ANALYSIS OF TWO COLUMN MULTISTORY BENTS WITH SIDESWAY INCLUDED BY THE CARRY-OVER MOMENT PROCEDURE

By

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PREFACE

This study is a continuation in a series of studies to develop the analysis of structures by the carry-over joint moment method. This investigation consists of the analysis of multistory rigid frames with columns of unequal stiffness, and with the effects of joint translation included in the distribution of moments.

The author would like to express his sincere gratitude to all those who have assisted him in his graduate study, especially Professor Jan J. Tuma for his advice and encouragement in the preparation of this paper and for his instruction in the theory of structural analysis.

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CHAPTER I

INTRODUCTION

A method for the analysis of multistory one-bay unsymmetrical frames is presented. The effects of sidesway are included directly in the analysis.

The carry-over joint moment equation is used as a mathematical tool for the solution of the problem. This is a process of iteration which may be carried out to any degree of accuracy.

As in most methods of structural analysis for this type of frame, the effects of shear and normal forces are neglected. All members are assumed to have a constant moment of inertia.

The carry-over moment procedure was first published in 1958 (1). Many other studies have been made since then for its use in the analysis of various types of structures (2,3,4).

A comprehensive list of references and a complete history of the development of the carry-over moment procedure is given in the work by Tseng (5).

The author's contribution is the extension of the carry-over procedure, with correction for sidesway included, to the analysis of one-bay, unsymmetrical multistory frames with rectangular panels and members of constant cross-section.

CHAPTER II

GENERAL SLOPE DEFLECTION EQUATIONS

1. Member Equations

A one-bay multistory unsymmetrical frame acted on by loads as shown in Fig. 1 is considered. This is approximately the effect of wind loads on buildings using curtain wall construction with panels fastened to the spandrel beams.

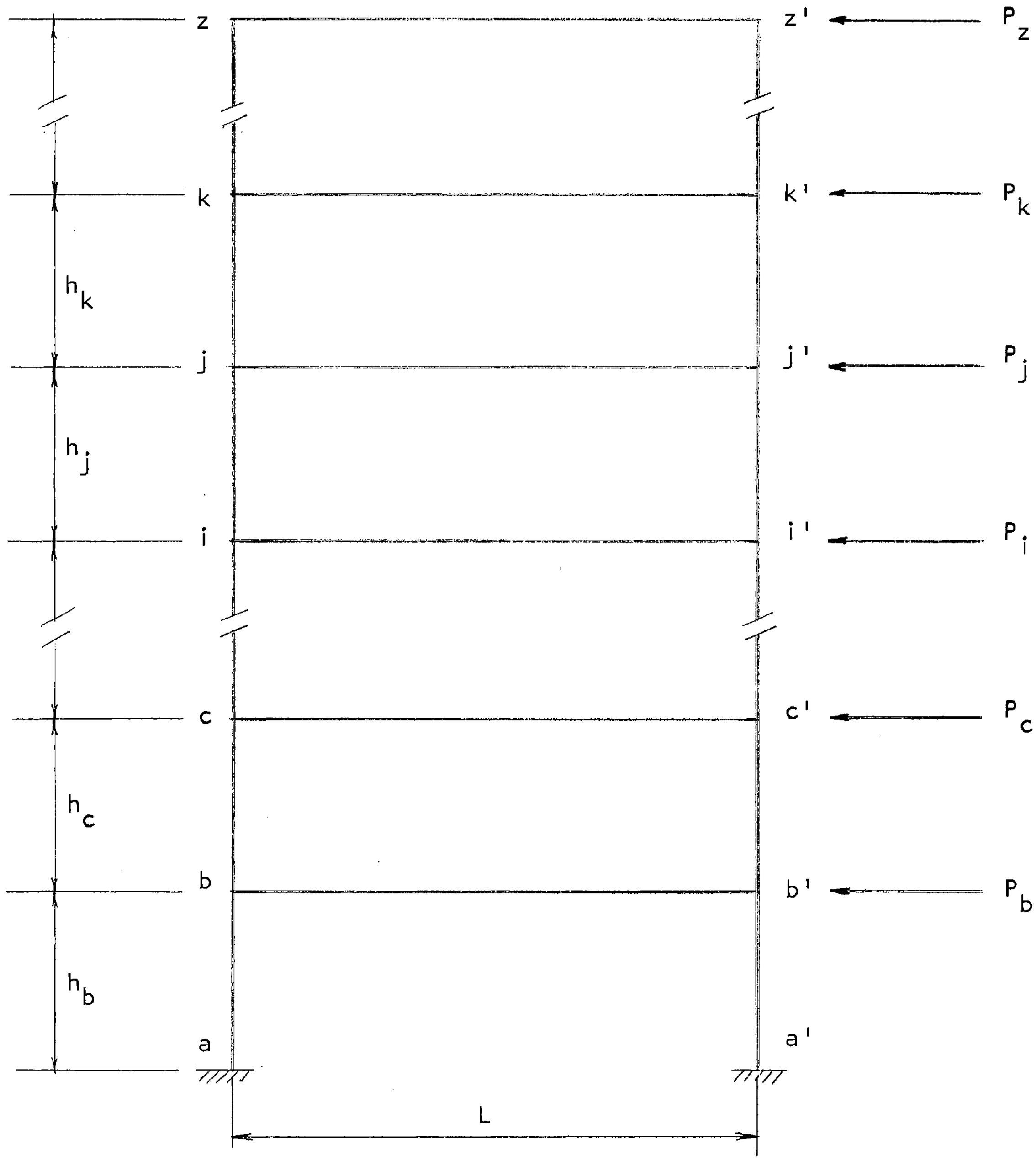
The general slope deflection equation for a member subject to end rotations and relative displacements of the ends is written for any member jk.

$$M_{jk} = K_{jk}\,\theta_j + C_{kj}\,K_{kj}\,\theta_k + (K_{jk} + C_{kj}\,K_{kj})\psi_k \qquad (1a)$$

where C_{kj} is the carry-over factor from joint k to joint j and is equal to one-half for straight members of constant cross-section. The value ψ_k is the relative displacement of end k from end j divided by the length of member jk, and the value K_{jk} is the stiffness factor of member jk.

Using the carry-over value of one-half, and K_{jk} equal K_{kj} for straight members with constant cross-section, the slope deflection equation (1a) becomes

$$M_{jk} = K_{jk}\,\theta_j + \frac{K_{jk}\,\theta_k}{2} + \frac{3\,K_{jk}\,\psi_k}{2} \qquad (1b)$$



One Bay Multistory Frame

Fig. 1

The general slope deflection equation for any girder jj' is written.

$$M_{jj'} = K_{jj'}\ \theta_j + C_{j'j}\ K_{j'j}\ \theta_{j'} = K_{jj'} + \frac{K_{j'j}\ \theta_{j'}}{2} \qquad (1c)$$

2. <u>Joint Equation</u>

By the laws of static equilibrium, the summation of moments at any joint j must equal zero.

$$\sum M_j = 0 = M_{ji} + M_{jj'} + M_{jk} \qquad (2a)$$

Using equations (1a) and (1b), the moments ~~and~~ at joint j may be written.

$$M_{ji} = K_{ij}\ \theta_j + \frac{K_{ij}\ \theta_i}{2} + \frac{3\ K_{ij}\psi_j}{2}$$

$$M_{jj'} = K_{jj'}\ \theta_j + \frac{K_{jj'}\ \theta_{j'}}{2} \qquad (2b)$$

$$M_{jk} = K_{jk}\ \theta_j + \frac{K_{jk}\ \theta_k}{2} + \frac{3\ K_{jk}\psi_k}{2}$$

The substitution of Eqs. (2b) into Eq. (2a) yields the joint equation at joint j.

$$\sum M_j = 0 = K_{ij}\ \theta_j + \frac{K_{ij}\ \theta_i}{2} + \frac{3\ K_{ij}\psi_j}{2} + K_{jj'}\ \theta_j + \frac{K_{jj'}\ \theta_{j'}}{2} + K_{jk}\ \theta_j + \frac{K_{jk}\ \theta_k}{2} + \frac{3\ K_{jk}\psi_k}{2}$$

Combining terms

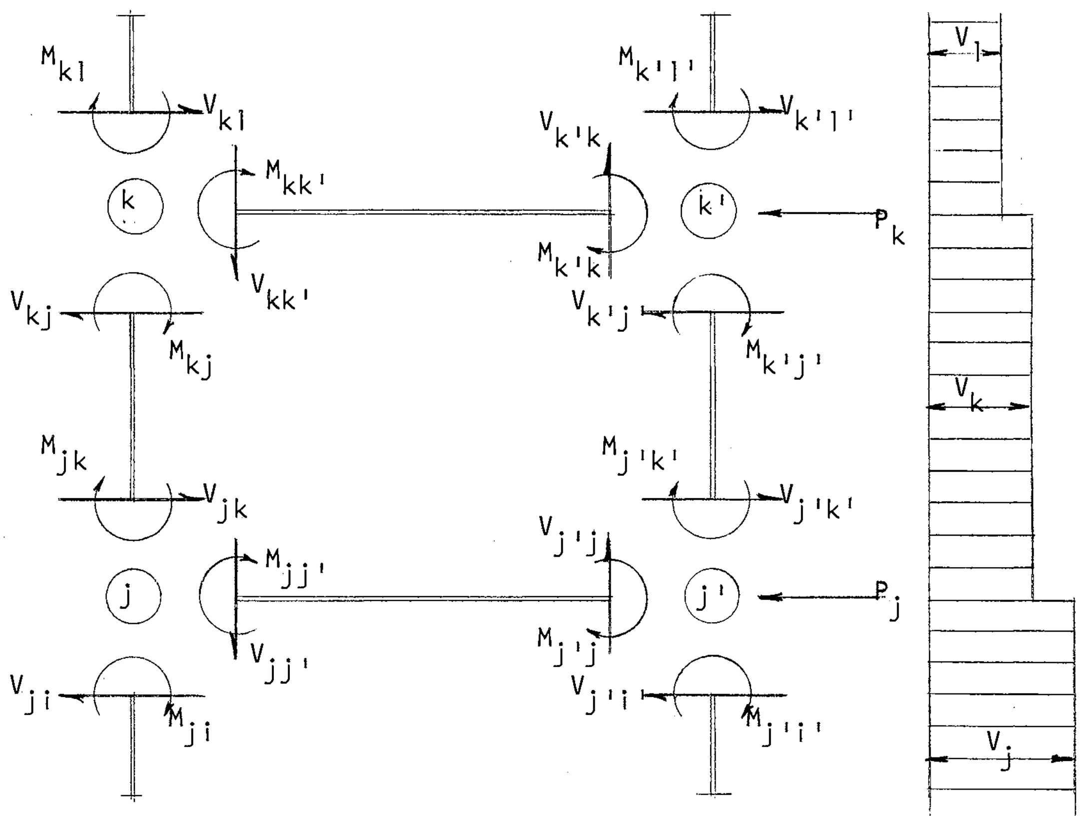
$$\frac{K_{ij}\ \theta_i}{2} + \theta_j\ (K_{ij} + K_{jj'} + K_{jk}) + \frac{K_{jj'}\ \theta_{j'}}{2} + \frac{K_{jk}\ \theta_k}{2} + \frac{3\ K_{ij}\psi_j}{2} + \frac{3\ K_{jk}\psi_k}{2} = 0 \qquad (2c)$$

CHAPTER III

STORY SHEAR EQUATIONS

1. <u>Shear Equations in terms of Moments</u>

A portion $jkj'k'$ of the frame in Fig. (1) is isolated as shown in Fig. (2). The shear equation for any story j may be written in terms of the column end moments.



Isolated Story

Fig. 2

Since there are no transverse loads acting on the columns between joints

$$V_{jk} = V_{kj} = \text{Shear in column } jk$$

$$V_{kl} = V_{lk} = \text{Shear in column } kl\text{, etc.}$$

Then

$$V_j = V_{ij} + V_{i'j'} = \text{Shear in story } j$$

$$V_k = V_{jk} + V_{j'k'} = \text{Shear in story } k\text{, etc.}$$

By the laws of static equilibrium, the story shear times the story height must equal the sum of the column end moments in the story.

$$\begin{aligned} V_j\, h_j &= M_{ij} + M_{ji} + M_{i'j'} + M_{j'i'} \\ V_k\, h_k &= M_{jk} + M_{kj} + M_{j'k'} + M_{k'j'} \end{aligned} \qquad (3)$$

2. <u>Shear Equation in terms of Slopes and Translations</u>

By substituting the required Eqs. (1) into Eqs. (3), the following shear equations in terms of slopes and translations may be written.

$$\begin{aligned} V_j\, h_j &= M_{ji} + M_{ij} + M_{i'j'} + M_{j'i'} \\ &= K_{ij}\,\theta_i + \frac{K_{ij}\,\theta_j}{2} + \frac{3\,K_{ij}\,\psi_j}{2} \\ &\quad + K_{ij}\,\theta_j + \frac{K_{ij}\,\theta_i}{2} + \frac{3\,K_{ij}\,\psi_j}{2} \\ &\quad + K_{i'j'}\,\theta_{i'} + \frac{K_{i'j'}\,\theta_{j'}}{2} + \frac{3\,K_{i'j'}\,\psi_j}{2} \\ &\quad + K_{i'j'}\,\theta_{j'} + \frac{K_{i'j'}\,\theta_{i'}}{2} + \frac{3\,K_{i'j'}\,\psi_j}{2} \\ &= \frac{3\,K_{ij}\,\theta_i}{2} + \frac{3\,K_{ij}\,\theta_j}{2} + \frac{3\,K_{i'j'}\,\theta_{i'}}{2} + \frac{3\,K_{i'j'}\,\theta_{j'}}{2} \\ &\quad + 3(K_{ij} + K_{i'j'})\,\psi_j\,. \end{aligned}$$

Also

$$V_k h_k = \frac{3 K_{jk} \theta_j}{2} + \frac{3 K_{jk} \theta_k}{2} + \frac{3 K_{j'k'} \theta_{j'}}{2} + \frac{3 K_{j'k'} \theta_{k'}}{2} + 3(K_{jk} + K_{j'k'})\psi_k . \tag{4a}$$

Solving for ψ_j and ψ_k ,

$$\begin{aligned} \psi_j &= \frac{1}{3(K_{ij} + K_{i'j'})} \left[V_j h_j - \frac{3 K_{ij} \theta_i}{2} - \frac{3 K_{ij} \theta_j}{2} - \frac{3 K_{i'j'} \theta_{i'}}{2} - \frac{3 K_{i'j'} \theta_{j'}}{2} \right] \\ \psi_k &= \frac{1}{3(K_{jk} + K_{j'k'})} \left[V_k h_k - \frac{3 K_{jk} \theta_j}{2} - \frac{3 K_{jk} \theta_k}{2} - \frac{3 K_{j'k'} \theta_{j'}}{2} - \frac{3 K_{j'k'} \theta_{k'}}{2} \right] \end{aligned} \tag{4b}$$

3. Joint Equation in terms of Slopes

Substituting the results of Eqs. (4b) into Eq. (2c) a new form of joint equation is derived.

$$\begin{aligned} 0 = {} & \frac{K_{ij} \theta_i}{2} + (K_{ij} + K_{jj'} + K_{jk})\theta_j + \frac{K_{jj'} \theta_{j'}}{2} + \frac{K_{jk} \theta_k}{2} \\ & + \frac{K_{ij}}{2(K_{ij} + K_{i'j'})} \left[V_j h_j - \frac{3 K_{ij} \theta_i}{2} - \frac{3 K_{ij} \theta_j}{2} - \frac{3 K_{i'j'} \theta_{i'}}{2} - \frac{3 K_{i'j'} \theta_{j'}}{2} \right] \\ & + \frac{K_{jk}}{2(K_{jk} + K_{j'k'})} \left[V_k h_k - \frac{3 K_{jk} \theta_j}{2} - \frac{3 K_{jk} \theta_k}{2} - \frac{3 K_{j'k'} \theta_{j'}}{2} - \frac{3 K_{j'k'} \theta_{k'}}{2} \right] \end{aligned} \tag{5a}$$

This equation may be reduced to

$$0 = \theta_j \left[\sum K_j - \frac{3(K_{ij})^2}{4(K_{ij} + K_{i'j'})} - \frac{3(K_{jk})^2}{4(K_{jk} + K_{j'k'})} \right]$$

$$+ \theta_{j'} \left[\frac{K_{jj'}}{2} - \frac{3(K_{ij})(K_{i'j'})}{4(K_{ij} + K_{i'j'})} - \frac{3(K_{jk})(K_{j'k'})}{4(K_{jk} + K_{j'k'})} \right]$$

$$+ \theta_i \left[\frac{K_{ij}}{2} - \frac{3(K_{ij})^2}{4(K_{ij} + K_{i'j'})} \right] + \theta_{i'} \left[- \frac{3(K_{ij})(K_{i'j'})}{4(K_{ij} + K_{i'j'})} \right]$$

$$+ \theta_k \left[\frac{K_{jk}}{2} - \frac{3(K_{jk})^2}{4(K_{jk} + K_{j'k'})} \right] + \theta_{k'} \left[- \frac{3(K_{jk})(K_{j'k'})}{4(K_{jk} + K_{j'k'})} \right]$$

$$+ \frac{K_{ij}\ h_j\ V_j}{2(K_{ij} + K_{i'j'})} + \frac{K_{jk}\ h_k\ V_k}{2(K_{jk} + K_{j'k'})} \quad . \qquad (5b)$$

Eq. (5b) is referred to as the six-slope equation because of its similarity to the well known three-moment equation (6).

CHAPTER IV

PHYSICAL INTERPRETATION OF CONSTANTS

It can be shown that the coefficients of the θ's in Eq. (5b) are constants for any given frame, and may be derived individually by considering the physical properties of the frame.

1. Modified Joint Stiffness Factor

A portion of the frame in Fig. (1) is isolated as shown in Fig. (3). The modified joint stiffness factor is defined as the moment required at joint j to produce a unit rotation at that joint with translation allowed and all other joints locked.

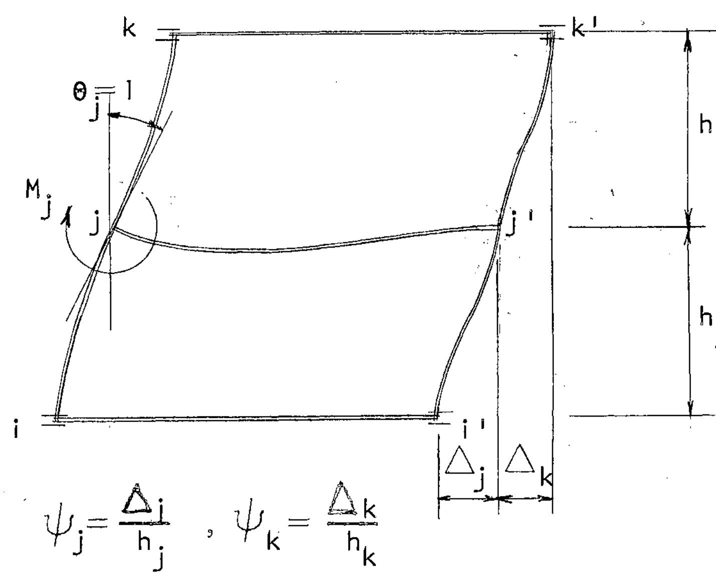


Fig. 3

From Eqs. (1b) and (1c) the moments at joint j are written.

$$M_{jk} = K_{jk}\,\theta_j + \frac{3\,K_{jk}\,\psi_k}{2}$$

$$M_{jj'} = K_{jj'}\,\theta_j$$

$$M_{ji} = K_{ij}\,\theta_j + \frac{3\,K_{ij}\,\psi_j}{2}$$

The summation of internal moments at joint j must then equal the applied moment M_j.

$$\begin{aligned}\sum M_j = M_j &= M_{jk} + M_{jj'} + M_{ji} \\ &= \theta_j(K_{jk} + K_{jj'} + K_{ij}) + \frac{3\,K_{ij}\,\psi_j}{2} + \frac{3\,K_{jk}\,\psi_k}{2} \\ &= \sum K_j\,\theta_j + \frac{3\,K_{ij}\,\psi_j}{2} + \frac{3\,K_{jk}\,\psi_k}{2}\;. \end{aligned} \qquad (6a)$$

Since there are no externally applied horizontal forces, the sum of the column end moments in story k must equal zero.

$$M_{kj} = \frac{K_{jk}\,\theta_j}{2} + \frac{3\,K_{jk}\,\psi_k}{2}$$

$$M_{j'k'} = \frac{3\,K_{j'k'}\,\psi_k}{2}$$

$$M_{k'j'} = \frac{3\,K_{j'k'}\,\psi_k}{2}$$

The sum of the column end moments in story k is written.

$$\begin{aligned} 0 &= M_{jk} + M_{kj} + M_{j'k'} + M_{k'j'} \\ &= K_{jk}\,\theta_j + \frac{K_{jk}\,\theta_j}{2} + 3(K_{jk} + K_{j'k'})\,\psi_k\;. \end{aligned}$$

Solving for ψ_k ,

$$\psi_k = - \frac{K_{jk}\,\theta_j}{2(K_{jk} + K_{j'k'})} \tag{6b}$$

By the same method, the unknown ψ_j is found to be

$$\psi_j = - \frac{K_{ij}\,\theta_j}{2(K_{ij} + K_{i'j'})} \tag{6c}$$

Substituting the values for ψ_j and ψ_k into Eq. (6a), the following equation expressing joint equilibrium is written. With $\theta_j = 1$ as assumed

$$\begin{aligned} \sum M_j = M_j &= \sum K_j + \left(\frac{3\,K_{ij}}{2}\right)\left[- \frac{K_{ij}}{2(K_{ij} + K_{i'j'})}\right] \\ &\quad + \left(\frac{3\,K_{jk}}{2}\right)\left[- \frac{K_{jk}}{2(K_{jk} + K_{j'k'})}\right] \\ &= \sum K_j - \frac{3(K_{ij})^2}{4(K_{ij} + K_{i'j'})} - \frac{3(K_{jk})^2}{4(K_{jk} + K_{j'k'})} \,. \end{aligned} \tag{6d}$$

This is the modified joint stiffness and will be referred to as $\sum K_j^*$.

2. Modified Joint Moment Carry-Over Factor

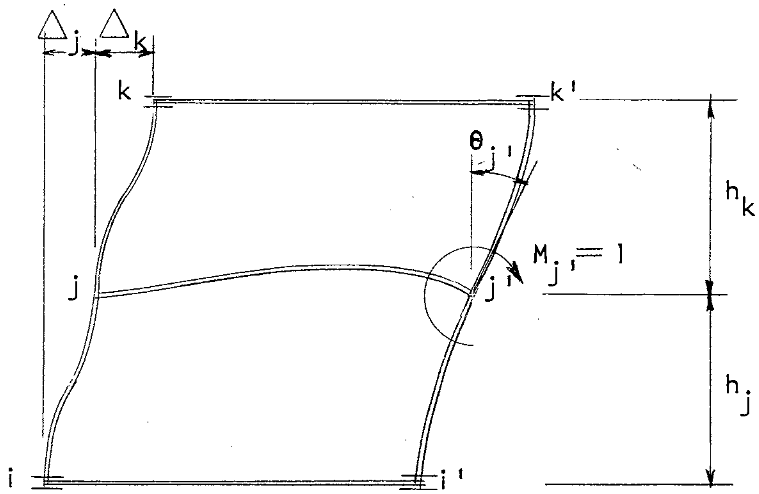
The modified joint moment carry-over factor from joint j' to joint j is defined as the joint moment at j due to a unit joint moment applied at j' with translation allowed and all other joints locked. See Fig. (4).

The end moments for the columns in story k are written.

$$M_{jk} = M_{kj} = \frac{3\,K_{jk}\,\psi_k}{2}$$

$$M_{j'k'} = K_{j'k'}\,\theta_{j'} + \frac{3\,K_{j'k'}\,\psi_k}{2}$$

$$M_{k'j'} = \frac{K_{j'k'}\,\theta_{j'}}{2} + \frac{3\,K_{j'k'}\,\psi_k}{2}$$



$$\psi_j = \frac{\Delta_j}{h_j}, \quad \psi_k = \frac{\Delta_k}{h_k}$$

Fig. 4

and are summed to equal zero.

$$\begin{aligned} 0 &= M_{jk} + M_{kj} + M_{j'k'} + M_{k'j'} \\ &= \frac{3\,K_{j'k'}\,\theta_{j'}}{2} + 3(K_{jk} + K_{j'k'})\,\psi_k \end{aligned}$$

Solving for ψ_k,

$$\psi_k = -\frac{K_{j'k'}\,\theta_{j'}}{2(K_{jk} + K_{j'k'})} \tag{7a}$$

By the same method, the unknown ψ_j is found to be

$$\psi_j = -\frac{K_{i'j'}\,\theta_{j'}}{2(K_{ij} + K_{i'j'})} \tag{7b}$$

The moment at joint j is written

$$\begin{aligned} M_j &= M_{ji} + M_{jj'} + M_{jk} \\ &= \frac{3\,K_{ij}\,\psi_j}{2} + \frac{K_{jj'}\,\theta_{j'}}{2} + \frac{3\,K_{jk}\,\psi_k}{2} \end{aligned} \tag{7c}$$

Substitution of the values from Eqs. (7a) and (7b) into Eq. (7c) gives

$$M_j = \theta_{j'}\left[\frac{K_{jj'}}{2} + \frac{3K_{ij}}{2}\left[-\frac{K_{i'j'}}{2(K_{ij}+K_{i'j'})}\right] + \frac{3K_{jk}}{2}\left[-\frac{K_{j'k'}}{2(K_{jk}+K_{j'k'})}\right]\right]$$

$$= \theta_{j'}\left[\frac{K_{jj'}}{2} - \frac{3(K_{ij})(K_{i'j'})}{4(K_{ij}+K_{i'j'})} - \frac{3(K_{jk})(K_{j'k'})}{4(K_{jk}+K_{j'k'})}\right] . \qquad (7d)$$

The moment at joint j' is

$$M_{j'} = \theta_{j'} \sum K^*_{j'} = 1$$

with $M_{j'}$ equal one as assumed. Solving for $\theta_{j'}$,

$$\theta_{j'} = \frac{M_{j'}}{\sum K^*_{j'}} = \frac{1}{\sum K^*_{j'}} \qquad (7e)$$

Thus the joint moment carry-over factor from joint j' to joint j, referred to as $r_{j'j}$, is

$$r_{j'j} = M_j = \frac{1}{\sum K^*_{j'}}\left[\frac{K_{jj'}}{2} - \frac{3(K_{ij})(K_{i'j'})}{4(K_{ij}+K_{i'j'})} - \frac{3(K_{jk})(K_{j'k'})}{4(K_{jk}+K_{j'k'})}\right] \qquad (7f)$$

The modified joint moment carry-over factor from joint i to joint j is defined as the moment at joint j due to a unit moment applied at joint i with translation allowed and all other joints locked. See Fig. (5). The end moments for the columns in story j are written

$$M_{i'j'} = M_{j'i'} = \frac{3\,K_{i'j'}\,\psi_j}{2}$$

$$M_{ij} = K_{ij}\,\theta_i + \frac{3\,K_{ij}\,\psi_j}{2}$$

$$M_{ji} = \frac{K_{ij}\,\theta_i}{2} + \frac{3\,K_{ij}\,\psi_j}{2}$$

and are summed to equal zero.

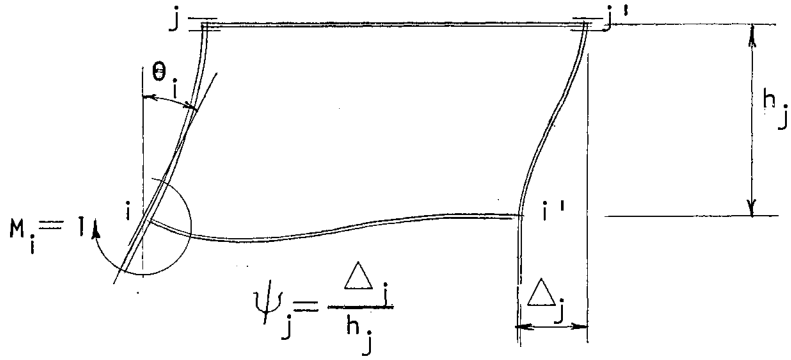


Fig. 5

$$0 = M_{ij} + M_{ji} + M_{i'j'} + M_{j'i'}$$

$$= \frac{3\,K_{ij}\,\theta_i}{2} + 3(K_{ij} + K_{i'j'})\,\psi_j \;.$$

Solving for ψ_j

$$\psi_j = -\frac{K_{ij}\,\theta_i}{2(K_{ij} + K_{i'j'})} \;. \qquad (7g)$$

The moment at joint j is written

$$M_j = M_{ji} = \frac{K_{ij}\,\theta_i}{2} + \frac{3\,K_{ij}\,\psi_i}{2}$$

$$= \theta_i\left[\frac{K_{ij}}{2} + \frac{3\,K_{ij}}{2}\left[-\frac{K_{ij}}{2(K_{ij} + K_{i'j'})}\right]\right]$$

$$= \theta_i\left[\frac{K_{ij}}{2} - \frac{3(K_{ij})^2}{4(K_{ij} \quad K_{i'j'})}\right] \;. \qquad (7h)$$

The moment at joint i is

$$M_i = \theta \sum K_i^* = 1$$

with M_i equal one as assumed. Solving for θ_i

$$\theta_i = \frac{M_i}{\sum K_i^*} = \frac{1}{\sum K_i^*} \quad . \tag{7i}$$

Thus the joint moment carry-over factor from joint i to joint j, referred to as r_{ij}, is

$$r_{ij} = M_j = \frac{1}{\sum K_i^*}\left[\frac{K_{ij}}{2} - \frac{3(K_{ij})^2}{4(K_{ij} + K_{i'j'})}\right] \quad . \tag{7j}$$

By the same method it can be shown that the joint moment carry-over factor from joint k to joint j, referred to as r_{kj}, is

$$r_{kj} = \frac{1}{\sum K_k^*}\left[\frac{K_{jk}}{2} - \frac{3(K_{jk})^2}{4(K_{jk} + K_{j'k'})}\right] \quad . \tag{7k}$$

The modified joint moment carry-over factor from joint i' to joint j is defined as the moment at joint j due to a unit joint moment applied at joint i' with translation allowed and all other joints locked. See Fig. 6.

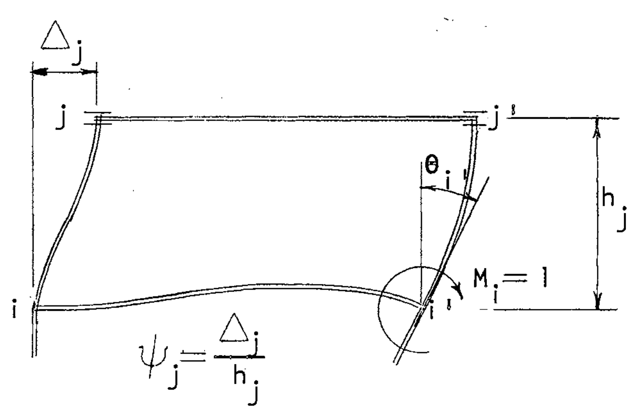


Fig. 6

The end moments for the columns in story j are written

$$M_{ij} = M_{ji} = \frac{3\,K_{ij}\,\psi_j}{2}$$

$$M_{i'j'} = K_{i'j'}\,\theta_{i'} + \frac{3\,K_{i'j'}\,\psi_j}{2}$$

$$M_{j'i'} = \frac{K_{i'j'}\,\theta_{i'}}{2} + \frac{3\,K_{i'j'}\,\psi_j}{2}$$

and are summed to equal zero.

$$0 = M_{ij} + M_{ji} + M_{i'j'} + M_{j'i'}$$

$$= \frac{3\,K_{i'j'}\,\theta_{i'}}{2} + 3(K_{ij} + K_{i'j'})\,\psi_j \;.$$

Solving for ψ_j

$$\psi_j = -\frac{K_{i'j'}\,\theta_{i'}}{2(K_{ij} + K_{i'j'})} \tag{7l}$$

The moment at joint j is written.

$$M_j = M_{ji} = \frac{3\,K_{ij}\,\psi_j}{2}$$

$$= \frac{3\,K_{ij}}{2}\left[-\frac{K_{i'j'}\,\theta_{i'}}{2(K_{ij} + K_{i'j'})}\right]$$

$$= -\frac{3(K_{ij})(K_{i'j'})\,\theta_{i'}}{4(K_{ij} + K_{i'j'})} \;. \tag{7m}$$

The moment at joint i' is

$$M_{i'} = \theta_{i'}\sum K^*_{i'} = 1$$

with $M_{i'}$ equal one as assumed. Solving for $\theta_{i'}$

$$\theta_{i'} = \frac{M_{i'}}{\sum K^*_{i'}} = \frac{1}{\sum K^*_{i'}}$$

Thus the joint moment carry-over factor from joint i' to joint j, referred to as $r_{i'j}$, is

$$r_{i'j} = M_j = -\frac{1}{K^*_{i'}}\left[\frac{3(K_{ij})(K_{i'j'})}{4(K_{ij}+K_{i'j'})}\right] . \qquad (7n)$$

By the same method it can be shown that the joint moment carry-over factor from joint k' to joint j, referred to as $r_{k'j}$, is

$$r_{k'j} = -\frac{1}{K^*_{k'}}\ \frac{3(K_{jk})(K_{j'k'})}{4(K_{jk}+K_{j'k'})} . \qquad (7o)$$

3. Fixed End Joint Moments

The fixed end joint moment is defined as the moment at joint j due to translation of adjacent joints. See Fig. 7.

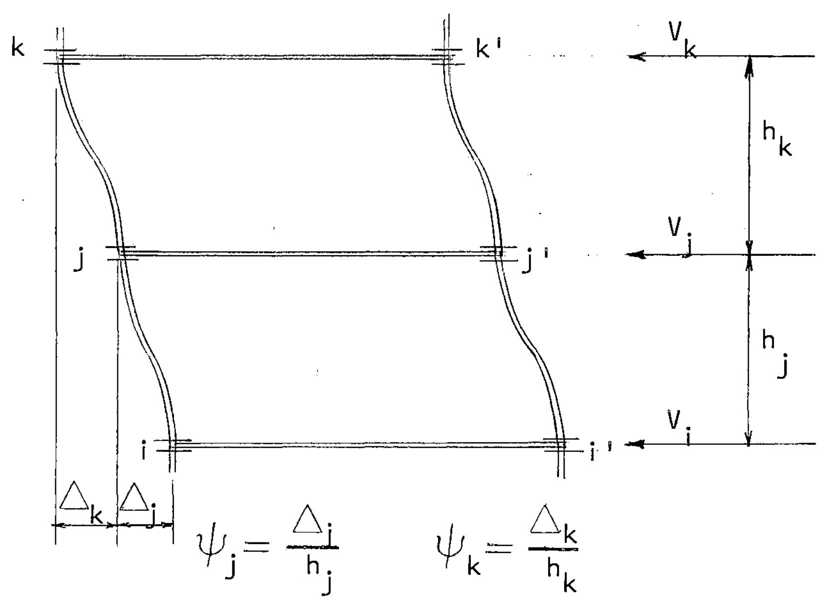


Fig. 7

For static equilibrium the story shear times the story height must equal the sum of the column end moments. For story j

$$
\begin{aligned}
V_j\, h_j &= M_{ij} + M_{ji} + M_{i'j'} + M_{j'i'} \\
&= \frac{3\,K_{ij}\,\psi_j}{2} + \frac{3\,K_{ji}\,\psi_j}{2} + \frac{3\,K_{i'j'}\,\psi_j}{2} + \frac{3\,K_{j'i'}\,\psi_j}{2} \\
&= 3(K_{ij} + K_{i'j'})\,\psi_j \quad .
\end{aligned}
$$

Solving for ψ_j ,

$$
\psi_j = \frac{V_j\, h_j}{3(K_{ij} + K_{i'j'})} \quad . \tag{8a}
$$

The column end moments may now be written in terms of known values.

$$
\begin{aligned}
M_{ji} &= \frac{3\,K_{ij}\,\psi_j}{2} \\
&= \frac{3\,K_{ij}}{2}\left[\frac{V_j\, h_j}{3(K_{ij} + K_{i'j'})}\right] \\
&= \frac{K_{ij}\,V_j\,h_j}{2(K_{ij} + K_{i'j'})}
\end{aligned} \tag{8b}
$$

The other column end moments, M_{ji}, $M_{i'j'}$, and $M_{j'i'}$, may be determined in a similar manner.

The coefficient of $V_j h_j$ in Eq. (8b),

$$
\frac{K_{ij}}{2(K_{ij} + K_{i'j'})} \quad ,
$$

occured quite frequently in previous derivations. It is obvious from inspection of Eq. (8b) that this value is a constant for each column in each story, such that when the story moment, $V_j h_j$, is multiplied by this constant the result is the guided end moment for column ij. This constant will be referred to as Q_{ij}.

The fixed end moment for joint j is written

$$\sum FM_j = M_{ji} + M_{jj'} + M_{jk}$$

$$= \frac{K_{ij}\ V_j\ h_j}{2(K_{ij} + K_{i'j'})} + 0 + \frac{K_{jk}\ V_k\ h_k}{2(K_{jk} + K_{j'k'})}$$

$$= Q_{ij}\ V_j\ h_j + Q_{jk}\ V_k\ h_k\ . \qquad (8c)$$

CHAPTER V

JOINT MOMENT EQUATION

1. Joint Moment Notation

The following notation will be introduced

$$JM_j = \text{Joint Moment at } j = \sum K^*_j \, \theta_j$$

$$JM_k = \text{Joint Moment at } k = \sum K^*_k \, \theta_k \text{ , etc.,}$$

where the joint moment is defined as the product of the modified joint stiffness and the angle of rotation of the joint.

Rewriting Eq. (5b), and making the substitution $\theta = \frac{JM}{\sum K^*}$,

$$0 = \frac{JM_j}{\sum K^*_j}\left[\sum K_j - \frac{3(K_{ij})^2}{4(K_{ij} + K_{i'j'})} - \frac{3(K_{jk})^2}{4(K_{jk} + K_{j'k'})}\right]$$

$$+ \frac{JM_{j'}}{\sum K^*_{j'}}\left[\frac{K_{jj'}}{2} - \frac{3(K_{ij})(K_{i'j'})}{4(K_{ij} + K_{i'j'})} - \frac{3(K_{jk})(K_{j'k'})}{4(K_{jk} + K_{j'k'})}\right]$$

$$+ \frac{JM_i}{\sum K^*_i}\left[\frac{K_{ij}}{2} - \frac{3(K_{ij})^2}{4(K_{ij} + K_{i'j'})}\right] \quad \frac{JM_{i'}}{\sum K^*_{i'}}\left[- \frac{3(K_{ij})(K_{i'j'})}{4(K_{ij} + K_{i'j'})}\right]$$

$$+ \frac{JM_k}{\sum K^*_k}\left[\frac{K_{jk}}{2} - \frac{3(K_{jk})^2}{4(K_{jk} + K_{j'k'})}\right] \quad \frac{JM_{k'}}{\sum K^*_{k'}}\left[- \frac{3(K_{jk})(K_{j'k'})}{4(K_{jk} + K_{j'k'})}\right]$$

$$+ \frac{K_{ij} \; V_j \; h_j}{2(K_{ij} + K_{i'j'})} + \frac{K_{jk} \; V_k \; h_k}{2(K_{jk} + K_{j'k'})} \quad . \tag{9a}$$

2. Joint Moment Equation

Substituting into Eq. (9a) the respective constants from Eqs. (6d), (7f), (7j), (7k), (7n), (7o), and (8c), and transposing terms, the joint moment equation for joint j is written.

$$JM_j = -r_{j'j}\,JM_{j'} - r_{ij}\,JM_i - r_{i'j}\,JM_{i'} - r_{kj}\,JM_k - r_{k'j}\,JM_{k'} - \sum FM_j \,. \qquad (9b)$$

A joint moment equation similar to Eq. (9b) is written for each joint in the structure. This system of equations may be solved by a carry-over procedure as illustrated in the numerical example.

CHAPTER VI

FINAL END MOMENTS

The final end moments are found from the joint moments after the carry-over procedure is completed. Using Eq. (2b) the final end moments at joint j are written.

$$M_{jk} = K_{jk}\,\theta_j + \frac{K_{jk}\,\theta_k}{2} + \frac{3\,K_{jk}\,\psi_k}{2}$$

$$M_{jj'} = K_{jj'}\,\theta_j + \frac{K_{jj'}\,\theta_{j'}}{2} \qquad (10a)$$

$$M_{ji} = K_{ij}\,\theta_j + \frac{K_{ij}\,\theta_i}{2} + \frac{3\,K_{ij}\,\psi_j}{2}$$

The values of ψ_j and ψ_k are obtained from Eqs. (6c), (7b), (7g), (7l), and (8a).

$$\psi_j = -Q_{ij}\,\theta_j - Q_{i'j'}\,\theta_{j'} - Q_{ij}\,\theta_i - Q_{i'j'}\,\theta_{i'} + \frac{V_j\,h_j}{3(K_{ij} + K_{i'j'})}\ .$$

$$\psi_k = -Q_{jk}\,\theta_j - Q_{j'k'}\,\theta_{j'} - Q_{jk}\,\theta_k - Q_{j'k'}\,\theta_{k'} + \frac{V_k\,h_k}{3(K_{jk} + K_{j'k'})}\ .$$

Substituting the values of ψ_j and ψ_k into Eq. (10a) and using the joint moment notation, a new form of end moment equation is written.

$$M_{jk} = JM_j \frac{K_{jk}}{\sum K^*_j} + JM_k \frac{K_{jk}}{2\sum K^*_k} + \frac{3\ K_{jk}}{2}\left[- JM_j \frac{Q_{jk}}{\sum K^*_j} - JM_{j'} \frac{Q_{j'k'}}{\sum K^*_{j'}} - JM_k \frac{Q_{jk}}{\sum K^*_k} - JM_{k'} \frac{Q_{j'k'}}{\sum K^*_{k'}} + \frac{V_k\ h_k}{3(K_{jk} + K_{j'k'})}\right]$$

$$= \frac{JM_j}{\sum K^*_j}\left[K_{jk} - \frac{3\ K_{jk}\ Q_{jk}}{2}\right] + \frac{JM_k}{\sum K^*_k}\left[\frac{K_{jk}}{2} - \frac{3\ K_{jk}\ Q_{jk}}{2}\right]$$

$$+ \frac{JM_{j'}}{\sum K^*_{j'}}\left[-\frac{3\ K_{jk}\ Q_{j'k'}}{2}\right] + \frac{JM_{k'}}{\sum K^*_{k'}}\left[-\frac{3\ K_{jk}\ Q_{j'k'}}{2}\right]$$

$$+ V_k\ h_k\ Q_{jk}\ . \qquad (10b)$$

Using nomenclature previously introduced and that in Table I, Eq. (10b) is written

$$M_{jk} = D^*_{jk}\ JM_j + r_{kj}\ JM_k + r_{j'k}\ JM_{j'} + r_{k'j}\ JM_{k'} + V_k\ h_k\ Q_{jk}$$

$$M_{jj'} = D^*_{jj'}\ JM_j + \frac{D^*_{j'j}}{2}\ JM_{j'} \qquad (10c)$$

$$M_{ji} = D^*_{ji}\ JM_j + r_{ij}\ JM_i + r_{j'i}\ JM_{j'} + r_{i'j}\ JM_{i'} + V_j\ h_j\ Q_{ij}$$

and for joint j'

$$M_{j'k'} = D^*_{j'k'} JM_{j'} + r_{k'j'} JM_{k'} + r_{jk'}\ JM_j + r_{kj'}\ JM_k + V_k\ h_k\ Q_{j'k'}$$

$$M_{j'j} = D^*_{j'j}\ JM_{j'} + \frac{D^*_{jj'}}{2}\ JM_j \qquad (10d)$$

$$M_{j'i'} = D^*_{j'i'} JM_{j'} + r_{i'j'} JM_{i'} + r_{ji'}\ JM_j + r_{ij'}\ JM_i + V_j\ h_j\ Q_{i'j'}\ .$$

TABLE 1

CONSTANTS FOR ANY JOINT j

$$Q_{ij} = \frac{K_{ij}}{2(K_{ij} + K_{i'j'})} \qquad Q_{i'j'} = \frac{K_{i'j'}}{2(K_{ij} + K_{i'j'})}$$

$$Q_{jk} = \frac{K_{jk}}{2(K_{jk} + K_{j'k'})} \qquad Q_{j'k'} = \frac{K_{j'k'}}{2(K_{jk} + K_{j'k'})}$$

$$K^*_{ij} = K_{ij} - (1.5)(K_{ij})(Q_{ij}) \qquad D^*_{ij} = \frac{K^*_{ij}}{\sum K^*_j}$$

$$K^*_{jj'} = K_{jj'} \qquad D^*_{jj'} = \frac{K^*_{jj'}}{\sum K^*_j}$$

$$K^*_{jk} = K_{jk} - (1.5)(K_{jk})(Q_{jk}) \qquad D^*_{jk} = \frac{K^*_{jk}}{\sum K^*_j}$$

$$r_{jj'} = \frac{1}{\sum K^*_j}\left[(0.5)(K_{jj'}) - (1.5)(K_{ij})(Q_{i'j'}) - (1.5)(K_{jk})(Q_{j'k'})\right]$$

$$r_{jk} = \frac{1}{\sum K^*_j}\left[(0.5)(K_{jk}) - (1.5)(K_{jk})(Q_{jk})\right]$$

$$r_{ji} = \frac{1}{\sum K^*_j}\left[(0.5)(K_{ij}) - (1.5)(K_{ij})(Q_{ij})\right]$$

$$r_{jk'} = \frac{1}{\sum K^*_j}\left[-(1.5)(K_{jk})(Q_{j'k'})\right]$$

$$r_{ji'} = \frac{1}{\sum K^*_j}\left[-(1.5)(K_{ij})(Q_{i'j'})\right]$$

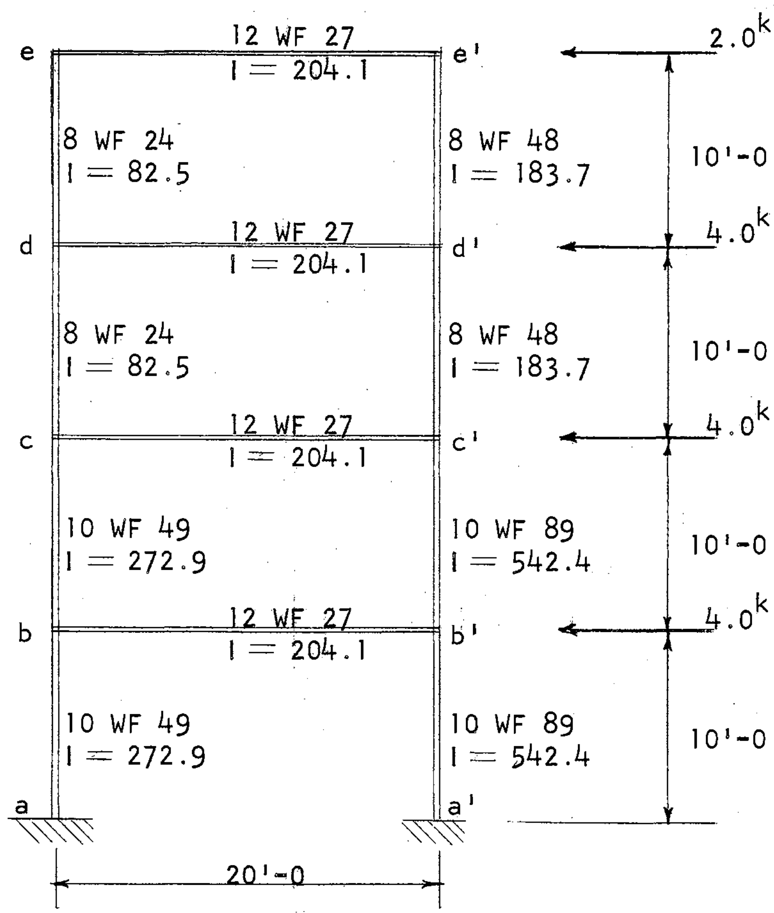
$$FM_j = V_j \, h_j \, Q_{ij} + V_k \, h_k \, Q_{jk}$$

CHAPTER VII

NUMERICAL EXAMPLE

1. Statement of Problem

To illustrate the use of the procedure derived in the predeeding chapters, the frame in Fig. (8) will be analysed. The assumed sections and moments of inertia are shown on the frame.



Given Structure

Fig. 8

2. <u>Calculation of Constants</u>

The relative value of $\frac{I}{L}$ will be used for the stiffness factor K.

$$K_{ab} = K_{ba} = K_{bc} = K_{cb} = \frac{272.9}{10(12)} = 2.275$$

$$K_{a'b'} = K_{b'a'} = K_{b'c'} = K_{c'b'} = \frac{524.4}{10(12)} = 4.518$$

$$K_{cd} = K_{dc} = K_{de} = K_{ed} = \frac{82.5}{10(12)} = 0.687$$

$$K_{c'd'} = K_{d'c'} = K_{d'e'} = K_{e'd'} = \frac{183.7}{10(12)} = 1.530$$

$$K_{aa'} = K_{bb'} = K_{cc'} = K_{dd'} = \frac{204.1}{20(12)} = 0.850$$

Calculation of constants for the carry-over moment procedure follows in a tabular form similar to Table 1.

<u>Joint a</u>

$$Q_{ab} = \frac{K_{ab}}{2(K_{ab} + K_{a'b'})} = \frac{2.275}{2(2.275 + 4.518)} = 0.167$$

$$Q_{a'b'} = \frac{K_{a'b'}}{2(K_{ab} + K_{a'b'})} = \frac{4.518}{2(2.275 + 4.518)} = 0.333$$

$$K^*_{ab} = K_{ab} - (1.5)(K_{ab})(Q_{ab}) = 2.275 - (1.5)(2.275)(0.167) = 1.701$$

$$\sum K^*_a = \infty$$

$$D^*_{ab} = \frac{K^*_{ab}}{\sum K^*_a} = \frac{1.701}{\infty} = 0$$

$$r_{aa'} = r_{ab} = r_{ab'} = 0$$

$$\sum FM_a = V_b\, h_b\, Q_{ab} = (10)(14)(0.167) = 23.3^{'k}$$

Joint a'

$$K^*_{a'b'} = K_{a'b'} - (1.5)(K_{a'b'})(Q_{a'b'}) = 4.518 - (1.5)(4.518)(0.333)$$
$$= 2.258$$

$$\sum K^*_{a'} = \infty$$

$$D^*_{a'b'} = \frac{K^*_{a'b'}}{\sum K^*_{a'}} = 0$$

$$r_{a'a} = r_{a'b'} = r_{a'b} = 0$$

$$\sum FM_{a'} = V_b\ h_b\ Q_{a'b'} = (10)(14)(0.333) = 46.7^{'k}$$

Joint b

$$Q_{ab} = 0.167$$

$$Q_{a'b'} = 0.333$$

$$Q_{bc} = \frac{K_{bc}}{2(K_{bc} + K_{b'c'})} = \frac{2.275}{2(2.275 + 4.518)} = 0.167$$

$$Q_{b'c'} = \frac{K_{b'c'}}{2(K_{bc} + K_{b'c'})} = \frac{4.518}{2(2.275 + 4.518)} = 0.333$$

$$K^*_{ba} = K_{ab} - (1.5)(K_{ab})(Q_{ab}) = 2.275 - (1.5)(2.275)(0.167) = 1.701$$

$$K^*_{bb'} = K_{bb'} = 0.850$$

$$K^*_{bc} = K_{bc} - (1.5)(K_{bc})(Q_{bc}) = 2.275 - (1.5)(2.275)(0.167) = 1.701$$

$$\sum K^*_b = K^*_{ba} + K^*_{bb'} + K^*_{bc} = 4.252$$

$$D^*_{ba} = \frac{K^*_{ba}}{\sum K^*_b} = \frac{1.701}{4.252} = 0.400$$

$$D^*_{bb'} = \frac{K^*_{bb'}}{\sum K^*_b} = \frac{0.850}{4.252} = 0.200$$

$$D^*_{bc} = \frac{K^*_{bc}}{\sum K^*_b} = \frac{1.701}{4.252} = 0.400$$

$$r_{bb'} = \frac{1}{\sum K^*_b}\left[(0.5)(K_{bb'}) - (1.5)(K_{ab})(Q_{a'b'}) - (1.5)(K_{bc})(Q_{b'c'})\right]$$

$$= [0.235]\left[(0.5)(0.850) - (1.5)(2.275)(0.333) - (1.5)(2.275)(0.333)\right]$$

$$= -0.431$$

$$r_{bc} = \frac{1}{\sum K^*_b}\left[(0.5)(K_{bc}) - (1.5)(K_{bc})(Q_{bc})\right]$$

$$= [0.235]\left[(0.5)(2.275) - (1.5)(2.275)(0.167)\right]$$

$$= +0.133$$

$$r_{ba} = \frac{1}{\sum K^*_b}\left[(0.5)(K_{ba}) - (1.5)(K_{ba})(Q_{ab})\right]$$

$$= [0.235]\left[(0.5)(2.275) - (1.5)(2.275)(0.167)\right]$$

$$= +0.133$$

$$r_{bc'} = \frac{1}{\sum K^*_b}\left[-(1.5)(K_{bc})(Q_{b'c'})\right] = [0.235]\left[-(1.5)(2.275)(0.333)\right]$$

$$= -0.266$$

$$r_{ba'} = \frac{1}{\sum K^*_b}\left[-(1.5)(K_{ab})(Q_{a'b'})\right] = [0.235]\left[-(1.5)(2.275)(0.333)\right]$$

$$= -0.266$$

$$\sum FM_b = V_b\, h_b\, Q_{ab} + V_c\, h_c\, Q_{bc} = (10)(14)(0.167) + (10)(10)(0.167)$$

$$= +40.0^{\,'k}$$

Joint b'

$$K^*_{b'a'} = K_{a'b'} - (1.5)(K_{a'b'})(Q_{a'b'}) = (4.518) - (1.5)(4.518)(0.333)$$

$$= 2.258$$

$$K^*_{b'b} = K_{b'b} = K_{bb'} = 0.850$$

$$K^*_{b'c'} = K_{b'c'} - (1.5)(K_{b'c'})(Q_{b'c'}) = (4.518) - (1.5)(4.518)(0.333)$$

$$= 2.258$$

$$\sum K^*_{b'} = K^*_{b'a'} + K^*_{b'b} + K^*_{b'c'} = 5.366$$

$$D^*_{b'a'} = \frac{K^*_{b'a'}}{\sum K^*_{b'}} = \frac{2.258}{5.366} = 0.420$$

$$D^*_{b'b} = \frac{K^*_{b'b}}{\sum K^*_{b'}} = \frac{0.850}{5.366} = 0.160$$

$$D^*_{b'c'} = \frac{K^*_{b'c'}}{\sum K^*_{b'}} = \frac{2.258}{5.366} = 0.420$$

$$r_{b'b} = \frac{1}{\sum K^*_{b'}} \left[(0.5)(K_{bb'}) - (1.5)(K_{a'b'})(Q_{ab}) - (1.5)(K_{b'c'})(Q_{bc})\right]$$

$$= \left[0.186\right] \left[(0.5)(0.850) - (1.5)(4.518)(0.167) - (1.5)(4.518)(0.167)\right]$$

$$= -0.342$$

$$r_{b'c'} = \frac{1}{\sum K^*_{b'}} \left[(0.5)(K_{b'c'}) - (1.5)(K_{b'c'})(Q_{b'c'})\right]$$

$$= \left[0.186\right] \left[(0.5)(4.518) - (1.5)(4.518)(0.333)\right]$$

$$= 0$$

$$r_{b'a'} = \frac{1}{\sum K^*_{b'}} \left[(0.5)(K_{a'b'}) - (1.5)(K_{a'b'})(Q_{a'b'})\right]$$

$$= \left[0.186\right] \left[(0.5)(4.518) - (1.5)(4.518)(0.333)\right]$$

$$= 0$$

$$r_{b'c} = \frac{1}{\sum K^*_{b'}} \left[-(1.5)(K_{b'c'})(Q_{bc})\right] = \left[0.186\right] \left[-(1.5)(4.518)(0.167)\right]$$

$$= -0.211$$

$$r_{b'a} = \frac{1}{\sum K^*_{b'}} \left[(1.5)(K_{a'b'})(Q_{ab}) \right] = \left[0.186\right]\left[-(1.5)(4.518)(0.167)\right]$$

$$= -0.211$$

$$\sum FM_{b'} = V_b \; h_b \; Q_{a'b'} + V_c \; h_c \; Q_{b'c'} = (10)(14)(0.333) + (10)(10)(0.333)$$

$$= +80.0 \; {}^{'k}$$

<u>Joint c</u>

$$Q_{cd} = \frac{K_{cd}}{2(K_{cd} + K_{c'd'})} = \frac{0.687}{2(0.687 + 1.530)} = 0.155$$

$$Q_{c'd'} = \frac{K_{c'd'}}{2(K_{cd} + K_{c'd'})} = \frac{1.530}{2(0.687 + 1.530)} = 0.345$$

$$K^*_{cb} = K_{bc} - (1.5)(K_{bc})(Q_{bc}) = (2.275) - (1.5)(2.275)(0.167)$$

$$= 1.701$$

$$K^*_{cc'} = K_{cc'} = 0.850$$

$$K^*_{cd} = K_{cd} - (1.5)(K_{cd})(Q_{cd}) = (0.687) - (1.5)(0.687)(0.155)$$

$$= 0.527$$

$$K^*_c = K^*_{cb} + K^*_{cc'} + K^*_{cd} = 3.078$$

$$D^*_{cb} = \frac{K^*_{cb}}{\sum K^*_c} = \frac{1.701}{3.078} = 0.554$$

$$D^*_{cc'} = \frac{K^*_{cc'}}{\sum K^*_c} = \frac{0.850}{3.078} = 0.275$$

$$D^*_{cd} = \frac{K^*_{cd}}{\sum K^*_c} = \frac{0.527}{3.078} = 0.171$$

$$r_{cc'} = \frac{1}{\sum K^*_c} \left[(0.5)(K_{cc'}) - (1.5)(K_{bc})(Q_{b'c'}) - (1.5)(K_{cd})(Q_{c'd'}) \right]$$

$$= \left[0.325\right] \left[(0.5)(0.850) - (1.5)(2.275)(0.333) - (1.5)(0.687)(0.345)\right]$$

$$= 0.347$$

$$r_{cd} = \frac{1}{\sum K_c^*}\left[(0.5)(K_{cd}) - (1.5)(K_{cd})(Q_{cd})\right]$$
$$= \left[0.325\right]\left[(0.5)(0.687) - (1.5)(0.687)(0.155)\right]$$
$$= +\ 0.060$$

$$r_{cb} = \frac{1}{\sum K_c^*}\left[(0.5)(K_{bc}) - (1.5)(K_{bc})(Q_{bc})\right]$$
$$= \left[0.325\right]\left[(0.5)(2.275) - (1.5)(2.275)(0.167)\right]$$
$$= +\ 0.184$$

$$r_{cd'} = \frac{1}{\sum K_c^*}\left[-(1.5)(K_{cd})(Q_{c'd'})\right] = \left[0.325\right]\left[-(1.5)(0.687)(0.345)\right]$$
$$= -\ 0.116$$

$$r_{cb'} = \frac{1}{\sum K_c^*}\left[-(1.5)(K_{bc})(Q_{b'c'})\right] = \left[0.325\right]\left[-(1.5)(2.275)(0.333)\right]$$
$$= -\ 0.370$$

$$\sum FM_c = V_c\ h_c\ Q_{bc} + V_d\ h_d\ Q_{cd} = (10)(10)(0.167) + (10)(6)(0.155)$$
$$= 26.0^{'k}$$

<u>Joint c'</u>

$$K^*_{c'b'} = K_{b'c'} - (1.5)(K_{b'c'})(Q_{b'c'}) = (4.518) - (1.5)(4.518)(0.333)$$
$$= 2.258$$

$$K^*_{c'c} = K_{c'c} = K_{cc'} = 0.850$$

$$K^*_{c'd'} = K_{c'd'} - (1.5)(K_{c'd'})(Q_{c'd'}) = (1.530) - (1.5)(1.530)(0.345)$$
$$= 0.738$$

$$\sum K^*_{c'} = K^*_{c'b'} + K^*_{c'c} + K^*_{c'd'} = 3.846$$

$$D^*_{c'b'} = \frac{K^*_{c'b'}}{\sum K^*_{c'}} = \frac{2.258}{3.846} = 0.588$$

$$D^*_{c'c} = \frac{K^*_{c'c}}{\sum K^*_{c'}} = \frac{0.850}{3.846} = 0.221$$

$$D^*_{c'd'} = \frac{K^*_{c'd'}}{\sum K^*_{c'}} = \frac{0.738}{3.846} = 0.191$$

$$r_{c'c} = \frac{1}{\sum K^*_{c'}} \left[(0.5)(K_{cc'}) - (1.5)(K_{b'c'})(Q_{bc}) - (1.5)(K_{c'd'})(Q_{cd})\right]$$
$$= \left[0.260\right] \left[(0.5)(0.850) - (1.5)(4.518)(0.167) - (1.5)(1.530)(0.155)\right]$$
$$= -0.276$$

$$r_{c'd'} = \frac{1}{\sum K^*_{c'}} \left[(0.5)(K_{c'd'}) - (1.5)(K_{c'd'})(Q_{c'd'})\right]$$
$$= \left[0.260\right] \left[(0.5)(1.530) - (1.5)(1.530)(0.345)\right]$$
$$= -0.007$$

$$r_{c'b'} = \frac{1}{\sum K^*_{c'}} \left[(0.5)(K_{b'c'}) - (1.5)(K_{b'c'})(Q_{b'c'})\right]$$
$$= \left[0.260\right] \left[(0.5)(4.518) - (1.5)(4.518)(0.333)\right]$$
$$= 0$$

$$r_{c'd} = \frac{1}{\sum K^*_{c'}} \left[-(1.5)(K_{c'd'})(Q_{cd})\right] = \left[0.260\right]\left[-(1.5)(1.530)(0.155)\right]$$
$$= -0.092$$

$$r_{c'b} = \frac{1}{\sum K^*_{c'}} \left[-(1.5)(K_{b'c'})(Q_{bc})\right] = \left[0.260\right]\left[-(1.5)(4.518)(0.167)\right]$$
$$= -0.294$$

$$\sum FM_{c'} = V_c\, h_c\, Q_{b'c'} + V_d\, h_d\, Q_{c'd'} = (10)(10)(0.333) + (10)(6)(0.345)$$
$$= +54.0^{'k}$$

Joint d

$$Q_{de} = \frac{K_{de}}{2(K_{de} + K_{d'e'})} = \frac{0.687}{2(0.687 + 1.530)} = 0.155$$

$$Q_{d'e'} = \frac{K_{d'e'}}{2(K_{de} + K_{d'e'})} = \frac{1.530}{2(0.687 + 1.530)} = 0.345$$

$$K^*_{dc} = K_{cd} - (1.5)(K_{cd})(Q_{cd}) = (0.687) - (1.5)(0.687)(0.155)$$
$$= 0.527$$

$$K^*_{dd'} = K_{dd'} = 0.850$$

$$K^*_{de} = K_{de} - (1.5)(K_{de})(Q_{de}) = (0.687) - (1.5)(0.687)(0.155)$$
$$= 0.527$$

$$\sum K^*_d = K^*_{dc} + K^*_{dd'} + K^*_{de} \qquad 1.904$$

$$D^*_{dc} = \frac{K^*_{dc}}{\sum K^*_d} = \frac{0.527}{1.904} = 0.277$$

$$D^*_{dd'} = \frac{K^*_{dd'}}{\sum K^*_d} = \frac{0.850}{1.904} = 0.446$$

$$D^*_{de} = \frac{K^*_{de}}{\sum K^*_d} = \frac{0.527}{1.904} = 0.277$$

$$r_{dd'} = \frac{1}{\sum K^*_d}\left[(0.5)(K_{dd'}) - (1.5)(K_{cd})(Q_{c'd'}) - (1.5)(K_{de})(Q_{d'e'})\right]$$
$$= [0.525]\left[(0.5)(0.850) - (1.5)(0.687)(0.345) - (1.5)(0.687)(0.345)\right]$$
$$= -0.151$$

$$r_{de} = \frac{1}{\sum K^*_d}\left[(0.5)(K_{de}) - (1.5)(K_{de})(Q_{de})\right]$$
$$= [0.525]\left[(0.5)(0.687) - (1.5)(0.687)(0.155)\right]$$
$$= +0.096$$

$$r_{dc} = \frac{1}{\sum K^*_d}\left[(0.5)(K_{cd}) - (1.5)(K_{cd})(Q_{cd})\right]$$

$$= \left[0.525\right]\left[(0.5)(0.687) - (1.5)(0.687)(0.155)\right]$$

$$= +\,0.096$$

$$r_{de'} = \frac{1}{\sum K^*_d}\left[-(1.5)(K_{de})(Q_{d'e'})\right] = \left[0.525\right]\left[-(1.5)(0.687)(0.345)\right]$$

$$= -\,0.187$$

$$r_{dc'} = \frac{1}{\sum K^*_d}\left[-(1.5)(K_{cd})(Q_{c'd'})\right] = \left[0.525\right]\left[-(1.5)(0.687)(0.345)\right]$$

$$= -\,0.187$$

$$\sum FM_d = V_d\,h_d\,Q_{cd} + V_e\,h_e\,Q_{de} = (10)(6)(0.155) + (10)(2)(0.155)$$

$$= +12.4^{'k}$$

<u>Joint d'</u>

$$K^*_{d'c'} = K_{c'd'} - (1.5)(K_{c'd'})(Q_{c'd'}) = (1.530) - (1.5)(1.530)(0.345)$$

$$= 0.738$$

$$K^*_{d'd} = K_{d'd} = K_{dd'} = 0.850$$

$$K^*_{d'e'} = K_{d'e'} - (1.5)(K_{d'e'})(Q_{d'e'}) = (1.530) - (1.5)(1.530)(0.345)$$

$$= 0.738$$

$$\sum K^*_{d'} = K^*_{d'c'} + K^*_{d'd} + K^*_{d'e'} = 2.326$$

$$D^*_{d'c'} = \frac{K^*_{d'c'}}{\sum K^*_{d'}} = \frac{0.738}{2.326} = 0.317$$

$$D^*_{d'd} = \frac{K^*_{d'd}}{\sum K^*_{d'}} = \frac{0.850}{2.326} = 0.366$$

$$D^*_{d'e'} = \frac{K^*_{d'e'}}{\sum K^*_{d'}} = \frac{0.738}{2.326} = 0.317$$

$$r_{d'd} = \frac{1}{\sum K^*_{d'}} \left[(0.5)(K_{dd'}) - (1.5)(K_{c'd'})(Q_{cd}) - (1.5)(K_{d'e'})(Q_{de})\right]$$
$$= [0.430] \left[(0.5)(0.850) - (1.5)(1.530)(0.155) - (1.5)(1.530)(0.155)\right]$$
$$= -0.123$$

$$r_{d'e'} = \frac{1}{\sum K^*_{d'}} \left[(0.5)(K_{d'e'}) - (1.5)(K_{d'e'})(Q_{d'e'})\right]$$
$$= [0.430] \left[(0.5)(1.530) - (1.5)(1.530)(0.345)\right]$$
$$= -0.012$$

$$r_{d'c'} = \frac{1}{\sum K^*_{d'}} \left[(0.5)(K_{c'd'}) - (1.5)(K_{c'd'})(Q_{c'd'})\right]$$
$$= [0.430] \left[(0.5)(1.530) - (1.5)(1.530)(0.345)\right]$$
$$= -0.012$$

$$r_{d'e} = \frac{1}{\sum K^*_{d'}} \left[-(1.5)(K_{d'e'})(Q_{de})\right] = [0.430]\left[-(1.5)(1.530)(0.155)\right]$$
$$= -0.153$$

$$r_{d'c} = \frac{1}{\sum K^*_{d'}} \left[-(1.5)(K_{c'd'})(Q_{cd})\right] = [0.430]\left[-(1.5)(1.530)(0.155)\right]$$
$$= -0.153$$

$$\sum FM_{d'} = V_d h_d Q_{c'd'} + V_e h_e Q_{d'e'} = (10)(6)(0.345) + (10)(2)(0.345)$$
$$= +27.6^{'k}$$

<u>Joint e</u>

$$K^*_{ed} = K_{de} - (1.5)(K_{de})(Q_{de}) = (0.687) - (1.5)(0.687)(0.155) = 0.527$$

$$K^*_{ee'} = K_{ee'} = 0.850$$

$$\sum K^*_{e} = K^*_{ed} + K^*_{ee'} = 1.377$$

$$D^*_{ed} = \frac{K^*_{ed}}{\sum K^*_e} = \frac{0.527}{1.377} = 0.383$$

$$D^*_{ee'} = \frac{K^*_{ee'}}{\sum K^*_e} = \frac{0.850}{1.377} = 0.617$$

$$r_{ee'} = \frac{1}{\sum K^*_e}\left[(0.5)(K_{ee'}) - (1.5)(K_{de})(Q_{d'e'})\right]$$
$$= \left[0.727\right]\left[(0.5)(0.850) - (1.5)(0.687)(0.345)\right]$$
$$= +\ 0.050$$

$$r_{ed} = \frac{1}{\sum K^*_e}\left[(0.5)(K_{de}) - (1.5)(K_{de})(Q_{de})\right]$$
$$= \left[0.727\right]\left[(0.5)(0.687) - (1.5)(0.687)(0.155)\right]$$
$$= +\ 0.133$$

$$r_{ed'} = \frac{1}{\sum K^*_e}\left[-(1.5)(K_{de})(Q_{d'e'})\right] = \left[(0.727)\right]\left[-(1.5)(0.687)(0.345)\right]$$
$$= -\ 0.259$$

$$\sum FM_e = V_e\ h_e\ Q_{de} = (10)(2)(0.155)$$
$$= +\ 3.1'^{k}$$

Joint e'

$$K^*_{e'd'} = K_{d'e'} - (1.5)(K_{d'e'})(Q_{d'e'}) = (1.530) - (1.5)(1.530)(0.345)$$
$$= 0.738$$

$$K^*_{e'e} = K_{e'e} = K_{ee'} = 0.850$$

$$\sum K^*_{e'} = K^*_{e'd'} + K^*_{e'e} = 1.588$$

$$D^*_{e'd'} = \frac{K^*_{e'd'}}{\sum K^*_{e'}} = \frac{0.738}{1.588} = 0.465$$

$$D^*_{e'e} = \frac{K^*_{e'e}}{\sum K^*_{e'}} = \frac{0.850}{1.588} = 0.535$$

$$r_{e'e} = \frac{1}{\sum K^*_{e'}} \left[(0.5)(K_{e'e}) - (1.5)(K_{d'e'})(Q_{de})\right]$$

$$= \left[0.630\right]\left[(0.5)(0.850) - (1.5)(1.530)(0.155)\right]$$

$$= +0.043$$

$$r_{e'd'} = \frac{1}{\sum K^*_{e'}} \left[(0.5)(K_{d'e'}) - (1.5)(K_{d'e'})(Q_{d'e'})\right]$$

$$= \left[0.630\right]\left[(0.5)(1.530) - (1.5)(1.530)(0.345)\right]$$

$$= -0.017$$

$$r_{e'd} = \frac{1}{\sum K^*_{e'}} \left[-(1.5)(K_{d'e'})(Q_{de})\right] = \left[0.630\right]\left[-(1.5)(1.530)(0.155)\right]$$

$$= -0.224$$

$$\sum FM_{e'} = V_e \, h_e \, Q_{d'e'} = (10)(2)(0.345)$$

$$= +6.9^{'k}$$

3. <u>Carry-Over Moment Procedure</u>

A joint moment equation is written for each joint in the form of Eq. (9b). Column headings in TABLE II are for individual joints and the respective carry-over factors are shown in the top eight rows of the table. The carry-over procedure is started at joint b with the fixed end joint moment of $-(+40.0)$. This value is carried over to the other joints using the carry-over factors from joint b as shown. A double line is drawn under the value (-40.0) so that it may be summed when the distribution is completed to obtain the final joint moment at b. The fixed end joint moment and the carry-over moment at joint b' are then summed to obtain the value (-97.3). This is carried over to other joints and the procedure is continued until the carry-over moments approach zero, or it may be stopped at any time depending on the accuracy desired. The final joint moments are found by summing the fixed end joint moments and the carry-over moments. This may be facilitated by summing only the values that are immediately above each double line.

4. Numerical Check

A numerical check may be performed by substituting the final joint moments from **TABLE II** in the right side of Eq. (9b). The joint moment obtained must check with that in **TABLE II**.

$$JM_a = -r_{ba}JM_b - r_{b'a}JM_{b'} - \sum FM_a$$

$$= -(0.133)(-114.0) - (-0.211)(-159.1) - (23.3) \qquad = -41.8^{'k}$$

$$JM_{a'} = -r_{b'a'}JM_{b'} - r_{ba'}JM_b - \sum FM_{a'}$$

$$= -(0)(-159.1) - (-0.266)(-114.0) - (46.7) \qquad = -77.0^{'k}$$

$$JM_b = -r_{b'b}JM_{b'} - r_{cb}JM_c - r_{c'b}JM_{c'} - \sum FM_b$$

$$= -(-0.342)(-159.1) - (0.184)(-81.3) - (-0.294)(-117.3) - (40.0)$$

$$= -114.0^{'k}$$

$$JM_{b'} = -r_{bb'}JM_b - r_{cb'}JM_c - r_{c'b'}JM_{c'} - \sum FM_{b'}$$

$$= -(-0.431)(-114.0) - (-0.370)(-81.3) - (0)(-117.3) - (80.0)$$

$$= -159.3^{'k}$$

$$JM_c = -r_{bc}JM_b - r_{b'c}JM_{b'} - r_{c'c}JM_{c'} - r_{dc}JM_d - r_{d'c}JM_{d'} - \sum FM_c$$

$$= -(0.133)(-114.0) - (-0.211)(-159.1) - (-0.276)(-117.3)$$

$$-(0.096)(-25.5) - (-0.153)(-43.6) - (26.0) \qquad = -81.2^{'k}$$

TABLE 11

CARRY-OVER MOMENT SOLUTION

Joint	a	a'	b	b'	c	c'	d	d'	e	e'
Carry-Over Factors	-.133	+.266	(b)	+.431	-.133	+.266				
" "	+.211	0	+.342	(b')	+.211	0				
" "			-.184	+.370	(c)	+.347	-.060	+.116		
" "			+.294	0	+.276	(c')	+.092	+.007		
" "					-.096	+.187	(d)	+.151	-.096	+.187
" "					+.153	+.012	+.123	(d')	+.153	+.012
" "							-.133	+.259	(e)	-.050
" "							+.224	+.017	-.043	(e')
Fixed End Joint Mom.	- 23.3	- 46.7	- 40.0	- 80.0	- 26.0	- 54.0	- 12.4	- 27.6	- 3.1	- 6.9
	+ 5.2	- 10.6	- 40.0	- 17.3	+ 5.2	- 10.6				
	- 20.6	- 0	- 33.3	- 97.3	- 20.6	0				
			+ 7.6	- 15.3	- 41.4	- 14.3	+ 2.5	- 4.8		
			- 23.2	0	- 21.8	- 78.9	- 7.2	- 0.6		
					+ 1.6	- 3.2	- 17.1	- 2.6	+ 1.6	- 3.2
					- 5.4	- 0.4	- 4.4	- 35.6	- 5.4	- 0.4
	+ 6.5	- 13.0	- 48.9	- 21.1	+ 6.5	- 13.0	+ 0.9	- 1.8	- 6.9	+ 0.3
	- 7.7	- 0	- 12.5	- 36.4	- 7.7	0	- 2.3	- 0.2	+ 0.4	- 10.2
			+ 4.8	- 9.9	- 26.8	- 9.2	+ 1.6	- 3.1		
			- 7.6	0	- 7.1	- 25.8	- 2.4	- 0.2		
					+ 0.6	- 1.2	- 6.6	- 1.0	+ 0.6	- 1.2
					- 1.0	- 0.1	- 0.8	- 6.3	- 1.0	- 0.1
	+ 2.0	- 4.1	- 15.3	- 6.6	+ 2.0	- 4.1	0	0	0	0
	- 3.5	0	- 5.4	- 16.5	- 3.5	0	- 0.3	0	+ 0.1	- 1.3
			+ 1.7	- 3.3	- 9.0	- 3.1	+ 0.5	- 1.0		
			- 2.5	0	- 2.3	- 8.5	- 0.8	- 0.1		
					+ 0.1	- 0.3	- 1.4	- 0.2	+ 0.1	- 0.3
					- 0.2	0	- 0.2	- 1.3	- 0.2	0
	+ 0.8	- 1.6	- 6.2	- 2.6	+ 0.8	- 1.6	0	0	0	0
	- 1.2	0	- 2.0	- 5.9	- 1.2	0	- 0.1	0	0	- 0.3
			+ 0.5	- 1.0	- 2.8	- 1.0	+ 0.2	- 0.3		
			- 0.8	0	- 0.7	- 2.6	- 0.2	0		
					0	- 0.1	- 0.3	0	0	- 0.1
					0	0	0	- 0.3	0	0
	+ 0.3	- 0.6	- 2.3	- 1.0	+ 0.3	- 0.6	0	0	0	0
	- 0.4	0	- 0.7	- 2.0	- 0.4	0	0	0	0	- 0.1
			+ 0.2	- 0.3	- 0.8	- 0.3	0	- 0.1		
			- 0.3	0	- 0.3	- 1.0	- 0.1	0		
					0	0	- 0.1	0	0	0
					0	0	0	0.1	0	0
	+ 0.1	- 0.2	- 0.8	- 0.3	+ 0.1	- 0.2	0	0	0	0
	- 0.1	0	- 0.2	- 0.6	- 0.1	0	0	0	0	0
			0	- 0.1	- 0.2	- 0.1	0	0		
			- 0.1	0	- 0.1	- 0.3	0	0		
					0	0	0	0	0	0
					0	0	0	0	0	0
	0	- 0.1	- 0.3	- 0.1	0	- 0.1	0	0	0	0
	- 0.1	0	- 0.1	- 0.3	- 0.1	0	0	0	0	0
			0	- 0.1	- 0.2	- 0.1	0	0		
			- 0.1	0	- 0.1	- 0.2	0	0		
					0	0	0	0	0	0
			- 0.2	- 0.1	- 0.1	0	0	0	0	0
Final Joint Moment	- 42.0	- 76.9	- 114.0	- 159.1	- 81.3	- 117.3	- 25.5	- 43.6	- 6.9	- 11.8

$$JM_{c'} = -r_{bc'}JM_b - r_{b'c'}JM_{b'} - r_{cc'}JM_c - r_{dc'}JM_d - r_{d'c'}JM_{d'} - FM_{c'}$$

$$= -(-0.266)(-114.0) - (0)(-159.3) - (-0.347)(-81.3)$$

$$-(-0.187)(-25.5) - (-0.012)(-43.6) - (54.0) \qquad = -117.7^{'k}$$

$$JM_d = -r_{cd}JM_c - r_{c'd}JM_{c'} - r_{d'd}JM_{d'} - r_{ed}JM_e - r_{e'd}JM_{e'} - FM_d$$

$$= -(0.060)(-81.3) - (-0.092)(-117.3) - (-0.123)(-43.6)$$

$$-(0.133)(-6.9) - (-0.224)(-11.8) - (12.4) \qquad = -25.4^{'k}$$

$$JM_{d'} = -r_{cd'}JM_c - r_{c'd'}JM_{c'} - r_{dd'}JM_d - r_{ed'}JM_e - r_{e'd'}JM_{e'} - FM_{d'}$$

$$= -(-0.116)(-81.3) - (-0.007)(-117.3) - (-0.151)(-25.5)$$

$$-(-0.259)(-6.9) - (-0.017)(-11.8) - (27.6) \qquad = -43.7^{'k}$$

$$JM_e = -r_{de}JM_d - r_{d'e}JM_{d'} - r_{e'e}JM_{e'} - FM_e$$

$$= -(0.096)(-25.5) - (-0.153)(-43.6) - (0.043)(-11.8) - (3.1)$$

$$= -6.9^{'k}$$

$$JM_{e'} = -r_{de'}JM_d - r_{d'e'}JM_{d'} - r_{ee'}JM_e - FM_{e'}$$

$$= -(-0.187)(-25.5) - (-0.012)(-43.6) - (0.050)(-6.9) - (6.9)$$

$$= -11.9^{'k}$$

5. Final End Moments

The final end moments are found by substituting the final joint moments from TABLE II into Eq. (10c).

$$M_{ab} = D^*_{ab}JM_a + r_{ba}JM_b + r_{a'b}JM_{a'} + r_{b'a}JM_{b'} + V_b h_b Q_{ab}$$

$$= (0)(-42.0) + (0.133)(-114.0) + (0)(-76.9) + (-0.211)(-159.1)$$

$$+(14)(10)(0.167) \qquad = +41.7^{'k}$$

$$M_{a'b'} = D^{*}_{a'b'} + r_{b'a'}JM_{b'} + r_{ab'}JM_{a} + r_{ba'}JM_{b} + V_{b}h_{b}Q_{a'b'}$$

$$= (0)(-76.9) + (0)(-159.1) + (0)(-42.0) + (-0.266)(-114.0) + (14)(10)(0.333) = +77.1'^{k}$$

$$M_{ba} = D^{*}_{ba}JM_{b} + r_{ab}JM_{a} + r_{b'a}JM_{b'} + r_{a'b}JM_{a'} + V_{b}h_{b}Q_{ab}$$

$$= (0.400)(-114.0) + (0)(-42.0) + (-0.211)(-159.0) + (0)(-76.9) + (14)(10)(0.167) = +11.2'^{k}$$

$$M_{bb'} = D^{*}_{bb'}JM_{b} + \frac{D^{*}_{b'b}}{2}JM_{b'}$$

$$= (0.200)(-114.0) + (0.5)(0.160)(-159.1) = -35.5'^{k}$$

$$M_{bc} = D^{*}_{bc}JM_{b} + r_{cb}JM_{c} + r_{b'c}JM_{b'} + r_{c'b}JM_{c'} + V_{c}h_{c}Q_{bc}$$

$$= (0.400)(-114.0) + (0.184)(-81.3) + (-0.211)(-159.1) + (-0.294)(-117.3) + (10)(10)(0.167) = +24.4'^{k}$$

$$M_{b'a'} = D^{*}_{b'a'}JM_{b'} + r_{a'b'}JM_{a'} + r_{ba'}JM_{b} + r_{ab'}JM_{a} + V_{b}h_{b}Q_{a'b'}$$

$$= (0.420)(-159.1) + (0)(-76.9) + (-0.266)(-114.0) + (0)(-42.0) + ((14)(10)(0.333) = +10.2'^{k}$$

$$M_{b'b} = D^{*}_{b'b}JM_{b'} + \frac{D^{*}_{bb'}}{2}JM_{b}$$

$$= (0.160)(-159.1) + (0.50)(0.200)(-114.0) = -36.9'^{k}$$

$$M_{b'c'} = D^{*}_{b'c'}JM_{b'} + r_{c'b'}JM_{c'} + r_{bc'}JM_{b} + r_{cb'}JM_{c} + V_{c}h_{c}Q_{b'c'}$$

$$= (0.420)(-159.1) + (0)(-117.3) + (-0.266)(-114.0) + (-0.370)(-81.3) + (10)(10)(0.333) = +26.7'^{k}$$

$$M_{cb} = D^{*}_{cb}JM_{c} + r_{bc}JM_{b} + r_{c'b}JM_{c'} + r_{b'c}JM_{b'} + V_{c}h_{c}Q_{bc}$$

$$= (0.554)(-81.3) + (0.133)(-114.0) + (-0.294)(-117.3) + (-0.211)(-159.1) + (10)(10)(0.167) = +24.7'^{k}$$

$$M_{cc'} = D^{*}_{cc'}JM_{c} + \frac{D^{*}_{c'c}}{2}JM_{c'}$$

$$(0.275)(-81.3) + (0.50)(0.221)(-117.3) = -35.3'^{k}$$

$$M_{cd} = D^*_{cd}JM_c + r_{dc}JM_d + r_{c'd}JM_{c'} + r_{d'c}JM_{d'} + V_d h_d Q_{cd}$$

$$= (0.171)(-81.3) + (0.096)(-25.5) + (-0.092)(-117.3)$$

$$+ (-0.153)(-43.6) + (6)(10)(0.155) \qquad = +10.5^{'k}$$

$$M_{c'b'} = D^*_{c'b'}JM_{c'} + r_{b'c'}JM_{b'} + r_{cb'}JM_c + r_{bc'}JM_b + V_c h_c Q_{b'c'}$$

$$= (0.588)(-117.3) + (0)(-159.1) + (-0.370)(-81.3)$$

$$+ (-0.266)(-114.0) + (10)(10)(0.333) \qquad = +24.8^{'k}$$

$$M_{c'c} = D^*_{c'c}JM_{c'} + \frac{D^*_{cc'}}{2}JM_c$$

$$= (0.221)(-117.3) + (0.50)(0.275)(-81.3) \qquad = -37.2^{'k}$$

$$M_{c'd'} = D^*_{c'd'}JM_{c'} + r_{d'c'}JM_{d'} + r_{cd'}JM_c + r_{dc'}JM_d + V_d h_d Q_{c'd'}$$

$$= (0.191)(-117.3) + (-0.012)(-43.6) + (-0.116)(-81.3)$$

$$+ (-0.187)(-25.5) + (6)(10)(0.345) \qquad = +13.0^{'k}$$

$$M_{dc} = D^*_{dc}JM_d + r_{cd}JM_c + r_{d'c}JM_{d'} + r_{c'd}JM_{c'} + V_d h_d Q_{cd}$$

$$= (0.277)(-25.5) + (0.060)(-81.3) + (-0.153)(-43.6)$$

$$+ (-0.092)(-117.3) + (6)(10)(0.155) \qquad = +14.8^{'k}$$

$$M_{dd'} = D^*_{dd'}JM_d + \frac{D^*_{d'd}}{2}JM_{d'}$$

$$= (0.446)(-25.5) + (0.50)(0.366)(-43.6) \qquad = -19.4^{'k}$$

$$M_{de} = D^*_{de}JM_d + r_{ed}JM_e + r_{d'e}JM_{d'} + r_{e'd}JM_{e'} + V_e h_e Q_{de}$$

$$= (0.277)(-25.5) + (0.133)(-6.9) + (-0.153)(-43.6)$$

$$+ (-0.224)(-11.8) + (2)(10)(0.155) \qquad = +4.4^{'k}$$

$$M_{d'c'} = D^*_{d'c'}JM_{d'} + r_{c'd'}JM_{c'} + r_{dc'}JM_d + r_{cd'}JM_c + V_d h_d Q_{c'd'}$$

$$= (0.317)(-43.6) + (-0.007)(-117.3) + (-0.187)(-25.5)$$

$$+ (-0.116)(-81.3) + (6)(10)(0.345) \qquad = +21.9^{'k}$$

$$M_{d'd} = D^*_{d'd} JM_{d'} + \frac{D^*_{dd'}}{2} JM_d$$

$$= (0.366)(-43.6) + (0.50)(0.446)(-25.5) \qquad = -21.7'^k$$

$$M_{d'e'} = D^*_{d'e'} JM_{d'} + r_{e'd'} JM_{e'} + r_{de'} JM_d + r_{ed'} JM_e + V_e h_e Q_{d'e'}$$

$$= (0.317)(-43.6) + (-0.017)(-11.8) + (-0.187)(-25.5)$$

$$+ (-0.259)(-6.9) + (2)(10)(0.345) \qquad = -0.1'^k$$

$$M_{ed} = D^*_{ed} JM_e + r_{de} JM_d + r_{e'd} JM_{e'} + r_{d'e} JM_{d'} + V_e h_e Q_{de}$$

$$= (0.383)(-6.9) + (0.096)(-25.5) + (-0.224)(-11.8)$$

$$+ (-0.153)(-43.6) + (2)(10)(0.155) \qquad = +7.2'^k$$

$$M_{ee'} = D^*_{ee'} JM_e + \frac{D^*_{e'e}}{2} JM_{e'}$$

$$= (0.617)(-6.9) + (0.50)(0.535)(-11.8) \qquad = -7.3'^k$$

$$M_{e'd'} = D^*_{e'd'} JM_{e'} + r_{d'e'} JM_{d'} + r_{ed'} JM_e + r_{de'} JM_d + V_e h_e Q_{d'e'}$$

$$= (0.465)(-11.8) + (-0.012)(-43.6) + (-0.259)(-6.9)$$

$$+ (-0.187)(-25.5) + (2)(10)(0.345) \qquad = +8.5'^k$$

$$M_{e'e} = D^*_{e'e} JM_{e'} + \frac{D^*_{ee'}}{2} JM_e$$

$$= (0.535)(-11.8) + (0.50)(0.617)(-6.9) \qquad = -8.4'^k$$

CHAPTER VIII

SUMMARY AND CONCLUSIONS

In this investigation the joint moments due to sidesway are found directly rather than resorting to the solution of a number of simultaneous equations. This approach was used by Heller (4) for two column multistory bents with equal column stiffness.

Although the numerical example required the calculation of thirty-six 'r' values that would not be required in moment distribution, only one distribution table was required whereas in moment distribution a distribution table would be necessary for each story in the structure. Also the solution of a set of simultaneous equations would be required.

A count of the arithmetical operations performed in the numerical example (the example was also solved by moment distribution) resulted in the following comparison.

	Approximate number of operations	
Method	Multiply, Divide	Add, Subtract
Carry-Over Joint Moment	550	285
Moment Distribution	775	1,040

The solution of the simultaneous equations was included in the moment distribution total.

The use of approximate methods of analysis such as the portal method, cantilever method, K-percentage, etc., does not give results sufficiently accurate for design in this type of structure due to the unequal column stiffness.

The author is an instructor in a course on multistory steel buildings in the School of Architecture at Oklahoma State University and has had occasion to use the various types of analysis in common use for tall building frames. It is believed that the carry-over moment procedure as presented in this study has an excellent potential for use in the analysis of irregular multistory frames, and that with further investigation it may be adapted to multibay frames.

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Candidate for the Degree of

Master of Architectural Engineering

Thesis: ANALYSIS OF TWO COLUMN MULTISTORY BENTS WITH SIDESWAY INCLUDED BY THE CARRY-OVER MOMENT PROCEDURE

Major Field: Architectural Engineering

Biographical:

Personal Data: Born at Chandler, Oklahoma, December 19, 1932, the son of Audie A. and Bernice M. Orr.

Education: Attended various elementary schools in Texas and Oklahoma from 1938 to 1945. Graduated from Central High School at Muskogee, Oklahoma in June, 1949. Attended Muskogee Junior College from 1949 to 1951. Attended the Oklahoma State University from September, 1951, to June, 1954. Completed the requirements for and received the degree Bachelor of Architectural Engineering. Attended the Oklahoma State University from September, 1957, to June, 1958. Completed the requirements for the degree Master of Architectural Engi~~neering in May, 1960.~~

Professional Experience: Employed as structural engineer for Roof Structures, Inc., Webster Groves, Missouri, during the summer of 1954. Served in various engineering and administrative capacities as Commissioned Officer in the Civil Engineer Corps, U.S. Navy, from September, 1954, to September, 1957. Employed as engineer and area representative in Dallas, Texas, for Roof Structures, Inc., Webster Groves, Missouri, from June, 1958, to August, 1959. Employed as Assistant Professor of Architecture at the Oklahoma State University from September, 1959, to June, 1960.