AN INVESTIGATION OF THE DYNAMIC PROPERTIES OF A ROTATING UNBALANCED RIGID BODY

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PREFACE

A note of explanation with reference to gravitational effects is in order. The reader may assume that the described motion takes place in a space which is free of gravitational forces or that these forces do exist but their lines of action are parallel to the fixed vertical axis. In either case, the motion would not be affected.

The term "axis of rotation" is used in this study to denote the line about which the body is to be balanced.

I wish to thank Dr. James H. Boggs for the opportunity to work as a half-time graduate assistant in the School of Mechanical Engineering of Oklahoma State University, which reduced considerably the expense of an added year of study.

I would like to acknowledge my indebtedness to Professor L. J. Fila, my thesis advisor, for his guidance and understanding throughout this study. Also I wish to thank my wife, Dana, for her constant encouragement and undying faith that this study would be a success.

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LIST OF SYMBOLS

C	Constant of Integration
d	Radial Distance of a Point Mass from Z, Feet
H	Angular Momentum, Slug - Ft.^{2}/Sec.
K	Damping Coefficient
L, U	Derived Vectors
m	Point Mass, Slugs
M	Summation of the Point Masses, Slugs
r	Radial Distance of a Point Mass from the Axis of Rotation, Feet
t	Time, Seconds
V_i^j	A Vibration Vector for which the Subscript Denotes the Plane of the Vector and the Superscript Denotes the Plane of the Trial Mass
X, Y	Cartesian Coordinates of the Axis of Rotation
Z	A Fixed Vertical Axis through the Mass Center
α	Angular Position of d Relative to an Arbitrary Reference, Radians
θ	Angular Position of r Relative to an Arbitrary Reference, Radians
ρ	Eccentricity of the Mass Center from the Axis of Rotation, Feet
Ψ_{ij}	Dynamic Influence Coefficient for which the First Subscript Denotes the Plane of the Vibration Vector and the Second Subscript Denotes the Plane of the Unbalanced Mass

CHAPTER I

INTRODUCTION

It is known that a body which is rotated about one of its principal axes will not experience translation. Because of manufacturing imperfections and non-homogeneity of the material it is difficult to insure that the desired axis of rotation will coincide with a principal axis of the body. Therefore it is necessary to alter the mass distribution of the body, if the axis of rotation is to remain fixed in space. This process is known as balancing.

There are several different balancing methods in use. One method is to provide a movable fulcrum by which the effect of balance weights in the arbitrary balance planes may be nullified. In operation, the fulcrum is fixed at one of the chosen balance planes and the object is balanced by a weight in the other plane. The fulcrum is then moved to the second plane and the object is balanced by a weight in the first plane.

Another method is employed by the Gisholt-Westinghouse balancing machines. In operation the effect of a trial weight in each of the balance planes is observed, thus establishing two vector equations in two unknowns. These equations are then solved with the aid of electrical circuits.

These balancing methods are suitable for production balancing of single components. With the advent of multi-stage space vehicles it becomes necessary to insure the balance of the complete assembly as well as the individual components, since the vehicle may depend upon spin stabilization as a means of counteracting thrust unbalance and thus maintaining the desired trajectory.

The purpose of the study contained in this thesis is to investigate the effect of an additional unbalance upon the motion of the axis of rotation and thus show that a two plane balance problem may be reduced to two single plane balance problems.

CHAPTER II

DERIVATION OF THE EQUATIONS OF MOTION OF THE MASS CENTER

Any condition of unbalance may be represented by two masses in two planes which are perpendicular to the axis of rotation. In Fig. 1, a general unbalance condition is represented by the masses m_1 and m_2 at radial distances r_1 and r_2 with phase angles Θ_1 and Θ_2 respectively.

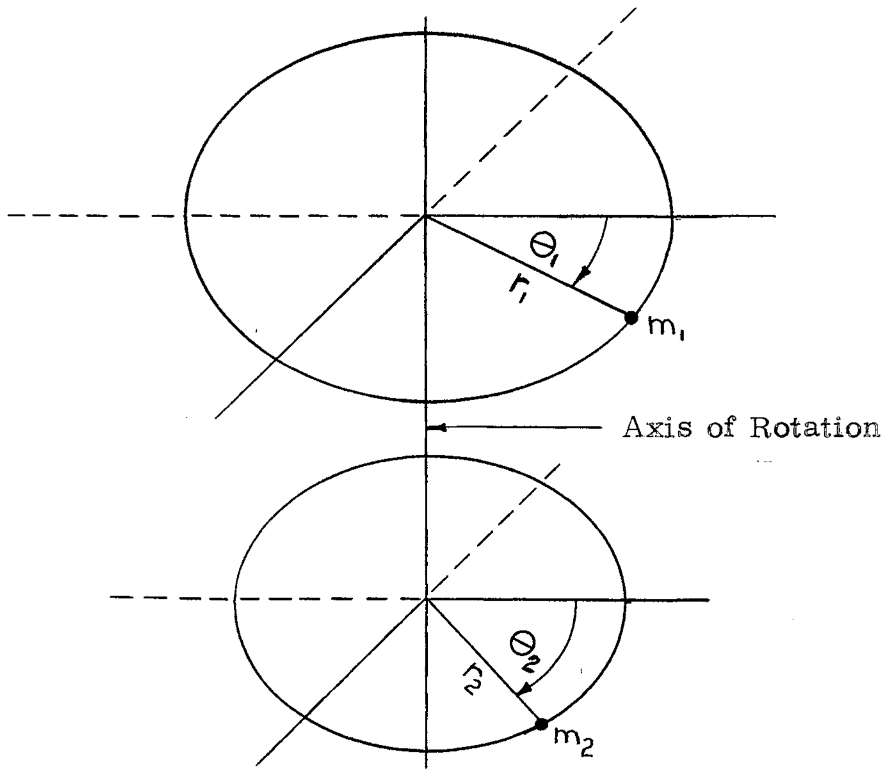

Fig. 1. General Unbalance

The body will be rotated about the axis of rotation with a constant angular velocity $\dot{\Theta}$ and the resulting motion of the axis of rotation will be determined.

The first step will be to determine the motion of the mass center. In Fig. 2 a cartesian coordinate system is established in a horizontal plane through the mass center. The coordinates of the axis of rotation are taken as x and y. The eccentricity of the mass center from the axis of rotation is taken as ρ with the phase angle Θ.

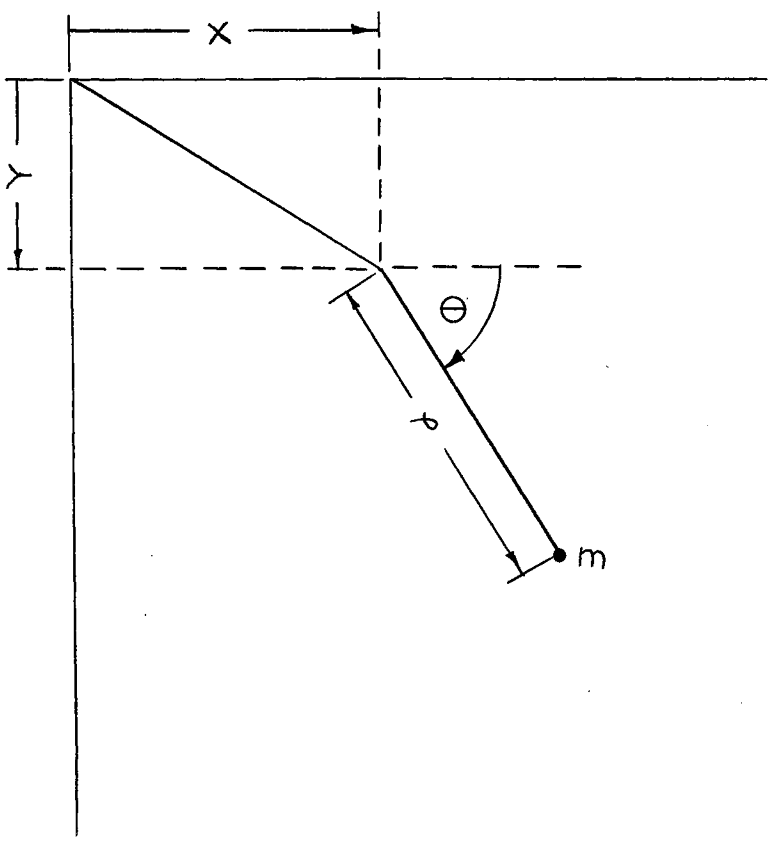

Fig. 2. Coordinate System for Motion of the Mass Center

Thus the coordinates of the mass center are

$$X_{cm} = X + \rho\cos\Theta\ , \qquad Y_{cm} = Y + \rho\sin\Theta\ . \tag{2-1}$$

The velocity and acceleration of the mass center are

$$\dot{X}_{cm} = \dot{X} - \rho\dot{\Theta}\sin\Theta\ , \qquad \dot{Y}_{cm} = \dot{Y} + \rho\dot{\Theta}\cos\Theta\ ,$$
$$\ddot{X}_{cm} = \ddot{X} - \rho\dot{\Theta}^2\cos\Theta\ , \qquad \ddot{Y}_{cm} = \ddot{Y} - \rho\dot{\Theta}^2\sin\Theta\ .$$

To obtain the steady state equations of motion, a damping force which is proportional to the velocity is placed at the mass center. Thus the differential equations of motion are:

$$M\ddot{X}_{cm} + K_x\dot{X}_{cm} = 0, \qquad M\ddot{Y}_{cm} + K_y\dot{Y}_{cm} = 0. \tag{2-2}$$

After substituting for $\dot{X}_{cm}$, $\ddot{X}_{cm}$ and $\dot{Y}_{cm}$, $\ddot{Y}_{cm}$ the equations are

$$\begin{aligned} M(\ddot{X} - \rho\dot{\Theta}^2\cos\Theta) + K_x(\dot{X} - \rho\dot{\Theta}\sin\Theta) &= 0,\\ M(\ddot{Y} - \rho\dot{\Theta}^2\sin\Theta) + K_y(\dot{Y} + \rho\dot{\Theta}\cos\Theta) &= 0, \end{aligned} \tag{2-3}$$

which may be written as

$$\begin{aligned} M\ddot{X} + K_x\dot{X} &= M\rho\dot{\Theta}^2\cos\Theta + K_x\rho\dot{\Theta}\sin\Theta\ ,\\ M\ddot{Y} + K_y\dot{Y} &= M\rho\dot{\Theta}^2\sin\Theta - K_x\rho\dot{\Theta}\cos\Theta\ ; \end{aligned} \tag{2-4}$$

and the reduced equations are

$$\begin{aligned} M\ddot{X} + K_x\dot{X} &= 0,\\ M\ddot{Y} + K_y\dot{Y} &= 0. \end{aligned} \tag{2-5}$$

Multiplying through by dt and integrating, the reduced equations are

$$\begin{aligned} M\dot{X} + K_x X &= C_1\ ,\\ M\dot{Y} + K_y Y &= C_2, \end{aligned} \tag{2-6}$$

in which the variables may be separated and the equations become

$$\frac{M\,dx}{K_x X - C_1} + dt = 0, \qquad \frac{M\,dy}{K_y Y - C_2} + dt = 0. \tag{2-7}$$

After integrating the equations are

$$\begin{aligned} &\frac{M}{K_x} \ln (x - C_1) + t = C_3, \\ &\frac{M}{K_y} \ln (y - C_2) + t = C_4; \end{aligned} \tag{2-8}$$

which may be solved for X and Y

$$\begin{aligned} X &= C_1 + e^{\frac{K_x}{M}(C_3 - t)}, \\ Y &= C_2 + e^{\frac{K_y}{M}(C_4 - t)}. \end{aligned} \tag{2-9}$$

Particular solutions to Equation (2-4) will be taken as

$$\begin{aligned} X &= A \cos\Theta + B \sin\Theta, \\ Y &= C \cos\Theta + D \sin\Theta; \end{aligned} \tag{2-10}$$

with velocities and accelerations

$$\dot{X} = -A\,\dot{\Theta}\sin\Theta + B\,\dot{\Theta}\cos\Theta, \qquad \dot{Y} = -C\,\dot{\Theta}\sin\Theta + D\,\dot{\Theta}\cos\Theta,$$

$$\ddot{X} = -A\,\dot{\Theta}^2\cos\Theta - B\,\dot{\Theta}^2\sin\Theta, \qquad \ddot{Y} = -C\dot{\Theta}^2\cos\Theta - D\dot{\Theta}^2\sin\Theta.$$

Now substituting into the differential equations of motion

$$\begin{aligned} &-MA\,\dot{\Theta}^2\cos\Theta - MB\,\dot{\Theta}^2\sin\Theta - K_x A\,\dot{\Theta}\sin\Theta + K_x B\,\dot{\Theta}\cos\Theta = \\ &M\rho\,\dot{\Theta}^2\cos\Theta + K_x\rho\,\dot{\Theta}\sin\Theta, \end{aligned} \tag{2-11}$$

$$\begin{aligned} &-MC\,\dot{\Theta}^2\cos\Theta - MD\,\dot{\Theta}^2\sin\Theta - K_y C\,\dot{\Theta}\sin\Theta + K_y D\,\dot{\Theta}\cos\Theta = \\ &M\rho\,\dot{\Theta}^2\sin\Theta - K_y\rho\,\dot{\Theta}\cos\Theta; \end{aligned}$$

and equating coefficients of like terms

$$-MA\,\dot{\Theta}^2 + K_x B\dot{\Theta} = M\rho\dot{\Theta}^2, \quad -MB\,\dot{\Theta}^2 - K_x A\dot{\Theta} = K_x\rho\dot{\Theta},$$
$$-MC\,\dot{\Theta}^2 + K_y D\dot{\Theta} = K_y\rho\dot{\Theta}, \quad -MD\,\dot{\Theta}^2 - K_y C\dot{\Theta} = M\rho\dot{\Theta}^2. \qquad (2\text{-}12)$$

Thus the unknown constants are

$$A = -\rho\,, \quad B = 0, \quad C = 0, \quad D = -\rho\;; \qquad (2\text{-}13)$$

and the equations of motion are

$$X = C_1 + e^{\frac{K_x}{M}(C_3 - t)} - \rho\cos\Theta\,,$$
$$Y = C_2 + e^{\frac{K_y}{M}(C_4 - t)} - \rho\sin\Theta\,. \qquad (2\text{-}14)$$

The steady state motion is

$$X = C_1 - \rho\cos\Theta\,,$$
$$Y = C_2 - \rho\sin\Theta\,. \qquad (2\text{-}15)$$

Therefore the motion of two points of the system is described. One point, which is the center of mass, remains fixed in space. The second point is the intersection of the axis of rotation with a plane perpendicular to this axis passing through the mass center.

This second point describes a circular orbit about the mass center with a velocity of constant magnitude. This magnitude is the product of the radius of the path and the rotational velocity of the system about its axis of rotation.

CHAPTER III

DERIVATION OF THE EQUATIONS OF MOTION OF THE AXIS OF ROTATION

In Fig. 3, a fixed vertical axis through the mass center is taken as the reference and is denoted by Z. The horizontal distances from Z to m_1 and m_2 are denoted by d_1 and d_2 with phase angles α_1 and α_2 respectively.

Since the mass center remains fixed in space, m_1 and m_2 must be diametrically opposite and

$$m_1 d_1 + m_2 d_2 = 0, \tag{3-1}$$

$$\dot{\alpha}_1 = \dot{\alpha}_2 . \tag{3-2}$$

When the motion becomes steady, damping has no effect as was shown in the previous chapter. Thus, the angular impulse about the Z axis is zero and the angular momentum is constant. Denoting the angular momentum by H

$$H_Z = m_1 d_1^2 \dot{\alpha}_1 + m_2 d_2 \dot{\alpha}_2, \tag{3-3}$$

which may be written as

$$\frac{H_Z}{\dot{\alpha}_1} = m_1 d_1^2 + m_2 d_2^2 . \tag{3-4}$$

Now d_1 may be eliminated by Equation (3-1), and the resulting

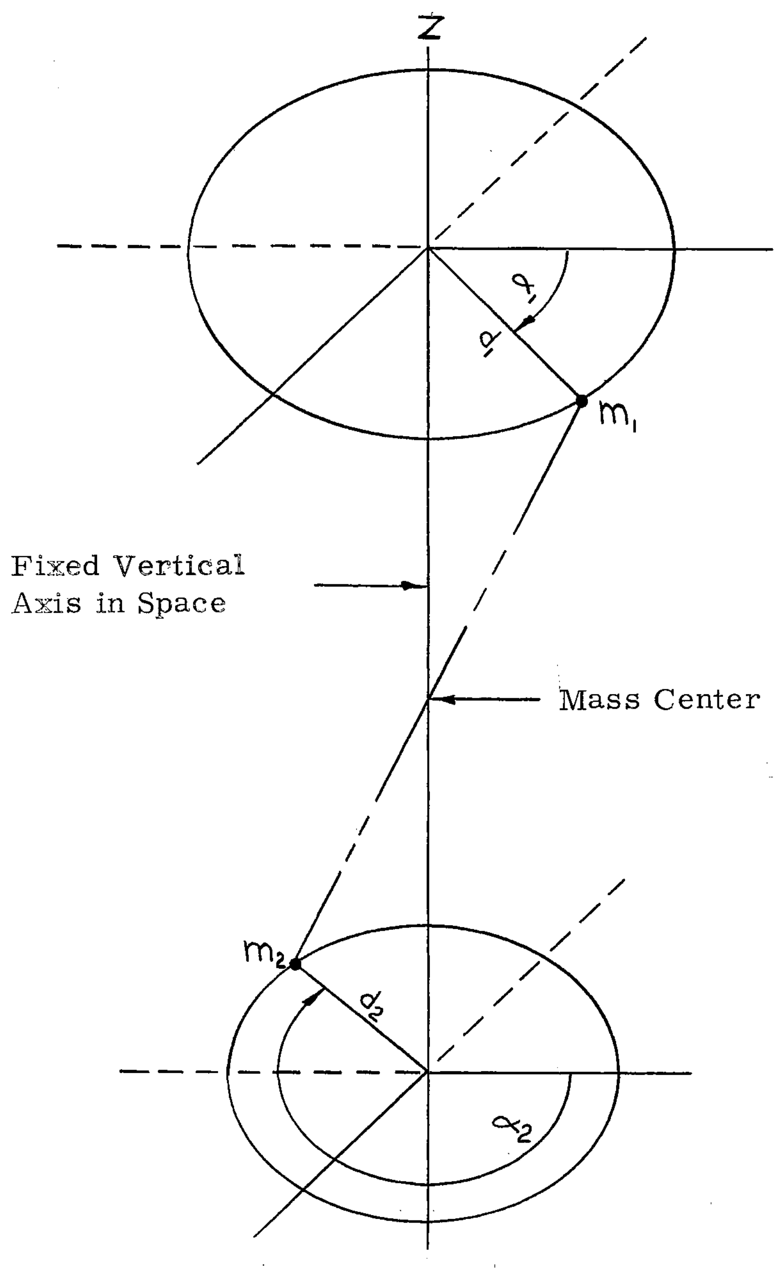

Fig. 3. Coordinate System for Motion of the Axis of Rotation

expression is

$$\frac{H_Z}{\dot{\alpha}_1} = m_1 \left(-\frac{m_2 d_2}{m_1}\right)^2 + m_2 d_2^2 , \qquad (3\text{-}5)$$

which may be solved for d_2^2

$$d_2^2 = \frac{H_Z m_1}{\dot{\alpha}_1(m_1 m_2 + m_2^2)} \quad .$$

Therefore, d_2 remains constant and from Equation (3-1) d_1 also remains constant.

The configuration of m_1, m_2 and the axis of rotation may be visualized as a frustum of a cone, whose axis coincides with the axis of rotation, with concentrated masses m_1 and m_2 in planes normal to the axis of the cone.

Now m_1 and m_2 rotate about Z in a circular path with the same constant angular velocity and the line connecting m_1 and m_2 must possess this same motion. Therefore, all lines through the cone rotate about Z in a circular path with a constant angular velocity, and one of these lines is the axis of rotation.

It was shown in Chapter II that the axis of rotation in the plane of the mass center rotated about Z with the same angular velocity that the masses rotated about the axis of rotation. Therefore, the axis of rotation rotates about Z with the same angular velocity that the masses rotate about the axis of rotation.

The motion of the axis of rotation may be summarized as follows. The axis of rotation seeks an inclination relative to the vertical such that the mass center will remain fixed in space as the axis of rotation rotates about a vertical axis through the mass center.

CHAPTER IV

REDUCTION OF A TWO PLANE BALANCE PROBLEM TO TWO SINGLE PLANE BALANCE PROBLEMS

It has been shown that any rigid body can be balanced by appropriate masses in two planes.[1] In Fig. 4, two arbitrary balance planes are chosen. The displacement, velocity, or acceleration of any two points on the axis of rotation will be measured to define the motion. Since any point on the axis of rotation travels about the fixed axis Z in a circular path, the displacement velocity and acceleration of the point are related. Therefore, if any one of the three quantities is reduced to zero, the other two must also be reduced to zero. This measured quantity will be referred to as the vibration vector and denoted by V_i^j where the subscript denotes the plane of the vibration vector and the superscript denotes the location of a trial mass.

The vibration vectors are related to the unbalanced masses by dynamic influence numbers. During the steady state motion the system is free of damping and the influence coefficients are real

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[1] J. P. Den Hartog, Mechanical Vibrations (4th ed., New York, 1956) p. 233.

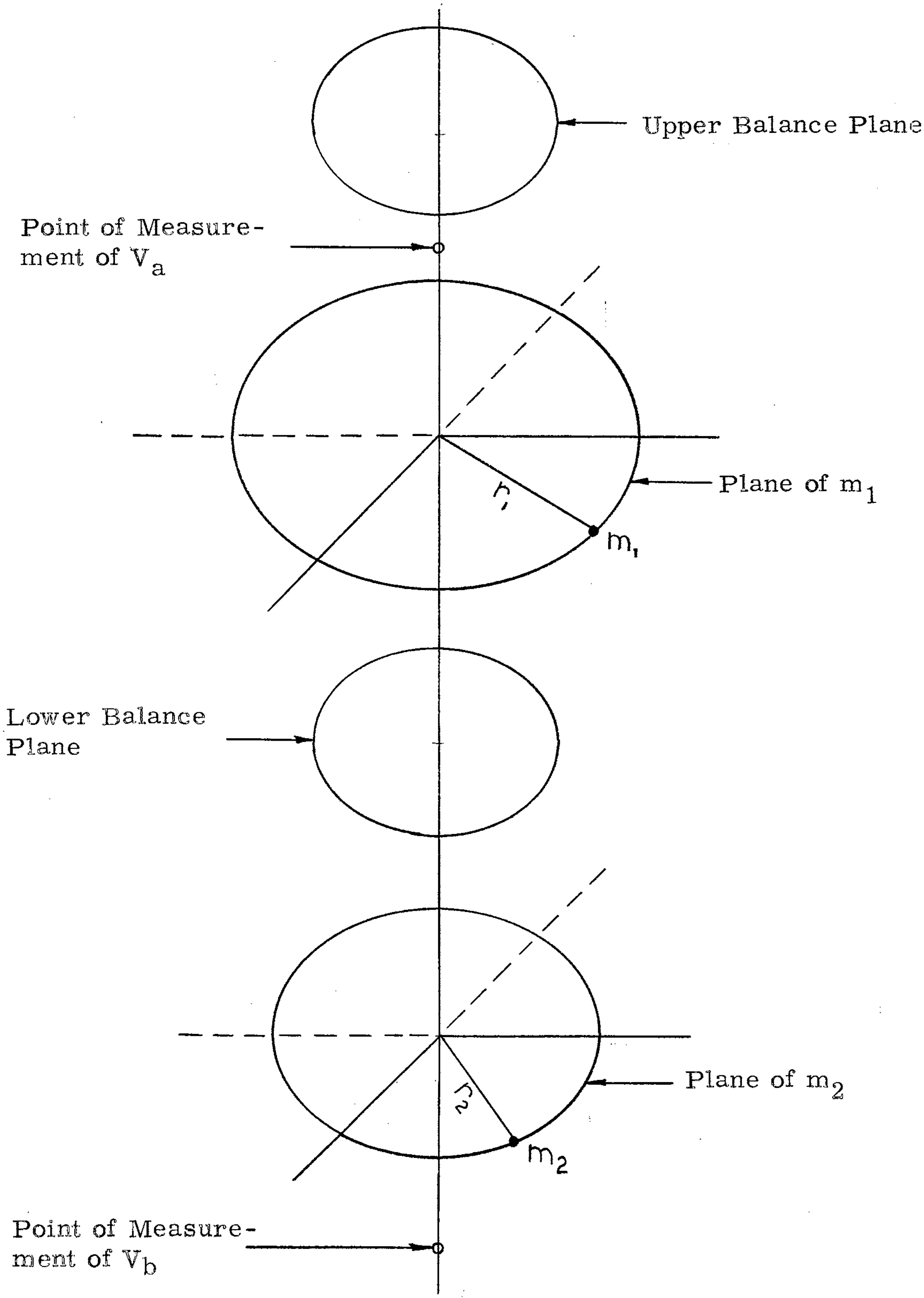

Fig. 4. Location of Balance Planes and Vibration Measuring Points

numbers which are entirely independent of the amount of unbalance present.[2] The vibration vectors may be written as complex numbers as

$$V_a^o = \psi_{a1} m_1 r_1 e^{i\theta_1} + \psi_{a2} m_2 r_2 e^{i\theta_2} ,$$
$$V_b^o = \psi_{b1} m_1 r_1 e^{i\theta_1} + \psi_{b2} m_2 r_2 e^{i\theta_2} . \tag{4-1}$$

The symbol ψ_{ij} is the dynamic influence number in which the first subscript denotes the plane of the vibration vector and the second subscript denotes the plane of the unbalanced mass.

When a trial mass is placed in the upper balance plane, the vibration vectors are

$$V_a^u = \psi_{a1} m_1 r_1 e^{i\theta_1} + \psi_{a2} m_2 r_2 e^{i\theta_2} + \psi_{a3} m_3 r_3 e^{i\theta_3} ,$$
$$V_b^u = \psi_{b1} m_1 r_1 e^{i\theta_1} + \psi_{b2} m_2 r_2 e^{i\theta_2} + \psi_{b3} m_3 r_3 e^{i\theta_3} ; \tag{4-2}$$

and the vector difference, which is the effect of the trial mass, is

$$V_a^u - V_a^o = \psi_{a3} m_3 r_3 e^{i\theta_3} ,$$
$$V_b^u - V_b^o = \psi_{b3} m_3 r_3 e^{i\theta_3} . \tag{4-3}$$

Thus, the vector differences are collinear and directly proportional to the dynamic influence coefficients. Consequently there is some point on the axis of rotation whose vibration vector was not affected

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[2] J. P. Den Hartog, Mechanical Vibrations (4th ed., New York, 1956) p. 239.

by the trial mass in the upper balance plane. This implies that there is some point on the axis of rotation for which the vibration vector was not affected by placing the trial mass in the lower balance plane.

In Fig. 5, a set of vibration vectors is plotted in which V_i^o denotes the vibration vector of the unbalanced body without a trial mass. The vibration vector of the point on the axis of rotation which was not affected by a trial mass in the upper balance plane is denoted by **L**.

The relationship of **L** to the measured vector is as follows. In Fig. 5, two possible cases exist. The vector **L** may be exterior to the region bounded by the measured vectors (5-a) or it may be interior to the region (5-b). In both cases the vector differences $V_i^u - V_i^o$ are collinear but in the latter case the vector differences are in opposite senses.

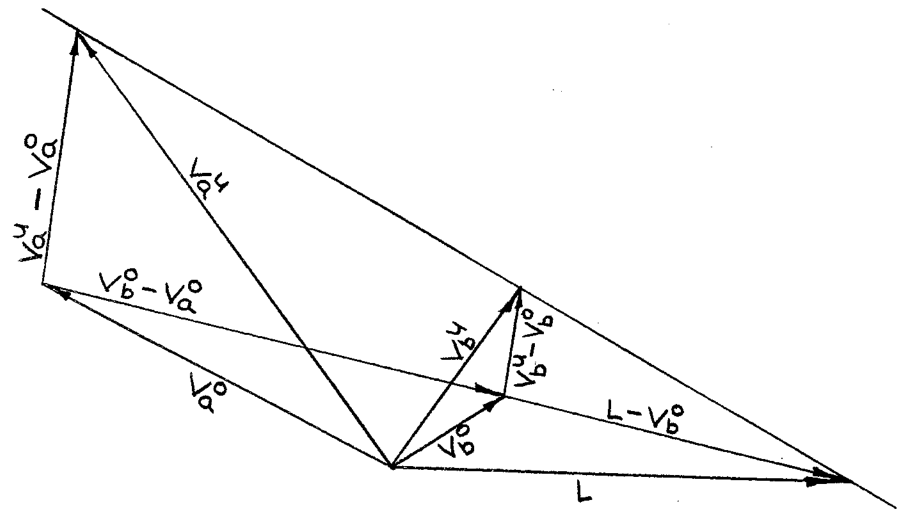
In the first case the relationship of the vectors may be expressed as

$$\frac{V_a^u - V_a^o}{V_b^u - V_b^o} = \frac{L - V_a^o}{L - V_b^o} \quad . \tag{4-4}$$

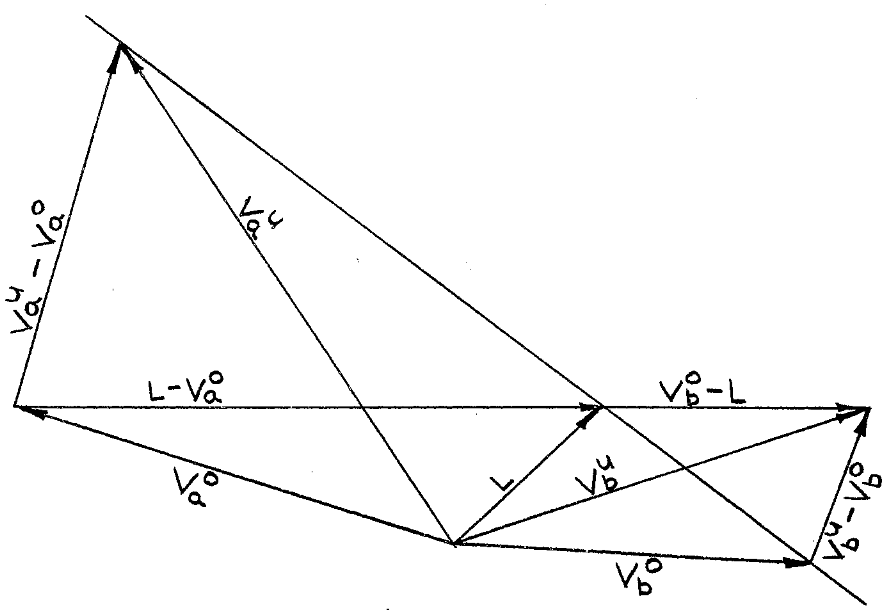
The vector **L** may be expressed as

$$L = V_b^o + L - V_b^o = V_b^o + \left| L - V_b^o \right| \frac{(V_b^o - V_a^o)}{\left| V_b^o - V_a^o \right|} \;, \tag{4-5}$$

where the symbol $\left| V_i^j \right|$ denotes the absolute value of the vector which is the length of the vector. Equation (4-4) may be rearranged and expressed as



(a)



(b)

Fig. 5. The Vector **L**

$$\frac{L - V_b^o}{V_b^u - V_b^o} = \frac{L - V_a^o}{V_a^u - V_a^o} = \frac{V_b^o - V_a^o}{V_a^u - V_a^o + V_b^o - V_b^u} ,$$

which may be further rearranged to

$$\frac{L - V_b^o}{V_b^o - V_a^o} = \frac{V_b^u - V_b^o}{V_a^u - V_a^o + V_b^o - V_b^u} .$$

This expression may be substituted into Equation (4-5) to obtain L as a function of the measured vectors

$$L = V_b^o + \left| V_b^u - V_b^o \right| \frac{(V_b^o - V_a^o)}{\left| V_a^u - V_a^o + V_b^o - V_b^u \right|} . \tag{4-6}$$

In the second case the measured vector differences are in opposite senses. In Fig. (5-b), the vector relationship is

$$\frac{V_a^u - V_a^o}{V_b^o - V_b^u} = \frac{L - V_a^o}{V_b^o - L} , \tag{4-7}$$

and the expression for L becomes

$$L = V_b^o - (V_b^o - L) = V_b^o - \left| V_b^o - L \right| \frac{(V_b^o - V_a^o)}{\left| V_b^o - V_a^o \right|} . \tag{4-8}$$

Equation (4-7) may be rearranged as

$$\frac{V_b^o - L}{V_b^o - V_b^u} = \frac{L - V_a^o}{V_a^u - V_a^o} = \frac{V_b^o - V_a^o}{V_b^o - V_b^u + V_a^u - V_a^o} ,$$

which may be further rearranged to

$$\frac{V_b^o - L}{V_b^o - V_a^o} = \frac{V_b^o - V_b^u}{V_b^o - V_b^u + V_a^u - V_a^o} ,$$

and substituting in Equation (4-8) the expression for L becomes

$$L = V_b^o - \left| V_b^o - V_b^u \right| \frac{(V_b^o - V_a^o)}{\left| V_b^o - V_b^u + V_a^u - V_a^o \right|} . \tag{4-9}$$

A general expression for L is

$$L = V_b^o \pm \left| V_b^o - V_b^u \right| \frac{(V_b^o - V_a^o)}{\left| V_b^o - V_b^u + V_a^u - V_a^o \right|} , \tag{4-10}$$

where the sign is chosen to agree with the vector differences. If the vector differences are in the same direction, the positive sign is chosen and conversely the negative sign is chosen when the vector differences are in opposite senses.

Now let the trial mass be placed in the lower balance plane and a third set of vectors will be formed. The vibration vector of the point on the axis of rotation which was not affected by the trial mass in the lower balance plane is denoted by U .

The expression for U as a function of the measured vectors could be derived by the same method of analysis as was used to determine the expression for L . The equation would be as follows:

$$U = V_b^o \pm \left| V_b^o - V_b^1 \right| \frac{(V_b^o - V_a^o)}{\left| V_b^o - V_b^1 + V_a^1 - V_a^o \right|} , \tag{4-11}$$

where the sign convention is the same as used in Equation (4-9).

To balance the body, it is necessary to reduce the vibration vectors of any two points on the axis of rotation to zero. Therefore, if the balance masses are placed so that they form the vectors $-L$ and $-U$ the body will be balanced.

Since the vector L was not affected by a mass in the upper balance plane, it will be necessary to place a mass in the lower balance plane to produce the vector $-L$. The mass that is placed in the lower balance plane will not affect the vector U.

The effect of a mass in the lower balance plane upon the vibration vector for the point on the axis of rotation denoted by L must be determined.

A complete set of vibration vectors is plotted in Fig. 6. When the trial mass was placed in the lower balance plane the vector corresponding to L was changed. This vector difference, which is denoted by $L' - L$ is collinear with the measured vector differences. Thus, the vector $L' - L$ is the effect of the trial mass in the lower balance plane upon the motion of the point on the axis of rotation whose motion was not affected by a mass in the upper balance plane.

From Fig. 6, the expression for $L' - L$ is

$$L' - L = \left| L - U \right| \frac{V_b^1 - V_b^o}{\left| V_b^o - U \right|} \quad . \tag{4-12}$$

In a similar manner the vector $U' - U$ is the effect of the trial mass in the upper balance plane upon the motion of the point on the

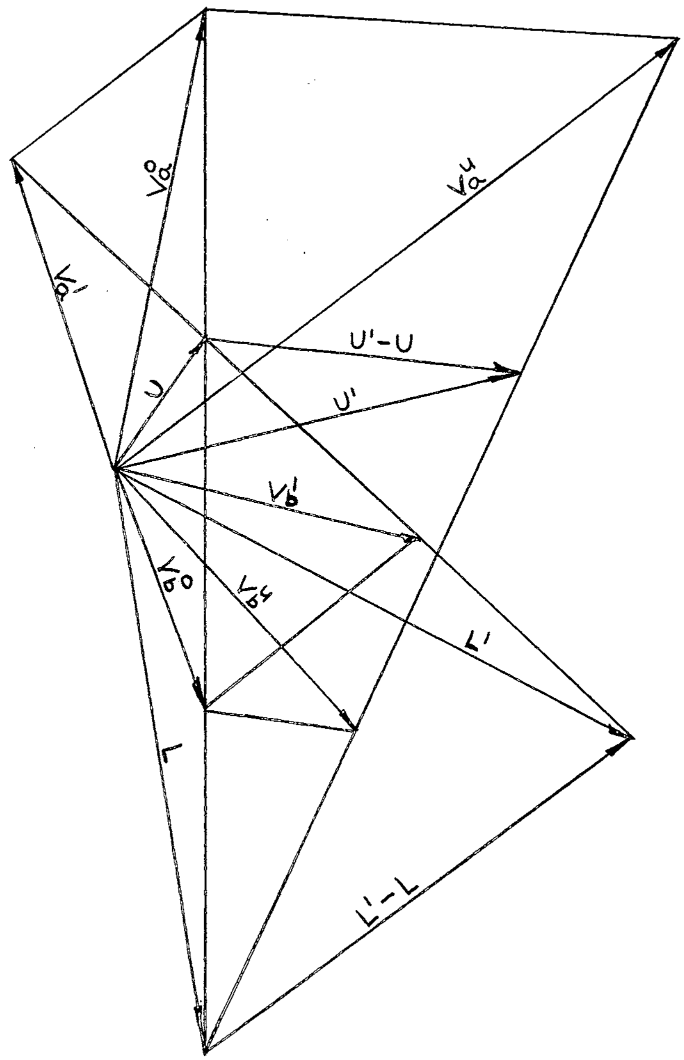


Fig. 6. A Complete Set of Vibration Vectors

axis of rotation whose motion was not affected by a mass in the lower balance plane. Again, the vector differences are collinear and the expression for $U' - U$ is

$$U' - U = |L - U| \frac{V_b^u - V_b^o}{|L - V_b^o|} \quad . \tag{4-13}$$

The vector $L' - L$ must be rotated through the angle between $-L$ and $L' - L$ to produce a vector in the direction of $-L$ and the trial mass must be increased by the ratio of the length of L to the length of $L' - L$ to produce the vector $-L$. This may be stated in equation form as

$$M_l = \frac{|L|}{|L' - L|} M_t , \qquad \Theta_l = \Theta_{-L} - \Theta_{L' - L} + \Theta_{iL} , \tag{4-14}$$

where l denotes the lower plane balance mass, M_t the trial mass and Θ_i the angular location of the trial mass. The upper plane balance mass is determined in a similar manner and the equations are

$$M_u = \frac{|U|}{|U' - U|} M_t , \quad \Theta_u = \Theta_{-U} - \Theta_{U' - U} + \Theta_{iU} . \tag{4-15}$$

Thus the two plane balance problem has been reduced to two single plane balance problems for a system free of damping. Also, it is not necessary to determine the dynamic influence coefficients for the body.

CHAPTER V

SUMMARY AND CONCLUSIONS

The motion of the axis of rotation of a rotating unbalanced rigid body may be described as follows. The axis of rotation seeks an inclination relative to the vertical such that the mass center remains fixed in space as the axis of rotation rotates about a vertical axis through the mass center.

The dynamic influence coefficients for a system free of damping are real numbers. Thus, the vectors which represent the effect of a trial mass are collinear and their magnitudes are proportional to the dynamic influence coefficients. Therefore, there is some vibration vector which was not affected by the trial mass, and this vector must be reduced to zero by a mass in the other balance plane. Thus, the two plane balance problem is reduced to two single plane balance problems.

Some of the advantages of this balancing method are as follows:

1. The balancing machine would be capable of balancing a wide variety of parts.

2. The machine would be ideal for a configuration such as a multi-stage space vehicle where it is necessary to balance each component as well as the complete assembly.

3. It is not necessary to know the mass, inertia properties, or the dynamic influence numbers of the body to be balanced, thus eliminating the need for a balanced specimen.

4. The balance problem is easier to understand since it has been reduced from a two plane balance problem to two single plane balance problems.

A trivial disadvantage is the time required to balance an object. The calibrated balance machines require a single set of vibration readings as opposed to three sets of readings required by this method.

The following recommendation for future study is suggested -- the design of a balancing machine with a suspension system in which no forces are introduced in the horizontal plane and which has the ability to measure the vibration vectors.

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