

A SERVO SYNTHESIZER

By

DONALD DEAN TIPTON

Bachelor of Science in Electrical Engineering

Oklahoma State University

Stillwater, Oklahoma

1959

Submitted to the Faculty of the Graduate School
of the Oklahoma State University in
partial fulfillment of
requirements for the degree of
MASTER OF SCIENCE
May, 1960

SEP 2 1960

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Thesis Approved:


Thesis Advisor


Faculty Representative


Dean of Graduate School

452862

PREFACE

In the last few years, feedback control systems have come to play a very important role in many engineering applications. Since the end of World War II there has been much literature published on the subject, and it is now being taught at the undergraduate level in most engineering colleges. This has developed a need for some means of demonstrating a feedback control system. Also, since many of the design procedures now in use are somewhat of a "trial and error" process, any technique or equipment capable of aiding the engineer in his design efforts would be of great value.

It is the object of this work to develop a unit capable of demonstrating a servo system and the compensation of one. Its primary use will be as an aid to classroom instruction; however, it could also be used as an aid in the design of a servo system.

I would like to express my gratitude to Professor Paul A. McCollum for the suggestions and assistance which he has furnished. I am also indebted to the National Science Foundation for granting me a fellowship which has made it financially possible for me to complete my graduate work at Oklahoma State University.

Finally, I would like to thank my wife, Mary Lou, for her diligent work in the preparation and typing of this thesis.

TABLE OF CONTENTS

Chapter	Page
I. INTRODUCTION.	1
Definitions	1
Objective	4
II. ANALYSIS OF A SERVO SYSTEM.	6
Transient Analysis.	6
Transfer Function Analysis.	10
Summary.	16
III. SYNTHESIS OF A SERVO SYSTEM	17
Analogous Systems	17
Synthesis Using Passive Networks	19
Synthesis Using an Operational Amplifier	20
IV. DESCRIPTION AND OPERATION	26
Servo Synthesizer Chassis.	26
Operational Amplifiers	29
Impedance Chassis.	34
Power Requirements	34
Recording or Indicating Device	36
Method of Operation.	37
V. RESULTS	39
Results of Tests on Single Impedance Units.	39
Results of the Solution of a Servo Problem.	41
Suggestions for Additional Applications.	49
VI. SUMMARY	52
BIBLIOGRAPHY	54

LIST OF TABLES

Table	Page
I. Analogous Elements	18
II. Circuits for the Generation of Transfer Functions	23
III. Circuits for the Generation of Non-Linearities	51

LIST OF FIGURES

Figure	Page
1. Open Loop Positioning System	2
2. Closed Loop Positioning System	2
3. Proportional Error Servomechanism	3
4. Block Diagram Notation for a Transfer Function	3
5. The Step Function	8
6. Response Curves for the Proportional Error Servomechanism of Figure 3	9
7. Series RLC Circuit	10
8. Typical Plots of the KG Locus	15
9. Analogous Systems	18
10. Network for Synthesizing a Simple Proportional Error Servomechanism	19
11. Block Diagram of an Operational Amplifier	21
12. Amplifier Circuit Having the Transfer Function of Equation (50)	24
13. Amplifier Circuit for Synthesizing a Simple Proportional Error Servomechanism	25

Figure		Page
14.	Top View of the Servo Synthesizer.	27
15.	Schematic Demonstrating the Function of a Single Relay	28
16.	Power Receptacle	29
17.	Schematic of the Servo Synthesizer	30
18.	Schematic of Philbrick K2-W Amplifier.	32
19.	Impedance Chassis.	35
20.	Transfer Function Units.	39
21.	Actual Response of the Units of Figure 20 to a Step Input.	40
22.	Positioning Servomechanism	42
23.	Schematic Diagram of Unit Used to Synthesize the System of Figure 22	43
24.	System Response to a Step Displacement Input.	45
25.	Response to Different Gain Settings.	46
26.	Plot of the KG Vector Locus.	47
27.	Amplifier Network to Provide Phase Lead	48
28.	Result of Series Compensation.	48

CHAPTER I

INTRODUCTION

During the past decade, feedback systems have become increasingly important due to the need in almost all fields of endeavor for more exacting control over the various related processes. Only a few years ago, the field of automatic control was poorly organized, and many of the design methods were tedious and only too often results were unsatisfactory. However, the military applications for automatic control brought on by World War II made necessary the rapid assembly of all available material on the subject. Consequently, better analysis and design methods were discovered and adapted for use by the control engineer. Thus, the field of automatic control has become one of the most important branches of electrical engineering.

Definitions

In general, automatic control systems can be classified into two types, open loop systems and closed loop systems. Figure 1 shows a block diagram of an open loop positioning system. θ_i represents the input variable, θ_o the output variable. Turning the input shaft will set the output according to some precalibrated chart. The principal characteristic of the open loop system is that there is no

communication between the controlled quantity and the controlling quantity.

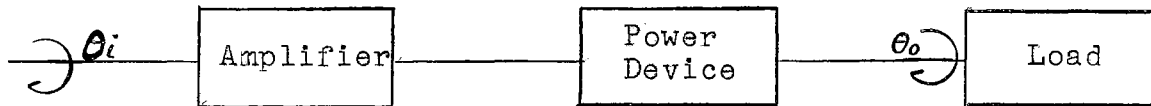


Figure 1. Open Loop Positioning System

Figure 2 shows the block diagram of a type of closed loop positioning system. Instead of using a calibration chart to set the controlled quantity to the desired position, the output information is available at the input station, thus the input shaft can be varied until the desired position for the controlled quantity is attained.

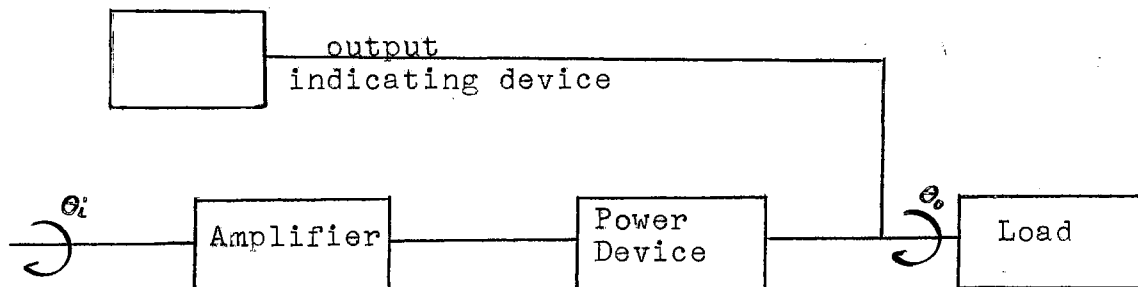


Figure 2. Closed Loop Positioning System

Figure 3 depicts the block diagram of another closed loop positioning system. This system continuously and automatically corrects for any errors in the system. Any control system which automatically compares the output with a reference quantity, and uses the resulting error to correct the output is known as a servomechanism.

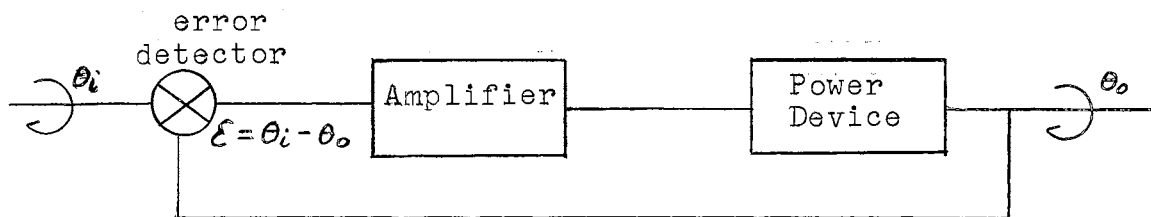


Figure 3. Proportional Error Servomechanism

Another term which should be defined before proceeding further is the transfer function. Consider the block diagram of Figure 4. The block has three characteristics; an input " θ_i ", an output " θ_o ", and some function " $Y(s)$ " which relates the output to the input. In general, $Y(s)$ operating on θ_i will cause a shift in both phase and amplitude, thus it will be complex. $Y(s)$ is defined as the transfer function of the system of Figure 4.

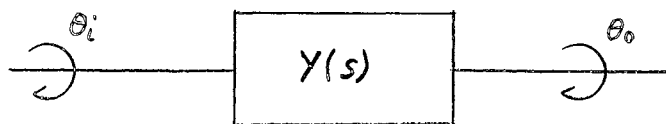


Figure 4. Block Diagram Notation for a Transfer Function

Now, consider Figure 1. Suppose " K " represents the constant factor and " G " the complex factor relating the output to the input, thus the equation

$$\frac{\theta_o}{\theta_i} = KG \quad (1)$$

can be written, where KG symbolizes the transfer function for the open loop system of Figure 1. If the amplifier and power device of Figure 3 is the same as that of Figure 1, then

from Figure 3,

$$\frac{\theta_o}{\epsilon} = KG \quad (2)$$

Substituting $\theta_i - \theta_o$ for ϵ , and solving for the ratio of θ_o to θ_i results in

$$\frac{\theta_o}{\theta_i - \theta_o} = KG \quad (3)$$

$$\frac{\theta_o}{\theta_i} = \frac{KG}{1 + KG} \quad (4)$$

Thus, the right side of Equation(4) is the general expression for the transfer function of a servomechanism where "KG" represents the open loop transfer function of the system.

The transfer function is an important tool in the study of servomechanisms. It is possible to determine directly from the transfer function whether the system it represents will be stable. Also, a graphical analysis of the transfer function furnishes the shortest and most direct method of compensating unstable systems.¹

Objective

The purpose of this thesis is to develop, test, and show typical solutions of problems on a unit capable of synthesizing a servo system. To accomplish this task, block or unitized setups are developed for synthesizing all the common transfer

¹Harold Chestnut and Robert W. Mayer, Servomechanisms and Regulating Systems Design (New York 1951), pp. 245-290.

functions encountered in the usual study of linear servomechanisms. The unit will primarily be used in the classroom to serve as a teaching aid; and to demonstrate the properties of typical systems and their compensation. It could also be used to an extent for the selection of suitable compensation elements associated with design.

CHAPTER II

ANALYSIS OF A SERVO SYSTEM

Transient Analysis

Figure 3 is a block diagram of a proportional error servomechanism. The differential equation for the system is²

$$A\mathcal{E} = J \frac{d^2\theta_o}{dt^2} + f \frac{d\theta_o}{dt} \quad (5)$$

where θ_o = output of the system

A = gain of the amplifier and power device

f = viscous damping coefficient of the system

J = inertia of the system

\mathcal{E} = error

t = time

Equation (5) may be rewritten in terms of the input and error by substituting $\theta_i - \mathcal{E}$ for θ_o ; the result is

$$J \frac{d^2\mathcal{E}}{dt^2} + f \frac{d\mathcal{E}}{dt} + A\mathcal{E} = J \frac{d^2\theta_i}{dt^2} + f \frac{d\theta_i}{dt} \quad (6)$$

To analyze the servo system represented by Figure 3, Equation (6) is solved for the error as a function of time.

²George J. Thaler and Robert G. Brown, Servomechanism Analysis (New York 1953), pp. 72-74.

The Laplace transform method is generally used for the solution of these equations. There are a number of reasons for this, one being that the labor is simplified because the differential equation above is reduced to an algebraic expression. Another reason is that the transfer function can be written directly from the transform of the differential equation.

As an example of the use of the Laplace transform in the solution of the equation of a servo system, suppose Equation (6) is solved for a step displacement input. The initial conditions would be as follows:

$$\begin{array}{lll}
 \text{At } t=0-; & \theta_i' = 0 & \frac{d\theta_i'}{dt} = 0 \\
 & \theta_o = 0 & \frac{d\theta_o}{dt} = 0 \\
 & \mathcal{E} = 0 & \frac{d\mathcal{E}}{dt} = 0 \\
 \text{At } t=0+; & \theta_i = \beta & \frac{d\theta_i}{dt} = 0 \\
 & \theta_o = 0 & \frac{d\theta_o}{dt} = 0
 \end{array}$$

where $t = 0-$ is the time preceding and up to time zero, and $t = 0+$ is the time starting at and following time zero as illustrated in Figure 5. Taking the Laplace transform of Equation (6) with the initial conditions as given, the result is

$$\begin{aligned}
 J [s^2 E(s) - sB] + f [sE(s) - B] + AE(s) & \quad (7) \\
 = J [s^2 \frac{B}{s} - sB] + f [s \frac{B}{s} - B] &
 \end{aligned}$$

where $E(s)$ represents the Laplace transform of the variable \mathcal{E} . Equation (7) is an algebraic expression and can be handled as such. Solving for $E(s)$ yields

$$E(s) = B \frac{sJ + f}{s^2J + sf + A} = B \frac{s + f/J}{s^2 + s f/J + A/J} \quad (8)$$

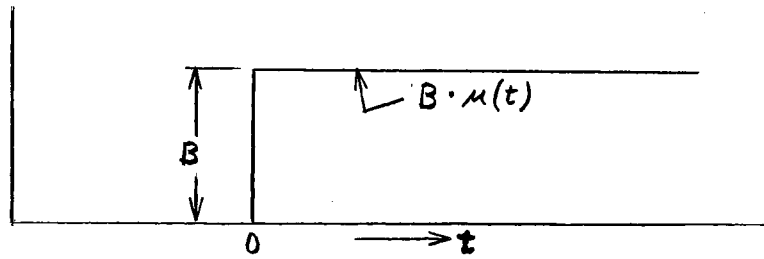


Figure 5. The Step Function

Equation (8) can be rewritten in the form

$$E(s) = B \frac{s + a}{[s + \alpha + \beta][s + \alpha - \beta]} \quad (9)$$

where

$$a = f/J \quad (10)$$

$$\alpha = f/2J \quad (11)$$

$$\beta = \sqrt{[f/2J]^2 - A/J} \quad (12)$$

and β may be real, zero, or imaginary. The inverse transform of Equation (9) has the form

$$\varepsilon = e^{-\alpha t} [c_1 e^{\beta t} + c_2 e^{-\beta t}] \quad (13)$$

When β is real, the response is that of an over-damped system as portrayed by the response curve of Figure 6. Physically, this represents a system that is sluggish and slow to respond to the input displacement. An examination of Equation (12) reveals that the condition for β to be real is that

$$\left[\frac{f}{2J}\right]^2 > \frac{A}{J} \quad (14)$$

From this, it can be seen that the system response can be improved either by decreasing the friction, or by increasing the amplification in the system.

If

$$\left[\frac{f}{2J}\right]^2 = \frac{A}{J} \quad (15)$$

β becomes zero and Equation (13) reduces to

$$\varepsilon = c e^{-\alpha t} \quad (16)$$

This corresponds to the critically damped response curve shown in Figure 6.

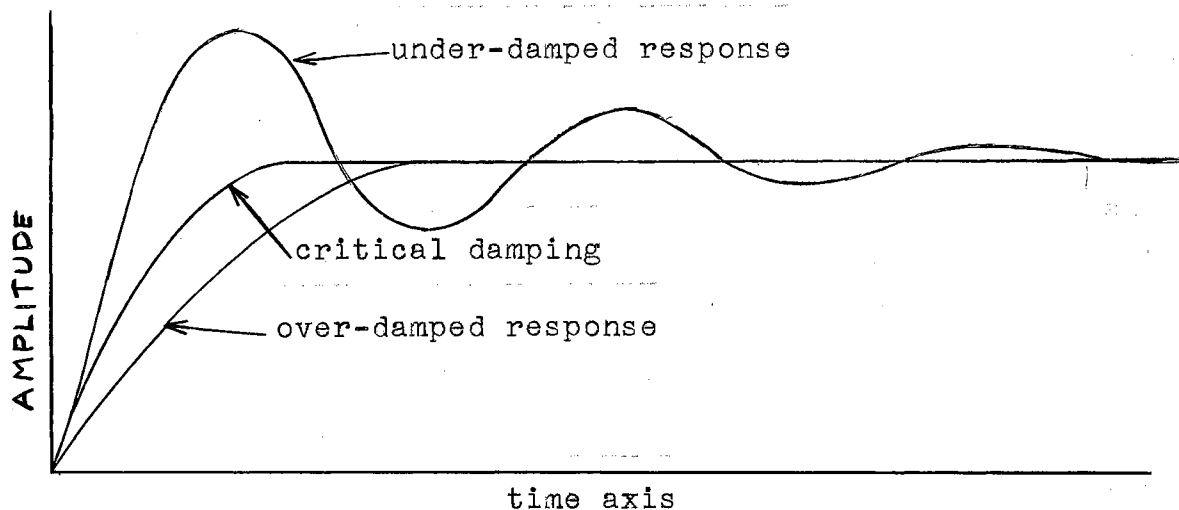


Figure 6. Response Curves for the Proportional Error Servomechanism of Figure 3.

Now, consider the case where

$$\left[\frac{f}{2J}\right]^2 < \frac{A}{J} \quad (17)$$

In this case, β is imaginary and if

$$\beta = j b \quad (18)$$

where

$$b = \sqrt{A/J - [f/2J]^2} \quad (19)$$

then Equation (13) can be written as

$$\varepsilon = e^{-\alpha t} [c_1 e^{jbt} + c_2 e^{-jbt}] \quad (20)$$

which reduces to

$$\varepsilon = e^{-\alpha t} [C_3 \cos bt + C_2 \sin bt] \quad (21)$$

or

$$\varepsilon = e^{-\alpha t} [D \sin (bt + \phi)] \quad (22)$$

This corresponds to the under-damped response curve of Figure 6.

The transient analysis of a servomechanism predicts the performance of the system, and so indicates whether improvement is needed. However, this type of solution does not indicate a direct means of altering the system in order to improve the performance. A method of analysis which lends itself more readily to design work is the transfer function method.

Transfer Function Analysis

The concept of a transfer function was introduced in Chapter I. The purpose of this section is to indicate how the transfer function is used in the design and analysis of servo systems.

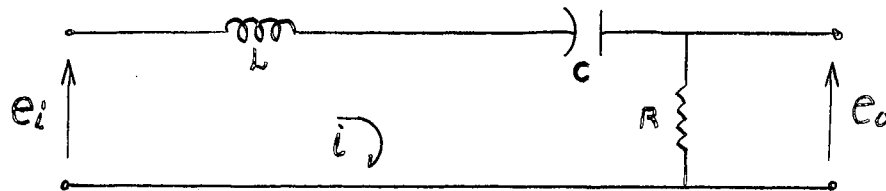


Figure 7. Series RLC Circuit

To illustrate the development of a transfer function for a system, consider the series RLC circuit of Figure 7. The differential equation for this system is

$$e_i = L \frac{di}{dt} + Ri + \frac{1}{C} \int i dt \quad (23)$$

With zero initial conditions, the transform of Equation (23) is

$$E_i(s) = Ls I(s) + R I(s) + I(s)/Cs \quad (24)$$

For Figure 7 it is determined that

$$I(s) = E_o(s) / R \quad (25)$$

Now, substituting Equation (25) into Equation (24), and solving for the ratio of $E_o(s)$ to $E_i(s)$ results in

$$\frac{E_o(s)}{E_i(s)} = \frac{R}{L} \left[\frac{s}{(s + s_1)(s + s_2)} \right] \quad (26)$$

where

$$s_1 = \frac{R}{2L} + \sqrt{\left[\frac{R}{2L}\right]^2 - \frac{1}{LC}} \quad (27)$$

and

$$s_2 = \frac{R}{2L} - \sqrt{\left[\frac{R}{2L}\right]^2 - \frac{1}{LC}} \quad (28)$$

Equation (26) is the transfer function of the complex ratio of output to input for any type of input function. The transfer function is thus obtained directly from the Laplace transform of the differential equation by assuming zero initial conditions. For sinusoidal excitation, the complex variable (s) may be replaced by the frequency variable ($j\omega$). The result is

$$\frac{e_o}{e_i}(j\omega) = \frac{R}{L} \left[\frac{j\omega}{(s_1 + j\omega)(s_2 + j\omega)} \right] \quad (29)$$

where s_1 and s_2 have the values as given by Equations (27) and (28). Equation (29) can thus be checked by normal circuit theory.

The transfer function of a servomechanism system can be written in much the same manner as the transfer function for the series RLC circuit above was written. For example, the differential equation for the system of Figure 3 is

$$A\mathcal{E} = J \frac{d^2\theta_o}{dt^2} + f \frac{d\theta_o}{dt} \quad (5)$$

Now if $\theta_i - \theta_o$ is substituted for ϵ in this equation the result becomes

$$A\theta_i - A\theta_o = J \frac{d^2\theta_o}{dt^2} + f \frac{d\theta_o}{dt} \quad (30)$$

This transforms into

$$A\theta_i(s) + A\theta_o(s) = J s^2 \theta_o(s) + f s \theta_o(s) \quad (31)$$

Solving for the ratio of θ_o to θ_i yields

$$\frac{\theta_o}{\theta_i}(s) = \frac{A}{J} \left[\frac{1}{s^2 + s \frac{f}{J} + \frac{A}{J}} \right] \quad (32)$$

Equation (32) is the transfer function of the proportional error servomechanism of Figure 3.

Consider the equation

$$s^2 + s \frac{f}{J} + \frac{A}{J} = 0 \quad (33)$$

which is obtained by equating the denominator of Equation (32) to zero. This is the characteristic equation of the system of Figure 3. A study of this equation can yield some significant facts relative to the system it represents. Solving for the roots of the equation would yield the following possible conditions:

- (1) roots are real and positive;
- (2) roots are real and negative;
- (3) roots are complex, with the real part positive;
- (4) roots are complex with real part negative;
- (5) roots are complex with real part zero.

When either condition (1) or (5) exists, the inverse transform of Equation (8) contains a growing exponential function, therefore the system is unstable. In a physical system,

instability could cause the system to destroy itself. If condition (5) existed, then the inverse transform of Equation (8) would contain an undamped sine function. Again, this would make the system unsuitable for control purposes. Under conditions (2) and (4), in which the real parts of the roots are negative, the transformed equation will contain factors of the type $e^{-\alpha t}$ where α is the real part of the roots of Equation (33). Therefore, conditions (2) and (4) result in a stable system. Thus, the real parts of the roots of the characteristic equation of a system determine the stability or the instability of the system.

It is possible to determine from the characteristic equation of a system whether the system will be stable; however, it is not always possible to tell just how stable the system is, or what should be done to affect the stability of the system. In general, it is easier to analyze and design servomechanisms when graphical methods are used. One method is to use the transfer function, KG , expressed in terms of the variable, $(j\omega)$, and plot the vector locus of the vector, KG , in the complex plane from $\omega = -\infty$ to $\omega = +\infty$.

From complex variable theory it can be shown that the transfer function of a system expressed in terms of $(j\omega)$ can be obtained directly from the transfer function, expressed in terms of the complex variable (s) , by replacing (s) with $(j\omega)$.³ Thus, Equation (32) can be rewritten

³Stanford Goldman, Transformation Calculus and Electrical Transients (New York 1949), pp. 226-233.

$$\frac{\theta_o}{\theta_i}(j\omega) = \frac{A}{J} \left[\frac{1}{(j\omega)^2 + \frac{F}{J}(j\omega) + \frac{A}{J}} \right] \quad (34)$$

Now, consider the equation

$$\frac{\theta_o}{\theta_i} = \frac{KG}{1 + KG} \quad (3)$$

developed in Chapter I. It is obvious that as KG approaches the value $-1 \neq j0$, the magnitude of the transfer function will become infinite. Under this condition, the system will be unstable. Suppose now that Equation (3) is substituted into Equation (34) and the resulting equation solved for KG . The result is

$$KG = A \left[\frac{1}{j\omega(j\omega \frac{J}{F} + 1)} \right] \quad (35)$$

If the locus of the KG vector, plotted in the complex plane, passes through the point $(-1, j0)$, then instability occurs in the system. If the KG vector encircles the point $(-1, j0)$, this also indicates an unstable system.

The plot of the KG locus in the complex plane using the frequency variable (ω) as a parameter is known as the "Nyquist" plot, and is widely used in design and analysis work. Figure 8 shows two Nyquist plots and their associated transfer functions. Figure 8(a) is the Nyquist plot of Equation (35). Since the KG locus neither passes through nor encircles the point $(-1, j0)$, this represents a stable system. Figure 8(b) represents a stable system if the gain is small enough so that the KG locus passes inside the point $(-1, j0)$. If the locus encircles the point, the system would be unstable; however, it would take only a gain adjustment to stabilize it.

The advantages in using graphical methods, such as the Nyquist plot, lie not only in the fact that it is possible to predict the performance of a system, but it is also possible to predict where the system should be altered and by how much in order to bring it to the desired performance standards. An example of this was pointed out for the system represented by Figure 8(b) where the gain may be altered to stabilize the system. In some cases it may be desirable or necessary to alter the shape of the KG locus. Again, it is much easier to choose the desired compensation from the graphical plots, than it would be from the system equations.

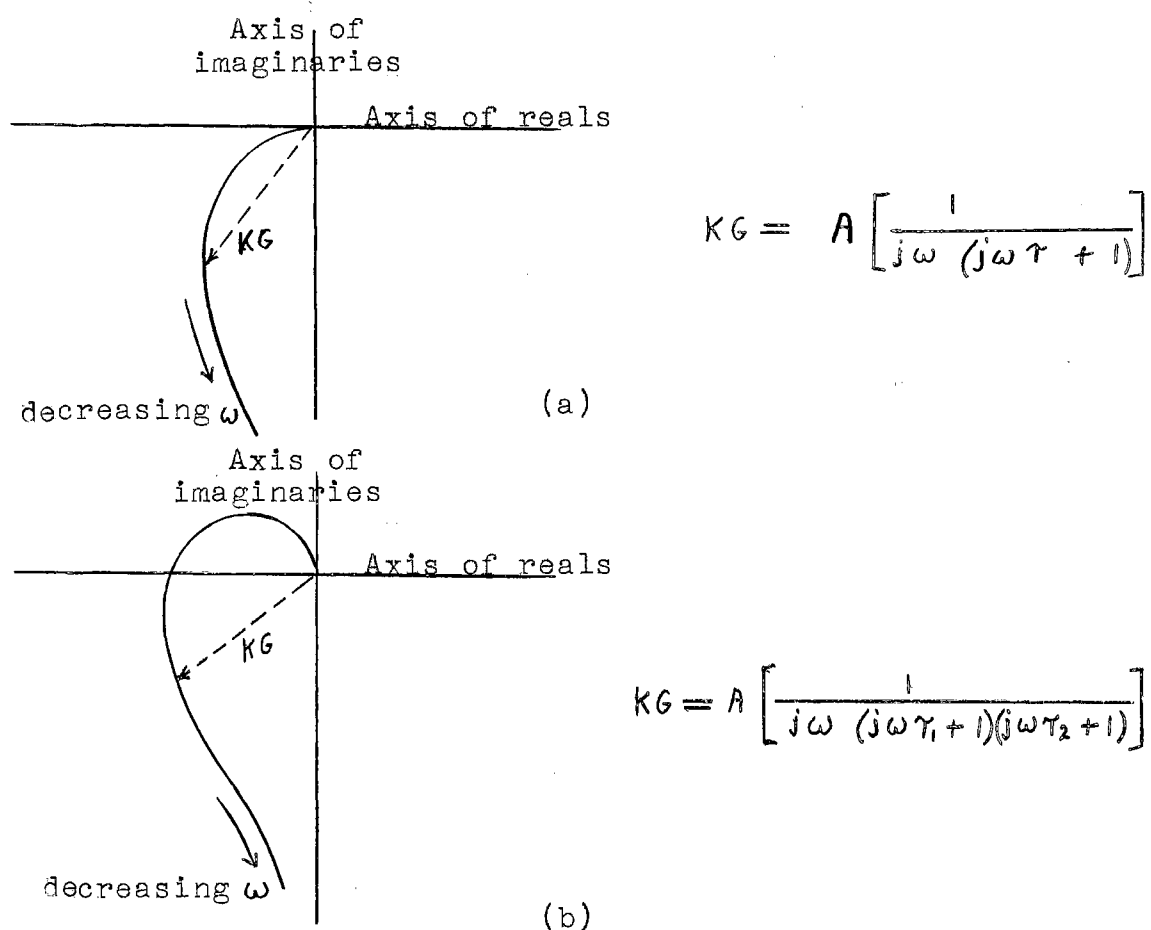


Figure 8. Typical Plots of the KG Locus

Summary

In general, differential equations may be written describing a servomechanism. The performance of the system may be predicted very accurately, within the limits of the accuracy of the system equations themselves, by solving the differential equations. However; in many cases the solution of differential equations becomes a long and tedious process and also there is no clear cut method of determining, from the solution of the equations, what should be done in order to improve the performance of the system.

A graphical analysis of the transfer function of a system has proven to be the most all-around satisfactory method for determining the system performance, and for suggesting alterations in order to improve the system. Because of this, it is apparent that the transfer function plays an important role in the study of servomechanisms.

CHAPTER III

SYNTHESIS OF A SERVO SYSTEM

Analogous Systems

One avenue of physically synthesizing control systems, makes use of the analogies that exist between physical systems. For instance, consider the systems of Figure 9. The differential equation for the mechanical system of Figure 9(a) is

$$F(t) = M \frac{d^2x}{dt^2} + f \frac{dx}{dt} + kx \quad (36)$$

where: $F(t)$ = applied force

k = spring constant

f = friction

M = inertia

x = displacement

The differential equation of the electrical circuit in terms of charge is

$$e(t) = L \frac{d^2q}{dt^2} + R \frac{dq}{dt} + \frac{1}{C} q \quad (37)$$

where: $e(t)$ = applied EMF

c = capacitance

R = resistance

L = inductance

q = charge

It will be observed that Equations (36) and (37) have the same mathematical form. Thus Figure 9(b) is said to be the electrical analogue of the mechanical system of Figure 9(a). Table I gives the analogous elements between a mechanical and an electrical system based on the analogy of force to EMF. By using analogies, such as those in Table I, it is possible to synthesize almost any linear physical system. In cases where gain is involved, an amplifier would have to be included in the electrical system.

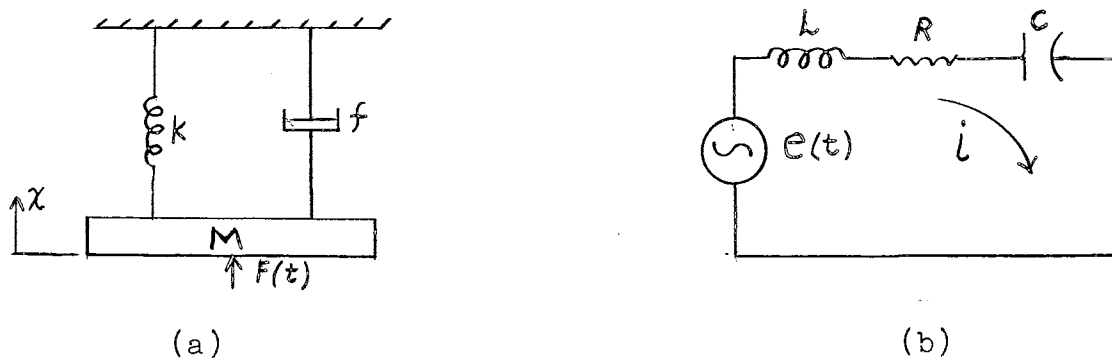


Figure 9. Analogous Systems

TABLE I

Analogous Elements

Mechanical Elements	Electrical Elements
$F(t)$ = force	$e(t)$ =EMF
M = inertia	L =inductance
k = spring constant	C =capacitance
f = friction	R =resistance
x = displacement	q =charge
v = velocity	i =current
a = acceleration	di/dt =rate of change of current

Synthesis Using Passive Networks

The transfer function for the mechanical system of Figure 9(a) in terms of the applied force and velocity is

$$\frac{v}{F}(s) = \frac{1}{M} \left[\frac{s}{s^2 + \frac{f}{M}s + \frac{K}{M}} \right] \quad (38)$$

Suppose it were desirable to find an electrical system analogous to this system. The problem then would be to find a network which would yield the transfer function of Equation (38). Thus, it can be said that the network of Figure 9(b) synthesizes the transfer function represented by Equation (38).

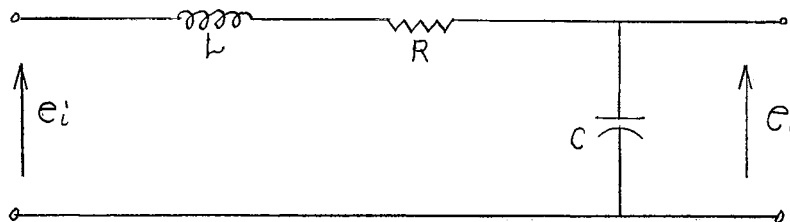


Figure 10. Network for Synthesizing a Simple Proportional Error Servomechanism

The transfer function for the proportional error servomechanism of Figure 3, Chapter I was

$$\frac{\theta_o}{\theta_i}(s) = \frac{A}{J} \left[\frac{1}{s^2 + s \frac{f}{J} + \frac{A}{J}} \right] \quad (32)$$

To synthesize this system, a network is to be found that has a transfer function of the same form as Equation (32). The circuit of Figure 10 gives the desired results. Using either of the methods discussed in Chapter II, the transfer function for the circuit of Figure 10 is

$$\frac{e_o}{e_i}(s) = \frac{1}{LC} \left[\frac{1}{s^2 + \frac{R}{L}s + \frac{1}{LC}} \right] \quad (39)$$

An inspection of Equations (32) and (39) will show that they have the same form. If R , L , and C in Figure 10 are chosen so that

$$1/LC = A/J \quad (40)$$

and

$$R/L = f/J \quad (41)$$

then the relation between e_i and e_o in Figure 10 is the same as the relation between the input and output shafts in the proportional error servomechanism. By this method, it would then be possible to check the performance of the servomechanism by running tests on the network of Figure 10.

It would not be possible, however, to simulate an unstable servo system with a passive network such as that of Figure 10. To do this, it would require a circuit which could add enough energy to the excitation to overcome the losses in the circuit elements and thus cause sustained oscillations. Another factor that discourages the use of passive networks for the simulation of servo systems is the loading effects that occur when either a signal source is attached to the input terminals, or a measuring or recording device is attached to the output terminals. The next section describes a method of synthesis with active networks which, to a great extent, eliminates the disadvantages pointed out above.

Synthesis Using an Operational Amplifier

Figure 11 shows a detailed block diagram of an operational amplifier. The blocks labeled Z_f and Z_i represent feedback and input impedances respectively. The triangular shaped

block represents a high gain d-c amplifier that has a gain of $-K$. If it is assumed that the grid of the amplifier draws no current, then the following equations can be written from Figure 11.⁴

$$i_i = i_f \quad (42)$$

$$i_i = (e_i - e_i) z_i \quad (43)$$

$$i_f = (e_i - e_o) z_f \quad (44)$$

$$e_o = -K e_i \quad (45)$$

Now, if substitutions are made and the ratio e_o to e_i solved for, the result is

$$\frac{e_o}{e_i} = -\frac{z_f}{z_i} \left[\frac{1}{1 + \frac{1}{K} \left(1 + \frac{z_f}{z_i}\right)} \right] \quad (46)$$

and if the relation

$$K \gg 1 \quad (47)$$

holds, then Equation (47) can be further simplified to

$$e_o/e_i = -z_f/z_i \quad (48)$$

From this relation then, it is seen that the transfer function of the operational amplifier is dependent solely on the input and feedback impedances.

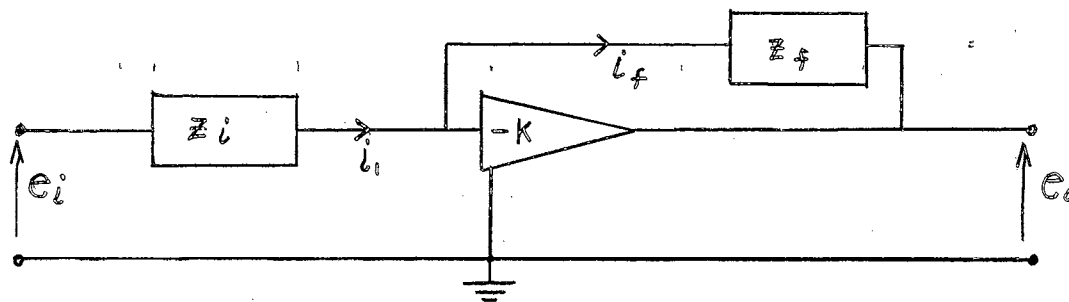


Figure 11. Block Diagram of an Operational Amplifier

⁴J. R. Ragazzini, R. H. Randall, and F. A. Russell, "Analysis of Problems in Dynamics by Electronic Circuits", Proc. IRE, Vol. 35, May 1947, pp. 444-452.

Two assumptions were made in developing this relation, one being that there was no grid current. In a well designed amplifier, the error resulting from this assumption would be negligible. The other was that the gain of the amplifier was much greater than unity. Obviously, then, it will be desirable to keep the amplifier gain as high as possible when using the amplifier to synthesize a transfer function with the relation given by Equation (48).

To realize a transfer function using the operational amplifier, input and output impedances would be chosen to yield a transfer function of the same form as the one being considered. It would be possible to synthesize any transfer function with one amplifier by finding suitable feedback and input impedance.⁵ However, this requires a knowledge of network synthesis techniques. A somewhat easier method would be to develop a table, such as Table II. The desired transfer function could then be found in this table and the proper impedances chosen to yield the desired result. More complicated transfer functions could be synthesized by using several amplifiers in cascade.

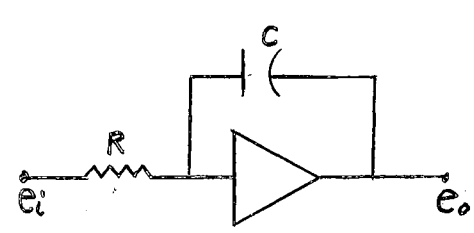
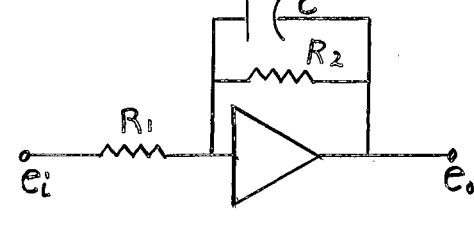
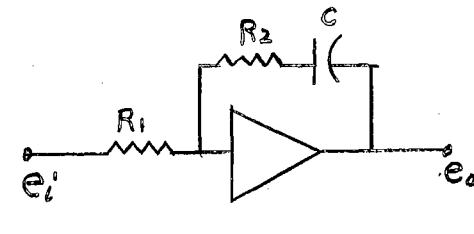
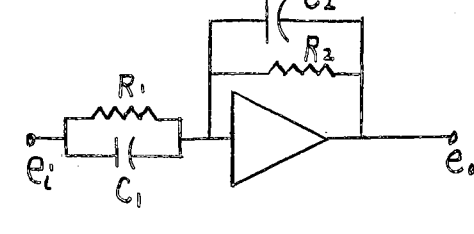
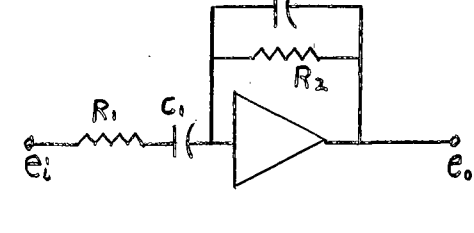
Using Equation (48) and the feedback and input impedances from Figure 12, the transfer function of the circuit of Figure 12 is found to be

$$\frac{e_o}{e_i} = - \frac{R_2 + 1/sC_2}{R_1 + 1/sC_1} \quad (49)$$

⁵Clarence L. Johnson, Analog Computer Techniques (New York 1956), p. 62.

TABLE II

Circuits for the Generation of Transfer Functions

<p>1.</p> 	$\frac{e_o}{e_i} = - \frac{1}{R C s}$
<p>2.</p> 	$\frac{e_o}{e_i} = - \frac{R_2}{R_1} \left[\frac{1}{1 + s R_2 C} \right]$
<p>3.</p> 	$\frac{e_o}{e_i} = - \frac{1 + s R_2 C}{s R_1 C}$
<p>4.</p> 	$\frac{e_o}{e_i} = - \frac{R_2}{R_1} \left[\frac{1 + s R_1 C_1}{1 + s R_2 C_2} \right]$
<p>5.</p> 	$\frac{e_o}{e_i} = - \frac{s R_2 C_1}{(1 + s R_1 C_1)(1 + s R_2 C_2)}$

or

$$\frac{e_o}{e_i} = - \frac{C_1}{C_2} \left[\frac{1 + s R_2 C_2}{1 + s R_1 C_1} \right] \quad (50)$$

This illustrates the method used in developing Table II.

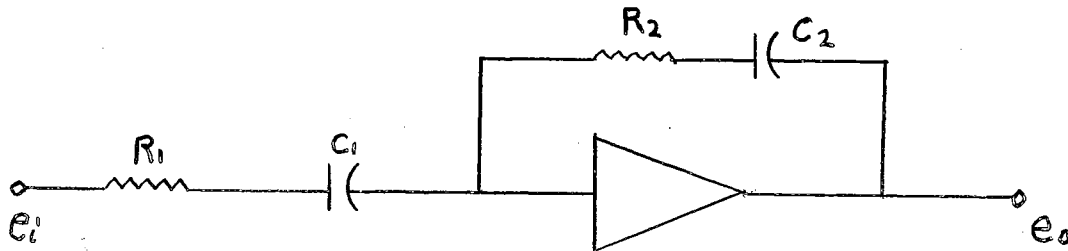


Figure 12. Amplifier Circuit Having the Transfer Function of Equation (50)

As an example of the use of the operational amplifier, suppose the transfer function of the proportional error servomechanism, Equation (32), is to be synthesized. The first step would be to factor Equation (32) and write it in the form

$$\frac{e_o}{e_i}(s) = K \left[\frac{1}{(1 + s \tau_1)(1 + s \tau_2)} \right] \quad (51)$$

No transfer function of this exact form can be found in Table II; however, if two amplifiers are used, then number 2 of Table II will furnish the desired results. Figure 13 shows the diagram for the two amplifiers. The resulting transfer function is

$$\frac{e_o}{e_i}(s) = \left[- \frac{R_2}{R_1} \frac{1}{s R_2 C + 1} \right] \left[- \frac{R_2'}{R_1'} \frac{1}{s R_2' C' + 1} \right] \quad (52)$$

or

$$\frac{e_o}{e_i}(s) = \frac{R_2 R_2'}{R_1 R_1'} \left[\frac{1}{(1 + s R_2 C)(1 + s R_2' C')} \right] \quad (53)$$

Note that Equation (53) has the same form as Equation (51), thus the proportional error servomechanism has been synthesized by the circuit of Figure 13.

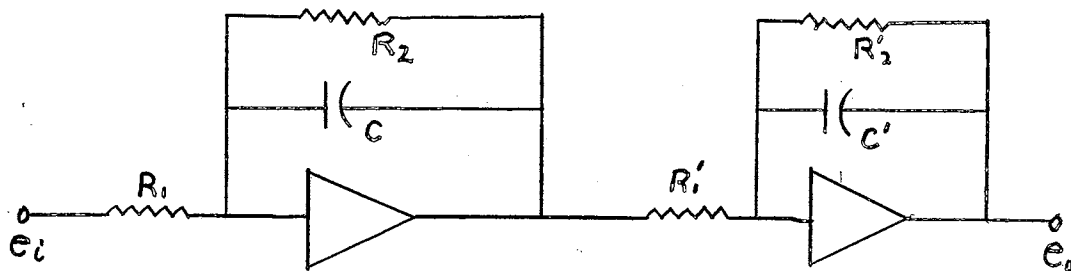


Figure 13. Amplifier Circuit for Synthesizing a Simple Proportional Error Servomechanism

It should be noted that a closed loop transfer function was used in the procedure discussed above. This procedure would apply in cases where the roots of the characteristic polynomial are not complex. A somewhat more general approach would be to synthesize the open loop transfer function using the same procedure as above and then connect a feedback loop from the output of the last amplifier to the input of the first amplifier. An additional amplifier would be used to combine the input and feedback signals, thus, functioning as an error detector for the closed loop system. Care would have to be exercised to insure that the feedback signal had the proper polarity.

This, in general, demonstrates the use of the operational amplifier for synthesizing a servo system. The remainder of this thesis will be devoted to developing a unit capable of synthesizing servo systems using the principle described above.

CHAPTER IV

DESCRIPTION AND OPERATION


The overall equipment consists of the following sub-units:

- (1) The servo synthesizer chassis
- (2) Operational amplifiers
- (3) Impedance units
- (4) Power supply
- (5) Performance indicating or recording device

The following sections present a physical description and a discussion of the function of each of the sub-units listed above.

Servo Synthesizer Chassis

The principal sub-unit consists of an aluminum chassis seventeen inches long, ten inches wide, and three inches deep. Figure 14 presents a top view of the chassis. There are ten octal sockets mounted in two rows across the back portion of the unit. Module type operational amplifiers plug into these sockets. Another row of five octal sockets is located on the front part. The impedance units plug into these sockets.

The symbol, , on Figure 14 represents a screw driver adjustment. In this case, they are bias adjustments for the

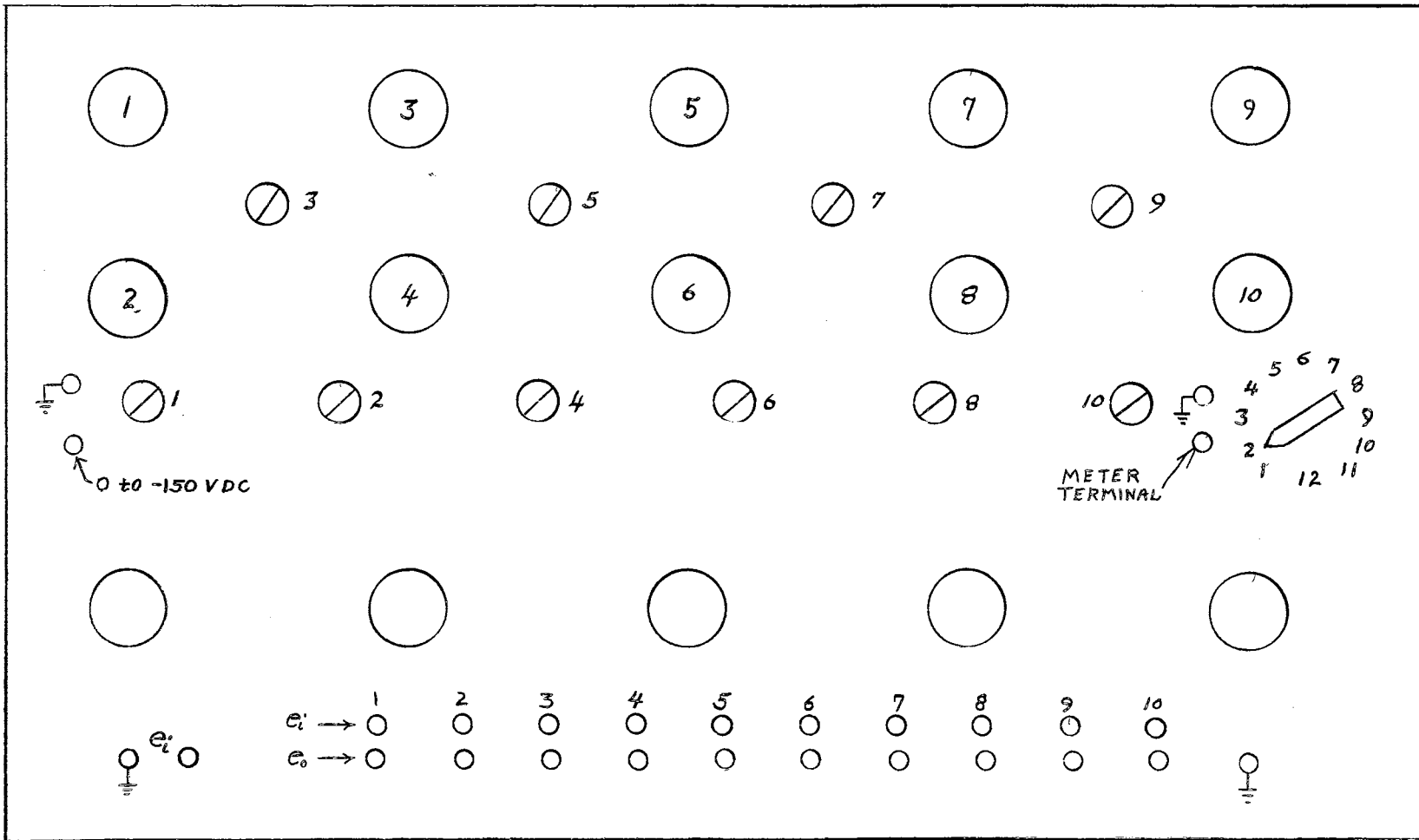


Figure 14. Top View of the Servo Synthesizer

operational amplifiers. The numbers beside the adjustments correspond to the numbers on the amplifier sockets.

The selector switch, shown on the right side of the chassis, connects any one output of the ten operational amplifiers to the terminals shown alongside the switch. Again, the numbers correspond to the numbers on the amplifier sockets. The last two positions on the switch are not used.

The symbol, "O", represents a terminal on the chassis. All black terminals are grounded. The terminal pair located on the front left corner is connected to the input of number one amplifier. The first row of terminals across the front is connected to the output of the amplifiers. The second row is connected to the input of each amplifier through a one megohm resistor. Available on the left side of the unit is a negative d-c voltage, which can be used as a step function input. The magnitude adjustment for this voltage is on the front panel of the power supply.

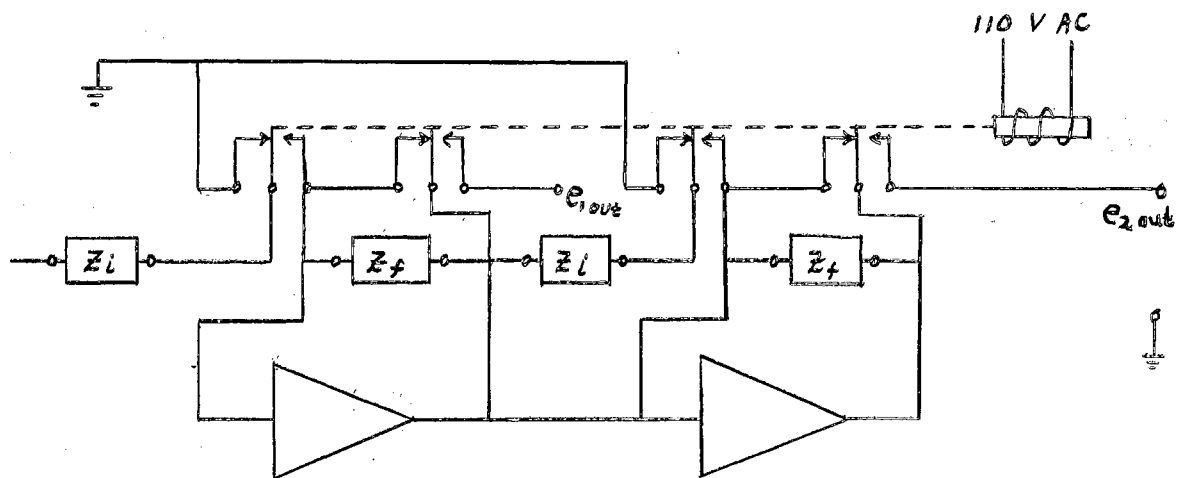


Figure 15. Schematic Demonstrating the Function of a Single Relay

Mounted underneath the chassis are five, four-pole double-throw relays. Each relay serves the purpose of initiating and stopping the problem solution on two amplifiers. Figure 15 shows how one relay is connected into the circuit. Aside from starting and stopping the problem solution, the relays, in the de-energized state, discharge any charge which may be built up on capacitors in the feedback loop during previous use. The relays are designed for operation on 110 volts a-c. The coils are wired in parallel and are energized through a two-pole switch located on the front side of the chassis.

All power is supplied to the unit through a receptacle located on the back side of the chassis. Figure 16 shows a view of the power receptacle from underneath the chassis. The proper voltages available at the receptacle are shown on the drawing.

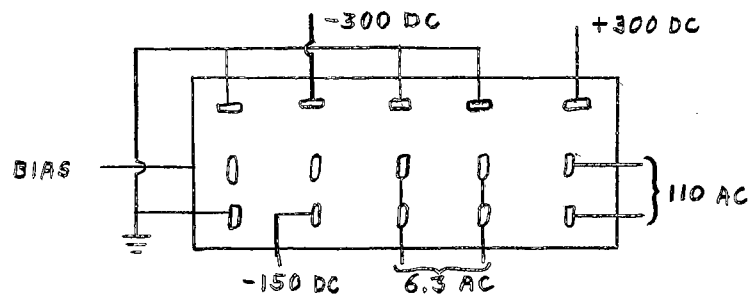


Figure 16. Power Receptacle

A complete schematic diagram of the unit is shown in Figure 17.

Operational Amplifiers

The unit described above is designed to use the Philbrick K2-W octal plug-in amplifier. General specifications for the

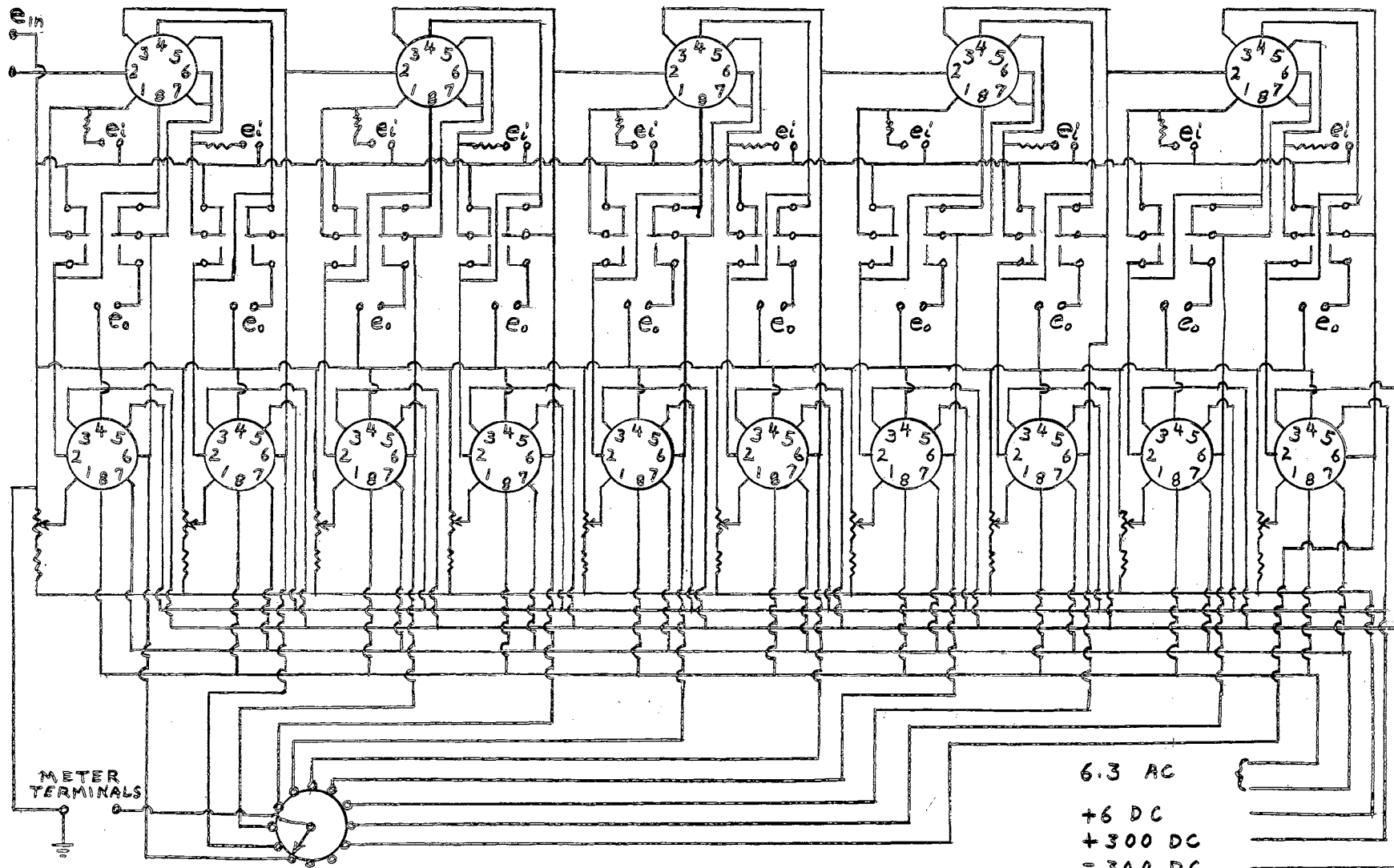


Figure 17. Schematic of the Servo Synthesizer

amplifier are as follows:⁶

Gain:

15,000 d-c open loop

Power requirements:

4.5 milliamperes at \pm 300 volts d-c.

4.5 milliamperes at - 300 volts d-c.

0.6 amperes at 6.3 volts a-c.

Tubes:

2 12AX7 twin triodes

Impedances:

Input impedance above 100 megohms.

Output impedance below one kilohm open loop and below one ohm fully fed back.

Voltage range:

-50 volts d-c to \pm 50 volts d-c on input

-50 volts d-c to \pm 50 volts d-c on output

Bias:

For balance, a positive bias adjustable from 0 to 3 volts d-c is used in series with pin number one.

Response:

The amplifier has a two microsecond rise time with band width over 100 kc.

Physical Properties:

The amplifier has a molded plastic sealed casing with an octal plug base. The overall height is four and one half inches and weight is three ounces.

⁶George A. Philbrick Researches, Inc., Bulletin; Model K2-W Operational Amplifier.

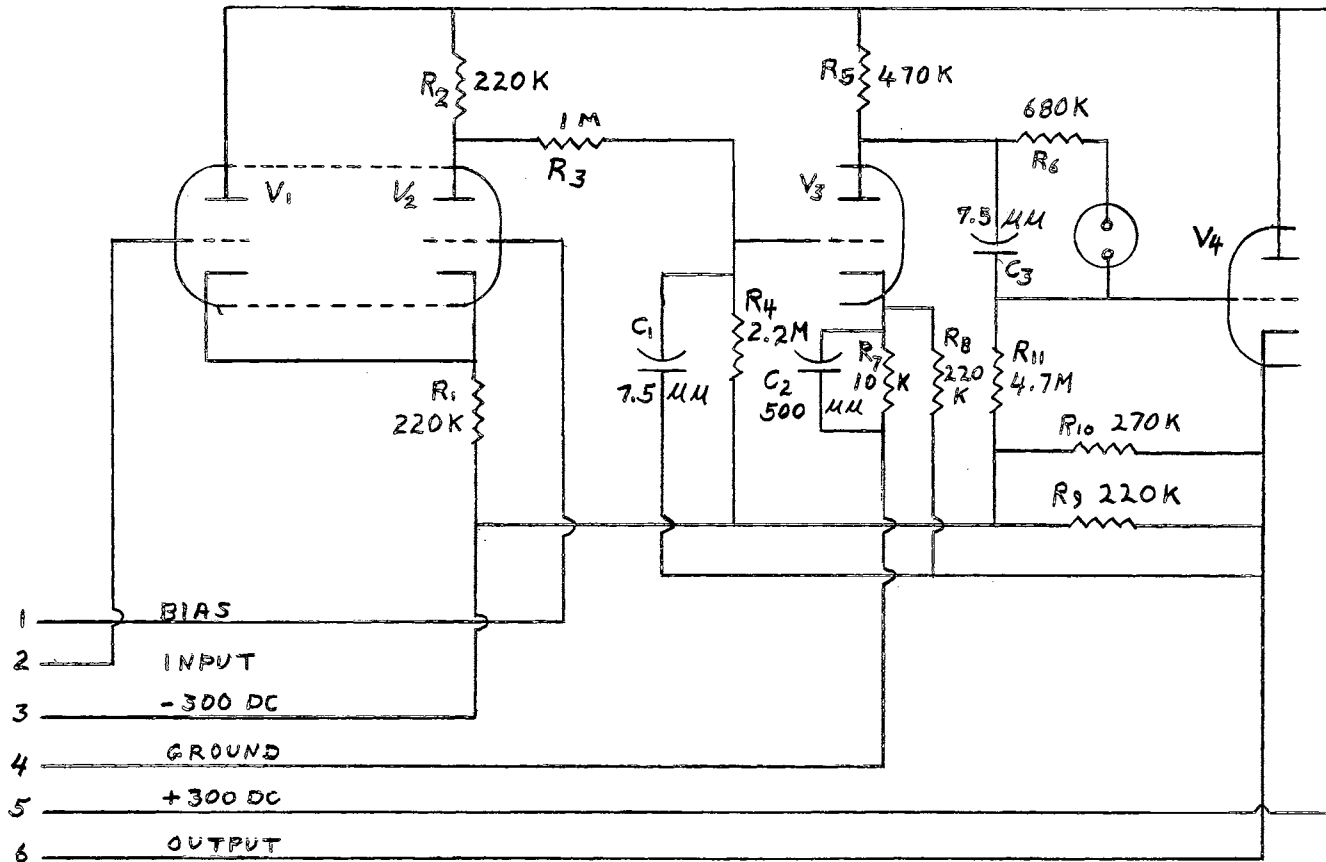


Figure 18.⁷ Schematic of Philbrick K2-W Amplifier

⁷George A. Philbrick Researches, Inc., Bulletin; Model K2-W Amplifier

Figure 18 shows the schematic diagram of the Philbrick K2-W operational amplifier. This is a conventional d-c amplifier circuit using three stages of amplification and a cathode follower on the output. In this unit, the positive input is biased and only the negative input is used for signal.

The operation of the d-c amplifier may best be described by tracing a signal through the circuit of Figure 18. A negative signal on the grid of V_1 decreases the current in that tube, and thus causes the voltage across the common cathode resistor, R_1 , to drop. This has the effect of decreasing the bias and increasing the plate current of V_2 . The plate voltage on V_2 decreases and this decrease is directly coupled through R_3 to the grid of V_3 . The decrease in grid voltage on V_3 causes a decrease in plate current and an increase in plate voltage. The increase in plate voltage on V_3 is directly coupled through R_6 and the gas tube to the grid of V_4 . The resulting increase in current through V_4 produces a positive voltage across R_9 and R_{10} , thus, giving a positive output from the amplifier circuit. In the same manner it can be ascertained that a positive signal on the grid of V_1 will produce a negative output from the amplifier circuit.

The resistors R_2 , R_3 and R_4 form a voltage divider network to maintain the proper bias on V_3 . R_5 , R_6 , R_{11} and the gas tube perform the same function for V_4 . R_2 and R_5 also serve as plate load resistors for amplifiers V_2 and V_3 respectively. The capacitors in the circuit are for high

frequency equalization. The overall gain is increased by using positive feedback. This is accomplished by coupling a portion of the output signal through R_8 to the cathode of V_3 .

Impedance Chassis

The physical properties and dimensions of an impedance chassis are illustrated in the isometric drawing of Figure 19(a). A standard octal plug is mounted in the hole shown at the bottom of the chassis. Each of these chassis contains the input and feedback elements for two of the operational amplifiers. Each chassis represents some transfer function of the form given in Table II, and is labeled with that function.

Figure 19(b) presents a larger view of the terminal board. The numbers correspond to pin numbers on the octal plug. The impedance elements connect to the terminals as shown, where:

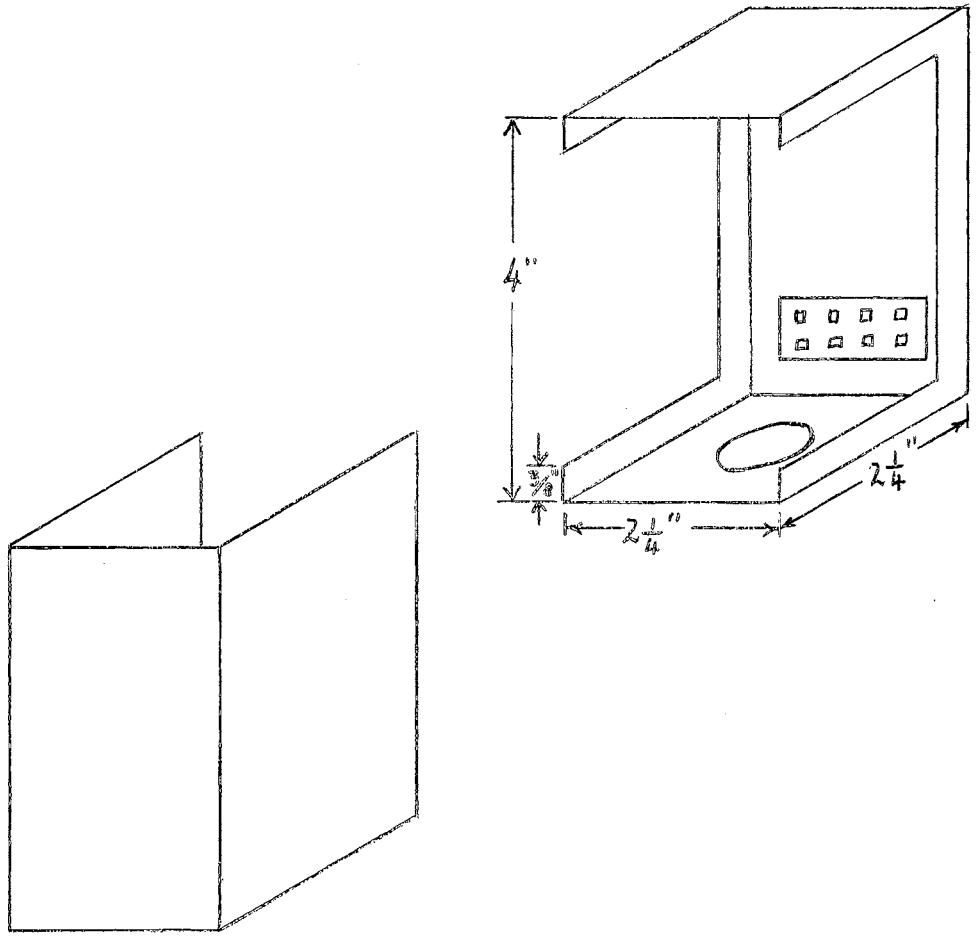
- Z_{i1} represents input impedance to the first amplifier;
- Z_{f1} represents feedback impedance on the first amplifier;
- Z_{i2} represents input impedance to the second amplifier;
- Z_{f2} represents feedback impedance on the second amplifier.

Power Requirement

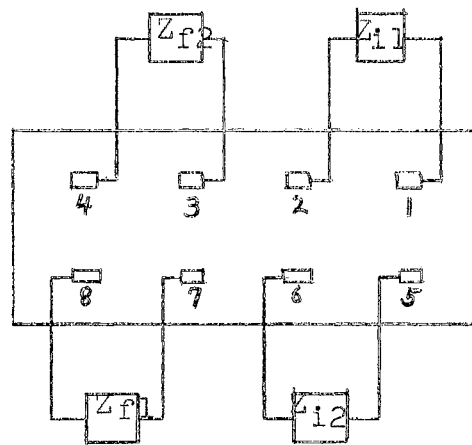
The power requirements for the unit are as follows:

E_f supply:

50 milliamperes at \neq 300 volts d-c regulated.



(a)



(b)

Figure 19. Impedance Chassis

B- supply:

50 milliamperes at -300 volts d-c regulated.

Filament:

6 amperes at 6.3 volts a-c.

Bias supply:

6 milliamperes at 6 volts d-c.

Relays:

110 volts a-c.

Function voltage:

0 to -50 volts d-c.

The desired power is obtained by using two Kepco 815B regulated power supplies. All voltages listed above are obtained from these two power supplies except the bias voltage and the 110 volts a-c.

The Kepco power supply is equipped with a power on-off switch and a B supply switch. The B supply is adjustable from zero to 600 volts d-c. Mounted on the front of the power supply is a voltmeter by which this adjustment can be made. Also available is a C supply, which can be adjusted from zero to -150 volts d-c. A selector switch allows the voltmeter to be switched to either the B supply or the C supply. The filament rating is adequate for the required power listed above.

The second power supply is used to obtain the negative 300 volts d-c required for proper operation of the amplifiers.

Recording or Indicating Device

To complete the servo synthesizer, a device to record or monitor the output is needed. The output can be observed

visually on either a voltmeter or an oscilloscope. Since the output impedance of the operational amplifiers is very low, neither of these are likely to cause any loading.

Another instrument available for this purpose is the pen recorder.⁸ This instrument gives a permanent record of the output. The input impedance to the pen recorder is low; therefore, it may be necessary to use a resistor in series with it, to prevent loading the amplifier circuits, to prevent damaging the recorder, and to properly damp the recorder.

Method of Operation

Power is supplied to the unit through individual cords that are connected to a common plug on the servo synthesizer end. The free ends of the cords are labeled as to the voltages they are to supply and thus, they can be readily connected to the proper terminals on the power supply. The cord, which furnishes 110 volts a-c to the relay coils, plugs into a standard a-c outlet.

Power should be applied to the unit for a period of 20 to 30 minutes before attempting any problem solution. At the end of this period, the amplifiers should be stable and bias adjustments can be made. This is done by connecting a voltmeter to the meter terminal and adjusting the bias of each amplifier to zero output on the voltmeter. The relays should be in a de-energized state while these adjustments are being made.

⁸Texas Instruments Rect/riter is recommended.

The recording device may be connected to any of the outputs on the amplifiers by plugging into the jack on the chassis, or it could be plugged into the meter terminal and the output of any of the ten amplifiers selected by means of the selector switch. The method used would depend upon the particular problem.

The response to a step or ramp function can be examined merely by connecting a single patch cord from the function terminal to the input terminal on the chassis. For a ramp, the first amplifier would have to be used as an integrator. The response to any kind of input function could be determined by using an additional unit to generate the desired input function and connecting it to the input terminals on the chassis.

An input to each amplifier through a one megohm resistor is provided on the amplifier chassis. This makes it possible to use any amplifier as an adder. Also, it provides a method of using an amplifier as an error detector and thus completing a closed loop system. The output of any amplifier can be fed back to the input of any amplifier merely by making the proper feedback connections on the chassis.

CHAPTER V

RESULTS

Results of Tests on Single Impedance Units

Figure 20 shows the schematic diagram of three of the impedance units built for use with the servo synthesizer. The associated transfer functions for each are as shown on the figure. For each of the three cases shown, the response to a step input is calculated below, and compared to the actual response as recorded by the servo synthesizer.

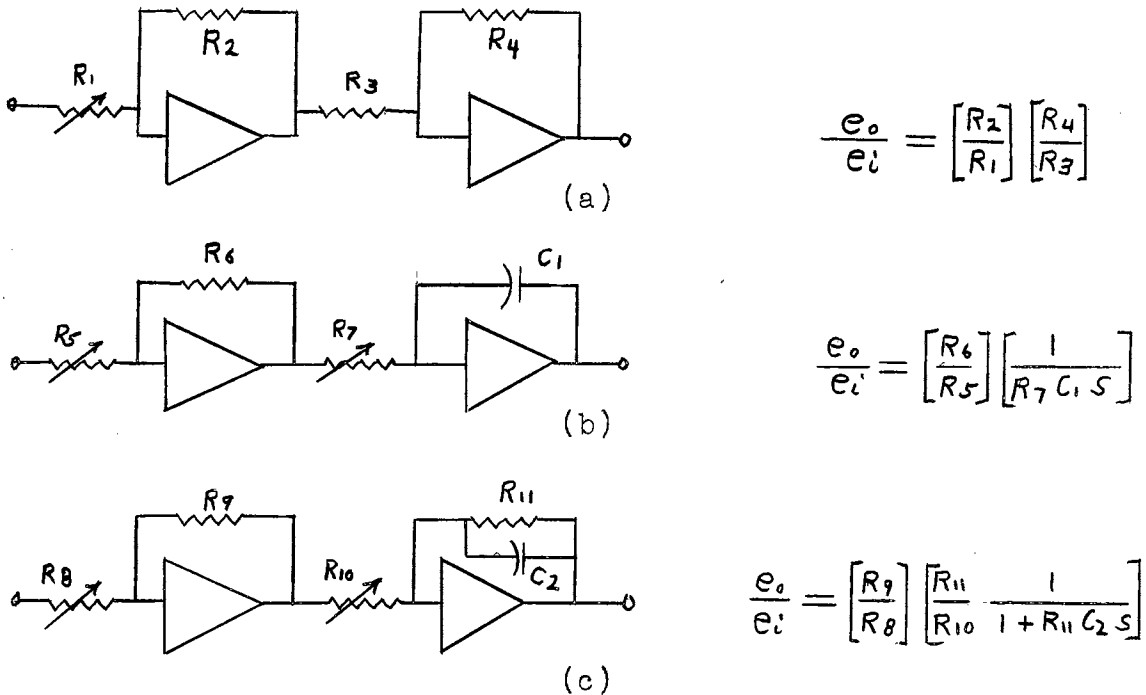


Figure 20. Transfer Function Units

The transfer function for the unit of Figure 20(b) is

$$\frac{e_o}{e_i} = \frac{R_6}{R_5} \frac{1}{R_7 C_1 s} = K \frac{1}{s \tau} \quad (54)$$

The response to a unit step input would be

$$e_o(s) = \frac{K}{\tau} \frac{1}{s^2} \quad (55)$$

Taking the inverse transform of this yields

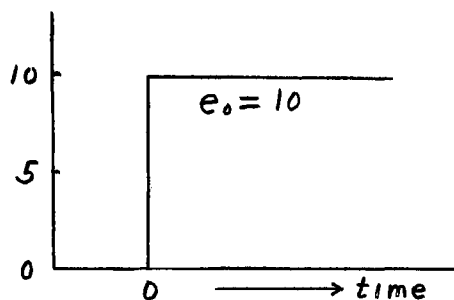
$$e_o(t) = \frac{K}{\tau} t \quad (56)$$

In the same manner the response for the units of Figure 20(a) and 20(c) is calculated to be

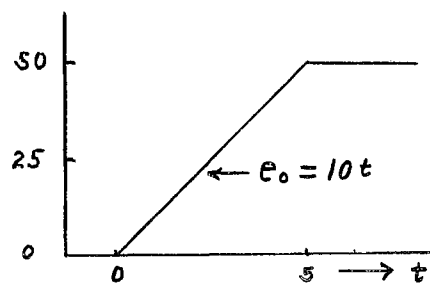
$$e_o(t) = K \quad (57)$$

$$e_o(t) = K [1 - e^{-t/\tau}] \quad (58)$$

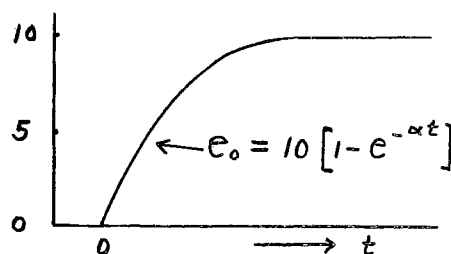
respectively.



(a)



(b)



(c)

Figure 21. Actual Response of the Units of Figure 20 to a Step Input

To verify these calculated results on the servo synthesizer, the following component sizes were used:

$$R_1 \text{ through } R_{11} = 1 \text{ megohm}$$

$$C_1 = C_2 = 1 \mu\text{f}$$

Figure 21 shows the response of each of the above units to a ten volt step input, as recorded by a pen recorder connected to the output of the servo synthesizer.

It will be observed that the response curves of Figure 21 agree with the calculated responses, Equations (56) through Equation (58). The break in the ramp of Figure 21(b) is caused by the amplifier saturating at 50 volts.

Results of the Solution of a Servo Problem

To test the servo synthesizer on a servo problem, consider the servo system represented by Figure 22. The transfer function for the individual blocks of Figure 22 are as follows:⁹

$$K_1 G_1 = \mathcal{M}_a \quad (59)$$

$$K_2 G_2 = \frac{K_g}{R_f} \frac{1}{1 + s \frac{L_f}{R_f}} \quad (60)$$

$$K_3 G_3 = \frac{1}{K_v} \frac{1}{s \left(1 + s \frac{J R_a}{K_t K_v}\right)} \quad (61)$$

where:

\mathcal{M}_a = amplifier gain

K_g = generator constant, volts/ampere

R_f = generator field resistance

⁹Gordon S. Brown and Donald P. Campbell, Principals of Servomechanisms (New York 1948), pp. 127-133.

L_f = generator field inductance

K_v = motor back-emf constant, volts/rad/sec

K_t = torque constant, ft.-lb./amp.

J = system inertia, slug-ft.²

R_a = motor armature resistance

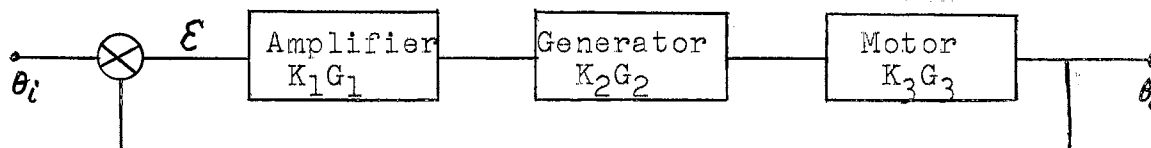


Figure 22. Positioning Servomechanism

The open loop transfer function for the system is

$$KG = \frac{\theta_o}{\epsilon} = [K_1 G_1] [K_2 G_2] [K_3 G_3] \quad (62)$$

or

$$KG = M_a \left[\frac{K_g}{R_f} \frac{1}{1 + s \frac{L_f}{R_f}} \right] \left[\frac{1}{K_v} \frac{1}{s \left(1 + s \frac{J R_a}{K_T K_v} \right)} \right] \quad (63)$$

The values of the system constants are:

$$M_a = 11.3$$

$$K_g = 220 \text{ volts/amp.}$$

$$R_f = 150 \text{ ohms}$$

$$L_f = 6 \text{ henrys}$$

$$K_v = 1.335 \text{ volts/radian/sec.}$$

$$K_t = 0.667 \text{ ft.-lb./amp.}$$

$$J = 0.053 \text{ slug-ft.}^2$$

$$R_a = 0.667 \text{ ohms}$$

Substituting these values into Equation (63) yields

$$KG = \frac{12.4}{s(1 + 0.04s)(1 + 0.04s)} \quad (64)$$

which is the overall open loop transfer function for the system of Figure 22.

An analytical solution of this problem for a step displacement input yields a response similar to that of the underdamped response curve of Figure 6.

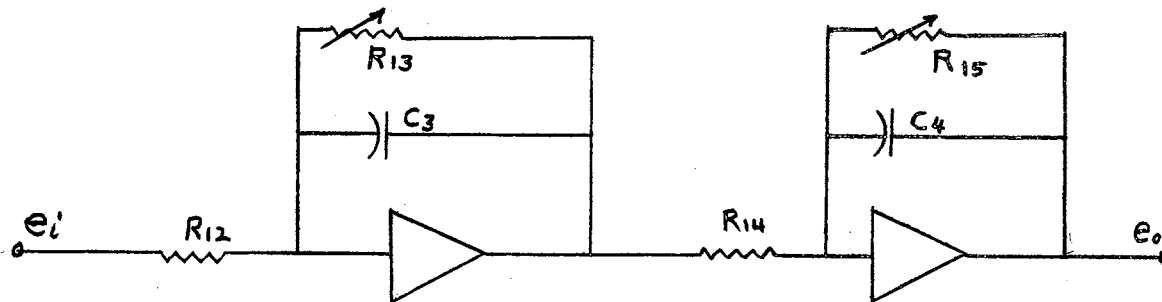


Figure 23. Schematic Diagram of Unit Used to Synthesize the System of Figure 22.

Figure 23 shows an operational amplifier configuration that can be used along with those of Figure 20(a) and 20(b) to synthesize this problem. The transfer function for the configuration of Figure 23 is

$$\frac{e_o}{e_i} = \left[\frac{R_{13}}{R_{12}} \frac{1}{1 + R_{13} C_3 s} \right] \left[\frac{R_{15}}{R_{14}} \frac{1}{1 + R_{15} C_4 s} \right] \quad (65)$$

For purposes of this problem the units are chosen with fixed components that have the following values:

$$R_2 = R_3 = R_4 = R_6 = R_{12} = R_{14} = 1 \text{ megohm}$$

$$C_1 = 1 \text{ microfarad}$$

$$C_3 = C_4 = 0.01 \text{ microfarad}$$

This leaves R_1 , R_5 , R_7 , R_{13} , and R_{15} to be set arbitrarily. The following three steps present a method by which this can be accomplished fairly easily.

- (1) Set R_{13} and R_{15} so that the time constants will correspond to those of Equation (64). In this case $R_{13} = R_{15} = 4$ megohms. Substitution of the component values into Equation (65) yields the following transfer function:

$$\frac{e_o}{e_i} = \frac{16}{(1 + 0.04s)(1 + 0.04s)} \quad (66)$$

- (2) Arbitrarily set R_7 . In this case R_7 can be set to one megohm yielding the transfer function

$$\frac{e_o}{e_i} = \frac{1}{s} \quad (67)$$

for the unit of Figure 20(b).

- (3) Set R_1 and R_5 so that the overall gain of the function corresponds to that of Equation (64). This may be done by setting R_5 arbitrarily and calculating the size of R_1 . In this case, set R_5 equal to one megohm, then the following relation must hold for R_1 :

$$\frac{1}{R_1} \cdot 16 = 12.4 \quad (68)$$

Solving for R_1 yields a value of 1.29 megohms. Therefore, the transfer function for the unit of Figure 20(a) is

$$\frac{e_o}{e_i} = 0.775 \quad (69)$$

The combination of Equations (66), (67) and (69) yields:

$$\begin{aligned} \frac{e_o}{e_i} &= \left[\frac{16}{(1 + 0.04s)(1 + 0.04s)} \right] \left[\frac{1}{s} \right] [0.775] \\ &= \left[\frac{12.4}{s(1 + 0.04s)(1 + 0.04s)} \right] \end{aligned} \quad (70)$$

A comparison of Equations (64) and (70) shows that they are identical; thus, the servo system of Figure 22 has been synthesized.

To investigate the closed loop response of this system, a feedback loop has to be connected from the output of the last unit back to an error detecting device. Another amplifier can be utilized for this purpose. Care has to be exercised to insure that the feedback is of the proper polarity.

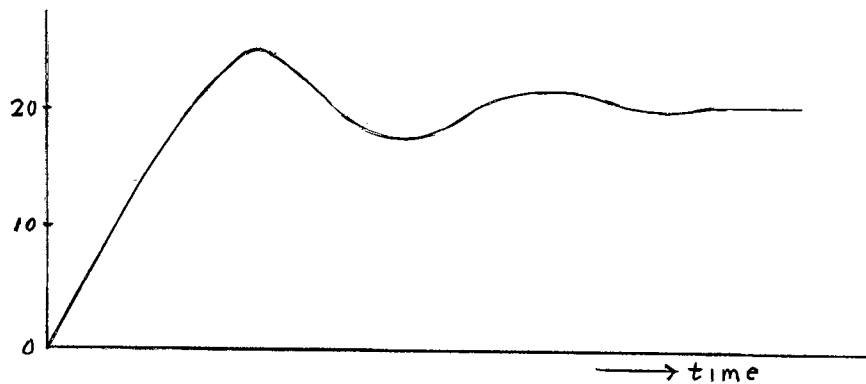


Figure 24. System Response to a Step Displacement Input

Figure 24 shows the actual response, as observed on an oscilloscope, to a step displacement input of twenty volts. The input and output voltages could be scaled to represent any desired radian equivalent on the positioning system. Notice that this is similar to the under-damped response curve of Figure 6. The oscilloscope was calibrated and the magnitudes shown on Figure 24 were determined.

The gain in the system can be varied by changing the value of either R_1 , R_5 , or R_7 of Figure 20. Figure 25 illustrates the result of changing the gain in the system.

Curves a and b of Figure 25 correspond to increased gain in the system while curve c corresponds to a lower gain setting.

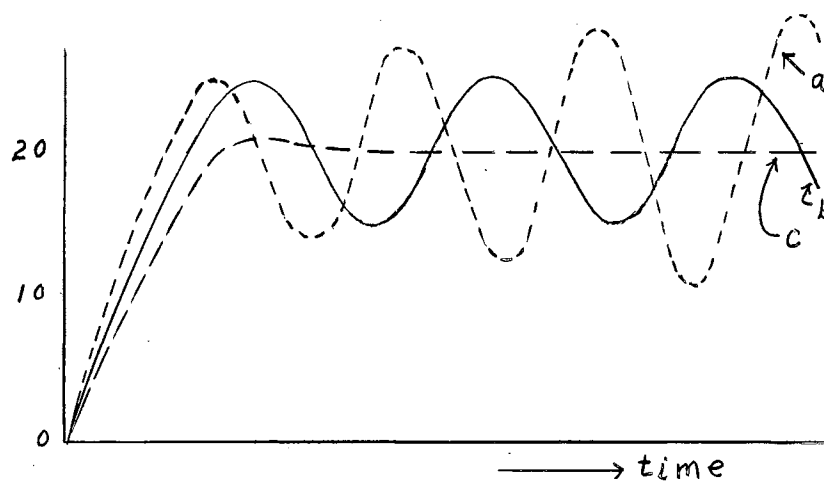


Figure 25. Response to Different Gain Settings

Changing the gain in the system has the effect of changing the magnitude of the KG vector as plotted in the complex plane. Figure 26 shows three plots of the KG vector corresponding to the gain settings of Figure 25. It will be observed that varying the gain does not alter the shape of the locus of the KG vector; but merely lengthens or shortens the vector for each value of the complex quantity. It is obvious, both from Figures 25 and 26, that the gain settings corresponding to curves a and b result in an unstable system.

The time constants in the system can be varied by changing the settings of R_{13} and R_{15} . Changing these settings had about the same effect as changing the gain in the system. It changed both the frequency and period of transient response. It should be noted that in the usual system it would probably not be possible to change these time constants since they

depend on the generator, motor, and load characteristics. This can be seen by an inspection of Equation (63).

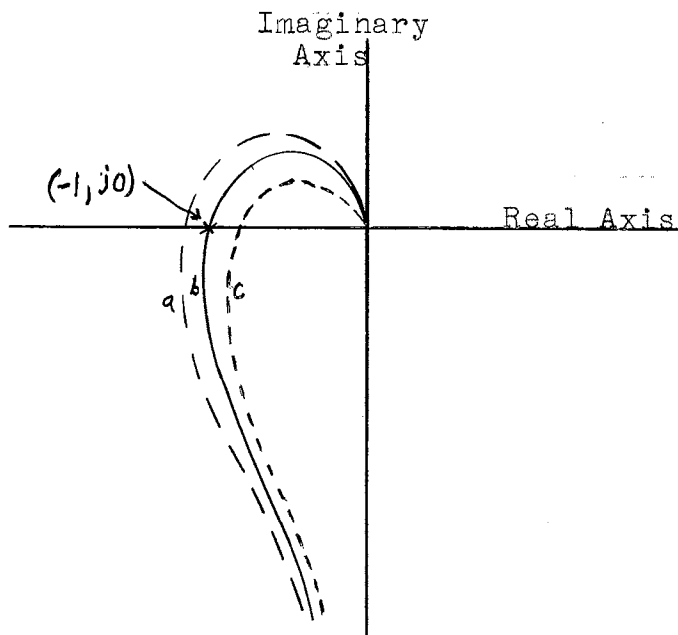


Figure 26. Plot of the KG Vector Locus

An inspection of Figure 25 will show that increased gain in the system has the effect of causing the output of the system to reach its steady state value at an earlier time, as compared to the time it takes with reduced gain. This, in most cases, is a desirable situation; however, the instability caused by the increased gain cannot be tolerated and must be compensated for. This can be accomplished by inserting a phase lead type network in series with the KG function. Figure 27 shows a network that could be used to compensate an unstable system. The transfer function for this network is

$$\frac{e_o}{e_i} = \left[\frac{R_1}{R_2} \right] \left[\frac{C_1}{C_2} \frac{1 + R_4 C_2 S}{1 + R_3 C_1 S} \right] \quad (71)$$

If $R_1 = R_2$ and $C_1 = C_2$ then R_4 should be much larger than R_3 in order to provide a phase lead. The transfer function for the system becomes

$$KG = \left[\frac{K}{s(1+0.045s)(1+0.045s)} \right] \left[\frac{1+\tau_1 s}{1+\tau_2 s} \right] \quad (72)$$

when the phase lead network is inserted. τ_1 and τ_2 must be determined in order to provide the desired compensation. The phase lead network has the effect of altering the shape of the KG vector as shown in Figure 28.

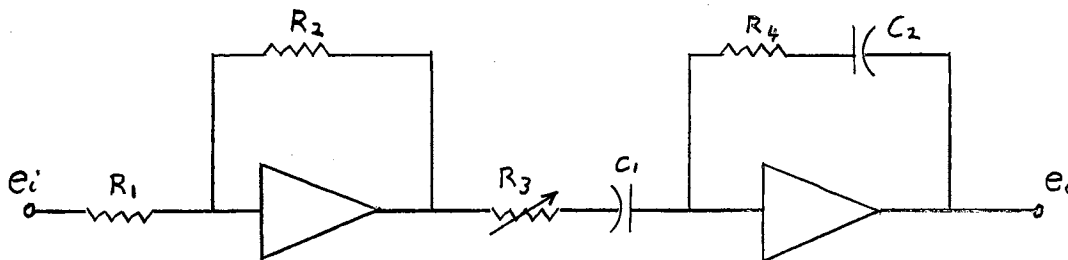


Figure 27. Amplifier Network to Provide Phase Lead

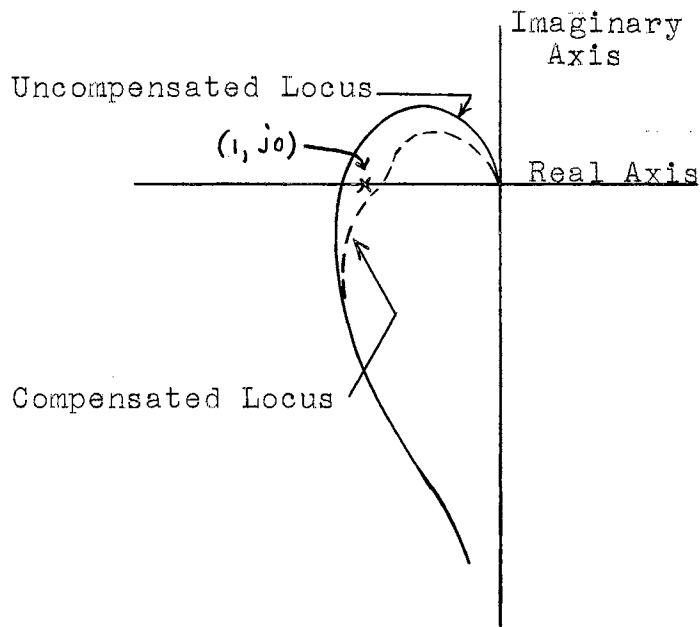


Figure 28. Result of Series Compensation

To improve the steady state performance of the system of Figure 22, a gain of approximately eighteen was chosen, which resulted in an unstable system. The network of Figure 27 was used to compensate the system. Values of R_3 and R_4 required to stabilize the system were determined by testing the response on the servo synthesizer. The final response of the system to a step function input was similar to that of Figure 24.

Suggestions for Additional Applications

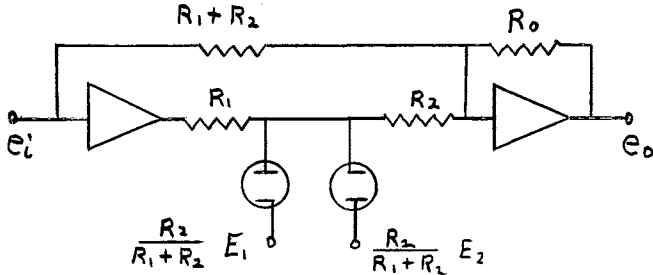
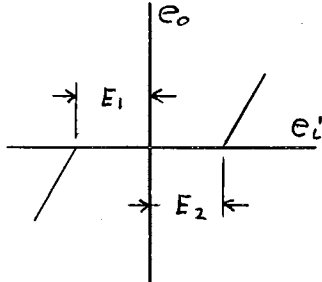
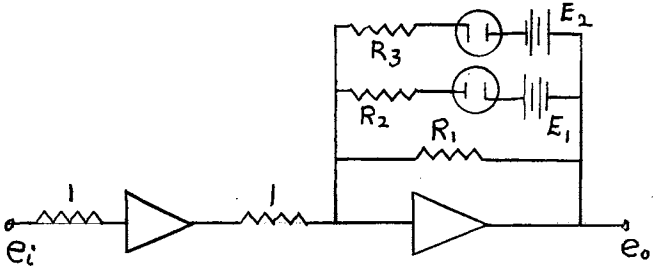
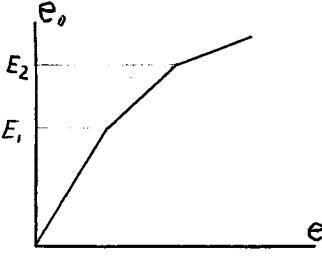
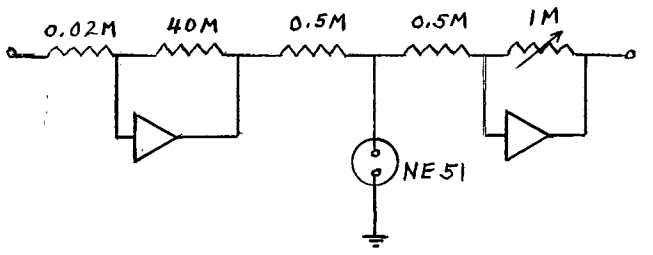
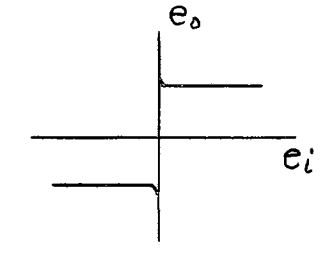
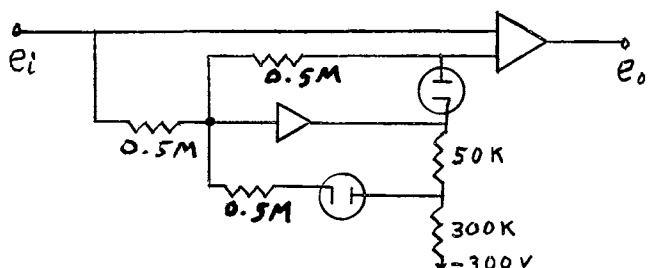
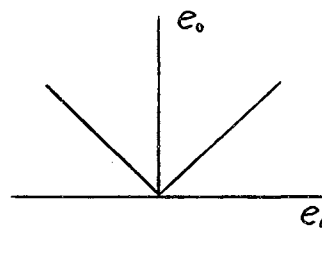
Although the principal purpose in developing this unit was to be able to synthesize a servo system, other uses can be readily found for it. For instance, in any system where the input-output relation can be represented by a linear transfer function, the function could be set up and investigated on this unit. To point out a specific example, suppose it were desired to find the limit of the high frequency response of an audio amplifier. By setting up an equivalent circuit and determining the transfer function, the problem could be investigated on the servo synthesizer. To determine the amplitude response, an audio oscillator would be used on the input and either an a-c voltmeter or an oscilloscope would be used on the output. To determine the phase shift, again an audio oscillator would be used on the input, and a phase meter on the output. The amplifiers in the unit itself are not likely to introduce any appreciable error since they have a band-width of over 100 kilocycles. If the

investigation were to be carried into the frequency range where the amplifiers would introduce an error, then frequency or time scaling could be used to move back into the useable frequency range.

Since this unit is basically an analog computer, it would take very little modification in order to use it as such, and all the analog computer techniques for generating non-linearities, multiplying, dividing, etc. would be applicable. The unit can be used as it is for certain non-linear cases by making use of diode function generators. Some sample circuits using diodes are given in Table III.

TABLE III

Circuits for the Generation of Non-Linearities¹⁰

Circuit	Function Generated
	
	
	
	

¹⁰ Harold Chestnut and Robert W. Mayer, Servomechanisms and Regulating System Design (2d Ed., New York, 1959), pp. 580-581

CHAPTER VI

SUMMARY

It was the object of this thesis to develop a unit capable of synthesizing all the common transfer functions encountered in the study of servo systems. This was accomplished by using operational amplifiers and utilizing the relation between the input and feedback impedances as developed in Chapter III.

A typical servo problem was set up and investigated on the unit. The effect of varying the gain and the different time constants in the system was observed. Also, the unit was used to observe the effect of series compensation on an unstable system.

The primary purpose in developing this unit was to be able to demonstrate the time response of a servo system to an external excitation. This was accomplished, as was demonstrated in the investigation of the servo problem. The unit could also be used to a certain extent for the selection of suitable compensation for a servo design, thus saving some work on the trial and error methods necessary by a strict analytical or graphical method.

Any of the common transfer functions can be synthesized by using the techniques described previously; therefore, the applications for this unit need not be limited to just the

solution or demonstration of servo problems. Any problem, where the input-output relation can be described by a linear transfer function, can be synthesized and investigated on this unit. Also, as pointed out in the last section, its useful applications can be extended into the non-linear regions by the use of certain network techniques as shown in Table III.

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VITA

Donald Dean Tipton

Candidate for the Degree of

Master of Science

Thesis: A SERVO SYNTHESIZER

Major Field: Electrical Engineering

Biographical:

Personal Data: Born at Fairland, Oklahoma, January 22, 1932, the son of Loil W. and Reathel Tipton.

Education: Attended grade school at Zena, Oklahoma, graduated from Delaware High School, Delaware, Oklahoma in 1949; graduated from Northeastern Oklahoma A & M College, Miami, Oklahoma in 1956; received a Bachelor of Science degree from Oklahoma State University, with a major in Electrical Engineering, in May, 1959; completed requirements for the Master of Science degree in Electrical Engineering in January, 1960.

Professional Experience: Entered the United States Navy in April, 1950, and served as an Electronics Technician aboard the U. S. S. Pickaway. Discharged in March, 1955 with the rating, Electronics Technician Second Class. While in school at Oklahoma State University, worked part time as a radio and television repairman at Crisp's Radio and TV, Stillwater, Oklahoma. Three months with the Continental Oil Company, Ponca City, Oklahoma as an assistant engineer during summer of 1957. Three months with Rocketdyne Company at Neosho, Missouri as an assistant engineer during the summer of 1958. Graduate assistant in the electrical engineering department of Oklahoma State University during Spring semester of 1959.