THERMODYNAMIC ASPECT OF PNEUMATIC ACCUMULATOR PERFORMANCE

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PERFORMANCE

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PREFACE

No available method presently exists for an engineer to make a proper selection of a pneumatic accumulator and specify the charging conditions to insure a given power output. The need for an adequate solution to this problem in modern fluid power applications has prompted this investigation.

I sincerely wish to thank Dean M. R. Lohmann, Dr. Clark Dunn, and Dr. J. H. Boggs not only for making this study possible, but also, for affording me an opportunity to receive my graduate study here at Oklahoma State University. Their generous award of an Honor's Fellowship to me was indeed an honor for which I am most grateful. I find it very difficult to express my appreciation to Professor E. C. Fitch for his continued advice and encouragement throughout the course of this study. I feel that it would be very difficult to find a more efficient and conscientious advisor. My gratitude is extended to Professor B. S. Davenport, Mr. John McCandless and Mr. George Cooper for their aid in constructing the testing apparatus. I am indebted to Mr. Charles Freeny for his advice and assistance with the electrical instrumentation. I wish to say thanks to Mrs. Mildred Avery for her many suggestions and criticisms after reading the entire thesis and for her efficient and capable services as my typist. Thanks are also extended to Professor John Wiebelt for his suggestions and criticisms. To my wife, Mary Ann, and my girls, thanks for the sacrifice you have made in order that I might receive my education.

NOMENCLATURE

P	Pressure, psi
v	Specific volume, cu in. per lb
n	Polytropic constant, dimensionless
V	Volume, cu in.
V_T	Total Volume of the Accumulator, cu in.
V_1	Volume of the compressed gas after the charging operation, cu in.
V_2	Volume of the expanded gas after the discharging operation, cu in.
V_C	Volume of hydraulic fluid displaced from the accumulator during the discharging operation, cu in.
P_1	Pressure of the compressed gas after the charging operation, psi
P_2	Pressure of the expanded gas after the discharging operation, psi
P_3	Effective load pressure on the Accumulator, psi
P_R	Preload pressure on the gas, psi
Q	Flow rate, cu in. per sec
t	Time of discharging operation, sec
C	Orifice discharge coefficient, dimensionless
A	Orifice area, sq in.
g	Gravitational acceleration constant, 32.2 ft per sec per sec
w	Specific Weight, lb per cu ft
G	Specific Gravity, dimensionless
D	Diameter, in.
ID	Inside diameter, in.
LMP	Log-mean-pressure, psi
P_f	Pressure loss, psi

f	Friction Coefficient, dimensionless
U	Velocity, ft per sec
L_e	Equivalent length of pipe, ft
K	Coefficient, dimensionless
R	Reynold's number, dimensionless

Greek Letters

ν	Nu, Kinematic viscosity, centistokes
γ	Gamma, Ratio of the specific heats of the gas, dimensionless
π	Pi

Abbreviations

lb	Pounds
cu	Cubic
psi	Pounds per square inch
sq	Square
ft	Feet
sec	Seconds
in.	Inches

Symbols

%	Per cent

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CHAPTER I

INTRODUCTION

One of the first applications of an accumulator was made in the Middle Ages, when a waterfall was harnessed in such a manner as to fill a large container balanced on one end of a lever, the other end of which was connected to a heavy stone door. When the container was full of water, it was heavy enough to overbalance the weight of the door.

Modern application of hydraulic power provides maximum efficiency in many hydraulically operated machines by placing accumulators at strategic locations in the hydraulic system. The most well-known accumulator function is that of storing hydraulic fluid under pressure to alleviate intermittent peak-demand pump requirements. The accumulator is also employed in a charged state for emergency or standby service in the event of partial circuit failure or pump disablement. Other important functions of the hydraulic accumulator are shock absorption, pulsation damping, static pressure-volume compensation, and pressure transfer between unlike fluids.

The earliest form of accumulator was classified as a gravity- or weight-loaded accumulator and is still being used today. The basic design of this type consists of a heavy-wall steel cylinder with a smoothly ground bore into which is fitted a piston. A large container, mounted on top of the piston, holds high-density material and is used for the counterweight. Thus the force of gravity provides the energy.

Another type accumulator used in the hydraulic power field is the spring-loaded accumulator. This type accumulator consists of a cylinder into which is fitted a piston and a set of compression springs, acting in such a manner so as to supply the dynamic load.

One of the most popular accumulators used today is the pneumatic accumulator in which potential energy is stored in an enclosed gas chamber contained within a fluid chamber. Most of the pneumatic accumulators are of the separator type and are further divided into solid piston separator and flexible separator types. Both of these accumulators separate the compressible gas from the incompressible hydraulic fluid. Urgent requirements of lightness and reliability have intensified the use of the pneumatic accumulator in the operation of aircraft systems and in many other industrial applications. Today, the work-cycle motions in many machine operations are intermittent. Thus, in these operations a proper approach through application of an accumulator can result in a lower cost and a simpler, smaller hydraulic system.

Although energy is usually stored in a pneumatic accumulator by pumping fluid into it under pressure, actually it is the potential energy of the compressed gas that fulfills the ultimate power demand. Therefore, the behavior of the accumulator must be studied from the standpoint of gas thermodynamics.

In the past, it has been a common practice to consider the compressible preload fluid as a perfect gas. However, work undertaken at Oklahoma State University has revealed that the compression of the gas under high pressure and low temperature conditions should be taken into consideration. By employing the compressibility factor, the state of the gas and the available energy of the system can be determined at any given time.

Although the available energy of a charged accumulator can be predicted, the time required for the accumulator to discharge a given volume of fluid has required a trial and error solution. If the time for discharge could not be tolerated for the chosen accumulator, then another one would have to be selected. The dynamics of the fluid system has been considered either too complicated to yield an adequate solution to the problem or has been completely avoided. In many applications, the time required for an accumulator to discharge fluid is just as important as its available energy consideration. No reported method presently exists for an engineer to make a proper selection of a pneumatic accumulator and specify the charging conditions to insure a given power output. It was the recognition of this need that prompted this investigation.

CHAPTER II

PREVIOUS INVESTIGATIONS

Very little work has been published relative to the proposed investigation. A few papers have been published which base the selection of a pneumatic accumulator on an energy basis, but only one (9) was found which considered both the available energy and the time required to dissipate the stored energy of a pneumatic accumulator.

Edward M. Greer (5) presents a method of selecting an accumulator in a hydraulic press application. In his work he does not indicate how the precharge pressure is selected. He merely states that "if 2000 psi pressure is the lowest pressure available after fluid has been withdrawn from the accumulator, then it is necessary to precharge the accumulator to 1500 psi to satisfy the flow requirement and provide extra fluid as a safety factor on the required flow." To determine the size of the accumulator required, he applied Boyle's law for gases (PV = constant) for both charging and discharging processes.

Applied Hydraulics (2) caused an interesting and valuable controversy with the Data Sheet appearing in the October, 1952 issue. This sheet presented a method for selecting hydropneumatic accumulators using Boyle's law also. A reader, whose name was not given in the article, suggested that Boyle's law would not hold where the expansion of the gas was very rapid. His analysis was sent to Greer Hydraulics Company, which manufactures hydropneumatic accumulators. As a result, in the June, 1953 issue of Applied Hydraulics, R. Hemeon (8) gave an explanation of factors which should be considered when selecting the correct accumulator size, but he gave no indication as to how the time could be related to his selection.

Applied Hydraulics (1) in the February, 1958 issue suggested that where fluid is discharged rapidly and the pressure drop must be kept within close limits, a polytropic expansion should be assumed. This particular statement does not contribute much to the solution of the problem. The general case where the pressure (P), specific volume (v), and temperature (T) all vary, is commonly called a polytropic process. The general equation for a polytropic process is

$$Pv^n = \text{constant} \qquad (2\text{-}1)$$

in which n is some constant for the particular process under consideration. The isothermal process might be considered as a special case of the polytropic process in which $n = 1$.

Another process that is frequently studied is the adiabatic process. For this case $n = \gamma$, in which

$$\gamma = \frac{\text{specific heat at constant pressure}}{\text{specific heat at constant volume}}$$

Perhaps the author of this article in Applied Hydraulics meant to imply a special case of the polytropic process being used, but no implication was observed.

George R. Keller (9) has presented an approach to selecting an accumulator in aircraft hydraulic designs that is very similar to the approach made by the author in his analysis. Keller states that it is common practice to assume that for dry air or nitrogen, the formulas PV = constant, for an isothermal expansion, and $PV^{1.4}$ = constant, for an isentropic expansion, hold true over the whole temperature and pressure range encountered. He further states that this is not strictly true though, and that considerable error may result from the application of these equations in some portion of these ranges. Therefore, he uses a plot of the actual paths of isothermal and isentropic volume changes of dry nitrogen to select the properties of the gas at the various states during the expansion process. It is very difficult to evaluate his work as presented in this article. He offers a solution to the problem by working with a volume of fluid required for the application which is determined by taking the product of the desired flow rate and time and then using his charts to select the size of accumulator required for the job. It would appear that any and every flow rate and time combination for which the product of the two were equal, would give the same solution to the problem. Therefore, it would be impossible to guarantee which combination of the two variables would be obtained from the accumulator selection based on the product of the flow rate and the time.

CHAPTER III

STATEMENT OF PROBLEM

The purpose of this investigation was to develope and verify experimentally analytical expressions for pneumatic accumulator selection on a power basis. This basis would demand not only a consideration of the available energy in an accumulator, but would also demand that the time required for the dissipation of this energy be investigated.

Although energy is usually stored in a pneumatic accumulator by pumping fluid into it under pressure, actually it is the total internal energy of the compressed gas that fulfills the ultimate power demand. Therefore, the behavior of the accumulator had to be studied from the standpoint of gas thermodynamics.

After the analytical expressions were developed, the necessary experimental testing had to be determined and initiated. Necessary processing of the collected data was then accomplished and the results were evaluated in order that the analytical work might be verified or disproved.

CHAPTER IV

ANALYTICAL DEVELOPEMENT

When a pneumatic accumulator is to be used as an auxiliary power source, the choice of the optimum size requires a detailed understanding of the system flow as a function of time and a knowledge of the high pressure thermodynamics of gases. Previous investigations in this field have been done by considering only the available energy of an accumulator and not considering the time necessary to dissipate this energy. Due to the fact that the time necessary to discharge the fluid from the accumulator is as important as the available energy stored in the accumulator in modern fluid power systems, the following analysis was developed.

In the analysis which follows, four basic assumptions were made.

(1) Isothermal compression of the gas--This is justified since the time period is generally of sufficient duration during the compression process to allow ambient temperature to prevail.

(2) Gas expansion is reversible and adiabatic (Described as isentropic)--The expansion would approach an isentropic process because the discharge of the fluid would be very rapid. (Idealization)

(3) Discharge pressure is invariant.

(4) The hydraulic oil is incompressible. Hydraulic fluid, as

normally used in industry, when raised to 5000 psi will have only about 1.7 per cent volume decrease.

Figure 1 shows the state of the system during the charging and discharging operation of the accumulator. The accumulator is preloaded initially with some compressible gas (usually nitrogen). The gas is then compressed by pumping the incompressible hydraulic oil into the accumulator at some higher pressure than the precharge pressure. This constitutes the charged state. The accumulator now has stored energy in the compressed gas and can be utilized as an auxiliary energy source. After the energy required for the application is dissipated from the accumulator, the final state exists.

Let V_T represent the total volume of the accumulator less the volume of the piston; P_R the gas preload pressure; V_1 and P_1 the volume and pressure, respectively, of the compressed gas after the charging operation; V_2 and P_2 the volume and pressure, respectively, of the expanded gas after the discharging operation; and P_3 the back pressure or load pressure on the accumulator. During the discharging operation, the equation for isentropic expansion is valid and states that

$$P_1V_1^{\gamma} = P_2V_2^{\gamma} = PV^{\gamma} = \text{constant} \tag{4-1}$$

where γ is the ratio of the specific heat capacity of the gas at constant pressure to the specific heat capacity of the gas at constant volume. Differentiation of Equation (4-1) gives

$$V^{\gamma}\,dP + \gamma PV^{\gamma-1}\,dV = 0$$

$$\text{or}\quad dV = -V/\gamma \quad dP/P \tag{4-2}$$

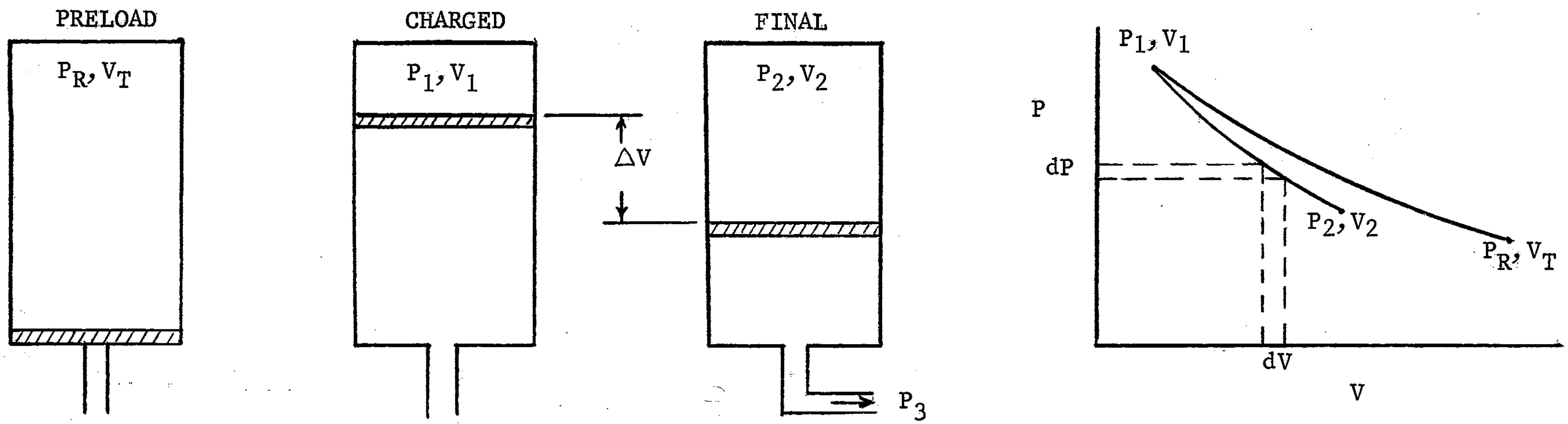

Fig. 1. State of the System

Substituting $V = \left(\frac{P_1}{P}\right)^{\frac{1}{\gamma}} V_1$ from Equation (4-1) gives

$$dV = \frac{-P_1^{\frac{1}{\gamma}} V_1}{\gamma} \quad \frac{dP}{P^{1+\frac{1}{\gamma}}}$$

and then integrating, Equation (4-2) becomes

$$\int_{V_1}^{V_2} dV = \frac{-P_1^{\frac{1}{\gamma}} V_1}{\gamma} \int_{P_1}^{P_2} P^{-1-\frac{1}{\gamma}} \, dP$$

$$V_2 - V_1 = P_1^{\frac{1}{\gamma}} V_1 \left[P_2^{-\frac{1}{\gamma}} - P_1^{-\frac{1}{\gamma}} \right] \qquad (4\text{-}3)$$

During the charging operation, the equation for isothermal compression of the gas is valid and states that

$$PV = \text{constant} \qquad (4\text{-}4)$$

Therefore,

$$P_R V_T = P_1 V_1$$

$$\text{or} \quad V_1 = P_R V_T / P_1 \qquad (4\text{-}5)$$

Equating the change in volume of the gas $(V_2 - V_1)$ to the volume of hydraulic fluid (V_C) displaced from the accumulator during the discharging process and combining Equations (4-5) and (4-3) gives the following form

$$V_C = \frac{P_R V_T}{P_1^{1-\frac{1}{\gamma}}} \left[P_2^{-\frac{1}{\gamma}} - P_1^{-\frac{1}{\gamma}} \right]$$

and hence

$$P_R V_T = \frac{V_C P_1^{1-\frac{1}{\gamma}}}{\left[P_2^{-\frac{1}{\gamma}} - P_1^{-\frac{1}{\gamma}} \right]} \qquad (4\text{-}6)$$

If V_C is substituted in Equation (4-3) for $V_2 - V_1$ and set equal to the product of the flow rate Q and the time of discharge t, then the following expression is obtained

$$Qt = P_1^{\frac{1}{\gamma}} V_1 \left[P_2^{-\frac{1}{\gamma}} - P_1^{-\frac{1}{\gamma}} \right] \tag{4-7}$$

The flow rate Q from the accumulator can be related to the equation which expresses the flow rate through an orifice. The actual discharge rate would then be

$$\text{actual } Q = CA \sqrt{2g \left(\frac{\Delta P}{w} \right)} \tag{4-8}$$

where C = the dimensionless discharge coefficient

A = the orifice area in sq ft

ΔP = the pressure drop across the orifice in lb per sq ft

w = the specific weight of the fluid flowing through the orifice in lb per cu ft and is equal to the specific weight of water (62.4) times the specific gravity of the oil (G)

g = the gravitational acceleration and is equal to 32.2 ft per sec per sec

If Q is expressed in cu in. per sec, A in sq in., w in lb per cu in., Equation (4-8) becomes

$$\text{actual } Q = 146.4 \frac{CA}{\sqrt{G}} \sqrt{\Delta P} \tag{4-9}$$

Equating ΔP to the average pressure upstream of the orifice, $(P_1 + P_2)/2$ minus the load pressure P_3, and combining Equations (4-7) and (4-9) gives

$$t = \frac{P_1^{\frac{1}{\gamma}} V_1 \left[P_2^{-\frac{1}{\gamma}} - P_1^{-\frac{1}{\gamma}} \right] \sqrt{G}}{146.4\ CA\ P_1^{1-\frac{1}{\gamma}} \sqrt{\frac{P_1+P_2}{2} - P_3}} \tag{4-10}$$

Replacing V_1 by $P_R\ V_T/P_1$ as given in Equation (4-5) yields

$$t = \frac{P_R\ V_T \left[P_2^{-\frac{1}{\gamma}} - P_1^{-\frac{1}{\gamma}}\right] \sqrt{G}}{146.4\ CA\ P_1^{1-\frac{1}{\gamma}} \sqrt{\frac{P_1+P_2}{2} - P_3}}$$

$$\text{or} \quad P_R\ V_T = \frac{146.4\ CA\ P_1^{1-\frac{1}{\gamma}} \sqrt{\frac{P_1+P_2}{2} - P_3}\ \ t}{\sqrt{G} \left[P_2^{-\frac{1}{\gamma}} - P_1^{-\frac{1}{\gamma}}\right]} \qquad (4\text{-}11)$$

Equations (4-6) and (4-11) give the expressions necessary for the proper selection of a pneumatic accumulator on a power basis. If these two equations are set equal to each other and simplified, an equation for P_2 would be obtained.

$$P_2 = 2P_3 - P_1 + 9.38 \times 10^{-5}\ G \left[\frac{V_C}{CA\ t}\right]^2 \qquad (4\text{-}12)$$

If Equations (4-6) and (4-12) are solved, the proper accumulator for a specific job can be chosen. The following example illustrates the selection of an accumulator using the Equations (4-6) and (4-12).

Example: It is necessary to select an accumulator for a hydraulic system which requires that 100 cu in. of fluid be discharged in 0.3 sec. A pump is available to charge the accumulator to 2000 psi pressure. The load pressure P_3 is 1000 psi. If the accumulator orifice is an AN tube fitting with an ID of 0.385 in. and the coefficient of discharge is assumed to be 0.80 for the flow conditions, what would be the optimum size of the accumulator? The specific gravity of the hydraulic oil in the system is 0.86 and the γ of the gas is 1.40.

Solution: $A_{orifice} = \pi D^2/4 = 0.785\ (0.385)^2 = 0.1165$ sq in.

$$P_2 = 2\ P_3 - P_1 + 9.38\ (10^{-5})\ G \left[\frac{\ddot{V}_C}{C\ A\ t}\right]^2 \qquad (4\text{-}12)$$

$$P_2 = 2(1000) - 2000 + 9.38(10^{-5})(.86)\left[\frac{100}{.8(.1165)(.3)}\right]^2$$

$$P_2 = 1035 \text{ psi}$$

$$P_R\ V_T = \frac{V_C\ \ P_1^{\,1-\frac{1}{\gamma}}}{\left[P_2^{-\frac{1}{\gamma}} - P_1^{-\frac{1}{\gamma}}\right]} \qquad (4\text{-}6)$$

$$= \frac{100(2000)^{.286}}{\left[1035^{-.714} - 2000^{-.714}\right]} = 335{,}000$$

It is common practice to select P_R less than P_2 by 10 to 20 per cent. If P_R is chosen to be 20 per cent less than P_2 , then it will be

$$P_R = 1035(.80) = 825 \text{ psi}$$

and

$$V_T = P_R\ V_T/P_R = \frac{335{,}000}{825} = 405 \text{ cu in.}$$

This is the necessary accumulator volume or size required to perform the specified task.

CHAPTER V

EXPERIMENTAL VERIFICATION

After the analytical analysis for the selection of a pneumatic accumulator was made, it was necessary to determine and conduct the necessary experimental testing required to support the expressions developed in the analysis. It was decided that the best way to test the equations for their validity was to select a cylinder (V_C = 134 cu in.) with which to apply an effective load pressure on the accumulator, choose the P_R V_T conditions for each test, and then determine P_3 and t by measurement. Figure 2 shows the graphical hydraulic circuit and Fig. 3 is a photograph of the test set-up. An accumulator (V_T = 410 cu in.) was chosen and tests were conducted at preload pressures P_R = 200, 400, 600, and 800 psi on the accumulator. A charge pressure P_1 equal to 2000 psi was selected for all four tests. No load was placed on the cylinder, but a load on the accumulator, P_3, existed due to the friction losses in the line and cylinder. This load pressure was determined by placing a calibrated pressure pickup in the line immediately downstream of the accumulator orifice. The pressure pickup used a metal diaphragm which deflected into a spherical shape when pressure was applied. The deflection of the disc was sensed by a capacitive type element, which sent an electrical signal to a pressuregraph. The pressuregraph transformed the signal into a voltage which was placed across the

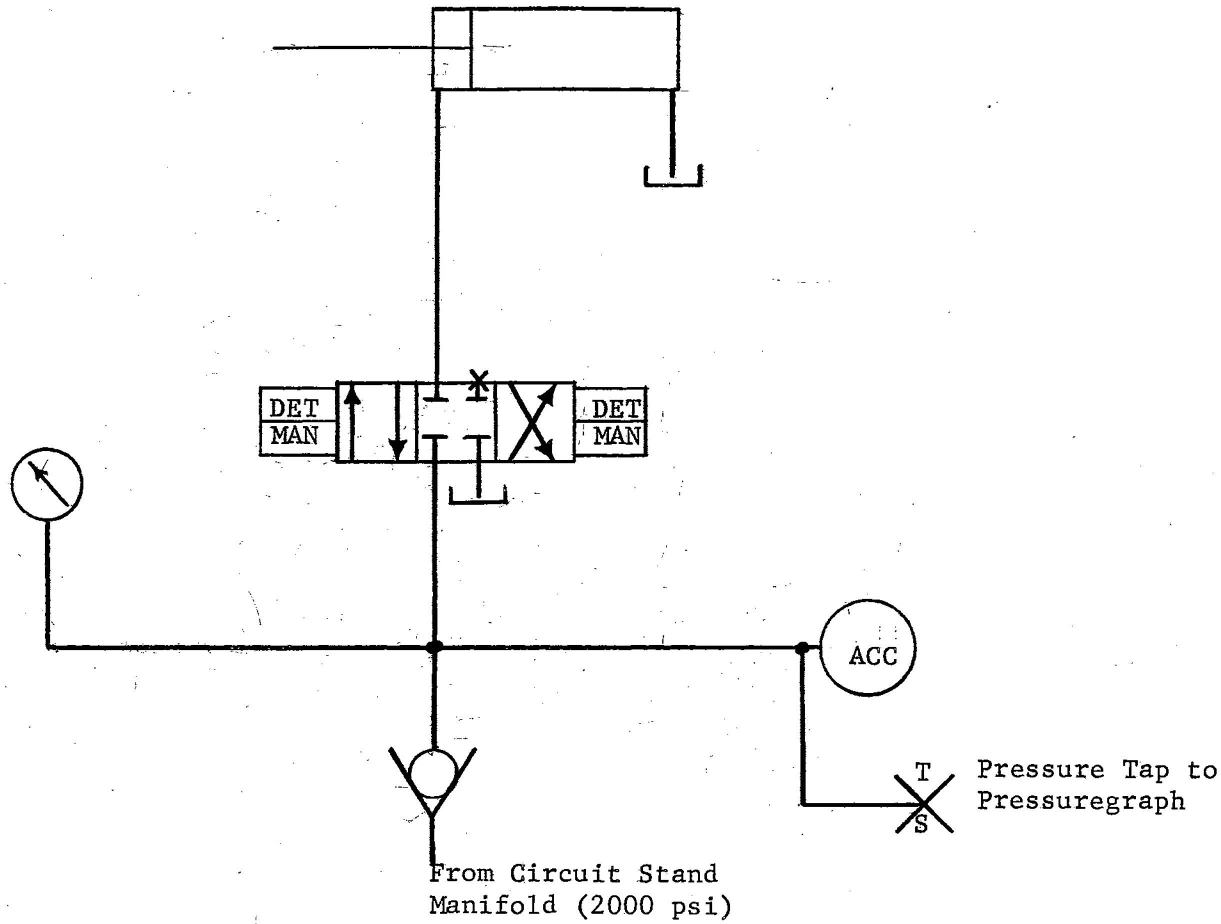

Fig. 2. Hydraulic Circuit for Test Set-Up

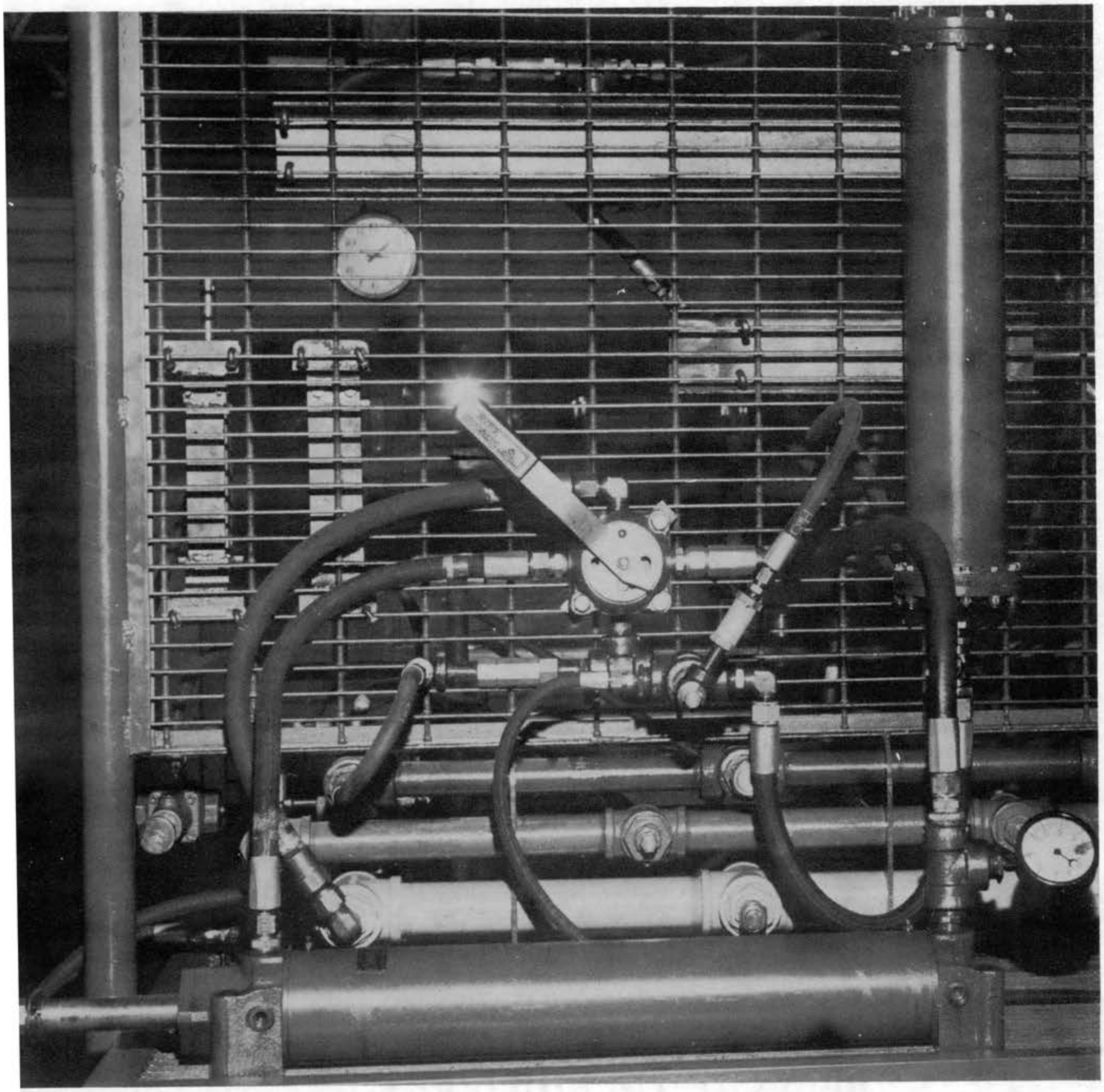

Figure 3. Test Set-up

vertical plates of an oscilloscope to indicate the pressure magnitude. The sweep of the oscilloscope was set at approximately one second by installing a capacitor between the sawtooth and ground connections on the scope. During the testing operation, a single sweep of the pressure beam was obtained by shifting the X-Selector on the oscilloscope to the "Driven" position and providing a triggering mechanism to send the sweep across the screen. The pressure profile was obtained by taking photographs of the single pass. While the sweep was moving across the screen, the shutter of the camera was held open so that the image appearing on the negative would be a continuous curve. The sweep of the beam across the screen was triggered with a limit switch as shown in Fig. 4.

The operational time of the cylinder stroke was measured with an electronic timer which was started with the same triggering mechanism that started the sweep of the pressure beam across the screen of the oscilloscope. The timer was stopped when the piston rod contacted another limit switch at the end of the stroke of the cylinder. See Fig. 4 for the graphical electrical circuit and Fig. 5 for a picture of the instrument arrangement.

The accumulator orifice was calibrated to give the discharge coefficients for various flow rates. (Details of the calibration can be found in Appendix C). A representative coefficient C for the testing conditions was determined to be 0.80. The diameter of the orifice was measured and the area calculated to be 0.1165 sq in.

Tests were run for different precharge pressures and the time and pressure profiles were determined. Figures 6, 7, 8, and 9 show

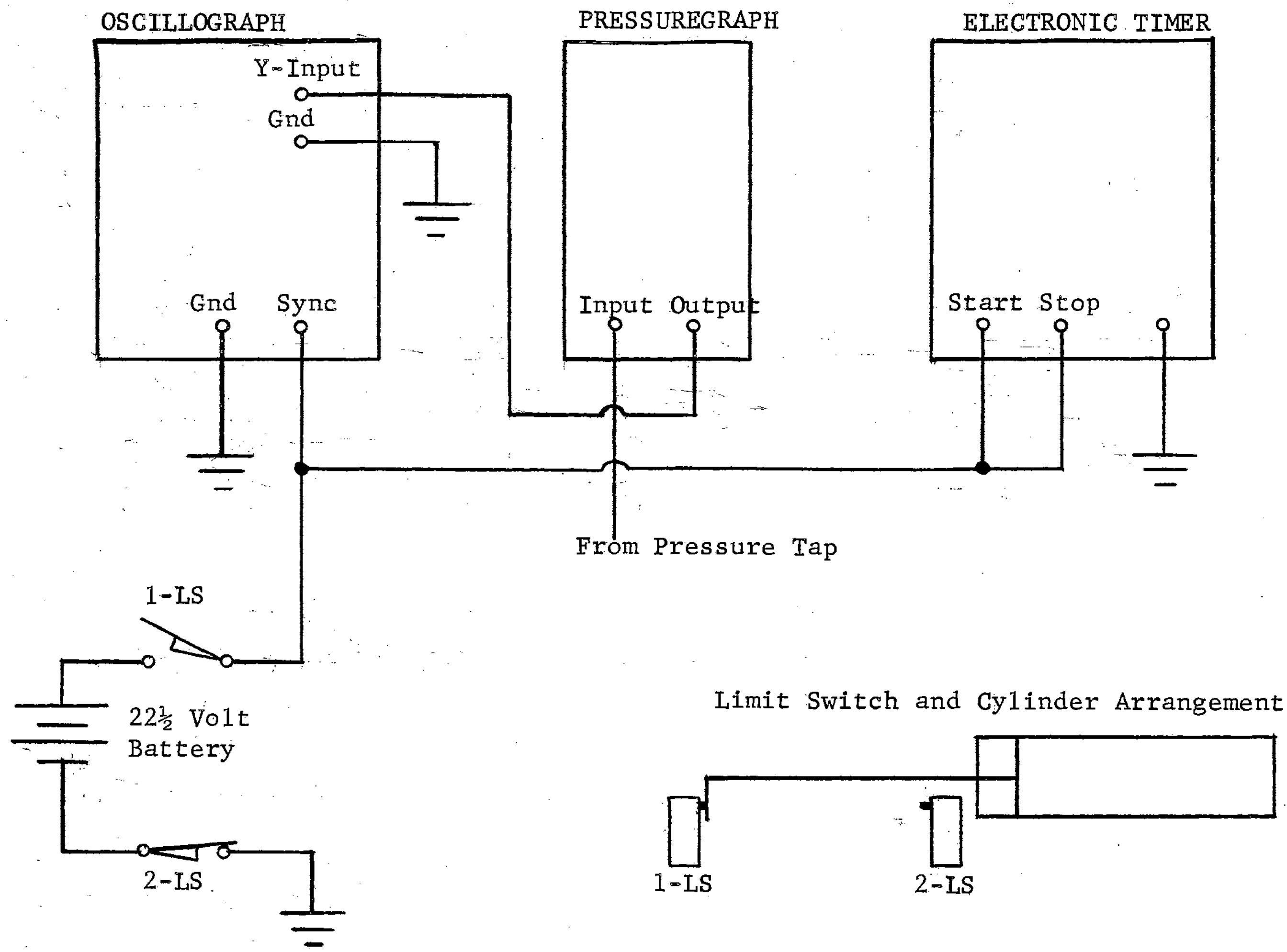

Fig. 4. Electrical Instrumentation Circuit

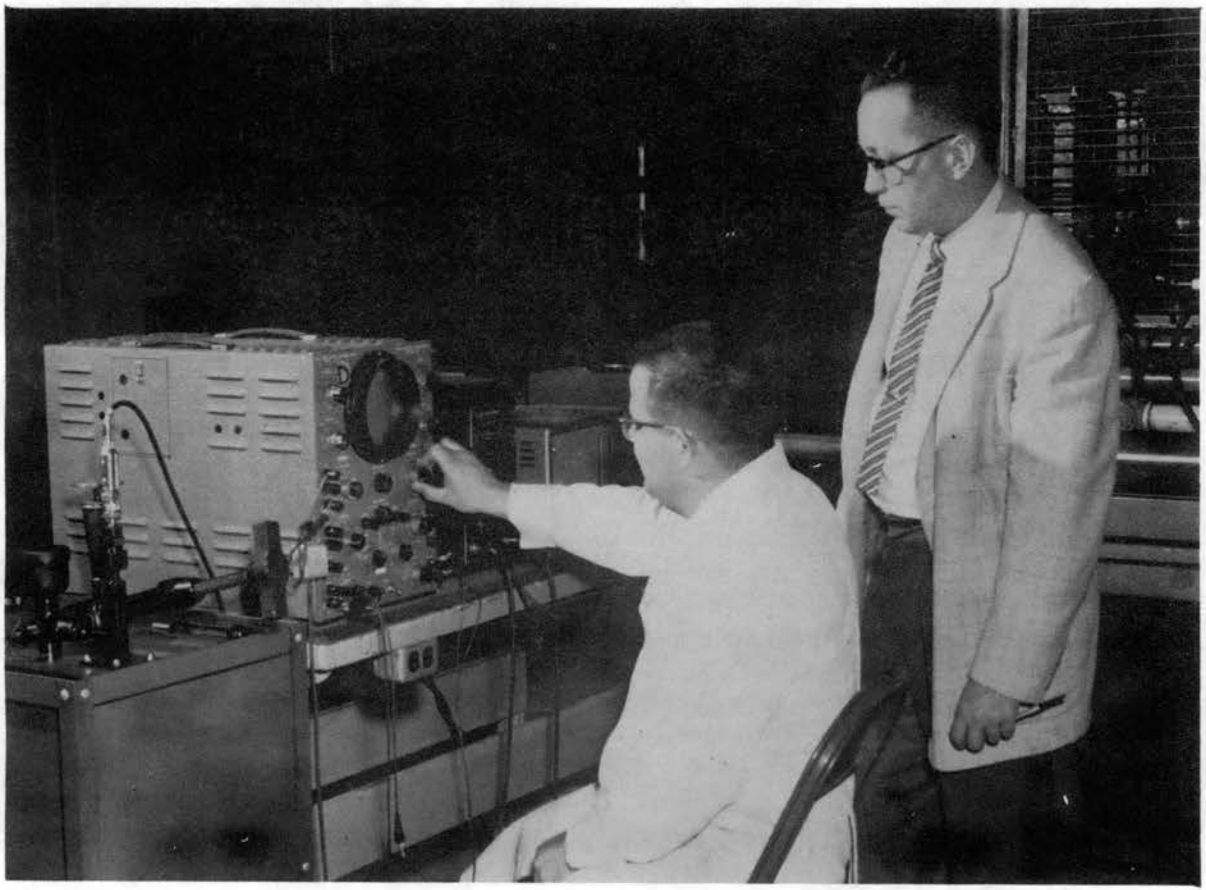

Figure 5. Instrumentation Set-up

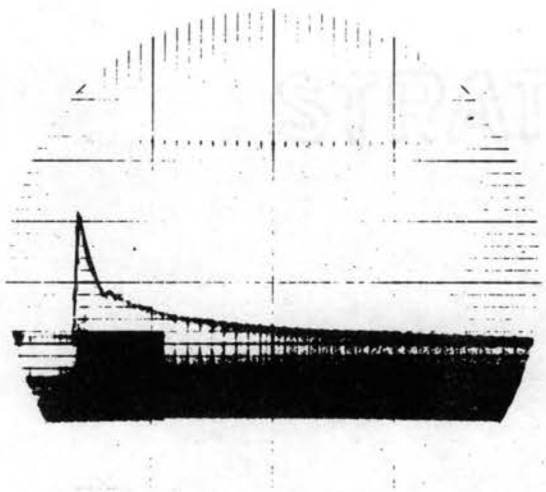

Figure 6. Pressure Curve ($P_R = 200$ psi)

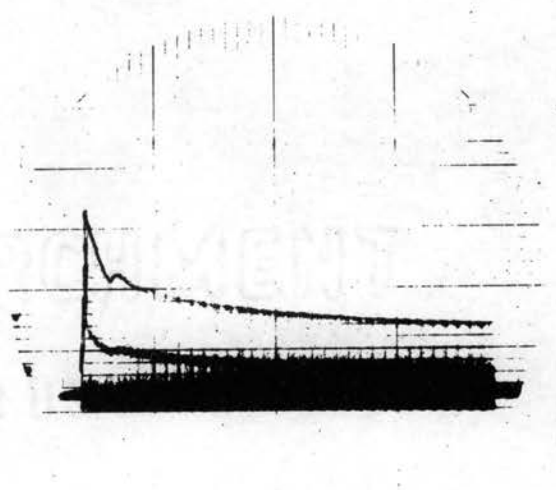

Figure 7. Pressure Curve ($P_R = 400$ psi)

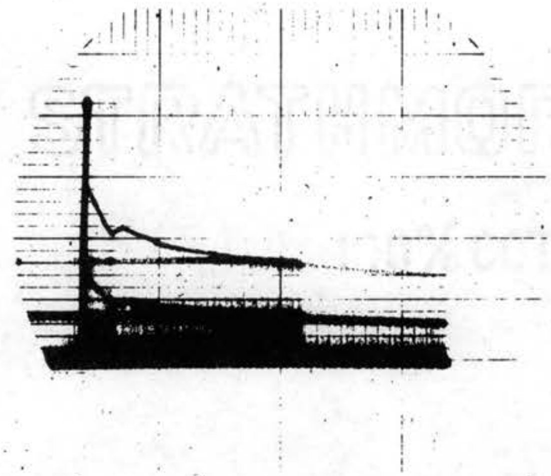

Figure 8. Pressure Curve (P_R = 600 psi)

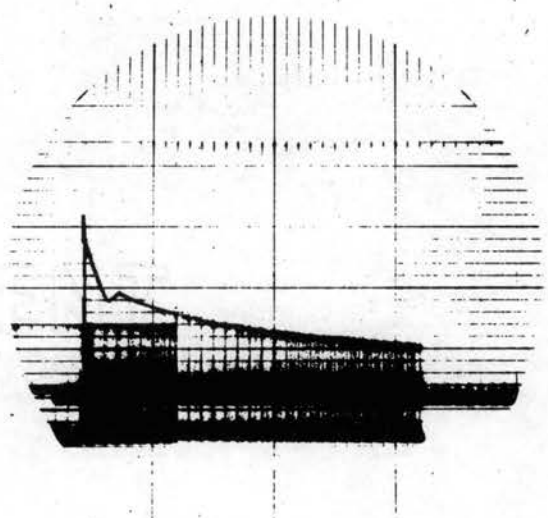

Figure 9. Pressure Curve (P_R = 800 psi)

the pressure curves for P_R = 200, 400, 600, and 800 psi, respectively. In Fig. 6 (P_R = 200 psi), the complete pressure curve was not obtained due to the fact that the time set for the pressure beam to sweep across the screen was of a shorter duration than that required to complete the stroke of the cylinder. The complete pressure-time curve was obtained when P_R exceeded 200 psi as evidenced by Figs. 7, 8, and 9. The superimposed curve on the figures was caused by a new sweep which was started by a pulse received from the limit switch that was opened at the end of the stroke of the cylinder. This pulse stopped the initial sweep and started the second one. The electrical circuit was wired in such a manner that the first sweep would be stopped when the piston rod contacted the limit switch, thereby establishing a correlation between the grid coordinates on the screen and the measured time of the stroke. With this correlation, a pressure-time determination was made.

The pressure-time determination was made from Figs. 6, 7, 8, and 9. By counting the number of vertical grid spaces across the pressure sweep and correlating this number with the measured time of the stroke, the time scale was determined (1 division = 0.0155 sec). With this time scale, the pressure was determined for various time divisions on the grid coordinates. Table I gives the pressure-time values over the entire range of the pressure sweep for the different tests run for each value of P_R. Working Figs. 10, 11, 12, and 13 were plotted from this data.

The discontinuity in each pressure-time curve at the beginning of the sweep was probably due to a shock wave which originated when

TABLE I

PRESSURE-TIME DATA

P_R (psi)	Time Divisions	P_3 (psi)
200	4	1650
200	6.5	940
200	7	1000
200	10	910
200	20	750
200	30	690
200	40	650
200	44.7	620
400	4	1650
400	6.5	1040
400	7	1100
400	10	1000
400	20	825
400	30	750
400	37.5	700
600	4	1650
600	6	1100
600	7	1150
600	10	1050
600	20	850
600	33.7	750
800	4	1700
800	6	1150
800	7	1250
800	10	1120
800	20	900
800	32.2	800

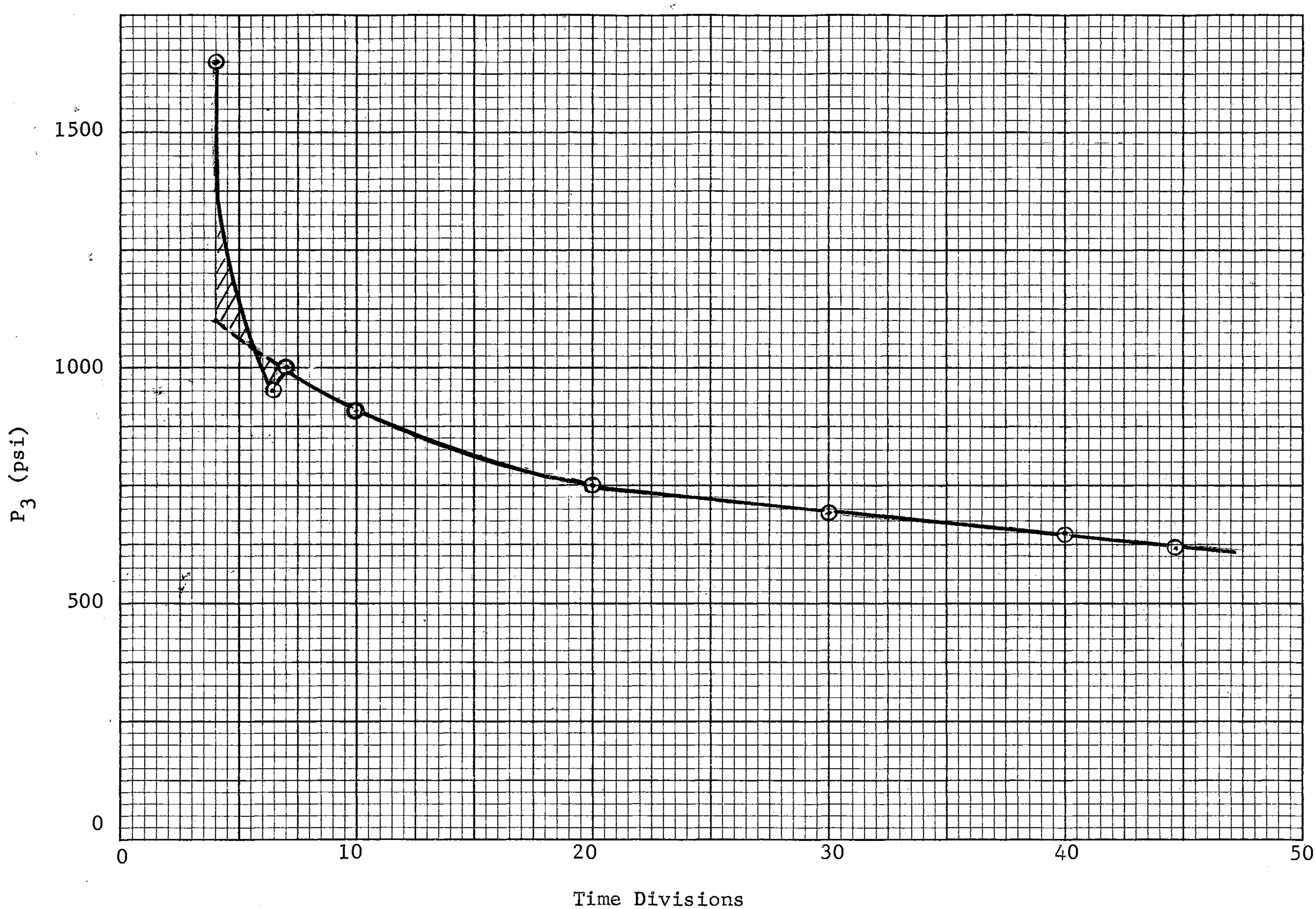

Fig. 10. Pressure Curve (P_R = 200 psi)
(Working)

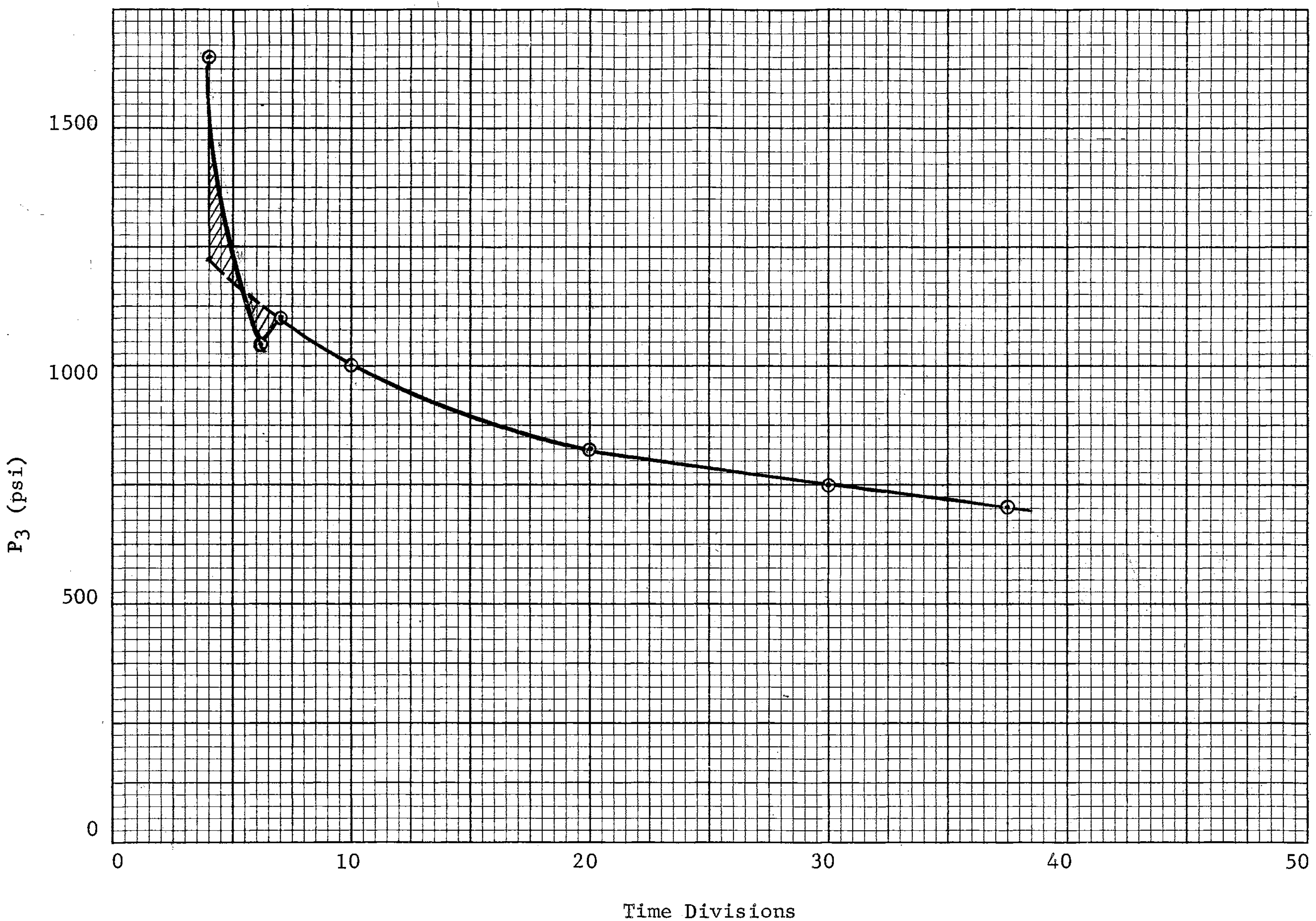

Fig. 11. Pressure Curve (P_R = 400 psi)
(Working)

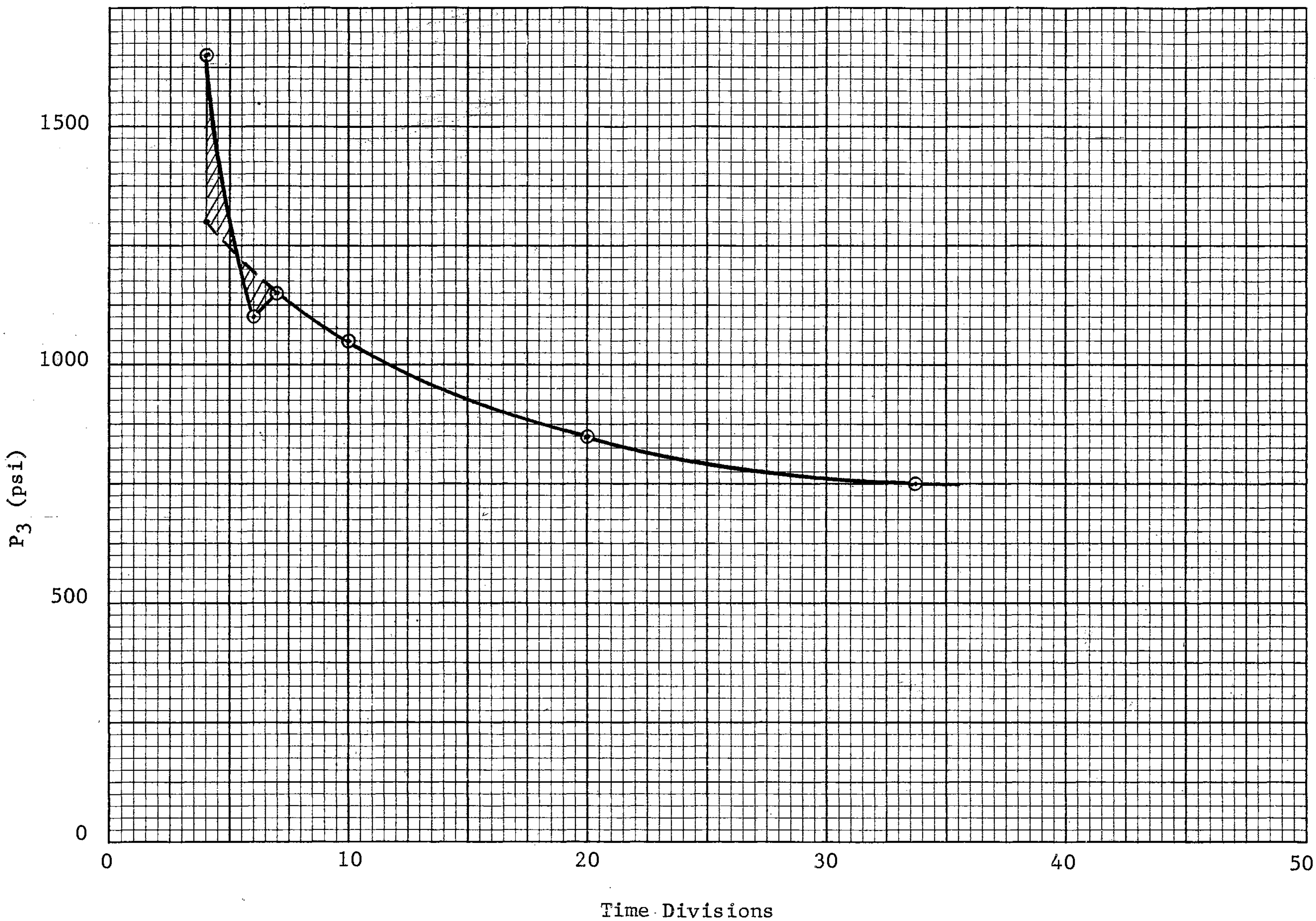

Fig. 12. Pressure Curve (P_R = 600 psi)
(Working)

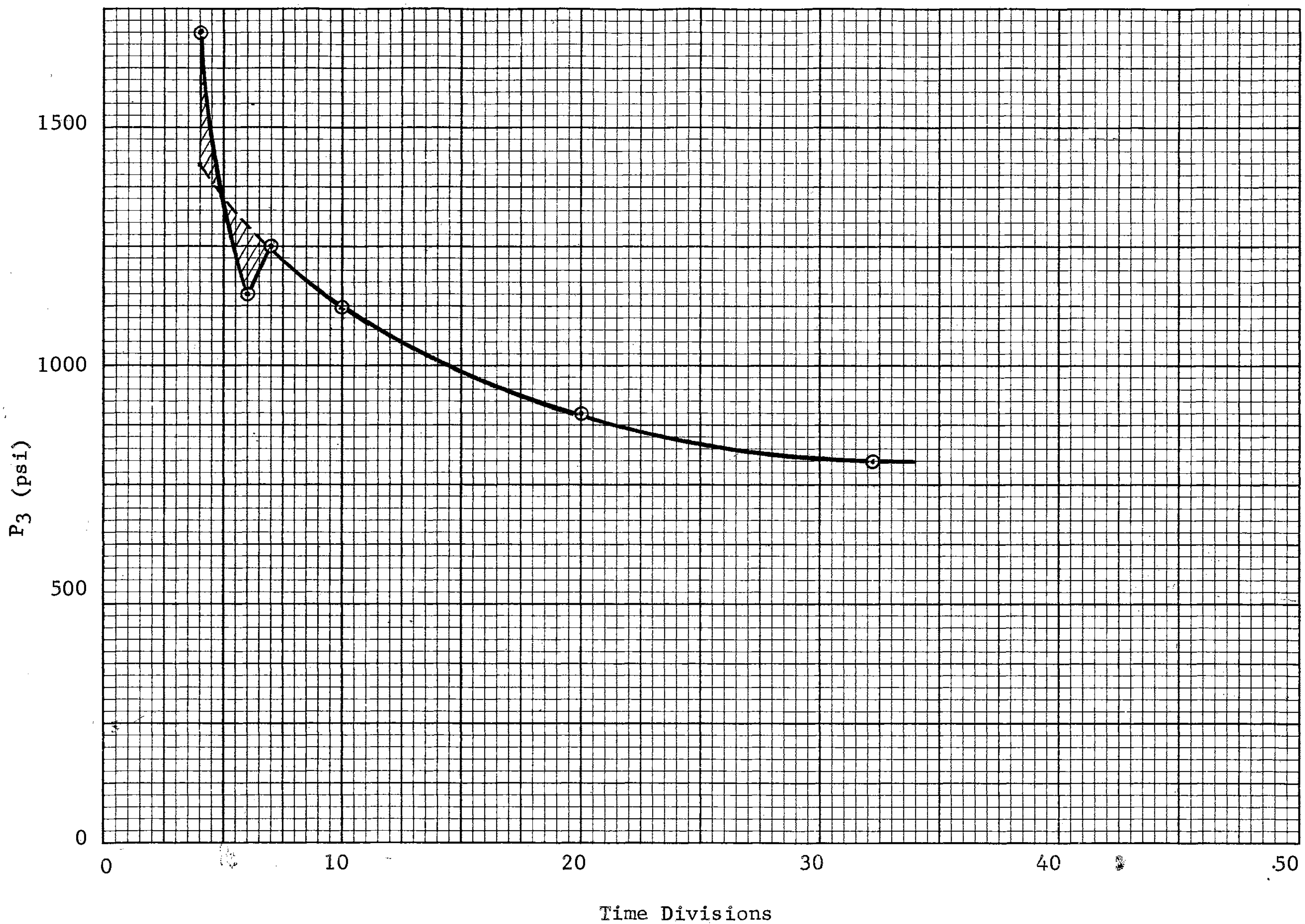

Fig. 13. Pressure Curve (P_R = 800 psi)
(Working)

the first pressure surge hit the piston of the cylinder and then bounced back. No other surges appeared on the curve. If this small discontinuity is neglected and the curve from $t = 4$ to $t = 7$ time divisions is connected with a smooth continuous curve to the rest of the curve, then the area under the original curve that is left out should equal the small increment of area that is added just above the discontinuity without introducing an appreciable error. These two areas in question are shown cross-hatched in the working figures. The pressure-time data from these curves (Table II) were then used to plot Figs. 14, 15, 16, and 17. These plots showed that log t varied linearly with P_3 for every test. Therefore, the log-mean-pressure for each curve was determined and used as the average load pressure during the operation.

Using the average time scale discussed earlier in this chapter (1 division = 0.0155 sec) and the measured time to complete the stroke of the cylinder, the final time was found to be 44.7 time divisions when P_R was equal to 200 psi. Then from Fig. 14 (P_R = 200 psi), the corresponding pressure at the end state was found by extending the curve upward and past the 44.7 time division line. The log-mean-pressure was then determined by applying the following equation to the properties of the end states.

$$LMP = \frac{(P_1 - P_2)}{\ln P_1/P_2} \qquad (5\text{-}1)$$

where LMP = log-mean-pressure. The pressures for all the tests were determined and tabulated as the average load pressures (P_3) in Table III.

Table II

PRESSURE-TIME DATA

P_R (psi)	Time Divisions	P_3 (psi)
200	4	1100
200	7	1000
200	10	910
200	20	750
200	30	690
200	40	650
200	44.7	620
400	4	1225
400	7	1100
400	10	1000
400	20	825
400	30	750
400	37.5	700
600	4	1300
600	7	1150
600	10	1050
600	20	850
600	33.7	750
800	4	1420
800	7	1250
800	10	1120
800	20	900
800	32.2	800

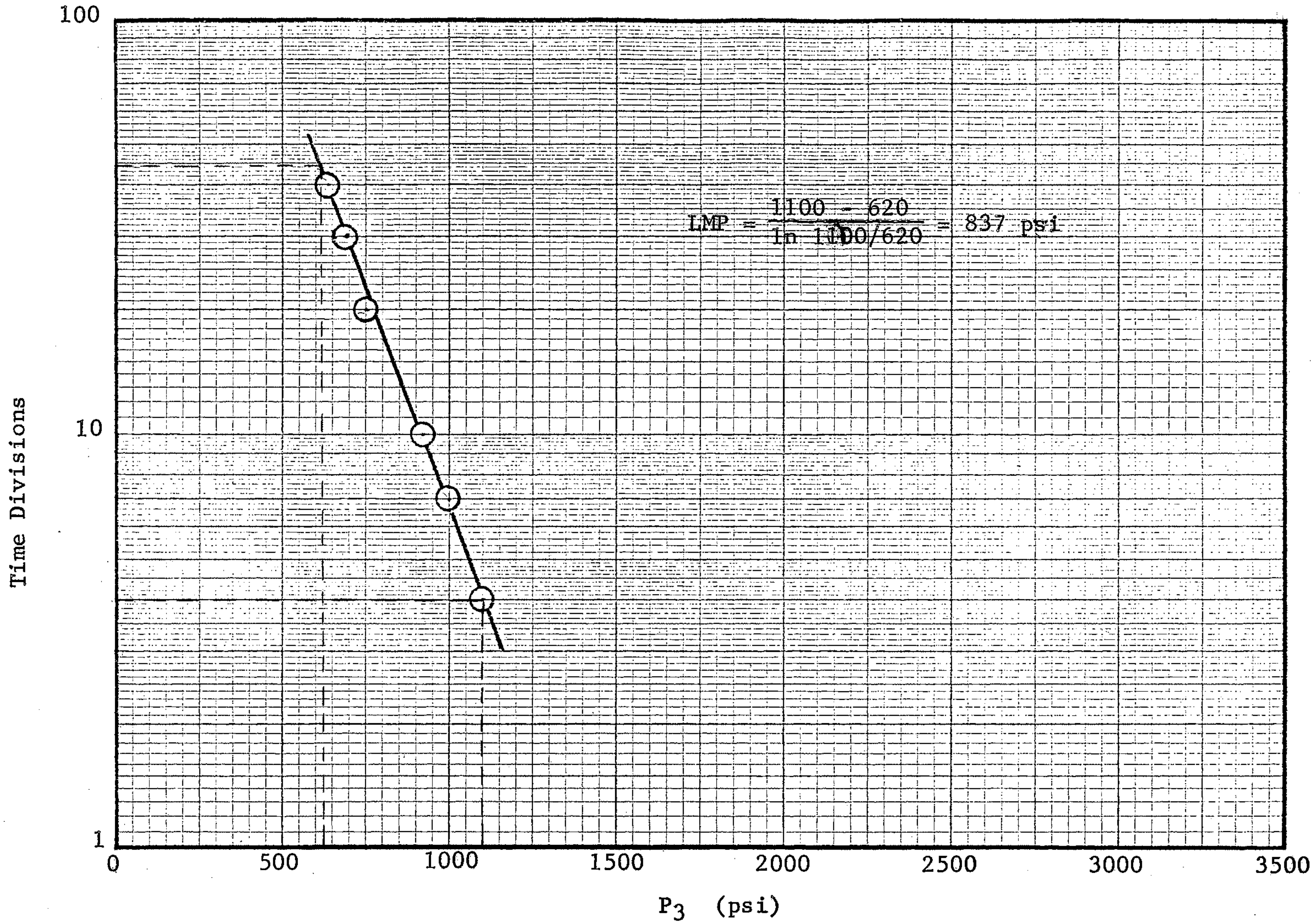

Fig. 14. Pressure Curve (P_R = 200 psi)

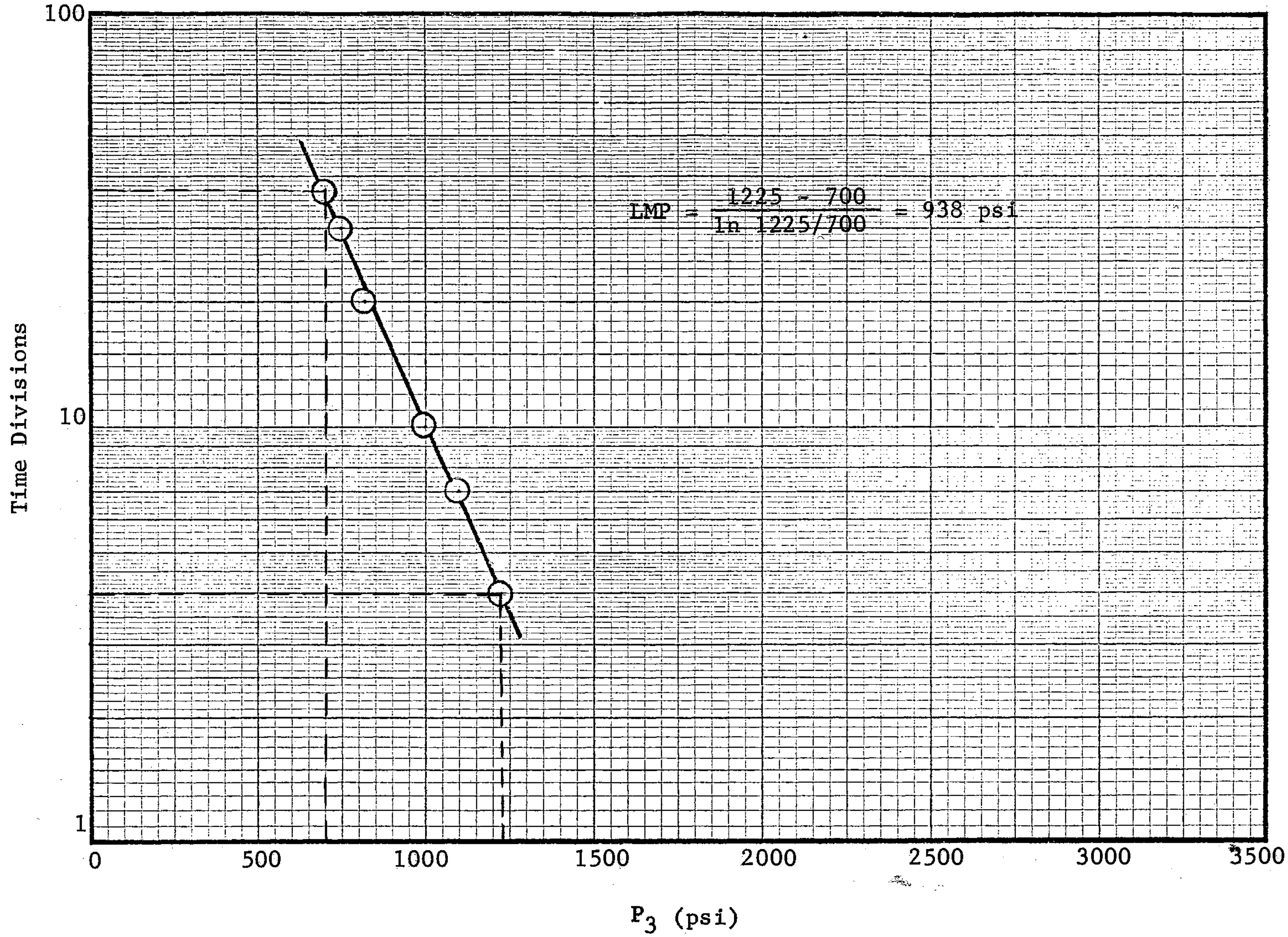

Fig. 15. Pressure Curve (P_R = 400 psi)

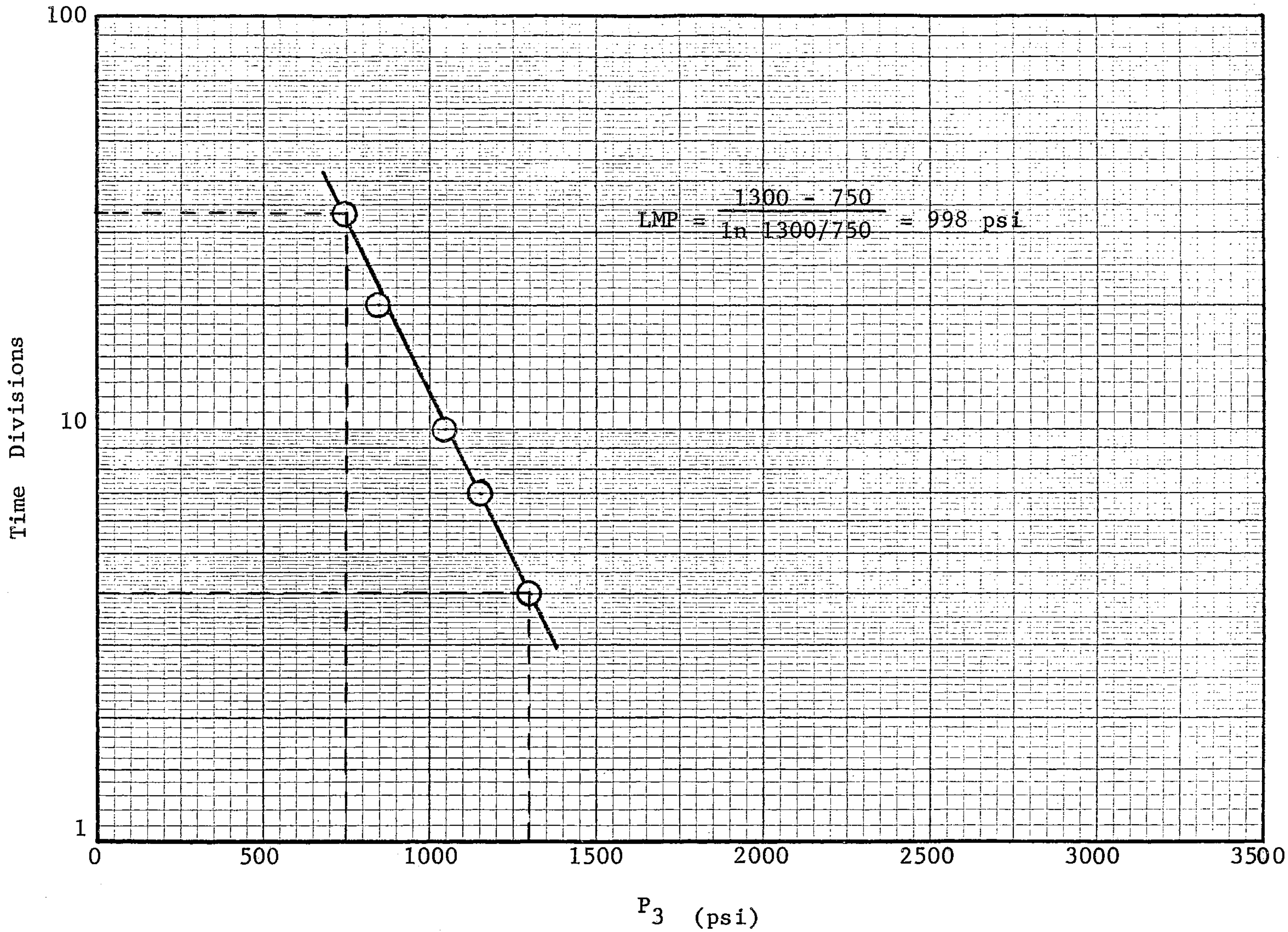

Fig. 16. Pressure Curve (P_R = 600 psi)

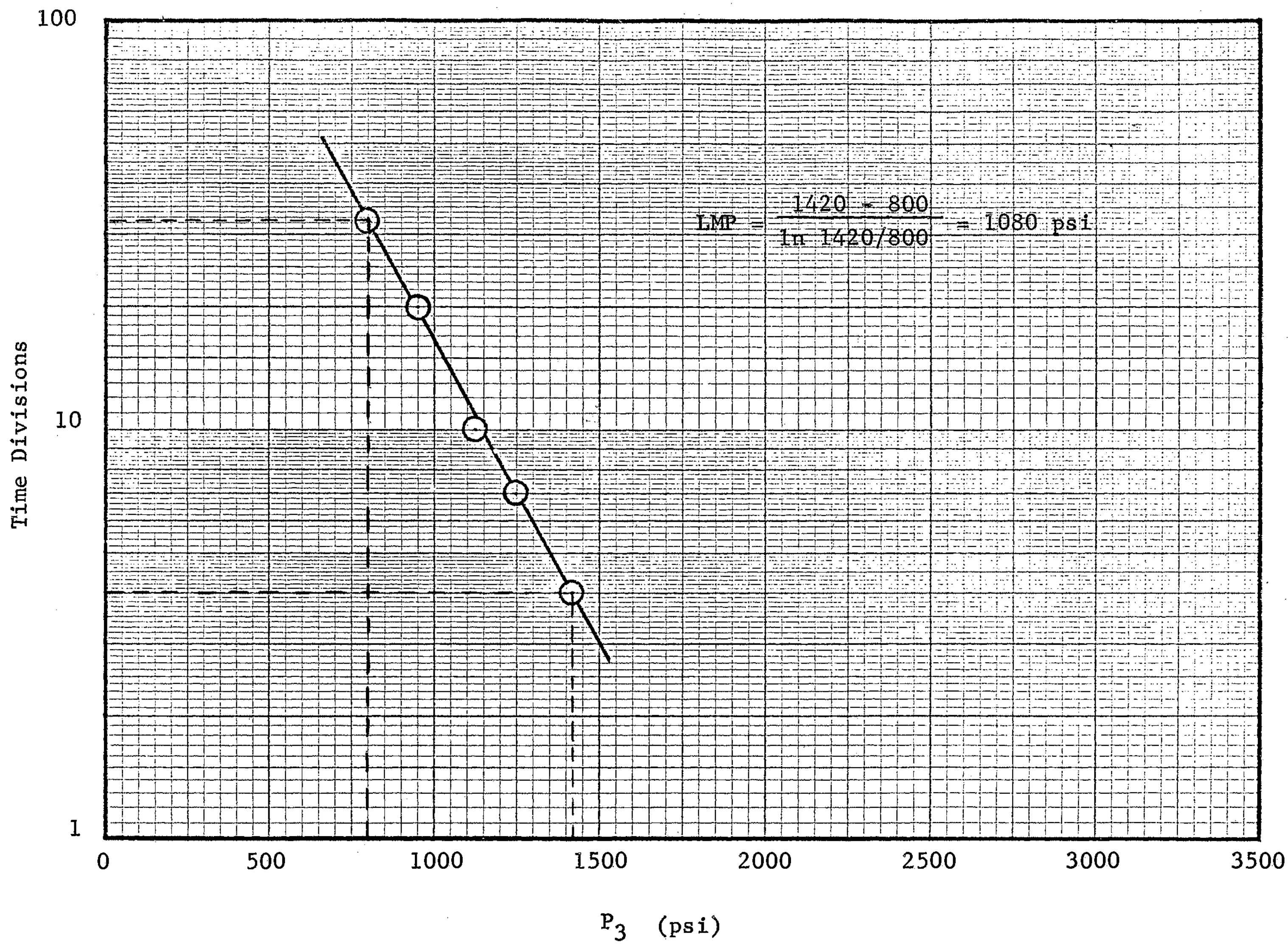

Fig. 17. Pressure Curve (P_R = 800 psi)

TABLE III

PRE-LOAD PRESSURE—AVERAGE P_3 DATA

P_R (psi)	Average P_3 (psi)
200	837
400	938
600	998
800	1080

From Equation (4-6), P_2 was determined for the P_R = 200 psi test

$$P_R\ V_T = \frac{V_C\ P_1^{\,1-\frac{1}{\gamma}}}{\left[P_2^{-\frac{1}{\gamma}} - P_1^{-\frac{1}{\gamma}}\right]} \qquad (4\text{-}6)$$

$$200(410) = \frac{134(2000)^{.286}}{\left[P_2^{-.714} - 2000^{-.714}\right]}$$

$$P_2 = 260 \text{ psi}$$

then applying Equation (4-11)

$$t = \frac{P_R\ V_T\ \left[P_2^{-\frac{1}{\gamma}} - P_1^{-\frac{1}{\gamma}}\right]\sqrt{G}}{146.4\ CA\ P_1^{\,1-\frac{1}{\gamma}}\ \sqrt{\frac{P_1+P_2}{2} - P_3}}$$

$$t = \frac{200(410)\ (.0144)\ \sqrt{.86}}{146.4(.8)(.1165)(8.8)\ \sqrt{1130-837}} = 0.533 \text{ sec}$$

In a similar manner, t was determined for the P_R = 400, 600, and 800 psi tests and the results were tabulated in Table IV. The actual time for each test, determined with the electronic timer, and the per cent error as determined by the equation

$$\% \text{ Error} = \frac{(\text{theoretical } t - \text{actual } t)(100)}{\text{theoretical } t} \qquad (5\text{-}2)$$

are also tabulated in Table IV.

TABLE IV

THEORETICAL-ACTUAL TIME AND % ERROR DATA

P (psi)	Theoretical t (sec)	Actual t (sec)	% Error
200	0.533	0.6929	-26.30
400	0.509	0.5257	- 3.28
600	0.480	0.4545	5.32
800	0.4850	0.4344	10.40

CHAPTER VI

APPLICATION OF ANALYSIS

The application of the analytical work for the selection of a pneumatic accumulator for a specific job would require that the power demand of the system be known (which requires that the amount of oil per cycle which must be supplied to the cylinder of the machine and the time allowed for the discharging process be known). Also, because the oil pressure delivered from an accumulator is not a constant, (it varies from the charged pressure P_1 to the final pressure P_2), the work which the machine performs must be completed in this pressure range. Therefore, the designer must know the minimum oil pressure (P_2) required for his job. It is necessary to determine the specific gravity of the hydraulic fluid to be used in the system and the ratio of the specific heats at constant pressure and volume, respectively, (γ) of the gas used for the precharging operation.

It is required that the type fitting used in the accumulator discharge line be selected and the cross-sectional area and the coefficient of discharge of the fitting be determined. A representative coefficient of discharge can be found by applying flow conditions through the orifice that are required for the specific application, determining the flow rate by some suitable method such as weighing a volume of fluid in a certain length of time, and then applying Equation (4-8).

$$Q = \frac{V}{t} = C\ A\ \sqrt{2g\,\Delta P/w} \qquad (4\text{-}8)$$

An average value of ΔP should be taken as $\left[(P_1 + P_2)/2\right] - P_3$. Therefore, P_3 must be determined.

The load pressure (P_3) on the accumulator or the pressure just downstream of the orifice is equal to the sum of the pressure required to move the effective load on the cylinder and the pressure drop in the hydraulic lines and fittings caused by frictional losses. The effective load on the cylinder must include both the static and inertia forces, which would be known. Therefore, the frictional losses must be determined if P_3 is to be found. One method used widely today (8) is that of expressing the losses in equivalent lengths of pipe and applying the Darcy equation:

$$P_f = \frac{0.0808\ f\ L\ G\ U^2}{D} \qquad (6\text{-}1)$$

where P_f = pressure loss, psi

f = dimensionless friction coefficient

D = pipe diameter, in.

U = velocity of flow, ft per sec

G = specific gravity of the fluid flowing

The equivalent length method expresses individual losses in fittings, etc. in equivalent lengths of straight pipe, having the same loss as the particular fitting. To do this, the following equation is applied:

$$L_e = KD/12\ f \qquad (6\text{-}2)$$

where L_e = equivalent length, ft

K = dimensionless coefficient. See Table V for values of K.

TABLE V

K VALUES (EQUIVALENT LENGTH COEFFICIENT)

Pipe Sections

d1/d2	K
4	0.45
3.5	0.43
3.0	0.42
2.5	0.40
2.0	0.37
1.5	0.28
1.25	0.19
1.1	0.10
1.0	0

Valves

Valve	Diameter Dimension, in.	coefficient K
Globe	1	5.3 (flow against bottom of seat) 4.4 (flow against top of seat)
Angle	1¼	2.1 (flow against bottom) 2.6 (flow against top)
Gate	1½	0.3

Bends and Fittings

Pipe bends for radii between 2½ and 5 pipe dia.	K = 0.3
Screwed elbows up to 2 in.	K = 0.72
Right angle bends	K = 1.20
45° bends	K = 0.263
T's	K = 1.500

The friction coefficient for the corresponding straight length of pipe is determined by Darcy's formula

$$f = 64/R \quad \text{for } R < 2000$$

$$f = 0.316/R^{.25} \quad \text{for } R > 2000 \text{ (Blasius Law)} \qquad (6\text{-}3)$$

where R is the Reynold's number and is obtained by applying the equation

$$R = 7740\ UD/\nu$$

where ν is the kinematic viscosity in centistokes.

To illustrate the computation of friction losses and substitution of equivalent lengths, an example of the calculation of P_3 for the P_R = 800 psi test (outlined in Chapter V) will be given and a comparison made of the calculated P_3 and the actual average P_3 determined with the pressure pick-up during the test run.

Example: P_1 = 2000 psi, t = 0.4344 sec, V_C rod end = 134 cu in., V_C blank end = 174 cu in., ν oil = 32.1 ct., D = 0.625 in., and G oil = 0.86.

Solution: The hydraulic circuit is divided into two sections in order that the pressure line losses might be determined. The first section to be considered is upstream of the cylinder and the second is downstream of the cylinder. The equivalent lengths of pipe will be expressed in straight lengths of pipe with D = 0.625 in. The analysis of losses is made as follows:

(a) Upstream section

Inside diameter = 0.625 in.

V_C = 134 cu in.

t = 0.4344 sec

$U = V_C/t\ A = 134/.4344(.785)(.625)^2(12) = 84$ ft/sec

$R = 7740\ UD/\nu = 7740(84)(.625)/32.1 = 12,680$

$f = 0.316/R^{.25} = 0.316/(12,680)^{.25} = 0.0298$

Losses:

Straight length of pipe	= 5.000 ft
1 in. pipe $L_e = 1(.625/.957)^5$	= 0.112 ft
T's (3 ea.) $L_e = 3(1.5)(.742)/12(.0298)$	= 9.330 ft
Elbows (2 ea.) $L_e = 2(.72)(.742)/12(.0298)$	= 2.980 ft
Right Angle bends in Valve (4 ea.) $L_e = 4(1.20)(.742)/12(.0298)$	= 9.920 ft
L_e total	=27.342 ft

Total loss upstream of cylinder:

$$P_f = \frac{0.0808\ f\ L\ G\ U^2}{D}$$

$$= \frac{0.0808(.0298)(27.34)(.86)(84)^2}{.625} = 640 \text{ psi}$$

(b) Downstream section

Inside diameter = 0.625 in.

$V_C = 174$ cu in.

$t = 0.4344$ sec

$U = 174/.4344(.785)(.625)^2(12) = 109.2$ ft/sec

$R = 7740(109.2)(.625)/32.1 = 16,480$

$f = 0.316/(16,480)^{.25} = 0.0279$

Losses:

Straight length of pipe = 2.00 ft

Total losses downstream of cylinder

$$P_f = \frac{.0808(.0279)(2)(.86)(109.2)^2}{.625} = 75 \text{ psi}$$

(c) Losses due to flow out of cylinder

Assume C (cylinder) = 0.50

D orifice = 0.625 in.

Applying Equation (4-9)

$$\Delta P = \left[\frac{Q\ \sqrt{G}}{146.4\ C\ A}\right]^2 = \left[\frac{174/.4344\ \ \sqrt{.86}}{146.4\ (.50)(.785)(.625)^2}\right]^2 = 275 \text{ psi}$$

The total pressure drop in the system is equal to

P_f (total) = P_f(upstream) + P_f(downstream) + P (cylinder)

= 640 + 75 + 275 = 990 psi

Due to the fact that no external load was applied to the cylinder and the inertia effects of the piston, piston rod, and fluid were assumed negligible

$$P_3 = P_f \text{ (total)} = 990 \text{ psi}$$

This compares very well with the measured average P_3 (1080 psi) obtained during the test.

After all the variables listed above (V_C, t, P_1, γ, P_2, ν, C, A, G, and P_3) are found, the method as outlined in Chapter IV can be followed to determine the size accumulator necessary to do a specific job.

CHAPTER VII

SUMMARY AND CONCLUSIONS

The object of this investigation was to develop and verify experimentally, analytical expressions for pneumatic accumulators on a power basis. The available energy of an accumulator and the time required to dissipate the energy was determined by considering the behavior of the accumulator from the standpoint of gas thermodynamics. By making four basic assumptions

(1) The compression of the gas is an isothermal process. (PV = constant)

(2) The expansion of the gas is an isentropic process. (PV^{γ} = constant)

(3) The discharge pressure is invariant.

(4) The hydraulic oil is incompressible.

the following equations were developed.

$$P_R \, V_T = \frac{V_C \, P_1^{1-\frac{1}{\gamma}}}{\left[P_2^{-\frac{1}{\gamma}} - P_1^{-\frac{1}{\gamma}}\right]} \qquad (4\text{-}6)$$

and

$$P_R \, V_T = \frac{146.4 \; C \; A \; P_1^{1-\frac{1}{\gamma}} \sqrt{\frac{P_1 + P_2}{2} - P_3} \;\; t}{\sqrt{G} \left[P_2^{-\frac{1}{\gamma}} - P_1^{-\frac{1}{\gamma}}\right]} \qquad (4\text{-}11)$$

where P_R = preload pressure, psi

V_T = accumulator size, cu in.

V_C = volume of fluid discharged from the accumulator during the discharging process, cu in.

P_1 = charged pressure on accumulator, psi

P_2 = final pressure on the accumulator after the discharging process, psi

γ = ratio of the specific heats of the gas at constant pressure and constant volume, respectively

C = dimensionless coefficient of discharge of the orifice

A = area of orifice, sq in.

P_3 = effective load pressure on the accumulator, psi

t = time of discharging process, sec

G = specific gravity of the hydraulic oil

A simultaneous solution of Equations (4-6) and (4-11) gives the following expression for P_2.

$$P_2 = 2\ P_3 - P_1 + 9.38(10^{-5})G \left[\frac{V_C}{C\ A\ t}\right]^2 \qquad (4\text{-}12)$$

Calculating P_2 from Equation (4-12) and substituting its value in Equation (4-6) yields the product $P_R\ V_T$. Establishing the preload pressure (P_R) 10 to 20 per cent less than P_2, gives the necessary accumulator volume required to discharge the volume V_C in time t.

A method for calculating P_3 was given in which the losses were expressed in equivalent lengths of pipe and obtained by applying Darcy's Equation.

$$P_f = \frac{0.0808\ f\ L\ G\ U^2}{D} \qquad (6\text{-}1)$$

where P_f = pressure loss, psi

f = dimensionless friction coefficient

D = pipe diameter, in.

U = velocity of flow, ft per sec

The equivalent length method expresses the individual losses in fittings, etc. in terms of equivalent lengths of straight pipe having the same loss as the particular fitting. This was done by applying the equation

$$L_e = \frac{K\ D}{12\ f} \tag{6-2}$$

where L_e = equivalent length, ft

K = dimensionless coefficient

The calculated value of P_3 must be less than the average upstream pressure $(P_1 + P_2)/2$ in order that the minimum pressure requirements on the system be met for the entire discharging process.

It was found that the experimental results supported the analytical analysis with a reasonable degree of accuracy (as shown in Table V) except for the P_R = 200 psi test. Here a 26.30 per cent error was noted between the theoretical time calculated by applying Equations (4-6) and (4-11) and the actual time as measured by an electronic timer. This indicates that assumption number two in Chapter IV, which assumes an isentropic discharging process, is limited to those applications where the value of P_R is equal to at least 25 per cent of the maximum working pressure. For P_R values much less than this, the gas expansion process is not rapid enough to support the assumption that an isentropic expansion exists. The 10.40 per cent error noted in the P_R = 800 psi test requires an explanation. Theoretically, according to the assumption that an isentropic process exists when a rapid expansion takes place, the error should be less than those determined for the P_R = 600 psi and P_R = 400 psi tests. Two or three reasons might explain these results.

First, the time measurements are more critical for the more rapid discharging process and the prevailing instrument errors would be more noticeable for this process. The time lag in the actuation of the electronic timer by a pulse being received from the limit switches would be more critical for this process. Other errors might be more appreciable for this test because of the existence of a higher initial pressure surge or due to the fact that inertia forces would be greater for the case of the higher rate of flow.

The only solution to the problem heretofore has been a "fit and miss" proposition with the actual components. Selection of hardware and conditions on experience alone would result in initial errors of several hundred per cent in many applications.

Considering the fact that the experimental results in this investigation compare very favorably with the theoretical values obtained from the application of the analytical expressions developed herein, it is felt that the analysis is a satisfactory solution to the design problem.

CHAPTER VIII

RECOMMENDATIONS FOR FUTURE STUDY

It is the belief of the author that the analytical approach of selecting a pneumatic accumulator on a power basis is sound and worthy of further consideration. Perhaps the weakest part of the analysis lies in the determination of the effective load pressure on the accumulator. I would recommend that an analysis be made to determine the effect of inertia forces on the effective load pressure. The analysis should consider the inertia forces on both the moving fluid and the moving piston. A study of the initial pressure surge and the effects that it might have on P_3 might also prove to be valuable.

It might also be worthwhile to consider the gas expansion process as a polytropic one and pick a value of n greater than one but less than γ. Perhaps a different value of n for different flow conditions should be used. This investigation might prove to be difficult in that a lot of testing would be required to determine the values of n that would give the best results.

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APPENDIX A

APPARATUS AND EQUIPMENT

Hydraulic Circuit

1. Cylinder: Manufacturer, Logansport Machine Co.; model no. 11020-3-24; size, 3 in. bore and 24 in. stroke; type, non-cushioned; maximum operating pressure, 1500 psi; port size, 3/4 in.
2. Accumulator: Manufacturer, Boeing Aircraft Co.; model no. AOS - 522; capacity, 410 cu in.; type, hydro-pneumatic, piston; port size, 3/8 in.
3. Check Valve: Manufacturer, unknown; model no. AN 6249-10; maximum operating pressure, 3000 psi; port size, 1/2 in. male tube connection.
4. Check Valve: Manufacturer, James-Pond-Clark; model no. 247A; maximum operating pressure, 3000 psi; port size, 3/8 in. female tube connection.
5. Limit Switch; Manufacturer, Microswitch Corp.; catalog no. WZE7RQ9 TN; capacity, 10 amperes at 125 volts.
6. Rotary Four-Way Valve: Manufacturer, Barksdale; Type, manually operated, block centered, detent; maximum operating pressure, 3000 psi; port size, 3/4 in.

Instruments

1. Oscillograph: Manufacturer, Allen B. DuMont Laboratories, Inc.; model, dual-beam cathode-ray; type, 322; serial no. 9X78.
2. Pressure Pickup: Manufacturer, Commercial Research Laboratory; model, Cox quartz pressure element; type 3; serial no. 1328.
3. Dead Weight Tester: Manufacturer, Ashcroft Gauge Division of Manning, Maxwell and Moore, Inc.; type no. 1313A.
4. Universal Counter and Timer: Manufacturer, Berkeley; model 5500 C.
5. Electro Pressuregraph: Manufacturer, Electro Products Laboratories; model 3700 A.
6. Pressure Gauge: Manufacturer, Ashcroft; pressure range, 0 to 3000 psi.
7. Pressure Gauge: Manufacturer, Ashcroft; pressure range, 0 to 5000 psi.

APPENDIX B

FLUID POWER TEST STAND FACILITIES

A portable hydraulic power unit Model X-003 was designed and fabricated in order to provide a main fluid power source at regulated temperature, pressure, and flow to the manifold on a circuit stand (Model X-004). The power unit is an electric-motor-driven machine designed to provide a universal all-purpose unit incorporating in one housing all hydraulic, mechanical, and electrical components together with the instruments and controls necessary for testing many hydraulic circuits. Figure 18 is a graphical circuit of the hydraulic power system. All components of the system are housed within the cabinet assembly as shown in Figure 19.

An operator's manual providing the necessary operating and maintenance instructions for the power unit was written and is available for use with the machine. Reference should be made to the operator's manual for a further discussion of the component parts of the power unit and the necessary calibration curves.

The circuit stand modified for use in conjunction with the power unit was originally designed and constructed by W. R. Matthews under Research Project Number 152, Oklahoma State University. This stand provided a support for the circuit under investigation, a fluid manifold system for connection with the hydraulic circuit, a drip pan for receiving leakage fluids, and a storage area for tools, equipment, and instruments. Figure 3 shows the modified circuit stand.

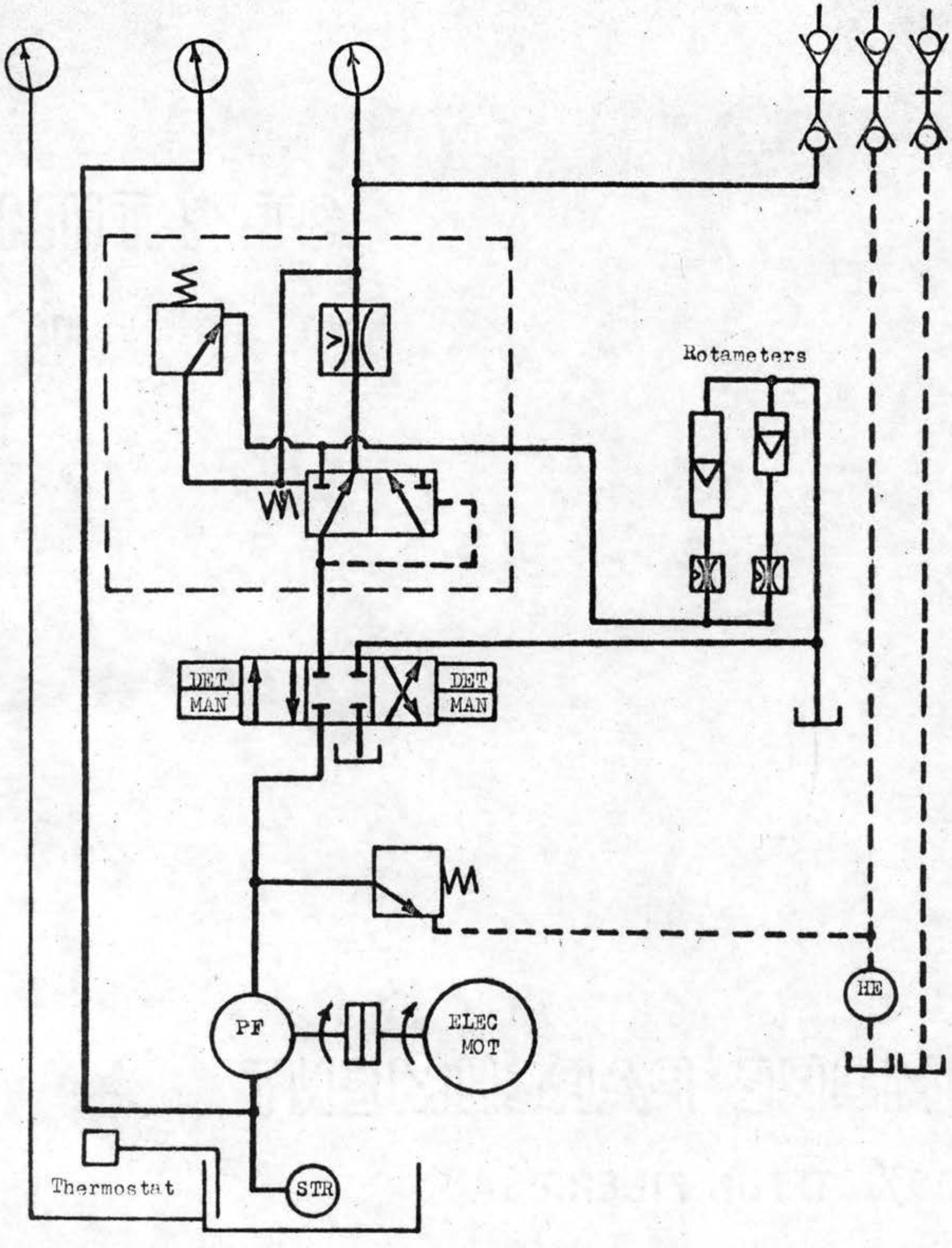

Fig. 18. Graphical Circuit of the Hydraulic Power System

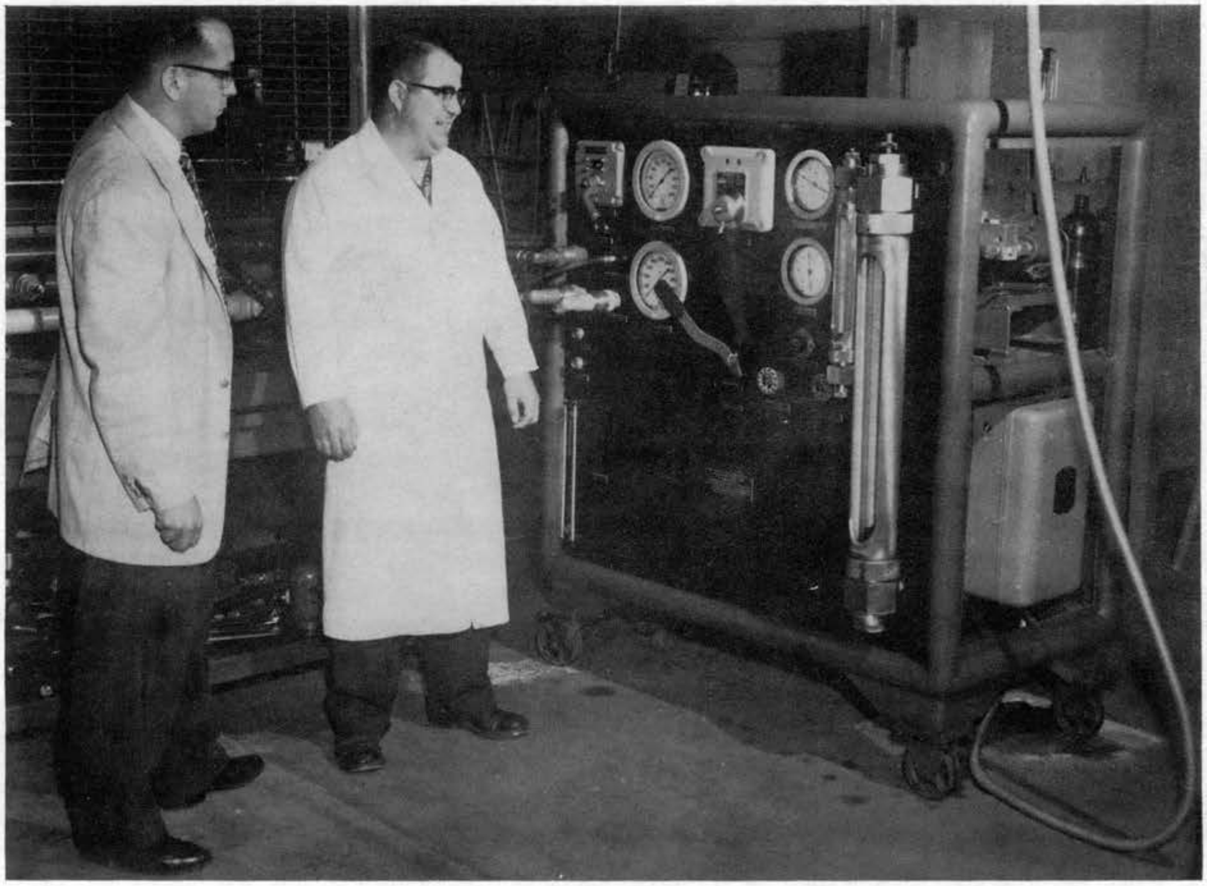

Figure 19. Power Unit Cabinet Assembly

APPENDIX C

ACCUMULATOR ORIFICE CALIBRATION

The coefficient of discharge was determined by measuring the flow rate and the pressure drop across the orifice for various flow conditions, calculating the area of the orifice, and applying the equation

$$Q = 146.4 \frac{C\,A}{\sqrt{G}} \sqrt{\Delta P} \qquad (4\text{-}9)$$

or

$$C = \frac{Q \sqrt{G}}{146.4\, A \sqrt{\Delta P}}$$

The piston was taken out of the accumulator for the calibration tests and a regulated flow from the power unit was furnished to the accumulator. The accumulator orifice was a size 8, AN tube fitting.

The flow rate Q was determined with a Stabl-Vis Rotameter (No. B-35625) which had been calibrated by the balanced beam weighing method. The rotameter was connected in the discharge line just downstream of the orifice.

The pressure drop across the orifice was obtained with a Mercury Manometer. One side of the manometer was connected to a static pressure tube that was inserted into the accumulator just upstream of the orifice. The other side was connected to a pressure tap just downstream of the orifice. The flow rate was regulated to give different pressure drops for the various calibration runs.

Data from the calibration runs are listed in Table VI. Figure 20 shows the calibration curve.

TABLE VI

ORIFICE CALIBRATION DATA

P (psi)	Q (gpm)	C
2	5.00	.742
4	7.30	.767
6	9.27	.795
8	11.00	.816
10	12.47	.830
12	13.66	.828
14	14.70	.828

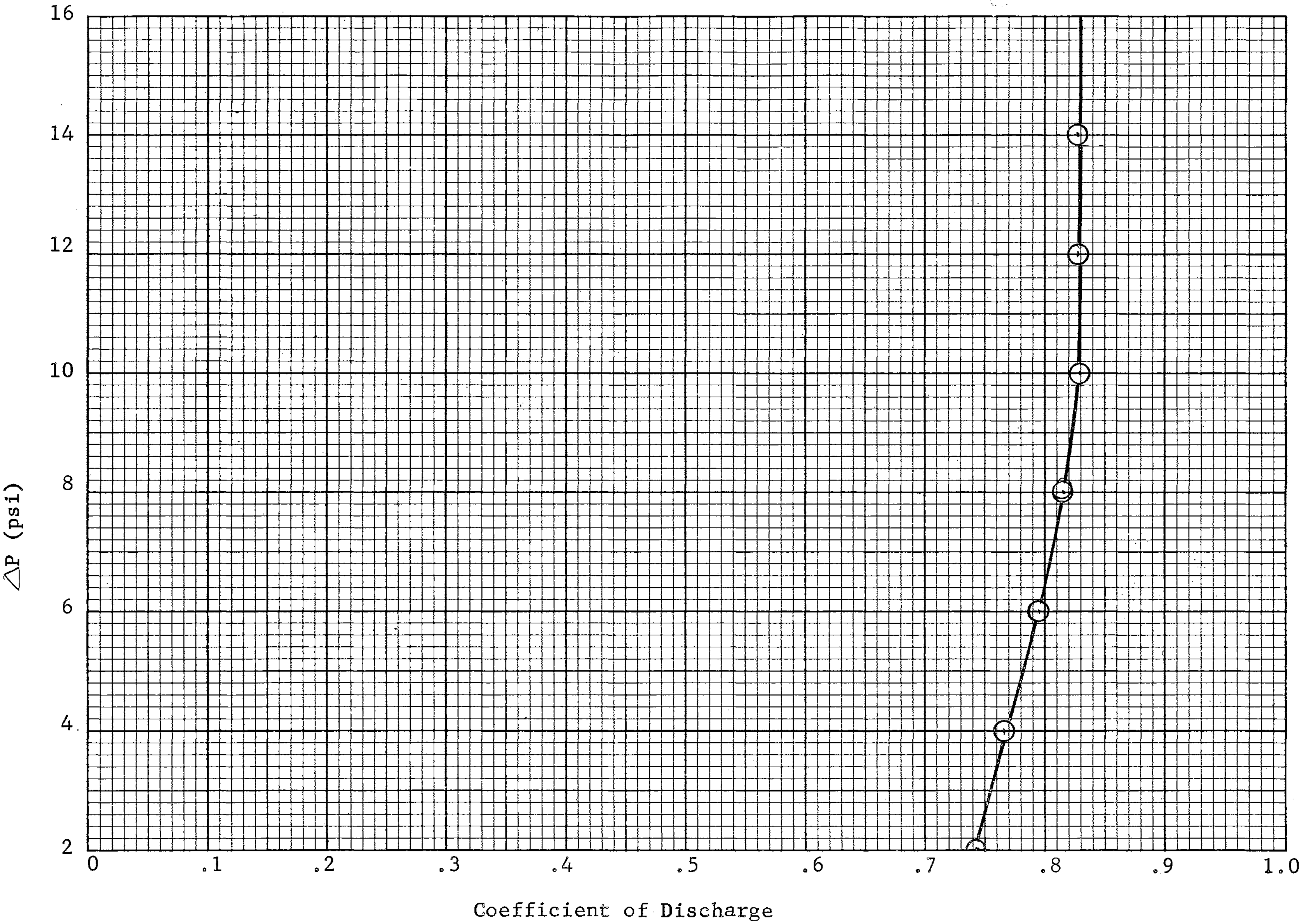

Fig. 21. Orifice Calibration Curve

VITA

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Experience: Employed by the Tidewater Oil Co. from June, 1950, to December, 1952, as a pipe fitter; employed by the Sinclair Oil and Gas Co. for the summer of 1956, as a roustabout; employed by Kerr and McGee Oil, Inc. for the summer of 1957 and the summer of 1958, as an oilfield relief pumper.

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