Values at an Interior Point

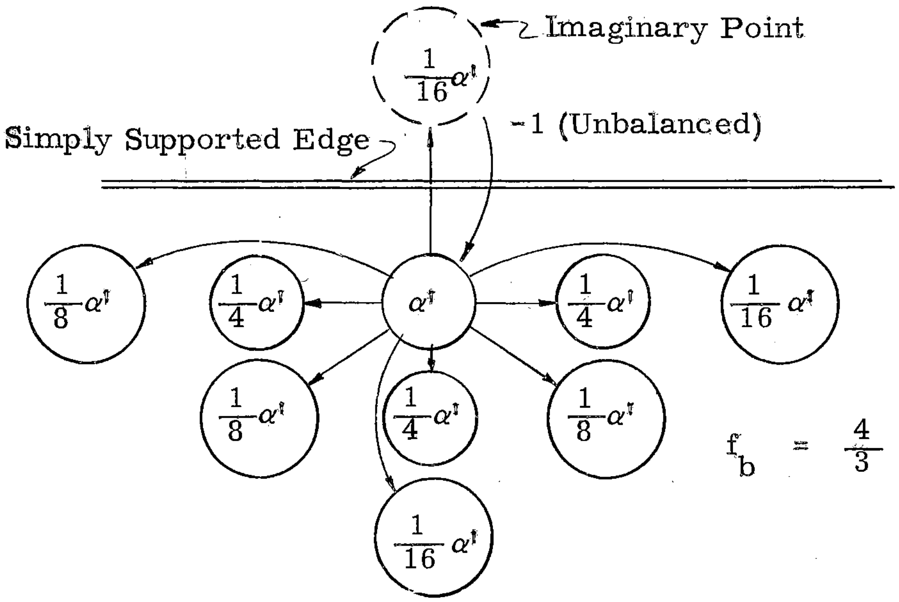


Fig. 21

Carry-over Values at a Perimeter Point

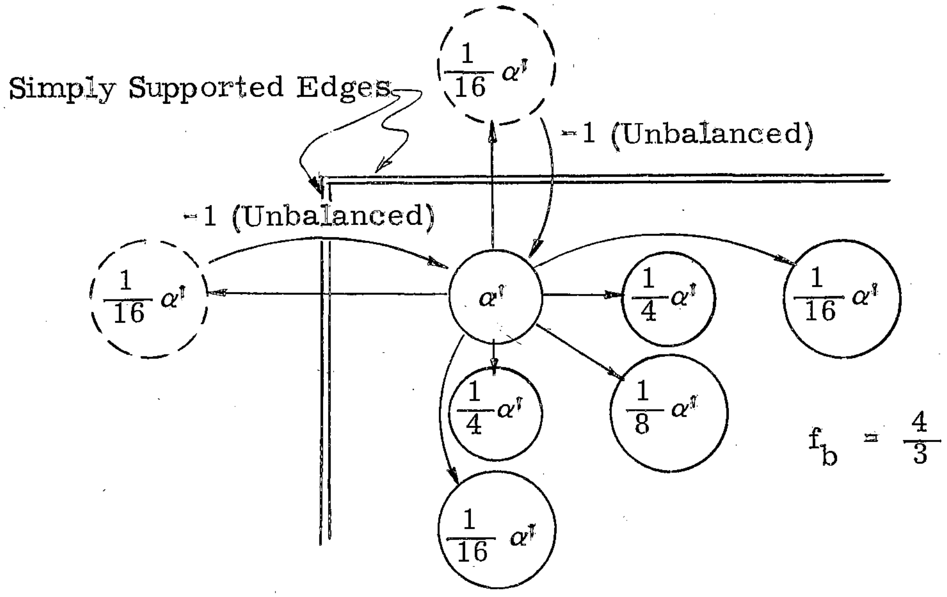


Fig. 22

Carry-over Values at a Corner Point

Illustrative Example

The moment values of the plate of Chapter II, part 2, are again computed in Table 4, where use is made of the improved convergency afforded by the algebraic series. The balance factors and carry over factors are shown in Fig's. 20, 21 and 22. The procedure followed in Table 4 is :

1. The starting loads are unbalanced values, and unbalanced values are entered to the left of the mesh line at the point where they occur. (For this example, there is only one starting value, which is 1. It is entered to the left of this mesh line at point 13.)
2. Pick out the largest unbalance value. Balance the value (multiply by f_b.) and enter the balanced value to the right of the mesh line opposite the unbalanced values. Cross out or underline the starting value to indicate it has been balanced. ($\frac{4}{3}$ is entered to the right of the mesh line at point 13, and the starting value 1 is underlined.)
3. Carry-over to the relaxation set by multiplying by the appropriate a or b. These values are balanced values, and are entered to the right of the mesh line. (These are the values 1.333 x $\frac{1}{4}$ = .333 to the right of the mesh line at points 8, 12, 14, and 18) .

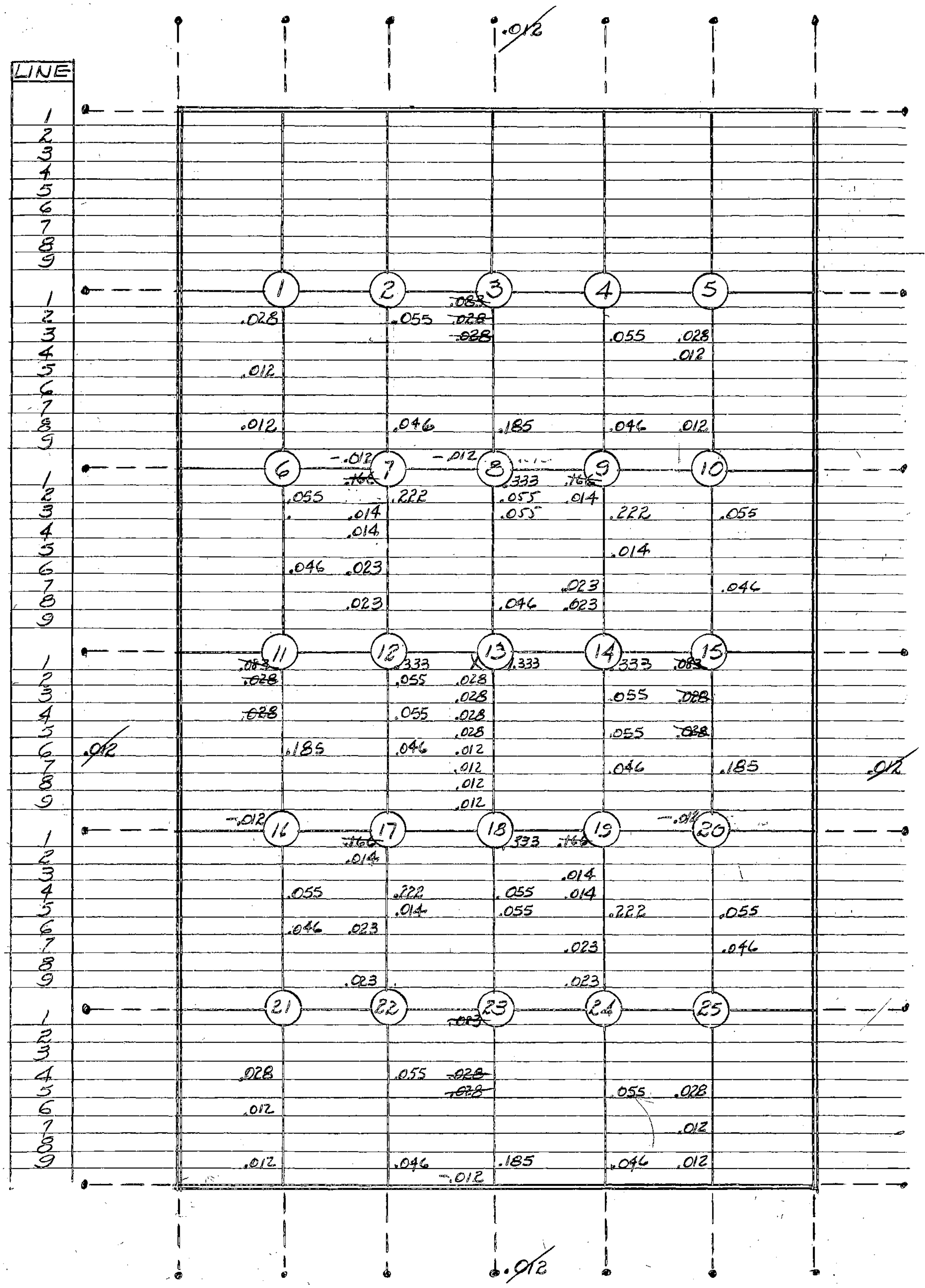


Table 4

M-ITERATION WITH IMPROVED CARRY-OVER FACTORS

4. Carry to the carry-over set by multiplying by the appropriate A or B. These values are unbalanced, and are entered to the left of the mesh line (these are the values $1.333 \times \frac{1}{16} = .083$ at points 3, 11, 15, and 19).
5. Repeat steps 2, 3, and 4 until the error is within limits. (Any part, or all, of a group of unbalanced values at a point can be used as a starting value.)
6. Sum the balanced values for the final value at the point.

Particular attention should be paid to the imaginary points outside the boundaries of the plate in Table 4. A value carried into one of these imaginary points is "stored". When desired, it is carried back (without being balanced) into its corresponding real point by the factor -1 .

Table 4 is the iteration in part. The table was completed to an unbalance of .03 and summed. The order in which those unbalances shown in Table 4 were released is :

1. Point 13 - Line 1
2. Point 7 - Line 2
3. Point 9 - Line 3
4. Point 17- Line 4
5. Point 19- Line 5
6. Point 11 - Line 6

7. Point 15 - Line 7
8. Point 3 - Line 8
9. Point 23 - Line 9
10. The values in the imaginary points opposite the real points 3, 11, 15, and 23 were all carried back into the real points on line 11.

The final sums are :

$M_1 = .084\ \lambda$
$M_2 = .191\ \lambda$
$M_3 = .253\ \lambda$

$M_6 = .185\ \lambda$
$M_7 = .407\ \lambda$
$M_8 = .617\ \lambda$

$M_{11} = .253\ \lambda$
$M_{12} = .611\ \lambda$
$M_{13} = 1.601\ \lambda$

where

$$\lambda = \frac{1}{n} \quad .$$

Values at other points are symmetrical to these.

The results may be checked as before by substituting them into Eq. (II-1) . For example:

At point 13

$$\left[-1.601 + \frac{2}{4}(.611) + \frac{2}{4}(.617)\right]\lambda = -\lambda$$

$$.987 \doteq 1$$

Error = 1.3 %

At point 8

$$\left[-.617 + \frac{1}{4}(.253) + \frac{2}{4}(.407) + \frac{1}{4}(1.601)\right]\lambda \doteq 0$$

$$-.617 + .667 \doteq 0$$

Error = 7.5%

At point 1

$$\left[-.084 + \frac{2}{4} (.191) \right] \lambda \doteq 0$$

$$- .084 + .096 \doteq 0$$

Error = 12.5%

These results of Table 4 are used as starting values for the deflection iteration, as in the numerical example of Chapter II, part 2. The same procedure as used in Table 4 applies also to the deflection table.

From Table 4, the following observations are made :

(1) The relaxation set, once calculated, served no function until the final summation.

(2) Each value (when balanced by multiplying by f_b) was carried into the carry-over set by multiplying by the appropriate A or B, where it was again multiplied by f_b.

From (1) it is deduced that the relaxation set need not be entered in the iteration table. The iteration can be completed, the values summed, and these sums carried into the relaxation set by the appropriate a or b. This is a single-step procedure, and no unbalances are incurred.

From (2) it is deduced that the unbalance carry-over factors A, B, and D can be modified to include the balance factor f_b. Thus when a balanced value (such as the value 1.333 at point 13 in the first step in Table 4) is carried to the carry-over set it is balanced and ready to be carried-over again (such as the value .166 at point 7 of Table 4).

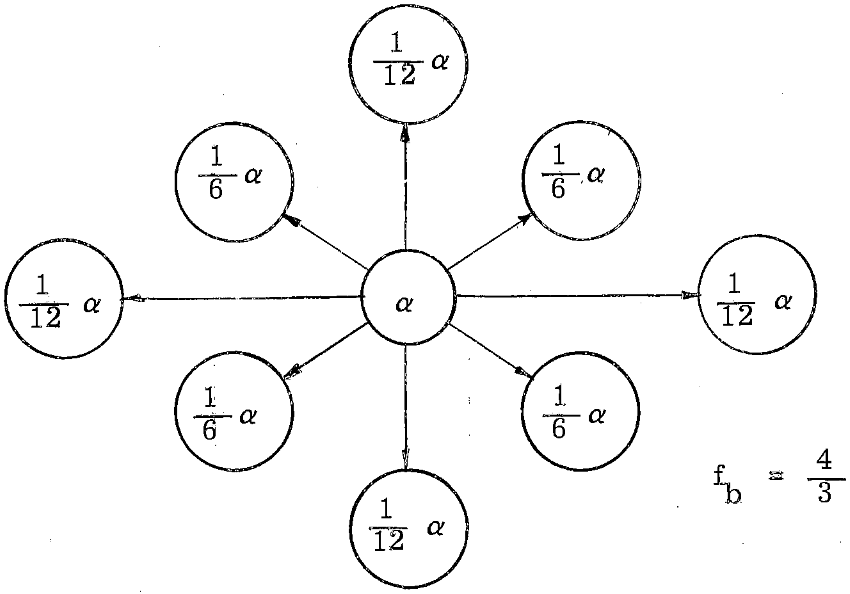
With these two refinements, the solution of the same problem ia accomplished in Table 5. The modified carry-over factors for $\Delta x = \Delta y$ are :

$$A' = A f_o = \frac{1}{16} \times \frac{4}{3} = \frac{1}{12}$$

$$B' = B f_o = \frac{1}{16} \times \frac{4}{3} = \frac{1}{12}$$

$$D' = D f_o = \frac{1}{8} \times \frac{4}{3} = \frac{1}{6}$$

The diagrams for modified carryovers for an interior, perimeter, and corner points are shown in Fig. 23, 24, and 25, respectively.



$f_b = \frac{4}{3}$

Fig. 23

Modified Carry-over Values at an Interior Point

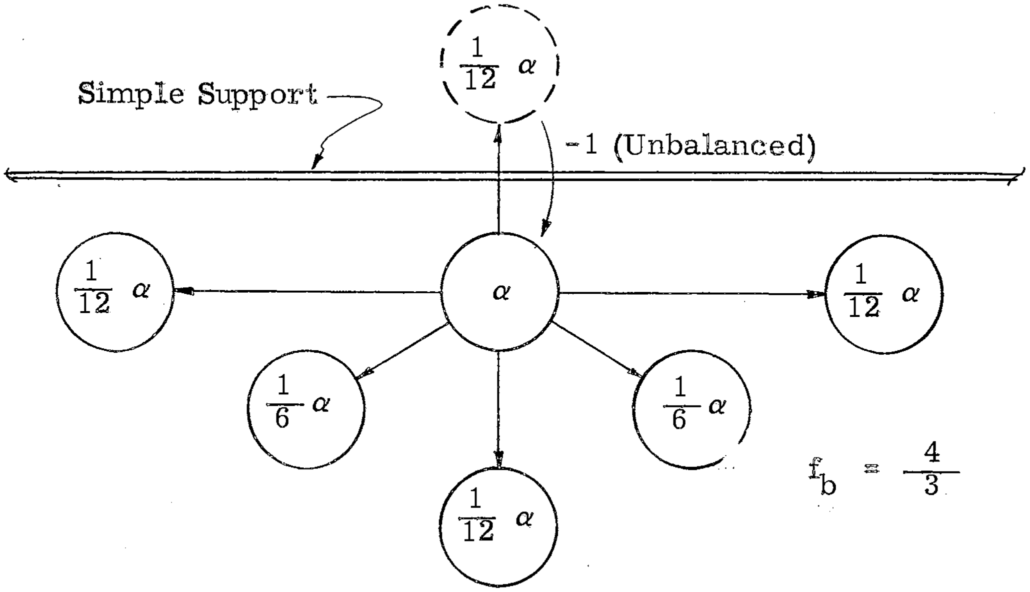


Fig. 24

Modified Carry-over Values at a Perimeter Point

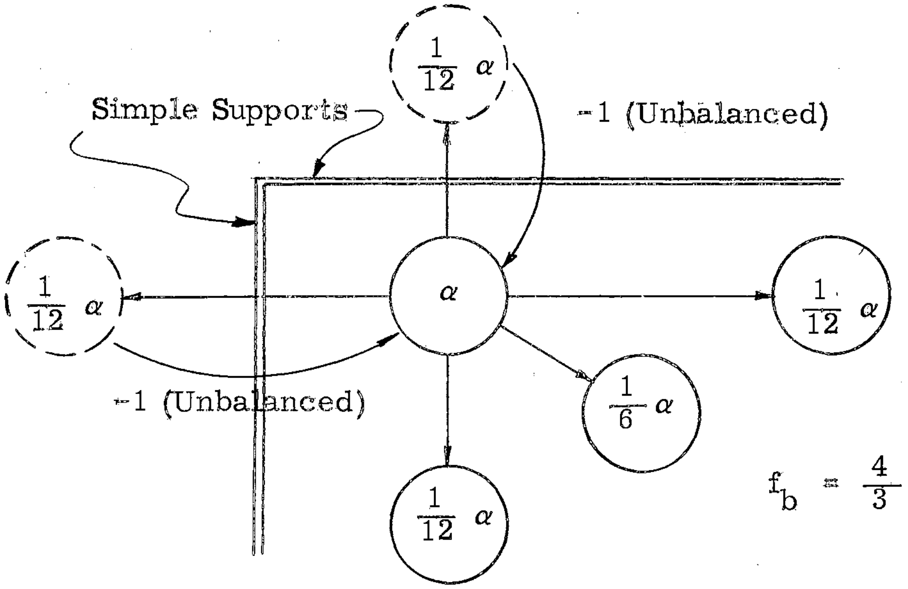


Fig. 25

Modified Carry-over Values at a Corner Point

The procedure followed in Table 5 is :

1. Enter all starting values to the left of the mesh line. Multiply each in turn by f_b and cross out the starting value, since it has no further use. Enter the balanced values to the right of the mesh line. (In Table 5, the only starting value is 1 at point 13, which is multiplied by 1.333 and this is entered to the right; the 1 is crossed out.)
2. Distribute the largest value to the carry-over set by the appropriate A', B', or D'. For the sake of uniformity, these values are entered to the right of the mesh line. Underline the value just carried over, so it will not be used again. (The value 1.333 is carried to 3, 11, 15, and 23 by the factor $\frac{1}{12}$. This is the value .111 at those points. It is carried to 7, 9, 17, and .9 by the factor $\frac{1}{6}$. This is the value .222 at those points. The value 1.333 at point 13 is then underlined.)
3. Repeat step 2 for all values which have not been distributed (those which have not been underlined) until the error is within limits. If several values occur at a point and have not been distributed, any part or all of them may be summed and distributed (the order in which the points were released are successively : 13, 7, 9, 17, 19, 11, 15, 23, 3 and imaginary points)

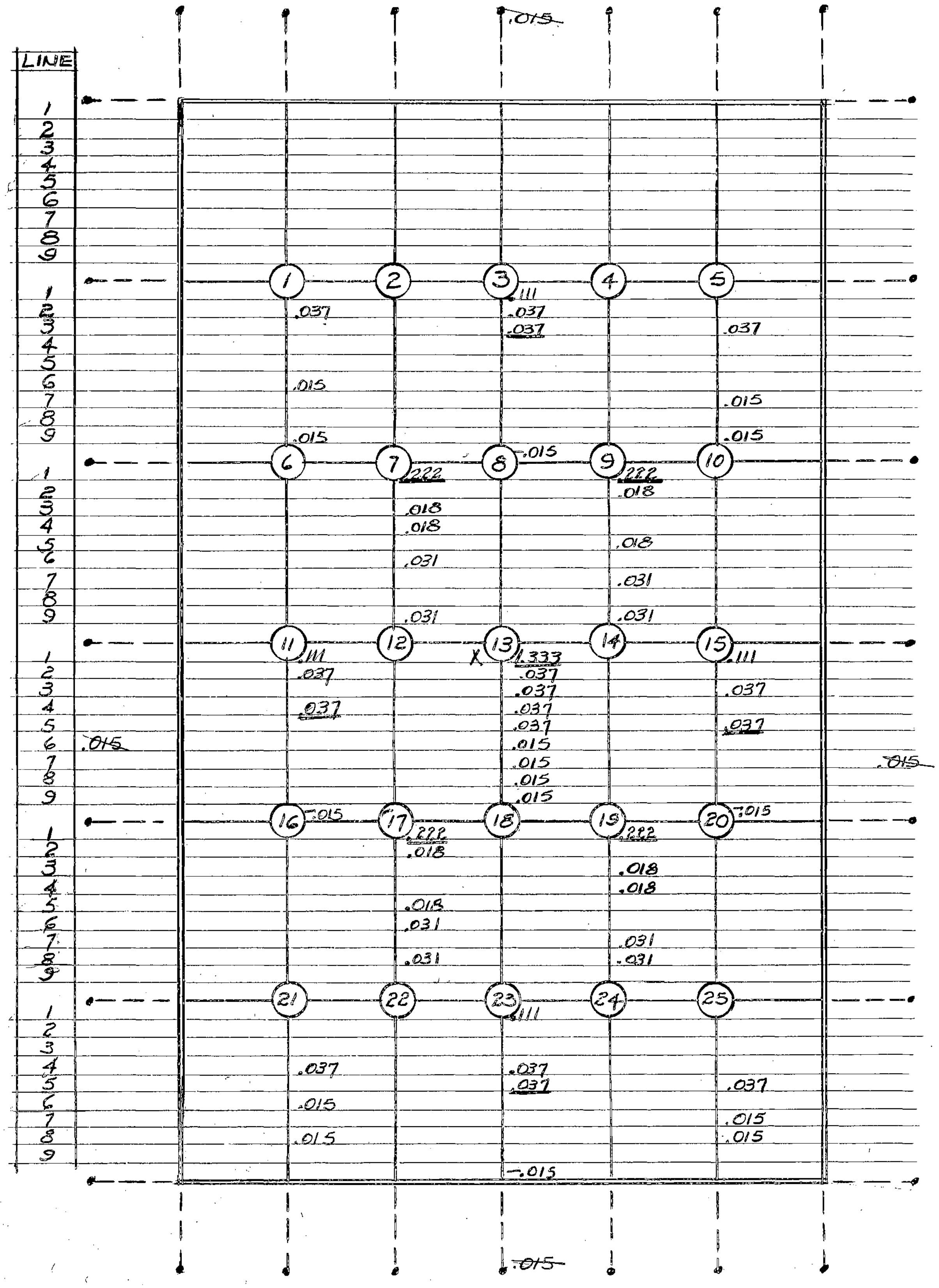


Table 5

M-ITERATION WITH MODIFIED CARRY-OVER FACTORS

4. When error is within limits, sum the value, and carry over to the relaxation set by the appropriate a or b. Sum all the resulting values at a point, which is then the final value for that point. (As in Table 4, only the first few cycles are shown in Table 5. Table 5 was run out to an unbalance of .02 and summed. The sums of Table 5 are entered into Table 6, then each of the starting values, in turn, is carried into the relaxation set. For example in Table 6, the starting value .474 λ at point 7 is carried into points 2, 6, 8, and 12 by $\frac{1}{4}$, which is the value .119 λ at those points.)

Note that in using the refinements, the need for keeping balanced and unbalanced columns was eliminated, thus eliminating the possibility of entering a number in the wrong column. All values in the modified method are distributed. Too, the need for multiplying each unbalance by the balance factor was eliminated, thus cutting down on the number of arithmetic operations and hence reducing the possibility of mistakes. Also, the carryover into the relaxation set at each cycle was reduced to a one-step operation, cutting down further on the arithmetic operations.

The final values from Table 6 are :

$M_1 = .104 \lambda$
$M_2 = .220 \lambda$
$M_3 = .229 \lambda$
$M_6 = .220 \lambda$
$M_7 = .474 \lambda$
$M_8 = .747 \lambda$
$M_{11} = .299 \lambda$
$M_{12} = .747 \lambda$
$M_{13} = 1.737 \lambda$

	① .104	②	③ .299	④	⑤ .104
		.026		.075	
		.075		.026	
		.119		.119	
Σ	.104	.220	.299	.220	.104
	⑥	⑦ .474	⑧	⑨ .474	⑩
	.026		.075		.026
	.119		.119		.119
	.075		.119		.075
			.434		
Σ	.220	.474	.747	.474	.220
	⑪ .299	⑫	⑬ 1.737	⑭	⑮ .299
		.119		.119	
		.075		.434	
		.434		.075	
		.119		.119	
Σ	.299	.747	1.737	.737	.299
	⑯	⑰ .474	⑱	⑲ .474	⑳
	.075		.434		.075
	.119		.119		.119
	.026		.119		.026
			.075		
Σ	.220	.474	.747	.474	.220
	㉑ .104	㉒	㉓ .299	㉔	㉕ .104
		.119		.119	
		.026		.075	
		.075		.026	
Σ	.104	.220	.299	.220	.104

Table 6

CARRY-OVER TO THE RELAXATION SET

The results are again checked by Eq (I-10) .

At point 13

$$\left[-1.737 + \frac{4}{4}(.747) \right] \lambda \doteq -1\,\lambda$$

$$.990 \doteq 1$$

Error = 1 %

At point 8

$$\left[-.747 + \frac{2}{4}(.474) + \frac{1}{4}(1.737) + \frac{1}{4}(.299) \right] \lambda \doteq 0$$

$$-.747 + .746 \doteq 0$$

Error = 0 %

At Point 1

$$\left[-.104 + \frac{2}{4}(.220) \right] \lambda = 0$$

$$-.104 + .110 = 0$$

Error = 5.8%

6. CROSS SECTIONAL ELEMENTS

The equations of Chapter I, Part 3 are expressed such that the iteration may be run directly for the load P, where

$$P = q\,\Delta x\,\Delta y\ .$$

The M- values are then :

$$M_{(x,y)} = \frac{k_{(x,y)}}{N}\ ,$$

where $k_{(x,y)}$ is the coefficient from the M- iteration. Having the values of k from the M- iteration, calculate shears from Eq's (I-15 and 16). Expressed in more usable form. Eq's (I-15 and 16) are :

$$Q_{x(x,y)} = \frac{1}{2N\,\Delta x}\left[k_{(x-1,y)} - k_{(x+1,y)}\right] \qquad \text{(III-1)}$$

$$Q_{y(x,y)} = \frac{1}{2N\,\Delta y}\left[k_{(x,y-1)} - k_{(x,y+1)}\right] \qquad \text{(III-2)}$$

The deflection iteration is then run for Eq. (I-8) using only the values $k_{(x,y)}$, yielding for deflection :

$$w_{(x,y)} = \frac{j_{(x,y)}}{N\,\psi}$$

where

$$\psi = \frac{D_f N}{\Delta x\,\Delta y}$$

and $j_{(x,y)}$ is the coefficient from the w- iteration.

When these values for w are substituted into Eq's (I-12), (I-13) and (I-14), the resulting equations are :

$$M_{x(x,y)} = \frac{-1}{N}\left[aj_{(x-1,y)} + \mu bj_{(x,y-1)} - 2(a+\mu b)\,j_{(x,y)} + \mu bj_{(x,y+1)} + aj_{(x+1,y)}\right] \qquad (\text{III-3})$$

$$M_{y(x,y)} = -\frac{1}{N}\left[\mu aj_{(x-1,y)} + bj_{(x,y-1)} - 2(\mu a+b)\,j_{(x,y)} + bj_{(x,y+1)} + \mu aj_{(x+1,y)}\right] \qquad (\text{III-4})$$

$$M_{xy(x,y)} = \frac{1-\mu}{4N^2}\left[j_{(x-1,y-1)} - j_{(x+1,y-1)} - j_{(x-1,y+1)} + j_{(x+1,y+1)}\right] \qquad (\text{III-5})$$

7. TABLES OF CARRY-OVER AND BALANCE FACTORS

The carry-over factors and balance factors for various values of n are listed in Table 7. Expressed mathematically, the factors are:

$$n = \frac{\Delta x}{\Delta y}$$

$$N = \frac{2+2n^2}{n}$$

$$a = \frac{1}{2+2n^2}$$

$$b = \frac{n^2}{2+2n^2}$$

$$f_o = \frac{1}{1-2a^2-2b^2}$$

$$A = a^2$$

$$B = b^2$$

$$D = 2ab$$

n	N	a	b	A	B	D	f_b
.5	5.00	.400	.100	.160	.0100	.0800	1.515
.6	4.53	.367	.133	.135	.0177	.0976	.1430
.7	4.26	4.26	.165	.112	.0272	.1106	.1385
.8	4.10	.305	.195	.0930	.0380	.1190	.1355
.9	4.02	.276	.224	.0762	.0502	.1236	1.340
1.0	4.00	.250	.250	.0625	.0625	.1250	1.333
1.1	4.02	.226	.274	.0511	.0751	.1240	.1338
1.2	4.07	.205	.295	.0420	.0870	.1210	1.348
1.3	4.14	.186	.314	.0346	.0986	.1168	1.362
1.4	4.23	1.69	.331	.0285	.1095	.1118	1.381
1.5	4.33	.154	.346	.0237	.1198	.1066	1.401

Table 7

CARRY-OVER AND BALANCE FACTORS

The modified carry-over factors are listed with their regular balance factor for various values of n in Table 8. Expressed mathematically, the factors are :

$$f_b = \frac{1}{1-2a^2-2b^2} \qquad B' = f_b B$$

$$A' = f_b A \qquad D' = f_b D \quad .$$

The factors a, b, A, B, D and f_b are listed in Table 7.

n	f_b	A'	B'	D'
.5	1.515	.2420	.0152	.1210
.6	1.430	.1930	.0253	.1395
.7	1.385	.1550	.0377	1.530
.8	1.355	.1260	.0515	.1610
.9	1.340	.1020	.0673	.1655
1.0	1.333	.0833	.0833	.1667
1.1	1.338	.0684	.1005	.1660
1.2	1.348	.0566	.1170	.1630
1.3	1.362	.0471	.1340	.1590
1.4	1.381	.0394	.1512	.1541
1.5	1.401	.0332	.1680	.1492

Table 8

MODIFIED CARRY-OVER FACTORS

CHAPTER

III

NUMERICAL EXAMPLES

CHAPTER III NUMERICAL EXAMPLES

The procedure will be demonstrated in two typical problems. The moments and shears will be calculated at specified points.

Example 1

A square steel plate, 10' each way and $\frac{1}{2}''$ thick, is simply supported at all four edges. It is loaded by a concentrated load of 1250 lbs. at its center. Compute the maximum stress in each direction, the shear at the midpoint of each side, and the corner reactions. Take $\mu = .3$, and neglect dead load in this computation.

Solution

A fair degree of accuracy can be obtained by a six-strip mesh (25 points). For this mesh, with unit load at the center, the M-values are those previously calculated. The final values are shown in Table 6 for unit load.

For P = 1250 lbs.,

$$M_1 = \frac{130}{N} \qquad M_7 = \frac{593}{N}$$

$$M_2 = \frac{275}{N} \qquad M_8 = \frac{934}{N}$$

$$M_3 = \frac{374}{N} \qquad M_{13} = \frac{2171}{N}$$

The deflection iteration is run for these values of M. (M-values at other points are symmetrical.)

For a square plate with six divisions,

$$\Delta x = \Delta y = \frac{10 \times 12}{6} = 20''$$

Figs. 23, 24, and 25 show the modified carry-over factors.

The M-values are entered in Table 9 and balanced (multiplied by f_b). The procedure outlined on page 50 is followed.

Final deflections, from Table 9 are :

$$w_1 = 1468 \frac{1}{N\psi}$$
$$w_2 = 2669 \frac{1}{N\psi}$$
$$w_3 = 3206 \frac{1}{N\psi}$$
$$w_6 = 2669 \frac{1}{N\psi}$$
$$w_7 = 5049 \frac{1}{N\psi}$$
$$w_8 = 6187 \frac{1}{N\psi}$$
$$w_{11} = 3206 \frac{1}{N\psi}$$
$$w_{12} = 6187 \frac{1}{N\psi}$$
$$w_{13} = 8034 \frac{1}{N\psi}$$

From inspection, the maximum value of deflection is seen to occur at the center. Its value is :

$$w_{(Max)} = 8034 \frac{1}{N\psi}$$

where

$$\psi = \frac{D_f N}{\Delta x \, \Delta y}$$

$$\Delta x = \frac{L}{6} = 20''$$

$$\Delta y = \frac{L}{6} = 20''$$

$$D_f = \frac{E h^3}{1 - \mu^2}$$

$$N = 4$$

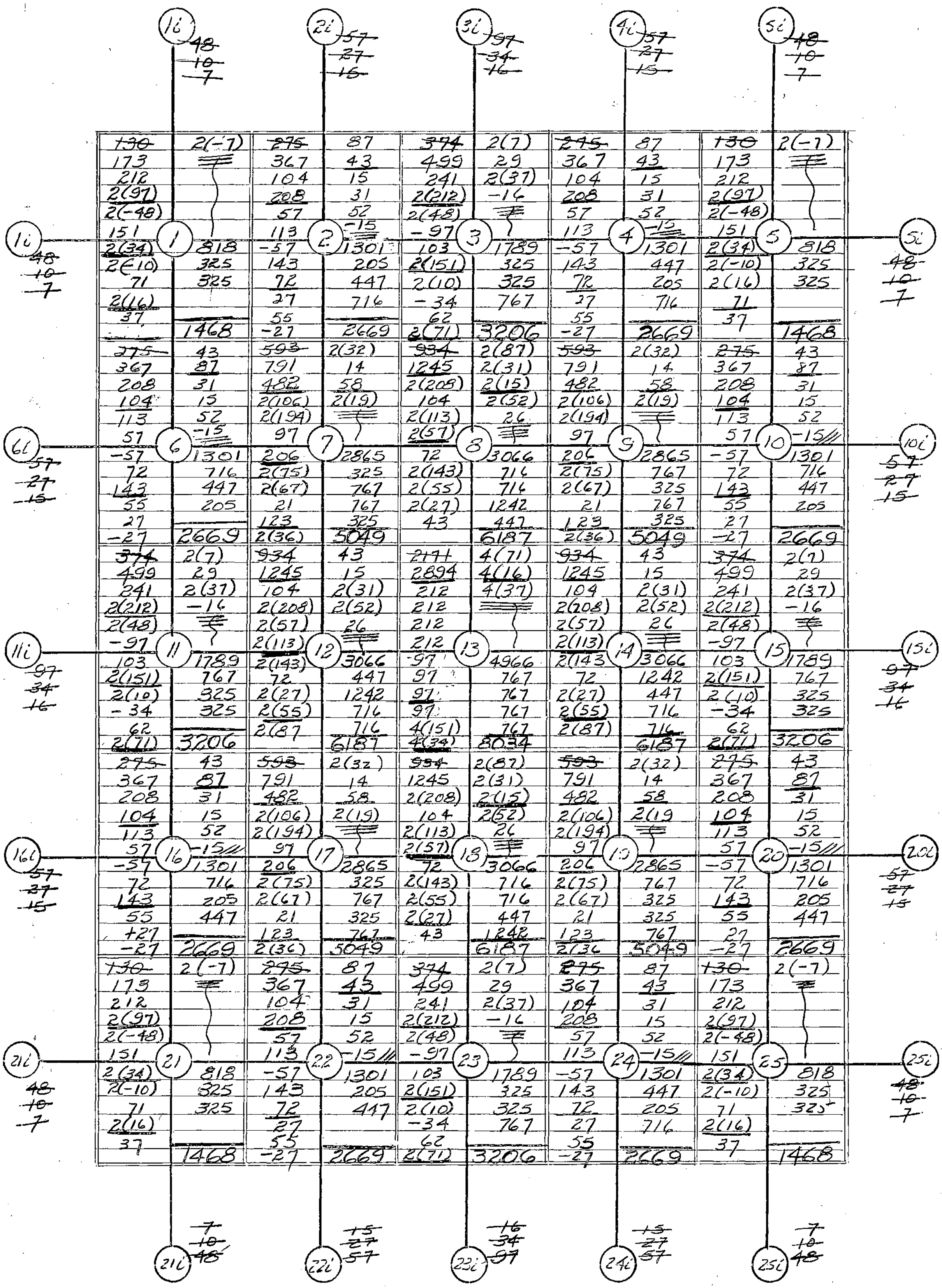


Table 9

DEFLECTION ITERATION–CONCENTRATED LOAD

whence :

$$w_{(Max)} = 152 \frac{L^2}{E h^3}$$

$$= .583''$$

The maximum moment occurs at the center, and is equal in both directions x and y. Substituting into Eq. (II-11a),

$$M_x = M_y = \frac{1}{4}\left\{\left[(\frac{2}{4})(6187) + (.3)(\frac{2}{4})(6187)\right.\right.$$

$$\left.\left. -2\left[\frac{1}{4} + (.3)(\frac{1}{4})\right](8034)\right]\right\}$$

$$= \quad 300 \text{ lb. in. per. in.}$$

The shears at the midpoints of each side are equal, and are obtained by substitution into Eq. (II-10a).

$$Q_k = \frac{1}{(2)(4)(20)}\left[374 - (-374)\right] = \frac{374}{80}$$

$$= 4.68 \text{ lb. per. in.}$$

The corner reaction is obtained from Eq. (II-13) and Eq. (II-12) :

$$R = \frac{2(1-.3)}{4(4)^2}\left[+(-1468)-(+1468)-(+1468)+(-1468)\right]$$

$$= -\frac{1.4}{16}(1468)$$

$$= - 128 \text{ Lb. per in. (downward)}.$$

Example 2

A rectangular steel plate 10 ft. x 15 ft., $\frac{1}{2}''$ thick, simply supported on all sides, is loaded by a transverse uniform load of 100 $\frac{\text{lbs.}}{\text{ft.}^2}$. Compute the center deflection and the corner reactions. Take $\mu = .3$ and the weight of steel at 490 $\frac{\text{lbs.}}{\text{ft.}^3}$

Solution

For the purpose of illustrating the use of unequal carry-over factors, the plate is divided into 30" strips in the X-direction and 24" strips in the Y-direction. The resultant 20-point mesh has no numbered point at the center of the plate; the central deflection will have to be interpolated.

The starting values are the equivalent concentrated loads P at each point of the mesh,

$$P_{(x,y)} = q_{(x,y)}\,\Delta x\,\Delta y = 720\,q$$

For the moment iteration, all starting values P are taken equal to unity; the results must then be multiplied by 720 q for the final values.

The plate with its mesh of points and equivalent concentrated loads is shown in Fig. 26.

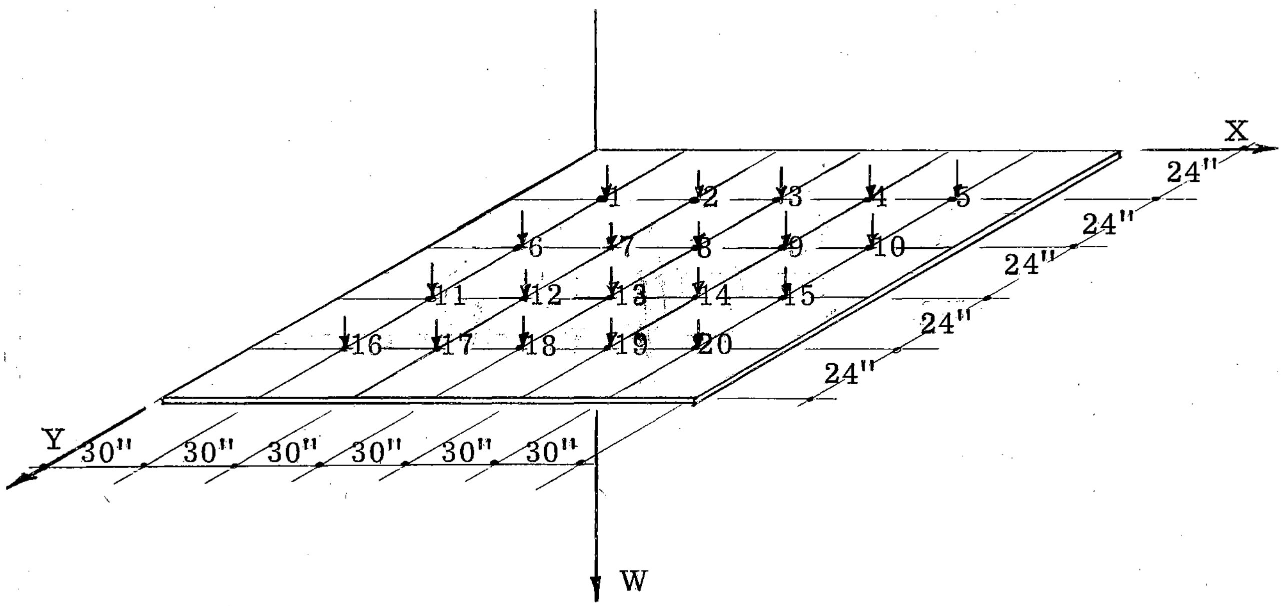


Fig. 26

Point Numbers for Example 2

The carry-over factors and balance factors may be calculated or interpolated from Tables 7 and 8.

$$n = \frac{\Delta x}{\Delta y} = 1.25$$

$N = 4.100$

$f_b = 1.355$

$a = .195$

$b = .305$

$A' = .0515$

$B' = .1260$

$D' = .1612$

The diagram for these factors is shown in Fig. 27.

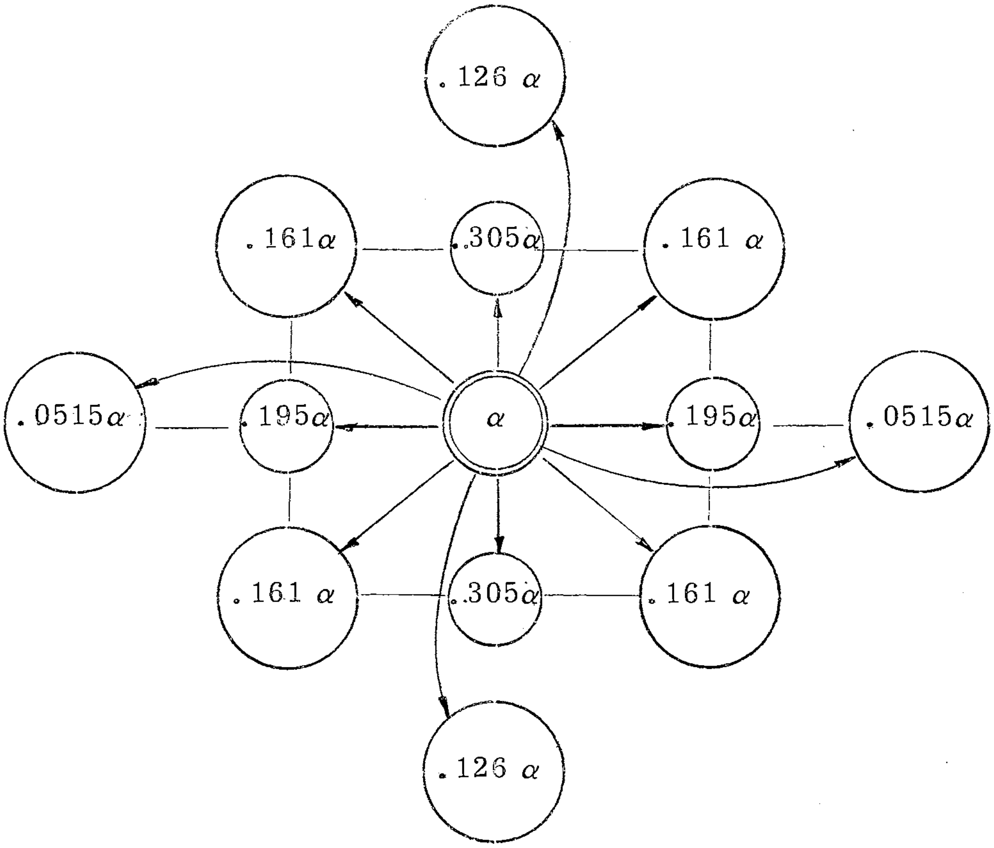


Fig. 27

Carry-over Diagram for n=1.25

Table 10 is the moment iteration for starting values of unity. The results of Table 10 are then the starting values for Table 11.

The results of Table 11 are, (for P = 1 lb.) :

$$\left| \begin{array}{lcl} w_1 & = & 16.194 \frac{1}{N\psi} \\ w_2 & = & 25.758 \frac{1}{N\psi} \\ w_3 & = & 29.245 \frac{1}{N\psi} \end{array} \right. \qquad \left| \begin{array}{lcl} w_6 & = & 25.457 \frac{1}{N\psi} \\ w_7 & = & 40.995 \frac{1}{N\psi} \\ w_8 & = & 47.133 \frac{1}{N\psi} \end{array} \right.$$

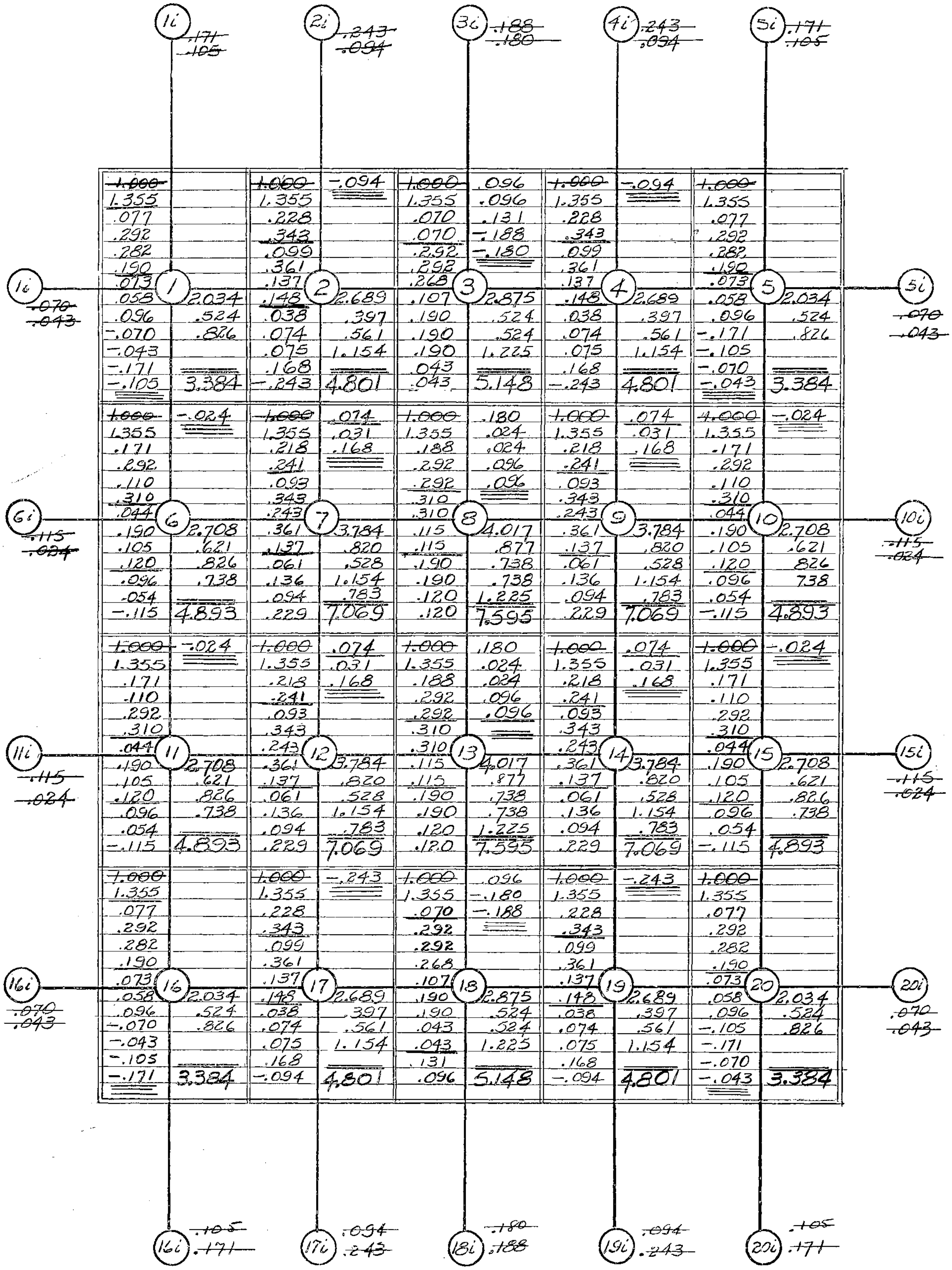
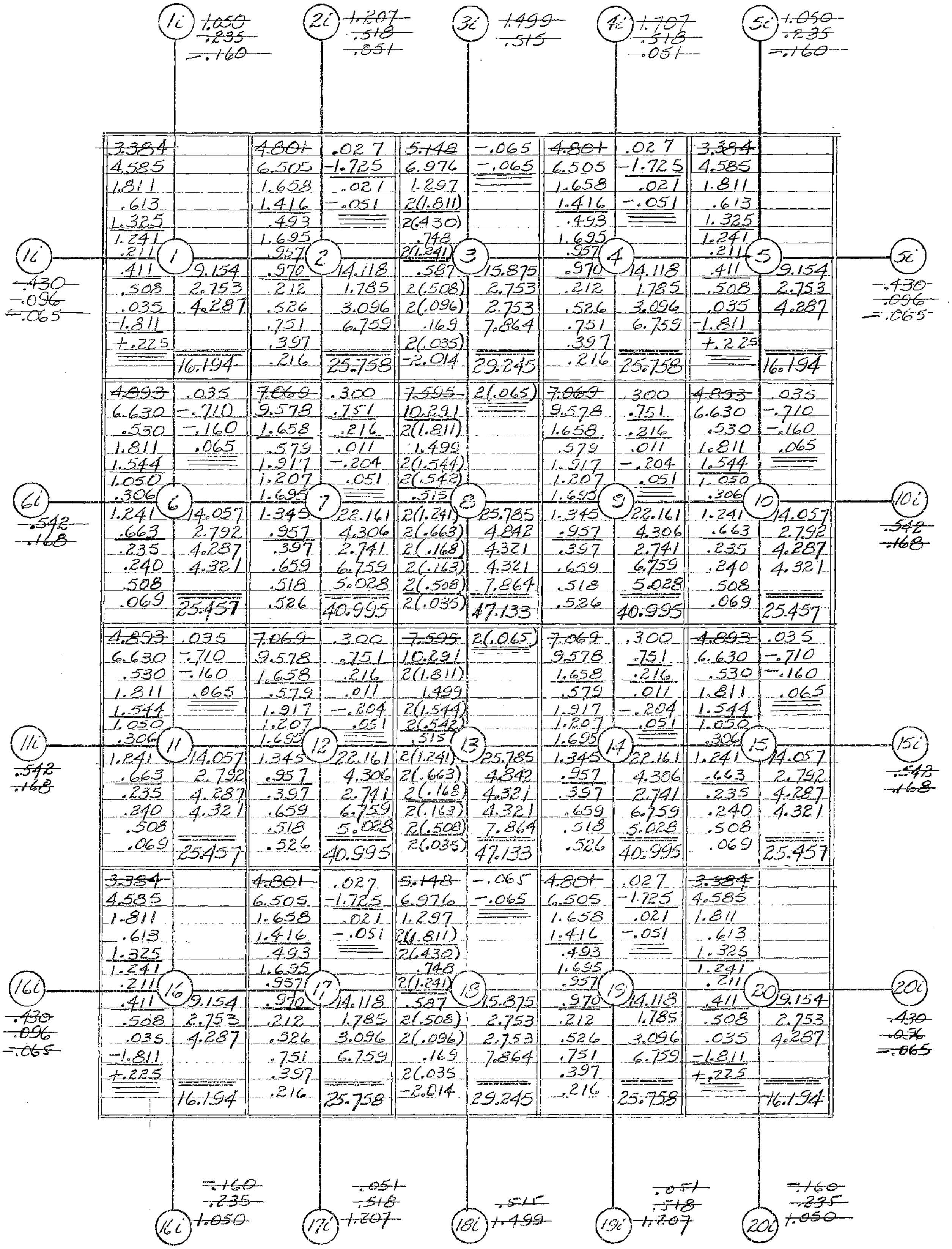


Table 10

MOMENT ITERATION-UNIFORM LOAD



(1i) ~~1.050~~ ~~.235~~ ~~-.160~~
(2i) ~~1.207~~ ~~.518~~ ~~.051~~
(3i) ~~1.499~~ ~~.515~~
(4i) ~~1.707~~ ~~.518~~ ~~.051~~
(5i) ~~1.050~~ ~~.235~~ ~~-.160~~

~~3.384~~ 4.585 1.811 .613 1.325 1.241 .211 ① .411 .508 .035 -1.811 +.225	9.154 2.753 4.287 16.194	~~4.801~~ 6.505 1.658 1.416 .493 1.695 .957 ② .970 .212 .526 .751 .397 .216	.027 -1.725 .021 -.051 14.118 1.785 3.096 6.759 25.758	~~5.148~~ 6.976 1.297 2(1.811) 2(.430) .748 2(1.241) ③ .587 2(.508) 2(.096) .169 2(.035) -2.014	-.065 -.065 15.875 2.753 2.753 7.864 29.245	~~4.801~~ 6.505 1.658 1.416 .493 1.695 .957 ④ .970 .212 .526 .751 .397 .216	.027 -1.725 .021 -.051 14.118 1.785 3.096 6.759 25.758	~~3.384~~ 4.585 1.811 .613 1.325 1.241 .211 ⑤ .411 .508 .035 -1.811 +.225	9.154 2.753 4.287 16.194
~~4.893~~ 6.630 .530 1.811 1.544 1.050 .306 ⑥ 1.241 .663 .235 .240 .508 .069	.035 -.710 -.160 .065 14.057 2.792 4.287 4.321 25.457	~~7.069~~ 9.578 1.658 .579 1.917 1.207 1.695 ⑦ 1.345 .957 .397 .659 .518 .526	.300 .751 .216 .011 -.204 .051 22.161 4.306 2.741 6.759 5.028 40.995	~~7.595~~ 10.291 2(1.811) 1.499 2(1.544) 2(.542) .515 ⑧ 2(1.241) 2(.663) 2(.168) 2(.163) 2(.508) 2(.035)	2(.065) 25.785 4.842 4.321 4.321 7.864 47.133	~~7.069~~ 9.578 1.658 .579 1.917 1.207 1.695 ⑨ 1.345 .957 .397 .659 .518 .526	.300 .751 .216 .011 -.204 .051 22.161 4.306 2.741 6.759 5.028 40.995	~~4.893~~ 6.630 .530 1.811 1.544 1.050 .306 ⑩ 1.241 .663 .235 .240 .508 .069	.035 -.710 -.160 .065 14.057 2.792 4.287 4.321 25.457
~~4.893~~ 6.630 .530 1.811 1.544 1.050 .306 ⑪ 1.241 .663 .235 .240 .508 .069	.035 -.710 -.160 .065 14.057 2.792 4.287 4.321 25.457	~~7.069~~ 9.578 1.658 .579 1.917 1.207 1.695 ⑫ 1.345 .957 .397 .659 .518 .526	.300 .751 .216 .011 -.204 .051 22.161 4.306 2.741 6.759 5.028 40.995	~~7.595~~ 10.291 2(1.811) 1.499 2(1.544) 2(.542) .515 ⑬ 2(1.241) 2(.663) 2(.168) 2(.163) 2(.508) 2(.035)	2(.065) 25.785 4.842 4.321 4.321 7.864 47.133	~~7.069~~ 9.578 1.658 .579 1.917 1.207 1.695 ⑭ 1.345 .957 .397 .659 .518 .526	.300 .751 .216 .011 -.204 .051 22.161 4.306 2.741 6.759 5.028 40.995	~~4.893~~ 6.630 .530 1.811 1.544 1.050 .306 ⑮ 1.241 .663 .235 .240 .508 .069	.035 -.710 -.160 .065 14.057 2.792 4.287 4.321 25.457
~~3.384~~ 4.585 1.811 .613 1.325 1.241 .211 ⑯ .411 .508 .035 -1.811 +.225	9.154 2.753 4.287 16.194	~~4.801~~ 6.505 1.658 1.416 .493 1.695 .957 ⑰ .970 .212 .526 .751 .397 .216	.027 -1.725 .021 -.051 14.118 1.785 3.096 6.759 25.758	~~5.148~~ 6.976 1.297 2(1.811) 2(.430) .748 2(1.241) ⑱ .587 2(.508) 2(.096) .169 2(.035 -2.014	-.065 -.065 15.875 2.753 2.753 7.864 29.245	~~4.801~~ 6.505 1.658 1.416 .493 1.695 .957 ⑲ .970 .212 .526 .751 .397 .216	.027 -1.725 .021 -.051 14.118 1.785 3.096 6.759 25.758	~~3.384~~ 4.585 1.811 .613 1.325 1.241 .211 ⑳ .411 .508 .035 -1.811 +.225	9.154 2.753 4.287 16.194

(1i) ~~.430~~ ~~.096~~ ~~-.065~~
(6i) ~~.542~~ ~~.168~~
(11i) ~~.542~~ ~~.168~~
(16i) ~~.430~~ ~~.096~~ ~~-.065~~

(5i) ~~.430~~ ~~.096~~ ~~-.065~~
(10i) ~~.542~~ ~~.168~~
(15i) ~~.542~~ ~~.168~~
(20i) ~~.430~~ ~~.036~~ ~~-.065~~

(16i) ~~-.160~~ ~~.235~~ ~~1.050~~
(17i) ~~.051~~ ~~.518~~ ~~1.207~~
(18i) ~~.515~~ ~~1.499~~
(19i) ~~.051~~ ~~.518~~ ~~1.707~~
(20i) ~~-.160~~ ~~.235~~ ~~1.050~~

Table 11

DEFLECTION ITERATION-UNIFORM LOAD

The total uniform load q_t is :

$$q_t = q_{dl} + q_{ll} = (450)(\frac{1}{1728})(\frac{1}{2}) + (100)(\frac{1}{144})$$

$$= .836 \frac{lb}{in.^2}$$

whence:

$$P = (.836)(720) = 602 \text{ lbs.}$$

The final deflection coefficients for use in the equations of Chapter II, Part 6, are :

$j_1 = 9,749$

$j_2 = 15,506$

$j_3 = 17,605$

$j_6 = 15,325$

$j_7 = 24,679$

$j_8 = 28,374$

The corner reaction is computed from Eq. (II-13 and -12) .

$$R_c = 2 (\frac{1 - .3}{4(4.10)^2}) \left[-9749 - 9749 - 9749 - 9749 \right]$$

$$= -812 \frac{lbs.}{in.}$$

The central deflection can be found by linear or parabolic interpolation. The ordinates for parabolic interpolation are shown in Fig. 28.

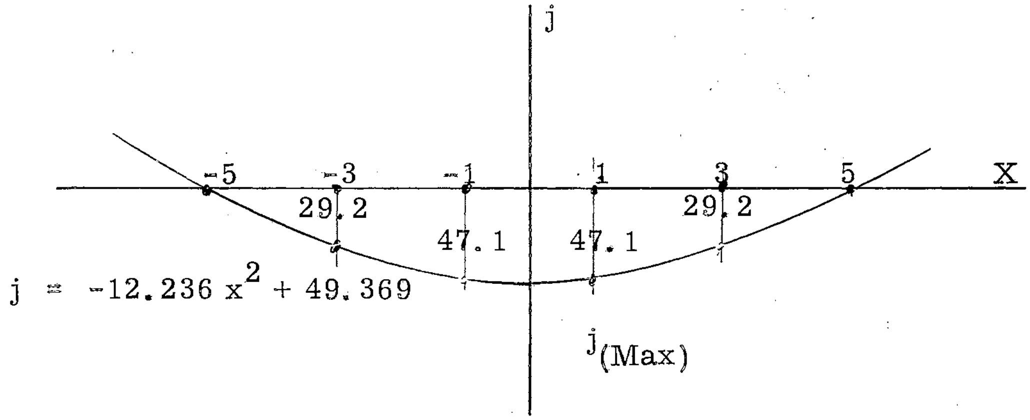


Fig. 28

Ordinates for Parabolic Interpolation

The curve is fitted to the second order parabola:

$$j = -12.236x^2 + 49.369 .$$

At x = 0, $j_{(Max)} = 49.269$.

In formula form,

$$w_{(Max)} = \frac{49.369}{N\psi} (q \, \Delta x \, \Delta y)$$

$$= .0802 \frac{qL_s^4}{E h^3}$$

where L_s is the length of the shorter side.

In inches,

$$w_{(Max)} = .0802 \left(\frac{.836 \times 120^4}{30 \times 10^6 \times (\frac{1}{2})^3} \right)$$

$$= 3.71''$$

COMPARISON OF RESULTS

The results of Example 1 are recorded in Table 12 and compared with the results obtained from Ref. (5).

Line	Point	1	3	13	Time
1	Moment-Ref. (5)	.115	.308	1.769	
2	Moment-Table 1b	.088	.250	1.653	150 min.
3	Error	23.5 %	18.8 %	6.56 %	
4	Moment-Table 6	.104	.299	1.737	55 min.
5	Error	9.6 %	2.9 %	1.8 %	
6	Deflection-Ref. (5)	1.322	2.911	7.355	
7	Deflection-Table 2b	.892	2.042	5.626	190 min.
8	Error	32.5 %	29.9 %	43.5 %	
9	Deflection-Table 9	1.174	2.564	6.427	70 min.
10	Error	11.2 %	11.9 %	12.6 %	

Table 12

COMPARISON OF RESULTS

The deflection values shown in line 9 are those in Table 9 divided by 1250 to reduce them to unit load for the comparison.

The last column indicates the time required for the solution.

Table 12 indicates that a greater degree of accuracy was obtained in less than half the time when compared to the straight iteration.

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Date of Final Examination; August, 1958.