ANALYSIS OF INTERNALLY INDETERMINATE TRUSSES BY CARRY-OVER FORCES

By

MICHEL KYRIACO ANTARAKIS

Bachelor of Science

University of Alexandria

Alexandria, Egypt

1957

Submitted to the Faculty of the Graduate School
of the Oklahoma State University of
Agriculture and Applied Science
in partial fulfillment of
the requirements for
the degree of
MASTER OF SCIENCE
1959

ANALYSIS OF INTERNALLY INDETERMINATE

TRUSSES BY CARRY-OVER FORCES

Thesis Approved:

Thesis Adviser

Dean of the Graduate School

PREFACE

I wish to acknowledge my indebtedness to the following indivi-viduals and organizations.

To Professor Jan J. Tuma, Head of the School of Civil Engineering, for his invaluable guidance, inspiration, and constructive criticism in the preparation of this thesis, and for acting as my major adviser during my graduate studies. I am also indebted to him for giving me the opportunity to teach in the School of Civil Engineering.

To the faculty of the School of Civil Engineering for awarding me a teaching and research assistantship which made my dream to come to the United States for graduate work a reality, and for awarding me the Robberson Steel Fellowship.

To the Robberson Steel Company for their fellowship which helped me complete my present graduate work and start work toward my Doctor's Degree.

To Professor R. L. Flanders, Head Emeritus, School of Civil Engineering, for careful reading of the manuscript.

To Mrs. Robby Tollison, for her excellent work in typing the manuscript.

M. K. A.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

NOMENCLATURE

Major Symbols

d_m	Length of Bar
d or d'	Distance
ω_m	Angle
A_m	Cross-Sectional Area of Bar
A'_m	Relative Cross-Sectional Area of Bar
a	Axial Deformation
a'	Relative Axial Deformation
E	Modulus of Elasticity in Tension or Compression
N	Axial Force
BN	Axial Force in Member of Primary System Due to External Loads
X	Redundant Force
S	Influence Number
P	External Load
U	Strain Energy
λ_m	Axial Flexibility
λ'_m	Relative Axial Flexibility

Subscripts

m	Panel
T	Top

PART I

INTRODUCTION

A general systematic procedure for the analysis of internally statically indeterminate coplanar trusses with any number of redundant members is presented in this thesis. The truss may be of constant or varying depth.

Trusses with internal redundants are being analyzed either by approximate methods (4), or by the exact stress analysis.

The approximate analysis is performed by means of equations of statics and certain assumptions. Some of these assumptions are:

a. All diagonals resist only tension

b. In each panel the shear is equally divided between diagonals

c. The indeterminate truss is assumed equivalent to two or more statically determinate ones superimposed on each other.

Assumptions (a) and (b) are frequently used in analyzing parallel-chord trusses with two diagonals in each panel. However these assumptions do not prove to be useful for trusses of variable depth.

In the exact stress analysis the procedure is as follows.

1. A primary system (statically determinate), is chosen by removing the redundant members. For the given external loads, the forces in the members of this system are then determined from statics.

2. Using the method of least work (5) or the method of consistent deformation (6) a set of deformation equations equal in number

to the unknown redundants can be set up. A solution of these simutaneous equations gives the values of the redundants.

3. The final axial forces are obtained by the superposition of the forces due to loads and redundants.

When the number of redundants is large the classical solution of the deformation equations is very laborious. A method of successive approximation is suggested by Timoshenko (3). According to his suggestion the first approximation for the unknowns is obtained by dividing the shear in each panel equally between diagonals. This procedure is limited to trusses of constant depth.

The deformation equation is a general three-force equation, similar to the three-moment equation used in the analysis of continuous beams (3). The similarity of the three-force equation to the three-moment equation and an approach similar to the carry-over moment procedure (2) for the solution of these trusses was presented by professor J. J. Tuma in his course CE-5A4- Theory of Structures III in the summer of 1958.

In this thesis the author has endeavored to explain the physical interpretation of the carry-over force procedure and to present a systematic analysis which can be a very useful tool for the investigation of indeterminate trusses. Expressing the influence numbers in terms of bar lengths (7) is extended to panels of a truss of varying depth. This enables a quick determination of these influence numbers which are used to find relative deformations and the carry-over force factors.

Two examples are presented which illustrate the numerical carry-over force procedure.

PART II

EQUATIONS OF DEFORMATION-THREE FORCE EQUATION

1. Equations of Deformation

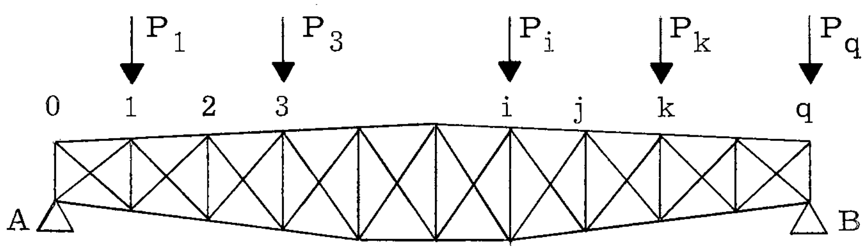

Fig. 1a
Simply Supported Indeterminate Truss $\overline{AB}$

A simply supported truss (Fig. 1a) subjected to a general system of loads is considered.

The panels are denoted by $0, 1, 2, 3, \ldots, i, j, k, \ldots, q$, and m will denote any panel of the truss.

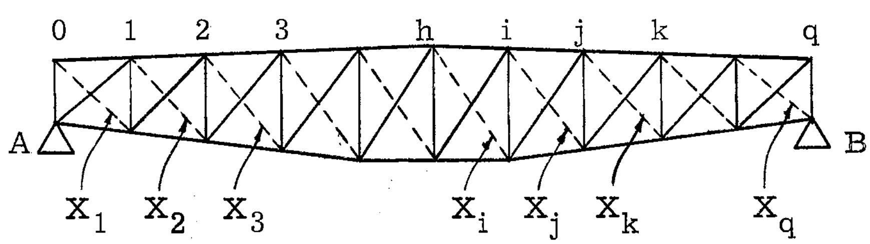

Fig. 1b
Redundant Members (Shown by Dotted Lines)

Redundant members are denoted by $X_1, X_2, X_3, \ldots, X_i, X_j, X_k \ldots,$ X_q corresponding to panels $1, 2, 3, \ldots, i, j, k, \ldots, q$ respectively, as shown in (Fig. 1b).

The total axial force in any bar of panel m

$$N_m = BN_m + X_1 S_{m1} + X_2 S_{m2} + \ldots + X_i S_{mi} + X_j S_{mj} + X_k S_{mk} + \ldots + X_q S_{mq} \quad . \tag{1}$$

Where

BN_m = Axial force in bar m of primary system due to loads

S_{mj} = Axial force in bar m due to $X_j = 1$.

The expression S_{mj} is hereafter called the influence number.

The strain energy of the truss

$$U = \frac{1}{2} \sum_{m=1}^{m=q} N_m^2 \lambda_m \tag{2}$$

where

$$\lambda_m = \frac{d_m}{A_m E} \quad . \tag{3}$$

From the principle of least work

$$\frac{\partial U}{\partial X_j} = 0 = \sum_{m=1}^{m=q} N_m \frac{\partial N_m}{\partial x_j} \lambda_m \quad . \tag{4j}$$

Substituting for N_m from Eq. (1)

$$\sum_{m=1}^{m=q} (BN_m + \ldots + X_i S_{mi} + X_j S_{mj} + X_k S_{mk} + \ldots + X_q S_{mq}) S_{mj} \lambda_m = 0$$

$$\sum_{m=1}^{m=q} BN_m S_{mj} \lambda_m + \dots + X_i \sum_{m=1}^{m=q} S_{mj} S_{mi} \lambda_m + X_j \sum_{m=1}^{m=q} S_{mj} S_{mj} \lambda_m$$

$$+ X_k \sum_{m=1}^{m=q} S_{mj} S_{mk} \lambda_m + \dots + X_q \sum_{m=1}^{m=1} S_{mj} S_{mq} \lambda_m = 0 , \qquad (5j)$$

Similarly from $\frac{\partial U}{\partial X_i} = 0$

$$\sum BN_m S_{mi} \lambda_m + \dots + X_i \sum S_{mi} S_{mi} \lambda_m + X_j \sum S_{mi} S_{mj} \lambda_m + X_k \sum S_{mi} S_{mk} \lambda_m$$

$$+ \dots + X_q \sum S_{mi} S_{mq} \lambda_m = 0 \quad . \qquad (5i)$$

From $\frac{\partial U}{\partial X_k} = 0$

$$\sum BN_m S_{mk} \lambda_m + \dots + X_i \sum S_{mk} S_{mi} \lambda_m + X_j \sum S_{mk} S_{mj} \lambda_m + X_k \sum S_{mk} S_{mk} \lambda_m$$

$$+ \dots + X_q \sum S_{mk} S_{mq} \lambda_m = 0 \quad . \qquad (5k)$$

And from $\frac{\partial U}{\partial X_q} = 0$

$$\sum BN_m S_{mq} \lambda_m + \dots + X_i \sum S_{mq} S_{mi} \lambda_m + X_j \sum S_{mq} S_{mj} \lambda_m + X_k \sum S_{mq} S_{mk} \lambda_m$$

$$+ \dots + X_q \sum S_{mq} S_{mq} \lambda_m = 0 \quad . \qquad (5q)$$

Denoting in equation (5j)

$$\sum_{m=1}^{m=q} BN_m S_{mj} \lambda_m = a_{jo} =$$ Deformation of the primary system in the direction of X_j due to loads

$$\sum_{m=1}^{m=q} S_{mj} S_{mi} \lambda_m = a_{ji} =$$ Deformation of the primary system in the direction of X_j due to $X_i = 1$

$$\sum_{m=1}^{m=q} S_{mj} S_{mj} \lambda_m = a_{jj} =$$ Deformation of the primary system in the direction of X_j due to $X_j = 1$ plus the deformation of redundant bar j due to $X_j = 1$

$$\sum_{m=1}^{m=q} S_{mj} S_{mk} \lambda_m = a_{jk} =$$ Deformation of the primary system in the direction of X_j due to $X_k = 1$

$$\sum_{m=1}^{m=q} S_{mj} S_{mq} \lambda_m = a_{jq} =$$ Deformation of the primary system in the direction of X_j due to $X_q = 1$

and using similar notation for equations (5i), (5k) and (5q)

$$a_{io} + \ldots + X_i a_{ii} + X_j a_{ij} + X_k a_{ik} + \ldots + X_q a_{iq} = 0 \quad (6i)$$

$$a_{jo} + \ldots + X_i a_{ji} + X_j a_{jj} + X_k a_{jk} + \ldots + X_q a_{jq} = 0 \quad (6j)$$

$$a_{ko} + \ldots + X_i a_{ki} + X_j a_{kj} + X_k a_{kk} + \ldots + X_q a_{kq} = 0 \quad (6k)$$

$$\vdots$$

$$a_{qo} + \ldots + X_i a_{qi} + X_j a_{qj} + X_k a_{qk} + \ldots + X_q a_{qq} = 0 . \quad (6q)$$

There are as many such deformation equations as redundants. From these Eq's the general three-force Eq's can be derived.

2. General Three-Force Equation

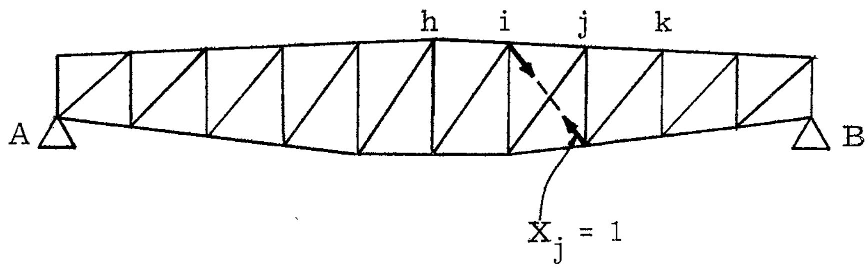

Fig. 2

Primary System with Redundant Member in Panel j

If the axial force in the redundant member of panel j (Fig. 2) is assumed to be equal to unity, the reactions and the axial forces become zero with the exception of those listed below:

S_{Tjj} = Axial force in top bar of panel j due to $X_j = 1$

S_{Bjj} = Axial force in Bottom bar of panel j due to $X_j = 1$

S_{Djj} = Axial force in Diagonal bar of panel j due to $X_j = 1$

S_{Vjj} = Axial force in vertical bar of panel j due to $X_j = 1$

S_{Vij} = Axial force in vertical bar of panel i due to $X_j = 1$

S_{Xjj} = 1

where:

1st subscript denotes position of bar

2nd subscript denotes panel of the bar

3rd subscript denotes position of unit loads .

If the axial force in the redundant member of panel i is assumed to be equal to unity, the reactions and the axial forces become zero with the exception of those listed below:

S_{Tii} = Axial force in top bar of panel i due to $X_i = 1$

S_{Bii} = Axial force in bottom bar of panel i due to $X_i = 1$

S_{Dii} = Axial force in diagonal bar of panel i due to $X_i = 1$

S_{Vii} = Axial force in vertical bar of panel i due to $X_i = 1$

S_{Vhi} = Axial force in vertical bar of panel h due to $X_i = 1$

$S_{Xii} = 1$.

Similarly, due to a unit force in the redundant member of panel k, the reactions and the axial forces are zero with the following exceptions:

S_{Tkk} = Axial force in top bar of panel k due to $X_k = 1$

S_{Bkk} = Axial force in bottom bar of panel k due to $X_k = 1$

S_{Dkk} = Axial force in diagonal bar of panel k due to $X_k = 1$

S_{Vkk} = Axial force in vertical bar of panel k due to $X_k = 1$

S_{Vjk} = Axial force in vertical bar of panel j due to $X_k = 1$

$S_{Xkk} = 1$.

In terms of this notation the deformations in Eq. (6j) become:

$$a_{jo} = BN_{Tj}S_{Tjj}\lambda_{Tj} + BN_{Bj}S_{Bjj}\lambda_{Bj} + BN_{Dj}S_{Djj}\lambda_{Dj} + BN_{Vj}S_{Vjj}\lambda_{Vj} + BN_{Vi}S_{Vij}\lambda_{Vi} \qquad (7j)$$

$$a_{ji} = S_{Vij}S_{Vii}\lambda_{Vi} \qquad (8j)$$

$$a_{jj} = S_{Tjj}^2\lambda_{Tj} + S_{Bjj}^2\lambda_{Bj} + S_{Djj}^2\lambda_{Dj} + S_{Xjj}^2\lambda_{Xj} + S_{Vjj}^2\lambda_{Vj} + S_{Vij}^2\lambda_{Vi} \quad (9j)$$

$$a_{jk} = S_{Vjk}S_{Vjj}\lambda_{Vj} \quad (10j)$$

And equation (6j) reduces to a three-force equation in terms of three adjacent panel redundants and loads.

$$a_{jo} + X_i a_{ji} + X_j a_{jj} + X_k a_{jk} = 0 \quad . \quad (11j)$$

Dividing equation (11j) by a_{jj} and introducing new equivalents

$$C_{ij} = -\frac{a_{ji}}{a_{jj}} \qquad C_{kj} = -\frac{a_{jk}}{a_{jj}} \quad (12j)$$

$$x_j = -\frac{a_{jo}}{a_{jj}} \quad (13j)$$

The three-force equation becomes

$$X_j = C_{ij}X_i + x_j + C_{kj}X_k \quad (14j)$$

The interpretation of the new equivalents follows.

3. Definition of Equivalents

a) Starting force x_j is the redundant force X_j due to loads, if the redundants X_i and X_k are equal to zero.

It must be noted that x_j is the redundant force X_j due to loads, if the truss has only X_j redundant (Fig. 3).

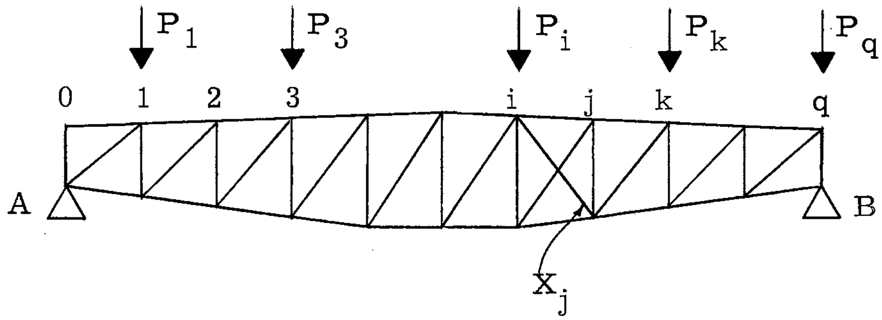

Fig. 3
Truss $\overline{AB}$ with One Redundant X_j

b) <u>Carry-over force factor C_{ij}</u> is the force X_j due to $X_i = 1$ and $X_k = 0$ (Fig. 4) .

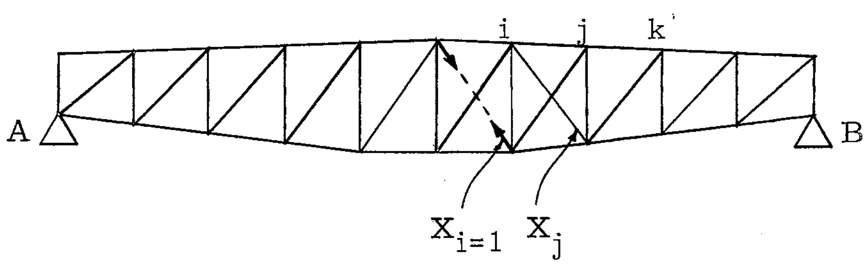

Fig. 4
Truss $\overline{AB}$ with $X_i = 1$ and $X_k = 0$

c) <u>Carry-over force factor C_{kj}</u> is the force X_j due to $X_k = 1$ and $X_i = 0$ (Fig. 5) .

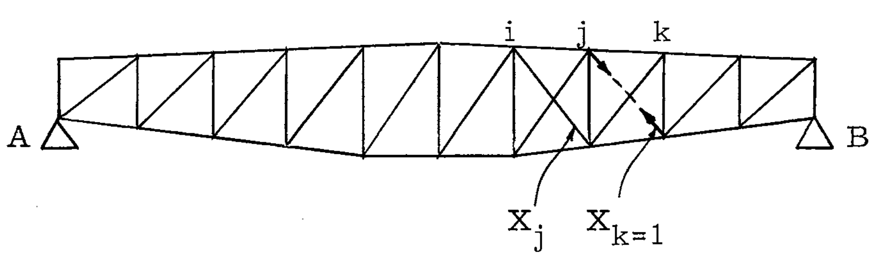

Fig. 5
Truss $\overline{AB}$ with $X_k = 1$ and $X_i = 0$

4 Physical Interpretation

Reference is made to Figs. (3), (4) and (5).

Assuming X_i and X_k temporarily equal to zero the force in X_j called starting force is determined from

$$x_j = - \frac{a_{jo}}{a_{jj}} .$$

If X_i is placed in position, a force is induced in $X_j = C_{ij}X_i$.

If X_k is placed in position, a force is induced in $X_j = C_{kj}X_k$.

This reasoning holds true for all panels. It can be seen that the redundant force in a panel is affected by other panel redundants only if its adjacent panels have redundant members.

At the start the truss is assumed to have no redundants.

Then the starting forces are computed. (The starting force is the redundant in a panel if the redundants in the adjacent panels do not exist).

Once the starting forces have been determined the carry-over force procedure may now be started by tying the redundant bars in the adjacent members of a chosen panel. The influence of each of these redundants on the redundant of the chosen panel is known as the carry-over force. Two such forces exist, the carry-over force to the left and the carry-over force to the right.

Once the carry-over force in the adjacent bars is established it may be easily observed that this force is also a new starting force. The previously described process may now be repeated with this new starting force.

Performing this procedure with all starting forces and repeating it a number of cycles the final value of the redundants is gradually obtained.

The degree of the accuracy of the final answer depends on the rate of convergence and the technique of application. In many cases three to four cycles yield a satisfactory result.

The final answer must satisfy the initial three-force equations.

5. Algebraic Expressions for Influence Numbers

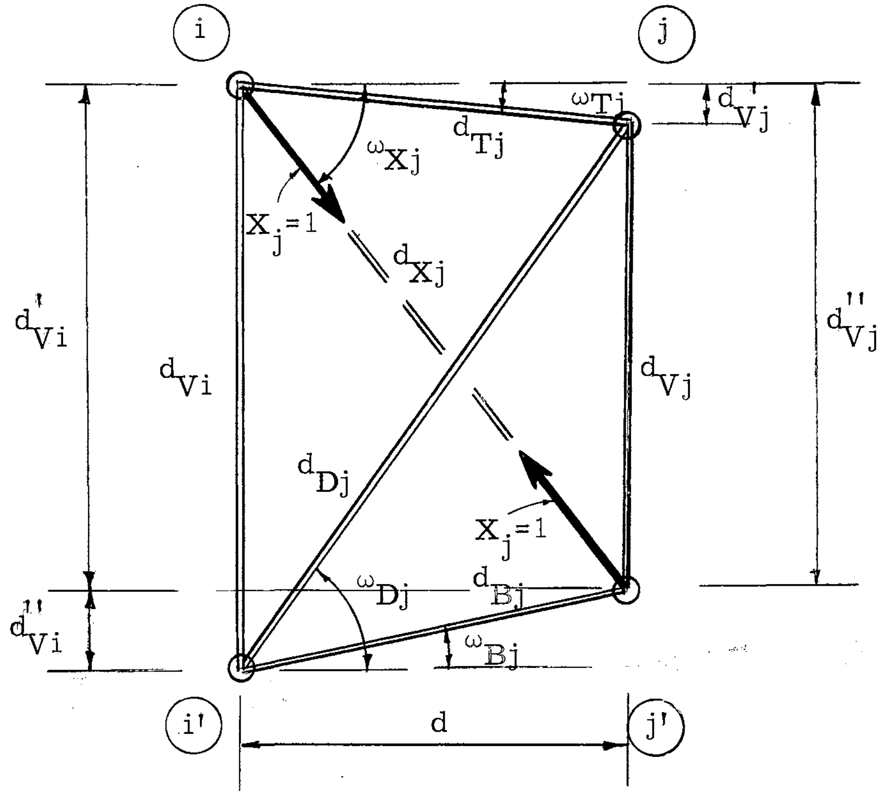

Fig. 6
Panel j of Truss $\overline{AB}$

The derivation of the algebraic expressions for influence numbers of the bars shown in (Fig. 6) is based on the geometric assumptions that

a) The bars $\overline{ii'}$ and $\overline{jj'}$ are parallel

b) The slopes of all bars are measured from the normal to these parallels.

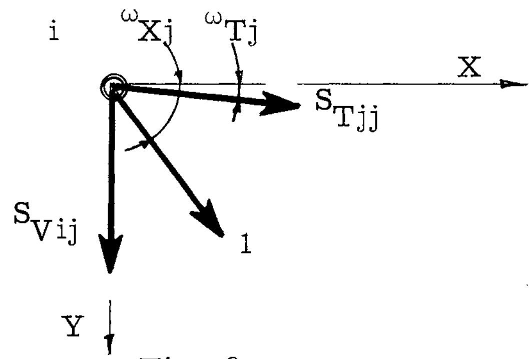

Fig. 6a

Joint i

From static equilibrium (Fig. 6a)

$$\sum F_x = 0$$

$$S_{Tjj} \cos \omega_{Tj} + (1) \cos \omega_{Xj} = 0$$

$$S_{Tjj} = - \frac{\cos \omega_{Xj}}{\cos \omega_{Tj}} = - \frac{\frac{d}{d_{Xj}}}{\frac{d}{d_{Tj}}} = - \frac{d_{Tj}}{d_{Xj}} . \qquad (15j)$$

$$\sum F_y = 0$$

$$S_{Vij} + (1) \sin \omega_{Xj} + S_{Tjj} \sin \omega_{Tj} = 0$$

$$S_{Vij} = - \sin \omega_{Xj} - S_{Tjj} \sin \omega_{Tj} .$$

Substituting

$$S_{Tjj} = - \frac{\cos \omega_{Xj}}{\cos \omega_{Tj}}$$

from equation (15j)

$$S_{Vij} = - \text{Sin } \omega_{Xj} + \text{Cos } \omega_{Xj} \text{ Tan } \omega_{Tj}$$

$$= - \text{Cos } \omega_{Xj} \ (\text{Tan } \omega_{Xj} - \text{Tan } \omega_{Tj})$$

$$= - \frac{d}{d_{Xj}} \left(\frac{d_{Vj}''}{d} - \frac{d_{Vj}'}{d} \right)$$

$$= - \frac{d_{Vj}'' - d_{Vj}'}{d_{Xj}}$$

but $d_{Vj}'' - d_{Vj}' = d_{Vj}$. (Fig. 6)

thus
$$S_{Vij} = - \frac{d_{Vj}}{d_{Xj}} \quad . \qquad (16j)$$

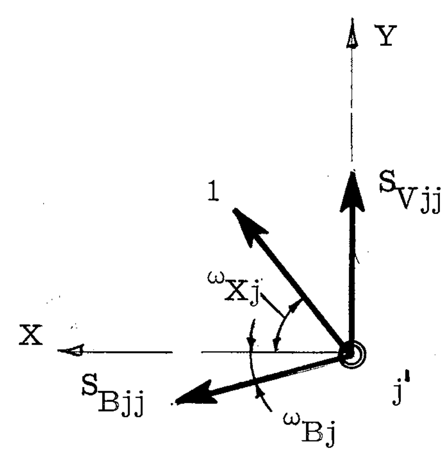

Fig. 6b
Joint j'

From static equilibrium (Fig. 6b)

$$\sum F_X = 0$$

$$S_{Bjj} \text{ Cos } \omega_{Bj} + (1) \text{ Cos } \omega_{Xj} = 0$$

$$S_{Bjj} = - \frac{\text{Cos } \omega_{Xj}}{\text{Cos } \omega_{Bj}} = - \frac{\frac{d}{d_{Xj}}}{\frac{d}{d_{Bj}}} = - \frac{d_{Bj}}{d_{Xj}} \quad . \qquad (17j)$$

$$\sum F_y = 0$$

$$S_{Vjj} + (1)\ \mathrm{Sin}\ \omega_{Xj} - S_{Bjj}\ \mathrm{Sin}\ \omega_{Bj} = 0 \quad .$$

Substituting

$$S_{Bjj} = - \frac{\mathrm{Cos}\ \omega_{Xj}}{\mathrm{Cos}\ \omega_{Bj}} \quad \text{from equation (17j)}$$

$$S_{Vjj} = - \mathrm{Sin}\ \omega_{Xj} - \mathrm{Cos}\ \omega_{Xj}\ \mathrm{Tan}\ \omega_{Bj}$$

$$= - \mathrm{Cos}\ \omega_{Xj}\ (\mathrm{Tan}\ \omega_{Xj} + \mathrm{Tan}\ \omega_{Bj})$$

$$= - \frac{d}{d_{Xj}} \left(\frac{d'_{Vi}}{d} + \frac{d''_{Vi}}{d} \right)$$

$$= - \frac{d'_{Vi} + d''_{Vi}}{d_{Xj}}$$

but $d'_{Vi} + d''_{Vi} = d_{Vi}$ (Fig. 6) .

Thus $$S_{Vjj} = - \frac{d_{Vi}}{d_{Xj}} \quad . \qquad (18j)$$

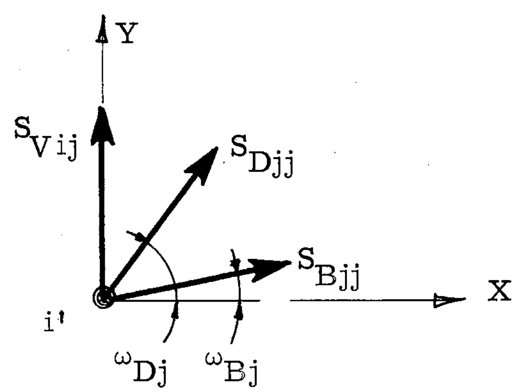

Fig. 6c
Joint i'

From static equilibrium (Fig. 6c)

$$\sum F_x = 0$$

$$S_{Djj} \operatorname{Cos} \omega_{Dj} + S_{Bjj} \operatorname{Cos} \omega_{Bj} = 0$$

$$S_{Djj} = - S_{Bjj} \frac{\operatorname{Cos} \omega_{Bj}}{\operatorname{Cos} \omega_{Dj}}$$

Substituting

$$S_{Bjj} = - \frac{\operatorname{Cos} \omega_{Xj}}{\operatorname{Cos} \omega_{Bj}} \quad \text{from equation (17j)}$$

$$S_{Djj} = + \frac{\operatorname{Cos} \omega_{Xj}}{\operatorname{Cos} \omega_{Dj}} = \frac{\frac{d}{d_{Xj}}}{\frac{d}{d_{Dj}}} = + \frac{d_{Dj}}{d_{Xj}} \tag{19j}$$

Also $S_{Xjj} = 1$.

6. Procedure of Analysis

1. A primary system is chosen and the axial forces due to loads (BN_m) are computed.

2. The carry-over force factors (C's) are determined by means of:

a. Influence numbers [Eq's (15j), (16j), (17j), (18j), (19j)]

b. Axial flexibilities λ_m (Eq. 3)

c. Deformations a [Eq's (7j), (8j), (9j), (10j)] .

From these values according to Eq's (12j)

$$C_{ij} = - \frac{a_{ji}}{a_{jj}}$$

$$C_{kj} = - \frac{a_{jk}}{a_{jj}} .$$

3. The starting force x for each panel is computed from Eq. (13j).

$$x_j = - \frac{a_{jo}}{a_{jj}} .$$

4. The final values of the redundants are obtained by means of the carry-over procedure.

5. The results must satisfy all three force equations (Eq. 14j and similar).

6. The true force in any member (Eq. 1) is

$$N_m = BN_m + \sum_{j=1}^{j=q} X_j S_{mj} .$$

PART III

ILLUSTRATIVE EXAMPLES

Two examples are presented to demonstrate the application of the carry-over force procedure. Relative values of cross-sectional areas, axial flexibilities and deformations are used to calculate the carry-over factors and starting forces. All values are given in inches, kips and kip-inches.

Example 1

A simply supported truss with horizontal top and bottom chords is investigated. The truss has 8 panels and is 8 times internally statically indeterminate. The loading is as shown in Fig. 7.

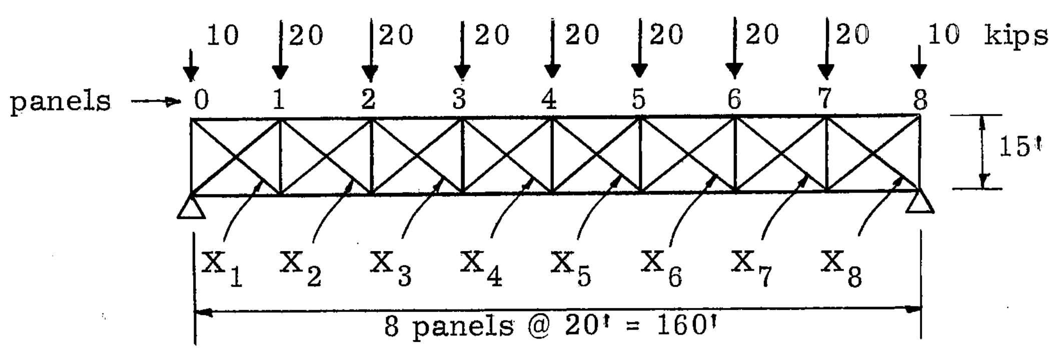

Fig. 7
Simple Indeterminate Truss of Constant Depth

The area of the top and bottom chord is taken equal to twice the area of the vertical and diagonal members.

1. Primary System

The redundant bars are removed and thus the primary system shown in Fig. 8 is introduced.

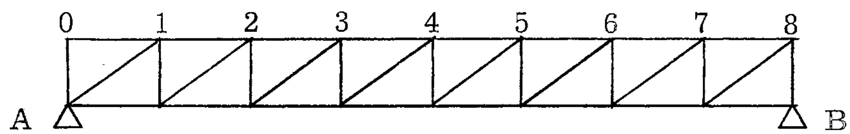

Fig. 8
Primary System (Example 1)

2. Carry-Over Force Factors

Since all panels are alike, all carry-over factors have the same numerical value.

Panel 3 is considered in (Fig. 9)

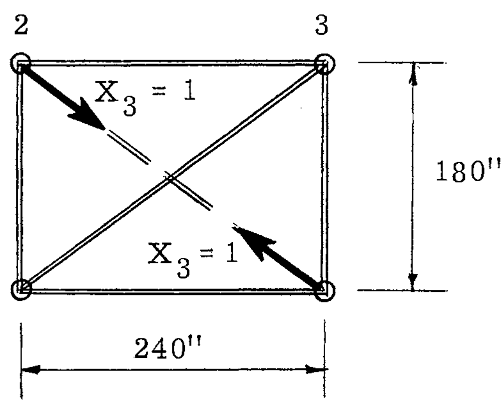

Fig. 9
Panel 3 (Example 1)

a. Influence numbers (Eq's 15j, 16j, 17j 18j, 19j) :

$$S_{T33} = - \frac{d_{T3}}{d_{X3}} = - \frac{240}{300} = - .8$$

$$S_{B33} = - \frac{d_{B3}}{d_{X3}} = - \frac{240}{300} = - .8$$

$$S_{D33} = + \frac{d_{D3}}{d_{X3}} = + \frac{300}{300} = + 1.0$$

$$S_{X33} = + 1.0$$

$$S_{V23} = - \frac{d_{V3}}{d_{X3}} = - \frac{180}{300} = - .6$$

$$S_{V33} = - \frac{d_{V2}}{d_{X3}} = - \frac{180}{300} = - .6$$

b. Relative axial flexibilities (Eq. 3) :

$$A'_{Top} = A'_{Bottom} = 1.00$$

$$A'_{Diagonal} = A'_{Redundant} = A'_{Vertical} = .50 \quad .$$

Thus

$$\lambda'_{T3} = \lambda'_{B3} = \frac{d_{T3}}{1} = \frac{240}{1} = 240$$

$$\lambda'_{D3} = \lambda'_{X3} = \frac{d_{D3}}{.5} = \frac{300}{.5} = 600$$

$$\lambda'_{V3} = \lambda'_{V3} = \frac{d_{V3}}{.5} = \frac{180}{.5} = 360 \quad .$$

The influence numbers and axial flexibilities are tabulated in Table (1-1) .

c. Relative deformations (Eq's 7j, 8j, 9j, 10j and Table 1-1)).

$$a'_{10} = (0.00 - 17920 - 70000 + 2160 - 10800) = - 96560$$

$$a'_{20} = (17920 - 30720 - 50000 - 10800 - 6480) = - 80080$$

INFLUENCE NUMBERS AND RELATIVE AXIAL FLEXIBILITIES

TABLE (1-1)

	m	d_m	λ'_m	BN_m	S_{m1}	S_{m2}	S_{m3}	S_{m4}	S_{m5}	S_{m6}	S_{m7}	S_{m8}	$BN_m S_{mi} \lambda'_m$	
Top	1	240	240	0.00	-.800								0.00	
Top	2	240	240	- 93.33		-.800							+ 17920	
Top	3	240	240	- 160.00			-.800						+ 30720	
Top	4	240	240	- 200.00				-.800					+ 38400	
Top	5	240	240	- 213.33					-.800				+ 40960	
Top	6	240	240	- 200.00						-.800			+ 38400	
Top	7	240	240	- 160.00							-.800		+ 30720	
Top	8	240	240	- 93.33								-.800	+ 17920	
Bottom	1	240	240	+ 93.33	-.800								- 17920	
Bottom	2	240	240	+ 160.00		-.800							- 30720	
Bottom	3	240	240	+ 200.00			-.800						- 38400	
Bottom	4	240	240	+ 213.33				-.800					- 40960	
Bottom	5	240	240	+ 200.00					-.800				- 38400	
Bottom	6	240	240	+ 160.00						-.800			- 30720	
Bottom	7	240	240	+ 93.33							-.800		- 17920	
Bottom	8	240	240	+ 0.00								-.800	0.00	
Diagonal	1	300	600	- 116.67	+1.00								- 70000	
Diagonal	2	300	600	- 83.33		+1.00							- 50000	
Diagonal	3	300	600	- 50.00			+1.00						- 30000	
Diagonal	4	300	600	- 16.67				+1.00					- 10000	
Diagonal	5	300	600	+ 16.67					+1.00				+ 10000	
Diagonal	6	300	600	+ 50.00						+1.00			+ 30000	
Diagonal	7	300	600	+ 83.33							+1.00		+ 50000	
Diagonal	8	300	600	+ 116.67								+1.00	+ 70000	
Diagonal Redundant	1	300	600	0.00	+1.00								0.00	
Diagonal Redundant	2	300	600	0.00		+1.00							0.00	
Diagonal Redundant	3	300	600	0.00			+1.00						0.00	
Diagonal Redundant	4	300	600	0.00				+1.00					0.00	
Diagonal Redundant	5	300	600	0.00					+1.00				0.00	
Diagonal Redundant	6	300	600	0.00						+1.00			0.00	
Diagonal Redundant	7	300	600	0.00							+1.00		0.00	
Diagonal Redundant	8	300	600	0.00								+1.00	0.00	
Vertical	0	180	360	- 10.00	-.600									2160
Vertical	1	180	360	+ 50.00	-.600	-.600							-10800	-10800
Vertical	2	180	360	+ 30.00		-.600	-.600						- 6480	- 6480
Vertical	3	180	360	+ 10.00			-.600	-.600					- 2160	- 2160
Vertical	4	180	360	- 10.00				-.600	-.600				+ 2160	+ 2160
Vertical	5	180	360	- 30.00					-.600	-.600			+ 6480	+ 6480
Vertical	6	180	360	- 50.00						-.600	-.600		+10800	+10800
Vertical	7	180	360	- 70.00							-.600	-.600	+15120	+15120
Vertical	8	180	360	- 80.00								-.600	+17280	

$$a'_{30} = (30720 - 38400 - 30000 - 6480 - 2160) = -46320$$

$$a'_{40} = (38400 - 40960 - 10000 - 2160 + 2160) = -12560$$

$$a'_{50} = (40960 - 38400 + 10000 + 2160 + 6480) = 21200$$

$$a'_{60} = (38400 - 30720 + 30000 + 6480 + 10800) = 54960$$

$$a'_{70} = (30720 - 17920 + 50000 + 10800 + 15120) = 88720$$

$$a'_{80} = (17920 + 0.00 + 70000 + 15120 + 17280) = 120320$$

$$a'_{32} = (-.6)\ (-.6)\ (360) = 129.6$$

$$a'_{33} = (-.8)^2\ (240) + (-.8)^2\ (240) + (1)^2\ (600) + (1)^2\ (600) + (-.6)^2\ (360) + (-.6)^2\ (360) = 1766.4$$

$$a'_{34} = (-.6)\ (-.6)\ (360) = 129.6$$

Substituting in Eq's (12j) the carry-over force factors are:

$$C_{23} = -\frac{a_{32}}{a_{33}} = -\frac{129.6}{1766.4} = -.0734$$

$$C_{43} = -\frac{a_{34}}{a_{33}} = -\frac{129.6}{1766.4} = -.0734$$

3. Starting Forces (Eq's 13j)

$$x_1 = -\frac{a_{10}}{a_{11}} = -\frac{-96560}{1766.4} = 54.665$$

$$x_2 = -\frac{a_{20}}{a_{22}} = -\frac{-80080}{1766.4} = 45.335$$

$$x_3 = -\frac{a_{30}}{a_{23}} = -\frac{-46320}{1766.4} = 26.223$$

$$x_4 = -\frac{a_{40}}{a_{44}} = -\frac{-12560}{1766.4} = 7.111$$

$$x_5 = -\frac{a_{50}}{a_{55}} = -\frac{21200}{1766.4} = -12.002$$

$$x_6 = -\frac{a_{60}}{a_{66}} = -\frac{54960}{1766.4} = -31.114$$

$$x_7 = -\frac{a_{70}}{a_{77}} = -\frac{88720}{1766.4} = -50.226$$

$$x_8 = -\frac{a_{80}}{a_{88}} = -\frac{120320}{1766.4} = -68.116$$

4. Carry-Over Force Procedure

The calculation of the redundants by means of the carry-over procedure is performed in Table (1-2) .

CARRY-OVER FORCE PROCEDURE								TABLE 1-2
Panel	1	2	3	4	5	6	7	8
Carry-Over Force Factors	→ -.0734	← -.0734 → -.0734	← -.0734 → -.0734	← -.0734 → -.0734	← -.0734 → -.0734	← -.0734 → -.0734	← -.0734 → -.0734	← -.0734
Starting Force	+54.665	+45.335	+26.223	+ 7.111	-12.002	-31.114	-50.226	-68.116
1. C.O.F.	 - 3.328	- 4.012 - 1.925	- 3.328 - 0.522	- 1.925 + 0.881	- 0.522 + 2.284	+ 0.881 + 3.687	+ 2.284 + 5.000	+ 3.687
2. C.O.F.	 + 0.436	+ 0.244 + 0.283	+ 0.436 + 0.077	+ 0.283 - 0.129	+ 0.077 - 0.335	- 0.129 - 0.535	- 0.335 - 0.271	- 0.535
3. C.O.F.	 - 0.039	- 0.032 - 0.038	- 0.039 - 0.011	- 0.038 + 0.019	- 0.011 + 0.049	+ 0.019 + 0.044	+ 0.049 + 0.039	+ 0.044
Redundants	+51.734	+39.855	+22.836	+ 6.202	-10.460	-27.147	-43.460	-64.920

5. Check of Redundants :

The redundants are checked by means of Eq. (14j).

$$X_1 = (54.665) + (-.0734)(+39.855) = +51.740$$

$$X_2 = (-.0734)(+51.734) + (+45.335) + (-.0734)(+22.836) = +39.862$$

$$X_3 = (-.0734)(+39.855) + (+26.223) + (-.0734)(+6.202) = +22.843$$

$$X_4 = (-.0734)(+22.836) + (+\ 7.111) + (-.0734)(-10.460) = +\ 6.203$$

$$X_5 = (-.0734)(+6.202) + (-12.002) + (-.0734)(-27.147) = -10.464$$

$$X_6 = (-.0734)(-10.460) + (-31.114) + (-.0734)(-43.460) = -27.156$$

$$X_7 = (-.0734)(-27.147) + (-50.226) + (-.0734)(-64.920) = -43.468$$

$$X_8 = (-.0734)(-43.460) + (-68.116) = -64.926$$

6. True Forces in Members (Eq. 1)

The true forces in the members of the indeterminate system are tabulated in Table (1-3) .

TRUE FORCES IN MEMBERS TABLE (1-3)

	m	BN_m	X_1S_{m1}	X_2S_{m2}	X_3S_{m3}	X_4S_{m4}	X_5S_{m5}	X_6S_{m6}	X_7S_{m7}	X_8S_{m8}	N_m
Top	1	0.00	-41.39								- 41.39
	2	- 93.33		-31.88							-125.21
	3	-160.00			-18.27						-178.27
	4	-200.00				-4.96					-204.96
	5	-213.33					+8.37				-204.96
	6	-200.00						+21.72			-178.28
	7	-160.00							+34.77		-125.23
	8	- 93.33								+51.94	- 41.39
Bottom	1	+ 93.33	-41.39								+ 51.94
	2	+160.00		-31.88							+128.12
	3	+200.00			-18.27						+181.73
	4	+213.33				-4.96					+208.37
	5	+200.00					+8.37				+208.37
	6	+160.00						+21.72			+181.72
	7	+ 93.33							+34.77		+128.10
	8	0.00								+51.94	+ 51.94
Diagonal	1	-116.67	+51.73								- 64.94
	2	- 83.33		+39.86							- 43.47
	3	- 50.00			+22.84						- 27.16
	4	- 16.67				+6.20					- 10.47
	5	+ 16.67					-10.46				+ 6.21
	6	+ 50.00						-27.15			+ 22.85
	7	+ 83.33							-43.46		+ 39.87
	8	+116.67								-64.92	+ 51.75
Diagonal Redundant	1	0.00	+51.73								+ 51.73
	2	0.00		+39.86							+ 39.86
	3	0.00			+22.84						+22.84
	4	0.00				+6.20					+ 6.20
	5	0.00					-10.46				- 10.46
	6	0.00						-27.15			- 27.15
	7	0.00							-43.46		- 43.46
	8	0.00								-64.92	- 64.92
Vertical	0	- 10.00	-31.04								- 41.04
	1	+ 50.00	-31.04	-23.91							- 4.95
	2	+ 30.00		-23.91	-13.70						- 7.61
	3	+ 10.00			-13.70	-3.72					- 7.42
	4	- 10.00				-3.72	+6.28				- 7.44
	5	- 30.00					+6.28	+16.29			- 7.43
	6	- 50.00						+16.29	+26.08		- 7.63
	7	- 70.00							+26.08	+38.95	- 4.97
	8	- 80.00								+38.95	- 41.05

Example 2

In this example a vertical truss with sloping sides is investigated. The truss is externally statically determinate and 8 times internally indeterminate. The loading is as shown in Fig. 10.

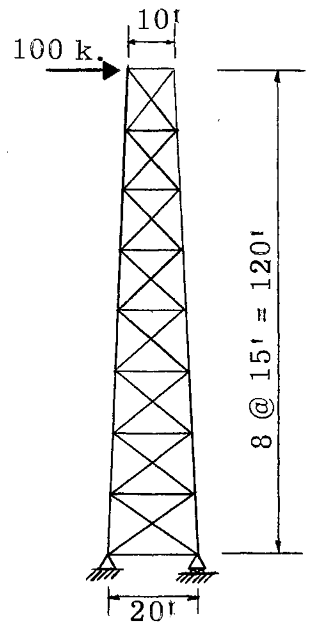

Fig. 10
Indeterminate Truss with Sloping Sides

The area of the left and right chords is taken equal to twice the area of the horizontal and diagonal members.

1. Primary System:

The redundant bars are removed and thus the primary system shown in Fig. 11 is introduced .

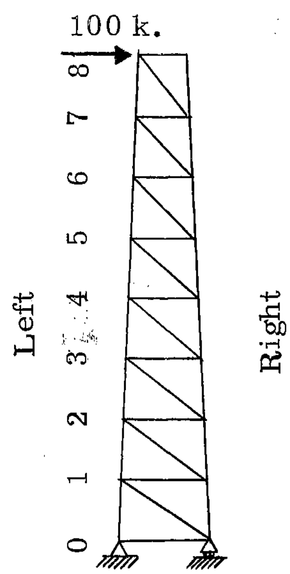

Fig. 11
Primary System (Example 2)

2. Carry-Over Force Factors

Geometry:

$d_{H0} = 240''$

$d_{H1} = 225''$	$d_{L1} = d_{R1} = 180.16''$	$d_{D1} = 294.04''$
$d_{H2} = 210''$	$d_{L2} = d_{R2} = 180.16''$	$d_{D2} = 202.32''$
$d_{H3} = 195''$	$d_{L3} = d_{R3} = 180.16''$	$d_{D3} = 270.93''$
$d_{H4} = 180''$	$d_{L4} = d_{R4} = 180.16''$	$d_{D4} = 259.92''$
$d_{H5} = 165''$	$d_{L5} = d_{R5} = 180.16''$	$d_{D5} = 249.31''$
$d_{H6} = 150''$	$d_{L6} = d_{R6} = 180.16''$	$d_{D6} = 239.18''$
$d_{H7} = 135''$	$d_{L7} = d_{R7} = 180.16''$	$d_{D7} = 229.79''$
$d_{H8} = 120''$	$d_{L8} = d_{R8} = 180.16''$	$d_{D8} = 220.57''$

The influence numbers and relative axial flexibilities under sections (a) and (b) that follow are tabulated in Table (2-1).

(a) Influence numbers (Eq's 15j, 16j, 17j, 18j, 19j)

(b) Relative axial flexibilities (Eq. 3)

(c) Relative deformations (Eq's 7j, 8j, 9j, 10j and Table 2-1)

$a'_{10} = -89979.71$	$a'_{11} = +1892.26$
$a'_{20} = -82721.23$	$a'_{22} = +1791.73$
$a'_{30} = -85223.07$	$a'_{33} = +1694.91$
$a'_{40} = -88511.02$	$a'_{44} = +1602.40$
$a'_{50} = -92912.30$	$a'_{55} = +1515.02$
$a'_{60} = -98937.06$	$a'_{66} = +1433.69$
$a'_{70} = -107432.37$	$a'_{77} = +1359.23$
$a'_{80} = -104776.23$	$a'_{88} = +1292.49$

INFLUENCE NUMBERS AND RELATIVE AXIAL FLEXIBILITIES

TABLE (2-1)

		m	d_m	λ'_m	BN_m	S_{m1}	S_{m2}	S_{m3}	S_{m4}	S_{m5}	S_{m6}	S_{m7}	S_{m8}	$BN_m S_{mj} \lambda'_m$	
Left		1	180.16	180.16	+600.53	-.6127								-66288.92	
		2	180.16	180.16	+560.49		-.6381							-64433.98	
		3	180.16	180.16	+514.74			-.6650						-61669.15	
		4	180.16	180.16	+461.95				-.6931					-57683.18	
		5	180.16	180.16	+400.35					-.7226				-52119.01	
		6	180.16	180.16	+327.56						-.7532			-44448.75	
		7	180.16	180.16	+240.21							-.7840		-33928.57	
		8	180.16	180.16	+133.45								-.8168	-19637.79	
Right		1	180.16	180.16	-560.49	-.6127								+61869.14	
		2	180.16	180.16	-514.74		-.6381							+59174.56	
		3	180.16	180.16	-461.95			-.6650						+55344.57	
		4	180.16	180.16	-400.35				-.6931					+49991.26	
		5	180.16	180.16	-327.56					-.7226				+42642.94	
		6	180.16	180.16	-240.21						-.7532			+32595.66	
		7	180.16	180.16	-133.45							-.7840		+18849.20	
		8	180.16	180.16	0.00								-.8168	0.00	
Diagonal		1	294.04	588.08	- 65.34	+1.00								-38425.15	
		2	282.32	564.64	- 71.70		+1.00							-40484.69	
		3	270.93	541.86	- 79.40			+1.00						-43023.68	
		4	259.52	519.84	- 88.86				+1.00					-46192.98	
		5	249.31	498.62	-100.73					+1.00				-50225.99	
		6	239.18	478.36	-115.97						+1.00			-55475.41	
		7	229.79	459.58	-136.18							+1.00		-62585.60	
		8	220.57	441.14	-163.40								+1.00	-72082.28	
	Redundant	1	294.04	588.08	0.00	+1.00								0.00	
		2	282.32	564.64	0.00		+1.00							0.00	
		3	270.93	541.86	0.00			+1.00						0.00	
		4	259.92	519.84	0.00				+1.00					0.00	
		5	249.31	498.62	0.00					+1.00				0.00	
		6	239.18	478.36	0.00						+1.00			0.00	
		7	229.79	459.58	0.00							+1.00		0.00	
		8	220.57	441.14	0.00								+1.00	0.00	
Horizontal		0	240	480	+ 75.00	-.7652									-27547.20
		1	225	450	+ 53.33	-.8162	-.7438							-19587.58	-17850.08
		2	210	420	+ 57.14		-.7970	-.7197						-19127.04	-17271.94
		3	195	390	+ 61.54			-.7751	-.6925					-18602.87	-16620.42
		4	180	360	+ 66.67				-.7502	-.6618				-18005.70	-15883.99
		5	165	330	+ 72.73					-.7219	-.6271			-17326.25	-15050.96
		6	150	300	+ 80.00						-.6899	-.5875		-16557.60	-14100.00
		7	135	270	+ 88.89							-.6528	-.5440	-15667.40	-13056.16
		8	120	240	0.00								-.6121	0.00	

a'_{12}	=	273.19	a'_{54}	=	178.73
a'_{21}	=	273.19	a'_{56}	=	149.39
a'_{23}	=	240.91	a'_{65}	=	149.39
a'_{32}	=	240.91	a'_{67}	=	121.59
a'_{34}	=	209.34	a'_{76}	=	121.59
a'_{43}	=	209.34	a'_{78}	=	95.88
a'_{45}	=	178.73	a'_{87}	=	95.88 .

Substituting in Eq's (12j) the carry-over force factors are

C_{01}	=	0	C_{45}	=	-.1180
C_{21}	=	-.1444	C_{65}	=	-.0986
C_{12}	=	-.1525	C_{56}	=	-.1042
C_{32}	=	-.1345	C_{76}	=	-.0848
C_{23}	=	-.1421	C_{67}	=	-.0895
C_{43}	=	-.1235	C_{87}	=	-.0705
C_{34}	=	-.1306	C_{78}	=	-.0742
C_{54}	=	-.1115	C_{98}	=	0 .

3. Starting Forces (Eq's 13j)

x_1	=	47.551	x_5	=	61.327
x_2	=	46.168	x_6	=	69.009
x_3	=	50.280	x_7	=	79.039
x_4	=	55.236	x_8	=	81.065 .

4. Carry-Over Force Procedure

The calculation of the redundants by means of the carry-over procedure is performed in Table (2-2) .

CARRY-OVER FORCE PROCEDURE TABLE 2-2

Panel	1	2	3	4	5	6	7	8
Carry-Over Force Factors	-.1525	-.1444 -.1421	-.1345 -.1306	-.1235 -.1180	-.1115 -.1042	-.0986 -.0895	-.0848 -.0742	-.0705
Starting Force	+47.551	+46.168	+50.280	+55.236	+61.327	+69.009	+79.039	+81.065
1. C.O.F.	- 6.667	- 7.252 - 6.763	- 6.560 - 6.822	- 6.567 - 6.838	- 6.518 - 6.804	- 6.390 - 6.703	- 6.176 - 5.715	- 5.865
2. C.O.F.	+ 2.024	+ 1.017 + 1.800	+ 1.992 + 1.656	+ 1.748 + 1.485	+ 1.582 + 1.291	+ 1.388 + 1.008	+ 1.172 + .413	+ .882
3. C.O.F.	- .407	- .309 - .491	- .400 - .399	- .476 - .320	- .381 - .236	- .299 - .134	- .214 - .062	- .118
4. C.O.F.	+ .116	+ .062 + .107	+ .114 + .098	+ .104 + .069	+ .094 + .043	+ .064 + .023	+ .039 + .008	+ .020
5. C.O.F.	- .024	- .018 - .029	- .024 - .021	- .028 - .015	- .020 - .009	- .014 - .004	- .008 - .001	.003
6. C.O.F.	+ .007	+ .004 + .006	+ .007 + .005	+ .006 + .003	+ .005 + .003	+ .003 + .001	+ .003 + .000	+ .001
Redundants	+42.780	+34.302	+39.926	+44.407	+50.377	+57.942	+68.498	+75.982

$$X_1 = (+47.551) + (-.1444)(+34.302) = +42.598$$

$$X_2 = (-.1525)(+42.780) + (+46.168) + (-.1345)(+39.926) = +34.274$$

$$X_3 = (-.1421)(+34.302) + (+50.280) + (-.1235)(+44.407) = +39.921$$

$$X_4 = (-.1306)(+39.926) + (+55.236) + (-.1115)(+50.377) = +44.405$$

$$X_5 = (-.1180)(+44.407) + (+61.327) + (-.0986)(+57.942) = +50.374$$

$$X_6 = (-.1042)(+50.377) + (+69.009) + (-.0848)(+68.495) = +57.951$$

$$X_7 = (-.0895)(+57.942) + (+79.039) + (-.0705)(+75.982) = +68.496$$

$$X_8 = (-.0742)(+68.498) + (+81.065) = +75.982$$

6. True Forces in Members (Eq. 1)

The true forces in the members of the indeterminate system are tabulated in Table (2-3).

TRUE FORCES IN MEMBERS — TABLE (2-3)

		m	BN_m	X_1S_{m1}	X_2S_{m2}	X_3S_{m3}	X_4S_{m4}	X_5S_{m5}	X_6S_{m6}	X_7S_{m7}	X_8S_{m8}	N_m
Left		1	+600.53	-26.21								+574.32
		2	+560.49		-21.89							+538.60
		3	+514.74			-26.55						+488.19
		4	+461.95				-30.78					+431.17
		5	+400.35					-36.40				+363.95
		6	+327.56						-43.64			+283.92
		7	+240.21							-53.70		+186.51
		8	+133.45								-62.06	+ 71.39
Right		1	-560.49	-26.21								-586.7
		2	-514.74		-21.89							-536.63
		3	-461.95			-26.55						-488.50
		4	-400.35				-30.78					-431.13
		5	-327.56					-36.40				-363.96
		6	-240.21						-43.64			-283.85
		7	-133.45							-53.70		-187.15
		8	0.00								-62.06	- 62.06
Diagonal		1	- 65.34	+42.78								- 22.56
		2	- 71.70		+34.30							- 37.40
		3	- 79.40			+39.93						- 39.47
		4	- 88.86				+44.41					- 44.45
		5	-100.73					+50.38				- 50.35
		6	-115.97						+57.94			- 58.03
		7	-136.18							+68.50		- 67.68
		8	-163.40								+75.98	- 87.42
	Redundant	1	0.00	+42.78								+ 42.78
		2	0.00		+34.30							+ 34.30
		3	0.00			+39.93						+ 39.93
		4	0.00				+44.41					+ 44.41
		5	0.00					+50.38				+ 50.38
		6	0.00						+57.94			+ 57.94
		7	0.00							+68.50		+ 68.50
		8	0.00								+75.98	+ 75.98
Horizontal		0	+ 75.00	-32.74								+ 42.26
		1	+ 53.33	-34.92	-25.51							- 7.10
		2	+ 57.14		-27.34	-28.73						+ 1.07
		3	+ 61.54			-30.95	-30.75					- 0.16
		4	+ 66.67				-33.31	-33.34				+ 0.02
		5	+ 72.73					-36.37	-36.34			+ 0.02
		6	+ 80.00						-39.97	-40.24		- 0.21
		7	+ 88.89							-44.72	-41.33	+ 2.84
		8	0.00								-46.51	- 46.51

PART IV

SUMMARY AND CONCLUSIONS

The objective of this thesis was:

1. The derivation of the three force equation for trusses with internal redundants.

2. The presentation of the carry-over force functions.

3. The description of the carry-over force procedure and its physical interpretation.

4. The application of the carry-over force procedure to the analysis of these trusses.

From the performed investigations the following conclusions were made:

1. The carry-over force procedure is a method of numerical successive approximation (special form of iteration) which offers a fast self-checking means for solving trusses with any number of internal redundants by the exact stress analysis.

2. The convergency of this procedure is rapid and in most cases three to four cycles are necessary to reach a satisfactory solution.

3. This procedure requires considerably less numerical labor if compared to the simultaneous solution of the deformation equations by elimination or determinants.

4. The carry-over force philosophy has a physical significance which can be easily explained by cutting and tying the redundant members of the truss according to the convenience of the investigator.

A SELECTED BIBLIOGRAPHY

1. "Analysis of Continuous Trusses by Carry-Over Moments, by J.J. Tuma, (in preparation).

2. "Analysis of Continuous Beams by Carry-Over Moments," by J.J. Tuma, Proceedings, A.S.C.E., Vol. 84, 1958.

3. "Theory of Structures", by S. Timoshenko and D.H. Young, McGraw-Hill Book Company, Inc. New York, New York, 1945 pp. 293-306.

4. "Elementary Structural Analysis," by J.B. Wilbur and C.H. Norris, McGraw-Hill Book Co., Inc., New York, New York., 1948, pp. 260-266, 270-271.

5. "Indeterminate Structural Analysis," by J.S. Kinney, Addison-Wesley Pub. Co., Inc., Reading, Mass., 1957, pp. 198-209, 254-257.

6. "Statically Indeterminate Structures," by Chu-Kia Wang, McGraw-Hill Book Co., Inc., New York, New York, 1953, pp. 110-118.

7. "Modern Framed Structures," Part II, by J.B. Johnson, C.W. Bryan and F.E. Turneaure, John Wiley and Sons, Inc., New York, New York, 1929, pp. 297-308.

8. "Analysis of Statically Indeterminate Structures," by J. Parcel and R. Moorman, John Wiley and sons, Inc., New York, New York, 1955, pp. 113-114, 139-140, 146.

9. "Theory of Structural Analysis and Design," by J. Michalos, Roland Press Co., New York, New York, 1958, pp. 150-153, 174-179.

10. "The Analysis of Engineering Structures," by A.J.S. Pippard and J.F. Baker, Edward Arnold (publishers) LTD., London, 1957, pp. 78-90, 107-111.

11. "Structural Analysis," by W.F. Cassie, Longmans, Green and Co., London, 1956, pp. 237-248.

12. "Stresses in Framed Structures," by G.A. Hool and W.S. Kinne, McGraw-Hill Book Co., Inc., New York, New York, 1942 pp. 398-402.

13. "Statically Indeterminate Stresses," by J.I. Parcel and G.A. Maney, John Wiley and sons, Inc., New York, New York, 1926, pp. 111-115.

14. "Theory of Modern Steel Structures," Vol. II, by L.E. Grinter, the MacMillan Co., New York, New York, 1949, pp. 81-91.

15. "Determination of the Stresses in Elastic Systems by the Method of Least Work," by W. Cain, Transactions, A.S.C.E., Vol. 24, April, 1891.

VITA

Michel Kyriaco Antarakis

Candidate for the Degree of

Master of Science

Title: ANALYSIS OF INTERNALLY INDETERMINATE TRUSSES BY CARRY-OVER FORCES

Major Field: Civil Engineering

Biographical:

Personal Data: Born in Alexandria, Egypt, February 22, 1935, the son of Kyriaco and Athina Antarakis.

Education: Graduated from the St. Andrew Boy's High School and received the London University General Certificate of Education, June 1952; received the Bachelor of Science Degree in Civil Engineering from the University of Alexandria, Alexandria, Egypt, June, 1957; completed requirements for the Master of Science Degree in May, 1959.

Professional Experience: Design Engineer, Bridge Department, Oklahoma State Highway Department, Oklahoma City, Oklahoma; Teaching and research assistant; Part-time instructor, School of Civil Engineering, Oklahoma State University.