INTERNAL COMBUSTION ENGINE ANALYSIS USING AN ANALOG COMPUTER

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INTERNAL COMBUSTION ENGINE ANALYSIS USING

AN ANALOG COMPUTER

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PREFACE

It is common practice to determine the indicated work or horsepower of an internal combustion engine by measuring the net area of an indicator card diagram. This is essentially a plot of Pressure vs. Piston Position. This particular method requires some manual means of measuring the area and usually a planimeter is used. Using such a procedure, some time elapses between the engine cycle during which the indicator card is taken and the measurement of the net area.

A means of measuring the indicated work which could be made automatic and also which could average this indicated work over many engine cycles would be valuable. The electronic analog computer was selected for investigation of the possibility of using such a device for determining the indicated work.

The major part of this investigation was devoted to arriving at an equation for the indicated work and which could be reduced to a form suitable for application to the computer. Considerable time was spent in setting up, in theory and in practice, an electronic analog computer system that would solve the desired problem. This required the design and construction of an arbitrary function generator. To maintain as simple a construction job as possible, a manually operated function generator was constructed. It was realized that an improved function generator would be ultimately required in a completed system used in the determination of indicated work in a real and practical case. The manually operated function generator dictates that the Pressure vs. Crank Angle or Time curve, which is introduced into the computer by means of the function generator, be one that can be followed manually with reasonable accuracy. For purposes of this study this will be quite satisfactory since such a curve can be used to check the validity of the equation and the computer system.

The results of the investigation were within the accuracy expected and it is quite clearly demonstrated that the electronic analog computer can be used on a problem of this type to advantage.

I wish to acknowledge indebtedness to Professor P. A. McCollum for his helpful suggestions, the procurement of essential equipment items and for his great patience; and to Messrs. Wilson and Harris of the RAD Laboratory for their excellent machine work on the arbitrary function generator.

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CHAPTER I

INTRODUCTION

The Problem

The determination of indicated work or horsepower of an internal combustion engine is generally accomplished by measuring the net area included in a conventional indicator card diagram. This area is usually measured by use of a planimeter and thus information on indicated work or horsepower is not immediately available as a test is being run on an engine. It would be of considerable benefit if this indicated work could be computed by some automatic means and could be available while the engine is being tested. The indicated work obtained over any one cycle of the engine is not necessarily the same as would be obtained over some other cycle, and it would be advantageous if indicated work could be averaged over many cycles to obtain a more accurate measure of the indicated work.

It is the purpose of this study to investigate the use of an electronic analog computer for computation of the indicated work done by an internal combustion engine. This investigation is limited to the following considerations:

(a) The determination of an equation or equations for the indicated work in such a form that an electronic analog computer may be used in their solution.

(b) The development of such auxiliary equipment as may be necessary for use with the computer. This equipment will be limited to the minimum needed to test the validity of the equations and computer system.

(c) The design of a computer arrangement that will solve the problem in question including the proper scaling of the computer components.

In order to test the validity and the accuracy of the method used to determine indicated work, the engine constants and the Pressure versus Crank Angle will be hypothetical rather than values obtained from an actual engine. This will be satisfactory for testing the equations and the method used and will simplify the design and building of the auxiliary equipment required for use with the basic elements of the computer.

It will not be the object of this study to develop an automatic means for computing the indicated work done by an engine, but merely to investigate the feasibility of applying an electronic analog computer to this type of problem.

General Theory

The equivalence between the differential equations describing a mechanical system to those describing an electrical system may be easily demonstrated (see Appendix A). As a result of the equivalence which exists between the two sets of equations, it is possible to establish analogies between electrical and mechanical quantities. One system of analogies is to let mass in the mechanical system be analogous to inductance in the electrical system; force analogus to voltage; velocity analogous to current; viscous friction analogous to resistance; etc.

Therefore, in principle, it would be possible to assemble a passive electrical network which would represent an analogous mechanical system and which would allow the determination of the action of the mechanical system under various conditions. Such an arrangement of electrical components would constitute an analog computer and could perhaps be of value due to the ease of changing the parameters of the electrical circuit as compared to the mechanical system.

A particular type of analog computer is the electronic analog computer. In this computer, the equivalence between the differential equation describing electrical and mechanical systems is not utilized. The fundamental component of the electronic analog computer is the high gain amplifier (gain greater than 10,000) and the following is an attempt to demonstrate how the electronic computer can be used in the solution of differential equations.

The high gain amplifier with a pure resistive feedback and input will be considered first.

Referring to Figure 1 where

K represents the gain of the amplifier,

R_1 represents the input resistance,

R_f represents the feedback resistance,

e_i represents the input voltage,

e_g represents the voltage on the grid of the amplifier,

e_o represents the output voltage,

and with currents i_1, i_2, and i_3 as indicated in Figure 1, it is possible to write voltage and current equations describing this circuit.

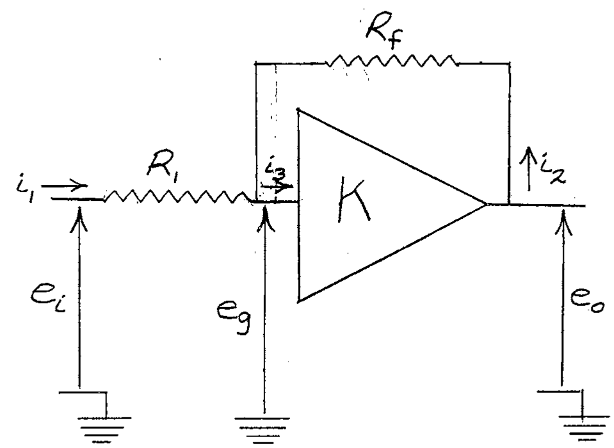

Figure 1. A High Gain Amplifier With Pure Resistive Input and Feedback Impedances.

The current equation may be written as

$$i_1 + i_2 = i_3 \tag{1}$$

and the voltage equation is

$$e_i - e_o = i_1 R_1 - i_2 R_f . \tag{2}$$

If the gain, K, of the amplifier is sufficiently high, 10,000 or more, then e_g may be assumed to be zero. In this case one may write that

$$R_1 = \frac{e_i}{i_1} \tag{3}$$

and

$$R_f = \frac{e_o}{i_2} . \tag{4}$$

In addition it may be assumed that the grid current, i_3, of the amplifier is so small so that it may be neglected. Thus

$$i_1 = -i_2 \tag{5}$$

and recalling that

$$R_1 = \frac{e_i}{i_1}, \tag{3}$$

one may substitute into equation (4) to obtain

$$R_f = \frac{e_o}{-i_1}. \tag{6}$$

Then dividing (6) by (3) obtain

$$\frac{e_o}{e_i} = -\frac{R_f}{R_1} \tag{7}$$

or

$$e_o = -\frac{R_f}{R_1} e_i. \tag{8}$$

Thus, since R_f and R_1 are fixed, their ratio becomes a constant and e_o is equal to a constant times the input voltage, e_i. It should be noted that the output voltage is the negative of the input voltage.

It is possible to arrange the high gain amplifier with a pure resistance for the feedback impedance and to have more than one input resistance. In this case it is possible to show that

$$e_o = -\left(\frac{R_f}{R_1} e_1 + \frac{R_f}{R_2} e_2 + \frac{R_f}{R_3} e_3 + --- + \frac{R_f}{R_m} e_m\right) \tag{9}$$

and from this one can see that the output voltage, e_o, is the negative of the sum of input voltages, e_1, e_2, etc., times a constant factor. This constant factor may be different for each input voltage since it is the ratio of feedback resistance to input resistance and the input resistance may be different for each input voltage.

Therefore, it may be seen that the electronic analog computer can be used to add several voltages and if the voltages and multiplying factors are properly selected, the input and output voltages may represent other quantities in a physical system.

Now, consider the case where the feedback impedance is a capacitor rather than a pure resistance. The input impedance will be a pure resistance as before. Such an arrangement is shown in Figure 2.

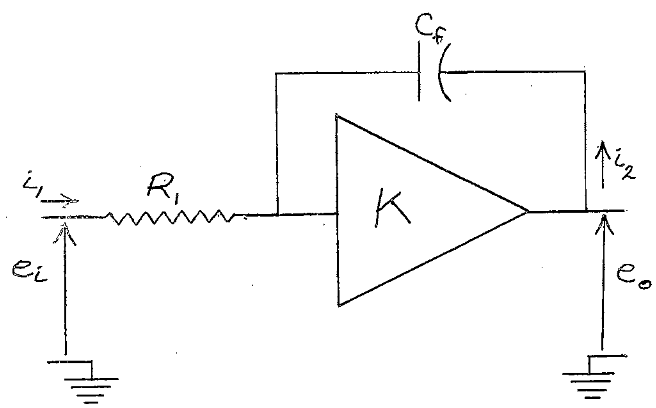

Figure 2. A High Gain Amplifier With a Pure Capacitive Feedback and a Pure Resistive Input Impedance.

Writing the current and voltage equations as before one obtains

$$i_1 = -i_2 , \tag{10}$$

$$e_i = i_1 R_1 , \tag{11}$$

$$e_o = \frac{1}{C_f}\int i_2 \, dt, \tag{12}$$

or

$$e_o = -\frac{1}{C_f}\int i_1 \, dt. \tag{13}$$

Substituting for i_1 from equation (11) it is seen that equation (13) becomes

$$e_o = -\frac{1}{R_1 C_f}\int e_i \, dt. \tag{14}$$

This equation indicates that the output voltage, e_o, is the negative of the integral of the input voltage multiplied by a constant, $1/R_1C_f$. These values are fixed for any particular problem or situation.

It can be shown that with a number of inputs, the output voltage is the negative of the sum of the integrals of the input voltages multiplied by constant factors which, in general, will be different for each input voltage.

This theory can be extended to include a capacitor in the input and a pure resistance for the feedback impedance. In such an arrangement the output voltage would be the negative of the derivative of the input voltage times a constant. This type of circuit is seldom used due to the noise which it is apt to generate. (1). In addition various configurations of capacitors and resistors may be used in the input and feedback paths to produce the output voltage input voltage ratio as various transfer functions. In this study only the adding and integrating features of the electronic analog computer will be used.

Since a high gain amplifier with the proper feedback and input impedance arrangement can be made to add and to integrate, then it should be useful in the solution of differential equations. As an example, consider the equation

$$A\frac{d^2x}{dt^2} + B\frac{dx}{dt} + C = f(t). \tag{15}$$

As a first step in representing this equation on an electronic analog computer, solve for $\frac{d^2x}{dt^2}$ to obtain

$$\frac{d^2x}{dt^2} = \frac{f(t)}{A} - \frac{B}{A}\frac{dx}{dt} - \frac{C}{A}. \tag{16}$$

Equation (16) indicates that $\frac{d^2x}{dt^2}$ may be obtained by the proper combination of the terms on the right hand side of the equality sign. Figure 3 indicates an amplifier arrangement to solve for $\frac{d^2x}{dt^2}$.

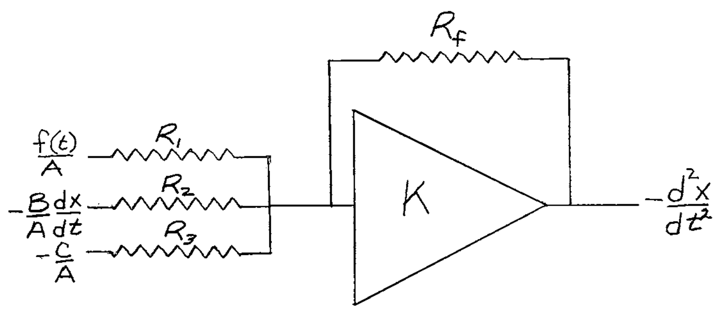

Figure 3. An Electronic Analog Computer Arrangement to Represent Equation (16).

It should be remembered that the quantities measured on an electronic computer are voltages and thus the input voltages and the output voltage shown in Figure 3 would be related to the corresponding quantity in equation (16) by some scale factor which is not shown in Figure 3 for the sake of simplicity.

One sees that the first derivative, $\frac{dx}{dt}$, can be obtained by integrating $\frac{d^2x}{dt^2}$. Further it is seen that $\frac{C}{A}$ is a constant and if $\frac{f(t)}{A}$ were obtainable from some other source, then the necessary inputs to the amplifier would be available for solving for $\frac{d^2x}{dt^2}$.

Figure 4 indicates an electronic analog computer arrangement which may be used to represent equation (16).

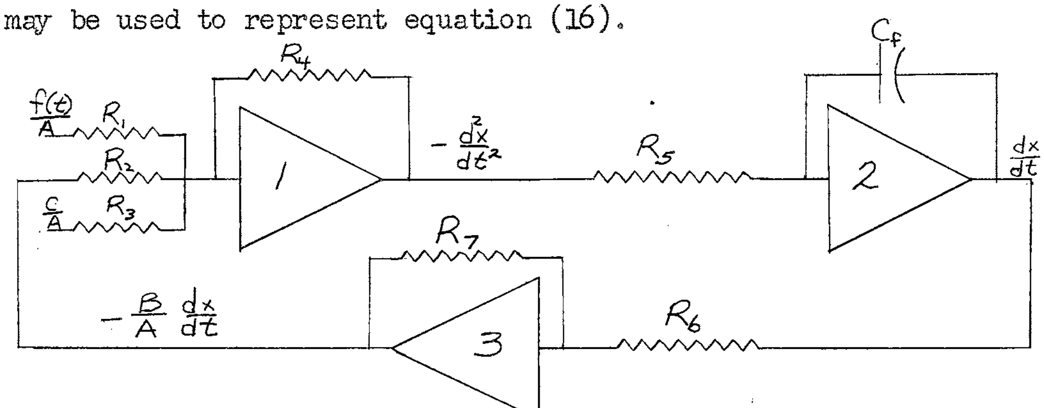

Figure 4. A Complete Electronic Analog Computer Arrangement to Represent Equation (16).

If $\frac{d^2x}{dt^2}$ is not desired in the solution, then it would be possible to combine the operations of amplifiers 1 and 2 into a single amplifier. That is to say that the summing and integration would be performed on one amplifier. This example is intended to illustrate how the electronic analog computer may be utilized in connection with the solution of differential equations. It is not intended that it should demonstrate anything about the very important phase of scaling the computer. The scaling of the computer entails the selection of the input and feedback impedances so that there is a known relationship between the quantities in the equation describing the physical system and the voltages which appear in the computer. The subject of scaling the computer will be dealt with in more detail in a subsequent chapter.

It should also be mentioned that the electronic analog computer may be used in the solution of equations of order higher than 2 and can be utilized in the solution of simultaneous equations. A very simple example has been included here simply for the purpose of illustration.

CHAPTER II

STATEMENT OF THE PROBLEM

This study is concerned with using an electronic analog computer to determine the indicated work done by an internal combustion engine based upon pressure in the combustion chamber as a function of crank angle position. The pressure versus crank angle position function may be obtained by means of an electric resistance strain gauge or other suitable pressure measuring device.

A four stroke per cycle engine will be considered for purposes of this study and a brief summary of the operation is given here for review purposes. The operation is as follows: (2).

(a) Gas is compressed as piston moves from BDC (bottom dead center) to TDC (top dead center).

(b) After the ignition spark (about TDC) the gas detonates, pressure in the combustion chamber increases, and a force is exerted on the piston returning it to BDC.

(c) On the return from BDC to TDC the burned gases are forced from the combustion chamber.

(d) A fuel and air mixture is drawn into the combustion chamber as the piston moves from TDC to BDC.

It can be seen from this that work is done by the expanding gases on the stroke caused by the detonation. On strokes described in (a) and (c) above, work is done by the engine to compress the gas and to remove the exhaust gases respectively. This then is work done by the engine and is to be subtracted from that developed by the expanding gases during the power stroke. The work required to charge the cylinder should not be subtracted from that developed during the power stroke. (2).

If a plot of pressure versus piston position was available, then the indicated work during the various portions of the cycle could be obtained. Work is defined as

$$\text{Work} = \text{Force} \times \text{Distance (in appropriate units)} \tag{17}$$

The force will be equal to the pressure per unit area multiplied by the area of the piston. The distance moved will be a function of the piston position. For a force which varies with distance or piston position, as in the case of an internal combustion engine, it will be necessary to perform an integration in order to obtain work. This equation will be

$$W = \int_0^l F\,dx \tag{18}$$

where W = work, ft. lbs.,

F = force on piston, lbs.,

x = piston position from BDC, ft.,

l = length of stroke, ft.

Figure 5 will be used in the development of an expression for piston position in terms of crank angle.

Let θ = crank angle at any time, radians,

r = radius of crankshaft, ft.,

l = length of connecting rod, ft., and

x = piston position from BDC, ft.

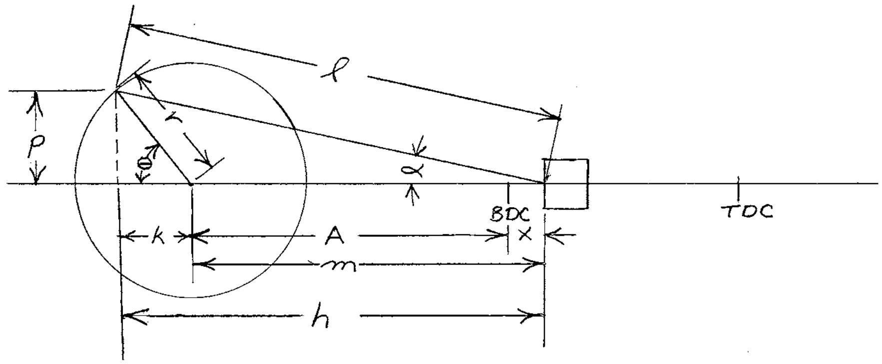

Figure 5. A Diagram Showing the Relationship Between Piston Position and Crank Angle.

Referring to Figure 5 one sees that

$$p = r \sin \theta \tag{19}$$

and also that

$$p = \ell \sin \alpha . \tag{20}$$

Thus

$$r \sin \theta = \ell \sin \alpha \tag{21}$$

and

$$\sin \alpha = \frac{r}{\ell} \sin \theta . \tag{22}$$

In addition one can write

$$h = \ell \cos \alpha , \tag{23}$$

$$k = r \cos \theta , \tag{24}$$

and

$$m = h - k . \tag{25}$$

Substituting equations (23) and (24), equation (25) becomes

$$m = l\cos\alpha - r\cos\theta . \tag{26}$$

Now

$$x = m - l + r \tag{27}$$

and so

$$x = l\cos\alpha - r\cos\theta - l + r, \tag{28}$$

or rearranging

$$x = l(\cos\alpha - 1) + r(1 - \cos\theta). \tag{29}$$

Now, from trigonometric idenities, it is known that

$$\cos\alpha = \sqrt{1 - \sin^2\alpha} . \tag{30}$$

Substituting equation (22) into equation (30) one obtains

$$\cos\alpha = \sqrt{1 - \frac{r^2}{l^2}\sin^2\theta} , \tag{31}$$

and utilizing this equation one can write equation (29) in terms of crank angle θ as

$$x = l\left[\left(1 - \frac{r^2}{l^2}\sin^2\theta\right)^{\frac{1}{2}} - 1\right] + r(1 - \cos\theta). \tag{32}$$

This equation for piston position, x, can be simplified if the term $\left(1 - \frac{r^2}{l^2}\sin^2\theta\right)^{\frac{1}{2}}$ is expanded by means of the binomial theorem to become

$$\left(1 - \frac{r^2}{l^2}\sin^2\theta\right)^{\frac{1}{2}} = 1 - \frac{r^2}{2l^2}\sin^2\theta - \frac{r^4}{8l^4}\sin^4\theta + --- . \tag{33}$$

Since r/l is a fraction and will become smaller when raised to the fourth power, then the last term in equation (33) will be small and in fact will be small enough so that it may be neglected. (2). Based upon this, the expression for piston position becomes

$$x = \ell\left(1 - \frac{r^2}{2\ell^2}\sin^2\theta - 1\right) + r(1 - \cos\theta), \tag{34}$$

$$x = \ell\left(-\frac{r^2}{2\ell^2}\sin^2\theta\right) + r(1 - \cos\theta). \tag{35}$$

This equation for piston position, x, can be differentiated with respect to crank angle, θ, to obtain dx. If, however, an electronic analog computer is to be used to perform the integration then time, t, will be the desirable independent variable of the problem. Thus it will be necessary to examine equation (35) to determine if piston position can be written as a function of time.

Since crank angle, θ, can be expressed as

$$\theta = 2\pi s t \tag{36}$$

where θ = crank angle in radians,

s = speed in revolutions per second,

and t = time in seconds,

then equation (35) for piston position may be written in terms of time as

$$x = \ell\left(-\frac{r^2}{2\ell^2}\sin^2 2\pi s t\right) + r(1 - \cos 2\pi s t). \tag{37}$$

Differentiating equation (37) with respect to time obtain

$$\frac{dx}{dt} = -\frac{2\pi r^2 s}{\ell}\cos 2\pi s t \sin 2\pi s t + 2\pi r s \sin 2\pi s t \tag{38}$$

or

$$dx = \left(2\pi r s \sin 2\pi s t - \frac{\pi r^2 s}{\ell}\sin 4\pi s t\right)dt. \tag{39}$$

Substituting this expression for dx into equation (18) the equation for indicated work becomes

$$W = \int_0^t F\left(2\pi rs \sin 2\pi st - \frac{\pi r^2 s}{\ell} \sin 4\pi st\right) dt \tag{40}$$

or

$$W = 2\pi rs \int_0^t F \sin 2\pi st\, dt - \frac{\pi r^2 s}{\ell} \int_0^t F \sin 4\pi st\, dt. \tag{41}$$

Equation (41) is in a form which could be used with an electronic analog computer. Figure 6 indicates an arrangement of a computer to represent equation (41).

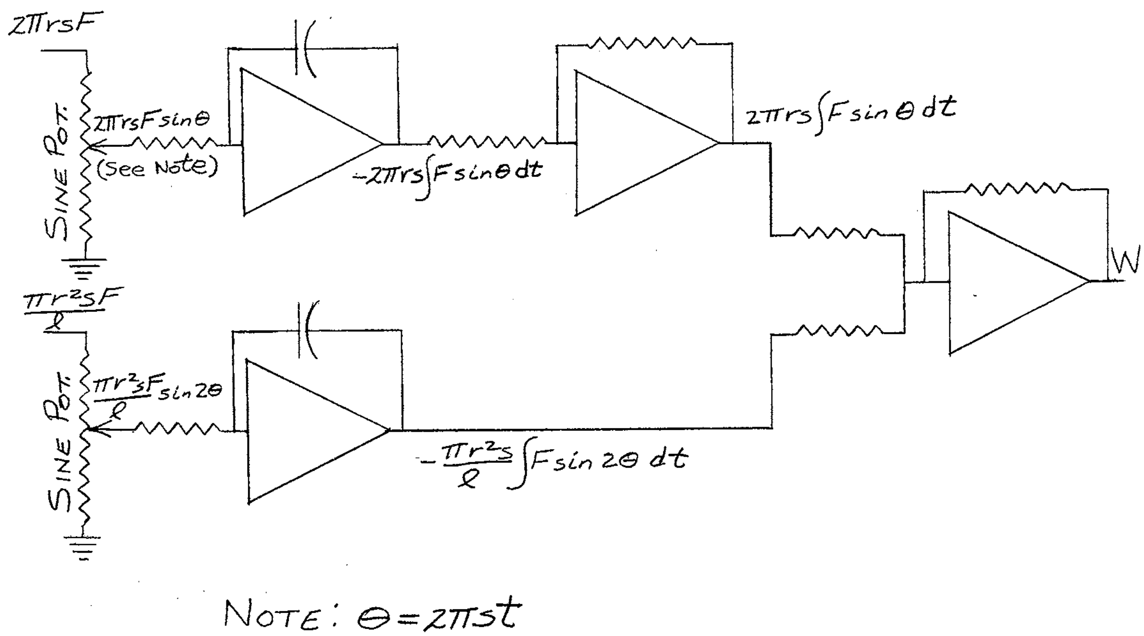

Figure 6. An Electronic Analog Computer Arrangement to Represent Equation (41).

The remainder of this study will be concerned with the details of this basic arrangement in an attempt to make it possible to measure the indicated work of the engine.

CHAPTER III

THE ELECTRONIC ANALOG COMPUTER ARRANGEMENT

General

In the preceeding chapters, the fundamentals of electronic analog computers have been explored and a problem has been set forth with a first approximation of how an electronic analog computer may be used in solving this problem. It is the purpose of this chapter to discuss in detail the computer arrangement which has been developed in connection with the problem at hand.

The basic electronic analog computer available for use on this problem was a Donner Model 30 computer. This computer has ten operational amplifiers with a selection of plug in resistors and capacitors for use as the input and feedback impedances. There were ten Philbrick amplifiers which were also available for this study.

Arbitrary Function Generator

In many applications of the electronic analog computer the driving function in the problem under consideration is a function which may be represented conveniently by conventional voltage generators. The square wave and sine wave generators are two examples of this type.

On the other hand there are those problems where the driving function is some arbitrary function of the independent variable. In cases of this kind it becomes necessary to devise a means to faithfully reproduce the driving function as a voltage so that this voltage may be introduced into the computer. In the problem under consideration, pressure is an arbitrary function of crank angle so it is necessary to develop a means to generate a voltage which will represent this pressure versus crank angle.

There are a number of ways that the arbitrary function may be generated and a number of function generators are available commercially. (1). For reasons of economy it was decided that a function generator would be developed as a part of this study utilizing items of material presently available.

In considering the arbitrary function generator to be developed, it was noted that the equations to be represented on the computer contain $\sin\theta$ and $\sin 2\theta$ functions, where θ is the crank angle and is the independent variable. It, therefore, seemed reasonable to consider a function generator which included rotation so that this rotation would represent the independent variable θ and could possibly be used in the generation of the $\sin\theta$ and $\sin 2\theta$ functions.

Suppose that a drum or cylinder be constructed and mounted in such a manner that it is free to rotate. Now, if a plot of pressure versus crank angle is placed on the circumference of the drum so that the pressure axis on the chart is along or parallel to the axis of the cylinder and so that the crank angle axis of the chart is around the circumference of the cylinder, then it would be possible for a follower to follow the pressure curve as the drum rotates. Figure 7 depicts this type of arrangement.

As the drum rotates, a follower which is free to move along a line that is parallel to the axis of the cylinder could follow the pressure variations with crank angle. If a slider on a rheostat is caused to move with the follower, then the voltage which is available at the slider of

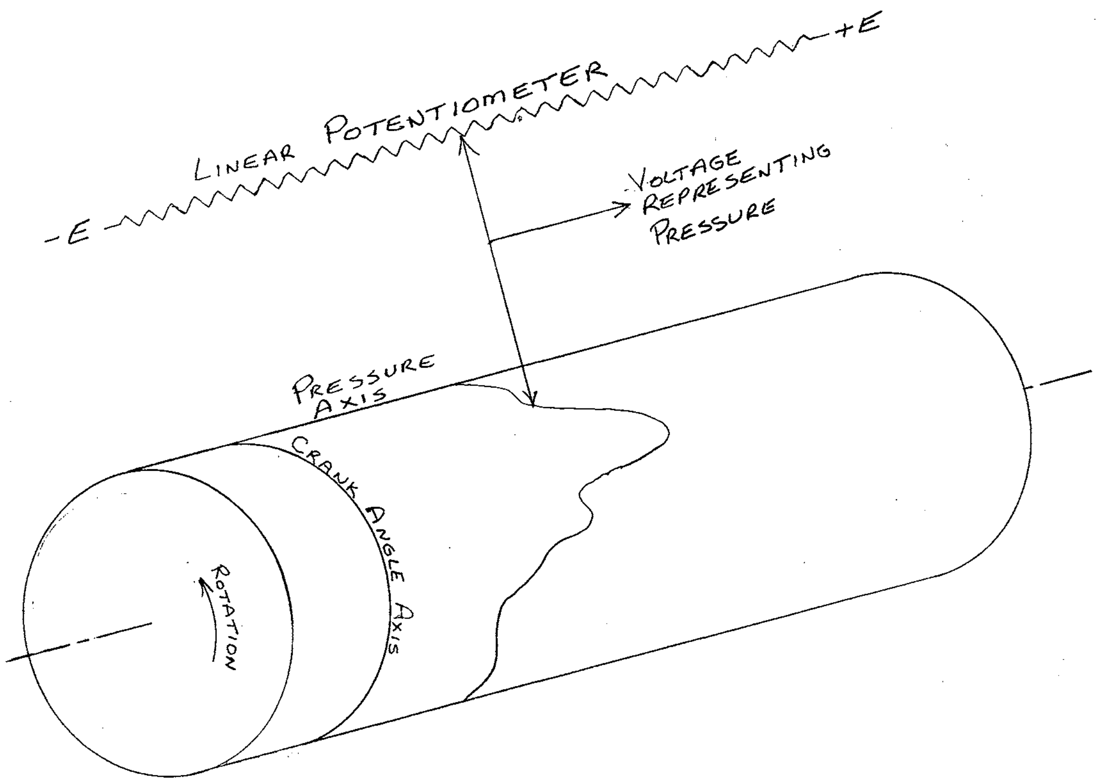

Figure 7. A Diagram of a Possibly Arbitrary Function Generator

the rheostat will vary with the rotation of the drum in the same way that pressure varies with crank angle. This voltage could then be applied to the electronic analog computer as the driving voltage.

It would be possible to design the follower system so that it would follow the pressure curve automatically. However, in this particular case the follower is to be driven manually and this will be a determining factor in the speed at which the drum is to rotate. The drum must be driven at a speed consistent with a person's ability to guide the follower along the pressure curve. Another factor which must receive consideration in selecting a drum speed is the frequency response of the recorder which will be used to measure the output of the computer. It will be necessary to select a drum speed which will allow the recorder to respond to the computer output accurately. The recorder available for use is a Rectilinear Recording Milliammeter manufactured by Texas Instruments, Inc. The trade name for this recorder is recti/riter and it will be referred to by this name in subsequent discussions. This recorder has a maximum frequency response of about 0.2 cps and thus it is necessary to complete the cycle in $1/0.2 = 5$ seconds. (3). This corresponds to 12 revolutions per minute or 0.5 revolutions per second. This speed would be considerably higher than that which would allow a person to guide the follower along a pressure curve so that the controlling consideration in the drum speed problem will be the speed at which a person can follow the pressure curve.

Before further consideration is given to the drum speed, the motor to drive the drum should be selected. Since it will be necessary for the drum to rotate at a constant speed then it will be necessary to select a motor which has essentially a constant speed. For this reason a synchronous motor was chosen to drive the drum and the rest of the apparatus connected with the function generator. The motor speed is 1200 revolutions per minute. A 668 to 1 gear reduction unit was made available for use on this problem. With this unit the drum speed would be 2.7 revolutions per minute or 0.0449 revolutions per second. This corresponds to 22.26 seconds per revolution and at this speed it will be feasible to follow the pressure curve manually although a further reduction in speed would be desirable. After some work with this apparatus it appears that a drum speed of less than 1 revolution per minute would be suitable for accurate curve following, but for purposes of this study it was decided that the 2.7 revolutions per minute speed would not detract from the accuracy to an extent that would warrant the speed reduction at this time.

An attempt was made to mount a wire wound resistance parallel to the axis of the drum and to use a follower sliding on the resistance to obtain a voltage that varies with drum rotation in the same manner that pressure varies with crank angle. It soon became apparent that a considerable amount of work would be required to make this method perform in a satisfactory way because of the variation in contact resistance as the slider moves along the resistance. To overcome this difficulty a screw thread and gear arrangement with a standard potentiometer (20,000 ohms) was developed and constructed. In this arrangement the curve follower is moved by means of turning the screw thread and this rotation of the screw thread causes the arm of the potentiometer to move.

The arrangement described proved satisfactory for developing a voltage to represent the pressure versus crank angle curve. Figure 8 is a picture of the completed function generator.

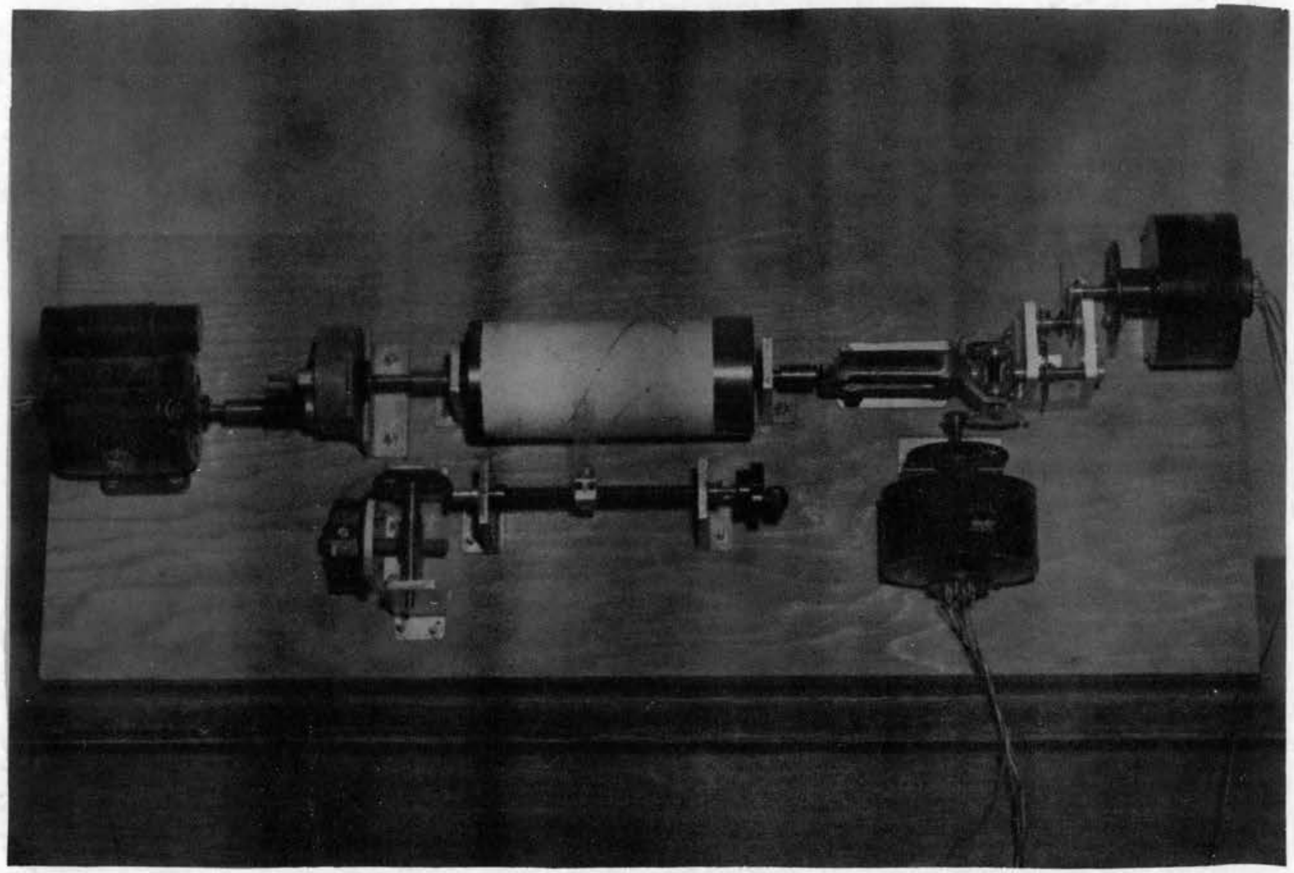

Figure 8. The Completed Function Generator.

The Sine Potentiometer

Referring to equation (41) in Chapter II, it is seen that the equation to be solved by means of the electronic analog computer contains the sine of an angle and the sine of twice the angle. This means that sine θ and sine 2θ functions must be available. There are a number of choices available for the generation of the sine function. One method that presents itself is to use a synchro generator and a demodulator. The output of the generator will vary sinusoidally as the shaft of the generator is rotated and a sine wave can be obtained if the output is demodulated. This type of sine wave function generator is termed the a-c type since a-c energy is supplied to the generator.

One of the most straightforward methods is to use a potentiometer so constructed that the voltage at the brushes of the potentiometer is the voltage applied to the potentiometer multiplied by the sine of the angle of potentiometer shaft rotation. Such a function generator is a d-c type. The fact that the output voltage of such a potentiometer is a sine function of the angle of shaft rotation is due to the way in which the potentiometers are wound and the manner in which the brushes are positioned on the resistance element. The type selected for this study is of the rectangular card type. Two of this type are used, and they are mechanically connected so that one of them makes two revolutions and the other one makes four revolutions for each revolution of the drum. This is necessary because one revolution of the drum corresponds to two revolutions of the crank shaft of a four stroke per cycle engine.

In order that the sine potentiometers of the rectangular card type may be applied properly, taking into account the possible errors, it is necessary to consider how a voltage at the brush of the potentiometer is equal to the voltage applied to the winding multiplied by the sine of the angle of rotation of the shaft. For purposes of analysis consider Figure 9.

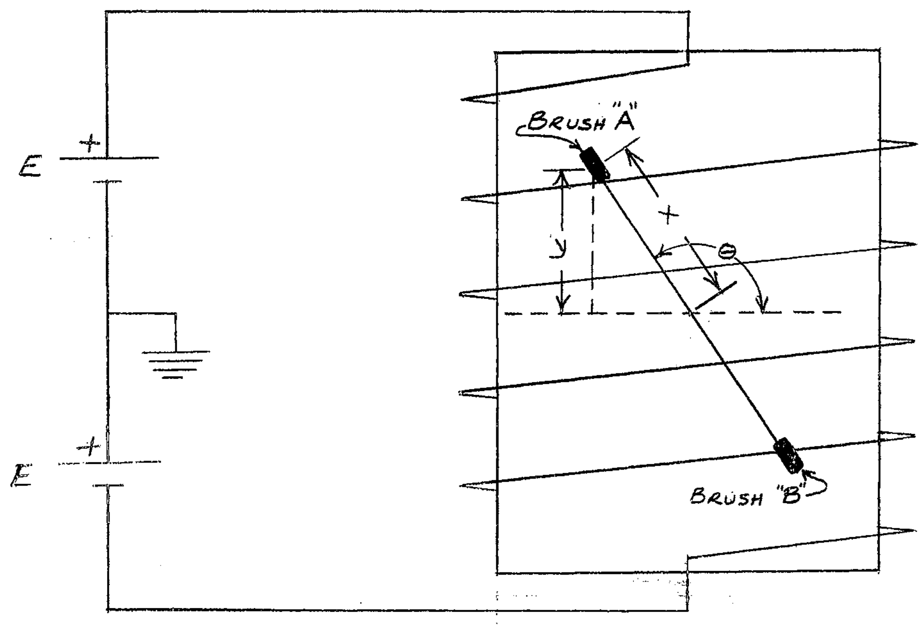

Figure 9. A Rectangular Card Sine Potentiometer

In Figure 9 let

E = voltage applied to the winding, volts

θ = angle of shaft rotation, radians,

n = number of turns between brushes "A" and "B",

T = resistance of one turn of the winding, ohms,

R = total resistance of the winding on the card, ohms,

r = resistance between brushes "A" and "B", ohms,

and I = current in the winding, amperes.

Let it be assumed that brushes "A" and "B" are diametrically opposite each other and also that the center of their rotation is the electrical center of the card.

One may write that

$$I = \frac{2E}{R} \tag{42}$$

and also that the voltage between brushes "A" and "B" is

$$V_{AB} = Ir = \frac{2E}{R}r. \tag{43}$$

In order to obtain an expression for V_{AB} as a function of the sine of the angle of shaft rotation, it will be helpful to develop an expression for r in terms of R and shaft rotation.

It is seen that

$$r = n\,T \tag{44}$$

and also that brushes "A" and "B" describe a circle as the shaft is rotated.

If N is the maximum number of turns included between the brushes, then the number of turns between the center and bottom end of the winding and also between the center and top end of the winding becomes N/2 since it has been assumed that the center of rotation is at the electrical center.

Referring to Figure 8 and noting that y is the perpendicular distance of brush "A" from the centerline and that x is the distance (measured along a radius) from the center to brush "A". It may be written that

$$y = x \sin \theta \tag{45}$$

and if m is the turns per unit length on the card, then one may also write that

$$ym = \frac{n}{2} \tag{46}$$

and thus

$$ym = xm \sin \theta ; \tag{47}$$

also

$$x_m = \frac{N}{2}, \tag{48}$$

therefore

$$y_m = \frac{N}{2} \sin \theta \tag{49}$$

and

$$\frac{n}{2} = \frac{N}{2} \sin \theta \tag{50}$$

or

$$n = N \sin \theta . \tag{51}$$

Referring back to equation (44)

$$r = nT = NT \sin \theta, \tag{52}$$

but

$$NT = R, \tag{53}$$

thus

$$r = R \sin \theta \tag{54}$$

and

$$V_{AB} = \frac{2E}{R} r = \frac{2E}{R} R \sin \theta \tag{55}$$

or

$$V_{AB} = 2E \sin \theta . \tag{56}$$

The voltage may be taken from brush "A" to ground and it will then be $E \sin \theta$ or from brush "B" to ground and it will be $- E \sin \theta$.

The preceeding development was based on the assumption that the center of rotation of the brushes is in the electrical center of the card. If this is not the case, then when the angle θ is zero degrees the voltage from brush "A" or brush "B" will not be zero. This effect is the same as a sine wave which is displaced above or below the zero axis.

The sine potentiometers used in connection with solving the problem under consideration in this study have their center of brush rotation removed from the electrical center. This may be seen in Figure 10.

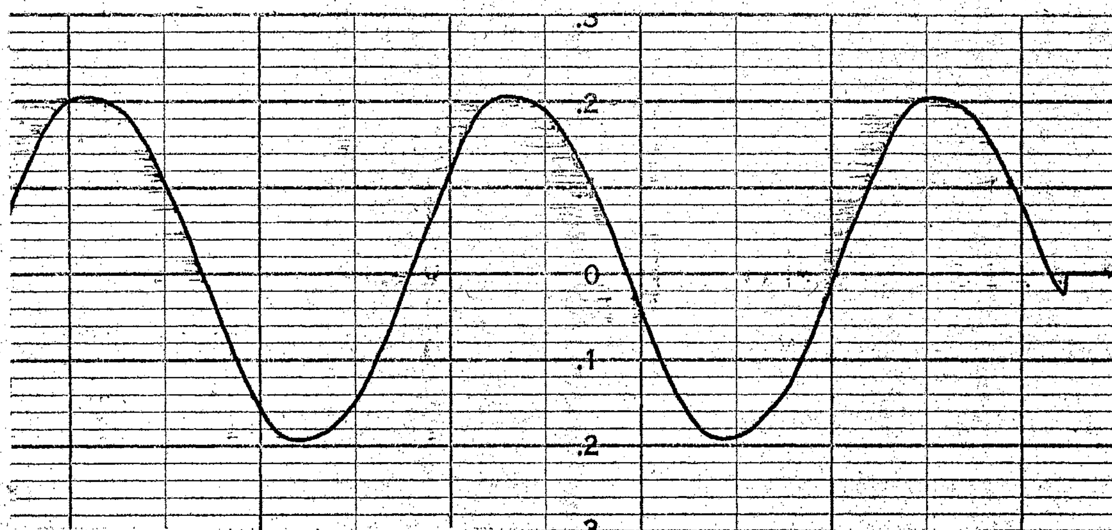

Figure 10. Output of Sine Potentiometer Showing That the Center of Rotation is not the Electrical Center of the Winding.

Figure 10 was obtained by applying a constant voltage to one of the sine potentiometers and connecting one of the brushes to the input of an operational amplifier with input and feedback resistances which were equal. The output of the amplifier was then connected to the recti/riter through an appropriate matching network (see Appendix C). The recti/riter with its matching network requires 4.16 volts per division. Using this information one notes that the maximum deflection in one direction in Figure 10 is 42.4 volts and is 40.4 volts in the other direction. The difference between these two is 2.0 volts and this is an indication of the fact that the electrical center and the center of shaft rotation do not correspond.

Two methods of compensating for this inaccuracy will be considered here. The first method uses the operational amplifiers which are a part of the Donner computer.

Referring to Figure 9, let the voltage from brush "A" be

$$E \sin\theta + k \tag{57}$$

and that from brush "B" be

$$-E \sin\theta + k, \tag{58}$$

where k is a constant voltage resulting from the displacement of the center of shaft rotation from the electrical center.

These two voltages will be applied to an amplifier arrangement as shown in Figure 11.

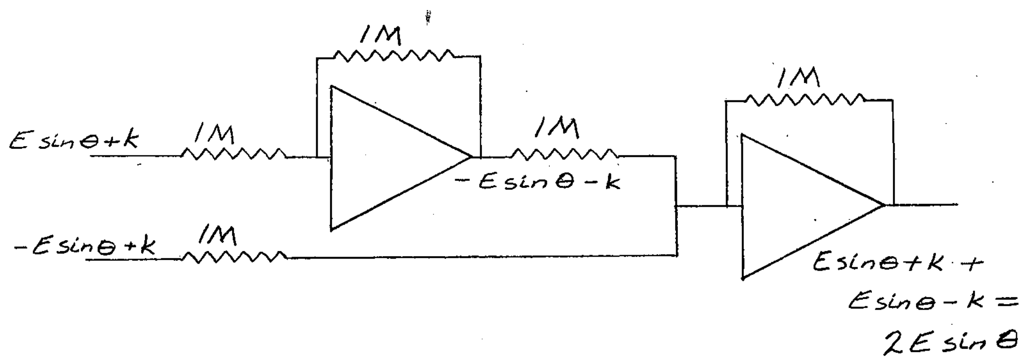

Figure 11. A Method of Correcting for the Displacement of Center of Shaft Rotation from Electrical Center of Rectangular Card Type Potentiometer.

As is seen in Figure 11 the results of such an arrangement is 2 sin θ. Thus it is seen that the constant voltage k has been removed from the output of the sine potentiometer.

A second method which may be used is to apply the voltages from brushes "A" and "B" to the inputs of a difference amplifier. A difference amplifier is used when it is desired to produce a signal referred to ground which is proportional to the difference between two input signals. (4). In the case of the sine potentiometers, the two signals are $E \sin \theta + k$ and $- E \sin \theta + k$. The difference between these two input signals would be $2E \sin \theta$. This is the desired result.

It has been shown that a sine function can be obtained from a rectangular card wound potentiometer and that certain circuit additions are required in order to adjust for errors which arise as a result of manufacturing tolerances. Another problem arises in the use of potentiometers. This is discussed in detail in the following section.

Isolation of Potentiometers

A potentiometer is a device consisting of a resistance having terminals at each end provided with a sliding contact or brush arranged so that it can be moved over the resistance from one end to the other. The relationship of follower position and resistance between the follower and one end may be linear or the potentiometer may be designed to follow some particular function such as logarithmic or trigonometric.

If a linear potentiometer is connected to a circuit which has infinite input impedance then the voltage developed between the follower and the lower end of the potentiometer will be a linear function of the voltage applied to the potentiometer.

Consider the potentiometer shown in Figure 12. This potentiometer is linear with a total resistance of R ohms. The resistance between the follower and the lower end may be called R_1 and between the follower and the upper end R_2.

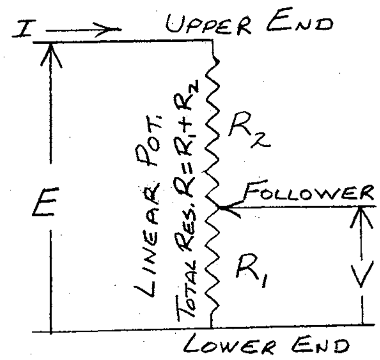

Figure 12. A Circuit Showing a Linear Potentiometer Connected To a Circuit of Infinite Input Impedance.

It is obvious that

$$R_1 + R_2 = R \tag{59}$$

and that

$$I = \frac{E}{R}; \tag{60}$$

one may write further that

$$V = IR_1, \tag{61}$$

and substituting equation (60) one obtains

$$V = \frac{ER_1}{R}, \tag{62}$$

which is the well known potential divider expression.

If the resistance ratio in equation (62) is denoted by, "a", which shall be referred to as potentiometer setting, then one may write equation (62) as

$$V = Ea. \tag{63}$$

If the potentiometer winding is linear then one sees that the voltage, V, available at the follower is a linear function of the voltage applied to the potentiometer. The foregoing applies only when the circuit connected to the follower has infinite input impedance, or is not loading the potentiometer.

A different result is obtained if the input impedance of the circuit connected to the follower is not infinite but rather is a finite impedance. In this case the circuit loads the potentiometer.

Consider the circuit in Figure 13.

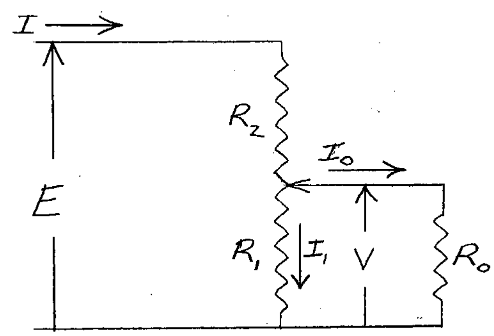

Figure 13. A Potentiometer With a Finite Impedance Loading it.

If the voltage and current equations are written for the circuit of Figure 13, the following expression for V will be obtained.

$$V = E\frac{R_0 R_1}{R_2(R_1 + R_0) + R_1 R_0}. \tag{64}$$

Examining equation (64), it is seen that as R_0 approaches infinity V approaches $ER_1/R_2 + R_1$, the voltage to be expected for a potentiometer that is not loaded. It is also seen that the voltage V is zero if R_0 is zero. This again is the expected voltage.

Equation (64) may be rewritten in a manner that will be helpful in determining how the voltage at the arm of a potentiometer differs in the loaded condition from that in the unloaded condition. In order to accomplish this, let

$$a = \frac{R_1}{R} \tag{65}$$

and recall that

$$R_2 = R - R_1. \tag{66}$$

Utilizing equation (65) one may rewrite equation (66) in the form

$$R_2 = R(1 - a). \tag{67}$$

Equation (64) may now be written in terms of R, R_o, and "a" to become

$$V = E \frac{R_o a}{(1-a)(aR+R_o)+aR_o} \, . \tag{68}$$

Expanding the denominator and dividing numerator and denominator by R_o, equation (68) becomes

$$V = \frac{E a}{\frac{Ra}{R_o}(1-a)+1} \, . \tag{69}$$

It is readily seen from equation (69) that the voltage at the arm of the potentiometer, V, is zero when a is zero (that is when the arm is at the lower end of the potentiometer). It is also seen that the voltage V is equal to E when a is equal to one (that is when the arm is at the upper end of the potentiometer). It has been shown earlier that the voltage at the arm of a linear potentiometer varies linearly with arm movement if the potentiometer is unloaded. This is shown graphically in Figure 14.

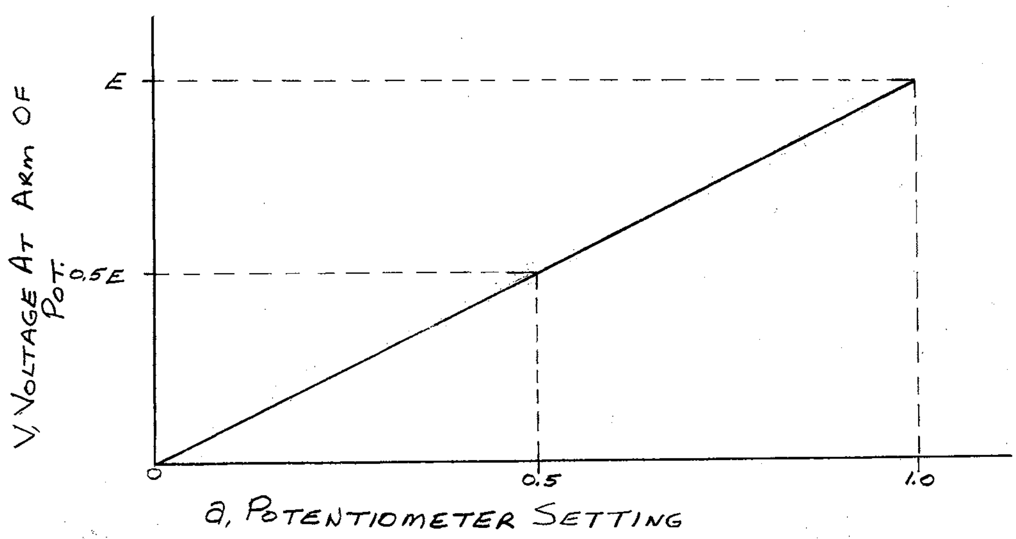

Figure 14. The Variation of Voltage at the Arm of a Potentiometer as the Arm Setting is Changed Assuming the Potentiometer is Unloaded.

Referring to equation (69) one sees that the voltage V will be less in the loaded condition than in the unloaded condition. This is because the denominator of equation (69) will be greater than one if the load resistor has a value less than infinity. Table I tabulates the voltage at the arm of a potentiometer for various positions of the arm using two different R/R_o ratios.

TABLE I

THE VOLTAGE AT THE ARM OF A POTENTIOMETER AS THE ARM POSITION IS VARIED

a	V (R/R_o = 1)	V (R/R_o = 0.5)
0.1	0.092E	0.069E
0.2	0.172E	0.185E
0.3	0.248E	0.271E
0.4	0.322E	0.357E
0.5	0.400E	0.445E
0.6	0.484E	0.536E
0.7	0.578E	0.634E
0.8	0.690E	0.740E
0.9	0.825E	0.860E
1.0	1.000E	1.000E

A curve of V versus a would appear as shown in Figure 15.

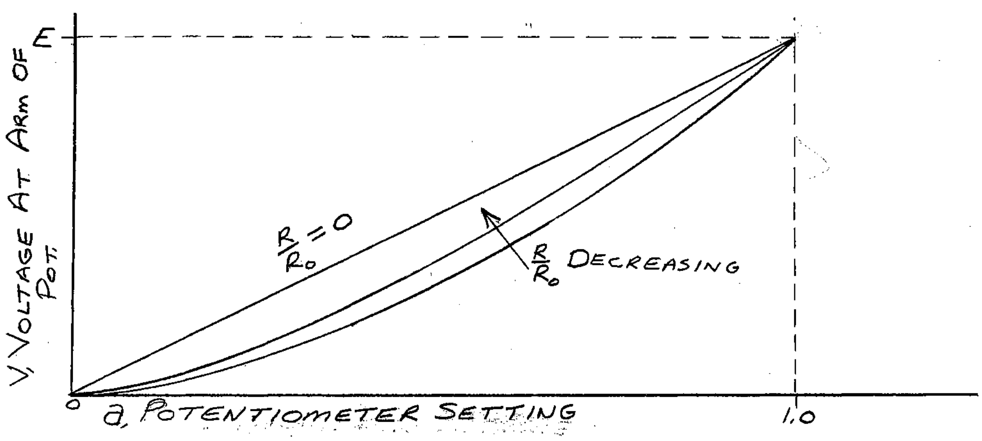

Figure 15. A Graphical Representation of the Effect of Loading on the Voltage at the Arm of a Potentiometer.

As R_o is increased with relation to the total resistance of the potentiometer resistance R (the ratio R/R_o decreases) then the deviation of the loaded from the unloaded condition becomes less and less. This can be seen from the equation for the loaded condition, equation (69). As $R/R_o\ a(1 - a)$ becomes much less than 1 then equation (69) approaches the unloaded equation which is

$$V = Ea \,. \tag{63}$$

It is possible to write an expression for the loaded potentiometer which is in terms of the error introduced by the loading. The potentiometer setting, "a," has been defined as the ratio between R_1 and R (see Figure 11) and one must bear in mind that this is the ratio between output and input voltages for an unloaded potentiometer. Thus if one takes the difference between a and the V/E ratio then an expression for the error will be obtained. Using equation (69) one sees that

$$\frac{V}{E} = \frac{a}{\frac{R}{R_o}a(1-a) + 1} \tag{70}$$

and thus error, ϵ, may be written as

$$\epsilon = a - \frac{V}{E} = a - \frac{a}{\frac{R}{R_o}a(1-a) + 1} \,. \tag{71}$$

Figure 16 represents the error versus potentiometer setting graphically.

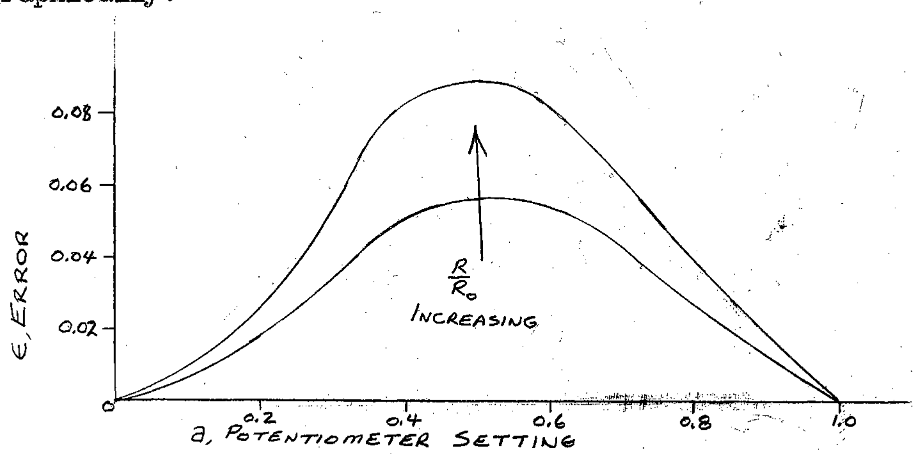

Figure 16. A Plot of Potentiometer Error Versus Potentiometer Setting.

Potentiometers are an important device in electronic analog computer application. It has been shown how potentiometers are used in the arbitrary function generator which has been developed for this project. In addition, it has been shown how potentiometers are used in the generation of trigonometric functions for use in representing the equation at hand. One other very important use to which the potentiometer is commonly put in the computer is that of multiplying a voltage by a factor less than unity.

The preceeding development indicates the effect of loading on the voltage at the arm of the potentiometer. One sees that the voltage at the arm of the potentiometer will be reduced by loading and it is necessary to consider the steps which may be taken to compensate for the loading effect.

The problem of compensating potentiometers for the loading effect may be broken into two parts. These are

(a) The condition where the arm of the potentiometer does not move during the solution of the problem (such as when the potentiometer is used to multiply the input voltage by a constant factor which is less than unity).

(b) The condition where the arm of the potentiometer moves during the solution of the problem (such as in the case of a sine potentiometer or other function generating potentiometer).

In the case of the condition as described in (a), the effect of loading may be taken into account in a relatively simple manner. One method which may be used is to employ a vacuum tube voltmeter to measure the voltage at the arm of the potentiometer with the load connected. The arm can then be adjusted so that the output voltage to input voltage ratio is the one desired. There are other ways in which the ratio may be set but in any case the important feature is that the load is connected and then the ratio is set. It is obvious that this takes care of the loading.

If the potentiometer arm is to rotate as a part of the problem, then the matter of compensating for the load on the potentiometer is somewhat more complicated. The loading error can be reduced to a reasonable value by adding end resistors, sometimes called compensating resistors. (5) (6).

Another approach to the isolation problem is to connect an isolation circuit between the arm of the potentiometer and the load. A circuit of this kind should have an infinite impedance input, zero impedance output and unity gain. McCoy and Bradley (7) describe an isolation circuit utilizing operational amplifiers. This circuit is shown in Figure 17.

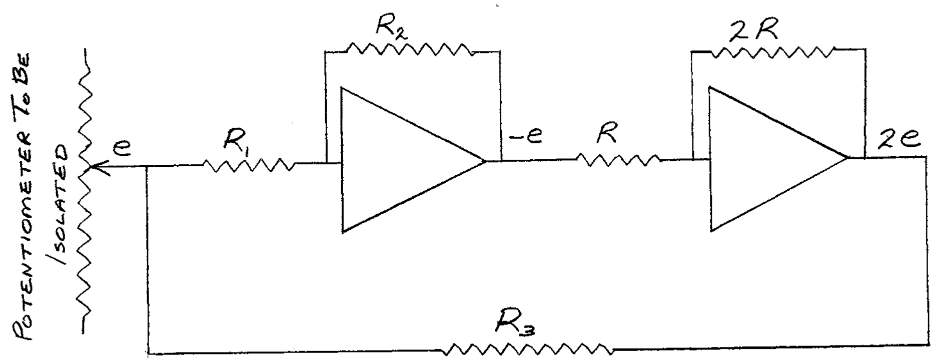

Figure 17. A Potentiometer Isolation Circuit Utilizing Operational Amplifiers.

In Figure 17, R_1, R_2, and R_3 are equal. Since the operational amplifiers have very high gains the junction between R_1 and R_2 is virtually at ground potential. Therefore the voltage 2e divides across R_1 and R_3 to give a voltage of e at the arm of the potentiometer.

In this circuit the current drawn from the potentiometer is essentially zero. The second amplifier provides the current through R_1.

Use may be made of the Philbrick amplifiers to achieve a very high degree of isolation. If the amplifiers are connected as shown in Figure 18 then the input impedance is more than 100 megohms which essentially isolates the potentiometer. (8).

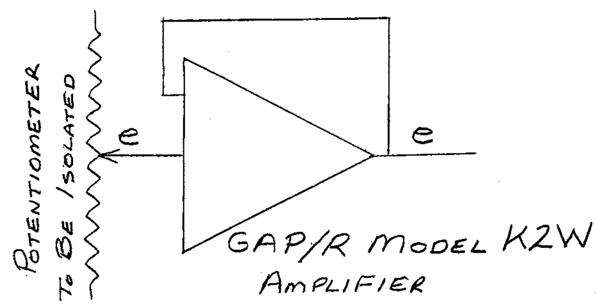

Figure 18. The Connection of a GAP/R Model K2W Amplifier to Isolate a Potentiometer.

Scaling

It has been shown in Chapter I that with the proper input and feedback impedances for an operational amplifier, the output voltage can be made to be a function of the input voltage which is similar to the relationship between quantities in equations describing a physical system. In order that a relationship between the computer voltages and the physical units may be established, it will be necessary to perform an operation which shall be called scaling the computer. By means of this operation the voltages at various points in the computer can be measured and by utilizing scale factors, these voltages can be converted into units found in the equations describing the system.

The values of the input and feedback impedances (and thus the gain of the amplifier) and the potentiometer settings are determined by the scaling operation. In scaling an electronic analog computer it is necessary to consider the following factors:

(a) The maximum value the physical quantity is estimated to attain.

(b) The maximum voltage obtainable from the operational amplifiers without overdriving.

(c) The minimum voltage at the output of the operational amplifiers which is well above the noise level.

An example of the scaling the electronic analog computer utilizing the arbitrary function generator described in this chapter is given in Appendix B.

The Computer Arrangement

In the preceeding sections of this chapter, the considerations involved in the problem at hand were discussed in some detail. It will be the purpose of this section to correlate the previous information into an electronic analog computer arrangement which will be satisfactory for representing the necessary equations.

As a first step in the problem solution assume that the following items will not affect the solution:

(a) Potentiometer loading.

(b) Location of the electrical center of the sine potentiometers with respect to the center of shaft rotation.

If these assumptions can be made then it will be possible to begin with a relatively simple computer arrangement to check the validity of the equations which are to be used on the computer. It will also be possible to check the performance of the arbitrary function generator. After this has been done, the computer arrangement can be modified to include such other steps as will be necessary to allow for the assumptions made in the foregoing.

It has been shown in Chapter II that the equation for the indicated work done is as given in equation (41) which is repeated here for convenience.

$$W = 2\pi r s \int_0^t F \sin 2\pi s t \, dt - \frac{\pi r^2 s}{l} \int_0^t F \sin 4\pi s t \, dt. \tag{41}$$

Since the curve that will be available will be pressure versus crank angle this equation should be changed to include pressure times area to replace force. The equation then becomes

$$W = 2\pi r s a \int_0^t P \sin 2\pi s t \, dt - \frac{\pi r^2 s a}{l} \int_0^t P \sin 4\pi s t \, dt, \tag{72}$$

where a is the cross sectional area.

It will be necessary to assume values for the constants in order that the problem may be placed on the computer. Assume that these constants have the following values:

P (max.) --	500 psi.
s ---------	25 rps.
a ---------	3.5 sq. in.
r ---------	2.125 in.
l ---------	6 in.

With these constants having values as shown it will be possible to evaluate the constants of the equation as follows:

$$2\pi r s a = \frac{(2\pi)(2.125)(25)(3.5)}{12} = 97.5$$

$$\frac{\pi r^2 s a}{l} = \frac{(\pi)(2.125)^2(25)(3.5)}{(\frac{6}{12})(144)} = 17.2$$

Thus the equation may then be written

$$W = 97.5 \int_0^t P \sin 2\pi s t \, dt - 17.2 \int_0^t P \sin 4\pi s t \, dt. \tag{73}$$

This equation is in problem or real units and in order to represent the equation on the computer it will be necessary to express it in machine or computer units.

Let

$$W = \alpha_1 \overline{W}, \tag{74}$$

$$P = \alpha_2 \overline{P}, \tag{75}$$

$$s = \alpha_3 \overline{s}, \tag{76}$$

and

$$t = \alpha_4 \overline{t}, \tag{77}$$

where α_1, α_2, α_3 and α_4 are scale factors and $\overline{W}$, $\overline{P}$, $\overline{s}$ and $\overline{t}$ are machine units.

The next step will be to evaluate these scale factors. It is known that 100 volts is the maximum voltage which may be obtained from the Donner operational amplifiers. (9). Let it be assumed that P maximum will be 500 psi., as indicated previously, and let the arbitrary function generator be so arranged that 10 volts is obtained from the potentiometer when the follower is at its maximum excursion. This voltage is adjustable by adjusting the d-c voltage applied to the potentiometer. Based on these assumptions one sees that the scale factors become

$$\alpha_1 = \frac{W_{max.}}{\overline{W}_{max.}} = \frac{200}{100} = 2, \tag{78}$$

$$\alpha_2 = \frac{P_{max.}}{\overline{P}_{max.}} = \frac{500}{10} = 50 \tag{79}$$

$$\alpha_3 = \frac{s}{\overline{s}} = \frac{25}{0.0898}, \tag{80}$$

(Where 0.0898 is the number of $\frac{1}{2}$ revolutions per second. One half of a revolution of the drum on the function generator corresponds to one revolution of the engine.)

$$\alpha_4 = \frac{t}{\bar{t}} = \frac{\frac{1}{25}}{\frac{1}{0.0898}} = 0.00359. \qquad (81)$$

The equation for work may be written in problem units as

$$\alpha_1\overline{W} = 97.5\int_0^{\bar{t}} \alpha_2 \overline{P} \sin 2\pi\alpha_3\bar{s}\alpha_4\bar{t}\ \alpha_4 d\bar{t} - 17.2\int_0^{\bar{t}} \alpha_2 \overline{P} \sin 4\pi\alpha_3\bar{s}\alpha_4\bar{t}\ \alpha_4 d\bar{t}. \qquad (82)$$

Now α_3 multiplied by α_4 is unity since one is the reciprocal of the other. Thus the equation becomes

$$\alpha_1\overline{W} = 97.5\int_0^{\bar{t}} \alpha_2 \overline{P} \sin 2\pi\bar{s}\bar{t}\alpha_4 d\bar{t} - 17.2\int_0^{\bar{t}} \alpha_2 \overline{P} \sin 4\pi\bar{s}\bar{t}\alpha_4 d\bar{t}. \qquad (83)$$

Substituting the values of the scale factors one obtains

$$2\overline{W} = 17.5\int_0^{\bar{t}} \overline{P} \sin 2\pi\bar{s}\bar{t}\ d\bar{t} - 3.1\int_0^{\bar{t}} \overline{P} \sin 4\pi\bar{s}\bar{t}\ d\bar{t}, \qquad (84)$$

$$\overline{W} = 8.75\int_0^{\bar{t}} \overline{P} \sin 2\pi\bar{s}\bar{t}\ d\bar{t} - 1.55\int_0^{\bar{t}} \overline{P} \sin 4\pi\bar{s}\bar{t}\ d\bar{t}. \qquad (85)$$

It is equation (85) which will be represented on the electronic analog computer.

First consider $\overline{P}$ which is to be obtained from the arbitrary function generator. It is readily seen from the equation that this function is to be multiplied by sin $2\pi\ \overline{st}$ and sin $4\pi\ \overline{st}$. Sine potentiometers are to be used to effect this multiplication, and in order for the sine potentiometer to be able to furnish both positive and negative values there must be a positive voltage applied to one end of the potentiometer winding and a negative voltage applied to the other end. Referring to the section of this chapter which discussed the sine potentiometer, it is seen that the voltage applied to the sine potentiometer is the voltage multiplied by the sine function. Thus, both plus and minus values of $\overline{P}$ must be applied to the sine potentiometers.

Referring to Figure 19 it is shown how the voltage representing pressure is to be obtained from the function potentiometer.

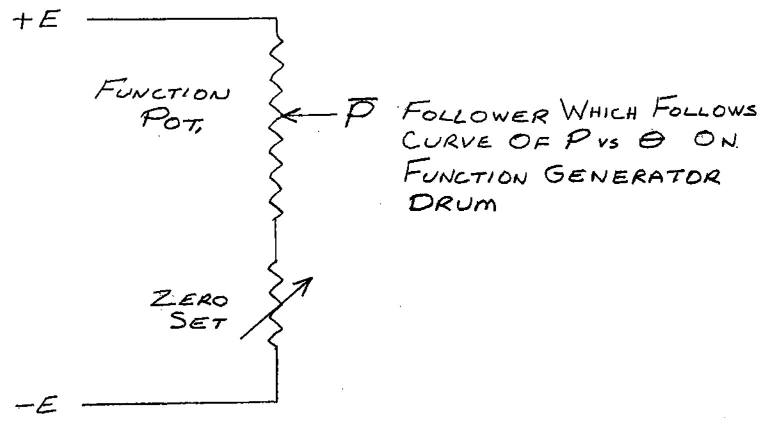

Figure 19. A Circuit Showing How the Voltage Representing Pressure is to be Obtained.

With this arrangement a positive voltage for $\overline{P}$ will be obtained. It will be necessary to utilize a sign changing amplifier to obtain a negative voltage equal in magnitude to the positive voltage for application to the sine potentiometer.

A sign changing amplifier is one with a pure resistance for both the input and feedback impedances and which has unity gain.

A first thought might be to arrange the circuit as shown in Figure 20 to obtain the positive and negative values of $\overline{P}$.

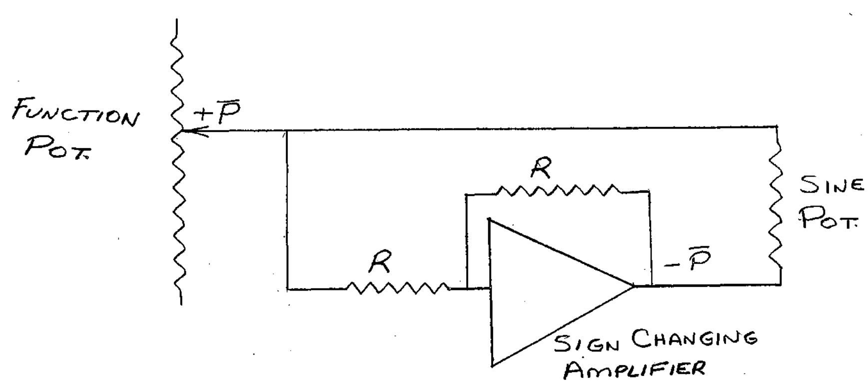

Figure 20. A First Attempt to Obtain Both Positive and Negative Values of $\overline{P}$ for Application to a Sine Potentiometer.

This curcuit, however, is not satisfactory as one can see that the resistance in the sine potentiometer winding forms a part of the feed-back loop for the sign changing amplifier.

A circuit such as the one depicted in Figure 20 will be necessary in order to obtain the appropriate voltages for application to the sine potentiometers. By means of this curcuit the function $\overline{P}$ with appropriate signs is obtained for use in representing the equation in question. Actually two such arrangements shown in Figure 20 will be necessary as one will be used in connection with the sine 2π st potentiometer and the other will be used in connection with the sine 4π st potentiometer.

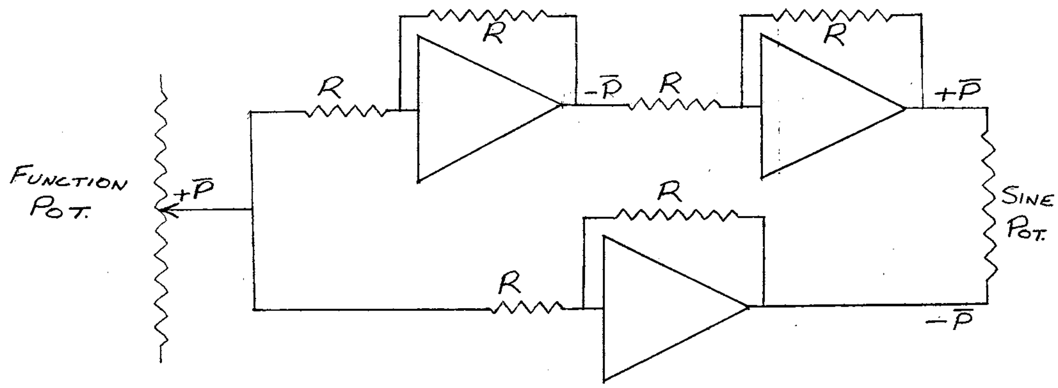

Figure 21. A Circuit Which is Satisfactory for use in Obtaining Both Positive and Negative Values of $\overline{P}$.

The method shown in Figure 21 results in some loading of the function generating potentiometer as a result of the amplifier input resistances. In this particular case there will be four resistances in parallel since the grids of the amplifiers are essentially at ground potential, and there will be two arrangements like that shown in Figure 21 connected to the function potentiometer. If the input resistances are made as large as possible then the loading error will be minimized. For example, if the input resistances are each 4×10^6 ohms then the equivalent resistance loading the function potentiometer would be 1 x 10^6 ohms. The function potentiometer used in this case has a resistance of 2 x 10^4 ohms. This gives a R/R_o ratio of 0.02 and results in a maximum potentiometer error of approximately 0.003. This is a reasonably small error and would for most purposes be satisfactory. A method will be shown later which more effectively isolates the function potentiometer.

Now that both $\overline{P} \sin 2\pi \overline{st}$ and $\overline{P} \sin 4\pi \overline{st}$ are available all that remains is to integrate these functions and to apply them to an adder in the proper way.

Referring to equation (85) it is seen that the integral of $\overline{P} \sin 4\pi \overline{st}$ is to be subtracted from the integral of $\overline{P} \sin 2\pi \overline{st}$. A computer arrangement which may be used to accomplish this is shown in Figure 22.

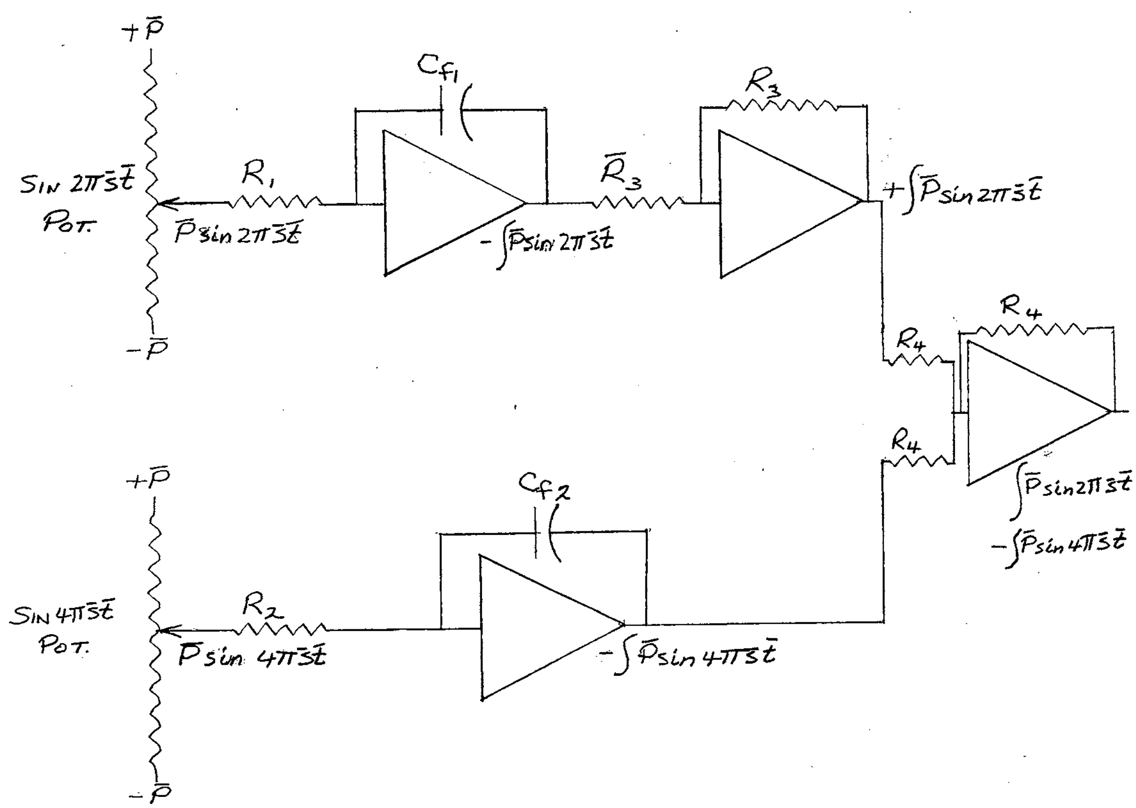

Figure 22. A Computer Circuit to Integrate $\overline{P} \sin 2\pi \overline{st}$ and $\overline{P} \sin 4\pi \overline{st}$ and to Add the Results.

This circuit is intended to show only a possible arrangement of computer components without regard to their magnitudes. The selection of R_1, R_2, C_{f1}, and C_{f2} will be made later.

One sees in Figure 22 that a sign changing amplifier is necessary. This amplifier would not be required if the sin $2\pi\overline{st}$ potentiometer is oriented with respect to the other sine potentiometer and the chart on the drum so that it is actually $-\sin 2\pi\overline{st}$.

With the computer circuit arrangement based on equation (85) available, the next step is to select the impedances to be used in the computer so that the constants of equation (85) are satisfied.

Actually, the only scaling that will be necessary is in the integrating portion of the computer. It will not be necessary to scale the impedances in the amplifiers used to apply the function $\overline{P}$ to the sine potentiometers since their only function is sign changing. Likewise, it will not be necessary to scale the adder impedances.

Consider the portion of equation (85)

$$8.75 \int \overline{P} \sin 2\pi \overline{s} \overline{t} \, d\overline{t}. \qquad (86)$$

This portion of the equation is to be represented by an integrating amplifier similar to the one shown in Figure 23.

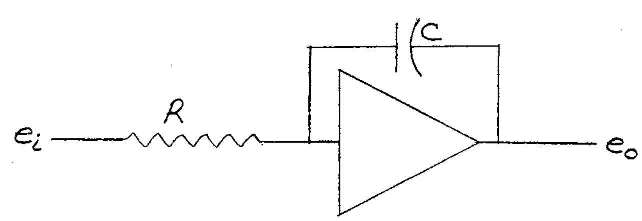

Figure 23. An Integrating Amplifier.

It is seen from equation (14) in Chapter I that

$$e_o = -\frac{1}{RC}\int e_i \, dt \, . \qquad (14)$$

In the case at hand

$$e_i = \bar{P} \sin 2\pi \xi \bar{t} \quad (87)$$

and therefore

$$e_o = -\frac{1}{RC}\int \bar{P} \sin 2\pi \xi \bar{t} \, d\bar{t}. \quad (88)$$

It is seen then, by comparing equations (86) and (88), that

$$\frac{1}{RC} = 8.75. \quad (89)$$

Now, since the variety of magnitudes of the Donner impedance components available for this problem is somewhat restricted, it will be necessary to utilize potentiometers. Figure 24 indicates how a potentiometer would be placed in the integrating circuit.

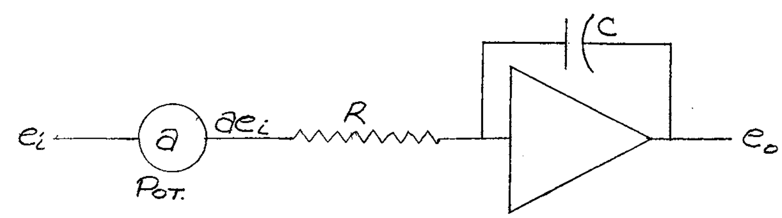

Figure 24. An Integrating Amplifier With a Potentiometer.

With the potentiometer in the circuit equation (89) becomes

$$\frac{a}{RC} = 8.75 \quad (90)$$

and if R is made 1 megohm and C is made 0.1 micro farad than we see that a will be 0.875. The potentiometer setting will always be less than one since its output voltage can never be greater than the input voltage.

Now, considering the remaining portion of equation (85),

$$1.55 \int \bar{P} \sin 4\pi \xi \bar{t} \, d\bar{t} \quad (91)$$

and using the same procedure as before to obtain

$$\frac{a}{RC} = 1.55. \quad (92)$$

If R is 0.5 megohm and C is 1 micro farad then one sees that "a" will be 0.775.

Thus the scaling is accomplished and it will be possible to arrive at a final computer circuit complete with impedance values and potentiometer settings. This circuit is shown in Figure 25.

One should realize that this circuit does not provide for complete isolation of the potentiometers and also that it does not correct for the fact that the center of rotation of the sine potentiometers is not at the electrical center of the potentiometer. A computer circuit which will provide for more effective potentiometer isolation and which will provide a means for correcting for the inaccuracy in the sine potentiometers is shown in Figure 26. This circuit provides very effective potentiometer isolation since the input impedance of the GAP/R amplifiers is more than 100 megohms. (8).

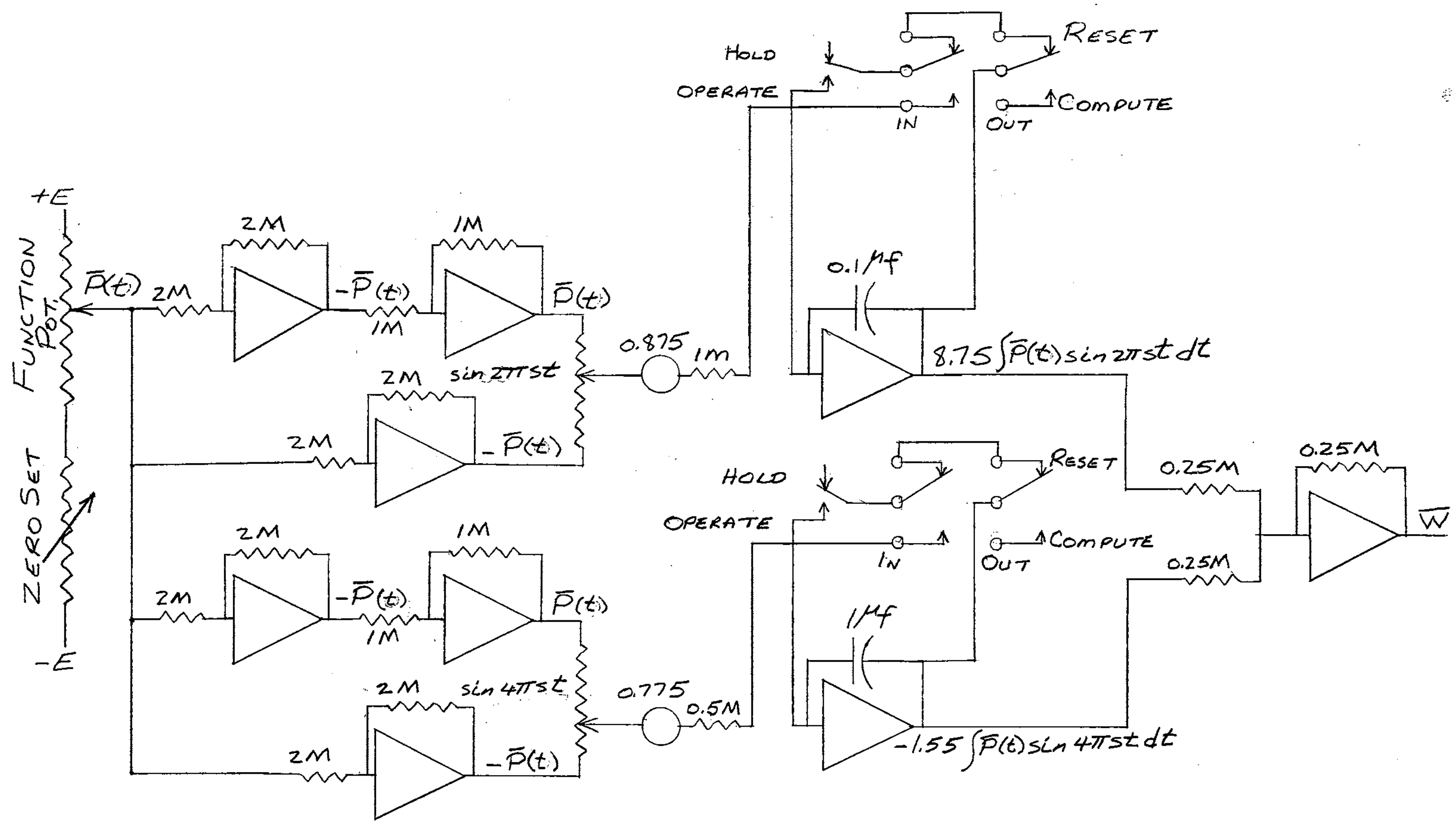

Figure 25. A Complete Computer Arrangement to Represent the Problem Neglecting Potentiometer Loading and Inaccuracies in the Sine Potentiometer.

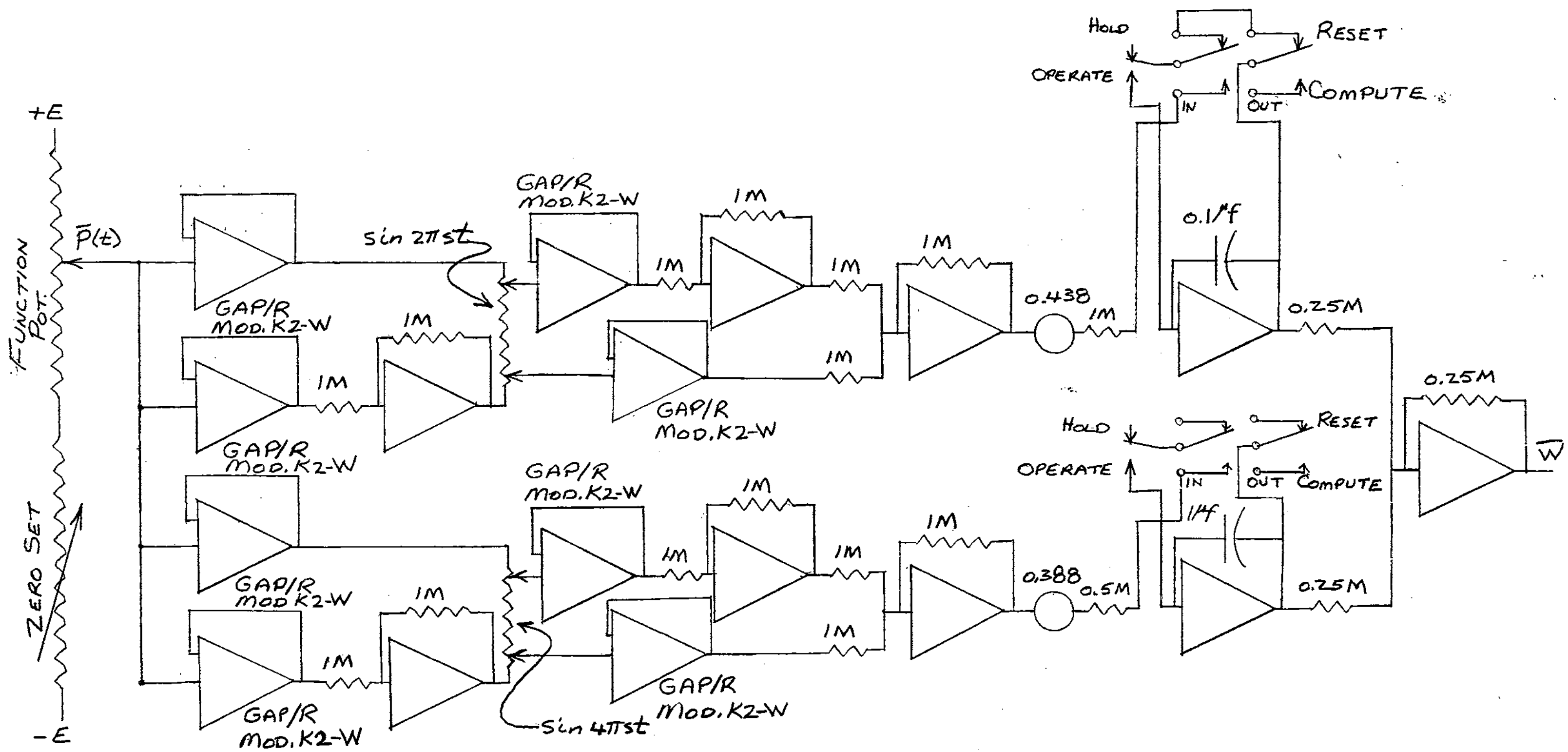

Figure 26. A Computer Arrangement Similar to Figure 25 But Designed to Provide for More Effective Potentiometer Isolation and to Compensate for the Sine Potentiometer Inaccuracies.

CHAPTER IV

ANALYSIS OF RESULTS

The next step is to verify that acceptable results may be obtained from the analog computer arrangement which has been devised. The method of accomplishing this in this particular case was to assume a pressure versus time (or crank angle) curve of such a shape so as to make it possible to determine the work done by analytical means.

The pressure versus time curve chosen is that depicted in Figure 27. The time 0.08 seconds represents the time for two complete strokes of the engine at a speed of 25 revolutions per second.

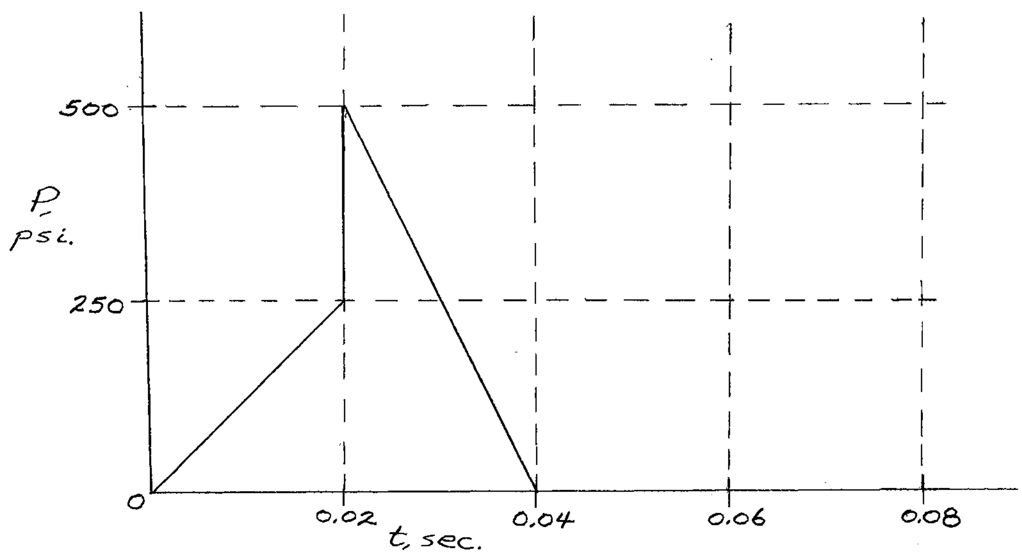

Figure 27. An Assumed Pressure Versus Time Curve.

The time interval from 0 - 0.02 seconds represents the compression stroke, from 0.02 - 0.04 seconds the power stroke, from 0.04 - 0.06

seconds the exhaust stroke, and from 0.06 - 0.08 seconds the intake stroke.

The equation to be evaluated analytically is

$$W = 97.5\int_0^t P \sin 2\pi s t \, dt - 17.2\int_0^t P \sin 4\pi s t \, dt, \qquad (73)$$

and to evaluate this equation divide it into two sections, namely 0 - 0.02 seconds and 0.02 - 0.04 seconds.

In the interval 0 to 0.02 seconds

$$P = \frac{250}{0.02} t \qquad (93)$$

and the equation for work becomes

$$W = 97.5\int_0^{0.02} \frac{250}{0.02} t \sin 2\pi s t \, dt - 17.2\int_0^{0.02} \frac{250}{0.02} t \sin 4\pi s t \, dt. \qquad (94)$$

Integrating this equation one obtains

$$W = \frac{(97.5)(250)}{0.02}\left[\frac{1}{(2\pi 25)^2}\sin 2\pi 25 t - \frac{t}{2\pi 25}\cos 2\pi 25 t\right]_0^{0.02} - \frac{(17.2)(250)}{0.02}\left[\frac{1}{(4\pi 25)^2}\sin 4\pi 25 t - \frac{t}{4\pi 25}\cos 4\pi 25 t\right]_0^{0.02}, \qquad (95)$$

and evaluating at the limits

W = 168.72 ft.-lbs.

This is the work done in compressing the gas in the combustion chamber.

Now, considering the interval from 0.02 - 0.04 seconds

$$P = -\frac{500t}{0.02} + 1000, \qquad (96)$$

and the equation for work becomes

$$W = 97.5\int_{0.02}^{0.04}\left(-\frac{500t}{0.02} + 1000\right)\sin 2\pi s t \, dt - 17.2\int_{0.02}^{0.04}\left(-\frac{500t}{0.02} + 1000\right)\sin 4\pi s t \, dt. \qquad (97)$$

Integrating this equation it is seen that

$$W = \frac{(97.5)(-500)}{0.02}\left[\frac{1}{(2\pi 25)^2}\sin 2\pi 25t - \frac{t}{2\pi 25}\cos 2\pi 25t\right]_{0.02}^{0.04} + (97.5)(1000)\left[-\frac{1}{2\pi 25}\cos 2\pi 25t\right]_{0.02}^{0.04} - \frac{(17.2)(-500)}{0.02}\left[\frac{1}{(4\pi 25)^2}\sin 4\pi 25t - \frac{t}{4\pi 25}\cos 4\pi 25t\right]_{0.02}^{0.04} - (17.2)(1000)\left[-\frac{1}{4\pi 25}\cos 4\pi 25t\right]_{0.02}^{0.04}, \quad (98)$$

and evaluating it is found that

$$W = -337.44 \text{ ft.-lbs.}$$

This work is opposite in sign from the work obtained during the interval from 0 to 0.02 seconds which, of course, is to be expected since the piston is moving in opposite directions in the two cases.

One readily sees that for the curve shown in Figure 27 that the work done in the interval 0.04 to 0.08 seconds is zero since the pressure is zero during this time. Thus the net indicated work is

$$W = 337.44 - 168.72 = 168.72 \text{ ft.-lbs.}$$

Using a computer arrangement as shown in Figure 25 in Chapter III, the problem was solved six times. The results are shown in Figure 28. It is seen that the maximum percentage error is 9.5% with a minimum error of 1.36%. The average error is 4.92%. Considering the fact that the computer arrangement used does not completely isolate the potentiometers and does not allow for the inaccuracies inherent in the sine potentiometers, these results seem to be acceptable. Another contributing factor to the errors in the computer answer is the manual follower used on the function generator. This verifies that the computer will

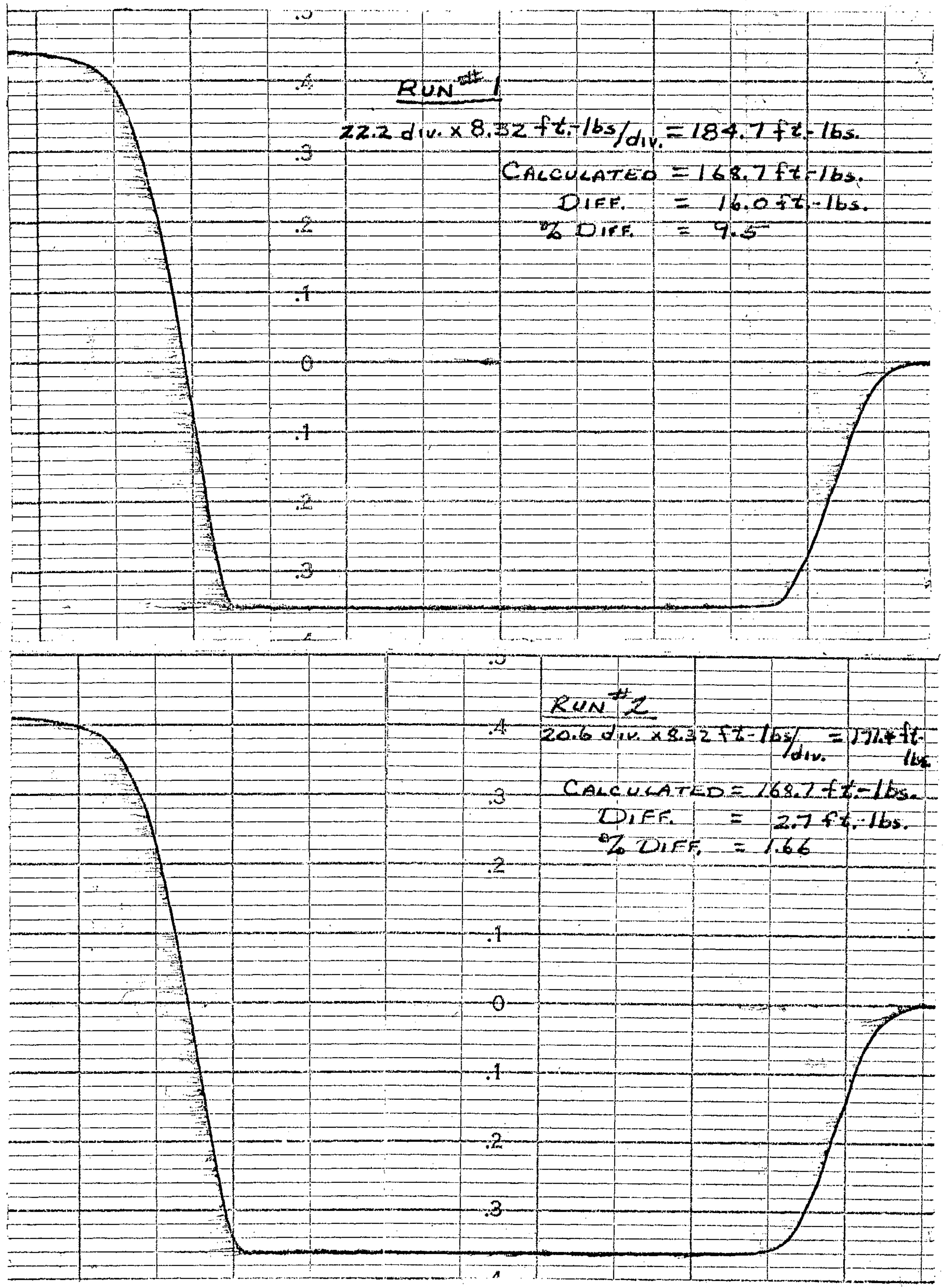

Figure 28. The Results of Using the Computer Arrangement of Figure 26 to Solve Equation 73.

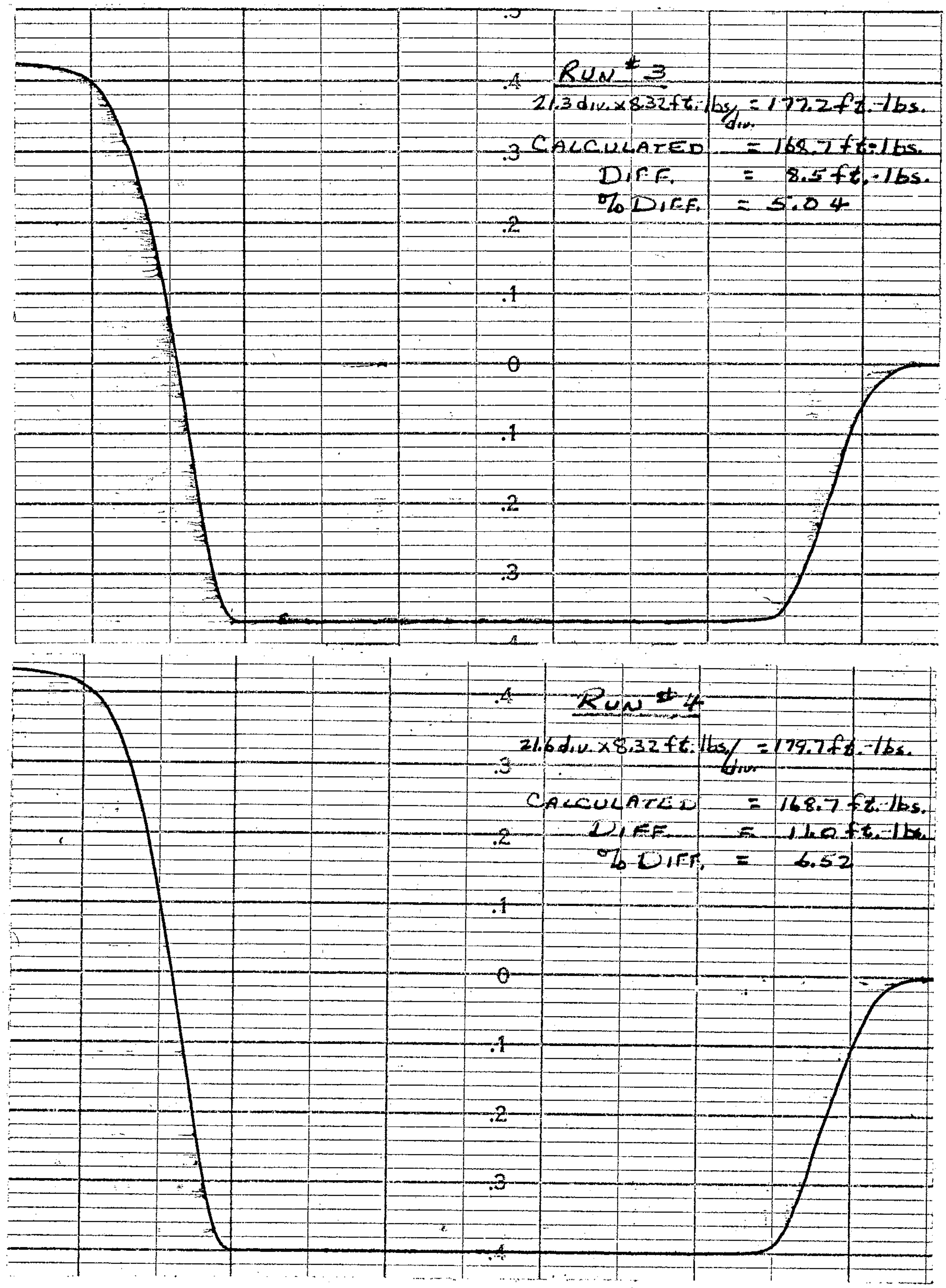

Figure 28. The Results of Using the Computer Arrangement of Figure 26 to Solve Equation 73.

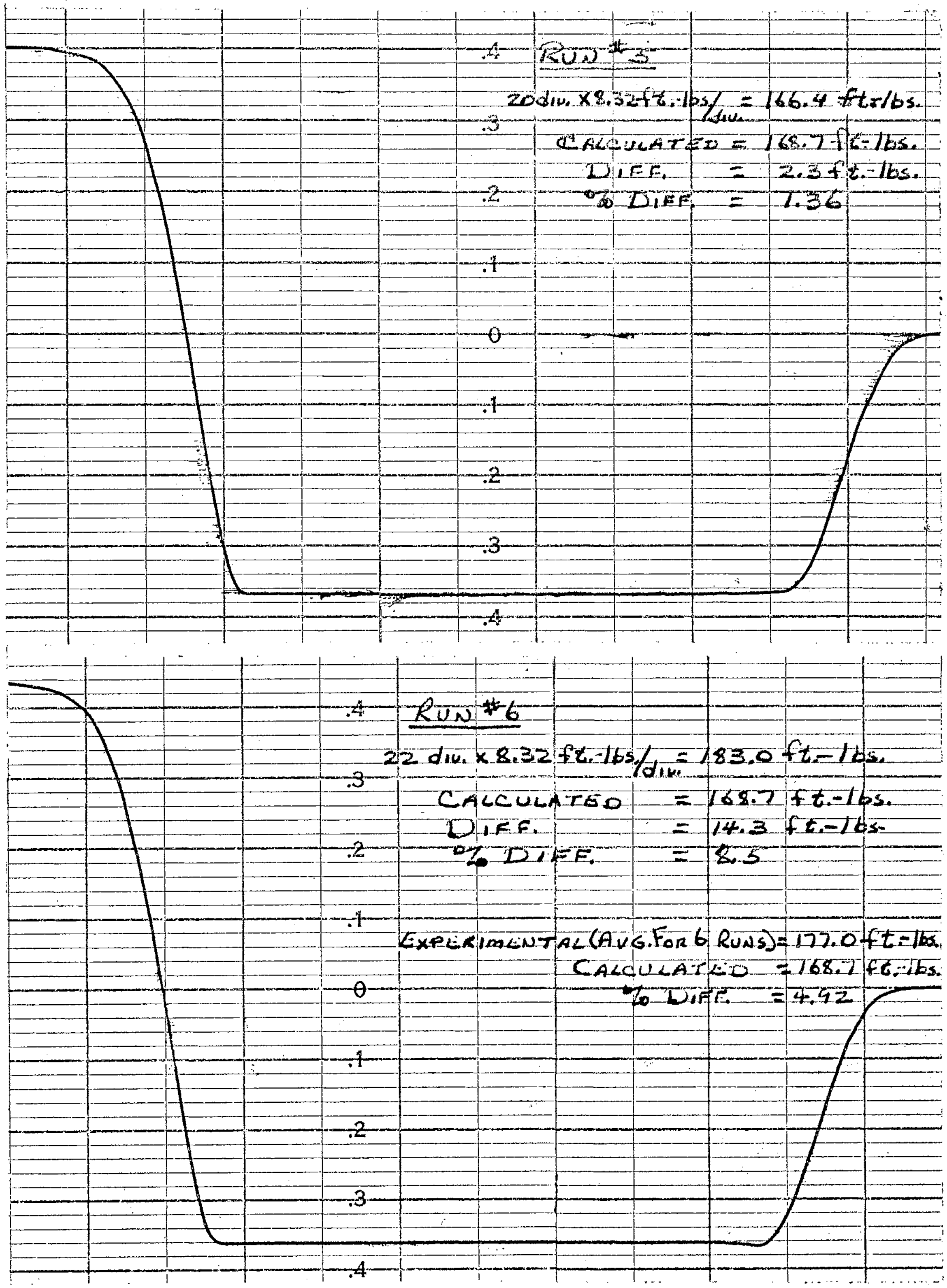

Figure 28. The Results of Using the Computer Arrangement of Figure 26 to Solve Equation 73.

represent the equations developed to a reasonable degree of accuracy. It, however, does not verify that the work obtained in this manner will agree closely with that obtained by a more conventional method of internal combustion engine analysis.

In order to compare the indicated work obtained by the electronic computer with that obtained by more conventional means, the plot of Pressure vs Time shown in Figure 27 is plotted as Pressure vs Piston Position for the compression stroke. This plot was made with the aid of Table II and is shown in Figure 29. This will be recognized as a part of the more familiar indicator card diagram ordinarily obtained from an internal combustion engine. By means of counting squares the area under the compression portion of the curve was obtained to be 293.15 squares which is 171 ft.- lbs. as compared with 168.72 determined analytically for the compression stroke. The indicated work obtained by counting squares is within 1.36% of that obtained analytically. It is obvious that the plot of the Pressure vs Piston Position for the power stroke would contain twice the area that was contained under the curve for the compression stroke since in the hypothetical case selected the pressure in the power stroke is twice that in the compression stroke. Thus the net indicated work would be 171 ft.-lbs. and this is in good agreement with the net indicated work obtained by means of the electronic analog computer.

Although the arbitrary function generator constructed in connection with this problem is crude in principle, it is the author's opinion that it serves the purpose quite well and that it is demonstrated that the results will agree favorably with those obtained analytically. Referring to Appendix B, it is seen that the electronic analog computer may be used

TABLE II

A TABULATION OF PISTON POSITIONS FOR VARIOUS CRANK ANGLES

Compression Stroke

Crank Angle θ, degrees	Piston Position x, in.
0	0
10	0.021
20	0.084
30	0.190
40	0.342
50	0.538
60	0.780
70	1.066
80	1.389
90	1.750
100	2.132
110	2.521
120	2.907
130	3.270
140	3.600
150	3.871
160	4.080
170	4.210
180	4.250

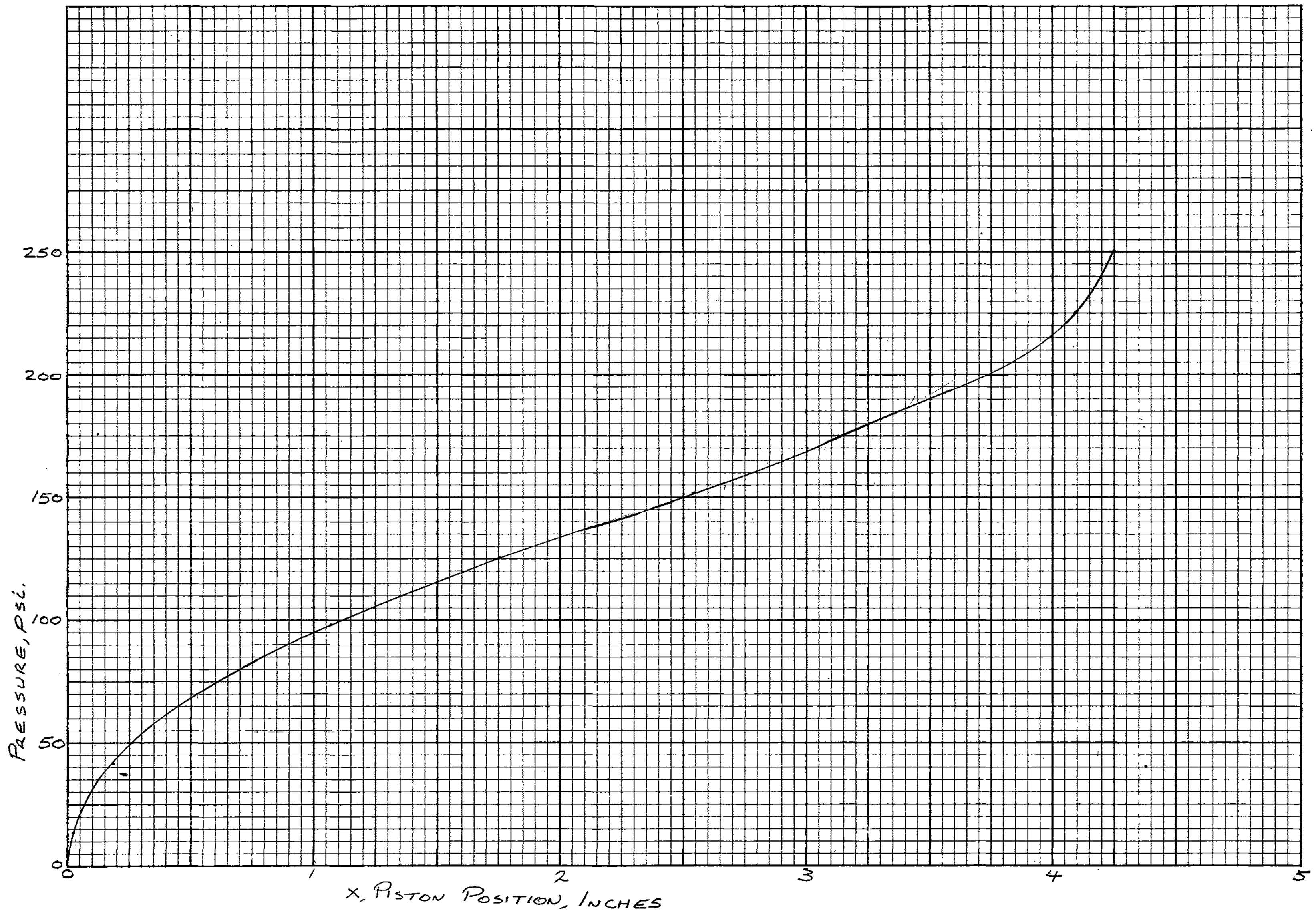

Figure 29. A Plot of Pressure Versus Piston Position Corresponding to the Curve Shown in Figure 27.

to measure an area with an error of less than 0.5%. This error is, of course, a function of the area to be measured due to the limitations inherent in manually following a curve. It is interesting that the computer can be employed in the determination of an area since the electronic analog computer is most often thought of in terms of the solution of problems wherein the independent variable is time. It is seen in the equations developed for an integrating amplifier that the independent variable is indeed time. By proper scaling, as demonstrated in Appendix B, it is possible to relate the time base on the computer to some other unit (such as length in Appendix B).

It is interesting to consider the possibility of representing the work equation in terms of crank angle rather than time on the electronic analog computer. This will be attempted by starting with the equation for piston position presented in Chapter II as equation (35). This equation is

$$x = \ell\left(-\frac{r^2}{2\ell^2}\sin^2\theta\right) + r(1-\cos\theta). \qquad (35)$$

Differentiate x with respect to θ and obtain

$$\frac{dx}{d\theta} = \left[\left(\frac{-r^2}{2\ell}\right)\cos\theta\sin\theta + \left(-\frac{r^2}{2\ell}\right)\cos\theta\sin\theta\right] + (-r)(-\sin\theta) \qquad (99)$$

and rearranging

$$dx = \left(-\frac{r^2}{\ell}\cos\theta\sin\theta + r\sin\theta\right)d\theta. \qquad (100)$$

Substituting this into equation (18) an expression for work is obtained

$$W = \int_0^\theta P_a\left(r\sin\theta - \frac{r^2}{2\ell}\sin 2\theta\right)d\theta. \qquad (101)$$

The scale factors for work and pressure will be the same as used in Chapter III.

These are

$$\alpha_1 = \frac{W_{max.}}{\overline{W}_{max.}} = \frac{200}{100} = 2 \tag{78}$$

and

$$\alpha_2 = \frac{P_{max.}}{\overline{P}_{max.}} = \frac{500}{10} = 50. \tag{79}$$

It will be necessary to develop a scale factor to relate crank angle to computer time. The drum on the function generator completes one revolution in 22.26 seconds and this one revolution represents 720 degrees. Thus the relationship between computer or machine time may be developed as

$$\theta = \alpha_3 \bar{t} \tag{102}$$

where

θ = crank angle (radians),

$\bar{t}$ = computer or machine time (seconds),

and α_3 = scale factor.

Now when the drum has rotated one time this represents two revolutions of the internal combustion engine so that it may be written

$$4\pi = \alpha_3\, 22.26 \tag{103}$$

or

$$\alpha_3 = \frac{4\pi}{22.26} = 0.565. \tag{104}$$

Using this scale factor along with those for pressure and work the equation for work becomes

$$\alpha_1 \overline{W} = \int ar\alpha_2 \overline{P} \sin \alpha_3 \bar{t}\; \alpha_3 d\bar{t} - \int \frac{ar^2}{2\ell} \alpha_2 \overline{P} \sin 2\alpha_3 \bar{t}\; \alpha_3 d\bar{t}, \tag{105}$$

$$2\overline{W} = (ar)(50)(0.565)\int \overline{P} \sin \alpha_3 \bar{t}\; d\bar{t} - \frac{(ar^2)(50)(0.565)}{2\ell}\int \overline{P} \sin 2\alpha_3 \bar{t}\; d\bar{t}. \tag{106}$$

If a = 3.5 sq. in.,

r = 2.225 in., and

l = 6 in.

as before then equation (106) may be written as

$$\overline{W} = 8.75\int \overline{P} \sin \alpha_3 \overline{t}\, d\overline{t} - 1.55\int \overline{P} \sin 2\alpha_3 \overline{t}\, d\overline{t}. \qquad (107)$$

Comparing this with the work equation in machine units (equation 85) which was developed in Chapter III from an equation in time, it is noted that the two equations are the same. Thus, a problem with an independent variable other than time may be represented on the computer.

CHAPTER V

SUMMARY AND CONCLUSIONS

While the computer arrangement used in this investigation would not be desirable for use in determining the indicated work of an actual internal combustion engine, it is felt that the purpose of the investigation has been accomplished.

The object of this study was to demonstrate that the calculation of indicated work by an internal combustion engine can be done by the electronic analog computer.

The results were quite satisfactory in view of the manually operated arbitrary function generator which was used. The accuracy obtained was well within the limits expected and it seems certain that with a servo-driven function generator, proper isolation of the potentiometers, and the use of difference amplifiers on the sine potentiometers, a much greater accuracy could be obtained.

Based on the results of this study it is apparent that the electronic analog computer can be used to advantage on problems of this nature. The major shortcoming of the system used in this investigation is the manually operated arbitrary function generator. The accuracy of the system using this function generator is dependent upon how accurately a person can follow the Pressure vs Crank Angle curve. Even with the simple hypothetical curve used, it was impossible to follow the curve accurately; and this, of course, contributed a major part of the errors.

Much could be gained in this particular problem if the Pressure vs Crank Angle data from the engine did not require plotting for application to an arbitrary function generator. In other words, the data would be fed directly from the engine into the computer. This would necessitate a high-speed computer including the recording equipment. With a system of this kind, the indicated work information could be obtained as the test on the engine was being carried out. It appears that this type of system would also lend itself to averaging the indicated work over many cycles of engine operation.

Another avenue of development is the application of analog-digital conversion techniques to the data so that the Pressure vs Crank Angle information could be converted to digital form, and then a digital computer used to compute the indicated work.

This investigation does not include work in these areas, but it is felt that the results of this study should encourage further efforts in the determination of indicated work by automatic means.

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APPENDIX A

If a simple mechanical system such as shown in Figure 1A is analyzed, the following equations will be obtained.

$$F \cdot U(t) = k(x_1 - x_2) = kx_1 - kx_2 \qquad (1A)$$

$$0 = M\frac{d^2x_2}{dt^2} + k(x_2 - x_1) + f\frac{dx_2}{dt}. \qquad (2A)$$

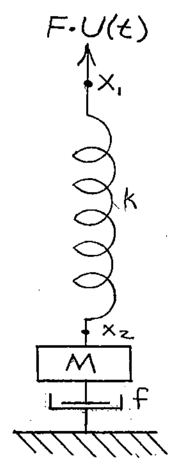

Figure 1A. A Simple Mechanical System.

In these equations F.U(t) = a force applied at time t = 0, (pounds),

M = mass, (lbs./ft./sec.^{2}),

k = spring constant, elastance, (lbs./ft.),

x = displacement, (ft.),

and f = viscous friction, (lbs./ft./sec.),

(coulomb friction is neglected).

Equations (1A) and (2A) may be rearranged as

$$F \cdot U(t) = kx_1 - kx_2 \qquad (1A)$$

$$0 = M\frac{d^2x_2}{dt^2} + f\frac{dx_2}{dt} - kx_1 + kx_2 \qquad (2A)$$

Now, suppose that the mesh equations for an electrical circuit such as the one in Figure 2A are written.

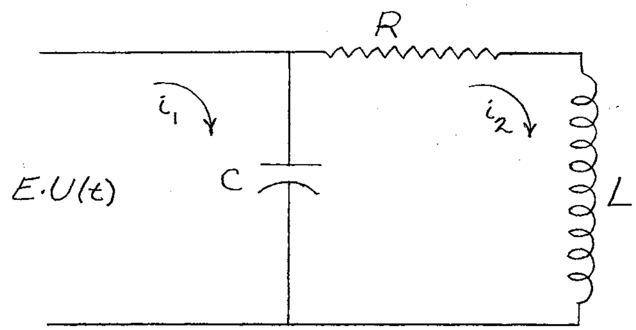

Figure 2A. A Two Mesh Electrical Circuit.

The equations which will result are

$$E \cdot U(t) = \frac{1}{C}\int i_1\,dt - \frac{1}{C}\int i_2\,dt \tag{3A}$$

$$0 = -\frac{1}{C}\int i_1\,dt + \frac{1}{C}\int i_2\,dt + i_2 R + L\frac{di_2}{dt}\,. \tag{4A}$$

These equations may be written in terms of charge, q, since

$$q = \int i\,dt\,. \tag{5A}$$

In terms of q the equations become

$$E \cdot U(t) = \frac{q_1}{C} - \frac{q_2}{C} \tag{6A}$$

$$0 = -\frac{q_1}{C} + \frac{q_2}{C} + R\frac{dq_2}{dt} + L\frac{d^2q_2}{dt^2}\,. \tag{7A}$$

Rearranging

$$E \cdot U(t) = \frac{q_1}{C} - \frac{q_2}{C} \tag{6A}$$

$$0 = L\frac{d^2q_2}{dt^2} + R\frac{dq_2}{dt} - \frac{q_1}{C} + \frac{q_2}{C}\,. \tag{7A}$$

Examining equations (1A), (2A), (6A), and (7A), one sees that an equivalence does exist between the equations describing the mechanical system and those describing the electrical system. Note that equivalence exists between mass and inductance, friction and resistance, displacement and charge, and elastance and the reciprocal of capacitance.

Table IA lists the electrical quantities and the analagous mechanical quantity under the mass-inductance analogy system. There are other analogy systems such as the mass-capacitance system but those will not be listed here.

TABLE IA

ANALOGIES BETWEEN ELECTRICAL AND MECHANICAL SYSTEMS--MASS-INDUCTANCE SYSTEM

Electrical System	Mechanical System
Voltage, (volt), e. E.	Force, (pound), F.
Charge (coulomb), q, Q.	Displacement, (feet), x.
Current, (amperes), i.	Velocity, (ft./sec.), v.
$\frac{di}{dt}$, (amperes/sec.).	Acceleration, (ft./sec.^{2}), a.
Resistance, (ohms), R.	Viscous friction, (lbs./ft./sec.),f
Inductance, (henry), L.	Mass, (lbs./ft./sec.^{2}), M.
Capacitance, (farads), C.	Compliance, (ft./lb.), 1/k.
Elastance, (volts/coulomb), S.	Spring Constant, (lbs./ft.), k.

APPENDIX B

A simple example of scaling the electronic analog computer when using the arbitrary function generator is given in this appendix.

The example chosen is the measurement of an area. The geometric configuration chosen is one that can be followed reasonably well on the arbitrary function generation and also one for which the area can be computed by analytical means.

The area between a curve and the x-axis is given by

$$A = \int f(x)\, dx \qquad (1B)$$

where A is area in square units.

In the example the following items were determined concerning the equipment to be used:

(a) Measurements indicated that the recti/riter required 30.5 volts for full scale deflection. (This is before the matching network described in Appendix C was constructed.)

(b) The maximum voltage obtainable from the function generator posentiometer was 34 volts.

(c) The time for the drum to complete one full revolution is 22.26 seconds.

For the example the geometrical configuration shown in Figure 1B was chosen. The base of 22.26 problem units was chosen to correspond to the time required for the drum to complete one revolution for the sake of simplicity.

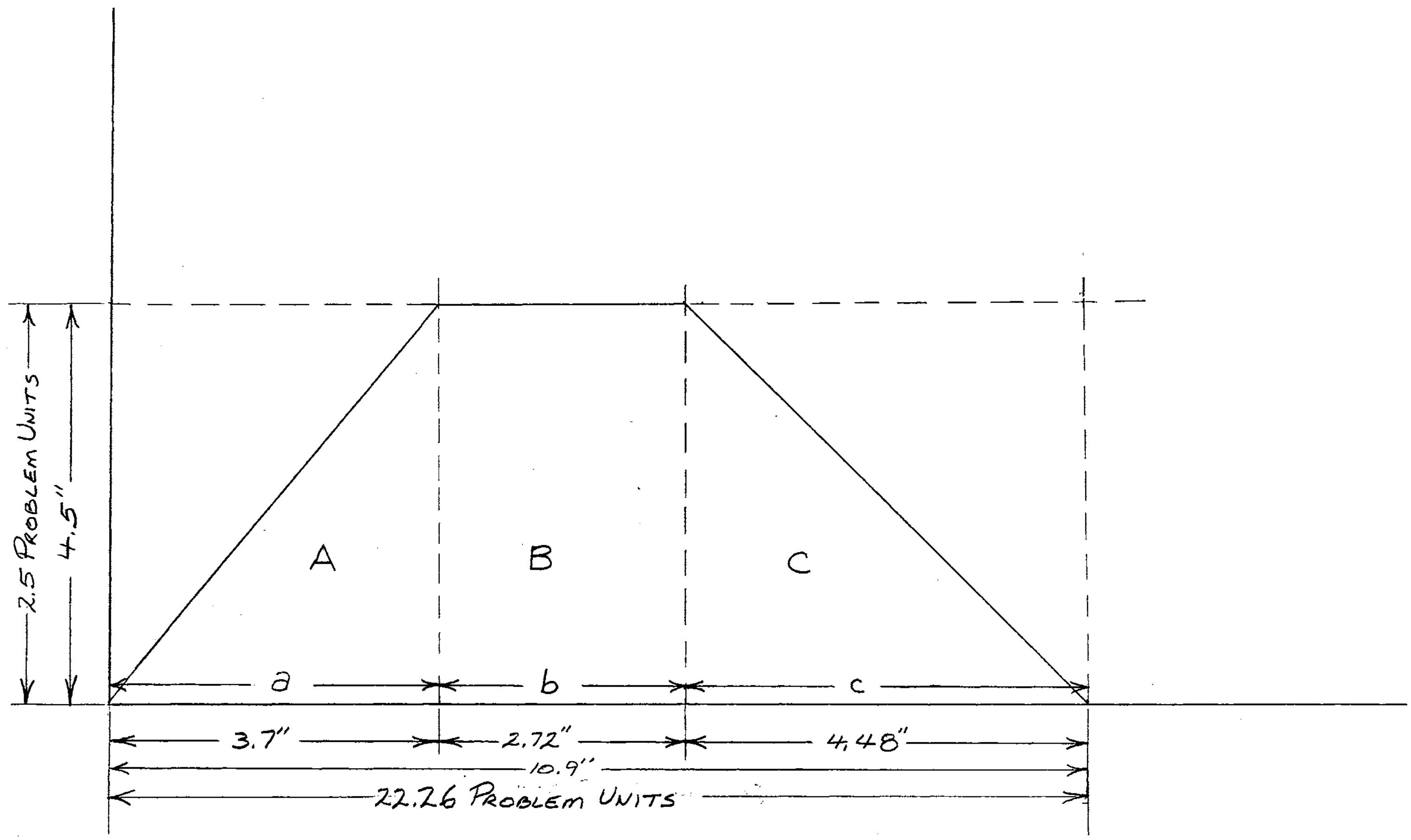

Figure 1B. The Figure Used in the Area Determination by the Electronic Analog Computer.

The lengths a, b, and c may be determined as

$$\frac{a}{22.26} = \frac{3.7}{10.9},\ a = 7.56 \text{ units}, \tag{2B}$$

$$\frac{b}{22.26} = \frac{2.72}{10.9},\ b = 5.56 \text{ units}, \tag{3B}$$

and

$$\frac{c}{22.26} = \frac{4.48}{10.9},\ c = 9.15 \text{ units}. \tag{4B}$$

The horizontal and vertical scales are found to be

$$\text{Horiz.} = \frac{22.26}{10.9} \text{ units/in.}, \tag{5B}$$

$$\text{Vert.} = \frac{2.5}{4.5} \text{ units/in.}, \tag{6B}$$

and it is seen that

$$1 \text{ sq. in.} = \frac{22.26}{10.9} \times \frac{2.5}{4.5} \text{ sq. units}. \tag{7B}$$

Now the area of sector A is

$$A_A = \frac{4.5 \times 3.7}{2} \text{ sq. in.}, \tag{8B}$$

$$A_A = \frac{4.5 \times 3.7}{2} \times \frac{22.26}{10.9} \times \frac{2.5}{4.5} \text{ sq. units}, \tag{9B}$$

$$A_A = 9.45 \text{ sq. units}. \tag{10B}$$

Likewise the areas of sectors B and C are determined to be

$$A_B = 13.9 \text{ sq. units}, \tag{11B}$$

$$A_C = 11.45 \text{ sq. units}. \tag{12B}$$

Adding the areas of the individual sectors together it is seen that the total area is 34.80 sq. units.

In order to obtain the area from the computer consider the equation

$$A \doteq \int f(x)\, dx . \tag{1B}$$

This may be written as

$$A = \int y\, dx . \tag{11B}$$

This equation must be written in machine units before it can be represented on the electronic analog computer. This means that the problem units must be expressed in terms of computer units which is volts. In order to do this a relationship must be established between the problem units and the machine units. This is accomplished by means of scale factors. The relationship between problem units and machine units is

$$A = \alpha_1 \bar{A}, \tag{12B}$$

$$y = \alpha_2 \bar{y}, \tag{13B}$$

and

$$x = \alpha_3 \bar{t}. \tag{14B}$$

In equations (12B), (13B), and (14B), the scale factors are α_1, α_2, and α_3, and $\bar{A}$, $\bar{y}$, and $\bar{t}$ are machine units.

Now evaluate the scale factors. Suppose that it is estimated that the maximum value of the area will be 50 problem units, and also that the maximum voltage to be obtained from the integrating amplifier is 25 volts (this is taken because measurements indicated that 30.5 volts resulted in full scale deflection). One can then evaluate α_1, as

$$\alpha_1 = \frac{A_{max.}}{\bar{A}_{max.}} = \frac{50}{25} = 2. \tag{15B}$$

Referring to Figure 1B it is seen that the maximum value of y is 2.5 problem units. From measurement it was found that the maximum voltage available from the function potentiometer was 34 volts and thus α_2 may be evaluated as

$$\alpha_2 = \frac{y_{max.}}{\bar{y}_{max.}} = \frac{2.5}{34} = 0.0735. \tag{16B}$$

It is noted from Figure 1B that the base in problem units is 22.26 and it has been determined that the drum completes one revolution in 22.26 seconds, thus α_3 is determined as

$$\alpha_3 = \frac{x}{\bar{t}} = \frac{22.26}{22.26} = 1. \tag{17B}$$

Writing equation (11B) in machine units obtain

$$\alpha_1 \bar{A} = \int \alpha_2 \bar{y} \alpha_3 \, d\bar{t} \tag{18B}$$

and substituting the values of the scale factors this becomes

$$2\bar{A} = \int 0.0735 \, \bar{y} \, d\bar{t}, \tag{19B}$$

$$\bar{A} = \frac{0.0735}{2} \int \bar{y} \, d\bar{t} \, . \tag{20B}$$

The relationship between the input and output voltages on an integrating amplifier such as shown in Figure 2B is

$$e_o = \frac{a}{RC_f} \int e_i \, dt. \tag{21B}$$

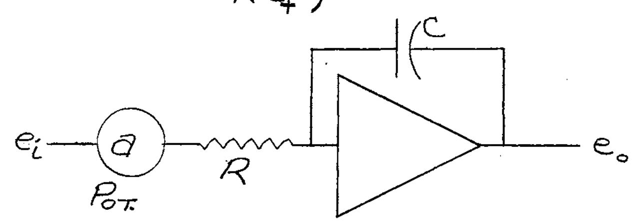

Figure 2B. An Integrating Amplifier.

Comparing equations (20B) and (21B) one sees that $\bar{A}$ corresponds to e_o and $\bar{y}$ corresponds to e_i and that 0.0735/2 corresponds to a/RC_f. Thus

$$\frac{a}{RC_f} = \frac{0.0735}{2} \tag{22B}$$

and if R is made 10 megohms and C_f is made 1 micro farad then "a," the potentiometer setting, will be 0.3675.

The computer circuit to measure the area then becomes that shown in Figure 3B. A chart with Figure 1B drawn on it was placed on the drum of the function generator and the figure was traced while the drum rotated. The voltage obtained from the function potentiometer was applied to the recti/riter. The deflection on the recti/riter was 5.7 divisions (see Figure 4B).

This corresponds to

$$\frac{30.5 \text{ volts}}{10 \text{ div.}} \times 5.7 \text{ div.} = 17.385 \text{ volts.} \qquad (23B)$$

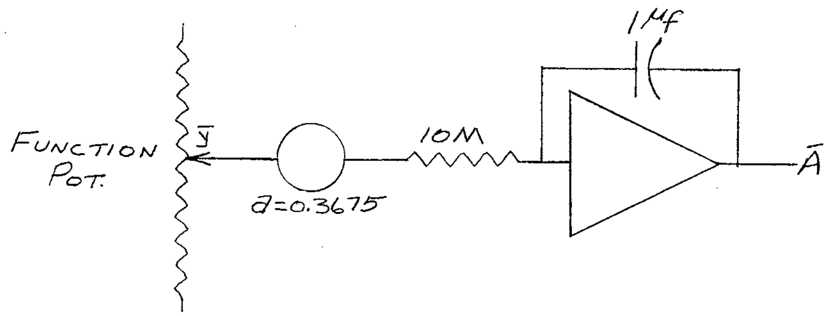

Figure 3B. The Computer Circuit to Compute the Area Shown in Figure 1B.

Thus it is seen that $\bar{A}$ is 17.385 volts and in order to convert this to problem units it must be multiplied by the appropriate scale factor. The scale factor, α_1, is 2 so that the area in problem units is

$$A = (2)(17.385) = 34.770 \text{ sq. units.} \qquad (24B)$$

Comparing this to the area obtained analytically, it is seen that the difference is 0.09 sq. units which is an error of 0.259%.

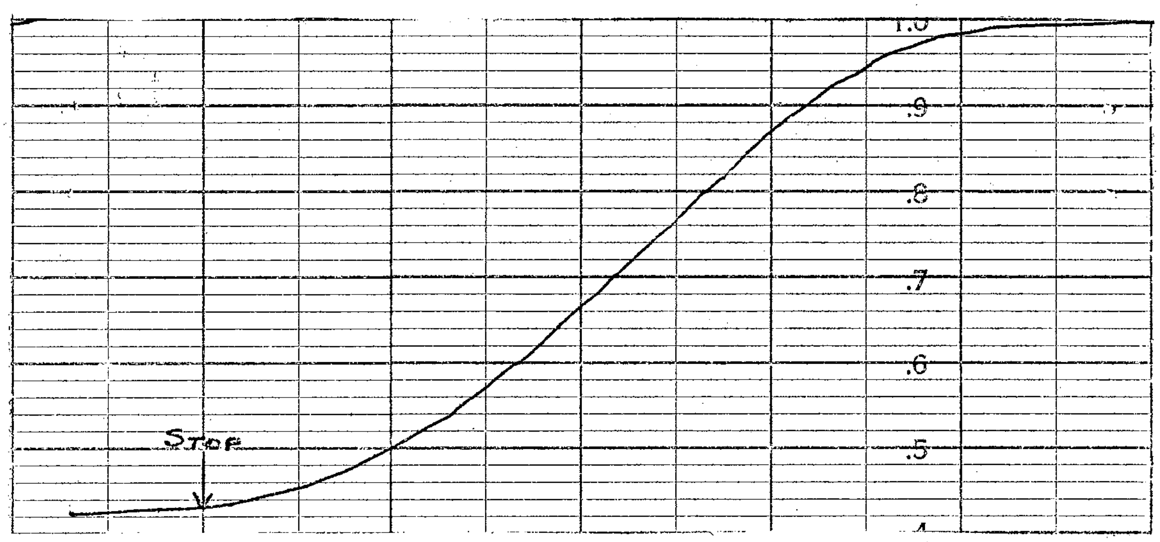

Figure 4B. The Output of the Integrating Amplifier of Figure 3B with a Voltage Shaped Like That Shown in Figure 1B Applied.

Now, consider the case where the independent variable is taken as something other than the time for the drum to complete one revolution, 22.26 seconds. Assume that the base of Figure 1B is taken to be 5 units long. On this basis the areas of sectors A, B, and C become

Sector A - 2.12 sq. units,

Sector B - 3.12 sq. units,

Sector C - 2.57 sq. units.

The total area is 7.81 sq. units.

Now the base used on the computer is 22.26 seconds and voltage output of the amplifier must be multiplied by the factor 5/22.26. The result of this multiplication is 3.9 volts which gives 7.8 sq. units when the scale factor is applied.

This could have been accomplished by reducing the gain of the amplifier as a result of changing the input and feedback impedances. In this case the scale factors α_1, and α_2 would remain the same, but the scale factor α_3 would become

$$\alpha_3 = \frac{x}{t} = \frac{5}{22.26} . \qquad (22B)$$

The area equation in machine units would be

$$\bar{A} = \frac{\alpha_2 \alpha_3}{\alpha_1} \int \bar{y} \, d\bar{t} . \qquad (23B)$$

Now

$$\frac{a}{RC_f} = \frac{\alpha_2 \alpha_3}{\alpha_1} = \frac{(0.0735)(5)}{(2)(22.26)} = 0.00825, \qquad (24B)$$

and if R is made 10 megohms and C_f is made 4 micro farads then the potentiometer setting, "a," will become 0.33.

The computer circuit based on these calculations is shown in Figure 5B.

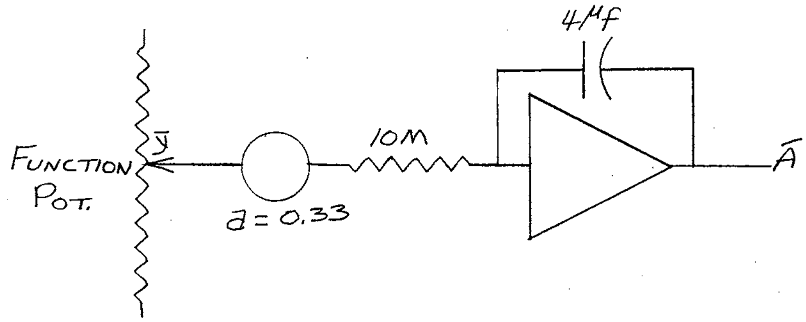

Figure 5B. A Computer Circuit to Compute the Area of Figure 1B if the Base is Taken as 5 Units.

APPENDIX C

This appendix will be devoted to the design of a resistance attentuator for matching the operational amplifiers of the Donner electronic analog computer to the Texas Instruments recti/riter.

The characteristics of the two items of equipment are as follows:

Donner Operational Amplifiers (9)

Output Voltage ----------------------	Any value between +100 and -100 volts.
Output Impedance ----------------------	Less than 1 ohm.
Load Current ----------------------	Up to 5 ma. (minimum load resistance 20,000 ohms).

Texas Instruments Rectillinear Recording Milliammenter, recti/riter (3)

Source Resistance for Critical Damping	--------	25,000 ohms.
D-C Resistance of Meter Movement	--------	1500 ohms.
Current for Full-Scale Deflection	--------	1 ma.

In this particular case it is desired to design the matching network so that 100 volts input will cause deflection from a zero center position on the recti/riter. In other words, with 100 volts input to the matching network it is desired to have a current of 0.5 ma. in the meter movement.

A "T" network of pure resistances as shown in Figure 1C will be used. Referring to Figure 1C, when the input voltage is 100 volts, it is desired that I be 0.5 ma.

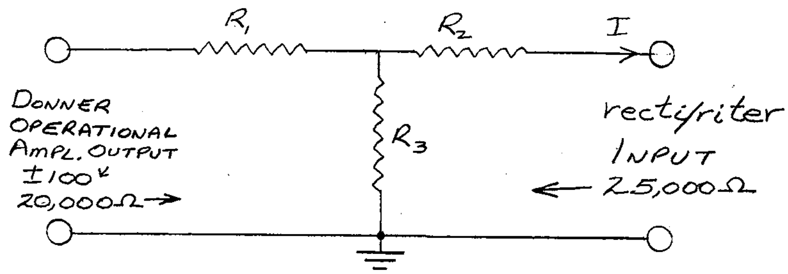

Figure 1C. A Matching Network for the Donner Operational Amplifiers and the Texas Instruments Recti/Riter.

Since a resistance of no less than 20,000 ohms is desired for the amplifier load, it may be written

$$20{,}000 = R_1 + \frac{R_3(R_2 + 1500)}{R_3 + R_2 + 1500} \quad . \tag{1C}$$

In addition, it is desired for the recti/riter to see 25,000 ohms looking back into the network so

$$25{,}000 = R_2 + \frac{R_1 R_3}{R_1 + R_3} \quad . \tag{2C}$$

One may also write a voltage equation assuming 100 volts applied to the network as

$$100 = (5\times10^{-3})R_1 + (4.5\times10^{-3})R_3 \tag{3C}$$

and the current division expression is

$$0.5\times10^{-3} = \frac{5\times10^{-3} R_3}{R_3 + R_2 + 1500} \quad . \tag{4C}$$

Clearing equations 1C and 2C of fractions, obtain

$$R_3 20{,}000 + R_2 20{,}000 + 30\times10^6 = R_1R_3 + R_1R_2 + R_1 1500 + R_3R_2 + R_3 1500 \tag{5C}$$

$$R_1 25{,}000 + R_3 25{,}000 \qquad = R_1R_2 + R_2R_3 + R_1R_3 \quad . \tag{6C}$$

Rearranging equations (5C) and (6C) and collecting terms

$$30\times 10^6 = R_1R_2 + R_2R_3 + R_1R_3 + R_1 1500 - R_2 20{,}000 - R_3 18{,}500 \quad (7C)$$

$$0 = R_1R_2 + R_2R_3 + R_1R_3 - R_1 25{,}000 \qquad - R_3 25{,}000\,. \quad (8C)$$

Subtracting equation (8C) from (7C) it is seen that

$$30\times 10^6 = R_1 26{,}500 - R_2 20{,}000 + R_3 6500\,. \quad (9C)$$

Now one may solve equation (3C) for R_1 to obtain

$$R_1 = 20{,}000 - R_3 0.9\,. \quad (10C)$$

Substituting this into equation (9C)

$$500\times 10^6 = R_2 20{,}000 + R_3 17{,}350\,. \quad (11C)$$

This equation may be used in combination with equation (4C) to solve for R_2 and R_3. Using these two equations together it is found that R_3 = 2686 ohms while R_2 = 22,674 ohms. Now solve for R_1 = 17,583 ohms.

The matching network becomes as shown in Figure 2C.

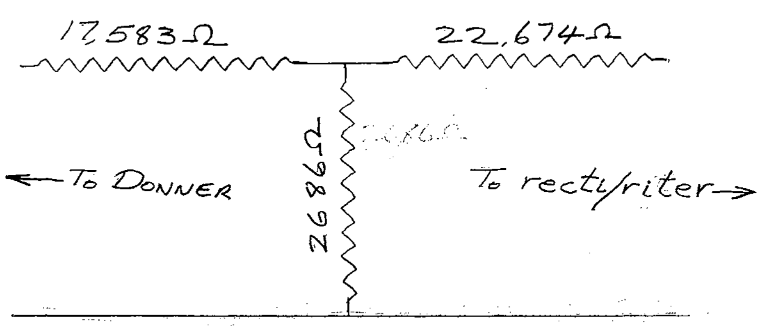

Figure 2C. A Matching Network With Resistance Values Shown.

This design has been checked and has been found to have a resistance of 20,000 ohms looking into the network with a 1500 ohm termination, a resistance of 25,000 ohms when looking back into the network with the input short circuited and 0.1 of the current in the input arm of the network flows in the output arm. These are the desired results.

A matching network was constructed having resistance values as shown in Figure 3C. With this network, 100 volts output from a Donner operational amplifier resulted in a deflection of 24 divisions of the recti/riter (see Figure 4C). Thus there are 4.16 volts/division.

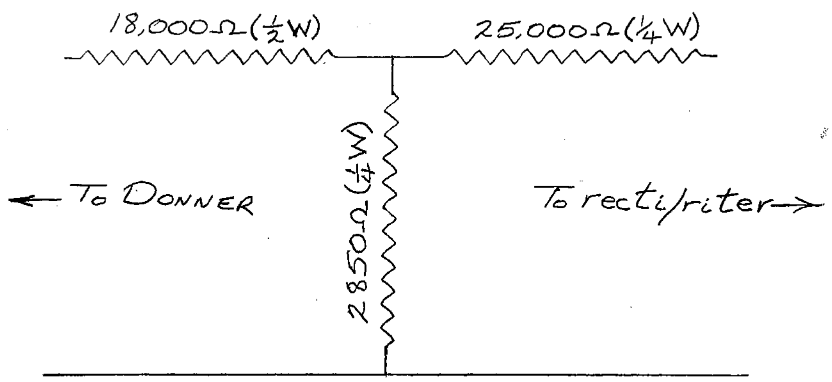

Figure 3C. The Actual Matching Network That Was Constructed.

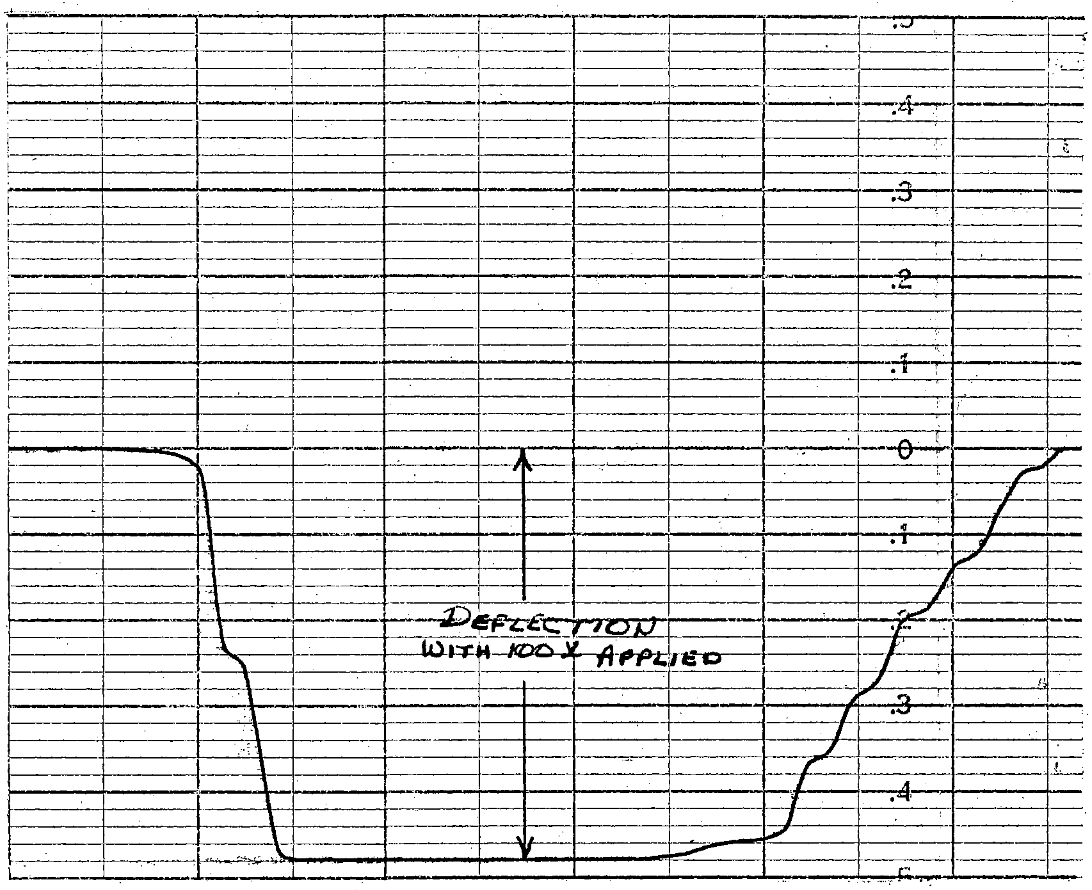

Figure 4C. A Recording Showing the Deflection of the Recti/Riter With 100 Volts Applied From an Operational Amplifier Through the Matching Network Shown in Figure 3C.

VITA

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