

PRODUCTION AND INVENTORY CONTROL
IN A PORTLAND CEMENT PLANT.

By

JOE MARION BOHANNON

"

Bachelor of Science

Oklahoma State University

Stillwater, Oklahoma

1958

Submitted to the Faculty of the Graduate School of
the Oklahoma State University
in partial fulfillment of the requirements
for the degree of
MASTER OF SCIENCE
May, 1959

NOV 18 1959

PRODUCTION AND INVENTORY CONTROL
IN A PORTLAND CEMENT PLANT

Thesis Approved:


Charles E. Thompson

Thesis Adviser

W. J. Bentley

James K. Anderson

Dean of the Graduate School

430716

PREFACE

The area of inventory and production control plays a very important role in the activity of today's industry. There are many books that have been published on this subject and also considerable research has been performed in the area of economic lot size and production control. These resources are usually written in a general nature to cover a large segment of interest. It is very difficult to find specific information on any given product in the field of inventory and production control. The purpose of this thesis is to present an approach and solution to an inventory and production control case study in the portland cement industry.

Much of the information and data used in this research comes from a particular cement plant, but the techniques used can be applied to other cement plants and also to companies producing other types of products.

I wish to express my indebtedness to the Ideal Cement Company, whose research fellowship at Oklahoma State University helped to make this study possible. Many people have helped in the writing of this paper by giving their views and opinions concerning the portland cement industry and inventory theory. I would like to thank the following men, who are officials of the Ideal Cement Company, for allowing me their valuable time during this research: Mr. John W. Hand, Director of Business Research; Mr. D. O. Howe, plant manager,

TABLE OF CONTENTS

Chapter	Page
I. INTRODUCTION	1
Statement of the Problem	2
Scope of the Study	3
II. THE DEVELOPMENT OF PORTLAND CEMENT AND THE PORTLAND CEMENT INDUSTRY	4
Discovery of Portland Cement	4
Development of the Portland Cement Industry	5
III. ORGANIZATION OF PLANT PRODUCTION AND STORAGE FACILITIES	7
IV. SALES PREDICTION ANALYSIS	11
Multiple Linear Regression Procedure	12
Economic Indicator Description and Usage	12
V. SCHEDULING OF GRINDING FACILITIES	19
Development of the Economic Run Equation for the Grinding Facilities	20
Determination of Values for Use in the Economic Run Equation	21
Least Squares Analysis of Sales	22
Inventory Carrying Charges	23
Type I Cement	24
Type IA Cement	25
Type III Cement	25
Masonry Cement	27
VI. SCHEDULING AND INVENTORY ANALYSIS OF CLINKER PRODUCTION	29
Production Alternatives	29
Derivation of the Production Cost Equation	30
Monthly Sales Prediction	33
Levels of Confidence	35
Sales Determination	37
Transportation Solution of Clinker Production	37
Production Analysis	39

TABLE OF CONTENTS (CONTINUED)

Chapter	Page
VII. SUMMARY AND CONCLUSIONS	41
SELECTED BIBLIOGRAPHY	43
APPENDICES	45
Appendix A	46
Appendix B	48
Appendix C	50
Appendix D	51
Appendix E	53

LIST OF TABLES

Table	Page
I. Production and Storage Statistics	9
II. Economic Indicators Used in Sales Prediction	13
III. Regression Constants and Multiple R Values of Linear Regression Analysis	16
IV. Results of Multiple Regression Analysis for Predicted Monthly Sales	36
V. Calculation of Safety Stock for Four Levels of Confidence	54
VI. Analysis for Production - 90% Level of Confidence . . .	55
VII. Analysis of Transportation Solution - 90% Level of Confidence	57
VIII. Analysis for Production - 95% Level of Confidence . . .	58
IX. Analysis of Transportation Solution - 95% Level of Confidence	60
X. Analysis for Production - 97.5% Level of Confidence . . .	61
XI. Analysis of Transportation Solution - 97.5% Level of Confidence	63
XII. Analysis for Production - 99% Level of Confidence . . .	64
XIII. Analysis of Transportation Solution - 99% Level of Confidence	66

LIST OF FIGURES

Figure	Page
1. Manufacture of Portland Cement - Isometric Flow Chart	8
2. Comparison of Past Industry Production and Plant Sales	17
3. Graphical Test for Inventory Overflow	34
4. Transportation Solution of Production - 90% Level of Confidence	56
5. Transportation Solution of Production - 95% Level of Confidence	59
6. Transportation Solution of Production - 97.5% Level of Confidence	62
7. Transportation Solution of Production - 99% Level of Confidence	65

CHAPTER I

INTRODUCTION

"How much should we produce this month?" This could be a typical question asked by the operating officials of a manufacturing concern. Perhaps many would try to answer this question on the basis of their past experience; others might make an educated guess. There are many ways in which this question could be evaluated and answered.

Today's management is not only faced with problems such as this, but many others of a similar nature. How much are we going to sell? Should we build up our inventories or deplete them this month? These problems are not independent of one another. Inventory depends on sales, and production depends on future demands. The basic problem of inventory policy is to strike a balance between operating savings and the cost and capital requirements associated with larger stocks.¹ As each business grows it becomes more complex, and as the officials become specialized in their jobs, it becomes increasingly difficult to solve the problem of economic balance. The problem of planning, scheduling, and controlling inventories and production is a universal one. It involves the control of internal operations of producing goods in respect to outside demands and constraints. The objective of sound production planning, scheduling, and control of inventories is

¹John F. Magee, Production Planning and Inventory Control, (New York, 1958) p. 2.

to minimize frictions of internal-external relationships or to adapt them to the advantage of the company.² Management has a wide range of techniques available to solve these problems. Some methods of solution range from simple graphical solutions to complicated operations research studies which involve higher mathematics.

Statement of the Problem

This thesis presents a method which the management of a local cement plant could use as an aid in production planning and inventory control. In a study of this type the complete operation must be analyzed before an approach to the problem can be made. The author has visited the Ideal cement plant in Ada, Oklahoma, several times and also its Divisional Sales offices located in Oklahoma City, Oklahoma. The solution has been developed in such a way as to apply to any industry. It is felt that by substituting data and using their own information, others may be able to use the ideas developed in this paper.

The first step in this study was to make an analysis of the present plant operations and sales program. Cement production can be broken down into two main processes: up through the clinker stage and the finish grinding operation. The problems involved in these two stages are quite different, yet they are closely related to each other. Each of these stages has been treated as a separate problem and analyzed. A brief history of portland cement and the cement industry is included along with a short description and layout of the present operations.

²Ibid. p. 3.

Scope of the Study

The information required for this study involved the sales figures for the last five years of the Ada plant, the description and capacity of the present production facilities, and familiarity with the operating policies of the company.

Each part of the company's management looks at its problems from its own viewpoint. Top management is concerned with the overall efficiency and operation of the company. The production officials are interested in making the highest quality product at the cheapest cost. The sales personnel are concerned with having enough stock on hand to meet the demand and the actual sales of the products. Each of these groups evaluates its problems in light of its own needs and requirements. A basis is needed by which to weigh and judge the various ideas and alternatives. This procedure will enable the officials to judge these alternatives on a mutual basis. The basis used in this research is the economic cost.

The area of sales prediction is covered in this study because it is felt that there should be some mathematical means other than experience by which one can predict sales and judge their accuracy. After sales are estimated, the problem can then be reduced to an inventory and production control problem. How much of the various types of cement should be produced, and on what facilities? Cement sales are seasonal, and this fact requires some special considerations. The final results are an evaluation of the cost of different production plans to meet desired levels of confidence.

CHAPTER II

THE DEVELOPMENT OF PORTLAND CEMENT AND THE PORTLAND CEMENT INDUSTRY

Portland cement occupies a major role in the modern civilization of today. It is now regarded as indispensable in highways, sidewalks, bridges and dams, in the construction of virtually all large buildings, and in airport runways, harbors, and a multitude of other major and minor projects. The American Society of Testing Materials' definition of portland cement is as follows:¹

Portland cement is the product obtained by pulverizing clinker consisting essentially of hydraulic calcium silicates to which no addition has been made subsequent to calcination other than water and/or untreated calcium sulphate.

Discovery of Portland Cement

The earliest cementing materials about which we have any historical records are common lime and hydraulic lime. The common lime will not harden under water, while hydraulic lime will. Proof of the use of these materials has been found as far back as the time of Emperor Galius Caesar, 12-41 A. D.² The ancient Greeks and Romans made real advances in the development of cementitious materials.

¹Robert F. Blanks and Henry L. Kennedy, The Technology of Cement and Concrete, (New York, 1958) p. 13.

²Ibid. p. 5.

Portland cement was first discovered and named by Joseph Aspdin, an English stone mason about 1824. It was called portland cement because it resembled the color and texture of a natural stone quarried on the Isle of Portland in the English Channel. Aspdin's method was to carefully proportion limestone and clay, then pulverize them and burn the mixture into clinker, which was then ground into finished cement.

During the years before portland cement was discovered, and some years after, large quantities of natural cement were used. Natural cement was produced by burning a naturally-occurring mixture of lime and clay. The natural cement gave way to portland cement, which is a predictable, known product of consistently high quality.

Development of the Portland Cement Industry

The first portland cement produced in the United States was made by D. Saylor in 1871. He found that by burning a certain rock to a higher temperature, the finished ground clinker made an excellent product.

The portland cement industry ranks first among the producers of heavy chemicals. Roughly speaking, about 625 pounds of raw materials goes into one barrel of cement which weighs 376 pounds. The industry has risen from 300,000 barrels of output per year to over 255 million barrels per year, with a corresponding improvement in the quality of the product. Of the original six plants started in the United States before 1881, only two were successful, those at Wampum and Coplay, Pennsylvania. By 1890, sixteen plants in the United States were

producing 335,000 barrels of cement per year. In 1955, 155 plants were producing 292,634,000 barrels of cement per year.

A main factor in the increase in size of the cement industry was the development of the rotary kiln.³ In the early days vertical kilns were used, and some of these can still be seen in the Lehigh Valley in Pennsylvania. The rotary kiln could produce clinker in greater amounts and was much more efficient to operate. Thomas Edison was a pioneer in the early development of the rotary kiln. In the beginning these kilns spanned from 60 to 150 feet, and today some are longer than 550 feet. Development of grinding and crushing equipment has also influenced the growth of the industry. The production curve of the portland cement industry in the United States since the early 1900's has closely coincided with the volume curve of the construction industry.

A recent development in the cement industry is air-entraining portland cement which contains a small amount of organic material in the form of natural resins or acid fats. These added materials entrap millions of microscopic air bubbles in concrete, thereby increasing its plasticity and resistance to damaging frost action. Normally, portland cement is gray in color because of the iron in the mixture, and some portland cement is even white.

The cement industry has its own research and development group, the Portland Cement Association. This is a national organization whose aim is to improve and extend the uses of portland cement and concrete through scientific research and engineering field work. It was founded in 1916, and the main offices are located in Chicago, Illinois.

³Ibid. p. 15.

CHAPTER III

ORGANIZATION OF PLANT PRODUCTION AND STORAGE FACILITIES

The cement plant used in this study is located in Ada, Oklahoma, and is owned and operated by the Ideal Cement Company, head offices in Denver, Colorado. Any information referring to operating figures such as sales, costs, or production facilities, have been adjusted so as to protect the company. The Ada plant is a wet process plant. A diagram of a typical cement plant is Figure 1. This represents both wet and dry process plants. Adjusted storage and production information is shown in Table I. In the plant used in this case study there are new and old facilities which can be used to produce the finished product. The new plant is much more efficient and has a higher production capacity than the old plant. It is assumed the plants usually operate twenty-four hours a day, seven days a week, twelve months of the year. This is common practice for most cement plants because of the high costs associated with firing up and cooling down the large kilns. The kiln is the heart of the plant, and there are no scheduled shutdowns on this particular line of equipment. There are breakdowns and necessary repairs, but these are handled as they occur.

Raw materials are crushed at the quarry, which is located five miles from the plant, and are transported by motor carrier or rail to the plant. When the materials reach the plant they are blended and

MANUFACTURE OF PORTLAND CEMENT

Isometric Flow Chart

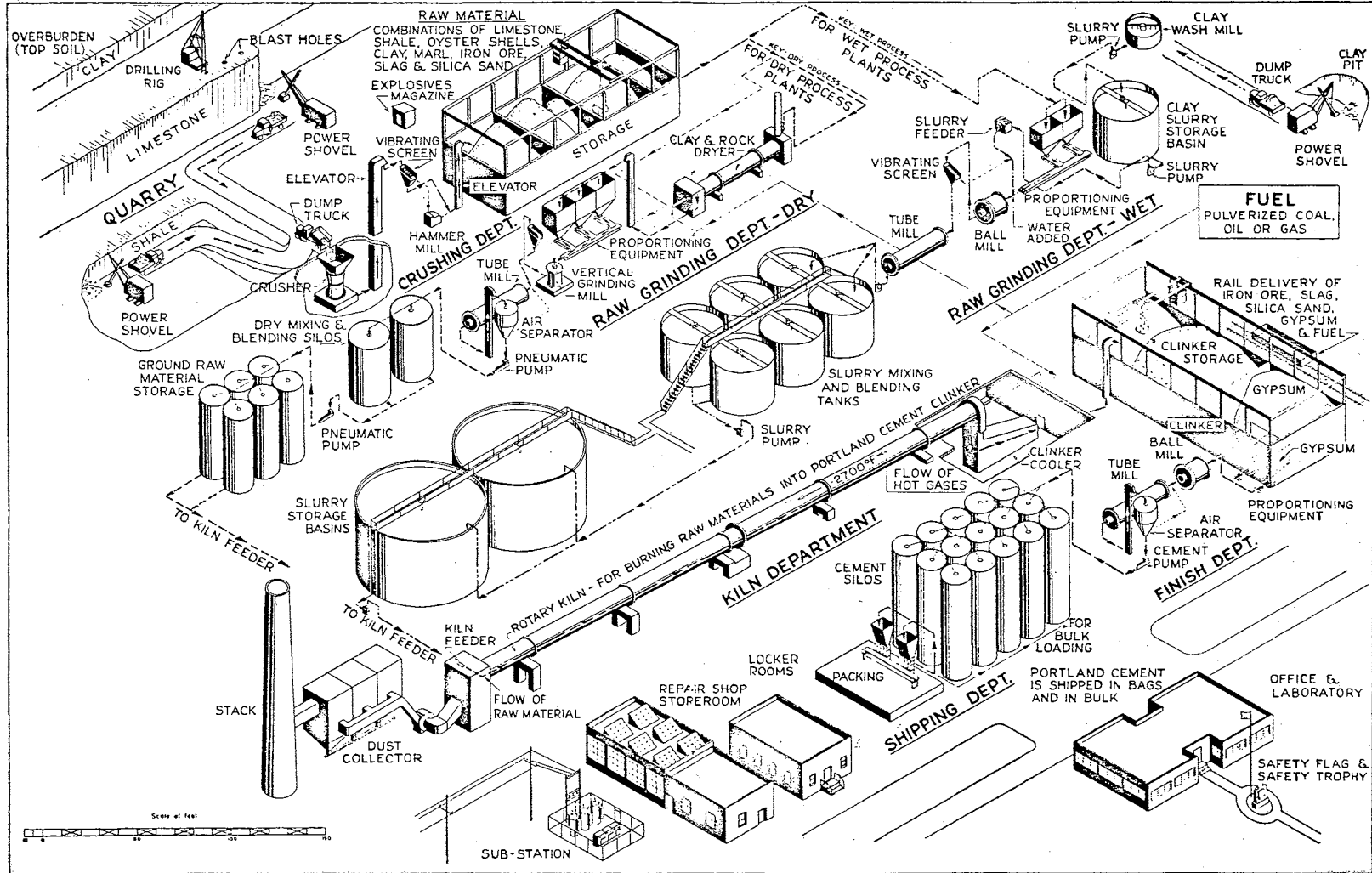


Figure 1.

TABLE I
 PRODUCTION AND STORAGE STATISTICS
 PORTLAND CEMENT PLANT
 ADA, OKLAHOMA

Plant	Facility	Number	Capacity	Units of Measure	Total Capacity
New	Kilns	2	4,500	Barrels per day	9,000
New	Raw Mills	2	5,500	Barrels per day	11,000
New	Finish Mills	2	5,500	Barrels per day	11,000
New	Silos	10	20,000	Barrels	200,000
New	Interstice Silos	4	11,000	Barrels	44,000
Old	Kilns	4	1,900	Barrels per day	7,600
Old	Raw Mills	4	2,100	Barrels per day	8,400
Old	Finish Mills	5	1,900	Barrels per day	9,500
Old	Silos	12	16,000	Barrels	192,000
Old	Interstice Silos	6	3,500	Barrels	21,000

ground together in large tube mills, and at this point a slurry is produced. The slurry is stored in large tanks until it is ready for use in the kilns. In the new plant there are two 450-foot kilns, and in the old, four smaller kilns. The kilns rotate and are built on a slight grade. The slurry is fed in at the high end, and as the kiln rotates, the material flows to the opposite end. Temperatures up to 2700° F. are reached in this stage, and the output of the kiln is called clinker. Clinker resembles fine gravel and is black in color and very hard. After cooling, the clinker is stored in silos until it is ground with other materials to produce the finished cement. The majority of all portland cement is made from the same clinker, and only the materials which go into the grinding process are varied in amounts and types.

After being ground in large tube mills, the finished product is stored in silos until it is shipped to the customer. Cement is sold in both the bulk and bag form. Units are expressed in barrels of cement which weigh 376 pounds. The majority of sales is in bulk form, and shipping is generally by rail. Because of the bulk form, the high weight, and the low market value of the product, the sales area is limited by transportation costs.

CHAPTER IV

SALES PREDICTION ANALYSIS

In any inventory study, the demand must be known, and if it is not, an estimate must be made for the future demand. There are many methods by which one can predict the sales or demand of a product. It is assumed that the term demand means the particular demand which the supplier will have as his part of the total market for the product. The problem is to have enough stock on hand to meet these demands so that the sales are equal to the demand. One way of prediction is straight statistical extrapolation of past information on the particular demand and sales. Another popular method is using economic indicators. Modigliani and Marks have done considerable research in this area, and they have developed some statistical information which gives the accuracy which has been achieved in using these methods.¹ They used three groups of indicators to find how well they forecasted sales of different products and also general business trends. The percentage of absolute error ranged from 3.8% to 13.1% on various products and industries.

It is thought that there are two major sets of factors influencing the level of inventories in any particular industry. One is the existence of seasonal demand. This fact is very important to a manufacturing

¹Franco Modigliani and Bernard J. Marks, "Economic Expectations and Plans of Firms in Relation to Short Term Forecasting" Short Term Economic Forecasting (Princeton, 1955) pp. 261-289.

industry which has marked variations due to seasonal demand. Second, the fact that the firm needs to supply current orders promptly will cause a lower limit to the desired level of inventories.

Multiple Linear Regression Procedure

In order to understand and utilize these two factors, a reasonable estimate of the sales must be known. The type of solution in this case study is the use of selected economic indicators to develop a multiple linear regression equation by which to predict future sales.

In multiple linear regression there can be K independent variables, denoted as X, and one dependent variable, denoted as Y, in the equation

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \dots + \beta_k X_k .$$

The β 's represent unknown regression constants which can be positive or negative, Y represents the sales, the X's represent selected economic indicators for the same periods. The problem is to find estimates of $\beta_0, \beta_1, \dots, \beta_k$ for these unknown parameters. To aid in this solution, because of the large number of independent variables, it was necessary to utilize the IBM 650 digital computer. Appendix B contains the theory of this solution.

Economic Indicator Description and Usage

In the solution to this problem past sales were grouped by months and quarters. In the monthly sales seven indicators were used and for the quarters, six were used.² Table II shows the indicators used and their respective units.

² Economic Indicators, U. S. Government Printing Office, February, 1959, (Washington, 1959), pp. 4-26.

TABLE II
ECONOMIC INDICATORS USED IN
SALES PREDICTION

Indicator	Units	Notation
<u>Monthly Series</u>		
Average Weekly Hours in Building Construction Industry	Hours	X ₁ ¹
Bank Loans, Investments, and Reserves	Billions of Dollars	X ₂ ¹
Employment	Millions of Persons	X ₃ ¹
Housing Starts	Thousands of Homes	X ₄ ¹
New Construction	Billions of Dollars	X ₅ ¹
Production and Business Activity Index (1947-49 = 100)	Index	X ₆ ¹
Selected Stock Prices	Dollars	X ₇ ¹
<u>Quarterly Series</u>		
Corporate Profits	Billions of Dollars	X ₁ ^{II}
Expenditures for New Plant and Equipment	Billions of Dollars	X ₂ ^{II}
Gross National Product	Billions of Dollars	X ₃ ^{II}
Gross Private Domestic Investment	Billions of Dollars	X ₄ ^{II}
National Income	Billions of Dollars	X ₅ ^{II}
Per Capita Disposable Income	Billions of Dollars	X ₆ ^{II}

In solving this problem on the IBM 650 it was necessary to use two different programs: the Beaton Correlation Routine and the Granet Multiple Regression Routine, which was designed by Professor William Granet, Oklahoma State University.

The sales for each period of time, along with the values of the selected economic indicators for the corresponding period, were punched onto data cards. The data cards were placed behind the Beaton Routine and then loaded into the computer. The output of this program consisted of the means, standard deviations, and the correlation matrix, of the input data. This information was then integrated into the Granet Routine and again loaded into the computer. The results of this run were the final answers. These were the regression coefficients: $\beta_0, \beta_1, \dots, \beta_k$, and the multiple regression coefficient R, which is the proportion of the sum of the squares of the dependent variable which are explained by our multiple regression equation. From these values the linear regression equation can be set up as follows:

$$\hat{Y} = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \dots + \beta_k X_k .$$

With known or predicted values of the X's, \hat{Y} can then be estimated.

The results obtained from the Ideal Cement Company sales figures have been forwarded to the company, and the results using the same indicators and the total industry sales for the same periods of time are presented in this thesis. The forwarded results are not presented in this thesis because the information is of a confidential nature. There is a high degree of correlation between these two sets of data.

On the first tests, the monthly and quarterly sales were matched with the indicators for the same period of time. The periods covered

were from 1954 to 1958, inclusive. The results of this test are presented in Table III. If this solution is used to predict sales, one must in turn predict the indicators for the period of time when the estimate of sales is desired. An experiment was made whereby the indicators were shifted back in time, either one month or one quarter. An example would be to use October indicators with November sales and derive the equation based on this data. Then it would be possible to predict December sales with November indicators. This method does away with the necessity of predicting the indicators as mentioned above. The same experiment was performed on the quarterly sales, and the results of these two tests are contained in Table III.

By using the monthly sales with the same monthly indicators a high multiple regression coefficient was obtained, which suggested that this method is a possible solution for sales prediction. Because of the high degree of correlation on the monthly basis, it could be implied that certain indicators predict sales very well for the same period of time. When the indicators are moved back in time, they no longer have the same relationship as in the first case. The goal is to find indicators which will predict sales several periods in advance for use in sales forecasting. This type of indicator could be called a leading indicator and could be very useful in this type of solution. When the months and quarters were shifted back in time, a still lower coefficient resulted, which further bears out this point. Graphs of the past sales of the company (in coded form) and total industry production are presented in Figure 2. The similarity between these two sets of curves should be noted. The national production graph is derived from a monthly index, and the company sales graph is

TABLE III
REGRESSION CONSTANTS AND MULTIPLE R VALUES OF
LINEAR REGRESSION ANALYSIS

	R	β_0	β_1	β_2	β_3	β_4	β_5	β_6	β_7
<u>Monthly Sales</u>									
Indicators and sales on the same basis	.9520	-1956.4012	1.6346	22.3920	1.6873	-.01259	1.9821	.31888	-.04264
Indicators shifted back one period	.8590	-5060.4036	1.3015	5.7725	-34.7371	-3.09190	2.1389	-2.83640	.7287
<u>Quarterly Sales</u>									
Indicators and sales on the same basis	.64487	-3631.3927	-30.7169	39.2654	2.1764	-27.9357	2.6939	-2.8559	--
Indicators shifted back one period	.42596	-5792.0832	-23.8620	38.7283	1.1637	-27.4201	17.6027	-5.0613	--
Monthly Linear Regression Equation	$Y' = \beta_0 + \beta_1 X_1' + \beta_2 X_2' + \beta_3 X_3' + \beta_4 X_4' + \beta_5 X_5' + \beta_6 X_6' + \beta_7 X_7'$								
Quarterly Linear Regression Equation	$Y'' = \beta_0 + \beta_1 X_1'' + \beta_2 X_2'' + \beta_3 X_3'' + \beta_4 X_4'' + \beta_5 X_5'' + \beta_6 X_6''$								

COMPANY SALES

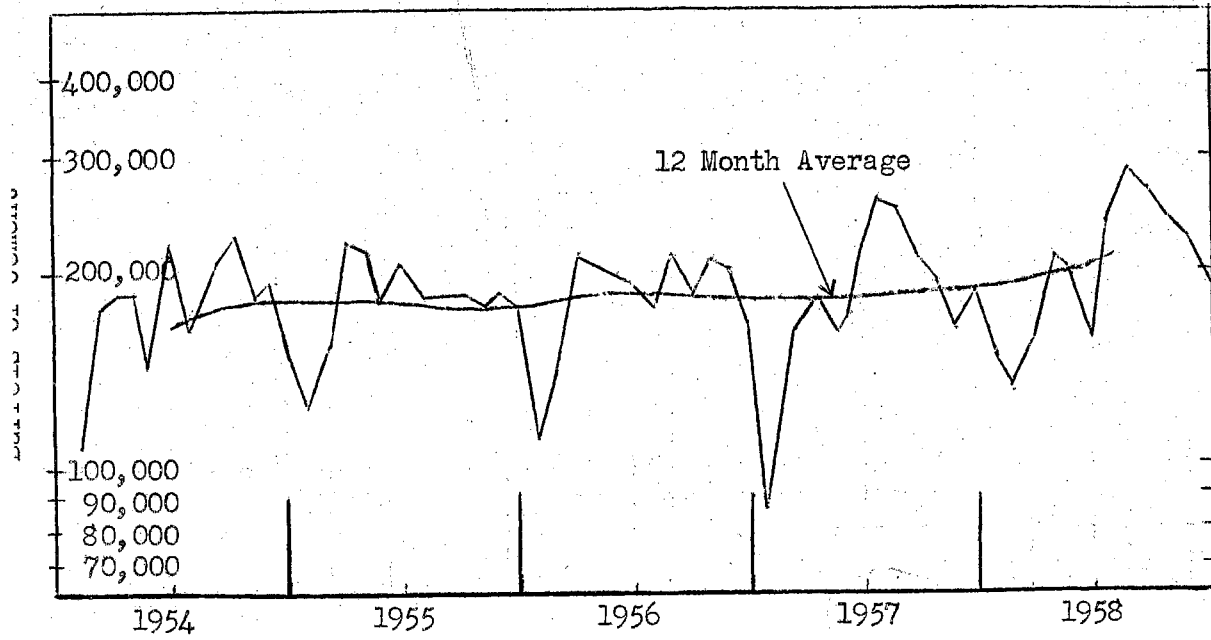
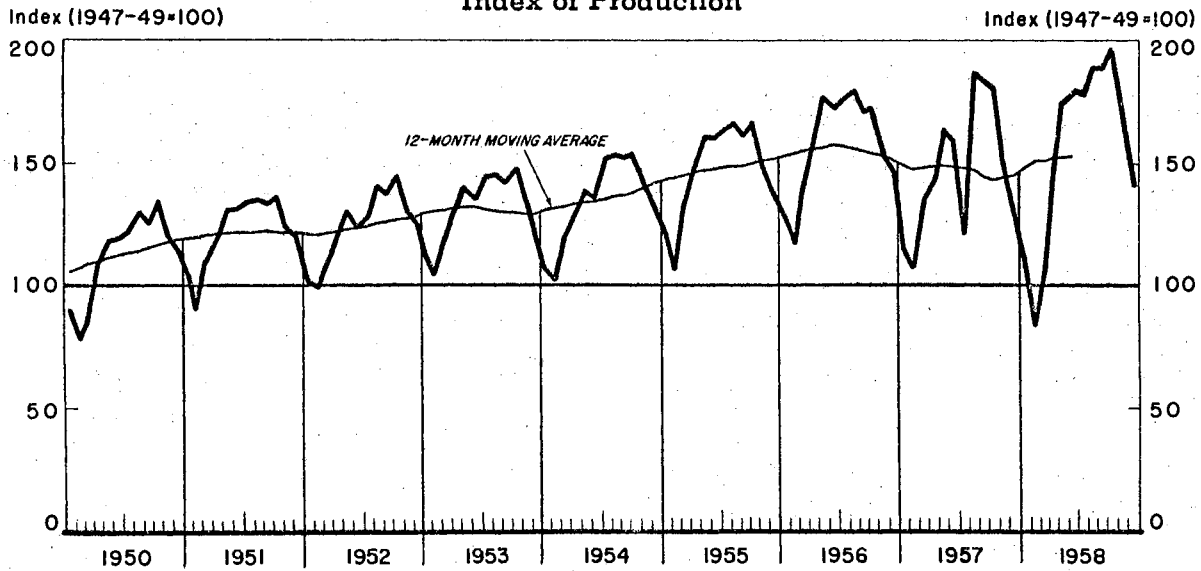


Chart II.

Portland Cement Index of Production



SOURCE: DEPARTMENT OF COMMERCE.

CONSTRUCTION REVIEW C.D.59-10-K

Figure 2.

Comparison of Past Industry Production and Plant Sales

based on monthly sales. The smooth line represents the moving average based on a twelve month period.

The value of these results is a method whereby the company can predict sales, which is necessary for a complete inventory study. It should be recognized that this method is by no means foolproof and can produce inaccurate estimates.

CHAPTER V

SCHEDULING OF GRINDING FACILITIES

The Ada, Oklahoma, plant produces four types of portland cement and also masonry cement. The types of portland cement produced are Types I, IA, II, and III. Type II is made only for special orders and it is not kept in stock. When an order is received, the item is produced. Sales figures for this particular cement are not complete, and several years have passed with no sales being made during the year. For these reasons this type is not included in this study. The present analysis considers only Types I, IA, III, and masonry cement. All four of these types are made from the same basic clinker. After the clinker comes from the kiln and is cooled, it is put in storage until it is ready for the finish grinding operation.

It is assumed in this study that Type I cement will be produced on the grinding facilities unless one of the other types is needed. The reason for this is that Type I composes about 88% to 96% of the total sales per year. The problem of how much clinker is needed for each period of time is covered in the next chapter. This chapter will be concerned with the finish grinding operations of the four types of cement. A method is presented in which an equation is developed to determine how much of each type to make on a production run of the grinding facilities.

The economic run size equation can be modified to take into account many special situations by altering the basic cost equation. This equation is patterned after the economic purchase order equation contained in various texts.¹ A method was needed in which from past data over a five year period a linear curve could be fitted which would represent the best estimate of the data and also the future values. A Least Squares method of curve fitting was chosen to make this analysis. Examination of the sales pattern of the three smaller types showed the patterns followed a normal distribution rather than a seasonal pattern as might be expected. Because of this it was decided to make the sales analysis on a yearly basis. Each of the types is analyzed later in this chapter and examples are given using the economic run equation. Type III and masonry cement require certain change-over costs in order that they can be ground on the finishing mills.

Two alternatives are available to the company: one is to produce frequently and thereby lower the inventory carrying charges; the other is to produce at longer intervals of time, which will cut down on the set-up costs. A balance should be made between these two plans whereby the total cost of meeting the requirements is at a minimum.

Development of the Economic Run Equation for Grinding Facilities

Let q denote the quantity to be made on each run. The average inventory between runs would be $q/2$. Assume:

1. Sales or expected demand for the year equal d .

¹ Thomas M. Whitin, The Theory of Inventory Management (Princeton, 1955) p. 31.

2. The cost of changing equipment to produce a different product is A.
3. The annual cost of carrying an item in stock for one year is equal to c.
4. The unit cost of an item is b'.

The total annual cost of grinding and holding inventory is given by the following equation:

$$C = Ad/q + b'd + cq/2$$

Differentiating with respect to q gives:

$$dC/dq = -Ad/q^2 + c/2$$

In order to minimize this equation set dC/dq equal to zero and solve for q.

$$q = \sqrt{\frac{2Ad}{c}}$$

This is a minimum value because

$$\frac{d^2C}{dq^2} = \frac{2Ad}{q^3}$$

is positive when

$$q = \sqrt{\frac{2Ad}{c}}$$

Determination of Values for use in the Economic Run Equation

A, the set-up cost, is determined by the value of the lost production time during the change-over period. This time is valued as lost cost of manufacturing for the regular grinding operations. It has been established that a change-over is needed for Type III and masonry cement. It requires four hours of time to change the settings on the separators and to change them back after the run is made for each of these two cements. The capacity of the grinding mill is 5500

barrels per day, with an average efficiency of 96%; the value of the grinding time to the total cost of the product is about 15%, and a barrel of cement is worth approximately \$3.75. The cost of a change-over would be

$$\frac{5500 \text{ bbl./day}}{24 \text{ hrs./day}} \times .96 \times \$3.75 \times 15\% \times 4 \text{ hrs.} = \$495.00$$

Labor is not included in this charge because a standby crew of personnel is on hand regardless of a change-over. The viewpoint taken was that the value of the lost production time during a change-over was the cost of the down time.

D, the yearly demand, is determined by using the Least Squares method of curve fitting to the past data to determine the percentage of the total sales for the year which can be attributed to each type. This percentage times the expected sales is equal to d.

Least Squares Analysis of Sales

A linear regression method is used with one independent and one dependent variable. If Y equals the dependent variable and X equals the independent variable, the functional relationship can be expressed as follows:

$$Y = a + bX \quad (1)$$

The problem is to estimate the values of a and b from the sample data. The method of Least Squares is used to provide this estimate. Appendix C contains the theory for the Least Squares Analysis.

Inventory Carrying Charges

A method to calculate the cost of carrying inventory has been suggested for use in the economic lot size equation.² The solution is general in nature and could be applied to various situations. Let c represent the cost of carrying inventory as a percentage of the inventory value. The charge can be broken down into three main parts: one, the storage cost; two, the desired return on investment in storage facilities; and three, the value of money in inventory. It is assumed that the risk of carrying inventories is about twice that of short term government bonds. On this basis, 6.5 per cent is a reasonable rate to assign to the value of money invested in inventories.

The value of money invested in storage facilities, considering the risk and liquidity associated with this type of investment, is assumed to be approximately 18 per cent.

Data

Storage cost (handling, taxes, maintenance, insurance, and depreciation)	= \$.0475 per cubic foot per year
Value of inventory stored per cubic foot	= \$.95 per cubic foot
Construction cost of storage facilities	= \$.45 per cubic foot
Value of money invested in inventory	= 6.5% per year
Value of money invested in storage facilities	= 18% per year

An example follows:

²William N. Hosley, "The Use of a Value-of-Money Concept in Operations Research." Paper presented at the 5th Annual Meeting of the Operations Research Society of America, Philadelphia, Pennsylvania, May 10, 1957.

$$\begin{aligned}
 \text{Storage cost} &= \frac{\text{Annual storage cost per cubic foot}}{\text{Value of inventory per cubic foot}} \\
 &= \frac{\$.0475}{.9500} = 5\%
 \end{aligned}$$

Desired return investment in storage facilities

$$\begin{aligned}
 &= \frac{\text{Construction cost x Value of money}}{\text{Value of inventory per cubic foot}} \\
 &= \frac{\$.45 \times 18\%}{.95} = 8.5\%
 \end{aligned}$$

Value of money in inventory = 6.5%

The total charge is the sum of these three items, which is equal to 20%. This figure is used as the cost of carrying inventory in this chapter as well as the next. An analysis of each of the four types of cement is now made, using the economic run equation to determine the best run size. The sales for the experimental year are assumed to be 2.8 million barrels of cement. Techniques were presented in Chapter IV which offer a procedure that the company can use to predict future sales.

Type I Cement

Since Type I cement comprises the largest percentage of the total sales - from 88% to 96% - it will be in production unless one of the other three stocked cements are needed. Type I cement is used by three major buyers: the federal government, the state, and commercial contractors and builders. The company makes its own quality checks on

the finished product as it is produced. When cement is sold to either the state or government, a special test is required before they will accept shipment. In performing this test, samples must be taken as the cement is placed in final storage, and these samples are sent to a testing laboratory. Within one to two weeks the results are available to the company. Because of the large amount of storage space available - over 400,000 barrels - this holding time does not present a problem. Perhaps in a situation where there was considerably less storage space, this fact would be of importance.

Type IA Cement

Since there is no set-up cost on Type IA cement and it is only a small part of the total sales, it can be made at almost any time. The only difference between Type I and IA cement is the addition of an air-entraining agent during the grinding operations. An analysis of the past sales figures indicated that the sales on a monthly basis followed a normal distribution. Since there is no set-up cost involved with this particular type, the scheduling would be left up to the operating officials. One suggestion would be to analyze the storage facilities and make maximum use of the storage involved with this type.

Type III Cement

Type III cement has a set-up time and a take-down time which totals four hours. The method of Least Squares, which is presented in Appendix C, is used to determine the estimate of the sales percentage which Type III cement is of the total yearly sales. Once

the demand, set-up cost, and inventory carrying charges are known, these values can be used in the economic run equation to determine the optimum run amount.

From the Least Squares analysis it is determined that the estimate for the percentage of the year's sales is .217 per cent. Therefore, the expected demand is

$$\hat{y}_{III}(\text{Sales}) = 2.8 \text{ Million (Expected Sales for Year)} \\ \times .217\% = 5,880 \text{ barrels.}$$

This figure is d in the equation. A is equal to four hours, which is valued at \$495.00; c is equal to 20%, and the incremental cost of going from clinker to cement is 15% of the total value; therefore, the inventory cost is based on this incremental cost. So, by taking $20\% \times 15\% \times \3.75 , the value of a barrel of cement, one gets \$.1125 as the inventory carrying charge for one unit of product for a period of one year. The equation would be solved as follows:

$$q = \sqrt{\frac{2Ad}{c}} = \sqrt{\frac{2(495)(5880)}{.1125}} = \sqrt{51,800,000} = 7,200 \text{ barrels.}$$

The number of runs would be the yearly requirements divided by the amount in the economic run.

$$R' = d/q = 5880/7200 = .817 \text{ runs per year.}$$

These figures provide the solution to the problem. If the company wants to include safety stock in this solution, it should analyze the past data to determine the mean and standard deviation. From this

information the level of safety stock can be determined in the manner discussed in Chapter VI.

Masonry Cement

Masonry cement falls in the same category as Type III. It has the same set-up and take-down, which is four hours. The demand is different, and the same method of solution is used to determine the percentage of sales as used for Type III cement. The Least Square solution predicted a percentage value of 1.37%. The solution would be

$$d = 1.37\% \times 2.8 \text{ million} = 38,390 \text{ barrels}$$

$$A = \$495.00$$

$$c = 20\% \times 15\% \times \$3.75 = \$.1125$$

$$q = \sqrt{\frac{2Ad}{c}} = \sqrt{\frac{2(495)(38,390)}{.1125}} = \sqrt{338,000,000}$$

$$= 18,380 \text{ barrels.}$$

The number of runs would be the yearly requirements divided by the amount in the economic run.

$$R' = \frac{d}{q} = \frac{38,390}{18,380} = 2.09 \text{ runs per year.}$$

The same policy discussed in the previous sections can be used in determining safety stock.

As the different factors change, they can be adjusted by a corresponding change in the economic equation. The scheduling of these

products will be left to the discretion of the company. A method has been presented to determine only the optimum run amount.

A suggestion is to keep accurate records of the amount of stock on hand of each of the different products, and when one of the types approaches a certain level, a run should be made on this item.

CHAPTER VI

SCHEDULING AND INVENTORY ANALYSIS OF CLINKER PRODUCTION

The production of clinker is the first stage of cement production. Since all four types of cement in this study are made from the same basic clinker, the only problem in the first stage is the amount of clinker to make each month. There are two plans available; one is to make clinker on the new facilities; the other is combining the new and the old facilities. The purpose of this chapter is to present a method whereby the company officials can schedule production over a period of time, once the demand is known or a reasonable estimate is available. Included in the analysis is a method to calculate the safety stock required for different levels of confidence, and also a procedure to evaluate the cost of various production plans.

Production Alternatives

The present clinker production alternatives of the two plants will now be described. There are two production sections in the old plant. The new plant consists of two 450-foot kilns and the old plant has four smaller kilns. The capacities and operation information of these two plants are located in Table I. The new plant is very efficient and can produce clinker at a larger volume and lower cost than the old plant. The company foregoes a certain profit

on the higher cost of clinker production in the old plant if it could have been made in the new plant. One approach is to make excess clinker in the new plant during the slack periods to use during the peak seasons when the sales exceed the capacity of production.

Another solution is to make clinker for each month's requirements as they occur, and if the requirements exceed the capacity of the new plant during the month, the excess requirements can be produced on the old facilities. Each of these plans has certain advantages over the other, but to use only one would result in a higher cost. There should be some combination of these two plans which would result in the lowest cost and still meet the demand for the product.

Derivation of the Production Cost Equation

In order to schedule production an equation should be developed which expresses the total cost of these two plans and then this equation will be minimized. Let x_1, x_2, \dots, x_n denote production for any given period of time; an example is January production equaling x_1 . Let $\alpha_1, \alpha_2, \alpha_n$ denote the production capacity of both the new and the old plant for each period. Therefore

$$x_1 \leq \alpha_1, \quad x_2 \leq \alpha_2, \quad \dots, \quad x_n \leq \alpha_n \quad (1)$$

Let X_i^c equal the cumulative production through each period; therefore

$$X_1^c = x_1, \quad X_2^c = x_1 + x_2, \quad \dots, \quad X_n^c = \sum_{i=1}^n x_i \quad (2)$$

Let R_i^c equal the cumulative sales requirement through each production period, μ_0 equal the inventory at the beginning of the year, and μ_n equal the year end inventory. At the end of each production period

certain requirements must be met. This expression is as follows:

$$\mu_0 + x_i^c \geq R_i^c \quad (3)$$

The ending inventory for each period can be expressed as follows:

$$\mu_i = \mu_0 + x_i^c - R_i^c \quad (4)$$

Let μ_0 and μ_n both be given as constants.

$$\mu_0 = (c), \quad \mu_n = (c) \quad (5)$$

The production cost is expressed as a function of x in the form, production cost = $f(x)$. The production costs for each period are based upon which plant the clinker is produced in and how much is made on either the new, or the new and the old combined. The production cost will be assumed to be a linear function, but actual production costs are linear within limits of production. The possibility of higher production costs at reduced operation is not included in this analysis. The total production cost is expressed in the following equation: let P_c equal total production cost

$$P_c = f(x_1) + f(x_2) + \dots + f(x_n) = \sum_{i=1}^n f(x_i) \quad (6)$$

The inventory carrying charge for each period is based upon the average inventory for that particular period. The average inventory for

period one is $\mu_1 = \frac{\mu_0 + \mu_1}{2}$; for the i 'th period, $\mu_i = \frac{\mu_{i-1} + \mu_i}{2}$.

The inventory for all the periods is equal to

$$\sum_{i=1}^n \mu_i = \frac{\mu_0 + \mu_1}{2} + \frac{\mu_1 + \mu_2}{2} + \dots + \frac{\mu_{n-1} + \mu_n}{2}$$

$$= \frac{\mu_0}{2} + \sum_{i=1}^n \mu_i + \frac{\mu_n}{2} . \quad (7)$$

The inventory carrying charge for each period for each unit is c' ; therefore, the total inventory cost for the year is: let P_I equal total inventory cost

$$P_I = 1/2 c' (\mu_0 + \mu_1) + 1/2 c' (\mu_1 + \mu_2) + \dots + 1/2 c' (\mu_{n-1} + \mu_n) . \quad (8)$$

The total cost of a given production plan would be the sum of the inventory carrying charges for the year and the total production cost for the year. The total cost equation would equal

$$P_T = P_C + P_I = 1/2 c' \sum_{i=1}^n (\mu_{i-1} + \mu_i) + \sum_{i=1}^n f(x_i) . \quad (9)$$

The problem is to minimize equation nine under the constraints of equations one to five in this chapter. This equation can be solved by using linear programming techniques, since all costs are assumed to be linear. By assigning cost to the production on both the old and new plants and also a cost for carrying inventory for each period of time and by having a common unit of production, the transportation method of linear programming can be used, which will greatly simplify the solution to this problem.

A test can be made of any production plan to determine if it falls within the storage capacity of the plant. On any plan a graphical solution can be made using the following equation as a guide:

$$\text{Initial stock} + \text{Production} - \text{Sales} \leq \text{Capacity of Storage.}$$

A graph should be made containing curves for production, sales, and inventory. An example of this graph is Figure 3. The total inventory curve should not intersect the capacity line.

Monthly Sales Prediction

The next step in this solution is to predict the sales requirements for each month of the year. Since cement sales were seasonal in nature and the total yearly sales were on an upward trend, the method of solution selected was predicting the percentage a particular month's demand would be of the total sales for the year. A linear regression equation was used in the analysis. It contained one dependent variable, the percentage of annual sales, and one independent variable, the particular month. This method was very similar to the procedure outlined and used in Chapter IV. The mathematical theory and solution to the linear regression equation is presented in Appendix B, and it will not be repeated in this section. During the solution to this phase of the problem, the IBM 650 digital computer was used to solve the linear regression equation for the predicted sales percentages. Data was supplied for the past five years, 1954 through 1958, on the total sales during each year on a monthly basis. This information was converted to a monthly percentage basis. From this data it was possible to derive a regression equation for each month of the year, which in turn made it possible to predict the percentage

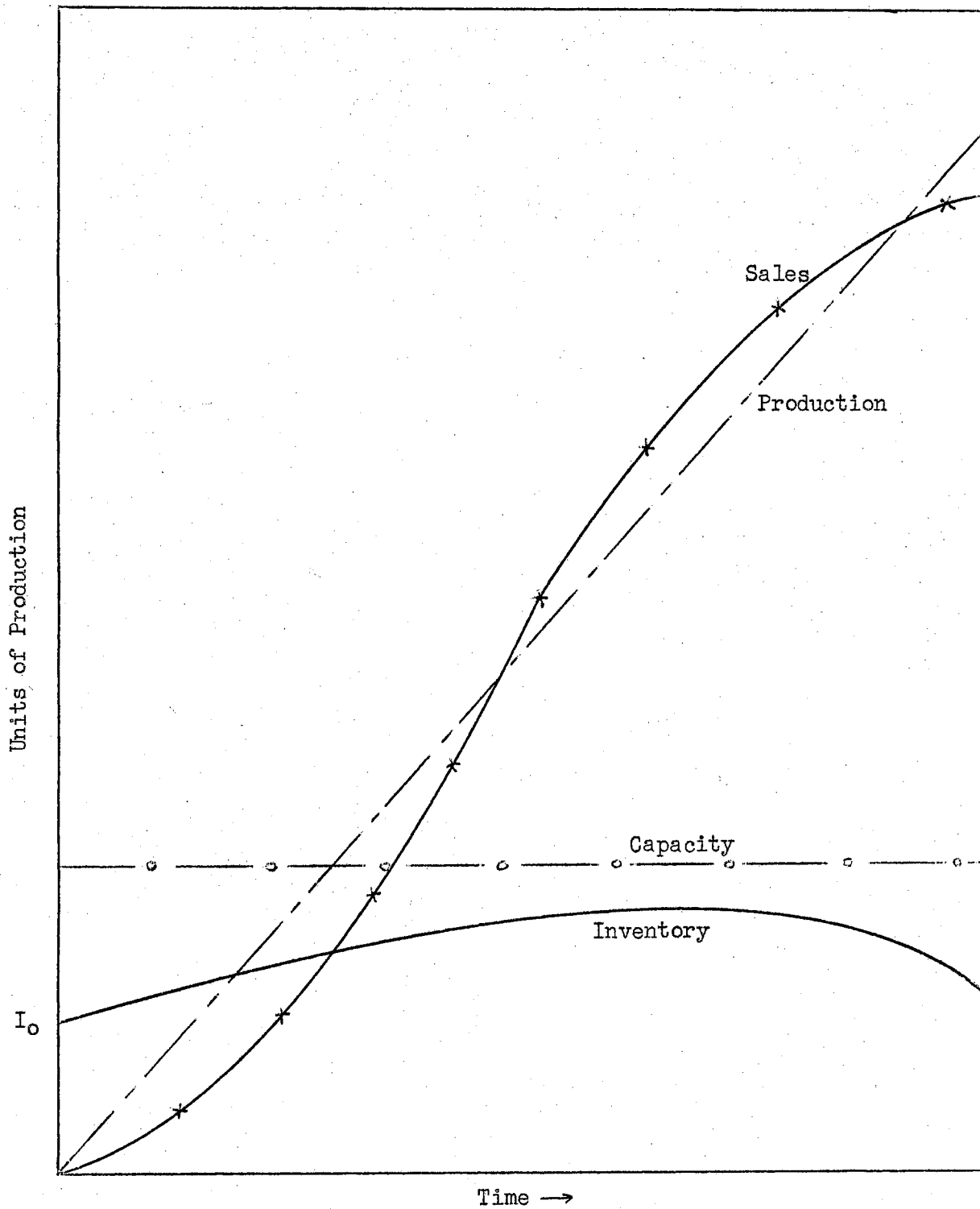


Figure 3.

Graphical Test for Inventory Overflow.

of the total sales each month would have as its part of the yearly demand. The results of the past data are located in Table IV. The derived equation was then used to calculate the expected percentage for the past five years in order to make a comparison of the test results and the actual data. This information made it possible to calculate the standard deviation of the past predictions and was of great use in setting the confidence levels for the different production plans. A normal distribution of prediction error was assumed for this test. Sample calculations for the month of January are presented in Appendix D to better understand the solution using the above information.

Levels of Confidence

Four levels of confidence were chosen to use in this study. The term "level of confidence" is used to denote the probability of the demand not exceeding production for each month in question. The levels selected were 90%, 95%, 97.5%, and 99%. From a normal distribution table it was possible to determine the value of k , which is the distance away from the average of the sample data, expressed as a multiple of the standard deviation, for the desired level of confidence.¹ For a level of 90% the table would be checked for a value of .90. The location of this value in the table will give the desired value of k , which is a function of the standard deviation. The table values are from that portion of the population extending from minus

¹Irving R. Burr, Engineering Statistics and Quality Control, (New York, 1953) p. 404.

TABLE IV

RESULTS OF MULTIPLE REGRESSION ANALYSIS FOR
PREDICTED MONTHLY SALES

Month	β_0	β_1	R	\mathcal{Y}_6	\mathcal{T}_y
J	5.133	-.078	.1760	4.665	.679
F	9.034	-.820	.9744	4.114	.308
M	10.629	-.709	.6743	6.375	1.413
A	9.604	-.332	.5238	7.612	.982
M	7.160	.240	.4908	8.600	.778
J	11.053	-.741	.7291	6.607	1.247
J	6.484	.958	.7643	12.232	1.475
A	7.781	.943	.8554	13.439	1.042
S	9.075	.187	.2363	10.197	1.676
O	7.813	.321	.7468	9.739	.3806
N	9.083	-.193	.3247	7.925	1.026
D	7.358	-.096	.3084	6.782	1.013

infinity up to the level desired. In the example, 90% means that 90% of the time the population lies within the selected limit. In the method used in this study, a 90% level means the demand will only exceed the supply 10% of the time, i.e. one month in ten. The value of k was found for each level and used in the expression of the upper limits as $+k\sigma_y$. This equation would be the required safety stock for each level of confidence. The safety stock level is expressed in percentages in the same manner as the monthly expected sales. The upper limit determines the level of stock necessary for each month in order to meet the predicted demand and also have enough safety stock on hand to meet possible errors in the original forecast.

Sales Determination

The sales for the test year were assumed to be 2.8 million barrels of cement. A procedure was presented in Chapter IV whereby the company could predict sales using economic indicators. With all the required information available it is now possible to construct the transportation tableau.

Transportation Solution of Clinker Production

A tabular array was set up like the ones in Figures 4, 5, 6, and 7, of Appendix E, for the solution to the production problem. It was assumed the cost for holding one barrel of cement in storage for one year was equal to \$.75, and therefore for each month the charge would be equal to \$.0625. Since it was more expensive to produce clinker on the old facilities instead of the new, a production cost of \$.10 was assigned to each barrel produced in the old plant, and a cost of

zero was assigned to each barrel produced on the new facilities. Across the top of each chart are the production capacities for each month on both the new and the old facilities. These figures are from the sample calculations in Appendix E.

The cost figures shown in the table represent the different costs of production in the periods represented by the column for use or sale in the period represented by the row. For example, the excess cost of production in January in the new plant for January production is zero, but to produce in January for use in March costs \$.1250. The figures in the column at the left represent the production requirements for each month as found in Tables VI, VIII, X, and XII, of Appendix E. Production in January on the old kilns costs \$.10 per unit and to produce on the old in January for use in March costs \$.2250. This figure is composed of the cost for production on the old in January and the cost of carrying inventory for two months. The table is completed in the following manner. The first row of the table opposite available is copied from the heading above, which is the production capacity for that period of time. The cost figures in the next row are checked to find the minimum cost. In the next row is entered the production to be planned. This figure is the lesser of the required or available. If the required figure is greater than the available, the procedure is to enter the available figure as above, then go to the next column with the lowest cost and repeat the same steps as above. This is continued until all of the month's requirements are met horizontally. The requirements for one period can never be met from the available in a later period, bringing out the fact that planned inventories cannot be negative.

When the planned row for the first period is completed, the available row for the second period can be completed. This is done by subtracting the planned from the available in January and placing this figure in the available row in February. These figures represent production capacity not yet assigned. The above procedure is repeated until all the production requirements are satisfied. The total production can be found by the addition of the plan figures in each column. Illustrations of production planning problems formulated in linear programming terms can be found in technical publications on this subject.²

Production Analysis

After the transportation tableau is solved, the cost of each plan must be evaluated. The completed production plans are presented in table form in Tables VII, IX, XI, and XIII, of Appendix E. From these tables the total cost which is a combination of the inventory carrying costs and the production cost can be computed. Since a cost of zero was assigned to production on the new, the only concern is the amount produced on the old facilities. The inventory cost is based on the average inventory during the month times the inventory carrying cost. Equation (7) presented earlier in this chapter gives a method to calculate the cost for carrying inventory based on the balance at the end of the month. When production exceeded sales, inventory was increased, and when sales exceeded production, the opposite was true. This caused the inventory balance to vary each month. The equation for the sum of the monthly averages is

²A. Charnes, W.W. Cooper, D. Farr, "Linear Programming and Profit Preference Scheduling for a Manufacturing Firm," Journal of the Operations Research Society of America, May, 1953, p. 114.

$$\sum \mu_i = \frac{\mu^0}{2} + \mu_1 + \mu_2 + \dots + \mu_{n-1} + \frac{\mu^n}{2}.$$

This total was multiplied times the cost per unit per month for carrying inventory and was equal to the inventory charge for the year for each plan.

The results in Appendix E show that as the level of confidence is increased; the cost of carrying inventory and production also increases, but at a much higher rate. An increase in the confidence level of five percent going from 90% to 95% costs approximately \$5,500.00. To go from 95% to 97.5%, which is an increase of only 2.5% costs about \$9,500.00. These figures show that the relationship is not linear, as might be assumed.

This type of solution is designed to enable the officials to evaluate the cost of various production plans for the different levels of confidence. Instead of relying on experience or guess work, a qualitative analysis can be made for each plan, and a decision can be reached using economic cost as a basis for evaluation. The information concerning capacities, production cost, and inventory charges can be changed to fit various situations without altering the method of solution.

CHAPTER VII

SUMMARY AND CONCLUSIONS

In this thesis an attempt has been made to set up a method by which the operating officials of a cement plant can predict sales and from this information schedule the amount of cement needed for each production period. Another problem was scheduling for production of the three types of cement, which represented only a small percentage of the sales.

In sales prediction, economic indicators were used to develop a multiple regression equation which would be useful in predicting future sales. Sales were grouped by months and quarters, and the results showed that the accuracy is fairly good, but by no means without error. Also, it was noticed that greater accuracy resulted when monthly forecasts were used rather than quarterly forecasts.

In determining how to allocate the grinding facilities, an economic run equation was developed by which to schedule the grinding facilities for the three small-use types; otherwise, Type 1 cement is produced on these mills. The equation contains three variables, and by substitution of their values it is possible to determine the economic run length. A method was suggested by which safety stock could be included in the analysis.

The main problem concerned the clinker-making process, which could be performed in both the new and the old plant. Various costs

were associated with the facilities and also inventory carrying charges played a very important role. The transportation method of linear programming was used to develop a method of allocation of the production facilities, depending on the desired confidence level. The solution was to minimize the cost associated with meeting the desired production schedule for four different plans.

Each plan consisted of a different confidence level, ranging from ninety to ninety-nine percent. It was found that as the desired confidence level was raised, the cost of carrying inventory increased greatly. A hand method of solution was presented because it is easily applied and understood. Included in this section was a discussion and example by which it was possible to determine the amount of safety stock required to meet the various demand levels based on past data and the deviation of the calculated values of sales. A system of predicting any month's percentage of sales to the yearly total was necessarily incorporated because of the seasonal nature of the product in question.

SELECTED BIBLIOGRAPHY

- Arrow, Kenneth J., Samuel Karlin, and Herbert Scarf. Studies in the Mathematical Theory of Inventory and Production. Stanford: Stanford University Press, 1958.
- Blanks, Robert F., and Henry L. Kennedy. The Technology of Cement and Concrete. New York: John Wiley, 1958.
- Bowman, Edward H., and Robert B. Fetter. Analysis for Production Management. Homewood, Illinois: Richard D. Irwin, 1957.
- Burr, Irving R. Engineering Statistics and Quality Control. New York: McGraw-Hill, 1953.
- Charnes, A., W. W. Cooper, and D. Farr. "Linear Programming and Profit Preference Scheduling for a Manufacturing Firm." Journal of the Operations Research Society of America. I, 3. (May, 1953), 114.
- Cowden, Dudley J., and Frederick E. Croxton. Applied General Statistics. New York: Prentice Hall, 1939.
- Hosley, William N. "The Use of a Value-of-Money Concept in Operations Research." Paper presented at the 5th Annual meeting of the Operations Research Society of America, Philadelphia, Pennsylvania, May 10, 1957.
- Magee, John F. Production Planning and Inventory Control. New York: McGraw-Hill, 1958.
- Modigliani, Franco, and Bernard J. Marks. "Economic Expectations and Plans of Firms in Relation to Short Term Forecasting." Short Term Economic Forecasting. National Bureau of Economic Research, Princeton: Princeton University Press, 1955.
- Ostle, Bernard. Statistics in Research. Ames: Iowa State College Press, 1954.
- Vazonyi, Andrew. Scientific Programming in Business and Industry. New York: John Wiley, 1958.
- Whitin, Thomas M. The Theory of Inventory Management. Princeton: Princeton University Press, 1955.

Construction Review, Published by the Superintendent of Documents,
U. S. Government Printing Office, Washington, D. C.

Economic Indicators, Prepared for the Joint Economic Committee by
the Council of Economic Advisers. U. S. Government Printing
Office, Washington, D. C.

APPENDIX

APPENDIX A

LIST OF SYMBOLS

A	Cost of change-over of equipment for grinding run.
a	A constant.
∞	Production capacity of kilns for a month.
b	A constant.
b'	Cost per barrel of cement.
β	Regression coefficient.
C	Total annual production cost for grinding operation.
c	Cost of carrying one unit in inventory for one year.
c'	Cost of carrying one unit in inventory for one month.
d	Expected sales for one year for each type of cement.
k	Confidence level factor from Normal Distribution Table.
k'	Number of independent variables.
ν	Inventory for each period of time.
n	Number of observational units in sample.
P_c	Total annual production cost of clinker operations.
P_I	Total annual inventory cost of finished cement.
P_T	Total annual cost of production plan.
Q	Error squared between predicted and actual value.
q	Economic run quantity.
R	Multiple regression coefficient.
R^c	Cumulative sales requirements.
R'	Number of production runs per year.
σ_y	Standard deviation of error of the predicted monthly percentage.

- X Independent variable.
- X^c Cumulative production requirements.
- X' Monthly economic indicator.
- X'' Quarterly economic indicator.
- x Clinker production for one period of time.
- Y Dependent variable.
- \hat{Y} Predicted dependent variable.
- \hat{Y}' Predicted total sales for month.
- \hat{Y}'' Predicted total sales for a quarter.
- y Actual monthly percentage of total yearly sales.
- \hat{y} Predicted monthly percentage of total yearly sales.
- y' Predicted yearly demand of one type of cement as a percentage of the expected sales.

APPENDIX B

MULTIPLE LINEAR REGRESSION ANALYSIS

In the solution to the sales prediction problem a procedure was needed to analyze past sales data and develop an equation which would express company sales as a function of selected economic indicators. The method selected was a linear regression analysis using the IBM 650 digital computer to solve the system of equations. Let Y equal the dependent variable, sales, and the X's represent the independent variables, selected economic indicators. The functional equation would be

$$Y = f(X_1, X_2, + \dots + X_k)$$

The general equation would be

$$\hat{Y} = \beta_0 + \beta_1 X_1 + \dots + \beta_k X_k$$

where $\beta_0, \beta_1, \dots, \beta_k$ are unknown parameters, and X_1, X_2, \dots, X_k are known economic indicators for periods of time from 1 to n. Using the Least Squares method of solution, the equation to minimize would be

$$Q = \sum_{i=1}^n (y_i - \beta_0 - \beta_1 X_{1i} - \dots - \beta_k X_{ki})^2$$

Where Q represents the error squared between the predicted and actual values. All summations are from 1 to n, the number of observational units in the sample. The above equation was differentiated with respect to $\beta_0, \beta_1, \dots, \beta_k$, and the following systems of equations was obtained:

$$\sum Y = n\beta_0 + \beta_1 \sum X_1 + \beta_2 \sum X_2 + \dots + \beta_k \sum X_k$$

$$\sum X_1 Y = \beta_0 \sum X_1 + \beta_1 \sum X_1^2 + \beta_2 \sum X_1 X_2 + \dots + \beta_k \sum X_1 X_k$$

$$\vdots$$

$$\sum X_k Y = \beta_0 \sum X_k + \beta_1 \sum X_1 X_k + \beta_2 \sum X_2 X_k + \dots + \beta_k \sum X_k^2$$

The solution is to solve these equations simultaneously for the values of $\beta_0, \beta_1, \dots, \beta_k$.

APPENDIX C

LEAST SQUARES ANALYSIS

This method provides a procedure to calculate a linear curve which will be the best estimate of past data and also predict future data. The functional relationship can be expressed as $Y = a + bX$. The equation to be minimized is

$$Q = \sum_{i=1}^n (Y_i - a - bX_i)^2 \quad (1)$$

where Q represents the error squared between the actual data and the predicted values. All summations are from 1 to n , where n is the number of observations. The first step is to differentiate Q with respect to a and b .

$$\frac{\partial Q}{\partial a} = -2 \sum (Y_i - a - bX_i) = 0 \quad (2)$$

$$\frac{\partial Q}{\partial b} = -2 \sum X_i (Y_i - a - bX_i) = 0 \quad (3)$$

Equations 2 and 3 can be rewritten as follows

$$\sum Y_i = na + b \sum X_i \quad (2a)$$

$$\sum X_i Y_i = a \sum X_i + b \sum X_i^2 \quad (3a)$$

These equations can be solved simultaneously for a and b , giving the estimation equation

$$\hat{Y} = a + bX$$

This equation represents a least squares fit to the data.

APPENDIX D

SAMPLE CALCULATION FOR THE MONTH OF JANUARY

Given: $\beta_0 = 5.133$ $\beta_1 = -.078$ $\hat{y} = \beta_0 + \beta_1 x_n$ (1)

Past data of sales percentages for the month of January

$y = \% \text{ of years' sales}$ $\hat{y} = \text{expected } \% \text{ of years' sales using equation 1 above}$

$x = \text{period of time} = 1, 2, 3, \dots n.$

x	y	\hat{y}	$y - \hat{y}$	$(y - \hat{y})^2$
1954	5.040	5.055	-.015	.000225
1955	5.520	4.977	.543	.294849
1956	4.780	4.899	-.119	.014161
1957	3.760	4.821	-1.061	1.125720
1958	4.590	4.743	-.153	.023409
1959		4.665		

$\Sigma = 1.448364$

The variance and standard error of the estimate can be found by using the equation

$$\sigma_{\hat{y}|2..k}^2 = \sum_{i=1}^n \frac{(y_i - \hat{y}_i)^2}{n - k' - 1}$$

for the particular case, $k' = 1$, and therefore the equation would be

$$\sigma_{\hat{y}}^2 = \frac{\sum (y_i - \hat{y}_i)^2}{n - 2} = \sqrt{\frac{\sum (y_i - \hat{y}_i)^2}{n - 2}} = \sigma_{\hat{y}}$$

$\sigma_{\hat{y}}$ is the same as σ_y for the notation, the solution would be

$$\sigma_y = \sqrt{\frac{1.448364}{3}} = \sqrt{.48612} = .6972$$

From a table of areas under the normal distribution curve the value of k can be found, which is the distance away from the mean, of the desired confidence level. The four selected levels are

Confidence Level	k	σ_y	$k\sigma_y$
.900	1.281	.6972	.892
.950	1.645	.6972	1.146
.975	1.960	.6972	1.366
.990	2.323	.6972	1.624

The equation for the safety stock percentage would be

$$+ k\sigma_y$$

Using the assumed expected sales figure of 2.8 million barrels of cement, the required production figure and safety stock for each level can be calculated.

January Sales = \hat{y} = 4.665 % x 2,800,000 = 130,620 barrels. The safety stock equation is $+ k\sigma_y$ and the amount needed for the four levels would be

Level	$k\sigma_y$		Expected Sales	=	Safety stock
90 %	.892	x	2,800,000	=	24,976 barrels.
95 %	1.146	x	2,800,000	=	32,088 barrels.
97.5 %	1.366	x	2,800,000	=	38,248 barrels.
99 %	1.624	x	2,800,000	=	45,472 barrels.

APPENDIX E

CLINKER PRODUCTION PLAN ANALYSIS

Calculation of Monthly Capacities of the
New and Old Plants

Equation:

Monthly capacity = Number of kilns x Average efficiency x daily capacity
x days in month.

New Plant: 28 day month = 2 x .96 x 4500 x 28 = 241,900 barrels.
30 day month = 2 x .96 x 4500 x 30 = 259,200 barrels.
31 day month = 2 x .96 x 4500 x 31 = 267,800 barrels.

Old Plant: 28 day month = 4 x .92 x 1900 x 28 = 195,800 barrels.
30 day month = 4 x .92 x 1900 x 30 = 209,800 barrels.
31 day month = 4 x .92 x 1900 x 31 = 216,800 barrels.

Calculations of Expected Monthly Sales

Equation:

Expected sales = Predicted monthly percentage x predicted yearly sales.

Month	Expected %	Yearly sales	Expected monthly sales
Jan.	4.665	2,800,000	130,620 barrels.
Feb.	4.114	2,800,000	115,192 barrels.
Mar.	6.375	2,800,000	178,500 barrels.
Apr.	7.612	2,800,000	213,136 barrels.
May	8.600	2,800,000	240,800 barrels.
June	6.607	2,800,000	184,996 barrels.
July	12.232	2,800,000	342,496 barrels.
Aug.	13.439	2,800,000	376,292 barrels.
Sept.	10.197	2,800,000	285,516 barrels.
Oct.	9.739	2,800,000	272,692 barrels.
Nov.	7.925	2,800,000	221,900 barrels.
Dec.	6.782	2,800,000	189,896 barrels.

TABLE V

CALCULATIONS OF SAFETY STOCK FOR FOUR LEVELS OF CONFIDENCE

Month	90% Level		95% Level		97.5% Level		99% Level	
	% Stock Required	Stock (Barrels)	% Stock Required	Stock (Barrels)	% Stock Required	Stock (Barrels)	% Stock Required	Stock (Barrels)
J	.892	24,976	1.146	32,088	1.366	38,248	1.624	45,472
F	.394	11,032	.507	14,196	.603	16,884	.718	20,104
M	1.809	50,652	2.324	65,072	2.769	77,532	3.292	92,204
A	1.257	35,196	1.615	45,220	1.925	53,900	2.288	64,064
M	.995	27,860	1.279	35,812	1.524	42,672	1.812	50,736
J	1.596	44,688	2.051	57,428	2.444	68,432	2.905	81,340
J	1.888	52,864	2.426	67,928	2.891	80,948	3.436	96,208
A	1.333	37,324	1.714	47,992	2.042	57,176	2.427	67,956
S	2.145	60,060	2.757	77,196	3.280	91,840	3.905	109,340
O	.487	13,636	.626	17,528	.745	20,860	.886	24,808
N	1.313	36,764	1.687	47,236	2.010	56,280	2.390	66,920
D	1.296	36,288	1.666	46,648	1.985	55,580	2.360	66,080
M		431,340		554,344		666,352		785,232

TABLE VI

ANALYSIS FOR PRODUCTION
90% LEVEL OF CONFIDENCE

Month	A Expected Sales	B Cumulative Sales Requirements (All Figures in Barrels of Cement)	C Required Inventory	D Cumulative Produc- tion Requirements	$(B_n + C_n) - (C_{n-1})$ Required Produc- tion for Month
D			32,000		
J	130,620	130,620	24,976	123,596	123,596
F	115,192	245,812	11,032	224,844	101,248
M	178,500	424,312	50,652	442,964	218,120
A	213,136	637,448	35,196	640,644	197,680
M	240,800	878,248	27,860	874,108	233,464
J	184,996	1,063,244	44,688	1,075,932	201,824
J	342,496	1,405,740	52,864	1,426,604	350,672
A	376,292	1,782,032	37,324	1,787,356	360,752
S	285,516	2,067,548	60,060	2,095,608	308,252
O	272,692	2,340,240	13,636	2,321,876	226,268
N	221,900	2,562,140	36,764	2,566,904	245,028
D	189,896	2,752,036	36,288	2,756,324	189,420

Month Required	Amount Required	Type of Production	Month of Production																														
			Jan.		Feb.		Mar.		Apr.		May		June		July		Aug.		Sept.		Oct.		Nov.		Dec.		Total						
			New	Old	New	Old	New	Old	New	Old	New	Old	New	Old	New	Old	New	Old	New	Old	New	Old	New	Old	New	Old							
		Capacity	267,800	216,800	241,900	195,800	267,800	216,800	259,200	209,800	267,800	216,800	259,200	209,800	267,800	216,800	267,800	216,800	259,200	209,800	267,800	216,800	259,200	209,800	267,800	216,800	259,200	209,800	267,800	216,800			
Jan.	123,596	Available Cost Plan	267,800 0 123,596	216,800 10																													
Feb.	101,248	Available Cost Plan	141,204 6.25	216,800 16.25	241,900 101,248	195,800 10																											
Mar.	218,120	Available Cost Plan	12.50	22.50	140,652 6.25	195,800 16.25	267,800 0 218,120	216,800 10																									
Apr.	197,680	Available Cost Plan	18.75	28.75	12.50	22.50	19,680 6.25	216,800 16.25	259,200 0	209,800 10																							
May	233,464	Available Cost Plan	25.00	35.00	18.75	28.75	12.50	22.50	61,520 6.25	209,800 16.25	267,800 0 233,464	216,800 10																					
June	201,824	Available Cost Plan	31.25	41.25	25.00	35.00	18.75	28.75	12.50	22.50	34,336 6.25	216,800 16.25	259,200 0 201,824	209,800 10																			
July	350,672	Available Cost Plan	37.50	47.50	31.25	41.25	25.00	35.00	18.75	28.75	12.50	22.50	57,376 6.25 57,376	209,800 16.25	267,800 0 267,800	216,800 10 25,196																	
Aug.	360,752	Available Cost Plan	43.75	53.75	37.50	47.50	31.25	41.25	25.00	35.00	18.75	28.75	0 12.50	22.50	0 6.25	191,304 16.25	267,800 10 267,800	216,800 10 92,952															
Sept.	308,252	Available Cost Plan	50.00	60.00	43.75	53.75	37.50	47.50	31.25	41.25	25.00	35.00	18.75	28.75	12.50	22.50	0 6.25	123,818 16.25	259,200 0 259,200	209,800 10 19,052													
Oct.	226,268	Available Cost Plan	56.25	66.25	50.00	60.00	43.75	53.75	37.50	47.50	31.25	41.25	25.00	35.00	18.75	28.75	12.50	22.50	0 6.25	160,718 16.25	267,800 0 226,268	216,800 10											
Nov.	215,028	Available Cost Plan	62.50	72.50	56.25	66.25	50.00	60.00	43.75	53.75	37.50	47.50	31.25	41.25	25.00	35.00	18.75	28.75	12.50	22.50	41,532 6.25	216,800 16.25	259,200 0 215,028	209,800 10									
Dec.	189,420	Available Cost Plan	68.75	78.75	62.50	72.50	56.25	66.25	50.00	60.00	43.75	53.75	37.50	47.50	31.25	41.25	25.00	35.00	16.75	28.75	12.50	22.50	11,172 6.25	209,800 16.25	267,800 0 189,420	216,800 10							
		Total Production Plan	123,596		101,248		218,120		197,680		233,464		259,200		267,800	25,196	267,800	92,952	259,200	19,052	226,268		215,028		189,420								

Figure 4. Transportation Solution of Production - 90% Level of Confidence.

TABLE VII
ANALYSIS OF TRANSPORTATION SOLUTION
90% LEVEL OF CONFIDENCE

Month	New Plant	Old Plant	Total	Sales	Balance † Inventory	Inventory
(All Figures in Barrels of Cement)						
D						32,000
J	123,596	0	123,596	130,620	-7,024	24,976
F	101,248	0	101,248	115,192	-13,944	11,032
M	218,120	0	218,120	178,500	39,620	50,652
A	197,680	0	197,680	213,136	-15,456	35,196
M	233,464	0	233,464	240,800	-7,336	27,860
J	259,200	0	259,200	184,996	74,204	102,064
J	267,800	25,496	293,296	342,496	-49,200	52,864
A	267,800	92,952	360,752	376,292	-15,540	37,324
S	259,200	49,052	308,252	285,516	22,736	60,060
O	226,268	0	226,268	272,692	-46,424	13,636
N	245,028	0	245,028	221,900	23,128	36,764
D	189,420	0	189,420	189,896	-476	36,288

TABLE VIII

ANALYSIS FOR PRODUCTION
95% LEVEL OF CONFIDENCE

Month	A Expected Sales	B Cumulative Sales Requirements	C Required Inventory	D Cumulative Produc- tion Requirements	$(B_n + C_n) - (C_{n-1})$ Required Produc- tion for Month
		(All Figures in Barrels of Cement)			
D			32,000		
J	130,620	130,620	32,088	130,708	130,708
F	115,192	245,812	14,196	228,008	97,300
M	178,500	424,312	65,072	457,384	229,376
A	213,136	637,448	44,220	649,668	192,284
M	240,800	878,248	35,812	882,060	232,392
J	184,996	1,063,244	57,428	1,088,672	206,612
J	342,496	1,405,740	67,928	1,441,668	352,996
A	376,292	1,782,032	47,992	1,798,024	356,356
S	285,516	2,067,548	77,196	2,112,744	314,720
O	272,692	2,340,240	17,528	2,325,976	213,024
N	221,900	2,562,140	47,236	2,577,584	251,608
D	189,896	2,752,036	46,648	2,766,892	189,308

Month Required	Amount Required	Type of Production	Month of Production																											
			Jan.		Feb.		Mar.		Apr.		May		June		July		Aug.		Sept.		Oct.		Nov.		Dec.		Total			
			New	Old	New	Old	New	Old	New	Old	New	Old	New	Old	New	Old	New	Old	New	Old	New	Old	New	Old	New	Old				
Capacity	267,800	216,800	241,900	195,800	267,800	216,800	259,200	209,800	267,800	216,800	259,200	209,800	267,800	216,800	267,800	216,800	259,200	209,800	267,800	216,800	259,200	209,800	267,800	216,800	259,200	209,800		267,800	216,800	
Jan.	130,708	Available Cost Plan	267,800 0 130,708	216,800 10																										
Feb.	97,300	Available Cost Plan	137,092 6.25	216,800 16.25	241,900 0 97,300	195,800 10																								
Mar.	229,376	Available Cost Plan	12.50	22.50	144,600 6.25	195,800 16.25	267,800 0 229,376	216,800 10																						
Apr.	192,284	Available Cost Plan	18.75	28.75	12.50	22.50	37,424 6.25	216,800 16.25	259,200 0 192,284	209,800 10																				
May	232,392	Available Cost Plan	25.00	35.00	18.75	28.75	12.50	22.50	66,716 6.25	209,800 16.25	267,800 0	216,800 10																		
June	206,612	Available Cost Plan	31.25	41.25	25.00	35.00	18.75	28.75	12.50	22.50	35,408 6.25	216,800 16.25	259,200 0 206,612	209,800 10																
July	352,996	Available Cost Plan	37.50	47.50	31.25	41.25	25.00	35.00	18.75	28.75	12.50	22.50	52,588 6.25 52,588	209,800 16.25	267,800 0 267,800	216,800 10 32,608														
Aug.	356,356	Available Cost Plan	43.75	53.75	37.50	47.50	31.25	41.25	25.00	35.00	18.75	28.75	0 12.50	22.50	0 6.25	184,192 16.25	267,800 0 267,800	216,800 10 88,556												
Sept.	311,720	Available Cost Plan	50.00	60.00	43.75	53.75	37.50	47.50	31.25	41.25	25.00	35.00	18.75	28.75	12.50	22.50	0 6.25	128,244 16.25	259,200 0 259,200	209,800 10 55,520										
Oct.	213,024	Available Cost Plan	56.25	66.25	50.00	60.00	43.75	53.75	37.50	47.50	31.25	41.25	25.00	35.00	18.75	28.75	12.50	22.50	0 6.25	154,280 16.25	267,800 0 213,232	216,800 10								
Nov.	251,608	Available Cost Plan	62.50	72.50	56.25	66.25	50.00	60.00	43.75	53.75	37.50	47.50	31.25	41.25	25.00	35.00	18.75	28.75	12.50	22.50	54,568 6.25	216,800 16.25	259,200 0 251,608	209,800 10						
Dec.	189,308	Available Cost Plan	68.75	78.75	62.50	72.50	56.25	66.25	50.00	60.00	43.75	53.75	37.50	47.50	31.25	41.25	25.00	35.00	18.75	28.75	12.50	22.50	7,392 6.25	209,800 16.25	267,800 0 189,308	216,800 10				
		Total Production Plan	130,708		97,300		229,376		192,284		232,392		259,200		267,800	32,608	267,800	88,556	259,200	55,520	213,024		251,608		189,308					

Figure 5. Transportation Solution of Production - 95% Level of Confidence.

TABLE IX
ANALYSIS OF TRANSPORTATION SOLUTION
95% LEVEL OF CONFIDENCE

Month	New Plant	Old Plant	Total	Sales	Balance [±] Inventory	Inventory
(All Figures in Barrels of Cement)						
D						32,000
J	130,708	0	130,708	130,620	88	32,088
F	97,300	0	97,300	115,192	-17,892	14,196
M	229,376	0	229,376	178,500	50,876	65,072
A	192,284	0	192,284	213,136	-20,852	44,220
M	232,392	0	232,392	240,800	- 8,408	35,812
J	259,200	0	259,200	184,996	74,204	110,816
J	267,800	32,608	300,408	342,496	-42,088	67,928
A	267,800	88,556	356,356	376,292	-19,936	47,992
S	259,200	55,520	314,720	285,516	29,204	77,196
O	213,024	0	213,024	272,692	-59,668	17,528
N	251,608	0	251,608	221,900	29,708	47,236
D	189,308	0	189,308	189,896	- 588	46,648

TABLE X
ANALYSIS FOR PRODUCTION
97.5% LEVEL OF CONFIDENCE

Month	A Expected Sales	B Cumulative Sales Requirements	C Required Inventory	D Cumulative Produc- tion Requirements	$(B_n + C_n) - (C_{n-1})$ Required Produc- tion for Month
(All Figures in Barrels of Cement)					
D			32,000		
J	130,620	130,620	38,248	136,868	136,868
F	115,192	245,812	16,884	230,696	93,828
M	178,500	424,312	77,532	469,844	239,148
A	213,136	637,448	53,900	659,348	189,504
M	240,800	878,248	42,672	888,920	229,572
J	184,996	1,063,244	68,432	1,099,676	210,756
J	342,496	1,405,740	80,948	1,454,688	355,012
A	376,292	1,782,032	57,176	1,807,208	352,520
S	285,516	2,067,548	91,840	2,127,388	320,180
O	272,692	2,340,240	20,860	2,329,100	201,712
N	221,900	2,562,140	56,280	2,586,420	257,320
D	189,896	2,752,036	55,580	2,775,616	189,196

Month Required	Amount Required	Type of Production	Month of Production																												
			Jan.		Feb.		Mar.		Apr.		May		June		July		Aug.		Sept.		Oct.		Nov.		Dec.		Total				
			New	Old	New	Old	New	Old	New	Old	New	Old	New	Old	New	Old	New	Old	New	Old	New	Old	New	Old	New	Old					
		Capacity	267,800	216,800	211,900	195,800	267,800	216,800	259,200	209,800	267,800	216,800	259,200	209,800	267,800	216,800	267,800	216,800	259,200	209,800	267,800	216,800	259,200	209,800	267,800	216,800	259,200	209,800	267,800	216,800	
Jan.	136,868	Available Cost Plan	267,800 0 136,868	216,800 10																											
Feb.	93,828	Available Cost Plan	130,932 6.25	216,800 16.25	211,900 0 93,828	195,800 10																									
Mar.	239,118	Available Cost Plan	12.50	22.50	118,072 6.25	195,800 16.25	267,800 0 239,118	216,800 10																							
Apr.	189,504	Available Cost Plan	18.75	28.75	12.50	22.50	28,652 6.25	216,800 16.25	259,200 0 189,504	209,800 10																					
May	229,572	Available Cost Plan	25.00	35.00	18.75	28.75	12.50	22.50	69,696 6.25	209,800 16.25	267,800 0 229,572	216,800 10																			
June	210,756	Available Cost Plan	31.25	41.25	25.00	35.00	18.75	28.75	12.50	22.50	38,228 6.25	216,800 16.25	259,200 0 210,756	209,800 10																	
July	355,012	Available Cost Plan	37.50	47.50	31.25	41.25	25.00	35.00	18.75	28.75	12.50	22.50	48,114 6.25 48,114	209,800 16.25	267,800 0 267,800	216,800 10 38,768															
Aug.	352,520	Available Cost Plan	43.75	53.75	37.50	47.50	31.25	41.25	25.00	35.00	18.75	28.75	0 12.50	22.50	0 6.25	176,032 16.25	267,800 0 267,800	216,800 10 81,720													
Sept.	320,180	Available Cost Plan	50.00	60.00	43.75	53.75	37.50	47.50	31.25	41.25	25.00	35.00	18.75	28.75	12.50	22.50	0 6.25	132,080 16.25	259,200 0 259,200	209,800 10 60,980											
Oct.	201,712	Available Cost Plan	56.25	66.25	50.00	60.00	43.75	53.75	37.50	47.50	31.25	41.25	25.00	35.00	18.75	28.75	12.50	22.50	0 6.25	118,820 16.25	267,800 0 201,712	216,800 10									
Nov.	257,320	Available Cost Plan	62.50	72.50	56.25	66.25	50.00	60.00	43.75	53.75	37.50	47.50	31.25	41.25	25.00	35.00	18.75	28.75	12.50	22.50	66,088 6.25	216,800 16.25	259,200 0 257,320	209,800 10							
Dec.	189,196	Available Cost Plan	68.75	78.75	62.50	72.50	56.25	66.25	50.00	60.00	43.75	53.75	37.50	47.50	31.25	41.25	25.00	35.00	18.75	28.75	12.50	22.50	1,880 6.25	209,800 16.25	267,800 0 189,196	216,800 10					
		Total Production Plan	136,868		93,828		239,118		189,504		229,572		259,200		257,800	38,768	267,800	81,720	259,200	60,980	201,712		257,320		189,196						

Figure 6. .Transportation Solution of Production - 97.5% Level of Confidence.

TABLE XI
ANALYSIS OF TRANSPORTATION SOLUTION
97.5% LEVEL OF CONFIDENCE

Month	New Plant	Old Plant	Total	Sales	Balance †	Inventory	Inventory
(All Figures in Barrels of Cement)							
D							32,000
J	136,868	0	136,868	130,620	6,248		38,248
F	93,828	0	93,828	115,192	-21,364		16,884
M	239,148	0	239,148	178,500	60,648		77,532
A	189,504	0	189,504	213,136	-23,632		53,900
M	229,572	0	229,572	240,800	-11,228		42,672
J	259,200	0	259,200	184,996	74,204		116,876
J	267,800	38,768	306,568	342,496	-35,928		80,948
A	267,800	84,720	352,520	376,292	-23,772		57,176
S	259,200	60,980	320,180	285,516	34,664		91,840
O	201,712	0	201,712	272,692	-70,980		20,860
N	257,320	0	257,320	221,900	35,420		56,280
D	189,196	0	189,196	189,896	- 700		55,580

TABLE XII
ANALYSIS FOR PRODUCTION
99% LEVEL OF CONFIDENCE

Month	A Expected Sales	B Cumulative Sales Requirements (All Figures in Barrels of Cement)	C Required Inventory	D Cumulative Produc- tion Requirements	$(B_n + C_n) - (C_{n-1})$ Required Produc- tion for Month
D			32,000		
J	130,620	130,620	45,472	144,092	144,092
F	115,192	245,812	20,104	233,916	89,824
M	178,500	424,312	92,204	484,516	250,600
A	213,136	637,448	64,064	669,512	184,996
M	240,800	878,248	50,736	896,984	227,472
J	184,996	1,063,244	81,340	1,112,584	215,600
J	342,496	1,405,740	96,208	1,469,948	357,364
A	376,292	1,782,032	67,956	1,817,988	348,040
S	285,516	2,067,548	109,340	2,144,888	326,900
O	272,692	2,340,240	24,808	2,333,048	188,160
N	221,900	2,562,140	66,920	2,597,060	264,012
D	189,896	2,752,036	66,080	2,786,116	189,056

Month Required	Amount Required	Type of Production	Month of Production																									
			Jan.		Feb.		Mar.		Apr.		May		June		July		Aug.		Sept.		Oct.		Nov.		Dec.		Total	
			New	Old	New	Old	New	Old	New	Old	New	Old	New	Old	New	Old	New	Old	New	Old	New	Old	New	Old	New	Old		
		Capacity	267,600	216,800	211,900	195,800	267,800	216,800	259,200	209,600	267,800	216,800	259,200	209,600	267,800	216,800	267,800	216,800	259,200	209,600	267,800	216,800	259,200	209,600	267,800	216,800		
Jan.	114,092	Available Cost Plan	267,800 0 114,092	216,800 10																								
Feb.	89,824	Available Cost Plan	123,708 6.25	216,800 16.25	211,900 0 89,824	195,800 10																						
Mar.	250,600	Available Cost Plan	12.50	22.50	152,076 6.25	195,800 16.25	267,800 0 250,600	216,800 10																				
Apr.	184,996	Available Cost Plan	18.75	28.75	12.50	22.50	17,200 6.25	216,800 16.25	259,200 0 184,996	209,600 10																		
May	227,472	Available Cost Plan	25.00	35.00	18.75	28.75	12.50	22.50	74,204 6.25	209,600 16.25	267,800 0 227,472	216,800 10																
June	215,600	Available Cost Plan	31.25	41.25	25.00	35.00	18.75	28.75	12.50	22.50	10,328 6.25	216,800 16.25	259,200 0 215,600	209,600 10														
July	357,364	Available Cost Plan	37.50	47.50	31.25	41.25	25.00	35.00	18.75	28.75	12.50	22.50	43,600 6.25 43,600	209,600 16.25	267,800 0 267,800	216,800 10 45,964												
Aug.	348,010	Available Cost Plan	43.75	53.75	37.50	47.50	31.25	41.25	25.00	35.00	18.75	28.75	0 12.50	22.50	0 6.25	170,636 16.25	267,800 0 267,800	216,800 10 80,240										
Sept.	326,900	Available Cost Plan	50.00	60.00	43.75	53.75	37.50	47.50	31.25	41.25	25.00	35.00	18.75	28.75	12.50	22.50	0 6.25	136,560 16.25	259,200 0 259,200	209,600 10 67,700								
Oct.	188,160	Available Cost Plan	56.25	66.25	50.00	60.00	43.75	53.75	37.50	47.50	31.25	41.25	25.00	35.00	18.75	28.75	12.50	22.50	0 6.25	112,100 16.25	267,800 0 155,160	216,800 10						
Nov.	261,012	Available Cost Plan	62.50	72.50	56.25	66.25	50.00	60.00	43.75	53.75	37.50	47.50	31.25	41.25	25.00	35.00	18.75	28.75	12.50	22.50	79,640 6.25 4,612	216,800 16.25	259,200 0 259,200	209,600 10				
Dec.	189,056	Available Cost Plan	68.75	78.75	62.50	72.50	56.25	66.25	50.00	60.00	43.75	53.75	37.50	47.50	31.25	41.25	25.00	35.00	18.75	28.75	71,828 12.50	216,800 22.50	0 6.25	209,600 16.25	267,800 0 169,056	216,800 10		
		Total Production Plan	114,092		89,824		250,600		184,996		227,472		215,600		357,364		348,010		326,900		188,160		261,012		189,056			

Figure 7.. Transportation Solution of Production - 99% Level of Confidence.

TABLE XIII
ANALYSIS OF TRANSPORTATION SOLUTION
99% LEVEL OF CONFIDENCE

Month	New Plant	Old Plant	Total	Sales	Balance †	Inventory	Inventory
(All Figures in Barrels of Cement)							
D							32,000
J	144,092	0	144,092	130,620	13,472		45,472
F	89,824	0	89,824	115,192	-25,368		20,104
M	250,600	0	250,600	178,500	72,100		92,204
A	184,996	0	184,996	213,136	-28,140		64,064
M	227,472	0	227,472	240,800	-13,328		50,736
J	259,200	0	259,200	184,996	74,204		124,940
J	267,800	45,964	313,764	342,496	-28,732		96,208
A	267,800	80,240	348,040	376,292	-28,252		67,956
S	259,200	67,700	326,900	285,516	41,384		109,340
O	192,972	0	192,972	272,692	-79,720		29,620
N	259,200	0	259,200	221,900	37,300		66,920
D	189,056	0	189,056	189,896	- 840		66,080

Cost Analysis of Alternate Production Plans

General Solution:

Production cost per unit on New Facilities x Amount produced	= \$X.XX
Production cost per unit on Old Facilities x Amount produced	= X.XX
Sum of average inventories for each month x monthly charge	= <u>X.XX</u>
Total Cost	\$X.XX

Note: Because of a production charge of zero on new facilities, this calculation will not be shown.

Production Cost

Plan	New	Old	Cost	Total
90 %	-	167,500	\$.10	\$16,750.00
95 %	-	176,684	.10	17,668.40
97.5%	-	184,468	.10	18,446.80
99 %	-	193,904	.10	19,390.40

Inventory Cost

Plan	Σ Average Inventory	Cost	Total Cost
90 %	486,572	\$.0625	\$30,410.75
95 %	599,408	.0625	34,963.00
97.5%	697,006	.0625	43,562.87
99 %	816,604	.0625	51,037.75

Total Production Cost

Plan	Production Cost	Inventory Cost	Total Cost
90 %	\$16,750.00	\$30,410.75	\$47,160.75
95 %	17,668.40	34,963.00	52,631.40
97.5%	18,446.80	43,562.87	62,009.67
99 %	19,390.40	51,037.75	70,428.15

VITA

Joe Marion Bohannon

Candidate for the Degree of

Master of Science

Thesis: PRODUCTION AND INVENTORY CONTROL IN A PORTLAND CEMENT PLANT

Major Field: Industrial Engineering and Management

Biographical:

Personal Data: Born in Tulsa, Oklahoma, September 1, 1936, the son of Ralph M. and Irene Bohannon.

Education: Attended elementary and junior high school in Tulsa and Oklahoma City, Oklahoma; graduated from Classen High School, Oklahoma City, Oklahoma, in May, 1954; received the Bachelor of Science degree from the Oklahoma State University, with a major in Industrial Engineering and Management, in August, 1958; completed requirements for a Master of Science degree in May, 1959.

Professional Experience: Worked at Tinker Air Force Base located in Oklahoma City, Oklahoma, during the summers of 1956 and 1957 as an industrial engineering trainee; worked during the month of August, 1958, as an industrial engineer at the same location. Commissioned as a 2nd Lieutenant in the United States Army, Ordnance Branch.

College Activities: Member of Phi Kappa Phi, honorary scholastic; Sigma Tau, honorary general engineering; Alpha Pi Mu, honorary industrial engineering; Blue Key, honorary leadership; served as assistant editor of Oklahoma State Engineer magazine; student member of American Institute of Industrial Engineers; served as president of Kappa Alpha Order, social fraternity; received Alpha Pi Mu senior award; received St. Pat's Salute senior award.