EFFECT OF SECONDARY LOOP CONFIGURATION ON OVER-ALL RESPONSE OF A CASCADE CONTROL SYSTEM

By

PAUL U. WEBB

Bachelor of Science

Oklahoma State University

Stillwater, Oklahoma

1950

Submitted to the faculty of the Graduate School
of the Oklahoma State University
in partial fulfillment of the requirements
for the degree of
MASTER OF SCIENCE
1960

EFFECT OF SECONDARY LOOP CONFIGURATION

ON OVER-ALL RESPONSE OF A CASCADE

CONTROL SYSTEM

Thesis and Abstract Approved:

Paul A. McCollum
Thesis Adviser

Harold Fristoe
Faculty Representative

Robert MacVicar
Dean of the Graduate School

PREFACE

Cascade control systems in the processing industries came into use about 1947. Since that time numerous papers have been written concerning the performances attained. Principal attention was placed on the benefits of interlocking a primary process variable with a secondary variable and the improved control when disturbances fall within the secondary loop. The work presented here will show that still further advantages may be realized from a cascade control system by including significant process time constants in the secondary loop so that the over-all time constant (transfer lag) may be reduced. This can greatly improve the controllability of the primary variable when disturbances enter the process outside the secondary loop. Two methods will be used to analyze a system. (1) Simulation on an analog computer is used to analyze the benefits of including various portions of the process inside the secondary loop. (2) Root Locus techniques are employed to analyze various controller configurations for the secondary controller.

Indebtedness is acknowledged to Dr. Truet B. Thompson and Professor Paul A. McCollum for their valuable guidance and advice. The author wishes to express his appreciation to Phillips Petroleum Company for permission to publish this work.

TABLE OF CONTENTS

LIST OF FIGURES

Figure Page

CHAPTER I

INTRODUCTION

In visiting any of the modern chemical plants or petroleum refineries, one striking feature noticed is the complex layouts of processing equipment. This equipment with its interwoven, interconnecting pipe networks almost silently processes millions of tons of valuable products each year. A more subtle fact is that these huge plants operate continuously with very little need for adjustments by a human hand. This is the reward of hundreds of automatic process control systems on duty in the plant. Sensitive transducers located in the process streams throughout the plant sense output variables of many processes. Signals from these transducers are continuously transmitted to centrally located recorders and controllers that automatically keep the individual processes within operating limits. A real benefit of all this is not only the reduction of manpower but also the ability to safely produce a higher quality product at a greater profit and herein lies the purpose of any manufacturing plant.

Automatic control is now recognized by many industrial organizations as being an essential aid to efficient plant operation. Furthermore, many processes would not operate at all if it were not for automatic control systems. Synthesis of organic compounds like benzene and toluene, production of synthetic rubber, catalytic refining of gasoline, and the separation of the isotope of uranium for atomic energy could never have resulted without precise controls.

The principle of operation of most process control systems is based on the feedback theory that is so important to audio communications and servomechanisms. The essential requirement of feedback control is that the error between the state desired and the state existing is constantly measured and if there is an error, corrective action is taken through a power amplifying device to eliminate the error.[1] Figure 1 is a diagram of a feedback control system.

It is true that some process control systems are true servomechanisms; however, most would be classified as regulators, because only load disturbances (not changes in commands) account for the initiation of corrective action.

In closed loop process control there are principally two types of systems:

(1) Single loop

(2) Multiloop (cascade).

The single loop system is the workhorse for controlling the ordinary process. Figure 1 is an example of a single loop control system. If the characteristics of the process are such that a single loop system does not do a satisfactory job of control, the multiloop or cascade control system will many times provide the quality of control desired (see Figure 2). It is the author's purposes: (a) to relate some of the characteristics and benefits of cascaded systems used for process control and (b) to demonstrate by analog simulation and root locus analysis of an actual system, effects of transfer lags and controller configurations on the performance of such systems.

CHAPTER II

REVIEW OF CONTROL SYSTEM PARAMETERS

From an engineering standpoint, the objective of a control system in any process is to maintain a state of dynamic equilibrium despite disturbances which may occur in any part of the process. Optimization of the design and performance of automatic controls requires a technique known as systems engineering. W. E. Vannah and L. E. Slater had this to say about systems engineering:

> "There is no magic to systems engineering. It requires definition of the problem, statement of the preliminary specifications, detailed examination of the components of a system and organized (not necessarily routine) integration of all the components into a system in such a way that over-all performance will be known before the system is flanged up.[2]

Regardless of the type of feedback control system used, there are four basic components which are required and which will receive the most attention in the systems analysis. They are:

1. Controller (one, two or three mode)
2. Final Control Element (control valve, etc.)
3. Process
4. Measuring Device (thermocouple, etc.)

Again, Figure 1 is a block diagram of a simple loop using these components. They will appear in other more complex systems in greater numbers and various arrangements. It is not the purpose to give a detailed description of the hardware and its operation since it can be found in any book written on process control.[3]

In this chapter we will review different system parameters that must be recognized and investigated in order to complete a systems analysis.

Transfer Lags. In analyzing any system one of the most important parameters to investigate is the transfer lag. A component in a system will have a transfer lag (sometimes called residence time, capacity lag or time lag) when time is required for the element to follow a change in energy level made at its input. An example of a transfer lag is the time required to complete the mixing of two feeds in a reactor. Another example is the electrical circuit consisting of a resistance R and a capacitance C in shunt (see Figure 3a). This is an electrical analog of many other systems such as surge tanks or thermocouple probes.

Transfer lags are described mathematically in a transfer function expressed as a complex ratio in Heaviside or Laplacian notation. The RC transfer lag shown in Figure 3a would be described mathematically by equation 2-1

$$(2\text{-}1) \quad \frac{e_o}{e_1} = \frac{1}{\tau s+1} \quad \text{where } \tau = RC \text{ the system time constant.}$$

The response of e_o following the application of a step voltage e_1 is the well known first ordered exponential rise seen in Figure 3b.

Ceaglske writes that it is difficult to make an all-inclusive statement on the effects of the transfer lag upon the stability and controllability of a system. Large lags slow the response of the system to a change, and this is not desirable. However, generally speaking the relative values of the lags are of greater significance than the value of any particular lag. It is a truism that the system with the least controllability is one in which the lags in each block of process are equal.[3]

Some of the segments of a process control system that contribute transfer lags are: (1) the process itself such as reactors, heat exchangers or distillation columns, (2) measuring elements such as thermocouple probes, (3) final control elements such as control valve actuators and positioners. If the designer can eliminate or reduce transfer lags in any of these segments, he will improve the response of the system.

With today's increasing emphasis on improved dynamic performance of industrial control systems, intensive attention is being focused on the final control element, actuators and positioners; for aside from the process itself the final control element has been the least responsive element of the control loop. However, there are many temperature control systems whose main improvement would be in decrease of transfer lags in thermocouple probes and wells.

Many of the lags of a process control system are second ordered. Pneumatic valve actuators, bourdon tubes, and thermocouples are typical examples. The process is most notorious for its multi-ordered lags. For example, in a fractionation column where each tray represents a heat exchanger with thermal capacitance and resistance, as well as mass transfer, the transfer lag is multi-ordered. This greatly retards the effect of heat input to the column reboiler upon a temperature measurement at some point toward the top of the column.[4]

Pure Time Delays. When there is an interval of time following a change in a system input before the output starts to change, a pure time delay is present. (This is sometimes called transport lag, distance velocity lag or dead-time.) It may be contrasted with the single ordered transfer lag where the output starts to change immediately after an input change.

Field experience has shown that systems which have pure time delays are very difficult to control. Dominant quantities of it will cause the controller gain to be set quite low for the sake of stability. With even small amounts of pure time delay, controllability deteriorates significantly. This is because of the linearly increasing phase lag of the output with respect to the input when the system frequency is increased.[5] A pure time delay arises more generally in connection with fluid flow in a pipe, chemical process or heat exchanger (see Figure 4). However, the measuring system itself may introduce this undesirable characteristic. One of the problems of the application of an on-stream analytical instrument in a chemical process is the pure time delay introduced by sampling systems.[6] Several transfer lags in cascade contribute an apparent pure time delay which is as detrimental to control systems as delays arising from fluid flow. A pure time delay is expressed as e^{-Ts} in Laplacian calculus form where T is the delay time.

Nonlinearities. Most chemical or petroleum processes have nonlinear relationships between variables over the range in which they operate. As a consequence, if a load disturbance covers a considerable range of values, then the control system will not be able to correct properly with any one gain or set point.

Some of the control components in a system will have nonlinearities. Pneumatic controllers and transmitters are examples since they saturate at both ends of their operating ranges. The presence of dead zone in instruments contributes a nonlinearity. Or, if energy is fed into a system through a controlled valve, this valve usually constitutes a nonlinear element unless special provisions are made to insure linearity.

This is because the energy flow and the displacement of the valve are not linearly related.[7] Also all valves have in their system mechanical movement which produces a nonlinear effect called hysteresis; that is, the valve will not produce the same flow when a given stem position is approached from opposite directions.[3]

For practical purposes, the reader may assume that a system is linear if in a sinusoidal analysis with fixed frequency the amplitude of the output is directly proportional to that of the input. But this is seldom the case.

Self Regulation. A factor that has an important bearing on the type of response and ease of control of a process is its degree of self regulation. The term self regulation is used to describe that action (in a system) which tends to correct for load changes independently of the controller.[3] Notwithstanding the many factors involved in unit design which favor self regulation, a cascaded system of material residences arranged in progressively decreasing time constants is inherently self regulating and can be operated with simpler instrumentation than a system not having this feature.[8]

CHAPTER III

CASCADE CONTROL SYSTEMS

Cascade control is defined as a control combination where a primary variable is held closer to the desired value by interlocking a primary controller with a controller for a related secondary variable[4] (see Figure 2).

The primary variable governs the operation of the portion of the process to which the system is applied. The secondary variable, while not being a primary variable in the process, is related to the primary variable. The magnitude of this effect will be dependent upon the characteristics of the process, the sensitivity of the primary variable to small changes in the secondary variable, and the transfer lag between a change in the secondary variable and a corresponding change of the primary variable. The greater the sensitivity of the primary variable and the larger the lags in the process the more important it is that cascade systems be used. In such systems, not only is each variable controlled independently but the two are linked together to provide an integrated control.

To further define the cascade system, it is well to discuss the different loops involved. Most systems will have two or three loops. These loops are referred to as:

(1) Primary

(2) Secondary

(3) Tertiary

The primary loop has in its path every element in the over-all multi-loop system, including the secondary controller et al. It is shown by the outside loop in Figure 2. The secondary loop then is inside the primary loop and includes the secondary controller, final control element, the portion of the process up to where variable C' is measured and the associated measuring element. A tertiary loop would be any loop required inside the secondary loop. The primary loop must have the largest transfer lags and delays while the secondary loop has smaller amounts and the tertiary even smaller.

The cascade control system derives an advantage from the secondary control loop being around a significant transfer lag. The existence of this loop provides two principal effects. First, the disturbances which fall inside it are regulated out very quickly because of the single transfer lag and the higher gain permissible in the secondary controller. (In an actual process the disturbance referred to could be changes in ambient conditions, feed conditions or other sudden changes.) Secondly, the response time of the control system in the regulation of disturbances entering the process outside the secondary loop is substantially decreased.

There are other advantages gained from using cascade control. The following will cover the main reasons for using this type of system.[10]

1. Improve control by reducing the effective transfer lag.
2. Reduce load changes, nonlinearities and discontinuities near their source.
3. Maintain a desired relationship between variables.
4. Accurately limit a secondary variable.

Investmentwise it is economically sound to run several continuous processes in series with small or no surge capacity between plants and this has led to the use of cascade controls. The following are comments by Mr. Allen L. Chaplin:[8]

> "The instrument engineer has made available a technique of instrumentation which minimizes the necessity for surge-averaging control and also makes possible in some cases a reduction in volume inventory of processing equipment. The cascade control system can be applied to series or branch arranged processing units and can be so adjusted to transfer load changes of a preceding stage without the necessity of large volume surge."

The most rewarding cascade control systems are those which actually improve the control system by reducing the effective transfer lag. When one of the main system time constants lies within the loop of the secondary controller, its harmful effect on the over-all systems control can often be greatly reduced and result in higher gain of the primary controller and shorter period of oscillation of the primary variable. Investigations on this advantage will be reported on in a later chapter.

Mr. J. G. Ziegler reports that one should always look for the second largest time constant (transfer lag) in the process for inclusion in the secondary loop. The largest is the one usually to be controlled by the primary loop. But the second largest has a considerable effect on the over-all effective lag, which in turn determines the period of oscillation of the system. If the second largest one were eliminated, it is likely that the system could be represented by a single transfer lag. This is what the designer strives for. Thus by including a large transfer lag in the secondary loop this time constant can virtually be eliminated from the transfer lag of the over-all cascade control system, thereby increasing the frequency of the system and improving control.[10]

Mr. R. L. Day states this another way by saying that if the gain of the secondary controller is increased to infinity, the phase advance resulting will be the lag angle of the process which this controller controls, and the frequency response will be the same as the section between the measurement of the secondary variable and primary variable.[9] In an actual plant, of course, the system will become unstable before this can be realized. However, the more plant included in the secondary loop, the greater is the phase advance obtainable and the more the operating period may be reduced. Mr. Day goes on to say that the ratio of the two loop time constants

$$\frac{\dfrac{K_pG_pK_2G_2K_3G_3}{1+H_pK_pG_pK_2G_2K_3G_3}}{K_2G_2} \approx \frac{\tau_p}{\tau_s}$$

(see Figure 5) must be greater than 3 so as to avoid resonance effects; but desirably not greater than 10. The total loop gain (the product of the controller gains) must not be so high as to over-range the final control element.

Figure 5 is a typical case of cascade control and can be used to define mathematically why we expect to eliminate a transfer lag and thus improve the frequency response of the total system. K_sG_s and K_1G_1 represents the transfer function of the secondary controller and part of the process included in its loop. H_s represents the transfer function of the measurement element and transmitter for the secondary loop. The equation representing the transfer function of the secondary loop then is simply

$$(2\text{-}1)\quad K_2G_2 = \frac{M}{R_1} = \frac{K_sG_sK_1G_1}{1+H_sK_sG_sK_1G_1}$$

If $H_sK_sG_sK_1G_1$ is very much greater than unity for all frequency values of interest, equation 2-1 reduces to

$$(2\text{-}2)\quad K_2G_2 = \frac{1}{H_s}$$

If H_s is characterized by a transfer lag then $\frac{1}{H_s}$ would be a transfer lead. This would mean that G_2 would render lead compensation to the over-all system instead of a lag resulting in system improvement. Usually H_s has a relatively small time constant and K_sG_s cannot be increased so that equation 2-2 holds; consequently, very little lead compensation results. However, even though very little actual lead compensation is present, the transfer lag of G_s and G_1 is reduced which is in effect the same as lead compensation.[11]

When the time constant in the secondary loop is very small, the secondary controller can be thought of as a perfect valve positioner. In this case both the primary and the secondary controller can be adjusted independently with respect to their own closed loop.[12]

A vector diagram method may be used to deduce the frequency response of a cascade control system, and hence to allow deduction of appropriate controller settings by a method used for simple loops. Also the vector diagram clearly illustrates the phase-advance properties of the secondary loop. Through their use the performance of a cascade control system can be predicted when the frequency response of the process is known or can be estimated.[9] A series of vector diagrams is drawn for a number of frequencies. The vector diagram is started by drawing a line E_s of unit length, representing the error in the secondary control loop (see

Figure 6). Vector Θ_s, representing the controller output, is drawn at the appropriate angle to E_s (180° for a proportional controller) and with length equal to the controller gain. The angle and length of vector B, representing the measured variable, is known from the frequency response of the portion of the process included in the secondary loop. The diagram is completed by joining the ends of vectors E_s and B which represents the set-point of the secondary controller. The angle α can then be read off. This is the angle which must be subtracted from the over-all process phase lag to determine the "effective" phase lag when the cascade controller is used. The effective open loop gain of the process, as seen by the primary controller, is obtained by multiplying the actual process gain by the ratio Θ_s/Θ_p.

If the intermediate measuring point is chosen so that the portion of the process included in the secondary loop is one single transfer lag, obviously the gain of the secondary controller can be very large. Furthermore, the phase lag in this part of the process will approach 90°. The effective phase advance α then tends to approach 90° as shown in Figure 7. This is an appreciable phase advance when compared with that obtainable with derivative action in a controller, which is usually about 30°.

There are many examples of cascade control systems in industry. One of the more common places to find them is on a fractionation column in a petroleum refinery (see Figure 8). Here you will find level controllers cascaded onto flow controllers forming a single cascade and composition controllers cascaded on temperature controllers which in turn are cascaded onto steam flow controllers forming a double cascade. In this case the flow controller's time constant is so small in comparison that its loop acts more like a perfect valve positioner. The composition analyzer monitors the product to determine whether the temperature controller is maintaining the desired value. If not, the composition controller will transmit an error signal changing the set point of the temperature controller so that through temperature correction the desired composition may be attained. As a follow up the thermocouple monitors temperature to check it against the new set point; if a discrepancy exists, an error signal is generated and the set point of the steam flow controller is changed to call for a new steam flow rate.

Each of these variables is related. The primary variable is composition. The first intermediate variable is temperature, which is indicative of composition. The second intermediate variable is steam flow. This variable is related more to temperature and less to composition. One of the criteria of a well performing system is to have firm relationships between these measured variables.

CHAPTER IV

THE ANALYSIS OF TRANSFER FUNCTIONS

Information on the advantages of cascade control systems over single loop systems may be obtained by analyzing the differences in the transfer functions for two such systems. These transfer functions may be the process output with respect to: (1) set-point inputs or (2) load inputs. From the comparison of analysis of systems with these inputs improvements in transient responses and the natural frequency of the system can be seen.

In this chapter transfer functions will be analyzed first for set-point inputs and then load inputs.

Set Point Inputs

In the past the response of a process control system to a set point change was used primarily for finding out more about the process which the system controlled. However, with the advent of computer systems cascaded onto the conventional control loops, the importance of set-point disturbances takes on added importance. This is because the computer system will manifest itself by applying a step change to the set points of other loops. Of course, set point changes have always taken place within a cascade system, but these changes have not been discrete but rather continuous.

To write the transfer function of a single loop system refer to Figure 9.

The transfer function of the process output with respect to the set point is $\frac{C}{r}$. Equation 3-1 is this transfer function.

$$(3\text{-}1)\quad \frac{C}{r} = \left[\frac{HK_pG_pK_1G_1K_2G_2}{1+HK_pG_pK_1G_1K_2G_2}\right]\left[\frac{1}{H}\right]$$

The transfer function for a cascade control system for a set-point change is now derived. Refer to Figure 10. It is assumed that the transfer function for both measuring elements are equal. This may or may not be true but for these purposes it will be accurate enough to make the assumption. The transfer function of the secondary loop is:

$$(3\text{-}2)\quad \frac{C'}{r'} = \frac{K_sG_sK_1G_1}{1+HK_sG_sK_1G_1} = K_3G_3$$

then

$$(3\text{-}3)\quad \frac{C}{r} = \left[\frac{HK_pG_pK_3G_3K_2G_2}{1+HK_pG_pK_3G_3K_2G_2}\right]\left[\frac{1}{H}\right]$$

Substitute $\dfrac{K_sG_sK_1G_1}{1+HK_sG_sK_1G_1}$ for K_3G_3

$$(3\text{-}4)\quad \frac{C}{r} = \left[\frac{\dfrac{HK_pG_pK_sG_sK_1G_1K_2G_2}{1+HK_sG_sK_1G_1}}{\dfrac{1+HK_pG_pK_sG_sK_1G_1K_2G_2}{1+HK_sG_sK_1G_1}}\right]\left[\frac{1}{H}\right]$$

Simplify

$$(3\text{-}5)\quad \frac{C}{r} = \left[\frac{HK_pG_pK_sG_sK_1G_1K_2G_2}{1+HK_sG_sK_1G_1 + HK_pG_pK_sG_sK_1G_1K_2G_2}\right]\left[\frac{1}{H}\right]$$

If $HK_pG_pK_sG_sK_1G_1K_2G_2 \gg 1$ then

$$(3\text{-}6)\quad \frac{C}{r} \approx \left[\frac{K_pG_pK_sG_sK_1G_1K_2G_2}{K_sG_sK_1G_1 + K_pG_pK_sG_sK_1G_1K_2G_2}\right]\left[\frac{1}{H}\right]$$

Simplify

$$(3\text{-}7)\quad \frac{C}{r} \approx \left[\frac{K_pG_pK_2G_2}{1+K_pG_pK_2G_2}\right]\left[\frac{1}{H}\right]$$

Compare this with the transfer function for the single loop system which was equation 3-1.

$$(3\text{-}1) \quad \frac{C}{r} = \left[\frac{HK_pG_pK_1G_1K_2G_2}{1+HK_pG_pK_1G_1K_2G_2}\right] \qquad \left[\frac{1}{H}\right]$$

From this comparison it is readily noted that the transfer function representing the lag of process #1 and measuring element has been eliminated from the main transfer function. This is contingent on K_s being quite high as pointed out previously. The $\frac{1}{H}$ term can be neglected in the comparison since it appeared in both expressions. Actually this term would furnish lead compensation to the system.

Since equation 3-7 has a smaller number of transfer lags than equation 3-1, it is self-evident that the cascade system would follow a set point change closer and the frequency of the response would be higher.

Load Disturbance Inputs

One of the principal objectives of a process control system is to eliminate the effects of load disturbances. Thus, the figure of merit of a system depends on how successfully this is accomplished. The comparison of the transfer function for a load disturbance with respect to the process output for the two systems under consideration will enable a judgment on this relative merit.

The advantages of cascade control systems when the load disturbances fall within the secondary loop have been described in a number of different papers written on the subject. However, there seems to be little work done to show the merits of such a system when the load disturbance falls in the primary loop but not in the secondary loop. Obviously, we do not expect the same dramatic improvement that can be had for the secondary loop disturbances, but improvement is experienced since the natural frequency of the entire system is increased. Mr. J. G. Ziegler says that an improvement for disturbances in the outside loop should be one of the objectives of a cascade control system.[10] It is this type of disturbance which will be studied here. The transfer function we are interested in is C/U (see Figure 10).

Equation (3-8) is this basic transfer function for the cascade system.

$$(3\text{-}8) \quad \frac{C}{U} = \frac{K_2G_2}{1+HK_pG_pK_3G_3K_2G_2}$$

by rearranging,

$$(3\text{-}9) \quad \frac{C}{U} = \left[\frac{HK_pG_pK_2G_2K_3G_3}{1+HK_pG_pK_2G_2K_3G_3}\right] \left[\frac{1}{HK_pG_pK_3G_3}\right]$$

Substitute

$$\frac{K_sG_sK_1G_1}{1+HK_sG_sK_1G_1} \quad \text{for } K_3G_3$$

$$(3\text{-}10) \quad \frac{C}{U} = \left[\frac{HK_pG_pK_sG_sK_1G_1K_2G_2}{1+HK_sG_sK_1G_1 + HK_pG_pK_sG_sK_1G_1K_2G_2}\right] \left[\frac{1+HK_sG_sK_1G_1}{HK_pG_pK_sG_sK_1G_1}\right]$$

if $HK_sG_sK_1G_1 \gg 1$ then,

$$(3\text{-}11) \quad \frac{C}{U} \approx \left[\frac{HK_pG_pK_sG_sK_1G_1K_2G_2}{HK_sG_sK_1G_1 + HK_pG_pK_sG_sK_1G_1K_2G_2}\right] \left[\frac{1}{K_pG_p}\right]$$

Simplify

$$(3\text{-}12) \quad \frac{C}{U} \approx \left[\frac{K_pG_pK_2G_2}{1+K_pG_pK_2G_2}\right] \left[\frac{1}{K_pG_p}\right]$$

Compare this with the transfer function for the single loop system shown by equation 3-13.

$$(3\text{-}13) \quad \frac{C}{U} = \frac{K_2G_2}{1+HK_pG_pK_1G_1K_2G_2}$$

This comparison is more difficult to analyze than that involving the set point change. Perhaps block diagrams will aid in recognizing the outstanding differences.

The transfer function for the two systems as represented by equations 3-12 and 3-13 are diagrammed in Figures 11a and 11b.

A well performing control system will have a configuration which forces C to follow the set point r rather than a load disturbance U, i.e., we want $\frac{C}{U} = 0$, while we want $\frac{C}{r} = 1$.

In looking at the diagrams developed from the respective transfer functions, we see that the single loop system has one transfer lag K_2G_2 in the forward direction, but it has a multi-ordered lag in the feedback representing the measuring element, controller and process #1.

On the other hand the cascade system has transfer lags representing the primary controller and process #2 in the forward loop, but no transfer lag in the feedback. It also has a transfer lead acting after the closed loop.

Based on this information we can conclude the C is more tightly coupled to U in the single loop system than in the cascade system because a smaller lag exists between the two. This will mean that C will try to follow a load change at U more readily. Furthermore, the feedback of this change is slower because of the multiordered lag in the feedback circuit causing a larger value of U' to be maintained. This could cause E to overshoot the value of U resulting in an even greater disturbance in the output of the process C.

For the cascade system, C does not follow U so quickly because of looser coupling through the forward part of the system. However, because of the gain of K_pG_p any disturbance which passes will be of a greater magnitude; but any change at the point m is quickly fed back to reduce the value of the disturbance. Even if a disturbance is felt at the point m, the extended portion of the process $1/K_pG_p$ will attenuate this disturbance so that the output C is less affected. This attenuation is proportional to $1/K_p$ and can be quite important in maintaining C at the set point rather than following the disturbance.

CHAPTER V

ANALYSIS OF CASCADE CONTROL SYSTEMS

It has been discussed in previous chapters how a secondary loop in a control system could improve the response of a process output not only for disturbances falling inside the secondary loop but also for a disturbance falling outside. It will be the purpose now to investigate the benefits of including various portions of a process in the secondary loop when a disturbance enters outside the secondary loop; also a study of controller parameters will be made when the secondary loop contains multiple lags.

Two methods will be used to analyze and investigate the systems: (1) analog simulation and (2) Root Locus. For the study an idealized linear third order process will be used. It was dynamically represented by three transfer lags of .5, 1.0 and 1.67 minutes. These lags are shown as transfer functions in Figure 12.

Analog simulation will be used to study the effects of including various portions of the process in the secondary loop when a cascaded system controls the process. The Root Locus method will be used to study the benefits of different controller modes in the secondary loop. Both methods will be used to study a single loop control of the process so that its comparison with the cascade control system may be made.

Analysis by Analog Simulation

The analog computer after having been used so fruitfully as a reliant tool in the study of servomechanisms is now being accepted in the study of process control systems. It receives its greatest workout in the synthesis of control systems, but since the process is a part of the control loop, the simulation of its dynamic characteristics is necessary and has pointed to improvement in the design of the process itself. With the analog simulation of a process, the design of desired dynamic parameters is possible. As a consequence, the control loop around the process can be improved.

The electronic analog computer operates by means of an electrical model of the system described by the mathematical expressions being solved. The voltages at various points in the computer represent the values of the variables involved. Operation of the analog computer is on a continuous basis, with the electrical parameters behaving precisely as the continuous physical system does.

By analog simulation of a process its measurable behavior, both dynamically and at steady state, is reproduced. The simulation is quantitative and requires a model that is accurate enough for the kind of performance needed and simple enough to be quickly assembled, modified and operated.

When applying the classical methods of synthesis to process control systems, the frequency response of the system is determined. Obtaining the actual time response of the system is possible, but it entails a very considerable amount of calculations in translating the results from the frequency domain to the time domain. In engineering applications the end result desired in evaluating any process control system is its time response. Fortunately this is the result obtained from the analog computer.

The analog equipment used in this study consisted of special computing units built around the George A. Philbrick operational amplifiers. All integrators were stabilized. Resistors and capacitors used in computing were the external plug-in type with 1 per cent accuracy. Outputs were recorded on a 6 channel, Sanborn 150 Recorder, Model 156-1100R. Speed of recording was 2.5 mm per second. A Sanborn D-C coupling preamplifier, Model 150-1300Z was used in the input to the recorder.

The process was simulated first with a single loop control system and then with a cascade control system. The cascade system was studied with the secondary loop containing three different transfer lags of the process: (1) .5 minute, (2) 1 minute, (3) .5 and 1 minute lag. The major process lag was always outside the secondary loop. The disturbance introduced to test the system's regulating ability consisted of a step change which always entered the process just before the 1.67 minute lag. The controller simulated had proportional and reset modes. Its transfer function is given in equation 5-1.

$$(5\text{-}1) \quad \frac{\Theta_o}{\Theta_i} = \frac{K\left(s + \frac{1}{T_i}\right)}{\left(s + \frac{1}{200\,T_i}\right)}$$

T_i is the reset (integral) mode time constant. The 200 associated with the pole is the integral mode gain at zero frequency. K is the proportional gain of the controller.

The analog simulation diagram used to patch the cascade control systems into the computer is shown in Figure 16. A similar diagram was used for the single loop simulation, but without the secondary controller.

In each case approximate controller adjustments were arrived at by first tuning the secondary controller without the primary controller cascaded onto it. Then the primary controller was connected and approximate adjustments were made on it. Optimum settings of each mode was accomplished by minimizing the integral of the absolute value of the error in the primary variable. This is sometimes called IAE or integral of absolute error and can be mathematically expressed as $\int /E/dt$. Since this was an idealized process, the physical variables were not described other than in the dimensions of the analog domain, i.e. volts. Of the functions that were recorded, the two important ones were the values of the primary variable C and the IAE. Others are useful, however, in making a complete analysis.

Figure 12 is a record on performance of a single loop control of the process. Curve a is the response of the controlled (primary) variable after a step change load disturbance. Curve c is the IAE and its maximum value of 9 volts furnishes an index for evaluating the other configurations of control.

Figure 13 depicts the performance achieved by cascade control with the .5 minute transfer lag in the secondary loop. The best controller setting showed that no reset action was needed in the secondary controller since a very high gain is permissible. Theoretically, this 0.5 minute lag is reduced to about .01 minutes by the application of the high gain. It is this that caused a quite definite improved performance of this system over the single loop system, when a load disturbance enters the process at the designated location. Curve e shows that the maximum value of IAE was decreased from 9 volts to 3.5 volts when compared with the single loop system. Curve a shows the output of the secondary loop. This is also the output of the primary controller since the secondary loop transfer function has been reduced to approximately one. Based on this, it can be seen that the primary controller output is the error amplified with integral action added. At steady state, the 9 volts output of the primary controller shown in curve a is from the integral mode. Thus the integral mode output permits load changes while the output is still held very close to the set point value of the primary controller.

Figure 14 illustrates the further improvement when the 1 minute transfer lag of the process is inside the secondary loop and the .5 and 1.67 minute transfer lags are in the primary loop. To include the 1 minute lag in the secondary loop, it required that the position of it and the .5 minute lag be interchanged. This is theoretically possible in linear systems without altering the dynamic characteristics of the over-all process. Obviously, it would not be possible in an actual system. By applying a gain of 70 with a negative feedback loop around a 1 minute transfer lag, this lag would be reduced to .014 minutes. This reduction results in an IAE of 3, which was a decrease over Figure 13 as well as Figure 12.

Figure 15 shows a cascade control system with both the .5 and 1 minute transfer lag in the secondary loop. The IAE curve shows that control of the primary variable deteriorated slightly over that of Figure 14 but was better than Figure 12 or 13.

The response of the output has a small high frequency component which is added to a larger slightly under-damped component. The higher frequency component is caused by two complex poles in the secondary loop. The slightly undamped component is generated by the single process transfer lag and the pole and zero of the reset mode. If the secondary controller gain were reduced the higher frequency component would be reduced but a larger d-c component would result. It was found that the secondary controller settings were more critical in this case than in others.

Even though the time response of the system in Figure 15 is not so good as that of Figure 14 it still has a merit which the other does not have. That is, the probability of disturbances falling inside the secondary loop is greater. These disturbances can be regulated out very quickly with little effect on the primary variable.

Figure 17 shows the effect of Reset action (integral) in the primary controller on the IAE number with various secondary controller gains for the system shown in Figure 15. It is noted that for any secondary controller gain, some reset action improves control but additional reset may cause control to deteriorate.

Root Locus Analysis

The root locus method is a graphical method for finding the roots of the characteristic equation of a system. This equation must be linear and of the form $F(s) + 1 = 0$, in which $F(s)$ is a function of the complex variable s and is factored. A single loop control system has this form, where $F(s)$ is the transfer function around the loop for the system. To find the locus of roots the poles and zeros of $F(s)$ are plotted in the s plane and used as the basis for sketching the locus. Each point on the locus is a root of the characteristic equation for a particular gain.

This method of analysis was developed by Walter R. Evans and has been used quite extensively in the analysis of control system dynamics.[13]

The root locus method of analysis has advantages over the frequency response method in that the closed loop transient response can immediately be obtained where with the frequency response method, only a very tenuous correlation exists between it and the transient response of the closed loop system.[14] For cascade control systems a particular advantage of the root locus method of analysis is that, when changes are made in the secondary loop, the effect on the over-all loop is shown directly.

Frequent examples are found in the literature on obtaining the transient response, for a change in set point, using root locus techniques; however, nothing has been written showing the transient response obtained for a load change. It is this type of system change which is considered in the analog study and shall also be used in this analysis.

For cascade systems, the procedure is to plot the root locus of the inner loop to obtain its transfer function in factored form for a particular gain. The major loop root locus is then that of a single loop containing this inner loop transfer function.

The control system shown in Figures 12 and 15 will be analyzed using root locus methods. The single loop system will be studied using the same controller settings as used in the analog study. This will allow a check to be made between the two methods of analysis. It will also be used as a reference for determining the total improvement when the cascade control system is studied using root locus.

This particular cascade control system was chosen for root locus analysis since its secondary loop is more complex than the others and the analog studied showed its controller setting to be more critical. This system will be studied with various modes in the secondary controller to determine the improvement possible.

A Spirule was used to determine the locus of roots. This device is described very well in the literature.[13]

Single Loop System

Figure 19 is a plot of the loci (possible roots) of the single loop system under study (Figure 12). The only difference between the two systems is that the pole of the reset mode transfer function is assumed to be at the origin instead of at -.005. This infers that the integral mode gain at zero frequency is infinite instead of 200.

At zero gain the closed loop poles are also the open loop poles of the control system. When gain is applied to the closed loop system, the closed loop poles move away from the open loop poles to determine the loci.

The open loop pole at the origin and the zero at 0.3 are contributed by the reset action of the controller. When controller gain is increased the closed loop pole moves toward the zero. (This closed loop pole is a root of the characteristic equation for the system.) Calculations of the gain at points on this locus reveals that this pole reaches the zero when the gain is infinite; however, with a finite gain it moves quite close to the zero.

At a gain (K) of .126 note that the system is critically damped. This is indicated by the loci leaving the real axis.

The pole located at -2.0 (for zero gain) moves out on the real axis as the gain of the closed loop system is increased. This renders the transfer lag, which is associated with this pole, less significant to the dynamic response of the over-all system.

The net effect of the reset pole and zero mentioned above is to bend the locus toward the jw axis. This of course makes the system more unstable. However, the pole near the origin causes the steady state error of the system to be very small and is usually desired in a control system.

The basic closed loop transfer function of the system for the output C with respect to the disturbance U_b is:

$$(5\text{-}2)\quad \frac{C(s)}{U_b(s)} = \frac{.6(s+2)(s+1)(s)}{(s+.6)(s+2)(s+1)(s+.005) + 1.2K\,(s+.3)}$$

When the root loci are referred to, at a gain of five, the following closed loop transfer function can be written:

$$(5\text{-}3)\quad \frac{C(s)}{U_b(s)} = \frac{.6(s+2)(s+1)(s)}{(s+.282)(s+3.07)(s+.125+j\,1.43)(s+.125-j\,1.43)}$$

If a load step change $(\frac{9.87}{s})$ is substituted for U_b the equation for C in the time domain can be written by taking the inverse transform. The inverse transform is equation 5-4.

$$(5\text{-}4)\quad C(t) = 1.26e^{-.282t} - .435e^{-3.07t} + 3.4e^{-.125t}\sin(81.5t-14)$$

Figure 18 is a plot of equation 5-4. It is noted that this time response has a smaller damping factor than that shown in Figure 12. The assumption that the reset mode pole was exactly at the origin instead of at -.005 caused this difference. The reset mode consequently had a higher gain for any particular frequency. This moved the closed loop poles further out on the loci for the proportional gain considered.

Secondary Loop

The particular cascade control system to be studied with root locus plots has two transfer lags in the secondary loop; thus the control of the secondary loop is more significant to the over-all process than if only one process lag was included. When only one process lag is in the secondary loop proportional gain is usually all that is necessary to regulate for disturbances inside the loop and also to reduce the single time constant to a small value. (The latter is important for regulation when disturbances enter the process outside the secondary loop.) With two lags in the secondary loop, consideration should be given to several controller mode combinations.

Figures 20 through 25 are root locus plots of secondary loops similar to that in Figure 15 but with various controller modes. The combinations used were:

1. Proportional only
2. Proportional and Reset
3. Proportional, Rate and Reset

The controller, with three modes of operation, has the following transfer function.

$$(5\text{-}5)\quad \frac{\Theta_o(s)}{\Theta_i(s)} = \frac{KK_d(s+\frac{1}{T_d})(s+\frac{1}{T_i})}{(s+\frac{K_d}{T_d})(\;s\;)}$$

where T_d = rate or derivative action time constant

T_i = Reset or integral action time constant

K = proportional gain of controller

K_d = 5 (gain of rate action)

Again the reset mode of the controller is assumed to have a pure integrator which is not the case with commercial process controllers. If the transfer function of a commercial controller is used, highly sensitive plotting scales would need to be used to represent the finite integral gain at zero frequencies. This amounts to a pole very near the origin but not quite upon it.

Figure 20 is a plot of the secondary loop with proportional control only. With proportional control only the controller does not introduce any poles or zeros into the system. Note that the secondary loop will begin to oscillate after disturbances when the loop gain is increased above .2, i.e., the loop is critically damped at this gain. This is shown by the loci moving away from the real axis into the complex plane. Once the roots move into the complex plane they move in a straight line to high frequencies as the gain is increased. The real component of the complex roots remains constant, i.e., the real root is not a function of the loop gain. In the time domain this real component is the time constant of the decaying oscillation. Thus an increase in gain does not change the decay time of the disturbance but only increases the frequency and decreases the damping factor. This is important since the real components determine the time constant of the secondary loop and ultimately play an important part in determining the response of the over-all cascade control system. When the real component of any complex root is small, it is easy for the root to move into the unstable region, once it has another loop cascaded onto it.

Figures 21, 22 and 23 show plots of the secondary loop with proportional and reset in its controller. In Figure 20 the zero of the reset transfer function $(\frac{1}{T_i})$ lies at the same place on the real axis as a process pole, thus eliminating each from affecting the system. In Figures 22 and 23, the reset zero is smaller and larger, respectively. It is noted that the real component of each set of complex roots is smaller than that in Figure 20 when the damping factors are similar. Furthermore, an additional closed loop pole is added for cases in Figures 22 and 23. This pole will cause a slow decaying transient in the secondary loop and slower response in the outside loop. Consequently, none of these controller configurations would be better than that of Figure 19.

Of course, it would not be expected that reset action could help the transient response of a system. Its usefulness is after the transient has decayed, particularly on a system that has low gains and a constant load. Therefore, the above does not exclude the use of reset action to improve system performance.

Figures 24 and 25 are plots of the secondary loop when the controller has proportional, rate and reset action. There is a definite improvement in the system by the introduction of rate action since the real part of the complex roots is much larger. This will result in a faster decay of oscillations. Note that the controllers rate and reset zeros were located to cancel the process poles leaving only a pole at the origin and the pole out on the real axis. This plot clearly shows the system to be an improvement over Figure 20 since it results in a similar loci except having complex poles with real components three times larger.

Figure 25 is a plot of the secondary loop with less rate but more reset action. When the controller gain is 20 or more the complex poles have a real component that is four times that of the straight proportional system. As pointed out before, this will cause the oscillations to decay faster. However, the closed loop pole which moved in on the real axis will increase the secondary loop time constant and slowing down the response of the overall cascade system.

Cascade Control System

A rational closed loop transfer function of these various secondary loop configurations can now be determined. It can then be used with the open loop transfer function of the remainder of the primary loop to plot the locus of roots for various controller gains. Transient response will be found when disturbances enter the process between the secondary loop and the final part of the process.

Figure 26 are the results of a root locus plot of the cascaded system using proportional gain only in the secondary loop. This system is the one shown in Figure 15 except pure integral (reset) is used instead of that which is available in most commercial controllers. The secondary loop transfer function was determined for a gain of 30 from Figure 20. The transfer function is shown in equation 5-6.

$$(5\text{-}6)\quad \frac{C'(s)}{r'(s)} = \frac{60}{s^2+3s+62.5}$$

Referring to Figure 26a, it is noted that increases of gain in the primary controller move the two poles in the real axis into the complex plane and back to the real axis. This type of locus contributes to stable operation in the loop. To counteract this, the two complex "control" poles of the secondary loop move in unstable regions as gain is increased.

By referring to Figure 15 and using equations 5-6 and 5-5, the basic transfer function for the cascade system with respect to U_b is:

$$(5\text{-}7)\quad \frac{C(s)}{U_b(s)} = \frac{.6(s^2+3s+62.5)(s)}{(s)(s^2+3s+62.5)(s+.6)+36K(s+.9)}$$

For a gain of five, the roots of the denominator of equation 5-7 can be evaluated from the root locus plot. This enables equation 5-8 to be written

$$(5\text{-}8)\quad \frac{C(s)}{U_b} = \frac{.6(s)(s+3s+62.5)}{(s+1.07)(s+2.5)(s+.1+j7.8)(s+.1-j7.8)}$$

Note that the numerator is the same as in equation 5-7 but the denominator is in factored form. Thus an inverse transform can be easily taken.

When U_b is a 10 volt step change, equation 5-9 can be written by taking the inverse transform of equation 5-12.

$$(5\text{-}9)\quad C(t) = 4.08e^{-1.07t} - 3.84e^{-2.5t} + .262e^{-.1t}\sin(447t-69)$$

Equation 5-9 has two d-c components and a highly underdamped a-c component. The a-c component is of great enough magnitude to drastically affect controllability. This component is caused by the secondary loop poles. A decrease in gain would be necessary to increase the damping of the oscillation. For a gain of 2.5 the transient response is equation 5-10.

$$(5\text{-}10)\quad C(t) = 12e^{-1.05t}\sin .5t - .1508e^{-.75t}\sin(444t-90)$$

Figure 26b is the transient response for a gain of 5. Figure 26d is the response for a gain of 2.5. It is evident that when the oscillation is reduced, a greater overshoot and slower decaying low frequency oscillation exist. The low frequency oscillation is because the two poles which left the real axis are still in the complex area.

Figure 26c is analog simulator solution to this same system and disturbance for a primary controller gain of 5. The fact that it is similar to Figure 26b confirms the correctness of the root locus solutions.

Figure 27 is the root locus plot of the cascade system with the secondary loop using a three mode controller. The root locus plot of this secondary loop is shown in Figure 25. For a secondary controller gain of 20 its loop transfer function is equation 5-11.

$$(5\text{-}11)\quad \frac{C'(s)}{r'(s)} = \frac{200(s+3)}{(s+3.5)(s+6.5+j11.25)(s+6.5-j11.25)}$$

This results in the following basic transfer function for the cascade system. Again refer to Figure 15 for the general configuration under study.

$$(5\text{-}12)\quad \frac{C(s)}{U_b(s)} = \frac{.6(s+3.5)(s^2+13s+168.8)(s)}{(s)(s+3.5)(s^2+13s+168.8)(s+6)+200K(s+1)(s+3)}$$

The root locus plot indicates that a gain of 10 may give the roots for an optimum system. Based on these roots equation 5-13 is written.

$$(5\text{-}13)\quad \frac{C(s)}{U_b} = \frac{.6(s)(s+3.5)(s^2+13s+168.8)}{(s+1.25)(s+2.72)(s+9.5)(s+2+j11.5)(s+2-j11.5)}$$

When U_b is a 10 volt step change the following equation can be written for C by taking the inverse transform of equation 5-13.

$$(5\text{-}14)\quad C(t) = 1.3e^{-1.25t} - .5e^{-2.72t} - .455e^{-9.5t} + .346e^{-2t}\sin(658t-61)$$

Note that equation 5-14 shows three d-c transient components and an a-c component. The d-c components are from poles on the real axis and the a-c component is from the two complex poles. The time constant of the a-c component is small enough to reduce it to zero in about two minutes.

Figure 28 is a plot of equation 5-14.

It is quite clear at this point that this secondary loop controller configuration improves control over the system shown in Figure 26. The improvement came about, even though there were additional poles introduced on the real axis. The main reason for the better response was that the real components of the complex poles were increased through the use of derivative in the secondary loop.

Figure 29 is a plot of the root locus of the system with more rate and less reset action in the secondary control. Figure 24 is the root locus plot of the secondary loop. The transient response to a 10 volt step is shown by equation 5-15 and in Figure 30.

$$(5\text{-}15) \quad C(t) = 12.88e^{-1.07t} \sin 28.6t$$

This response is not as good as that shown in Figure 28. However, it is evident that additional gain would improve the response still further. At this point it is hard to say which adjustment of rate action is better but it has been shown that it definitely improves the over-all response of a cascade system when rate action is included in a secondary loop with at least two transfer lags. This is true when a load disturbance enters the system outside the secondary loop as well as inside the secondary loop.

It is apparent from the root locus plots that secondary loops with multiple transfer lags can easily cause rapid oscillations in the output of cascade control systems. If the complex poles which cause these oscillations do not have a fast time constant, very little gain is possible in the primary controller and fast over-all response may not be possible.

As the transfer lags in the secondary loop increase above two, the benefits of rate action decrease because of the over-balance in poles to the one zero of the rate action.

CHAPTER VI

CONCLUSIONS

The purpose of this work is to discuss some characteristics of cascade control systems and show the advantages of including larger proportions of the process in the secondary loop of these systems. The benefits of rate action in the secondary controller when two lags are included are demonstrated.

If a multilag process is to be controlled, very great improvement is realized when a cascade control system is used instead of a single loop system. This is true even though the disturbance enters the process outside of the secondary loop. By including larger portions of the process in the secondary loop, further improvements are possible.

The most effective way to obtain improvement of system response to a step disturbance is to include the second largest process transfer lag in the secondary loop. This is because the large lag can be practically eliminated with a simple proportional controller thus reducing its effect when disturbances enter the process outside the secondary loop.

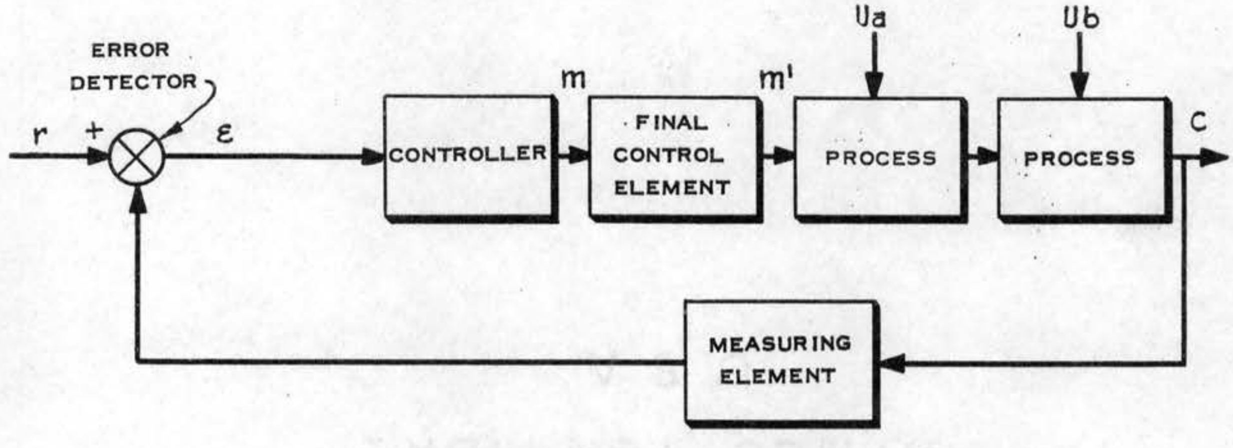
As the gain of the proportional controller is increased, the size of the transfer lag is reduced. However, there is a limit to the amount of gain used because the secondary controller gain is a multiplier of the primary controller proportional gain which causes the apparent gain of the system to be quite high. When this apparent gain is too high, unstable operation results.

When it is not possible to include only the second largest transfer lag in the secondary loop, next best improvement in response is possible by including this second largest together with a lesser but significant lag inside the secondary loop. Here the type of secondary controller used is more critical. A proportional-only controller can be used, but the gain must be kept rather low for over-all stability reasons. Better results are possible when rate action is used in the controller. This enables a higher gain setting and consequently faster response for the over-all system. Reset action will be needed only when a low controller gain is used. This is mainly for keeping steady state offset disturbances from entering the primary loop.

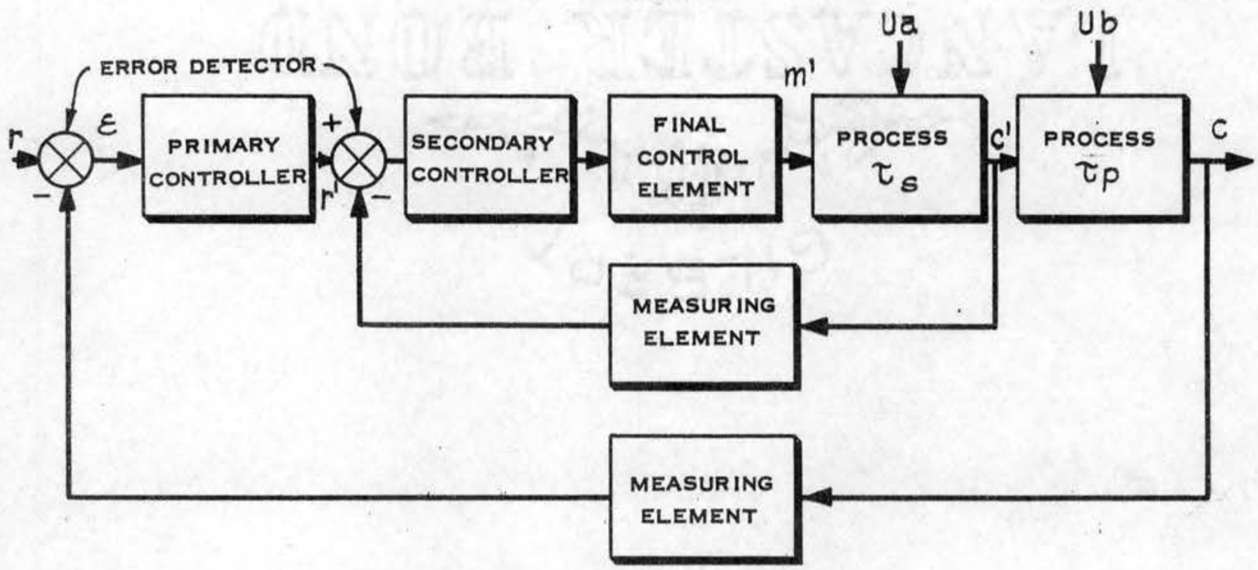
It is deduced from this investigation that when more than a second ordered lag is included in the secondary loop, the degree of control deteriorates and the complexity of controller settings increases. As more lags are added, the phase advance rendered by rate action becomes negligible and the controller gain must be lowered. Furthermore, the apparent pure time delay (dead time) associated with multi-ordered lags causes a drastic reduction in the controller gain. Thus the advantages of the secondary loop are greatly reduced.

Since the configuration of the secondary loop has so much effect on the response of the over-all system, the correct procedure is to first adjust the controller for the secondary loop without the primary controller cascaded onto it. Here the secondary controller is tuned for a disturbance inside the secondary loop and its response can be more oscillatory than usual. After an optimum operation is obtained, then cascade the primary controller onto the secondary controller and tune the primary controller. The primary controller is tuned for a disturbance outside the secondary loop and should be only slightly oscillatory. If unstable operation results with a very low primary controller gain, then the gain of the secondary controller should be reduced, because again it is the product of the two gains that is important to the over-all system. Thus a compromise will need to be made between the two settings.

It has been shown that both analog simulation and root locus methods are very effective methods of studying cascade control systems. Analog simulation is effective where a wide variety of controllers and settings are tried in order to arrive at a proper controller configuration. Root locus is useful in gaining an insight on what is happening to the system as the different parameters are changed.



FEEDBACK CONTROL SYSTEM
FIGURE 1



CASCADE CONTROL SYSTEM
FIGURE 2

C = PRIMARY VARIABLE (PROCESS OUTPUT)
C' = SECONDARY VARIABLE
τ_s = PROC. TIME CONSTANT OF SECONDARY LOOP
τ_p = PROC. TIME CONSTANT OF PRIMARY LOOP
r = DESIRED VALUE OF C (SET POINT)
ε = ERROR = (r − c)
m = OUTPUT OF CONTROLLER
m' = OUTPUT OF FINAL CONTROL ELEMENT
Ua = LOAD DISTURBANCE a
Ub = LOAD DISTURBANCE b
r' = DESIRED VALUE OF C' (SET POINT)

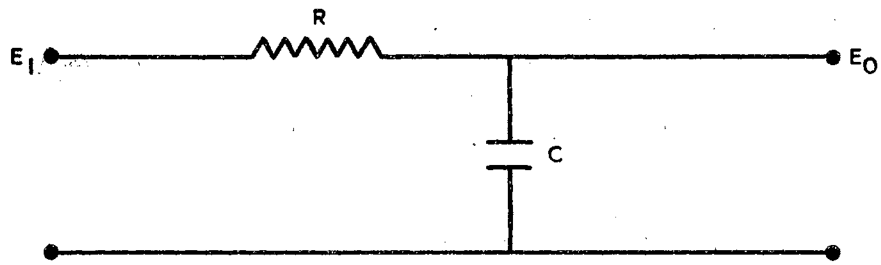


FIGURE 3A
RC LAG

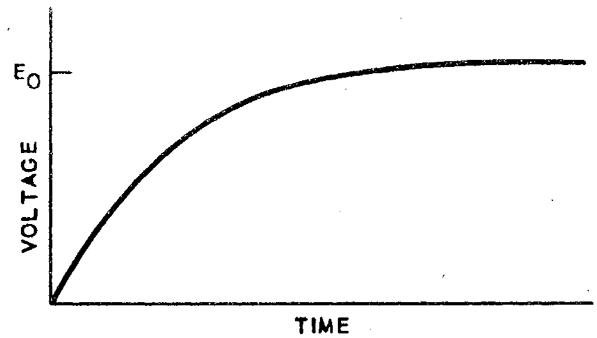


FIGURE 3B
STEP RESPONSE OF RC LAG

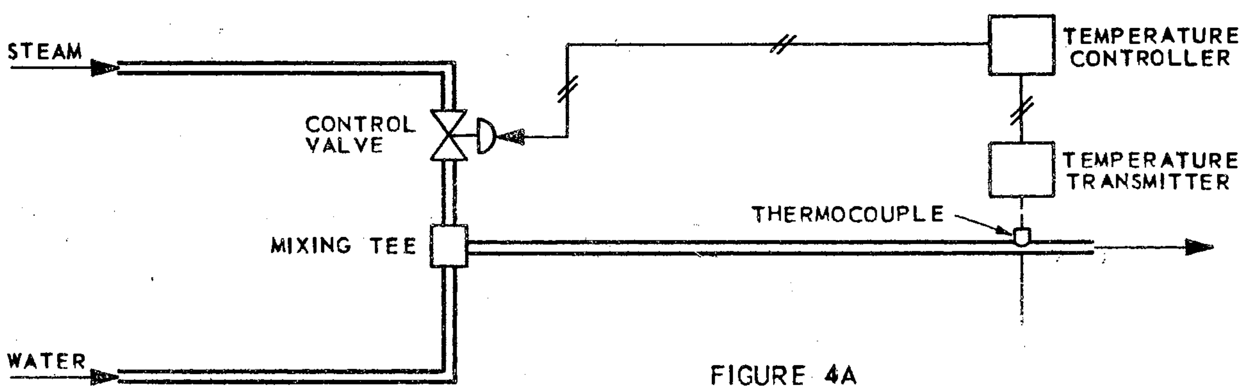


FIGURE 4A
SYSTEM WITH PURE TIME DELAY

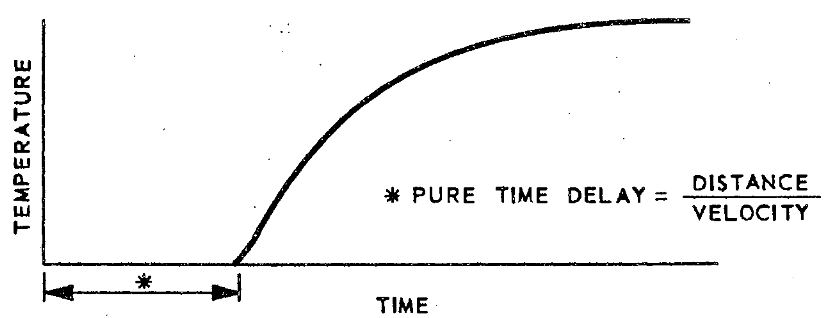


FIGURE 4B
STEP RESPONSE OF SYSTEM WITH
PURE TIME DELAY

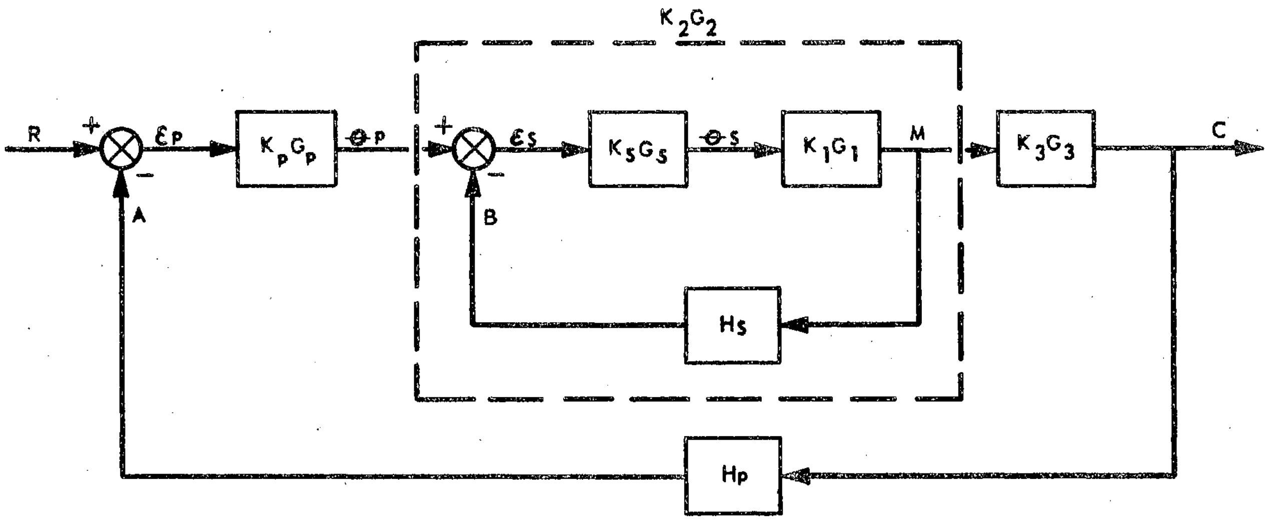


FIGURE 5
ILLUSTRATIVE CASCADE SYSTEM

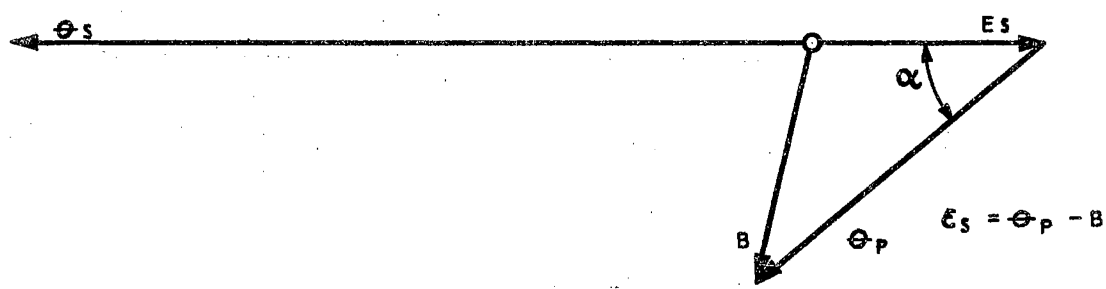


FIGURE 6
VECTOR DIAGRAM FOR SECONDARY LOOP
USING LIMITED GAIN (K_S)

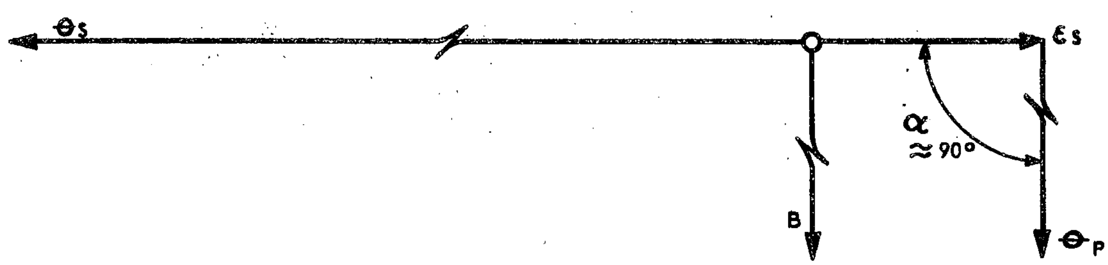


FIGURE 7
VECTOR DIAGRAM FOR SECONDARY LOOP
USING HIGH GAIN (K_S)

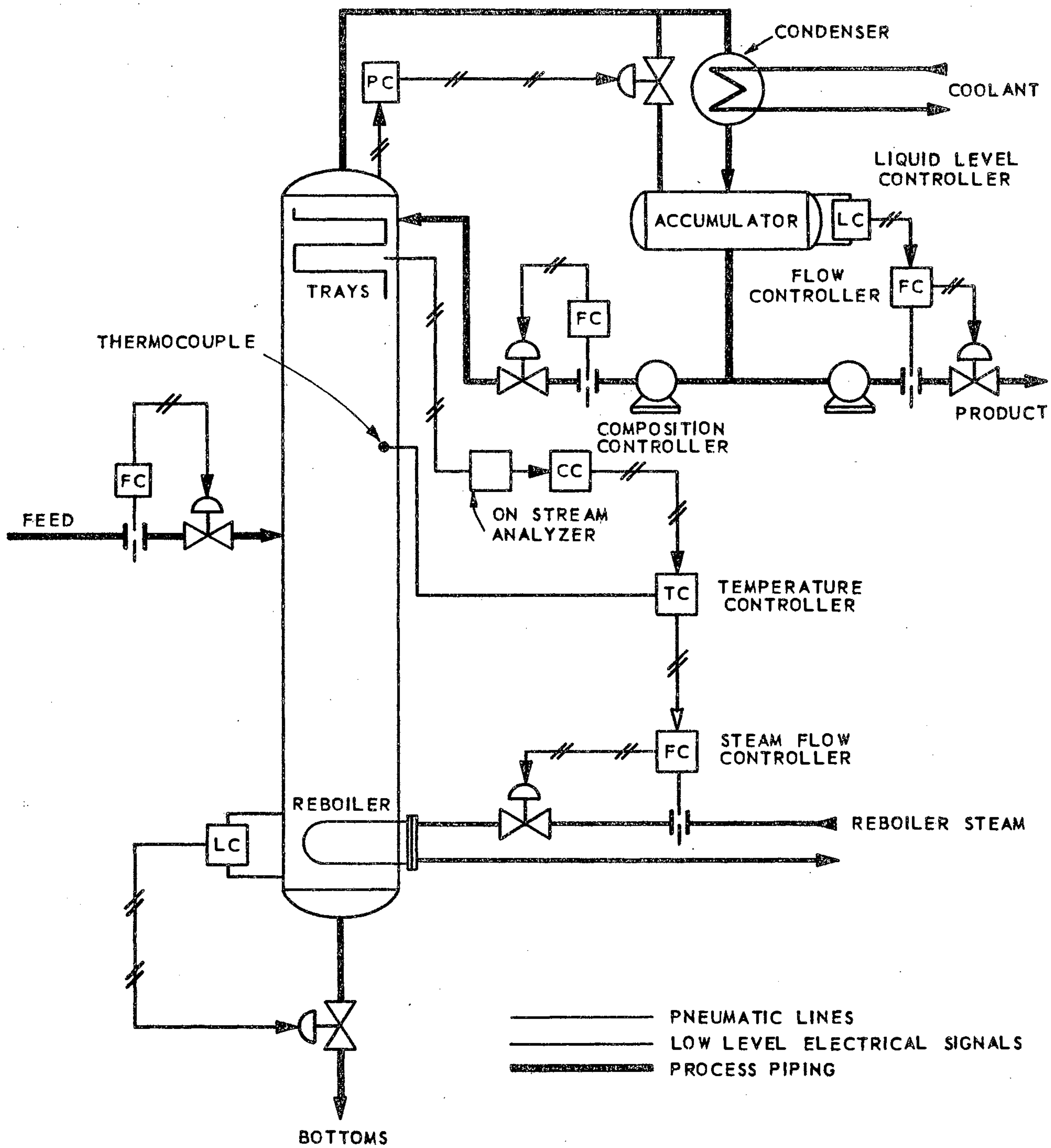


FIGURE 8
CONTROL SYSTEM ON A FRACTIONATION COLUMN

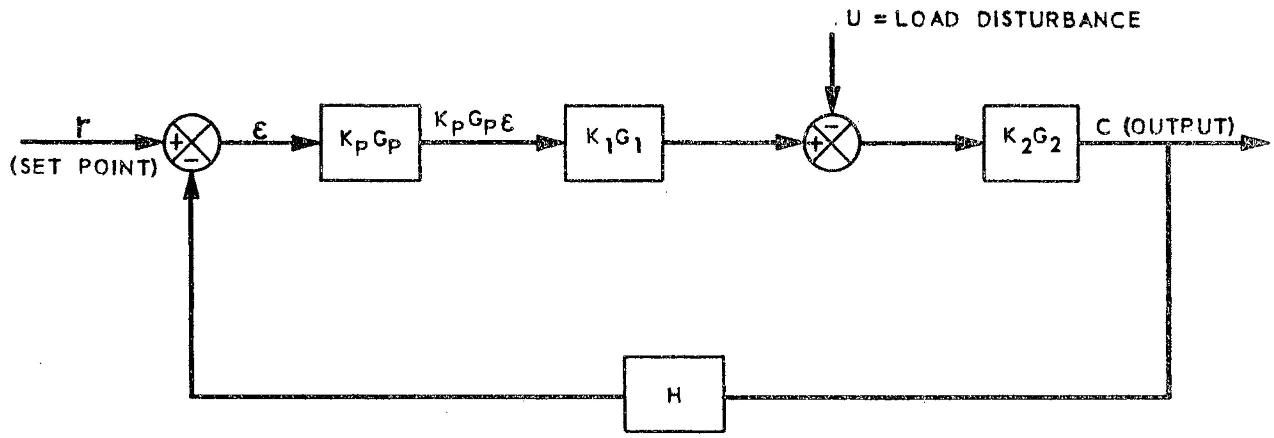


FIGURE 9
SINGLE LOOP SYSTEM USED TO GET TRANSFER FUNCTION

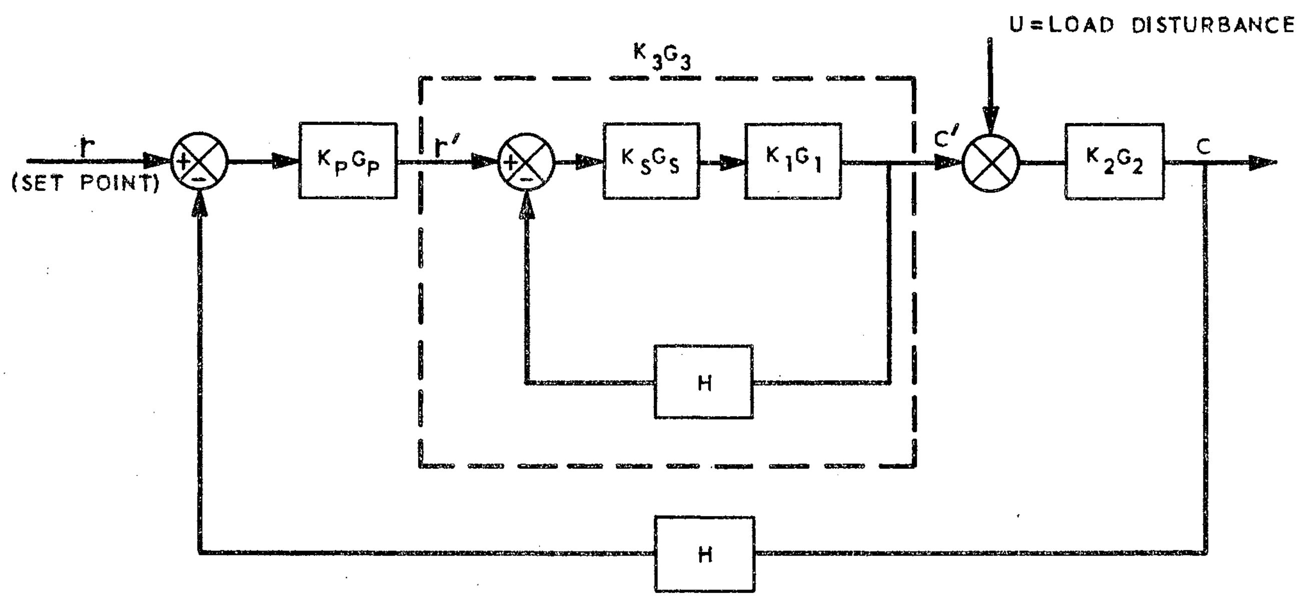


FIGURE 10
CASCADE SYSTEM USED TO GET TRANSFER FUNCTION

$K_p G_p$ = TRANSFER FUNCTION OF PRIMARY CONTROLLER
$K_S G_S$ = " " " SECONDARY CONTROLLER
$K_1 G_1$ = " " " PROCESS NO. 1
$K_2 G_2$ = " " " PROCESS NO. 2
H = " " " MEASURING ELEMENT

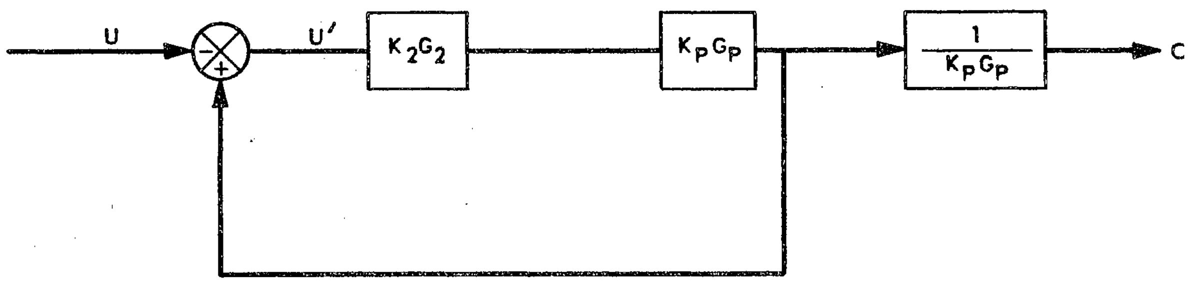


FIGURE 11A
DIAGRAM FROM EQUATION 3–12

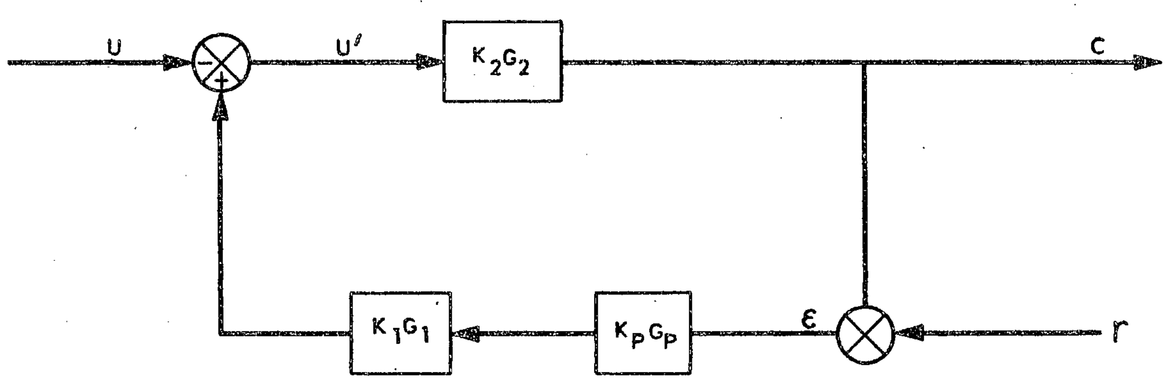


FIGURE 11B
DIAGRAM FROM EQUATION 3–13

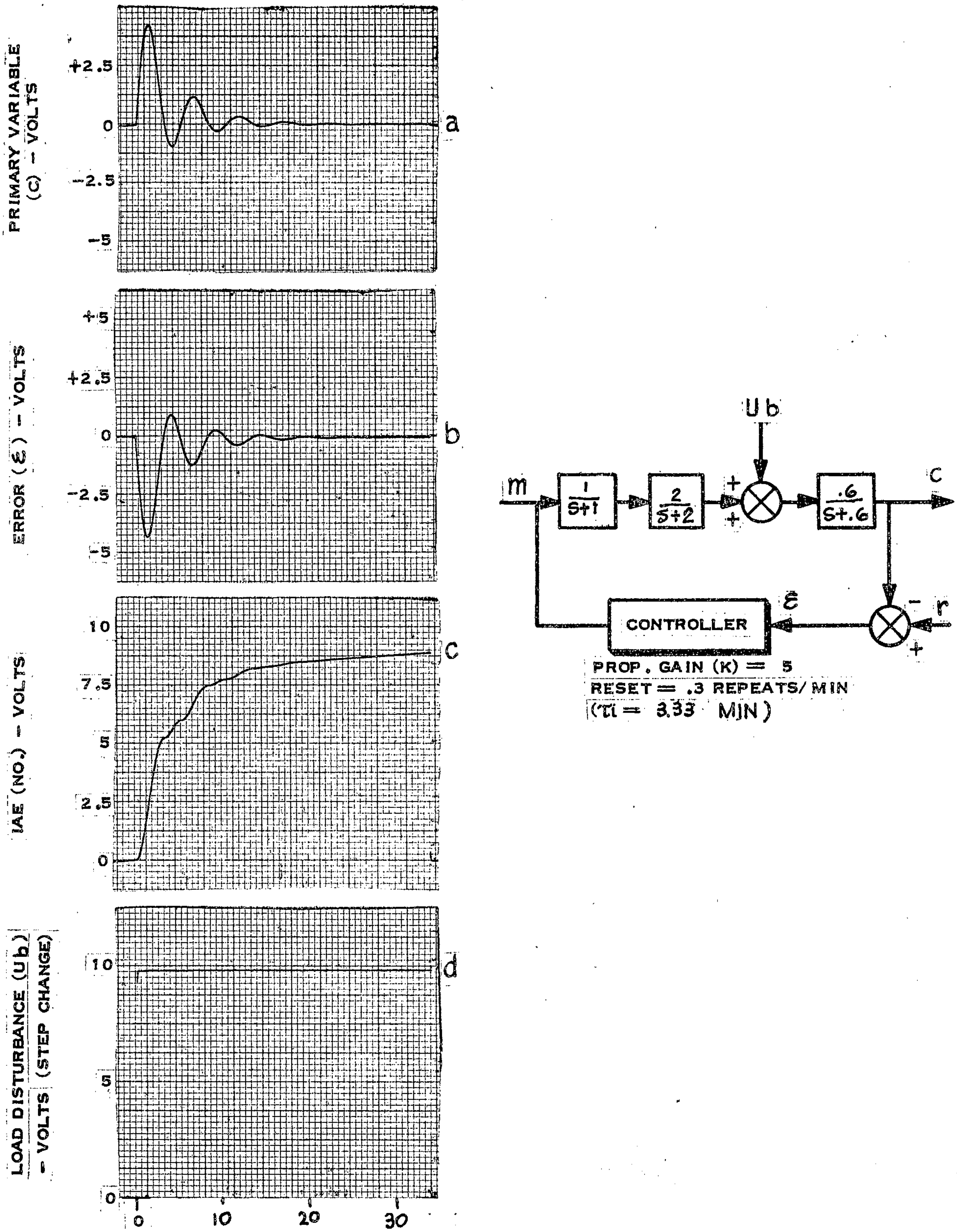


FIGURE 12

ANALOG RESULTS FOR SINGLE LOOP

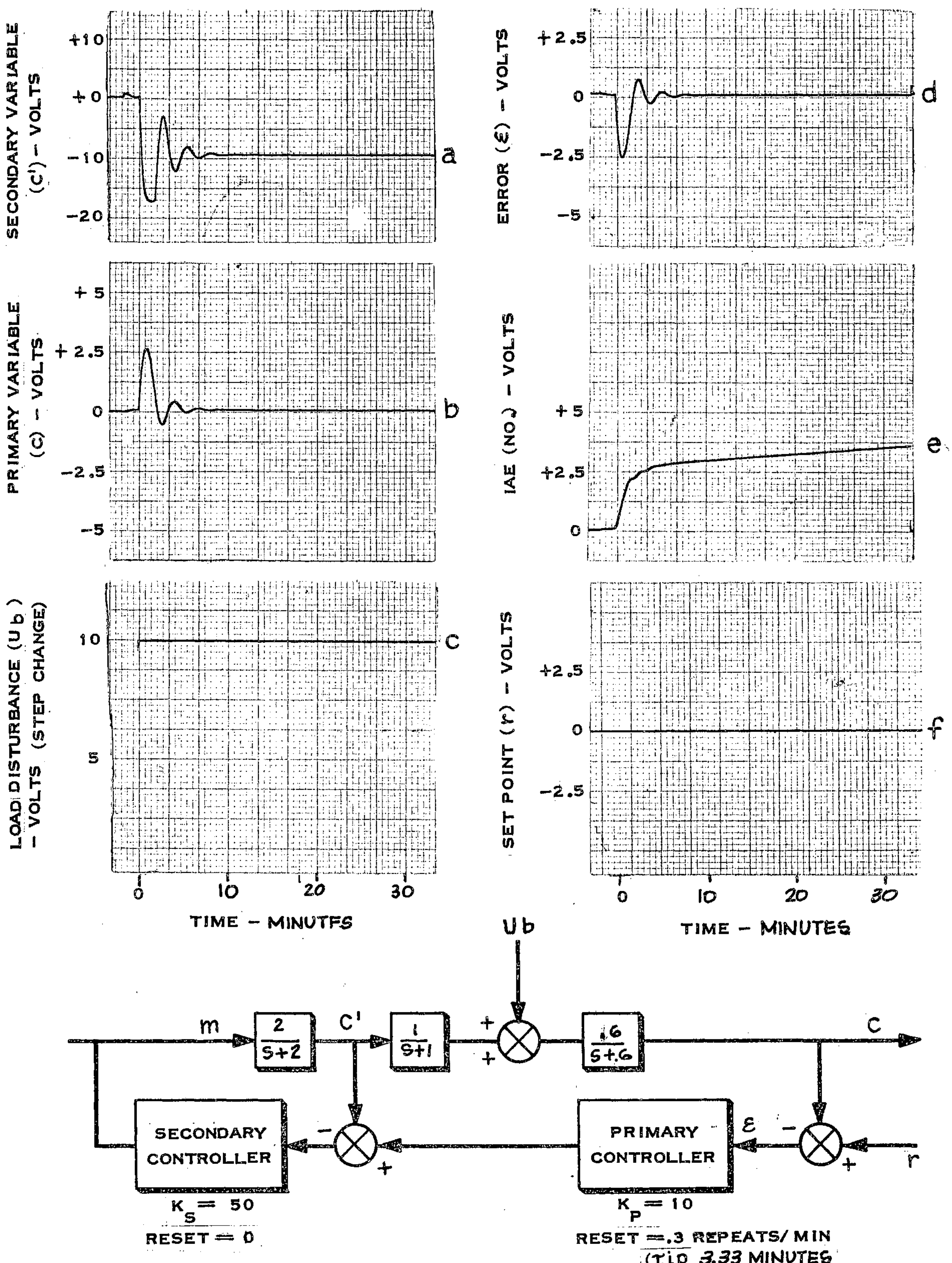


FIGURE 13

ANALOG RESULTS WHEN SMALLEST LAG IS IN SECONDARY LOOP

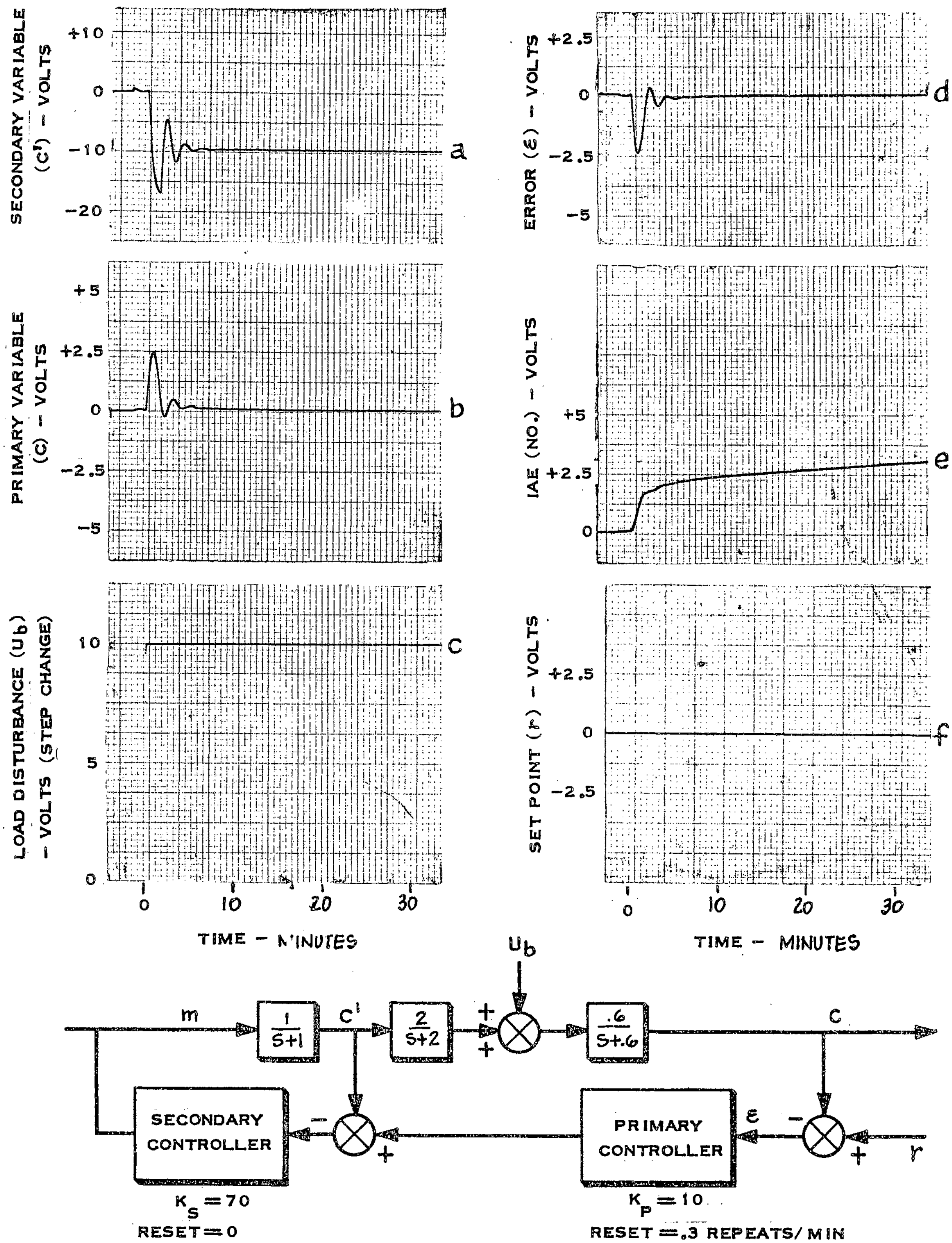


FIGURE 14

ANALOG RESULTS WHEN SECOND LARGEST LAG IS IN SECONDARY LOOP

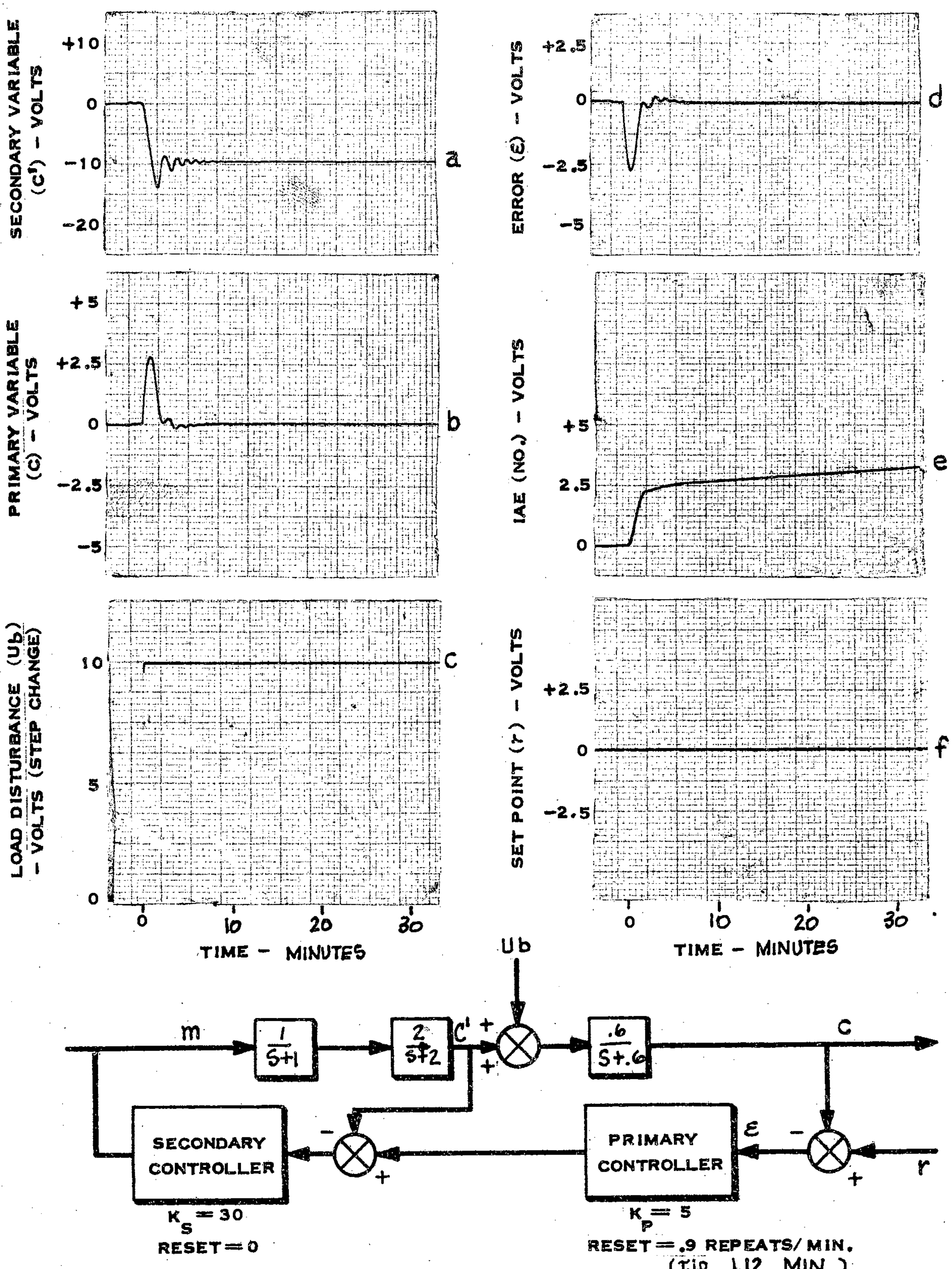


FIGURE 15

ANALOG RESULTS WHEN TWO SMALLEST LAGS ARE IN SECONDARY LOOP

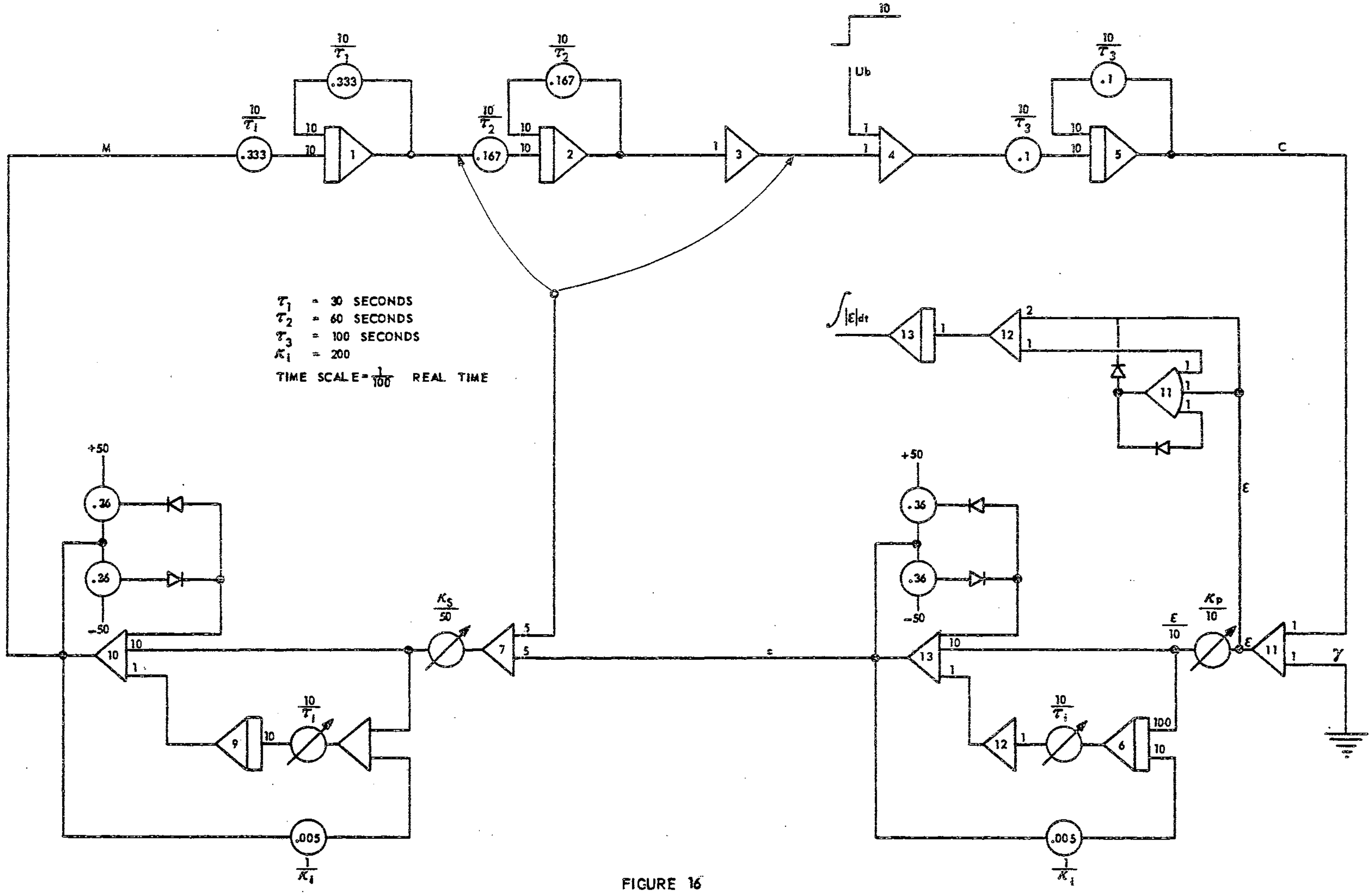


FIGURE 16
ANALOG SIMULATION DIAGRAM
CACADE CONTROL SYSTEM

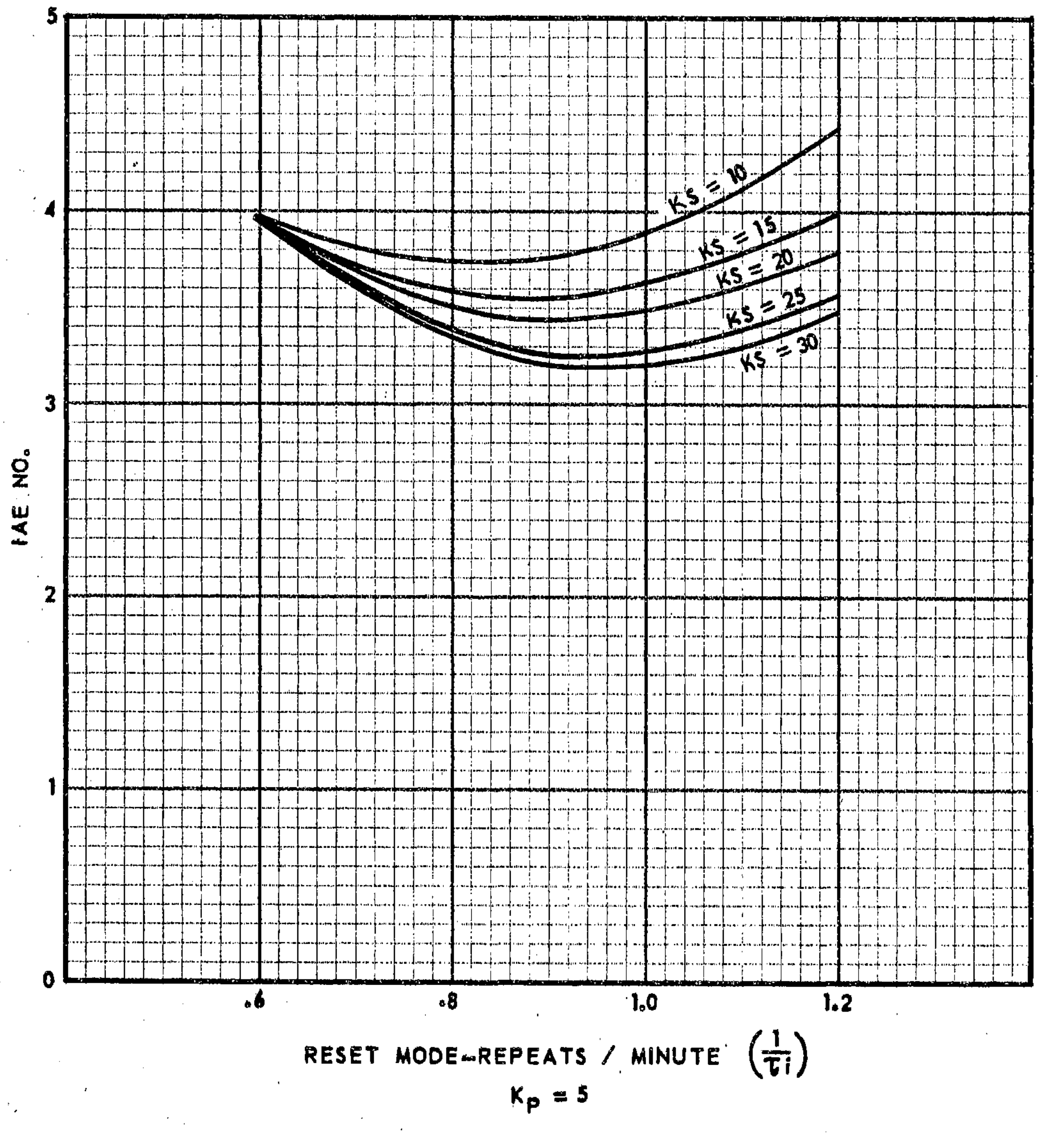


FIGURE 17
EFFECTS OF RESET ON IAE NO. FOR CONTROL SYSTEM SHOWN IN FIGURE 15

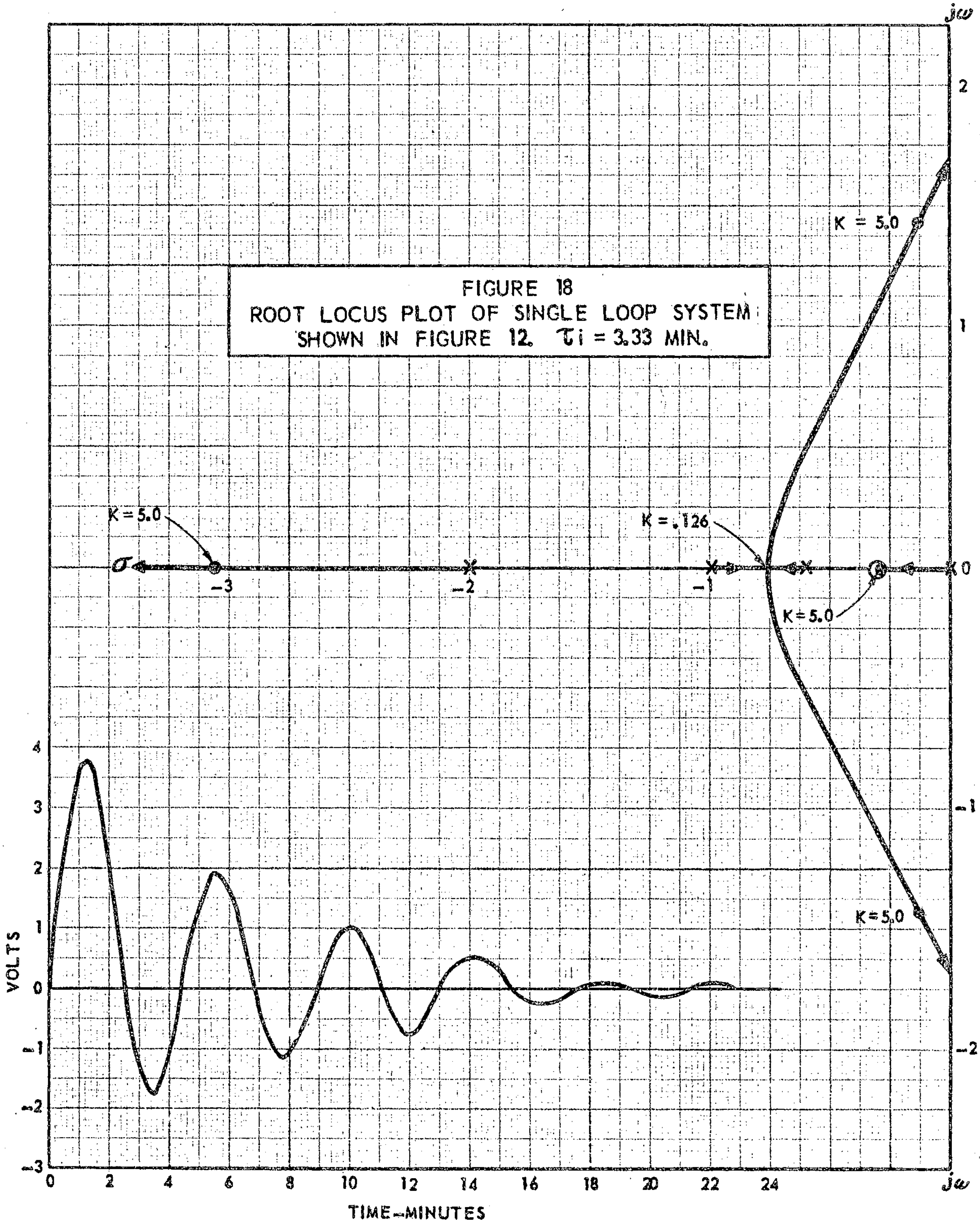


FIGURE 19
TIME RESPONSE OF SINGLE LOOP SYSTEM
WITH A GAIN OF FIVE

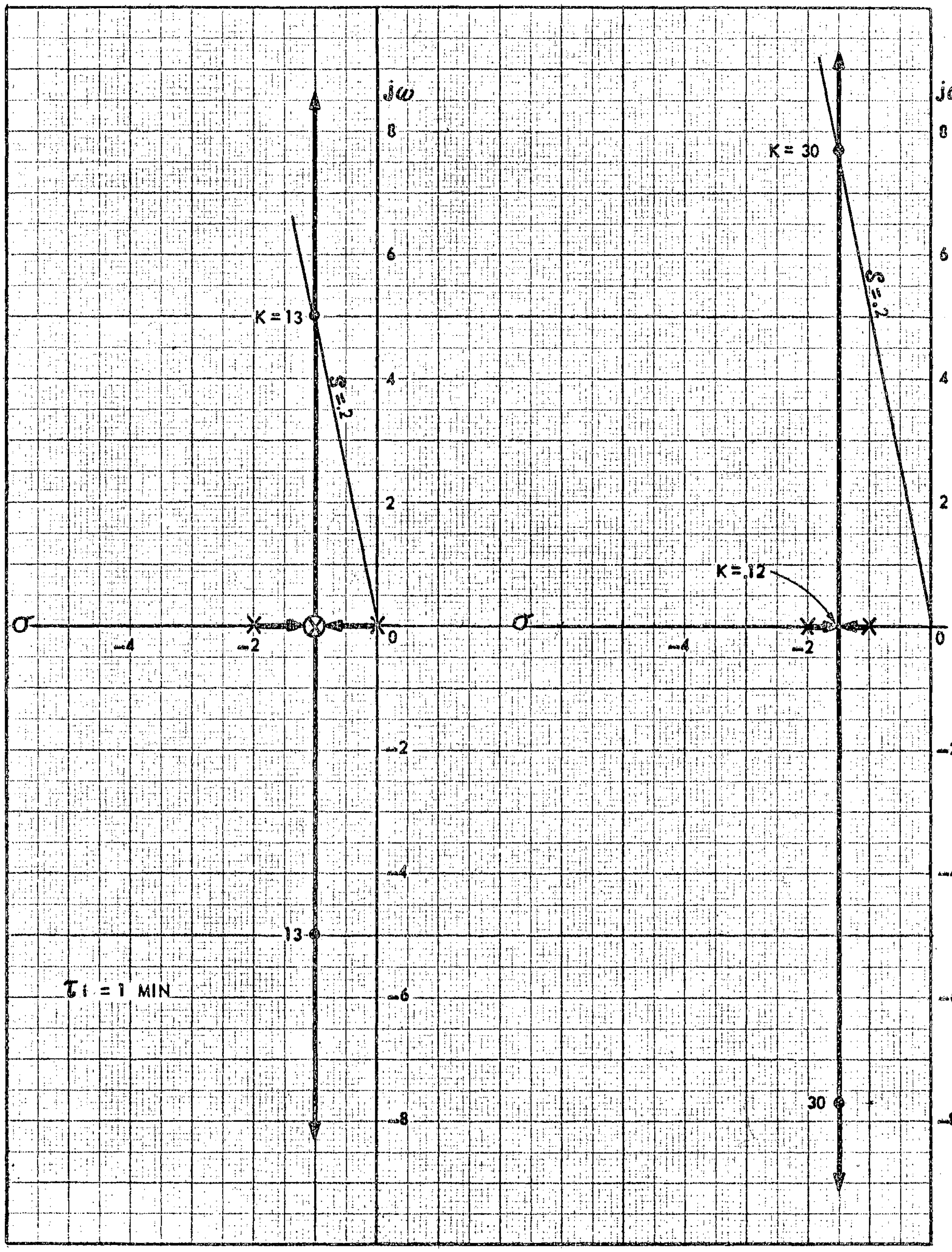


FIGURE 21
ROOT LOCUS OF SECONDARY LOOP
USING PROPORTIONAL AND RESET CONTROL

FIGURE 20
ROOT LOCUS OF SECONDARY LOOP
USING ONLY PROPORTIONAL CONTROL

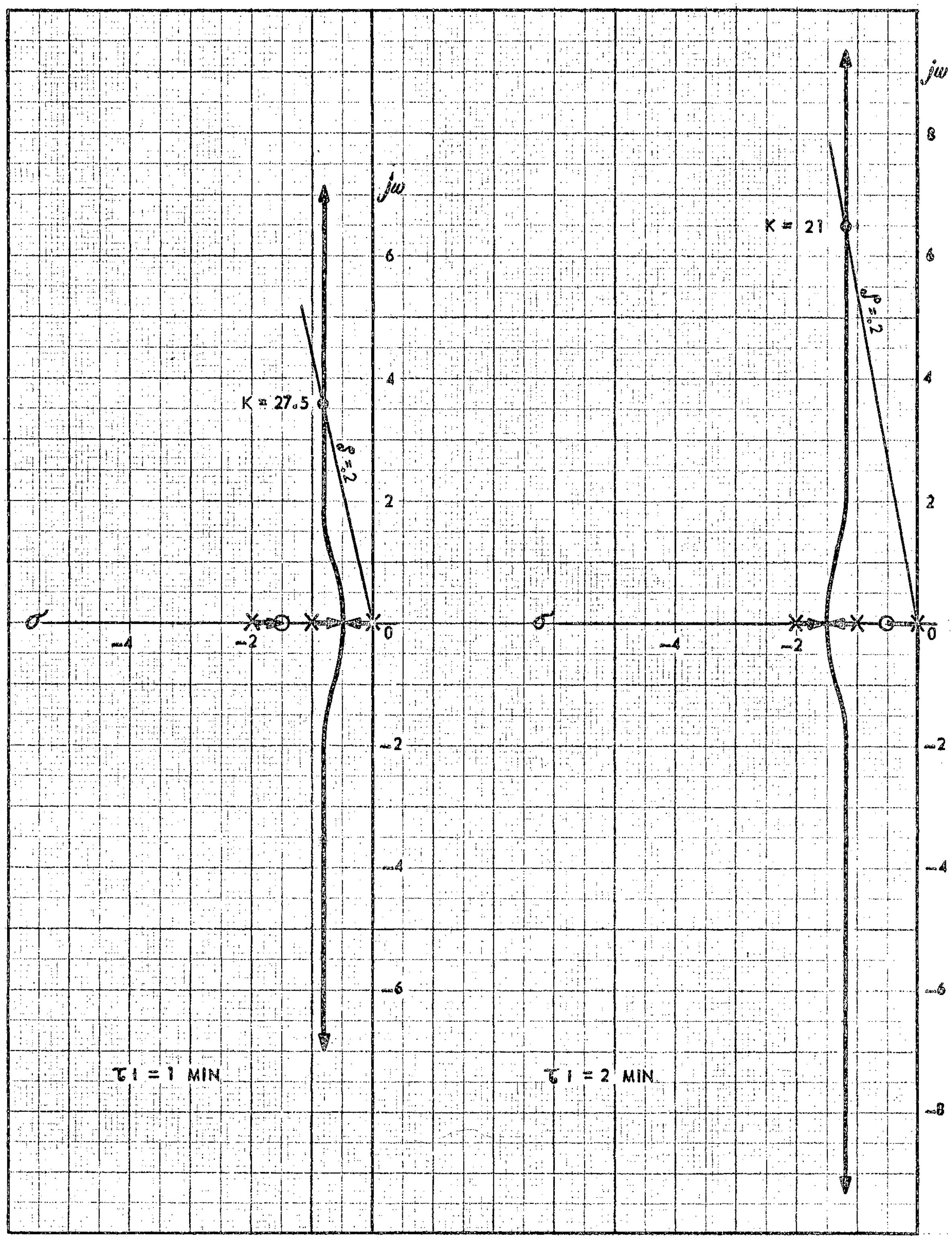


FIGURE 23
ROOT LOCUS PLOT OF SECONDARY LOOP WITH PROPORTIONAL AND RESET CONTROL

FIGURE 22
ROOT LOCUS OF SECONDARY LOOP WITH PROPORTIONAL AND RESET CONTROL

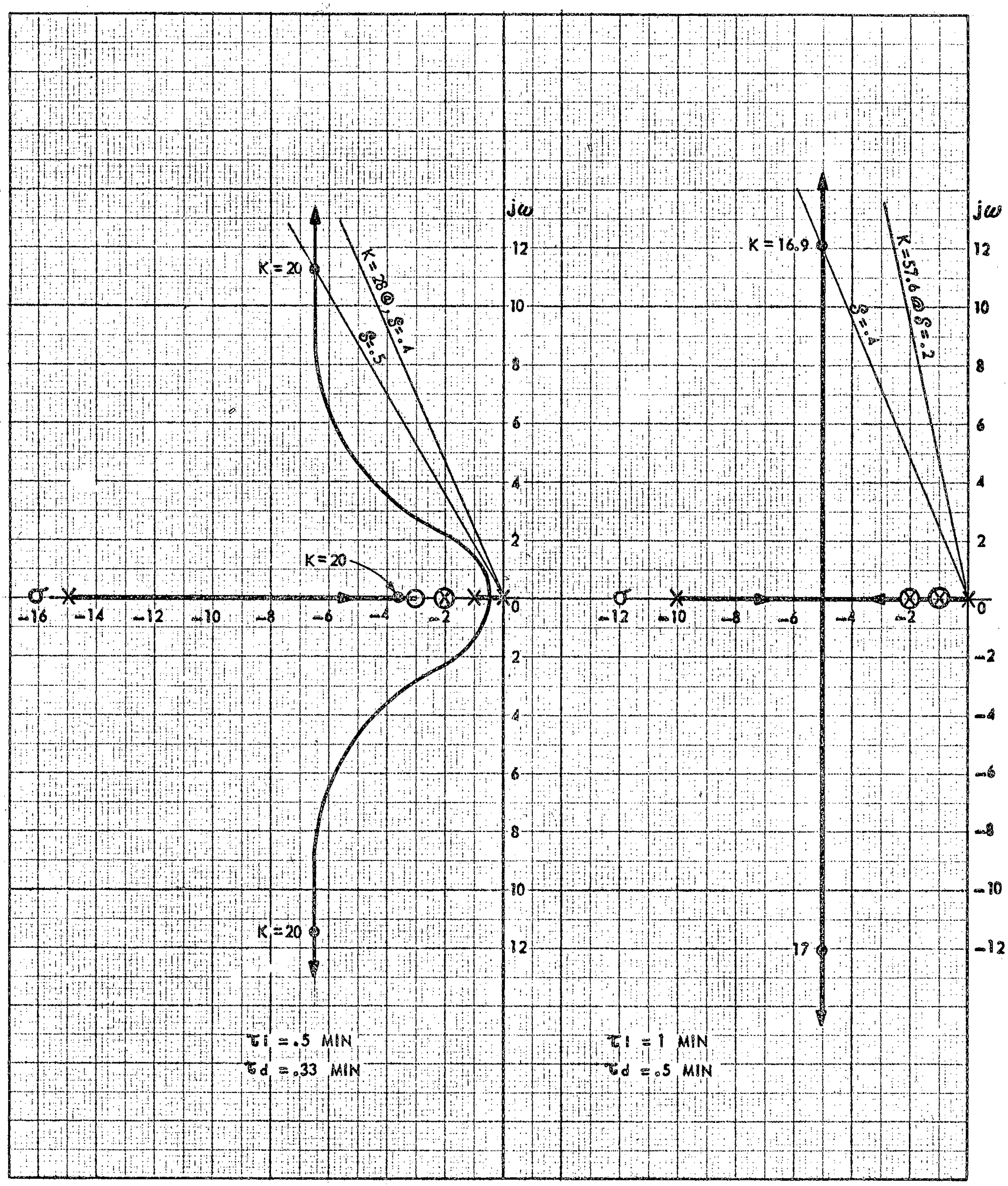


FIGURE 25
ROOT LOCUS PLOT OF SECONDARY LOOP WITH PROPORTIONAL, RATE AND RESET CONTROL

FIGURE 24
ROOT LOCUS OF SECONDARY LOOP WITH PROPORTIONAL, RATE & RESET CONTROL

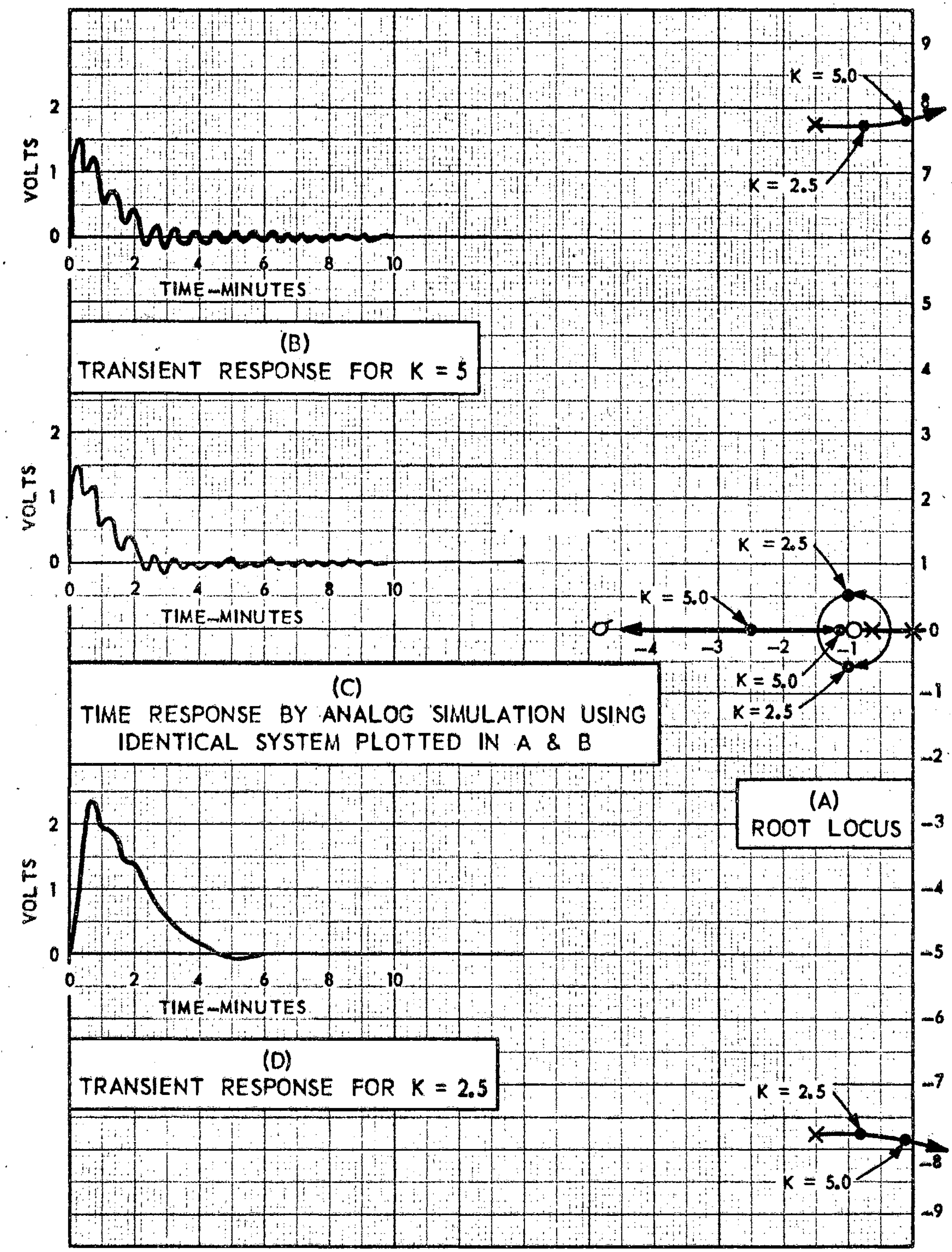


FIGURE 26
ROOT LOCUS SOLUTION OF CASCADE CONTROL WITH ONLY
PROPORTIONAL GAIN IN SECONDARY CONTROLLER

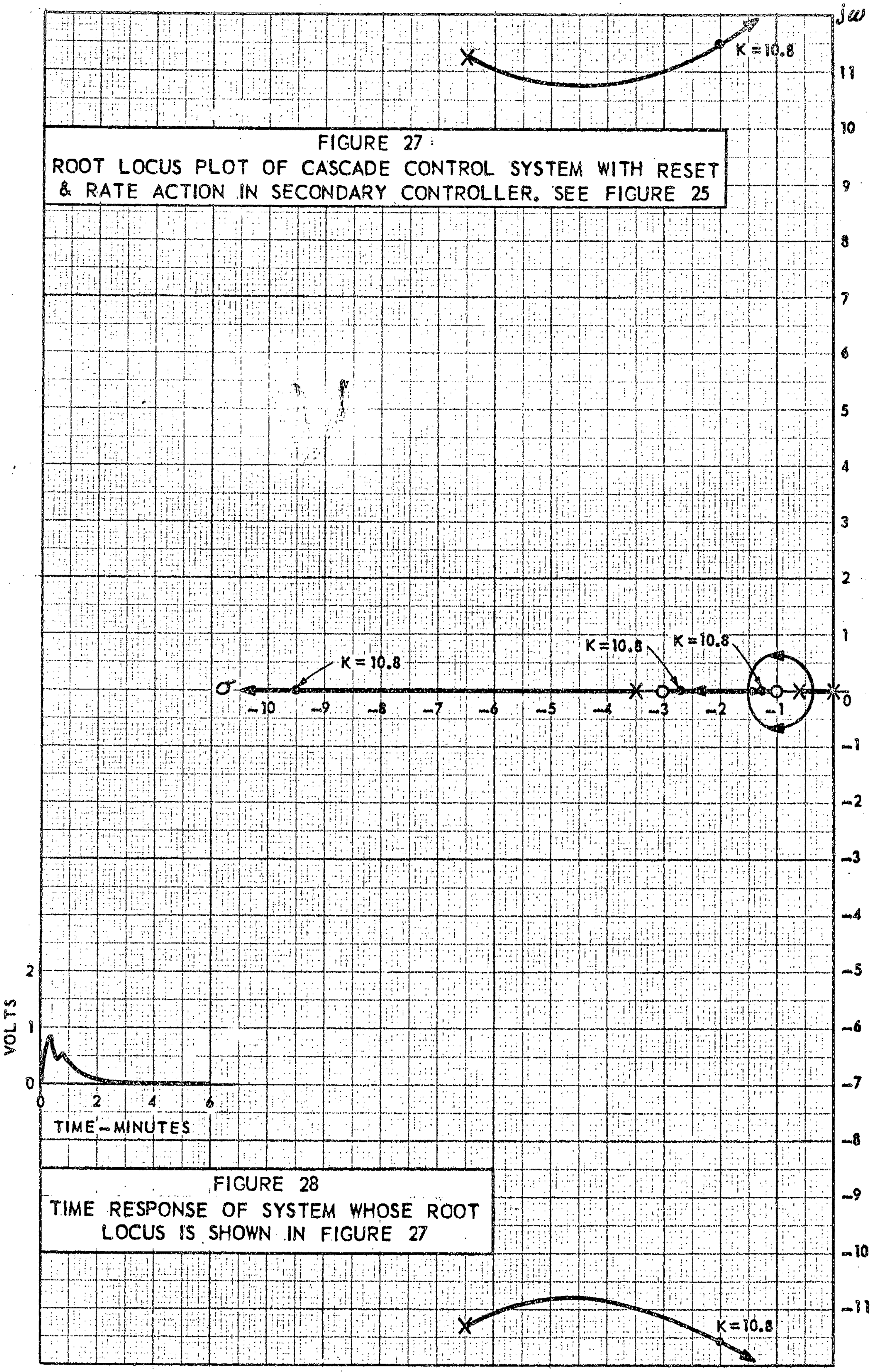


FIGURE 27
ROOT LOCUS PLOT OF CASCADE CONTROL SYSTEM WITH RESET & RATE ACTION IN SECONDARY CONTROLLER. SEE FIGURE 25

FIGURE 28
TIME RESPONSE OF SYSTEM WHOSE ROOT LOCUS IS SHOWN IN FIGURE 27

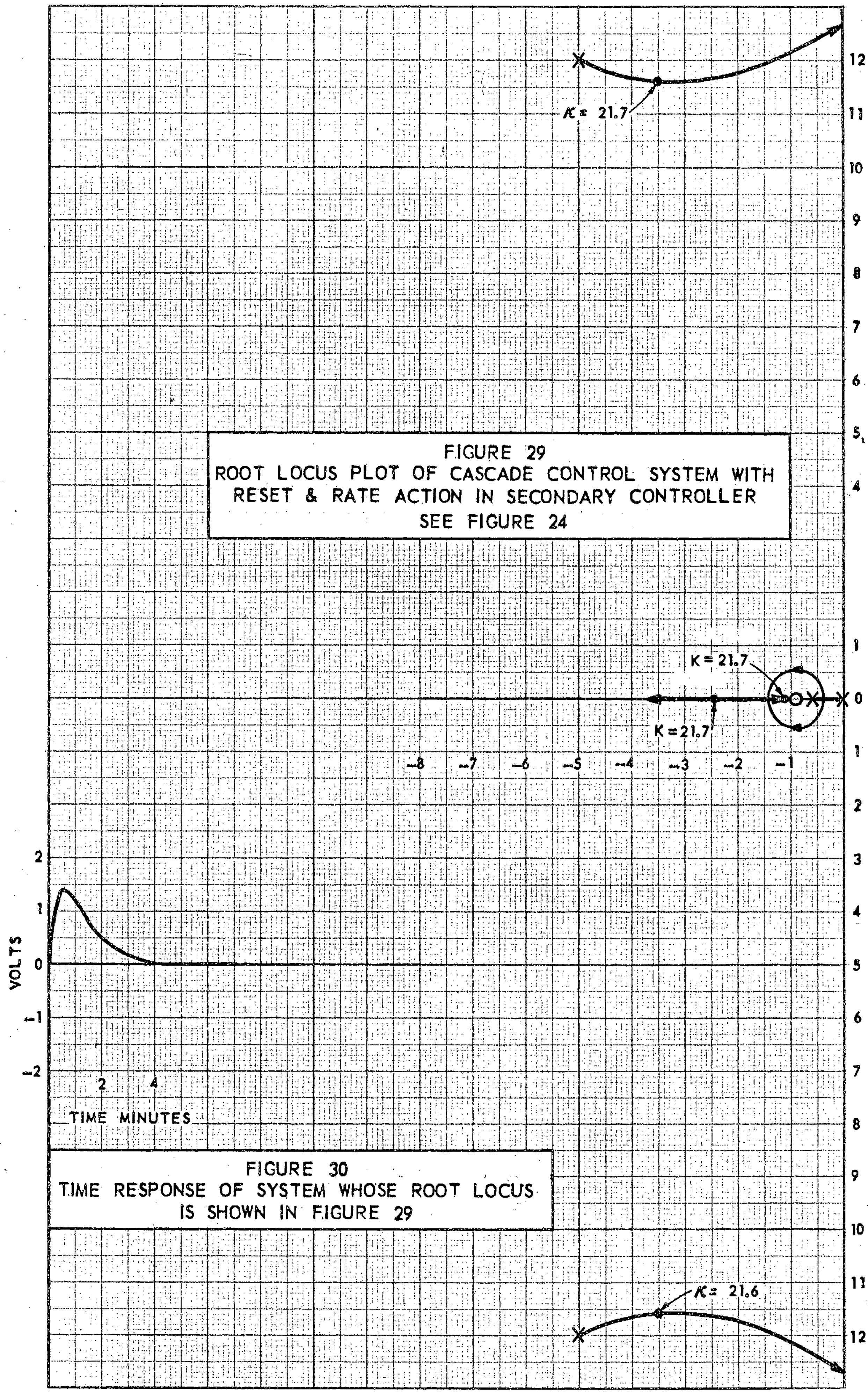


FIGURE 29
ROOT LOCUS PLOT OF CASCADE CONTROL SYSTEM WITH RESET & RATE ACTION IN SECONDARY CONTROLLER
SEE FIGURE 24

FIGURE 30
TIME RESPONSE OF SYSTEM WHOSE ROOT LOCUS IS SHOWN IN FIGURE 29

BIBLIOGRAPHY

1. Brown, G. S. and Campbell, D. P., Principles of Servomechanisms, John Wiley and Sons, Inc.; New York, 1953.

2. Vannah, W. E. and Slater, L. E., "Process Control Enters a New Era", Industrial and Engineering Chemistry, Vol. 48, No. 6, 1000-1001.

3. Ceaglske, N. H., Automatic Process Control for Chemical Engineers, John Wiley and Sons, Inc., New York, 1956.

4. Minneapolis-Honeywell, Manual on Fundamentals of Instrumentation for the Industries, Minneapolis-Honeywell Regulator Co., Philadelphia, 1952.

5. Coon, Geraldine, "Analyzing the Control Loop", Proceedings of Tenth Control Systems Symposium, ISA, Newark, N.J., April 1958, B1-B34.

6. Aikman, A. R., "Use of Analysis Instrumentation in Process Control", Proceedings of National Conference on Instrumentation Methods of Analysis, Chicago, June 1957, 1-7.

7. Ahrendt, W. R. and Taplin, J. F., Automatic Feedback Control, McGraw-Hill, New York, 1950.

8. Chapman, A. L., "Analyzing the Instrumentation of a Chemical Process", Instruments, Vol. 21, 1948, 532-539.

9. Day, R. L., "The Design of Cascade Control Systems", Transactions of the Society of Instrument Technology, Vol. 10, 1958, 81-86.

10. Ziegler, J. G., "Cascade Control Systems", Proceedings of Texas A and M College Symposium on Instrumentation for the Process Industries, 1954, 24-29.

11. Thaler, G. J. and Brown, R. G., Servomechanism Analysis, McGraw-Hill, New York, 1953.

12. Farquhar, J. C., "Some Aspects of Cascade and Multi-element Control Systems", Proceedings Third World Petroleum Congress, Section VI, 1951, 496-501.

13. Evans, W. R., Control System Dynamics, McGraw-Hill, New York, 1957.

14. Truxal, J. G., Automatic Feedback Control System Synthesis, McGraw-Hill, New York, 1955.

15. Grabbe et al., Handbook of Automation, Computation, and Control, John Wiley and Sons, Inc., New York, 1958.

APPENDIX A

DEVELOPMENT OF TIME RESPONSE EQUATIONS FROM ROOT LOCUS PLOTS

To write the transfer function of a system from the root locus plot, the basic transfer function which has a polynomial in the denominator must first be written. This transfer function is then normalized by reducing the coefficients of all highest powered Laplace operators to one. When this has been accomplished the resulting constant factor must be moved into the numerator. The numerator is now the numerator for the transfer function obtained by root locus. The denominator for the transfer function obtained from root locus is obtained by determining the roots of the denominator of the basic transfer function and is written simply as the factored form of the original transfer function. The following shows the development of various equations used in this paper.

Development of equation 5-4

$$C(s) = \frac{(5.85)(s+1)(s+2)\ s}{(s)(s+.282)(s+3.07)(s+.125\pm j1.43)}$$

Using theories of partial fractions

$$C(s) = \frac{K_1}{s+.282} + \frac{K_2}{s+3.07} + \frac{K_3}{s+.125-j1.43} + \frac{K_4}{s+.125+j1.43}$$

Calculate the residues

$$K_1 = \frac{(5.85)(1.72)(.72)}{(2.79)(-.157+j1.43)(-.157-j1.43)} = 1.264$$

$$K_2 = \frac{(5.85)(-1.07)(-2.07)}{(-2.79)(-2.94+j1.43)(-2.94-j1.43)} = -.435$$

$$K_3 = \frac{(5.85)(1.88+j1.43)(.88+j1.43)}{(.157+j1.43)(2.94+j1.43)(j2.86)} = -.403-j1.65$$

$$K_4 = -.403+j1.65$$

$$C(s) = \frac{1.264}{s+.282} - \frac{.435}{s+3.07} - \frac{.403+j1.65}{s+.125-j1.43} + \frac{-.403+j1.65}{s+.125+j1.43}$$

By taking the inverse transform the following equation in time is written.

$$C(t) = 1.26e^{-.282t} - .435e^{-3.07t} + 3.4e^{-.125t} \sin (81.5t-14)$$

Development of equation 5-9

$$(5\text{-}8) \quad C(s) = \frac{6(s+3s+62.5)(s)}{(s)(s+1.07)(s+2.5)(s+.1+j7.8)(s+.1-j7.8)}$$

Using theories of partial fractions,

$$C(s) = \frac{K_1}{s+1.07} + \frac{K_2}{s+2.5} + \frac{K_3}{s+.1-j7.8} + \frac{K_4}{s+.1+j7.8}$$

Solve for the residues,

$$K_1 = \frac{6(.43-j7.75)(.43+j7.75)}{(1.43)(.97+j7.8)(.97-j7.8)} = 4.08$$

$$K_2 = \frac{6(-1-j7.75)(-1+j7.75)}{-1.43(-2.4+j7.8)(-2.4-j7.8)} = -3.84$$

$$K_3 = \frac{6(1.4+j.05)(1.4+j15.55)}{(.97+j7.8)(2.4+j7.8)(j15.6)} = -.122-j.0469$$

$$K_4 = -.122+j.0469$$

$$C(s) = \frac{4.08}{s+1.07} - \frac{3.84}{s+2.5} + \frac{-.122-j.0469}{s+.1-j7.8} + \frac{-.122+j.0469}{s+.1+j7.8}$$

By using inverse transforms shown on page 37 of Truxal's Synthesis of Automatic Control System:

$$C(t) = 4.08e^{-1.07t} - 3.84e^{-2.5t} + .262e^{-.1t} \sin (447t-69)$$

Developing of equation 5-10

$$C(s) = \frac{6(s+1.5-j7.75)(s+1.5+j7.75)(s)}{(s)(s+1.05-j.5)(s+1.05+j.5)(s+.75-j7.75)(s+.75+j7.75)}$$

Using theories of partial fractions,

$$C(s) = \frac{K_1}{s+1.05-j.5} + \frac{K_2}{s+1.05+j.5} + \frac{K_3}{s+.75-j7.75} + \frac{K_4}{s+.75+j7.75}$$

Solve for the residues

$$K_1 = \frac{6(.45-j7.25)(.45+j8.25)}{(j1)(-.3-j7.25)(-.3+j8.25)} = -j6$$

$$K_2 = j6$$

$$K_3 = \frac{6(.75)(.75+j15.5)}{(.3+j7.25)(.3+j8.25)(j15.5)} = -.0754$$

$$K_4 = -.0754$$

$$C(s) = \frac{j6}{s+1.05+j.5} - \frac{j6}{s+1.05-j.5} - \frac{.0754}{s+.75-j7.75} - \frac{.0754}{s+.75+j7.75}$$

By taking inverse transform

$$C(t) = 12e^{-1.05t} \sin .5t - j.1508e^{-.75t} \sin (7.75t + \pi/2)$$

Development of equation 5-14

$$(5\text{-}13) \quad C(s) = \frac{6(s+3.5)(s^2+13s+168.8)(s)}{(s)(s+1.25)(s+2.72)(s+9.5)(s+2+j11.5)(s+2-j11.5)}$$

Using partial fraction theory

$$C(s) = \frac{K_1}{s+1.25} + \frac{K_2}{s+2.72} + \frac{K_3}{s+9.5} + \frac{K_4}{s+2-j11.5} + \frac{K_5}{s+2+j11.5}$$

Solve for the residues

$$K_1 = \frac{6(2.25)(5.25-j11.25)(5.25+j11.25)}{(1.47)(8.25)(.75+j11.5)(.75-j11.5)} = 1.3$$

$$K_2 = \frac{6(.78)(3.78-j11.25)(3.78+j11.25)}{(-1.47)(6.68)(-.72+j11.5)(-.72-j11.5)} = -.5$$

$$K_3 = \frac{6(-6)(-3-j11.25)(-3+j11.25)}{(-8.25)(-6.78)(-7.5+j11.5)(-7.5-j11.5)} = -.455$$

$$K_4 = \frac{6(1.5+j11.5)(4.5+j.25)(4.5+j22.75)}{(-.75+j11.5)(.72+j11.5)(7.5+j11.5)(+j23)} = -.161-j.062$$

$$K_5 = -.161+j.062$$

$$C(s) = \frac{1.3}{s+1.25} - \frac{.5}{s+2.72} - \frac{.455}{s+9.5} + \frac{-.161-j.062}{s+2-j11.5} + \frac{-.161+j.062}{s+2+j11.5}$$

Taking the inverse transform the following time response equation is written:

$$(5\text{-}14)\quad C(t) = 1.3e^{-1.25t} - .5e^{-2.72t} - .455e^{-9.5t} + .346e^{-2t}\sin(658t-61)$$

Development of equation 5-15

$$C(s) = \frac{6(s+5-j12)(s+5+j12)(s)}{(s)(s+1.07-j.5)(s+1.07+j.5)(s+4.25-j11.75)(s+4.25+j11.75)}$$

Using partial fractions

$$C(s) = \frac{K_1}{s+1.07-j.5} + \frac{K_2}{s+1.07+j.5} + \frac{K_3}{s+4.25-j11.75} + \frac{K_4}{s+4.25+j11.75}$$

Calculate the residues

$$K_1 = \frac{6(3.93-j11.5)(3.93+j12.5)}{(j1)(3.18-j11.25)(3.18+j12.25)} = -j6.44$$

$$K_2 = j6.44$$

$$K_3 = \frac{6(.75-j.25)(.75+j23.75)}{(3.18+j11.25)(3.18+j12.25)(j23.5)} = -.032-j.0068$$

$$K_4 = -.032+j.0068$$

Terms involving K_3 and K_4 can be neglected

$$C(s) = \frac{j6.44}{s+1.07+j.5} - \frac{j6.44}{s+1.07-j.5}$$

Inverse transform gives the following in (t)

$$C(t) = 12.88e^{-1.07t}\sin 38.6t$$

VITA

Paul Uyles Webb

Candidate for the Degree of

Master of Science

Thesis: EFFECT OF SECONDARY LOOP CONFIGURATION ON OVER-ALL RESPONSE OF A CASCADE CONTROL SYSTEM

Major Field: Electrical Engineering

Biographical:

Personal Data: Born near Lindsey, Oklahoma, July 22, 1924, the son of Firm and Myrtle Webb

Education: Attended grade school at Cox City, Oklahoma; graduated from Cox City High School in 1942; received the Bachelor of Science degree from Oklahoma State University, with a major in Electrical Engineering, in May, 1950; completed requirements for the Master of Science degree in May 1960.

Professional experience: Spent five years with the Oklahoma Gas and Electric Company designing electric power systems, particularly the automatic control features; worked two years with the C. H. Guernsey, Consulting Engineers firm designing and supervising installation of electric steam generating plants, transmission and distribution system; worked the last two years with Phillips Petroleum Company, in research and development of advanced types of control systems for petroleum and chemical processes.